Cooperative Reliability with Incomplete Trust

A Preliminary Discussion

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Abstract

Cooperation with incomplete trust is an established problem in international relations. Research on strategic interaction, institutional design, compliance, trust and relational power already explains important conditions of its possibility. This paper develops a Generative Relational investigation of the cooperative arrangement itself: the history-bearing organization through which participants, commitments, interpretive rules and practical capacities are maintained or transformed. Its central distinction concerns preservation of a commitment and preservation of the powers needed to interpret, contest and revise it. The philosophical argument examines delivered performance, generative capacity, participant continuity, heterogeneous commonality and interpretive authority. Several possible formal models make selected obligations explicit. Spacetime and two-complex representations retain historical incidence; continuation-sensitive records distinguish current compliance from future operational possibilities; interacting group-domain fields investigate shared mediation and an institutionally variable coupling kernel; history-indexed operator algebras separate commitment transport from the preservation of available operations. Exact nonlinear variational and moment identities supplement the generative field construction. A finite recourse model remains a restricted operational projection. Historical arms-control provisions and consolidated environmental-cooperation provisions ground the institutional discussion, alongside clearly stipulated cases. The results are conditional conceptual and formal investigations. They establish neither a universal model of international cooperation nor a scalar measure of trust, and they do not infer normative legitimacy from stable performance or mathematical preservation.

Keywords: international cooperation; incomplete trust; generative capacities; relational spacetime; institutional kernels; operator algebra

Research Scope

This paper develops the third undertaking in a compact research programme on trust and cooperation. Its primary object is the cooperative arrangement: the historically maintained organization through which a specified joint activity becomes possible and its consequences remain answerable to those affected. The preceding investigations concern near-critical trust relations and trust-formation trajectories. Here the question persists even when trust does not increase.

The argument combines an IR literature review, philosophical reconstruction, explicit mathematical models, counterexamples and institutional illustrations. Cooperation with incomplete trust is an established IR topic; the proposed GR contribution concerns preservation and transformation within the arrangement. Generative Relational theory is treated as an open programme. Its prior manuscripts provide concepts and formal architectures whose epistemic status remains provisional. The paper does not infer a physical ontology from spacetime, field, group or operator language. It distinguishes a relational being, a representation of its history, and the further assumptions required by a particular mathematical instantiation.

The mathematical results concern the stated finite or bounded settings. Their purpose is to make preservation obligations visible. They are not calibrated predictions of institutional conduct or claims to have invented the underlying mathematical techniques. The computational examples are reproducible illustrations. Two treaty texts ground the institutional discussion without establishing causal effects; three further cases are expressly hypothetical. The literature review records selected primary-page readings and bounded uses of publisher or author abstracts, rather than claiming full readings of every cited article. The contribution remains subject to the author's review and to wider literature comparison.

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Notices

Status. This working draft is prepared for the author's review. It records an evolving research argument; definitions, section structure, and conclusions remain subject to revision. Completion of the draft does not imply publication, acceptance, or final author approval.

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Statement on the use of language models. The author selected the research problem, working title, and paper queue and provided the exploratory discussion and corrections. OpenAI Codex assisted with source retrieval, conceptual analysis, formal reconstruction, drafting, computational examples, and technical checks. This completed working draft awaits the author's review; its new formulations and arguments have not yet received final author approval. Authorship credit remains with the human author. This disclosure should be reviewed before external circulation.

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1 Cooperative Arrangements

The practical need for cooperation often precedes a settled judgment that another participant is trustworthy. Parties may have little common history, disagree about the meaning of an undertaking, or possess justified reservations about each other's conduct. They may nevertheless need to exchange resources, maintain a shared facility, conduct joint inquiry, or coordinate a response to a common problem. The present investigation asks what can make such an undertaking sufficiently reliable and defensible while these reservations remain. Its method is to reconstruct the arrangement supporting cooperation, identify its philosophical commitments, and examine selected compositional problems through explicit formal models.

The possibility of cooperation without strong prior trust is established territory in international relations. Research on strategic cooperation, institutions, compliance and trust has already examined how actors cooperate amid conflict, uncertainty and vulnerability [2, 9, 31, 32]. Institutional cooperation also receives sustained treatment beyond IR [5]. Prior GR work discusses protected cooperation through bounded commitments, verification, reversibility and withdrawal [23]. The present paper does not introduce that possibility or discover that institutions can support it. Its problem concerns what a cooperative arrangement preserves and transforms while providing that support.

The analytical object is consequently a *cooperative arrangement*. This term denotes a historically maintained organization of participants, commitments, practical supports and interpretive procedures that enables a specified joint undertaking. It can include a formal institution, but it can also consist of distributed practices with no single institutional center. It extends beyond a contract's text and beyond the present psychological attitudes of its parties. Its persistence depends on activities such as recognizing authority, maintaining records, interpreting obligations, supplying resources, responding to departures and transmitting relevant capacities to successors.

This object changes the unit of assessment. Asking whether a participant trusts a provider can disclose something important about the participant's stance. It cannot by itself establish whether the provider's verifier is available, whether the proposed remedy affects the harm at issue, or whether evidence survives long enough to make a claim. Conversely, a functioning arrangement does not settle whether the participant trusts the provider. The participant might reasonably continue to rely on staged exposure and independent review while withholding the interpersonal stance that another account calls trust.

1.1 Cooperative transformation

An institution can preserve a promise while altering the powers through which it can be interpreted or challenged. It can improve current performance while increasing dependence on a mediator. It can continue to issue valid reports while losing a distinction that later becomes consequential. These possibilities concern transformations of the arrangement itself, including its participants and the operations available to them. They connect the cooperation problem to established relational IR and constitutive power analysis [27, 3].

The GR reconstruction treats the arrangement and its participants as historically extended relational objects, ordinarily represented as spacetimes. Two-complexes retain selected event, coupling and element histories; optional spin data add a more specific instantiation. The purpose is to investigate continuity of commitments and capacities through transformation. The representation does not establish a physical or ultimate social ontology.

The paper's guiding problem is the relation between local support and the whole undertaking. A safeguard is local when its success is specified relative to a limited task and a set of background conditions. A verifier checks a submitted item using available records. A custodian preserves deposited material under its retention policy. An adjudicator can issue a decision within its jurisdiction. These functions need to connect: the preserved material must be usable by the verifier; the verifier's output must bear on the commitment; the adjudicator's power must still reach the consequences. The connection is a substantive feature of the arrangement.

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The word *composition* therefore has both practical and formal uses here. Practically, it concerns how activities support, constrain and sometimes frustrate one another. Formally, later sections specify how histories glue, how interaction kernels generate joint structures, and how operators preserve a declared admissible sector. These mathematical constructions illuminate particular compositional questions. They do not make the whole practical problem a theorem.

Three distinctions organize the inquiry. First, a protection can exist in a description without becoming an available action for the relevant participant. Second, a support can contribute to reliable performance while making future challenge harder. Third, different supports can remain dependent on a common mediator even when they have separate organizational names. The paper develops these distinctions without requiring a single numerical index of reliability or trust.

1.2 Argument boundaries

The proposed contribution is a coordinated investigation of commitment preservation and cooperative capacity across institutional transformations. It joins philosophical analysis to several possible formal models: continuation-sensitive records, common descriptions, interacting group-domain fields, and history-indexed operator representations. The field extension makes a mediating kernel an interacting object, while the algebraic extension separates preservation of commitment observables from transport of available operations. Earlier models of shared mediation and finite recourse remain useful restricted investigations.

This contribution is a proposal for conceptual and formal coordination, not a claim that existing IR theory is incapable of representing the same mechanisms. The literature review identifies substantial inheritance and disagreement. Mathematical results establish consequences of stated assumptions, including distinctions that weaker criteria cannot capture. They do not establish the empirical relevance of every proposed variable or the justice of a preserved constraint.

This inquiry differs from an account of trust formation. Cooperation may generate a shared history through which trust becomes possible, but that trajectory is neither required nor assumed. An arrangement can remain worth maintaining precisely because participants retain bounded skepticism. Equally, persistent skepticism can be a symptom of avoidable institutional failure. The account must permit both possibilities rather than treating trust's absence as either inherently healthy or inherently defective.

The paper also leaves the independent resource and power projects bounded. Maintaining records, supplying review and preserving alternatives have costs that fall unevenly. Those costs enter the present analysis whenever they condition the effectiveness of a claimed safeguard. A complete theory of their distribution would require a further argument. Near criticality is likewise not presumed: ordinary institutional change, contested interpretation and dependence already generate the problems examined here.

1.3 Investigative sequence

Section 2 locates the problem within IR cooperation research. Sections 3–9 develop the philosophical distinctions, with Section 5 examining generative capacity, participant continuity and interpretive authority. Sections 10–11 establish history and observation structures. Sections 12–14 investigate field generation, shared mediation and an interacting institutional kernel. Sections 15–16 distinguish several algebraic preservation obligations. Section 17 retains a finite operational projection. Treaty provisions, hypothetical cases, objections and an empirical agenda then examine the constructions' meaning and limits.

2 International Cooperation Research

International cooperation with limited confidence in others is an established research problem. This section locates the paper within that literature, distinguishes competing explanations, and identifies the obligations any GR contribution must satisfy. The review is organized around strategic cooperation,

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institutional design, compliance, trust, relational constitution and power. It is a selective conceptual review of works bearing directly on the argument, rather than an exhaustive history of IR or a systematic empirical meta-analysis.

2.1 Strategic cooperation

Anarchy supplies a problem of political organization; it does not identify a single game, a universal degree of hostility, or an absence of institutions. Axelrod and Keohane distinguish cooperation from harmony and examine how actors adjust conduct amid partially conflicting interests. Their account concerns mutuality of interest, future interaction and the number of participants, while expressly allowing actors to reshape the context through institutions [2]. This paper therefore inherits the possibility of cooperation without identical purposes. It also inherits the need to examine institutional context rather than treating every episode as an isolated exchange.

The practical consequence is that incomplete trust cannot itself diagnose the obstacle. Two parties may agree on a desirable joint outcome but fear unilateral exposure. Alternatively, each may prefer a different cooperative settlement, or each may lack the capacity to implement the same settlement. These situations can exhibit similar hesitation while requiring different responses. Additional information may help identify intentions; it cannot by itself allocate a disputed benefit or construct a missing administrative capacity.

Stein's distinction between coordination and collaboration sharpens this point. His account separates dilemmas of common aversions from dilemmas of common interests and connects them to different institutional requirements. Coordination can be sustained through conventions, whereas collaboration can require monitoring and compliance mechanisms [42]. The distinction prevents the present paper from putting every cooperative difficulty into a single assurance category. A convention that selects compatible conduct may succeed without a sanctioning authority. A convention can also leave distributional disagreement unresolved.

Jervis's security-dilemma analysis adds a problem that a generic exchange model can obscure. An action that increases one side's security can reduce the other's; the implications depend partly on exposure, expectations and the distinguishability of defensive from offensive postures [28]. A protective device must therefore be assessed through its effects on the counterparty. Verification, reserve capacity and rapid suspension may reassure one participant while appearing threatening to another. This is a reason to model relational consequences, not evidence that protection is inherently counterproductive.

The GR question begins after these distinctions. Suppose an arrangement initially addresses an identified strategic obstacle. Does its operation leave the actors with the capacities and interpretive relations required for the same solution to remain effective? A repeated-game description can incorporate many such changes. The distinctive burden here is to specify what the description retains when the repertoire of moves, the authority of a signal or the identity of an authorized participant changes. Merely writing a nonlinear equation does not meet that burden.

2.2 Bargaining over cooperative terms

An undertaking can be feasible and still lack agreed terms. Fearon distinguishes bargaining over an agreement from enforcing it and demonstrates an interaction between the two. Anticipated future cooperation can strengthen incentives to uphold a settlement while also increasing incentives to bargain harder over its distribution, delaying agreement [9]. The prospect of a durable relation consequently supplies no general theorem of prompt convergence.

This result matters for the proposed emphasis on usable recourse. A party can demand a lengthy review because evidence is difficult to assess, because a delayed decision improves its bargaining position, or because delay shifts costs to its counterpart. These mechanisms can coexist. Calling a procedure deliberative does not identify which mechanism operates; calling delay strategic does not establish that the disputed claim lacks merit. The analysis must preserve both the procedural question and the distribution of burdens.

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The realist challenge also requires a substantive response. Grieco disputes the institutionalist interpretation of anarchy and the resulting account of cooperation [10]. The relative-gains controversy concerns more than whether each side receives something beneficial. A settlement may change the parties' future positions. Powell's analysis identifies commitment problems associated with changing power, including bargaining over matters that affect subsequent bargaining power [38]. These are established reasons why mutually beneficial present terms need not secure future cooperation.

The present reconstruction extends this concern to the repertoire through which later bargaining can occur. An institution can alter access to records, authorization to propose changes, or the ability to maintain an alternative supplier. These changes may affect future bargaining power, but the inquiry need not compress them immediately into a scalar power distribution. Keeping the underlying operations distinct can reveal which institutional act changes a participant's position. The field and algebraic models later in the paper are proposed ways of retaining that structure.

No implication follows that revisability should always override commitment. An agreement that each party can suspend costlessly after receiving its benefit may be worthless to the other. The relevant problem is how an arrangement differentiates opportunistic abandonment from justified reopening, and how participants can challenge that differentiation. A GR account must confront the same incentive problem as other approaches while making its constitutive and historical dimensions explicit.

2.3 Institutional design

Keohane explains demand for regimes partly through information provision and reduced transaction costs. He also considers institutional persistence after the structural circumstances associated with their creation have changed [30]. Accordingly, persistence, historical dependence and changing context are not missing discoveries that GR can supply to institutionalism. They are points of entry for a more specific comparison.

Abbott and Snidal examine the functions made possible by organizational centralization and independence. Their account also addresses distribution, norms and shared understandings [1]. Centralization can pool expertise or coordinate tasks; independence can support actions that individual members could not credibly perform in their own names. The same organization can nevertheless depend on member resources, delegated authority and acceptance of its classifications. Independence is a structured relation with a specified scope.

Koremenos, Lipson and Snidal investigate institutional variation through the cooperation problems actors face. Their analysis expressly includes distributional conflict, enforcement, numbers and uncertainty, and it allows nonstate actors and institutional modification [31]. The present paper shares the demand for explanations of particular arrangements. It differs in analytical emphasis when the arrangement changes the categories of participant, obligation or admissible evidence through which its own design is subsequently assessed.

This difference must be stated carefully. Rational design is not restricted by definition to immutable institutions, and a sufficiently rich strategic model can represent an actor choosing rules for later interaction. A GR representation earns its additional structure only when it makes a relevant transformation easier to state, compare or investigate. For example, distinguishing the production of a certificate from the production of the authority that makes certificates actionable can reveal a dependence concealed by a single verification variable. If a simpler institutional model preserves that dependence, the field representation has no automatic explanatory advantage.

Institutional substitution also needs a precise object. A monitor may reduce reliance on a counterparty's unobserved intentions, but doing so introduces dependence on the monitor's access, interpretation and accountability. That observation is compatible with institutionalist explanation. The further task is to trace how these dependencies compose and change through use. The result might support delegation, distributed checking or deliberate limits to delegation; it should not be decided by a general preference for either centralization or decentralization.

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2.4 Compliance research

Compliance is an indispensable but incomplete description of cooperation. Chayes and Chayes argue that departures from treaty obligations often cannot be understood as deliberate violations chosen for advantage. They also treat acceptable compliance as dependent on the purposes and concerns of the regime [4]. This opens questions about interpretation and implementation that a binary comply/defect classification can suppress. It does not establish that enforcement is generally unnecessary.

Downs, Rocke and Barsoom contest the inference from high observed compliance to the dispensability of enforcement. Selection and the depth of commitments matter: states may agree to obligations close to what they would otherwise have done [7]. A smoothly functioning arrangement can consequently provide weak evidence of a substantial cooperative achievement. Successful implementation and demanding cooperation require separate investigation.

These approaches generate rival diagnoses that should remain rival where evidence does not resolve them. Consider an unimplemented obligation. A managerial explanation directs attention to capability and interpretation; an enforcement explanation directs attention to incentives and the consequences of nonperformance. A third possibility is that the obligation has become performable only at a cost excluded from the original settlement. The proper response depends on which process is operating. An undifferentiated call for more trust or more verification can worsen the problem.

GR adds a further representational question without displacing these explanations. What historical transformations are erased when two arrangements receive the same present compliance classification? One may have preserved the recipient's ability to dispute an assessment; the other may have made that ability inaccessible while continuing to produce accepted reports. The difference concerns future operations and their enabling conditions. Section 11 formalizes this through continuation-sensitive records, drawing on prior GR work rather than claiming a new theory of behavioral equivalence.

Three empirical comparisons follow. First, compare conduct against a defensible baseline, rather than treating an institution's presence as its effect. Second, examine whether the monitored obligation captures the undertaking's purposes. Third, examine how the institution changes the capacities required for future performance and contestation. The third comparison does not replace the first two. It asks whether the observed achievement is being sustained by a transformation that the current indicator omits.

2.5 Trust research

The phrase incomplete trust traverses several theories of trust, and this paper cannot settle their differences by definition. Kydd develops a strategic account in which costly reassurance can communicate trustworthiness under specified conditions [32]. It asks when a signal can distinguish types and support cooperation. A record of cooperative behavior, however, may be compatible with different motivational and institutional explanations; its evidential meaning depends on the model and setting.

Rathbun argues that generalized trust can precede institution building, using the creation of American multilateral security arrangements to challenge accounts in which institutions first solve distrust [40]. This is a direct objection to any universal claim that assurance can be built independently of trusting dispositions. Some institutional commitments may require participants willing to accept vulnerability before extensive protection exists. The present paper investigates arrangements that limit the amount or scope of prior interpersonal confidence required; it does not establish the complete elimination of trust from institutional formation.

Ruzicka and Wheeler analyze the Nuclear Non-Proliferation Treaty through changing trusting relationships involving interests and promises [41]. Their approach makes it difficult to classify a treaty simply as a replacement for trust. An institutional relation can embody commitments whose significance includes the expectation that others will honor them. Michel further argues that trust scholarship remains incomplete when it excludes emotional and phronetic dimensions [34]. Strategic calculation and reliable behavior do not exhaust what participants may experience as trusting or as betrayed.

These disagreements affect the meaning of the paper's title. Incomplete trust can concern a particular actor, action, period or domain. It can also concern uncertainty about a shared interpretation rather than

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a judgment that the other is malicious. The title does not posit a measured deficit on one trust scale. Nor does it designate a mature stage at which institutional machinery has superseded personal or political relations.

The arrangement-centered approach deliberately leaves room for several combinations. Participants may trust each other's intentions while requiring verification of complex performance. They may doubt intentions while accepting a reversible exchange. They may trust a mediator in one role while contesting its authority in another. Which combination is present requires interpretation and evidence. The conditional models concern what specified supports preserve, not a general mechanism that manufactures the emotional or normative relation described by trust theorists.

2.6 Relational constitution

Wendt's argument makes the constitution of identities and interests through social practice central to explaining power politics [44, pp. 395, 397]. Jackson and Nexon distinguish substantialist approaches from analyses beginning with processes and relations, while recognizing that both can be fruitful for different questions [27]. Qin develops a relational theory centered on actors-in-relations and processes, explicitly locating its theoretical premises in a cultural background [39]. GR enters an already developed conversation about relational constitution.

The strongest possible overlap should be admitted. Describing an institution as constituting its participants, or describing cooperation as transforming identities, is insufficient to establish a GR contribution. The present paper instead asks for explicit relations among a historically extended object, a representation of that object, the operations the representation permits, and the observations through which a claim about cooperation is assessed. Its spacetime language is an analytical proposal for preserving historical constitution, not a proof of an ontology superior to other relational approaches.

The choice of starting point also has consequences. Beginning with named states is often appropriate, but the model should not treat their boundaries, administrative capacities and authorized representatives as timeless givens when these are the variables being investigated. A treaty organization may become an actor in some respects; a ministry may lose the authority through which earlier promises were effective; a transnational network may sustain a practice across changing membership. These are different questions of individuation, and no single state variable answers all of them.

The paper therefore retains both objects and relations. It does not require the dissolution of every relatively stable participant into an unlimited history. A representation can identify a participant at a chosen scale while recording the dependencies necessary for the question. The philosophical obligation is to justify that scale and to reopen it when the arrangement transforms the capacities it presupposes.

2.7 Power over cooperative conditions

Barnett and Duvall's distinctions among compulsory, institutional, structural and productive power expressly address effects on actors' capacities [3]. This inheritance rules out a claim that GR first moves power analysis beyond material stocks or direct commands. It also prevents a narrow account of assurance from treating domination as a residual problem appearing only after the technical design is complete.

Farrell and Newman explain how asymmetric economic networks can enable information gathering and denial of access [8]. A common infrastructure can thus facilitate exchange while providing coercive opportunities. The present paper's shared-mediator field is not a reconstruction or empirical test of their theory. It isolates one formal dependence issue: channels attached to a common generative mediator need not remain separable after that mediator is marginalized. Whether the shared node also supports coercion requires authority, jurisdiction and access conditions absent from the bare integral.

Power analysis changes the evaluation of reliability. A procedure can reliably deliver what the dominant actor recognizes as performance, while making other claims increasingly difficult to formulate or pursue. This is a proposed diagnostic for investigation, not a reason to infer domination from every asymmetry. Differentiated expertise and delegated authority can improve cooperation. The question

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Table 1: Established IR problems and the present representational obligations.

Problem	Established analytical concern	Present obligation
Strategic cooperation	Exposure, reciprocal response, bargaining and enforcement	Retain changes to feasible moves and their enabling relations.
Institutional design	Information, delegation, distribution and institutional modification	Represent the production and alteration of mediating kernels.
Compliance	Interpretation, implementation, commitment depth and selection	Separate current classification from continuation-sensitive capacity.
Trust	Reassurance, generalized dispositions, promises and emotion	Avoid equating protected performance with trust.
Relational power	Constitution of actors and unequal control of conditions	Test commitment preservation separately from preservation of usable powers.

is whether the allocation of powers remains contestable and whether the arrangement reproduces the capacities its participants need to act within it.

The position developed here is therefore cumulative and conditional. Strategic IR explains important incentive problems; institutional analysis identifies support functions and design variation; compliance research contests causal inferences; trust research preserves motivational and experiential differences; relational IR and power analysis address constitutive effects. GR contributes a proposed coordination of these questions through history-bearing spacetimes, generative kernels and operation-preserving maps. Its value depends on the distinctions those constructions actually preserve and on evidence that the distinctions matter in a case.

2.8 Contribution boundary

Table 1 records the boundary in a form that can guide the subsequent argument. The final column contains research obligations, rather than findings claimed against entire theoretical traditions.

The paper does not claim that these obligations are unavailable to existing IR theories. It proposes explicit constructions through which they can be inspected together in a particular cooperative arrangement. If a construction merely renames a familiar mechanism, its contribution is expository. If it identifies a previously suppressed dependence, that dependence must still be demonstrated historically or empirically. The formal results below establish conditional distinctions and counterexamples; they do not establish an independently verified new law of international cooperation.

3 Trust Distinctions

This section establishes the distinctions needed to investigate cooperation without reducing trust to an unspecified confidence level. It positions the problem within selected trust and institutional literatures, separates different forms of incompleteness, and explains why observable cooperation cannot settle the participants' attitudes. These distinctions determine what the later formal variables may legitimately represent.

3.1 Interpersonal attitudes

Trust is contested even before its relation to cooperation is considered. Jones's introductory characterization emphasizes an affective orientation involving optimism about goodwill and competence [29]. Holton distinguishes a participant stance from mere reliance and separates deciding to trust from deciding to believe someone trustworthy [12]. Hawley's account organizes trust and distrust around commitments, including commitments that need not be explicit promises [11]. These approaches identify different features of interpersonal life. Their coexistence supplies a reason to state the local use of trust rather than silently translate them into one probability.

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The present paper does not decide among these accounts. It uses *interpersonal trust* for a family of stances directed toward another participant that exceed the bare fact of depending on an outcome. Particular applications must specify whether the relevant stance concerns goodwill, responsible acknowledgment, commitment or another feature. The arrangement's reliability can then be examined without pretending that a formal success criterion measures all these attitudes.

The distinction is consequential. A participant who uses an escrow-like mechanism may believe that the partner will probably perform while remaining unwilling to regard that performance as an expression of goodwill. Another may trust the partner's intentions while doubting its capacity to meet a deadline. A third may judge the partner reliable in one task and untrustworthy in another. Calling all three cases low trust conceals the different supports they require.

3.2 Forms of incompleteness

Incomplete trust can concern domain, evidence, interpretation or commitment. A domain limitation restricts the activities for which trust is extended. An evidential limitation concerns the grounds available for a judgment. Interpretive incompleteness arises when parties cannot confidently determine what an action signifies or how a commitment applies. A commitment limitation concerns the absence or uncertainty of the obligation on which reliance would rest. These dimensions can coexist without admitting a common ordering.

Active distrust adds a further possibility. Lewicki, McAllister and Bies challenge a simple one-dimensional treatment of trust and distrust [33]. An arrangement may contain positive expectations concerning routine performance and negative expectations concerning disputed accounts. It may consequently support ordinary cooperation while reserving certain decisions to a protected procedure. Nothing in this configuration requires a globally trusting or globally distrusting relationship.

Incompleteness also need not be a temporary defect. Participants can retain stable differences in values, interpretive traditions and interests. A bounded undertaking may be successful without eliminating these differences. Requiring a comprehensive interpersonal settlement before every cooperation would make the scope of assurance unnecessarily narrow. Yet the opposite inference is also unwarranted: an arrangement that works for one purpose provides no general warrant for expanding exposure into another domain.

3.3 Institutional support

The institutional literature already places cooperation beyond a simple opposition between trust and force. Cook, Hardin and Levi examine how institutional arrangements can support cooperation without interpersonal trust and can sometimes provide settings in which trust develops [5]. Ostrom, Walker and Gardner's experiments compare communication and sanction arrangements in common-pool settings [36]. Their relevance here is the existence of multiple routes to credible commitments. The consulted evidence does not establish a universal recipe or identify all cooperation with compliance under external enforcement.

The present paper extends this problem at the level of support relations. A rule that makes performance credible changes dependence rather than abolishing it. A participant may rely less on a partner's goodwill and more on the availability of a record, a custodian's competence, an adjudicator's authority or the continued operation of an infrastructure. These dependencies can carry different epistemic and normative requirements. It would be unhelpful either to call every dependency trust or to declare the resulting arrangement wholly independent of trust.

Assurance consequently names a practical and evidential function. A practice supplies assurance when it contributes grounds or capacities for undertaking a specified exposure. The function can be performed by a person, a technical device, a procedure or a distributed combination. Whether the participant also trusts any of those supports remains a further question.

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3.4 Evidence of cooperation

Observable participation underdetermines its own explanation. A party may participate because it expects a benefit, fears exclusion, accepts a norm, anticipates a sanction, trusts a person, or combines these considerations. Cox's experimental work motivates caution about interpreting a transfer or response as a pure measure of trust or reciprocity [6]. Here that caution extends to the inference from sustained institutional participation to an interpersonal achievement.

Repeated success still matters. It can provide evidence concerning the arrangement's performance within the conditions actually encountered. Its scope depends on what was tested. Routine success may reveal little about appeal access, extraordinary departures or changes in the interpretive interface. An arrangement whose participants never invoke recourse has not thereby demonstrated that recourse works; neither has it demonstrated that recourse is unnecessary.

The implication for formalism is restrictive and productive. Later fields represent coupling modes and selected support interactions. Projections represent declared history classifications. Reachability sets represent available strategies. None is a numerical amount of trust. Their narrower meanings allow exact questions to be asked without claiming to have captured the entire relation.

4 Historical Relational Constitution

The GR reconstruction concerns how the entities participating in cooperation acquire and preserve their capacities. This section develops that philosophical basis before introducing mathematical representations. It distinguishes relational reality from manifestation, explains the constitutive role of assurance, and separates a historically effective nomos from a justified one. The objective is to show why the arrangement's supports belong within its dynamics.

4.1 History-bearing participants

A participant enters an undertaking through a name, credential, role, account or other manifestation. The relations making that manifestation effective can extend far beyond the moment of entry. A laboratory's capacity depends on accumulated practices, personnel, equipment, inherited records and recognition by others. An association's representative authority depends on a history of authorization. A participant seeking a remedy may possess formal standing while lacking the social or material conditions for exercising it.

GR's history-bearing conception of relational being retains these relations within the analytical object. Prior work represents subjects as spacetimes that persist through transformations of their active organization, with two-complexes supplying one possible formal realization [21, 14]. This does not assert that the subject is fundamentally a mathematical complex. It asks a representation to preserve enough historical constitution to explain its present capacities and possible continuations.

The distinction matters when an assurance mechanism appears to be external. A guarantor's promise is supported by its own history of authority and resources. A verification service depends on a maintained practice of calibration and interpretation. Their apparent externality to a transaction can be a useful organizational distinction; it is not independence from relational history. A model that treats them as unchanging background conditions may be adequate for a narrow calculation, but it should identify the dependence that it brackets.

Historical constitution also changes the interpretation of identity. A cooperative partner can retain its name while replacing its personnel, revising its procedures or losing the supports on which earlier performance depended. Continuity of the label alone does not justify transferring every past warrant. Conversely, a changed label need not erase all relevant continuity. The inquiry must identify which capacities and obligations are carried through the transition.

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4.2 Manifestation limits

The prior GR distinction among ontological, relational and manifestational reality provides an explicit discipline [21]. Ultimate ontology remains suspended. Relational reality concerns structures with modeled persistence and generative efficacy. Manifestation concerns the context-dependent presentations through which those structures become accessible. A record can therefore be real and causally important while remaining a selective presentation of the relation it records.

This distinction prevents a common assurance error. A verification system tests a manifestation according to a selected criterion. It may establish that a signature matches an authorized credential, that an item satisfies a documented specification, or that a record has remained unaltered. Further claims require further relations: that the credential corresponds to legitimate authority, that the specification captures the relevant obligation, or that the original record was adequate. Technical success at one layer cannot silently certify all of them.

Selection is not necessarily a failure. Cooperation requires finite descriptions, and preserving every distinction can be impractical or harmful. The analytical question is which distinctions matter to the undertaking and how an excluded distinction can become consequential. If the specification changes after exposure, participants may need access to earlier versions and to the process through which the change acquired authority. Prior GR work on archival power expressly distinguishes record preservation from retrieval, versioning and contestability [19].

4.3 Constitutive assurance

Assurance practices do more than reveal an independently complete relationship. They can generate new roles and alter what actions mean. Establishing a custodian can create a new party with duties over deposited material. Introducing a review procedure can make a previously private complaint institutionally actionable. Defining a reporting category can make a condition visible to allocation rules. These are changes in the effective possibilities of the relation.

This constitutive claim has a straightforward argument. First, a support practice defines or stabilizes distinctions such as authorized and unauthorized, accepted and disputed, retained and released. Second, participants can act on those distinctions through available procedures. Third, those acts alter subsequent access, obligation and exposure. Where these three steps hold, the practice belongs to the process producing later relational configurations. It cannot be represented adequately as a purely detached observer if the question concerns those changes.

The conclusion does not eliminate observation or independent review. Independence can refer to a specific separation of powers, incentives, records or personnel. The reviewer's involvement in the wider relational field does not make impartiality meaningless. It makes the relevant separation an achievement to be specified rather than a metaphysical exteriority to be assumed. A reviewer can be independent of a disputed party for the purpose at issue while still depending on a common archive or shared professional vocabulary.

Prior public-trust work already treats intervention within the interacting field [23]. The present paper uses that inheritance to ask a narrower question: what interaction terms or operational dependencies are introduced by a selected assurance practice? A safeguard that reduces one vulnerability can generate another. Requiring centralized records may improve consistency while concentrating control over future evidence. The net evaluation requires more than naming the practice protective.

4.4 Nomos formation

In the consulted GR formalism, a nomos is a relatively stabilized relational organization conditioning recognition, admissibility, interpretation and obligation [21]. It can determine whose submission counts, which evidence is relevant and what response becomes available. Its efficacy can arise through coordination, endorsement, dependency, power or combinations of these processes. The fact that it works supplies no sufficient account of why it should govern.

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Background independence, at this level of the programme, motivates attention to historically generated possibility spaces rather than an unconditional assumption that all relevant roles and transformations are given in advance [15]. A concrete model still needs a specification. The models below therefore declare their fields, types and interaction vocabulary. They represent some generated changes while leaving the revision of that vocabulary as an explicit further obligation. A finite formal carrier is a chosen analytical boundary, not a proof that social possibilities are permanently closed.

The philosophical payoff is a twofold responsibility. The analyst must explain how a cooperative arrangement maintains its operative distinctions, and must assess those distinctions using additional evaluative premises. These tasks are connected because a remedy's efficacy depends on the operative nomos. They remain distinct because a stable and efficient arrangement can reproduce exclusion. Reliability becomes a valuable property only after its object, conditions and consequences have been made sufficiently explicit.

5 Generative Cooperative Capacities

The philosophical problem concerns what a cooperative arrangement enables its participants to become and to do. This section develops that problem through prior GR arguments about historically constituted objects, heterogeneous knowledge and the reproduction of capacity. It first distinguishes delivered performance from the capacities sustaining it, then examines commonality, interpretation, power and the normative standing of revision. These distinctions supply the questions for the later models; mathematical preservation alone cannot settle their normative interpretation.

5.1 Delivered performance

Consider an arrangement that supplies a participant with a functioning service. Its delivery can improve that participant's position immediately. Yet the arrangement can also determine who may alter the service, inspect its operation, replace its provider or challenge its classification of a problem. Delivery and these capacities are different objects of inquiry. An increase in delivered output does not logically imply either their increase or their preservation.

The prior GR discussion of power and value in IR makes this distinction central. It contrasts provision that leaves a recipient dependent on controlled inputs with provision that develops capacities for further independent activity, while acknowledging unresolved measurement, boundary and aggregation problems [17, pp. 15–18]. The present paper inherits the distinction. It does not inherit a universal inference that differentiated relations must become domination, or that every beneficial transfer conceals extraction.

The argument can be stated without a capacity score. First, an undertaking consists of more than its current product: it includes the practices through which that product can be maintained, interpreted and changed. Second, some of those practices may be controlled by a counterparty. Third, cooperation can alter the recipient's access to them. It follows that an evaluation confined to current product delivery can omit a transformation relevant to future cooperation. The conclusion is a claim about the incompleteness of a particular evaluation, not a verdict on every dependent arrangement.

Dependence is not itself the failure condition. Cooperation often works because participants specialize and rely on each other. Requiring every participant to reproduce every function independently would destroy many advantages of cooperation and could be materially impossible. The relevant capacity is situated: the ability to invoke, contest, redirect or replace the dependencies that matter to the undertaking, through some adequately supported route. A jointly maintained alternative can sometimes supply this capacity more effectively than individual self-sufficiency.

This qualification changes the meaning of generativity. A generative cooperative relation need not maximize the number of imaginable actions. It may deliberately restrict destructive actions while enabling new forms of inquiry, production or response. The analytical task is to identify which capacities are created, which are lost, who bears the loss and how that allocation can be justified. A claim that

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cooperation creates possibilities is incomplete until it identifies whose possibilities and their relations to others.

5.2 Participant continuity

A participant's continuity through cooperation cannot be inferred from a stable name alone. An institution may retain its legal identity while changing the agencies through which it acts. A state may acquire a technical capability through a network whose maintenance depends on another state. A civil society organization may become an accredited interlocutor through procedures that also restrict which claims it can present. Each remains identifiable in some respects while its effective organization changes.

GR's spacetime object is useful here because it includes formation, continuation and transformation within the represented object. A present boundary identifies currently available interfaces; its history records how those interfaces became available. The two-complex representation then distinguishes events, coupling histories and element histories. It can record the creation of an office or an authorization alongside the later acts that depend on it. A graph of current contacts can be a legitimate projection, but it need not retain those dependencies.

Two qualifications prevent this account from becoming an unlimited demand for historical detail. A history-bearing object need not carry an exhaustive record into every calculation. Prior GR work distinguishes historical constitution from the sufficiency of a present record for a selected prediction [18, pp. 46–47]. Moreover, a continuity criterion is question-relative. Continuity of an obligation, an administrative role and a community's self-understanding can diverge. A representation should state which of these it preserves.

For cooperative reliability, the decisive issue is often the continuity of powers rather than the identity of the nominal parties. If a new administrative body inherits a treaty commitment but no procedure for receiving a counterparty's evidence, documentary continuity can coexist with practical discontinuity. Conversely, a changed institution may preserve an effective route through which an earlier claim can still be heard. The algebraic transport model in Section 16 treats these as separate preservation tests.

This argument does not require an ontological thesis that all objects are ultimately relations. It adopts a representational discipline: when a capacity is historically and relationally constituted, the explanation should not erase its enabling relations while continuing to rely on its effects. Relatively stable actors remain available as analytical units. Their stability becomes a scoped achievement to be examined when the problem concerns institutional transformation.

5.3 Commonality

A cooperative undertaking requires some compatibility, but the nature of that compatibility remains open. Participants can share an operational interface without sharing a worldview; they can accept a procedure while disagreeing over its moral foundation; they can recognize the same prohibition while assigning it different historical significance. An account that requires complete interpretive convergence would exclude these possibilities by definition.

The GR diplomacy manuscripts offer two distinct formal ideas. A common dynamical factor is a shared description related to each participant's dynamics by an appropriate map [16, pp. 26–29]. An invariant of a declared local transformation group is a feature unchanged by those transformations [20, pp. 13–16]. A description and an invariant are not interchangeable. The first must support the distinctions the joint practice uses; the second depends on which transformations have been declared equivalent.

Neither construction establishes political agreement by itself. Two maps to the same description space can assign different values to the participants' actual states. An invariant can be mathematically well-defined but inaccessible to the parties or irrelevant to their obligations. The present paper therefore supplies an explicit compatibility relation for common descriptions and treats access to a shared feature as a separate evidential problem. These qualifications preserve the useful distinctions without converting mathematical commonality into achieved cooperation.

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Commonality also has a political cost when it determines what cannot be said. A shared reporting language can enable coordination while excluding a locally meaningful distinction. The exclusion may be a justified simplification; it may also make a harmful consequence institutionally invisible. The relevant inquiry concerns both the selected common object and the routes through which excluded distinctions can return as claims. A common vocabulary is valuable partly through what it enables, and contestable partly through what it suppresses.

Preserved difference does not mean preserving every existing classification. Some classifications sustain exclusion or make obligations impossible to discharge fairly. Nor does difference have an intrinsic positive value in every setting. The paper's proposal is more limited: the success of a cooperative interface should not be inferred from uniform public output when that uniformity may have been obtained by removing participants' capacities to articulate consequential differences.

5.4 Interpretive authority

The diplomatic knowledge manuscript places attention on the conditions under which a claim can be admitted, rather than only on disagreements over its content [16, pp. 4–6]. Applied to cooperation, this directs inquiry toward who can supply evidence, which forms count as evidence, and which institution can transform a contested description into an actionable finding.

Suppose two parties exchange reports about the same undertaking. The reports can differ because one party misstates a fact, because the parties observe different aspects, because their classifications differ, or because a shared classification has changed. These possibilities require different investigations. A verification procedure that assumes only the first can mistakenly treat an interpretive dispute as dishonesty. A procedure that assumes only the third can obscure deliberate deception. GR's distinction between manifestation and relational reality requires discrimination among these mechanisms; it supplies no automatic preference for the most charitable interpretation.

Interpretive authority enters when one classification becomes binding for subsequent action. If an institution alone decides what counts as completion, it can settle uncertainty efficiently. It can also make its own continued operation the condition for questioning its decisions. That dependency becomes especially important when a challenge requires records held by the challenged institution, recognition granted by it, or a vocabulary it alone may revise.

The dependence is recursive in a specific sense. A present classification affects the actions participants can take; those actions affect the records and capacities available in the next episode; the resulting distribution influences whose classifications can later become authoritative. Each link requires evidence in an application. The chain is not a general law of institutional capture, and an external review, distributed archive or protected right of initiation can interrupt it.

The later institutional kernel field supplies one possible representation of this reciprocity between a mediation practice and its coupling rules. It does not assign numerical values to legitimacy. Its purpose is to make a previously fixed mediating relation into an interacting formal object. The separate operator model asks whether a change in that relation preserves a participant's ability to perform an operation relevant to the undertaking.

5.5 Reproduction of cooperative conditions

An arrangement that works today can consume conditions required for tomorrow's cooperation. It may rely on unpaid translation work, temporary technical expertise or a participant's capacity to wait. Repeated successful performance can conceal the depletion because the visible output remains acceptable until a supporting practice fails. Conversely, cooperation can reproduce those conditions through training, shared archives, distributed authority or practices of reciprocal explanation.

This is a philosophical reason to treat maintenance as part of the object. The inference does not begin from the metaphor that institutions are living things. It begins from the dependence of performance on activities that themselves require renewal. If the research question concerns sustained cooperation, an explanation omitting their renewal is incomplete whenever that renewal affects continuation.

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Preserving conditions does not imply freezing them. A procedure can become less reliable precisely because it faithfully preserves a classification after the undertaking has changed. An arrangement must sometimes transform its own supports to remain answerable to its purposes. The relevant continuity then concerns the undertaking and its justified constraints, rather than literal repetition of every procedure.

This distinction also explains why stable output is a weak test of relational stability. Two histories can produce the same present report while differing in the operations that remain available. One can include unused but accessible review powers; another can contain only nominal rights whose enabling institution has disappeared. A continuation-sensitive comparison can distinguish them without requiring a speculative trust magnitude. It asks what specified future test or intervention could expose the difference.

Not all such tests are neutral. Requesting review can consume resources, expose information, alter expectations or terminate an informal accommodation. The inquiry should therefore distinguish a hypothetical comparison from an intervention in the actual relation. A successful mathematical witness shows that two modeled configurations differ under a declared continuation. It does not prove that performing the test would be prudent or that the difference has the asserted historical cause.

5.6 Normative standing

The preservation of agency and meaningful contestation is a normative proposal in this paper. It does not follow from the fact that a system is relational, generative or mathematically compositional. A stable domination can possess all three properties. An operator algebra can preserve a badly chosen constraint perfectly. The normative question concerns which constraints deserve preservation and who may participate in choosing them.

The proposal has a conditional justification. When an arrangement claims mutually beneficial cooperation among participants who remain entitled to contest its terms, it should retain effective means of presenting and examining relevant challenges. If the arrangement removes those means while invoking the participants' continued acceptance as its warrant, it undermines the conditions under which that acceptance can serve as evidence. The criticism applies to the arrangement's own justificatory claim; it does not establish an unrestricted right to block every collective decision.

Legitimate commitment can narrow future options. Arms limitation, environmental protection and reliable delegation all involve constraints. A participant's earlier assent may justify some of them, subject to the scope and validity of that assent. Meaningful revisability concerns access to an appropriate process for examining changed conditions, breaches or contested interpretation. It does not promise unilateral release from every obligation or restoration of every foregone advantage.

The relevant subjects also extend beyond governments. A state-level agreement can be mutually advantageous to its negotiators while imposing unrepresented costs on a region, class or affected population. Prior GR power analysis explicitly identifies the problem of aggregation across internal inequalities [17, p. 18]. The present paper therefore treats participant boundaries as substantive modeling decisions. Its finite examples do not establish that the selected parties exhaust the people whose claims matter.

These qualifications produce a demanding but bounded analytical object. Evaluating the cooperative arrangement concerns expectations of performance together with the historically maintained capacities needed to interpret, contest and adapt the undertaking. Operational reliability and normative justification remain distinct assessments. Some questions can be answered in a fixed-rule model. Others require the rules, mediating structures and participant capacities to become variables of the investigation. The following constructions distinguish these cases and make their preservation assumptions inspectable.

6 Cooperative Criteria

An arrangement cannot be assessed for reliability until the relevant achievement and conditions are specified. This section separates four evaluative dimensions and explains how they bear on a decision to

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cooperate. The distinctions are philosophical prerequisites for the later admissibility predicates; those predicates record choices whose justification remains outside the algebra.

6.1 Performance

Operational reliability concerns the warranted expectation that an arrangement will perform specified functions within a declared range of circumstances. The range matters as much as the function. A process can reliably deliver during ordinary operation while offering no reliable response to disputed performance. It can be reliable for one category of participant while failing for another whose submissions require a different interpretive interface.

The warranted expectation need not be a numerical probability. It can rest on demonstrable capacities, historical evidence, controlled procedures and explicit limits. A probability model can be added when the events, sampling conditions and inference are justified. In the absence of those conditions, a precise scope statement is often more informative than a fabricated confidence level.

Reliability is also indexed to the function selected. Efficient enforcement of an exploitative commitment is still operational effectiveness. Calling it reliable does not endorse its object. This point prevents the analysis from acquiring a positive moral conclusion simply because a formal system preserves an invariant.

6.2 Exposure

Safety concerns consequences that the arrangement is required to avoid, limit or address. A bounded exposure is one whose relevant consequences have been specified sufficiently to support a decision about undertaking them. The bound need not be monetary, and distinct losses need not be commensurable. Disclosure, bodily risk, loss of standing, irreversible environmental change and exclusion from future opportunities can require different protections.

A safeguard can reduce the probability of a departure without limiting its consequence when it occurs. Another can leave the departure possible while preserving a workable remedy. Neither function should be inferred from the other. A highly reliable delivery mechanism may leave an exceptional failure catastrophic; a less certain undertaking may remain defensible because exposure is small and a practical alternative survives.

Reversibility also needs an object. Reversing a transfer can restore a balance while leaving disclosure, delay or dependency unchanged. A rollback in a technical system does not erase the social history of the action. The present account therefore uses reversibility locally: a specified effect can be undone or compensated to a stated extent. It avoids a global promise to restore the entire relation to its earlier condition.

6.3 Benefit

Mutual benefit concerns the improvement that each relevant participant has reason to seek through the undertaking. Its assessment requires an explicit comparison. A party may benefit relative to immediate nonparticipation because its alternatives have been severely restricted. That fact can explain entry without justifying the restrictions or the terms imposed by the party controlling access.

Nor does agreement establish benefit to everyone affected. An arrangement can advantage its direct participants while imposing costs on workers, neighbors, future users or ecological systems excluded from the agreement. A bounded paper cannot supply a universal decision procedure for those effects. It can require that the choice of affected parties be defended and that excluded effects not disappear merely because the formal participant set is small.

Benefit must also be distinguished from successful completion. A process can deliver exactly what was specified after the purpose that made delivery valuable has expired. It can resolve a dispute when the claimant has already lost an opportunity that the remedy cannot recover. The history of the undertaking

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Table 2: Distinct criteria for a cooperative arrangement. Each requires its own scope and evidence.

Criterion	Primary question	Insufficient substitute
Performance	Which functions remain dependable in the declared circumstances?	Routine success alone
Exposure	Which consequences are avoided, bounded or remediable?	A low frequency of failure alone
Benefit	What valuable possibilities remain for each affected participant?	Formal completion alone
Justification	Are the terms and distributions of power defensible?	Continued participation alone

therefore belongs to benefit assessment rather than entering only as a correction to an otherwise complete endpoint comparison.

6.4 Justification

Normative justification concerns whether the arrangement's terms, powers and consequences are defensible. It cannot be read directly from stable participation. A participant can remain because exit is prohibitively costly, because its dependents would suffer, or because no other provider exists. These circumstances do not automatically invalidate every agreement. They do make participation alone insufficient evidence of free endorsement.

The present paper adopts a limited evaluative orientation: affected participants should be able to understand the relevant undertaking sufficiently, contest consequential classifications through workable procedures, and avoid being required to assume unbounded or concealed exposure as the price of cooperation. These are proposed premises for this investigation, not deductions from GR's relational ontology. Their application remains open to disagreement about necessity, proportionality and the treatment of third parties.

Table 2 separates the four dimensions. An arrangement can improve on one dimension while worsening another. Their plurality is a reason to preserve a profile of assessments rather than aggregate them into a single trust score.

6.5 Undertaking decisions

A defensible undertaking combines these assessments without requiring certainty. The participant needs reasons that connect the proposed activity to the available supports and to the consequences of their failure. An explicit residual uncertainty can be acceptable; an uncertainty concealed by a misleading assurance cannot obtain that status simply through successful performance to date.

Safe refusal or termination can be an important achievement. It preserves the ability to avoid an unacceptable continuation. Yet an arrangement that always refuses every task is not thereby a successful cooperative system. Later models accordingly distinguish preserved admissibility, access to a remedy and completion of the joint purpose. A safety theorem alone does not establish cooperative productivity.

7 Assurance Practices

This section examines what assurance practices contribute and what they presuppose. Its purpose is to replace an undifferentiated institutional substitute for trust with a functional account of commitments, verification, custody and response. Each function is assessed through the relation it changes and the dependency it introduces.

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7.1 Commitment articulation

A commitment gives participants an object around which expectations and responses can be organized. Articulation can identify an activity, a responsible role, a condition of completion and a procedure for departures. It reduces some ambiguity without guaranteeing agreement about every future application. A useful commitment can consequently remain incomplete while supplying a workable route for handling the incompleteness.

Articulation also creates authority. Someone determines the operative wording, classifies submissions and decides whether a departure falls within a response procedure. Treating these activities as neutral administration can hide a consequential part of the undertaking. A specification becomes a support for cooperation when its authority and revision process are intelligible enough for the participants to rely on them within the selected scope.

This point extends Hawley's attention to commitment without identifying her theory with GR [11]. The GR question concerns how a commitment acquires persistent relational efficacy through histories of recognition and execution. A stated obligation can have interpersonal significance while lacking institutional enforcement, and an enforceable requirement can operate without interpersonal trust. Their relation must be established locally.

7.2 Verification

Verification connects an accessible manifestation to a declared criterion. It can reduce uncertainty about a selected feature of performance. It cannot independently establish that the criterion is adequate, that the source material is complete, or that a detected departure will receive an effective response. These are separate links in the arrangement.

The division becomes visible when participants disagree about the object of the undertaking. A system may verify that an output was delivered while the recipient disputes whether the output is usable. A record may demonstrate that a notice was transmitted while leaving open whether the addressee could understand or act on it. More accurate verification of transmission will not by itself resolve the latter problem.

Verification can also affect the behavior it examines. Participants may redirect effort toward auditable outputs, adopt the verifier's categories, or avoid activity that the records represent poorly. O'Neill's discussion of intelligent accountability warns that accountability processes can impair the activities they are meant to assess and emphasizes evidence that audiences can assess rather than information merely disseminated [35]. The present inquiry uses that bounded insight to ask how the verification practice changes the cooperation it supports.

7.3 Custody

Custody concerns the maintained availability of resources, records or decision powers needed for later operations. Separating custody from a directly interested party can protect against some forms of unilateral change. The separation requires substance: who controls access, who can revise the classification, which costs accompany retrieval, and what happens if the custodian becomes unavailable?

Custody is temporal. A record can be accurate at creation yet unavailable when a dispute becomes intelligible. It can remain stored while becoming difficult to retrieve or relate to an earlier version of a specification. Prior GR analysis of archival capture locates the problem in disproportionate control over the evidential basis of later reconstruction, including cases without falsification [19]. An honest archive can remain an inadequate basis for independent challenge.

Plural custody is therefore a possible response, not a sufficient condition. Multiple offices may reproduce the same filtered record. Conversely, a single archive may provide genuinely contestable access under effective controls. The relevant assessment concerns preserved distinctions, available powers and shared dependencies rather than the number of organizational boxes.

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7.4 Response

A response procedure connects a recognized departure to action. It can pause exposure, replace a performer, restore a resource, revise a classification, impose a consequence or authorize exit. These responses have different temporal and material conditions. The ability to issue a favorable decision does not imply the ability to restore what mattered.

The procedure also changes incentives and standing before any dispute occurs. A credible capacity to pause release can alter negotiations. A burdensome complaint process can discourage challenge even when its final decisions are impartial. An arrangement's response architecture is therefore part of its ordinary constitution, not merely an exceptional repair added after normal cooperation fails.

Protected cooperation in prior public-trust work already combines these elements [23]. The present analysis asks how they connect. An authoritative commitment needs an interpretable criterion; verification needs accessible evidence; a response needs power and time; each support needs maintenance. The missing link is often more consequential than the presence of any single device.

7.5 Residual dependence

Assurance cannot eliminate every dependency without eliminating the undertaking itself. Cooperation involves acting through capacities that are not wholly one's own. The relevant demand is consequently a bounded and revisable account of residual dependence. Participants should be able to identify which supports remain assumptions, why reliance on them is reasonable, and what is possible if they fail.

This account need not become an infinite chain of guarantors. Inquiry can stop provisionally at warranted background capacities whose uncertainty is acceptable for the defined exposure. The stopping point should remain visible. Calling a support institutional, technical or independent supplies no automatic warrant to treat it as infallible.

8 Interpretive Interfaces

Participants can cooperate without sharing a complete framework of meaning. This section examines the narrower commonality that an undertaking requires. It draws on prior GR work on inter-nomic mediation to distinguish translation, operative formation and local recognition, then identifies the authority questions that remain after a common interface has been established.

8.1 Partial commonality

A shared interface is a stabilized set of distinctions and procedures sufficient for a selected joint activity. It can connect participants whose wider practices remain different. The interface might identify a deliverable, a period of use, a custody arrangement and a response to specified departures. Agreement about those matters need not imply convergence about the participants' broader purposes or accounts of value.

The social-contract manuscript already distinguishes local capability from the existence of a shared formative route [21]. Two parties can each possess well-developed local procedures while lacking a common way to recognize one undertaking. The problem is relational rather than a deficiency attributed to one side. The manuscript's partial translation maps are especially useful here because an interface need not represent every locally meaningful distinction.

For cooperation, the significance of partiality depends on what is omitted. A distinction can be irrelevant to the selected task and remain outside the interface without difficulty. Another can bear on a consequence that only one side initially recognizes. The arrangement then needs a way to raise that distinction without treating its initial absence as proof that it does not matter.

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8.2 Formation

Mutual legibility does not yet create an operative commitment. The participants can understand each other's descriptions while disagreeing about who can authorize the undertaking or how departures will be handled. Formation adds a relational act: the selected roles, obligations and procedures become effective for the activity.

This act can generate a new cooperative object. A joint archive, review panel or shared facility can persist across changes in individual membership. Its identity depends on maintained practices and recognition rather than the mere persistence of its founding text. Its later actions can alter the possibilities of the original participants and of people who did not take part in formation.

The interface consequently has a history of its own. Changes to it should be evaluated as changes to an operative relation, not just updates to a neutral description. A new evidence rule can close a previously available challenge; a new interpretation can expand one party's exposure. These effects can occur without any alteration to the headline commitment.

8.3 Recognition

An interface-generated undertaking can acquire different local effects. One participant may treat a successful review as sufficient authorization for release, while another requires an additional act. One may recognize a remedy as binding while another treats it as advisory. The prior social-contract formalism separates shared formation from local recognition and from the fields of effect generated by recognition [21]. The same separation prevents an agreement on paper from being mistaken for equivalent practical consequences.

Literal equivalence of every local effect is unnecessary. What matters is compatibility on the distinctions material to the undertaking. If both routes preserve a specified right to pause release, different local procedures may be adequate. If one route turns the same pause into a breach exposing the claimant to additional liability, the difference is material. The relevant comparison must be argued, not inferred from a common label.

The categorical constructions available in prior work express stronger forms of coherence when functoriality and the comparison domain have been justified. This paper does not require that lift. Its formal model retains partial maps and declared compatibility predicates. A failure of total translation remains analytically visible rather than being repaired by an unsupported assertion that every local practice belongs to one category.

8.4 Interface authority

Who can revise the interface becomes central once an excluded distinction is raised. A procedure that permits only correction of clerical errors may be incapable of addressing disagreement about the categories themselves. An appeal limited to whether existing rules were followed can be effective within its scope while offering no remedy for a harmful rule.

This limit does not make every bounded appeal illegitimate. Different forums can have different functions. The arrangement needs an intelligible account of where a category-level challenge can be raised, what consequences can be paused during review, and whose interpretation governs in the meantime. Where no such path exists, participants should not be told that a narrow error-correction procedure supplies comprehensive contestability.

Temporal asymmetry sharpens the issue. The participant controlling the interface may continue operating while another bears the costs of waiting for a revision. A favorable later interpretation can leave those costs untouched. This is one reason recourse must be evaluated through available histories rather than through the existence of a favorable terminal statement alone.

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8.5 Persistent difference

A successful interface can preserve disagreement. Participants may accept a limited criterion because it makes a bounded undertaking possible, while retaining objections to its wider use. The permission to cooperate within that scope should not be converted into endorsement of the entire institutional order that supports it.

This interpretation also limits the paper's ambition. It does not propose a universal translation mechanism. Some differences may make the selected cooperation unavailable or indefensible. Recognizing that boundary is part of a reliable arrangement: the ability to decline a relation whose material terms remain unresolved can be as important as the ability to complete one.

9 Compositional Obligations

This section gathers the preceding argument into explicit obligations for evaluating a cooperative arrangement. It distinguishes sequential compatibility, shared dependence and practical closure, then explains how the formal investigations address them. These obligations organize analysis rather than functioning as an exhaustive certification checklist.

9.1 Sequential compatibility

A support ordinarily works from particular starting conditions. Verification presupposes an identifiable submission and an operative criterion. Release presupposes a recognized authorization. Recourse presupposes a departure that can be represented and a participant who can invoke the procedure. When operations are composed, the output of an earlier operation must supply the conditions required by the next.

Checking each operation only in the initial situation does not establish this compatibility. An earlier act can alter standing, records or timing in a way that makes a later safeguard unavailable. A process may let a participant review an item before acceptance and request reimbursement after acceptance, yet fail to preserve the evidence needed for the latter request. The two protections are locally intelligible while their sequence lacks the promised coverage.

The algebra in Section 15 formalizes one restricted version of this problem. It asks whether permitted operators preserve a declared admissible sector for every admissible input in that representation. This is stronger than checking a favorable initial state. It remains weaker than establishing that the sector captures every relevant social concern.

9.2 Shared dependence

Separate safeguards can share a support that fails or changes. The shared element can be material, organizational or interpretive: a power supply, funding stream, archive, classification rule or credential authority. Two reviewers can reach independent judgments from the same selectively preserved record. Their independence of decision does not make the evidential basis independent.

Shared dependence need not be harmful. A common interface can enable coordination and reduce conflicting interpretations. A shared archive can preserve continuity. The question is which connections are created and whether the arrangement's assurance claims acknowledge them. Treating every common support as a defect would ignore the very relations that make cooperation possible.

The field calculation in Section 13 addresses this ambiguity. Integrating a shared mediator generates mixed interaction terms between distinct channels. Those terms can reinforce or oppose particular configurations depending on their mode relation and the selected parameters. The calculation demonstrates dependence within the model, not a universal negative effect of mediation.

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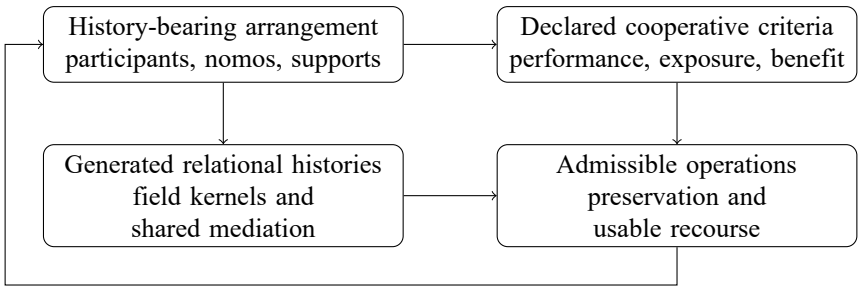

Figure 1: Analytical dependencies of the proposed investigation. Implemented safeguards can change the arrangement to which its criteria and models apply.

9.3 Practical closure

A remedy needs a connected course of action from the participant's current position to an effect that matters. Recognition, access, evidence, decision power and timing all contribute. A formal entitlement can remain intact while one link disappears. Such a remedy is nominally present and practically closed.

Closure can occur gradually or at an identifiable event. A deposit can be released beyond the adjudicator's reach. A record can expire. A participant can lose authorization to submit. An opportunity can end while review continues. Different histories can therefore contain the same final verdict but different effective remedies.

The finite model in Section 17 makes a further distinction: the existence of a favorable path is not the existence of a strategy a participant can execute despite the modeled adverse choices. This stronger condition depends on the distribution of control and on what the participant can observe. It is one precise operational meaning of usable recourse, not a general theory of legal rights.

9.4 Maintenance

The supports must remain available long enough to perform their roles. Maintenance includes personnel, material resources, interpretive competence and the continued recognition of obligations. It can be invisible when routines work. Its interruption reveals that a seemingly static institution was an ongoing accomplishment.

Maintenance also has a distribution. An arrangement may preserve one party's flexibility by requiring another to monitor continuously, retain records or wait through uncertain review. Those requirements can undermine the very recourse the arrangement advertises. The relevant costs enter the present inquiry when they determine whether an action is actually available. A complete account of their just distribution remains beyond this paper.

9.5 Semantic accountability

Every formal assurance result needs a declared relation to the cooperative object. A field variable must represent a stated mode, not trust in general. A protected subspace must correspond to an explicit criterion. A terminating path must identify what has been remedied and what persists. Without this semantic accountability, formal precision can conceal rather than resolve the practical problem.

Figure 1 locates the models within the argument. The arrows indicate dependencies of assessment, not a universal historical sequence. The feedback path records that implementing a support can alter the arrangement and require renewed assessment.

These obligations justify the combination of models without asserting their equivalence. The space-time construction preserves incidence and history; the field construction investigates generated couplings; the algebra studies preservation; the finite game studies executable response. Their interfaces must be stated because a result in one representation does not automatically transfer to another.

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10 Relational Spacetimes

The formal investigation begins with the represented history of an arrangement. This section fixes primitive types, historical composition and manifestation before any field or operator is introduced. It follows the incidence convention of the heterogeneous GR framework and distinguishes a two-complex representation from an optional spin-foam instantiation [14, 25]. The construction has a representational status; incidence alone supplies neither probabilities nor social dynamics.

10.1 Boundary data

Let P_0 be a finite set of elements and C_0 a finite set of couplings. The subscript distinguishes these sets from an operator projection introduced later. Elements are primitive only at the selected resolution. A role, record or resource can require a finer history representation in another inquiry. In the graph sector, each coupling joins at least two elements and each element belongs to one or two couplings. The boundary data are defined by the following expression.

$$\Gamma = (P_0, C_0, \iota), \qquad \iota : C_0 \longrightarrow \mathcal{P}(P_0), \qquad |\iota(c)| \geq 2, \quad 1 \leq |\{c : p \in \iota(c)\}| \leq 2. \tag{1}$$

Equation (1) makes couplings the graph vertices and elements the edges or half-edges. It does not draw people as points connected by trust-valued lines. A participant ordinarily occupies several couplings and has a history extending beyond the boundary. Higher incidence requires a richer representation or an explicitly justified refinement.

Let T_P and T_C be declared element and coupling type sets. Type maps send P_0 to T_P and C_0 to T_C. Labels can carry a commitment identifier, a specification version or an evidence status, with each label's domain declared by the application. Group variables in the next section provide additional compatibility data; they do not replace these semantic assignments.

10.2 Historical cells

Let V be events, E coupling histories and Φ element histories. Incidence data specify which coupling histories meet at an event and which element histories attach to them. A selected arrangement spacetime has the following finite representation.

$$\mathcal{S} = (F, \Gamma_-, \Gamma_+, \tau, \jmath, \preceq), \qquad F = (V, E, \Phi, \mathrm{inc}). \tag{2}$$

In Equation (2), Γ_- and Γ_+ are incoming and outgoing boundaries, τ supplies types, $\jmath$ supplies continuation identifications, and $\preceq$ is an acyclic event order. Vertices of the two-complex are events, edges are coupling histories, and faces are element histories. A continuing role can therefore cross several events without being identified with any one of them.

The order $\preceq$ represents precedence within this model. It need not establish a fundamental temporal ontology, and it supplies no calibrated duration. A deadline enters later only through an additional operational representation. Histories with no compatible acyclic orientation remain outside this ordered sector.

At a cut Σ, let H_Σ denote the accumulated complex, Γ_Σ the active boundary and N_Σ a represented nomos. The configuration and its contextual manifestation have the following types.

$$\omega_\Sigma = (H_\Sigma, \Gamma_\Sigma, N_\Sigma) \in \Omega, \qquad m_c : \Omega \rightharpoonup X_c. \tag{3}$$

Equation (3) uses Ω for the selected configuration family and X_c for the records accessible in context c. The arrow permits undefined manifestations. Two different histories can have the same m_c value. An operator on records must therefore not be presumed to preserve every distinction in Ω.

Historical extension retains the realized cells under a declared embedding; it can add later classifications and consequences without erasing their provenance. This is the conservative historical convention inherited from the prior framework, not a theorem about physical time [21]. Silent replacement of an earlier label would lose the difference between an original classification and a later revision.

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10.3 Composition

Suppose two histories have an explicit type-, incidence- and continuation-preserving boundary isomorphism $\alpha : \Gamma_{1,+} \to \Gamma_{2,-}$. Choose fresh interior identifiers. Their gluing is the quotient of the disjoint union by that identification.

$$F_2 \circ_\alpha F_1 = (F_1 \sqcup F_2)/\!\sim_\alpha. \tag{4}$$

Equation (4) identifies only the specified boundary. The combined precedence relation must also remain acyclic. A matching visual shape without matching role or version data is insufficient. This condition captures one way that a local assurance can fail to attach to a later operation.

Proposition 10.1 (Compatible historical composition)**.** *For three histories with compatible boundary identifications, fresh interiors and an acyclic combined order, the two bracketings of gluing are canonically isomorphic as typed history complexes.*

Proof. Form the disjoint union of all three cell sets. Both bracketings take its quotient by the equivalence relation generated by the two boundary maps. Sending a class in one bracketing to the class of the same original cells in the other defines a bijection. The boundary maps preserve types, incidence and continuation, so the bijection does also. The combined order is the same transitive closure in both constructions and is acyclic by assumption. □

Proposition 10.1 is a standard gluing consequence of the declared data. It permits regrouping a sequence, not reordering its events. Review followed by release can differ from release followed by review even when both bracketings of a longer valid sequence agree.

10.4 Admissible histories

Let I be a declared finite set of assessment standpoints, which can include affected parties beyond the direct cooperators. For each $i \in I$, let $a_i : \Omega \to \{0, 1\}$ be a criterion. Criteria can inspect history, so a past breach need not become invisible after a favorable current outcome. Define the selected admissible family as follows.

$$A = \{\omega \in \Omega : \ a_i(\omega) = 1 \text{ for every } i \in I\}. \tag{5}$$

Equation (5) represents a conjunction, not an aggregation of interests into one utility. The criterion's meaning and the choice of I require the philosophical assessment of Section 6. If the criteria are disputed, the model should preserve alternative A families rather than disguise a contested choice as mathematical necessity.

A record-only classifier can reproduce admissibility only under a specific preservation condition.

Proposition 10.2 (Manifestation sufficiency)**.** *On a domain $D_c \subseteq \mathrm{Dom}(m_c)$, a map $\widehat{a} : m_c(D_c) \to \{0, 1\}$ satisfying $\widehat{a}(m_c(\omega)) = \mathbf{1}_A(\omega)$ exists if and only if $\mathbf{1}_A$ is constant on every fibre of m_c restricted to D_c.*

Proof. If $\widehat{a}$ exists, equal manifestations have equal images under it, hence equal admissibility values. Conversely, if each fibre has a constant value, assign that value to its manifestation. The assignment is well defined and has the required equality. □

Proposition 10.2 is an elementary factorization criterion, also developed in prior GR work on historical records [18]. Its practical consequence is exact: when a record collapses an admissibility-relevant distinction, no reclassification of that same record alone can recover it in every case. Additional evidence, a narrower claim or a different criterion is necessary. This does not imply that every lost distinction deserves preservation.

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10.5 Spin data

A spin-foam instantiation adds representation-theoretic structure. Choose a compact group G, a finite label set J_G of finite-dimensional unitary irreducible representations, and spaces V_j. Assign a representation to each face and an invariant tensor to each edge, with orientations determining duals.

$$j_f \in J_G, \qquad \eta_e \in \mathrm{Inv}_G\left(\bigotimes_{f \supset e} V_{j_f}^{\epsilon_{ef}}\right), \qquad V_j^+ = V_j, \quad V_j^- = V_j^*. \tag{6}$$

Equation (6) supplies compatibility conditions beyond bare incidence. Choosing finite bases in the intertwiner spaces and local weights gives a possible fixed-complex amplitude.

$$Z_F(b) = \sum_{\text{internal labels compatible with } b} \prod_f A_f \prod_e A_e \prod_v A_v. \tag{7}$$

In Equation (7), b fixes boundary labels and the A factors have declared dependence on adjacent labels. Local factorization is an assumption. Gluing contracts shared boundary data with a convention counting each boundary factor once. Complex amplitudes are not probabilities merely because they can be summed.

The additional data in Equations (6) and (7) would require a target interpretation before an empirical application. The present paper retains them as one possible instantiation, consistent with prior GR work [24]. The next construction specifies a finite group-domain generator rather than presupposing a single completed complex.

11 Continuation-Sensitive Commitments

The spacetime representation retains histories, but an institution usually works with selected records. This section asks what a record must preserve when commitments are assessed through possible continuations. It adapts the continuation construction of prior GR work and then distinguishes a common dynamical description from actual compatibility [18, 16]. These are restricted deterministic models. Their role is to identify information and composition requirements before field weights or operator representations are introduced.

11.1 Continuation domains

Choose a nonempty family $\Omega_0 \subseteq \Omega$ of configurations from Equation (3). Let $\mathscr{U}$ be a declared alphabet of operations, with partial maps $T_u : \Omega_0 \rightharpoonup \Omega_0$. Operations may append a review, change an active interface or create an authorization, while retaining the inherited history. The alphabet describes modeled possibilities; it is not a list of actions every participant can execute.

Introduce a formal blocked configuration $\perp$ and extend each map to a total map $\widetilde{T}_u$ on $\widetilde{\Omega}_0 = \Omega_0 \cup \{\perp\}$ by sending undefined applications and $\perp$ to $\perp$. Let Y be an observation set and let $o : \Omega_0 \to Y$ be a selected readout, such as the current compliance classification. Set $\widetilde{o}(\perp) = \dagger$ for a fresh symbol $\dagger \notin Y$. These conventions distinguish blocked execution from successful execution with an unfavorable result.

For a word $w = u_1 \cdots u_\ell$ executed from left to right, put $\widetilde{T}_w = \widetilde{T}_{u_\ell} \cdots \widetilde{T}_{u_1}$ and use the identity for the empty word. The continuation signature and its equivalence are defined as follows.

$$\sigma_\omega(w) = \widetilde{o}(\widetilde{T}_w \omega), \qquad \omega \equiv_o \omega' \iff \sigma_\omega(w) = \sigma_{\omega'}(w) \quad \text{for all } w \in \mathscr{U}^*. \tag{8}$$

Equation (8) compares complete histories only relative to declared operations and observations. It does not identify relational identity with behavioral equivalence. Two historically different arrangements can be equivalent for this task; enlarging the operation alphabet can separate them.

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Proposition 11.1 (Continuation record)**.** *The relation $\equiv_o$ is the coarsest equivalence contained in the kernel of $\widetilde{o}$ and preserved by every $\widetilde{T}_u$. Its quotient supports well-defined induced observations and updates.*

Proof. Equality of signatures is an equivalence. The empty word gives equality of current observations. If $\omega \equiv_o \omega'$, then the signatures after u agree because every later word w tests the original word uw. Hence the relation is preserved. Conversely, any equivalence preserving the current observation and every update preserves the observation after every finite word, by induction on word length. It is therefore contained in $\equiv_o$. The induced maps $[\omega] \mapsto \widetilde{o}(\omega)$ and $[\omega] \mapsto [\widetilde{T}_u\omega]$ are independent of representatives by these two preservation properties. □

Proposition 11.1 is an inherited behavioral construction, restated with an explicit blocked-execution convention. Its new use here concerns cooperative records. A finite record that reports current compliance can be inadequate for predicting what a later review will disclose or whether the review can be initiated.

11.2 Dormant capacities

A three-configuration example isolates this difference. Let x and y have the same public performance readout, but let only x retain an executable review operation r. Let z record a completed review. The operation and observations are stipulated by the following data.

$$o(x) = o(y) = \text{performed}, \qquad T_r(x) = z, \qquad T_r(y) \text{ undefined}, \qquad o(z) = \text{reviewed}. \tag{9}$$

Equation (9) gives equal present manifestations but unequal signatures after r. The model does not infer that y is unjust or that a review would reveal a breach. It shows that the current performance record cannot certify equal review capacity. The capacity can be dormant while still affecting possible continuations.

The distinction becomes historical when x and y result from different institutional transformations. Both may have delivered the promised item, while only one retained the records and authority needed for later challenge. A retrospective summary that merges them loses a feature relevant to sustained cooperation. Adding an unused right as a label is insufficient: the transition rule must represent the conditions under which it can be exercised.

The example also identifies a limit of the deterministic representation. Actual review may depend on uncertain decisions by others. The recourse game in Section 17 adds that issue explicitly. Here totalization is a bookkeeping device for specified possibilities. It neither supplies a probability of blockage nor treats all imaginable continuations as practically accessible.

11.3 Common dynamical descriptions

To model a scoped shared description, choose two nonempty state spaces X_A, X_B, deterministic maps $f_i : X_i \to X_i$, a nonempty space Z and a map $g : Z \to Z$. Let $\pi_i : X_i \to Z$ be surjective. The common-factor condition and the compatible-pair set are the following.

$$\pi_i f_i = g\pi_i \quad (i = A, B), \qquad \mathcal{R}_\pi = \{(x_A, x_B) : \pi_A(x_A) = \pi_B(x_B)\}. \tag{10}$$

Equation (10) adapts the GR diplomacy construction while stating a necessary qualification: having a common codomain does not make arbitrary readings equal. Agreement concerns pairs in $\mathcal{R}_\pi$. The factor condition says that each local transition is compatible with the declared shared transition.

Proposition 11.2 (Compatible description persistence)**.** *If Equation* (10) *holds, then $(f_A \times f_B)(\mathcal{R}_\pi) \subseteq \mathcal{R}_\pi$. If the local maps change to f'_A, f'_B while the description maps are retained, persistence requires $\pi_A f'_A(x_A) = \pi_B f'_B(x_B)$ for every $(x_A, x_B) \in \mathcal{R}_\pi$.*

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Proof. For a compatible pair, both original projected successors equal $g(\pi_A(x_A)) = g(\pi_B(x_B))$. They are therefore compatible. For the changed maps, membership of the successor pair in $\mathcal{R}_\pi$ is exactly the stated equality. This proves necessity and sufficiency for persistence of the relation, without requiring a common factor map on every point outside the relation. □

Proposition 11.2 distinguishes two obligations: preserving a relation of agreement and realizing a particular shared dynamics. The former can hold even if the latter changes. A treaty can continue to coordinate conduct while its interpretation evolves; conversely, a familiar text can remain in force while the operations it is supposed to coordinate cease to fit together.

For example, let $X_i = \{0, 1\} \times D_i$ for nonempty sets D_i, and let π_i select the first coordinate. Take $f_i(c, d) = (c, h_i(d))$ with arbitrary maps $h_i : D_i \to D_i$. The common reading c remains compatible while local differences develop. If only A later changes its update to $(c, d) \mapsto (1 - c, h_A(d))$, compatibility fails after one step from every previously compatible pair. No inference about hostile intent is needed to establish that formal failure.

The first coordinate is not intrinsically a commitment. An application must identify what it records, why both maps represent the same undertaking, and how their alignment is maintained. The example separates shared operational content from complete identity of the parties' dynamics. It does not claim that all diplomatic agreement has a deterministic factor representation.

11.4 Invariance obligations

The unitary-orbit construction in the GR diplomacy companion addresses another question: which features remain unchanged by a specified class of local redescriptions [20, pp. 13–16]. Its invariance claim is restricted to that transformation class. A general local channel, an alteration of authority or a change in admissible evidence need not preserve the same features.

This distinction matters politically. If an investigator declares a reinterpretation to be mere redescription, the resulting invariants will ignore whatever that transformation changes. The decision can be appropriate for coordinate changes and inappropriate for a change in the right to initiate review. Consequently, a symmetry assumption needs a preservation argument tied to the paper's analytical object. Mathematical elegance cannot establish that the ignored difference is socially irrelevant.

The three constructions now have separate roles. A continuation signature records future observational distinctions. A common factor supplies a compatible shared description for selected local dynamics. A group invariant identifies features preserved by declared redescriptions. None alone establishes legitimacy, mutual benefit or trust. Together they help specify what the generative and algebraic models must retain when an institution changes.

12 Generative Assurance Fields

The field model investigates a particular way that support channels can generate joint relational structure. It begins with group-domain fields and explicit strand contractions, establishes a stable interacting action, and explains how formal diagrams produce two-complexes. The model extends the generator architecture of prior GR manuscripts [13, 24]; its assurance interpretation and parameter choices are hypotheses of this paper.

12.1 Field domain

Choose a finite group G of order n, equipped with normalized counting measure. The rank-two field space is the following finite-dimensional Hilbert space.

$$\mathfrak{h} = \{U : G^2 \to \mathbb{C} : \; U(ih, jh) = U(i, j) \; \forall i, j, h \in G\}, \quad \langle U, V \rangle = n^{-2} \sum_{i,j} \overline{U_{ij}} V_{ij}. \tag{11}$$

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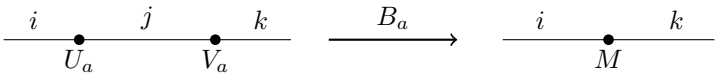

Figure 2: Boundary incidence of one cubic interaction. Dots represent couplings, strands represent elements, and the event joins the selected input coupling slots to a mediation slot. Both channel types can attach to the same field type M.

Equation (11) states simultaneous right invariance and fixes the norm $\|U\|^2 = \langle U, U\rangle$. The invariance concerns internal labels. It is not an assertion that every social reinterpretation is a gauge transformation. An application would need evidence that the selected relabeling preserves the target distinctions.

Let $U_a, V_a \in \mathfrak{h}$ for channel index $a \in \{1, 2\}$, and let $M \in \mathfrak{h}$ be a shared mediation field. These are coupling-mode fields, not individual actors or trust magnitudes. Define normalized composition and channel currents by the following contraction.

$$(U \star V)_{ik} = n^{-1} \sum_j U_{ij} V_{jk}, \qquad J_a = U_a \star V_a, \qquad J = g_1 J_1 + g_2 J_2. \tag{12}$$

Here $g_1, g_2 \in \mathbb{C}$ are declared coupling parameters. Equation (12) joins two coupling slots at a shared port. One hypothetical interpretation joins a departure report to a commitment and then joins that commitment to a response authority. The resulting composite channel can interact with a shared mediation practice. The interpretation requires matching versions, standing and semantic data in addition to the group contraction.

Proposition 12.1 (Invariant channel composition)**.** *Composition maps* $\mathfrak{h} \times \mathfrak{h}$ *into* $\mathfrak{h}$ *and satisfies* $\|U \star V\| \le \|U\|\|V\|$.

Proof. In $(U \star V)(ih, kh)$, substitute $j = j'h$ and use right invariance in both factors. This proves the output's invariance. As matrices, $U \star V = UV/n$ and $\|U\| = \|U\|_{\mathrm{F}}/n$, where $\|\cdot\|_{\mathrm{F}}$ is the Frobenius norm. The inequality $\|UV\|_{\mathrm{F}} \le \|U\|_{\mathrm{F}}\|V\|_{\mathrm{F}}$ then gives the stated bound after dividing by n^2. □

Proposition 12.1 ensures that the channel current stays within the declared domain and supplies the estimate required for stability. It does not prove the social compatibility of two practices bearing similar names.

12.2 Interaction kernel

The shared mediation coupling is the cubic polynomial obtained by attaching the outer ports of a channel to M. Its full index form is explicit.

$$B_a(M, U_a, V_a) = \langle M, U_a \star V_a\rangle = n^{-3} \sum_{i,j,k} \overline{M_{ik}} (U_a)_{ij} (V_a)_{jk}. \tag{13}$$

Equation (13) pairs the i strand between M and U_a, the j strand between U_a and V_a, and the k strand between V_a and M. This is an incidence-generating interaction. It is not merely a coefficient placed on an already chosen actor graph.

Figure 2 shows the elementary boundary move. Repeated contractions can create structures with different numbers of events and different attachments. The rank-two sector has ribbon combinatorics and is deliberately limited; no physical spacetime dimension follows from the number of field arguments.

12.3 Stable action

Write $r^2 = \sum_{a=1}^{2} (\|U_a\|^2 + \|V_a\|^2)$ and $m^2 = \|M\|^2$. Fix $k, \kappa, \epsilon, \lambda > 0$. The finite Euclidean action is the following dimensionless polynomial.

$$S = kr^2 + \kappa m^2 + \epsilon r^4 + \lambda m^4 - 2\,\mathrm{Re}\langle M, J\rangle. \tag{14}$$

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In Equation (14), the quadratic coefficients define a positive kinetic operator. Its inverse is the free covariance, preserving the convention of the later GR spacetime formalism [24]. The quartic terms are explicit stabilizing interactions. They also generate vertices if included in a diagram expansion; they have not been assigned a measured social-force interpretation.

Choose orthonormal coordinates in $\mathfrak{h}^5$, of complex dimension D, and use Lebesgue measure $d^{2D}z$. The partition integral is defined as follows.

$$Z = \int_{\mathbb{C}^D} e^{-S(z,\bar{z})}\, d^{2D}z. \tag{15}$$

Equation (15) is an actual finite-dimensional integral. Its existence does not establish the convergence of a perturbative diagram series or the empirical validity of its field distribution.

Proposition 12.2 (Finite interacting ensemble)**.** *For the stated positive parameters and finite couplings g_a, $0 < Z < \infty$, and every polynomial field moment is finite.*

Proof. Let $b = \frac{1}{2}\max_a |g_a|$. Proposition 12.1 and $xy \le (x^2+y^2)/2$ give $\|J\| \le br^2$. Thus the magnitude of the coupling is at most $2bmr^2$. Maximizing $2bmr^2 - (\lambda/2)m^4$ over $m \ge 0$ gives $Cr^{8/3}$ for a finite C. Hence $S \ge (\lambda/2)m^4 + \epsilon r^4 - Cr^{8/3} + kr^2 + \kappa m^2$. The r^4 term dominates $r^{8/3}$, so $S \ge (\lambda/2)m^4 + (\epsilon/2)r^4 - C'$ for some finite C'. This bound makes the integral and all polynomial moments finite. The integrand is strictly positive on every bounded open set, so $Z > 0$. □

Proposition 12.2 permits exact marginalization and moment calculations below. Appendix A gives the maximizing constant and the Gaussian-sector restriction. Finiteness is achieved relative to a finite group and finite type vocabulary. A continuum limit or an unrestricted sum over social distinctions remains unproved.

12.4 Generated complexes

Expand interaction exponentials formally about the positive quadratic part. Gaussian pairings join each field factor to a conjugate factor of a compatible type. The corresponding propagator carries its two strands, while Equation (13) determines their pairing at interaction events. Vertices become events, propagator edges become coupling histories, and strand paths or cycles supply element-history faces. Unpaired external slots define a boundary.

For a chosen boundary insertion, this organization has the following schematic form.

$$Z_\partial \sim \sum_{F \in \mathfrak{C}_\partial} \frac{\prod_\beta c_\beta^{n_\beta(F)}}{|\operatorname{Aut}_\partial F|} \mathcal{A}_F(K^{-1}, V_\beta). \tag{16}$$

In Equation (16), β runs over cubic, conjugate and stabilizing interaction types; c_β includes the sign and normalization from the exponential; $n_\beta(F)$ counts its events; K^{-1} is the free covariance; and V_β specifies its port pairings. The automorphism group fixes the boundary. The symbol $\sim$ marks a formal expansion. The positive measure on field coordinates does not imply positive individual diagram weights.

The semantic extraction requires additional work. Some diagrams are disconnected or have orientations unsuitable for the acyclic history convention. Selecting only histories that satisfy a causal or normative predicate changes the represented ensemble unless that restriction has already been encoded in the generator. It is therefore impermissible to retain the unrestricted integral while interpreting every diagram as a valid cooperative history.

The framework also distinguishes generation from ontology. Prior GR work separates representation, generative dynamics, historical criteria and observation [26]. Here the action generates a family of possible combinatorial attachments relative to chosen primitives. It does not prove that the primitives are final or that an institutional process literally performs a field integral.

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12.5 Nonlinear field equations

The stationary equations make the nonlinear interaction explicit. Let $U_{ij}^{\dagger} = \overline{U_{ji}}$; matrix adjoints preserve $\mathfrak{h}$. Differentiating the real action with respect to the conjugate field coordinates gives the following equations.

$$\begin{aligned}(\kappa + 2\lambda\|M\|^2)M &= J,\\ (k + 2\epsilon r^2)U_a &= \overline{g_a}\, M \star V_a^{\dagger},\\ (k + 2\epsilon r^2)V_a &= \overline{g_a}\, U_a^{\dagger} \star M.\end{aligned} \tag{17}$$

Equation (17) follows from the adjoint identities for normalized matrix multiplication. It couples each channel to the shared field and to the norms of all retained channels. It is not a linear response equation on a fixed interaction graph.

These stationary equations do not supply a social clock, select a stable historical trajectory or justify replacing ensemble moments by a stationary point. In a finite symmetric ensemble, a one-field mean can vanish while correlations remain nonzero. A kinetic social model would need additional transition, observation and calibration rules. The present generative dynamics concerns histories produced by the interaction structure and their formal weights, with stationary configurations providing only one possible diagnostic.

13 Shared Mediation

This section derives the effect of integrating out the shared field. Its objective is to identify the dependence retained between support channels after the mediator disappears from an explicit description. The calculation is exact at the finite integral level, followed by a controlled local expansion in a selected one-mode sector. It addresses representational reduction, not the physical removal of an institution.

13.1 Exact marginalization

Write $S_{\text{ret}} = kr^2 + \epsilon r^4$ and $S_M = \kappa\|M\|^2 + \lambda\|M\|^4$. Let $d = \dim_{\mathbb{C}} \mathfrak{h}$. Define the normalized uncoupled mediator measure and its source function as follows.

$$d\mu_M = Z_M^{-1} e^{-S_M}\, d^{2d}M, \qquad F(J) = \int e^{2\,\mathrm{Re}\langle M, J\rangle}\, d\mu_M. \tag{18}$$

Equation (18) gives $F(0) = 1$ and $F(J) > 0$. Its moments and derivatives exist for all finite J when $\lambda > 0$, by quartic tail domination. Integrating M in the full action yields a retained-field integral with an effective action.

$$Z = Z_M \int e^{-S_{\text{eff}}}\, dU_1\, dV_1\, dU_2\, dV_2, \qquad S_{\text{eff}} = S_{\text{ret}} - \log F(g_1 J_1 + g_2 J_2). \tag{19}$$

The notation $dU_a\, dV_a$ in Equation (19) means Lebesgue measure on their real and imaginary orthonormal coordinates. No mediator variable remains in this description, but its effects remain in the logarithm. The reduction integrates over possible mediator configurations; setting $M = 0$ or deleting the coupling would define a different model.

13.2 Shared source dependence

The measure in Equation (18) is invariant under unitary changes of the d field coordinates. Consequently F depends on $\|J\|$, and its Taylor expansion at zero has a quadratic leading term. Let $\chi = d^{-1}\mathbb{E}_{\mu_M}\|M\|^2 > 0$. Then the leading dependence is the following expression.

$$\log F(J) = \chi\|J\|^2 + O(\|J\|^4), \qquad \|J\|^2 = |g_1|^2\|J_1\|^2 + |g_2|^2\|J_2\|^2 + 2\,\mathrm{Re}(\overline{g_1} g_2 \langle J_1, J_2\rangle)\,. \tag{20}$$

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To obtain Equation (20), phase invariance removes linear terms and pairs of unconjugated coordinates. Unitary invariance gives $\mathbb{E}(M_\alpha \overline{M_\beta}) = \chi\delta_{\alpha\beta}$. The quadratic term of the exponential is therefore $\chi\|J\|^2$; taking the logarithm leaves it unchanged at that order. The remainder is local near zero, with parameters fixed.

The mixed term can be positive, negative or zero according to the selected mode relation. Its existence is the relevant result. Organizational separation of the two channels does not imply factorization after both couple to the same mediator. This is a model of a common support, not a theorem that every shared service produces the same dependence.

Proposition 13.1 (Independent mediator factorization)**.** *Suppose instead that channels couple separately to M_1, M_2 with product uncoupled measure $\mu_1 \otimes \mu_2$. Their integrated mediation contribution is* $\log F_1(g_1J_1) + \log F_2(g_2J_2)$. *A shared mediator generally does not yield this sum.*

Proof. With product measure, the integrand is a product of a function of M_1 and a function of M_2. Both factors are positive and integrable, so the integral is $F_1(g_1J_1)F_2(g_2J_2)$ and logarithms add. For a shared mediator with the invariant measure above, choose $g_1J_1 = g_2J_2 = u \neq 0$ small. Equation (20) gives $\log F(2u) - 2\log F(u) = 2\chi\|u\|^2 + O(\|u\|^4)$, which is nonzero for sufficiently small u. □

Proposition 13.1 concerns the mediation contribution only. In the present action, ϵr^4 already couples the retained channels. Even independent mediators therefore do not imply independence of the full retained-field distribution. This qualification is essential: isolating one shared support cannot certify independence of an entire arrangement.

13.3 Non-Gaussian interaction

A solvable sector displays an effect beyond the Gaussian contribution. Restrict the mediator to one normalized complex mode, write its coordinate as $m \in \mathbb{C}$ and the source as $j \in \mathbb{C}$, and take $\kappa = 0$, $\lambda > 0$ in that sector. This changes the selected mediator model while retaining a finite quartic integral. There is no Gaussian free covariance for this isolated choice; the following calculation is nonperturbative in its uncoupled measure.

Let $t = |m|^2$. Angular integration makes the normalized density of t proportional to $e^{-\lambda t^2}$ on $[0, \infty)$. Its moments are the following gamma-function ratios.

$$\mu_q := \mathbb{E}|m|^{2q} = \lambda^{-q/2}\,\frac{\Gamma((q+1)/2)}{\Gamma(1/2)}, \qquad q = 0, 1, 2, \ldots. \tag{21}$$

Equation (21) follows by substituting $s = \lambda t^2$ in the numerator and denominator of the radial integral. It gives $\mu_1 = 1/\sqrt{\pi\lambda}$ and $\mu_2 = 1/(2\lambda)$. Expanding both factors of $e^{\bar{m}j + m\bar{j}}$ and integrating the phase leaves only equal powers, yielding the source function and its local logarithm.

$$\begin{aligned} F(j) &= \sum_{q=0}^{\infty} \frac{\mu_q |j|^{2q}}{(q!)^2}, \\ \log F(j) &= \frac{|j|^2}{\sqrt{\pi\lambda}} + \frac{\pi - 4}{8\pi\lambda}|j|^4 + O(|j|^6). \end{aligned} \tag{22}$$

The series for F in Equation (22) is justified by the angular integral and nonnegative radial terms, with finiteness supplying convergence for every finite source. The logarithmic expansion is local. Its quartic coefficient is negative because $\pi < 4$; this is a statement about a cumulant coefficient, not about the sign of every derivative of $\log F$.

Proposition 13.2 (Residual coupling of orthogonal real directions)**.** *In the one-mode quartic sector, there exist arbitrarily small nonzero sources with vanishing quadratic interference and a nonzero mixed mediation contribution.*

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Proof. Take $j_1 = s$ and $j_2 = it$ for small nonzero real s, t. Then $\mathrm{Re}(\bar{j}_1 j_2) = 0$. Write $c = (\pi - 4)/(8\pi\lambda) \neq 0$. Equation (22) gives $\log F(j_1 + j_2) - \log F(j_1) - \log F(j_2) = 2cs^2t^2 + O((s^2 + t^2)^3)$. Along $s = t \to 0$ the leading coefficient is nonzero, establishing the claim. □

Proposition 13.2 shows why a quadratic reduction can miss a shared dependence. Since each j_a represents a selected component of the bilinear current $g_a(U_a \star V_a)$, the $|j|^4$ term is eighth degree in the retained field coordinates. The non-Gaussian mediator induces higher interactions rather than merely changing one pairwise coefficient.

The example is deliberately a selected field mode. Its scalar coordinate is an amplitude of that mode, not a scalarized trust relation. Neither the complex phase nor the coefficient is assigned an unverified social measurement. The result establishes a structural possibility that a calibrated model would need to identify.

13.4 Convexity

The source function also has a useful exact property. In real coordinates put $X = (\sqrt{2}\,\mathrm{Re}\,M, \sqrt{2}\,\mathrm{Im}\,M)$ and $y = (\sqrt{2}\,\mathrm{Re}\,J, \sqrt{2}\,\mathrm{Im}\,J)$. Their dot product is $2\,\mathrm{Re}\langle M, J\rangle$. Let μ_y be the normalized measure tilted by $e^{X\cdot y}$. Differentiation under the integral gives the following identities.

$$\nabla_y \log F = \mathbb{E}_{\mu_y} X, \qquad \nabla_y^2 \log F = \mathbb{E}_{\mu_y}(XX^{\mathsf{T}}) - \mathbb{E}_{\mu_y} X\,(\mathbb{E}_{\mu_y} X)^{\mathsf{T}}. \tag{23}$$

Equation (23) is a covariance matrix and is positive semidefinite. This follows because its quadratic form on a real vector v is the variance of $v \cdot X$. Quartic tail domination permits the required differentiations on compact source sets. Thus the exact $\log F$ is convex even though one coefficient in its local expansion is negative. Using that truncated polynomial outside its local range could falsely suggest otherwise.

13.5 Interpretive consequence

The calculation distinguishes three claims. A shared support creates a possible route for dependence. A specified interacting model determines particular induced terms. Empirical evidence is required to establish that those terms describe a concrete arrangement. The first claim motivates inquiry, the second is the mathematical result, and the third remains open.

For example, two review channels using a common evidence service may interact through capacity constraints, common classifications or access rules. The present field does not identify which of these mechanisms applies. It provides an architecture in which shared mediation can remain present after the service is hidden in a reduced description. A graph showing only the two reviewers could therefore omit a consequential interaction.

The philosophical consequence concerns assurance claims. An arrangement cannot warrant independence merely by presenting its supports separately. It must investigate what the separation preserves, what shared dependencies remain and whether a change in those dependencies affects the cooperative criteria. Removing the mediator from a diagram is not the same action as removing the dependency from the relation.

14 Institutional Kernel Fields

The preceding field model allows mediation to fluctuate while its coupling rule is fixed. This section makes that rule an interacting field. It follows the GR distinction between a history ensemble, a complex-generating action and a semantic realization [24, pp. 35–37]. The objective is to investigate a possible representation of institutions whose mediating practices and coupling rules condition each other. This is a second model with its own action; it does not silently change the assumptions behind the earlier marginalization results.

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14.1 Kernel domain

Retain the finite group G, invariant Hilbert space $\mathfrak{h}$, fields U_a, V_a, M and current J from Equations (11)–(12). Let $d = \dim_{\mathbb{C}} \mathfrak{h}$. Introduce an operator-valued field $\mathsf{N} \in \mathrm{End}(\mathfrak{h})$, with Hilbert–Schmidt inner product $\langle A, B\rangle_{\mathrm{HS}} = \mathrm{Tr}(A^\dagger B)$. It is a coupling kernel, distinct from the semantic nomos N_Σ in Equation (3).

Equivalently, represent N by a four-argument kernel, separately right invariant in its two pairs. Its action and norm are fixed by normalized counting measure.

$$\begin{aligned} (\mathsf{N}W)_{ij} &= n^{-2} \sum_{k,l} \mathsf{N}_{ij;kl} W_{kl}, \\ \mathsf{N}_{ih,jh;ks,ls} &= \mathsf{N}_{ij;kl} \quad (h, s \in G), \\ \|\mathsf{N}\|_{\mathrm{HS}}^2 &= n^{-4} \sum_{i,j,k,l} |\mathsf{N}_{ij;kl}|^2. \end{aligned} \tag{24}$$

Equation (24) supplies a unique separately invariant kernel for each endomorphism of $\mathfrak{h}$. To see the normalization, choose the orbit basis indexed by ij^{-1}, with value $\sqrt{n}$ on its orbit. This basis is orthonormal; the operator matrix is the corresponding orbit-kernel matrix divided by n. Its squared Hilbert–Schmidt norm is therefore the last expression in Equation (24).

The kernel can represent how a channel current becomes available to a mediation practice: which reported differences enter, how several modes are combined, and what information remains effective. These are candidate interpretations. A complex matrix element is not a degree of legitimacy, a legal permission or a probability. Connecting N to a particular nomos requires a realization specifying those distinctions.

14.2 Coupled action

Set $s^2 = \|\mathsf{N}\|_{\mathrm{HS}}^2$, retain $r^2 = \sum_a (\|U_a\|^2 + \|V_a\|^2)$, and choose $k, \epsilon, \kappa, \lambda, \nu, \tau > 0$. The following dimensionless action penalizes a mismatch between mediation and the institutionally routed current.

$$S_{\mathrm{I}} = kr^2 + \epsilon r^4 + \kappa \|M - \mathsf{N}J\|^2 + \lambda \|M\|^4 + \nu s^2 + \tau s^4. \tag{25}$$

Equation (25) is a modeling proposal. Its terms impose a tractable positive ensemble; they do not express an empirically established tendency of institutions to minimize a social energy. The action concerns possible generative configurations and their weights. It supplies no calendar-time trajectory.

The square contains both a coupling term and its stabilizing completion.

$$\kappa \|M - \mathsf{N}J\|^2 = \kappa \|M\|^2 - 2\kappa \,\mathrm{Re}\langle M, \mathsf{N}J\rangle + \kappa \|\mathsf{N}J\|^2. \tag{26}$$

Equation (26) exposes a four-field vertex involving M, N, U_a, V_a, and six-field terms in the final square. Freezing N to the identity does not recover Equation (14): the coefficient and the positive $\|J\|^2$ term differ. Results from the two models must therefore retain their model labels.

In orthonormal coordinates on $\mathfrak{h}^5 \oplus \mathrm{End}(\mathfrak{h})$, let $d\mu_{\mathrm{I}}$ be Lebesgue measure and define the partition integral.

$$Z_{\mathrm{I}} = \int e^{-S_{\mathrm{I}}} \, d\mu_{\mathrm{I}}, \qquad \langle F\rangle_{\mathrm{I}} = Z_{\mathrm{I}}^{-1} \int F e^{-S_{\mathrm{I}}} \, d\mu_{\mathrm{I}}. \tag{27}$$

Equation (27) introduces expectations only for the declared field ensemble. They become empirical expectations only after an independently supported measurement and sampling interpretation.

Proposition 14.1 (Finite institutional field ensemble)**.** *The integral Z_{I} is positive and finite, and all polynomial moments exist.*

Proof. All terms in Equation (25) are nonnegative. In particular, $S_{\mathrm{I}} \geq \epsilon r^4 + \lambda \|M\|^4 + \tau s^4$. In finite-dimensional orthonormal coordinates, the exponential of the negative right-hand side is integrable, including after multiplication by any polynomial. This bounds the required integrals. Positivity on every bounded open set gives $Z_{\mathrm{I}} > 0$. □

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Proposition 14.1 justifies differentiation identities and marginalization in this finite model. It does not establish a continuum limit, an infinite type vocabulary or convergence of the diagram expansion.

14.3 Nonlinear reciprocal equations

Write $D = \mathsf{N}J - M$. For vectors $D, J \in \mathfrak{h}$, the rank-one endomorphism $DJ^\dagger$ sends W to $D\langle J, W\rangle$. Differentiation in orthonormal complex coordinates gives the following stationary equations.

$$\begin{aligned} 0 &= \kappa(M - \mathsf{N}J) + 2\lambda\|M\|^2 M, \\ 0 &= (\nu + 2\tau s^2)\mathsf{N} + \kappa DJ^\dagger, \\ 0 &= (k + 2\epsilon r^2)U_a + \kappa\overline{g_a}\,(\mathsf{N}^\dagger D) \star V_a^\dagger, \\ 0 &= (k + 2\epsilon r^2)V_a + \kappa\overline{g_a}\,U_a^\dagger \star (\mathsf{N}^\dagger D). \end{aligned} \tag{28}$$

Equation (28) follows by differentiating the squared norm and using the adjoints of normalized multiplication. For example, varying N changes D by $(\delta\mathsf{N})J$, whose adjoint derivative is $DJ^\dagger$. Varying U_a changes J by $g_a\,\delta U_a \star V_a$, producing the third line. Appendix C supplies the directional-derivative check.

The reciprocal structure is substantive within the model. N changes how currents affect mediation, while the current and mediation together enter the equation for N. This extends the earlier model's fluctuating mediator to a fluctuating rule of mediation. Nevertheless, a stationary point is not an account of institutional learning over time. The model requires no assumption that political actors optimize S_{I}, and none is inferred from these equations.

The zero configuration is always a stationary point because the action has no external source. Consequently the model does not demonstrate spontaneous emergence of a nonzero cooperative institution. Its nontrivial content lies in the interaction structure, correlations and generated complexes. Boundary insertions or sources could investigate selected nonzero configurations, but doing so would add a specified model choice.

14.4 Exact moment hierarchy

An exact ensemble equation differs from a stationary-field equation. Let N_α be an orthonormal coordinate of N, for $\alpha \in \{1, \ldots, d^2\}$, and let $(DJ^\dagger)_\alpha$ denote the matching coordinate. Integration by parts gives an identity for every α, β.

$$\begin{aligned} \delta_{\alpha\beta} = \nu\langle\overline{N}_\beta N_\alpha\rangle_{\mathrm{I}} + 2\tau\langle s^2\overline{N}_\beta N_\alpha\rangle_{\mathrm{I}} \\ + \kappa\langle\overline{N}_\beta (DJ^\dagger)_\alpha\rangle_{\mathrm{I}}. \end{aligned} \tag{29}$$

To derive Equation (29), integrate $\partial_{\overline{N}_\alpha}(\overline{N}_\beta e^{-S_{\mathrm{I}}})$. The boundary term vanishes by the quartic lower bound in Proposition 14.1. The derivative of $\overline{N}_\beta$ is $\delta_{\alpha\beta}$, and the derivative of the action is the second line of Equation (28). Rearranging yields the displayed identity.

Equation (29) relates second moments to fourth and mixed moments. Replacing it by an equation for $\langle\mathsf{N}\rangle_{\mathrm{I}}$ alone would require a closure approximation. Such an approximation could erase the joint variation through which two apparently stable channels depend on a common institutional rule. The field model's additional content should be sought in these coupled quantities, rather than in a scalar index of cooperative confidence.

Even averaging a fixed quadratic mismatch illustrates the distinction. For fixed M, J and any probability law of N with finite second moment, put $\overline{\mathsf{N}} = \mathbb{E}[\mathsf{N}]$. Direct expansion gives the following identity.

$$\mathbb{E}\|M - \mathsf{N}J\|^2 = \|M - \overline{\mathsf{N}}J\|^2 + \mathbb{E}\|(\mathsf{N} - \overline{\mathsf{N}})J\|^2. \tag{30}$$

The cross term in Equation (30) vanishes because the centered kernel has zero mean. Equal mean kernels therefore need not yield equal mismatch expectations. Moreover, a marginalized effective action uses a logarithm of an exponential integral, not either side of this averaged-action identity. These are distinct operations with distinct approximations.

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14.5 Generated institutional incidence

The kernel field adds combinatorial structure to the generator. Its propagator carries four group strands, while U_a, V_a, M retain their paired strands. The mixed term in Equation (26) attaches a channel current through a kernel instance to a mediation instance. Pairing compatible field factors across repeated interaction events can produce histories with different numbers and arrangements of these attachments.

As in Section 12, the vertices, propagators and strand continuations determine a two-complex in the declared combinatorial sector. This is a change in the family of generated incidence structures, not merely a changed coefficient on one predetermined actor graph. The representation still uses a finite primitive vocabulary and fixed field arities. It does not generate every possible institutional category or establish the emergence of its own group domain.

The corresponding social reading requires more than naming the new field institutional. An application would have to identify events that produce or alter a rule of mediation, specify which records retain that change, and show how later actions depend on it. Historical and normative admissibility gates would then have to be built into the chosen kernels or imposed through a separately justified realization. The unrestricted integral does not certify such gates.

The algebra in the next sections treats that missing preservation problem explicitly. It represents institutional transformations as maps among history-bearing sectors and asks which observables and usable operations survive. This supplies a conditional interface between generative structure and cooperative interpretation. It is not a derivation of the entire algebra from the present action.

14.6 Redescription symmetry

A restricted symmetry makes the representational status explicit. Let $\Theta : \mathfrak{h} \to \mathfrak{h}$ be unitary and preserve convolution: $\Theta(U \star V) = (\Theta U) \star (\Theta V)$. Transform every channel field and M by Θ, and transform the kernel by conjugation.

$$U_a \mapsto \Theta U_a, \quad V_a \mapsto \Theta V_a, \quad M \mapsto \Theta M, \quad \mathsf{N} \mapsto \Theta \mathsf{N} \Theta^{-1}. \tag{31}$$

Equation (31) includes relabelings induced by group automorphisms of G, acting simultaneously on both arguments. A general unitary on $\mathfrak{h}$ need not preserve convolution and is not automatically an allowed symmetry.

Proposition 14.2 (Kernel redescription invariance)**.** *Under the stated assumptions on Θ, the transformation in Equation* (31) *preserves S_{I}, Lebesgue measure and Z_{I}.*

Proof. The current transforms as $J \mapsto \Theta J$. Therefore $M - \mathsf{N}J$ transforms as $\Theta(M - \mathsf{N}J)$ and retains its norm. All field norms and the Hilbert–Schmidt norm of the conjugated kernel are unchanged. Every term of the action is preserved. The coordinate transformations are unitary, so their real Jacobians have absolute value one. Changing variables in the integral proves invariance of the partition function. □

Proposition 14.2 concerns a declared redescription class. An institutional act that removes access to evidence or changes the set of authorized participants need not belong to it. Treating such an act as a gauge change would assume away precisely the political transformation this paper seeks to investigate.

15 Preservation Algebras

The field calculation concerns generated dependence. The present section asks which operations preserve a declared admissibility condition when composed. It constructs operators from complete historical incidence data and states their domain and closure assumptions. It then distinguishes preservation, compression and physical intervention. This is an operator-algebra construction within a specified linear representation; order sensitivity alone would not supply that structure.

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15.1 Boundary replacements

Choose a countable set $\mathcal{B}$ of finite typed configurations of Equation (3), with countable semantic labels and explicit incidence. If an application requires uncountable labels, the following counting representation must be replaced or discretized with a preservation argument. Let the Hilbert space and its dense finite-support domain be the following spaces.

$$\mathcal{H} = \ell^2(\mathcal{B}), \qquad \mathcal{D} = \operatorname{span}_{\text{finite}}\{|b\rangle : b \in \mathcal{B}\}. \tag{32}$$

The basis vector $|b\rangle$ in Equation (32) represents a whole incidence-complete configuration at the chosen resolution. It is not one person or an occupation number stripped of attachments. The scalar field of the representation does not imply a quantum social state.

A replacement rule r lists permissible matches of a typed boundary fragment and supplies a finite continuation for each match, retaining the inherited history. Let $\mathcal{M}_r(b)$ be the finite match set and $a_r(b,\mu) \in \mathbb{C}$ an assigned coefficient. Its linear action is defined on basis vectors as follows.

$$R_r|b\rangle = \sum_{\mu \in \mathcal{M}_r(b)} a_r(b,\mu)|r_\mu b\rangle. \tag{33}$$

Equation (33) maps $\mathcal{D}$ into itself. The coefficients may be formal weights or matrix elements; they are not transition probabilities unless an additional stochastic interpretation is supplied. If reverse incidence is locally finite too, the matrix adjoint also maps $\mathcal{D}$ into itself. Neither condition makes the operator bounded on $\mathcal{H}$.

Composition can create or remove active coupling slots and change the incidence of the boundary. It thus acts on a family of generated structures. The formal linear carrier $\ell^2(\mathcal{B})$ is fixed by the investigator, while the structures represented by its basis vectors can vary. This distinction avoids identifying a convenient ambient vector space with a permanently fixed social organization.

15.2 Admissibility projection

Restrict the declared family A of Equation (5) to $\mathcal{B}$. Let P project onto its basis vectors and let $Q = I - P$.

$$P|b\rangle = \mathbf{1}_A(b)|b\rangle, \qquad Q = I - P, \qquad \mathcal{A}_P = \{T \in \mathcal{B}(\mathcal{H}) : QTP = 0\}. \tag{34}$$

In Equation (34), $\mathcal{B}(\mathcal{H})$ denotes bounded linear operators, distinct from the configuration set $\mathcal{B}$. The condition $QTP = 0$ says that T maps the admissible subspace into itself. It does not say that T is executable, norm preserving, positive on probabilities or a successful completion of cooperation.

Proposition 15.1 (Preservation algebra)**.** *The set $\mathcal{A}_P$ is a norm-closed unital subalgebra of $\mathcal{B}(\mathcal{H})$.*

Proof. The identity satisfies $QIP = 0$. The condition is linear, so sums and scalar multiples preserve it. If $S, T \in \mathcal{A}_P$, then $SP = PSP$ and $TP = PTP$, hence $QTSP = QTPSP = 0$. Finally, if $T_n \to T$ in operator norm, then $\|Q(T - T_n)P\| \le \|T - T_n\| \to 0$, proving $QTP = 0$ when each $QT_nP = 0$. □

Proposition 15.1 gives an exact compositional result for the chosen representation. Every intermediate admissible vector is covered. By contrast, checking each operator on a single initial vector establishes no such closure.

The block form makes the limitation transparent.

$$\mathcal{H} = P\mathcal{H} \oplus Q\mathcal{H}, \qquad T \in \mathcal{A}_P \iff T = \begin{pmatrix} T_{AA} & T_{AQ} \\ 0 & T_{QQ} \end{pmatrix}. \tag{35}$$

Equation (35) permits an inadmissible input to enter the admissible sector. Whether this represents repair depends on what the criterion records. A criterion requiring an entirely breach-free history cannot be restored by merely changing a current label; a criterion concerning presently available recourse can be restored through a later intervention.

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Proposition 15.2 (Adjoint closure)**.** *The preservation algebra need not be closed under adjoints. Its intersection with its adjoint algebra is the commutant of P, a norm-closed unital star subalgebra.*

Proof. In Equation (35), a nonzero T_{AQ} becomes a nonzero lower-left block in $T^\dagger$, violating preservation. Both T and $T^\dagger$ preserve exactly when both off-diagonal blocks vanish, equivalently $TP = PT$. Operators commuting with a projection form a norm-closed unital algebra closed under adjoints. □

Proposition 15.2 is also a historical warning. The algebraic adjoint of a forward operation is not automatically an executable historical reversal. Treating both as available can introduce a backward move that the arrangement has no power to perform.

15.3 Local tests

A three-state counterexample separates initial testing from compositional preservation. Take an orthonormal basis e_0, e_1, e_2, let P project onto span$\{e_0, e_1\}$, and define the following two bounded operators.

$$R_1 = |e_1\rangle\langle e_0|, \qquad R_2 = |e_0\rangle\langle e_0| + |e_2\rangle\langle e_1|, \qquad R_2 R_1 e_0 = e_2. \tag{36}$$

Equation (36) gives admissible outputs for both separate tests on e_0: $R_1 e_0 = e_1$ and $R_2 e_0 = e_0$. Their composition leaves the admissible sector. The missing test concerns R_2 on the state produced by R_1.

One interpretation is that preparation changes a record status, after which release bypasses a safeguard that was operative before preparation. The example is not a model of an entire institution. It proves that the weaker testing procedure cannot establish the advertised closure even in a finite representation.

15.4 Compression

Given an arbitrary bounded R, the compressed operator PRP always preserves the selected sector. That fact must not be mistaken for an implemented safeguard. The distinction is exact.

$$RP = PRP + QRP, \qquad PRP = RP \iff QRP = 0. \tag{37}$$

Equation (37) shows what compression removes: the inadmissible output component from admissible inputs. If such a component exists, replacing R by PRP changes the operation. In a probabilistic interpretation it might discard failed runs; in another it might represent a real gate that prevents them. The matrix alone does not decide which interpretation is justified.

Sequential compression can also differ from checking only the end of a sequence. Inserting P after each operation excludes intermediate departures that a final P could hide after apparent recovery. A historically sensitive admissibility predicate can represent the distinction. If the basis records only present status, the algebra cannot recover an erased breach.

15.5 Approximate preservation

Exact preservation is often too strong for a chosen approximate representation. Define an operator-norm leakage $\delta(T) = \|QTP\|$. This is a mathematical approximation measure, not a measured risk or a trust score. Products satisfy the following bound.

$$\delta(TS) \le \delta(T)\|S\| + \|T\|\delta(S). \tag{38}$$

To derive Equation (38), insert $I = P + Q$ between T and S. Then $QTSP = QTPSP + QTQSP$, and the triangle inequality and submultiplicativity give the bound. If $R_1, \ldots, R_N$ are contractions, induction yields a cumulative estimate.

$$\delta(R_N \cdots R_1) \le \sum_{j=1}^{N} \delta(R_j). \tag{39}$$

Equation (39) can become uninformative for long sequences. It is an upper bound, not a theorem that every local error accumulates monotonically. Cancellation or stronger structure can improve it; a meaningful application must specify why its norm controls a consequence of interest.

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15.6 Observable effects

For a bounded contraction R and an effect $0 \le E \le I$, the pulled-back effect is well defined.

$$E' = R^\dagger E R, \qquad 0 \le E' \le I. \tag{40}$$

Equation (40) follows by evaluating its quadratic form: $\langle x, E'x\rangle = \langle Rx, ERx\rangle$ lies between zero and $\|Rx\|^2 \le \|x\|^2$. For a general bounded non-contraction, positivity remains but the upper bound becomes $\|R\|^2 I$. A probability interpretation therefore requires normalization conditions beyond algebraic composition.

This bounded observable layer is compatible with prior GR use of boundary algebras [25]. It is deliberately separate from the unbounded replacement operators on $\mathcal{D}$. A finite truncation or bounded realization can connect them only if it preserves the relevant incidence and admissibility distinctions. Numerical convenience supplies no automatic bridge.

15.7 History realization

A connection to the field generator requires an explicit realization of its typed events as boundary replacements. For a finite cutoff and a selected chronological diagram, suppose that its semantic extraction yields a word $R_{\beta_N} \cdots R_{\beta_1}$ on the history-bearing basis. Internal sums must use the same label and gluing conventions in both constructions. This is a stronger assumption than recognizing similar diagrams.

Proposition 15.3 (Finite realized-history preservation)**.** *Suppose each realized event operator in a finite family belongs to $\mathcal{A}_P$. Every finite chronological word and every finite linear combination of these words preserves $P\mathcal{H}$.*

Proof. Apply multiplicative closure in Proposition 15.1 repeatedly to each word, then apply linear closure to the finite sum. □

Proposition 15.3 supplies a conditional bridge. It does not establish that the unrestricted field action realizes those operators. The action in Section 12 investigates mediation couplings without encoding the full historical admissibility predicate. Adding history-dependent gates to every event kernel changes the generator and must be treated as a new model.

A concrete gate illustrates the distinction. Suppose a release event can match only a configuration containing a retained item, a certificate for that item's current specification, and the relevant participant's authorization. Its output appends the release and preserves the linked evidence. Let A require that every represented release have those antecedents in its recorded history. This gated event preserves A provided that other events retain the antecedents and cannot overwrite versions. Deleting a historical certificate or permitting release after a specification change can violate the condition. The guarantee belongs to that entire rule family, not to the word *certificate*.

The algebra establishes one part of assurance composition. It does not provide evidence that a proposed operation can be invoked by a participant, that its semantic predicate is justified, or that it reaches a valuable outcome. The next model isolates those operational questions.

16 Institutional Transformation Algebras

The preservation algebra of Section 15 holds its admissibility projection fixed. An institution can change the representation of a commitment and the operations through which participants exercise it. This section constructs history-indexed representations, derives a transport condition and separates preserved commitment readings from preserved practical capacities. It continues the GR use of history-bearing boundary spaces and operator algebras while treating the social interpretation of each operator as an additional obligation.

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16.1 Institutional history sectors

Let $\mathscr{N}$ be a countable set of institutional history types. A type includes a specified rule configuration and its provenance; it need not identify a country, organization or political regime. For each $\eta \in \mathscr{N}$, choose a countable set $\mathcal{B}_\eta$ of incidence-complete history configurations and let $\mathcal{H}_\eta = \ell^2(\mathcal{B}_\eta)$. The spaces can have different dimensions and different active boundaries.

To represent selected commitment observables, choose a finite-dimensional unital star algebra $\mathfrak{C} = \bigoplus_{j=1}^{q} M_{d_j}(\mathbb{C})$, with its usual operator norm, and unital star representations $\pi_\eta : \mathfrak{C} \to \mathcal{B}(\mathcal{H}_\eta)$. Assume $\|\pi_\eta(c)\| \le \|c\|$. The common algebra records the distinctions being transported; its existence is a modeling premise, not a theorem that political commitments share a universal language.

The global history representation and the associated commutant are defined as follows.

$$\mathcal{H}_{\mathscr{N}} = \bigoplus_{\eta \in \mathscr{N}} \mathcal{H}_\eta, \qquad \Pi(c) = \bigoplus_{\eta \in \mathscr{N}} \pi_\eta(c), \qquad \mathfrak{T} = \Pi(\mathfrak{C})'. \tag{41}$$

In Equation (41), the prime denotes bounded operators commuting with every represented element of $\mathfrak{C}$. The norm assumption ensures that $\Pi(c)$ is bounded on the direct sum. A basis vector still denotes a whole history configuration, rather than a participant stripped of its relations.

A bounded block $T_{\eta'\eta} : \mathcal{H}_\eta \to \mathcal{H}_{\eta'}$ transports the selected commitment algebra when it satisfies the intertwining condition.

$$\pi_{\eta'}(c) T_{\eta'\eta} = T_{\eta'\eta} \pi_\eta(c) \qquad (c \in \mathfrak{C}). \tag{42}$$

Equation (42) compares an observable before and after institutional transformation. It permits different carriers and different representations. A same-named document or office supplies no automatic intertwiner.

Proposition 16.1 (Commitment transport algebra)**.** *The set $\mathfrak{T}$ is a norm-closed unital star subalgebra of $\mathcal{B}(\mathcal{H}_{\mathscr{N}})$. A bounded global operator belongs to it exactly when each block satisfies Equation* (42)*. Composable intertwining blocks remain intertwiners.*

Proof. Commutation is preserved under linear combinations, products and norm limits. It contains the identity. If T commutes with every $\Pi(c)$, apply this to $c^\dagger$ and take adjoints to obtain commutation for $T^\dagger$. Blockwise multiplication by the diagonal operator $\Pi(c)$ gives precisely Equation (42). Finally, for $T : \mathcal{H}_\eta \to \mathcal{H}_{\eta'}$ and $S : \mathcal{H}_{\eta'} \to \mathcal{H}_{\eta''}$, the two intertwining equalities yield $\pi_{\eta''}(c)ST = S\pi_{\eta'}(c)T = ST\pi_\eta(c)$. □

Proposition 16.1 establishes an algebraic preservation property. It does not imply that every member of $\mathfrak{T}$ is an executable operation. In particular, the algebraic adjoint of a historical extension is not automatically a politically or physically available reversal.

16.2 History embeddings

Suppose an institutional transformation is represented by an isometry $V : \mathcal{H}_\eta \to \mathcal{H}_{\eta'}$ that retains the old history while adding new structure. Isometry is a strong assumption: irreversible loss of a represented distinction requires another model. If V intertwines the representations, the corresponding observable compression has an exact form.

$$V^\dagger \pi_{\eta'}(c) V = \pi_\eta(c) \qquad (c \in \mathfrak{C}). \tag{43}$$

Equation (43) follows from Equation (42) and $V^\dagger V = I$. It expresses preservation of selected readings along the embedding. The target can contain additional distinctions not in the source. This is consistent with the GR distinction between enlargement of a representational space and change in an already selected invariant [20, p. 35].

If an admissibility projection is itself represented by a projection $p \in \mathfrak{C}$, so that $P_\eta = \pi_\eta(p)$, an intertwiner sends its admissible sector into the corresponding target sector. This connects the two operator

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constructions. The additional premise matters: a historically relevant criterion omitted from $\mathfrak{C}$ is not preserved merely because all its chosen observables are.

There are also transformations for which no adequate common $\mathfrak{C}$ has yet been constructed. Participants may dispute whether a new obligation continues an old one. Declaring an intertwiner in that case would settle the contested issue by stipulation. The model instead makes the obligation visible: identify a justified correspondence, preserve alternative correspondences or acknowledge that commitment continuity remains unresolved.

16.3 Practical operation transport

For participant i, let $\mathscr{R}_i(\eta)$ be a declared set of bounded operators on $\mathcal{H}_\eta$ representing operations available to that participant in the scoped model. Availability includes authorization, access and relevant material conditions. It must not be inferred merely because an operator belongs to a generated algebra. Assume that the identity and the compositions being used are available; other products may require an enlarged operational model.

Relative to an isometry V, define preservation of the participant's selected repertoire by an implementation condition.

$$\text{For every } R \in \mathscr{R}_i(\eta) \text{ there exists } R' \in \mathscr{R}_i(\eta') \text{ such that } R'V = VR. \tag{44}$$

Equation (44) states that an old operation can be reproduced on embedded histories by an available new operation. It imposes no obligation on the complement of $V\mathcal{H}_\eta$. It also does not assert that every old capacity deserves preservation: the selected repertoire must be normatively and analytically justified.

Proposition 16.2 (Composable capacity transport)**.** *Suppose Equation* (44) *holds along $V : \mathcal{H}_\eta \to \mathcal{H}_{\eta'}$ and along $W : \mathcal{H}_{\eta'} \to \mathcal{H}_{\eta''}$ for the relevant repertoires. Then it holds along WV. Within one transport, available implementations of individual operations implement their available finite compositions.*

Proof. For an old R, choose R' with $R'V = VR$, then R'' with $R''W = WR'$. Thus $R''WV = WR'V = WVR$. For two old operations and chosen implementations, $R_2'R_1'V = R_2'VR_1 = VR_2R_1$. Induction gives the finite-composition statement whenever the stipulated products are available. □

Proposition 16.2 supplies a structural comparison rather than a scalar capacity measure. Two institutions can each preserve some operations the other lacks. The relation need not yield a total ranking. Nor does algebraic implementability establish acceptable cost or timing unless those constraints have been included in the selected histories and operation sets.

The political reading is specific. A treaty amendment might preserve the substance of a claim while changing who can initiate its review. An institutional successor might retain all records while removing a participant's permission to access them. In either case, Equation (43) can be an adequate description of one preserved feature while Equation (44) fails. The two questions should be investigated separately.

16.4 Selective Capacity Loss

A finite example proves that commitment transport does not entail capacity transport. Use $\mathcal{H}_\eta = \mathcal{H}_{\eta'} = \mathbb{C}^2 \otimes \mathbb{C}^2$ and $V = I$. Let $\mathfrak{C} = \mathbb{C} \oplus \mathbb{C}$, represented identically in both sectors by the following diagonal operators.

$$\pi_\eta(c_0, c_1) = \pi_{\eta'}(c_0, c_1) = \begin{pmatrix} c_0 & 0 \\ 0 & c_1 \end{pmatrix} \otimes I_2, \qquad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{45}$$

In Equation (45), the first factor records a selected commitment classification. The second records two distinct procedural configurations. It is not an unobserved scalar amount of agency.

Give a participant two available old operations, but only the identity after the transformation.

$$\mathscr{R}_i(\eta) = \{I_4, I_2 \otimes X\}, \qquad \mathscr{R}_i(\eta') = \{I_4\}. \tag{46}$$

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The sets in Equation (46) are closed under composition. V intertwines all commitment observables, and every such reading is preserved. However, the old operation $I_2 \otimes X$ has no implementation in the target repertoire: with $V = I_4$, implementation would require $I_4 = I_2 \otimes X$, which is false.

The example can be interpreted as a recorded obligation surviving a withdrawal of a procedural power, provided an application supplies that interpretation. Its mathematical result is narrower and exact: commitment-preserving representation data alone cannot determine operation preservation. Increasing the dimension of the target would not repair the missing authorization automatically. Capacity depends on implemented operations, not only on available coordinates.

16.5 Observable compression

An operator-algebra model can also distinguish preserving selected observations from preserving all distinctions. In the same four-dimensional space, let $Q_0 = I_2 \otimes |0\rangle\langle 0|$ and $Q_1 = I_2 \otimes |1\rangle\langle 1|$. Define a map on the full matrix algebra.

$$\mathcal{E}(A) = Q_0 A Q_0 + Q_1 A Q_1, \qquad A \in M_4(\mathbb{C}). \tag{47}$$

Equation (47) retains block-diagonal observables. It provides a distinct model of representational compression; it is not identified with the withdrawal of an executable operation in Equation (46).

Proposition 16.3 (Selective algebraic preservation)**.** *The map $\mathcal{E}$ is unital, completely positive and idempotent. It fixes every $\pi_\eta(c)$ but sends $I_2 \otimes X$ to zero.*

Proof. The projections are orthogonal and sum to I_4, which proves unitality and idempotence. For every auxiliary dimension, the amplified map is a sum of compressions by $I \otimes Q_j$, so it preserves positive matrices; this is complete positivity. Every $\pi_\eta(c)$ commutes with the Q_j, giving $\mathcal{E}(\pi_\eta(c)) = \pi_\eta(c)$. Finally, $Q_j(I_2 \otimes X)Q_j = 0$ for each j because X exchanges the two procedural basis vectors. □

Proposition 16.3 shows that exact preservation of a chosen subalgebra can coexist with loss of other observables. It does not imply that international actors possess quantum states. The matrix representation is one possible formal tool for separating preserved tests from erased distinctions. A classical record model may suffice when its own continuation structure captures the intended question.

16.6 Generative realization

The institutional kernel field and the transport algebra address related but distinct problems. A field action supplies weighted generative structure; the algebra supplies conditions on represented operations. A realization would assign each admitted generated event to a block operator, each boundary to a history sector, and each selected commitment test to an element of the represented algebra. Its contractions would have to agree with the chosen boundary gluing.

Under that realization, Propositions 16.1 and 16.2 can be applied event by event and then composed. They do not prove that the unrestricted action in Equation (25) realizes the conditions. A kernel that permits an event removing a protected review power fails the relevant capacity requirement even if its field integral is finite and its commitment reading remains invariant.

This separation is useful for IR inquiry. It permits an arrangement to be assessed as strategically enforceable, administratively functional and commitment-preserving while leaving open whether it reproduces participants' capacities for meaningful contestation. Each assessment has its own evidence and normative premises. GR's contribution here is the proposed coordination of those assessments through explicit historical and operational structures, rather than a general assertion that relationality guarantees good cooperation.

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17 Practical Recourse

The existence of a remedy is a statement about a possible response; its usability concerns a participant's actual position within a history. This section gives a finite operational model to separate those claims. It is a restricted projection of the spacetime account, useful when a finite state description preserves the distinctions needed for a particular response. It does not replace the generative field or provide a complete theory of institutional rights.

17.1 Operational projection

Let X be a finite set of states recording the relevant history, active powers, remaining time and resource availability. An extraction $\pi : \Omega_0 \to X$ is defined on a selected family $\Omega_0 \subseteq \Omega$. Its adequacy requires that histories assigned the same state support the same response distinctions at the chosen resolution. Proposition 10.2 gives a necessary criterion for preserving a binary assessment; preserving actions and their consequences requires additional conditions.

For each $x \in X$, let $\mathcal{U}(x)$ be the participant's available actions. Each action has a nonempty set $\mathrm{Succ}(x,u) \subseteq X$ of modeled possible outcomes. The participant chooses u; an adverse environment may choose any listed successor. This represents robust controllability within the declared possibilities. It does not assert that every listed event has equal probability or that an actual institution is adversarial.

Let $S \subseteq X$ be an admissible operational region and $D \subseteq S$ the selected remedial target. A target could restore an item, stop a pending release or make an alternative available. The model stops when D is reached. The initial winning set and its recursion are defined as follows.

$$\begin{aligned} W_0 &= D, \\ W_{n+1} &= W_n \cup \{x \in S : \exists u \in \mathcal{U}(x),\ \mathrm{Succ}(x,u) \subseteq W_n\}. \end{aligned} \tag{48}$$

Equation (48) asks for a choice whose every modeled outcome remains on a route to the target. An empty action set contributes nothing. Nonempty successor sets exclude a vacuous universal condition that would incorrectly count an action with no executable result.

Proposition 17.1 (Bounded effective recourse)**.** *Under full observation of x, a state belongs to W_n exactly when the participant has a strategy that reaches D in at most n steps while remaining in S, for every sequence of modeled adverse choices.*

Proof. The assertion holds for $n = 0$ by definition. Suppose it holds for n. A state already in W_n has the required strategy. A newly included state has an action whose successors all belong to W_n; execute it and use the corresponding inductive strategy after observing the successor. Conversely, a strategy reaching the target within $n+1$ steps either starts in D or chooses an action from which every possible successor admits a strategy within n steps. Those successors belong to W_n, placing the state in W_{n+1}. All included states and successors lie in S. □

Proposition 17.1 is finite backward induction. Its significance is the quantifier order: one available choice must work against all modeled subsequent outcomes. A right written in a policy document supplies neither the action set nor the required inclusion by itself.

17.2 Nominal paths

For comparison, define an existential reachability family.

$$E_0 = D, \qquad E_{n+1} = E_n \cup \{x \in S : \exists u \in \mathcal{U}(x),\ \mathrm{Succ}(x,u) \cap E_n \neq \varnothing\}. \tag{49}$$

Equation (49) requires only one favorable successor. Induction gives $W_n \subseteq E_n$. Strict inclusion occurs when a participant can reach the target only if an uncontrollable delay or refusal does not occur.

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Consider a procedure with three mandatory stages: obtain an accessible record, file a recognized challenge and execute a remedial decision. Suppose the environment can impose at most one additional step of delay. With three steps remaining, a favorable path exists but robust completion is unavailable. With four steps and all required actions accessible, the target is robustly reachable. If the record cannot be accessed at all, extending the deadline does not repair the missing first action.

These statements are about the constructed procedure. They do not identify three stages as a universal institutional form or supply a natural numerical threshold for effective correction. The example exposes the difference between a short path in a diagram and a strategy available under the declared conditions.

17.3 Observation

Full observation is a substantive assumption. Suppose a participant receives only $o(x)$ for a map $o : X \to O$. A strategy must choose the same action in histories it cannot distinguish, subject to the actions available there. Statewise membership in W_n need not produce such a uniform strategy.

Proposition 17.2 (Observation obstruction)**.** *Two states can each have effective recourse under full observation while their common observation admits no guaranteed one-step remedy.*

Proof. Take states x_0, x_1 with $o(x_0) = o(x_1)$ and common actions α, β. Let α lead to D from x_0 and to a failure state outside S from x_1; let β have the reverse effects. Each state belongs to W_1 when known. No single action succeeds from both. Randomization assigns a positive failure probability in at least one state unless it selects an action that is already uniformly successful, which neither is. □

Proposition 17.2 connects epistemic access to practical remedy without scalarizing epistemic lag. More records help only if they preserve the distinction needed to choose an effective action. An inaccessible or irrelevant archive can leave the observation obstruction unchanged.

17.4 Deadline closure

Time is represented here through a remaining-step component, not a universal social clock. A deadline removes an operation once the state crosses a declared boundary. It can represent the expiry of a retained resource, a decision power or a meaningful alternative. The closure is practical even if a later institution can acknowledge the earlier error.

Deleting actions or adding adverse successors can only shrink the recursively computed winning family when the state space, S and D are fixed. This follows by induction from Equation (48): every surviving witness must still meet the stronger requirements. Such comparisons do not apply unchanged when an intervention also changes the state description or the meaning of the target.

The target itself needs care. Reaching a state labeled *remedied* can restore one interest while leaving another lost. A procedural victory after a missed opportunity is not equivalent to preserving that opportunity. A defensible model either distinguishes those targets or states that its conclusion covers only the narrower restoration.

17.5 Cooperative productivity

Recourse can end in safe termination. That can satisfy a remedial target while failing to deliver the original joint benefit. The arrangement therefore needs a separate completion criterion. A rule that permits only refusal may preserve safety and guarantee exit while making every productive undertaking impossible.

This separation prevents a formal safety result from absorbing the paper's whole question. Reliable cooperation requires an appropriately supported route to a valuable joint activity, together with responses to specified departures. Whether the residual uncertainty is acceptable remains a situated judgment supported by evidence and evaluative premises.

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18 Cooperative Cases

This section connects the analytical distinctions to international cooperation and to scoped institutional examples. It begins with two documented treaty provisions, then develops three explicitly hypothetical cases. Treaty texts establish features of institutional design; they do not by themselves establish successful implementation or the causal effects of those features. The subsequent model interpretations are proposals for investigation.

18.1 Arms-control verification

The 1972 Anti-Ballistic Missile Treaty provides a historically specific arrangement of observation and consultation. Article XII recognizes national technical means of verification and includes obligations against interference and specified concealment. Article XIII establishes a Standing Consultative Commission with functions including consideration of compliance, ambiguous situations, unintended interference, changed strategic circumstances and possible amendments [43, Arts. XII–XIII]. These provisions are cited as historical text, without a claim about current treaty status.

The design is analytically useful because it places several distinct operations within one arrangement. Observation supplies a possible manifestation of conduct. Consultation supplies a forum in which its significance can be considered. Attention to changing circumstances makes the interpretation of continued cooperation an explicit institutional concern. The text therefore already contains more than a simple instruction to verify instead of trust.

A GR investigation would reconstruct the histories through which an observation acquires evidential standing and a consultation acquires practical consequences. It would ask whether the relevant observation can be made, whether the interpretation is admitted, and whether a response remains available. These questions correspond respectively to manifestation, institutional mediation and an executable continuation. They are not claims that the treaty failed on any of those dimensions.

The common-factor model gives one possible language for a bounded part of the inquiry. Each party can retain its own internal description while projecting selected conduct onto a common compliance vocabulary. Persistence requires compatibility after the parties' relevant transitions. If a new capability changes what an old classification signifies, the earlier projection may require revision. Establishing that this occurred in a historical episode would require technical, diplomatic and archival evidence beyond the treaty.

The transformation algebra suggests another distinction. An institutional successor could preserve the wording of a commitment while altering the route through which a concern reaches the consultative body. Commitment transport would then be insufficient evidence of capacity transport. A historical study could inspect the rules and actual use of initiation, information exchange and response. The mathematical distinction identifies what to look for; it does not settle how the actors understood those powers or whether they were sufficient.

The case also limits the normative argument. A right of consultation cannot reasonably be equated with a unilateral right to void a commitment whenever it becomes inconvenient. Reliability requires some protection against opportunistic reinterpretation. The proposed analysis asks how an arrangement sustains both an intelligible commitment and an effective procedure for examining relevant challenges. Neither requirement can simply absorb the other.

18.2 Environmental implementation capacity

The consolidated Montreal Protocol's Article 10 connects compliance to financial and technical cooperation, including technology transfer and agreed incremental costs. It provides for a Multilateral Fund, relevant support functions, balanced representation on its Executive Committee and beneficiary concurrence in disbursement [37, Art. 10, paras. 1–5, 8–9]. The citation refers to the consolidated text accessed for this paper; it does not attribute every provision to the unamended 1987 instrument.

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This design directs attention to the production of an ability to comply. A participant can sincerely accept an obligation while lacking a needed technical or administrative capacity. In that situation, monitoring alone cannot perform the missing work. Cooperation that supplies a capability changes the conditions under which subsequent performance becomes possible. This is consistent with the implementation concerns of compliance research reviewed in Section 2.

The GR question concerns the form of that capacity change. Does assistance support local ability to maintain, adapt and replace a technical practice, or does performance remain dependent on a narrow authorization or supply relation? The prior output/capacity distinction makes these different possibilities visible. The treaty provision alone does not establish which occurs in a project, and dependence on a supplier is not by itself evidence of extraction.

Institutional kernel fields offer a proposed representation of how contributions, recognized needs and technical practices become coupled through a mediating arrangement. The formal kernel would need a defended relation to observable institutional operations. Its entries cannot simply be labeled aid, trust or fairness. An actual application would also have to distinguish the agency deciding eligibility, the organization disbursing resources and the actors providing technical support.

The operator model gives a separate test for an institutional change. If a revised procedure preserves a nominal entitlement but makes a previously available request impossible to submit, the commitment and capacity tests diverge. If it preserves the request through a different accessible route, the changed form need not amount to diminished capacity. Investigating these alternatives requires rules, records of attempted use and evidence about access costs.

Balanced representation and beneficiary concurrence are relevant design provisions, but their existence does not establish equal effective influence. Representation can be exercised under unequal expertise, time or information. Conversely, those provisions can constrain a stronger actor in ways that a crude power ratio omits. The appropriate inquiry concerns their operation and historical development. It should permit evidence of successful protection as well as evidence of failure.

Together the treaty illustrations show why the paper's contribution cannot be the discovery that institutions combine verification, interpretation, adaptation and capacity support. Such combinations are already part of international cooperation. The proposed GR investigation concerns the relations through which those capacities are generated, transformed and preserved, and the tests needed to distinguish formal continuity from practical continuity.

18.3 Staged technical exchange

Two small organizations agree to exchange a specialized component for a sequence of payments. The buyer lacks a long history with the supplier and is uncertain about the supplier's willingness to disclose defects. The supplier doubts whether the buyer will pay after receiving the component. Both nevertheless expect gains from the exchange. They arrange staged delivery, an independent test, retained funds and a procedure for disputed performance.

The initial arrangement appears strong because each fear has a named protection. The supplier sees retained funds; the buyer sees testing and recourse. Yet the relationship depends on the connections among these protections. The test must apply to the relevant component version. Its result must reach the custodian before release. The dispute procedure must have authority over the retained funds. The participants must preserve the information needed to distinguish a defect from later misuse.

Suppose the component specification changes after testing but before release. A gate that checks only whether *a* certificate exists can authorize release using an obsolete certificate. Each local routine may operate as coded. The resulting history nevertheless violates the intended link between the current undertaking and its verification. This is the semantic counterpart of the history-sensitive gate in Section 15.

The bounded response is to bind the certificate to the item and specification version, preserve those bindings, and make a material revision reopen the relevant authorization. This response is not universally appropriate: repeated reopening can impose substantial delay, and some changes may be immaterial. The

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arrangement needs a procedure for deciding materiality and allocating the costs of review. A technical version identifier makes the decision traceable; it does not decide its normative content.

Now suppose the independent tester and the dispute panel both receive evidence through a platform controlled by the supplier. Their personnel may be independent while their evidence remains jointly dependent. A common record filter can hide the same distinction from both. Adding a second panel using the same filter may improve deliberation without resolving the shared evidential limitation.

The shared mediation model identifies the structural question: which dependencies remain after the platform disappears from the parties' simplified description? It does not establish a numerical coupling coefficient for this case. Such a claim would require evidence about access, retention, interpretation and the platform's operation.

Trust may form after successful exchanges, but it need not. The buyer may continue to use versioned tests because it regards disclosure as uncertain. The supplier may continue to require retained funds because payment remains a concern. If the terms remain beneficial and defensible, persistent assurance need not be a failed stage on the way to a more intimate relation.

18.4 Shared research infrastructure

Three research groups maintain a common archive and analysis service. Their methods differ, and no group is willing to accept the others' interpretations without scrutiny. They agree to deposit selected materials, document transformations and allow independent reanalysis. Cooperation promises access to complementary expertise and resources while interpretive trust remains limited.

The common interface identifies a dataset version, a transformation record and an attribution of responsibility. It does not require agreement about every theoretical interpretation. This is a thin operative commonality of the kind examined in Section 8. Its value lies partly in preserving disagreement while making selected work jointly usable.

Suppose one group's categories become the archive's mandatory metadata schema. Materials that fit those categories become searchable, while other distinctions survive only in unindexed attachments. The archive remains accurate in a narrow sense: it has not falsified the deposited files. Yet its ordinary retrieval process makes one interpretation practically dominant. Users may then treat the dominant classification as a property of the material rather than a result of the interface's history.

This failure concerns manifestation and retrieval rather than simple dishonesty. Proposition 10.2 identifies the relevant limit when a record-only view collapses a distinction needed for an assessment. The practical response may include alternative metadata, linked provenance, versioned classifications and protected access to original traces. It may also require acknowledging that some materials cannot be shared without violating privacy or prior obligations.

The response should not become a demand for total memory. Preserving every raw trace indefinitely could expose participants or third parties to serious risks. The prior GR critique of archival power expressly leaves room for legitimate deletion and opacity [19]. The cooperative question is which materials must remain available for the claims the archive makes, who can access them, and what limits those conditions place on later conclusions.

Maintenance creates another dependency. If one group performs most of the indexing and support work, the other groups may enjoy flexibility at its expense. The formal right to request reanalysis can become ineffective when the responsible group lacks capacity to respond. A participation agreement that omits maintenance can therefore promise a remedy without sustaining its first action.

An appropriate response can reduce the promised scope, supply shared maintenance, rotate responsibility or preserve an alternative route. Which response is defensible depends on the parties' capacities and the significance of the inquiry. The present paper does not derive a universal cost allocation. It identifies the inconsistency in treating an unfunded capacity as a functioning safeguard.

Success in this case is not convergence on one interpretation. It is the preservation of a productive, accountable practice through which heterogeneous interpretations can be compared and consequential claims challenged. Interpersonal trust might deepen, remain partial or decline in a particular domain. None of those developments alone determines whether the shared practice remains worth maintaining.

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Table 3: Hypothetical cases and their distinct assurance failures. Proposed responses remain context-dependent.

Undertaking	Failure location	Bounded response
Technical exchange	Certificate and current specification cease to match	Preserve version bindings and reopen material authorization
Research infrastructure	Retrieval suppresses a consequential distinction	Preserve appropriate provenance and alternative access
Community facility	Evidence and remedial power arrive after closure	Preserve an effective interim response or alternative

18.5 Community facility

A community association and an outside provider jointly operate a facility. The provider has technical capacity; the association supplies local coordination and access. Some residents distrust the provider's priorities, and the provider doubts that the association can resolve internal disagreements. The arrangement includes published service standards, a review forum and a right to end the service relationship.

Routine performance improves. Both sides can point to delivered services, and the review forum issues reasoned decisions. Yet ending the relationship would interrupt a necessary activity for residents who have no ready alternative. The formal exit right therefore has limited practical value. The arrangement's success cannot be assessed simply by observing continued participation.

Suppose a disputed practice can be challenged within thirty days, while the records needed to recognize its cumulative effect are produced only after a longer cycle. This constructed timing mismatch illustrates the difference between a nominal path and effective recourse. The number of days has no theoretical significance here; the relevant relation is that the evidence becomes usable after the consequential window closes.

Extending the filing window alone may be insufficient. The review forum might lack power to suspend the practice or restore the lost alternative. A favorable decision can recognize the problem without changing the exposure. The arrangement must identify the effect the remedy is supposed to achieve and the conditions under which that effect remains possible.

A bounded response could preserve an interim alternative, enable an early protective pause on a lower evidential threshold, or give an independent actor authority to secure relevant records. Each option has costs and risks. A pause can interrupt valuable service; preserving an alternative requires resources; independent custody can raise privacy concerns. These tradeoffs require judgment rather than being resolved by the existence of a winning state in an abstract game.

The provider's and association's agreement also does not exhaust the affected parties. Residents unable to attend the forum can remain exposed to its decisions. Their absence from the formal interface is not evidence that their interests are absent. The assessment standpoint set introduced in Equation (5) therefore needs a justification beyond the signatures on the agreement.

This case shows why mutual benefit and justification remain distinct. The arrangement can improve residents' situation relative to the available alternatives while still requiring correction of its dependency structure. Criticizing that structure need not mean that immediate termination is best. It can mean that the arrangement's continued defensibility depends on building capacities that its present success does not automatically provide.

18.6 Comparative interpretation

Table 3 compares the failures. The common analytical move is to follow a support through the history of its use. The cases differ in whether the missing relation concerns a version, a preserved distinction or a practical alternative.

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No case demonstrates that more safeguards always improve cooperation. Each response can introduce a new support relation, a new burden and a new authority. The analysis is recursive in that limited practical sense: assess the added support by the same standards of dependence, scope and usable response, while allowing a justified stopping point for the particular undertaking.

19 Philosophical Objections

The account risks becoming too broad if every dependency is called assurance, and too narrow if formal admissibility replaces the practical relation. This section addresses those concerns together with objections about trust substitution, regress, control and established mathematical techniques. The responses delimit the argument rather than claiming to resolve every dispute about institutional cooperation.

19.1 Institutional substitution

The first objection is that the paper redescribes familiar propositions: institutions can support cooperation without strong prior trust, and their effects depend on how they are organized. Both propositions are inherited, from IR institutional analysis, wider institutional research and prior GR work [31, 5, 23]. The paper's task is to specify preservation obligations across transformations of the arrangement, not to present institutional support as an unexplored possibility.

The difference becomes substantive when adding a safeguard fails to improve the advertised property. A second reviewer sharing the first reviewer's inaccessible record does not necessarily provide a second evidential route. An appeal that loses its practical object before judgment does not provide the same protection as an earlier effective remedy. A locally tested operation can violate admissibility after another operation changes its input. These claims concern the arrangement's structure and history, not merely the presence of an institution.

This response also limits novelty. The individual insights have analogues in other disciplines, and the mathematical techniques are standard. The paper proposes a GR reconstruction linking historical constitution, generative dependence and practical recourse. It does not claim priority over every account of common-cause failure, compositional verification or institutional design.

19.2 Trust relocation

The literature creates a stronger version of the objection. Generalized trust may help actors create the very institutions that later limit vulnerability [40]. Institution building cannot therefore be assumed to start from a completely trust-free position. This paper's bounded claim concerns the scope and distribution of trust required for particular undertakings. It leaves the historical formation of those supports open to explanations in which trust matters.

A second objection holds that cooperation always depends on trust somewhere, so incomplete interpersonal trust merely relocates it. Some arrangements do relocate trust: a participant may withhold trust from a provider while trusting a custodian or reviewer. That possibility is compatible with the account. It is not the only interpretation of every dependency.

If trust is defined so broadly that reliance on any support counts, the claim of universal trust dependence becomes close to a conceptual stipulation. It then cannot distinguish reliance on a testable mechanism from a participant stance toward a person without additional vocabulary. The present paper retains that distinction because it affects the warrants, responses and interpersonal meanings involved.

At the other extreme, declaring an institutional arrangement trust-free can obscure the dependence on competence, maintenance and interpretation. The defensible conclusion is more specific: strong prior mutual interpersonal trust need not be a universal entry condition, but the arrangement still requires an account of its supports. It may contain trust, warranted reliance, limited confidence and deliberate skepticism in different places.

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19.3 Justification regress

If every safeguard requires another safeguard, the analysis appears to generate an infinite regress. The objection correctly identifies a danger in demanding absolute assurance. The present account makes no such demand. It requires a scoped statement of the undertaking, its supports, its residual uncertainties and its response options.

A provisional stopping point can be warranted when the exposure is sufficiently bounded, the background evidence is adequate for that scope, and further verification would impose costs disproportionate to the remaining question. The justification of those judgments remains defeasible. What matters is that the stopping point is visible and revisable rather than concealed inside a claim of institutional infallibility.

GR's historical perspective changes the form of the question without solving it by ontology. A support can be sufficiently persistent and effective to serve as a background condition for one inquiry while becoming the object of another. Historical emergence explains how reliance becomes possible; it does not supply an unlimited warrant for reliance. The separation of generative dynamics and criteria in prior GR work supports this restraint [26].

19.4 Control

A fourth objection is that assurance replaces cooperation with control. Some assurance practices can do so: intrusive monitoring, unilateral classification and prohibitive exit conditions may produce compliance while eroding meaningful agency. The account does not identify all controlled behavior with a cooperative achievement.

The distinction depends on the criteria of Section 6. An arrangement can be operationally reliable and normatively defective. A procedure that preserves a formal admissible sector can still preserve domination if the sector's predicate embodies an unjust order. The algebra cannot correct that choice merely by proving closure.

At the same time, safeguards need not be hostile to agency. A bounded commitment, an accessible appeal or an alternative provider can enlarge a participant's practical possibilities. The relevant question is what the practice enables and forecloses for the affected participants. The category *control* is too broad to answer that question without examining its form and distribution.

Assurance can also communicate distrust. Requiring a guarantee from one party while exempting another can express unequal standing. That expressive consequence belongs to the relation and may affect its defensibility. It does not follow that every request for verification insults the other party or that removing protection is the proper way to demonstrate respect. The interpersonal meaning requires interpretation within the history at issue.

19.5 Self-maintaining relations

An objection from within GR might ask whether the arrangement is simply another trust or distrust attractor. Prior work already investigates self-reinforcing relational trust, distrust and poetic composition [22]. The present paper neither rejects that vocabulary nor treats persistence as its contribution.

Its object can persist for reasons that do not amount to trust. Repeated cooperation may be supported by limited exposure, reciprocal need, institutional authority or constrained alternatives. A self-maintaining configuration can therefore be beneficial, exploitative or mixed. The compositional analysis asks which relations sustain it and whether its advertised protections remain operative.

An attractor interpretation would require a specified dynamics, state or history space, and an appropriate persistence criterion. The finite field integral and boundary algebra do not automatically supply those ingredients. Calling an arrangement stable because its operator family is closed would confuse an algebraic property with a dynamical one.

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19.6 Relational redescription

A further objection is that the proposed GR contribution could be expressed entirely within existing relational IR. This is a serious possibility. The constitutive role of practices, historically formed actors and differential capacities already appears in the literature reviewed in Section 2. A new vocabulary or a more elaborate mathematical carrier does not establish an explanatory increment.

The response is to make the increment assessable. The continuation model distinguishes current agreement from agreement under selected future operations. The kernel field distinguishes fluctuation of a mediator from fluctuation of a rule of mediation. The transformation algebra distinguishes a preserved commitment observable from a preserved implementable operation. If another model preserves these distinctions more economically, it can be preferred. GR's programme offers a coordinated way of constructing and comparing such representations; it does not require exclusive ownership of the mechanisms.

Even the conjunction of these distinctions may prove primarily expository. A stronger contribution would require showing that a historically consequential dependence was previously omitted, and that retaining it improves explanation or interpretation. The present conditional results make such a demonstration possible to formulate. They do not claim to have completed it through two treaty illustrations.

19.7 Commitment discipline

Preserving revisability can conflict with credible commitment. An actor may use every opportunity for reopening to demand new concessions after the other has incurred costs. The bargaining and commitment literature supplies reasons to take this problem seriously [9, 38]. A framework that identifies every closed option with loss would be unable to explain the value of binding agreements.

The response distinguishes unrestricted option preservation from protected standing to invoke an appropriate process. The repertoire in Equation (44) is selected, justified and limited. It can include access to evidence and review while excluding unilateral appropriation or costless abandonment. Its preservation requirement does not determine that selection. When an old power is legitimately withdrawn, the analysis should record the change and its justification rather than disguising it as no loss.

This distinction leaves difficult cases unresolved. A review procedure itself can be used strategically; a justified restriction can be imposed through an unequal process; a credible commitment can become harmful after unanticipated transformation. No algebra settles the balance among these considerations. The formalism clarifies which capacities change and which claimed continuities survive, so that the normative disagreement concerns an explicit object.

19.8 Formal excess

The final objection questions whether the mathematical constructions add enough to justify their complexity. A simpler model may suffice for many cases. The paper's methodological pluralism requires that possibility to remain open.

The spacetime representation earns its place when current status fails to preserve relevant history and incidence. The field construction earns its place when the generation of attachments and mediation kernels matters, particularly when a quadratic or pairwise description omits induced higher interactions. The history-indexed algebra earns its place when commitment transport and operation transport differ. The recourse game earns its place when control, observation and deadlines determine whether a remedy can be executed.

If an application cannot identify these distinctions, invoking the richer formalism would add terminology without analytical gain. Conversely, using only a fixed graph could erase the creation of roles or response channels that the investigation needs to explain. The proper model is selected by preservation and explanatory obligations, not by a preference for either mathematical simplicity or mathematical prestige.

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The present models are therefore several possible investigations. They are not competing proofs of GR's ultimate ontology, and their coexistence does not establish their equivalence. The interfaces among them remain explicit research obligations. That limitation is part of the argument's content rather than a reason to collapse the programme into one conventional dynamical system.

20 Empirical Investigations

The formal models become useful beyond illustration only when their semantic assignments and preservation claims can be investigated. This section proposes a bounded empirical programme organized around observable failures and competing explanations. It separates descriptive reconstruction from intervention and does not treat the simulations as data about actual trust relations.

20.1 History reconstruction

A first study can reconstruct an undertaking through its commitments, versions, evidence, response actions and outcomes. The objective is to identify the relations required by a claimed safeguard. Relevant materials can include procedure descriptions, timestamps, authorized changes, access histories and participant accounts. Each source is itself a manifestation with a production context.

The reconstruction should retain competing descriptions where they matter. An institution's record of successful notice can coexist with a participant's account of inaccessible language or unavailable response. The study needs criteria for assessing these descriptions without assuming either that official records exhaust the event or that every retrospective account is equally supported.

A useful output is a sequence of tested connections: which specification a certificate addressed, which evidence a reviewer could inspect, what powers remained when a decision arrived, and which alternatives existed at that time. This sequence can reveal a compositional failure even when all local procedures have favorable performance reports.

20.2 Shared supports

A second study can map dependencies among nominally separate safeguards. It should distinguish personnel, funding, information, infrastructure and classification. Organizational separation is evidence about some of these relations, not all. A common archive may be visible while a common selection rule is harder to identify.

The field model suggests a question about omitted interactions, not a ready-made estimator. One can compare descriptions that treat review channels separately with descriptions that include their shared support. The richer model should improve the explanation of observed joint failures or changes under stated conditions. If the shared field supplies no identifiable distinction beyond an ordinary common-cause model, the latter may be sufficient.

Empirical use of the higher-order terms would require more. The variables must have defensible domains, the observations must identify the relevant interactions, and the parameter regime must justify the approximation. A fitted quartic term alone would not establish the generative history interpretation. Similar response patterns can arise from different mechanisms.

20.3 Recourse availability

A third study can distinguish nominal procedures from practically available response paths. It can compare advertised steps with the costs, permissions, evidence and elapsed time encountered by differently situated participants. Successful users alone supply a selected sample: people unable to enter the procedure can disappear from completion statistics.

The finite game offers a disciplined way to state a limited counterfactual. Given an explicitly reconstructed action set and outcome family, does a participant have a strategy reaching the stated remedial

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Table 4: Evidence obligations for the proposed model classes. These are application tests, not claims of current validation.

Representation	Required connection	Evidence of inadequacy
History complex	Cells and gluing preserve relevant continuity	Distinct obligations collapse under the chosen identification
Mediation field	Variables and kernels represent identifiable support interactions	Equivalent fit with incompatible semantics and no discriminating evidence
Institutional kernel	Changes of mediation rules have distinct observable consequences	Kernel variation merely recodes an ordinary omitted variable
Preservation algebra	The admissible sector and operations match the declared criterion	Actual operations reach excluded histories or erase material distinctions
Transformation algebra	Commitment correspondences and available implementations are separately supported	Claimed intertwiners preserve a record while the asserted operation is inaccessible
Recourse game	Actions, observation and deadlines reflect practical access	Claimed actions cannot be invoked or outcomes omit consequential obstruction

target before closure? The answer is conditional on those families and on observation. It should not be presented as an empirical probability when no probability model has been supplied.

Alternative assumptions should be reported. A larger delay budget, an inaccessible record or a different target can change the result. Such sensitivity is informative when it identifies which support the assurance claim actually needs. It becomes misleading if the analyst tunes the assumptions merely to produce a favorable certificate.

20.4 Model comparison

IR research supplies competing explanations that should be compared directly. A failed undertaking may reflect an incentive to defect, disagreement over distribution, limited implementation capacity, a contested interpretation or a transformation of authority. A historical study should trace the sequence linking the proposed mechanism to the outcome and seek evidence that discriminates among alternatives. Simply finding interdependence cannot select the GR interpretation.

The treaty illustrations suggest two bounded study designs. An arms-control study could compare episodes in which an observation reached a consultative process with episodes in which a disputed classification obstructed that route. An environmental-cooperation study could compare formally preserved entitlements across procedural changes, examining whether the relevant participants retained usable request and review powers. These are proposed comparisons, not completed empirical findings.

Institutional selection requires particular care. High compliance can reflect shallow commitments, and observed users of recourse can exclude those unable to enter the procedure. The critique developed by Downs, Rocke and Barsoom remains relevant to claims that an arrangement's apparent success demonstrates its effectiveness [7]. The new capacity criterion needs the same discipline: choose a defensible baseline, identify the relevant affected population and avoid selecting only successful continuations.

The models make distinct failure claims. Table 4 states what would count against a proposed application. An application should retain the simplest representation that preserves the distinctions needed for its question, while remaining willing to move to a richer representation when those distinctions fail.

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20.5 Intervention

An intervention can change the relation being investigated. Adding a record requirement alters work; introducing review alters authority; preserving an alternative reallocates resources. The intervention should therefore be evaluated through its effects on the undertaking and its affected parties, not only through the metric used to motivate it.

For this paper, non-disruptive reconstruction, procedure review and controlled illustrative exercises are more directly supported than experiments that remove protections from exposed participants. Any actual intervention would require its own ethical and practical justification. The present mathematics does not authorize withholding recourse in order to test whether trust increases.

The research programme also needs negative results. A safeguard may prove unnecessary within a bounded domain; an added layer may impair performance; a proposed group symmetry may lack a meaningful target interpretation. Recording these failures would improve the model choice. The aim is a more accurate account of cooperative possibilities, not the universal installation of the richest available formalism.

21 Research Limits

The paper has investigated cooperative arrangements whose participants can continue a worthwhile undertaking while trust remains partial, differentiated or contested. This possibility is inherited from a mature literature. Strategic IR, institutional design, compliance research, trust scholarship and relational power analysis already supply explanations and objections that the GR reconstruction must engage. The contribution proposed here concerns the coordination of those problems through explicit historical and preservation structures.

The philosophical center is the distinction between an arrangement's current performance and the capacities it reproduces or transforms. A participant can receive a benefit while losing a practical route for challenging future decisions. An institution can retain a commitment while changing the operations through which it can be exercised. These possibilities motivate an analysis of participants and arrangements as historically extended objects, ordinarily represented within GR as spacetimes. The representation remains a research choice with ontological questions suspended.

The formal investigations establish several conditional distinctions. Continuation signatures retain operational differences that a current record can erase. A common-factor construction requires compatible state pairs before it supports a claim of shared readings. The original field model exhibits dependence induced by shared non-Gaussian mediation. The institutional kernel model additionally treats a coupling rule as an interacting field, with a finite positive ensemble, nonlinear stationary equations and an exact moment hierarchy. It does not establish a calendar-time dynamics of institutional learning.

The algebraic construction gives the paper's central separation a precise form. Intertwiners can preserve a represented commitment algebra across history sectors. Preservation of a participant's selected repertoire requires a further implementation condition. A finite counterexample proves that the first does not entail the second. Observable compression supplies another distinct loss mechanism, while the recourse game adds constraints of access, opposition, observation and timing. No one of these representations exhausts practical cooperation.

Several bridges remain open. The field action does not independently implement every historical or normative gate assumed by the operator interpretation. The common algebra presupposes a justified translation of selected commitments. Isometric history embeddings preserve distinctions by assumption and must be replaced when the target problem concerns irreversible loss. The finite operation repertoire cannot represent unprecedented institutional categories without enlargement. Continuum limits, richer interaction types and empirical measurement require further work.

The normative premises also remain contestable. Effective challenge is defended where an arrangement invokes the standing and continued acceptance of its participants, but this does not imply costless unilateral exit or preservation of every possible action. Legitimate commitments can close harmful alter-

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natives. Judging their scope requires attention to affected outsiders and internal inequalities as well as the interests of formal parties. Neither a field minimum nor an invariant subspace supplies that judgment.

The treaty illustrations establish design provisions, not institutional effectiveness or a causal test of GR. Empirical work must distinguish strategic incentives, implementation capacity, contested interpretation and changes in authority, rather than treating every failure as evidence for a single relational mechanism. The proposed models are useful only where their added distinctions improve that inquiry.

Cooperative reliability can thus be investigated without requiring complete trust or reducing cooperation to technical assurance. The relevant assessment follows commitments, supporting relations and available powers through the history of their use. It asks what continues, what changes, and which justified responses remain practically possible. The present discussion supplies a set of arguments and conditional constructions for that investigation, while leaving their wider explanatory and normative adequacy open to criticism.

A Field Calculations

This appendix records intermediate steps and numerical checks for the field model. It distinguishes exact finite integrals from formal diagram expansions and local series truncations. All computational parameters are illustrative and dimensionless.

A.1 Coercive estimate

The estimate used in Proposition 12.2 follows by optimizing a one-variable polynomial. For fixed $r \geq 0$ and $b \geq 0$, the required maximum is the following quantity.

$$\sup_{m \geq 0} \left(2bmr^2 - \frac{\lambda}{2} m^4 \right) = \frac{3}{2} b^{4/3} \lambda^{-1/3} r^{8/3}. \tag{50}$$

For $br = 0$, Equation (50) is immediate. Otherwise the derivative vanishes at $m = (b/\lambda)^{1/3} r^{2/3}$, the expression tends to minus infinity, and substitution gives the stated value. The resulting negative $r^{8/3}$ term is dominated by any positive multiple of r^4 outside a bounded interval. This proves the tail bound without requiring the coupling parameters to be small.

The Gaussian mediator sector requires a separate condition when $\lambda = 0$. For $\kappa > 0$, completing the square gives the exact identity.

$$\kappa \|M\|^2 - 2 \operatorname{Re} \langle M, J \rangle = \kappa \|M - J/\kappa\|^2 - \frac{\|J\|^2}{\kappa}. \tag{51}$$

Equation (51) yields $\log F(J) = \|J\|^2/\kappa$. Since $\|J\| \leq br^2$, the condition $\epsilon > b^2/\kappa$ is sufficient for a coercive retained quartic term. It is not claimed necessary for every restricted current family. Simply setting the quartic mediator coefficient to zero while retaining arbitrary parameters would therefore lose the general stability guarantee.

Gaussian elimination already produces mixed terms in the currents. The non-Gaussian investigation adds higher current interactions. Their difference is a mathematical distinction within the selected model class, not a claim that every common-support problem needs the richer model.

A.2 Radial moments

For the one-mode pure quartic sector, polar coordinates give $d^2 m = r \, dr \, d\theta = \frac{1}{2} dt \, d\theta$ with $t = r^2$. The radial integral for any integer $q \geq 0$ is the following expression.

$$\int_0^\infty t^q e^{-\lambda t^2} \, dt = \frac{1}{2} \lambda^{-(q+1)/2} \Gamma\left(\frac{q+1}{2} \right). \tag{52}$$

Dividing Equation (52) by its $q = 0$ value gives Equation (21). Expanding $e^{\bar{m}j}e^{m\bar{j}}$ produces terms indexed by two nonnegative integers. Integration over θ annihilates unequal exponents. The surviving term of degree $2q$ is $\mu_q|j|^{2q}/(q!)^2$.

Put $x = |j|^2$. The logarithm of $1 + \mu_1 x + \mu_2 x^2/4 + O(x^3)$ supplies the fourth-order coefficient explicitly.

$$c_4 = \frac{\mu_2}{4} - \frac{\mu_1^2}{2} = \frac{1}{8\lambda} - \frac{1}{2\pi\lambda} = \frac{\pi - 4}{8\pi\lambda}. \tag{53}$$

Equation (53) establishes the sign used in Proposition 13.2. The local expansion controls small sources. Its truncation is not a replacement for the exact logarithm across an unrestricted source range.

A.3 Strand contraction

The mixed quadratic current term has a direct two-vertex interpretation. A mediator propagator joins a cubic vertex of channel 1 to a conjugate vertex of channel 2. The normalized external-index contraction is the following expression.

$$\langle J_1, J_2 \rangle = n^{-4} \sum_{i,j,k,l} \overline{(U_1)_{ij}(V_1)_{jk}}\,(U_2)_{il}(V_2)_{lk}. \tag{54}$$

Equation (54) follows by substituting the two normalized convolutions into the field inner product. It retains four external field factors and their attachments. This is the sense in which elimination induces an interaction among the retained couplings rather than merely deleting a mediation node from a graph.

The source code checks this contraction against direct matrix composition for randomly sampled invariant fields on $G = \mathbb{Z}/3\mathbb{Z}$. It also checks the nonlinear action derivative against symmetric finite differences in independently sampled invariant directions. These checks can detect index, normalization or conjugation errors; they do not replace the derivations or validate a social interpretation.

A.4 Numerical illustration

The one-mode illustration fixes $\lambda = 1$. The exact source series is evaluated with 80 terms over the plotted range. Independent quadrature uses 320 Gauss–Legendre nodes on $t \in [0, 12]$ and 768 equally spaced angular samples. The omitted radial tail is exponentially suppressed in this parameter range; convergence is also checked against the analytic moments and source series. No Monte Carlo estimate or empirical dataset enters the figure.

Figure 3 compares the exact source function with its local approximations and shows the mixed contribution for aligned and orthogonal real source directions. The horizontal axes parameterize field modes, not amounts of trust, institutional strength or observed risk.

The complete check report includes moment comparisons through order eight, source-integral comparisons at 17 source magnitudes, decreasing small-source remainders, a local non-Gaussian mixed term, and numerical convexity diagnostics. The exact covariance identity in Equation (23) supplies the proof of convexity; the diagnostics only check the implementation.

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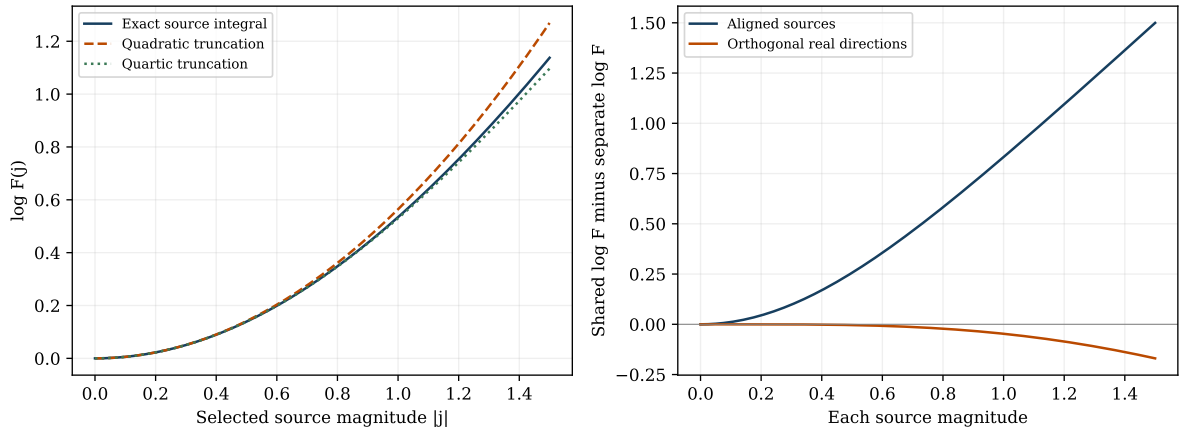

Figure 3: Illustrative one-mode quartic mediator, $\lambda = 1$. Left: the exact source function and its quadratic and quartic local truncations. Right: $\log F(j_1 + j_2) - \log F(j_1) - \log F(j_2)$ for equally sized aligned sources and orthogonal real directions in the complex plane. The latter retains a nonzero higher-order interaction despite vanishing quadratic interference. Truncations are displayed for comparison beyond the immediate small-source region; their global adequacy is not asserted.

B Operational Countermodels

This appendix describes the finite checks accompanying the algebra and recourse models. Their purpose is to expose the difference between the assumptions of a theorem and weaker checks that might be mistaken for those assumptions. The examples are deliberately small enough to enumerate.

B.1 Exact composition

On the three-state set $\{0, 1, 2\}$, take $\{0, 1\}$ as admissible. There are $3^3 = 27$ deterministic total maps. Exactly $2^2 \cdot 3 = 12$ preserve the admissible set: the first two inputs must map into it, while the third input can map anywhere. The script enumerates all 144 ordered pairs of preserving maps and verifies preservation by their compositions.

This finite result illustrates Proposition 15.1 in the set-map representation embedded as basis transformations. It does not establish that all preserving maps are contractions in the counting Hilbert norm; several inputs can map to one output and increase that norm. The approximate leakage checks therefore evaluate the full bound of Equation (38) rather than silently applying the contraction-only estimate.

The script separately verifies the initial-state counterexample, failure of adjoint closure and the change introduced by compression. Forty pairs of random complex matrices check the norm estimate numerically. The universal algebraic statements follow from the proofs, not from the sampled matrices.

B.2 Versioned release

A finite history begins with a retained item at specification version zero. Three event types are available: certify the current version, revise to a new version, and release. Release terminates the selected history. A version-aware gate permits release only when a certificate for the current version exists in the retained history.

The script enumerates every permitted history through depth six, obtaining 158 histories including the empty history and shorter prefixes. Each release has the required antecedent. A comparison gate checks only whether some certificate exists. It admits histories in which certification is followed by revision and release without a certificate for the revised version.

The countermodel does not claim that all revisions should invalidate every certificate. It records the declared criterion that a certificate must address the operative specification. An application allowing cer-

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tain changes without recertification would need to define the relevant equivalence relation and preserve that relation in the gate.

Historical evidence is retained in this model. If another rule could erase or reassign certificates, the proof obligation would extend to that rule. The exercise consequently illustrates a rule-family guarantee rather than a property of one favorable release example.

B.3 Delay budget

The finite recourse model has state (s, t, b): completed stage $s \in \{0, 1, 2, 3\}$, remaining step allowance t, and remaining adverse delay budget b. A productive response advances s and decreases t by one. If $b > 0$, the environment may instead consume one step and one delay unit without advancing the stage. Stage three is the target.

When all initial actions are accessible, the initial state has a favorable path exactly when the allowance is at least three. It has robust recourse exactly when the allowance is at least three plus the delay budget. The sufficiency follows because every nonproductive step consumes a finite delay unit, leaving three productive steps. Necessity follows because the environment can spend its entire delay budget before allowing completion.

The implementation checks all allowances from zero through eight and all delay budgets from zero through four. It also removes the record-access action and verifies that no initial state then reaches the target, regardless of the enlarged allowance. This comparison distinguishes a timing obstruction from an access obstruction.

These are bounded operational claims. An unbounded possibility of delay would defeat the finite guarantee unless further assumptions, such as an enforceable timing rule, constrained it. Calling an institution ordinarily prompt is not the same assumption as providing a bound that a participant can rely on.

B.4 Reproducibility

The source script is `simulations/formal_checks.py`; its machine-readable report is `simulations/results/formal_checks.json`. The fixed random seed is 61001. The completed run passes 432 checks across the field, integral, algebraic, historical and recourse examples. Counts are implementation checks, not an independent measure of philosophical or empirical support.

The report records the exact finite cases separately from sampled numerical diagnostics. Reproduction requires Python with NumPy and Matplotlib. The manuscript build and audit scripts are included with the source packet. Changes to the models or their parameters require rerunning the corresponding checks and rebuilding the affected figure and manuscript.

C Institutional Field Derivatives

This appendix checks the nonlinear equations in Section 14. Its role is to expose the adjoints and coordinate conventions needed for the variation, without interpreting stationarity as a social evolution law.

All complex inner products are conjugate-linear in their first argument. For a real-valued differentiable action, the conjugate-coordinate gradient $G_z = \partial S/\partial \overline{z}$ satisfies $dS[\delta z] = 2\,\mathrm{Re}\langle G_z, \delta z\rangle$. Define linear maps $B_V(U) = U \star V$ and $C_U(V) = U \star V$. Their adjoints satisfy the following identities.

$$B_V^\dagger(W) = W \star V^\dagger, \qquad C_U^\dagger(W) = U^\dagger \star W, \qquad L_J^\dagger(D) = DJ^\dagger, \quad L_J(\mathsf{N}) = \mathsf{N}J. \tag{55}$$

The first two identities in Equation (55) follow from the cyclic property of the finite matrix trace with the normalized inner product. For the third, expand $\langle D, (\delta\mathsf{N})J\rangle = \mathrm{Tr}((DJ^\dagger)^\dagger \delta\mathsf{N})$. The Hilbert–Schmidt pairing therefore identifies $DJ^\dagger$ as the adjoint image.

The differential of the squared residual is $d\|D\|^2[\delta D] = 2\,\mathrm{Re}\langle D, \delta D\rangle$. For M, $\delta D = -\delta M$; for N, $\delta D = (\delta\mathsf{N})J$; for U_a, $\delta D = g_a \mathsf{N} B_{V_a}(\delta U_a)$; and for V_a, $\delta D = g_a \mathsf{N} C_{U_a}(\delta V_a)$. Taking the respective

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adjoints yields all residual terms of Equation (28). Differentiating ϵr^4, $\lambda\|M\|^4$ and τs^4 supplies their factors of two.

The accompanying script tests these directional derivatives against centered finite differences for generic complex fields and kernels in the invariant group sector. It also checks kernel-coordinate normalization, the redescription symmetry and the variance identity. These are implementation checks of exact formulas, not empirical calibration or a substitute for the derivations.

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