

Beyond Unity and Fragmentation

Relational Crystallization and the Emergence of Generative Plurality

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Abstract

A polity holding several coherent traditions faces two opposite dangers. It may dissolve them into a single order, and it may allow them to separate until nothing passes between them. Governance is ordinarily theorized as the avoidance of the second, with the first taken as the ideal it approaches. This paper examines a picture in which both are failures of the same kind. Polycrystalline materials hold their form through many domains of differing orientation, joined at boundaries that are structures in their own right, and such materials are neither single crystals nor powders. The paper asks what transfers from that picture to the governance of plural relational domains, states carefully what fails to transfer, and builds a formal model on the part that survives. The model is deliberately non-optimizing: it states two positivity conditions, one requiring that each domain retain the capacity to reproduce its own meaning system and one requiring that each boundary permit exchange without imposition, and it defines injustice as the loss of either. Four failure modes follow as a theorem rather than a list, of which unity and fragmentation are two. A three-domain case study is worked through with the dynamics displayed at each step. The survey concedes that the productivity of interfaces is established in the study of scientific subcultures, that plural centres without an apex are established in the study of polycentric governance, and that the rejection of maximization is settled in political philosophy. What appears to remain open is the boundary itself, treated as a structure with properties of its own, and the nucleation of novelty at boundaries, which is the point at which the crystallographic source supplies something its rivals do not. The account's propositions hold of the model that generates them, and the conditions under which they would bear on a polity are set out with the questions that would have to be settled first. Those questions are held to be the more valuable half of what is offered.

Keywords: relational plurality; grain boundaries; viability conditions; co-creation; generativity.

A note on the standing of this paper. This is a discussion paper and the fourth of a series. It identifies a structure, surveys what existing scholarship has made of it, marks where each account appears to run out, and proposes a direction for work. It does not complete a theory. Two features require notice at the outset. The paper borrows from a natural science, and §4 states what licenses the borrowing and where it fails, since an undisciplined transfer of physical vocabulary into social theory is a known and deserved target of criticism. And the paper offers a formal model whose propositions are consequences of stipulated dynamics; they are true of the model, and their bearing on any polity is a further question the paper does not settle. Where a claim entered here as open is in fact occupied, or where a source has been read wrongly, correction is welcome at huangwanhong@serendip.ngo.

1 INTRODUCTION

A polity containing several coherent traditions of practice may fail in two opposite ways. Its traditions may be dissolved into a single order, whether by assimilation, by the imposition of one tradition's categories on the rest, or by the ordinary administrative preference for uniform procedure. Or its traditions may separate, until each is internally intact and nothing of consequence passes between them. The first failure leaves a polity that decides easily and has lost what it might have decided among. The second leaves a polity whose parts survive and which cannot act as one.

Political theory has treated these asymmetrically. Fragmentation is the recognized danger, and the literatures on cohesion, integration, and shared identity are directed against it. Unity, by contrast, tends to appear as the limiting ideal, approached where circumstances allow. A polity that achieved perfect agreement would be counted fortunate.

The picture examined here treats both as failures, and it takes its structure from a class of materials in which neither occurs. Most solids of practical importance are polycrystalline. They consist of many domains, each internally ordered, whose orientations differ, joined along boundaries where the two orders meet. A polycrystal is not a single crystal, since no orientation extends through the whole. It is not a powder, since the domains are joined and the material holds together and bears load. Its properties depend as much on the boundaries as on the domains, and in several respects the boundaries are where the interesting behaviour occurs.

The suggestion pursued below is that this structure supplies a picture of plural governance that is available in neither of the received alternatives. A polity may be ordered locally without being ordered globally. Difference of orientation may be a condition of its properties. The boundary between two domains may be a structure with characteristics of its own, and the site at which something arises that neither domain contained.

Three difficulties motivate the enquiry. The first concerns the form of the normative question. A theory of plural governance that proceeds by maximizing a measure of the good imports commitments that the plurality was introduced to resist: that there is a measurable good, that arrangements are rankable, that losses in one domain are compensable by gains in another. The second concerns the treatment of boundaries. Where boundaries between communities are theorized at all, they tend to appear as obstacles to be reduced or as defects to be managed, and the possibility that a boundary is a productive structure is seldom pursued formally. The third concerns the conditions of novelty. Accounts of plural governance that stop at coexistence describe an arrangement in which nothing new arises from the plurality, and an arrangement of that kind is difficult to distinguish from an elaborate separation.

Five bodies of work bear on these difficulties and each is treated below. The study of scientific subcultures has established that groups without a shared global framework coordinate at

interfaces where new intermediate structures develop (Galison, 1997; Star & Griesemer, 1989). Postcolonial and ethnographic work has argued that encounter at zones of difference produces forms reducible to neither party (Bhabha, 1994; Pratt, 1992; Tsing, 2005). The study of polycentric governance has established that order may be sustained by many centres of decision with overlapping jurisdictions and no apex (Polanyi, 1951; Ostrom et al., 1961; Ostrom, 2010). Political philosophy has, since the rejection of aggregative utilitarianism, developed conceptions of justice as thresholds, capabilities, and relational conditions in place of maxima (Rawls, 1971; Frankfurt, 1987; Anderson, 1999; Sen, 1999; Pettit, 1997; Berlin, 1969). Viability theory has, finally, developed a mathematics of systems constrained to remain within a set, with no optimization performed (Aubin, 1991; Aubin et al., 2011). These literatures settle a considerable proportion of the subject, and one purpose of this paper is to record how much; the survey below accordingly proceeds by conceding first and locating what remains afterwards.

The account is developed within a framework the author has set out elsewhere, and which may be stated in a sentence:

Generative Relational Being. *A theory of how subject, meaning, value, creation, and normativity co-emerge through generative relational processes.*

Three features of the framework bear here. Values are treated as generated within relations, so that a plurality of relational structures yields values differing in kind. Relations are treated as generative, in that a relation in working order produces further determinations of meaning, of value, and of what the relation itself requires. The framework carries, third, a methodological commitment governing how its claims are advanced, its keynote being historical dialectics, so that propositions are entered as revisable within a historical frame, the finitude of knowledge and of practice is acknowledged as a standing condition, and philosophical argument is joined to operable formulation where the subject admits of it. A paper written under that commitment states its claims so that they may be given up.

Four contributions are offered, claimed with differing degrees of confidence. The first is a disciplined statement of the analogy, given in §4, which enumerates what transfers from crystallography and what fails to. The second is the pair of conditions stated in §9, which express plural governance as constraint satisfaction with no aggregation across domains. The third is the formal model of §10, whose propositions include a derivation of four failure modes from a single requirement, and a demonstration that the conditions leave the arrangement underdetermined. The fourth is the treatment of the boundary developed in §13, where the crystallographic source supplies a structure that the surveyed rivals leave thin. The questions set down in §16 are held to be the more valuable half of what is offered.

2 SCOPE AND METHOD

2.1 *The Containment Device*

Three questions organize the discussion, and material earns its place by discharging one of them:

- (i) *The transfer*. What may be carried from the behaviour of polycrystalline materials to the governance of plural relational domains, and on what licence.
- (ii) *The conditions*. What must hold of a plural arrangement for it to be defensible, given that maximization is unavailable.
- (iii) *The boundary*. What a boundary between domains is, and under what conditions it produces something that neither domain contained.

2.2 *The Conceding Register*

A theory whose components are individually unoriginal may still contribute through the manner in which it joins them, and the joining becomes visible only where the components are correctly attributed. The argument here is architectural, and an architectural argument is weakened by any claim to have quarried its own stone. Three sections consist largely of concessions and are placed before the constructive material for that reason.

2.3 *The Boundaries of the Undertaking*

Three matters lie outside the enquiry, and marking them fixes what the conditions of §9 are conditions upon.

The object is a condition, with arrangements left aside. A condition specifies what an arrangement must leave possible; which arrangements would secure it in a given polity is a question of institutional design, and §16 enters it as such.

The object is a model, and the polity such a model might describe is a further matter. The case study of §11 works through the model's behaviour under varied parameters, and its labels are chosen for legibility.

The individuation of domains is taken as given. Which relational structures a polity comprises, and by what procedure they are identified, is the presupposition on which every condition below rests, and §16 treats it as the account's first outstanding question.

2.4 *A Note on Sources*

The survey draws on materials science, science and technology studies, political theory, philosophy of science, and applied mathematics. The transfer of concepts across such a range is the paper's principal methodological risk, and §4 is written to contain it.

3 CRYSTALLIZATION AS A SOURCE PHENOMENON

The source is set out before anything is drawn from it, since a transfer from a misdescribed source is worthless. The account is given in the terms the physical literature uses, with the governing relations stated, because a qualitative paraphrase would conceal exactly the conditions under which each relation holds, and those conditions bear on what may be transferred.

3.1 Nucleation and the Existence of a Barrier

A liquid cooled below its freezing point does not order itself at once. For a spherical region of the ordered phase of radius r , the free-energy change relative to the disordered parent is a competition between a volume term that favours the new phase and a surface term that opposes it:

$$\Delta G(r) = -\frac{4}{3}\pi r^3 \Delta g_v + 4\pi r^2 \gamma, \quad (1)$$

where $\Delta g_v > 0$ is the magnitude of the bulk free-energy reduction per unit volume of the new phase and $\gamma > 0$ is the interfacial free energy per unit area. The surface term dominates at small r and the volume term at large r , so ΔG rises, attains a maximum, and falls. Small regions therefore lower their free energy by disappearing, and only a region carried past the maximum by fluctuation can lower its free energy by growing. Setting $d\Delta G/dr = 0$,

$$r^* = \frac{2\gamma}{\Delta g_v}, \quad \Delta G^* = \frac{16\pi\gamma^3}{3\Delta g_v^2}. \quad (2)$$

Panel (a) of Figure 1 plots (1) with its two components; the marked maximum agrees with (2) to the resolution of the plot. Two features matter below. Order begins at scattered points, leaving the rest of the field unchanged, and a threshold must be crossed before a new arrangement can persist at all.

3.2 Why New Order Appears at Boundaries

A region forming on a pre-existing surface need not create its whole interface anew. For a spherical cap meeting a flat substrate at contact angle θ ,

$$\Delta G_{\text{het}}^* = f(\theta) \Delta G_{\text{hom}}^*, \quad f(\theta) = \frac{2 - 3\cos\theta + \cos^3\theta}{4} = \frac{1}{4}(1 - \cos\theta)^2(2 + \cos\theta), \quad (3)$$

with $f(180^\circ) = 1$, $f(90^\circ) = \frac{1}{2}$, and $f \rightarrow 0$ as $\theta \rightarrow 0$. Panel (b) of Figure 1 plots it. One point of precision should be recorded, because it is often mistaken: $f(\theta)$ reduces the *barrier* and leaves the critical radius r^* of (2) unchanged.

The case that matters here is formation on a boundary between two existing domains, treated by Clemm & Fisher (1955) and Cahn (1956). A region forming there destroys a patch

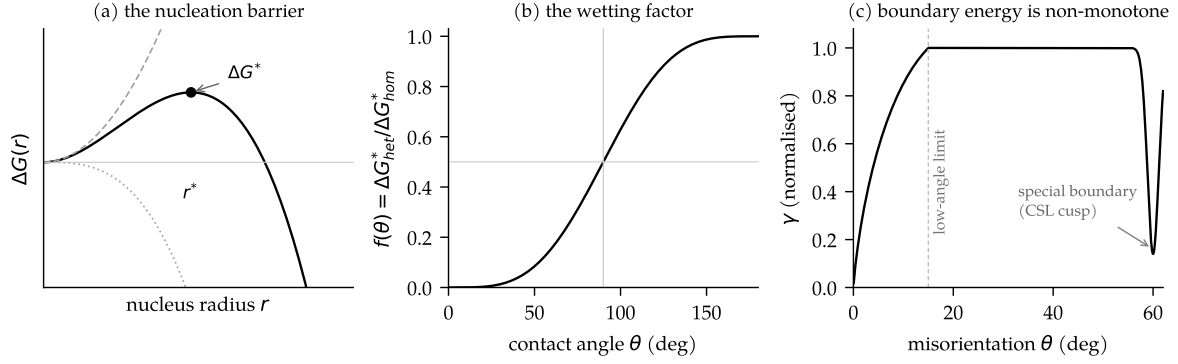


Figure 1: The three relations of §3 that carry weight later, computed from the expressions given in the text. In (a) the free energy of a forming region, equation (1), with the volume and surface contributions dotted and dashed and the critical point marked. In (b) the factor $f(\theta)$ of equation (3), by which a pre-existing surface reduces the barrier. In (c) boundary energy against misorientation: the Read–Shockley form (5) below the low-angle limit, an approximate plateau above it, and a cusp at a special misorientation. Panel (c) is schematic in its vertical scale and is drawn to display the shape, in particular the non-monotonicity on which §4 turns.

of the boundary, and the energy of the destroyed boundary is a credit against the interface that must be created. With $k = \gamma_{\alpha\alpha}/2\gamma_{\alpha\beta}$, the ratio of the energy of the existing boundary to twice the energy of the new interface, the barrier falls as the credit rises, and it falls further at junctions where more boundary area is destroyed per unit volume formed. The ordering established there is

$$\Delta G_{hom}^* > \Delta G_{face}^* > \Delta G_{edge}^* > \Delta G_{corner}^*, \quad (4)$$

with formation on a given class of site occurring only below a critical k for that class. New order therefore appears preferentially where existing domains meet, and most readily where several meet at once. This is the single relation from which §13 draws most, and it is stated here in the form its authors gave it.

3.3 Boundary Structure and Misorientation

A boundary between domains whose orientations differ by a small angle θ may be described as an array of dislocations of spacing b/θ . Summing their elastic energies gives the Read–Shockley relation

$$\gamma(\theta) = \gamma_0 \theta (A - \ln \theta), \quad \gamma_0 = \frac{Gb}{4\pi(1-\nu)}, \quad (5)$$

with G the shear modulus, b the Burgers vector magnitude, ν Poisson’s ratio, and A a constant absorbing the dislocation core. The relation holds for low-angle boundaries only, conventionally $\theta \lesssim 15^\circ$; above that the description fails, and boundary energy approximately saturates.

Two facts about γ as a function of misorientation are decisive for this paper and are recorded here in the body of the text. Boundary energy is **not monotone** in misorientation: it rises steeply at small angles, saturates at large ones, and possesses deep local minima at particular misorientations at which the two lattices share a high proportion of sites. The coherent twin is the standard instance, with an energy in copper roughly an order of magnitude below that of a general boundary. A boundary is specified, second, by five macroscopic parameters, three for the relative orientation and two for the plane on which the boundary lies, so that misorientation underdetermines boundary properties. Panel (c) of Figure 1 displays the shape. Section 4 states what this forbids.

3.4 *Boundaries as Obstacle and as Weakness*

Boundaries impede the propagation of deformation, and the yield stress of a polycrystal rises as domain size d falls:

$$\sigma_y = \sigma_0 + k d^{-1/2}. \quad (6)$$

A material of many small domains is stronger than one of few large domains, which is the physical sense in which plurality of domains is not a defect. The relation reverses below a crossover size, where deformation passes to boundary-mediated mechanisms and further refinement weakens the material; simulations of copper place the maximum near ten to fifteen nanometres.

Boundaries are at the same time paths of enhanced transport, with diffusion along them exceeding diffusion through domain interiors by several orders of magnitude, and they are the sites at which impurities collect and at which fracture propagates when they do. Boundaries are accordingly neither defects nor advantages as such: what they do depends on the property in question and on what has collected at them.

3.5 *Coarsening and the Status of Plurality*

Boundary migration is driven by curvature. With mobility M and driving pressure P ,

$$v = MP, \quad P = \gamma\kappa = \frac{2\gamma}{r}, \quad (7)$$

so a boundary moves toward its centre of curvature and boundary area falls. In two dimensions the von Neumann–Mullins relation makes the consequence exact for an isotropic domain of n sides:

$$\frac{dA}{dt} = \frac{\pi M \gamma}{3} (n - 6), \quad (8)$$

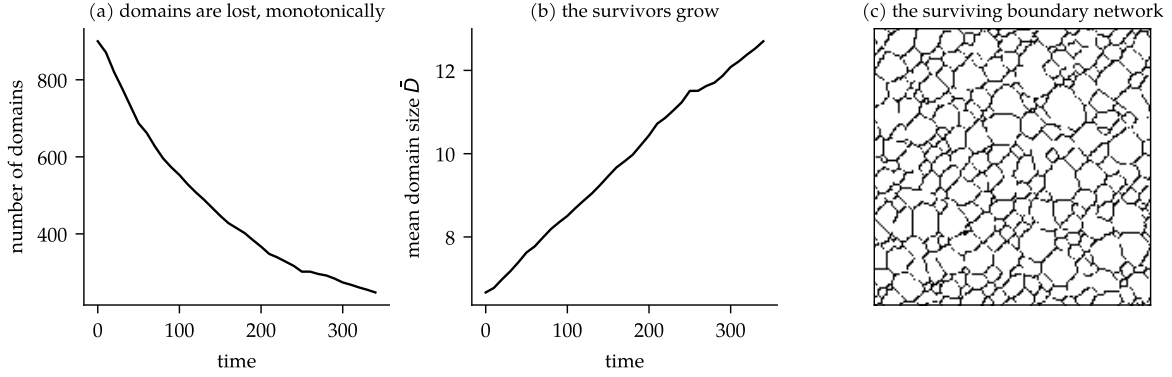


Figure 2: Coarsening in a simulated field of 900 domains under a local curvature-reducing rule. In (a) the number of surviving domains falls monotonically; in (b) the mean size of the survivors rises; in (c) the boundary network that remains at the end of the run. The simulated rule coarsens somewhat faster than curvature flow, giving a best-fit exponent near 1.5 in (9) where the ideal value is 2; the exponent carries no weight below. The simulation is used for the monotone and unforced loss of domains, which is robust to the choice of local rule.

so that domains with more than six neighbours grow, those with fewer shrink, and six is neutral. Domains are lost, the survivors enlarge, and the mean size advances as

$$\bar{D}^n - \bar{D}_0^n = \kappa t, \quad (9)$$

with $n = 2$ in the ideal isotropic case and values between two and four usual in real materials.

Figure 2 shows the process in simulation. Two consequences follow, and the second bears directly on any normative reading of the source.

Coarsening requires no agent. No domain intends to consume its neighbours; each boundary responds to its own curvature, and the loss of plurality follows from local rules operating everywhere.

Plurality is, second, not the favoured state. Every boundary carries positive excess free energy, so the single crystal is the thermodynamic ground state and a polycrystal is metastable, held in being by kinetic obstruction: low mobility, and obstacles that pin boundaries. A dispersion of second-phase particles of radius r and volume fraction f exerts a pinning pressure that arrests growth at a limiting size of order

$$R_{\text{lim}} \sim \frac{r}{f}, \quad (10)$$

with the standard estimate $R_{\text{lim}} = 4r/3f$ and prefactors varying with convention. Where such obstruction is absent or is removed, coarsening resumes; and where normal coarsening is pinned but a few boundaries escape, a small number of domains may grow to consume

the rest, a condition known as abnormal growth, which is the route by which a polycrystal is deliberately converted into something approaching a single crystal.

One qualification belongs with this, since it will matter in §15. The metastability of plurality is not absolute. If a segregating species accumulates at boundaries it lowers their energy, in the simplest treatment

$$\gamma = \gamma_0 - \Gamma_s (\Delta H_{\text{seg}} + k_B T \ln X), \quad (11)$$

and where γ is driven toward zero the driving force for coarsening vanishes and a plural state of finite domain size becomes a genuine equilibrium in place of a delay. This route is predicted and simulated; it is not experimentally settled, and it is reported here as a possibility whose confirmation is outstanding.

3.6 Findings and Residue

Order arises at scattered points and only past a threshold, extends by local rule with no global calculation and nothing optimized, and produces boundaries where independently oriented regions meet. New order forms preferentially at those boundaries, and most readily where several meet, because forming there destroys existing boundary and recovers its energy. Boundary energy rises with misorientation at small angles, saturates at large ones, and dips at special misorientations, so that difference and boundary property are not monotonically related. Boundaries both obstruct deformation and provide the paths along which transport and failure travel. Plurality of domains is metastable and decays by coarsening unless something obstructs it, and may in principle be stabilized by a species that accumulates at the boundaries themselves.

4 THE DISCIPLINE OF THE ANALOGY

Transfers of physical vocabulary into social theory have a poor record, and the criticism they have attracted is largely deserved (Sokal & Bricmont, 1998). The apparatus for conducting such a transfer responsibly is available, and this section applies it.

4.1 The Status Claimed

Hesse (1966) distinguishes, within an analogy, the properties known to be shared, the properties known to differ, and the properties whose status is undetermined. Inference proceeds from the first, is blocked by the second, and is licensed as conjecture in the third. Black (1962) treats a model as an instrument that organizes a subject by importing a structure, and holds that its value lies in the questions it makes askable.

The status of the analogy. *Crystallography is used here as a source of structure and as a generator of hypotheses. Its role is exhausted by those two functions: the claims advanced*

below concern relational domains and their boundaries, and they are answerable to grounds internal to political theory and to the model of §10. The formal model of §10 is a stipulated system whose propositions are true of it, and whose relation to any polity is conjectural.

4.2 The Positive Analogy

Four structural features are carried forward.

Local rules producing extended order. Coherent arrangements may extend across a population through interactions each of which is local, with no participant possessing the structure of the whole.

Independent nucleation producing differing orientations. Where order begins at several points independently, the resulting domains will differ in orientation as a matter of course, and their difference requires no explanation beyond the independence of their origins.

Impingement producing boundaries. Where two coherent domains meet, the surface of meeting is a structure determined by both and identical with neither.

Boundaries as sites of formation. The conditions at a boundary differ from those within either domain, and formation of something new occurs preferentially there.

4.3 The Negative Analogy

Eight disanalogies are entered. Each marks a feature of the source that is absent from the target, and the arguments below are constructed so as to turn on features common to both. Two of them correct a transfer the source will not license and are given first.

Units of a material have no interests, purposes, or interpretations. An atom does not understand its neighbours, and nothing in the physical case corresponds to a party's reading of what another party is doing. Every argument below concerning imposition and rendering has no physical counterpart.

Grain boundaries are not evaluable. A physical boundary is neither just nor unjust. The normative content of every condition below is supplied by argument in political theory and is imported into the model, never derived from the physical source.

There is no social free energy. The physical account has thermodynamic potentials that make certain evolutions determinate. No quantity of that kind is claimed here, and the model below is stipulated, with no derivation from any such potential.

Orientation is not measurable. A crystallographic orientation is a physical quantity. The orientation of a relational domain, used below, is a placeholder for a mode of articulation and admits of no measurement procedure at present.

Domains are not sharply bounded. A grain has a definite extent. A relational domain does not, its membership is contested, and parties belong to several at once. The model treats domains as discrete because a tractable model requires it, and §15 records the cost.

Coarsening is not deliberate; homogenization often is. The physical process has no agent. The corresponding social process frequently has one, and the difference matters at every point where responsibility is at issue.

Difference does not vary monotonically with the property of the boundary it produces. This is the disanalogy most likely to be assumed away, and §3 recorded the physical fact that forbids it. Boundary energy rises with misorientation only at small angles, saturates thereafter, and falls sharply at special misorientations where the two orders happen to share structure. Two consequences follow for the model of §10. Where co-creation is made to increase with misorientation, that is a stipulation of the model and is licensed by the physical source only within the low-angle regime. The physical source positively suggests, second, something the model does not represent: that particular pairs of widely differing domains may stand in an unusually accommodating relation, cheap to maintain and correspondingly unproductive of anything new, which would be a special boundary in the sense of (5) and its exceptions. Whether relational analogues of such boundaries exist is entered among the open questions.

Boundary properties are underdetermined by misorientation. A physical boundary requires five parameters and misorientation supplies three. The model below characterizes a boundary by a single difference and two further quantities, and does so for tractability; nothing in the physical source suggests that so coarse a description is adequate.

4.4 Findings and Residue

Four structural features transfer and six do not. The transferred features concern the emergence of plural coherent domains, the structure of what lies between them, and the preferential formation of novelty there. The normative content of the account is imported from political theory, and the physical source supplies structure and vocabulary alone.

5 INTERFACES AS PRODUCTIVE SITES

The largest concession of this paper is owed to work that reached its central observation some decades ago.

5.1 Trading Zones

Galison (1997) examines the coordination of scientific subcultures that hold incompatible commitments and share no global framework. Theorists, experimenters, and instrument builders coordinate, on his account, within a trading zone, where a local interlanguage develops for the purposes of exchange. The interlanguage begins as a restricted jargon adequate to a particular transaction, thickens into a pidgin as the exchange becomes routine, and may stabilize into a creole rich enough to support a practice of its own. His summary formulation, that what is

achieved is local coordination without global meaning, states the position this paper's picture arrives at from another direction.

The consequence for the present account should be stated without qualification. The proposition that coherent domains lacking a common framework generate, at their interface, a structure belonging to neither and capable of becoming a domain in its own right is not new. It is the central finding of the trading zone literature, and a paper presenting it as a discovery would be mistaken.

5.2 *Boundary Objects*

Star & Griesemer (1989) identify objects that inhabit several communities at once, plastic enough to be adapted to local requirements and robust enough to preserve a common identity across sites. Their treatment establishes that an interface may be inhabited by stable structures with characteristics of their own, and it develops a typology of such structures.

5.3 *Contact, Hybridity, Friction*

Three further treatments occupy the same terrain from the humanities. Pratt (1992) names the contact zone, in which parties previously separated encounter one another under conditions of asymmetry. Bhabha (1994) argues that cultural statements are constructed in a third space of enunciation, and that what emerges from encounter is irreducible to either source. Tsing (2005) treats friction at zones of difference as productive of new arrangements.

5.4 *Coexistence and Co-Creation*

A parallel debate has been conducted in policy terms. The critique of multiculturalism developed by Cantle (2005, 2012) holds that arrangements securing the standing of communities may produce parallel lives, in which groups occupy the same territory with little contact and develop separately. Interculturalism, developed by Bouchard (2012) and others, proposes that diversity be treated as a resource generating something new, and Parekh (2000) argues that cultures are transformed through dialogue.

The move from coexistence to co-creation is accordingly the constitutive move of an existing position, and this paper inherits it.

5.5 *The Residue*

Two things appear to remain.

The literature theorizes interlanguages, boundary objects, and third spaces without supplying a typology of boundaries by structure. Whether a boundary strengthens or weakens what it joins, and what distinguishes the two cases, is not the question these accounts were built to answer.

The literature is, second, largely descriptive. It establishes that interfaces are productive and examines the conditions under which coordination succeeds. A criterion distinguishing a defensible interface from an indefensible one is not among its products.

5.6 *Findings and Residue*

The productivity of interfaces is established, the move from coexistence to co-creation is the constitutive move of interculturalism, and the vocabulary of interlanguage, boundary object, contact zone, and third space is available. The residue lies in the structural typology of boundaries and in a normative criterion for them.

6 PLURAL CENTRES WITHOUT AN APEX

The polycrystalline picture has a close relative in governance theory, and the relation must be stated plainly.

6.1 *Polycentricity*

Polanyi (1951) introduced polycentric order for arrangements in which many participants adjust to one another under general rules with no central direction. Ostrom et al. (1961) carried the term into political science, describing metropolitan governance as many formally independent centres of decision with overlapping jurisdictions and no single apex. Ostrom (2010) developed polycentric governance for common-pool resources, establishing that arrangements of this kind sustain themselves and explain outcomes that a dichotomy of market and state does not.

6.2 *The Concession*

The structural claim of the polycrystalline picture, that a stable order requires local coherence within domains together with workable relations among them and requires no single order extending through the whole, is the claim of polycentricity. The two words describe the same structure, and a reader familiar with the polycentric literature will find the picture familiar.

6.3 *The Residue*

One asymmetry between the two literatures leaves room, and it is the asymmetry this paper works in.

Polycentricity theorizes centres. Its unit is the decision centre, its questions concern the autonomy of centres, the overlap of jurisdictions, and the rules under which centres adjust to one another. What lies between centres is treated as relation, adjustment, or contract, with no structure of its own.

The crystallographic source theorizes boundaries as intensively as domains. A boundary has a structure determined by the orientations meeting at it, that structure varies, and the variation accounts for much of the material's behaviour. Section 13 develops this asymmetry into the paper's principal claim.

6.4 *Findings and Residue*

Plural centres without an apex are established ground, and the polycrystalline picture adds nothing to that claim. The residue lies in the treatment of what stands between centres, which polycentricity leaves thin and which the crystallographic source treats as a structure.

7 JUSTICE WITHOUT MAXIMIZATION

The refusal to write justice as an optimization problem is the paper's normative starting point and is a settled position in political philosophy.

7.1 *The Established Refusal*

Rawls (1971) rejects aggregative utilitarianism on the ground that it fails to take seriously the distinction between persons, since a maximizing criterion permits a loss to one to be compensated by a gain to another as though the two occurred within a single life. The rejection is foundational to the subsequent literature.

Four developments carry it further. Frankfurt (1987) argues that what matters is whether each has enough, so that the criterion is a threshold and no maximum is required. Sen (1999) and Nussbaum (2011) frame justice in terms of capabilities secured, which is a condition on what a person is able to do and be. Anderson (1999) grounds equality in the character of the relations in which persons stand, so that the object of concern is relational in character. Pettit (1997) treats freedom as non-domination, which is a property of a relationship and admits of no natural maximand. Berlin (1969) argues, finally, that fundamental values are plural and incommensurable, so that no single scale is available on which arrangements could be ranked.

7.2 *The Concession*

A paper claiming credit for rejecting maximization would be claiming credit for the settled consensus of a discipline. The refusal is inherited here, and the sources above are its authors.

7.3 *The Residue*

What is comparatively less developed is the formal expression of the refusal. A criterion stated as a threshold or as a relational condition is ordinarily left in prose, and the mathematics ordinarily reached for in modelling work is optimization, which reintroduces what the prose

refused. Section 10 states the conditions in a form that permits a model without reintroducing a maximand, and §14 records that the mathematics required already exists.

7.4 Findings and Residue

The rejection of maximization is settled and is inherited here. The residue lies in expressing plural conditions formally without smuggling an objective function back in.

8 THE RELATIONAL FIELD

The apparatus is stated here. The definitions are stipulative and are chosen for tractability.

Definition 1 (Relational domain). *A relational domain D_i is a set of parties together with the relations among them, such that the relations sustain a common mode of articulation: a way of stating what is at issue, what counts as a reason, and what would settle a question.*

Definition 2 (Coherence). *The coherence $c_i \in [0, 1]$ of a domain is its capacity to reproduce its own mode of articulation across occasions and across changes of membership. A domain with $c_i = 0$ has lost that capacity: its mode of articulation is no longer transmitted, and questions arising within it are settled in the terms of some other domain.*

Definition 3 (Orientation and misorientation). *The orientation θ_i of a domain is its mode of articulation, represented as a point in a space Θ . The misorientation $\Delta_{ij} \in [0, 1]$ between two domains is their separation in Θ , normalized so that $\Delta_{ij} = 0$ where the modes coincide and $\Delta_{ij} = 1$ where they are maximally distinct.*

Definition 4 (Contact graph). *The contact graph Γ has the domains as vertices, with $(i, j) \in \Gamma$ where the two domains stand in a relation such that determinations in one bear on the other. Domains not in contact have no boundary, and no condition below applies to them.*

Definition 5 (Exchange and asymmetry). *For $(i, j) \in \Gamma$, the exchange intensity $e_{ij} \in [0, 1]$ is the volume of traffic across the boundary, and is symmetric. The asymmetry $\sigma_{ij} \in [-1, 1]$ records the extent to which one domain's terms govern the exchange, with $\sigma_{ij} > 0$ where i 's terms govern, $\sigma_{ij} = -\sigma_{ji}$, and $|\sigma_{ij}| = 1$ where one domain's terms govern entirely.*

Definition 6 (Boundary viability). *The viability of the boundary (i, j) is*

$$v_{ij} = e_{ij} (1 - |\sigma_{ij}|).$$

A boundary is viable where $v_{ij} > 0$, which holds precisely where exchange occurs and neither domain's terms govern it entirely.

Two features of the last definition should be marked. It is a stipulation chosen because it makes the two ways of failing visible in one expression. It renders, second, the two failures distinguishable: $v_{ij} = 0$ with $e_{ij} = 0$ is separation, and $v_{ij} = 0$ with $|\sigma_{ij}| = 1$ is domination. The vanishing of viability is therefore the meeting point of two distinct conditions.

Definition 7 (Imposition load). *The load borne by domain i is*

$$\Lambda_i = \sum_{j:(i,j) \in \Gamma} e_{ij} \max(0, \sigma_{ji}),$$

the summed traffic across boundaries at which another domain's terms govern.

9 THE TWO CONDITIONS AND THE FOUR FAILURE MODES

9.1 The Conditions

Proposed conditions. *A plural relational arrangement is defensible when both hold:*

$$(C1) \quad c_i > 0 \text{ for every } i, \quad (C2) \quad v_{ij} > 0 \text{ for every } (i, j) \in \Gamma.$$

Condition C1 requires that every domain retain the capacity to reproduce its own mode of articulation. Condition C2 requires that every boundary permit exchange without the terms of either side governing entirely.

The form of the conditions carries the normative content. Both are universally quantified over domains and boundaries, and neither is an aggregate. No sum, average, or weighted total appears, and nothing in the conditions permits a deficiency at one domain to be offset by a surplus at another.

9.2 The Independence of the Conditions

Proposition 1 (Independence). *Neither condition implies the other. There exist arrangements satisfying C1 and violating C2, and arrangements satisfying C2 and violating C1.*

The proposition is established by the two constructions in §11. Scenario B has every domain at maximal coherence with every boundary dead, satisfying C1 and violating C2. Scenario A has every boundary viable with one domain extinguished, satisfying C2 and violating C1. The two conditions are therefore doing separate work, and an account stating only one of them would miss the failures the other catches.

The first construction deserves emphasis. Under separation, coherence attains its maximum for every domain: a domain subject to no imposition reproduces itself perfectly. Isolation is optimal by the measure of C1 alone. A theory of plural governance built on the protection of domains, with no condition on what passes between them, therefore recommends the arrangement that the critique of parallel lives was directed against.

9.3 The Four Failure Modes

Definition 8 (Co-creation). *Co-creation occurs at a boundary where a determination arises there that belongs to neither adjoining domain's prior mode of articulation. The rate of co-creation at (i, j) is stipulated as*

$$n_{ij} = \eta v_{ij} \Delta_{ij} \min(c_i, c_j), \quad \eta > 0.$$

The stipulation carries the crystallographic transfer of §4: formation occurs preferentially at boundaries, requires a difference across the boundary, and requires that both sides be capable of contributing.

Proposition 2 (The four failure modes). *$n_{ij} > 0$ holds if and only if all four of the following hold:*

$$e_{ij} > 0, \quad |\sigma_{ij}| < 1, \quad \Delta_{ij} > 0, \quad \min(c_i, c_j) > 0.$$

Co-creation therefore fails in exactly four ways: by separation ($e_{ij} = 0$), by domination ($|\sigma_{ij}| = 1$), by homogenization ($\Delta_{ij} = 0$), and by extinction ($\min(c_i, c_j) = 0$).

Proof. n_{ij} is a product of $\eta > 0$ with v_{ij} , Δ_{ij} , and $\min(c_i, c_j)$, each non-negative. A product of non-negative factors is positive if and only if every factor is positive. By Definition 6, $v_{ij} > 0$ if and only if $e_{ij} > 0$ and $|\sigma_{ij}| < 1$. $\square$

The proposition earns the paper's title. Homogenization and separation are the two failures named there, and the derivation places them alongside two others as consequences of one requirement. A polity avoiding fragmentation by pursuing unity moves from the second failure to the first, and the model registers no improvement.

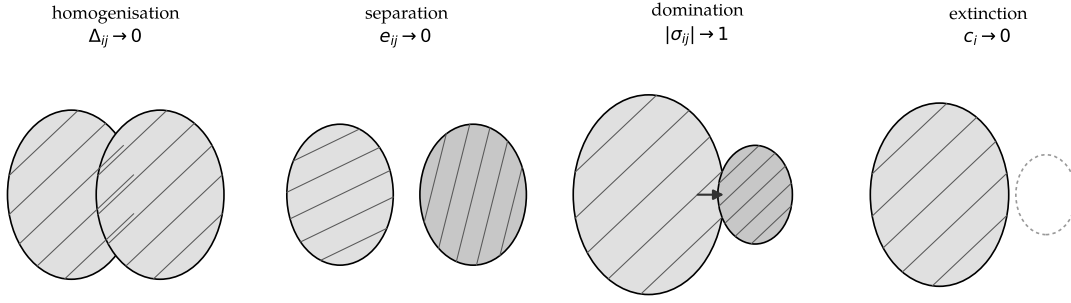


Figure 3: The four ways in which co-creation fails, shown schematically. Shading and internal rulings represent a domain’s orientation. Under homogenization the two orientations coincide and the boundary is no longer a meeting of anything. Under separation the domains are intact and unconnected. Under domination one domain’s terms govern the exchange, drawn as a single arrow. Under extinction one domain has lost the capacity to reproduce its mode of articulation.

9.4 Findings and Residue

Two conditions are stated, in a form that quantifies over domains and boundaries and aggregates nothing. The conditions are logically independent, and the isolation result shows why both are required. Four failure modes follow from a single requirement by a two-line argument, and two of them are the extremes the paper is named against.

10 THE FORMAL MODEL

The conditions of §9 are static. This part supplies dynamics, so that the loss of a condition may be described as a process as well as a state.

10.1 The Dynamics

Definition 9 (Coherence dynamics). *For each domain i , with intrinsic reproduction rate $\rho_i > 0$ and susceptibility $\delta > 0$,*

$$\dot{c}_i = \rho_i c_i (1 - c_i) - \delta c_i \Lambda_i. \quad (12)$$

The first term is logistic: a domain reproduces its mode of articulation at a rate proportional to the coherence already attained and to the room remaining. The second term is the cost of rendering: a domain obliged to state itself in another’s terms transmits its own less effectively, at a rate proportional to its coherence and to the load it bears.

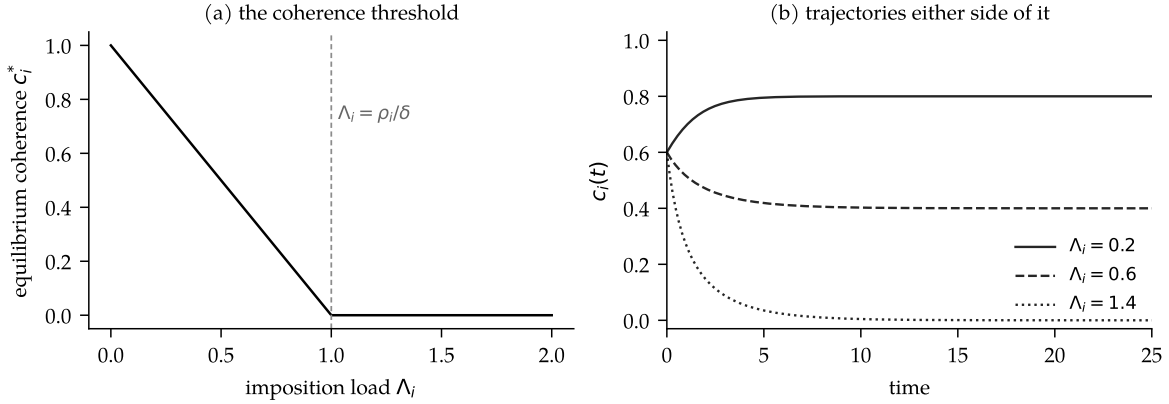


Figure 4: The coherence threshold, with $\rho_i = \delta = 1$. Panel (a) gives equilibrium coherence as a function of imposition load: coherence declines linearly and reaches zero at $\Lambda_i = \rho_i/\delta$, beyond which the domain cannot reproduce itself. Panel (b) shows trajectories either side of the threshold from a common initial condition.

The model is deliberately minimal. Nothing is optimized, no agent chooses, and the controls e and σ enter only through Λ .

10.2 The Threshold

Proposition 3 (Coherence threshold). *Fix Λ_i . If $\delta\Lambda_i \geq \rho_i$ then $c_i(t) \rightarrow 0$ from any $c_i(0) \in (0, 1]$. If $\delta\Lambda_i < \rho_i$ then $c_i(t) \rightarrow c_i^* = 1 - \delta\Lambda_i/\rho_i$, which is asymptotically stable on $(0, 1]$.*

Proof. Write $\dot{c}_i = c_i(\rho_i - \delta\Lambda_i - \rho_i c_i)$. On $c_i > 0$ the sign of $\dot{c}_i$ is the sign of $\rho_i - \delta\Lambda_i - \rho_i c_i$. If $\rho_i - \delta\Lambda_i \leq 0$ this is negative for all $c_i > 0$, so c_i decreases monotonically and is bounded below by 0; the only equilibrium in $[0, 1]$ is $c_i = 0$. If $\rho_i - \delta\Lambda_i > 0$ the expression vanishes at $c_i^* = (\rho_i - \delta\Lambda_i)/\rho_i \in (0, 1]$, is positive below it and negative above it, so c_i^* attracts every trajectory from $(0, 1]$. $\square$

Figure 4 displays the result. Two features bear on the argument. The approach to extinction is continuous in the load, so a domain under increasing imposition loses coherence gradually and without any threshold being visibly crossed until it is. Extinction is reached, second, at a finite load, so imposition need not be total to be fatal: it suffices that Λ_i exceed ρ_i/δ .

Definition 10 (Degenerative transformation). *A degenerative transformation is a change in the arrangement that raises some Λ_i , lowering c_i^* . It is extinguishing where the resulting load satisfies $\delta\Lambda_i \geq \rho_i$.*

The definition supplies the model's account of injustice. Linguistic suppression, epistemic exclusion, obligatory translation into an official register, the administration of a category that a

domain does not use, and the absence of any procedure in which a domain's terms are receivable are each descriptions of a raised Λ_i .

10.3 The Refusal of Aggregation

Proposition 4 (Non-aggregability). *Condition C1 is not implied by the maximization of any weighted aggregate $\sum_i w_i c_i$ with $w_i > 0$. There exist states violating C1 that attain a strictly greater aggregate than states satisfying it.*

Proof. Take three domains and $w_i = 1$. The state $(0.34, 0.34, 0.34)$ satisfies C1 with aggregate 1.02. The state $(0, 0.75, 0.75)$ violates C1 with aggregate 1.50. The aggregate ranks the second above the first while the second extinguishes a domain. $\square$

The proposition is elementary and it is the formal statement of the position inherited in §7. A maximizing criterion permits the extinction of a domain to be purchased by gains elsewhere, and the universally quantified condition does not.

10.4 Underdetermination

An objection arises at this point and should be met. Constraining a system to remain within a set may look like optimization in another notation, with the set playing the part of an objective. The reply is that a maximum selects and a constraint does not.

Proposition 5 (Underdetermination). *Consider three domains in a triangle, $\rho_i = \delta = 1$, uniform exchange e on every boundary, and asymmetry $\sigma \geq 0$ oriented so that domain 0's terms govern at both its boundaries and domain 1's govern at its boundary with domain 2. Then $\Lambda_0 = 0$, $\Lambda_1 = e\sigma$, $\Lambda_2 = 2e\sigma$, and conditions C1 and C2 hold jointly if and only if*

$$e > 0, \quad \sigma < 1, \quad 2e\sigma < 1.$$

This set has non-empty interior in (e, σ) , and the conditions therefore do not determine the arrangement.

Proof. By Proposition 2, $c_i > 0$ if and only if $\Lambda_i < 1$; the binding case is $\Lambda_2 = 2e\sigma$. By Definition 6, $v > 0$ if and only if $e > 0$ and $\sigma < 1$. The conjunction is the stated region, which contains the open neighbourhood of $(e, \sigma) = (1/2, 1/2)$. $\square$

Figure 5 displays the region. Its shape carries the philosophical point of §7 in operational form. A criterion of this kind excludes arrangements without prescribing one, and the question of which admissible arrangement a polity adopts is left open for reasons the criterion does not supply. A polity is bounded, and the direction it takes within those bounds is its own.

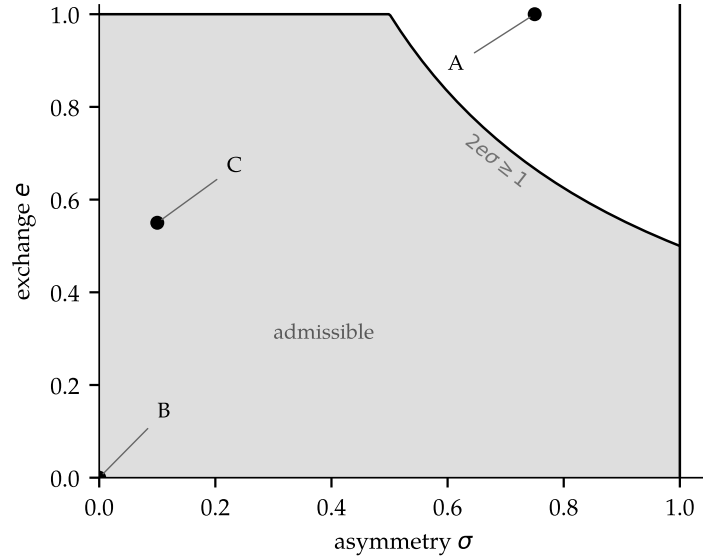


Figure 5: The admissible region in the control plane for the arrangement of Proposition 4. The shaded set is bounded on the right by total imposition ($\sigma = 1$), below by the absence of exchange ($e = 0$), and above by the extinction locus $2e\sigma = 1$. The three scenarios of §11 are marked. The region is an open set with interior, so the conditions admit a continuum of arrangements and single out none.

10.5 Capture of Orientation

The model as stated has domination reduce coherence, so that a dominated domain approaches extinction. That representation is incomplete, and correcting it yields the model's least obvious result.

A domain subjected to sustained imposition need not cease to reproduce itself. It may continue to reproduce, and to do so in terms it has taken from the domain imposing on it, so that what is lost is the distinctness of its articulation, with its capacity to articulate preserved. Let orientation accordingly move under the same pressure that generates the imposition load:

$$\dot{\theta}_i = \kappa \sum_{j: (i,j) \in \Gamma} e_{ij} \max(0, \sigma_{ji}) (\theta_j - \theta_i), \quad (13)$$

so that a domain drifts toward the orientation of whichever domain imposes on it, at a rate set by the intensity and the asymmetry of the boundary between them, and does not drift at all where the boundary is symmetric.

Proposition 6 (Capture). *Under (13) with σ_{ij} of fixed sign and $e_{ij} > 0$, the orientations of dominated domains converge on that of the dominating domain, so that $\Delta_{ij} \rightarrow 0$ across the affected boundaries. Where the imposition load remains below the threshold of*

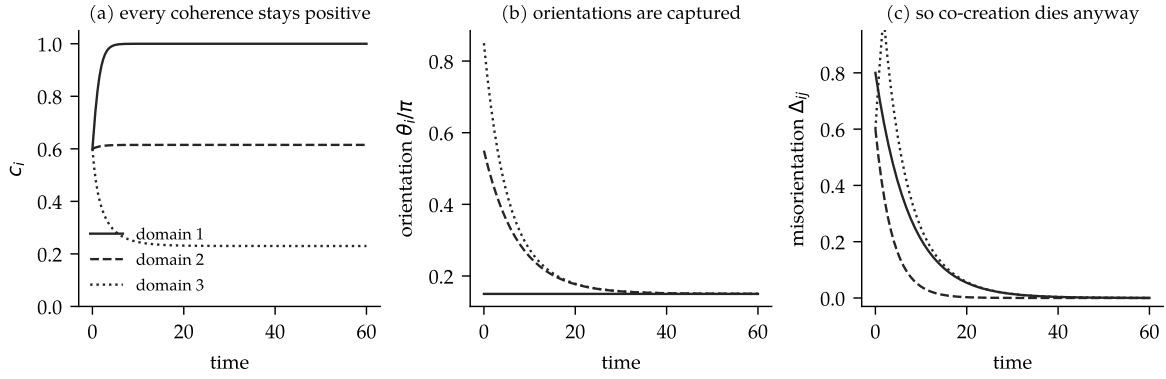


Figure 6: Capture of orientation under sustained asymmetric exchange, computed from (12) and (13). In (a) every coherence remains positive, the three domains settling at 1.00, 0.62 and 0.23. In (b) the orientations of the two subordinated domains are drawn to that of the third. In (c) the misorientations across all three boundaries fall to zero. Co-creation therefore ceases although no domain has been extinguished and every boundary has remained open.

Proposition 2, this occurs with every coherence positive and every boundary viable.

The simulation reported in Figure 6 exhibits it: the coherences settle at 1.00, 0.62 and 0.23, all positive; every boundary remains open; and every misorientation falls to zero, so that the co-creation rate of §9 vanishes through its third factor.

Three consequences follow.

Capture is a path from domination to homogenization. The four failure modes of §9 were established as logically independent, and (13) supplies a dynamical route from one to another: a polity that imposes need not extinguish anything, since sustained imposition converts difference into sameness and sameness ends co-creation without any domain being lost.

Capture is invisible to a test on coherence alone. An observer monitoring whether each domain continues to reproduce itself would record no failure whatever along the trajectory of Figure 6. Every domain is intact, every channel is open, and the capacity that has been lost is registered by neither measurement.

Capture is, third, the form in which the account represents a claim the framework makes elsewhere: that pressure on a relational subject characteristically redirects what it can generate, leaving the generating itself intact. A domain under capture is generating as much as before and generating in another's terms. The distinction between a condition on generation and a condition on the distinctness of what is generated is the distinction between the first of the two conditions and the third factor of the co-creation rate, and §9 requires both for that reason.

A caution belongs with this, and it is a caution about the source and leaves the model untouched. In the physical case a domain's orientation is ordinarily fixed, and the microstructure evolves by boundaries migrating so that one orientation replaces another. Continuous

reorientation of a domain does occur, by rotation coupled to shear, but it is prominent only in fine-grained and high-temperature regimes. Equation (13) therefore has a physical counterpart in a restricted regime, falling short of a general feature of the source, and it is offered here as a stipulation about relational domains for which the physical case supplies a suggestive precedent, falling short of a warrant.

11 A CASE STUDY

The model is exhibited on three illustrations, chosen on a stated principle, with the arrangement varied within the first and the consequences traced.

11.1 *Why These Illustrations*

Illustrations in a paper of this kind are chosen for what they discriminate, and the principle of selection is stated below in place of leaving a reader to infer it. The four failure modes of §9 are distinguished by which factor of the co-creation rate vanishes, and an illustration earns its place by exhibiting a failure that the others do not, under conditions that differ in the respects the model treats as relevant: whether the domains are territorially separated, whether one of them administers the arrangement, and whether the boundaries are institutionally recognized at all.

Three illustrations are accordingly used. The first is a resource decision among domains sharing a territory and answering to an authority that is itself one of them, which exhibits domination and separation and the passage between them. The second is a scientific collaboration among domains with no shared authority and boundaries that are recognized and staffed, which exhibits the productive case and the special-boundary difficulty of §4. The third is a professional standard-setting body in which the boundaries are unrecognized, which exhibits capture, the failure that no coherence test detects.

They are illustrations. Their function is to display what the propositions assert, in settings whose institutional features differ enough that a reader may judge whether the assertions are worth taking to a case where measurement might be attempted.

11.2 *The First Setting*

Three domains bear on the disposition of a watershed. The first is customary, sustained by inherited practice and articulating the watershed in terms of obligation and standing. The second is regulatory and scientific, articulating it in terms of measured condition against stated criteria. The third is commercial, articulating it in terms of valuation and return. Each is in contact with both others, so the contact graph is a triangle.

Orientations are placed at $\theta = (0.15\pi, 0.55\pi, 0.85\pi)$ on a circle of period π , giving misorientations $\Delta_{01} = 0.8, \Delta_{02} = 0.6, \Delta_{12} = 0.6$. Reproduction rates and susceptibility are set to one,

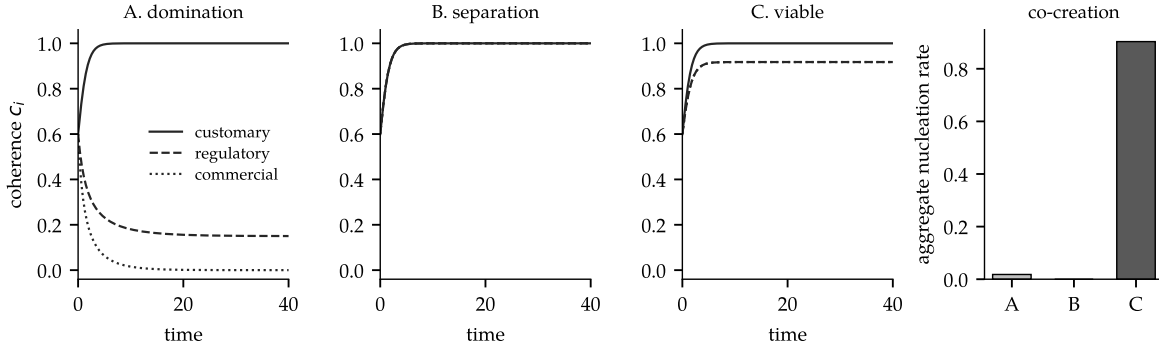


Figure 7: Coherence trajectories under the three arrangements, with aggregate co-creation at right. Under domination the commercial domain crosses the extinction threshold and the regulatory domain is heavily suppressed, while the domain whose terms govern rises to full coherence. Under separation every domain attains maximal coherence and no co-creation occurs. Under viable exchange all domains persist near full coherence and co-creation is an order of magnitude greater than under either failure.

and every domain begins at $c_i = 0.6$. The labels carry no empirical claim, and the numbers are chosen for legibility.

11.3 The Arrangements Compared

Scenario A, domination. Exchange is intense across every boundary, $e_{ij} = 1$, and the terms of the customary domain are largely displaced: $\sigma_{01} = 0.85$, $\sigma_{02} = 0.75$, $\sigma_{12} = 0.50$, with the regulatory and commercial domains governing the exchange. Loads follow as $\Lambda = (0, 0.85, 1.25)$.

Scenario B, separation. No exchange occurs, $e_{ij} = 0$, and asymmetry is undefined and set to zero. Loads are $\Lambda = (0, 0, 0)$.

Scenario C, viable exchange. Exchange is moderate, $e_{ij} = 0.55$, and asymmetry is small: $\sigma_{01} = 0.15$, $\sigma_{02} = 0.05$, $\sigma_{12} = 0.10$. Loads are $\Lambda \approx (0, 0.082, 0.083)$.

11.4 Outcomes

Scenario	Coherence at rest	C1 holds	C2 holds	Aggregate co-creation
A. domination	(1.00, 0.15, 0.00)	no	yes	0.018
B. separation	(1.00, 1.00, 1.00)	yes	no	0.000
C. viable ex- change	(1.00, 0.92, 0.92)	yes	yes	0.903

Four observations follow from the table and from Figure 7.

First, the two failures are distinguished by different conditions, which establishes Proposition 1. Domination preserves every boundary in a viable state while extinguishing a domain.

Separation preserves every domain while leaving every boundary dead.

Second, separation attains the highest coherence available. Every domain reaches $c_i = 1$, and by the measure of domain integrity alone the arrangement is optimal. Co-creation is nevertheless zero, and the arrangement is a set of intact domains with nothing between them.

Third, the domain whose terms govern gains under domination. Its load is zero, and its coherence rises to the maximum while its neighbours decline. The model reproduces without further assumption the observation that imposition is not costly to the imposer and is invisible from that position.

Fourth, the viable arrangement is not the one that maximizes any single quantity in the table. Coherence is lower than under separation, and the coherence of the governing domain is no higher than under domination. What distinguishes it is that both conditions hold, and co-creation follows.

11.5 *The Path from C to A*

A final observation concerns how an arrangement moves between these states. Holding exchange at $e = 0.55$ and raising asymmetry continuously from $\sigma = 0.05$, the load on the most burdened domain rises continuously, its equilibrium coherence falls linearly, and nothing discontinuous occurs until the threshold of Proposition 2 is reached. Before that point the arrangement satisfies both conditions and displays a domain in decline. A polity monitoring only whether its domains persist would observe no failure until the failure was complete.

11.6 *The Second Setting: Recognized and Staffed Boundaries*

The second illustration differs from the first in three respects the model treats as relevant. The domains occupy no common territory, no one of them administers the arrangement, and the boundaries between them are recognized and provided with people whose work is conducted there.

Consider three research communities bearing on a common problem, one experimental, one theoretical, and one computational, joined by shared instruments, joint appointments, and a literature that each reads. Exchange is high, asymmetry is low because no community can compel another, and misorientation is substantial because each articulates the problem in terms the others do not use. On the model this is the productive configuration: with e large, $|\sigma|$ small, Δ substantial and every c_i positive, all four factors of the co-creation rate are bounded away from zero, and the arrangement generates what §13 calls new domains at the boundaries, which is what a shared instrument or a joint method in fact is.

The illustration is included for a second reason, which is that it displays the difficulty entered in §4. Two of these communities may develop an accommodation so practised that exchange between them becomes cheap and unremarkable: a shared formalism that each can

use without translation. On the physical side such a relation is a special boundary, low in energy and correspondingly poor as a site at which new order forms. The model as written does not represent it, since a low-energy accommodation between widely differing domains would require boundary property to depend on more than misorientation, and §16 records the gap. What the illustration shows is that the gap is not artificial: relations of this kind are common, and an account that predicted high novelty wherever difference is high would misdescribe them.

11.7 The Third Setting: Unrecognized Boundaries

The third illustration is chosen because it exhibits capture, and because capture is the failure that the first two settings would not reveal.

Consider a professional body setting a standard, whose members are drawn from several practices with different formations. The body has procedures, its deliberations are open, no practice is excluded, and no participant is silenced. The arrangement would satisfy any test framed in terms of participation. What the body lacks is any recognition that the practices articulate the matter differently: it treats its members as individuals holding views, and the differences among their formations appear within it only as disagreement to be resolved.

Under that description the boundaries carry exchange and are asymmetric, since one formation supplies the vocabulary in which the standard is written and the others must render their concerns in it to be heard. Equation (13) then applies, and the trajectory is the one shown in Figure 6. Each practice continues to reproduce itself; none is extinguished; the body's records show continuous participation by all. What falls is the misorientation, as the practices come to articulate the matter in the terms the standard uses, and with it the rate at which the arrangement generates anything the standard did not already contain.

The illustration turns on the mismatch between what such a body would monitor and what would be occurring. Participation is recorded and is intact. Coherence is intact. The quantity that has gone is the one for which the body has no measure and, having no recognition of its boundaries, no reason to seek one.

11.8 Findings and Residue

The three arrangements separate the two conditions, exhibit the isolation result, and show co-creation depending on both. Separation maximizes coherence and produces nothing. Domination is costless to the domain whose terms govern. The approach to extinction is, finally, continuous, so the loss of a domain is not signalled in advance by the condition that its coherence be positive.

12 THE WALL BETWEEN DESCRIPTION AND JUSTIFICATION

An account written in the vocabulary of generativity is exposed to a specific failure, and the exposure is greatest at exactly the point this paper has now reached. The failure is set out here, together with what in the account prevents it.

12.1 *The Sentence That Must Remain Refusable*

Consider the proposition that destroying a domain may release the generative capacity of the rest. In a coupled system this is not rhetoric; it is frequently true, and it is observable. Where a domain is removed, coordination costs among the survivors fall, contested boundaries disappear, and what remains may generate more freely than before.

An account that took the free unfolding of generativity as its end could not refuse that proposition, since the proposition would be true in the account's own terms. It would have replaced the question whether a party is useful with the question whether a party contributes to the system's generative capacity, and those are the same question in different vocabulary. The account offered here must therefore be able to assert the following.

The sentence. *Even where extinguishing a domain would in fact release the generative capacity of the remaining configuration, extinguishing it is impermissible on the conditions stated.*

12.2 *Why the Conditions Permit That Assertion*

Three features of the conditions of §9, taken together, secure it.

The conditions are universally quantified and are not aggregates. Proposition 5 establishes that $\forall i (c_i > 0)$ is not implied by the maximization of any weighted sum, and exhibits a configuration in which a domain is extinct and the aggregate is nonetheless higher. An arrangement that extinguished a domain would fail the condition whatever happened to the total, because there is no total in the condition. The refusal of aggregation is therefore a property of the form in which the conditions are written, holding independently of any preference of the author.

The conditions are constraints, and they function as such throughout. Proposition 4 establishes that they admit an open region of arrangements and select none. A constraint of that kind cannot be traded against, since there is no scale on which the trade would be conducted, and it cannot be optimized, since it is satisfied or unsatisfied and does not admit of degree at the point where it binds.

The normative force of the conditions is, third, imported and never derived. Nothing in §3 entails that a domain ought to persist; a physical account entails nothing of the kind about anything. The reasons for holding that a domain's capacity to reproduce its own terms may

not be extinguished are the reasons given in the tradition surveyed in §7, and they are reasons about the standing of parties, with the productivity of configurations set aside. The model states those reasons in a form that permits their consequences to be traced; it does not supply them.

12.3 *Three Statements the Account Does Not Make*

That generativity is a purpose. The relations described here exhibit a tendency to sustain the conditions of their own continued generation. That is a description of what is observed, and this paper treats it as a primitive of the framework, declining to explain it. To explain it would be to name whatever it serves, and whatever were named would become the more fundamental good, with generativity reduced to its means. The tendency is accordingly descriptive throughout, and the arguments below draw their reasons from §7.

That what conduces to generativity is thereby justified. Section 3 records that coarsening proceeds without an agent and that plurality is metastable. Neither observation is a reason. That a configuration tends toward the loss of its domains explains why sustaining plurality requires something and does not establish that the tendency is legitimate, nor that resisting it is.

That a domain's claim rests on what it contributes. A domain that generated nothing of value to any other would stand under the conditions exactly as one that generated much. The conditions are indifferent to output by construction, and a version of them sensitive to output would be the utilitarianism this paper set out to avoid, reconstructed one level up.

12.4 *The Point at Which the Wall Is Thinnest*

The weakest point may now be named. The co-creation rate of §9 is a quantity, it is defined so as to be greater or smaller, and the case study reports it as a number. Nothing prevents a reader from treating that number as a maximand, and an arrangement chosen to maximize it would be an arrangement chosen on precisely the grounds this section rejects.

The defence available is structural. The co-creation rate is a diagnostic and appears in no condition. The conditions are the two positivity requirements, and an arrangement satisfying them is admissible whether its co-creation rate is high or low. Should a reader nonetheless wish to maximize the rate, the conditions constrain that maximization exactly as they constrain any other objective, and an arrangement that raised the rate by extinguishing a domain would be inadmissible for the reason given above. The rate is reported in §11 because it makes the failure modes legible, and it carries no normative weight anywhere in the account.

12.5 *Findings and Residue*

The conditions permit the refusal of the sentence that an account of this kind is most likely to be unable to refuse, and they do so because they are universally quantified, because they func-

tion as constraints, and because their normative content is imported. The account treats the tendency of relations to sustain their own conditions of generation as a primitive that justifies nothing. The thinnest point is the co-creation rate, which is a quantity and might be mistaken for a maximand; it enters no condition, and §15 records the risk.

13 THE CONTRIBUTION OF THE CRYSTALLOGRAPHIC SOURCE

Sections 5 through 7 conceded that interfaces are productive, that plural centres without an apex are established, and that the refusal of maximization is settled. This part states what the source supplies beyond them.

13.1 *The Boundary as a Structure*

Polycentricity theorizes centres and treats what lies between them as relation or adjustment. The trading zone literature theorizes what develops at an interface and treats it as a language. The crystallographic source treats the boundary as a structure whose characteristics are determined by the orientations meeting at it, and it distinguishes boundaries by those characteristics.

Two distinctions carry over as hypotheses. Boundaries between domains of small misorientation differ in kind from boundaries between domains of large misorientation, and the two behave differently in every respect that matters physically. Boundaries differ, second, in whether the two orders meeting at them are accommodated by a common arrangement or whether the meeting is abrupt, which bears on whether the boundary strengthens or weakens the material.

Hypothesis on boundary typology. *Boundaries between relational domains admit of classification by the misorientation across them and by the degree to which the arrangement at the boundary accommodates both modes of articulation. The classification bears on whether the boundary transmits, resists, or generates, and the classes are not ordered by a single scale.*

The hypothesis states what a typology would require, and the typology itself is outstanding. Section 16 enters it as a question.

13.2 *The Triple as the Unit of Analysis*

A second consequence concerns what is analysed. Where boundaries are structures, the unit of analysis is neither the domain nor the pair of domains, and is the triple consisting of two domains together with the boundary between them. The triple is not decomposable: the properties of the boundary follow from both orientations and belong to neither domain.

The proposal is a departure from network analysis, in which an edge carries a weight and the structure resides in the vertices and the pattern of connection. An edge carrying a structure of its own, with its own conditions of persistence, is a different object.

13.3 *Nucleation at Boundaries*

The third contribution carries the most weight in the argument above. In materials, the formation of a new phase begins preferentially at boundaries, because the energy already invested in the boundary lowers the barrier to forming something new there. Formation in the interior of a domain requires more.

Transferred, the proposition is that novelty arises preferentially at boundaries and requires both that a difference exist across the boundary and that the boundary permit exchange. Definition 7 states this, and Proposition 3 draws out its consequences. The transferred proposition is a hypothesis, and §4 governs the standing of the physical fact behind it.

The formulation supplies a reason, absent from an account resting on coexistence, why plurality is worth sustaining. Domains held apart produce nothing between them, and domains dissolved into one another have no boundary at which anything could form. Plurality is a condition of novelty on this account, and the two failures the paper is named against are the two ways of removing the condition.

13.4 *Findings and Residue*

The crystallographic source supplies a treatment of the boundary as a structure, a unit of analysis that is a triple, and a mechanism locating novelty at boundaries. The first is a hypothesis awaiting a typology, the second is a proposal about method, and the third is stipulated in the model of §10 and carries its central result.

14 RELATION TO VIABILITY THEORY AND TO SYSTEMS THEORY

Two existing bodies of work supply the mathematics and the vocabulary the account requires, and neither was reached for in constructing it.

14.1 *Viability Theory*

Aubin (1991) develops the study of systems required to remain within a constraint set, with no optimization performed. The viability kernel of a set is the collection of states from which some admissible evolution remains within it indefinitely, and the theory concerns the computation of such kernels and the characterization of the evolutions that maintain them (Aubin et al., 2011).

The correspondence is exact enough to be adopted. Conditions C1 and C2 define a constraint set. The controls are the arrangements e and σ . The region computed in Proposition 4 is a section through the set of admissible controls. The theory's characteristic result, that viable controls are generically multiple, is the formal statement of the underdetermination that §10 demonstrates by construction.

Adopting the vocabulary strengthens the account, since it supplies a developed mathematics for a normative form that political philosophy has left in prose. The programme it suggests is stated among the open questions.

Two further frameworks share the form. Ecological resilience distinguishes the capacity of a system to persist within a domain of attraction from its speed of return to a single equilibrium (Holling, 1996), and the distinction is the one required here. The framing of environmental governance by boundaries within which operation is admissible (Rockström et al., 2009), developed normatively as a space bounded by floors and ceilings (Raworth, 2017), is an existing example of a criterion of the present form applied at scale.

14.2 Self-Reproduction

Condition C1 requires that a domain reproduce its own mode of articulation, and the requirement has an established name. Luhmann (1995), developing the account of self-producing systems given by Maturana & Varela (1980), treats a social system as reproducing its own elements through its own operations, so that the system's continuation is an achievement of those operations. Coherence as defined here is operational closure under another description, and the correspondence should be recorded.

One difference bears on the argument. The systems-theoretic account emphasizes closure, and treats what crosses a system's boundary as perturbation to be processed internally. The account here requires that boundaries be viable, which is a requirement about what passes across them, and Proposition 3 makes novelty depend on it. Closure is a condition of a domain's persistence, and the arrangement requires more to be defensible.

14.3 The Precedent for the Term

Mueller & Hechter (2021) develop an account of social order in the European Union under the name of crystallization, in which cooperation at lower levels of organization sustains order at a higher level. The account is set in a plural and multilingual polity and connects to the literature on the commons. The present paper's use of the term is accordingly not a coinage in an empty field, and the difference lies in the level of the mechanism: the prior account concerns the emergence of order from lower-level cooperation, and the account here concerns the structure of boundaries between domains once order has emerged.

An older usage runs the other way. Cultural crystallization, in the sense given by Gehlen

(1961), names a condition of exhaustion in which the possibilities of a culture have been worked through and nothing further arises. The sense used here is opposed to that one: crystallization is where structure is generated, and the failure modes of Proposition 3 are the conditions under which generation ceases.

14.4 Findings and Residue

Viability theory supplies the mathematics for a non-optimizing criterion, ecological resilience and the boundary framings of environmental governance supply precedents for its normative form, and systems theory supplies the established name for condition C1. The term crystallization has a prior use in a closely related setting, and an older use with the opposite valence.

15 THE LIMITS OF THE ACCOUNT

The analogy is disciplined by declaration. Section 4 states what transfers and what does not, and the statement is a declaration of method. Nothing establishes that the four transferred features hold of relational domains, and a reader who denies the transfer is left with a stipulated model bearing a crystallographic vocabulary.

The propositions are true of the model. Propositions 1 through 4 are consequences of Definitions 1 through 8. They establish nothing about any polity, and their interest depends entirely on whether the definitions capture anything.

The quantities are not measurable. Coherence, orientation, exchange, and asymmetry have no measurement procedures. A model whose variables cannot be measured cannot be tested, and the account is accordingly a proposal for work.

Domains are treated as discrete and given. Parties belong to many domains at once, domains overlap and nest, and their boundaries are contested. The model treats them as a fixed finite set with a fixed contact graph, and every result depends on that simplification.

The dynamics are stipulated. Logistic reproduction with a linear cost of rendering was chosen for tractability and for the sharpness of its threshold. Other dynamics would yield other thresholds, and nothing here shows the choice to be the right one.

Nucleation is asserted, not derived. Definition 7 makes co-creation a product of four factors, and Proposition 3 follows from that form alone. The proposition is therefore a consequence of how co-creation was defined, and its force depends on whether the definition is a defensible reconstruction of what co-creation is.

The core intuitions are inherited. As §§5 through 7 concede, the productivity of interfaces, plural centres without an apex, and the refusal of maximization are all established elsewhere.

Two claims are entered against this list. The derivation of four failure modes from a single requirement places unity and fragmentation in one frame with two further failures, which the

surveyed accounts do not do. The treatment of the boundary as a structure with a typology of its own addresses an asymmetry that the polycentric literature leaves open.

16 OPEN QUESTIONS

The first group concerns the model.

- Q1.** What would a measurement procedure for coherence look like? The condition C1 turns on a domain's capacity to reproduce its mode of articulation across changes of membership, and an operationalization would convert the account from a proposal into a framework.
- Q2.** Are the failure modes of Proposition 3 exhaustive of the ways co-creation fails, or exhaustive only of the ways permitted by Definition 7? The proposition is a consequence of a stipulated product form, and a different reconstruction would yield a different list.
- Q3.** Does the threshold of Proposition 2 survive more realistic dynamics? Reproduction that is not logistic, costs of rendering that are not linear in load, and domains that adapt their own reproduction rates under pressure would each alter the result.

The second group concerns the boundary, which is where the account claims most.

- Q4.** What is the typology of relational boundaries? The hypothesis of §13 proposes classification by misorientation and by accommodation. Constructing the classification, and establishing that its classes behave differently, is the work the account most requires.
- Q5.** Which boundaries strengthen and which weaken what they join? In materials the answer depends on structure and is well studied. The corresponding question for relational domains is open and appears answerable.
- Q6.** Can a boundary become a domain? The trading zone literature describes interlanguages that stabilize into practices of their own. The model has no mechanism by which a boundary acquires coherence and enters the contact graph as a vertex, and supplying one would close the account's principal structural gap.

The third group concerns the normative content, and the questions here are sharpest because the claims are strongest.

- Q7.** Do the two conditions exhaust what a defensible plural arrangement requires? An arrangement may satisfy both while distributing the benefits of co-creation in a manner no account of justice would accept, and the conditions are silent about distribution.

- Q8.** How is the contact graph itself to be assessed? The conditions apply to boundaries that exist. An arrangement that never brings two domains into contact violates nothing, and the question of which domains ought to be in contact is not posed by the account.
- Q9.** What follows where the conditions cannot be jointly satisfied? Proposition 4 exhibits a case where the admissible region is non-empty. Where the region is empty, the account states that every available arrangement is indefensible and offers no guidance on choosing among them.

The last question is the one the account is directed toward.

- Q10.** Under what conditions does a plural arrangement resist coarsening? Materials coarsen because boundaries carry energy and their reduction is favoured; polities exhibit a corresponding tendency, since uniform procedure is cheaper to administer than plural procedure and the pressure toward it is continuous. An arrangement satisfying both conditions when established may fail them a generation later with no act of imposition by anyone. What sustains a plurality of domains against that pressure is the question on which the practical value of the account depends, and it is entirely open.

A closing observation about the shape of the list. The questions of the first group could be settled by anyone willing to work the model. The questions of the second are where the account claims novelty and where its rivals leave room. The questions of the third concern whether the conditions are the right ones, and a reader persuaded of the survey and unpersuaded of the conditions may take the first and leave the second.

A note on method. The formal model is stipulated and its propositions are elementary. The propositions were verified numerically before being stated, and Proposition 2 was checked against integration of the dynamics across randomized parameters, with all apparent discrepancies falling within a small neighbourhood of the bifurcation and attributable to slow convergence at criticality. Proposition 4 was checked against the closed-form equilibrium across randomized controls with no discrepancies. The figures are generated from the model and from a growing-front schematic, and their sources accompany the paper.

A note on the analogy. The physical facts reported in §3 are stated as they are understood in materials science and are used as a source of structure. No physical result is offered as evidence for any claim about a polity, and §4 enumerates the points at which the transfer fails. A reader who judges the enumeration incomplete is invited to extend it, since the value of the exercise depends on the enumeration being honest.

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