

Attributing Decision Roles in Human–Device Procedures for Allocating Shared Resources

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Abstract

A device can determine a resource recipient while people control eligibility, the selection rule, repeated draws, authorization and review. Describing the device’s contribution requires several distinct questions about the procedure. This paper develops a role-attribution framework for shared-resource allocation involving people and nonhuman devices. It separates causal pivotality, functional contribution, interpretive attribution, procedural control, institutional authority and responsibility. A staged model identifies where those roles enter. Established Boolean influence supplies a limited causal comparison; worked examples show why its values provide neither exclusive shares of responsibility nor an authority test. Further derivations characterize the effect of changing a random-allocation threshold, accepting the first favorable result across repeated draws, and intervening under limited information and permitted actions. A conditional information-and-action criterion clarifies the difference between a formal override and an effective opportunity. The English school-admissions and appeals codes provide a bounded institutional setting for examining random allocation, supervision and review, without attributing a particular device deployment to those texts. Constructed comparisons preserve the source discussion’s questions about stones, ceremonial interpretation and distributed procedures while suspending conclusions about personhood. A prospective audit specifies evidence for each role and conditions for revising an attribution.

Keywords: human-device procedures; decision roles; causal influence; procedural control; institutional authority; random allocation

Research Note

This paper develops a bounded argument through conceptual distinctions, stated assumptions, conditional derivations, worked examples, and objections. Its length follows the requirements of that argument. Formal statements concern the specified models; the application of those models to persons and institutions requires independent interpretation and evidence. Constructed examples are explicitly identified and carry illustrative status.

The manuscript follows the repository’s paper template for typography, numbered formal statements, section structure, citations, and navigable contents. The associated project files preserve research provenance, source verification, derivation records, unresolved objections, and validation results. Philosophical commitments about the ultimate nature of social facts remain suspended where the descriptive framework is sufficient for the argument.

Authorship and Research Assistance

Wanhong Huang directed the research questions and authorized this paper’s preparation. An AI research assistant helped organize the material, develop explicit formal constructions, check primary sources, and prepare the text. Agent-added arguments and formulations remain subject to the author’s review. Source positions, repository proposals, and model consequences are distinguished throughout the manuscript.

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1 Introduction

A procedure can use a device to select a resource recipient while people determine the eligible set, the selection rule, the conditions of acceptance and the available review. This paper examines how decision roles should be attributed across that organization. Its scope is shared-resource allocation involving people and nonhuman devices, with constructed formal examples and a bounded institutional comparison drawn from English school admissions.

Consider a single place allocated between two eligible applicants. A device produces a bit, and a rule maps that bit to one applicant. A human operator announces the result. Another person chose the mapping, an authority established eligibility, and an independent reviewer can examine whether the procedure was applied correctly. Asking who “made the decision” can refer to several contributions. The device may be causally pivotal; the operator may implement the result; the authority may settle the applicable rule; and the reviewer may possess a distinct power of correction.

The source discussion raises the question through stones, divination, courts and supported decisions. A stone’s physical stability supplies a weak basis for excluding it from every decision-related role: a stone participates in changing physical relations, and an interpreted token can occupy a consequential place in a practice. Equally, the fact that someone treats a stone’s fall as decisive leaves its other roles open. This paper preserves those challenges while suspending conclusions about consciousness, personhood and ultimate social ontology.

The central position is that attribution should follow a declared role and the evidence appropriate to it. Causal pivotality, functional contribution, interpretive attribution, procedural control, institutional authority and responsibility can vary independently. A single output or outcome distribution can conceal materially different arrangements. The proposed framework therefore asks a sequence of scoped questions about a procedure and records unresolved attributions where its evidence is insufficient.

The argument first confronts established distinctions in automation and a mathematical antecedent for pivotality. It then defines a staged allocation model and derives three limited results. Boolean examples show why influence values fail to provide exclusive responsibility shares. A repeated-draw calculation shows how control over acceptance can reshape the output of a nominally fixed random device. An information-and-action criterion identifies when an effective intervention policy exists under declared constraints.

The institutional comparison uses the English admissions framework to distinguish selection, supervision and review. The relevant code provides rules for random allocation in its scope; it does not specify the fictional software or device modeled here. The examples therefore examine possible implementations under declared assumptions, while compliance and actual deployment remain separate questions.

This approach preserves a substantive sense in which devices can participate in decisions. It also prevents that participation from silently acquiring every status associated with a human decision-maker. An influential device need not possess authority; a formally authorized person may lack usable information or time to intervene; and a distributed procedure can require accountability at several points.

The proposed contribution is an attribution method with explicit warrants and counterconditions. Its value lies in making the organization of a decision inspectable, especially where familiar labels such as “automated,” “human-controlled” or “random” conceal the conditions under which a recipient is selected and a result becomes binding.

2 Antecedents in Automation and Procedural Allocation

This section identifies the verified antecedents used in the argument. The source positions support specific distinctions, while the proposed role framework and its normative interpretation remain separate constructions. The review is selective and carries no claim to exhaust the literature on agency, responsibility or collective decision-making.

Parasuraman, Sheridan and Wickens distinguish functions of automation involving information acquisition, information analysis, decision or action selection, and action implementation. They also con-

sider levels of automation within those functions [4]. This is a direct antecedent for decomposing a human–device procedure. The present paper’s additional focus is the warrant for attributing a particular role, especially the difference between affecting an output, possessing a feasible opportunity and holding an institutional power. The cited article’s abstract and relevant primary indexed material support this bounded use; the present study makes no claim to replicate its empirical implications.

O’Donnell defines a coordinate’s influence on a Boolean function through the probability that flipping it changes the output under a specified uniform-input convention. His treatment includes majority-of-three and total influence [3]. These established constructions provide a transparent mathematical comparison. The present paper reproduces small calculations to expose the limits of interpreting the quantity as authority or responsibility. The calculations are applications of an existing definition, with the institutional interpretation requiring independent argument.

The English School Admissions Code supplies a different kind of antecedent: a rule framework for an actual institutional domain. Its random-allocation provisions address scope, transparency, applicable priority and supervision, while the School Admission Appeals Code identifies distinct review questions [1, 2]. These texts give content to institutional roles that an output function alone cannot establish. The manuscript uses the texts’ stated rules and constructed implementations; it reports no observation of a particular school’s device or practice.

The sources help locate a gap in the source discussion’s broad question. A taxonomy of automated functions can describe where a device contributes. A pivotality measure can quantify one counterfactual feature of that contribution. A governing rule can identify an authorized office and review route. None of those descriptions should be silently substituted for the others. Their combination can improve an attribution only when the relation among their different questions is explicit.

The repository’s original formulation emphasizes that a decision episode can involve an interpretation of alternatives, a settlement, reaffirmation, refusal or revision within a relational history. That formulation is retained here as a working framework. It is not attributed to the reviewed scholars, and its similarity to their distinctions remains a matter for further comparison.

A fuller literature confrontation would be needed before advancing a general account of distributed agency or moral responsibility. The present paper instead develops a bounded method for allocation procedures, where roles can be tied to explicit stages, interventions and governing rules. That scope permits substantial formal and philosophical analysis while keeping its source commitments reviewable.

3 Distinct Warrants for Decision Roles

This section defines the proposed attribution vocabulary and the evidence each role requires. Its objective is to preserve distinctions that ordinary decision language can compress. The categories describe analytical questions and can apply simultaneously to one participant or device.

Let q designate a candidate bearer, c a procedure context, e an episode and i an attribution standpoint or inquiry. For a declared role r , write

$$\text{Attr}_r(q, c, e, i) \in \{1, 0, ?\}. \quad (1)$$

The values indicate supported attribution, supported exclusion within the declared scope, and unresolved status. An inquiry’s standpoint selects its question and evidence; it does not make the answer true merely by preferring it.

Table 1 gives these questions different warrants. A random bit can be causally pivotal in a stipulated selection rule. Its functional contribution can consist in generating an input with a required distribution. Participants can treat its result as decisive. An authority can nevertheless retain the power to define eligibility and the acceptance conditions. Whether anyone bears responsibility for a defect requires further grounds concerning duties, opportunities and the applicable standard.

The functional question needs particular care. A useful prediction can motivate a functional description, but convenience alone supplies weak evidence that the described contribution realizes the task. The

Table 1: Proposed decision roles and their distinct evidential warrants

Role	Attribution question	Evidence required by the question
Causal	Can a specified change alter an outcome?	Intervention, background and causal model
Functional	Does the contribution realize a declared task relation?	Task, mechanism and performance conditions
Interpretive	How do participants treat the contribution?	Accounts, practices and relevant history
Control	Which interventions are feasible and permitted?	Information, action set, time and resources
Institutional	Which act or office has recognized powers?	Applicable rule, authority and validity conditions
Responsibility	Who owes reasons or bears a consequence?	Normative grounds, duties and case-specific facts

task itself may be contested. A selector can reliably implement a rule that excludes a group unjustifiably. Functional adequacy under the rule and justification of the rule then diverge.

Interpretive attribution is also substantive. If participants treat a stone’s fall as an instruction, that treatment can organize subsequent action and expectations. Evidence about the practice supports this role. Further claims about the stone’s capacity to understand alternatives, revise a policy or possess rights require separate targets and warrants. The ceremonial example remains a constructed possibility, with ethnographic attribution left to verified empirical work.

The source framework’s six descriptive categories remain available throughout. A device can be an entity with an attribute, occupy a relation to an authority, have a state, participate in a selection event and form part of an allocation process. A decision episode can admit several of these descriptions. Its occurrence need not change the final resource state: reaffirmation or an authorized refusal to reopen can still change the procedural history.

The resulting framework is deliberately plural without making every attribution optional. A role can be unsupported or excluded when its declared conditions fail. Recording the reason for that verdict, together with unresolved evidence, makes the framework more demanding than a free choice of metaphors.

4 A Staged Model of Resource Allocation

This section specifies a constructed allocation procedure and locates the roles within it. The equations are a transparent schema whose causal and institutional interpretation requires the stated application. Their stages can overlap in practice; the model separates dependencies needed by the argument.

Let $h \in H$ describe a procedure history. Let x be application information, E an eligible set, p a selection rule or its parameter, S a device input, D a proposed recipient, A an authorized settlement and Y an implemented allocation. A review result is denoted R . A simple arrangement is

$$\begin{aligned}
 E &= e(x, g), & p &= r(E, g), \\
 D &= d(E, p, S), & A &= a(D, x, g), \\
 Y &= y(A, x), & R &= v(Y, D, x, g),
 \end{aligned} \tag{2}$$

where g records applicable governance. Domains must ensure that a proposed recipient belongs to the eligible set or that an explicit failure outcome is available. The schema allows an authorized settlement to accept, reject, defer or request a rerun where the governing arrangement permits those acts.

Figure 1 locates the separate inputs to this part of the argument. The lower input supplies conditions whose justification remains distinct from the relation represented along the upper path.

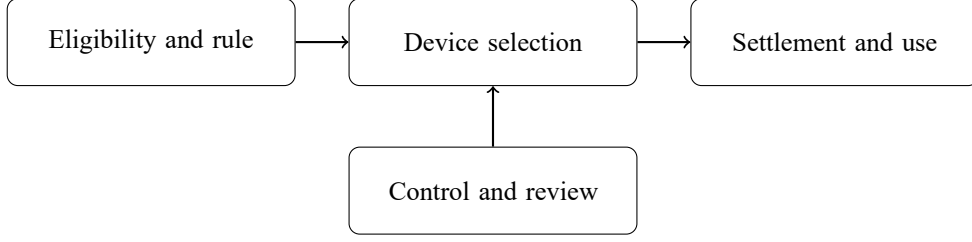


Figure 1: Selection within a governed allocation procedure. Arrows show the declared organization of the argument; their empirical or institutional warrant requires the accompanying analysis.

An arrow in this dependency description means that a variable enters a declared map. Establishing the map’s causal adequacy for an actual institution requires evidence about interventions, omitted conditions and execution. Establishing the settlement’s validity requires the governing rule and its facts. The same numerical function can occur in arrangements with different authority relations.

Definition 4.1 (Scoped procedural control). For a bearer i and an observed situation o , let $\Omega_i(o)$ be the set of feasible, permitted interventions within a declared stage and time horizon. Procedural control concerns this set and its consequences under the model. A formal permission and a feasible intervention are recorded separately when they differ.

Control over E can determine whose chances exist. Control over p can change the rule among eligible applicants. Control over S can select or manipulate a device input. Control over acceptance can determine whether an unfavorable draw is repeated. Control over A can settle the output’s institutional status, and control over implementation can prevent a settled allocation from occurring. Review adds a further stage whose power and evidence may differ from the original decision.

This decomposition explains why a device can be decisive within one stage while a wider procedure remains shaped elsewhere. Calling a selection “random” identifies a property of a particular mechanism or distribution. It leaves eligibility, seeding, repetition and acceptance open. Calling a procedure “human-controlled” similarly requires identifying which person can do what, with which information and within which time.

The model also distinguishes alternatives. A recipient can be physically selectable, considered by an operator, eligible under a rule or available to an authorized decision-maker. These sets may differ. A device that samples only from a restricted list can function exactly as specified while the construction of the list remains the central issue.

Finally, a procedure has a history beyond its terminal output. The same recipient can result from a valid draw, a rerun after an unfavorable draw or a human override. An audit interested in rule compliance must retain those differences. Reducing the episode to Y alone would make the relevant procedural query unrecoverable by construction.

5 Causal Pivotality and Its Attribution Limits

This section uses established Boolean influence to examine one limited meaning of causal contribution. The derivations make the reference distribution and intervention explicit. They then show why the resulting quantity leaves authority, task fidelity and responsibility unanswered.

Let $f : \{0, 1\}^n \rightarrow \{0, 1\}$ be an outcome function and $x^{\oplus i}$ the input obtained by flipping coordinate i . Under a declared distribution μ , define

$$I_i^\mu(f) = \Pr_{X \sim \mu} \{f(X) \neq f(X^{\oplus i})\}. \quad (3)$$

For uniform inputs this is the standard Boolean influence discussed by O’Donnell [3], expressed with zero–one coding. A causal application requires that flipping the coordinate represent the intended in-

tervention while the stipulated background remains appropriate. A coordinate may be an input or stage variable rather than a person.

Example 5.1 (Majority-of-three influence). Let $f(x) = 1$ when at least two of three input bits equal one, with uniform independent bits. Coordinate i is pivotal exactly when the other two bits differ. Two of their four equally likely configurations satisfy that condition, so $I_i(f) = 1/2$ for each i and $\sum_i I_i(f) = 3/2$.

The example is an established illustration reproduced here to clarify interpretation. The sum exceeds one because pivotal events need not define exclusive shares. Dividing by the sum would create a normalized quantity, but its interpretation as responsibility would require a new argument. The normalization alone supplies none.

For two-input conjunction $f(x) = x_1x_2$, coordinate one is pivotal exactly when $x_2 = 1$; under uniform inputs both influences equal $1/2$. Under a different input distribution, $I_1^\mu(f) = \Pr(X_2 = 1)$ and $I_2^\mu(f) = \Pr(X_1 = 1)$. The outcome function stays fixed while the scores change. A report should consequently identify whether it concerns a mathematical reference distribution, an observed task population or a policy-relevant intervention distribution.

Proposition 5.2 (Influence under output reversal). *For binary f and every μ , $I_i^\mu(1 - f) = I_i^\mu(f)$.*

Proof. The inequality $1 - f(x) \neq 1 - f(x^{\oplus i})$ holds exactly when $f(x) \neq f(x^{\oplus i})$. The defining events and therefore their probabilities coincide. $\square$

Thus $f(x) = x_1$ and $\tilde{f}(x) = 1 - x_1$ give coordinate one maximal influence even if the task requires following the first input and regards its reversal as a defect. Pivotality captures sensitivity to the input; correctness relative to the task depends on the intended relation.

Authority is still further removed. Construct two arrangements with identical f and input distribution. In the first, a rule gives the device's output a stipulated binding role once validity conditions are satisfied. In the second, the same output is advisory and a person routinely accepts it. Their observed outcomes can agree exactly. The authority question differs because their governing rules differ. This is a logical construction, with actual institutional validity requiring evidence about the applicable rule.

The influence quantity is therefore useful when assigned its proper question. It can reveal dependence hidden by a procedural description or compare specified interventions. It supplies no general test of understanding, entitlement or blame. A role audit should preserve the mathematical result and then identify the additional evidence needed for each further attribution.

5.1 Group interventions and the description of bearers

This extension examines how a causal attribution depends on the intervention assigned to a group. The purpose is to prevent a process-level role from being inferred by summing individual influence values without specifying how the group can act.

For $C \subseteq \{1, \dots, n\}$, let $x^{\oplus C}$ flip every coordinate in C and define $I_C^\mu(f) = \Pr\{f(X) \neq f(X^{\oplus C})\}$. This is a declared simultaneous-flip comparison. It differs from allowing the group to choose any new vector on its coordinates, and it differs from the sum of individual flip probabilities.

For the parity function $f(x_1, x_2) = x_1 \oplus x_2$, each single coordinate has influence one under every input distribution. Flipping both coordinates preserves parity, so $I_{\{1,2\}}^\mu(f) = 0$. The calculation follows because adding two modulo two leaves the output unchanged. Summed individual influence equals two while this joint intervention has zero influence.

The apparent reversal has a simple explanation: the intervention changed. A group permitted to alter either coordinate separately can change the parity outcome; a group constrained to flip both together cannot do so through that operation. The label “group influence” therefore needs an action set and a comparison, just as individual control does.

Bearer description also matters. Two coordinates may describe inputs controlled by one office, while one coordinate may aggregate several people's contributions. A coordinate's mathematical influence cannot be attributed to a person until the relation between coordinate and bearer is justified. A model

that divides one person's contribution into many bits can change an inventory of influences without changing the person's actual opportunities.

These observations support a process-level extension of the role framework. An integrated procedure may possess a functional capacity through coordination that no constituent realizes alone. To justify that attribution, specify the coordinated operation and the conditions sustaining it. Then examine the individual permissions, knowledge and duties through their own questions. The group description can explain organization while preserving the distinctions needed for an accountable procedure.

The example also limits comparisons across models. Influence values computed with different coordinate definitions or intervention schemes require a translation argument before being ranked. A larger number can reflect a different question rather than a stronger contribution in any institutionally meaningful sense.

6 Delegation, Rule Choice and Repeated Selection

This section derives how selection probabilities depend on control over a rule and over repeated draws. The examples distinguish a device's immediate contribution from the wider procedure that makes its output consequential. Each result assumes a deliberately simple two-recipient allocation.

Let S be uniform on $[0, 1]$ and let $p \in [0, 1]$. Define

$$D_p(S) = \mathbf{1}\{S < p\}. \quad (4)$$

The outcome one denotes a specified recipient, and zero the other. The choice of p belongs to the rule-setting stage; generating S belongs to the device stage. Uniformity and the mapping to recipients are assumptions of the construction.

Proposition 6.1 (Shared-input rule contrast). *Using the same random input for two parameter settings,*

$$\Pr\{D_p(S) \neq D_{p'}(S)\} = |p - p'|. \quad (5)$$

Proof. For $p \leq p'$, the outputs differ exactly on $[p, p']$, whose probability under the uniform distribution is $p' - p$. Reversing the order yields the absolute value. $\square$

The shared input is a specified coupling of counterfactual outcomes. Using independent inputs would answer a different comparison: two independent Bernoulli draws disagree with probability $p(1 - p') + (1 - p)p'$. A causal interpretation should state which intervention and background justify its coupling. Otherwise the phrase “effect of changing the rule” can conceal different mathematical quantities.

Now suppose an operator can repeat independent draws and accept the first favorable outcome, up to $n \geq 1$ attempts. If every attempt is unfavorable, zero is recorded. Let Y_n be the eventual result. Then

Proposition 6.2 (Selective repetition). *For independent draws with success probability p ,*

$$\Pr(Y_n = 1) = 1 - (1 - p)^n. \quad (6)$$

Proof. The final result equals zero exactly when all n potential draws fail. Independence gives probability $(1 - p)^n$ for that conjunction. Subtracting from one yields the stated probability. Potential draws after an earlier success need not be executed; they represent the equivalent fixed sample construction. $\square$

At $p = 1/2$ and $n = 3$, the probability becomes $7/8$. A perfectly implemented single-draw device can consequently participate in a procedure with a very different eventual distribution. The relevant control lies in the stopping and acceptance rule. If the procedure instead fixes the number of draws and uses only the last independent result, the probability remains p . Dependence between draws or intervention in their inputs changes the calculation again.

These results distinguish constrained delegation from an unrestricted transfer of control. An authority may delegate a particular selection operation while retaining the power to define eligible alternatives,

approve reruns or invalidate a defective process. Whether that retention is legitimate and effective requires its own assessment. The device’s reliable performance within its assigned operation does not settle the wider procedure’s fairness.

The examples also identify records needed for an audit. A final recipient alone cannot distinguish a single valid draw from selective repetition. A procedure interested in that distinction must preserve appropriately governed evidence of draw count, acceptance and relevant inputs or commitments. The precise record design depends on the system and its constraints; the mathematical example supplies the target that such a design would need to cover.

6.1 History-dependent acceptance and procedural records

This subsection generalizes the repeated-draw example just enough to identify its policy dependencies. Its aim is to distinguish a fixed random mechanism from a procedure that conditions acceptance on its history.

Let $D_1, D_2, \dots$ be potential device outcomes. Let τ be the attempt at which the procedure stops, determined by a declared acceptance policy and a maximum horizon where applicable. The implemented selection is $Y = D_\tau$. Its distribution depends jointly on the generating law of the draws and the stopping rule. Specifying the marginal law of a single draw leaves that distribution unresolved.

With independent Bernoulli draws, the first-favorable policy gives Equation 6. If instead the draws are perfectly repeated copies of one initial bit, $D_t = D_1$ for every t , the same policy yields $\Pr(Y = 1) = p$. More attempts add no new opportunity for success. This countercondition shows exactly where independence enters the earlier proof.

If the procedure permits a rerun only after a documented technical failure, the target event differs again. Modeling that arrangement requires a failure variable and a rule connecting it to acceptance. A failure correlated with recipient identity can alter the eventual distribution even when each validly generated raw draw has a desired marginal law. The present paper leaves such a mechanism as a proposed research problem; a claim of bias would require specifying and verifying it.

The history also matters for attribution of control. One operator may be able to request a rerun, while another office approves the request and a supervisor observes the next draw. Their contributions can be distributed across the stopping rule. Calling the device the sole selector would miss those dependencies; calling any single operator fully controlling would also exceed the described opportunities.

A useful procedural record should therefore be designed against declared history queries. An audit might ask how many attempts occurred, which were accepted, who authorized a repetition and which reason was recorded. The record must preserve the distinctions needed by those questions. A cryptographic or logging mechanism would require its own technical specification and validation before receiving credit for doing so. The present model identifies the target without inventing an implementation guarantee.

This analysis connects allocation to the repository’s work on retention. A final outcome can preserve recipient identity while losing the grounds for assessing the process. A justified record must balance the need for those grounds with access, privacy and the applicable retention conditions. The cross-paper connection is an analytical dependency, not an assumption that the same legal regime governs every allocation.

7 Information and Time for Effective Intervention

This section formalizes a limited sense of effective control. A person can have permission to override a device while lacking the information, time or permitted action needed to achieve an acceptable result. The derivation states when an observation-based intervention policy exists under finite declared conditions.

Let H be a finite set of possible procedure histories and $o : H \rightarrow O$ the information available to an operator. Write $H_z = \{h : o(h) = z\}$ for an attainable observation fiber. Let $A(z)$ be the feasible and permitted actions available after observation z . Let $G(h)$ be the actions meeting a declared acceptability

criterion in history h . The criterion may be procedural or task-specific; its justification remains separate from the existence result.

Assumption 7.1 (Information and action closure). The operator’s policy can depend only on z , the action sets include the relevant time and resource limits, and acceptability is represented by $G(h)$ for every admitted history. Each history’s consequences under an action are already incorporated in that criterion.

Proposition 7.2 (Compatible effective intervention). *There exists a deterministic policy α with $\alpha(z) \in A(z)$ and $\alpha(o(h)) \in G(h)$ for every $h \in H$ if and only if*

$$A(z) \cap \bigcap_{h \in H_z} G(h) \neq \emptyset \quad \text{for every attainable } z. \quad (7)$$

Proof. A successful policy selects one action at each observation. That action must belong to the permitted set and every acceptability set in the corresponding fiber, proving necessity. Conversely, choose an action from each nonempty intersection. Since the attainable observation set is finite, these choices define a policy satisfying the conditions for every history. $\square$

The result adapts the repository’s earlier finite branchwise compatibility argument to procedural intervention. It establishes existence within the model. It leaves training, actual execution, motivation and the justification of acceptability open. Randomization can change expected performance objectives, but cannot guarantee an acceptable action in every history when the required supports have an empty intersection.

For a constructed counterexample, suppose the operator sees one identical display in two possible histories. In h_1 , only accepting is acceptable; in h_2 , only rejecting is acceptable. Even if both actions are formally allowed, the intersection is empty. An additional discriminating observation can split the fiber and make a successful policy possible. This is an informational change, distinct from adding a permission already present.

Timing can create a separate failure. In a simplified sequential task, let T_a, T_u, T_e be durations for obtaining information, understanding it and executing an action, with deadline D . A required action is feasible under these assumptions only if

$$T_a + T_u + T_e \leq D. \quad (8)$$

Concurrent activities or uncertain durations require a different timing model. The inequality is a declared feasibility condition, not a universal psychology of human response. A nominal override window shorter than the required path can remove the intervention from $A(z)$ even when permission exists on paper.

Observed acceptance also leaves ability unresolved. A person who always accepts may lack capacity to intervene, choose acceptance under the circumstances or follow a defensible policy. Inferring powerlessness from agreement alone repeats the mistake of inferring control from a formal button alone. Evidence should examine feasible alternatives, information and timing together.

The proposed audit therefore distinguishes a right or permission to act, a model-relative opportunity and an actually exercised intervention. Preserving these distinctions makes claims about human oversight both more precise and more demanding.

7.1 Information refinement and action expansion

This extension distinguishes two ways to improve a model-relative opportunity: providing more discriminating information and enlarging the feasible permitted action set. The comparison keeps the acceptability target fixed, then identifies why real interventions can violate that assumption.

Suppose o' refines o , meaning $o = v \circ o'$ for a map v . Every finer observation fiber lies inside an earlier one. If the new action set at z' includes the old set $A(v(z'))$, a previously successful policy

remains available: define $\alpha'(z') = \alpha(v(z'))$. The action remains permitted and acceptable in each history because it implements the earlier successful choice.

The same argument proves that expanding $A(z)$ while holding observations and acceptability fixed cannot remove the existence of a successful policy. These are existence results. They do not establish that an actual operator will discover the policy, choose it or execute it accurately. Extra options can increase practical burden even when they preserve mathematical feasibility.

Example 7.3 (Separate repairs of an empty intersection). Let two histories share an observation. Suppose $G(h_1) = \{a, c\}$ and $G(h_2) = \{b, c\}$, while the permitted set is $\{a, b\}$. The intersection in Equation 7 is empty. Either a separating observation permits choosing a or b appropriately, or adding c to the permitted set permits a common choice without further information.

The two repairs have different institutional meanings. One supplies knowledge; the other supplies an alternative. Their costs and legitimacy can differ. If c represents deferral, adding it might preserve an acceptable possibility under uncertainty while creating a burden for applicants awaiting a resource. The model records its stipulated acceptability, and the normative assessment must examine whether that stipulation is warranted.

Real changes can also alter the target or opportunities. A more detailed interface can consume the time previously available for intervention. A newly offered action can change the expectations or rights against which acceptability is assessed. In those cases, the fixed sets assumed by the inclusion argument have changed. Reporting that change prevents a mathematical monotonicity result from becoming an unsupported claim that every increase in information or choice improves control.

The distinction provides a practical diagnostic for proposed safeguards. Identify whether the obstacle lies in indistinguishable histories, unavailable actions, insufficient time or contested acceptability. Then evaluate a repair against that obstacle and its side effects. A generic promise of “more human oversight” leaves those questions unanswered.

8 Random Allocation in the English School-Admissions Framework

This section uses a verified institutional setting to examine the relation between random selection, supervision and review. The discussion concerns the English codes’ stated scope and a constructed implementation. It leaves actual device deployment and case-specific compliance to further investigation.

The School Admissions Code’s paragraphs 1.34–1.35 address random allocation. Their conditions include clarity and transparency, applicable priority for looked-after and previously looked-after children, and independent supervision of the random process. The Code restricts a local authority’s use of random allocation as the principal oversubscription criterion across all schools for which it is the admission authority. It also requires a fresh random round when offering a waiting-list place under the specified arrangement [1]. These provisions make the procedure’s scope and organization material to the role analysis.

Imagine a compliant arrangement has already established an eligible tied group under its applicable priorities. A hypothetical device samples a recipient from that group. An independent person supervises the process, the admission authority remains responsible for the relevant arrangements, and an authorized communication follows. This description is an analytic implementation proposal. The code itself supplies no evidence that this particular device, sampling algorithm or interface is in use.

The selected group’s construction belongs to a different stage from its random selection. A mathematically uniform draw over the wrong eligible set can be procedurally defective. A correct eligible set can also be used in a manipulated acceptance process. Equations 5 and 6 explain possible differences in a simple model; they cannot establish compliance with the full admissions framework.

The waiting-list provision is especially useful for distinguishing authorized repetition from selective rerunning. A fresh round for a newly available place has a stated procedural role. Repeating an unfavorable draw for the same allocation until a preferred result appears is a different constructed policy. Both

involve multiple draws, yet their rules, targets and justifications differ. An audit must identify which event creates the occasion for a new round.

The School Admission Appeals Code introduces further questions. Its ordinary staged analysis includes the compliance of arrangements and their correct and impartial application, together with further conditions concerning prejudice and the appeal; infant-class-size appeals have a separate route [2]. A procedural concern consequently requires the appropriate review analysis and does not automatically produce an offered place. This distinction separates identifying a defect, holding review authority and possessing grounds for a particular remedy.

The role framework asks for evidence at each stage. Which rule established eligibility? Who could alter the candidate list? What mechanism generated the selection? Who could request another draw? What did the supervisor observe? Which record supports a challenge, and which office can act on it? These are proposed audit questions derived from the model and the bounded institutional comparison. Their answers depend on the actual arrangement.

The example makes a general analytical point without claiming a universal institutional pattern. Randomness is a property of a specified mechanism within a procedure. Authority, supervision and review are relations whose warrant comes from rules and facts beyond that mechanism's output distribution. Their combination is central to understanding how a device can participate in a consequential decision.

9 Comparisons across Human–Device Procedures

This section compares constructed arrangements with similar outputs and different roles. The purpose is to test the framework against cases that a single outcome measure would merge. Each comparison changes a specified feature while preserving other parts of the construction.

First compare an advisory selector with a binding selector. Both compute the same recipient and both are always accepted in the observed episodes. In the advisory arrangement, a person has a permitted and feasible alternative after considering relevant information. In the binding arrangement, the applicable rule assigns settlement to the validated device output. Observed agreement leaves this difference unidentified. Examining the rule and the person's counterfactual opportunities supplies evidence relevant to authority and control.

Second compare a uniform sampler over a justified eligible set with the same sampler over an unjustifiably narrowed set. Their conditional behavior within the displayed list can be identical. Functional performance of the sampler is therefore compatible with a defect at the eligibility stage. An audit confined to the sampled output would miss the source of the exclusion. The relevant intervention targets the construction of E in Equation 2.

Third compare a single authorized draw with selective repetition. The same final recipient can occur in both. Proposition 6.2 distinguishes their distributions under the stipulated assumptions, while a particular episode's history requires evidence of attempts and acceptance. A population-level difference and a historical attribution are different targets. The distribution alone cannot establish which policy produced one observed result.

Fourth compare two operators with identical permission to reject a proposal. One receives a clear display early enough to examine the relevant information; the other sees an ambiguous display moments before the deadline. Their formal authority can coincide while their feasible sets or observation fibers differ. Proposition 7.2 explains how either difference can affect effective intervention. The actual interface and timing must be measured before applying the model.

Fifth consider the source discussion's ceremonial stone. In a constructed practice, participants agree that one orientation selects the first claimant and another orientation the second. The stone's movement affects the recorded outcome, while a recognized interpreter identifies its orientation and participants treat the result as decisive. The case supports several possible role descriptions. It leaves understanding, independent policy revision and moral standing open. A verified account of an actual practice would require its own historical and ethnographic evidence.

Sixth consider a software selector that updates a policy from prior allocations. The procedure now contains a changing rule p_t and a training history. This can expand the device’s functional role beyond executing one fixed draw. It also makes control over training inputs, objectives and update permissions relevant. The example is a proposed type of system, with performance and deployment unspecified. The framework can describe the added roles without inferring institutional authority from adaptation alone.

These comparisons show why distributional equality is a weak stopping point for a procedural inquiry. A target concerning recipient probabilities can be identified while a target concerning valid authorization or feasible challenge remains open. The required record and evidence depend on the latter query. The comparisons also prevent a blanket verdict about nonhuman devices: their roles vary with their organization and the procedure in which they participate.

10 Standing, Authority and Distributed Responsibility

This section examines the normative boundaries of role attribution. The central issues are whether a functional role supports standing, whether distributed contributions dissolve responsibility, and how control should affect an obligation to answer for a procedure. These questions require reasons beyond the mathematical models.

A device’s participation can matter without settling its moral or legal status. A procedure may depend on a selector, preserve its output and assign it a named role. Those facts establish consequential organization under appropriate evidence. Claims about rights, personhood or interests require further criteria and arguments. The source discussion deliberately leaves those matters open, and the present framework preserves that suspension.

Institutional authority also differs from moral justification. A rule can confer a power whose exercise remains open to ethical criticism. Conversely, someone may have a strong reason to challenge an allocation while lacking authority to revise it directly. The framework therefore records an authority attribution and the justification of that authority separately. This separation prevents the applicable rule from becoming the complete standard of criticism.

Distributed contribution raises a familiar practical danger within the constructed model: each participant may point to another stage. The operator accepted the device’s result; the designer implemented the specified rule; the authority relied on the operator’s supervision. The decomposition should make these dependencies visible. It should not make a finding of multiple contributions an excuse to stop asking who owed which task, who could obtain relevant knowledge and who had a feasible opportunity to act.

Claim 10.1 (Role-specific answerability). Where a person or office holds a justified duty within an allocation procedure, an assessment of answerability should examine that duty together with relevant knowledge, feasible opportunities and the contribution actually made. A causal influence score alone leaves those conditions unresolved.

This is a proposed normative orientation, with particular duties and consequences requiring their own source and case support. It allows several actors to owe different explanations for one outcome. The person choosing eligibility may owe reasons for exclusions; the operator may owe an account of repetitions; a reviewing office may owe reasons for its treatment of a challenge. These proposed examples identify questions, without assigning legal liability to a hypothetical actor.

Control can also be unequal within a nominally shared procedure. A participant may be permitted to object while lacking access to the rule or enough time to formulate a challenge. The finite intervention model supplies a way to describe that gap. Whether the arrangement is unjust depends on further considerations, including the stakes, available alternatives and the institution’s responsibilities. A mathematically empty intersection can reveal a constraint without supplying its moral verdict.

An objection to the framework is that role separation fragments an integrated act. Some procedures may have an organized collective contribution that individual stage descriptions miss. The proposed response is to allow a compound bearer or process-level role when its warrant is stated. Such a description should explain coordination and dependencies, then preserve the distinct questions about the constituent

stages. A collective label becomes explanatory when it identifies organization beyond an inventory of parts.

The resulting position permits distributed decision roles while resisting automatic promotion from participation to standing or from causal sensitivity to blame. Its normative demand is for an argument appropriate to the status being attributed, with the procedure's information and opportunities visible enough for that argument to be assessed.

11 A Prospective Audit of Role Attribution

This section proposes a method for evaluating the framework. The audit would examine whether separate role questions yield more accurate and useful descriptions of constructed procedures. It remains a research proposal, with empirical validation and institutional deployment open.

The initial materials would be fictional allocation vignettes whose rules, stages and available records are fully specified. Matched pairs would preserve the same final recipient or distribution while varying authority, eligible-set control, access to information, permitted reruns or intervention time. Other pairs would preserve authority while changing causal influence. These comparisons directly test the distinctions developed in the paper.

An audit record would begin by naming the target role, bearer, episode and scope. It would then identify the relevant rule or task, the information available, the feasible and permitted interventions, and the evidence supporting the attribution. Unresolved fields would remain explicit. An entry that merely labels a procedure "automated" would be insufficient for questions about effective override or valid settlement.

For the finite mathematical cases, independent enumeration can establish the proposed output function and influence values. A separate procedural assessment can classify the stipulated authority and action sets. Comparing those results tests whether a single measure would merge cases the framework distinguishes. It also checks the analyst's fidelity to the construction before introducing human judgment data.

Participants in a prospective study could assign roles and explain their reasons after reading the vignettes. Outcomes might include conflation of influence with authority, failure to notice selective repetition, recognition of inadequate information, and appropriate use of unresolved status. These are proposed measures whose coding requires validation. Agreement with the research team's preferred label is weak evidence if the label's warrant has been concealed or the vignette remains ambiguous.

Research Obligation 11.1 (Audit validation). Predeclare the role targets, stipulated rules, intervention backgrounds, feasible action sets and conditions for unresolved attribution. Test the coding against adversarial examples, including equal outputs with different authority and equal authority with different practical opportunities.

An institutional pilot would require actual workflow evidence: governing documents, interface observations, timing, available records and the relevant participants' accounts. Each source answers different questions. An operator's account can support interpretive attribution while leaving the technical output law unverified. A system log can show attempts while leaving the operator's permitted alternatives unresolved. A rule can establish a power while leaving its practical usability open.

The audit should also examine its own burden. An exhaustive stage inventory can overwhelm a simple procedure. The appropriate level of detail depends on the disputed role and consequences. A compact analysis may suffice when the rule, output and opportunities are clear. Greater complexity is justified when a consequential attribution depends on hidden stages, information asymmetry or a contested intervention.

The framework would receive support if it reliably exposes material differences, preserves uncertainty and helps identify the evidence needed to resolve a dispute. It should be revised when its role categories repeatedly overlap without explanatory benefit, omit an integrated contribution or encourage false certainty. Those are substantive failure conditions, not merely opportunities to add more labels.

12 Further Research and the Scope of the Results

This section identifies the further work required by the proposed attribution method. The paper has supplied a staged vocabulary, limited derivations and comparative cases for shared-resource allocation. Broader theories of agency and institutionally deployed audits require additional conceptual, empirical and normative work.

The formal contribution is a set of explicit distinctions. Boolean influence depends on a function, an intervention and a reference distribution. A rule contrast depends on its counterfactual coupling. Selective repetition depends on independence and an acceptance policy. Effective intervention depends on information, feasible actions and a justified acceptability target. Each result becomes useful through those conditions and can mislead when they disappear from its interpretation.

The staged model also requires extension for richer resources and collective outcomes. Multiple places, ranked preferences and constraints across allocations change the task relation. Repeated decisions can create path dependence, and the eligible set can evolve through prior outcomes. A useful extension should specify the additional dependencies before importing a binary influence quantity into the enlarged setting.

Adaptive devices present another research branch. When the selector changes its policy through interaction, identifying its role requires a history of objectives, updates, permissions and responses. A device can acquire broader functional discretion while its institutional power remains constrained by an external arrangement. Studying that difference would connect the present paper to the repository's work on interpretation and retained organization, while preserving the distinct warrants of each inquiry.

The normative branch concerns the relation among duties, opportunities and collective organization. The paper's answerability claim is a limited orientation. A substantive account of responsibility must confront verified normative and legal antecedents, identify the relevant standards and explain how they apply to actual cases. A numerical contribution measure can be one piece of evidence while leaving that larger task open.

The ceremonial and stone examples also deserve careful treatment. Their value here is to expose an exclusion based merely on material type or apparent staticness. Actual practices may organize interpretation and authority in ways that the constructed case fails to capture. Further development requires primary historical or ethnographic work, with the repository's questions kept distinct from the participants' concepts.

Finally, the audit should remain responsive to its own target. Some inquiries concern probabilities, some concern valid procedures, and others concern a person's opportunity to challenge a result. A model that answers one can leave the others unresolved. The framework's continuing value depends on making that division visible without losing the integrated procedure in which the roles interact.

The resulting research position is that decision participation is organized through relations whose types of warrant matter. A device can make a consequential contribution, a person can hold a constrained power, and a process can coordinate both. Describing those roles carefully provides a basis for criticism and further inquiry while leaving unsupported conclusions about personhood, authority and responsibility open.

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