

A Dynamic Model of Resource Capture in Familial Dependency Relations

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Discussion paper — 10 September 2026

Abstract

Resources acquired outside a family relationship can enlarge a recipient's opportunities while also becoming available to an actor who exercises control over the recipient. This paper develops a conditional account of the resulting interaction. A resource budget distinguishes retained resources from captured transfers, and a dynamic model allows invested transfers to maintain control and influence subsequent capture. Under a saturating capture technology, the model identifies a threshold above which positive control persists. On that branch, an increase in external resources can reduce stationary independent capacity even while the recipient retains a positive resource flow. The result depends on specified conversion and suppression mechanisms. A fixed-capture counterexample and an extension with background control clarify its limits. A separate increasing-returns specification establishes sufficient conditions for multiple stable equilibria and a threshold separating their basins. The analysis distinguishes these mathematical results from hypotheses about the emergence of familial control. It situates the framework within household economics, psychological control, economic abuse, and kinship sharing; develops observable implications; and separates developmental dependency from restrictions on agency. The contribution is a tractable account of resource-financed control and its evidential requirements. The model provides neither a prevalence estimate nor a general claim that external assistance reduces autonomy.

Keywords: familial dependency; resource capture; household political economy; control; independent capacity; nonlinear dynamics.

Discussion Paper Note

This note explains the status, scope, and intended scholarly use of the paper. It also identifies questions on which criticism and further research would be particularly valuable.

This is a discussion paper offered for critical examination and revision. Its purpose is to organize a possible mechanism linking resource capture and the maintenance of control in specified familial dependency relations. The equations establish conditional properties of a constructed model. Their application to actual relationships requires independent evidence about capture, conversion into control, and effects on practical agency. The numerical illustrations use hypothetical parameters and provide no estimates of prevalence or effect size.

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The analysis concerns relationships satisfying stated conditions. It provides no basis for presuming that family support, interdependence, or continuing support needs are intrinsically coercive. Its examples are hypothetical, and the paper makes no finding about an identifiable person or family. The companion discussion paper examines the normative significance of hidden control for recipients and outside contributors; agreement with its normative analysis is not an assumption of the mathematical results presented here.

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1 Introduction

This section defines the resource-control problem, specifies the paper's contribution, and explains the relationship between the formal analysis and its proposed familial application. The central question concerns the conditions under which resources acquired by a person contribute to the maintenance of another actor's control over that person. The analysis proceeds through conceptual distinctions, a tractable dynamic model, and a research design for assessing the mechanism.

Consider an adult recipient who obtains income or support through an external relationship. Some of the resulting resources remain available to the recipient; another part becomes accessible to a family actor who exercises substantial influence over housing, communication, productive activity, or the practical consequences of refusal. If that actor uses captured resources to maintain those control instruments, the external inflow can affect both the recipient's opportunities and the constraints governing their use. The relevant comparison concerns the subsequent configuration of control and independent capacity, alongside the amount received.

This configuration can arise through several institutional arrangements. A resource transfer may be compelled through credible sanctions, governed by an account arrangement that the recipient cannot effectively revise, or obtained through pressure sustained by dependence on essential support. A family actor may also control decisions without receiving money, and may receive money without controlling decisions. Resource-financed control denotes the narrower conjunction of extraction, conversion into effective control, and a continuing effect on the recipient's opportunities. Establishing that conjunction requires evidence for each link.

The economic literature already treats household members as actors whose interests and control over resources can differ. Lundberg and Pollak (1993) locate bargaining threats within an ongoing marriage, illustrating that distributional power can operate through alternatives short of dissolution. Doepke and Zilibotti (2017) model parental influence over both preferences and choice sets. Research on economic abuse explicitly examines control over material resources (Adams et al., 2008). The present analysis builds on these precedents by concentrating on the feedback from captured resources to the future capacity to capture and constrain.

The paper makes three limited contributions. First, it separates the accounting of resources from their conversion into agency-relevant capacities. Second, it establishes a conditional threshold and a comparative static in a model with endogenous capture and depreciation. Third, it specifies the additional assumption required for multiple stable regimes. These results organize a mechanism that can be assessed empirically. Their interpretation depends on whether the proposed control technology and its effects adequately describe a particular relationship.

The model represents an institutional process through reduced-form equations. It does not solve an optimizing game between a parent, an adult child, and an outside contributor. Resource supply, the investment share of captured funds, and the institutional environment are held fixed for the main comparative statics. This choice permits transparent analysis of the feedback while limiting claims about equilibrium strategy, welfare, or policy. The term political economy refers here to the distribution and reproduction of effective control over resources and activity.

The familial application concerns a class of relationships meeting the specified conditions. Childhood dependency may help explain an initial asymmetry, but the recipient in the external transaction is treated as an adult. Continuing support needs, including those associated with disability, remain compatible with agency and legitimate care. An adult recipient can exercise meaningful choice within an interdependent relationship; a financial measure of self-sufficiency would inadequately describe that situation.

Section 2 positions the mechanism in adjacent scholarship. Section 3 defines the relational structure and its boundaries. Sections 4 and 5 establish the formal results. Section 6 examines possible antecedents; Section 7 develops an empirical strategy; and Section 8 evaluates the contribution and its limitations. Detailed proofs appear in the appendices.

2 Literature and Analytical Position

This section compares the proposed mechanism with scholarship on relational power, household allocation, economic abuse, and psychological control. The review identifies direct precedents and specifies the narrower work performed by the present model. It is a targeted review based on verified publications and accessible source material, with no claim to systematic coverage.

2.1 Relational Power and Household Allocation

The relational-power literature provides the conceptual basis for treating control as specific to an actor and a domain. Emerson (1962) ties power to dependence within a relationship and examines balancing processes. Accordingly, the variable introduced below represents effective control by one actor over another in specified activities. A person's wealth, status, or forcefulness alone provides an inadequate measure of this relation.

Household bargaining directs attention to decision rules and disagreement positions. The separate-spheres model shows that a transfer's distributional consequences depend on the organization of bargaining within the household (Lundberg and Pollak, 1993). The present model examines a complementary issue: a resource flow may help reproduce the instruments that influence future allocation. It leaves the bargaining process itself unmodeled.

Parental choice-set restriction also has an explicit economic precedent. Doepke and Zilibotti (2017) distinguish altruistic and paternalistic motives and analyze instruments through which parents influence children's decisions. Their framework makes the economic environment consequential for parenting. The present paper therefore makes no novelty claim for treating a child's choices as subject to parental constraints. Its focus is the resource-financed persistence of a specified constraint after external acquisition becomes possible.

2.2 Economic Abuse and Kinship Sharing

Research on economic abuse supplies a close substantive comparison for capture and control. Adams et al. (2008) develop a measurement instrument using interviews with 103 survivors of domestic abuse. Their work establishes that economic practices belong within the study of controlling relationships. Extending particular items to adult parent-child relations would require new validation. Stark (2007) provides a broader account of coercive entrapment in intimate relationships; its gendered and institutional setting should remain explicit when concepts travel across domains.

Kinship-sharing research shows that resource transfers and visibility can alter incentives. Baland et al. (2011) combine observations and interviews with a signaling model of costly borrowing as a means of resisting demands for financial help. Jakiela and Ozier (2016) experimentally vary the observability of returns and examine participants' willingness to sacrifice earnings to conceal income. These contributions substantially anticipate the idea that acquiring resources within a kin network may carry costs. The additional link studied here is the reinvestment of captured resources in control over the same recipient.

A further comparator is the use of coercion to obtain resources through family relationships. Bloch and Rao (2002) study dowry violence as a bargaining instrument. Their setting cautions against treating indirect family extraction as an unexplored topology. The present framework abstracts from that specific institution and examines a dynamic mechanism whose operation would need independent evidence in other settings.

2.3 Psychological Control and Epistemic Conditions

The psychological-control literature helps distinguish ordinary guidance from intrusive manipulation. Barber (1996) develops the construct and differentiates it from behavioral control; Soenens and Vansteenkiste (2010) clarifies its relation to self-determination theory. Green and Werner (1996) distinguish intrusiveness from closeness and caregiving. These distinctions support a scope restriction:

shared residence, emotional closeness, expectations, and assistance are insufficient indicators of the mechanism analyzed here.

Interpretive conditions may affect whether an imposed restriction can be identified or contested. Sweet (2019) links gaslighting to social inequalities and institutional vulnerabilities, while Fricker (2007) analyzes wrongs affecting people as knowers. The present model leaves these processes outside its state equations. Possible effects on capture, credibility, or control depreciation are hypotheses for a richer specification. Treating interpretation as endogenous in principle does not identify its actual causal role in a case.

2.4 Empirical Counterweights and Contribution Boundaries

Evidence on assistance provides a necessary check on the direction of the proposed effect. The mixed-method review by Buller et al. (2018) reports predominantly reductions in intimate partner violence following cash transfers, with limited evidence of overall adverse effects. Partner violence differs from the independent-capacity index used here, and adult parent-child relations differ from the reviewed settings. Nevertheless, that evidence precludes presenting harmful assistance as a general empirical expectation.

The contribution is thus an explicit conditional mechanism. The paper neither infers the sign of an actual transfer's effect from a relationship label nor treats a mathematical possibility as an empirical regularity. It specifies measurements and comparisons through which the adverse mechanism could be distinguished from beneficial sharing, ordinary allocation conflict, and exogenous hardship.

3 Relational Structure and Scope Conditions

This section defines the actors, resource categories, and control relations needed for the model. It then distinguishes resource capture from voluntary sharing and specifies the meaning of independent capacity. These distinctions prevent the accounting assumptions from carrying an implicit moral judgment.

3.1 Actors, Transactions, and Effective Control

Let A denote an actor with effective control over a domain of B 's activity, B an adult recipient, and C an external contributor or transaction partner. The roles describe positions in a relationship. They do not imply particular genders, clinical characteristics, or a unified interest among all members of a family. A coalition could occupy the role of A , provided its decision process were specified in an application.

The transaction between B and C may involve wages, a gift, assistance, or a joint project. The legal and normative character of these transactions differs. The formal model considers a flow that has become available for B 's use and asks how it is subsequently allocated. In a wage case, that flow represents earnings after the relevant work. The formulation assigns C no continuing authority over those earnings. In a joint project, the modeled flow must be B 's allocated portion or another clearly identified resource stream; it cannot simply be equated with the entire project budget.

Effective control means a continuing capacity to alter the costs or feasibility of B 's decisions in the relevant domain. Its indicators might include control of access to funds, enforceable restrictions on relocation, or credible penalties for independent transactions. Formal account ownership and expressed preference are insufficient proxies. A person may own an account but face coercive demands concerning its use; another person may delegate account management voluntarily while retaining effective revision and refusal.

3.2 Capture, Investment, and Resource Conservation

Resource capture denotes a transfer obtained under effective restrictions on refusal or revision in the domain being studied. A consensual contribution to household expenses can have an identical monetary value while belonging to a different relational process. Classification therefore requires evidence about the transfer conditions. The model begins after those conditions have been assessed.

Let $R > 0$ be a constant external flow, T the captured portion, and X the portion retained by B . These quantities are measured in the same resource units per unit time. Let $p \geq 0$ be a control stock, $\lambda(p)$ the captured fraction, and $\rho \in [0, 1]$ the fraction of T invested in maintaining control. The resource and investment accounts appear in Equation 1.

$$T = \lambda(p)R, \quad X = [1 - \lambda(p)]R, \quad R = X + T, \quad Z = \rho T. \quad (1)$$

The remainder $(1 - \rho)T$ is available for other uses by A . Consequently, a transfer to A establishes neither the size of Z nor the effectiveness of that investment. Expenditure, substitution of other funds, and changes in control instruments would need to be traced separately.

The model concerns a single commensurable resource. Time, income, reputation, and emotional support cannot be added without a defensible measurement rule. Applications involving several resources should use separate accounts and specified conversion processes. Fungibility also matters: externally provided funds may replace expenses that A would otherwise finance, indirectly freeing other resources for control. Such substitution changes the empirical interpretation of Z and cannot be inferred from the recipient's bank balance alone.

3.3 Independent Capacity and Reachable Possibilities

The capacity variable $g \geq 0$ represents an operational index of resources and competencies that B can effectively use in a specified domain. It might concern maintaining independent housing, choosing among feasible employment arrangements, or sustaining communication under conditions of refusal. Its empirical components and units would need to be defined before estimation. The variable is neither a diagnostic scale nor a complete welfare measure.

The distinction between resources and capabilities has a substantial precedent in Sen (1992). Relational approaches likewise locate autonomy within social conditions that can support or impede agency (Mackenzie and Stoljar, 2000). The present interpretation concentrates on the time path by which usable capacity changes. An individual may acquire money while losing the ability to direct work or movement, or may receive extensive care while retaining substantial decision authority.

The phrase *generative landscape* can describe a set of feasible future trajectories under specified support, institutional, and relational conditions. A defensible comparison must name those conditions. Comparing actual dependence with an imagined world of unlimited independence would provide an unsuitable counterfactual. The scalar g is a limited indicator relevant to some trajectories, with no claim to measure the volume or moral value of a person's possible futures.

4 Baseline Model and Stationary Resource Effects

This section presents the baseline dynamics, proves the existence and stability of their stationary states, and derives the conditional resource effect. It then examines two changes in assumptions. The purpose is to identify the precise mechanism responsible for the adverse result.

4.1 Dynamic Specification and Assumptions

Control investments produce effective control at rate κZ , while the existing stock depreciates at rate δp . Set $\alpha = \kappa\rho$. The main results assume $\alpha, \delta, K, \beta, \mu, \gamma > 0$ and $s \geq 0$. The parameter K controls the scale at which capture becomes effective; β converts retained resources into capacity; μ represents ordinary capacity depreciation; and γ represents the additional attrition of usable capacity under control. The constant s supplies an independent baseline of capacity formation.

A saturating capture function and the dynamic equations are specified in Equation 2.

$$\lambda(p) = \frac{p}{K + p}, \quad \dot{p} = \alpha R \frac{p}{K + p} - \delta p, \quad \dot{g} = s + \beta R \frac{K}{K + p} - (\mu + \gamma p)g. \quad (2)$$

The time derivatives describe adjustment under a sustained resource flow. The first equation closes the feedback from control to capture and from invested capture to control. The second permits retained resources to form capacity while control raises the difficulty of maintaining its independent use. Such attrition could represent repeated displacement of self-directed activity or the additional resources required to preserve access. Its substantive interpretation requires evidence.

The positive investment share and the effect of control on g are assumptions. They are the mechanisms being investigated. Neither follows from kinship, dependency, or the observation of a transfer. In particular, care financed by a family actor may raise s , improve β , or reduce effective attrition. Those possible benefits are held fixed in the baseline comparison, and must be measured in an application.

Equation 2 is triangular: p affects g , while g does not feed back into capture. The system therefore captures one form of endogenous control reproduction. It leaves bargaining responses, strategic withdrawal, and capacity-dependent resistance for extensions. This restricted structure is valuable because the principal result can be located in identifiable assumptions.

4.2 Feasibility and Equilibrium Selection

The feasibility analysis establishes that the model preserves nonnegative states and has bounded trajectories. At $p = 0$, control remains zero. At $g = 0$, the derivative of capacity is nonnegative. Moreover, $\dot{p} \leq \alpha R - \delta p$ and $\dot{g} \leq s + \beta R - \mu g$. These inequalities bound both stocks for finite nonnegative initial states.

Define the persistence threshold as $R_c = \delta K / \alpha$. The stationary states selected by positive initial control are given in Equation 3.

$$(p^*, g^*) = \begin{cases} \left(0, \frac{s + \beta R}{\mu}\right), & R \leq R_c, \\ \left(\frac{\alpha R}{\delta} - K, \frac{s + \beta \delta K / \alpha}{\mu + \gamma(\alpha R / \delta - K)}\right), & R > R_c. \end{cases} \quad (3)$$

Proposition 1 (Persistence threshold). *For every $p(0) > 0$ and $g(0) \geq 0$, the baseline system converges to the corresponding state in Equation 3. Below the threshold, control converges to zero. Above the threshold, the positive control equilibrium is asymptotically stable and the zero-control equilibrium is unstable to positive perturbations. At the threshold, control converges to zero at a non-exponential rate.*

The proof appears in Appendix A. The threshold expresses the balance between reproduction and depreciation near zero control. Its interpretation concerns the persistence of a control technology already present in the relationship. When $p(0) = 0$, the baseline remains on that boundary even above R_c . A background-input extension below removes this absorbing boundary.

The selected equilibrium is continuous at R_c . Stability changes there, but the baseline has no pair of stable attractors at the same R . Differences in initial states affect transient experience and convergence speed. They do not generate distinct long-run outcomes for positive initial control under fixed baseline parameters.

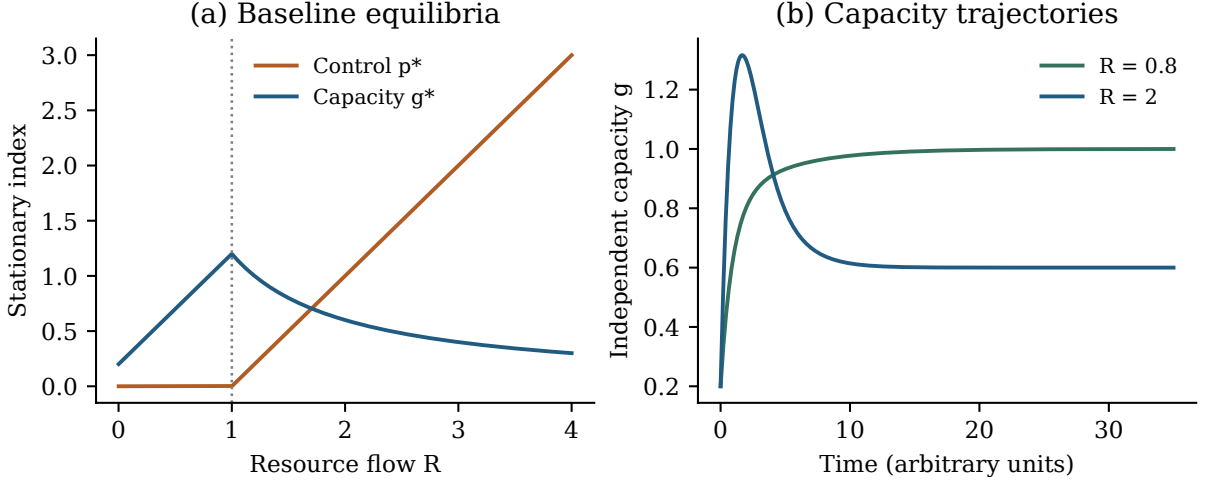
4.3 Resource Inflows and Independent Capacity

The comparative-static analysis examines a sustained increase in R while holding the other parameters fixed. On the positive-control branch, the equilibrium condition implies a constant retained flow, $X^* = \delta K / \alpha$. Additional resources then increase control and the captured amount. The resulting derivative is shown in Equation 4.

$$\frac{dg^*}{dR} = -\frac{\gamma \alpha}{\delta} \frac{s + \beta \delta K / \alpha}{[\mu + \gamma(\alpha R / \delta - K)]^2} < 0, \quad R > R_c. \quad (4)$$

Proposition 2 (Conditional adverse resource effect). *In the baseline model, stationary independent capacity increases with the resource flow below R_c and decreases with it above R_c . Above the threshold,*

Figure 1: Hypothetical baseline equilibria and adjustment paths. Panel (a) shows the selected stationary states for positive initial control. Panel (b) starts both paths at $p = 0.1, g = 0.2$. Parameters are $\alpha = \delta = K = \beta = \mu = \gamma = 1, s = 0.2$. All units are illustrative.



control increases, the captured fraction increases, and retained resources remain positive and constant. The adverse capacity effect requires $\gamma > 0$ and the maintained resource-to-control feedback.

This proposition follows directly from Equation 3. It describes a possible configuration in which beneficial retained resources coexist with an adverse marginal effect of a larger total inflow. It does not state that receiving resources is worse than receiving none. With $s = 0$, every finite positive-control equilibrium still has positive g^* . Comparing a transfer with complete withdrawal is a different question from comparing two sustained positive flows.

Figure 1 illustrates the stationary comparison and two transition paths. The illustration uses arbitrary normalized parameters and contains no empirical observations. For $\alpha = \delta = K = \beta = \mu = \gamma = 1$ and $s = 0.2$, the persistence threshold is one. A flow of 0.8 produces limiting capacity one, whereas a flow of two produces limiting capacity 0.6. The larger flow initially accelerates capacity formation from the stated initial condition, followed by a decline as control accumulates. This is a time-dependent effect, rather than a claim about every short-run response.

4.4 Capture Technology and Comparative-Static Generalization

The generalization identifies the condition behind the baseline result without relying on its particular capture function. Let λ be differentiable and consider a positive stationary state with local stability margin $D = \delta - \alpha R\lambda'(p^*) > 0$. Differentiating the equilibrium equations yields Equation 5.

$$p_R^* = \frac{\alpha\lambda(p^*)}{D}, \quad X_R^* = 1 - \lambda(p^*) - R\lambda'(p^*)p_R^*, \quad g_R^* = \frac{\beta X_R^* - \gamma g^* p_R^*}{\mu + \gamma p^*}. \quad (5)$$

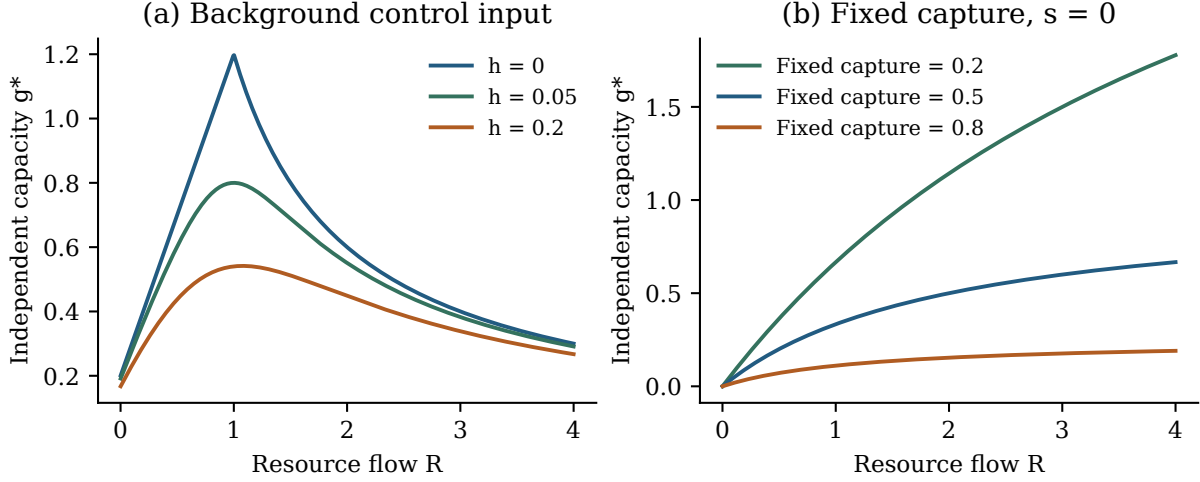
The adverse effect occurs when increased control-related attrition, $\gamma g^* p_R^*$, exceeds the contribution of additional retained resources, βX_R^* . This is a comparison of specified mechanisms. A larger external flow may improve both retention and capacity when the feedback is weak or the retained flow grows sufficiently quickly.

For a concrete counterexample, hold capture at a constant $\ell \in (0, 1)$ and set $s = 0$. The stationary control stock is $\alpha\ell R/\delta$, and capacity is given in Equation 6.

$$g^*(R) = \frac{\beta(1-\ell)R}{\mu + \gamma\alpha\ell R/\delta}, \quad \frac{dg^*}{dR} = \frac{\beta(1-\ell)\mu}{(\mu + \gamma\alpha\ell R/\delta)^2} > 0. \quad (6)$$

Resource-financed control remains present in this example, yet greater resources increase independent capacity. The comparison demonstrates the analytical importance of endogenous capture. With $\rho = 0$,

Figure 2: Sensitivity to maintained assumptions. Panel (a) varies background input h with the baseline parameters used in Figure 1. Panel (b) fixes capture at the indicated fraction and sets $s = 0$, with all other coefficients equal to one. The specifications illustrate different signs of the stationary resource effect.



control receives no investment from transfers and decays in the baseline. With $\gamma = 0$, baseline capacity is constant on the positive-control branch. These limiting cases also identify mechanisms an empirical study would need to distinguish.

4.5 Background Control and Local Robustness

The background-input extension evaluates the assumption that control can reproduce only from captured resources. Add a constant input $h > 0$ to the control equation. This input may represent resources originating elsewhere, without assigning it a particular social source. The positive stationary stock is shown in Equation 7.

$$p_h^* = \frac{h + \alpha R - \delta K + \sqrt{(\delta K - h - \alpha R)^2 + 4\delta h K}}{2\delta}, \quad g_h^* = \frac{s + \beta R K / (K + p_h^*)}{\mu + \gamma p_h^*}. \quad (7)$$

This extension has a unique positive control equilibrium, which is attracting. The adverse effect on the interior baseline branch persists for sufficiently small h by continuity away from the threshold. It is also present at sufficiently large R for fixed h , as demonstrated in Appendix B. The sharp baseline corner is smoothed, and a universal threshold equal to R_c no longer characterizes the full system.

Figure 2 displays the background-input extension alongside the fixed-capture counterexample. Taken together, the cases show that the adverse effect survives one relaxation while changing under another. They are a sensitivity analysis of assumptions, with no calibration to a population.

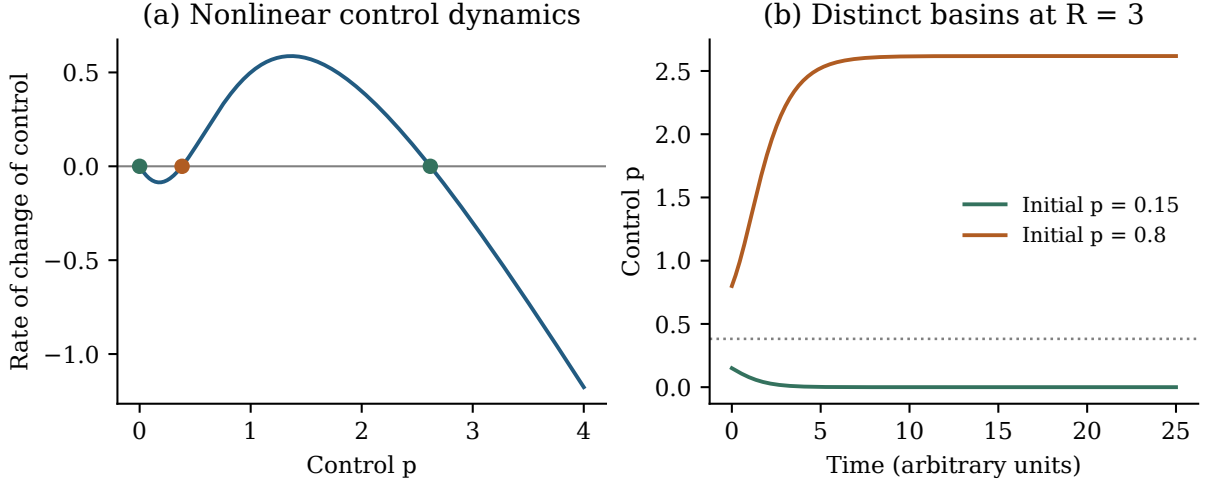
5 Increasing Returns and Multiple Stable Regimes

This section establishes the additional conditions under which control can exhibit distinct basins of attraction. It changes the capture technology while preserving the resource account and capacity equation. The distinction matters because positive feedback alone leaves the baseline with a single attracting outcome for positive initial control.

Suppose that capture requires complementary control instruments. Access to a resource account, for example, may become substantially more effective when accompanied by influence over housing or communication. A smooth representation of increasing returns at low control is $\lambda_2(p) = p^2 / (K^2 + p^2)$. Substituting this function yields Equation 8.

$$\dot{p} = \alpha R \frac{p^2}{K^2 + p^2} - \delta p, \quad \dot{g} = s + \beta R \frac{K^2}{K^2 + p^2} - (\mu + \gamma p)g. \quad (8)$$

Figure 3: Multiple attractors in the increasing-returns extension. Parameters are $\alpha = \delta = K = \beta = \mu = \gamma = 1$, $s = 0.2$, and $R = 3$. Green markers in panel (a) denote stable control equilibria and the orange marker denotes the unstable threshold. The dotted line in panel (b) indicates that threshold. This property belongs to the nonlinear extension.



Complementarity is an additional substantive assumption. The equation does not establish that such complementarity characterizes familial control. Its value is to specify what would make the stronger dynamic claim possible.

The nonzero stationary control stocks are given in Equation 9.

$$p_{\pm} = \frac{\alpha R \pm \sqrt{\alpha^2 R^2 - 4\delta^2 K^2}}{2\delta}, \quad R > R_{\text{SN}} = \frac{2\delta K}{\alpha}. \quad (9)$$

Proposition 3 (Multiple stable control regimes). *For $R > R_{\text{SN}}$, Equation 8 has stable control equilibria at zero and p_+ , separated by the unstable equilibrium p_- . Initial control below p_- converges to zero; initial control above p_- converges to p_+ . Capacity converges to the corresponding value determined by its stationary equation. At $R = R_{\text{SN}}$, the positive equilibria meet at a saddle-node; below it, all nonnegative initial control converges to zero.*

Appendix C proves the result and describes the equality case. In the full two-dimensional system, the state associated with p_- is a saddle. Its stable set is the vertical line $p = p_-$. This geometry follows from the absence of feedback from g into p .

Figure 3 illustrates the distinct regimes at $R = 3$ with normalized coefficients. The positive roots are approximately 0.382 and 2.618. Two trajectories with different initial control reach different long-run control stocks despite an identical external flow. This is a precise form of dependence on initial conditions.

A temporary perturbation can have persistent consequences if it changes the state sufficiently to cross the separating threshold while the post-perturbation parameters again support two attractors. The model does not establish a conventional hysteresis loop between two finite parameter thresholds. Zero control remains locally stable for every finite R in this specification. Claims about a complete hysteresis cycle would require a further model and proof.

The distinction also qualifies intervention language. Moving a stock below a computed threshold is a mathematical statement about this particular specification. Implementing an intervention requires knowledge of the control instruments, possible responses, costs, and the recipient's priorities. None of those quantities can be recovered from the diagram alone.

6 Genesis and Risk Conditions

This section separates the emergence of a controlling relationship from the reproduction process described by the equations. It reviews selected findings concerning psychological and relational antecedents, then formulates hypotheses about institutional opportunity. The analysis concerns possible mechanisms and avoids diagnostic classification.

6.1 Initial Asymmetry and Relational Development

Childhood dependence provides one route to an initial asymmetry in access to resources, information, and decisions. Whether that asymmetry persists into a particular adult domain depends on subsequent opportunities, support needs, institutions, and relationship practices. The model represents a given initial condition; it supplies no account of its historical origin.

Boundary-dissolution research provides differentiated constructs for examining relational development. Thompson et al. (2024) review 478 studies and distinguish enmeshment, disorganization, caregiving, and coerciveness, with differing associations with psychological difficulties. Their findings support careful separation of these patterns. They do not identify the probability that any pattern leads to the resource-investment loop studied here.

The direction of control may also change across the life course. A younger relative can acquire power over an older family member through financial or care arrangements. The actor labels permit that reversal, but evidence about one direction cannot establish prevalence or mechanisms in the other. An empirical application should define the relationship, developmental period, and domain before measurement.

6.2 Unfulfilled Goals and Self-Related Motives

The literature on parental aspirations provides a possible motivational link between a child's outcomes and a parent's interests. In an experiment involving 73 parents, Brummelman et al. (2013) find that reflecting on unfulfilled ambitions affects reported wishes for a child to fulfill those ambitions among parents who include the child more strongly in their sense of self. The study measures a desire; its results leave the translation into controlling practices and extraction unresolved.

Ng et al. (2014) examine contingent self-worth and psychological control using two waves of reports from mothers and children in China and the United States. Their study supplies evidence relevant to the connection between a parent's self-evaluation and controlling behavior. Its sampling, measures, and design constrain population and causal inferences. Cultural group differences in the study provide no basis for classifying individual families by nationality.

The corresponding hypothesis is conditional: when an actor values another person's achievements as a source of status or compensation, and possesses effective instruments for directing those achievements, the private incentive to preserve control may increase. Shared ambitions can also motivate supportive investments chosen by the recipient. Motive, instrument, and effect must therefore be investigated separately.

6.3 Institutional Opportunity and Feedback

Institutional opportunity determines whether a motive can be converted into persistent control. Relevant conditions may include access to funds, control of indispensable services, dependence on a single network, weak possibilities for revising arrangements, and sanctions for refusing transfers. These conditions could affect K , ρ , or δ , but the parameter mapping remains an empirical task.

A proposed genesis sequence should also accommodate alternative explanations. Limited employment may arise from local labor-market conditions; an adverse health event may increase both assistance and decision support; poverty may constrain both generations. Such common causes can produce correlations between dependence and family involvement without control-generated deprivation. A temporal

Table 1: Measurement domains and competing interpretations. Indicators are research proposals and require validation in the selected population.

Model component	Candidate observation	Competing interpretation
External flow R	Recipient earnings or support, dated and classified by transaction	Joint funds or borrowed money incorrectly recorded as individual income
Capture T	Transfer accompanied by credible penalties for refusal or limited ability to revise	Voluntary sharing, repayment, or agreed cost allocation
Control investment Z	Captured resources finance access restrictions or substitute for their prior funding	Consumption by A or care that improves B 's opportunities
Control p	Effective influence over specified decisions and consequences of refusal	Consensual delegation or justified decision support
Independent capacity g	Ability to sustain selected activities under refusal or changed arrangements	Income, satisfaction, and account ownership used as insufficient substitutes

sequence of control followed by reduced opportunity is informative, yet still requires a credible counterfactual.

Interpretive pressure may reduce reporting or make resource transfers appear voluntary in administrative records. It may also be weak, contested, or irrelevant to a particular mechanism. A study should assess access to independent advice and the consequences of disagreement directly. The framework's descriptive vocabulary must remain open to accounts from the recipient that revise the researcher's initial interpretation.

7 Measurement and Empirical Assessment

This section translates the model into observable implications and distinguishes the evidence needed for description, causal explanation, and model testing. It proposes a longitudinal strategy rather than treating hypothetical examples as an empirical case. The full mechanism remains a research hypothesis.

7.1 Observable Components and Rival Interpretations

An empirical study would record transaction types, effective allocation powers, imposed transfers, control investments, and changes in independently usable capacity. Repeated observations are essential because an immediate benefit can coexist with a later adverse adjustment. Table 1 connects the principal model components to possible indicators and competing interpretations.

Capture should be assessed with sensitivity to coercion and limited alternatives. A recipient may describe a contribution as voluntary while reporting severe consequences of refusal; another may freely endorse obligations despite substantial material costs. Both descriptions require attention. The researcher's preference for financial separation cannot serve as the coding rule.

7.2 Temporal Predictions and Identification

The baseline proposes a sequence: an external increase raises the amount available for capture; invested capture subsequently maintains control; control increases the capture fraction and affects usable capacity. Failure at any link weakens the complete mechanism. An increase in family transfers alone provides evidence only about allocation.

Several predictions distinguish the model from a fixed-sharing account. Capture fractions should respond to effective control in the relevant domain. Transferred resources should contribute to the maintenance of control instruments. Capacity changes should follow the predicted adjustment pattern after

accounting for direct benefits. If a credible change reduces appropriability while preserving support, the allocation and capacity response should differ from an otherwise similar increase in gross resources.

Identifying these effects requires more than correlating earnings and dependence. Earnings can respond to control; anticipated extraction can change effort or concealment; and a common shock can affect all measured variables. Longitudinal within-person evidence can improve temporal resolution but remains vulnerable to time-varying confounding. Changes in payment arrangements, account access, or institutional support may offer useful comparisons only if their accompanying changes are understood. The ethical and practical case for any intervention must be assessed independently of its analytical convenience.

The nonlinear extension makes a stronger demand. Observing persistence does not establish multiple attractors. Evidence would need to support the proposed nonlinearity, distinguish transient adjustment from durable regimes, and address unobserved heterogeneity. Two families at different outcomes under similar incomes may have different parameters. They need not occupy separate basins of the same dynamic system.

7.3 Research Ethics and Evidential Status

The present paper contains no participant data and reports no empirical causal estimate. Its examples are constructed to explain concepts. Personal experiences can generate research questions, but converting them into a case study would require a defined evidential method, independent corroboration where appropriate, and careful consideration of confidentiality and consent.

Prospective data collection should minimize information that could expose a participant to retaliation or identify a private relationship through a distinctive combination of events. Participants' disagreement with the model must remain reportable. A design restricted to people already endorsing the proposed mechanism could describe those accounts while providing a weak test of its scope. Comparison cases involving supportive interdependence and voluntary sharing are analytically necessary.

8 Discussion and Analytical Limits

This section evaluates the model's explanatory reach and distinguishes its results from normative and intervention claims. It concentrates on what the formal structure clarifies, what remains assumed, and what further work would materially strengthen the argument.

The principal result is transparent because the retained flow saturates on the positive-control branch while control continues to rise. That transparency is an advantage for interpretation and a limitation on novelty. A mathematical derivation does not establish that the functional form is empirically apt. The general condition in Equation 5 and the counterexample in Equation 6 locate the argument at the level of mechanisms rather than a preferred result.

A stronger political-economic account would endogenize investment in control, resource production, and recipient responses. An actor who extracts excessively may reduce future production, induce exit, or incur enforcement costs. A recipient may conceal resources, change collaborators, accept some family obligations, or negotiate different terms. Existing kinship-sharing studies make those responses especially salient (Baland et al., 2011; Jakiela and Ozier, 2016). Their incorporation could change the stationary resource effect and the stability analysis.

The capacity index also limits interpretation. A decline in g concerns the selected domain under the stated measurement rule. It cannot establish that total welfare, subjective well-being, or every available future declines. Family care may improve some dimensions while restricting others. A multidimensional extension should preserve those differences rather than aggregate them through arbitrary weights.

The model offers one explanation of persistence without requiring a comprehensive plan by the controlling actor. Yet that explanatory economy cannot establish motive or excuse conduct. The use of a reduced-form control process leaves individual knowledge, justification, and responsibility open. Normative evaluation requires premises concerning agency, coercion, fair obligations, and the recipient's

standing.

The introduction of C identifies a possible route through which resources enter the relationship. Whether C has an independent complaint depends on the transaction and the conduct affecting it. Unknown spending after an unconditional transfer differs from interference with a shared project or deception about an undertaking. The economic mechanism is compatible with these different normative configurations. A separate jurisprudential treatment is therefore warranted.

Intervention implications are limited to identifying parameters and links worth investigating. Reduced appropriability, increased access to independent support, and lower conversion of resources into control could alter the dynamics. In practice, each may be offset by substitution, retaliation, withdrawal of beneficial care, or new dependence on the outside provider. A general recommendation to reduce resources would not follow from the adverse marginal effect. The recipient's chosen relationships and goals must inform any evaluation.

Finally, colonial vocabulary performs no necessary role in the derivation. Directed activity and extraction have analogues across several institutional structures. Historical classification would require additional evidence about the features that make a relation colonial. The familial model can be assessed on its specified mechanisms independently of that comparison.

9 Conclusion

This section consolidates the formal and empirical implications of the analysis. Resource capture can contribute to the persistence of familial control when a portion of captured resources sustains instruments that influence subsequent capture. Under a specified saturating technology, the model has a persistence threshold and an adverse stationary resource effect on a limited index of independent capacity. A fixed-capture comparison yields a beneficial effect, while a separate increasing-returns extension supports multiple stable regimes.

These findings define a conditional theoretical account. Their application requires evidence about transfer conditions, resource conversion, control instruments, and capacity formation over time. Existing work on household bargaining, economic abuse, psychological control, and kinship sharing supplies essential precedents and competing interpretations. The next substantive advance would be a design that can distinguish the proposed feedback from voluntary sharing, beneficial care, and common external constraints.

A Proof of Baseline Convergence

This appendix establishes convergence by analyzing the scalar control equation and then the driven linear capacity equation. It also identifies the stability change at the persistence threshold.

For $p > 0$, the sign of $\dot{p}$ is the sign of $\alpha R/(K + p) - \delta$. If $R < R_c$, that expression is negative everywhere, and p decreases to zero. If $R > R_c$, it is positive below $p^* = \alpha R/\delta - K$ and negative above it. Every positive initial state therefore converges monotonically to p^* . At equality, the control equation reduces to Equation 10.

$$\dot{p} = -\frac{\delta p^2}{K + p}, \quad R = R_c. \quad (10)$$

Every positive solution decreases to zero, with an asymptotic rate proportional to $1/t$. This explains the non-exponential threshold behavior.

The Jacobian is lower triangular. Its eigenvalues at the zero-control equilibrium are $\alpha R/K - \delta$ and $-\mu$. At the positive equilibrium the first eigenvalue is $-\delta p^*/(K + p^*)$, while the second is $-(\mu + \gamma p^*)$. Both are negative. At R_c , linearization has a zero eigenvalue and the nonlinear sign argument supplies convergence.

Write the capacity equation as $\dot{g} = b(t) - a(t)g$, where $a(t) = \mu + \gamma p(t) \geq \mu$ and $b(t) = s + \beta R K/(K + p(t))$. These coefficients converge to $a^* > 0$ and b^* . The error $e = g - b^*/a^*$ satisfies a

stable linear equation with a forcing term tending to zero. Variation of constants gives convergence of e to zero. Combined with the control analysis and boundedness, this proves the stated result.

B Background-Input Equilibrium and Derivative

This appendix derives the positive equilibrium under background input and evaluates its comparative static. Multiplication of the stationary control equation by $K + p$ yields Equation 11.

$$\delta p^2 + (\delta K - h - \alpha R)p - hK = 0. \quad (11)$$

For $h > 0$, the product of the two roots is negative, so exactly one root is positive. The control derivative is positive at zero and negative above the positive root. This proves attraction and yields Equation 7.

At equilibrium, $p > h/\delta$ and the retained flow satisfies $X^* = K(\delta - h/p)/\alpha$. Differentiating capacity with respect to this positive equilibrium stock gives Equation 12.

$$\frac{dg_h^*}{dp} = \frac{\frac{\beta K h}{\alpha p^2}(\mu + \gamma p) - \gamma \left[s + \frac{\beta K}{\alpha} \left(\delta - \frac{h}{p} \right) \right]}{(\mu + \gamma p)^2}. \quad (12)$$

The positive root increases with R . As p becomes large, the numerator in Equation 12 tends to $-\gamma(s + \beta K \delta / \alpha) < 0$. Thus the adverse effect holds for sufficiently large R in this extension. Continuity of the interior root and its derivative as h approaches zero establishes local persistence of the negative derivative at every fixed baseline point with $R > R_c$.

C Proof of the Nonlinear Regime Structure

This appendix derives the positive equilibria and their stability in the increasing-returns extension. For $p > 0$, setting the control derivative to zero gives $\delta p^2 - \alpha R p + \delta K^2 = 0$, whose roots are Equation 9. The discriminant is positive exactly when $R > R_{SN}$.

The derivative of the control flow at a positive root can be simplified using the equilibrium relation. Its value is given in Equation 13.

$$f'(p^*) = \delta \frac{K^2 - (p^*)^2}{K^2 + (p^*)^2}. \quad (13)$$

The roots satisfy $p_- < K < p_+$, so the lower root is unstable and the upper root stable. At zero the slope is $-\delta$, giving local stability. Signs of the flow establish the claimed basins. The capacity equation converges by the same linear-equation argument used in Appendix A.

At $R = R_{SN}$, the repeated positive root is K . The flow equals $-\delta p(p - K)^2/(K^2 + p^2)$. Initial values in $(0, K)$ tend to zero; those above K approach K from above; and the root itself is stationary. Below the threshold the flow is negative for every positive p . These cases complete the proposition and identify the one-sided stability at the saddle-node.

D Numerical Reproducibility

This appendix records how the illustrations can be reproduced and what numerical checking establishes. The accompanying script `research/model_analysis.py` uses NumPy and Matplotlib, evaluates the analytical stationary states, and integrates the equations using a fixed-step fourth-order Runge–Kutta method. The standard plotted step is 0.01 in arbitrary time units. A halved-step comparison checks the displayed transient.

The script verifies resource conservation, equilibrium residuals, nonnegative simulated states, convergence away from the critical point, finite-difference agreement with analytical derivatives, and the two basins of the nonlinear extension. It also generates a stationary-value CSV and a machine-readable verification report. These checks concern implementation and algebraic consistency. The parameter values remain hypothetical and provide no calibration or empirical validation.

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