Braid Groups and Procedural Justice

Path-Dependence in Generative Relational Being

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Abstract

Two procedures may issue in the same distribution and yet not be equally just. Theories of procedural justice have long recognized this, and each has registered it in its own way: Rawls in the doctrine of pure procedural justice, Tyler in the finding that judgments of fairness track something the outcome does not record, deliberative theory in its attention to sequence and agenda, Sen in the turn toward comparison. What none of them supplies is a formal object whose elements are the procedural differences themselves—a structure in which two histories that project to the same endpoint remain distinguishable, and in which the question of whether they are the same procedure has an answer. This paper proposes the braid group as one candidate structure. Under the projection $\pi\colon B_n \to S_n$, which retains the endpoint permutation and discards the path, the pure braid group $P_n = \ker \pi$ is the group of transformations that preserve outcome while altering relational history; two histories are outcome-equivalent precisely when they differ by an element of P_n. The paper argues that this is the object procedural justice has described without possessing, and argues equally that it is not by itself an object of justice. Whether a given procedural difference matters is not settled by the topology, and an account that claims otherwise attributes to a structure of path-dependence a normative content it does not carry. The paper therefore introduces an interpretive operator, written Φ, that assigns relational significance to elements of P_n in context; it does not construct Φ, and argues that the contribution of a formalism of this kind is to locate where interpretation must enter rather than to dispense with it. Four constructed cases test the resulting model for sensitivity, specificity, interpretive underdetermination, and boundary—the last exhibiting a case the formalism cannot represent at all. A closing section places procedural justice among the layers of evaluation that generative relational being requires, and marks what braids cannot carry.

Keywords: procedural justice; braid groups; path-dependence; relational history; formal modeling; generative relational being.

A note on the standing of this paper. This is a modeling paper, and the model is offered for what it forbids rather than for what it permits. A formalism imported into social theory earns its place by ruling things out: by making some claims unavailable, some questions answerable that were not, and some limits explicit that were previously matters of taste. The exposition is arranged accordingly. The mathematical structure is set out on its own terms before any interpretation is placed on it, and the ceiling of that structure—what it cannot represent under any interpretation—is stated in the same section, before the model is built rather than after it is defended. The constructed cases include one in which the model succeeds, one in which it produces a difference that carries no normative weight, one in which it cannot distinguish two situations that differ in every respect that matters, and one in which it fails outright. Should the mapping proposed here be shown to license inferences it should not, or to be recoverable from an existing framework at lower cost, the argument loses its ground. Objection, correction, and counter-example are welcome at `huangwanhong@serendip.ngo`.

1 Introduction

Consider two committees that reach the same decision. The membership is the same, the vote is the same, the distribution of benefits and burdens that follows is the same, and the reasons entered into the record are the same. In the first committee, an objection was raised early and the proposal was altered in response. In the second, the same objection was raised after the proposal had been settled, was recorded, and altered nothing. Assessed at the endpoint, the two committees are indistinguishable. Assessed by their participants, they are not: something occurred in the first that did not occur in the second, and it is not obvious that the difference is merely psychological.

This is not a marginal observation. It is the recurring finding of the empirical literature on procedural justice and the persistent intuition behind its normative wing. People care about how a decision was reached in ways that survive the observation that the decision would have been the same regardless. Theories of procedural justice have recognized this and have registered it, each in its own idiom. Rawls's doctrine of pure procedural justice makes the procedure the source of the outcome's justice rather than its instrument. Tyler's empirical program isolates factors—voice, neutrality, trustworthiness, respect—that predict judgments of fairness independently of the favorability of the result. Deliberative theory attends to sequence, agenda, and the order in which claims are heard. Sen turns from the ranking of institutions toward the comparison of realized states.

Each of these registers that procedure carries weight the outcome does not record. None of them supplies an object whose elements are the procedural differences themselves. This is a different lack from the one it is easily mistaken for. It is not that these accounts fail to represent history; a range of frameworks represent history well, among them process tracing, event-history modeling, causal graphs, the histories of extensive-form games, and the path-dependent institutionalism that made lock-in a term of art. What is missing is narrower and more specific: a structure in which the histories that project to a given outcome form a determinate collection, in which the differences among them are themselves elements of something, and in which the question *are these two procedures the same* has an answer that does not depend on the analyst's description. Without such a structure, the claim that procedure matters beyond outcome remains a claim about salience. With one, it becomes a claim about a space.

This paper proposes the braid group as one candidate for that structure. The proposal rests on a single feature. There is a natural map $\pi\colon B_n \to S_n$ from the braid group on n strands to the symmetric group, which retains where each strand ends and forgets how it got there. The map is surjective and is not injective, and its kernel—the pure braid group P_n—consists of exactly those braids that return every strand to its own position while having, in general, done something in between. Two braids have the same image under π if and only if they differ by an element of P_n. If strands are relational positions, crossings are interaction events, the

braid is the history and its image the outcome, then P_n is the group of transformations that alter procedural history while preserving everything an outcome-based evaluation can see.

> **Central claim.** *The pure braid group supplies one candidate structure for the object that theories of procedural justice have described without possessing. It is a group of transformations that preserve endpoint outcome while altering relational history, within which outcome-invariant procedural differences are determinate elements and procedural identity is decidable. It does not follow that these differences are thereby justice-relevant, and §5.3 exhibits one that is not. Which of them matter is settled by an interpretive operator that the formalism requires, locates, and does not supply.*

The two halves of this claim are equally load-bearing, and the second is the one more easily lost. A formalism that represents procedural difference invites the inference that the differences it represents are the differences that matter. The inference is false. Some procedural differences are justice-relevant; some are recorded by the topology and are of no interest to anyone; and some—the environment of fear as against the environment of safety, the concession extracted by threat as against the concession reached by learning—matter greatly and are invisible to the topology altogether. A model that could not accommodate all three would be worse than no model. What the braid supplies is a space in which the first two are distinguishable from one another; what it cannot supply is the criterion by which they are distinguished, or any purchase at all on the third.

That criterion is written here as an operator Φ, carrying elements of P_n together with context to relational significance. The paper does not construct Φ. It argues something weaker and, if correct, more useful: that an account of procedural justice which discriminates among procedural differences thereby employs an operator of this type, whether or not it is written down, and that a formalism which makes the operator explicit has performed a service even when it leaves the operator empty. The contribution claimed is the localization of a problem, not its solution.

A further limit must be entered before the argument begins, since it bounds what the whole can be taken to establish. Procedural justice asks whether relational generation occurred through legitimate relations. It does not ask what is being generated, or toward what. A procedure may satisfy the criteria the literature imposes—equal voice, transparency, revisability, freedom from coercion—and issue in the exclusion of a minority; procedural legitimacy does not entail substantive justice, and no formalization of the procedural layer alters this. Generative relational being poses a sharper version of the same difficulty: a procedurally impeccable process may consume the conditions of its own future generation, leaving a relation that was fairly conducted and is no longer capable of generating anything. The braid represents the memory of a process. It does not determine whether the process deserves continuation. What

this paper treats is one layer among several, and §6 sets out the others and their relations rather than leaving the scope to be inferred.

The novelty claimed may accordingly be stated with some precision. It is not that procedural history matters, which is established. It is not that P_n contains hidden histories, which is a loose way of speaking. It is that P_n characterizes the transformations among histories that are observationally equivalent under endpoint evaluation, that membership in P_n is decidable, and that the separation of this representational question from the interpretive one is available to any theory of procedural justice and has not been taken.

Two features of the exposition follow from the character of the argument, and are stated here in advance. First, the mathematics is presented before any interpretation is placed upon it, and in a section that contains no vocabulary of justice; the account of procedural justice that follows contains no vocabulary of braids. The join is made only in §4, where each correspondence is proposed and then tested. The discipline is not stylistic. A formalism assembled alongside the phenomenon it is to model will come to fit it, and the fit will establish little. Second, the structural ceiling of the formalism—fixed n, no creation or destruction of strands, discreteness of events, silence on the content of an interaction—is stated in §2, before the model is built. A reader should know what the apparatus cannot do before being shown what it can.

The remainder proceeds as follows. §2 sets out the braid group: its presentation, the relations among its generators, the projection to the symmetric group and its kernel, the decidability of the word problem, and the ceiling. §3 sets out the theories of procedural justice and isolates the specific gap this paper addresses. §4 constructs the correspondence, treating in turn outcome-invariant difference, sequencing and procedural indifference, accumulated asymmetry, revisability, and degradation, and closes by fixing the semantics of the mapping together with the conditions under which each part of it would fail. §5 presents four constructed cases, testing the model for sensitivity, specificity, interpretive underdetermination, and boundary. §6 states what the model buys, what braids cannot carry, where procedural justice stands among the layers of evaluation that generative relational being requires, and the work left open.

2 The Braid Group

This section sets out the mathematical structure on its own terms. No interpretation is placed on it here, and no vocabulary of justice appears; the reader who already knows the material may read to the end of §2.7, where the structural ceiling is stated, and pass on. The presentation is standard and follows Artin's, whose original treatment established both the geometric definition and the algebraic presentation that will be used throughout (Artin, 1925, 1947). Textbook accounts may be found in Birman (1974) and Kassel & Turaev (2008).

2.1 *Definition*

Fix an integer $n \geq 2$ and consider two horizontal planes, one above the other, each carrying n marked points in a row. A *geometric braid* on n strands is a collection of n disjoint curves running from the marked points of the upper plane to those of the lower, subject to one condition: each curve descends monotonically, meeting every intermediate horizontal plane exactly once. The condition forbids a strand from doubling back upward. What it permits is that the strands pass around one another, and it is this passing that the structure records.

Two geometric braids are regarded as the same if one may be deformed continuously into the other without moving the endpoints and without any strand passing through another. This relation is *isotopy*, and a *braid* is an isotopy class of geometric braids. The definition already carries a commitment worth marking: everything about a configuration that a continuous deformation can remove is discarded, and everything it cannot remove is kept. How far apart two strands run, how sharply a curve bends, at what height a crossing occurs—none of this survives. What survives is which strands crossed which others, in what order, and in which sense.

Braids compose. Given braids a and b on the same number of strands, stack a above b, identify the lower endpoints of a with the upper endpoints of b, and rescale; the result is a braid, written ab. Composition is associative on isotopy classes. The braid whose strands descend without crossing is an identity for this operation, written e. And every braid has an inverse: reflect it in a horizontal plane, and the composite of a braid with its reflection may be deformed, crossing by crossing, into the identity. The isotopy classes of braids on n strands accordingly form a group, the *braid group* B_n.

2.2 *Generators and Orientation*

Any braid may be decomposed into a sequence of elementary crossings, each involving one adjacent pair of strands. Write σ_i for the braid in which the strand in position i crosses over the strand in position $i+1$, all other strands descending untouched, where $1 \leq i \leq n-1$. Its inverse σ_i^{-1} is the braid in which the strand in position i crosses *under* the strand in position $i+1$.

The pair σ_i and σ_i^{-1} differ in exactly one respect. The strands involved are the same, the positions exchanged are the same, and the sole distinguishing datum is which strand passed over the other. This is the orientation of the crossing, and it is the only asymmetry the formalism records at the level of a single event. What does not follow should be noted. That $\sigma_i \neq \sigma_i^{-1}$ is a fact about the structure; that one of them is in any sense better, stronger, or prior is not a fact about the structure at all, and no relation in B_n supplies one.

Every element of B_n can be written as a finite product of the σ_i and their inverses. Such a product is a *braid word*, and it is an ordered record: the word $\sigma_1\sigma_2\sigma_1^{-1}$ specifies not only

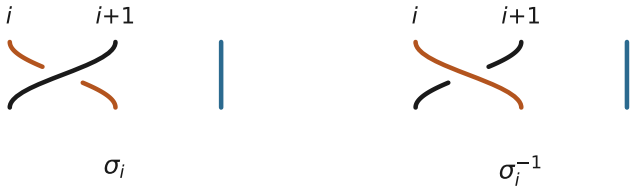

Figure 1: The generator σ_i and its inverse. The strands involved and the positions exchanged are the same in both; the sole distinguishing datum is which strand passes over the other. Breaks in a strand mark where it passes underneath.

Figure 2: The two relations. Left pair: far commutation, where the crossings involve disjoint pairs of strands and their order leaves no trace. Right pair: the braid relation, where three strands interact pairwise and one particular reordering yields the same braid. In each pair the two diagrams are the same element of B_n.

which crossings occurred but the sequence in which they occurred. The number of letters is the *length* of the word, and a word of minimal length among all words representing a given braid is *reduced*. A braid typically admits many words, and the relations of the next subsection say which.

2.3 *The Relations*

Two families of relation hold among the generators, and together with the generators they present the group completely:

$$B_n = \left\langle \sigma_1, \ldots, \sigma_{n-1} \;\middle|\; \sigma_i\sigma_j = \sigma_j\sigma_i \;\; (|i-j| \geq 2), \quad \sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1} \right\rangle. \tag{1}$$

The two are illustrated in Figure 2. The first family is *far commutation*. Where two crossings involve disjoint pairs of strands—positions $i, i+1$ and $j, j+1$ with $|i-j| \geq 2$—the order in which they occur makes no difference to the resulting braid. The two events do not interact, and the structure records no trace of which came first. This is a statement of independence, and it is exact. The order does not matter little; the order has no representation at all.

The second is the *braid relation*, also known in its wider setting as the Yang–Baxter relation (Yang, 1967; Baxter, 1972). Where two crossings share a strand—positions $i, i+1$ and $i+1, i+2$—the order is partly constrained. The relation says that one particular reordering leaves the braid unchanged: performing σ_i, then σ_{i+1}, then σ_i yields the same braid as σ_{i+1}, then σ_i,

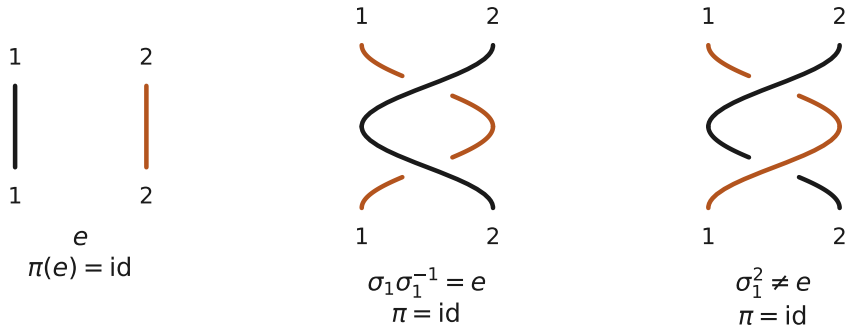

Figure 3: What the projection cannot see. All three braids send every strand back to its own position and so have the same image in S_n. The first two are the identity braid; the third is not, the strands having wound once fully around one another. The permutation records the endpoints and discards the winding.

then σ_{i+1}. Geometrically, this is the statement that a strand may be slid past the crossing of the other two. What the relation supplies is thus a criterion of *indifference*: among sequences of overlapping crossings, some reorderings change nothing and some change everything, and the relation says which are which.

The presentation is complete, and its completeness has a consequence about what the group does *not* say. Whatever is not a consequence of the two relations does not hold in B_n. In particular no relation identifies σ_i with σ_i^{-1}: a crossing and its reverse are distinct elements, and no amount of composition will conflate them. Nor does any relation permit a crossing to be deleted except by composition with its own inverse. Words grow, and they reduce only in the ways the relations allow.

2.4 Projection to the Symmetric Group

A braid determines a rearrangement of positions: strand beginning at position 1 ends somewhere, strand beginning at 2 ends somewhere else, and the assignment is a permutation of $\{1, \ldots, n\}$. Sending each braid to this permutation defines a map

$$\pi \colon B_n \longrightarrow S_n, \tag{2}$$

which is a group homomorphism, and which sends the generator σ_i to the transposition $(i\ i{+}1)$. On the presentation, π discards the distinction between σ_i and σ_i^{-1}—both map to the same transposition—and retains the exchange of positions.

The map is surjective: every permutation is realized by some braid, since the transpositions generate S_n. It is not injective, and the failure of injectivity is what this paper takes up. Figure 3 exhibits the failure. Both $\sigma_i\sigma_i^{-1}$ and $\sigma_i^{-1}\sigma_i$ map to the identity permutation, as they must, being the identity braid; but so does σ_i^2, which is not the identity braid at all—two strands have

wound fully around one another and returned to their starting positions. The permutation cannot see the winding.

Writing P_n for the kernel of π, we obtain the short exact sequence

$$1 \longrightarrow P_n \longrightarrow B_n \xrightarrow{\pi} S_n \longrightarrow 1, \tag{3}$$

which is the structure on which everything later in the paper rests. Its content may be put as follows. B_n records the full history of a configuration; S_n records only where the strands ended; and P_n is what the second forgets of the first.

The relation between the kernel and the classes it acts upon is easy to state loosely, and is stated here carefully. For a permutation $s \in S_n$, the set of braids realizing s is the fiber $\pi^{-1}(s)$, and these fibers partition B_n. The kernel P_n is the fiber over the identity, and is a group; the other fibers are its cosets and are not. What is true in general, and is the operative fact, is this:

$$\pi(b_1) = \pi(b_2) \quad \Longleftrightarrow \quad b_1 b_2^{-1} \in P_n. \tag{4}$$

P_n is thus not itself the collection of same-outcome histories. It is the group of transformations that carry any such history to any other with the same outcome—the group acting within each fiber. This distinction will matter later, when what is evaluated turns out to be differences between histories rather than histories.

2.5 The Pure Braid Group

The elements of P_n are the *pure braids*: those in which every strand returns to its own position. A pure braid is not thereby trivial. The strands may have wound around one another arbitrarily on the way, and the group is correspondingly rich.

It is generated by the elements A_{ij} for $1 \leq i < j \leq n$, where A_{ij} is the pure braid in which strand i passes across the intervening strands, encircles strand j once, and returns. In terms of the Artin generators,

$$A_{ij} = \sigma_{j-1} \cdots \sigma_{i+1}\, \sigma_i^2\, \sigma_{i+1}^{-1} \cdots \sigma_{j-1}^{-1}, \tag{5}$$

and one sees the σ_i^2 at the center: a full winding, which the projection to S_n annihilates. Figure 4 shows the case A_{13}. Structurally, P_n decomposes as an iterated semidirect product of free groups, a result due to Artin (1947); the reason is that forgetting the last strand defines a surjection $P_n \to P_{n-1}$ whose kernel is free on $n-1$ generators. The pure braid group is thus large, non-abelian for $n \geq 3$, and possessed of considerable internal structure. It is not a degenerate remainder.

To each pure braid and each pair of positions $i < j$ one may associate an integer, the *linking number* $\mathrm{lk}_{ij}(b)$, obtained by counting the crossings between strands i and j with sign—positive where i passes over j, negative where it passes under—and dividing by two. This is

Figure 4: A generator of the pure braid group: strand 1 crosses the intervening strand, encircles strand 3 once, and returns. Every strand ends where it began, so the braid lies in $\ker \pi$; the full winding at the centre is what the projection annihilates.

an isotopy invariant, and it is a homomorphism from P_n to $\mathbb{Z}$; the total linking numbers give the abelianization of P_n. What the linking number measures is the net winding of one strand about another, accumulated over the whole history. It is defined here in these terms and in no others. Whether such a quantity admits an interpretation, and what interpretation, is not a question the topology answers, and §4.4 takes it up only after the model has been built.

2.6 Decidability

Two braid words that look different may represent the same braid, the relations of §2.3 permitting a great many rewritings. It is not obvious in advance that one can always tell. In fact one can: the word problem for B_n is solvable, a result established by Artin (1925) and since given several algorithmic treatments, among them the combing procedure implicit in the semidirect product decomposition, Garside's normal form (Garside, 1969), and the handle-reduction algorithm of Dehornoy (1997), which is efficient in practice.

The consequence is unusual among the structures that social theory borrows. Given two histories represented as braid words, the question *are these the same* has an answer, and the answer is obtainable by a procedure that does not depend on the judgment of the analyst. A formalism that represented differences but could not adjudicate identity would be of little use for the purposes of this paper; it would relocate an interpretive difficulty rather than remove one. Decidability is the reason the braid group is a candidate at all.

2.7 The Structural Ceiling

What follows is stated here, before any interpretation has been proposed, so that the reader knows the boundary of the apparatus in advance of seeing its use. Each item is a feature of the

mathematics, not a limitation of the present treatment; no refinement of the model removes them, and each will be revisited in §6 as a constraint on what the model can be asked to bear.

Fixed n. The number of strands is fixed for the whole of a braid. Strands cannot be created, destroyed, merged, or split. B_n and B_m are different groups for $n \neq m$, and there is no operation within the theory carrying a configuration from one to the other. Whatever a braid describes, it describes a fixed population of positions persisting throughout.

Discreteness. A braid is a finite word in crossings. Crossings are events, individuated and countable, and there is no representation of a continuously varying influence between strands, of a field, or of a gradual change in the relation between two positions unpunctuated by an event.

Orientability without direction. A braid distinguishes over from under. It does not rank them, and the group contains no element and no relation that would let one say a braid is more advanced, more developed, or further along than another. Composition supplies an order in time; it supplies nothing normative. This is a limitation and, as §6 argues, also a protection.

Isotopy invariance. Everything that a continuous deformation can remove has been discarded by construction. The distance between strands, the sharpness of a bend, the timing of a crossing within the descent—none is represented. What the model sees is exactly the topological residue.

Silence on content. This limit is the least visible of the five. Two crossings of the same pair of strands in the same sense are the same generator, full stop. Whatever distinguished the two occasions—if anything did—has no representation whatever in B_n. The formalism records that positions i and $i+1$ interacted with a given orientation. It records nothing about what the interaction was.

With the structure and its boundary set down, we turn to the domain, and to the specific gap this structure is proposed to fill.

3 Procedural Justice

This section sets out the domain on its own terms. No braid appears in it, and the accounts reviewed are not treated as deficient versions of something the previous section supplies; each is treated as an answer to the question it set itself. What the section is for is to locate, as precisely as possible, a question that none of them poses, and to distinguish that question from several nearby ones that have been answered well.

3.1 Rawls and the Three Kinds of Procedural Justice

The distinction from which most subsequent discussion proceeds is Rawls's, and it is a distinction in the relation between a procedure and an independent criterion of the just outcome (Rawls, 1971, §14).

Where such a criterion exists and a procedure reliably realizes it, the case is one of *perfect* procedural justice. Rawls's example is the division of a cake among equals: the criterion is an equal division, and the rule that the person who cuts takes the last piece achieves it. The procedure is instrumental, and it is judged by whether it delivers what the criterion specifies.

Where the criterion exists but no procedure reliably realizes it, the case is *imperfect*. A criminal trial is the example: the criterion is that the guilty and only the guilty be convicted, and no procedure of evidence and argument guarantees this. Here too the procedure is instrumental, and its imperfection is a shortfall measured against something outside it.

Where no independent criterion exists, the case is *pure*. A fair gamble is the example: no distribution of winnings is just in advance of the procedure, and whatever distribution results from a properly conducted gamble is just because it so resulted. The procedure is constitutive rather than instrumental, and this is why the doctrine bears the weight it does in Rawls's larger argument, where the outcome of a suitably specified original position is just in virtue of the procedure rather than by correspondence to a prior standard.

Pure procedural justice is the case closest to what this paper is concerned with, since it is the case in which the procedure carries the whole normative burden. It is also, for present purposes, the case in which a specific insufficiency is clearest. The doctrine tells us that a correctly conducted procedure confers justice on its outcome. It does not tell us how to compare two procedures both of which are correctly conducted. If two gambles are both fair, the doctrine has nothing further to say about the difference between them; and if two deliberative procedures both satisfy whatever conditions are imposed, the doctrine treats them as equivalent. The apparatus is built to relate procedures to outcomes. It is not built to relate procedures to one another.

This is not a criticism of Rawls, who was answering a different question. It is an observation about what the trichotomy leaves undetermined. A theory that individuates procedures only by their conformity to conditions, and outcomes only by their distributions, has no resources for a difference between two conforming procedures that issue in the same distribution. Whether there is any such difference worth marking is precisely what the empirical literature went on to establish.

3.2 Tyler and the Empirical Turn

The finding that people evaluate procedures independently of the outcomes those procedures deliver was established first by Thibaut & Walker (1975) in experimental settings and then, at scale and in the field, by Tyler and his collaborators (Lind & Tyler, 1988; Tyler, 1990, 2006). The central result is robust and has been replicated across legal, organizational, and political settings: judgments of the fairness of a procedure predict compliance, acceptance, and the legitimacy accorded to an authority, and they do so with the favorability of the outcome controlled. People who lost accept losing, when they judge the procedure fair.

The factors that carry this judgment are conventionally given as four: *voice*, the opportunity to state one's case; *neutrality*, the absence of bias in the decision-maker; *trustworthiness*, the perception that the authority is acting in good faith; and *respect*, treatment consistent with one's standing as a person entitled to consideration. These are not features of the distribution. Two procedures identical in what they distribute may differ in every one of them.

The literature thus establishes, as securely as such things are established, that something outcome-invariant is being tracked. What it does not do—and does not attempt—is supply a structure for what is tracked. The four factors are properties of a procedure as experienced, measured by instruments that ask participants how they experienced it. This is entirely appropriate to the research programme, which is a programme in social psychology. But it means that the outcome-invariant residue is characterized by its effects on judgment rather than by anything about the procedure itself. If two procedures elicit different fairness ratings, we know that they differed; we do not have an account of what they differed *in*, other than by the same instruments that detected the difference. The finding is a finding about people. The question of what property of the procedures the people were responding to remains open.

3.3 Deliberative Theory and Sequence

Deliberative democratic theory is the strand that has come closest to the concern of this paper, because it is the strand that already knows that order matters.

The recognition arrives from two directions. From social choice, there is the long-established result that outcomes depend on agenda: McKelvey (1976) showed that under majority rule with multidimensional preferences, an agenda-setter can drive the assembly to any point whatever by a suitable sequence of pairwise votes. From the theory of deliberation itself, there is sustained attention to who speaks, when, and with what effect on what follows—the concern with inclusion in Young (2000), the treatment of deliberative conditions in Cohen (1989) and Habermas (1996), and the empirical study of deliberative settings in Mansbridge (1983) and Fung (2004), where the order in which perspectives enter is repeatedly found to shape what the assembly is subsequently able to consider.

The literature is therefore not innocent of the phenomenon. Its characteristic treatment of it, however, is diagnostic rather than descriptive: order effects appear as distortions to be identified and neutralized. Agenda manipulation is a pathology; the late arrival of a marginalized perspective is a failure of inclusion; the remedy in each case is a procedural constraint that removes the dependence on order. This is a reasonable response to the problem as posed. But it treats sequence as a source of error rather than as a dimension along which procedures differ, and consequently it produces no account of the differences themselves. Two procedures that differ only in sequence, and in which neither sequence is manipulative, are not a case the diagnostic framing has anything to say about. There is no positive characterization of the space of sequences—only a set of conditions that sequences ought not to violate.

3.4 Sen and the Comparative Alternative

Sen (2009) argues against what he calls transcendental institutionalism—the project of specifying perfectly just institutions—in favor of comparative assessments of realized social states. The argument has a component directly relevant here: Sen insists that the assessment of a state must include how it was arrived at, and offers the example of a flute to be assigned among three children with different claims, where the identity of the just assignment depends on considerations that no ranking of final distributions captures.

This is a substantial widening. The object of assessment is enlarged from distributions to realizations, and realizations carry information about process. Yet the structure of the assessment is unchanged: what is compared is still a pair of states, and the process information enters as a further descriptor of the state that resulted. Two realizations reached by different routes are two states to be compared. The comparison does not take as its object the difference between the routes; it takes as its object the states, more richly described. The comparative move accordingly relocates the endpoint rather than dispensing with it.

3.5 The Gap

It would be a mistake to state the gap as an absence of attention to history. History is well represented across the social sciences, and a claim to be supplying it would be false. Process tracing reconstructs causal sequences within cases (Beach & Pedersen, 2013). Event-history models represent the timing and order of transitions. Causal graphs represent the structure of dependence among events (Pearl, 2009). The extensive form of a game is precisely a representation of histories, complete with the information available at each point. Path-dependent institutionalism has made lock-in, sequence, and critical juncture into standard analytical vocabulary (David, 1985; Arthur, 1994; Pierson, 2000; Mahoney, 2000). Each of these represents processes, and represents them well.

What is missing is narrower, and it survives all of the above. Consider the collection of procedural histories that issue in a given distribution. The frameworks just named will describe each of these histories individually, and describe them with as much fidelity as one likes. None of them turns the *collection* into an object. None of them, consequently, has anything to say about the differences among its members considered as differences: whether they form a structure, whether that structure has parts, whether two of them are the same difference, whether the difference between the first and second history is the same difference as that between the third and fourth. These are questions one can ask only if the differences are themselves elements of something.

There is a second component to the lack, and it is what makes the first component consequential rather than merely abstract. Suppose one wished to claim of two procedures that they are the same procedure differently described, or that they are genuinely different. In the

frameworks named, this question is settled by the analyst's description: two process traces are the same if the analyst has described them the same way, and the individuation of the events is part of what the analyst supplies. There is no independent criterion of procedural identity. A structure in which such a criterion exists would put the claim that two procedures differ beyond the reach of redescription, and would make the assertion that a difference is merely notational into something refutable.

The gap may accordingly be stated as follows.

> **The gap.** *Every account reviewed here registers that procedure carries weight the outcome does not record, and several represent procedural history in detail. None supplies a structure in which the histories issuing in a given outcome form a determinate collection, in which the differences among them are themselves elements admitting composition and comparison, and in which the identity of two procedures is decidable independently of how the analyst has chosen to describe them.*

Whether such a structure exists is a mathematical question, and §2 has exhibited a candidate. Whether the candidate can be brought to bear on the domain just surveyed—and at what cost, with what license, and with what refusals—is the subject of what follows.

4 Formal Modeling

The two preceding sections were written not to meet. This one makes them meet, and the manner of the meeting is the substance of what follows. Each correspondence below is proposed, and then immediately tested: what it licenses is stated, and what it does not license is stated with equal care, since a mapping that licenses everything licenses nothing.

4.1 The Base Correspondence

Strands. A strand is a *role*: a relational position within a procedure, occupied by whoever occupies it. Three candidates present themselves, and the choice among them is forced rather than free.

Strands might be individuals. This fails immediately against the fixed n of §2.7: a procedure in which a participant arrives late or withdraws is not representable, and such procedures are common. Strands might be relational identities in the fuller sense that generative relational being gives that term—the marginalized participant, the community, the institution as a constituted thing. This is closer to the framework's own ontology and fails for a subtler reason: relational identities are generated and transformed by the very interactions the braid would record, so the strands would have to change during the braid, which the structure does not permit.

Roles are what remains, and their adequacy is bounded in a way that should be stated rather than concealed. A role in a procedure—the proposer, the objector, the chair, the party with the technical claim—persists through the procedure while the individual occupying it may not, and a procedure with a stable role structure and a changing membership is representable where a procedure with changing roles is not.

This has a consequence for the relation between the model and the wider framework.

> **Claim 1 (Domain of application).** *Fixed-n braid theory begins after role stabilization. It models transformations among established relational positions, not the emergence of those positions. Where the roles are themselves in formation, the model does not apply, and the point at which it ceases to apply is detectable.*

Events. A crossing is an interaction event, but not every occurrence within a procedure is one. The individuation requires a criterion, and the following three conditions are proposed jointly: an occurrence is a crossing when (i) it modifies the relational state of at least two roles; (ii) it is recognized within the procedure as an interaction, rather than being incidental to it; and (iii) it alters the space of transitions subsequently available.

The third condition is the one that does the work, and it is supplied by the framework rather than by the mathematics. An interaction, on this account, is contact that changes what can happen next. Two participants who are simultaneously present in a room have not thereby crossed. Two participants one of whom has advanced a claim the other must now answer have.

The condition carries an implication that should be entered explicitly.

> **Claim 2 (Inherited ontology).** *Event individuation is performed before topological encoding, and by criteria the braid does not contain. Condition (iii) refers to a space of future possibility of which the braid has no representation whatever. The formalism therefore inherits an event ontology it does not generate.*

The shape of this dependence is familiar: a coordinate system does not generate the geometry it describes, and a notation does not generate the meanings it records. What follows for the model is that its outputs are only as well-founded as the individuation supplied to it, and that disagreement about a case may be disagreement about the encoding rather than about the topology. Such disagreement is at least locatable, which is more than the alternative offers.

Orientation. The generator σ_i and its inverse σ_i^{-1} differ in which role passed over the other. This is the *orientation* of the interaction, and it is precisely that: which of the two roles was, in the event, the one whose position prevailed. Figure 1 shows the two. It is not a measure of power, of domination, of advantage, or of injustice, and the mapping does not license the

inference from orientation to any of these. Two parties may exchange positions in a hundred ways of which the topology sees only one bit, and a role that prevails in an interaction may be the weaker party prevailing, the stronger party conceding, or neither. Asymmetry, where the paper needs it, is defined separately in §4.4 and is not read off the generator.

History and outcome. A braid word is the ordered record of the interaction events, and the braid is the equivalence class of such records under the relations of §2.3. Its image $\pi(b)$ is the *terminal configuration of roles*: the rearrangement of relational positions that the procedure effected.

It is tempting to call this the distributional outcome, and the temptation should be resisted. The two coincide only where the distribution is a function of terminal relational position—where, that is, what each party receives is determined by where each party ends. This condition is often satisfied and is not trivial, and §5.2 exhibits a procedure in which it fails: two assemblies may approve the same allocation while leaving the roles in different terminal arrangements, or leave the roles as they were while altering what is allocated. Where the condition fails, outcome-equivalence in the model's sense and outcome-equivalence in the distributional sense come apart, and the model tracks the former.

What the braid does not contain. The correspondence must be stated with a negative half, or it will be read as claiming more than it does. The braid is not the procedure. It is the relational event history *generated by* a procedure. A procedure comprises rules, authorizations, the information available to each party, the intentions with which parties act, the meanings the parties attach to what occurs, and an institutional setting that constrains all of these. None of this is in the braid. The procedure generates the braid; the braid does not recover the procedure, and two procedures differing in every respect just enumerated may generate the same braid.

4.2 Outcome-Invariant Procedural Difference

We may now state what the structure supplies against the gap of §3.5.

Let a procedure generate a braid $b \in B_n$. Its outcome is $\pi(b) \in S_n$, and the procedures issuing in a given outcome s are those whose braids lie in the fiber $\pi^{-1}(s)$. By (4), two procedures are outcome-equivalent exactly when their braids differ by an element of P_n. The differences among outcome-equivalent procedures are therefore not a heterogeneous collection of dissimilarities; they are the elements of a group, they compose, they invert, and two of them may be compared and found equal or unequal by a decidable procedure.

The first consequence concerns Rawls. Pure procedural justice, as §3.1 observed, has no resources for comparing two correctly conducted procedures. In the present terms, the doctrine evaluates in S_n. This is a statement of where its instruments look, and no slight against

them. And it makes precise what was earlier said loosely: an evaluation conducted in S_n is structurally blind. It is not incomplete in the way a coarse measurement is incomplete. There is no refinement of outcome evaluation that recovers P_n, because P_n is by construction what the projection annihilates.

The second consequence concerns identity. The claim that two procedures differ, in the frameworks of §3.5, is settled by the analyst's description. Here it is settled by (4) together with the decidability of §2.6: given encodings, whether $b_1 b_2^{-1} = e$ has an answer, and the answer does not depend on who is asking. The encoding remains the analyst's, by Claim 2; but the analyst who has supplied an encoding has thereby given up the freedom to redescribe.

> **Claim 3 (The object).** *The pure braid group is a structure of the kind §3.5 found missing: the histories issuing in a given outcome form the fiber $\pi^{-1}(s)$; the differences among them are the elements of P_n, acting within the fiber; and the identity of two procedures is decidable independently of description. Outcome-based evaluation is blind to P_n by construction, not by coarseness.*

What does not follow must be stated in the same breath. It does not follow that the elements of P_n are justice-relevant, singly or collectively. P_n is a space of candidate differences, and the claim that a candidate matters is a claim of a different kind, requiring an argument the topology cannot supply. A reader who takes Claim 3 to have established that procedural differences matter has read into it the conclusion §4.7 exists to withhold.

4.3 Sequence and Procedural Indifference

Deliberative theory, by §3.3, knows that order matters and treats order effects as distortions. It has no positive characterization of which reorderings are substantive. The relations of §2.3 supply one, and it has two parts.

Far commutation says that crossings involving disjoint pairs of roles may be reordered freely. Where the objection of one party to a second and the objection of a third party to a fourth are the events in question, the sequence between them is not represented at all: the two orders generate the same braid, and no evaluation sensitive only to the relational history can distinguish them. This is a strong statement, and it is falsifiable. If participants reliably distinguish such orders, and the distinction is not attributable to something outside the relational history, then either the encoding has mis-individuated the events—the two objections were not in fact disjoint—or the model is wrong at this point.

The braid relation says that among overlapping crossings, one specific reordering is likewise indifferent: $\sigma_i\sigma_{i+1}\sigma_i$ and $\sigma_{i+1}\sigma_i\sigma_{i+1}$ are the same braid. Where three roles interact pairwise in sequence, there are two orders that produce the same relational history and others that do not. The relation thus divides reorderings of overlapping events into the indifferent and the substantive, which is exactly the division §3.3 found the literature unable to draw.

The limit of this test is as important as the test. That two procedures are the same braid establishes that they are the same relational history. It does not establish that they are the same in any other respect, and the respects in which they may differ are unbounded, since by §4.1 the braid contains no rules, intentions, information, or meanings. Two Yang–Baxter-equivalent sequences may differ in whether the interactions were conducted with respect or with humiliation. The braid sees one object; the participants may see two. The model's verdict of indifference is a verdict about relational topology and about nothing else.

4.4 Asymmetry and Persistence

The linking number was defined in §2.5 as the signed count of crossings between two strands. Its interpretation is now proposed, and the proposal is deliberately weak.

> **Claim 4 (Persistence observable).** *For a pure braid b and roles i, j, the quantity* $\mathrm{lk}_{ij}(b)$ *is an observable of* relational persistence*: the extent to which the interaction between two roles has accumulated in a consistent orientation over the history rather than cancelling. It is not a measure of dominance, inequality, advantage, or injustice, and no such reading is licensed by the topology.*

The reason for the weakness is that a stronger reading fails on cases. A high linking number between two roles is consistent with sustained domination of one by the other; it is equally consistent with a productive and repeated engagement in which one party consistently carries a particular function. The topology does not distinguish these, and a definition asserting that it does would be false.

The connection to the framework's account of asymmetry and absorption may now be made. Generativity, in that account, absorbs asymmetry rather than accumulating it: a relation in which asymmetries arise and are worked through differs from one in which they arise and persist. In braid terms, absorption appears as reduction—a history whose word, however long, reduces; whose crossings meet their inverses; whose linking numbers return toward zero. Persistence appears as its failure. The correspondence is exact enough to be useful and must be handled carefully, because reduction is a property of the braid and absorption is a claim about the relation, and the second does not follow from the first.

A further limit applies here, and it constrains the model more than any other. Winding has no direction. The braid σ_i^k records that two roles wound about one another k times; it does not record whether the winding was a deepening or a deadlock. A negotiation that returns twenty times to the same issue because each return teaches the parties something, and a negotiation that returns twenty times because neither party can move, generate the same topological signature. The distinction between them is real and consequential, and the braid does not carry it.

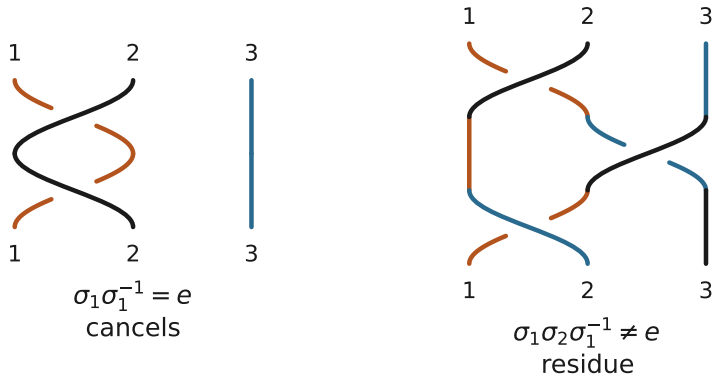

Figure 5: Reversal and residue. Left: a crossing immediately followed by its inverse cancels, and the relational history is as if neither had occurred. Right: the same reversal separated from its original by an interaction touching one of the same roles does not cancel, and the history retains a trace of an event formally undone.

The situation admits a precise statement. What a braid supplies is the kinematics of a relational history—the structure of the trajectory—and not its dynamics. A trajectory does not determine the forces that produced it, and no amount of refinement of the trajectory's description will supply them. Anything the model is asked to say about whether a history was generative or degenerative therefore requires a state variable the braid does not carry, together with an interpretation of the sort §4.7 formalizes.

4.5 Revisability and Reversal

That σ_i and σ_i^{-1} are distinct and mutually inverse gives the model a natural image of correction: an interaction that reverses an earlier one. Appeal, redress, the reconsideration of a decision, the retraction of a claim—each is an event whose orientation is opposite to one that preceded it.

The structure then says something that is not obvious and is not trivial. Where a crossing is immediately followed by its inverse, the two cancel: $\sigma_i\sigma_i^{-1} = e$, and the relational history is as if neither had occurred. Where they are separated in the word by other crossings, they need not cancel, and in general do not. Reversal at a distance leaves residue.

Figure 5 contrasts the two cases. The reason is structural. Between the crossing and its reversal, other interactions have occurred, and the relations of §2.3 permit the reversal to be brought back to meet its original only when the intervening crossings commute past it—which, by far commutation, requires that they involve roles disjoint from the pair in question. Where the intervening interactions touched the same roles, the cancellation is blocked, and the history retains a trace of an event that has been formally undone.

The reading for appeals procedures is direct, and it is a genuine output of the model rather than a restatement of its input. A decision reversed on appeal is not thereby a decision that

was never taken, and whether the reversal restores the prior relational state depends on what occurred in between and on whom it involved. The model does not merely permit this observation; it predicts when the residue occurs and when it does not.

4.6 Degradation

The concept treated here does not come from the literature of §3. It is supplied by generative relational being, where degradation names the failure of a relation to continue generating, and it is entered here because the braid structure sharpens a distinction that framework draws informally. It is treated only to the depth the present argument requires.

The framework distinguishes two levels. At the first, generation is blocked while the grammar of the relation remains intact: the parties continue to interact in the terms they have, and those terms cease to produce anything new. At the second, the grammar itself is rewritten: what counts as an interaction, who the parties are, what moves are available—these change, and the relation that results is a different relation, rather than a degraded version of the earlier one.

In the model, the second level is unambiguous. A rewriting of the grammar is an alteration of the generators or the relations among them, and a configuration so altered is not an element of B_n at all. Level 2 degradation is exit from the group.

The first level is not unambiguous, and the previous subsection has already shown why. The braid signature of blocked generation—a word that grows without reducing, winding accumulating between a fixed pair of roles—is exactly the signature of productive repetition. The signature is necessary and it is not sufficient. To assert Level 1 degradation of a history exhibiting it, one requires in addition a judgment about whether the space of relational possibility contracted or expanded, and the braid contains no such space.

The asymmetry between the levels may be stated as follows.

> **Claim 5 (Detectability asymmetry).** *Level 2 degradation is detectable as failure of the model: the encoding breaks, and the breaking is the finding. Level 1 degradation is not detectable within the model at all, its topological signature being shared with generative repetition; it requires an interpretation supplied from outside.*

4.7 The Interpretive Operator

Everything above has been withheld from the same conclusion, and the withholding may now be given a form.

The model represents outcome-invariant procedural differences as elements of P_n. It does not evaluate them. Some of these differences are justice-relevant; some are not; and the assign-

ment of significance is made on grounds that are not topological. Write

$$\Phi \colon P_n \times C \longrightarrow \mathcal{R} \tag{6}$$

for the operator effecting that assignment, where $\mathcal{R}$ is a space of relational significance and C carries what the braid does not: role meanings, institutional setting, the information available to the parties, the content of the interactions.

The context argument is not a formality. The same element of P_n may signify differently in different settings, and the case that shows it most economically is the reversal of §4.5. A crossing followed by its inverse is one element of the group. Occurring in one setting it is a healthy correction; in another the public humiliation of the party reversed; in a third a forced retreat under pressure. The topology is identical in all three. Any operator assigning significance to the topology alone would have to give them the same value, and would be wrong twice.

For readability the paper writes Φ as acting on P_n where the context is fixed and understood. The full form is (6), and the abbreviation should not be mistaken for a claim that significance is recoverable from topology.

This paper does not construct Φ. The claim entered is weaker and is not, on that account, empty.

> **Claim 6 (Necessity and localization).** *An account of procedural justice that distinguishes the procedural differences that matter from those that do not thereby employs an operator of the type* (6)*, whether or not it is written down. A formalism that separates the representation of differences from their evaluation has therefore performed a service even where the evaluative half is left empty: it converts a difficulty distributed throughout an account into a single identified object.*

The objection that the hard part has been moved into Φ is correct, and is the point. What has been gained is that it has been moved somewhere, and can now be examined.

4.8 Semantics and Failure Conditions

Table 1 fixes the mapping. The third and fourth columns are the operative ones: a correspondence that licenses nothing specific is decoration, and one whose failure conditions are unstated cannot be wrong.

Three failure conditions apply to the mapping as a whole, and each is testable in principle. First, if participants in procedures reliably distinguish far-commuting reorderings, and the distinction is not traceable to mis-individuated events or to content outside the relational history, the treatment of independence in §4.3 is wrong. Second, if procedures whose braids are equal are found to differ in respects the model attributes to the braid, the encoding is under-specified

Domain	Braid object	Licenses	Does not license
Relational position	Strand	Fixed role structure through a procedure	Individuals; emerging or dissolving identities
Interaction event	Crossing	Countable, ordered events meeting the three conditions	Continuous influence; incidental co-presence
Orientation of interaction	σ_i vs. σ_i^{-1}	Which role's position prevailed	Power, dominance, advantage, injustice
Procedural history	Braid word	Ordered record; comparison; decidable identity	The procedure itself, with its rules and meanings
Distributional outcome	$\pi(b) \in S_n$	Terminal configuration of roles	The distribution, except where it is a function of that configuration
Procedural difference	Element of P_n	Composition, inversion, comparison of differences	That the difference is justice-relevant
Accumulated interaction	$\mathrm{lk}_{ij}(b)$	Persistence of orientation across a history	Domination; inequality; injustice
Absorption of asymmetry	Word reduction	Formal image of asymmetry worked through	That reduction was generative
Correction, appeal	σ_i^{-1} after σ_i	Cancellation when adjacent; residue at distance	That the correction was adequate or fair
Level 1 degradation	Unreduced growth	A necessary signature	Sufficiency; generative repetition shares it
Level 2 degradation	Exit from B_n	Detection as encoding failure	Representation of what replaced the grammar
Relational significance	$\Phi(p, c)$	Location of the evaluative step	Any particular assignment of value

Table 1: The mapping, with what each correspondence licenses and refuses.

or the correspondence is too coarse. Third, if the elements of P_n arising in practice turn out to be uniformly justice-relevant, then Φ is constant and the elaborate apparatus of representation without evaluation is unmotivated—the model would then be claiming a distinction that does no work. §5 takes up the third directly, since a model that cannot produce an insignificant difference is a machine for confirming itself.

5 Constructed Cases

5.1 Method

The four cases below are constructed rather than observed. The choice is deliberate and follows the practice of modeling in general: a constructed case can be specified so that the quantity under test is the only thing varying, which no observed case permits. Historical instances carry

uncontrolled variation in the dimensions the model claims to be sensitive to and in those it claims to be blind to, and a model tested against them fails or succeeds for reasons that cannot be isolated. The cases here are accordingly fictional, and are built to be realistic in the sense that each could occur, not in the sense that each did.

The cases are built for discrimination rather than illustration. A model that produces a difference wherever a difference exists has established nothing, since by Claim 3 nontrivial elements of P_n are guaranteed and their exhibition is arithmetic rather than evidence. The four cases accordingly test four distinct things. Case A tests *sensitivity*: whether the difference the model isolates is the difference participants contest. Case B tests *specificity*: whether the model can produce a procedural difference that carries no significance, which by §4.8 it must be able to do or be unfalsifiable. Case C tests *underdetermination*: whether procedures the model identifies may differ in ways that matter. Case D tests the *boundary*: whether the model fails where §2.7 says it must, and whether the failure is legible when it comes.

The encoding protocol is uniform. In each case the roles are named and fixed; the events are individuated by the three conditions of §4.1; the orientation of each crossing is stated; and the resulting word is given. Where two variants are compared, their images under π are computed and the separating element of P_n is exhibited.

5.2 Case A: The Village Water System

Two villages decide whether to build a community water system. In both, the decision is taken by an assembly; in both, the same system is approved, with the same budget, the same allocation of access, and approval by the same margin.

Three roles are relevant, and they are stable throughout in each village: the council, 1; the technical advisor, 2; and the households whose water access is at stake, 3. Individuals vary and the roles do not, which is the condition Claim 1 requires.

Village A. The council raises the proposal, and the advisor's assessment modifies it: σ_1. The households then object that the proposed siting will disadvantage the outlying settlements, and the advisor revises the design in response: σ_2. The revised design returns to the council, which accepts the revision: σ_1^{-1}. The assembly approves. The word is

$$b_A = \sigma_1 \, \sigma_2 \, \sigma_1^{-1}.$$

Village B. The council raises the proposal and the advisor's assessment modifies it: σ_1. The council then settles the design: the advisor's revision is accepted before consultation: σ_1^{-1}. The households are consulted afterward, raise the same objection about the outlying settlements,

and it is recorded: σ_2. The assembly approves. The word is

$$b_B = \sigma_1 \, \sigma_1^{-1} \, \sigma_2 = \sigma_2.$$

Outcome. $\pi(b_A) = (1\,2)(2\,3)(1\,2) = (1\,3)$ and $\pi(b_B) = (2\,3)$. These are not equal, and the case as stated does not yet exhibit what it was built to exhibit.

This is a finding about the encoding rather than an oversight to be repaired quietly. Two procedures with the same distributional outcome must have the same permutation of roles, and the permutation records which role ends in which position. The difficulty is that "the same decision was taken" is not the same claim as "the roles ended in the same relation." In Village B, the households did not exchange position with the advisor in the course of the decision; their objection was recorded after the design was fixed. In Village A they did.

The repair is to encode the recording of an objection that alters nothing as what it is: an event that fails condition (iii) of §4.1, since it does not alter the space of transitions subsequently available. It is not a crossing. Village B's word is then

$$b_B = \sigma_1 \, \sigma_1^{-1} = e,$$

and the two procedures now have different outcomes for a reason that is substantive rather than technical: in Village B, no relational rearrangement occurred at all.

What the case establishes. Not what it was designed to establish, and something more useful. The case was constructed to show two procedures with the same outcome differing by a pure braid. It shows instead that the difference between deliberation-before-decision and consultation-after-decision is not an outcome-invariant difference at all. It is a difference in outcome, in the model's sense of outcome—which is the rearrangement of relational positions, and not the distribution of water. The households who altered the design occupy a different terminal position from the households whose objection was filed.

This is a result about the model's sense of *outcome*, and it sharpens §4.2. The projection π does not record the distribution. It records the terminal configuration of roles, and a procedure may leave the distribution untouched while rearranging the roles, or leave the roles untouched while altering the distribution. The identification of $\pi(b)$ with "what outcome evaluation sees" in §4.1 was too quick, and the honest statement is narrower: $\pi(b)$ is what an evaluation of terminal relational position sees, which coincides with distributional evaluation only where the distribution is a function of that position.

Sensitivity, retested. A case that does test sensitivity may be built from the same material. Suppose in both villages the households do alter the design, so that $\pi(b_A) = \pi(b_{A'})$, but in Village A' the alteration comes after the advisor and the council have twice exchanged and

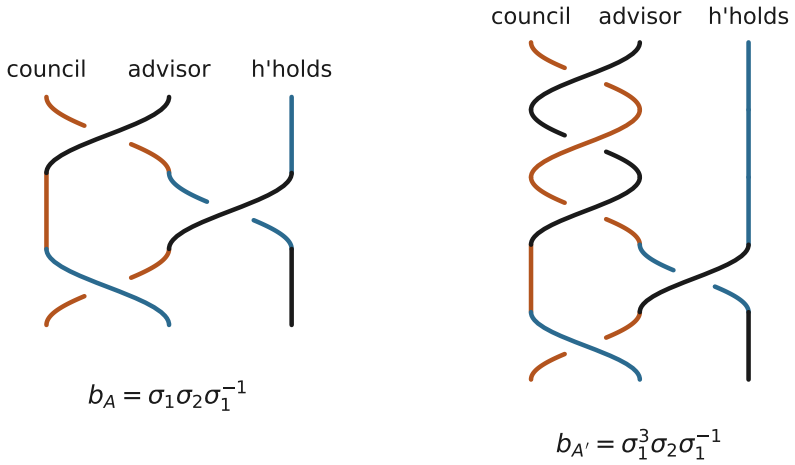

Figure 6: Case A, retested for sensitivity. Both procedures leave the roles in the same terminal arrangement, so no evaluation of outcome distinguishes them. In the second, council and advisor exchange and re-exchange position before the households are heard. The procedures differ by σ_1^2, a pure braid of linking number one.

re-exchanged positions over the siting question:

$$b_{A'} = \sigma_1^2\,\sigma_1\,\sigma_2\,\sigma_1^{-1}, \qquad b_A = \sigma_1\,\sigma_2\,\sigma_1^{-1}.$$

The two are drawn in Figure 6. Then $b_{A'}b_A^{-1} = \sigma_1^2 \in P_3$, a pure braid of linking number one between council and advisor. The procedures are outcome-equivalent and differ by a determinate element of the kernel. Whether participants contest this difference is the substantive question, and the answer is not obvious: an extended exchange between council and advisor before the households are heard may be experienced as due diligence or as the settling of the matter in advance. The model isolates the difference and hands the question on. That is the whole of what Claim 3 promised, and the case shows the promise kept and its narrowness.

5.3 Case B: Two Sittings of a Committee

A committee of three roles—chair 1, first member 2, second member 3—considers two motions in a single sitting. The motions are unrelated: the first concerns the maintenance schedule, the second the wording of an annual report. In each, one member raises a point and the chair rules on it.

In sitting X, the maintenance point is raised and ruled on, then the report point: $b_X = \sigma_1\sigma_2$. In sitting Y, the order is reversed: $b_Y = \sigma_2\sigma_1$. Both sittings dispose of both motions identically.

Here $\pi(\sigma_1\sigma_2)$ and $\pi(\sigma_2\sigma_1)$ are the two distinct three-cycles, so this pair is not outcome-equivalent, and the far commutation of §4.3 does not apply either, the two crossings sharing the middle strand. Construct instead the comparison the test requires. Let each sitting com-

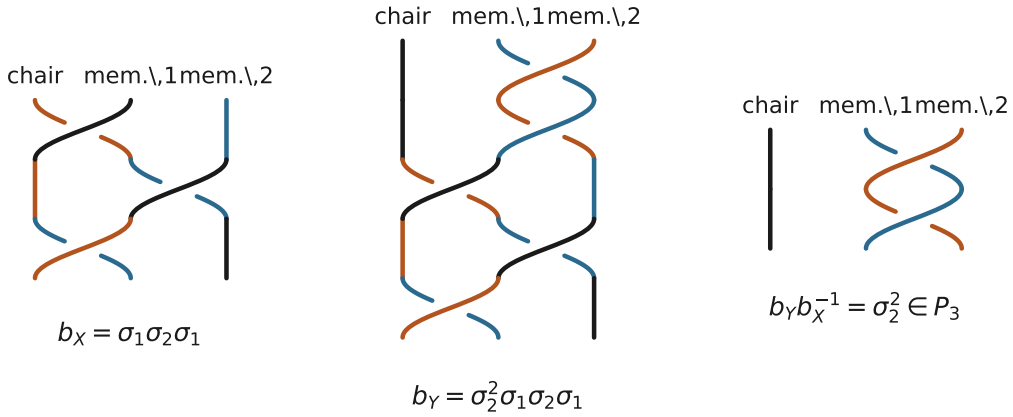

Figure 7: Case B. The two sittings leave the roles identically arranged and differ by the pure braid on the right: a full winding between the two members, arising from a preliminary consultation on a matter about which they agreed. The model reports a difference; no participant has reason to care about it.

prise the same three substantive exchanges, and let sitting Y open with a preliminary consultation between the two members in which each defers to the other in turn and the matter is left where it stood:

$$b_X = \sigma_1\,\sigma_2\,\sigma_1, \qquad b_Y = \sigma_2^2\,\sigma_1\,\sigma_2\,\sigma_1.$$

Then $\pi(b_X) = \pi(b_Y) = (1\,3)$, and

$$b_Y\,b_X^{-1} = \sigma_2^2 \in P_3,$$

a pure braid of linking number one between the two members. Figure 7 shows the two sittings and the element separating them. The element is nontrivial: sitting Y contains a full winding between the members that sitting X does not, and by §2.6 the fact that the two sittings are different procedures is decidable rather than a matter of the analyst's description.

Assessment. No participant regards the sittings as differing in fairness, and there is no evident ground on which one should. The additional exchange in sitting Y was a consultation between two members about a matter on which they agreed, conducted in the ordinary course, affecting no one's standing and no one's opportunity to be heard. The model reports a difference; the difference is of no interest.

> **Claim 7 (Specificity).** *The case exhibits a nontrivial $p \in P_n$ for which $\Phi(p, c) = 0$ on any reasonable assignment. The model is therefore not a machine for converting topological difference into normative difference, and the third failure condition of §4.8 is not met here.*

Omitting this case would be a serious fault. Without it, every difference the model isolates would be available for treatment as significant, and the apparatus of §4.7—the insistence that

representation and evaluation are separate—would be unmotivated. A model that cannot produce an insignificant difference has no need of an operator to filter differences, and its author's protestations that topology carries no normative weight would be belied by a practice in which topology always did.

Toward a criterion. What distinguishes Case A's contested pure braid from Case B's uncontested one? A conjecture, offered as such: the winding in Case A occurred between roles whose relative position bore on the matter under decision, and altered the sequence in which a third role could enter; the winding in Case B occurred between roles whose exchange bore on nothing at issue and foreclosed no subsequent move. The suggestion is that the relevant feature is the pure braid together with its relation to the possibility space—which is to say, precisely the content that condition (iii) of §4.1 appeals to and the braid does not contain. If this is right, Φ draws on the same information that event individuation draws on, and the two problems are one problem. The conjecture is not established here.

5.4 Case C: The River Treaty

Two states share a river and negotiate its allocation. Three roles: the upstream state 1, the downstream state 2, and the technical commission 3 that certifies the hydrological basis of any agreement. In both versions the outcome is a fifty-fifty allocation, certified, on the same schedule.

Version I. The upstream state opens with a demand for the greater share, which the downstream state refuses: σ_1. The commission's assessment establishes the downstream state's agricultural dependency, and the upstream state revises: $\sigma_2\sigma_1^{-1}$. The downstream state, learning the upstream state's energy constraint, concedes on timing: $\sigma_1\sigma_2^{-1}$. The parties converge. The word is

$$b_{\mathrm{I}} = \sigma_1\,\sigma_2\,\sigma_1^{-1}\,\sigma_1\,\sigma_2^{-1} = \sigma_1.$$

Version II. The upstream state opens with the same demand and the same refusal follows: σ_1. The upstream state signals that a trade agreement under separate negotiation may be affected; the commission's assessment is entered and the downstream state's position shifts: $\sigma_2\sigma_1^{-1}$. The downstream state accepts the timing terms: $\sigma_1\sigma_2^{-1}$. The word is

$$b_{\mathrm{II}} = \sigma_1\,\sigma_2\,\sigma_1^{-1}\,\sigma_1\,\sigma_2^{-1} = \sigma_1.$$

Assessment. The braids are equal. Not outcome-equivalent, not differing by a pure braid: equal. The model reports that these are the same procedure, and by (4) together with the decidability of §2.6, the report is not a matter of judgment.

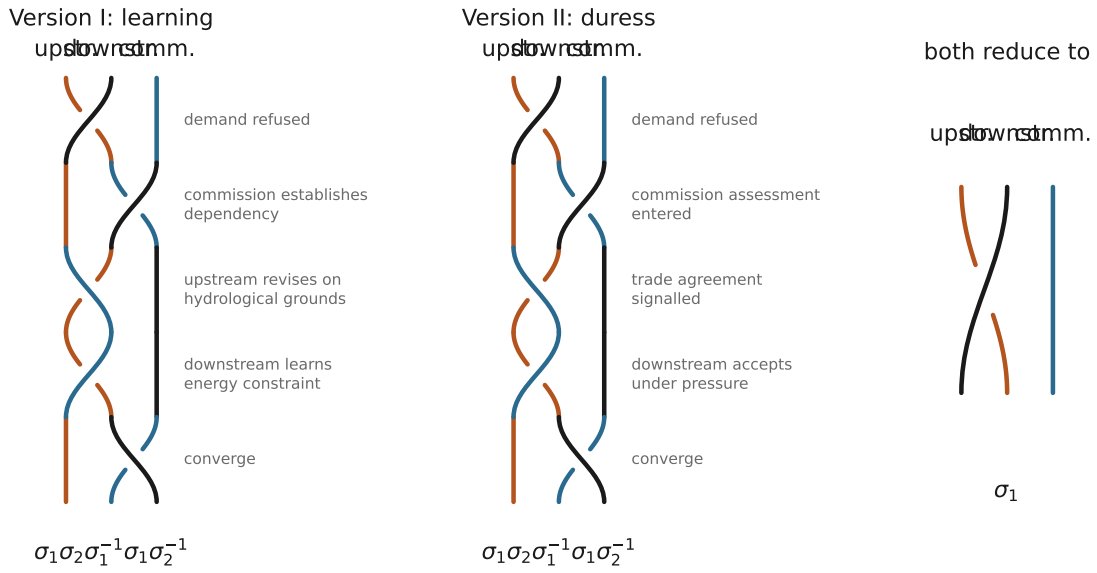

Figure 8: Case C. The two panels are identical, and that is the finding. Each crossing is annotated with what occurred on that occasion; the annotations differ throughout, and the braid records none of them. Both words reduce to the single crossing at right. The difference between agreement reached by learning and agreement reached under duress is not an element of P_n awaiting evaluation—it is absent from the representation.

Figure 8 draws the two encodings side by side, annotating each crossing with what took place on that occasion. The diagrams are indistinguishable and the annotations are not. The procedures are not the same. In Version I the downstream state's shift followed from information about the river; in Version II it followed from a threat concerning something else entirely. The parties know the difference, any observer knows it, and the relation the two treaties leave behind is not the same relation. A treaty reached by mutual learning and a treaty reached under duress may be word-for-word identical and are not the same achievement.

Nothing in the encoding is wrong. The roles are correctly individuated, the events meet all three conditions, the orientations are correct. The model has done what it does, and what it does not do is carry the content of an interaction. By the last item of §2.7, two crossings of the same pair in the same sense are the same generator, whatever occurred on the two occasions. Version I's third event and Version II's third event are both the downstream state shifting its position in response to something the upstream state did. The topology sees a shift.

What would be required. The distinction demands a state variable the braid does not carry: minimally, whether the shift was produced by a change in the shifting party's information about the object of the negotiation or by a change in its expectations about matters outside it. This is not a refinement of the topology and cannot be obtained by encoding more carefully. It is the content of the crossing, and the model's silence on content is structural.

The case accordingly confirms §4.1's negative half in a strong form. Not that the braid omits some of the procedure, but that it omits enough of it that two procedures differing in what is arguably the most important respect available—whether agreement was reached through understanding or through pressure—are formally indistinguishable. The winding observation of §4.4 is a corollary rather than the point: here the words do not merely wind alike, they are the same word.

5.5 Case D: The Union

A firm negotiates terms with its workers. Initially each worker negotiates individually, and the relevant roles are the firm 1, worker A as 2, and worker B as 3. The firm makes an offer to A, who accepts a variation: σ_1. The firm makes a different offer to B: σ_2, and B refuses: σ_2^{-1}. So far the encoding proceeds.

The workers then form a union. Subsequent negotiation is between the firm and the union, and the union bargains for both workers together.

The point of failure. The encoding cannot continue, and where it stops can be stated exactly. A report of general breakdown would conceal the structure of the failure. Three distinct things have occurred, and §2.7 treats them differently.

Two strands have merged into one. The union is the role in which the several positions of A and B are now jointly occupied, and not a fourth role alongside them. The configuration after formation has fewer relational positions than before, and B_3 and B_2 are different groups with no operation of the theory carrying an element of one to the other. This is the fixed-n ceiling, and it is reached exactly at the formation.

A role has been created. The union did not exist before and exists after, and its existence is a product of the interactions the braid was recording. By Claim 1 the model applies after role stabilization; here the procedure destabilized the roles, doing the thing the model's domain excludes.

The grammar has changed. Before formation, a refusal by one worker left the other's negotiation untouched, and the two were independent in the sense far commutation formalizes. After formation there is no such independence, because there is no longer a pair of positions to be independent. What counts as an interaction, and which interactions are available, have both been altered. This is Level 2 degradation in the sense of §4.6. The term formally denotes a rewriting of the grammar, and carries no implication of deterioration: union formation may be an improvement in the respects that matter.

The finding. These three are separable, and separating them is the case's contribution. Strand merging, strand creation, and relation redefinition are distinct failures requiring distinct extensions: a structure permitting merges, a structure permitting births, and a structure in which

the relations among generators may themselves vary. A single gesture at "the model breaks here" would conceal that three different repairs are needed. §6 takes up what the repairs would involve.

The legibility of the failure is itself a result. The model does not degrade gracefully into wrongness; it stops, at an identifiable event, for reasons that name themselves. A formalism whose domain boundary is visible from inside is more useful than one that continues to produce output after its assumptions have lapsed.

5.6 Cross-Case Comparison

Case A tested sensitivity and returned, first, a correction: the model's outcome is the terminal configuration of roles, not the distribution, and the two coincide only under conditions the paper had not stated. With the correction in hand, the case exhibits a contested pure braid and hands the question of its significance on, which is what the model claims to do and the limit of what it claims.

Case B tested specificity and returned a nontrivial element of P_n that no one has reason to care about. This is the case without which the model would be unfalsifiable, and it also produced the beginning of a criterion for Φ: the difference between the two windings appears to lie in their relation to the space of subsequent possibility, which is content the braid does not contain and event individuation already requires.

Case C tested underdetermination and returned a negative result stronger than the one it was built for. Two procedures differing in whether agreement was reached by learning or under duress generate the same word, not merely words differing by an element of the kernel. The model's silence on content is a wall, and not a residual imprecision.

Case D tested the boundary and returned a legible failure decomposing into three distinct ceiling violations. The model stopped where §2.7 said it would, at an identifiable event, and the manner of stopping distinguished three different extensions that would be required.

Taken together the four establish the necessity of Φ from two directions at once. Case B shows that topological difference does not suffice for significance: there are differences Φ must send to zero. Case C shows that topological identity does not suffice for insignificance: there are significant differences Φ cannot see at all, because they are not in P_n to be evaluated. An operator is required, and it must draw on information the braid does not carry—which is why (6) carries a context argument, and why the conjecture of §5.3 locates that information in the same place event individuation finds it.

The boundary the cases jointly draw may then be stated. Within its domain the model represents procedural differences determinately and adjudicates procedural identity decidably. It does not evaluate what it represents, it does not represent the content of what it records, and it ceases to apply when the roles it presupposes are themselves in formation. Each of these three is a limit of a different kind, and §6 treats them in turn.

6 Discussion

6.1 Yield

Five things, of unequal weight.

A *decidable criterion of procedural identity*. By §2.6 and (4), whether two encoded procedures are the same is answerable by a procedure independent of the analyst. This is the component that answers the second half of the gap in §3.5, and it is what distinguishes the proposal from the frameworks that already represent history well. Encoding remains the analyst's, by Claim 2; but an analyst who has encoded has forfeited the freedom to redescribe.

A *criterion of procedural indifference*. Far commutation and the braid relation divide reorderings into those that change the relational history and those that do not. Deliberative theory wanted this division, as §3.3 argued, and its diagnostic framing could not supply it. What the model gives is narrow—indifference of relational topology, not of experience—and it is a positive characterization where none existed.

A *prediction about reversal*. §4.5 is the one place the formalism generates a result rather than restating an intuition in new notation. Cancellation of a crossing by its inverse is complete when they are adjacent and incomplete when separated by interactions touching the same roles; and this is derived, not stipulated. Appeal restores the prior relational state under a condition the model specifies. Whether the condition is empirically right is testable and is not tested here.

An *observable of persistence*. The linking number, under the deliberately weak reading of Claim 4, measures accumulation of orientation across a history. That it is not a measure of injustice is what makes it usable: a quantity defined as injustice would have prejudged the questions worth asking.

A *location for the evaluative step*. Before (6), the interpretive move in accounts of procedural justice is distributed—some of it in the individuation of what counts as a procedural feature, some in the selection of which features to measure, some in the normative argument proper. The model does not remove the move. It collects it.

6.2 Direction and Its Absence

A formal structure imported into normative theory carries a risk of smuggling a direction of travel. The risk bears naming because the framework this paper belongs to has committed itself against it, and a formalism that reintroduced it through the back door would undo the commitment more effectively than an argument could.

The braid group supplies no direction. There is no partial order on B_n under which some braids are more advanced than others; length is not such an order, since a longer word may represent a shorter braid; and composition orders events in time without ranking configura-

tions. No braid is progress. Two procedures related by an element of P_n stand in no relation of priority whatever, and the model has no resources for saying otherwise.

This limitation of the formalism is simultaneously a protection. Because no normative ordering is available in B_n, none can enter the account by way of B_n; whatever normative content the account carries must be carried by Φ, where it is visible and can be argued with. The conatus back door—the move by which persistence or elaboration or increase becomes self-justifying—is closed here by absence rather than by prohibition. There is nothing in the structure that could be mistaken for a direction.

6.3 Limits

Six limits, each traced to §2.7 and each now furnished with a case or an argument.

Emergence of roles. Fixed n forbids strand creation, and Case D reached the prohibition at an identifiable event. The framework's account of subject production—relational positions generated by the interactions they then participate in—is thus outside the model, not partially represented within it.

Merging and dissolution. Distinct from creation, and requiring a distinct repair. Case D exhibited two workers becoming one bargaining position; nothing in B_n carries B_3 to B_2.

Alteration of the grammar. Distinct again. Where the relations among generators change—where what counts as an interaction, or which interactions are available, is itself rewritten—the configuration is not an element of the group. This is Level 2 degradation in the sense of §4.6, and its detectability is entirely negative: the model reports it by failing.

Continuous influence. Crossings are discrete events. A relation that shifts gradually without punctuating events has no representation, and the three conditions of §4.1 will find no crossings to individuate.

The content of an interaction. Case C is the demonstration. Two procedures identical in topology may differ in whether agreement was reached by learning or under duress, and the difference is substantial and is not recoverable by more careful encoding. The braid records that an interaction occurred with an orientation. It records nothing of what the interaction was.

The direction of winding. By §4.4, repetition that deepens and repetition that deadlocks share a signature. Any claim about degradation at Level 1 accordingly requires a state variable the model does not contain. A concept central to the framework is therefore one the topology cannot adjudicate.

6.4 Layers of Evaluation

The scope declaration of §1 may now be developed, since the model is in hand and its boundary is known. This subsection states where the contribution sits and is not itself the contribution,

entered so that the reader does not take a treatment of one layer for a treatment of the whole.

The architecture runs: a relational process generates a history; the history is represented as a braid; the projection π discards the path and retains the terminal configuration; what it discards is characterized by P_n; and Φ assigns significance to elements of P_n in context. Everything this paper has done lies at or below Φ. Above it lie questions of evaluation, and they are several questions of different kinds.

Generativity (G) asks whether an interaction expands the space of relational possibility available afterward. This is descriptive, not normative—a fact about a process, ascertainable in principle without any judgment about whether the expansion is welcome. It is also, and this is the awkward part, presupposed by the model already: condition (iii) of §4.1 individuates events by whether they alter the space of subsequent transitions, which is to say that the encoding requires a notion of possibility space that the braid does not represent. The gap is load-bearing. A formal account of G would be a precondition for a fully specified encoding, not merely an extension of it.

Procedural justice (J_p) asks whether generation occurred through legitimate relations. This is the layer treated here, and it is a question about legitimacy rather than about capacity or about value.

Generative justice (J_g) asks whether a process preserves and expands the capacity of its participants to generate. This is the distinctively framework-specific question, and it is independent of J_p in both directions. A procedure impeccable by the criteria of §3 may leave its participants less able to generate than it found them—consuming trust, foreclosing positions, exhausting the conditions on which future interaction depends—and the case is neither exotic nor rare. That $J_p = 1$ is compatible with $J_g = 0$ is the reason procedural justice cannot be the whole of what the framework requires.

Normative orientation (V) asks toward what. Neither generativity nor procedural legitimacy answers it, and two examples establish it. A hurricane is a highly generative dissipative structure, producing organization across scales; nothing follows about its value. A community may fairly, transparently, and with full participation design a more efficient system of exploitation; nothing about the fairness of the design redeems what was designed. Generativity is not value, and legitimate procedure is not value, and an account that lets either stand in for the third commits the error this paper's guardrail was erected against.

These four are not four species of one thing, and the differences among them are differences of kind: G descriptive, J_p a matter of legitimacy, J_g a matter of sustained capacity, V a matter of direction. Collapsing them is the characteristic failure. Collapse J_p into V and legitimate procedure becomes self-justifying, which is the teleology §6.2 excludes. Collapse G into V and generation becomes its own warrant, which is the conatus back door the framework has consistently refused. The separation is the same guardrail, positioned where the pressure to breach it is greatest.

As to how the layers might be investigated: G invites treatment in terms of reachability and the size of the option set available after an interaction as against before. J_g admits a candidate formal criterion—whether the set of relational states reachable after a procedure contains the set reachable before—which is offered here as a conjecture and is not developed. V is not, on the present evidence, amenable to formal treatment of the kind attempted here, and naming it as such is preferable to gesturing at a formalism that does not exist.

6.5 Open Problems

Constructing Φ. The central one. The paper has argued that an account discriminating among procedural differences thereby employs an operator of type (6), and has declined to build it. What Case B produced is a starting point and no more: the conjecture that a pure braid's significance turns on its relation to the space of subsequent possibility, and that Φ therefore draws on the same information that event individuation draws on. If that is right, the two problems are one, and a treatment of G would supply both.

Structures permitting birth, death, merging, and splitting. Cobordism is the natural direction. Where a braid is a collection of strands running between two fixed configurations, a cobordism is a manifold between two boundaries whose components may join, divide, appear, and vanish; the four failures Case D separated correspond to the four operations a cobordism permits and a braid does not. The framework's account of subject production requires exactly these, which suggests that the extension is a precondition for treating emergence formally at all.

Compositional settings. Braided monoidal categories generalize the braid group to a setting where objects may be composed, and would be the appropriate home for procedures built from sub-procedures. Higher categorical structures allow the relations themselves to vary, which is what Level 2 degradation would require to be represented rather than merely detected as failure.

Other candidate structures. The braid group is one candidate and the paper has claimed no more. Fundamental groups and groupoids of configuration spaces, mapping class groups, and path spaces more generally are alternatives with different strengths, and a comparison would be informative. It is possible that some other structure supplies decidable procedural identity at lower cost in assumptions; if so, the argument of §3.5 survives and the candidate changes.

Operationalization. The persistence observable of Claim 4 is defined and unmeasured. Whether it can be computed from records of actual procedures—minutes, transcripts, negotiation logs—is an empirical question, and the encoding problem of Claim 2 is where the difficulty would concentrate.

Degradation at length. The two-level distinction of §4.6 has been treated only as far as the present argument required. The asymmetry of Claim 5—one level detectable only as model failure, the other not detectable at all without interpretation—calls for a fuller treatment than

the present argument affords it.

6.6 Conclusion

Procedural justice has long described a class of differences that lacked a formal object adequate to represent them. Rawls's doctrine of pure procedural justice made the procedure constitutive of the justice of its outcome and gave no way to compare two procedures both correctly conducted. Tyler's programme established that something outcome-invariant is tracked and characterized it by its effects on judgment. Deliberative theory knew that sequence mattered and treated the knowledge diagnostically. Sen widened the object of assessment and left it an object still assessed at its terminus.

The pure braid group supplies one candidate structure for what these accounts described. It is a group of transformations preserving terminal configuration while altering relational history, within which outcome-invariant differences are determinate elements and procedural identity is decidable. That is the positive claim, and the four cases have shown its reach and where it stops. It does not evaluate what it represents. It does not represent the content of what it records—Case C being the demonstration. It ceases to apply where the roles it presupposes are still in formation, which is where the framework's own deepest questions begin.

What remains, then, is a structural substrate on which evaluations may operate, together with a specification of where each of them must enter. The braid carries the memory of a process. Whether the process was legitimate, whether it left its participants able to continue generating, and whether what it generated was worth generating are three further questions, and the value of a formalism that answers none of them is that it makes the asking of each unavoidable and separate.

Acknowledgments

The argument was refined through successive rounds of critical reading, and several of its central corrections—the status of P_n as a transformation group rather than a collection, the context-dependence of Φ, the directionlessness of winding, and the necessity of a case in which the model produces a difference of no significance—originated as objections. Errors that remain are the author's.

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