

Preliminary Notes on Heterogeneous Value Circulation and a Mathematical Representation

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Abstract

This preliminary discussion paper examines one possible mathematical representation of heterogeneous value circulation. It distinguishes typed transfer records, local interpretations, admissible transformations, changing conditions of circulation, and the independent choice of evaluative criteria. Two motivating cases are a bilateral exchange of money and labour service and a three-node route in which delayed receipt can affect a stipulated task threshold. A finite directed graph with locally heterogeneous state spaces supplies a deliberately limited starting representation. Additive inventory laws are separated from service events, replicable artifacts, interpretive assessments, and non-conservative transformation. Typed event accounts, in-transit stocks, local update maps, and set-valued uncertainties are specified explicitly. Equality within a unit, conversion by convention, contextual equivalence, similarity, and incomparability are considered separately. Reproducible synthetic Python studies illustrate receipt delay, threshold-dependent responses, path-composed internal transformations, conflicting sufficiency requirements, selective two-resource diffusion, and conductance feedback with parameter and time-step sensitivity checks. These calculations make formal assumptions inspectable; they are not calibrated economic or biological models and do not establish an ethically privileged objective. The paper compares established mathematical possibilities and identifies conditions for more responsible modeling of circulation and recognition.

Keywords: heterogeneous value; value circulation; local recognition; network dynamics; partial comparison; generative capacity

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Reading orientation. Sections 1–2 develop conceptual and literature background and the distinction between accounting, recognition, and value. Sections 3–4 introduce a locally structured graph model, typed transfers, conversions, delayed transit, and transport feedback. Sections 5–6 examine contextual comparisons, epistemic uncertainty, and plural evaluation. Section 7 presents reproducible synthetic experiments; Section 8 assesses the scope of the representation. The appendix records mathematical and computational assumptions, reproducibility details, and validation boundaries.

Discussion Paper Note

This discussion paper explores one possible mathematical representation of heterogeneous value circulation in relational systems. It is an early modelling exercise. The paper does not claim a novel geometry, a general account of value, an identified social mechanism, or a theory of just allocation. Its main contribution at this stage is to make modelling choices inspectable and to exhibit some of their consequences in constructed examples.

The object of discussion is the circulation of differently typed resources and services among participants whose interpretive conditions may differ. A monetary transfer and an episode of labour, for example, can be recorded within their respective units while the relationship between them remains contingent on exchange conventions, contracts, situated interpretations, and normative judgment. The proposed local state spaces are representational devices; their coordinates are not exhaustive descriptions of a participant's experience or worth.

The present inquiry adopts ontological suspension concerning the ultimate nature of persons, institutions, values, and generative capacities. It distinguishes consequential relational arrangements from the records and interpretations through which they may be known. Local recognition operators and comparison relations are illustrative specifications, potentially inaccessible to direct observation, and not validated psychological measurements.

The paper explores several separate elementary consequences of stipulated flow laws and thresholds with deterministic Python examples. They are synthetic numerical demonstrations, without original human-participant data, empirical calibration, estimates of social effect, or conclusions about the efficacy of a real intervention. In the simulations, the words “resource,” “conductance,” “distance,” and “geometry” have expressly restricted mathematical meanings.

Different questions demand different evaluative approaches. Conservation of a resource, successful completion of a task, a balanced account, voluntary consent, and distributive justice are distinct predicates. A model that discriminates among feasible trajectories does not by itself authorize an ethical ranking. The paper deliberately leaves plural and contested normative criteria open to further inquiry.

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Notices

Status. Independent theoretical discussion paper, preliminary expanded working draft (v0.2), 18 October 2026. This manuscript has not been peer reviewed or released as a validated mathematical, economic, or social-scientific model. Examples and numerical experiments are constructed for methodological exploration.

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Statement on the use of language models. OpenAI’s ChatGPT contributed to research dialogue, literature discovery, mathematical and editorial drafting, Python simulation design and execution, LaTeX preparation, and preliminary consistency checks. The model and writing remain a human-directed exploratory work. The author selected the research problem, constrained its epistemic claims, and retains responsibility for the definitions, interpretations, computations, citations, revisions, and errors. Bibliographic verification drew on publisher, conference, institutional, and preprint records, often at abstract or metadata level; full texts were not consistently examined. ChatGPT is not listed as an author.

Research ethics and provenance. The constructed examples refer to hypothetical participants. No original human-subject observations, experimental intervention, institutional approval, or third-party endorsement are asserted. Simulation source code, fixed parameters, numerical results, source verification ledger, and an audit note accompany the manuscript.

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1 Introduction

Section 1 introduces the scope of an inquiry that explores a limited mathematical description of heterogeneous value circulation. Its motivating difficulty arises when transfers among participants have an order, when the objects transferred are of different kinds, and when participants interpret them under different practical and epistemic conditions. The paper first separates these aspects conceptually, then introduces a discrete, locally structured network model, and finally examines selected implications through constructed examples. The resulting framework is a candidate representational vocabulary, not a claim that the economic or social world has a particular underlying geometry.

Consider a hypothetical circulation sequence among participants A, B, and C. A possesses a resource that may be transferred first to B and subsequently to C. C might require access to the resource before a particular deadline to begin an activity. The final amount delivered to C could be identical along two pathways, while the timing changes whether the activity can occur. Equally, B may need the resource urgently or may incur substantial costs as an intermediary. The example therefore supports an inquiry into *temporal feasibility* but does not, by itself, establish which arrangement is just. Relevant ethical considerations include claims over the resource, the needs of participants, the legitimacy of control over pathways, and the consequences of delaying circulation.

A second hypothetical case concerns an exchange in which A pays B \$100 and B performs twelve hours of labour for A. The transaction can be recorded with separate monetary and labour units. Whether either participant regards the exchange as advantageous may depend on alternatives, living costs, work conditions, recognition, and expectations. A contractual exchange rate supplies one convention for comparison; it does not automatically supply an exhaustive measure of effort, benefit, or justice. The example draws attention to the construction of comparison rules, a matter with substantial antecedents in analyses of commensuration [7] and distributive evaluation [12].

Generative justice supplies a further motivation. In Eglash's formulation, questions of value circulation concern the possibility of retaining and circulating value in less alienated arrangements, together with the conditions of its generation [5]. The present inquiry borrows this orientation as a motivation while limiting its immediate task to elementary representations of flows, local states, and comparison. It leaves distributive entitlements and the content of a generative-justice standard open to independent argument.

Existing mathematics already provides relevant structures. Network transport models address the movement of quantities through vertices and edges [4]. Resource theories permit the study of transformations without assuming that resources share a single numerical unit [8]. Cellular sheaves assign local spaces and compatibility maps to graphs and support more structured forms of diffusion [2]. Multicriteria optimization studies alternatives under several objectives [6]. We draw upon these established directions selectively and do not claim that their combination, as sketched here, is mathematically novel.

A circulation model concerns pathways, events, and states, whereas an account of value must also say what the transferred item is and how its significance can be investigated. The distinction is substantial in settings where a single event simultaneously moves a material object, creates a contractual claim, changes someone's ability to perform a task, and affects trust. A transfer of money is normally quantified in monetary units, while a service may be recorded as hours, evaluated by its quality, and experienced as effort or obligation. A diagram that treats these quantities as a common substance has already made substantive decisions before optimization begins.

The literature provides useful but differently situated tools. Network optimization studies capacities, costs, and feasible flows, including multicommodity and discrete constraints [1]. Dynamic optimal transport addresses quantities moving through a network while accounting for motion along edges and exchange between edges and vertices [4]. These methods provide descriptions of transport conditional on what is treated as mobile and what costs are assigned. Category-theoretic resource approaches examine permissible conversions and compositionality without imposing a uniform monetary unit [8]. None of these tools by itself selects a distributive conception of justice.

The comparison question has an independent history. Espeland and Stevens locate commensuration

within institutional and cognitive practices, including its implications for power [7]. Sen’s analysis of equality emphasizes the evaluative importance of the dimension in which persons are compared [12]. Nussbaum’s capabilities approach draws attention to opportunities for action and flourishing that are not exhausted by aggregate monetary output [9]. These are background motivations for maintaining separate levels of accounting, capability, and normative assessment; the proposed mathematical examples do not operationalize any one of these theories.

The geometry analogy also requires care. A graph can carry local state spaces, and some models of diffusion on graphs use a cellular sheaf to encode how neighboring local data are compared [2]. A smooth geometric model may be appropriate when a continuum limit and its regularity assumptions are independently justified. In this paper the finite directed graph is the primary object: it permits explicit conservation checks, selective paths, time-indexed transfers, and computable counterexamples. The mathematical question is whether these elementary distinctions remain stable under more expressive local spaces and transformations.

Four aspects will be treated separately throughout. The *descriptive* aspect concerns the recording of events, amounts, transformations, and their temporal order. The *epistemic* aspect concerns which local states and interpretations are available to each participant and to an external observer. The *structural* aspect concerns permitted pathways, delays, conversion interfaces, and changes in access. The *normative* aspect concerns the selection and justification of evaluation criteria. These aspects may interact, and their separation prevents an algorithm for transfer from silently claiming authority over values and obligations.

For clarity, the proposed representation has three levels of specification. A *circulation record* describes a transfer event and its type, location, quantity, and time. A *local state rule* specifies how the event may affect a receiving or supplying participant. An *evaluation rule* relates a state or trajectory to some explicitly identified criterion. These levels can be coupled, but one does not determine the others. In particular, an allocation feasible under a flow law need not satisfy a sufficiency criterion, and satisfaction of a modeled criterion is not equivalent to a justified normative judgment.

The paper remains under ontological suspension. Nodes may represent participants, organizations, functional units, or spatial sites depending on the application. Values may be treated as resources, services, interpreted meanings, capabilities, or recorded manifestations where the relevant distinctions have been stated. The model’s coordinates, edges, and flow parameters are chosen representations of aspects of a situation. Their ability to support justified inferences is contingent on evidence and appropriate identification of what can be observed.

Sections 2–4 develop the minimal vocabulary and flow law. Sections 5–6 consider comparisons and candidate evaluative criteria. Section 7 reports deterministic numerical illustrations. Section 8 records boundaries of the proposal. The small computational examples make assumptions inspectable; they do not supply empirical confirmation of any social mechanism.

2 Heterogeneity and Local Recognition of Value

This section specifies the distinctions needed before heterogeneous circulation can be represented. Its purpose is to separate the type of a transferred object, a record of that object, a participant’s interpretation, and a judgment about the exchange. The bilateral money–labour case serves as a constructed example. The formal categories are deliberately minimal and may require revision in an empirical setting.

2.1 Typed objects and recorded transfers

Suppose participant A transfers \$100 to participant B, while B provides twelve hours of labour service to A. The units “dollars” and “hours” belong to different measurement practices. The exchange can be entered in two typed accounts without defining a conversion between them. Figure 1 displays the two transfers separately.

A precise statement about equality therefore needs a declared domain. Within a monetary account, the outgoing sum of \$100 at A equals the incoming sum of \$100 at B, assuming the event is fully recorded

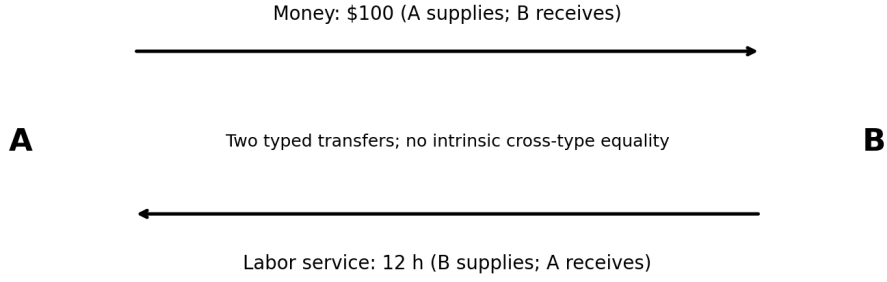


Figure 1: Bilateral transfer of two differently typed items. Arrows show accounting directions; the diagram implies neither common utility nor fairness of compensation.

and no fee or leakage is introduced. The labour account records twelve hours supplied by B and twelve hours received by A. This is an incidence identity for two records of a transaction. It does not assert that labour time constitutes a conserved physical stock, that hours represent homogeneous effort, or that the monetary and labour entries are equal in value.

Real transactions often contain additional features: quality, preparation, fatigue, liability, dependence, waiting time, and implicit rights of control. Even the apparently simple term “twelve hours of labour” can conceal variation in task intensity, skill, safety, and availability. A typed representation is consequently a starting specification and not a complete ontology of value.

2.2 Local interpretations and epistemic access

Let the relevant recorded events be r and the context of participant i at time t be $h_i(t)$. We may introduce a local recognition map, understood provisionally, that assigns a representation or assessment to an event. Equation 1 supplies the notation.

$$\mathcal{E}_{i,t} : \mathcal{R} \times H_i \longrightarrow Q_i, \quad (r, h_i(t)) \longmapsto q_i(r, t). \quad (1)$$

The spaces Q_i need not contain numerical scores. They may contain categories, partial descriptions, qualitative reasons, or several dimensions of assessment. The domain H_i can stand for prior knowledge, commitments, alternatives, and situated constraints. The operator is a modelling choice; in a real setting its inputs and outputs might be observable only in part, and the participant’s own articulated account may be indispensable.

For A, the service received could be assessed relative to a project deadline, the availability of other workers, or the usefulness of the task. For B, the payment may matter through immediate expenses, future options, and labour conditions. The same transfer record can therefore support different local evaluations without a contradiction. Differences in interpretation may themselves change as the exchange takes place and as participants learn about its effects.

The epistemic challenge extends to observers. A researcher may observe the amount, hours, and contractual terms while lacking access to the participants’ alternatives, private evaluations, or relevant histories. Representations that presume these values are known can introduce an unrecognized epistemic assumption. The model should mark such quantities as stipulated, measured, elicited, contested, or unknown, rather than assigning numerical precision by default.

2.3 Forms of value and distinct modes of circulation

A useful preliminary typology separates at least four operational situations. A *conserved stock* can be counted in a unit and transferred with a corresponding debit, such as a stipulated divisible resource

Table 1: Illustrative resource and value-related forms with distinct accounting requirements.

Represented form	Possible observation	Candidate update and principal caution
Conserved stock	Typed quantity at places and times	Debit–credit transfer; losses and boundary terms require explicit accounting
Service event	Duration, task, performance record	Creates an event and changes capacities; hours do not capture quality or effort
Replicable artifact	Version, access rights, provenance	Copy or transform with persistent source; access may differ from usable knowledge
Relational state	Reports, actions, institutional records	Context-dependent state change; measurement may be incomplete or contested

in the numerical experiment. A *service event* records performed activity over a specified interval; its completion does not imply the transfer of an independently stored physical substance called labour. A *replicable information object* may be copied, leaving the source in possession of the original, and may be useful only to a recipient with the relevant interpretive or technical capabilities. A *relational state* such as trust may change through interaction without a corresponding conserved quantity leaving one participant and entering another.

Table 1 gives the operational distinction without proposing an exhaustive theory of value. The entries are properties of *representations* under the indicated conventions, and an empirical case can involve several rows at once.

A single labor contract can consequently generate a monetary-stock event, a service event, a legal obligation, and subsequent assessments. Separate types allow accounting consistency, while translation or conversion relations must be introduced through additional assumptions. Introducing a money-per-hour contract parameter records one local contractual convention. It does not settle whether working conditions, hidden preparation time, care responsibilities, dependence, or long-term effects are fully recognized by that convention.

2.4 A more detailed bilateral exchange

Consider an imagined twelve-hour technical service delivered by B to A in return for \$100. A basic event ledger contains parties, timestamps, nominal amounts, task specification, contractual obligations, and their fulfillment status. A second descriptive layer can distinguish the actual service trajectory: preparation, communication, task performance, possible rework, and whether output was usable to A. A third layer records B’s alternatives, immediate financial needs, fatigue, and prospects for subsequent activity insofar as legitimate evidence permits. None of these layers is inferred from the first.

A may regard the service as exceptionally useful for a deadline, while B may regard the payment as urgently required for subsistence. Even if both parties acknowledge an implicit contractual rate of \$100/12 hours, their valuations need not coincide. Both might consent and still possess different preferences over alternative exchanges. Consent itself has degrees of informational and institutional support. Moreover, B may be unable to decline under conditions of scarcity. The descriptions “accepted,” “contractually valid,” and “ethically justified” therefore require separate evidence and separate evaluation rules.

A different illustration concerns a public research result. A group can publish a non-rival mathematical argument. Multiple readers can access the result without proportionally reducing the authors’ stock of knowledge, although attention, verification, communication, and infrastructure remain scarce. This is a poor fit to the diffusion law later used for an additive conserved resource. It can instead be represented as a copying event with an origin record, an access condition, and potential recipient-specific transformations.

2.5 The observer’s epistemic problem

An observer of a circulation system commonly sees a proper subset of relevant information. Formal transaction records may show payments and receipts, while productive opportunities, constraints, informal obligations, and the interpretation of a received object remain latent or only partly elicitable. Different mechanisms can generate the same visible transfer history. For example, B may accept \$100 because it is a preferred exchange under available alternatives, because alternatives are absent, or because B is misinformed about the task. The payment record alone cannot distinguish these explanations.

We can represent epistemic access as an observation map, distinct from the participant’s actual local state. Equation 2 specifies the distinction.

$$y_i(t) = O_{i,t}(s_i(t), r_{0:t}) + \epsilon_i(t), \quad (2)$$

where $O_{i,t}$ is a context-dependent observation operator and ϵ_i stands for measurement or reporting uncertainty where a numerical model is meaningful. A qualitative representation might instead record a set of admissible descriptions. The existence of $O_{i,t}$ is a modeling assumption; a formal observer map must not be mistaken for permission to collect sensitive personal information.

A useful representation should distinguish an explicitly reported interpretation, an estimate made by a researcher, a contested assessment, and information that remains unavailable. The same numerical transfer can then support multiple local accounts without forcing them into a single scalar score.

2.6 Exchange, equivalence, and justice

A voluntarily accepted exchange can reveal that a particular action was chosen under a given set of opportunities and constraints. Acceptance alone does not establish a unique comparison of the transferred objects or the absence of coercive pressures. Market exchange ratios, accounting conventions, and institutional valuation rules provide forms of commensuration. Espeland and Stevens examine how such forms of commensuration are social processes with cognitive and political consequences [7]. The present inquiry therefore treats a conversion rule as an object of analysis when one is supplied, rather than as an intrinsic property of heterogeneous values.

The same caution applies to fairness. Equal monetary entries may have different consequences for participants with different needs. Equivalent opportunity costs may leave other forms of vulnerability unaddressed. Determining the relevant dimensions of equality is a substantive normative question, as illustrated by Sen’s discussion of the evaluative basis of equality [12]. These concerns motivate the separation of comparison relations from circulation dynamics in Section 5.

The remainder of the paper uses the word *resource* for a typed, modelled quantity or service event and *value* for a broader evaluative or relational question. This terminological separation is provisional, but it prevents a bookkeeping equality from being mistaken for an equality of value.

3 A Locally Structured Relational Representation

This section develops a small discrete model for typed circulation and local state change. The aim is to provide a representation sufficient for the constructed examples, with room for heterogeneous interpretation, rather than to prescribe an encompassing formalism. The section introduces local spaces, event records, admissible transfers, and an update rule, then situates the representation among existing graph and geometric approaches.

3.1 Nodes, states, and typed events

Let V be a finite set of nodes and let $E_t \subseteq V \times V$ be the directed connections available at time t . For each node $i \in V$, let F_i be a local state space and $s_i(t) \in F_i$ a state. The local spaces may differ in dimension and interpretation. They need not be manifolds or vector spaces. A state might contain

typed resource stocks, institutional permissions, task readiness, or records of previously received events. Under a particular application, some components may be unobservable.

Let K be a finite set of resource types for a toy model. For each $k \in K$, we can distinguish a nonnegative recorded amount $x_{ik}(t)$, when amounts are defined, from other aspects of $s_i(t)$. A transfer event is specified by a tuple containing origin, destination, resource type, amount, and time. Equation 3 fixes its notation.

$$r = (i, j, k, a, t), \quad (i, j) \in E_t, \quad k \in K, \quad a \geq 0. \quad (3)$$

The event record is descriptive and does not assign an economic or ethical value to a . For non-additive resources such as knowledge, the amount field may need to be replaced by a typed object, versioned artifact, or process descriptor. The additive notation will be used only in the explicitly conserved diffusion demonstration.

An admissibility relation $A_t(i, j, k, s_i, s_j)$ identifies transfers permitted under the chosen model. Its role is to express resource-specific connectivity, local readiness, capacity, or a stipulated barrier. It may also depend on rights, rules, or permissions if those can be operationally specified; an observed obstruction and a legitimate restriction should remain analytically distinguishable.

3.2 State changes and interpretive rules

We represent the effects of a local event history by an update map. Equation 4 expresses a general discrete-time form.

$$s_i(t+1) = \Phi_{i,t}(s_i(t), R_i^{\text{in}}(t), R_i^{\text{out}}(t), h_i(t)). \quad (4)$$

Here R_i^{in} and R_i^{out} are the collections of recorded events entering and leaving i during the interval; h_i contains stipulated or recorded context. The map $\Phi_{i,t}$ may encode conservation where justified, non-conservative transformation, task thresholds, or other specified effects. It should not be read as a psychology of human agents without further evidence. The recognition operator $\mathcal{E}_{i,t}$ in Equation 1 can be one part of this update or an associated interpretation of its inputs and effects.

A transmission interface may be represented as a partial map between typed components of neighboring local spaces. For example, an input accepted by B may activate a local task, whereas an ostensibly identical input to C may be ignored or rejected under different conditions. Such a map can be specified independently for each edge and resource type. This makes selection explicit without presuming that neighboring nodes share the same internal coordinates.

3.3 Typed inventories, event traces, and model signatures

For computations involving finite types, we separate additive inventories from the wider local state. Let K_{stock} index only those quantities with a defensible stock interpretation. Each node carries an inventory vector $x_i \in \mathbb{R}_+^{K_{\text{stock}}}$ and possibly nonnumeric variables z_i for capability, interpretation, rights, and history. Equation 5 defines a candidate state layout.

$$s_i = (x_i, z_i, h_i) \in F_i, \quad F_i \subseteq \mathbb{R}_+^{K_{\text{stock}}} \times Z_i \times H_i. \quad (5)$$

The product is a convenient data design, not a claim that human states factor into independent resources, interpretations, and histories. The local spaces Z_i and H_i may have different dimensions, categorical structure, or observational availability at each node. Even the meaning of a resource type may require a dictionary specifying units and admissible operations.

An event trace records identified source, destination, time of initiation, arrival time, type, quantity or artifact identifier, permission status, and transformation provenance. A service event may record start and completion, with an agreed service descriptor, while information copying records both persistence of the origin and creation of a recipient instance. The general event trace is an append-only description under an explicit logging convention. Such records are distinct from the inferred mechanism that produced them.

For a conserved type, feasibility of a proposed transfer can be expressed through constraints on inventory, rate, and edge capacity. Equation 6 provides one schematic admissible-flow set.

$$\mathcal{U}_t(x) = \left\{ f \geq 0 : \sum_j f_{ij}^k \leq x_{ik}, \quad \sum_k a_{ij}^k f_{ij}^k \leq c_{ij}(t), \quad f_{ij}^k = 0 \text{ if } (i, j, k) \notin \mathcal{P}_t \right\}. \quad (6)$$

Here a_{ij}^k is the capacity usage per unit, c_{ij} the stipulated edge capacity, and $\mathcal{P}_t$ the set of type-permitted directed movements. The inventory condition assumes no within-period receipt can be forwarded in the same step. Other scheduling conventions are possible and should be documented.

3.4 Conversions, replication, and coupled local change

Transport and transformation are different operators. In a local conversion process indexed by r , inputs and outputs may be recorded by a signed vector ν_r and nonnegative event extent ξ_r . Equation 7 gives a linear special case for genuinely measurable resource types.

$$x_i^+ = x_i^- + \sum_r \nu_r \xi_{ir}, \quad x_i^+ \geq 0. \quad (7)$$

For example, a local work process might consume a specified stock of electrical energy and materials while producing an item, and an associated service event can be recorded separately. The vector ν_r represents a stipulated transformation recipe in compatible engineering units. It does not imply that labor, trust, and knowledge are interconvertible at stable numerical coefficients. A copying operation for knowledge requires another operator, since preservation of the source alongside new access at the destination is inconsistent with simple one-unit debit-credit accounting.

A more general description uses typed partial maps between a domain of admissible inputs and local states. Equation 8 exhibits the possibility of local selection.

$$T_{ij,t}^k : D_{ij,t}^k \subseteq F_i \times F_j \times R_k \longrightarrow F_i \times F_j, \quad (8)$$

where R_k is the type- k event space and the domain specifies whether the recipient is prepared, authorized, or capable of accepting the event. The map need not conserve a common scalar, nor need it be invertible. If its domain is unknown, the mapping is an exploratory possibility rather than an empirically identified causal law.

3.5 Local uncertainty and partially specified mechanisms

In an early-stage inquiry, an observer often cannot specify a unique $\Phi_{i,t}$. One possible alternative is a set-valued transition correspondence $\Gamma_{i,t}$. Equation 9 retains this uncertainty.

$$s_i(t+1) \in \Gamma_{i,t}(s_i(t), R_i^{\text{in}}(t), R_i^{\text{out}}(t), h_i(t)). \quad (9)$$

Such models are familiar in variational analysis and mathematical optimization [11]. In this paper Γ is a device for retaining several possible consequences when the mechanism or state is incompletely known. If the true consequence is represented by a set, it is essential to distinguish a structural range of possibilities from an epistemic confidence or credible region: they answer different questions.

3.6 Connections with established network and geometric languages

A structured local graph may carry linear node spaces and restriction maps, in which case cellular sheaf methods become available for compatibility and diffusion [2]. Our broad F_i spaces do not necessarily form a sheaf. Moreover, the maps in Equation 8 can be irreversible, nonlinear, and state dependent. Resource theories of conversion are helpful for describing admissible processes, whereas graph transport

methods are more directly suited to additive quantities [8, 1]. This division of mathematical labor should remain visible.

For a smooth-base alternative, a state field might be a section of a bundle $\pi : E \rightarrow M$ with local fiber F_p . Movement from p along $m \in T_p M$ can be represented locally by $q = \exp_p(\varepsilon m)$ whenever an exponential map is defined in the chosen geometry. Relating F_p and F_q requires a transport rule. In general, local state transformations could also change effective movement costs, but changing a weight on a fixed network is weaker than proving a smooth metric evolution. A continuum limit would require a sequence of discretizations, a scaling regime, and convergence claims absent from the present simulations.

The model therefore treats geometry as an optional representation of accessibility and internal state, to be adopted when it adds testable or explanatory structure. A flexible mathematical language alone does not confer empirical identifiability.

3.7 Relation to geometric and sheaf representations

A smooth manifold M equipped with a local state space above each point suggests fiber-bundle language, and an infinitesimal displacement from p in a tangent direction $m \in T_p M$ may be expressed using a local chart or an exponential map when its hypotheses hold. Transport between different local fibers then requires an additional identification or connection. Such structures are mathematically established; the present examples contain neither empirical evidence of an underlying smooth manifold nor a need to assume one.

A discrete analogue can be expressed through local vector spaces and edge maps when the necessary linearity is appropriate. Cellular sheaves provide a developed framework for this kind of local compatibility, with sheaf-based diffusion studied in contemporary graph learning [2]. The general local spaces in the present model are intentionally broader and need not support linear sheaf operators. The relation is consequently methodological rather than a statement of equivalence.

Resource theories offer another relevant perspective: what can be transformed into what can be investigated through structured conversion relations, including preorders and compositional operations [8]. A resource transform and a participant’s normative judgment about its desirability remain distinct objects here. The choice among graph, sheaf, resource-theoretic, and geometric variants should follow the data and mechanism at issue.

The subsequent dynamics use the elementary graph special case because typed flows and conservation checks can be represented transparently. The flexibility of the general update rule should not be mistaken for explanatory power: specifying an arbitrary $\Phi_{i,t}$ can describe many trajectories without identifying why a particular one occurs.

4 Circulation, Temporal Order, and State Change

This section specifies elementary dynamical mechanisms available within the local representation. Its purpose is to show where the order of receipt, type-dependent connectivity, and feedback can enter the model. A conservation law provides a particularly transparent special case, followed by an optional conductance update. Such rules are modelling assumptions; different kinds of value may require distinct laws.

4.1 Typed stock balances

For a conserved and additively recorded resource type k , let $f_{ij}^k(t)$ denote the amount transferred from node i to node j during time interval t , and let $b_{ik}(t)$ represent a specified external source or sink. Under the convention that each interval’s transfers are completed before the next time index, a simple balance is given in Equation 10.

$$x_{ik}(t+1) = x_{ik}(t) + b_{ik}(t) + \sum_{j:(j,i) \in E_t} f_{ji}^k(t) - \sum_{j:(i,j) \in E_t} f_{ij}^k(t). \quad (10)$$

Flows are nonnegative and constrained by outgoing availability, connection capacity, and any type-selective admissibility rule. With $b_{ik} = 0$ and no cross-type conversion, summing Equation 10 across all nodes conserves the total of each resource type separately. This is a mathematical property of the specified model, not a universal law governing social value. A service, idea, or recognition event can change participants' capacities without behaving as an additively conserved stock.

Transport delays may be represented by events entering a transit state and arriving later. For the three-node example, the two paths A–B–C and A–C can deliver equal final quantities but different intermediate stocks. If a task at C is feasible only after the arrival of a resource before some deadline, the order has consequences for the modeled opportunity. A loss of opportunity is not equivalent to a morally attributable wrong unless the surrounding claims, alternatives, and responsibilities have been specified.

4.2 Diffusion and selective barriers

For an undirected graph with symmetric, nonnegative conductance $w_{ij}^k(t)$, a standard discrete diffusion special case assigns a net flow proportional to the difference in neighboring stocks. Equation 11 states the rule for one arbitrary orientation of each edge, with time step Δt .

$$f_{i \rightarrow j}^k(t) = w_{ij}^k(t)(x_{ik}(t) - x_{jk}(t)), \quad x_k(t+1) = x_k(t) - \Delta t L_{w^k(t)} x_k(t). \quad (11)$$

The signed first expression represents a net flux and may have either sign; it is consequently different from the nonnegative directed event amounts in Equation 10. L_{w^k} is the weighted graph Laplacian. For the explicit scheme, Δt is chosen sufficiently small to keep the stocks nonnegative. Type-specific conductances can represent selective openings and barriers. A zero conductance prevents modeled diffusion along that edge, irrespective of the difference between endpoint stocks.

Diffusion is only one candidate mechanism. In institutional circulation, participants can make decisions, reserve resources, distinguish obligations, and intentionally route transfers against a concentration gradient. Directed flow control, multicommodity capacity constraints, and state-dependent action rules would generally be needed to represent such cases. Dynamic optimal transport on networks provides a relevant established mathematical setting [4].

4.3 Feedback and effective transport geometry

To examine a narrow form of geometry change, the numerical illustration updates edge conductances according to the magnitude of recent modeled flux. Equation 12 describes the stipulated feedback, where $w_{ij,0}^k$ is a baseline, η the adaptation coefficient, and λ a relaxation coefficient.

$$w_{ij}^k(t+1) = \text{clip}_{[0, w_{\max}]} \left[w_{ij}^k(t) + \eta \Delta t |f_{i \rightarrow j}^k(t)| - \lambda (w_{ij}^k(t) - w_{ij,0}^k) \right]. \quad (12)$$

In the implemented simulation, edges with zero baseline conductance are constrained to remain closed. For positive edges, increased flux can increase conductance up to a cap. This functional form has been selected for the demonstration and is not fitted to a physical or economic process. More complicated adaptation rules could include wear, congestion, antagonism, institutional learning, or direct changes to edge existence.

Given positive conductances, a derived edge cost $c_{ij}^k(t) = 1/w_{ij}^k(t)$ permits the calculation of shortest-path transport cost. A change in this distance represents a change in *effective network geometry*, in a restricted weighted-graph sense. The graph's vertices, edges, embedding coordinates, and topology remain fixed. The local spaces F_i also remain unchanged in the present numerical diffusion experiment. Claims about curvature of a smooth manifold or changing physical spatial geometry would require a different model.

4.4 Incidence structure, conservation, and transit stocks

The stock law becomes clearer in matrix form. Let $B_t \in \{-1, 0, 1\}^{|V| \times |E_t|}$ be a directed incidence matrix with -1 at the sending vertex and $+1$ at the receiving vertex. For a separately conserved type k , the inventory vector obeys Equation 13 when transfers complete within the interval.

$$x^k(t+1) = x^k(t) + B_t f^k(t) + b^k(t). \quad (13)$$

Because each column of B_t sums to zero, the internal transfer term vanishes on summation. With $b^k = 0$, total recorded stock is constant. The identity depends on type-specific units and a correctly specified system boundary. Even when the accounting identity holds, an unmodeled use or loss can create a discrepancy between the recorded and material worlds.

For transfers with delay, an immediate deposit at the destination would be premature. The quantity should instead enter an in-transit account. Let $d_e^k(t)$ denote dispatch onto edge e , and let $a_e^k(t)$ be its arrival at the destination. If $z_e^k(t)$ denotes the in-transit stock, Equation 14 gives the elementary conservation rule.

$$\begin{aligned} x_i^k(t+1) &= x_i^k(t) - \sum_{e: \text{src}(e)=i} d_e^k(t) + \sum_{e: \text{dst}(e)=i} a_e^k(t), \\ z_e^k(t+1) &= z_e^k(t) + d_e^k(t) - a_e^k(t). \end{aligned} \quad (14)$$

For a closed network $\sum_i x_i^k + \sum_e z_e^k$ remains constant, provided dispatch, arrival, and update timing follow one convention and in-transit amounts remain nonnegative. The destination's inventory can be zero before an arrival even while the transfer has left the sender. This distinction is important whenever deadlines, queueing, custody, or responsibility for losses in transit are studied.

Proposition 4.1 (Conservation of a typed inventory). *Under a finite incidence graph, zero external source, complete accounting for transfers in transit, and no conversion, a type- k stock is conserved across nodes and transit accounts.*

Argument. The statement of Proposition 4.1 follows by summing the two lines of Equation 14, which cancels each dispatch and arrival exactly once. The result is a bookkeeping identity derived from the stated assumptions. It says nothing about the value, social significance, or physical validity of the type.

4.5 Local conversion and generative opportunity

When activities transform resources, the stock balance can include the local conversion term from Equation 7, as in Equation 15.

$$x^k(t+1) = x^k(t) + B_t f^k(t) + b^k(t) + \sum_r \nu_r^k \xi_r(t). \quad (15)$$

The conversion term depends on measurable processes and an explicit choice of accounting boundary. Materials may be converted to products; a service may produce a completed task without being meaningfully represented as transfer of a conserved labor stock. A value related to know-how may enable several future conversions without being depleted when used. For these latter cases a typed event rule is more suitable than Equation 15.

Let $g_i(t)$ be a task-specific indicator of a node's prospective capacity for a stipulated process, with all underlying assumptions stated. A stylized update can include capability depreciation and a nonlinear increment, as shown in Equation 16.

$$g_i(t+1) = (1 - \delta_i)g_i(t) + \varphi_i(x_i(t), z_i(t), R_i^{\text{in}}(t)). \quad (16)$$

The function φ_i may activate only after several necessary inputs are simultaneously available, and δ_i is a stipulated decay parameter when appropriate. In practice, translating future generative capacity to

a scalar would require a defensible operational definition and observations over relevant time horizons. The existence of a scalar g_i is a modeling convenience, not a settled definition of generativity.

Timing matters because a threshold may be attached to a deadline or because missed opportunities can affect later states. For instance, C may require six units before time one to initiate a task; receiving eight units at time two restores its inventory but may not restore the elapsed opportunity. Conversely, B may require an immediate allocation to maintain the intermediary pathway, in which case an apparent shortcut to C could impair future circulation. Both effects can be represented, though the relative urgency and legitimacy of claims require further study.

4.6 Discrete diffusion and sufficient positivity conditions

The weighted graph Laplacian L_w has diagonal entries equal to weighted degree and off-diagonal entries $-w_{ij}$ for adjacent nodes. The explicit Euler diffusion step in Equation 11 can be written as a linear map. If the time step obeys Equation 17, each updated stock is a nonnegative weighted combination of the previous stocks for a closed undirected graph with $w \geq 0$.

$$\Delta t \max_i \left(\sum_j w_{ij}^k(t) \right) \leq 1, \quad \text{for every represented type } k. \quad (17)$$

Indeed, the diagonal coefficient equals $1 - \Delta t \sum_j w_{ij}^k$ and neighbor coefficients equal $\Delta t w_{ij}^k$. All coefficients are nonnegative and each row sums to one. For symmetric conductances the global sum is preserved. This is a sufficient numerical condition, rather than an empirical law of circulation. It can be enforced under conductance feedback by checking the degree bound at each time step.

Network diffusion has limitations even before value-related interpretation. It tends to follow modeled gradients, while persons and institutions may deliberately route resources toward a low-stock destination, retain an emergency reserve, negotiate permissions, or create new pathways. The same resource may carry a different usefulness at different nodes. A diffusion model therefore helps investigate selective accessibility and baseline transport, while a policy model is needed for purposive allocation. Open Markov and reaction-network methods also illustrate why equilibrium and through-flow must be distinguished, particularly when boundary sources and sinks are present [10].

4.7 Access geometry and distinct kinds of change

A type-indexed edge cost can be derived from positive conductance, for example $c_e^k = 1/w_e^k$, and the corresponding path cost is the sum across an admissible route. Equation 18 defines this graph-derived accessibility measure.

$$d_k(i, j; t) = \inf_{\gamma \in \mathcal{P}_{ij}^k(t)} \sum_{e \in \gamma} c_e^k(t), \quad c_e^k(t) > 0. \quad (18)$$

If there is no permitted path, the distance is defined as $+\infty$. When c is symmetric and the graph is connected, the construction is a standard path metric on vertices; on directed or disconnected structures, the result may be asymmetric or extended-valued. The term *geometry change* refers here to changing effective accessibility. It should be distinguished from embedding deformation, local fiber transformation, and topological rewiring.

An addition of a permitted path can weakly decrease the shortest-path cost between any two existing nodes, provided original edge costs remain fixed and nonnegative. This elementary monotonicity property follows because the previous paths remain feasible after an additional path is allowed. It cannot establish improvement under a fairness or sufficiency criterion, since a new path can impose costs, redirect control, or carry unwanted material. It also ceases to apply in that elementary form when edge costs themselves depend on congestion and other users' decisions.

The feedback process in Equation 12 supplies a different mechanism: weights change while the adjacency pattern is fixed. The numerical experiment includes a parameter sensitivity check because a

chosen feedback coefficient can strongly influence the magnitude of the effect. The assumption that flux strengthens future conductance is particularly fragile as a social analogy: repeated use can also congest, deteriorate, monopolize, or destabilize an interface. A balanced research programme should compare positive, zero, and negative feedback under identical task conditions.

4.8 An optional evolving metric on local state spaces

The preceding simulations alter transport weights on graph edges. The original motivating idea also concerned geometry *within* a point's state space. We can construct a separate, deliberately invented example by treating a local two-component additive inventory as coordinates on $F_i = \mathbb{R}_+^2$. Let the coordinates be (x_i^α, x_i^β) . A state-dependent positive-definite metric is specified by Equation 19.

$$g_i(t) = \begin{pmatrix} (1 + \gamma x_i^\alpha(t)/100)^{-2} & 0 \\ 0 & (1 + \gamma x_i^\beta(t)/100)^{-2} \end{pmatrix}, \quad \gamma \geq 0. \quad (19)$$

This illustrative rule makes a unit change along a locally abundant coordinate have a smaller squared tangent cost than the same coordinate change when that resource is scarce. The metric depends on the state and changes as the diffusion system evolves. The numerical example fixes $\gamma = 2.5$. Each matrix remains positive definite on F_i and defines an instantaneous local inner product. It represents a chosen sensitivity or accessibility convention; the physical or ethical interpretation of that convention has not been established.

Unlike updating the graph conductance, updating $g_i(t)$ in Equation 19 changes the local internal geometry. Yet the resulting geometry is still stipulated by a stock-dependent formula, and a plot of metric unit balls does not establish a curvature effect, phase transition, or neural mechanism. In particular, a metric can be proposed without supplying an empirically warranted way to compare coordinates across heterogeneous participants. The two geometrical variants should therefore be tested and criticized separately.

4.9 Capacity change and explanatory boundaries

A local task may require a threshold in an observable input or a combination of inputs. Let g_i denote a tentative capacity indicator only when a concrete task and measurement procedure are supplied. A threshold such as $x_{ik}(t) \geq \theta_i$ can gate a task, but the size of g_i and its ethical meaning remain additional modelling choices. The experiments distinguish feasibility of a task from its desirability and from the participant's own assessment of it.

The key point of the proposed dynamics is limited: temporal order and selective connectivity can alter modeled opportunities even when aggregate resource totals are unchanged. That statement follows from the stipulated rules. Whether these rules approximate social circulation is an empirical and interpretive question reserved for future work.

5 Relations of Comparison and Incomparability

This section investigates how the representation might express comparisons among heterogeneous states or transfers. Its analytical role is to prevent a typed record or a transport trajectory from silently becoming an evaluative ranking. Equality within a measurement type, equivalence under a stipulated transformation, similarity under a selected metric, and partial normative ordering are treated separately.

5.1 Typed equality and specified equivalence

Within one defined quantity space, equality can describe identical amounts, records, or objects under a specified identity convention. For example, the monetary amount of an outgoing entry at A may coincide with that of the corresponding incoming entry at B. There is no parallel intrinsic identity between a dollar

quantity and a number of hours. Cross-type equivalence requires a convention or relation, such as a contractual exchange rate, a permissible conversion process, or an explicitly justified evaluative rule. These conventions may vary across institutions, historical periods, and participants.

An equivalence relation $\sim_\alpha$ can be declared relative to a context α if the intended domain and axioms are specified. It may express substitutability for a particular task, equality in an account, or practical indifference under particular assumptions. The scope of the resulting assertion is the chosen relation. A statement that two transfers are equivalent under an accounting convention should not be read as a claim of equal social or moral significance.

5.2 Comparison spaces and direct relations

Suppose two objects belong to heterogeneous spaces F_A and F_B . A comparison context α might supply a third ordered space C_α together with explicitly chosen maps. Equation 20 specifies this possible construction.

$$\psi_A^\alpha : F_A \longrightarrow C_\alpha, \quad \psi_B^\alpha : F_B \longrightarrow C_\alpha. \quad (20)$$

If both maps and an order on C_α are justified, their images can be compared according to that order. Such maps may be many-to-one, lossy, uncertain, contested, or unavailable. The image space represents the particular comparison selected, rather than a universal dimension of value.

A different construction defines a relation directly between local spaces: $R_{AB}^\alpha \subseteq F_A \times F_B$. It can represent whether a pair is mutually usable, institutionally convertible, or comparable by a stated criterion without first mapping both objects to a numerical magnitude. Resource-theoretic approaches supply established examples of conversion relations and structured resource comparisons [8]. At this stage, the model does not choose universally between comparison maps and direct relations.

5.3 Preorders and missing comparisons

When an evaluative relation $\succeq_\alpha$ on a set of alternatives is reflexive and transitive, it is a preorder. It can express weak preference or weak dominance under explicit assumptions. Equation 21 distinguishes the induced equivalence, strict part, and incomparability relation.

$$\begin{aligned} x \sim_\alpha y &\iff (x \succeq_\alpha y) \wedge (y \succeq_\alpha x), \\ x \succ_\alpha y &\iff (x \succeq_\alpha y) \wedge \neg(y \succeq_\alpha x), \\ x \parallel_\alpha y &\iff \neg(x \succeq_\alpha y) \wedge \neg(y \succeq_\alpha x). \end{aligned} \quad (21)$$

Incomparability under a declared preorder does not automatically mean that comparison is impossible in every respect. A researcher may also fail to establish a relation because evidence is missing. That latter case is better represented as an *epistemically unresolved* status rather than as an ontological assertion of incomparability. A completed model would need to distinguish a demonstrated absence of ordering under its rules from an unknown ordering.

Similarity is distinct again. A metric, divergence, or other similarity function requires a domain and structural assumptions that may be inappropriate across different value types. A small distance under a selected representation does not necessarily entail indifference, substitutability, or ethical equivalence.

5.4 Four distinct comparative tasks

The question “does input equal output?” can refer to at least four operations. First, *incidence equality* checks whether the debit recorded for a transfer matches its credit in the same type and unit. Second, *contractual conversion* records a mutually specified exchange relation such as a stipulated dollars-per-hour term. Third, *functional substitutability* asks whether two resources can satisfy a particular task under the recipient’s available technology and constraints. Fourth, *normative comparison* asks which configuration is ethically, legally, or institutionally preferable. A positive result at one level supplies no general logical implication for the others.

A distance or similarity score supplies a fifth, mathematically different kind of comparison. It may register how alike two recorded profiles are in selected coordinates. A metric is silent about whether near profiles are equally desirable or morally interchangeable. Similarity can also be undefined across incompatible units unless a feature representation and weighting scheme are chosen. This makes the epistemic status of dimension-reduction or embedding methods particularly important.

5.5 Observation-relative comparisons

An evaluation can be specified for a fully identified local state or for an observer’s uncertainty about that state. Suppose $S_i(y)$ is the set of states consistent with the available observation y . A criterion J_i on those states induces an identified image set, as in Equation 22.

$$\mathcal{J}_i(y) = \{J_i(s) : s \in S_i(y)\}. \quad (22)$$

If $\mathcal{J}_i(y)$ contains several outcomes, a ranking based on a single imputed state may be unjustifiably precise. A robust comparison could require an alternative to perform at least as well for all admissible states, while a weaker, possible comparison might require just one. The two statements have quite different meanings, and neither is automatically ethically preferred. Set-valued analysis supplies a general mathematical vocabulary for correspondences and partial identification [11].

For a qualitative evaluation one could preserve interpreted accounts and explicit reasons, rather than convert every description into a fixed number. The inability to obtain an ordered representation may then be an important substantive result: the comparison is undefined under the available information or rejected by a participant’s legitimate evaluative commitments.

5.6 A worked example of contextual equivalence

Suppose B needs six units of a specified material to begin a task. Receiving six units of that material and receiving eight units both meet the eligibility predicate. Under the task-feasibility criterion, the states are equivalent. In an inventory account the quantities remain different. If additional stock helps later tasks, a longer-horizon evaluation may distinguish them again. Thus a single ordered criterion can identify a coarse equivalence class without implying equality of resources, overall welfare, or future prospects.

The money–labor case is analogous but more difficult. A contract may establish the accepted ratio between a payment and labor service for one transaction; an external observer can reproduce the ledger without thereby identifying a welfare equivalence for either person. The selection of a social comparison space can require judgments about rights, bargaining power, opportunity, and historically accumulated disadvantages. Any such space should therefore carry provenance, versioning, an explicit domain of use, and a means of recording disagreement.

5.7 The evaluator as a participant in the model

Comparison procedures have owners, revision processes, and effects. An authority choosing a metric may influence whether a contribution is recorded, whether a resource is considered fungible, and which trajectories appear efficient. The act of scoring can thereby alter future incentives and, in some contexts, the relation that produces the scored value. This concern follows from the social analysis of commensuration [7]; it is especially relevant to a system that is explicitly dynamic.

We may therefore associate a criterion with a tuple describing its authorizing procedure, evidentiary basis, scope, version, and contestability. This is a prospective documentation practice, not a solved formal representation of political legitimacy. It permits the mathematical model to retain which comparisons were assumed, which were negotiated, and which remain unresolved.

5.8 The politics and history of comparison

The use of a common metric can alter the institutional meaning of a transaction. Commensuration is therefore more than a technical reduction of variables; it is a social process with possible power effects [7]. Participants may differ in their ability to specify the conversion rule, challenge a metric, or preserve unrecognized dimensions of their contributions. These differences matter even when the mathematical comparison is internally well-defined.

The proposed formalism can expose comparison assumptions by recording their domain, origin, purposes, time, and evidentiary basis. It cannot by itself justify those assumptions. This boundary is especially relevant when a comparison becomes part of optimization, because an apparently neutral objective may inherit the normative choices made in constructing its comparison space.

6 Candidate Criteria for Circulation Optimization

This section distinguishes possible evaluation criteria for complete circulation trajectories. Its aim is to explain why equilibrium, balanced accounting, task feasibility, generative capacity, and normative legitimacy need separate treatment. The discussion introduces a plural comparison of trajectories without proposing a universally valid notion of good optimization.

6.1 Stability, balance, and opportunity

A stationary stock distribution is an equilibrium relative to specified flow rules and boundary conditions. Such a condition can aid analysis of persistence or system stability. Its attainment has no immediate normative implication. A stable allocation could maintain deprivation or support productive cooperation, depending on the setting.

Likewise, a balanced ledger describes entries that satisfy a chosen accounting identity. If one participant provides money and receives labour, the paired records can each be balanced within type even when the parties disagree about equivalent benefit. A proposed local reciprocity criterion would require additional rules for legitimate expectations and an explicit account of which value dimensions are relevant. Strict equality of incoming and outgoing amounts is not generally meaningful for cross-type transfers, care, public goods, or gifts.

A future-generativity criterion adds another object of interest. If a task is feasible only when a resource arrives before a deadline, a trajectory may preserve an opportunity that another trajectory forecloses. The value assigned to that opportunity depends on the task, the affected participants, competing needs, and the uncertainty of its future effects. A capacity metric can be informative within a specified task yet incomplete as an ethical criterion.

6.2 Plural trajectory orders

Let $\mathcal{X}$ be a set of feasible trajectories under a specified dynamical model and time horizon. Each evaluation context $\alpha \in \mathcal{A}$ may provide an ordered output Q_α and a mapping $J_\alpha : \mathcal{X} \rightarrow Q_\alpha$. Equation 23 defines a possible joint dominance relation when each constituent order is available.

$$X \succeq_{\mathcal{A}} Y \iff J_\alpha(X) \succeq_\alpha J_\alpha(Y) \text{ for all } \alpha \in \mathcal{A}. \quad (23)$$

Under the usual preorder assumptions for each $\succeq_\alpha$, the joint relation is a preorder. It can leave trajectories incomparable. In a fully numerical case this construction is related to the established study of Pareto dominance and multicriteria optimization [6]. The selection of $\mathcal{A}$, the measurement maps, and the admissibility conditions are substantive choices. A Pareto-efficient trajectory may still be ethically unacceptable.

The relevant criteria might include resource availability at deadlines, continuing access to a basic task, disruption imposed on intermediaries, ecological effects, autonomy, or participants' opportunities

to contest a rule. The model would need evidence and argument before assigning these measures to a real setting. Aggregation into a single scalar requires a further rationale, including rules governing tradeoffs among differently situated participants.

6.3 Justice as a separate level of argument

The concept of justice may enter the model through rights-based constraints, procedural requirements, sufficiency claims, legitimate expectations, or historically situated demands for recognition. None follows solely from the existence of a path or the value of its throughput. Sen’s discussion of competing dimensions of equality is pertinent because a comparison of persons changes with the chosen evaluative space [12]. The same issue applies to a comparison of circulation trajectories: the mathematical objective reflects a preceding choice concerning what is worth comparing.

One could adopt an admissible set $\mathcal{X}_{\text{adm}} \subseteq \mathcal{X}$ representing a justified set of constraints, then seek nondominated trajectories within it. This may be appropriate when the rights or conditions are sufficiently specified. The model remains silent about who has authority to choose the constraints, whether the requirements are legitimate, and how conflicts should be adjudicated. These are ethical, legal, and institutional questions for separate investigation.

6.4 Feasible trajectories and alternative objectives

The elementary optimization problem begins with a feasible trajectory set determined by the chosen dynamics, capacity limits, admissibility rules, and relevant initial conditions. Let X collect time-indexed stocks, dispatches, arrivals, transformations, and local states. Let $\mathcal{X}_T$ denote the resulting feasible histories over a specified horizon T . A designer may then select a criterion J and seek an optimizing trajectory. Equation 24 records the distinction between feasibility and objective.

$$X^* \in \arg \min_{X \in \mathcal{X}_T} J(X). \quad (24)$$

For a network-flow special case, J may penalize transport cost, delay, unused capacity, or inventory shortfall. Standard network and convex optimization supply developed methods for such problems [1, 3]. By contrast, a criterion that quantifies the generative potential of human subjects would need an independently justified operationalization. A mathematically solvable objective is not, by that fact, the right objective.

Three different criteria may lead to different policies. A throughput objective prefers larger delivered amounts by a horizon. A timing objective may prefer an expensive direct edge if an urgent deadline can be met. A reciprocity objective may consider burdens and returns to intermediaries. Additional criteria may concern sufficient resources, autonomy, ecological effects, or the ability of participants to contest transfer rules. Some admit scalar representations in a constrained toy problem; others may remain qualitative or partial.

6.5 Incompatible local sufficiency thresholds

Consider two recipients B and C sharing eight divisible units, with hypothetical task thresholds of five and six units respectively. If each must receive its threshold simultaneously, the required total exceeds the available eight units. The condition cannot be satisfied under the stipulated resource constraint. No optimizer can satisfy both requirements without an additional resource source, a changed task, a later time horizon, or a permissible transformation. This is a feasibility issue rather than a failure of a particular algorithm.

If the designer maximizes the number of activated tasks, either B or C can receive priority and the other task remains inactive. If the criterion is the maximal minimum proportion of each threshold achieved, it may spread resources differently. If a rights-based constraint protects an individual claim, some of the mathematically feasible allocations may become inadmissible. These cases illustrate how

a formal model can make normative disagreements and resource scarcity explicit while leaving their resolution open.

6.6 Partial dominance and incomparable alternatives

Suppose a trajectory is described by two numerical attributes: resource supplied to B and resource supplied to C. For eight units divided between them, increasing one decreases the other. Every point on the fully utilized allocation frontier is nondominated under coordinatewise more-is-better comparison, and the order offers no unique solution. The Pareto vocabulary identifies such limits of ordering, as established in multicriteria optimization [6]. A scalar weighted sum would select among points only after weights were chosen, while a procedural allocation rule could select without attempting an interpersonal utility sum.

For broader value types, the evaluation outputs may belong to different partially ordered spaces, as in Equation 23. Even if one criterion ranks two trajectories, another may leave them incomparable. This warrants careful distinction between a failure of identification, an explicitly incomplete evaluative relation, and a conflict between two complete rankings. Each leads to different requirements for further evidence or institutional judgment.

6.7 Structural choice and feedback from optimization

One may also choose to modify the admissible network, creating or removing a connection or changing local capacities. Such a design problem has a feasible set over graph structures as well as flows. Any comparison needs to include the cost of construction, control over new interfaces, disruption to existing relationships, and future changes in access. An attractive direct path in the three-node toy model is thus a counterfactual intervention whose practical permission and expense are left unspecified.

If circulation changes future edge conductance, the optimization problem becomes coupled to state evolution. A short-horizon path may increase transport accessibility but deplete an intermediary's capacity, or a longer path may develop useful future infrastructure. Horizon selection and uncertainty over long-run outcomes can therefore change a recommended policy. These are ordinary issues of dynamic control in mathematical form; their application to justice still requires a separate account of legitimate objectives and constraints.

6.8 Criteria as evolving relational objects

Comparison rules may themselves evolve. An institution can change how it prices labour, recognizes contributions, or defines eligibility. Participants may dispute the existing metric, withdraw from a transaction, or establish an alternative comparison practice. A time-indexed family of criteria could record such developments, but the legitimacy of the process would remain to be assessed on its own terms. The present paper therefore proposes a description of potential evaluation structures rather than an optimization procedure guaranteed to find a just outcome.

7 Constructed Examples and Representational Boundaries

This section reports small numerical experiments designed to make the earlier modelling choices inspectable. The first checks typed transaction records and a deadline-sensitive circulation path. The second examines conservative two-resource diffusion through a selective barrier and a stipulated adaptation rule. Both simulations are deterministic, synthetic, and uncalibrated. Numerical observations below are conditional consequences of the chosen equations and parameters, rather than estimates of a social mechanism. The accompanying Python source and machine-readable results make the statements reproducible.

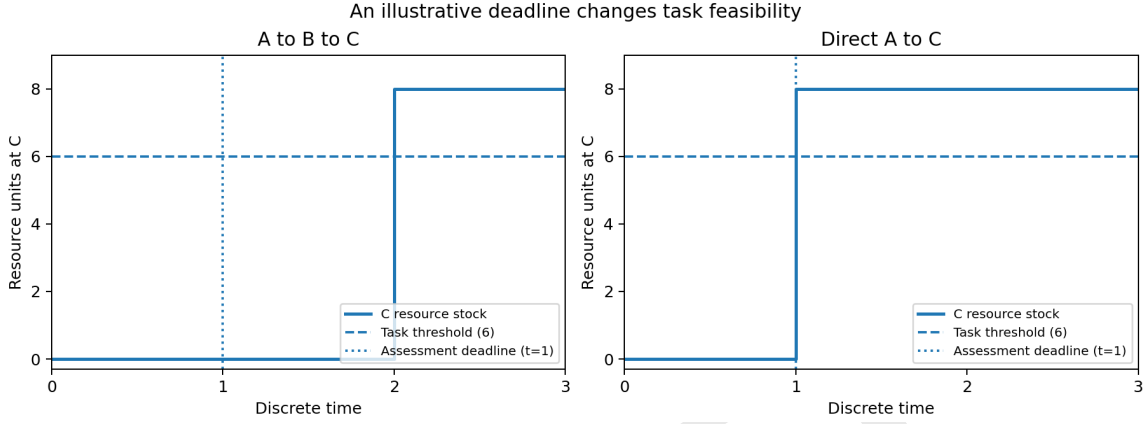


Figure 2: Constructed deadline-dependent task feasibility. Both pathways deliver eight units to C by $t = 2$, but only the direct path supplies C by the stipulated assessment time $t = 1$. All assumptions, including B’s ability to forward resources without delay beyond one interval, are chosen for illustration.

7.1 Bilateral exchange and typed bookkeeping

For the constructed exchange described in Section 2, A sends B \$100 and B supplies A with twelve hours of labour service. In the type-specific incidence ledger, the sum of monetary inflow minus outflow is zero and the sum of service-hours received minus supplied is also zero. Those two bookkeeping checks are performed independently. The calculation does not add dollars to hours, infer their equality, or treat labour effort as a conserved physical substance. Figure 1 represents the exchange; the accompanying `heterogeneous_ledger.csv` stores the separate entries.

To assess the agreement, one would need information about its circumstances, participants’ alternatives, task conditions, and their own descriptions of benefit or burden. Our ledger provides none of those elements. It therefore illustrates the distinction between the successful recording of heterogeneous transfers and an evaluative finding about the exchange.

7.2 A deadline-sensitive three-node path

The second constructed experiment starts with eight units of one conserved abstract resource at A, and zero units at B and C. C can begin a stipulated task if it holds at least six units at time $t = 1$. Under the path A–B–C, A transfers eight units to B during the first interval and B forwards the units to C during the second. Under an alternative graph with direct edge A–C, A transfers eight units directly to C during the first interval. Figure 2 shows C’s resource stock over time for the two trajectories.

The path through B delivers C’s stock at $t = 1$ as zero; direct transfer delivers eight units at $t = 1$. The corresponding task predicate is false for the first path and true for the second. Both trajectories contain eight units in total at each time. This constructed contrast illustrates that a deadline can make the ordering of transfers relevant even when total conserved resources and eventual delivery coincide.

The example does not establish that the direct edge is available, costless, permissible, or normatively preferable. B could have a claim to the resource or perform an indispensable transformation that the simple model omits. The condition “task feasible” also stands apart from evidence that the task would have produced a particular amount of future value. The example thus identifies a modelling question, rather than an empirical relationship between flow order and generative justice.

7.3 Two-type selective diffusion

The third experiment uses a fixed rectangular graph containing $13 \times 9 = 117$ nodes. Two additive resource types, α and β , each begin with total stock 100 concentrated at a different source node. A separating barrier has two openings: one has high conductance for α and low conductance for β , while

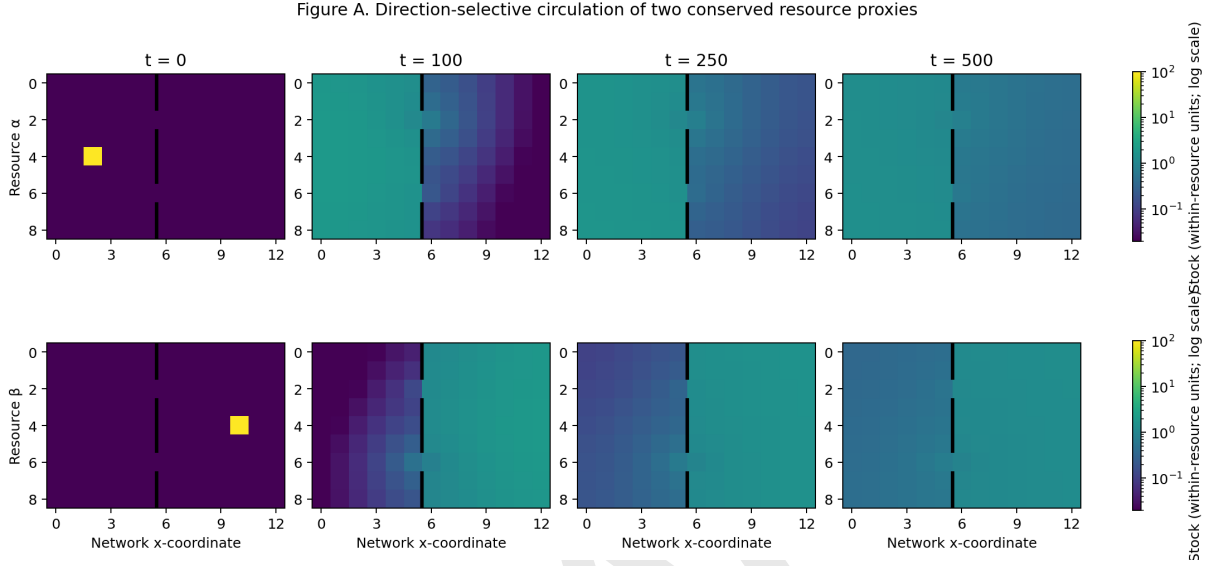


Figure 3: Two-resource diffusion on a fixed grid with type-selective openings, shown at four discrete times. A conserved-resource toy model produces different spatial distributions for types α and β under stipulated conductances.

the other reverses these type-specific values. Other crossings are closed. All interior edges begin with conductance 0.42 for both resource types. The time step is $\Delta t = 0.16$, and 500 updates are performed. These values are experimental settings rather than calibrated measurements.

Figure 3 records snapshots of both resource stocks under the adaptive conductance rule of Equation 12. The colors represent stock magnitude within each type on a logarithmic scale. Differences between the distributions result from source locations, initial conductances, and the stipulated update rule. The figure should not be read as a depiction of social actors, market transactions, human neurons, or a physical manifold.

We compare static conductances with an adaptive rule using $\eta = 0.014$, $\lambda = 0.0012$, and a cap of 1.30 for each edge conductance; originally closed edges remain closed. Figure 4 plots the share of each initial resource stock found across the barrier. At the final step, the share of α on the opposite side is 27.22% with fixed conductances and 30.97% with adaptation. Corresponding shares for β are 22.15% and 24.88%. The direction of these differences is an expected effect of the chosen positive flux feedback under these parameters. It is not evidence that adaptive institutional pathways generally improve welfare or justice.

The reported largest absolute mass drift is below 6×10^{-14} units across all time steps and both variants; numerical stocks remain nonnegative. These checks establish appropriate numerical behaviour for the toy conservation law. They do not validate interpretation of resources or parameter selection in an empirical population.

7.4 Receipt timing with explicit transit delay

The original three-node calculation treats each edge traversal as one interval. To expose the timing assumption, an additional script uses an arrival-time description over four observed intervals: an indirect path reaches C at time two; an allowed direct edge reaches C at time one; and a waiting policy reaches C at time three. Each delivers eight abstract units eventually, while only the direct route meets the stipulated six-unit threshold at time one. Figure 5 reports the three destination stock curves. The delay representation is equivalent to a transit queue in the sense that the resource leaves its source before becoming available to the destination; the figure plots recipient inventory rather than all system stock.

For a complete conservation check, the remaining units reside with A, B, or in transit according to the schedule. In particular, the indirect route can have no units at C at time one without implying that eight

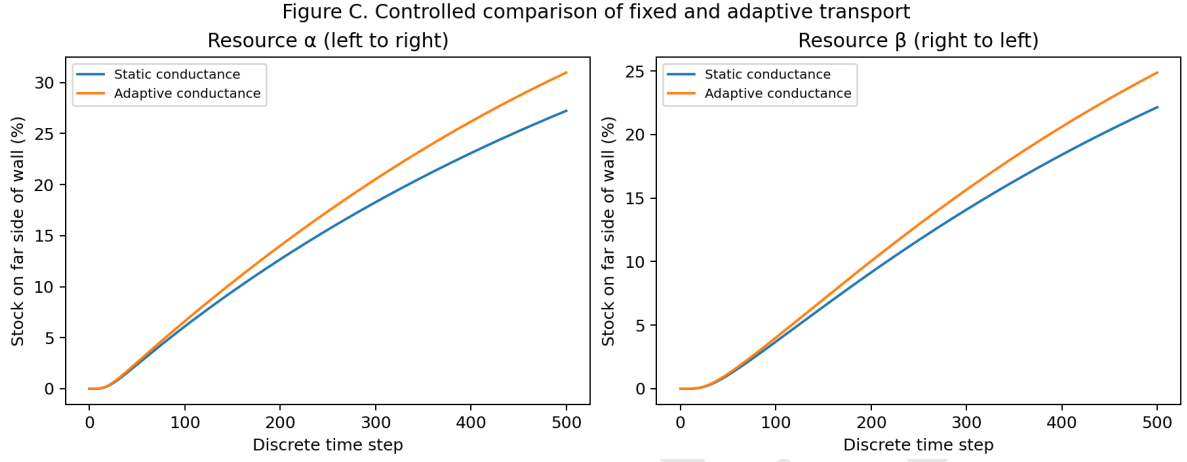


Figure 4: Cross-barrier fractions of each conserved resource with static and flux-adaptive conductances. Percentages are fractions of each type’s initial total stock, under two stipulated deterministic variants.

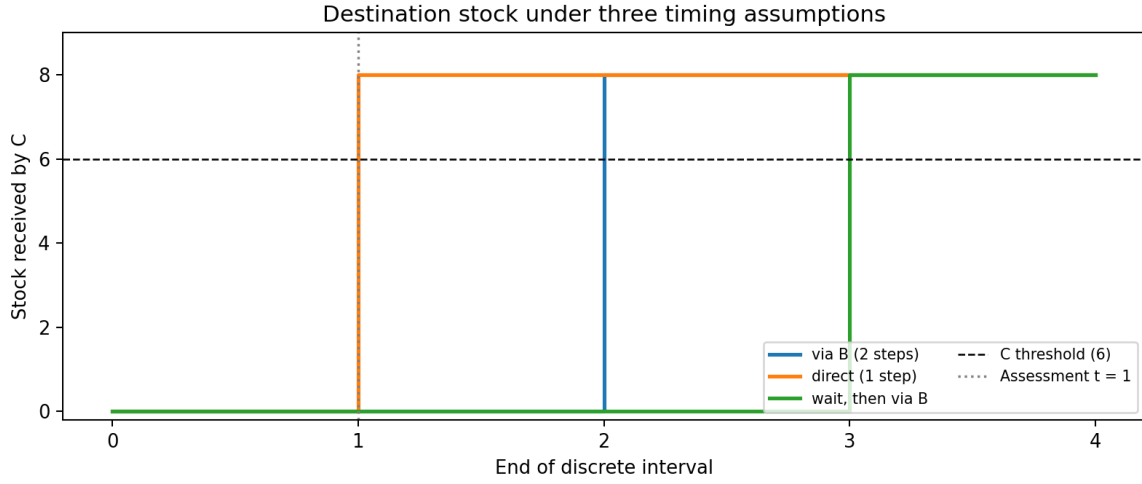


Figure 5: Destination availability under three constructed transfer schedules with equal eventual delivery. The threshold at time one is a stipulated task requirement; the graph ignores legitimacy and costs of adding or using a shortcut.

units have disappeared. This is why the update in Equation 14 includes an edge inventory. A schedule optimizing early receipt may also require extra capacity or incur greater transfer cost. The constructed event traces contain no evidence about those competing considerations.

7.5 Local epistemic context and latent task thresholds

The next illustration separates a transfer quantity from its local consequence. Two imagined recipients receive the same five units of an otherwise identical resource. The first has a task activation threshold of two units, and the second has a threshold of seven units. Under the stipulated response function $r(q; \theta) = \max\{0, q - \theta\}$, their numerical task-output indices at receipt of five units are three and zero. Figure 6 displays both response curves and a second panel of hypothetical intervals within which an outside observer might believe local thresholds lie.

The curves are deliberately simple and discontinuous in derivative at their thresholds. They cannot establish that social generativity follows a threshold law. They demonstrate a narrower representational fact: knowledge of a transfer record alone is insufficient to infer a local response when the receiving context matters. An observer with only an uncertainty interval for the threshold can derive a range

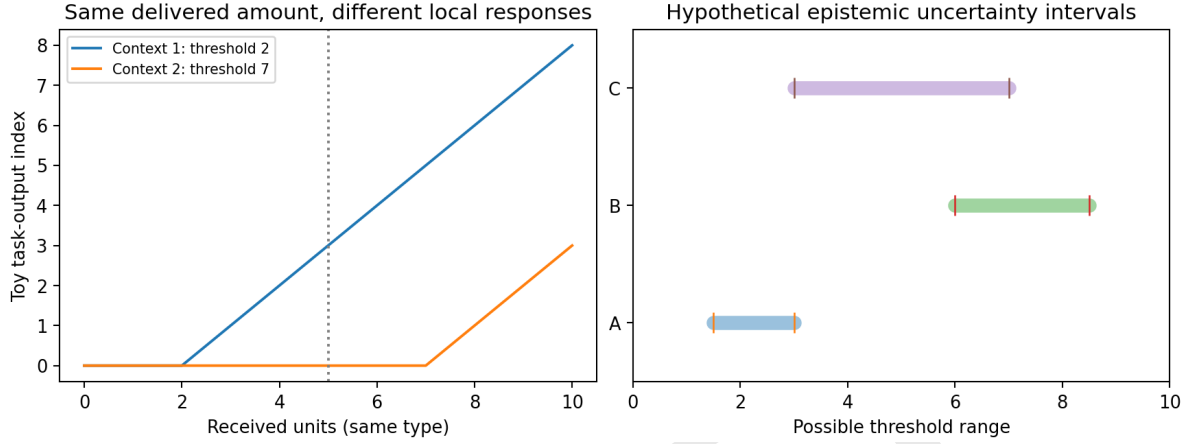


Figure 6: Two stipulated local response laws and separate illustrative observer uncertainty intervals. Identical recorded input may be associated with different task responses; the intervals are constructed epistemic inputs, not statistically estimated confidence regions.

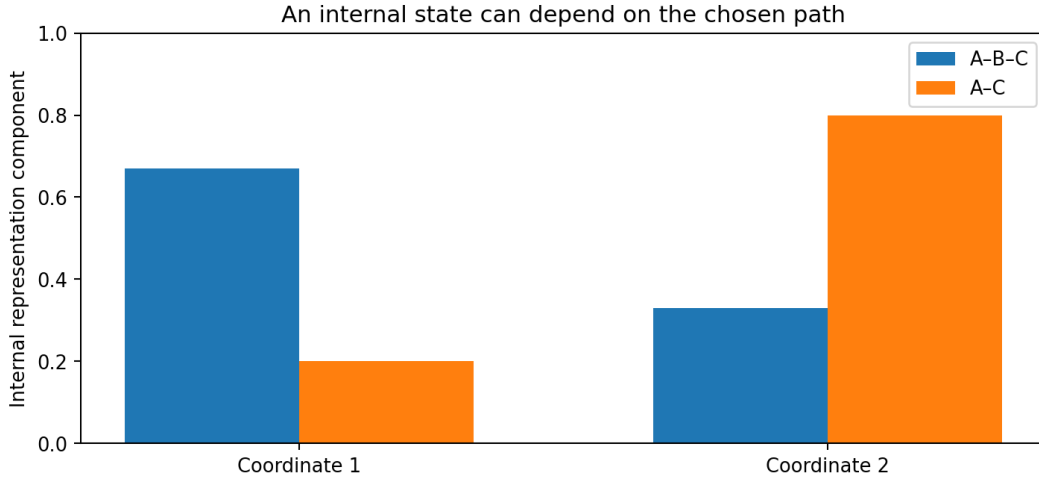


Figure 7: Alternative internal representations after two constructed paths. The matrices preserve the two-component total but can produce different component compositions at the destination; components have no calibrated economic meaning.

of compatible consequences rather than a single warranted prediction. The right panel is therefore an illustration of incomplete knowledge *about* the local mechanism, distinct from disagreement between participants about its significance.

7.6 Path-dependent internal-state transformations

We also constructed an example of locally transported, two-component internal representations. All transfer matrices are nonnegative and column-stochastic, so the component total is conserved in this special case. The initial vector at A is $(1, 0)$, and the indirect path applies a matrix T_{AB} followed by T_{BC} , while a direct edge applies T_{AC} . Their outputs are $(0.67, 0.33)$ and $(0.20, 0.80)$ respectively. Figure 7 shows the resulting components at C.

The example illustrates the role of composition rather than identifying a geometric phase. Each matrix is stipulated, and the representation could stand for a harmless classification channel, a technical conversion convention, or a probabilistic interpretation under different assumptions. For actual heterogeneous values, transformations may be nonlinear, non-invertible, and nonconservative. A fiber-bundle interpretation would require additional transport structure, while these path-dependent output differences

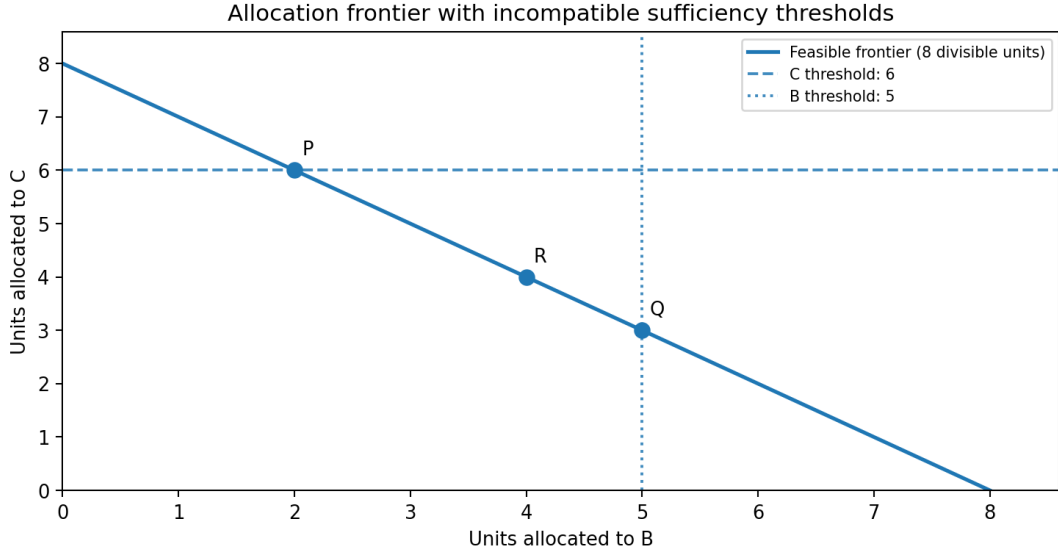


Figure 8: A constructed feasible allocation frontier for eight divisible units. The stipulated sufficiency requirements of B and C cannot both be met; points P and Q each activate a different threshold and R activates neither.

are already possible on an ordinary discrete graph.

7.7 Plural allocation criteria and incompatible requirements

A resource-allocation illustration gives B and C an eight-unit divisible stock, with thresholds of five and six respectively. Figure 8 plots the complete-utilization frontier in the space of recipient allocations and indicates the two separate thresholds. Their joint satisfaction is arithmetically infeasible under the specified conditions. Three examples on the frontier are P with $(B, C) = (2, 6)$, Q with $(5, 3)$, and R with $(4, 4)$.

Under a criterion focused on C's activation, P satisfies the threshold, while Q does not. Under a criterion focused on B, the roles are reversed. R lies between them in the raw allocation coordinates, but its status depends on the evaluative dimension selected. Every complete-utilization point is Pareto nondominated when more units for each person is treated as better, making the Pareto set too large to identify a unique allocation. This simple problem is a useful check against descriptions of Pareto efficiency as a sufficient theory of justice.

7.8 Conductance feedback sensitivity and numerical resolution

The initial diffusion study compared static conductance with one arbitrarily selected adaptive coefficient. We have now rerun the same 13×9 network with feedback values $\eta \in \{0, 0.007, 0.014, 0.03\}$, while retaining the type-specific openings, relaxation parameter, conductance cap, and final time. Figure 9 shows the cross-barrier resource fractions. The curve varies with the stipulated feedback coefficient, making the parameter dependence visible. The script also repeats the $\eta = 0.014$ run with half the time step and twice the number of steps at the same final physical time. The maximal difference in a final node stock is approximately 0.01984 resource units; this comparison is a limited numerical refinement check, not a convergence proof.

The dependence of final stock fractions on feedback is a result of the specified numerical system. Figure 10 additionally shows spatially heterogeneous changes in horizontal conductance for the adaptive run compared with the static baseline. Such heterogeneity follows from unequal initial source locations, selective openings, and different flux histories. The image displays a change in effective edge weights, not a smooth manifold changing curvature or an observed social network changing institutional rules.

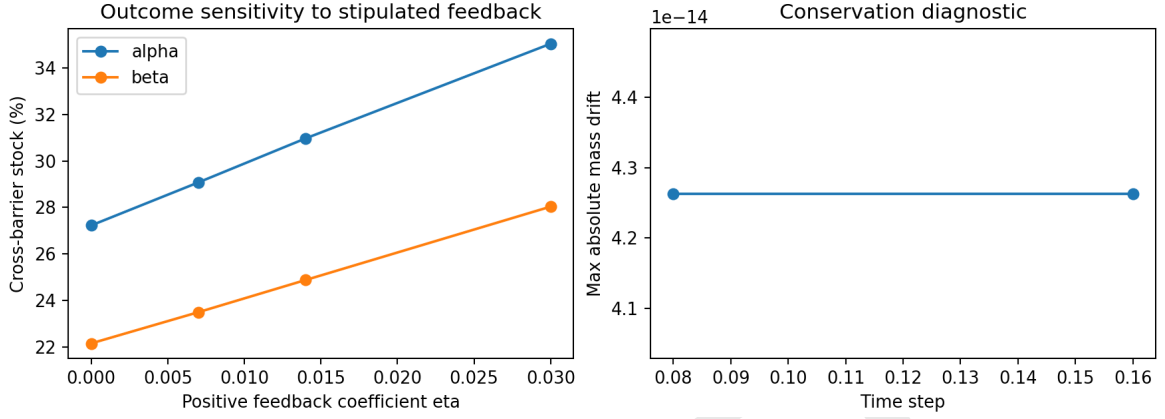


Figure 9: Sensitivity to feedback strength and an elementary mass-conservation check. Left: cross-barrier fractions at the final time for two independently conserved types. Right: numerical mass drift under two step sizes, whose small values follow the conservative discretization.

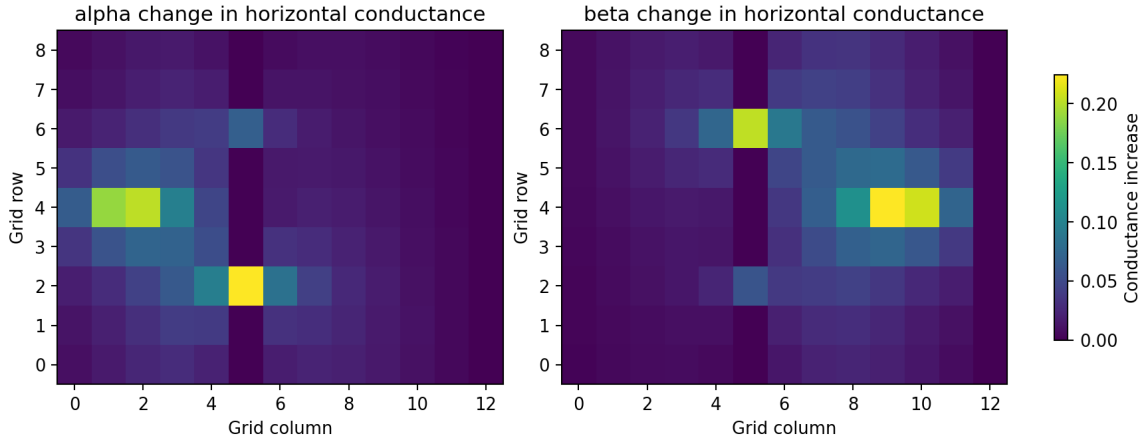


Figure 10: Difference in horizontal edge conductance between adaptive and static variants at the final simulation time. The two resource types produce distinct local feedback patterns on the same fixed network.

The numerical studies thus represent several different mathematical mechanisms: time delay; local threshold interpretation; compositional state transport; plural allocation criteria; and type-specific diffusion with feedback. Their juxtaposition reveals a limitation as well as a possibility. A single generic term such as *value flow* does not specify which mechanism is intended. Further modeling should begin with observed event kinds, practical questions, and justifiable parameter descriptions, then select the corresponding model class.

7.9 An illustrated changing local metric

The optional local metric in Equation 19 is evaluated at three selected network sites, initially and after the adaptive diffusion simulation, using the two stock coordinates as inputs. Figure 11 represents the corresponding metric unit balls. A unit ball here consists of tangent directions u with $u^T g_i(t) u \leq 1$; its changing outline makes the imposed local geometry visible. The quantity $\gamma = 2.5$ is selected solely for legibility of the change.

The internal directions are labels in an illustrative two-coordinate state space. They do not assert a universal coordinate system for money, labor, knowledge, or trust. In a genuinely heterogeneous system, local fibers could have different dimensions and one would need comparison or transport maps to relate

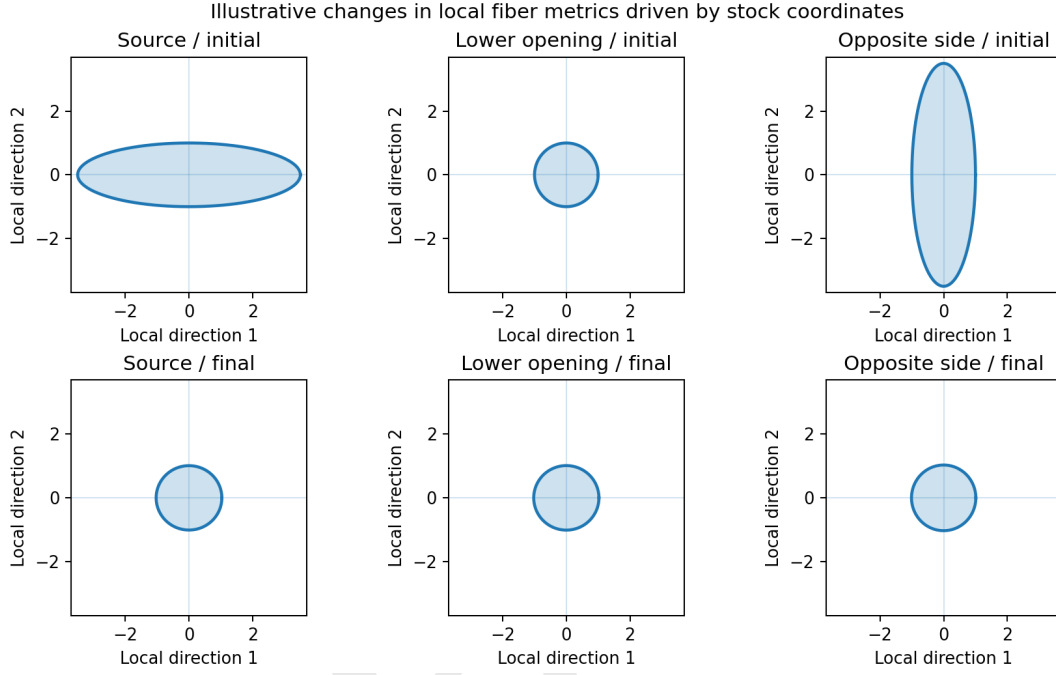


Figure 11: State-dependent internal metric unit balls at three selected sites, before and after diffusion. The local metric changes because it is explicitly defined as a function of typed resource stocks; the figure is a constructed geometrical analogy without empirical calibration.

Table 2: Constructed examples and the boundaries of inference.

Construction	Established within the model	Unaddressed question
Money and labour ledger	Each type's incidence entries balance	Equivalent benefit, consent quality, and justice
Three-node deadline	Route timing changes a stipulated feasibility predicate	Claims on resources, alternative costs, future outcomes
Selective diffusion	Type-specific paths yield different stock trajectories	Purposeful behaviour, recognition, transformation of value
Adaptive weights	Stipulated feedback changes shortest-path costs	Changing social topology or physical geometry

their tangent vectors. The example clarifies how a local geometry *could* coevolve with a circulation process while also exposing the additional assumptions required to specify that geometry.

7.10 Effective distance and representational limits

To illustrate the limited sense of geometric change adopted in this experiment, Figure 12 displays resource-specific shortest-path costs computed from reciprocal conductances. In the α network, the computed cost between the two initial source nodes decreases from approximately 27.37 to 23.86. This is a change in derived accessibility on a fixed weighted graph. Vertex positions, edge existence, and the spaces of local interpretation remain unchanged.

Table 2 summarizes what the illustrations establish within their specifications and what they leave unresolved. It is particularly important that the money–labour ledger and the diffusion model use different mathematical meanings of resource: an incidence record of performed service in the former and an additive conserved stock in the latter. The shared language of circulation does not make those special cases interchangeable.

The constructed results therefore demonstrate the feasibility of reproducing certain formal distinc-

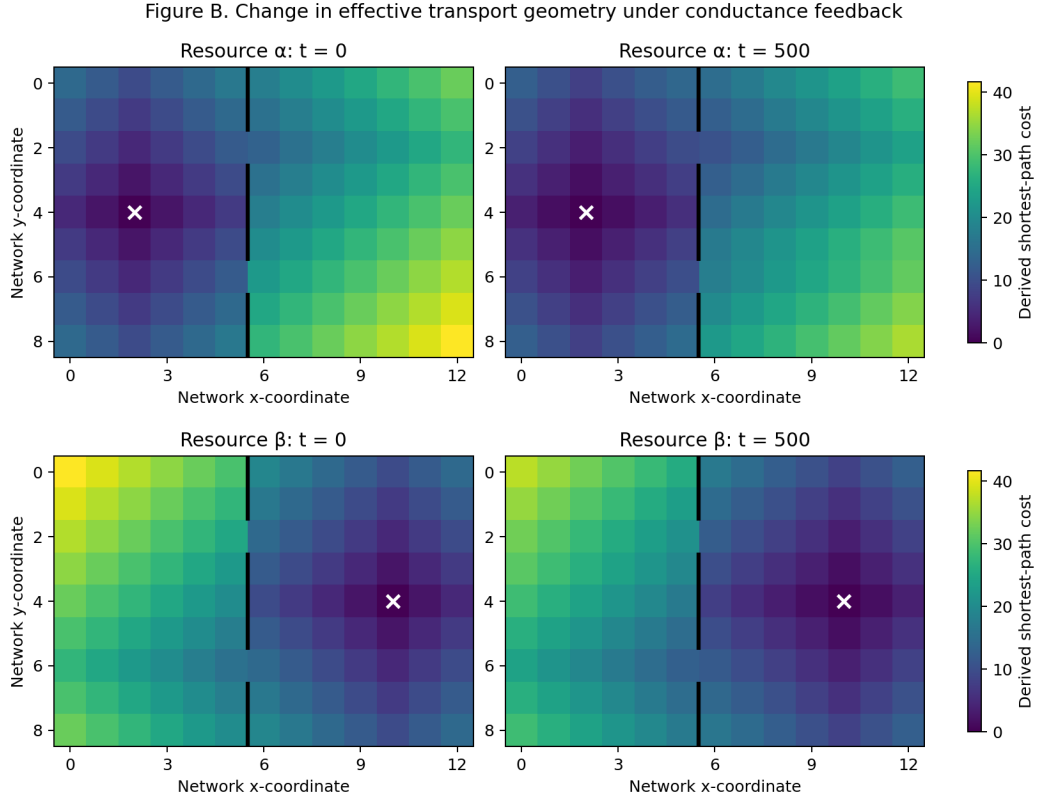


Figure 12: Effective network transport cost before and after conductance feedback, measured as shortest-path sums of reciprocal positive conductances from each resource’s source node. The embedding and topology are fixed.

tions. Their principal use is diagnostic: they expose how little an additive flow model can tell us about locally recognized value, justification of comparison, or the ethics of circulation without additional specifications.

8 Discussion and Further Questions

This final section assesses the scope of the preliminary representation and identifies the assumptions that would need to be strengthened before further analytical or empirical work. The proposed model is deliberately elementary. Its purpose is to keep differences among transfers, local states, comparisons, and normative judgments visible while permitting controlled demonstrations of selected dynamics.

8.1 What the current representation permits

The model can distinguish several objects often compressed into the word “value”: a recorded transfer, a typed quantity, a receiving participant’s state change, an interpretation of an event, and a comparison across outcomes. In a synthetic circulation problem, type-specific accounting and delays can be represented while preserving the possibility that some cross-type comparisons remain absent or contested. The conductance example also supplies a reproducible illustration of selective transfer and changing effective network accessibility.

Each of these capacities follows from established mathematical methods and our particular specifications. The paper has not shown that the proposed formalism captures the most consequential determinants of value circulation in a real institution or community. The interpretive operators and update maps are placeholders requiring substantive empirical development. Even when a local state is represented

numerically, its relation to the subjective and social meaning of an event remains open to validation.

8.2 Geometric and discrete extensions

The locally structured graph of Section 3 and the initial motivating idea of a manifold with internal state spaces has useful mathematical relatives in fiber and field representations. It can be examined when a smooth spatial or relational approximation is demonstrably appropriate. A locally structured graph is currently easier to audit because transfer events, capacities, and typed resource quantities can be specified with clear incidence rules. The two constructions need not be equivalent. Heterogeneous node spaces are also not automatically smooth fibers of a standard bundle, and a change in shortest-path cost does not entail a change in curvature or topology.

A neuron-like interpretation suggests local processing and selective communication, but the paper provides no account of neural anatomy, evolutionary selection, action potentials, or biological information coding. Nervous systems combine multiple signaling mechanisms, and a proposed analogy to selective routing would need task-specific comparative models and biological evidence before bearing on explanations of neural organization. The present study neither tests nor endorses an evolutionary explanation.

8.3 Recognition, comparison, and power

The most consequential representational difficulty may concern evidence for local interpretations. What an observer can record about an event can differ substantially from what participants understand or experience. The chosen representation could obscure this difference if it treats a researcher’s measurement as the participant’s own value. An empirical extension would need a defensible procedure for collecting, preserving, or contesting descriptions of local conditions, including uncertainty and missing information.

The choice of a comparison space is similarly consequential. A common metric may support coordination, but its introduction can obscure dimensions important to some participants and favor those able to define the metric. This concern connects the formal question of comparability to the social analysis of commensuration [7]. The existence of a common representation is therefore an analytical condition, not a conclusion that the relevant parties accept the same evaluative standard.

8.4 Representation adequacy and alternatives

The added examples show that a graph with typed local events can accommodate several independent distinctions, but it does not yet establish an explanatory model of social value creation. In particular, a node’s local interpretation is not identified by observing its inbound transfers; the numerical response curves are stipulated functions, and the transfer matrices are arbitrary normalized channels. A future application would need a protocol for defining event types, eliciting or independently observing local conditions, documenting contested meanings, and assessing whether repeated observations support any proposed update law.

The literature suggests several narrower alternatives to a single encompassing geometry. Multicommodity network programming is appropriate for inventories and capacities with shared constraints [1]. Optimal transport supplies a principled treatment of mobility and transport costs when the transferred quantity and geometry have been specified [13]. Resource-theoretic orders are suited to questions about permissible transformations between heterogeneous objects [8]. Sheaf-type structures are promising when local vector spaces and explicit compatibility maps make sense [2]. Set-valued analysis can retain uncertainty about states and conversion relations [11]. The present constructions combine elementary features of these traditions for exposition. Demonstrating a common mathematical framework with useful general results remains a separate undertaking.

8.5 Three testable distinctions for future refinement

First, *temporal access* should be distinguished from eventual receipt. The transit-accounting example preserves material conservation while allowing a deadline-sensitive opportunity to expire. Empirical evaluation would require observing deadlines, substitutes, claim priority, and any irreversibility of the task. Second, *recognition* should be distinguished from *conversion*. A recipient can understand the significance of an object yet lack the resources required to use it, while a mechanism can convert inputs without the represented actor possessing an articulated theory of that conversion. Third, *changing accessibility* should be distinguished from *changing social structure*. Edge-weight adjustment alters a calculated path cost; a new legal right, altered governance structure, or emerging trust relation is a different kind of event that needs its own data and assumptions.

The toy outputs could support falsifiable local predictions if a real mechanism and measurement process were supplied. For example, a specified routing rule predicts an arrival-time distribution; a genuinely conserved stock implies an accounting identity within a validated boundary; a proposed threshold law predicts a particular discontinuity in the derivative of task response. In their present form the examples simply follow from stipulated mechanisms, so none serves as empirical evidence for the wider generative-relational programme.

8.6 Evaluation, responsibility, and limits of optimization

Optimization needs an admissible set and an evaluative relation. Both may be institutionally contested, and the latter may be incomplete. Even within the simple two-recipient allocation, Pareto nondominance leaves many candidates. An institution might prioritize urgency, minimum sufficiency, rights, reciprocity, prospective value creation, or transparent procedures. Each choice can be studied without pretending that its mathematical consequences alone justify the choice. Moreover, a participant's ability to contest a comparison rule can be a relevant concern in its own right.

A promising modest research practice would be to publish, alongside a simulation, the typed-event specification, admissibility assumptions, unobserved state variables, comparison rule, and alternatives that the formalism excludes. This would support critical review of the results and expose where ethical and epistemic judgment enters. Such documentation would be especially important when a simulation is used in institutions that allocate scarce resources among differently situated people.

8.7 Future model development

Several modest next steps can be pursued without assuming that the first construction is privileged. One is to replace conserved resource stocks with typed processes that can be copied, jointly used, or transformed into different products. Another is to distinguish permission to transfer from material ability to receive and use an item. A third is to allow nodes and edges to appear, disappear, or change type while accounting for the cost and legitimacy of those modifications. A fourth is to construct small cases where two evaluation relations produce incomplete rankings, and to test whether the incomparability is structural or arises from insufficient information.

A further programme would connect constructed transfer rules to independently documented examples, with an audit of measurement practices, uncertainty, and rival causal explanations. Such work requires attention to the historical formation of value categories and to the situated obligations of participants. It may draw on qualitative records as well as computational methods. The formal representations should support inquiry rather than preempt questions of ethics, law, or institutional authority.

The computational protocol is specified in Appendix A. The preliminary discussion has therefore produced a set of inspectable candidate definitions and demonstrations, alongside a record of their limitations. Whether they form a useful research method for heterogeneous value circulation remains an open question that can be examined through further mathematical comparison and carefully scoped empirical studies.

A Computational Specification and Validation

This appendix records the implementation boundaries of the examples presented in Section 7. Three Python scripts are supplied with the manuscript source, including `experiments/local_exchange_and_routing.py` and `experiments/preliminary_diffusion.py`. Their saved numerical outputs are `toy_results.json`, `run_summary.json`, and `heterogeneous_ledger.csv`. A GIF gives a non-evidential animated view of the latter process. The figure set is generated from the archived code.

A.1 Three-node transfer procedure

The graph has nodes A, B, and C and an initial stock vector $(8, 0, 0)$ for a single resource. Each transfer uses one complete discrete interval. In the indirect version A transfers eight units to B in interval 0, and B transfers eight units to C in interval 1. In the direct version A transfers eight units to C in interval 0. The feasibility predicate at $t = 1$ is $x_C(1) \geq 6$. All resource quantities are hypothetical and nonnegative. The script asserts conservation of eight total units at each time, nonnegativity, and the distinct outcomes of the deadline predicate. It does not model the costs of constructing a direct edge.

A.2 Grid diffusion procedure

The network contains 117 vertices and nearest-neighbour undirected edges, represented by an arbitrary edge orientation for net flux. The barrier divides columns 5 and 6 and admits crossings only at rows 2 and 6. At row 2, the baseline conductances for α and β are 0.85 and 0.055; at row 6, the values are reversed. Other horizontal and vertical edges have baseline conductance 0.42. Initial stocks are 100 units of α at coordinate $(2, 4)$ and 100 units of β at $(10, 4)$. Each resource evolves independently in the diffusion special case, with the update in Equation 11.

The adaptive variant uses Equation 12 with $\Delta t = 0.16$, $\eta = 0.014$, $\lambda = 0.0012$, and $w_{\max} = 1.30$. Zero-baseline edges remain zero. The static variant retains the baseline conductances. Both are run for 500 steps. Distances in Figure 12 are calculated using the minimum summed edge cost $1/w$ over positive-conductance paths. No pseudometric is claimed for a continuously evolving physical space.

A.3 Validation and reproduction limits

The scripts assert or report nonnegativity, per-type conservation, and the final cross-barrier stock fractions. For the rerun accompanying this draft, both grid variants have maximum mass drift below 6×10^{-14} relative to initial total 100 for each resource, and the smallest resource stock is zero. These are checks of deterministic numerical consistency, not comparisons against measured data. The code does not perform parameter inference, sensitivity analysis across a distribution of settings, causal identification, or an independent verification of any institutional mechanism. Reproduction requires Python with NumPy, SciPy, and Matplotlib; the source archive retains the exact implemented assumptions.

A.4 Extended Python studies

The independent script `experiments/extended_studies.py` creates seven additional figures, including receipt timing with alternate policies (Figure 5), distinct local response laws and observer intervals (Figure 6), path-composed internal transformations (Figure 7), a two-recipient allocation frontier (Figure 8), feedback sensitivity (Figure 9), localized edge-weight change (Figure 10), and the local metric experiment (Figure 11). It writes numerical outputs to `experiments/extended_results.json`. The original diffusion and typed-event scripts are retained unchanged for comparison with the first draft.

A.5 Matrix and parameter specification

The internal-state example starts from $(1, 0)^\top$ and uses column-stochastic maps. Equation 25 records the complete matrices, permitting direct reproduction without reliance on the code.

$$T_{AB} = \begin{pmatrix} .9 & .1 \\ .1 & .9 \end{pmatrix}, \quad T_{BC} = \begin{pmatrix} .7 & .4 \\ .3 & .6 \end{pmatrix}, \quad T_{AC} = \begin{pmatrix} .2 & .7 \\ .8 & .3 \end{pmatrix}. \quad (25)$$

The indirect result is $T_{BC}T_{AB}(1, 0)^\top = (.67, .33)^\top$, whereas direct transport gives $T_{AC}(1, 0)^\top = (.20, .80)^\top$. Both column sums are one and components remain nonnegative. The matrices are chosen for illustrative path dependence, not estimated conversion interfaces.

For the threshold experiment, the two response laws use thresholds two and seven with $r(q; \theta) = \max\{0, q - \theta\}$. The uncertainty intervals in Figure 6 are A: $[1.5, 3.0]$, B: $[6.0, 8.5]$, and C: $[3.0, 7.0]$. They represent invented ranges an observer might entertain and have no statistical coverage interpretation. In Figure 8, total available resource is eight, B's threshold is five, and C's threshold is six. The inability to satisfy both is a direct arithmetic consequence of these values.

The sensitivity study uses the same grid and baseline conductances as the original experiment, with 500 time steps of length 0.16 (final time 80). The adaptive cases set η to 0, 0.007, 0.014, or 0.03, maintain $\lambda = 0.0012$ in the discrete relaxation term, and cap positive conductances at 1.30. A refinement at $\Delta t = 0.08$ uses 1000 steps and the same feedback coefficient. Both settings satisfy the programmed nonnegativity and mass-drift tests; the reported maximum absolute difference at final time is 0.01984 units. The code makes the arbitrary grid orientation, edge selection, and per-type conservation explicit.

A.6 Computational interpretation and further validation requirements

These deterministic experiments are numerically reproducible under compatible Python, NumPy, SciPy, and Matplotlib versions. Their correctness checks concern bookkeeping, nonnegative stocks, normalized transformation matrices, and sensitivity to selected time steps and coefficients. Numerical reproducibility does not establish structural identification. A more developed validation programme would include parameter distributions, alternative feedback laws, direct event-level calibration where ethically obtainable, and comparison with baseline network models. A true geometric evolution experiment would also require specifying the geometric object that changes, rather than describing an edge-weight update as manifold curvature.

A.7 State-dependent local metric experiment

The extended script also produces Figure 11 from the stock vectors of the baseline and adaptive two-type simulations. Equation 19 with $\gamma = 2.5$ specifies the metric completely. The selected sites are grid locations $(2, 4)$, $(5, 2)$, and $(10, 4)$. For a diagonal metric with positive entries, the tangent unit ball is an ellipse with radii $1 + \gamma x_i^\alpha / 100$ and $1 + \gamma x_i^\beta / 100$. The program checks positivity of the eigenvalues and archives metric entries in `experiments/extended_results.json`. These are prescribed geometrical parameters and their plotted consequences, not a measured metric on any social system.

Research Ethics and Source Provenance

Scope. All cases, flow rules, thresholds, and numerical displays are constructed; none reports an observed social exchange, an intervention, a neurological mechanism, or a calibrated economic model. No personal, confidential, or human-subject data are used. The source archive contains the scripts, their outputs, and a description of limitations.

Source access. External citations were checked against publisher, conference, institutional, or preprint metadata and abstracts before inclusion. The source ledger documents what was verified, not comprehensive full-text review. Interpretations of the proposed model and any mistakes remain the author’s responsibility.

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