Proposal: A Group Field Theory-Inspired Computational Modeling of MRSI Spacetime Structures

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Note: This is currently a conceptual work and is currently unevaluated.

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Abstract

Group Field Theory (GFT) is a quantum field theoretical framework that offers a general language for describing the emergence of complex structures, particularly in the context of quantum gravity. Its foundational ontology is rooted in abstract algebraic structures—groups—on which fields are defined. The dynamics of these fields, encoded in the action function, give rise to discrete spacetime structures. Through processes such as condensation and coarsening, described by an effective field theory, a smooth spacetime geometry emerges. GFT provides several significant advances both in physics and philosophy: (1) it describes the emergence of structure without assuming a preexisting spacetime, (2) it adopts a non-entity ontology, offering a relational framework where entities are defined solely by their interactions, and (3) it is algebraically grounded.

In this work, we explore the potential for GFT to be generalized for modeling complex structures beyond quantum gravity, specifically in the domain of Magnetic Resonance Spectroscopic Imaging (MRSI). We develop a computational model that captures the spatio-spectro-temporal dynamics observed in MRSI data, offering a novel physics-based and mathematically grounded approach to understanding brain activity. By leveraging the principles of GFT, we aim to provide new insights into how brain activity emerges from interactions within complex systems, offering both a powerful model for MRSI and a fresh practice philosophical framework for studying the dynamics of the brain.

1 Introduction

Group Field Theory (GFT) [2, 4], which originated in the 1990s [1, 3], is a quantum field theoretical framework formulated for quantum gravity. It provides a general language to

describe the generation of spacetime structures from pre-geometric objects. Unlike many other approaches to quantum gravity, GFT is philosophically distinct in two key aspects: (1) it removes the fundamental role of a pre-existing spacetime continuum, which instead emerges from the dynamics of the theory, and (2) it relies on an ontology of abstract algebraic structures—specifically, groups—rather than assuming any entity as the fundamental building block.

In GFT, the fundamental degrees of freedom are fields defined on group manifolds. These fields and their interactions give rise to system dynamics, which are represented in an *action* within the quantum field theory context. A perturbative expansion of the action generates discrete spacetime structures known as spin foams. Through processes of coarse-graining and condensation, these discrete structures are proposed to give rise to continuous spacetime structures.

While GFT is primarily a theoretical framework explaining the emergence of spacetime, this work aims to explore its potential in an applied, computational context. Specifically, we propose adapting the mathematical structure of GFT to model the complex, high-dimensional data acquired via *Magnetic Resonance Spectroscopic Imaging* (MRSI). MRSI data is inherently 5-dimensional, comprising three spatial dimensions, one spectral frequency dimension, and one temporal dimension. In this work, we investigate a computational model derived from GFT to fit the MRSI data.

This framework offers several potential advantages for the computational model and analysis of MRSI: (1) It provides a physical and mathematical-grounded foundation for modeling the structure of MRSI data, (2) As a relational ontological model, GFT has the potential to naturally model the coupling relationships between brain regions in parameter space, and (3) GFT offers a philosophically advanced methodological approach that can help us understand the generation of complex structures. Our work represents an attempt to translate the theoretical philosophy of GFT into a practical philosophy, offering a practically feasible approach for understanding the spatiotemporal structure generation of brain metabolism.

This manuscript presents a conceptual framework to answer the following research questions: (1) How can a computational GFT-inspired model be derived to fit MRSI data? (2) How can the computational model parameters be estimated? (3) How might the application of MRSI, such as brain region connectome extraction, perform using the estimated model?

It should be noted that the proposed framework is preliminary, conceptional, and has not yet been empirically evaluated. In the following sections, we will outline the fundamentals of GFT, describe how a computational model can be derived from it to fit 5D MRSI data, and discuss its potential application in extracting the metabolic connectome of brain regions.

2 Preliminary and Method

2.1 Overview of Spacetime Generation in Group Field Theory

Group Field Theory (GFT) is a quantum field theoretical framework that extends traditional quantum field theory to describe the emergence of spacetime in the context of quantum gravity. It use algebric groups as the ontology, and describe how field are build from such abstract structures, and how spacetime structures are emerged from the fields and their interactions.

Specifically, in GFT, the field $\varphi(g_1, \ldots, g_D)$ is defined on D algebraic groups, where each g_i represents an element of a group. The fields and their interactions give rise to the system dynamics, which are encoded in the action $S[\varphi]$, defined within the context of quantum field theory. This action governs the evolution of the system and serves as the fundamental object in GFT. A perturbative expansion of $S[\varphi]$ leads to Feynman diagrams, which, in the framework of GFT, are generalized into spin foams. These spin foams correspond to discrete spacetime structures.

Through processes such as coarse-graining, which averages over the finer details of these discrete structures, and condensation, which involves the organization of these structures into larger-scale configurations, the theory proposes that these discrete building blocks can give rise to continuous spacetime structures. In this way, the quantum dynamics represented by the discrete spin foams may eventually lead to the classical spacetime we observe at macroscopic scales.

2.2 Choice of Groups

The spacetime structure of MRSI is indexed by the five variables (x, y, z, f, t), where x, y, z represent spatial coordinates, f represents frequency indexing, and t repzresents time indexing. To encode these variables, we select the groups $SU(2), SU(2), SU(2), U(1)$ and $U(1)$.

Specifically, the three $SU(2)$ groups are used to encode the angular components of the spatial coordinates x, y, z, since these coordinates are associated with rotations in a three-dimensional space. The $U(1)$ groups encode the frequency f and time t indices, where one $U(1)$ encodes the frequency and time, mapping to the variable $s = k\theta + s_0$, with s representing t or f. In which θ is described by $U(1)$, and k and s_0 are constants.

2.3 Emergence of Fields

The field $\varphi(g_1, g_2, g_3, g_4, g_5) : SU(2) \times SU(2) \times SU(2) \times U(1) \times U(1) \to \mathbb{C}$ is defined on the product of groups described in Section 2.2. We denote an element in this product of groups as $\mathbf{g}_i$ in the rest of this manuscript, i.e., $\mathbf{g}_i \in SU(2) \times SU(2) \times SU(2) \times U(1) \times U(1)$.

2.4 Emergence of the Action

The action $S[\varphi]$ in *quantum field theory* (QFT) encodes the dynamics of fields by integrating over the fields and their coupling structures. Specifically, $S[\varphi] = S_0[\varphi] + S_{\text{int}}[\varphi]$, where S_0 is the *free action* [2], and S_{int} is an *interaction term* of the action, which encodes the coupling structures of the fields.

2.5 Phase Transition and Condensate Phase

Building upon the GFT action described in Section 2.4, we recognize that the field configurations give rise to discrete spacetime structures. The expansion of the GFT action leads to spin foams, which represent quantum spacetime in the discrete regime. However, to transition from discrete to continuous spacetime, a process of coarsening or condensation is required.

In the low-temperature phase, the field configurations become coherent and exhibit collective behavior, which facilitates the emergence of continuous spacetime. Given this nature, and building upon the discrete spacetime dynamics described by the GFT action, we assume that the system undergoes a phase transition into a *Bose-Einstein Condensate* (BEC) phase. The BEC phase is a well-known phenomenon in quantum mechanics, where, at low temperatures, particles (bosons) condense into the lowest quantum state, leading to the formation of a coherent, macroscopic quantum state.

The phase transition to BEC involves spontaneous symmetry breaking, where the system moves from a disordered phase (with no manifest spacetime) to an ordered phase, in which spacetime emerges as a classical geometry. In the condensate phase, the expectation value of the quantum field operator $\hat{\varphi}$ becomes nonzero, i.e., $\sigma(\mathbf{g}_i) = \langle\hat{\varphi}(\mathbf{g}_i)\rangle \neq 0.$, where $\langle\hat{\varphi}(\mathbf{g}_i)\rangle = \frac{1}{Z}\int \mathcal{D}\varphi\ \varphi(\mathbf{g}_i)e^{-S[\varphi]}$. Here, $\hat{\varphi}$ is the quantum field operator, and σ represents the classical field, which serves as the *order parameter* that conveys the occurrence of symmetry breaking. Z is a *partition function*, with the form of $Z = \int \mathcal{D}\varphi\ e^{-S[\varphi]}$.

In the condensate phase, the system can be approximated by a coherent state $|\text{BEC}\rangle$, which is an eigenstate of the quantum field operator. This means $\hat{\varphi}|\text{BEC}\rangle \approx \sigma|\text{BEC}\rangle$. This implies that quantum fluctuations $\delta\hat{\varphi} = \hat{\varphi} - \sigma$ are small, and the system can be effectively described by the classical field σ.

2.6 Definition of the Effective Action

The partition function with the source field J coupled to φ is given by Equation 1. In which, $\bar{\varphi}$ is the conjugate field of field φ.

$$Z[J] = \int \mathcal{D}\varphi\mathcal{D}\bar{\varphi}\, e^{-S[\varphi]+\int(\bar{J}\varphi+\bar{\varphi}J)} \tag{1}$$

The generating functional $W[J] = \ln Z[J]$ connects quantum theory with statistical mechanics. The first derivative of $W[J]$ gives the expectation value of the field, which

described in Equation 2.

$$\frac{\delta W[J]}{\delta \bar{J}(g_i)} = \langle \varphi(g_i) \rangle_J \equiv \sigma_J(g_i) \tag{2}$$

The *effective action* (Equation 3) is obtained through the Legendre transformation, switching from the source J to the expectation value σ.

$$\Gamma[\sigma] = \sup_J \left(\int (\bar{J}\sigma + \bar{\sigma}J) - W[J] \right) \tag{3}$$

$\Gamma[\sigma]$ is our effective action $S_{\text{eff}}[\sigma]$. It has the following key properties: (1) It is the 1-particle irreducible (1PI) generating functional, containing all essential interactions., and (2) Its classical equation of motion, $\frac{\delta\Gamma[\sigma]}{\delta\sigma} = 0$, gives the system's quantum exact expectation values.

In the macroscopic limit of long wavelengths and low energies, the field variations are slow. To describe this behavior, we perform a derivative expansion (see Appendix A) of the effective action $\Gamma[\sigma]$ and retain only the lowest-order terms. The resulting expansion is given by Equation 4.

$$\Gamma[\sigma] = \int d^D X \left[U(\sigma) + \frac{1}{2} Z(\sigma)(\partial\sigma)^2 + \mathcal{O}(\partial^4) \right] \tag{4}$$

Here, $U(\sigma)$ is the *effective potential* of the field σ, which describes the energy of the field when it is constant. It reflects how the field's value influences the system's energy. The minima of $U(\sigma)$ correspond to the stable configurations of the field in the absence of kinetic effects.

The term $\frac{1}{2}Z(\sigma)(\partial\sigma)^2$ represents the *kinetic term*, with $Z(\sigma)$ being a field-dependent factor that modulates the propagation of the field across spacetime. A field redefinition can be performed to set $Z(\sigma) = 1$, simplifying the expression for the kinetic term.

The higher-order terms $\mathcal{O}(\partial^4)$ involve derivatives of higher order, but at low energies and long wavelengths, these contributions are negligible and are therefore omitted.

In this expression, D represents the number of spacetime dimensions in the theory. In our case, we consider $D = 5$.

This derivative expansion is valid in the low-energy limit, where field variations are slow and higher-order derivatives contribute only minimally. The effective potential $U(\sigma)$ and the kinetic term $\frac{1}{2}Z(\sigma)(\partial\sigma)^2$ are the leading contributions, and higher-order terms in ∂^4 is neglected.

2.7 Computational Model Derivation

Since our microscopic theory is separable on the $SU(2)$ and $U(1)$ groups, the kinetic term of the effective action must also be separable, which indicates in Equation 5). In which, $\supset$ indicates the right-hand side (kinetic term) is part of the effective action $\Gamma(\sigma)$. c_t, c_x, and c_f are coefficients associated with the temporal, spatial, and frequency components of the field σ, respectively.

$$\Gamma[\sigma] \supset \int d^5X \left[c_t |\partial_t \sigma|^2 + c_x |\nabla_x \sigma|^2 + c_f |\partial_f \sigma|^2 + \dots \right] \tag{5}$$

By re-scaling the fields, we can absorb the coefficients c_t, c_x, and c_f, leading to the simplified kinetic term $(\partial_t \sigma)^2 + |\nabla_v \sigma|^2 + (\partial_f \sigma)^2$.

The effective potential $U(\sigma)$ can be expanded in a Taylor series around $\sigma = 0$. Symmetry allows the presence of terms like $|\sigma|^2$ and $|\sigma|^4$ (due to $U(1)$ symmetry $\sigma \to e^{i\theta}\sigma$), and we come to Equation 6.

$$U(\sigma) = \mu |\sigma|^2 + \lambda |\sigma|^4 + \dots \tag{6}$$

The microscopic non-local interactions $S_{\text{int}}[\varphi]$ do not disappear upon coarse-graining. In the effective action $\Gamma[\sigma]$, the microscopic interaction term $S_{\text{int}}[\varphi]$ and all quantum fluctuations contribute corrections, resulting in an *effective non-local interaction* (Equation 7).

$$\Gamma[\sigma] \supset \frac{1}{2} \iint d^5X \, d^5Y \, \sigma(X) \, J(X, Y) \, \sigma(Y) \tag{7}$$

Here, the kernel function $J(X, Y)$ encapsulates the net effect of all interactions.

Since the interactions in the microscopic GFT are combinatorial and separable (e.g., space components bind independently), the macroscopic effective kernel $J(X, Y)$ naturally takes a separable form $J(X, Y) \approx J_S(v, v') \cdot J_F(f, f') \cdot J_T(t, t')$. In which, v, f, t are spatial, frequency, and time components of field respectively.

To make the coupiling structures, i.e., the interaction J learnable, we make the J associates with the parameter Θ, and the J with parameter Θ nodes as J_Θ. The S_{eff} with such parameter denotes as $S_{eff}[\sigma; \Theta]$.

Combining all the steps above, and we obtain the most general form of the effective action (Equation 8).

$$S_{\text{eff}}[\sigma; \Theta] = \int d^5X \left[(\partial_t \sigma)^2 + |\nabla_v \sigma|^2 + (\partial_f \sigma)^2 + \mu |\sigma|^2 + \lambda |\sigma|^4 \right] + \iint d^5X \, d^5Y \, \sigma(X) \, J_\Theta(X, Y) \, \sigma(Y) \tag{8}$$

2.8 Model Estimation

As indicated in Section 2.7, we make the effective action learnable (Equation 9). In which, the interaction term $S_{\text{int}}[\sigma; \Theta]$ is given by Equation 10.

$$S_{\text{eff}}[\sigma; \Theta] = \int dX \left[(\partial_t \sigma)^2 + |\nabla_v \sigma|^2 + (\partial_f \sigma)^2 + \mu \sigma^2 + \lambda \sigma^4 \right] + S_{\text{int}}[\sigma; \Theta] \tag{9}$$

$$S_{\text{int}}[\sigma; \Theta] = \iint dX \, dY \, \sigma(X) \, J_\Theta(X, Y) \, \sigma(Y) \tag{10}$$

The $J_\Theta(X, Y)$ (Equation 11) is a learnable kernel function, encoding the coupling strength between the spacetime points X and Y. The parameter Θ governs the specific form of the coupling.

$$J_\Theta(X, Y) = J_S(\Theta_s; v, v') \cdot J_F(\Theta_f; f, f') \cdot J_T(\Theta_t; t, t') \tag{11}$$

For a given set of parameters Θ, we generate the predicted MRSI data σ_Θ by solving the classical field equations $\frac{\delta S_{\text{eff}}[\sigma;\Theta]}{\delta\sigma} = 0$.

We then compare the predicted field σ_Θ with the true MRSI data field D using the following loss function $L(\Theta)$ (Equation 12).

$$L(D;\Theta) = \text{Mean Squared Error}(\sigma_\Theta, D) = \int |\sigma_\Theta(X) - D(X)|^2 \, dX \tag{12}$$

A gradient descent-based algorithm can be used to minimize the loss $L(D, \Theta)$. For a simplest optimization rule is $\Theta \leftarrow \Theta - \eta \cdot \nabla_\Theta L$, where η is the learning rate.

The optimal learned parameters Θ^* define the effective coupling rules present in the data.

3 Application

3.1 Connectome Analysis

The brain voxel space is divided into N brain regions $\{R_1, R_2, \ldots, R_N\}$. Using the learned spatial kernel $J_S(\Theta_s^*; v, v')$, we compute the average coupling strength between brain regions R_i and R_j (Equation 13).

$$C[i, j] = \frac{1}{|R_i||R_j|} \sum_{v \in R_i} \sum_{v' \in R_j} J_S(\Theta_s^*; v, v') \tag{13}$$

This results in an $N \times N$ symmetric matrix, which is the brain region coupling connectivity matrix learned from MRSI data.

A Proof. Effective Action Derivation Expension Form

Step 1: Representing $\Gamma[\sigma]$ as a Functional Integral

The quantum effective action $\Gamma[\sigma]$ is defined by the following relation:

$$e^{-\Gamma[\sigma]} = \int \mathcal{D}\phi\, e^{-S[\phi]} + \int (\phi - \sigma)\, \frac{\delta\Gamma}{\delta\sigma}$$

This formula shows that $\Gamma[\sigma]$ is the weighted sum over all field configurations ϕ, where their expectation value is constrained to be σ. This is a self-consistent definition.

For calculation purposes, we perform a change of variables $\phi = \sigma + \eta$, where η represents quantum fluctuations:

$$e^{-\Gamma[\sigma]} = \int \mathcal{D}\eta\, e^{-S[\sigma+\eta]} + \int \eta \frac{\delta\Gamma}{\delta\sigma}$$

Step 2: Expanding the Micro Action around the Classical Solution

Now, we expand the microscopic action $S[\sigma + \eta]$ around the uniform field σ to first order in fluctuations η:

$$S[\sigma+\eta] = S[\sigma] + \int d^D X\, \frac{\delta S}{\delta\phi(X)}\Big|_{\phi=\sigma} \eta(X) + \frac{1}{2}\iint d^D X d^D Y\, \eta(X) \frac{\delta^2 S}{\delta\phi(X)\delta\phi(Y)}\Big|_{\phi=\sigma} \eta(Y) + O(\eta^3)$$

We assume σ is close to the classical solution, so the first-order variation $\frac{\delta S}{\delta\phi}$ is small. Higher-order terms $O(\eta^3)$ represent interactions.

Step 3: Performing the Gaussian Integral and Identifying the Effective Action

Substituting the expanded form into the functional integral, the main contribution comes from the quadratic terms (Gaussian integrals), and higher-order terms $O(\eta^3)$ can be treated as perturbations:

$$e^{-\Gamma[\sigma]} \approx e^{-S[\sigma]} \cdot \int \mathcal{D}\eta\, \exp\left(-\frac{1}{2}\iint \eta(X) G^{-1}(X, Y; \sigma)\eta(Y)\, d^D X d^D Y\right)$$

Here, we define the fluctuation operator:

$$G^{-1}(X, Y; \sigma) = \frac{\delta^2 S}{\delta\phi(X)\delta\phi(Y)}\Big|_{\phi=\sigma}$$

Now, we compute the Gaussian integral. The result of the functional Gaussian integral is the square root of the determinant:

$$\int \mathcal{D}\eta\, e^{-\frac{1}{2}\iint \eta(X) G^{-1} \eta(Y)} = [\det(G^{-1})]^{-1/2} = [\det(G)]^{1/2}$$

Therefore, we have:

$$e^{-\Gamma[\sigma]} \propto e^{-S[\sigma]} \cdot [\det(G)]^{1/2}$$

Taking the logarithm, we obtain the expression for the effective action:

$$\Gamma[\sigma] = S[\sigma] + \frac{1}{2}\mathrm{Tr}\ln G(\sigma) + \text{constant} + O(\text{higher-order perturbations})$$

This formula is the bridge between the microscopic and macroscopic descriptions:

$$S[\sigma] : \text{The classical action. This is the tree-level contribution.}$$

$$\frac{1}{2}\mathrm{Tr}\ln G(\sigma) :$$

The one-loop contribution. It contains the main quantum fluctuation corrections.

Step 4: Performing the Derivative Expansion

Now, we focus on the term $\frac{1}{2}\mathrm{Tr}\ln G(\sigma)$. The operator $G(\sigma)$ is:

$$G^{-1}(X, Y; \sigma) = [-\partial_X^2 + V''(\sigma)]\delta^{(D)}(X - Y)$$

Here, ∂_X^2 is the Laplacian (or d'Alembert operator), and $V''(\sigma)$ is the second derivative of the potential function.

We now compute the trace $\mathrm{Tr}\ln G(\sigma) = \mathrm{Tr}\ln\left[-\partial^2 + V''(\sigma)\right]^{-1} = -\mathrm{Tr}\ln\left[-\partial^2 + V''(\sigma)\right]$.

The key to evaluating this trace is performing it in momentum space:

$$\mathrm{Tr}\ln\left[-\partial^2 + V''(\sigma)\right] = \int d^D X \int \frac{d^D p}{(2\pi)^D} \ln\left[p^2 + V''(\sigma)\right]$$

This integral is over all momentum p. However, this is a divergent integral, which needs to be renormalized.

We do not need the exact value, but only its form as a functional of σ. The integrand $\ln[p^2 + V''(\sigma)]$ is a function of σ. We can expand it around low momentum (or around $V''(\sigma)$).

Thus, the result of the trace $\mathrm{Tr}\ln G(\sigma)$ is a functional of σ and its derivatives. Since we assume σ is uniform, $V''(\sigma)$ is a constant, and the result of $\mathrm{Tr}\ln G(\sigma)$ is a local term of the form $\int d^D X U_1(\sigma)$, where $U_1(\sigma)$ is some function of $V''(\sigma)$.

If σ is not strictly uniform but varies slowly, $V''(\sigma)$ will depend on position. In this case, the momentum integral will generate derivative terms of σ. The low-momentum part (long-wavelength limit) contributes terms like $(\partial\sigma)^2$.

Therefore, the one-loop correction can be expanded as:

$$\frac{1}{2}\mathrm{Tr}\ln G(\sigma) = \int d^D X \left[U_1(\sigma) + \frac{1}{2}Z_1(\sigma)(\partial\sigma)^2 + O(\partial^4)\right]$$

Step 5: Combine Results to Obtain Final Form

Now, combining the classical and quantum correction parts:

$$\Gamma[\sigma] = S[\sigma] + \frac{1}{2}\mathrm{Tr}\ln G(\sigma) + \dots$$

The microscopic action $S[\sigma]$ itself contains kinetic and potential terms: $S[\sigma] = \int \left[\frac{1}{2}(\partial\sigma)^2 + V(\sigma)\right]$. Adding both parts together:

$$\Gamma[\sigma] = \int d^D X \left[\frac{1}{2}(\partial\sigma)^2 + V(\sigma) + U_1(\sigma) + \frac{1}{2}Z_1(\sigma)(\partial\sigma)^2 + O(\partial^4)\right]$$

Now, combining like terms:

$$\Gamma[\sigma] = \int d^D X \left[(V(\sigma) + U_1(\sigma)) + \frac{1}{2}\left(1 + Z_1(\sigma)\right)(\partial\sigma)^2 + O(\partial^4)\right]$$

Thus, we obtain the final derivative expansion form:

$$\Gamma[\sigma] = \int d^D X \left[U(\sigma) + \frac{1}{2}Z(\sigma)(\partial\sigma)^2 + O(\partial^4)\right]$$

Where:

$$U(\sigma) = V(\sigma) + U_1(\sigma) + \dots$$

is the effective potential, which includes both classical potential and quantum fluctuation corrections (e.g., the famous Coleman-Weinberg potential).

$$Z(\sigma) = 1 + Z_1(\sigma) + \dots$$

is the wavefunction renormalization factor, which corrects the "stiffness" or kinetic coefficient of the field. It can often be set to 1 by a field redefinition.

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