

From Fluid to Braid: A Relational-Ontological Model of Value and Power, and Its Computational Horizon

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Abstract

We develop a formal model of how value and power circulate in relational systems, in which relations are prior to the entities they relate. A fluid-dynamical description yields one exact result—the appropriative, gradient part of value transport contributes nothing to circulation, so genuine relational value is non-possessive—but is shown to be ontologically self-undermining, since it presupposes a background and conserves what value must be free to generate. We resolve this by taking a symmetry pre-structure G as primary and letting spontaneous symmetry breaking $G \rightarrow H$ generate the rest: the order-parameter space arises as $M = G/H$, subjects are its topological defects $\pi_k(G/H)$, the surplus of a closed cycle is the holonomy on G/H (matching the Marxian circuit $M-C-M'$, with the local field value as stock), and power is its non-commutativity, which holds exactly when H is non-abelian—whence power, resonance, and generativity flow from one condition. The braid group of defect statistics is derived rather than posited. We prove that normative discriminating power requires a representation that is non-abelian yet non-universal, draw the corollary that power is ineliminable wherever there is generativity (so justice is the shaping of its geometry, not its removal), add a dynamical layer and a computational horizon, and give a falsifiable, scale-invariant operationalization with a case study in educational ethics.

Keywords: relational ontology; spontaneous symmetry breaking; holonomy; non-abelian braid representations; theory of computation.

1 Introduction: the computational form of a relational ontology

The dominant metaphors through which modernity understands information, mind, and society are entity-first. The Turing machine reads and writes discrete symbols on a tape; the symbol is the primitive, and any relation among symbols is a derived fact about an external read-write head. Classical information theory counts distinguishable states of a substrate that is assumed already to exist. Graph-theoretic models of social structure begin with a vertex set V and append an edge set $E \subseteq V \times V$: vertices are given first, edges depend on them. In each case the formalism encodes a metaphysical ordering—*entities, then relations*—and that ordering is not innocent. It determines what the formalism can and cannot say.

This paper is concerned with systems whose value and power cannot be faithfully expressed within an entity-first formalism, because in such systems the ordering is reversed: the relation is prior, and what we call an entity—a subject, an institution, a stable locus of value—is a secondary condensation of relational activity. We take this *relational ontology* as a working commitment rather than arguing for it from first principles here; the argument for it belongs to a separate body of work on relational presence and generative justice. Our question is narrower and, we think, more tractable: *what is the right mathematical language for a relational ontology, and what model of computation does that language imply?*

The paper proceeds by a deliberate failure. We first build the most natural continuous model of value and power circulation—a fluid-dynamical one (Section 2)—and extract from it one

genuine and, we believe, philosophically important result about the non-possessive character of relational value. We then show (Section 3) that the fluid model undermines itself: its dynamics are wrong for value (which is not conserved) and its tacit ontology, though closer to the relational commitment than the graph's, still smuggles in a fixed background space. The diagnosis of *why* the fluid and the neural/graph formalisms each meet one of the two requirements and violate the other (Section 3) points to the resolution: not a new formalism bolted on, but a relational *pre-structure*—a symmetry group G —from which structure arises by spontaneous symmetry breaking. We develop this foundation (Section 4): breaking $G \rightarrow H$ generates the order-parameter space $M = G/H$, subjects are its defects $\pi_k(G/H)$, value is the holonomy of the canonical connection on G/H (the surplus of a closed cycle, as against the local value stock), power is its non-commutativity when H is non-abelian, and the braid group of defect statistics is derived as the fundamental group of the defect configuration space. We do not explain why the symmetry breaks, marking this as out of scope, so the theory is post-breaking. We then show that on the resulting fixed-number braid representation three features of relational value—power, resonance-without-fusion, and generativity—each independently force the representation to be non-abelian, and trace their common structural source to the single condition that H be non-abelian (Section 5.3). Generation being only half of a complete model, Section 5 adds the dynamical layer—how value, once generated, flows and circulates on the structure—and it is here that the fluid description of Section 2 is reinstated in the role it is fit for, with Proposition 2.1 as the founding theorem of the flow. We then prove the discriminating-power constraint: the representation must also be non-universal (Section 6). Section 7 introduces the skeleton/flesh distinction that prevents the model from over-reaching, and Section 8 locates its computational horizon. Section 9 pushes the construction to its limit, making the base space itself relational—a group, and more properly a category—in contact with group field theory, and states with a toy model both what this buys and the three open walls it meets. Section 10 turns to evaluation, giving a scale-invariant quantitative operationalization—measuring the sign, order, and non-commutativity of the holonomy rather than its forbidden magnitude—and a falsifiable case study in educational ethics. Section 12 returns the abstract apparatus to the concrete question that motivates it: how two meaning-worlds can couple without being merged.

A note on method. Because the paper moves repeatedly between mathematical statements and philosophical ones, we fix a discipline for the bridge between them and hold to it throughout. We distinguish three grades of claim. A *theorem* (or proposition) is a mathematical fact about the formalism, true or false independently of interpretation; Proposition 2.1 and Proposition 6.1 are of this kind. An *argument from expressive adequacy* is a conditional of the form “if feature X of relational value is to be representable in the formalism, then the formalism must have property P ”; its mathematical half is exact, but its force depends on a substantive premise—that X is real and non-trivial—which the formalism cannot establish and which an opponent may deny. The three results of Section 5.3 are of this kind, and we state each premise explicitly. A *conjecture* is a philosophical thesis the formalism motivates but does not establish, such as the proportionality thesis about the good in Section 7. We do not let a theorem’s exactness silently transfer to the philosophical thesis it is paired with; where the formalism proves less than the philosophy wants, we say so and locate the gap rather than paper over it. The two places where the formalism falls visibly short of the philosophy—the existence of a normatively interpretable invariant (Section 6) and the token-identity of same-type subjects (Section 4, Section 7)—are treated as the paper’s principal results about its own limits, not as embarrassments to be hidden.

2 A fluid model of value and power

We begin with the most natural continuous model of value and power, a fluid-dynamical one, and treat it as a first attempt to write the relational *field theory* that Section 4 will make

primary—an attempt that fails instructively because it chooses the wrong kind of field theory, one that conserves and fixes its background. Let the relational medium be, provisionally, a domain $\Omega \subseteq \mathbb{R}^d$ with a value density $\rho(x, t) \geq 0$ and a value velocity field $\mathbf{u}(x, t)$. The choice to make the *velocity*, rather than the density, the primary object is deliberate: it is the first concession to the relational ontology, since on that ontology the flow is prior and the “amount” of value is a quantity derived from the flow. We will find that the fluid field theory is wrong on two counts—it conserves what should be generated, and it fixes the background Ω that should itself be dynamical—and that locating these two faults is what fixes the requirements on the correct relational field theory of Section 4.

2.1 Continuity with sources: value is not conserved

Value genuinely emerges and is genuinely extinguished. We do not model this as the transport of an antecedently conserved stuff, but we postpone its correct treatment (as a phase transition against a background manifold) to Section 3; here we record only that the continuity equation must carry a source term,

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = S(\rho, \psi), \quad (1)$$

where S is not an externally imposed influx but a generation/extinction rate governed by an order parameter ψ encoding the degree to which background potential is locally realized. The mere presence of $S \neq 0$ already marks the departure from fluid dynamics proper, whose entire mathematical legitimacy rests on $S \equiv 0$. We return to this in Section 3.

2.2 Momentum: appropriation, inertia, and the power force

The flow is driven by three contributions:

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla P(\rho) + \nu \nabla^2 \mathbf{u} + \mathbf{f}_{\text{pow}}. \quad (2)$$

We read each term in the relational register.

- $-\nabla P(\rho)$, the *pressure gradient*, is the appropriative drive: value flows “downhill” from where it is dense to where it is sparse. It is, by construction, the gradient of a scalar potential.
- $\nu \nabla^2 \mathbf{u}$, the *viscosity*, is relational inertia or historical drag. Large ν smooths the flow toward laminar, frozen structure; small ν permits turbulence—the continual generation of new structure. The viscosity is the solidification parameter.
- $\mathbf{f}_{\text{pow}}$ is the *power force*: the power field acts as an external body force biasing the direction of value flow. It is the coupling channel between value and power.

2.3 Power as the history of value flux

Power is not a conserved stock that is transported; it is a gradient continually rewritten by the history of value flux. Writing $\mathbf{J} = \rho \mathbf{u}$ for the value flux, the minimal law is

$$\partial_t \mathbf{p} = \alpha \mathbf{J} - \beta \mathbf{p}, \quad (3)$$

with $\mathbf{f}_{\text{pow}} = \mathbf{f}(\mathbf{p})$. Equation (3) is the continuous analogue of a temporally asymmetric, local update rule: where value has recently and repeatedly flowed, the power gradient there is reinforced ($\alpha \mathbf{J}$), subject to a natural forgetting ($-\beta \mathbf{p}$). The ratio α/β is an order parameter for solidification: when β dominates, power cannot remember its own history and the system keeps flowing; when α dominates, value flux reinforces power gradients that in turn advect more value along the same direction, a positive feedback that locks the flow into a frozen, appropriative channel.

2.4 The central result: appropriation generates no circulation

The object of normative interest is not the velocity but its circulation. Define the value vorticity

$$\boldsymbol{\omega} := \nabla \times \mathbf{u}, \quad (4)$$

and, for a closed relational loop $\gamma = \partial\Sigma$, the value holonomy

$$\text{hol}(\gamma) := \oint_{\gamma} \mathbf{u} \cdot d\boldsymbol{\ell} = \int_{\Sigma} \boldsymbol{\omega} \cdot d\mathbf{A}, \quad (5)$$

the net value generated by traversing the loop once. The holonomy is the formal criterion distinguishing *generative* (good) circulation, in which a loop returns more than it took, from *degenerate* (bad) circulation, in which it returns to a fixed point.

Proposition 2.1 (Appropriation is irrotational). The pressure-gradient (appropriative) term in (2) makes no contribution to the evolution of vorticity, and a purely appropriative flow $\mathbf{u} = -\nabla\phi$ has $\text{hol}(\gamma) = 0$ for every contractible loop γ .

Proof. Taking the curl of (2) and using $\nabla \times \nabla P = \mathbf{0}$ for any scalar field P , the pressure term is annihilated, yielding the vorticity equation

$$\partial_t \boldsymbol{\omega} + (\mathbf{u} \cdot \nabla) \boldsymbol{\omega} = (\boldsymbol{\omega} \cdot \nabla) \mathbf{u} + \nu \nabla^2 \boldsymbol{\omega} + \nabla \times \mathbf{f}_{\text{pow}}. \quad (6)$$

The pressure gradient is absent. For the second claim, if $\mathbf{u} = -\nabla\phi$ then $\boldsymbol{\omega} = -\nabla \times \nabla\phi = \mathbf{0}$, and by Stokes' theorem $\text{hol}(\gamma) = \int_{\Sigma} \boldsymbol{\omega} \cdot d\mathbf{A} = 0$ on any surface Σ bounded by a contractible γ . $\square$

Proposition 2.1 is the fluid model's one exact contribution, but its force depends on an interpretive premise that must be stated rather than hidden. The mathematics—that a gradient field is curl-free and contributes no circulation—is an identity, true by the structure of vector calculus. What converts the identity into a substantive claim about value is the prior modeling decision, made in (2), to identify *appropriation* with the pressure-gradient term, i.e. with the part of the driving force derivable from a scalar potential. That decision is defensible on independent grounds: appropriation is, conceptually, the reduction of value relations to a single scalar ordering (more or less, higher or lower), and a drive so reducible is by definition the gradient of that scalar. Granting the premise, the proposition states that circulation—hence the holonomy on which the good/bad distinction will rest—cannot be produced by any drive reducible to a scalar ordering. Inspecting (6), circulation can arise from only two sources: the vortex-stretching term $(\boldsymbol{\omega} \cdot \nabla) \mathbf{u}$, by which existing circulation amplifies itself (the self-maintenance of a generative cycle), and the curl of the power force $\nabla \times \mathbf{f}_{\text{pow}}$, by which an asymmetry not reducible to a scalar ordering injects circulation. The claim is therefore not that “possession is bad” as a moral primitive, but the sharper structural one: *whatever in a relation reduces to a single scalar comparison cannot, by itself, generate value around a closed loop*. This is informative precisely because much of what passes for relational value—accumulation, ranking, exchange at a price—is scalar-reducible, and the proposition denies that any amount of it sums to generation.

Remark 2.1. The result already shows why “power” and “the good” are not opposites in this framework. Power enters the vorticity equation only through its *curl*: a power field that is itself a pure gradient (a strict, totally ordered dominance hierarchy) is irrotational and generates no value circulation, whereas a power field with non-trivial curl—an asymmetry that does not reduce to a single potential—is a genuine source of generation. The distinction between dominating and generative power is thus, at this level, the distinction between the gradient and the solenoidal parts of $\mathbf{f}_{\text{pow}}$ under a Helmholtz decomposition, which is unique once suitable boundary or decay conditions are fixed on the domain.

3 The self-undermining of the fluid model

The fluid model yields one real result and then fails, and the manner of its failure is informative. The failure is ontological, and it has two layers, corresponding to the two halves of the model that pull in opposite directions.

3.1 The dynamical layer: non-conservation forbids fluid dynamics

Fluid dynamics is, mathematically, the theory of a conserved substance transported on a fixed space. Its continuity equation $\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0$ asserts that what is not present cannot flow in: mass is neither created nor destroyed, only moved. But value is created and destroyed. Encoding this required the source term S in (1), and $S \neq 0$ is precisely the violation of the conservation law that makes fluid dynamics fluid dynamics. In the fluid formalism the source is an external patch, a term one writes by hand because the dynamics cannot generate it; in an entity-first *dynamical* formalism such as neural dynamics, by contrast, generation is intrinsic, since a threshold unit produces an output whose energy comes from local metabolism rather than from the input. On the dynamical question—does the formalism allow value to be generated?—the entity-first neural picture is correct and the fluid picture is not.

3.2 The ontological layer: the relational commitment forbids entity-first

Yet on the *ontological* question the verdict reverses. Neural and graph formalisms are entity-first: the neuron, the vertex, is the primitive, posited with a location and an identity, and the synapse, the edge, is a derived relation depending on the prior existence of its endpoints. The graph $G = (V, E)$ encodes this in its very signature, V before E . A relational ontology cannot be founded on such a formalism without contradiction, for it holds that relations are prior to relata. The fluid picture, whatever its dynamical defects, is ontologically closer: a vortex is not a thing with a persisting substantial core but a pattern of the flow, with no interior that can be separated from its relational surround; as it moves, it is not the same water that moves but a pattern propagating through a medium whose constituents are continually exchanged. Form, here, is the secondary and temporary condensation of a prior formlessness—the *form of the formless*, in the classical figure of water. On the ontological question—is form prior to relation, or relation to form?—the fluid picture is correct and the entity-first picture is not.

3.3 The diagnosis

The two formalisms each meet one of the two requirements set out in Section 1—a dynamics admitting generation and a relation-prior ontology—and violate the other, and the requirements they meet are complementary.

	Dynamical requirement (generation, asymmetry, plasticity)	Ontological requirement (relation prior to entity)
Neural / graph	satisfied	violated
Fluid	violated	satisfied

The entries record not a verdict of correctness but whether each formalism’s constitutive commitments *satisfy* or *violate* the corresponding requirement: the graph posits entities prior to relations, violating the ontological requirement while admitting generation and plasticity; the fluid imposes a conservation law and supplies no causal attribution, violating the dynamical requirement while treating form as the condensation of a prior flow.

A finer reading of the dynamical column is needed, because “admits generation” conceals two grades. Fluid dynamics generates at no level: it conserves and transports. Neural and

graph dynamics generate at the level of *activity*—a spike is produced and extinguished—but on a fixed population of carriers; the neurons do not come and go, only their firings do. Neither, then, generates at the level of the *carrier*, which is what a relation-prior ontology ultimately demands: if subjects are not antecedent but themselves condensations of relation, their own coming-to-be and passing-away must be within the dynamics, not merely the coming-and-going of activity on fixed subjects. This third grade—generation of the carrier itself—is beyond both standard formalisms and is exactly what the field construction of Section 4 supplies, by making the subject a defect of the field rather than a node bearing activity.

The required formalism must therefore be *ontologically* of the fluid kind (form from formlessness, relation prior to entity) and *dynamically* stronger than the neural kind (generating not only activity but carriers). Neither standard fluid dynamics nor standard graph dynamics meets both requirements; each satisfies exactly the one the other violates, and even the one that gets the dynamics least wrong stops a grade short. This is the structural reason a single off-the-shelf formalism has felt inadequate, and it fixes the target precisely: a relational field theory whose topological defects are its subjects, whose defect creation is value generation, and whose background is not presupposed but—in the dynamical layer left open in Section 4.9—emergent. The remainder of the paper builds the kinematics of that field theory and extracts its normative consequences.

4 The relational field as primary; braiding as defect statistics

The reconciliation is to make the relational *field*, not any collection of nodes and not the braid group, the primary object, and to derive everything else—the order-parameter space, subjects, their statistics, value, and power—as secondary structure generated from a pre-structure by symmetry breaking. This ordering matters and was, in earlier formulations of this material, inverted twice over: beginning from the braid group presupposes a fixed number of strands, and beginning from a freely chosen order-parameter space M presupposes M itself as a given. We remove both by descending to a pre-structure and letting breaking generate the rest.

4.1 Pre-structure and the generation of the order-parameter space

We do not take the order-parameter space as given. We take as the pre-structure a symmetry group G —pure relation in the sense that its elements are individuated only by their composition, with no intrinsic content—and we let structure arise by *spontaneous symmetry breaking* of G to a subgroup $H \subseteq G$. The order-parameter space is then not chosen but generated, as the coset space

$$M = G/H, \tag{7}$$

the space of directions the broken state can take. This inverts the dependency of the earlier account: rather than positing M and asking what defects it supports, we posit the symmetry G and the broken subgroup H , and $M = G/H$ follows. Figure 1 shows the whole construction as a single chain with the epistemic status of each layer marked. Three points fix the epistemic status of this move.

First, the pre-structure G is homogeneous—every point alike, no point distinguished. This was, against a *background*, an objection; against a *pre-structure* it is exactly right, since “no distinguished point” is precisely what it means to be prior to all structure. Distinguished points—defects, subjects—appear only after breaking.

Second, we do *not* explain why the breaking $G \rightarrow H$ occurs. The cause of symmetry breaking is a metaphysical question on which traditions differ and which we do not adjudicate; in physics likewise the breaking is encoded in the shape of a potential or the crossing of a critical parameter, themselves taken as given. This paper is accordingly a *post-breaking* theory: the breaking is its input, not its conclusion. Everything to the right of the breaking arrow— M , subjects, value,

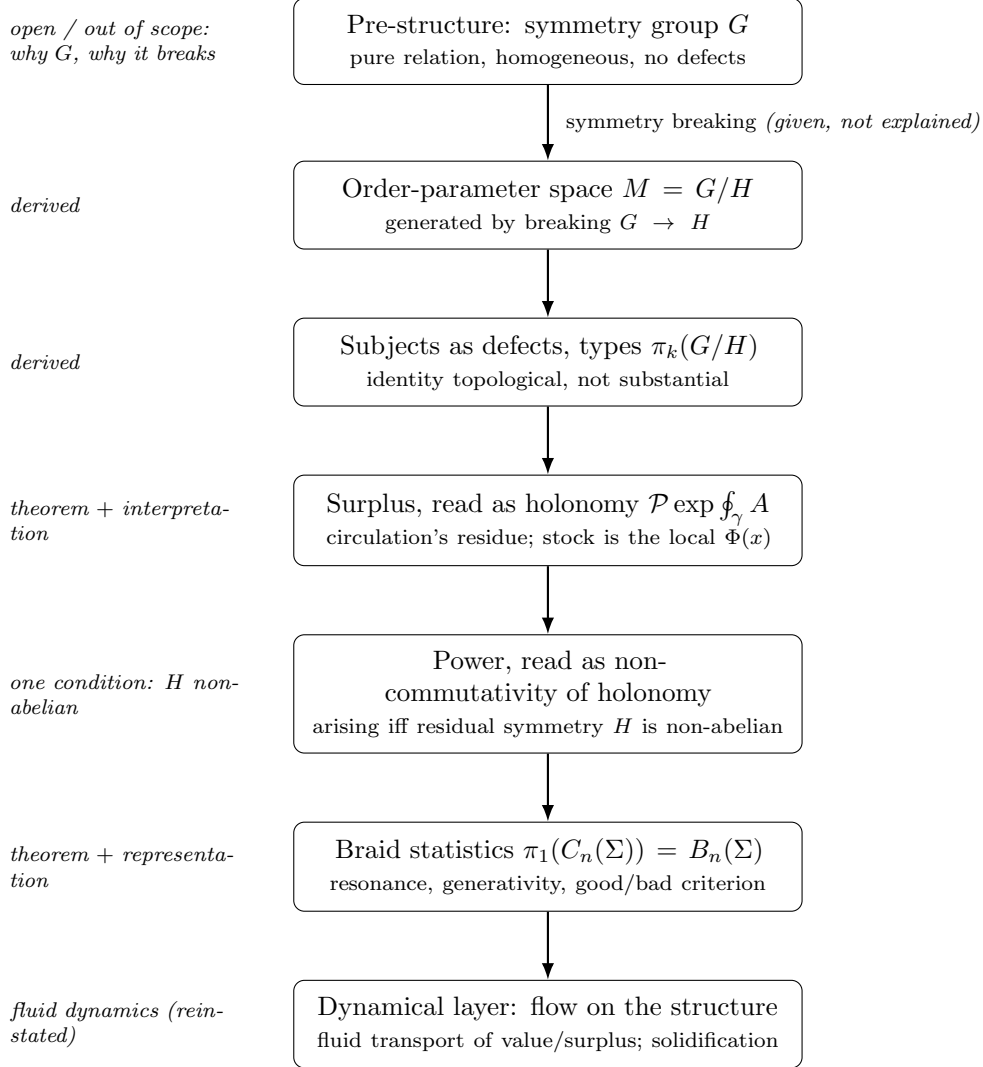


Figure 1: The model as a single chain. A symmetry pre-structure G breaks to H ; everything below the breaking arrow is generated structure, and the right-hand boundary of the theory is the breaking arrow itself, above which (the origin of G and the cause of breaking) lies outside the paper's scope. The first six layers are the *generation* of structure (the cosmogony); the final layer is its *running*—the dynamical flow of value on the generated structure, where the fluid description of Section 2 is reinstated in the role it is fit for (Section 5). The italic tags record the epistemic status of each layer: *derived* (a consequence of the layer above), *theorem* (a mathematical fact), *interpretation* (a philosophical identification the formalism cannot establish on its own), and *open*. The three premises that Section 5.3 would otherwise assume separately—power, resonance, generativity—contract to the single condition that H be non-abelian.

power—is its subject matter; everything to the left—why G , why it breaks—is marked as outside its scope. This single stipulation absorbs all the “where does it come from” regresses into one declared boundary.

Third, the generation is real but partial: $M = G/H$ is generated from G and the choice of H , but G itself is not generated here, and its own origin falls on the unexamined left of the breaking arrow. We claim only that no *order-parameter space*, and hence no structure of value, is presupposed—not that relation has been grounded without remainder. The remainder is named, not hidden (see Section 9).

4.2 Subjects as defects of the broken state

Once G has broken to H , the broken state is described by a field valued in the generated space,

$$\Phi: \Sigma \rightarrow M = G/H, \quad (8)$$

where Σ carries the relational connectivity of the configuration (not assumed a fixed physical space, and in particular not assumed two-dimensional). The decisive gain is that the classification of subjects is now *derived* rather than chosen: defects of the broken state are labelled by the homotopy of the generated space,

$$\text{defect types} = \pi_k(G/H), \quad (9)$$

for the codimension-appropriate k . This is the standard topological classification of defects in a system with broken symmetry: the order-parameter space is G/H and its homotopy groups enumerate the stable defects, a result with mature physical standing in the theory of ordered media and early-universe defect formation.

Definition 4.1 (Subject as defect). A *subject* is a localized excitation of the broken field Φ carrying a non-trivial class in $\pi_k(G/H)$. Its persistence is the conservation of that class; its type is the class itself.

A semi-concrete instance, now *derived* rather than stipulated: break G so that the residual relevant factor is H with $G/H \simeq S^1$; then $\pi_1(S^1) = \mathbb{Z}$ and subjects are labelled by an integer winding, a unit of relational winding whose persistence is the conservation of that integer rather than the endurance of any inner core—congenial to the no-self of Buddhist analysis and to the subject as a structural effect in Lacanian theory, but here a precise statement about $\pi_1(G/H)$. The integer is no longer an assumption about M but a consequence of which symmetry broke; a different H yields a different, possibly non-abelian, classification, and we will see that the non-abelian case is exactly where power and resonance live.

4.3 Value and power as generated structure on G/H

The coset space $M = G/H$ is not a bare set; the bundle $G \rightarrow G/H$ is canonically an H -bundle and so carries a canonical connection A . This is the point at which value and power cease to be posited and become generated structure. A first distinction must be drawn here, sharper than in earlier formulations. There are two quantities, not one, and they live at different levels. The *value stock* is the field value at a point, $\Phi(x)$: local, present at a place, in principle countable there. The *circulated surplus* is the holonomy a subject accumulates around a closed relational loop γ ,

$$\text{hol}(\gamma) = \mathcal{P} \exp \oint_{\gamma} A, \quad (10)$$

which is of an entirely different character: it is not present at any point but is the irreducible residue of traversing the whole loop, the failure of the state to return exactly to itself after

one circuit. We identify this holonomy not with value as such but with the *surplus generated by circulation*—what a closed relational cycle produces over and above what was put in. The structural template is the circuit $M \rightarrow C \rightarrow M'$ with $M' = M + \Delta M$: a passage that returns to its starting form yet not to its starting magnitude, the increment ΔM being exactly a holonomy, a non-return after a closed loop. The stock $\Phi(x)$ is what can be appropriated and held; the surplus $\text{hol}(\gamma)$ is what only a non-possessive circulation can generate.

The identification of surplus with holonomy is not a free choice of metaphor; three considerations constrain it. First, it is forced by Proposition 2.1. If one asks which mathematical quantity has the property that appropriation cannot produce it and only non-possessive circulation can, the answer is essentially unique: a quantity that vanishes on gradients and is sourced only by curvature is a holonomy. The interpretive freedom is thus narrower than it appears—having already proved (in the fluid model, independently of this section) that the good/bad quantity must be insensitive to the gradient/appropriative part, one is committed to a holonomy, because that is what “insensitive to gradients, sourced by circulation” means. Second, holonomy has the defining character of a surplus rather than a stock: it is path-dependent and irreducible to any point, present nowhere along the loop yet produced by the loop as a whole. This matches the economic notion of surplus as what a complete cycle yields over its inputs, not anything localizable in a single moment or commodity; and the closure of the loop—return to the starting form—encodes the non-possessive character directly, since an open, one-way path is the figure of appropriation (carrying off) whereas a closed circuit that nonetheless fails to return to its starting magnitude is the figure of generation (giving and receiving back with increase). Third, the identification is continuous with a long usage of geometric phase as the carrier of accumulated, non-local relational meaning; it relocates an object already in service rather than importing a new one. None of this proves that relational surplus *is* holonomy—no formalism can prove an interpretive identification—but it shows the identification is constrained on three sides rather than stipulated.

This two-level reading is what makes Proposition 2.1 precise rather than paradoxical. The earlier slogan “appropriation generates no value” was awkward, since appropriation plainly transfers value. The correct statement is about surplus: the part of the connection that is a pure gradient (reducible to a scalar—the form taken by appropriation, by ranking, by equivalent exchange) contributes nothing to (10), exactly as in Proposition 2.1. Appropriation transfers the stock but produces no surplus; only the non-flat, circulating, non-possessive part of the connection generates a non-zero $\text{hol}(\gamma)$. The good/bad criterion of the paper is accordingly a criterion on the *surplus*—its sign and its geometry—not on the stock, and this is the surplus that the non-possessive holonomy of Section 2 was tracking all along, now intrinsic to the generated space rather than imposed on it.

Power concerns not the magnitude of the surplus but its algebra. Whether the accumulated holonomy (10) is a mere phase or a genuine transformation—whether the order of traversal matters—is fixed by the residual symmetry H :

$$H \text{ abelian} \Rightarrow \text{hol commutes (no power)}; \quad H \text{ non-abelian} \Rightarrow \text{hol}(\gamma_1)\text{hol}(\gamma_2) \neq \text{hol}(\gamma_2)\text{hol}(\gamma_1). \quad (11)$$

Power—the order-dependence that Proposition 5.1 had to posit as premise P—is here *derived* from a single structural condition: that the symmetry surviving the breaking is non-abelian. The identification of power with non-commutativity is, like that of surplus with holonomy, constrained rather than chosen. Power, as distinct from mere resource or influence, has a definitional formal mark: it is the feature whereby the order of acts changes the outcome—an act lying upstream of another’s realization, so that “ A before B ” and “ B before A ” are not the same history. “Order changes outcome” has a precise mathematical name, non-commutativity; to accept the former characterization of power is already, on pain of inconsistency, to accept the latter as its form. The identification also has the right ontological grain: power so construed is not a stock held at a node but an algebraic property of how relational traversals compose—it is not possessed but

exercised, dispersed in the structure rather than accumulated at a point, which is precisely the relational, non-substantive conception of power familiar from Foucault. Non-commutativity is the algebra of the exercised, not of the owned.

This is the central economy of the symmetry-breaking foundation. The three features that Section 5.3 will otherwise install as independent premises—power (order-dependence), resonance-without-fusion (a degenerate multiplet under H), and generativity (multiple fusion channels of H -representations)—all flow from the single condition that H be non-abelian, supplemented only by the interpretive identifications (holonomy as surplus, H -multiplet as joint state) that no formalism can establish on its own. We do not claim the interpretive identifications are thereby proved; we claim that the *structural source* of the three features contracts from three assumptions to one symmetry condition.

4.4 A corollary: the inevitability of power, and what justice can mean

The identifications of Section 4.3 have a consequence for political philosophy that we draw out, because it follows from the structure rather than from any further premise. It takes the form of a structural dilemma and a corresponding reconstrual of what justice could be.

Power, on the present account, requires asymmetry twice over. The pre-structure G is homogeneous: in a state of full symmetry there is no distinction, no “this rather than that,” and hence no place for power, which needs an upstream and a downstream to obtain. The first asymmetry is therefore the breaking $G \rightarrow H$ itself, which is what lets the world contain distinctions at all—and with them, subjects. The second asymmetry is internal to what survives: only if the residual symmetry H is non-abelian does the holonomy fail to commute (11), so that order matters and power genuinely arises. Full symmetry yields no power because it yields nothing; broken symmetry with abelian residue yields subjects but no power, only commuting phases.

Proposition 4.1 (Power–generativity co-origination). Power (non-commutativity of holonomy), resonance-without-fusion (a degenerate multiplet), and generativity (multi-channel fusion) are three faces of one condition, that H be non-abelian (Proposition 5.1–Proposition 5.3). Consequently any relational structure capable of generativity necessarily carries power; a generative structure with strictly commuting holonomy is impossible, in the same way that a non-trivial holonomy with everywhere-vanishing curvature is impossible.

This yields a trilemma: of the three desiderata—(i) subjects and generativity, (ii) the absence of power, (iii) not collapsing to the structureless pre-state—no two can be had together with the third. The single variable of the degree of symmetry breaking binds them. To secure (i) one must break the symmetry, since the unbroken state contains neither subjects nor surplus. But breaking far enough to support rich subjects and generativity brings, by Proposition 4.1, a non-abelian residue and hence power, defeating (ii). The only routes to a strict (ii) are two, and each forfeits more than it saves. The first returns to full symmetry, $H = G$: no power, but no subjects, no surplus, no generativity—the equality of the void. The second breaks but holds H strictly abelian: subjects exist, but with commuting holonomy there is no multi-channel fusion and no degenerate resonance, so the encounter of two meaning-worlds can only superpose, never generate—a barren equality in which things exist but cannot make new things between them. Power and generativity share a single root, the non-abelian residue; to pull up the one is to pull up the other.

It is essential not to let this rest as fatalism, for it does not say that *domination* is inevitable, only that *power*—asymmetry, order-dependence—is. The distinction is the one drawn in earlier work between power-to and power-over, and it survives here with a formal hold. That H is non-abelian is not negotiable in a generative world; but the *geometry* of the resulting holonomy is. The same non-commutativity can accumulate as a generative asymmetry—an upstream position that serves the downstream’s own capacity, as in teaching, nurturing, protecting, guiding, where

the holonomy returns positive surplus that flows back and the asymmetry works toward its own eventual dissolution—or as a dominative asymmetry, where the upstream position fixes the downstream in dependence and the surplus is retained rather than returned. Both are equally cases of non-abelian H , equally power; they differ in the sign and the circulation of the surplus, exactly the good-cycle/bad-cycle distinction of Section 2.

Justice, on this picture, cannot consist in the abolition of power, since that is the abolition of generativity and reduces, by the trilemma, to one of the two equalities of privation. Justice is instead the shaping of an ineliminable asymmetry: the condition that the unavoidable power be generative rather than dominative—that its holonomy carry positive, returned surplus rather than retained, that no party be rendered mere fuel. The utopia of a powerless society is, in this formalism, not merely hard but structurally impossible; the achievable aim is a society whose power is shaped into the generative geometry. This converts the ethical distinction between power-to and power-over into a corollary of a structural theorem: because power cannot be removed, the only coherent form justice can take is the governance of its geometry.

4.5 What the identifications explain, and what they do not

Because the central moves of Sections 4.3 and 4.4 are interpretive identifications carrying real weight, it is worth stating plainly what they do and do not account for, so that their reach is neither overestimated nor quietly assumed.

They explain several things that an unstructured notion of value and power leave obscure. They explain why appropriation can transfer without generating—the gradient part of the connection moves stock but contributes no holonomy—and so why accumulation and equivalent exchange, however large, never by themselves amount to generation. They explain why value of the generative kind is non-localizable and path-dependent, residing in the cycle rather than in any holder. They explain why power is not a quantity one can abolish by redistribution: as an algebraic property of relational composition it is removed only by removing the structure that composes, which is generativity itself. And they explain, through the power–generativity co-origination of Proposition 4.1, why the dream of a generative yet powerless order is not merely unrealized but structurally closed, and why the achievable form of justice is the shaping of power’s geometry rather than its elimination.

They do not explain, and should not be read as explaining, the following. They do not derive which symmetry G or which broken subgroup H a given concrete relation realizes; the formalism classifies and constrains, but the identification of a real relationship with a particular G, H is not supplied and may have no canonical answer. They do not measure: the holonomy’s magnitude is defined relative to the connection A , and the present work fixes no empirical scale on which surpluses of different relations could be numerically compared. They do not reduce value to holonomy or power to non-commutativity as a matter of fact; the identifications are constrained, as argued above, but remain interpretive, and a reader who rejects the construal of power as order-dependence, or of surplus as cyclic residue, is not refuted by the mathematics. And, by the standing restriction of Section 4.1, they do not explain why the symmetry breaks at all, so the very event that generates value and power is taken as given. The identifications illuminate the structure downstream of breaking; they neither ground the breaking nor convert their own interpretive content into theorem. We take their value to lie in making a family of previously separate intuitions—about appropriation, surplus, the inevitability of power, and the meaning of justice—follow from a single structural condition, not in any claim to have measured or metaphysically secured what they organize.

4.6 Value generation and the variability of subject-number

Because subjects are defects of a field rather than antecedent entities, their creation and annihilation are intrinsic to the field: defects may be pair-created from the uniform configuration

and may mutually annihilate, with no external source term required. This cures, at the level of *kinematics*, the defect that sank the fluid model in Section 2: the source S that had to be inserted by hand into the continuity equation (1) is here the intrinsic rate of defect creation and extinction. Two layers of generation must now be distinguished, sharpening a point left blurred in the fluid treatment. Fluid dynamics generates nothing: it conserves and transports. Neural and graph dynamics generate at the level of *activity*—a spike is produced and extinguished—but the activity is borne by a fixed population of nodes; the carriers do not come and go. The relational field generates at the level of the *carrier itself*: the subject is the excitation, so the generation of value and the generation of the subject are one event, and the number n of subjects is a dynamical quantity read off the field’s state, not a constant fixed in advance.

4.7 Defect statistics: the braid group as a derived object

The number n of subjects being given by the field’s current state, fix it momentarily and ask how n indistinguishable defects may be moved around one another. Their collective configuration space is

$$C_n(\Sigma) = (\Sigma^n \setminus \Delta) / S_n, \quad (12)$$

where Δ is the diagonal (coincident defects, excluded) and the quotient by the symmetric group S_n enforces indistinguishability. The statistics of the defects—how their joint relational state transforms when they are carried around one another and returned—is governed by the fundamental group of this space:

$$\pi_1(C_n(\Sigma)) = B_n(\Sigma), \quad (13)$$

the *surface braid group* of Σ . This is the pivot of the entire construction. The braid group is not posited; it is computed, as the fundamental group of the field’s defect configuration space. When Σ is a plane, (13) returns the classical braid group B_n of Definition 4.2 below; but two-dimensionality is here a *sufficient*, not a necessary, condition for non-trivial braid statistics, and enters only as the most familiar instance. For a general relational Σ the relevant object is whatever $\pi_1(C_n(\Sigma))$ happens to be, and the formalism asks only that this group be non-abelian—a condition on the topology of Σ , not a commitment to its dimension. The vortex-in-a-plane picture is retained throughout as heuristic illustration, never as an assumption.

Definition 4.2 (Braid group). For Σ a plane, the braid group on n strands, B_n , is generated by $\sigma_1, \dots, \sigma_{n-1}$ subject to

$$\sigma_i \sigma_j = \sigma_j \sigma_i, \quad |i - j| \geq 2, \quad (14)$$

$$\sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1}, \quad 1 \leq i \leq n - 2. \quad (15)$$

A generator σ_i is the elementary process of carrying the i -th defect around the $(i+1)$ -th, i.e. a generator of $\pi_1(C_n(\Sigma))$ via (13). Relation (14) states that braidings of disjoint pairs commute; relation (15), the braid (Yang–Baxter) relation, is the non-trivial content.

4.8 Variable subject-number: categorification

Equation (13) holds at fixed n . To let n vary—to let subjects be created and annihilated, as Section 4.6 requires—the fixed- n braid group is insufficient and must be embedded in a structure that carries fusion and splitting of defect types. The appropriate object is a *braided monoidal* (indeed modular tensor) category: its objects are defect types, its tensor product is the fusion of defects, its fusion rule

$$a \otimes b = \bigoplus_c N_{ab}^c c \quad (16)$$

records the multiple channels by which two defects may combine or a defect may split, and its braiding isomorphism $c_{a,b}: a \otimes b \rightarrow b \otimes a$ supplies the statistics. The fixed- n representations of B_n used in the remainder of the paper are exactly the restrictions of this categorical structure to a fixed configuration of objects. Thus the braid group is doubly secondary: it is the fundamental group of the field's defect configuration space at fixed n (13), and a fixed-object section of the braided category that governs variable n (16).

4.9 What the field layer supplies, and what is left open

It is essential to be exact about how far this descends. The construction above is purely *kinematic*: it specifies the field's value space (M), its subjects (defects), their statistics ($\pi_1(C_n(\Sigma))$), and the variability of their number (the braided category). It does *not* specify the field's *dynamics*: there is as yet no action functional or equation of motion $\mathcal{L}[\Phi]$ determining which configurations are realized, at what rate defects are created and annihilated, or how a relational *background*—the effective geometry of Σ itself—might emerge from the field's correlations rather than being presupposed. The last of these is the relational analogue of background independence in theoretical physics, where spacetime geometry is not a fixed stage but a dynamical or emergent quantity. We claim no solution to it. We claim only to have located it correctly: the generation of background and of subject-number belongs to the dynamical layer of the field, a layer this paper leaves as an open programme, while the statistics of subjects at fixed number—which is what the normative results of the following sections require—belongs to the kinematic layer, which is in hand. The braid representation is the kinematic shadow of a field theory whose dynamics remain to be written.

4.10 Subjects, types, and the limit of topological identity

One consequence of Definition 4.1 must be confronted now rather than concealed, because it bears directly on the philosophy the model serves. A homotopy class individuates a defect only up to *type*: it records that this is a defect of charge τ , not that it is *this* defect rather than another of the same charge. Two defects of identical charge are, within the topological data alone, interchangeable. For the physics this is correct and required. For a philosophy of intimacy it is a threat, since the guiding intuition there is that the beloved is not substitutable by a qualitative duplicate—that token identity, the *haecceity* of *this* relation, is not exhausted by any specification of type. The field's topological data cannot express token-distinctness of same-type defects. We do not patch this by fiat; we mark it, and Section 7 will argue that token identity is precisely what topological protection cannot carry, and so belongs to the local flesh rather than the skeleton—numerical, non-substitutable identity living exactly where topology is blind.

4.11 The relational state and its holonomy

Definition 4.3 (Relational state and resonance). At fixed defect-number, a configuration of n excitations is assigned a finite-dimensional complex Hilbert space $\mathcal{H}_n$, the *fusion space*, carrying a unitary representation $\varrho: \pi_1(C_n(\Sigma)) \rightarrow U(\mathcal{H}_n)$, written $\varrho: B_n \rightarrow U(\mathcal{H}_n)$ in the planar instance. The act of two meaning-worlds resonating is the braiding of the corresponding defects, represented by the unitary $\varrho(\sigma_i)$; the cumulative effect of a closed sequence of braidings is the holonomy $\varrho(b)$.

In this language the holonomy of Section 2 reappears as the geometric phase accumulated by the relational state under braiding, now a unitary on $\mathcal{H}_n$ rather than a scalar flux—and, one level down, as the holonomy of the field's connection over its defect configuration space, of which the braid holonomy is the fixed-sector shadow. The good/bad distinction is carried by $\varrho(b)$.

5 The dynamical layer: flow on the generated structure

Everything to this point has been an account of *generation*: how a symmetry pre-structure, by breaking, brings forth a space, subjects, value, power, and their braid statistics. That account answers the question of where the structure comes from, but it is only half of a complete model. Once the structure is in place there is a second question, logically posterior and equally necessary: how does value—the stock Φ and the surplus it can generate—then *flow, circulate, and redistribute* across the structure in time? Generation is the cosmogony of the model; this section is its running. The distinction matters because the two halves speak different languages. Generation is topological and group-theoretic; flow is the province of a continuous dynamics, and it is here, at the dynamical layer, that the fluid description of Section 2 is reinstated—no longer as a candidate foundation, which it failed to be, but in the role it is genuinely fit for.

5.1 The fluid description belongs here and only here

The fluid model was set aside as a foundation for two faults (Section 3): it conserves what should be generated, and it presupposes its background. Both faults are disqualifying for the *generation* layer and harmless for the *flow* layer. The background is no longer presupposed but generated, supplied to this layer by the construction of Section 4; the fluid dynamics now runs *on* G/H and its defect configuration, not on an assumed stage. And conservation, fatal when the question was how value comes to be, is appropriate when the question is how already-existing value moves: generation has been handed to defect creation and annihilation (Section 4.6), so the flow layer is free to treat transport without having to manufacture being. The fluid language describes exactly what it is good at—the continuous transport, diffusion, and circulation of a quantity—once the quantity and its arena have been generated elsewhere. This is the precise sense in which the title’s passage from fluid to braid is not an abandonment: the fluid returns as the dynamics of the very structure the braid layer generated.

5.2 The transport equations on the generated space

Let $\rho(x, t)$ be the value-stock density and $\mathbf{u}(x, t)$ its flow velocity on the generated space, with $\mathbf{p}(x, t)$ the power field. The dynamics are those of Section 2, now read as the running of the generated structure:

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = S, \quad (17)$$

$$\partial_t \boldsymbol{\omega} + (\mathbf{u} \cdot \nabla) \boldsymbol{\omega} = (\boldsymbol{\omega} \cdot \nabla) \mathbf{u} + \nu \nabla^2 \boldsymbol{\omega} + \nabla \times \mathbf{f}(\mathbf{p}), \quad (18)$$

$$\partial_t \mathbf{p} = \alpha \rho \mathbf{u} - \beta \mathbf{p}, \quad (19)$$

where $\boldsymbol{\omega} = \nabla \times \mathbf{u}$. Three points connect this layer to the generation layer above it and prevent it from floating free.

First, the source S in (17) is no longer an external patch, the embarrassment that sank the fluid model as a foundation. It is now *supplied* by the generation layer: S is the rate of defect creation and annihilation (Section 4.6), so the flow layer imports its non-conservation from below rather than stipulating it. The two layers join exactly at S .

Second, the circulation that the flow generates is the surplus of Section 4.3. The vorticity $\boldsymbol{\omega}$ and its loop integral are the dynamical, time-resolved version of the holonomy: where (10) gave the surplus accumulated around a fixed loop, (18) governs how that circulating surplus is produced, advected, and dissipated as the system runs. The surplus is not a static label but a quantity with a flow.

Third—and this is the reason the fluid layer is indispensable rather than ornamental—the one exact theorem of the paper lives here. Proposition 2.1 is a statement about *flow*: the appropriate, gradient part of the motion drops out of the vorticity equation (18), so that no

amount of appropriative transport generates surplus, and only the circulating, non-possessive part does. It is the founding theorem of the dynamical layer, and it is the same theorem that, propagated upward, constrained the identification of surplus with holonomy in Section 4.3. The generation layer and the flow layer are thus locked together by Proposition 2.1 at one end and by the source S at the other: the flow layer receives its non-conservation (S) from generation and returns to it the theorem (Proposition 2.1) that fixes what circulation can and cannot do.

5.3 Solidification as a dynamical phase

The flow layer also supplies something the generation layer cannot: a notion of how a living structure can *die* into a fixed one. The ratio α/β in (19) governs whether the power field forgets its history (β dominant, flow maintained) or accumulates it without limit (α dominant, positive feedback locking the flow into a frozen, appropriative channel). The latter is *solidification*: the dynamical collapse of a circulating, surplus-generating structure to a fixed point at which vorticity, and hence surplus, goes to zero. In the language of the whole model, solidification is the running-down of a generative cycle into a merely conservative one—a structure that still exists but no longer generates, its power frozen into pure gradient. This is the dynamical face of the bad cycle, and it can only be stated at this layer, because it is a statement about flow ceasing, not about structure failing to be generated. Justice as the governance of power’s geometry (Section 4.4) therefore has a dynamical reading as well: it is the maintenance of the system away from the solidified fixed point, the continual reopening of a flow that would otherwise freeze.

The three results of this section are *arguments from expressive adequacy*: each shows that *if* a certain feature of relational value is to be representable within the formalism of Definition 4.3, *then* the representation ϱ must be non-abelian. Their status has, however, changed in light of Section 4.3. There the structural source of all three features was traced to a single condition—that the residual symmetry H surviving the breaking is non-abelian (11)—so the three antecedents are no longer independent assumptions but consequences of one symmetry fact, together with the interpretive identifications (holonomy as value, H -multiplet as joint state) that no formalism can establish on its own. This section therefore serves two purposes: it records, for readers who do not grant the symmetry-breaking foundation, that each feature *independently* forces non-commutativity (so the conclusion is robust to rejecting Section 4); and it isolates precisely the residual interpretive premises that even the derivation of Section 4.3 cannot discharge. We state each premise explicitly, since the argument’s force is the force of that premise.

Proposition 5.1 (Power requires non-commutativity). Suppose (premise P) that relational power is real and consists in the asymmetry of causal credit—the condition of an act’s lying repeatedly upstream of another’s realization of value—so that the relational history “ A before B ” is in general a different history from “ B before A .” Then any representation faithful to this asymmetry satisfies $\varrho(\sigma_i\sigma_j) \neq \varrho(\sigma_j\sigma_i)$ for some i, j , and hence is non-abelian.

Proof. By premise P the two orderings are distinct relational histories with, in general, distinct value-consequences. An abelian representation satisfies $\varrho(\sigma_i)\varrho(\sigma_j) = \varrho(\sigma_j)\varrho(\sigma_i)$ for all i, j and so assigns the two orderings the same holonomy, collapsing a distinction that premise P holds to be real. A representation that does not collapse it must therefore be non-abelian. The premise is doing the work: an eliminativist who denies that order of relational history ever bears on value (rejecting P) is untouched by the argument, and for such an opponent an abelian representation suffices. We take P to be independently motivated by the role of causal asymmetry in any account of power, but we do not prove P here. $\square$

Proposition 5.2 (Resonance-without-fusion requires a degenerate fusion space). Suppose (premise R) that two meaning-worlds can be coupled into a shared state that is (i) not reducible to the pair of their separate states, yet (ii) does not erase the distinctness of either—resonance

that is neither mere juxtaposition nor merger. Then the fusion space $\mathcal{H}_n$ must have dimension greater than one, and ϱ is non-abelian.

Proof. In the abelian case the joint state of a collection of excitations is fixed uniquely by their individual charges; $\dim \mathcal{H}_n = 1$, there is no residual joint freedom, and the only available coupling is relative phase, whose nontrivial limit is phase identity, i.e. merger. Such a formalism cannot realize a state satisfying both (i) and (ii) of premise R: condition (i) requires joint freedom beyond the separate charges, so $\dim \mathcal{H}_n > 1$, which is precisely the non-abelian case; condition (ii) is then met because the individual charges are retained as labels even as the joint state fails to factor. An opponent who holds that all genuine union is merger (rejecting the coherence of R) is again untouched; the argument shows that R is *representable* only non-abelianly, not that R obtains. $\square$

Proposition 5.3 (Generativity requires multi-channel fusion). Suppose (premise G) that the encounter of two meaning-worlds can be genuinely generative, its outcome not fixed in advance by the two relata alone. Then the fusion of two charges, in the sense of the categorical fusion rule (16), must admit more than one outcome, $\sum_c N_{ab}^c > 1$ for some a, b , which is a non-abelian fusion rule.

Proof. Abelian fusion is single-channel: $a \times b$ has a unique outcome, so the result of an encounter is a function of its inputs and nothing arises that the inputs did not already determine—the negation of premise G. For G to hold, fusion must be multi-channel, $\sum_c N_{ab}^c > 1$, which is the defining mark of a non-abelian fusion rule. A determinist about relational outcomes (rejecting G) may keep abelian fusion; the argument fixes the cost of G, not its truth. $\square$

Non-commuting braiding, a degenerate fusion space, and multi-channel fusion are the three defining features of non-abelian anyonic statistics; the three propositions recover them one-to-one from three independent commitments—P, R, and G—about relational value. The correspondence is exact, but its evidential weight is conditional on the premises: it shows that a formalism representing power, resonance-without-fusion, and generativity *must* be non-abelian, not that relational systems exhibit these features. We regard the convergence of three independent premises on a single algebraic demand as the strongest available abductive reason to locate the sought-after object in this structure, while granting that an opponent who denies all three premises incurs no contradiction.

6 Discriminating power requires non-universality

Non-abelianness is necessary but not sufficient, and an uncritical embrace of it is a trap. Among unitary representations of the braid group, computational power and normative discriminating power stand in tension, and the model requires the latter.

Definition 6.1 (Universality and discrimination). A family of representations $\{\varrho_n\}$ is *computationally universal* if, for n large, the image $\varrho_n(B_n)$ is dense in $PU(\mathcal{H}_n)$ (the projective unitary group, i.e. $U(\mathcal{H}_n)$ up to overall phase). A normative predicate Φ on closed braids is *holonomy-factoring* if it depends on the braid only through its holonomy, i.e. $\Phi(b) = \hat{\Phi}(\varrho(b))$ for some $\hat{\Phi}: PU(\mathcal{H}_n) \rightarrow \{0, 1\}$. Such a Φ has *robust discriminating power* if $\hat{\Phi}$ is non-constant and its decision boundary is *stable*: there is some $\delta > 0$ such that $\hat{\Phi}$ is constant on every ball of radius δ meeting the reachable holonomy set.

Proposition 6.1 (Universality precludes robust holonomy-factoring discrimination). If $\{\varrho_n\}$ is computationally universal, then no holonomy-factoring predicate Φ has both non-trivial content and robust discriminating power. Concretely, for any non-constant $\hat{\Phi}$ and any $\delta > 0$, some δ -ball meeting the reachable set contains holonomies on which $\hat{\Phi}$ takes both values.

Proof. By universality the reachable set $\varrho(B_n)$ is dense in $PU(\mathcal{H}_n)$. Suppose $\hat{\Phi}$ is non-constant, so there exist U_0, U_1 with $\hat{\Phi}(U_0) = 0$ and $\hat{\Phi}(U_1) = 1$. Fix any $\delta > 0$. Density supplies reachable holonomies V_0, V_1 with $\|V_i - U_i\| < \delta/2$, and density again supplies, in any neighborhood of a point on a path from U_0 to U_1 , reachable holonomies of either value arbitrarily close together; in particular there is a point W and reachable holonomies within distance δ of W taking both values, so $\hat{\Phi}$ is not constant on that δ -ball. Hence the stability condition of robust discriminating power fails for every δ , and Φ cannot both be non-constant and decide robustly. $\square$

Remark 6.1 (Scope of the result, and what it does *not* say). Two qualifications keep Proposition 6.1 honest. First, it is restricted to *holonomy-factoring* predicates with *robust* (perturbation-stable) boundaries; it does *not* say that under universality no predicate whatever is well-defined. A predicate reading off an exactly conserved quantity—e.g. “ $\varrho(b)$ lies in a fixed closed subgroup $K \subsetneq PU(\mathcal{H}_n)$ ”—remains perfectly well-defined even under density, because membership in K is a closed condition. What such a predicate lacks is *robustness*: it is destroyed by arbitrarily small perturbations, so it cannot serve as a stable normative criterion in a system with any noise. The proposition’s real content is thus that universality forces a dilemma—a holonomy criterion is either non-robust (a measure-zero exact condition) or non-discriminating (constant)—and neither horn yields a usable good/bad distinction. Second, the result says nothing about predicates that depend on the braid beyond its holonomy; those are exactly the non-topological data that Section 7 assigns to the local flesh, and the present proposition is silent on them by design.

Remark 6.2 (The normative cost of omnipotence). Proposition 6.1 has a philosophical reading worth stating plainly. A relational structure powerful enough to compute anything is, for that very reason, unable to tell good circulation from bad: discriminating power requires that the reachable set of holonomies be constrained, and a constrained reachable set is a non-universal one. The capacity for normative judgment and the capacity for unlimited computation are, in this setting, inversely related. The correct formal object is therefore a representation that is *non-abelian yet non-universal*: non-abelian to admit power, resonance, and generativity (Section 5.3); non-universal to retain the discriminating power on which the good/bad distinction depends.

Remark 6.3 (The admissible band is non-empty; its normative occupancy is open). The two boundaries exclude the strictly abelian representations (too weak: no power, by Proposition 5.1) and the computationally universal ones (too strong: no robust discrimination, by Proposition 6.1), selecting an intermediate band of non-abelian but non-universal unitary representations of B_n . That this band is non-empty is not in doubt: any representation factoring through a finite non-abelian quotient of B_n is non-abelian yet has finite, hence non-dense, image, and the Jones representations at certain roots of unity (those whose associated braiding is non-universal, away from the universal Fibonacci-type cases) furnish infinite-image examples. What the argument does *not* establish—and what we therefore mark as the central open problem of the formal development—is that any representation in this band carries a holonomy-factoring predicate that is at once robust (Definition 6.1) and *normatively interpretable*, i.e. that some stable invariant of the non-universal holonomy actually coincides with the good/bad distinction rather than merely having the right logical shape. We have fixed the two-sided boundary within which such a predicate must live and shown the region is inhabited; whether it is inhabited by a *normative* predicate is unproven, and we do not claim otherwise.

7 Skeleton and flesh: the protected remainder

Topological protection—the insensitivity of the braiding holonomy to local perturbation—is the chief virtue of the formalism and also its chief danger. The holonomy depends only on the topological class of the braid, not on local detail, which is exactly right for the *relational skeleton*

of value: the good/bad criterion should not depend on accidental specifics. But it is exactly wrong as a complete theory of value, because in intimate and singular relations it is often the local, non-topological detail—a particular encounter, a particular word—that generates meaning. A formalism that classified all such detail as computationally irrelevant noise would contradict the incommensurable singularity that a serious account of value must preserve.

We therefore decline the totalizing reading on which value is exhausted by topology. Value has two components: a *topological skeleton*, captured by the non-abelian holonomy and carrying the normative good/bad criterion, and a *local flesh*, the singular detail that topological protection by construction cannot see, carrying the non-substitutable particularity of each relation. The two are not rivals; they are the protected and unprotected parts of one object. This is also where the token-identity problem flagged in Section 4 is discharged. Topological charge fixes type but not token; the numerical, non-substitutable identity of *this* relation—its haecceity, the fact that the beloved is not replaceable by a qualitative duplicate of the same type—is by construction invisible to the holonomy, and so falls on the flesh side of the division. The model thus does not merely tolerate the irreplaceability of the particular relation; it predicts the exact formal location of that irreplaceability, namely the perturbation-sensitive data that topological protection discards. What would have been an objection to the formalism (it cannot tell two same-type subjects apart) becomes a structural claim about where intimacy’s non-substitutability must reside.

It is worth collecting, at this point, the several things the braid layer cannot see, for they prove to have a common home. Three have arisen: the token identity just discussed; the generation of subjects and the variability of their number (Section 4.6); and the emergence of the background itself (Section 4.9). None is a defect of the analysis; each is invisible to the braid representation for the same structural reason—the braid group is the fixed-sector, perturbation-robust shadow of the field, and all three live below that shadow, in the field’s dynamical and local data. Token identity is the field’s local, sub-topological detail; subject-number is set by the field’s defect content; the background is the field’s emergent geometry. The braid layer carries the normative skeleton and is blind to all of them by the same construction that makes it robust. The skeleton is the field seen at fixed defect-number and up to local perturbation; the flesh, the subject-number, and the background are the field seen without those two restrictions. The division of labour is therefore not ad hoc but follows the single seam between the field and its fixed-sector shadow.

Remark 7.1 (A conjecture about the good). This division suggests a thesis we state as a conjecture rather than a result. Pure skeleton is hollow—formal synchrony without flesh, a relation with structure but no particularity; pure flesh is structureless contingency—particularity with no relational form. The good may be neither, but the appropriate proportion of the two: a relation whose topological skeleton supports it as genuinely generative *and* whose local flesh makes it this irreplaceable relation rather than any other instance of the same class. On this reading the earlier holonomy criterion is necessary but not sufficient for the good, and the present paper isolates the necessary part while marking the remainder as the protected complement it cannot reach.

8 The computational horizon

We close the technical development by locating the model’s relation to computation, which is also the sense in which the title’s “emerging computing” is meant.

8.1 The entity-first residue in reservoir and morphological computing

Reservoir computing exploits a fixed macroscopic dynamical system—often a physical one—as a high-dimensional non-linear map, training only a linear readout. The substrate is borrowed complexity, not a computational subject: it does not know what it computes, and the semantics of the computation reside entirely in the external readout and its target. This is an entity-first

arrangement in disguise. There is a trainer, a readout, an objective; the relational medium is a passive tool. Morphological computing goes further, distributing computation into the form-dynamics of a physical system with no central processor, which is closer to the relational commitment; but it still lacks a clean semantics of correctness, an account of what it is for the form-dynamics to have computed rightly.

8.2 Computation as holonomy

The relational-ontological limit of these tendencies is a model of computation in which there is no antecedent node or bit serving as a substrate of information, in which information is the pattern itself, and computation is the creation, fusion, and braiding of patterns—and in the field picture of Section 4 these are literally the creation, fusion, and braiding of the field’s defects, not metaphors. In the braid formalism this is exact rather than aspirational: the computational state is the relational state $\mathcal{H}_n$, a step of computation is a braiding $\varrho(\sigma_i)$ or a fusion (16), and the result is the holonomy $\varrho(b)$, which is simultaneously a physical quantity, a computational output, and—by Section 5.3–Section 7—the normative criterion. The identification of these three is not a coincidence to be explained but the single content of the geometric phase seen from three directions.

Remark 8.1. This yields a thesis about the natural model of computation for a relational ontology: it is not the Turing machine (entity-first, discrete states, an external read–write head) but braiding—a computation in which the holonomy *is* the result. Reservoir computing falls short precisely because it retains an external, entity-first readout; a thoroughly relational computation requires that even the readout be internalized into the relations—the braiding—among excitations. Whether a physical, macroscopic “fluid computer” realizing this without an external readout exists as a mature form, we leave as an open problem; the braid representation shows at least that the target is mathematically well-posed.

9 The aggressive limit: a relational base space and group field theory

The construction of Section 4 took a symmetry group G as pre-structure and generated the order-parameter space $M = G/H$ from it, but it still presupposed a second, distinct background: the configuration space Σ over which the broken field $\Phi: \Sigma \rightarrow G/H$ lives, whose very dimension silently re-imported the kind of given stage a relational ontology is meant to refuse. This section takes the final step the ontology demands and tests how far it can be carried. The step is to make Σ *itself* relational—to let the configuration space be not a manifold but a pure relational structure, a group, which to avoid collision with the symmetry G we denote Γ . We present the move, give a minimal model with explicit formulae, and then state plainly the three walls it runs into. We do not claim to have crossed them; we record the attempt and invite its continuation.

9.1 A group as the right kind of base space

A group is, in a precise sense, pure relation. An element $\gamma \in \Gamma$ has no intrinsic content; it is individuated entirely by how it multiplies, i.e. by the relations $\gamma\gamma'$ it participates in. Cayley’s theorem makes this exact: every group is isomorphic to a group of permutations of its own underlying set, so a group element simply *is* its position in the web of multiplications, with no inner core. This is the algebraic archetype of the paper’s guiding commitment that entities are secondary effects of relations: here the “entity” γ is nothing over and above the relation $\gamma\gamma'$. Taking $\Sigma = \Gamma$ therefore replaces the last presupposed background—the configuration stage—by a relational structure, so that relations become primary not only for the subjects (defects) and the order-parameter space (G/H) but for the very arena in which subjects are arranged.

9.2 A minimal model

Let Γ be a group, taken now as the configuration “space,” and let the relational field be a map

$$\Phi: \Gamma \longrightarrow M, \quad (20)$$

assigning to each relational location $\gamma \in \Gamma$ a value $\Phi(\gamma) \in M$ in the order-parameter space $M = G/H$ of Section 4.1. This is the form taken by group field theory, in which the field’s domain is a group rather than a spacetime, introduced in quantum gravity precisely to let geometry emerge rather than be assumed. Three structures of Section 4 have natural relational analogues on Γ .

Subjects as relational defects. A subject is a configuration of Φ that cannot be deformed to the constant one. With Γ discrete the homotopy notion is replaced by its combinatorial shadow: fix a generating set S of Γ and form the Cayley graph $\text{Cay}(\Gamma, S)$, whose loops are the relational closed paths. A defect is a value assignment whose restriction to some loop γ winds nontrivially in M ,

$$w(\gamma) = [\Phi|_\gamma] \in \pi_1(M), \quad w(\gamma) \neq 0, \quad (21)$$

exactly as in Section 4.7 but with the loop now a word in the generators of Γ rather than a path in a manifold. The base space’s own topology is replaced by the relational connectivity encoded in Γ .

Statistics from the group itself. The configuration space of n indistinguishable defects, formerly $C_n(\Sigma)$, is here built from Γ acting on itself; the braid statistics of (13) are replaced by the relevant fundamental group of that relational configuration, and the demand of Section 5.3—that the statistics be non-abelian—becomes a demand on Γ : that Γ (or the automorphism structure it induces on defect configurations) be non-abelian. Power, in the sense of Proposition 5.1, is then literally the non-commutativity of the relational base: $\gamma\gamma' \neq \gamma'\gamma$.

Dynamics by convolution. A group carries a canonical way to write an interaction without reference to any external coordinate: convolution. A minimal action for the field (20) is

$$\mathcal{S}[\Phi] = \sum_{\gamma \in \Gamma} V(\Phi(\gamma)) + \lambda \sum_{\gamma, \gamma' \in \Gamma} K(\gamma^{-1}\gamma') \Phi(\gamma) \Phi(\gamma'), \quad (22)$$

where V is a local potential on M and the kernel $K(\gamma^{-1}\gamma')$ couples two relational locations through the relation $\gamma^{-1}\gamma'$ between them—never through any external position, only through their relative group element. The entire dynamics is thus written in relations alone. (For non-discrete Γ the sums become integrals against Haar measure.) This is the sense in which the model is aggressive: not even the coupling refers to a background; it refers only to $\gamma^{-1}\gamma'$.

9.3 Three walls

We now state, without softening, the three obstacles this model faces. Each is, in our view, a genuine open problem rather than a technicality, and we present the model in full awareness of them.

Wall 1: a group may be too rigid to be generative. A group is relation, but *already-closed* relation: associative, invertible, with a determinate multiplication table. The result $\gamma\gamma'$ is fixed in advance, which is exactly the single-channel determinism that Proposition 5.3 identified as the negation of generativity. A purely group-based base space thus risks purchasing “relation first” at the cost of “generation,” the very property the paper’s account of the good requires. The natural repair is to weaken the group to a structure that is relation-first but not everywhere invertible or everywhere defined—a *groupoid* or, more generally, a *category*, whose objects are defined by their morphisms (the categorical form of “ γ is defined by $\gamma\gamma'$ ”) while admitting partial composition and multiple channels. The aggressive limit, pursued consistently, therefore

points not to group field theory alone but to a *categorical* field theory—and this rejoins the braided monoidal category already introduced in Section 4.8 to handle variable subject-number. The two arrive at the same place from opposite directions, which we take as mild evidence that the category, not the bare group, is the correct relational base.

Wall 2: the regress of the base. Replacing the configuration stage by a group Γ sharpens, but does not answer, the question of where the background comes from. If Γ is presupposed, the background has merely changed costume: a fixed group is as much a given stage as a fixed plane, and the same objection the paper raised against starting from the braid group (Section 4) now applies to starting from Γ . If instead Γ is itself to be generated, it must be generated by some yet more basic relational structure, and the question recurs. The same regress attends the symmetry pre-structure G of Section 4, whose own origin we placed on the unexamined left of the breaking arrow; the two open ends—where Γ comes from, where G comes from—are the same wall seen twice. This is not an incidental difficulty but the wall that any thoroughgoing relational ontology meets: either it admits one ungenerated relational structure (in tension with the claim that all is generated) or it regresses without end. We do not resolve this. We note only that the honest options are few—a self-generating, self-referential base that is its own ground (a fixed point of its own construction), or an explicit acceptance of one primitive relational given—and that choosing between them is a metaphysical commitment the formalism can frame but not force.

Wall 3: group field theory is itself unfinished. The emergence of geometry from a field on a group is an active but incomplete programme; that a background space and its dimension genuinely arise from (22), rather than being put in through the choice of G and K , is not established in general. To rest the paper’s foundation on this would be to build on a frontier. We therefore present (20)–(22) as a *toy model*: formally well-posed, relationally written, and adequate to display how power ($gh \neq hg$), subjects (21), and background-free dynamics (22) can all be stated without a geometric stage—but not as a demonstration that geometry, or the normative holonomy of the earlier sections, provably emerges from it. What can be claimed is the conditional: *if* a categorical field theory in the spirit of (22) yields, in some sector, the non-abelian non-universal braid representations of Section 5.3, *then* the entire construction—value, power, resonance, the good/bad criterion—descends from a base that is pure relation, and the relational ontology is realized without remainder.

9.4 Status and invitation

We regard this section as the programmatic summit of the paper and, equally, as its least finished part. The trajectory of the whole—fluid to field to braid to relational base—points unmistakably here, to a setting in which not merely the subjects but the arena is relational; yet the three walls are real, and we have crossed none of them. The honest position is that the aggressive limit is correct in direction and open in execution. We state the toy model so that the proposal has formal content rather than rhetorical gesture, and we mark its limits so that no exactness is claimed beyond what is shown. The questions it leaves—whether the right base is a group, a groupoid, or a category; whether a relational base can be self-grounding without regress; whether geometry and the normative holonomy provably emerge—are offered as genuinely open, and we would welcome their development by others as much as by ourselves.

10 Evaluation: quantitative operationalization and a case study in educational ethics

A theory of value and power should say what, in data, would count for or against it. This section gives a quantitative operationalization that respects the limits declared in Section 4.5—in particular that holonomy has no absolute scale—and applies it to a domain where the framework’s distinctions are unusually legible: education. Education is a natural test case because good teaching has the rare property of aiming at its own dissolution (the student ceasing to need the teacher), which is exactly the signature of a generative asymmetry whose geometry works toward the decay of its own power edge (Section 4.4).

10.1 What is measurable, and what is not

By Section 4.5 the magnitude of the holonomy is not measurable, since it is defined relative to a connection with no fixed empirical scale. Three quantities, however, are scale-invariant and can be operationalized: the *sign* of the holonomy (generative, appropriative, or dominative), the *order* among relations (which of two is more generative), and the *degree of non-commutativity* (the strength of power). The programme measures only these. It does not attempt to assign an absolute value to a relationship, and any reading that does so would contradict Section 4.5.

10.2 Discretization: from the field to estimable matrices

The continuous field $\Phi: \Sigma \rightarrow G/H$ is not directly sampled; we discretize it. Nodes are individuals in a teaching relation (students, teachers). Each node i carries a relational state vector $s_i(t)$, and the connection becomes a state-transition operator $T_{i \rightarrow j}$ along each relational edge. The continuous holonomy $\mathcal{P} \exp \oint_{\gamma} A$ becomes the ordered product of transitions around a closed interaction loop,

$$\hat{H}(\gamma) = T_{i \rightarrow j} T_{j \rightarrow i}, \quad (23)$$

an estimable matrix rather than an abstract integral. This is the bridge from formalism to data: holonomy is realized as a product of measured transition operators around a cycle.

10.3 The state vector and its data proxies

For the educational application we take a three-component state vector, each component a quantity educational research already records:

$$s_i(t) = (a_i(t), d_i(t), g_i(t)), \quad (24)$$

where $a_i(t)$ is *autonomy*—the rate at which the student initiates questions or tasks unprompted (codable from classroom video or from the self-initiated/assigned ratio in learning-management-system logs); $d_i(t)$ is *dependence*—the fraction of tasks requiring teacher intervention, or the performance drop when scaffolding is removed; and $g_i(t)$ is *generative output*—the rate of new, non-reproductive content the student produces (original questions, transfer to novel contexts, rubric-coded so as to distinguish it from reproductive responses a recording could replace). The data of interest is the longitudinal trajectory of these over a term.

10.4 The three computed quantities

(1) Sign of the holonomy (good/bad). A scale-free proxy for the sign of the holonomy accumulated over one completed teaching cycle is the net change in autonomy against dependence,

$$\Theta(\gamma) = (a_i(t_2) - a_i(t_1)) - (d_i(t_2) - d_i(t_1)). \quad (25)$$

Then $\Theta > 0$ (autonomy up, dependence down) reads as positive holonomy—generative power, power-to, a good cycle; $\Theta \approx 0$ (no net change, mere information transfer) reads as zero holonomy—the appropriative, pure-gradient case that generates no surplus; and $\Theta < 0$ (dependence up, autonomy down) reads as negative holonomy—power-over, a bad cycle. This is falsifiable: it predicts that good teaching appears in data as a $\Theta > 0$ trajectory, dependence decaying and autonomy rising over time, whatever the test scores, and that teaching which fixes the student in dependence has $\Theta \leq 0$ however high its scores.

(2) Strength of power (non-commutativity). Power is the order-dependence of traversal, operationalized as the failure of the two interaction orders to commute,

$$\mathcal{C} = \|T_{i \rightarrow j}T_{j \rightarrow i} - T_{j \rightarrow i}T_{i \rightarrow j}\|, \quad (26)$$

the norm of the difference between “teacher acts first” and “student acts first.” $\mathcal{C} \approx 0$ marks a near-symmetric relation (weak power, as in peer learning); large $\mathcal{C}$ marks a strongly asymmetric one. The transition operators T are estimated from the time-stamped interaction sequence (who acts first, how the other’s state responds), available from coded classroom dialogue or LMS interaction logs.

(3) Surplus against stock. Stock is testable, countable knowledge (exam scores, appropriable and transferable); surplus is what a completed cycle leaves that is not localizable in either party, proxied by the non-attributable part of generative output g and by Θ . The framework predicts that a system optimizing stock alone (test preparation) exhibits systematically low Θ and surplus even at high stock—a prediction standard score-based evaluation cannot make.

10.5 Case study: lecture-drill versus inquiry-based instruction

Consider two cohorts over one term, one taught in a lecture-and-drill mode, one in an inquiry-based (broadly Socratic) mode, with (a, d, g) sampled at fortnightly points and the transition operators T estimated from time-coded classroom interaction. The framework issues predictions *before* the data, which is what makes the study falsifying rather than fitting.

It predicts: the inquiry cohort shows a $\Theta > 0$ trajectory (dependence decaying, autonomy rising), positive holonomy, power-to; the lecture-drill cohort shows $\Theta \leq 0$, and where students are held in dependence, $\Theta < 0$, power-over. Crucially it predicts that the two cohorts may have comparable exam scores—comparable stock—while diverging in Θ and in transfer ability—the surplus—so that the difference the model identifies is exactly the one standard scores miss. The verdict rule is explicit: if the Θ trajectories diverge as predicted *and* the divergence is independent of exam score, the model is supported; if Θ covaries completely with exam score, or the inquiry cohort’s Θ is not higher, the model is falsified.

10.6 Limits of the operationalization

We mark the limits without softening, consistently with Section 4.5. Estimating the transition operators T requires substantial interaction data, and (23) is only as good as that estimation. The proxies (a, d, g) have construct validity that must be established independently rather than assumed; Θ is a first-order proxy for the holonomy’s sign, not a derivation of it. Causal identification—whether the instructional mode, rather than cohort selection, produces the Θ divergence—requires a design that controls selection, which the present sketch does not supply. And, by Section 4.1, the correspondence between a real teaching situation and a particular G, H has no canonical form, so the discretization is an operational stipulation, not a measurement of the underlying symmetry. What the section establishes is therefore not a validated instrument but a falsifiable operationalization: a route by which the framework’s central distinctions—

generative versus dominative power, surplus versus stock—become, in their sign and order if not their magnitude, answerable to longitudinal data. The full empirical validation is left as future work.

11 Recapitulation of the principal formulae

Because the argument crosses three formal languages—fluid dynamics, braid-group representation theory, and the theory of computation—we collect here the load-bearing expressions in one place, each with a one-line gloss and its epistemic status (modeling stipulation, theorem, or premise-dependent argument). Nothing in this section is new; it is a reading aid.

Stage I: the fluid model (Section 2). *Status: the entry point and a failed first field theory, not the foundation—but reinstated at the dynamical layer (Section 5) as the language of flow on the generated structure, and retained throughout for the one theorem it yields (Proposition 2.1).*

- **Value continuity with sources.**

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = S(\rho, \psi).$$

Value density ρ is transported by the flow $\mathbf{u}$ but is *not* conserved: the source S encodes genuine emergence and extinction, governed by an order parameter ψ . (*Modeling stipulation; the non-zero S is exactly what later forbids a pure fluid reading.*)

- **Value momentum.**

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla P(\rho) + \nu \nabla^2 \mathbf{u} + \mathbf{f}_{\text{pow}}.$$

The flow is driven by appropriation $-\nabla P$ (downhill toward scarcity), damped by relational inertia $\nu \nabla^2 \mathbf{u}$ (large ν = solidification), and biased by the power force $\mathbf{f}_{\text{pow}}$ (the value–power coupling). (*Modeling stipulation.*)

- **Power as accumulated value flux.**

$$\partial_t \mathbf{p} = \alpha \mathbf{J} - \beta \mathbf{p}, \quad \mathbf{J} = \rho \mathbf{u}.$$

Power is a gradient rewritten by the history of value flow: reinforced where value recently flowed ($\alpha \mathbf{J}$), forgotten otherwise ($-\beta \mathbf{p}$). The ratio α/β is the solidification order parameter—large α/β locks the flow into a frozen channel. (*Modeling stipulation; continuous analogue of a temporally asymmetric local update.*)

- **Value vorticity and holonomy.**

$$\boldsymbol{\omega} = \nabla \times \mathbf{u}, \quad \text{hol}(\gamma) = \oint_{\gamma} \mathbf{u} \cdot d\boldsymbol{\ell} = \int_{\Sigma} \boldsymbol{\omega} \cdot d\mathbf{A}.$$

The holonomy is the net value generated around a closed relational loop γ ; it is the formal criterion separating generative cycles ($\text{hol} \neq 0$) from degenerate ones ($\text{hol} \rightarrow 0$). (*Definition.*)

- **The central identity (Proposition 2.1).**

$$\nabla \times \nabla P = \mathbf{0} \implies \partial_t \boldsymbol{\omega} + (\mathbf{u} \cdot \nabla) \boldsymbol{\omega} = (\boldsymbol{\omega} \cdot \nabla) \mathbf{u} + \nu \nabla^2 \boldsymbol{\omega} + \nabla \times \mathbf{f}_{\text{pow}}.$$

Appropriation drops out of the vorticity equation entirely: whatever in a relation reduces to a single scalar ordering cannot, by itself, generate value around a loop. Circulation can come only from self-amplifying existing circulation $(\boldsymbol{\omega} \cdot \nabla) \mathbf{u}$ or from the curl of power $\nabla \times \mathbf{f}_{\text{pow}}$. (*Theorem, given the identification of appropriation with the gradient term.*)

Stage II: the relational field (Section 4).

- **Pre-structure, breaking, and generated order-parameter space.**

$$G \xrightarrow{\text{symmetry breaking}} H, \quad M = G/H, \quad \text{defect types} = \pi_k(G/H).$$

The pre-structure is a symmetry group G (pure relation); breaking to H generates the order-parameter space $M = G/H$ rather than positing it, and subjects are its defects, classified by the homotopy of the generated space. The breaking itself is taken as given: the paper is post-breaking, and does not explain why G breaks. (*Foundation; the cause of breaking is declared out of scope.*)

- **Surplus, power, and the value stock (Section 4.3).**

$$\text{hol}(\gamma) = \mathcal{P} \exp \oint_{\gamma} A, \quad H \text{ non-abelian} \Rightarrow \text{hol}(\gamma_1)\text{hol}(\gamma_2) \neq \text{hol}(\gamma_2)\text{hol}(\gamma_1).$$

The bundle $G \rightarrow G/H$ carries a canonical connection A . Two levels: the value *stock* is the local field value $\Phi(x)$; the *surplus* generated by a closed cycle is the holonomy a subject accumulates (gradient/appropriative part contributing zero, as in Proposition 2.1), structurally the increment ΔM of the circuit $M - C - M'$. Appropriation transfers stock but generates no surplus; the good/bad criterion is on the surplus. Power is the non-commutativity of that holonomy, holding exactly when H is non-abelian. The three premises of Section 5.3 thereby contract to “ H non-abelian” plus interpretive identifications. (*Derivation; identifications interpretive.*)

- **Power–generativity co-origination and justice (Section 4.4).**

generativity $\Rightarrow H$ non-abelian $\Rightarrow$ power; justice = shaping power’s geometry, not removing it.

Power, resonance, and generativity are three faces of H non-abelian, so any generative structure necessarily carries power (trilemma: subjects-and-generativity, no-power, and not-collapsing-to-the-void cannot all hold). Justice is therefore not the abolition of power—which would abolish generativity—but the shaping of the ineliminable asymmetry so its surplus is returned, not retained (power-to vs. power-over). (*Corollary; rests on the interpretive identifications.*)

- **Braid group as derived defect statistics (Section 4.7).**

$$C_n(\Sigma) = (\Sigma^n \setminus \Delta)/S_n, \quad \pi_1(C_n(\Sigma)) = B_n(\Sigma).$$

The braid group is not posited but computed: it is the fundamental group of the field’s n -defect configuration space. Two-dimensionality is a sufficient, not necessary, condition for non-trivial statistics; in general the relevant group is whatever $\pi_1(C_n(\Sigma))$ is. (*Theorem; the basis for “braiding is secondary to the field”.*)

- **Variable subject-number by categorification (Section 4.8).**

$$a \otimes b = \bigoplus_c N_{ab}^c c.$$

To let defects be created and annihilated (n variable), the fixed- n braid group is embedded in a braided (modular tensor) category whose fusion rule changes defect number; the braid representations used below are its fixed-object sections. (*Definition; kinematic.*)

- **Open dynamical layer (Section 4.9).**

action $\mathcal{L}[\Phi]$, defect rates, emergent background Σ = open programme.

The field's kinematics (value space, defects, statistics, variable number) are in hand; its dynamics—what configurations are realized, at what rate defects form, and how the background Σ itself emerges—are not supplied here, by analogy with background independence in physics. (*Open problem, explicitly marked.*)

Stage III: the braid representation (Section 5.3–Section 6).

- **Braid relations.**

$$\sigma_i \sigma_j = \sigma_j \sigma_i \quad (|i - j| \geq 2), \quad \sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1}.$$

The generator σ_i is the elementary act of one defect braiding around the next (a generator of $\pi_1(C_n(\Sigma))$); the second relation (Yang–Baxter) carries the non-trivial content. (*Definition.*)

- **Relational state and holonomy.**

$$\varrho: B_n \rightarrow U(\mathcal{H}_n), \quad \text{good/bad criterion carried by } \varrho(b).$$

A configuration of defects is a vector in a fusion space $\mathcal{H}_n$; a closed sequence of couplings b acts as the unitary $\varrho(b)$, the braid-language reappearance of Stage I's holonomy and the fixed-sector shadow of the field's connection. (*Definition.*)

- **The three non-abelian demands (Section 5.3).**

$$\underbrace{\varrho(\sigma_i \sigma_j) \neq \varrho(\sigma_j \sigma_i)}_{\text{power}}, \quad \underbrace{\dim \mathcal{H}_n > 1}_{\text{resonance without fusion}}, \quad \underbrace{a \times b = \bigoplus_c N_{ab}^c c, \sum_c N_{ab}^c > 1}_{\text{generativity}}.$$

Order-dependence of coupling expresses power; a degenerate (> 1 -dimensional) joint state expresses coupling that neither juxtaposes nor merges; multi-channel fusion expresses outcomes not fixed by the relata. Each *independently* forces ϱ to be non-abelian. (*Three premise-dependent arguments; premises P, R, G stated in Section 5.3.*)

- **The discrimination constraint (Proposition 6.1).**

$$\overline{\varrho(B_n)} = PU(\mathcal{H}_n) \implies \text{no robust, non-constant holonomy-factoring criterion exists.}$$

If braiding can approximate every unitary (universality), then any perturbation-stable good/bad criterion read from the holonomy must be constant: omnipotence and normative discrimination are incompatible. The admissible representation is therefore *non-abelian yet non-universal*. (*Theorem, scoped to robust holonomy-factoring predicates.*)

Stage III': the dynamical layer (Section 5).

- **Flow of value on the generated structure.**

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = S, \quad \partial_t \boldsymbol{\omega} + (\mathbf{u} \cdot \nabla) \boldsymbol{\omega} = (\boldsymbol{\omega} \cdot \nabla) \mathbf{u} + \nu \nabla^2 \boldsymbol{\omega} + \nabla \times \mathbf{f}(\mathbf{p}).$$

Generation (Stages II–III) gives the structure; this layer gives its running—how value flows, circulates, and redistributes. The fluid description of Stage I is reinstated here, now on the generated space rather than an assumed one. The source S is supplied from below as the defect creation/annihilation rate; Proposition 2.1 is the founding theorem of this layer (appropriation, being gradient, generates no circulating surplus); and α/β controls solidification, the dynamical collapse of a generative cycle to a frozen fixed point. (*Dynamics; joins generation at S and at Proposition 2.1.*)

Stage IV: value’s two components (Section 7).

- **Skeleton and flesh.**

$$\text{value} = \underbrace{\text{topological skeleton}}_{\varrho(b), \text{ type, normative criterion}} + \underbrace{\text{local flesh}}_{\text{token identity, singular detail}}.$$

The non-abelian holonomy carries the good/bad criterion but only type identity; the non-substitutable particularity of *this* relation (its haecceity) is precisely what topological protection discards, and so lives in the flesh. The conjecture of Section 7 is that the good is the appropriate proportion of the two. (*Decomposition; the proportionality thesis is a conjecture.*)

Stage V: the aggressive relational limit (Section 9).

- **Group as base; background-free field and dynamics.**

$$\Phi: \Gamma \rightarrow M, \quad \mathcal{S}[\Phi] = \sum_{\gamma} V(\Phi(\gamma)) + \lambda \sum_{\gamma, \gamma'} K(\gamma^{-1}\gamma') \Phi(\gamma)\Phi(\gamma').$$

The configuration “space” is a group Γ (pure relation: γ is individuated only by $\gamma\gamma'$); the coupling refers only to the relative relation $\gamma^{-1}\gamma'$, never to an external position, so the dynamics is written in relations alone. Power is the non-commutativity of the base, $\gamma\gamma' \neq \gamma'\gamma$. (*Toy model; formally well-posed, presented with three explicit open walls—group rigidity vs. generativity, the regress of the base, and the unfinished state of group/categorical field theory.*)

Stage VI: empirical operationalization (Section 10).

- **Scale-invariant measurement and educational case study.**

$$\hat{H}(\gamma) = T_{i \rightarrow j} T_{j \rightarrow i}, \quad \Theta = \Delta a - \Delta d, \quad \mathcal{C} = \|T_{i \rightarrow j} T_{j \rightarrow i} - T_{j \rightarrow i} T_{i \rightarrow j}\|.$$

Magnitude is unmeasurable (no scale); sign, order, and non-commutativity are not. The continuous holonomy discretizes to a product of estimated transition operators (23); with the state vector $s_i = (a, d, g)$ (autonomy, dependence, generative output), the sign proxy Θ reads good/bad ($\Theta > 0$ generative/power-to, $\Theta < 0$ dominative/power-over, $\Theta \approx 0$ appropriative), and $\mathcal{C}$ measures power strength. Case study: lecture-drill vs. inquiry-based instruction, with the falsifiable prediction that Θ trajectories diverge *independently of exam score*. (*Falsifiable operationalization, not a validated instrument; limits explicitly marked.*)

12 Conclusion: coupling without merger

The argument has traveled from a fluid intuition to a theory of structure generated by symmetry breaking, and the journey was the point. The fluid model gave one exact result—that appropriation generates no circulation, so that the holonomy carrying the good/bad distinction is necessarily non-possessive (Proposition 2.1)—and then failed in a way that diagnosed the requirement on any successor: a relational ontology in which not even the order-parameter space is presupposed. The resolution took a symmetry group G as pre-structure and let spontaneous breaking $G \rightarrow H$ generate the rest—the order-parameter space as $M = G/H$, subjects as its defects $\pi_k(G/H)$, the surplus of a closed cycle as the holonomy of the canonical connection on G/H with its appropriative part contributing nothing (the local value stock being the field value itself), and power as the non-commutativity of that holonomy when H is non-abelian—so that the three features otherwise installed as separate premises descend from one symmetry condition.

The braid group of defect statistics is then the computed fundamental group of the configuration space, $\pi_1(C_n(\Sigma)) = B_n(\Sigma)$, not an axiom; on its non-abelian, non-universal representation, power, resonance-without-fusion, and generativity unify in the non-abelian holonomy, while Section 7 restrains the unification by isolating what that holonomy cannot reach—the local flesh, the number of subjects, and the background. Why the symmetry breaks at all we do not say: the theory is post-breaking, its input a generation event it does not explain, and the aggressive limit of Section 9—making the base itself relational—marks the open frontier where that question, and the origin of G , would have to be met.

It is worth returning the apparatus to the concrete relation that motivates it. The thesis that shared happiness is the resonance of two meaning-worlds rather than their merger receives, in this language, an exact form: two excitations may be braided so that their joint state occupies a degenerate fusion space, coupled and irreducible to the sum of its parts, yet without merging, since each retains its own topological charge. The charge secures only that the two remain of distinct *type* within the coupling; that they remain *these two*, irreplaceable, is carried not by the charge but by the local flesh of Section 7, so that the formal account of resonance and the account of irreplaceability are handled by complementary parts of one model rather than competing for the same one. Coupling without merger is the braiding of charges that do not fuse into one; and the order of that coupling matters, which is to say that even here, in the gentlest relation, the asymmetry that the formalism calls power is not an intrusion but a condition of there being a determinate shared state at all. The same structure scales from the intimate to the political: because power and generativity share the single root of a non-abelian residue (Proposition 4.1), a world with subjects and generativity cannot be a world without power, and the abolition of power would be the abolition of generativity—one of the two equalities of privation in the trilemma of Section 4.4. What is achievable is not the removal of asymmetry but the shaping of its geometry, so that the unavoidable power returns surplus rather than retaining it, serves the other’s capacity rather than fixing it in dependence. Justice, in this formalism, is the governance of power’s geometry, not its elimination. What the model finally offers the philosophy of intimacy, and of justice, is therefore not a reduction of relation to physics but a vocabulary precise enough to say what was always meant: that two, or many, can be joined in a way that generates rather than consumes, and that being so joined need not cost either its own form.

We end by naming the source this formal structure has, in the end, only retranslated. The *Daodejing* (ch. 51) gives four phrases: *sheng er bu you*, *wei er bu shi*, *zhang er bu zai*, *shi wei xuan de*—“generating yet not possessing, acting yet not presuming upon it, fostering growth yet not ruling over it: this is called the dark virtue (*xuan de*).” Each phrase has, in the present formalism, a precise body. *Generating yet not possessing* is the holonomy itself: the surplus of a closed cycle is genuinely produced yet, being non-local and irreducible to any point, has no possessor (Section 4.3); a value that is real yet without an owner is, mathematically, a non-local circulating residue, and the classical commentators’ gloss on *xuan de*—“virtue real yet whose master cannot be found”—is the same thought. *Acting yet not presuming* is the distinction between power-to and power-over (Section 4.4): asymmetry is unavoidable wherever there is generation, but its holonomy may return the surplus rather than retain it. *Fostering yet not ruling* is the generative asymmetry whose geometry works toward its own dissolution, the upstream position that serves the downstream’s autonomy rather than fixing it in dependence. What the formalism adds to the ancient phrase is only a reason for its necessity: the generative surplus *cannot* be possessed, because possession is the gradient, single-valued, scalar-reducible mode that contributes nothing to the holonomy (Proposition 2.1). To grasp at the surplus is to collapse it back into stock and halt the generation; in the instant of appropriation, creation stops. “Generating yet not possessing” is therefore not a counsel of restraint but the mode of existence of generative value as such—and this, perhaps, is what was always meant by the dark virtue.