

A Generalised Heterogeneous Dynamical Network and a Spinfoam Model of Knowledge Emergence and Evolution

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Abstract

Accounts of how knowledge spreads ordinarily treat difficulty as loss in transit, and take the ideal case to be one in which what arrives is what was sent. On a relational account that ideal is incoherent: what a party holds is reconstructed from the representations available to it under the interpretation it holds, so what crosses between parties is a perturbation and heterogeneity is primitive. This paper provides two preliminary formal models of knowledge emerging from dynamics over heterogeneous structures. The first couples components whose state spaces differ in structure and in whether they carry a metric, whose laws differ in kind, and whose events are their own, by propagation operators that carry occurrences and presuppose no common space; a coupling is itself a component with a state of its own, and a coupled assembly is a component of the same class under stated conditions. The second takes the combinatorial structure of spin foam models and reassigns it: faces are the histories of primitive elements, edges are couplings persisting across intervals, vertices are events at which couplings rearrange, and a boundary is a state, so that a history is a two-complex whose one-dimensional shadow is a record of transitions between identified states. The justification for the physical borrowing is background independence, which this account requires twice, and the amplitudes that carry the physical content are declined with the reason given. Knowledge is located within a design: for a party it is the set of couplings surviving reconstruction under every perturbation that party can undergo, for a pair it is an invariant of the coupled system that belongs to neither component, and at an event it is the couplings present after it and absent from everything it received. The definitions yield tests. Deleting a coupling settles whether a capacity belongs to a party or to the coupling; a difference between two sets of couplings settles whether a causal attribution was made at an event or carried through it, which gives the construction of a social fact an operational form. The models also expose a commitment the account had carried unnoticed: a weight over histories presupposes recurrence, while an account holding interpretive conditions to be historically produced is committed to the singularity of a trajectory, so the position itself withholds the dynamics such a weight would require. A news-reporting case runs through both models and a simulation exhibits the design.

Keywords. background independence; heterogeneous dynamical networks; spin foam models; knowledge emergence; relational epistemology

Draft. This is a working draft. It is circulated for comment, and its arguments, formulations, and open questions are subject to revision. Correction and criticism are welcome at huangwanhong@serendip.ngo.

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Use of generative artificial intelligence. Generative AI assistants (Anthropic’s Claude and OpenAI’s ChatGPT) were used extensively in preparing this manuscript: for literature survey, for drafting prose from the author’s specifications, for adversarial review of the argument, for the construction of the figures, for the implementation of the simulation and the computation of the quantities reported from it, for consistency and citation auditing, and for typesetting. Formulations and objections arose in the course of that exchange as well as from the author. Every claim, argument, and citation was reviewed and decided by the author, who is responsible for the content and for any errors. Bibliographic details are to be verified against the primary sources before reliance.

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1 Introduction

This section states the refusal from which the paper proceeds, the formal problem the refusal creates, the two models it proposes, the standard against which they are to be judged, and the structure of the argument. Its method is expository, and every claim made here is established later and is marked accordingly.

Accounts of how knowledge spreads ordinarily proceed from a picture in which something is held by one party, is conveyed to another, and arrives more or less intact. The picture is so ordinary that it survives in the vocabulary of writers who reject it: knowledge is said to be transferred, disseminated, shared, communicated, and the difficulties treated by such accounts are difficulties of fidelity, being losses, distortions and noise incurred in transit. The ideal case is one in which what arrives is what was sent.

The relational account this series develops makes that ideal incoherent. What a party holds is not a content that could be conveyed. It is a structure reconstructed from the representations available to that party under the interpretive rules in force for it, and a second party receiving a representation reconstructs from it under rules that are their own. Nothing content-like crosses. What crosses is a perturbation, and what results is generated at the receiving side.

The refusal is easy to state and hard to hold. Its difficulty is that the transmission picture reinstalls itself in a formalism even where the prose has rejected it, and the mechanism by which it does so is worth naming at the outset. A model that places the states of several parties in one space, so that their states can be compared, has supposed that differences between parties are distances in a common medium. Distances in a common medium are what a signal traverses. The picture returns as a property of the mathematics, and no amount of care in the surrounding prose removes it.

So the refusal has a formal consequence, and the consequence is the paper's first constraint: a model of the kind this account requires may not assume a space in which the states of distinct parties are located. Their state spaces may differ in dimension, in structure, and in whether they carry a metric at all, and a formalism adequate to the account must couple them without embedding them.

That constraint is unusually demanding, and the paper's contribution is an attempt to meet it twice, in two different respects, with two models that are complementary rather than alternative.

The first concerns the coupling of processes. Parties to a knowledge relation are heterogeneous not by accident but by their material situation: what a party can observe, afford to record, and be exposed to differs, and those differences produce different internal dynamics and different representational systems. A model adequate to this is a network whose nodes carry their own state spaces and their own laws of evolution, coupled by operators that map events emitted at one node into admissible inputs at another. Such an operator requires no distance, no similarity measure and no common coordinates. It requires only that what one node emits be presentable to another, which is the minimum any coupling requires and less than any embedding supposes.

The second concerns the structure of a change. The companion papers of this series

specify a record whose unit is the transition between two identified states of an understanding, and that record is a directed graph with typed arcs. It can say that a state became another state. It cannot say which elements of the earlier state survived the change, which couplings among them were broken, and which were created. If a state is not a blob of content but a relational structure of primitive elements and their couplings, then a change is a rearrangement of that structure, and a representation adequate to the account must show the rearrangement. A history is then not a sequence but an object of two dimensions: the histories of elements, the couplings that persist between events, and the events at which couplings rearrange.

The second model borrows its combinatorial structure from spin foam models in quantum gravity, and the borrowing needs a justification stronger than resemblance. The justification is background independence. In those models there is no background metric on which the theory is defined: geometry is a result of combinatorial and algebraic data rather than a stage on which the data sit. In the account developed here there is no pre-given space of meanings on which parties are located: relational structure is generated by representations and by the interpretations in force upon them. Both are theories in which the space is a consequence. That is a structural correspondence and not a metaphor, and it is the reason the borrowing is from this quarter of physics in place of from the general stock of graph formalisms, whose members ordinarily presuppose a background of states, places or positions.

The paper declines one part of the borrowed apparatus, and the declining is stated in the opening because it bounds everything that follows. Spin foam models assign amplitudes, and their amplitudes derive their content from the representation theory of a specific group and exhibit interference under summation. Neither has a counterpart here. A paper claiming amplitudes without an interpretation would be exposed to the objection its own analogy discipline exists to prevent, and this one accordingly stops at the combinatorial structure and records the amplitude assignment as an open problem.

The paper also declines to locate knowledge in the models, and this is a matter of architecture rather than caution. A model of the kind proposed here is a formalism: it fixes what kinds of object there are and how they compose, and it asserts nothing about the world until an interpretation is assigned. Knowledge is located only once a design fixes the reconstruction maps and the class of perturbations under which a party's structure is assessed. The answer therefore appears in the design Part and is stated for that design: knowledge for a party is the set of couplings that survive reconstruction under every admissible perturbation; relational knowledge is an invariant of a coupled system that is not the image of any component's invariant; and emergence at an event is the set of couplings present in an outgoing boundary and in no incoming one. That last is where the terminological commitment of this series receives a formal statement rather than a stipulation, since a coupling in an outgoing boundary was either already in an incoming one, in which case it was re-instantiated by the receiver's own dynamics, or was not, in which case it was made at that event. There is no third case for a word naming conveyance to name.

The standard by which the models are to be judged is fixed here in advance, so that they are not assessed by elegance. The transition graph of the companion papers is the existing representation, it is adequate to everything the specified protocol requires, and it

can be implemented with a text editor and a version-control system. A richer formalism is easy to construct. Either model in this paper earns its place only where some question about knowledge is answerable within it and is not answerable in that graph, and the paper states three candidate questions early and returns to them in its assessment.

A standing illustration runs through both models. It is a news-reporting case: an occurrence, witnesses, a reporter, an editor, an archive, readers and an institution issuing statements, with the reporter's work treated as the encapsulation it is. The case is chosen because the construction of a social fact is visible in it, because the parties are heterogeneous in exactly the material way the account requires, and because the question of what a report conveys and what it makes is the question the models are supposed to render structural. A musical example appears once, in the design Part, where a reader meeting a two-complex needs something concrete with elements, couplings and events and none of the difficulties a knowledge example carries.

The argument proceeds in seven parts. The first fixes the method, the terms and the analogy discipline. The second states the ontology and epistemology, imported from the author's prior work as revisable posits rather than re-derived, and ends by deriving the two representational requirements the models are answers to. The third states the premises: background independence, spin foam models and group field theory, event-based computation and the nearest existing formalisms, with the boundary of each transfer. The fourth and fifth develop the two models. The sixth states their correspondence and gives a design, together with the alternative designs the models admit and the sense in which the transition graph is a coarse-graining of them. The seventh states what the models answer, what they do not, and the conditions under which either would be shown inadequate.

One caution belongs at the opening rather than in a final section. The transfer of heavy mathematical apparatus into a domain that lacks the empirical constraints giving it content elsewhere is a recognised failure, and this paper is exposed to it. What limits the exposure is that the borrowing is justified structurally rather than by resemblance, that the part of the apparatus which carries the physical content is explicitly declined, that the standard of adequacy is fixed before the models are stated, and that the representation the companion specification actually uses is the directed graph, which neither model here replaces.

2 Method, Terms, and the Analogy Discipline

This section fixes what kind of object the paper offers, what it borrows and from where, what it refuses, and the words it uses in a fixed sense. Its role is to make the two models assessable: a formalism proposed without a stated standard, a stated boundary on its borrowings and a stated vocabulary cannot be judged, and is accordingly not a proposal but a picture. Its structure treats the distinction between a model and a design, the modal status of what follows, the transfer from physical theory, the relation to event-driven computation, the terminological rule, the terms, and the historical qualification on the imported primitives. Its method is stipulative.

2.1 The Distinction between a Model and a Design

Definition 2.1. (Model and design). *A model fixes what kinds of object there are, how they compose, and what operations are defined on them, and asserts nothing about any domain. A design assigns interpretations to the objects of a model, fixes the parameters the model leaves open, and thereby asserts something that may be right or wrong.*

The distinction is not pedantic here, since it determines where the paper's substantive claims may appear. A model that located knowledge would be asserting, and would be a design under another name. The two models of Parts IV and V accordingly locate nothing, and the location of knowledge appears in Part VI, stated for the design given there and holding for no other.

The distinction also creates an obligation the paper must discharge. A pair of models with no design is an offer of notation, and the reader is entitled to ask what either is for. Part VI therefore supplies one design in full, and states the alternatives the models admit so that the design is not mistaken for the models' only reading.

2.2 The Modal Status of What Follows

Requirement 2.2. (Independence of the companion specification). *Nothing in the companion specification of this series depends on either model. No requirement, act, field or conformance condition is added or altered by this paper, and an implementation conforming to that specification remains conformant whatever becomes of the constructions proposed here.*

The requirement is a practical arrangement rather than a modesty. A specification whose adequacy waited on an unformalised model would be unimplementable until the model was settled, and the series is arranged so that the engineering does not wait on the mathematics.

Within the paper, the two models are offered as formal proposals with a bounded claim. The combinatorial structure is defined completely, the conditions a coupling must satisfy are stated precisely, and one correspondence is worked. What is not offered is a theory: no dynamics is derived, no quantity is computed, and no prediction about any actual knowledge process is made.

2.3 The Transfer from Physical Theory and the Refusal of Amplitudes

Requirement 2.3. (The physical transfer and its boundary). *From spin foam models this paper takes a combinatorial structure, namely a two-complex whose faces, edges and vertices carry content of different kinds, together with the composition of histories along shared boundaries. It takes no representation theory, no amplitude assignment, no summation over histories, and no physical claim of any kind. The justification for taking the structure is background independence, stated in Part III, and no further property of the physical theory transfers on the strength of that justification.*

The refusal of amplitudes has a cost and the cost is admitted here rather than buried. In physics the amplitude assignment is what makes a spin foam a theory rather than a

bookkeeping device: it is where the dynamics lives and where predictions come from. Declining it leaves this paper with a combinatorial skeleton, and a reader who holds that a skeleton is not worth having is holding a reasonable position that the paper does not refute. What the paper claims is narrower: that the skeleton represents something the existing representation cannot, and that the question of what an amplitude would mean here is better left open than answered badly.

Two properties of an amplitude assignment show why answering it badly is the likely outcome. Physical amplitudes are complex, and their sums exhibit interference, which is a substantive feature and not a convention; there is no evident counterpart to superposition in an account of reconstruction under interpretation. And their content derives from the representation theory of a specific group, which supplies the algebra by which the labels on faces and edges compose; the informational side has no such algebra at present, and inventing one to fill the slot would be the failure the analogy discipline exists to prevent.

2.4 The Relation to Event-Driven Computation

Requirement 2.4. (The computational transfer and its boundary). *From event-driven computation this paper takes three things: that a system may advance at occurrences in place of at intervals, that components with distinct internal descriptions may be coupled, and that a coupling should itself be an object of the same class as the components it couples. It takes no property of neural computation, no hardware property, and no claim that a knowledge relation resembles a nervous system. The network model of Part IV is not a spiking neural network, and is not inspired by one in the sense of adapting its mechanisms; a spiking network is one instance of a class that the model generalises, and the generalisation removes precisely the features that make a spiking network specific.*

The requirement is stated at length because the resemblance is the first thing a reader will notice and the most misleading. A spiking network has homogeneous units, a shared notion of time, a common state space across units, and a fixed form of coupling. The model of Part IV has none of these, and its interest lies in what happens when they are removed.

2.5 The Terminological Rule

Requirement 2.5. (Propagation, evolution, and the forbidden word). *Propagation is used of perturbations crossing a coupling, and of records moving between trajectories and implementations in the sense fixed by the companion papers. Evolution is used of relational structure changing at events. Transmission, and every cognate that names conveyance, is not used of knowledge anywhere in this paper.*

The ground of the rule is that vocabulary reinstates a picture that definitions have removed. A paper may define reconstruction carefully and then write that a finding was transmitted to a community, and the reader will supply the transmission picture from the word, because that is what the word is for. The rule is therefore stated as a prohibition on a term rather than as a preference, and Part II supplies the argument that there is

nothing for the term to name.

The rule has a cost in ordinary usability, recorded here and again in Part II. The account cannot say that one party learned what another knows, and must say instead that the second party reconstructed, under its own interpretation and following a perturbation originating with the first, a structure standing in some stated relation to what the first party's reconstruction contains. The formulation is longer and weaker, and its weakness is the intended content.

2.6 Terms Fixed for the Remainder

Definition 2.6. (The standing terms). *A primitive element is a representational atom of a design: what the design treats as not further decomposed. A coupling is a relation holding between primitive elements. A relational structure is a set of primitive elements together with the couplings holding among them. A reconstruction is a relational structure produced by a party from the representations available to it under the interpretation in force for it. A perturbation is what crosses a coupling between parties: an occurrence at the receiving side which its own dynamics take as input. An event is an occurrence at which couplings within a relational structure are created, destroyed or rearranged. A state is a relational structure at a boundary.*

Two of these depart from ordinary usage and the departures are marked. A state is not a condition of a party but a configuration of couplings, so a party does not have a state in the sense in which a system has one; it has a reconstruction, and the reconstruction has a boundary configuration. And an event is not something that happens to a party but a rearrangement of structure, so a party's activity may produce no event and an event may involve several parties.

2.7 The Historical Qualification on the Imported Primitives

Part II imports the relational primitives of this account from the author's prior work rather than re-deriving them, on the ground that a paper re-arguing its ontology is a philosophical paper carrying two models in place of a formal paper with a stated foundation. What is imported is stated there, and the import carries a qualification that does formal work later.

Requirement 2.7. (The historical qualification). *The imported primitives are revisable posits and not axioms. Relational structures are produced under material conditions and are transformed historically, so a party's representational system, its interpretive rules, and the class of perturbations to which it is exposed are historically produced rather than chosen. Two consequences are held throughout: the heterogeneity assumed in Part IV is a consequence of material differences among parties in place of a modelling convenience; and the class of admissible perturbations against which invariance is assessed in Part VI is fixed by conditions rather than stipulated by the analyst.*

The second consequence is the one that makes the qualification more than a preface. Knowledge is defined in Part VI relative to a class of admissible perturbations, and that class is set by what a party can be exposed to, which is a material matter. Two parties

with identical reconstruction maps and different exposure hold different knowledge of the same world, which is a formal statement of a materialist commitment rather than an illustration of one.

2.8 Summary of Results

This section has fixed the instruments. Definition 2.1 separates a model, which asserts nothing, from a design, which assigns interpretations and asserts, and places the location of knowledge in the design Part accordingly, while creating the obligation to supply a design in full. Requirement 2.2 makes the companion specification independent of both models. Requirement 2.3 states the transfer from spin foam models as combinatorial structure and composition alone, refuses amplitudes, and admits the cost of the refusal together with the two properties of physical amplitudes that have no counterpart here. Requirement 2.4 states the transfer from event-driven computation and denies that either model is or adapts a spiking neural network. Requirement 2.5 fixes the terminological rule and its ground, with its cost in ordinary usability recorded. Definition 2.6 fixes primitive element, coupling, relational structure, reconstruction, perturbation, event and state, with the two departures from ordinary usage marked. Requirement 2.7 fixes the historical qualification on the imported primitives and its two consequences: heterogeneity is material rather than convenient, and the perturbation class is set by conditions rather than stipulated.

3 Knowledge as Reconstruction from Relational Structure

This section states the ontological and epistemological commitments the models answer to. Its role is to establish that the transmission picture is not merely unattractive but unavailable, and to draw from its unavailability the formal constraint that governs Part IV. Its structure treats the imported posits, the argument by regress, what may still be said of two parties, the formal consequence, and the objection that the account applies to itself. Its method is argument, and the posits it argues from are imported rather than derived.

3.1 The Imported Posits

Three posits are imported from the author's prior work as revisable posits, in the sense fixed in Section 2. They are stated compactly and are not re-argued.

A representation is a material thing: a mark, an utterance, a file, a gesture, standing in relations to other representations and to the conditions under which it was produced.

An interpretation is a set of rules, held by a party and historically produced, under which representations yield a relational structure. Interpretations are not private in the sense of being arbitrary, since they are formed under conditions the party shares with others, and are not common in the sense of being identical, since the conditions differ.

Definition 3.1. (Reconstruction). *A reconstruction is the relational structure a party produces from the representations available to it under the interpretation in force for it. What a party holds is a reconstruction; what a party can make available to another is a representation.*

The definition already forecloses the transmission picture, and the remainder of the section establishes that the foreclosure is not a stipulation.

3.2 The Argument by Regress

Suppose it were said that two parties hold the same knowledge. The claim is a comparison, so something has compared two reconstructions and found them alike.

Claim 3.2. *A comparison of two reconstructions is itself performed under an interpretation, and the interpretation belongs to one of the two parties or to a third. There is no comparison performed from no position. This claim is formal.*

Two consequences follow immediately, and the second is the one that matters.

Where the comparison is performed by one of the parties, its result reports what that party's interpretation makes of the other's representations, which is a fact about the comparer.

Where it is performed by a third party, the same holds of the third, and the appeal to a third has produced a further reconstruction rather than a view of the two.

Claim 3.3. *The transmission picture requires a standpoint from which the content held by two parties can be compared without being reconstructed. No such standpoint is available under the imported posits, and the requirement is therefore not merely unmet but incapable of being met within the account. This claim is formal.*

The argument is a regress and not an empirical finding, and its force is correspondingly limited. It establishes that the account cannot support the picture; it does not establish that the account is true.

3.3 What May Still Be Said of Two Parties

Refusing the picture is not refusing all comparison, and the account would be useless if it were.

Claim 3.4. *Under the account the following are available and the following alone: that a party's reconstruction contains a coupling; that a second party's reconstruction contains a coupling standing in a stated relation to the first, as assessed under a stated interpretation; and that a perturbation originating with one party preceded a change in the other's reconstruction. Statements of the form that two parties hold the same content are unavailable. This claim is formal.*

The third item deserves emphasis because it is what survives of causal talk. The account permits a party to be said to have occasioned a change in another, and forbids that party to be said to have conveyed anything, and the difference is exactly the difference between a perturbation and a content.

3.4 The Formal Consequence

Claim 3.5. *A formalism adequate to this account may not place the states of distinct parties in a common space. A common space supplies positions, positions supply differences, and differences in a common medium are what a signal traverses, so an embedding reinstates the transmission picture as a property of the mathematics whatever the surrounding prose asserts. State spaces of distinct parties may therefore differ in dimension and in structure, may lack a metric, and must be coupled without being embedded. This claim is formal.*

The claim is the paper's principal constraint and it is severe. It excludes vector-space models of meaning in which parties are points or regions, since those are embeddings. It excludes similarity measures defined across parties, since a measure defined across parties presupposes the common space. And it excludes any account of communication as reduction of a distance.

What it leaves is a coupling that requires only that what one party emits be presentable to another, which is what Part IV constructs.

3.5 The Objection That the Account Applies to Itself

The objection is obvious and is answered here rather than left to a reader.

If every claim is a reconstruction under an interpretation, then so is this account, and the account appears to undercut its own standing.

The first half is conceded without qualification. The account is a reconstruction, produced under an interpretation the author holds, from representations the author had available. Nothing in it claims a standpoint it denies to others.

The second half does not follow. That a claim is made under an interpretation does not make it arbitrary, because an interpretation constrains: under a stated interpretation some structures are admissible and others are not, and a claim can be assessed against the interpretation stated. The account's own admissibility is assessable in exactly the way it says any claim is, which is a weaker warrant than the objection demands and is the warrant the account says is available.

Claim 3.6. *Self-application concedes that the account is a reconstruction and denies that reconstruction implies arbitrariness, since an interpretation constrains what is admissible under it. The account claims no exemption and offers the warrant it says is generally available. This claim is formal.*

3.6 Summary of Results

This section has stated the posits and drawn their consequences. Definition 3.1 fixes reconstruction as what a party holds, against representation as what a party can make available. Claim 3.2 establishes that a comparison of reconstructions is itself performed under an interpretation belonging to some party, and Claim 3.3 that the transmission picture requires a standpoint the account cannot supply. Claim 3.4 states what remains available: containment of a coupling, a stated relation between couplings under a stated

interpretation, and the precedence of a perturbation before a change. Claim 3.5 draws the formal consequence that governs Part IV, namely that distinct parties' state spaces may not be embedded in a common space, which excludes vector-space models of meaning, cross-party similarity measures, and accounts of communication as the reduction of a distance. Claim 3.6 answers self-application by conceding that the account is a reconstruction and denying that reconstruction implies arbitrariness.

4 Interpretation across Heterogeneous Representational Systems

This section establishes that heterogeneity among parties is primitive rather than residual. Its role is to supply the second constraint on the models: not only may parties not be embedded in a common space, they may not be assumed to differ only in degree. Its structure treats the sources of heterogeneity, its material production, the standardising response and what it costs, and the consequence for a formalism. Its method is argument from the imported posits together with the historical qualification.

4.1 The Sources of Heterogeneity

Parties to a knowledge relation differ in at least four respects, and the respects are not reducible to one another.

They differ in the representations available to them. What one party can observe, receive, retain and consult is not what another can.

They differ in the interpretive rules in force. Two parties reading the same representation produce different structures because the rules by which representations yield structure are not the same.

They differ in what counts as a primitive element. One party's atom is another party's composite, and there is no level at which the decomposition is settled independently of a party.

And they differ in the kind of process by which their reconstruction changes. A party whose structure changes by deliberation, a party whose structure changes by accumulation of instances, and a party whose structure changes by institutional decision are not doing the same kind of thing more or less well.

Claim 4.1. *Heterogeneity among parties is fourfold and is not a difference of degree along a single dimension. A formalism representing parties as instances of one kind, differing in parameter values, has assumed away three of the four respects. This claim is formal.*

4.2 The Material Production of Heterogeneity

The historical qualification of Section 2 does its first work here.

A representational system is not chosen by a community from a menu. It is shaped by what that community can afford to observe, what it has the means to record, what circulates within reach of it, and what it is exposed to. An observatory and a village both hold accounts of the sky, and the difference between their accounts is not a difference of care.

Claim 4.2. *The heterogeneity assumed by the models of Parts IV and V is a consequence of material differences among parties and not a modelling convenience adopted for generality. A model that removed it would be describing parties whose material conditions had been equalised, which is not the case the account concerns. This claim is formal, and the material premise is imported.*

The claim has a consequence for how the models should be read. Their generality is not the generality of a framework that could be specialised to a homogeneous case as a matter of taste. The homogeneous case is a limit that obtains where material conditions have converged, and it is not the ordinary case.

4.3 The Standardising Response and Its Cost

There is a well-established response to heterogeneity, and it is worth stating fairly before its cost is named.

Common vocabularies, controlled terminologies, standard formats, shared ontologies and reporting guidelines reduce the divergence between parties' representations, and they work. The companion papers of this series bind to several such standards and are better for it.

The cost is not that standardisation fails. It is that it is not free and that its price falls unevenly.

Claim 4.3. *A standard is adopted at a cost in effort, equipment and training, and parties unable to bear that cost are excluded from the arrangement the standard constitutes. Their exclusion appears from inside the arrangement as their absence rather than as an exclusion, since the arrangement's records contain only what conforming parties produced. This claim is formal, and the pattern it describes is documented in the companion paper on adoption.*

Two remarks bound the claim. It is not an argument against standards, which this series uses. And it is not an argument that heterogeneity is good, which would be sentimental: heterogeneity is costly, and the point is that its costs and the costs of removing it fall on different parties.

4.4 The Consequence for a Formalism

The section's constraint is stated for use in Part IV.

Requirement 4.4. (Irreducible heterogeneity). *A formalism adequate to this account admits parties whose state spaces differ in structure, whose laws of evolution differ in kind, whose primitive elements differ in what they decompose, and whose interpretive rules differ. It may not require that any of these be shared, and it may not represent the differences as parameter values within a single specification.*

The requirement is demanding in one specific respect that Part IV must handle. Where two parties' state spaces have no common structure, an operator carrying something from one to the other has no obvious type, and the temptation is to supply one by embedding both in something larger. That is the move Claim 3.5 forbids, and Part IV must couple without it.

4.5 Summary of Results

This section has established that heterogeneity is primitive. Claim 4.1 states that it is fourfold, covering available representations, interpretive rules, primitive elements and the kind of process by which reconstruction changes, and that these are not degrees along one dimension. Claim 4.2 states that the heterogeneity assumed by the models follows from material differences rather than from a wish for generality, so the homogeneous case is a limit obtaining under converged conditions in place of the ordinary case. Claim 4.3 states the cost of the standardising response: a standard excludes parties unable to bear its cost, and the exclusion is invisible from inside the arrangement because its records contain only what conforming parties produced. Requirement 4.4 fixes irreducible heterogeneity for Part IV and names the difficulty it creates, namely that coupling state spaces without common structure must be achieved without the embedding that Claim 3.5 forbids.

5 Emergence, Evolution, and the Refusal of Transmission

This section fixes the three words on which the paper's terminological rule rests, and establishes that the fourth word has nothing to name. Its role is to convert a stylistic prohibition into a result. Its structure treats the three terms, the exhaustiveness argument, the independence of propagation and evolution, and the costs of the position. Its method is definition followed by argument.

5.1 The Three Terms

Definition 5.1. (Propagation). *Propagation is the crossing of a coupling by a perturbation. What propagates is an occurrence at the receiving side, taken by that side's own dynamics as an input. Propagation is defined between parties and asserts nothing about what either party's reconstruction contains.*

Definition 5.2. (Evolution). *Evolution is the change of a relational structure at an event: couplings created, destroyed or rearranged. Evolution is defined within a reconstruction and asserts nothing about what crossed any coupling.*

Definition 5.3. (Emergence). *A coupling is emergent at an event where it is present in the structure the event produces and is present in no structure the event took as input. A coupling present in some input structure is re-instantiated at that event.*

The third definition is stated informally here and receives its formal statement in Part VI, where the boundaries of an event are available. What matters at this point is its exhaustive form.

5.2 The Exhaustiveness Argument

Claim 5.4. *Consider any coupling present in the structure produced at an event. Either it was present in some structure the event took as input, in which case it was re-instantiated*

by the receiving side's own operation, or it was not, in which case it was made at that event. The disjunction is exhaustive and exclusive. There is accordingly no third case in which a coupling arrived from elsewhere, and no work remains for a word naming conveyance. This claim is formal, given Definition 5.1 to 5.3.

The claim is the formal form of the terminological rule fixed in Section 2, and it changes the status of that rule. The prohibition on the word is not a preference for a certain register. It is the observation that the word would name a case the definitions do not admit, so its use would assert something the account denies.

Two clarifications forestall a misreading.

The claim does not deny that a party's reconstruction may come to contain a coupling that another party's reconstruction contained. It denies that the coupling travelled, and it locates the coupling's appearance in the receiver's operation under a perturbation.

And the claim does not make the sender irrelevant. Without the perturbation the receiver would not have produced what it produced, and the account permits that dependence to be stated, as Claim 3.4 records.

5.3 The Independence of Propagation and Evolution

A natural misreading treats the two as stages of one process: a perturbation propagates and then structure evolves. The account holds them independent, and two ordinary cases show why.

A reader receives an article and puts it down unmoved. A perturbation propagated and no coupling in the reader's reconstruction was created, destroyed or rearranged. Propagation occurred and evolution did not.

A researcher reconsiders overnight and wakes holding a structure they did not hold the evening before. Couplings were rearranged with no perturbation crossing any coupling between parties. Evolution occurred and propagation did not.

Claim 5.5. *Propagation and evolution are independent: either may occur without the other. They are therefore not stages of a single process and must be represented by different apparatus, which is one reason the paper proposes two models in place of one. This claim is formal, and the two cases above are illustrations rather than evidence.*

The claim has a structural consequence for the paper. The network model of Part IV represents propagation and represents evolution only as whatever the receiving node's dynamics do. The history model of Part V represents evolution and represents propagation not at all, since a foam records rearrangements and not crossings. Neither is the other's refinement.

5.4 The Costs of the Position

Three costs are recorded, and the first is the one readers will find most irritating.

The account cannot say that a party learned what another knows. It must say that the second party, under its own interpretation and following a perturbation originating with the first, reconstructed a structure standing in a stated relation to what the first

party's reconstruction contains. The formulation is longer, weaker, and relative to an interpretation under which the relation is assessed. Its weakness is the intended content, since the account holds that the stronger formulation asserts something unavailable.

The account cannot measure fidelity. Fidelity is a relation between what was sent and what arrived, and nothing is sent, so the ordinary vocabulary of loss, noise and distortion has no application. What can be said is that a receiver's reconstruction differs from a sender's in stated respects under a stated interpretation, and the difference is not thereby a defect.

And the account complicates ordinary description without supplying a replacement for every use of the words it forbids. Some sentences that a working researcher needs are simply longer under it, and the paper does not pretend otherwise.

5.5 Summary of Results

This section has fixed the three terms and established the fourth's vacancy. Definition 5.1 to 5.3 fix propagation as the crossing of a coupling by a perturbation, evolution as the change of a relational structure at an event, and emergence as a coupling present in a produced structure and in no input structure. Claim 5.4 establishes the exhaustive disjunction between re-instantiation and making, from which it follows that no case remains for a word naming conveyance, so the terminological rule of Section 2 is a result and not a preference. Claim 5.5 establishes that propagation and evolution are independent, with a reader unmoved by an article and a researcher who reconsiders overnight as the two cases, and draws the consequence that the two require different apparatus. Three costs are recorded: no party can be said to learn what another knows, fidelity cannot be measured, and ordinary description is lengthened without full replacement.

6 The Two Representational Requirements

This section derives the requirements that the two models are answers to. Its role is to make the models consequences of the account rather than proposals appended to it, and to establish that two are needed. Its structure treats each requirement with its derivation, the failure of the existing representation to satisfy either, and the failure of each model to satisfy both. Its method is derivation from the preceding sections.

6.1 The First Requirement

The account holds that reception is reconstruction: what a party produces on receiving a perturbation is generated by its own dynamics, under its own interpretation, in its own state space. Section 4 holds that those dynamics, interpretations and state spaces differ in kind among parties and differ for material reasons. Section 3 forbids embedding them in a common space.

Requirement 6.1. (Coupling without embedding). *A formalism satisfying this requirement admits components with distinct state spaces, distinct laws of evolution and distinct notions of what constitutes an event, and couples them by operators that carry occurrences from one to another without presupposing a common space, a metric, a similarity measure or*

a shared clock.

Three features of the requirement are worth marking, since each excludes a familiar apparatus.

No common space excludes representations of parties as points or regions in one geometry, and with them every account of communication as the reduction of a distance.

No shared clock excludes a global update rule, and with it the ordinary formulation in which all components are advanced together.

And distinct notions of event exclude the assumption that what counts as an occurrence is the same for all components, which is the assumption a common event alphabet makes.

6.2 The Second Requirement

The account holds that a state is a relational structure of primitive elements and their couplings. A change of state is therefore a rearrangement of that structure, and the interesting content of a change is which elements survived it, which couplings were destroyed, and which were created.

Requirement 6.2. (Representation of internal restructuring). *A formalism satisfying this requirement represents a change as a rearrangement of a relational structure, so that for any change it determines which primitive elements persisted through it, which couplings held before and not after, which held after and not before, and which held throughout.*

The requirement is what distinguishes a representation of a history from a record of a sequence. A sequence says that this was followed by that. A history in the sense required says what became of each element and each coupling in the passage.

6.3 The Failure of the Existing Representation

The companion papers specify a record whose unit is the transition between two identified states, forming a directed acyclic graph with typed arcs. The question is whether new models are needed at all, or whether that representation could be extended.

Claim 6.3. *The transition graph satisfies neither requirement. It satisfies neither the first, since it represents no dynamics at all and therefore neither couples nor embeds anything, so the question of how heterogeneous components interact is outside what it expresses. And it fails the second, since a transition names a prior state and a posterior state and does not decompose either, so two changes with the same endpoints and different internal structurings are the same object in it. This claim is formal.*

The second failure is the one that matters for the paper's adequacy standard, and it is worth making concrete before Part V. A report may take an occurrence and an actor, both present in a witness's structure, and produce a structure in which the actor is coupled to the occurrence as its cause. A second report may take the same two elements and produce a structure in which the actor is coupled to the occurrence as its observer. Both transitions have the same prior state and, at the level of which elements are present,

the same posterior state. In the transition graph they are indistinguishable. Under the second requirement they are different changes, and the difference is exactly the one an account of the construction of social facts needs.

6.4 The Failure of Each Model to Satisfy Both

Claim 6.4. *Neither model proposed in this paper satisfies both requirements. The network model of Part IV satisfies the first and not the second: it couples heterogeneous components without embedding them, and a node's state is an element of its own state space, which the model does not decompose into elements and couplings. The history model of Part V satisfies the second and not the first: it represents the internal restructuring of a change, and it contains no dynamics, no components and no coupling operators, so nothing in it propagates. This claim is formal.*

The consequence for the paper's architecture is stated here so that Parts IV and V are read correctly. The two models are complementary and not alternative, and neither is a refinement of the other. A reader asking which is the right model has misread the requirements, and Part VI accordingly owes a correspondence between them rather than an argument that one subsumes the other.

The consequence is also a limitation and is recorded as one. A single formalism satisfying both requirements would be preferable to two that divide them, and this paper does not supply one. Whether the two models can be unified, and what a unification would have to look like, is recorded among the open problems.

6.5 Summary of Results

This section has derived the requirements. Requirement 6.1 fixes coupling without embedding, excluding common spaces, shared clocks and common event alphabets. Requirement 6.2 fixes the representation of internal restructuring, which distinguishes a history from a sequence. Claim 6.3 establishes that the transition graph of the companion papers satisfies neither, since it represents no dynamics and does not decompose its states, so two changes with identical endpoints and different internal restructurings are one object in it; the reporting case makes the failure concrete. Claim 6.4 establishes that neither proposed model satisfies both requirements, the network model satisfying the first and the history model the second, so the two are complementary rather than alternative, Part VI owes a correspondence rather than a subsumption, and the absence of a unified formalism is recorded as a limitation.

7 Background Independence in Physical Theory and the Emergence of Space-time

This section states the property of certain physical theories that justifies the borrowing of Part V. Its role is to establish that the borrowing is structural rather than decorative, since a formalism taken from physics on the strength of resemblance is the failure this paper's analogy discipline exists to prevent. Its structure treats the property, its role in the physical theories, the correspondence with the account of Part II, what the correspon-

dence licenses, and what it does not. Its method is exposition of a physical notion at the level a non-specialist can use, followed by an argument about its transferability.

7.1 The Property

A physical theory is background independent where the structures it describes are not defined on a fixed stage supplied in advance. In a background-dependent theory a spacetime geometry is given, and fields are described as living on it. In a background-independent theory the geometry is itself among the things determined, and there is no prior arena (Smolin, 2006).

General relativity is the standing example: the metric is dynamical, and there is no fixed background against which motion is described. Approaches to quantum gravity that take this seriously accordingly face a problem their background-dependent rivals do not, since the usual apparatus for describing quantum fields presupposes the stage that has been removed. What replaces the stage is combinatorial and algebraic data, from which geometrical notions are recovered in place of being assumed (Rovelli, 2004).

7.2 The Correspondence with the Account

The account of Part II has the same shape at the level of its own subject matter, and the correspondence is exact enough to state as a claim rather than as an analogy.

Claim 7.1. *The account of Part II is background independent in the following sense. There is no pre-given space of meanings on which parties or their reconstructions are located. Relational structure is generated by representations and the interpretations in force upon them, so notions ordinarily treated as prior, including similarity between accounts, distance between positions, and the identity of a topic across parties, are results where they are available at all. Claim 3.5 is the formal statement of this property, and it was derived without reference to physics. This claim is formal.*

The second sentence of the claim matters for the paper's standing. The constraint was reached by an argument about comparison and interpretation, not by admiring a physical theory and looking for an application. The physics enters afterwards, as the place where formal apparatus adequate to a background-independent setting has actually been developed.

7.3 What the Correspondence Licenses

Claim 7.2. *Background independence licenses the choice of source. Formalisms developed for background-dependent settings presuppose what this account denies, since they are defined on a given space of states, places or positions, and adapting them requires supplying the background the account refuses. Formalisms developed for background-independent settings do not, and are accordingly the appropriate quarter from which to borrow combinatorial structure. This claim is formal.*

The claim can be made concrete by naming what it excludes, and the exclusions are all respectable formalisms that a reader might reasonably have expected this paper to

use.

Petri nets are defined over a fixed set of places, and a marking is a distribution over that set. The set is a background.

Event calculus and situation calculus are defined over a domain of fluents whose identity is given in advance, and a situation is characterised by which fluents hold. The domain is a background.

Process algebras are defined over an alphabet of actions shared by the processes composed, and composition presupposes agreement on that alphabet. The alphabet is a background.

Each is a good formalism and each supplies exactly the thing the account cannot have: a stage on which the parties are already commensurable.

7.4 What the Correspondence Does Not License

Requirement 7.3. (The boundary of the background-independence transfer). *The correspondence licenses the choice of formal source and licenses nothing further. No physical claim, no dynamical principle, no result about spacetime, and no property of any quantum theory is transferred to this account on the strength of it. In particular the account makes no claim that knowledge is physical, that meanings are quantum, or that any physical theory describes the phenomena treated here.*

Three further limits are recorded, and the third bounds the claim of Section 7 itself.

The correspondence is a correspondence of a single structural property. Background independence is one feature of the physical theories concerned, and those theories have many others, of which none transfers.

The physical theories are unsettled. Quantum gravity is an open research programme, its approaches disagree, and none is confirmed. A borrowing from an unsettled theory takes a formal structure whose physical adequacy is undecided, which is acceptable where the structure is used for its combinatorics and would not be acceptable where it was used for its physical content.

And the correspondence is not evidence that the borrowed structure is adequate here. It establishes that the source is appropriate and leaves the adequacy question to the standard fixed in Section 2, which asks whether some question is answerable in the borrowed structure and unanswerable in the transition graph.

7.5 Summary of Results

This section has established the justification for the borrowing. Background independence is the property of a theory whose structures are not defined on a stage supplied in advance, and in the physical case geometry is recovered from combinatorial and algebraic data in place of being assumed. Claim 7.1 states that the account of Part II has the same property at the level of its own subject matter, that similarity, distance and topic identity are results rather than priors, and that Claim 3.5 states this formally and was derived without reference to physics. Claim 7.2 states what the correspondence licenses, namely the choice of formal source, and names the exclusions: Petri nets presuppose a

set of places, event and situation calculus a domain of fluents, and process algebras a shared action alphabet, each of which is the background the account refuses. Requirement 7.3 fixes the boundary of the transfer, and three further limits are recorded: the correspondence concerns one structural property, the physical theories are unsettled, and the correspondence establishes the appropriateness of the source without establishing the adequacy of the borrowed structure.

8 Spin Foam Models and Group Field Theory

This section states the structure the paper borrows, at the level of detail the borrowing requires. Its role is to make Part V intelligible to a reader who does not work in quantum gravity, and to fix precisely which parts of the apparatus are taken. Its structure treats the boundary states, the two-complex and its three strata, the amplitude assignment, the generation of histories in the field-theoretic reformulation, and the elements taken and left. Its method is exposition, and no claim about physics is made or needed.

8.1 The Boundary States

In the approach concerned, a state of geometry is represented by a graph carrying algebraic labels: the graph's edges carry representations of a group and its vertices carry intertwining operators between them. Geometrical quantities are recovered from these labels rather than assumed (Rovelli, 2004).

Two features of this arrangement bear on the borrowing. The state is combinatorial together with algebraic labelling, so it has parts and relations rather than coordinates. And it is a boundary object: it is what a history has at its edge, which is what makes composition possible.

8.2 The Two-Complex and Its Three Strata

A history interpolating between such states is represented by a two-complex: a structure with vertices, edges joining vertices, and faces bounded by edges (Baez, 1998; Perez, 2013).

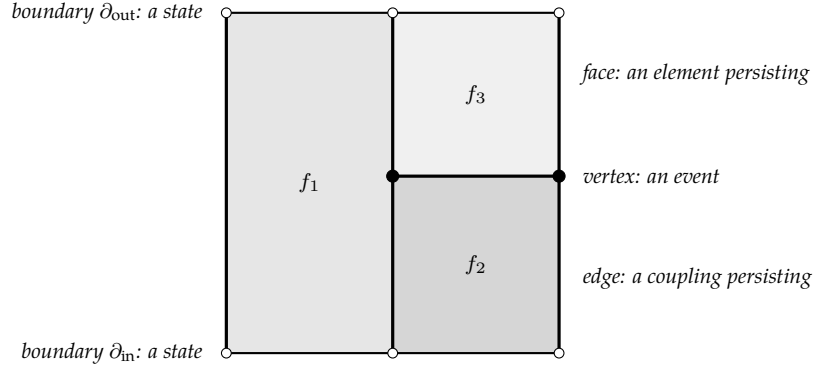
The assignment of content to the three strata is the part this paper takes.

A face is the history of an edge of the boundary graph. Where the boundary state has an edge carrying a label, the history has a face carrying it, and the face records that this element of the state persisted through an interval.

An edge of the two-complex is the history of a vertex of the boundary graph, and carries the corresponding intertwining data. It records that a junction among elements persisted through an interval.

A vertex of the two-complex is where the structure changes: edges meet, faces terminate or begin, and the combinatorial arrangement on one side differs from that on the other. Nothing persists at a vertex; a vertex is where persistence is interrupted.

Claim 8.1. *The three strata of a two-complex carry, respectively, the persistence of elements, the persistence of relations among elements, and the events at which persistence is interrupted*



The drawing is schematic: it shows the incidence of the three strata and not a metric arrangement. Below the vertices the boundary graph has one junction pattern; above them it has another, and the edge joining the two vertices is a coupling created there.

Figure 8.1: The two-complex and its three strata under the assignment of Part V. A *face* is the history of a primitive element and is two-dimensional; an *edge* is the history of a coupling among elements and is one-dimensional; a *vertex* is an event at which couplings rearrange, and nothing persists at it. A *boundary* is a state: a relational graph whose edges are the elements and whose nodes are the couplings among them, so that the faces meeting a boundary are its elements and the foam edges meeting it are its couplings.

and the arrangement changes. This is a structure for representing what became of each part of a configuration through a passage, and it is that structure, and not any content assigned to it in physics, that Part V borrows. This claim is formal.

8.3 The Dictionary Fixed

The assignment used in Part V is fixed here, since the naive reading of it is wrong in a way that would break the composition rule.

Definition 8.2. (The assignment). *In a boundary state the primitive elements are the edges of the boundary graph and the couplings are its nodes, each node being the junction at which the elements incident to it meet. In a history the face swept by a boundary edge is the persistence of an element, the foam edge swept by a boundary node is the persistence of a coupling, and a vertex is an event at which couplings rearrange.*

The inversion is what makes the dimensions agree. An element persisting through an interval sweeps a two-dimensional face; a junction persisting through an interval sweeps a one-dimensional edge; an event at which junctions are made and destroyed is a point. Under the naive reading, in which elements are nodes and couplings are edges, an element would sweep a line and a coupling a surface, and a coupling would then be the thing that persists most robustly, which is the opposite of what the account holds. Figures 8.1 and 8.2 show the assignment and the dictionary.

8.4 Composition along Shared Boundaries

A history has a boundary, and two histories sharing a boundary compose into one history. The composition is the reason boundary states are the objects they are: they are what

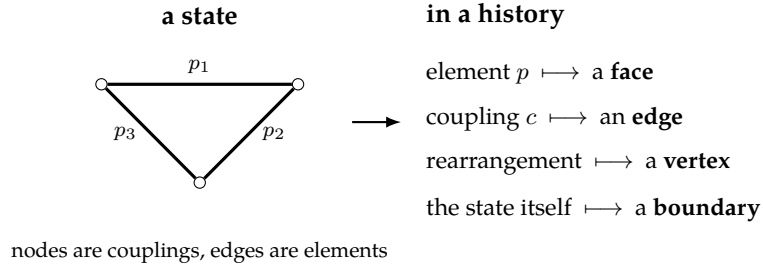


Figure 8.2: The dictionary. A state is a relational graph in which the nodes are couplings and the edges are the primitive elements they couple, which is the assignment that makes the dimensions match: an element persisting sweeps a two-dimensional face, a coupling persisting sweeps a one-dimensional edge, and an event at which couplings rearrange is a point. The reading inverts the naive one, in which elements would be nodes, and the inversion is what allows a coupling to be the junction at which several elements meet.

histories are glued along.

Composition is associative in the appropriate sense and behaves as a reader would expect: gluing a history to a second and the result to a third gives the same object as gluing the second to the third and the first to that.

This is the second element the paper takes, and Part V’s section on gluing is where it does its work. A formalism representing histories without a composition rule would represent episodes, and the account requires that a longer history be assembled from shorter ones.

8.5 The Amplitude Assignment and the Field-Theoretic Reformulation

In the physical theory an amplitude is assigned: faces, edges and vertices each contribute a factor, the factors are combined into an amplitude for the whole history, and a sum is taken over histories with given boundaries.

The group field theory reformulation changes the status of the sum. Rather than positing a class of histories and summing over it, one defines a field theory on a group manifold whose perturbative expansion generates the histories as its Feynman diagrams, with the amplitude arising from the expansion in place of being imposed (Freidel, 2005; Oriti, 2009).

The reformulation is worth stating because it names an ambition Part V’s closing section discusses and does not realise: a formalism in which histories are generated by a dynamics rather than enumerated.

8.6 From an Action to a Sum over Histories

The reformulation deserves a fuller statement than a sentence, since it is what turns a class of histories from something posited into something generated, and since the paper’s closing sections refer to it as an ambition.

One begins with a field φ on several copies of a group manifold, subject to invariances,

and posits an action of the schematic form

$$S[\varphi] = \frac{1}{2} \int \varphi \mathcal{K} \varphi + \frac{\lambda}{D!} \int \varphi^D,$$

in which $\mathcal{K}$ is a kinetic kernel and the second term is an interaction of order D . The partition function is the formal integral $Z = \int \mathcal{D}\varphi e^{-S[\varphi]}$, and its perturbative expansion in λ produces a sum over Feynman diagrams,

$$Z = \sum_{\Gamma} \frac{\lambda^{V_{\Gamma}}}{\text{sym}(\Gamma)} A_{\Gamma},$$

where V_{Γ} counts the interaction vertices of the diagram Γ and A_{Γ} is the amplitude the expansion assigns to it (Freidel, 2005; Oriti, 2009).

The point of the construction for this paper is structural rather than computational. The diagrams of the expansion are dual to two-complexes: an interaction vertex of the field theory corresponds to a vertex of a foam, a propagator to a foam edge, and the faces are recovered from the way the group arguments are traced through the diagram. So the class of histories, which the earlier formulation had to posit, is produced by the expansion of a posited dynamics, and the amplitude of a history is a consequence of the action in place of a separate assignment.

8.7 Condensates and the Formation of a Continuum

A second development is worth stating because it is the clearest instance of a space being a result and not a stage, which is the property Section 7 identified as the ground of the borrowing.

A quantum of the field is a single building block: in the geometric reading, an elementary chunk with a few faces. A state containing very many such quanta, all in the same single-quantum state, is a condensate, and it admits a description by a small number of collective variables in place of the vast number of combinatorial ones. Applying the field equations to such a state yields effective equations for the collective variables, and those equations have been argued to take the form of cosmological equations for a homogeneous continuum (Gielen and Sindoni, 2016; Oriti, 2017).

Claim 8.3. *In this construction a continuum is a phase of a system of very many discrete quanta, characterised by collective variables, and not a stage on which the quanta are placed. Where the condensate description fails, no continuum is available and the description in terms of the underlying combinatorics is the only one. This is the sense in which the emergence of spacetime is meant, and it is the strongest available illustration of the property Section 7 borrows.*

Figure 8.3 shows the route and marks what this paper takes from it.

Three cautions attach and are recorded because the temptation to transfer the whole picture is considerable.

The condensate construction rests on approximations whose validity is a matter of continuing argument within the physical literature, and it is not a settled derivation.

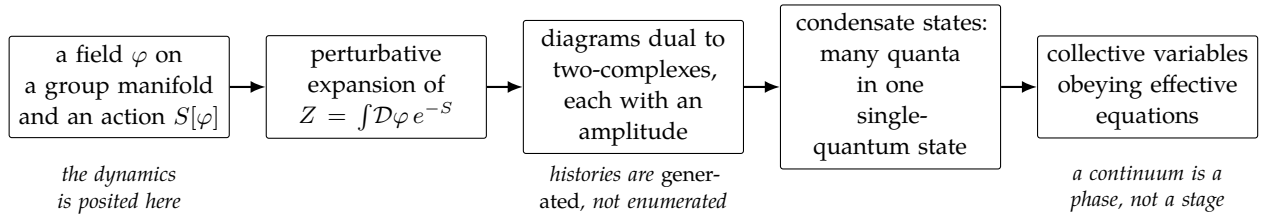


Figure 8.3: The route from a field theory to a continuum, in the reformulation described in Section 8. An action is posited for a field on a group manifold; the perturbative expansion of the partition function generates diagrams dual to two-complexes, so that the histories summed over are produced by the expansion in place of being assumed; and states with very many quanta in one single-quantum state admit a description by collective variables whose effective equations are read as the emergence of a continuum. This paper takes the third box for its combinatorial structure alone. It takes neither the first, since it posits no dynamics, nor the fourth and fifth, which are recorded in Section 8 as an ambition the paper does not realise.

Nothing analogous to a condensate is proposed here. This paper’s histories are not generated by a dynamics, so it has no expansion, no amplitudes, and therefore no collective description to take a limit of.

And the parallel that does hold is a parallel of ambition rather than of result: the physical construction shows what it would look like for a continuum to be a phase, and Section 24 records the corresponding question for this account, namely whether a shared space of meanings could ever be recovered as a collective description of very many reconstructions in place of being assumed.

Requirement 8.4. (What is taken and what is left). *Taken: the two-complex with its three strata and their reading as persistence of elements, persistence of relations, and events of rearrangement; the boundary state as the object along which histories compose; and the composition rule. Left: the group, the representations and the intertwiners, the amplitude factors, the sum over histories, the field-theoretic generation of histories, and every physical interpretation of any of these.*

The cost of what is left has been stated in Section 2 and is restated here in its proper place. Without amplitudes there is no dynamics on the space of histories, so the borrowed structure represents histories and does not weight, generate or select among them. A reader who holds that a formalism without a dynamics is not a model is holding a defensible position, and the paper’s response is that a structure adequate to represent what the transition graph cannot is worth having before the question of dynamics is opened.

8.8 Summary of Results

This section has stated the borrowed structure. Boundary states are combinatorial objects with algebraic labels, having parts and relations in place of coordinates, and serving as the edge along which histories compose. A two-complex represents a history, with faces carrying the persistence of elements, edges the persistence of relations among them, and vertices the events at which arrangement changes, which Claim 8.1 states as the property the paper borrows. Definition 8.2 fixes the assignment used throughout Part V, in which the elements of a state are the edges of its graph and the couplings are its nodes, and

records why the naive reading would break the correspondence of dimensions. Composition along shared boundaries is the second element taken. The route from an action to a sum over histories is stated, with the point that the class of histories is generated by an expansion in place of being posited, and Claim 8.3 states the condensate reading, in which a continuum is a phase of very many discrete quanta and not a stage, together with three cautions. Requirement 8.4 fixes the inventory of what is taken and what is left, and the cost of leaving the amplitudes is restated: the borrowed structure represents histories without weighting, generating or selecting among them.

9 Event-Based Computation and Heterogeneous Dynamical Systems

This section states the prior art for the model of Part IV. Its role is to establish what already exists, so that the model's contribution is the difference and not the whole. Its structure treats event-advanced computation, the discrete-event tradition and its coupling construction, the formalisms that couple two kinds of dynamics, the formalisms that couple many components of one kind, and what each withholds. Its method is survey with the boundary of each transfer stated.

9.1 Event-Advanced Computation

A system is event-advanced where its state changes at occurrences rather than at intervals fixed independently of them. The contrast is with a clocked system, which updates on a schedule and represents occurrences by sampling.

Spiking neural networks are the best-known instance: units communicate by discrete events, computation is driven by their arrival, and time enters through the events rather than through a global step (Maass, 1997). Neuromorphic hardware realises the arrangement with asynchronous circuits and event-based communication between units (Indiveri et al., 2011).

What the tradition supplies to this paper is the idea that a system may be specified by what happens at occurrences, with no schedule and no global update. What it does not supply is anything about units, since its units are homogeneous, share a state description, and are coupled in one way.

Claim 9.1. *A spiking neural network is one instance of the class the model of Part IV generalises, and the generalisation proceeds by removing exactly the features that make it specific: homogeneity of units, a common state description, a single form of coupling, and a shared notion of time. No property of neural computation is carried into the model, and the resemblance is a resemblance of class membership. This claim is formal.*

9.2 The Discrete-Event Tradition

The nearest prior art is the discrete-event system specification, which formalises systems evolving at occurrences over a continuous time base, distinguishes atomic components from coupled ones, and establishes that a coupling of components is itself a component of the same class (Zeigler et al., 2018).

Two of its features are exactly what Part IV needs. The first is closure under coupling, which the tradition establishes for its own class and which Part IV must establish for a wider one. The second is the explicit ambition of multi-formalism coupling: the framework is designed so that components specified in different formalisms, continuous, discrete-time and discrete-event among them, can be brought into one coupled model.

Claim 9.2. *The discrete-event tradition is the nearest existing formalism to the model of Part IV and is the one that model must improve upon rather than the one it replaces. What it withholds is the case in which components have state spaces of different mathematical types with no common structure, since its coupling construction operates on components whose interfaces are expressed in a common input and output vocabulary. This claim is formal.*

The withholding is the whole of Part IV’s difficulty and is stated here so that the model is read as an attempt on a known problem in place of a fresh invention.

9.3 Formalisms Coupling Two Kinds of Dynamics

Hybrid systems couple continuous flow with discrete jumps, and the coupling is specified: a flow condition, a jump condition, and a reset map. The apparatus is mature and is adequate to its case.

Its limitation for the present purpose is that the case is two kinds, fixed in advance, with the relation between them given by the formalism. A setting with several kinds, not fixed in advance, and with the relation between any two supplied by whoever specifies the coupling, lies outside it.

Co-simulation standards address the practical version of the same problem, orchestrating simulators with different internal descriptions through defined interfaces and time-stepping protocols. What they supply is engineering practice for exchanging quantities at agreed points; what they withhold is a formal account of when the composed object is well defined.

9.4 Formalisms Coupling Many Components of One Kind

Multilayer and multiplex network formalisms treat systems whose components are related by several kinds of relation, and admit different dynamics on different layers (Kivela et al., 2014; Boccaletti et al., 2014). Graph dynamical systems formalise local update rules applied over a graph under sequential or parallel schedules.

These are the nearest formalisms on the network side, and what they withhold is the heterogeneity that matters here. A layer is ordinarily homogeneous within itself, node states are ordinarily drawn from a common set, and the update rule is ordinarily of one form. The variation they accommodate is variation in topology and in relation type rather than in the mathematical type of a node’s state space.

9.5 The Inventory of Withholdings

Four statements assemble the survey, and together they state what Part IV must supply.

Event-advanced computation supplies advance at occurrences and withholds hetero-

geneity of units.

The discrete-event tradition supplies closure under coupling and multi-formalism ambition, and withholds coupling across components whose state spaces have no common structure.

Hybrid systems supply a rigorous coupling of two kinds and withhold the case of many kinds not fixed in advance.

Multilayer and graph dynamical formalisms supply many components with varied relations and withhold variation in the mathematical type of a component's state.

Claim 9.3. *No surveyed formalism couples components whose state spaces differ in mathematical type, without embedding them in a common space, while establishing that the coupled object is itself a component of the same class. The absence is a claim about the formalisms surveyed here and is not a claim about the literature as a whole, and a formalism meeting the description would remove the ground for the model of Part IV. This claim is formal, and its scope is limited to the sources surveyed.*

9.6 Summary of Results

This section has surveyed the prior art for Part IV. Claim 9.1 states that a spiking network is one instance of the class the model generalises, and that the generalisation removes homogeneity of units, a common state description, a single coupling form and a shared notion of time. Claim 9.2 identifies the discrete-event tradition as the nearest formalism and the one to improve upon, supplying closure under coupling and multi-formalism ambition while withholding the case of components without common interface structure. Hybrid systems supply a rigorous coupling of two fixed kinds; co-simulation supplies engineering practice without a well-definedness account; multilayer and graph dynamical formalisms supply varied relations without variation in the type of a component's state. Claim 9.3 states the absence that Part IV addresses, with its scope limited to the sources surveyed.

10 The Boundaries of the Transfers

This section assembles the borrowings and fixes what each carries and what it does not. Its role is to make the analogy discipline of Section 2 checkable in one place, so that a reader who suspects an unlicensed transfer can find the licence or its absence. Its structure is the table, the general rule, the falsifier for the analogy claim, and the residual exposure. Its method is assembly.

10.1 The Table

Table 10.1 states each transfer.

Table 10.1: The transfers, what each carries, and what each leaves

Source	Carried	Left
Background independence	The justification for choosing a formal source in which structure is generated rather than posited	Every physical claim; any suggestion that the subject matter is physical
Spin foam structure	The two-complex with faces as element histories, edges as persisting relations, vertices as events of rearrangement; boundary states; composition by gluing	The group, representations and intertwiners; amplitude factors; the sum over histories; every physical interpretation
Group field theory	The description of an ambition: histories generated by a dynamics in place of enumerated	The field, the expansion, the amplitudes, and the claim that any such generation is available here
Event-advanced computation	Advance at occurrences in place of at intervals	Homogeneity of units; any property of neural computation or of hardware
Discrete-event specification	The requirement that a coupling be an object of the same class as what it couples	The common interface vocabulary on which its coupling construction depends
Multilayer network dynamics	Components related by several kinds of relation	The assumption that component states are drawn from a common set

10.2 The General Rule

Requirement 10.1. (Licence for a transfer). *A property is transferred from a source only where the transfer is licensed by an argument stated at the point of transfer. Structural correspondence licenses the choice of a source and licenses no property of it. A property carried without such an argument is an error to be corrected, and any passage in the remainder of this paper that relies on an unlicensed property is to be struck rather than defended.*

The requirement is written in that form because the failure it guards against is not usually deliberate. Borrowed vocabulary carries its home connotations, and a term used repeatedly begins to do work its licence did not grant. The requirement makes the check a matter of finding an argument at the point of use.

10.3 The Falsifier for the Analogy Claim

The paper’s justification for borrowing from spin foam models is that both settings are background independent in the sense of Section 7. That justification is defeasible and the defeating observation is stated so a reader may look for it.

Claim 10.2. *The justification fails if the account of Part II can be given a formalism in which parties or their reconstructions occupy positions in a common space, without reinstating the transmission picture that Claim 3.5 says such a formalism reinstates. Were that available, the*

borrowing would lose its ground, since a background would be admissible and the formalisms excluded in Section 7 would become available. This claim is formal.

The claim identifies the load-bearing step precisely. Everything the paper does with the two-complex rests on the refusal of a common space, and the refusal rests on the regress argument of Section 3. A reader who defeats that argument defeats the borrowing along with it.

10.4 The Residual Exposure

Two exposures remain after the boundaries are fixed, and neither is removed by them.

The first is that a structure borrowed for its combinatorics may be adequate to nothing here. The boundaries prevent illegitimate transfer and do not establish that the transferred structure represents anything worth representing, which is the question the adequacy standard of Section 2 asks and Part VII answers as far as the paper can.

The second is that the source theories are unsettled, so the paper borrows a structure whose own standing is undecided in its home domain. This is acceptable for a combinatorial borrowing and would not be acceptable for a physical one, and the distinction is the one Requirement 10.1 exists to police.

Claim 10.3. *The boundaries stated in this section prevent unlicensed transfer and establish nothing about adequacy. A reader who accepts every boundary may still hold that neither model represents anything the account needs, and that position is answered, so far as this paper answers it, in the assessment of Part VII. This claim is formal.*

10.5 Summary of Results

This section has assembled the transfers in Table 10.1, with what each carries and what each leaves. Requirement 10.1 fixes that a property transfers only under an argument stated at the point of transfer, that structural correspondence licenses the choice of source alone, and that unlicensed reliance is to be struck rather than defended. Claim 10.2 states the falsifier for the analogy claim: the justification fails if the account admits a formalism placing parties in a common space without reinstating the transmission picture, so a reader who defeats the regress argument of Section 3 defeats the borrowing. Claim 10.3 records the residual exposure, that boundaries prevent illegitimate transfer and establish nothing about adequacy, and that the source theories are themselves unsettled.

11 Nodes, State Spaces, and Heterogeneous Dynamics

This section defines the components of the first model. Its role is to give heterogeneity a formal expression, so that the differences established in Section 4 appear in the mathematics rather than in the commentary. Its structure treats the definition of a node, the absence of assumptions on a state space, the treatment of time, and the standing illustration. Its method is definition, and every assumption withheld is named where it is withheld.

11.1 The Definition

Definition 11.1. (Node). A node is a tuple $n_i = (S_i, A_i, E_i, \varphi_i, \varepsilon_i)$ in which S_i is a set of states, A_i is a set of admissible inputs, E_i is a set of emissions, $\varphi_i : S_i \times A_i \rightarrow S_i$ is an update map, and $\varepsilon_i : S_i \times A_i \rightarrow E_i \cup \{\emptyset\}$ is an emission map, with $\emptyset$ denoting that nothing is emitted.

The definition is deliberately thin, and what it omits is the content of the model.

S_i is a set. It carries no metric, no topology, no linear structure and no order unless a design supplies one, and two nodes' state sets may have nothing in common.

A_i and E_i are the node's own. What can perturb a node and what a node can emit are properties of the node, not of the network, and there is no common alphabet.

φ_i and ε_i are the node's own laws. Nothing requires that two nodes' laws be of the same kind, be given by the same sort of expression, or be defined by the same mathematics.

Claim 11.2. *Definition 11.1 satisfies Requirement 6.1, since it admits components with distinct state sets, distinct input and emission sets, and distinct laws, and it introduces no space in which states of distinct nodes are located. This claim is formal.*

11.2 The Assumptions Withheld

Four assumptions common in models of interacting agents are withheld here, and each withholding has a reason in Part II.

No common state space, by Claim 3.5.

No metric or similarity on states, since a metric on a common space is what would let differences between nodes be read as distances, and a metric within a node's own space is permitted where a design supplies one and is required by nothing.

No shared alphabet, since a common E and A would presuppose that what one node can say is the kind of thing another can receive, which is the assumption that heterogeneity denies.

And no requirement that φ_i be of any particular mathematical type. It may be a map on a finite set, a flow on a manifold, a stochastic kernel, or an operator on a function space, and different nodes may differ in which.

Requirement 11.3. (Type heterogeneity). *A network conforming to this model may contain nodes whose state sets differ in mathematical type. No construction in the model may require that the state sets be of one type, be embedded in a common object, or be related by any map other than those the couplings supply.*

11.3 Runs and Causal Order

Time is treated with the same reticence, and the treatment requires a notion of a run, since an order among occurrences presupposes occurrences.

Definition 11.4. (Occurrence and run). An occurrence at node n_i is a triple $u = (i, s, a)$ with $s \in S_i$ and $a \in A_i$, recording that n_i in state s consumed input a ; the occurrence produces the state $\varphi_i(s, a)$ and the emission $\varepsilon_i(s, a)$. A run of a network is a set U of occurrences together with a delivery relation $\delta \subseteq U \times U$ such that $(u, u') \in \delta$ only where the emission produced by u is carried by some operator to the input consumed by u' , each occurrence consumes at most one delivered input, and the state consumed by an occurrence is the state produced by the preceding occurrence at the same node, or that node's initial state where there is none.

Definition 11.5. (Causal order). For occurrences u, u' in a run, $u \prec u'$ where $(u, u') \in \delta^+$, the transitive closure of the delivery relation together with the same-node succession relation. Occurrences unrelated by $\prec$ are concurrent.

Three consequences are recorded and the third is a difficulty the following sections must face.

The relation $\prec$ is a strict partial order on the occurrences of a run whenever it is irreflexive, and irreflexivity is not automatic: it fails exactly where a chain of deliveries returns to its own origin. The condition excluding this is stated in Section 13 as one of the conditions on a well-formed network.

A network has no state at a time. What plays the role of a state is a *cut*: a subset $D \subseteq U$ downward closed under $\prec$, together with the states each node has produced within it. Questions about a network are asked relative to a cut.

And concurrency is not a defect to be removed. Two occurrences at different nodes with no delivery chain between them are unordered, and any formalism that ordered them would be supplying the global clock that Claim 3.5 forbids in its temporal form.

11.4 The Standing Illustration

The news case is introduced here and runs through the rest of the paper.

Six kinds of node are enough for the argument. A *witness* whose states are perceptual and whose updates follow occurrences in its vicinity. A *reporter* whose states include a working account together with a phase comprising deadline, assignment and standing commitments, and whose updates transform received perturbations into a publishable account. An *editor* whose states include a stock of accounts awaiting decision and whose updates select and alter. An *archive* whose states are accumulations and whose updates append. A *reader* whose states include a structure built from prior reading. And an *institution* whose states include positions it has taken and whose updates issue statements consistent with them.

The heterogeneity here is not stipulated for the model's convenience. A witness's state set and an archive's have no common structure: one is perceptual and transient, the other is accumulative and indexed. An institution's update is constrained by its prior positions in a way a witness's is not. And what can perturb an archive, namely a deposit, is not the kind of thing that can perturb a witness.

Claim 11.6. In the standing illustration no two of the six node kinds share a state set, an input set or a law of the same mathematical type, and the differences follow from what each

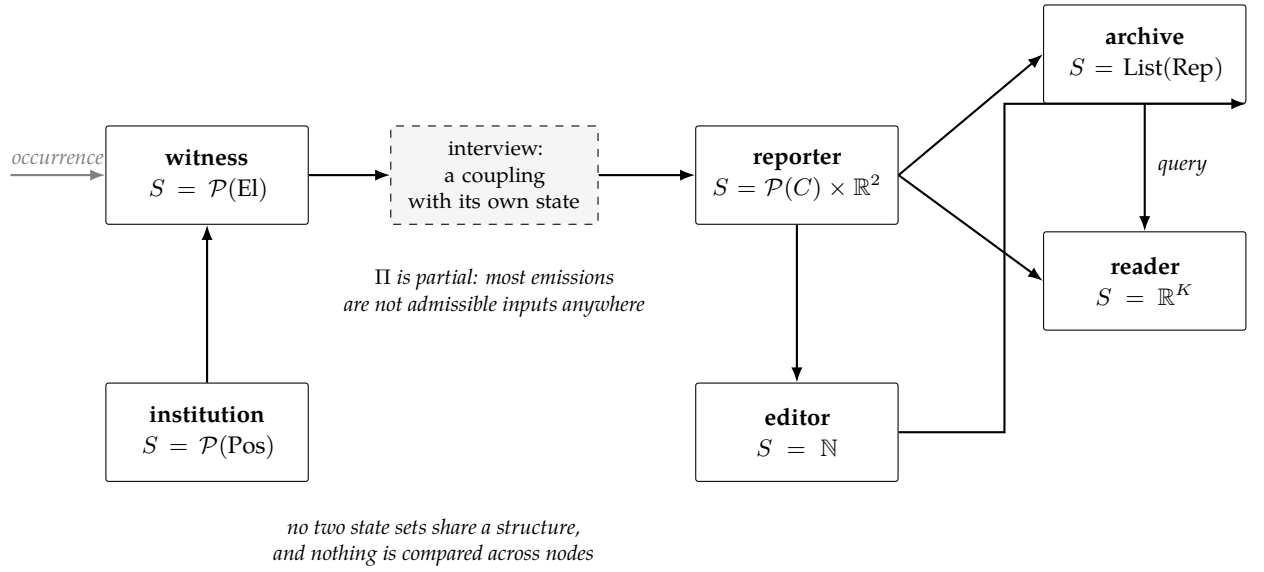


Figure 11.1: The structure of the standing illustration, as a network in the sense of Definition 13.1. Each node carries its own state set, written beneath its name, and the sets differ in mathematical type: a finite set of observed elements at the witness, a set of couplings together with a real phase vector at the reporter, a counter at the editor, an accumulating list at the archive, a real vector at the reader, and a set of positions at the institution. The interview is drawn as a generalised coupling under Definition 12.3, that is, a node in its own right with a state of its own, so that what reaches the reporter depends on the coupling's history and not on the witness alone. Arrows are propagation operators and are partial: most of what any node emits is not an admissible input anywhere.

party can observe, retain and be exposed to. The illustration therefore exhibits the material heterogeneity of Claim 4.2 rather than a variation contrived to exercise the model. This claim is formal as to the model and illustrative as to the case.

11.5 Summary of Results

This section has defined the components. Definition 11.1 fixes a node as a state set, an input set, an emission set, an update map and an emission map, all of them the node's own, and Claim 11.2 establishes that the definition satisfies the first representational requirement. Four assumptions are withheld with their reasons: no common state space, no cross-node metric or similarity, no shared alphabet, and no restriction on the mathematical type of a node's law, with Requirement 11.3 fixing type heterogeneity. Definition 11.4 fixes occurrences and runs, and Definition 11.5 fixes causal order over the occurrences of a run, from which it follows that a network has no state at an instant, that questions are asked relative to a cut, that irreflexivity requires a condition excluding delivery cycles, and that concurrency is a feature in place of a defect. The news case is introduced with six node kinds, and Claim 11.6 records that their heterogeneity is material rather than contrived.

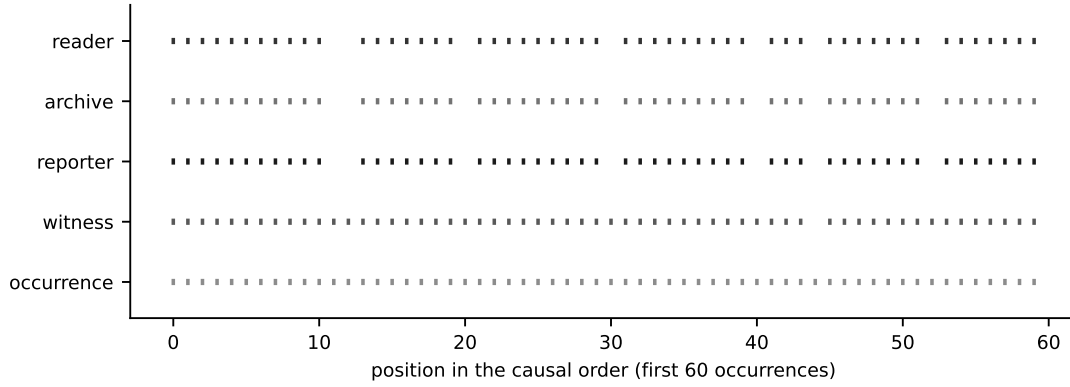


Figure 11.2: A run of the design instance of Section 11, showing the first sixty occurrences in the causal order of Definition 11.5. Each mark is an occurrence at a node. The six node kinds carry state sets of different mathematical type, a finite set at the witness, a pair of a coupling set and a real phase vector at the reporter, an accumulating list at the archive and a real vector at the reader, and no quantity is compared across them. Gaps in the reporter row are occurrences the reporter’s own law declined to publish, and gaps in the reader row are the reader’s limited exposure. The horizontal axis is position in the causal order and not a global time, since occurrences at distinct nodes are ordered only through delivery.

12 Propagation Operators and Reconstruction at Reception

This section defines the couplings of the first model and states what does and does not cross them. Its role is to give the refusal of Part II a formal form: the operator carries an occurrence and nothing else, and what a receiving node holds afterwards is produced by that node. Its structure treats the operator, the generalised coupling, reconstruction, the formal statement of the refusal, and the standing illustration. Its method is definition followed by claim.

12.1 The Operator

Definition 12.1. (Propagation operator). *For an ordered pair of nodes (n_i, n_j) , a propagation operator is a partial map $\Pi_{ij} : E_i \rightharpoonup A_j$. Where Π_{ij} is undefined on an emission, that emission does not perturb n_j .*

The definition is as thin as the node definition and for the same reason. An operator of this kind requires no metric, no similarity measure, no shared alphabet and no common space: it requires only that some of what n_i can emit be presentable as something n_j can receive.

Partiality is essential and is not a technical convenience. Most of what a node emits is not receivable by most other nodes, and a total map would assert that everything emitted anywhere is admissible everywhere, which is the assumption of a common medium in another form.

Claim 12.2. *A propagation operator couples two nodes without embedding either. It presupposes no object containing both state sets, induces no comparison between states, and supports*

no notion of distance between nodes. The model therefore couples without embedding, which is what Requirement 6.1 demands and what the formalisms surveyed in Section 9 supply only where interfaces share a vocabulary. This claim is formal.

12.2 The Generalised Coupling

A coupling need not be a fixed map, and the generalisation is what the model's title refers to.

Definition 12.3. (Generalised coupling). *A generalised coupling from n_i to n_j is itself a node $c_{ij} = (S_c, A_c, E_c, \varphi_c, \varepsilon_c)$ together with propagation operators $\Pi_{ic} : E_i \rightarrow A_c$ and $\Pi_{cj} : E_c \rightarrow A_j$. A coupling in the sense of Definition 12.1 is the degenerate case in which S_c is a singleton.*

The definition permits a coupling to have its own dynamics, its own state, and its own delay, and to be of any mathematical type. A coupling may be continuous where the nodes it joins are discrete, may accumulate, may attenuate, and may itself be perturbed by a third node.

The generalisation costs nothing structurally, since a coupling is a node, and it buys the case the account requires: what happens between parties is not a transparent channel but a process with its own conditions. An editorial pipeline, a citation practice, a platform's ranking, a translation, a mailing list: each is a coupling with a state and a law, and none is a wire.

12.3 Reconstruction

The model as so far defined has states and says nothing about what a state is of. Reconstruction is where that is supplied, and it is supplied as a map rather than as an assumption about S_i .

Definition 12.4. (Reconstruction map). *A reconstruction map for node n_i is a function $R_i : S_i \rightarrow \Gamma$, where Γ is the class of relational structures (P, C) of primitive elements and couplings among them in the sense of Section 2. The map is the node's own, and no relation between R_i and R_j is assumed.*

Three properties of the definition are load-bearing.

R_i is a map from states to structures, so a node's reconstruction is determined by its state and not by what it received. Two nodes in different states receiving identical perturbations reconstruct differently, and the same node in different states reconstructs differently from the same perturbation.

R_i is not required to be injective, so distinct states may reconstruct alike, and it is not required to be surjective, so structures may exist that a node cannot reconstruct.

And R_i is not observable. What another party can observe is what the node emits, which is ε_i applied to a state and an input, and no inverse of R_i is available to anyone.

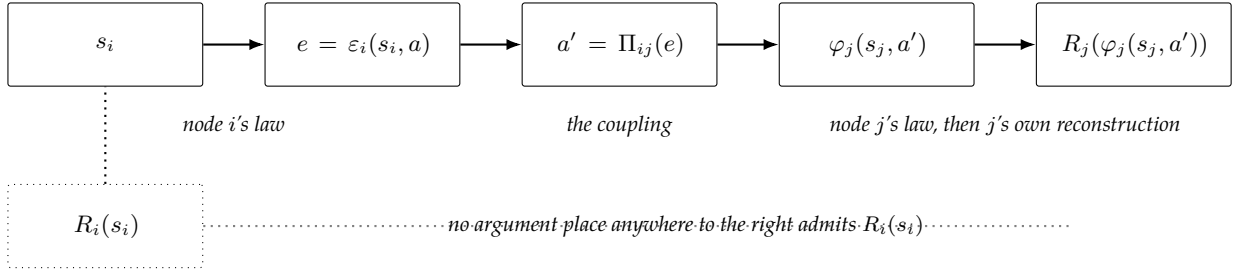


Figure 12.1: Reception, as a property of the signature. What the receiving node holds after reception is a function of its own prior state, its own law and the received input, so the sender's reconstruction $R_i(s_i)$ appears in no expression to the right of the coupling. The refusal of Part II is therefore a consequence of the types and not a side condition imposed on them. Dependence on the sender is preserved, since a' depends on e and e depends on s_i , which is what permits the statement that the receiver would not have produced what it produced without the perturbation.

12.4 The Formal Statement of the Refusal

Claim 12.5. *Let n_i emit e in state s_i under input a , and let $\Pi_{ij}(e) = a'$ be defined. The state of n_j after reception is $\varphi_j(s_j, a')$ and its reconstruction is $R_j(\varphi_j(s_j, a'))$. Neither expression contains R_i , s_i , or any structure reconstructed by n_i . What n_j holds after reception is therefore a function of n_j 's prior state, n_j 's law and the received input alone, which is the refusal of Part II stated as a property of the model. This claim is formal.*

The claim is what makes the model an expression of the account rather than a formalism decorated with it. The refusal is not imposed by a side condition or a warning in the prose. It is a consequence of the signature of φ_j : there is no argument place in which another node's structure could appear.

Two remarks bound the claim.

Dependence on the sender is preserved, since a' depends on e , which depends on s_i . The account permits the statement that n_j would not have produced what it produced without the perturbation, which is what Claim 3.4 allows.

And nothing prevents a design from arranging that $R_j(\varphi_j(s_j, a'))$ stands in a close relation to $R_i(s_i)$. Where a node's law and another's emissions are well adapted to one another, reconstructions may correspond closely, and the account calls this correspondence rather than transmission because the correspondence is produced twice and conveyed never.

12.5 The Standing Illustration

An occurrence perturbs a witness. The witness emits an account whose content is fixed by the witness's own state, which includes what it was attending to and what it was in a position to see.

A generalised coupling carries the account to a reporter: an interview, with its own state, since what a reporter is told depends on what was asked, and what was asked depends on the interview's prior course.

The reporter's update is the case the model exists to represent. Its state includes a

phase, comprising the deadline, the assignment and the outlet's standing commitments, and the account it emits is ε applied to that state and that input. An element present in the witness's account may be dropped, an element absent from it may be supplied from the reporter's prior state, and a coupling between two elements may be created that no input contained. This is the encapsulation the case is chosen for, and in this model it appears as an ordinary application of a node's own law.

Further couplings carry the emitted account to an editor, an archive and readers, whose state sets share nothing with the reporter's, and each of whom reconstructs under its own map.

Claim 12.6. *In the illustration nothing is conveyed at any step, and the account that reaches a reader is the reader's reconstruction of a state produced by the reader's law under an input produced by a chain of nodes each applying its own law. The model represents the chain and represents no content passing along it. This claim is formal as to the model.*

12.6 Summary of Results

This section has defined the couplings. Definition 12.1 fixes a propagation operator as a partial map from one node's emissions to another's admissible inputs, requiring no metric, similarity, shared alphabet or common space, with partiality essential rather than convenient, and Claim 12.2 establishes that such an operator couples without embedding. Definition 12.3 generalises a coupling to a node in its own right with its own state, law and type, so that what lies between parties is a process and not a wire. Definition 12.4 fixes the reconstruction map as a node's own function from its states to relational structures, not required to be injective or surjective and not observable. Claim 12.5 states the refusal formally: the receiving node's state and reconstruction after reception are functions of its own prior state, its own law and the received input, and no argument place exists in which another node's structure could appear, while dependence on the sender is preserved. The standing illustration exhibits encapsulation as an ordinary application of a node's own law.

13 Closure under Coupling

This section states the condition that makes the model a formalism in place of a picture. Its role is to establish, or where it cannot establish then to state precisely, that a coupled assembly of nodes is itself a node, so that composition may be iterated and reasoning about a network may proceed compositionally. Its structure treats the network, the composite construction, the conditions the construction requires, the status of the result, and what closure buys. Its method is construction followed by an honest statement of what has and has not been proved.

13.1 The Network

Definition 13.1. (Network). *A network is a pair $N = (V, \Pi)$ in which V is a finite set of nodes and Π is a set of propagation operators, at most one for each ordered pair. Generalised*

couplings are members of V under Definition 12.3, so a network contains only nodes and operators between them.

13.2 The Composite Construction

Given a network N , the candidate composite node $n_N = (S_N, A_N, E_N, \varphi_N, \varepsilon_N)$ is constructed as follows.

The state set is the product $S_N = \prod_{i \in V} S_i$ together with, for each ordered pair carrying an operator, a queue of inputs delivered and not yet consumed. Writing Q for the queue component, $S_N = (\prod_i S_i) \times Q$.

The input set is the disjoint union $A_N = \coprod_i A_i^{\text{ext}}$, where $A_i^{\text{ext}} \subseteq A_i$ is the set of inputs to n_i not produced by any operator within N . The composite is perturbed only from outside.

The emission set is the disjoint union $E_N = \coprod_i E_i^{\text{ext}}$, where $E_i^{\text{ext}} \subseteq E_i$ is the set of emissions on which no operator within N is defined. What the composite emits is what the network does not consume.

The update map φ_N takes a composite state and an external input, delivers the input to its node, applies that node's update, places any emission on which an internal operator is defined into the appropriate queue, and repeats until no queue contains an entry, at which point the resulting composite state is returned.

The emission map ε_N collects the external emissions produced in the course of that resolution.

13.3 The Conditions the Construction Requires

The construction is well defined only under conditions, and the conditions are the substance of this section.

Requirement 13.2. (Conditions for closure). *The composite construction yields a node of the class of Definition 11.1 where the following hold for every composite state and external input. Termination: every maximal sequence of $\rightarrow$ steps from σ_0 is finite. Confluence: where $\sigma_0 \rightarrow^* \sigma_1$ and $\sigma_0 \rightarrow^* \sigma_2$, there is a σ_3 with $\sigma_1 \rightarrow^* \sigma_3$ and $\sigma_2 \rightarrow^* \sigma_3$. Acyclicity of delivery: no cycle of channels exists along which an emission produced during a resolution returns to its origin within that resolution. Locality: every operator is a map on emissions alone, so no operator reads a node's state.*

Termination and confluence together give a unique quiescent state, by the standard argument: termination supplies a normal form on every path, confluence makes it unique, and φ_N is accordingly a total function.

Each condition earns its place and none is decorative.

Without termination the update map is partial in a way that leaves the composite undefined on some inputs, so the composite is not a node.

Without confluence the composite has no update map at all, since the quiescent state reached would depend on the order in which pending inputs were drawn and φ_N would not be a function.

Without acyclicity the resolution may not terminate, and where it does the result may depend on an arbitrary choice; the condition is stated separately because a network may satisfy termination through some other structural feature and because cycles are the ordinary case in which it fails.

And locality is what keeps the construction faithful to the refusal, since an operator reading a state directly would supply exactly the access that Claim 12.5 denies.

13.4 The Status of the Result

Claim 13.3. *Under the conditions of Requirement 13.2 the composite construction yields a tuple of the form required by Definition 11.1, whose update and emission maps are total functions of the composite state and the external input. Composition may therefore be iterated: a network of composites is a network, and its composite is a node. This claim is formal.*

The status of the claim must be stated exactly, and this paper is a proposal rather than a completed piece of mathematics.

What is established is that the construction produces an object of the right shape under the stated conditions, which follows from the construction by inspection.

What is not established is that the conditions are satisfiable in interesting cases, that they can be checked, or that they are the weakest conditions sufficient. In particular, confluence is undecidable for systems of this generality; a design will therefore either restrict the class of laws until confluence is decidable or impose an ordering, and the second is a retreat toward the global clock that Section 11 refused.

Claim 13.4. *The tension between determinacy and the absence of a global clock is the model's principal unresolved difficulty. Imposing an order restores determinacy and reintroduces a background time; leaving the order partial preserves the refusal and leaves the composite's update map dependent on a resolution order. The paper states the tension and does not resolve it. This claim is formal.*

13.5 What Closure Buys

Three things, and the third is why the section is not merely technical.

Iterated composition. A laboratory, a journal, a field and a discipline are all networks whose components are networks, and without closure the model would describe one level and stop.

Boundary reasoning. A composite has external inputs and external emissions, so a network can be treated from outside as a node, which is what permits an account of what an arrangement does without an account of everything within it.

And the correspondence of Part VI. The history model represents a change to a structure without representing components at all, and the composite construction is what lets a whole arrangement be treated as the single locus at which a change occurs.

13.6 Summary of Results

This section has treated composition. Definition 13.1 fixes a network as nodes with operators between them, and Definition ?? fixes channels, with emissions broadcast on every channel where the operator is defined and externally visible without exception. The composite construction is given: a product state set carrying a multiset of pending inputs on each channel, disjoint unions of tagged external inputs and emissions, a step relation consuming one pending input at a time, and an update map returning the quiescent state reached. Requirement 13.2 fixes four conditions: termination, confluence, acyclicity of delivery, and locality, the last of which preserves the refusal by denying operators access to states, with termination and confluence together yielding a unique normal form. Claim 13.3 states that under those conditions composition may be iterated, and states the limits of what has been established. Claim 13.4 records the principal unresolved difficulty, that confluence and the absence of a global order pull against one another and the paper resolves neither. Closure buys iterated composition, boundary reasoning, and the correspondence developed in Part VI.

14 Emergence as a Property of the Coupled System

This section defines emergence within the first model. Its role is to give the word a formal sense that can be checked, in place of the informal sense in which any surprising outcome is called emergent. Its structure treats invariants, the definition, the removal test, the standing illustration, and the limits. Its method is definition followed by claim, and the section deliberately stops short of calling anything knowledge, which is deferred to Part VI.

14.1 Invariants

Emergence is defined here in terms of what a system preserves, and preservation must be relative to something, since a system preserves everything under no perturbation at all.

Definition 14.1. (Perturbation family and invariant). *Let N be a network with node set V . A perturbation family is a choice $\mathcal{A} = \{A'_i\}_{i \in V}$ with $A'_i \subseteq A_i$, together with the induced external class $A'_N = \coprod_i (\{i\} \times A'_i) \subseteq A_N$. For a node n with admissible class A' , a predicate π on its state set is an invariant of n relative to A' where $\pi(s)$ implies $\pi(\varphi(s, a))$ for every $a \in A'$ and every s satisfying π . The set of such predicates is written $\text{Inv}_{A'}(n)$.*

Two properties of the definition matter for what follows.

An invariant is relative to a perturbation class, and there is no absolute notion. A structure that survives everything a party ordinarily meets may not survive what it rarely meets, and the account has no reason to privilege one class over another except that a party's actual exposure is a fact about its conditions, which is where the historical qualification of Section 2 enters.

And an invariant is a predicate rather than a part of a state. Nothing in the definition locates an invariant anywhere, which is what makes it available for the relational case.

14.2 The Definition

Definition 14.2. (Lifting of a component predicate). For $i \in V$ let $\text{pr}_i : S_N \rightarrow S_i$ be the projection of a composite state to node i 's component. For a predicate π on S_i , its lift is the predicate $\iota_i(\pi)$ on S_N defined by $\iota_i(\pi)(\sigma) = \pi(\text{pr}_i(\sigma))$.

Definition 14.3. (Emergent invariant). Let N satisfy the conditions of Requirement 13.2, let $\mathcal{A}$ be a perturbation family, and let n_N be the composite. An invariant $\pi \in \text{Inv}_{A'_N}(n_N)$ is emergent relative to $\mathcal{A}$ where π is not logically equivalent to any member of $\{\iota_i(\rho) : i \in V, \rho \in \text{Inv}_{A'_i}(n_i)\}$, nor to any finite Boolean combination of such lifts.

The clause on Boolean combinations is required and its absence would make the definition trivial in the wrong direction: a conjunction of component invariants is preserved whenever each conjunct is, so without the clause every such conjunction would count as emergent merely for not being a single component's invariant.

The definition says that the coupled system preserves something that no component preserves and that no combination of component invariants expresses.

Claim 14.4. *An emergent invariant is a property of the coupling and of no component. It is not the lift of any component's invariant, nor a Boolean combination of such lifts, and it is not a property of the product state set independently of the operators, since altering an operator alters the step relation and may destroy the invariant. This claim is formal.*

14.3 The Removal Test

The definition yields an operational check, and the check is what makes the notion usable rather than decorative.

Definition 14.5. (Removal test). Let π be an invariant of the composite of N relative to A'_N , and let $N \setminus j$ be the network obtained by deleting node j and every channel incident to it. Let $r_j : S_N \rightarrow S_{N \setminus j}$ delete the j component and the multisets on incident channels, and let $\pi \upharpoonright_j$ be any predicate on $S_{N \setminus j}$ with $\pi \upharpoonright_j \circ r_j$ logically equivalent to π on states in which the deleted multisets are empty, where such a predicate exists. The test is passed at j where either no such $\pi \upharpoonright_j$ exists, or $\pi \upharpoonright_j$ fails to be an invariant of the composite of $N \setminus j$ relative to the restricted family.

The first disjunct is not a technicality. Where no predicate on the smaller state set corresponds to π at all, the property was not expressible without j , which is the strongest form of dependence.

Passing the test for some j shows that the invariant depended on j 's participation. Passing it for every j shows that it depended on every participant, which is the strongest relational case. Failing it for every j shows that the invariant is carried by a proper sub-network, and the correct description locates it there.

Two cautions attach.

Removal changes the network, so the test compares two systems rather than examining one, and the comparison is legitimate only where the perturbation class is held fixed.

And the test is a test of dependence rather than of location. An invariant may depend on a node without being in it, which is the whole point, and the test is worded to avoid suggesting otherwise.

14.4 The Standing Illustration

A reader who has followed a field for years, an archive that retains what was published, and the reader's habit of consulting it form a small network.

The composite preserves something under the arrival of further reports: the reader, on receiving a new account, produces a state in which certain expectations about the outlet hold. Call the predicate that this correspondence obtains π .

π is not an invariant of the reader alone. Remove the archive and the correspondence fails on the next report that would have required consulting it, so the removal test is passed for the archive.

π is not an invariant of the archive alone, since an archive without a reader preserves records and no correspondence.

So π is emergent in the sense of Definition 14.2. The capacity is in the coupled system, it is preserved under the perturbations the reader ordinarily meets, and it belongs to no participant.

Claim 14.6. *The illustration exhibits an invariant that depends on two participants and is located in neither, and the emergence is checkable by the removal test rather than asserted. Nothing in the illustration establishes that the invariant is knowledge, which requires the interpretive apparatus of Part VI. This claim is formal as to the model and illustrative as to the case.*

14.5 The Design Instance and Its Simulation

The definitions above are exhibited on a design instance, so that the removal test is applied rather than described. The instance fixes six nodes of the kinds named in Section 11, with state sets of different mathematical type: a finite set at the witness, a pair of a coupling set and a real phase vector at the reporter, an integer backlog at the editor, an accumulating list at the archive, a real vector at the reader, and a finite set of positions at the institution. Couplings are partial maps, and the reporter's law omits an actor with a probability depending on the actor's type and on its own phase, which is the regularity at issue.

The predicate π is that the reader's expectations about omission by actor type stand within a fixed tolerance of the reporter's actual rates. The perturbation family is the arrival of further occurrences of the kinds the world produces.

The process of a single run is as follows, and it is stated step by step because the causal order of Definition 11.5 makes a narrative more useful than a diagram.

An occurrence is drawn: an actor of one of four types together with a small set of element identifiers. The witness consumes it and retains a subset, since not everything within reach is observed. The interview coupling, which is a node in its own right, consumes the witness's emission, appends to its own buffer, and passes on a subset whose size depends on that buffer, so what reaches the reporter is a function of the coupling's history as well as of the witness. The reporter consumes what arrives, raises its phase, and either publishes an account in which the actor is coupled to the occurrence or omits the actor, with the probability of omission depending on the actor's type and on the reporter's phase; publishing lowers the phase. Where an account is published, the archive appends it. The reader consumes the account with probability equal to its exposure, and where it does so it updates its expectation for that actor type and, if the archive is present, further updates every expectation from the archive's summary of a recent window. Between receipts the reader's expectations decay toward an uninformed prior. The run repeats for eight hundred occurrences.

Figure 11.2 shows the occurrences in causal order, Figure 14.1 shows the individual node and coupling dynamics of one run, and Figure 14.2 shows the test. With the archive in the network, π comes to hold and continues to hold under further perturbation. With the archive deleted, it does not, because the reader is exposed to a small fraction of what is published and its expectations decay between receipts. Deleting the reader destroys π trivially, since the predicate is not expressible on the remaining state set, which is the first disjunct of Definition 14.5.

Claim 14.7. *In the design instance π is an invariant of the composite relative to the stated family, is not the lift of any component's invariant, and fails on deletion of either the archive or the reader. It is therefore emergent under Definition 14.3, and the emergence is exhibited by simulation in place of being asserted. This claim is formal as to the definitions and is a property of the stated design instance, which supplies its own numerical apparatus; the model itself supplies no metric and no measure of discrepancy.*

Three cautions attach to the simulation and are recorded so that it is not over-read. It exhibits the definitions on one instance and establishes nothing about knowledge processes. Its parameters were chosen to make the contrast legible, and a different choice yields a different picture. And it is a design in the sense of Definition 2.1, so every number in it belongs to the interpretation and not to the formalism.

14.6 The Limits

Four, and the third is the one that shapes Part VI.

Emergence is relative to a perturbation class, so a system may preserve an invariant relative to what it ordinarily meets and lose it under conditions it rarely meets, and the account offers no privileged class.

Emergence as defined concerns preservation and not production. A network that produces something novel and preserves nothing has no emergent invariant under this definition, and the word in its ordinary use covers cases the definition excludes.

The definition says nothing about content. An invariant is a predicate on composite states, and whether it corresponds to anything a party would recognise as understanding

depends on the reconstruction maps, which is why the location of knowledge is deferred.

And the definition inherits the conditions of Requirement ??, since it is stated for a composite. Where those conditions fail there is no composite and the definition does not apply.

14.7 Summary of Results

This section has defined emergence within the first model. Definition 14.1 fixes a perturbation family and an invariant relative to a class, stated as a predicate in place of a part of a state, and records that the class is a matter of a party's actual exposure. Definition 14.2 fixes the lift of a component predicate through the projection, and Definition 14.3 fixes an emergent invariant as one preserved by the composite and equivalent to no lift and to no Boolean combination of lifts, the last clause being required to prevent conjunctions of component invariants from counting as emergent. Claim 14.4 states that such an invariant is a property of the coupling and of no component. Definition 14.5 fixes the removal test with the restriction map made explicit, and records that inexpressibility on the smaller state set is the strongest form of dependence. The illustration exhibits an emergent invariant in a reader, an archive and a consultation habit, checkable by the test. Four limits are recorded: relativity to a perturbation family, the restriction to preservation in place of production, silence about content, and dependence on the closure conditions.

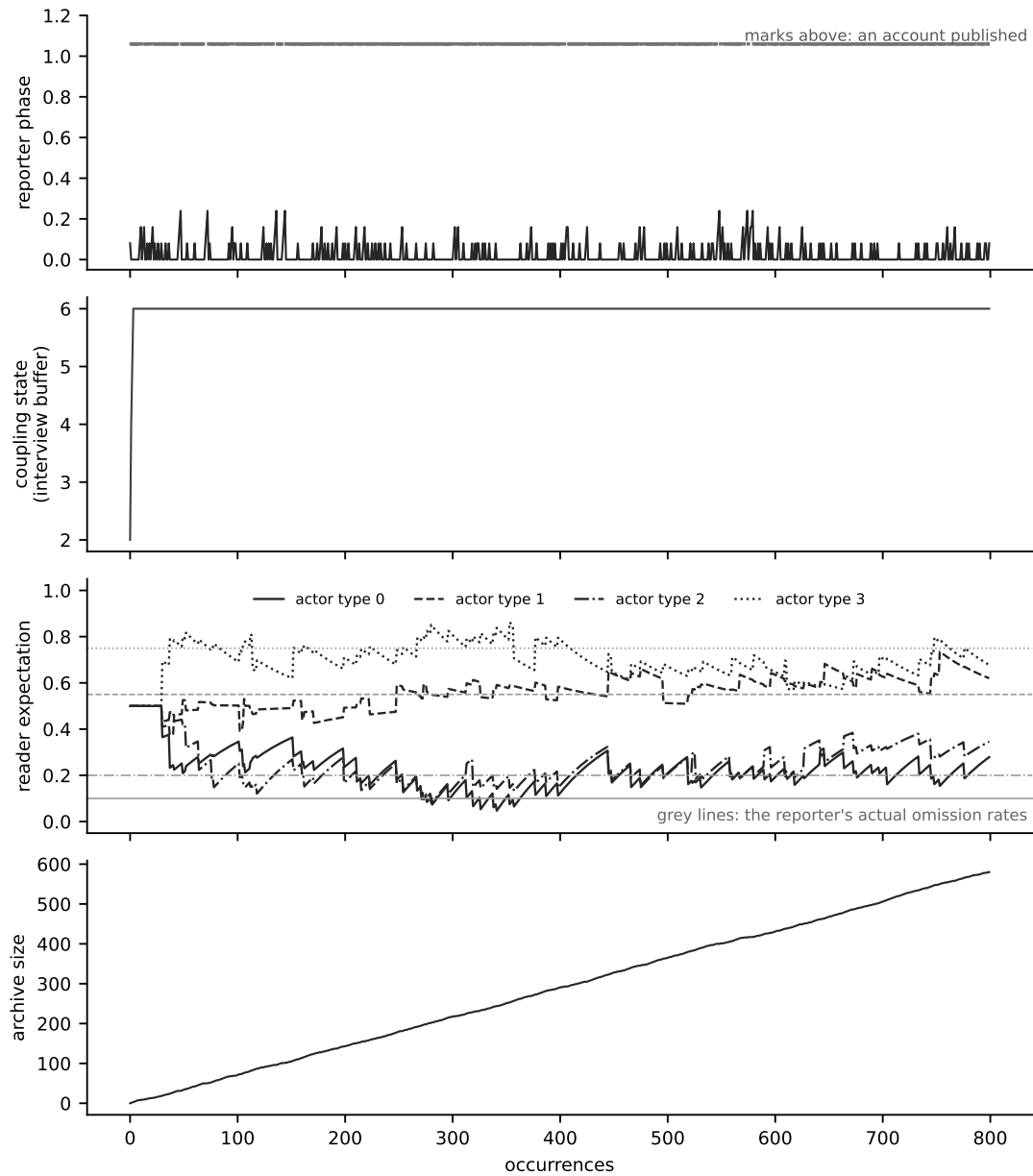


Figure 14.1: Individual node and coupling dynamics in one run of the design instance. The first panel shows the reporter’s phase, which rises with pending work and falls when an account is published, with marks above recording publications. The second shows the state of the interview coupling, which is a node in its own right and whose buffer governs what reaches the reporter. The third shows the reader’s expectation for each actor type against the reporter’s actual omission rates, drawn in grey: the expectations approach the rates and do not reach them, since the reader’s exposure is partial and its expectations decay between receipts. The fourth shows the archive accumulating. No quantity in one panel is compared with a quantity in another, and each belongs to its own node’s state space.

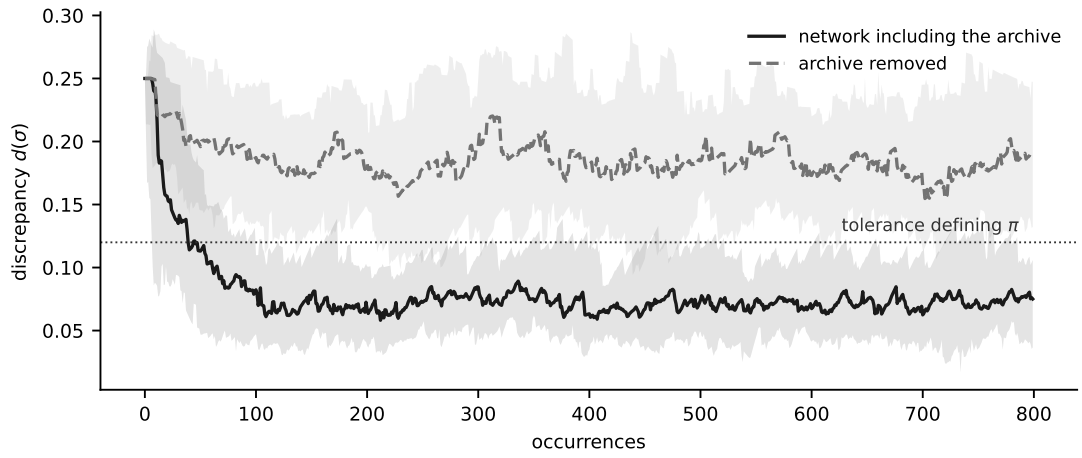


Figure 14.2: The removal test of Definition 14.5 applied to the design instance. The discrepancy $d(\sigma)$ is the mean absolute difference between the reader’s expectation vector and the reporter’s actual omission rates by actor type, and the predicate π holds where $d(\sigma)$ lies below the marked tolerance. Curves are medians over thirty runs with the interdecile band shaded. With the archive present the discrepancy falls below the tolerance and stays there under continuing perturbation; with the archive deleted it does not, since the reader’s exposure is partial and its expectation decays between receipts. The test is therefore passed at the archive, and by the symmetric argument at the reader, so π is emergent in the sense of Definition 14.3. The numerical quantities belong to this design and not to the model, which supplies no metric.

15 States as Relational Graphs of Primitive Elements

This section defines the objects the second model takes as states. Its role is to replace the undecomposed state of the transition graph with a structure whose parts can be tracked through a change, which is what the second representational requirement demands. Its structure treats the definition, the incidence condition, the identity of elements across states, the standing illustration, and the assumptions withheld. Its method is definition, with each condition justified by what the history model will need of it.

15.1 The Definition

Definition 15.1. (State). *A state is a triple $\Gamma = (P, C, \iota)$ in which P is a finite set of primitive elements, C is a finite set of couplings, and $\iota : C \rightarrow \mathcal{P}(P)$ assigns to each coupling the set of elements it joins, subject to two conditions: every coupling joins at least two elements, and every element belongs to $\iota(c)$ for at least one c .*

The definition places elements and couplings on different footings, and the asymmetry is the content. An element is a representational atom of the design: what that design treats as not further decomposed. A coupling is a junction at which several elements meet, and it is not itself an element, so a state is not a graph whose nodes and edges are the same kind of thing seen twice.

15.2 The Incidence Condition

Requirement 15.2. (Incidence). *For the correspondence with the history model of Section 16 to hold, each element belongs to at most two couplings. A state is then presented as a graph whose nodes are the couplings and whose edges are the elements, with an element belonging to one coupling drawn as a half-edge.*

The requirement is what makes Definition 8.2 exact rather than suggestive. Under it a state is a graph in the ordinary sense, with couplings at the nodes and elements on the edges, so an element persisting through an interval sweeps a surface and a coupling persisting sweeps a line.

The condition is a restriction and its cost is stated. An element participating in three or more couplings must be represented by introducing an auxiliary coupling that joins it to the others, which changes the presentation without changing what is represented. Whether the restriction can be lifted, and what a history model over general incidence structures would look like, is recorded among the open problems.

15.3 The Identity of Elements across States

A history model that tracks what became of each element needs elements to be identifiable across states, and identity of this kind is not free.

Definition 15.3. (Element identity). *A design fixes an identification of elements across the states of a trajectory: a partial injection between the element sets of successive states, recording which element of the earlier state each element of the later one continues. An element of the*

later state not in the image continues nothing and is introduced; an element of the earlier state not in the domain is terminated.

Three remarks bound the definition and the second is the one a critic will press.

The identification is supplied by a design and not by the formalism, which is consistent with the position of Section 2 that models locate nothing.

The identification is contestable, and two designs may identify differently over the same material. This is not a defect to be removed: whether the actor named in one account is the actor named in another is exactly the kind of question the account of Part II says has no answer independent of an interpretation.

And the identification is what the history model records. A face is the history of an element under the identification in force, so a dispute about identity appears in the model as a dispute about which faces there are, which is where such a dispute belongs.

15.4 The Standing Illustration

In the news case a design fixes the primitive elements as representational atoms: an identified actor, an identified occurrence, an attributed property such as a role or a time.

Couplings join them. The junction at which the element *actor A* and the element *occurrence O* meet under the relation of causing is one coupling; the junction at which *occurrence O* meets *time T* is another; the junction at which a source vouches for a claim is a third.

A witness's state, on this design, is a small graph: a few elements, a few junctions, most of them of the observing rather than the causing kind. A report's state is a different graph over some of the same elements, with junctions the witness's state did not contain.

Claim 15.4. *On this design the difference between two accounts of the same occurrence is a difference of couplings over a partly shared set of elements, and is therefore representable within a state in the sense of Definition 15.1. The transition graph of the companion papers represents each account as an undecomposed state and can represent no such difference. This claim is formal.*

15.5 The Assumptions Withheld

Three, each corresponding to a temptation.

No type discipline on elements. The formalism does not say what kinds of thing may be elements, and a design that treated sentences, propositions or entities as elements would all be admissible; the account has no view about which is correct and says so.

No ordering or weighting of couplings. A state records which junctions exist and nothing about their strength, and a design wanting strength adds it as a labelling, which the history model then has to carry.

And no requirement that distinct parties' states be over a common element set. Two parties may hold states with no element in common, and the identification of Definition 15.3 is defined within a trajectory and not across parties, which is the second representational requirement respecting the first.

15.6 Summary of Results

This section has defined the object the history model takes as a state. Definition 15.1 fixes a state as elements, couplings and an incidence assignment, with elements and couplings on different footings. Requirement 15.2 fixes the incidence condition under which a state is a graph with couplings at nodes and elements on edges, which is what makes the dimensional correspondence of Definition 8.2 exact, at the cost of restricting elements to at most two couplings. Definition 15.3 fixes element identity across states as a partial injection supplied by a design, with introduction and termination as its residues, and records that the identification is contestable and that disputes about it appear as disputes about which faces there are. The news design fixes elements as representational atoms and couplings as junctions, and Claim 15.4 states that differences between accounts become differences of couplings over shared elements, which the transition graph cannot represent. Three assumptions are withheld: no type discipline on elements, no weighting of couplings, and no common element set across parties.

16 Histories as Two-Complexes

This section defines the second model. Its role is to represent a change as a rearrangement of a state, which the transition graph cannot do, and to make the parts of a change addressable. Its structure treats the definition, the boundaries of a vertex, the three residues of an event, the vertex types of the standing illustration, and what the model does not represent. Its method is definition followed by the case.

16.1 The Definition

Definition 16.1. (History). *A history is a two-complex $F = (V, E, \Phi)$ together with boundary data, in which V is a finite set of vertices, E a finite set of edges each with an ordered pair of endpoints in V or a free end, and Φ a finite set of faces each bounded by a cycle or a path of edges and free ends. Under the assignment of Definition 8.2, each face is the history of one primitive element, each edge is the history of one coupling, and each vertex is an event.*

Definition 16.2. (Boundary of a history). *The boundary of a history is the state $\Gamma = (P, C, \iota)$ whose elements are the free ends of faces, whose couplings are the free ends of edges, and whose incidence is inherited: a coupling joins the elements whose faces are bounded by the edge in question at that end.*

The boundary of a history is a state in the sense of Definition 15.1, which is what makes composition possible and what makes the two models parts of one apparatus in place of two drawings.

16.2 The Boundaries of a Vertex

Definition 16.3. (Incoming and outgoing boundaries). *Let a history carry a partial order on vertices induced by the orientation of its edges. For a vertex v , the incoming bound-*

ary $\partial_{\text{in}}v$ is the state formed by the edges and faces incident to v from below, and the outgoing boundary $\partial_{\text{out}}v$ the state formed by those incident from above.

A vertex therefore has two states associated with it, and the difference between them is what the event did.

16.3 The Three Residues of an Event

Definition 16.4. (Created, destroyed and persisting couplings). For a vertex v write $C(\Gamma)$ for the coupling set of a state Γ . Then

$$\text{Cr}(v) = C(\partial_{\text{out}}v) \setminus C(\partial_{\text{in}}v), \quad \text{De}(v) = C(\partial_{\text{in}}v) \setminus C(\partial_{\text{out}}v),$$

and the persisting couplings are those in the intersection, which are exactly the couplings whose edges pass through v without terminating.

The definition is combinatorial and asserts nothing about knowledge, which is deferred to Part VI in accordance with Definition 2.1. What it supplies is an addressable answer to a question the transition graph cannot pose: given a change, which junctions were made at it and which were unmade.

Claim 16.5. Two histories may have the same initial and final boundaries and differ in the residues of their vertices. The transition graph of the companion papers identifies such histories, since it records only the endpoints of a change, and the history model distinguishes them. This is the sense in which the transition graph is the coarser representation, and it is the second representational requirement satisfied. This claim is formal.

16.4 The Vertex Types of the Standing Illustration

A design fixes which events it recognises, and the news design fixes seven. Each is a vertex type in the sense that a design's charter names it; the model itself recognises vertices and not types.

Occurrence: elements become available. Faces begin, and few couplings are created.

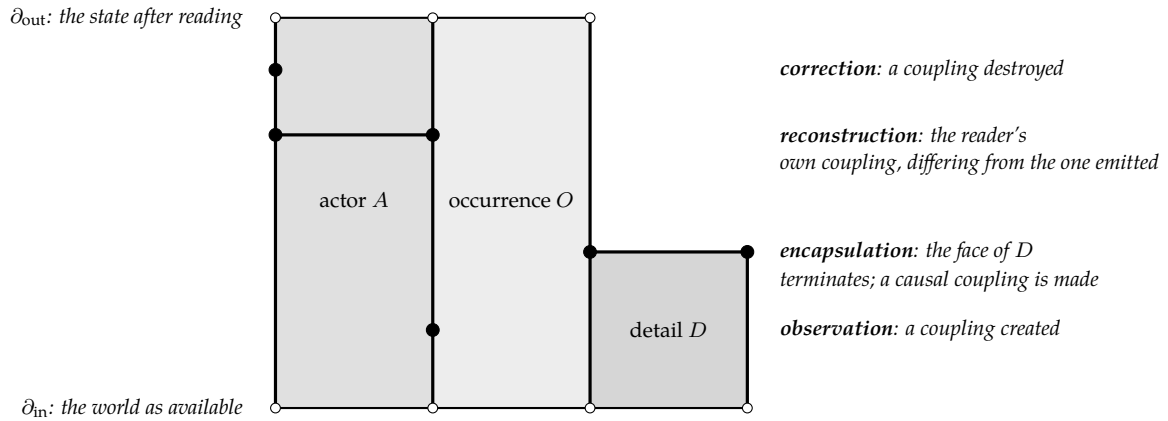
Observation: a coupling is created between a witness's element and an element of the occurrence. Faces continue; one edge begins.

Encapsulation: the reporter's law rearranges. Some faces terminate, which is the dropping of elements; some couplings are destroyed; and one or more couplings are created, characteristically a causal junction between an actor and an occurrence.

Emission: the account is published as a perturbation. In the history model nothing is created or destroyed here, which is worth noting: publication is an event in the network model and, on its own, no rearrangement.

Reconstruction: a receiving party's own couplings are created, and they may differ from those in the emitted account, which is where divergent boundaries appear.

Corroboration: a second coupling is created to an element already coupled, from an independent source.



Schematic. Faces are shaded, foam edges are heavy lines, vertices are filled discs, and the two boundaries are the horizontal graphs at bottom and top.

Figure 16.1: The news case as a history. Three elements are followed: an actor, an occurrence and a detail. The detail’s face terminates at the encapsulation vertex, which is the reporter’s own law dropping it, and the same vertex creates the coupling that joins the actor to the occurrence as its cause. That coupling is present in the outgoing boundary of the vertex and in no incoming boundary, which is the structural signature the companion paper on journalism argues for in prose: the causal attribution was made at that event and was not received. Later vertices show a reader creating a coupling of its own and a correction destroying one.

Correction: couplings are destroyed, and faces may terminate.

Figure 16.1 shows a history containing several of these. The detail’s face terminates at the encapsulation vertex and the causal coupling is created there, so that coupling lies in $\text{Cr}(v)$ for that vertex. That is the structural form of a claim the companion paper on journalism argues in prose, and it is checkable: inspect the incoming boundaries of the vertex and see whether the coupling is present in any of them.

Claim 16.6. *On the news design, a causal attribution present in a published account and absent from every incoming boundary of the encapsulation vertex lies in $\text{Cr}(v)$, and was therefore made at that event. Where it is present in an incoming boundary it lies in the persisting set and was carried through. The distinction between the construction of an attribution and its carriage is accordingly a set difference in this model. This claim is formal as to the model and depends, as to any case, on the identification of elements fixed by the design.*

16.5 The Limits of the Representation

Four, and the third bears on the paper’s architecture.

The model represents no dynamics. Nothing in it says which histories occur, which are likely, or what an event will do; it represents what an event did.

The model represents no parties. A vertex is an event, and which party’s law produced it is not in the history, so an account of who did what requires the network model alongside.

The model therefore represents no propagation. A perturbation crossing a coupling between parties is not a rearrangement of a state and has no image here, which is Claim 6.4

appearing as a property of the construction.

And the model depends on the identification of elements. Where a design's identification is contested, the faces are contested, and the model records the consequences of an identification in place of adjudicating it.

16.6 Summary of Results

This section has defined the history model. Definition 16.1 fixes a history as a two-complex with faces as element histories, edges as coupling histories and vertices as events, and Definition 16.2 fixes its boundary as a state in the sense of Part V, which is what makes composition possible. Definition 16.3 fixes the incoming and outgoing boundaries of a vertex, and Definition 16.4 the created, destroyed and persisting couplings, which are combinatorial and assert nothing about knowledge. Claim 16.5 establishes that histories with identical endpoints may differ in their residues, which the transition graph identifies and this model distinguishes. Seven vertex types are fixed for the news design, with emission notable for rearranging nothing, and Claim 16.6 states the set-difference test separating an attribution made at an event from one carried through it. Four limits: no dynamics, no parties, no propagation, and dependence on the identification of elements.

17 Gluing, Boundaries, and Composition

This section fixes how histories compose. Its role is to establish that a long history is assembled from short ones, which a representation of episodes would not support, and to state the conditions under which the assembly is well defined. Its structure treats the gluing operation, its conditions, the identity histories, the algebraic properties, and the consequence for the account. Its method is definition followed by claim.

17.1 The Gluing Operation

Definition 17.1. (Composable histories and their gluing). *Histories F and F' are composable where the outgoing boundary of F and the incoming boundary of F' are isomorphic as states, by an isomorphism that identifies elements with elements and couplings with couplings and preserves incidence. Their gluing $F' \circ F$ is the two-complex obtained by identifying, along that isomorphism, each free face end of F with the corresponding free face end of F' and each free edge end with the corresponding free edge end.*

The isomorphism is part of the data and not a fact discovered about the two histories. Two histories may be composable in several ways, and a design that glues has chosen one, which is the same kind of choice as the identification of elements in Definition 15.3 and is contestable in the same way.

17.2 The Conditions

Requirement 17.2. (Conditions on gluing). *The gluing of composable histories is a history where the following hold. Boundary agreement: the chosen isomorphism preserves*

incidence in both directions. Face consistency: an element identified across the seam has exactly one face on each side, so that the glued face is well formed. Edge consistency: a coupling identified across the seam has exactly one edge end on each side. Freshness: vertices, edges and faces of the two histories are otherwise disjoint, so that gluing creates no unintended identification.

Freshness is the condition an implementer will forget. Two histories over the same design will name the same elements, and gluing them without renaming would identify faces that merely share a name, which is a different operation and usually a mistake.

17.3 Identity Histories

Definition 17.3. (Identity history). *For a state Γ the identity history id_Γ has no vertices, one face for each element of Γ and one edge for each coupling, with both boundaries equal to Γ . It represents an interval in which nothing rearranged.*

The identity history is not a technicality. It is the formal statement that time passing is not by itself an event, which the account requires: a state persists until something rearranges it, and an interval of persistence is representable without positing an event to fill it.

17.4 The Algebraic Properties

Claim 17.4. *Under Requirement 17.2, gluing is associative up to the isomorphism identifying the two ways of bracketing a triple, and the identity histories are neutral: $F \circ \text{id}_{\partial_{\text{in}} F}$ and $\text{id}_{\partial_{\text{out}} F} \circ F$ are isomorphic to F . States and histories therefore form a category in which states are the objects, histories are the morphisms, and gluing is composition. This claim is formal, and its proof is by inspection of the construction rather than by any argument this paper supplies.*

Two consequences are worth drawing.

A history may be decomposed as well as assembled. Cutting along any state that appears as a boundary within it yields two composable histories, which is what permits an account of a long process to be given in parts.

And the categorial reading supplies a discipline the paper uses in Part VI. A correspondence between the two models is then a statement about structure-preserving maps rather than an informal parallel, which is the difference between a correspondence and a resemblance.

17.5 The Consequence for the Account

Claim 17.5. *The composition rule is what makes the borrowing from Section 8 more than an assignment of names to strata. A two-complex whose boundaries were not states, or whose gluing were not defined, would represent episodes and would leave the assembly of a longer history to the reader. The account requires assembly, since an inquiry is not a sequence of unrelated changes, and the borrowed structure supplies it. This claim is formal.*

One limit is recorded. Composability requires isomorphic boundaries, and in practice two histories will meet at states that are similar and not isomorphic: the party who continues an account will hold a state differing in some element or coupling from the state the previous party left. Gluing then requires an intermediate history representing the adjustment, and the account has no procedure for constructing one. This is the formal form of a fact the account insists on elsewhere, that continuation between parties is reconstruction and not resumption.

17.6 Summary of Results

This section has fixed composition. Definition 17.1 makes histories composable where a boundary isomorphism exists, and records that the isomorphism is chosen and contestable. Requirement 17.2 states four conditions, of which freshness is the one most easily overlooked. Definition 17.3 fixes identity histories and records that they express the persistence of a state without an event. Claim 17.4 states associativity and neutrality, so that states and histories form a category with gluing as composition, which supplies the discipline used for the correspondence in Part VI and permits decomposition as well as assembly. Claim 17.5 states that the composition rule is what makes the borrowing structural. The limit recorded is that boundaries met in practice are similar and not isomorphic, so gluing requires an intermediate history the account does not construct, which is the formal form of the claim that continuation is reconstruction.

18 The Field-Theoretic Reading and the Generation of Histories

This section states what the model does not have and what having it would require. Its role is to make the absence of a dynamics precise, so that it is an identified gap in place of an unnoticed one. Its structure treats the difference between enumeration and generation, what a generative account would require here, the two obstacles, the condensate question, and the position the paper takes. Its method is analysis, and the section proposes nothing.

18.1 Enumeration and Generation

As defined in Section 16, the model supplies a class of objects and a composition. It does not say which histories occur.

The distinction matters because a representation without a dynamics answers only questions of the form: given this history, what became of each element. It cannot answer questions of the form: given this state, which rearrangements are available, or which are more probable, or which the system tends toward.

The reformulation described in Section 8 is the standing example of the difference. There the class of histories is not posited: an action is posited, and the histories appear as the terms of an expansion, each with an amplitude the action determines.

18.2 What a Generative Account Would Require Here

Three things, and the paper has none of them.

A space of configurations, with a measure or a weight, over which something is summed. In the physical case this is a field on a group manifold, and its content comes from the algebra of the group.

An action, which is a functional on that space determining the weight of each history. The account of Part II supplies no candidate: there is no quantity of which a trajectory of reconstruction could be the stationary point, and inventing one to fill the slot is the failure Requirement 2.3 exists to prevent.

And an interpretation of the weight. Amplitudes in the physical case are complex and interfere, and the interference is substantive. An account in which the weight of a history were a probability would be a different mathematics with different consequences, and an account in which it were an arbitrary positive number would be a labelling.

Claim 18.1. *The absence of a dynamics in the history model is not a gap that a further definition would close. It requires a configuration space, a functional determining weights, and an interpretation of those weights, and the account of Part II supplies no candidate for any of the three. This claim is formal.*

18.3 The Two Obstacles

The obstacles are worth separating, since they are of different kinds.

The first is the absence of an algebra. In the physical case the labels on faces and edges compose, and the composition is what makes an amplitude assembled from local factors well defined. Primitive elements and couplings as defined in Section 15 carry no algebra: there is no operation composing two elements into a third, and none is available without a design supplying one.

The second is the absence of a repeated trial. A weight is interpretable where something recurs, whether as a frequency, a propensity or an ensemble. A trajectory of reconstruction occurs once, under conditions produced historically, and the account of Part II holds that the conditions are not reproducible in the way a weight would require.

Claim 18.2. *The second obstacle is the deeper one, since a design could supply an algebra by fiat and could not supply a repeated trial. An account committed to the historical production of interpretive conditions is committed to the singularity of a trajectory, and a weight over histories presupposes what that commitment denies. This claim is formal, and it identifies a tension internal to the account rather than a technical difficulty.*

18.4 The Condensate Question

The physical construction described in Section 8 recovers a continuum as a phase of very many quanta, and the corresponding question for this account is worth stating precisely, since it is the paper's most speculative open problem.

The question is whether a shared space of meanings, which Part II refuses as a background, could be recovered as a collective description of very many reconstructions. On such a reading, the appearance that parties occupy positions in a common space would be a property of a regime with very many densely coupled reconstructions, and would

fail outside that regime, exactly as a continuum fails where the condensate description fails.

Claim 18.3. *The condensate question, if answered affirmatively, would reconcile the refusal of Part II with the manifest usefulness of talking about parties as though they occupied a common space: the talk would be a valid collective description within a regime and invalid outside it. The paper does not answer the question, has no construction that would answer it, and records that answering it would require a dynamics it lacks. This claim is formal as to what an answer would require and is conjectural as to the prospect.*

18.5 The Position Taken

The paper stops at the combinatorial structure, and the reasons are stated together here.

An amplitude assigned without an interpretation would be a decoration, and the analogy discipline forbids it.

A structure that represents what an event did is worth having before the question of which events occur is opened, since the second question presupposes an answer to the first.

And the absence is stated as an absence, with its requirements enumerated, so that a later paper attempting a dynamics can be judged against them in place of being credited for using the vocabulary.

18.6 Summary of Results

This section has stated what the history model lacks. Enumeration is distinguished from generation, and the reformulation of Section 8 is the example in which histories are produced by an expansion of a posited action. Claim 18.1 states that a generative account here would require a configuration space, a functional determining weights, and an interpretation of those weights, and that the account supplies no candidate for any of the three. Two obstacles are separated: the absence of an algebra on elements and couplings, and the absence of a repeated trial, and Claim 18.2 identifies the second as deeper, since an account committed to the historical production of interpretive conditions is committed to the singularity of a trajectory. Claim 18.3 states the condensate question, whose affirmative answer would make talk of a common space a valid collective description within a regime, and records that the paper cannot answer it. The position taken is to stop at the combinatorial structure and to state the absence with its requirements enumerated.

19 The Correspondence between the Two Models

This section states how the two models relate. Its role is to discharge the obligation created by Claim 6.4, which established that neither model satisfies both requirements, so that the paper owes a correspondence in place of an argument that one subsumes the other. Its structure treats what a design must supply for the correspondence to exist, the construction, the structural properties, what each model contains that the other does not,

and the limits. Its method is construction followed by claim.

19.1 What a Design Must Supply

The correspondence is not a relation between the models, which locate nothing, but between the objects a design produces under them.

Requirement 19.1. (Data for the correspondence). *A design supports the correspondence where it supplies, in addition to a network in the sense of Definition 13.1, a reconstruction map R_i for each node, and an identification of elements across the successive reconstructions of each node in the sense of Definition 15.3.*

Both additions are substantive. Without reconstruction maps a run produces states in state sets and no relational structures; without an identification there are no faces, since a face is the history of an element under an identification.

19.2 The Construction

Fix a design satisfying Requirement 19.1, a node i , and a run U of the network.

The occurrences of U at node i are linearly ordered by same-node succession, and each produces a state s from which the design's reconstruction map yields a relational structure $R_i(s)$.

Definition 19.2. (Induced history). *The history induced at node i by the run U is the two-complex $H_i(U)$ whose vertices are the occurrences of U at i , whose boundaries between successive vertices are the reconstructions $R_i(s)$ of the states between them, whose faces are the maximal intervals over which an element persists under the identification, and whose edges are the maximal intervals over which a coupling persists.*

The definition is well posed under the incidence condition of Requirement 15.2 and requires no further assumption. What it says informally is that a node's own history, as seen through its reconstructions, is a foam whose events are that node's occurrences.

19.3 The Structural Properties

Claim 19.3. *The induced history respects composition. Where a run U is the concatenation of U_1 followed by U_2 at node i , the histories satisfy $H_i(U) \cong H_i(U_2) \circ H_i(U_1)$, and a run with no occurrence at i induces the identity history on i 's reconstruction. The construction is accordingly a functor from the runs of a network at a node, composed by concatenation, to the category of histories of Claim 17.4. This claim is formal, and its verification is by inspection of the construction.*

The functorial statement is what makes this a correspondence rather than a parallel. It says that the two models agree wherever they both apply, so a claim established in one about a composite process is available in the other.

19.4 What Each Model Contains and the Other Does Not

Claim 19.4. *The functor loses and gains as follows. It loses the propagation structure: the operators Π , the emissions and the couplings between nodes have no image, since a history contains no components and nothing crosses within it. It loses the other nodes: $H_i(U)$ is a history of one node's reconstructions, and a run involving many nodes induces many histories with no object joining them. And it gains the internal structure of a change: the residues $\text{Cr}(v)$ and $\text{De}(v)$ of Definition 16.4 are properties of the induced history and are not recoverable from the run, since a node's state set is not decomposed by the network model. This claim is formal.*

The third item is the one that vindicates the paper's architecture. The network model can say that node i consumed an input and produced a state; only the history model, through the design's reconstruction map and identification, can say which couplings that event made and which it unmade.

19.5 The Correspondence in the Standing Illustration

The reporter's encapsulation appears in both models and appears as different objects.

In the network model it is one occurrence: a state and an input are consumed, a new state and an emission are produced, and the emission is what the couplings carry onward. Which elements were dropped is not visible, since the reporter's state set is not decomposed.

In the induced history it is one vertex, and the elements dropped appear as faces terminating there while the causal coupling appears in $\text{Cr}(v)$.

Claim 19.5. *The two descriptions of the encapsulation are of the same event and neither contains the other. The network description supplies the party, the law and the perturbation; the history description supplies the rearrangement. A design wanting both must supply the data of Requirement 19.1, and an account of the construction of a social fact requires both, since it must say who did it and what was done. This claim is formal as to the models and illustrative as to the case.*

19.6 The Limits

Three.

The correspondence is per node. There is no construction here yielding a single history for a whole network, and whether one exists, and what its boundaries would be, is recorded among the open problems.

The correspondence depends on the identification of elements, which a design supplies and which is contestable, so two designs over one run yield different histories.

And the correspondence is one-directional in content. A history may be induced from a run; a run may not be recovered from a history, since the propagation structure is not in the image.

19.7 Summary of Results

This section has stated the correspondence. Requirement 19.1 fixes the additional data a design must supply, namely reconstruction maps and an identification of elements. Definition 19.2 constructs the history induced at a node by a run, with the node's occurrences as vertices and its reconstructions as boundaries. Claim 19.3 establishes that the construction is functorial with respect to concatenation and gluing, which makes it a correspondence rather than a parallel. Claim 19.4 states what is lost, namely the propagation structure and the other nodes, and what is gained, namely the residues of a change, which are not recoverable from the run. Claim 19.5 shows both descriptions of the reporter's encapsulation and states that an account of the construction of a social fact requires both. Three limits: the correspondence is per node, depends on a contestable identification, and does not run backwards.

20 A Heterogeneous Event-Based Design Scheme

This section gives one design and locates knowledge within it. Its role is to discharge the obligation created by Definition 2.1: two models with no design are an offer of notation, and knowledge is locatable only once interpretations are fixed. Its structure treats the design's commitments, knowledge for a party, relational knowledge, emergence at an event, the musical reading, and the limits of the definitions. Its method is definition, and everything defined here holds for this design and for no other.

20.1 The Commitments

Definition 20.1. (The design). *The design fixes, for each party i : a state set S_i with no metric assumed, an update map φ_i , an emission map ε_i , and a reconstruction map $R_i : S_i \rightarrow \Gamma$; for each ordered pair, a propagation operator Π_{ij} , possibly a generalised coupling in the sense of Definition 12.3; an identification of elements across each party's successive reconstructions; and a class $A'_i \subseteq A_i$ of admissible perturbations, being those the party can in fact undergo.*

The last item is the one that carries the historical commitment of Requirement 2.7. The class is not stipulated by an analyst. It is fixed by what a party can be exposed to, which is a matter of its material conditions, and two parties with identical reconstruction maps and different classes hold different knowledge of the same world.

20.2 Knowledge for a Party

Definition 20.2. (Knowledge for a party). *For a party i in state $\sigma_i(t)$, the knowledge of i at t is*

$$K_i(t) = \bigcap_{p \in A'_i} C(R_i(\varphi_i(\sigma_i(t), p))),$$

the set of couplings present in i 's reconstruction under every admissible perturbation.

Three features of the definition are the reasons for it.

Knowledge is a set of couplings and not a state. Nothing is stored: the definition quantifies over what the party would reconstruct, so knowledge is a stability property of reconstruction in place of a content held somewhere.

Knowledge is what survives. A coupling appearing in one reconstruction and absent under some admissible perturbation is not knowledge on this design but a reading, and the distinction is exactly the one an intersection makes.

And knowledge is indexed to a party and to a perturbation class. There is no unindexed object, and the account has no way of forming one.

Claim 20.3. *On this design, shared knowledge is a coupling lying in K_i for several parties i . This is weaker than agreement, since the parties need not be able to compare their reconstructions and the account of Part II says they cannot, and it is the strongest notion of sharing available under a background-independent account. This claim is formal.*

20.3 Relational Knowledge

Definition 20.4. (Relational knowledge). *For parties i and j coupled by Π , the relational knowledge of the pair is*

$$K_{i \oplus j} = \text{Inv}(F_i \oplus_{\Pi} F_j) \setminus \iota(\text{Inv}(F_i) \cup \text{Inv}(F_j)),$$

where F_i denotes the party as a node, $\oplus_{\Pi}$ the coupled composite of Section 13, Inv the invariants relative to the induced family, and ι the lifting of Definition 14.2, with Boolean combinations of lifts excluded as in Definition 14.3.

The definition is the emergence of Part IV read as knowledge, which the design's interpretation licenses and the model alone did not. Its test is the removal test: an invariant in $K_{i \oplus j}$ fails when either party is deleted, which is what shows it was in neither.

20.4 Emergence at an Event

Definition 20.5. (Emergence at an event). *For a vertex v of an induced history,*

$$E(v) = C(\partial_{\text{out}} v) \setminus \bigcup C(\partial_{\text{in}} v),$$

the couplings present in the outgoing boundary and in no incoming boundary.

Claim 20.6. *The definition vindicates the terminological rule of Requirement 2.5 rather than merely respecting it. A coupling in an outgoing boundary either lies in some incoming boundary, in which case it was re-instantiated by the party's own operation, or lies in $E(v)$, in which case it was made at that event. Nothing is ever conveyed. Perturbations propagate along the couplings of the network model, and structure evolves at the vertices of the history model, and there is no third thing for a word naming conveyance to name. This claim is formal.*

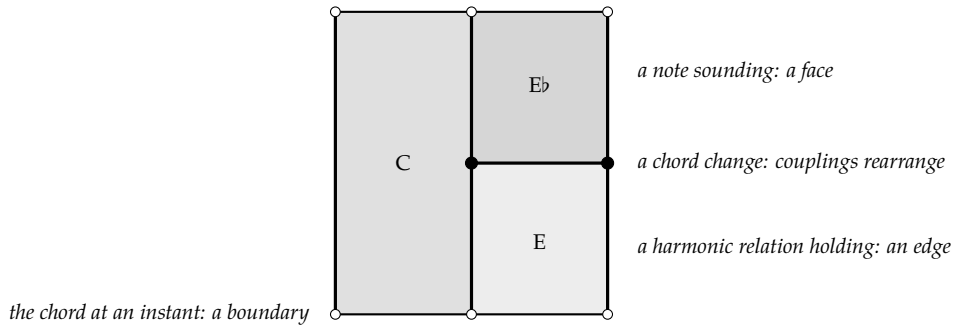


Figure 20.1: The musical reading, given once because a reader meeting a two-complex needs a case with elements, couplings and events and none of the difficulties a knowledge example carries. A note sounding through an interval is the history of an element and is a face; a harmonic relation holding across a span is the history of a coupling and is an edge; a chord change is an event at which couplings rearrange and is a vertex; and the configuration at an instant is a boundary, which is a state. The reading also shows why the assignment of Definition 8.2 is the correct one: it is the notes that persist and the relations among them that are made and unmade, and the naive assignment would have the relation persist and the note be a junction.

20.5 The Musical Reading

Figure 20.1 gives the assignment on a case with no interpretive difficulty. A note sounding is a face, a harmonic relation holding across a span is an edge, a chord change is a vertex, and the configuration at an instant is a boundary.

The reading is included for one reason beyond legibility. It shows why the assignment of Definition 8.2 is the right one: the notes persist and the relations among them are made and unmade, so it is the notes whose histories are two-dimensional. The naive assignment would make the relation persist and the note a junction, which is audibly wrong.

An event-based design over such a case is immediate: the events are onsets, releases and changes, and nothing is updated between them. This is the sense in which the design scheme of this section is event-based, and Section 21 records that it is one strategy among several.

20.6 The Limits of the Definitions

Four, and the fourth is the account's own position in place of a defect.

A'_i must be specified before K_i is defined, and there is no universal class. A design that omitted it would have defined nothing, since an intersection over an unspecified index is not a set.

R_i is not observable. Only its outputs are, and those only where the party emits, so K_i is not measurable by another party and is at best inferable.

The definitions are relative to a design. Another design over the same material yields different knowledge, and the account has no procedure for choosing between designs.

And that relativity is the epistemological claim of Part II restated at the level of the formalism. On this account there is no view from which knowledge is located independently of an interpretation in force, so a definition that was not design-relative would

contradict the position it formalises.

20.7 Summary of Results

This section has given one design and located knowledge within it. Definition 20.1 fixes the design's commitments, of which the class of admissible perturbations carries the historical commitment, being fixed by material conditions in place of stipulation. Definition 20.2 fixes knowledge for a party as the couplings surviving reconstruction under every admissible perturbation, which makes knowledge a stability property in place of a stored content, and Claim 20.3 fixes shared knowledge as a coupling lying in several parties' knowledge, which is weaker than agreement and is the strongest notion available. Definition 20.4 fixes relational knowledge as an invariant of the coupled pair that is no lift of a component's invariant, tested by removal. Definition 20.5 fixes emergence at an event, and Claim 20.6 states that it vindicates the terminological rule, since a coupling was either re-instantiated locally or made at the event and there is no third case. The musical reading exhibits the assignment and shows why the naive one is wrong. Four limits: the perturbation class must be specified, the reconstruction map is unobservable, the definitions are design-relative, and that relativity is the account's position rather than a defect.

21 The Alternative Design Strategies the Models Admit

This section states that the design of Section 20 is one strategy and names others. Its role is to prevent the paper from being read as proposing a system when it proposes a formalism, which is the ordinary fate of a paper that gives one design and stops. Its structure treats the underdetermination, four alternative strategies with what each buys and costs, the criterion by which a design is chosen, and the consequence for the paper's claims. Its method is enumeration.

21.1 The Underdetermination

Claim 21.1. *The two models underdetermine a design. A design must fix state sets, laws, couplings, reconstruction maps, an identification of elements and a perturbation class, and the models constrain none of these beyond the conditions of Requirements 13.2 and 15.2. Distinct designs over the same material are therefore admissible, will locate knowledge differently, and are not in competition as readings of the models. This claim is formal.*

21.2 The Event-Based Strategy

The design of Section 20 advances a party's state at occurrences and updates nothing between them.

It buys the property the account requires most directly, namely that nothing happens without an occasion, and it makes the causal order of Definition 11.5 the only order in the design.

It costs the ability to represent processes that proceed without occasions: matura-

tion, forgetting, drift and habituation are all changes without an input, and the design of Section 20 represents them only by introducing artificial occurrences, which is what the simulation of Section 14 does with its decay.

21.3 Three Alternatives

A continuous strategy. Nodes carry flows in place of update maps, with events at threshold crossings. This buys maturation and drift directly, and costs the property that the design has no order beyond the causal one, since a flow requires a parameter along which to flow, and a shared parameter across nodes reintroduces exactly what Definition 11.5 refused.

An ensemble strategy. A party is represented by a distribution over states in place of a state, and reconstruction yields a distribution over relational structures. This buys an account of variation within a party, and of what a party might have reconstructed, and it costs the definiteness of K_i : knowledge becomes a set of couplings above a threshold of probability, and the threshold is a further design parameter with no principled value.

A history-only strategy. A design fixes histories and no network, taking rearrangements as primitive and declining to represent parties at all. This buys a great deal of simplicity and is adequate wherever the question concerns what became of a structure. It costs everything the network model supplies: no propagation, no parties, and no account of who did what, so it cannot represent the construction of a social fact, which requires an agent.

Claim 21.2. *Each alternative buys a property the event-based design lacks and costs a property it has, and none dominates. The choice among them is a choice about which questions a study intends to ask, and the models are neutral between them. This claim is formal.*

21.4 The Criterion for Choosing a Design

The models are neutral and a study is not, so a criterion is owed.

Requirement 21.3. (Design adequacy). *A design is adequate to a study where the questions the study asks are answerable from the objects the design produces, and where the parameters the design fixes are fixed by something other than the convenience of the answer. A design whose perturbation class was chosen so that a desired coupling would lie in K_i has not defined knowledge; it has described a conclusion.*

The second clause is the one that matters and it is easily violated. The class of admissible perturbations determines what counts as knowledge on the design, so a study free to choose it can obtain any result it likes, and the historical commitment of Requirement 2.7 is what constrains the choice: the class is what a party can be exposed to, which is a question about conditions and not about the analyst's purposes.

21.5 The Consequence for the Paper's Claims

Claim 21.4. *Every claim of Section 20 is a claim about that design. The definitions of knowledge, relational knowledge and emergence hold under it, hold in modified form under the*

alternatives, and hold in no form for the models alone. A reader who finds the definitions unconvincing has an objection to a design and not to the formalisms, and may construct another. This claim is formal.

21.6 Summary of Results

This section has stated that the models underdetermine a design, so distinct designs are admissible and are not competing readings. The event-based strategy buys the property that nothing happens without an occasion and costs the representation of change without an input, which the simulation handles by an artificial decay. Three alternatives are named: a continuous strategy, which buys drift and costs the absence of a shared parameter; an ensemble strategy, which buys variation within a party and costs the definiteness of knowledge; and a history-only strategy, which buys simplicity and costs the representation of parties and hence of the construction of a social fact. Requirement 21.3 fixes design adequacy and forbids fixing the perturbation class to obtain a desired conclusion, with the historical commitment supplying the constraint. Claim 21.4 records that every claim of the design section is a claim about that design.

22 The Transition Graph as a Coarse-Graining

This section states the relation between the models of this paper and the record specified in the companion papers. Its role is to establish consistency: the series must not contain a specification whose representation this paper's models contradict. Its structure treats the map, its properties, what the fibre contains, the consequence for the specification, and the limits. Its method is construction followed by claim.

22.1 The Map

Definition 22.1. (Coarse-graining). *Let F be a history with initial boundary Γ and final boundary Γ' . Its coarse-graining $\kappa(F)$ is the transition of the companion specification whose prior state is Γ , whose posterior state is Γ' , and whose typed act is supplied by the design.*

Claim 22.2. *κ respects composition: $\kappa(F' \circ F)$ is the transition from the initial boundary of F to the final boundary of F' , which is the composite of the two transitions in the transition graph. The map therefore carries the category of histories to the transition graph and is a functor onto its image. This claim is formal.*

22.2 The Fibre

Claim 22.3. *κ is not injective. Two histories with the same boundaries and different residues at their vertices have the same image, as Claim 16.5 established and as Figure 22.1 shows. The fibre of κ over a transition accordingly contains everything about a change other than its endpoints: which elements persisted, which couplings were made, which were unmade, at how many events, and in what order. This claim is formal.*

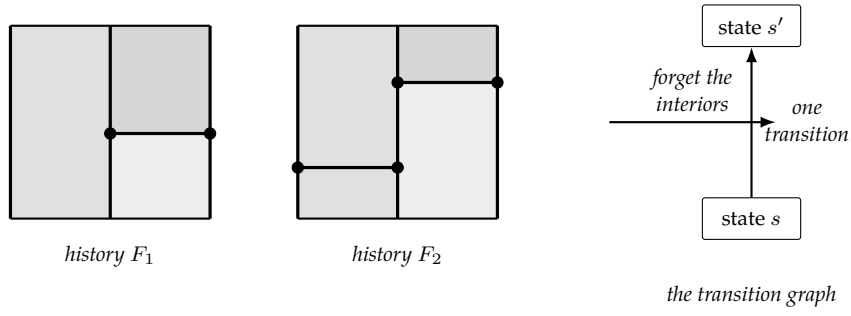


Figure 22.1: The transition graph as a coarse-graining. Two histories with the same initial and final boundaries, differing in which couplings were made and unmade and at which events, have the same image under the map that forgets the interiors of histories. The companion specification records the image; this paper's second model records the fibre. Nothing in the companion papers is contradicted by the finer representation, and nothing in the finer representation is required by them.

The fibre is exactly what the second representational requirement demanded and the transition graph could not supply, so the map states the relation between the two representations rather than merely comparing them.

22.3 The Consequence for the Companion Specification

Claim 22.4. *The companion specification is not contradicted by this paper. Its record is the image of κ , so every transition it can record is the coarse-graining of some history, and no history is inconsistent with the transition it induces. The specification remains implementable and conformant exactly as it stands, and this paper's models add a representation it does not require. This claim is formal.*

Two practical consequences follow for a reader of the series.

An implementation conforming to the specification loses the fibre, and loses it silently. Two changes that differed in what they made and unmade appear in a conformant record as one kind of object, and nothing in the record signals the loss.

And a design wishing to retain the fibre must record more than the specification requires: the elements, the couplings, and the identification across states. That is an extension of a conformant record and not a departure from it, since the extra material sits alongside the transition without altering it.

22.4 The Limits

Three.

The map is defined for histories with a designated initial and final boundary. A history with several boundary components, which the composition rule permits, has no single transition as its image, and the account has no treatment of that case.

The typed act of the image is supplied by the design and is not determined by the history. Two designs may coarse-grain one history to transitions of different act types, which is a further instance of the design-relativity recorded throughout.

And the direction of the map is fixed. A transition determines no history, so no procedure recovers the fibre from a conformant record, and a study wanting the finer representation must have recorded it at the time.

22.5 Summary of Results

This section has fixed the relation to the companion specification. Definition 22.1 defines the coarse-graining of a history to a transition, and Claim 22.2 establishes that the map respects composition. Claim 22.3 establishes that it is not injective and that its fibre contains everything about a change other than its endpoints, which is what the second representational requirement demanded. Claim 22.4 establishes that the specification is not contradicted, since its record is the image of the map, with two consequences: a conformant implementation loses the fibre silently, and a design wishing to retain it extends a conformant record rather than departing from it. Three limits: histories with several boundary components have no image, the act type of the image is design-supplied, and the map does not run backwards.

23 The Questions the Models Answer and the Questions They Do Not

This section applies the standard fixed in Section 2 to the two models. Its role is to judge them against a criterion stated before they were constructed, in place of judging them by their appearance. Its structure treats the three candidate questions in turn, the questions the models answer that were not anticipated, the verdict, and the respects in which the verdict is weak. Its method is assessment.

23.1 The Criterion

Requirement 2.7 fixed the standard: a representation proposed for an object already representable is adequate only where it answers a question the existing representation cannot, and the existing representation is the transition graph of the companion papers.

Three candidate questions were named there, and each is now answered or not.

23.2 The First Question: Structural Similarity across Vocabularies

The question is whether two trajectories are structurally similar in a sense surviving differences of vocabulary and of field.

Claim 23.1. *The models make the question well posed and do not answer it. Under the history model two changes are comparable as two-complexes, and comparison up to isomorphism of subcomplexes is a definite relation that requires no common vocabulary, since it concerns the arrangement of faces, edges and vertices and not the labels on them. What the models withhold is any graded notion: they support identity of structure and its absence, and they support no measure of nearness, since a measure would be a metric on a space of structures and Claim 3.5 forbids the space. This claim is formal.*

The consequence for the companion papers is worth stating, since it corrects an expectation those papers created. The cross-field matching they describe as an open problem

is not made tractable here. What is supplied is a formulation under which the problem is about the structure of changes in place of the similarity of topics, which is a reformulation and not a solution.

23.3 The Second Question: Events Altering Several Elements at Once

The question is whether an event that changed several elements and their couplings at once is representable as one object.

Claim 23.2. *The models answer this question and the transition graph cannot. A vertex is one object with an incoming and an outgoing boundary, the residues of Definition 16.4 are properties of it, and Claim 16.5 established that histories identified by the transition graph are distinguished here. This is the question on which the paper's adequacy rests, and the news case exhibits it: the reporter's encapsulation drops elements and makes a coupling at one event, and the transition graph records only that one state became another. This claim is formal.*

23.4 The Third Question: Dynamics Constraining Successors

The question is whether the dynamics by which a state was reached constrain which states can follow.

Claim 23.3. *The models do not answer this question and the network model does not answer it either, notwithstanding that it contains dynamics. A node's law determines what follows from a state and an input, so within a design the question has an answer for each node; what neither model supplies is a constraint on histories, since the history model has no dynamics at all and the network model's dynamics are internal to nodes and say nothing about which rearrangements of a relational structure are possible. This claim is formal.*

The absence is the one Section 18 treats, and it is the paper's principal gap.

23.5 The Questions Answered That Were Not Anticipated

Three, and they are the paper's substantive additions beyond the criterion.

Whether a coupling was made at an event or carried through it. Claim 16.6 makes this a set difference, and it is the formal statement of a distinction the companion paper on journalism argues in prose.

Whether a capacity is located in a party or in a coupling. The removal test of Definition 14.5 makes this checkable, and the simulation exhibits it.

And what the record specified in the companion papers loses. Claim 22.3 identifies the fibre of the coarse-graining, so the loss is characterised in place of being suspected.

23.6 The Verdict

Claim 23.4. *Of the three questions fixed in advance, the models answer one, make one well posed without answering it, and leave one untouched. Under Requirement 2.7 the history*

model is accordingly adequate, since the second question is answerable within it and unanswerable in the transition graph, and the network model is adequate to the first representational requirement without being assessed by the criterion, which concerns representations of changes. This claim is formal.

23.7 The Respects in Which the Verdict Is Weak

Four, and the fourth is the one a referee will press.

One answered question out of three is a modest result, and the paper does not present it as more.

The answered question was chosen by the author, and a reader who holds that the interesting questions lie elsewhere is not answered by a criterion the paper set for itself.

The models are assessed against the transition graph and against no other representation. A different existing formalism might answer the second question, and Section 9 surveyed the network side more thoroughly than the history side.

And adequacy in the sense of the criterion is not usefulness. A representation may answer a question no other representation answers and still be used by nobody, which is a fact about proposals of this kind that the companion paper on adoption treats at length.

23.8 Summary of Results

This section has assessed the models against the criterion fixed in advance. Claim 23.1 states that structural similarity across vocabularies is made well posed and not answered, since comparison up to isomorphism is available and no graded measure is, and records that the cross-field matching of the companion papers is reformulated in place of being solved. Claim 23.2 states that the representation of an event altering several elements at once is answered, which is the question the paper's adequacy rests on. Claim 23.3 states that dynamics constraining successors is untouched, which is the paper's principal gap. Three unanticipated questions are answered: whether a coupling was made or carried, whether a capacity is in a party or a coupling, and what the companion record loses. Claim 23.4 gives the verdict, and four weaknesses are recorded, of which the fourth is that adequacy is not usefulness.

24 Open Problems and the Conditions of Falsification

This section states what remains and what would show the paper wrong. Its role is to close with a list a later paper can be judged against, in place of a summary. Its structure treats the open problems in three groups, the conditions of falsification, the status of the paper, and the conclusion. Its method is enumeration.

24.1 The Problems Internal to the Formalisms

Amplitudes and a dynamics. The history model represents histories and neither weights nor generates them. Section 18 states what a generative account would require and identifies the absence of a repeated trial as the deeper obstacle, since a design could supply

an algebra by fiat and cannot supply recurrence.

A whole-network history. The correspondence of Section 19 induces a history at each node and joins them by nothing. Whether a single history exists for a network, what its boundaries would be, and whether the propagation structure could appear in it are open.

General incidence. Requirement 15.2 restricts each element to at most two couplings so that a state is a graph. Whether a history model exists over general incidence structures, in which an element may join several couplings directly, is open, and the restriction is currently discharged by introducing auxiliary couplings.

Decidability of the closure conditions. Confluence is undecidable at the generality of Definition 11.1, so a design must either restrict the class of laws until it is decidable or impose an order, and the second retreats toward the global clock the model refuses. Which restrictions are both decidable and wide enough to be useful is open.

Unification. Claim 6.4 established that neither model satisfies both representational requirements, and a single formalism satisfying both would be preferable to two dividing them. Whether one exists is open, and Section 6 records the absence as a limitation rather than as a design choice.

24.2 The Problems That Await a Design

The identification of elements. Definition 15.3 places identity across states with the design, and no procedure is given for fixing it or for adjudicating between designs that fix it differently.

The perturbation class. Definition 20.2 defines knowledge relative to a class fixed by material conditions, and no method is given for determining the class in a real case, so the definition is presently applicable to constructed instances and not to observed ones.

Gluing across parties. Section 17 records that boundaries met in practice are similar and not isomorphic, so composition requires an intermediate history the account does not construct.

24.3 The Speculative Problem

The condensate question. Whether a shared space of meanings could be recovered as a collective description of very many densely coupled reconstructions, valid within a regime and failing outside it, in the way a continuum is recovered in the physical construction described in Section 8. An affirmative answer would reconcile the refusal of Part II with the manifest usefulness of speaking as though parties occupy a common space. The paper has no construction that would answer it and records that answering it requires the dynamics it lacks.

24.4 The Conditions of Falsification

Five findings would show parts of the paper wrong, and each is stated so that a reader may look for it.

A formalism placing distinct parties' states in a common space, which does not re-

instate the transmission picture, would defeat Claim 3.5 and with it the justification for borrowing from a background-independent source, as Claim 10.2 records.

A single formalism satisfying both representational requirements would defeat Claim 6.4 and would make the two models an unnecessary division.

An extension of the transition graph representing the residues of a change, without the apparatus of Part V, would defeat Claim 16.5 and would remove the paper's adequacy argument.

A demonstration that the identification of elements is determined by the material rather than supplied by a design would defeat the design-relativity recorded throughout, and would strengthen the models by making their outputs comparable across studies.

And a demonstration that confluence fails in every network of interest would make Requirement 13.2 vacuous and the closure result empty, since a condition satisfied by nothing establishes nothing.

24.5 The Status of the Paper

The paper proposes two formalisms, gives one design, exhibits the design on a constructed case with a simulation, and claims no result about any actual process of knowledge.

Its models are unimplemented in the sense that matters: no study has used them, no data has been represented in them, and the simulation of Section 14 is a demonstration of definitions in place of evidence about anything.

Its borrowings are bounded in Section 10 and its declined apparatus is stated where it is declined, so a reader can check at each point whether a property was carried under a licence or under a resemblance.

24.6 Conclusion

The paper began from a refusal. If what a party holds is reconstructed under an interpretation that party holds, then nothing content-like passes between parties, and the ordinary vocabulary of transfer names nothing.

Taking the refusal seriously has a formal price, and the paper's contribution is mainly an accounting of that price. A model may not place parties in a common space, so it may not compare their states, so it may not measure the distance a message travels or the fidelity with which it arrives. What remains is coupling without embedding, which the network model supplies, and the rearrangement of a relational structure, which the history model supplies, and a design in which knowledge is the part of a reconstruction that survives what a party can be exposed to.

What the paper does not have is a dynamics, and the reason is not technical. A weight over histories presupposes recurrence, and an account committed to the historical production of interpretive conditions is committed to the singularity of a trajectory. That tension is internal to the position and is the most interesting thing the formal work has exposed, since it was not visible while the account remained in prose.

24.7 Summary of Results

This section has listed what remains. Five problems are internal to the formalisms: amplitudes and a dynamics, a whole-network history, general incidence, the decidability of the closure conditions, and unification of the two models. Three await a design: the identification of elements, the determination of a perturbation class in a real case, and gluing across parties. One is speculative: the condensate question, whose affirmative answer would reconcile the refusal of Part II with the usefulness of common-space talk. Five conditions of falsification are stated, concerning a common space without transmission, a unified formalism, an extension of the transition graph, a determinate identification of elements, and the vacuity of the closure conditions. The status of the paper is fixed as two formalisms with one design, exhibited on a constructed case, claiming no result about any actual process, and the conclusion records that the absence of a dynamics follows from the account's own commitment to the singularity of a trajectory.

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