

Comparing Interpretive Distinctions across Changes of Category Criteria in Generative Relational Models

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Abstract

A changed classification can reveal a previously unrecorded distinction, replace one distinction with another, or accompany a change in social practice. This paper develops a comparison framework for revisions of interpretive category criteria in generative relational models. Fixed-criterion quotient consistency supplies a baseline. Pullback relations place comparisons across carriers on a common pair domain; joint classifications and counterexamples distinguish refinement, translation and renaming. A sequential derivation shows how local changes cancel at an endpoint, and a weighted bound states the reference-population transport required for cumulative comparison. A finite revision procedure uses an explicit evaluation corpus, potential, feasible menu and improvement threshold. Its termination is proved, with counterexamples separating termination from global adequacy and changing-data revision from fixed-potential descent. A further factorization result characterizes when retained records contain enough information to implement a prescribed revision. A constructed institutional-response case connects these results to interpretation, criterion authority and historical consequences. The philosophical discussion preserves the difference between representational refinement, practical uptake and ontological commitment. A prospective study and revision record identify the evidence and objections required for application. The resulting position supports local, accountable criterion revision while leaving open-ended category formation and empirical validation as further obligations.

Keywords: category revision; interpretive criteria; relational models; classification refinement; record sufficiency

Research Note

This paper develops a bounded argument through conceptual distinctions, stated assumptions, conditional derivations, worked examples, and objections. Its length follows the requirements of that argument. Formal statements concern the specified models; the application of those models to persons and institutions requires independent interpretation and evidence. Constructed examples are explicitly identified and carry illustrative status.

The manuscript follows the repository’s paper template for typography, numbered formal statements, section structure, citations, and navigable contents. The associated project files preserve research provenance, source verification, derivation records, unresolved objections, and validation results. Philosophical commitments about the ultimate nature of social facts remain suspended where the descriptive framework is sufficient for the argument.

Authorship and Research Assistance

Wanhong Huang directed the research questions and authorized this paper’s preparation. An AI research assistant helped organize the material, develop explicit formal constructions, check primary sources, and prepare the text. Agent-added arguments and formulations remain subject to the author’s review. Source positions, repository proposals, and model consequences are distinguished throughout the manuscript.

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1 Introduction

An institution can change its account of an episode by changing what occurred, recovering additional evidence, or revising the distinctions through which the episode is described. These changes can occur together, yet they support different conclusions. This paper develops comparisons across revisions of interpretive category criteria in generative relational models. It supplies a typed account of lost and gained distinctions, a finite criterion-revision procedure, and an information requirement for implementing that procedure. The discussion moves from conditional mathematics to the warrants needed for social interpretation.

Consider a review procedure that initially records whether an organization responded to a request. A later procedure distinguishes an acknowledgment, an explanation and an effective remedy. An unchanged archive can receive a more differentiated coding. The organization can also introduce a remedy that was previously unavailable. Finally, participants can learn to treat a previously routine acknowledgment as a challengeable refusal. These examples concern classification, practical possibilities and interpretive uptake respectively. A suitable model should let an investigator connect them through explicit assumptions while preserving their separate evidential requirements.

The generative relational starting point consists of a carrier of configurations, a collection of composable mechanisms, their action and an equivalence relation used to identify configurations for an inquiry. Within a fixed preserved equivalence, equivalent configurations remain equivalent after a common action. That elementary result creates a useful discipline: a model cannot simultaneously assume that preservation and claim that the same action splits an equivalence class. An account of emerging distinctions must identify a changed criterion, a changed model, or an earlier failure of the preservation assumption.

The substantive difficulty begins after this observation. Writing a subscript on an equivalence relation leaves the revision process unspecified. A useful account needs to identify the information on which revision depends, the alternatives under consideration, the authority through which a revision becomes operative, and the comparisons that survive it. The present construction addresses these requirements within a finite, declared catalogue of regimes. Open-ended creation of previously unformulated meanings remains a further problem.

Three contributions organize the argument. First, comparisons across different carriers are defined through a declared transport map, so that every pair comparison has a common domain. Second, a sequence of revisions is analyzed through pair-status changes; cancellations explain why an endpoint comparison can conceal substantial intermediate reorganization. Third, a revision rule is given substantive content through a fixed evaluation corpus, a loss function, a complexity term and a feasible menu of changes. Its termination is proved under explicit assumptions. A separate factorization result characterizes when compressed records contain enough information to execute the prescribed revision.

The intended contribution is methodological and philosophical. The elementary set-theoretic results are established within the paper, and the revision mechanism is a proposed model. A fitted score provides evidence about its declared target. The further judgments that a target is appropriate, a criterion is intelligible, or a revision is legitimate require arguments and evidence of their own. This separation permits formal criticism to sharpen a conceptual proposal without turning a convenient mathematical representation into a settlement of ontology.

The next section situates the proposal and defines its research status. Sections 3–6 develop fixed-criterion inheritance, transport, joint comparison and sequential change. Sections 7–8 provide the revision mechanism and its information obstruction. A constructed institutional example then links the results. The final sections examine criterion authority, outline an evaluation design and identify the conditions under which the proposal should be revised.

2 Antecedents and Analytical Scope

This section identifies the inherited framework and the additions required by the present question. It separates a mathematical representation from its interpretation, and distinguishes local criterion revision

from the broader formation of units and social meanings.

Huang's Part I proposes a carrier, composable mechanisms, an action and a preserved equivalence for organizing social dynamics [1]. The present treatment inherits this representational starting point while making its algebraic commitments explicit. Identities and inverses are permitted where a chosen mechanism family contains them. A reversible action can change a configuration, and an irreversible action can preserve every distinction relevant to a particular quotient. Consequently, historical change, absence of an inverse and loss of distinctions occupy separate positions in the argument.

Huang's conformance specification asks analyses of generated units to declare commitments concerning units, partitions and scale [2]. That demand motivates the present insistence on recording the criterion and carrier before comparing results. A finite catalogue of candidate regimes provides a tractable local construction. A claim about the emergence of an unanticipated unit would require an account of how the catalogue itself is extended, together with evidence that the extension has interpretive or practical significance.

Partition refinement also has an established algorithmic literature. Paige and Tarjan develop algorithms for several partition problems, including relational coarsest partition [3]. The present paper uses refinement in a broader research-design question concerning changing criteria, information availability and authorization. Its elementary proofs are given directly. It makes no claim to improve the algorithms in that work, and algorithmic efficiency supplies no independent warrant for choosing a social classification.

2.1 Representational commitments

The representation has three declared layers. A configuration domain specifies what is being compared. A criterion identifies configurations for a particular inquiry. An interpretation states why that identification is useful and what evidence supports applying it. The layers can constrain one another. For example, a criterion that asks whether a request received an explanation requires configurations rich enough to distinguish an explanation from a mere receipt confirmation.

The six analytical categories developed in the underlying research programme are entity, relation, event, attribute, state and process. They name different representational jobs. A criterion revision can affect several jobs at once: a new account of continuing membership can alter entity tracking, relation classification and process description. Their coordinated use requires explicit dependencies. Their availability provides an analytical repertoire while ontology remains suspended.

A distinction between two records also needs an inquiry-specific interpretation. Differences in file encoding can be irrelevant to a review question; a missing qualification can be decisive. The mathematical relation records the investigator's identification commitments. Establishing that those commitments capture the intended social distinction requires comparison with cases, practices, participant understandings and objections. Treating every representable difference as equally important would conceal this work.

2.2 Kinds of result

The paper uses four research statuses. Definitions stipulate domains and comparisons. Propositions prove consequences of those stipulations and stated assumptions. Constructed examples demonstrate possibility, incompatibility or a failure of inference. Empirical hypotheses concern whether a proposed mechanism describes actual practice. A fifth, normative register enters when the paper evaluates who should be able to revise a criterion or demand an explanation.

The distinction matters particularly for endogenous revision. Here an update is endogenous relative to an enlarged state containing a regime, an evaluation corpus and a revision procedure. The procedure's catalogue, loss and authority rules remain specified inputs. This relative sense supplies a reproducible transition mechanism. It leaves the origin and justification of those inputs available for subsequent investigation, including the possibility that participants contest the framework itself.

3 Dynamics under Fixed Category Criteria

The fixed-criterion model provides the baseline against which revisions can be identified. This section introduces the symbols, derives quotient consistency and isolates the precise sense in which a common mechanism preserves a previous collapse.

Definition 3.1 (Action and criterion). Let S be a nonempty set, M a semigroup, and $\alpha : M \times S \rightarrow S$ an action satisfying $\alpha(nm, x) = \alpha(n, \alpha(m, x))$. Write $mx = \alpha(m, x)$. A criterion $E \subseteq S \times S$ is an equivalence relation. It is preserved by the action when xEy implies $mxEmy$ for every $m \in M$.

The elements of S are model configurations. The semigroup law concerns composability, and a word in mechanisms can record a history richer than its resulting transformation. An identity may be adjoined when a zero-step observation is useful. The value of E depends on the declared inquiry. Equality of current outcomes and equality under every admissible future observation can therefore define different relations on the same carrier.

Proposition 3.2 (Quotient consistency). *If E is preserved, the expression $\bar{\alpha}(m, [x]_E) = [mx]_E$ defines an action on S/E .*

Proof. Two representatives of $[x]_E$ satisfy xEy . Preservation gives $mxEmy$, so the proposed image is independent of the representative. The action equation follows from $n(mx) = (nm)x$ by taking equivalence classes. Associativity in M supplies the corresponding consistency across three or more operations. $\square$

Definition 3.3 (Collapse and split sets). For $m \in M$, define

$$C_E(m) = \{(x, y) : \neg xEy, mxEmy\}, \quad (1)$$

$$G_E(m) = \{(x, y) : xEy, \neg(mxEmy)\}. \quad (2)$$

Here C_E records a loss of distinctions and G_E a gain relative to the same criterion and pair domain.

Preservation gives $G_E(m) = \emptyset$. If $(x, y) \in C_E(m)$, then $mxEmy$, and applying any common subsequent n gives $nmxEEnmy$. The pair therefore remains collapsed at the endpoint. The common-operation qualification is essential. Applying different operations to the two configurations can separate their outcomes even when each operation individually preserves E .

Two counterexamples prevent stronger readings. On the integers with equality, the shift $x \mapsto x + 1$ changes each configuration and has an inverse. Its distinction-loss set is empty. On the nonnegative integers with equality, the same shift is injective and lacks an inverse within that carrier, yet its distinction-loss set is again empty. The examples separate changing a state, being invertible, and collapsing distinctions. An inquiry into historical irreversibility must specify which of these properties it intends.

The fixed-criterion result is useful even where preservation fails. Failure identifies a modeling obligation: either retain finer state information, restrict the mechanism family, or revise the equivalence. Each repair changes a different component. Refining observations until future responses become class-consistent is one possible construction already available in the underlying programme. The current question concerns what happens when the category criteria themselves are subsequently reconsidered.

A normative interpretation adds another layer. A collapse matters differently if it erases a qualification required for a challenge, compresses redundant formatting, or protects an irrelevant personal detail. The set $C_E(m)$ has no intrinsic welfare sign. A protected family of distinctions, a reason for protecting it and an account of affected interests must be specified separately. This observation carries into every later comparison.

4 Transport between Criterion Domains

Changing carriers introduces a typing problem before it introduces a substantive theory of change. This section defines a common pair domain, derives loss and gain sets, and examines the information needed to interpret them.

Definition 4.1 (Transported successor criterion). Let S_0, S_1 be nonempty carriers with equivalences E_0, E_1 , and let $T : S_0 \rightarrow S_1$ be a declared transition or alignment. Its pullback criterion is

$$T^*E_1 = \{(x, y) \in S_0^2 : (Tx, Ty) \in E_1\}. \quad (3)$$

Define the gain and loss of distinctions on the initial pair domain by

$$G = E_0 \setminus T^*E_1, \quad C = T^*E_1 \setminus E_0. \quad (4)$$

The pullback is an equivalence: reflexivity follows from TxE_1Tx , symmetry from symmetry of E_1 , and transitivity from applying transitivity to the three transported points. Consequently, both relations being compared occupy S_0^2 . Directly subtracting $E_0 \subseteq S_0^2$ from $E_1 \subseteq S_1^2$ would generally lack the intended pairwise meaning.

The same expression covers several cases. If T is the identity, changes arise entirely from the criterion. If $E_0 = E_1$ on a fixed carrier, changes arise through T under that fixed comparison. If both vary, the endpoint loss and gain combine those changes. Their separation into material and criterion contributions requires additional intermediate comparisons. There can be several defensible intermediate paths, and the chosen path belongs in the interpretation.

The kernel of T exposes a limit. Whenever $Tx = Ty$, the pair belongs to T^*E_1 for every successor equivalence. Thus an initial distinction between two points that T maps to the same point cannot survive through this successor representation. A subsequent refinement of E_1 still treats a point as equivalent to itself. Recovering the distinction requires a richer successor carrier, such as a retained provenance coordinate, or a different alignment. A changed label alone cannot supply the missing information.

Proposition 4.2 (Descent through the initial quotient). *There is a map $\bar{T} : S_0/E_0 \rightarrow S_1/E_1$ with $\bar{T}([x]_{E_0}) = [Tx]_{E_1}$ exactly when $E_0 \subseteq T^*E_1$.*

Proof. Necessity follows because equal initial classes must have equal images. For sufficiency, inclusion guarantees that any two representatives give the same successor class, which defines $\bar{T}$. The result applies to the declared T ; it supplies no evidence that a disputed alignment is the right one. $\square$

If an inquiry contains births, mergers or uncertain reidentifications, a total single-valued T can overstate the available alignment. A partial map permits comparison on its declared domain, with unaligned configurations reported separately. A correspondence permits several candidate alignments, yielding several pullback comparisons or robust statements across them. Choosing one representative correspondence without justification can manufacture apparent continuity. The present proofs use a total map so that their assumptions remain transparent; applications must either establish that map or retain the weaker result.

Social interpretation introduces one further caution. An analyst may change coding criteria while the participants' practices remain stable. Participants may change their interpretation while the analyst's coding remains fixed. An institution may give a category new consequences through an authorized procedure. The map and equivalences locate representational differences. A claim about which of these processes occurred requires evidence about actors, records, uptake and consequences.

5 Joint Classifications and Translation

This section examines the information retained by two classifications of a common domain. It derives a joint comparison, distinguishes refinement from renaming, and gives a small counterexample to comparison by category counts.

Let E and F be equivalences on S , with $F = T^*E_1$ when transport is required. Smaller equivalence relations distinguish more pairs. Accordingly, H refines E when $H \subseteq E$. This convention concerns pair identification; its direction should remain explicit whenever the language of “more categories” is used.

Proposition 5.1 (Least additional discrimination for joint retention). *The relation $H = E \cap F$ is an equivalence and the coarsest equivalence refining both E and F . The map $x \mapsto ([x]_E, [x]_F)$ has kernel H .*

Proof. Reflexivity, symmetry and transitivity hold in both constituent relations and therefore in their intersection. Any common refinement K is included in each relation, hence in H . Finally, two points have equal ordered pairs of classes exactly when they are related by both E and F . $\square$

The joint representation therefore retains both declared classifications with the fewest further pair distinctions. This is an informational statement relative to the two criteria. It leaves storage format, interpretive burden and disclosure risk open. The product of class sets can contain unrealized combinations; the representation uses its actual image, which is determined by the common carrier.

Consider $S = \{1, 2, 3, 4\}$ with partitions

$$E : \{1, 2\} \mid \{3, 4\}, \quad F : \{1, 3\} \mid \{2, 4\}. \quad (5)$$

Both have two classes. Yet $1E2$ and $\neg(1F2)$, whereas $1F3$ and $\neg(1E3)$. Neither relation includes the other. There is no deterministic translation from an E -class to the correct F -class of an unspecified member, and the converse translation also fails. Their intersection is equality, with four classes. A common coarsening must connect 1 to 2 through E , 1 to 3 through F , and 2 to 4 through F ; transitivity then puts all four points together.

This example has a direct methodological interpretation. A classification by response timing and a classification by response content can have identical numbers of categories while cutting across each other. A summary reporting only the class count conceals the replacement of distinctions. The coarsest common coarsening can make the two procedures comparable only by discarding precisely the differences that motivate the inquiry.

Renaming is a different operation. If $q : S \rightarrow L$ is a label map and $b : L \rightarrow L'$ is injective on $q(S)$, then $\ker(b \circ q) = \ker q$. This follows because equality after b is equivalent to equality before b on the realized labels. More elaborate wording, an additional code prefix or a translated label therefore leaves the represented partition unchanged when the relabeling is injective. Semantic change can still occur through use, but evidence for it must concern that use.

The joint-classification construction is also limited by availability. Given only an E -class, an observer generally cannot reconstruct its joint $E \cap F$ class. A later comparison can require access to original configurations or another record that supplies the missing distinction. The mathematics of a desirable comparison and the informational resources available for conducting it are separate obligations. Section 8 returns to this issue for the revision rule itself.

6 Sequential Revision and Cumulative Difference

A final classification can conceal changes that occurred and were subsequently reversed. This section derives the relation between local revisions and endpoint disagreement, then introduces a weighted comparison with explicit transport of its reference population.

Let S_t carry E_t , and let $T_t : S_t \rightarrow S_{t+1}$ for $t = 0, \dots, k-1$. Define U_0 to be the identity on S_0 and $U_{t+1} = T_t U_t$. For an initial pair $p = (x, y)$, put

$$b_t(p) = \mathbf{1}\{U_t x \ E_t \ U_t y\}, \quad d_t(p) = b_t(p) \oplus b_{t+1}(p), \quad (6)$$

where $\oplus$ is addition modulo two. The value $b_t = 1$ means that the transported pair is identified at stage t .

Proposition 6.1 (Endpoint parity and cumulative exposure). *For every initial pair,*

$$b_0 \oplus b_k = \bigoplus_{t=0}^{k-1} d_t, \quad \mathbf{1}\{b_0 \neq b_k\} \leq \sum_{t=0}^{k-1} d_t. \quad (7)$$

Proof. Substituting the definition of each d_t makes every interior b_t appear twice. Addition modulo two cancels those appearances and leaves the endpoint values. If the endpoints differ, the number of local changes is odd and therefore at least one, proving the inequality. $\square$

A coarse criterion, followed by equality, followed by the original coarse criterion has zero endpoint disagreement for every pair while some pairs change twice. The cumulative sum detects exposure to revision. It still leaves the direction and significance of each change to the signed sequence of gains and losses. This matters if an intermediate classification affected an actual decision: returning to an earlier partition cannot itself undo a sanction or restore a lost opportunity.

For a finite carrier, let w be a probability distribution on ordered pairs S_0^2 , and define

$$D_w(E, F) = \sum_{p \in S_0^2} w(p) |\mathbf{1}_E(p) - \mathbf{1}_F(p)|. \quad (8)$$

This is a pseudometric on equivalences: symmetry and nonnegativity follow term by term, while the triangle inequality follows from the triangle inequality for real absolute values. If some pairs have zero weight, distinct equivalences can have distance zero. Full support makes the comparison a metric on the finite set of equivalences. Neither version ranks social importance without a justification of the weights.

For two transitions $T : S_0 \rightarrow S_1$ and $V : S_1 \rightarrow S_2$, write $T^{(2)}(x, y) = (Tx, Ty)$ and let $T_*^{(2)}w$ be the induced pair distribution. Then

$$D_w(E_0, (VT)^*E_2) \leq D_w(E_0, T^*E_1) + D_{T_*^{(2)}w}(E_1, V^*E_2). \quad (9)$$

To derive this expression, insert T^*E_1 into the triangle inequality on S_0^2 . The second term is a sum of a function of $T^{(2)}p$, so regrouping the weights of pairs with the same image gives the pushforward term. Using an unrelated uniform distribution on S_1^2 would change the question, especially when T merges many initial configurations.

The weights can represent a declared case population, a balanced experimental corpus or protected comparison pairs. These choices have different interpretations. A frequent category can dominate a population-weighted score while a rare but consequential distinction disappears. Reporting protected pairs separately can preserve that objection. The mathematical bound clarifies how much endpoint discrepancy is compatible with local discrepancy; it supplies no universal tolerance for acceptable revision.

7 A Finite Procedure for Revising Criteria

This section gives criterion revision an explicit transition rule. It specifies the inputs and decision sequence, proves termination under fixed conditions, and identifies the points where validity and authority enter independently of optimization.

Definition 7.1 (Declared revision problem). Let R be a nonempty finite catalogue. Each regime r supplies a criterion E_r and a prediction or interpretation map f_r . Let $D = \{(x_j, y_j) : 1 \leq j \leq N\}$ be a fixed evaluation corpus with $N \geq 1$. A nonnegative loss ℓ and complexity value $K(r)$ define

$$\Phi_D(r) = \frac{1}{N} \sum_{j=1}^N \ell(f_r(x_j), y_j) + \lambda K(r), \quad \lambda \geq 0. \quad (10)$$

For each r , a feasible revision menu $A(r) \subseteq R$ specifies the allowed successors. A threshold $\delta > 0$ specifies a minimum accepted improvement.

The regime contains more than an unnamed equivalence. The map f_r connects the regime to an explicit target and enables evaluation. Two regimes can induce the same partition while interpreting its classes differently. The pair-comparison results concern E_r ; the evaluation concerns f_r , its target and the selected loss. These distinctions prevent a change in prediction quality from being automatically described as a gain of categories.

At regime r_t , form the admissible improving set

$$B_D(r_t) = \{r' \in A(r_t) : \Phi_D(r') \leq \Phi_D(r_t) - \delta\}. \quad (11)$$

If this set is empty, stop. Otherwise select a member by a declared deterministic tie rule, or record the realized selection when a permitted random procedure is used, and set r_{t+1} to that member. The proof below covers every sequence satisfying the displayed improvement condition.

Proposition 7.2 (Finite accepted revision). *Suppose D , R , Φ_D and δ remain fixed. If $\Phi_{\min} = \min_{r \in R} \Phi_D(r)$, a trajectory starting at r_0 contains at most*

$$\min \left\{ |R| - 1, \left\lfloor \frac{\Phi_D(r_0) - \Phi_{\min}}{\delta} \right\rfloor \right\} \quad (12)$$

accepted revisions.

Proof. Every accepted revision decreases the potential by at least δ . After n accepted steps, repeated substitution gives $\Phi_D(r_n) \leq \Phi_D(r_0) - n\delta$. The lower bound $\Phi_D(r_n) \geq \Phi_{\min}$ yields the second bound. Strict decrease prevents a repeated regime, so a catalogue with $|R|$ elements permits at most $|R| - 1$ transitions. Taking the smaller bound proves the claim. $\square$

Termination means that the procedure reaches a regime with no allowed improvement of at least δ . It establishes neither a global minimum nor a socially adequate classification. A lower-potential regime can be inaccessible through the menu. A smaller improvement can remain below the threshold. A menu that omits an affected person's proposed criterion can terminate rapidly while preserving the substantive dispute.

For example, let three regimes have potentials 3, 2, 0. From the first allow only the second, and from the second allow no successor. With $\delta = 1$ the procedure terminates at potential 2. Adding the third regime to the second menu changes its result. This comparison separates the effect of a search or authorization structure from the quality of the evaluation target. An observed final score cannot identify which restriction produced it.

The fixed-data assumption is equally substantial. Take two regimes predicting constant binary values 0 and 1, zero complexity, and a corpus containing a single target value. With zero-one loss and $\delta = 1/2$, a target 0 favors the first regime and a target 1 favors the second. Alternating the corpus target permits indefinite alternation. Each local revision improves the currently used potential, while there is no single decreasing potential over the whole sequence. An expanding corpus therefore requires a new convergence argument or a weaker monitoring objective.

Allowing zero improvement also permits cycles among tied regimes. A deterministic global tie convention can sometimes prevent them, but that is an added assumption. Revising the loss function or the authority menu during the process likewise changes the problem. Such revisions may be warranted; their reasons and effects should be recorded as new transitions in the enlarged research state.

The procedure is consequently a local model of criterion revision under declared evaluative and institutional commitments. Its constructive value is that it tells an investigator what must be supplied and what its guarantee actually covers. It gives a reasoned alternative to an unexplained sequence $E_t \rightarrow E_{t+1}$, while leaving participant acceptance, target validity and institutional authorization open to criticism.

7.1 Protected distinctions and admissible improvement

The revision procedure can preserve a declared family of distinctions while evaluating fit. This extension makes one possible relation between a normative commitment and the optimization problem explicit. It also shows why feasibility should be checked before a favorable loss is interpreted.

Let $P \subseteq S \times S$ be a family of pairs that the inquiry has independently decided to keep distinguishable. Define

$$R_P = \{r \in R : P \cap E_r = \emptyset\}, \quad A_P(r) = A(r) \cap R_P. \quad (13)$$

Use A_P in the update rule. If the initial regime belongs to R_P , every accepted successor remains in R_P by construction. Induction establishes preservation throughout the sequence. The previous termination proof is unchanged, because every accepted step still decreases the same potential. A tighter catalogue bound uses $|R_P| - 1$ when the initial regime and every successor lie there.

This extension has an explicit consistency condition. If P includes a diagonal pair (x, x) , then R_P is empty because every equivalence is reflexive. Even when P is irreflexive, the declared catalogue may contain no regime that preserves all protected distinctions. In that case the procedure should report the incompatibility. Relaxing a protection, expanding the catalogue or revising the carrier are distinct responses requiring separate reasons.

The protected-pair formulation also limits what the constraint achieves. It ensures representational separation of the specified cases. It does not guarantee that their labels are intelligible, that decision-makers attend to the difference, or that the resulting treatment is justified. A regime can preserve the distinction between a qualified and an unqualified statement while applying an inappropriate consequence to both. Protection of a pair is therefore one component of the intended safeguard.

The extension illustrates a general discipline for combining formal and normative reasoning. The value judgment selects a protected family, the feasibility check determines whether the available representation can honor it, and the proof describes the consequence of enforcing the constraint. A conflict appears as a conflict between explicit commitments. The potential function remains an instrument for comparison within those commitments; its minimum cannot adjudicate their justification.

8 Information Required to Implement Revision

A revision rule can be well defined on detailed evidence while being impossible to execute from the records retained by an institution. This section characterizes that obstruction and derives a sufficient repair within the declared information model.

Fix the current regime, catalogue, loss, menu and tie rule. Let $\mathcal{D}$ be a nonempty set of possible detailed corpora. The resulting prescribed next regime is a function $U : \mathcal{D} \rightarrow R$, with stopping represented by retaining the current regime. Let $z : \mathcal{D} \rightarrow Z$ be the information actually supplied to the revision procedure.

Proposition 8.1 (Implementation through a retained record). *A function $\hat{U} : z(\mathcal{D}) \rightarrow R$ satisfying $U = \hat{U} \circ z$ exists exactly when*

$$z(D) = z(D') \implies U(D) = U(D') \quad (14)$$

for every pair of possible corpora.

Proof. If the factorization exists, equal records give equal values under $\hat{U}$ and hence equal prescribed updates. Conversely, assign to each realized record a the value $U(D)$ of any corpus with $z(D) = a$. The condition makes this assignment independent of the chosen corpus, proving the factorization on the image of z . $\square$

Consider the two constant-prediction regimes from the previous section. Let $K(r_0) = 0$, $K(r_1) = 1$, $\lambda = 0.2$, current regime r_0 and $\delta = 0.5$. The menu permits switching to r_1 . Let D_0 contain target 0 and D_1 contain target 1. Their potential pairs are $(0, 1.2)$ and $(1, 0.2)$ respectively. The rule stays at r_0 for

D_0 and switches to r_1 for D_1 . A record storing only the old prediction, which is 0 in both cases, merges the corpora and violates the condition.

The example concerns information about an evaluation target. It does not establish that either target label is true. If the target is disputed, the possible-corpus domain should contain the rival admissible interpretations and the procedure may appropriately remain set-valued. A deterministic tie rule cannot turn missing warrant into knowledge. It merely chooses among the values that the declared procedure permits.

For a fixed update function, retaining $z'(D) = (z(D), U(D))$ is formally sufficient. This expression is a characterization, not generally an operational recommendation: computing the second coordinate already requires the information whose availability is at issue. A usable repair instead identifies a measurable statistic $a(D)$ from which U factors and establishes a procedure for collecting and checking that statistic. In the binary example, the validated target value suffices. In a social inquiry, collecting it can require testimony, interpretation and a protected opportunity to disagree.

Several update questions can be retained jointly. If $\mathcal{U}$ is a declared family of future update functions, the equivalence

$$D \equiv_{\mathcal{U}} D' \iff U(D) = U(D') \text{ for every } U \in \mathcal{U} \quad (15)$$

is the coarsest record identification that preserves all those update answers. The proof repeats the joint-kernel argument: every sufficient record must separate a pair distinguished by some member of the family. If the family contains every function from $\mathcal{D}$ to a set with at least two values, the equivalence is equality. Thus unrestricted future revision can require retaining the entire relevant corpus distinction structure.

The practical conclusion is a declared revision budget. An institution can preserve information for specified future reconsiderations, document which other comparisons remain unavailable, and provide routes for obtaining additional evidence. The cost of that information can include privacy exposure, interpretive burden and uneven access. These costs prevent informational sufficiency from becoming an unconditional demand for maximal retention.

9 A Constructed Review of Institutional Responses

This section applies the comparison and revision constructions to a fictional coding exercise. The case is designed to expose the assumptions required by each inference; its numerical values and institutional arrangements are stipulated for analysis.

Suppose four archived episodes record a request and a subsequent institutional message. Episode a contains a timely acknowledgment without an explanation. Episode b contains a timely explanation without a remedy. Episode c contains a delayed acknowledgment without an explanation. Episode d contains a delayed explanation and a remedy. A first criterion groups episodes by timing, giving $\{a, b\} \mid \{c, d\}$. A second groups them by explanatory content, giving $\{a, c\} \mid \{b, d\}$. These are the crossing partitions of Section 5.

Table 1 records the feature declarations. Each column has a local role: timing and explanation instantiate the crossing comparison, while remedy shows a further distinction that both classifications can miss. Calling every response “completed” would add yet another criterion, whose practical interpretation would require a rule connecting response content to case closure.

Three inquiries can now be distinguished. An analyst can ask which episodes meet a timing standard. A participant can ask whether a reason was supplied. A reviewer can ask whether an identified problem was remedied. The first two have the same class count and different kernels. The third distinguishes d from the other episodes. Combining all relevant answers retains a joint classification, but deciding that all three matter is an interpretive and normative judgment preceding the combination.

To instantiate the revision rule, suppose a frozen corpus contains separately checked target judgments about whether a message supplies an explanation. One regime uses the presence of any response as its predictor; a second requires an articulated reason. The target-checking protocol should specify how

Table 1: Stipulated episode features for comparing response criteria. The entries are constructed and supply no evidence about an actual institution.

Episode	Timing category	Explanatory content	Remedy recorded
<i>a</i>	Timely	Acknowledgment only	Absent
<i>b</i>	Timely	Explanation	Absent
<i>c</i>	Delayed	Acknowledgment only	Absent
<i>d</i>	Delayed	Explanation	Present

qualifications, ambiguous language and disagreements are recorded. The loss function then evaluates those predictors against that declared protocol. A lower loss supports a better fit to this target. It leaves open whether explanation is sufficient for a fair response or whether remedy should enter a different target.

The information obstruction appears if the archive contains only the first regime’s code. All four episodes can be stored as “response received.” The later explanatory criterion cannot be reconstructed from that code alone. A retained message may supply the distinction, provided it is accessible and interpretable. An alternative witness account may create a new evidential route, with its own uncertainties. The required repair therefore concerns both record detail and access to that detail.

Now suppose the institution changes its practice by enabling a reviewer to order a remedy. This changes the action repertoire and possibly the carrier of future episodes. It can be represented by a transition with an added remedy-status coordinate. A coding revision and a practice revision can occur on the same date. To attribute the resulting change, the inquiry should retain the old criterion applied to the new practice where meaningful, and the new criterion applied to earlier episodes where evidence permits. Missing cross-comparisons should be reported explicitly.

Finally, imagine that the institution later returns to the initial response code. An endpoint partition comparison can suggest continuity, while an intervening remedy rule has changed people’s options or produced effects that persist. The sequential pair trace records the intermediate revision, and the practical history records its consequences. Their coordination supplies a richer account than either an endpoint score or an unstructured narrative alone.

10 Interpretive Warrant and Authority over Criteria

This section examines the philosophical commitments that the formal construction leaves open. It considers the reality of distinctions, the formation of categories, the role of disagreement and the institutional authority through which a criterion can acquire consequences.

10.1 Distinctions and their social use

A relation E identifies configurations for an inquiry. This is a commitment of the representation. Its adequacy depends on why the identified cases can be treated alike for that purpose. A response-timing criterion may be adequate for measuring delay while concealing the absence of an explanation. The criterion’s local success gives a reason to use it within that scope. Extending it to the adequacy of the institutional response requires another argument.

The mathematical possibility of refining a partition is especially weak evidence for the emergence of a social category. Any finite carrier admits equality, and equality distinguishes every different element. A social category usually involves more: an intelligible basis for grouping, a way of applying the grouping across cases, and some relation to inquiry or practice. These requirements can themselves be disputed. The framework should expose the dispute by recording target interpretations and competing criteria.

Category formation can also change the world to which the category applies. A recognized distinction between acknowledgment and reasoned response may guide staff training, participant expectations

and appeal procedures. To model this process, the regime becomes an input to subsequent actions, which affect later configurations and evidence. The update then participates in a feedback system. Establishing that feedback requires evidence of uptake and practice change; the existence of a parameter named “regime” establishes only a place where such a mechanism could enter.

10.2 Evaluation and disagreement

An evaluation corpus contains judgments that can be revisable. If its labels are treated as unquestionable, a potential-minimizing procedure can stabilize an error or a partial standpoint. The mathematical proof still holds because it concerns a fixed function. The philosophical criticism concerns the function’s interpretation and authority. A rigorous model should retain both statements without allowing either to erase the other.

One response is to preserve a family of defensible target interpretations and compare regimes across the family. A regime that improves every declared target has a stronger comparative warrant than one that improves a single favored target. Yet the family also has a boundary: including every conceivable target would often prevent a useful ordering. The selection of relevant objections and targets requires a publicly assessable research purpose and an opportunity for affected participants to challenge exclusions.

Disagreement can concern application, meaning or priority. Two coders may agree on what an explanation is while disagreeing about whether a particular message contains one. Participants may disagree about the criterion itself. They may agree on both application and meaning while assigning different importance to timing and remedy. These disputes require different remedies: further evidence, conceptual clarification, or normative deliberation. A single aggregate error statistic can conceal their difference.

10.3 Authority and revisability

The feasible menu $A(r)$ can reflect technical feasibility, organizational authorization or a negotiated research protocol. These interpretations should be separated even when they induce the same mathematical set. A technologically available revision may be institutionally forbidden; an authorized revision may be informationally impossible. An analyst who collapses these cases into “unavailable” loses the basis for identifying an appropriate response.

Criterion authority includes the ability to propose a distinction, secure its consideration, influence its adoption and challenge its subsequent application. These abilities can be distributed among participants, analysts, managers and reviewers. A formal revision opportunity can therefore coexist with a practical inability to supply the evidence needed to use it. The information-factorization result gives one precise form of this concern: a procedure can promise reconsideration while retaining only records on which the promised revision cannot be implemented.

The normative proposal is to make criterion revisions answerable through an explicit record of purpose, evidence, alternatives and effects on affected comparisons. This proposal rests on the value of accountable inquiry and contestable consequential classification. It is not deduced from a set inclusion. A justified exception may arise where disclosure would expose a participant to harm, provided a suitable protected route preserves the relevant review task. The framework supports analysis of such tradeoffs by specifying the distinction whose availability is at stake.

Ontology remains suspended throughout this discussion. The model organizes claims about configurations, distinctions and practices under explicit commitments. Evidence may support particular claims about actual social relations or institutions. That inquiry can proceed without turning the six-category analytical repertoire or the current equivalence relation into a comprehensive account of what exists.

10.4 Criterion adoption and the order of practice

A criterion revision can occur before or after a practical transition, and the order can affect what follows. To make this point explicit, consider an enlarged state (x, r) with a configuration and an operative regime.

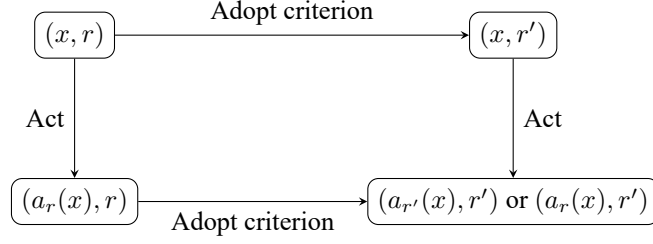


Figure 1: Order of criterion adoption and practice. The two paths can produce different configurations when action depends on the operative criterion.

Let $J(x, r) = (x, r')$ adopt a new regime while holding the immediate configuration fixed. Let $A(x, r) = (a_r(x), r)$ apply a practice whose behavior depends on the operative regime. Then

$$AJ(x, r) = (a_{r'}(x), r'), \quad JA(x, r) = (a_r(x), r'). \quad (16)$$

The endpoints can differ whenever $a_{r'}(x) \neq a_r(x)$. This is an elementary order comparison, supplied here as a proposed representation of a criterion's possible practical role.

For the response example, adopting a criterion that treats a reasoned answer as necessary for closure may lead a staff member to seek an explanation before closing the request. Closing under the earlier criterion and revising the criterion afterward can leave a different practical history. Whether a later review reopens the case is another operation. The model therefore distinguishes changing the interpretive rule from actually repairing its earlier consequences.

The notation also exposes a confounding possibility. If the regime is merely an analyst's code and staff behavior is independent of it, then $a_r = a_{r'}$ and the displayed operations commute. An empirical claim that a criterion shapes practice requires evidence that the action depends on it. Simultaneous changes in staffing, resources or case composition can otherwise explain a difference attributed to the criterion.

This extension gives a precise place to the idea that interpretation can participate in social development. The criterion's role is represented through a dependence of action on the operative regime. Evidence of adoption, understanding and implementation is needed to warrant that dependence. The formal representation can then help identify which order comparison would be informative and which unresolved alternative mechanisms should be retained.

Figure 1 displays the two orders and the conditional divergence of their endpoints.

11 Prospective Evaluation and Revision Records

The proposed evaluation tests whether the framework makes criterion changes more intelligible and criticizable in a bounded inquiry. This section specifies the materials, comparisons and failure conditions before any empirical effectiveness claim is made.

Begin with a jointly specified domain of episodes and a frozen record of the questions being investigated. Preserve original observations, their provenance and declared access conditions. Construct at least two defensible criteria whose differences matter to those questions. Cases that expose crossing partitions are particularly informative because a class-count summary will conceal their disagreement. Ambiguous cases should retain their alternative interpretations.

The first comparison evaluates the same material under both criteria. It records the pairwise gain and loss sets, the joint classification and any failure of translation. A second comparison varies the record supplied to the reviewer: original material, the earlier code alone, and a purpose-designed supplementary record. This tests the implementation condition by checking whether corpora merged by a supplied record require different revisions. The result concerns the declared task family and available evidence.

The update procedure should then be run with a preregistered catalogue, target-checking method, loss, complexity convention, revision menu and threshold. Each accepted transition should have a reproducible potential calculation and a record of the permitted alternatives. The mathematical termination bound can be checked directly. Empirical interpretation requires additional outcomes, such as the intelligibility of distinctions, the visibility of disagreement and whether participants can identify omitted questions.

Several sensitivity comparisons are necessary. Vary the weighting of rare and common pairs. Compare a fixed target protocol with an explicitly revised one. Distinguish restrictions imposed by the research design from restrictions imposed by the institution. Repeat the comparison on held-out episodes where that design is suitable, while recording any criterion changes induced by the new material. A result dependent on one favorable corpus should retain that scope.

The revision record should contain the previous and successor criteria, the carrier alignment, the target and loss, the information used, the authority for adoption, the objections considered and the unresolved comparisons. It should also identify consequences already produced under earlier criteria. This last entry prevents a return to an old partition from being mistaken for the restoration of an earlier practical situation.

The framework would need revision if independent reviewers cannot reconstruct its pair comparisons, if an intended application repeatedly requires unrecorded carrier alignments, or if its evaluation targets exclude the distinctions motivating participants' objections. A further failure would occur if the administrative cost of maintaining the record defeats the review task. These possibilities are empirical and design questions. The conditional theorems remain assessable independently of whether a particular procedure proves useful.

11.1 Auditability across missing and disputed records

An evaluation should report the domain on which each comparison is actually supported. If the new criterion requires explanatory content but older records retain only timing, the complete pair-gain set is unavailable. Restricting the calculation to recoverable episodes can introduce a selection effect: episodes with richer records may differ systematically from the others. The domain restriction should therefore be reported alongside the result, with the uncovered questions preserved.

One defensible output is a family of comparisons across admissible completions of missing information. A pair belongs to a robust gain set if every admissible completion treats it as a gain. It belongs to a possible gain set if at least one completion does. The robust set is included in the possible set by the meanings of the universal and existential conditions, provided the completion family is nonempty. An empty family signals inconsistent constraints and should not produce a vacuous robust claim.

This extension preserves a useful distinction between uncertainty and lack of a target. Missing evidence can leave several answers to a well-defined criterion question. Disagreement about the criterion can leave several questions. The first invites additional evidence or bounded completion assumptions; the second invites conceptual and normative discussion. Recording both through an undifferentiated confidence interval would obscure the different work required.

The audit should also identify reversibility of the research procedure. A revised code can often be changed again if original evidence remains available. If the update overwrites the only detailed record, later reconsideration may be impossible. A revision record should therefore state what was transformed, what was retained and which future comparisons became unavailable. That information is part of the model's practical accountability.

12 Research Obligations and Further Development

The final section identifies the work required to extend the construction and the conditions governing its present use. The scope is a declared local revision problem with explicit informational and interpretive commitments.

The first obligation concerns open-ended category development. A finite catalogue supports a termination theorem because it fixes the candidate space. A broader model must explain how a proposed new distinction becomes available, how it connects to earlier observations, and what counts as a warranted extension. Merely allowing an arbitrary new set at each stage would enlarge the notation while leaving those questions unanswered.

The second obligation concerns uncertain alignment. Social histories can involve disputed continuity, collective reorganization and incomplete records. A correspondence-valued transport could preserve rival reidentifications and characterize which comparisons hold across all admissible alignments. Such an extension should distinguish uncertainty about the alignment from plurality in the criterion and from actual multiplicity in the process being represented.

The third obligation concerns reflexive dynamics. A criterion can affect conduct, evidence production and future evaluation labels. The fixed-potential proof then requires replacement or restriction to intervals over which the relevant inputs remain stable. A useful extension would state an explicit coupled update and analyze its equilibria, cycles or continuing revision under assumptions appropriate to the chosen institution. Mathematical stability would still require an independent interpretation of its desirability.

The fourth obligation concerns the distribution of revisability. A retained distinction can be available to one actor and inaccessible to another. A criterion can be formally contestable while the evidence needed to challenge it remains controlled by its beneficiary. Connecting the present information requirement to actor-specific access and review procedures is therefore a substantive next step. The corresponding normative argument must address confidentiality, burdens and the authority of affected participants.

The present position supports a disciplined use of formalism: compare typed pairs, preserve the history of criterion changes, give revision a specified mechanism, and test whether the available record can implement that mechanism. These commitments make the model reviewable. Their practical and philosophical value remains answerable to the purposes, evidence and objections of the inquiry in which they are used.

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