Symbol Emergence and Symbolic Mediation in a Background-Independent Relational Ontology: A Preliminary Discussion

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Abstract

This paper constructs a vocabulary for symbols and symbolic mediation on a combinatorial substrate of the kind that background-independent approaches to quantum gravity, group field theory among them, have developed as models of the emergence of a discrete spacetime, in which no arena is given in advance and every structure, extent and order included, is carried by the relations themselves, and applies it to the elementary structure of production. The substrate is taken as a formal template and no physical claim is made of the objects described. On this substrate, time is generated by the system's own records, and the division into system and environment is a choice the describer makes and can vary; a subsystem is a region of a combinatorial complex, and its dynamics is given by an admissibility relation on configurations. Within that setting the paper defines discrimination by equality of induced admissible continuations over a class of contexts, and defines a symbol as a persistent product that substitutes for a discrimination class in the dynamics. A three-step ladder distinguishes traces, products and symbols, and separates a product that participates in a subsystem's continuations from one that stands in for something else. The reverse pass through which a persistent structure conditions the activity that generated it is defined as selection among admissible continuations, and two derived relations are introduced, the set of subsystems a structure mediates and the set of subsystems able to revise it, with definitions that use no magnitude. The paper's contributions are: a repaired discrimination relation that is an equivalence and is indexed to the subsystem's current configuration; a substitution relation with an explicit interaction class and an explicit residue; a state-sufficiency criterion for the decomposition of a subsystem into relational and symbolic sectors; a result that a coupling which has ceased can leave a persistent structure standing where it stood, so that parties who were never party to it are mediated by it without acquiring any means of revising it; and a mapping of the elementary categories of production onto these objects, with the conditions of failure and the open problems stated.

Keywords: symbol emergence; symbolic mediation; background independence; relational ontology; relations of production

Draft. This is a working draft. It is circulated for comment, and its arguments, formulations, and open questions are subject to revision. Correction and criticism are welcome at huangwanhong@serendip.ngo.

Use of generative artificial intelligence. Generative AI assistants (Anthropic's Claude, OpenAI's ChatGPT, and Google's Gemini) were used extensively in preparing this manuscript: for literature survey, for drafting prose from the author's specifications, for adversarial review of the argument, and for consistency and citation auditing and typesetting. No simulation or empirical figure is used in this paper, and its arguments are discursive throughout. Formulations and objections arose in the course of that exchange as well as from the author. Every claim, argument, and citation was reviewed and decided by the author, who is responsible for the content and for any errors. Bibliographic details are to be verified against the primary sources before reliance.

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1 Introduction

A great deal of social theory is written with a metaphor of production at its centre: something is made, the thing made persists, and what persists comes to shape the making. The metaphor is old and serviceable, and it has resisted formalisation for an identifiable reason. To say that a product comes to stand over its producer, one needs a vocabulary in which the boundary between producer and product is itself a variable. Formalisms that fix a subject in advance, and then ask what happens to it, cannot express the movement in which a region of activity passes out of a subject's reach while remaining coupled to it. The boundary must itself belong to the description.

Background-independent formalisms in physics were developed for an unrelated reason [3, 5], and they have exactly this property. The programmes in question, of which group field theory is one, model spacetime as emerging from a discrete combinatorial substrate: a field over a group manifold whose perturbative expansion generates complexes of cells, with geometry carried by labels on those cells. No arena is given in which events are placed; there are relations, and what is called a region is a choice of subcomplex made by whoever is describing. Time is a structure read off the relations themselves, and extent is carried by the incidence structure. A formalism of this kind offers social theory a template in which the partition into subsystem and environment is a description choice with consequences, and in which a subsystem's history is what fixes what may follow.

This paper takes up that template and asks a narrow question. Given such a substrate, what would it take to say that a subsystem has a symbol? The question is narrow deliberately. Claims about ideology, value, class and alienation all presuppose that some configuration is functioning symbolically for some party, and the presupposition is normally left unexamined. If it can be made precise without importing an interpreter, a meaning relation, or a semantic primitive, then a range of further claims become statable; if it cannot, assertion will not supply what the definition could not.

The construction proceeds in the order of dependence. A complex and its subsystems are introduced first, with the partition explicitly relative. A record relation on cells is defined, and the causal order is generated from it, so that a clock is a function compatible with an order that the system's own history has produced. An admissibility relation on configurations supplies the dynamics. On that base, a subsystem's capacity to discriminate is defined by equality of the admissible continuations that different boundary configurations induce, quantified over a class of contexts. A persistent configuration is a trace; a trace that makes a difference to admissible continuations is a product; a product that stands in for a discrimination class across a stated interaction class is a symbol. Persistence, participation and substitution are three separate conditions, and each is checkable on the objects already introduced.

Two further steps complete the elementary picture. A subsystem is decomposed into a relational sector and a symbolic sector, with the criterion that the pair is sufficient for the subsystem's continuations while the relational sector alone is not. And the reverse direction is defined: a persistent structure mediates a subsystem when the subsystem's admissible continuations depend on it. From mediation and its complement, two sets are derived, the subsystems a structure mediates and the subsystems whose own activity can alter it, and these carry the weight of the later analysis without any scalar being introduced.

The last part of the paper applies the vocabulary to production. Activity, objectification, the product, the means of production and the relations of production receive formal counterparts, and one substantive result is established: a coupling that ceases can leave behind a structure that occupies its place in the dynamics, mediating parties who were never party to the coupling and supplying them, by default, with no channel of revision.

This is a conceptual and discussion paper. It offers definitions, a small number of propositions with short proofs, and an explicit register of what remains open. It offers no empirical test and no worked model, and it does not derive quantitative predictions. The absence of these is a limitation of scope and does not relieve the paper of the obligation to say what would count against it, which is discharged in Section 15.

Two objections deserve an answer at the outset, because they will otherwise be read into every subsequent section.

Remark 1.1 (The template objection)**.** The paper does not claim that social structures are quantum-gravitational, that persons are physical subsystems in the sense the physics intends, or that any measurement in physics bears on any claim made here. What is borrowed is a formal template: a combinatorial substrate with no background, an internally generated order, and a relative partition. Requirement 2.2 states the discipline governing each borrowing, and every borrowing in the paper is argued at the point where it is made.

Remark 1.2 (The computation objection)**.** A complex with configurations, a record relation and an admissibility relation is a computational structure under another description, and the apparatus is not offered as an alternative to that description. Its contribution is that the partition, the order and the clock are internal and relative, so that the questions this paper is built to ask (whose product, mediating whom, revisable by whom) have formal expressions. A computational description with a fixed machine and a fixed tape does not have a variable boundary, and the movement this vocabulary exists to describe is a movement of the boundary.

2 Method, Levels, and the Discipline of Transfer

Three requirements govern the whole paper. They are stated once and are in force throughout; later sections cite them by number.

2.1 Preliminaries

The paper borrows its setting from physics and its bookkeeping from elementary mathematics. Both are set out here in plain terms, and Section 2.2 resumes the argument.

The physical picture. Several current programmes in quantum gravity build spacetime in place of assuming it. Their elementary object is a network: dots, called vertices; links joining the dots, called edges; and surfaces spanning closed circuits of links, called faces. A history of such a network, in which edges and faces persist, appear and recombine, is called a *spin foam*, the name recalling the walls of soap bubbles, where faces meet along edges and edges meet at vertices. *Group field theory* is a machinery for generating such histories: a single mathematical object, a field defined over a space of symmetry operations, whose systematic expansion produces foams in the way that the expansion of an ordinary particle theory produces interaction diagrams. Geometric quantities, lengths, areas and durations, are carried as labels attached to the network's parts, and are read off the relations. *Background independence* names the resulting situation: there is no stage on which events are placed, the network of relations is all there is, and space, time and extent are features of it.

What this paper takes. Only the skeleton is used: a *complex*, meaning cells of the three kinds with the record of which touches which, and *configurations*, meaning assignments of labels to cells. Every quantum-mechanical and geometric result of the physics is left behind, and Requirement 2.2 governs the little that is borrowed. The skeleton serves as bookkeeping for relations, and a reader may picture a vast diagram of dots, links and panels, carrying labels, and growing.

The mathematical vocabulary. Four notions from elementary mathematics recur. A *relation* pairs some objects with others, and $x \in S$, $S \subseteq T$ and $S \setminus T$ read: x belongs to the collection S; every member of S belongs to T; the members of S outside T. An *equivalence relation* is

Table 1: Notation. Each symbol is introduced where indicated; the table is a lookup and replaces no definition.

Symbol	Reading	Introduced
[L1], [L2], [L3]	level tags: generating theory; combinatorial; effective	Requirement 2.1
$\mathcal{A}$	amplitude, where a model supplies one	Requirement 2.3
$\mathcal{C}$, $\mathcal{C}_I$, $\partial\mathcal{C}_I$	complex; subsystem; its boundary	Definition 3.1
$\mathcal{W}_I$	worldtube: a subsystem whose boundary persists	Definition 3.2
$u \preceq v$	u precedes v in the record-generated order	Definition 4.1
χ, $X_I(\chi)$	clock; the configuration on its level set, a cut	Definition 4.3
$X = (E, X_I)$	joint configuration: environment on the boundary, and subsystem	Definition 5.1
$X \rightsquigarrow X'$	X' is an admissible continuation of X	Definition 5.1
$\mathrm{Con}_I(X)$, $\mathrm{Con}_I^h(X)$	continuation set; admissible h-step continuation sequences	Definition 5.1
$U \otimes V$, $\cdot$	composition on disjoint regions; a component held fixed	Remark 5.2
$\mathcal{P}_I$	context class: the admissible boundary variations compared	Definition 6.2
$E_a \sim_{I,X} E_b$, $\approx_{I,X}$	discrimination; indistinguishability on any region	Definition 6.2, Definition 6.6
$[E]_{I,X}$	a discrimination class	Definition 6.2
$T \sqsubseteq X$, $T \not\sqsubseteq X$	T is, is not, a subconfiguration of X	Definition 7.1
π	product: a trace whose presence alters Con_I	Definition 7.4
$\sigma \dashv_{\mathcal{Y}} [E]_{I,X}$	σ substitutes for the class over $\mathcal{Y}$	Definition 7.5
ϱ	residue: the interactions at which substitution fails	Definition 7.6
Σ_I	repertoire: I's set of symbols	Definition 7.5
(X^R, X^S)	relational and sufficient-state sectors	Definition 8.1
$\mathrm{Med}(\pi)$	the subsystems π mediates	Definition 13.1
$\mathrm{Rev}_h(\sigma)$	the subsystems able to revise σ within h steps	Definition 13.2

a sorting of objects into classes: every object is sorted with itself, sorting is mutual, and two objects sorted with a common third are sorted together; a *class* is one cell of the sorting, and the *quotient* is the collection of the cells, the objects considered up to the sorting. A *partial order* is a ranking in which some pairs are comparable and others are left unranked, in contrast with a total order, which ranks every pair; much of this paper's discipline consists in claiming partial orders where a stronger vocabulary would claim numbers. A function is *monotone* along an order when it never decreases along it. A *horizon* h is a stated number of steps, and claims made at a horizon are claims about at most that many steps.

2.2 Levels

Statements in this paper belong to one of three levels, and the level is marked wherever confusion is possible.

Requirement 2.1 (Levels)**.** [L1] is the fundamental description: a field over a group manifold with an action, whose perturbative expansion generates complexes. [L2] is a complex and its combinatorics: cells, configurations, the record relation, admissible continuations. [L3] is an effective description on emergent structure: fields, couplings, potentials, and the quantities definable in a regime where such a description applies. Every definition and every claim carries a level. A term introduced at [L3] does not appear in a definition at [L2]. Social and economic readings are [L2] where they are combinatorial and [L3] otherwise, and no claim in this paper asserts that a social structure is an [L1] field.

The requirement is not a formality. The temptation it guards against is specific: having defined a structure at [L2] and observed that it resembles an object in the [L1] theory, to say that the social structure is that object. Statements of the form "domination is a curvature" or "an institution is a phase of the foam" arise this way. They are ruled out here by construction, and where an [L3] dependence genuinely holds, the word *effective* appears.

2.3 Transfer

Requirement 2.2 (Transfer)**.** A property of a physical theory is used in this paper only where an argument is given at the point of transfer for why the property holds of the object under discussion, and only where the transfer names a condition under which it would fail. A resemblance between an equation in one domain and a structure in another is not an argument. A borrowing that cannot name its own failure condition is not made.

Three borrowings occur in the paper and each is argued where it is made: the combinatorial substrate itself (Section 3), the generation of an order from records (Section 4), and the use of a sufficiency condition to characterise a decomposition (Section 8). Statements about vacua, condensation and effective couplings appear only as remarks pointing to work outside this paper's scope.

2.4 Admissibility and enrichment

The dynamics used here is an admissibility relation, and not a weight over histories.

Requirement 2.3 (Enrichment)**.** The relation $\rightsquigarrow$ of Definition 5.1 is primitive. Where a model supplies an amplitude $\mathcal{A}$ over continuations, the relation is recovered as its support: $X \rightsquigarrow X'$ exactly when $\mathcal{A}[X \to X'] \neq 0$. No definition or proposition in this paper uses more of $\mathcal{A}$ than its support. Constructions requiring a weight (transition probabilities, entropies over histories, expectation values) are marked as enrichments and are not used in any result.

The reason for the restriction is a commitment about trajectories. A weight over histories carries an implicit reference class: to say that a continuation has a given amplitude is to place it among alternatives that are, in the relevant respect, comparable. Where the object of study is a singular historical trajectory whose conditions were themselves produced along the way, the reference class is not available, and a formalism that presupposes one has assumed what is in question. The admissibility relation carries the modal content that the analysis requires, namely what may follow from what, and carries nothing further.

Remark 2.4 (Cost of the restriction)**.** The restriction has a cost. Without a weight there are no rates, no likelihoods, no comparison between more and less probable continuations, and no entropy. Several attractive statements are therefore unavailable, including any account of how quickly a structure changes. Section 15.2 records this among the open problems.

2.5 Standing constraints

Four constraints hold throughout, and are stated here so that their observance can be checked.

1. **Structural comparison.** Comparisons between subsystems proceed by structure: by membership, inclusion and position. The framework treats the heterogeneity of parties as primitive, and a common measure over their configurations would be an addition to the substrate.

Table 2: Reading of the strata. The dictionary fixes what a cell is taken to represent, and is used in Sections 4 to 7. Nothing below depends on the reading being unique; it is a design assignment in the sense of Remark 3.4.

Stratum	Reading	Introduced
Face	History of an element	Definition 3.1
Edge (in the foam)	A coupling that persists over an interval	Definition 3.5
Vertex	An event at which couplings are rearranged	Definition 4.1
Boundary graph	A state: elements as its edges, couplings as its nodes	Definition 3.2

2. **Partial order.** Orderings, where available, are partial and are declared as such. The vocabulary supports the comparison of positions, and the aggregation a scalar would license belongs to a framework with a common measure.
3. **Actual configurations.** Every condition is stated on configurations the dynamics admits. A definition earns its place by conditions that obtain, checkable against what the design contains.
4. **Description.** The vocabulary describes structures and their consequences, and evaluative conclusions require premises supplied elsewhere.

3 The Complex and Its Subsystems

3.1 Cells, labels, configurations

Definition 3.1 (Complex and configuration, [L2])**.** $\mathcal{C}$ is a two-complex: a set of cells consisting of vertices, edges and faces, with incidence relations among them. A *configuration* X of a region assigns to each of its cells a label drawn from a fixed label set. Two configurations of the same region are distinct when they differ on at least one cell.

The complex is generated, at [L1], as a term in the perturbative expansion of a field theory over a group manifold; that generation is not used in what follows, and every construction below is stated on the complex and its labels. This is the first borrowing under Requirement 2.2, and the argument for it is the one given in Section 1: the setting supplies a combinatorial substrate whose extent is carried by its own incidence structure, and whose decomposition into regions is left to the describer. It fails as a template for any object whose parts are individuated prior to and independently of their relations, and Section 15 records that condition.

3.2 Subsystems and the relativity of the partition

Definition 3.2 (Subsystem, [L2])**.** A *subsystem* is a subcomplex $\mathcal{C}_I \subset \mathcal{C}$ together with its boundary $\partial\mathcal{C}_I$, the set of cells of $\mathcal{C}_I$ incident to cells outside it. The *environment* of $\mathcal{C}_I$ is $\mathcal{C} \setminus \mathcal{C}_I$. A *worldtube* $\mathcal{W}_A$ is a subsystem whose boundary persists across an interval of the order of Definition 4.3.

Remark 3.3 (Relativity)**.** The partition is a description choice and it is symmetric. For any two subsystems I and J of a common complex, each lies in the environment of the other. There is no subsystem whose environment is privileged, and there is no description made from no subsystem at all. Every statement in this paper of the form "for I" is therefore a statement relative to a partition, and the qualification is not repeated after this point.

Remark 3.4 (The partition as a design act)**.** Nothing at [L2] selects a partition. The complex admits every subcomplex, and the formalism supplies no criterion distinguishing those that constitute subjects from those that do not. This paper does not supply one. It marks the selection as a design act: a model asserts nothing, and a design assigns interpretations to a model's terms and thereby incurs the obligation to state its assignment. Candidate criteria (persistence of a boundary across the record order, closure of a subsystem's continuations under variation of its environment) are listed in Section 15.2 as open problems and are used nowhere in the results below.

One consequence of Remark 3.3 requires statement, because the later analysis depends on it. Where a partition is fixed in advance, a region generated by a subsystem's activity is either inside that subsystem or outside it, permanently. Where the partition is relative and can change along the order, a region generated by I can come to lie outside $\partial\mathcal{C}_I$ while remaining coupled across it. A theory of production requires that movement, and a fixed decomposition cannot state it.

3.3 Couplings and non-factorisation

Definition 3.5 (Coupling, [L2]). A *coupling* J_{AB} between subsystems A and B is a persisting relation carried by cells incident to both ∂W_A and ∂W_B. A coupling may itself carry a configuration; a pipeline, a contract and a practice are relations with states, and are represented as such.

Proposition 3.6 (Non-factorisation, [L2]). *Let A and B be coupled. Then in general the admissible continuations of the pair are a proper subset of the pairs of separately admissible continuations:*

$$\mathrm{Con}_{A\cup B}(X) \ \subsetneq\ \mathrm{Con}_A(X) \times \mathrm{Con}_B(X).$$

Proof. Con is defined at Definition 5.1; the argument uses only that a coupling constrains the joint labelling. Take a coupling cell whose label must agree with labels on both sides. Any pair of separately admissible continuations assigning incompatible labels to that cell is excluded from the joint set, and such a pair exists whenever the label set on the coupling has more than one element. Equality holds exactly when the coupling imposes no joint constraint, in which case A and B are not coupled. $\square$

Proposition 3.6 is stated combinatorially and requires nothing beyond Requirement 2.3. Its content is that the description of a coupled pair is not assembled from the descriptions of the parties, and that the structure carried on the boundary belongs to neither of them. This is the formal counterpart of the claim that a relation is not reducible to its relata, and it is used in Section 11 to locate the relations of production. Discussion in the physics literature of degrees of freedom carried on a boundary is an enrichment of this observation and is not required for it.

4 Records, Causal Order, and Clocks

4.1 The record relation

Definition 4.1 (Record relation, [L2]). For cells u, v of $\mathcal{C}$, write $u \ R\ v$ when u contributed to a persistent configuration on which the admissible continuations at v depend. Let $\preceq$ be the reflexive transitive closure of R.

This is the second borrowing under Requirement 2.2. The argument for it is that a background-independent setting supplies no order from outside, so an order must be read off the structure, and the structure that does the reading is the one by which what has happened conditions what may happen. The transfer fails, and $\preceq$ is unavailable, for any system in which persistent configurations do not condition continuations at all; Proposition 6.5 shows that such a system also has no symbols, so the two failures coincide.

Requirement 4.2 (Acyclicity)**.** $\preceq$ as defined is a preorder. A design is admissible in this paper only where R is acyclic on it, so that $\preceq$ is a partial order. Whether acyclicity can be derived from conditions on the complex, and so cease to be required of a design, is open; Section 15.2 records it.

The requirement is stated as a requirement because the alternative would be a claim the paper cannot support. A persistent structure that conditions continuations at a vertex which in turn contributed to that structure gives a cycle, and cycles are not excluded by anything introduced so far. Designs containing them are outside the scope of the results below.

4.2 Clocks

Definition 4.3 (Admissible clock and cut, [L2])**.** A function $\chi : \mathcal{C}_I \to \mathcal{O}_\chi$ into a totally ordered set is an *admissible clock* for $\mathcal{C}_I$ when it is non-decreasing along $\preceq$. Its level sets Σ_χ are then *cuts*: downward-closed sets with respect to $\preceq$ restricted to $\mathcal{C}_I$. Write $X_I(\chi)$ for the configuration of $\mathcal{C}_I$ restricted to the cut at χ, and $[\mathcal{C}_I]_\chi$ for the quotient of $\mathcal{C}_I$ by the induced equivalence.

4.3 Two formulations of internal time

Two formulations of internal time are available on this substrate, and both are carried, with the first adopted as primary. Stating both, and the reason for the adoption, belongs here because later constructions depend on which is in force.

The first is the *relational clock* of Definition 4.3: a function monotone along the record-generated order, whose level sets are cuts. On this formulation time is generated by the production of records. What has left a record constraining a cell precedes it; a state is a cut; and two cells of one cut stand in the record relation in neither direction, which is what simultaneity amounts to here. The formulation carries structure: cuts, states, the quotient $[\mathcal{C}_I]_\chi$, and the comparison of branches meeting a common cut. Its costs are equally definite: it presupposes the acyclicity of Requirement 4.2, and it requires a monotone function covering the region, which a sufficiently branched or sufficiently disconnected order may fail to admit.

The second is the *traversal*: a parameter τ along a single admissible continuation sequence, ordering the steps of one trajectory. A traversal exists wherever there is a continuation at all, needs neither acyclicity of the whole region nor a covering function, and its invariant content is exactly the order of steps, any reparametrisation carrying the same information. Its poverty is the counterpart of its generality: a traversal supplies neither cuts nor states nor any comparison between branches, and two traversals of one region are related only where the record relation relates their steps.

The two formulations are compatible and unequally strong. Every admissible clock induces a traversal along each realised sequence, by restriction; a traversal extends to a clock exactly where a monotone covering function exists, which Requirement 4.2 makes possible and does not guarantee. The relational formulation is adopted as primary for a reason internal to the paper's subject: the constructions to come, states, discrimination at a cut, the sufficient-state sector, all live on cuts, and the clock formulation is the one that supplies them. The adoption also keeps the dependence of time on records explicit, which is the substantive point: a region generating no persistent structure supports no clock beyond the traversal of its own steps, so that internal time, in the strong form, is itself a product of the production of records. Where a clock fails to cover a region the traversal remains, and every construction below stated at a cut holds, in weakened form, along a traversal step.

Remark 4.4 (Orientation)**.** An *orientation* criterion $\mathfrak{A}$, distinguishing a direction along $\preceq$, is a separate structure and is not fixed in this paper. Both formulations above supply precedence; a direction is a further assignment. That a subsystem's states are ordered is a weaker claim than that one end of the order is the past, and constructions requiring the second are marked and deferred.

5 Admissible Continuation

Definition 5.1 (Admissible continuation, [L2])**.** Let E denote a configuration of the environment on $\partial\mathcal{C}_I$. The relation $\rightsquigarrow$ holds between joint configurations at successive cuts:

$$(E, X_I)(\chi_n) \;\rightsquigarrow\; (E, X_I)(\chi_{n+1}).$$

For a joint configuration $X = (E, X_I)$, the *continuation set* is

$$\mathrm{Con}_I(X) \;=\; \{\, X'_I \;:\; X \rightsquigarrow (E', X'_I) \text{ for some } E' \,\},$$

and $\mathrm{Con}_I^h(X)$ denotes the set of admissible h-step continuation sequences from X, each taken together with its realising joint sequence.

Remark 5.2 (Notation for composed configurations)**.** Configurations on disjoint regions compose: for configurations U, V on disjoint sets of cells, $U \otimes V$ is the configuration assigning each cell the label it receives from the component containing it. Where the arguments of Con_I are listed, as in $\mathrm{Con}_I(X, E, p)$ or $\mathrm{Con}_I(X \otimes \sigma \otimes Y)$, they denote the joint configuration composed of the listed components, with any component written "$\cdot$" held fixed by the context. Nothing beyond composition is intended: $\otimes$ carries no algebraic structure and in particular no factorisation claim, which Proposition 3.6 excludes.

Three features of Definition 5.1 are deliberate and are recorded so that later sections can rely on them.

First, the relation is stated on the joint configuration, and the environment appears at every cut. A formulation in which a subsystem's configuration continues into a subsystem's configuration, with the environment entering only as an initial condition, would make the boundary a source and would contradict Remark 3.3. Second, Con_I projects the joint continuation onto I's sector, and the projection is where the environment's contribution ceases to be visible; every later claim about what a subsystem can and cannot distinguish rests on this projection. Third, no structure beyond the relation is assumed: no factorisation of the joint configuration into a product across $\partial\mathcal{C}_I$, which Proposition 3.6 rules out; no invertibility, which would prejudge the orientation left open in Remark 4.4; and no metric, by the first standing constraint of Section 2.

6 Boundary Configurations and Dynamical Discrimination

6.1 The discrimination relation

A subsystem is said, informally, to distinguish two configurations when it responds to them differently. The formal definition must specify what counts as a different response, and must do so without appeal to a threshold of similarity, because a relation defined by sufficient similarity is not transitive and therefore has no classes.

Definition 6.1 (Context class, [L2])**.** A *context class* $\mathcal{P}_I$ for I is a set of boundary variations under which I's continuations are to be compared. $\mathcal{P}_I$ is a feature of the situation and not of the analyst: it is produced historically, and two subsystems with identical dynamics and different context classes discriminate differently.

Definition 6.2 (Discrimination, [L2])**.** Fix I, a configuration $X = X_I(\chi)$, and a context class $\mathcal{P}_I$. For boundary configurations E_a, E_b, write

$$E_a \sim_{I,X} E_b \quad \text{iff} \quad \mathrm{Con}_I(X, E_a, p) \ = \ \mathrm{Con}_I(X, E_b, p) \ \text{ for every } p \in \mathcal{P}_I.$$

The classes are written $[E_a]_{I,X}$.

Proposition 6.3. *$\sim_{I,X}$ is an equivalence relation.*

Proof. The condition is a universally quantified equality of sets. Equality is reflexive, symmetric and transitive, and universal quantification preserves each. □

Proposition 6.3 is elementary and is stated because the alternative formulation is not elementary and does not hold. Defining discrimination by the requirement that two boundary configurations induce *sufficiently similar* continuations yields a relation that is reflexive and symmetric and fails transitivity, so that its classes are not well defined and no object can be said to substitute for one. Equality quantified over a class of contexts is the weakest repair that leaves the notion usable.

6.2 Consequences of the definition

Proposition 6.4 (Configuration indexing)**.** *In general $\sim_{I,X} \neq \sim_{I,X'}$ for $X \neq X'$.*

Proof. Con_I depends on X by Definition 5.1. Choose X and a pair E_a, E_b whose induced continuation sets agree for every $p \in \mathcal{P}_I$, and X' at which they differ for some p; the dependence of Con_I on its first argument permits such a choice whenever I's configuration enters the constraint on continuations non-trivially. □

The partition of boundary configurations into classes is therefore not a fixed property of a subsystem. It is a property of a subsystem in a configuration, and the configuration changes. Since the symbolic sector isolated in Section 8 is part of X_I, it follows that what a subsystem carries alters what it can distinguish. Proposition 6.4 is the formal hook on which a later paper's treatment of coarsening and refinement hangs; nothing evaluative is claimed here.

Proposition 6.5 (Triviality under closure)**.** *If $\partial\mathcal{C}_I$ admits no variation, in that the boundary configuration and the context class are both singletons, then $\sim_{I,X}$ has exactly one class.*

Proof. With a single boundary configuration E, the quantified equality holds vacuously for the only available pair (E, E), and there is no pair in distinct classes. □

6.3 Extension to arbitrary configurations

Definition 6.2 is stated on boundary configurations because that is where the notion is needed first. The same schema extends to configurations of any region, and the extension is required in Section 7, where what must be compared are the continuations that substitution produces.

Definition 6.6 (Indistinguishability, [L2])**.** For configurations Z, Z' of any region presented to I at X, write

$$Z \approx_{I,X} Z' \quad \text{iff} \quad \mathrm{Con}_I(X, Z, p) = \mathrm{Con}_I(X, Z', p) \ \text{ for every } p \in \mathcal{P}_I.$$

$\sim_{I,X}$ of Definition 6.2 is the restriction of $\approx_{I,X}$ to boundary configurations, and Proposition 6.3 holds of $\approx_{I,X}$ by the same proof.

Proposition 6.5 fixes a limiting case that recurs in the analysis. A subsystem that absorbs everything crossing its boundary, in the sense that no variation on the boundary makes any difference to what may follow, discriminates nothing. By Definition 7.5 it therefore has no symbols. Total absorption and total insensitivity coincide, and a structure that has reached this condition cannot be described as interpreting its environment at all.

7 Persistence, Products, and Symbols

Three conditions are separated in this section: that a configuration persists, that it makes a difference, and that it stands in for something. Each is checkable on the objects already introduced, and the third is the pivot on which the rest of the paper turns.

7.1 Traces

Definition 7.1 (Trace, [L2]). A configuration tr on $\mathcal{C}_I$ is a *trace* over an interval $[\chi_1, \chi_2]$ when tr $\sqsubseteq X_I(\chi)$ for every χ in the interval, where $\sqsubseteq$ denotes restriction to a subconfiguration.

Remark 7.2. Persistence is here exhibited and not explained. Mechanisms by which a configuration persists (the depth of a well in an effective potential, redundancy of copies, an institution maintaining a record) belong to [L3] or to substantive social analysis, and none is invoked in the definitions below.

7.2 Generation and products

Definition 7.3 (Generation, [L2]). A trace tr is *generated by I* over $[\chi_0, \chi_1]$ when tr $\not\sqsubseteq X_I(\chi_0)$, tr $\sqsubseteq X_I(\chi_1)$, and the admissible continuation realising the difference runs through cells of $\mathcal{C}_I$.

Definition 7.4 (Product, [L2]). A trace π is a *product* when it participates in the dynamics: there exist configurations Y with

$$\mathrm{Con}_I(\,\cdot \otimes \pi \otimes Y\,) \;\neq\; \mathrm{Con}_I(\,\cdot \otimes Y\,).$$

Definition 7.4 applies to a product the same test the paper applies to its own categories. A persistent configuration that makes no difference to what may follow is not counted, whatever else may be true of it; a product is a persistent configuration that alters the set of admissible continuations of the subsystem in whose region it lies.

7.3 Symbols

Definition 7.5 (Symbol, [L2]). Let $\mathcal{Y}$ be a class of interaction configurations. A product σ is a *symbol for I over $\mathcal{Y}$*, written $\sigma \dashv_{\mathcal{Y}} [E_A]_{I,X}$, when for every $Y \in \mathcal{Y}$ the two continuation sets coincide in the quotient by indistinguishability:

$$\mathrm{Con}_I(X \otimes \sigma \otimes Y)\,/\!\approx_{I,\cdot} \;=\; \mathrm{Con}_I(X \otimes E_A \otimes Y)\,/\!\approx_{I,\cdot}\,,$$

where E_A is a representative of the class and $\approx$ is Definition 6.6. Substitution is thus exact only up to what I can itself distinguish, which is what the informal notion requires: a stand-in whose deviations the subsystem could not register is, for that subsystem, a stand-in without deviation.

The definition states that σ occupies, across the stated interaction class, the place that a member of the class would occupy. The definition turns on the dynamics of I alone, and is checked by comparing continuation sets; resemblance, reference, information, intent and an interpreter are all resources it leaves aside.

Definition 7.6 (Residue). For a substitution holding over $\mathcal{Y}$, the *residue* $\varrho(\sigma, [E_A], \mathcal{Y}')$ over a wider class $\mathcal{Y}' \supseteq \mathcal{Y}$ is the set of $Y \in \mathcal{Y}'$ at which the quotient equality of Definition 7.5 fails, together with the classes on which the two quotients differ. A substitution with $\varrho = \emptyset$ over every wider class is not required by anything in this paper and is generically unavailable: the interaction class over which substitution holds is part of what a symbol is.

Proposition 7.7. *$\dashv$ is in general neither symmetric nor transitive, and it is subsystem-relative: $\sigma \dashv_{\mathcal{Y}} [E_A]_{I,X}$ does not entail $\sigma \dashv_{\mathcal{Y}} [E_A]_{J,X'}$ for a distinct subsystem J.*

Proof. Each clause follows from the quantifier structure of Definition 7.5. Symmetry fails because the class $[E_A]$ and the single configuration σ enter the definition asymmetrically, and a class member need not substitute for the class. Transitivity fails because the interaction classes over which two substitutions hold need not coincide. Relativity to the subsystem holds because Con_I and $\sim_{I,X}$ are both indexed to I and to its configuration. □

The third clause of Proposition 7.7 has consequences beyond its brief proof. The same configuration may be a symbol for one subsystem and a material obstacle for another, with no contradiction and no privileged verdict, because there is no description made from outside every partition. A mark on paper that substitutes for a class of obligations in the dynamics of one party, and that merely occupies space in the dynamics of another, is correctly described by both.

Proposition 7.8 (Composition up to residue)**.** *If $\sigma \dashv_{\mathcal{Y}} [E]$ and $\sigma' \dashv_{\mathcal{Y}} [E']$, then substitution for the composite holds on $\mathcal{Y}$ up to the accumulated residue. It is exact when $\approx_{I,\cdot}$ is a congruence for the composition in question.*

Proof. Substituting in turn yields agreement of quotients at each step; agreement for the composite requires that $\approx_{I,\cdot}$ be preserved under the composition, which is the congruence condition. □

Whether a given system's discrimination relation is such a congruence is a property of that system and is not settled here. Section 15.2 records it. The content of Proposition 7.8 is that substitution is not preserved under composition without loss, and that a symbolic system in which composition is nearly exact is a contingent achievement with identifiable conditions.

Corollary 7.9 (Products and symbols)**.** *Every symbol is a product, and not every product is a symbol.*

Proof. A symbol satisfies Definition 7.5, which presupposes participation, so the first clause holds. For the second, a persistent configuration may alter Con_I without there being any discrimination class for which it substitutes across an interaction class. □

Corollary 7.9 separates two conditions that are conflated in ordinary usage and that a theory of production needs apart. A machine that conditions what its operator may do next participates without standing in for anything; a price that occupies the place of a class of activities stands in for something. Both are products of prior activity, and the difference between them is not a difference of importance but a difference of role.

Remark 7.10 (Three independent axes)**.** Symbols vary independently along three axes, and conflating them produces errors that are hard to detect afterwards. *Privacy*: whether the configuration lies within a single worldtube or is carried across several. *Timescale*: whether it changes on the scale of a subsystem's internal continuations or on the scale of exchanges across its boundary. *Substitution class*: whether the class substituted for consists of environmental configurations, of the subsystem's own configurations, or of configurations at later cuts. Definition 7.5 admits internal symbols, sharedness being a further and separable condition. The axes are used in later work and are recorded here because the definitions fix them.

7.4 Related constructions

The construction in Definitions 6.2 and 7.5 stands close to existing work, and the debts are stated here where the definitions are made.

Equivalence of states by equality of their induced futures, and the identification of a minimal structure sufficient for prediction, are the substance of computational mechanics: causal states are defined by exactly such an equivalence, and the resulting machine is minimal among sufficient predictors. That construction owns the structure used in Definition 6.2 and in Section 8. What is altered here is the setting: computational mechanics quantifies over an ensemble and defines equivalence by equality of conditional distributions, and the present setting has a singular trajectory and no ensemble, so distributions are replaced by admissible-continuation sets in accordance with Requirement 2.3. The replacement weakens the results and preserves the structure.

Teleosemantic and biosemiotic accounts address the question of what makes a physical structure carry content, and answer it by appeal to function, history or interpretation. Definition 7.5 does not answer that question; it declines it, and defines a role in the dynamics that content-bearing structures would also occupy. Whether the role suffices for the explanatory purposes those accounts serve is a fair objection and is registered in Section 15.

The claim that abstraction is performed in practice before it is performed in thought belongs to Sohn-Rethel [6], and the counterpart here, that $\sim$ is a relation constituted by what a subsystem's dynamics actually distinguishes, is a formal restatement of his position [4], and no part of it is an independent discovery. Its application to the classification of activities is taken up in a companion paper.

8 The Effective Decomposition and Its Criterion

8.1 The criterion

A subsystem carrying symbols is customarily described as having two aspects: what it is doing, and what it is carrying. The description is useful and is usually offered as an appeal to what is effective or functional, which leaves open whether the division is a fact about the subsystem or a convenience of the describer. The criterion adopted here is sufficiency.

Definition 8.1 (Admissible decomposition, [L2])**.** A decomposition $X_I = (X^R, X^S)$ is *admissible* when

1. $\mathrm{Con}_I(E, X^R, X^S)$ at a cut is a function of (E, X^R, X^S) at that cut, and
2. Con_I is not a function of (E, X^R) at that cut alone.

X^S is then a *sufficient-state sector* and X^R the *relational sector*.

The two clauses assert, respectively, that the pair carries everything the continuations depend on, and that the relational sector by itself does not. The second clause is what makes the decomposition non-trivial: were the relational sector already sufficient, the symbolic sector would be redundant and nothing would be gained by naming it. This is the third borrowing under Requirement 2.2. What is transferred is the use of a sufficiency condition to characterise a state; the transfer would fail for any system whose continuations depend on unbounded stretches of its history in a way no finite adjunct can summarise, and Section 15 records the condition.

Definition 8.2 (Minimality)**.** An admissible decomposition is *minimal* when no proper sub-configuration of X^S is itself sufficient in the sense of Definition 8.1. The *symbolic sector* of I is a minimal sufficient sector.

Remark 8.3 (Uniqueness)**.** Whether a minimal sufficient sector is unique is open. Nothing established here excludes two minimal sectors that are not subconfigurations of one another, and no result below assumes uniqueness. Section 15.2 records the problem.

8.2 Consequences of the criterion

Definition 8.1 settles a question left open by informal presentations, which describe the symbolic sector as effective and functional without stating a test. The test is now available: a candidate division is admissible when the continuations are a function of the pair and are not a function of the relational part alone.

The criterion also fixes the direction of dependence that Section 10 requires. On this account the symbolic sector renders the joint configuration sufficient; the manner in which it conditions the relational sector's continuations is not an additional postulate but the content of clause (i) taken together with clause (ii).

Remark 8.4 (Effective form)**.** [L3]. In an effective description in which the symbolic sector is represented by a field and integrated out, the same fact appears as a retarded kernel acting on the relational field: the continuations of that field at a cut depend on earlier cuts through the kernel. The kernel and the sufficient sector are one object described at two levels. No result in this paper uses the effective form, and it is recorded here so that later work may cite the identification without restating it.

9 Repertoire, Composition, and Local Symbolic Systems

Definition 9.1 (Repertoire and order sensitivity, [L2])**.** Σ_I is the set of configurations that are symbols for I. Composition is *order-sensitive* when there are $\sigma_A, \sigma_B \in \Sigma_I$ and a context with

$$\mathrm{Con}_I(\sigma_A\sigma_B \otimes \cdot) \;\neq\; \mathrm{Con}_I(\sigma_B\sigma_A \otimes \cdot).$$

Definition 9.2 (Grammar and local system, [L2])**.** $\mathcal{G}_I$ is the constraint structure on admissible compositions and continuations of elements of Σ_I: which composites occur, and what they permit. The pair $\mathrm{L}_I = (\Sigma_I, \mathcal{G}_I)$ is the *local symbolic system* of I.

Order sensitivity is the minimal condition under which a repertoire has syntax at all, and it is stated as a condition satisfied or not, with no claim that human language is the only structure satisfying it. Weights on compositions, understood as a measure of how strongly one symbol constrains the next, require an amplitude and are available only as an enrichment under Requirement 2.3; they appear in no definition here.

Remark 9.3 (Shared order without a shared object)**.** Where several subsystems have local systems that agree, the agreement is a relation among them and not a further object above them. Two forms of the relation can be stated with the machinery introduced: dynamically, the parties' trajectories are mutually constraining in a way that persists, so that each is predictable to the other within its context class; statically, the same discrimination class is substituted for by configurations carried redundantly across several partitions. The second is the sense in which a structure may be called objective for a set of parties, and it is a matter of degree. Neither form requires a common system of which the local systems are instances, and this paper does not introduce one.

The restraint in Remark 9.3 is deliberate. A construction that begins from local systems and then posits their union as the real symbolic order reintroduces exactly the shared arena that Remark 3.3 removed, and it does so at the point where the analysis of who can revise what is about to be made. Whatever agreement obtains must be exhibited as a relation among the parties.

10 Mediation and the Reverse Pass

Definition 10.1 (Mediation, [L2])**.** A product π *mediates* I at X when Con_I depends on it: there are joint configurations X, X' agreeing except on π with $\mathrm{Con}_I(X) \neq \mathrm{Con}_I(X')$.

The mechanism is selection. The presence of the product removes some continuations from the admissible set and admits others, and this exhausts the reverse pass at [L2]. Descriptions in which a persistent structure modifies the phase of a subsystem's dynamics, or biases it toward a region, require an amplitude and belong to [L3] or [L1]; they are excluded here by Requirement 2.3.

Remark 10.2 (Mediation carries no verdict)**.** That π mediates I asserts nothing adverse about either. Every tool, every grammar, every road and every institution mediates the subsystems whose continuations depend on it, and a subsystem mediated by nothing would be one for which nothing persistent makes any difference. The conditions under which mediation acquires the structure that a critical vocabulary describes are stated in Section 13 and are additional to Definition 10.1.

11 Activity, Product, and the Means of Production

The vocabulary of the preceding parts was built without reference to any social content. This part applies it to the elementary categories of production. The application is a mapping and it is offered as such: each category is assigned a formal counterpart, the assignment is stated, and the record includes both what the assignment preserves and what it does not. The mapping leaves the standing of the categories as it found it, vindicating and replacing neither.

11.1 The assignments

Activity, living labour, is the dynamics of a subsystem's relational sector: the continuations that run through X^R and alter the configuration of $\mathcal{C}_I$ and its boundary.

Objectification is generation in the sense of Definition 7.3: a trace present at a later cut and absent at an earlier one, with the continuation realising the difference running through the subsystem's cells. This is the forward direction, and Definition 7.3 is the whole of it.

The product is π of Definition 7.4. By Corollary 7.9 the products of a subsystem divide into those that participate without substituting and those that substitute: the second are its symbols, and the symbolic order it produces is a subclass of what it produces, not a separate kind of thing. This is the assignment on which the rest of the mapping depends, and its consequence is drawn in Section 11.2.

The means of production is a product that re-enters as a condition on its producer's continuations: π generated by I that mediates I in the sense of Definition 10.1. The definition is deliberately broad and covers an instrument, a road and a trained habit alike. It does not yet distinguish the case in which the condition is set by another party, which requires Section 13.

Externalisation is the movement made expressible by Remark 3.3: a product generated by I comes to lie outside $\partial\mathcal{C}_I$ while remaining coupled across it. On a fixed partition the movement cannot be stated, since a region is inside or outside once and for all. On a relative partition it is a change in where the boundary falls, and the coupling that persists across the new boundary is what makes the product still matter to its producer.

11.2 Dead labour and the value form

Corollary 7.9 yields a distinction that ordinary usage conflates. A product that participates in a subsystem's continuations without substituting for any discrimination class conditions what may be done without standing in for anything: a machine, a built environment, an accumulated technique. A product that substitutes for a class occupies the place of what it stands in for: a price, a title, a record of account.

Both are traces of prior activity conditioning present activity, and the vocabulary of dead labour weighing on the living covers both. The ladder adds that the two condition activity by different routes, and that an analysis treating them alike will misdescribe at least one. The first constrains by participation; the second constrains by substitution, and therefore inherits the residue of Definition 7.6, which the first does not have. Where the substituted class and the substitute diverge, a structure that constrains by substitution constrains on the basis of a class it no longer tracks, and the divergence goes unregistered in the dynamics. This is a property of the second route alone.

11.3 The relations of production

The relations of production are not assigned to a single object here. Three distinct structures are available in the construction, and the category is composite.

1. **The placement of the boundary.** Which cells lie within $\partial\mathcal{C}_I$ is a fact about the partition, and property is a placement: what falls within a party's region and what falls outside it. Dispossession is a displacement of the boundary, and Remark 3.3 is what makes that statable.
2. **Structure carried on the boundary.** By Proposition 3.6 the coupled pair carries structure belonging to neither party, and by Definition 3.5 a coupling may carry its own configuration. A contract, a wage relation and an established procedure are structures of this kind: they are not properties of either party and they constrain both.
3. **The form of the coupling.** How the parties' continuations constrain one another, that is, which of I's continuations depend on which of J's configurations, is a third structure, distinct from the first two, and at [L3] it appears as an interaction term with its own couplings.

Two received distinctions are then differences of which structure has changed. Where a party is brought under a coupling while its own continuation map is untouched, the change is in (3) alone. Where the party's own continuation map is restructured, so that what it can do at all is different, the change reaches the party's own terms. The distinction between formal and real subsumption is, on this reading, a distinction between the term of the description that has moved, and is checkable once the three structures are separated.

Remark 11.1 (What is deferred)**.** Three categories require machinery this paper does not supply and are deferred. *Abstract labour* requires a discrimination relation over activities and a warrant for saying that a practice performs the quotient; the warrant is Sohn-Rethel's and the formal statement belongs with it. *Value* requires a magnitude, and magnitudes are available only at [L3] and only in a regime where a universal equivalent exists; a magnitude introduced at [L2] would violate the standing constraints of Section 2. *Fetishism* requires both Section 13 and a condition on the accessibility of generation records, and is stated in a companion paper.

12 Condensation of a Coupling into a Persistent Structure

A coupling may cease: a live relation between two parties may end through the withdrawal of one of them, through the completion of their joint activity, or through the passage of the interval over which the coupling persisted. This section concerns what remains.

Definition 12.1 (Condensate, [L2]). Let J_{AB} be a coupling active over $[\chi_0, \chi_1]$ and absent after χ_1. A product σ_{AB} is a *condensate* of J_{AB} when

1. σ_{AB} is a trace over an interval beginning at or before χ_1 and extending beyond it, and
2. $\sigma_{AB} \dashv_{\mathcal{Y}}$ [configurations in which J_{AB} is active] for a non-empty interaction class $\mathcal{Y}$.

The second clause carries the content of the definition. The structure that remains occupies, across some class of interactions, the place that the live coupling occupied: what could be done because the parties were in relation can now be done because the structure persists. A record of an agreement standing where the agreeing stood, a procedure standing where a founder's practice stood, and a title standing where an act of appropriation stood are instances.

Proposition 12.2 (Mediation of non-parties). *Let σ_{AB} be a condensate of J_{AB}. Any subsystem K whose continuations depend on σ_{AB} is mediated by it, including $K \notin \{A, B\}$.*

Proof. Immediate from Definition 10.1, which is indexed to dependence of Con_K and places no condition on K's participation in the coupling. □

Proposition 12.3 (The structural default). *Let σ_{AB} be a condensate of a coupling absent after χ_1, and let $K \notin \{A, B\}$ with Con_K depending on σ_{AB} at cuts after χ_1. Then $K \in \mathrm{Med}(\sigma_{AB})$, and membership in $\mathrm{Rev}_h(\sigma_{AB})$ for K requires an admissible continuation through K's own sector that alters σ_{AB}. The constitution of σ_{AB} by a coupling to which K was not party supplies no such continuation. Any such continuation is therefore a separately instituted channel, itself a product with its own mediation and revision sets.*

Proof. The first clause is Proposition 12.2. The second is Definition 13.2. For the third, σ_{AB} was generated, in the sense of Definition 7.3, by continuations running through the cells of A and B, and that generation leaves every cell of K off the continuations that alter it. The existence of a continuation through K's sector altering σ_{AB} is thus an additional fact about the design, and whatever carries it is a persistent configuration conditioning continuations, hence a product by Definition 7.4. □

Proposition 12.3 is stated as a default and a burden, and not as an impossibility. Structures do come to be revisable by parties who did not produce them, and the proposition states the cost: a channel, itself produced, itself mediating, and itself revisable or not. The proposition also identifies where unrevisable structure comes from, which the vocabulary of the preceding parts described without explaining. A relation is live; it condenses; the relation ends; the structure remains and mediates those who come after. This sequence is the elementary mechanism, and it requires no adverse intent, no imposition and no decision by any party.

Remark 12.4 (Prior owners). The sequence is not a discovery of this paper. Weber [7]'s account of the routinization of charisma describes it for the case where the ceased coupling is a following, and Berger [1] and Luckmann's account of habitualization and institutionalization describes it in general. What is added here is a formal statement in which the mediation of non-parties and the default absence of a revision channel are consequences of definitions and not observations. Marx's claim [2] that relations between persons take on the form of relations between things describes the same transition from the side of what the resulting structure looks like to those who encounter it, and Definition 12.1 is one precise reading of it.

13 Mediation Sets and Revision Sets

Definition 13.1 (Mediation set). $\mathrm{Med}(\sigma) = \{\, I : \sigma \text{ mediates } I \text{ in the sense of Definition 10.1} \,\}$.

Definition 13.2 (Revision set, horizoned)**.** $I \in \mathrm{Rev}_h(\sigma)$ at X when there are admissible h-step continuations $c, c' \in \mathrm{Con}_I^h(X)$ such that

1. the contributions from outside I across $\partial\mathcal{C}_I$ are identical along c and c',
2. c and c' differ within I's sector, and
3. the configurations of σ at step h differ between them.

$\mathrm{Rev}(\sigma) = \bigcup_h \mathrm{Rev}_h(\sigma)$.

The first clause carries the weight of the definition. Holding the environment's contribution fixed along both continuations isolates difference-making *through* I, and does so on continuations that the dynamics admits, without introducing an intervention, a manipulation or a counterfactual world. The third standing constraint of Section 2 is thereby observed.

Remark 13.3 (Only bounded horizons are testable)**.** Rev is defined as a union over horizons and is not decidable in general. Whether a structure is ever revisable by a party, over an unbounded horizon, is a question about the eventual behaviour of a system whose dynamics may be rich enough to make such questions undecidable. Statements in this paper and its companions are therefore made at stated horizons, and a claim that a structure is unrevisable is always a claim about some h.

Proposition 13.4 (Structure without magnitude)**.** Med *and* Rev_h *induce a directed structure on subsystems, on which only partial orders are defined. No magnitude of mediation or of revisability is definable from the constructions of this paper.*

Proof. Both sets are defined by existence conditions on continuations, which yield membership and not degree. An ordering by inclusion is available and is partial: two subsystems may each be mediated by structures the other is not. A total ordering would require a comparison across subsystems, which the first standing constraint of Section 2 excludes. □

Proposition 13.4 states a limitation and defends it. A vocabulary in which the extent of mediation could be summed or compared across parties would permit aggregation, and a framework that treats heterogeneity as primitive must decline aggregation over parties. What the sets support is the comparison of positions, and that is what the next paper requires.

Remark 13.5 (Definitions to come)**.** Three configurations of Med and Rev are the subject of a companion paper and are named here without being defined, so that the present paper's definitions may be cited exactly. The case in which a subsystem is mediated by a structure it generated and cannot revise; the case in which a structure is revisable by no subsystem; and the case in which a structure generated by one subsystem is revisable only by another. Their evaluation requires premises supplied elsewhere.

14 Summary of Results

The paper set out to determine the conditions under which a subsystem may be said to have a symbol, on a substrate with no arena, no external time and no privileged partition. Six results have been established; they are separated here from what has been assumed and from what has been deferred.

A discrimination relation is available that is an equivalence, and its classes are therefore well defined (Proposition 6.3). The relation is indexed to the subsystem's configuration, so that what a subsystem carries alters what it can distinguish (Proposition 6.4), and it degenerates exactly in the case where nothing crossing the boundary makes a difference (Proposition 6.5).

Table 3: The mapping of Section 11, with the level at which each assignment is made and its status in this paper. Entries marked DEFERRED require machinery introduced elsewhere and are named in Remark 11.1.

Category	Formal counterpart	Level	Status
Activity, living labour	dynamics of the relational sector X^R	[L2]	fixed
Objectification	generation, Definition 7.3	[L2]	fixed
Product	π, Definition 7.4	[L2]	fixed
Symbolic order	the substitutive subclass of products, Corollary 7.9	[L2]	fixed
Dead labour	product participating without substituting	[L2]	fixed
Means of production	product re-entering as a condition on its producer	[L2]	fixed
Externalisation	generated product passing beyond $\partial\mathcal{C}_I$ while coupled	[L2]	fixed
Relations of production	composite: boundary placement; boundary-borne structure; coupling form	[L2]/[L3]	fixed, Section 11.3
Formal and real subsumption	which of the three structures has changed	[L2]/[L3]	fixed
Mode of production	regime in which the coupling form may be treated as fixed	[L3]	fixed
Abstract labour	quotient of activities by a practice-borne $\sim$	[L2]	deferred
Value	effective magnitude, universal-equivalent regime	[L3]	deferred
Fetishism	$\mathrm{Rev} = \emptyset$ with generation record inaccessible	[L2]	deferred

A substitution relation is available that introduces no interpreter and no semantic primitive (Definition 7.5). It carries an explicit interaction class and an explicit residue, it is neither symmetric nor transitive, it is relative to the subsystem, and its composition is exact only under a congruence condition that is a property of a design (Propositions 7.7 and 7.8).

Persistence, participation and substitution are three conditions and not one, so that a product that conditions activity without standing in for anything is distinguished from one that stands in for a class (Corollary 7.9). The distinction is what allows an accumulated instrument and a price to be described as products of the same kind of process, conditioning activity by different routes.

The decomposition of a subsystem into relational and symbolic sectors has a criterion (Definition 8.1), where informal presentations appeal to what is effective or functional, and the criterion is checkable.

The reverse pass has a mechanism, selection among admissible continuations (Definition 10.1), and mediation carries no verdict of its own (Remark 10.2). Two derived relations, mediation and revision, are defined by membership conditions and support partial orderings (Definitions 13.1 and 13.2 and Proposition 13.4).

A coupling that has ceased can leave a structure standing in its place, mediating parties who were never party to it and supplying them by default with no channel of revision (Definition 12.1 and Proposition 12.3). This identifies an elementary mechanism by which unrevisable structure is produced, and it requires no imposition by anyone.

15 Limitations and Open Problems

15.1 Conditions of failure

The paper offers no empirical test, and the obligation that remains is to say what would count against it. Five conditions would.

The template. If the objects to be described have parts individuated prior to and independently of their relations, the substrate is the wrong one and its advantages are irrelevant. The construction is committed to the claim that the boundary between a producer and a product is a variable of the situation, and a domain in which it is not is a domain this vocabulary should not be applied to.

The pivot. If a structure that plays the role of Definition 7.5 in a subsystem's dynamics can nonetheless fail to be a symbol in any sense the surrounding theory requires, the definition is too weak, and the objection is the standing one from accounts that take content to be the essential question. The reply available here is that the dynamical claims of the theory require only the role; the objection maintains that the theory requires more, and it is not answered in this paper.

Sufficiency. If a subsystem's continuations depend on unbounded stretches of its history in a way that no finite adjunct summarises, Definition 8.1 has no admissible instance and the symbolic sector is not definable by this route.

Acyclicity. If the designs of interest contain cycles in the record relation, Requirement 4.2 fails for them, the order is a preorder, and the clock construction of Definition 4.3 is unavailable.

Congruence. If the indistinguishability relations of interest are generically not congruences, Proposition 7.8 leaves composite substitution inexact by an amount that accumulates, and any symbolic system with more than a few composition steps is described by this vocabulary only loosely.

15.2 Open problems

1. **Selection of the partition.** Nothing at [L2] distinguishes the subcomplexes that constitute subjects. Candidate criteria are named in Remark 3.4 and none is adopted. This is the load-bearing gap of the construction, and every relativity in the paper inherits from it.

2. **Acyclicity.** Whether acyclicity of the record relation can be derived from conditions on the complex, so that it need not be required of a design (Requirement 4.2).

3. **Uniqueness of the minimal sufficient sector** (Remark 8.3).

4. **Congruence of the indistinguishability relation** under composition (Proposition 7.8).

5. **Orientation.** The order supplies precedence and not direction. What at [L2] would supply an orientation criterion is open; a candidate at [L3], and the observation that reduction to a subsystem is what makes a direction available at all, are recorded and not developed (Remark 4.4).

6. **Undecidability of eventual revisability** (Remark 13.3), which bounds every claim about what a party can and cannot alter.

7. **Routes to a symbol.** Three routes by which a product comes to substitute for a class are visible in the construction: generation by a subsystem whose continuations come to depend on it, condensation of a ceased coupling (Definition 12.1), and the formation of a record by a third party observing an interaction. Whether these are three mechanisms or one mechanism seen from three positions is open.

8. **Identity of elements across cuts.** That the same element is present at two cuts is an assignment made by a design, and a contestable one. The construction inherits the problem and does not solve it.

9. **The cost of Requirement 2.3.** Rates, likelihoods and any account of how quickly a structure changes are unavailable without an amplitude, and the conditions under which an amplitude could be reintroduced without presupposing a reference class are open.

15.3 Subsequent work

Three lines of work depend on the definitions fixed here and are pursued elsewhere. The configurations of Med and Rev named in Remark 13.5 carry the analysis of production for which Section 11 is the preparation, together with the deferred categories of Remark 11.1. The axes of Remark 7.10, and in particular the case in which the class substituted for consists of a subsystem's own configurations, carry an account of subject formation. And the limits on what a subsystem can establish about a complex containing it, of which Proposition 6.5 is the degenerate case, carry an account of the observer internal to the complex it observes.

Recapitulation

The principal objects of the paper, collected for reference. Levels: [L1] the generating field theory, [L2] a complex and its combinatorics, [L3] effective descriptions; everything below is [L2].

- **Substrate.** Complex $\mathcal{C}$ of labelled cells; subsystem $\mathcal{C}_I$ with boundary $\partial\mathcal{C}_I$; the partition relative and symmetric. Record relation: $u\ R\ v$ when u contributed to a persistent configuration conditioning continuations at v; $\preceq$ its closure, acyclic by requirement. Clock χ: monotone along $\preceq$; states are cuts; the traversal τ is the general, weaker formulation, and the relational clock is primary.

- **Dynamics.** Admissibility $\rightsquigarrow$ on joint configurations (E, X_I) at successive cuts; $\mathrm{Con}_I(X)$ the continuation set; $\mathrm{Con}_I^h(X)$ the admissible h-step sequences with their joint realisations; composition $U \otimes V$, a fixed component written $\cdot$.

- **Discrimination.** $E_a \sim_{I,X} E_b$ iff $\mathrm{Con}_I(X, E_a, p) = \mathrm{Con}_I(X, E_b, p)$ for every $p \in \mathcal{P}_I$; an equivalence; extended to arbitrary configurations as $\approx_{I,X}$. Indexed to X: what a subsystem carries alters what it distinguishes. A boundary admitting no variation yields one class and hence no symbol.

- **The ladder.** Trace: persistent configuration. Product π: participating trace, $\mathrm{Con}_I(\cdot \otimes \pi \otimes Y) \neq \mathrm{Con}_I(\cdot \otimes Y)$ for some Y. Symbol: $\sigma \dashv_{\mathcal{Y}} [E_A]_{I,X}$ iff $\mathrm{Con}_I(X \otimes \sigma \otimes Y)/\approx\ =\ \mathrm{Con}_I(X \otimes E_A \otimes Y)/\approx$ for every $Y \in \mathcal{Y}$; residue ϱ: the failures over a wider class. $\dashv$ is asymmetric, intransitive, subsystem-relative, and composes up to residue, exactly under congruence of $\approx$.

- **Sectors.** $X_I = (X^R, X^S)$ admissible when the pair is sufficient for Con_I and X^R alone is insufficient; the symbolic sector is a minimal sufficient sector, uniqueness open.

- **Mediation and revision.** π mediates I when Con_I depends on it, by selection; $\mathrm{Med}(\sigma)$ the mediated set. $I \in \mathrm{Rev}_h(\sigma)$: two admissible h-step continuations, outside contributions identical, differing in I's sector, yielding different σ at step h. Partial orders only.

- **Condensation.** A ceased coupling may leave σ_{AB} substituting for its active configurations; the condensate mediates non-parties, and their membership in Rev requires a separately instituted channel, itself a product.

- **Economic mapping.** Activity: dynamics of X^R. Objectification: generation. Product: π; the symbolic order its substitutive subclass. Dead labour: participation lacking substitution. Means of production: a product re-entering as condition on its producer. Externalisation: a generated product beyond $\partial\mathcal{C}_I$, still coupled. Relations of production: boundary placement, boundary-borne structure, coupling form. Formal and real subsumption: which of the three moved. Abstract labour, value, fetishism: deferred, with their requirements stated (Table 3).

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