

Generative Interface Substitution

A Preliminary Study Of Linguistic Continuation Conditions

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Abstract

A replacement can reproduce a linguistic practice's familiar outputs while altering the continuations available to its participants. This preliminary study examines substitution of a supporting region relative to declared interfaces, surrounding contexts and selected continuation tasks. The Generative Relational framework supplies history-bearing configurations, partial operations and changing realization carriers; its prior preservation and field constructions are inherited explicitly. Philosophical analysis distinguishes functional contribution, historical identity, participant choice and justified replacement. A finite interaction model gives a sufficient fixed-translator condition for bounded policy transfer and a counterexample to successful-trace matching. A nonlinear regional model derives a matrix memory kernel and a separate initial response. Exact response equality permits different internal realizations, whereas aggregate equality can conceal local support redistribution that extinguishes a selected activity. Further results bound ordered and approximate substitution. History-operator intertwining, compression defects and observable-algebra closure expose separate algebraic conditions. Primary interaction studies constrain the interpretation without being presented as replacement trials. The resulting criteria are conditional models of a bounded problem; practical handover, evidence, authority and open-ended linguistic formation remain additional responsibilities.

Keywords: linguistic continuation; support substitution; historical interfaces; contextual refinement; memory fields; operator algebra

Scope Note

This preliminary study examines a selected linguistic practice whose supporting arrangement is replaced. Its question concerns the continuation capacities that remain available to participants after the replacement, relative to declared surrounding contexts. An interface is the specified locus of interaction between a support region and the rest of the represented practice. The title's use of generative refers to the possibilities of further situated activity sustained through that interaction. It does not identify generativity with output novelty or with a single numerical property.

The Generative Relational research programme supplies a revisable language for history-bearing configurations, manifestations, partial operations, regional fields and changes of representational carrier. Its existing preservation taxonomy already treats capacity-condition coupling and functional substitution. The present inquiry specializes that inheritance through three possible models: contextual strategy transfer in finite interaction systems, memory-mediated coupling in nonlinear regional fields, and domain-sensitive transport in a history-operator representation. Their distinct assumptions remain visible. They are neither a universal mathematical theory of language nor interchangeable descriptions of one empirically established object.

Conditional proofs, constructed counterexamples and illustrative calculations are distinguished from public-source findings and philosophical arguments. The manuscript also separates preservation of a selected function from historical identity, acceptable burden and the authority to replace a supporting arrangement. Its detailed exposition is intended for discussion and further revision. The source packet retains actual reading scopes, source verification, reproducible calculations and the checks on the compiled document.

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Status. This discussion draft presents an evolving inquiry. The definitions, translations, formal constructions, empirical interpretations, and conclusions remain open to criticism and author review. It investigates the preservation of selected linguistic continuation capacities when supporting arrangements are replaced. Its formal models are conditional representations; the interface criteria, memory fields and operator constructions do not establish universal substitutability or an empirically calibrated preservation rule.

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Statement On The Use Of Language Models. The author selected the research direction, supplied prior manuscripts and a reference exposition, and requested consistency with the Generative Relational research programme. AI-assisted research, source comparison, drafting, formal checking, and document preparation contributed substantially to this draft. These contributions include proposed arguments and interpretations that remain subject to the author's review. Preparation of a checked draft does not establish the author's approval of every claim. Authorship credit remains with the human author; the use of these tools is disclosed to support critical assessment of the manuscript and its limitations.

1 Linguistic Support Replacement

A practice can retain its familiar expressions while the arrangement that supports their use changes substantially. Participants might move from shared annotated materials to a searchable archive, from one correction arrangement to another, or from a locally maintained support channel to a distributed service. Each replacement can reproduce examples that were previously available. The more demanding question concerns what participants can subsequently do: request clarification, develop an unfamiliar continuation, correct a misleading account, or sustain activity after an initial demonstration. This paper investigates the conditions of that more limited and practical preservation claim.

Consider a constructed narrative workshop. Its participants use a collection of examples, an annotation system and an arrangement for returning disputed interpretations to further discussion. A proposed replacement supplies the same prepared narratives and recognizes the same initial requests. One version also retains an effective route for revising an account. Another silently assigns each request to a fixed response before the participant has chosen which issue needs clarification. The initial display can be identical while the continuation possibilities differ. The example is hypothetical; its role is to identify a distinction that a replacement assessment would need to test.

The primary analytical object is a *context-relative support substitution*. A support region of a historically formed practice is replaced by a different arrangement, and selected participant-executable continuations are compared across the resulting configurations. This object includes more than the replaced component. It includes its attachments, the surrounding contexts admitted by the comparison, the participants' accessible information, their available operations, and a justified account of successful continuation. The inquiry remains bounded because none of these selections is claimed to exhaust the language or the community's concerns.

1.1 Preservation Commitments

The existing preservation taxonomy distinguishes retained records, historical witnesses, capacities, conditions and their coupling [8]. It already makes room for changed realizations and functional condition substitution. The field-theoretic power manuscript develops historical anchors, regional realization, inter-algebra transport and correspondences between operation families on changed carriers [7]. These are inherited resources. Repeating that supports can change while a selected function persists would supply little reason for an independent paper.

The additional investigation concerns the consequence of placing a candidate replacement back into a surrounding practice. An arrangement can pass a standalone comparison while failing after a participant chooses an action that the original arrangement supported. It can also have the same aggregate response while redistributing local support in a way that prevents a selected activity from continuing. The comparison therefore requires an account of composition: which interactions are represented at the interface, which choices remain available, and which consequences follow when those choices feed back into the supporting region.

The metamodel already distinguishes present observations from descriptions that remain sufficient through selected continuations [6]. It also inherits the earlier distinction between a successful path and a policy that participants can select from their actual information [10]. The present paper changes the comparison object. It compares alternative support realizations and asks when a selected family of usable policies survives their exchange. It does not claim to originate dynamic equivalence, partial-domain analysis or controller refinement.

1.2 Methodological Architecture

The philosophical argument begins with historical individuation and situated capacity. A boundary is treated as a declared analytical interface rather than an automatically given edge of the practice. The formal investigations then offer three different ways of making the replacement claim inspectable. They share an interpretive problem while retaining their own mathematical assumptions.

The first investigation represents a history-bearing configuration through a labelled two-complex and attaches alternative future support regions to a retained context. A finite open interaction model then specifies partial operations and possible outcomes. A directed replacement relation is shown to preserve bounded winning policies across contexts that interact through the declared ports. The result is a sufficient condition within a restricted model class. Its proof exposes why a translator that depends on hidden analyst knowledge would be inadequate.

The second investigation gives the support region an internal field realization. Regional activity generates several support channels, which decay and diffuse before returning their effects to the activity. Eliminating those channels produces a matrix-valued memory kernel and a separate response carrying their initial history. This yields an exact replacement criterion for a specified feedback class. A two-site counterexample shows how aggregate response equality can conceal a local loss of sustained activity.

The third investigation studies a deterministic history-operator sector. An embedding that transports operation domains and effects supports word transport. Generic operator compression has a precisely displayed multiplicativity defect, and a boundary-observable algebra need not remain closed under the field dynamics. These calculations identify additional assumptions needed by algebraic claims of preserved operation. They do not turn a Hilbert-space embedding into evidence of semantic identity.

1.3 Contribution Boundaries

The paper's contribution is a scoped comparison architecture and a collection of connected conditional investigations. Its strongest mathematical statements are consequences of explicit assumptions. Their mathematical methods have established antecedents, reviewed in Section 2. The work offered here lies in aligning those methods with a historical linguistic substitution problem, developing a regional memory example, and identifying which practical and philosophical claims remain unsupported by each result.

The scope does not include a general measure of generativity, a theory of linguistic identity, or a universal specification for institutional replacement. The field variables describe selected activity and support channels. The finite operations represent a declared repertoire. The histories retain selected provenance rather than every feature of a life. These limits make the representations criticizable: a missing interaction, an inaccessible choice or an unjustified target can challenge a proposed application.

The remaining sections develop that criticism constructively. The literature review situates the problem; the philosophical sections establish the object and its scope; the formal sections derive the comparison criteria and their failures. Published interaction studies and a separately labelled constructed dossier then test the interpretive obligations. The concluding discussion returns to the difference between a warranted conditional substitution and the wider judgment that a replacement should occur.

2 Support Substitution Research

The problem of replacing a linguistic support crosses several established inquiries without coinciding with any one of them. Studies of situated interaction help identify what a support contributes to an activity. Refinement theories specify conditions under which choices and guarantees survive a change of implementation. Compositional dynamics provides methods for connecting systems and separating internal realization from selected boundary behavior. The present section compares these contributions by the problem each resolves. Its purpose is to identify the inherited methods and the additional interpretation required before applying them to linguistic continuation. The empirical sources supply documented distinctions; the mathematical sources supply conditional constructions. Neither kind of source, by itself, certifies a proposed replacement of a living practice.

2.1 Situated Operation Identification

Goodwin's analysis of situated interaction treats talk, bodily orientation, and material graphic resources as jointly relevant to action. His archaeological episode shows participants using a color chart in an

ongoing classification activity. Understanding their English utterances alone does not explain the work accomplished through the chart and the surrounding practice [5, pp. 1505–1511].

For the current inquiry, the initial challenge is therefore to identify an operation at a resolution appropriate to substitution. Consider an analyst who describes a support as something that supplies a color word. That description admits a lookup list as a possible replacement. A second analyst describes the support as something through which participants compare a sample, indicate alternatives, challenge a selection, and record an agreed classification. The second description imposes a different continuation test. Neither description is automatically correct. Their adequacy depends on the declared activity and on evidence about what participants can do with the support. The difference cannot be settled merely by counting the words that both arrangements produce.

This makes operation identification a substantive part of modeling. An action called *compare* needs a domain: the relevant materials must be available and distinguishable to the participant performing the comparison. An action called *correct* needs a consequential route: issuing a new label must be able to alter the operative classification when the target requires such alteration. A model that includes these action names while omitting their practical conditions has specified a vocabulary rather than established a capacity. The formal sections consequently retain partial domains and separate an analyst's description from a participant's executable repertoire.

Bloch and Wilkinson examine selected conversations involving acquired dysarthria. Their analyses locate understandability problems and their resolution in sequences involving both speakers. The recipient's repair initiation helps specify what the next repair must address. Their four analyzed extracts come from two dyads; communication-aid extracts were excluded [3, pp. 512–513, 515–516, 521].

This comparison directs our attention to the changing problem that an operation addresses. A correction made after a broad request for repetition is not necessarily interchangeable with the same correction made after a specific misunderstanding has been identified. In a constructed replacement model, those histories can place the participants in different operation domains even if their most recent utterance is identical. A support that replays a prior expression may serve one domain and fail in the other. This is one reason to retain history in the represented configuration. History is not added as a decorative record of everything that happened; the model needs those earlier distinctions on which later availability or interpretation depends.

The situated approach also places a limit on the present formal translation. An observed repair sequence supplies evidence about actual action and uptake. It does not enumerate the complete successor set of a finite interaction model. A formal universal guarantee therefore requires additional justification for the alternatives admitted by that model. The appropriate relation is one of disciplined construction: empirical analysis helps constrain the action distinctions, while the theorem makes the consequences of a proposed abstraction explicit.

2.2 Temporal Repair Opportunities

Kendrick's investigation distinguishes the timing of several repair practices. Its discussion separates a delay's public consequences from possible personal motives and examines visible signals preceding verbal repair. Findings from a small other-correction sample complicate a uniform account of delayed repair initiation [12, Methods; Discussion].

The current substitution question is consequently not exhausted by asking whether a replacement transmits the same eventual sentence. Suppose a constructed interface immediately supplies a completed interpretation whenever the speaker pauses. A second interface permits the speaker to hold the floor while a partner displays uncertainty. If the task is only to deliver a specified sentence, the first arrangement may be adequate. If the task includes revising an interpretation before it becomes operative, the same immediate completion can remove a relevant opportunity. Which evaluation applies depends on the selected continuation, not on an unqualified ranking of speed.

Timing can enter a model in several ways. It can be recorded explicitly in the observation, represented by intermediate states, or included in a cost or deadline. Each choice preserves a different aspect of the episode. A single time penalty cannot substitute for an account of what becomes possible during

an interval. Two delays of equal duration can differ because one permits a challenge while the other requires passive waiting. Conversely, two different durations can preserve the same bounded operation if the surrounding practice accommodates both. The formal choice should follow this practical distinction rather than impose a universal measure of conversational efficiency.

These considerations do not require an infinitely detailed temporal model. They require an explicit decision about the distinctions relevant to the claim. A replacement theorem for an untimed action model remains useful when the application justifies disregarding timing. Where timing changes the domain of a repair operation, that theorem needs a richer model before it addresses the intended question. Section 12 develops this distinction through a constructed support comparison whose assumptions are separate from the documented interactional findings.

2.3 Controlled Continuation Comparisons

Alur, Henzinger, Kupferman, and Vardi develop alternating refinement for systems whose transitions depend on several agents. Their simulation condition distinguishes a coalition's selected move from the possible responses of other agents. Alternating quantifiers preserve that distinction when matching successors; strategies are defined on finite histories [1, Sections 2–3].

This is established mathematical territory for the present policy-transfer investigation. Our question is narrower than the development of a general refinement theory. Once a linguistic support has been represented by a declared action and observation structure, we ask which replacement conditions preserve a specified family of executable continuations. The order of choice matters. A participant must select an action before an uncontrolled response is known. A relation that selects the matching action only after seeing that response may match completed stories while failing to supply a usable method for generating them.

There is also a distinction between a proof witness and an instruction. A proof can associate each replacement path with some reference path. The participant need not know that reference path if the actual instruction depends only on the accessible history. By contrast, a proposed strategy that branches on a hidden feature available only to the analyst does not become executable because each branch individually succeeds. The current model therefore uses a fixed public action translator and policies defined on observation histories. These restrictions make its contribution inspectable while leaving more general adaptive translations open.

De Alfaro and Henzinger's interface automata distinguish input assumptions from output guarantees. Compatibility is optimistic: a suitable environment can enable components to work together. Refinement relaxes input assumptions and strengthens output guarantees. Their compositional replacement theorem includes a condition on the inputs connected to the environment [4, pp. 109–110, 118].

The relevant lesson for our model is to state its context class before announcing a replacement guarantee. An environment that always accepts the first answer may make two supports indistinguishable. An environment that requests clarification can distinguish them. Expanding the context class thus changes the question, even when the internal supports remain fixed. The present paper does not identify success in one accommodating context with preservation across all surrounding practices.

A linguistic context also contains participants whose possible responses have their own significance. Treating every response as uncontrolled nondeterminism can be useful for a worst-case task guarantee. It can obscure the difference between environmental disturbance and another person's exercise of a protected choice. A refinement that removes uncertain outcomes may improve the first guarantee while narrowing the second person's opportunities. The appropriate remedy is to state whose operations are selected for preservation and how their exercise is represented. No formal ordering can choose that analytical commitment without the application's substantive argument.

2.4 Compositional Dynamical Descriptions

Vagner, Spivak, and Lerman specify open dynamical systems through internal state, input, and readout data, and construct an algebra of their wiring diagrams. The composition result makes the resulting

dynamics coherent with the declared wiring. Their formulation also distinguishes the inclusion of initial conditions as additional structure [13, Definition 2.13; Definition 4.2; Remark 4.4; Proposition 4.5].

This provides a disciplined alternative to using a diagram merely to illustrate connectedness. A composition law must say what is attached to what, which variables are shared, and how one component's response enters another's evolution. Such a law has explanatory value even before a preservation theorem is available. It exposes circumstances in which the same internal component behaves differently after a change of attachment. The current historical construction similarly distinguishes a support region from the interface through which it contributes to a surrounding formation.

Composition and substitution nevertheless remain different problems. Coherently composing a replacement does not establish that it performs the selected role of the earlier support. Two well-defined composite systems can have very different continuations. Conversely, two systems can preserve one selected boundary response while having different internal decompositions. A useful comparison must name the property retained across composition rather than infer preservation from the mere existence of a well-typed diagram.

Baez and Pollard develop compositional descriptions of open reaction networks and associated dynamics. Their black-boxing construction records the relation among boundary concentrations and flows at steady state. Its functoriality concerns that selected steady-state semantics [2, Definitions 19–21; Theorem 23].

Our comparison must therefore distinguish the boundary feature being retained. Equality of eventual response levels does not, by itself, settle whether a support reacts quickly enough for a bounded operation or whether its initial condition permits the same next step. The memory-sensitive field example in Section 9 investigates a stronger temporal question in its own restricted setting. This is not a criticism of steady-state semantics. Selecting steady behavior is a legitimate abstraction when the inquiry concerns steady behavior. The error would be to use that abstraction as a certificate for properties it deliberately leaves unspecified.

Nor does compositional resemblance identify chemical and linguistic objects. A reaction-network species, a conversational participant, and an institutional support do not acquire common empirical properties by being placed in diagrams with similar arrows. In the current application, the relevant bridge is an explicit assignment of observed or hypothesized support processes to model variables, accompanied by an account of what the projection omits. The framework can organize this bridge without supplying the missing empirical identification.

2.5 Boundary Response Realizations

Kalman's state-space investigation distinguishes internal dynamical descriptions from their response realization. The convolution representation discussed there concerns a system initially at rest; the general state solution separately carries its initial condition. Response realization is therefore a classical comparison problem, not a new consequence of the present framework [11, pp. 154–159].

This distinction matters for a support that stores effects of earlier activity. A present output can be the same even when the internal organization retains different dispositions to respond to a future input. The proposed field investigation asks which selected aspects of that disposition must agree for replacement in a declared surrounding dynamics. Its linear support channels are a tractable specialization. Their role is to permit an explicit derivation of memory at the boundary, while the surrounding activity can have nonlinear dynamics. They are not offered as a law equating linguistic capacity with a linear state variable.

There are two distinct restrictions on the resulting claim. First, the surrounding system must interact through the boundary variables that the comparison retains. If it also responds to an omitted internal feature, equality at the chosen boundary does not settle its behavior. Second, the replacement must be initialized in a configuration satisfying the stated comparison. A response certificate for a prepared component cannot establish that a practical handover can prepare it. Reproducing a future response and acquiring the ability to reproduce it are separate tasks.

This separation gives a constructive role to internal diversity. A replacement need not copy every internal component if the selected continuation depends on a less detailed response organization. Yet the

claim remains conditional on that organization and its context. It does not follow that the internal history has no value, that all targets admit the same substitution, or that a practically unavailable initialization should be ignored. The representational reduction is useful precisely when those boundaries are retained.

2.6 Admissible Test Executions

Van Glabbeek’s investigation of must-testing makes the completeness criterion explicit: the test must specify which executions count as complete. Replacing a progress assumption with justness changes the resulting comparison. In this setting, justness is a technical condition on executions, not an ethical characterization of a social relation [14, Sections 1, 4, 8].

The present bounded model avoids much of the machinery of infinite executions, but it still needs an explicit account of stopping and continuing. A claimed guarantee can become artificially easy if every unsuccessful interaction is classified as unfinished and excluded from assessment. It can become impossibly demanding if the environment is allowed indefinitely to prevent an action that the intended context would permit. Neither choice is dictated by the word *continuation*. The finite-horizon criterion states an available-action requirement, an explicit stop decision, and a target at stopping.

The corresponding empirical question concerns the reach of the test. A successful demonstration is one admitted execution. It can establish that a particular continuation occurred, subject to the quality of its record, without establishing a guarantee for every modeled response. Conversely, a counterexample can defeat a universal claim without showing that the replacement is generally useless. The strength of the conclusion must follow the quantifier in the preservation specification. This avoids turning either isolated success or isolated failure into a total judgment of a support.

The literature comparison thus locates the current investigation within established work rather than beyond it. The prior Generative Relational taxonomy already permits multiple realization, partial substitution, and functional comparison of generative conditions [8]. The present paper develops a selected contextual comparison with explicit domains, accessible policies, historical attachment, and temporal support response. Its formal statements are conditional results in declared models. Its linguistic cases test how those declarations should be interpreted. The contribution sought is a more accountable analysis of a particular preservation problem, with empirical certification remaining a further task.

3 Generative Interface Objects

A support replacement changes the relations through which a practice can continue. Identifying the replaced object therefore precedes comparison of its outputs. This section specifies a support region, its interface and its historical bearer, then examines how different realizations can sustain a selected contribution. The argument inherits the preservation taxonomy’s distinction between internal organization, relational position, capacity and conditions [8, Sections 6–8]. Its present purpose is to establish what a contextual substitution claim concerns before the subsequent constructions provide sufficient conditions for one.

3.1 The Selected Support Region

A support region is an analytically identified part of a larger historical formation whose organization contributes to a declared continuation capacity. The region can include materials, access procedures, communication routes, arrangements of work and their history. Its boundary follows the inquiry’s selected dependencies. A room or database can provide a convenient location without exhausting the support region, while a geographically dispersed arrangement can form one relevant region.

Consider a constructed narrative workshop. Participants use a distinction between an interrupted attempt and a completed event, extend it to unfamiliar accounts and correct misleading descriptions. A support arrangement connects annotated examples, opportunities to ask for clarification and a procedure for circulating accepted corrections. Replacing that arrangement could involve another medium,

a reorganized collection or a different division of assistance. The selected object is the arrangement's contribution to those operations. It is not an individual participant considered independently of their purposes, commitments and relations.

This choice of boundary has substantive consequences. If correction depends on a contributor who can explain an annotation, retaining only the annotation file omits a relevant connection. If participants already understand the convention and can correct one another, the same contributor may be unnecessary for the narrower operation. Both descriptions can be defensible in different inquiries. The dependency should be established for the retained task rather than assigned from an institution's official organizational chart.

The taxonomy treats a support family as a declaration of regions whose modification can affect a selected operation. It explicitly separates that declaration from individual necessity or a uniquely minimal support set [8, Section 8]. A substitution study inherits this restraint. Finding one effective support arrangement establishes an enabling route. It leaves open whether another route exists, whether several supports are redundant, and whether the proposed boundary omits conditions supplied elsewhere.

3.2 Relational Attachments

An interface is the selected organization of relations through which the support region interacts with its surroundings. It includes what can enter, what becomes available in response, and which operations remain possible at successive stages. A technical representation may use ports and labels. Their substantive meaning comes from the activities, distinctions and access conditions they represent.

In the workshop, a request to reconsider an account differs from a request to replay an example. Both might produce another sentence, but their place in the activity differs. Reconsideration can require preserving the contested interpretation, identifying the reason for challenge and allowing a response that changes the shared account. An interface that retains only a text-input and text-output label may omit those distinctions even when its physical connections are intact.

The interface also locates agency. A participant can initiate a question, reject an offered interpretation or defer a response. A support arrangement can make one of these moves available while another restricts it to a coordinator. Matching the vocabulary of the moves does not establish that the same actors can use them. The comparison must retain the relevant allocation of initiative when that allocation belongs to the selected capacity.

The field-theoretic source distinguishes transport comparing local representations from an operation changing the represented possibilities [7, pp. 34–35]. This distinction applies directly to substitution. A correspondence between request labels establishes how descriptions are compared. A working interface additionally requires an available act that implements the correspondence. Translating a request into an internal code does not show that the replacement can respond to it, or that the participant can use the response.

3.3 Constitutive Support

The word support can suggest an external accessory added to an otherwise complete capacity. Some linguistic operations have a different structure. A joint correction can consist partly in the participants' responsive relation: one account is challenged, a reason becomes intelligible and an amended account becomes available for further use. The contribution of the surrounding arrangement then enters the operation being identified, rather than merely increasing the rate at which an independently specified act occurs.

This observation does not make every surrounding condition constitutive. Electricity may enable a particular recording session without belonging to the interpretation of the corrected account. A standing invitation to challenge an account can play a different role. The inquiry should distinguish a condition that allows the episode to occur, a relation partly organizing the episode, and evidence by which the episode is recognized. These roles can overlap, but their overlap requires an argument.

Changing a constitutive relation can still preserve a selected capacity at another level. Participants might move from marginal annotations passed around a table to an accessible shared display, while retaining the ability to identify the contested passage and contribute to its revision. The concrete joint act changes. A justified comparison can nevertheless identify a continuing correction role. Its adequacy depends on what the task retains, including the forms of participation that make correction meaningful.

The taxonomy's capacity-condition coupling makes this point formally available without settling it empirically. Preserving the two component descriptions separately does not establish preservation of their joint realization [8, Section 8]. The current paper studies a particular consequence: an interface replacement must sustain the operative relation between participants, support and continuation. A collection of preserved ingredients is evidence about that relation only when their post-replacement interaction is also warranted.

3.4 Multiple Realizations

A useful substitution claim permits variation within the replacement. Requiring the same internal organization would defeat the purpose of investigating how a support role can continue through change. A distributed annotation arrangement and a single coordinated service could, under specified conditions, supply the same interpreted correction history to participants. Their internal number of steps or division of storage need not coincide.

Multiple realization is relative to a retained role. It does not mean that internal differences are unimportant in every respect. One arrangement may depend on a single maintainer, another on shared work; one may retain dissenting annotations, another only the current accepted version. If those differences affect the declared continuation task, they belong in the interface or in additional state. If they do not, the selected functional comparison can remain valid while another historical or normative comparison differs.

The positive argument for abstraction is consequently task-specific. Participants sometimes need to know whether a supported activity remains possible after a medium becomes unavailable. Full identity of the replacement's internal history is an excessive requirement for that question. A comparison retaining the relevant requests, responses, operation domains and continuation choices can establish something useful. It enables transformation without pretending that the whole practice is interchangeable with a diagram of outputs.

A replacement can also add operations. Participants may gain a way to inspect an earlier version or combine two accounts. Retention of the older selected capacity then coexists with enlargement of the repertoire. Conversely, preserving that capacity can coexist with loss of an operation outside the retained family. Reporting both findings avoids treating either an innovation count or an unchanged demonstration as a complete assessment of generative continuity.

3.5 Historical Bearers

The support region, the larger practice and the individual participants are different possible bearers of a preservation claim. A particular annotation service may end while the workshop continues using another arrangement. The claim that the practice retains a selected capacity does not imply that the earlier service survives, or that its workers have been historically replaced without remainder. One transition can constitute continuation for one target and termination for another.

Historical comparison therefore requires more than assigning the replacement the earlier name. The provenance of materials, the sequence of adopted changes and the relation of current participants to earlier activity help identify the transition actually being studied. An independently formed arrangement with an equivalent response can be a candidate substitute without already being a continuation of the earlier arrangement. The replacement event supplies a further historical relation whose interpretation must be stated.

The anchored regime in the field-theoretic manuscript provides one possible representation. A selected historical subcomplex can remain identifiable while subsequent attachments, admissible opera-

tions and larger carriers change [7, pp. 46–48]. That formal possibility separates retention of past occurrence from fixation of future organization. It does not prove the identity of a social practice or require every application to represent history by one immutable anchor.

For the workshop, the record that a participant challenged a description can remain attributable after the communication medium changes. Preserving that occurrence need not preserve the earlier medium. At the same time, a replacement that drops the challenge from the currently available account can change present interpretation even if an archival copy survives. Historical retention and current usability are linked questions with distinct evidential requirements.

3.6 The Replacement History

Comparing two ready arrangements leaves a further question: whether the later arrangement can be established through an available transition. A replacement can require copying materials, learning conventions, obtaining permissions, coordinating contributions or retaining temporary access to the earlier service. Those acts form part of the historical process. Their feasibility cannot be inferred from a favorable comparison of endpoints.

Suppose a new arrangement can support correction only after its annotations have been interpreted by the previous coordinator. If that coordinator has already left, the stipulated ready state may be unattainable by the currently available procedure. Alternatively, an overlap period can make the interpretation possible, but its work must be included in the comparison. The later arrangement's capacity is real within the constructed case; its immediate availability remains conditional on the transition.

A replacement can also disrupt the operation it will eventually support. During transfer, participants may temporarily lose the ability to identify which account is current. The endpoint can satisfy a continuation criterion while the transition fails a stronger requirement of uninterrupted correction. A study should distinguish these targets rather than describing the same evidence alternately as endpoint recovery and continuous preservation.

This historical account gives the later formal comparison a precise role. The interface construction can state which continuation capacities are retained once corresponding configurations are established. A transition account can then ask how those configurations are reached and what happens along the way. Together they permit a positive, bounded substitution finding whose bearer, conditions and remaining obligations are visible.

4 Contextual Continuation Capacity

A replacement preserves a selected continuation capacity when participants retain a usable way to pursue the declared activity across the surrounding situations included in the comparison. The claim concerns possible interaction, including responses to what happens next. This section develops its task, context and agency conditions. It distinguishes a capacity from a demonstrated outcome, identifies the role of an implementable strategy correspondence and explains why exact response agreement and one-way preservation answer different questions. The discussion supplies the philosophical interpretation for the later restricted constructions.

4.1 The Retained Activity

A continuation target states what the activity must still permit after the support changes. In the workshop example, the relevant aim can include interpreting a new account, asking whether the described event was completed, challenging a misleading use of the distinction and carrying an accepted correction into further discussion. A target framed only as producing an earlier sentence would retain less. Neither formulation acquires priority from its mathematical simplicity; the choice should follow the undertaking being assessed.

The target should also identify its bearer and horizon. Enabling an established group to complete several agreed episodes differs from forming new competent participants or sustaining a practice indefinitely. A finite investigation can legitimately study the former. Its findings should retain that boundary even if the broader reason for undertaking it concerns long-term linguistic continuation.

The preservation taxonomy already treats generative capacity as an organized operation family with partial composition and state-dependent realizations [8, Section 7]. This provides a richer starting point than an inventory of outputs. A participant may know how to produce a conventional form yet lack the support needed to correct its application. Conversely, a changed form may perform a corresponding correction role after the medium and local procedure change.

An adequate target comparison therefore specifies which changes count as successful continuation. It can permit different wording, another route for clarification or a reorganized record of revision. What matters for the selected claim is whether participants can still complete the retained interpreted operation. The comparison does not become informative simply by calling every later performance an equivalent realization.

4.2 Surrounding Contexts

A surrounding context includes the interaction conditions outside the replaced region that can affect the selected task. These conditions can involve interlocutors' requests, available time, feedback conventions, access arrangements and relevant previous exchanges. The context class declares which variations the substitution claim covers. It supplies the scope of the claim rather than a complete ontology of the environment.

In the workshop, a narrow class might contain only requests for familiar examples. A broader class can include an unfamiliar account, a correction request and a participant's decision to withdraw an earlier interpretation. A replacement passing the first class can fail the second. The difference does not make the narrow result logically false. It makes its relevance to a capacity for correction questionable if that capacity motivated the substitution.

Contexts should retain responsive behavior where the task requires it. An interlocutor can change the next request after hearing an answer. Comparing two systems against one fixed sequence of questions may suppress precisely that dependence. A useful contextual test asks whether the replacement remains adequate when the surrounding activity reacts in the permitted ways, rather than merely replaying the sequence that produced a favorable demonstration.

The class should nevertheless remain bounded by a defensible inquiry. Requiring success against every imaginable interlocutor, unlimited delay or arbitrary removal of support would make many modest continuation claims impossible by construction. A declared class can exclude such cases when the exclusions match the practical undertaking. Its evidential defense should explain both the retained challenges and the omissions, including who can contest that boundary.

The historical setting can change the appropriate class. A communication arrangement adequate for participants sharing familiar conventions may be unsuitable when a new group joins. The original result can remain valid within its earlier scope while the new participation task requires further inquiry. Context sensitivity permits this distinction without treating either the earlier participants or the later demands as deviations from a universal standard.

4.3 Responsive Correction

Correction provides a demanding example because the next operation depends on a recognized difficulty. A participant can hear an account, identify a misleading implication and request a response that changes the shared understanding. Preserving only the final revised sentence omits the route through which the participant can raise the problem. If that route disappears, the arrangement may retain a product of correction while losing the selected corrective capacity.

Suppose the old workshop support permits a participant to mark a disputed description and ask for its reason. A proposed replacement stores several polished accounts and returns one whenever the topic

is recognized. Its demonstration can reproduce every previously displayed answer. Yet an unfamiliar objection may have no available destination. The missing continuation is not an inferior wording of a known answer; it is the absence of a usable act when a challenge arises.

A different replacement can satisfy the narrower correction task without recreating the old internal organization. It might preserve the disputed passage, route the participant's reason to a competent respondent and make the resulting revision available to the group through another medium. Those are stipulated features of a constructed alternative. If they are available in the retained contexts, they supply a positive basis for comparing the correction role despite different internal steps.

The interpretation of correction remains substantive. A response that automatically overwrites the participant's account may resolve a technical discrepancy while failing the intended participatory operation. Likewise, receiving an objection does not establish that it can affect the subsequent account. A model should retain whichever features distinguish the correction at issue: initiative, intelligibility, a consequential response and an opportunity to continue after the response.

These features do not imply that every disagreement must prevail or that every exchange must remain open indefinitely. A practice can have justified stopping conditions. The present concern is whether the selected route for correction remains usable within them. Its legitimacy and the reasons for closure remain further questions.

4.4 Implementable Strategy Correspondence

A continuation strategy gives participants a way to select their next move using what they have observed and the operations available to them. Preserving a strategy therefore requires more than an analyst's pairing of successful paths. The participant must be able to enact the corresponding moves without access to hidden information, future responses or a skill that the substitution removes.

This condition exposes a quantifier difference. An analyst might find one replacement procedure for each possible surrounding situation. If the participant cannot distinguish those situations when a choice is required, the family does not yet supply a usable strategy. A single observation-compatible procedure is needed on the unresolved cases, unless an available preliminary interaction supplies the missing distinction.

The formal policy-transfer construction uses an explicit correspondence between public actions and equal retained observations at related configurations. Its purpose is to keep the next choice available at the same informational stage. The replacement can have different internal states, but the translator cannot select its action by consulting an internal difference that the participant cannot read. A correspondence that violates this restriction would preserve an analyst's solution rather than the participant's capacity.

Uncertain responses require another discipline. After the participant chooses a move, the replacement may admit several outcomes. Retention of a successful old strategy requires the relevant guarantee across the new outcomes covered by the model. Pointing to one favorable branch establishes a possibility, while a reliable strategy claim concerns the admitted branches against which success is required. The appropriate standard can differ across applications, but its quantifiers should remain explicit.

Implementation also includes comprehension and timing. A correct code translation can still arrive after the next move has become unavailable. A participant may be able to understand an instruction only with additional assistance. Such assistance can legitimately form part of the available strategy; it should then appear in the support arrangement and its conditions. The inquiry should attribute success to the supported procedure actually available.

4.5 Comparison Strength

Exact contextual agreement and preservation of a selected ability are different achievements. Exact agreement requires the retained response behavior to coincide throughout the declared comparison class. A weaker one-way claim can preserve the participant's ability to achieve the selected goal while permitting new responses, internal structure or additional operations. The latter may be sufficient when the task concerns keeping a practice possible rather than reproducing every feature of its earlier interaction.

The direction of the comparison matters. A replacement can retain the old successful procedures and add another means of clarification. The reverse substitution can then remove a newly available capacity. Calling the two arrangements simply equivalent would conceal that asymmetry. A one-way statement can report the preserved family precisely without turning every additional possibility into either an automatic improvement or a defect.

Greater response in a numerical channel is still another comparison. Within the selected cooperative field model, with the same prescribed input, a stronger support response can preserve a target that requires activity to remain above a declared floor. A linguistic task involving listening, waiting, revision or refusal need not be ordered by the amount of activity. Numerical response domination acquires its practical interpretation only through the stated target and coupling assumptions.

The inherited taxonomy leaves the comparison relation to the preservation specification and permits exact, functional, simulation-based or approximate comparisons [8, Sections 7–8]. The present paper follows that pluralism. The task is to establish consequences of a chosen comparison, including counter-cases showing what weaker data fail to establish. There is no single ordering that automatically settles historical identity, meaningful participation and every future use.

4.6 Bounded Positive Preservation

A positive substitution finding can be substantial while remaining limited. Suppose the participants' retained task is defended, the relevant contexts are identified, the replacement is prepared through an available transition, and an implementable correspondence preserves their goal-directed continuations. The later activity can then count as retention of the selected capacity despite changes in its supporting realization. Full recovery of the earlier internal organization is unnecessary for that particular conclusion.

The finding can inform a real choice among arrangements. A support service may be unavailable in its earlier form, while another can sustain the correction practices participants wish to continue. Demonstrating the retained capacity can help distinguish a feasible transformation from a substitution that merely presents familiar material. That practical significance does not depend on claiming that the replacement preserves everything.

Participants may still reject the available alternative. Its costs, ownership arrangements, demands for disclosure or allocation of work can matter independently of its functional adequacy. Their refusal does not refute a properly scoped capacity result. It shows that preserving the selected capacity is one consideration within an undertaking whose authority and value remain subject to deliberation.

Conversely, a failed comparison can guide repair without condemning the replacement as a whole. It can identify a missing observation, an unavailable action, an unsupported transition or a context outside the warranted domain. Those differences suggest different responses. The analytical value lies in making the specific continuation condition visible, so that further work can address it rather than treating resemblance or difference as a complete verdict.

5 Historical Attachment Architecture

A replacement comparison needs a place to retain the history that makes its components and operations intelligible. This section specifies such a place through one possible two-complex representation. It first constructs the attachment of an active support region to a surrounding context, then distinguishes comparison of future templates from an actual replacement event. The construction establishes structural consistency. The following sections add the operational conditions required for continuation.

5.1 Selected Historical Cells

Let a represented history consist of a finite regular cellular complex of dimension at most two, together with labels and declared event precedence. Vertices can mark selected situations, edges record transitions or dependencies, and faces retain witnessed relations among represented paths. An application must

justify each interpretation. In particular, a face need not declare that two linguistic operations commute. It may record a comparison, a correction episode or a joint constraint whose content belongs to its label.

The choice of dimension is a representational restriction. A two-complex can retain attachment and overlap information that a point-state summary omits, while still leaving much of a practice unrepresented. A spin-foam realization would add its own representation labels, amplitudes and boundary structure. Nothing in the present substitution argument requires that additional realization. The history-bearing object is conceptually prior to either mathematical choice, in accordance with the inherited GR architecture [7].

Select a surrounding complex C , an active support complex R , and a shared interface subcomplex B . Injective cellular maps $i_C : B \rightarrow C$ and $i_R : B \rightarrow R$ identify the cells through which the selected attachments occur. Labels agree on identified cells. Their precedence restrictions agree on B , and their union is assumed acyclic if event precedence is represented by a strict partial order. The last requirement is substantive: locally consistent orders can produce a cycle after gluing. It must be checked in the combined representation. When it is acyclic, its transitive closure supplies the glued strict precedence relation.

The attached carrier identifies precisely the paired interface cells.

$$K = C \sqcup_B R := (C \sqcup R) / \{i_C(b) \sim i_R(b) : b \in B\}. \quad (1)$$

Equation (1) is used for subcomplex inclusions with compatible cellular attachments. It does not authorize arbitrary quotients of a historical record. A replacement R' carries an interface map $i_{R'} : B \rightarrow R'$ with the same declared port types, giving a candidate $K' = C \sqcup_B R'$. The existence of these two carriers establishes that both arrangements can be represented at the chosen boundary. Their linguistic adequacy remains untested.

5.2 Cellular Consistency

For a complex X , let $C_q(X; \mathbb{R})$ be the real vector space generated by its oriented q -cells and let ∂_X be its cellular boundary. The same symbol C without a subscript continues to denote the surrounding complex. Compatible orientations are chosen for shared cells. The inclusions induce chain maps, written i_{C*} and i_{R*} . At each degree the chain spaces of the attachment satisfy the following sequence.

$$0 \longrightarrow C_q(B; \mathbb{R}) \xrightarrow{b \mapsto (i_{C*}b, -i_{R*}b)} C_q(C; \mathbb{R}) \oplus C_q(R; \mathbb{R}) \xrightarrow{p_q} C_q(K; \mathbb{R}) \longrightarrow 0. \quad (2)$$

In Equation (2), p_q sends the two copies of a shared cell to their common class and otherwise retains the distinct cells. The first arrow is injective because both interface inclusions are injective. Every cell of K comes from one of the two pieces, so p_q is surjective. Its kernel consists exactly of differences between the two copies of shared cells. This proves exactness directly from the cellular bases; no claim about capacity follows from the exact sequence.

The direct-sum boundary preserves that kernel because the interface maps are chain maps. It therefore induces a boundary on the quotient. Applying the boundary twice gives zero on every representative because it gives zero in each summand. The attached cellular model thus retains the required chain-complex identity. A finite implementation can check the corresponding boundary matrices before considering dynamical or semantic claims.

The significance of this calculation is limited but useful. A diagram with matching ports can conceal incompatible attaching maps; those would undermine even the proposed historical representation. Passing the cellular check removes that particular defect. It does not show that the same response remains possible at a port, that an old operation remains executable, or that participants can interpret its output. Those requirements are carried by the open interaction model in Section 6.

5.3 Historical Retention

An actual support replacement occurs after a history that includes use of the old arrangement. It cannot be represented faithfully merely by deleting that use from the past. Let $A \subseteq C$ be the retained anchor,

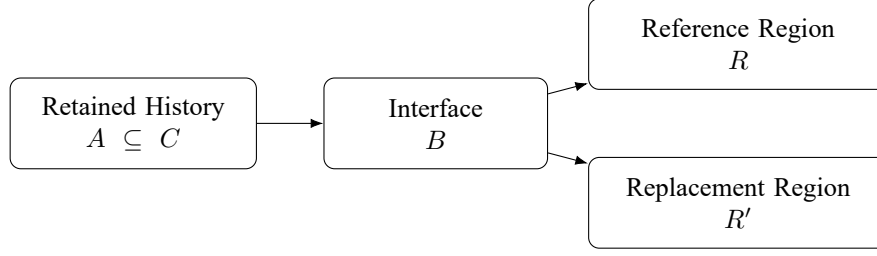


Figure 1: Alternative Support Attachments. The branches compare continuation templates from a common historical context. Actual replacement adds a historical event; it does not remove the old arrangement’s earlier occurrences.

including selected earlier uses of R where those uses have already occurred. The R attached in Equation (1) then represents an active or prospective continuation template, distinguished from the historical occurrences already in A .

The two candidates $C \sqcup_B R$ and $C \sqcup_B R'$ compare continuations from a shared historical situation. Executing the replacement appends a handover event and its consequences to that history. The old arrangement may cease to provide active support while its contributions and failures remain historically represented. Retention of those occurrences allows later recognition or criticism to refer to them. Their present interpretation can change through new events without erasing the distinction between earlier occurrence and later interpretation.

When a structural map $f : K \rightarrow K'$ is additionally supplied, anchor retention has its inherited form. Let $j : A \rightarrow K$ and $j' : A \rightarrow K'$ be the anchor embeddings.

$$f \circ j = j'. \quad (3)$$

Equation (3) is recalled from the existing anchor-preserving construction [7]. A replacement comparison need not supply such an f on every cell. Alternative future templates may instead be compared by a relation between their operations or responses. The common anchor and interface embeddings should remain explicit even in that case.

Figure 1 illustrates the distinction. The two future support realizations share an attachment specification and a retained context. The figure does not identify the interiors or declare their continuation behavior equivalent.

5.4 Interface Selection

A cellular boundary and an interaction interface have related but distinct roles. The former records where pieces attach in the chosen representation. The latter records which actions, observations and consequences cross that attachment. A model can have perfectly compatible cells while omitting a communication channel that matters to the practice. It can also declare a port for correction while failing to represent who is permitted to use it. Structural inclusion alone does not settle either issue.

This distinction guides refinement. If a context responds to delays, provenance or rejection, those must enter the interface data relevant to the intended comparison. Enriching a port can reveal that a proposed replacement fails a previously hidden requirement. Alternatively, an application can defend a narrower context class that does not respond to that difference. Neither response is automatically correct. The selection must remain answerable to the linguistic capacity being investigated.

The resulting architecture separates three claims that can now be checked in order. The candidate attachment must first be a coherent historical representation. Its declared ports must then cover the interactions on which the comparison depends. Finally, a behavioral or dynamical argument must show that the relevant continuation survives through those ports. The following sections supply several possible arguments for the third claim while retaining the first two as explicit premises.

6 Open Interaction Models

The historical attachment supplies a carrier for comparison, but it leaves the effects of interaction unspecified. This section introduces a finite operational specialization. Its purpose is to distinguish available choices, possible outcomes and accessible records before stating a replacement criterion. The specialization can describe a bounded inquiry episode within a larger history. It does not identify the linguistic practice with a finite automaton.

6.1 Actions And Possible Outcomes

For each support realization indexed by i , let S_i be a nonempty finite set of represented local configurations. A state can retain a bounded historical record or designate a class of histories justified for the task. Such a class is a modeling decision. It must not silently discard a distinction on which later availability or interpretation depends.

Let U_i be a finite action alphabet. Each action $a \in U_i$ has a domain $D_i(a) \subseteq S_i$ and a nonempty set of possible successors at every point of its domain. A public observation $o_i : S_i \rightarrow O$ has values in a common finite record alphabet O . The resulting open model is specified by the following data.

$$\mathcal{M}_i = (S_i, U_i, D_i, F_i, o_i), \quad F_i(a) : D_i(a) \longrightarrow \mathcal{P}(S_i) \setminus \{\emptyset\}. \quad (4)$$

Equation (4) separates choosing an available action from controlling its outcome. The successor set can represent several allowed responses, unresolved variation, or an explicitly chosen adversarial interpretation. It does not specify probabilities. A later claim about expected success or likely performance would require a further law and a different preservation argument.

The domain is also an interpreted object. It can encode an implemented operation together with required access, competence and permission, provided the application has evidence for those conditions. A formal domain does not make a requested action feasible by notation. Conversely, a rule absent from the selected model need not be impossible in the wider practice. The present criteria compare the represented domains and retain that limitation.

Observed refusal can be represented by an available request followed by a refusal output. An unavailable operation can instead remain outside $D_i(a)$. These conventions answer different questions. They must not be switched midway through a comparison. For sure-executable policies below, the chosen operation must be available at every hidden configuration still compatible with the participant's information. This follows the existing partial-domain discipline [6, 10].

6.2 Surrounding Contexts

The surrounding practice contributes its own state and response. Let Z be a finite context-state carrier. For transparency, the controller knows its own context state; hidden context variation can instead be included in the joint information state. A context receives a requested reference action a , followed by a module observation y' , and has a nonempty set $\Gamma(z, a, y') \subseteq Z$ of possible next context states. A fixed context can be deterministic or nondeterministic. Its rule is the same in the two compared systems.

A fixed action translator $\tau : U_0 \rightarrow U_1$ implements the reference request in the replacement. It is available to the participant or to a declared public adapter. It does not depend on the analyst's knowledge of the hidden pair (s, t) . In the replacement, a requested semantic action a executes $\tau(a)$, while the external action record remains a . An implementation involving several steps, delay or adapter memory would require an enlarged model containing those events. The one-step translator used here is an explicit restriction.

The product transition for the reference model uses an available a , and the replacement product uses $\tau(a)$. Their typed rules are written together.

$$\begin{aligned} (s, z) &\xrightarrow{a} (s', z') \iff s \in D_0(a), s' \in F_0(a)(s), z' \in \Gamma(z, a, o_0(s')), \\ (t, z) &\xrightarrow{a} (t', z') \iff t \in D_1(\tau(a)), t' \in F_1(\tau(a))(t), z' \in \Gamma(z, a, o_1(t')). \end{aligned} \quad (5)$$

Equation (5) declares what port-mediated means for this model. The context reacts through the action and observation records retained by the rule. If it also reacts to a hidden local variable, to a different timing convention or to the name of the supplier, the stated product is incomplete unless those factors enter its state or observations. The later transfer theorem cannot compensate for an omitted port.

6.3 Observable Continuation Policies

A policy π takes a finite record of observed pairs $(o_i(s_k), z_k)$ and previously requested actions to either a new action in U_0 or a stop instruction. Its dependence is on the accessible record, not the hidden module state. Identical records therefore receive the same request. The replacement executes the translated request and uses the same external policy.

Specify a family of initial joint configurations and a target $G_i \subseteq S_i \times Z$. A policy is *sure-winning within horizon N* when every path admitted by that initial family and the transition rules executes available actions, stops by N actions, and stops in G_i . The criterion includes all possible outcomes. It permits a policy to stop earlier on some records, but does not count an unnoticed visit to the target followed by failure as success. The certification convention is part of the task.

This demanding criterion is chosen to make the transfer statement exact. Some linguistic tasks tolerate occasional error or incomplete completion. They would require an appropriate probability model, loss specification or weaker target. The sure-winning criterion also concerns only the declared target and horizon. It says nothing about the desirability of reaching that target or the preservation of unlimited future creativity.

The distinction between retained policies and retained outcomes is philosophically important. A successful continuation can exist without being identifiable from the available record. Several continuations can also share the same final expression while differing in whether a participant controls a correction. Retaining a policy family preserves a selected relation between information, choice and consequence. It is stronger than retaining a list of possible successful endings, yet still narrower than preserving the whole practical relation.

6.4 Initial Handover Conditions

A replacement assessment must identify its admitted initial configurations. The reference may begin with prepared materials or already established coordination. The replacement may require a migration, interpretation or training event to reach a comparable configuration. Such an event has its own domain, possible failures and history.

The formal comparison starts after a stated handover. It can therefore establish what follows from a prepared replacement state without establishing that preparation is available. An application should report both claims. This separation will recur in the field model, where equality of the present boundary output fails to determine the full future response of the internal support state.

The next section supplies a sufficient relation between the post-handover states. The relation respects public observations and possible choices. It can be checked independently of a particular winning policy, which is why it supports transfer across a declared family of contexts rather than only replay of one successful demonstration.

7 Contextual Strategy Transfer

A useful replacement criterion must preserve an executable choice before its outcome is known. This section gives a sufficient relation for the finite models just defined and proves its contextual consequence. The criterion is a restricted refinement construction using a fixed action translator. Its value here lies in exposing the assumptions of the linguistic comparison, not in claiming a new general refinement theorem.

Alternating refinement supplies established methods for separating a component's choices from those of its environment [1]. Interface automata likewise make compatibility and refinement depend on explicit input, output and environment conditions [4]. The construction below uses its own fixed-translator

premises and gives the full proof. It does not identify compatibility with one environment with success in every environment.

7.1 Directed Replacement Relation

Let $E \subseteq S_0 \times S_1$ compare reference and replacement configurations. The following conditions are imposed for every pair $(s, t) \in E$.

$$\begin{aligned} o_0(s) &= o_1(t), \\ s \in D_0(a) &\implies t \in D_1(\tau(a)) & (a \in U_0), \\ s \in D_0(a), t' \in F_1(\tau(a))(t) &\implies \exists s' \in F_0(a)(s) \text{ with } (s', t') \in E. \end{aligned} \quad (6)$$

Equation (6) retains the public record, preserves each represented reference choice, and requires every possible replacement outcome to have a matching reference outcome. The direction of the last quantifiers matters. Requiring merely one favorable replacement outcome would not protect a policy against another admitted outcome. Requiring a translator after seeing that outcome would reverse the temporal order of choice.

Definition 7.1 (Fixed-Translator Replacement). A fixed-translator replacement relation is a relation E and a public map τ satisfying Equation (6). Its contextual target compatibility requires $(s, z) \in G_0 \implies (t, z) \in G_1$ whenever $(s, t) \in E$.

Definition 7.1 is directed. A replacement can add operations or remove some uncertain outcomes while preserving all the reference choices needed by the relation. No reverse replacement follows without a separate argument. The target implication also has an independent interpretation. Matching a label named completed does not establish that the relevant linguistic activity has been completed.

The relation may be stronger than a particular task needs. It checks every available reference action, even one unused by the selected policy. A task-specific relation could restrict that repertoire, but must state the restriction. This stronger formulation is useful because its certificate can serve more than one bounded policy and more than one admissible context without rebuilding the proof for each example.

7.2 Context Lifting

For any context of Equation (5), lift E to joint states by pairing the same context coordinate.

$$\hat{E} = \{((s, z), (t, z)) : (s, t) \in E, z \in Z\}. \quad (7)$$

Equation (7) preserves the joint public observation. If a replacement successor is (t', z') , the local matching condition gives s' with the same module observation. The same $\Gamma(z, a, o_0(s'))$ therefore admits z' . The reference can match the complete joint successor.

This reasoning explains the benefit and the restriction of port composition. The proof does not require the context to be passive. It may choose a different later request after a response, and its state can record previous interactions. It must, however, receive the same information and follow the same response rule in the two systems. An unrepresented dependence on a hidden support variable invalidates the lifting step rather than producing a mysterious exception to it.

Proposition 7.2 (Bounded Policy Transfer). *Assume Definition 7.1, the common port-mediated context, and target compatibility. Let $\mathcal{I}_0 \subseteq S_0 \times Z$ and $\mathcal{I}_1 \subseteq S_1 \times Z$ be initial families such that each member of $\mathcal{I}_1$ is $\hat{E}$ -related to some member of $\mathcal{I}_0$. Every reference observation-history policy that is sure-winning from $\mathcal{I}_0$ within N transfers through τ to a policy sure-winning from $\mathcal{I}_1$ within N .*

Proof. Fix a replacement initial state and choose a related reference initial state in the stated family. Their public histories agree initially. Suppose a finite replacement path has already been matched by a reference path with the same public record and related current states.

If the policy requests an action a , reference sure-executability gives availability at the matched reference state. The second condition in Equation (6) gives availability of $\tau(a)$ at the replacement state. For each admitted replacement successor, the third condition chooses a related reference successor. Equal observations permit the identical context successor by Equation (5). The extended paths again have the same public record.

If the policy stops, it stops on that same record in the reference. Sure-winning implies that the matched reference state belongs to G_0 . Target compatibility places the replacement state in G_1 . Finally, a replacement path failing to stop by N would have a matched reference prefix of the same length and the same stopping decisions. It would contradict the bounded reference guarantee. The construction applies to every admitted initial state and every replacement outcome, proving the result. $\square$

Proposition 7.2 does not require participants to construct the hidden reference path. The path is a witness in the proof. The executable policy uses only the common public history, and the translator was fixed before the hidden state or outcome was known. This distinction prevents a mathematical existence argument from quietly giving participants information they do not possess.

7.3 Scope Of The Guarantee

The guarantee transfers all reference policies meeting the stated bounded criterion, but only their selected success property. It need not preserve how a participant values every intermediate experience. A delay, cost, burden or refusal relevant to the comparison should be represented in the public record, the state, the domain or the target as appropriate. If such a difference is omitted, the proof concerns the resulting restricted task.

Likewise, the policy may rely on a context state that is publicly available in the formal model. Where participants cannot distinguish context states, the policy domain must instead identify their common observation histories. A family of context-specific strategies selected by an analyst does not establish one strategy executable by a participant who cannot identify the context. The uniform translator and public policy address that quantifier problem only after the observation architecture is specified correctly.

The bounded horizon supplies a clear stopping requirement rather than a claim that linguistic practice has a natural fixed length. It is appropriate to an episode such as revising a selected narrative or finishing a particular clarification. Indefinite viability, repeated learning or the emergence of a new action vocabulary demands a corresponding infinite-horizon or changing-repertoire model. The present result does not silently extend to those settings.

7.4 Counterfactual Evidence

The criterion quantifies over unexecuted actions and outcomes. Establishing it for an empirical replacement therefore requires evidence beyond a successful demonstration. Documentation of operation domains, participant trials, failure analysis or independently justified mechanism knowledge could contribute. A finite model can make those obligations clear without providing the evidence itself.

The target compatibility premise requires particular care. Suppose an archive returns an item labeled correction while participants cannot revise its classification. Matching the label does not show preservation of the target if that target includes effective correction. The substantive interpretation should constrain the target set before the relation is checked. Redefining success after observing a replacement's limitations would change the preservation claim.

The result thus provides a conditional route to a positive conclusion. Where the interface is adequate, the translator is implementable, the initial states are related and the target is justified, a whole class of bounded continuation strategies survives. The next section examines composition of such certificates and a small failure case that an output-based comparison would miss.

8 Substitution Composition

A supporting arrangement can be replaced more than once, and a preservation argument may need to survive that sequence. This section proves a compositional property of the stated replacement relation, then contrasts it with a successful-trace comparison. The first result permits reuse of a certificate under explicit conditions. The counterexample shows why a weaker certificate would not suffice.

8.1 Sequential Certificates

Suppose three models $\mathcal{M}_0, \mathcal{M}_1, \mathcal{M}_2$ share a public observation alphabet. Relations $E_{01} \subseteq S_0 \times S_1$ and $E_{12} \subseteq S_1 \times S_2$ satisfy Equation (6) with translators $\tau_{01} : U_0 \rightarrow U_1$ and $\tau_{12} : U_1 \rightarrow U_2$. Their proposed composite has the following definition.

$$\begin{aligned} E_{02} &= \{(s_0, s_2) : \exists s_1, (s_0, s_1) \in E_{01}, (s_1, s_2) \in E_{12}\}, \\ \tau_{02} &= \tau_{12} \circ \tau_{01}. \end{aligned} \tag{8}$$

Equation (8) is a relation between the first and third models, not a path of actual installation events. The intermediate witness need not be accessible to the participant, whereas the composite translator must remain implementable.

Proposition 8.1 (Composition Of Replacement Relations). *The pair (E_{02}, τ_{02}) in Equation (8) satisfies Equation (6). Target compatibility and initial coverage compose when they hold through the same intermediate model and the same context interpretation.*

Proof. Choose an intermediate witness s_1 for (s_0, s_2) . The two observation equalities give equality of the first and third observations. If a is available at s_0 , domain preservation gives availability of $\tau_{01}(a)$ at s_1 , followed by availability of $\tau_{12}(\tau_{01}(a))$ at s_2 .

For any successor s'_2 of that translated action, E_{12} provides a matching successor s'_1 . Applying the matching condition of E_{01} to this particular s'_1 provides s'_0 . The pair (s'_0, s'_2) therefore belongs to E_{02} . This proves all three conditions.

Target membership at (s_0, z) implies membership at (s_1, z) and then at (s_2, z) . For initial coverage, choose the admitted intermediate initial state provided by the second certificate and then the admitted first initial state provided by the first certificate. They witness composite coverage. $\square$

Proposition 8.1 gives a closure property for analytical certificates. A participant can continue using the original semantic requests through the composed adapter. The proof does not require matching internal dimensions or structural isomorphism of the support regions.

Actual sequential replacement carries further requirements. The second certificate may assume an intermediate initialization that the first installation does not produce. A migration may temporarily remove access to the operation whose later preservation is proved. Two translators may each be implementable while their combination exceeds a participant's time or coordination resources. Such failures concern the handover or the model of feasibility. They should be represented explicitly rather than hidden behind relational composition.

Multiple simultaneously replaced regions require another check. A certificate for one region treats the rest as a context. Replacing another region can change whether that surrounding arrangement belongs to the certified context class. Individual certificates compose only when the corresponding interface and context assumptions remain satisfied. Pairwise resemblance cannot establish this closure by itself.

8.2 Hidden Commitment Counterexample

Use a one-state context with a self-transition after every request and observation, and the identity translator on action labels. Target subsets of module states below therefore designate their products with that single context state.

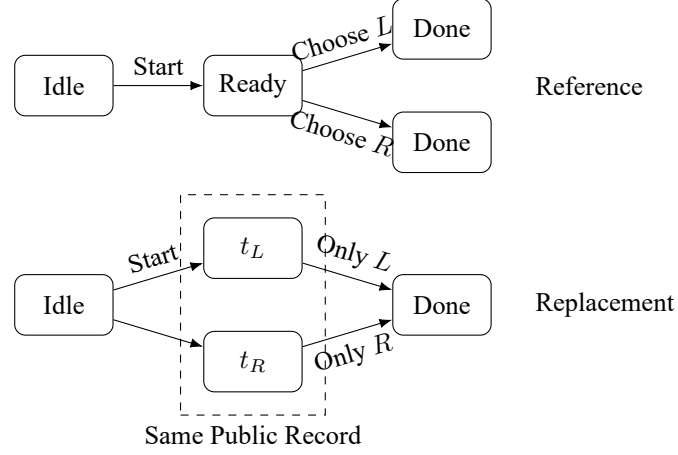


Figure 2: Hidden Commitment After A Shared Observation. The reference retains both next choices at its ready state. The replacement commits to a branch before the participant chooses, while reporting the same ready observation.

A finite example isolates the difference between possible successful traces and retained choices. The reference has four states s_0, s_f, s_L, s_R , with public observations idle, ready, done and done. A start action takes s_0 to s_f . At s_f , both L and R are available and lead to their respective done states. The target contains those two final states.

The replacement has states t_0, t_L, t_R, t_d , with observations idle, ready, ready and done. The start action has two possible successors, t_L and t_R , which participants cannot distinguish. At t_L , only L is available; at t_R , only R is available. Either available action leads to t_d . All actions not just specified are unavailable. There is no inspection action that distinguishes the two ready states.

The successful visible traces are start- L -done and start- R -done in both models, with the same visible prefixes. Here a successful trace reports only an actually executable path. It omits refusal and unavailable requests. Under that deliberately weak semantics, the trace sets agree.

The reference has a sure-winning policy that requests L after ready. It also has one that requests R . In the replacement, either choice fails sure-executability for one possible hidden ready state. Stopping at ready fails the target, and no other operation is available. Thus no sure-winning policy exists from t_0 within two actions, or within a larger horizon in this model. Randomizing the choice does not make every outcome successful.

Figure 2 displays the different location of choice. This is a constructed countermodel, not an empirical claim about a support service.

This failure appears in Equation (6). Matching either replacement ready state to s_f violates one of the two domain-preservation conditions. The relation cannot certify this replacement under the two-action repertoire. Restricting the repertoire or target might make a different comparison valid, but it would express a different preservation claim.

8.3 Trace Interpretation

The counterexample depends on the declared weak trace semantics. A domain-sensitive semantics, an observable refusal record or an appropriate testing semantics can distinguish the systems. The example does not show that every behavioral semantics misses participant choice. It shows which information is lost by one particular use of successful traces.

Testing accounts distinguish possible success from success required across admitted executions, with additional completeness assumptions affecting the latter [14]. The present finite example avoids importing an infinite-run completeness convention: failure already occurs at the next request. Its relevance is local and explicit.

An empirical assessment based on successful demonstrations can have the same weakness if it records only what happened to work. The appropriate response is to investigate alternatives and availability. For the workshop example, a scripted answer to one anticipated clarification would leave open whether participants can choose a different correction after encountering an unfamiliar continuation.

8.4 Generative Scope

The example gives a modest formal sense to preserving meaningful alternatives. An alternative belongs to the selected continuation capacity when a participant can request it at the relevant record and the model retains the required guarantee. Placing both endings somewhere in the analyst's transition graph does not meet that criterion.

The construction still represents an anticipated repertoire. Participants can form new expressions, requests or criteria absent from U_i . Those developments may require a new model and comparison relation. The present account supports a bounded preservation claim while making that boundary visible. It does not define the full generativity of a language by a finite set of branches.

The nonlinear investigation that follows addresses a complementary limitation. Even a support module with a well-defined response interface can have a current output that conceals a consequential internal history. Its continuation comparison must then retain a temporal response, not merely a state label.

9 Boundary Memory Fields

A replacement support arrangement can have a different internal organization while supplying the same historically responsive interface to a surrounding practice. This section investigates that possibility through a selected nonlinear field realization. It derives the full causal boundary response of several support channels, identifies the role of their initialized state, and proves a conditional substitution result. The comparison concerns histories of incoming activity and outgoing support. A current output value alone supplies a smaller record.

The construction inherits the distinction between capacity, enabling conditions and their joint realization from the preservation taxonomy [8, Section 8]. It also inherits the field-realization requirement: a regional datum needs an interpreted response and compatible restrictions, while comparison transport remains distinct from substantive dynamical action [7, pp. 27–31, 33–35]. The activity law specializes the prior reactivation model [9, Section 6]. Its present use concerns substitution of a support realization, rather than another initiation threshold. The convolution calculation below makes that comparison explicit within a restricted family of internal laws.

9.1 Declared Regional Ports

Let G be a finite undirected graph with site set $\{1, \dots, n\}$ and symmetric weights $w_{ij} \geq 0$, where $w_{ii} = 0$. Its Laplacian is defined by $(L_G x)_i = \sum_j w_{ij}(x_i - x_j)$ for $x \in \mathbb{R}^n$. Sites represent selected operational interfaces. Their meaning, historical attachment and eligibility for comparison require the preceding architecture; a graph label does not itself identify a person or preserve an institution. Write $\mathbf{1}$ for the all-one vector and Id_n for the identity matrix.

A support realization $\mathcal{M}$ contains $N \geq 1$ internal channels $v_\ell(t) \in \mathbb{R}_{\geq 0}^n$. For each channel choose $\varepsilon_\ell > 0$, $d_\ell \geq 0$, $\kappa_\ell \geq 0$ and $c_\ell \geq 0$. Its incoming port is a nonnegative vector history $r(t)$; its outgoing port is a nonnegative vector $y(t)$ supplied at the identified regional locations. Define $A_\ell = \varepsilon_\ell \text{Id}_n + d_\ell L_G$. The open support equations are Equation 9.

$$\dot{v}_\ell = -A_\ell v_\ell + \kappa_\ell r, \quad y = \sum_{\ell=1}^N c_\ell v_\ell, \quad v_\ell(0) = v_{\ell 0} \geq 0. \quad (9)$$

In Equation 9, ε_ℓ governs decay, d_ℓ governs regional exchange, κ_ℓ governs renewal from the incoming activity signal, and c_ℓ specifies contribution to the retained output. Different channels can represent

selected preparation processes with different response times or locations. Interpreting them requires evidence about those processes. None is a scalar amount of linguistic competence or an inventory of every enabling condition.

The chosen internal subsystem is linear in r . That restriction permits its entire causal response to be calculated. It does not assert that supporting institutions, material arrangements or human relations are generally linear. The surrounding activity and its renewal signal will be nonlinear. More general internal laws would require comparison of their causal input-output maps, without the particular kernel representation derived here.

The interface comprises both the input vector and the location-resolved output vector, with their meanings and timing retained. A scalar total of the outputs is a different interface. A surrounding arrangement that receives y_i at a particular site depends on information that can disappear from that total. Section 10 develops a counterexample in which equality of aggregate responses conceals a consequential redistribution.

9.2 A Nonlinear Surrounding Field

Let $u(t) \in \mathbb{R}_{\geq 0}^n$ record an identified execution-activity channel. Put $h(x)_i = x_i^2/(1+x_i^2)$ for nonnegative vectors x , and close the incoming port through $r = h(u)$. Choose $a, b > 0$, $\sigma \geq 0$, $D_u \geq 0$ and a nonnegative, locally bounded, piecewise-continuous input $I(t)$. The surrounding field is Equation 10.

$$\dot{u} = -D_u L_G u + u \odot [-a\mathbf{1} + b(\sigma\mathbf{1} + y) - u \odot u] + I(t), \quad r = h(u). \quad (10)$$

The symbol $\odot$ denotes coordinatewise multiplication. Equation 10 retains the earlier activity-support feedback: support enables activity, and some activity renews support. The saturation of h limits the renewal rate. The support channels here describe additions to the baseline σ . With one channel, $c_1 = 1$, and the earlier support variable written as $\sigma\mathbf{1} + v_1$, the law recovers the corresponding reactivation specialization.

The activity amplitude is not the retained operation grammar. A field history supports a linguistic conclusion only when an independently justified realization connects the channel to the selected operation. The same positive amplitude could otherwise describe imitation, newly acquired performance, or an unwanted activity. Likewise, the baseline and every omitted maintenance condition remain present when a module is replaced. Equal output histories cannot establish that burdens hidden outside the declared interface have stayed equal.

The closed system has an explicit boundedness argument. On a finite interval with $I_i \leq I_{\max}$, choose numbers $V_\ell, U > 0$ containing the initial coordinates and satisfying Equation 11.

$$V_\ell \geq \frac{\kappa_\ell}{\varepsilon_\ell}, \quad Y = \sum_\ell c_\ell V_\ell, \quad U^3 + [a - b(\sigma + Y)]U \geq I_{\max}, \quad B = [0, U]^n \times \prod_\ell [0, V_\ell]^n. \quad (11)$$

At a zero coordinate, diffusion, input and renewal point into the nonnegative carrier. At an upper support face $v_{\ell i} = V_\ell$, diffusion is nonpositive and the remaining derivative is at most $-\varepsilon_\ell V_\ell + \kappa_\ell \leq 0$. At an upper activity face, the derivative is at most $U[-a + b(\sigma + Y) - U^2] + I_{\max} \leq 0$. Hence the box in Equation 11 is invariant by first-contact comparison. Smooth state dependence gives local uniqueness, and the bounds prevent finite-time escape. The solution is global and absolutely continuous through input changes. This establishes the mathematical realization for the prescribed inputs; it does not establish the availability of a physical replacement procedure.

9.3 Causal Support Memory

Classical linear-system analysis already distinguishes a state realization, its input-output response and the contribution of an initialized state [11, pp. 156, 158]. The following derivation applies that machinery to the declared support ports.

Let $E_\ell(t) = \exp(-A_\ell t)$. These matrices preserve the nonnegative cone. To see this directly, choose α large enough that $\alpha \text{Id}_n - A_\ell$ is entrywise nonnegative. The series for its exponential is entrywise

nonnegative, and multiplication by $e^{-\alpha t}$ gives $E_\ell(t)$. Moreover, $L_G \mathbf{1} = 0$, so $E_\ell(t) \mathbf{1} = e^{-\varepsilon_\ell t} \mathbf{1}$. Each row consequently sums to $e^{-\varepsilon_\ell t}$, and the induced infinity norm of $E_\ell(t)$ equals that value.

Multiplying each support equation by its integrating factor and integrating gives Equation 12.

$$v_\ell(t) = E_\ell(t)v_{\ell 0} + \kappa_\ell \int_0^t E_\ell(t-s)r(s) ds. \quad (12)$$

Equation 12 separates the remaining contribution of initialized support from the response to later incoming activity. Summing through the output coefficients yields the boundary response in Equation 13.

$$\begin{aligned} y(t) &= g_{\mathcal{M}}(t) + \int_0^t K_{\mathcal{M}}(t-s)r(s) ds, \\ g_{\mathcal{M}}(t) &= \sum_{\ell} c_\ell E_\ell(t)v_{\ell 0}, \quad K_{\mathcal{M}}(t) = \sum_{\ell} c_\ell \kappa_\ell E_\ell(t). \end{aligned} \quad (13)$$

Here $g_{\mathcal{M}} : [0, \infty) \rightarrow \mathbb{R}_{\geq 0}^n$ is the free initial response, and $K_{\mathcal{M}} : [0, \infty) \rightarrow \mathbb{R}_{\geq 0}^{n \times n}$ is the matrix-valued memory kernel. Its entry (i, j) specifies the retained response at output location i to earlier input at location j . Equation 13 therefore retains timing and regional alignment. A total response or a single stationary gain can omit either.

The initial response is substantive. Equality of $y(0)$ only fixes $g_{\mathcal{M}}(0)$, not its later values. Internally allocated support with different decay rates can give equal current output and different future output before any new input occurs; Section 11 gives the corresponding observable-closure counterexample. Preparing a replacement with the appropriate $g_{\mathcal{M}}$ can therefore require an actual state-setting or handover process. An existence proof for such a state does not deliver the resources or permission needed to reach it.

For this open subsystem, equality of g and K is exactly equality of its output map on the declared family of all continuous nonnegative port inputs. Sufficiency follows by substitution in Equation 13. For necessity, zero input establishes equality of g . A constant input $r = \beta e_j$, with $0 < \beta \leq 1$ and e_j the j th coordinate vector, then establishes equality of the integrals of the j th kernel columns. Differentiation gives equality of those columns at every time because the kernels are continuous. Repeating for each j gives equality of the matrices. The probing family belongs to the formal input-output comparison; a real inquiry must separately establish which such probes it can realize.

9.4 Exact Contextual Substitution

Proposition 9.1 (Exact Boundary Response Substitution). *Let two initialized support realizations have the same input and output port types and satisfy $g_{\mathcal{M}} = g_{\widetilde{\mathcal{M}}}$ and $K_{\mathcal{M}} = K_{\widetilde{\mathcal{M}}}$. Coupling either to Equation 10, with the same initial activity and prescribed input, gives the same activity and boundary-output histories. The same conclusion holds for a deterministic surrounding context coupled solely through those ports, with the same initial context state and decision convention, whenever both interconnections have unique well-posed evolution.*

Proof. Take the original interconnection and its incoming history r . Drive the replacement's internal channels with that history using Equation 12. Equality of g and K gives the original output y . The original activity history therefore satisfies the replacement's surrounding equation as well, with the same input I and initial activity. Together with the constructed replacement channels, it is a solution of the replacement interconnection. Uniqueness identifies it with that interconnection's solution.

For the stated broader context, the same construction supplies exactly the history of boundary outputs received by the original context. Its state and decisions therefore satisfy the same deterministic context rule and produce the same incoming history. The constructed pair is again a solution of the new interconnection, and its assumed uniqueness gives the conclusion. This argument requires the context to use only the declared ports; an additional dependence on an internal channel would violate its premise. $\square$

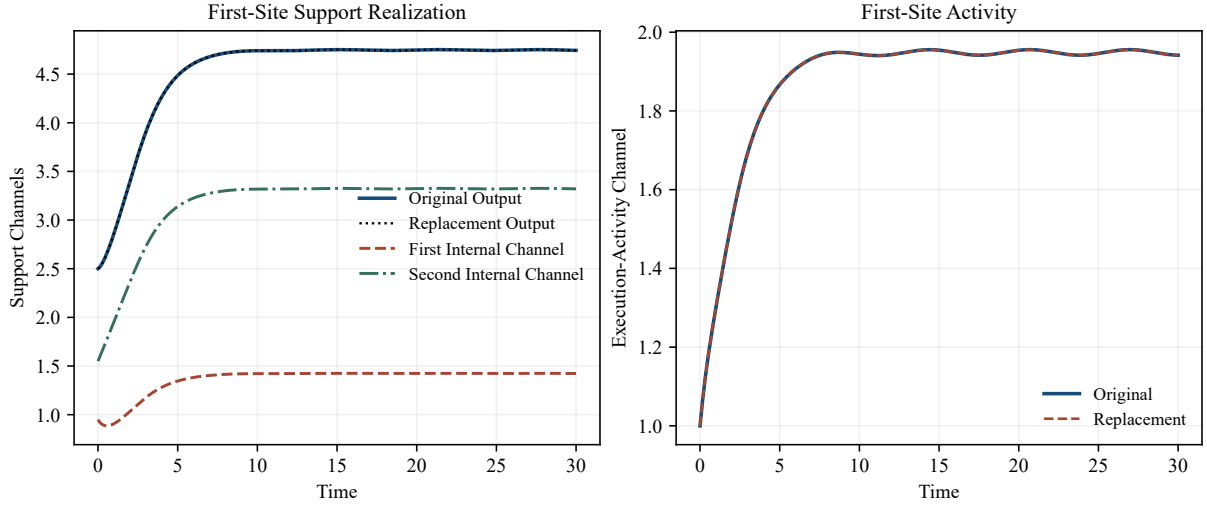


Figure 3: Different-Dimensional Support Realizations. A single support channel and a two-channel split have matching causal kernels and initialized free responses. Their internal allocations differ while the retained output and nonlinear activity histories coincide. The illustrated fields are dimensionless model channels; the equality does not establish an available handover procedure or unchanged internal burdens.

Proposition 9.1 permits different internal dimensions. For example, replace a channel with parameters $(\varepsilon, d, \kappa, c)$ by two channels with the same ε, d, c , renewal coefficients $\theta\kappa$ and $(1 - \theta)\kappa$, and initial states θv_0 and $(1 - \theta)v_0$, where $0 < \theta < 1$. Their summed kernel and free response equal those of the original. More generally, initial states $\theta v_0 + z$ and $(1 - \theta)v_0 - z$ work whenever both are nonnegative. The redistribution z changes internal allocation while leaving its sum, and hence the full port response, unchanged. This is a comparison between initialized realizations, not merely an invertible coordinate renaming.

Figure 3 illustrates that construction in a nonlinear spatial context. Internal traces differ, while their output and the surrounding activity agree. The agreement preserves any target depending only on those retained histories. It leaves the identity of the internal bearers, their burdens and their historical relations open. A participant whose task depends on an internal allocation needs a richer interface and a different comparison.

The context premise includes practical readout and timing conventions. A nominally identical feedback rule is insufficient if the participant can no longer interpret its signal, receive it in time, or execute the associated operation after replacement. Those changes belong to the historical substitution problem even though the present response equations leave them fixed. The proposition supplies a positive dynamical bridge when that interface is complete for the selected context. It does not decide that the chosen interface is complete.

9.5 Regional Restriction Data

The kernel formula was derived on the specified full graph. Restricting observation to a smaller region does not automatically produce the same formula with the exterior deleted. For a site i inside a region R , activity restriction retains $D_u \sum_{j \notin R} w_{ij}(u_j - u_i)$, and channel ℓ retains $d_\ell \sum_{j \notin R} w_{ij}(v_{\ell j} - v_{\ell i})$. These are actual crossing histories. A regional response can incorporate them as declared incoming ports, or an application can justify a closed boundary. Dropping them without either step changes the dynamics.

This restriction discipline follows the prior field construction [7, pp. 27–28, 33–35]. It matters especially for substitution: a replacement that matches an internally computed kernel can still alter an omitted exchange with surrounding regions. Equality of the selected response is then a correct but incomplete comparison. The next section uses a fully specified two-site example to locate that incompleteness and distinguish exact substitution from one-way and approximate preservation.

10 Field Substitution Conditions

Exact boundary response provides one sufficient substitution condition. A practical comparison can possess weaker information: aggregate responses, an ordered response bound, or a finite error estimate. This section distinguishes those cases within the nonlinear field of Section 9. It first proves that a complete aggregate response can omit the regional alignment needed for continuation. It then derives one-way preservation in a cooperative context and a conservative finite-horizon approximation bound. Each result identifies the context and target to which it applies.

The distinction between response timing and stationary support strength is already inherited. The reactivation paper shows that identical stationary parameters can coexist with different support-building times and different continuation histories [9, Section 8]. The counterexample below concerns a different omission: two support arrangements agree on every aggregate response to a common imposed input, while their local responses support different nonlinear continuations. Its purpose is to test the completeness of the declared interface.

10.1 Aggregate-Matched Support Replacement

Take two sites with unit edge weight, one support channel, $c = 1$, $\varepsilon = 1$ and $\kappa = 6$. The original realization has support diffusion $d = 0$; the replacement has $d = 2$. Their Laplacian and kernels are given by Equation 14.

$$\begin{aligned} L_G &= \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, & K_0(t) &= 6e^{-t} \text{Id}_2, \\ K_2(t) &= 3e^{-t} \begin{pmatrix} 1 + e^{-4t} & 1 - e^{-4t} \\ 1 - e^{-4t} & 1 + e^{-4t} \end{pmatrix}. \end{aligned} \tag{14}$$

To derive Equation 14, the constant vector is a Laplacian eigenvector with eigenvalue zero, and $(1, -1)$ has eigenvalue two. The replacement's generator $\text{Id}_2 + 2L_G$ therefore has eigenvalues one and five. Resolving its exponential into those two eigenspaces gives the displayed entries. The original kernel follows from its generator Id_2 .

Choose the same initial support $v_0 = (5/2, 0)$ in both realizations. Summing the two outputs gives the identities in Equation 15.

$$\mathbf{1}^\top K_0(t) = \mathbf{1}^\top K_2(t) = 6e^{-t} \mathbf{1}^\top, \quad \mathbf{1}^\top g_0(t) = \mathbf{1}^\top g_2(t) = \frac{5}{2}e^{-t}. \tag{15}$$

Equation 15, together with the convolution formula, proves equality of aggregate outputs for every common imposed vector input r . This is stronger than a matching snapshot or matching stationary gain. The local kernel entries nevertheless differ. Support originating at one location remains there in the original realization and spreads to the other location in the replacement.

Now use the nonlinear context with $a = 2$, $b = \sigma = 1$, $D_u = 0$, $I = 0$ and $u(0) = (1, 0)$. Activity at the second site remains zero because its equation has a factor u_2 and receives no external input. The selected target is continuation of activity $u_1 \geq 1$ at the first site for all subsequent times. Interpreting that target as continuation of a linguistic operation still requires its independently established realization; the field floor alone supplies no such interpretation.

Proposition 10.1 (Aggregate Response Substitution Obstruction). *The original realization satisfies the selected activity target. In the replacement, activity and support tend to zero, despite the aggregate input-output equality in Equation 15.*

Proof. In the original system, v_2 also stays zero. The remaining equations are $\dot{u}_1 = u_1(-1 + v_1 - u_1^2)$ and $\dot{v}_1 = -v_1 + 6h(u_1)$. The lower faces $u_1 = 1$, $v_1 = 5/2$ have derivatives at least $1/2$ and $1/2$, respectively. With upper bounds from Equation 11, their rectangle is forward invariant. The stipulated initial state therefore retains $u_1 \geq 1$.

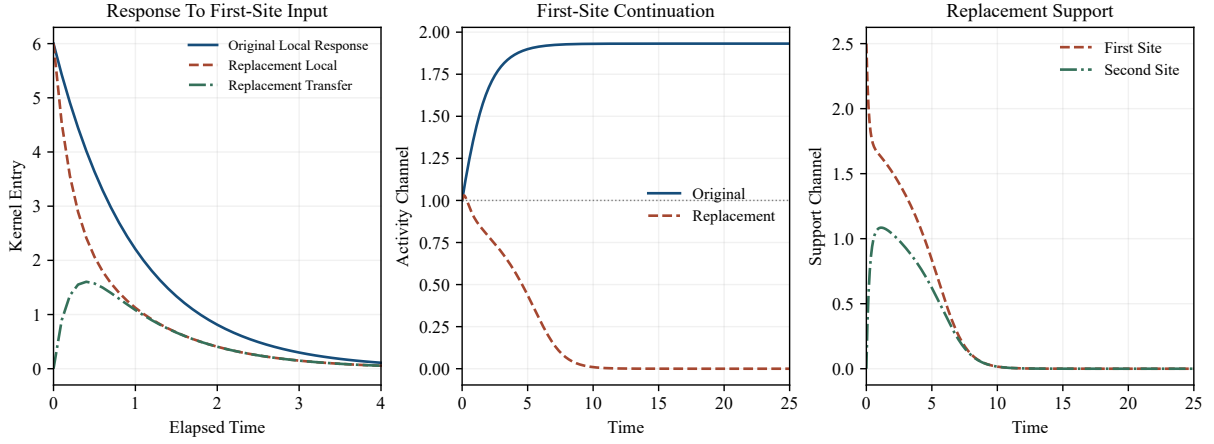


Figure 4: Aggregate-Matched Field Substitution. The replacement redistributes the response to a local input while retaining its aggregate kernel. In the specified nonlinear context, the original arrangement sustains first-site activity and the replacement extinguishes it. Kernel equality concerns a common imposed input; the later endogenous input histories differ. The example tests interface completeness, not the value of connection in general.

For the replacement, the active subsystem is Equation 16.

$$\dot{u}_1 = u_1(-1 + v_1 - u_1^2), \quad \dot{v}_1 = -3v_1 + 2v_2 + 6h(u_1), \quad \dot{v}_2 = 2v_1 - 3v_2. \quad (16)$$

At equilibrium, Equation 16 requires $v_2 = 2v_1/3$ and $v_1 = (18/5)h(u_1)$. If $u_1 > 0$, putting $w = u_1^2$ gives $w^2 - (8/5)w + 1 = 0$. Its discriminant is $-36/25$, so it has no real root. The only nonnegative equilibrium is consequently the zero state.

Global extinction requires more than this equilibrium calculation. The system is cooperative: its off-diagonal derivatives are nonnegative, including u_1 , $6h'(u_1)$ and the two exchange coefficients. Its flow preserves coordinatewise order by first-contact comparison. Choose a support bound $V \geq 6$ above both initial support coordinates, and an activity bound U above the initial activity with $U^2 \geq V - 1$. At the upper corner (U, V, V) all derivatives are nonpositive. Its trajectory remains below that constant upper bound by comparison. If ϕ_t denotes the flow and c that corner, then $\phi_s(c) \leq c$ implies $\phi_t(\phi_s(c)) \leq \phi_t(c)$. The semigroup identity makes the upper trajectory coordinatewise nonincreasing.

Positivity gives a lower bound, so that trajectory has a limit. For each fixed s , continuity of the flow gives $\phi_s(\lim_{t \rightarrow \infty} \phi_t(c)) = \lim_{t \rightarrow \infty} \phi_{t+s}(c)$, equal to the same limit. It is therefore an equilibrium and must be zero. Every nonnegative trajectory below the selected corner lies between zero and that upper trajectory. Thus it also tends to zero, including the stipulated initial state. This completes the extinction argument. $\square$

The comparison method in Proposition 10.1 extends the cooperative upper-bound argument already used for regional extinction [9, Section 9]. Here the support is redistributed inside an explicitly modeled two-site realization. The conclusion does not infer a general harmful effect of diffusion or connection. A different activity context could use the redistributed support, and a different target could evaluate that continuation differently.

The exact substitution proposition remains intact. This context receives the two local outputs separately, whereas Equation 15 retains only their sum. The comparison has omitted a port distinction that the context actually uses. Once the distinct nonlinear activity histories feed back, their incoming signals $h(u)$ differ as well; their aggregate closed-loop outputs need not remain equal. The equality proved above concerns common imposed inputs. Figure 4 displays both that response comparison and the resulting nonlinear continuation contrast.

10.2 Ordered Support Responses

A replacement can preserve a selected lower activity requirement without matching the original response exactly. Define componentwise order for vectors and entrywise order for matrices. Consider two initialized realizations on the same external graph. Their internal dimensions and channel parameters may differ. Assume the ordered-response conditions in Equation 17.

$$g_{\widetilde{\mathcal{M}}}(t) \geq g_{\mathcal{M}}(t), \quad K_{\widetilde{\mathcal{M}}}(t) \geq K_{\mathcal{M}}(t) \quad (t \geq 0), \quad \widetilde{u}(0) \geq u(0). \quad (17)$$

Equation 17 is a strong response comparison. A greater total support output is insufficient: every relevant kernel entry must satisfy the order. The two kernels in Equation 14, for example, are not ordered in this way; redistribution decreases diagonal entries while increasing off-diagonal entries.

Proposition 10.2 (Ordered Support Substitution). *Under Equation 17, the activity contexts with the same a, b, σ, D_u, L_G and the same prescribed input $I(t)$ satisfy $\widetilde{u}(t) \geq u(t)$ for every time. Hence every upward-closed target of their activity histories satisfied by the original is satisfied by the replacement.*

Proof. As long as the activity histories are ordered through time t , monotonicity of h and the nonnegative kernels give $\widetilde{y}(t) \geq y(t)$ by Equation 13. At a first contact of ordered activity coordinates, say $\widetilde{u}_i = u_i$, all neighboring differences remain nonnegative. The diffusion contribution to the derivative difference is then nonnegative. The own-coordinate loss and cubic terms agree, while the support contribution is $bu_i(\widetilde{y}_i - y_i) \geq 0$.

To make the weak comparison rigorous, add a positive constant η to every replacement initial activity coordinate and to its prescribed input. Before any first contact, the preceding history comparison applies. At that contact the additional input makes the derivative difference strictly positive, contradicting approach to contact from above. Apply this argument between input discontinuities; the fields remain continuous there. For each fixed finite horizon, the bounds in Equation 11 can be chosen uniformly for $0 < \eta \leq 1$. Continuous dependence of the two finite-dimensional systems then permits $\eta \downarrow 0$, proving the desired order. Since the horizon was arbitrary, it holds for all forward times.

An upward-closed activity-history target contains a history whenever it contains a pointwise smaller one on the same sites and times. Applying this definition to the proved order gives the final statement. $\square$

Proposition 10.2 can transfer a previously justified activity floor. It does not transfer arbitrary operation domains or targets. A task imposing upper workload limits, preserving an opportunity to refuse, or retaining a choice among several activities need not be upward closed in these coordinates. More activity is therefore not a general measure of linguistic or normative improvement. Even for a lower-floor target, the realization connecting activity to the selected operation remains necessary.

The context restriction also matters. Allow the delivered input at one site to follow the decreasing feedback rule $I(y) = 1/(1+y)$. Compare equal kernels but free initial responses $g_0(t) = 0$ and $g_1(t) = e^{-t}$, with initial activity zero. Such responses arise from the same channel parameters and different initial support. At time zero the multiplicative activity term vanishes. The original activity derivative is one and the replacement's is one-half. The latter activity is therefore smaller for sufficiently small positive times, even though the support response order holds. This context lies outside the prescribed-input comparison. Exact response equality would still be sufficient in that well-posed port-only context, but the one-way order condition is not.

10.3 Finite-Horizon Response Errors

A physical replacement may approximate a response or initialize its support only approximately. A finite-horizon estimate makes the resulting claim different from exact equivalence. Keep the same graph, context parameters and prescribed input, and suppose both compared solutions remain in $[0, U]^n$ with outputs in $[0, Y]^n$ on $[0, T]$. Use the vector infinity norm and its induced matrix norm. Define the errors

and integrated response bound by Equation 18.

$$\begin{aligned} \eta_0 &= \|u(0) - \tilde{u}(0)\|_\infty, & \eta_g &= \sup_{0 \leq t \leq T} \|g(t) - \tilde{g}(t)\|_\infty, \\ \eta_K &= \int_0^T \|K(t) - \tilde{K}(t)\|_\infty dt, & M_T &= \int_0^T \|\tilde{K}(t)\|_\infty dt, & \eta &= \eta_g + \eta_K. \end{aligned} \quad (18)$$

In Equation 18, untilded and tilded responses refer to the original and replacement realizations. The free-response error includes initialization. Matching the kernel while preparing the wrong stored-support state can leave η_g positive.

The scalar derivative of h is $2x/(1+x^2)^2$, whose maximum on the nonnegative line is $L_h = 3\sqrt{3}/8$, attained at $x = 1/\sqrt{3}$. Since $0 \leq h \leq 1$, subtracting the two convolution expressions and adding and subtracting $\tilde{K} * h(u)$ gives Equation 19.

$$\|y(t) - \tilde{y}(t)\|_\infty \leq \eta + L_h \int_0^t \|\tilde{K}(t-s)\|_\infty \|u(s) - \tilde{u}(s)\|_\infty ds. \quad (19)$$

The kernel-error term in Equation 19 uses the bound one on the incoming renewal signal. This is one reason the declared channel law matters to the estimate.

Proposition 10.3 (Bounded Field Substitution Error). *Under the stated finite-horizon assumptions, put $L_0 = a + b(\sigma + Y) + 3U^2$ and $C = L_0 + bUL_hM_T > 0$. Then Equation 20 holds for $0 \leq t \leq T$.*

$$\sup_{0 \leq s \leq t} \|u(s) - \tilde{u}(s)\|_\infty \leq \eta_0 e^{Ct} + \frac{bU\eta}{C} (e^{Ct} - 1). \quad (20)$$

Proof. For the local activity term $F_i(u, y) = u_i[-a + b(\sigma + y_i) - u_i^2]$, the absolute derivative with respect to u_i is at most L_0 on the stipulated box, and that with respect to y_i is at most bU . The activity diffusion semigroup $e^{-D_u L_G t}$ is nonnegative and preserves $\mathbf{1}$, so it has infinity norm one. Subtract the two variation-of-constants formulas for activity and use these bounds. Write $E(t) = \sup_{0 \leq s \leq t} \|u(s) - \tilde{u}(s)\|_\infty$. Equation 19 bounds the output error at time s by $\eta + L_h M_T E(s)$. Thus Equation 21 follows.

$$E(t) \leq \eta_0 + \int_0^t [CE(s) + bU\eta] ds. \quad (21)$$

The right side of Equation 21 is nondecreasing in t , which justifies taking the running supremum. If it is denoted by $Q(t)$, then $E \leq Q$ and $\dot{Q} \leq CQ + bU\eta$, with $Q(0) = \eta_0$. Multiplication by e^{-Ct} and integration give Equation 20. $\square$

Proposition 10.3 gives a conservative error certificate. If the original activity history exceeds a justified floor by more than its right side at T , the replacement retains that floor over the horizon. A target touching its boundary, or an operation whose domain changes discontinuously under small perturbations, needs additional reasoning. Approximate trajectory agreement alone does not preserve such a domain.

The exponential estimate can become too large to be useful over long horizons. It is neither an infinite-time preservation result nor a practical error tolerance supplied by the model. An application must justify its measurement norm, target margin and horizon. The value of the estimate is that it keeps kernel mismatch, initialization mismatch and contextual amplification visible in one inspectable comparison. Exact equality, one-way response order and approximate finite continuation are consequently three distinct substitution claims, each with its own evidential and interpretive obligations.

11 Operation Algebra Transport

An algebraic comparison can make the composition of operations inspectable, provided that its map class and domains are stated. This section develops two possible realizations. The first represents deterministic history extension by partial isometries and asks when a replacement transports selected words. The second represents field dynamics by pullback on continuous observables and asks whether a boundary record has an autonomous update. The two constructions address different questions. Neither is assumed to encode every linguistic operation.

11.1 History Extension Operators

Let $\mathcal{H}_i$ be a countable family of declared finite histories for realization i . A history retains its operation sequence and successive configurations, including the selected historical anchor. It is not identified merely with its terminal configuration. For this subsection, each available action has a deterministic history extension $T_{i,a} : D_{i,a} \rightarrow \mathcal{H}_i$. Appending the labeled occurrence preserves the preceding history, so the extension is injective on its domain. This is a stricter specialization than the set-valued model of Section 6.

The Hilbert space $\ell^2(\mathcal{H}_i)$ has orthonormal basis $\{e_h : h \in \mathcal{H}_i\}$. For $D \subseteq \mathcal{H}_i$, let P_D project onto the closed span of its basis vectors. Define the extension operator on basis elements, then extend linearly and continuously.

$$S_{i,a}e_h = \begin{cases} e_{T_{i,a}h}, & h \in D_{i,a}, \\ 0, & h \notin D_{i,a}. \end{cases} \quad S_{i,a}^*S_{i,a} = P_{D_{i,a}}. \quad (22)$$

Injectivity proves the identity in Equation (22): the images of distinct available basis histories are orthonormal, while unavailable basis histories are annihilated. Thus the norm is at most one and the extension is a partial isometry. Without injectivity, a generic map on an infinite basis need not have this bounded operator realization. Retaining the occurrence history supplies the stated property here.

Zero denotes non-execution in this representation. It is not an empirical state of silence or loss. A recorded failed request can instead be a distinct history extension with its own domain. This choice follows the same distinction between request refusal and unavailable action used earlier.

Let $j : \mathcal{H}_0 \rightarrow \mathcal{H}_1$ be an injection preserving the selected anchor and public record. It induces an isometry $V : \ell^2(\mathcal{H}_0) \rightarrow \ell^2(\mathcal{H}_1)$ by $Ve_h = e_{j(h)}$. For the same public translator τ , a sufficient transport condition is stated on the source operation domain.

$$\begin{aligned} P_{D_{1,\tau(a)}}VP_{D_{0,a}} &= VP_{D_{0,a}}, \\ S_{1,\tau(a)}VP_{D_{0,a}} &= VS_{0,a}. \end{aligned} \quad (23)$$

The first line of Equation (23) certifies availability of the translated operation on the embedded source domain. The second line certifies its exact historical effect there. This condition allows additional replacement operations and histories outside the embedded sector. It is stronger than merely obtaining a matching public output.

11.2 Composite Domains

For a word $w = a_1 \cdots a_m$, execution proceeds from left to right. Its domain $D_{0,w}$ consists of histories for which every prefix executes and leaves the next action available. The composite operator is $S_{0,w} = S_{0,a_m} \cdots S_{0,a_1}$. The translated word follows the same convention.

Proposition 11.1 (Domain-Sensitive Word Transport). *If Equation (23) holds for every letter, then every source history in $D_{0,w}$ has an executable translated word, and the following equality holds.*

$$S_{1,\tau(w)}VP_{D_{0,w}} = VS_{0,w}. \quad (24)$$

For selected terminal-history target subsets $G_i \subseteq \mathcal{H}_i$, let $Q_i = P_{G_i}$. If these target projections also satisfy $Q_1VQ_0 = VQ_0$, target membership at a source terminal history is retained by the translated terminal history.

Proof. The empty word is the identity. For an available first action, Equation (23) gives its availability and sends e_h to the embedded basis vector of the extended source history. That history belongs to the next required domain by the definition of $D_{0,w}$. Applying the same argument to the next letter proves availability and equality for the next prefix. Induction gives Equation (24) on every basis vector in the domain. Both sides vanish on its complement, and continuity extends the identity to the Hilbert space. The target equation fixes every embedded source target vector under Q_1 , giving the last claim. $\square$

Proposition 11.1 is an exact deterministic certificate. The embeddings and domain conditions must be supplied from a justified history comparison; they cannot be inferred from similar names for operations. A fixed word also differs from a policy that responds to observations. If each branch of a deterministic observation policy obeys these conditions, the branchwise argument can be repeated. For genuine set-valued outcomes the relational proof of Section 7 is the appropriate stated result.

The generated operator algebra can retain more information than the diagonal record algebra. Products retain selected order effects; adjoints encode a mathematical relation to predecessor histories. An adjoint is not automatically an available reversal operation. Likewise, a linear combination of history vectors is part of this representation, not evidence that participants physically occupy a quantum superposition. The model's use of operator tools remains separate from that stronger ontology.

11.3 Compression Defects

An injection between history spaces also induces a natural compression of replacement observables. Set $P = VV^*$, the projection onto the embedded source sector, and let $\Phi(T) = V^*TV$ for bounded operators T on the replacement space. This map preserves the identity and adjoint and is positive. It is generally not multiplicative.

For bounded replacement operators A_1, B_1 , inserting P between them yields the exact defect.

$$\Phi(A_1B_1) - \Phi(A_1)\Phi(B_1) = V^*A_1(I - P)B_1V. \quad (25)$$

Equation (25) follows because $\Phi(A_1)\Phi(B_1) = V^*A_1VV^*B_1V$. It identifies an excursion outside the embedded sector between the two operations. Separate compressed effects can lose that excursion while the compressed composite retains its return.

For a finite witness, embed $\mathbb{C}^2$ as the span of e_1, e_2 in $\mathbb{C}^3$. Take $A_1 = |e_1\rangle\langle e_3|$ and $B_1 = |e_3\rangle\langle e_1|$. Both compressed factors vanish. Their product is the projection onto e_1 , whose compression is nonzero. This is a direct matrix calculation, and it prevents compression from being called an algebra homomorphism without further assumptions.

The defect vanishes for a chosen product if $(I - P)B_1V = 0$, or if the return term annihilates that excursion. For a chosen star algebra, a reducing embedded subspace is a sufficient condition for multiplicativity of its compression. The exact intertwining conditions already give forward invariance for selected executable source histories. They do not, by themselves, prove a reducing-subspace condition for every adjoint and every operator in an enlarged replacement algebra.

The philosophical consequence is specific. A comparison of isolated operations can miss a new intermediate route whose return matters to the practice. Conversely, not every matrix excursion represents an admissible linguistic act. That interpretation requires an operation realization and its domain. The calculation provides a test of an algebraic claim, not an independent account of meaning.

11.4 Boundary Observable Closure

The field model supplies a second algebraic question. Can the present boundary record evolve autonomously, or must it retain additional support history? Close the chosen deterministic field and context into an autonomous system on a compact forward-invariant carrier $\mathcal{B}$, with jointly continuous forward flow φ_t . Constant inputs and a compact context realization give possible examples. Arbitrary externally prescribed time dependence is not automatically covered by this autonomous specialization.

The algebra $C(\mathcal{B})$ consists of complex-valued continuous functions with the supremum norm, pointwise multiplication and complex conjugation as adjoint. The pullback $\alpha_t f = f \circ \varphi_t$ is a unital star endomorphism. The semigroup law follows from the flow law; strong continuity follows from joint continuity and compactness. Invertibility is not required.

Let $p : \mathcal{B} \rightarrow Y = p(\mathcal{B})$ be a continuous boundary readout into a Hausdorff space. The corresponding record algebra is the pulled-back subalgebra.

$$\mathcal{A}_p = \{g \circ p : g \in C(Y)\} \subseteq C(\mathcal{B}). \quad (26)$$

Equation (26) specifies which observables are mathematically determined by that record. Actual interpretable readouts require a separate realization. It distinguishes the algebra from the full field carrier and makes a dynamic closure question precise.

Proposition 11.2 (Boundary Observable Closure). *The inclusion $\alpha_t(\mathcal{A}_p) \subseteq \mathcal{A}_p$ holds for every $t \geq 0$ if and only if $p(x) = p(x')$ implies $p(\varphi_t x) = p(\varphi_t x')$ for every t . Under these conditions, a continuous forward semigroup on Y is defined by $\psi_t(p(x)) = p(\varphi_t x)$.*

Proof. Suppose the algebra is invariant and $p(x) = p(x')$. For every $g \in C(Y)$, the function $\alpha_t(g \circ p)$ belongs to $\mathcal{A}_p$, so it has equal values at x, x' . Continuous functions separate points of the compact Hausdorff space Y ; the two evolved readouts therefore agree.

Conversely, fibre compatibility makes ψ_t well defined. The continuous surjection from compact $\mathcal{B}$ to Hausdorff Y is a quotient map. Since $\psi_t \circ p = p \circ \varphi_t$ is continuous, ψ_t is continuous. On each compact time interval, the map $\text{id} \times p$ is likewise a compact-to-Hausdorff quotient map, so the descended time-dependent map is jointly continuous. The flow identities descend through p . Finally, $\alpha_t(g \circ p) = (g \circ \psi_t) \circ p$ belongs to $\mathcal{A}_p$, proving invariance. $\square$

Proposition 11.2 is a continuous-observable version of the inherited continuation-consistency problem [6]. It is not a new general quotient theorem. Its field application clarifies why present output matching is too weak. At one site, take two support channels with decay rates 1 and 2, unit output weights, and otherwise fixed parameters. Let the readout retain u and $y = v_1 + v_2$. The initial channel allocations $(1, 0)$ and $(0, 1)$ have the same readout. Their output derivatives differ by one under the same incoming activity, because the decay term is $-v_1 - 2v_2$. Thus their later output differs, and the record algebra is not invariant.

The kernel parameters in this example are identical. What differs is the free initial response: one allocation contributes e^{-t} , the other e^{-2t} . The complete causal representation in Section 9 retains that distinction. If all channels share the same decay-diffusion matrix, their weighted sum instead satisfies a closed aggregate equation. That is a special closure result supplied by the law, not by the word boundary.

11.5 Algebraic Interpretation

The three algebraic conditions developed here have separate uses. Domain-sensitive intertwining transports selected deterministic histories. Multiplicativity of compression requires control of intermediate excursions. Dynamic closure of an observable algebra requires that its fibres remain indistinguishable under the chosen flow. None follows merely from equality of a boundary diagram.

These distinctions also preserve the historical dimension of the inquiry. A field state can summarize past influence adequately for a selected response while omitting the provenance required for attribution. A history embedding can retain that provenance while failing to make a new operation practically available. The GR formalism is useful when those responsibilities remain separately testable and their connections are argued, rather than when one algebraic equality is asked to settle them all.

12 Linguistic Support Tests

This section examines how the substitution criteria can guide the description of concrete linguistic activities. Three primary studies provide documented distinctions concerning material classification, conversational repair, and visible recognition. The replacement scenarios developed from them are constructed cases, not interventions reported by those studies. Their purpose is to expose what a preservation specification would need to retain and what evidence would be required to assess it. No case establishes a general ranking of technologies, interlocutors, or forms of support.

12.1 Color Classification Support

In Goodwin's archaeological episode, participants classify soil using a Munsell chart. He distinguishes three sign systems on the chart: color samples, coordinates organized through hue, chroma, and value, and color names. These support different operations. In particular, the sample allows a direct visual comparison that the phonetic or graphic form of a color name does not supply. The classification also enters an excavation record [5, pp. 1509–1511].

Consider a constructed replacement of the chart by a device that returns a color name after receiving a code. Fix the selected operation as *retrieving an already assigned name from a supplied code*. If the participant can enter the code, obtain the corresponding name, and correct an entry error, this replacement may satisfy the operation's requirements. There is no reason to reject it merely because it has a different physical carrier. The task-specific comparison can be positive without claiming that every earlier chart use survives.

Now change the selected operation to *jointly classifying an unfamiliar sample*. The device's code-to-name correspondence does not yet provide an action for comparing the sample with a reference. An observer may be able to supply the correct code, making the eventual output match the earlier result. That observer's intervention does not establish that the participants can perform the required classification through the proposed interface. It changes the source of the discrimination on which success depends. If an external classifier is part of the replacement, its own inputs, reliability assumptions, and challenge procedure belong in the declared support region.

A richer constructed replacement might display comparison samples, allow alternative selections, and preserve the participants' ability to revisit an entry. Such an arrangement has a plausible route to a positive assessment, although its actual adequacy would need evidence. The important distinction is between altering the carrier and eliminating an operation. A claim that the carrier is indispensable requires more than showing that one impoverished substitute fails. Conversely, a claim of successful substitution requires more than showing that the final color word can be produced somehow.

The common interface must also include the selected opportunity to challenge. Suppose both arrangements initially select the same name, but only one allows a second participant to point to a competing sample and request reconsideration. A test that ends at the first name will miss this difference. A test that includes the subsequent challenge can reveal it. The formal issue is not an ineffable quality beyond representation; it is an operation omitted from the earlier specification. Whether that operation belongs in the practical preservation target requires a substantive decision about the activity being maintained.

The historical interpretation remains distinct. A new display could retain the selected comparison and correction operations while changing the provenance of its reference values or the authority responsible for them. If provenance is relevant to the task, it must be represented and checked. If the bounded inquiry concerns only an agreed local operation, a successful result remains correspondingly limited. Functional adequacy does not silently settle continuity of the scientific practice that uses the support.

12.2 Repair Sequence Continuation

One extract analyzed by Bloch and Wilkinson contains two names in a speaker's turn. The recipient's repair initiation identifies a person reference as troublesome without specifying which name. The speaker repeats a larger portion of the turn and modifies both names; the recipient's uptake subsequently identifies the difficulty. Their study also examines longer repair sequences and emphasizes participants' contributions to their resolution. It does not test the efficacy of a replacement communication device [3, pp. 515–516, 521].

Construct a support that helps a speaker revise a short narrative. Its reference interface permits selection of a troublesome segment, repetition of a larger passage, and acceptance or rejection of a proposed interpretation. The replacement stores the same narrative but permits only whole-message replay. The two supports can produce exactly the same complete message in an uncomplicated demonstration. Their continuation capacities differ if the selected task requires revising a specific interpretation while preserving the rest of the account.

This difference should be formulated at the level of actions and their domains. Whole-message replay may remain an adequate action after a general request to hear the message again. It may be inadequate after a partner has singled out a detail whose revision requires selecting an alternative. A model that treats both requests as the same generic *failure* observation can erase the condition that makes the distinction important. Refining the observation record does not automatically solve the problem, but it makes the missing branch of the participant's strategy visible.

A positive replacement could use a different sequence of actions while retaining the selected capability. For example, the participant might navigate a menu before revising the troublesome segment. That construction would require a multistep adapter and an account of its accessibility. The one-step translator in Section 6 would not directly certify it. Enlarging the model to include navigation, intermediate observations, and possible failure would make the comparison more accurate. The need for such enlargement is a limitation of the chosen certificate, not evidence that every multistep substitute fails.

The case also exposes an ambiguity in the treatment of uncertainty. Suppose a replacement always approves the first candidate interpretation. The set of possible unsuccessful responses becomes smaller, so an outcome-based completion guarantee can improve. Yet a participant who wishes to reject a misinterpretation has lost a relevant operation. To assess that loss, rejection must be represented as an available participant choice, or its preservation must enter the target. Treating it only as noise that the replacement beneficially removes changes the object of preservation.

This is an analytical reason for keeping a participant's practical position visible inside a support comparison. The same person can be an uncontrolled source of responses for one narrowly defined task and a bearer of selected correction operations for another. A model may legitimately adopt either perspective, but it cannot claim the guarantees of the second after proving only the first. For the present linguistic object, preserving continuation usually requires enough detail to identify who can initiate, diagnose, revise, and acknowledge the repair. The exact allocation is part of the proposed application rather than an empirical universal established by the illustrative model.

12.3 Visible Recognition Signals

Kendrick describes an episode in which a recipient visibly displays nonrecognition after a place reference. The speaker then repairs the reference, and subsequent uptake closes the sequence. The broader discussion treats visible actions and delayed verbal repair as potentially interacting resources. It distinguishes what a delay publicly permits from a claim about why an individual delayed; the study does not establish a universal conversational timeout [12, pp. 11–12].

Consider a constructed change from a communication channel that makes a declared recognition signal available to one that transmits only words. The task is to help an interlocutor identify a previously unfamiliar reference while retaining a chance to request clarification. If the recipient has an accessible verbal alternative and sufficient opportunity to use it, losing the visual signal need not destroy the selected continuation. The replacement may preserve the task through a different operation. A successful comparison would need to establish that the alternative is available in the relevant states, rather than assume that any participant can switch channels without cost or instruction.

Now stipulate that the replacement automatically advances after the initial reference and provides no route to reopen it. A transcript comparison ending before the recipient's next contribution can show identical words in both arrangements. The continuation test distinguishes them because the second arrangement lacks the required clarification branch. Merely storing a record of the initial reference does not restore that branch when the practical task requires correction before the advance becomes consequential.

The test need not prefer a visible signal in every context. A constructed textual interface could retain a request for clarification longer than a fleeting bodily signal remains noticeable. For a participant who can use the textual interface, that persistence might enlarge the selected action domain. Such a possibility prevents the case from becoming a general argument that a particular medium possesses an irreplaceable essence. Its point is to compare the operations made available by the whole arrangement, including timing, access, and the partner's response rule.

The surrounding context matters directly. A partner who waits for an explicit acknowledgment produces a different comparison from a partner who proceeds whenever no challenge is immediately audible. A replacement certificate proved for the first rule does not apply automatically to the second. If participants can renegotiate that rule, the negotiation itself is an additional operation requiring representation. Otherwise a model could credit the replacement with success that actually depends on an unacknowledged adaptation of the surrounding practice.

The memory model in Section 9 addresses a related formal distinction, although it is not fitted to this interaction. The present boundary appearance does not determine the later response unless the retained response structure and initialization support that inference. In the constructed recognition case, the analogous modeling obligation is to record whether an unresolved request remains actionable after the visible display has disappeared. The analogy identifies a question for representation; it does not derive a psychological mechanism or a response coefficient from the conversational study.

12.4 Handover Evaluation

Each constructed case permits a distinction between testing a prepared replacement and testing the transition that installs it. A participant may use a new classification interface effectively after training while being unable to perform the same task during migration. A repair menu may have suitable actions once a narrative has been segmented, although no available participant can perform that segmentation. A recognition channel may support an explicit acknowledgment convention after it has been agreed, while the current partners have not yet established that convention. These are different initial conditions for the comparison.

A responsible application can therefore conduct two linked assessments. The first fixes the prepared configurations and checks the selected continuation relation. The second asks whether an admissible handover can reach those configurations while respecting the relevant ongoing tasks. The first can be successful and the second unsuccessful. That combination does not invalidate the conditional preservation theorem; it restricts what the theorem currently warrants about practical replacement. It also identifies where further design or support is required.

An assessment should preserve negative and positive evidence at the same resolution. A failure to challenge a classification is evidence against a challenge-preserving specification in that tested setting. It does not establish the failure of simple name retrieval. A successful revision after a fully prepared handover supports the tested continuation but does not establish unassisted migration. Keeping these conclusions separate makes the inquiry more informative than issuing one undifferentiated verdict of equivalence or loss.

The cases thus give the formal claims a concrete interpretive burden. The support boundary must include the resources credited with producing success. The operation repertoire must include the selected correction and extension opportunities. Observations must be available to the participants who use the policy, and the context rule must represent the responses on which the comparison depends. These conditions permit substitution to be investigated without treating material identity as necessary or output similarity as sufficient. Whether a particular replacement satisfies them remains a question for the specified practice and its participants.

13 Substitution Objections

The substitution account makes its strongest claim when a specified replacement retains usable continuation across a defensible context class. That formulation also concentrates its vulnerabilities. This section develops objections concerning the selection of contexts, constitutive relations, hidden adapter work, transition history, changing repertoires and the explanatory force of the formal results. Each objection tests a substantive premise of a positive preservation claim. The replies identify conditions for a narrower defensible conclusion rather than treating formal correctness as sufficient evidence of practical adequacy.

13.1 Selected Contexts

A critic can object that contextual preservation is too easy to manufacture. An assessor selects requests the replacement already handles, declares those requests the relevant contexts and concludes that continuation is preserved. A playback arrangement can pass if the test includes only familiar accounts. The same arrangement can fail when participants question an annotation or introduce an unfamiliar use. The formal test then appears to reward an impoverished description of the activity.

The objection concerns the justification of the comparison domain. A theorem covering a declared class remains conditional on that class, even when its internal quantifiers are exact. Expanding the number of tests does not repair a systematic omission if every test shares the same restricted structure. For example, many varied requests for examples can leave the ability to challenge an example entirely unexamined.

A defensible response begins with the activity's consequential alternatives. Participants can identify the kinds of interruption, disagreement and extension that the support is expected to handle. Historical episodes can reveal dependencies absent from an official specification. These sources do not produce an exhaustive universal class, but they provide reasons for including specific challenges. An assessment should retain those reasons alongside the class itself.

The boundary can also be criticized after a successful test. A newly encountered request may reveal that the original class was too narrow for the practical claim. The earlier conditional result need not be discarded; its application can be restricted while a broader inquiry begins. This is more informative than protecting the original claim by classifying every difficult encounter as irrelevant.

There is a corresponding danger of excessive demands. A comparison admitting unlimited disruption or removing every available support may defeat both arrangements. Such a result can identify an ambitious objective, but it does not establish that a modest replacement is useless. Context criticism should examine the intended undertaking in both directions: it should resist convenience-driven omissions and requirements disconnected from what participants are trying to preserve.

13.2 Constitutive Relations

A second objection challenges the functional framing itself. Some linguistic practices concern a relation to particular people, places or histories. Addressing an account to its original contributor can differ from obtaining an equally fluent account elsewhere. If the earlier relation partly constitutes the activity, replacement of that relation may change the activity even when selected response patterns are identical.

This objection establishes a genuine limit. A task defined as receiving a particular person's acknowledgment cannot be satisfied merely by generating the same words through another source. The relevant identity is part of the target. The framework should record that condition and the resulting failure of the proposed comparison, rather than weakening the target until replacement becomes possible.

The limit does not establish universal non-substitutability. Participants can also seek a transferable capacity: understanding a selected distinction, correcting its application or maintaining an accessible record of shared decisions. The prior source can remain historically significant while another arrangement supports those later acts. A claim about that retained role need not assert that the new relation duplicates the earlier one.

The taxonomy separates historical targets, retained interfaces and selected operation families [8, Sections 6–8]. That separation allows an assessment to say that a particular relation ends, a larger activity continues and some contributions remain historically attributable. These statements can all be true of the same transition. The philosophical work lies in establishing which bearer and which comparison answer the participants' concern.

The objection therefore remains an empirical and interpretive test of the functional description. An assessor should investigate whether the alleged replaceable role omits what makes the activity intelligible. Neither a broad declaration that meaning is relational nor a formal response match decides the matter. The target-specific account should explain why the retained relations suffice for the particular continuation being claimed.

13.3 Hidden Adapter Work

A third objection concerns the apparent location of the retained capacity. A replacement may pass its tests because another actor quietly performs the work it cannot do. A coordinator can reinterpret every difficult request, select the appropriate response and repair the result before participants see it. The visible interface then appears adequate while the supposed replacement contributes much less than the assessment suggests.

The existence of assistance does not invalidate supported capacity. Many activities are legitimately distributed. The problem is a false attribution of independence, or an omitted condition whose loss would defeat the claim. An adequate description includes the adapter within the tested arrangement and states the continuing work it performs. The bearer is then the supported configuration rather than an isolated device or procedure.

This can change the practical comparison sharply. If the new arrangement requires a scarce interpreter on every occasion while the old arrangement allowed participants to proceed themselves, both may reach the same selected endpoint. Their available domains, resilience to absence and distribution of work differ. Whether those differences defeat preservation depends on the declared capacity and context class; whether they are acceptable requires further judgment.

Information hidden inside the adapter creates a second risk. An analyst may know which internal replacement state is actual and choose a successful translation accordingly. A real coordinator may lack that knowledge. The construction must either provide an available observation or use the same admissible choice on indistinguishable cases. Otherwise the adapter is a description of successful outcomes rather than an executable procedure.

An explicit adapter consequently strengthens a substitution claim only when its prerequisites are included. Its timing, access, memory and interpretation are part of the relevant support. This is an application of the inherited distinction between representational transport and dynamical action [7, pp. 34–35]. A well-typed correspondence establishes a comparison resource. A warranted procedure establishes that someone can use it.

13.4 Transition Dependence

A fourth objection concerns the feasibility and effects of changeover. Matching two initialized systems gives no guarantee that the replacement's initial memory can be established. The new arrangement may require interpretations, permissions or resources that disappeared when the earlier support ended. Even where initialization is possible, transferring the work can interrupt the very capacity being preserved.

The boundary-memory construction makes this concern especially precise. Matching the stored-response term is a condition on the compared initial preparations. It does not identify who supplies that preparation or how it is produced. A practical claim must connect the specified state to an admissible history. Declaring the term equal without that connection would place a significant part of the preservation task in an unexplained initial condition.

An overlap period can be a positive remedy. Participants can use both arrangements while checking the meaning of transferred records and learning the new request conventions. That period changes the resources required by substitution and may create additional obligations. The comparison should include those conditions rather than treating them as negligible merely because the final arrangement is smaller.

The strength of the historical claim should also remain clear. A replacement that restores correction after a temporary interruption can satisfy an endpoint capacity target. It can fail a target requiring uninterrupted access throughout the transition. The earlier field-theoretic account already distinguishes anchored history from transformed continuation structure [7, pp. 46–48]; neither form of historical representation automatically supplies an uninterrupted practical corridor.

This objection leaves a useful division of labor. The current constructions can certify selected post-replacement continuations, while an assessment states whether the transition is established, hypothesized or still unavailable. A later practical workflow may investigate the missing steps. Treating that work as

unfinished is preferable to either denying the endpoint result or announcing a completed substitution from it.

13.5 Changing Repertoires

A further objection concerns generativity. If a model fixes its action alphabet and target in advance, it can preserve only operations already described. Linguistic activity can generate new distinctions, new forms of challenge and changed criteria of success. An account of substitution might therefore preserve a frozen selection while overlooking the capacity to transform the practice itself.

The objection limits the scope of a fixed-repertoire result. A bounded model can establish that selected clarification and correction operations survive replacement. It cannot thereby establish preservation of every operation participants may later form. Treating a generic action called *innovate* as a complete representation of that open possibility would merely conceal the problem in a label.

Some extension can nevertheless be represented within a declared operation family. An existing correction procedure can apply to an unfamiliar account without requiring a wholly new kind of act. A proposal operation can admit new data while retaining its established role. The distinction between a new instance and a new operation kind needs interpretation; novelty of the output alone does not decide it.

Where the repertoire itself changes, the comparison needs additional data. The field-theoretic source already allows operations to disappear, split or acquire descendants with different composability conditions [7, pp. 55–56]. A substitution inquiry can use that architecture to describe a changed grammar, but a prior fixed-interface certificate does not automatically cover it. The resulting extension should be argued on its own domain.

The present preliminary study remains useful if it states that boundary honestly. Retention of selected operations can protect meaningful possibilities even without guaranteeing indefinite future invention. It can also expose a restrictive replacement that preserves prepared answers while removing current correction. Such a finding concerns an identifiable generative loss without claiming an exhaustive measure of generativity.

13.6 Explanatory Contribution

A final objection asks what the construction explains beyond familiar refinement and response theory. Contextual comparison, policy simulation, memory kernels and partial operators are established mathematical resources. A new application vocabulary does not establish a new general result. The paper's contribution must therefore be assessed through the connected substitution question and the distinctions it makes operationally consequential.

The exact results can support that bounded purpose. Matching an immediate response or steady response can coexist with different subsequent support. Matching the full declared causal response, including its initialized contribution, preserves the selected nonlinear history in the well-posed port-mediated context of Section 9. Internal realizations can differ. A suitable one-way relation can transport bounded successful policies while a weaker demonstration cannot. These comparisons explain why particular evidence is insufficient and which stronger conditions can provide a positive certificate.

Their explanatory force remains conditional on the representation. If the chosen ports omit a relevant route of influence, response equality need not survive an enlarged context. If action labels lack participant-understood realizations, a strategy theorem lacks its practical bridge. These are identifiable failures of application, rather than reasons to treat all formal comparison as either decisive or empty.

An empirical application would be strongest where competing arrangements satisfy different conditions and the resulting difference matters to actual continuation. It might discover that a modest interface description is adequate, or that the proposed decomposition hides constitutive relations. Either finding would constrain the present proposal. The study offers a disciplined way to investigate those alternatives while leaving general novelty, empirical fit and normative endorsement as separate questions.

14 Contextual Substitution Assessment

A substitution assessment should explain which continuation remains possible, how the replacement supplies it and which conclusions the comparison leaves open. This section develops that account through a constructed workshop inquiry. It follows the same selected linguistic distinction through demonstration, responsive challenge, adapter use and changeover. The exercise is a proposal for organizing evidence and criticism, rather than a validated protocol or a report of a replacement trial. Its purpose is to make the earlier philosophical and formal distinctions consequential for an intelligible assessment.

14.1 The Declared Comparison

The workshop's established participants use annotated accounts to distinguish an interrupted attempt from a completed event. Their support arrangement combines an example collection with a way to mark disputed descriptions, consult a relevant source note and circulate a revision. The proposed change concerns this arrangement. Retention of the participants' relevant linguistic contribution is an independently stipulated premise of the case; the existence of annotated recordings alone does not establish it.

The selected task covers three agreed workshop meetings. At each meeting participants should be able to interpret an unfamiliar account within the retained distinction, raise a specified kind of ambiguity and use the available clarification to correct a misleading description before the account is finalized. The unfamiliar accounts can vary in content while remaining within the agreed operation family. Cases requiring a different linguistic distinction or unavailable historical evidence lie outside this initial assessment.

The surrounding class includes both unchallenged accounts and accounts challenged by an interlocutor. In the latter cases, an accessible source note can resolve the stipulated ambiguity. The response may arrive at either of two agreed stages before finalization. This restriction makes the case finite without treating every conceivable disagreement as resolvable. A later task including unresolved evidence or longer delays would require an enlarged comparison.

Two candidate arrangements are considered. The first supplies an indexed collection of prepared responses. The second combines a revised collection with a shared correction register and a route for obtaining clarification. These descriptions are constructed analytical alternatives. Their capacities, timing and domains are assumed for comparison rather than inferred from a software feature list or an actual community's practice.

14.2 A Demonstration Result

The initial demonstration asks both candidates to return familiar examples. Each produces the expected visible answer. The result establishes agreement for those requests and that stage of interaction. It can confirm that the transferred collection is readable and that the lookup labels function. It supplies no observation of how either arrangement responds when the displayed interpretation is contested.

This limited finding has legitimate value. If access to examples were the entire task, the indexed collection could be an adequate substitute. The assessment should preserve that result rather than treating the simpler arrangement as worthless. The difficulty arises because the workshop's declared target also includes correcting a misleading interpretation during further use.

A challenge test changes the evidence. A participant marks a description whose event-status implication conflicts with the source note and asks for clarification. The indexed arrangement returns another prepared example without retaining the disputed passage or connecting the challenge to a revision. Under the stipulated operation description, its available path fails the selected correction task. Repeating the lookup more accurately would not supply the missing branch.

The finding identifies a specific failure rather than a general judgment about mediated practice. A richer collection might contain relevant explanations, and participants might already know how to use them to correct the account independently. Either change would alter the available operation family. The

Table 1: Constructed Substitution Comparisons. Each judgment concerns the declared workshop task and stipulated operation conditions. These are analytical cases, not observed replacement outcomes.

Comparison	Supported Finding	Remaining Question
Familiar example requests	Both candidates reproduce the selected demonstration	Whether a participant can initiate usable correction
Responsive challenge	The indexed candidate lacks the stipulated corrective continuation	Whether another participant-supported route exists
Shared correction register	The responsive candidate retains the selected branches	Whether its interpretation and delivery are warranted in practice
Prepared-state comparison	Corresponding initialized arrangements support the retained task	Whether the replacement can actually be prepared
Completed transition	An available changeover can establish the candidate’s initial conditions	Whether its burdens and authority are acceptable

present failure depends on the declared participants, context and support arrangement, which exclude that additional route.

14.3 The Responsive Alternative

The second candidate retains a marked account while clarification is sought. It makes the relevant source note accessible, records the distinction between a pending challenge and an accepted revision, and allows the participant to reopen the account before finalization. Suppose those operations are available at both response stages in the retained context class. The candidate then supplies the interactive structure missing from the indexed arrangement.

The comparison should examine a whole permissible continuation. If the clarification arrives early, participants can revise immediately. If it arrives at the later allowed stage, the pending marker prevents premature finalization and the same correction remains available. A candidate that works only on the early branch would not meet the declared guarantee. This is the practical role of retaining all admitted responses in the policy comparison.

The action translation is public. Marking a passage in the earlier arrangement corresponds to a recognizable request in the new register; consulting the source note and accepting a revision have similarly understood counterparts. The participant follows these conventions from the observed status of the account. The assessment does not permit a coordinator to select a hidden route using knowledge unavailable to the participant.

Internal identity is unnecessary for this bounded finding. The earlier arrangement might use several physical cards, while the replacement uses one shared display with a versioned entry. Their internal organization differs. If the comparison retains the disputed passage, clarification route, responsive control and resulting account across the declared contexts, it can support preservation of the selected correction capacity. It does not establish that the materials have the same historical or experiential significance.

Table 1 separates the resulting judgments. The rows compare claims supported by different parts of the constructed inquiry; they do not rank the arrangements along one universal quality scale.

14.4 Adapter Dependence

Suppose a trial reveals that only one coordinator understands the replacement’s pending and accepted labels. The formal response structure may be suitable, but the participants cannot yet follow its action conventions. There are several possible responses. The labels could be redesigned, participants could

learn the conventions, or an agreed interpreter could remain part of the arrangement. These responses change different conditions and should be reported accordingly.

If participants learn the conventions, the later capacity can be attributed to the revised supported configuration. The learning episode does not establish that the initial replacement was already usable. Its time and resources belong to the transition. If interpretation remains delegated, the continuing availability of the interpreter belongs to the post-replacement context and operation domains.

The dependence can affect the preserved family. An arrangement adequate when the coordinator attends every meeting may fail when participants require independent use during an absence. Those are different context classes. Retaining the dependence explicitly permits an informative comparison: the proposed arrangement preserves one supported capacity while failing the stronger independence target. It also prevents a successful assisted trial from becoming evidence for unsupervised operation.

A participant can have reasons to prefer either design. Shared interpretation may distribute work effectively, while dependence on a particular coordinator may create fragility or an unwanted authority relation. The functional finding informs these considerations but does not order them automatically. Describing the actual bearer and support commitments gives participants a basis for assessing the change.

14.5 Transition Assessment

The prepared responsive candidate contains interpreted annotations and a correctly initialized revision status. Establishing those conditions requires an available process. A transferred file may preserve its text while losing the correspondence between a correction and the passage it revised. A new display can show the right current account while omitting an unresolved challenge. Those losses can matter before any later interaction begins.

A bounded transition inquiry would therefore examine the passage from the earlier support to the candidate, including whatever interpretation work makes the new state usable. In the constructed case, assume an overlap meeting permits participants to compare pending items, verify the request conventions and confirm which revisions have been accepted. The meeting provides an initialization route only because the required materials and participants are available.

The overlap can preserve some continuity while introducing costs. It may require maintaining two arrangements temporarily or revisiting disputed annotations. A favorable endpoint comparison supplies no argument that this work is free, fairly distributed or obligatory. If the resources are unavailable, the prepared-state result remains a conditional possibility while immediate substitution remains unestablished.

The historical record should also preserve the selected old occurrences without confusing them with current operation. Earlier contributions can retain their attribution in the new arrangement even when a convention is revised. Conversely, copying provenance does not show that a former correction remains usable. The inherited distinction between anchored history and changing continuation provides a representation for separating these claims [7, pp. 46–48].

The result can take several forms. The inquiry may establish an available transition and retained bounded correction; it may establish only prepared-state adequacy; or it may identify a transition failure that can be repaired. These statements describe different achievements. Combining them into a single unqualified declaration of preservation would obscure the work still required.

14.6 Participant Assessment

The final assessment returns to the participants' undertaking. A report can state that the responsive candidate preserves the selected three-meeting correction task within the declared response and access conditions. It should distinguish the evidence supporting those conditions from the conditional model result. In the present case the evidence is stipulated, so the report is an analytical demonstration of what such a finding would require.

Participants can question both the result and its scope. They may identify an omitted request, reject the proposed coordinator role or decide that retaining the earlier physical materials matters independently.

These responses have different implications. An omitted relevant request challenges coverage; unavailable interpretation challenges implementation; attachment to the earlier materials can supply a further preservation target or a reason against replacement even when the original functional finding stands.

The inquiry can also reveal gains that matter. The replacement may make pending disagreement easier to inspect or enable a participant previously excluded by the earlier medium. Those possibilities deserve assessment rather than being treated as irrelevant because they depart from exact reproduction. A one-way preservation result can coexist with newly available capacities, while the value and distribution of those gains remain open to deliberation.

The proposed assessment thus supports a bounded form of practical judgment. It identifies what continues, what changes, how the corresponding configuration can be reached and which dependence remains. It does not authorize replacing people, prescribe a universal interface specification or settle the later obligations of a mentorship handover. Its usefulness would need to be tested in actual substitution inquiries, where participants and historical evidence can challenge the analytical boundary as well as the candidate arrangement.

15 Constructed Continuation Dossier

The formal investigations become useful only when their objects can be connected to a situated claim. This section assembles a constructed dossier for the narrative workshop introduced earlier. The case is fictional and supplies no evidence about an actual community. Its purpose is to show how the historical, strategic, field and algebraic questions can organize an inquiry without being collapsed into one measure of preservation.

15.1 Declared Task

The task is the three-meeting correction inquiry specified in Section 14, including an unfamiliar account, an available source note and correction before finalization. This dossier maps that established case to the technical constructions. It does not introduce a further candidate sequence. The selected support region contains examples, retrieval rules and correction routing; the participants and their attributed contributions remain distinct historical bearers.

The historical dossier records which examples came from which earlier episodes, who added disputed annotations, and what agreement initially gave the register its role. These records supply possible anchor data for Section 5. Their adequacy is a historical and evidential question. A valid cellular representation of the records does not establish their truth.

The declared interface retains retrieval, a proposed continuation, a pending challenge, clarification and the outcome of a revision. The case assumes that the selected ambiguity can be resolved. Refusal or unresolved disagreement would be legitimate enlarged-task variants, requiring corresponding choices or target conditions. They are not silently included in the original guarantee.

15.2 Candidate Comparisons

Consider a diagnostic variant of the responsive register, rather than the adequate candidate stipulated in Section 14. Suppose each request is assigned in advance to one of two incompatible correction tracks, while the public record reports only ready. This is the role of the hidden commitment in Section 8. Both corrections can appear among observed successful cases while a participant facing the shared record lacks a guaranteed available choice.

One repair exposes the track before the participant chooses, or retains both tracks until the choice is made. The two repairs differ. Exposing the track can support a policy for a goal that accepts either correction, but it may still fail a goal that requires the participant to choose a particular kind of revision. Retaining both tracks can preserve that stronger choice if their domains and effects meet the stated conditions. The target, observation and domain therefore need to be assessed together.

Table 2: Constructed Substitution Dossier. Each comparison concerns the declared workshop task and requires its own evidence. Formal success does not settle historical or normative claims in another row.

Comparison	Evidence Or Construction	Warranted Conditional Conclusion
Historical attachment	Provenance, retained occurrences, compatible interface cells	Candidate continuation has a coherent selected historical attachment
Participant choices	Public records, operation domains, fixed adapter, all admitted outcomes	Certified bounded policies transfer
Support response	Initialized channel state, local ports, justified renewal dynamics	Declared exact, ordered or finite-horizon field comparison holds
Operation composition	Domain-sensitive history embedding and intertwining	Selected deterministic words transport
Replacement transition	Feasible migration, preparation and continued access during handover	Certified initial arrangement can actually be reached
Normative assessment	Participant standing, burdens, refusals and reasons for the selected target	A separately argued judgment about whether replacement is justified

For an arrangement that retains the required choices, the positive certificate has a precise application. Its initialized states must be related; its adapter must be public; availability and every admitted outcome must satisfy Definition 7.1; and the interpreted target must be compatible. The bounded-policy conclusion then follows. A different internal realization is permitted by those premises.

15.3 Support History

A suitable arrangement can still require continuing maintenance. Participants may make successful requests initially while the resources supporting later responses decay or migrate to another location. The field model supplies a possible investigation of that dependence if activity and support channels can be operationally interpreted. It is not a second empirical description automatically attached to the fictional case.

Under that optional realization, the boundary response includes the effect of the initial support history and the subsequent renewal kernel. Matching both can establish exact continuation of a well-posed port-mediated context. Matching only the current response leaves open a different future decay. Matching an aggregate response leaves open a redistribution of support across the local ports that participants actually use. The two-site example makes that latter failure mathematically explicit.

The field and finite models consequently test different parts of the replacement claim. A renewal trajectory can remain positive while a correction request is prohibited. A formally available correction can cease to be practically executable when its support channel fails. Combining those accounts requires a realization that connects field conditions to the relevant domains and participant evidence. The current paper identifies that bridge as an obligation rather than assuming it from the shared word support.

15.4 Assessment Record

Table 2 organizes the dossier by the conclusion each piece of evidence can support. The rows are complementary inquiries, not a scoring scheme. A strong result in one row cannot cancel a missing premise in another.

The mathematical checks are reproducible within their model class. The source packet compares finite policy trees and replacement relations, verifies the small matrix obstruction, and checks the field

calculations by independent formulas and numerical refinement. These computations test the constructions. They do not convert the fictional dossier into empirical confirmation.

The dossier separates failure diagnoses. A missing correction domain calls for an operation-level revision; hidden commitment calls for an information or choice revision; aggregate agreement with local failure calls for an interface revision. An unavailable initialization challenges the handover, while disputed authority challenges the target or its justification. These diagnoses locate different unfinished arguments instead of treating every failed comparison as one undifferentiated loss.

16 Research Boundaries

This study has examined a bounded substitution problem: replacement of a supporting region while retaining selected participant-executable linguistic continuations in declared surrounding contexts. The analytical center is the relation among support realization, interface, context and usable continuation. Equality of a present output leaves that relation underdetermined.

The philosophical argument distinguishes the encompassing practice from the replaced region and its own historical identity. A changed carrier can sustain a selected contribution while its earlier occurrences remain part of the practice's history. Such preservation does not make contributors interchangeable or establish that the replacement is justified. Participant standing, burden and authority require their own arguments.

The finite investigation supplies a sufficient fixed-translator condition for bounded policy transfer. It preserves the relevant choice before uncertainty is resolved and matches every replacement outcome to an admitted reference outcome. Its contextual proof depends on complete declared ports, compatible targets and related initial states. The hidden-commitment counterexample shows why successful traces alone can miss the loss of a usable choice. Sequential certificates compose under the stated conditions, while actual handover remains a separate process to investigate.

The field investigation supplies a different sufficient condition. A matrix memory kernel together with the free initial response determines the chosen support module's causal response. Exact equality permits changed internal realizations within a well-posed port-mediated context. Aggregate equality can still omit local differences that determine nonlinear continuation. Ordered response and finite-horizon approximation yield narrower guarantees with explicitly restricted contexts and targets.

The algebraic investigation identifies further distinctions. Domain-sensitive intertwining transports deterministic history words. Compression can lose intermediate excursions and therefore fail to preserve multiplication. A boundary-observable algebra evolves autonomously only when its fibres remain compatible with the flow. These results use established mathematical structures to expose premises; they do not derive semantic identity or practical access from algebra alone.

Several research tasks remain. Empirical applications need justified interfaces and evidence about alternatives, not merely a record of successful demonstrations. Models of multiple participants need to distinguish their controlled choices from unresolved outcomes, so that removing another person's refusal is not mistaken for an uncomplicated gain in reliability. Handover models need to represent preparation, temporary loss of access and the maintenance of the adapter itself. Approximate substitution needs tolerances connected to the task and to robust operation domains.

The open-ended formation of new requests and interpretive criteria requires an additional comparison problem. The present repertoire models anticipated continuations, while the inherited GR architecture permits changing operation families and carriers. Connecting those levels would require explicit descendant relations and evidence for newly formed operations. Merely enlarging a state space after the event would not explain their formation.

These limits preserve the provisional role of the framework. A meta-language is useful here when it coordinates historical, philosophical and mathematical responsibilities without converting one discipline's result into another's conclusion. The present constructions make a selected replacement claim more explicit and more open to criticism. Their adequacy depends on the practices, participants and evidence to which a future application gives them substantive meaning.

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