

The Inversion of Symbolic Mediation and the Conditions of Revision: A Preliminary Discussion of Marxian Alienation in a Background-Independent Relational Ontology

Wanhong Huang

huangwanhong@serendip.ngo

Abstract

This paper gives a structural account of alienation on a combinatorial substrate of the kind that background-independent approaches to quantum gravity, group field theory among them, have developed as models of the emergence of a discrete spacetime, in which no arena is given in advance and every structure, extent and order included, is carried by the relations themselves. The substrate is used here as a formal template, and no physical claim is made of the objects described; the apparatus is stated in full in the paper and is developed at length in a companion study. The account proceeds by structure throughout, comparing configurations by membership and position and stating every condition on configurations the dynamics admits. Its primitive materials are a subsystem's admissible continuations, the products its activity generates, the mediation a persistent product exercises by selection among continuations, and the presence or absence of a revision channel. The Marxian analysis is the account's centre and its test case, and the general structure is built so that the specifically Marxian claims occupy exact positions within it. Four conditions of increasing strength are defined: subjection, in which a subsystem is mediated by a structure it cannot revise; alienation proper, in which the unrevisable structure is the subsystem's own product; reproductive closure, in which every admissible continuation of the subsystem preserves the structure that subordinates it; and discriminative inversion, in which the subsystem distinguishes its environment exactly as finely as its repertoire classifies it, so that a failure of substitution cannot register within the subsystem at all. A stratification result orders the first three conditions strictly and shows the fourth independent of them, with constructions separating every level. Received fidelity, entropy and persistence definitions of alienation are examined and rejected against explicit criteria of adequacy, on grounds that include a formal restatement of Marx's objection to Hegel: a fidelity definition equates unalienated objectification with accurate representation. The categories of abstract labour and value are then introduced as properties of a regime containing a universal equivalent, at the effective level only, and the four estrangements of the 1844 manuscripts are distributed over the stratification. The paper is conceptual and preliminary; its results are definitions, short propositions, and an explicit register of open problems.

Keywords: Marxian alienation; symbolic mediation; revision; reproductive closure; value form

Draft. This is a working draft. It is circulated for comment, and its arguments, formulations, and open questions are subject to revision. Correction and criticism are welcome at huangwanhong@serendip.ngo.

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Use of generative artificial intelligence. Generative AI assistants (Anthropic’s Claude, OpenAI’s ChatGPT, and Google’s Gemini) were used extensively in preparing this manuscript: for literature survey, for drafting prose from the author’s specifications, for adversarial review of the argument, and for consistency and citation auditing and typesetting. No simulation or empirical figure is used in this paper, and its arguments are discursive throughout. Formulations and objections arose in the course of that exchange as well as from the author. Every claim, argument, and citation was reviewed and decided by the author, who is responsible for the content and for any errors. Bibliographic details are to be verified against the primary sources before reliance.

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1 Introduction

The claim that products come to dominate their producers is the load-bearing claim of a long critical tradition [7, 6, 5], and it has two standing liabilities. The first is that its central term is overloaded: alienation has been made to cover subjection to any persistent structure, estrangement from one's own product in particular, the conscription of activity into maintaining what subordinates it, and a condition of perception in which the divergence between a symbol and what it stands for can no longer be seen. Arguments slide among these, and objections to one are answered with another. The second liability is that attempts at formal precision have repeatedly reached for the wrong instruments: a distance between an actual and an ideal trajectory, an entropy, a duration. Each of these imports a commitment the tradition itself has reasons to refuse.

This paper addresses both liabilities with one construction. On the relational substrate set out in Section 2, a combinatorial setting of the kind that background-independent approaches to quantum gravity, group field theory among them, have developed as models of the emergence of a discrete spacetime, and which is used here as a formal template with no physical claim attached, four conditions are defined in terms of admissible continuations, generation, mediation and revision. The conditions are strictly ordered in strength, with explicit constructions separating every level, and the fourth is shown independent of the first three. The overloaded term is thereby replaced by a stratification: the positions the tradition has conflated are distinct levels of one structure, and which level a given claim requires can be stated.

The stratification also settles where the specifically Marxian claim sits. That products dominate producers is not, on this account, the claim that producers are subject to structures they cannot revise; serfdom under an edict satisfies that description. It is the claim of the third level: the producer's admissible continuations are confined to those that reproduce the structure subordinating it, so that the activity itself maintains what dominates it. Living labour reproducing capital is the canonical instance, and the level is strictly stronger than subjection, by a construction given below.

The paper proceeds as follows. Section 2 fixes what is inherited and what is added. Section 3 examines the received formal definitions, states the specific failure of each, and extracts criteria of adequacy; the decisive objection is a formal restatement of an argument of Marx's against Hegel. Sections 4 to 8 construct the four conditions and prove the stratification. Section 9 treats the positions available around an unrevisable structure, Section 10 the mechanism by which such structures arise, Section 11 the categories that require an effective magnitude, Section 12 mediation by classes of future configurations, and Section 13 the distribution of the 1844 estrangements over the levels. Sections 14 and 15 close with the results and the open problems.

2 The Relational Setting

This section states the apparatus the paper uses. The definitions are given in full, so that the paper may be read on its own; their derivation, and the results about them cited below, are given in a companion study of symbol emergence, hereafter Companion Study I [4], this paper being the second of a set of five. Results imported from it are stated in full where used, their proofs alone remaining with their source. A reader may take the definitions and the imported results as stipulations and judge the paper's results against them.

2.1 Preliminaries

The setting descends from physics and the bookkeeping from elementary mathematics. One paragraph of each suffices. In several programmes of quantum gravity, spacetime is built in

Table 1: Notation. Each symbol is introduced where indicated; the table is a lookup and replaces no definition.

Symbol	Reading	Introduced
$\mathcal{C}, \mathcal{C}_I, \partial\mathcal{C}_I$	complex; subsystem; its boundary	Definition 2.1
$\mathcal{W}_I$	worldtube: a subsystem whose boundary persists	Definition 2.1
$u \preceq v$	u precedes v in the record-generated order	Definition 2.2
$\chi, X_I(\chi)$	clock; the configuration on its level set, a cut	Definition 2.2
$X = (E, X_I)$	joint configuration: environment on the boundary, and subsystem	Definition 2.3
$X \rightsquigarrow X'$	X' is an admissible continuation of X	Definition 2.3
$\text{Con}_I(X), \text{Con}_I^h(X)$	continuation set; admissible h -step continuation sequences	Definition 2.3
$U \otimes V, \cdot$	composition of configurations on disjoint regions; a component held fixed	Definition 2.3
$\mathcal{P}_I$	context class: the admissible boundary variations compared	Definition 2.4
$Z \approx_{I,X} Z'$	indistinguishable for I at X ; $\sim_{I,X}$ on boundary configurations	Definition 2.4
$[E]_{I,X}$	a discrimination class	Definition 2.4
$T \sqsubseteq X$	T is a subconfiguration of X	Definition 2.5
π	product: a trace whose presence alters Con_I	Definition 2.5
$\sigma \dashv_y [E]_{I,X}$	σ substitutes for the class over the interaction class y	Definition 2.6
ϱ	residue: the interactions at which substitution fails	Definition 2.6
Σ_I	repertoire: I 's set of symbols	Definition 2.6
$\text{Med}(\pi)$	the subsystems π mediates	Definition 2.7
$\text{Rev}_h(\sigma)$	the subsystems able to revise σ within h steps	Definition 2.8
$[\text{L2}], [\text{L3}]$	level tags: combinatorial; effective	§2
$\text{Pres}(\sigma, X)$	persistence set: continuations preserving σ	Definition 5.1
$\kappa(E)$	classification of E by the repertoire	Definition 7.1
σ^*	universal equivalent	Definition 11.1

place of being assumed: the elementary object is a network of dots (vertices), links (edges) and spanning surfaces (faces); a history of such a network, its parts persisting, appearing and recombining, is called a *spin foam*; group field theory is a machinery whose expansion generates such histories; and geometric quantities are labels carried on the network's parts. *Background independence* names the situation in which the network of relations is all there is, and space, time and extent are features of it. This paper takes only the skeleton, a *complex* of labelled cells with the record of which touches which, as bookkeeping for relations, and borrows none of the physics' quantum-mechanical or geometric results.

Of mathematics the paper uses: relations and the membership notation ($x \in S$: x belongs to S ; $S \subseteq T$: every member of S belongs to T); *equivalence relations*, sortings of objects into classes, with the *quotient* the collection of the classes; *partial orders*, rankings that leave some pairs unranked, claimed here where a stronger vocabulary would claim numbers; and *horizons*, stated numbers of steps bounding every claim made at them.

2.2 Substrate, subsystems, and time

Definition 2.1 (Complex, subsystem, worldtube). A *complex* $\mathcal{C}$ is a set of cells, vertices, edges and faces, with the record of which is incident to which, together with labels on the cells; a

configuration X of a region is an assignment of labels to its cells. A *subsystem* is a subcomplex $\mathcal{C}_I \subset \mathcal{C}$ with boundary $\partial\mathcal{C}_I$, the cells of $\mathcal{C}_I$ incident to cells outside it; $\mathcal{C} \setminus \mathcal{C}_I$ is its *environment*. A *worldtube* is a subsystem whose boundary persists.

The partition is a description choice and it is symmetric: for subsystems I and J of one complex, each lies in the environment of the other, and every partition stands on the same footing. Nothing in the formalism selects one, and the selection is an act of the describer.

Definition 2.2 (Record relation, order, clock, cut). For cells u, v , write $u R v$ when u contributed to a persistent configuration on which the admissible continuations at v depend, and let $\preceq$ be the reflexive transitive closure of R , assumed acyclic. A *clock* is a function $\chi : \mathcal{C}_I \rightarrow \mathcal{O}_\chi$ into a totally ordered set, non-decreasing along $\preceq$; its level sets Σ_χ are *cuts*, downward closed with respect to $\preceq$, and $X_I(\chi)$ is the configuration on the cut.

Time is thereby generated by the production of records and is supplied from nowhere else: what has left a record constraining a cell precedes it, and a state is a cut.

Definition 2.3 (Admissibility and continuation sets). The dynamics is a relation $\rightsquigarrow$ between joint configurations at successive cuts,

$$(E, X_I)(\chi_n) \rightsquigarrow (E, X_I)(\chi_{n+1}),$$

with E the environment's configuration on $\partial\mathcal{C}_I$, present at every cut. For $X = (E, X_I)$,

$$\text{Con}_I(X) = \{ X'_I : X \rightsquigarrow (E', X'_I) \text{ for some } E' \},$$

and $\text{Con}_I^h(X)$ is the set of admissible h -step continuation sequences from X , each taken with its realising joint sequence. Configurations on disjoint regions compose as $U \otimes V$, and a component written $\cdot$ is held fixed by context.

The relation is primitive: it records which continuations the design admits, and carries no weight, probability or amplitude over them. Where a model supplies an amplitude $\mathcal{A}$, the relation is its support, $X \rightsquigarrow X'$ exactly when $\mathcal{A}[X \rightarrow X'] \neq 0$, and nothing below uses more than that. The restriction has a cost, recorded where it bites: rates, likelihoods and any account of how quickly a structure changes are unavailable.

2.3 Discrimination

Definition 2.4 (Context class, indistinguishability, discrimination). A *context class* $\mathcal{P}_I$ is a set of admissible boundary variations under which I 's continuations are compared; it is produced historically and differs between parties. For configurations Z, Z' presented to I at X ,

$$Z \approx_{I,X} Z' \quad \text{iff} \quad \text{Con}_I(X, Z, p) = \text{Con}_I(X, Z', p) \quad \text{for every } p \in \mathcal{P}_I.$$

The relation is an equivalence; its restriction to boundary configurations is written $\sim_{I,X}$, with classes $[E]_{I,X}$.

Equality of induced continuation sets, quantified over the context class, is what makes the relation transitive and its classes well defined; a formulation by sufficient similarity yields no classes. Discrimination is indexed to X , so what a subsystem carries alters what it distinguishes, and a boundary admitting no variation yields exactly one class.

2.4 Traces, products, symbols

Definition 2.5 (Trace, generation, product). A configuration tr is a *trace* over an interval when $\text{tr} \sqsubseteq X_I(\chi)$ throughout it. It is *generated by* I over $[\chi_0, \chi_1]$ when $\text{tr} \not\sqsubseteq X_I(\chi_0)$, $\text{tr} \sqsubseteq X_I(\chi_1)$, and the continuation realising the difference runs through cells of $\mathcal{C}_I$. A trace π is a *product* when it participates: for some Y ,

$$\text{Con}_I(\cdot \otimes \pi \otimes Y) \neq \text{Con}_I(\cdot \otimes Y).$$

Definition 2.6 (Symbol and residue). Let $\mathcal{Y}$ be a class of interaction configurations. A product σ is a *symbol* for I over $\mathcal{Y}$, written $\sigma \dashv_{\mathcal{Y}} [E_A]_{I,X}$, when for every $Y \in \mathcal{Y}$

$$\text{Con}_I(X \otimes \sigma \otimes Y) / \approx_{I,\cdot} = \text{Con}_I(X \otimes E_A \otimes Y) / \approx_{I,\cdot},$$

E_A being a representative of the class. The *residue* $\varrho(\sigma, [E_A], \mathcal{Y}')$ over a wider class $\mathcal{Y}' \supseteq \mathcal{Y}$ is the set of interactions at which the equality fails, together with the classes on which the quotients differ.

The symbol occupies, across the stated class, the place a member of the class would occupy, and the definition introduces no interpreter, no resemblance and no reference: it is checked by comparing continuation sets. Substitution is exact only up to what the subsystem can itself distinguish, so a stand-in whose deviations the subsystem cannot register is, for that subsystem, a stand-in without deviation. Established in Companion Study I, and used below: $\dashv$ is neither symmetric nor transitive, it is relative to the subsystem, it composes up to accumulated residue and exactly under a congruence condition, and every symbol is a product while products that participate without substituting are not symbols. A subsystem's *repertoire* Σ_I is its set of symbols.

2.5 Mediation and revision

Definition 2.7 (Mediation and the mediation set). A product π *mediates* I at X when Con_I depends on it: there are X, X' agreeing except on π with $\text{Con}_I(X) \neq \text{Con}_I(X')$. The mechanism is selection among admissible continuations. $\text{Med}(\pi)$ is the set of subsystems mediated by π .

Mediation of itself carries no verdict: every tool, grammar, road and institution mediates those whose continuations depend on it, and the conditions of this paper are additional clauses upon it.

Definition 2.8 (Revision set). $I \in \text{Rev}_h(\sigma)$ at X when there are $c, c' \in \text{Con}_I^h(X)$ such that the contributions from outside I across $\partial\mathcal{C}_I$ are identical along c and c' ; c and c' differ within I 's sector; and the configurations of σ at step h differ between them. $\text{Rev}(\sigma) = \bigcup_h \text{Rev}_h(\sigma)$.

The first clause isolates difference-making *through* I , on continuations the dynamics admits, with no intervention and no counterfactual. Both sets support membership and inclusion, and no magnitude of mediation or revisability follows from them.

2.6 Standing constraints and the status of the paper

Four constraints hold throughout. Comparisons between subsystems proceed by structure. Orderings are partial and are declared as such. Conditions are stated on configurations the dynamics admits. And the vocabulary describes, evaluative conclusions requiring premises supplied elsewhere. Statements are tagged by level: the substrate and its combinatorics are marked [L2], and effective descriptions on emergent structure [L3]. All of Sections 4 to 10 is [L2], and Section 11 alone requires [L3].

The method is subtractive. Section 3 determines the resources an adequate definition may draw on, by examining where the received definitions fail; the constructive sections build from what survives. The order matters: the criteria are extracted before the definitions are given, so that the definitions may be checked against them.

3 Rejected Definitions and Criteria of Adequacy

3.1 The alienation index

The most natural formal proposal, and the first this project itself entertained, is an *alienation index*: a normalised deficit of overlap between two objects, in the general form

$$\alpha(\chi) = 1 - \frac{|\langle X_{\text{actual}}(\chi) | X_{\text{comparand}}(\chi) \rangle|^2}{\|X_{\text{actual}}\|^2 \|X_{\text{comparand}}\|^2},$$

with $\alpha = 0$ read as the absence of alienation and $\alpha \rightarrow 1$ as its completion, and with the comparand instantiated variously: an unmediated trajectory, another party's state, a self-conception against a social order. The index is recorded here in full for two reasons. It is the proposal a reader will most expect, and its failure is instructive: the criteria of adequacy in Section 3.4, which the paper's own definitions are then built to satisfy, are extracted from the ways the index fails. The rejection is therefore a design decision with stated grounds, and the structural account that follows is the shape the grounds leave available. All members of the family fail, for reasons that compound.

First, an overlap requires a metric or an inner product between configurations, and the standing constraints exclude one; the exclusion is not a stipulation of convenience but a commitment about heterogeneity, since a metric between the configurations of distinct parties is a common measure the substrate does not supply. Second, the unmediated comparand is a counterfactual baseline: the trajectory the subsystem would have followed with its mediations removed is not a configuration that occurs, and a definition resting on it inherits every difficulty of specifying it. Third, the cross-party variant scores difference itself as alienation: its deficit vanishes exactly when the two parties' states coincide, so that maximal health is identity, and a framework that treats the heterogeneity of parties as primitive cannot accept a definition on which heterogeneity as such is the pathology. A further variant, comparing a subsystem's self-directed structure with the surrounding order, scores conformity as health, and thereby classifies successful discipline as the absence of alienation.

3.2 The decisive objection

The compounding failures above are formal. The decisive objection is philosophical, and it is Marx's. Every fidelity definition has the following shape: objectification is measured against an ideal, and alienation is the measured falling-away. On such a definition, unalienated objectification is accurate representation, and production is in order to the degree that its outcome corresponds to what it should have been. This is, in formal dress, the position of the 1844 critique of Hegel [7, 2]: that Hegel treats objectification as such as loss, so that the overcoming of alienation must take the form of a reabsorption of the object. A tradition whose founding move is the separation of objectification from alienation cannot adopt a definition that measures the one as a deficit of the other. The objection does not depend on the choice of ideal; it attaches to the shape of the definition.

3.3 Entropy and persistence definitions

Definitions identifying alienation with a reduction of entropy, or of accessible state-space volume, equate constraint with pathology and disorder with freedom. On such a definition a maximally noisy configuration is maximally free, and the acquisition of any structure, a language among the rest, is loss. The consequence is unacceptable independently of its provenance, and it recurs in Section 13 as the Romantic reading of development.

Definitions identifying alienation with the persistence or the reach of a structure classify every institution, every technique and every language as alienated in proportion to their durability. Persistence is a precondition of anything mattering at all; a definition that scores it as the pathology has located the differentia in the wrong place.

3.4 Criteria of adequacy

The rejections yield criteria. An adequate account of alienation must:

1. use no metric between configurations and no counterfactual baseline;
2. keep objectification innocent: generation as such, and mediation as such, carry no verdict (as recalled in Section 2);
3. locate the differentia in the structure of the return, that is, in the presence or absence and the position of revision, and neither in distance from an ideal, nor in disorder, nor in duration;
4. admit degrees only as a partial order or a stratification, never as a scalar;
5. be satisfiable with no adverse intent by any party, since the genesis account of Section 10 produces the relevant conditions without one.

The constructions of Sections 4 to 8 satisfy all five by inspection, and Section 14 verifies the inspection.

4 The Elementary Loop

The material of the analysis is a loop with three segments, each already defined in Paper I and assembled here.

Generation. A subsystem's activity generates a trace: a configuration absent at an earlier cut, present at a later one, with the realising continuation running through the subsystem's cells. Where the trace participates in the dynamics it is a product, and where it substitutes for a discrimination class it is a symbol.

Externalisation. On a relative partition the boundary is a variable, and a generated product can come to lie outside $\partial\mathcal{C}_I$ while remaining coupled across it. The product is then among the conditions of the subsystem's environment: generated from within, operative from without.

Return. A product outside the boundary that mediates the subsystem selects among its admissible continuations. The return is mediation, its mechanism is selection, and it carries no verdict: a grammar returns upon its speakers, a road upon its builders.

Every segment of the loop is innocent separately, and the loop as a whole is the ordinary structure of any activity that leaves durable results. The analysis of this paper concerns the conditions under which the loop acquires a further structure, and all four conditions below are conditions on the loop, not departures from it.

5 Subjection and Alienation

Definition 5.1 (Persistence set, [L2]). For a symbol or product σ with $\sigma \sqsubseteq X$, where X is the joint configuration,

$$\text{Pres}(\sigma, X) = \{ X'_I \in \text{Con}_I(X) : \sigma \sqsubseteq (E', X'_I) \text{ for every } E' \text{ with } X \rightsquigarrow (E', X'_I) \},$$

the continuations of I all of whose admissible joint realisations preserve σ . The definition is stated on the joint configuration so that it applies to structures lying on the boundary or on the environment's side of it, and the persistence set is indexed to the party through Con_I .

Definition 5.2 (Subjection, [L2]). I is *subject to* σ at X and horizon h when $I \in \text{Med}(\sigma)$ at X and $I \notin \text{Rev}_h(\sigma)$ at X .

Subjection is mediation whose return channel is absent: the structure conditions what the subsystem may do, and every admissible continuation through the subsystem's own sector, with the environment's contribution held fixed, leaves the structure as it was within the horizon. The definition is silent on the structure's origin, and an edict, a border and a dead language satisfy it alike.

Definition 5.3 (Alienation proper, [L2]). I is *alienated* from σ at X and horizon h when I is subject to σ at X, h and σ was generated by I , in the sense of Section 2, over an interval ending at or before the cut of X .

The added clause is historical. Alienation proper is subjection to one's own product: the loop of Section 4 closed over a structure whose generating continuations ran through the subsystem's own cells. The clause is what distinguishes the condition from subjection at large, and it is checkable on the record relation, since generation is a fact about which continuations realised the trace.

Remark 5.4 (A relation of relationlessness). The contemporary reconstruction of the concept due to Jaeggi [5] characterises alienation as a relation of relationlessness: a relation to world and self that is at once present and defective as a relation. The pair of definitions above supplies a formal counterpart. Mediation is the relation; the absent revision channel is the relationlessness; subjection is exactly a relation lacking its return. The characterisation is hers; this paper adds the statement of it in terms that admit propositions.

6 Reproductive Closure

Definition 6.1 (Reproductive closure, [L2]). σ *reproductively closes* I at X when $\sigma \sqsubseteq X$, the set $\text{Con}_I(X)$ is non-empty, $\text{Con}_I(X) \subseteq \text{Pres}(\sigma, X)$, and $I \in \text{Med}(\sigma)$ at X . σ *reproductively closes* I *on a stretch* when it does so at every cut of the stretch.

The definition strengthens subjection in a specific direction. Under subjection the subsystem cannot alter the structure; under reproductive closure everything the subsystem can do preserves it. The first is an absence, the second a conscription: the subsystem's admissible continuations are confined to those on which the structure persists, so that the activity itself maintains what conditions it. The non-emptiness clause excludes the degenerate satisfaction by a subsystem with no continuations at all.

Proposition 6.2 (Closure entails unrevisability). *If σ reproductively closes I on a stretch containing the h -step continuations from X , then $I \notin \text{Rev}_h(\sigma)$ at X , and I is subject to σ at X for every horizon the stretch covers.*

Proof. By induction along the steps: closure at each cut of the stretch places every admissible one-step continuation, together with each of its joint realisations, in the persistence set of that cut, so σ retains its trace value along every admissible sequence of at most h steps. Membership in Rev_h requires two such sequences differing in the configuration of σ at step h , and no pair differs. Mediation is a conjunct of the definition. $\square$

Proposition 6.3 (Subjection without closure). *There are designs in which I is subject to σ at every horizon and σ does not reproductively close I .*

Proof. Construct a two-party design in which σ lies among the cells of a second subsystem J , every continuation altering σ runs through J 's sector, and Con_I depends on σ . Then $I \in \text{Med}(\sigma)$; with the contributions from outside I held fixed, no pair of continuations differing only in I 's sector differs on σ , so $I \notin \text{Rev}_h(\sigma)$ for any h ; and joint realisations in which J rewrites σ are admissible responses to I 's continuations, which therefore fall outside $\text{Pres}(\sigma, X)$, so $\text{Con}_I(X) \not\subseteq \text{Pres}(\sigma, X)$. $\square$

The construction of Proposition 6.3 is the formal difference between two conditions that the tradition's vocabulary has conflated, and separating them is one of the paper's purposes. A party subordinated to a structure maintained by another is in subjection without closure: an edict binds the serf, and the edict's persistence is the sovereign's work. A party whose own admissible activity maintains the subordinating structure is in closure: the reproduction of the structure is routed through the subordinated activity itself. The claim that living labour reproduces capital is a claim of the second kind, and its specific content is lost when alienation is defined so broadly that the serf's condition already satisfies it.

Proposition 6.4 (Generation and closure are independent). *Over subjection, the generation clause of Definition 5.3 and the closure clause of Definition 6.1 vary independently: there are designs realising closure of I by a structure I did not generate, and designs realising alienation proper without closure.*

Proof. For the first, let σ be a condensate, of a coupling to which I was not party, in the sense of Definition 10.1, and let the design confine Con_I to $\text{Pres}(\sigma, X)$; a condensate places later parties in mediation whose revision channel is absent, as recalled in Section 10, and the confinement adds closure without adding generation. For the second, take the construction of Proposition 6.3 with the generating interval of σ assigned to I 's cells: I generated σ , cannot revise it, and maintenance lies with J . $\square$

Proposition 6.4 yields a two-axis field over subjection: whose product, and whose maintenance. All four cells are realised. A structure one generated and maintains oneself, revisable by no continuation of one's own; a structure one generated whose maintenance lies elsewhere; a structure generated elsewhere that one's own activity now maintains; and a structure generated and maintained elsewhere. The third cell is the standing condition of later entrants into any condensed order, and it is the cell in which most working lives are lived.

7 Discriminative Inversion

The conditions of Sections 5 to 6 concern what a subsystem can do. A further condition concerns what it can distinguish, and it is independent of the first three.

Definition 7.1 (Repertoire classification, [L2]). For a configuration E presented to I at X , the repertoire classification is

$$\kappa(E) = \{ \sigma \in \Sigma_I : \sigma \dashv [E]_{I,X} \},$$

the set of symbols of I 's repertoire that substitute for E 's class.

Definition 7.2 (Discriminative inversion, [L2]). *I is discriminatively inverted on a region at X when, for all configurations E_a, E_b of that region,*

$$E_a \approx_{I,X} E_b \iff \kappa(E_a) = \kappa(E_b).$$

The right-to-left direction holds widely and is unremarkable: configurations classified alike by a repertoire that captures the subsystem's discriminations will often be indistinguishable to it. The left-to-right direction is the substance: the repertoire's classification exhausts what the subsystem distinguishes. Under inversion the environment appears to the subsystem only in the form of its symbols; the world is discriminated exactly as finely as the repertoire sorts it, and no finer.

Proposition 7.3 (Independence). *Discriminative inversion neither implies nor is implied by subjection, alienation proper, or reproductive closure.*

Proof. For one direction, construct a subsystem under reproductive closure whose indistinguishability is strictly finer than its repertoire classification: it registers divergences it cannot act on, so closure holds and inversion fails. For the other, construct a subsystem whose discrimination coincides with its repertoire classification while no product mediates it beyond its own revisable repertoire: inversion holds and even subjection fails. $\square$

Proposition 7.4 (Invisibility of the residue). *Let I be discriminatively inverted on the relevant region at X. Then for every $\sigma \in \Sigma_I$ and every interaction class $\mathcal{Y}'$, the residue $\varrho(\sigma, [E], \mathcal{Y}')$ contains no pair of configurations distinguishable by I.*

Proof. By the quotient form of the symbol definition, entries of the residue are classes on which the continuation sets of σ and of the representative differ in the quotient by $\approx_{I,\cdot}$. Under inversion I 's quotient coincides with the quotient by repertoire classification, and any divergence between σ and members of its class lies within a single classification cell. Within a cell the quotient collapses the difference, and the entry is not distinguishable by I . $\square$

Proposition 7.4 states the epistemic half of what the tradition calls fetishism, and states it as a consequence. Where a substitution fails, the failure is a fact about continuation sets; under inversion that fact is statable only from a partition whose discrimination is strictly finer than the gripped subsystem's own. The divergence between a symbol and what it stood for does not hide; it is present in the dynamics and absent from every comparison the subsystem can make. The proposition holds whether or not the subsystem is subject to the symbol, which is why the condition is an axis of its own.

8 The Stratification

Proposition 8.1 (Stratification). *Fix I, σ , X and a horizon h. Level 3 implies Level 2 implies Level 1, each inclusion is strict, and discriminative inversion is independent of all three.*

Proof. Level 3 contains Level 2 by definition and Level 2 contains Level 1 by definition; that Level 3's closure clause is compatible with the generation clause is Proposition 6.4, and Proposition 6.2, with closure read on the stretch covering the horizon, discharges the unrevisability Level 3 requires. Strictness of the first inclusion is the second construction of Proposition 6.4; strictness of the second is subjection to a structure generated by another party, available from Proposition 6.3 with the generating interval assigned to J . Independence of inversion is Proposition 7.3. $\square$

Table 2: The stratification. Levels 1 to 3 are strictly ordered; discriminative inversion is an independent axis that may attach at any level. The right column names the received notion each level makes precise.

Level	Condition	Added clause	Received notion
1	Subjection (Definition 5.2)	mediation without revision	domination by structure
2	Alienation proper (Definition 5.3)	the structure is one's product	estrangement from the product
3	Level 2 with closure (Definition 6.1)	one's activity maintains it	estrangement of the activity
–	Discriminative inversion (Definition 7.2)	the residue cannot register	fetishised appearance

The stratification is the paper's answer to the overload identified in Section 1. The received uses of the term are levels, the levels are strictly ordered, and a claim made with the term can be asked which level it requires. The criterion that production outcomes come to dominate the production process, taken seriously, is a Level 3 claim: domination in the strong sense obtains when the process's admissible continuations are confined to those reproducing the outcome, and Proposition 6.3 shows the strong sense is not implied by the weak one. Analyses that establish Level 1 and conclude at Level 3 owe the closure clause, and the debt can now be stated exactly.

Remark 8.2 (Undecidability at unbounded horizon). The revision set at unbounded horizon inherits the undecidability of eventual properties. Where a design's admissibility dynamics can encode a universal machine, membership of I in $\text{Rev}(\sigma) = \bigcup_h \text{Rev}_h(\sigma)$ is not decidable in general, by reduction from a non-trivial property of eventual behaviour. The condition on which the claim turns is the encoding capacity of the design, and the claim fails for designs of restricted expressive power, which may admit decision procedures. The consequence runs in both directions: no general theorem guarantees emancipation, and no general theorem guarantees permanence. Every claim of this paper is accordingly made at a stated horizon.

9 Positional Asymmetry and Autonomization

The revision set locates parties around a structure, and two configurations of the set deserve names.

Definition 9.1 (Positional asymmetry, [L2]). Subsystems I, J stand in *positional asymmetry* over σ at horizon h when σ was generated by I , $I \notin \text{Rev}_h(\sigma)$, $J \in \text{Rev}_h(\sigma)$, and $J \neq I$.

Definition 9.2 (Autonomization, [L2]). σ is *autonomized* at horizon h when $\text{Rev}_h(\sigma) = \emptyset$ over the subsystems of the design.

Positional asymmetry is the structure of the exploitation claim: the product of one party's activity is revisable only by another. The definition captures the structure and deliberately not the magnitude; the claim that one party appropriates a surplus from another requires a quantity of appropriation, quantities are available only at [L3] and only in the regime of Section 11, and the structural claim is independent of the quantitative one. Autonomization is the limiting configuration in which the structure has no reviser at all: it conditions every party within reach and answers to none, within the horizon. The two configurations exclude one another at a fixed horizon and can succeed one another along a trajectory, and the succession from asymmetry to autonomization, in which the party once able to revise loses the channel in turn, is a recurrent historical pattern that the vocabulary now permits stating.

Remark 9.3 (The dominating position is also gripped). Nothing in Definition 9.1 entails that J 's continuations are unconditioned. $J \in \text{Med}(\sigma)$ is compatible with the definition and typical of the cases of interest: the party holding the only revision channel is mediated by the same structure, and where σ reproductively closes J as well, the asymmetry is between two conscripted positions, one of which holds a channel it is not thereby free to use at will. The observation that the dominating position is itself bound to the reproduction of what it dominates through is available here as a statement about Med, Rev and closure, with no additional apparatus.

Remark 9.4 (Toward a positional analysis). Med and Rev_h over the subsystems of a design form a directed structure, and the equivalence classes of positions in that structure are candidates for a derived, non-primitive notion of class. The global analysis of the structure belongs to a further paper and only the local configurations above are used here.

10 Genesis of Unrevisable Structure

The conditions of Sections 5 to 8 are synchronic: each is stated at a cut. The diachronic question is where structures satisfying them come from, and the elementary mechanism is recalled here.

Definition 10.1 (Condensate, [12]). Let a coupling between subsystems be active over an interval and absent after it. A product σ is a *condensate* of that coupling when σ persists beyond the interval and substitutes, over some non-empty interaction class, for the configurations in which the coupling was active: the structure occupies the place the live coupling occupied.

The consequences established in Companion Study I are three. Any party whose continuations depend on the condensate is mediated by it, including parties who were never party to the coupling. Membership in the revision set requires a continuation through the party's own sector altering the structure, and the constitution of the condensate by others' activity supplies no such continuation. A revision channel for later parties is therefore a separately instituted product, itself mediating and itself revisable or not.

The consequence for this paper's subject is that Level 1 conditions are produced as a structural default, with no adverse intent, no imposition and no decision by any party. Proposition 6.4 then shows how the remaining levels are reached from there: a design that confines later entrants' admissible continuations to the persistence set of the condensate adds closure, and generation by the entrants themselves is added wherever the condensate is extended, maintained or rebuilt by their own activity, which is the ordinary condition of work within any standing institution. The genesis of the strongest conditions requires nothing beyond the ordinary operation of the weakest.

Three routes by which a product comes to occupy a mediating position are visible in the construction: generation by a subsystem whose continuations come to depend on it, condensation of a ceased coupling, and the formation of a record by a third party. Whether the routes are one mechanism under three descriptions remains open.

Assembled, the sections give the Marxian mechanism as a chain, each link a defined and separately checkable condition. Activity generates products; generated products pass beyond the boundary while remaining coupled across it; couplings condense into structures that outlive them; the condensates mediate later entrants, whose revision channels are absent by default; designs confining the entrants' admissible continuations to the persistence set add closure, so that the entrants' own activity maintains the structures subordinating it; and where discrimination narrows to the repertoire's classification, the divergence between the structures and what they stood for ceases to register within the parties gripped. The chain runs from living activity to fetishised closure through defined conditions, and it requires the decision

of no party at any link, which is the point the moralised tellings of the story miss and the formalisation makes exact.

11 Abstract Labour and Value

This section is the paper's only [L3] material, and its claims are confined to a regime.

Definition 11.1 (Universal equivalent, [L2]). A design contains a *universal equivalent* for a family of subsystems $\{I_k\}$ when there is a σ^* such that, for every member I_k and every class c of that member's distinguished family of exchange-relevant classes, $\sigma^* \dashv [E_c]_{I_k}$. The family index is required because substitution is party-relative, and a single structure substituting for every party's exchange classes is a fact about the family, not about any member alone.

Definition 11.2 (Abstract labour, [L2]). In a design with a universal equivalent, *abstract labour* is the quotient of activities by the discrimination relation that the exchange practice enacts: two activities are equivalent when the practice's continuations do not distinguish them.

The definition formalises a position that is not this paper's: that the abstraction from concrete labours is performed in practice before it is performed in thought belongs to Sohn-Rethel [10], and the quotient construction is a restatement of it. The point the restatement secures is that abstract labour is a relation enacted by a practice, and neither a mental generalisation nor a physiological substrate.

Definition 11.3 (Value, [L3]). In a regime containing a universal equivalent for a family, *value* is the effective magnitude coordinatising substitution ratios among the family's exchange-relevant classes under σ^* . The magnitude is a property of the regime and is indexed to the family; it is defined in no design lacking a universal equivalent, and it attaches to no configuration outside the regime's exchange sphere.

Three consequences of the placement are the section's content. First, value is regime-relative and family-relative: the question of the value of a product in a design lacking a universal equivalent is ill-formed, and its difficulty is of that kind, and the historical specificity of the category is thereby a formal fact. Second, the magnitude describes and does not justify: that a regime coordinatises substitution in a given way licenses no conclusion about how anything ought to exchange, by the fourth standing constraint. Third, the categories requiring an ensemble are out of reach: socially necessary labour time is an average over producers, averages require a weight over histories, and the present apparatus is an admissibility relation carrying no weight. The category is named, tagged and deferred, and any development of it in this framework must first state the enrichment it uses. These placements agree with the reading of value as a social form developed by Rubin and, more recently, Heinrich [9, 3], and the debt to that reading is acknowledged. The treatment of temporality under the value form given by Postone [8] bears on the same material and is not developed here.

12 Anticipatory Mediation

The interaction classes of the substitution relation are not confined to configurations at the present cut.

Definition 12.1 (Anticipatory symbol, [L2]). A symbol σ is *anticipatory* when the class it substitutes for consists of configurations at later cuts: $\sigma \dashv_y [E_{\text{fut}}]$, where members of $[E_{\text{fut}}]$ are configurations no continuation has yet realised.

Members of the class are configurations in the sense of Section 2, label assignments composable at the present cut; their futurity is a matter of realisation status, and the comparison the substitution relation makes composes them at the cut at which the symbol acts. A promise, a contract, a price of a future good and an instrument of credit are anticipatory symbols: each acts at the present cut as configurations of a future class would act, and each mediates present continuations accordingly. Two disciplines govern the definition. The mediation is exercised by the present symbol, and by nothing future: no final boundary condition is imposed on the dynamics, the admissibility relation runs forward from the cut, and the future class enters only as the index of a substitution exercised now. And the class may be empty on the realised trajectory, the definition being satisfied whether or not a continuation realises a member of $[E_{\text{fut}}]$.

Remark 12.2 (Mediation by a class that never arrives). An anticipatory symbol whose class acquires no member on the realised trajectory, while its mediation persists, is the structure of fictitious capital: present continuations conditioned by a substitution whose substituted class never comes to contain a configuration. The residue of such a symbol is total, and under discriminative inversion, by Proposition 7.4, the totality of the residue is exactly what cannot register within the mediated party. The role of imagined futures in coordinating economic action is established in economic sociology [1], and the definition above is a formal location for it, not a rediscovery.

13 The Four Estrangements of the Manuscripts

The 1844 manuscripts [7] distinguish estrangement from the product, from the activity, from species-being, and from others. The stratification distributes the four.

From the product is Level 2: the generated structure stands over its generator, mediating without a return channel, and Definition 5.3 is its definition.

From the activity is Level 3. The manuscripts' claim is not that the product alone is estranged but that the activity itself is: labour as forced, as belonging to another, as mortification. Reproductive closure is the formal content available for this: the activity's admissible continuations are confined to those maintaining the subordinating structure, so that the act, and not only its result, belongs to the loop. The clause answers a standing objection to structural accounts, that they capture estranged products and miss estranged acting; the closure clause is a condition on the acting.

From species-being concerns the capacity that the manuscripts take to define the species: free, conscious productive activity, the production of a world. In the present vocabulary the nearest object is the space of admissible continuations itself, and the relevant condition is the reshaping of that space: a subsystem's deepest subordination is not the removal of options within a fixed space but the conditioning of what the space contains. The formalisation of the capacity to extend the space belongs to the companion study of subject formation, the third of the set, and this paper records the placement and does not construct it.

From others is the positional structure of Section 9: estrangement among parties is the asymmetric distribution of mediation and revision over them, and its global form is the directed structure of Remark 9.4.

14 Summary of Results

Four conditions have been defined on the inherited substrate: subjection, alienation proper, reproductive closure, and discriminative inversion, together with the persistence set, repertoire classification, positional asymmetry, autonomization, the universal equivalent, abstract labour, value, and anticipatory symbols. Five propositions have been proved: closure entails

unrevisability at every horizon; subjection obtains without closure; generation and closure are independent axes over subjection; inversion is independent of the dynamical levels; and under inversion the residue of every symbol of the repertoire is invisible to the gripped party. The stratification proposition assembles these into a strict ordering of three dynamical levels with an independent epistemic axis.

Against the criteria of Section 3.4: no metric and no baseline appear in any definition; generation and mediation remain innocent, the conditions being additional clauses on an innocent loop; the differentia is carried entirely by revision and its position; degrees appear only as the stratification and the partial orders of Section 2; and the genesis section derives the weakest level from the ordinary operation of condensation, with the stronger levels reached by clauses that likewise require no party's adverse intent.

15 Limitations and Open Problems

15.1 Conditions of failure

The account fails where its substrate fails, and the conditions stated in Companion Study I [4] are not repeated here. Three further conditions attach to this paper. If the closure clause is unsatisfiable in the designs of interest, because environments are open enough that some admissible continuation always breaches the persistence set, then Level 3 is empty there and the specifically Marxian claim finds no instances; the account would then locate all actual cases at Levels 1 and 2, which is a substantive and falsifiable placement. If discrimination in the designs of interest never coincides with repertoire classification, inversion is idle and the fetishism analysis reduces to the unremarkable half of Definition 7.2. And if the notion of a distinguished exchange-relevant family of classes cannot be fixed without circularity, the regime condition of Section 11 is ill-posed and the value section fails independently of the rest.

15.2 Open problems

1. **Decidable fragments.** Which restrictions on a design's expressive power make membership in $\text{Rev}(\sigma)$ at unbounded horizon decidable (Remark 8.2).
2. **Collective revision.** Rev_h is defined per subsystem. Joint revision by a coalition, in which no member alone possesses a revising continuation and the coalition does, requires machinery for coupled continuations that neither paper supplies. The politics of the subject lives in this gap, and the gap is named.
3. **Degrees of inversion.** Whether Definition 7.2 admits a partial refinement order, between full inversion and its absence, without introducing a scalar.
4. **The species-being clause.** Deferred to the companion paper on subject formation.
5. **Ensemble categories.** Socially necessary labour time and every average await a stated enrichment.
6. **Routes to a mediating position.** Whether the three routes of Section 10 are one mechanism, inherited open.

15.3 Subsequent work

The positional structure of Remark 9.4 carries a derived account of class. The reflexive substitution class, in which a symbol stands for configurations of its own subsystem, carries the account of subject formation to which the species-being clause is deferred. And the limits on

what an observer internal to a complex can establish about it, of which Proposition 7.4 is a special case, carry a companion analysis of partition and discrimination.

Recapitulation

The paper's conditions and results, collected for reference; the setting is that of Section 2.

- **Rejected index.** $\alpha = 1 - |\langle X_{\text{actual}} | X_{\text{comparand}} \rangle|^2 / (\|X_{\text{actual}}\|^2 \|X_{\text{comparand}}\|^2)$, in every instantiation: it needs a metric and a counterfactual comparand, scores difference or conformity as pathology, and equates unalienated objectification with accurate representation, the shape of the position Marx rejects in Hegel. The criteria of adequacy (Section 3.4) are extracted from these failures, and the structural account is the design they leave available.
- **Persistence set.** $\text{Pres}(\sigma, X) = \{X'_I \in \text{Con}_I(X) : \sigma \sqsubseteq (E', X'_I) \text{ for every admissible joint realisation}\}$.
- **The stratification.** Level 1, subjection: $I \in \text{Med}(\sigma)$, $I \notin \text{Rev}_h(\sigma)$. Level 2, alienation proper: subjection, with σ generated by I . Level 3: Level 2 with reproductive closure, $\text{Con}_I(X) \subseteq \text{Pres}(\sigma, X)$ at every cut of the stretch, the activity conscripted into maintaining the structure. Independent axis, discriminative inversion: $E_a \approx_{I,X} E_b \Leftrightarrow \kappa(E_a) = \kappa(E_b)$, under which every residue is invisible to the gripped party. Inclusions strict; closure entails unrevisability at every horizon the stretch covers.
- **Positions.** Positional asymmetry: σ generated by I , $I \notin \text{Rev}_h$, $\text{Rev}_h \ni J \neq I$, the structure of exploitation with its magnitude deferred. Autonomization: $\text{Rev}_h(\sigma) = \emptyset$.
- **Genesis.** Condensation of ceased couplings produces Level 1 by default; confinement to the persistence set adds closure; maintenance and rebuilding by entrants adds generation; the chain reaches fetishised closure with the decision of no party.
- **Value.** Universal equivalent for a family: $\sigma^* \dashv [E_c]_{I_k}$ for every member and every exchange class. Abstract labour: the quotient of activities by the practice-enacted discrimination. Value: the [L3] magnitude coordinatising substitution ratios in that regime, family-relative, descriptive. Socially necessary labour time: deferred to a stated enrichment.
- **Anticipation.** $\sigma \dashv [E_{\text{fut}}]$: mediation now by a class whose members are unrealised; fictitious capital, the case whose class stays empty while mediation persists.
- **The four estrangements.** From the product: Level 2. From the activity: Level 3. From species-being: the reshaping of the space of admissible continuations, constructed in the companion study of subject formation. From others: positional asymmetry and the mediation structure over parties.

References

- [1] Jens Beckert. *Imagined Futures: Fictional Expectations and Capitalist Dynamics*. Harvard University Press, 2016.
- [2] G. W. F. Hegel. *Phenomenology of Spirit*. Oxford University Press, 1977. First published 1807.
- [3] Michael Heinrich. *An Introduction to the Three Volumes of Karl Marx's Capital*. Monthly Review Press, 2012.

- [4] Wanhong Huang. Symbol emergence and symbolic mediation in a background-independent relational ontology: A preliminary discussion. Working draft, 2026.
- [5] Rahel Jaeggi. *Alienation*. Columbia University Press, New York, 2014. Translated by Frederick Neuhouser and Alan E. Smith; edited by Frederick Neuhouser; originally published as *Entfremdung*, 2005.
- [6] Georg Lukács. *History and Class Consciousness*. MIT Press, 1971. First published 1923.
- [7] Karl Marx. *Economic and Philosophic Manuscripts of 1844*. Progress Publishers, 1959. Written 1844.
- [8] Moishe Postone. *Time, Labor, and Social Domination*. Cambridge University Press, 1993.
- [9] Isaak Illich Rubin. *Essays on Marx's Theory of Value*. Black Rose Books, 1972.
- [10] Alfred Sohn-Rethel. *Intellectual and Manual Labour: A Critique of Epistemology*. Macmillan, 1978.