

Causal Sufficiency and Structural Vulnerability

*Rethinking Responsibility in Generative Relational Systems:
A Preliminary Position Paper on the Foundations of Structural Attribution*

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Abstract

Two companion papers establish that in one class of injury the classical apparatus of causal attribution fails, and that the failure is of two kinds, one of resources and one of what a determinate contribution would require of any tribunal. Where causal sufficiency is unattainable, responsibility theory may turn to the structure of the relational system and ask which transformations of that structure a party brought about. This paper recapitulates those problems, separates the operation of law from the justice of its results, and examines whether the turn can be made. It offers no rule for deriving a share of responsibility from a structural change, and it holds that none is presently available. Its claims are four. The object of an attributive enquiry may be relocated from the production of an outcome to the production of a vulnerability, and the relocation is intelligible independently of any measure of contribution. A description of structural change is by itself normatively inert, since organisational reform alters connectivity, feedback and reachability in the same registers as degradation does, and no quantity computed on the structure distinguishes the two. A criterion is therefore required, and the paper proposes a candidate: a structural change carries normative weight where it removes the paths by which that very change could be contested and reversed. The candidate is self-referential where the alternatives are not, since a change leaving the channel of its own correction intact remains corrigible whatever else it does, and two changes to one structure are exhibited that agree on size, density and connectedness and differ only in whether the path of contestation survives. The invariants employed are then defined in the standard way, as quantities equal on topologically equivalent flows, and a pair of dynamics is exhibited whose basins differ in the number of holes they contain, so that the register of loops is shown to carry structural and not quantitative information. A constructed case is then carried through both enquiries: over the weightings the record admits, the share attributed to each of two withdrawals occupies most of the available range, while membership of a minimal cut is settled for all three. Finally the paper states what a finding on an invariant contributes and what it withholds. It contributes a finite evidentiary task, robustness to the strengths, rates and couplings a record leaves open, order-independent composition, and an identification of the parties concerned through the minimal sets of relations whose removal destroys the path. It withholds a share, it moves the requirement of closure from variables

to positions rather than escaping it, it purchases invariance by discarding the difference between a channel that functions and one that is nominal, it leaves the minimal set undetermined where the record is incomplete, and it is silent on time. The proposal is offered as a constraint on attribution rather than as a procedure for it.

Keywords: structural attribution; causal sufficiency; vulnerability; relational systems; contestability; revisability; organisational structure; responsibility theory.

A note on the standing of this paper. This paper is diagnostic and not attributive. It states a direction for a theory of responsibility and does not state a theory of responsibility, and the distinction is maintained throughout because the temptation to overstate is strong in this material. Where a structural account is offered, what is offered is a candidate criterion together with the conditions under which it would fail. The two companion papers, on the representation of trajectories and on the observability of the condition of a system, are assumed and are not re-argued. Objection, correction, and counter-evidence are welcome at huangwanhong@serendip.ngo.

Draft. This is a preprint draft. It is circulated for comment, and its arguments, formulations, and open questions are subject to revision. Correction and criticism are welcome at huangwanhong@serendip.ngo.

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Use of generative artificial intelligence. An AI assistant (Anthropic’s Claude) was used extensively in preparing this manuscript: for literature survey, for drafting prose from the author’s specifications, for adversarial review of the argument, for the computation underlying the figure, for consistency auditing, and for typesetting. Formulations and objections arose in the course of that exchange as well as from the author. Every claim, argument, and citation was reviewed and decided by the author, who is responsible for the content and for any errors. Bibliographic details are to be verified against published sources before reliance.

Suggested citation: Wanhong Huang, *Causal Sufficiency and Structural Vulnerability: Rethinking Responsibility in Generative Relational Systems*, draft preprint, 2026.

1 INTRODUCTION

This section states what the companion papers establish, states the question that follows from them, and marks the distance between that question and the one this paper answers. The method is to fix the claim at the outset at the level the argument can support, since the material invites a stronger claim than the argument delivers.

Two results are assumed. In a class of injury where factors individually incapable of destabilising a system act over a long period and bring it near a boundary of stability, the classical apparatus of causal attribution fails, and it fails at both ends of the process for reasons belonging to different doctrines. The failure is of two kinds. One concerns the resources available for recording a history and for interpreting what is recorded, and it would be relieved by the extension of those resources. The other concerns what a finding of determinate contribution would require of any tribunal whatever, namely closure of the set of variables bearing on the case, a declared class of admissible trajectories, and identifiability of the couplings among the variables, and it is unaffected by any capacity of the tribunal.

The second kind is the one that generates the present question. Where a determinate contribution is unavailable in principle, an attributive enquiry that insists upon it has no object, and a theory of responsibility for such systems must either abandon the field or find a different object.

Claim 1.1. (The relocation of the object). *An attributive enquiry may take as its object the production of a vulnerability rather than the production of an outcome. On this construction the question put to a party is whether that party brought about a transformation of the relational structure after which the system was less able to absorb what befell it, and the question is intelligible without any measure of the party's contribution to the outcome that followed.*

The relocation is intelligible and it is not thereby a theory. Three things are absent from it. It supplies no rule by which a share of responsibility follows from a structural transformation. It supplies no criterion distinguishing the transformations that matter from those that do not. And it supplies no method of establishing that a transformation occurred, which is the subject of the second companion paper and is not treated here.

Claim 1.2. (The standing of what follows). *This paper addresses the second of the three absences and leaves the first and the third open. What it offers is a constraint upon attribution and not a procedure for attribution, and the difference is that a constraint excludes candidates while a procedure selects one.*

The order of what follows is: the problems that a generative relational system presents to legal form, and the justice problems that no advance in legal form addresses (§2); the literatures that already describe structural change in organisations (§3); the shift from sufficiency to vulnerability and what it costs (§4); the three registers in which structural change is ordinarily

described, and the reason that a description in those registers settles nothing (§5); a candidate criterion and the demonstration that it selects where description does not (§6); the definition of the invariants employed, a worked case, and what a finding on an invariant contributes and withholds (§7); the questions the criterion leaves unsettled (§8); and the conditions under which the argument fails (§9).

2 THE PROBLEMS OF LAW AND OF JUSTICE IN GENERATIVE RELATIONAL SYSTEMS

This section assembles the problems that a generative relational system presents to legal form, and separates those concerning the operation of law from those concerning the justice of its results. The method is to state, for each problem, the requirement that legal form imposes and the property of such a system that defeats it, and to record where each is treated at length. The section is a recapitulation and establishes nothing new, and it is included because the argument of §4 onwards is intelligible only against the whole set.

2.1 *Problems of legal operation*

Five problems arise in the operation of law, and they are independent in the sense that a doctrinal advance meeting one leaves the others standing.

The first concerns the subject. Attribution presupposes a subject antecedent to the act, and in a generative system the responsible subject is produced by the process to which it is to be held. The temporal order that attribution requires is reversed, and the doctrines that assume otherwise operate upon a fiction whose protective value is real and whose factual claim is false.

The second concerns causation. Legal causal doctrine imposes representational requirements upon the causal relation: that contributions be apportionable, that the duty-bearer sit at the level of the harm, that the effect of removing a cause be the restoration of the prior state, that a wrong have a place, that durations rather than rates be the temporal quantity, and that a change of state carry a date. Circular causation, level-crossing, hysteresis, non-locality, the relative rate of two processes, and the approach to a boundary of stability defeat these severally.

The third concerns attribution. Law attaches duties and rights to loci of control, and a distributed generative system produces its grammars, labels and norms at a level occupied by no locus of control, so it can be neither obligor nor obligee. The available responses are to freeze the process and treat a produced subject as antecedent, to relocate the duty to an exogenous party and assign it control, or to confer personality upon the process, and the three are asymmetric between rights and duties.

The fourth concerns evidence. What reaches a tribunal is a finite sample of a continuous history, taken under a threshold of salience, and the reconstruction is an interpolation supplied

from a repertoire that contains chains over events and no term for a change in the stability of a system. Where the threshold governing the record and the threshold governing legal significance coincide, the absence of evidence and the insignificance of the acts are one filter counted twice.

The fifth concerns remedy. The structure of legal causal representation is a projection of the structure of the available judgment: acyclicity follows from finality, a binary gain from a binary verdict, a fractional gain from divisible damages, and locality from territorial jurisdiction. The question what law can represent is therefore answered by asking what remedial forms are available, and forms that are continuing rather than terminal relax the requirements accordingly.

2.2 Problems of justice

Three further problems concern the justice of the results rather than the operation of the machinery, and they are not resolved by any advance in the machinery.

The first is standing. The occupant of a generated position retains an interest in entering an account of his own situation into a common record and in having it revised there, and a process that closes that possibility injures him in a manner no compensation reaches. This is the interest on which the criterion of §6 is founded.

The second is the protection of relationally generated value. Value produced by a relational process has no holder at the level at which it is produced, so it is protected where it can be assigned to a party and unprotected where it cannot, and the pattern of protection follows the availability of a holder rather than the worth of what is produced.

The third is the burden of translation. Where the parties to a relational system do not share the terms in which harm may be stated, the burden of rendering an experience in the terms the system recognises falls unequally, and it falls most heavily upon the party whose terms are furthest from those of the record. The discounting of an account for reasons bearing on the credibility of its author, and the absence of shared resources in which an experience could be rendered at all, are the two forms of that burden (Fricker, 2007).

Claim 2.1. (Two orders of difficulty). *The problems of legal operation admit of doctrinal advance, and several have received it. The problems of justice are not addressed by such advance, since a machinery that attributes accurately may still afford no standing, protect no relationally generated value, and impose the whole burden of translation upon one party. An account of responsibility for these systems must therefore be answerable at both levels, and a proposal that improves the machinery alone is to be assessed as such.*

Table 1 collects the eight problems with the requirement each engages.

Table 1: Problems of legal operation and of justice in generative relational systems

Problem	Requirement of legal form	Property that defeats it
Subject	A subject antecedent to the act	The subject is an output of the process
Causation	Apportionable, co-level, reversible, local, dated contributions	Circularity, level-crossing, hysteresis, non-locality, rate, undated transitions
Attribution	A locus of control at the level of the harm	The object is produced at a level no party occupies
Evidence	A record of the history	A sample under a threshold, interpolated from a repertoire
Remedy	A judgment that terminates	Continuing processes and non-terminal harms
Standing	A party able to state a claim	The position that would state it has been closed
Relational value	An assignable holder	Value produced at a level with no holder
Translation	Harm stated in the terms of the record	The terms are unequally available

3 RELATION TO PRIOR WORK ON STRUCTURAL DESCRIPTION

This section identifies what is already available for describing the structure of a relational system, so that the residue claimed here is narrow. The concession is extensive and the contribution of this paper lies entirely outside it.

The description of social structure by the relations among positions is a mature field. The consequences of the pattern of ties for the flow of information are established, and the distinction between ties that reproduce what is already available within a group and ties that connect otherwise separate groups is long settled (Granovetter, 1973). The advantage accruing to a position that spans a gap between groups has been analysed at length (Burt, 1992), and the formal apparatus for measuring connectivity, centrality, community structure and path length is standard (Newman, 2018).

The claim that responsibility may attach to participation in a structural process rather than to the causation of an outcome is likewise not new. The social connection model locates responsibility in participation in structures that produce unjust outcomes and distinguishes it explicitly from the liability model that requires a causal link to a harm (Young, 2011). The organisational literature on safety treats the maintenance of adaptive capacity as an object of management and describes the erosion of that capacity in structural terms (Woods, 2015). The design of arrangements that permit a group to govern a shared resource has produced principles that concern the standing of participants to alter the arrangements under which they act (Ostrom, 1990).

Two further bodies of work bear on the argument. The analysis of the injustices done to a

knower, in the discounting of credibility and in the absence of shared resources for rendering an experience intelligible, supplies the vocabulary in which §6 states what is lost when a channel of contestation is closed (Fricker, 2007). And the requirement that a private law claim be bipolar, so that the plaintiff's right and the defendant's duty are two aspects of one relation, marks the boundary that any structural account must confront (Weinrib, 1995).

Claim 3.1. (The residue after concession). *The description of structural change is available and is not claimed here. The proposition that responsibility may attach to participation in a structure is available and is not claimed here. What is absent from both is a criterion that separates the structural changes carrying normative weight from those that do not, and the absence is consequential because without such a criterion a structural account condemns reform and degradation alike.*

4 THE SHIFT FROM CAUSAL SUFFICIENCY TO STRUCTURAL VULNERABILITY

This section states the shift in the form of the attributive question, states what the shift purchases, and states the two things it gives up. The method is to set the two forms side by side and to identify the conditions each requires.

The classical form runs from an actor to a damage, and it requires that the damage be traced to the actor through a chain each link of which is established. The form proposed here runs from an actor to a transformation of structure and from that transformation to an increased vulnerability of the system, and it requires that the transformation be established and that the vulnerability follow from it.

Claim 4.1. (What the shift purchases). *The second form dispenses with the conditions that the first form cannot meet in this class of case. It requires no closure of the variable set, since a transformation of structure is stated over the relations present and not over everything bearing on the outcome. It requires no reconstruction of a continuous trajectory, since the transformation is a difference between two structures. And it requires no identification of the couplings, since the object is the structure through which couplings run and not the couplings themselves.*

Claim 4.2. (What the shift gives up). *The second form gives up the connection to the outcome. A party who brought about a transformation after which the system was more vulnerable has brought about a vulnerability, and whether the vulnerability was realised in the injury remains unaddressed. The second form therefore establishes a wrong of a different kind, and any account that presents it as an easier route to the original conclusion misdescribes it.*

Claim 4.3. (The bipolarity difficulty). *A structural transformation is a change to a system and the injured party occupies a position within it, so a claim founded on such a transformation must show that the change was a wrong to that party and not merely a change to the system in which*

that party stood. The requirement of bipolarity is therefore the first obstacle a structural account must clear, and the criterion of §6 is stated with reference to the position of the injured party for that reason (Weinrib, 1995).

5 THE REGISTERS OF STRUCTURAL CHANGE AND THEIR DESCRIPTIVE SILENCE

This section sets out the three registers in which a change to a relational system is ordinarily described, gives an instance of each, and states why a description in these registers settles nothing on its own. The method is to exhibit a benign instance alongside a harmful one in each register.

The first register is connectivity. A structure in which a member reaches those with authority through several distinct paths is transformed into one in which those paths are reduced to one or to none. The harmful instance is the isolation of a member; the benign instance is the removal of a reporting line in a reorganisation that consolidates a function.

The second register is the composition of loops. A structure in which conflict issues in communication, communication in repair, and repair in a restored relation is transformed into one in which conflict issues in retaliation, retaliation in silence, and silence in further conflict. The harmful instance is the establishment of a self-reinforcing cycle; the benign instance is the closure of a loop that reproduced an inefficiency.

The third register is the set of reachable states. A structure from which several arrangements were attainable is transformed into one from which a single arrangement is attainable. The harmful instance is the foreclosure of every settlement other than the departure of the injured party; the benign instance is the standardisation of a process that had proliferated without benefit.

Figure 1 sets the two instances of each register side by side.

Proposition 5.1. (Descriptive silence). *Each register admits benign and harmful instances that are indistinguishable within the register. Connectivity falls in a reorganisation and in an isolation; loops are broken in an improvement and in a degradation; the set of reachable states contracts under a standardisation and under a foreclosure. No quantity computed on the structure alone separates the pairs, so a structural description settles nothing until a criterion is supplied from outside it.*

Claim 5.2. (Against a coinage). *A term such as generative structure earns its place only where it names the invariants it selects. Where it names the intention with which a structure is regarded, it renames the problem of Proposition 5.1 and leaves it standing. Any such term used in this programme is to be introduced together with the criterion that gives it extension.*

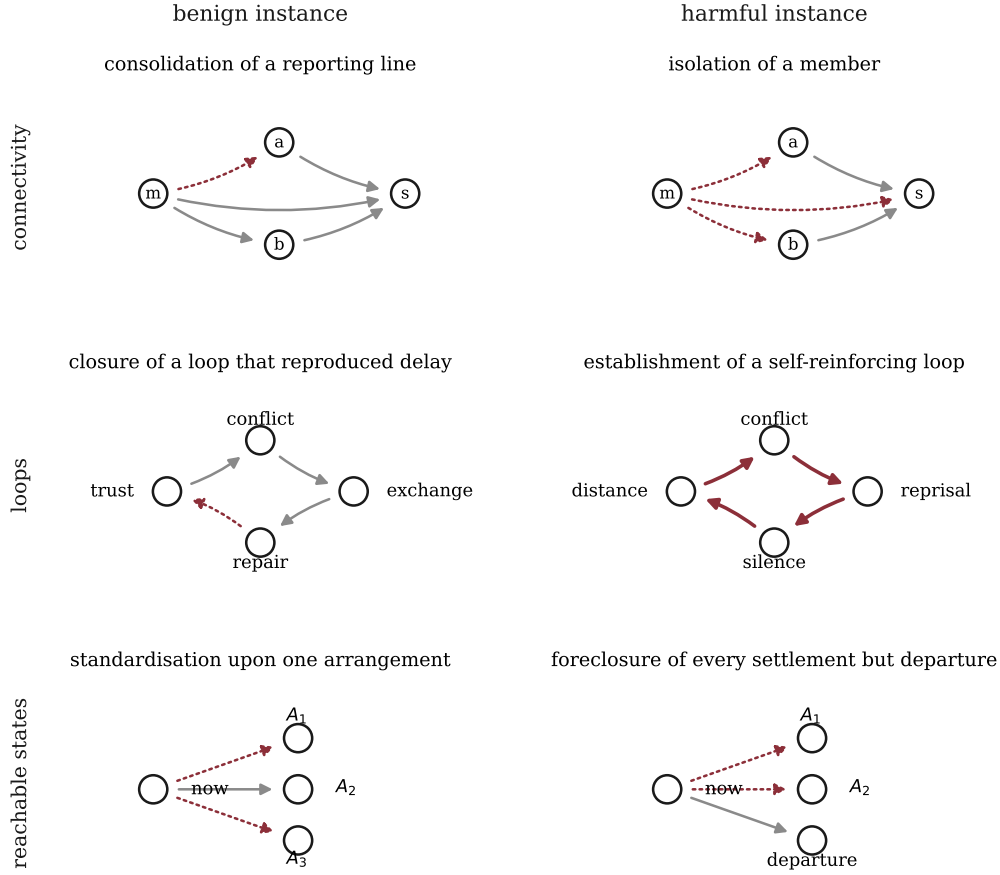


Figure 1: Benign and harmful instances within one register. Dotted arrows mark relations withdrawn and heavy arrows a loop established. In the first row connectivity falls in both columns; in the second a loop is broken on the left and a loop is formed on the right; in the third the set of reachable arrangements contracts in both. Within a register the two columns are described in the same terms.

6 A CANDIDATE CRITERION FOUNDED ON THE PATH OF CONTESTATION

This section proposes a criterion for Proposition 5.1, states the property that recommends it over the alternatives, and exhibits a pair of changes that the criterion separates and that structural description does not. The method is to state the criterion, to state the reason for preferring it, and then to test it against a constructed pair.

Definition 6.1. (The path of contestation). *Let the relational structure be represented by the relation holding between positions when the occupant of the first may raise a matter with the occupant of the second and have it entered into a record that the second may act upon. A path of contestation for a position is a sequence of such relations running from that position to a position competent to revise the arrangement in question.*

Claim 6.2. (The criterion). *A structural transformation carries normative weight where it*

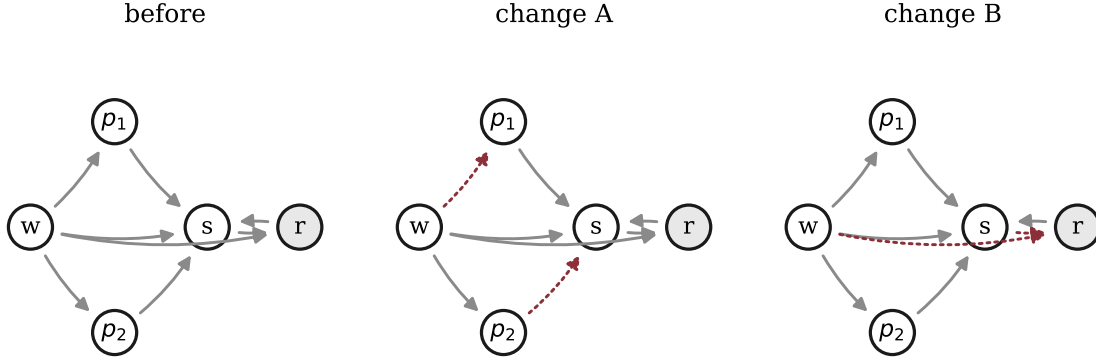


Figure 2: One structure and two transformations of it. Dotted arrows mark the relations removed. Change A withdraws a peer relation and a reporting line; change B withdraws the member's direct access to the position competent to revise and the supervisor's referral to it. The two resulting structures have five positions, six relations, a density of 0.3 and one weakly connected component apiece. Under change A a path of contestation from the member survives, and under change B none survives.

removes the paths of contestation available to a position, and it carries none on this ground where those paths survive it. The transformation that closes the channel through which it could itself be raised and reversed is the transformation that requires justification, and a transformation that leaves that channel intact remains corrigible whatever else it does.

The property recommending the criterion is its self-reference, and the alternatives lack it. A criterion stated on the number of ties, on the density of the structure, or on the size of the reachable set requires an independent judgement that the smaller value is worse, and that judgement is what Proposition 5.1 shows to be unavailable from the structure. The present criterion requires no such judgement, since the value it protects is the capacity of the structure to be corrected, and a structure retaining that capacity may be reformed and reformed again without the reformer being called upon to establish that each step was an improvement.

Claim 6.3. (Asymmetry of reversal). *A second consideration accompanies the first. Where the party who brought a transformation about is not among the parties who can reverse it, the transformation is irreversible from the position of those who bear it, and the two considerations coincide in the case of interest, since removing a path of contestation is one way of placing the reversal beyond the reach of the party affected.*

Figure 2 exhibits a structure and two transformations of it. The nodes are a member, two peers, a supervisor and a position competent to revise the arrangement. Each transformation removes two relations. The resulting structures agree in the number of positions, in the number of relations, in density and in connectedness, and they differ in whether the member retains a path to the position competent to revise.

Proposition 6.4. (Selection beyond the structural measures). *In the pair of Figure 2 the standard measures of size, density and connectedness take identical values after either transformation, so no attributive rule stated on those measures distinguishes them. The criterion of Claim 6.2 distinguishes them, since the path of contestation survives one and not the other. The criterion therefore has extension beyond the structural measures and is not a redescription of them.*

Claim 6.5. (Where the normative layer enters). *The criterion is not derived from the structure. It is supplied to the structural description from an account of what a relational system owes those who occupy positions within it, on which the occupant of any position retains the standing to enter an account of his situation into a common record and to have it revised there. The structural apparatus renders that standing checkable by inspection of the relations, and it supplies neither the standing nor the reason for protecting it.*

7 THE CONTRIBUTION OF INVARIANTS AND ITS LIMITS

This section states what a finding stated on an invariant of the structure contributes to an attributive enquiry, and states what it withholds. The method is to compare the finding with the quantitative finding it replaces, taking in turn the evidentiary burden, the robustness of the result, the composition of several contributions, and the identification of the parties concerned, and then to record the four respects in which the replacement is a loss.

7.1 *The invariants and the equivalence under which they are invariant*

This subsection fixes the sense in which the word invariant is used, states the equivalence relation the invariants are invariant under, names the particular invariants the argument employs, and records the relation between the invariants of a flow and those of a graph. The method is definitional, and the definitions are standard.

Definition 7.1. (Topological equivalence of flows). *Let φ and ψ be flows on manifolds M and N . They are topologically equivalent where a homeomorphism $h : M \rightarrow N$ carries the orbits of φ onto the orbits of ψ and preserves the direction of time. A quantity is a topological invariant of a flow where it takes equal values on topologically equivalent flows (Guckenheimer & Holmes, 1983).*

The definition fixes what a structural claim about a dynamical system can assert. Rates, distances, eigenvalues and the shapes of orbits are altered by a homeomorphism and are therefore no part of such a claim. What survives is the arrangement of the invariant sets and the manner in which the space is divided among them, and it is that arrangement the present argument uses.

Definition 7.2. (The invariants employed). *Three are employed. The number of equilibria*

together with the index of each, where the index of an isolated equilibrium is the degree of the normalised vector field on a small circle about it, and where the sum of the indices over a region is fixed by the boundary. The homology of the basin of an attractor, of which b_0 counts its connected components and b_1 the independent one-dimensional cycles it contains, so that b_1 counts the holes (Hatcher, 2002). And the existence of a periodic orbit together with the homotopy class it represents in the basin.

The general apparatus for invariants of this kind associates to an isolated invariant set a homotopy type that is unchanged under continuous deformation of the system, and it is the setting in which the three above are special cases (Conley, 1978). Nothing below requires that apparatus, and it is named so that the invariants employed are seen to belong to a developed theory and not to an analogy.

Figure 3 exhibits two dynamics that differ in the second of the three. The system is

$$\dot{r} = r(\mu - r^2), \quad \dot{\theta} = \omega, \quad (1)$$

in polar coordinates on the plane. For $\mu < 0$ the origin is asymptotically stable and its basin is the whole plane, which is contractible, so $b_0 = 1$ and $b_1 = 0$. For $\mu > 0$ the origin is unstable, a stable periodic orbit sits at $r = \sqrt{\mu}$, and the basin of that orbit is the plane with the origin removed, which retracts to a circle, so $b_0 = 1$ and $b_1 = 1$. The two flows are therefore not topologically equivalent, and the quantity separating them is the first Betti number of the basin.

Claim 7.3. (The example and the second register). *The passage from the left panel to the right is the formal counterpart of the second register of §5. A relational system whose ordinary operation is a cycle of conflict, exchange and repair has a basin containing a hole, and one that has settled into a single state has a basin without one. The transformation between them is a change in a topological invariant and is not a change of degree, so the claim that the second register carries structural rather than quantitative information is exact rather than figurative.*

The step from the invariants of a flow to the invariants of a graph is the one at which an argument of this kind most easily loses its footing, so it is taken in three stages.

Definition 7.4. (Recurrent sets and connections in a graph). *Let G be a directed graph of positions and relations. Its strongly connected components are the sets of positions each reachable from every other, and they are the recurrent sets of the discrete system in which a matter passes along a relation in one step. Contracting each to a point yields the condensation of G , which is acyclic and therefore carries a partial order, and a path in G between positions in distinct components is a connection between the corresponding elements of that order.*

Proposition 7.5. (One construction in two settings). *A Morse decomposition of a flow is a finite collection of isolated invariant sets containing all recurrence, ordered by the existence of*

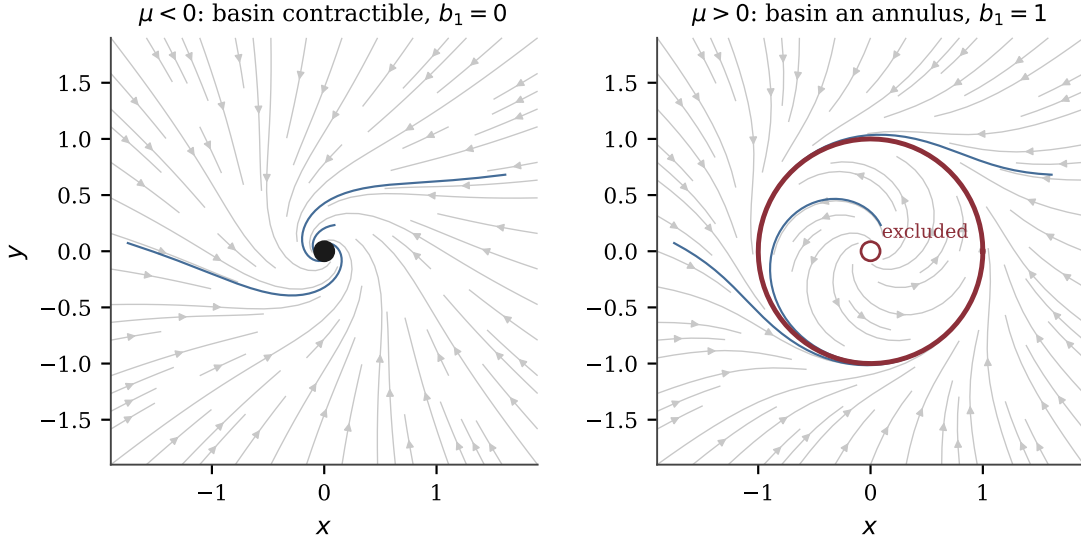


Figure 3: Two dynamics separated by a topological invariant. Left: for $\mu < 0$ every orbit approaches the equilibrium and the basin is contractible. Right: for $\mu > 0$ the equilibrium repels, orbits from within and from without approach the periodic orbit drawn heavy, and the basin of that orbit excludes the equilibrium, so it contains one hole. Light curves give the flow and the darker curves are three orbits integrated from the initial conditions marked by their endpoints. The limiting radius was computed as 1.001 against the predicted $\sqrt{\mu} = 1$.

connecting orbits between them, and the collection together with that order is unchanged under topological equivalence. The condensation of Definition 7.4 is the same construction carried out on a discrete system: the strongly connected components hold the recurrence, the partial order records which may be reached from which, and both are unchanged by any relabelling of the positions (Conley, 1978; Forman, 1998). The reachability of one position from another is therefore an invariant of the same kind as the existence of a connecting orbit, computed on a system whose steps are discrete.

Corollary 7.6. (The cut and the connection). Under Proposition 7.5 the path of contestation of Definition 6.1 is a connection between two elements of the order, and a cut is a set of relations whose removal destroys that connection. The passage from the invariants of §7.1 to the cuts of §7.2 is therefore a passage within one notion, and it requires no analogy.

Claim 7.7. (The limit of the correspondence). Proposition 7.5 places the two invariants in one family and establishes no numerical identity between them. The cycle rank of a relational graph and the first Betti number of a basin are quantities of two systems, and the graph employed here is a record of which relations obtain and is no discretisation of any flow. What the argument takes from the flow is the form of the invariant, and what it computes on the graph is an invariant of that form.

7.2 The contribution of a finding on an invariant

This subsection states the four respects in which a finding on the existence of a path improves upon a finding on a quantity. The method is to take in turn the evidentiary burden, the robustness of the result, the composition of several contributions, and the identification of the parties concerned.

Proposition 7.8. (Reduction of the evidentiary burden). *Establishing that a path of contestation is absent requires showing, for each candidate path over the declared positions, that one relation in it is missing. That is a finite search over an enumerable object. Establishing a share of contribution requires the three conditions of the first companion paper, of which the first is unavailable in principle. The two enquiries therefore differ in kind and not in degree of difficulty.*

Proposition 7.9. (Robustness to what the record leaves open). *The existence of a path is unchanged by the strength of the relations composing it, by the rate at which matters pass along them, and by the couplings among the positions. Those are the quantities that a finite record leaves underdetermined. A finding stated on the existence of a path therefore survives the uncertainties that defeat a finding stated on a quantity, and it survives them by construction and not by good fortune.*

Proposition 7.10. (Order-independent composition). *Reachability is a monotone function of the set of relations, since the removal of a relation can destroy a path and can create none. Contributions to the destruction of a path therefore compose without regard to the order in which they are counted. The order dependence that attends the apportionment of a concave quantity, recorded in the first companion paper, does not arise here, and this is the principal formal advantage of the change of object.*

Proposition 7.11. (The minimal cut and the parties concerned). *Call a set of relations a cut where its removal destroys every path of contestation from a position, and minimal where no proper subset of it is a cut. Every member of a minimal cut is necessary within that set and no member is sufficient by itself, so a minimal cut has the structure that the first companion paper found in the factors of a criticality case, with the difference that membership of a cut is decidable. In the structure of Figure 2 the minimum cut is unique and consists of the member's direct access to the revising position together with the supervisor's referral to it, which is change B; five inclusion-minimal cuts exist in that structure in all.*

The four propositions together give the sense in which an invariant may serve where a quantity may not. Each finding is moreover binary, and a verdict is binary, so a finding of this form is carried by the existing remedial apparatus without modification, which is the argument developed at length in the first companion paper and assumed here.

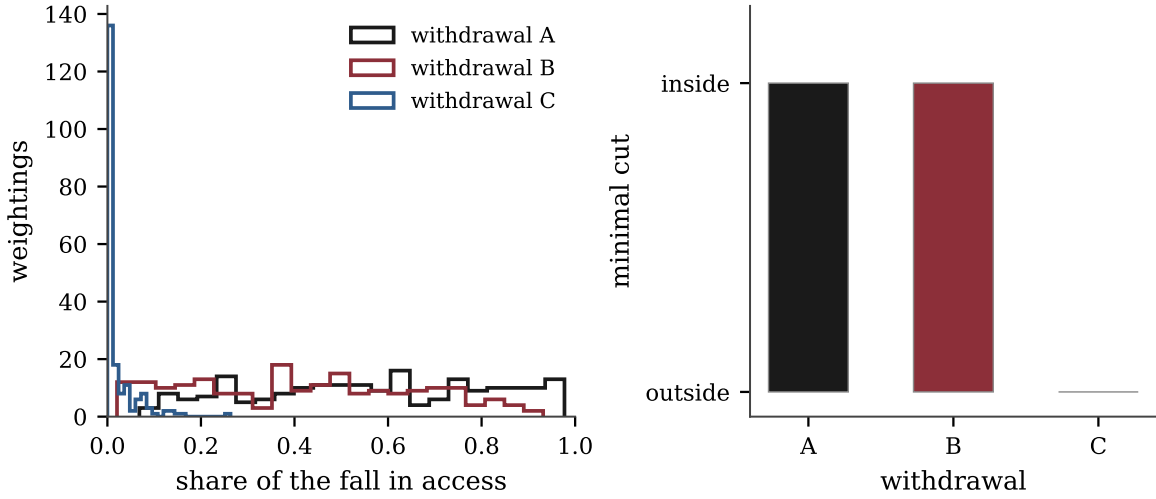


Figure 4: One case under the two enquiries. Left: the share of the fall in access attributed to each withdrawal, over two hundred weightings consistent with the record. The share of withdrawal A runs from 0.07 to 0.98 and that of B from 0.02 to 0.93, so the quantitative answer is settled by the weighting and by the order of counting. Right: membership of a minimal cut, which the record settles. Withdrawals A and B belong to the unique minimum cut and withdrawal C belongs to none, although a weighted measure charges C a positive share in about half of the weightings.

7.3 A constructed case

This subsection carries one constructed case through the two enquiries, so that the advantage claimed in the four preceding propositions may be inspected rather than accepted. The method is to fix a structure, to fix three withdrawals of relations, and then to compute the quantitative attribution over the weightings that the record leaves open and the cut membership over the same record.

The structure is that of Figure 2. Three parties act. Party A withdraws the member’s direct access to the revising position, party B withdraws the supervisor’s referral to it, and party C withdraws one of the member’s relations to a peer. The record establishes which relations obtained before and after and establishes nothing about the strength of any of them. Two enquiries are then put.

The first asks what share of the fall in access each withdrawal produced. Answering it requires a weighting of the relations, which the record withholds, so the computation was run over two hundred weightings drawn at random from the admissible range, with the withdrawals counted in a random order in each. The second asks which withdrawals belong to a minimal cut of the paths of contestation, which the record settles. Figure 4 reports both.

Proposition 7.12. (The case as computed). *Over the weightings admitted by the record the share attributed to withdrawal A ranged from 0.07 to 0.98 and that attributed to B from 0.02 to 0.93, so each occupied most of the available range. Withdrawal C received a positive share in about*

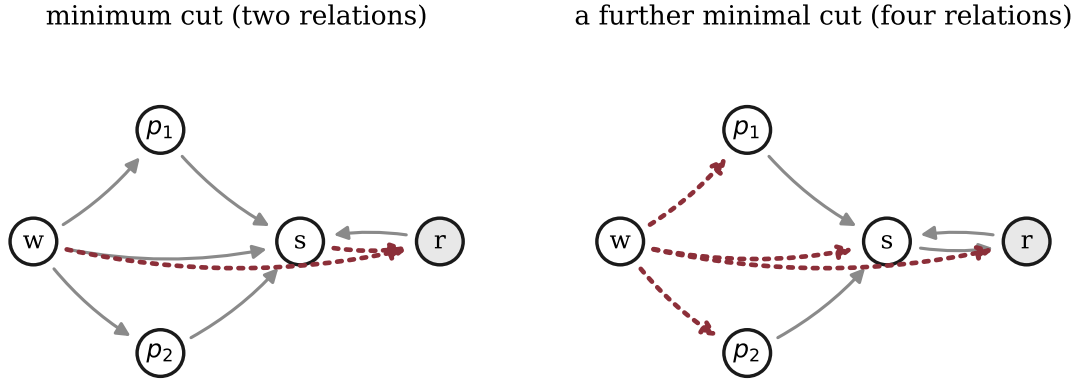


Figure 5: Two of the five inclusion-minimal cuts of the structure of Figure 2, with the relations of each marked by dotted arrows. The minimum cut, on the left, consists of two relations and is unique. The cut on the right consists of four and is minimal in the sense that no proper subset of it is a cut. Proposition 7.11 identifies the sets and selects among them only where the removals that occurred are known.

half of the weightings, with a median of zero. Over the same record, *A* and *B* belong to the unique minimum cut and *C* belongs to no minimal cut whatever. The two enquiries therefore return an answer of a different character on one and the same evidence, and the second returns the same answer for every weighting the first requires and lacks.

Claim 7.13. (The case and the two procedures). *A tribunal proceeding on shares would divide responsibility among the three parties in proportions settled by a weighting it has no means of establishing, and would charge party C in half of them. A tribunal proceeding on the cut would place the burden of justification upon A and B and would leave C outside it. The gain of the second procedure is the determinacy of its answer, and the case exhibits it on a structure small enough to be checked by hand.*

7.4 The residue such a finding withholds

Figure 5 shows the minimum cut of the structure and one of the further minimal cuts, which bears on the concession stated below.

Claim 7.14. (The displacement of closure). *The existence of a path is invariant under re-belling of the positions and under changes in the strength of the relations, and it is not invariant under the addition or removal of positions. A finding is therefore relative to a declared set of positions, and the condition of closure returns at the level of the structure. What is gained is that closure is required over positions rather than over variables, and the positions of an organisation are enumerable where the variables bearing on its margin are not. The gain is real and it is a reduction of the difficulty and no escape from it.*

Claim 7.15. (Coarseness is the price of invariance). *A relation may be present in the structure and answered by no one, and the existence of a path counts it. The invariance of Proposition 7.9 is purchased by discarding the information that would distinguish a channel that functions from one that is nominal, and repairing the omission requires weights on the relations, which reintroduces the quantities the change of object was made to avoid.*

Claim 7.16. (The multiplicity of cuts). *A structure admits several minimal cuts, five in the five-position structure of Figure 2, and the number rises with the size of the structure. Proposition 7.11 identifies the sets of relations whose removal would destroy the path and selects among them only where the removals that actually occurred are known. Where several parties acted and the record is incomplete, the identification of the set concerned is underdetermined in the same manner as a share.*

Claim 7.17. (The silence of a static invariant on time). *A structure is an object at a time and an invariant computed upon it carries no account of when a relation was withdrawn or by whom. Recovering that account requires a sequence of structures, and a sequence recovered from a record is a sample, so the difficulties of evidentiary sampling recorded in the first companion paper return in full at that point. The proposal of this paper is therefore complete only where the structure before and the structure after are both established, and it says nothing about the interval between them.*

8 THE LIMITS OF THE CRITERION

This section states the questions the criterion does not answer, so that the proposal is not read as more than it is. The method is to take in turn the elements of an attributive finding and to mark which the criterion supplies.

Claim 8.1. (Attributability in place of apportionment). *The criterion identifies transformations that require justification and assigns no share of responsibility to those who brought them about. Where several parties contributed to the removal of a path, the criterion is silent on the division among them, and the difficulty of division is the one the companion papers show to be intractable in this class. The proposal is therefore a constraint on the set of candidates and no procedure for selecting within it.*

Claim 8.2. (Mixed transformations). *A transformation may close one path of contestation and open another, and the criterion as stated does not order such cases. An ordering would require a measure over paths, which reintroduces the difficulty of Proposition 5.1 at one remove, and the honest position is that the criterion applies cleanly to transformations that remove without replacing and is undeveloped for the rest.*

Claim 8.3. (The burden of justification). *A transformation that removes a path of contestation*

is not thereby wrongful, since a path may be removed for reasons that a tribunal would accept. What the criterion does is to place the transformation among those calling for justification and to put the burden of justification upon the party who brought it about, which is a shift in the order of argument rather than a conclusion about the merits.

Claim 8.4. (The measurement problem is untouched). *Establishing that a path of contestation existed and was removed requires a representation of the relations actually obtaining, which a formal organisational chart does not supply. The instruments for obtaining such a representation, and the ways in which the accounts on which it would rest are discounted, are the subject of the second companion paper, and nothing in the present criterion relieves that difficulty.*

9 CONDITIONS OF REFUTATION AND OPEN QUESTIONS

This section states, for each principal claim, what would defeat it, and records what the paper leaves open.

Claim 1.1 fails if the production of a vulnerability is shown to be unintelligible as an object of attribution independently of an outcome, which would follow if every account of a vulnerability were shown to presuppose a specification of the harm against which the system is vulnerable.

Proposition 5.1 fails upon the exhibition of a quantity computed on the structure alone that takes different values across the benign and harmful instances in each of the three registers. A candidate would be a strong result and would remove the need for the criterion.

Claim 6.2 fails in either of two ways. It fails if a transformation is exhibited that removes every path of contestation from a position and is nonetheless of no normative interest, which would show the criterion to be too broad. And it fails if a transformation is exhibited that carries evident normative weight while leaving every path of contestation intact, which would show it to be too narrow. The second is the more likely and the more instructive, and instances of it are invited.

Proposition 6.4 is a claim about one constructed pair. It establishes that the criterion is not a function of the three measures named and establishes nothing about measures not named, so a structural quantity that separates the pair would narrow the proposition without defeating it.

Proposition 7.10 fails if a structure is exhibited in which the withdrawal of a relation creates a path of contestation, which would occur where the representation admits relations whose presence blocks others. Proposition 7.11 fails as a description of the parties concerned where the cut that actually occurred is not among the minimal ones, which happens whenever more was removed than was needed. Claims 6.5 to 6.8 are concessions and are refuted by being met rather than by being contradicted.

Three questions are left open. The first is the ordering of mixed transformations, recorded in Claim 8.2. The second is the relation between the criterion and the requirement of bipolarity, since a path of contestation belongs to a position and the wrong must be a wrong to its occupant, and the argument for that step is sketched in Claim 4.3 and is not completed. The third is whether a criterion of this form generalises beyond organisations to the relational structures of political and international life, where the position competent to revise an arrangement is frequently absent by design and the criterion would then condemn the arrangement as a whole rather than any transformation of it.

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