

Generative Relational Freedom: Situated Hermeneutics and Relational Symmetry Breaking

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Abstract

This discussion paper develops a generative relational approach to freedom for subjects whose agency is always situated within non-zero relations of interaction. An analytical free dynamics may be represented by S_0 , while the situated dynamics of an embodied and historically formed subject take the form $S = S_0 + S_{\text{int}}$, with $S_{\text{int}} \neq 0$. The paper therefore treats freedom as a phenomenon requiring interpretation within relationally structured dynamics rather than through an assumed zero-interaction baseline.

The analysis introduces a spatiotemporally situated relational language system L_{st} , through which actions, constraints, possibilities, and claims of freedom become intelligible within particular historical and relational configurations. Material relations, institutions, symbolic orders, religious and normative structures, and accumulated practices participate in the formation and transformation of L_{st} . Drawing on historical materialism and dialectical accounts of historical change, the paper examines freedom meanings as historically generated and revisable relational forms rather than as temporally invariant semantic objects.

Formal resources from dynamical systems, geometry, group theory, and symmetry analysis are then used to articulate the locality and transformation of these interpretive structures. A bounded spin-foam-inspired construction motivates the passage from local relational substructures to effective geometry and situated hermeneutic organization. Local language systems constrain recognizable and admissible trajectories within such relational patches. Group actions, stabilizers, and invariants provide a framework for examining how a situated relational configuration can select and stabilize particular interpretations of freedom through relational symmetry breaking.

The paper further develops the problem of local-to-global interpretation. Freedom meanings associated with different relational patches may overlap, translate partially, transform through historical contact, or fail to admit a unique global gluing. Lexical correspondence across languages therefore does not establish hermeneutic identity. Cross-relational inquiry is redirected toward partial translatability, local compatibility, transformation, and possible invariants rather than a presumed universal scalar of freedom.

This framework also clarifies the epistemic limits of measuring and ranking freedom. Indicators are treated as situated projections that select observable dimensions from a richer relational and hermeneutic configuration. The paper ultimately proposes Generative Relational Freedom as a revisable framework for studying how freedom becomes meaningful, operative, contested, and transformed within historically situated relational fields.

Keywords: generative relational freedom; freedom; situated hermeneutics; relational symmetry breaking; relational subjectivity; historical materialism; spatiotemporal language systems; cross-relational invariance

Discussion Paper Note

This paper is a conceptual and formal discussion of freedom within the Generative Relational framework (GR). Its purpose is not to provide a final, context-independent definition of freedom. It instead examines the conditions under which freedom becomes intelligible, operative, contested, and historically transformable for subjects whose existence is already relational.

The paper begins from a simple analytical decomposition:

$$S = S_0 + S_{\text{int}}, \quad S_{\text{int}} \neq 0. \quad (1)$$

Here S_0 denotes an analytical free reference and S_{int} denotes relational interaction. Equation 1 does not claim that human agency is literally governed by a known physical action functional. Nor does S_0 represent an authentic, pristine, or historically prior self. The decomposition is used to distinguish an analytically isolated reference dynamics from the interacting conditions within which an embodied and historically situated subject actually develops and acts.

For the purposes of the present argument, the important implication is that a relational subject cannot ordinarily be understood through a zero-interaction baseline. Bodily conditions, other persons, language, institutions, material relations, symbolic orders, historical inheritance, and environmental conditions already participate in the formation of possible trajectories. The removal of one constraint therefore does not remove the relational field within which agency continues to occur.

The paper consequently does not treat freedom as the inverse of constraint. It rejects both of the following reductions:

$$\text{Freedom} \neq \text{AbsenceOfRelation}, \quad \text{Freedom} \neq -\text{Constraint}. \quad (2)$$

Constraint can organize, coordinate, enable, redirect, foreclose, or dominate. The relevant question concerns the relational organization of constraint and the trajectories that become meaningful and generable within it.

A generative grammar provides a limited illustration. An unconstrained string space permits indefinitely many combinations, while meaningful linguistic generation depends upon structured rules, distinctions, histories, and shared practices. The example does not establish that constraint produces freedom. Its narrower purpose is to show that unrestricted combinatorial possibility and meaningful generativity are different objects.

The paper therefore approaches freedom through a spatiotemporally situated relational language system,

$$L_{st}, \quad (3)$$

where s and t indicate relational and historical locality. L_{st} is broader than natural language. It includes historically sedimented concepts, practices, norms, institutions, symbolic distinctions, material relations, religious forms, political vocabularies, and other structures through which a relational configuration becomes interpretable.

Freedom is consequently not assumed to possess a fully specified meaning prior to L_{st} . A lexical token such as *freedom*, *liberty*, or another translated term acquires significance within an already situated interpretive field. Translation at the lexical level therefore does not establish semantic identity:

$$T_{\text{lexical}}(w_i) = w_j \not\Rightarrow \llbracket w_i \rrbracket_{L_i} = \llbracket w_j \rrbracket_{L_j}. \quad (4)$$

The same caution applies within a single natural language. Two speakers may employ the same word while participating in different historical, institutional, religious, political, or personal relational configurations. The paper therefore treats linguistic communities as internally heterogeneous and avoids assigning one static meaning of freedom to an entire civilization, religion, language, or political tradition.

Historical materialism enters the analysis through this situated account of meaning. Material production, labor relations, property relations, institutions, technologies, legal forms, and practical activity

participate in the historical formation of the relational fields within which political concepts become meaningful. The paper does not reduce language to material production, nor does it reduce historical materialism to a deterministic causal sequence. It instead examines reciprocal formation among material relations, practices, institutions, subjectivity, and interpretive structures.

Historical change can therefore affect more than the location of a subject within an unchanged conceptual space. It can also alter the space within which particular trajectories become recognizable and interpretable:

$$(\mathcal{R}_{s,t}, L_{s,t}) \longrightarrow (\mathcal{R}_{s,t+1}, L_{s,t+1}). \quad (5)$$

The freedom language of one historical configuration need not remain semantically unchanged after such a transformation. The paper accordingly uses dialectical analysis to examine relational configurations that generate conditions for their own reinterpretation and reorganization. Dialectics is not treated as a fixed three-stage formula.

The mathematical and theoretical-physical constructions in the paper serve a different role. Concepts from dynamical systems, geometry, group theory, symmetry analysis, and spin-foam approaches provide formal resources for thinking about locality, transformation, invariance, and the relation between local and global structures. Their use does not establish a physical identity between quantum geometry and political or hermeneutic processes.

A bounded source-domain architecture may be written schematically as

$$\mathfrak{F}_U \longrightarrow (\mathcal{M}_U, g_U) \longrightarrow L_U \longrightarrow \mathcal{A}_{L_U} \longrightarrow \gamma, \quad (6)$$

where $\mathfrak{F}_U$ denotes a local substructure of a spin-foam description, $(\mathcal{M}_U, g_U)$ an effective local geometry, L_U a situated interpretive structure, $\mathcal{A}_{L_U}$ a set of hermeneutically admissible directions or trajectories, and γ an enacted trajectory.

Equation 6 is a formal scaffold rather than a claim of literal social quantum gravity. In particular, the paper does not claim that social relations possess spin labels, that political transitions are quantum amplitudes, or that human interpretation is generated by a physical spin foam. The transferable structures are locality, relational composition, emergence of effective description, transformation between local descriptions, and the possible failure of a simple global representation.

Within an effective local state space, the paper distinguishes mathematical possibility from hermeneutic admissibility. If $x \in \mathcal{M}_U$, then

$$\mathcal{A}_{L_U}(x) \subseteq T_x \mathcal{M}_U \quad (7)$$

may be used provisionally to represent directions that become recognizable within the situated interpretive structure. This inclusion does not assert that every socially meaningful trajectory possesses a literal tangent vector. It supplies a disciplined geometric model for distinguishing a wider possibility space from the subset rendered meaningful within a particular relational language system.

The symmetry construction follows the same methodological restriction. Suppose an interpretive state space $\mathcal{X}_F$ admits an action of a transformation group G_F :

$$G_F \curvearrowright \mathcal{X}_F. \quad (8)$$

A situated relational configuration may stabilize a particular interpretive form f_{st} . The subgroup preserving that form is

$$H_{st} = \text{Stab}(f_{st}) = \{g \in G_F \mid g \cdot f_{st} = f_{st}\}. \quad (9)$$

The corresponding reduction

$$G_F \longrightarrow H_{st} \quad (10)$$

is what the paper calls *relational symmetry breaking*. The expression is retained only where the transformation space, relevant action, and preserved structure can be stated with sufficient clarity. Where these conditions cannot be established, symmetry breaking is treated as a bounded formal analogy rather than a demonstrated mathematical isomorphism.

Relational symmetry breaking addresses the stabilization of situated interpretations. A separate problem concerns whether locally stabilized interpretations can be combined into a coherent global form. The paper therefore introduces a local-to-global hermeneutic problem.

For relational patches U_i , let

$$f_i \in \mathcal{F}(U_i) \quad (11)$$

denote a local freedom interpretation. Where patches overlap, compatibility requires agreement after appropriate restriction or translation. Local interpretations need not always satisfy the conditions required for a unique global gluing.

The paper consequently does not assume the existence of a unique global section

$$f_{\text{global}} \in \mathcal{F}(\mathcal{M}) \quad (12)$$

whose local restrictions exhaust every historically situated meaning of freedom. Failure of global gluing is considered as one possible structural source of persistent disagreement. It is not proposed as a complete explanation of political disagreement, which may also involve material interests, domination, strategic behavior, incomplete information, and other causal processes.

Translation between interpretive patches may also be historically dependent. For transition maps T_{ij} , the paper does not presuppose

$$T_{ik} = T_{jk} \circ T_{ij} \quad (13)$$

in every hermeneutic setting. Where composition depends upon the path of historical interpretation, the resulting phenomenon is discussed as *hermeneutic path dependence*. More developed geometric language, including holonomy, is introduced only when the required formal structure has been specified.

The failure of universal gluing does not imply that every interpretation is equally adequate or that cross-relational judgment is impossible. The paper distinguishes global semantic identity from partial translatability, overlapping compatibility, and cross-relational invariance. Its comparative programme therefore seeks structures that may survive transformations among situated descriptions without assuming in advance that all relational language systems encode one hidden scalar quantity called freedom.

This distinction also governs the treatment of measurement. A freedom index, institutional classification, or political declaration selects particular observable dimensions from a richer relational configuration. Such a representation may be useful and empirically rigorous while remaining a projection:

$$(\mathcal{R}_{st}, L_{st}) \xrightarrow{P} \mathbf{z} \xrightarrow{M} F_{\text{rep}}. \quad (14)$$

The paper does not infer from this limitation that measurement is futile. It instead distinguishes measurement of selected freedom-related conditions from exhaustive identification of freedom itself. The choice of observables, their aggregation, and their interpretation already occur within a relational language system.

Generative Relational Freedom is therefore not presented as the final definition that resolves earlier theories of liberty, autonomy, non-domination, capability, emancipation, or religious freedom. Those traditions become interlocutors within a wider inquiry into how meanings of freedom arise, stabilize, conflict, translate, and change.

The framework is reflexive in the same respect. GR itself is articulated within a historically situated language system:

$$L_{\text{GR}}^{(t)} \longrightarrow L_{\text{GR}}^{(t+1)}. \quad (15)$$

Its categories of relation, generativity, symmetry, interpretation, and freedom therefore remain revisable. A framework that rejected final, context-independent definitions of freedom while treating its own vocabulary as historically invariant would reproduce the closure that the present analysis seeks to examine.

The orientation of the paper can be summarized through four methodological commitments:

Freedom is examined within relation rather than from a presumed relational vacuum. Meaning is examined within situated language systems rather than from a presumed view outside history. Symmetry is used only where transformations and invariants can be specified. Cross-relational comparison seeks translatability and invariance without presupposing a universal scalar of freedom.

The Introduction develops these commitments through the persistent plurality of freedom claims, the relational formation of subjectivity, the historical generation of interpretive structures, and the problem of comparing situated freedom meanings without prematurely converting them into a single global definition.

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1 Introduction

Freedom occupies an unusual position in political, philosophical, and religious language. It is invoked across sharply different traditions, institutions, and political projects, often with apparently incompatible meanings. Modern political philosophy alone includes influential accounts of liberty in terms of non-interference, self-direction, non-domination, and substantive opportunity [4, 23, 28]. Relational approaches to autonomy have further challenged models of agency that isolate the subject from the social relations through which capacities, identities, and possibilities develop [13]. These disagreements have generated extensive attempts to distinguish, refine, compare, and operationalize different forms of freedom.

The present paper begins from a different difficulty. Persistent disagreement over freedom may arise partly because freedom is frequently approached as though there were a sufficiently specified object prior to the historical, relational, and linguistic conditions through which that object becomes intelligible. Competing theories can then appear to offer rival measurements or definitions of the same underlying entity. The present inquiry suspends that assumption. It asks instead how freedom becomes interpretable for a relational subject whose agency is already embedded in material, symbolic, historical, interpersonal, institutional, and environmental relations.

The argument proceeds through three connected commitments. First, an embodied relational subject is not approached through an actual state of zero interaction. Second, the meaning of freedom is interpreted within a spatiotemporally situated relational language system L_{st} , whose formation depends upon historical practices and relations. Third, different situated interpretations may exhibit local compatibility, transformation, and invariance without necessarily admitting a unique global semantic identification. These commitments motivate the paper’s use of hermeneutics, historical materialism, dynamical description, geometry, group actions, symmetry breaking, and local-to-global analysis.

1.1 Plurality and Historical Situatedness of Freedom

The conceptual plurality surrounding freedom is well established. Berlin’s distinction between negative and positive liberty remains a major point of reference in modern political theory [4]. Republican theory identifies freedom with a condition of non-domination and therefore distinguishes freedom from the mere absence of actual interference [23]. The capability approach shifts attention toward the substantive opportunities through which persons are effectively able to do and to be particular things [28]. Feminist work on relational autonomy, meanwhile, has examined how capacities for self-direction are socially constituted and can be supported or impaired by relations and institutions [13].

The present paper does not attempt to reduce these traditions to competing values on a common axis. Their disagreements may concern different analytical objects, different relations, and different historical problems. A concept developed to diagnose arbitrary power need not occupy the same theoretical position as one developed to distinguish formal permission from substantive opportunity. The lexical recurrence of terms such as *freedom* and *liberty* therefore supplies insufficient grounds for assuming semantic identity.

This problem becomes more pronounced across historical and linguistic contexts. A dictionary may establish a lexical correspondence between an English expression and a translated expression in another language. Such a correspondence does not establish that the relational histories through which those expressions have acquired political, religious, legal, and ethical significance are identical. Even within a single natural language, the same term can participate in distinct institutional, theological, philosophical, and political configurations.

The paper therefore distinguishes lexical correspondence from situated meaning. For expressions w_i and w_j , a translation relation

$$T_{\text{lex}}(w_i) = w_j \tag{16}$$

does not by itself establish

$$\llbracket w_i \rrbracket_{L_i} = \llbracket w_j \rrbracket_{L_j}. \quad (17)$$

Equations 16 and 17 state a methodological distinction rather than a claim that translation is impossible. The relevant problem is how much structure survives translation, which relations permit partial compatibility, and which differences remain after apparently corresponding vocabulary has been identified.

1.2 Relational Subjectivity and Non-Zero Interaction

The second difficulty concerns the subject to whom freedom is attributed. Some philosophical representations of freedom can be read through an analytical contrast between an agent and constraints subsequently imposed upon that agent. Relational approaches to autonomy have supplied important reasons to resist an excessively atomistic representation of agency [13]. The present paper extends that concern through the generative relational framework.

A compact analytical representation is

$$S = S_0 + S_{\text{int}}, \quad S_{\text{int}} \neq 0, \quad (18)$$

where S_0 denotes an analytically isolated reference dynamics and S_{int} denotes interaction-dependent structure. Equation 18 is not a physical law of human agency and does not identify S_0 with a true or authentic self. Its role is methodological. The representation makes explicit that the situated subject under examination is already embodied, linguistically mediated, historically formed, and relationally coupled.

The consequence for freedom is significant. Removing one interpersonal, institutional, or symbolic constraint does not produce a relational vacuum. The subject remains situated within bodily conditions, material resources, languages, histories, institutions, environments, commitments, and other relations. Freedom therefore cannot be inferred solely from the disappearance of a particular interaction term.

The same point prevents a simple inverse relation between constraint and freedom. An increase in the number of unconstrained formal possibilities does not necessarily produce a corresponding increase in meaningful generative possibility. Linguistic grammar offers a limited illustration. A grammar excludes indefinitely many strings while enabling recognizable, compositional, and revisable expression. The analogy does not establish that constraint creates freedom. It demonstrates only that unstructured combinatorial possibility and situated generative agency are analytically different.

The paper consequently treats constraint as structurally heterogeneous. Material, bodily, symbolic, interpersonal, institutional, and transcendent forms of ordering can affect trajectories in different ways. Some relations coordinate or stabilize practices. Others close pathways, establish dependency, or limit the subject's ability to revise the relations through which action occurs. The quantity of constraint therefore cannot serve as a general scalar measure of freedom.

1.3 Situated Hermeneutics and Relational Language

The third difficulty concerns interpretation. Philosophical hermeneutics has long emphasized the historical situatedness of understanding and the role of historically formed horizons in interpretation [8]. Generative Relational Freedom develops a related concern through the concept of a spatiotemporally situated relational language system L_{st} .

The term *language system* is intentionally broader than natural language. L_{st} includes symbolic distinctions, practical norms, institutional arrangements, religious vocabularies, political categories, historical memories, material practices, and other relations through which a trajectory can become recognizable and interpretable. The subscripts s and t indicate that this interpretive organization is spatially, relationally, and historically situated.

The resulting relation is provisionally represented by

$$(\mathcal{R}_{st}, \mathcal{P}_{st}, \mathcal{I}_{st}, H_{st}) \longrightarrow L_{st}, \quad (19)$$

where $\mathcal{R}_{st}$ denotes relational organization, $\mathcal{P}_{st}$ material and practical conditions, $\mathcal{I}_{st}$ institutional and symbolic organization, and H_{st} historical dependence. The arrow in Equation 19 does not assert a one-directional determinism. The components interact recursively, and the language system can itself reshape later practices, institutions, and relations.

Historical materialism supplies an important interlocutor at this point. Marx's materialist conception of history located social and intellectual forms within historically changing relations of production and material life [16]. The present framework does not adopt a deterministic reduction of interpretation to economic structure. It retains the stronger methodological lesson that political concepts and forms of consciousness have histories embedded in practical and material relations. A freedom language can therefore change when the relational organization through which it has been generated changes.

The historical process is represented schematically by

$$(\mathcal{R}_{s,t}, L_{s,t}) \longrightarrow (\mathcal{R}_{s,t+1}, L_{s,t+1}). \quad (20)$$

This formulation allows historical transformation to alter both trajectories within an interpretive field and the interpretive field through which trajectories become meaningful. The paper therefore avoids treating freedom as a semantic object whose complete meaning remains fixed while societies move around it.

1.4 Local Interpretation, Symmetry, and Global Structure

Situated interpretation introduces a local-to-global problem. If meanings of freedom are generated within relationally differentiated patches, philosophical comparison cannot simply presume that every local interpretation is a partial measurement of one already specified global object. The relationship among local interpretations must itself be examined.

The formal development begins from a local relational region U . Later sections use concepts from dynamical systems, geometry, group theory, and spin-foam approaches to articulate locality, emergence, transformation, and invariance. Spin-foam theory supplies a particularly useful source-domain example because effective geometrical description is developed from relational combinatorial structures rather than assumed as an elementary smooth background [22]. The paper transfers only specified structural features of that formal setting. It does not identify political relations with quantum geometry.

The corresponding philosophical construction proceeds from local relational organization toward local interpretive structure:

$$\mathfrak{F}_U \longrightarrow (\mathcal{M}_U, g_U) \longrightarrow L_U \longrightarrow \mathcal{A}_{L_U}, \quad (21)$$

where $\mathfrak{F}_U$ denotes a bounded relational source-domain structure, $(\mathcal{M}_U, g_U)$ an effective local geometry, L_U a situated interpretive system, and $\mathcal{A}_{L_U}$ a provisional structure of hermeneutically admissible directions. The successive arrows identify analytical transitions. Their detailed interpretation and limits are developed in Sections 3 and 4.

The title's reference to *relational symmetry breaking* arises from a second formal problem. Suppose a transformation group G_F acts upon an interpretive state space $\mathcal{X}_F$. A situated relational configuration can stabilize an interpretive form f_{st} , leaving a stabilizer subgroup

$$H_{st} = \text{Stab}(f_{st}) \subseteq G_F. \quad (22)$$

The reduction

$$G_F \longrightarrow H_{st} \quad (23)$$

provides the formal template for relational symmetry breaking. The paper uses this language only where the relevant transformation, equivalence, and invariance structures can be specified. Symmetry

breaking therefore concerns the situated stabilization of an interpretation rather than the mere observation that different communities disagree.

A further problem remains after local interpretations have stabilized. Let $\mathcal{F}(U_i)$ denote freedom interpretations available over relational patch U_i . Local interpretations

$$f_i \in \mathcal{F}(U_i) \quad (24)$$

may agree on overlaps, require non-trivial translation, or fail to satisfy the compatibility conditions required for a unique global section. Persistent disagreement over freedom can therefore be investigated partly as a local-to-global problem. Such non-gluability is proposed as one possible structural source of disagreement. Material conflict, political interest, domination, strategic action, and ordinary epistemic error remain additional explanations.

1.5 Research Contribution and Method

The paper develops *Generative Relational Freedom* as a provisional framework rather than a final definition of freedom. Its contribution is organized around five analytical moves.

First, it removes the zero-interaction subject from the position of an assumed empirical baseline. Freedom is examined within relational dynamics in which interaction is already present.

Second, it introduces L_{st} to represent the historical and relational locality through which claims, experiences, and practices of freedom become interpretable. This move distinguishes lexical equivalence from hermeneutic identity and permits freedom meanings to change with historical relational configurations.

Third, it develops a formal account of relational symmetry and symmetry breaking. The framework asks which transformations preserve an interpretive structure, how situated configurations stabilize particular forms, and what residual symmetries remain after contextual specification.

Fourth, it formulates the comparison of freedom meanings as a local-to-global problem. Local compatibility, translation, path dependence, possible non-gluability, and cross-relational invariance replace the presumption that every situated freedom language must be reducible to a single scalar object.

Fifth, the paper draws an epistemic consequence for representation and measurement. Freedom indicators may capture important institutional and practical conditions while remaining situated projections. The methodological problem precedes aggregation: the choice of observables already depends upon a language system within which particular phenomena have become recognizable as freedom-related.

The inquiry is conceptual and formal. Political philosophy, relational autonomy, philosophical hermeneutics, historical materialism, theoretical physics, geometry, and group theory provide different resources. Their conceptual roles remain distinguished throughout the paper. Formal structures are introduced only where they perform identifiable analytical work, and source-domain structures from theoretical physics are accompanied by explicit limits on transfer.

Section 2 develops the philosophical and historical background. Section 3 establishes the dynamical, geometrical, and group-theoretical premises. Section 4 introduces the bounded spin-foam source domain and the emergence of local effective geometry. Subsequent sections develop relational subjectivity, L_{st} , the historical generation and hermeneutics of freedom, relational constraint architecture, interpretive geometry, relational symmetry breaking, local-to-global gluing, translation, cross-relational invariance, and the limits of scalar representation. The final conceptual and discussion sections examine rival accounts, counterexamples, relativism, domination, and the reflexive status of the framework itself.

2 Freedom, Relation, and Historical Situatedness

This section establishes the philosophical background required for the later formal account. Its purpose is not to reconstruct the entire history of freedom, autonomy, hermeneutics, Marxism, or dialectical philosophy. The section instead identifies several conceptual resources that bear directly on the present

problem: the plurality of freedom concepts, the relational formation of agency, the historical and material constitution of social meaning, the transformation of conceptual forms through practice, and the situated character of interpretation. These resources are subsequently reorganized within the Generative Relational framework rather than treated as components of a single inherited doctrine.

The guiding difficulty is that a theory of freedom can begin too late. If an already constituted subject, an already specified set of possibilities, and an already stabilized meaning of *freedom* are presupposed, then much of the relational and historical work through which those objects became intelligible has already disappeared from view. The present paper therefore moves backward to the conditions under which subjects, constraints, possibilities, and freedom languages become available for interpretation in the first place.

2.1 Major Traditions of Freedom

Modern political philosophy contains several influential ways of locating the object of freedom. Berlin's distinction between negative and positive liberty places one major contrast between freedom from interference and forms of self-direction or self-mastery [4]. Republican theory locates a different object in domination. On Pettit's account, a person may remain unfree when subject to another's arbitrary power even when actual interference does not occur [23]. The capability approach shifts analytical attention toward substantive opportunities and the effective possibilities available for living and acting [28].

These traditions cannot be reduced to different measurements of one uncontroversially specified variable. Non-interference, non-domination, self-direction, and substantive opportunity identify different relational features. A person can therefore occupy different positions under these accounts without logical inconsistency. Low interference does not entail low domination; formal permission does not entail substantive capability; and an expanded set of options does not independently establish self-direction.

The present paper does not attempt to adjudicate among these traditions by constructing a more comprehensive scalar. Such an aggregation would already presuppose that the relevant objects can be translated into commensurable dimensions. The prior question concerns the historical and relational conditions under which these dimensions themselves become recognizable as dimensions of freedom.

The distinction is particularly important because political vocabularies frequently compress different analytical relations into the same lexical term. A claim that a society is "free" may concern constitutional protection, market choice, political participation, absence of arbitrary power, religious practice, mobility, personal autonomy, or some combination of these. The recurrence of a common term does not determine the relation among the underlying objects.

Generative Relational Freedom therefore begins from conceptual plurality without treating plurality as the final result. The task is to examine how different freedom meanings are generated, stabilized, translated, contested, and transformed within historically situated relational structures.

2.2 Relational Subjectivity and Agency

The plurality of freedom concepts is accompanied by a second problem concerning the bearer of freedom. A strongly atomistic representation begins with a substantially constituted individual and subsequently asks which external relations interfere with that individual's choices. Relational approaches to autonomy have challenged this picture by examining the social conditions through which autonomy, agency, identity, and self-understanding are formed [13].

The present framework adopts the relational challenge while avoiding the stronger conclusion that the subject can be exhaustively reduced to social relations. Embodiment, material processes, personal history, non-human environments, symbolic structures, and relations with others can all participate in subject formation. Relation therefore names an analytical structure of dependence, interaction, and co-formation rather than an ontological assertion that only interpersonal relations exist.

Ricoeur's hermeneutics of selfhood offers an additional point of contact. His treatment of selfhood, action, narrative identity, and the relation between self and other resists a simple model of an entirely self-transparent subject [25]. The present framework develops a different formal vocabulary, but shares

the concern that agency cannot be adequately described through an isolated subject whose meaning and identity are already complete before interpretation and relation.

This position changes the initial freedom problem. If the subject is partly formed through relations, then relation cannot be classified in advance as an external reduction of an antecedently complete freedom. Language, education, care, bodily dependence, social recognition, and institutional participation can simultaneously constrain and contribute to the capacities through which agency becomes possible.

The relevant opposition is therefore not

individual freedom versus relation.

The analytical problem concerns the organization of relations through which a subject forms, interprets, revises, and enacts trajectories. Some relational structures enlarge meaningful agency; some stabilize it; some distribute agency asymmetrically; some generate dependency; and some restrict the subject's capacity to alter the relations themselves.

This distinction also prevents relationality from becoming intrinsically normative. A relation is not justified merely because agency is relational. Relations of domination, exploitation, coercion, exclusion, and dependence are relations as well. A relational theory of freedom therefore requires an account of relational structure rather than a general affirmation of relational embeddedness.

2.3 Historical Materialism and Social Relations

Historical materialism contributes a different but complementary displacement. The abstract individual cannot simply be detached from the historical organization of production, practical life, institutions, and social relations within which that individual acts. Marx's *Theses on Feuerbach* rejects an account of the human essence located as an abstraction inside an isolated individual and instead situates human life within social relations and practical activity [15]. The 1859 *Preface to A Contribution to the Critique of Political Economy* similarly connects forms of social consciousness with historically changing material and social conditions [16].

Marx and Engels develop this historical orientation further in *The German Ideology*, where production, cooperation, material life, social relations, language, and consciousness are treated within an evolving historical process rather than as independently fixed domains [18]. The present paper does not adopt every substantive claim associated with later traditions of historical materialism. It takes from this lineage a methodological requirement: concepts of agency and freedom should not be detached from the historically specific relations through which social life is reproduced.

This requirement has direct consequences for the analysis of freedom. Material conditions do more than determine whether an independently defined freedom can be exercised. They can participate in the historical production of the language through which freedom itself becomes politically and socially intelligible.

Property relations, labor organization, technologies, legal institutions, forms of exchange, familial structures, religious practices, and political organization can therefore enter the analysis at two levels. They can alter the practical trajectories available to subjects, and they can contribute to the interpretive structures through which those trajectories acquire significance.

The distinction may be represented schematically by

$$\Delta \mathcal{R}_{st} \longrightarrow \{ \Delta \mathcal{T}_{st}, \Delta L_{st} \}, \quad (25)$$

where $\mathcal{R}_{st}$ denotes a situated relational configuration, $\mathcal{T}_{st}$ a structured field of trajectories, and L_{st} the corresponding relational language system. Equation 25 expresses possible dependence rather than deterministic sufficiency. Changes in material relations need not produce a unique conceptual outcome, and language, institutions, and political practice can recursively reshape material relations.

This reciprocal structure is important for GR. Historical materiality is retained without reducing interpretation to an epiphenomenon. Symbolic and normative structures have causal consequences of their own, while remaining historically situated within wider relational processes.

2.4 Dialectical Transformation and Historical Becoming

The historical situatedness of freedom also requires an account of change. A static relational model would merely replace an isolated individual with a fixed network of relations. Such a model would remain insufficient because subjects, institutions, material conditions, conceptual distinctions, and relations themselves change over time.

Dialectical traditions supply resources for thinking about forms whose determinations are developed through relation, mediation, tension, and transformation. Hegel's *Science of Logic* is especially relevant as a philosophical precedent for treating conceptual determination as something developed through a structured movement rather than supplied once and for all at the beginning of inquiry [10]. Marx's later materialist reorientation places transformative practice and historically changing social relations at the center of analysis [15, 18].

The present paper does not use dialectic to denote a mechanical sequence of thesis, antithesis, and synthesis. Such a formula would obscure the forms of historical change relevant here. The narrower interest concerns relational configurations that generate tensions, differences, and practical conditions through which the configuration itself can subsequently be transformed.

A freedom language can, for example, stabilize a political order while simultaneously generating experiences that cannot be adequately articulated within its existing distinctions. New forms of labor, technology, dependency, religious practice, migration, social organization, or political struggle may produce demands that exceed an inherited vocabulary. The resulting process can affect both the relations being interpreted and the interpretive categories through which those relations are understood.

A minimal representation is

$$(\mathcal{R}_t, L_t) \longrightarrow \Delta(\mathcal{R}_t, L_t) \longrightarrow (\mathcal{R}_{t+1}, L_{t+1}), \quad (26)$$

where the middle term represents historically generated tensions, interactions, and transformations rather than a predetermined logical negation.

Equation 26 is deliberately open. The resulting configuration need not be morally better, more free, or more generative than the earlier one. Historical transformation can produce emancipation, closure, new domination, conceptual expansion, semantic loss, or configurations that remain normatively indeterminate.

This non-teleological treatment is essential for the later symmetry analysis. A historically stabilized interpretation of freedom can persist for a period, lose stability, interact with competing interpretations, and become reconfigured. Symmetry breaking therefore cannot be treated as one irreversible event that reveals a final meaning. Historical processes can repeatedly alter the relational structures that sustain a particular interpretive configuration.

2.5 Hermeneutics and Situated Meaning

Historical change alone does not explain how a concept becomes meaningful. The paper therefore draws on philosophical hermeneutics, especially the emphasis on historically situated understanding, interpretive horizons, tradition, language, and the transformation of understanding through encounter [8]. Hermeneutics is relevant here because freedom is encountered through inherited vocabularies and practices rather than from a position outside language and history.

The present framework extends this concern through L_{st} . A relational language system includes natural language while exceeding it. Legal categories, institutional routines, religious symbols, economic practices, forms of recognition, embodied habits, collective memories, political narratives, and normative distinctions can all participate in the interpretability of a trajectory.

The resulting account distinguishes three levels that are frequently compressed.

First, there is a lexical expression:

$$w = \text{freedom.}$$

Second, there is an interpretive configuration within which the expression acquires situated meaning:

$$\llbracket w \rrbracket_{L_{st}}.$$

Third, there are practical and relational consequences associated with using that interpretation:

$$\llbracket w \rrbracket_{L_{st}} \longrightarrow \Delta \mathcal{R}_{s,t+1}.$$

Meaning is therefore neither reducible to a lexical token nor confined to an internal mental representation. Interpretive forms can enter institutions, laws, rituals, expectations, classifications, and collective practices and thereby participate in later relational change.

This structure also introduces reflexivity. A society that repeatedly describes itself as free does more than report an external condition. The description can stabilize expectations, political identities, institutional standards, and boundaries of legitimate criticism. Conversely, challenges to an inherited freedom language can alter what experiences become recognizable as domination, constraint, autonomy, obligation, or emancipation.

The hermeneutic problem is consequently dynamic:

$$\mathcal{R}_{st} \longleftrightarrow L_{st} \longleftrightarrow \mathcal{P}_{st}, \quad (27)$$

where $\mathcal{P}_{st}$ denotes situated practices. None of the three terms is treated as a uniquely determining first cause.

2.6 Generativity and Relational Organization

The Generative Relational framework brings the preceding traditions together through a specific analytical interest: how relational configurations produce and transform further trajectories, meanings, practices, relations, and conditions of generation.

Generativity in this paper does not denote unrestricted production or the maximization of possible outputs. It concerns the situated organization through which further trajectories can be formed. A generative system is therefore structured. Rules, histories, resources, relations, interpretations, and constraints participate in determining which transitions become recognizable, viable, revisable, and capable of generating further change.

This position supplies an important distinction for the freedom inquiry:

$$\text{unrestricted possibility} \not\equiv \text{generative freedom}. \quad (28)$$

Equation 28 does not define generative freedom. Its purpose is to reject one possible reduction. The mere cardinality of formally available options does not capture the relational organization through which a subject interprets, enters, sustains, revises, or transforms a trajectory.

Language again supplies a useful structural illustration. A generative grammar restricts the set of admissible combinations while enabling indefinitely extensible meaningful expression. The point is not that grammatical constraint constitutes political freedom. The example shows that generativity can depend upon structured relations and that removal of structure does not automatically produce an enhanced generative state.

GR therefore approaches freedom from within relational organization. The subject, the relevant trajectories, the interpretive language, and the constraint architecture can all change. Freedom cannot be assigned in advance to any one of these components. The later sections investigate the relations among them.

This perspective also explains why the framework remains cautious about a single universal freedom function

$$F : \mathcal{X} \longrightarrow \mathbb{R}. \quad (29)$$

The paper does not prove that every scalar representation is impossible or useless. It questions whether a context-independent mapping such as Equation 29 can be assumed before the relevant relational and hermeneutic equivalences have themselves been established.

The analytical alternative is to begin locally, identify the relational language system within which freedom becomes meaningful, examine the transformations among such local structures, and only then ask whether cross-relational invariants or legitimate forms of comparison can be found.

2.7 Scope of the Present Inquiry

The present inquiry is foundational and deliberately bounded. It does not offer a comprehensive history of freedom, a complete exegesis of Marx, Hegel, Gadamer, Ricoeur, republican theory, the capability approach, or relational autonomy. These traditions provide conceptual resources and counterpoints for a narrower problem concerning the relational and historical interpretability of freedom.

The paper also stops short of a complete normative theory of effective freedom. Questions concerning whether a particular generative landscape provides adequate material, institutional, relational, or ecological conditions for the exercise of freedom require a subsequent analysis. The present paper addresses the logically prior problem of how freedom becomes an interpretable object within relationally situated dynamics.

Three boundaries are especially important.

First, historical situatedness does not entail historical determinism. L_{st} is conditioned by history without being uniquely deducible from a fixed set of material variables.

Second, situated interpretation does not entail cultural relativism. Differences among local freedom languages do not establish that comparison, criticism, translation, or cross-relational judgment is impossible. Later sections investigate partial equivalence, overlap, transformation, and invariance precisely because semantic identity cannot be assumed.

Third, relationality does not legitimate constraint. The fact that a subject always develops within relations supplies no justification for domination, coercion, exclusion, or irreversible closure. The normative evaluation of a constraint requires further analysis of its structure, distribution, revisability, effects, and the position of the affected subject.

The resulting research sequence is therefore

$$\begin{aligned}
 \text{relational and historical situatedness} &\longrightarrow \text{local interpretive structure} \\
 &\longrightarrow \text{situated freedom meaning} \\
 &\longrightarrow \text{transformation and symmetry analysis} \\
 &\longrightarrow \text{local-to-global comparison.}
 \end{aligned} \tag{30}$$

Section 3 next establishes the mathematical and theoretical-physical vocabulary required for this programme. The formalism is introduced before its application to freedom so that concepts such as state space, manifold, group action, invariance, stabilizer, and symmetry breaking retain explicit mathematical meanings before their bounded transfer into the relational and hermeneutic analysis.

3 Formal and Physical Premises

This section establishes the mathematical and theoretical-physical vocabulary used in the later relational analysis. Its role is preparatory. States, trajectories, interaction terms, manifolds, tangent spaces, group actions, orbits, stabilizers, invariants, symmetry breaking, and gauge redundancy are introduced first in their ordinary formal meanings. Their application to subjectivity, hermeneutics, and freedom is deferred until the relevant correspondence can be stated explicitly.

The distinction is methodologically important. Mathematical structure can clarify locality, transformation, equivalence, and persistence without establishing that a social or interpretive process is literally

governed by the same physical mechanism. The present section therefore separates three levels throughout: the established mathematical object, its use in theoretical physics, and its later bounded transfer into Generative Relational analysis.

3.1 States, Trajectories, and Dynamical Description

A dynamical description begins with a space of possible states. Let $\mathcal{X}$ denote such a state space and let

$$x_t \in \mathcal{X} \quad (31)$$

denote the state of a system at time t . A trajectory is a map

$$\gamma : I \longrightarrow \mathcal{X}, \quad t \longmapsto \gamma(t), \quad (32)$$

where $I \subseteq \mathbb{R}$ is a temporal interval.

In a smooth deterministic system, evolution may be generated locally by a vector field V on the state space,

$$\dot{\gamma}(t) = V(\gamma(t)). \quad (33)$$

Classical mechanics supplies familiar examples in which trajectories evolve through phase space under equations generated by a Hamiltonian or Lagrangian structure [1]. The present paper requires only the more elementary distinction among a state space, a state, and an evolution through that space.

These objects should remain separate in the later philosophical transfer. A subject is not identified with a single coordinate x , and a life history is not assumed to satisfy a known autonomous differential equation. The dynamical vocabulary is used to express the weaker idea that a relational configuration can constrain, redirect, stabilize, or transform possible trajectories over time.

3.2 Free and Interacting Dynamics

The distinction between free and interacting dynamics supplies a second formal premise. In field theory and related physical descriptions, an action functional can often be decomposed schematically into a free part and an interaction part,

$$S[\phi] = S_0[\phi] + S_{\text{int}}[\phi]. \quad (34)$$

The free part describes a chosen non-interacting reference theory, while S_{int} introduces couplings beyond that reference description [27]. Equations of motion follow from the relevant variational structure rather than from the decomposition alone.

The word *free* in Equation 34 has no normative meaning. It does not denote political freedom, autonomy, liberation, or the absence of every physical constraint. It identifies a reference theory in which specified interactions have been removed or neglected.

This distinction is essential for the present paper. The expression

$$S = S_0 + S_{\text{int}} \quad (35)$$

will later serve as an analytical scaffold for distinguishing an isolated reference dynamics from relationally coupled dynamics. It does not imply that a human subject possesses a recoverable pre-relational state corresponding to S_0 . Nor does increasing the magnitude or complexity of S_{int} supply a scalar measure of reduced freedom.

The relevant transfer is therefore structural:

$$\text{reference dynamics} + \text{interaction-dependent dynamics}. \quad (36)$$

The philosophical argument begins only after this distinction has been established.

3.3 Relational Interaction Terms

For several coupled degrees of freedom, an interaction term can depend jointly upon variables that would otherwise belong to distinguishable subsystems. Schematically,

$$S_{\text{int}} = S_{\text{int}}[\phi_A, \phi_B, \chi, \dots], \quad (37)$$

where ϕ_A and ϕ_B may represent subsystem variables and χ additional mediating degrees of freedom.

The presence of a non-zero interaction term means that the dynamics cannot be fully reconstructed by treating the designated components as mutually independent free systems. It does not, by itself, establish domination, restriction, dependence, or any other normative relation. Interaction is a structural condition; its consequences depend upon the form and dynamics of the coupling.

This distinction will later constrain the use of relational language. The claim

$$S_{\text{int}} \neq 0 \quad (38)$$

means that the situated system is not represented by the isolated reference alone. It does not mean that every relation is restrictive or that every interaction diminishes freedom.

A relational system may also contain interaction terms at several scales. Pairwise coupling, institutional mediation, environmental forcing, and historically accumulated structures need not be reducible to one homogeneous interaction. The later GR model therefore treats S_{int} as a schematic aggregate unless a more specific decomposition is warranted.

3.4 State Spaces and Manifold Structure

A smooth manifold provides a mathematical setting in which a system can be locally described by coordinates while possessing a global structure that need not be equivalent to a single Euclidean space. Standard treatments define a smooth n -manifold through compatible local charts and smooth transition maps [11, 20].

Let

$$\mathcal{M} \quad (39)$$

denote a smooth manifold used as an effective state space. For a point $x \in \mathcal{M}$, a chart on a neighborhood U_α is

$$\varphi_\alpha : U_\alpha \longrightarrow \varphi_\alpha(U_\alpha) \subseteq \mathbb{R}^n. \quad (40)$$

The manifold is not identified with any one chart. Where two charts overlap, their coordinate descriptions are related by transition maps

$$\varphi_\beta \circ \varphi_\alpha^{-1}. \quad (41)$$

This local-to-global structure will later be important for the analysis of situated interpretation. At this stage, however, no hermeneutic conclusion follows from manifold theory. A cultural or historical context is not thereby a mathematical chart, and semantic translation is not automatically a smooth coordinate transformation.

The transferable insight is narrower: a system may admit valid local descriptions without possessing one privileged global coordinate representation.

3.5 Local Coordinates and Contextual Description

Coordinates represent points; they are not the points themselves. If $x \in U_\alpha \cap U_\beta$, then

$$x \mapsto \varphi_\alpha(x), \quad x \mapsto \varphi_\beta(x) \quad (42)$$

provides two coordinate descriptions of the same underlying manifold point. Their numerical components may differ even though the represented point is unchanged.

This distinction motivates an important discipline for later sections:

$$\text{difference of description} \not\equiv \text{difference of represented structure}. \quad (43)$$

The converse caution is equally important. Two descriptions that use corresponding words or coordinates need not represent the same underlying structure. Establishing equivalence requires a transformation relation and a criterion specifying what is preserved.

The paper will therefore avoid treating every cross-context semantic difference as analogous to a coordinate transformation. Coordinate changes become useful models only where an invariant object and admissible transformation between descriptions can be specified.

3.6 Tangent Spaces and Admissible Directions

For each $x \in \mathcal{M}$, the tangent space

$$T_x \mathcal{M} \quad (44)$$

contains tangent vectors representing local directions of smooth variation through x [11]. A differentiable trajectory $\gamma(t)$ passing through x determines a tangent vector

$$\dot{\gamma}(t) \in T_{\gamma(t)} \mathcal{M}. \quad (45)$$

The full tangent space should not be confused with the directions generated by a particular dynamical system. If evolution is governed by a vector field V , the actual local velocity is selected by $V(x)$. Additional constraints can further restrict dynamically admissible directions.

The later hermeneutic analysis will introduce a provisional subset

$$\mathcal{A}_L(x) \subseteq T_x \mathcal{M} \quad (46)$$

to represent directions rendered interpretable or admissible within a situated language system L . Equation 46 is not a theorem of differential geometry. It is a later modeling choice built upon the mathematical distinction between a tangent space and a restricted family of local directions.

This separation prevents a recurring error. A mathematically describable direction, a dynamically reachable direction, a socially recognizable direction, and a practically viable trajectory are different analytical objects. Later sections retain these distinctions rather than placing them inside one undifferentiated possibility set.

3.7 Groups and Group Actions

Symmetry requires a specified family of transformations. Group theory provides the basic formal structure. A group G acting on a space $\mathcal{X}$ assigns to each pair $(g, x) \in G \times \mathcal{X}$ an element

$$g \cdot x \in \mathcal{X} \quad (47)$$

such that the identity acts trivially and composition of group elements is compatible with successive action. Lie groups supply continuous transformation groups with compatible smooth structure [9, 11].

Writing

$$G \curvearrowright \mathcal{X} \quad (48)$$

is therefore incomplete unless both G , $\mathcal{X}$, and the action are specified. The existence of several possible interpretations of a concept does not itself establish a group action among them.

This requirement will matter considerably later. The expression *relational symmetry* will be used only after the relevant transformations have been identified and shown to possess sufficient composition, identity, and inverse structure for the intended analysis.

Where these conditions are absent, the paper will use weaker language such as *transformation family*, *correspondence*, or *translation relation*.

3.8 Orbits, Stabilizers, and Equivalence Classes

Given an action $G \curvearrowright \mathcal{X}$, the orbit of $x \in \mathcal{X}$ is

$$\mathcal{O}_x = G \cdot x = \{g \cdot x \mid g \in G\}. \quad (49)$$

The orbit identifies states accessible from x through the specified group action. The stabilizer, or isotropy subgroup, is

$$G_x = \text{Stab}_G(x) = \{g \in G \mid g \cdot x = x\}. \quad (50)$$

These objects perform different analytical roles. The orbit describes transformation-related configurations, whereas the stabilizer describes the transformations that leave a selected configuration unchanged [9].

An equivalence relation can sometimes be induced by the group action:

$$x \sim_G y \iff y = g \cdot x \text{ for some } g \in G. \quad (51)$$

The quotient

$$\mathcal{X}/G \quad (52)$$

then collects the corresponding orbits. Whether this quotient is itself a well-behaved manifold requires additional conditions and cannot be assumed from the notation alone.

These distinctions will later support three different questions concerning freedom interpretations: which interpretations are transformation-related, which transformations preserve a situated interpretation, and which differences can legitimately be treated as representational rather than substantive.

3.9 Symmetry and Invariance

A symmetry is defined relative to a structure that remains invariant under a specified transformation. For an action functional $S[\phi]$, a transformation $g \in G$ is a symmetry when

$$S[g \cdot \phi] = S[\phi]. \quad (53)$$

Other contexts may instead require invariance of equations of motion, observables, geometric structures, probability laws, or another explicitly specified object [27, 20].

Consequently, the statement

“the system has a symmetry”

is analytically incomplete. One must specify:

1. the transformation set or group;

2. the space upon which it acts;
3. the structure claimed to remain invariant.

The later freedom analysis adopts the same discipline. Similarity among interpretations does not establish symmetry. A proposed symmetry must identify what transformations are permitted and what relational or interpretive structure is preserved.

An invariant quantity or structure I satisfies, where appropriate,

$$I(g \cdot x) = I(x) \quad \text{for all } g \in G. \quad (54)$$

The search for cross-relational invariants later in the paper is modeled on this formal requirement. It is not satisfied merely by identifying recurring vocabulary across traditions.

3.10 Symmetry Reduction and Symmetry Breaking

Two distinct operations must be separated.

The first is *reduction by symmetry*. If configurations related by a group action are treated as equivalent for a stated purpose, analysis may pass from $\mathcal{X}$ to a quotient such as

$$\mathcal{X} \longrightarrow \mathcal{X}/G. \quad (55)$$

This procedure removes distinctions associated with the specified group orbits.

The second is *symmetry breaking*. In the standard physical pattern, the governing structure can possess a symmetry group G while a particular state or configuration is preserved only by a proper subgroup $H \subset G$. For a selected configuration x_* ,

$$H = \text{Stab}_G(x_*), \quad H \subsetneq G. \quad (56)$$

The resulting pattern is represented by

$$G \longrightarrow H. \quad (57)$$

Spontaneous symmetry breaking in field theory provides a prominent physical instance of this structure: the underlying theory retains a symmetry that is not preserved by the selected vacuum or state [27].

The present paper does not assume that relational symmetry breaking is *spontaneous* in this technical physical sense. Historical institutions, material relations, symbolic orders, and political action can explicitly participate in selecting and stabilizing an interpretation. The transferable formal feature is therefore the relation between a broader transformation structure and the smaller stabilizer of a situated configuration.

This distinction prevents the phrase *symmetry breaking* from becoming a general synonym for difference, disagreement, or historical change.

3.11 Gauge-Like Transformations and Descriptive Redundancy

Gauge theory introduces a further distinction between physically meaningful difference and descriptive redundancy. In gauge formulations, different field configurations related by a gauge transformation may represent the same physical configuration, so physical description is associated with gauge-equivalence classes rather than every raw representative separately [20, 27].

Schematically, for a gauge group $\mathcal{G}$,

$$\phi \sim_{\mathcal{G}} g \cdot \phi, \quad g \in \mathcal{G}, \quad (58)$$

and the physically relevant configuration space is represented abstractly by

$$\mathcal{X}_{\text{phys}} \sim \mathcal{X}/\mathcal{G}. \quad (59)$$

The symbol $\sim$ is used deliberately. Quotient constructions can possess technical complications, and a complete gauge theory requires substantially more structure than Equation 59 displays.

The later paper uses the phrase *gauge-like* only for a bounded analytical question: whether two descriptions differ while preserving the relational content relevant to the comparison. A change of natural language, political vocabulary, or historical interpretation is not automatically a gauge transformation.

A legitimate gauge-like comparison therefore requires an independently specified invariant:

$$I(D_1) = I(D_2), \quad (60)$$

where D_1 and D_2 are alternative descriptions and I identifies the structure claimed to survive the transformation. Without such an invariant, the language of gauge equivalence adds no analytical content.

3.12 Formal Scope and Analogy Limits

The formal vocabulary established above supplies the mathematical premises for the remainder of the paper. Its transfer into Generative Relational analysis is intentionally restricted.

Several non-identities govern that transfer:

$$\begin{aligned} \text{social relation} &\neq \text{physical interaction term,} \\ \text{historical context} &\neq \text{coordinate chart,} \\ \text{interpretive possibility} &\neq \text{tangent vector,} \\ \text{semantic translation} &\neq \text{gauge transformation,} \\ \text{political differentiation} &\neq \text{physical symmetry breaking.} \end{aligned} \quad (61)$$

The value of the formalism lies instead in a smaller set of structural questions. It permits the paper to ask whether a relational analysis can specify:

1. a local state or interpretive space;
2. transformations among configurations;
3. structures preserved under those transformations;
4. stabilizers associated with situated configurations;
5. local descriptions and their transition relations;
6. admissible local directions or trajectories;
7. conditions under which local descriptions can be compared or combined.

A physical term is retained only when this additional structure can be supplied. Otherwise, the corresponding philosophical argument is stated in ordinary relational language.

The distinction also establishes a burden of proof for later sections. Introducing G , H , $\mathcal{M}$, a quotient, or a local section does not by itself explain freedom. Each formal object must acquire an explicit interpretation, and the conclusions licensed by the formal model must be distinguished from conclusions requiring historical, hermeneutic, empirical, or normative premises.

The next section turns to the spin-foam source domain. Its purpose is narrower than constructing a physical model of society. It examines how a fundamentally relational and locally combinatorial description can support an effective geometrical account, and which aspects of that local-emergence architecture can subsequently inform the construction of situated relational language systems.

4 Spin-Foam Locality and Emergent Relational Geometry

This section develops the bounded physical source domain used later to motivate the locality of relational language systems. Its purpose is not to construct a quantum-gravitational theory of subjectivity or political freedom. The section instead isolates several structural features of spin-foam approaches: combinatorial locality, relational labeling, boundary dependence, coarse-graining, scale-dependent effective description, and the non-trivial passage from local discrete structures toward continuum or semiclassical geometry.

The physical construction is introduced before its philosophical transfer. This ordering is important because a spin foam is not, in general, a smooth spacetime manifold. Nor does every labeled two-complex determine a unique classical geometry. The later Generative Relational construction therefore uses spin-foam theory as a source domain for local emergence and relational composition while retaining explicit limits on what can be transferred.

4.1 Spin-Foam Complexes and Local Substructures

Spin foams were introduced as higher-dimensional counterparts of spin networks. In a standard description, a spin foam is based on a two-dimensional combinatorial complex whose faces are labeled by group representations and whose edges carry intertwining data [3, 22]. Spin-network boundary states can then be related through amplitudes associated with the intervening foam.

Let a finite two-complex be denoted by

$$\mathcal{C} = (V, E, F), \quad (62)$$

where V , E , and F denote its vertices, edges, and faces, respectively. A labeled spin-foam configuration may be represented schematically by

$$\mathfrak{F} = (\mathcal{C}, \{j_f\}_{f \in F}, \{\iota_e\}_{e \in E}), \quad (63)$$

where j_f denotes representation labels and ι_e denotes intertwiner data. The precise representation theory and admissibility conditions depend upon the spin-foam model under consideration [22, 26].

A generic spin-foam amplitude can be written schematically as

$$Z_{\mathcal{C}} = \sum_{\{j_f, \iota_e\}} \prod_{f \in F} A_f \prod_{e \in E} A_e \prod_{v \in V} A_v, \quad (64)$$

where the face, edge, and vertex amplitudes are model-dependent. Equation 64 is used only to identify the compositional structure. The paper does not rely upon a particular EPRL, FK, Barrett–Crane, or other spin-foam amplitude.

The relevant feature for the present inquiry is locality within a combinatorial structure. A bounded subcomplex

$$\mathcal{C}_U \subseteq \mathcal{C} \quad (65)$$

can be considered together with the labels and boundary data associated with that region. The corresponding restricted configuration is denoted

$$\mathfrak{F}_U. \quad (66)$$

The notation U does not presuppose an already existing open subset of a smooth spacetime manifold. At this stage, U denotes a bounded relational or combinatorial region of the discrete structure.

This distinction is central to the later analogy. Relational locality need not begin from a globally given smooth background. A local structure can be specified through relations among components before an effective continuum description is available.

4.2 Local Coarse-Graining and Effective Geometry

A spin-foam model defined on a fine discretization contains more microscopic degrees of freedom than a corresponding coarse description. Coarse-graining attempts to construct effective amplitudes on coarser boundary data while retaining, within a chosen approximation, the physical information relevant at the coarser level [29]. The problem is especially important in background-independent approaches because ordinary notions of scale cannot simply be imported from a fixed metric background.

Let

$$\mathfrak{F}_U^{(\ell)} \tag{67}$$

denote a fine description associated with a relational resolution ℓ , and let

$$\mathfrak{E}_U^{(L)} \tag{68}$$

denote a coarser effective description at L . A coarse-graining map is written schematically as

$$\mathcal{C}_{\ell \rightarrow L} : \mathfrak{F}_U^{(\ell)} \longrightarrow \mathfrak{E}_U^{(L)}. \tag{69}$$

Equation 69 does not imply that $\mathcal{C}_{\ell \rightarrow L}$ is unique, exact, or invertible. In practical spin-foam research, coarse-graining involves choices of boundary data, truncation, embedding maps, effective amplitudes, and approximation schemes [29].

The important structural lesson is therefore weaker than a one-to-one micro-to-macro correspondence:

$$\text{fine relational description} \longrightarrow \text{scale-dependent effective description}. \tag{70}$$

Information can be integrated out, encoded differently, or become relevant only through effective couplings at the coarser level. The effective description need not reproduce the fine labels individually.

This point will later constrain the interpretation of L_{st} . A spatiotemporal language system is not proposed as a transparent transcription of every microscopic relation. It is an effective interpretive organization that can preserve some relational distinctions while suppressing, combining, or reorganizing others.

4.3 Emergent Manifold Structure

The passage from a discrete spin-foam description to a smooth classical spacetime is a substantive physical problem. Spin-foam models are motivated in part by the expectation that suitable semiclassical and continuum regimes recover general-relativistic geometry, but that recovery cannot be assumed for an arbitrary two-complex or arbitrary labeling [22, 26]. Coarse-graining and renormalization are among the tools used to investigate the relation between microscopic spin-foam dynamics and larger-scale effective behavior [29].

For that reason, the paper does not write

$$\mathfrak{F}_U = (\mathcal{M}_U, g_U).$$

The stronger identity would conceal the very emergence problem that supplies the source-domain insight.

Instead, the physical relation is represented provisionally by

$$\mathfrak{F}_U^{(\ell)} \xrightarrow{\mathcal{C}_{\ell \rightarrow L}} \mathfrak{E}_U^{(L)} \rightsquigarrow (\mathcal{M}_U, g_U), \tag{71}$$

where the final symbol $\rightsquigarrow$ denotes a possible effective or semiclassical reconstruction under additional conditions.

Those conditions are not supplied by the notation itself. Depending upon the model and regime, one would need evidence that the effective data admit an appropriate geometric interpretation and that the relevant continuum or semiclassical behavior is recovered. The present paper therefore treats

$$(\mathcal{M}_U, g_U) \quad (72)$$

as an effective geometric object whose availability must be justified rather than as a primitive component of the spin foam.

This distinction supplies a useful philosophical discipline. If a smooth interpretive geometry is later associated with a relational system, that geometry should likewise be understood as an effective descriptive level. It should not be projected backward as though the underlying relations had always possessed a unique global manifold representation.

4.4 Relational Patches and Local State Spaces

Once an effective local geometric description is available, a bounded region may be represented through a local state space

$$\mathcal{M}_U. \quad (73)$$

The subscript U emphasizes that the description is local. It does not follow that all local effective geometries belong to one already established global manifold, nor that transition maps among every pair of regions are known.

This local perspective is especially important in a background-independent setting. Spin-foam amplitudes are naturally associated with boundary data, and coarse-graining can be formulated in terms of relations between finer and coarser boundary descriptions [29]. The local region is therefore characterized partly through its relational boundary and the amplitudes connecting boundary configurations.

For the later philosophical transfer, this suggests a bounded notion of a relational patch:

$$U = \{\text{relations and histories relevant to a specified local inquiry}\}. \quad (74)$$

Equation 74 is not a definition in spin-foam theory. It marks the point at which the physical source domain and the Generative Relational construction begin to diverge.

The corresponding GR object will eventually contain a situated language system

$$L_U, \quad (75)$$

but L_U is not a physical spin-foam label. It is introduced later as an effective hermeneutic structure associated with a historically and relationally bounded domain.

The intended sequence is therefore

$$\text{relational substrate} \longrightarrow \text{bounded effective description} \longrightarrow \text{local interpretive construction}. \quad (76)$$

The first two stages are motivated by the physical source domain. The third belongs to the philosophical model developed in subsequent sections.

4.5 Local Geometry and Trajectory Formation

If an effective manifold $\mathcal{M}_U$ is available, a state $x \in \mathcal{M}_U$ possesses a tangent space

$$T_x \mathcal{M}_U. \quad (77)$$

As established in Section 3, the tangent space represents locally available geometric directions. It does not identify which directions are dynamically realized.

A local trajectory

$$\gamma : I \longrightarrow \mathcal{M}_U \quad (78)$$

has tangent

$$\dot{\gamma}(t) \in T_{\gamma(t)}\mathcal{M}_U. \quad (79)$$

Dynamics then selects or weights particular trajectories according to additional structure. In the physical source domain, amplitudes and constraints supply information that cannot be inferred from the manifold alone.

This separation is essential for the later account of freedom. Even if an effective relational geometry describes a field of local possibilities, the existence of a geometric direction does not imply that the direction is recognized, meaningful, legitimate, practically reachable, or sustainable for a particular subject.

The later hermeneutic construction therefore introduces

$$\mathcal{A}_{L_U}(x) \subseteq T_x\mathcal{M}_U, \quad (80)$$

where $\mathcal{A}_{L_U}(x)$ denotes directions provisionally interpreted as hermeneutically admissible within L_U . Equation 80 belongs to the GR model, not to spin-foam physics.

The full analytical ordering is thus

$$\begin{aligned} T_x\mathcal{M}_U &\supseteq \mathcal{A}_{L_U}(x) \\ &\supseteq \mathcal{A}_{B,U}^{\text{eff}}(x), \end{aligned} \quad (81)$$

where the final set anticipates directions that remain effectively generable for a bearer B after material, institutional, bodily, and relational conditions are considered.

Equation 81 is a modeling scaffold rather than a general theorem. In particular, the subset relation can require refinement where interpretive change itself generates previously unrepresented directions. Its present role is to prevent geometric possibility, hermeneutic intelligibility, and effective agency from being collapsed into one object.

4.6 Physical Locality and Philosophical Transfer

The source-domain contribution can now be stated more precisely. Spin-foam theory contributes several structural ideas that are useful for the later philosophical model.

First, relational structure can precede a smooth global background. A two-complex is specified combinatorially and relationally, while geometric information is encoded through labels and amplitudes rather than introduced through a fixed classical spacetime [3, 22].

Second, locality can be represented through bounded substructures and their boundaries. A local region need not initially be defined through coordinates on a pre-given global manifold.

Third, effective descriptions can arise through coarse-graining. The fine-scale structure need not remain explicitly represented at the coarse level [29].

Fourth, the passage to smooth or classical geometry is non-trivial. Effective geometry requires additional conditions and cannot be identified with the microscopic combinatorial object by stipulation.

Fifth, local descriptions raise a subsequent global problem. Even when several local effective regions are individually well defined, their compatibility, transition relations, and possible integration into a larger coherent description require further structure.

These source-domain observations motivate the later philosophical sequence

$$\mathfrak{R}_U \longrightarrow \mathfrak{E}_U \longrightarrow L_U \longrightarrow \mathcal{A}_{L_U}, \quad (82)$$

where $\mathfrak{R}_U$ denotes a bounded relational configuration and $\mathfrak{E}_U$ an effective relational description.

The notation deliberately differs from the spin-foam notation $\mathfrak{F}_U$. Once the analysis moves into hermeneutics, institutions, history, and subjectivity, it is no longer analyzing a physical spin foam.

4.7 Limits of the Spin-Foam Analogy

The spin-foam construction is useful only if its limits remain explicit. Several non-identities therefore govern the remainder of the paper:

$$\begin{aligned}
&\mathfrak{R}_U \neq \mathfrak{F}_U, \\
&L_U \neq \{j_f, \iota_e\}, \\
&\text{historical relation} \neq \text{spin-foam edge or face}, \\
&\text{social event} \neq \text{spin-foam vertex}, \\
&\text{interpretive change} \neq \text{quantum-gravitational transition amplitude}, \\
&\text{freedom meaning} \neq \text{quantum geometry}.
\end{aligned} \tag{83}$$

The paper makes no claim that persons, institutions, religions, languages, or political systems are literally spin foams. It also makes no claim that spin-foam labels should be assigned empirically to social entities.

The transferable content is restricted to the following architecture:

$$\begin{aligned}
&\text{relationally specified local structure} \\
&\longrightarrow \text{effective local description} \\
&\longrightarrow \text{possible emergent geometry} \\
&\longrightarrow \text{local-to-global compatibility problem}.
\end{aligned} \tag{84}$$

Even this transfer remains conditional. If later hermeneutic analysis cannot specify what constitutes a local patch, what transformations connect patches, or what structures are preserved across those transformations, the geometric language should be weakened rather than retained for rhetorical effect.

The physical source domain therefore performs a limited methodological role. It demonstrates that local relational organization, effective geometry, and global reconstruction can be analytically separated. The next section applies that separation to the relational subject. It asks how a subject can be represented when interaction is already non-zero and when the relevant trajectory space is itself historically and relationally structured.

5 Generative Relational Subjectivity

This section develops the account of subjectivity required by Generative Relational Freedom. The formal premises established in Sections 3 and 4 distinguished reference dynamics from interaction, local state spaces from global descriptions, and geometric possibility from dynamically selected trajectories. The present section transfers only those structural distinctions needed to analyze an embodied, historically situated, and relationally developing subject.

The principal claim is limited. A situated subject is not represented as a completed entity to which relations are subsequently added. Relations, embodiment, material conditions, language, institutional organization, and history participate in the formation of the subject's practical and interpretive organization. This relational formation does not eliminate agency, nor does it establish that every interaction is constitutive, beneficial, or normatively justified. It changes the location of the analytical problem. Agency must be studied within interacting dynamics rather than by treating zero interaction as the ordinary condition of subjecthood.

5.1 Analytical Free Reference Dynamics

The decomposition

$$S = S_0 + S_{\text{int}} \quad (85)$$

provides the initial formal scaffold. As established in Section 3.2, S_0 denotes a selected free or isolated reference description, while S_{int} represents interaction-dependent structure. The adjective *free* retains its technical analytical meaning at this stage.

For a subject B , an analogous local dynamical representation can be written schematically as

$$\dot{x}_B = F_B^{(0)}(x_B) + F_B^{(\text{int})}(x_B, \mathcal{R}_{B,t}, \mathcal{E}_t), \quad (86)$$

where x_B denotes a situated state, $F_B^{(0)}$ an analytically isolated reference dynamics, $\mathcal{R}_{B,t}$ a relevant relational configuration, and $\mathcal{E}_t$ additional material or environmental conditions.

Equation 86 is not proposed as a literal differential equation of human behavior. Its function is to preserve a distinction between what an analytical model attributes to a selected reference system and what becomes dependent upon interaction.

The reference term should not be interpreted in any of the following ways:

$$F_B^{(0)} \neq \begin{cases} \text{the authentic self,} \\ \text{the original self,} \\ \text{the morally preferable self,} \\ \text{the historically pre-social self.} \end{cases} \quad (87)$$

The free reference is therefore methodological rather than genealogical. Nothing in the decomposition establishes that the corresponding isolated state ever existed.

This restriction is particularly important for theories of freedom. If $F_B^{(0)}$ were treated as the subject's true dynamics, every relational effect could be interpreted as a deviation from an antecedent freedom. The present framework rejects that inference. A relational subject can acquire capacities, concepts, commitments, identities, and forms of agency through relations that have no meaningful pre-relational counterpart.

The reference dynamics remains useful because counterfactual analysis often requires isolation of particular interactions. One may ask how a trajectory would differ without a specified institutional constraint, interpersonal dependency, or environmental condition. Such a comparison does not establish that the resulting counterfactual describes the subject's true or natural state.

5.2 Non-Zero Relational Interaction

For the situated subject considered in this paper, the relevant analytical condition is

$$S_{\text{int}} \neq 0. \quad (88)$$

The statement is intentionally modest. It means that the actual system cannot be represented by the selected isolated reference alone. It does not imply that every relation contributes equally, that all interaction is constraining, or that interaction necessarily reduces freedom.

A person's relational field may include, among other structures,

$$\mathcal{R}_{B,t} = \left\{ \mathcal{R}^{\text{interpersonal}}, \mathcal{R}^{\text{institutional}}, \mathcal{R}^{\text{symbolic}}, \mathcal{R}^{\text{material}}, \mathcal{R}^{\text{technical}}, \mathcal{R}^{\text{environmental}}, \dots \right\}_{B,t}. \quad (89)$$

The components in Equation 89 are analytical categories rather than mutually exclusive ontological substances. A legal institution, for example, can simultaneously possess material, symbolic, interpersonal, and institutional dimensions.

The significance of Equation 88 lies partly in what it prevents. The removal of one relation,

$$\mathcal{R}_{B,t}^{(k)} \longrightarrow 0, \quad (90)$$

does not imply

$$S_{\text{int}} \longrightarrow 0. \quad (91)$$

A subject released from one form of control remains embodied, historically formed, materially situated, linguistically mediated, and connected to other relations. This distinction will later be important when the paper examines claims that the removal of an identifiable restriction is sufficient to establish freedom.

The same structure applies to voluntary relation. Entering a friendship, language community, religious practice, profession, family relation, or political association changes relational dynamics even when the relation is chosen. Voluntariness therefore does not make the interaction term disappear.

Conversely, non-zero interaction does not establish unfreedom:

$$S_{\text{int}} \neq 0 \not\Rightarrow \text{Unfree}(B). \quad (92)$$

This non-implication is foundational for the paper. The later task is to analyze the organization, interpretation, and transformation of interaction, rather than to use interaction itself as a negative quantity.

5.3 Embodiment and Material Situatedness

Relational subjectivity is not exhausted by social interaction. The subject considered here is embodied and materially situated. Bodily capacities, spatial location, available resources, infrastructures, environmental conditions, technologies, and temporal demands participate in the practical organization of trajectories.

Embodiment is especially important because it complicates any representation of freedom as unlimited abstract choice. A subject can formally conceive an action that is physically impossible, biologically inaccessible, materially unsupported, or incompatible with other necessary conditions of continued existence. Formal conceivability and embodied reachability are therefore different objects.

The capability tradition has emphasized the importance of distinguishing formal resources or permissions from substantive opportunities [28]. The present framework retains that insight while placing it within a wider relational account. Material conditions do more than supply resources to an otherwise complete chooser. They can participate in the formation of habits, expectations, skills, dependencies, temporal horizons, and interpretive possibilities.

A material condition can therefore affect both current action and subsequent subject formation. Schematically,

$$\mathcal{E}_t \longrightarrow \{x_{B,t+1}, \mathcal{R}_{B,t+1}, \Gamma_{B,t+1}\}, \quad (93)$$

where $\Gamma_{B,t}$ provisionally denotes the subject's generative organization.

The term *generative organization* refers to the historically developed structure through which a subject interprets situations, forms ends, generates responses, establishes relations, and revises prior organization. It should not be understood as an immutable internal program. Its more specific relation to the language system L_{st} is developed in Section 6.

Material situatedness also prevents the later theory from reducing freedom to semantics. Freedom meanings emerge within language systems, but the subject who interprets those meanings remains exposed to bodily and material conditions that cannot be transformed merely through redescribing them.

5.4 Temporality and Historical Dependence

A situated subject is temporally extended. Present organization contains effects of prior interactions, learning, institutions, injuries, commitments, practices, and environments. A snapshot of current state can therefore be insufficient for reconstructing the dynamics through which future trajectories are generated.

Let

$$H_{B,t} = (x_{B,\tau}, \mathcal{R}_{B,\tau}, \mathcal{E}_\tau)_{\tau < t} \quad (94)$$

denote schematically the relevant history of subject B prior to time t . The present analysis does not assume that the entire history can be observed or represented without loss.

Historical dependence means that two systems with superficially similar current observations need not possess identical generative organization:

$$O_{B,t} = O_{C,t} \not\Rightarrow H_{B,t} = H_{C,t}, \quad (95)$$

and consequently does not establish identical future response.

A more explicit update relation can be represented by

$$\Gamma_{B,t+1} = \mathcal{U}_B(\Gamma_{B,t}, x_{B,t}, \mathcal{R}_{B,t}, H_{B,t}, \eta_t), \quad (96)$$

where η_t represents contingencies or disturbances not already included in the declared variables.

Equation 96 performs several analytical tasks. It permits present interaction to depend upon prior organization. It allows two subjects exposed to similar conditions to update differently. It also prevents relation from becoming deterministic: the presence of a relation does not uniquely specify its generative consequence.

Historical dependence further means that removal of a present source does not necessarily erase prior relational effects:

$$\mathcal{R}_{B,t}^{(k)} = 0 \not\Rightarrow \Delta\Gamma_{B,t} = 0. \quad (97)$$

The significance of this possibility was developed in earlier GR work on relational alteration and carry-over. In the present paper it has a narrower function. Freedom cannot always be inferred from the current inventory of external constraints because the subject's present organization can remain historically dependent upon relations that are no longer active.

Historical dependence should not be confused with historical determinism. Past relations condition later states without uniquely fixing them. Learning, reinterpretation, new relations, material change, and contingent events can alter trajectories that initially appear stable.

5.5 Subject Formation within Relational Fields

The preceding premises support a relational account of subject formation. Relational autonomy and hermeneutic accounts of selfhood have already challenged representations of a fully self-sufficient and fully self-transparent subject [13, 25]. GR develops this problem through the dynamics of generative organization.

The subject at time t may be represented provisionally by

$$\mathfrak{S}_{B,t} = \langle x_{B,t}, \Gamma_{B,t}, \mathcal{R}_{B,t}, H_{B,t} \rangle. \quad (98)$$

The components are analytical distinctions within one evolving system. $x_{B,t}$ denotes a situated state, $\Gamma_{B,t}$ generative organization, $\mathcal{R}_{B,t}$ relevant current relations, and $H_{B,t}$ relational and developmental history.

This representation rejects two opposite reductions.

The first reduction identifies the subject with an isolated internal state:

$$\mathfrak{S}_{B,t} \neq x_{B,t}. \quad (99)$$

The second reduction identifies the subject exhaustively with the surrounding relational field:

$$\mathfrak{S}_{B,t} \neq \mathcal{R}_{B,t}. \quad (100)$$

Relational formation therefore does not erase differentiation between a bearer and the relations in which that bearer participates. The distinction is analytically necessary for examining heterogeneous responses, responsibility, conflict, revision, and transformation.

A relation can contribute to subject formation without supplying a complete copy of its own structure. If subject A influences subject B , the result need not satisfy

$$\Gamma_{B,t+1} = \Gamma_{A,t}. \quad (101)$$

The resulting organization can instead emerge through interaction among prior organization, relational exposure, other concurrent relations, and contingency.

This generative view is important for the later hermeneutics of freedom. Freedom meanings do not enter a passive subject as completed semantic objects. They encounter subjects whose histories and generative organization affect how those meanings are recognized, resisted, transformed, or incorporated.

5.6 Constraint and Trajectory Formation

Constraint enters the framework at several levels. It can alter which trajectories are physically possible, materially reachable, institutionally permitted, socially recognized, interpretively available, or practically sustainable. These forms should not be collapsed into one scalar quantity.

Let $\mathcal{T}_{B,t}$ denote a broad trajectory space relevant to a specified inquiry. A relational configuration can modify this space through several operations, including

$$\mathcal{R}_{B,t} \rightarrow \left\{ \begin{array}{l} \text{closure of trajectories,} \\ \text{creation of trajectories,} \\ \text{change of transition cost,} \\ \text{change of stability,} \\ \text{change of recognizability,} \\ \text{change of revisability.} \end{array} \right. \quad (102)$$

The list is analytical rather than exhaustive.

Constraint therefore should not be represented solely through subtraction. A language rule can prohibit some combinations while enabling comprehensible expression. A legal procedure can close some actions while creating stable expectations through which other actions become possible. A relationship can restrict some trajectories while providing resources, knowledge, trust, or coordination required for others.

Conversely, a relation can produce severe closure without issuing an explicit prohibition. Economic dependency, inaccessible infrastructure, social exclusion, informational isolation, or accumulated relational history can make a trajectory practically unavailable even where formal permission remains.

The following non-implications are therefore important:

$$\begin{array}{l} \text{formal permission} \not\Rightarrow \text{effective reachability,} \\ \text{absence of prohibition} \not\Rightarrow \text{absence of constraint,} \\ \text{presence of constraint} \not\Rightarrow \text{absence of agency.} \end{array} \quad (103)$$

The paper consequently avoids defining freedom through a simple difference between an unconstrained possibility set and a constrained one. The organization of trajectories, their meanings, and the subject's relation to the structures that organize them require separate examination.

5.7 Generative Agency within Structured Dynamics

Agency in the present framework is not identified with causally unconditioned motion. Such an identification would return the analysis to the zero-interaction baseline rejected above.

Generative agency instead concerns the subject's situated participation in forming responses, trajectories, interpretations, relations, and subsequent conditions under structured dynamics. The concept is intentionally broader than momentary choice. A subject can exercise agency through learning, deliberation, refusal, commitment, reinterpretation, experimentation, coordination, institutional participation, and the generation of new relations.

This perspective distinguishes a generated trajectory from a merely permitted trajectory. A formal option may exist without becoming integrated into the subject's practical or interpretive organization. Conversely, agency can sometimes generate a trajectory that was previously weakly represented or unrecognized.

The relation may be represented schematically as

$$(\Gamma_{B,t}, \mathcal{R}_{B,t}, x_{B,t}, H_{B,t}) \longrightarrow \mathcal{P}_{B,t+1}, \quad (104)$$

where $\mathcal{P}_{B,t+1}$ denotes generated practices, actions, interpretations, or relational changes.

No one component independently determines $\mathcal{P}_{B,t+1}$. Generative agency therefore occupies neither an isolated internal location nor an entirely external field. It is expressed through the coupling between organized subjectivity and situated relations.

The framework also permits reflexive agency. A subject can direct activity toward the conditions under which activity itself is generated:

$$\Gamma_{B,t} \longrightarrow \Delta\Gamma_{B,t+1}, \quad \mathcal{R}_{B,t} \longrightarrow \Delta\mathcal{R}_{B,t+1}. \quad (105)$$

Self-revision and relational revision are therefore distinct but potentially coupled forms of generative activity.

This distinction will later become central to Generative Relational Freedom. A subject may possess possibilities within a relational structure while having little capacity to question or modify that structure. Another subject may operate within a relatively narrow set of current trajectories while retaining substantial capacity for reinterpretation and relational reorganization. Option cardinality alone cannot distinguish these cases.

5.8 Transformation of Relational Structure

The relational field should not be treated as a fixed background against which subjects move. Subjects participate in reproducing, contesting, and transforming the relations that condition later action.

A minimal coupled representation is

$$(\mathfrak{S}_{B,t}, \mathcal{R}_t) \longrightarrow (\mathfrak{S}_{B,t+1}, \mathcal{R}_{t+1}). \quad (106)$$

Equation 106 expresses co-evolution rather than symmetry between the two terms. A subject and an institution, for example, can possess radically unequal capacities to alter the relational field.

This possibility introduces a distinction between two forms of movement:

$$\begin{aligned} &\text{movement within a relational configuration,} \\ &\text{transformation of the relational configuration.} \end{aligned} \quad (107)$$

The first concerns trajectories available under a given organization. The second concerns changes to the rules, relations, interpretations, boundaries, or institutions that structure subsequent trajectories.

The distinction is particularly important for political and religious examples. A person may possess substantial discretion within an established institution while lacking meaningful influence over the institution's ordering principles. Conversely, a disciplined practice may contain strong constraints on

immediate conduct while permitting significant participation in reinterpretation, communal deliberation, or transformation of the practice itself.

The capacity to transform relation should not be assumed to be individually held. Relational structures are frequently reproduced through collective action, accumulated history, institutional power, technical infrastructure, and symbolic organization. A single subject may alter only a small local component while collective processes generate larger structural change.

This observation also prevents GR from converting agency into sovereign control. Generative agency can operate under uncertainty, dependence, constraint, and limited causal reach. A subject may participate in a change without determining its final form.

The resulting account of subjectivity can be summarized by three non-reductions:

$$\begin{aligned} \text{subject} &\neq \text{isolated state,} \\ \text{subject} &\neq \text{relational field,} \\ \text{agency} &\neq \text{absence of interaction.} \end{aligned} \tag{108}$$

A generative relational subject is instead historically formed within interaction while remaining capable, to varying degrees, of generating responses that reproduce, revise, or transform the relations through which later trajectories become possible.

This account provides the bearer required for the remainder of the paper. Section 6 next develops the relational language system L_{st} . The central question shifts from how a relational subject is constituted to how a historically situated relational field organizes what can become meaningful, recognizable, and interpretable for that subject.

6 Spatiotemporal Relational Language Systems

This section develops the spatiotemporal relational language system L_{st} , which provides the hermeneutic medium required by the later account of Generative Relational Freedom. The preceding section established a subject whose agency is historically formed within non-zero interaction. The present section addresses the corresponding interpretive problem: how actions, relations, constraints, possibilities, obligations, and claims of freedom become recognizable and meaningful within a particular relational world.

The term *language system* is used in an extended sense. It includes natural language while also including historically sedimented symbols, norms, institutional categories, practical distinctions, legal forms, religious vocabularies, material practices, conventions of recognition, and recurrent patterns of interpretation. L_{st} is therefore neither a dictionary nor a purely mental representation. It is a relationally generated organization through which a situated subject can distinguish, interpret, and respond to possible trajectories.

The subscripts s and t indicate that such a system is spatially, relationally, and historically situated. The notation does not assume that s is reducible to geographic location or that t is reducible to calendar time. A relational space may include institutional position, religious community, legal order, linguistic environment, technological infrastructure, and other structures that affect interpretive access.

6.1 The Relational Language System L_{st}

Let

$$L_{st} \tag{109}$$

denote the interpretive organization associated with a specified relational and historical region. A provisional representation is

$$L_{st} = \Psi(\mathcal{R}_{st}, \mathcal{P}_{st}, \mathcal{I}_{st}, \mathcal{S}_{st}, H_{st}), \tag{110}$$

where $\mathcal{R}_{st}$ denotes relational organization, $\mathcal{P}_{st}$ material and practical structures, $\mathcal{I}_{st}$ institutional organization, $\mathcal{S}_{st}$ symbolic and normative structures, and H_{st} relevant historical dependence.

Equation 110 should not be read as a closed functional law. The operator Ψ indicates a generative relation rather than a deterministic mapping. The components can interact recursively, and L_{st} can itself participate in later changes to relations, practices, institutions, and symbolic structures.

The language system therefore occupies an intermediate analytical position. It is generated through relation, but once stabilized it also helps organize subsequent relation.

A minimal recursive structure is

$$\mathcal{R}_{st} \longrightarrow L_{st} \longrightarrow \mathcal{R}_{s,t+1}, \quad (111)$$

with the understanding that the arrows summarize reciprocal processes rather than a single causal sequence.

This recursive position is important for freedom. A concept such as *freedom* does not merely describe an already completed relational field. Its interpretation can become part of the field by shaping expectations, claims, institutions, identities, and the recognition of particular trajectories.

6.2 Material and Institutional Formation of L_{st}

A situated language system develops through more than discourse. Material conditions and institutional structures can affect which distinctions become salient, which practices recur, and which categories acquire practical importance.

Historical materialism provides one important precedent for this orientation. Social consciousness and political categories are historically connected to material life, labor, production, and social relations [16, 18]. The present framework retains this historical and relational insight without treating L_{st} as a mechanically determined reflection of economic structure.

Material and institutional relations can enter language formation through repeated practice. Property relations, employment arrangements, religious institutions, legal classifications, educational systems, technologies, and forms of political organization can stabilize distinctions that later appear natural or self-evident.

A schematic relation is

$$(\mathcal{P}_{st}, \mathcal{I}_{st}) \longrightarrow \Delta L_{st}. \quad (112)$$

The reverse relation is also possible:

$$\Delta L_{st} \longrightarrow (\Delta \mathcal{P}_{s,t+1}, \Delta \mathcal{I}_{s,t+1}). \quad (113)$$

For example, a new legal category can reorganize material relationships, while changes in production or technology can generate new interpretive categories. The two directions should therefore be analyzed as historically coupled.

This structure prevents the paper from locating freedom only in ideas. Interpretations of freedom can have material and institutional consequences, while material and institutional structures participate in determining which freedom interpretations become practically intelligible.

6.3 Symbolic and Normative Mediation

Symbolic and normative structures constitute another dimension of L_{st} . A subject encounters relations through distinctions such as legitimate and illegitimate, permitted and forbidden, sacred and profane, private and public, dependent and independent, voluntary and compulsory, or free and unfree.

These distinctions are not merely labels attached after action occurs. They can affect which actions are contemplated, which experiences are recognized, and which responses become socially or personally intelligible.

Let

$$\mathcal{S}_{st} = \{\sigma_1, \sigma_2, \dots, \sigma_n\} \quad (114)$$

denote a provisional collection of symbolic and normative distinctions active within a relational region.

A trajectory τ can then be interpreted through these distinctions:

$$\tau \mapsto \text{Interpret}_{L_{st}}(\tau). \quad (115)$$

The resulting interpretation can affect later generative response:

$$\text{Interpret}_{L_{st}}(\tau) \longrightarrow \Delta\Gamma_{B,t+1}. \quad (116)$$

A similar external event can therefore generate different consequences in different language systems, or for subjects differently positioned within the same language system.

This point is especially important for religious and political freedom. Practices that appear restrictive under one interpretive structure can acquire a different relation to agency under another, while practices described as liberating in one political vocabulary can be experienced as alienating or dominating within another. The framework does not resolve these disagreements by selecting one vocabulary in advance. It first asks how the relevant interpretive structures were generated and how they organize possible trajectories.

6.4 Historical Sedimentation and Interpretive Memory

A language system is temporally layered. Present meanings can contain traces of earlier institutions, religious traditions, political conflicts, legal forms, economic relations, and prior translations. The history of a concept can therefore remain operative even after the material conditions associated with its earlier use have changed.

Let

$$H_{st}^L \quad (117)$$

denote the historical dependence relevant to the formation of L_{st} . Then a present language system can be represented as

$$L_{st} = \Psi(\mathcal{R}_{st}, H_{st}^L), \quad (118)$$

where the historical term includes prior interpretive configurations rather than only present relational conditions.

Historical sedimentation means that similar contemporary relations can be interpreted differently because they enter different inherited symbolic orders. It also means that apparent lexical continuity can conceal semantic change.

For a lexical expression w ,

$$\llbracket w \rrbracket_{L_{s,t_1}} \not\equiv \llbracket w \rrbracket_{L_{s,t_2}} \quad (119)$$

can hold even when the visible word remains unchanged.

The persistence of vocabulary should therefore not be confused with the persistence of interpretive structure.

This historical dimension also provides a bridge to dialectical change. An inherited language system can stabilize a relational configuration while accumulating tensions that later support reinterpretation. The memory carried by L_{st} can therefore contribute both to persistence and to transformation.

6.5 Recognizable Possibilities and Interpretive Horizons

A central function of L_{st} is to organize recognizability. The total set of mathematically conceivable or physically possible trajectories is wider than the set a situated subject can identify as meaningful possibilities.

Let

$$\mathcal{T}_U \quad (120)$$

denote a broad trajectory space associated with a local relational region U . The language system induces a provisional family

$$\mathcal{T}_U^L \subseteq \mathcal{T}_U, \quad (121)$$

where $\mathcal{T}_U^L$ contains trajectories that become interpretable within L_U .

This construction parallels, without identifying itself with, the tangent-space model introduced earlier:

$$\mathcal{A}_{L_U}(x) \subseteq T_x \mathcal{M}_U. \quad (122)$$

The language system can affect at least several kinds of recognizability:

$$\mathcal{A}_{L_U}(x) = \left\{ \begin{array}{l} \text{conceivable directions,} \\ \text{nameable directions,} \\ \text{normatively recognizable directions,} \\ \text{socially legible directions,} \\ \text{practically interpretable directions} \end{array} \right\}. \quad (123)$$

These components need not coincide.

A trajectory can be conceptually imaginable but socially illegible. It can be legally recognized but personally unthinkable. It can be linguistically expressible while remaining institutionally unsupported.

The later theory of freedom will therefore distinguish a wide possibility space from the situated interpretive horizon through which a subject can recognize possible action.

This distinction also prevents freedom from being reduced to option count. Increasing the number of formal possibilities does not necessarily expand the set of meaningful trajectories available within the subject's interpretive field.

6.6 Locality of Relational Meaning

The notation L_{st} indicates that interpretive structure is local in a relational and historical sense. This locality should not be reduced to national culture, natural language, or geographic region.

A relational patch U may be specified through a combination of

$$U = U(\text{institution, history, practice, language, relation, material setting}). \quad (124)$$

Different patches can overlap. A person can participate simultaneously in a family, profession, religious tradition, political system, legal order, linguistic community, and transnational network. The corresponding interpretive structures can therefore coexist without forming a single homogeneous language system.

For subject B , the effective interpretive environment may be represented as

$$L_{B,t}^{\text{eff}} = \Phi_B \left(L_{st}^{(1)}, L_{st}^{(2)}, \dots, L_{st}^{(m)}, H_{B,t} \right). \quad (125)$$

Equation 125 emphasizes that L_{st} is not necessarily shared uniformly by every person located in the same society.

This point is especially important for cross-cultural analysis. Expressions such as “Western freedom,” “Chinese freedom,” “Islamic freedom,” or “Asian freedom” can conceal substantial internal heterogeneity. The relevant analytical unit is the specified relational language system, not an assumed civilizational essence.

The local approach therefore replaces broad semantic attribution with a more restricted question:

Within which relational and historical configuration does a particular interpretation become intelligible?

This question will later support the patchwise and local-to-global treatment of freedom meanings.

6.7 Transformation of L_{st} through Practice

A relational language system is generative rather than static. Practices can reproduce inherited distinctions, introduce new distinctions, modify old ones, or alter the relations among them.

Let P_t denote a situated practice. A minimal update structure is

$$L_{s,t+1} = \mathcal{U}_L(L_{s,t}, P_t, \mathcal{R}_{s,t}, H_{s,t}). \quad (126)$$

The update can be gradual or discontinuous. Repeated institutional practice can stabilize a distinction slowly, while political crisis, religious reform, technological change, legal transformation, or collective mobilization can produce more rapid interpretive reorganization.

A practice can also generate vocabulary for a trajectory that was previously poorly represented. In that case,

$$\tau \notin \mathcal{T}^{L_t} \longrightarrow \tau \in \mathcal{T}^{L_{t+1}}. \quad (127)$$

The reverse can occur as well. Interpretive disappearance, suppression, forgetting, or institutional closure can remove a previously recognizable trajectory from the active language system.

This structure gives historical change a hermeneutic dimension. The relevant state space is not fixed while subjects merely move through it. Practices can change the interpretive organization through which the space itself is recognized.

Such transformation will later provide one mechanism for relational symmetry breaking. A particular historical configuration can stabilize some meanings while making others less available, and subsequent practice can reorganize the stabilizer structure.

6.8 Reflexivity between Language and Relation

The relation between L_{st} and the relational field is reflexive. A language system emerges within relations and then becomes one of the conditions through which later relations are interpreted and organized.

The recursive structure can be represented by

$$\mathcal{R}_{s,t} \longrightarrow L_{s,t} \longrightarrow \mathcal{P}_{s,t} \longrightarrow \mathcal{R}_{s,t+1}, \quad (128)$$

where $\mathcal{P}_{s,t}$ denotes practices generated within the relevant interpretive organization.

This cycle does not imply closure. New encounters, material disturbances, translation, conflict, technological change, external relations, and contingency can introduce information not generated by the prior cycle.

The reflexive structure is especially important for political descriptions of freedom. A declaration such as

“This is a free society”

can function simultaneously as a description, a normative claim, a symbolic classification, and an intervention into the language system through which later political relations are judged.

Likewise, institutional measurements of freedom can become performative. Once incorporated into governance, education, diplomacy, or public discourse, a classificatory scheme can affect the relational field that it purportedly describes.

The paper therefore distinguishes

representation of relation (129)

from

representation acting within relation. (130)

The second relation will become important when the paper later examines measurement, classification, and political claims of freedom.

The resulting architecture of L_{st} can be summarized as

historically situated relations $\longrightarrow$ relational language system
 $\longrightarrow$ recognizable trajectories
 $\longrightarrow$ practice and interpretation
 $\longrightarrow$ relational and semantic transformation. (131)

No stage in Equation 131 is treated as a final state. The system remains historically open.

The construction developed in this section provides the interpretive medium required for the analysis of freedom. Section 7 next examines how historically changing material, institutional, symbolic, and practical relations generate different freedom languages and how those languages can become unstable, contested, and reorganized over time.

7 Historical Generation of Freedom

This section introduces the historical dimension of Generative Relational Freedom. Section 6 developed L_{st} as a situated interpretive organization through which relations and trajectories become recognizable. The present section asks how such interpretive structures acquire, stabilize, and transform meanings of freedom through material relations, production, institutions, practice, and historical conflict.

The argument is neither a semantic historicism in which concepts simply change with time nor a deterministic materialism in which meanings can be derived directly from economic conditions. Its methodological premise is relational. Material organization, institutions, practical activity, symbolic forms, and interpretation participate in historically evolving couplings. Freedom languages arise within these couplings and can subsequently enter the relations that generated them.

Historical materialism and dialectical philosophy provide important interlocutors for this construction. Marx's emphasis on practical activity, social relations, and historically specific material organization challenges accounts that begin from an abstract isolated individual [15, 18, 16]. Dialectical analysis contributes a concern with forms whose internal relations can generate tensions and transformations rather than remaining permanently fixed [10]. GR retains these methodological resources while rejecting an automatic historical teleology and while treating its own categories as historically revisable.

7.1 Material Relations and Freedom Languages

Freedom is frequently discussed through explicitly political or normative vocabulary, but the conditions under which such vocabulary becomes meaningful are also material. Labor organization, property relations, access to land and infrastructure, technologies, patterns of dependence, systems of exchange, forms of care, and distributions of temporal resources affect the relations through which subjects act and interpret action.

Historical materialism is relevant here because it directs attention away from concepts considered independently of social practice. Marx and Engels locate forms of consciousness within historically changing forms of material life and social intercourse while also recognizing reciprocal relations among the different dimensions of social organization [18]. The present framework adopts this relational orientation without assuming that a material configuration determines one uniquely corresponding freedom concept.

Let the material-relational configuration of a local historical region be denoted by

$$\mathcal{Q}_{st} = \langle \mathcal{P}_{st}, \mathcal{R}_{st}, \mathcal{I}_{st} \rangle, \quad (132)$$

where $\mathcal{P}_{st}$ denotes material and productive arrangements, $\mathcal{R}_{st}$ relational organization, and $\mathcal{I}_{st}$ institutional structure.

A freedom interpretation f_{st} is then situated within, rather than external to, this configuration:

$$f_{st} \in L_{st} [\mathcal{Q}_{st}, H_{st}]. \quad (133)$$

The notation does not mean that f_{st} can be calculated from $\mathcal{Q}_{st}$. It states a dependence relation: a situated meaning of freedom develops within a language system whose history includes material and institutional organization.

This dependence operates in more than one direction. A material relation may make a distinction politically salient, while a stabilized freedom language can later motivate institutional reforms that alter material relations. Consequently,

$$\mathcal{Q}_{s,t} \longleftrightarrow L_{s,t} \longleftrightarrow \mathcal{Q}_{s,t+1} \quad (134)$$

is a more appropriate provisional architecture than a one-way determination from material structure to political concept.

This recursive account matters because a society can retain the same lexical term while the material relations surrounding its use change substantially. The historical persistence of a word therefore provides no guarantee that the relational object organized through that word remains unchanged.

7.2 Production, Practice, and Concept Formation

Historical generation involves practice as well as material structure. Marx's *Theses on Feuerbach* emphasizes practical activity rather than a purely contemplative relation to the world [15]. For the present inquiry, the important methodological consequence is that concepts can develop through participation in practices rather than solely through detached theoretical reflection.

A subject who enters a labor relation, religious discipline, legal procedure, political movement, migration process, or technological environment encounters forms of organization that may produce distinctions unavailable at the level of abstract speculation alone. Practice can therefore modify both the relational field and the vocabulary through which that field is interpreted.

Schematically,

$$(L_{s,t}, \mathcal{P}_{s,t}) \longrightarrow (L_{s,t+1}, \mathcal{P}_{s,t+1}). \quad (135)$$

The relation is generative because practice can produce new interpretive objects. A distinction that was previously unavailable can become salient through repeated experience, conflict, organization, or institutional change.

For a trajectory τ ,

$$\tau \notin \mathcal{T}^{L_t} \longrightarrow \tau \in \mathcal{T}^{L_{t+1}} \quad (136)$$

can represent the emergence of a newly recognizable possibility.

The change need not involve invention of a new lexical item. An existing word can acquire a new relational significance when practices change. Conversely, a concept can remain linguistically available while losing practical connection to the relations through which it previously operated.

Production should therefore be understood broadly in this section. It includes production of material goods, institutions, relations, symbols, practices, expectations, and interpretive distinctions. The relevant GR question concerns how these productions alter the conditions for further generation.

This position also complicates the division between interpreting and changing the world. Interpretation is itself a practice capable of entering later relations, while practical transformation can alter the horizon within which future interpretation occurs.

7.3 Institutional Mediation of Freedom

Institutions provide relatively durable structures through which freedom meanings are reproduced, specified, and operationalized. Constitutions, courts, markets, schools, religious organizations, workplaces, families, administrative agencies, and international institutions can each stabilize different distinctions concerning permission, obligation, autonomy, dependence, membership, participation, and legitimate authority.

Institutional mediation should be distinguished from simple institutional definition. An institution does not possess exclusive authority over the meaning of freedom merely because it uses the term. Institutional categories interact with wider histories, practices, symbolic systems, and forms of contestation.

Let an institution I participate in a relational patch U . Its mediation can be represented by

$$L_U \xrightarrow{\mathcal{I}} L_U^{(I)}, \quad (137)$$

where $L_U^{(I)}$ denotes an institutionally stabilized articulation within the wider language system.

The institutional form can influence which freedom claims become legally actionable, administratively measurable, publicly recognizable, or materially supported. It can also exclude claims that remain meaningful in other parts of the relational field.

Accordingly,

$$\text{institutional recognition} \not\equiv \text{exhaustive hermeneutic recognition}. \quad (138)$$

The reverse non-implication also matters. A freedom claim can be intelligible within a social or religious language system while lacking institutional recognition.

Institutional mediation introduces persistence. Once embedded in law, administration, measurement, education, or ritual, an interpretation can outlast some of the relations that originally generated it. This persistence contributes to historical sedimentation and can generate tensions when an institutionally stabilized language no longer corresponds closely to changing practices.

Institutions therefore function simultaneously as repositories of historical meaning and sites of possible reinterpretation.

7.4 Historical Transformation of Freedom Meanings

The historical character of L_{st} means that freedom meanings can change without requiring disappearance of the lexical term through which they are expressed.

For a word w_F used to signify freedom,

$$\llbracket w_F \rrbracket_{L_{s,t_1}} \neq \llbracket w_F \rrbracket_{L_{s,t_2}} \quad (139)$$

can hold even when

$$w_F(t_1) = w_F(t_2). \quad (140)$$

Lexical continuity and hermeneutic continuity are therefore different historical claims.

A changing material or political order may reorganize the relations among concepts already present in the language system. A term previously connected primarily with legal status may become connected with economic independence, political participation, religious emancipation, personal autonomy, or other relations. The transformation can involve expansion, contraction, differentiation, or recombination rather than simple replacement.

Historical transformation can also create several concurrent freedom languages. Older meanings need not disappear when new interpretations emerge. A relational field can contain overlapping or competing configurations:

$$\mathcal{F}_{st} = \{f_{st}^{(1)}, f_{st}^{(2)}, \dots, f_{st}^{(n)}\}. \quad (141)$$

The elements of $\mathcal{F}_{st}$ need not be mutually exclusive or fully commensurable. Their coexistence can generate political disagreement, translation problems, hybrid interpretations, and later conceptual reorganization.

This multiplicity is one reason the paper avoids assigning a single historical period one homogeneous definition of freedom. Historical situatedness produces differentiation within periods as well as differences between them.

7.5 Contradiction and Relational Reconfiguration

A historically stabilized freedom language can enter into tension with the relations through which it operates. The term *contradiction* is used here in a bounded dialectical sense. It refers to relational configurations in which practices, institutions, or normative commitments generate pressures that cannot be accommodated without modifying some part of the existing organization.

Consider a relational configuration

$$\mathfrak{C}_t = (\mathcal{R}_t, L_t, \mathcal{I}_t, \mathcal{P}_t). \quad (142)$$

A tension can arise when a stabilized freedom claim within L_t depends upon practices whose consequences produce experiences or relations that the same interpretive structure cannot coherently absorb.

The resulting process is represented schematically by

$$\mathfrak{C}_t \longrightarrow \Delta \mathfrak{C}_t \longrightarrow \mathfrak{C}_{t+1}. \quad (143)$$

Equation 143 does not imply that every tension generates transformation. Institutions can suppress, redistribute, reinterpret, or endure contradictions for long periods. Nor does transformation imply normative improvement.

A contradiction can produce several trajectories:

$$\Delta \mathfrak{C}_t \longrightarrow \left\{ \begin{array}{l} \text{reinterpretation,} \\ \text{institutional reform,} \\ \text{suppression,} \\ \text{fragmentation,} \\ \text{new domination,} \\ \text{partial accommodation.} \end{array} \right. \quad (144)$$

The absence of a predetermined outcome distinguishes the present model from a teleological reading of dialectic. Historical tension generates a field of possible reconfiguration rather than a guaranteed higher synthesis.

This structure is important for freedom because an inherited freedom language can participate simultaneously in stabilization and critique. A concept used to legitimate an existing order can later supply vocabulary through which subjects challenge exclusions generated within that order.

7.6 Dialectical Change in Interpretive Structures

Dialectical change concerns more than movement among positions inside a fixed interpretive space. The relations defining the space of intelligible positions can themselves change.

Suppose a situated language system at time t supports an interpretive configuration f_t . Historical transformation can produce

$$(L_t, f_t) \longrightarrow (L_{t+1}, f_{t+1}). \quad (145)$$

The transformation can alter both the meaning assigned to freedom and the relations among competing interpretations.

This distinction is important. A debate within a stable L_t may concern which interpretation should prevail. A deeper historical transition changes the vocabulary, distinctions, and practical relations through which the debate itself is formulated.

The latter case can be represented as

$$\mathcal{X}_t^F \longrightarrow \mathcal{X}_{t+1}^F, \quad (146)$$

where $\mathcal{X}_t^F$ denotes the effective interpretive state space within which freedom configurations are distinguished.

Equation 146 anticipates the later symmetry analysis. If the interpretive state space changes, the transformation group and stabilizer structure relevant to freedom may also change. Historical symmetry breaking therefore cannot always be represented by one fixed group acting on one fixed space for all time.

A more general structure is

$$(\mathcal{X}_t^F, G_t^F, f_t) \longrightarrow (\mathcal{X}_{t+1}^F, G_{t+1}^F, f_{t+1}). \quad (147)$$

The later formal sections will impose stricter conditions before referring to these transformations as symmetries. The present equation identifies the historical problem that the formalism must eventually represent.

Dialectical change is therefore treated as transformation of relational and interpretive organization rather than as a fixed logical sequence imposed on history.

7.7 Alienation and Generative Closure

Alienation provides an important boundary case for a relational theory of freedom. Marx's early writings analyze forms of estrangement in relation to labor, its products, social relations, and human activity [17]. The present subsection does not attempt a complete reconstruction of Marx's theory of alienation. Its narrower purpose is to examine how alienation can illuminate a relation between historically organized activity and generative closure.

A difficulty arises if alienation is defined solely through deviation from a fixed and fully specified human essence. Such a formulation would sit uneasily with the present framework, which rejects an untouched pre-relational subject as an empirical baseline.

GR therefore approaches alienation through relational organization rather than through restoration of a pristine subject. A candidate alienating configuration can involve a subject participating in the reproduction of relations that progressively reduce the subject's ability to recognize, revise, redirect, or transform the conditions through which activity is generated.

Let the capacity for relational revision be represented provisionally by

$$Rev_{B,t} = \{\text{recognition, reinterpretation, revision, exit, reorganization}\}_{B,t}. \quad (148)$$

A candidate process of generative closure may then involve

$$\mathcal{Rev}_{B,t+1} \prec_D \mathcal{Rev}_{B,t+1}^q, \quad (149)$$

relative to an independently defended comparison q and declared domain D .

This condition alone does not establish alienation in a Marxian sense. It identifies one GR-compatible diagnostic feature: the relational process reduces the subject's capacity to participate in the interpretation and reorganization of the conditions governing later generation.

Such closure can occur even while immediate choice remains available. A subject can choose among many actions within a structure while possessing little capacity to question the categories, dependencies, or institutions that organize those choices.

Conversely, strong discipline or limited immediate options do not automatically establish alienation. A demanding practice can remain reflectively endorsed, revisable, interpretable within a broader life project, and open to relational renegotiation.

The distinction will become important later:

$$\text{restricted immediate choice} \not\equiv \text{generative closure}. \quad (150)$$

Alienation therefore enters the paper as a historically and relationally mediated problem of subject, activity, and control over generative conditions, rather than as a synonym for every form of constraint.

7.8 Historical Revisability of Freedom

The preceding analysis has a reflexive consequence. If freedom meanings are generated within historically situated relational language systems, then the present theory cannot claim an exemption from the same condition.

Generative Relational Freedom is itself articulated within a particular historical and intellectual configuration. Its concepts of relation, generativity, language system, trajectory, symmetry, and freedom draw upon available philosophical, political, mathematical, and scientific vocabularies. The framework therefore has no legitimate position outside history from which to issue a final semantic determination of freedom.

This does not make the inquiry arbitrary. A situated theory can still be evaluated for conceptual coherence, explanatory usefulness, fidelity to evidence, comparative reach, capacity to survive counterexamples, and revisability under encounters with other relational systems.

The appropriate relation is therefore

$$\text{GRF}_t \longrightarrow \text{GRF}_{t+1}, \quad (151)$$

where later versions can alter concepts, distinctions, and formal structures when new arguments or relational encounters require revision.

Historical revisability also changes the objective of the freedom inquiry. The paper does not seek a semantic terminal point

$$f_t \longrightarrow f^* \quad \text{for all future } t, \quad (152)$$

where f^* would constitute a final historically independent definition.

The alternative programme examines

$$\{f_{st}, L_{st}, T_{ij}, I_{\text{cross}}, \dots\}, \quad (153)$$

including situated interpretations, their language systems, transformations among relational contexts, and structures that may remain invariant across some of those transformations.

The distinction between a final definition and cross-relational invariance is essential. Rejecting a context-independent semantic endpoint does not require rejecting every possibility of common structure. It changes where such common structure must be sought.

The historical thesis of this section can therefore be stated narrowly. Freedom meanings develop within changing relations of material life, institutional organization, practice, and interpretation. Those meanings can stabilize and acquire substantial causal force, yet they remain susceptible to historical reconfiguration. Neither lexical continuity nor institutional persistence guarantees semantic invariance.

Section 8 now turns from the historical generation of freedom languages to the paper's central conceptual object: Generative Relational Freedom. The task is to specify what can meaningfully be claimed about freedom when the subject is already relational, interaction is non-zero, and the language through which freedom becomes intelligible is historically situated.

8 Generative Relational Freedom

The preceding sections established three premises for the present account. First, the subject under examination is historically formed within non-zero interaction. Second, trajectories become meaningful within a situated relational language system L_{st} . Third, both the relational field and the interpretive structures through which that field is understood can change historically.

This section brings those premises together under the concept of *Generative Relational Freedom* (GRF). The objective is not to supply a context-independent definition that assigns every subject a determinate quantity of freedom. GRF instead identifies a structured field of relations among situated interpretation, effective generative capability, revision, exit, and transformation.

Freedom is therefore approached within relational dynamics rather than from a presumed state prior to relation. The central question concerns the subject's position within a relationally and historically organized field: which trajectories can become intelligible, which can be effectively generated, which relations can be revised, which can be left, and to what extent the conditions governing subsequent generation can themselves be transformed.

These dimensions need not move together. A subject can possess many immediate options while having little capacity to alter the structure organizing those options. Another subject can inhabit a highly disciplined practice while retaining substantial interpretive participation, revisability, and capacity to transform the relation. GRF therefore resists both option counting and a simple inverse relation between freedom and constraint.

8.1 Analytical Location of Freedom

The first task is to identify the analytical location of freedom. If the subject were treated as complete prior to interaction, freedom could be represented as a property possessed by the isolated subject and subsequently reduced by external constraints. The relational account developed in Section 5 does not support this sequence.

The situated subject is represented by

$$\mathfrak{S}_{B,t} = \langle x_{B,t}, \Gamma_{B,t}, \mathcal{R}_{B,t}, H_{B,t} \rangle, \quad (154)$$

while interpretation occurs through a historically situated language system

$$L_{st}. \quad (155)$$

Freedom therefore cannot be located exclusively in $x_{B,t}$, exclusively in $\mathcal{R}_{B,t}$, or exclusively in L_{st} . It concerns relations among these structures and the trajectories they support.

For a declared domain D and horizon H , let

$$\mathcal{G}_{B,t}^{D,H} = \left\{ \tau \in \mathcal{T}_B^{D,H} \mid \tau \text{ is effectively generable from } \mathfrak{S}_{B,t} \right\} \quad (156)$$

denote the subject's effective generative capability. The term *effectively generable* is stronger than formal permission and weaker than guaranteed realization. It concerns trajectories that the situated sub-

ject can practically enter or generate under the declared relational, material, interpretive, and temporal conditions.

Equation 156 does not itself define freedom. A large effective capability set can coexist with domination, irreversibility, manipulation, or severe inability to transform the conditions through which those trajectories are generated.

GRF therefore requires additional dimensions. A provisional freedom profile can be represented by

$$\mathfrak{F}_{B,t}^{GR}(D, H) = \left\langle \mathcal{G}_{B,t}^{D,H}, \mathcal{R}ev_{B,t}, \mathcal{E}x_{B,t}, \mathcal{T}r_{B,t} \right\rangle_{L_{st}, \mathcal{R}_{B,t}, H_{B,t}}, \quad (157)$$

where

- $\mathcal{G}_{B,t}^{D,H}$ concerns effective generative trajectories;
- $\mathcal{R}ev_{B,t}$ concerns capacities for revision and reinterpretation;
- $\mathcal{E}x_{B,t}$ concerns meaningful possibilities of leaving or suspending relevant relations;
- $\mathcal{T}r_{B,t}$ concerns capacities to transform the relational conditions themselves.

The profile in Equation 157 is not proposed as exhaustive. Its purpose is to prevent several analytically distinct dimensions from being compressed into one quantity.

The subscript attached to the profile is equally important. The dimensions are interpreted relative to L_{st} , the relevant relational field, and history. A claimed possibility of exit, for example, can have very different meaning depending upon whether the subject can recognize the alternative, whether leaving is materially survivable, whether the relation is replaceable, and whether accumulated history continues to organize the subject after formal exit.

Generative Relational Freedom is therefore located in a structured relation among subject, language, trajectory, and relational transformability.

8.2 Freedom under Non-Zero Interaction

The formal premise

$$S_{\text{int}} \neq 0 \quad (158)$$

changes the initial freedom problem. Freedom cannot be understood through the empirical achievement of

$$S_{\text{int}} = 0. \quad (159)$$

For an embodied and relationally formed subject, such a condition would remove not only particular forms of domination but also language, social learning, care, coordination, material dependence, institutional support, and many of the relations through which meaningful agency develops.

The central GRF question is consequently not whether interaction exists. It concerns how interaction is organized.

This distinction can be expressed through the non-implications

$$\begin{aligned} S_{\text{int}} \neq 0 &\not\Rightarrow \text{Unfree}(B), \\ S_{\text{int}} \rightarrow 0 &\not\Rightarrow \text{Free}(B). \end{aligned} \quad (160)$$

A subject abandoned in an environment without direct interpersonal interference may possess little meaningful agency if the material, interpretive, or relational conditions required for viable trajectories are absent. Conversely, participation in a structured practice can involve substantial interaction while sustaining meaningful forms of agency.

Liberation is therefore more adequately represented as possible relational reconfiguration,

$$S_{\text{int}}^{\text{old}} \longrightarrow S_{\text{int}}^{\text{new}}, \quad (161)$$

than as a universal movement toward zero interaction.

Equation 161 does not imply that every replacement relation is emancipatory. The normative status of the new configuration depends upon its consequences for subjects, generative conditions, domination, revisability, and other relevant values.

This position places GRF in conversation with theories of non-interference and non-domination while preserving a distinct analytical problem [4, 23]. Removing interference can matter profoundly, and eliminating arbitrary power can matter even in the absence of actual interference. GRF adds a further question concerning the generative organization that remains before, during, and after such changes.

8.3 Constraint and Situated Generativity

Constraint is unavoidable in any sufficiently structured relational system. The relevant distinction is therefore not between constraint and a wholly unconstrained state. It concerns the different operations constraints perform within generative dynamics.

Let C denote a constraint acting within a relational configuration. Depending upon its structure, C can

$$C : \mathcal{T} \longrightarrow \left\{ \begin{array}{l} \text{exclusion,} \\ \text{selection,} \\ \text{coordination,} \\ \text{stabilization,} \\ \text{redirection,} \\ \text{generation,} \\ \text{closure.} \end{array} \right. \quad (162)$$

The list does not imply that these effects are mutually exclusive.

A grammar illustrates the distinction at a limited structural level. If Σ^* denotes all finite strings over an alphabet Σ , a grammar Γ selects only a structured subset:

$$\mathcal{L}(\Gamma) \subseteq \Sigma^*. \quad (163)$$

The reduction in formally admissible strings does not entail a reduction in meaningful linguistic generativity. Structured rules can enable compositionality, recognizability, and indefinitely extensible expression.

The analogy is deliberately narrow. Political and social constraints do not acquire legitimacy merely by resembling grammatical rules. The example establishes only

$$\text{fewer formal possibilities} \not\Rightarrow \text{less generative freedom.} \quad (164)$$

The reverse implication is equally invalid:

$$\text{constraint} \not\Rightarrow \text{greater generative freedom.} \quad (165)$$

Constraint architecture must therefore be analyzed relationally. The important questions include who participates in generating the constraint, who can interpret it, who bears its costs, whether it can be revised, whether exit is meaningful, and whether the constraint preserves or forecloses future generative trajectories.

This account also separates GRF from a maximization principle. Generativity itself is not an unconditional normative objective. A system capable of generating indefinitely many harmful, dominating, or destructive trajectories is not thereby more free in the relevant normative sense.

Consequently,

$$\text{Maximize } |\mathcal{G}_{B,t}^{D,H}| \quad (166)$$

is not adopted as the objective of GRF.

The quantity, quality, structure, distribution, reversibility, and relational consequences of generative possibilities remain distinct concerns.

8.4 Generative Trajectories and Relational Agency

The distinction between a possible trajectory and a generative trajectory is central to GRF. A trajectory can exist formally within a state space while remaining inaccessible to the situated subject.

The preceding sections identified several nested analytical levels:

$$\mathcal{T}_U \supseteq \mathcal{T}_{B,U}^L \supseteq \mathcal{G}_{B,t}^{D,H}, \quad (167)$$

where $\mathcal{T}_U$ denotes a broad local trajectory space, $\mathcal{T}_{B,U}^L$ trajectories recognizable within the relevant language system for subject B , and $\mathcal{G}_{B,t}^{D,H}$ trajectories effectively generable under the declared conditions.

Equation 167 remains a provisional modeling relation. Interpretive innovation can expand the represented trajectory space, and actual practice can transform the conditions under which trajectories are classified.

The distinction nevertheless identifies an important failure mode in option-based accounts. A formally available trajectory can fail to enter $\mathcal{G}_{B,t}^{D,H}$ because it is materially inaccessible, institutionally blocked, socially unintelligible, historically unavailable to the subject, or incompatible with necessary conditions of continued action.

Conversely, generative agency can sometimes alter the trajectory field:

$$\mathcal{G}_{B,t}^{D,H} \longrightarrow \mathcal{G}_{B,t+1}^{D,H}. \quad (168)$$

This transformation can occur through learning, institution building, collective organization, technological change, reinterpretation, creation of new relations, or modification of existing constraints.

GRF therefore concerns more than choosing among already listed alternatives. It includes the situated capacity to participate in the generation of possibilities themselves.

This distinguishes

$$\text{Choose}(\tau) \quad (169)$$

from

$$\text{Generate}(\mathcal{T}, L, \mathcal{R}). \quad (170)$$

The latter concerns changes to the space of possible action, the interpretive system through which actions become recognizable, and the relational field within which they become viable.

The distinction is one reason GRF cannot be reduced to preference satisfaction. Preferences themselves can be historically and relationally formed, and the language through which a subject recognizes an alternative can change. Work on adaptive preferences within the capability tradition provides an important neighboring concern [21].

GRF does not infer that every socially formed preference is defective. The relevant problem is whether subjects retain meaningful opportunities for recognition, reinterpretation, revision, and engagement with alternative relational configurations.

8.5 Revision and Reinterpretation

A relational system can provide numerous trajectories while remaining interpretively rigid. For this reason, GRF treats revisability as a distinct dimension.

Let

$$\mathcal{Rev}_{B,t} \quad (171)$$

denote a structured capacity to reconsider and modify interpretations, commitments, practices, and relevant relational arrangements.

Revisability should not be equated with continual change. A subject can reflectively retain an existing practice. The important distinction is between persistence that remains open to re-examination and persistence sustained through conditions that make meaningful reconsideration unavailable.

The distinction can be represented by

$$\text{Persistence} \not\equiv \text{Irrevisability} . \quad (172)$$

Likewise,

$$\text{Change} \not\equiv \text{Revision} . \quad (173)$$

A subject can be repeatedly redirected by external pressures without possessing meaningful interpretive participation in those changes.

Reinterpretation is especially important because L_{st} itself is historically generative. A subject can acquire a new interpretation of a previously accepted relation, and that reinterpretation can alter later trajectories:

$$[\mathcal{R}]_{L_t} \longrightarrow [\mathcal{R}]_{L_{t+1}} \longrightarrow \Delta \mathcal{G}_{B,t+1}^{D,H} . \quad (174)$$

The relation can also operate in reverse. New practices and trajectories can produce reinterpretation of the language system.

This reflexivity makes freedom temporally extended. The relevant question is not limited to whether a subject can act now. It includes whether present relations preserve the possibility of later re-recognition and revision.

GRF therefore treats irreversible interpretive closure with particular caution. A relation that systematically removes the subject's capacity to recognize alternative interpretive worlds may reduce freedom even where substantial immediate choice remains.

8.6 Transformation of Relational Conditions

Revision of interpretation should be distinguished from transformation of the relations being interpreted. A subject can understand a relation differently while remaining unable to alter its institutional, material, or interpersonal structure.

Let

$$\mathcal{Tr}_{B,t} \quad (175)$$

denote the subject's situated capacity to participate in transformation of relevant relational conditions. This capacity can operate at several levels:

$$\mathcal{Tr}_{B,t} = \left\{ \begin{array}{l} \text{local adjustment,} \\ \text{rule revision,} \\ \text{institutional transformation,} \\ \text{relational reorganization,} \\ \text{creation of alternative structures.} \end{array} \right. \quad (176)$$

The capacity need not be individually sovereign. Many relational structures can be transformed only through collective organization or institutional processes. GRF therefore includes participation in collective transformability rather than requiring individual control over the entire field.

This distinction separates movement within a structure from transformation of the structure:

$$\mathcal{G}_{B,t}^{D,H} \neq \mathcal{T}r_{B,t}. \quad (177)$$

A person may have broad effective possibilities within an institution while having almost no influence over the rules governing those possibilities. Another person can temporarily possess a narrower trajectory set while participating meaningfully in transforming the institution.

Neither condition alone settles the freedom judgment.

Exit provides an additional but distinct possibility. Let $\mathcal{E}x_{B,t}$ denote meaningful exit from a specified relation. Formal exit is insufficient when leaving destroys the material or relational conditions required for continued agency.

Thus,

$$\text{LegalExit}(B, \mathcal{R}) \not\equiv \text{EffectiveExit}(B, \mathcal{R}). \quad (178)$$

Exit and voice are also not interchangeable. The ability to leave a relationship does not by itself supply the ability to participate in changing it, while transformability does not automatically provide meaningful exit.

GRF therefore retains both dimensions.

8.7 Freedom across Relational Scales

Freedom judgments can change with scale. A relation beneficial at one local level can participate in closure at another, while a locally restrictive rule can sustain a wider structure that enables substantial agency.

Let

$$U_1 \subseteq U_2 \subseteq \dots \subseteq U_n \quad (179)$$

denote nested relational regions. A freedom profile evaluated within U_i need not coincide with one evaluated within U_j :

$$\mathfrak{F}_{B,t}^{GR}(U_i) \neq \mathfrak{F}_{B,t}^{GR}(U_j). \quad (180)$$

For example, a local relation may increase the subject's immediate generative capability while producing dependency on a larger institutional structure that reduces long-horizon transformability. Conversely, a local restriction may preserve shared infrastructure upon which wider generative capabilities depend.

Temporal scale produces the same problem. A configuration that expands current options can degrade later generative conditions:

$$\mathcal{G}_{B,t}^{D,H_1} \uparrow \quad \text{while} \quad \mathcal{G}_{B,t+\Delta t}^{D,H_2} \downarrow, \quad H_2 > H_1. \quad (181)$$

The arrows indicate directional comparison rather than scalar magnitude.

This possibility is particularly important for exploitation, education, environmental governance, dependency, and institutional design. Immediate choice and future generative capability can diverge.

GRF therefore requires declared domain and horizon. A freedom claim that omits both can conceal substantial scale dependence.

This requirement does not imply that every analysis must model the entire social system. It requires only that local conclusions remain local unless cross-scale compatibility has been established.

8.8 Limits of Option-Based Accounts

Option-based representations provide useful information in many practical settings. The present framework does not reject the importance of having alternatives. It rejects the stronger inference that the number of available alternatives exhausts freedom.

Let

$$N_B = |\mathcal{O}_B| \quad (182)$$

denote the number of options available under a particular operational definition. An increase

$$N_B(t+1) > N_B(t) \quad (183)$$

does not establish an increase in GRF.

Several reasons follow from the preceding analysis.

First, options can be formally distinct while generatively equivalent. A large menu can contain alternatives that reproduce the same underlying relation.

Second, an option can exist formally without becoming recognizable or effectively reachable.

Third, current options can be generated through relations that reduce later revisability, exit, or transformability.

Fourth, a subject can acquire meaningful freedom through the capacity to generate new trajectories rather than through selection among a fixed list.

Fifth, option expansion can coexist with domination. Republican theory has already shown why absence of actual interference does not exhaust the freedom problem [23]. GRF adds that a wide menu of choices also says little about the subject's relation to the architecture that produces the menu.

The relevant comparison is therefore multidimensional:

$$|\mathcal{O}_B| \quad \text{and} \quad \mathfrak{F}_{B,t}^{GR} \quad (184)$$

describe different objects.

An option count can remain one valuable observable inside a broader inquiry. It should not be promoted into a complete ontology of freedom.

8.9 Limits of Scalar Representations

The preceding distinctions culminate in a broader methodological problem. Suppose a measurement system represents freedom through observables

$$\mathbf{z} = (z_1, z_2, \dots, z_n) \quad (185)$$

and aggregates them through

$$F_{\text{index}} = f(z_1, z_2, \dots, z_n). \quad (186)$$

Such a representation can be useful for a declared institutional or empirical purpose. GRF does not infer that the index is meaningless merely because it is selective.

The deeper problem is that the selection of $\mathbf{z}$ already requires an interpretation of what counts as freedom-relevant. The measurement procedure therefore has the structure

$$(\mathcal{R}_{st}, L_{st}, H_{st}) \xrightarrow{P} \mathbf{z} \xrightarrow{f} F_{\text{index}}, \quad (187)$$

where P is an interpretive and observational projection.

The projection can be rigorous while remaining partial. More data do not automatically remove this hermeneutic selection. Expanding the observable set

$$(z_1, \dots, z_n) \longrightarrow (z_1, \dots, z_n, z_{n+1}, \dots, z_m) \quad (188)$$

does not independently establish convergence toward a context-free object called freedom.

The issue is therefore deeper than insufficient measurement resolution. Before aggregation, one must ask which relational language system made the selected variables intelligible as indicators of freedom.

Optimization introduces the same limitation. If a governing institution defines

$$x^* = \arg \max_{x \in \mathcal{X}} F_{\text{index}}(x), \quad (189)$$

the optimization identifies a state that performs best under the declared representation. It does not prove that F_{index} exhausts freedom.

This distinction also prevents the framework from moving to the opposite extreme. Freedom is not reduced to subjective feeling. A subject can report feeling free under conditions of domination, adaptive preference, severe dependency, or absence of meaningful alternatives. Conversely, a subject can experience frustration within a relation that nevertheless preserves substantial effective capability and revisability.

GRF therefore separates at least four objects:

$$\begin{aligned} &\text{experienced freedom,} \\ &\text{formal freedom,} \\ &\text{effective generative capability,} \\ &\text{relational freedom structure.} \end{aligned} \quad (190)$$

Their relations are empirical, hermeneutic, and normative questions rather than identities.

The resulting concept of Generative Relational Freedom is thus intentionally non-scalar. It concerns the situated organization through which a relational subject can generate meaningful trajectories, reinterpret commitments, revise or exit relevant relations, and participate in transforming the conditions governing further generation.

This formulation remains incomplete in one important respect. The section has identified dimensions of GRF while relying upon the situated meanings through which those dimensions are recognized. Section 9 therefore turns directly to the hermeneutics of freedom. It examines how a particular freedom meaning becomes intelligible, stabilizes within L_{st} , encounters competing interpretations, and changes through historical and relational practice.

9 Hermeneutics of Freedom

Section 8 located Generative Relational Freedom within a structured relation among situated subjectivity, effective generative capability, revisability, exit, and relational transformation. The present section addresses a prior interpretive dependence within that structure. Before a trajectory can be classified as liberating, restrictive, voluntary, dominating, autonomous, or otherwise freedom-relevant, the categories through which the trajectory is recognized must themselves become intelligible within a situated relational language system.

The purpose of this section is therefore hermeneutic rather than measurement-oriented. It examines how freedom meanings emerge within L_{st} , how they become recognizable and temporarily stabilized, how competing interpretations coexist and contest one another, and how historical practice can alter the interpretive structures through which freedom is understood.

The argument draws on philosophical hermeneutics, especially the historical situatedness of understanding and the dependence of interpretation upon language, inherited horizons, and encounters with other interpretations [8]. Ricoeur's treatment of interpretation, selfhood, and narrative mediation supplies a related resource for avoiding an account of the subject as fully self-transparent prior to interpretation [25]. GR reorganizes these concerns through the spatiotemporal language system L_{st} and the dynamics of relational generation.

9.1 Situated Interpretation

An interpretation of freedom is never generated from an entirely unspecified position. It occurs for a subject, within a relational region, under a historically formed language system.

Let B denote a subject situated in relational patch U at time t . For a freedom-relevant event, trajectory, or relation q , the interpretive operation may be written schematically as

$$\mathcal{I}_{B,U,t}^F(q) = \text{Interpret}_{L_{U,t}}(q; \mathfrak{S}_{B,t}, H_{B,t}). \quad (191)$$

Equation 191 is not a numerical function. The notation identifies the dependencies of interpretation. The same observable event can acquire different freedom significance when the language system, relational position, or historical trajectory changes.

Accordingly,

$$q_1 = q_2 \not\Rightarrow \mathcal{I}_{B,U_1,t_1}^F(q_1) = \mathcal{I}_{C,U_2,t_2}^F(q_2). \quad (192)$$

The non-implication does not imply that interpretation is arbitrary. It means that equality of observable events is insufficient to establish equality of their situated meaning.

Conversely, different practices can sometimes occupy similar interpretive positions inside different relational systems. Cross-relational comparison must therefore examine structure rather than rely only upon surface similarity or lexical correspondence.

This position also changes the meaning of the question

Is this subject free?

In GRF, the question is underspecified until the relevant domain, relations, historical horizon, and interpretive language are identified. The underspecification is semantic before it is statistical. Additional observations cannot resolve a question whose interpretive object has not yet been adequately specified.

9.2 Freedom Meanings within L_{st}

Let

$$f_{st} \in \mathcal{X}_{st}^F \quad (193)$$

denote a situated freedom interpretation within an effective interpretive space $\mathcal{X}_{st}^F$.

The object f_{st} should not be confused with the lexical expression through which it is communicated. A word such as *freedom* can function as one linguistic vehicle for several historically differentiated interpretive structures.

For a lexical item w_F ,

$$w_F \longrightarrow \llbracket w_F \rrbracket_{L_{st}} = f_{st} \quad (194)$$

expresses the dependence of situated meaning upon the language system.

The relation is not one-to-one. A single lexical token can support several interpretations:

$$w_F \longrightarrow \left\{ f_{st}^{(1)}, f_{st}^{(2)}, \dots, f_{st}^{(n)} \right\}, \quad (195)$$

while different lexical expressions can participate in partially overlapping interpretive structures.

This distinction prevents lexical identity from being treated as semantic identity:

$$w_i = w_j \not\Rightarrow \llbracket w_i \rrbracket_{L_i} = \llbracket w_j \rrbracket_{L_j}. \quad (196)$$

Even two subjects using the same expression within the same natural language may occupy different relational language systems:

$$\llbracket \text{freedom} \rrbracket_{L_{B,t}} \neq \llbracket \text{freedom} \rrbracket_{L_{C,t}}. \quad (197)$$

The difference may arise from institutional position, political history, religious formation, social experience, professional practice, generation, translation history, or other relational factors.

The unit of analysis is therefore not “English,” “Chinese,” “Western thought,” or another civilizational totality. It is the historically and relationally specified L_{st} .

9.3 Recognition and Interpretive Stabilization

A freedom interpretation can become relatively stable through repeated recognition. Such stabilization can occur through ordinary language, education, law, institutions, ritual, political discourse, scholarly theory, media, or repeated practical experience.

Let

$$f_{st} \in \mathcal{X}_{st}^F \quad (198)$$

be one possible interpretation. Repeated relational reproduction may produce

$$f_{s,t} \longrightarrow f_{s,t+1} \longrightarrow \cdots \longrightarrow f_{s,t+k}, \quad (199)$$

with sufficient continuity that the interpretation functions as a relatively stable component of L_{st} .

Stability does not require perfect semantic identity across time. A concept can remain recognizable while undergoing gradual internal modification. The relevant relation may therefore be one of structured continuity:

$$f_{s,t} \approx_L f_{s,t+1} \quad (200)$$

rather than strict equality.

Interpretive stabilization can occur through several mutually reinforcing processes:

$$\left\{ \begin{array}{l} \text{lexical repetition,} \\ \text{institutional codification,} \\ \text{normative reinforcement,} \\ \text{practical reproduction,} \\ \text{symbolic legitimation,} \\ \text{collective recognition.} \end{array} \right. \longrightarrow \text{Persistence}(f_{st}). \quad (201)$$

No individual process is assumed to be necessary or sufficient in every context.

This distinction will later support the formal language of symmetry breaking. At the present stage, however, interpretive stabilization merely denotes historical persistence under relational reproduction. It should not yet be identified with a mathematical stabilizer subgroup.

Stabilized interpretations can become phenomenologically unobtrusive. Once embedded deeply in institutions and everyday distinctions, a particular meaning of freedom may appear to be the meaning of freedom rather than one historically generated configuration among several possible ones.

Hermeneutic analysis makes this historical stabilization visible without thereby invalidating the interpretation.

9.4 Contestation and Semantic Reconfiguration

A relational language system need not stabilize one freedom interpretation. Several interpretations may coexist within the same relational patch:

$$\mathcal{F}_{U,t} = \left\{ f_{U,t}^{(1)}, f_{U,t}^{(2)}, \dots, f_{U,t}^{(n)} \right\}. \quad (202)$$

Relations among these interpretations can include compatibility, partial overlap, tension, translation, exclusion, or competition for institutional recognition.

Contestation therefore occurs within language systems as well as between them. A political community can disagree internally about whether freedom is better articulated through non-interference, independence, non-domination, collective self-rule, substantive opportunity, religious obligation, personal autonomy, or another locally intelligible configuration.

The existence of disagreement does not imply that the competing interpretations are entirely disconnected. They may share lexical resources, historical references, institutions, or practical concerns while assigning different relational significance to them.

A contestation event can be represented by

$$\left(f_{U,t}^{(1)}, f_{U,t}^{(2)}\right) \longrightarrow \Delta L_{U,t+1}. \quad (203)$$

Several outcomes are possible:

$$\Delta L_{U,t} \longrightarrow \begin{cases} \text{stabilization of one interpretation,} \\ \text{continued plurality,} \\ \text{hybridization,} \\ \text{semantic differentiation,} \\ \text{institutional separation,} \\ \text{emergence of a new interpretation.} \end{cases} \quad (204)$$

The framework does not presume convergence.

This non-teleological structure is important because disagreement can be generative. Contestation can expose distinctions that were previously hidden inside a stabilized vocabulary and produce new ways of recognizing relational conditions.

The possibility of semantic reconfiguration also prevents interpretive stability from being mistaken for conceptual finality.

9.5 Historical Horizons of Interpretation

Hermeneutic interpretation is historically conditioned. Gadamer's account of historically effected consciousness and horizons provides an important philosophical precedent for treating understanding as situated within traditions and histories rather than generated from a neutral position outside them [8].

GR represents this historical dependence through the history terms already introduced:

$$f_{st} = \Phi_F \left(L_{st}, H_{st}^L, H_{B,t}, \mathcal{R}_{B,t} \right), \quad (205)$$

where H_{st}^L denotes the history sedimented into the relevant language system and $H_{B,t}$ the subject's own relational history.

The notation does not imply that the interpretation can be predicted from these histories. Historical dependence supplies conditions of intelligibility rather than deterministic semantic output.

This distinction explains why retrospective interpretation can differ from interpretation available at the time of an event. Later conceptual resources can make visible relational structures that an earlier L_{st} did not readily distinguish.

For an event q ,

$$\mathcal{I}_{L_{t_1}}^F(q) \neq \mathcal{I}_{L_{t_2}}^F(q), \quad t_2 > t_1. \quad (206)$$

A later interpretation can therefore identify domination, dependency, exclusion, autonomy, or generative possibility differently from an earlier one.

This does not establish that later interpretations are automatically more correct. Historical distance can disclose some relations while obscuring others. The relevant comparison requires evidence, explicit interpretive premises, and attention to the relational conditions of both interpretive positions.

The same principle applies prospectively. Present freedom concepts cannot be assumed to exhaust every distinction through which future relational systems may interpret freedom.

9.6 Lexical Equivalence and Hermeneutic Non-Identity

Cross-linguistic translation provides a particularly clear illustration of the distinction between lexical correspondence and situated semantic identity.

Suppose a dictionary or translation convention provides

$$T_{\text{lex}}(\text{freedom}) = \text{自由}. \quad (207)$$

Equation 207 is used only as a schematic translation example. It does not assign one homogeneous historical meaning to either English or Chinese.

Let

$$f_{\text{EN}} = \llbracket \text{freedom} \rrbracket_{L_{\text{EN}, s_1, t_1}} \quad (208)$$

and

$$f_{\text{ZH}} = \llbracket \text{自由} \rrbracket_{L_{\text{ZH}, s_2, t_2}}. \quad (209)$$

Lexical translation does not establish

$$f_{\text{EN}} = f_{\text{ZH}}. \quad (210)$$

The translation operation is better represented as

$$T_{\text{EN} \rightarrow \text{ZH}} : f_{\text{EN}} \longrightarrow \tilde{f}_{\text{EN}}^{(\text{ZH})}, \quad (211)$$

where $\tilde{f}_{\text{EN}}^{(\text{ZH})}$ denotes the interpretation generated through translation into the receiving language system.

In general, the framework does not assume

$$\tilde{f}_{\text{EN}}^{(\text{ZH})} = f_{\text{ZH}}. \quad (212)$$

Translation therefore generates an interpretive bridge rather than proving that the two relational systems contain one identical semantic object.

The distinction applies within languages as well. A legal theorist, religious practitioner, political activist, economist, and ordinary speaker can all use the same lexical expression while participating in partially different language systems.

The general principle is

$$T_{\text{lexical}}(w_i) = w_j \not\Rightarrow \llbracket w_i \rrbracket_{L_i} = \llbracket w_j \rrbracket_{L_j}. \quad (213)$$

Translation permits comparison. It does not eliminate the need for hermeneutics.

9.7 Reflexivity of Freedom Discourse

Freedom discourse can modify the relational field that it interprets. A political classification, philosophical distinction, religious teaching, or institutional index can become part of the practices through which subjects subsequently understand themselves and others.

The interpretive loop can be represented by

$$\mathcal{R}_{st} \longrightarrow L_{st} \longrightarrow f_{st} \longrightarrow \mathcal{P}_{st} \longrightarrow \mathcal{R}_{s,t+1}. \quad (214)$$

Here f_{st} is not merely an observation of $\mathcal{R}_{st}$. Once expressed and socially operative, it can alter practice $\mathcal{P}_{st}$ and thereby participate in later relational organization.

This reflexivity is especially significant when an interpretation receives institutional authority. If a state, court, international institution, religious authority, school, or measurement system adopts a specific freedom classification, that classification can change access to resources, legitimacy, political recognition, and available forms of critique.

Thus,

$$\text{Classify}_F(\mathcal{R}) \longrightarrow \Delta\mathcal{R} \quad (215)$$

is possible.

The classificatory act can therefore possess both epistemic and generative effects. An interpretation may describe some relations accurately while simultaneously reorganizing other relations through its institutional use.

This distinction will become important in Section 17, where freedom indicators are examined as relational projections with possible reflexive consequences.

Reflexivity also applies to GRF. Once the framework supplies categories such as generative capability, revisability, or transformability, those categories can influence how subjects interpret their own relations. GRF therefore cannot claim to remain external to the systems it helps describe.

9.8 Interpretive Change through Relational Practice

Interpretive change does not occur only through philosophical argument. Practice itself can create new distinctions and expose previously obscured relations.

Let P_t denote a practice that interacts with the current language system:

$$(L_t, P_t) \longrightarrow L_{t+1}. \quad (216)$$

The resulting language system can contain freedom interpretations unavailable or weakly represented in L_t .

Such changes can arise through political mobilization, migration, new forms of labor, religious reform, technological change, legal struggle, interpersonal encounter, education, or ordinary experimentation with alternative ways of living.

Interpretive innovation can therefore alter the boundary between recognized and unrecognized trajectories:

$$\mathcal{T}_B^{L_t} \longrightarrow \mathcal{T}_B^{L_{t+1}}. \quad (217)$$

A newly recognized trajectory need not be materially reachable. Hermeneutic expansion and effective capability remain distinct.

The converse process is also possible:

$$\mathcal{T}_B^{L_{t+1}} \subset \mathcal{T}_B^{L_t}. \quad (218)$$

Institutional suppression, loss of vocabulary, destruction of cultural practice, enforced classification, or historical forgetting can narrow the interpretive field through which alternatives become recognizable.

This form of contraction is particularly significant for GRF because the subject can lose possibilities before those possibilities disappear physically. A trajectory may remain technically possible while becoming linguistically, symbolically, or institutionally difficult to recognize.

Hermeneutic freedom therefore concerns more than possessing words. It concerns maintaining generative access to interpretive structures through which alternatives can be encountered, compared, and reconsidered.

9.9 Revisability of Freedom Languages

The hermeneutic account culminates in the revisability of the language system itself. If L_{st} organizes what counts as a recognizable freedom possibility, then the inability to revise L_{st} can produce a deeper form of closure than restriction within an otherwise open vocabulary.

A language system can therefore be analyzed with respect to

$$\text{Revise}(L_{st}), \quad (219)$$

understood as the situated possibility of questioning, extending, reinterpreting, translating, or reorganizing its operative distinctions.

Revisability does not require indiscriminate semantic instability. Stable shared language is often necessary for communication, institutional coordination, and collective action. The relevant concern is whether stabilization becomes sufficiently rigid that alternatives can no longer be recognized or meaningfully articulated.

This produces an important distinction:

$$\text{Stability}(L) \not\Rightarrow \text{Closure}(L). \quad (220)$$

Likewise,

$$\text{Change}(L) \not\Rightarrow \text{Openness}(L). \quad (221)$$

A language system can change rapidly under coercive or manipulative conditions while becoming less capable of sustaining alternative interpretations.

For GRF, the relevant property is therefore generative revisability rather than change alone. A revisable language system permits subjects, under some conditions, to encounter distinctions not exhausted by the currently stabilized freedom interpretation.

The relation can be represented by

$$L_{st} \longrightarrow \{L_{s,t+1}^{(1)}, L_{s,t+1}^{(2)}, \dots\}, \quad (222)$$

where the multiplicity represents possible reinterpretive developments rather than a requirement that all developments remain simultaneously available.

This revisability also establishes an epistemic norm for the present paper. A theory of situated freedom that rendered its own categories unrevisable would contradict its account of historical hermeneutics. GRF must therefore remain open to transformation when encounters with other language systems, historical evidence, or counterexamples expose limitations in its own interpretive structure.

The hermeneutic analysis developed in this section can be summarized through the following sequence:

$$\begin{aligned} \text{relational and historical situation} &\longrightarrow L_{st} \\ &\longrightarrow \text{recognition of freedom meanings} \\ &\longrightarrow \text{stabilization and contestation} \\ &\longrightarrow \text{practice and relational effects} \\ &\longrightarrow \text{reinterpretation of } L_{st}. \end{aligned} \quad (223)$$

Freedom is therefore not treated as a semantically complete object that travels unchanged across relational systems. Its interpretations become locally meaningful through historical language systems and can subsequently be translated, contested, and transformed.

Section 10 next turns from the hermeneutics of freedom to the architecture of constraint. The objective is to distinguish material, transcendent, symbolic, interpersonal, institutional, and self-regulatory forms of constraint and to examine how their distribution, revisability, exit conditions, and generative effects alter the relational structure within which freedom is interpreted.

10 Relational Constraint Architecture

The preceding sections rejected a simple inverse relation between constraint and freedom. A relational subject exists within material, symbolic, institutional, interpersonal, embodied, and interpretive structures, and many of these structures constrain possible trajectories. The existence of constraint therefore cannot by itself distinguish freedom from unfreedom.

This section develops the concept of *relational constraint architecture*. The analytical object is not the total quantity of constraint imposed upon a subject. It is the organization of constraint across sources, relations, scales, interpretive systems, and temporal horizons. The analysis asks where a constraint is generated, through which relations it operates, who can interpret or revise it, who bears its consequences, what forms of exit remain available, and how the constraint affects subsequent generative capability.

This architecture is particularly important for comparing social, political, and religious configurations. Two systems can contain very different numbers of explicit rules while generating similar practical closure. Conversely, a system containing extensive explicit discipline can preserve substantial revisability, interpretive participation, or collective transformability. Counting prohibitions therefore provides little information about the relational structure of freedom.

For subject B in relational region U at time t , let

$$\mathfrak{C}_{B,U,t} = \left\langle C^{\text{emb}}, C^{\text{mat}}, C^{\text{tr}}, C^{\text{sym}}, C^{\text{int}}, C^{\text{inst}}, C^{\text{self}} \right\rangle_{B,U,t} \quad (224)$$

denote a provisional constraint profile. The components refer respectively to embodied, material, transcendent, symbolic, interpersonal, institutional, and self-regulatory constraint. They are analytical dimensions rather than mutually exclusive kinds. One concrete practice can operate through several dimensions simultaneously.

10.1 Sources and Locations of Constraint

A constraint can be classified by its observable source, by the relation through which it is transmitted, or by the location at which its effects appear. These classifications need not coincide.

An institutional rule, for example, may originate within an organization, be communicated through interpersonal authority, acquire symbolic legitimacy, be internalized as self-regulation, and eventually alter material or embodied trajectories. Describing the rule simply as an “institutional constraint” therefore identifies only one level of the architecture.

Let a constraint-generating process be represented by

$$c_k : A_k \longrightarrow B, \quad (225)$$

where A_k denotes a source or mediating configuration and B the subject whose trajectories are affected. The arrow does not imply intentional action. A_k may represent another subject, an institution, a material environment, a symbolic order, an inherited practice, or a compound relational structure.

The same observable restriction can arise from different architectures:

$$c^{(1)} \neq c^{(2)} \quad \text{while} \quad \Delta \mathcal{T}_B^{(1)} \approx \Delta \mathcal{T}_B^{(2)}. \quad (226)$$

Conversely, apparently similar constraints can produce very different effects for differently situated subjects:

$$c_B = c_C \not\Rightarrow \Delta \mathcal{G}_B = \Delta \mathcal{G}_C. \quad (227)$$

The latter possibility follows from the dependence of effective generative capability upon embodiment, history, resources, relations, and L_{st} .

Constraint architecture therefore requires at least three distinctions: source, transmission, and effect. Normative judgment enters only after these structures have been specified.

10.2 Material and Embodied Constraint

Material and embodied conditions place some of the most immediate limits upon possible trajectories. Bodies require time, energy, spatial access, and material support. Physical infrastructure, income, housing, transportation, technology, environmental conditions, and bodily capacities can substantially alter whether formally permitted trajectories remain effectively reachable.

This dimension connects GRF with the capability approach's distinction between resources or formal rights and substantive opportunities [28]. GRF extends the distinction by examining how such conditions participate in the continuing generation of the subject and the relational field.

Let

$$C_{B,t}^{\text{mat}} = C^{\text{mat}}(x_{B,t}, \mathcal{E}_t, \mathcal{P}_t) \quad (228)$$

denote the material component of constraint, where $\mathcal{E}_t$ represents relevant environmental conditions and $\mathcal{P}_t$ material or practical organization.

The effect of material constraint need not be binary. A trajectory can remain physically possible while becoming costly, unstable, dangerous, or unsustainable. Effective reachability should therefore not be equated with logical possibility.

Embodied constraint also complicates the idea of liberation through simple removal. A subject cannot ordinarily be liberated from embodiment itself. Agency must instead develop through bodily and material conditions. The relevant problem concerns whether those conditions sustain, differentially distribute, or foreclose meaningful generative trajectories.

Material constraint also interacts with other dimensions. A legal right of exit can become practically irrelevant when the material consequences of exit destroy the conditions required for continued agency. Conversely, material resources can reduce the force of an otherwise restrictive interpersonal or institutional relation.

Constraint dimensions must therefore be studied relationally rather than separately summed.

10.3 Transcendent Ordering

Some relational language systems organize conduct through reference to a transcendent, divine, sacred, cosmological, or otherwise supra-individual order. The present framework refers to this analytical dimension as *transcendent ordering*.

The term does not require the paper to decide the ontological status of the transcendent referent. The relevant object is the relational configuration in which obligations, prohibitions, disciplines, or permissions acquire force through a recognized relation to transcendent authority.

Let

$$C_{B,t}^{\text{tr}} = C^{\text{tr}}(L_{st}, \mathcal{R}_{B,t}^{\text{tr}}), \quad (229)$$

where $\mathcal{R}_{B,t}^{\text{tr}}$ denotes the subject's situated relation to the relevant transcendent order.

Transcendent constraint should not be interpreted automatically as external coercion. A religious discipline can be experienced as obligation, orientation, devotion, identity, self-cultivation, communal membership, or several of these simultaneously. The same practice can also operate through institutional authority, interpersonal pressure, symbolic legitimacy, and self-regulation.

For this reason, the number of explicit religious prescriptions does not supply a direct measure of freedom:

$$|C^{\text{tr}}| \uparrow \not\Rightarrow \mathfrak{F}_{B,t}^{GR} \downarrow. \quad (230)$$

The reverse inference is equally unsupported. A relational system with fewer explicit transcendent rules can contain extensive interpersonal, institutional, symbolic, or self-regulatory constraint.

This distinction provides a safer basis for comparative religious inquiry. Different traditions can distribute normative ordering across different relational locations. The relevant comparison concerns

the architecture, interpretation, revisability, and consequences of those constraints rather than a ranking based on the visible number of prescriptions.

10.4 Symbolic and Normative Ordering

Constraints can operate through symbolic distinctions even when no identifiable agent issues a command. Categories of honor, shame, respectability, normality, success, duty, purity, adulthood, family obligation, professional conduct, or citizenship can organize trajectories by altering what actions appear meaningful, legitimate, or recognizable.

Let $\mathcal{S}_{st}$ denote the symbolic and normative structure introduced in Section 6.3. A symbolic constraint acts through the relation

$$C_{B,t}^{\text{sym}} : \mathcal{T}_{B,t} \longrightarrow \mathcal{T}_{B,t}^L, \quad (231)$$

in the limited sense that interpretive structure can affect which trajectories remain recognizable within the subject's situated language system.

The notation does not imply that every trajectory outside $\mathcal{T}_{B,t}^L$ has been intentionally suppressed. Some possibilities may simply lack stable symbolic representation within the relevant L_{st} .

Symbolic ordering therefore differs from explicit prohibition. A trajectory can remain legal and physically reachable while becoming socially unintelligible or normatively difficult to articulate.

The converse is also possible. A symbolic vocabulary can make a previously obscured trajectory recognizable and thereby enlarge the interpretive field.

Accordingly, symbolic structure can both close and generate:

$$\Delta C^{\text{sym}} \longrightarrow \Delta \mathcal{T}_B^L. \quad (232)$$

The direction of change must be established in the particular case.

This dual capacity is one reason GRF avoids defining constraint as inherently negative. Symbolic organization is necessary for recognition and communication, while rigid symbolic organization can foreclose alternative interpretations.

10.5 Interpersonal and Institutional Constraint

Interpersonal and institutional constraints differ in scale and durability but frequently interact. A person can possess power over another through direct dependency, while institutions can stabilize asymmetric relations beyond the intentions or presence of particular individuals.

Republican theories of freedom are especially important here because they distinguish actual interference from exposure to arbitrary or uncontrolled power [23]. This distinction helps prevent GRF from identifying constraint only with observable intervention.

Let an asymmetric relation between A and B be represented by

$$A \xrightarrow{\rho} B, \quad (233)$$

where ρ denotes a capacity of A or an institution associated with A to alter relevant conditions of B 's trajectory generation.

The normative significance of ρ depends upon more than whether it is currently exercised. Relevant properties include predictability, contestability, reciprocity, revisability, accountability, and the subject's capacity to leave or transform the relation.

Institutional constraint can also persist after the persons who originally created it disappear. Rules, procedures, property arrangements, classifications, infrastructures, and organizational routines can reproduce relational asymmetry over time.

The architecture therefore distinguishes

$$\text{Source}(c) \neq \text{CurrentCarrier}(c) \quad (234)$$

where historically generated constraint can be reproduced by later actors who did not create its original structure.

This distinction will matter for responsibility. Structural diagnosis should precede the allocation of blame to particular agents.

10.6 Internalization and Self-Regulatory Constraint

A constraint can become self-regulatory. Through learning, repeated practice, social recognition, institutional participation, religious discipline, or historical adaptation, a subject can incorporate rules and distinctions into its own generative organization.

Let

$$C_{B,t}^{\text{ext}} \longrightarrow \Delta\Gamma_{B,t+1} \longrightarrow C_{B,t+1}^{\text{self}} \quad (235)$$

represent one possible pathway from externally mediated constraint to self-regulatory organization.

Equation 235 does not imply that self-regulation is deceptive or pathological. Deliberately cultivated habits, ethical commitments, professional discipline, religious practice, and self-imposed routines can be integral to agency.

The analytical problem concerns the process through which self-regulation was formed and the conditions under which it remains interpretable and revisable.

This is particularly important because externally enforced compliance and reflectively sustained self-discipline can produce similar observable behavior while possessing very different relational structures.

Thus,

$$O_B = O_C \not\Rightarrow \mathfrak{C}_B = \mathfrak{C}_C. \quad (236)$$

Observed conduct alone does not reveal whether a trajectory was produced through coercion, adaptation, commitment, identification, habit, or a mixture of these processes.

Work on relational autonomy and adaptive preferences provides neighboring resources for this problem [13, 21]. GRF does not classify socially formed preferences as inherently unfree. Social formation is unavoidable under the relational premises of the paper.

The sharper concern is whether the conditions of formation progressively destroy the subject's capacity for later recognition and revision.

Self-regulation can therefore coexist with freedom when it remains integrated into a sufficiently revisable and generative relational structure. It can also become a mechanism of closure when alternatives become systematically unrecognizable or practically unreachable.

10.7 Distribution and Asymmetry of Constraint

Constraint architecture is distributive. The same rule can constrain several subjects unequally, and the power to create or revise the rule can itself be distributed asymmetrically.

For constraint c , distinguish at least four relational positions:

$$\mathcal{D}(c) = \langle A_{\text{generate}}, A_{\text{interpret}}, A_{\text{revise}}, A_{\text{bear}} \rangle, \quad (237)$$

where the components denote actors or relational positions associated with generating, interpreting, revising, and bearing the constraint.

These positions can coincide, but they need not.

A collectively generated rule in which affected subjects participate in interpretation and revision has a different relational architecture from a rule whose authors, interpreters, and beneficiaries are separated from those who bear its consequences.

This distinction provides a more precise treatment of asymmetry. Asymmetry alone does not establish domination. Many functional relations contain differences in expertise, responsibility, information, or role. The relevant problem concerns the structure and consequences of the asymmetry.

A particularly important configuration occurs when

$$A_{\text{bear}} \cap A_{\text{revise}} = \emptyset, \quad (238)$$

or when participation in revision exists only formally while practical access remains negligible.

Such separation does not independently prove unfreedom, but it identifies a relational condition requiring further analysis.

Temporal asymmetry also matters. One generation can establish structures whose costs are borne primarily by later subjects who had no role in their formation. Constraint architecture must therefore permit historical distribution across time as well as distribution among contemporaneous actors.

10.8 Revisability, Exit, and Transformability

The same constraint can have different implications for GRF depending upon whether it can be interpreted, revised, left, or transformed.

For constraint c , define a provisional relational profile

$$\Pi_C(c) = \langle \text{Rev}(c), \text{Exit}(c), \text{Trans}(c) \rangle, \quad (239)$$

where the three components concern revisability, effective exit, and transformability.

They remain distinct.

A subject may be able to leave an institution while possessing no meaningful voice in its rules. Another subject may participate substantially in rule revision while facing high material costs of exit. A third may be unable to change individual rules while participating in collective processes capable of transforming the wider institution.

The presence of constraint therefore becomes especially significant when several dimensions close together:

$$\text{Rev}(c) \downarrow, \quad \text{Exit}(c) \downarrow, \quad \text{Trans}(c) \downarrow. \quad (240)$$

The arrows indicate comparative reduction within a declared domain rather than measurement on a universal scale.

Such a configuration begins to distinguish ordinary structured constraint from relational closure. The subject is constrained and has diminishing capacity to alter, leave, or reinterpret the architecture through which that constraint is generated.

This pattern connects directly to the earlier discussion of generative closure. The crucial issue is not that a rule exists. It is that the relational system increasingly removes the pathways through which its own organization can be questioned or transformed.

10.9 Constraint across Relational Scales

Constraint architecture varies across spatial, institutional, relational, and temporal scales. A constraint that appears locally restrictive can participate in sustaining a wider generative structure. A locally permissive arrangement can conversely depend upon wider relations that create severe closure elsewhere.

Let

$$U_1 \subseteq U_2 \subseteq \dots \subseteq U_n \quad (241)$$

denote nested relational regions. The constraint profile evaluated in U_i need not be preserved in U_j :

$$\mathfrak{C}_{B,U_i,t} \neq \mathfrak{C}_{B,U_j,t}. \quad (242)$$

A workplace rule, for example, can be analyzed locally as an organizational constraint while also participating in broader legal, economic, family, or technological relations. Similarly, a religious practice can be organized through personal commitment at one scale, communal recognition at another, and institutional regulation at a third.

Temporal scale matters as well. A constraint can increase current coordination while reducing later revisability, or impose current discipline while preserving long-horizon generative conditions.

Consequently,

$$\Delta \mathfrak{F}_{B,t}^{GR}(H_1) \neq \Delta \mathfrak{F}_{B,t}^{GR}(H_2) \quad (H_1 \neq H_2). \quad (243)$$

Constraint judgments should therefore declare their domain and horizon. Without that declaration, a local effect can be mistaken for a global property.

This principle also limits cross-cultural comparison. A highly visible rule in one relational patch cannot be meaningfully compared with an apparently absent rule elsewhere unless the wider distribution of material, interpersonal, institutional, symbolic, and self-regulatory constraints is considered.

10.10 Generative and Foreclosing Configurations

The preceding distinctions permit a more precise separation between constraint that participates in generative organization and constraint that contributes to relational foreclosure.

Let a constraint c modify the subject's freedom profile

$$\mathfrak{F}_{B,t}^{GR} = \langle \mathcal{G}_{B,t}, \mathcal{R}ev_{B,t}, \mathcal{E}x_{B,t}, \mathcal{T}r_{B,t} \rangle. \quad (244)$$

A constraint is *generatively organizing* in a declared domain when its operation helps sustain or create meaningful trajectories while preserving sufficient conditions for interpretation and revision. A constraint is *foreclosing* when its operation progressively removes relevant trajectories or the capacities through which the relational architecture can be recognized, revised, exited, or transformed.

The distinction is comparative and domain-dependent. It should therefore not be represented through a universal binary classification

$$c \in \{\text{good constraint, bad constraint}\}.$$

The same rule can possess different generative consequences for different subjects, scales, or historical periods.

A more useful relation is

$$c : \mathfrak{F}_{B,t}^{GR} \longrightarrow \mathfrak{F}_{B,t+1}^{GR}, \quad (245)$$

followed by an examination of which components and relational conditions have changed.

This formulation also separates structural diagnosis from normative judgment. A constraint can be shown to reduce revisability or effective generative capability without that fact alone settling every question of permissibility. Public safety, coordination, protection of others, ecological sustainability, and other normative considerations can enter later evaluation.

Accordingly, the analytical sequence is

$$\text{constraint architecture} \longrightarrow \text{generative effects} \longrightarrow \text{normative evaluation}. \quad (246)$$

The arrows indicate an order of inquiry rather than automatic implication.

The concept of relational constraint architecture now clarifies why explicit restriction cannot be used as a universal proxy for unfreedom. Constraint can be distributed across transcendent, symbolic, institutional, interpersonal, material, embodied, and self-regulatory relations. Removing one visible form can displace constraint into another location without necessarily changing the subject's overall generative position.

A schematic displacement can be represented by

$$\Delta C^{\text{tr}} < 0 \quad \text{while} \quad \Delta (C^{\text{sym}} + C^{\text{int}} + C^{\text{self}}) > 0, \quad (247)$$

where the expression represents directional redistribution rather than commensurable numerical quantities.

This possibility is especially important for comparative studies of religious, political, and cultural systems. A relational configuration with fewer explicit commands need not contain less constraint. It may instead distribute normative ordering through social expectation, institutional structure, symbolic distinction, market dependence, or self-regulation. Conversely, highly explicit rules need not independently establish stronger generative closure.

The appropriate object of comparison is therefore the architecture through which constraint is generated, interpreted, distributed, revised, and transformed.

Section 11 next returns to the geometric vocabulary established earlier. Having distinguished the relevant structures of constraint, the paper can now examine how situated language systems and constraint architectures organize local interpretive spaces, admissible directions, and trajectories without identifying those spaces with the underlying physical manifold.

11 Interpretive Geometry of Freedom

The preceding sections established the relational subject, the situated language system L_{st} , the historical generation of freedom meanings, and the architecture through which constraints affect generative trajectories. The present section develops an effective geometric representation of these structures. Its purpose is to clarify locality, direction, differentiation, and transformation within an interpretive field before the group-theoretical analysis of symmetry in the following sections.

The construction is intentionally conditional. A historically situated interpretive system is not assumed to possess a literal smooth manifold structure. The manifold language becomes appropriate only when the relevant interpretive configurations admit sufficiently stable local variation and transition structure to support such an approximation. Where this condition fails, a graph, stratified space, partially ordered structure, or another non-smooth representation may be more appropriate. The geometric model should therefore be understood as an effective analytical representation rather than an ontological claim about the constitution of meaning.

When the approximation is defensible, the paper writes

$$\mathcal{M}_{U,t}^F \quad (248)$$

for a local interpretive manifold associated with relational patch U at time t . Points of $\mathcal{M}_{U,t}^F$ represent locally distinguishable interpretive configurations relevant to freedom. The manifold does not represent all conceivable meanings of freedom and does not imply the existence of one global freedom space containing every historical language system.

11.1 Local Interpretive Manifolds

The local-manifold construction begins from the effective character of $L_{U,t}$. Section 6 treated a relational language system as an historically generated organization through which trajectories become recognizable. If a family of nearby interpretive configurations can be varied continuously within a specified domain, their local organization may be represented by a manifold patch

$$\mathcal{U}_\alpha \subseteq \mathcal{M}_{U,t}^F. \quad (249)$$

A coordinate chart is then

$$\varphi_\alpha : \mathcal{U}_\alpha \longrightarrow \varphi_\alpha(\mathcal{U}_\alpha) \subseteq \mathbb{R}^n. \quad (250)$$

The coordinates supplied by φ_α can represent selected dimensions of local interpretive variation. Depending upon the inquiry, these may concern distinctions such as formal permission, dependency, participation, revisability, recognition, or other locally meaningful relations.

The coordinates are analytical constructions. They are not presumed to be natural dimensions already present in the world. Choosing a chart therefore requires a prior hermeneutic decision concerning which local differences are relevant enough to represent.

This produces an immediate distinction between the interpretive space and its coordinate representation:

$$x \in \mathcal{M}_{U,t}^F, \quad \varphi_\alpha(x) \in \mathbb{R}^n. \quad (251)$$

The coordinate vector is not the interpretation itself.

The same local configuration may admit several descriptions:

$$\varphi_\alpha(x) \neq \varphi_\beta(x) \quad (252)$$

without requiring the represented configuration x to differ.

This ordinary geometric distinction will later become important when the paper examines translation and cross-relational comparison. Difference in descriptive coordinates can sometimes preserve relational structure, although semantic translation should not be presumed to possess the exact properties of smooth coordinate transformation.

The local construction also preserves the paper's refusal of a global definition of freedom. The existence of one chart

$$\varphi_\alpha : \mathcal{U}_\alpha \rightarrow \mathbb{R}^n \quad (253)$$

does not establish a global chart

$$\varphi : \mathcal{M}^F \rightarrow \mathbb{R}^n \quad (254)$$

covering every historically situated interpretation.

The global problem must be established separately.

11.2 Freedom Meanings over Relational Patches

A relational patch U identifies the local domain within which an interpretive construction is being made. As established earlier, such a patch can contain material, institutional, linguistic, historical, symbolic, and interpersonal structure. It should not be reduced to a geographical region.

Within U , let

$$f_{U,t} \in \mathcal{M}_{U,t}^F \quad (255)$$

represent a locally situated freedom configuration.

The object $f_{U,t}$ does not represent a universal definition restricted to a smaller territory. It is generated within the local language system:

$$f_{U,t} = \Phi_F(L_{U,t}, \mathcal{R}_{U,t}, H_{U,t}), \quad (256)$$

where Φ_F again denotes a generative relation rather than a deterministic function.

If two relational patches U_i and U_j overlap, the corresponding freedom configurations need not be identical:

$$f_{U_i,t} \neq f_{U_j,t}. \quad (257)$$

The overlap nevertheless provides a domain in which comparison can become possible.

For

$$U_{ij} = U_i \cap U_j, \quad (258)$$

one may ask whether the two interpretations induce compatible structures over U_{ij} . At this stage, compatibility remains an open relation:

$$f_{U_i,t} \overset{U_{ij}}{\sim} f_{U_j,t}. \quad (259)$$

The symbol $\sim$ does not yet denote an equivalence relation. Its precise meaning may involve lexical translation, preservation of relational structure, practical correspondence, normative overlap, or another declared criterion.

Section 14 will later formalize this local-to-global problem through restriction and gluing. The present section establishes only the geometric prerequisite: freedom meanings are first represented locally, and relationships among local representations must be constructed rather than presumed.

11.3 Tangent Directions and Hermeneutic Admissibility

If $\mathcal{M}_{U,t}^F$ admits a smooth local approximation, each point

$$x \in \mathcal{M}_{U,t}^F \quad (260)$$

has a tangent space

$$T_x \mathcal{M}_{U,t}^F, \quad (261)$$

whose vectors represent infinitesimal directions of local variation [11, 20].

The entire tangent space should not be interpreted as the set of trajectories available to a subject. A direction can exist in the effective geometric representation without being recognizable within the subject's language system.

For subject B , define provisionally

$$\mathcal{A}_{B,U,t}^H(x) \subseteq T_x \mathcal{M}_{U,t}^F \quad (262)$$

as the set of locally hermeneutically admissible directions.

The superscript H denotes hermeneutic admissibility rather than historical horizon. A direction belongs to $\mathcal{A}_{B,U,t}^H(x)$ when the corresponding change can be rendered sufficiently recognizable within the relevant $L_{U,t}$ for the subject to interpret it as a possible transition.

This produces a distinction among

$$T_x \mathcal{M}_{U,t}^F, \quad \mathcal{A}_{B,U,t}^H(x), \quad \mathcal{A}_{B,U,t}^G(x), \quad (263)$$

where $\mathcal{A}_{B,U,t}^G(x)$ denotes directions that remain effectively generable after material, institutional, embodied, and relational conditions are considered.

Under a sufficiently regular local approximation, one may write

$$\mathcal{A}_{B,U,t}^G(x) \subseteq \mathcal{A}_{B,U,t}^H(x) \subseteq T_x \mathcal{M}_{U,t}^F. \quad (264)$$

Equation 264 is a local modeling convention rather than a universal law. Interpretive innovation can generate a direction not already represented in the prior tangent approximation, in which case the effective manifold itself must be updated.

The distinction remains useful because it separates three questions:

1. whether a local variation is represented geometrically;
2. whether the variation is interpretable within the current language system;

3. whether the situated subject can effectively generate the corresponding trajectory.

A theory that collapses these questions can mistake abstract possibility for freedom.

11.4 Admissible Generative Trajectories

A local direction becomes dynamically significant only when it can participate in a trajectory.

Let

$$\gamma : I \longrightarrow \mathcal{M}_{U,t}^F \quad (265)$$

be a differentiable local trajectory. Hermeneutic admissibility requires

$$\dot{\gamma}(\tau) \in \mathcal{A}_{B,U,t}^H(\gamma(\tau)) \quad (266)$$

for the relevant portion of the trajectory.

Effective generative admissibility imposes the stronger local condition

$$\dot{\gamma}(\tau) \in \mathcal{A}_{B,U,t}^G(\gamma(\tau)). \quad (267)$$

A trajectory satisfying Equation 267 remains only *locally* admissible. Long-horizon viability can depend upon later regions of the relational field, accumulated costs, path dependence, or changes in the subject itself.

Accordingly,

$$\gamma \text{ locally generable} \not\equiv \gamma \text{ globally sustainable}. \quad (268)$$

This distinction is important for freedom. A subject can enter a trajectory that subsequently removes the conditions required for its continuation or for later revision. Immediate reachability therefore does not exhaust long-horizon generative freedom.

Conversely, an initially difficult trajectory can generate new interpretive and relational resources:

$$\gamma_{[t_0,t_1]} \longrightarrow \Delta L_{U,t_1} \longrightarrow \Delta \mathcal{A}_{B,U,t_1}^G. \quad (269)$$

The geometry is therefore reflexive. Enacted trajectories can modify the space through which later trajectories are represented.

This relation prevents the model from treating $\mathcal{M}_{U,t}^F$ as a permanent container. In GRF, the interpretive geometry is generated historically and can itself be transformed by movement through the relational field.

11.5 Local Metrics and Relational Distinction

A manifold structure provides locality and direction but does not by itself specify how local differences should be compared. Where the analysis requires such comparison, an additional local metric structure may be introduced.

Let

$$g_{U,t}^F \quad (270)$$

denote a metric tensor on a suitable region of $\mathcal{M}_{U,t}^F$. In local coordinates x^μ , the infinitesimal interval is represented by

$$ds^2 = g_{\mu\nu}^F(x) dx^\mu dx^\nu. \quad (271)$$

The metric should not be interpreted as a numerical measure of freedom. Its role is to model relational differentiation among nearby interpretive configurations.

Depending upon the application, $g_{\mu\nu}^F$ may encode, for example, how strongly a local distinction is weighted, how difficult a transition is relative to another, how sharply two interpretive states are separated within a specified representation, or how relational structure makes some directions more consequential than others.

A direction $v \in T_x \mathcal{M}_{U,t}^F$ can therefore acquire a context-dependent local magnitude

$$\|v\|_{g^F}^2 = g_{\mu\nu}^F v^\mu v^\nu. \quad (272)$$

The quantity is meaningful only after the interpretation of the metric has been declared. A metric chosen to represent material transition cost will not automatically measure normative significance, semantic difference, or experienced difficulty.

This restriction is essential because otherwise the geometric language would silently restore the scalar freedom model rejected in Section 8.9.

In particular,

$$d_{g^F}(x, y) \neq |\text{Freedom}(x) - \text{Freedom}(y)|. \quad (273)$$

A local distance can describe one specified form of relational separation without producing a universal freedom ordering.

The metric may also vary among subjects or relational positions:

$$g_{B,U,t}^F \neq g_{C,U,t}^F. \quad (274)$$

This possibility formalizes an intuition already present in the earlier GR framework: the same nominal transition can possess very different practical or interpretive significance for differently situated subjects.

Such variation should not be used casually. A subject-relative metric must be supported by an explicit account of what relational differences the metric represents. Otherwise the notation should be omitted.

11.6 Coordinate Dependence and Interpretive Description

A geometric representation distinguishes the underlying local structure from the coordinates used to describe it. Suppose two charts φ_α and φ_β cover an overlapping region. Their coordinates are related by

$$x'^\mu = x'^\mu(x^1, \dots, x^n). \quad (275)$$

The tensor components of a metric transform accordingly:

$$g_{\alpha\beta}^F = \frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial x^\nu}{\partial x'^\beta} g_{\mu\nu}^F. \quad (276)$$

In ordinary differential geometry, these transformations allow the same geometric object to be represented through different coordinate systems [11, 20].

The philosophical transfer requires additional caution. Two linguistic or cultural descriptions should be treated as coordinate transformations only when an underlying invariant structure has already been established. The presence of two descriptions alone does not prove that they are merely alternative coordinates on the same semantic object.

Accordingly,

$$\text{different descriptions} \not\Rightarrow \text{coordinate equivalence}. \quad (277)$$

A translation can alter the represented meaning itself, as established in Section 9.6.

The coordinate analogy is therefore useful for distinguishing two possible cases.

In the first case, two descriptions differ while preserving the structure relevant to the inquiry:

$$D_\alpha \xleftrightarrow{T} D_\beta, \quad I(D_\alpha) = I(D_\beta). \quad (278)$$

Such a relation may later support a gauge-like treatment.
In the second case, transformation changes the interpretive structure:

$$I(D_\alpha) \neq I(D_\beta). \quad (279)$$

The change is then substantive for the declared invariant I and should not be removed as descriptive redundancy.

This distinction is foundational for the later analysis of cross-relational invariance. The objective is to identify which differences are representational and which alter the relational structure through which freedom becomes meaningful.

11.7 Geometric Change under Historical Transformation

The interpretive geometry constructed above is historically generated. It must therefore be permitted to change.

A weaker transformation alters the location of the subject within a stable interpretive manifold:

$$x_t \longrightarrow x_{t+1}, \quad \mathcal{M}_{U,t}^F = \mathcal{M}_{U,t+1}^F. \quad (280)$$

A stronger historical transformation changes the effective geometric structure itself:

$$(\mathcal{M}_{U,t}^F, g_{U,t}^F) \longrightarrow (\mathcal{M}_{U,t+1}^F, g_{U,t+1}^F). \quad (281)$$

The latter can correspond to emergence of new distinctions, disappearance of old categories, reorganization of institutional boundaries, transformation of constraint architecture, or changes in the relative significance of local directions.

Historical change can therefore alter

$$\{\mathcal{M}^F, T_x \mathcal{M}^F, \mathcal{A}^H, \mathcal{A}^G, g^F\}. \quad (282)$$

This possibility is more consequential than movement through a fixed state space. A newly generated language system can render previously absent directions interpretable. Conversely, destruction or suppression of an interpretive structure can remove directions from the effective local geometry.

A particularly important transition is

$$v \notin \mathcal{A}_{B,U,t}^H(x) \longrightarrow v' \in \mathcal{A}_{B,U,t+1}^H(x'), \quad (283)$$

where historical transformation produces a newly recognizable direction.

The reverse process,

$$v \in \mathcal{A}_{B,U,t}^H(x) \longrightarrow v' \notin \mathcal{A}_{B,U,t+1}^H(x'), \quad (284)$$

represents interpretive contraction.

These transformations can occur without any simple change in the number of physical actions available. They concern the geometry through which actions are differentiated and recognized.

The metric can also evolve:

$$\partial_t g_{\mu\nu}^F \neq 0. \quad (285)$$

Equation 285 expresses historical variation in the relational significance assigned to local distinctions. It does not introduce a gravitational field equation for social life. No analogue of Einstein's equations is assumed.

This restriction illustrates the general method of the paper. Geometric formalism is retained where it clarifies local structure, direction, comparison, and historical transformation. Physical dynamics are not imported when the corresponding social or hermeneutic mechanism has not been established.

The interpretive geometry of freedom can therefore be summarized through the local architecture

$$\begin{aligned}
L_{U,t} &\longrightarrow \mathcal{M}_{U,t}^F \\
&\longrightarrow T_x \mathcal{M}_{U,t}^F \\
&\longrightarrow \mathcal{A}_{B,U,t}^H(x) \\
&\longrightarrow \mathcal{A}_{B,U,t}^G(x) \\
&\longrightarrow \gamma_B.
\end{aligned} \tag{286}$$

The arrows identify analytical dependencies rather than an irreversible causal chain. Enacted trajectories can feed back into the language system and alter the geometry itself.

Three limits remain important.

First, the existence of a local geometric representation does not establish a global freedom manifold.

Second, a local metric does not provide a scalar measure of freedom.

Third, difference between descriptions does not establish that those descriptions are coordinate-equivalent representations of the same object.

These unresolved problems lead directly to symmetry. Once a local interpretive space and a family of transformations have been specified, it becomes possible to ask which transformations preserve relevant freedom structure, which configurations belong to common orbits, and which transformations leave a situated interpretation invariant. Section 12 develops those questions through group actions, equivalence, and relational invariance.

12 Relational Symmetry of Freedom Interpretations

Section 11 developed a local geometric representation of freedom interpretations and distinguished interpretive configurations from the coordinates used to describe them. The present section adds a transformation structure. Its objective is to determine when different freedom interpretations can be regarded as related through a specified family of transformations and which features remain invariant under those transformations.

The term *symmetry* is used conditionally. A plurality of meanings does not itself constitute a symmetry, and the existence of translations among languages does not establish a transformation group. A symmetry claim requires at least three objects: a space upon which transformations act, a family of transformations with adequate algebraic structure, and a declared structure that remains invariant under those transformations [9, 27].

For a relational patch U , historical location t , and declared analytical domain D , let

$$\mathcal{X}_{U,t,D}^F \tag{287}$$

denote the effective interpretive state space relevant to the inquiry. A candidate transformation group is written

$$G_{U,t,D}^F. \tag{288}$$

When a defensible group action exists,

$$G_{U,t,D}^F \curvearrowright \mathcal{X}_{U,t,D}^F. \tag{289}$$

The dependence on U , t , and D is essential. The paper does not assume one universal transformation group acting upon every historical meaning of freedom.

12.1 Interpretive State Spaces

An interpretive state space contains configurations that can be meaningfully distinguished within a specified inquiry. A point

$$f \in \mathcal{X}_{U,t;D}^F \quad (290)$$

represents a situated configuration of freedom-related distinctions rather than a scalar quantity of freedom.

The domain D determines which features are retained in constructing the space. For example, one inquiry may concern institutional non-domination, another may concern interpretive revisability, and another may concern effective exit from a relational structure. These inquiries need not generate the same $\mathcal{X}_{U,t;D}^F$.

A more explicit representation is

$$f = \langle \mathcal{R}_f, L_f, \mathfrak{C}_f, \mathfrak{F}_f^{GR} \rangle_D, \quad (291)$$

where $\mathcal{R}_f$ denotes relevant relational organization, L_f the situated interpretive system, $\mathfrak{C}_f$ the relevant constraint architecture, and $\mathfrak{F}_f^{GR}$ the corresponding Generative Relational Freedom profile.

Equation 291 is schematic. A concrete application may require only a subset of these components.

The state space should therefore be interpreted as constructed rather than discovered in a completely theory-independent form:

$$\mathcal{X}_{U,t;D}^F = \Xi_D(L_{U,t}, \mathcal{R}_{U,t}, H_{U,t}), \quad (292)$$

where Ξ_D denotes a domain-dependent construction procedure.

This dependence prevents the later symmetry group from becoming universal by assumption. If the interpretive space changes historically, the admissible transformation group may change with it.

12.2 Transformation Groups

A transformation group must contain transformations that are meaningful for the declared interpretive space. Let

$$g : \mathcal{X}_{U,t;D}^F \longrightarrow \mathcal{X}_{U,t;D}^F \quad (293)$$

denote one candidate transformation.

Examples may include changes of representation, reparameterizations, permutations of relationally equivalent positions, or other transformations that preserve the structure relevant to D . The exact content must be specified in the application. Arbitrary reinterpretation is not sufficient.

A set of transformations qualifies as a group only when it satisfies the ordinary structural conditions of closure, associativity, identity, and inverse [9]. Accordingly,

$$g_1, g_2 \in G_{U,t;D}^F \implies g_2 g_1 \in G_{U,t;D}^F, \quad (294)$$

together with an identity element e satisfying

$$e \cdot f = f, \quad (295)$$

and an inverse g^{-1} for each g .

This requirement immediately excludes many ordinary translation relations. A historical translation can be lossy, path-dependent, or non-invertible. Such a relation may still be analytically valuable, but it should not be called a group transformation without further structure.

The distinction is represented by

$$T_{ij} : \mathcal{X}_i \longrightarrow \mathcal{X}_j \quad (296)$$

for a general cross-context map, compared with

$$g \in G_{U,t;D}^F \quad (297)$$

for a transformation belonging to a specified symmetry group.

Later sections will therefore treat cross-relational translation more generally than the local symmetry group considered here.

12.3 Orbits of Freedom Interpretations

Given the action in Equation 289, the orbit of an interpretation $f \in \mathcal{X}_{U,t;D}^F$ is

$$\mathcal{O}_f = \{g \cdot f \mid g \in G_{U,t;D}^F\}. \quad (298)$$

The orbit contains configurations connected to f through the declared transformation group.

This provides a formal distinction between two kinds of variation. Variation inside an orbit may preserve the relational structure selected by the symmetry criterion, whereas variation between different orbits may represent a difference that cannot be removed through the declared transformations.

Thus,

$$f_i, f_j \in \mathcal{O}_f \quad (299)$$

indicates transformation-related interpretations under $G_{U,t;D}^F$.

This relation does not imply complete semantic identity. It means only that the two configurations are connected under the transformations declared relevant for the specified domain.

A different domain D' may generate a different group and therefore a different orbit decomposition:

$$\mathcal{O}_f^D \neq \mathcal{O}_f^{D'}. \quad (300)$$

Relational equivalence is therefore always equivalence under a declared transformation structure.

This domain dependence is important when comparing freedom theories. Two interpretations may be equivalent with respect to one structural property while remaining importantly different with respect to another.

12.4 Stabilizers of Interpretive Configurations

For an interpretation $f \in \mathcal{X}_{U,t;D}^F$, its stabilizer is

$$H_f = \text{Stab}_{G_{U,t;D}^F}(f) = \{g \in G_{U,t;D}^F \mid g \cdot f = f\}. \quad (301)$$

The stabilizer contains transformations that leave the selected interpretation unchanged.

The object should be distinguished from the historical stabilization discussed in Section 9.3. Historical stabilization concerns persistence through institutions, practices, discourse, and relational reproduction. The mathematical stabilizer concerns invariance under a specified group action.

The two notions become connected only when the later analysis can show that a historically stabilized configuration corresponds to preservation under a well-defined transformation subgroup.

The stabilizer can reveal residual interpretive symmetry. Even after a particular freedom interpretation has been selected, some transformations may continue to preserve it:

$$H_f \subseteq G_{U,t;D}^F. \quad (302)$$

If

$$H_f = G_{U,t;D}^F, \quad (303)$$

the configuration is invariant under the entire declared group.

If instead

$$H_f \subsetneq G_{U,t;D}^F, \quad (304)$$

the configuration preserves only part of the available symmetry.

This latter relation provides the formal premise required for relational symmetry breaking in Section 13.

12.5 Relational Equivalence

A group action induces an equivalence relation through orbit membership. For interpretations f_i and f_j ,

$$f_i \sim_{G,D} f_j \iff f_j = g \cdot f_i \text{ for some } g \in G_{U,t;D}^F. \quad (305)$$

When the group action is well defined, $\sim_{G,D}$ is reflexive, symmetric, and transitive.

The quotient

$$\mathcal{X}_{U,t;D}^F / G_{U,t;D}^F \quad (306)$$

then collects the corresponding equivalence classes.

This quotient should not be interpreted immediately as the space of “true meanings of freedom.” It is only the space obtained after differences generated by the declared group have been treated as equivalent.

The philosophical significance of the quotient therefore depends entirely upon the justification for the group action.

For example, if G contains only descriptive transformations that preserve a declared relational structure, the quotient may help separate representational difference from structural difference. If G incorrectly identifies substantive historical differences as representational redundancies, the quotient will erase precisely the distinctions the hermeneutic analysis is meant to preserve.

Accordingly,

$$f_i \sim_{G,D} f_j \not\Rightarrow f_i \equiv_{\text{semantic}} f_j. \quad (307)$$

Nor does orbit equivalence establish normative equivalence:

$$f_i \sim_{G,D} f_j \not\Rightarrow f_i \equiv_{\text{normative}} f_j. \quad (308)$$

These stronger claims require independent arguments.

12.6 Interpretive Invariants

A symmetry becomes analytically useful only when the preserved structure can be identified. Let

$$I_D : \mathcal{X}_{U,t;D}^F \longrightarrow \mathcal{Y}_D \quad (309)$$

denote an observable or structural descriptor relevant to domain D .

I_D is invariant under $G_{U,t;D}^F$ when

$$I_D(g \cdot f) = I_D(f) \quad \text{for all } g \in G_{U,t;D}^F. \quad (310)$$

Several invariants can be considered jointly:

$$\mathbf{I}_D(f) = (I_1(f), I_2(f), \dots, I_m(f)). \quad (311)$$

Possible candidates in GRF may concern relational topology, distribution of revision authority, effective exit structure, dependence relations, transformability, or another explicitly specified feature. Whether

any of these structures is genuinely invariant under a proposed transformation is an analytical or empirical question.

The existence of a recurring lexical item is insufficient:

$$w_i = w_j \not\Rightarrow I_D(f_i) = I_D(f_j). \quad (312)$$

Likewise, different vocabulary does not preclude structural invariance:

$$w_i \neq w_j \not\Rightarrow I_D(f_i) \neq I_D(f_j). \quad (313)$$

This distinction is central to the later cross-relational programme. A potential invariant should survive a declared transformation even when surface descriptions change.

Relational invariance therefore offers a possible middle path between two extremes. One extreme assumes that every tradition contains the same hidden semantic object. The other treats every local interpretation as incomparable. GRF instead asks whether specific structures remain stable across specified transformations.

No invariant is presumed in advance.

12.7 Symmetry under Underspecified Context

The formal symmetry construction also clarifies a recurring problem in discussions of freedom. Before a relational and historical context is specified, several interpretations may remain admissible within an underspecified interpretive space.

Let

$$\mathcal{X}_{\emptyset}^F \quad (314)$$

denote schematically a freedom-interpretive space before sufficient contextual specification. The symbol $\emptyset$ indicates underspecification rather than absence of all relations.

Suppose a transformation group

$$G_{\emptyset}^F \curvearrowright \mathcal{X}_{\emptyset}^F \quad (315)$$

preserves the structure currently declared relevant. At this stage, several interpretations can remain related without one being selected as the locally stabilized meaning.

Contextual information then restricts the admissible structure:

$$\emptyset \longrightarrow (U, t, L_{U,t}, \mathcal{R}_{U,t}, H_{U,t}), \quad (316)$$

and can produce a corresponding change

$$G_{\emptyset}^F \longrightarrow G_{U,t,D}^F. \quad (317)$$

A situated interpretation

$$f_{U,t} \in \mathcal{X}_{U,t,D}^F \quad (318)$$

may then possess a stabilizer

$$H_{U,t} = \text{Stab}(f_{U,t}) \subseteq G_{U,t,D}^F. \quad (319)$$

This construction should be read cautiously. The paper is not claiming that an abstract universal freedom concept exists in a perfectly symmetric state and is subsequently broken by culture. The underspecified space is an analytical representation of unresolved interpretive alternatives.

The temporal and relational context participates in constructing the state space itself. Consequently, historical specification may alter

$$(\mathcal{X}^F, G^F, I_D) \quad (320)$$

rather than merely select a point inside a permanently fixed space.

A more general historical relation is therefore

$$(\mathcal{X}_t^F, G_t^F, \mathbf{I}_{D,t}) \longrightarrow (\mathcal{X}_{t+1}^F, G_{t+1}^F, \mathbf{I}_{D,t+1}). \quad (321)$$

This possibility distinguishes GRF from a simple application of spontaneous symmetry breaking in a fixed physical theory. The interpretive space, transformation structure, and relevant invariants can themselves be historically generated.

The concept of symmetry nevertheless remains useful because it forces the analysis to state explicitly what remains unchanged during a claimed transformation.

The resulting formal architecture is

$$\begin{aligned} L_{U,t} &\longrightarrow \mathcal{X}_{U,t;D}^F \\ &\longrightarrow G_{U,t;D}^F \curvearrowright \mathcal{X}_{U,t;D}^F \\ &\longrightarrow \mathcal{O}_f, \text{Stab}(f), \mathbf{I}_D. \end{aligned} \quad (322)$$

Each arrow represents an additional burden of construction. A language system does not automatically determine a manifold, a manifold does not automatically determine a meaningful transformation group, and a transformation group does not automatically determine philosophically significant invariants.

Where the required algebraic conditions cannot be established, the paper retains weaker terminology such as transformation, translation, partial correspondence, or structural similarity.

The symmetry analysis therefore introduces no claim that all local freedom meanings are equivalent. It establishes a formal method for distinguishing transformational variation from structural difference when adequate invariants can be specified.

Section 13 next examines the complementary problem of selection. Once a broader transformation structure has been identified, a historically situated relational configuration can preserve only a subgroup of those transformations. Relational symmetry breaking will be used to describe that reduction, while retaining a strict distinction between mathematical stabilizer structure and the historical mechanisms that produce interpretive stabilization.

13 Relational Symmetry Breaking

Section 12 introduced a conditional symmetry structure for situated freedom interpretations. Given a declared interpretive state space $\mathcal{X}_{U,t;D}^F$, a transformation group $G_{U,t;D}^F$, and an action

$$G_{U,t;D}^F \curvearrowright \mathcal{X}_{U,t;D}^F, \quad (323)$$

the present section examines how a historically situated relational configuration can stabilize an interpretation that preserves only a subgroup of the previously available transformation structure.

The term *relational symmetry breaking* refers to this reduction. It does not imply that political or hermeneutic processes instantiate spontaneous symmetry breaking in the technical physical sense. In field theory, a symmetric dynamical structure can possess states or vacua whose stabilizer is smaller than the symmetry group of the theory [27]. GRF transfers the group-theoretical relation between a broader transformation structure and the stabilizer of a selected configuration. The historical mechanism responsible for selection remains material, institutional, symbolic, practical, interpretive, or otherwise relational.

The section therefore separates three questions:

1. which transformations are initially available within the declared interpretive model;

2. which situated configuration becomes stabilized;
3. which transformations continue to preserve that configuration.

Only after these objects have been specified does the language of symmetry breaking perform analytical work.

13.1 Spatiotemporal Specification

An underspecified discussion of freedom can leave several interpretations compatible with the information currently supplied. Section 12.7 represented this condition through an analytical space

$$\mathcal{X}_{\emptyset;D}^F, \quad (324)$$

where the symbol $\emptyset$ indicates insufficient relational specification rather than absence of history or interaction.

Contextual specification introduces a relational patch, historical location, language system, and relevant relational organization:

$$\emptyset \longrightarrow (U, t, L_{U,t}, \mathcal{R}_{U,t}, H_{U,t}). \quad (325)$$

The resulting interpretive structure is

$$\mathcal{X}_{\emptyset;D}^F \longrightarrow \mathcal{X}_{U,t;D}^F. \quad (326)$$

This transformation may involve more than selection of one point inside a fixed space. Historical specification can alter the available distinctions, the relevant transformations, and the structures treated as invariant.

Accordingly,

$$(\mathcal{X}^F, G^F, \mathbf{I}_D)_{\emptyset} \longrightarrow (\mathcal{X}^F, G^F, \mathbf{I}_D)_{U,t} \quad (327)$$

is more general than a model in which only f changes.

This distinction is central to GRF. A situated freedom meaning is generated within a language system whose historical formation contributes to the construction of the interpretive space itself. Context is therefore not an external parameter added after a universal semantic space has already been fixed.

Spatiotemporal specification can include material organization, legal institutions, religious ordering, political history, linguistic practice, technological infrastructure, interpersonal relations, and embodied conditions. Which components matter depends upon the domain D .

The formal claim should therefore remain local:

$$G_{U,t;D}^F \curvearrowright \mathcal{X}_{U,t;D}^F. \quad (328)$$

No universal G^F is assumed to govern every freedom interpretation across all histories.

13.2 Selection through L_{st}

The situated language system $L_{U,t}$ participates in the selection and recognition of interpretive configurations. This role follows from Section 6, where $L_{U,t}$ was introduced as an effective structure organizing recognizability, symbolic distinction, and interpretive possibility.

Suppose several configurations

$$f_1, f_2, \dots, f_n \in \mathcal{X}_{U,t;D}^F \quad (329)$$

remain available under the declared transformation structure. The relational language system can participate in assigning different degrees of recognizability, legitimacy, recurrence, or institutional support to these configurations.

The resulting process is represented schematically by

$$(L_{U,t}, \mathcal{R}_{U,t}, H_{U,t}) \longrightarrow f_{U,t}^*, \quad (330)$$

where $f_{U,t}^*$ denotes a locally stabilized interpretation.

The star does not denote an optimum. In particular,

$$f_{U,t}^* \neq \arg \max_f \text{Freedom}(f). \quad (331)$$

It marks a selected or stabilized configuration within the present formal analysis.

Selection can occur through repeated discourse, institutional codification, legal interpretation, ritual practice, political struggle, education, material reproduction, or combinations of these processes. The framework does not require one unique mechanism.

This historical selection should also be distinguished from conscious choice. A freedom interpretation can become stabilized without any subject or institution deliberately choosing it as a conceptual standard.

Thus,

$$\text{Stabilized}(f) \not\Rightarrow \text{IntentionallySelected}(f). \quad (332)$$

The distinction prevents symmetry breaking from being reduced to individual decision.

13.3 Historical and Material Symmetry Reduction

Historical and material relations can reduce the effective range of transformations that remain relevant to an interpretive configuration.

Let the wider candidate group before local stabilization be

$$G_{U,t;D}^F. \quad (333)$$

A situated configuration $f_{U,t}^*$ possesses stabilizer

$$H_{U,t;D}^F = \text{Stab}_{G_{U,t;D}^F}(f_{U,t}^*). \quad (334)$$

When

$$H_{U,t;D}^F \subsetneq G_{U,t;D}^F, \quad (335)$$

the selected interpretation preserves only part of the original transformation structure.

The formal reduction is

$$G_{U,t;D}^F \longrightarrow H_{U,t;D}^F. \quad (336)$$

Equation 336 is the central formal relation of relational symmetry breaking.

Its historical interpretation requires an additional layer. Material and institutional relations can make some transformations unavailable, unrecognized, costly, illegitimate, or unstable. Repeated practice can then reproduce the selected configuration and its residual symmetry.

A schematic historical mechanism is

$$(\mathcal{P}_{U,t}, \mathcal{I}_{U,t}, \mathcal{R}_{U,t}, L_{U,t}) \longrightarrow f_{U,t}^* \longrightarrow H_{U,t;D}^F. \quad (337)$$

The first arrow is historical and relational. The second is group-theoretical.

Keeping these arrows distinct is essential. Group theory describes the stabilizer structure after a configuration has been specified. It does not explain why a legal institution, religious order, political economy, or historical community produced that configuration.

Historical explanation therefore cannot be replaced by the statement $G \rightarrow H$.

13.4 Stabilization of Situated Freedom Meanings

Once an interpretation becomes sufficiently stabilized, it can organize later judgment. Other practices, institutions, and concepts may then be evaluated through distinctions supplied by the stabilized configuration.

Let

$$f_{U,t}^* = \llbracket w_F \rrbracket_{L_{U,t}}^* \quad (338)$$

represent one such locally stabilized meaning associated with a freedom lexeme w_F .

The interpretation can acquire persistence through recursive reproduction:

$$f_{U,t}^* \longrightarrow \mathcal{P}_{U,t} \longrightarrow \mathcal{R}_{U,t+1} \longrightarrow L_{U,t+1} \longrightarrow f_{U,t+1}^*. \quad (339)$$

This recursion provides a possible mechanism through which a historically generated interpretation comes to appear self-evident.

The stability of f^* should not be confused with truth, universality, or normative justification:

$$\text{Stable}(f^*) \not\Rightarrow \begin{cases} \text{Universal}(f^*), \\ \text{True}(f^*), \\ \text{Just}(f^*). \end{cases} \quad (340)$$

A concept can be highly stable because it is embedded in institutions, material dependencies, education, symbolic authority, or relations of power.

Conversely, instability does not imply falsity. A newly emerging interpretation can remain weakly institutionalized while identifying relational distinctions that older language systems obscured.

The symmetry-breaking analysis therefore diagnoses stabilization structure. Normative evaluation remains a separate stage.

13.5 Residual Symmetries

Symmetry breaking rarely requires disappearance of every transformation. The selected configuration can remain invariant under a residual subgroup.

For

$$H_{U,t;D}^F = \text{Stab}(f_{U,t}^*), \quad (341)$$

each

$$h \in H_{U,t;D}^F \quad (342)$$

satisfies

$$h \cdot f_{U,t}^* = f_{U,t}^*. \quad (343)$$

Residual symmetry is philosophically important because a stabilized freedom meaning can tolerate considerable variation while retaining its organizing structure.

Different institutions, practices, or lexical formulations can therefore remain inside the same residual equivalence structure for a declared domain.

This gives a more precise meaning to interpretive flexibility. Stability need not require exact repetition:

$$D_1 \neq D_2 \quad \text{while} \quad I_D(D_1) = I_D(D_2). \quad (344)$$

The differences between D_1 and D_2 can remain admissible when the relevant invariant is preserved.

Residual symmetries can therefore help distinguish a living interpretive tradition from a completely rigid one. A tradition may preserve a recognizable freedom structure while allowing substantial variation in expression and practice.

The size of a residual subgroup should not, however, be treated as a scalar measure of freedom. A large stabilizer can indicate extensive descriptive redundancy or structural symmetry without any necessary normative implication.

Thus,

$$|H^F| \uparrow \not\Rightarrow \mathfrak{F}^{GR} \uparrow. \quad (345)$$

The significance of residual symmetry depends upon what transformations the group contains.

13.6 Historical Persistence and Path Dependence

Once a symmetry-breaking event or process has stabilized an interpretive configuration, later development can depend upon that historical selection.

Suppose two relational systems begin from comparable candidate spaces but stabilize different configurations:

$$f_A^* \neq f_B^*. \quad (346)$$

Their subsequent language systems can evolve along different trajectories:

$$L_{t_0}^A \longrightarrow L_{t_1}^A \longrightarrow L_{t_2}^A, \quad (347)$$

and

$$L_{t_0}^B \longrightarrow L_{t_1}^B \longrightarrow L_{t_2}^B. \quad (348)$$

Even when later relational conditions become similar, the historical trajectories need not converge semantically:

$$\mathcal{R}_{t_2}^A \approx \mathcal{R}_{t_2}^B \not\Rightarrow f_{t_2}^A = f_{t_2}^B. \quad (349)$$

Earlier selections can remain sedimented in institutions, vocabulary, practices, and symbolic structures.

This is one form of hermeneutic path dependence. The meaning reached at t_2 depends partly upon the interpretive trajectory through which the system arrived there.

Path dependence also complicates translation. Translating an interpretation through an intermediate language system can generate a result different from direct translation:

$$T_{AC} \not\equiv T_{BC} \circ T_{AB}. \quad (350)$$

The stronger geometric implications of this relation are deferred to Section 14. At the present stage it establishes only that historical mediation can matter to the result of interpretive transformation.

13.7 Reconfiguration of Stabilizer Structure

Historical transformation can alter the residual symmetry itself.

At time t ,

$$H_t = \text{Stab}_{G_t}(f_t), \quad (351)$$

while at a later time,

$$H_{t+1} = \text{Stab}_{G_{t+1}}(f_{t+1}). \quad (352)$$

Several forms of change are possible.

The interpretation can change while the transformation group remains approximately stable:

$$(G_t, f_t, H_t) \longrightarrow (G_t, f_{t+1}, H_{t+1}). \quad (353)$$

The transformation group itself can change:

$$(G_t, f_t, H_t) \longrightarrow (G_{t+1}, f_{t+1}, H_{t+1}). \quad (354)$$

The interpretive state space can also be reconstructed:

$$(\mathcal{X}_t^F, G_t, f_t, H_t) \longrightarrow (\mathcal{X}_{t+1}^F, G_{t+1}, f_{t+1}, H_{t+1}). \quad (355)$$

The final case is the most important for historical GRF. New political, religious, technological, material, or linguistic relations can generate distinctions that were absent from the earlier state space. The later system cannot then be adequately described as simple movement among points already available at t .

This distinction provides a formal expression for conceptual innovation. Historical change can modify the symmetry structure through which freedom itself is interpreted.

13.8 Repeated Symmetry Breaking

A situated interpretation need not undergo only one symmetry-breaking event. Historical development can proceed through successive reductions and reconfigurations.

A simple sequence is

$$G_0 \longrightarrow H_1 \longrightarrow H_2 \longrightarrow \cdots \longrightarrow H_n, \quad (356)$$

where each stage preserves a smaller transformation subgroup.

Such a sequence can represent progressively stronger stabilization within a fixed group structure. Historical processes, however, may be more complex. New interactions can enlarge the transformation space:

$$H_t \longrightarrow G'_{t+1} \longrightarrow H'_{t+1}. \quad (357)$$

The first transition can correspond to an expansion or reconstruction of available interpretations, while the second represents subsequent restabilization.

For example, encounter with another linguistic, religious, legal, or political system can introduce distinctions that destabilize the existing freedom vocabulary. The relational system can temporarily support a wider family of interpretations before new institutional or practical stabilization occurs.

A general historical sequence can therefore take the form

$$\begin{aligned} G_t &\longrightarrow H_t \\ &\longrightarrow G'_{t+\Delta} \\ &\longrightarrow H'_{t+\Delta}. \end{aligned} \quad (358)$$

This process provides a formal vocabulary for repeated conceptual reorganization without assuming a final interpretive equilibrium.

Repeated symmetry breaking is also compatible with local plurality. Different subregions of the same wider relational field can stabilize different subgroups:

$$G \longrightarrow \{H_1, H_2, \dots, H_n\}. \quad (359)$$

This possibility leads directly toward the local-to-global problem developed in the following section. Several locally coherent freedom interpretations can emerge from related structures without sharing one common global stabilizer.

13.9 Formal Limits of Symmetry Transfer

The mathematical terminology developed in this section should be retained only where its formal requirements are satisfied.

A defensible claim of relational symmetry breaking requires at least

$$\{\mathcal{X}, G, G \curvearrowright \mathcal{X}, f^*, \text{Stab}_G(f^*)\} \quad (360)$$

to be stated with adequate clarity.

If the proposed transformation family lacks inverses, closure, or a stable domain of action, the group-theoretical language should be weakened. A historical transition can remain important without constituting symmetry breaking in the formal sense.

Similarly,

$$\text{historical differentiation} \not\Rightarrow \text{symmetry breaking}, \quad (361)$$

and

$$\text{semantic stabilization} \not\Rightarrow \text{Stab}_G(f) \text{ has been constructed.} \quad (362)$$

The paper therefore distinguishes three levels of claim:

1. *historical stabilization*, where an interpretation becomes recurrent within a relational field;
2. *structural symmetry analogy*, where a broader family of alternatives narrows around a stable form but a complete group action is unavailable;
3. *formal relational symmetry breaking*, where the state space, group action, selected configuration, and residual stabilizer can be specified.

This hierarchy prevents physical terminology from carrying more explanatory weight than the model can justify.

The distinction from spontaneous symmetry breaking should also remain explicit. GRF does not assume a symmetric fundamental law together with a degenerate vacuum manifold of the type encountered in field theory [27]. Historical actors, institutions, material relations, and language systems can participate directly in constructing the interpretive space and selecting the stabilized configuration.

The transfer from physics is therefore structural:

$$\text{broader transformation structure} \longrightarrow \text{selected configuration} \longrightarrow \text{residual stabilizer.} \quad (363)$$

The historical explanation of each arrow remains within the relational domain.

Relational symmetry breaking consequently provides a formal account of local interpretive stabilization without requiring a context-independent meaning of freedom. A relational patch can stabilize one

freedom configuration from a broader family of transformation-related possibilities, preserve a residual subgroup, and later undergo further reconfiguration.

The result remains local. If several relational patches stabilize distinct interpretations, a further question arises concerning their compatibility. Local symmetry breaking does not establish that the resulting interpretations can be combined into a single global freedom structure.

Section 14 therefore turns from selection to composition. It develops local freedom interpretations as patchwise structures, introduces restriction and transition relations, and examines the conditions under which local interpretations can or cannot be glued into a coherent global form.

14 Local Hermeneutics and Global Gluing

The preceding sections developed freedom interpretations locally. A relational patch U supports a situated language system L_U , an effective interpretive space, and potentially a locally stabilized freedom configuration. The present section addresses a different problem. Even when local interpretations are individually coherent, it does not follow that they combine into one coherent global interpretation.

This local-to-global problem motivates a sheaf-like formalism. In mathematics, a sheaf assigns local data to open regions together with restriction maps and requires compatible local data to glue uniquely into global data [5, 12]. The present paper uses this structure conditionally. A family of situated freedom interpretations is called a sheaf only when the corresponding restriction and gluing conditions can be specified. Otherwise, the weaker language of a presheaf-like or patchwise interpretive system is retained.

The distinction is important because the philosophical conclusion of the section is not that freedom “is a sheaf.” The claim is narrower: freedom meanings are locally generated, and their relation across overlapping historical and relational contexts raises a mathematically recognizable local-to-global compatibility problem.

14.1 Local Interpretive Patches

Let $\mathcal{M}^F$ denote an effective interpretive space in a domain where the geometric approximation introduced in Section 11 is defensible. Let

$$\mathcal{U} = \{U_i\}_{i \in I} \quad (364)$$

be a family of local relational patches satisfying

$$\mathcal{M}^F = \bigcup_{i \in I} U_i. \quad (365)$$

The notation resembles an open cover in topology, although the philosophical construction requires an additional interpretive step. A relational patch is not merely a geographical region. Its boundaries can depend upon relevant institutional, linguistic, historical, religious, material, or practical relations.

The cover therefore represents a declared analytical decomposition:

$$U_i = U_i(\mathcal{R}, L, H, \mathcal{I}, \mathcal{P}, D), \quad (366)$$

where D denotes the domain of inquiry.

Different decompositions can be appropriate for different questions. A study of legal freedom may require patches organized around jurisdictions and legal institutions, while a study of religious discipline may require patches organized around communities, practices, and interpretive traditions.

Consequently,

$$\mathcal{U}_D \neq \mathcal{U}_{D'} \quad (367)$$

can hold for different domains D and D' .

The choice of cover is therefore part of the analytical construction. It should not be treated as an innocent partition of an already fully specified global semantic object.

14.2 Freedom Meanings as Local Sections

For each relational patch U , let

$$\mathcal{F}(U) \quad (368)$$

denote a collection of freedom interpretations available over U .

An element

$$f_U \in \mathcal{F}(U) \quad (369)$$

is called a *local section* provisionally. It represents an interpretation that is coherent over the declared relational patch.

The object f_U can contain more structure than a lexical meaning. Depending upon the domain, it may include

$$f_U = \langle L_U, \mathcal{R}_U, \mathfrak{C}_U, \mathfrak{F}_U^{GR}, \mathbf{I}_U \rangle_D, \quad (370)$$

where the components represent the local language system, relational organization, constraint architecture, GRF profile, and any declared comparison structures relevant to the inquiry.

Equation 370 is schematic. A local section need not contain every component explicitly.

The use of section terminology emphasizes that the interpretation is attached to a domain:

$$f_U \neq f \text{ without specification of } U. \quad (371)$$

This is especially important for freedom. A locally coherent interpretation should not be silently promoted into a universal definition merely because it is internally stable.

The assignment

$$U \mapsto \mathcal{F}(U) \quad (372)$$

is the first component of a possible presheaf structure. Additional restriction relations are required before the construction becomes mathematically meaningful.

14.3 Restriction across Nested Contexts

Suppose

$$V \subseteq U. \quad (373)$$

A restriction map

$$\rho_V^U : \mathcal{F}(U) \longrightarrow \mathcal{F}(V) \quad (374)$$

takes an interpretation defined over the wider patch U and asks what interpretive structure remains when attention is restricted to V .

For a presheaf-like construction, restriction should satisfy

$$\rho_U^U = \text{id}_{\mathcal{F}(U)} \quad (375)$$

and, whenever

$$W \subseteq V \subseteq U, \quad (376)$$

the composition condition

$$\rho_W^U = \rho_W^V \circ \rho_V^U. \quad (377)$$

These conditions are standard in presheaf theory [5, 12]. Their philosophical interpretation requires caution.

A wider political or religious interpretation does not necessarily possess a well-defined restriction to every subcontext. Restricting the domain can alter which relations are visible and can therefore change the interpretation itself.

For this reason,

$$\rho_V^U(f_U) \quad (378)$$

should be understood as the interpretation induced on V under a declared restriction procedure, rather than as a mere deletion of information.

If no coherent restriction map can be defined, the family $U \mapsto \mathcal{F}(U)$ should not be treated as a presheaf.

This gives the formalism a useful falsification condition. The mathematical language is retained only when the contextual relations support the required maps.

14.4 Overlap and Local Compatibility

Consider two patches U_i and U_j with non-empty overlap

$$U_{ij} = U_i \cap U_j. \quad (379)$$

Let

$$f_i \in \mathcal{F}(U_i), \quad f_j \in \mathcal{F}(U_j). \quad (380)$$

The two local interpretations are compatible on the overlap when

$$\rho_{U_{ij}}^{U_i}(f_i) = \rho_{U_{ij}}^{U_j}(f_j). \quad (381)$$

Equation 381 is strong. It requires equality after restriction under the declared interpretive construction.

In many hermeneutic settings, strict equality may be unavailable. Translation or another comparison map may be required. One may then introduce

$$T_{ij} : \mathcal{F}(U_i)|_{U_{ij}} \longrightarrow \mathcal{F}(U_j)|_{U_{ij}} \quad (382)$$

and ask whether

$$T_{ij} \left(\rho_{U_{ij}}^{U_i}(f_i) \right) \approx_D \rho_{U_{ij}}^{U_j}(f_j), \quad (383)$$

where $\approx_D$ denotes a declared criterion of structural compatibility.

The latter relation is weaker than the ordinary sheaf axiom. A system requiring non-trivial translation on overlaps may therefore belong more naturally to a bundle-like, stack-like, groupoid-valued, or other generalized local-to-global structure. The present paper does not develop those stronger mathematical constructions.

This limitation should remain explicit:

$$\text{translated local compatibility} \not\equiv \text{ordinary sheaf compatibility}. \quad (384)$$

The sheaf model is therefore retained as the simplest formal benchmark for the local-to-global problem rather than assumed to exhaust every form of hermeneutic composition.

14.5 Transition Maps between Interpretive Patches

Where two local language systems differ, comparison may require explicit transition maps. Let

$$T_{ij} : L_i \longrightarrow L_j \quad (385)$$

denote a translation or interpretive transformation from the language system associated with U_i into that associated with U_j .

At the level of freedom interpretations,

$$T_{ij}^F : \mathcal{F}(U_i) \longrightarrow \widetilde{\mathcal{F}}_j(U_i \cap U_j) \quad (386)$$

maps an interpretation from one local system into a representation available within the receiving system.

The tilde is important. The translated interpretation need not coincide with the interpretation generated locally inside L_j :

$$T_{ij}^F(f_i) \not\equiv f_j. \quad (387)$$

Likewise, invertibility cannot be presumed:

$$T_{ji}^F \circ T_{ij}^F \not\equiv \text{id}. \quad (388)$$

This is the local-to-global version of the lexical and hermeneutic distinction developed in Section 9.6.

Where transition maps are invertible and satisfy suitable cocycle conditions, a stronger geometric construction may become possible. For three overlapping patches, one would expect a compatibility relation of the form

$$T_{ik} = T_{jk} \circ T_{ij} \quad (389)$$

on the relevant common domain.

The paper does not assume Equation 389. Its possible failure becomes analytically significant because translation histories can alter the interpretation produced at the destination.

14.6 Failure of Global Gluing

Suppose a region U is covered by local patches

$$U = \bigcup_{i \in I} U_i \quad (390)$$

and suppose local sections

$$f_i \in \mathcal{F}(U_i) \quad (391)$$

are given.

For an ordinary sheaf, if the sections agree on every overlap,

$$\rho_{U_i \cap U_j}^{U_i}(f_i) = \rho_{U_i \cap U_j}^{U_j}(f_j) \quad \text{for all } i, j, \quad (392)$$

there exists a unique section

$$f \in \mathcal{F}(U) \quad (393)$$

such that

$$\rho_{U_i}^U(f) = f_i \quad \text{for every } i. \quad (394)$$

These locality and gluing properties are central to the mathematical concept of a sheaf [5, 12]. The philosophical significance lies partly in the possibility that the conditions fail. Failure can occur at several stages:

$$\begin{aligned}
& \text{restriction failure,} \\
& \text{overlap incompatibility,} \\
& \text{translation non-invertibility,} \\
& \text{cocycle failure,} \\
& \text{non-existence of a global section,} \\
& \text{non-uniqueness of global reconstruction.}
\end{aligned} \tag{395}$$

These failures should not be collapsed.

In particular, local disagreement does not itself establish absence of a global structure. Apparently different local sections may become compatible after a legitimate translation. Conversely, pairwise similarities do not guarantee a coherent global interpretation.

A central GRF possibility is therefore

$$\{f_i \in \mathcal{F}(U_i)\}_{i \in I} \not\Rightarrow \exists! f \in \mathcal{F}(U). \tag{396}$$

The failure of a unique global section provides one possible structural account of persistent disputes over freedom. Different relational patches may support locally coherent meanings whose relations on overlaps do not satisfy the conditions required for a unique global reconstruction.

This is only one source of disagreement. Political interest, domination, strategic behavior, unequal information, historical violence, and empirical error can independently generate conflict. Non-gluability should therefore not be used as a complete explanation of political disagreement.

14.7 Hermeneutic Path Dependence

Transition among relational language systems can depend upon the route through which interpretation travels.

Consider three patches U_1 , U_2 , and U_3 . A direct transformation

$$T_{13}^F : f_1 \longrightarrow \tilde{f}_1^{(3)} \tag{397}$$

can be compared with the composite transformation

$$T_{23}^F \circ T_{12}^F : f_1 \longrightarrow \tilde{f}_1^{(2,3)}. \tag{398}$$

Hermeneutic path dependence occurs when

$$T_{13}^F(f_1) \neq T_{23}^F \circ T_{12}^F(f_1). \tag{399}$$

The discrepancy can arise because translation through L_2 introduces distinctions, suppresses distinctions, changes associations, or embeds the interpretation within a different historical structure before it reaches L_3 .

This path dependence is especially plausible when translation is generative rather than purely representational. An interpretation entering another language system can acquire relations that were absent from its previous configuration.

The transformation history then becomes part of the resulting meaning:

$$f_{\text{result}} = \Phi(f_{\text{initial}}, T_{12}, T_{23}, H_T), \tag{400}$$

where H_T denotes the relevant translation history.

This construction gives a formal expression to historical semantic sedimentation. Meanings do not necessarily move through relational systems without being changed by the route.

Path dependence also prevents translation from being treated automatically as a coordinate transformation. Ordinary coordinate changes preserve the underlying geometric object. Hermeneutic transition can modify the object being interpreted.

14.8 Interpretive Holonomy

Path dependence suggests a stronger geometric analogy when additional structure is available. In differential geometry, holonomy concerns the result of parallel transport around closed loops in a space equipped with an appropriate connection [20]. Returning to the original base point need not return the transported object to its original state.

A hermeneutic analogue can be considered for a closed sequence of interpretive transitions

$$U_1 \longrightarrow U_2 \longrightarrow \cdots \longrightarrow U_n \longrightarrow U_1. \quad (401)$$

The composite transformation is

$$\mathcal{H}_\gamma^F = T_{n1}^F \circ T_{n-1,n}^F \circ \cdots \circ T_{12}^F, \quad (402)$$

where γ denotes the corresponding interpretive loop.

If

$$\mathcal{H}_\gamma^F(f_1) \neq f_1, \quad (403)$$

the interpretation has changed after returning to the initial relational patch.

This structure captures a recognizable hermeneutic phenomenon. A concept can leave one language system, pass through several others, and return through translation carrying distinctions or associations generated during that journey.

The term *holonomy* is used only analogically at this stage. A genuine geometric holonomy construction requires a bundle, a connection, a notion of parallel transport, and loops in the appropriate base space. The present paper has not constructed all of these objects for freedom interpretation.

Consequently,

$$\mathcal{H}_\gamma^F(f) \neq f \quad (404)$$

should presently be read as an *interpretive-holonomy-like effect*, unless a future formalization supplies the required differential-geometric structure.

The bounded claim is therefore path-sensitive transformation around an interpretive loop, rather than literal equality with physical or differential geometric holonomy.

14.9 Global Sections and Their Limits

The local-to-global construction culminates in the question of whether a global freedom interpretation exists.

If the sheaf conditions are satisfied, a compatible family of local sections can determine

$$f_{\text{global}} \in \mathcal{F}(\mathcal{M}^F). \quad (405)$$

Such a global section would represent a coherent interpretation whose restrictions reproduce the declared local sections.

Its existence would be philosophically significant, but its interpretation must remain modest.

First,

$$\exists f_{\text{global}} \not\Rightarrow f_{\text{global}} \text{ is a universal metaphysical definition of freedom.} \quad (406)$$

A global section is global only relative to the space, cover, category of local data, and restriction structure that have been constructed.

Second,

$$\exists! f_{\text{global}} \not\Rightarrow \text{NormativelyFinal}(f_{\text{global}}). \quad (407)$$

Mathematical uniqueness under a specified gluing structure does not establish moral truth.

Third, failure of a global section does not imply that comparison is impossible:

$$\nexists f_{\text{global}} \not\Rightarrow \nexists \{\text{overlap, translation, invariant, partial comparison}\}. \quad (408)$$

This third point is especially important for GRF. The absence of a single globally glued freedom meaning can coexist with substantial local translatability and cross-relational structure.

The relevant hierarchy is therefore

$$\begin{aligned} \text{local intelligibility} &\longrightarrow \text{overlap comparison} \\ &\longrightarrow \text{partial compatibility} \\ &\longrightarrow \text{possible gluing} \\ &\longrightarrow \text{possible global section.} \end{aligned} \quad (409)$$

No later stage is assumed merely because an earlier one exists.

This hierarchy also provides a more precise account of the paper's rejection of a context-independent freedom scalar. The difficulty is not simply that available measurements contain too few variables. The deeper possibility is that locally meaningful freedom structures may fail to constitute one global semantic object on which a universal scalar could be defined.

Thus,

$$\nexists f_{\text{global}} \implies \text{a global scalar representation requires additional justification.} \quad (410)$$

The implication is epistemic rather than strictly mathematical. Different scalar constructions may still be useful for declared purposes. What fails is the inference that aggregation alone establishes access to a pre-existing global semantic object.

The local-to-global analysis can therefore be summarized by the conditional structure

$$\begin{aligned} U &\mapsto \mathcal{F}(U), \\ V \subseteq U &\mapsto \rho_V^U, \\ \{f_i\} &\mapsto \text{overlap compatibility,} \\ \text{compatible local sections} &\mapsto \text{possible global gluing.} \end{aligned} \quad (411)$$

The formalism yields three distinct outcomes. Local interpretations can glue uniquely, can glue only after additional translation structure is supplied, or can fail to admit a coherent global reconstruction.

None of these outcomes should be presumed in advance.

For Generative Relational Freedom, this result changes the form of cross-context inquiry. The immediate task is no longer to identify one hidden meaning behind every local freedom vocabulary. The more disciplined questions concern which local meanings can be restricted, which can be translated, which agree on overlaps, which fail to glue, and which structures remain stable despite semantic difference.

Section 15 develops the translation problem in greater detail. It turns from the abstract gluing structure to lexical correspondence, cross-linguistic transformation, historical semantic sedimentation, and the non-identity of meanings generated within different relational language systems.

15 Language, Translation, and Hermeneutic Non-Identity

Section 14 established that locally coherent freedom interpretations need not automatically glue into a unique global semantic structure. The present section examines one of the principal mechanisms through which such local interpretations encounter one another: translation.

Translation is particularly important for freedom because political, religious, legal, and philosophical discourse frequently proceeds through apparently straightforward lexical correspondences. A term in one language is assigned an equivalent expression in another, after which discussion can proceed as though both expressions referred to one already identical semantic object. Such correspondence is often practically indispensable. It is not, however, sufficient to establish hermeneutic identity.

The argument of this section is that translation occurs between historically and relationally situated language systems. A lexical expression carries relations to practices, institutions, texts, political struggles, religious forms, legal distinctions, and prior translations. When an expression enters another language system, these relations cannot be presumed to transfer without transformation.

Translation is therefore treated as a generative relational process:

$$(w_i, L_i, H_i) \xrightarrow{T_{ij}} (\widetilde{w}_i, L_j, H_j), \quad (412)$$

where w_i denotes an expression situated within language system L_i , T_{ij} a translation process, and $\widetilde{w}_i$ the translated form as it becomes interpretable within L_j .

The tilde indicates that the result is neither assumed to preserve every relation of the source interpretation nor assumed to coincide with a meaning already generated locally in the receiving system.

15.1 Lexical Correspondence and Semantic Difference

The first distinction concerns lexical correspondence. Let

$$w_i \in L_i, \quad w_j \in L_j \quad (413)$$

denote expressions occurring in two situated language systems.

A dictionary, translator, legal instrument, or established convention may provide the lexical relation

$$T_{\text{lex}}(w_i) = w_j. \quad (414)$$

Equation 414 can be accurate and practically useful while remaining weaker than semantic identity. Let

$$f_i = \llbracket w_i \rrbracket_{L_i} \quad (415)$$

and

$$f_j = \llbracket w_j \rrbracket_{L_j}. \quad (416)$$

The central non-implication is

$$T_{\text{lex}}(w_i) = w_j \not\Rightarrow f_i = f_j. \quad (417)$$

A schematic example is the ordinary translation relation

$$T_{\text{lex}}(\text{freedom}) = \text{自由}. \quad (418)$$

Equation 418 does not claim that English possesses one freedom concept or that Chinese possesses one concept of 自由. Both natural languages contain multiple historical, philosophical, legal, political, and ordinary usages. The example isolates the difference between a lexical bridge and a proof of semantic identity.

More precisely, one would need to compare expressions situated in specified language systems:

$$f_1 = \llbracket \text{freedom} \rrbracket_{L_{s_1, t_1}}, \quad f_2 = \llbracket \text{自由} \rrbracket_{L_{s_2, t_2}}. \quad (419)$$

The appropriate question is then which relations, distinctions, and consequences associated with f_1 can be reconstructed within L_{s_2, t_2} , and conversely.

Translation permits that inquiry to begin. It does not settle it.

15.2 Translation between Relational Language Systems

Translation in GRF operates between relational language systems rather than between isolated words.

Let

$$T_{ij} : L_i \dashrightarrow L_j \quad (420)$$

denote a possibly partial translation relation. The dashed arrow is used because translation need not be defined for every structure within L_i .

For a freedom interpretation f_i , the induced translation is

$$T_{ij}^F : f_i \mapsto \tilde{f}_i^{(j)}, \quad (421)$$

where $\tilde{f}_i^{(j)}$ denotes the source interpretation as re-generated within the receiving relational language system.

The translated configuration can depend upon more than the source meaning:

$$\tilde{f}_i^{(j)} = \Theta_{ij}(f_i, L_j, H_j, \mathcal{R}_j, C_{ij}), \quad (422)$$

where C_{ij} denotes features of the translation context, including the purpose, medium, institutional setting, translator, audience, or other declared mediating conditions.

Equation 422 is schematic. Its role is to make explicit that translation does not operate upon semantics in a relational vacuum.

The same source interpretation can therefore generate several receiving configurations:

$$T_{ij,1}^F(f_i) \neq T_{ij,2}^F(f_i). \quad (423)$$

A legal translation, philosophical translation, diplomatic translation, and ordinary conversational translation may preserve different structures even when they employ the same target lexeme.

The unit of analysis should consequently include the translation relation itself:

$$(L_i, T_{ij}, L_j). \quad (424)$$

Translation is part of the relational system rather than an invisible bridge between two independently complete semantic domains.

15.3 Partial Translatability

Hermeneutic non-identity does not imply untranslatability. Two situated freedom interpretations can differ while retaining substantial structures that permit comparison.

For a declared domain D , let

$$\mathcal{K}_D(f_i) \quad (425)$$

denote the aspects of interpretation relevant to the current inquiry.

A translation may preserve part of this structure:

$$\mathcal{K}_D(f_i) \xrightarrow{T_{ij}} \mathcal{K}_D(\tilde{f}_i^{(j)}). \quad (426)$$

Partial translatability exists when some declared relations can be reconstructed with sufficient fidelity while others cannot.

One may represent this through a decomposition

$$\mathcal{K}_D(f_i) = \mathcal{K}_D^{\text{trans}} \cup \mathcal{K}_D^{\text{res}}, \quad (427)$$

where $\mathcal{K}_D^{\text{trans}}$ denotes the portion for which an adequate translation relation can be constructed and $\mathcal{K}_D^{\text{res}}$ denotes residual structure that remains untranslated, underdetermined, or transformed beyond the declared tolerance.

The decomposition is domain-relative. A translation can be sufficient for one purpose while insufficient for another.

For example,

$$T_{ij}^{D_1} \text{ adequate} \not\Rightarrow T_{ij}^{D_2} \text{ adequate}. \quad (428)$$

A translation sufficient for identifying a formal legal permission may fail to preserve the historical or religious associations relevant to a hermeneutic inquiry.

This leads to an important alternative to the binary distinction between perfect equivalence and complete incomparability:

$$\text{semantic non-identity} + \text{partial translatability}. \quad (429)$$

GRF treats this combination as an ordinary possibility.

The later search for cross-relational invariants begins precisely here. Structures can sometimes remain comparable without requiring that the local meanings themselves be identical.

15.4 Semantic Transformation through Translation

Translation can generate semantic change. When a concept enters another relational language system, it encounters distinctions and associations not identical to those of its source environment.

Consequently,

$$T_{ij}^F(f_i) = \tilde{f}_i^{(j)} \neq f_i \quad (430)$$

can hold even when the translation is competent and useful.

The inequality need not indicate error. It can arise because the receiving system reorganizes the translated material.

Similarly,

$$\tilde{f}_i^{(j)} \neq f_j, \quad (431)$$

where f_j denotes an interpretation generated historically within the receiving system itself.

The resulting configuration can therefore occupy a third position:

$$f_i \xrightarrow{T_{ij}} \tilde{f}_i^{(j)} \quad \text{with} \quad \tilde{f}_i^{(j)} \neq f_i, \quad \tilde{f}_i^{(j)} \neq f_j. \quad (432)$$

This possibility is theoretically important. Cross-cultural encounter need not consist only of selecting between two pre-existing semantic systems. Translation can generate hybrid or newly differentiated interpretations.

The translated interpretation may then enter the receiving language system:

$$\tilde{f}_i^{(j)} \longrightarrow \Delta L_j \quad (433)$$

and subsequently affect locally generated meanings:

$$\Delta L_j \longrightarrow \Delta f_j. \quad (434)$$

Translation is therefore potentially transformative for both the transferred concept and the receiving interpretive system.

The source system can also change through return translation, commentary, institutional uptake, or later cross-relational encounter. Translation should therefore be modeled as a potentially reciprocal historical process rather than as one-directional semantic transportation.

15.5 Historical and Cultural Sedimentation

A translated expression enters a language system already containing history. Its later meaning can accumulate traces of both local and translated traditions.

Let

$$H_{w,t}^T \quad (435)$$

denote the translation history associated with expression w up to time t . A situated interpretation can then depend upon

$$\llbracket w \rrbracket_{L_{st}} = \Phi(L_{st}, H_{st}^L, H_{w,t}^T), \quad (436)$$

where H_{st}^L denotes the wider history sedimented within the local language system.

This formulation permits translation itself to become part of semantic history. A translated concept can be incorporated into scholarship, law, religion, political movements, education, or ordinary discourse and thereby develop relations unavailable at the time of its initial transfer.

The historical trajectory may take the form

$$f_i \xrightarrow{T_{ij}} \tilde{f}_{i,t_1}^{(j)} \longrightarrow \tilde{f}_{i,t_2}^{(j)} \longrightarrow \tilde{f}_{i,t_3}^{(j)}. \quad (437)$$

Later stages need not remain equivalent to the initial translation.

A concept can therefore become locally naturalized. After sufficient historical sedimentation, speakers may no longer experience it primarily as a translated import. It becomes one component of the receiving L_{st} .

This possibility complicates sharp contrasts between “indigenous” and “foreign” meanings. Relational language systems are historically permeable. Their vocabularies can contain layers produced through earlier encounters.

The relevant analytical question is therefore historical:

Through which relational and translational processes did the present interpretive configuration become available?

The question is more precise than assigning a contemporary meaning directly to a large civilizational category.

15.6 Intra-Linguistic Interpretive Difference

The problem of hermeneutic non-identity is not restricted to translation between natural languages.

Two subjects can employ the same lexical expression while occupying substantially different relational language systems:

$$w_B = w_C \quad \text{while} \quad \llbracket w_B \rrbracket_{L_B} \neq \llbracket w_C \rrbracket_{L_C}. \quad (438)$$

The difference can arise from professional vocabulary, legal position, religious formation, political history, generation, class relations, institutional membership, personal history, or participation in different communities of practice.

Even within one subject, semantic continuity is not guaranteed across time:

$$\llbracket w \rrbracket_{L_{B,t_1}} \neq \llbracket w \rrbracket_{L_{B,t_2}}. \quad (439)$$

A person can reinterpret freedom after migration, political experience, religious transformation, education, institutional participation, or other relational change.

This observation prevents natural language from becoming the fundamental unit of hermeneutic analysis.

The more appropriate relation is

$$\text{natural language} \supset \{L_1, L_2, \dots, L_n\}, \quad (440)$$

where multiple relational language systems can operate through overlapping lexical resources.

The notation is schematic and should not be understood as a literal set-theoretical ontology of language. Its purpose is to emphasize internal heterogeneity.

Consequently, the statement

“Both speakers use the same word”

cannot substitute for evidence that they inhabit the same interpretive configuration.

Political disagreement can therefore appear even where lexical translation is unnecessary. Participants may believe they disagree over the application of a shared concept while actually operating with partially different freedom meanings.

The reverse can also occur. Speakers can employ different vocabularies while preserving similar relational distinctions.

This reinforces the need for structural comparison developed in the following section.

15.7 Translation Loops and Non-Identity

Translation becomes especially revealing when an interpretation returns to its initial language system.

Consider

$$f_i \xrightarrow{T_{ij}} \tilde{f}_i^{(j)} \xrightarrow{T_{ji}} \tilde{f}_i^{(j,i)}. \quad (441)$$

There is no general reason to assume

$$\tilde{f}_i^{(j,i)} = f_i. \quad (442)$$

The intermediate language system can introduce distinctions that survive the return process.

More generally, for a sequence

$$i_1 \rightarrow i_2 \rightarrow \dots \rightarrow i_n \rightarrow i_1, \quad (443)$$

the composite transformation is

$$\mathcal{T}_\gamma = T_{i_n i_1} \circ T_{i_{n-1} i_n} \circ \dots \circ T_{i_1 i_2}. \quad (444)$$

A non-trivial return effect occurs when

$$\mathcal{T}_\gamma(f_{i_1}) \neq f_{i_1}. \quad (445)$$

Section 14.8 described the stronger geometric analogy of interpretive holonomy. The present subsection keeps the weaker and more general claim: translation can be path-dependent and repeated translation can generate semantic transformation.

This also means that reverse translation is not an epistemically neutral test of equivalence. A successful return to the original lexical expression does not establish recovery of the original relational meaning:

$$T_{\text{lex}}^{-1}(T_{\text{lex}}(w_i)) = w_i \not\equiv \mathcal{T}_\gamma(f_i) = f_i. \quad (446)$$

Lexical reversibility and hermeneutic reversibility are distinct properties.

The distinction is particularly important when concepts circulate repeatedly through academic, diplomatic, legal, religious, and political networks. Historical circulation can become part of the concept's subsequent relational structure.

15.8 Freedom across Linguistic Boundaries

The preceding analysis changes the objective of cross-linguistic inquiry into freedom.

The objective cannot simply be

$$f_i \stackrel{?}{=} f_j, \quad (447)$$

because such equality presupposes a common semantic space and criterion of identity that may itself require construction.

A more disciplined comparison begins with several distinct questions:

$$\begin{aligned} &\text{which structures are lexically transferable,} \\ &\text{which relations remain semantically recognizable,} \\ &\text{which distinctions are transformed,} \\ &\text{which elements resist translation,} \\ &\text{which relational structures remain invariant.} \end{aligned} \quad (448)$$

These questions permit comparison without requiring total equivalence.

For two situated interpretations f_i and f_j , cross-linguistic comparison may therefore produce a structured result

$$\mathcal{C}_D(f_i, f_j) = \langle \mathcal{E}_D, \mathcal{P}_D, \mathcal{D}_D, \mathcal{R}_D \rangle, \quad (449)$$

where

- $\mathcal{E}_D$ denotes structures judged equivalent under the declared domain;
- $\mathcal{P}_D$ denotes partially translatable structures;
- $\mathcal{D}_D$ denotes substantive differences;
- $\mathcal{R}_D$ denotes residual or presently indeterminate structure.

The profile is analytical rather than numerical. It prevents comparison from being forced into a binary classification of “same” or “different.”

This structure also provides a response to semantic relativism. If local meanings are historically situated, it does not follow that no cross-relational structure can be discovered. Partial translatability itself is evidence that relational overlap may exist.

Conversely, successful communication does not prove universal semantic identity:

$$\text{CommunicationSuccessful}(f_i, f_j) \not\equiv f_i = f_j. \quad (450)$$

Human communication routinely operates under partial overlap, approximation, clarification, correction, and iterative reinterpretation.

The possibility of such interaction is central to GRF. Different relational language systems need not first collapse into one vocabulary before their freedom structures can be compared.

The relevant sequence is instead

$$\begin{aligned}
 \text{local interpretation} &\longrightarrow \text{translation} \\
 &\longrightarrow \text{partial comparison} \\
 &\longrightarrow \text{identification of preserved structures} \\
 &\longrightarrow \text{possible cross-relational invariants.}
 \end{aligned} \tag{451}$$

Translation therefore occupies an intermediate position between local hermeneutics and cross-relational invariance. It makes encounter possible while preserving the possibility that some semantic structure changes in the encounter itself.

The resulting account can be summarized through three non-identities:

$$\begin{aligned}
 \text{lexical equivalence} &\not\equiv \text{semantic identity,} \\
 \text{semantic non-identity} &\not\equiv \text{incomparability,} \\
 \text{successful translation} &\not\equiv \text{translation without transformation.}
 \end{aligned} \tag{452}$$

These distinctions allow the paper to move beyond both universal semantic identity and isolated cultural particularism.

Section 16 develops the next step. Having established that translation can be partial, generative, and path-dependent, the paper asks which structures can nevertheless remain stable across relational transformations and whether such structures can support comparison without presupposing a universal scalar or globally identical meaning of freedom.

16 Cross-Relational Transformation and Invariance

Section 15 established that translation between situated language systems can preserve some structures, transform others, and generate interpretations that coincide with neither the source meaning nor the meaning historically generated in the receiving system. The present section develops the comparative consequence of that result.

The central problem is how freedom interpretations can be compared across relational systems without first assuming that they are local expressions of one globally identical semantic object. Generative Relational Freedom approaches this problem through transformation and invariance. The relevant question is whether some declared relational structures survive movement between situated interpretive systems even when vocabulary, historical associations, institutional forms, or local semantic organization change.

The formalism therefore begins with local interpretive systems and cross-relational transformations. It then asks which structures remain stable under those transformations. Equality of a cross-relational invariant is weaker than semantic identity and stronger than superficial lexical similarity. This intermediate level provides a possible basis for comparison under conditions of hermeneutic plurality.

The construction remains conditional throughout. A proposed invariant must identify its domain, transformation class, and preserved structure. Recurring terms, intuitively similar practices, or externally imposed classifications do not independently establish cross-relational invariance.

16.1 Transformation between Situated Freedom Systems

Let two situated freedom systems be represented by

$$\mathfrak{F}_i = \langle \mathcal{X}_i^F, L_i, \mathcal{R}_i, H_i \rangle \tag{453}$$

and

$$\mathfrak{F}_j = \langle \mathcal{X}_j^F, L_j, \mathcal{R}_j, H_j \rangle. \quad (454)$$

A cross-relational transformation is written provisionally as

$$T_{ij}^D : \mathcal{X}_i^F \dashrightarrow \mathcal{X}_j^F, \quad (455)$$

where D denotes the declared comparison domain and the dashed arrow permits a partial transformation.

The use of a partial map is important. Some distinctions available in $\mathcal{X}_i^F$ may lack an adequate image in $\mathcal{X}_j^F$, while other structures can be translated only after additional contextual reconstruction.

For

$$f_i \in \text{Dom}(T_{ij}^D), \quad (456)$$

the transformed interpretation is

$$\tilde{f}_i^{(j)} = T_{ij}^D(f_i). \quad (457)$$

As established in the previous section,

$$\tilde{f}_i^{(j)} \not\equiv f_j \quad (458)$$

can hold, where f_j denotes a freedom interpretation generated locally within $\mathfrak{F}_j$.

The transformation T_{ij}^D can encode translation, contextual reinterpretation, structural comparison, or another explicitly specified relation. These operations should not be treated as interchangeable. A lexical translation and a transformation designed to preserve institutional dependency structure answer different comparative questions.

Cross-relational analysis therefore requires the comparison protocol itself to be declared:

$$\Pi_{ij}^D = \langle D, T_{ij}, \mathcal{K}_D, \mathcal{E}_D \rangle, \quad (459)$$

where $\mathcal{K}_D$ denotes the structures selected for comparison and $\mathcal{E}_D$ the criteria used to evaluate preservation, difference, or indeterminacy.

The transformation relation is consequently part of the epistemic object. Different comparison protocols can generate different conclusions from the same pair of situated systems.

16.2 Partial Equivalence across Relational Contexts

Cross-relational comparison rarely requires complete equivalence. Two interpretations can preserve some relevant structures while differing substantially in others.

For a declared domain D , define a relation

$$f_i \approx_D f_j \quad (460)$$

when the interpretations satisfy an explicitly specified set of domain-relevant compatibility conditions.

The relation $\approx_D$ need not be an equivalence relation in the strict mathematical sense. Symmetry or transitivity can fail when translation is partial or path-dependent. The weaker notation is therefore retained unless those properties are demonstrated.

A comparison profile may be written as

$$\mathcal{C}_D(f_i, f_j) = \langle \mathcal{K}_D^{\text{pres}}, \mathcal{K}_D^{\text{trans}}, \mathcal{K}_D^{\text{diff}}, \mathcal{K}_D^{\text{ind}} \rangle, \quad (461)$$

where

- $\mathcal{K}_D^{\text{pres}}$ contains structures judged preserved under the declared transformation;
- $\mathcal{K}_D^{\text{trans}}$ contains structures retained only after identifiable transformation;
- $\mathcal{K}_D^{\text{diff}}$ contains substantive differences;
- $\mathcal{K}_D^{\text{ind}}$ contains structures for which the present comparison remains indeterminate.

This representation prevents uncertainty from being silently classified as difference and prevents partial similarity from being upgraded to complete semantic identity.

Domain dependence remains essential:

$$f_i \approx_{D_1} f_j \not\Rightarrow f_i \approx_{D_2} f_j. \quad (462)$$

Two institutional systems may be comparable with respect to formal contestability while remaining difficult to compare with respect to religious self-understanding or historically accumulated symbolic meaning.

Partial equivalence therefore identifies a local result of comparison rather than a universal relation between entire cultures or traditions.

16.3 Cross-Relational Invariants

The strongest comparative object developed in this section is a cross-relational invariant.

Let

$$I_i^D : \mathcal{X}_i^F \longrightarrow \mathcal{Y}_D \quad (463)$$

and

$$I_j^D : \mathcal{X}_j^F \longrightarrow \mathcal{Y}_D \quad (464)$$

be relational descriptors taking values in a common comparison space $\mathcal{Y}_D$.

The descriptor is preserved under T_{ij}^D when

$$I_j^D (T_{ij}^D(f_i)) = I_i^D(f_i) \quad (465)$$

for the relevant domain of the transformation.

Equation 465 is the central compatibility condition of cross-relational invariance.

It can be represented through the commuting structure

$$\begin{array}{ccc} \mathcal{X}_i^F & \xrightarrow{T_{ij}^D} & \mathcal{X}_j^F \\ \downarrow I_i^D & & \downarrow I_j^D \\ \mathcal{Y}_D & \xrightarrow{\text{id}} & \mathcal{Y}_D. \end{array} \quad (466)$$

The diagram states that transformation between local interpretive systems and evaluation of the declared invariant produce compatible results.

An invariant need not be numerical. Possible forms include

$$I_D(f) \in \left\{ \begin{array}{l} \text{a relation graph,} \\ \text{an order structure,} \\ \text{a stabilizer property,} \\ \text{a distribution of revision authority,} \\ \text{an exit-dependency structure,} \\ \text{a relational topology,} \\ \text{a logical or categorical pattern.} \end{array} \right. \quad (467)$$

This flexibility is important because reducing invariance to a scalar would reintroduce the measurement problem identified earlier.

For GRF, a candidate invariant might concern the relational distinction between a subject who can participate in revising the conditions of action and one whose conditions are unilaterally fixed. The institutional and linguistic forms through which that distinction appears can vary substantially while the declared relational pattern remains recognizable.

Such a candidate must still be tested. The recurrence of an abstract pattern across selected cases does not establish its invariance under every cross-relational transformation.

The appropriate claim has the form

$$I_D \text{ is invariant under } \mathcal{T}_D, \quad (468)$$

where $\mathcal{T}_D$ is a declared class of transformations.

No invariant is global merely because it survives one comparison.

16.4 Gauge-Like Descriptive Transformations

Section 3.11 introduced gauge equivalence as a physical and geometric source-domain distinction between changes of description and changes of physical state [20, 27]. A bounded analogue can now be formulated for cross-relational interpretation.

Suppose a family of transformations

$$G_i^{\text{desc}} \curvearrowright \mathcal{X}_i^F \quad (469)$$

changes the representation of a freedom configuration while preserving every structure declared relevant to domain D :

$$I_k^D(g \cdot f) = I_k^D(f) \quad \text{for all } I_k^D \in \mathbf{I}_D. \quad (470)$$

Such transformations may be treated as *gauge-like* for the limited purpose of the comparison.

A descriptive equivalence class is then

$$[f]_{\text{desc}} = \left\{ g \cdot f \mid g \in G_i^{\text{desc}} \right\}. \quad (471)$$

The quotient

$$\mathcal{X}_i^F / G_i^{\text{desc}} \quad (472)$$

represents configurations after declared descriptive redundancies have been removed.

This construction carries a demanding burden of proof. A transformation between natural languages, religious vocabularies, or political traditions cannot be classified as gauge-like merely because two observers judge the results similar.

The analysis must establish which structure is preserved.

In particular,

$$T_{ij} \text{ changes vocabulary} \not\Rightarrow T_{ij} \in G^{\text{desc}}. \quad (473)$$

Likewise,

$$I_D(T_{ij}(f_i)) \neq I_D(f_i) \quad (474)$$

means that the transformation is substantive relative to D .

Gauge-like language is therefore reserved for cases where the transformation can defensibly be interpreted as representational redundancy under a declared set of invariants. Semantic change, historical reinterpretation, and translation-induced transformation remain outside that class.

16.5 Relationally Stable Structures

Cross-relational invariance can be extended across a family of situated systems.

Let

$$\{\mathfrak{F}_i\}_{i \in I} \quad (475)$$

denote a collection of local freedom systems and let $\mathcal{T}_D$ contain declared transformations among them.

A structure I_D is relationally stable over this transformation family when

$$I_j^D \circ T_{ij}^D = I_i^D \quad (476)$$

whenever the relevant transformation is defined.

Relational stability is therefore relative to

$$(D, \{\mathfrak{F}_i\}, \mathcal{T}_D). \quad (477)$$

The qualifier prevents a locally robust structure from being declared universally invariant.

A family of such structures can be written

$$\mathbf{I}_D = \{I_1^D, I_2^D, \dots, I_m^D\}. \quad (478)$$

The family can contain mutually independent or partially coupled relational features.

This representation is preferable to constructing a single universal index:

$$\mathbf{I}_D \not\equiv F_{\text{universal}}. \quad (479)$$

An invariant profile can preserve several qualitatively distinct structures without aggregating them into one magnitude.

The existence of relationally stable structures would provide a limited basis for cross-context criticism. If a relational pattern associated with severe closure remains identifiable across several linguistic and institutional transformations, criticism of that pattern need not depend upon semantic identity among the corresponding local freedom concepts.

This possibility is philosophically important. Situated hermeneutics need not terminate in isolated local worlds. Comparison can proceed through structures that survive declared relational transformations.

At the same time, the absence of a currently identified invariant does not prove that no invariant exists. It may reflect inadequate models, poor translation, insufficient evidence, or genuine structural heterogeneity.

16.6 Incomparability and Interpretive Indeterminacy

Cross-relational analysis must distinguish incomparability from epistemic indeterminacy.

Suppose no adequate transformation

$$T_{ij}^D \quad (480)$$

has been established between two local systems.

Several explanations are possible:

- the available translation is inadequate,
 - relevant historical evidence is missing,
 - the declared domain is poorly specified,
 - the chosen representation is inappropriate,
 - the structures are genuinely non-equivalent,
 - the structures may be incomparable under D .
- (481)

These outcomes should not be conflated.

Let

$$f_i ?_D f_j \quad (482)$$

denote a comparison whose status remains unresolved.

The distinction is epistemically important:

$$\text{UnknownComparable}_D(f_i, f_j) \not\equiv \text{Incomparable}_D(f_i, f_j). \quad (483)$$

A declaration of incomparability is itself a substantive result and requires justification.

The same discipline applies to absence of invariance:

$$\neg \text{Established}(I_D(f_i) = I_D(f_j)) \not\equiv I_D(f_i) \neq I_D(f_j). \quad (484)$$

GRF therefore permits an explicitly indeterminate category. Such indeterminacy is preferable to forcing every pair of local systems into equivalence or opposition.

Interpretive indeterminacy can also be productive. Failure to construct a mapping can reveal which distinctions remain unarticulated and can motivate new translation, historical research, or reformulation of the comparison domain.

16.7 Comparison without Global Semantic Identity

The local-to-global analysis in Section 14 allowed for the possibility

$$\nexists f_{\text{global}} \in \mathcal{F}(\mathcal{M}^F). \quad (485)$$

The present section shows why this possibility does not terminate comparison.

Two local interpretations can lack a common global semantic identity while still admitting

$$T_{ij}^D, \quad \mathcal{C}_D(f_i, f_j), \quad \mathbf{I}_D. \quad (486)$$

Translation, partial equivalence, and invariance occupy different levels of the comparative structure. The resulting hierarchy is

$$\begin{aligned} \text{semantic identity} &\implies \text{strong compatibility,} \\ \text{partial translatability} &\implies \text{restricted comparison,} \\ \text{cross-relational invariance} &\implies \text{preservation of declared structure.} \end{aligned} \quad (487)$$

The reverse implications are not assumed.

In particular,

$$I_i^D(f_i) = I_j^D(f_j) \not\equiv f_i = f_j. \quad (488)$$

A preserved relational structure can occur inside semantically different freedom systems.

This provides a formal basis for comparison across historical and cultural difference without requiring one vocabulary to become the semantic standard for all others.

The comparison can instead ask whether each system contains a locally expressed structure corresponding to a declared relational property. The linguistic and institutional manifestations can differ while the relational property remains comparable.

Cross-relational judgment therefore proceeds from

$$\text{local meaning} \longrightarrow \text{declared transformation} \longrightarrow \text{preserved relational structure} \quad (489)$$

rather than from lexical similarity directly to universal judgment.

This approach also clarifies why a global freedom ranking faces a stronger burden than a local comparative claim. A ranking usually requires commensurability across all selected cases and often requires aggregation. Cross-relational invariance requires only preservation of the particular structure being compared.

The latter can therefore survive where global scalarization remains unjustified.

16.8 Epistemic Humility in Cross-Relational Judgment

Cross-relational judgment is itself situated. The researcher, institution, translator, or political actor performing the comparison operates within a language system and selects the domain, transformations, and proposed invariants.

Let the judging system be represented by

$$L_J. \quad (490)$$

A judgment concerning two systems then takes the form

$$J_{L_J}^D (\mathfrak{F}_i, \mathfrak{F}_j). \quad (491)$$

The subscript makes explicit that comparison is not conducted from an interpretively empty position.

This does not invalidate judgment. It introduces a reflexive requirement: the categories used to compare other systems should themselves be open to examination.

A responsible comparison should therefore declare, where relevant,

$$\langle D, L_J, T_{ij}, \mathbf{I}_D, H, \mathcal{E} \rangle, \quad (492)$$

where H denotes the relevant temporal horizon and $\mathcal{E}$ the evidential basis.

The declaration limits the scope of the conclusion. A result obtained under one transformation protocol should not be generalized automatically to every dimension of the compared systems.

Epistemic humility also requires attention to asymmetry in the act of comparison. An institution with classificatory authority can impose its freedom vocabulary upon another relational system and subsequently treat failure to fit that vocabulary as evidence of deficiency.

Such a process can be represented schematically by

$$L_J \xrightarrow{\text{Classify}} L_i \longrightarrow \Delta L_i, \quad (493)$$

where the judgment becomes a relational intervention rather than remaining only an observation.

The classification can affect institutional legitimacy, resource access, political discourse, or the categories through which subjects interpret themselves. Cross-relational measurement and judgment therefore possess potentially reflexive effects.

This possibility does not prohibit external criticism. It requires the comparison to distinguish between

$$\text{identifying a preserved relational harm} \quad (494)$$

and

$$\text{requiring semantic assimilation to the evaluator's language.} \quad (495)$$

The former can potentially be supported through cross-relational invariants. The latter requires substantially stronger justification.

Epistemic humility also applies to GRF itself. The framework's proposed invariants are hypotheses subject to revision. A structure that appears cross-relationally stable within a limited set of comparisons may fail under encounter with additional systems.

Accordingly,

$$\mathbf{I}_D^{(t)} \longrightarrow \mathbf{I}_D^{(t+1)} \quad (496)$$

must remain possible.

The objective is therefore neither permanent semantic unification nor withdrawal from comparison. GRF seeks comparison that remains explicit about its transformation rules, preserved structures, unresolved differences, and historical position.

The architecture developed in this section can be summarized by

$$\begin{aligned} \mathfrak{F}_i &\xrightarrow{T_{ij}^D} \mathfrak{F}_j \\ &\longrightarrow \mathcal{C}_D(f_i, f_j) \\ &\longrightarrow \mathbf{I}_D \\ &\longrightarrow \text{bounded cross-relational judgment.} \end{aligned} \quad (497)$$

Each stage adds an evidential and formal burden. Translation does not establish equivalence; partial equivalence does not establish semantic identity; invariance does not establish universal normativity; and comparative judgment does not acquire a view outside history merely because it is formally expressed.

Cross-relational invariance nevertheless supplies a significant possibility. Freedom meanings can remain historically and hermeneutically local while some relational structures remain sufficiently stable to support comparison. The framework can therefore acknowledge semantic plurality without abandoning the search for common structure.

Section 17 next examines what occurs when such comparisons are converted into observables, indicators, rankings, and institutional representations. The central question shifts from cross-relational comparability to the epistemic and relational consequences of projecting situated freedom structures into measurable forms.

17 Representation and Measurement of Freedom

The preceding sections developed freedom through situated interpretation, relational transformation, local-to-global compatibility, and cross-relational invariance. The present section examines what occurs when these structures are converted into observable variables, indicators, indices, rankings, and institutional claims.

Measurement is indispensable in many domains of governance and social inquiry. Constitutional protections can be coded, institutional practices can be observed, material opportunities can be compared, and patterns of domination or exclusion can be investigated empirically. The present argument does not challenge the legitimacy of such measurement. It concerns the relation between a measurable representation and the richer relational object from which that representation is constructed.

Research on quantification has emphasized that numerical representation depends upon processes of classification, commensuration, and institutional standardization [24, 19]. Public measures can also become reactive, changing behavior and institutional organization after the objects being measured respond to the measurement system [7]. These problems acquire additional importance for freedom because the relevant observables are selected within historically situated language systems.

The analytical sequence developed here is

$$\begin{aligned} (\mathcal{R}_{st}, L_{st}, H_{st}) &\xrightarrow{P_D} \mathcal{O}_D \\ &\xrightarrow{M_D} \mathbf{z}_D \\ &\xrightarrow{A_D} F_D^{\text{rep}}, \end{aligned} \quad (498)$$

where P_D selects freedom-relevant observable structures, $\mathcal{O}_D$ denotes the selected observational domain, M_D operationalizes these structures into variables $\mathbf{z}_D$, and A_D optionally aggregates them into an institutional representation F_D^{rep} .

Each map performs analytical work. The final representation therefore cannot be understood independently of the selections and transformations that precede it.

17.1 Observability and Hermeneutic Selection

A relational property must first become recognizable before it can be measured. This requirement introduces an interpretive stage prior to quantification.

Let

$$\mathfrak{W}_{st} = \langle \mathcal{R}_{st}, L_{st}, H_{st}, \mathfrak{C}_{st}, \mathfrak{F}_{st}^{GR} \rangle \quad (499)$$

denote the wider relational configuration relevant to a freedom inquiry.

An observational projection

$$P_D : \mathfrak{W}_{st} \longrightarrow \mathcal{O}_D \quad (500)$$

selects structures that the inquiry recognizes as observable and relevant within domain D .

The projection depends upon prior distinctions. Before counting restrictions, for example, the inquiry must determine which relations qualify as restrictions. Before measuring political participation, it must determine which practices count as participation. Before coding domination, it requires a criterion for identifying the relevant form of power.

The process therefore has a hermeneutic component:

$$L_{J,t} \longrightarrow P_D \longrightarrow \mathcal{O}_D, \quad (501)$$

where $L_{J,t}$ denotes the language system of the researcher, institution, or evaluative framework.

This dependence does not make observation arbitrary. Empirical procedures can be transparent, replicable, and well supported while remaining selective.

The important distinction is between

$$q \notin \mathcal{O}_D \quad (502)$$

and

$$q \text{ does not exist.} \quad (503)$$

Failure to include a relational structure in an observational scheme does not establish its absence from the wider relational field.

Several reasons for observational absence should therefore be distinguished:

$$\text{Absent}_{\mathcal{O}}(q) \in \begin{cases} \text{outside the declared domain,} \\ \text{unrecognized by the current framework,} \\ \text{difficult to observe,} \\ \text{unavailable in the evidence,} \\ \text{deliberately excluded,} \\ \text{genuinely absent.} \end{cases} \quad (504)$$

These conditions have different epistemic implications.

The hermeneutic problem therefore precedes the statistical problem. Increasing measurement precision cannot recover a structure that the observational projection does not recognize.

17.2 Freedom Indicators as Relational Projections

An indicator transforms selected relational structures into an operational representation.

Let

$$M_D : \mathcal{O}_D \longrightarrow \mathbf{z}_D \quad (505)$$

denote an operationalization map, with

$$\mathbf{z}_D = (z_1, z_2, \dots, z_n). \quad (506)$$

Each z_k represents a declared observable, coded variable, estimate, or derived measure.

The indicator vector should therefore be interpreted as

$$\mathbf{z}_D = M_D \circ P_D(\mathfrak{W}_{st}), \quad (507)$$

rather than as an unmediated reading of freedom itself.

This distinction resembles Merry's analysis of indicators as techniques that translate complex social phenomena into standardized and comparable forms [19]. Standardization can make comparison possible while reducing contextual information.

For GRF,

$$\mathbf{z}_D \not\equiv \mathfrak{F}_{B,t}^{GR}. \quad (508)$$

The indicator vector can contain observations relevant to a GRF profile without exhausting the profile's historical, interpretive, and relational structure.

Projection can preserve some distinctions while erasing others. If two relational configurations satisfy

$$P_D(\mathfrak{W}_1) = P_D(\mathfrak{W}_2), \quad (509)$$

it does not follow that

$$\mathfrak{W}_1 = \mathfrak{W}_2. \quad (510)$$

The projection can be many-to-one.

This provides a formal account of representational loss. Distinct constraint architectures, histories, or interpretive conditions can produce the same indicator values.

The phenomenon is especially important when observable outcomes are similar. Two subjects may both perform the same action while one acts under reflectively sustained discipline and another under severe dependency. A measurement that records only the action cannot reconstruct the difference.

Indicator validity therefore requires correspondence between the operational representation and the relational structure the inquiry claims to measure.

17.3 Selection of Observable Dimensions

The construction of an indicator set requires a selection of dimensions.

Let the candidate feature family be

$$\mathcal{K}_D = \{k_1, k_2, \dots, k_m\}, \quad (511)$$

and let

$$S_D : \mathcal{K}_D \longrightarrow \mathcal{K}_D^{\text{obs}} \quad (512)$$

denote the selection procedure that identifies the structures to be operationalized.

The resulting set

$$\mathcal{K}_D^{\text{obs}} = \{k_{i_1}, \dots, k_{i_n}\} \quad (513)$$

depends upon the purpose of the inquiry, available evidence, conceptual framework, institutional constraints, and practical feasibility.

Selection can be legitimate while remaining partial.

A major epistemic risk arises when

$$\mathcal{K}_D^{\text{obs}} \quad (514)$$

is gradually treated as though it were identical to

$$\mathcal{K}_D. \quad (515)$$

The shift converts a pragmatic measurement boundary into an ontological boundary.

The distinction can be stated as

$$\text{Measured}(k) \not\leftrightarrow \text{FreedomRelevant}(k). \quad (516)$$

A structure can be freedom-relevant while remaining difficult to measure. Likewise, a highly measurable variable can provide only indirect information about freedom.

This is particularly important for revisability and transformability. Formal constitutional procedures may be straightforward to code, while the effective capacity of differently situated subjects to participate in revision can require more contextual evidence.

The choice of observable dimensions therefore carries a representational ethics. Excluded dimensions can systematically correspond to relations that are difficult for dominant institutions to observe.

A measurement framework should consequently record its exclusions as well as its inclusions:

$$\Pi_D^{\text{meas}} = \langle \mathcal{K}_D^{\text{obs}}, \mathcal{K}_D^{\text{excluded}}, \mathcal{K}_D^{\text{unknown}} \rangle. \quad (517)$$

The third category preserves uncertainty instead of silently converting it into absence.

17.4 Aggregation and Scalar Compression

After observable variables have been selected and measured, an institutional framework may aggregate them into a scalar or ordinal representation.

Let

$$A_D : \mathbb{R}^n \longrightarrow \mathbb{R} \quad (518)$$

be an aggregation rule, producing

$$F_D^{\text{rep}} = A_D(z_1, \dots, z_n). \quad (519)$$

The aggregation can involve weights,

$$F_D^{\text{rep}} = \sum_{k=1}^n w_k z_k, \quad (520)$$

or substantially more complex procedures.

The mathematical validity of the aggregation does not independently justify the conceptual commensurability of its components. Weighting requires an account of how dimensions that may represent different relational structures are permitted to compensate for one another.

Suppose

$$z_1 \uparrow, \quad z_2 \downarrow. \quad (521)$$

A compensatory scalar model may leave F_D^{rep} unchanged. The resulting equality

$$F_D^{\text{rep}}(A) = F_D^{\text{rep}}(B) \quad (522)$$

can therefore conceal different relational profiles.

More generally,

$$A_D(\mathbf{z}_A) = A_D(\mathbf{z}_B) \not\Rightarrow \mathbf{z}_A = \mathbf{z}_B. \quad (523)$$

Scalar compression introduces an additional many-to-one mapping after the earlier observational projection.

The total sequence can thus contain at least two forms of representational loss:

$$\mathfrak{W} \xrightarrow{P_D} \mathbf{z} \xrightarrow{A_D} F_D^{\text{rep}}. \quad (524)$$

Different relational worlds can generate the same indicator vector, and different indicator vectors can generate the same scalar.

Quantification scholarship has examined how commensuration allows heterogeneous objects to be compared through common metrics [24, 7]. For GRF, this capacity is useful precisely because it is selective. The resulting number should retain the declared scope of the transformation that produced it.

Scalar compression becomes problematic when

$$F_D^{\text{rep}} \longrightarrow \text{Freedom} \quad (525)$$

occurs without preserving the distinction between representation and object.

17.5 Institutional Representations of Freedom

Measurements of freedom are frequently produced within institutions. The institution selects the conceptual framework, data sources, coding procedures, aggregation rules, temporal horizon, and comparison units.

Let institution J produce

$$F_{J,D}^{\text{rep}} = A_{J,D} \circ M_{J,D} \circ P_{J,D}(\mathfrak{W}). \quad (526)$$

The institutional subscript matters because two organizations can produce different representations from the same relational system:

$$F_{J_1,D}^{\text{rep}} \neq F_{J_2,D}^{\text{rep}}. \quad (527)$$

The difference can arise from distinct normative purposes, observable dimensions, evidence standards, coding practices, or aggregation procedures.

Institutional origin does not invalidate measurement. Institutions can develop specialized expertise, stable procedures, and valuable longitudinal datasets. The relevant requirement is transparency concerning the transformation from relational world to institutional representation.

A representation should therefore ideally be accompanied by a provenance structure

$$\mathcal{P}rov(F_{J,D}^{\text{rep}}) = \langle P_{J,D}, M_{J,D}, A_{J,D}, \mathcal{E}, H, V \rangle, \quad (528)$$

where $\mathcal{E}$ denotes evidence, H temporal coverage, and V version information.

The provenance structure makes the representation revisable. New evidence or changes in conceptual assumptions can be traced to particular stages of the measurement process.

This is especially important for freedom because institutional classifications can acquire political authority. A numerical representation can travel beyond the methodological context in which it was originally constructed and become a general description of an entire society.

The scope relation should therefore remain visible:

$$F_{J,D}^{\text{rep}} \text{ valid for } D \not\Rightarrow F_{J,D}^{\text{rep}} \text{ exhaustive outside } D. \quad (529)$$

17.6 Political Claims and Symbolic Stabilization

Institutional representations can become political symbols.

A numerical score, category, or ranking can enter public discourse and acquire a semantic role beyond its original measurement protocol:

$$F_{J,D}^{\text{rep}} \longrightarrow \sigma_F \in L_{st}, \quad (530)$$

where σ_F denotes a symbolic representation of freedom within the situated language system.

For example, a classificatory statement of the form

“This society is free”

can function simultaneously as an empirical summary, political identity, normative claim, institutional classification, and legitimating symbol.

Once incorporated into L_{st} , the representation can contribute to interpretive stabilization:

$$F_{J,D}^{\text{rep}} \longrightarrow \sigma_F \longrightarrow \Delta L_{st}. \quad (531)$$

The measurement therefore participates in the hermeneutic field it describes.

A particular freedom vocabulary can become privileged because its operational dimensions have acquired institutional authority. Alternative relational structures that are weakly represented by the metric can consequently become less visible in political discourse.

This process does not require deliberate manipulation. Standardized classification itself can influence which distinctions become administratively and publicly salient.

The effect can be especially strong when an indicator is repeatedly cited without its domain declaration:

$$F_{J,D}^{\text{rep}} \longrightarrow F_J \longrightarrow \text{“freedom”}. \quad (532)$$

The successive removal of qualifiers produces semantic expansion beyond the original operationalization.

GRF therefore treats preservation of measurement context as part of responsible representation.

17.7 Reflexive Effects of Measurement

Measurement can alter the object measured. Espeland and Sauder describe *reactivity* as the process through which actors change behavior in response to evaluation and ranking [7]. This problem is especially relevant when metrics influence resources, reputation, institutional legitimacy, or policy.

Let

$$F_{J,D,t}^{\text{rep}} = \mathcal{M}_J(\mathfrak{M}_t) \quad (533)$$

denote a representation generated at time t .

Publication or institutional use of the representation can produce

$$F_{J,D,t}^{\text{rep}} \longrightarrow \Delta \mathcal{P}_t \longrightarrow \Delta \mathcal{R}_{t+1}, \quad (534)$$

where $\mathcal{P}_t$ denotes practices and $\mathcal{R}_{t+1}$ later relational organization.

The next measurement then observes a system partly transformed by prior measurement:

$$\mathfrak{M}_t \xrightarrow{\mathcal{M}_J} F_{J,D,t}^{\text{rep}} \longrightarrow \mathfrak{M}_{t+1} \xrightarrow{\mathcal{M}_J} F_{J,D,t+1}^{\text{rep}}. \quad (535)$$

The measurement system has entered the dynamics of the object.

Several forms of response are possible:

$$\begin{aligned} &\text{substantive institutional reform,} \\ &\text{strategic adaptation to measured dimensions,} \\ &\text{reclassification of existing practices,} \\ &\text{resource redistribution,} \\ &\text{symbolic conformity,} \\ &\text{critique and rejection of the metric.} \end{aligned} \quad (536)$$

These responses can have different implications for freedom.

An institution can improve conditions genuinely captured by the indicator. It can also optimize observable variables while leaving unmeasured relational structures unchanged.

Thus,

$$F_{J,D,t+1}^{\text{rep}} > F_{J,D,t}^{\text{rep}} \not\asymp \mathfrak{F}_{t+1}^{GR} \succ \mathfrak{F}_t^{GR}. \quad (537)$$

The comparison relation $\succ$ is intentionally unspecified because GRF is not globally scalar.

Reactivity therefore introduces a governance problem. Once an indicator becomes a target, the relations between measurement, behavior, and the underlying freedom structure require renewed examination.

17.8 Limits of Comparative Freedom Rankings

Comparative rankings require stronger assumptions than local measurement. They require enough common structure among cases for their representations to be placed within a shared order.

Suppose a ranking procedure compares systems

$$\{\mathfrak{M}_1, \dots, \mathfrak{M}_n\} \quad (538)$$

through a common representation

$$F_D^{\text{rep}}(\mathfrak{M}_i). \quad (539)$$

The procedure implicitly requires cross-relational comparability of the selected dimensions.

Section 16 showed that such comparability should itself be established through transformations and preserved structures. A common lexical label does not suffice.

For each pair of systems, the ranking therefore carries a hidden requirement:

$$I_j^D \circ T_{ij}^D = I_i^D \quad (540)$$

for the structures being compared, or an adequately justified weaker correspondence.

Without such compatibility, numerical equality can conceal semantic or relational non-equivalence.

Ranking introduces a further step:

$$F_D^{\text{rep}}(\mathfrak{M}_i) > F_D^{\text{rep}}(\mathfrak{M}_j). \quad (541)$$

This relation establishes ordering under the declared representation. It does not independently establish

$$\text{Freedom}(\mathfrak{W}_i) > \text{Freedom}(\mathfrak{W}_j) \quad (542)$$

under every meaningful interpretation of freedom.

Comparative rankings can nevertheless remain valuable. They can identify differences in declared institutional conditions, reveal temporal change, support accountability, and direct attention toward severe observable problems.

The relevant epistemic discipline is to preserve the qualification:

higher or lower under this declared representation of these specified freedom-related structures.

This qualification prevents a local comparative instrument from becoming a claim of global semantic authority.

Rankings should also preserve multidimensional profiles when aggregation would hide important divergence. If

$$\mathbf{z}_i = (z_{i1}, \dots, z_{im}), \quad (543)$$

then retaining $\mathbf{z}_i$ alongside the scalar F_D^{rep} can reveal cases in which similar total scores arise from substantially different relational structures.

The ranking problem is therefore a special case of the wider local-to-global problem developed earlier in the paper.

17.9 Representation without Exhaustion

The critique developed in this section does not require abandonment of measurement. A representation can be partial and still be informative.

The appropriate relation is

$$F_D^{\text{rep}} = \text{Rep}_D(\mathfrak{W}), \quad (544)$$

with the declared condition

$$F_D^{\text{rep}} \neq \mathfrak{W}. \quad (545)$$

The distinction permits a constructive measurement programme.

A responsible representation of freedom should seek at least five forms of explicitness:

$$\mathfrak{M}_D = \langle D, P_D, M_D, A_D, Prov \rangle, \quad (546)$$

where D declares the domain, P_D the observational projection, M_D the operationalization, A_D any aggregation rule, and $Prov$ the evidential and methodological provenance.

Where possible, the representation should also preserve its unmeasured and indeterminate margins:

$$(\mathcal{K}_D^{\text{obs}}, \mathcal{K}_D^{\text{excluded}}, \mathcal{K}_D^{\text{unknown}}). \quad (547)$$

Such disclosure makes revision possible when new relational structures become recognizable.

Representation can then be understood as one epistemic interface with a situated freedom system:

$$\mathfrak{W} \longrightarrow F_D^{\text{rep}} \longrightarrow K_J(\mathfrak{W}), \quad (548)$$

where $K_J(\mathfrak{W})$ denotes knowledge of the relational system produced for evaluator J .

The interface can improve knowledge substantially while remaining non-exhaustive.

This distinction also applies to qualitative representation. Narrative, ethnography, legal analysis, historical reconstruction, and philosophical interpretation select and organize relational structures as well.

Quantitative measurement is therefore not uniquely situated. Its distinctive power lies in standardization, aggregation, reproducibility, and portability across institutional contexts [24, 19].

The epistemic objective is consequently neither unrestricted quantification nor a rejection of quantitative representation. It is the preservation of the relation between a representation and the situated relational structure from which it was generated.

For Generative Relational Freedom, the complete measurement sequence can be represented as

$$\begin{aligned}
\mathfrak{W}_{st} &\xrightarrow{P_D} \mathcal{O}_D \\
&\xrightarrow{M_D} \mathbf{z}_D \\
&\xrightarrow{A_D} F_D^{\text{rep}} \\
&\longrightarrow L'_{st} \\
&\longrightarrow \mathfrak{W}_{s,t+1}.
\end{aligned} \tag{549}$$

The first three stages concern representation. The latter two acknowledge that the representation can enter discourse and participate in later relational change.

Freedom measurement is therefore itself a generative relational process.

The principal conclusion of this section is limited. Indicators, indices, and rankings can provide rigorous knowledge about declared freedom-related structures. Their epistemic status depends upon the projections, operationalizations, aggregation rules, cross-relational correspondences, and institutional contexts through which they are constructed. Increasing the quantity of data does not remove these prior interpretive operations.

Section 18 next returns to concrete illustrative domains. Linguistic grammar, religious discipline, symbolic authority, political institutions, material relations, and translation provide contrasting cases through which the distinctions among constraint, generativity, situated interpretation, and relational freedom can be examined without assuming that one visible form of constraint or one institutional indicator exhausts the freedom structure.

18 Comparative Illustrations

The preceding sections developed Generative Relational Freedom through a sequence of increasingly formal constructions: relational subjectivity, situated language systems, constraint architecture, interpretive geometry, symmetry breaking, local-to-global gluing, translation, cross-relational invariance, and measurement. The present section returns to several concrete domains in order to illustrate how these distinctions alter the analysis of freedom.

The examples are deliberately limited. They are not offered as comprehensive empirical studies of language, religion, political institutions, or historical change. Nor are they intended to rank traditions according to a common freedom scale. Their role is diagnostic. Each example isolates a different relation among constraint, interpretation, generativity, historical formation, and relational transformation.

The comparative method therefore proceeds through

$$\text{local configuration} \longrightarrow \text{declared analytical distinction} \longrightarrow \text{relational comparison}, \tag{550}$$

rather than through direct classification of whole societies or traditions.

18.1 Linguistic Grammar and Structured Generation

Linguistic grammar provides the simplest illustration of the distinction between unrestricted possibility and structured generativity. Generative grammar was developed precisely to characterize systems capable of generating structured linguistic expressions through a finite formal organization [6]. The present

paper does not transfer a particular linguistic theory into political philosophy. It uses the example to isolate a formal relation between constraint and generative capacity.

Let Σ denote a finite alphabet. The unrestricted set of finite strings is

$$\Sigma^*. \quad (551)$$

A grammar Γ determines a structured language

$$\mathcal{L}(\Gamma) \subseteq \Sigma^*. \quad (552)$$

The grammar therefore excludes combinations. The resulting reduction in formal string possibilities does not imply a corresponding reduction in meaningful linguistic generation.

The relevant distinction is

$$|\Sigma^*| > |\mathcal{L}(\Gamma)| \not\Rightarrow \text{Generativity}(\Sigma^*) > \text{Generativity}(\mathcal{L}(\Gamma)). \quad (553)$$

The comparison is schematic because the two notions of generativity require different specifications. Its purpose is to show that cardinality of formal possibilities and structured capacity for meaningful production are different objects.

The analogy becomes more useful when grammar itself can change. If

$$\Gamma_t \longrightarrow \Gamma_{t+1}, \quad (554)$$

then a generative system can support both production within an established rule structure and transformation of the structure through which later production occurs.

The corresponding GRF distinction is

$$\begin{aligned} &\text{generation within a relational organization,} \\ &\text{transformation of the relational organization.} \end{aligned} \quad (555)$$

This analogy should not be pushed further. Social constraints are not grammatical rules, political freedom is not linguistic well-formedness, and institutional authority does not acquire legitimacy because grammar enables meaningful expression.

The example establishes only a negative result:

$$\text{constraint} \not\Rightarrow \text{loss of generativity.} \quad (556)$$

The character of the constraint and the subject's relation to its formation, interpretation, revision, and transformation remain decisive.

18.2 Religious Discipline and Transcendent Constraint

Religious practice provides a more difficult illustration because discipline can be organized through transcendent authority, embodied practice, institutional relations, communal norms, interpersonal expectations, and self-regulation at the same time.

A comparison based only on the visible number of prescriptions would therefore be structurally incomplete.

Let two local religious configurations possess constraint profiles

$$\mathfrak{C}_A = \langle C_A^{\text{tr}}, C_A^{\text{sym}}, C_A^{\text{int}}, C_A^{\text{inst}}, C_A^{\text{self}} \rangle \quad (557)$$

and

$$\mathfrak{C}_B = \langle C_B^{\text{tr}}, C_B^{\text{sym}}, C_B^{\text{int}}, C_B^{\text{inst}}, C_B^{\text{self}} \rangle. \quad (558)$$

Suppose configuration A contains more explicit transcendent prescriptions:

$$C_A^{\text{tr}} > C_B^{\text{tr}}. \quad (559)$$

No ordering of GRF follows:

$$C_A^{\text{tr}} > C_B^{\text{tr}} \not\Rightarrow \mathfrak{F}_A^{\text{GR}} < \mathfrak{F}_B^{\text{GR}}. \quad (560)$$

The second configuration may distribute normative ordering through interpersonal expectation, symbolic status, institutional dependence, or self-regulatory discipline rather than explicit transcendent command.

This possibility should be understood as an analytical hypothesis concerning constraint architecture rather than an empirical law:

$$\Delta C^{\text{tr}} < 0 \quad \text{can coexist with} \quad \Delta (C^{\text{sym}} + C^{\text{int}} + C^{\text{self}}) > 0. \quad (561)$$

The terms in Equation 561 are not assumed to be numerically commensurable. The notation indicates possible directional redistribution.

Anthropological work provides important reasons for such caution. Asad's genealogical analysis challenges the treatment of religion as a historically invariant universal category and examines discipline, power, and religious formation within particular histories [2]. Mahmood's ethnographic study of the women's mosque movement in Cairo similarly questions an analysis of agency that treats resistance to norms as its privileged expression [14].

For GRF, the relevant comparison therefore concerns the subject's relation to discipline:

$$\mathfrak{D}_B = \langle \text{Interpret, Endorse, Revise, Exit, Transform} \rangle_B. \quad (562)$$

A demanding religious practice can be deeply integrated into the subject's own interpretive and ethical organization. Another practice with fewer explicit prescriptions can operate through social exclusion or material dependency that leaves little meaningful possibility of revision.

Neither structure can be diagnosed from rule count alone.

This does not make every religiously interpreted constraint compatible with freedom. Coercion, domination, irreversible closure, or destruction of effective exit remain possible inside religious systems just as they remain possible inside secular systems.

The narrower conclusion is that

$$\text{religious discipline} \neq \text{unfreedom}, \quad (563)$$

while

$$\text{absence of explicit religious prescription} \neq \text{freedom}. \quad (564)$$

Constraint architecture, rather than the religious or secular label itself, is the relevant GRF object.

18.3 Symbolic Authority and Relational Ordering

Constraint can operate through symbolic authority without appearing as a formal command. A subject may encounter expectations concerning respectability, professionalism, family role, success, citizenship, gender, status, or legitimate political conduct. Such expectations can organize trajectories by influencing which forms of action become socially recognizable and which appear unintelligible or costly.

Taylor's concept of the social imaginary provides a neighboring account of the broader understandings through which people imagine social existence, expectations, and normative order [30]. GRF gives this problem a more explicitly generative and relational formulation.

Let a symbolic configuration be

$$\mathcal{S}_{U,t} = \{\sigma_1, \sigma_2, \dots, \sigma_n\}. \quad (565)$$

A subject does not simply observe these symbols. Their relations can affect the interpretive trajectory set:

$$\mathcal{S}_{U,t} \longrightarrow \mathcal{T}_{B,U,t}^L. \quad (566)$$

Consider a simple case in which no explicit rule prevents trajectory τ :

$$\text{LegalProhibition}(\tau) = 0. \quad (567)$$

The trajectory may nevertheless be strongly disfavored by symbolic and interpersonal organization:

$$\tau \in \mathcal{T}_B^{\text{formal}} \quad \text{while} \quad \tau \notin \mathcal{G}_{B,t}^{D,H}. \quad (568)$$

Formal permission and relational freedom therefore diverge.

The reverse process is also possible. A new symbolic category can make a previously weakly articulated trajectory recognizable:

$$\Delta \mathcal{S}_{U,t} \longrightarrow \tau \in \mathcal{T}_B^{L_{t+1}}. \quad (569)$$

Symbolic ordering is consequently generative in both expansive and foreclosing directions.

This example is relevant to the broader freedom problem because highly visible formal constraints can coexist with low symbolic closure, while systems with few explicit restrictions can maintain strong conformity through distributed symbolic relations.

A comparative inquiry therefore requires examination of

$$C^{\text{formal}}, \quad C^{\text{sym}}, \quad C^{\text{int}}, \quad C^{\text{self}} \quad (570)$$

rather than identification of freedom with one component alone.

18.4 Political Institutions and Freedom Languages

Political institutions provide an illustration of how distinct freedom languages can become embedded in durable organizational forms.

A legal system can emphasize protection against interference. A republican institutional vocabulary can emphasize exposure to arbitrary power [4, 23]. A capability-oriented evaluation can emphasize substantive opportunities [28]. These objects are related, but they are not interchangeable.

Suppose three institutional descriptions focus respectively upon

$$I_1 = \text{NonInterference}, \quad I_2 = \text{NonDomination}, \quad I_3 = \text{Capability}. \quad (571)$$

A political system can perform differently under each description:

$$I_1(\mathcal{R}) \neq I_2(\mathcal{R}) \neq I_3(\mathcal{R}) \quad (572)$$

in the sense that the corresponding evaluations need not induce the same ordering.

GRF does not solve the disagreement by defining

$$F = w_1 I_1 + w_2 I_2 + w_3 I_3. \quad (573)$$

Such aggregation would require a prior justification of commensurability and weights.

Instead, the institution is treated as one contributor to a situated freedom language:

$$\mathcal{I}_t \longrightarrow L_t^F \longrightarrow \mathcal{P}_{t+1}. \quad (574)$$

Once embedded in law and governance, a freedom concept can organize later claims. Institutional language determines which grievances can be legally articulated, which rights are actionable, and which relational structures remain outside the institution's principal vocabulary.

Institutional reform can consequently alter both practical conditions and the interpretive space:

$$\Delta\mathcal{I}_t \longrightarrow (\Delta\mathcal{G}_{B,t+1}, \Delta L_{t+1}^F). \quad (575)$$

A new right can open an effective trajectory while also supplying a conceptual category through which subjects reinterpret prior relations.

Political freedom is therefore generated partly through recursive interaction between institutions and freedom languages.

18.5 Material Relations and Historical Freedom

Material relations illustrate why freedom meanings cannot be separated from historical conditions of practical life.

Historical materialism directs attention toward relations of production, property, labor, social organization, and practical activity [16, 18]. For GRF, their relevance operates at two levels.

First, material relations affect effective trajectory generation:

$$\Delta\mathcal{P}_t \longrightarrow \Delta\mathcal{G}_{B,t}^{D,H}. \quad (576)$$

Second, material transformation can alter the language through which freedom itself is articulated:

$$\Delta\mathcal{P}_t \longrightarrow \Delta L_{t+1}^F. \quad (577)$$

These two effects need not occur simultaneously or in the same direction.

Consider a technological or economic change that greatly enlarges the subject's immediate menu of transactions. The same transformation can create new dependencies on infrastructures or institutions over which the subject possesses little control:

$$\mathcal{G}_{B,t}^{\text{short}} \uparrow \quad \text{while} \quad \mathcal{T}r_{B,t}^{\text{long}} \downarrow. \quad (578)$$

The example is schematic. It illustrates the possibility that current capability and long-horizon transformability diverge.

Material relations can also generate new political categories. Historical changes in labor, property, communication, migration, or technology can produce relations for which inherited freedom vocabularies are initially poorly adapted.

In such a case,

$$\mathcal{R}_t \longrightarrow \Delta\mathcal{R}_t \longrightarrow \Delta L_t \quad (579)$$

expresses the emergence of interpretive pressure from changing practice.

The resulting concept of freedom is historical because the relational problem to which it responds is historical.

This does not reduce freedom language to material relations. Political thought, religious concepts, legal traditions, and symbolic organization can reshape material relations in return:

$$L_t \longrightarrow \Delta\mathcal{P}_{t+1}. \quad (580)$$

The relation is recursive rather than unidirectional.

18.6 Translation across Religious and Political Contexts

Translation becomes particularly difficult when a term crosses both linguistic and institutional boundaries.

A freedom expression originating in one political language can enter a religious language system, while a religious conception of obligation or liberation can enter legal and political discourse. The resulting interpretation is generated within the receiving relations rather than preserved automatically from the source.

Let

$$f_P \in L_P \quad (581)$$

denote a freedom interpretation within political language system L_P , and let

$$T_{PR} : L_P \dashrightarrow L_R \quad (582)$$

denote a partial transformation into religious language system L_R .

The translated interpretation

$$\tilde{f}_P^{(R)} = T_{PR}(f_P) \quad (583)$$

need not coincide with a locally generated religious conception

$$f_R \in L_R. \quad (584)$$

Thus,

$$\tilde{f}_P^{(R)} \not\equiv f_R. \quad (585)$$

The same problem appears when religious freedom becomes an object of state or international governance. Comparative scholarship on religious freedom has emphasized that the category itself has historically and politically variable meanings rather than operating everywhere as a self-evident universal object [2].

The GRF question is therefore not whether one side has translated the word correctly in isolation. It concerns the relational structures preserved, altered, or generated by the translation.

A useful comparison may examine

$$\mathcal{C}_D(f_P, f_R) = \langle \mathcal{K}^{\text{pres}}, \mathcal{K}^{\text{trans}}, \mathcal{K}^{\text{diff}}, \mathcal{K}^{\text{ind}} \rangle. \quad (586)$$

The framework therefore permits partial comparison while preserving semantic difference.

This is especially important for external judgment. A political institution can identify a relational structure that it regards as coercive or dominating without assuming that every locally meaningful religious distinction must first be translated into the institution's own complete vocabulary.

Conversely, invocation of local semantic difference does not automatically invalidate external criticism.

The relevant burden is to show which relational structure is being compared and whether the transformation used in that comparison preserves it.

18.7 Relational Reorganization and Semantic Change

The final illustration combines the preceding cases. Freedom meanings can change when relational organization changes, and the new meaning can subsequently participate in further relational reorganization.

Let a local system at time t be

$$\mathfrak{W}_t = \langle \mathcal{R}_t, L_t, \mathcal{I}_t, \mathcal{P}_t, \mathfrak{C}_t \rangle. \quad (587)$$

A disturbance or historical transformation produces

$$\mathfrak{W}_t \longrightarrow \mathfrak{W}_{t+1}. \quad (588)$$

The resulting freedom interpretation can also change:

$$f_t \longrightarrow f_{t+1}. \quad (589)$$

The stronger possibility is that the interpretive space itself changes:

$$\mathcal{X}_t^F \longrightarrow \mathcal{X}_{t+1}^F. \quad (590)$$

A relational transformation can therefore generate a distinction that the earlier system did not contain. The new interpretation then feeds back into practice:

$$f_{t+1} \longrightarrow \mathcal{P}_{t+1} \longrightarrow \mathcal{R}_{t+2}. \quad (591)$$

This recursive relation is consistent with both the historical-material and hermeneutic dimensions developed earlier. Material and institutional change can transform interpretation, while interpretation can enter practical activity and alter subsequent relations [16, 8, 30].

The process can also involve symmetry reconfiguration:

$$(\mathcal{X}_t^F, G_t^F, H_t^F) \longrightarrow (\mathcal{X}_{t+1}^F, G_{t+1}^F, H_{t+1}^F). \quad (592)$$

A historical transition may change which interpretations are available, which transformations connect them, and which structures remain stabilized.

Different local patches can undergo this process differently:

$$U_i : f_{i,t} \longrightarrow f_{i,t+1}, \quad U_j : f_{j,t} \longrightarrow f_{j,t+1}. \quad (593)$$

Their later meanings can therefore diverge, converge, hybridize, or remain partially translatable.

The comparative examples in this section consequently support a common diagnostic principle. Visible constraint, lexical correspondence, formal permission, institutional classification, and option count are each partial observables. None independently determines the relational freedom structure.

The relevant analysis proceeds through

$$\begin{aligned} \text{constraint architecture} &\longrightarrow \text{situated interpretation} \\ &\longrightarrow \text{effective generative trajectories} \\ &\longrightarrow \text{revision, exit, and transformation} \\ &\longrightarrow \text{historical relational change.} \end{aligned} \quad (594)$$

The examples also show why GRF should not be used to produce rapid cultural rankings. The framework increases the information required before a freedom judgment can be made. A religious discipline, a political institution, a symbolic norm, and a material dependency can occupy different locations in the constraint architecture while producing partially overlapping effects.

The comparative contribution of GRF therefore lies in making those architectures explicit before attempting normative or quantitative comparison.

Section 19 next subjects the framework itself to conceptual boundary tests. The analysis compares GRF with non-interference, non-domination, capability, autonomy, and emancipation and examines difficult cases involving voluntary discipline, severe constraint, residual agency, relativism, and the possible misuse of relational analysis to legitimate domination.

19 Conceptual Boundaries and Countercases

The preceding sections developed Generative Relational Freedom through relational subjectivity, situated hermeneutics, constraint architecture, interpretive geometry, symmetry breaking, local-to-global composition, translation, cross-relational invariance, measurement, and comparative illustration. The present section tests the boundaries of that construction.

Its purpose is diagnostic. A framework of freedom should remain distinguishable from neighboring concepts while surviving cases that place pressure on its central premises. The discussion therefore compares GRF with non-interference, non-domination, capability, autonomy, and emancipation, then examines voluntary discipline, severe constraint, residual agency, relativism, normative nihilism, and the possible misuse of relationality to legitimate domination.

These boundary tests are especially important because several claims developed earlier can be misunderstood in normatively dangerous ways. The propositions that subjects are relational, that constraint can participate in generativity, and that freedom meanings are historically situated do not imply that domination is unavoidable, that every constraint is justified, or that local interpretation places practices beyond criticism.

The section therefore preserves the distinction

$$\text{relational description} \longrightarrow \text{structural diagnosis} \longrightarrow \text{normative evaluation}, \quad (595)$$

without treating any transition as automatic.

19.1 Freedom and Non-Interference

Negative liberty provides one of the clearest neighboring concepts. In Berlin's influential formulation, one central sense of political liberty concerns the area within which a person can act without interference from others [4].

GRF accepts the importance of interference. Direct prevention, coercive intervention, censorship, confinement, and similar forms of obstruction can substantially reduce the trajectories available to a subject.

The relation

$$\text{Interference}(A, B, \tau) \longrightarrow \Delta \mathcal{G}_{B,t}^{D,H} \quad (596)$$

can therefore describe one important mechanism of freedom reduction.

GRF nevertheless distinguishes absence of interference from the wider relational structure of freedom.

A subject may experience little direct interference while possessing very limited effective generative capability:

$$\text{Interference}(B) \approx 0 \not\Rightarrow \left| \mathcal{G}_{B,t}^{D,H} \right| \text{ is extensive.} \quad (597)$$

The cardinality notation is illustrative only. GRF does not use set size as a complete freedom measure.

The subject may lack material conditions, interpretive resources, institutional access, effective exit, or the capacity to transform surrounding relations. An isolated subject can therefore be free from another agent's interference while remaining severely constrained in other dimensions.

The converse also matters. Some forms of interaction can expand effective generative trajectories. Education, care, infrastructure, coordination, translation, and institutional support all involve interaction.

Accordingly,

$$S_{\text{int}} \neq 0 \not\Rightarrow \text{Interference} \quad (598)$$

and

$$\text{NonInterference} \not\equiv \mathfrak{F}^{GR}. \quad (599)$$

Non-interference remains a potentially important component of a freedom analysis. GRF places it inside a broader relational architecture rather than treating it as an exhaustive description.

19.2 Freedom and Non-Domination

Republican freedom shifts attention from actual interference to the possession of arbitrary or uncontrolled power over another [23]. This move is especially close to GRF because both approaches recognize that observable intervention does not exhaust relational unfreedom.

A subject can remain vulnerable to another's power even when that power is not currently exercised. Let

$$\rho_{AB} \quad (600)$$

denote a relation through which A possesses significant capacity to alter the conditions of B 's action.

The relation can affect B 's generative organization through anticipation, dependency, self-censorship, strategic adaptation, or uncertainty even in periods of non-interference.

Thus,

$$\text{ActualInterference}_{AB} = 0 \not\Rightarrow \rho_{AB} = 0. \quad (601)$$

GRF shares this concern while introducing additional distinctions.

A relation can be non-dominating in the republican sense while remaining generatively restrictive because material, interpretive, or historical conditions limit trajectories without being reducible to another agent's arbitrary power.

Conversely, broad current capability does not remove domination:

$$\mathcal{G}_{B,t}^{D,H} \text{ extensive} \not\Rightarrow \text{NonDomination}(B). \quad (602)$$

A subject may enjoy many choices only at the discretion of another actor.

The GRF profile therefore keeps transformability, revisability, and exit distinct from current generative capability:

$$\mathfrak{F}_{B,t}^{GR} = \langle \mathcal{G}_{B,t}, \text{Rev}_{B,t}, \text{Ex}_{B,t}, \text{Tr}_{B,t} \rangle. \quad (603)$$

Non-domination can strongly affect several components without being identical to the entire profile.

This distinction should be read as conceptual extension rather than replacement. Republican theory identifies a specific relational pathology that GRF must preserve rather than dissolve into generic relational constraint.

19.3 Freedom and Capability

The capability approach provides another close neighboring framework because it directs attention toward substantive opportunities rather than formal entitlements alone [28].

The GR concept

$$\mathcal{G}_{B,t}^{D,H} \quad (604)$$

shares with capability-oriented analysis a concern with what a situated subject can effectively do rather than what is merely permitted.

The two concepts should nevertheless remain distinguished.

GR generative capability explicitly includes the capacity to generate and transform future trajectories, interpretive structures, and relational conditions. It therefore emphasizes temporal and recursive organization in addition to current substantive possibility.

For example,

$$\mathcal{G}_{B,t} \longrightarrow \Delta \mathcal{G}_{B,t+1} \quad (605)$$

represents a process in which present activity alters future generative capability.

GRF also separates current capability from the subject's ability to revise the relational conditions through which capability is generated:

$$\mathcal{G}_{B,t} \not\equiv \mathcal{T}r_{B,t}. \quad (606)$$

A subject can possess substantial substantive opportunities inside an institution while remaining excluded from meaningful participation in changing the institution itself.

The framework further retains the hermeneutic problem developed in earlier sections. A trajectory cannot function as an effective freedom possibility for a subject if the relevant interpretive system renders it unavailable for recognition, even when an external observer can list the trajectory formally.

Work on adaptive preferences demonstrates why preference and opportunity analysis may need attention to the conditions through which aspirations and evaluations are formed [21]. GRF extends this concern toward the revisability of the interpretive language itself.

Capability is therefore one important dimension of GRF while the broader framework additionally examines language, relation, historical generation, and transformation of the capability-producing structure.

19.4 Freedom and Autonomy

Autonomy concerns forms of self-direction, self-government, endorsement, authorship, and related capacities. Relational autonomy scholarship has shown why these capacities cannot be understood adequately through an isolated subject detached from social formation [13].

GRF shares the relational premise while retaining a broader object of inquiry.

A subject can exercise considerable autonomy over a specific decision while possessing limited effective exit or structural transformability. Conversely, a subject can participate in a relational structure with extensive collective transformability while accepting strong constraints on particular acts.

Thus,

$$\text{Autonomy}_D(B) \not\equiv \mathfrak{F}_{B,t}^{GR}. \quad (607)$$

The distinction is especially important for voluntarily adopted practices. Reflective endorsement can provide evidence relevant to autonomy while remaining insufficient to settle every GRF question.

An endorsed relation may still create long-horizon dependency or reduce effective exit:

$$\text{Endorse}_t(c) \not\equiv \text{Exit}_{t+\Delta}(c) \text{ remains viable.} \quad (608)$$

Conversely, the fact that a preference or commitment was socially formed does not independently invalidate it:

$$\text{SociallyFormed}(p) \not\equiv \text{Heteronomous}(p). \quad (609)$$

Under the relational premises of the paper, all meaningful commitments are formed within some history of interaction.

The stronger concern is whether the subject retains meaningful capacities for re-recognition, reinterpretation, revision, and engagement with alternatives.

GRF therefore treats autonomy as one locally important structure whose relation to generative freedom must be examined rather than presumed.

19.5 Freedom and Emancipation

Emancipation is temporally directional. It concerns movement from a problematic relational configuration toward another configuration.

The simplest representation is

$$\mathfrak{R}_t \longrightarrow \mathfrak{R}_{t+1}. \quad (610)$$

GRF resists identifying emancipation with the disappearance of relational structure:

$$S_{\text{int}} \longrightarrow 0. \quad (611)$$

The more plausible relational form is

$$S_{\text{int}}^{(1)} \longrightarrow S_{\text{int}}^{(2)}. \quad (612)$$

The normative status of the transition depends upon the properties of the resulting configuration.

Removal of a dominating relation can constitute a major emancipatory change while leaving material or interpretive conditions severely damaged. The subject can therefore be liberated from one relation without yet acquiring viable effective freedom.

Schematically,

$$\text{RemoveDomination} \not\Rightarrow \text{Restore}(\mathcal{G}, \mathcal{R}ev, \mathcal{E}x, \mathcal{T}r). \quad (613)$$

This distinction motivates the later GR inquiry into generative landscapes. A transition out of domination and the construction of conditions capable of supporting continued agency are related but analytically separable problems.

Emancipation can therefore be understood as one class of relational transition relevant to freedom rather than as a synonym for the complete GRF profile.

19.6 Voluntary Discipline and Chosen Constraint

Voluntary discipline is an important countercase to any account that treats constraint as direct evidence of unfreedom.

A subject can deliberately enter a practice involving substantial restrictions:

$$B \xrightarrow{\text{consent}} c. \quad (614)$$

Examples include demanding education, athletic training, artistic practice, religious discipline, professional commitments, collective projects, and voluntary institutional roles.

The existence of consent matters. It does not independently settle the relational analysis.

A chosen constraint may remain compatible with substantial GRF when

$$\left\{ \begin{array}{l} \text{Interpret}(c), \\ \text{Revise}(c), \\ \text{Exit}(c), \\ \text{Transform}(c) \end{array} \right\} \quad (615)$$

remain sufficiently available under the declared domain and horizon.

The constraint can even enable trajectories that would otherwise remain difficult to generate:

$$c \longrightarrow \mathcal{G}_{B,t+1}^{D,H} \text{ expands in some dimensions.} \quad (616)$$

This does not establish a general principle that discipline creates freedom. The same formal discipline can become foreclosing when exit, reinterpretation, or relational transformation progressively disappear.

Consent also has temporal structure:

$$\text{Consent}_{t_0}(c) \not\Rightarrow \text{Consent}_{t_1}(c). \quad (617)$$

A freedom analysis should therefore avoid treating an earlier act of acceptance as permanent authorization for every later state of the relation.

Likewise, the possibility of revision should not require continual reconsideration. A subject can stably maintain a commitment while retaining the capacity to revisit it.

The counterexample therefore supports the GRF distinction between structured discipline and relational foreclosure.

19.7 Severe Constraint and Residual Agency

The opposite counterexample concerns agency under severe constraint.

A subject can retain meaningful forms of agency even where the relational environment is highly restrictive. Resistance, reinterpretation, concealment, solidarity, imagination, or local choice can persist under domination.

Consequently,

$$\mathcal{C}_{B,t} \text{ severely restrictive} \not\Rightarrow \mathcal{G}_{B,t}^{D,H} = \emptyset. \quad (618)$$

The existence of residual agency, however, should not be used to deny unfreedom.

A subject's ability to resist oppression does not transform oppression into an acceptable relation:

$$\text{ResidualAgency}(B) \not\Rightarrow \text{NonDomination}(B). \quad (619)$$

Nor does successful survival establish adequate generative conditions:

$$\text{Survival}(B) \not\Rightarrow \mathfrak{S}_B^{GR} \text{ is adequate}. \quad (620)$$

This boundary is normatively important. The framework's emphasis on generativity should not romanticize the capacity of subjects to generate responses under harmful conditions.

Agency and structural freedom must therefore remain distinct analytical objects.

The same distinction applies retrospectively. Observing that individuals created valuable cultural, intellectual, spiritual, or political practices under oppressive conditions does not justify the conditions under which those practices emerged.

Generativity can survive injustice.

Its survival is not evidence that injustice was generatively beneficial.

19.8 Situated Interpretation and Cultural Relativism

The claim that freedom becomes meaningful within L_{st} raises an obvious objection. If meanings are historically and relationally situated, the theory may appear unable to criticize practices outside the evaluator's own language system.

The local-to-global and cross-relational analyses were developed partly to avoid this conclusion.

Situated interpretation establishes

$$f_i \not\equiv f_j \quad (621)$$

as a possible relation. It does not establish

$$\text{NoComparison}(f_i, f_j). \quad (622)$$

Sections 15 and 16 supplied intermediate structures:

$$T_{ij}^D, \quad \mathcal{C}_D(f_i, f_j), \quad \mathbf{I}_D. \quad (623)$$

These structures permit partial translation and comparison without requiring complete semantic identity.

Suppose a relational structure

$$I_D \quad (624)$$

captures a form of irreversible dependency or exclusion from revision, and the same structure survives a defensible transformation between two language systems:

$$I_j^D \circ T_{ij}^D = I_i^D. \quad (625)$$

The relational structure can then become a legitimate object of cross-contextual criticism even if the local freedom vocabularies remain semantically different.

The conclusion is therefore neither cultural absolutism nor isolation of local worlds. Cross-relational criticism requires more work than lexical classification. It requires translation, structural comparison, evidence, and explicit declaration of the relevant invariant.

GRF thus replaces immediate universalization with a revisable comparative procedure.

19.9 Historical Situatedness and Normative Nihilism

A second objection concerns normativity itself. If freedom meanings and even GRF's analytical categories are historically revisable, one might conclude that no normative judgment can retain validity.

Historical revisability does not entail this conclusion.

A judgment can be situated and still be supported by reasons, evidence, relational consequences, and comparison with alternatives. The absence of an atemporal standpoint does not eliminate the distinction between better and worse arguments within a historical inquiry.

Let a normative judgment at time t be

$$J_t = J(\mathcal{E}_t, L_t, D, \mathbf{I}_D), \quad (626)$$

where $\mathcal{E}_t$ denotes the relevant evidence.

Revisability means

$$J_t \longrightarrow J_{t+1} \quad (627)$$

when new evidence, conceptual distinctions, or relational encounters provide sufficient reason for revision.

It does not require

$$J_t = \text{arbitrary}. \quad (628)$$

Normative humility and normative paralysis are different positions.

GRF therefore preserves a provisional structure of judgment:

$$\text{evidence} + \text{declared values} + \text{relational diagnosis} + \text{revisability}. \quad (629)$$

The framework does not claim that these components produce one automatic moral answer. It claims that historical situatedness is compatible with disciplined normative reasoning.

This point is especially important because the framework itself must remain open to criticism. Reflexivity would be empty if revisability meant that every claim could simply be abandoned without reasons.

19.10 Relational Analysis and Legitimation of Domination

The most serious boundary problem concerns possible misuse of relationality.

The argument developed throughout the paper states that subjects are formed within relations and that constraint can sometimes participate in meaningful generativity. A dominating institution could misuse these propositions to claim that dependency, hierarchy, or coercive discipline is justified because all agency is relational.

That inference is invalid:

$$\text{Relational}(B) \not\Rightarrow \text{Justified}(\mathcal{R}_{AB}). \quad (630)$$

Likewise,

$$\text{GenerativeEffect}(c) \not\Rightarrow \text{Justified}(c). \quad (631)$$

A coercive institution can generate skills, identities, social coordination, or valuable outputs while simultaneously producing domination or serious harm.

Positive generation in one dimension does not erase closure in another.

The normative analysis must therefore retain disaggregated effects:

$$c \longrightarrow \{ \Delta\mathcal{G}, \Delta\mathcal{R}ev, \Delta\mathcal{E}x, \Delta\mathcal{T}r, \Delta H_{\text{harm}}, \Delta D_{\text{power}} \}, \quad (632)$$

where H_{harm} and D_{power} provisionally denote harm and domination-related structures.

A gain in one component does not automatically compensate for deterioration in another.

The relevant non-compensation principle is

$$\Delta\mathcal{G} > 0 \not\Rightarrow \Delta\mathfrak{F}^{GR} > 0. \quad (633)$$

Indeed, the paper has deliberately avoided defining $\Delta\mathfrak{F}^{GR}$ as a scalar quantity.

The framework also rejects retrospective legitimation. If an oppressive relation produced later knowledge, solidarity, resistance, or cultural creation, those outcomes do not retroactively justify the relation.

Formally,

$$\text{ValuableOutcome}(c) \not\Rightarrow \text{JustifiedCause}(c). \quad (634)$$

This boundary matters directly for the concept of generativity. Generativity is descriptive before it is normative. Systems can generate domination, harmful institutions, destructive technologies, and increasingly rigid relations.

A normative GR analysis must therefore ask what is generated, for whom, under which conditions, at whose cost, with what degree of revisability, and with what consequences for other subjects and later generative conditions.

The same discipline applies to claims of cultural situatedness. A harmful relation cannot be insulated from criticism merely by asserting that it is locally meaningful:

$$\text{LocallyMeaningful}(c) \not\Rightarrow \text{NormativelyPermissible}(c). \quad (635)$$

Local interpretation is evidence required for understanding the relation. It is not a normative veto against external inquiry.

The boundary analysis can therefore be summarized through the following non-identities:

$$\begin{aligned} \mathfrak{F}^{GR} &\neq \text{NonInterference}, \\ \mathfrak{F}^{GR} &\neq \text{NonDomination}, \\ \mathfrak{F}^{GR} &\neq \text{Capability}, \\ \mathfrak{F}^{GR} &\neq \text{Autonomy}, \\ \mathfrak{F}^{GR} &\neq \text{Emancipation}. \end{aligned} \quad (636)$$

These theories and concepts identify structures that can enter a GRF analysis. Their non-identity does not imply opposition.

The counterexamples also establish several further limits:

$$\begin{aligned}
&\text{chosen constraint} \not\Rightarrow \text{unfreedom}, \\
&\text{residual agency} \not\Rightarrow \text{adequate freedom}, \\
&\text{situated meaning} \not\Rightarrow \text{relativism}, \\
&\text{historical revisability} \not\Rightarrow \text{normative nihilism}, \\
&\text{relational constitution} \not\Rightarrow \text{legitimate domination}.
\end{aligned} \tag{637}$$

Together, these distinctions define the conceptual perimeter of Generative Relational Freedom. GRF studies freedom within already relational conditions, while preserving independent analysis of domination, capability, autonomy, effective exit, historical interpretation, and normative justification.

Section 20 next brings these results together. It evaluates the ontological and epistemic status of GRF, the role of non-zero interaction, the locality and possible non-gluability of freedom meanings, the explanatory limits of symmetry breaking, the prospects for cross-relational invariance, and the relation between the present foundational account and a subsequent theory of generative landscapes and effective freedom.

20 Discussion

The preceding sections developed Generative Relational Freedom through relational subjectivity, historically situated language systems, constraint architecture, interpretive geometry, symmetry and symmetry breaking, local-to-global composition, translation, cross-relational invariance, and representational measurement. The present section evaluates the resulting framework rather than introducing another major formal layer.

Several conclusions require particular clarification. First, GRF does not supply a context-independent substance called freedom. It provides an analytical framework for examining how freedom becomes intelligible and practically consequential within relational systems. Second, relational constraint is treated as constitutive of situated agency without being granted normative legitimacy merely by virtue of that constitutive role. Third, historical and hermeneutic locality does not eliminate cross-contextual comparison. Translation, partial equivalence, and cross-relational invariance provide increasingly demanding forms of comparison without requiring global semantic identity.

The strongest contribution of the framework therefore lies in changing the structure of the freedom problem. The central inquiry shifts away from locating a single quantity possessed by a subject and toward examining the relational conditions under which trajectories become recognizable, generable, revisable, exitable, and transformable.

20.1 Ontological Status of Generative Relational Freedom

GRF does not begin from a commitment to a relational ontology in which relation is declared the fundamental substance of reality. The term *relational* performs a more modest analytical role. It directs attention toward the dependence of subject formation, interpretation, action, and institutional possibility upon structured interactions.

The relational subject introduced earlier can therefore be represented through

$$\mathfrak{S}_{B,t} = \langle x_{B,t}, \Gamma_{B,t}, \mathcal{R}_{B,t}, H_{B,t} \rangle, \tag{638}$$

without requiring the claim that the subject has no reality outside this formal decomposition.

The same restraint applies to freedom. The provisional GRF profile

$$\mathfrak{F}_{B,t}^{GR}(D, H) = \langle \mathcal{G}_{B,t}^{D,H}, \mathcal{R}ev_{B,t}, \mathcal{E}x_{B,t}, \mathcal{T}r_{B,t} \rangle \tag{639}$$

is an analytical organization of freedom-relevant relations. It should not be read as an ontological decomposition of freedom into four fundamental substances.

This distinction permits the framework to remain compatible with several stronger philosophical positions. A realist, pragmatist, phenomenological, materialist, or process-oriented account may supply a deeper ontology while still making use of some GRF distinctions.

The framework therefore makes a weaker claim:

$$\text{GRF} \longrightarrow \text{relational analytical commitment}, \quad (640)$$

rather than requiring

$$\text{GRF} \longrightarrow \text{complete relational ontology}. \quad (641)$$

This restraint is also consistent with the paper's treatment of mathematical and physical formalisms. Manifolds, group actions, stabilizers, spin-foam locality, and sheaf-like constructions supply models for specified structural problems. Their usefulness does not establish that political or hermeneutic reality is literally constituted by the corresponding physical objects.

The ontological status of GRF is therefore deliberately open. Its present claim concerns the explanatory adequacy of relational description for the problem of situated freedom.

20.2 Freedom without a Zero-Interaction Baseline

One of the central departures of GRF concerns the baseline from which freedom is evaluated.

The analytical decomposition

$$S = S_0 + S_{\text{int}} \quad (642)$$

was introduced to distinguish free reference dynamics from interacting dynamics. Within the philosophical transfer, S_0 functions only as an analytical reference. It does not represent an authentic, pristine, pre-social, or morally preferable subject.

For embodied relational subjects,

$$S_{\text{int}} \neq 0 \quad (643)$$

is the practically relevant starting condition.

The consequence is substantial. Freedom cannot be defined through the gradual removal of every relation until a hypothetical isolated subject remains. Such a procedure would remove many of the conditions through which agency itself is generated.

The relevant transformation is instead

$$S_{\text{int}}^{(1)} \longrightarrow S_{\text{int}}^{(2)}. \quad (644)$$

The evaluative problem concerns what changes between the two relational configurations.

This perspective explains why constraint requires architectural analysis. Relations can coordinate, sustain, educate, protect, enable, discipline, exclude, dominate, or foreclose. The existence of interaction alone supplies insufficient information.

The framework therefore rejects both simple monotonic relations

$$\text{constraint} \uparrow \implies \text{freedom} \downarrow \quad (645)$$

and

$$\text{constraint} \uparrow \implies \text{freedom} \uparrow. \quad (646)$$

Neither direction follows generally.

The more informative questions concern which relations constrain, which subjects bear the constraint, how constraint is distributed, and whether the relational structure remains interpretable, revisable, extensible, and transformable.

This formulation also protects the framework against a romanticization of constraint. The fact that some constraints support generativity does not supply a general defense of discipline, hierarchy, or coercion. As established in Section 19.10,

$$\text{GenerativeEffect}(c) \not\Rightarrow \text{Justified}(c). \quad (647)$$

Relational constitution and normative legitimacy remain separate questions.

20.3 Hermeneutic Locality and Conceptual Stability

The spatiotemporal language system L_{st} supplies the framework's central hermeneutic object. Freedom becomes recognizable through historically formed relations among language, institutions, practices, symbols, norms, material conditions, and interpretive memory.

A situated freedom meaning can therefore be represented by

$$f_{st} = \llbracket w_F \rrbracket_{L_{st}}, \quad (648)$$

where the notation emphasizes dependence upon the relevant language system.

The framework does not infer from this locality that every freedom meaning is unstable. Historically generated concepts can become remarkably persistent. Legal systems, philosophical traditions, religious practices, education, and ordinary language can reproduce interpretive configurations across long periods.

The relevant distinction is between stability and context independence.

A meaning can remain stable over a substantial historical interval:

$$f_{s,t} \approx_L f_{s,t+\Delta}, \quad (649)$$

while its intelligibility continues to depend upon the relational system that sustains it.

Stability therefore does not require semantic timelessness.

This position helps reconcile two observations that are often treated as opposites. Freedom concepts possess enough continuity to support institutions, political argument, scholarship, and translation. They also undergo historical reinterpretation and can become differentiated across relational systems.

GRF treats both phenomena as outcomes of relational reproduction.

The framework consequently avoids identifying historical situatedness with semantic volatility. A relational language system can preserve a concept while remaining capable of revision.

The analytically important property is therefore generative revisability:

$$L_t \longrightarrow \left\{ L_{t+1}^{(1)}, L_{t+1}^{(2)}, \dots \right\}. \quad (650)$$

The possibility of revision does not require constant revision. It preserves the capacity for reinterpretation when history, experience, evidence, or cross-relational encounter makes revision necessary.

20.4 Historical Materiality and Interpretive Change

The hermeneutic account is inseparable from material and institutional history. The paper therefore incorporated insights from historical materialism without reducing semantic change to economic determination [16, 18].

A relational language system develops within material practice:

$$\mathcal{R}_t \longrightarrow L_t, \quad (651)$$

while language can subsequently participate in changing practice:

$$L_t \longrightarrow \mathcal{R}_{t+1}. \quad (652)$$

The combined relation is recursive.

This matters for freedom because historical change can modify more than the availability of actions. It can modify the categories through which actions are interpreted.

A transformation can therefore affect both

$$\mathcal{G}_{B,t} \quad (653)$$

and

$$L_{st}. \quad (654)$$

The first concerns practical generability. The second concerns interpretability.

This distinction suggests that historical liberation can occur at several levels. A legal reform can remove a prohibition. A material transformation can make an existing right practically usable. A new interpretive vocabulary can make a previously obscured form of domination recognizable. Institutional change can subsequently stabilize the new distinction.

No single level exhausts the process.

Dialectical transformation is useful here when understood through internally generated tension and historical reorganization rather than a fixed sequence of thesis, antithesis, and synthesis. A language system can contain competing interpretations whose practical conflicts contribute to later conceptual change.

The resulting process

$$(L_t, f_t) \longrightarrow (L_{t+1}, f_{t+1}) \quad (655)$$

does not guarantee progress.

Historical transformation can enlarge, redistribute, or contract generative conditions. The GR framework therefore contains no teleological law according to which later freedom meanings must be superior to earlier ones.

20.5 Explanatory Role of Relational Symmetry Breaking

Relational symmetry breaking supplies one of the most formally ambitious components of the paper. Its explanatory role should therefore be stated precisely.

When a state space, transformation group, group action, and situated configuration can be specified, the formal structure

$$G \longrightarrow H, \quad H = \text{Stab}_G(f^*) \subseteq G \quad (656)$$

describes the reduction from a broader transformation structure to the residual symmetry preserving the stabilized configuration [9, 27].

The formalism explains the *structure* of stabilization. It does not supply the historical cause of stabilization.

The causal or genealogical explanation remains located in relations such as

$$(\mathcal{P}, \mathcal{I}, \mathcal{R}, L, H) \longrightarrow f^*. \quad (657)$$

Institutional reproduction, political conflict, religious practice, material dependency, education, trans-lation, and symbolic authority can all participate in this process.

This division of labor is important. Group theory does not explain why a particular political meaning became dominant. It can clarify which transformations remain after that configuration has been specified.

Accordingly, three levels remain distinct:

1. historical stabilization;
2. structural symmetry analogy;
3. formally specified relational symmetry breaking.

The terminology should be weakened whenever the relevant group structure cannot be defended.

A second limitation concerns the state space itself. In many physical models, symmetry breaking occurs inside a theory whose state space and symmetry group are already specified. GRF permits historical transformation to alter the space and the group:

$$(\mathcal{X}_t^F, G_t^F, f_t) \longrightarrow (\mathcal{X}_{t+1}^F, G_{t+1}^F, f_{t+1}). \quad (658)$$

The analogy with physical symmetry breaking is therefore deliberately bounded.

Its value lies in distinguishing broader transformational possibility, situated selection, and residual invariance. It should not replace historical explanation with physical vocabulary.

20.6 Non-Gluability and Persistent Disagreement

The local-to-global analysis provides another important consequence. Locally coherent freedom interpretations do not necessarily determine a unique global interpretation.

For an assignment

$$U \longmapsto \mathcal{F}(U), \quad (659)$$

local sections

$$f_i \in \mathcal{F}(U_i) \quad (660)$$

can fail to satisfy the compatibility conditions required for global gluing.

The resulting possibility

$$\nexists f_{\text{global}} \in \mathcal{F}(U) \quad (661)$$

has a significant philosophical interpretation.

Persistent disagreement about freedom need not always arise because one side possesses an incorrect measurement of a shared semantic object. In some cases, the relevant local interpretations may possess different historical organization and fail to admit a unique global reconstruction.

This conclusion should remain limited.

Non-gluability is one possible structure of disagreement. Political conflict, strategic interest, misinformation, domination, conceptual confusion, and empirical error can also generate disagreement.

The framework therefore does not infer

$$\text{Disagreement} \implies \text{NonGluability}. \quad (662)$$

Nor does failure of global gluing imply complete semantic isolation.

Local sections can still possess overlaps, translations, and invariant structures.

The important result is therefore

$$\nexists f_{\text{global}} \not\Rightarrow \nexists \text{meaningful comparison}. \quad (663)$$

This relation is central to avoiding cultural relativism.

The sheaf-like formalism also remains conditional. If restriction maps cannot be coherently specified, the mathematical language should be weakened to a more general patchwise structure. If translation on overlaps requires non-invertible or path-dependent transformations, ordinary sheaf theory may be insufficient.

Future formalization may therefore require richer structures. The present paper establishes the local-to-global problem without claiming that a single mathematical category already captures every hermeneutic case.

20.7 Cross-Relational Invariance and Comparative Judgment

Cross-relational invariance provides the principal route from local hermeneutics toward comparative judgment.

For a transformation

$$T_{ij}^D : \mathcal{X}_i^F \dashrightarrow \mathcal{X}_j^F, \quad (664)$$

an invariant satisfies

$$I_j^D (T_{ij}^D(f_i)) = I_i^D(f_i). \quad (665)$$

The significance of this condition is methodological. It allows a relational structure to remain comparable even when the corresponding local meanings are semantically non-identical.

Thus,

$$I_i^D(f_i) = I_j^D(f_j) \not\Rightarrow f_i = f_j. \quad (666)$$

This distinction creates a middle position between universal semantic identity and radical incomparability.

Potential invariants might concern distributions of revision authority, effective exit structures, dependency relations, forms of unilateral closure, or other relational properties. Their validity must be demonstrated rather than presumed.

The framework therefore places a substantial burden upon comparative judgment. A cross-contextual claim should specify at least

$$\langle D, T_{ij}, I_D, \mathcal{E} \rangle, \quad (667)$$

where D is the comparison domain, T_{ij} the transformation, I_D the proposed invariant, and $\mathcal{E}$ the evidential basis.

This requirement is more demanding than applying one externally selected lexicon to every context. It is also more constructive than refusing cross-contextual judgment entirely.

The resulting comparative stance can support criticism of domination or closure without requiring semantic assimilation.

The possibility remains provisional. A candidate invariant that appears stable across several relational systems may fail under later comparison.

Cross-relational invariants therefore inherit the revisability of the wider GR framework.

20.8 Reflexivity of the Present Framework

GRF is itself formulated within a historical language system. Terms such as relation, generativity, freedom, agency, symmetry, translation, and revisability carry intellectual histories and are shaped by the theoretical resources available to the present inquiry.

The framework cannot consistently treat

$$L_{st}^{GR} \quad (668)$$

as an interpretively neutral metalanguage outside history.

The same hermeneutic analysis applied to other freedom theories must therefore apply reflexively to GRF.

This entails at least three forms of revisability.

First, the conceptual vocabulary may require revision:

$$L_t^{GR} \longrightarrow L_{t+1}^{GR}. \quad (669)$$

Second, the formal representation may require revision. A manifold approximation may fail for a discontinuous interpretive space; a group action may prove unavailable; a sheaf-like construction may require replacement by a richer structure.

Third, the normative interpretation may require revision when encounters with additional relational systems reveal overlooked harms, capabilities, or forms of agency.

Reflexivity does not require abandonment of the present concepts. A framework must retain enough stability to support cumulative reasoning.

The desired relation is

$$\text{stability} + \text{revisability}. \quad (670)$$

GRF should therefore be treated as a working theoretical architecture whose claims can be refined through historical research, comparative analysis, formal clarification, and counterexamples.

This position also limits the paper's novelty claim. Many individual components have substantial predecessors: relational autonomy, non-domination, capabilities, hermeneutics, historical materialism, dynamical-systems reasoning, symmetry, and sheaf-theoretic local-to-global analysis each arise from established traditions. The contribution lies in their particular organization around the problem of historically situated, relationally generated freedom.

The present paper should therefore be evaluated by the explanatory usefulness and coherence of that organization rather than by a claim to have replaced existing theories of freedom.

20.9 Relation to Generative Landscapes and Effective Freedom

The present paper deliberately stops before a full theory of generative landscapes.

GRF has introduced the set

$$\mathcal{G}_{B,t}^{D,H} \quad (671)$$

of effectively generable trajectories and distinguished it from merely formal, geometrically represented, or hermeneutically recognizable possibilities.

The present analysis, however, does not yet provide a sufficiently developed account of the landscape through which these trajectories evolve.

A subsequent paper can therefore begin from the distinction

$$\mathcal{T}_B^{D,H} \supseteq \mathcal{G}_{B,t}^{D,H} \quad (672)$$

and investigate the structure governing reachability within $\mathcal{G}_{B,t}^{D,H}$.

Such a theory may require consideration of local barriers, basins, bifurcations, critical regions, path dependence, attractor structure, transition costs, and changing accessibility across time.

The present paper provides several prerequisites for that development.

First, the landscape cannot be assumed to be the same for every subject:

$$\mathcal{L}_{B,t}^G \not\equiv \mathcal{L}_{C,t}^G. \quad (673)$$

Second, interpretive change can alter the effective landscape. A previously unrecognized trajectory can become available after transformation of L_{st} .

Third, the landscape itself can be historically dynamic:

$$\mathcal{L}_{B,t}^G \longrightarrow \mathcal{L}_{B,t+1}^G. \quad (674)$$

Fourth, freedom cannot be inferred from the number of points or paths in the landscape. Stability, revisability, returnability, exit, and long-horizon effects may all matter.

The subsequent inquiry can therefore address a more specific question:

Given a situated relational subject and a historically generated field of possibilities, which trajectories are effectively reachable, sustainable, revisable, and capable of altering the landscape through which later action occurs?

That question is narrower than the foundational problem addressed here and requires additional dynamical machinery.

Keeping the two papers separate prevents the present account from becoming overloaded and preserves a distinction between the conceptual conditions of freedom and the detailed dynamics of effective trajectory generation.

20.10 Directions for Further Development

Several directions follow from the limitations identified throughout the paper.

A first direction concerns empirical reconstruction of constraint architectures. The framework currently distinguishes material, embodied, transcendent, symbolic, interpersonal, institutional, and self-regulatory dimensions analytically. Comparative research would be required to determine how these dimensions interact in particular historical settings.

A second direction concerns cross-relational invariance. Candidate invariants should be tested across linguistic, religious, legal, institutional, and historical systems rather than introduced only through conceptual illustration.

A third direction concerns the mathematical structure of local hermeneutics. Ordinary sheaf theory supplies a useful benchmark, but path-dependent, non-invertible translation may require groupoids, stacks, categories, connections, or other structures capable of representing transformations between heterogeneous local systems.

A fourth direction concerns the geometry of interpretation. The present manifold and metric language is conditional. Future work should determine when smooth approximation is appropriate and when discrete, stratified, non-Hausdorff, graph-based, or other representations better capture the relevant structure.

A fifth direction concerns formal symmetry. Concrete applications should identify actual transformation classes and invariants before invoking symmetry breaking. This would allow a clearer separation between cases where $G \rightarrow H$ provides genuine formal structure and cases where symmetry remains only heuristic.

A sixth direction concerns measurement. GRF may support multidimensional representations preserving provenance, uncertainty, excluded dimensions, and cross-relational comparability without requiring a universal freedom index.

A seventh direction concerns normativity. The present paper establishes that generativity alone cannot justify a relation. A fuller Generative Relational Ethics would need to investigate the normative conditions under which generative capability, justice, non-domination, harm, reciprocity, and the generative conditions of other subjects should be evaluated together.

Finally, the relation between human and non-human generativity remains open. The formal vocabulary of relational conditions, response, trajectory, and generative closure may eventually support analysis beyond interpersonal human relations, but such extensions require independent epistemic and normative arguments.

These directions can be organized through three levels:

$$\begin{aligned} &\text{conceptual refinement} \longrightarrow \text{formal development} \\ &\hspace{15em} \longrightarrow \text{empirical and normative application.} \end{aligned} \tag{675}$$

The sequence is methodological rather than mandatory. Empirical countercases can force conceptual revision, while conceptual developments can alter what future empirical work should observe.

The discussion therefore returns to the paper’s central methodological commitment. Freedom should be investigated within the relational and historical conditions through which subjects, meanings, trajectories, and constraints are generated. The resulting framework remains local enough to preserve hermeneutic difference and structured enough to permit cross-relational comparison.

The remaining task is to state this result compactly. The concluding section therefore summarizes the conceptual architecture of Generative Relational Freedom, its principal limitations, and the transition from the present foundational inquiry toward subsequent work on effective generative dynamics.

21 Conclusion

This paper developed a preliminary framework of *Generative Relational Freedom* for examining freedom under conditions of relational constitution, historical situatedness, and interpretive plurality. Its starting point was that embodied subjects ordinarily exist within non-zero interaction. Relations to other persons, institutions, material environments, symbolic structures, histories, technologies, and inherited practices participate in the formation of both agency and the possibilities through which agency can be exercised.

This starting point changes the analytical baseline of freedom. An abstract zero-interaction condition can remain useful as a formal reference, but it cannot serve as the normative image of an authentic subject from which every relation is interpreted as a loss. Freedom must instead be investigated within already structured relational fields. The relevant transformations therefore concern changes in relational organization and their consequences for generative trajectories.

The paper represented this premise schematically through

$$S = S_0 + S_{\text{int}}, \quad S_{\text{int}} \neq 0, \quad (676)$$

while maintaining a strict distinction between the physical source formalism and its philosophical use. The analytical S_0 does not denote a pristine or pre-relational self.

From this premise, the paper developed a provisional freedom profile

$$\mathfrak{F}_{B,t}^{GR}(D, H) = \left\langle \mathcal{G}_{B,t}^{D,H}, \mathcal{R}ev_{B,t}, \mathcal{E}x_{B,t}, \mathcal{T}r_{B,t} \right\rangle_{L_{st}, \mathcal{R}_{B,t}, H_{B,t}}, \quad (677)$$

where the components concern effective generative trajectories, revisability, meaningful exit, and transformation of relational conditions. The profile is deliberately non-scalar and non-exhaustive. Its purpose is to separate several freedom-relevant structures that can vary independently.

This separation helps clarify the role of constraint. Constraint accompanies relational existence, but its mere presence does not determine the freedom status of a subject. What matters is its architecture: where constraint is generated, how it is transmitted, who bears it, who can interpret or revise it, whether exit is effectively available, and how the constraint reorganizes later generative capability.

The resulting analysis rejects a universal monotonic relation between the quantity of visible restriction and freedom. A reduction in explicit rules can coexist with increasing symbolic, interpersonal, institutional, material, or self-regulatory constraint. Conversely, structured discipline can participate in the generation of meaningful agency when the wider relational conditions remain sufficiently revisable and transformable.

This conclusion does not grant normative legitimacy to constraint merely because some generative effect can be identified. Generativity remains analytically distinct from justification. Harmful and dominating systems can also generate identities, knowledge, resistance, coordination, or cultural forms. The value of an outcome does not retroactively justify the conditions under which it emerged.

A second central claim concerned the hermeneutic status of freedom. Freedom meanings become intelligible within historically situated relational language systems L_{st} . Such systems include linguistic vocabulary together with institutions, practices, norms, symbols, histories, material relations, and other structures participating in interpretation.

Accordingly, lexical correspondence does not establish semantic identity. Expressions such as freedom and 自由 can be connected through ordinary translation while their situated meanings remain dependent upon different relational and historical configurations. The appropriate unit of analysis is therefore a specified L_{st} , rather than an undifferentiated natural language or civilizational category.

Translation nevertheless permits comparison. The paper distinguished lexical correspondence, partial translatability, semantic transformation, and cross-relational invariance. A translated interpretation can become a new configuration within the receiving system, and repeated translation can produce path-dependent semantic change.

The resulting comparative position can be summarized by

$$\text{semantic non-identity} \not\Rightarrow \text{incomparability}. \quad (678)$$

Comparison can proceed through explicitly specified transformations and preserved relational structures without requiring complete semantic identity.

The paper formalized this possibility through cross-relational invariance. For a declared transformation T_{ij}^D , a candidate invariant satisfies

$$I_j^D (T_{ij}^D(f_i)) = I_i^D(f_i). \quad (679)$$

The significance of this relation lies in its limited scope. An invariant is defined relative to a comparison domain and a transformation class. Equality of an invariant does not establish that two freedom meanings are identical, nor does it establish a universal moral ranking. It identifies a structure that survives a declared relational transformation.

The local character of interpretation also generated a local-to-global problem. Freedom interpretations can be associated with relational patches, restricted to overlaps, translated across neighboring systems, and tested for compatibility. When the relevant mathematical conditions are satisfied, a sheaf-like construction can represent the possibility of gluing local interpretations into a global section.

The existence of such a global section is not assumed. Local interpretations may remain coherent while failing to support a unique global reconstruction. This possibility offers one structural explanation for persistent disagreement about freedom, while leaving room for other causes such as political conflict, domination, misinformation, and empirical error.

The relevant conclusion is therefore

$$\nexists f_{\text{global}} \not\Rightarrow \nexists \text{ cross-relational comparison}. \quad (680)$$

Local comparison, translation, partial equivalence, and relational invariance can remain possible even where global semantic gluing fails.

A related formal contribution concerned relational symmetry breaking. Where an interpretive state space, transformation group, group action, and stabilized configuration can be defensibly specified, a situated freedom interpretation may preserve only a subgroup of a wider transformation structure:

$$G^F \longrightarrow H^F, \quad H^F = \text{Stab}_{G^F}(f^*). \quad (681)$$

The formalism describes the structural relation between broader transformational possibility and residual invariance. Historical explanation remains separate. Institutions, material relations, political struggles, religious practices, symbolic systems, and language histories participate in producing the stabilized configuration.

For this reason, the paper distinguishes formal symmetry breaking from a looser symmetry analogy. Where a transformation group or invariant cannot be constructed, weaker terminology should be retained.

The geometric and physical formalisms used throughout the paper are subject to the same restriction. State spaces, manifolds, tangent spaces, metrics, spin-foam locality, group actions, and sheaf-like structures are introduced only where they clarify a specific analytical problem. The paper does not claim that social or hermeneutic reality literally possesses the ontology of a physical theory.

This bounded use of formalism supports one of the broader methodological claims of the paper: mathematical structure can organize philosophical inquiry without requiring an ontological identification between the source theory and the philosophical domain.

The analysis of measurement produced a further consequence. Indicators and rankings arise through a sequence of interpretive selection, operationalization, and possible aggregation:

$$\mathfrak{W}_{st} \xrightarrow{P_D} \mathcal{O}_D \xrightarrow{M_D} \mathbf{z}_D \xrightarrow{A_D} F_D^{\text{rep}}. \quad (682)$$

A representation can be rigorous and useful while remaining partial. More precise measurement of selected variables cannot recover relational structures excluded by the preceding projection. Increasing the number of indicators therefore does not by itself establish convergence toward one context-independent quantity of freedom.

Measurement can also become reflexive. Once indicators enter institutions and public discourse, they can change practices and relational conditions. Freedom measurement is consequently part of the relational dynamics it seeks to describe.

Taken together, these arguments suggest a shift in the form of freedom analysis. The guiding question becomes less concerned with how much freedom a subject possesses in the abstract and more concerned with the structure of the relational field within which freedom becomes possible and intelligible.

Such inquiry asks which trajectories a situated subject can effectively generate, which possibilities can be recognized, how constraint is distributed, whether interpretive and institutional structures can be revised, whether meaningful exit remains possible, and whether subjects can participate in transforming the conditions under which future possibilities are produced.

The framework remains preliminary. Its formal structures require testing in concrete cases, its candidate cross-relational invariants require comparative investigation, and its normative implications require development beyond the descriptive concept of generativity. GRF itself remains historically situated and revisable.

The next theoretical problem concerns the dynamics lying underneath the effective generative set introduced in the present paper. Once freedom has been located within situated relational conditions, a further inquiry can ask how available trajectories are dynamically organized across time.

Such an inquiry would examine generative landscapes, barriers, basins, critical regions, path dependence, attractors, transition costs, and the capacity of trajectories to modify the landscape through which later action takes place. The relation between the present framework and that subsequent problem can be expressed schematically as

$$\begin{aligned} \text{relational conditions of freedom} &\longrightarrow \text{effective generative trajectories} \\ &\longrightarrow \text{dynamics of generative landscapes.} \end{aligned} \quad (683)$$

The present paper therefore closes with a deliberately limited proposition. Freedom can be investigated as a historically situated and relationally generated structure without reducing it to absence of constraint, subjective experience, option count, or a universal scalar. Local meanings can remain different while supporting translation and bounded comparison. Relational structures can constrain agency while also participating in its generation, and those structures remain open, under some conditions, to reinterpretation and transformation.

Generative Relational Freedom names this field of inquiry. Its central object is the continuing relation among subjects, relational conditions, interpretive systems, and the trajectories through which each can participate in the generation and revision of the others.

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