

Generative Closure in Generative Relational Being

A Preliminary Reflection on Turbulence Models

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Abstract

Research on culture, norms, trust, and institutions routinely writes the macro-level as a system with dynamics of its own—an equation in which the future of a collective variable is a function of that variable alone. This paper argues that the self-containedness such writing presupposes is an assumption rather than a finding, and that a mature physical case, fully developed turbulence, shows how and why the assumption fails. The failure has a precise name in statistical physics: the closure problem. When a system with exact microscopic dynamics is projected onto the coarse variables one wishes to study, the projected dynamics are in general not closed—each order depends on a higher one, and the coarse variable is not by itself a sufficient description of its own future. The Mori–Zwanzig formalism gives the projected evolution exactly: alongside its drift it carries a memory kernel and a noise, and reduces to a self-contained macroscopic equation only when a separation of scales lets the memory decay and the noise whiten. Turbulence is the case in which that separation is absent across the range that matters, and it serves here as a calibration rather than a metaphor: it exhibits the closure problem on a fixed state space under the simultaneous loss of every feature that ordinarily rescues it, and thereby marks what a relational difficulty of merely this kind would amount to. The renormalization group, the second instrument for the passage across scales, sharpens the caution from the other side: where it succeeds, universality forbids reading a closed macroscopic law back to its microscopic mechanism; where it fails, as on the anomalous scaling of turbulence, it shows that a scaling law is no warrant for a universal macroscopic theory. The paper carries these into the study of relational emergence as a methodological caution—that a macro-level’s autonomy is to be established, not assumed—and distinguishes the closure it warns about, a closure of dynamics, from the notion the framework will require, a closure of generation. The latter is named and delimited, not established: what would ground it, an invariant of generation not underwritten by any conservation law, is set down as the open problem the reflection leaves standing.

Keywords: closure problem; Mori–Zwanzig; turbulence; relational emergence; generative closure.

A note on the standing of this paper. The paper takes one physical case—the closure problem of fully developed turbulence—and asks what it warns against in the study of how macroscopic order arises from microscopic relation. The order of exposition follows the order of the argument: the physical mechanism is set out first, in its own terms, and the transfer to relational emergence follows as a caution that mechanism licenses. One notion the framework will require, a closure of generation as against a closure of dynamics, is introduced toward the end and left as an open problem, together with the invariant that would ground it. Should the caution be shown to be either already common ground or unsupported by the physical case, the corresponding sections lose their ground. Objection, correction, and counter-evidence are welcome at huangwanhong@serendip.ngo.

1 INTRODUCTION

A collective phenomenon studied over time invites a certain notation. Let C stand for a culture, a body of norms, a level of institutional trust; one then writes

$$\frac{dC}{dt} = F(C),$$

and asks how C evolves. The notation is natural, it is often useful, and it carries an assumption that is rarely examined: that the future of the macro-level is a function of the macro-level alone—that culture is determined by culture, norms by norms, the collective variable by itself. Why a macro-level should possess dynamics of its own, closed upon its own variables, is not a question the notation raises; the closedness is built in before inquiry begins.

This paper argues that the closedness is an assumption rather than a finding, and that it can fail. The argument is not made in the abstract. It is borrowed from a physical case in which the same assumption is made, is precisely stated, and is known to fail in a way that has resisted resolution for the better part of a century: fully developed turbulence, and the closure problem to which it gives rise. In that case the microscopic dynamics are known exactly—the Navier–Stokes equations, written down in the nineteenth century—and the difficulty lies entirely in passing from those dynamics to a self-contained description at the scale one wishes to study. The passage does not close. What the coarse variable does next depends on information the coarse variable does not contain.

The transfer of this difficulty to the study of relational emergence is the work of the paper, and two misreadings would each void it. The first would treat turbulence as a metaphor, a suggestive picture of complexity from which one helps oneself to conclusions; it is not so used here. Turbulence serves as a calibration: it exhibits the closure problem on a fixed state space under the loss of every feature that ordinarily rescues it, and so provides a benchmark against which a relational difficulty can be placed. The second misreading would let the borrowed notion of closure stand in for the notion the framework needs. The closure turbulence teaches is a closure of *dynamics*; generative relational being will require a closure of *generation*, and the two differ. This paper reaches that distinction and stops: the notion that would complete it is identified as an open problem, not delivered.

The account this paper offers may be stated as a single claim.

Central claim. *The generation of a macro-level from microscopic relation does not by itself confer upon that macro-level a closed dynamics of its own. Whether a collective variable has dynamics in its own terms—whether $dC/dt = F(C)$ may be written at all—is a condition to be established, not a default to be assumed; and the closure problem of turbulence supplies both the precise form of the condition and a case in which the condition demonstrably does not hold. For generative relational being the consequence is a reordering of questions: prior*

to asking how a culture or a norm evolves stands the question of whether a culture or a norm constitutes a level at which evolution in its own terms can be written.

The claim proceeds in stages. The next section states the closure problem in its technical form, and introduces the renormalization group as the second instrument for the passage across scales, so that what follows rests on real mechanisms rather than a loose analogy (§2). The third justifies the choice of turbulence and fences it: turbulence is one case among the many that might have been chosen, singled out for its maturity, and confined to the role of a calibration (§3). The fourth gives the exact content of the failure through the Mori–Zwanzig formalism, whose memory and noise terms are what a closed macroscopic equation must be shown to be free of (§4). The fifth carries both instruments into relational emergence as four cautions, three from the failure of closure and one from its success (§5). The sixth introduces the distinction between a closure of dynamics and a closure of generation, and marks the boundary of what is here established (§6). The seventh states the limit of the turbulence case—the step it cannot take, which is the step generative relational being most needs—and the caution that the renormalization group’s own limit on turbulence yields (§7). A closing section sets down what has and has not been shown, and the work the reflection leaves open.

2 THE CLOSURE PROBLEM

The closure problem is a definite object in statistical physics, and the caution drawn below depends on its being that rather than a figure of speech. This section states it in its least technical form.

Take a system whose complete state u evolves by known dynamics that are nonlinear. The incompressible Navier–Stokes equations (Landau & Lifshitz, 1987) are the standard instance:

$$\partial_t u_i + u_j \partial_j u_i = -\partial_i p + \nu \partial^2 u_i, \quad \partial_i u_i = 0,$$

in which the term $u_j \partial_j u_i$ is quadratic. Suppose that what one wishes to study is not the full field u but an averaged quantity—the mean flow $U = \langle u \rangle$, say, the average taken over an ensemble of realizations. Writing $u = U + u'$ with $\langle u' \rangle = 0$ and averaging the equation, one obtains an evolution equation for U that contains, through the quadratic term, a new unknown: the second moment $\langle u'_i u'_j \rangle$, the Reynolds stress. The dynamics of the first moment depend on a second moment.

One then seeks an equation for the second moment. It is formed by applying the quadratic dynamics to the product $u'_i u'_j$ and averaging; the quadratic term $u \partial u$ acting on a second-order product yields a third-order correlation, so the equation for the second moment contains a third moment $\langle u'_i u'_j u'_k \rangle$. The equation for the third moment, formed the same way, contains a fourth. Writing M_n for the n -th moment—itsself a multi-point tensor field, not a scalar—the

structure is a hierarchy

$$\partial_t M_n = \mathcal{F}(M_n, M_{n+1}),$$

in which the equation for each order introduces the order above it. The hierarchy does not terminate. Truncating it at a finite order N —retaining $M_1, \dots, M_N$ and their N evolution equations—leaves M_{N+1} as an unknown for which the truncation supplies no equation; the system is in this sense underdetermined, not through a mismatch of scalar counts but because the chain never closes upon itself.

The closure problem. *In the moment hierarchy generated by a nonlinear dynamics, the equation for the n -th moment depends on the $(n+1)$ -th; no finite truncation is self-contained. A closure is an externally supplied relation expressing the highest retained moment as a functional of lower ones, $M_{N+1} = \Phi(M_1, \dots, M_N)$, imposed to make the truncated system determinate. The relation Φ does not follow from the underlying dynamics; it is an added assumption, and the behavior of the truncated system inherits whatever the assumption puts in.*

The essential point for what follows is the standing of Φ . It is not derived; it is posited. The dynamics fix the hierarchy but do not fix where to cut it or how to fold the discarded orders back onto the retained ones. Every operative theory that works with a finite set of coarse variables—the eddy-viscosity models of engineering turbulence, the truncations of kinetic theory, the mean-field decouplings of statistical mechanics—is at bottom a choice of Φ , licensed by whatever combination of physical argument and empirical fit the modeler can muster. Where the choice is well licensed, the closure is reliable and the coarse description earns its autonomy. Where it is not, the autonomy is borrowed, and the coarse equation reports as much about the modeler’s assumption as about the system. The closure problem is not that closures cannot be found; it is that they must be supplied from outside, and that their supply is where the substantive commitments hide.

This structure is not peculiar to fluids. The same hierarchy appears in field theory, where the equation for an n -point correlation function involves the $(n+1)$ -point function, and in the kinetic theory of gases, where the equation for the one-particle distribution involves the two-particle distribution and so up the chain (Zwanzig, 2001). Perturbation theory, mean-field approximation, and the assumption of molecular chaos are, in these settings, the corresponding closures. The problem is generic to the passage from a nonlinear microscopic dynamics to a description in terms of a few averaged quantities. It is this generality that makes it worth carrying elsewhere; and it is turbulence that shows it in its least escapable form.

There is a second, and stronger, instrument for the passage across scales, and it must be introduced because it carries a caution of its own. Where the scales of a system are related not by a small parameter but by self-similarity, the systematic method is the renormalization

group (Goldenfeld, 1992): one coarse-grains—integrates out the shortest-wavelength degrees of freedom—and rescales, and studies the flow this induces on the space of possible descriptions. At a critical point the flow runs to a fixed point, and near that fixed point all but a few directions in the space of descriptions are *irrelevant*: they contract away under repeated coarse-graining, leaving a low-dimensional effective theory governed by the few relevant directions that remain. This is closure achieved not by an imposed Φ but by the dynamics of coarse-graining itself, which discards the microscopic detail for us. It is among the strongest available accounts of why a macroscopic level should have laws of its own, and it is exact where it holds.

The success has a price, and the price is the first of the two cautions this instrument yields. What the renormalization group establishes at a fixed point is that the surviving macroscopic law is reached by systems with different microscopic constitutions alike—the content of universality, that liquid–gas critical points, magnets, and binary alloys share critical exponents. Universality is the statement that the macroscopic behaviour is blind to almost everything about the microscopic: the irrelevant directions, which is to say most of the microscopic detail, have been flowed away and cannot be recovered from the macroscopic law. A successful reduction of this kind is therefore not a bridge from the macroscopic back to the microscopic but a wall: it certifies that the information needed to make the return journey has been destroyed. This bears directly on the study of macroscopic social order, and §5 states how.

3 THE CHOICE OF TURBULENCE, AND ITS LIMITS AS ONE CASE

Turbulence is chosen here for a specific reason and confined to a specific role. A physical system invoked in an argument about social order is not thereby claimed to be identical to one; the reason for the choice and the limits of the role are set out so that the invocation is read as what it is.

The reason for the choice is that turbulence presents the closure problem stripped of every feature that would make it tractable. In many systems the hierarchy can be truncated because a small parameter renders the higher orders negligible: a weak coupling, a small amplitude, a ratio of scales that is large or small. Where such a parameter exists, truncation is licensed by it—the discarded orders are demonstrably smaller than the retained ones, and the closure is an approximation with a controlled error. Where scales are related by self-similarity instead, the renormalization group of §2 is the instrument; but turbulence, as §7 sets out, is the case in which that instrument reaches only part way. Fully developed turbulence has no small parameter. At high Reynolds number the nonlinear term dominates rather than perturbs the linear one; the moments do not decrease with order in a way that would justify neglecting the higher ones; and energy is transferred across the inertial range through a scale-local cascade, with no internal gap at which that range might be severed. The features usually available

to rescue closure—a small parameter, an exploitable separation of scales within the range of interest, proximity to equilibrium—are the ones fully developed turbulence lacks.

This is what qualifies turbulence as a calibration. Throughout, the velocity field u inhabits one and the same state space—for a fluid, the function space of admissible velocity fields, which serves here as the system's space of states and does not itself change as the dynamics run. The difficulty is entirely that a low-dimensional projection of a dynamics on that fixed space fails to close, not that the space itself is in question. Turbulence therefore serves as a benchmark rather than a bound in any ordered sense: the point is not that no fixed-state-space system could present a harder closure problem—difficulty of this kind admits no evident total order, and the claim of a maximal element would require an argument not offered here—but that turbulence already exhibits the closure problem under the simultaneous loss of every feature that ordinarily rescues it, so that a phenomenon recalcitrant in just this way (strong coupling, no separation of scales, non-equilibrium) has an established physical counterpart and calls for no resource beyond what the study of turbulence already contains. What turbulence cannot calibrate is difficulty of a different kind, arising not on a fixed space but from the change of the space itself; that distinction is the subject of §7.

Claim 2. *Turbulence exhibits the closure problem under the simultaneous loss of small parameter, scale separation, and equilibrium, all while its state space stays fixed, and so serves as a calibration benchmark: a relational difficulty consisting only in the failure of a low-dimensional projection to close on a fixed space has an established physical counterpart and requires no new apparatus. A relational difficulty that turbulence cannot calibrate must therefore differ in kind, by a feature turbulence lacks—which, as §7 argues, is the evolution of the state space itself.*

The role to which turbulence is confined follows from this. It is one case, not the case. Other systems would expose other facets of the same problem: the development of an organism, in which the variables that matter are brought into being over the course of the process; the change of a language, in which the medium that constrains individual expression is itself remade by that expression; ecological succession, in which the community that selects for traits is assembled from the traits it selects. Each of these might have furnished the case, and each would show something the others do not. Turbulence is taken up first because its closure problem is the most fully worked—stated with a precision the others have not yet received. It is a first illustration, chosen for that maturity, and not privileged beyond it. Nothing in the framework requires that turbulence, rather than some other multi-scale system, be the model through which the problem is posed; the requirement is only that some such model serve, and this one is taken first.

4 THE EXACT FORM OF THE FAILURE

Two forms of the closure problem must be separated before the argument proceeds, since they are related in substance but distinct as objects, and the transfer of §5 depends on not confusing them. The first, of §2, is the truncation of a moment hierarchy: the closure is an algebraic constitutive relation Φ expressing a high moment through lower ones. The second, developed now, is the question whether the dynamics projected onto a coarse variable can be written in that variable alone—a closure not of a truncated hierarchy but of the reduced equation of motion. The two are two faces of one difficulty, but the objects differ: a truncation Φ is an algebraic relation among moments, whereas the obstruction to a closed reduced equation is an integral memory term, and no choice of Φ is the same object as that memory. Where the sequel speaks of an imposed closure it means whichever of the two the context fixes; the present section treats the second.

The closure problem as stated in §2 leaves the impression that the difficulty is one of bookkeeping—too many unknowns, an unclosed chain—and that a sufficiently clever truncation might dispatch it. The Mori–Zwanzig formalism (Mori, 1965; Zwanzig, 1961) removes this impression by giving the projected dynamics exactly, with no truncation and no approximation, and locating what a self-contained macroscopic equation would require in a term that is in general non-vanishing.

Let the microscopic dynamics be written $\dot{u} = \mathcal{L}u$, where $\mathcal{L}$ is the Liouville operator that generates the flow: acting on functions of the microstate it is linear, even though the underlying velocity dynamics are nonlinear, and this linear representation on the space of observables is what makes the construction below available. Let P be a projection that maps an observable onto the part expressible through the coarse variable one retains, and $Q = I - P$ the complementary projection onto what is discarded. Applying the Dyson identity to the propagator $e^{t\mathcal{L}}$ split by P and Q yields, for the coarse variable $X = Pu$, an exact equation of motion of the form

$$\dot{X}(t) = \underbrace{v(X(t))}_{\text{drift}} + \underbrace{\int_0^t K(t-s) X(s) ds}_{\text{memory}} + \underbrace{\eta(t)}_{\text{noise}}.$$

The drift $v(X) = P\mathcal{L}X$ is the instantaneous, memoryless part; the noise $\eta(t) = e^{tQ\mathcal{L}}Q\mathcal{L}X$ is the projected-out dynamics evolving under the orthogonal propagator $e^{tQ\mathcal{L}}$; and the memory kernel K is fixed by the autocorrelation of that noise, $K(t) \sim \langle \eta(t) \eta(0) \rangle$, the relation between friction and fluctuation that is the second fluctuation–dissipation theorem. Two qualifications attach to the display. First, the drift is in general a nonlinear function of X —a mean force—and reduces to a linear term ΩX only under the linear (Mori) projection onto X itself; the general (Zwanzig) projection retains the nonlinearity. Second, the memory term is written here in its linear-response form for legibility; in the nonlinear case the convolution is against a function of

the past coarse trajectory rather than against $X(s)$ directly. Subject to these, the decomposition is exact: it is the coarse dynamics, not a model of them. What the decomposition establishes is the standing of the closed equation. The clean form $\dot{X} = v(X)$, in which the future of the coarse variable is a function of its present value alone, is the special case in which the memory kernel and the noise are both negligible; where they are not, the coarse variable's future depends on its own history and on the degrees of freedom the coarse description discarded.

The condition under which the exception holds is a separation of time scales. If the discarded degrees of freedom relax much faster than the retained ones, the memory kernel collapses toward a delta function and the noise toward white noise; the convolution becomes instantaneous, the fluctuating force becomes memoryless, and one recovers a closed—if now dissipative and stochastic—Markovian equation in the coarse variable. This is the circumstance under which a macro-level has dynamics of its own: when the scales below it relax fast enough to be summarized by an effective friction and an effective noise. Where the fast and slow scales are not separated—where the coarse variable and the fine ones evolve on comparable time scales and remain strongly coupled—the memory does not decay and the noise does not whiten, and the reduction to a closed equation in the coarse variable is no longer available. Turbulence is the case in which this separation is absent within the range that matters: across the inertial range the energy cascade is scale-local and self-similar, with no internal spectral gap at which the coarse and fine motions could be cleanly divided.

Claim 3. *The self-contained macroscopic equation $dC/dt = v(C)$ is the special case of an exact projected dynamics in which memory and noise vanish, and it is licensed only by a separation of scales that lets the sub-macroscopic degrees of freedom relax quickly relative to the macroscopic ones. Absent that separation, the macro-level's evolution depends on its own history and on the degrees of freedom the coarse description omits, and no closed Markovian dynamics in the macro-variable alone is licensed by the standard reduction—which is not to prove that none exists, but to withdraw the ground on which its existence is ordinarily assumed.*

Two things follow that matter for the transfer to come. The first is that non-closure is not a defect of a bad choice of coarse variable that a better choice would cure; for a genuinely multi-scale system without a spectral gap, it is a feature of the passage to any low-dimensional description. The second is that history enters not as an ornament but as a term. When the memory kernel does not decay, the past of the macro-variable is part of the specification of its future, and two systems in the same present macroscopic state may have different futures because their pasts differ. What is called path dependence in other vocabularies is, in this one, a memory kernel that fails to collapse.

5 THE CAUTION, CARRIED TO RELATIONAL EMERGENCE

The transfer of the foregoing to the study of relational emergence is made now, and it is made as a caution rather than as a result. What the closure problem licenses is not a new theory of culture or norms but a set of questions that a claim about macroscopic social order must be able to answer, and a corresponding set of places at which such claims are prone to hide an unlicensed assumption. Four are set out; each takes the form of a question to put to a claim of the shape “the macro-level emerges from the micro-level and evolves in its own terms.” The first three concern the failure of closure and follow from §4; the fourth concerns the success of closure and follows from the renormalization group of §2, and cuts in the opposite direction.

The first concerns the smuggled closure. A common form of argument sets out rules for individuals, allows them to interact, and reports that a macroscopic regularity—a norm, a convention, a distribution—emerges; the macro-level is then treated as the statistical consequence of the micro-level, and its evolution studied in macroscopic terms. The closure problem locates the unstated step. Wherever the interaction among individuals is nonlinear and the relevant scales fail to separate—the conditions under which, as §4 showed, the projected dynamics carry a non-vanishing memory—the evolution of the norm level is not guaranteed to close in the norm alone: it depends on the higher-order relational information that the coarse description discarded. An account that nonetheless writes a closed dynamics for the norm has supplied a closure from outside the individual rules, whether as an algebraic decoupling of the higher-order relations or as the tacit assumption that the memory may be dropped; either way the substantive commitments are lodged in that supplied closure rather than derived. The question to put is accordingly: *where, in this account, is the closure, and what licenses it?* An account that cannot exhibit its closure has not dispensed with one; it has left it implicit.

The second concerns sufficiency of the macroscopic state. Where the memory kernel does not decay, the present macroscopic configuration is not a sufficient variable for its own future. Two societies with the same present distribution of a norm may diverge because their histories differ, and the divergence is not noise to be averaged away but a term in the exact dynamics. The question to put is: *is the present state of the macro-variable being treated as sufficient for its evolution, and if so, what warrants ignoring the memory?* A Markovian model of culture—current state plus transition rule—assumes a collapse of memory that, in a system whose scales are not separated, is exactly what cannot be assumed. History, in such a system, is not a prior condition that the present has absorbed; it is ontological, a standing part of the specification.

The third concerns the absence of scale separation itself. The license for a closed macro-dynamics was a gap between fast and slow scales. In social systems the relevant scales—individual, relational, group, institutional, cultural—do not evidently separate; they act on one another across comparable spans and remain mutually coupled, which is the condition under which no finite-dimensional coarse description closes. The question to put is: *does this*

system exhibit a separation of scales, and if not, on what ground is a closed macro-description expected to exist at all? The force of the question is that in the no-separation case the right expectation is the opposite of the usual one. One should expect the macroscopic description to be unstable, to require repeated repair, to fail to close—and a theory that reports a clean closed macro-dynamics for such a system should be read as having imposed one, not found one.

The fourth cuts the other way, and concerns not the failure of closure but its success. Suppose that a macroscopic social regularity does close—that a level of norm or convention settles into a description governed by a few parameters, stable under the coarsening of individual detail. The renormalization group shows what such a success would cost. A macroscopic law reached by coarse-graining to a fixed point is universal: it is the same law for microscopically different systems, because the flow to the fixed point has flowed away all but a few directions, and the microscopic detail that distinguishes one constituent system from another sits in the directions it discarded. Two consequences follow, and each is a caution. The first is against reduction: where a macroscopic regularity is genuinely of this kind, deriving it from a particular microscopic mechanism is not merely hard but foreclosed, since the regularity retains none of the information that would identify which mechanism produced it—a programme that would read individual rules off the macroscopic norm is asking the norm for what the flow to the fixed point has destroyed. The second is against the inference from likeness: two societies exhibiting the same macroscopic regularity are not thereby shown to share a microscopic mechanism, for universality is precisely the statement that unlike microscopics yield like macroscopics. The question to put is accordingly: *if this macroscopic regularity closes, is its closure being read as a bridge back to the mechanism that produced it, or as evidence of shared mechanism across cases—either of which universality forbids?* A successful macroscopic law is a wall, not a window, and an account that treats its success as license to reason back through it has mistaken the one for the other.

Claim 4. *For the study of how norms, culture, and institutions arise from individual relation, the two fixed-space instruments yield a methodological caution in four parts. From the failure of closure: that a closed macro-dynamics conceals an imposed closure—whether an algebraic decoupling of higher-order relations or a dropped memory—whose license must be exhibited; that where memory does not decay the present macro-state is an insufficient variable, and history enters as a term of the dynamics, not as background; and that where scales do not separate the default expectation should be non-closure, so that a reported clean macro-dynamics is evidence of an imposed assumption rather than of a discovered law. From the success of closure: that where a macroscopic regularity does close in the manner of a renormalization-group fixed point, its universality forbids both the derivation of the regularity from a particular microscopic mechanism and the inference of shared mechanism from shared macroscopic form.*

The caution supplies no way to close the social hierarchy; it withdraws the presumption that the hierarchy closes, and converts that presumption into a question with a determinate form. Its content is the reordering stated at the outset: before “how does the macro-level evolve” stands “does the macro-level evolve in its own terms, and if so by what license.”

6 FROM DYNAMICAL CLOSURE TO GENERATIVE CLOSURE

The caution of the previous section is stated in the vocabulary of the closure it borrows—a closure of dynamics, the existence or non-existence of $dC/dt = F(C)$. Generative relational being will require a second notion. This section introduces it, marks how it differs from the first, and states how far the present paper carries it.

The closure that Mori–Zwanzig characterizes is a closure of dynamics: the question whether the coarse variable has a self-contained equation of evolution, answered by whether memory and noise vanish. There is a second question that this one does not reach. A system may lack a closed dynamics in its coarse variable and yet sustain something across time—not a fixed macroscopic law but a persisting circuit of generation that does not break. That a structure can be maintained precisely by continual throughput rather than by rest is familiar from the study of systems held far from equilibrium (Prigogine, 1980). A community subject to continual turnover of members, resources, and information may hold no closed dynamics at the level of any aggregate variable and nonetheless remain a community, its constitutive relation regenerated as fast as it dissipates. What persists there is not a state and not a law over states; it is a generation that continues. The closure at issue is a closure of that generation—the condition under which a generative circuit sustains itself—and it is not the same as the closure of a dynamics.

Claim 5 (proposed). *I propose to distinguish generative closure from dynamical closure. A system may fail to admit a closed macroscopic dynamics—memory undecaying, noise uncolored, no reduction to an equation in the coarse variable alone—and yet, the proposal runs, exhibit a generative closure, a persistence of the circuit that generates its constitutive relation rather than of any state or law over states. The proposal is not yet a definition: the predicate that would make it one, and would keep it from collapsing into the statement that a persisting system persists, is the invariant identified as open below. It is entered here as the distinction the framework requires, pending that invariant.*

To assert that generative closure is a distinct and real condition is not yet to have said what it is. Stated only through instances—the community that persists, the institution that renews, the practice that regenerates—the notion is exposed to a circularity that would empty it: if a generative closure is nothing but the fact that a system continues to generate without breaking, then to say a system has generative closure is only to say it has not yet broken, and the notion

predicts nothing it was not given. Turbulence sharpens the danger: it too persists without a closed dynamics, so if generative closure cannot be told apart from what turbulence already displays, it has no content beyond the closure problem it was meant to exceed.

What would give it content is an invariant. In turbulence the persistence of the cascade is anchored by a quantity that stays constant through it: the mean rate of energy dissipation, equal across the inertial range to the flux of energy through it (Kolmogorov, 1941; Frisch, 1995). That flux is constant because the inertial range, by definition, neither receives energy directly from the forcing nor loses it directly to viscosity, so whatever energy enters one end must pass to the next scale rather than accumulate; the constancy follows from the conservation of energy through a range with no source or sink of its own. Two features of this quantity are what a candidate for relational use must reproduce. It is defined by constancy along a definite dimension—across scale, not across time—so that the invariant is a statement about how a quantity behaves as one passes through the levels of the cascade. And it is a flux, a globally averaged rate of passage, which is why it survives the intermittency that breaks the naive self-similarity of the field (Frisch, 1995): the higher-order statistics fluctuate strongly from place to place while the mean flux does not. An *invariant of generation* would need both features—a quantity stationary along a specified dimension of the generative circuit, and a rate of passage robust to the local fluctuation of what it aggregates—in virtue of which the circuit is a sustained thing one can point to. The difficulty is that a social or relational system affords no conservation law to underwrite such a quantity. Whatever might play the role energy dissipation plays cannot be delivered by a conservation principle, and would have to be grounded, if at all, in something proper to relation—in what the parent framework terms a relational grammar, the standing constraints on how relations may combine and pass into one another, rather than in a conserved substance.

This paper does not supply that invariant. It identifies the need for it, locates the circularity its absence leaves open, and fixes three conditions on any candidate: it must be stationary along a specified dimension of the generative circuit, on the model of the flux constant across scale; it must not be underwritten by a conservation law, since relation affords none; and it must not reduce to the statement that a persisting system persists. A physical precedent for an invariant meeting the first two conditions—stationary across scale, yet fixed by global rather than local-conservation considerations—appears in the zero modes that govern anomalous scaling in turbulence, discussed in §7. Within these constraints the notion of generative closure has room to become a definite claim; within the present reflection it remains a named problem rather than a solved one.

7 THE LIMIT OF THE TURBULENCE CASE

A case leaned on owes an account of what it cannot reach, the more so here because what turbulence cannot reach is the thing generative relational being was framed to address. This section states that limit, and states alongside it a caution that the limit of the renormalization group on turbulence yields in its own right.

Through every difficulty rehearsed above, turbulence retains one feature: its state space is fixed. The velocity field lives in a space of admissible fields that does not itself change as the dynamics proceed; the projection P onto coarse variables is a fixed operator on that fixed space; the Mori–Zwanzig equation can be written because the space on which it is written does not move. Turbulence is, in this sense, a problem posed on a fixed state space. Every resource it lacks—small parameter, scale separation, equilibrium—is a resource internal to the analysis of a fixed space. The space of states itself is never in question.

The renormalization group meets its limit on turbulence at the same fixed-space boundary, and the manner of its failure carries a caution of its own. Turbulence is, on its face, the ideal object for the renormalization group: it is scale-related, it cascades, and its energy spectrum follows a power law that looks like a scaling law inviting a fixed-point treatment. The instrument does capture the scale-invariant, dimensional part of the behaviour—the Kolmogorov spectrum—but reaches only part way, for the part of turbulence that resists it is the part that matters. The higher-order structure functions do not follow the exponents dimensional analysis predicts; they scale anomalously, and this anomaly, the signature of intermittency, is not what a local fixed point delivers. Its source, in the cases where it has been computed, is not the local self-similarity a fixed point encodes but a set of *zero modes*: quantities invisible to dimensional analysis, fixed not within the inertial range but by the large scales and boundaries that feed it (Falkovich et al., 2001). The caution this yields for the study of social order is direct. A system may display every scaling signature that invites a universal, fixed-point account—the power laws, the scale-free networks, the self-similar cascades of influence that social systems present in abundance—and still possess no clean closed macroscopic theory, because scale invariance in the observable statistics does not entail a fixed point in the dynamics. Turbulence is the standing counterexample: it holds a textbook scaling law together with a stubborn non-closure. To read a power law in a social system as a warrant for a universal macroscopic theory is to make the inference turbulence refutes.

The residue of the renormalization group’s failure bears on the invariant of §6. The zero modes are quantities held fixed not by a local conservation law but by the global conditions under which the cascade is driven—a physical instance of the invariant that account requires: one stationary across the levels of a generative process, not underwritten by a conservation of substance, and decisive for the high-order behaviour that local self-similarity misses. What the renormalization group leaves unresolved in turbulence is, in this respect, a closer model

for generative relational being than what it resolves.

The distinctive claim of generative relational being is that the state space itself is in question. Where relation generates being, the space of possible states is not given in advance and held fixed while dynamics unfold upon it; it is enlarged and reshaped by the relations it hosts, so that which states are available at a later time is a function of what was generated earlier. In the notation of the framework this is the difference between writing $\dot{x} = F(x, R)$, which adds a relational variable R to a fixed space and remains within the reach of a standard, if enlarged, dynamics, and writing an evolution of the space of states itself, $\Omega_{t+1} = H(\Omega_t, R_t)$, in which the set of available states at time $t+1$ is generated from the set at time t together with the relations R_t then obtaining. The first is the turbulence class: a larger but fixed state space, on which the Mori–Zwanzig construction still applies and closure is the familiar problem. The second lies outside it: when the state space itself evolves, the projection P has no fixed space to act on, and the Mori–Zwanzig derivation loses the fixed space and fixed inner product it assumes. Whether an analogous closure problem can even be posed in this second case is not settled here; that it cannot be posed in the form turbulence gives it is what the failure of the construction shows, and the positive question is left open to §8.

Claim 6. *Turbulence exhibits the closure problem of a fixed state space and, being confined to a fixed state space, can exhibit no more than that. The evolution of the state space itself—the generation of new dimensions of possibility by the relations a system hosts—lies outside the fixed-space setting the turbulence case occupies, and so outside what that case can calibrate. Turbulence therefore calibrates generative relational being without grounding it: it shows what a difficulty on a fixed space is when that difficulty is at its most severe, and thereby marks, by contrast, where the framework’s distinctive difficulty must be sought—in the evolution of the state space. Whether a difficulty so located is genuinely irreducible to a dynamics on some fixed enlarged space is not settled by this contrast; it is the open question recorded in §8. Two further points attach to the same limit: that the renormalization group’s failure on turbulence shows a scaling law to be no warrant for a universal macroscopic theory, and that the boundary-fed zero modes in which that failure resides are the nearest physical model for the invariant of generation §6 requires.*

For this reason the turbulence case cannot be the foundation the framework needs, and the open problem of §6 cannot be discharged by refining the turbulence analogy. An invariant of generation, if it exists, must be an invariant of a process that remakes its own state space; turbulence, whose state space never changes, supplies the idea of an invariant but not an invariant of the required kind. The case gives the framework its calibration and its vocabulary—memory, cascade, sustained flux—but not the step past the fixed state space. That step begins where a harder dynamics on a fixed space, which is what turbulence exemplifies, leaves off: in the evolution of the space itself.

8 THE ESTABLISHED AND THE OPEN

This section separates what the foregoing establishes from what it leaves standing.

What is established is a reordering of questions. The passage from microscopic relation to macroscopic order does not confer on the macro-level a closed dynamics of its own; whether such a dynamics exists is a condition licensed, when it is licensed, by a separation of scales that lets the sub-macroscopic degrees of freedom be summarized as friction and noise. Where that separation is absent—as it is in turbulence, and as it appears to be across the relational scales of social life—the macroscopic description does not close, its history enters as a term rather than a background, and a reported clean macro-dynamics is to be read as an imposed closure rather than a discovered law. For the study of culture, norms, and institutions the consequence is the caution of §5: before asking how a macro-level evolves, one must ask whether it constitutes a level at which evolution in its own terms can be written, and by what license. The renormalization group adds two edges to the same caution: where a macroscopic regularity does close as a fixed point, its universality forbids reading it back to a mechanism or across cases; and where a scaling law is present without a fixed point, as in the anomalous scaling of turbulence, the scaling law is no evidence that a closed macroscopic theory exists. This much the two fixed-space instruments support, and this much the paper claims.

What is not established is generative closure. It has been distinguished from dynamical closure and shown to be the condition the framework will require—a persistence of generation rather than of state—but it has not been given content sufficient to keep it from collapsing into the statement that a system which persists has persisted. What would give it content, an invariant of generation not underwritten by any conservation law and not reducible to the fact of persistence, has been named as a requirement and left unmet. This limit follows from the case chosen: turbulence, confined by its fixed state space to the region the framework must exceed, cannot furnish the invariant of a process that remakes the space of its own states. The turbulence case calibrates the problem and supplies its language; it does not supply that invariant.

Two lines of work follow, and the paper leaves both open. The first is the invariant: to determine whether a quantity conserved or held stationary across a generative circuit can be defined in relational terms—grounded in a relational grammar rather than a conserved substance—and thereby to convert generative closure from a named distinction into an operational condition. The second is the evolving state space: to give $\Omega_{t+1} = H(\Omega_t, R_t)$ a form definite enough to be reasoned about, and in particular to settle whether such an evolution can be shown irreducible to a dynamics on any fixed enlarged space—for only if it can is the framework's distinctive difficulty a real one rather than a fixed problem in disguise. Both lie ahead of the present paper, which goes only as far as the two open questions and the reordering that motivates them: it makes the assumption of macro-level autonomy visible as an assumption,

gives through a physical case the condition under which the assumption holds and fails, and fixes what a fixed-state-space case can and cannot carry over to a framework whose subject is a changing state space.

A note on method. The moment hierarchy and the Mori–Zwanzig decomposition are used as the theorems they are; where the argument passes to relational emergence, no equation is carried across, only the structure of the question. No invariant of generation is defined here: the notion is left as the open problem of §6.

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