Constraint Dynamics on an Evolving Manifold: A Preliminary Field-Theoretic Discussion of the Emergence of Knowledge in a Background-Independent Relational Ontology

Wanhong Huang

huangwanhong@serendip.ngo

Abstract

This paper proposes a philosophical model of knowledge and states it in field-theoretic terms. The proposal is that knowledge is constraint dynamics on an evolving manifold: at any time, a subject's admissible continuations are shaped by a structure of constraints, figurable as a surface whose stable regions are the subject's settled capacities; and the surface itself evolves, its evolution driven by the subject's couplings to what it is not, the environment, other subjects, and the condensed symbolic structures its history has deposited. Applying knowledge is the fast dynamics upon the surface; learning is the slow dynamics of the surface; and the two form one system, since applications leave records and records feed the couplings that reshape the surface. The setting is a combinatorial substrate of the kind that background-independent approaches to quantum gravity, group field theory among them, have developed as models of the emergence of a discrete spacetime, in which no arena is given in advance and structure is carried by the relations themselves; the substrate is used as a formal template, and no physical claim is made of the objects described. The paper works in two registers under an explicit discipline: a combinatorial base defines knowledge as the constraint structure on a subsystem's admissible continuations, proves that acquisition requires prior structure and that constraint structures pass between subjects only through the recipient's own continuations; a declared enrichment then introduces the manifold, the surface and the two-time dynamics, with each effective object anchored to a combinatorial counterpart, its anchored and its merely posited features separated, and its conditions of failure named. Inheritance, the coarsening and refinement of discrimination, and the reconstruction of the past are treated within the model. The paper is conceptual, preliminary and offered for revision: its scope, the considerations behind its modelling choices, its limitations, and its open questions are stated in dedicated sections, and its debts to prior work, from the epigenetic landscape to structural coupling, are conceded where they arise.

Keywords: knowledge emergence; constraint dynamics; evolving manifold; effective potential; background independence

Draft. This is a working draft. It is circulated for comment, and its arguments, formulations, and open questions are subject to revision. Correction and criticism are welcome at `huangwanhong@serendip.ngo`.

Use of generative artificial intelligence. Generative AI assistants (Anthropic's Claude, OpenAI's ChatGPT, and Google's Gemini) were used extensively in preparing this manuscript: for literature survey, for drafting prose from the author's specifications, for adversarial review of the argument, and for consistency and citation auditing and typesetting. No simulation or empirical figure is used in this paper, and its arguments are discursive throughout. Formulations and objections arose in the course of that exchange as well as from the author. Every claim, argument, and citation was reviewed and decided by the author, who is responsible for the content and for any errors. Bibliographic details are to be verified against the primary sources before reliance.

Suggested citation: Wanhong Huang, *Constraint Dynamics on an Evolving Manifold: A Preliminary Field-Theoretic Discussion of the Emergence of Knowledge in a Background-Independent Relational Ontology*, working draft, 2026.

1 Introduction

The keynote of this paper can be stated in three sentences, and stating it first is the honest order, since everything after is elaboration, qualification and defence. Knowledge is not a stock of representations held by a subject; it is constraint dynamics on an evolving manifold. At any time, what a subject can do, distinguish and continue into is shaped by a structure of constraints, and that structure is usefully figured as a surface: its stable regions are the subject's settled capacities, and its shape is what the subject knows. And the surface is not fixed: it evolves, and its evolution is driven by the subject's couplings to what the subject is not, the environment it acts in, the other subjects it interacts with, and the condensed symbolic structures, languages, techniques and institutions, that history has deposited around it.

Two consequences of the keynote organise the paper. First, knowing and learning become two timescales of one dynamics: applying knowledge is motion upon the surface, under the constraints as they stand; learning is deformation of the surface, a change in the constraints themselves; and the separation of the two timescales is a substantive condition that can fail. Second, knowledge becomes constitutively relational: since the surface evolves only through couplings, a subject's knowledge is at every point the sediment of its relations, and the question what a subject knows is inseparable from the question what it has been coupled to.

The paper's register requires an equal candour. The keynote is a philosophical proposal, and its field-theoretic dress, a manifold, a potential, couplings, a slow flow, is a model, adopted because it makes the proposal precise enough to be examined and revised. The paper therefore works under a two-register discipline. A combinatorial base, inherited from four companion studies, hereafter Companion Studies I to IV, and restated in full in Section 3, defines knowledge without geometric commitments, as the constraint structure on a subsystem's admissible continuations, and proves what can be proved there. A declared enrichment, stated object by object in Section 4, then introduces the manifold and the surface, and for each effective object separates what its combinatorial anchor supports from what the model merely posits. Claims resting only on anchored features are marked as such; claims resting on posited features are proposals. Section 4 also states what the model is a model of and what it leaves out, and Section 12 collects the limitations and the open questions, which are substantial.

The intellectual debts are old and are conceded where they arise. That development runs on a shaped landscape, itself underpinned by a network, is Waddington's picture [16]. That a system's structure changes through recurrent coupling while its organisation is conserved is the structural coupling of Maturana and Varela [9, 10], and the keynote's second half is that thesis in field-theoretic dress. That intelligent capacity is prior to representation is Ryle's and Polanyi's ground [13, 12]. The two-time reading of application and learning restates Piaget's pair of assimilation and accommodation [11], and is standard in connectionist modelling as the split between fast activations and slow weights [4]. Three further lineages bear on the model and are conceded here. The systematic treatment of slow and fast variables, in which the slow enslave the fast, is developed in synergetics [3]; the study of dynamics on deforming potential surfaces belongs to the morphogenetic tradition [14]; and a large contemporary literature models learning as the minimisation of a variational quantity [2], whose apparatus is probabilistic where the present one is an admissibility relation, a difference of instrument and no claim of superiority. The paper's claim to novelty is confined: it places these positions on one substrate, in continuity with the four companion studies, of symbols, alienation, subject formation and observation, so that knowledge, its inheritance, its application and its revision of the past are configurations of the same few relations that carry those accounts.

2 Preliminaries

The paper borrows its setting from physics, its central image from dynamical systems, and its bookkeeping from elementary mathematics. All three are set out here in plain terms, and Section 3 resumes the formal development.

The physical picture. Several current programmes in quantum gravity build spacetime, taking none as given. Their elementary object is a network: dots, called vertices; links joining the dots, called edges; and surfaces spanning closed circuits of links, called faces. A history of such a network, in which edges and faces persist, appear and recombine, is called a *spin foam*, the name recalling the walls of soap bubbles. *Group field theory* is a machinery for generating such histories: a single mathematical object, a field defined over a space of symmetry operations, whose systematic expansion produces foams in the way that the expansion of a particle theory produces interaction diagrams. Geometric quantities are labels carried on the network's parts. *Background independence* names the resulting situation: there is no stage on which events are placed, the network of relations is all there is, and space, time and extent are features of it. This paper takes from the physics only the skeleton, a *complex* of labelled cells, as bookkeeping for relations, and borrows the field-theoretic imagery a second time, at the effective level, under the discipline of Section 4.

The dynamical image. A *potential surface* is a landscape over a space of possible states: valleys are stable arrangements, in which a system settles; ridges separate valleys; and a ball rolling on the landscape pictures a system evolving under its constraints. Two kinds of change are then distinct: the ball moving on a fixed landscape, and the landscape itself deforming, valleys deepening, merging or appearing, so that where the ball can settle changes. When the landscape deforms much more slowly than the ball moves, the two dynamics separate cleanly, and this *slow and fast* separation is the image behind the paper's treatment of applying and learning. The image commits, taken literally, to more than the paper can support, and Section 4 states exactly how much of it is meant.

The mathematical vocabulary. A *relation* pairs objects; $x \in S$, $S \subseteq T$ and $S \setminus T$ read: x belongs to S; every member of S belongs to T; the members of S outside T. An *equivalence relation* sorts objects into classes, the *quotient* being the collection of the classes. A *partial order* ranks some pairs and leaves others unranked, and much of the series' discipline consists in claiming partial orders where a stronger vocabulary would claim numbers. A *horizon* h is a stated number of steps bounding a claim. A *manifold* is a space that near any point looks like ordinary coordinate space; the paper uses the word for its effective state space and states in Section 4 which of the word's commitments it takes on.

3 The Relational Setting

This section states the combinatorial apparatus in full, so that the paper may be read on its own. The derivations are given in four companion studies, cited by number throughout: Companion Study I on symbol emergence [8], II on Marxian alienation [5], III on subject formation [7], and IV on the internal observer [6]; the present paper is the fifth of the set. Results imported from them are stated in full where they are used, as numbered Established Results, their proofs alone remaining with their sources. A reader may take the definitions and the imported results as stipulations and judge the paper against them.

3.1 Substrate, subsystems, and time

Definition 3.1 (Complex, subsystem, worldtube)**.** A *complex* $\mathcal{C}$ is a set of cells, vertices, edges and faces, with the record of which is incident to which, together with labels on the cells; a *configuration* of a region assigns labels to its cells. A *subsystem* is a subcomplex $\mathcal{C}_I \subset \mathcal{C}$ with

Table 1: Notation. Each symbol is introduced where indicated; the table is a lookup and replaces no definition.

Symbol	Reading	Introduced
$\mathcal{C}, \mathcal{C}_I, \partial\mathcal{C}_I$	complex; subsystem; its boundary	Definition 3.1
$\mathcal{W}_I$	worldtube: a subsystem whose boundary persists	Definition 3.1
$u \preceq v$	u precedes v in the record-generated order	Definition 3.2
χ, $X_I(\chi)$	clock; the configuration on its level set, a cut	Definition 3.2
$X = (E, X_I)$	joint configuration: environment on the boundary, and subsystem	Definition 3.3
$X \rightsquigarrow X'$	X' is an admissible continuation of X	Definition 3.3
$\mathrm{Con}_I(X)$, $\mathrm{Con}_I^h(X)$	continuation set; admissible h-step continuation sequences	Definition 3.3
$U \otimes V$, $\cdot$	composition of configurations on disjoint regions; a component held fixed	Definition 3.3
$\mathcal{P}_I$	context class: the admissible boundary variations compared	Definition 3.4
$Z \approx_{I,X} Z'$	indistinguishable for I at X; $\sim_{I,X}$ on boundary configurations	Definition 3.4
$[E]_{I,X}$	a discrimination class	Definition 3.4
$T \sqsubseteq X$	T is a subconfiguration of X	Definition 3.5
π	product: a trace whose presence alters Con_I	Definition 3.5
$\sigma \dashv_{\mathcal{Y}} [E]_{I,X}$	σ substitutes for the class over the interaction class $\mathcal{Y}$	Definition 3.6
ϱ	residue: the interactions at which substitution fails	Definition 3.6
Σ_I	repertoire: I's set of symbols	Definition 3.6
$\mathrm{Med}(\pi)$	the subsystems π mediates	Definition 3.7
$\mathrm{Rev}_h(\sigma)$	the subsystems able to revise σ within h steps	Definition 3.8
[L2], [L3]	level tags: combinatorial; effective	§4
X^S	sufficient-state sector	Definition 3.10
$K_I(\chi)$	constraint structure: knowledge	Definition 5.1
$\mathcal{M}_I$, V_I	effective manifold; constraint surface	Section 4
w, χ_{slow}	slow variables; the slow timescale	Section 4

boundary $\partial\mathcal{C}_I$; $\mathcal{C} \setminus \mathcal{C}_I$ is its *environment*. A *worldtube* $\mathcal{W}_I$ is a subsystem whose boundary persists.

The partition is a description choice and it is symmetric: for any two subsystems each lies in the environment of the other, and the selection of one is an act of the describer.

Definition 3.2 (Record relation, order, clock, cut)**.** For cells u, v, write $u \ R \ v$ when u contributed to a persistent configuration on which the admissible continuations at v depend; $\preceq$ is its reflexive transitive closure, assumed acyclic. A *clock* χ is a function into a totally ordered set, non-decreasing along $\preceq$; its level sets are *cuts*, and $X_I(\chi)$ is the configuration on the cut.

Time is generated by the production of records: what has left a record constraining a cell precedes it, and a state is a cut.

Definition 3.3 (Admissibility and continuation sets)**.** The dynamics is a relation $\rightsquigarrow$ between joint configurations at successive cuts,

$$(E, X_I)(\chi_n) \ \rightsquigarrow \ (E, X_I)(\chi_{n+1}),$$

with E the environment's configuration on $\partial\mathcal{C}_I$, present at every cut. For $X = (E, X_I)$,

$$\mathrm{Con}_I(X) \;=\; \{\, X'_I \;:\; X \rightsquigarrow (E', X'_I) \text{ for some } E' \,\},$$

and $\mathrm{Con}^h_I(X)$ is the set of admissible h-step continuation sequences from X with their realising joint sequences. Configurations on disjoint regions compose as $U \otimes V$; a component written $\cdot$ is held fixed by context.

The relation is primitive and carries no weight over continuations. Where a model supplies an amplitude, the relation is its support, and nothing below uses more than that.

3.2 Discrimination

Definition 3.4 (Context class, indistinguishability, discrimination)**.** A *context class* $\mathcal{P}_I$ is a set of admissible boundary variations, produced historically and differing between parties. For configurations Z, Z' presented to I at X,

$$Z \approx_{I,X} Z' \quad \text{iff} \quad \mathrm{Con}_I(X, Z, p) \;=\; \mathrm{Con}_I(X, Z', p) \;\text{ for every } p \in \mathcal{P}_I,$$

an equivalence relation; its restriction to boundary configurations is $\sim_{I,X}$, with classes $[E]_{I,X}$.

Discrimination is indexed to X: what a subsystem carries alters what it distinguishes. A boundary admitting no variation yields exactly one class.

3.3 Traces, products, symbols

Definition 3.5 (Trace, generation, product)**.** A configuration is a *trace* over an interval when it is a subconfiguration of $X_I(\chi)$ throughout it, and is *generated by I* when it is absent at an earlier cut, present at a later one, and the realising continuation runs through cells of $\mathcal{C}_I$. A trace π is a *product* when it participates: for some Y,

$$\mathrm{Con}_I(\,\cdot \otimes \pi \otimes Y\,) \;\neq\; \mathrm{Con}_I(\,\cdot \otimes Y\,).$$

Definition 3.6 (Symbol and residue)**.** For a class $\mathcal{Y}$ of interaction configurations, a product σ is a *symbol* for I over $\mathcal{Y}$, written $\sigma \dashv_{\mathcal{Y}} [E_A]_{I,X}$, when for every $Y \in \mathcal{Y}$

$$\mathrm{Con}_I(X \otimes \sigma \otimes Y)\,/\!\approx_{I,\cdot} \;=\; \mathrm{Con}_I(X \otimes E_A \otimes Y)\,/\!\approx_{I,\cdot}\,,$$

E_A representing the class. The *residue* over a wider class is the set of interactions at which the equality fails. The *repertoire* Σ_I is I's set of symbols.

The symbol occupies, across the stated class, the place a member of the class would occupy; no interpreter, resemblance or reference enters, and the condition is checked by comparing continuation sets. Substitution is exact only up to what the subsystem itself distinguishes. Established in [8] and used below: $\dashv$ is neither symmetric nor transitive, is relative to the subsystem, composes up to residue, and every symbol is a product while products participating without substituting are not symbols. Symbols vary independently along three axes: privacy, whether the configuration is carried within one worldtube or across several; timescale, whether it changes on the scale of internal steps or of boundary exchanges; and the class substituted for, whether of environmental configurations, of configurations at later cuts, or of the subsystem's own.

3.4 Mediation and revision

Definition 3.7 (Mediation and the mediation set)**.** A product π *mediates* I at X when Con_I depends on it: there are X, X' agreeing except on π with $\mathrm{Con}_I(X) \neq \mathrm{Con}_I(X')$, the mechanism being selection among admissible continuations. $\mathrm{Med}(\pi)$ is the set of subsystems mediated.

Mediation of itself carries no verdict: every tool, grammar and institution mediates those whose continuations depend on it.

Definition 3.8 (Revision set)**.** $I \in \mathrm{Rev}_h(\sigma)$ at X when there are $c, c' \in \mathrm{Con}_I^h(X)$ whose contributions from outside I are identical, which differ within I's sector, and whose configurations of σ at step h differ.

The first clause isolates difference-making through I on continuations the dynamics admits, with no intervention and no counterfactual. Both sets support membership and inclusion, and no magnitude follows from them.

3.5 Condensates, sectors, and separation

Definition 3.9 (Condensate)**.** Let a coupling between subsystems be active over an interval and absent after it. A product σ is a *condensate* of that coupling when σ persists beyond the interval and substitutes, over some non-empty interaction class, for the configurations in which the coupling was active. A condensate mediates any party whose continuations depend on it, parties never party to the coupling included, and supplies such parties with no revising continuation by its constitution alone.

Definition 3.10 (Sufficient-state sector, separation)**.** A decomposition $X_I = (X^R, X^S)$ is admissible when Con_I at a cut is a function of (E, X^R, X^S) there while failing to be a function of (E, X^R) alone; X^S is the *sufficient-state sector*, the part of the configuration that carries the subsystem's persistent structure. A worldtube has *s-separation* on a stretch when at least s internal vertices lie between consecutive boundary-incident vertices; internal continuations, confined to internal cells, then run between exchanges.

The condensate and the decomposition are established in Companion Study I [8], separation and its consequences in Companion Study III [7]; they are used below in Section 8, wherever a subsystem's persistent structure is spoken of, and as the combinatorial anchor of the slow and fast decomposition of Section 7. One result of Companion Study I is imported here in full, since three later sections stand on it.

Established Result 3.11 (Configuration indexing; Companion Study I)**.** *Discrimination is indexed to the subsystem's configuration: for fixed boundary data and context class, the relation $\approx_{I,X}$ varies with X_I, so that acquiring or losing persistent structure alters which configurations the subsystem distinguishes. In particular the acquisition of a symbol can strictly refine, and can strictly coarsen, the discrimination on a region.*

4 Levels, the Declared Enrichment, and the Scope of the Model

4.1 Levels and the rule of the paper

Statements in this paper carry one of two tags. [L2] marks the combinatorial level: the complex, its configurations, and the admissibility relation, with everything defined from them. [L3] marks the effective level: descriptions of emergent, coarse-grained structure in the vocabulary of manifolds, potentials and couplings. The rule of the paper is that every [L3] object is introduced with three declarations: its *anchor*, the [L2] structure it coarse-grains; its *anchored* *features*, the properties the anchor supports; and its *posited features*, the properties the model adds for tractability and vividness. Claims resting only on anchored features hold as combinatorics in effective dress. Claims resting on posited features are proposals of the model, and the paper marks them.

4.2 The declared enrichment

Three effective objects are used, and their declarations follow.

The manifold $\mathcal{M}_I$. The effective state space of a subsystem's persistent structure, with coordinates the slow variables w: the sufficient-state sector and the repertoire at coarse grain. *Anchor*: the set of constraint structures (Definition 5.1) reachable by the subsystem, with the partial order of admissible reachability. *Anchored*: the connectivity of that set and the reachability order. *Posited*: smoothness, dimension, and any coordinate system. *Fails when*: the reachable set is too discrete or too disconnected for a continuum picture to organise anything.

The surface V_I. An effective potential over $\mathcal{M}_I$ whose local minima are the metastable constraint structures, the settled capacities; the hypothesis carried from the source of this project is that acquired symbols are occupied minima of the shared sector. *Anchor*: the stability of repertoires and traces under admissible perturbation, the invariance formulation of Proposition 5.2. *Anchored*: which structures are stable, and the adjacency of basins as an accessibility order. *Posited*: depth, curvature, any metric, and gradient flow. *Fails when*: stability is maintained by ongoing external input, so that basin talk misattributes to shape what is due to driving.

The two-time dynamics. Fast flow on the surface at fixed V_I, and slow deformation of V_I through coupling terms, developed in Section 7. *Anchor*: the separation of internal steps from boundary exchanges (Definition 3.10), and the [L2] distinction between continuation under a constraint structure and change of the constraint structure. *Anchored*: that distinction. *Posited*: adiabaticity, and the functional form of the couplings. *Fails when*: the timescales mix, which is a substantive and interesting failure, recorded in Section 12 as the crisis of learning.

4.3 What is modelled and what is not

The model is a model of the *structure* of knowing: of the relation between capacity and constraint, application and learning, inheritance and reconstruction. It models a subject as a subsystem of a relational complex, knowledge as the constraint structure on the subject's admissible continuations, and learning as the evolution of that structure through coupling.

The model does not treat, and the paper claims nothing about: the content of any particular knowledge; truth, justification or warrant, which belong to epistemology proper and are compatible with several positions there; consciousness or experience, on which the substrate is silent; the neural or physiological realisation of any capacity; and any quantitative prediction, since the admissibility relation carries no weight and every magnitude in the paper is a posited feature of the enrichment. Where the paper's vocabulary touches established empirical fields, developmental psychology, the learning sciences, connectionist modelling, the relation claimed is one of formal location, and priority is theirs.

5 Knowledge as Constraint Structure

The combinatorial base of the model is one definition and two propositions, and everything in this section is [L2].

Definition 5.1 (Constraint structure, [L2])**.** The *constraint structure* of I at the cut χ is the map

$$K_I(\chi) \;:\; (E,p) \;\longmapsto\; \mathrm{Con}_I(X;\, E,\, p),$$

assigning to each boundary configuration and context the induced continuation set: the restriction of the admissibility relation around the subsystem's configuration. The paper's base proposal is that *knowledge is the constraint structure*: to know is for one's admissible continuations to be shaped thus, and an item of knowledge, where the phrase is useful at all, is a feature of the shape.

The definition places capacity before representation, in a lineage the introduction conceded, and it makes knowledge relational twice over: the structure is defined on the boundary, so its content concerns the subsystem's couplings; and it is indexed to the subsystem's configuration, so what is carried shapes what is known.

Proposition 5.2 (Two formulations, one object, [L2])**.** *The invariant content of K_I across the context class is the discrimination structure: the quotient of boundary configurations by $\sim_{I,X}$, together with the induced map on continuation sets, determines and is determined by K_I restricted to $\mathcal{P}_I$.*

Proof. From K_I the quotient and the induced map are constructed by Definition 3.4, since $\sim$ is defined by equality of the very continuation sets K_I assigns. Conversely, the quotient and the induced map recover K_I on $\mathcal{P}_I$ up to the labelling of class representatives, which is all that determination over the context class can mean. □

The proposition reconciles the project's two source formulations. Knowledge as constraint, the shape of what may follow, and knowledge as invariance, what survives every admissible perturbation, describe one object, and the choice between them is a choice of emphasis: the first foregrounds the shaping, the second the stability of the shape.

Proposition 5.3 (Acquisition requires structure, [L2])**.** *If K_I is trivial, in that the assigned continuation sets are independent of (E, p), then I has exactly one discrimination class, no product of I's is a symbol, and I acquires no symbol at that cut. An initial constraint structure is a precondition of acquisition.*

Proof. Independence of the continuation sets from the boundary data collapses Definition 3.4 to a single class; a symbol substitutes for a discrimination class, and substitution for the sole class is trivial; acquisition adds a symbol to the repertoire, and there is none to add. □

The proposition derives, from the definitions alone, a conclusion usually defended empirically: a blank slate cannot learn, since learning is reshaping and a trivial shape offers no purchase. The specification of an initial condition as maximal plasticity with an acquisition mechanism and an empty repertoire, given in Companion Study III [7], is hereby closed from the other side: the repertoire may be empty, and the constraint structure may not.

6 The Effective Manifold and the Constraint Surface

This section is [L3] throughout, under the declarations of Section 4.

The space of constraint structures reachable by a subsystem, ordered by admissible reachability, is coarse-grained as a manifold $\mathcal{M}_I$, with coordinates the slow variables w: the sufficient-state sector and the repertoire at coarse grain. Over $\mathcal{M}_I$ the model posits a surface, an effective potential $V_I(w)$, whose local minima are the metastable constraint structures: the arrangements of the subsystem's persistent structure that admissible perturbation within the context class leaves in place. The anchored content of the surface is exactly the invariance formulation of Proposition 5.2: which structures are stable, and which are reachable from which, the adjacency of basins. Depth, curvature and flow are posited.

On this surface the hypothesis that initiated the present project is stated as a modelling proposal. Acquired symbols are occupied minima: a symbol held in the repertoire is a stable arrangement of the shared sector of the surface, its stability being the persistence and substitution that the combinatorial definition requires, and the repertoire at a cut is the set of minima the subsystem's history has settled. The proposal earns its place by what it organises: acquisition becomes the appearance or deepening of a basin followed by settlement; forgetting becomes the flattening of a basin; interference between acquisitions becomes the interaction of basins; and the difficulty of unlearning becomes the depth of what must be climbed out of. Each of these is a redescription, anchored only as far as Section 4 states, and offered because redescription in one connected image is itself a contribution when the phenomena are usually treated piecemeal.

7 The Evolution of the Surface through Coupling

The keynote's second half is that the surface is not fixed, and this section states its dynamics, [L3] under the declarations, with the combinatorial anchor made explicit at each step.

The model separates two timescales. On the fast scale the surface is fixed and the subsystem's state flows upon it: continuation under the constraint structure as it stands, which is the effective description of acting, distinguishing and applying what one knows. On the slow scale the surface itself deforms:

$$\frac{\partial V_I}{\partial \chi_{\text{slow}}} = F[\, g_e\, \Phi_{\text{env}},\;\; g_c\, \Sigma_{\text{cond}},\;\; g_s\, \Psi_J \,],$$

where Φ_{env} stands for the environment fields the subsystem is coupled to, Σ_{cond} for the condensed symbolic structures within reach, languages, techniques, institutions, in the sense of Definition 3.9, and Ψ_J for the fields of other subjects; the g's are coupling strengths and F a functional the model does not fix. The equation is a schema and is offered as one: its content is the keynote's claim that the surface evolves only through coupling, so that a subject's knowledge is at every point the sediment of its relations. The anchored content is the [L2] distinction between continuation under K_I and change of K_I, together with the separation requirement of Definition 3.10; adiabaticity and the form of F are posited, and their failure is the crisis case of Section 12.

Learning, on this model, is the slow dynamics: the deformation of the surface through the couplings. Applying is the fast dynamics: motion upon it. The pair restates, as the introduction conceded, an old and well-supported distinction, and the model's addition is the closed loop between them: fast motion leaves records, records condense, condensates enter Σ_{cond}, and the slow drive is thereby fed by the fast history, one's own and others'. A subject deforms the surfaces of those it couples to, and its own surface is deformed in turn, which is the field-theoretic form of the claim that knowing is constitutively social.

One result constrains every reading of the schema, and it is [L2].

Proposition 7.1 (No transfer of structure, [L2])**.** *Let I and J be distinct subsystems, and let a persistent change of the configuration-indexed part of K_J be attributable to I. Then the change is realised through continuations through J's own cells.*

Proof. K_J is determined by the design and by J's configuration. The admissible route from I to J is coupling across $\partial\mathcal{C}_J$, which enters K_J as boundary dependence; a persistent change of the part indexed to J's configuration requires a change of that configuration persisting across cuts, hence the generation of a trace in J's sector, and generation runs through the cells of the generating subsystem. □

The proposition bounds what any coupling can do: what crosses a boundary is products, and a product reshapes the recipient's constraint structure only through the recipient's own continuations. Knowledge, on this model, cannot be transmitted, only generated, and the slogan is hereby a proposition with stated assumptions. Teaching is thereby relocated, and ennobled: a teacher cannot install a structure and can arrange couplings, supply products, and shape the environment fields, under which the student's own continuations regenerate the structure; pedagogy is the art of the drive term, exercised without access to the surface it deforms.

8 Inheritance

A subject's surface is deformed by more than its contemporaries. The condensed structures Σ_{cond} of Section 7 include the deposits of parties long absent, and coupling to them is inheritance: the shaping of a constraint structure by structures whose generation lies outside the inheritor's records.

The combinatorial base is imported from Companion Study III [7].

Established Result 8.1 (Continuation without memory; Companion Study III). B continues *A when the generating continuations of B's constitution pass through cells conditioned, in the sense of the record relation, by traces A generated; the degree of continuation is ordered by inclusion of the conditioning trace sets, and branching is admitted in both directions. A conditioning trace is a* memory *for B only when it is discriminable by B within $\mathcal{P}_B$, and continuation to a maximal degree with no conditioning trace a memory is constructible and ordinary.*

The present model adds the effective form. In the effective action of the slow variables, the past enters as a memory kernel,

$$S_{\text{eff}} \supset \iint K(\chi, \chi')\, w(\chi)\, w(\chi')\, d\chi\, d\chi',$$

an influence of earlier configurations on present dynamics carried by the accumulated structure itself, with no stored representation anywhere. The formula is [L3] and schematic; its anchor is the conditioning relation, and the kernel's form is posited. It organises the phenomenology of inheritance: a speaker is shaped by generations of a language none of whose speakers it met, a practitioner by the sedimented technique of a craft, and in each case the shaping is real, the records are largely inaccessible to the shaped party, and no transfer of structure occurred at any point, by Proposition 7.1: each generation's structure was regenerated through its own continuations, under couplings the deposits arranged.

Two consequences are worth drawing. Inheritance without memory is the ordinary case, and accounts that model tradition as stored and retrieved content mislocate it: what persists is products and the constraint structures they keep regenerating. And the impossibility of handover dignifies the machinery of transmission: texts, tools, curricula and institutions are not containers of knowledge but arrangements of coupling, and their quality is the quality of the regeneration they drive. The debts here are conceded: the habitus of Bourdieu [1] is the sociological form of the kernel, the tacit dimension of Polanyi [12] its epistemological form, and internalisation through interaction in Vygotsky [15] its developmental form; the model contributes a location in which the three are one mechanism.

9 Application: Coarsening and Refinement

Acquired structure does not only enable; it filters. The combinatorial base is configuration-indexing: discrimination is indexed to what the subsystem carries, so acquisition reshapes the classes, and the reshaping has two directions.

Definition 9.1 (Coarsening and refinement, [L2]). An acquisition *coarsens* I's discrimination on a region when, at the later cut, the discrimination relation there has strictly fewer classes; it *refines* when strictly more. Both are checkable by comparing the quotients before and after.

On the surface picture, a coarsening is a widening of basins: configurations formerly leading to different continuations now settle alike, and the subject responds to the class where it once responded to the case. A refinement is the splitting of a basin: distinctions become available that were not. Neither direction is defined by truth or falsity, and neither carries a verdict of itself: a diagnostic training refines a clinician's discrimination of presentations, and a notation coarsens a musician's discrimination of what it deliberately abstracts from, each to the purpose. The evaluative questions, which coarsenings serve whom, belong to Companion Study II [5], where the limiting case is analysed under the name of discriminative inversion, and the imported statement is this: with $\kappa(E) = \{\sigma \in \Sigma_I : \sigma \dashv [E]_{I,X}\}$ the classification of a configuration by the repertoire, the subsystem is discriminatively inverted on a region when

$$E_a \approx_{I,X} E_b \quad \Longleftrightarrow \quad \kappa(E_a) = \kappa(E_b),$$

the world discriminated exactly as finely as the repertoire sorts it, under which condition the residue of every symbol of the repertoire is invisible to the party gripped; the present section supplies only the mechanism, and supplies it for both directions at once, which is its point: the same dynamics of the surface that opens distinctions closes them, and a theory of knowledge that treats only refinement has modelled half the phenomenon.

10 The Reconstruction of the Past

The evolving surface bears on the past as well as the future, and the bearing is the model's final application.

Records are traces: configurations that persist, and as traces they are what they are, the past as history. The *reconstructed* past is another object: what a subsystem can now recover from its records, the image of the record set under the current constraint structure, since recovering is itself continuation and continuation runs under K_I as it stands. The two objects diverge in a definite way.

Proposition 10.1 (Reconstruction varies with structure, [L2]). *Let two cuts of one subsystem carry identical record sets and distinct constraint structures whose discrimination differs on the records' region. Then the reconstructed pasts at the two cuts differ: some class of records supports continuations at one cut that it supports at neither the other, with no new record anywhere.*

Proof. Recovery from records is continuation conditioned on them; by configuration-indexing, the induced continuation sets differ where discrimination differs; the difference in what is recoverable is the difference in the reconstructed pasts. □

Reinterpretation without new evidence is thereby the expected case, and its familiar instances are ordinary: a diary reread after a decade yields a different past from the same pages; a discipline's archive, reanalysed under a refined structure, yields findings the depositors could not have extracted; and learning in general revises what one's own history was, since it revises the structure under which that history is recoverable. On the surface picture, the past a subject can have is a function of where on the surface it stands, and the slow dynamics of learning is equally a slow dynamics of the recoverable past.

Two boundary stones close the section. First, the records themselves are untouched by any of this: reconstruction acts at the present cut, on and under present structure, and the proposition asserts revision of the recoverable, never of the traces; readings of the registered past as itself created by present acts, of the kind associated with Wheeler's participatory formulations

[17], are hereby declined, and the modest form of the observation that registration involves the present apparatus is the whole of what is kept. Second, the apparatus of reconstruction, the records held, the structures trained, the access arranged, is itself a set of products with mediation and revision sets, so the questions of who can recover what, and who can revise the conditions of recovery, are instances of a condition defined in Companion Study IV [6]: a party is *observationally subject* at horizon h when the placement and structure of its boundary, and thereby the domain of its discrimination, are conditioned by products with the party outside Rev_h and some other party within it. Applied to records and to the structures trained for their recovery, the condition states exactly when the reconstructable past of one party is a product revisable only by another; the present paper notes the location and develops it no further.

11 Summary of Results

The paper has proposed a model and disciplined it. The proposal: knowledge is constraint dynamics on an evolving manifold, the surface's stable regions the settled capacities, its evolution driven by coupling to environment, others and condensates, application the fast dynamics upon it and learning the slow dynamics of it. The discipline: a combinatorial base defining the constraint structure (Definition 5.1), with the equivalence of the constraint and invariance formulations (Proposition 5.2), the derivation that acquisition requires prior structure (Proposition 5.3), and the bound that constraint structures pass between subjects only through the recipient's own continuations (Proposition 7.1); a declared enrichment separating, for the manifold, the surface and the two-time dynamics, anchored features from posited ones, with conditions of failure named (Section 4); and applications to inheritance, with the memory kernel as the effective form of conditioning without memory, to the two directions of discrimination change, and to the reconstruction of the past, with Proposition 10.1 placing reinterpretation without new evidence as the expected case.

12 Limitations and Open Problems

12.1 Conditions of failure

The model fails where its substrate fails, and the substrate's conditions are stated in Companion Study I [8] and are not repeated. Its own conditions of failure are those declared in Section 4: a reachable set too discrete for the manifold to organise; stability by driving, which basin talk misattributes; and the mixing of timescales, under which the separation of applying from learning, and with it the model's central distinction, degrades. Each failure is checkable in principle against a stated design, and the second and third name phenomena of independent interest.

12.2 What remains unmodelled

The scope section bears repeating in one line: content, truth, warrant, experience, realisation and quantity are outside the model, and nothing in the paper constrains them.

12.3 Open problems

1. **Derivation of the enrichment.** Deriving $\mathcal{M}_I$ and V_I from a stated generating field theory, so that the posited features become computed ones, is the genuinely field-theoretic continuation of this paper and is future work; the present paper is its conceptual layer.
2. **The geometric residue.** Which of the paper's effective claims survive on topology and

order alone, with every metric posit removed.

3. **The crisis of learning.** The dynamics under timescale mixing, where application and learning interpenetrate; the adiabatic boundary from the inside.
4. **Collective structures.** A family's joint constraint structure, with aggregation excluded by the standing constraints; the gap is the coalition gap recorded in Companion Studies II, III and IV, as collective revision, co-formation and the joint observer, and the four are one problem.
5. **The acquirer.** What structure makes a subsystem an acquisition mechanism, inherited from Companion Study III.
6. **Kernels with content.** Any empirical development of the memory kernel or the drive functional requires enrichments beyond this paper, and the requirement is recorded so that no reader takes the schemata for more.

Recapitulation

The paper's objects and results, collected for reference; the setting is that of Section 3.

- **Keynote.** Knowledge as constraint dynamics on an evolving manifold: capacities as stable regions of a surface; the surface deformed only through coupling; applying as fast motion upon it, learning as slow deformation of it.
- **Constraint structure.** $K_I(\chi) : (E, p) \mapsto \mathrm{Con}_I(X; E, p)$; knowledge is the structure itself. Equivalent to the invariance formulation over the context class. Trivial structure admits no acquisition.
- **Enrichment, declared.** $\mathcal{M}_I$: reachable constraint structures, anchored in connectivity and reachability order, smoothness posited. V_I: stability anchored, depth and flow posited; symbols as occupied minima, a modelling proposal. Two-time dynamics: the applying and learning distinction anchored, adiabaticity posited; the deformation schema $\partial V_I/\partial\chi_{\text{slow}} = F[\, g_e\Phi_{\text{env}},\, g_c\Sigma_{\text{cond}},\, g_s\Psi_J\,]$.
- **No transfer.** A persistent change of another's constraint structure is realised through that party's own continuations; products cross boundaries, structures regenerate. Teaching as the art of the drive term.
- **Inheritance.** Conditioning without memory; the effective memory kernel of Section 8; tradition as arranged coupling, its machinery arrangements of regeneration.
- **Application.** Coarsening: strictly fewer discrimination classes; refinement: strictly more; both checkable, neither evaluative of itself; the same slow dynamics opens and closes distinctions.
- **Reconstruction.** Records unrevisable as traces; the reconstructed past the image of the records under current structure, revised by learning without new evidence; the apparatus of reconstruction a set of products with mediation and revision sets.

References

[1] Pierre Bourdieu. *Outline of a Theory of Practice*. Cambridge University Press, 1977.

[2] Karl Friston. The free-energy principle: A unified brain theory? *Nature Reviews Neuroscience*, 11:127–138, 2010.

[3] Hermann Haken. *Synergetics: An Introduction*. Springer, 1983.

[4] John J. Hopfield. Neural networks and physical systems with emergent collective computational abilities. *Proceedings of the National Academy of Sciences*, 79(8):2554–2558, 1982.

[5] Wanhong Huang. The inversion of symbolic mediation and the conditions of revision: A preliminary discussion of marxian alienation in a background-independent relational ontology. Working draft, 2026.

[6] Wanhong Huang. Partition, discrimination, and the internal observer in a background-independent relational ontology: A preliminary discussion. Working draft, 2026.

[7] Wanhong Huang. Plasticity, generativity, and the acquisition of constraint: A preliminary discussion of subject formation in a background-independent relational ontology. Working draft, 2026.

[8] Wanhong Huang. Symbol emergence and symbolic mediation in a background-independent relational ontology: A preliminary discussion. Working draft, 2026.

[9] Humberto R. Maturana and Francisco J. Varela. *Autopoiesis and Cognition: The Realization of the Living*. Reidel, 1980.

[10] Humberto R. Maturana and Francisco J. Varela. *The Tree of Knowledge: The Biological Roots of Human Understanding*. Shambhala, 1987.

[11] Jean Piaget. *The Origins of Intelligence in Children*. International Universities Press, 1952.

[12] Michael Polanyi. *The Tacit Dimension*. Doubleday, 1966.

[13] Gilbert Ryle. *The Concept of Mind*. Hutchinson, 1949.

[14] René Thom. *Structural Stability and Morphogenesis*. Benjamin, 1975.

[15] Lev S. Vygotsky. *Mind in Society: The Development of Higher Psychological Processes*. Harvard University Press, 1978.

[16] Conrad H. Waddington. *The Strategy of the Genes*. Allen and Unwin, 1957.

[17] John Archibald Wheeler. Law without law. In *Quantum Theory and Measurement*. Princeton University Press, 1983.