

Generativity and the Justice of Return: Theory, Practice, and an Atlas of Equivalence

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Abstract

This paper develops a comprehensive inquiry into the justice of return when harm, deprivation, extraction, or intervention has altered the conditions through which subjects and relational systems generate value, capacities, relations, and future possibilities. The inquiry begins from historically and relationally constituted subjects whose generativity continues to evolve during and after harm. Under such conditions, the return of a resource, status, opportunity, or formal right need not restore its prior generative significance. The paper therefore distinguishes the return of objects and resources from the restoration of relations, capacities, trajectories, conversion structures, and generative processes. It examines these problems across pre-harm prevention, intervention during ongoing harm, and post-harm restitution, compensation, repair, assistance, and future-directed support, while considering the respective roles and limits of law, governance, political economy, and assistance practices. The central analytical contribution is an atlas of restorative comparison. A claim that something has been restored is treated as an indexed comparative judgment requiring specification of a reference system, an object of comparison, a comparative relation, relevant observations or interventions, a temporal horizon, an approximation tolerance, and an epistemic basis. The paper surveys comparison objects including resources, attributes, relations, networks, states, trajectories, reachable sets, transition structures, attractors, basins, stability properties, perturbation responses, causal structures, conversion mechanisms, generative processes, meanings, and historical configurations. It then examines comparative relations including identity, isomorphism, homomorphism, simulation, bisimulation, observational and trace equivalence, topological and dynamical correspondences, statistical indistinguishability, causal and interventional equivalence, approximation, dominance, and refinement. Particular attention is given to dynamical and generative comparison, including the non-equivalence of delayed return, robustness under admissible perturbations, and changes in the processes through which future possibilities are generated. The paper further connects these comparative structures to theories of value and justice, including resource, capability, relational, recognition-based, rights-based, care-oriented, reparative, and generative approaches. Substantive justice in the achieved condition is distinguished from procedural justice in the means through which a proposed equivalence is produced. Idealized technologies and hypothetical powers are used to separate current practical limitations from epistemic, structural, and normative limits. The resulting framework does not identify a universally sufficient form of return. It instead maps the choices concealed within claims of restitution, compensation, repair, and restoration, and provides a common structure for asking what should be compared, in what sense equivalence matters, which values warrant preservation, and which processes of return can themselves be regarded as just.

Keywords: generativity; justice of return; restorative comparison; equivalence; restitution; reparation; compensation; generative justice; capability approach; relational justice; dynamical systems; bisimulation; structural equivalence; procedural justice; assistance; irreversibility

Discussion Paper Note

This paper is a conceptual, philosophical, formal, and practice-oriented inquiry into generativity and the justice of return. Its central concern is broader than compensation for a particular class of harms. The inquiry begins from a recurring problem across interpersonal relations, employment, education, research, health, institutional life, political economy, historical injustice, and public governance: a subject or relational system can lose not only an identifiable object, resource, entitlement, or opportunity, but also conditions through which subsequent value, relations, capacities, knowledge, and possibilities would otherwise have been generated.

The distinction is consequential because generative conditions participate in processes that continue through time. A research position can generate publications, collaborations, reputation, new questions, institutional access, and subsequent positions. Health can condition work, relationships, mobility, learning, and participation. Social recognition can influence entry into later relations. A productive resource can generate outputs that become resources for further production. A particular encounter can redirect an intellectual, professional, or personal trajectory. Interruption of such conditions can therefore affect more than the object initially removed. It can alter the subsequent processes through which new conditions and values would have emerged.

The paper uses *generativity* in a deliberately broad sense. Generativity concerns the capacity of a subject, relation, institution, community, or other system to produce, sustain, transform, or open further states, relations, capacities, values, meanings, and possibilities. The term does not imply that every form of generation is normatively desirable. Destructive, exploitative, dominating, and exclusionary systems can also reproduce themselves. The normative status of a generative process therefore remains distinct from the fact that the process is generative.

The expression *justice of return* is similarly broader than restitution in the narrow sense of returning an identifiable object. Return can concern resources, status, access, capabilities, social relations, control over conditions of production, opportunities, institutional positions, value flows, future possibilities, or support for renewed generativity. Depending upon the context, return can take the form of restitution, compensation, reparation, rehabilitation, recognition, institutional correction, assistance, renewed access, redistribution, or the creation of alternative enabling conditions. The paper does not assume in advance that these forms are mutually reducible or that they can be represented by a common scalar measure.

This inquiry is motivated in part by the limits of a simple reversible model of harm. In such a model, a condition is removed and subsequently returned, and the two operations are implicitly treated as approximate inverses. This model can be useful for some objects. It becomes increasingly inadequate when the affected subject and surrounding relational system continue to develop during the period of deprivation. A position restored after several years is not necessarily the same generative condition for the historically changed subject. A correction of a false reputation does not recreate all relations that failed to form while the false belief circulated. Restoration of equipment does not produce the research that was never conducted, the collaborators who were never met, or the subsequent opportunities that would have depended upon those events.

The paper therefore distinguishes the return of a condition from the return of its generative significance. The same nominal object, resource, institutional status, or formal entitlement can occupy a different place within a changed relational and historical system. This distinction is especially important for subjects understood as historically and relationally constituted. Harm can change the subject who later receives the returned condition, the relations within which the condition is used, the meaning associated with the condition, and the processes through which the condition can be converted into subsequent generativity.

The resulting problem should not be reduced to the familiar observation that some harms are difficult to compensate. Irreparable harm, lost opportunities, counterfactual uncertainty, incommensurable values, relational injury, and the limitations of monetary compensation are longstanding subjects of legal and philosophical inquiry. The present paper takes these problems as important starting points rather than as novel discoveries. Its principal contribution lies elsewhere: it asks what a claim of return or restoration

means once the object and standard of comparison are made explicit.

A statement that a subject or system has been “restored” is incomplete unless it specifies what has been compared. Possible objects of comparison include identifiable entities, resources, attributes, relations, network positions, institutional statuses, present states, probability distributions, trajectories, sets of possible traces, reachable future states, transition structures, dynamical rules, attractors, basins of attraction, stability properties, responses to perturbation, causal structures, conversion mechanisms, generative processes, meaning structures, and historical configurations. Different choices can produce substantially different judgments concerning whether return has occurred.

The comparative relation must also be specified. Exact identity is only one possibility and is often neither attainable nor normatively necessary. Depending upon the object under consideration, comparison can involve isomorphism, homomorphism, embedding, simulation, mutual simulation, bisimulation, trace equivalence, observational equivalence, behavioral equivalence, topological correspondence, conjugacy, distributional equivalence, statistical indistinguishability, causal or interventional equivalence, metric proximity, dominance, refinement, or other forms of approximate or order-based comparison. Not all of these are equivalence relations in the strict mathematical sense. The paper therefore uses the broader expression *comparative relations of restoration* when discussing the full family of possible comparisons.

The resulting framework treats restorative judgment as indexed rather than binary. Whether a restoration claim is defensible depends upon a reference system, an object of comparison, a comparative relation, a relevant temporal horizon, a set of admissible observations or interventions, an approximation standard where exact correspondence is unnecessary, and an epistemic basis for making the comparison. A person can consequently be restored in one sense while remaining substantially unrestored in another. Equal wages can coexist with altered professional relations. Formal reinstatement can coexist with different future opportunities. Similar observable performance can coexist with different responses to stress. Similar current states can coexist with different attractor structures or reachable futures.

This distinction motivates the *atlas of equivalence* developed in the paper. The atlas is not a ranking in which every stronger mathematical relation automatically represents a more just form of return. It is a map of comparison possibilities. Some standards are stronger with respect to particular structures while remaining incomparable with others. Structural isomorphism need not imply observational equivalence under every environment. Similar outputs need not imply similar causal mechanisms. Identical resource holdings need not generate identical capabilities. Similar capability sets need not imply similar processes of value generation. Comparable long-run distributions can conceal different individual histories. The relevant space is therefore better understood as a partially ordered and multidimensional family of restorative criteria than as a single hierarchy.

Dynamical comparison receives particular attention. For a generative system, the present state is only one possible object of concern. Justice can also involve the transitions available from that state, the future regions that are reachable, the stability of desirable modes of activity, the structure of attractors and their basins, the system’s resilience under disturbance, and the way interventions are converted into subsequent capacities and relations. A restored subject can appear similar under ordinary conditions while responding very differently to later institutional stress, uncertainty, conflict, or resource change. Comparison under perturbation can therefore reveal differences that ordinary observational comparison conceals.

The paper also distinguishes observation from intervention. Two systems can be observationally similar while responding differently to the same policy, resource transfer, treatment, institutional opportunity, or social support. This possibility is particularly important for assistance. A monetary payment can provide a highly transferable resource without ensuring that the recipient can convert that resource into the same relations, capabilities, or future possibilities that would have been available under another history. The conversion process itself can therefore become an object of restorative comparison.

The use of money and other general equivalents is nevertheless treated seriously rather than dismissed as intrinsically inadequate. General equivalents can perform an important institutional function precisely because public authorities frequently lack sufficient knowledge to determine the future a subject should

pursue. Transferable resources can preserve a domain of subject-directed choice and allow a historically changed person to allocate resources toward rest, relocation, education, care, new work, family life, research, or other priorities. Fungibility can therefore express a form of institutional epistemic restraint. Its limits arise when the relevant generative conditions are non-fungible, relationally singular, historically time-sensitive, or unavailable for purchase.

The loss of a particular collaboration illustrates this difficulty. A different collaborator with similar credentials need not constitute an equivalent generative relation. The significance of a relation can depend upon the particular persons involved, their questions, the surrounding intellectual environment, the timing of the encounter, and subsequent contingencies. Similarly, a publication opportunity that disappears during a period of exclusion cannot necessarily be recreated by publishing another work years later. The later work enters another literature, another network, and another historical situation. The paper therefore distinguishes newly created generative opportunities from equivalent restoration of lost opportunities.

The inquiry is organized around the temporal structure of harm. Pre-harm justice concerns the protection of conditions whose destruction can generate large, difficult, or irreversible downstream effects. During an ongoing harm, the central problem often becomes interruption: stopping a negative generative cycle before deprivation of one condition produces additional losses of relations, capacities, reputation, access, confidence, health, or future opportunity. Post-harm justice confronts the most difficult comparative questions because the relevant subject and system may already have changed.

This temporal organization also clarifies the different roles of law, governance, political economy, organizational practice, and assistance. Law can be comparatively effective at establishing prohibitions, rights, procedural requirements, responsibility, restitution of identifiable objects, formal status correction, injunctions, and monetary compensation. Governance can address monitoring, institutional safeguards, early intervention, risk distribution, evidence preservation, and procedures for contesting harmful decisions. Political economy can organize transfers, insurance, public infrastructure, redistribution, and collective absorption of risks that cannot reasonably be assigned to a single actor. Assistance can become increasingly important after the original generative trajectory can no longer be reconstructed.

The paper does not assume that law ought to absorb all of these functions. Adjudication requires closure, administrability, evidentiary thresholds, bounded responsibility, and decisions within finite institutional procedures. A generative relational system can remain open long after a judgment is issued. Later consequences can emerge through relations that were not foreseeable at the time of adjudication. The inability of legal institutions to represent the entire generative history of a subject is therefore not treated simply as a defect that sufficiently advanced doctrine must eventually eliminate. Some compression is constitutive of adjudication itself.

At the same time, the paper does not assume that present limitations establish permanent limits. Improvements in longitudinal data, causal inference, simulation, computational social science, institutional records, biomedical knowledge, and other technologies can enlarge the range of harms and consequences that can be identified. Future institutions may become able to model aspects of generative change that are currently inaccessible. The paper therefore distinguishes current practical limitations from stronger claims about epistemic, mathematical, structural, or normative impossibility.

Idealized technologies and deliberately unrealistic thought experiments are used for this purpose. Some discussions assume unusually complete information, extremely powerful simulation, perfect medical intervention, reconstruction of social networks, manipulation of memory, or other capacities beyond current technical practice. In some cases the analysis goes further and grants a hypothetical actor powers approaching unrestricted physical intervention. These cases are not predictions about future technology. They function as analytical devices for separating limitations caused by present resources from limitations that remain once practical constraints are removed.

Such idealization exposes several distinct problems. One is epistemic: the counterfactual history in which the harm did not occur was never observed. Another is structural: social and generative dynamics need not admit a unique inverse. Another is reflexive: observation, documentation, judgment, and repair can themselves become new objects and events within the relational system. Another is normative: even

if a counterfactual state could be reproduced with great fidelity, reproducing it might require interventions that violate consent, agency, dignity, privacy, historical continuity, or the subject's present preferences.

The last problem motivates a strict distinction between substantive or outcome justice and procedural justice. A proposed restoration can be evaluated by the degree to which the resulting system satisfies a chosen comparative standard. The process through which that condition is produced requires an independent normative evaluation. Stronger equivalence is not automatically more just. Deleting memories, manipulating third parties, compelling treatment, restricting future choices, or reconstructing relations without consent might increase similarity to a selected counterfactual state while simultaneously creating severe procedural injustice.

The subject's right to reject restoration therefore becomes an important problem. A historically changed person may reasonably prefer a new trajectory to the trajectory that an institution regards as the best approximation of the life that would otherwise have occurred. Return cannot consequently be reduced to optimization of counterfactual fidelity. The present subject is not merely an imperfect copy of a hypothetical uninjured subject. The present subject is also a bearer of agency whose current judgments enter the justice of any proposed intervention.

This problem is especially visible when the pre-harm state was itself unjust. A framework that treats the previous state as the automatic reference point can reproduce prior domination, exclusion, dependency, or inequality. The paper therefore distinguishes the pre-harm actual state, the no-harm counterfactual state, a statistically expected state, a normatively preferred state, a minimum sufficient state, a best feasible state, and a subject-selected future as different possible reference systems. Choosing among them is itself a normative act.

Value theory consequently occupies a central place in the atlas. A comparison cannot determine justice until it is clear which features are considered valuable and why. Different theories can prioritize welfare, resources, capabilities, opportunities, rights, recognition, autonomy, relationships, equality, non-domination, care, control over productive conditions, generativity, historical continuity, or other values. Some values can be represented approximately by scalar quantities. Others require vectors, partial orders, thresholds, lexical priorities, or qualitative judgments. Still others may resist stable commensuration.

The paper therefore places utilitarian, resource-oriented, capability, relational, recognition-based, rights-based, non-domination, care-oriented, restorative, reparative, and generative approaches into dialogue with the comparative framework. The purpose is not to reduce each tradition to a mathematical relation. Rather, the framework is used to identify which objects of comparison each theory tends to foreground, which differences it treats as normatively salient, and which forms of equivalence, sufficiency, dominance, or non-equivalence are compatible with its account of justice.

Generative justice is especially important to the inquiry because it directs attention from completed products toward the conditions and circulations through which value continues to be generated. The present paper takes this orientation seriously while asking a subsequent question. If extraction, deprivation, exclusion, or domination has already altered the generator, the relations surrounding the generator, or the mechanisms through which returned value would become productive, the meaning of "return" becomes underdetermined. Returning value to a changed generative system need not reproduce the generative process that would have existed without the earlier interruption.

The paper consequently distinguishes justice in the circulation of generated value from justice after alteration of generativity. These questions overlap but are not identical. A system can establish a formally circular return of resources while failing to restore relevant control, relations, conversion capacity, resilience, or future reachability. Conversely, a just response to past alteration might require creating new conditions rather than reproducing a former circulation.

This distinction leads to a broader account of post-harm assistance. Assistance is not treated as a residual category used only when compensation fails. It addresses a different normative question: given a subject whose history cannot be undone, what resources, relations, institutional protections, time, recognition, care, opportunities, and decision rights can support the development of a viable future? Assistance can therefore be future-directed without pretending that new possibilities are equivalent substitutes for what was lost.

The paper remains cautious about any theory that claims to know the future a subject should generate. Subject-directed regeneration is accordingly treated as an important constraint. Institutions can provide money, access, education, care, safe environments, network opportunities, legal correction, and other conditions, but these supports do not automatically justify institutional control over the trajectory that follows. A theory of generative assistance must remain attentive to the possibility that attempts to restore generativity can themselves become new relations of domination.

The analysis also considers the distribution of burdens created by return. Restoration of one subject can require resources, institutional changes, or alterations in relationships that affect other subjects. Historical reconstruction can impose burdens upon innocent third parties. Reputational correction can conflict with speech, privacy, or archival interests. Reinstatement can alter opportunities available to others. Public compensation can distribute costs across people who did not cause the original harm. Generative return therefore cannot be evaluated exclusively through a dyadic relation between an injurer and an injured party.

Collective and intergenerational cases make this problem more pronounced. Communities, institutions, cultures, and ecological systems contain multiple overlapping generative processes whose boundaries are contestable. A restorative intervention can increase the generativity of one subsystem while reducing that of another. The paper treats these problems as extensions of the comparative framework rather than assuming that a criterion adequate for a single individual can be transferred unchanged to collective systems.

The atlas is intended to support rather than replace practical judgment. Different contexts can justify different comparison objects and different standards of adequacy. Returning a fungible commodity, correcting a legal record, rebuilding a professional network, supporting recovery from prolonged institutional abuse, responding to historical dispossession, and restoring an ecological system need not share one criterion of equivalence. A plural framework is therefore not a temporary concession to incomplete theory. It may reflect the heterogeneous structure of the objects and values involved.

The paper nevertheless seeks systematic relations among these standards. Questions of refinement, implication, incompatibility, partial ordering, and common comparison are examined where possible. Some restoration criteria may imply others under specified assumptions. Some may be mutually incomparable. Some may become jointly satisfiable only under strong conditions. Some may be mathematically coherent while remaining normatively irrelevant. Others may be normatively compelling while currently resistant to measurement. The atlas makes these distinctions explicit.

The resulting project is intentionally expansive. It is closer to a problem-space map and theoretical atlas than to a narrowly bounded model with a single theorem or policy prescription. The length of the inquiry is not restricted by the conventions of a standard journal article. Topics are developed to the degree required to distinguish their conceptual structures, relations, practical implications, and unresolved problems. Extensive classification is therefore a methodological feature of the paper rather than an attempt to imply that every entry can presently be operationalized.

The formal components of the paper should be read accordingly. Mathematical language is used where it clarifies distinctions among states, relations, trajectories, mappings, dynamics, interventions, robustness, and comparative standards. Formalization is not treated as an exhaustive representation of the social world. A social description can itself become part of the social system it describes. Records alter decisions, classifications alter treatment, predictions alter behavior, and restorative judgments alter subsequent relations. The paper therefore rejects the assumption that a sufficiently large symbolic representation would necessarily stand outside the generative system as a complete and inert copy.

This reflexivity creates a further limit on claims of complete restoration. Documentation of harm becomes part of history. Correction of the documentation becomes another event. Knowledge that restoration occurred can affect how subjects interpret the restored condition. Attempts to erase such knowledge raise additional questions of identity and procedural justice. The world in which harm occurred and was subsequently repaired is therefore generally not identical to the world in which the harm never occurred.

The paper does not infer from these difficulties that justice is futile. Limited reparability can instead strengthen the importance of prevention, timely intervention, institutional humility, and future-directed

assistance. Where a class of harm can alter highly singular or weakly replaceable generative conditions, reliance upon later compensation becomes less satisfactory. The degree of irreversibility can therefore become relevant to the design of ex ante safeguards and the urgency of intervention during an ongoing harmful process.

The resulting orientation can be summarized through three connected claims. First, restoration requires a theory of comparison. A claim that something has been returned must specify the object, reference, and relation through which the return is evaluated. Second, a theory of comparison requires a theory of value. Mathematical or empirical similarity cannot establish which similarities matter for justice. Third, realization of a selected restorative standard requires a theory of just process. A desirable endpoint does not independently justify the means used to produce it.

These claims do not yield a universal algorithm for return. They provide a framework within which competing restorative claims can be stated more precisely, compared more transparently, and subjected to both substantive and procedural evaluation. The intended contribution is therefore neither a complete solution to historical irreversibility nor a universal measure of generative loss. It is a systematic decomposition of the conceptual, mathematical, normative, institutional, and practical choices concealed within ordinary assertions that a loss has been compensated, a condition has been restored, a harm has been repaired, or generativity has been returned.

The paper remains open about several questions that cannot presently be settled. It does not assume that every relevant generative process can be observed, represented, or reconstructed. It does not assume that stronger technological capacity will eliminate counterfactual uncertainty. It does not assume that different theories of value can be reduced to a common metric. It does not assume that the strongest available equivalence relation is the most just restorative target. It does not assume that assistance can replace what has been irreversibly lost. It also does not assume that the present atlas is exhaustive.

The inquiry instead treats these limitations as part of the subject matter. The justice of return begins precisely where the apparent simplicity of “giving back” breaks down. Once value, capacity, meaning, relation, and possibility are understood as generated through time, return becomes a problem of comparison across changing systems. The purpose of this paper is to expose that problem space in sufficient detail that legal theory, political economy, governance, assistance theory, philosophy, and formal analysis can engage with the same underlying questions without prematurely reducing them to a single notion of equivalence or repair.

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1 Introduction

This section establishes the problem structure, conceptual motivation, principal claims, analytical strategy, and scope of the paper. The inquiry begins from a general difficulty shared by restitution, compensation, reparation, rehabilitation, restorative practice, public assistance, and other responses to harm: a condition that is returned at a later time may enter a subject and a relational system that have already changed. The central objective is to examine what *return* can mean under these circumstances and to identify the comparative, normative, epistemic, and procedural commitments required by a claim that generativity has been restored. The section first introduces the relation among generative conditions, historical change, and return. It then locates the problem in relation to capability-oriented, restorative, and generative approaches to justice, develops the temporal distinction among prevention, interruption, and post-harm response, introduces the comparative problem that motivates the atlas developed later in the paper, and concludes by stating the paper's contributions and organization.

1.1 Generative Conditions and the Problem of Return

Many familiar harms can be described through the loss of an identifiable object. Property can be taken, income withheld, a legal status revoked, access denied, equipment destroyed, or an entitlement withheld. Such descriptions are important because they identify objects upon which legal and institutional responses can operate. Property can sometimes be returned. Money can be paid. A record can be corrected. Access can be reopened. A position can be restored.

A second structure becomes visible when the affected object also participates in the production of later possibilities. Productive equipment can generate goods, income, skills, and subsequent investment. A research position can generate publications, collaborations, reputation, institutional access, new questions, and later appointments. Education can create capacities through which further education becomes easier to acquire. Health can condition work, mobility, social participation, care, attention, and learning. Reputation can affect whether later relations become available. A particular professional or intellectual encounter can redirect a trajectory whose later consequences could not have been specified in advance.

The relevant object in such cases is therefore connected to a process of generation. This paper uses the term *generative condition* for a condition that participates in the production, maintenance, transformation, or opening of subsequent capacities, relations, values, states, meanings, or possibilities. A generative condition can be material, institutional, relational, symbolic, cognitive, bodily, ecological, informational, or organizational. Its generative significance depends upon the system within which it operates. The same material resource can have very different consequences for different subjects, and it can have different consequences for the same subject at different historical moments.

This dependence places the problem near the capability approach. Sen's work directed attention away from exclusive concentration on holdings of resources and toward the substantive freedoms that people possess to achieve valuable beings and doings [55, 56]. Capability-oriented analysis also recognizes that personal, social, and environmental conversion conditions affect what a person can actually achieve with a given bundle of resources, a theme developed extensively in subsequent capability scholarship and in Nussbaum's account of human capabilities [39]. The present inquiry shares this concern with the difference between possession and effective possibility. It extends the question in a specifically diachronic direction by examining how the processes that convert conditions into later capabilities, relations, and values can themselves be altered by harm.

The problem also lies near restorative approaches. Restorative justice has emphasized processes through which harms are addressed through restoration, repair, participation, accountability, and the involvement of affected parties, rather than treating justice exclusively through punitive response [7]. These approaches make the process of repair normatively salient and therefore provide an important point of contact with the present paper's distinction between the condition eventually produced and the process through which that condition is reached.

Generative justice provides an especially direct intellectual antecedent. Eglash develops generative justice around the circulation of unalienated forms of value, the capacity of value generators to participate

in the benefits of their production, their control over conditions of production, and the self-sustaining circulation of value within generative communities [15]. More recent work on computational reparations makes the connection to repair explicit by describing reparative transformation as a movement toward systems in which value generators regain control over production and value can circulate in unalienated forms [17]. This orientation substantially enlarges the meaning of reparation beyond a discrete transfer of money.

The present paper begins from a subsequent problem. Suppose that the conditions of generation have already been removed, damaged, redirected, or placed under another actor's control. Suppose further that the affected subject continues to live, learn, adapt, withdraw, reorganize relationships, lose opportunities, acquire new ones, and change during the period in which those conditions remain unavailable. Return then occurs within a later system. The recipient of the returned condition can be historically different from the recipient who lost it. Surrounding relations can be different. The social meaning of the condition can be different. The condition's capacity to participate in further generation can also be different.

This observation motivates the paper's basic distinction between *condition return* and *generative return*. Condition return concerns the reappearance, replacement, or renewed availability of an identifiable condition. Generative return concerns the stronger family of questions concerning whether, and in what sense, the capacities and processes through which values and possibilities are generated have become equivalent, sufficiently comparable, newly viable, or otherwise just.

The distinction is intentionally open. Generative return does not presuppose that an original generative state can always be recovered. It also does not presuppose that justice always requires the closest possible reconstruction of a counterfactual past. The expression identifies a field of normative and comparative problems whose possible responses include restitution, compensation, structural repair, renewed access, re-entry, rehabilitation, assistance, redistribution, recognition, institutional transformation, and the creation of new enabling conditions.

1.2 Historical Subjects and Diachronic Generativity

The problem becomes especially pronounced when subjects are treated as historical and relationally constituted. A subject enters later moments with memories, learned responses, altered expectations, accumulated skills, relationships, losses, commitments, and interpretations that have developed through preceding events. The generative significance of a returned condition therefore depends upon more than the formal identity of the returned object.

Consider a person who has experienced prolonged workplace bullying. An institution can later restore employment, salary, access to the workplace, and formal professional status. These measures can be significant and necessary. Their presence does not establish that the workplace has regained the same generative significance for the subject. Trust, attention, expectations, responses to authority, willingness to initiate collaboration, interpretations of ambiguous organizational signals, tolerance for uncertainty, and patterns of social participation may all have changed. These changes need not be pathologized in order to matter. They can arise from ordinary learning within a history of adverse relations.

A similar structure appears in academic and intellectual life. A researcher who loses access to a laboratory, a community, a publication opportunity, or a particular collaborator can later receive equipment, funding, or institutional access of comparable market value. The later conditions may support substantial new work. They cannot be assumed to reproduce the sequence of questions, encounters, publications, readers, collaborations, and institutional openings that could have developed during the missing interval. The difference concerns the recursive structure of generation: outputs from one period frequently become conditions for subsequent periods.

This recursive structure creates a distinction between the initial loss and the loss of subsequent generation. The latter should be treated cautiously because future histories are open and counterfactual outcomes are uncertain. A missed publication cannot be equated with every later achievement that might possibly have followed from it. A missed encounter with a particular scholar cannot be represented as a deterministic claim that a specific collaboration or discovery would have occurred. The relevant concern

is more modest and more structural: the intervention can alter the set and organization of paths through which later possibilities could emerge.

The paper therefore treats generativity as diachronic. A generative system has a present state, yet its normative significance can depend upon transition possibilities, conversion mechanisms, reachable regions, resilience, relationships, and longer patterns of development. This perspective becomes important for the later comparison between state equivalence and stronger forms of dynamical equivalence. A returned resource can reproduce a state variable while leaving transition structures, reachable sets, attractors, response functions, or conversion mechanisms substantially changed.

The same reasoning applies at collective scales. A community deprived of land, productive infrastructure, cultural institutions, environmental conditions, or knowledge-generating practices can later receive resources whose market value is large while remaining differently positioned within surrounding economic, political, ecological, or epistemic relations. Institutional repair at a later time operates upon a changed collective history. The relevant analysis can therefore involve both individual and collective generativity, while remaining sensitive to the different ontological and normative problems raised by each.

1.3 Temporal Non-Equivalence of Deprivation and Return

A simple model of return can represent deprivation and restoration as inverse operations. Let H denote an intervention that removes or impairs a generative condition, let R denote a later restorative operation, and let Φ denote the evolution of the relevant system through time. For a historically changing system, the comparison of interest is generally between the history that includes the intervention and the history associated with a reference trajectory. A schematic expression of this distinction is given in Equation 1.

$$R_{t_1} \circ \Phi_{t_1, t_0}^H \circ H_{t_0} \neq \Phi_{t_1, t_0}^{\text{ref}}. \quad (1)$$

Equation 1 does not assert a universal physical law. It states the comparison problem that motivates the paper. The operation of return at t_1 acts upon the system that developed through the interval $[t_0, t_1]$ under altered conditions. The reference history on the right-hand side can denote several different objects: a no-harm counterfactual trajectory, a continuation of the pre-harm system, a statistically expected development, a normatively preferred condition, or another explicitly selected baseline. Different choices lead to different restorative judgments.

The temporal separation between deprivation and return has several consequences. First, value generation during the interval can be changed. Second, outputs that would have become conditions for subsequent rounds of generation may never appear. Third, subjects can adapt to altered conditions. Fourth, surrounding actors can respond to the changed subject or changed institutional position. Fifth, social meanings can change. Sixth, the restorative intervention itself becomes a new event with its own consequences.

These features produce what the paper later describes as the *non-equivalence of delayed return*. The expression does not deny the importance of returning resources, rights, or statuses. It identifies the additional question of whether later return bears the relevant relation to the system against which restoration is being evaluated.

The point can be illustrated through reputation. Suppose a false accusation reduces a person's professional standing. A later correction can remove the formal accusation and publicly recognize the error. The correction matters. During the intervening period, however, some collaborations may have failed to form, some invitations may not have been issued, some institutions may have made decisions using the earlier information, and some observers may never encounter the correction. The corrective statement becomes another event in the history of the reputational system. It does not erase the fact that the earlier statement circulated and produced effects.

The same temporal structure applies to health, education, employment, incarceration, environmental harm, institutional exclusion, and many other domains. The concrete mechanisms vary substantially.

The common problem lies in the possibility that later return operates upon a system whose relations and generative processes have already changed.

1.4 Temporal Domains of Justice and Governance

The temporal structure of generative harm suggests a corresponding distinction among pre-harm, during-harm, and post-harm responses. These domains overlap in practice, yet they raise different normative and institutional problems.

The pre-harm domain concerns protection, precaution, institutional design, and the governance of interventions capable of damaging generative conditions. Where later repair is expected to be difficult, uncertain, or structurally limited, prevention acquires additional importance. Relevant mechanisms can include rights, prohibitions, procedural safeguards, due process, occupational protections, safety standards, institutional checks, review requirements, and constraints upon actors capable of imposing high-impact changes on another subject's generative environment.

The during-harm domain concerns interruption. Once a harmful process begins, time can become normatively significant because continuing exposure can produce additional relational and generative consequences. Workplace harassment can affect participation and professional relations. Exclusion from education can affect later educational opportunities. Reputational harm can propagate across networks. Loss of access to productive infrastructure can interrupt projects that would otherwise generate further resources. Institutional responses during this period can include temporary protection, separation of parties, independent review, suspension of contested decisions, evidence preservation, interim resource support, monitoring, and mechanisms designed to reduce the continued expansion of harm.

The post-harm domain contains the most difficult restorative comparisons. At this stage, the problem includes an existing history whose consequences may be partly reversible, partly substitutable, partly compensable, partly transformable, and partly unavailable for reconstruction. Legal restitution, monetary compensation, institutional correction, rehabilitation, recognition, re-entry, and assistance can each address different components of this situation.

This temporal division also clarifies an institutional division of labor. Law has strong capacities for articulating rights, prohibitions, duties, procedural requirements, responsibility, formal restitution, injunctions, status correction, and monetary awards. Governance mechanisms can address ongoing monitoring, risk management, institutional design, appeals, review procedures, and early intervention. Political economy can organize redistribution, insurance, public provision, compensation funds, and other mechanisms for mobilizing fungible resources. Assistance practices can address needs that remain after formal liability and resource transfer have been determined. Organizational practices can support safe re-entry, changes of environment, renewed access, and alternative pathways.

The paper treats these capacities as complementary and limited. A legal judgment requires sufficient closure for adjudication. A generative trajectory can remain open after judgment. A political-economic transfer can enlarge the resources available to the subject while leaving relational conditions unchanged. Assistance can create new possibilities while remaining unable to reproduce a singular opportunity that disappeared. Governance can reduce risk while introducing new forms of monitoring or control. Each institutional response therefore implements an implicit theory of what matters and what counts as sufficient repair.

1.5 General Equivalents and Subject-Directed Regeneration

Money occupies a distinctive position in post-harm response because it is a general equivalent with unusually broad convertibility. Legal systems and political-economic institutions can transfer money without specifying in detail how the recipient must use it. This feature provides an important reason for taking monetary compensation seriously within a theory of generative return.

A historically changed subject may have priorities that differ from those that would have existed before the harm. A person who lost a professional trajectory may prefer retraining, relocation, rest, care, new research, family time, a different occupation, or another form of life. A monetary transfer can leave

a substantial portion of this choice with the recipient. In this sense, fungibility can function as a form of institutional epistemic restraint. The institution acknowledges a claim while limiting its attempt to determine the recipient's preferred future.

This advantage coexists with important limitations. Money can purchase some conditions and cannot directly reproduce others. A particular relationship, historical moment, intellectual encounter, period of health, developmental window, community, ecological condition, or social meaning may have no close market substitute. The same amount of money can also produce different generative effects for subjects with different health, knowledge, networks, institutional access, locations, responsibilities, and histories. The conversion from general resources into generative possibility therefore becomes an independent object of analysis.

The paper accordingly distinguishes at least three components of post-harm support: general equivalents, specific enabling conditions, and subject-directed regeneration. General equivalents provide flexible resources. Specific enabling conditions address constraints that flexible resources alone may not remove, including legal status, institutional access, treatment, education, safety, documentation, professional re-entry, or network access. Subject-directed regeneration concerns the preservation of the subject's agency in determining how future possibilities should be developed.

This tripartite structure also clarifies the place of assistance. Assistance is not treated as evidence that a previous loss has become equivalent to a newly created opportunity. It can instead acknowledge historical non-equivalence while supporting the development of a viable future. This distinction becomes especially important where the lost condition was singular, temporally sensitive, or relationally irreproducible.

1.6 Restorative Claims as Comparative Claims

The central analytical move of the paper is to treat restoration as a comparative judgment whose meaning depends upon explicitly specified parameters. Ordinary language often compresses these parameters into statements such as "the position was restored," "the victim was compensated," "the community was repaired," or "the lost opportunity was replaced." Each statement can be institutionally useful. Each also leaves open several questions.

The first concerns the reference. A restored system can be compared with the pre-harm actual state, a no-harm counterfactual trajectory, a statistically expected trajectory, a normatively desirable state, a threshold of sufficiency, a best feasible alternative, or a future selected by the affected subject. These reference systems can diverge substantially. Returning to a pre-harm state can be inappropriate when that state already contained domination, exclusion, deprivation, or structural inequality.

The second concerns the object of comparison. A restorative judgment can compare resources, legal statuses, attributes, relations, networks, institutional positions, present states, probability distributions, trajectories, possible traces, transition systems, reachable sets, attractors, basins, stability properties, perturbation responses, causal structures, conversion mechanisms, generative mechanisms, meanings, or historical configurations.

The third concerns the comparative relation. Exact identity represents only one possibility. Depending upon the object, relevant comparison can involve isomorphism, homomorphism, embedding, simulation, mutual simulation, bisimulation, observational equivalence, trace equivalence, topological correspondence, conjugacy, distributional equivalence, statistical indistinguishability, causal or interventional equivalence, metric proximity, dominance, refinement, or another order relation. Some of these relations are formal equivalence relations. Others are mappings, preorders, approximation criteria, or one-directional preservation relations. The later atlas therefore uses *comparative relations* as its broader category.

The fourth concerns tests and environments. Two systems can appear equivalent under ordinary observation while responding differently to stress, institutional conflict, uncertainty, resource loss, social pressure, or another perturbation. Two systems can produce similar outcomes under passive observation while responding differently to the same intervention. Restoration can therefore depend upon the class of observations, perturbations, and interventions regarded as normatively relevant.

The fifth concerns time. Instantaneous similarity can coexist with divergent long-run dynamics. A subject can appear to have resumed normal professional performance while retaining a very different response structure under later organizational stress. A restored ecological system can have a similar present composition while possessing different resilience or basin structure. A restored institution can reproduce current outputs while generating a different future distribution of opportunities. Temporal horizon is therefore part of the restorative claim.

The sixth concerns approximation. Exact equivalence will often be unavailable or unnecessary. A theory must then determine which differences can be tolerated, under which metric or observational criterion, and for what purpose. Such choices already contain normative commitments.

These elements motivate the indexed representation introduced in Equation 2.

$$\text{Rest}(\mathcal{G}_R, \mathcal{G}_{\text{ref}}; Q, E, \mathcal{T}, \tau, \varepsilon, K). \quad (2)$$

In Equation 2, $\mathcal{G}_R$ denotes the system after a proposed restorative response, $\mathcal{G}_{\text{ref}}$ denotes the selected reference system, Q identifies the object of comparison, E identifies the comparative relation, $\mathcal{T}$ identifies relevant observations, perturbations, tests, or interventions, τ identifies the temporal horizon, ε identifies an admissible approximation where appropriate, and K represents the epistemic basis upon which the comparison is made. The notation is a framework for decomposition rather than a claim that every restorative judgment can be reduced to a single computable predicate.

This decomposition supplies the organizing logic for the paper’s *Atlas of Equivalence*. The atlas maps the space of restorative comparisons by separating the object being compared from the relation used to compare it. The distinction matters because the same object can support several forms of comparison, and the same comparative relation can acquire different meanings when applied to different objects.

For example, a professional network can be compared through graph isomorphism, reachability preservation, centrality, functional substitutability, or statistical opportunity structure. Observable behavior can be compared through output similarity, trace equivalence, simulation, or bisimulation. Dynamical systems can be compared through trajectories, attractor correspondence, topological conjugacy, stability properties, basin structure, or statistical response under perturbation. Each criterion makes some structures visible and places others outside the comparison.

1.7 Dynamical and Generative Comparison

Dynamical systems provide one family of tools for expressing the diachronic problem with greater precision. Their role in this paper is analytical. Human subjects and social relations are not assumed to be exhaustively representable by low-dimensional deterministic systems. Dynamical language is useful because it distinguishes current state from transition structure, trajectory, reachability, stability, attractor organization, perturbation response, and long-run behavior.

This distinction reveals several levels of restorative comparison. State restoration concerns whether relevant present variables have returned to specified values. Trajectory comparison concerns whether later evolution follows a comparable path. Reachability comparison concerns whether relevant future states remain available. Attractor comparison concerns whether the system retains comparable long-run modes of organization. Basin comparison concerns the range of conditions from which those modes remain reachable. Stability and resilience comparisons concern how the system responds when it is perturbed.

These distinctions are particularly relevant where the visible state has returned while latent response structure has changed. A person can hold the same position, receive the same salary, and produce similar work under ordinary conditions while reacting differently to organizational uncertainty after a history of coercion or harassment. A firm can regain the same productive assets while losing network relationships that previously made those assets valuable. A community can regain legal access to land while confronting an ecological or economic system that has moved into a different regime.

The paper later develops a robust form of restorative comparison in which responses are assessed across a class of admissible perturbations. A schematic version is given in Equation 3.

$$D(P_R(O \mid \delta), P_{\text{ref}}(O \mid \delta)) \leq \varepsilon \quad \text{for all } \delta \in \Delta_{\text{adm}}. \quad (3)$$

Equation 3 represents one possible criterion. O denotes an observable of interest, δ denotes a perturbation, Δ_{adm} denotes the class of perturbations treated as relevant, D denotes a statistical or other divergence measure, and ε specifies an admissible degree of difference. The normative questions remain substantial: which observables matter, which perturbations are admissible, what degree of difference matters, who selects the threshold, and whether statistical similarity adequately represents the values at stake.

Dynamical equivalence is therefore one component of the atlas rather than its normative conclusion. A technically strong form of equivalence can preserve structures that a theory of justice regards as morally irrelevant. A weaker form of comparison can preserve precisely the freedom or relationship that matters in a particular case. Formal strength and normative importance must remain analytically distinct.

1.8 Value Theory and the Selection of Equivalence

A comparison becomes normatively meaningful only after the relevant values are specified. Equal market value, equal welfare, equal resources, equal capabilities, equal legal status, equal recognition, equivalent social relations, equal freedom from domination, equivalent control over productive conditions, and equivalent generativity are different normative objects.

The capability approach illustrates this point clearly. Equal resource bundles do not ensure equal substantive freedoms because conversion conditions differ [55, 39]. A capability-oriented restoration can therefore reject resource equality as an adequate endpoint where resources translate into materially different opportunities.

Generative justice adds another layer by directing attention to value circulation and control over generative conditions [15, 17]. Two systems can distribute a similar quantity of resources while differing sharply in who controls the conditions through which value is produced, whether value is extracted, and whether productive communities can sustain recursive circulation.

Relational and recognition-oriented approaches can emphasize a different comparison again. A monetary transfer may leave a relation of subordination unchanged. Formal equality can coexist with social relations in which one party remains dependent upon another's discretionary power. A restored entitlement can therefore satisfy one comparison while leaving another normative structure unrepaired.

The atlas developed in this paper consequently does not assign a universal ranking to mathematical relations. Isomorphism is not inherently more just than simulation. Bisimulation is not inherently more just than capability sufficiency. Exact state equality is not inherently preferable to the creation of a new trajectory selected by the affected subject. The normative force of a comparative criterion depends upon the value theory through which the compared structure matters.

This leads to a central principle of the paper:

A theory of restoration requires a theory of comparison, and a theory of comparison requires a theory of value.

The principle has two implications. First, ordinary restorative claims contain implicit normative selections even when expressed in apparently technical language. Second, formalizing a comparison does not settle the justice of the comparison. Formalization can expose the choice that requires normative justification.

1.9 Substantive and Procedural Justice of Return

A further distinction separates the justice of an achieved condition from the justice of the process used to produce it. Let a restorative intervention transform a post-harm system into a system that satisfies a selected comparative criterion. The achievement of that criterion does not establish that the intervention was permissible.

An idealized example makes the distinction clear. Suppose a hypothetical technology could erase a subject's memory of a traumatic institutional history and thereby make later observable behavior more similar to a selected counterfactual reference. Such an intervention might increase behavioral similarity. It could simultaneously violate consent, agency, identity, historical continuity, privacy, or dignity. A technically stronger restorative equivalence can therefore coexist with a normatively worse restorative process.

The same structure can arise without speculative technology. An institution might obtain closer behavioral conformity through coercive treatment, compelled relocation, restrictions on the subject's choices, manipulation of social relations, suppression of records, or burdens imposed upon third parties. Outcome fidelity and procedural legitimacy must therefore be evaluated separately.

The procedural dimension includes questions of participation, consent, transparency, contestability, revisability, proportionality, privacy, non-domination, dignity, epistemic fairness, burdens upon third parties, and the right to reject a proposed restoration. The last of these is especially important where the affected subject has changed. A person may reasonably prefer a newly chosen trajectory over the institution's reconstruction of the life the person would supposedly have led without the harm.

The framework therefore resists an optimization objective that simply maximizes similarity to a counterfactual state. A just response can accept a lower degree of counterfactual fidelity in order to preserve the agency of the present subject. Conversely, a subject can request forms of restoration that require more substantial institutional effort than a minimal compensation standard would provide. The relation between substantive and procedural justice must therefore remain open to context and value theory.

This yields a second central principle:

The realization of a selected restorative equivalence requires an independent theory of just process.

Together, the two principles organize the normative architecture of the paper. The first concerns the selection of the restorative object and criterion. The second concerns the legitimacy of the transformation through which the selected condition is pursued.

1.10 Epistemic and Structural Limits

The difficulty of generative return is often described as a limitation of knowledge. Counterfactual histories cannot be directly observed. Long causal chains are difficult to identify. Many social relations are incompletely recorded. Future opportunities are uncertain. Individual responses vary. Current institutions therefore operate under substantial epistemic constraints.

The paper treats these constraints seriously while distinguishing several forms of limitation. Some are technological and may diminish with improved data, longitudinal observation, causal inference, computational modeling, medical knowledge, institutional records, and simulation. Others concern the openness of counterfactual history. The history in which a harm did not occur is not obtained mechanically by subtracting the harmful event from the actual history. Later encounters, choices, relationships, and environmental events could themselves have changed.

A further difficulty concerns non-invertibility. Multiple histories can produce similar present states. A harmful intervention can alter parameters, transition rules, network structures, or the subject's relation to later conditions. A unique inverse operation that reconstructs the relevant reference history may therefore be unavailable even under extensive knowledge.

Reflexivity introduces another layer. A record about a social system can enter the system and influence later decisions. A correction of the record can become another event. A judgment can alter reputation. Compensation can alter relationships. Public recognition of harm can change collective memory. Attempts at restoration therefore participate in the history they seek to repair.

These considerations motivate the paper's use of idealized and "superpower" thought experiments. The purpose is methodological. By gradually removing practical constraints, the analysis can distinguish

limitations arising from present technology from limitations associated with counterfactual identification, structural non-invertibility, reflexive non-closure, value conflict, subject identity, and procedural justice. A hypothetical ability to alter reality does not by itself specify the reality that ought to be produced.

The paper remains deliberately open concerning the strongest possible epistemological conclusions. It does not require the claim that complete social symbolization is metaphysically impossible. It establishes a weaker analytical point sufficient for the present project: any practically or theoretically available representation selects objects, relations, observables, and boundaries, and the representation can itself become socially consequential. Claims of complete restoration therefore require careful specification of what has been represented, compared, and omitted.

1.11 Contributions

The paper makes seven connected contributions.

First, it develops a diachronic account of the justice of return centered on generativity. The account distinguishes return of identifiable conditions from return of their generative significance and examines the consequences of historical change between deprivation and later response.

Second, it organizes justice and governance across three temporal domains: pre-harm prevention, intervention during ongoing harm, and post-harm response. This organization clarifies why law, governance, political economy, organizational practice, and assistance have different comparative advantages and limitations across the temporal development of harm.

Third, it develops a general framework of restorative comparison. A restoration claim is represented as an indexed judgment involving a reference system, an object of comparison, a comparative relation, a class of relevant tests or interventions, a temporal horizon, an approximation tolerance, and an epistemic basis. This framework makes explicit the choices compressed within ordinary claims that a condition, status, capability, relation, or system has been restored.

Fourth, it constructs an extensive atlas of comparison objects and comparative relations. Comparison objects range from resources and present states to relations, networks, trajectories, reachable sets, attractors, causal structures, conversion mechanisms, generative processes, meanings, and histories. Comparative relations include identity, isomorphism, homomorphism, simulation, bisimulation, observational and trace relations, dynamical correspondences, statistical criteria, causal and interventional relations, approximation, dominance, and refinement. The atlas distinguishes formal equivalence relations from broader comparative structures.

Fifth, it develops the dynamic dimension of restorative comparison. The paper examines state, trajectory, reachability, attractor, basin, stability, resilience, perturbation-response, causal, interventional, and generative-process comparisons. These distinctions provide a formal vocabulary for cases in which present conditions appear restored while the system's later responses and future possibilities remain substantially altered.

Sixth, it connects restorative comparison to theories of value and justice. Resource, capability, relational, recognition-based, rights-based, non-domination, care-oriented, reparative, restorative, and generative approaches can privilege different comparison objects and different standards of adequacy. The paper uses the atlas to make these normative selections explicit while preserving the irreducibility of the underlying traditions.

Seventh, it separates substantive restorative equivalence from procedural justice. Stronger similarity to a selected reference does not automatically produce a more just restorative process. Consent, agency, participation, privacy, dignity, contestability, reversibility, proportionality, non-domination, third-party burdens, and the right to pursue a different future remain independent considerations.

These contributions are designed to support systematic inquiry rather than to produce a universal metric of generative loss. The atlas is intended to show the range of questions that must be answered before a claim of sufficient return can be defended.

1.12 Scope and Method

The paper is a comprehensive conceptual and formal discussion. Its method combines philosophical analysis, comparison of justice traditions, legal and institutional reasoning, political-economic analysis, dynamical and computational concepts, idealized thought experiments, and structured case analysis. The objective is to map the problem space and clarify relations among its components.

The formalism is deliberately plural. Different domains require different mathematical representations. Graph structures can be useful for relational networks. Transition systems can support simulation and bisimulation analyses. Dynamical systems can express trajectories, attractors, basins, and stability. Probability distributions can support statistical comparison. Causal models can distinguish observation from intervention. Order relations can represent dominance or sufficiency where equality is unnecessary. The paper therefore does not force all restorative questions into a single mathematical language.

The paper also distinguishes conceptual possibility from present operationalizability. Some comparison criteria can be measured with existing methods. Others function as ideal-theoretical constructs. Some may remain impossible to identify empirically even if they are conceptually coherent. Their inclusion in the atlas remains useful because they clarify what weaker criteria omit and expose the normative assumptions made when institutions select tractable substitutes.

The inquiry is intentionally broad in domain. Examples are drawn from work, education, research, reputation, health, productive resources, institutional exclusion, historical injustice, community life, and other settings. The paper does not assume that a single restorative standard applies uniformly across these cases. Their inclusion serves the comparative objective of showing how different objects and forms of generativity generate different restorative problems.

The project is also intentionally open-ended. The atlas can be extended as new comparison objects, mathematical relations, governance technologies, and theories of value become relevant. An exhaustive final taxonomy is therefore not required for the framework to be useful. The aim is to provide a sufficiently systematic architecture that additions can be located, distinguished, and evaluated without collapsing them into a single undifferentiated concept of restoration.

1.13 Organization of the Paper

The remainder of the paper proceeds from conceptual and historical foundations toward the formal atlas and then returns to normative and institutional practice.

The next group of sections examines the intellectual traditions surrounding return, repair, capabilities, relational justice, reparative and restorative justice, and generative justice. It develops the conceptual structure of generative conditions and generative harm, followed by the historical and relational constitution of subjects and the temporal organization of prevention, interruption, and post-harm response. Law, governance, political economy, organizational practice, and assistance are then considered as partially overlapping institutional responses.

The paper subsequently examines the practical, epistemic, and structural boundaries of restoration. Counterfactual uncertainty, historical openness, non-invertibility, reflexivity, and normative underdetermination provide the transition to the formal theory of restorative comparison.

The central analytical portion develops the atlas. It first specifies possible reference systems and then constructs a taxonomy of objects of restorative comparison. A separate taxonomy examines comparative relations, followed by an object–relation atlas that studies combinations of comparison objects and comparison criteria. Temporal horizons, approximation, and admissible tests are incorporated into this structure.

The dynamical portion then examines state, trajectory, reachability, attractor, basin, stability, resilience, and perturbation-response comparisons. Causal and interventional equivalence are developed separately in order to distinguish observational similarity from similarity under policy, assistance, or other interventions. These discussions culminate in generative-process and historical-reflexive forms of comparison.

The normative portion identifies possible value objects and studies how major families of justice theory select and prioritize different restorative features. Substantive justice is then separated from procedural justice, with special attention to cases in which greater restorative fidelity conflicts with consent, agency, dignity, privacy, historical continuity, or the interests of third parties.

Idealized restoration cases remove progressively larger technological and resource constraints in order to isolate epistemic, structural, and normative limits. The later institutional sections return to law, political economy, assistance, and governance and analyze their practices through the comparative framework. Applied cases demonstrate how different restorative judgments arise from different selections of reference systems, comparison objects, comparative relations, values, temporal horizons, and procedures.

The final sections collect open mathematical, philosophical, legal, and governance problems and state the limits of the framework. The paper concludes with a modest claim. Restoration becomes intelligible only after the relevant comparison has been specified; the significance of that comparison depends upon a theory of value; and the pursuit of the selected restorative condition remains subject to an independent theory of just process.

2 Intellectual Lineages of Restoration and Generative Justice

This section situates the problem of generative return within several traditions that have developed distinct accounts of loss, rectification, resources, capabilities, relationships, recognition, domination, repair, reparation, and value circulation. Its purpose is neither to assimilate these traditions into a single theory nor to present their differences as merely terminological. Each tradition foregrounds a different object of normative concern and therefore suggests a different answer to what should be restored after harm. The method of this section is comparative. It examines the characteristic normative object, the corresponding form of repair or return, the institutional practices with which the theory is naturally associated, and the remaining questions that become visible when the affected subject or system is treated as historical, relational, and continuously generative.

Several themes recur across these traditions. Corrective justice concerns the relation between wrongful loss, responsibility, and rectification. Reparative theories extend the temporal and political horizon to historical wrongs, collective responsibility, reconciliation, recognition, and public programs of redress. Restorative justice directs attention toward harms to persons and relationships and toward participatory processes of accountability and repair. Resource-oriented theories clarify the distributive significance of holdings and transferable means. Capability approaches distinguish possession of means from substantive freedom and emphasize the conditions through which resources are converted into valuable beings and doings. Relational and republican approaches examine oppression, social standing, and domination. Recognition theory adds the symbolic and intersubjective organization of social value. Generative justice directs attention toward the production and circulation of value, control over generative conditions, and the self-sustaining dynamics of social and ecological systems.

The present inquiry inherits concerns from all of these traditions. It also encounters a problem that cuts across them. Statements such as “the loss has been repaired,” “the capability has been restored,” “the relationship has been repaired,” “value has been returned,” or “the system has become generative again” require a comparison between at least two conditions. The object being compared, the reference condition, the comparative relation, the relevant temporal horizon, and the values that make the comparison normatively significant are often supplied implicitly by the surrounding theory. Later sections make these components explicit. The role of the present section is to show why such an explicit comparative framework becomes necessary.

2.1 Corrective Justice and Dyadic Rectification

Corrective justice provides one of the clearest philosophical models of justice after a wrong. Its characteristic structure is relational and retrospective. A wrong has occurred between parties; a wrongful loss

or alteration can be attributed to an agent under the relevant theory of responsibility; and a duty of repair arises in response. Coleman describes corrective justice in terms of wrongfulness and responsibility and develops the idea that wrongdoers can have agent-relative duties to repair wrongful losses for which they are responsible [11]. Weinrib's account of private law similarly places corrective justice within the juridical relation between the doer and sufferer of an injury and emphasizes the correlativity of the parties' positions [69].

This structure supplies several important resources for a theory of return. First, it identifies the normative importance of the relation between a particular wrong and a corresponding duty. A theory of post-harm response cannot automatically replace responsibility with a general concern for welfare. Second, corrective justice preserves the historical fact that the claimant's present disadvantage can matter because of how it arose. Third, the theory provides an intelligible institutional reason for distinguishing compensation owed by a wrongdoer from assistance supplied for independent humanitarian or social-policy reasons.

At the same time, the object of correction can become difficult to specify when the consequences of a wrong propagate through a generative system. A damaged object, withheld payment, or cost of treatment can provide a relatively tractable object of rectification. A changed professional trajectory, lost network position, interrupted research process, altered trust structure, or reduced capacity to convert later resources into opportunities presents a more complex target. The relational logic of corrective justice remains relevant, while the content of the wrongful loss becomes difficult to delimit.

A further difficulty concerns temporal closure. Corrective justice must identify a sufficiently bounded relation between wrong, loss, responsibility, and remedy for adjudication to proceed. Generative processes can continue beyond this boundary. A wrongful exclusion from a productive environment can first generate a lost income claim, then affect professional relationships, then alter later opportunities, then change the subject's own choices and expectations. Each later development can depend jointly upon the original wrong and upon subsequent events that are independent of it.

This does not render corrective justice conceptually defective. It reveals the importance of its institutional function and the limits of the object it can reasonably be asked to represent. A court can require a tractable account of causation and responsibility. A theory of generative return can examine a wider space of effects while preserving the distinction between effects that ground legal liability and effects that remain normatively relevant in other domains.

Corrective justice therefore contributes an indispensable responsibility dimension to the present inquiry. It also raises an early version of the comparison problem. Repair of a wrongful loss presupposes some account of the condition against which the loss is measured. In simple cases, the relevant comparison can concern an identifiable asset or expense. In generative cases, the reference can involve expected income, professional capacity, social relations, opportunity structures, or other counterfactual developments. Different specifications of the object can produce substantially different accounts of what rectification requires.

2.2 Restitution and the Structure of Return

Restitution supplies an especially intuitive model for the language of *return*. An identifiable thing has been transferred, withheld, or possessed under circumstances that justify restoring it to another party. Where the relevant object is stable and identifiable, the structure of return can appear comparatively straightforward. The object that leaves one party's possession can later be returned to that party.

The simplicity of this image has substantial normative appeal. It links remedy to an identifiable history, resists arbitrary substitution where the original object matters, and can avoid reducing every claim to a monetary valuation. The image also supports a familiar intuition that a person deprived of an object can possess a stronger claim to that object than a person making an independent distributive request.

Generative conditions complicate this image because the significance of an object can depend upon its temporal position within a process. A productive resource returned after a long interval is materially the same resource while its possible uses can be different. A legal entitlement restored after a critical

educational period can possess the same formal content while the developmental context has changed. Access to a professional institution can be restored after the relations that made the access valuable have dissolved.

The resulting distinction between object identity and generative significance will recur throughout this paper. Restitution can succeed according to one comparison and remain incomplete according to another. A theory that asks only whether the object has been returned evaluates a different proposition from a theory that asks whether the subject can again use the object within a similar productive, relational, or developmental structure.

This distinction also shows why a theory of generative return should preserve restitution as one important mode of response. The conclusion that return of an object does not reconstruct an entire history should not weaken a legitimate claim for return of the object itself. Multiple restorative dimensions can be simultaneously relevant. Object restitution can be required even where trajectory restoration is impossible. Monetary compensation can be required in addition to object restitution. Assistance can remain appropriate after both have occurred.

The broader framework developed later therefore treats object identity as one possible comparison relation applied to one possible comparison object. Its normative importance depends upon the type of wrong and the value attached to the object. This placement preserves the force of restitution while preventing its structure from becoming an implicit model for every form of restoration.

2.3 Historical Injustice and Reparative Responsibility

The literature on historical injustice enlarges the temporal scale of repair. It confronts circumstances in which the original victims and perpetrators may no longer be alive, institutions persist across generations, property and advantages have changed hands, identities have developed through the history of the wrong, and the present distribution cannot be understood without reference to past events. These features make historical reparation an important intellectual predecessor of the present inquiry.

Waldron's discussion of superseding historic injustice directly addresses the possibility that changes in circumstances can alter the normative implications of historical claims [67]. Thompson develops an account of reparation for historical injustice that addresses intergenerational responsibility, inherited claims, reconciliation, and the continuing significance of past wrongs [62]. Boxill's defense of Black reparations develops inheritance and counterfactual arguments within a Lockean framework and illustrates the importance of counterfactual reasoning for reparative claims [6].

Historical injustice makes several difficulties unavoidable. First, the reference condition is contestable. A claimant can compare the present with a world in which a specific wrong never occurred, with a world in which a legitimate transaction occurred, with a normatively fair present, or with a minimum condition of justice. These references need not coincide.

Second, historical change can make literal reconstruction normatively problematic. Persons, communities, institutions, identities, and social relations can arise partly through histories that include injustice. Removing every causal consequence of the injustice, were such an operation imaginable, would raise questions about the continuity and identity of present subjects. Historical reparation therefore contains an early form of the tension between counterfactual fidelity and respect for present subjects.

Third, the parties to repair can differ from the parties to the original wrong. Responsibilities can be institutional, inherited, collective, civic, or based upon continuing benefit rather than personal perpetration. This feature complicates a purely dyadic model of return.

Fourth, historical harms can produce advantages and disadvantages through long chains of social reproduction. A transferred asset can appreciate, support education, create political influence, finance later investments, and alter future inheritance. Conversely, dispossession can remove the conditions through which such cumulative processes would have occurred. The problem therefore already contains a generative dimension.

The present framework does not attempt to resolve the philosophical literature on historical reparations through a single formal device. It draws a more limited lesson. Historical reparation demonstrates that a restorative claim requires explicit treatment of temporal reference, causal history, present identity,

responsibility, and the structure of the object being restored. These issues later reappear in the atlas as separate dimensions rather than as one composite question.

2.4 Institutional Vocabulary of Reparation

International human-rights practice provides a useful example of an already plural institutional vocabulary of reparation. The United Nations Basic Principles and Guidelines on the Right to a Remedy and Reparation distinguish among forms of reparation that include restitution, compensation, rehabilitation, satisfaction, and guarantees of non-repetition [65]. The significance of this plurality for the present paper is conceptual. Institutional practice does not require every consequence of serious harm to be represented by one remedial instrument.

Restitution can address restoration of rights, liberty, status, residence, employment, or property where such restoration is possible. Compensation can address economically assessable damage. Rehabilitation can involve medical, psychological, legal, and social services. Satisfaction can include measures concerning truth, acknowledgment, apology, commemoration, and dignity. Guarantees of non-repetition concern institutional and future-oriented changes.

This plurality contains an implicit decomposition of harm. Material holdings, formal status, bodily or psychological condition, recognition, public truth, institutional structure, and future risk are treated as distinct enough to justify distinct forms of response. The structure is already incompatible with a purely scalar account in which every relevant injury is translated into one common quantity.

Large-scale reparations programs make the plurality even clearer. De Greiff argues that the ideal of compensation proportional to harm, familiar from individual judicial cases, cannot simply be transplanted as the sole standard for programs addressing large numbers of victims. His account emphasizes recognition, civic trust, and social solidarity as goals relevant to the justice of reparations programs [13]. The broader *Handbook of Reparations* documents the institutional diversity and design problems of reparations across contexts [12].

For the present inquiry, these practices show that the problem of return already has multiple institutional dimensions. They also reveal a distinction that will later become formal. Different remedial instruments compare different objects. Restitution can compare possession or legal status. Compensation can compare monetized loss. Rehabilitation can compare functional capacity or need. Satisfaction can compare recognition, public record, or social meaning. Guarantees of non-repetition can compare institutional transition rules and future risk.

These objects do not automatically share a common equivalence relation. Restoring employment is different from restoring health. Correcting a public record is different from restoring a professional network. Reducing recurrence risk is different from reproducing a past state. The plurality of reparative practice therefore provides an institutional precursor to the object-relation atlas developed later in this paper.

2.5 Restorative Justice and Relational Repair

Restorative justice changes the orientation of post-wrong response by placing greater emphasis on concrete harm, affected persons, relationships, accountability, participation, and repair. Zehr's influential restorative framework contrasts a lens centered on rule violation and punishment with one that directs attention toward harms, needs, obligations, and the persons and relationships affected by wrongdoing [72]. Braithwaite develops restorative justice alongside responsive regulation and argues for institutional processes capable of restoring victims, offenders, and communities [7].

This tradition is especially important to the present inquiry for three reasons. The first concerns the object of harm. A wrong can damage a relationship or community in ways that are poorly captured by a transfer of resources. Trust, standing, safety, participation, accountability, and social connection can become legitimate objects of repair.

The second concerns process. Restorative justice makes the manner of response part of the justice of the response. Participation, voice, acknowledgment, responsibility, dialogue, and community involve-

ment can matter independently of the final distribution of material goods. This orientation directly supports the later distinction between substantive restorative equivalence and procedural justice.

The third concerns plurality of outcomes. A restorative process can produce apology, restitution, agreement, changed behavior, community reintegration, recognition of harm, practical support, or institutional change. Its success cannot always be represented by proximity to a single pre-harm state.

The relational orientation also introduces complications. The phrase “restoring a relationship” can describe substantially different normative goals. It can mean reopening communication, removing domination, re-establishing trust, recognizing responsibility, restoring a social role, recreating an institutional relation, or enabling parties to separate under safe and non-dominating conditions. A prior relationship can itself have contained coercion or inequality. Restoration of the previous relational form can therefore conflict with justice.

This point is crucial for the later analysis of reference systems. The pre-harm condition cannot automatically serve as the normative baseline. A workplace relation preceding overt harassment can already contain arbitrary dependence. A family relationship preceding a visible conflict can already contain domination. A colonial relation preceding a specific expropriation can already be unjust. A theory of return therefore needs to distinguish historical reference from normative reference.

Restorative justice also helps clarify the temporal structure of intervention. A participatory restorative process conducted soon after harm can operate upon a different relational system from one conducted after years of conflict, litigation, public accusation, institutional exclusion, or social withdrawal. The passage of time changes the parties and the relevant relational environment. The process itself can then become generative, producing new recognition, commitments, relationships, and institutions whose value does not depend upon their replication of the past.

2.6 Moral Repair and the Reconstruction of Normative Relations

The language of moral repair adds another dimension by emphasizing the normative meaning of acknowledgment and responsibility. Walker’s account of moral repair examines the reconstruction of moral relations after wrongdoing and treats making amends as involving more than the production of a beneficial outcome. Acceptance of responsibility and acknowledgment of wrongdoing are integral to what makes an action count as amends [68].

This distinction is directly relevant to the separation between assistance and reparation developed later in this paper. A third party can provide resources, care, or opportunities to a harmed person without thereby making amends for the wrong. The same material transfer can possess a different normative meaning when it is offered by a responsible party as acknowledgment of a wrong, by a public institution as social assistance, by an insurer under a contractual scheme, or by a community as solidarity.

The comparison object therefore includes more than the material content of the transfer. Normative relations among parties can matter. A payment accompanied by denial of responsibility can be materially identical to a payment accompanied by acknowledgment, while the two interventions differ in their effects on recognition, trust, dignity, and the social interpretation of the history.

Moral repair also demonstrates why outcome-based equivalence cannot exhaust the justice of return. If the normative relation created by wrongdoing includes unanswered responsibility, then reproduction of the claimant’s material position leaves part of that relation unaddressed. Conversely, acknowledgment without material repair can also remain inadequate where the wrong caused substantial material or generative losses.

The present framework consequently treats responsibility, recognition, and material restoration as potentially distinct dimensions. Their relationship is a matter for normative theory rather than an assumption built into the formal comparison.

2.7 Resources, Distribution, and General Equivalents

Distributive theories provide another major lineage because many practical responses to harm operate through resources. Dworkin’s equality of resources offers a sophisticated account of equality in the

distribution of resources and makes resource holdings an explicit object of egalitarian evaluation [14]. Resource-oriented approaches are especially relevant to institutions because resources can often be identified, valued, transferred, taxed, insured, or redistributed.

The practical strength of resource comparison should not be underestimated. Money and other fungible resources allow institutions to respond across heterogeneous contexts. They provide a common medium through which losses in income, property, treatment costs, replacement costs, relocation, training, and other needs can be addressed. A transferable resource can also leave significant discretion with the recipient.

This discretionary feature becomes important for historically changed subjects. An institution attempting to reconstruct a claimant's hypothetical future must make substantive judgments about what that future should contain. A monetary transfer can reduce the need for such judgments. The claimant can allocate the resource among health, education, housing, relocation, professional change, care, leisure, research, family responsibilities, or other priorities. The general equivalent can therefore support agency precisely because it is less specific about the future to be generated.

The same feature explains its limitations. A general equivalent relies upon the existence of conversion pathways through which the transferred resource can be turned into goods, capabilities, relationships, or opportunities that matter. Some pathways are unavailable. Some goods are scarce. Some relationships are singular. Some historical moments cannot be purchased later. Some forms of recognition cannot be produced by market exchange. Some ecological conditions cannot be reconstructed at relevant scales.

Resource equality and resource compensation therefore identify one important comparison object without settling the status of other objects. The relation between resource restoration and generativity becomes an empirical and normative question concerning conversion. This question provides the transition to the capability approach.

2.8 Capabilities, Conversion, and Substantive Possibility

The capability approach offers one of the closest established frameworks to the problem developed in this paper. Sen argues that evaluation should attend to what people are substantively able to be and do, and emphasizes capabilities rather than relying exclusively on resources, utility, or formal rights [55, 56]. Nussbaum develops a capabilities framework in relation to human development and political principles [39]. Robeyns provides a systematic account of the capability approach and its use across disciplines [50].

The concept of conversion factors is particularly significant. Possession of a resource does not determine what a person can achieve with it. Personal, social, and environmental conditions affect the conversion of resources into functionings and capabilities [50]. A bicycle has different mobility significance depending upon physical ability, knowledge, social norms, infrastructure, safety, and the surrounding environment. Similar reasoning applies to money, employment, education, legal rights, technology, and institutional access.

This insight is foundational for generative return. A returned condition can have changed conversion properties. A professional position restored after harassment can be formally identical while the subject's relation to the organization has changed. Funding restored after exclusion can enter a research network that is differently organized. A legal right restored after a critical developmental period can coexist with opportunities that have disappeared.

Capability analysis therefore supplies a powerful critique of simple resource equivalence. It also provides a vocabulary for real opportunity and human diversity that any theory of generative return should preserve.

The present inquiry extends the analysis along several dimensions. First, it places conversion factors explicitly within time. Harm can alter the conversion mechanism itself. The issue can therefore involve more than a fixed difference between two persons' abilities to use the same resource. The same person can acquire a different conversion structure through history.

Second, capabilities can themselves become generative conditions. Education is a capability and a source of later capabilities. Mobility can produce employment and social relations. Political partici-

pation can alter institutions that affect future participation. Research capacity can produce knowledge that generates later research opportunities. The relation between capability and future capability can consequently be recursive.

Third, the set of capabilities available at one moment may not fully describe the dynamical organization of future possibilities. Two subjects can possess similar current opportunity sets while differing in the stability of those opportunities, their response to perturbation, their ability to generate new opportunities, or the institutional dependence through which the opportunities are sustained.

Fourth, a capability set does not by itself determine which comparative relation should count as adequate restoration. Exact equality of capability sets, set inclusion, threshold sufficiency, dominance, weighted similarity, and dynamic reachability can produce different restorative standards. The later atlas treats these possibilities separately.

The relation between the capability approach and the present inquiry is therefore complementary. The capability approach establishes that means should be distinguished from substantive possibilities. The theory of generative return adds sustained attention to the historical evolution, recursive generation, dynamical robustness, and comparative structure of those possibilities.

2.9 Relational Equality, Oppression, and Social Position

Relational theories of equality shift the normative focus from distributions alone toward the social relations through which persons stand to one another. Anderson's critique of luck egalitarianism develops a conception of democratic equality concerned with a social order in which persons relate as equals [2]. Young's account of justice and difference criticizes the reduction of social justice to distribution and directs attention toward domination, oppression, decision-making, cultural expression, and institutional divisions of labor [71].

These approaches are important for generative return because access to a resource can coexist with an unjust social relation. An employee can receive adequate pay while remaining subject to arbitrary humiliation. A researcher can possess funding while depending upon a gatekeeper with unchecked power over publication, access, or professional standing. A community can receive material benefits while remaining excluded from decisions concerning the conditions of its own production.

A resource-centered restoration can therefore leave the relational architecture of the harm substantially intact. If domination, marginalization, or exclusion affected the subject's generativity, returning resources without changing these relations can reproduce the conditions that generated vulnerability.

Young's critique also warns against treating every social good as a distributive object [71]. Decision-making power, division of labor, culture, and social processes require forms of analysis that exceed the allocation of divisible goods. This observation strongly supports the comparison-object distinction developed later in the paper. Some objects of justice are resources; others are relations, institutional structures, processes, powers, or modes of participation.

Relational equality further complicates the meaning of restoration. Suppose a subject regains every resource held before a wrong while the institutional relationship remains hierarchical and stigmatizing. Resource equivalence can be high while relational equivalence remains low. Suppose instead that an institution is transformed so that the previous relation no longer exists. Relational justice can improve even though restoration to the previous relational structure becomes neither possible nor desirable.

The latter case demonstrates again that restorative justice cannot be defined solely by similarity to the past. A normatively superior transformation can produce a system that is less historically similar and more just.

2.10 Non-Domination and Control over Generative Conditions

Republican theories of freedom add a related concern with domination. Pettit's account defines freedom through non-domination and emphasizes exposure to another actor's capacity for arbitrary interference [44]. Domination can exist even when interference is not continuously exercised because the relevant asymmetry concerns the power structure within which choices are made.

This perspective is especially useful for generative conditions. A subject can retain access to resources while another actor holds discretionary power to remove them. A researcher can possess laboratory access that remains dependent upon unreviewable personal authority. An employee can retain a position while knowing that arbitrary retaliation is possible. A community can use land or infrastructure while lacking secure control over the conditions that make the use possible.

Current output can therefore conceal domination. Two systems can generate similar observable production while differing in the security and independence of the underlying conditions. A restoration criterion based exclusively on outputs or present resource possession can overlook this difference.

Non-domination also connects directly with procedural justice. A restorative intervention can increase the subject's material resources while increasing institutional control over the subject. Conditional assistance, intrusive monitoring, discretionary eligibility, or dependence upon a powerful benefactor can expand some options while creating new domination.

The later atlas therefore includes control relations, security of access, institutional dependence, and intervention structure among possible objects of comparison. A generative condition matters partly through its availability and partly through the structure governing that availability.

2.11 Recognition, Redistribution, and Social Meaning

Recognition theory adds the social interpretation of persons, groups, and relations to the problem of justice. Honneth places recognition at the center of social conflict and develops a normative account of patterns of recognition and misrecognition [25]. Fraser distinguishes socioeconomic injustice from cultural or symbolic injustice for analytical purposes and examines the tensions and interactions between redistribution and recognition [20].

For generative return, this distinction is essential because material and symbolic conditions can interact recursively. Stigmatization can reduce access to employment, collaboration, credit, education, or participation. Material marginalization can in turn reinforce cultural interpretations of inferiority. A program of redistribution can alleviate material deprivation while simultaneously producing stigmatizing social classifications. Fraser's discussion of affirmative and transformative remedies is especially useful because it shows that a remedy can alter one dimension of injustice while generating adverse effects in another [20].

This interaction complicates any claim of equivalence. A person can receive monetary compensation whose amount corresponds to an estimated economic loss while remaining publicly associated with the accusation that produced the loss. A formal correction can remove the accusation from an official record while leaving social knowledge unchanged. A public apology can improve recognition while leaving the material consequences of exclusion unaddressed.

Social meaning can also change through the remedial process itself. A special program can mark recipients as dependent, deficient, privileged, suspect, or permanently victimized. A public correction can draw renewed attention to an old accusation. A reparative classification can become a new social category. The act of repair therefore enters the symbolic system that it seeks to change.

Recognition theory consequently contributes two insights to the present framework. First, meaning and standing are legitimate comparison objects. Second, restorative interventions possess reflexive effects because their classification and public interpretation can change the social relations under comparison.

2.12 Generative Justice and Unalienated Value Circulation

Generative justice provides the most direct theoretical foundation for the present inquiry. Eglash develops generative justice through a critique of extractive and centralized forms of value circulation and through attention to bottom-up systems in which value can remain in unalienated forms and circulate among the communities that generate it [15]. The framework connects justice to control over production, the circulation of generated value, and the capacity of communities to sustain their own generative processes.

This perspective changes the object of justice. A finished allocation is only one part of the relevant structure. The system through which value is produced, extracted, transformed, circulated, and regenerated becomes normatively important. The difference is substantial for the present project because a resource transfer that appears distributively favorable can leave an extractive generative architecture unchanged.

Generative justice also already contains an explicitly dynamical lineage. Eglash and Garvey use the language of attractors and basins of attraction to analyze social ecologies and to conceptualize self-generating forms of social and environmental justice [16]. Their work considers the possibility of social ecologies whose dynamics support adaptive metastability, sustainability, and egalitarian value generation.

This antecedent is important for positioning the present paper. The use of dynamical systems, attractors, or basins in relation to justice is therefore not presented here as a novel contribution. The present inquiry asks a different question concerning comparison after alteration. Even when a generative system is represented dynamically, a claim that generativity has been *returned* requires a standard by which the post-repair system is compared with a reference system. Equality of current states, similarity of attractors, isomorphism of basin structures, similarity of perturbation responses, preservation of reachability, or some weaker functional relation can all produce different judgments.

The distinction becomes even clearer in Eglash and colleagues' later work on computational reparations. That work explicitly interprets reparations through the etymological and conceptual idea of becoming generative again and argues that extraction cannot be adequately addressed by monetary transfer alone. The proposed alternative seeks transitions toward systems in which value generators control their systems of production, resist extraction, and circulate value in unalienated forms [17].

The present paper strongly shares this orientation. It accepts that the justice of repair can concern the restoration or reconstruction of generative conditions and value circulation. Its distinctive problem begins when the phrase "generative again" is treated as a comparative claim.

Several possibilities then arise. A repaired system can regenerate the same amount of value through a different social structure. It can reproduce similar value flows while assigning control differently. It can restore local control while generating a different set of outputs. It can possess an attractor with similar long-run properties while having different basin geometry. It can return control over production while historical changes have made the original network of relations unavailable. It can become more generative than the pre-harm system while remaining less similar to the no-harm counterfactual system.

These cases reveal that generativity is multidimensional. The quantity of value generated, the type of value, the identities of generators, control over conditions, circulation structure, recursive reinvestment, relational effects, ecological effects, robustness, adaptability, and future option space can all matter. A statement that a system is generative therefore cannot by itself determine whether a particular restoration has achieved justice.

The relation between generative justice and the present paper can accordingly be stated with precision. Generative justice supplies a normative orientation toward unalienated value generation, control over generative conditions, and self-sustaining circulation. The theory developed here examines the *comparative grammar of return*: which features of a generative system are being compared, which relation among them is treated as sufficient, which reference system is selected, and which process of transformation can legitimately produce the resulting condition.

2.13 Generative Contexts and Recursive Value Formation

The literature surrounding generative justice also helps move analysis beyond a simple producer–product relation. Generative value can circulate across educational, technological, cultural, ecological, and community contexts. Practices can create artifacts, skills, identities, social connections, knowledge, and further conditions of production. The value created at one stage can become infrastructure for later stages.

This recursive structure matters for return because loss can propagate through the same architecture. Removal of a productive condition can reduce an initial output. The missing output can then fail to

become a later condition. The absence of that condition can change still later relations and outputs. The resulting process can be described as a loss of generative recursion without requiring a deterministic claim concerning the precise future that would have occurred.

The distinction between deterministic prediction and structural possibility is important. Generative justice need not promise exact counterfactual knowledge. A social system can be recognized as having lost a productive feedback structure even when the exact outputs of that structure remain uncertain. A community deprived of a learning institution can lose recursive educational effects without allowing precise calculation of every career that would otherwise have emerged.

This observation will later support comparisons based on transition structures, reachable sets, robustness, and generative mechanisms. Such comparisons ask whether the architecture through which new possibilities arise has been restored, rather than claiming knowledge of every missing outcome.

2.14 Relations Among Distribution, Capability, Recognition, and Generativity

The traditions considered above can be arranged according to different objects of normative attention, although none of them is reducible to a single object. A distributive account can foreground holdings. A capability account can foreground substantive opportunity. A relational account can foreground social standing and institutional relations. A recognition account can foreground social interpretation and status. A republican account can foreground arbitrary power and dependence. A corrective account can foreground the wrongdoer–victim relation and rectification of wrongful loss. A restorative account can foreground harm, accountability, participation, and relational repair. A reparative account can foreground historical responsibility, recognition, reconciliation, and institutional response. A generative account can foreground control of productive conditions and the circulation of value.

These objects can interact while remaining analytically distinguishable. Resources affect capabilities. Capabilities affect the ability to enter relations. Relations affect recognition. Recognition affects access to resources. Domination affects how capabilities can be exercised. Control over production affects value circulation. Value circulation affects future resources and institutional power. Historical wrongs can alter every one of these dimensions.

This interaction creates a strong temptation to select a single master concept. The present paper takes a different methodological approach. The dimensions are kept separate long enough to determine what each contributes to restorative judgment. Their relations can then be studied explicitly.

For example, monetary compensation can restore a resource dimension while leaving recognition unchanged. Public acknowledgment can improve recognition while leaving capabilities constrained. Institutional reform can reduce domination while leaving a particular past loss uncompensated. Assistance can expand present capabilities without satisfying a wrongdoer’s duty of corrective justice. A generative intervention can improve future value circulation while producing no close equivalence to the trajectory that was lost.

The possibility of these cross-dimensional divergences is one of the central reasons for developing an atlas. An institution that reports “restoration” after satisfying one dimension can inadvertently convert a local success into a global claim. The framework developed later allows the statement to remain local: a resource has been restored under a specified resource comparison; a formal status has been restored under a specified institutional comparison; a capability threshold has been reached; a relation of domination has been removed; or a generative process has become self-sustaining. Each can be valuable without implying the others.

2.15 Affirmative and Transformative Modes of Response

A further cross-cutting distinction concerns whether a response operates within an existing structure or transforms the structure that generates the relevant injustice. Fraser’s distinction between affirmative and transformative remedies is especially useful here [20]. An affirmative measure can correct outcomes while leaving deeper structures substantially intact. A transformative measure can alter the institutional or cultural processes that repeatedly produce the problem.

Generative justice also places strong emphasis on transformation of value circulation rather than repeated downstream redistribution [15, 17]. The connection with the present inquiry is direct. Return can target an output, a condition, a conversion mechanism, or the generative structure that produces future outputs.

These levels should remain distinguishable. Compensation for lost income can be appropriate even where institutional transformation is also required. Structural reform does not automatically satisfy an individual's claim for a past wrongful loss. A transformed institution can prevent recurrence while leaving a previously harmed subject without resources for rebuilding a life. Conversely, generous individual compensation can coexist with a structure that continues to expose others to the same harm.

This distinction is particularly important for the temporal architecture developed in later sections. Post-harm rectification, current assistance, and future prevention can require different interventions. A theory of justice that collapses them into one restorative objective risks losing the function of each.

2.16 The Persistence of Singular Value

Several traditions considered above also reveal a difficulty concerning non-fungible and singular goods. Resource theories benefit from the comparability and transferability of resources. Capability approaches enlarge the analysis by examining what resources make possible. Relational and recognition theories show that some goods arise only within particular social relations. Historical reparation shows that objects can derive significance from their provenance and history. Moral repair shows that acknowledgment by a responsible party cannot always be replaced by a materially identical transfer from a stranger.

Generative systems amplify this difficulty. A particular teacher, collaborator, community, ecosystem, historical moment, or intellectual environment can participate in a process whose value emerges from the specific relation among its components. Functional substitutes can exist without being relationally equivalent. A later opportunity can be highly valuable while remaining historically different from the opportunity that disappeared.

The existence of singular value prevents a simple inference from replacement to equivalence. It also prevents the opposite inference that absence of equivalence renders replacement worthless. A new collaboration can be valuable without erasing the loss of an earlier collaboration. A new career can become generative without being equivalent to the career that was interrupted. A new community can support flourishing without reproducing the relations of a destroyed community.

This distinction will later become central to generative assistance. Assistance can create valuable new conditions while openly acknowledging that some lost conditions have no available equivalent.

2.17 Procedural Commitments Across the Traditions

The traditions also differ in the normative importance they assign to process. Corrective justice emphasizes responsibility and the relational structure of the wrong. Restorative justice emphasizes participation, accountability, and the involvement of affected parties. Moral repair emphasizes acknowledgment of responsibility. Relational equality and non-domination constrain the authority through which interventions can be imposed. Capability approaches preserve the importance of agency and substantive freedom. Generative justice emphasizes bottom-up control by value generators.

Taken together, these traditions support a strong conclusion concerning the process of return. A restorative endpoint cannot be evaluated independently of the relations through which it is produced.

A transfer imposed without participation can increase resources while reinforcing domination. A treatment can increase functional capacity while violating bodily autonomy. An institutional reconstruction can improve aggregate output while excluding the persons whose lives are being reorganized. A computational system can increase the efficiency of compensation while centralizing control over data and judgment. A restoration program can recreate a prior relation that the affected subject no longer wishes to enter.

The present paper therefore treats process as an independent object of justice. Later sections distinguish the degree of equivalence achieved by a restorative intervention from the legitimacy of the interven-

tion itself. This distinction draws support from several traditions at once and prevents the mathematical language of equivalence from being converted into an optimization mandate.

2.18 Comparative Structure Across Justice Traditions

The preceding traditions can now be read through a common comparative lens. This lens does not claim that the theories themselves are reducible to formal relations. It identifies a recurring structure implicit in their judgments.

Corrective justice requires a comparison between a wrongful condition and a condition in which the relevant wrongful loss is repaired. Restitution compares possession or entitlement across time. Compensation compares a loss with a transfer judged sufficiently commensurate under some valuation. Capability theory compares substantive possibility. Relational equality compares social standing and the organization of relations. Non-domination compares structures of discretionary power. Recognition theory compares social meaning and status. Restorative justice compares conditions of harm, accountability, relationship, and participation. Reparative justice compares historical and present conditions through a theory of responsibility. Generative justice compares structures of production, control, circulation, and recursive value formation.

The comparison remains incomplete until several further elements are supplied. A theory must identify the reference condition. It must specify the object whose relevant features are being compared. It must specify which differences matter and which can be ignored. It must specify the time over which the comparison is valid. It must identify the value that makes the comparison normatively important. Where the comparison is approximate, it must determine a permissible degree and form of difference.

The omission of these elements is often harmless in ordinary cases because the surrounding practice supplies them tacitly. Returning a borrowed object can make the relevant object and relation obvious. Paying an unpaid invoice can make the monetary quantity obvious. Reinstating a revoked license can make the formal status obvious.

The omission becomes consequential in complex generative cases. “Restoring research capacity” can refer to equipment, funding, access, skill, network position, reputation, institutional authority, output, future opportunity, or some combination. “Restoring a community” can refer to population, territory, governance, cultural practice, economic self-sufficiency, ecological relations, recognition, or collective agency. “Restoring generativity” can refer to value output, control of conditions, recursive circulation, reachability, resilience, or the creation of new possibilities.

Theories of justice can disagree even when they use the same language because they select different comparison objects. They can also agree on an object and disagree on the comparative relation. Two capability theorists can agree that capabilities matter while disagreeing over equality, sufficiency, priority, or another standard. Two relational theorists can agree that domination matters while disagreeing over the institutional transformation required to remove it. Two generative approaches can agree that control over production matters while differing over how much structural similarity to a prior system is desirable.

The later atlas is designed to make these disagreements explicit.

2.19 The Comparative Gap in Theories of Return

The literature reviewed in this section already contains many of the substantive insights required for a sophisticated theory of repair. It would therefore be misleading to present irreversibility, conversion, relational harm, non-fungibility, historical responsibility, recognition, domination, participatory repair, or generative value circulation as newly discovered problems.

The distinctive problem developed in this paper lies at a different level. It concerns the grammar through which claims of return are made across these domains.

A restorative claim can be represented informally as a judgment that a post-response system bears an acceptable relation to some reference system. Existing theories supply rich accounts of what may matter in that judgment. They provide less reason to expect that one comparative relation can represent all of

these objects. Equality of resources, capability sufficiency, relationship repair, removal of domination, recognition, and restoration of generative circulation are heterogeneous comparisons.

The problem becomes especially visible once mathematical and computational relations are introduced. Equality, isomorphism, homomorphism, simulation, bisimulation, observational equivalence, topological conjugacy, statistical indistinguishability, causal equivalence, approximate similarity, dominance, and refinement preserve different structures. None possesses normative priority merely by being mathematically stronger.

The same holds for the object of comparison. State, trajectory, network, reachable set, attractor, basin, response function, causal graph, conversion mechanism, value flow, and historical configuration are different objects. A system can be equivalent under one and inequivalent under another.

The resulting gap can be stated as a need for a systematic theory of *restorative comparison*. Such a theory should be capable of representing at least four distinctions.

First, it should separate the *reference* from the *object of comparison*. The world used as a benchmark and the feature examined within that world are different choices.

Second, it should separate the *object of comparison* from the *comparative relation*. Comparing networks does not determine whether the relevant relation is isomorphism, reachability preservation, similarity of centrality, or another criterion.

Third, it should separate *formal equivalence* from *normative adequacy*. A mathematically precise relation can be ethically irrelevant, and a normatively important relation can resist exact formalization.

Fourth, it should separate *outcome comparison* from *procedural justice*. A highly faithful reconstruction can be achieved through an impermissible process.

These distinctions provide the conceptual bridge from the existing traditions to the atlas developed later in the paper.

2.20 Position of the Present Inquiry

The present inquiry can therefore be positioned as an extension across several established lines of thought.

From corrective justice, it retains the significance of wrongful history, responsibility, and duties of repair. From restitution, it retains the importance of identifiable objects and historical entitlement. From reparative theory, it retains attention to historical injustice, collective responsibility, recognition, civic trust, and institutional redress. From restorative justice, it retains the significance of participation, accountability, affected relationships, and repair processes. From moral repair, it retains the normative importance of acknowledgment and responsibility.

From distributive theory, it retains the importance of resources and the institutional power of transferable means. From the capability approach, it retains the distinction between resources and substantive possibility and the importance of conversion factors. From relational equality and theories of oppression, it retains attention to social structure and institutional position. From republican theory, it retains attention to arbitrary power and secure control. From recognition theory, it retains the irreducibility of social meaning and standing.

From generative justice, it retains the deepest structural orientation of the paper: justice concerns the conditions and processes through which value is generated and circulated, together with the control of those conditions by value-generating communities. The present inquiry further accepts the relevance of dynamical concepts to generative justice already developed in work on attractors and social ecologies [16, 15].

Its additional task is to develop a comprehensive comparative architecture for the problem of return after generativity has been altered. This architecture asks which system is used as the reference, which object is compared, which formal or informal relation counts as adequate correspondence, which tests and perturbations are relevant, which temporal horizon matters, which values justify the comparison, and which processes of transformation remain permissible.

This positioning also limits the novelty claim of the paper. The paper does not claim to originate the insight that money can fail to repair some harms. It does not claim to originate the idea that justice concerns capabilities, relationships, recognition, non-domination, or generative circulation. It does not

claim to introduce dynamical systems into generative justice. It does not claim that historical harms are always fully recoverable through counterfactual calculation.

The contribution lies in bringing these concerns into a common theory of restorative comparison and in expanding that theory into an atlas of possible comparison objects, comparative relations, dynamic structures, value interpretations, and procedural constraints.

This architecture changes the form of the central problem. The relevant inquiry is no longer exhausted by determining whether something has been returned. It requires specifying what has been returned, what reference condition gives the return meaning, which structure has been preserved or reconstructed, which differences remain normatively relevant, and whether the path toward the resulting condition is itself just.

The following sections develop the conceptual structure required before the formal atlas is introduced. They examine generative conditions and modes of generative harm, the historical and relational constitution of subjects, the temporal architecture of prevention, interruption, and post-harm response, the institutional division of labor among law, governance, political economy, and assistance, and the epistemic and structural limits that make a more explicit theory of comparison necessary.

3 Generative Conditions, Value Circulation, and the Structure of Harm

This section develops the conceptual vocabulary required for the later theory of restorative comparison. Its role is analytical rather than classificatory in the sense of the later equivalence atlas. The immediate objective is to distinguish the principal components of a generative system and to identify the different locations at which harm can alter its subsequent development. The discussion proceeds from generative conditions to capacities, conversion structures, generative processes, outputs, recursive value circulation, relations, and future possibilities. It then examines several modes of harm, including deprivation, extraction, obstruction, destruction, capture, distortion, and alteration of conversion structures. The final subsections consider recursive loss, singular opportunities, path dependence, resilience, manufactured vulnerability, and multi-party effects.

The distinctions introduced here are intentionally broader than the categories used by any single field of law, economics, political theory, or social policy. Their purpose is to provide a common vocabulary for later comparison. A loss of property, a loss of professional standing, an interruption of education, a change in health, destruction of an ecosystem, exclusion from a research network, and capture of the value generated by a community can differ greatly in institutional form. Each can nevertheless be examined in terms of the conditions through which subsequent states, relations, capacities, and values are produced.

The concept of generativity used throughout the paper is descriptive before it is normative. A system is generative insofar as its present organization can participate in producing, sustaining, transforming, or opening subsequent states, relations, capacities, values, meanings, or possibilities. Generativity therefore does not by itself imply justice, welfare, equality, or moral desirability. Extractive institutions, systems of domination, monopolies, violent organizations, and ecologically destructive industries can all be highly generative in the descriptive sense that they reproduce structures, relations, incentives, identities, and material outputs. The normative evaluation of what is generated, for whom, under whose control, through which relations, and with which consequences remains a separate task.

3.1 Analytical Representation of a Generative System

A minimal representation of a generative system should distinguish the subject or system whose generativity is under consideration from the conditions, relations, transformations, and environments through which generation occurs. For introductory purposes, let a generative system at time t be represented schematically by Equation 4.

$$\mathcal{G}_t = (X_t, C_t, R_t, \Gamma_t, O_t, \Xi_t, H_t). \quad (4)$$

In Equation 4, X_t denotes the relevant state of the subject or system, C_t denotes generative conditions, R_t denotes relations, Γ_t denotes the processes through which conditions and states generate subsequent

changes, O_t denotes outputs or other generated objects of interest, Ξ_t denotes the surrounding environment and exogenous or partially exogenous influences, and H_t denotes the history relevant to the present configuration.

Equation 4 is not offered as an exhaustive ontology. Some applications will require institutional rules, control structures, stochastic variables, information states, value flows, causal graphs, or multiple interacting agents to be represented separately. Other applications can combine several elements. The representation functions as a decomposition interface. It prevents a statement such as “the system has been restored” from concealing the possibility that one component has been restored while another has changed.

The components are also mutually dependent. Conditions influence state, relations influence how conditions can be used, history influences the meaning of relations, and generated outputs can later become new conditions. A more explicit temporal representation is given in Equation 5.

$$\mathcal{G}_{t+1} = \Phi_t(\mathcal{G}_t, \Xi_t). \quad (5)$$

Equation 5 emphasizes that generativity concerns transition as well as possession. Two systems can contain similar resources at time t while possessing different transition structures. Conversely, two systems with different current resources can generate similar future possibilities through different mechanisms. These possibilities motivate the later distinction among state, functional, structural, behavioral, dynamical, and generative forms of restorative comparison.

3.2 Generative Conditions

A *generative condition* is a condition that participates in the production, maintenance, transformation, or opening of subsequent states, relations, capacities, values, meanings, or possibilities within a specified system and temporal horizon. This definition is deliberately relational. An object does not possess a fixed quantity of generativity independently of its user, environment, relations, institutions, and historical position.

A laboratory illustrates this dependence. Its equipment can constitute a generative condition for a research group possessing relevant knowledge, institutional authorization, collaborators, funding, and research questions. The same equipment can have little immediate generative significance for an actor without these complementary conditions. Conversely, a researcher can retain substantial knowledge while losing the institutional access required to convert that knowledge into experiments, publications, collaborations, and subsequent opportunities.

Generative conditions can therefore be complementary. Let $C_t = \{c_1, c_2, \dots, c_n\}$. The generative contribution of one element cannot generally be assumed to be independent of the others. A simple additive representation of the form

$$G(C_t) = \sum_i G(c_i)$$

will often be inadequate. Complementarity, threshold effects, substitution, interaction, sequencing, and institutional dependence can matter. A more general representation is given in Equation 6.

$$G_t = \mathcal{F}(X_t, C_t, R_t, H_t, \Xi_t). \quad (6)$$

Equation 6 states that the generative significance of conditions is evaluated within a subject–relation–history–environment configuration. The same nominal condition can consequently produce different generative consequences after the system has changed.

This relational definition also prevents an overly narrow association between generative conditions and material resources. Relevant conditions can include at least the following families.

3.2.1 Material and infrastructural conditions

Material generative conditions include land, tools, equipment, housing, transportation, laboratories, computing resources, energy, communications infrastructure, and other physical means through which activities are performed. Their generative significance often depends upon maintenance, location, complementary resources, access rights, and the surrounding institutional system.

3.2.2 Economic and financial conditions

Income, savings, credit, insurance, liquidity, capital, and predictable access to financial resources can support planning, mobility, education, health, entrepreneurship, research, care, and the capacity to absorb shocks. Financial resources are distinctive because their fungibility permits conversion across many domains, while the success of conversion remains dependent upon prices, markets, legal access, health, information, and available substitutes.

3.2.3 Institutional and legal conditions

Licenses, citizenship, professional status, employment rights, academic affiliation, property rights, procedural protections, contractual rights, and access to public institutions can condition which actions are available to a subject. Their significance can exceed the formal content of the entitlement. An affiliation can generate network access, legitimacy, information, professional recognition, and opportunities that are difficult to reproduce through a monetary substitute.

3.2.4 Relational conditions

Teachers, collaborators, colleagues, family relations, communities, professional networks, trusted institutions, mentors, audiences, and reciprocal relations can constitute generative conditions. Their value often emerges from the relation itself and can therefore resist substitution by another actor with similar observable attributes.

3.2.5 Epistemic and informational conditions

Knowledge, education, language, archives, data, methods, access to evidence, interpretive resources, and participation in epistemic communities can generate further knowledge and action. Their effects can be recursive because knowledge changes the questions a subject can formulate, the information the subject can recognize, and the communities within which later learning becomes possible.

3.2.6 Bodily and psychological conditions

Health, mobility, sensory capacities, cognitive resources, attention, energy, sleep, safety, and patterns of learned response can condition participation in other domains. These conditions interact strongly with social and institutional environments. The present framework does not require that every change in behavior or response be medicalized. Historically learned changes can matter for generativity even when they fall outside any clinical category.

3.2.7 Symbolic and reputational conditions

Reputation, credentials, public recognition, trust, classification, identity documents, records, rankings, and social meanings can influence whether other actors enter relations with a subject. Symbolic conditions can therefore have material consequences without being reducible to material objects. Recognition and misrecognition can participate in the production of later access, authority, credibility, and opportunity [25, 20].

3.2.8 Environmental and ecological conditions

Climate, land quality, water, biodiversity, ecological stability, and other environmental conditions can support human and non-human generative systems. Ecological analysis is particularly instructive because the same observable composition can coexist with different resilience, stability, and proximity to regime change. Holling’s distinction between stability and resilience provides an important precedent for separating return to an observable state from the capacity of a system to persist under disturbance [24].

These families are neither exhaustive nor mutually exclusive. Their purpose is to establish that a generative condition can occupy multiple layers of a system, and that restorative comparison will depend upon which layer is selected as the relevant object.

3.3 Generative Capacity, Opportunity, and Process

Generative conditions should be distinguished from *generative capacity*. A condition can be externally available while the subject lacks the present capacity to use it. Conversely, a subject can retain capacities for which the necessary external conditions are unavailable.

This distinction parallels the capability approach’s separation between resources and substantive opportunities [55, 50]. The present framework adds a further distinction between the capacity to realize an available option and the capacity to generate additional options through action.

Let K_t denote a set of capabilities or available action possibilities at time t . A static comparison can ask whether a subject can perform a particular action. A generative comparison asks additionally whether exercise of that capacity creates subsequent conditions, relations, or capabilities. Equation 7 expresses this distinction schematically.

$$K_t \xrightarrow{\Gamma_t} (O_{t+1}, C_{t+1}, R_{t+1}, K_{t+1}). \quad (7)$$

Equation 7 shows why some opportunities are generatively significant beyond their immediate exercise. Education can produce knowledge and credentials that expand later educational and professional options. Participation in a research collaboration can produce both a paper and new relationships. Political participation can change institutions that affect later participation. A period of secure employment can produce savings, professional standing, skills, and network ties.

The distinction also allows generativity to be separated from realized output. A system can possess substantial generative capacity without producing a large observable output during a particular interval. Research exploration, learning, relationship formation, ecological recovery, or institutional preparation can have generative significance before measurable products appear. A theory that evaluates restoration entirely through short-term outputs can therefore underestimate recovered generativity.

3.4 Conversion Structures

The relation between a condition and its generative consequences is mediated by a *conversion structure*. The term extends the capability approach’s attention to conversion factors into a dynamic setting [50]. Conversion structures include the personal, relational, institutional, environmental, informational, and temporal mechanisms through which a condition can be transformed into capabilities, relations, outputs, or subsequent conditions.

Let M_t represent a transferable resource supplied at time t , and let $\mathcal{K}_t$ represent the relevant conversion structure. A schematic conversion is given in Equation 8.

$$M_t \xrightarrow{\mathcal{K}_t} (K_{t+1}, R_{t+1}, O_{t+1}, C_{t+1}). \quad (8)$$

Equation 8 makes explicit why equality of resource transfer does not establish equality of generative effect. If $\mathcal{K}_t$ changes through injury, exclusion, institutional change, loss of networks, or environmental transformation, the same input can produce a different set of consequences.

Conversion structures are especially important for post-harm justice because harm can act upon them directly. A subject can receive compensation while remaining unable to access a professional field. A community can receive funds while lacking institutional authority over the land, infrastructure, or data required to use them effectively. A person can regain formal employment while the relational conditions that previously supported productive work have changed.

The object of restoration can therefore be the resource, the conversion structure, the resulting capability, or some combination. These alternatives will later correspond to different comparison objects in the equivalence atlas.

3.5 Value Generation and Recursive Circulation

Generative justice places particular emphasis on the relation between production and circulation of value [15, 17]. This orientation is important because value generated at one stage can become a condition for further generation.

Consider a simplified process in which a condition C_t supports a generative process Γ_t that produces value V_{t+1} . A portion of that value can then become part of the conditions for subsequent production. Equation 9 represents this recursive structure.

$$C_t \xrightarrow{\Gamma_t} V_{t+1} \xrightarrow{\Lambda_t} C_{t+1} \xrightarrow{\Gamma_{t+1}} V_{t+2}, \quad (9)$$

where Λ_t represents the mechanisms through which generated value is retained, transformed, reinvested, circulated, distributed, or extracted.

Equation 9 makes several distinct normative questions visible. One concerns the amount of value produced. Another concerns who controls the conditions of production. Another concerns who receives the generated value. Another concerns whether value is extracted from the system or returned in forms capable of supporting later production. Another concerns whether the resulting cycle expands or contracts the generativity of other subjects.

These distinctions are central to generative justice. They are also central to the present inquiry because a later return can act at different points in the cycle. An institution can return the original condition C_t . It can return an estimated amount of value corresponding to V_{t+1} . It can provide a new condition C'_{t+1} . It can alter Λ_t so that future value remains within the generating community. These interventions are not equivalent.

A recursive system also makes delayed deprivation more consequential. If V_{t+1} would have become part of C_{t+1} , then the absence of the first output can affect later conditions. The resulting structure is described here as *generative recursion*. A loss that interrupts this recursion can have effects that extend beyond the value of the first missing output.

3.6 Recursive Generative Loss

A useful distinction can be drawn between first-order and higher-order generative losses. A first-order loss concerns the immediate condition or output affected by an intervention. A higher-order loss concerns subsequent generation that depends upon the earlier condition or output.

Suppose an intervention at t_0 removes a condition C_{t_0} . A first-order consequence can be the loss of an output O_{t_1} . If O_{t_1} would also have served as a condition for later processes, its absence changes the inputs available at t_1 . The structure can be represented schematically by Equation 10.

$$\Delta C_{t_0} \longrightarrow \Delta O_{t_1} \longrightarrow \Delta C_{t_1} \longrightarrow \Delta O_{t_2} \longrightarrow \dots \quad (10)$$

Equation 10 should not be read as a deterministic account of missing future value. Each arrow can represent uncertain, nonlinear, contingent, and relational effects. The equation identifies the architecture through which an initial alteration can propagate.

Academic research provides a simple example. Loss of laboratory access can prevent completion of a project. The absent project can mean that a paper is never written. The absent paper can mean that

some readers never encounter the work. Some collaborations that might have followed from the paper therefore do not arise. Later questions, grants, appointments, or methods that would have depended upon those relations may also fail to arise. None of these later outcomes is guaranteed. The loss concerns an altered generative tree rather than a list of certain missing products.

This distinction protects the theory from an inflated counterfactual claim. A subject cannot demand that every favorable future imaginable from a lost condition be treated as a realized loss. The theory instead identifies the alteration of a structure through which possible futures could have been generated.

3.7 Modes of Generative Harm

Generative harm can occur through several structurally distinct mechanisms. Their separation matters because different mechanisms can require different forms of prevention, interruption, responsibility, and post-harm response.

3.7.1 Deprivation

Deprivation removes or withholds a condition that a subject would otherwise possess or access. Examples include loss of employment, equipment, land, institutional access, education, funding, mobility, or legal status. Deprivation primarily acts upon C_t , although its downstream consequences can alter other components of the generative system.

3.7.2 Extraction

Extraction permits generation to occur while transferring a substantial portion of generated value away from the generating subject or community. Generative justice gives this mechanism particular importance [15]. Extraction can leave productive activity intact in the short term while weakening later generative conditions if the extracted value would otherwise support reproduction, maintenance, learning, care, or collective investment.

3.7.3 Obstruction

Obstruction preserves nominal possession of a condition while interfering with its use. A researcher can formally retain laboratory access while administrative barriers make research impracticable. A worker can retain a formal right while retaliation makes exercise of the right costly. A community can possess legal access to resources while procedural complexity prevents effective use.

Obstruction illustrates the importance of distinguishing formal access from effective generative access.

3.7.4 Destruction

Destruction eliminates or severely degrades a condition, relation, or system. The destroyed object can be material, ecological, relational, institutional, bodily, or symbolic. Restoration becomes especially difficult where the destroyed condition is singular or where reconstruction requires a long developmental process.

3.7.5 Capture

Capture alters control over a generative condition or process. The subject can remain productive while another actor acquires power over access, decision-making, distribution, publication, ownership, or the direction of future generation. Capture is therefore closely related to domination and to the generative-justice concern with control over conditions of production [44, 15].

3.7.6 Distortion

Distortion changes incentives, meanings, signals, classifications, or feedback in ways that redirect generation. A reputational system can reward conformity at the expense of exploration. A metric can redirect research toward measurable outputs. A public classification can alter how others interact with a subject. The relevant harm concerns the direction and organization of generation rather than complete loss of the capacity to generate.

3.7.7 Conversion impairment

Conversion impairment leaves a resource nominally available while weakening the mechanisms through which the resource can become capability, relation, output, or further condition. Loss of professional networks, institutional trust, health, legal recognition, or complementary infrastructure can produce this effect.

Conversion impairment is especially important for return because the original resource can later be restored while its earlier conversion pathway remains damaged.

3.7.8 Relational displacement

Relational displacement changes the subject's location within a network or relation structure. The subject can lose collaborators, become dependent on a new gatekeeper, move from a central to peripheral institutional position, or lose access to communities in which information and opportunities circulate. The subject can retain many individual resources while experiencing substantial generative contraction through changed relational position.

3.7.9 Temporal displacement

Temporal displacement delays access to a condition or opportunity beyond the period in which its generative significance was greatest. A delayed scholarship, diagnosis, publication, legal status, professional license, or educational opportunity can eventually be granted while the relevant developmental, professional, or historical window has changed.

Temporal displacement is one reason that return at t_1 cannot be assumed to be equivalent to uninterrupted possession from t_0 .

3.7.10 Recursive suppression

Recursive suppression occurs when a loss reduces later generative conditions and the weakened later conditions further reduce subsequent generation. This mechanism can produce self-reinforcing contraction. It is analytically distinct from the original deprivation because subsequent losses can arise through the changed system rather than through repeated direct intervention by the original actor.

These modes can occur simultaneously. Workplace bullying can combine relational displacement, distortion of social signals, conversion impairment, obstruction, and recursive suppression. Colonial dispossession can combine deprivation, extraction, capture, destruction, symbolic reclassification, and intergenerational recursive effects. A digital blacklist can combine reputational distortion, relational displacement, institutional obstruction, and automated propagation.

The point of the taxonomy is not to assign one label to each case. It is to identify the mechanisms through which generativity can be altered.

3.8 Loss of Generative Conditions and Alteration of Generativity

A central distinction of the paper is the difference between losing a generative condition and experiencing an alteration of generativity.

Let C_t denote a condition and Γ_t the generative process through which the relevant system develops. A deprivation can initially be represented as a change

$$C_t \rightarrow C'_t.$$

If the deprivation persists, however, the system can change such that

$$\Gamma_t \rightarrow \Gamma'_t.$$

This second transition is conceptually different. The problem has moved from the absence of an input to an alteration of the process through which inputs are converted into future states and possibilities.

The distinction is visible in professional life. A person excluded from work loses income and occupational access. A prolonged exclusion can also change skills, relationships, reputation, confidence in institutions, geographic location, family responsibilities, or willingness to return to the same field. Restoration of the original employment condition then operates upon a changed generative process.

The distinction can be summarized by Equation 11.

$$C'_t = C_t \not\Rightarrow \Gamma'_t = \Gamma_t. \quad (11)$$

Equation 11 expresses one of the central motivations for the later atlas. Equality of conditions is one restorative criterion. Equality, similarity, simulation, or sufficient correspondence of generative processes constitutes another.

3.9 Generative Significance of a Condition

The previous distinction can be expressed through the concept of *generative significance*. Let the generative significance of a condition c for subject or system A at time t be represented schematically by Equation 12.

$$\Sigma_t(c; A) = \Sigma(c, X_t^A, R_t^A, H_t^A, \Xi_t). \quad (12)$$

Equation 12 emphasizes that generative significance is indexed by the condition's relation to a particular system and history. Returning an object c at a later time can satisfy object identity:

$$c_{t_1} = c_{t_0},$$

while failing to restore its earlier generative significance:

$$\Sigma_{t_1}(c; A) \neq \Sigma_{t_0}(c; A).$$

This distinction provides a more precise formulation of the intuition that the same workplace, laboratory, social status, or financial resource can become a different condition for a historically changed subject.

Generative significance also depends upon horizon. A condition can have low immediate output and high long-term significance. Education, preventive health, basic research, trust, ecological resilience, and institutional legitimacy can all have this structure. Short-horizon restorative comparison can therefore misrepresent the significance of conditions whose effects accumulate through longer processes.

3.10 Singular Generative Conditions

Some generative conditions possess high substitutability. A standard tool can be replaced by an equivalent model. Fungible money can be replaced by another unit of the same currency. A generic service can often be purchased from multiple providers.

Other generative conditions derive substantial value from singular relations, timing, provenance, identity, or historical configuration. The paper calls these *singular generative conditions*. Their significance depends upon features that cannot be preserved merely by replacing them with another object of similar market value or apparent function.

A particular mentor can be a singular generative condition because the relation depends upon the mentor's questions, experience, trust, intellectual style, network, timing, and interaction with the student. A particular ecological site can be singular because of its historical and biological composition. A cultural archive can be singular because provenance is part of its meaning. A period of childhood education can be singular because the same educational input provided decades later does not occupy the same developmental position.

Singularity should be treated as graded rather than absolute. Many conditions have both substitutable and non-substitutable dimensions. A lost mentor can be partly replaced with access to other excellent scholars. The new relations can generate valuable futures. The existence of valuable substitutes does not establish equivalence with the lost relation.

This distinction allows the theory to avoid two extremes. One extreme treats every loss as monetizable and replaceable. The other treats every historically specific condition as absolutely incomparable. A more useful approach examines which relevant structures can be preserved by substitution and which cannot.

3.11 Temporal Windows and Opportunity Structure

Generative significance can depend upon timing. A condition can become available after a temporal window in which its effects would have been different. This is particularly important in education, health, research, employment, political participation, family life, and technological development.

Let c be a condition and let $\Sigma_t(c)$ denote its generative significance at time t . There is no general reason to expect

$$\Sigma_{t_0}(c) = \Sigma_{t_1}(c).$$

The difference can arise because the subject has changed, because other actors have changed, because institutions have changed, because complementary conditions have disappeared, or because the surrounding field has moved.

Academic publication provides an illustrative case. A manuscript published in one period enters a particular literature, set of debates, readership, institutional environment, and network of possible collaborators. Publication of a similar manuscript years later can remain valuable while entering a different generative environment. The later publication is therefore not naturally represented as the same event shifted along a time axis.

Temporal windows also complicate monetary compensation. A large later transfer can finance new opportunities while remaining unable to reopen an earlier window. Post-harm justice can consequently support new generation without claiming that the original opportunity has been returned.

3.12 Path Dependence and Historical Sequence

The importance of timing and sequence has a substantial literature outside justice theory. Arthur's analysis of increasing returns and technological lock-in shows how contingent historical events can influence later outcomes in processes characterized by self-reinforcement [3]. Pierson develops increasing-returns accounts of path dependence in political analysis and emphasizes the importance of sequence, timing, critical junctures, and the increasing difficulty of reversal once a particular path develops [45].

The present framework uses path dependence in a broad but disciplined sense. Historical sequence matters when the effect of a condition depends upon earlier states and when present configurations influence which future transitions are available or probable. Generative harm can then alter later possibilities without determining one unique counterfactual future.

Path dependence also clarifies why delayed correction can have limited reach. Suppose a false classification causes an institution to reject an applicant. The applicant then enters another environment, acquires different experience, forms different relations, and makes later choices under the new trajectory. Removal of the original classification years later does not reproduce the sequence that would have followed from the earlier acceptance.

This conclusion does not imply that every small event creates unlimited liability for every subsequent difference. The legal problem of attribution remains distinct from the structural problem of historical dependence. The framework identifies possible causal and generative propagation; later normative and institutional sections determine which portions of that propagation should matter for responsibility, compensation, assistance, or prevention.

3.13 Attractors, Basins, and Regime Change

Some generative alterations concern the organization of long-run dynamics rather than immediate output. Dynamical systems concepts can help distinguish these cases. Eglash and Garvey already introduce attractors and basins into generative-justice analysis and use them to conceptualize social ecologies and self-generating structures [16]. The present section uses these concepts to clarify different locations of generative harm.

Let a system possess an attractor $\mathcal{A}$ with basin $\mathcal{B}(\mathcal{A})$. A harmful intervention can affect at least four different structures.

It can displace the current state while leaving the attractor and basin largely unchanged. In that case, ordinary recovery can return the system toward its previous mode.

It can shrink the basin while preserving the attractor. The system can still support the earlier mode, yet a smaller class of disturbances can be absorbed without transition.

It can alter the attractor itself. The system can then stabilize around a different pattern of behavior or relation.

It can alter the wider attractor landscape, creating new stable regimes, removing old ones, or changing the thresholds at which transitions occur.

These possibilities show why restoration of a current state does not determine restoration of the surrounding dynamic structure. A subject can appear to have returned to ordinary functioning while occupying a system with lower resilience to subsequent disturbance.

3.14 Resilience and Generative Robustness

Holling's classic distinction between stability and resilience is useful for this analysis [24]. Stability can concern the tendency to return toward an equilibrium following disturbance, while resilience concerns the capacity of a system to absorb disturbance without undergoing a fundamental change in structure or regime. Later ecological work continues to distinguish alternative senses of resilience and their relation to nonlinear system dynamics.

For generative return, the relevant lesson is methodological. A system can match a reference under ordinary conditions while differing substantially in its response to perturbation. A restored worker can perform normally under stable conditions while having a narrower margin for tolerating organizational conflict. A restored research group can produce similar output while depending upon a fragile funding arrangement. A restored community can regain economic activity while remaining more exposed to environmental or political shocks.

The paper calls the corresponding property *generative robustness*: the capacity of a generative system to continue producing or reopening relevant possibilities across a specified class of disturbances.

Generative robustness is explicitly indexed to the disturbance class. Robustness to economic shocks does not imply robustness to reputational shocks, health shocks, political intervention, ecological disturbance, or relational conflict. The later equivalence atlas therefore treats perturbation classes as part of the specification of restorative comparison.

3.15 Adaptive Change and the Ambiguity of Recovery

Adaptation creates a further complication. A subject can respond to deprivation by developing new capacities, preferences, relationships, strategies, and identities. Some adaptations increase later generativity. Others reflect constraints imposed by the harmful environment. Many contain both features.

A simplistic restoration theory can treat every deviation from the counterfactual trajectory as evidence of remaining harm. This approach risks denying the reality and agency of the present subject. A person who develops a new career after exclusion from an earlier profession can value the new life. Restoration should not require that this development be erased.

An opposite error treats successful adaptation as proof that no serious loss occurred. The existence of a valuable new trajectory does not establish that the earlier deprivation was harmless or fully compensated.

The distinction can be represented by separating historical value from current value. A present state can be valuable in its own right while the process through which it became necessary remains unjust. The theory of return must therefore accommodate both acknowledgment of irreversible loss and respect for newly generated value.

This point becomes particularly important for procedural justice. A restorative institution cannot assume that the subject should prefer the closest available approximation to a no-harm counterfactual. The present subject must retain a role in deciding which future to pursue.

3.16 Negative Generative Cycles

Generativity is often associated with positive creation, yet harmful processes can also be recursively generative. An initial disadvantage can create conditions that produce further disadvantage. The resulting structure can be called a *negative generative cycle* from the perspective of the affected subject.

A simplified example is given in Equation 13.

$$\text{loss of standing} \rightarrow \text{loss of access} \rightarrow \text{loss of opportunity} \rightarrow \text{weaker future standing} \rightarrow \text{further loss of access.} \quad (13)$$

Equation 13 illustrates why early interruption can possess a value distinct from later compensation. A process that remains active can continuously produce new conditions of disadvantage. Intervention during the process can therefore prevent later harms that would be difficult to identify or reverse individually.

Negative generative cycles can involve material, symbolic, relational, and institutional components simultaneously. A reputational injury can reduce employment, reduced employment can reduce network participation, reduced participation can weaken reputation, and weakened reputation can further reduce employment. The categories interact recursively.

This structure supplies an important bridge to the later temporal division between prevention, interruption, and post-harm response. Prevention seeks to avoid entry into a harmful generative cycle. Interruption seeks to stop its continued self-production. Post-harm response operates upon the system after some portion of the cycle has already occurred.

3.17 Manufactured Vulnerability

A particularly important structure arises when an actor or institution first reduces another subject's generative conditions and later treats the resulting weakness as an independent property of that subject. This paper refers to this structure as *manufactured vulnerability*.

Let A exercise control over conditions relevant to B 's generativity. Suppose an intervention H_A reduces B 's access, relations, capacity, or conversion structure. At a later time, the altered state of B can be observed without reference to the intervention that helped produce it. The resulting judgment can then be used to justify further exclusion or dependency.

The structure is represented schematically in Equation 14.

$$H_A \rightarrow \Delta \mathcal{G}_B \rightarrow \text{observed vulnerability of } B \rightarrow \text{further restriction of } B. \quad (14)$$

Equation 14 describes a feedback structure, not a complete account of responsibility. The vulnerability observed at the later stage can have multiple causes. The normative concern arises when a historically produced condition is interpreted as evidence of an intrinsic deficiency while the relational history that contributed to the condition is ignored.

Manufactured vulnerability is relevant to employment, dependency, institutional exclusion, colonial relations, disability, poverty, educational inequality, and other settings. Its significance for generative return lies in the possibility that the subject who later receives assistance is judged only through the post-harm state. A purely present-state assessment can then obscure the process through which the need for assistance arose.

The concept also creates a connection between justice of return and epistemic justice. Accurate normative evaluation requires attention to the history through which present capacities and vulnerabilities were generated.

3.18 Control, Dependency, and Conditional Generativity

Generativity can depend upon conditions controlled by another actor. Such dependence is not automatically unjust. Cooperation, employment, education, public administration, and collective production require many forms of conditional access and coordinated authority.

The normative problem concerns the structure of control. A subject can possess substantial productive capacity while another actor retains discretionary power to remove the conditions required for its exercise. Republican theories of non-domination make this distinction particularly clear [44]. The relevant freedom can depend upon security against arbitrary interference as well as upon the absence of actual interference.

For generative systems, control can therefore be represented separately from current access. Let $A_t(c)$ represent present access to condition c , and let $D_t(c)$ represent the structure governing the discretion to maintain, alter, or revoke that access. Equality of $A_t(c)$ across two systems does not imply equality of $D_t(c)$.

This distinction is important for restoration. Reopening access while leaving the subject exposed to the same arbitrary control can reproduce the generative insecurity associated with the original harm. Conversely, changing the control structure can improve justice even when the material condition itself remains unchanged.

Generative justice similarly emphasizes control over conditions of production as part of the normative architecture of value circulation [15, 17]. The present framework therefore treats control structure as a possible independent object of restorative comparison.

3.19 Meaning, Interpretation, and Relational Usability

A condition can be formally available and functionally intact while acquiring a different meaning for the subject. This difference can affect whether and how the condition is used.

Consider again a workplace after prolonged harassment. A meeting room, supervisor's message, performance review, or request for collaboration can possess one meaning before the history of harassment and another after it. The physical objects and formal procedures can remain unchanged. Their position within the subject's relational interpretation has changed.

The paper refers to the capacity of a condition to be used within the subject's current relational and interpretive structure as its *relational usability*. Relational usability differs from formal access and from physical functionality.

A condition can therefore satisfy

$$A_t(c) = 1$$

in the sense that access exists, while its effective generative significance remains low because the relation in which the condition is embedded has changed.

This concept should be treated carefully. It does not imply that subjective interpretation alone determines justice. Other actors, institutions, public reasons, and external constraints remain relevant. The point is narrower: formal return cannot by itself establish that a condition occupies the same generative role in the current relational system.

3.20 Multi-Party Generative Systems

Many generative systems involve more than an injurer and an injured party. Employers, colleagues, customers, governments, families, insurers, taxpayers, professional communities, future participants, and unrelated third parties can all be affected by restoration.

Let a social system contain subjects $A_1, A_2, \dots, A_n$, with coupled generative processes. A restorative intervention directed toward A_i can change the conditions available to A_j . The corresponding system cannot be represented as a collection of independent individual restoration problems.

A simple coupled representation is given in Equation 15.

$$\mathcal{G}_{i,t+1} = \Phi_i(\mathcal{G}_{i,t}, \mathcal{G}_{-i,t}, \Xi_t), \quad (15)$$

where $\mathcal{G}_{-i,t}$ denotes the relevant states and actions of other participants.

Equation 15 makes third-party burdens visible. Reinstating one employee can alter another employee's assignment. Returning land can affect current occupants. Correcting a public historical narrative can change collective identities. Public compensation can distribute costs through taxation. Environmental restoration can constrain present economic activity.

These consequences do not automatically defeat restoration. They establish that the justice of return can require a multi-party analysis. The comparison object can include the affected subject's generativity, third-party generativity, institutional structure, distribution of burdens, and system-level effects.

This feature will later complicate any attempt to maximize restorative equivalence for one subject independently of others.

3.21 Collective Generativity

The concept of generativity can also apply to collective systems. Communities, research groups, institutions, professions, cultures, and political organizations can possess structures through which knowledge, practices, resources, roles, and identities are reproduced and transformed.

Collective generativity should not be treated as a simple sum of individual generativities. A language, legal institution, scientific community, or ecological-management practice can depend upon relations and distributed knowledge that no individual possesses alone. Damage to collective structure can therefore survive even when individual resources are compensated.

Collective restoration introduces difficult boundary questions. Which persons constitute the relevant collective? Which historical configuration is the reference? Which internal differences should be preserved? Who has authority to speak for the collective? Can restoration of collective generativity constrain individual exit or dissent? These questions prevent the collective level from becoming an automatic extension of individual restoration.

Generative justice is particularly relevant here because its emphasis on community control and circular value flows directly addresses collective production [15]. The present paper extends the comparison problem to collective systems without presuming that every collective has a single coherent generative interest.

3.22 Generativity Across Scales

Generative processes operate across multiple scales. A household can generate care and education. An organization can generate knowledge and employment. A city can generate economic and cultural rela-

tions. A legal system can generate expectations and institutional pathways. An ecosystem can generate biological and material conditions upon which other systems depend.

A process that is generative at one scale can be destructive at another. A firm can increase its internal productive capacity by externalizing environmental costs. A university can increase publication output through practices that reduce the long-term generativity of junior researchers. A state can increase short-term administrative efficiency through surveillance that weakens trust and civic autonomy.

The normative evaluation of generativity therefore requires explicit scale. Let $G^{(s)}$ denote generativity evaluated at scale s . There is no general reason to expect

$$G^{(s_1)} \uparrow \Rightarrow G^{(s_2)} \uparrow .$$

The possibility of cross-scale conflict is another reason generativity itself cannot function as a simple maximand.

For restorative comparison, scale determines the object of concern. A remedy can restore an individual's income while leaving community generativity unchanged. It can restore a community's productive infrastructure while reducing ecological resilience. It can improve system-level efficiency while reducing a particular subject's autonomy.

The later value-theory sections therefore examine which scale and whose generativity a theory of justice treats as normatively relevant.

3.23 Normative Neutrality of Generativity

The broad definition of generativity makes an explicit normative qualification necessary. Increased generativity is not inherently better. A system can generate inequality, propaganda, ecological destruction, coercive dependency, weapons, discriminatory classifications, or increasingly effective mechanisms of extraction.

A theory of justice cannot therefore adopt

$$\max G$$

as a sufficient normative objective.

This point distinguishes the descriptive concept of generativity from *generative justice*. Generative justice introduces normative concerns about control, alienation, circulation, and the distribution of generative benefits [15]. The present theory similarly requires independent criteria concerning rights, equality, capabilities, recognition, non-domination, process, and other values.

The normative neutrality of generativity also limits the concept of “restoring generativity.” Restoring the capacity of an unjust institution to reproduce itself can strengthen injustice. Restoration to a prior generative structure can be normatively inappropriate where that structure was itself extractive or dominating.

This consideration reinforces the distinction among historical, counterfactual, and normative reference systems. The system that would have existed without a particular harm need not be the system that justice should create.

3.24 Generative Harm and Normative Harm

A parallel distinction should be maintained between *generative harm* and *normative harm*. A change can reduce a subject's generativity without being unjust. Aging, ordinary competition, natural disasters, voluntary sacrifice, lawful institutional change, and unavoidable scarcity can all reduce some generative possibilities. Conversely, an unjust act can violate rights or dignity without producing a measurable long-term reduction in generativity.

The paper therefore does not define injustice as generativity loss. Generative analysis identifies a dimension of consequence. Normative theory determines whether the intervention, distribution, relation, or process is unjust.

This distinction matters for responsibility. A person can require assistance because of severe generative contraction even when no wrongdoer can be identified. A wrongdoer can owe acknowledgment or restitution even when the victim later adapts successfully and experiences little measurable generative loss.

The separation between generative and normative analysis allows the later institutional discussion to distinguish corrective duties from social assistance, prevention from liability, and recognition from compensation.

3.25 Problem Structure of Generative Harm

The distinctions developed in this section can be assembled into a general problem structure. A harmful intervention H_t can act upon one or more components of a generative system:

$$H_t : (X_t, C_t, R_t, \Gamma_t, \Xi_t) \longrightarrow (X'_t, C'_t, R'_t, \Gamma'_t, \Xi'_t) .$$

The altered system then continues to evolve. Generated outputs and missed outputs affect later conditions. Relations respond to changed states. Subjects adapt. External actors react. Institutional classifications propagate. Subsequent opportunities appear or disappear.

A later restorative intervention R_{t+k} consequently operates on

$$\mathcal{G}'_{t+k},$$

rather than on the earlier system

$$\mathcal{G}_t.$$

The central restorative problem is therefore not exhausted by asking whether the initially altered component can be replaced. The theory must determine which component or structure at $t + k$ should be compared with which reference, and under which criterion.

The possibilities already visible at this stage include:

- return of an identical object;
- restoration of resource value;
- restoration of formal access;
- restoration of effective access;
- restoration of capability;
- restoration of relational position;
- restoration of control over generative conditions;
- restoration of conversion capacity;
- restoration of recursive value circulation;
- restoration of reachable future possibilities;
- restoration of dynamical resilience;
- creation of new generative conditions without equivalence to the lost conditions.

These possibilities are not interchangeable. Their separation is the immediate conceptual result of this section.

3.26 Implications for the Theory of Return

Several implications follow from the conceptual structure developed above.

First, the object initially taken and the object ultimately requiring normative attention can differ. A deprivation of a material condition can later alter relations, capabilities, conversion structures, or generative dynamics.

Second, return of an original condition can be valuable without constituting complete restoration. The significance of restitution should therefore be preserved without treating object return as a universal model of repair.

Third, generative loss can propagate recursively. The theory should recognize this architecture while avoiding unsupported claims about specific counterfactual outcomes.

Fourth, singular and temporally sensitive conditions create limits on substitution. New opportunities can support valuable futures without becoming equivalent replacements for lost opportunities.

Fifth, robustness and resilience can matter independently of ordinary observed performance. A system that appears restored under baseline conditions can have a different response to later perturbations.

Sixth, control over conditions can matter independently of current access. A subject whose productive activity remains dependent upon arbitrary power can remain generatively insecure.

Seventh, present adaptation creates both value and difficulty for restoration. The current subject possesses agency and interests that cannot be subordinated automatically to reconstruction of a hypothetical earlier trajectory.

Eighth, generativity is normatively plural and scale-dependent. Increased generation at one level can coexist with domination, extraction, or destruction at another.

Ninth, post-harm assistance can remain normatively important even where no well-defined equivalent of the lost condition exists. Its purpose can be the opening of a viable future rather than replication of a vanished past.

These implications motivate the next stage of the paper. Before the formal atlas can specify what counts as equivalence, the historical and relational constitution of the subject must be examined in greater depth. The same external condition can acquire a different generative significance because the subject, surrounding relations, and interpretive environment have changed. Section 4 therefore turns from the structure of generative conditions to the structure of the historically constituted subject that receives, loses, uses, and later encounters those conditions.

4 Historical and Relational Constitution of Generative Subjects

This section develops the philosophical account of the subject presupposed by the theory of generative return. Its role is to clarify why restoration cannot be evaluated solely by comparing external resources or institutional conditions across two dates. A subject who loses a generative condition continues to develop during the interval of deprivation. Relationships change, memories accumulate, expectations are revised, bodily dispositions develop, capacities are exercised or left unused, social classifications affect later encounters, and new commitments can arise. The subject who later receives a returned condition is therefore historically continuous with the earlier subject while also being historically changed.

The discussion proceeds through several analytical layers. It first locates the subject within relational and process-oriented accounts of social life. It then examines social constitution, historical continuity, embodiment, memory, expectation, narrative identity, and situated autonomy. Subsequent subsections consider adaptive transformation, the relation between actual and counterfactual subjects, continuity across restoration, the relational usability of returned conditions, and the normative importance of preserving the agency of the present subject. The section concludes by specifying the consequences of historical and relational subjecthood for later restorative comparison.

The argument does not require a claim that subjects are completely determined by social relations or historical circumstances. Agency, reflection, resistance, novelty, reinterpretation, and self-transformation remain central. The relevant claim is more limited: the capacities through which a subject acts are ex-

exercised within histories and relations that can themselves be altered by harm. A theory of return must therefore evaluate both the conditions offered to the subject and the historically developed subject who encounters those conditions.

4.1 Relational and Processual Conceptions of the Social Subject

A substantial tradition in social theory challenges descriptions of the social world that begin from fully formed, self-contained entities and treat relations as secondary connections among them. Emirbayer contrasts substantialist approaches with a relational perspective centered on dynamic, unfolding processes and transactions [18]. The significance of this distinction for generative return is methodological. If a subject's properties, opportunities, identities, and capacities are partly constituted through relations, alteration of those relations can alter the subject's effective generativity even where many internal or material attributes remain unchanged.

Mead's social account of the self provides an earlier and particularly relevant formulation. In *Mind, Self, and Society*, the self develops through social processes in which the attitudes and responses of others become incorporated into the organization of conduct [35]. The self is therefore connected to patterns of interaction, communication, expectation, and social response. This orientation helps explain why reputational, institutional, and relational harms can enter the later organization of action without requiring the affected person to lose any single material object.

A relational analysis can be expressed schematically without treating the subject as reducible to an externally observable network. Let the state of a subject A at time t be represented by Equation 16.

$$A_t = \Psi(B_t, M_t, K_t, R_{0:t}, H_{0:t}, I_t, \Xi_{0:t}), \quad (16)$$

where B_t denotes relevant bodily conditions, M_t denotes memory and interpretive organization, K_t denotes capacities, $R_{0:t}$ denotes the history of relevant relations, $H_{0:t}$ denotes experienced events, I_t denotes institutional positions and classifications, and $\Xi_{0:t}$ denotes relevant environmental conditions.

Equation 16 is an analytical representation. It does not define an exhaustive metaphysics of personhood. Its purpose is to preserve distinctions that become normatively important in restoration. A subject's present capacity can depend upon bodily state, institutional standing, learned interpretation, social relations, and previous events simultaneously.

The representation also makes temporal dependence explicit. A subject at t_1 cannot generally be reconstructed from a list of resources currently held at t_1 . The pathway through which the subject arrived at those resources can matter for their meaning, usability, and future generative effects.

4.2 Social Constitution and Individual Agency

Relational constitution should be distinguished from social determination. A subject can be socially constituted while retaining meaningful agency, reflection, resistance, and the capacity to revise inherited relations. This distinction is essential for the normative use of relational theory.

The literature on relational autonomy provides a particularly useful framework. Mackenzie and Stoljar collect approaches that examine how identity, agency, self-conception, self-worth, memory, imagination, and autonomy are shaped through social relations while preserving autonomy as an important normative ideal [31]. Barclay similarly develops an account of autonomy that takes the social formation of identity and aspiration seriously [5].

This tradition is important for generative return because a purely atomistic account of the subject creates two distortions. The first appears when external conditions are treated as detachable inputs supplied to an otherwise unchanged individual. The second appears when support for a relationally constituted subject is interpreted as requiring external actors to decide what the subject should become.

A relational theory of generative return should avoid both distortions. Relations participate in the formation of agency, while agency remains relevant to the justice of later interventions. The social constitution of preferences, skills, identities, and expectations does not authorize an institution to replace the subject's present judgment with the institution's preferred counterfactual reconstruction.

This point becomes especially important after severe disruption. The subject's current preferences can reflect adaptation to constrained conditions. They can also reflect genuine learning, new commitments, moral development, changed relationships, or discoveries that the subject now values. These possibilities can coexist. A restorative theory therefore requires a more careful analysis than either complete deference to current preference or complete replacement of current preference by a hypothetical pre-harm preference.

4.3 Historical Constitution and Temporal Accumulation

A historical subject carries forward consequences from previous periods. Skills accumulate through practice. Knowledge develops through prior learning. Trust can develop through repeated reliable interaction. Expectations can change through repeated institutional response. Bodily capacities can improve or deteriorate. Social networks can deepen, weaken, or disappear. Opportunities can open because of earlier outputs, and later opportunities can depend upon relations formed through earlier ones.

Historical constitution therefore involves more than memory of past events. Past conditions participate in the organization of the present system.

Let the evolution of the subject be represented by Equation 17.

$$A_{t+1} = \Phi_t(A_t, C_t, R_t, E_t), \quad (17)$$

where C_t denotes relevant generative conditions, R_t denotes relations, and E_t denotes events occurring during the interval.

Equation 17 shows that a temporary change in C_t can enter later states of the subject even if the condition is restored at a later time. The subsequent state carries the accumulated consequences of the intervening evolution.

Suppose that a harmful intervention modifies the available conditions during the interval $[t_0, t_1]$. The subject developing under the altered conditions can be represented by $A_{t_1}^H$. A counterfactual subject developing without the intervention can be represented by $A_{t_1}^C$. The two states can diverge:

$$A_{t_1}^H \neq A_{t_1}^C. \quad (18)$$

Equation 18 does not imply that $A_{t_1}^C$ is fully knowable. It identifies the conceptual distinction between the actual historical subject and the counterfactual subject invoked by many restorative claims. Later sections examine the epistemic status of such counterfactuals in greater detail.

The distinction also reveals why delayed return can fail to reproduce earlier generative relations. A condition restored at t_1 interacts with $A_{t_1}^H$, while the counterfactual comparison concerns a condition interacting with $A_{t_1}^C$. Equality of the external condition therefore does not entail equality of the resulting subject-condition system.

4.4 Embodiment and Historical Carryover

Historical change is partly embodied. Human generativity depends upon health, energy, sensory and motor capacities, attention, stress responses, learned skills, habits, and other bodily conditions. These conditions can change through experience and can influence how later environments are encountered.

Embodiment matters for generative return in at least three ways.

First, bodily change can affect conversion. A financial resource, employment opportunity, educational program, or physical environment can become easier or harder to use depending upon health and bodily capacity. Resource restoration therefore interacts with the embodied state of the recipient.

Second, bodily history can alter responses to later environments. A person returning to a previously harmful workplace can encounter physical spaces, organizational signals, interpersonal styles, or institutional procedures through learned patterns that differ from those present before the harm. Restoration of the external environment does not by itself determine the subject's later response.

Third, some bodily changes have their own temporal structure. Rehabilitation can require time. Skills can decay through non-use. Developmental opportunities can be age-sensitive. Chronic impairment can alter occupational and social possibilities. A delayed remedy can therefore confront an embodied subject for whom the original condition has a different practical meaning.

The theory developed here does not require a medical interpretation of all such changes. Embodiment includes ordinary processes of learning, fatigue, adaptation, aging, habituation, and skill formation. The normative question concerns the effect of these processes on present and future generativity.

4.5 Memory and the Historical Availability of the Past

Memory creates another form of historical carryover. Harm can persist in the present through remembered events, yet memory also supports learning, responsibility, identity, testimony, and practical judgment. A theory of restoration therefore cannot treat memory solely as a residual trace whose removal would increase restorative fidelity.

Memory has several roles in generative systems. It can preserve knowledge of risk. It can support avoidance of previously harmful relations. It can motivate institutional reform. It can influence trust. It can provide evidence in later legal or political processes. It can become part of personal and collective identity. It can also constrain participation in environments associated with earlier harm.

These roles create an important asymmetry between a world in which harm never occurred and a world in which harm occurred and was later repaired. In the second world, the subject can know that the harm occurred. Other actors can know it as well. That knowledge can become part of subsequent decision-making.

A hypothetical technology capable of deleting every memory associated with the harm would therefore alter more than distress or behavioral response. It could remove evidence, learning, commitments, relationships, and parts of the subject's present interpretation of self and world. Stronger similarity to a selected counterfactual behavioral state could thereby conflict with autonomy, historical continuity, epistemic justice, and procedural legitimacy.

Memory consequently becomes both a component of the subject and a possible object of restorative controversy. Later sections use this structure to examine conflicts between restorative fidelity and procedural justice.

4.6 Expectation and Anticipatory Structure

Generative action depends partly upon expectations about future response. A researcher chooses whether to begin a long project partly on the expectation that institutional access will remain available. An employee decides whether to report misconduct partly on expectations concerning retaliation and review. A student chooses whether to invest effort in a field partly on expectations about educational and professional opportunity. A community invests in shared infrastructure partly on expectations concerning future control and benefit.

Harm can change these anticipatory structures.

Suppose an institution restores a formal right after previously violating it. The current entitlement can be identical to the earlier entitlement, while the subject's expectation concerning its security has changed. The generative significance of the right can therefore remain different.

Let $p_t(a \mid s)$ represent a subject's practical expectation that an action a , undertaken in situation s , will remain supported by the surrounding system. An institutional history can alter this expectation:

$$p_{t_1}^H(a \mid s) \neq p_{t_1}^C(a \mid s). \quad (19)$$

Equation 19 illustrates one mechanism through which identical formal conditions can generate different behavior. The difference need not arise from irrationality. A changed expectation can be an appropriate response to observed institutional history.

This point is especially important for theories of non-domination. The existence of a right or resource can have different generative significance depending upon whether continued access is secure against

arbitrary withdrawal [44]. Restoration can therefore require attention to the credibility and security of conditions in addition to their nominal availability.

4.7 Trust as a Generative Relation

Trust is a relational condition with distinctive generative properties. It can reduce the cost of cooperation, facilitate disclosure, support learning, permit delegation, and make long-horizon coordination possible. Trust can therefore generate possibilities that are difficult to reproduce through contractual specification alone.

Damage to trust can have recursive effects. A subject who expects opportunistic or arbitrary behavior can reduce disclosure, collaboration, investment, or participation. These changes can then reduce the frequency of successful cooperative experiences through which trust might otherwise have developed.

Institutional restoration frequently confronts this problem. An organization can replace a manager, correct a record, change a policy, or provide compensation. These interventions can improve objective conditions. Trust can recover more slowly because its formation depends upon subsequent interaction.

This temporal difference shows why a condition can be legally restored before a relation becomes generatively usable again. It also suggests that trust should sometimes be represented as a distinct comparison object rather than inferred from formal status.

The same reasoning applies to collective trust. Repeated institutional failure can change how a community responds to later public programs. Additional resources can then produce limited effects because the relation required for their use has been weakened. Post-harm assistance may consequently require both material resources and credible institutional change.

4.8 Narrative Identity and Continuity Through Change

The problem of restoration also concerns how a subject maintains continuity through change. Ricoeur's account of selfhood and narrative identity provides a useful philosophical resource because it distinguishes dimensions of identity that cannot be reduced to numerical persistence or sameness of attributes [49]. A person can remain the same person through extensive changes in character, relationships, commitments, and circumstances.

This distinction matters for generative return because restoration often invokes two forms of continuity simultaneously. One concerns the continuing claimant: the person who experienced the harm remains entitled to recognition, remedy, or assistance. Another concerns the reconstructed reference: the institution seeks to approximate the state or trajectory the person would have possessed without the harm.

These two continuities can diverge. The actual claimant has developed through the history containing the harm. The counterfactual reference describes a different possible development. Treating the latter as the unique authentic version of the person can diminish the normative status of the actual subject.

Narrative identity also illuminates the importance of acknowledgment. A public response to harm can affect how an event is incorporated into personal and collective narratives. Recognition of wrongdoing does not remove the event. It can change its social interpretation and the subject's relation to public institutions.

A theory of generative return can therefore distinguish historical continuity from historical replication. Justice can respect continuity of the subject while supporting substantial transformation of the conditions in which future life develops.

4.9 Moral Frameworks and the Situated Self

Taylor's account of modern identity emphasizes the relation between selfhood and frameworks of moral significance within which persons orient themselves [60]. The relevance for generative return lies in the possibility that harm and subsequent adaptation change the practical significance of available options.

An opportunity has value for a subject partly within a wider orientation toward goods, commitments, projects, relationships, and forms of life. A professional position can be highly valuable while it supports a project the subject regards as central. The same position can become less valuable after the subject's commitments change. A particular community can provide recognition and meaning that another community with greater material resources does not reproduce.

This does not imply that justice should compensate every change in personal valuation. It establishes that the normative significance of a returned condition cannot always be inferred from its external description.

The situated character of valuation also complicates counterfactual reasoning. A counterfactual subject can value different options because that subject would have developed through a different history. The actual subject's present values can therefore differ from the values attributed to the counterfactual subject.

A restorative institution that selects the counterfactual subject's presumed preferences as the sole optimization target can reproduce a form of paternalism. The problem is especially severe where the reconstruction requires speculative claims about what the person "would really have wanted."

4.10 Relational Autonomy and Conditions of Self-Direction

Relational autonomy provides a useful normative response to the preceding difficulty. The social formation of agency does not eliminate the value of self-direction. It changes the conditions under which autonomy should be evaluated.

Mackenzie and Stoljar's relational-autonomy framework emphasizes that autonomy can be affected by social relations, self-conception, self-worth, memory, imagination, and oppressive conditions [31]. Mackenzie's analysis of imagination further highlights how social conditions can affect the repertoire through which subjects imagine alternative selves and possibilities [30].

This insight is particularly important for generativity. A subject's future option space depends partly upon the capacity to recognize and imagine possible actions. Harm can therefore contract generativity without formally prohibiting an option. A person who has experienced repeated institutional retaliation can stop treating a formally available path as practically viable. A subject who has been repeatedly excluded from a field can cease to imagine participation in that field as a realistic future.

A relational theory of autonomy consequently supports attention to enabling conditions for self-direction. These can include information, security, recognition, social support, absence of coercion, opportunities for reflection, and credible alternatives.

The same framework also constrains assistance. Assistance intended to restore generativity can become autonomy-reducing if access to resources is made conditional upon the recipient adopting a trajectory designed by the assisting institution. An intervention can therefore expand material options while contracting self-direction.

This tension will later become one component of procedural justice in restoration.

4.11 Imagination and the Generation of Future Possibility

Generativity involves both currently available options and the capacity to formulate new possibilities. Imagination is therefore relevant to the structure of future generation.

Mackenzie's account of imagining oneself otherwise is useful because it treats imagination as connected to autonomous reflection and to the ability to conceive alternative forms of selfhood and action [30]. Oppressive conditions can narrow the repertoire of alternatives a subject can seriously consider.

Within the present framework, this suggests a distinction between *available possibilities* and *recognized possibilities*. A legal or institutional option can exist objectively while remaining outside the subject's practical horizon because the subject lacks information, confidence, social support, or an interpretive framework through which the option becomes actionable.

Let $\mathcal{R}_t$ denote a set of states reachable from the current system, and let $\hat{\mathcal{R}}_t^A$ denote the subset of possibilities that subject A can presently identify and treat as practically actionable. The relation is

represented in Equation 20.

$$\hat{\mathcal{R}}_t^A \subseteq \mathcal{R}_t. \quad (20)$$

Equation 20 expresses a distinction that will become important in the later atlas. Restoration of objective reachability does not necessarily restore the subject’s epistemic or imaginative access to the same future possibilities.

Assistance can therefore include more than provision of resources. Education, information, mentoring, safe experimentation, temporary support, and access to communities can enlarge the set of futures that a subject can recognize and test.

Such assistance also requires caution. Institutions can manipulate aspiration under the language of empowerment. A theory of generative assistance should support the expansion of possible self-direction while avoiding institutional ownership of the recipient’s future.

4.12 Adaptive Transformation

Historical subjects adapt. Adaptation can preserve life and agency under constraint, generate new skills, create unexpected relationships, alter preferences, and produce values that did not exist before the harm. Adaptation can also encode the effects of domination, exclusion, deprivation, or fear. The normative interpretation of adaptation is therefore context-sensitive.

The capability literature has long recognized the difficulty created by adaptive preferences, especially where deprivation alters expectations and desires [55, 50]. Generative return adds a temporal and relational dimension to this concern. The subject’s present preferences can emerge through a history in which the original generative field was constrained, while those preferences can simultaneously become genuine parts of the subject’s current life.

Three forms of adaptive change are analytically useful.

4.12.1 Protective adaptation

Protective adaptation reduces exposure to a harmful environment. A worker can avoid particular organizational interactions, a researcher can leave a field, or a community can withdraw from an institution perceived as unsafe. Such adaptation can be rational and agency-preserving within the altered environment.

4.12.2 Generative adaptation

Generative adaptation creates valuable new capacities, relations, or projects. A person excluded from one profession can enter another and develop a life that the person values. A community responding to dispossession can build new institutions and practices. These new values should not be treated merely as defects relative to a counterfactual past.

4.12.3 Constraint-shaped adaptation

Constraint-shaped adaptation occurs when preferences, expectations, or behaviors develop around restricted conditions. The resulting pattern can be stable and internally coherent while reflecting a diminished set of practical possibilities.

These forms can overlap. A protective adaptation can later become generative. A generative adaptation can remain partly shaped by an unjust constraint. A restorative theory therefore cannot classify every difference from the counterfactual subject as damage or every present preference as conclusive evidence of justice.

4.13 The Actual Subject and the Counterfactual Subject

The distinction between the actual and counterfactual subject is one of the central philosophical problems of generative return.

Suppose a harmful intervention H occurs at t_0 . At a later time t_1 , the actual subject is $A_{t_1}^H$. A model of the history without H produces some counterfactual subject $A_{t_1}^C$. A restorative theory can then be tempted to define success through minimization of the difference between the actual subject after intervention R and the counterfactual subject:

$$R^* = \arg \min_R d(R(A_{t_1}^H), A_{t_1}^C). \quad (21)$$

Equation 21 represents an intuitive restorative objective. It is also normatively incomplete.

The metric d requires a theory of which dimensions of the subject matter. The counterfactual $A_{t_1}^C$ requires an epistemic theory. The admissible interventions R require a theory of procedural justice. The actual subject $A_{t_1}^H$ possesses present preferences and agency that can conflict with the reconstructed interests of $A_{t_1}^C$.

This last conflict is fundamental. The counterfactual subject never became an actual historical person. The present claimant did. A counterfactual can inform assessment of loss without automatically acquiring authority over the future of the actual subject.

Consequently, generative return should distinguish at least two normative questions:

1. Which losses become intelligible through comparison with a counterfactual subject or trajectory?
2. Which restorative interventions are justified for the actual subject who now exists?

The answers can differ. A large historical loss can be recognized even where the actual subject chooses a future that bears little resemblance to the counterfactual path.

4.14 Counterfactual Fidelity and Present Agency

The previous distinction produces a tension between counterfactual fidelity and present agency.

Counterfactual fidelity measures the extent to which a restorative outcome resembles a selected no-harm reference. Present agency concerns the authority of the current subject to participate in selecting the future toward which restorative resources are directed.

These objectives can align. A person wrongfully excluded from a profession can want reinstatement, restored credentials, renewed access, compensation, and support for returning to the original field.

They can also diverge. The person can prefer a new profession, a different city, reduced working hours, family care, education in another field, or a period of rest. A restoration program that conditions compensation on return to the earlier trajectory can then increase counterfactual fidelity while reducing present self-direction.

This conflict provides one reason that money and other broadly convertible resources can possess normative value. They permit institutions to acknowledge loss while leaving some selection of future direction with the subject. Specific enabling conditions remain necessary where general resources cannot remove important barriers.

The resulting model of assistance should therefore combine flexibility and targeted support without assuming that the institution knows the unique future that constitutes restoration.

4.15 Continuity Without Replication

A theory of generative return requires a concept of continuity that can survive substantial change. Exact replication of a previous or counterfactual state would make historical transformation appear as a defect to be eliminated. Ordinary personal continuity has a different structure. People remain continuous subjects through learning, aging, migration, relationships, conversion, illness, recovery, loss, and changing commitments.

Ricoeur's distinction between dimensions of sameness and selfhood provides a philosophical vocabulary for this problem [49]. The relevant implication for restoration is that preservation of the claimant's identity does not require preservation of every attribute.

This distinction enables a future-directed theory of repair. The subject can remain the bearer of claims arising from an earlier wrong while pursuing a future that differs substantially from the earlier trajectory. Historical continuity grounds responsibility and recognition. Present agency permits new generation.

A restorative process should therefore be capable of saying both:

The earlier loss remains attributable and normatively significant.

and:

The present subject can choose a future whose value does not depend upon reproducing the life that was interrupted.

These statements are compatible once restoration is separated from historical replication.

4.16 Identity-Preserving and Identity-Altering Restoration

Restorative interventions themselves can alter the subject. Education changes knowledge. Rehabilitation changes capacities. Therapy can change interpretation and response. Relocation changes relationships. Institutional reintegration creates new roles. Compensation changes material possibility. Public recognition can alter social identity.

The existence of such effects does not make restoration suspect. It means that restoration is itself generative.

A useful distinction can therefore be drawn between *identity-preserving constraints* and *identity-altering effects*. Identity-preserving constraints concern features of intervention required to respect the continuing subject, such as consent, bodily integrity, agency, memory rights, and continuity of legal personhood. Identity-altering effects concern ordinary changes that can emerge through legitimate assistance, learning, healing, and new relationships.

The boundary between the two cannot be determined purely mathematically. A memory intervention, personality-altering treatment, or manipulation of social relations can produce strong behavioral equivalence while raising profound questions concerning whether the subject whose condition is improved is being treated as an agent or as an object of reconstruction.

These cases motivate the later distinction between equivalence of outcome and justice of process.

4.17 Relational Usability of Returned Conditions

Section 3 introduced relational usability as the capacity of a condition to function within the subject's present relational and interpretive structure. The historical account developed here provides its philosophical foundation.

Let c denote a returned condition. Formal access can be represented by an indicator $A_t(c)$, while generative usability can be represented by a relation-dependent function $U_t(c; A, R, H)$. A system can satisfy

$$A_t(c) = 1$$

while the usable generative contribution of c remains substantially different from the reference condition.

A more explicit comparison is given in Equation 22.

$$U_{t_1}(c; A_{t_1}^H, R_{t_1}^H, H) \neq U_{t_1}(c; A_{t_1}^C, R_{t_1}^C, H^C). \quad (22)$$

Equation 22 shows why object identity alone cannot establish generative restoration. The returned condition enters a different subject–relation–history configuration.

This problem appears in employment, education, housing, public institutions, family relations, medical settings, and community life. Formal re-entry can be available while practical re-entry remains constrained by changed trust, status, security, interpretation, accessibility, or relations with others.

Restorative practice can therefore require enabling conditions around the returned object. These can include transition time, protection from retaliation, independent supervision, accessible procedures, alternative channels, rehabilitation, mentoring, renewed network access, or opportunities to develop new relations.

Such measures should be evaluated as assistance to present generativity rather than evidence that the past has been reproduced.

4.18 Reputation and Socially Distributed Subjecthood

Reputation provides a particularly clear example of relational subjecthood because part of its causal significance exists in the beliefs and responses of other actors. A subject cannot restore reputation unilaterally.

Suppose a false classification z about subject A circulates through a network. Each recipient can update beliefs, make decisions, transmit the classification, or generate derivative records. A later correction z' does not simply replace z in every relevant relation. Some actors encounter z' , others retain z , and others observe both.

The subject's social position at a later time therefore depends partly upon states distributed across other actors. A simplified representation is given in Equation 23.

$$R_t^A = \mathcal{R}(b_{1,t}^A, b_{2,t}^A, \dots, b_{n,t}^A, L_t), \quad (23)$$

where $b_{i,t}^A$ denotes relevant beliefs or classifications concerning A held by actor i , and L_t denotes the relational network through which these states acquire practical effects.

Equation 23 illustrates why reputational restoration can require more than correction of the originating record. The relevant object is partly distributed across a social network.

This distributed structure also creates epistemic limits. The institution responsible for correction may be unable to identify every downstream use of the earlier information. A declaration that reputation has been fully restored can therefore exceed what the institution can establish.

4.19 Institutional Identity and Path-Dependent Standing

Subjects also occupy institutionally produced identities. Student, employee, researcher, citizen, patient, licensed professional, debtor, beneficiary, member, and other statuses influence access to resources and relations. These statuses can accumulate historical meaning.

A formal status can be restored quickly. The standing associated with that status can develop more slowly. Reinstatement as an employee does not automatically reproduce prior seniority relations, informal authority, network position, or expectations of advancement. Restoration of academic affiliation does not recreate missed conferences, collaborative histories, or accumulated institutional trust.

Institutional identity can also be changed by records concerning earlier exclusion. A person can become institutionally legible as a “reinstated employee,” “wrongfully disciplined researcher,” or “compensated claimant.” These categories can be protective and stigmatizing at the same time.

The restoration event therefore becomes part of the institutional identity of the subject. This is one instance of the reflexivity examined in greater detail later in the paper.

4.20 Recognition and the Social Confirmation of Agency

Recognition theory deepens the relational account of subjecthood by emphasizing that social standing and self-relation can depend upon forms of recognition [25]. The effects of recognition are particularly

relevant where generativity requires credibility, authority, trust, or social membership.

A researcher can possess knowledge without having the institutional recognition required for the knowledge to circulate. A professional can possess skill while a damaged reputation restricts opportunities to exercise it. A community can possess cultural resources while public institutions deny their legitimacy. Recognition can therefore function as a generative condition.

Restoration of recognition differs from simple declaration. A formal apology or correction can be necessary while the social relations through which standing is enacted develop over a longer period.

Recognition also has a process dimension. An institution that offers resources while continuing to deny the validity of the claimant's experience can produce a different normative relation from one that acknowledges responsibility. Walker's account of moral repair similarly emphasizes the role of acknowledgment and responsibility in making amends [68].

The theory of generative return therefore treats recognition as both a possible object of restoration and a condition affecting the conversion of other forms of return into future generativity.

4.21 Historical Subjects and Adaptive Preferences

The existence of adaptive preferences creates a difficult problem for any subject-directed theory of return. If present preference were always decisive, severe and prolonged deprivation could reduce the obligations owed to a subject by reducing what the subject expects or requests. Capability theory has long identified this danger [55, 50].

The opposite rule is equally problematic. If institutions always replace present preference with an externally reconstructed account of what the subject would have wanted under ideal conditions, current agency can disappear.

The theory developed here therefore requires a distinction among at least three objects:

1. the subject's currently expressed preferences;
2. the set of capabilities and relational conditions within which those preferences are formed and exercised;
3. the historical process through which the current preference structure developed.

These objects can be evaluated jointly. A preference can be respected as the current subject's judgment while institutions also expand the conditions under which revision is possible. Support for education, information, temporary security, alternative relations, and reflection can increase the subject's capacity to reconsider a constrained choice without imposing a predetermined replacement.

This approach is especially suitable for generative assistance because the objective becomes expansion of revisable possibility rather than institutional selection of a final life plan.

4.22 Revisability and Future Self-Transformation

A historical subject is capable of further change. Justice after harm should therefore consider whether the conditions supplied through repair preserve the capacity for future revision.

A restorative settlement that locks a claimant into one occupation, institution, location, or identity can provide immediate security while reducing later generativity. An assistance program can support present needs while making continued eligibility dependent upon maintaining a fixed status. A public reparative category can secure recognition while making exit from the category socially difficult.

The concept of revisability addresses this problem. A generative condition is more supportive of agency when the subject can later reinterpret, redirect, or leave the trajectory it enables without disproportionate loss.

Revisability does not imply unlimited reversibility. Many life choices create commitments and path dependence. The normative concern is whether an institutional response unnecessarily converts assistance into control over future development.

This principle will later become relevant to process justice, where reversibility, contestability, and the right to revise participation are evaluated alongside outcome equivalence.

4.23 The Right to a Different Future

The preceding analysis supports a stronger proposition concerning post-harm justice: the subject can possess a legitimate interest in a future that differs from both the actual damaged trajectory and the reconstructed no-harm trajectory.

This third possibility is essential.

Let $\mathcal{T}^H$ denote the actual trajectory after harm and $\mathcal{T}^C$ denote a reconstructed no-harm trajectory. A subject-directed restorative process can generate a third trajectory $\mathcal{T}^N$ through newly available resources, relations, and choices. The relevant possibility is represented in Equation 24.

$$\mathcal{T}^N \neq \mathcal{T}^H, \quad \mathcal{T}^N \neq \mathcal{T}^C. \quad (24)$$

Equation 24 represents a future whose legitimacy does not depend upon its similarity to either previous path. It can incorporate acknowledgment of loss, compensation, assistance, and new values generated by the present subject.

This possibility transforms the meaning of post-harm justice. Counterfactual comparison remains relevant to responsibility and valuation. It ceases to function as the sole destination of repair.

The right to a different future also limits paternalistic forms of rehabilitation. A person whose original professional path was unjustly interrupted can receive compensation and enabling resources without assuming a duty to re-enter the same profession. A community can receive reparative resources without being required to reproduce an idealized version of an earlier culture. A worker can accept institutional acknowledgment while choosing separation from the workplace.

Generative assistance should therefore support reopening of future possibility while preserving the subject's authority to select among viable paths.

4.24 Subject Continuity and the Problem of Perfect Restoration

The historical account of the subject creates a conceptual limit for the idea of perfect restoration.

Suppose a hypothetical intervention possesses unlimited technical power and can reconstruct every external condition associated with a no-harm reference. If the actual subject retains memory, relationships, knowledge, and adaptations generated through the history of harm, the resulting system remains historically different from the no-harm system.

Suppose the intervention goes further and modifies the subject until every relevant internal property matches the reconstructed counterfactual subject. The intervention then raises a different problem. The process can remove features through which the actual subject currently understands identity, commitment, memory, and agency.

The difficulty therefore changes form as restorative power increases. Limited technology creates a problem of incomplete reconstruction. Unlimited intervention creates a problem concerning the normative status of the actual subject.

This thought experiment suggests that perfect restoration cannot be defined solely through maximal similarity. A complete theory must also specify which features of present subjecthood are protected against restorative alteration.

The resulting constraint can be represented schematically by separating an equivalence objective $E(R)$ from an agency and continuity constraint $J_A(R)$. An admissible restorative intervention must satisfy both a selected comparative objective and a minimum normative condition:

$$R \in \{R' \mid E(R') \geq E_{\min}, J_A(R') \geq J_{\min}\}. \quad (25)$$

Equation 25 is illustrative. It does not claim that agency or continuity can always be represented by scalar functions. Its purpose is to show that restorative fidelity and respect for the present subject constitute distinct requirements.

4.25 The Subject as Participant in Restorative Knowledge

The subject is also an epistemic participant in restoration. Institutions can possess records, statistics, professional assessments, causal models, and comparative data. The affected subject can possess experiential and relational knowledge unavailable in those sources.

A person can know that a formally restored workplace remains practically unusable because of informal relations invisible to administrative records. A community can know that a reconstructed service does not occupy the same social position as the institution that was lost. A patient can report changes in function that standardized measures do not capture. A researcher can identify lost network relations that publication counts fail to represent.

Subject testimony is neither infallible nor dispensable. It should enter a plural epistemic process alongside other forms of evidence.

This point connects procedural justice with epistemic justice. If the affected subject is excluded from defining the relevant comparison object, an institution can declare successful restoration according to a variable that fails to represent the experienced generative loss.

Participation in restorative knowledge therefore includes participation in questions such as:

- which conditions matter;
- which relations have changed;
- which future possibilities remain relevant;
- which forms of support are usable;
- which differences from the reference matter to the subject;
- which forms of intervention would create new burdens.

Participation does not eliminate disagreement. It changes the epistemic and procedural status of the restorative judgment.

4.26 Relational Boundaries of the Subject

A relational theory also confronts a boundary problem. If every relation affecting a person is treated as part of the subject, the object of restoration can expand indefinitely. If the subject is represented too narrowly, important generative dependencies disappear from analysis.

The framework therefore uses relational constitution without collapsing the subject into the entire social system. Relevant relations are selected according to the restorative question under consideration.

For employment restoration, professional networks, supervisory relations, organizational authority, and family consequences can be relevant. For educational restoration, teachers, peer communities, credentials, institutional access, and developmental timing can matter. For reputational restoration, the relevant network can include actors whose beliefs influence opportunities. For collective reparation, historical, territorial, cultural, and institutional relations can become central.

The boundary is therefore indexed to the problem. This indexing parallels the later selection of comparison objects and admissible tests.

The need for bounded analysis also provides an institutional reason for legal compression. A court cannot reconstruct every relation causally connected to a harm. Legal relevance requires boundaries. A philosophical theory can examine the costs and consequences of those boundaries without assuming that an institution can eliminate them.

4.27 Distributed Consequences of Subject Restoration

Because subjects are relationally embedded, changing one subject's conditions can alter the conditions of others. Restoration is therefore capable of creating new relational effects.

Reinstatement can alter organizational roles. Return of property can affect current occupants. Rehabilitation can change care relations. Public correction can affect the reputation of accusers, institutions, or third parties. Restoration of community authority can change the distribution of political power. Reparative transfers can impose fiscal burdens on persons who did not participate in the original wrong.

These effects do not establish an objection to restoration. They demonstrate that the subject of justice cannot always be modeled through a closed dyad.

A relationally adequate framework must therefore distinguish:

- the claimant's interest in return;
- the responsible actor's duties;
- the generative consequences of restoration for third parties;
- public or institutional interests;
- procedural rights of persons affected by the restorative process.

The later sections on substantive and procedural justice examine how these interests can conflict.

4.28 Historical Subjects and Singular Relations

The historical constitution of subjects also explains why some relational conditions are singular.

A relationship develops through accumulated interaction. Trust, shared language, tacit knowledge, familiarity, mutual expectations, and joint projects can emerge over time. Another relationship can perform similar functions while possessing a different history and a different generative structure.

This observation is especially important in education and research. A later mentor can provide excellent guidance. The later relation can produce greater academic value than the lost relation would have produced. It still does not establish historical equivalence with the earlier relation.

The same distinction applies to friendship, professional collaboration, community membership, family relations, cultural transmission, and other relations whose value partly emerges through their particular history.

A theory of return should therefore distinguish functional substitution from relational equivalence. Functional substitution can be highly valuable and can form an important part of assistance. Its value does not require the theory to declare the lost relation replaceable in every relevant sense.

4.29 Historical Meaning of Generated Value

Generated value can itself possess historical meaning. Two outputs with similar observable properties can differ in significance because of the processes through which they were produced.

A publication produced through an independent research trajectory can have a different meaning for the author from a publication produced under coercive institutional conditions. Income earned through autonomous work can have a different relational meaning from an equivalent transfer made through dependency. A community artifact produced through inherited practice can differ from an externally manufactured replica even where physical form is similar.

This observation extends the comparison problem beyond the quantity of generated value. Provenance, agency, authorship, relation, and control can form part of the value under comparison.

Generative justice already emphasizes the distinction between alienated and unalienated forms of value and the importance of value generators retaining control over generative processes [15, 17]. The

historical-subject perspective adds that the meaning of generated value can also depend upon the subject's relation to the process through which it emerged.

Restorative equivalence therefore cannot always be established by reproducing an output. The process and relation of production can matter independently.

4.30 Historical Irreversibility and the Preservation of Meaning

Historical irreversibility should not be interpreted solely as an obstacle. The fact that an event cannot be erased is also what makes responsibility, learning, testimony, memory, forgiveness, institutional reform, and moral development possible.

A world in which every harmful consequence could be deleted immediately would raise a different set of normative problems. Erasure could remove evidence of wrongdoing, dissolve responsibility, prevent collective learning, and destroy meanings formed through response to the harm.

This observation suggests that some dimensions of historical difference can possess positive normative value after repair. A public record of an injustice can remain important even after material consequences have been addressed. A subject can choose to preserve memory of a harmful period as part of identity. An institution can preserve records of failure in order to prevent recurrence.

The aim of justice therefore cannot be assumed to be maximal reduction of historical difference from a no-harm world.

This conclusion will become important when the paper later considers historical-generative equivalence. Very strong equivalence can require the removal of differences that justice itself has reasons to preserve.

4.31 Restoration as a New Relational Event

A restorative intervention enters the history of the subject as a new event. It can create gratitude, resentment, renewed trust, dependency, recognition, stigma, public attention, legal precedent, financial opportunity, or new institutional identity.

Consequently, restoration is itself generative.

Let R_t denote a restorative intervention. The subsequent subject develops through a history containing both the original harm and the restorative event. A schematic representation appears in Equation 26.

$$A_{t+1}^{HR} = \Phi_t \left(A_t^H, R_t, R_t^{\text{social}}, \Xi_t \right), \quad (26)$$

where R_t^{social} represents the relational and social responses generated by the restorative intervention.

Equation 26 shows that the restored subject develops through a history containing the restoration itself. The resulting state generally differs from a history in which neither harm nor restoration occurred.

This structure is one source of the reflexive non-closure developed later in the paper. Attempts to repair the system produce new system states that can themselves become objects of evaluation.

4.32 Present Subjecthood and the Ethics of Assistance

The historical and relational account developed above supports a distinctive ethics of post-harm assistance. Assistance begins from the subject who exists at the time assistance is provided. It can acknowledge a counterfactual loss while orienting practical support toward the present subject's capacities, relations, needs, commitments, and future possibilities.

This orientation avoids two reductions.

One reduction treats assistance as a substitute payment that closes the moral account once an equivalent amount of value has been transferred. Such an approach can overlook non-fungible relations, altered conversion structures, and present barriers.

A second reduction treats the subject as an object to be reconstructed toward a model selected by an institution. Such an approach can overlook present agency and the value of adaptations and commitments that emerged after the harm.

Generative assistance occupies a different normative space. It can provide resources, safety, health-care, education, legal correction, institutional access, time, mentoring, network opportunities, relocation, or other conditions while preserving substantial authority for the subject to determine the direction in which those conditions are used.

The objective can be expressed as expansion of viable and revisable future generativity rather than reproduction of a fixed historical state.

4.33 Subject-Directed Regeneration

The concept of *subject-directed regeneration* names this future-oriented component. A restorative response is subject-directed when the affected subject possesses meaningful authority over the selection, interpretation, and use of the conditions supplied for future generation, subject to the legitimate rights and interests of others.

Subject direction requires more than formal choice. A person presented with one practically viable option and several unusable alternatives has limited effective control. Information, security, time, material resources, social support, and accessible procedures can therefore be necessary conditions for meaningful direction.

Subject direction also remains compatible with public norms. A subject cannot claim an unlimited right to impose costs, violate others' rights, or demand any resource necessary for a preferred trajectory. The concept concerns the distribution of decision-making authority within the set of legitimate restorative possibilities.

The distinction will later support a three-component model of post-harm response:

general resources + specific enabling conditions + subject-directed regeneration.

Each component addresses a different problem. General resources preserve flexibility. Specific enabling conditions address barriers that fungible resources cannot remove. Subject direction protects the agency of the present historical subject.

4.34 Normative Risks of Relational Restoration

A relational account of harm can produce its own normative dangers. If relationships are treated as constitutive of persons, an institution can be tempted to preserve or reconstruct relationships against a subject's wishes. Family, workplace, community, religious, educational, and cultural relations can all be generative while also containing domination or abuse.

The value of relationality therefore does not imply a general duty to maintain relations.

A person can reasonably seek restoration through separation. A worker can prefer compensation and exit. A student can prefer transfer to another institution. A community member can reject reintegration. A person can decline contact with a party responsible for harm.

The relevant normative object can therefore be the capacity to form and leave relations under just conditions, rather than preservation of a particular relation.

This distinction is especially important for restorative justice practices. Participation and dialogue can be valuable while remaining dependent upon consent, safety, power structure, and the interests of affected parties [7].

A theory of generative return should consequently treat relational repair, relational replacement, relational transformation, and relational exit as distinct possible responses.

4.35 Relational Equality and Generative Standing

Relational equality adds a further dimension to subjecthood. A subject's generativity depends partly upon the standing from which the subject can enter social interaction. Anderson's account of democratic equality and Young's analysis of oppression show why equality cannot be reduced to divisible resources [2, 71].

A person can possess sufficient material resources while occupying a subordinate social position that constrains voice, authority, or access to cooperation. Restoration of resources without restoration or transformation of standing can therefore leave important generative limitations intact.

Generative standing can include credibility, authority to participate, recognized competence, secure membership, freedom from humiliation, and the ability to contest decisions. These features influence whether the subject can convert capacities into social action.

The concept is useful for later comparison because it identifies another object that differs from resources, capabilities, and outputs. Two subjects with similar capabilities can possess different generative standing within the same institution.

4.36 Historical Subjects and the Interpretation of Resilience

The historical account of the subject also complicates the interpretation of resilience. A subject who continues functioning under adverse conditions can appear highly resilient. Continued functioning can reflect skill, support, adaptation, necessity, suppressed alternatives, or some combination.

Resilience therefore should not automatically be treated as evidence that a harm produced little loss. A person can maintain output by expending more effort, sacrificing other life domains, drawing down savings, relying on unpaid care, or avoiding longer-term projects.

A restorative assessment that observes only final output can consequently underestimate the changed internal and relational cost of maintaining that output.

This point motivates later comparisons of response functions and perturbation robustness. The relevant question can include the resources required to sustain an observed behavior, the range of shocks the system can absorb, and the opportunity costs associated with continued functioning.

4.37 The Historical Subject in Collective Context

Individual historical subjects are embedded within collective histories. Families, professions, institutions, communities, states, and cultural systems provide languages, roles, practices, archives, expectations, and opportunities through which individual generativity develops.

Collective history can therefore affect individual restoration. A person returning to a community after displacement encounters a community that has also changed. A researcher returning to a field after exclusion encounters a field whose questions, methods, and participants have developed. A worker reinstated after several years returns to an organization with different colleagues and structures.

This creates a double historical movement:

$$(A_{t_0}, S_{t_0}) \longrightarrow (A_{t_1}, S_{t_1}), \quad (27)$$

where A_t denotes the subject and S_t denotes the surrounding social system.

Equation 27 makes clear that restoration cannot generally hold the world fixed while changing only the injured subject. Both sides of the relation have developed.

This point becomes especially important for long-delayed reparation. The object of return can remain identifiable while the social ecology in which it once functioned has disappeared.

4.38 Implications for Restorative Comparison

The historical and relational constitution of subjects yields several consequences for the theory developed in later sections.

First, equality of external conditions does not entail equality of subject–condition relations. The same resource, status, workplace, legal right, or institutional opportunity can possess different generative significance for a historically changed subject.

Second, the present subject and the counterfactual subject are distinct objects of analysis. Counterfactual comparison can inform assessment of loss while the actual subject remains the bearer of present agency and the recipient of restorative intervention.

Third, restoration of capability requires attention to conversion structures. The subject's capacity to transform resources into valuable action can itself change through history.

Fourth, relational restoration cannot be inferred from resource restoration. Trust, standing, recognition, network position, and security from domination can develop on different timescales.

Fifth, subjective adaptation requires a dual form of respect. Constraint-shaped preferences should not automatically reduce claims of justice, while valuable new commitments should not be erased in pursuit of counterfactual fidelity.

Sixth, memory and historical knowledge can be constitutive of present agency. Their removal can increase some forms of behavioral equivalence while decreasing procedural justice and historical continuity.

Seventh, present agency places limits on reconstructive intervention. The affected subject can possess a legitimate interest in choosing a new trajectory that differs from both the damaged actual path and the reconstructed no-harm path.

Eighth, restoration is itself a relational event. It changes the history it enters and can generate new meanings, identities, dependencies, opportunities, and social responses.

Ninth, the surrounding system changes alongside the subject. Long-delayed restoration therefore compares two historically evolving configurations rather than a changed subject against a static world.

Tenth, stronger similarity to a counterfactual subject does not automatically represent greater justice. Any theory of restorative equivalence must therefore be supplemented by constraints concerning agency, consent, dignity, identity, recognition, and process.

These consequences establish the philosophical basis for the temporal analysis that follows. Section 5 examines how the justice of generative conditions changes across the periods before harm, during an ongoing harmful process, and after the relevant subject and relational system have already been altered.

5 Temporal Architecture of Prevention, Interruption, and Post-Harm Response

This section develops the temporal architecture of generative justice across the period before harm, the period in which a harmful process is occurring, and the period after relevant generative conditions, subjects, and relations have already been altered. Its objective is to distinguish the normative objects, epistemic conditions, institutional tasks, and practical instruments that arise at each stage. The discussion proceeds through three temporal domains: pre-harm prevention, intervention during ongoing harm, and post-harm response. It then examines transitions among these domains, the changing distribution of uncertainty and evidentiary burdens, the temporal value of early action, the risks created by excessive intervention, and the requirements for coherence across the entire sequence.

The temporal distinction developed here is stronger than a chronological classification of remedies. The object of justice itself can change as a harmful process unfolds. Before harm, justice can concern preservation of valuable generative conditions and protection against unjustified contraction of future possibility. During an ongoing harmful process, justice can concern interruption of a developing negative generative cycle, containment of further propagation, preservation of evidence, and maintenance of viable alternatives. After substantial alteration has occurred, justice increasingly concerns restitution, compensation, recognition, rehabilitation, re-entry, institutional transformation, assistance, and support for future-directed regeneration.

The three domains therefore correspond to different practical questions:

1. Which interventions and institutional arrangements should protect generative conditions before substantial alteration occurs?
2. Which processes should be interrupted, contained, reviewed, or suspended while harm is developing?

3. Which forms of return, repair, compensation, recognition, and assistance remain appropriate after the relevant historical system has changed?

The distinction is analytical. Real cases frequently contain overlap. A reparative program can also prevent recurrence. An injunction can both interrupt an ongoing harm and preserve conditions for later restoration. Compensation can finance immediate protection from continuing harm. Institutional reform can respond to earlier wrongs while altering future risk. The temporal framework therefore identifies dominant functions at different stages without requiring every measure to belong exclusively to one stage.

5.1 Temporal Decomposition of Generative Harm

Let a potentially harmful intervention begin at t_0 , develop through an interval, and produce consequences that remain relevant at a later time t_2 . A simple temporal decomposition is given in Equation 28.

$$\mathcal{J} = (\mathcal{J}_{\text{pre}}, \mathcal{J}_{\text{ongoing}}, \mathcal{J}_{\text{post}}), \quad (28)$$

where $\mathcal{J}_{\text{pre}}$ concerns the normative architecture before substantial harm, $\mathcal{J}_{\text{ongoing}}$ concerns intervention while a harmful process remains active, and $\mathcal{J}_{\text{post}}$ concerns response after relevant alteration has occurred.

Equation 28 does not define three independent theories. Decisions made in one phase change the problem inherited by later phases. Weak prevention can increase the burden placed upon interruption. Delayed interruption can increase the burden placed upon post-harm repair. Effective assistance can reduce the future recurrence of vulnerability. Institutional learning from post-harm review can alter future prevention.

The temporal architecture is consequently recursive:

$$\mathcal{J}_{\text{pre}} \longrightarrow \mathcal{J}_{\text{ongoing}} \longrightarrow \mathcal{J}_{\text{post}} \longrightarrow \mathcal{J}'_{\text{pre}}. \quad (29)$$

Equation 29 represents the possibility that post-harm experience changes later rules, safeguards, institutional knowledge, risk classifications, and preventive practices. Justice across time therefore contains both restorative and learning functions.

This recursive relation is especially important for generative systems. Institutional responses become part of the conditions under which future subjects act. A well-designed review process can generate trust and preventive knowledge. A failed response can generate silence, avoidance, normalization of harm, or expectations that reporting will be ineffective. The governance of one case can therefore alter the generative environment of later cases.

5.2 Temporal Asymmetry of Prevention and Repair

Prevention and repair are temporally asymmetric. Before an intervention occurs, some possible histories remain open. After the intervention and the resulting system evolution, some of those histories cease to be available as actual histories. Later institutions can create new possibilities, yet they cannot convert those new possibilities into the historical fact that the earlier process never occurred.

This asymmetry becomes especially important for singular generative conditions. A specific developmental period, professional encounter, ecological configuration, institutional opportunity, or historical relation can disappear as time passes. The later availability of substantial resources can expand the future while remaining unable to recreate the earlier temporal location.

Let $\mathcal{P}_t$ denote the set of feasible preventive actions available before a harmful transition and let $\mathcal{R}_{t+k}$ denote the set of available restorative interventions after the system has developed for an interval k . In cases involving temporally singular conditions, the relation can take the form shown in Equation 30.

$$\mathcal{R}_{t+k} \not\supseteq \mathcal{P}_t \quad \text{with respect to preservation of the original historical path.} \quad (30)$$

Equation 30 does not claim that prevention is always superior to every later response. Preventive measures can be costly, intrusive, mistaken, or themselves unjust. The equation identifies a structural difference: some options available before irreversible or highly path-dependent change cannot later be reproduced as restorative options.

This temporal asymmetry motivates greater attention to prevention where the potential loss involves weakly substitutable generative conditions. The normative implication remains conditional upon probability, severity, uncertainty, rights, proportionality, distribution of burdens, and the risks of the preventive intervention itself.

5.3 Pre-Harm Justice as Protection of Generative Possibility

Pre-harm justice concerns the conditions under which actors and institutions may place another subject's generativity at risk. Its primary objects include rights, security of access, procedural safeguards, environmental and occupational safety, protection from arbitrary interference, institutional redundancy, informed participation, and maintenance of options whose later reconstruction would be difficult.

This domain has a long institutional history outside the vocabulary of generativity. Accident law and regulation address prevention and allocation of accident costs; Calabresi's economic analysis of accident law explicitly treats accident prevention and the costs associated with accidents as objects of institutional design [8]. Regulatory theory similarly examines how institutional structures can shape conduct before severe harm occurs. Ayres and Braithwaite's account of responsive regulation emphasizes adaptive regulatory strategies, participation, and escalation across different forms of regulatory response [4].

The present framework adds a generative interpretation. Some preventive rules protect against immediate injury. Others preserve conditions through which future capacities, relations, and values can continue to develop. Due process before dismissal protects more than current salary when employment also conditions professional standing and future opportunity. Research-integrity procedures protect more than a single publication when allegations can alter entire professional networks. Environmental safeguards protect more than current output when ecological changes can alter future regimes and intergenerational possibilities.

Pre-harm justice can therefore be understood partly as protection of *option-generating structures*. The relevant concern is not unlimited preservation of every existing condition. Social systems require change, competition, experimentation, institutional reform, and termination of harmful relations. The concern is the legitimacy of interventions capable of producing large contractions in another subject's future generative space.

5.4 Prevention and the Burden of Irreversibility

Irreversibility changes the significance of preventive judgment. Where a potential loss is cheaply reversible, institutions can tolerate a greater role for post-event correction. Where a loss is difficult to reverse, uncertainty concerning later repair becomes more consequential.

This intuition resembles precautionary reasoning. The 1998 Wingspread Statement on the Precautionary Principle formulates precaution in the context of threats to human health and the environment under scientific uncertainty and emphasizes anticipatory action, consideration of alternatives, participation, and attention to burdens of proof [70]. The present paper does not adopt that statement as a universal principle for all generative harms. It draws from it a more general structural insight: uncertainty about complete causation need not always justify postponement of protective action where potential harms are severe and difficult to reverse.

A generativity-sensitive preventive analysis should consider at least:

- the severity of the possible contraction of generative conditions;
- the substitutability of the conditions at risk;

- the temporal sensitivity of the relevant opportunities;
- the likelihood of recursive propagation;
- the availability of later repair;
- uncertainty concerning causal mechanisms;
- the cost and intrusiveness of precaution;
- the distribution of burdens created by preventive measures;
- the rights and agency of the affected parties;
- the possibility that prevention itself constrains legitimate generativity.

This list prevents a simple inference from low reparability to maximal restriction. A preventive system can itself become dominating when every possible future loss becomes a justification for surveillance, prohibition, or control.

5.5 Option-Preserving Governance

A useful objective for pre-harm governance is preservation of important options while uncertainty remains unresolved. *Option-preserving governance* refers to institutional arrangements that avoid unnecessarily closing high-value or weakly reversible paths before adequate review has occurred.

Examples include temporary rather than permanent exclusion, provisional rather than irreversible classification, appeal before destructive enforcement, preservation of access pending review, data-retention rules that permit later correction, and staged implementation of high-impact institutional decisions.

The logic can be represented through reachable sets. Suppose an intervention a changes the subject's reachable future set from $\mathcal{R}_t$ to $\mathcal{R}_t^a$. Where uncertainty concerning the justification of a remains high, an option-preserving design can prefer an intervention a' that satisfies the immediate legitimate objective while producing a smaller contraction of relevant future possibilities.

This can be expressed schematically by Equation 31.

$$\mathcal{R}_t^a \subset \mathcal{R}_t^{a'} \subseteq \mathcal{R}_t, \quad (31)$$

where a' achieves the required protective function with a less extensive contraction of future options.

Equation 31 is not a universal decision rule. Some threats require decisive action. Its purpose is to identify option preservation as an independent governance value where uncertainty and irreversibility interact.

5.6 Procedural Protection Before Generative Contraction

Procedural justice has an important ex ante function. Procedures do more than evaluate a completed loss. They can prevent institutions from creating severe generative contractions without sufficient evidence, participation, or review.

Relevant mechanisms can include:

- notice before adverse action;
- access to the evidence used in decision-making;
- opportunity to contest material factual claims;
- independent review;
- conflict-of-interest controls;

- reason-giving requirements;
- proportionality review;
- staged sanctions;
- appeal mechanisms;
- temporary protection of contested rights or access;
- safeguards against retaliation.

These mechanisms are often justified through familiar legal and administrative values. A generative analysis adds attention to the downstream structure of an error. A mistaken decision concerning a highly replaceable benefit can be correctable at moderate cost. A mistaken decision that destroys a professional network, closes a developmental window, removes legal status, or irreversibly alters an ecosystem can create a qualitatively different problem.

Procedural intensity can therefore reasonably vary with the generative stakes of the intervention.

5.7 Redundancy and Resilience Before Harm

Prevention can also operate through resilience rather than prohibition. A system with multiple pathways to important conditions can absorb failure of one path without producing severe generative contraction.

Examples include multiple reporting channels, portable professional credentials, diversified funding, independent data backups, alternative care providers, decentralized decision authority, appeal to external institutions, and transferable benefits.

Redundancy can reduce dependency upon a single gatekeeper. It therefore has a close relation to non-domination. A subject whose entire professional generativity depends upon one unreviewable authority is more vulnerable to arbitrary intervention than a subject possessing several legitimate pathways.

Preventive resilience can consequently be represented through multiplicity of paths in a transition or network structure. A system in which every important future requires passage through one node possesses a different vulnerability profile from a system containing several independent routes.

This structural perspective will later become relevant to network, reachability, and robustness equivalence.

5.8 Ongoing Harm as an Evolving Generative Process

Once a harmful process begins, the normative problem changes. The central question is no longer confined to whether the original intervention was justified. The system is now evolving under altered conditions.

A continuing harmful process can produce additional outputs, classifications, relationships, learned responses, resource losses, and institutional decisions. The passage of time therefore becomes part of the harm architecture.

Let H_t denote an ongoing harmful process and $\mathcal{G}_t^H$ the affected generative system. The development can be represented by Equation 32.

$$\mathcal{G}_{t+1}^H = \Phi_t^H(\mathcal{G}_t^H). \quad (32)$$

Equation 32 emphasizes that continued inaction can have substantive consequences. The system at $t + 1$ can differ because the harmful process remained active during the interval.

This observation creates a distinct domain of justice: *interruption*.

5.9 Interruption as an Independent Normative Objective

Interruption aims to stop or reduce the continued generation of harm before final responsibility, valuation, or long-term remedy is fully determined.

This objective is analytically distinct from punishment and compensation. An institution can have insufficient evidence to impose a final sanction while possessing sufficient reason to adopt a temporary protective measure. A court or regulator can preserve a contested condition while the underlying claim is adjudicated. An organization can separate parties while investigating harassment. A platform can pause dissemination of contested material while provenance is reviewed.

The justification of interruption therefore depends upon a balance among continuing risk, uncertainty, proportionality, reversibility, and burdens created by the interim action.

Interruption can be represented as a change in the active transition mechanism:

$$\Phi_t^H \xrightarrow{I_t} \Phi_t^I, \quad (33)$$

where I_t denotes an interim intervention and Φ_t^I denotes the transition structure after interruption.

Equation 33 clarifies the objective. The intervention need not reconstruct the prior state immediately. It can first alter the mechanism through which further harm is being generated.

5.10 Temporary Measures Under Uncertainty

The ongoing-harm phase often contains the strongest conflict between urgency and epistemic incompleteness. Waiting for complete knowledge can permit further propagation. Acting on incomplete knowledge can impose serious burdens on persons whose responsibility has not been established.

Temporary measures provide one institutional response to this conflict. Their design can incorporate:

- limited duration;
- periodic review;
- reversible implementation where feasible;
- minimum necessary interference;
- preservation of the rights of all affected parties;
- separation between protective action and final findings;
- clear evidentiary thresholds;
- opportunities for contestation;
- documentation of reasons;
- explicit expiration or transition conditions.

The distinction between interim protection and final judgment is especially important. A protective measure should not automatically become a durable social classification. A temporary reassignment made for safety should not silently become evidence of guilt. A provisional flag should not become a permanent reputational object simply because it was recorded.

This concern connects interruption with the later analysis of reflexive records. Interim governance can generate new information that subsequently changes the system.

5.11 Interruption of Negative Generative Cycles

Section 3.16 described processes in which an initial disadvantage generates conditions for further disadvantage. During an ongoing harm, interruption seeks to break one or more links in such a cycle.

Consider the sequence

stigma → loss of access → withdrawal → weaker network position → additional stigma.

An effective intervention need not solve every component simultaneously. It can prevent the next transition. Protection against retaliation can preserve access. Temporary reassignment can interrupt hostile interaction. Public correction can reduce propagation of false information. Income support can prevent an employment dispute from immediately producing housing insecurity. Independent network access can reduce dependence upon a gatekeeper.

This produces a practical principle:

During an ongoing generative harm, an important object of intervention is the next harmful transition, even where complete restoration of the existing state remains unavailable.

The principle gives intermediate governance a constructive role. Justice need not wait until every loss can be calculated.

5.12 Evidence Preservation During Ongoing Harm

Evidence has temporal properties. Records can disappear, memories can change, digital systems can overwrite logs, organizations can restructure, witnesses can leave, and social interpretations can stabilize around incomplete accounts.

Evidence preservation is therefore part of interruption and future restorability.

Relevant practices can include preservation of correspondence, decision logs, access records, versions of policies, provenance metadata, witness statements, financial records, health records, and documentation of institutional actions.

Preservation should remain subject to privacy, proportionality, data-security, and procedural constraints. The objective is not indiscriminate surveillance. It is preservation of information whose later disappearance would make responsibility, correction, or restoration substantially harder to evaluate.

The generative significance of evidence also extends beyond litigation. Preserved records can support institutional learning, public acknowledgment, historical research, and prevention of recurrence.

5.13 Provenance and the History of Institutional Decisions

A generative system can be altered through a chain of decisions rather than one identifiable act. Provenance records can help reconstruct this chain.

A useful record can distinguish:

- the original claim or observation;
- the source of the information;
- the evidence available at the time;
- the decision made from that evidence;
- the actors authorized to make the decision;
- later challenges;
- corrections;

- downstream uses of the earlier decision;
- institutional responses to those corrections.

This event-oriented structure becomes especially important where a later correction does not erase the earlier event. The history can preserve both the fact that a claim was made and the fact that it was later rejected, qualified, or superseded.

Provenance therefore supports revisable knowledge without requiring archival erasure. It can also reduce the tendency to treat an outdated classification as a timeless property of the subject.

The reflexive consequences of such records require separate analysis later in the paper. Recording history can itself influence later relations.

5.14 Anti-Retaliation and the Preservation of Voice

Ongoing harm can become self-sustaining when reporting or contesting the harm creates additional risk. A worker who reports harassment can fear dismissal. A researcher who contests misconduct can fear exclusion from collaboration. A student who challenges a decision can fear academic retaliation. A community that opposes a project can fear withdrawal of services or resources.

Anti-retaliation protections preserve more than individual speech. They preserve the epistemic conditions through which an institution can detect and correct its own failures.

A system in which reporting reliably produces retaliation can generate silence. Silence then reduces information available to governance, which can permit the harmful process to continue. The cycle can be represented by Equation 34.

$$\text{retaliation risk} \rightarrow \text{reduced reporting} \rightarrow \text{reduced institutional knowledge} \rightarrow \text{continued harm} \rightarrow \text{greater retaliation risk.} \quad (34)$$

Equation 34 identifies voice protection as a generative governance mechanism. It supports the production of corrective knowledge.

5.15 Intermediate Resource Support

The period between allegation and final resolution can itself generate severe loss. A person suspended from work can lose income. A student excluded pending review can lose a semester. A patient awaiting a determination can lose access to treatment. A community awaiting a land decision can be unable to make long-term investments.

Interim support can preserve the subject's capacity to remain within a viable range while the institutional process continues. Such support can include temporary income, healthcare, housing protection, legal assistance, access to alternative facilities, continuation of credentials, or temporary institutional affiliation.

The normative significance of interim support is distinct from final compensation. It protects generativity during uncertainty.

Where support is later shown to have benefited a party whose claim was not ultimately established, institutional design can address allocation of those costs separately. The possibility of error does not eliminate the temporal problem created by leaving all risk with the potentially harmed subject.

5.16 The Cost of Delay

Delay is often treated as a procedural inconvenience. In generative systems, delay can become substantively transformative.

Let the effect of unresolved harm during the interval $[t_0, t_1]$ be represented by the evolving system $\mathcal{G}_t^H$. An institutional decision made at t_1 acts upon $\mathcal{G}_{t_1}^H$, not upon $\mathcal{G}_{t_0}$. Where the relevant conditions are path-dependent, the cost of delay includes the generation that occurred under the unresolved condition.

A schematic cumulative loss expression is given in Equation 35.

$$L(t_1) = L_0 + \int_{t_0}^{t_1} \ell(\mathcal{G}_t^H, H_t) dt, \quad (35)$$

where L_0 denotes the initial loss and $\ell(\mathcal{G}_t^H, H_t)$ denotes the rate at which additional relevant loss develops during the unresolved interval.

Equation 35 is conceptual. The relevant loss can be multidimensional and need not admit a common scalar measure. Its purpose is to make delay analytically visible.

This structure provides a reason for time-sensitive procedures in cases where continuing uncertainty itself contracts generative possibility.

5.17 Temporal Proportionality

Proportionality across time differs from proportionality at a single moment. An intervention that appears modest when viewed as a one-day restriction can become severe when prolonged for years. Conversely, a strong short-term intervention can prevent a much larger continuing harm.

The duration of a measure should therefore enter its normative evaluation.

This principle applies to:

- suspension;
- provisional exclusion;
- asset freezing;
- restrictions on access;
- monitoring;
- emergency separation;
- confidentiality rules;
- temporary protective arrangements.

A temporally proportionate system should periodically reassess whether the conditions justifying an interim measure remain present and whether a less restrictive alternative has become available.

This requirement connects procedural justice with the generative effects of time.

5.18 Transition from Interruption to Post-Harm Response

The boundary between ongoing harm and post-harm response is often ambiguous. The original act can have ceased while its consequences continue to generate new effects.

A harassing supervisor can leave the organization while reputational and relational consequences remain active. A false record can be corrected while downstream copies remain in use. Pollution can cease while ecological effects continue. An exclusionary policy can be abolished while institutional distributions formed under the policy persist.

The transition therefore should be defined functionally. A process enters the post-harm domain when the principal governance objective shifts from stopping an active harmful mechanism toward addressing the altered system that remains.

This transition can be gradual. Interruption and repair can occur simultaneously.

The distinction matters because different evidentiary and practical tools become relevant. Immediate protective action can tolerate a provisional account of causation under appropriate safeguards. Long-term compensation and responsibility generally require stronger specification. Assistance based upon present need can sometimes proceed even where complete historical attribution remains unresolved.

5.19 Post-Harm Justice as a Plural Response

Post-harm justice begins from an altered system. Its central problem is not the selection of a single remedy. It is the coordination of several possible responses to different dimensions of harm.

International human-rights practice already provides an important plural vocabulary. The United Nations Basic Principles and Guidelines on the Right to a Remedy and Reparation identify restitution, compensation, rehabilitation, satisfaction, and guarantees of non-repetition as forms of reparation [65]. The categories demonstrate that serious harm can require responses directed toward different objects and temporal functions.

The present framework expands the analytical interpretation of this plurality. Post-harm response can include:

- cessation of continuing effects;
- return of identifiable objects or rights;
- restoration of formal status;
- monetary compensation;
- rehabilitation;
- correction of records;
- acknowledgment and recognition;
- re-entry into institutions or networks;
- reconstruction of enabling conditions;
- assistance with new trajectories;
- institutional transformation;
- prevention of recurrence.

These measures can coexist because they address different comparison objects. A monetary payment does not perform the same function as correction of an official record. Reinstatement does not perform the same function as rehabilitation. Institutional reform does not perform the same function as individual compensation.

The later atlas will represent these differences formally.

5.20 Cessation and the End of Continuing Harm

Even after a case is described as historical, some consequences can remain actively generated. A post-harm response should therefore distinguish historical loss from continuing violation or continuing harmful mechanism.

Examples include:

- a false record that remains publicly accessible;
- a discriminatory rule that continues to govern access;
- continued possession of wrongfully taken property;
- ongoing environmental contamination;
- continuing publication of false information;

- continuing exclusion from an institution;
- recurring automated use of an invalid classification.

Where an active mechanism remains, cessation retains temporal priority. Compensating a subject while permitting the harmful mechanism to continue can generate repeated need for compensation.

This distinction also prevents post-harm assistance from becoming a substitute for stopping an ongoing injustice.

5.21 Restitution of Identifiable Conditions

Restitution addresses conditions that can still be returned or restored in an identifiable form. Property, legal status, employment, access, documentation, licenses, or institutional membership can sometimes be restored.

Restitution has strong normative significance because it reconnects the remedy to the object affected by the wrong. The historical-subject analysis in Section 4 nevertheless shows that return of the condition leaves a further question concerning its present generative significance.

Post-harm justice should therefore permit two judgments to coexist:

The relevant condition has been restituted.

and:

The subject's broader generative system remains historically altered.

The second judgment does not invalidate the first. It prevents object restitution from becoming an unsupported claim of complete repair.

5.22 Compensation and the Use of General Equivalents

Compensation uses transferable value to address losses that cannot or need not be repaired through literal return. Monetary compensation is especially important because of its administrative tractability, divisibility, and broad convertibility.

A generative account adds another reason for its importance: money can support subject-directed future formation. Where institutions cannot know which new trajectory the subject should pursue, a general equivalent allows the subject to direct resources toward personally relevant combinations of care, education, relocation, rest, research, housing, family responsibilities, new employment, or other purposes.

This flexibility should not be confused with perfect commensuration. A payment can be justified even where the lost condition is not equivalent to money in every relevant sense. Compensation can acknowledge loss, distribute resources, support future action, and partially discharge legal responsibility without proving that the loss and payment are structurally interchangeable.

The difference between justified compensation and complete equivalence will be important in the later value-theory sections.

5.23 Rehabilitation and Restoration of Usability

Rehabilitation addresses bodily, psychological, functional, social, or professional conditions through which resources become usable.

Its role becomes especially important after conversion structures have changed. Returning employment can have limited value where the subject lacks health, confidence, transportation, credentials, or practical support required for re-entry. Monetary compensation can have limited effect where treatment, accessibility, or institutional accommodation remains unavailable.

Rehabilitation can therefore be understood as work upon the conversion layer of generative return.

This interpretation also clarifies why rehabilitation should remain subject-sensitive. The objective is not necessarily reproduction of a previous body, psychology, identity, or performance level. A legitimate rehabilitation program can support new ways of acting and living that the present subject values.

5.24 Recognition and Satisfaction

Some post-harm needs concern social meaning. A formal finding of wrongdoing, acknowledgment, correction of the historical record, apology, commemoration, truth recovery, or public recognition can alter the normative interpretation of a past event.

The United Nations reparation framework includes satisfaction among relevant forms of reparation [65]. Moral-repair theory similarly emphasizes acknowledgment and responsibility [68].

Recognition can affect generativity because standing influences later relations. A public correction can improve credibility. An institutional acknowledgment can change how colleagues interpret an earlier dispute. A historical finding can alter collective memory and future governance.

Recognition also has limits. A symbolic act can coexist with continuing material deprivation. Public acknowledgment can increase exposure or reopen conflict. The design of recognition therefore remains subject to procedural and relational analysis.

5.25 Re-Entry into Generative Systems

Re-entry concerns the subject's return to institutions, professions, communities, educational systems, networks, or other environments from which the subject was excluded or withdrew.

Re-entry is analytically distinct from reinstatement. Reinstatement can restore formal status immediately. Re-entry concerns the practical reconstruction of participation.

Relevant conditions can include:

- current skills and training;
- changed institutional procedures;
- network access;
- reputation;
- safety;
- trust;
- accessibility;
- workload;
- transitional financial support;
- mentoring or independent supervision;
- protection against renewed retaliation.

A successful re-entry process can therefore require a bundle of enabling conditions around the formally restored status.

The subject's consent remains essential. Re-entry is one possible trajectory among several. A subject can legitimately prefer compensated exit or movement into another generative environment.

5.26 Generative Assistance

Some post-harm circumstances exceed the practical reach of restitution. Singular opportunities can have disappeared. Relationships can be unavailable. The subject can have changed. The surrounding system can have changed. The counterfactual future can remain uncertain.

These conditions motivate the category of *generative assistance*.

Generative assistance concerns support supplied to a historically altered subject or system for the purpose of expanding viable future generativity. It does not require a claim that the newly created possibilities are equivalent to the possibilities that were lost.

A basic structure is given in Equation 36.

$$\mathcal{A}_{\text{gen}} = \mathcal{M} + \mathcal{C}_{\text{specific}} + \mathcal{S}_{\text{direction}}, \quad (36)$$

where $\mathcal{M}$ represents flexible general resources, $\mathcal{C}_{\text{specific}}$ represents specific enabling conditions, and $\mathcal{S}_{\text{direction}}$ represents meaningful authority of the subject over the direction of future development.

Equation 36 is conceptual. The components cannot always be added numerically. The expression identifies three distinct functions of assistance.

Generative assistance can include time, income, healthcare, education, professional retraining, relocation, legal correction, access to facilities, network opportunities, childcare, independent advice, mobility, or other supports appropriate to the subject's current situation.

Its normative basis can differ from corrective liability. Assistance can remain appropriate where no individual wrongdoer exists, where responsibility is diffuse, where the responsible party cannot provide adequate repair, or where social solidarity supports collective provision.

5.27 Future-Directed Regeneration

The strongest temporal difference between restitution and assistance concerns orientation. Restitution looks toward a condition associated with the past. Generative assistance can look toward futures that have never previously existed.

Let $\mathcal{R}_t^H$ denote the future possibilities available after harm, and let an assistance intervention A_t produce a new reachable set $\mathcal{R}_{t+1}^{HA}$. The objective can be expansion of viable future possibility:

$$\mathcal{R}_t^H \subset \mathcal{R}_{t+1}^{HA}, \quad (37)$$

without requiring

$$\mathcal{R}_{t+1}^{HA} = \mathcal{R}_{t+1}^C,$$

where $\mathcal{R}_{t+1}^C$ denotes a reconstructed counterfactual set.

Equation 37 captures a central feature of future-directed regeneration. Justice can support the opening of new possibilities while preserving acknowledgment that the lost historical possibilities remain unrecovered.

This structure will later become important when comparing equality, simulation, dominance, sufficiency, and refinement as alternative restorative relations.

5.28 Guarantees of Non-Repetition and the Return to Prevention

Post-harm justice can contain a future preventive component. The United Nations reparation framework explicitly recognizes guarantees of non-repetition among forms of reparation [65]. Such measures connect the end of one case to the prevention of later harms.

Possible mechanisms include:

- institutional reform;
- training;

- independent oversight;
- changes in appointment or disciplinary procedures;
- stronger conflict-of-interest rules;
- improved reporting channels;
- legal reform;
- archival and documentation reform;
- redistribution of decision authority;
- transparency requirements;
- structural changes reducing dependency on single gatekeepers.

These measures can possess reparative meaning when they respond to a recognized past wrong. They also function prospectively by changing the transition structure faced by future subjects.

The temporal cycle of justice therefore closes only provisionally. Post-harm learning becomes pre-harm governance for the next period.

5.29 Responsibility and Assistance Across Temporal Domains

Different temporal domains support different forms of obligation.

Before harm, duties can concern reasonable care, non-domination, institutional safeguards, respect for rights, and avoidance of unjustifiable risk.

During an ongoing harm, duties can concern cessation, protective intervention, preservation of evidence, non-retaliation, temporary support, and timely review.

After harm, duties can concern restitution, compensation, acknowledgment, rehabilitation, assistance, re-entry, institutional reform, and prevention of recurrence.

The parties bearing these duties can also differ. The actor responsible for the original wrong can bear corrective obligations. An employer or institution can bear governance responsibilities because of its authority over the environment. An insurer can bear contractual obligations. The state can bear public-law or social-protection responsibilities. A professional community can bear institutional responsibilities. Society can provide assistance through solidarity even where individual responsibility remains unclear.

The temporal framework therefore resists reduction of all post-harm support to wrongdoer liability.

5.30 Changing Epistemic Standards Across Time

The quantity and quality of information available to institutions change across the temporal sequence. Pre-harm governance often operates through risk assessment. During an ongoing harm, institutions can possess incomplete but developing evidence. Post-harm adjudication can have access to a richer record while still confronting counterfactual uncertainty.

A single evidentiary threshold for all temporal functions would therefore be inappropriate.

Preventive rules can be justified by general knowledge of risk without proof that a specific future harm will occur. Temporary interruption can sometimes be justified by credible evidence of continuing risk under safeguards that preserve review. Final responsibility can require a more demanding evidentiary basis. Humanitarian or social assistance can sometimes respond to present need without settling every question of historical causation.

The distinction is important because excessive demand for certainty at an early stage can permit irreversible harm, while weak evidentiary requirements for final findings can create serious injustice.

Temporal justice therefore requires a differentiated epistemology of action.

5.31 Allocation of Uncertainty

Uncertainty itself can be distributed. An institution deciding whether to act under uncertainty effectively decides who bears the risk of error.

If no interim protection is provided, the potentially harmed subject can bear the risk that the allegation proves correct only after substantial generative loss has occurred.

If strong interim restrictions are imposed immediately, the accused or other affected parties can bear the risk that the allegation later proves unsupported.

A just governance architecture should therefore ask:

- which party is better able to absorb temporary error;
- which losses are more reversible;
- which interests are legally protected;
- what interim arrangement preserves the greatest legitimate option space;
- which safeguards can reduce the burden on each party;
- when review must occur.

This allocation problem is particularly important where the potential harms are asymmetric in reversibility.

5.32 The Temporal Distribution of Proof

The burden of proof can also have generative effects. A subject required to prove extensive downstream harm before receiving any protection can lose the very conditions required to assemble that proof. An institution required to justify every preventive measure with complete causal certainty can become incapable of acting before severe harm occurs.

At the same time, shifting burdens too aggressively can create arbitrary restrictions, reputational injury, and institutional overreach.

The appropriate distribution of proof therefore depends upon the function of the decision. Preventive, interim, adjudicative, compensatory, and assistance decisions can legitimately employ different evidentiary structures.

The Wingspread formulation of precaution places particular emphasis on the burden borne by proponents of activities that raise threats to health or the environment [70]. The present paper does not generalize this burden rule automatically to every social dispute. It treats burden allocation as one variable in a wider temporal architecture.

5.33 Temporal Value of Information

Information acquired earlier can have a different practical value from the same information acquired later.

A diagnosis after irreversible deterioration can explain what occurred while being unable to prevent it. A correction after a hiring cycle can establish error while being unable to reopen that cycle. Evidence of harassment after several employees have left can support responsibility while being unable to preserve the earlier team. Environmental data obtained after regime change can improve future policy while being unable to reconstruct the earlier ecosystem.

The value of information therefore includes its capacity to change action while relevant options remain available.

This temporal dimension supports investment in monitoring, review, independent reporting, and early warning where the expected cost of delayed knowledge is high.

Such monitoring remains constrained by privacy, autonomy, and the dangers of surveillance. More information is not automatically more just governance.

5.34 Institutional Learning and Error Correction

A temporal theory of justice should also examine how institutions learn from their own errors.

An institution can respond to failure by:

- correcting the individual case;
- identifying the mechanism that produced the error;
- changing procedures;
- preserving an auditable history of the change;
- monitoring whether the reform produces new harms;
- permitting later revision.

This sequence resembles responsive regulation in its emphasis on adaptation and institutional responsiveness [4].

For generative justice, institutional learning has additional importance because governance processes produce expectations. An institution that openly corrects error can generate a different relation with participants from one that conceals error. A system that repeatedly fails to correct known problems can generate distrust even among subjects who have never personally experienced the original harm.

Institutional learning is therefore a form of generative governance.

5.35 Temporal Closure and the Need for Institutional Decisions

Justice operates under a tension between open history and institutional closure. Subjects and relations continue to develop, while courts, agencies, organizations, insurers, and assistance programs must make decisions at finite times.

A legal case cannot remain open until every downstream consequence of a wrong has become observable. A compensation program must eventually determine eligibility and amount. An organization must decide whether a temporary measure continues. An assistance program must allocate finite resources.

Closure is therefore an institutional necessity.

The normative problem concerns how closure is designed. A decision can be final with respect to one object while leaving other forms of support open. A damages award can settle a legal claim while healthcare or social assistance continues. A disciplinary finding can close an investigation while institutional reform remains subject to later review. A historical commission can publish findings while archival research continues.

This layered closure is especially suitable for generative harms because it avoids the fiction that one institutional decision exhausts the history.

5.36 Revisability Across the Temporal Sequence

Revisability provides one response to the tension between timely action and incomplete knowledge. Institutions can make decisions that remain open to specified forms of reconsideration when relevant evidence or circumstances change.

Revisability can include:

- appeal;
- periodic reassessment;
- sunset clauses;
- adjustable assistance plans;

- correction of public records;
- reopening under newly discovered evidence;
- review of long-term institutional consequences.

Revisability does not eliminate the need for finality. Endless reopening can create insecurity and impose substantial burdens. The design problem is to identify which components require closure and which benefit from structured revision.

The same principle applies to assistance. A subject's needs and chosen future can change. A program designed around one assumed trajectory can become misaligned with later development. Periodic subject participation can help maintain fit between support and current generativity.

5.37 Over-Prevention and the Suppression of Generativity

A theory emphasizing irreversibility faces an obvious danger: fear of future harm can justify excessive preservation of existing conditions.

Innovation, migration, career change, institutional reform, experimentation, political contestation, artistic creation, and scientific research all involve risk. A system designed to prevent every possible contraction of existing generativity can freeze relations and protect incumbents.

Over-prevention can also create domination. Intensive monitoring can reduce privacy. Broad protective powers can silence dissent. Strong preclearance rules can centralize authority. Institutional reluctance to permit exit can transform care into control.

The preventive domain therefore requires a balance among protection, experimentation, reversibility, and agency.

The relevant goal is preservation of just generative possibility, not preservation of the status quo as such.

5.38 Over-Intervention During Uncertainty

The interruption phase carries a parallel danger. Interim measures can become punitive before responsibility is established. Public suspension can create reputational harm. Separation can isolate the person ostensibly being protected. Investigative processes can generate stigma. Protective monitoring can become surveillance.

A just interruption mechanism should therefore attend to the generative effects of the intervention itself.

This requires explicit analysis of:

- reversibility;
- confidentiality;
- duration;
- burden distribution;
- public interpretation;
- procedural voice;
- access to independent review;
- risks of institutional retaliation;
- effects on third parties.

Interruption is itself a generative event.

5.39 Over-Restoration and Counterfactual Paternalism

The post-harm domain has its own form of excess. A powerful institution can attempt to reconstruct the subject toward an imagined counterfactual state with greater fidelity than the present subject desires.

This paper refers to the problem as *counterfactual paternalism*. Counterfactual paternalism occurs when an institution treats a reconstructed no-harm subject as having stronger authority over restorative design than the actual historical subject who now exists.

Examples can include pressure to return to an earlier occupation, mandatory rehabilitation toward an institutional performance norm, reconstruction of a community according to an idealized historical image, or intrusive interventions intended to eliminate every behavioral trace of harm.

Counterfactual information remains relevant to responsibility and valuation. Its normative role requires limits.

The right to a different future developed in Section 4.23 therefore functions as a constraint on post-harm restoration.

5.40 Temporal Misclassification of Justice Problems

Institutions can make systematic errors by treating a problem from one temporal domain as though it belonged to another.

A continuing harm can be misclassified as a completed harm and addressed only through compensation.

A preventable risk can be treated exclusively as a future damages problem.

A subject in need of immediate interruption can be required to complete a lengthy final adjudication.

A historically altered subject can be treated as though literal restitution will automatically restore the prior generative relation.

A post-harm assistance need can be denied because corrective responsibility cannot be assigned to a single actor.

These errors arise partly because institutions possess different competencies and incentives. Courts, insurers, employers, regulators, welfare systems, and community organizations often encounter the same history through different temporal frames.

The next section examines this institutional division of labor directly.

5.41 Temporal Coordination Among Institutional Responses

A coherent response to generative harm requires handoffs among institutions and functions across time.

A preventive system can identify a risk and create a review trigger. An interruption mechanism can preserve evidence and maintain temporary support. A legal process can determine rights and responsibility. A compensation mechanism can transfer resources. A healthcare or rehabilitation system can address specific capacities. An organization can support re-entry. A public authority can implement structural reform. An assistance program can support alternative future trajectories.

Failure of coordination can create gaps. A court can recognize a wrong after the subject has lost access to the practical supports required for recovery. An employer can restore employment while healthcare or visa status remains unresolved. An assistance program can supply money while official records continue to reproduce the original stigma.

Temporal coordination therefore requires attention to dependencies among remedies.

The issue can be represented through a directed structure in which the effectiveness of one response depends upon completion of another. Later sections will treat such dependencies as possible objects of structural comparison.

5.42 Phase-Specific Conceptions of Success

The three temporal domains require different concepts of success.

For prevention, success can mean that a legitimate activity proceeds while unjustified contraction of important generative conditions is avoided or made less likely.

For interruption, success can mean that harmful propagation is stopped, essential options are preserved, evidence remains available, and interim burdens remain proportionate.

For post-harm response, success can involve a plural combination of restitution, compensation, recognition, rehabilitation, assistance, renewed access, institutional reform, and future generative expansion.

These criteria should not be collapsed into one metric.

A prevention regime can succeed even though no harm materializes from which to calculate damages. An interim measure can succeed by preserving options even when the underlying allegation later proves mistaken, provided the measure was proportionate and procedurally justified. A post-harm assistance program can succeed in expanding the subject's future without reconstructing the counterfactual trajectory.

Temporal justice therefore requires phase-sensitive evaluation.

5.43 Temporal Ordering of Normative Priorities

Some normative priorities also change with time.

Before harm, the preservation of agency can support informed choice and protection against unjustified risk.

During harm, safety and interruption can acquire temporary urgency while procedural protections remain active.

After harm, acknowledgment, responsibility, compensation, rehabilitation, assistance, and future direction become increasingly important.

This ordering should not be treated as a rigid lexical hierarchy. Rights and procedural constraints remain relevant throughout. The point is that the practical salience of different values changes as the system evolves.

A mature governance framework therefore needs temporal sensitivity without permitting emergency language to erase ordinary protections.

5.44 Prevention Under Limited Reparability

The analysis developed in this and previous sections supports a conditional relation between reparability and preventive significance.

Where a harm is highly reparable, institutions can rely more extensively on later correction, subject to the costs and rights involved.

Where a harm has low reparability, prevention and early interruption acquire greater weight.

This relation can be stated schematically as Equation 38.

$$\text{lower expected reparability} \implies \text{greater relevance of prevention and early interruption}, \quad (38)$$

with the implication conditioned by probability, severity, rights, proportionality, uncertainty, and the costs of preventive action.

Equation 38 is intentionally weaker than a decision rule. Low reparability does not automatically authorize intrusive preventive measures. It identifies a factor that should enter the justification of institutional safeguards.

5.45 Post-Harm Justice Under Limited Restorability

Limited restorability also changes the ethical meaning of post-harm response. Where historical equivalence cannot be attained, an institution should avoid presenting a weaker form of repair as though it had erased the loss.

A more transparent structure distinguishes:

1. what has been returned;
2. what has been compensated;
3. what has been acknowledged;
4. what has been rehabilitated;
5. what remains unavailable;
6. which new conditions are being supplied;
7. which future decisions remain with the affected subject.

This structure permits institutional humility without abandoning action.

The inability to restore a singular relation does not justify withholding financial compensation. The inability to calculate a full counterfactual does not justify ignoring an identifiable deprivation. The impossibility of historical reconstruction does not eliminate the value of recognition, rehabilitation, or assistance.

Justice under limited restorability therefore becomes plural and explicit about the scope of each measure.

5.46 Temporal Justice and Human Finitude

The temporal architecture also reflects a basic feature of human life: time is finite.

A delayed educational opportunity does not enter an indefinitely extended life. Years spent outside a profession consume part of a finite career. A health condition can alter later life stages. Care responsibilities change across time. Relationships and communities can disappear. People age and die.

This finitude strengthens the significance of delay without implying that later life lacks generative value. New possibilities can remain deeply valuable. Their existence does not make time fungible.

A year cannot always be treated as a resource that can be returned by adding a year at another point in life. The surrounding body, relationships, institutions, and historical world are different.

Temporal justice therefore requires attention to the location of opportunities within a life as well as their nominal duration.

5.47 Intergenerational Temporal Structure

At collective scale, the temporal architecture can extend across generations. A deprivation affecting one generation can alter education, wealth, institutional position, ecological conditions, cultural transmission, and political influence available to later generations.

Historical reparations literature has long confronted the difficulties created by such temporal distance [67, 62, 12].

A generative account highlights the mechanisms through which earlier conditions become later conditions. The relevant object can include inherited resources, institutional structures, social classifications, cultural practices, ecological systems, and accumulated network positions.

Intergenerational response can therefore involve both historical acknowledgment and present transformation. Literal reconstruction of an earlier social order can be impossible or undesirable. Current institutions can nevertheless address continuing structures generated through the earlier injustice.

The distinction between historical equivalence and present generative justice becomes especially important in these cases.

5.48 Temporal Architecture of Collective Repair

Collective harms also reveal the importance of sequencing.

Immediate humanitarian assistance can precede full historical investigation. Truth recovery can precede some reparative programs and continue after others begin. Institutional reform can occur while compensation claims remain open. Collective recognition can alter the context in which later individual claims are interpreted.

De Greiff's work on large-scale reparations emphasizes the institutional design problems that arise when reparative programs address large populations and broader goals such as recognition, civic trust, and social solidarity [13, 12].

The present temporal framework interprets these programs as multi-stage generative interventions. Their effects depend partly upon ordering. A payment before acknowledgment can carry a different social meaning from the same payment after public recognition. Institutional reform before re-entry can create a different environment from re-entry followed by reform.

Sequence is therefore itself a possible object of justice.

5.49 Temporal Justice Beyond Liability

The distinction among prevention, interruption, and post-harm assistance shows why justice cannot be reduced to assignment of liability.

Liability remains essential where a wrongdoer bears responsibility. It supports accountability, compensation, restitution, and expressive recognition of the wrong.

Other temporal functions can arise independently.

A public institution can prevent risk without identifying a future wrongdoer. An employer can interrupt a dangerous process before completing responsibility findings. A healthcare system can provide treatment without determining legal fault. A social-insurance system can support a person whose generative capacity has contracted through diffuse causes. A community can provide solidarity without claiming that the assistance constitutes reparation by the original wrongdoer.

This plurality avoids a harsh consequence of a liability-only system:

absence of fully attributable liability $\nRightarrow$ absence of every legitimate claim to assistance.

The distinction between responsibility and assistance will receive fuller treatment in later institutional sections.

5.50 Cross-Phase Procedural Justice

Procedural justice should operate across the full temporal sequence.

Before harm, procedure governs authorization of risky interventions.

During harm, procedure governs temporary protection, investigation, evidence, review, and burden distribution.

After harm, procedure governs responsibility, valuation, recognition, participation in restorative design, and access to assistance.

A coherent system should avoid procedural discontinuities. Information supplied during an interim process should not become an unchallengeable fact in later adjudication. Participation during restorative design should not require waiver of unrelated legal rights unless clearly justified. Temporary classifications should be reviewed before becoming permanent.

Cross-phase procedural design is therefore essential for preventing protective measures from becoming new sources of injustice.

5.51 Temporal Integrity of Records

Records across phases should preserve temporal context.

A record that states only a final conclusion can obscure the uncertainty present when earlier decisions were made. A record that preserves only the initial allegation can obscure later correction. A record that overwrites every earlier state can eliminate information needed to understand institutional history.

A temporally adequate record can preserve:

- event time;
- knowledge available at the event time;
- decision time;
- decision rationale;
- later evidence;
- revisions;
- superseded conclusions;
- downstream institutional actions.

Such a record does not make the social system epistemically complete. It improves traceability of the institution's own knowledge evolution.

This distinction will become important in the later analysis of reflexive non-closure and event-oriented provenance.

5.52 Temporal Multiplicity of Justice

The framework developed in this section yields a plural conception of temporal justice.

A single history can generate several legitimate claims at once:

- a claim that the original harm should have been prevented;
- a claim that institutional actors should have interrupted it earlier;
- a claim for responsibility and acknowledgment;
- a claim for restitution;
- a claim for compensation;
- a claim for rehabilitation;
- a claim for assistance;
- a claim for institutional reform;
- a claim for protection of future subjects.

These claims need not be reducible to one another.

Generous post-harm compensation does not prove that inadequate prevention was justified. Successful future reform does not eliminate the claimant's past loss. Effective assistance does not convert the original deprivation into an equivalent exchange. Recognition does not replace material response. Material response does not automatically repair social meaning.

Temporal justice is therefore cumulative in structure even where particular claims can legally or institutionally settle others.

5.53 Temporal Architecture and the Problem of Equivalence

The temporal framework also clarifies why the later atlas of equivalence is necessary.

At the pre-harm stage, comparison can concern the preservation of option sets, risk distributions, resilience, rights, and control structures.

During an ongoing harm, comparison can concern rates of propagation, transition mechanisms, exposure, remaining reachable possibilities, and the effects of interim intervention.

At the post-harm stage, comparison can concern resources, status, relations, capabilities, conversion structures, trajectories, attractors, robustness, generative mechanisms, recognition, or newly created opportunities.

The object of comparison therefore changes with temporal purpose.

A single phrase such as “restore the victim” obscures these distinctions. The subject can be protected successfully at one temporal layer and remain unrepaired at another. The system can achieve strong post-harm compensation after poor interruption. Excellent prevention can make post-harm comparison unnecessary in a particular case.

The atlas developed later must therefore remain temporally indexed.

5.54 Implications for Institutional Design

The temporal architecture developed in this section yields several implications for institutional design.

First, prevention, interruption, and post-harm response should be treated as distinct functions even where one institution performs more than one.

Second, high irreversibility increases the importance of procedures that preserve options before final determination.

Third, ongoing harm requires mechanisms capable of acting under uncertainty without converting temporary protection into premature final judgment.

Fourth, evidence preservation and provenance are themselves components of future restorability.

Fifth, delay can produce substantive generative effects and should enter the evaluation of institutional performance.

Sixth, post-harm response should remain plural. Restitution, compensation, rehabilitation, recognition, assistance, re-entry, and institutional reform address different objects.

Seventh, assistance can remain justified where complete liability or complete counterfactual reconstruction is unavailable.

Eighth, the present subject should retain meaningful authority over future trajectory formation.

Ninth, institutional closure should be layered so that one final decision does not imply that every dimension of the history has been resolved.

Tenth, every phase can itself generate harm. Preventive surveillance, over-intervention, stigmatizing interim measures, coercive rehabilitation, and counterfactual paternalism therefore require independent scrutiny.

Eleventh, post-harm learning should feed into subsequent preventive governance.

Twelfth, the effectiveness of a temporal response depends upon coordination among institutions whose competencies differ.

These implications lead directly to the next section. The temporal architecture identifies the functions that must be performed; it does not establish that one institution should perform all of them. Section 6 therefore examines the institutional division of labor among law, governance, political economy, organizational practice, social protection, and assistance. The analysis asks which temporal and restorative functions each institutional form performs comparatively well, which forms of information and authority each requires, and where coordination or supplementation becomes necessary.

6 Institutional Division of Labor Across the Harm Timeline

This section examines the institutional division of labor through which prevention, interruption, restitution, compensation, rehabilitation, recognition, assistance, and future-oriented support can be pursued. The temporal architecture developed in Section 5 identified several functions that justice can require at different stages of a harmful process. The present section asks which institutional forms are suited to those functions, which forms of knowledge and authority they possess, which objects they can represent effectively, and which dimensions of generative harm tend to exceed their ordinary capacities.

The analysis rejects the assumption that one institutional form should become a universal mechanism of generative justice. Law, administrative governance, markets, insurance, social protection, organizations, professional communities, health and rehabilitation services, civil society, and affected subjects possess different forms of competence. Their strengths arise partly from the same structures that produce their limitations. Courts obtain legitimacy from bounded procedures and authoritative decisions, yet those requirements constrain open-ended historical reconstruction. Markets and general equivalents enable flexible conversion across heterogeneous goods, yet singular relations and non-market values can resist substitution. Organizations possess detailed local knowledge and rapid intervention capacity, yet they can also reproduce the power structures implicated in the original harm. Social protection can respond to need without proving individual fault, yet generalized eligibility rules can be insensitive to singular histories. Assistance can respond to the actual subject's present condition, yet assistance alone cannot discharge duties of responsibility or acknowledgment.

The institutional problem is consequently one of differentiated competence, coordination, and limits. A mature theory of generative return should identify which institution is answering which question before evaluating whether the institution has succeeded.

6.1 Institutional Functions and Distinct Objects of Justice

The same harmful history can produce several institutionally distinct questions.

A court can ask whether a legal right was violated and what remedy follows. An employer can ask whether immediate protection or organizational change is required. A regulator can ask whether the institutional process creates an unacceptable risk of recurrence. An insurer can ask whether a covered loss has occurred. A social-protection system can ask whether the affected person currently requires income security or healthcare. A rehabilitation service can ask which capacities can be supported. A professional community can ask how access, standing, or network participation can be restored. A reparations program can ask how historical acknowledgment, material transfer, public recognition, and institutional reform should be coordinated. The affected subject can ask which future remains worth pursuing.

These questions overlap while remaining non-identical. Their institutional answers can therefore diverge without every divergence constituting inconsistency.

Let $\mathcal{I} = \{I_1, \dots, I_n\}$ denote a set of institutional actors and let $\mathcal{F} = \{F_1, \dots, F_m\}$ denote relevant restorative functions. An institutional architecture can be represented schematically by Equation 39.

$$\mu : \mathcal{I} \times \mathcal{F} \longrightarrow \{\text{primary, supporting, limited, unsuited}\}. \quad (39)$$

Equation 39 represents a functional mapping rather than a permanent assignment. Institutional competence depends upon jurisdiction, legal order, resources, organizational form, scale, and type of harm. The same institution can also occupy several roles.

The important point is that restorative functions should be distinguished before institutional responsibility is allocated. Failure to separate them can create two opposite errors. One institution can be expected to solve a problem outside its competence, or a problem can disappear institutionally because every actor regards it as belonging elsewhere.

6.2 Institutional Compression

Every institution simplifies the social world in order to act upon it. Courts translate complex histories into legally cognizable claims, evidence, causation, rights, responsibility, and remedies. Insurance systems translate events into covered risks and compensable losses. Administrative programs use eligibility categories, thresholds, standardized evidence, and procedural rules. Markets translate heterogeneous goods into prices where exchange is possible. Organizations translate interpersonal and productive relations into roles, policies, reporting structures, performance measures, and disciplinary categories.

This paper refers to such transformations as *institutional compression*. Compression is neither inherently unjust nor merely a technical defect. Institutions require bounded representations in order to decide, allocate, record, and coordinate.

Mashaw's study of Social Security disability administration demonstrates how bureaucratic justice depends upon institutional modes of decision and the organization of mass adjudication [33]. Fuller's analysis of legality similarly directs attention toward formal characteristics through which law can guide conduct, including generality, publicity, prospectivity, intelligibility, consistency, practicability, relative stability, and congruence between official action and declared rules [21]. These institutional virtues require abstraction from the unrestricted particularity of each life history.

The normative question is therefore not whether compression can be eliminated. The relevant questions concern which dimensions are compressed, which remain visible, which institutional decisions depend upon the compressed representation, and what complementary mechanisms exist for values that the institution does not represent well.

A theory of generative return can contribute by making these omitted dimensions explicit. The conclusion need not be that the institution must internalize every omitted variable. Another institution or practice can sometimes address the omitted dimension more appropriately.

6.3 Legal Adjudication and Bounded Rectification

Law provides several capacities of exceptional importance to generative justice. It can articulate rights, establish prohibitions, define authority, constrain arbitrary action, compel disclosure, preserve procedural standing, assign responsibility, order cessation, return identifiable property, correct formal status, award damages, and create enforceable duties.

These capacities are particularly valuable where another actor controls conditions essential to a subject's generativity. A legal right can transform access from discretionary permission into a claim. Due process can constrain the unilateral removal of status. Anti-retaliation rules can preserve the capacity to contest institutional action. Injunctions can interrupt harmful processes before final damages are calculated. Liability can force some costs of harmful activity back toward actors capable of influencing the underlying risk.

The economic analysis of accident law has long examined how liability rules affect both the allocation of accident costs and incentives for prevention [8]. The generative perspective adds that prevention can protect recursive and relational consequences that later damages cannot fully reproduce.

Legal adjudication also possesses strong expressive and historical functions. A judgment can establish that a wrong occurred, reject an invalid institutional classification, recognize responsibility, and create an authoritative public record. These functions can influence social standing and subsequent relations.

The strengths of law are inseparable from limits. Adjudication requires legal causes of action, jurisdiction, evidence, standards of proof, legally relevant causation, procedural closure, and administrable remedies. A court ordinarily cannot leave responsibility open indefinitely while observing every later development of a claimant's life.

Generative consequences can exceed these boundaries. A missed collaboration can alter future network position. A false classification can influence actors who never participate in litigation. Years outside a profession can change skills, preferences, and relationships. These consequences can remain normatively significant while exceeding what a court can establish with sufficient legal certainty.

The theory developed here therefore treats legal remedy as one institutionally bounded projection of generative repair. This description preserves its importance while avoiding the inference that legally unremedied dimensions are normatively nonexistent.

6.4 Legal Causation and Generative Propagation

Generative systems make causation institutionally difficult because one intervention can participate in long chains of subsequent events.

Suppose an unlawful exclusion causes loss of employment. The employment loss affects income, professional participation, geographic decisions, network relations, later applications, and the subject's willingness to remain within the field. Some later consequences may be strongly connected to the original wrong. Others arise through independent choices, environmental changes, and subsequent actors.

A complete generative graph can therefore contain far more causal paths than a legal system can reasonably attribute to the original defendant.

The distinction can be represented by separating the set of all causally relevant generative consequences $\mathcal{C}_G$ from the set of consequences institutionally attributed within a legal process $\mathcal{C}_L$:

$$\mathcal{C}_L \subseteq \mathcal{C}_G. \quad (40)$$

Equation 40 is schematic. Legal doctrine can recognize indirect and future consequences, and the boundary varies across fields and jurisdictions. The expression identifies a structural difference between causal relevance within a generative history and institutional attribution for a legally enforceable remedy.

This difference provides one reason post-harm assistance should not always depend upon successful attribution of every need to one legally responsible actor.

6.5 Legal Finality and Open Generative History

Legal systems also require forms of finality. Parties need to know whether a claim has been resolved, institutions need to allocate resources, and social relations cannot remain indefinitely subject to the re-opening of every past decision.

Generative history remains open. The effect of a professional exclusion can become more visible years later. A reputational event can reappear when old records are rediscovered. A health condition can develop in ways that were uncertain at the time of judgment.

This creates a structural tension:

$$\text{institutional finality} \leftrightarrow \text{open historical development}. \quad (41)$$

Equation 41 should not be resolved by eliminating either side. A workable legal order requires closure, while a theory of generative justice should resist treating legal closure as proof that all generative consequences have ceased.

Layered institutional response provides one solution. A legal damages claim can close while medical support continues. A formal judgment can become final while a social-protection program remains responsive to later need. A reparations program can distribute benefits while historical research continues.

6.6 Public Law and Preventive Architecture

Law also acts before individualized adjudication. Legislation, regulation, licensing, administrative procedure, occupational standards, environmental rules, anti-discrimination requirements, public-health measures, and institutional duties can govern conditions that affect generativity.

Preventive law has an important advantage over individualized post-harm litigation. It can act upon recurring structures without requiring every affected subject to experience severe harm and then prove the consequences separately.

This function is especially important where harms are diffuse, cumulative, or difficult to reverse. Environmental regulation provides an obvious example. Employment rules can similarly address retaliation, safety, discriminatory conditions, or due process at the level of institutional practice.

Preventive law can also become overinclusive or rigid. General rules necessarily operate across heterogeneous situations. Excessively restrictive safeguards can suppress experimentation, organizational adaptation, and legitimate risk-taking. Legal design therefore requires mechanisms for review, proportionality, exemptions where justified, and revision in response to evidence.

6.7 Administrative Governance and Iterative Decision

Administrative governance occupies an intermediate position between general legal norms and highly localized organizational practice. Agencies and public administrative bodies can collect information, monitor risk, issue standards, review decisions, administer benefits, conduct inspections, adapt rules, and coordinate across cases.

Mashaw's work on bureaucratic justice is particularly relevant because it examines how justice is produced through administrative structures that must decide large numbers of claims under organizational constraints [33]. Administrative systems cannot reproduce the individualized inquiry of an unconstrained philosophical ideal. Their design determines which evidence becomes visible, which categories are used, and what forms of review remain available.

Generative harms create both opportunities and risks for administration. Longitudinal data and repeated cases can reveal patterns that are invisible in individual litigation. Agencies can identify recurring risk factors, monitor institutional behavior, and intervene before severe loss becomes widespread.

At the same time, administrative classification can itself become generative. A risk score, disability category, professional flag, eligibility decision, or inspection result can influence later institutional action. Errors can propagate across systems.

Administrative governance therefore requires strong correction, provenance, appeal, and revision mechanisms when classifications affect future generative conditions.

6.8 Responsive Regulation and Graduated Intervention

Responsive regulation offers a useful model for matching institutional response to the conduct and context of regulated actors. Ayres and Braithwaite reject a simple opposition between strict regulation and deregulation and develop a model in which regulatory responses can escalate or adapt according to behavior and the effectiveness of less intrusive mechanisms [4].

This structure aligns naturally with the temporal architecture developed in Section 5. A governance system can begin with information, dialogue, or self-regulatory obligations, escalate toward stronger oversight when risks persist, and reserve more coercive interventions for circumstances in which weaker mechanisms fail.

For generative justice, responsiveness has two advantages. It reduces the need to choose permanently between *laissez-faire* and maximal intervention, and it allows the intensity of governance to change as evidence and harm evolve.

Responsive regulation remains subject to the same procedural requirements as other forms of intervention. Escalation should be reviewable, evidence-sensitive, and proportionate. An adaptive regulatory system can otherwise become an opaque mechanism through which discretionary power expands.

6.9 Polycentric Governance and Institutional Diversity

Complex generative systems frequently involve problems that exceed the competence of a single centralized institution. Ostrom's work on polycentric governance challenges the assumption that complex public problems must be organized through a simple market-versus-state dichotomy. Her analysis emphasizes diverse public, private, community, and self-governing arrangements operating across multiple scales [40].

The relevance for generative return is substantial. Different components of a harm can be governed at different levels. A national legal system can establish rights. A professional body can restore credentials. A local organization can alter workplace relations. A public healthcare system can support rehabilitation. A compensation fund can transfer resources. A community body can support social reintegration. Independent archives can preserve historical records.

Polycentricity can increase institutional redundancy and permit multiple routes to correction. A subject blocked by one institution can retain access to another forum. Local knowledge can coexist with external review. Different institutions can experiment with solutions.

Polycentric systems also create coordination problems. Responsibilities can become fragmented. Institutions can shift burdens to one another. Records can be incompatible. A claimant can be required to repeat the same history across multiple procedures. Conflicting decisions can create uncertainty.

The value of institutional diversity therefore depends upon interfaces among institutions as well as upon the number of institutions present.

6.10 Governance as Management of Generative Conditions

Governance extends beyond adjudication. It concerns the design and maintenance of the conditions within which action occurs.

From a generative perspective, governance can operate upon:

- access structures;
- decision authority;
- review procedures;
- information flows;
- institutional memory;
- risk distribution;
- network architecture;
- redundancy;
- standards;
- monitoring;
- funding;
- opportunities for participation;
- mechanisms for correction.

This breadth gives governance strong preventive and intermediate capacities. Governance can modify the field within which future harm would otherwise be generated.

The same breadth creates risks. Governance can become intrusive, technocratic, or paternalistic. Institutions can infer that preservation of generativity authorizes extensive monitoring of subjects. The framework developed here rejects that inference. Governance of generative conditions remains constrained by rights, privacy, autonomy, proportionality, and non-domination.

6.11 Political Economy and Resource Mobilization

Political economy addresses a function that normative theory can easily underestimate: resources required for prevention, interruption, and post-harm support must be produced, collected, pooled, and allocated.

Legal recognition without material capacity can produce nominal remedies. Rehabilitation requires healthcare and professional labor. Re-entry can require training, housing, transportation, childcare, or relocation. Compensation funds require financing. Environmental repair can require long-term investment. Social protection requires sustainable fiscal structures.

Political economy therefore enters the justice of return through the organization of material possibility.

It also determines who bears the cost of uncertainty. Risk can remain with the individual subject, be transferred contractually through insurance, be internalized by an organization, be socialized through taxation, or be shared through community institutions.

Different allocations express different theories of responsibility, solidarity, and risk.

6.12 General Equivalents and Institutional Fungibility

Money possesses unusual institutional usefulness because many heterogeneous claims can be represented and transferred through a common medium. It reduces the information required of the compensating institution because the institution does not need to supply every final good directly.

Let M denote a monetary transfer and let $\mathcal{K}_A$ denote the recipient's current conversion structure. The effect of the transfer depends upon

$$\Gamma_A(M) = \mathcal{K}_A(M; C_A, R_A, H_A). \quad (42)$$

Equation 42 expresses why the same financial transfer can produce different generative consequences across subjects and histories.

The institutional strength of money lies partly in this openness. The state, court, insurer, or organization can provide a flexible resource without making a complete decision concerning the recipient's future.

This can be interpreted as a form of institutional epistemic humility. The institution recognizes that its knowledge of the subject's preferred future is limited and transfers a broadly usable resource.

Fungibility becomes inadequate where the relevant barriers concern non-purchasable status, unique relationships, inaccessible institutions, health conditions, unavailable ecological states, time-sensitive opportunities, or social meaning. Monetary provision then requires supplementation rather than dismissal.

6.13 Insurance and the Socialization of Uncertain Loss

Insurance provides another institutional response to uncertainty. It pools risk across persons and time, converting uncertain individual losses into predictable collective financing requirements.

This function can support generative resilience. A person who experiences illness, unemployment, disability, property loss, or another covered event can receive resources before the full long-term generative consequences become known.

Insurance also illustrates the difference between responsibility and risk pooling. Payment can occur because a covered event happened, independently of whether another person committed a wrong.

The structure is valuable for generative assistance because many contractions of generativity arise from events without a single responsible actor.

Insurance has its own limits. Coverage definitions compress heterogeneous losses. Exclusions leave some risks individualized. Pricing can shift burdens toward vulnerable subjects. Moral hazard, adverse selection, administrative costs, and market access affect institutional design. Commercial insurance can also be unavailable for risks that are highly correlated, difficult to price, or socially unacceptable to leave to market allocation.

These limitations create a role for social insurance and public protection.

6.14 Social Protection and Need-Based Continuity

Social protection provides a particularly important institutional mechanism where present need does not map cleanly onto corrective responsibility. The International Labour Organization's Social Protection Floors Recommendation, 2012 (No. 202) defines nationally determined social protection floors around basic social security guarantees and emphasizes universality based on social solidarity, adequacy and predictability of benefits, dignity, and the primary responsibility of the state [26].

The ILO framework includes access to essential healthcare and basic income security across the life cycle, including circumstances such as sickness, unemployment, maternity, disability, childhood, and old age [26].

The relevance for generative return lies in the decoupling of immediate support from individualized fault. A person can require healthcare, income security, or other enabling conditions while a legal dispute concerning responsibility remains unresolved. A person can also require support where no wrongful act occurred.

Social protection can therefore stabilize the subject's generative conditions during periods in which other institutions are determining liability, rehabilitation, or long-term direction.

This role should remain conceptually distinct from reparation. A social benefit provided because a person meets a general need-based criterion does not necessarily acknowledge a historical wrong. A reparative payment can carry a different normative meaning even where its material amount is identical.

6.15 Social Protection as a Generative Floor

The concept of a social-protection floor can be interpreted generatively as the maintenance of a minimum set of conditions below which severe recursive contraction becomes more likely.

Income security can preserve housing, nutrition, transportation, and participation. Healthcare can preserve bodily capacities. Support during unemployment can preserve time for search and retraining. Child benefits can support education and development.

This interpretation does not imply that social protection should be justified only by its productive effects. Human dignity, rights, welfare, equality, and social solidarity provide independent reasons for protection. Generativity adds one further perspective: preventing collapse of enabling conditions can preserve future autonomy and possibility.

A generative floor also differs from complete restoration. It establishes a minimum continuity condition while leaving claims for corrective justice, reparation, recognition, and higher levels of assistance open.

6.16 Organizational Practice and Local Intervention

Organizations occupy a distinctive position because many generative harms arise within workplaces, universities, hospitals, schools, professional associations, platforms, and other institutions that possess immediate authority over the relevant conditions.

Organizations can often act faster than courts. They can separate parties, alter supervision, preserve access, continue salary, modify workload, transfer responsibility, correct internal records, restore credentials, provide temporary facilities, or create independent reporting channels.

They also possess local knowledge concerning roles, informal networks, workflow, and institutional history.

These advantages make organizations central to the interruption phase.

The same organizational proximity creates conflicts of interest. The organization can be implicated in the harm. Managers can have incentives to minimize reputational exposure. Internal investigators can depend upon the same authority structure they are evaluating. Informal retaliation can evade formal policy.

Effective organizational governance therefore benefits from external review, independent channels, transparent procedures, and preservation of appeal rights where stakes are high.

6.17 Organizational Re-Entry

Organizations are also essential to practical re-entry. Courts can order reinstatement, yet daily participation depends upon schedules, colleagues, supervision, workspace, information, role expectations, and informal interaction.

A meaningful re-entry process can require:

- restoration of formal authority and access;
- correction of internal records;
- clarification of reporting relationships;
- protection against retaliation;
- restoration or reconstruction of professional networks;
- transitional workload;
- training where skills or systems have changed;
- independent support;
- periodic review;
- a viable option for transfer or exit.

These measures illustrate the difference between legal status restoration and relational usability.

Organizational re-entry should also preserve the subject's right to decline return. Compensation or assistance should not automatically become conditional upon re-entry into the environment associated with the original harm.

6.18 Professional Communities and Distributed Re-Entry

Some generative conditions are distributed beyond the organization responsible for the original harm. Professional reputation, disciplinary knowledge, collaborative opportunities, conference access, journal participation, certification, and informal standing can belong to wider professional communities.

A university can restore a researcher's internal status while the wider field continues to act upon earlier information. An employer can correct a disciplinary finding while licensing or credentialing bodies retain derivative records.

Professional associations, journals, funders, credentialing bodies, and disciplinary networks can therefore possess roles in distributed re-entry.

This domain is difficult to govern because no single institution controls the whole network. Correction requires provenance and communication across organizational boundaries.

The later atlas will treat such cases through network comparison and distributed state representations.

6.19 Healthcare and Rehabilitation Institutions

Healthcare and rehabilitation institutions address bodily, psychological, and functional dimensions of generative capacity. Their epistemic object differs from that of courts and employers.

A clinician can evaluate present function, treatment options, adaptation, and health-related needs without resolving every question of legal fault. A rehabilitation program can support work, mobility,

communication, cognition, or daily functioning without claiming that the subject will return to the exact pre-harm state.

This future-oriented orientation makes rehabilitation an important bridge between remedy and assistance.

Medicalization also carries risks. A relational or institutional problem can be misrepresented as an individual pathology. A subject can be asked to adapt clinically to an environment that remains unjust.

A generative framework should therefore distinguish support for the subject from normalization of the environment that produced the harm.

6.20 Assistance Institutions and Present-Oriented Need

Assistance differs from corrective justice in its immediate normative question. Corrective justice asks what responsibility follows from a wrong. Assistance asks what support is justified given the subject's current condition, needs, and future possibilities.

The distinction becomes especially important where:

- responsibility is disputed;
- causation is diffuse;
- the wrongdoer no longer exists;
- the responsible party lacks sufficient resources;
- the loss exceeds available legal remedies;
- the subject requires support before adjudication is complete;
- some dimensions of the loss are structurally non-restitutable.

Assistance can be public, organizational, community-based, charitable, professional, or interpersonal.

Its legitimacy need not imply that the underlying wrong has been repaired. Assistance should therefore avoid language suggesting that receipt of support waives claims for responsibility unless such waiver is independently justified.

6.21 Assistance as Expansion of Conversion Capacity

The most distinctive function of assistance in the present framework is its ability to work upon conversion structures.

A monetary award supplies a resource. Assistance can make that resource usable. Examples include legal advice, translation, childcare, rehabilitation, transportation, mentoring, temporary housing, professional retraining, accessible technology, and network introduction.

Let M denote a general resource and $\mathcal{K}$ the recipient's conversion structure. Assistance A can operate upon $\mathcal{K}$:

$$\mathcal{K}^H \xrightarrow{A} \mathcal{K}^{HA}. \quad (43)$$

Equation 43 identifies a different object of intervention from the monetary transfer itself.

The normative objective should remain compatible with subject direction. Assistance that improves conversion only for an institutionally selected life plan can become paternalistic.

6.22 Care and Relational Support

Some forms of generative support are relational rather than transactional. Care, accompaniment, mentoring, social recognition, community support, and trusted interpersonal relations can reduce the burden of navigating recovery or re-entry.

These relations cannot generally be ordered through legal compulsion in the same manner as payment of money or correction of a record. Their value often depends upon voluntariness, trust, and continuity.

This limitation is important. A theory of justice can recognize the importance of relational support while acknowledging that institutions cannot manufacture authentic relationships on demand.

Institutions can nevertheless create conditions under which relations are more likely to form. They can provide safe spaces, mentoring programs, peer networks, community organizations, mediation, and opportunities for sustained participation.

The distinction between producing a relationship and supporting conditions for relation formation is a recurring generative theme.

6.23 Civil Society and Associational Capacity

Civil-society organizations can perform functions that state institutions and markets perform imperfectly. They can provide advocacy, community knowledge, peer support, representation, monitoring, public memory, professional assistance, and spaces for collective organization.

Their proximity to affected communities can provide epistemic advantages. Their independence can permit criticism of state or organizational actors implicated in harm.

Civil society can also reproduce internal inequalities, represent only part of a community, depend upon unstable funding, or lack procedural safeguards.

Its role should therefore be evaluated through the same comparative framework as other institutions. Community proximity does not automatically establish representative authority.

6.24 Reparations Programs and Multi-Object Response

Large-scale reparations programs illustrate the need for institutional coordination because they frequently combine several objects of justice. De Greiff's work emphasizes that programs addressing large populations cannot simply reproduce the logic of individualized judicial compensation and draws attention to recognition, civic trust, and social solidarity alongside material measures [13, 12].

A reparations program can include monetary transfers, restitution, rehabilitation, symbolic recognition, memorialization, institutional reform, and guarantees of non-repetition. The United Nations Basic Principles similarly identify several forms of reparation rather than a single universal remedy [65].

These programs provide an institutional precedent for the plural theory of return developed here. Different components address different objects.

Their design also reveals difficult distributive questions. Individualized precision can conflict with administrative feasibility. Uniform benefits can be manageable while failing to represent heterogeneous losses. Highly individualized assessment can consume resources, delay assistance, and create new evidentiary inequalities.

The design of large-scale generative-return programs will therefore require explicit decisions concerning where precision matters and where standardized support is justified.

6.25 Markets and Functional Substitution

Markets provide powerful mechanisms for substitution. Where goods and services are sufficiently fungible, a claimant can use money to obtain alternatives without requiring a public authority to identify the substitute directly.

This function is valuable for repair. A destroyed standard tool can be replaced. A lost service can be purchased elsewhere. Relocation can substitute for an unusable environment. Training can provide a new occupational route.

Market substitution becomes weaker where value is relational, singular, public, collective, or unavailable for purchase.

A lost historical archive, a particular mentor relationship, public recognition, legal status, ecological integrity, civic membership, and freedom from domination cannot be assumed to possess market substitutes.

Market price therefore represents one comparative relation among many. It is institutionally powerful because of exchangeability, while its normative domain remains limited.

6.26 Institutions of Knowledge and Epistemic Repair

Libraries, archives, scientific institutions, statistical agencies, professional repositories, investigative bodies, and public records systems play a distinct role in generative justice because future repair depends upon what can later be known.

An archive can preserve evidence that a classification was contested. A versioned record can distinguish an original claim from a later correction. A scientific repository can preserve provenance. An ombuds institution can record patterns across cases. Statistical systems can reveal structural disparities that individual proceedings cannot identify.

These institutions support what can be called *epistemic reparability*. A system is more reparable when relevant histories remain traceable enough for later correction, review, and learning.

Epistemic reparability does not justify total recording. Comprehensive surveillance can create severe privacy and domination risks. Records can themselves become harmful generative objects.

Knowledge institutions therefore require selective preservation, access controls, provenance, contestability, and rules governing correction.

6.27 Data Infrastructure and Distributed Evidence

Some governance problems involve multiple parties that cannot or should not share raw information freely. Evidence can contain health data, employment records, proprietary information, personal communications, or sensitive identity information.

A useful institutional architecture can therefore separate common evidentiary standards from centralized possession of all underlying data.

Local actors can maintain protected raw records while sharing verified claims, aggregates, transformations, attestations, or other limited outputs necessary for a common decision process.

This structure can support accountability while reducing unnecessary exposure.

The precise technical mechanisms lie beyond the present section. The important institutional point is that shared epistemic infrastructure does not require complete informational centralization.

6.28 Institutional Memory and Corrective Capacity

Institutions differ in their ability to remember their own errors.

An organization with no durable record of complaints can repeatedly interpret each case as isolated. A system that overwrites corrected classifications can lose information about the mechanism that produced earlier errors. A system that preserves every allegation without adequate supersession can create permanent stigma.

Institutional memory therefore requires more than retention. It requires temporally structured provenance.

Useful memory can preserve:

- what was known at a particular time;
- which decision followed;
- which evidence supported it;

- which later evidence challenged it;
- how the decision was revised;
- which downstream processes used the earlier state.

This structure supports both individual correction and institutional learning.

6.29 Institutional Handoffs

Generative repair often requires one institution to hand a problem to another.

A court can establish wrongful dismissal and award damages. The employer must implement reinstatement. A professional body may need to correct credentials. A healthcare provider can support rehabilitation. A social-protection program can provide income during transition. A regulator can investigate the organizational process that produced the wrong.

Each handoff creates potential information loss and delay.

A robust handoff requires at least:

- clear identification of the function being transferred;
- sufficient information for the receiving institution;
- protection against unnecessary disclosure;
- continuity of temporary support;
- clarity concerning responsibility for follow-up;
- mechanisms for the affected subject to contest errors;
- confirmation that the receiving institution can actually perform the requested function.

Institutional fragmentation becomes unjust when a subject must personally solve coordination failures created by the architecture of the response.

6.30 Restorative Interfaces

The concept of a *restorative interface* describes the rules and information structures through which different institutions coordinate without being merged into one institution.

A restorative interface can specify:

- which findings are authoritative for which purposes;
- which records can be shared;
- how corrections propagate;
- which institution funds interim support;
- which institution evaluates present need;
- which institution determines legal responsibility;
- which institution supports re-entry;
- how unresolved disputes are escalated;
- how the subject participates across the process.

This approach preserves institutional specialization while reducing gaps.

The interface itself becomes a governance object. A perfectly designed compensation program can fail if correction of legal status does not propagate to institutions controlling actual access.

Table 1: Illustrative institutional division of labor in generative return

Institutional form	Characteristic strengths	Common objects	Characteristic limitations
Legal adjudication	Rights, responsibility, authoritative findings, injunctions, restitution, damages	Legal status, property, attributable loss, enforceable duties	Finite causation, proof, jurisdiction, closure, administrability
Administrative governance	Monitoring, standards, repeated-case learning, review, prevention, adaptive intervention	Risk, compliance, institutional process, eligibility, public systems	Classification error, bureaucracy, opacity, discretionary power
Political economy	Resource mobilization, redistribution, insurance, risk pooling, public investment	Income, material resources, aggregate risk, infrastructure	Commensuration limits, allocation conflict, fiscal constraints
Organizations	Rapid local action, workplace or institutional redesign, practical re-entry	Access, supervision, roles, local networks, working conditions	Conflict of interest, internal power asymmetry, reputational incentives
Social protection	Need-based continuity, income security, healthcare, risk socialization	Minimum material and bodily conditions	Generalization, eligibility boundaries, weak historical specificity
Rehabilitation and assistance	Present-oriented support, conversion capacity, individualized enabling conditions	Function, usability, adaptation, future options	Resource intensity, paternalism, inability to discharge responsibility alone
Civil society and communities	Local knowledge, advocacy, solidarity, peer support, representation	Recognition, social relation, participation, collective support	Representation disputes, uneven capacity, unstable resources
Knowledge institutions	Evidence, provenance, public memory, revision, long-term learning	Records, claims, historical states, institutional knowledge	Privacy risk, surveillance, archival bias, downstream misuse

6.31 A Functional Matrix of Institutional Competence

Table 1 summarizes the characteristic strengths of several institutional forms. The entries are analytical tendencies rather than universal claims. Actual capacities vary across legal and political systems.

The principal purpose of Table 1 is to show that institutional strengths correspond to different objects of restorative comparison. The existence of multiple institutions therefore reflects more than administrative complexity. Different objects of justice require different forms of knowledge and action.

6.32 Institutional Fit Across Temporal Phases

Institutional competence also changes across the temporal sequence.

Before harm, legislation, regulation, organizational safeguards, professional standards, insurance design, public infrastructure, and social protection can reduce vulnerability and preserve options.

During ongoing harm, organizations and administrative institutions often have the greatest capacity for rapid interruption. Courts can provide urgent injunctive relief where legal mechanisms permit. Social protection and insurance can stabilize material conditions. Knowledge institutions can preserve evidence.

After harm, courts and reparations processes can address responsibility, restitution, compensation, and public recognition. Healthcare and rehabilitation services can address present capacities. Organizations and professional communities can support re-entry. Political-economic institutions can mobilize long-term resources. Assistance systems can support new trajectories. Regulators can transform the structural conditions that permitted the original harm.

No one sequence is universal. The framework supplies a method for asking whether the required function has an institution capable of performing it.

6.33 Institutional Gaps

A generative harm can remain practically unresolved even when every institution performs its conventional task correctly.

Suppose a court awards compensation for wrongful exclusion. The employer pays the award. The official employment record is corrected. The claimant remains outside the profession because professional relationships disappeared and no institution has responsibility for re-entry.

Every bounded institutional task can be completed while a significant generative dimension remains unsupported.

The paper calls such situations *institutional gaps*. A gap exists where a normatively relevant object of prevention, interruption, return, or assistance falls outside the effective mandate of available institutions.

Common gaps can involve:

- network reconstruction;
- long-term re-entry;
- non-market relational support;
- coordination of corrected records;
- support during unresolved legal proceedings;
- singular opportunity loss;
- long-term conversion impairment;
- harms with diffuse responsibility;
- transitions between compensation and new trajectory formation.

Identification of a gap does not automatically imply that the state should create a new bureaucracy. Civil society, professional associations, insurance, community institutions, or hybrid mechanisms can sometimes address the relevant function more appropriately.

6.34 Institutional Overlap

The opposite problem is institutional overlap. Several actors can claim authority over the same object while using different standards.

A court, employer, professional body, insurer, and administrative agency can reach different conclusions about the same event because each applies a different evidentiary rule and institutional purpose.

Overlap can provide redundancy and multiple correction pathways. It can also produce inconsistent records, repeated proceedings, and strategic forum use.

The existence of differing judgments does not always indicate failure. A legal finding of insufficient evidence for liability can coexist coherently with an organizational decision to alter future safeguards. An insurer can pay under a contractual standard without determining tort liability. A social-protection system can provide assistance without determining responsibility.

Institutional coherence therefore requires purpose-specific interpretation of findings.

6.35 Institutional Contradiction

Overlap becomes more problematic when institutions make claims that cannot practically coexist.

A court can order reinstatement while an independent licensing body continues to deny the credential required for work. An employer can correct an internal record while a shared database continues distributing the superseded classification. A reparations program can recognize a community's claim while another state institution continues the policy that generated the injury.

These contradictions require governance at the interface level.

The theory of generative return can help identify contradictions because it tracks dependencies among conditions. Restoration of C_1 can be ineffective when access to C_2 remains blocked.

6.36 The Problem of Institutional Sequencing

The order in which institutions act can affect the generative outcome.

Rehabilitation before stable housing can have limited effect. Reinstatement before protection against retaliation can recreate vulnerability. Compensation before legal-status correction can leave important uses of the money unavailable. Public acknowledgment before a reliable factual process can create new disputes. Long legal proceedings before interim support can permit severe material contraction.

Institutional sequence is therefore part of the restorative process.

Let $I_1, \dots, I_n$ denote institutional interventions. In a path-dependent system, there is no general reason to expect

$$I_2 \circ I_1 = I_1 \circ I_2. \quad (44)$$

Equation 44 states that institutional interventions can be non-commutative. Their order can alter the system upon which later interventions act.

This feature suggests that restorative design should consider sequencing explicitly rather than merely compiling a list of remedies.

6.37 Institutional Latency

Institutions also operate at different speeds. Emergency organizational measures can occur within hours. Administrative review can require weeks or months. Litigation can require substantially longer periods. Professional reputation can take years to rebuild. Social-protection systems can provide benefits on another schedule.

The mismatch between institutional time and generative time can itself produce harm.

Let τ_I denote the response time of an institution and τ_G denote the time scale on which a relevant generative condition deteriorates. A potentially serious mismatch arises where

$$\tau_I \gg \tau_G. \quad (45)$$

Equation 45 explains why long-form adjudication can require interim mechanisms. The institutional process can remain procedurally appropriate while being temporally incapable of preserving the relevant condition.

An effective architecture therefore combines institutions operating at different timescales.

6.38 Institutional Knowledge and Epistemic Specialization

Institutions differ not only in authority but also in what they can know.

Courts are structured around admissible evidence and contested claims. Regulators accumulate repeated-case information. Organizations possess local operational knowledge. Healthcare institutions possess clinical expertise. Professional communities possess disciplinary knowledge. Social-protection systems observe patterns of need. Subjects possess experiential knowledge. Communities possess local historical and relational knowledge.

These epistemic advantages can be complementary.

The corresponding danger is epistemic hierarchy. One institution can treat its own form of knowledge as exhaustive. A legal judgment can be interpreted as a complete clinical assessment. A medical diagnosis can be used to settle a question of institutional responsibility. An organizational performance metric can be treated as an exhaustive representation of professional value.

The institutional division of labor therefore requires epistemic boundaries as well as jurisdictional boundaries.

6.39 Institutional Authority and the Risk of Paternalism

Institutions capable of restoring generative conditions can also acquire power over the future of the subject.

A rehabilitation program can condition benefits on compliance with a selected life plan. An employer can offer re-entry only under intensive monitoring. A public assistance system can restrict how resources are used. A reparations program can define an authorized identity through which claimants must continue to present themselves.

These arrangements can increase some forms of support while reducing autonomy.

The problem is especially serious when institutions justify control by claiming expert knowledge of what the subject would have become without the harm. Such a claim converts counterfactual reconstruction into authority over the actual subject.

Institutional assistance should therefore be evaluated through the procedural principles developed later in the paper, including participation, contestability, proportionality, privacy, revisability, and the right to choose a different future.

6.40 Institutional Non-Domination

The institutional division of labor should also reduce dependence upon unreviewable power.

Polycentricity, appeal, independent review, portability of benefits, multiple reporting channels, and external oversight can reduce the extent to which one actor controls all conditions necessary for a subject's future.

The normative value of institutional multiplicity therefore includes non-domination as well as specialization.

Institutional diversity can itself become burdensome when claimants must navigate many procedures without support. A system can be formally polycentric and practically inaccessible.

Non-domination therefore requires both multiple loci of authority and usable interfaces among them.

6.41 Institutional Resilience

A restorative architecture should remain capable of functioning when one institution fails.

If an employer's internal process is compromised, external review should remain available. If a wrongdoer is insolvent, some forms of assistance should remain possible. If evidence is lost in one repository, independent records can preserve relevant history. If one funding mechanism collapses, essential care should not disappear immediately.

This redundancy is analogous to resilience in other generative systems. Institutional resilience concerns the capacity of the justice architecture to continue preserving essential functions under disturbance.

The design objective is not duplication of every function. It is avoidance of single points of failure for highly consequential conditions.

6.42 Institutional Feedback and Learning

Institutions should also learn across cases. Repeated individual claims can reveal a common structural mechanism. A compensation system can identify recurring categories of loss. A regulator can observe patterns in complaints. A healthcare service can detect common barriers to rehabilitation. Professional bodies can identify recurring gatekeeping practices.

Learning requires an information pathway from post-harm cases back into prevention.

Equation 46 represents this feedback.

case response → aggregated learning → institutional reform → changed future conditions. (46)

Equation 46 is a generative cycle of governance. The value produced by one case includes information capable of improving the institutional environment of later subjects.

The same process requires safeguards. Aggregation should not expose sensitive individual information unnecessarily.

6.43 Institutional Evaluation and Multiple Standards of Success

Because institutions perform different functions, their success should be evaluated by function-specific standards.

A court should not be judged solely by whether every dimension of the claimant's life becomes restored. Its performance can be evaluated through lawfulness, procedural justice, accuracy, timeliness, enforceability, and the adequacy of legally available remedies.

A social-protection system can be evaluated through coverage, adequacy, accessibility, predictability, dignity, and continuity.

An organization can be evaluated through prevention, response time, non-retaliation, quality of investigation, and practical re-entry.

An assistance program can be evaluated through usability, flexibility, conversion support, subject participation, and future option expansion.

A regulator can be evaluated through risk reduction, proportionality, adaptability, accountability, and institutional learning.

No single metric should silently replace these distinct objects.

6.44 Institutional Minimalism and Institutional Ambition

The theory of generative return creates pressure in two directions.

Institutional minimalism emphasizes epistemic humility. Institutions should avoid claiming an ability to reconstruct complete counterfactual lives or control every relational consequence.

Institutional ambition emphasizes that narrow remedies can leave substantial generative harms undressed.

A workable architecture requires both attitudes. Institutions should act where they possess justified competence and coordinate with others where the relevant object exceeds that competence.

This balance can be summarized through the principle of *bounded institutional responsibility*:

An institution should identify the restorative objects within its legitimate competence, act effectively upon those objects, disclose the limits of the resulting intervention, and maintain interfaces through which remaining generative needs can be addressed by other legitimate institutions or by the subject.

This principle avoids the fiction of comprehensive institutional repair while also avoiding abdication.

6.45 Institutional Humility

Institutional humility concerns the relationship between authority and knowledge.

A court can state that legally compensable damages have been awarded without claiming that the claimant's life has been made equivalent to the no-harm counterfactual. An organization can state that formal access has been restored without claiming that trust has been restored. A rehabilitation service can state that functional capacity has improved without claiming that professional opportunity has been restored. A social-protection system can provide income security without claiming that historical responsibility has been settled.

Such precision improves rather than weakens institutional legitimacy.

The language of completion can be harmful where it transforms a partial remedy into a denial of remaining loss. The theory of restorative comparison developed later provides a formal vocabulary for expressing partial success accurately.

6.46 Institutional Recognition of Irreparable Dimensions

An institution can recognize a loss even where it cannot replace it.

This distinction is important for singular generative conditions. A court, reparations program, organization, or public body can acknowledge that a historical opportunity or relation cannot be recreated. Compensation or assistance can then be offered without describing the new resource as an equivalent substitute.

Recognition of non-equivalence can itself possess moral value because it avoids rewriting the history of the claimant.

Institutional acknowledgment of limits should therefore form part of the theory of return.

6.47 Institutional Responsibility for New Generative Harm

Restorative institutions can themselves generate harm.

Litigation can expose private information. Investigation can create reputational effects. Compensation processes can require repeated narration of painful events. Means testing can produce stigma. Rehabilitation can become coercive. Public memorialization can conflict with the wishes of some affected persons. Digital records created for correction can later be repurposed.

The justice of institutional response therefore requires monitoring of second-order effects.

Let R denote a restorative intervention and H_R the new harms generated by the intervention. A complete evaluation cannot depend solely upon the benefit B_R :

$$J(R) = J(B_R, H_R, P_R), \quad (47)$$

where P_R represents the procedural structure of the intervention.

Equation 47 again emphasizes that restorative outcome and restorative process must remain distinct objects of evaluation.

6.48 Institutional Scale

Institutional scale should correspond to the scale of the generative problem.

An interpersonal dispute can be resolved locally where power asymmetry is low and external effects are limited. A systemic employment practice can require organization-wide or sectoral regulation. Historical dispossession can require national legislation, public financing, and community-level governance. Environmental restoration can require coordination across jurisdictions.

Ostrom's work is relevant because polycentric governance permits different functions to be organized at different scales [40].

Scale mismatch can create failure. A local organization cannot correct a national legal status. A national compensation scheme can lack local knowledge necessary for relational repair. A centralized rule can be too coarse for a heterogeneous community.

Institutional design should therefore distinguish which objects require common standards and which can remain locally adaptable.

6.49 Common Infrastructures and Distributed Decisions

Multi-institutional justice does not require every decision to be centralized. Some infrastructures can be shared while substantive decisions remain distributed.

Potentially shared infrastructures include:

- evidentiary standards;
- interoperable correction protocols;
- secure identity and authorization systems;

- provenance formats;
- appeal interfaces;
- public statistical monitoring;
- minimum procedural safeguards;
- referral mechanisms;
- benefit portability.

Local institutions can retain authority over context-specific decisions.

This structure resembles a governance interface. Common constraints and epistemic infrastructure provide coordination while preserving decentralized judgment.

Such arrangements require careful data governance. Interoperability can increase the speed with which an erroneous classification propagates. Correction mechanisms must therefore travel with the information architecture.

6.50 Correction Propagation

A distributed institutional environment creates a specific restoration problem: a correction made in one node may fail to propagate to other nodes that used the earlier information.

Suppose institution I_1 generates classification z , which is later used by institutions $I_2, \dots, I_n$. Institution I_1 later replaces z with z' . A complete correction process requires more than updating the originating database.

The relevant graph contains downstream uses:

$$I_1(z) \longrightarrow \{I_2(z), I_3(z), \dots, I_n(z)\}. \quad (48)$$

A correction protocol should provide a pathway for

$$I_1(z') \longrightarrow \{I_2(z'), I_3(z'), \dots, I_n(z')\}, \quad (49)$$

subject to legitimate data-retention and evidentiary rules.

Equations 48 and 49 identify correction propagation as a distinct governance function.

Even complete technical propagation does not erase the historical effects already produced by z . It prevents continued institutional use of a superseded state.

6.51 Institutional Interfaces with the Subject

A multi-institutional architecture can become unintelligible to the person it is intended to assist. The subject can face multiple forms, evidentiary standards, appeals, deadlines, and professional vocabularies.

Institutional complexity should therefore be evaluated from the perspective of the subject's ability to use the system.

Relevant design features include:

- a clear point of entry;
- explanation of institutional roles;
- assistance with referral;
- portability of evidence;
- avoidance of unnecessary repeated disclosure;

- accessible review procedures;
- transparent status tracking;
- continuity of essential support during transfers.

These features concern the generative usability of the justice architecture itself.

A remedy that exists formally yet cannot be navigated by the affected subject resembles the broader distinction between formal access and effective generative access.

6.52 Institutional Architecture and Subject-Directed Regeneration

The ultimate institutional challenge is to support future generation without collectively taking ownership of the subject's future.

Law can protect the right to choose. Political economy can supply resources. Social protection can preserve a minimum floor. Healthcare can support present capacity. Organizations can provide access. Professional communities can reopen networks. Civil society can support voice and solidarity.

The combination can become highly enabling.

It can also become highly controlling if every institution conditions support upon the same officially preferred trajectory.

Subject-directed regeneration therefore requires space among institutional functions. The system can specify legitimate boundaries and provide enabling conditions while leaving meaningful decisions concerning future life with the affected person.

6.53 Institutional Division of Labor and Generative Justice

Generative justice adds a further institutional question: who controls the conditions through which repair itself generates value?

A reparative program can contract with outside organizations that extract a large portion of its value. A technological platform for community restoration can centralize data and intellectual property. A training program can prepare participants for continued dependence upon the same institutional actors that controlled the original conditions.

Eglash's account of generative justice emphasizes control over production and unalienated circulation of value [15, 17]. Applied to repair, this implies that the architecture of restoration should itself be examined as a generative system.

The relevant questions include:

- who defines the restorative objective;
- who owns the infrastructure;
- who controls data;
- who receives the value produced through the process;
- whether affected subjects acquire durable capacities;
- whether the program produces continued dependency;
- whether knowledge generated through repair remains available to the community.

A program intended to restore generativity can reproduce extraction through its own institutional design.

6.54 From Institutional Division to Practice Architecture

The institutional analysis developed in this section yields several conclusions.

First, the justice of return is institutionally plural because its objects are plural. Rights, resources, capabilities, relations, recognition, bodily function, future opportunity, historical record, and control over generative conditions require different forms of institutional knowledge and authority.

Second, legal adjudication is indispensable for rights, responsibility, authoritative findings, cessation, restitution, and enforceable remedy. Its need for bounded causation, proof, jurisdiction, and finality limits its ability to represent an indefinitely evolving generative history.

Third, governance possesses stronger capacities for prevention, monitoring, iterative correction, and structural intervention. These capacities require constraints against opacity, surveillance, and discretionary domination.

Fourth, political economy provides the material architecture of return through resource mobilization, insurance, redistribution, and public investment. Its general equivalents preserve flexibility while remaining limited in relation to singular and non-market conditions.

Fifth, social protection preserves basic continuity independently of complete attribution of fault. It can therefore prevent present need from becoming additional generative contraction while other institutional processes continue.

Sixth, organizations possess local knowledge and rapid intervention capacity, which makes them especially important during ongoing harm and practical re-entry. Their proximity to the original power structure creates a need for independence and external review.

Seventh, rehabilitation and assistance address present conversion structures and future usability. Their orientation toward the present subject distinguishes them from retrospective liability.

Eighth, civil society and professional communities can address distributed relations, representation, solidarity, and network access that centralized institutions often cannot reproduce directly.

Ninth, knowledge institutions preserve the possibility of later correction and institutional learning. Their recording power requires privacy, provenance, and contestability.

Tenth, institutional gaps, overlaps, contradictions, latency, and failed handoffs can themselves become sources of generative harm.

Eleventh, institutional plurality requires interfaces. Shared standards, correction protocols, referral mechanisms, provenance, and benefit portability can coordinate specialized institutions without centralizing every decision.

Twelfth, no institution should convert success within its own bounded object into an unsupported claim that the subject or system has been completely restored.

These conclusions establish the institutional background for the practical architecture developed next. Section 7 examines concrete mechanisms for prevention, interruption, restitution, compensation, re-entry, assistance, and future-directed support. The objective shifts from identifying which institution is suited to a function toward examining how the relevant function can be designed in practice, which information it requires, how its effects can be evaluated, and how present technical and institutional limitations should be represented.

7 Practice Architectures for Prevention, Interruption, and Generative Assistance

This section translates the temporal and institutional analyses developed in Sections 5 and 6 into practical architectures. Its purpose is to identify how generative conditions can be protected before serious harm, how harmful processes can be interrupted while they are developing, and how resources, relations, capacities, and future possibilities can be supported after substantial alteration has already occurred. The discussion remains deliberately plural. No single procedural sequence is assumed to fit every form of generative harm. The objective is to identify reusable design components, their functions, their dependencies, their risks, and the kinds of restorative objects toward which they are directed.

The section distinguishes three levels of practical design. The first concerns *protective architecture*: rules, safeguards, redundancy, review mechanisms, and institutional arrangements intended to reduce the probability that severe generative contraction occurs. The second concerns *interruptive architecture*: mechanisms for acting while a harmful process remains active, under conditions of incomplete knowledge and potentially rapid propagation. The third concerns *post-harm architecture*: restitution, compensation, rehabilitation, recognition, re-entry, assistance, and support for new trajectory formation after the affected subject or system has already changed.

These architectures are evaluated through four recurrent considerations. First, the relevant generative object must be identified. Second, intervention should occur at a temporal scale appropriate to the rate at which harm propagates. Third, the intervention itself should remain subject to procedural justice. Fourth, every practical mechanism should disclose the limits of what it restores. A process that restores formal access should not thereby be described as restoring trust, capability, network position, or historical trajectory unless those claims are separately supported.

7.1 From Institutional Functions to Practice Design

Section 6 distinguished the functions performed by law, administrative governance, political economy, social protection, organizations, professional communities, rehabilitation services, civil society, and knowledge institutions. A practical architecture must connect these functions through an ordered process.

Let $\mathcal{F}_t = \{F_1, \dots, F_n\}$ denote the set of functions required at time t , and let $\mathcal{I}_t = \{I_1, \dots, I_m\}$ denote institutions capable of performing them. A practical architecture can be represented schematically by Equation 50.

$$\Pi_t : \mathcal{F}_t \longrightarrow (\mathcal{I}_t, \mathcal{P}_t, \mathcal{D}_t), \quad (50)$$

where $\mathcal{P}_t$ denotes the relevant procedures and $\mathcal{D}_t$ denotes the information required for those procedures.

Equation 50 emphasizes that assignment of an institution is insufficient by itself. A court, employer, regulator, welfare agency, or assistance provider requires procedures and information appropriate to its function. Failure can occur because the institution is unsuitable, because the procedure is poorly designed, or because the necessary information does not reach the actor capable of responding.

A practical architecture therefore includes both institutional assignment and interfaces among institutions.

7.2 Phase-Specific Practical Objectives

The practical objectives of the three temporal domains can be summarized as

$$\mathcal{P} = (P_{\text{protect}}, P_{\text{interrupt}}, P_{\text{support}}), \quad (51)$$

where P_{protect} concerns preservation of legitimate generative conditions before substantial alteration, $P_{\text{interrupt}}$ concerns interruption of active harmful dynamics, and P_{support} concerns restoration, assistance, and future-directed regeneration after alteration.

The phases differ in their characteristic errors.

A preventive system can intervene too early or too broadly. An interruptive system can convert provisional action into premature punishment. A post-harm system can claim equivalence where only partial support has been achieved. A coherent practical architecture should therefore contain phase-specific safeguards rather than applying one model of decision to the entire history.

7.3 Protective Mapping of Critical Generative Conditions

Prevention begins with identification of conditions whose alteration could produce large, weakly reversible, or recursively amplified consequences.

A *critical generative condition* is a condition whose removal or degradation can substantially alter later generative capacity, conversion structure, network access, reachable possibilities, or system stability.

Criticality can arise through several mechanisms:

- high dependence on a single condition;
- low substitutability;
- developmental or historical timing;
- strong network centrality;
- recursive value generation;
- limited institutional alternatives;
- high switching costs;
- high probability of downstream propagation;
- proximity to a threshold or regime transition;
- dependence of several other conditions upon the same resource.

Protective mapping does not require a complete model of the subject's future. Its purpose is to identify places where institutional error can have unusually large generative consequences.

For example, termination of employment can affect income, health insurance, professional status, immigration status, housing eligibility, network access, and future references simultaneously. A decision affecting one formal relation can therefore occupy a much more central position than its immediate description suggests.

A protective architecture should identify such dependency structures before a crisis occurs.

7.4 Generative Criticality Assessment

A practical risk assessment can distinguish ordinary conditions from those requiring stronger safeguards. Let c denote a condition and let its generative criticality be represented schematically by Equation 52.

$$\kappa(c) = \mathcal{K}(S_c, T_c, D_c, Q_c, R_c, U_c), \quad (52)$$

where S_c denotes severity of potential contraction, T_c denotes temporal sensitivity, D_c denotes dependence of other conditions upon c , Q_c denotes substitutability, R_c denotes expected reparability, and U_c denotes uncertainty.

Equation 52 is a conceptual decomposition rather than a universal scalar formula. Some applications will require qualitative classification, partial orders, or multidimensional risk profiles.

The purpose of criticality assessment is procedural. A condition with high generative criticality can justify stronger notice, review, evidence, proportionality analysis, or interim protection before irreversible action is taken.

7.5 Pre-Decision Safeguards

Where a decision can substantially contract another subject's generative conditions, preventive practice can require safeguards before implementation.

Relevant measures include:

- advance notice;
- disclosure of material reasons;
- access to relevant evidence;
- opportunity to correct factual error;
- conflict-of-interest screening;
- independent review;
- staged rather than immediate irreversible action;
- preservation of essential access during review;
- explicit proportionality analysis;
- documentation of alternative interventions;
- appeal before irreversible implementation where feasible.

The justification for stronger procedure increases where the consequences are difficult to reconstruct later.

These safeguards have familiar legal and administrative analogues. Their generative significance lies in preserving future options before a potentially erroneous classification becomes embedded in later processes.

7.6 Staged Intervention

A useful practical principle is to separate reversible from irreversible steps.

Suppose an institution has several possible responses to a perceived risk:

$$a_1, a_2, \dots, a_n.$$

Where several actions can achieve the immediate protective purpose, an institution can prefer an intervention that preserves a larger set of later options until stronger evidence becomes available.

A staged process can take the form

$$a_1 \rightarrow \text{review} \rightarrow a_2 \rightarrow \text{review} \rightarrow \dots, \quad (53)$$

where the severity and irreversibility of intervention increase only when the relevant conditions remain satisfied.

This architecture resembles the graduated logic of responsive regulation [4]. Within generative governance, staged intervention has an additional purpose: it reduces unnecessary closure of future possibilities while uncertainty remains substantial.

7.7 Redundancy of Access

Dependence upon one actor or pathway creates vulnerability. Prevention can therefore include redundant routes to conditions essential for generativity.

Examples include:

- multiple reporting channels;
- independent research access;
- portable professional credentials;
- benefits that survive change of employer;
- external appeal;
- multiple data repositories;
- diversified funding;
- alternative healthcare access;
- multiple authorized supervisors;
- interoperable institutional records.

The objective is not duplication for its own sake. It is reduction of unjustified single-point dependency.

Redundancy also contributes to non-domination. A subject has greater practical security where one gatekeeper cannot unilaterally destroy every route to an important generative condition.

7.8 Exit-Preserving Design

Generative systems can become unjust when continued participation is the only way to preserve essential resources. Exit-preserving design reduces the cost of leaving a harmful relation.

Relevant mechanisms include portability of pensions and insurance, transfer of academic credits, preservation of professional credentials, access to records, data portability, relocation support, and continuation of essential benefits during transition.

Exit preservation matters because a formal right to leave has limited practical value when departure destroys every condition required for future generativity.

A relation can therefore be assessed partly through the cost imposed upon legitimate exit.

7.9 Protective Data and Evidence Architecture

Prevention also depends upon the ability to reconstruct how important decisions were made.

Institutions can preserve:

- decision provenance;
- versions of policies;
- authorization records;
- material evidence;
- relevant communications;
- conflict-of-interest declarations;

- review histories;
- correction histories.

The objective is traceability rather than unrestricted retention.

Privacy-sensitive systems should preserve the minimum information necessary to support legitimate review. Access controls, retention limits, purpose limitations, and correction mechanisms are part of the protective architecture.

The absence of provenance can transform later correction into speculation. Excessive preservation can transform governance into surveillance. Practical design must therefore optimize neither deletion nor recording in isolation.

7.10 Early-Warning Mechanisms

Some harmful processes become visible through weak signals before severe generative contraction occurs.

Possible indicators include repeated complaints, turnover, unexplained exclusion, rapidly deteriorating participation, recurrent procedural reversal, patterns of retaliation, repeated access failures, or unusual concentrations of authority.

An early-warning mechanism aggregates signals without automatically converting them into findings of individual responsibility.

This distinction is essential. Detection and adjudication have different epistemic thresholds.

An early-warning system can trigger review, additional supervision, audit, or temporary support while preserving procedural protections for affected parties.

7.11 Distributed Reporting Channels

A reporting architecture should avoid requiring a subject to report harmful conduct exclusively through the actor controlling the contested condition.

Distributed reporting can include:

- internal supervisors;
- independent compliance units;
- ombuds institutions;
- unions or professional bodies where applicable;
- external regulators;
- confidential advisory channels;
- formal legal mechanisms.

The existence of several channels can reduce retaliation risk and improve institutional knowledge.

Channels should remain coordinated enough to prevent the subject from having to reconstruct the same history repeatedly. With appropriate consent and privacy controls, evidence should be reusable across compatible processes.

7.12 Interruption Triggers

Once a harmful process is plausibly active, institutions require criteria for moving from observation to intervention.

An interruption trigger can depend upon:

- severity of possible continued harm;
- speed of propagation;
- reversibility;
- reliability of available evidence;
- risk to third parties;
- availability of low-burden protective measures;
- probability that delay will close important options.

The trigger should be lower than the threshold for final responsibility where a temporary and reversible intervention can prevent severe continuing harm.

At the same time, interruption should remain procedurally distinct from final judgment.

7.13 Minimum Necessary Interruption

A practical interruptive architecture should seek the minimum intervention capable of stopping the relevant harmful transition.

Suppose a set of candidate interventions $\mathcal{I} = \{i_1, \dots, i_n\}$ can reduce the harmful propagation rate. A proportional strategy can select an intervention satisfying the protective objective while minimizing collateral contraction of legitimate generativity.

This can be represented schematically by Equation 54.

$$i^* \in \arg \min_{i \in \mathcal{I}} C_{\text{collateral}}(i) \quad \text{subject to} \quad H_{\text{remaining}}(i) \leq H_{\text{max}}. \quad (54)$$

Equation 54 is illustrative. The relevant costs and harms can be multidimensional and cannot always be reduced to scalar quantities. The structure expresses a proportionality requirement.

7.14 Protective Separation

Some ongoing harms arise through repeated interaction. Temporary separation can interrupt the generative cycle without determining final responsibility.

Examples include:

- temporary change of supervision;
- remote participation;
- alternative workspace;
- reassignment of decision authority;
- temporary suspension of direct contact;
- independent approval channels.

Protective separation should be designed carefully. Removing the harmed subject from the environment can itself produce exclusion, stigma, and loss of opportunity. Where possible, the burdens of separation should not default automatically to the person seeking protection.

The design should therefore ask which party controls the harmful mechanism and which arrangement preserves the greatest legitimate generative continuity.

7.15 Continuity of Essential Conditions During Review

Long review processes can produce independent harm if essential conditions are suspended throughout the investigation.

A practical architecture can preserve selected conditions temporarily:

- salary;
- healthcare;
- housing;
- institutional affiliation;
- library and data access;
- professional credentials;
- immigration-related documentation where legally possible;
- research materials;
- educational enrollment.

Continuation does not prejudice final responsibility. It allocates the risk of institutional delay.

Where continuation is impossible, substitute support can prevent collapse of related conditions.

7.16 Interim General-Equivalent Support

Money can play a distinct role during the interruptive phase. Interim financial support can preserve housing, food, transportation, healthcare, legal access, and time while a dispute remains unresolved.

This function differs from final damages.

Interim support is primarily stabilizing. It can be justified by the expected cost of allowing short-term resource collapse to generate larger downstream losses.

A later determination can address whether and how the cost should be allocated among responsible actors, insurers, public funds, or the recipient.

7.17 Evidence Freezing Without Interpretive Freezing

During an ongoing dispute, evidence should sometimes be preserved in its current state. The interpretation of that evidence should remain revisable.

This distinction can be expressed as

$$\text{preserve data state} + \text{preserve interpretive revisability.} \quad (55)$$

Equation 55 prevents two opposite errors. Alteration or deletion of evidence can make later review impossible. Treating the earliest recorded interpretation as permanently authoritative can make correction impossible.

Versioned records, explicit confidence levels, provenance, and supersession links can support this distinction.

7.18 Anti-Propagation Measures

Some harms propagate through information systems. A disputed classification, disciplinary flag, risk score, or reputational claim can be copied across databases and institutions.

During review, anti-propagation measures can limit new downstream uses while preserving evidence for legitimate investigation.

Possible mechanisms include:

- provisional status markers;
- restrictions on secondary dissemination;
- purpose limitations;
- automatic expiration of unconfirmed flags;
- notification of downstream users when a record is contested;
- later propagation of corrections.

The design should distinguish between preservation of historical evidence and continued operational use of a contested classification.

7.19 Temporary Independent Decision Channels

Where the contested actor controls conditions essential to the claimant, temporary independent channels can reduce continued dependency.

Examples include external approval of research access, independent scheduling, third-party authorization of leave, external review of performance decisions, or temporary transfer of funding control.

The purpose is to prevent the same contested relation from governing the subject's immediate survival while the relation itself is being reviewed.

This architecture can be especially important where retaliation risk is high.

7.20 Interruption Review Cycles

Temporary measures should contain review points.

Let i_t denote an interim measure. A review cycle can be represented by

$$i_t \rightarrow E_{t+\Delta} \rightarrow i_{t+\Delta}, \quad (56)$$

where $E_{t+\Delta}$ denotes updated evidence and circumstances.

Equation 56 emphasizes that continuation, relaxation, strengthening, or termination of the measure should depend upon new information rather than institutional inertia.

Without review, temporary protection can become indefinite punishment.

7.21 Transition Planning Before Final Resolution

Where a harmful process is likely to produce long-term consequences, planning for post-harm response can begin before final adjudication.

A subject who has been excluded from a profession for a prolonged period can require skill maintenance, networking, healthcare, or alternative access before the legal process ends. Waiting for complete resolution can make later re-entry substantially harder.

Transition planning does not require a final counterfactual valuation. It can preserve current options while responsibility remains under review.

7.22 Post-Harm Decomposition of Restorative Needs

Once the principal harmful process has ceased, a practical architecture should avoid treating the remaining problem as one undifferentiated loss.

A post-harm assessment can decompose needs into:

- continuing harmful mechanisms;
- identifiable objects or rights capable of restitution;
- monetizable losses;
- bodily or functional rehabilitation needs;
- formal-status correction;
- reputational and recognition needs;
- relational barriers;
- network-access barriers;
- altered conversion structures;
- professional or educational re-entry needs;
- present social-protection needs;
- singular losses that cannot be reproduced;
- new future possibilities selected by the subject.

This decomposition prevents one successful remedy from being mistaken for complete repair.

7.23 Restitution Layer

The first post-harm layer concerns identifiable conditions capable of direct return.

Examples include:

- property;
- salary withheld;
- employment status;
- institutional membership;
- access credentials;
- licenses;
- records;
- data;
- physical equipment.

Where direct restitution remains possible and normatively appropriate, it can preserve historical continuity and avoid unnecessary substitution.

The institution should nevertheless distinguish object return from the return of the broader generative environment.

7.24 Correction Layer

A second layer concerns correction of institutional information.

Correction can involve:

- removing an invalid current classification;
- marking an earlier decision as superseded;
- preserving the historical event while correcting its status;
- notifying downstream users;
- updating automated systems;
- correcting public-facing records;
- restoring credentials.

Correction should propagate across the same institutional pathways through which the earlier classification acquired practical effect where legally and technically feasible.

Complete propagation may remain impossible. The institution should therefore avoid presenting correction of the originating record as proof that all downstream consequences have been reversed.

7.25 Compensation Layer

Compensation supplies transferable resources for losses that cannot be repaired through direct return alone.

A practical compensation architecture should distinguish at least:

- direct economic loss;
- replacement cost;
- transition cost;
- treatment and rehabilitation cost;
- future income effects where institutionally cognizable;
- cost of reconstructing access or capability;
- non-economic damages where the relevant legal system recognizes them.

The use of money should be accompanied by explicit conceptual humility. A payment can be justified without establishing that the amount is structurally equivalent to every dimension of the loss.

This distinction is essential for singular generative conditions.

7.26 Flexible Resource Layer

Beyond legally calculated compensation, future-directed assistance can include flexible resources whose purpose is to enlarge subject-directed option space.

Possible uses include:

- relocation;
- retraining;

- rest;
- childcare;
- independent research;
- new equipment;
- professional transition;
- temporary reduction of work;
- language learning;
- mobility;
- creation of a new enterprise.

Flexible support is particularly valuable where institutions possess limited knowledge of which future the subject now prefers.

Its use should therefore remain less conditional than highly targeted rehabilitation where feasible.

7.27 Specific Enabling Conditions

Some barriers cannot be removed by money alone.

Specific enabling conditions can include:

- access to healthcare;
- credential correction;
- visa or legal-status correction;
- laboratory access;
- educational admission;
- professional licensing;
- accessible transportation;
- network introductions;
- independent supervision;
- protected workspace;
- communication assistance;
- specialized equipment.

The practical architecture should therefore combine fungible and non-fungible support. A simple representation is given in Equation 57.

$$S_{\text{post}} = M + C_{\text{specific}} + R_{\text{support}} + A_{\text{subject}}, \quad (57)$$

where M denotes flexible resources, C_{specific} denotes specific enabling conditions, R_{support} denotes relational support, and A_{subject} denotes meaningful authority of the affected subject over future direction.

Equation 57 is conceptual rather than additive in a numerical sense.

7.28 Rehabilitation Layer

Rehabilitation can address bodily, cognitive, psychological, occupational, and social conversion conditions.

A generative rehabilitation plan should specify:

- the present functional objective;
- the subject's own priorities;
- environmental barriers;
- required accommodations;
- expected duration;
- indicators for revision;
- exit or modification rights.

The objective should be usable future capability rather than forced replication of a prior performance profile.

Where an environment remains harmful, rehabilitation of the subject alone is an inadequate response.

7.29 Recognition Layer

Recognition can include:

- acknowledgment of wrongdoing;
- correction of public narrative;
- apology;
- formal exoneration;
- restoration of title or status;
- memorialization;
- publication of findings;
- recognition of institutional responsibility.

The United Nations reparation framework's category of satisfaction provides an important institutional precedent for this plural form of response [65].

Recognition should be designed with the subject's privacy and preferences in mind. Public acknowledgment can create new exposure.

7.30 Relational Re-Entry Architecture

Formal reinstatement can require a separate relational re-entry process.

A re-entry architecture can include:

- independent contact point;
- clarification of authority;
- restoration of access;

- communication to relevant colleagues;
- correction of organizational records;
- anti-retaliation monitoring;
- voluntary mentoring;
- transitional workload;
- opportunities for new collaboration;
- regular review with the returning subject;
- transfer or exit options.

The objective is restoration of practical usability rather than symbolic presence alone. Re-entry should remain voluntary. A claimant can prefer assistance toward a new trajectory.

7.31 Network Re-Entry

Where harm altered professional or social networks, direct organizational reinstatement can leave substantial generative loss.

A network re-entry architecture can support:

- renewed conference participation;
- introductions to collaborators;
- access to professional associations;
- restoration of mailing lists and information channels;
- invitations to relevant events;
- support for publication or dissemination;
- correction of network-distributed misinformation.

These interventions cannot guarantee reconstruction of singular relationships.

Their purpose is to reopen relational possibility.

This distinction is especially important for academic, scientific, artistic, and professional communities where future opportunity emerges through weak and strong network ties rather than formal employment alone.

7.32 Alternative Trajectory Architecture

Post-harm support should include the possibility that the subject does not wish to return to the previous system.

Alternative trajectory support can include:

- retraining;
- education;
- entrepreneurship;
- relocation;

- transition to another profession;
- independent practice;
- public or nonprofit work;
- reduced work intensity;
- family or care-oriented choices.

The existence of an alternative path should not be interpreted as evidence that the earlier loss has become equivalent.

A new trajectory can be independently valuable.

7.33 Subject-Directed Planning

Subject-directed planning provides a practical implementation of the principle developed in Section 4.33.

The process can begin from three separate questions:

1. Which present conditions constrain viable future choice?
2. Which resources or enabling conditions would expand the subject's option space?
3. Which future directions does the subject presently wish to explore, while retaining the ability to revise that choice?

The process should avoid requiring the subject to identify one final preferred future immediately. Exploration itself can be a legitimate use of restorative resources.

7.34 Revisable Assistance Plans

Assistance plans should remain revisable where the subject's capacities, preferences, environment, or opportunities change.

A plan can include periodic review at times $t_1, t_2, \dots, t_n$:

$$A_{t_k} \xrightarrow{\text{review}} A_{t_{k+1}}. \quad (58)$$

Equation 58 represents adjustment rather than continuous institutional supervision.

Review should be limited to information necessary for the legitimate purpose of the program. Revisability should not become a justification for intrusive monitoring.

7.35 Time as an Assistance Resource

Time itself can function as a generative condition.

A person affected by prolonged harm can require a period during which immediate economic pressure is reduced enough to permit recovery, reflection, retraining, or formation of new relationships.

Practical mechanisms can include:

- paid leave;
- transition grants;
- reduced workload;
- extended educational access;
- delayed performance evaluation;

- temporary income support;
- flexible deadlines.

These measures recognize that generativity does not always increase through maximal immediate output.

A period of reduced production can create conditions for later generation.

7.36 Safe Experimentation

Subjects recovering from major generative disruption can benefit from opportunities to test new trajectories without bearing the full cost of failure.

Safe experimentation can include:

- trial placements;
- temporary research affiliations;
- short courses;
- pilot projects;
- temporary relocation;
- part-time re-entry;
- reversible role changes.

The objective is to enlarge information available to the subject about current preferences and capacities.

This architecture is especially appropriate where the subject has changed substantially and cannot know in advance whether the previous trajectory remains desirable.

7.37 Conversion-Support Architecture

A practical assistance program should identify the barriers preventing resources from becoming effective generative conditions.

Suppose a monetary transfer M produces limited benefit because $\mathcal{K}^H$, the post-harm conversion structure, is impaired. Conversion support C_S can modify the pathway:

$$M \xrightarrow{\mathcal{K}^H} G_{\text{low}} \quad \longrightarrow \quad M \xrightarrow{\mathcal{K}^H + C_S} G_{\text{expanded}}. \quad (59)$$

Equation 59 is conceptual. It identifies why targeted support can complement general resources.

Relevant conversion barriers can include information, health, transportation, credentials, childcare, language, legal status, social isolation, or inaccessible technology.

7.38 Layered Support Rather Than Single Remedy

The practical architecture developed above can be summarized as a layered model.

$$\mathcal{R}_{\text{practical}} = (R_{\text{cessation}}, R_{\text{restitution}}, R_{\text{correction}}, R_{\text{compensation}}, R_{\text{rehabilitation}}, R_{\text{recognition}}, R_{\text{reentry}}, R_{\text{assistance}}, R_{\text{prevention}}) \cdot \quad (60)$$

Equation 60 identifies distinct functions. Not every case requires every layer.

The model rejects the assumption that one intervention should be interpreted as performing functions assigned to another.

7.39 Sequencing of Post-Harm Measures

Order matters among post-harm measures.

For example, re-entry before safety reform can recreate the harmful environment. Compensation before correction of legal status can limit the uses of the compensation. Public recognition before confidentiality preferences are established can create new exposure. Training before stabilization of housing and health can have limited effectiveness.

A practical plan should therefore identify dependencies among measures.

Let $R_i \prec R_j$ denote that intervention R_i should ordinarily precede R_j because R_j 's effectiveness depends upon R_i . The resulting response can be represented by a partial order rather than one universal sequence.

This structure is useful because many interventions can proceed in parallel while some require pre-requisites.

7.40 Parallel Support Tracks

A subject should not necessarily wait for one institutional process to finish before another begins.

Legal responsibility, healthcare, interim income, record correction, education, and network support can sometimes proceed on parallel tracks.

Parallelism is especially important where litigation or administrative review has a long time scale.

A practical architecture can therefore separate:

- liability track;
- immediate-support track;
- health and rehabilitation track;
- status-correction track;
- organizational reform track;
- future-planning track.

Coordination should ensure that findings are interpreted according to the purpose of each track.

7.41 Single Entry and Distributed Execution

Institutional plurality can be made more usable through a single entry point combined with distributed execution.

The subject can initially present the problem through one accessible interface. That interface does not decide every issue. It identifies relevant functions, preserves evidence, provides information, and routes the case to appropriate institutions.

A single-entry architecture can reduce repeated narration and navigation costs.

The risks include excessive centralization and concentration of sensitive information. The interface should therefore function as a router rather than a universal decision-maker where possible.

7.42 Case Coordination

Complex generative harms can benefit from a coordinating function responsible for tracking dependencies among institutional responses.

Coordination can include:

- identifying unresolved barriers;

- ensuring continuity of temporary support;
- tracking correction propagation;
- identifying contradictions among institutional decisions;
- scheduling re-entry steps;
- confirming subject consent to information sharing;
- identifying missing institutional functions.

Coordination should not create a new actor with unlimited authority over the subject's life. The coordinator's mandate should remain procedural and bounded.

7.43 Restorative Provenance

A practical restoration process should preserve the history of its own decisions.

Restorative provenance can record:

- the condition identified as lost;
- the reference used for valuation;
- the evidence considered;
- the remedy selected;
- the assumptions underlying the remedy;
- the limits acknowledged by the institution;
- later revisions;
- subject objections;
- downstream institutional actions.

This information allows later actors to distinguish a final factual finding from a pragmatic settlement, an estimated compensation value from a claim of equivalence, and a temporary assistance measure from a permanent classification.

Provenance therefore protects against overinterpretation of institutional outputs.

7.44 Correction Propagation Architecture

Where harmful information has propagated across systems, correction should be designed as a distributed process.

A correction architecture can include:

- source identification;
- downstream-use registry where feasible;
- versioning;
- supersession metadata;
- notification of authorized downstream users;

- removal of invalid operational flags;
- preservation of legally necessary historical records;
- verification that critical systems have updated.

The distinction between historical preservation and operational correction is central.

A corrected event can remain part of the historical archive while ceasing to operate as a valid present classification.

7.45 Propagation Limits and Institutional Honesty

Complete propagation can remain impossible.

Informal conversations, external copies, independent databases, screenshots, human memory, and derivative decisions can exceed the correcting institution's control.

A practical architecture should therefore distinguish:

$$\text{correction issued} \neq \text{all consequences reversed.} \quad (61)$$

Equation 61 is one example of the institutional humility required throughout this paper.

The institution should describe the scope of correction accurately.

7.46 Privacy-Preserving Restoration

Restorative processes can require sensitive information. The practical architecture should minimize unnecessary exposure.

Privacy-preserving design can include:

- purpose limitation;
- data minimization;
- local processing;
- access control;
- selective disclosure;
- aggregation;
- retention limits;
- audit trails;
- subject-controlled sharing where feasible.

A strong restoration process should not require creation of a permanent centralized dossier containing every dimension of the subject's history.

This concern becomes particularly important in later sections addressing reflexive recording and the limits of complete social representation.

7.47 Distributed Evidence Processing

Multi-party governance can share evaluative infrastructure without centralizing raw data.

Suppose actors $I_1, \dots, I_n$ each hold sensitive data D_i . A common decision system can, in principle, operate upon authorized transformations $T_i(D_i)$ rather than requiring every D_i to enter one global repository.

The architecture can be represented by Equation 62.

$$\{T_1(D_1), T_2(D_2), \dots, T_n(D_n)\} \xrightarrow{\text{common interface}} E, \quad (62)$$

where E denotes the evidence state available for the relevant institutional decision.

Equation 62 is intentionally technology neutral. Cryptographic, statistical, administrative, or procedural mechanisms can implement different versions of the architecture.

The normative principle is separation between the information required for a decision and unnecessary exposure of raw personal data.

7.48 Subject Access to Restorative Records

Affected subjects should ordinarily be able to understand the institutional representation of their case where legitimate confidentiality interests permit.

Access can include:

- current status;
- material findings;
- sources relied upon;
- correction history;
- pending institutional actions;
- downstream notifications;
- available appeal mechanisms.

This access supports contestability and reduces the possibility that a person is governed by classifications that remain invisible to them.

Access should be accompanied by intelligible explanation rather than raw data alone.

7.49 Third-Party Protection

Restorative action can impose burdens upon third parties. Practical design should identify those burdens early.

Relevant third-party interests can include:

- privacy;
- employment rights;
- property interests;
- procedural rights;
- reliance interests;
- freedom of expression;

- organizational continuity;
- public-resource constraints.

Protection of third parties does not eliminate the claimant's restorative interest. It affects the admissible process through which that interest is pursued.

Where direct restoration would impose severe injustice upon an innocent third party, compensation or alternative assistance can become more important.

7.50 Multi-Party Restoration Planning

Where several subjects have legitimate generative interests, restoration should be treated as a coupled problem.

Let $\mathcal{G}_1, \dots, \mathcal{G}_n$ denote affected generative systems. An intervention R produces a vector of effects:

$$\Delta_R = (\Delta\mathcal{G}_1, \Delta\mathcal{G}_2, \dots, \Delta\mathcal{G}_n). \quad (63)$$

Equation 63 prevents a design from treating the claimant's restoration as the only relevant output.

Later normative sections will examine how competing interests should be weighted or ordered. The practical requirement at this stage is visibility of the distribution of effects.

7.51 Collective Repair Architecture

Collective harms require additional practical structures.

A community-level architecture can include:

- representative participation;
- protection for internal dissent;
- community-controlled funds;
- collective infrastructure;
- land or environmental restoration;
- cultural and archival support;
- governance reform;
- education;
- public acknowledgment;
- long-term institutional capacity.

Large-scale reparations practice already combines multiple material and symbolic measures [12, 65].

A generative perspective adds attention to whether collective resources become durable conditions for self-directed future generation or create new dependency upon external administrators.

7.52 Community Control of Reparative Infrastructure

Where repair produces new infrastructure, governance of that infrastructure can be part of the reparation itself.

Examples include community archives, educational institutions, local funds, research facilities, cultural centers, environmental-monitoring systems, and productive assets.

A one-time transfer can have different generative significance depending upon who controls future decisions.

This concern directly connects practical restoration to generative justice's emphasis on control over productive conditions and circular value flows [15, 17].

7.53 Avoiding Extractive Repair

Repair can itself become extractive.

A consulting industry can absorb a large share of reparative funds. A digital platform can obtain ownership over community data generated through a repair project. Training programs can generate unpaid labor for outside institutions. Research about harmed communities can create academic value without returning knowledge or control.

An anti-extractive practical architecture should therefore ask:

- who owns the infrastructure;
- who controls the data;
- who receives financial and reputational value;
- what capacities remain after external actors leave;
- whether participants retain rights over generated knowledge;
- whether the program creates durable local control.

Repair should be evaluated as a generative process in its own right.

7.54 General-Equivalent Support and Epistemic Humility

The practical role of monetary support deserves explicit restatement.

General equivalents are valuable partly because institutions cannot know every feature of the subject's desired future.

A flexible transfer can therefore express epistemic humility:

The institution acknowledges that generative capacity has been impaired and provides resources while refraining from determining every use of those resources.

This principle supports broad discretion where risks of misuse do not justify stronger restrictions.

The principle also clarifies why money should be supplemented, rather than replaced wholesale, by specific assistance. The general equivalent preserves choice. Specific enabling conditions address barriers that money alone cannot remove.

7.55 Non-Fungible Support

Where the relevant condition is non-fungible, practical support must operate through other mechanisms.

Possible responses include:

- access to institutions;

- public correction;
- restoration of legal status;
- introductions to networks;
- protection of cultural heritage;
- ecological restoration;
- provision of specialized care;
- creation of new institutional opportunities.

These interventions can approximate some functions of the lost condition without claiming full equivalence.

The later atlas will provide a vocabulary for specifying the precise relation being asserted.

7.56 Singular-Loss Acknowledgment

Some losses remain singular even after extensive support.

A practical architecture should therefore permit an explicit category of *acknowledged non-restorability*.

This category can be appropriate where:

- the relevant person is no longer available;
- a historical opportunity has closed;
- an ecological configuration cannot be recreated;
- a developmental period has passed;
- a destroyed artifact or archive is irrecoverable;
- an earlier relationship cannot legitimately be reconstructed.

Acknowledgment of non-restorability should coexist with compensation, assistance, and prevention of recurrence.

It should not function as an excuse for institutional inactivity.

7.57 Restoration Statements with Explicit Scope

Institutions should describe restorative outcomes with explicit scope.

Instead of stating simply

The claimant has been restored.

a more accurate statement can specify:

Formal employment status and salary have been restored; professional-network effects and later opportunity effects remain outside the scope of this determination.

or:

The official classification has been corrected and downstream institutional users under this system have been notified; informal dissemination and historical effects cannot be fully reversed.

Such statements reduce conceptual overreach.

The later theory of restorative comparison will formalize this practice through indexed claims.

7.58 Practice-Level Reference Selection

Every practical intervention uses a reference even when the reference remains implicit.

Compensation can use market value, lost income, expected future earnings, replacement cost, or statutory schedules. Rehabilitation can use prior function, a clinical target, or a subject-selected goal. Re-entry can use the previous institutional role, a current comparable role, or a newly negotiated position.

Practical design should therefore document which reference has been selected.

Where several references are relevant, the intervention should avoid silently treating one as universally authoritative.

7.59 Practice-Level Comparison Objects

A practical plan can also identify its comparison object.

Examples include:

- resource amount;
- legal status;
- functional capacity;
- network access;
- institutional authority;
- health condition;
- observable work ability;
- future opportunity set;
- robustness under ordinary stress;
- control over productive conditions.

Making the object explicit improves coordination because different institutions can see which dimensions remain outside their mandate.

7.60 Outcome Monitoring Without Counterfactual Overreach

Post-harm support requires some evaluation of whether it is working.

Monitoring should focus on the objectives actually selected rather than claiming to establish the complete counterfactual future.

For example, a re-entry program can evaluate access, retention, workload, network participation, reported usability, and recurrence of retaliation. It need not claim that the person's career has returned to the exact path that would have existed without the harm.

This distinction allows practical learning without false epistemic precision.

7.61 Subject-Reported Generative Usability

Formal indicators should be supplemented by the subject's account of whether conditions are practically usable.

Relevant questions can concern:

- whether access feels secure;
- whether reporting channels are credible;

- whether the subject can use returned resources;
- whether institutional barriers remain;
- whether new support creates unwanted dependency;
- whether the current trajectory remains desirable.

Subject report is one source of evidence among several. It should neither be treated as automatically dispositive nor excluded from the assessment.

7.62 Robustness Testing of Restorative Arrangements

A practical restoration can appear successful under stable conditions while remaining fragile.

Where ethically and institutionally appropriate, arrangements can therefore be evaluated under ordinary foreseeable perturbations.

Examples include:

- change of supervisor;
- moderate workload increase;
- temporary income shock;
- organizational restructuring;
- ordinary disagreement;
- change of service provider;
- change in funding source.

The objective is not deliberate exposure to harmful stress. It is assessment of whether the restored condition depends upon unusually fragile arrangements.

This practical concern anticipates the later formal discussion of perturbation response and robustness equivalence.

7.63 Exit Criteria for Restorative Programs

A support program should specify when intensive institutional involvement can end.

Exit can occur when:

- the subject chooses to leave;
- the selected objectives have been reached;
- support has been transferred to ordinary public systems;
- another institution becomes responsible;
- continued intervention would create greater burden than benefit.

Exit should not be defined through a claim that every historical effect has been resolved.

A program can end because its legitimate function has been completed.

7.64 Long-Term Follow-Up

Some generative effects emerge over longer periods. Limited follow-up can therefore be appropriate after intensive support ends.

Follow-up can assess:

- recurrence;
- durability of institutional reform;
- continued access;
- correction propagation;
- new barriers;
- unintended effects of the restorative intervention.

Long-term follow-up should remain proportionate and consensual where personal data are involved.

Its purpose is institutional learning and detection of persistent structural failure, not indefinite supervision of the claimant.

7.65 Adaptive Institutional Learning

Case-level outcomes should feed back into institutional design.

A repeated pattern of re-entry failure can reveal that formal reinstatement is insufficient. A recurring need for emergency financial assistance can reveal that legal procedures operate too slowly. Repeated correction failures can reveal a defective information architecture. Recurrent retaliation can reveal concentrated power.

Learning can be represented as

$$\Pi_t \xrightarrow{\text{observed outcomes}} \Pi_{t+1}, \quad (64)$$

where Π_t denotes the current practical architecture.

Equation 64 treats governance itself as a revisable generative process.

7.66 Practice Architecture for Workplace Generative Harm

A workplace example can illustrate how the layers combine.

Before serious harm, the architecture can include clear conduct standards, multiple reporting channels, anti-retaliation rules, independent review, portable benefits, transparent performance criteria, and preservation of decision provenance.

During an ongoing conflict, the organization can separate reporting and supervisory authority, preserve salary and healthcare, create an independent contact point, restrict retaliation, preserve evidence, and review temporary measures periodically.

After a substantiated harm, the response can include correction of records, compensation, restoration of status, rehabilitation, protected re-entry, alternative supervision, network support, professional retraining, or support for exit into another career.

The appropriate combination depends upon the subject's present preferences and the structures affected by the harm.

The example demonstrates why a single remedy such as reinstatement cannot represent the entire practical architecture.

7.67 Practice Architecture for Academic and Research Exclusion

Research environments illustrate especially strong recursive generativity.

Protective architecture can include independent supervision, transparent authorship rules, data-access rights, portable research records, external appeal, clear procedures for misconduct allegations, and preservation of research provenance.

During exclusion or dispute, interim support can preserve library access, data, computing resources, affiliation where appropriate, immigration-related documentation where legally possible, and independent communication channels.

Post-harm response can include correction of disciplinary records, restored access, funding, publication support, network re-entry, conference participation, new mentorship, independent research time, or transition to another institution.

A missed collaboration or historical intellectual encounter can remain non-restorable. The architecture can support new generativity without declaring the original opportunity equivalent to later alternatives.

7.68 Practice Architecture for Reputational Harm

Reputational harm requires a distributed architecture.

Prevention can include provenance requirements, careful distinction between allegation and finding, purpose limitation, expiration of provisional classifications, and rights of contestation.

During review, disputed information can be preserved while its operational dissemination is limited.

Post-harm correction can include source correction, supersession metadata, notification of downstream institutional users, public clarification where appropriate, and support for re-entry into networks affected by the earlier classification.

Complete reputational restoration is rarely under the control of one institution because beliefs and derivative records can exist across distributed social systems.

This makes reputational harm a useful later case for observational, network, and distributed-state comparison.

7.69 Practice Architecture for Health-Related Generative Loss

Health-related loss often combines bodily, economic, relational, and occupational effects.

Prevention can involve safety standards, access to care, early diagnosis, and occupational protections.

During emerging harm, timely treatment and temporary income support can prevent secondary losses such as housing instability or occupational exit.

Post-harm response can combine treatment, rehabilitation, disability accommodation, assistive technology, income security, retraining, workplace modification, and alternative career support.

The objective should remain compatible with the person's present values and preferred form of life.

A health condition should not automatically become an institutional justification for narrowing the subject's future beyond what legitimate safety and capability considerations require.

7.70 Practice Architecture for Collective Historical Harm

Collective historical harm requires longer temporal horizons and plural institutional instruments.

A practical architecture can include:

- historical investigation;
- archival preservation;
- public acknowledgment;
- restitution where possible;

- individual or collective compensation;
- community-controlled infrastructure;
- education;
- institutional reform;
- cultural support;
- environmental repair;
- guarantees of non-repetition.

The plural categories recognized in international reparations practice are directly relevant here [65, 12].

A generative orientation adds particular attention to whether reparative resources become durable, community-controlled conditions capable of producing future value.

7.71 Present Technical Feasibility

The practical architectures described above vary greatly in present feasibility.

Some are already commonplace: monetary transfers, record correction, independent review, rehabilitation, income support, procedural safeguards, and organizational re-entry.

Others require stronger information infrastructure, interoperability, longitudinal monitoring, causal analysis, or distributed correction protocols.

Still others remain largely idealized, including reliable reconstruction of complex counterfactual trajectories or high-dimensional dynamic comparison of social systems.

The framework therefore separates conceptual design from present operational capacity.

A practical proposal should state which components can currently be implemented, which require institutional development, and which remain theoretical.

7.72 Resource Constraints

Comprehensive support can require substantial resources.

Institutions therefore face allocation problems. Intensive individualized re-entry support for one person can consume resources that could provide basic protection to many others.

A generative theory should not evade scarcity.

Possible design responses include:

- minimum universal floors;
- severity-sensitive additional support;
- standardized components combined with individualized modules;
- risk pooling;
- public financing;
- time-limited intensive support;
- prioritization of high-criticality barriers.

The appropriate distributive principles require later normative analysis.

7.73 Administrative Burden

Restorative programs can create significant administrative burden for claimants.

Repeated documentation, medical examinations, evidentiary requirements, interviews, deadlines, travel, and appeals can consume the resources needed for recovery.

Administrative burden should therefore be treated as part of the cost of the restorative process.

Practical design can reduce burden through:

- reuse of verified evidence;
- clear interfaces;
- assistance with documentation;
- presumptions where justified;
- coordinated scheduling;
- digital and non-digital access options;
- support for language and disability access.

A process intended to restore generativity should avoid consuming it unnecessarily.

7.74 Epistemic Burden

The claimant can also bear an excessive epistemic burden.

A person may be required to explain what would have happened in a complex counterfactual life, identify every downstream relation affected, quantify non-fungible losses, or prove causal chains beyond the subject's available knowledge.

Practical design should distinguish evidence necessary for a specific institutional function from impossible demands for complete historical reconstruction.

This distinction becomes central in Section 8.

7.75 Measurement Burden

Generative frameworks can themselves create a risk of overmeasurement.

Once states, networks, attractors, resilience, conversion capacity, and reachable sets are recognized as possible objects of comparison, institutions can be tempted to collect extensive data about subjects.

The atlas developed later in the paper is an analytical space, not an instruction that every object should be measured in every case.

A proportional practice should collect only information required for the selected restorative objective.

7.76 Practice Minimalism

Practical minimalism seeks the least intrusive intervention capable of achieving the legitimate restorative function.

Minimalism can reduce:

- surveillance;
- paternalism;
- administrative burden;
- third-party interference;

- dependency;
- unnecessary standardization.

Minimalism should not become under-response.

The relevant standard is sufficient intervention for the legitimate function, rather than minimal institutional expenditure.

7.77 Practice Pluralism

Different subjects can require different combinations of support after formally similar harms.

One person can prefer reinstatement. Another can prefer compensated exit. A third can require rehabilitation before either decision becomes feasible. A fourth can prefer education for a new field.

Pluralism is therefore a structural feature of future-directed assistance.

Institutions can provide a menu of legitimate pathways rather than one mandatory model of recovery.

7.78 Practice-Level Procedural Justice

Every practical mechanism described in this section remains subject to procedural requirements.

These include:

- consent where appropriate;
- notice;
- participation;
- reason-giving;
- contestability;
- privacy;
- proportionality;
- non-retaliation;
- revisability;
- independent review;
- protection against domination;
- ability to refuse or leave assistance.

A technically effective intervention can remain unjust if these requirements are ignored.

7.79 Practice-Level Substantive Evaluation

Procedural legitimacy also does not establish substantive adequacy.

A perfectly fair procedure can produce a remedy that restores only a small fraction of the relevant object.

Practical evaluation should therefore distinguish:

$$J_{\text{practice}} = (J_{\text{outcome}}, J_{\text{process}}). \quad (65)$$

Equation 65 anticipates the later formal distinction between substantive restorative equivalence and procedural justice.

The two dimensions can vary independently.

7.80 Practice Architecture Under Radical Uncertainty

Some cases involve uncertainty too deep for reliable counterfactual estimation.

A practical architecture can still proceed through identifiable present objects:

- stopping ongoing harm;
- returning known conditions;
- correcting known errors;
- compensating identifiable losses;
- supplying present healthcare or income;
- reopening viable future options.

This approach avoids requiring complete knowledge before any action can occur.

Uncertainty concerning the whole does not imply uncertainty concerning every part.

7.81 Practice Architecture Under Partial Irreversibility

Where some dimensions are irrecoverable and others remain recoverable, the response should distinguish them explicitly.

Let the relevant restorative objects be partitioned into

$$Q = Q_{\text{recoverable}} \cup Q_{\text{partially recoverable}} \cup Q_{\text{nonrecoverable}}. \quad (66)$$

Equation 66 supports a differentiated response.

Recoverable objects can be restored directly. Partially recoverable objects can receive targeted support. Nonrecoverable objects can be acknowledged while compensation, assistance, or prevention addresses other dimensions.

This structure prevents irreversibility in one domain from becoming a reason for inaction in another.

7.82 Practice Architecture Under Unknown Reparability

Sometimes the degree of reparability is itself unknown.

A practical response can then preserve optionality.

Examples include temporary support while recovery potential becomes clearer, staged rehabilitation, preservation of professional access, and assistance that can later be redirected.

Optionality is especially valuable when premature commitment to one restorative path could close alternatives.

7.83 Restorative Experimentation and Ethical Constraints

Institutions can learn from experimental or pilot restorative programs where appropriate.

Such experimentation should remain subject to ordinary ethical protections. Affected persons should not become involuntary experimental subjects merely because existing restoration methods are uncertain.

Useful experiments can concern institutional design rather than manipulation of the claimant. Examples include different referral systems, record-correction protocols, assistance interfaces, or re-entry supports.

Learning should feed into later practice while preserving privacy and procedural rights.

7.84 Current Practice and Idealized Design

The practical architectures in this section occupy different positions between current practice and ideal theory.

A useful distinction is:

$$\mathcal{P} = \mathcal{P}_{\text{available}} \cup \mathcal{P}_{\text{developable}} \cup \mathcal{P}_{\text{idealized}}. \quad (67)$$

$\mathcal{P}_{\text{available}}$ contains mechanisms already institutionally available in some jurisdictions or organizations. $\mathcal{P}_{\text{developable}}$ contains mechanisms requiring plausible institutional or technical development. $\mathcal{P}_{\text{idealized}}$ contains thought-experimental mechanisms used to clarify conceptual boundaries.

Equation 67 helps prevent ideal-theoretical analysis from being mistaken for an immediate policy recommendation.

7.85 Minimum Practical Architecture

Even where resources are limited, a minimum practical architecture can contain several basic elements:

- accessible reporting;
- evidence preservation;
- anti-retaliation protection;
- temporary interruption where severe continuing harm is plausible;
- correction of established factual errors;
- restitution of directly returnable conditions;
- basic financial or social protection where present need is severe;
- participation of the affected subject in future planning;
- appeal and review;
- institutional learning after resolution.

This minimum does not represent complete justice. It identifies components whose absence can make later repair substantially harder.

7.86 Expanded Practical Architecture

Where institutional capacity is greater, the architecture can additionally support:

- integrated case coordination;
- distributed correction propagation;
- individualized conversion support;
- network re-entry;
- long-term transition resources;
- community-controlled reparative infrastructure;
- privacy-preserving evidence exchange;

- longitudinal outcome evaluation;
- advanced risk and dependency mapping.

These mechanisms expand the range of restorative objects that institutions can address.

They also increase informational and governance power. Their use therefore requires stronger procedural safeguards.

7.87 Limits of Practice Architecture

No practical architecture described in this section can establish complete restoration merely by becoming sufficiently comprehensive.

Several limitations remain.

First, some relevant histories cannot be observed.

Second, some counterfactual trajectories are underdetermined.

Third, some generative conditions are singular.

Fourth, some social meanings cannot be directly controlled.

Fifth, interventions produce new historical events.

Sixth, complete measurement can itself alter the social system.

Seventh, stronger restorative fidelity can conflict with agency, privacy, and procedural justice.

Eighth, different value theories can disagree about which restored feature matters.

Ninth, multi-party systems can contain conflicts among legitimate generative interests.

These limitations motivate the transition from practice to the epistemic and structural analysis of restoration.

7.88 Implications for the Formal Theory of Restorative Comparison

The practice architectures developed in this section expose several variables that later require formal separation.

A practical claim of restoration depends upon:

- the temporal phase;
- the reference condition;
- the object targeted by the intervention;
- the institution performing the intervention;
- the comparative standard used to evaluate success;
- the time horizon;
- the relevant perturbations or future environments;
- the approximation tolerated;
- the epistemic basis of the judgment;
- the procedural conditions under which the intervention occurred.

These variables explain why apparently simple statements such as “employment has been restored,” “the loss has been compensated,” or “generativity has been rebuilt” can conceal several distinct claims.

The practical architecture therefore points directly toward the comparative framework developed later in the paper.

7.89 Transition to the Limits of Restoration

The preceding sections have progressively enlarged the problem of return.

Section 2 located the inquiry within established theories of corrective, restorative, reparative, capability, relational, recognition-based, non-domination, and generative justice.

Section 3 distinguished resources, generative conditions, conversion structures, capacities, recursive value cycles, and modes of generative harm.

Section 4 established that the recipient of return is a historically and relationally constituted subject whose generative relation to the returned condition can change.

Section 5 separated prevention, interruption, and post-harm response.

Section 6 distributed these functions across law, governance, political economy, social protection, organizations, knowledge institutions, and assistance.

The present section has translated those functions into practical architectures.

The resulting framework remains incomplete until the limits of restoration are made explicit. A more elaborate practice can reduce some limitations while leaving others untouched. Improved data can reduce uncertainty while failing to produce a unique counterfactual history. Better simulation can increase predictive capacity while leaving normative selection unresolved. Complete formal restoration of one object can coexist with non-equivalence of another. Recording can improve traceability while becoming a new event in the system.

Section 8 therefore distinguishes practical, epistemic, representational, mathematical, structural, reflexive, and normative limits of restoration. That analysis provides the final conceptual bridge to the formal theory of generative systems and the subsequent atlas of restorative comparison.

8 Epistemic, Structural, and Practical Boundaries of Restoration

This section examines the limits that arise when restitution, compensation, repair, rehabilitation, assistance, or another intervention is expected to restore a historically altered generative system. Its role is diagnostic. The preceding sections established that generative harm can affect resources, relations, conversion structures, capacities, trajectories, meanings, and future possibilities; that subjects continue to develop throughout the period of harm; that prevention, interruption, and post-harm response perform different functions; and that these functions are distributed across several institutions and practical architectures. The present section asks which remaining difficulties arise from present institutional and technical limitations and which arise at deeper epistemic, mathematical, structural, reflexive, or normative levels.

The distinction is important because very different conclusions follow from different kinds of limit. A currently expensive measurement can become inexpensive. Missing longitudinal data can become available in future cases. A weak causal model can improve. Institutional coordination can be redesigned. Such limitations provide reasons for technical and institutional development.

Other difficulties have a different structure. A counterfactual history may fail to be uniquely identified even under extensive observation. Several different histories can generate the same present state. A system's response rules can change through the intervention being evaluated. A record of the system can become an event within the system. A restorative intervention can change the subject whose state it seeks to restore. A perfectly specified counterfactual state can remain normatively inappropriate as the destination of repair.

The section therefore distinguishes six broad families of boundary:

1. practical and technological limits;
2. epistemic and counterfactual limits;
3. representational limits;
4. mathematical and structural limits;

5. reflexive limits;
6. normative limits.

These families overlap. Their separation serves an analytical purpose. It allows the later atlas of restorative comparison to state precisely which assumptions are required by a particular claim of equivalence and which limitations remain unresolved.

8.1 A Layered Account of Restorative Limits

Let $\mathcal{G}_t^H$ denote the actual generative system after a harmful history H , let $\mathcal{G}_t^{\text{ref}}$ denote a selected reference system, and let R denote a proposed restorative intervention. A general restorative objective can be represented by Equation 68.

$$R(\mathcal{G}_t^H) \sim_{E,Q} \mathcal{G}_t^{\text{ref}}, \quad (68)$$

where Q denotes the object of comparison and E denotes the selected comparative relation.

Equation 68 appears simple only after several problems have already been resolved. The reference system must be specified. The relevant object must be identified. The selected object must be observable or otherwise representable. The comparative relation must be defined. The restorative intervention must be available. The intervention must produce the relevant target. The resulting transformation must remain normatively permissible.

These requirements correspond to different forms of limitation. A useful schematic decomposition is given in Equation 69.

$$\mathcal{L}_R = (L_{\text{practical}}, L_{\text{epistemic}}, L_{\text{representational}}, L_{\text{structural}}, L_{\text{reflexive}}, L_{\text{normative}}). \quad (69)$$

Equation 69 does not imply that the six components are independent. An improved representation can reduce an epistemic limit. A reflexive system can produce new structural uncertainty. A normative constraint can rule out an intervention that is technically available. The decomposition prevents these different reasons for incompleteness from being described under a single undifferentiated concept of difficulty.

8.2 Practical and Technological Limits

The most immediate limits are practical. Institutions possess finite time, money, personnel, information, computational capacity, jurisdiction, and authority. Subjects have finite time and energy. Records can be unavailable. Witnesses can disappear. Relevant organizations can cease to exist. Some medical interventions remain unavailable. Some ecological restoration requires resources beyond feasible allocation. Some relationships cannot be recreated because the persons involved are unavailable.

These limits can be severe without establishing any deeper impossibility.

A professional network can currently be difficult to reconstruct because institutions lack reliable longitudinal network data. Future institutional records can improve. A distributed correction can currently fail because databases lack interoperable provenance. Better correction protocols can be designed. A causal effect can currently be estimated poorly because relevant data were never collected. Future systems can preserve better evidence.

Practical limits therefore belong to the domain of institutional and technical development.

The distinction can be expressed through a feasible-intervention set $\mathcal{R}^{\text{now}}$ and a conceptually admissible set $\mathcal{R}^{\text{possible}}$:

$$\mathcal{R}^{\text{now}} \subseteq \mathcal{R}^{\text{possible}}. \quad (70)$$

Equation 70 identifies a space in which technical development can enlarge practical capacity.

The paper therefore avoids inferring permanent impossibility from present institutional weakness.

8.3 Resource Limits and Opportunity Cost

Restoration also competes for scarce resources. Highly individualized counterfactual reconstruction, long-term professional re-entry, ecological repair, specialized rehabilitation, and extensive relational support can be resource-intensive.

A program that devotes unlimited resources to increasing the restorative fidelity of one case can reduce resources available for prevention, basic social protection, or assistance to other subjects.

Resource limitation therefore creates a distributive problem in addition to a technical problem.

Let B denote a finite restorative budget and $r_1, \dots, r_n$ candidate interventions. Feasibility requires Equation 71.

$$\sum_{i=1}^n \text{cost}(r_i) \leq B. \quad (71)$$

Equation 71 is elementary, while its normative implications are substantial. A theory of perfect individualized restoration can remain institutionally unusable when it lacks an account of resource distribution across claimants.

The later justice sections therefore distinguish the conceptual definition of a restorative target from the distributive principles governing how closely that target should be pursued under scarcity.

8.4 Institutional Limits of Observation

Institutions observe only parts of a generative history.

A court can observe submitted evidence. An employer observes organizational records and interactions. A healthcare provider observes health-related information. A professional association observes credential and disciplinary information. A statistical agency observes variables defined by its data system. The affected subject possesses experiential knowledge unavailable to these institutions.

The observable system can therefore be represented by a projection

$$O_I : \mathcal{G} \longrightarrow Y_I, \quad (72)$$

where O_I denotes the observation map available to institution I and Y_I denotes the institutional representation.

Equation 72 makes a simple point explicit: an institution acts upon Y_I , while the relevant social and generative system can contain structures outside that representation.

Two distinct systems can consequently satisfy

$$O_I(\mathcal{G}_1) = O_I(\mathcal{G}_2) \quad (73)$$

while differing in dimensions that matter under another observation map or theory of value.

Equation 73 anticipates the later distinction between observational equivalence and structural or generative equivalence.

8.5 Missing Historical Information

Historical restoration frequently requires information that was never recorded.

A person can lose opportunities through informal conversations that leave no archive. A professional network can change through decisions whose reasons were never documented. A community can lose ecological knowledge transmitted orally. A workplace can develop an informal reputation that no official record captures.

Missing information differs from inaccessible information. Inaccessible information can become available through improved access or technology. Unrecorded historical events may no longer possess any recoverable trace sufficient to identify their complete effects.

The resulting limit should remain specific. The absence of one form of historical knowledge does not imply that every restorative judgment becomes impossible. Identifiable losses, current needs, formal records, and observable continuing mechanisms can still support action.

This distinction supports a recurring methodological principle:

Incomplete historical knowledge should reduce the scope of unsupported claims, while preserving action on dimensions for which adequate evidence exists.

8.6 Counterfactual Reference and the Unobserved History

A stronger epistemic problem arises when restoration is defined through a counterfactual history.

Suppose harm H occurred at t_0 . The actual later history $\mathcal{H}_{t_0:t_1}^H$ is observed at least partially. The relevant no-harm history $\mathcal{H}_{t_0:t_1}^{-H}$ did not occur.

The counterfactual problem is therefore structurally different from recovery of a hidden historical record. The relevant trajectory is unobserved because it was never realized.

Counterfactual reasoning is a well-developed philosophical and causal domain. Lewis’s analysis of counterfactuals formalizes counterfactual evaluation through relations among possible worlds [28]. Structural causal approaches provide another formal language in which counterfactuals and interventions are represented through causal models [43].

The present paper does not adopt one universal semantics of counterfactual restoration. It draws a more basic distinction. A restorative theory that uses a no-harm trajectory as its reference requires assumptions concerning how the unobserved history is generated.

This requirement should be stated explicitly.

8.7 Counterfactual History Beyond Event Subtraction

The no-harm history cannot generally be obtained by removing one event from the actual event sequence while holding everything else constant.

Let

$$\mathcal{H}^H = (e_0, H, e_2, e_3, \dots, e_n)$$

represent the actual history. A naive subtraction model would construct the counterfactual by deleting H .

A generative system creates a different structure. Later events can depend upon earlier states. Removal of H can therefore alter $e_2, e_3, \dots, e_n$.

The relevant relation is expressed in Equation 74.

$$\mathcal{H}^{-H} \neq \mathcal{H}^H \setminus \{H\}. \quad (74)$$

Equation 74 is central to generative restoration. If the harmful event altered later encounters, choices, relationships, and institutional responses, the counterfactual trajectory must be regenerated under the altered assumptions.

The regenerated history can remain uncertain even where the immediate causal effect of H is well understood.

8.8 Counterfactual Multiplicity

The removal of one harmful event can open several possible no-harm histories.

Suppose the subject would have retained access to a research environment in the absence of exclusion. The subject could have completed one project, changed research direction, joined another collaboration, left voluntarily, become ill, received another offer, or experienced numerous other events.

A counterfactual reference can therefore be set-valued:

$$\mathfrak{H}^{\neg H} = \left\{ \mathcal{H}^{(1)}, \mathcal{H}^{(2)}, \dots \right\}. \quad (75)$$

Equation 75 prevents a model from treating one plausible counterfactual trajectory as the unique lost future without further justification.

The restorative problem then changes. A theory can compare expectations, distributions, reachable sets, robust features shared across several histories, or another summary of the counterfactual space. Each choice constitutes a different comparison.

8.9 Counterfactual Identification and Model Dependence

Counterfactual quantities are identified relative to assumptions encoded in a model. Pearl’s structural account of causality makes identification an explicit problem and distinguishes causal and counterfactual claims from ordinary associational observation [43].

For generative return, the implication is methodological. A statement such as

Without the exclusion, the claimant would have reached state x .

is stronger than a statement that exclusion reduced access to a set of opportunities. The stronger statement requires a model capable of supporting the specific counterfactual.

Where several models remain observationally compatible while producing different counterfactual histories, the target remains underidentified.

Let $\mathcal{M}(D)$ denote the set of models compatible with observed data D . A counterfactual quantity Q^{cf} is non-unique when

$$Q^{\text{cf}}(M_i) \neq Q^{\text{cf}}(M_j) \quad \text{for some } M_i, M_j \in \mathcal{M}(D). \quad (76)$$

Equation 76 identifies an epistemic limit that additional computation alone does not remove. Additional evidence or stronger assumptions are required.

8.10 Prediction, Causal Attribution, and Restoration

Prediction and causal attribution should remain distinct from restorative comparison.

A model can predict that a claimant has a low probability of returning to a profession without identifying the causes of that probability. A causal model can estimate the effect of a specific intervention without specifying which future state should count as just restoration. A restorative standard can identify a desirable object without predicting whether the available intervention will produce it.

These correspond to different mappings:

$$\begin{aligned} \text{prediction : } & X_t \longrightarrow P(X_{t+1}), \\ \text{causal analysis : } & do(H) \longrightarrow \Delta X, \\ \text{restorative comparison : } & (X_R, X_{\text{ref}}) \longrightarrow \text{comparative judgment}. \end{aligned} \quad (77)$$

Equation 77 separates three epistemic tasks. Strong performance in one does not automatically establish competence in the others.

8.11 Counterfactual Uncertainty and Institutional Action

Incomplete counterfactual knowledge does not eliminate every basis for institutional action.

A system can know with high confidence that a salary was withheld while remaining unable to know which investments would have followed from receipt of the salary. It can know that access to a laboratory was unlawfully denied while remaining unable to know which papers would have been published. It can know that a false record circulated while remaining unable to identify every relationship affected by the record.

The appropriate response can therefore decompose certainty across objects.

Let $Q_1, \dots, Q_n$ denote restorative objects and $K(Q_i)$ the epistemic status associated with each object. A practical system can act upon high-confidence objects while representing lower-confidence objects separately.

This structure avoids two symmetric errors: demanding complete counterfactual knowledge before any remedy and converting uncertainty into speculative precision.

8.12 Loss of Chance and Probabilistic Reference

Some restorative contexts already respond to uncertainty through probabilistic reasoning. The legal concept of loss of chance provides one family of examples, especially where a wrong reduces the probability of a favorable outcome without making the original outcome certain.

A generative framework can generalize the underlying structure without assuming that every future possibility has a reliable monetary value.

Rather than claim

$$X_{t_1}^{\text{cf}} = x^*,$$

a model can represent a distribution

$$P(X_{t_1}^{\text{cf}}).$$

Where such a distribution is justified, restoration can compare probability structures, expected capabilities, opportunity distributions, or another probabilistic object.

The choice among these objects remains normative as well as epistemic.

8.13 Historical Contingency

Generative systems can depend upon contingent encounters whose occurrence is neither necessary nor fully predictable.

A scholar reads a paper because another person happens to recommend it. The paper leads to a meeting. The meeting produces a collaboration. The collaboration changes a research question. The resulting work affects another institution.

Such chains are causally intelligible after they occur while remaining difficult to specify before they occur.

Historical contingency therefore complicates a restorative theory that attempts to enumerate every lost downstream event. The appropriate object can instead be the generative architecture within which contingent opportunities could arise.

This distinction motivates later attention to reachable sets, transition structures, network position, and generative mechanisms.

8.14 Epistemic Limits of Singular Relations

Singular relationships produce a special counterfactual problem because their future value often depends upon reciprocal adaptation.

The generative consequences of a mentorship, friendship, collaboration, or community relation emerge through interactions that have no predetermined output. The participants respond to one another, change questions, revise expectations, and create new joint states.

A model can estimate some features of such a relation. It cannot infer the full joint history merely from independent profiles of the participants.

This problem limits strong claims that another relation with similar observable attributes constitutes an equivalent substitute.

Functional substitution can remain valuable while historical relational equivalence remains unestablished.

8.15 Representational Selection

Every formal model selects variables, boundaries, scales, and relations.

Simon's work on the architecture of complex and artificial systems emphasizes the analytical importance of decomposition and representation in understanding complexity [59]. The relevance here is methodological: restorative analysis requires a representation through which comparison becomes possible.

Let

$$\pi : \mathcal{W} \longrightarrow \mathcal{M} \quad (78)$$

map a relevant portion of the social world $\mathcal{W}$ into a model $\mathcal{M}$.

Equation 78 necessarily determines which features are retained for the purpose of the analysis.

A legal model can preserve ownership, income, and causation while omitting informal network structure. A capability model can preserve opportunity sets while abstracting from some historical meanings. A dynamical model can preserve transition behavior while omitting narrative identity. A graph model can preserve relation topology while omitting the substantive meaning of each edge.

Such abstractions can be useful. Their legitimacy depends upon the question being asked.

8.16 Representational Incompleteness

A representation can be complete relative to a specified purpose while remaining incomplete relative to the social world.

This distinction is crucial. The paper does not require the metaphysical claim that social reality is absolutely unsymbolizable. It requires only the weaker claim that a representation used for restoration is indexed to selected objects, relations, observables, and scales.

Let Q denote a selected restorative object. A representation π_Q can preserve the structures relevant to Q :

$$\pi_Q(\mathcal{W}) = \mathcal{M}_Q. \quad (79)$$

Equation 79 can support a valid Q -relative judgment while leaving other restorative objects unresolved.

This purpose-relative account allows formal analysis without claiming an exhaustive symbolic duplicate of society.

8.17 Lossy Compression and Justice

Institutional and formal representations can be interpreted as forms of compression. They preserve some features and discard others.

Compression becomes normatively important when omitted dimensions systematically correspond to forms of harm that matter to affected subjects.

A damages model that preserves income while omitting professional-network effects is not necessarily invalid. Its output should be described as an income-loss estimate rather than as a complete valuation of the claimant's historical loss.

The practical norm is therefore scope discipline:

The strength of a restorative conclusion should not exceed the informational scope of the representation that supports it.

This principle will later become part of the indexed structure of restorative claims.

8.18 Boundary Selection

A model must also determine where the relevant system ends.

Should workplace restoration include family effects produced by employment loss? Should academic restoration include the careers of collaborators affected by an interrupted project? Should community reparation include ecological systems, neighboring communities, descendants, and future residents?

Expanding the boundary captures more relations while increasing complexity and the possibility of conflicting claims.

Boundary selection is therefore both analytical and normative.

Let $\partial\mathcal{G}$ denote the selected boundary of the generative system. Restorative comparison is always implicitly conditioned upon $\partial\mathcal{G}$.

The later atlas should therefore treat system boundary as part of the modeling context even where it is not included as a primary index.

8.19 Scale Dependence

A restorative conclusion can change with scale.

An intervention can restore the productive capacity of one organization while reducing the generativity of a profession. It can improve community-level economic output while reducing ecological resilience. It can increase aggregate welfare while leaving a particular subject under domination.

Consequently,

$$\text{Restored}^{(s_1)} \not\equiv \text{Restored}^{(s_2)}. \quad (80)$$

Equation 80 requires restorative claims to identify the scale at which equivalence or adequacy is asserted.

8.20 Observability and Hidden State

A system can contain variables that influence future behavior without being directly observable through the available measurement process.

In control-theoretic language, observability concerns whether internal state can be reconstructed from available outputs over time. The present paper uses the idea conceptually without assuming that human subjects can be represented by one fixed state-space model.

Suppose

$$x_{t+1} = F(x_t, u_t), \quad y_t = O(x_t). \quad (81)$$

Equation 81 distinguishes internal state x_t from observable output y_t .

Two systems can generate similar outputs over a relevant interval while possessing different internal structures that later respond differently to perturbation.

This provides a formal reason that apparent behavioral recovery at one time does not establish full dynamical restoration.

8.21 Many-to-One Dynamics

A second structural difficulty arises when different earlier states produce the same later state.

Let the evolution map satisfy

$$F(x_1) = F(x_2) = y, \quad x_1 \neq x_2. \quad (82)$$

Equation 82 shows that F lacks a unique inverse at y .

If a social system has analogous many-to-one dynamics, observing the present state does not uniquely identify the earlier state or history.

This structural non-uniqueness is stronger than ordinary measurement error. Perfect observation of y remains insufficient for distinguishing x_1 from x_2 .

Historical records can reduce the ambiguity by preserving information that the current state no longer contains. Where such records are absent, the inverse problem can remain underdetermined.

8.22 Structural Non-Invertibility

The previous example motivates the broader concept of *structural non-invertibility*.

A restorative theory often imagines an inverse operation capable of undoing a harmful transformation:

$$R \circ H \approx I, \quad (83)$$

where I denotes an identity operation.

Equation 83 becomes unreliable where the harmful period includes further system evolution.

The relevant composition is instead

$$R_{t_1} \circ \Phi_{t_1, t_0}^H \circ H_{t_0}. \quad (84)$$

Equation 84 acts upon a system that developed under altered conditions. Even if H_{t_0} itself admits a formal inverse, the entire historical composition can fail to do so.

This provides a mathematical interpretation of the non-equivalence of delayed return.

8.23 State Recovery and History Recovery

State recovery should be distinguished from history recovery.

Suppose a powerful intervention places the system at state x^* , identical under a selected representation to the state of a reference system:

$$x_R(t_1) = x_{\text{ref}}(t_1). \quad (85)$$

Equation 85 establishes state equality under the selected variables.

It does not establish

$$\mathcal{H}_R = \mathcal{H}_{\text{ref}}. \quad (86)$$

Equation 86 identifies a different object. The actual system reached $x_R(t_1)$ through harm and restoration. The reference system reached $x_{\text{ref}}(t_1)$ through another history.

Whether this historical difference matters depends upon the theory of value and the restorative claim being made.

8.24 Dynamics Recovery and State Equality

State equality also does not establish equality of future dynamics.

Two systems can satisfy

$$x_1(t) = x_2(t)$$

while differing in their effective transition rules.

A formal representation appears in Equation 87.

$$F_1(x, u) \neq F_2(x, u). \quad (87)$$

Equation 87 expresses the possibility that the same current condition responds differently to future inputs.

For generative return, this distinction is central. Restoration of a job, income, or institutional status can reproduce visible state variables while the subject's response to later organizational stress, uncertainty, or opportunity has changed.

The later atlas therefore separates state equivalence from dynamical and response equivalence.

8.25 Rule Change Through Harm

Some harms alter more than the state of a system. They can change the effective rules through which later action is generated.

A worker learns that formal reporting channels trigger retaliation. A researcher learns that access depends upon informal loyalty rather than written policy. A community learns that public consultation has little influence upon actual decisions.

The relevant change can be represented by

$$F \longrightarrow F^H. \quad (88)$$

Equation 88 is not a claim that human behavior follows a fixed mechanical law. It identifies a shift in the effective response structure of the subject or institution.

This possibility also has a precedent in policy analysis. Lucas's critique emphasizes that behavioral relations estimated under one policy regime cannot automatically be treated as invariant under policy change [29]. The present application is broader and more modest: interventions can change the processes generating the observations used for prediction.

Restoration therefore cannot always rely upon a model estimated entirely from the pre-intervention regime.

8.26 Intervention-Induced Model Change

A restorative intervention can itself change the system's effective dynamics.

Suppose a compensation program changes incentives, a public apology changes social meaning, or a new monitoring regime changes behavior. The model used to predict the effect of the intervention can cease to describe the system once the intervention is introduced.

Let M denote the model estimated under the pre-intervention system and R a restorative intervention. The post-intervention system can require a new model M_R :

$$M \xrightarrow{R} M_R. \quad (89)$$

Equation 89 limits naive extrapolation from historical data.

This problem is particularly important for ambitious technological restoration, where policy is selected on the basis of a model that subjects can observe and respond to.

8.27 Aggregate Outcomes and Micro-Level Difference

Aggregate restoration can conceal heterogeneous individual effects.

Schelling's analysis of micromotives and macrobehavior demonstrates how aggregate patterns can emerge from decentralized individual responses and how macro-level observations can differ substantially from the mechanisms visible at individual level [54].

For generative return, similar aggregate outputs can arise from different individual distributions and relations.

An organization can regain its total publication output after a harmful episode while the researchers producing the output are entirely different. A community can regain aggregate income while wealth distribution and social relations have changed. A labor market can restore employment totals while particular workers remain permanently displaced.

Aggregate equivalence therefore requires explicit justification where justice concerns individuals or subgroups.

8.28 Micro-Level Restoration and System-Level Difference

The reverse problem also occurs. Several individuals can be restored according to local criteria while the system-level generative structure remains changed.

Individual compensation can coexist with a continuing extractive institution. Restoration of several workers can coexist with a workplace culture that continues producing retaliation. Return of resources to several community members can coexist with ecological regime change.

The relation is therefore bidirectional:

$$\text{local equivalence} \not\Rightarrow \text{system equivalence.} \quad (90)$$

Equation 90 is one reason the later atlas must support multiple scales.

8.29 Sensitivity to Initial Conditions and Historical Divergence

Some nonlinear systems display strong sensitivity to small differences in initial condition or parameter values. Social systems need not satisfy a specific mathematical definition of chaos for historical divergence to matter. Small differences can be amplified through network effects, cumulative advantage, institutional thresholds, and recursive value generation.

The implication for restoration is epistemic. Small uncertainty about an early state can produce large uncertainty concerning long-horizon counterfactual trajectories.

This weakens claims of precise long-run reconstruction while leaving shorter horizon or more structural comparisons available.

For example, it can be impossible to know which specific scholar a researcher would have met ten years later while remaining reasonable to conclude that exclusion substantially reduced participation in the relevant scholarly network.

8.30 Branching and Reachable Futures

A generative system can be represented through branching possible transitions rather than one predetermined trajectory.

Let

$$\mathcal{R}(x_t) = \{x : x \text{ is reachable from } x_t\}. \quad (91)$$

Equation 91 defines a generic reachable set.

A harmful intervention can change this set without determining which element would otherwise have been realized.

This provides a more modest restorative object than one unique counterfactual biography.

The comparison of reachable sets will later receive formal treatment. At the present stage, the important point is epistemic: uncertainty about the selected future does not imply ignorance about changes in the structure of possible futures.

8.31 Open-Ended Generation

Generative systems can also create states that were not specified in the original restorative model.

A collaboration can produce a new field of inquiry. A community can develop a new institution. A technological innovation can create new occupations. A restorative process can generate an unexpected relationship or political movement.

Open-ended generation complicates restoration because the future state space itself can change.

Let X_t denote the set of recognized states at time t . In an open-ended process,

$$X_{t+1} \supset X_t \cup X_{\text{new}}. \quad (92)$$

Equation 92 expresses the possibility that new states become meaningful through the process of development.

A restorative model fixed entirely at t_0 can therefore fail to represent possibilities that later emerge.

This is another reason post-harm justice should preserve future openness rather than defining justice exclusively through return to a pre-specified state.

8.32 Reflexivity in Social Observation

Social observation can affect the system observed.

Merton's analysis of the self-fulfilling prophecy provides a classic account of situations in which beliefs and definitions influence behavior in ways that help produce the anticipated condition [36]. Hacking's analysis of the looping effects of human kinds develops a related problem for social and human-scientific classification: classifications can affect classified persons, and those changes can in turn alter the relevant classifications [23].

These traditions provide strong precedent for treating social description as potentially endogenous to the system described.

Let

$$Z_t = O(\mathcal{G}_t) \quad (93)$$

denote a classification, record, model, prediction, or institutional description generated from the system.

Where actors observe and respond to Z_t , subsequent dynamics can take the form

$$\mathcal{G}_{t+1} = F(\mathcal{G}_t, Z_t). \quad (94)$$

Equation 94 shows that the representation becomes an input into the next system state.

8.33 The Record as a Relational Object

A social record is therefore capable of becoming a new relational object.

An employment classification changes who can access a position. A diagnosis can change treatment. A credit score can change lending decisions. A disciplinary record can alter later institutional trust. A judicial finding can alter public reputation. A reparative designation can change eligibility and social identity.

The record has two simultaneous roles:

1. it represents a claim about the system;
2. it becomes part of the conditions influencing the system.

This dual role creates a fundamental difference between inert observation and socially operative representation.

A theory of restorative records should therefore evaluate both epistemic accuracy and downstream generative effects.

8.34 Correction as a New Event

When an earlier record is corrected, the correction does not erase the event that the earlier record existed.

Suppose Z_0 contains a false classification and Z_1 later records that Z_0 was incorrect. The history now contains both events.

A schematic sequence is given in Equation 95.

$$\mathcal{G}_0 \rightarrow Z_0 \rightarrow \mathcal{G}_1 \rightarrow Z_1 \rightarrow \mathcal{G}_2. \quad (95)$$

Equation 95 makes clear that Z_1 is not an algebraic cancellation of Z_0 .

Some actors can remember Z_0 . Some systems can retain derivative data. Some relationships can already have changed. The fact that a correction became necessary can itself acquire social meaning.

Correction remains essential. Its function is to alter present and future epistemic conditions rather than to make the earlier informational history literally nonexistent.

8.35 Recursive Description

The attempt to produce a complete social record generates a recursive problem.

Suppose Z_0 is intended as a complete description of social system $\mathcal{G}_0$. Once Z_0 becomes socially available, it can enter the system:

$$\mathcal{G}_1 = F(\mathcal{G}_0, Z_0). \quad (96)$$

A complete description of the new system would then require Z_1 :

$$Z_1 = O(\mathcal{G}_1). \quad (97)$$

Once Z_1 becomes operative, it can generate a further state.

The sequence takes the form

$$\mathcal{G}_0 \rightarrow Z_0 \rightarrow \mathcal{G}_1 \rightarrow Z_1 \rightarrow \mathcal{G}_2 \rightarrow Z_2 \rightarrow \dots. \quad (98)$$

Equation 98 expresses *reflexive non-closure*: the description of a socially responsive system can become a further event requiring inclusion in later description.

The result does not establish that every form of social representation is impossible. It limits the idea of a once-and-for-all representation that is simultaneously complete, socially operative, and historically inert.

8.36 Completeness, Externality, and Non-Intervention

The recursive structure suggests a tension among three desirable properties of a social representation:

1. completeness with respect to the relevant social system;
2. externality from the system represented;
3. absence of causal influence upon the represented system.

A record used for governance loses strict externality because actors respond to it. A representation kept completely external to the social world can avoid some feedback while providing limited governance value. A socially operative record can remain useful while changing the object being described.

The paper therefore avoids the stronger unsupported claim that social reality is absolutely unsymbolizable. The more defensible conclusion is conditional:

A socially operative representation should not be assumed to remain a complete and causally inert external duplicate of the system it represents.

This condition is sufficient for the restorative problem.

8.37 Classification Loops

Hacking's looping-effects framework provides a concrete form of reflexive non-closure [23]. A classification can alter how classified persons understand themselves, how institutions interact with them, or how other actors interpret their behavior. These changes can then alter the phenomena upon which the classification depends.

A restorative classification can produce similar loops.

Designating a person as "rehabilitated," "high risk," "wrongfully dismissed," "reparation beneficiary," or "vulnerable" can influence later institutional action.

The classification can be beneficial and remain generatively consequential.

A restorative system should therefore treat categories as interventions in addition to descriptions where the social context makes that interpretation appropriate.

8.38 Measurement Feedback

Measurement can become part of governance.

Campbell's work on evaluation of planned social change emphasizes the difficulty of attributing observed outcomes in complex historical environments and the ways social interventions and indicators interact within changing systems [9].

Where an indicator becomes a decision target, subjects and institutions can adapt behavior toward the indicator. A measure originally selected as evidence of generativity can therefore become part of the process that shapes generativity.

Publication count can alter research practice. Employment-retention metrics can alter hiring. Rehabilitation scores can alter service eligibility. Risk scores can alter institutional treatment.

A restorative metric should therefore be evaluated for both measurement validity and incentive effects.

8.39 Observer Participation

The restorative observer is often also an institutional participant.

A judge issues an authoritative classification. A clinician prescribes treatment. An employer determines access. A regulator changes permissible behavior. A researcher publishing an evaluation can alter public interpretation.

The observer therefore cannot always be modeled as a passive external sensor.

This does not invalidate observation. It requires an explicit distinction between descriptive and performative effects.

A restorative system can record:

- the observation;
- the authority of the observer;
- the action triggered by the observation;
- later changes associated with that action.

Such provenance makes observer participation more visible.

8.40 Reflexivity and Causal Estimation

Reflexivity also complicates causal estimation across changing governance regimes.

A relationship estimated before a public policy is implemented can change after subjects adapt to the policy. Lucas's critique provides a prominent economic example of the broader problem that policy interventions can change behavioral relations used in policy evaluation [29].

The present theory does not generalize one econometric result into a universal social law. It uses the structural lesson: interventions can alter response functions.

Consequently, a restoration model should distinguish:

$$P(Y | X, \text{pre-}R) \quad \text{and} \quad P(Y | X, \text{post-}R). \quad (99)$$

Equation 99 warns against assuming invariance across restorative regimes.

8.41 Reflexive Restoration

Restoration itself can become generative.

A compensation payment can change family relations. A public apology can change institutional trust. A legal judgment can create precedent. A re-entry program can produce new professional networks. A reparations program can produce new collective identities and organizations.

The restorative event therefore creates a history

$$\mathcal{H}^{H,R} \quad (100)$$

that differs from both the harm history $\mathcal{H}^H$ and the no-harm reference $\mathcal{H}^{-H}$.

Equation 100 identifies restoration as a source of new historical value and new risk.

This observation supports a future-oriented interpretation of justice: restoration can generate a good history without reproducing the history that would have occurred in the absence of harm.

8.42 Reflexive Non-Closure of Historical Equivalence

Historical equivalence becomes especially demanding under reflexivity.

Suppose an intervention reconstructs every state variable selected by the model. The fact that the intervention occurred remains part of the actual history. If subjects know that restoration occurred, the knowledge can influence subsequent interpretation. If the knowledge is erased, the erasure becomes an intervention with independent normative significance.

A strict requirement that

$$\mathcal{H}^{H,R} = \mathcal{H}^{-H} \quad (101)$$

therefore demands progressively stronger control over records, memories, relationships, and knowledge of the restorative process.

Equation 101 reveals why historical identity is qualitatively different from state or functional equivalence.

The later atlas treats historical-generative equivalence as an exceptionally strong comparison rather than the default meaning of restoration.

8.43 Normative Underdetermination

Suppose every practical and epistemic obstacle were removed. An ideal observer knows the complete actual history, every relevant causal relation, and the full space of counterfactual futures. A further problem remains:

Which state, relation, trajectory, capability set, value distribution, or generative process should justice seek to realize?

Knowledge alone does not answer this question.

A pre-harm state can contain injustice. A no-harm counterfactual can contain inequality or domination. The subject can prefer a newly developed trajectory. A maximally productive state can violate rights. A structurally similar state can distribute burdens unfairly.

This is *normative underdetermination*: facts and causal models can narrow the available options without uniquely determining the just restorative target.

8.44 The Reference-State Problem

Normative underdetermination first appears in selection of the reference.

Candidates include:

- the actual pre-harm state;
- the expected no-harm state;
- a distribution over no-harm trajectories;
- the best feasible present state;
- a sufficient capability threshold;
- a normatively ideal state;
- a subject-selected target;
- a state that improves upon both the actual and counterfactual histories.

These reference systems can generate different conclusions even under complete information.

The formal atlas therefore cannot define restoration before reference selection has been made explicit.

8.45 The Metric Problem

A second normative problem concerns the meaning of distance between systems.

Suppose a restorative system seeks to minimize

$$d(\mathcal{G}_R, \mathcal{G}_{\text{ref}}).$$

The function d determines which differences matter.

A metric based upon income can regard two lives as close while their relationships differ radically. A metric based upon capability sets can ignore historical provenance. A behavioral metric can ignore internal cost. A network metric can ignore the meaning of relations.

The issue can be represented more explicitly by Equation 102.

$$d_V(\mathcal{G}_1, \mathcal{G}_2), \tag{102}$$

where the subscript V indicates that the selected distance depends upon a theory of value or relevance.

Equation 102 anticipates a central thesis of the later atlas: comparative structure becomes normatively meaningful only after the value of preserved features is specified.

8.46 The Weighting Problem

Multidimensional restorative comparison can require weighting several objects.

Suppose a framework compares income, health, network access, autonomy, and recognition. A scalar aggregation would take a form such as

$$D = \sum_{i=1}^n w_i d_i. \tag{103}$$

Equation 103 introduces weights w_i .

The weights cannot be obtained from the distances alone. They express judgments about importance, priority, substitutability, and trade-offs.

Some theories will reject scalar aggregation entirely and use thresholds, lexical priority, incomparability, or partial orders.

The mathematical framework must therefore remain compatible with plural value structures.

8.47 Incommensurability

Some restorative values can resist representation on one common scale.

Loss of a unique relationship, bodily integrity, freedom from domination, income, historical recognition, and ecological integrity can all matter while lacking an agreed exchange rate.

Incommensurability does not imply that practical choice becomes impossible. Institutions regularly choose among heterogeneous values through rights, thresholds, procedural rules, deliberation, and bounded judgment.

It does imply that a single numerical restoration score should not be assumed to capture the full normative problem.

This observation is especially relevant to the role of money. Monetary compensation can be justified as one component of repair without requiring the claim that every relevant value is monetarily equivalent.

8.48 Subject Preference and Normative Authority

The affected subject possesses important epistemic and normative authority over restorative design. The subject knows aspects of current need, meaning, and future preference that institutions cannot fully observe.

Present preference is also historically formed. Severe constraint can influence what a subject expects or requests.

The theory therefore cannot resolve normative underdetermination by either excluding the subject's preference or treating every present preference as conclusive.

A legitimate process can combine subject participation, expansion of viable options, information, time for reflection, and protection against coercion.

The later procedural-justice analysis develops these requirements in greater detail.

8.49 Third-Party Normative Constraints

A target that maximizes restorative similarity for one claimant can violate the rights or generativity of other persons.

Returning property can displace an innocent current occupant. Reconstructing a professional role can affect another employee. Erasing public records can conflict with archival or speech interests. Recreating a historical social structure can revive earlier hierarchy.

The admissible restorative set is therefore constrained by third-party claims.

Let $\mathcal{R}_A$ denote interventions favorable to claimant A and $\mathcal{C}_{-A}$ denote legitimate constraints arising from others. The admissible set is

$$\mathcal{R}_A^{\text{adm}} = \{R \in \mathcal{R}_A : R \text{ satisfies } \mathcal{C}_{-A}\}. \quad (104)$$

Equation 104 makes clear why restorative fidelity cannot be optimized in isolation.

8.50 Procedural Limits on Technically Available Restoration

A technically possible intervention can remain procedurally impermissible.

Hypothetical memory modification can increase behavioral similarity while violating autonomy. Manipulation of third parties can recreate an earlier network while violating their freedom. Coercive treatment can improve measured function while violating bodily integrity. Comprehensive surveillance can improve causal inference while violating privacy.

The distinction can be represented by

$$\mathcal{R}_{\text{admissible}} \subseteq \mathcal{R}_{\text{technically possible}}. \quad (105)$$

Equation 105 will later support the independent evaluation of outcome equivalence and procedural justice.

8.51 Identity Constraints

Restoration can also encounter limits associated with continuity of the actual subject.

Suppose an intervention can transform the actual post-harm subject A^H into a state extremely similar to the counterfactual subject A^C . Achieving this state can require modifying memories, relationships, commitments, personality traits, or values acquired through the actual history.

The stronger the intervention becomes, the more important the following question becomes:

Which present features can be altered in the name of restoring the person who would otherwise have existed?

The answer requires a theory of identity, agency, consent, and continuity.

This problem survives hypothetical unlimited technical capacity.

8.52 Idealized Unlimited Observation

A useful thought experiment removes ordinary information constraints.

Suppose an observer has access to every event that actually occurred, every social interaction, every bodily change, every institutional decision, every record, and every relevant environmental state.

This idealization eliminates many practical epistemic gaps.

It does not directly provide the no-harm counterfactual history because that history did not occur. A model of alternative development remains necessary.

It also does not determine which dimensions matter normatively.

Unlimited observation therefore solves

missing actual data
while leaving
counterfactual generation
and
normative selection
as separate problems.

8.53 Idealized Perfect Causal Knowledge

A stronger thought experiment grants perfect causal knowledge of the relevant system.

Suppose the observer knows the complete transition mechanism governing every relevant variable during the period of interest.

This assumption removes ordinary causal-identification uncertainty.

Several questions remain.

If the system contains stochasticity, the no-harm future can remain a distribution rather than one state. If the state space can expand through new generation, the future can contain possibilities unavailable in the original description. If agents respond to the restoration process itself, later dynamics can differ from the modeled no-restoration history.

Perfect causal knowledge therefore improves counterfactual specification while leaving questions of realization, reflexivity, and value.

8.54 Idealized Perfect Simulation

A still stronger thought experiment assumes that every relevant counterfactual history can be simulated exactly according to a specified model.

The result is a family of possible futures

$$\mathfrak{H}^{-H}.$$

The simulation can identify the distribution of outcomes under the model.

A normative decision is still required concerning whether restoration should target:

- expected state;
- modal state;
- preservation of the full distribution;
- capability set;
- reachable set;
- worst-case protection;
- some subject-selected future.

Perfect simulation therefore does not collapse the distinction between description and justice.

8.55 Idealized Unlimited Intervention

The strongest practical idealization grants an actor the capacity to alter any physical or social variable required by the restorative plan.

The actor can restore wealth, health, institutions, professional status, relationships, infrastructure, environmental conditions, and other relevant states.

This thought experiment removes technological feasibility as a limitation.

The remaining problems become conceptually clearer.

The intervention still requires a target. Third parties possess independent claims. The actual subject possesses present agency. Restoration becomes part of history. Knowledge of restoration can change later relations. Erasure of that knowledge becomes another intervention.

Unlimited power therefore does not imply a uniquely defined just restoration.

This relation can be stated as

$$\text{unlimited intervention capacity} \not\Rightarrow \text{uniquely defined complete restoration.} \quad (106)$$

Equation 106 is one of the central boundary claims of the paper.

8.56 The Omniscience–Omnipotence Separation

The idealized cases also establish a distinction between knowledge and power.

An omnipotent restorer without sufficient knowledge cannot identify the target. An omniscient observer without power cannot realize the target. An omniscient and omnipotent actor still requires a normative criterion determining which target and which intervention are just.

The three components can be represented by

$$\text{knowledge} + \text{capacity} + \text{normative criterion.} \quad (107)$$

Equation 107 shows why technical progress alone cannot complete the theory of restoration.

8.57 The Perfect-State Thought Experiment

Suppose an idealized intervention can reproduce every state variable of a selected counterfactual subject at time t_1 .

Under state equality,

$$x_R(t_1) = x_C(t_1). \quad (108)$$

Equation 108 still leaves open:

- whether the transition structure is the same;
- whether memories are the same;
- whether surrounding actors possess the same beliefs;
- whether the same futures are reachable;
- whether the restoration process was just;
- whether the subject consented to the transformation.

The thought experiment demonstrates that state equality is one strong restorative achievement whose normative meaning depends upon the object selected for comparison.

8.58 The Perfect-Behavior Thought Experiment

Suppose the restored subject produces exactly the same observable behavior as the counterfactual subject across a specified observation class.

This establishes a form of observational or behavioral equivalence.

Internal cost, subjective meaning, causal structure, or robustness can still differ.

A worker can produce the same output while expending much greater effort. A subject can display the same public behavior while experiencing a different sense of security. Two systems can generate identical baseline output and diverge under future perturbation.

Behavioral equivalence therefore supports a bounded restorative claim.

8.59 The Perfect-Response Thought Experiment

A stronger idealization requires identical response across a class of perturbations.

Let Δ_{adm} denote the selected perturbation class. The criterion can require

$$P_R(O \mid \delta) = P_C(O \mid \delta) \quad \text{for all } \delta \in \Delta_{\text{adm}}. \quad (109)$$

Equation 109 establishes a much stronger dynamic similarity.

The criterion remains indexed to the chosen observables and perturbations. A difference outside Δ_{adm} remains untested. The selection of admissible perturbations also contains normative judgment concerning which future environments matter.

8.60 The Perfect-Structure Thought Experiment

Another idealization seeks structural equivalence.

A post-harm network can be reconstructed so that its graph is isomorphic to the reference network. A dynamical system can be reconstructed with corresponding attractors and basins. A transition system can be made bisimilar to the reference.

Such achievements can be mathematically strong while remaining silent about historical identity or substantive value.

An isomorphic network containing different persons is structurally equivalent under one mapping and historically different as a set of relationships.

The later atlas therefore treats structural equivalence and identity as separate relations.

8.61 The Perfect-History Thought Experiment

The strongest restorative ideal seeks to make the actual history equivalent to the history in which the harm never occurred.

This ideal creates the most severe reflexive difficulty.

If harm occurred, the fact of occurrence belongs to actual history. If the history is altered so that no trace of the harm remains, the intervention must also address memories, records, downstream effects, and knowledge of the intervention itself.

If the restoration event remains recorded, the restored history differs from the no-harm history. If the restoration event is erased, that erasure becomes part of the causal process used to produce the resulting state.

The concept of historical identity therefore approaches a requirement to replace the actual world history with another history.

Such an operation is substantially different from restitution, compensation, rehabilitation, or assistance.

This thought experiment clarifies why historical-generative equivalence should be treated as an extreme comparison standard rather than the implicit ideal of ordinary justice.

8.62 The Better-Than-Counterfactual Possibility

A restorative intervention can produce a future that is normatively preferable to the estimated no-harm counterfactual.

A person can receive resources and discover a field more valuable to the person than the career that was interrupted. Institutional reform can produce a safer workplace than the one that existed before the harm. Community reparations can create forms of democratic governance absent from the historical baseline.

Let V denote the relevant normative evaluation. It is possible that

$$V(\mathcal{G}_R) > V(\mathcal{G}_{\text{ref}}). \quad (110)$$

Equation 110 does not imply that the earlier wrong has disappeared. It demonstrates that normative improvement and historical equivalence are independent dimensions.

This possibility will later motivate comparison relations based on dominance and improvement in addition to equivalence.

8.63 The Worse-but-Closer Possibility

The reverse configuration is also possible.

An intervention can make the restored system more similar to the counterfactual reference while reducing present welfare, autonomy, or justice.

A subject can be pressured back into an earlier profession despite preferring a new one. A community can be reconstructed according to an idealized historical form that restricts present members. A person can undergo intrusive treatment to increase behavioral resemblance to a counterfactual self.

The resulting relation can be represented as

$$d(\mathcal{G}_{R_2}, \mathcal{G}_{\text{ref}}) < d(\mathcal{G}_{R_1}, \mathcal{G}_{\text{ref}}) \quad (111)$$

while

$$V(\mathcal{G}_{R_2}) < V(\mathcal{G}_{R_1}). \quad (112)$$

Equations 111 and 112 demonstrate that restorative fidelity and normative desirability can move in opposite directions.

8.64 Epistemic Precision and False Exactness

Advanced technical systems can create an appearance of precision that exceeds the underlying epistemic basis.

A model can produce a counterfactual salary estimate to several decimal places while relying upon uncertain assumptions concerning career progression. A simulation can generate a detailed professional network while several structurally different networks remain equally compatible with available evidence.

Numerical precision should therefore be distinguished from epistemic precision.

A responsible restorative model should represent:

- model assumptions;
- uncertainty intervals where appropriate;
- alternative plausible models;
- sensitivity to parameter choice;
- unmodeled dimensions;
- normative choices embedded in the output.

The purpose is not to discourage formalization. It is to preserve the distinction between computational detail and justified knowledge.

8.65 Model Uncertainty

Restorative uncertainty can arise from uncertainty about parameters within a model and uncertainty about the model structure itself.

Let θ denote parameters and M denote model structure. Parameter uncertainty concerns

$$P(\theta \mid M, D),$$

while model uncertainty concerns uncertainty over M .

A technically advanced restoration system can reduce parameter uncertainty while remaining highly sensitive to model selection.

This distinction is particularly important where the object is a historical subject whose relevant relations cannot be captured by one accepted formalism.

The later atlas therefore supports several formal representations rather than a single master model.

8.66 Normative Model Risk

Model selection can also embed normative assumptions.

A model using income as its target treats income differences as centrally relevant. A capability model selects substantive possibilities. A network model selects relational structure. A dynamical model selects transitions and stability. Each can be appropriate for a specific purpose.

Normative model risk arises when the selected formalism systematically excludes a value that matters to the restorative question and the exclusion remains unstated.

Explicit comparison objects reduce this risk.

8.67 Formal Completeness and Social Incompleteness

A model can be formally complete within its own specification.

Every variable can possess a value. Every transition can be defined. Every comparison can be computable.

Such formal completeness does not establish social completeness.

The model's ontology, boundary, and observables were selected before the calculation began.

This distinction should remain explicit throughout the later formal sections. Mathematical rigor concerns relations within the model. Normative adequacy concerns whether the model preserves structures relevant to the justice problem.

8.68 Restoration Under Unknown Unknowns

Generative systems can produce consequences that no actor anticipated and no current model represents.

The existence of such unknowns cannot serve as a universal reason for inaction. Every governance decision would become impossible under that standard.

The appropriate implication is institutional revisability.

A restorative system should preserve mechanisms through which new evidence, previously unrecognized harms, or unintended intervention effects can enter later evaluation.

Revisability is therefore an institutional response to epistemic humility.

8.69 The Boundary Between Epistemic and Normative Uncertainty

Disagreement about restoration can arise from lack of knowledge or from different values.

The distinction matters.

Two analysts can agree that a claimant lost a professional network and disagree about how much that network should matter relative to income. This is primarily a normative disagreement.

Two analysts can agree about the value of the network and disagree about whether the harm caused its dissolution. This is primarily an epistemic disagreement.

Many real disputes combine both.

A transparent framework should state which type of uncertainty is responsible for disagreement.

8.70 Bounds Rather Than Point Restoration

Where exact counterfactual reconstruction is unjustified, restorative analysis can sometimes identify bounds.

Instead of claiming that a claimant would have earned precisely Y , an analysis can identify a plausible range. Instead of identifying one lost network, it can identify minimum structural losses shared across several plausible reconstructions.

Let a restorative quantity Q^{cf} satisfy

$$Q_{\min} \leq Q^{\text{cf}} \leq Q_{\max}. \quad (113)$$

Equation 113 can support decisions that remain robust across the identified range.

Pearl's treatment of bounding causal and counterfactual effects under imperfect conditions illustrates the broader methodological value of bounds where point identification is unavailable [43].

8.71 Robust Restorative Decisions

An intervention can be justified because it performs acceptably across several plausible models or references rather than because one model is known to be correct.

Let $\mathcal{M}$ denote a set of plausible models. A robust intervention can satisfy a minimum criterion across the set:

$$J(R; M) \geq J_{\min} \quad \text{for all } M \in \mathcal{M}. \quad (114)$$

Equation 114 represents one form of robust decision.

The criterion can be especially useful where several counterfactual histories remain plausible while all support some common intervention, such as correction of a false record, return of property, provision of healthcare, or restoration of formal access.

8.72 Minimal Common Restoration

A related strategy identifies restorative claims shared across several theories or models.

Suppose different plausible accounts disagree about the full lost trajectory while agreeing that condition c was wrongfully removed and that present access should be restored.

The common claim can be pursued without resolving every higher-order counterfactual.

This produces a concept of *minimal common restoration*: the strongest restorative intervention justified across the relevant set of plausible epistemic and normative specifications.

The formal properties of such common refinements will remain an open question for the later atlas.

8.73 Partial Restoration as a Legitimate Category

The existence of deep limits supports an explicit category of partial restoration.

Partial restoration should not be treated as failed complete restoration by definition. It can be the appropriate outcome when:

- some objects are fully restored;
- others are only approximately restored;
- others are compensated;
- others receive future-directed assistance;
- some remain irrecoverable.

The institutional statement should identify which category applies to each object.

This vocabulary reduces pressure to overstate the completeness of a remedy.

8.74 Non-Restorable Loss and Continuing Justice

A loss can be non-restorable while continuing to generate normative obligations.

The death of a person, destruction of a historical archive, loss of a unique ecosystem, or disappearance of a singular opportunity can make literal restoration unavailable.

Justice can still involve:

- acknowledgment;
- compensation;
- support for affected persons;
- memorialization;
- institutional reform;
- preservation of remaining conditions;
- prevention of recurrence;
- creation of new generative possibilities.

Non-restorability therefore limits one family of remedies rather than justice as a whole.

8.75 Irreparability and Preventive Weight

The more difficult a class of loss is to repair, the more significant preventive design becomes, subject to proportionality and rights.

The relationship introduced in Equation 38 can now be interpreted more precisely.

Low reparability can arise from:

- physical destruction;
- temporal singularity;
- counterfactual underidentification;
- relational singularity;
- structural non-invertibility;
- reflexive effects;
- normative constraints on reconstruction.

These sources differ. Their practical implication can converge: exclusive reliance upon later compensation becomes less satisfactory.

8.76 Boundary Claims and Institutional Honesty

Institutions should make claims at the level supported by their knowledge and authority.

A court can establish legal liability. A compensation program can state the basis of its valuation. A rehabilitation provider can state the functional objectives achieved. An organization can state which records and access conditions have been corrected.

The institution should avoid global language where only a local comparison has been established.

A useful form is:

$$\text{Restored } [Q, E, \tau, \mathcal{T}, \varepsilon, K] . \quad (115)$$

Equation 115 anticipates the formal indexed restoration relation introduced in the later atlas. It makes the scope of the claim explicit.

8.77 Practical Improvement Without Epistemic Completion

A society can improve restorative practice substantially without solving every epistemic problem.

Better provenance can reduce uncertainty. Faster intervention can preserve options. Better social protection can prevent recursive collapse. Improved compensation can expand subject-directed possibility. More effective re-entry can restore usable access. Stronger rights can prevent arbitrary deprivation.

These improvements should be pursued even where complete counterfactual reconstruction remains unavailable.

The distinction prevents philosophical limits from becoming a rationale for practical pessimism.

8.78 Epistemic Improvement Without Normative Completion

The reverse distinction is equally important.

Future technology can produce extraordinary knowledge of social trajectories, causal relations, health states, networks, and behavioral response.

Such knowledge can improve restoration while leaving unresolved questions concerning:

- which values deserve priority;

- which historical reference should be selected;
- which third-party burdens are permissible;
- which interventions respect identity and agency;
- how much inequality is acceptable;
- whether a subject can reject counterfactual reconstruction.

Technical progress therefore increases normative responsibility. Greater capacity creates more possible interventions whose justice must be evaluated.

8.79 Structural Limits Without Fatalism

Structural non-invertibility and reflexive non-closure can sound pessimistic. Their practical implication is more limited.

Justice does not require a universal inverse operator for history.

It can require accurate acknowledgment of what can be returned, careful compensation for what can be valued, rehabilitation of current capacities, removal of continuing constraints, support for subject-directed futures, and stronger prevention of losses whose later reconstruction is structurally weak.

The inability to recreate one history therefore shifts the object of justice rather than eliminating it.

8.80 Restoration as an Indexed Family of Claims

The analysis of limits supports a conceptual transition.

The statement

The system has been restored.

is too coarse for the classes of harm considered in this paper.

A more adequate formulation is:

The system has been restored with respect to object Q , reference $\mathcal{G}_{\text{ref}}$, comparative relation E , test class $\mathcal{T}$, horizon τ , tolerance ε , and epistemic basis K .

The resulting judgment can be strong in one dimension and weak in another.

This indexed structure is the conceptual foundation of the atlas developed in the following sections.

8.81 Limits as Dimensions of the Atlas

The limits identified in this section will later appear as explicit dimensions or qualifications of restorative comparison.

Practical limits affect the feasible intervention set.

Epistemic limits affect confidence in the reference and comparison.

Representational limits determine which objects are visible.

Structural limits affect invertibility, reachability, and dynamic comparison.

Reflexive limits affect the stability of descriptions and interventions.

Normative limits determine which technically achievable equivalences are permissible or valuable.

The atlas should therefore be capable of distinguishing a criterion that is:

- conceptually coherent and currently measurable;
- conceptually coherent and currently unmeasurable;

- mathematically definable and epistemically unidentified;
- technically achievable and procedurally impermissible;
- normatively desirable and presently infeasible;
- formally strong and normatively irrelevant;
- partially identifiable and robustly actionable.

These distinctions are essential to prevent the formal theory from collapsing feasibility, knowledge, mathematics, and justice into one scale.

8.82 Transition to Formal Representation

The analysis has now reached the point at which a more explicit formal representation becomes necessary.

The preceding sections established the following structure.

Generative conditions participate in recursive processes through which values, relations, capacities, and possibilities are produced.

Subjects are historically and relationally constituted, so the recipient of a returned condition can differ from the subject who lost it.

Justice operates across prevention, interruption, and post-harm response.

Different institutions perform different functions and compress different dimensions of the generative system.

Practical architectures can improve protection, correction, compensation, re-entry, and assistance while remaining limited in scope.

The present section has shown that further limits arise from counterfactual multiplicity, model dependence, incomplete representation, many-to-one dynamics, non-invertibility, reflexivity, open-ended generation, normative underdetermination, and procedural constraints.

The resulting question is therefore more precise:

Which components of a generative system should be represented for a particular restorative inquiry, and which formal structure is sufficient to state the comparison without claiming an exhaustive ontology of the social world?

Section 9 develops such a representation. It introduces an explicitly modular formal description of generative systems that can support several different objects of comparison while remaining compatible with the representational and normative limits identified in this section. The formal system is treated as an analytical interface rather than a complete symbolic duplicate of social reality.

9 Formal Representation of Generative Systems

This section introduces a modular formal representation for the generative systems examined throughout the remainder of the paper. Its role is to provide a common analytical interface through which resources, relations, states, transitions, histories, observables, interventions, value flows, and future possibilities can be distinguished before particular restorative comparisons are defined. The representation is deliberately plural and extensible. Different restorative questions require different subsets of the formal structure, and some applications will require additional components that are absent from the baseline representation.

The formalism serves four objectives. First, it separates objects that ordinary restorative language frequently compresses into a single claim. Second, it provides explicit locations at which harm, return, assistance, and governance can act. Third, it permits later sections to distinguish several kinds of comparison, including state equality, structural correspondence, simulation, bisimulation, reachability, dynamical equivalence, causal equivalence, and generative-process comparison. Fourth, it preserves the

distinction between a formal representation and the social system represented. The latter distinction follows the representational and reflexive limits developed in Section 8 and is consistent with the broader methodological role of decomposition in the analysis of complex systems [59].

9.1 Formalization as an Analytical Interface

A formal representation is useful when it makes distinctions precise enough to support comparison, inference, and criticism. Its adequacy is therefore indexed to an analytical purpose.

Let $\mathcal{W}$ denote the relevant portion of the social, institutional, bodily, ecological, or technical world under consideration. A formalization is represented by a mapping

$$\Pi_Q : \mathcal{W} \longrightarrow \mathfrak{M}_Q, \quad (116)$$

where Q denotes the restorative inquiry and $\mathfrak{M}_Q$ denotes a model space constructed for that inquiry.

Equation 116 makes the purpose-relative character of formalization explicit. A model designed to compare employment income can preserve different features from a model designed to compare professional networks, bodily function, attractor structure, or control over productive conditions.

The formalism introduced below therefore supplies a vocabulary from which application-specific models can be constructed. It does not require every component to be instantiated in every case.

9.2 Baseline Generative-System Tuple

A generative system over a temporal domain $\mathbb{T}$ is represented by the tuple in Equation 117.

$$\mathcal{G} = (\mathcal{X}, \mathcal{A}, \mathcal{C}, \mathcal{R}, \mathcal{E}, \mathcal{U}, \Xi, \Phi, \mathcal{O}, \mathcal{V}, \Lambda, \mathcal{H}). \quad (117)$$

The components of Equation 117 are:

- $\mathcal{X}$: the state space;
- $\mathcal{A}$: the set of relevant agents or subsystems;
- $\mathcal{C}$: the space of generative conditions;
- $\mathcal{R}$: the relational structure;
- $\mathcal{E}$: the event space;
- $\mathcal{U}$: the intervention or action space;
- Ξ : the environment or disturbance space;
- Φ : the transition or generative evolution operator;
- $\mathcal{O}$: the family of observation maps;
- $\mathcal{V}$: the space of generated values or value-bearing outputs;
- Λ : the circulation, allocation, retention, or extraction structure governing generated value;
- $\mathcal{H}$: the historical structure.

The tuple is intentionally richer than an ordinary state-space model because the justice of return can concern several distinct structures. A resource can be restored while a relation remains altered. An observable output can be restored while the transition structure differs. Generated value can return while control over its circulation remains external. A present state can match a reference state while the histories leading to the states differ.

The subsequent subsections define these components in greater detail.

9.3 State Space

The state space $\mathcal{X}$ contains the variables selected to represent the current configuration of the system. A state at time t is denoted by

$$x_t \in \mathcal{X}. \quad (118)$$

Equation 118 can represent a single subject, an organization, a community, an ecological system, or a coupled multi-agent configuration depending upon the system boundary.

For an individual-centered application, a state can contain components such as

$$x_t = \left(x_t^{\text{body}}, x_t^{\text{capacity}}, x_t^{\text{resource}}, x_t^{\text{status}}, x_t^{\text{belief}}, x_t^{\text{position}} \right). \quad (119)$$

Equation 119 is illustrative. The representation should include only dimensions relevant to the selected restorative inquiry.

The distinction between state and other components is important. Relations, history, transition rules, and value circulation can influence future behavior without being reducible to current state variables in a particular model.

9.4 Agent and Subsystem Structure

The set $\mathcal{A}$ identifies the agents or subsystems whose interactions matter to the analysis:

$$\mathcal{A} = \{A_1, A_2, \dots, A_n\}. \quad (120)$$

Equation 120 can include individuals, organizations, public institutions, professional communities, technical systems, or other relevant units.

An agent need not be represented as ontologically primitive. The formalism requires only that the selected analysis can identify a unit whose state, relations, actions, or generative conditions are being tracked.

For collective systems, an agent can itself possess internal structure. The formalism therefore supports nested systems:

$$A_i = \mathcal{G}_i \subset \mathcal{G}. \quad (121)$$

Equation 121 permits analysis across scales without requiring collective generativity to be reduced to a sum of individual states.

9.5 Generative-Condition Space

The space $\mathcal{C}$ contains conditions that can affect subsequent generation.

At time t , the available condition configuration is

$$c_t \in \mathcal{C}. \quad (122)$$

Equation 122 can include material, financial, legal, institutional, relational, epistemic, bodily, symbolic, or ecological conditions.

A condition can be represented as a vector when its components are distinguishable:

$$c_t = \left(c_t^{\text{material}}, c_t^{\text{financial}}, c_t^{\text{institutional}}, c_t^{\text{relational}}, c_t^{\text{epistemic}}, c_t^{\text{bodily}}, c_t^{\text{symbolic}}, c_t^{\text{ecological}} \right). \quad (123)$$

Equation 123 should be interpreted as one possible decomposition. Some conditions occupy several categories simultaneously.

The formal separation between x_t and c_t permits the same condition to interact differently with different states. This distinction supports the concept of altered generative significance developed earlier in the paper.

9.6 Relational Structure

The relational structure $\mathcal{R}$ specifies relevant relations among agents, conditions, institutions, and other modeled objects.

For a graph-based representation, the relational system at time t can be written as

$$\mathcal{R}_t = (N_t, L_t, \omega_t, \ell_t), \quad (124)$$

where N_t is a node set, L_t is an edge set, ω_t assigns weights or strengths where relevant, and ℓ_t assigns relation types or labels.

Equation 124 can represent mentorship, authority, dependency, trust, information flow, collaboration, ownership, contractual relation, family relation, professional recognition, or another relation class.

A typed relation can be represented as

$$r_{ij}^{(k)}(t) : A_i \longrightarrow A_j, \quad (125)$$

where k denotes the relation type.

Equation 125 permits later comparison of relational structure independently of the identities of the participating nodes.

The relational formalism can also include hyperedges or higher-order relations where pairwise representation is insufficient.

9.7 Control Relations

Control over generative conditions requires separate representation because availability and control are distinct.

Let

$$\chi_t : \mathcal{A} \times \mathcal{C} \longrightarrow \mathcal{P}, \quad (126)$$

where $\mathcal{P}$ denotes a space of control rights, permissions, powers, or decision authorities.

Equation 126 can represent rights to access, modify, allocate, revoke, transfer, or govern a condition. The distinction between access and control can be expressed through separate maps:

$$\alpha_t(A_i, c) \neq \chi_t(A_i, c), \quad (127)$$

where α_t denotes effective access.

Equation 127 captures cases in which a subject currently uses a resource while another actor retains discretionary authority over continued use.

This distinction is especially relevant to non-domination and generative justice [44, 15].

9.8 Event Space

The event space $\mathcal{E}$ contains temporally localized changes treated as analytically relevant.

An event is represented by

$$e_k = (t_k, \eta_k, s_k, o_k, m_k), \quad (128)$$

where t_k denotes event time, η_k denotes event type, s_k denotes source or initiating actor, o_k denotes affected objects, and m_k denotes associated metadata.

Equation 128 can represent dismissal, accusation, correction, transfer, publication, diagnosis, institutional decision, legal judgment, environmental disturbance, compensation, or restorative intervention.

Events are distinguished from states. A state records a configuration at a time. An event records a transition or occurrence relevant to the production of later configurations.

This distinction becomes important for historical and provenance-based comparison.

9.9 Action and Intervention Space

The intervention space $\mathcal{U}$ contains actions through which agents or institutions deliberately alter the system.

An intervention at time t is denoted by

$$u_t \in \mathcal{U}. \quad (129)$$

Equation 129 can include harmful interventions, ordinary decisions, preventive measures, compensation, re-entry support, rehabilitation, regulatory action, or other policy changes.

The distinction between observation and intervention follows the causal framework developed by Pearl [43]. In the present model, an observation records some function of the system, while an intervention changes one or more components or mechanisms of the system.

The intervention can therefore act upon state, conditions, relations, control, transition rules, or value circulation.

9.10 Environmental and Disturbance Space

The environment Ξ contains influences not modeled as direct decisions of the focal actors.

A disturbance at time t is represented by

$$\xi_t \in \Xi. \quad (130)$$

Equation 130 can include economic shocks, changes in law, illness, environmental change, random events, changes in external demand, or actions by agents outside the modeled boundary.

The distinction between $\mathcal{U}$ and Ξ is model-relative. An event treated as exogenous in one analysis can become endogenous in a larger system.

This relativity should be documented because restorative conclusions can depend upon system boundary.

9.11 Transition Operator

The transition operator Φ specifies how the system evolves.

A discrete-time representation is given in Equation 131.

$$x_{t+1} = \Phi_t(x_t, c_t, \mathcal{R}_t, u_t, \xi_t, h_t), \quad (131)$$

where h_t denotes the historically relevant information carried into the transition.

Equation 131 is deliberately general. Φ_t can be deterministic, stochastic, set-valued, agent-based, rule-based, learned, or partially specified.

A stochastic version is represented by

$$X_{t+1} \sim P_{\Phi_t}(\cdot \mid x_t, c_t, \mathcal{R}_t, u_t, \xi_t, h_t). \quad (132)$$

Equation 132 permits uncertainty in later states without requiring uncertainty to be represented only through measurement error.

The distinction between state change and transition-rule change is central to the theory of generative harm.

9.12 Transition-Rule Change

A harmful or restorative intervention can change the transition operator itself.

Let θ_t denote parameters or structural features of the transition mechanism. The transition system can be represented by

$$x_{t+1} = \Phi(x_t, c_t, \mathcal{R}_t, u_t, \xi_t; \theta_t). \quad (133)$$

Equation 133 allows the system to change through both state evolution and parameter evolution. A structural intervention can then produce

$$\theta_t \xrightarrow{u_t} \theta_{t+1}. \quad (134)$$

Equation 134 captures cases in which harm changes the effective rules governing later generation.

Examples include institutional retaliation altering reporting behavior, exclusion altering professional conversion mechanisms, or regulatory reform altering the incentive structure of an organization.

9.13 Continuous-Time Representation

Some applications are better represented in continuous time.

A generic continuous-time form is given in Equation 135.

$$\dot{x}(t) = F(x(t), c(t), \mathcal{R}(t), u(t), \xi(t), h(t); \theta(t)). \quad (135)$$

Equation 135 provides a basis for later analysis of trajectories, stability, attractors, basins, perturbations, and control.

The paper does not assume that every social application admits a meaningful smooth differential representation. Continuous-time models are one member of the formal family.

9.14 Transition-System Representation

Other comparisons, especially simulation and bisimulation, are more naturally stated through labeled transition systems.

A generative transition system can be represented as

$$\mathfrak{T} = (S, L, \rightarrow), \quad (136)$$

where S is a set of states, L is a set of transition labels, and $\rightarrow \subseteq S \times L \times S$ is the transition relation.

Equation 136 permits the same generative system to be represented without assuming smooth dynamics or numerical state spaces.

A transition

$$s \xrightarrow{\ell} s' \quad (137)$$

can represent an action, event, institutional decision, interaction, or other generative step.

Equation 137 will later support formal definitions of simulation and bisimulation.

9.15 History Structure

The historical structure $\mathcal{H}$ records the sequence or partially ordered structure through which the current system emerged.

A simple sequential history is represented by

$$h_{0:t} = (x_0, e_1, x_1, e_2, \dots, e_t, x_t). \quad (138)$$

Equation 138 is appropriate where the relevant event ordering is approximately linear.

Many social systems require a partial order because several events occur concurrently or independently. A history can therefore also be represented as

$$\mathcal{H} = (\mathcal{E}, \prec_H), \quad (139)$$

where $\prec_H$ denotes a temporal, causal, or provenance ordering relation among events.

Equation 139 permits later comparison of historical structure independently of current state.

The distinction between state and history is essential for cases in which two systems reach similar states through different processes.

9.16 Historical Dependence

A process is history-sensitive when the current state is insufficient to determine the transition structure relevant to future evolution.

A history-sensitive transition can be represented by

$$x_{t+1} = \Phi(x_t, h_{0:t}, u_t, \xi_t). \quad (140)$$

Equation 140 differs from a Markovian representation in which the current state contains all information required for future evolution.

This distinction is important because some restorative analyses can compress history into a sufficiently rich state representation, while others treat history as an irreducible comparison object.

The choice should be explicit rather than assumed.

9.17 Observation Maps

The observation family $\mathcal{O}$ specifies which features of the system are visible to an observer or institution.

For observer j ,

$$y_t^{(j)} = O_j(x_t, c_t, \mathcal{R}_t, h_t). \quad (141)$$

Equation 141 permits several observers to access different projections of the same system.

A legal institution can observe one set of variables. A clinician can observe another. The subject can report another. A regulator can observe repeated-case statistics.

The family structure

$$\mathcal{O} = \{O_1, O_2, \dots, O_m\} \quad (142)$$

makes observational pluralism explicit.

Equation 142 will later permit definitions of observer-relative and test-relative equivalence.

9.18 Interventional Observation

Some system properties become visible only after intervention.

A passive observation asks about

$$P(Y \mid X = x),$$

while an interventional analysis asks about

$$P(Y \mid do(X = x)).$$

Within the generative-system formalism, an interventional response map can be represented by

$$\mathcal{I}_u(\mathcal{G}) = P(O_{t+\tau} \mid do(u_t = u)). \quad (143)$$

Equation 143 defines an object that can later be compared across restored and reference systems.

Two systems can be observationally similar while their interventional response maps differ.

9.19 Value Space

The space $\mathcal{V}$ represents generated outputs or states treated as value-bearing within the relevant analysis.
At time t ,

$$v_t \in \mathcal{V}. \quad (144)$$

Equation 144 does not imply that value is reducible to one scalar.
A multidimensional value representation can take the form

$$v_t = (v_t^{(1)}, v_t^{(2)}, \dots, v_t^{(k)}). \quad (145)$$

Equation 145 can represent economic value, capability, recognition, autonomy, relational value, ecological value, or another normatively relevant dimension.

The paper distinguishes generated value from normative evaluation. A system can generate large quantities of value in one dimension while remaining unjust in another.

9.20 Value-Generation Map

A generative process produces value from states, conditions, relations, and actions.

The value-generation map is represented by

$$\Gamma_t : (x_t, c_t, \mathcal{R}_t, u_t, \xi_t) \longrightarrow v_{t+1}. \quad (146)$$

Equation 146 identifies value generation as a distinct object from the complete state transition.

This separation is useful because a restorative intervention can preserve total value output while altering state, relation, control, or distribution.

Generative justice places particular emphasis on how value is produced and who controls the relevant conditions [15, 17].

9.21 Value Circulation

Generated value can be retained, transferred, extracted, reinvested, destroyed, or transformed.

Let

$$\Lambda_t : \mathcal{V} \times \mathcal{A} \times \mathcal{R}_t \longrightarrow \mathcal{D}_t \quad (147)$$

represent the circulation or allocation of generated value, where $\mathcal{D}_t$ denotes the resulting distribution or destination structure.

Equation 147 allows analysis of extraction and return separately from production.

A recursive generative cycle can then be represented by

$$c_t \xrightarrow{\Gamma_t} v_{t+1} \xrightarrow{\Lambda_t} c_{t+1}. \quad (148)$$

Equation 148 expresses the possibility that generated value becomes part of subsequent generative conditions.

This recursive structure is central to later comparison of generative processes.

9.22 Alienation and Control of Generated Value

A system can generate value while the generators lack control over its circulation.

Let $g(v)$ denote the set of actors whose activity generated value v , and let $q(v)$ denote the actors controlling its later allocation. A divergence

$$g(v) \neq q(v) \quad (149)$$

can be normatively significant depending upon the theory of justice.

Equation 149 provides one formal location for the alienation and extraction concerns emphasized in generative justice.

The equation itself does not establish injustice. Legitimate taxation, cooperative production, public provision, contractual transfer, and fiduciary arrangements can all separate generation from control. Normative interpretation requires additional criteria.

9.23 Conversion Map

The conversion structure developed in earlier sections can be represented through a map

$$\mathcal{K}_t : (c_t, x_t, \mathcal{R}_t, h_t) \longrightarrow \mathcal{Y}_t, \quad (150)$$

where $\mathcal{Y}_t$ denotes the set of capabilities, usable opportunities, or other relevant converted states.

Equation 150 permits comparison of the same resource under different histories.

A returned condition c can satisfy

$$c_R = c_{\text{ref}} \quad (151)$$

while

$$\mathcal{K}_R(c_R) \neq \mathcal{K}_{\text{ref}}(c_{\text{ref}}). \quad (152)$$

Equations 151 and 152 provide a compact formal expression of the paper's distinction between condition return and restoration of generative significance.

9.24 Meaning Map

Some restorative questions require representation of social or subjective meaning.

Let

$$\mathcal{M}_t : (x_t, c_t, \mathcal{R}_t, h_t) \longrightarrow \mathcal{S}_t \quad (153)$$

denote a meaning map into a space $\mathcal{S}_t$ of relevant symbolic, interpretive, or social meanings.

Equation 153 can represent the difference between formal availability of a condition and its meaning within a historical relation.

The formalization does not imply that meaning is completely measurable. $\mathcal{M}_t$ can remain qualitative, partially observed, or represented by a set of interpretations.

Its inclusion prevents the formal model from assuming that material identity captures every socially relevant feature.

9.25 Reputation State

Reputation can be represented as a distributed relational state.

For subject A_i , let

$$\rho_i(t) = (b_{1i}(t), b_{2i}(t), \dots, b_{ni}(t)), \quad (154)$$

where $b_{ji}(t)$ denotes a relevant belief, classification, or trust state held by actor A_j concerning A_i .

Equation 154 can be combined with the relation graph to determine which beliefs possess practical consequences.

A correction applied to one component does not imply equality of the full reputation vector.

This representation will later support network and distributed-state comparison.

9.26 Institutional Classification State

Institutional classifications can be represented separately from informal reputation.

Let

$$z_t^{(j)} = Z_j \left(x_t, D_t^{(j)} \right) \quad (155)$$

denote a classification produced by institution j from the subject state and evidence $D_t^{(j)}$.

Equation 155 can represent eligibility, disciplinary, risk, professional, health, or administrative classifications.

The classification becomes socially operative when it affects subsequent transitions:

$$x_{t+1} = \Phi \left(x_t, z_t^{(j)}, \dots \right). \quad (156)$$

Equation 156 formalizes the reflexive structure discussed in Section 8.

9.27 Record and Provenance Structure

The historical record of claims and revisions can be represented as a provenance graph

$$\mathcal{P} = (V_P, E_P, \tau_P, \sigma_P), \quad (157)$$

where V_P contains claims, evidence, decisions, corrections, and other recorded objects; E_P contains provenance relations; τ_P records temporal metadata; and σ_P records status such as active, contested, corrected, superseded, or withdrawn.

Equation 157 permits an earlier record to remain part of history while its current normative or operational status changes.

This structure is especially useful where overwriting would destroy evidence of institutional history and unqualified retention would preserve an invalid classification as though it remained current.

9.28 Reachable Set

For a system state x_t , intervention class $\mathcal{U}_{\text{adm}}$, disturbance class Ξ_{adm} , and horizon τ , the reachable set is represented by

$$\mathcal{R}_\tau(x_t) = \{x_{t+\tau} : x_{t+\tau} \text{ is reachable under } \mathcal{U}_{\text{adm}}, \Xi_{\text{adm}}\}. \quad (158)$$

Equation 158 is one of the most important formal objects for the later atlas because it represents future possibility without requiring one counterfactual trajectory to be selected.

A harmful intervention can contract, expand, or transform the reachable set.

The set can also remain similar in size while changing composition.

Accordingly, later comparison will distinguish cardinality, inclusion, topology, geometry, and other relations among reachable sets.

9.29 Recognized Reachable Set

The objectively reachable set can differ from the set of possibilities the subject can identify and treat as actionable.

Let

$$\hat{\mathcal{R}}_\tau^A(x_t) \subseteq \mathcal{R}_\tau(x_t) \quad (159)$$

denote the recognized reachable set for subject A .

Equation 159 supports analysis of information, imagination, expectation, social support, and practical confidence as generative conditions.

An intervention can restore objective access while leaving recognized possibility constrained.

9.30 Viable Set

Some reachable states are incompatible with relevant safety, rights, sustainability, or minimum capability constraints.

Let $\mathcal{K}_{\text{adm}} \subseteq \mathcal{X}$ denote the set of states satisfying selected admissibility conditions. The viability set is represented by

$$\mathcal{V} = \{x_0 \in \mathcal{K}_{\text{adm}} : \exists u(\cdot) \text{ such that } x_t \in \mathcal{K}_{\text{adm}} \text{ for the relevant horizon}\}. \quad (160)$$

Equation 160 distinguishes possibility from viable possibility.

This distinction is relevant to generative assistance. A nominal option can be reachable while requiring intolerable risk, domination, or resource depletion.

9.31 Trajectory Space

The trajectory space contains possible histories of state evolution under the selected dynamics:

$$\mathfrak{X} = \{\gamma : \mathbb{T} \longrightarrow \mathcal{X}\}. \quad (161)$$

Equation 161 provides the object required for later trajectory comparison.

An actual trajectory is denoted by γ^H , a reference trajectory by γ^{ref} , and a post-restoration trajectory by γ^R .

The relevant comparison can concern pointwise distance, qualitative shape, orbit structure, distribution over trajectories, or another relation.

9.32 Trace Space

Where internal states are unavailable or irrelevant, the system can be represented through observable traces.

For observation map O , a trace is

$$\text{tr}_O(\gamma) = (O(x_0), O(x_1), \dots). \quad (162)$$

Equation 162 permits comparison of externally visible behavior without requiring identity of internal state.

Trace comparison will later be distinguished from bisimulation because systems can generate the same traces while differing in branching structure.

9.33 Attractor Structure

For dynamical representations in which attractors are meaningful, let

$$\mathfrak{A}(\Phi) = \{\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_m\} \quad (163)$$

denote the family of attractors.

Equation 163 can represent stable equilibria, cycles, metastable regions, or other long-run structures depending upon the model.

Eglash and Garvey's use of attractors and basins in generative justice provides a direct intellectual precedent for employing such structures in normative analysis [16].

The present framework extends the analysis by treating attractor structure as one comparison object among many.

9.34 Basin Structure

For attractor $\mathcal{A}_i$, its basin is represented by

$$\mathcal{B}(\mathcal{A}_i) = \{x \in \mathcal{X} : \Phi^t(x) \longrightarrow \mathcal{A}_i \text{ under the selected dynamics}\}. \quad (164)$$

Equation 164 supports comparison of stability landscapes rather than current states alone.

Two restored systems can possess corresponding attractors while their basin sizes, boundaries, or accessibility differ.

This difference can represent altered resilience.

9.35 Perturbation Class

A robustness comparison requires a class of admissible perturbations.

Let

$$\Delta_{\text{adm}} = \{\delta : \delta \text{ belongs to the selected perturbation family}\}. \quad (165)$$

Equation 165 can include economic shocks, organizational conflict, resource fluctuation, environmental disturbance, policy change, or another domain-specific class.

The choice of Δ_{adm} is analytically and normatively significant. Robustness to one disturbance family does not establish robustness to all possible disturbances.

9.36 Response Operator

For perturbation δ , the response operator is represented by

$$\mathcal{Q}_\tau(\mathcal{G}, \delta) = \mathcal{O}(\Phi_\delta^\tau(\mathcal{G})). \quad (166)$$

Equation 166 maps a perturbation into an observable response over horizon τ .

A stochastic version can produce a response distribution

$$P_{\mathcal{G}}(O_{t+\tau} \mid \delta). \quad (167)$$

Equation 167 provides the object required for the later definition of statistical response equivalence.

9.37 Resilience Functional

Where a domain requires an explicit resilience measure, let

$$\varrho : (\mathcal{G}, \Delta_{\text{adm}}) \longrightarrow \mathcal{Z} \quad (168)$$

map the system and perturbation class into a resilience object $\mathcal{Z}$.

Equation 168 remains intentionally abstract because resilience can refer to recovery time, basin retention, persistence of function, maximum tolerable perturbation, or another domain-specific property.

Holling's distinction between stability and resilience motivates preserving these dimensions separately [24].

9.38 Generative Mechanism

The central object of the paper is the process through which present conditions produce further conditions, relations, capabilities, and values.

A generative mechanism is represented by

$$\Gamma_t^G : (x_t, c_t, \mathcal{R}_t, h_t, u_t, \xi_t) \longrightarrow (x_{t+1}, c_{t+1}, \mathcal{R}_{t+1}, v_{t+1}). \quad (169)$$

Equation 169 differs from a simple output function because it explicitly includes production of later generative conditions and relations.

A system can therefore be highly generative even when immediate observable output is modest.

The generative mechanism is the principal object for the later concept of generative-process equivalence.

9.39 Recursive Generative Operator

Recursive generation can be represented through repeated application of a generative operator.

Let

$$\mathfrak{G}_{t+1} = \Gamma_t^G(\mathfrak{G}_t), \quad (170)$$

where $\mathfrak{G}_t$ contains the relevant state, conditions, relations, and value structures at time t .

Equation 170 permits analysis of higher-order loss. A change at one stage can alter the inputs to every later stage.

This structure formalizes the recursive generative loss described in Section 3.6.

9.40 Generative Opportunity Graph

Some applications can represent future generative possibilities through a directed opportunity graph

$$\mathfrak{O}_t = (N_t^O, E_t^O), \quad (171)$$

where nodes represent reachable states, roles, relations, or opportunities, and edges represent feasible transitions.

Equation 171 provides a discrete alternative to a continuous reachable-set representation.

A harm can remove nodes, remove edges, change edge costs, or change the probabilities associated with transitions.

A restorative intervention can therefore be evaluated through several different graph relations rather than through node count alone.

9.41 Transition Cost Structure

Availability of a transition does not imply that it is equally usable across systems.

Let

$$\kappa_t : E_t^O \longrightarrow \mathbb{R}_{\geq 0}^m \quad (172)$$

assign multidimensional costs to transitions.

Equation 172 can include financial cost, time, psychological burden, institutional risk, social cost, uncertainty, or another relevant burden.

A restored system can reproduce the same opportunity graph while imposing much higher transition costs.

This distinction is relevant to effective capability and practical reachability.

9.42 Security of Conditions

A condition can be available with different degrees of security.

Let

$$\sigma_t(c) \in \Sigma \quad (173)$$

represent the security profile of condition c .

Equation 173 can include revocability, dependence upon another actor, procedural protection, expected duration, and vulnerability to arbitrary interference.

Two systems can satisfy equal current access while differing in $\sigma_t(c)$.

This structure provides a formal location for non-domination and anticipatory security.

9.43 Subjective Expectation Structure

The subject's expectation concerning future availability can be represented separately from objective security.

Let

$$\pi_t^A (c_{t+\tau} \text{ remains available} \mid h_t) \quad (174)$$

denote subject A 's practical expectation regarding future access.

Equation 174 can differ from the objective institutional probability.

The distinction matters because learned expectations can influence current investment and participation.

Restoration of objective security and restoration of subjective confidence can therefore be different objects.

9.44 Trust Structure

Trust can be represented through a relation-valued function

$$\tau_{ij}(t) = \text{Trust}(A_i, A_j, h_t, \mathcal{R}_t). \quad (175)$$

Equation 175 permits trust to evolve through interaction history.

The formalism should not assume one universal scalar trust measure. Depending upon the application, trust can be multidimensional or ordinal.

Its inclusion allows later relational comparison to distinguish formal authority from trusted usability.

9.45 Subjective Meaning and Public Meaning

Social meaning can be distributed across perspectives.

Let

$$m_t^A(c) \quad (176)$$

denote the meaning of condition c for subject A , and let

$$m_t^{\text{pub}}(c) \quad (177)$$

denote a public or institutionally dominant interpretation.

Equations 176 and 177 permit divergence between private and public meaning.

This divergence can matter for recognition, stigma, institutional re-entry, and historical interpretation.

9.46 Counterfactual Model Family

Counterfactual restorative analysis should represent model uncertainty explicitly.

Let

$$\mathfrak{M}^{-H} = \{M_1^{-H}, M_2^{-H}, \dots\} \quad (178)$$

denote the family of plausible models of the no-harm system.

Each model can generate a counterfactual distribution

$$P_{M_i}(\mathcal{G}_{t+\tau}^H). \quad (179)$$

Equation 179 avoids treating one counterfactual history as uniquely established where the evidence supports several models.

Later restorative comparison can therefore seek robustness across $\mathfrak{M}^H$.

9.47 Reference System

A restorative judgment requires a reference system $\mathcal{G}^{\text{ref}}$.

The reference can be generated by a reference-selection operator

$$\mathcal{B} : (\mathcal{G}^H, D, V, N) \longrightarrow \mathcal{G}^{\text{ref}}, \quad (180)$$

where D denotes evidence, V denotes the relevant value framework, and N denotes normative constraints.

Equation 180 makes explicit that reference selection can depend upon both epistemic and normative inputs.

Possible references include a pre-harm state, no-harm counterfactual, capability threshold, normatively preferred system, best feasible system, or subject-selected future.

Section 10 develops this problem in detail.

9.48 Restorative Intervention

A restorative intervention is represented as an operator

$$R : \mathcal{G}^H \longrightarrow \mathcal{G}^R. \quad (181)$$

Equation 181 can act upon any subset of the generative tuple.

For example,

$$R = (R_X, R_C, R_{\mathcal{R}}, R_{\chi}, R_{\Phi}, R_{\Lambda}, R_{\mathcal{H}}) \quad (182)$$

decomposes restoration into interventions on state, conditions, relations, control, transition dynamics, value circulation, and historical representation.

Equation 182 is not intended to imply that history itself can always be altered. $R_{\mathcal{H}}$ can represent correction of records, acknowledgment, reinterpretation, or other interventions upon historical representation.

The decomposition clarifies why one restorative measure can succeed locally while leaving other structures unchanged.

9.49 Restorative Intervention Sequence

Where several interventions are required, the restorative process is represented by a sequence

$$\mathcal{G}^H \xrightarrow{R_1} \mathcal{G}^{(1)} \xrightarrow{R_2} \dots \xrightarrow{R_n} \mathcal{G}^R. \quad (183)$$

Equation 183 makes sequencing explicit.

The interventions can be non-commutative:

$$R_i \circ R_j \neq R_j \circ R_i. \quad (184)$$

Equation 184 formalizes the sequencing problem identified in Section 6.36.

9.50 Restorative Policy

A restorative policy can select interventions conditionally upon observed states.

Let

$$\pi_R : Y_t \longrightarrow \mathcal{U}_R \quad (185)$$

map observable information Y_t into a restorative action set $\mathcal{U}_R$.

Equation 185 allows adaptive assistance and staged intervention.

The policy can itself affect later observables, creating the reflexive structure identified in Section 8.

9.51 Procedural State

Because the process of restoration is independently normatively significant, the formal system includes a procedural state.

Let

$$p_t \in \mathcal{P} \quad (186)$$

represent relevant procedural conditions at time t .

The procedural state can include consent, participation, notice, contestability, privacy, review, reversibility, and other process variables.

A restorative transition can therefore be represented jointly as

$$(\mathcal{G}_t, p_t) \xrightarrow{R_t} (\mathcal{G}_{t+1}, p_{t+1}). \quad (187)$$

Equation 187 preserves the distinction between substantive state and procedural path while allowing both to enter later evaluation.

9.52 Substantive and Procedural Evaluation

Let

$$J_S : \mathcal{G}^R \times \mathcal{G}^{\text{ref}} \longrightarrow \mathcal{J}_S \quad (188)$$

denote substantive restorative evaluation, and let

$$J_P : (R_{0:n}, p_{0:n}) \longrightarrow \mathcal{J}_P \quad (189)$$

denote procedural evaluation.

Equations 188 and 189 formalize the separation developed in earlier sections.

A restorative policy can score highly under one evaluation and poorly under the other.

The later justice sections will examine the relation between these two domains.

9.53 Feasible Restorative Set

The technically and institutionally feasible restorative set is represented by

$$\mathcal{F}_R \subseteq \mathcal{U}_R. \quad (190)$$

Equation 190 depends upon resources, law, technology, time, jurisdiction, and institutional competence.

The normatively admissible set is

$$\mathcal{A}_R \subseteq \mathcal{F}_R, \quad (191)$$

where $\mathcal{A}_R$ excludes interventions violating the selected procedural, rights-based, or third-party constraints.

Equations 190 and 191 make the distinction between technical capacity and normative permission explicit.

9.54 Idealized Restorative Set

For thought experiments, the technical feasibility constraint can be relaxed.

Let

$$\mathcal{U}_R^{\text{ideal}} \supseteq \mathcal{F}_R \quad (192)$$

denote the set of idealized interventions available under expanded technological or hypothetical capacities.

Equation 192 permits later sections to ask which limits disappear under stronger intervention capacity and which normative or structural limits remain.

The idealized set remains subject to normative evaluation unless a thought experiment explicitly suspends those constraints for analytical purposes.

9.55 Restorative Comparison Relation

A restorative comparison relation is defined over a selected comparison object Q .

Let

$$Q : \mathcal{G} \longrightarrow \mathcal{Q} \quad (193)$$

project the generative system into the object space $\mathcal{Q}$.

A comparative relation is then

$$E_Q \subseteq \mathcal{Q} \times \mathcal{Q}. \quad (194)$$

Equation 194 provides the formal structure for later equality, isomorphism, simulation, bisimulation, approximation, dominance, and other relations.

A restorative claim takes the form

$$Q(\mathcal{G}^R) E_Q Q(\mathcal{G}^{\text{ref}}). \quad (195)$$

Equation 195 is the elementary grammar of the atlas.

9.56 Indexed Restorative Relation

The full restorative relation requires additional indices.

Let

$$\mathcal{G}^R \sim_E^{Q, \mathcal{T}, \tau, \varepsilon, K} \mathcal{G}^{\text{ref}} \quad (196)$$

denote a restorative judgment under comparison object Q , relation E , test or intervention class $\mathcal{T}$, temporal horizon τ , approximation tolerance ε , and epistemic basis K .

Equation 196 formalizes the indexed concept of restoration developed conceptually in Sections 1 and 8.

The notation does not imply that every index is numerical. K can contain a set of evidence, assumptions, confidence statements, or admissible models. $\mathcal{T}$ can contain observations, interventions, or perturbations. ε can be omitted where an exact relation is required.

9.57 Local and Global Restorative Claims

A restorative claim can concern one component or the whole represented system.

A local claim can take the form

$$Q_i(\mathcal{G}^R) \sim_{E_i} Q_i(\mathcal{G}^{\text{ref}}). \quad (197)$$

A global claim requires a family of local relations:

$$\bigwedge_{i \in I} [Q_i(\mathcal{G}^R) \sim_{E_i} Q_i(\mathcal{G}^{\text{ref}})]. \quad (198)$$

Equation 198 shows why global restoration is much stronger than restoration of one dimension.

The paper will generally prefer scoped local claims unless the conditions for a global claim are explicitly established.

9.58 Restorative Profile

A restored system can be described through a restorative profile rather than a single binary value.

Let

$$\mathbf{R}(\mathcal{G}^R, \mathcal{G}^{\text{ref}}) = (r_1, r_2, \dots, r_n) \quad (199)$$

represent the degree or status of restoration across selected objects.

Equation 199 can contain qualitative labels, equivalence classes, distances, probabilities, or partial-order relations.

The profile representation prevents strong restoration in one dimension from concealing weak restoration in another.

9.59 Restorative Partial Order

Where comparison criteria cannot be reduced to one scalar, restorative outcomes can be ordered partially.

Let

$$\mathcal{G}_1 \succeq_R \mathcal{G}_2 \quad (200)$$

mean that $\mathcal{G}_1$ is at least as satisfactory as $\mathcal{G}_2$ under the selected restorative dimensions and normative constraints.

Equation 200 permits incomparability:

$$\mathcal{G}_1 \not\succeq_R \mathcal{G}_2 \quad \text{and} \quad \mathcal{G}_2 \not\succeq_R \mathcal{G}_1. \quad (201)$$

Equation 201 is important where one intervention improves autonomy and another improves income, with no accepted trade-off between the two.

9.60 Restorative Dominance

A stronger order can be defined where one outcome is at least as good across all selected dimensions and strictly better in at least one.

Let

$$\mathcal{G}_1 \succ_R \mathcal{G}_2 \quad (202)$$

denote restorative dominance under a specified family of value-relevant dimensions.

Equation 202 is useful because justice can sometimes prefer a system that exceeds the counterfactual reference in relevant ways without reproducing it.

This structure will later support the analysis of better-than-reference restoration.

9.61 Subject-Directed Target Set

The subject-directed future developed earlier can be represented as a target set

$$\mathcal{T}_A \subseteq \mathcal{X} \quad (203)$$

selected through a procedurally legitimate process involving subject A .

Equation 203 need not contain one final state. It can contain several acceptable trajectories or future regions.

A restorative policy can then seek to expand reachability toward $\mathcal{T}_A$ while satisfying third-party and public constraints.

This formulation respects the subject's right to a different future without requiring the subject to specify one exact endpoint.

9.62 Normatively Admissible Future Set

Subject preference operates within a broader normative environment.

Let

$$\mathcal{N} \subseteq \mathcal{X} \quad (204)$$

denote states satisfying selected rights, third-party, institutional, or collective constraints.

The feasible subject-directed target becomes

$$\mathcal{T}_A^{\text{adm}} = \mathcal{T}_A \cap \mathcal{N}. \quad (205)$$

Equation 205 formalizes the principle that subject direction is substantial while remaining compatible with legitimate claims of others.

9.63 Minimal Sufficient Restoration

Some institutional contexts require a threshold rather than equivalence.

Let $S_Q \subseteq \mathcal{Q}$ denote the set of states satisfying a sufficient condition for object Q . A restorative criterion can require

$$Q(\mathcal{G}^R) \in S_Q. \quad (206)$$

Equation 206 differs from equality to the reference.

A capability floor, minimum income security, safe work condition, or minimum health access can be represented through sufficiency.

Later normative sections will compare sufficiency with equality, priority, dominance, and stronger equivalence standards.

9.64 Restorative Refinement

One restorative criterion can preserve all distinctions of another while adding further structure.

Let E_1 and E_2 be comparative relations. E_2 refines E_1 when

$$xE_2y \implies xE_1y. \quad (207)$$

Equation 207 provides a formal basis for discussing stronger and weaker comparison standards.

The normative importance of refinement remains separate. A mathematically stronger criterion can preserve distinctions irrelevant to justice or conflict with procedural values.

9.65 Restorative Test Class

A comparative claim can depend upon the tests permitted to distinguish two systems.

Let

$$\mathcal{T} = \{T_1, T_2, \dots\} \quad (208)$$

denote a family of observations, interventions, or perturbations.

Two systems can be equivalent relative to $\mathcal{T}$ when no test in the selected class distinguishes them under the chosen criterion.

This structure is especially important for observational and behavioral equivalence.

Changing the test class can change the conclusion.

9.66 Temporal Horizon

Restorative comparison is indexed to time.

Let

$$\tau \in \mathbb{T}_{\geq 0} \quad (209)$$

denote the comparison horizon.

A system can satisfy short-horizon equivalence and fail long-horizon equivalence.

The later atlas will distinguish instantaneous, finite-horizon, asymptotic, long-run statistical, and pathwise comparisons.

9.67 Approximation Tolerance

Where exact equivalence is unnecessary or unavailable, a tolerance parameter can be introduced.

Let

$$d_Q \left(Q(\mathcal{G}^R), Q(\mathcal{G}^{\text{ref}}) \right) \leq \varepsilon. \quad (210)$$

Equation 210 defines approximate restoration under metric d_Q .

The choice of metric and tolerance is substantive. A small numerical difference can be normatively significant in one domain and irrelevant in another.

The later value sections therefore treat ε as partly a normative parameter.

9.68 Epistemic Basis

A restorative claim should record the evidence and assumptions supporting it.

Let

$$K = (D, M, A, U) \quad (211)$$

denote an epistemic basis containing data D , model family M , assumptions A , and uncertainty representation U .

Equation 211 permits two restorative claims with the same formal criterion to differ in epistemic strength.

The atlas can therefore distinguish conceptual equivalence from empirically identified equivalence.

9.69 Confidence in Restorative Claims

Where meaningful, a confidence statement can be associated with a restorative claim:

$$\text{Conf} \left[\mathcal{G}^R \sim_E^Q \mathcal{G}^{\text{ref}} \mid K \right]. \quad (212)$$

Equation 212 need not always be probabilistic. It can be categorical, interval-based, model-relative, or qualitative.

The purpose is to prevent a formal relation from appearing more epistemically certain than its supporting evidence warrants.

9.70 Multi-Model Restorative Robustness

Where several plausible models remain, a restorative relation can be required to hold across them.

Let $\mathfrak{M}$ denote the model family. A robust restorative claim can take the form

$$\mathcal{G}^R \sim_{E,M}^Q \mathcal{G}^{\text{ref}} \quad \text{for all } M \in \mathfrak{M}. \quad (213)$$

Equation 213 is stronger epistemically than a claim holding under one selected model.

Where the relation does not hold across all models, the analysis can identify which parts remain stable.

9.71 Minimal Common Restorative Object

Suppose several models disagree about higher-order trajectories while agreeing about a common loss.

Let Q_i denote objects identified by model M_i . A common restorative object can be represented by

$$Q_{\cap} = \bigcap_{M_i \in \mathfrak{M}} Q_i. \quad (214)$$

Equation 214 is schematic because the objects need not literally be sets. The concept identifies a core claim supported across several plausible models.

This provides a formal route to the minimal common restoration discussed in Section 8.72.

9.72 Harm Operator

Harm can be represented as an intervention operator

$$H : \mathcal{G} \longrightarrow \mathcal{G}^H. \quad (215)$$

Equation 215 can act upon state, conditions, relations, control, transition rules, value circulation, or historical representation.

A decomposed harm operator can therefore take the form

$$H = (H_X, H_C, H_R, H_{\chi}, H_{\Phi}, H_{\Lambda}, H_{\mathcal{O}}). \quad (216)$$

Equation 216 formalizes the modes of generative harm developed in Section 3.7.

A particular case can contain several nontrivial components simultaneously.

9.73 Extraction Operator

Extraction can be represented as a transformation of value circulation.

Let

$$\Lambda_t \xrightarrow{H_{\text{ext}}} \Lambda_t^{\text{ext}} \quad (217)$$

denote a change in the allocation of generated value.

Equation 217 permits production to continue while a greater share of value leaves the generating system.

This formal separation between generation and circulation is central to generative justice.

9.74 Obstruction Operator

Obstruction can preserve nominal condition ownership while changing effective access.

Let

$$\alpha_t \xrightarrow{H_{\text{obs}}} \alpha'_t \quad (218)$$

represent a reduction in effective access.

Equation 218 distinguishes formal possession from practical usability.

The distinction is useful in cases involving administrative delay, retaliation, procedural barriers, or inaccessible infrastructure.

9.75 Relational Displacement Operator

Relational displacement acts upon network structure:

$$\mathcal{R}_t \xrightarrow{H_{\text{rel}}} \mathcal{R}'_t. \quad (219)$$

Equation 219 can represent removal of edges, change of centrality, alteration of authority, or changes in network reachability.

A later return of material resources can leave $\mathcal{R}'_t$ unchanged.

9.76 Conversion-Impairment Operator

Conversion impairment acts upon the map through which conditions become usable capabilities or outputs:

$$\mathcal{K}_t \xrightarrow{H_{\text{conv}}} \mathcal{K}'_t. \quad (220)$$

Equation 220 formalizes the possibility that the same resource produces weaker generative effects after harm.

This structure is particularly important for assistance.

9.77 Temporal-Displacement Operator

Temporal displacement changes the time at which a condition becomes available.

Let

$$c(t_0) \xrightarrow{H_{\text{delay}}} c(t_1), \quad t_1 > t_0. \quad (221)$$

Equation 221 preserves nominal condition identity while changing temporal position.

Where generative significance depends upon timing,

$$\Sigma_{t_0}(c) \neq \Sigma_{t_1}(c). \quad (222)$$

Equation 222 formalizes one source of non-equivalence in delayed return.

9.78 Return Operator

A return operator acts upon a condition, value, status, relation, or another selected object.

Let

$$\mathfrak{R}_Q : Q(\mathcal{G}^H) \longrightarrow Q(\mathcal{G}^R) \quad (223)$$

represent return with respect to object Q .

Equation 223 separates return from the stronger claim that the entire generative system is restored.

A monetary payment can be a return in the value domain while leaving relational and historical objects unchanged.

9.79 Generation–Return Non-Commutativity

The formal system now permits the central temporal structure to be stated more precisely.

Let H denote deprivation at t_0 , let Φ_{t_1, t_0}^H denote evolution under the altered conditions, and let R denote later return. The actual repaired system is

$$\mathcal{G}_{t_1}^R = R_{t_1} \circ \Phi_{t_1, t_0}^H \circ H_{t_0}(\mathcal{G}_{t_0}). \quad (224)$$

The reference system is

$$\mathcal{G}_{t_1}^{\text{ref}} = \Phi_{t_1, t_0}^{\text{ref}}(\mathcal{G}_{t_0}). \quad (225)$$

The central comparison is therefore

$$R_{t_1} \circ \Phi_{t_1, t_0}^H \circ H_{t_0} \sim_E^Q \Phi_{t_1, t_0}^{\text{ref}}. \quad (226)$$

Equation 226 replaces the much stronger assumption that return acts as a simple inverse of deprivation.

Whether the relation in Equation 226 holds depends upon the selected comparison object Q and relation E .

9.80 Identity Return and Generative Return

An identical condition can be returned without restoring the generative mechanism.

Suppose

$$c_R = c_{\text{pre}}. \quad (227)$$

Equation 227 establishes condition identity.

Generative return would require an additional relation such as

$$\Gamma_R^G \sim_E \Gamma_{\text{ref}}^G. \quad (228)$$

Equation 228 is strictly conceptually distinct from Equation 227.

This distinction is one of the principal formal motivations for the atlas.

9.81 Resource Return and Capability Return

Similarly, resource equality can coexist with capability inequality.

Let $M_R = M_{\text{ref}}$. Capability equivalence requires

$$\mathcal{K}_R(M_R) \sim_E \mathcal{K}_{\text{ref}}(M_{\text{ref}}). \quad (229)$$

Equation 229 formalizes the relationship between resource return and conversion-sensitive restoration.

This structure connects the formal system with the capability approach [55, 50].

9.82 Relational Return

A relational return can compare networks or specific relations.

For a network representation, one can ask whether

$$\mathcal{R}_R \sim_E \mathcal{R}_{\text{ref}}. \quad (230)$$

Equation 230 leaves E open.

The relevant relation can be graph isomorphism, preservation of reachability, functional similarity, centrality correspondence, simulation, or another criterion.

The next several sections of the paper will unpack these alternatives.

9.83 Control Return

Restoration of access can be distinguished from restoration of control.

A control-return criterion can require

$$\chi_R \sim_E \chi_{\text{ref}}. \quad (231)$$

Equation 231 becomes relevant where generativity depends upon secure authority over productive conditions.

A subject can therefore be materially compensated while control relations remain extractive.

9.84 Value-Circulation Return

Generative justice can be represented through comparison of circulation structures:

$$\Lambda_R \sim_E \Lambda_{\text{ref}}. \quad (232)$$

Equation 232 allows analysis of whether generated value again circulates through the relevant community, institution, or subject under an appropriate structure of control.

This criterion can diverge from equality of total value output.

9.85 Reachability Return

A future-oriented restorative criterion can compare reachable sets:

$$\mathcal{R}_\tau(\mathcal{G}^R) \sim_E \mathcal{R}_\tau(\mathcal{G}^{\text{ref}}). \quad (233)$$

Equation 233 permits the justice of return to concern future possibility without specifying one lost future.

The relevant relation E can be equality, inclusion, approximate correspondence, dominance, or another set relation.

9.86 Robustness Return

A robustness criterion can compare response operators across perturbations:

$$\mathcal{Q}_\tau(\mathcal{G}^R, \delta) \sim_E \mathcal{Q}_\tau(\mathcal{G}^{\text{ref}}, \delta) \quad \forall \delta \in \Delta_{\text{adm}}. \quad (234)$$

Equation 234 formalizes the idea that restoration can be tested through response to relevant future disturbances.

This criterion is stronger than equality under one baseline condition.

9.87 Historical Return

A historical comparison concerns

$$\mathcal{H}^R \sim_E \mathcal{H}^{\text{ref}}. \quad (235)$$

Equation 235 is among the strongest possible restorative claims because the restorative event itself belongs to $\mathcal{H}^R$.

Later sections will distinguish historical identity, historical isomorphism, provenance correspondence, and weaker forms of historical-generative comparison.

9.88 Open-Ended State-Space Extension

Generative processes can create new categories of states.

Let the state space evolve according to

$$\mathcal{X}_{t+1} = \mathcal{X}_t \cup \Delta \mathcal{X}_{t+1}. \quad (236)$$

Equation 236 represents emergence of newly meaningful states, roles, technologies, relations, or possibilities.

A restorative comparison that assumes a fixed state space can therefore become inadequate over long horizons.

The formal framework remains extensible so that later state-space enlargement can be incorporated.

9.89 Model Revision

The representation itself can change through learning.

Let

$$M_t \xrightarrow{\mathcal{L}_t} M_{t+1} \quad (237)$$

denote revision of model M_t through learning process $\mathcal{L}_t$.

Equation 237 permits a restorative system to update its ontology, causal assumptions, or comparison criteria as new evidence becomes available.

This revisability is important for historically open and reflexive systems.

9.90 Representation Provenance

Model revision should preserve provenance.

Let

$$\mathfrak{P}_M = (M_0 \rightarrow M_1 \rightarrow \dots \rightarrow M_n) \quad (238)$$

record the sequence of model revisions.

Equation 238 allows later analysis to identify which restorative decisions relied upon which representation.

This structure becomes important when an earlier model is later rejected.

9.91 Reflexive Observation in the Formal System

The formal system can represent observation as an event that feeds back into the system.

Let

$$z_t = O(\mathcal{G}_t) \quad (239)$$

and let the next state satisfy

$$\mathcal{G}_{t+1} = \Phi(\mathcal{G}_t, z_t). \quad (240)$$

Equations 239 and 240 embed the reflexive structure directly within the formal representation.

Observation is thereby available as both representation and intervention.

9.92 Reflexive Restoration

A restorative intervention can similarly enter later dynamics:

$$\mathcal{G}_{t+1} = \Phi(\mathcal{G}_t, R_t, K(R_t)), \quad (241)$$

where $K(R_t)$ denotes socially available knowledge that the restoration occurred.

Equation 241 separates the physical or institutional effect of restoration from the effect of knowledge concerning the restoration.

This structure is useful for public apologies, legal judgments, reputation correction, reparations programs, and other interventions whose social meaning depends partly upon their visibility.

9.93 System Boundary

Every formalization requires a system boundary.

Let

$$\partial\mathcal{G} \quad (242)$$

denote the selected boundary separating endogenous components from the surrounding environment.

A boundary expansion can transform an exogenous variable into an endogenous agent or relation.

Restorative claims should therefore identify the boundary relevant to the analysis, especially where third-party or system-level effects matter.

9.94 Scale Index

Multi-scale analysis can be represented through an index s :

$$\mathcal{G}^{(s)} \quad (243)$$

for individual, organizational, community, societal, ecological, or another selected scale.

A restorative relation can then be scale-indexed:

$$\mathcal{G}^{R,(s)} \sim_E^Q \mathcal{G}^{\text{ref},(s)}. \quad (244)$$

Equation 244 prevents an equivalence established at one scale from being generalized automatically to another.

9.95 Coupled Generative Systems

For multi-party cases, each subsystem can possess its own dynamics while remaining coupled to others.
Let

$$x_{i,t+1} = \Phi_i(x_{i,t}, x_{-i,t}, c_t, \mathcal{R}_t, u_t, \xi_t), \quad (245)$$

where $x_{-i,t}$ denotes the states of relevant other subsystems.

Equation 245 captures the possibility that restoring one subject changes the conditions of another.

A multi-party restorative policy should therefore evaluate a vector of effects rather than one scalar claimant outcome.

9.96 Generative Externalities

An intervention directed toward subject A_i can produce effects on A_j .

Let

$$\epsilon_{ij}(R) = \Delta \mathcal{G}_j \mid R_i \quad (246)$$

denote the generative externality of intervention R_i on subsystem j .

Equation 246 can represent beneficial or harmful third-party effects.

Later substantive-justice analysis will determine how such effects should enter normative evaluation.

9.97 Collective State

A collective system can contain emergent variables that are absent from individual states.

Let

$$X_t^{\text{collective}} = \mathcal{C}(x_{1,t}, \dots, x_{n,t}, \mathcal{R}_t), \quad (247)$$

where $\mathcal{C}$ denotes a collective-state construction.

Equation 247 can represent institutional knowledge, collective trust, governance capacity, culture, ecological-management practice, or another emergent property.

The formalism therefore avoids equating collective restoration with restoration of every individual component separately.

9.98 Distributed Generative Knowledge

Some generative knowledge is distributed across agents.

Let

$$K_t^{\text{dist}} = \bigcup_{i=1}^n K_{i,t} \quad (248)$$

represent the union of locally held knowledge.

Equation 248 can overstate effective collective knowledge when agents cannot communicate. A more realistic representation conditions accessibility upon network structure:

$$K_t^{\text{eff}} = \mathcal{D}(K_{1,t}, \dots, K_{n,t}, \mathcal{R}_t). \quad (249)$$

Equation 249 permits a network disruption to reduce collective generativity even when individual knowledge remains.

This structure is useful for research teams, communities, and institutions.

9.99 Institutional Interface Structure

The restorative interfaces introduced in Section 6.30 can be represented as mappings among institutional subsystems.

Let

$$\mathcal{I}_{ij} : Y_i \longrightarrow Y_j \quad (250)$$

denote the information or decision interface from institution i to institution j .

Equation 250 can represent referral, correction propagation, evidence transfer, benefit portability, or recognition of another institution's finding.

A broken interface can prevent an otherwise valid restorative decision from having practical effect.

9.100 Correction Propagation

Suppose a classification z_i produced by institution i has propagated through an institutional network.

Let

$$\mathcal{P}_z = (N_z, E_z) \quad (251)$$

denote the propagation graph.

A correction z'_i can be represented by a propagation process

$$z'_i \xrightarrow{\mathcal{P}_z} \{z'_j\}_{j \in N_z}. \quad (252)$$

Equation 252 describes successful update of institutionally reachable copies or derivative states.

Historical effects produced before correction remain separate objects.

9.101 Assistance Operator

Generative assistance can be represented as an intervention that expands viable future possibilities without requiring equality to the historical reference.

Let

$$A : \mathcal{G}^H \longrightarrow \mathcal{G}^{HA}. \quad (253)$$

Equation 253 can act upon resources, conditions, conversion structures, networks, health, or institutional access.

A basic future-expansion criterion is

$$\mathcal{R}(\mathcal{G}^H) \subseteq \mathcal{R}(\mathcal{G}^{HA}). \quad (254)$$

Equation 254 captures one possible meaning of successful assistance.

The later value sections will examine whether simple set expansion is normatively sufficient.

9.102 Subject-Directed Assistance Policy

A subject-directed assistance policy can be represented as

$$\pi_A : (Y_t, P_A, \mathcal{N}) \longrightarrow \mathcal{U}_A, \quad (255)$$

where P_A represents the subject's expressed priorities, $\mathcal{N}$ represents legitimate normative constraints, and $\mathcal{U}_A$ is the assistance-action space.

Equation 255 makes subject preference an input to assistance without treating it as the only determinant.

This structure supports revisable, participatory future formation.

9.103 Revision Operator

Restorative policies can be revised through

$$R_t \xrightarrow{\mathcal{U}} R_{t+1}, \quad (256)$$

where $\mathcal{U}$ incorporates new evidence, subject preference, outcome data, or normative review.

Equation 256 formalizes the revisability principle developed in earlier sections.

A revisable system can adapt without treating every earlier decision as epistemically final.

9.104 Cessation Operator

Where a harmful mechanism remains active, cessation is represented separately from restoration.

Let

$$C_H : \Phi^H \longrightarrow \Phi^{\neg H_{\text{active}}} \quad (257)$$

denote removal or disabling of the continuing harmful mechanism.

Equation 257 clarifies why compensation without cessation can be inadequate. The system can continue generating the same class of loss.

9.105 Prevention Operator

Prevention acts before realization of the harmful transition.

Let

$$P : (\mathcal{G}_t, \mathcal{H}_{\text{risk}}) \longrightarrow \mathcal{G}_t^P \quad (258)$$

denote a preventive intervention modifying the system in response to a risk structure $\mathcal{H}_{\text{risk}}$.

Equation 258 can alter safeguards, control relations, redundancy, review, or access security.

Prevention therefore acts upon the transition architecture before the harmful event occurs.

9.106 Interruption Operator

Interruption acts during an active harmful process.

Let

$$I : \Phi^H \longrightarrow \Phi^I \quad (259)$$

denote a change that reduces or stops further harmful propagation.

Equation 259 formalizes the temporal distinction between interrupting a process and repairing the historical state already produced.

9.107 Three-Phase Operator Family

The three temporal domains can now be represented by the operator family

$$\mathfrak{J} = (P, I, R/A), \quad (260)$$

where P denotes prevention, I denotes interruption, and R/A denotes restorative or assistance interventions.

Equation 260 provides a compact formal summary of the temporal architecture developed in Section 5.

The operators can interact and overlap in practice.

9.108 Restorative Claims as Typed Statements

The formalism permits restorative claims to be typed according to their object.

Examples include:

$$\text{Restored}_{\text{resource}}, \quad \text{Restored}_{\text{relation}}, \quad \text{Restored}_{\text{capability}}, \quad \text{Restored}_{\text{reachability}}, \quad \text{Restored}_{\text{generative}}. \quad (261)$$

Equation 261 avoids the ambiguity of an unqualified restoration predicate.

A later institutional statement can therefore assert resource restoration while remaining agnostic about relational or generative restoration.

9.109 Formal Scope Discipline

The formal system supports a principle of scope discipline.

Let $\mathcal{S}(M)$ denote the representational scope of model M , and let $\mathcal{C}(M)$ denote the claims issued from that model. A defensible restorative analysis should satisfy

$$\mathcal{C}(M) \subseteq \mathcal{S}(M). \quad (262)$$

Equation 262 expresses the requirement that the strength of the conclusion remain within the scope of the representation.

A model of monetary loss should not silently issue claims about historical identity. A graph model should not infer subjective meaning without additional representation. A behavioral model should not infer identical causal structure without further assumptions.

9.110 Formal Pluralism

The baseline tuple supports several formal languages simultaneously.

State-space methods can represent continuous or discrete dynamics.

Graph models can represent relational topology.

Transition systems can represent branching behavior.

Probability models can represent uncertainty.

Causal models can distinguish intervention from observation.

Order structures can represent sufficiency, dominance, and refinement.

Provenance graphs can represent historical knowledge and correction.

The formal architecture therefore favors interoperability among representations over reduction to one mathematical language.

This pluralism corresponds to the heterogeneous restorative objects identified throughout the paper.

9.111 Mapping Between Formal Representations

Where several representations concern the same case, mappings among them become important.

Let

$$F_{ij} : M_i \longrightarrow M_j \quad (263)$$

translate information from model M_i into model M_j .

Equation 263 can represent conversion of a network model into a state variable, aggregation of individual states into a collective indicator, or transformation of event records into causal variables.

Information can be lost under F_{ij} .

The later comparison framework should therefore identify which structures are preserved by such transformations.

9.112 Abstraction Relations

One representation can be an abstraction of another.

Let

$$\alpha : M_{\text{fine}} \longrightarrow M_{\text{coarse}} \quad (264)$$

denote an abstraction map.

Equation 264 can collapse several detailed states into one coarse category.

Restorative equivalence at the coarse level can coexist with non-equivalence at the fine level.

This observation will later connect restorative comparison with abstraction, homomorphism, simulation, and refinement.

9.113 Observation-Relative Identity

Two systems can be identical relative to one observation map:

$$O(\mathcal{G}_1) = O(\mathcal{G}_2), \quad (265)$$

while

$$\mathcal{G}_1 \neq \mathcal{G}_2. \quad (266)$$

Equations 265 and 266 provide the simplest formal statement of the difference between observational identity and system identity.

This distinction is foundational for the later taxonomy.

9.114 Object-Relative Equivalence

Similarly, equivalence is indexed to the object projection Q :

$$Q_1(\mathcal{G}_1) \sim Q_1(\mathcal{G}_2) \quad (267)$$

can coexist with

$$Q_2(\mathcal{G}_1) \not\sim Q_2(\mathcal{G}_2). \quad (268)$$

Equations 267 and 268 formalize the paper's central claim that restoration is multidimensional.

9.115 The Formal Object–Relation Product Space

Let $\mathfrak{Q}$ denote the family of comparison objects and $\mathfrak{E}$ the family of comparative relations.

The formal comparison space is

$$\mathfrak{C} = \mathfrak{Q} \times \mathfrak{E}. \quad (269)$$

Equation 269 is the mathematical core of the later Atlas of Equivalence.

An element

$$(Q, E) \in \mathfrak{C} \quad (270)$$

defines one candidate restorative comparison.

The same object can support several relations, and the same relation can apply to several objects.

The atlas will examine which pairs are formally meaningful, normatively relevant, empirically identifiable, and practically usable.

9.116 Extended Restorative Index Space

The object–relation pair can be extended by the additional indices developed earlier.

Let

$$\mathfrak{R} = \mathfrak{Q} \times \mathfrak{E} \times \mathfrak{T} \times \mathbb{T} \times \mathfrak{Eps} \times \mathfrak{K} \times \mathfrak{S}, \quad (271)$$

where $\mathfrak{T}$ is the family of test or intervention classes, $\mathfrak{Eps}$ is the family of approximation specifications, $\mathfrak{K}$ is the family of epistemic bases, and $\mathfrak{S}$ is the family of scale or boundary specifications.

Equation 271 represents the broader parameter space in which restorative claims are located.

The formalism thereby replaces a binary notion of restoration with a family of indexed comparative statements.

9.117 Normative Layer

The formal system remains incomplete until a normative layer is added.

Let

$$\mathcal{N}_V : \mathfrak{R} \longrightarrow \mathfrak{J} \quad (272)$$

map restorative comparisons into normative judgments under value theory V .

Equation 272 formalizes the principle that a theory of comparison requires a theory of value.

The same formal equivalence can receive different normative interpretations under capability, relational, rights-based, utilitarian, recognition, non-domination, care-oriented, or generative theories.

The later value sections develop these mappings explicitly.

9.118 Procedural Layer

A separate procedural interpretation evaluates the path by which a restorative state is produced.

Let

$$\mathcal{P}_J : (R_{0:n}, p_{0:n}) \longrightarrow \mathfrak{J}_P. \quad (273)$$

Equation 273 can incorporate consent, participation, contestability, privacy, reversibility, dignity, non-domination, proportionality, and third-party rights.

A restorative outcome can therefore be substantively strong under $\mathcal{N}_V$ and procedurally weak under $\mathcal{P}_J$.

This distinction will become central in the later analysis of restorative fidelity and procedural conflict.

9.119 Formal Interpretation of Complete Restoration

The phrase *complete restoration* can now be given several distinct formal meanings.

A representation-relative complete restoration can require

$$Q_i(\mathcal{G}^R) \sim_{E_i} Q_i(\mathcal{G}^{\text{ref}}) \quad \text{for every } i \in I_{\text{selected}}, \quad (274)$$

where I_{selected} contains all comparison objects included in the chosen model.

Equation 274 can establish completeness relative to the representation.

It does not establish completeness relative to every possible description of the social world.

This distinction resolves part of the ambiguity identified in Section 8. Formal completeness is model-relative and can remain conceptually useful without becoming an ontological claim.

9.120 Strong Historical-Generative Restoration

A much stronger criterion would require correspondence across state, relation, dynamics, value circulation, future possibilities, and historical structure.

A schematic version is

$$\left(X_R, \mathcal{R}_R, \Phi_R, \Lambda_R, \mathcal{R}_R^{\text{reach}}, \mathcal{H}_R \right) \sim \left(X_{\text{ref}}, \mathcal{R}_{\text{ref}}, \Phi_{\text{ref}}, \Lambda_{\text{ref}}, \mathcal{R}_{\text{ref}}^{\text{reach}}, \mathcal{H}_{\text{ref}} \right). \quad (275)$$

Equation 275 represents a family of extremely demanding criteria.

The relation $\sim$ must still be specified for each component.

This formulation makes clear that strong historical-generative restoration is not a primitive concept. It is a compound claim constructed from several comparison choices.

9.121 Future-Directed Generative Justice

The formal system also supports a future-directed response where historical equivalence is unavailable or undesirable.

Let $\mathcal{T}_A^{\text{adm}}$ be the admissible subject-directed target set defined in Equation 205. A future-directed restorative policy can seek

$$\mathcal{R}_\tau(\mathcal{G}^R) \cap \mathcal{T}_A^{\text{adm}} \neq \emptyset. \quad (276)$$

Equation 276 requires the restored system to provide at least one viable path toward a legitimate subject-directed future.

A stronger criterion can require expansion of the viable target set.

This structure provides a formal basis for generative assistance without pretending that new possibility is historically equivalent to the lost possibility.

9.122 Formal Distinction Between Return and Assistance

The distinction between return and assistance can now be stated explicitly.

Return is reference-directed:

$$R : \mathcal{G}^H \longrightarrow \mathcal{G}^R \quad \text{with evaluation against } \mathcal{G}^{\text{ref}}. \quad (277)$$

Assistance is possibility-directed:

$$A : \mathcal{G}^H \longrightarrow \mathcal{G}^{HA} \quad \text{with evaluation through viable future expansion.} \quad (278)$$

Equations 277 and 278 describe different normative orientations.

A practical response can contain both.

9.123 Formal Distinction Between Compensation and Equivalence

A compensation payment can be represented as

$$M : \mathcal{G}^H \longrightarrow \mathcal{G}^{HM}. \quad (279)$$

Equation 279 identifies a resource transfer.

A separate claim is required to establish equivalence:

$$Q(\mathcal{G}^{HM}) \sim_E Q(\mathcal{G}^{\text{ref}}). \quad (280)$$

Equation 280 demonstrates formally that justified compensation does not logically entail equivalent restoration.

This distinction will later become central to the political economy of general equivalents.

9.124 Formal Distinction Between Recognition and Reconstruction

Recognition can alter public meaning or normative status without reproducing the lost system.

Let

$$\mathcal{M}^{\text{pub}} \xrightarrow{R_{\text{rec}}} \mathcal{M}^{\text{pub}'} \quad (281)$$

denote a recognition intervention.

Equation 281 can improve social standing or public interpretation while leaving material, relational, and historical structures partially unchanged.

Recognition should therefore be represented as a distinct restorative object.

9.125 Formal Distinction Between Re-Entry and Historical Restoration

Re-entry can be represented as restoration of access to a current institutional system:

$$\alpha_R(A, C_{\text{institution}}) = 1. \quad (282)$$

Equation 282 establishes present access.

Historical restoration would require substantially stronger correspondence among prior network position, accumulated experience, trajectory, and later opportunity.

Re-entry should therefore be treated as an important restorative achievement with a bounded formal scope.

9.126 Formal Distinction Between Prevention and Restoration

Prevention preserves a set of histories before the harmful transition occurs.

Restoration acts after one harmful history has already been realized.

Let

$$\mathfrak{H}_t^{\text{open}} \quad (283)$$

denote the set of histories still available at time t .

A preventive intervention can preserve elements of $\mathfrak{H}_t^{\text{open}}$.

A restorative intervention at $t + \tau$ operates after one branch has already been realized and cannot convert the later history into the fact that another branch was historically realized.

This distinction formalizes the temporal asymmetry developed in Section 5.2.

9.127 Formal Limits of the Baseline Representation

The baseline tuple in Equation 117 remains an analytical convenience.

Several limitations should remain explicit.

First, the tuple presupposes a selected system boundary.

Second, some components can be only partially observable.

Third, relations can be higher-order and context-dependent.

Fourth, state spaces can change over time.

Fifth, values can remain incommensurable.

Sixth, histories can be only partially ordered or incompletely known.

Seventh, observation can affect the system.

Eighth, the meaning of a condition can resist stable formalization.

Ninth, the same empirical case can support several legitimate representations.

The formalism is therefore designed for modular extension and comparison across representations.

9.128 Analytical Commitments of the Formal Framework

Despite its openness, the framework makes several analytical commitments.

The first commitment is separation of state, conditions, relations, history, transition, observation, control, value generation, and value circulation.

The second commitment is explicit representation of time.

The third commitment is preservation of the distinction between observation and intervention.

The fourth commitment is compatibility with stochastic and branching futures.

The fifth commitment is compatibility with several system scales.

The sixth commitment is explicit indexing of restorative claims by object and comparative relation.

The seventh commitment is separation of formal comparison from normative interpretation.

The eighth commitment is separation of substantive restorative outcome from procedural justice.

These commitments provide the common structure required by the later atlas.

9.129 Implications for Reference Selection

The formal representation reveals that selection of a reference system is more complex than choosing a date.

A reference can specify:

- reference state;
- reference conditions;
- reference relations;
- reference control structure;
- reference transition dynamics;
- reference value circulation;
- reference history;
- reference scale;
- reference time horizon.

Two theories can therefore use the same phrase, “the no-harm reference,” while constructing substantially different formal objects.

This observation motivates the next section.

9.130 Transition to Reference Systems

The formal architecture developed in this section provides the objects required for restorative comparison, while leaving the reference itself unspecified.

That omission is deliberate.

A comparison between the post-restoration system $\mathcal{G}^R$ and a reference system $\mathcal{G}^{\text{ref}}$ cannot be interpreted until the origin and normative status of $\mathcal{G}^{\text{ref}}$ are known.

The reference can be the actual pre-harm system. It can be a counterfactual system generated by removing a harmful intervention. It can be a statistical benchmark, capability threshold, normatively preferred configuration, best-feasible alternative, or subject-directed target.

These alternatives differ in both epistemic and normative structure.

Section 10 therefore develops a taxonomy of restorative reference systems before the paper turns to the larger taxonomies of comparison objects and comparative relations.

10 Reference Systems and Comparative Baselines

This section develops the reference-system problem that precedes every restorative comparison. The formal framework introduced in Section 9 represented a restorative judgment through comparison between a post-intervention system $\mathcal{G}^R$ and a reference system $\mathcal{G}^{\text{ref}}$. The notation can appear neutral while concealing a substantive choice. The reference can be the actual condition immediately before harm, a counterfactual history in which the harm did not occur, a statistical benchmark, a normatively preferred system, a threshold of sufficiency, the best feasible alternative, a subject-selected future, or a hybrid constructed from several of these sources.

These references answer different questions. The pre-harm state asks what existed before the relevant intervention. The no-harm counterfactual asks what might have developed had the intervention not occurred. A statistical baseline asks what tends to occur among a selected comparison population. A normative reference asks which condition justice has reason to regard as desirable. A sufficiency reference asks whether an adequate threshold has been reached. A best-feasible reference asks what can reasonably be achieved under present constraints. A subject-directed reference asks which viable future the present historical subject now has reason to pursue.

The choice among these references cannot be postponed until after the comparison. It determines the object against which equality, similarity, simulation, dominance, or another relation is evaluated. The same restorative intervention can therefore appear complete under one reference, incomplete under another, and normatively undesirable under a third.

The principal objective of this section is to construct a taxonomy of reference systems and to separate their epistemic and normative functions. The discussion first introduces a general formal structure for reference selection. It then examines historical, counterfactual, statistical, normative, threshold-based, feasible, and subject-directed references. Later subsections consider multi-reference comparison, uncertainty, temporal reference, reference updating, unjust baselines, counterfactual paternalism, and the relation between references and different theories of justice. The section concludes by specifying the reference metadata required by the Atlas of Equivalence developed later in the paper.

10.1 The Reference-System Problem

A restorative comparison takes the general form

$$Q(\mathcal{G}^R) \ E \ Q(\mathcal{G}^{\text{ref}}), \quad (284)$$

where Q denotes the comparison object and E denotes the comparative relation.

Equation 284 cannot be interpreted without specifying the origin of $\mathcal{G}^{\text{ref}}$. A statement that income has been restored to the pre-harm level differs from a statement that income has been restored to the expected no-harm level. A statement that capabilities exceed a minimum threshold differs from a statement that they match those of statistically similar persons. A statement that a new career is better than the best feasible alternative differs from a statement that the career reproduces the trajectory interrupted by harm.

The reference therefore performs at least three functions.

First, it determines the direction of comparison.

Second, it determines which differences count as deficits, improvements, or irrelevant variation.

Third, it implicitly identifies a theory of what the restorative process is trying to achieve.

The last function makes baseline choice normatively significant even where the comparison itself is mathematically precise.

10.2 Reference Selection as an Operator

Let $\mathfrak{B}$ denote the space of possible reference systems. A reference-selection operator can be represented by

$$\mathcal{B} : (D, H, M, V, N, P_A, F) \longrightarrow \mathcal{G}^{\text{ref}} \in \mathfrak{B}, \quad (285)$$

where D denotes available evidence, H denotes historical information, M denotes the model family, V denotes a value framework, N denotes normative constraints, P_A denotes the present subject's preferences or priorities, and F denotes feasibility constraints.

Equation 285 makes explicit that reference selection can contain both epistemic and normative inputs.

Some references are predominantly historical. Others are predominantly normative. Others combine empirical and normative components.

The formal architecture should therefore record the basis of reference selection rather than treating the reference as a primitive constant.

10.3 Historical Actual-State Reference

The most intuitive reference is the actual system immediately before the relevant harm.

Let

$$\mathcal{G}^{\text{pre}} = \mathcal{G}_{t_0^-} \quad (286)$$

denote the system at the last relevant pre-harm moment.

Equation 286 supports a straightforward restorative question:

Has the subject or system been returned to the condition that actually existed before the intervention?

This reference has several advantages. It is historically grounded. It can be supported by observable records. It avoids speculative claims about the future. It can be especially appropriate for return of property, formal status, physical access, or another identifiable condition.

Its limitations are equally important.

The pre-harm state can itself have been unstable, unjust, temporary, or already changing. A person about to receive a promotion does not have the same reference as the person's current salary merely because the harm occurred before the promotion was finalized. A community can have lived under unjust conditions before a specific additional deprivation. A workplace can have contained domination before overt harassment occurred.

Restoring the actual pre-harm state therefore answers a historical question without automatically answering the normative question of what justice now requires.

10.4 Pre-Harm Trajectory Reference

A stronger historical reference extrapolates the trajectory already underway before the harm.

Let the pre-harm system evolve according to a model Φ^{pre} . A trajectory reference can be represented by

$$\gamma^{\text{pre}}(t) = \Phi_{t,t_0}^{\text{pre}}(\mathcal{G}_{t_0}). \quad (287)$$

Equation 287 differs from restoration to the state at t_0 . It asks where the system would have developed under continuation of the pre-harm process.

This reference becomes relevant where the subject was already accumulating skills, income, seniority, publication record, educational progress, or another time-dependent condition.

Its epistemic burden is greater because continuation of the pre-harm trend is an assumption rather than an observed future.

The longer the horizon, the weaker the inference can become.

10.5 No-Harm Counterfactual Reference

A central restorative reference is the system that would have developed had the harmful intervention not occurred.

Let H denote the harmful intervention. A no-harm reference is represented by

$$\mathcal{G}_t^{-H} = \Phi_{t,t_0}^{-H}(\mathcal{G}_{t_0}). \quad (288)$$

Equation 288 appears natural for corrective and compensatory reasoning because it isolates the effect of the relevant harm.

The reference is nevertheless counterfactual. It requires assumptions about the transition process in the absence of H . As discussed in Section 8, the no-harm history cannot generally be obtained by subtracting the harmful event from the actual history.

A counterfactual reference therefore belongs to an epistemic structure including model assumptions, alternative plausible histories, and uncertainty [28, 43].

The no-harm reference is particularly important for valuation of lost income, lost chance, delayed development, and altered professional trajectory.

Its usefulness should not be converted into a claim of unique historical knowledge where several no-harm futures remain plausible.

10.6 Distributional Counterfactual Reference

Where the no-harm future is stochastic or underdetermined, the reference can be a distribution rather than one state.

Let

$$P_t^{-H} = P(\mathcal{G}_t^{-H} \mid D, M). \quad (289)$$

Equation 289 represents uncertainty over no-harm outcomes conditional upon evidence D and model M .

A restorative intervention can then be compared with the distribution through expected value, quantiles, stochastic dominance, probability of threshold attainment, distributional distance, or another relation.

This approach avoids arbitrary selection of one trajectory.

It also moves normative choice into the way the distribution is evaluated. Expected value, worst-case protection, median outcome, and preservation of the entire distribution express different priorities.

10.7 Reachable-Set Counterfactual Reference

A still more structural reference avoids assigning probabilities to every counterfactual state.

Let

$$\mathcal{R}_\tau^{-H}(x_{t_0}) \quad (290)$$

denote the set of future states reachable in the absence of the harm over horizon τ .

Equation 290 supports the question:

Which futures remained open before the harmful intervention, whether or not one can know which future would actually have been realized?

This reference is particularly compatible with generative analysis.

A harm can remove routes or regions from the reachable set without supporting a claim that one specific missing state would certainly have occurred.

Later sections will examine equality, inclusion, topology, and dominance relations among reachable sets.

10.8 Capability Reference

The capability approach suggests a reference defined through substantive opportunities rather than resource holdings alone [55, 39, 50].

Let

$$\mathcal{K}_A^{\text{ref}} \quad (291)$$

denote the reference capability set for subject A .

The reference can be historically grounded, counterfactual, threshold-based, or normatively constructed.

A capability reference therefore does not resolve baseline selection by itself. It changes the object being compared.

For example, the comparison can ask whether the person's post-harm capability set matches the capability set expected without the harm, exceeds a threshold, or dominates the capability set available under the actual damaged trajectory.

This illustrates the distinction between reference and comparison object that the atlas will later preserve systematically.

10.9 Statistical-Peer Reference

Another reference is constructed from outcomes among a selected comparison population.

Let $\mathcal{P}_A$ denote a peer set selected for subject A . A statistical reference can be represented by

$$\mathcal{G}^{\text{stat}} = \mathcal{S}(\{\mathcal{G}_j : j \in \mathcal{P}_A\}), \quad (292)$$

where $\mathcal{S}$ denotes a statistical summary such as a mean, median, distribution, percentile structure, or model-based expectation.

Equation 292 can be useful where individual counterfactual history is poorly identified.

A researcher excluded from employment can be compared with researchers at a similar career stage. A worker's expected earnings can be estimated from a relevant labor-market cohort. A patient's functional trajectory can be compared with persons possessing similar clinical characteristics.

The legitimacy of the result depends strongly upon peer selection.

10.10 Comparator Selection

A statistical reference requires a rule defining who counts as comparable.

Let

$$\mathcal{P}_A = \{j : d_C(z_j, z_A) \leq \epsilon_C\}, \quad (293)$$

where z contains selected comparator characteristics and d_C is a similarity criterion.

Equation 293 makes the construction of the peer group explicit.

Comparator variables can include age, profession, education, seniority, location, health, prior output, socioeconomic background, institutional position, or another factor.

Every inclusion or exclusion changes the benchmark.

A statistical baseline should therefore be interpreted as constructed rather than natural.

10.11 The Normality Problem

Statistical prevalence does not establish normative desirability.

Suppose the average member of a population experiences discrimination, insecurity, poor health access, or limited opportunity. Restoring a claimant to the statistical mean can reproduce structural injustice.

The relation can be represented by

$$\mathcal{G}^{\text{stat}} \neq \mathcal{G}^{\text{just}} \quad (294)$$

in general.

Equation 294 is fundamental.

Statistical baselines answer descriptive questions concerning what tends to occur. Normative baselines answer questions concerning what should occur.

The two can overlap without being identical.

10.12 Normative-Ideal Reference

A normative reference specifies a system regarded as just or desirable according to a selected theory of value.

Let

$$\mathcal{G}_V^* \quad (295)$$

denote an ideal or preferred system under value theory V .

The reference can be defined through rights, welfare, equality, capabilities, recognition, non-domination, generative justice, or another normative framework.

A normative reference can exceed both the pre-harm and no-harm systems.

For example, a worker harmed within an already arbitrary hierarchy can be restored into a transformed institution with stronger procedural rights. A community subjected to historical dispossession need not be returned to a preexisting social order that contained unrelated injustices. A reparative process can create democratic governance absent from the original baseline.

Normative reference therefore permits justice to be transformative rather than merely reconstructive.

10.13 The Unjust-Baseline Problem

The normative-reference framework exposes the unjust-baseline problem.

Let

$$\mathcal{G}^{\text{pre}} \notin \mathcal{N}_{\text{just}}, \quad (296)$$

where $\mathcal{N}_{\text{just}}$ denotes the set of normatively admissible systems.

Equation 296 describes a case in which literal restoration to the pre-harm state reproduces injustice.

The same problem can affect the no-harm counterfactual. The absence of one wrong does not imply absence of every other wrong.

Consequently,

$$\mathcal{G}^{-H} \notin \mathcal{N}_{\text{just}} \quad (297)$$

can also hold.

A theory that treats counterfactual reconstruction as the complete definition of justice therefore risks reproducing unjust background conditions.

This concern appears in relational, capability, recognition, and non-domination theories, all of which direct attention beyond possession of the same resources or statuses [2, 71, 44].

10.14 Transformative Reference

A transformative reference explicitly seeks a state that differs from the historical baseline because the historical structure itself requires change.

Let

$$\mathcal{G}^{\text{trans}} = T(\mathcal{G}^{\text{pre}}, V), \quad (298)$$

where T transforms the historical system according to a normative framework V .

Equation 298 can represent institutional reform, redistribution of authority, removal of domination, transformation of value circulation, or creation of new participatory structures.

Fraser's distinction between affirmative and transformative remedies provides a relevant conceptual precedent [20]. Generative justice similarly emphasizes transformation of extractive value structures rather than repeated downstream redistribution [15, 17].

A transformative reference therefore asks a different question from counterfactual restoration:

Which system should replace the structure within which the harm became possible?

10.15 Rights-Based Reference

A rights-based reference can be defined through satisfaction of a specified set of rights.

Let

$$\mathcal{N}_R = \{\mathcal{G} : r_i(\mathcal{G}) = 1 \text{ for all protected rights } r_i\}. \quad (299)$$

Equation 299 defines a set of admissible systems rather than one target state.

This feature is important. Rights can constrain restoration without specifying a unique ideal outcome.

Several post-harm futures can satisfy the same rights.

The subject can therefore retain substantial agency within the admissible set.

10.16 Non-Domination Reference

A republican reference can require that the subject not remain exposed to arbitrary control over relevant conditions [44].

Let χ denote the control structure. A non-domination reference can require

$$\chi \in \mathcal{N}_{\text{ND}}, \quad (300)$$

where $\mathcal{N}_{\text{ND}}$ contains control structures satisfying the selected non-domination conditions.

This criterion can be met by several materially different systems.

A subject can therefore be restored into a new institutional arrangement whose resources differ from the previous arrangement while dependence upon arbitrary power is substantially reduced.

10.17 Generative-Justice Reference

A generative-justice reference can focus on control over generative conditions and circulation of generated value [15, 17].

Let

$$\mathcal{N}_{GJ} = \{\mathcal{G} : \text{Control}(\mathcal{G}), \text{Circulation}(\mathcal{G}), \text{Generation}(\mathcal{G}) \text{ satisfy selected criteria}\}. \quad (301)$$

Equation 301 again defines a family of acceptable systems rather than one state.

A post-harm system can therefore be generatively more just than the historical baseline without being structurally identical to it.

This distinction becomes especially important where return of value to a changed generator cannot reproduce the original trajectory.

10.18 Sufficiency Reference

A sufficiency reference asks whether a minimum acceptable condition has been reached.

Let Q denote a restorative object and $q_{\min}$ a threshold. A simple criterion is

$$Q(\mathcal{G}^R) \geq q_{\min}. \quad (302)$$

Equation 302 does not require equality with the pre-harm or counterfactual system.

Capability theory can support threshold-oriented interpretations in which justice requires access to a sufficient range of substantive capabilities, although capability theorists differ substantially concerning specification and political use of such thresholds [39, 50].

Sufficiency is particularly relevant to social protection, emergency assistance, and minimum rehabilitation.

Its limitation is equally clear. Reaching a minimum floor can remain far below what corrective or reparative justice requires.

A person wrongfully deprived of substantial resources does not lose the claim for rectification merely because later support keeps the person above a minimum threshold.

10.19 Social-Protection Reference

A social-protection system can use a reference based upon minimum guarantees rather than historical counterfactuals.

The ILO Social Protection Floors Recommendation emphasizes basic social security guarantees, essential healthcare, income security, dignity, and social solidarity [26].

A corresponding reference can be represented as

$$\mathcal{G}^{\text{floor}} \in \mathcal{N}_{\text{SP}}, \quad (303)$$

where $\mathcal{N}_{\text{SP}}$ denotes the set of systems satisfying the relevant social-protection floor.

This reference is institutionally powerful because it does not require proof of a no-harm counterfactual.

It answers a different question from reparation.

The distinction should remain explicit so that provision of a universal floor does not extinguish a separate claim arising from wrongdoing.

10.20 Best-Feasible Reference

Practical constraints can make the ideal or counterfactual reference unattainable.

A best-feasible reference is selected from a feasible set $\mathcal{F}$:

$$\mathcal{G}^{\text{BF}} \in \arg \max_{\mathcal{G} \in \mathcal{F}} V(\mathcal{G}), \quad (304)$$

where V denotes the relevant evaluative criterion.

Equation 304 is schematic. The evaluative structure can be multidimensional or partially ordered rather than scalar.

Best-feasible reference is important where a lost condition is unavailable. A mentor has died. An ecosystem cannot be recreated. A career window has closed. A building no longer exists.

The relevant practical question becomes:

Given the irreversible history and current constraints, which available configuration now provides the strongest just response?

Best feasible does not mean historically equivalent.

10.21 Feasible-Set Construction

The feasible set is itself constructed.

Let

$$\mathcal{F} = \mathcal{F}(B, L, T, P, C, N), \quad (305)$$

where B denotes budget, L denotes law, T denotes technology, P denotes physical constraints, C denotes institutional capacity, and N denotes normative constraints.

Equation 305 shows that changes in technology, law, or institutional design can alter what counts as feasible.

A current best-feasible reference can therefore become obsolete.

The reference should be revisable where constraints change materially.

10.22 Subject-Selected Reference

The present historical subject can select or participate in selecting a future reference.

Let

$$\mathcal{G}^A \in \mathfrak{B}_A \quad (306)$$

denote a reference chosen through a procedurally legitimate subject-directed process.

The reference can be a target state, a target capability set, a target trajectory family, or a broad future region.

Subject-selected references are especially important after adaptation or major life change.

A person excluded from academia can choose a future in public service. A worker can choose compensated exit rather than reinstatement. A community can choose a new governance arrangement rather than reconstruction of a historical institution.

The legitimacy of these choices does not depend upon similarity to the counterfactual no-harm trajectory.

10.23 Subject-Selected Reference Set

The subject need not identify one exact future.

A subject-selected reference can be a set

$$\mathfrak{B}_A = \{\mathcal{G}^{A,1}, \mathcal{G}^{A,2}, \dots\} \quad (307)$$

of futures regarded as acceptable.

Equation 307 supports exploration, uncertainty, and revisability.

The restorative process can then aim to preserve access to several viable futures rather than prematurely optimize toward one endpoint.

This structure is particularly compatible with the paper's emphasis on generativity.

10.24 Present-Need Reference

Assistance can use the subject's present need as a reference rather than a past counterfactual.

Let

$$N_A(t) \quad (308)$$

denote the current need profile.

An assistance reference can specify the conditions required to meet those needs:

$$\mathcal{G}^{\text{need}} = \mathcal{N}(N_A(t)). \quad (309)$$

Equation 309 is especially relevant to healthcare, housing, income support, rehabilitation, and emergency assistance.

Need-based response and corrective response can coexist.

The first asks what the subject requires now. The second asks what is owed because of the history of the wrong.

10.25 Adaptive Reference

Some restorative processes require references that change through time.

Let

$$\mathcal{G}_{t+1}^{\text{ref}} = \mathcal{U} \left(\mathcal{G}_t^{\text{ref}}, D_{t+1}, P_{A,t+1}, F_{t+1} \right), \quad (310)$$

where new evidence, subject preference, and feasibility conditions update the reference.

Equation 310 is suitable for long-term rehabilitation, career transition, ecological recovery, and other processes where the relevant target cannot reasonably be fixed at the beginning.

Adaptive reference requires safeguards against moving the goalposts in ways that reduce legitimate claims.

Reference revision should therefore preserve provenance.

10.26 Reference Provenance

A restorative system should record how its reference was selected.

Let

$$\mathfrak{P}_B = (B_0 \rightarrow B_1 \rightarrow \cdots \rightarrow B_n) \quad (311)$$

represent the history of reference selection and revision.

The provenance record can include:

- evidence used;
- counterfactual assumptions;
- normative criteria;
- comparator population;
- feasibility constraints;
- subject participation;
- reasons for later revision.

Reference provenance reduces the risk that a benchmark appears objective merely because the decisions underlying its construction have become invisible.

10.27 Hybrid Reference Systems

Many practical restorative judgments use several references simultaneously.

A hybrid reference can combine:

- historical restitution for identifiable property;
- counterfactual estimation for lost income;

- capability threshold for present welfare;
- normative reform for institutional relations;
- subject-selected future for assistance.

Formally, let

$$\mathcal{B} = (B_1, B_2, \dots, B_k) \quad (312)$$

denote a reference bundle.

Each comparison object Q_i can be associated with its own reference B_i .

The resulting restorative profile becomes

$$Q_i(\mathcal{G}^R) \succeq_i Q_i(B_i) \quad i = 1, \dots, k. \quad (313)$$

Equation 313 is often more realistic than a single global baseline.

10.28 Reference Pluralism

Hybrid reference should be distinguished from unstructured baseline switching.

Reference pluralism means that different restorative objects can legitimately require different references when the choice is explicit and normatively justified.

For example, property identity can use a historical reference. Income loss can use a counterfactual reference. Current healthcare can use a needs-based reference. Institutional reform can use a normative reference.

The existence of several references therefore need not indicate theoretical incoherence.

Incoherence arises when institutions move between references opportunistically without explaining the change.

10.29 Reference Conflict

Different references can recommend incompatible interventions.

A no-harm counterfactual can support reinstatement. The present subject can prefer exit. A normative institutional reference can require transformation of the workplace rather than reconstruction of the previous role.

Let

$$R^C, \quad R^A, \quad R^N \quad (314)$$

denote interventions favored by counterfactual, subject-directed, and normative references respectively.

There is no general reason to expect

$$R^C = R^A = R^N. \quad (315)$$

Equation 315 shows why reference conflict requires a theory of priority or procedural resolution.

The later justice sections will address these conflicts.

10.30 Reference Hierarchy

Some theories can impose a hierarchy among references.

A rights-based system can first require that all candidate references satisfy basic rights.

Within the admissible set, subject preference can receive priority.

Where several subject-compatible options remain, counterfactual fidelity can serve as a secondary criterion.

A schematic hierarchy is

$$\text{rights} \succ \text{subject agency} \succ \text{counterfactual fidelity}. \quad (316)$$

Equation 316 is illustrative rather than a claim of the paper.

Different theories can order these considerations differently.

The important methodological point is that priority should be explicit.

10.31 Lexical and Non-Lexical Reference Rules

Priority among references can be lexical or tradeable.

A lexical rule can require satisfaction of one reference condition before another is considered.

For example,

$$B_1 \succ_{\text{lex}} B_2 \quad (317)$$

means that B_2 matters only after the constraints associated with B_1 have been satisfied.

A non-lexical rule can permit trade-offs.

The distinction matters because many conflicts among restitution, welfare, agency, and institutional transformation cannot be resolved merely by specifying several references.

A theory must also state how conflicts among them are handled.

10.32 Reference Weighting

Where references are aggregated, weights become normatively significant.

A composite reference score can take the form

$$D_R = \sum_{i=1}^k w_i d_i (Q_i(\mathcal{G}^R), Q_i(B_i)). \quad (318)$$

Equation 318 requires weights w_i .

The weights cannot be justified by mathematics alone.

They reflect judgments concerning the relative importance of historical fidelity, current need, capability, autonomy, recognition, relational equality, or another value.

The atlas will therefore separate mathematical comparison from normative weighting.

10.33 Reference Incomparability

Some references can be normatively incomparable.

A singular relationship lost through harm can lack a common scale with a new career opportunity generated through assistance. A public acknowledgment can lack a common measure with financial compensation.

In such cases, the theory can preserve multiple reference-relative judgments rather than forcing a synthetic scalar comparison.

The result is a restorative profile rather than one score.

10.34 Temporal Index of the Reference

Every reference requires a temporal index.

The comparison can concern:

- the moment immediately before harm;
- the moment at which return occurs;
- a future horizon;
- an entire trajectory;
- a long-run distribution;
- an asymptotic structure.

Let

$$\mathcal{G}^{\text{ref}}(\tau) \quad (319)$$

denote the reference defined over horizon τ .

A state can match the reference at one time and diverge later.

Reference time should therefore be stated separately from comparison time.

10.35 Instantaneous Reference

An instantaneous reference compares systems at a selected time t :

$$Q(\mathcal{G}_t^R) \ E \ Q(\mathcal{G}_t^{\text{ref}}). \quad (320)$$

This criterion is useful for current income, formal status, present health variables, or immediate access.

It provides limited information concerning future evolution.

10.36 Finite-Horizon Reference

A finite-horizon reference compares development over $[t, t + \tau]$.

Let

$$\gamma_{[t, t+\tau]}^R \ E \ \gamma_{[t, t+\tau]}^{\text{ref}}. \quad (321)$$

Equation 321 becomes relevant where stable re-entry or rehabilitation matters beyond the initial moment of return.

The choice of τ is itself substantive.

A one-month restoration can be inadequate where the relevant generative cycle operates across years.

10.37 Long-Run Reference

Some comparisons concern stable patterns or distributions.

A long-run reference can involve

$$\lim_{t \rightarrow \infty} Q(\mathcal{G}_t), \quad (322)$$

where such a limit is meaningful, or a long-run invariant distribution.

Attractor and resilience analysis frequently employ this type of reference.

Long-run comparison can conceal severe short-term injustice. A system can converge eventually while imposing years of unnecessary loss.

Temporal justice therefore requires both long-run and path-sensitive analysis.

10.38 Historical-Path Reference

A path reference compares the entire sequence of relevant events.

Let

$$\mathcal{H}^R E_H \mathcal{H}^{\text{ref}}. \quad (323)$$

Equation 323 is substantially stronger than endpoint comparison.

Its relevance depends upon whether historical provenance, process, memory, or identity matters to the value under consideration.

The later atlas will distinguish several possible historical comparison relations.

10.39 Reference Updating Under New Evidence

Counterfactual and statistical references can change as new evidence becomes available.

A Bayesian representation can take the form

$$P\left(\mathcal{G}^{\text{ref}} \mid D_{t+1}\right) \propto P\left(D_{t+1} \mid \mathcal{G}^{\text{ref}}\right) P\left(\mathcal{G}^{\text{ref}} \mid D_t\right). \quad (324)$$

Equation 324 is one possible update mechanism.

Reference revision raises procedural questions. A claimant should not be subject to arbitrary repeated reevaluation merely because an institution continually changes its model.

Rules concerning finality, reopenability, and evidentiary significance are therefore required.

10.40 Reference Updating Under Changed Values

The normative reference can also change because the subject or society revises its values.

A person can no longer desire the profession that formed the original counterfactual baseline. A community can reject a traditional governance form that earlier reparative discourse assumed should be restored. Society can recognize that the former institutional structure was discriminatory.

Let

$$V_t \longrightarrow V_{t+1} \quad (325)$$

denote a change in the evaluative framework.

The associated reference can change:

$$\mathcal{G}_{V_t}^* \neq \mathcal{G}_{V_{t+1}}^*. \quad (326)$$

Equation 326 demonstrates that normative reference can be historically dynamic.

This feature is especially important in long-delayed reparations.

10.41 Historical Change in the Reference Environment

The surrounding world changes independently of the claimant.

A reference constructed by simply restoring the subject's earlier state can therefore become incoherent if the relevant external environment no longer exists.

A researcher returning after ten years enters another literature and institutional field. A worker returns to another technology. A community returns to a changed economy and ecology.

A reference system should therefore include the environment appropriate to the comparison.

Let

$$\mathcal{G}_t^{\text{ref}} = \left(X_t^{\text{ref}}, \Xi_t^{\text{ref}}\right). \quad (327)$$

Equation 327 makes environmental reference explicit.

10.42 Static-World Fallacy

A common restorative error is to imagine that the subject changed while the rest of the world remained fixed.

This paper calls the assumption the *static-world fallacy*.

The error takes the form

$$\Xi_{t_1} = \Xi_{t_0} \quad (328)$$

when the relevant environment has in fact evolved.

The fallacy can make literal return appear more powerful than it is.

Restoration of the same credential, equipment, legal status, or social role can have different consequences in the changed environment.

10.43 Reference and Opportunity Cost

Baseline selection also determines which opportunities count as lost.

If the reference is the pre-harm state, opportunity cost can appear limited to the immediate condition.

If the reference is the expected no-harm trajectory, later opportunities enter the analysis.

If the reference is a capability threshold, losses above the threshold can receive less institutional attention.

If the reference is subject-selected, opportunities the subject no longer values can become less relevant to practical assistance while remaining relevant to corrective history.

The reference therefore shapes the apparent scale of loss.

10.44 Reference and Responsibility

Different references can also support different accounts of responsibility.

Corrective responsibility frequently relies upon comparison between actual and no-wrong conditions.

Social assistance can rely upon present need.

Transformative justice can rely upon a normative institutional reference.

Generative justice can rely upon a reference concerning control and value circulation.

A single institutional process should therefore avoid conflating the baseline used to determine responsibility with the baseline used to determine present support.

10.45 Reference and Compensation

Compensation amounts depend directly upon reference choice.

Let V be the valuation function. A compensatory amount can take the form

$$M = V \left(Q(\mathcal{G}^{\text{ref}}) \right) - V \left(Q(\mathcal{G}^H) \right). \quad (329)$$

Equation 329 is conceptually familiar.

Its apparent simplicity conceals several choices:

- which reference;
- which comparison object;
- which valuation;
- which temporal horizon;
- which uncertainty model.

Different reasonable choices can produce different amounts.

This is one reason compensation should not be mistaken for a theory-free measurement of loss.

10.46 Reference and Rehabilitation

Rehabilitation also requires a target.

Possible targets include:

- pre-harm function;
- current maximum feasible function;
- independent living threshold;
- subject-selected activity;
- ability to perform a specific occupational role;
- general capability expansion.

The appropriate target depends upon the purpose of rehabilitation and the subject's present priorities. A narrow pre-harm reference can become inappropriate when the subject prefers a new mode of life.

10.47 Reference and Re-Entry

Re-entry can use several references.

A formal reference can ask whether the original title or membership has been restored.

A functional reference can ask whether the subject can again participate effectively.

A network reference can ask whether access to relevant professional relations has been restored.

A subject-directed reference can ask whether re-entry remains part of the future the subject wishes to pursue.

The practical design of re-entry should identify which reference it uses.

10.48 Reference and Recognition

Recognition-based repair can compare public standing before and after harm, social meaning under a no-harm counterfactual, or a normative standard of equal recognition [25, 20].

The latter is especially important where the pre-harm social meaning was itself hierarchical.

Recognition can therefore be restorative and transformative simultaneously.

10.49 Reference and Generative Assistance

Generative assistance frequently requires less historical specificity than corrective compensation.

Its reference can be future-oriented:

$$\mathcal{R}_\tau(\mathcal{G}^{HA}) \supseteq \mathcal{T}_A^{\text{adm}}, \quad (330)$$

where $\mathcal{T}_A^{\text{adm}}$ denotes a subject-directed admissible target region.

Equation 330 asks whether assistance creates viable paths toward futures the present subject can legitimately pursue.

It does not require those futures to reproduce the lost trajectory.

10.50 Reference and Prevention

Preventive governance uses prospective references.

The reference can be a safe state, acceptable risk distribution, rights- preserving condition, or sufficiently resilient system.

Let

$$\mathcal{N}_{\text{safe}} \subseteq \mathcal{X} \quad (331)$$

denote the acceptable preventive region.

A preventive policy seeks to keep the system within or away from boundaries of $\mathcal{N}_{\text{safe}}$, depending upon the application.

Preventive reference therefore differs fundamentally from a no-harm historical counterfactual.

10.51 Reference and Interruption

During an ongoing harm, the relevant reference can be the condition required to stop further propagation.

An interruption target can therefore be comparatively modest:

$$H_{\text{rate}}(\mathcal{G}^I) \leq H_{\text{max}}. \quad (332)$$

Equation 332 does not require restoration of the whole system.

It defines success relative to the immediate temporal function.

This phase-specific baseline reinforces the need for temporal indexing.

10.52 Counterfactual Paternalism

The no-harm counterfactual can acquire excessive normative authority.

Suppose an institution estimates that subject A would have pursued career C without the harm. The institution can then design all assistance around return to C , even though the present subject prefers another future.

The paper calls this error *counterfactual paternalism*.

Formally, counterfactual paternalism treats

$$\mathcal{G}^{-H} \quad (333)$$

as both an epistemic model of lost opportunity and the uniquely authoritative normative target.

These are separate roles.

The counterfactual can help establish the magnitude or nature of a historical loss while the present subject retains authority over future-directed support.

10.53 Baseline Capture

Reference selection can also be influenced by institutional power.

An employer can choose a baseline minimizing apparent lost earnings. A state can select a historical date favorable to its property claim. A reparations program can define a comparator group that reduces eligibility. A beneficiary can select an exceptionally favorable counterfactual.

The paper refers to strategic control over benchmark construction as *baseline capture*.

Baseline capture is a procedural problem.

Reference selection should therefore be transparent, contestable, and supported by reasons.

10.54 Reference-Selection Procedure

A just reference-selection process can include:

- disclosure of the candidate references;
- explanation of their epistemic basis;
- participation of affected subjects;
- opportunity to propose alternatives;
- independent review;
- sensitivity analysis;
- documentation of uncertainty;
- explanation of normative assumptions;
- provenance of later revisions.

The process should be proportionate to the stakes.

A small standardized reimbursement need not require a philosophical tribunal. A multi-generational reparation or reconstruction of a career trajectory can justify much stronger procedural scrutiny.

10.55 Reference Sensitivity Analysis

Where several plausible references exist, a restorative conclusion can be tested for sensitivity.

Let $\mathfrak{B}$ denote the plausible reference family. A conclusion C_R is reference-robust when

$$C_R(B_i) = C_R(B_j) \quad \text{for all relevant } B_i, B_j \in \mathfrak{B}. \quad (334)$$

Equation 334 is a strong form.

More generally, one can examine which components of the conclusion change when the reference changes.

Reference sensitivity is particularly useful where compensation or priority depends heavily upon uncertain counterfactual assumptions.

10.56 Reference-Robust Minimum Claims

Even where strong restorative conclusions vary across references, weaker claims can remain stable.

Suppose every plausible reference supports:

- correction of a false record;
- restitution of an identifiable asset;
- current healthcare;
- protection against retaliation.

These measures can be justified without resolving the entire baseline dispute.

Let

$$\mathcal{R}_\cap = \bigcap_{B \in \mathfrak{B}} \mathcal{R}(B) \quad (335)$$

denote the set of interventions supported across the reference family.

Equation 335 provides one formal expression of reference-robust action.

10.57 Reference-Dependent Restorative Profiles

The restorative profile introduced in Equation 199 can itself depend upon the reference.
Let

$$\mathbf{R}^B = (r_1^B, r_2^B, \dots, r_n^B) \quad (336)$$

denote the profile under reference B .

Comparison of $\mathbf{R}^{B_1}$ and $\mathbf{R}^{B_2}$ reveals which dimensions are baseline-sensitive.

This structure is preferable to hiding sensitivity within one aggregate score.

10.58 Reference Families Rather Than Singular Baselines

In complex cases, the appropriate analytical object can be a reference family

$$\mathfrak{B} = \{B_{\text{pre}}, B_{\text{cf}}, B_{\text{stat}}, B_{\text{norm}}, B_{\text{suff}}, B_{\text{BF}}, B_A\}. \quad (337)$$

Equation 337 contains historical, counterfactual, statistical, normative, sufficiency, best-feasible, and subject-directed references.

The theory can ask how a proposed intervention performs across the family rather than pretending that one reference exhausts every restorative purpose.

10.59 Reference Relations

References themselves can be compared.

For example, the normative reference can dominate the historical reference:

$$\mathcal{G}^{\text{norm}} \succ_V \mathcal{G}^{\text{pre}}. \quad (338)$$

A best-feasible reference can be bounded by the normative ideal:

$$\mathcal{G}^{\text{BF}} \preceq_V \mathcal{G}^{\text{norm}}. \quad (339)$$

A subject-directed reference can lie outside the counterfactual trajectory:

$$\mathcal{G}^A \notin \mathfrak{H}^{-H}. \quad (340)$$

These relations help clarify the structure of competing restorative objectives.

10.60 Historical Fidelity

One evaluative dimension is fidelity to the historical or counterfactual reference.

Let

$$F_H(\mathcal{G}^R, B) \quad (341)$$

measure or classify historical fidelity under baseline B .

The paper treats F_H as one dimension rather than a master value.

High historical fidelity can be desirable where restitution of a specific object matters.

It can be undesirable where the historical system was unjust or the subject now rejects the earlier trajectory.

10.61 Normative Improvement

A separate dimension concerns whether the post-harm system improves upon the reference under a selected theory of value.

Let

$$I_V(\mathcal{G}^R, B) \quad (342)$$

represent normative improvement.

A system can satisfy

$$F_H \text{ low, } I_V \text{ high.} \quad (343)$$

Equation 343 describes transformative restoration.

Conversely, strong historical similarity can coexist with low normative improvement.

10.62 Subjective Acceptance

A third dimension concerns whether the present subject accepts the proposed reference and restorative direction.

Let

$$A_S(B) \quad (344)$$

represent the subject's acceptance of reference B , subject to the limitations of reducing complex preference to one formal variable.

Subject acceptance does not determine justice alone.

Its inclusion prevents the reference system from becoming entirely external to the person whose future is being organized.

10.63 Public Justifiability

Some restorative references must also be publicly justifiable.

A public compensation program cannot select baselines through criteria that are arbitrary, discriminatory, or inaccessible to similarly situated claimants.

Let

$$J_{\text{pub}}(B) \quad (345)$$

represent the public justifiability of reference B .

This dimension is particularly important for mass reparations, social protection, public employment, and administrative compensation.

Individualized counterfactual precision can conflict with administrability and equal treatment.

10.64 Reference Choice in Mass Reparations

Large-scale reparations make baseline selection especially difficult.

A program can attempt individualized counterfactual valuation for every claimant, use standardized categories, provide equal payments, restore specific rights, finance community infrastructure, or combine these mechanisms.

De Greiff's discussion of large-scale reparations emphasizes that proportional individual compensation cannot simply function as the sole organizing model for programs involving large populations and broader goals such as recognition, civic trust, and social solidarity [13, 12].

A mass program therefore often uses several references simultaneously:

- individualized history for eligibility;
- standardized schedule for payment;
- social minimum for rehabilitation;
- normative institutional reference for reform;
- community-selected reference for collective projects.

This is another practical example of reference pluralism.

10.65 Reference Choice in Historical Injustice

Historical injustice generates particularly strong baseline disputes.

Waldron's discussion of superseding historic injustice demonstrates how later changes can affect the normative significance of historical claims [67]. Thompson's work similarly examines historical responsibility across generations [62].

A historical claimant can invoke:

- the condition before dispossession;
- the expected development without dispossession;
- current descendants' position;
- present relational equality;
- a transformed political order;
- current community priorities.

No one of these references can be assumed to subsume the others.

The generative approach adds attention to how historical deprivation altered the processes through which later conditions were produced.

10.66 Reference Choice in Workplace Harm

A workplace case illustrates the plurality at smaller scale.

A subject harmed through prolonged bullying or wrongful exclusion can be compared with:

- the employee immediately before the harm;
- the expected employee trajectory without the harm;
- statistically similar employees;
- a minimum safe-work threshold;
- a transformed non-dominating workplace;
- a subject-selected future outside the organization.

Each reference supports a different practical question.

The same settlement can therefore be adequate under one object-reference pair and inadequate under another.

10.67 Reference Choice in Academic Harm

Academic and research harms make the difference between state and generative reference particularly clear.

The reference can be:

- the researcher's previous institutional status;
- projected publication trajectory;
- expected network development;
- opportunity set among comparable researchers;
- access required for independent research;
- a subject-selected transition to another institution or field.

A financial settlement can approximate lost income while remaining largely silent about lost network trajectory.

A new mentorship can expand future generativity while remaining historically non-equivalent to the relation that disappeared.

Reference precision is therefore essential for avoiding overstatement.

10.68 Reference Choice in Health-Related Harm

Health-related restoration can compare current condition with:

- pre-injury function;
- predicted no-injury health;
- age-adjusted population norms;
- minimum independent-living capacity;
- best feasible rehabilitation outcome;
- subject-selected functioning priorities.

These references need not agree.

Pre-injury function can become medically unattainable. Population norms can be irrelevant to the person's previous life. Best feasible recovery can differ from what the person values most.

A plural reference framework is therefore especially appropriate.

10.69 Reference Choice in Ecological Restoration

Ecological restoration illustrates the baseline problem at system scale.

Possible references include:

- a historical ecosystem state;
- a pre-disturbance composition;
- a range of historical variability;
- a resilient functional state;

- a climate-adjusted future ecosystem;
- a normatively preferred human–ecological relation.

Historical replication can be ecologically impossible under changed climate and land-use conditions. A functional or resilience reference can therefore become more relevant than species-by-species historical identity.

This case anticipates the later distinction among state, structural, functional, attractor, and robustness equivalence.

10.70 Reference Choice and Scale

Reference selection is scale-dependent.

An individual reference can favor restoration of one worker’s career. An organizational reference can favor overall functioning. A societal reference can favor distributional equality. An ecological reference can constrain all three.

Let

$$B^{(s)} \tag{346}$$

denote a reference at scale s .

There is no general reason to expect

$$B^{(s_1)} = B^{(s_2)}. \tag{347}$$

Equation 347 requires multi-scale restorative analysis where interventions generate substantial cross-scale effects.

10.71 Reference Choice and System Boundary

Reference choice also depends upon which actors and relations are included inside the system boundary.

A no-harm reference for one employee can hold the rest of the organization fixed.

A broader reference can model how colleagues, projects, and organizational decisions would also have changed.

The second reference can generate a very different counterfactual.

The system boundary should therefore accompany the reference metadata.

10.72 Reference Granularity

A reference can be coarse or fine-grained.

A coarse reference can specify annual income.

A finer reference can specify monthly income, role, network position, health, and professional output.

Greater granularity can reveal additional loss.

It can also increase uncertainty and measurement burden.

Reference granularity should therefore match the legitimate purpose of the comparison.

10.73 Reference Abstraction

Several detailed references can map to the same abstract reference.

Let

$$\alpha(B_i) = B^{\text{coarse}} \tag{348}$$

for several B_i .

Equation 348 permits institutions to use a coarse common standard while acknowledging that several detailed histories are consistent with it.

This structure can be useful under counterfactual uncertainty.

10.74 Reference Equivalence

Different references can themselves be equivalent under the comparison relevant to a particular object.

For example,

$$Q(B_1) = Q(B_2) \quad (349)$$

even where $B_1 \neq B_2$ globally.

If two baselines imply the same reference income, the income-restoration claim can be reference-robust even though other dimensions differ.

This distinction can reduce unnecessary disputes about irrelevant baseline differences.

10.75 Reference Dominance

One reference can dominate another under a selected value framework.

Let

$$B_1 \succeq_V B_2. \quad (350)$$

Equation 350 means that B_1 is at least as desirable as B_2 under V .

If a post-harm intervention can reach B_1 , a theory can question why restoration should stop at B_2 .

This argument must be used carefully. Corrective justice can preserve special historical claims even where another state has greater aggregate value.

Dominance under one value theory does not automatically extinguish historical entitlement.

10.76 Reference and the Meaning of Improvement

The concept of improvement is reference-relative.

A system can improve relative to the damaged actual state while remaining below the pre-harm state.

It can exceed the pre-harm state while remaining below a normative ideal.

It can be normatively superior to the no-harm counterfactual while historically less similar to it.

A precise restorative statement should therefore distinguish:

$$\text{improvement over actual} \quad (351)$$

from

$$\text{equivalence to reference} \quad (352)$$

and from

$$\text{normative superiority to reference.} \quad (353)$$

These are three different claims.

10.77 Reference and the Meaning of Failure

Failure is similarly reference-dependent.

A rehabilitation program can fail to restore pre-injury function while succeed in achieving a subject-selected independence goal.

A reparations program can fail to reconstruct a historical community while succeed in establishing new community-controlled institutions.

A compensation program can fail to reproduce lost opportunity while succeed in providing meaningful future options.

A theory should therefore specify the standard under which failure is asserted.

10.78 Multiple Success Criteria

A restorative intervention can have several success criteria:

$$S(R) = (S_{\text{historical}}, S_{\text{counterfactual}}, S_{\text{normative}}, S_{\text{sufficiency}}, S_{\text{subject}}) . \quad (354)$$

Equation 354 permits the same intervention to perform differently across reference dimensions.

This multidimensional representation is preferable to forcing the criteria into one score before the value theory has been specified.

10.79 Reference and Evidentiary Standard

Different references require different evidence.

A pre-harm actual state can rely upon records and testimony.

A counterfactual reference requires causal assumptions.

A statistical reference requires comparator data.

A normative reference requires philosophical and political justification.

A subject-selected reference requires meaningful participation and evidence concerning present feasibility.

The evidentiary status of the reference should therefore accompany the restorative claim.

10.80 Epistemic Confidence in References

Let

$$K_B \quad (355)$$

denote the epistemic basis of reference B .

A reference-confidence statement can take the form

$$\text{Conf}(B \mid K_B) . \quad (356)$$

Equation 356 prevents a counterfactual reference from appearing as epistemically equivalent to a directly documented historical state.

Confidence can be qualitative, interval-based, model-relative, or probabilistic.

10.81 Normative Confidence in References

Normative agreement can also vary.

A minimum safety reference can have broad public support.

A specific theory of ideal distributive justice can be deeply contested.

A public institution should distinguish epistemic certainty from normative consensus.

The fact that a benchmark is measurable does not make it normatively uncontroversial.

10.82 Reference Metadata

A complete restorative reference should therefore contain metadata describing its construction.
Let

$$B = (G_B, T_B, S_B, M_B, V_B, K_B, P_B), \quad (357)$$

where:

- G_B identifies the reference-generating method;
- T_B identifies the temporal index or horizon;
- S_B identifies system scale and boundary;
- M_B identifies the modeling assumptions;
- V_B identifies the normative framework;
- K_B identifies the epistemic basis and uncertainty;
- P_B identifies the procedural history of reference selection.

Equation 357 makes the reference auditable.

This metadata structure will become part of the later atlas.

10.83 A Taxonomy of Restorative References

Table 2 summarizes the principal reference families developed in this section.

Table 2 demonstrates why the phrase “the reference state” can be misleading. Complex restoration can involve a family of references with different epistemic and normative roles.

10.84 Reference Selection Across Temporal Phases

Reference selection also varies across the pre-harm, ongoing-harm, and post-harm domains.

Prevention frequently uses safety, rights, risk, resilience, or option- preservation references.

Interruption frequently uses cessation, containment, and preservation-of-options references.

Post-harm corrective justice frequently uses historical and counterfactual references.

Rehabilitation can use functional or subject-selected references.

Social protection can use sufficiency and present-need references.

Transformative governance can use normative and structural references.

Generative assistance can use future reachable sets and subject-directed target regions.

The reference is therefore partly determined by institutional function.

10.85 Reference Selection Across Institutions

Institutions can also be distinguished by characteristic reference types.

Courts often rely upon historical and counterfactual references.

Insurance frequently relies upon contractual and actuarial baselines.

Social protection relies upon eligibility and sufficiency references.

Healthcare can use functional, clinical, and subject-selected references.

Organizations can use role, performance, safety, and re-entry references.

Reparations programs combine historical, normative, recognition, and distributional references.

The same history can therefore generate multiple valid institutional baselines.

Coordination requires awareness of these differences.

Table 2: Principal reference systems for restorative comparison

Reference family	Primary question	Characteristic strength	Characteristic limitation
Pre-harm actual	What condition actually existed before harm?	Historical observability and object continuity	Can reproduce an unjust or temporary baseline
Pre-harm trajectory	Where was the existing process heading?	Captures ongoing development	Requires extrapolation
No-harm counterfactual	What would likely have occurred without the harm?	Relevant to causal loss and corrective reasoning	Unobserved and model-dependent
Counterfactual distribution	What range or distribution of futures was plausible?	Represents uncertainty explicitly	Requires a rule for evaluating the distribution
Reachable-set counterfactual	Which futures remained possible without the harm?	Avoids commitment to one lost future	Can be broad and difficult to identify
Statistical peer	What occurs among selected comparators?	Empirically tractable in some domains	Comparator selection and normality problems
Normative ideal	Which system would be just or desirable?	Supports transformation beyond historical restoration	Depends upon contested value theory
Rights-based	Which states satisfy protected rights?	Defines admissible boundaries without one endpoint	Can leave distributive and generative questions open
Sufficiency	Has an adequate threshold been reached?	Useful for floors and urgent support	Can understate corrective or reparative claims
Best feasible	What is the strongest attainable response now?	Adapts to irreversible constraints	Can normalize avoidable institutional weakness
Present need	What does the subject require now?	Suitable for assistance and social protection	Does not settle historical responsibility
Subject-directed	Which future does the present subject seek to pursue?	Protects agency and future openness	Preference can be historically constrained
Transformative	Which structure should replace the one implicated in harm?	Addresses structural causes and unjust baselines	Can diverge sharply from historical fidelity
Hybrid	Which reference is appropriate for each restorative object?	Matches plural structure of actual practice	Requires explicit coordination among standards

10.86 Baseline Translation Across Institutions

A reference constructed for one institutional purpose should not automatically be transferred into another.

An insurance valuation of earning loss does not establish the person's full social loss.

A clinical functional baseline does not establish legal responsibility.

A judicial damages estimate does not define the appropriate rehabilitation target.

An administrative social minimum does not define the magnitude of reparation.

Institutional handoffs therefore require translation of reference purpose.

10.87 Reference Persistence and Expiration

Some references lose relevance over time.

A pre-harm occupation can cease to exist. A statistical comparator group can change. A subject can revise priorities. A once-feasible rehabilitation target can become medically unavailable.

A reference architecture should therefore specify whether the reference is fixed or revisable.

Let

$$\text{valid}_B(t) \in \{0, 1\} \quad (358)$$

denote whether reference B remains valid for the specified restorative function at time t .
Equation 358 is conceptual. Reference validity can be graded rather than binary.

10.88 Reference Preservation for Historical Claims

The fact that a reference becomes practically obsolete does not necessarily eliminate its historical significance.

A person can no longer wish to return to a lost profession while the estimated no-harm career remains relevant to compensation.

A community can reject literal reconstruction while the historical baseline remains relevant to acknowledgment.

A reference can therefore remain valid for one function and expire for another.

This is another reason references should be indexed by purpose.

10.89 Reference Function

Let

$\text{Role}(B) \in \{\text{liability, valuation, restitution, rehabilitation, assistance, transformation, recognition, prevention}\}.$
(359)

Equation 359 records the function for which a reference is used.

The same reference can perform several roles, while different roles can require different references.

Explicit role metadata reduces baseline confusion.

10.90 Reference and the Atlas of Equivalence

The reference taxonomy developed here precedes the atlas of comparison objects and comparative relations because equivalence cannot be evaluated in an unreferenced space.

The complete comparison will eventually take the form

$$Q(\mathcal{G}^R) \sim_{E, \tau, \varepsilon, K}^T Q(B), \quad (360)$$

where B is a reference system with explicit provenance and normative role.

Equation 360 demonstrates that the reference is analytically distinct from the object Q and relation E .

This distinction prevents several common confusions.

A capability reference is not itself a comparative relation.

A counterfactual reference does not determine whether equality, sufficiency, or dominance is required.

A normative reference does not determine which object within that system should be compared.

A subject-directed reference does not determine the procedural legitimacy of the intervention used to reach it.

10.91 Reference Selection as a Normative Decision

Reference selection is therefore partly a decision about justice.

Selecting the pre-harm state prioritizes historical fidelity.

Selecting the no-harm counterfactual prioritizes causal rectification.

Selecting a social floor prioritizes sufficiency.

Selecting a normative ideal prioritizes transformation.

Selecting the best feasible state prioritizes practical attainability.

Selecting a subject-directed future prioritizes current agency.

None of these priorities is generated by mathematics alone.

A theory of restorative justice should therefore state why the selected reference is normatively appropriate for the institutional function under consideration.

10.92 Reference Selection as an Epistemic Decision

Reference selection is simultaneously an epistemic decision.

The theory must state what is known, inferred, modeled, assumed, or uncertain.

A richly specified no-harm history can be normatively attractive and epistemically weak.

A statistical baseline can be epistemically strong and normatively inadequate.

A subject-selected future can be normatively important while its practical feasibility remains uncertain.

Reference legitimacy therefore contains at least two dimensions:

$$L(B) = (L_{\text{epistemic}}(B), L_{\text{normative}}(B)). \quad (361)$$

Equation 361 resists the assumption that a reference becomes legitimate merely by being empirically measurable or normatively desirable.

10.93 Reference Selection as a Procedural Decision

A third dimension concerns procedure.

Who selected the baseline?

Was the subject heard?

Were alternative models considered?

Were comparator groups disclosed?

Could assumptions be challenged?

Was a later reference revision explained?

These questions matter independently of epistemic and normative quality.

A reference selected through opaque or dominating procedures can undermine the legitimacy of the restorative process.

Reference selection is therefore itself part of procedural justice.

10.94 Three-Dimensional Reference Legitimacy

The legitimacy of reference B can be represented schematically by

$$\mathcal{L}_B = (L_E, L_N, L_P), \quad (362)$$

where L_E denotes epistemic adequacy, L_N denotes normative justification, and L_P denotes procedural legitimacy.

Equation 362 does not require the dimensions to be reduced to numbers.

A reference can be epistemically strong and normatively weak.

It can be normatively attractive and epistemically speculative.

It can be substantively reasonable and procedurally imposed.

The atlas should preserve these distinctions.

10.95 Reference Indeterminacy

Some cases do not permit selection of one uniquely superior reference.

A reference family can remain legitimately plural after extensive analysis.

This condition is *reference indeterminacy*.

Reference indeterminacy differs from ignorance.

The institution can understand the available baselines and the values each represents while lacking a justified rule for collapsing them into one.

A plural restorative profile can be preferable to arbitrary selection.

10.96 Reference Contestation

Restorative disputes can therefore be disputes about baselines rather than facts within a shared baseline.

One party can argue for pre-harm actual state.

Another can argue for no-harm trajectory.

Another can argue for social sufficiency.

Another can argue for a transformed institution.

Recognizing the level of disagreement can improve deliberation.

Parties need not argue about the same number when they are actually answering different comparative questions.

10.97 Reference Transparency

A basic institutional principle follows:

Every consequential restorative comparison should identify the reference system, the purpose for which it is used, the evidence supporting it, and the normative reason for treating it as relevant.

Reference transparency is especially important where a quantitative model gives the appearance that the baseline was discovered rather than chosen.

10.98 Reference Humility

A second principle concerns humility.

An institution can say:

This reference is used for calculation of lost income.

without saying:

This reference represents the life the person truly should have lived.

The difference is substantial.

Reference humility permits a model to perform a bounded function without claiming authority over the entire meaning of the subject's history.

10.99 Reference Multiplicity and Restorative Honesty

Where several references matter, restorative honesty requires acknowledging the plurality.

A subject can be fully restored relative to formal employment status, partly restored relative to counterfactual earnings, above a social-protection floor, below a professional-network reference, and successfully moving toward a subject-selected new future.

These statements can all be true simultaneously.

The language of restoration becomes more informative when these distinctions are made explicit.

10.100 Implications for the Taxonomy of Comparison Objects

The taxonomy developed in this section yields several implications for the next stage of the paper.

First, a comparison object should never be interpreted independently of the reference against which it is evaluated.

Second, one reference system can provide several comparison objects.

Third, different objects within the same case can require different references.

Fourth, the same object can be compared against historical, counterfactual, statistical, normative, sufficiency, feasible, and subject-directed references.

Fifth, reference choice changes the normative meaning of equality.

Sixth, no-harm counterfactual comparison remains important without becoming the universal model of restoration.

Seventh, historical fidelity and normative improvement are independent dimensions.

Eighth, subject-directed reference is essential for future-oriented assistance but does not erase corrective claims grounded in historical reference.

Ninth, statistical references require independent normative justification and should not be equated with justice.

Tenth, best-feasible references should remain sensitive to the possibility that institutional or technological improvement can expand the feasible set.

Eleventh, reference selection is epistemic, normative, and procedural simultaneously.

Twelfth, reference pluralism is often a feature of adequate restorative analysis rather than evidence of theoretical failure.

10.101 Transition to Objects of Restorative Comparison

The formal theory now contains two of the elements required by a complete restorative claim.

Section 9 specified a modular representation of the generative system.

The present section specified the family of systems against which a post-restoration system can be compared.

The next question concerns the projection Q :

Which part, property, relation, dynamic, possibility structure, or historical feature of the two systems is actually being compared?

This question cannot be answered by the choice of reference alone.

A no-harm counterfactual contains resources, states, relations, networks, trajectories, transition rules, reachable sets, attractors, control structures, value flows, meanings, and histories. A theory must select which of these objects carries restorative significance.

Section 11 therefore develops an extensive taxonomy of objects of restorative comparison. The taxonomy begins with entities, resources, and attributes and proceeds through relations, networks, states, trajectories, traces, reachable sets, dynamical structures, causal structures, conversion mechanisms, generative processes, meanings, and historical configurations. Only after the comparison objects have been mapped will the paper turn to the distinct question of which comparative relations, including equality, isomorphism, simulation, bisimulation, approximation, dominance, and other relations, can be applied to them.

11 Objects of Restorative Comparison

This section develops the first major taxonomy of the Atlas of Equivalence: the objects that can serve as the terms of restorative comparison. The reference systems examined in Section 10 specify *against what* the post-harm or post-restoration system is evaluated. The present section asks a different question:

Which component, property, relation, trajectory, mechanism, or historical structure of the two systems is actually being compared?

The distinction is foundational. A statement that two systems are equivalent is incomplete until the object of equivalence has been identified. Equal resources do not imply equal capabilities. Equal current states do not imply equal future dynamics. Similar observable behavior does not imply similar causal structure. Isomorphic networks do not imply identical relationships. Equal value output does not imply equal control over the conditions of production. Similar capability sets do not imply similar histories or conversion structures.

The taxonomy developed here is intentionally broad. Its purpose is to expose the full range of objects that restorative theories, legal doctrines, political- economic practices, capability approaches, relational theories, dynamical models, and generative-justice frameworks can implicitly or explicitly treat as the object of return.

The objects are organized into several families:

1. entities and identifiable objects;
2. resources and holdings;
3. attributes and scalar or vector properties;
4. formal statuses and institutional positions;
5. bodily and functional states;
6. capabilities and opportunities;
7. relations and relational positions;
8. networks and higher-order structures;
9. distributions across persons or states;
10. present system states;
11. trajectories and histories;
12. traces and observable behavior;
13. transition structures;
14. reachable and viable future sets;
15. dynamical laws and parameter structures;
16. attractors, basins, stability, and resilience;
17. perturbation-response structures;
18. causal and interventional structures;
19. conversion mechanisms;
20. control and security structures;
21. value-generation and value-circulation structures;
22. meanings, recognition, and symbolic structures;
23. provenance and historical configuration;

24. composite and multi-object restorative profiles.

The list is not claimed to be ontologically exhaustive. It is an analytical catalogue designed to make comparison claims more precise. New objects can be added where an application requires them.

11.1 Comparison Objects as Projections

Let the generative system be represented by the formal structure introduced in Section 9. A comparison object is a projection

$$Q_i : \mathcal{G} \longrightarrow \mathcal{Q}_i, \quad (363)$$

where $\mathcal{Q}_i$ is the space containing the selected restorative object.

Equation 363 makes clear that the object of comparison is not automatically the entire system. A restorative analysis can project the system into resource space, relation space, trajectory space, capability space, network space, value space, or another domain.

Two systems can therefore satisfy

$$Q_i(\mathcal{G}_1) = Q_i(\mathcal{G}_2) \quad (364)$$

while

$$Q_j(\mathcal{G}_1) \neq Q_j(\mathcal{G}_2) \quad (365)$$

for another object Q_j .

Equations 364 and 365 formalize the multidimensionality of restoration.

11.2 Entity Identity

The most concrete restorative object is an identifiable entity.

Examples include:

- a specific physical object;
- a parcel of land;
- a document;
- an artwork;
- a dataset;
- a biological specimen;
- a piece of equipment;
- a digital file;
- a legal instrument.

Let

$$Q_{\text{entity}}(\mathcal{G}) = e \quad (366)$$

denote the selected entity.

Entity-level restoration asks whether the same entity, a legally recognized equivalent, or a substitute has been returned.

The object is especially relevant to restitution.

Its apparent simplicity should not obscure the distinction between entity identity and generative significance. Return of the same laboratory instrument, property, archive, or credential does not imply that the object now occupies the same relational, historical, or productive position.

Entity identity is therefore among the strongest local and weakest global restorative objects.

11.3 Entity Bundle

A restorative object can consist of a bundle of identifiable entities:

$$Q_{\text{bundle}}(\mathcal{G}) = \{e_1, e_2, \dots, e_n\}. \quad (367)$$

Equation 367 becomes relevant where generativity depends upon complementarity among several items.

A research environment can require equipment, data, software access, authorization, workspace, and funding simultaneously.

Returning each element separately can still fail if their coordination differs.

This motivates later structural comparisons among bundles and their relations.

11.4 Resource Holdings

A second family concerns resources rather than unique entities.

Let

$$Q_{\text{resource}}(\mathcal{G}) = r \in \mathcal{R}_{\text{res}}. \quad (368)$$

Resources can include:

- money;
- income;
- savings;
- credit;
- housing;
- productive capital;
- time;
- educational access;
- healthcare;
- infrastructure;
- information;
- energy;
- land.

Resource comparison is central to distributive justice, compensation, insurance, and social protection.

Its institutional strength lies in tractability and transferability.

Its limitation lies in the fact that resource possession does not determine what a subject can actually achieve with those resources [55, 50].

11.5 Resource Vector

Many restorative problems require several resources to be represented jointly.

Let

$$\mathbf{r} = (r_1, r_2, \dots, r_k) \quad (369)$$

denote a resource vector.

Equation 369 permits comparison of multidimensional holdings without forcing immediate monetary aggregation.

This is important where resources are only partially substitutable.

Income, healthcare, legal status, and network access cannot necessarily be collapsed into one scalar without normative assumptions.

11.6 Resource Security

A resource can be available with different levels of security.

Let

$$Q_{\text{security}}(\mathcal{G}) = \sigma(c) \quad (370)$$

denote the security profile of condition c .

Relevant dimensions can include:

- expected duration;
- revocability;
- dependence upon another actor;
- procedural protection;
- portability;
- exposure to arbitrary withdrawal.

Two systems can therefore possess equal current resources and unequal resource security.

This object is especially relevant to non-domination and anticipatory generativity.

11.7 Attributes

An attribute is a property associated with an entity, subject, relation, or system.

Let

$$Q_{\text{attribute}}(\mathcal{G}) = a. \quad (371)$$

Examples include:

- income level;
- health score;
- professional rank;
- seniority;
- educational qualification;

- network centrality;
- trust level;
- productivity;
- risk score;
- reputation indicator.

Attribute comparison can be convenient because it compresses complex systems into tractable variables.

Its normative weakness is the same source as its administrative strength: different structures can produce the same attribute.

Equality of salary does not imply equality of authority. Equality of publication count does not imply equality of research freedom. Equality of network centrality does not imply equality of relational meaning.

11.8 Attribute Vector

A multidimensional attribute object can be represented as

$$\mathbf{a} = (a_1, a_2, \dots, a_n). \quad (372)$$

Equation 372 permits richer comparison while introducing the later problem of weighting and aggregation.

A vector can preserve plurality more faithfully than one score.

It can also conceal structural relations among the dimensions.

11.9 Formal Legal Status

A formal legal or institutional status can itself be a restorative object.

Let

$$Q_{\text{status}}(\mathcal{G}) = s_{\text{formal}}. \quad (373)$$

Examples include:

- employment status;
- citizenship;
- membership;
- professional license;
- academic enrollment;
- legal title;
- property entitlement;
- benefit eligibility.

Status restoration is often institutionally powerful because status controls other conditions.

The object should remain distinct from effective access. A formally restored status can fail to recreate the relationships, opportunities, or conversion pathways associated with the earlier status.

11.10 Institutional Position

Institutional position extends beyond formal title.

Let

$$Q_{\text{position}}(\mathcal{G}) = p_I. \quad (374)$$

The position can contain:

- decision authority;
- reporting relation;
- access to information;
- influence;
- visibility;
- seniority;
- institutional dependence;
- eligibility for future advancement.

Two employees with the same job title can occupy different institutional positions.
Reinstatement of title therefore does not establish restoration of position.

11.11 Bodily State

A bodily state can be selected as the restorative object.

Let

$$Q_{\text{body}}(\mathcal{G}) = x^{\text{body}}. \quad (375)$$

The object can include mobility, pain, strength, sensory function, physiological state, fatigue, or another medically or practically relevant feature.

Bodily-state comparison is central to health-related restoration.

The object can be compared with pre-injury state, expected no-injury state, population norm, best-feasible rehabilitation, or subject-selected functioning.

Its selection does not determine which baseline is appropriate.

11.12 Functional Capacity

Functional capacity concerns what the subject can practically do.

Let

$$Q_{\text{function}}(\mathcal{G}) = f. \quad (376)$$

Examples include:

- ability to work;
- ability to walk;
- ability to communicate;
- ability to perform a professional task;

- ability to live independently;
- ability to participate in education.

Functional comparison is broader than bodily-state comparison because different bodily configurations can support similar function.

This object therefore already introduces a form of abstraction from internal structure.

11.13 Capability Set

A capability set represents substantive opportunities available to a subject.

Let

$$Q_{\text{cap}}(\mathcal{G}) = \mathcal{K}_A. \quad (377)$$

The capability approach emphasizes the distinction between possession of resources and substantive freedom [55, 39, 50].

Capability comparison can therefore reveal differences hidden by resource equality.

A capability set can itself be compared through:

- equality;
- inclusion;
- cardinality;
- threshold satisfaction;
- dominance;
- weighted similarity;
- reachability.

The object does not determine the comparative relation.

11.14 Capability Profile

Where a full set representation is impractical, a capability profile can be used:

$$Q_{\text{cap-profile}}(\mathcal{G}) = (k_1, \dots, k_m). \quad (378)$$

Equation 378 can represent mobility, education, health, political participation, professional access, or another selected family.

The profile is easier to operationalize than a complete opportunity set.

Its content depends heavily upon the dimensions selected.

11.15 Recognized Capability Set

The subject's recognized or actionable possibilities can differ from objectively available capabilities.

Let

$$Q_{\text{recognized}}(\mathcal{G}) = \hat{\mathcal{K}}_A. \quad (379)$$

Equation 379 represents options the subject can identify and treat as practically available.

The distinction is relevant where information, imagination, trust, or prior institutional experience constrains effective use of formal options.

11.16 Preference Structure

A restorative theory can compare preferences or priorities.

Let

$$Q_{\text{pref}}(\mathcal{G}) = P_A. \quad (380)$$

Preference comparison is potentially useful for subject-directed assistance.

It is normatively delicate because preferences can change through learning, adaptation, coercion, or deprivation.

The capability literature's concern with adaptive preferences makes this object especially important and especially non-self-validating [55, 50].

11.17 Expectation Structure

Expectation about future response can be a separate object.

Let

$$Q_{\text{expect}}(\mathcal{G}) = \Pi_A. \quad (381)$$

Examples include expectations that:

- access will remain available;
- reporting will produce retaliation;
- an institution will follow its rules;
- investment in a project will be supported;
- collaboration will be reciprocated.

Expectation can change even where objective rights are restored.

This object is especially relevant to trust, security, and future investment.

11.18 Dyadic Relation

A specific relation can be the object of restorative comparison.

Let

$$Q_{\text{rel}}(\mathcal{G}) = r_{ij}. \quad (382)$$

The relation can include:

- trust;
- authority;
- mentorship;
- collaboration;
- care;
- contractual dependence;
- professional recognition;

- friendship;
- family relation.

Relational comparison is especially important where the value arises from the specific interaction between participants.

A replacement relation can be functionally similar without being historically or relationally identical.

11.19 Relation Type

The object can be the type rather than the particular relation.

For example,

$$Q_{\text{relation-type}}(\mathcal{G}) = \ell(r_{ij}) \quad (383)$$

can identify a relation as supervisory, collaborative, fiduciary, equal, dominating, dependent, or reciprocal.

A restored system can preserve the relation type while changing participants.

This distinction becomes important for structural equivalence.

11.20 Relational Quality

A relation can also be compared through qualities such as trust, reciprocity, security, equality, or domination.

Let

$$Q_{\text{rel-quality}}(\mathcal{G}) = \mathbf{q}_{ij}. \quad (384)$$

Relational equality and non-domination theories make such qualities normatively central [2, 71, 44].

The object can differ sharply from material exchange between the same parties.

11.21 Relational Position

The subject's position within a relation system can be selected as the object.

Let

$$Q_{\text{rel-pos}}(\mathcal{G}) = \pi_A^R. \quad (385)$$

Relevant features can include:

- centrality;
- dependency;
- brokerage;
- access;
- authority;
- exposure;
- structural isolation.

The same person can retain every formal resource while moving to a peripheral network position.

11.22 Network Topology

A network itself can be the restorative object.

Let

$$Q_{\text{network}}(\mathcal{G}) = \mathcal{R} = (N, L). \quad (386)$$

The comparison can concern:

- node set;
- edge set;
- degree distribution;
- connectedness;
- communities;
- centrality;
- path structure;
- structural holes;
- hierarchy.

A professional network destroyed through exclusion can later be rebuilt with different persons while preserving some topological properties.

Whether that constitutes adequate restoration depends upon the selected comparative relation and value theory.

11.23 Typed Network Structure

Many social networks contain several relation types.

A multiplex or typed network can be represented by

$$Q_{\text{multi-rel}}(\mathcal{G}) = \left(N, L^{(1)}, L^{(2)}, \dots, L^{(k)} \right). \quad (387)$$

Equation 387 can distinguish professional collaboration, trust, authority, financial flow, communication, and friendship.

A system can restore one network layer while another remains altered.

This is especially relevant to organizations and communities.

11.24 Higher-Order Relation Structure

Pairwise edges can be insufficient where value is generated through groups.

A research team, family, committee, community, or coalition can contain higher-order relations represented through hypergraphs, simplicial complexes, or another structure.

Let

$$Q_{\text{higher}}(\mathcal{G}) = \mathcal{H}_R \quad (388)$$

denote the selected higher-order relational structure.

The object becomes relevant where a group interaction cannot be reconstructed from dyadic relations alone.

11.25 Distribution Across Persons

A restorative object can be a distribution rather than an individual holding.

Let

$$Q_{\text{dist}}(\mathcal{G}) = P(Z) \quad (389)$$

denote the distribution of resource, capability, health, income, status, or another variable Z .

Distributional comparison is central to theories concerned with equality, priority, or aggregate institutional effects.

Two systems can have equal means and different distributions.

An individual claimant can be restored while the distributional system remains unchanged.

The object therefore belongs primarily to system-level analysis.

11.26 Joint Distribution

A joint distribution can preserve correlations among several dimensions:

$$Q_{\text{joint}}(\mathcal{G}) = P(Z_1, \dots, Z_k). \quad (390)$$

Equation 390 becomes important where the same marginal distributions conceal different patterns of intersection.

Income and health can each have the same distribution while their correlation differs.

This object can capture structural concentration of disadvantage.

11.27 Conditional Distribution

A restorative object can also be conditional:

$$Q_{\text{cond}}(\mathcal{G}) = P(Y | X). \quad (391)$$

This can represent promotion probabilities conditional on qualification, health outcome conditional on treatment, or opportunity conditional on social position.

Conditional distributions are especially relevant where formal equality coexists with different transition probabilities.

11.28 Present System State

The complete represented state can itself be the object:

$$Q_{\text{state}}(\mathcal{G}_t) = x_t. \quad (392)$$

State restoration asks whether selected state variables now correspond to the reference.

This object is intuitive and frequently useful.

Its limitation is temporal. Equal present states can evolve differently.

State equivalence therefore provides no automatic claim about future dynamics.

11.29 State Subspace

Restoration can concern only selected state dimensions.

Let I be an index set. Then

$$Q_{\text{substate}}(\mathcal{G}_t) = x_t^I. \quad (393)$$

Equation 393 is common in practice.

A legal remedy can restore salary and title while leaving health and relations outside its scope.

The object makes that limitation explicit.

11.30 State Distribution

For stochastic systems, the object can be a distribution over states:

$$Q_{\text{state-dist}}(\mathcal{G}_t) = P(X_t). \quad (394)$$

Two systems can then be compared statistically even where individual state identity is neither possible nor meaningful.

This object is useful for populations and uncertain systems.

11.31 Trajectory

A trajectory records the evolution of system state through time:

$$Q_{\text{traj}}(\mathcal{G}) = \gamma_{[t_0, t_1]}. \quad (395)$$

Trajectory comparison is stronger than endpoint comparison because it preserves the path taken.

Two systems can share the same endpoint and differ greatly in intermediate burden, duration, instability, or exposure.

Trajectory is therefore especially relevant to procedural and temporal justice.

11.32 Trajectory Family

Under uncertainty, the object can be a family of possible trajectories:

$$Q_{\text{traj-family}}(\mathcal{G}) = \mathfrak{X}. \quad (396)$$

This object can represent the possible evolution of a system under alternative actions or disturbances.

It is more expressive than one expected trajectory and less committed than one deterministic prediction.

11.33 Trajectory Distribution

A stochastic model can assign a probability distribution over trajectories:

$$Q_{\text{traj-dist}}(\mathcal{G}) = P(\gamma). \quad (397)$$

This object becomes relevant where restoration concerns the probability of different life courses or institutional developments.

Its use requires substantial epistemic caution because long-horizon trajectory distributions can be highly model-dependent.

11.34 Orbit

In dynamical systems, the orbit of state x under evolution operator Φ can be represented by

$$Q_{\text{orbit}}(\mathcal{G}) = \{\Phi^t(x)\}_{t \geq 0}. \quad (398)$$

Orbit comparison abstracts from some details of temporal parameterization.

It can therefore provide a useful object where qualitative dynamical structure matters more than exact timing.

11.35 Observable Trace

An observable trace records external behavior under observation map O :

$$Q_{\text{trace}}(\mathcal{G}) = \text{tr}_O(\gamma). \quad (399)$$

Trace comparison is relevant where internal state is inaccessible or where the institution legitimately evaluates only observable behavior.

Its limitation is familiar: different internal and branching structures can produce identical traces.

11.36 Trace Language

A system can be represented through the set of traces it can generate:

$$Q_{\text{trace-lang}}(\mathcal{G}) = \mathcal{L}_{\text{tr}}(\mathcal{G}). \quad (400)$$

Equation 400 becomes important for later trace equivalence.

The object captures possible observable behavior rather than one realized history.

11.37 Transition Relation

The transition relation itself can be the object.

For transition system $\mathfrak{T} = (S, L, \rightarrow)$,

$$Q_{\rightarrow}(\mathcal{G}) = \rightarrow. \quad (401)$$

This object asks which states can lead to which other states under which actions or events.

Two systems can share the same states and differ in available transitions.

This difference is central to future generativity.

11.38 Labeled Transition System

The entire labeled transition system can be compared:

$$Q_{\text{LTS}}(\mathcal{G}) = (S, L, \rightarrow). \quad (402)$$

This object supports simulation and bisimulation.

It is particularly useful for branching systems in which the ability to respond to future actions matters.

11.39 Transition Probability Kernel

For stochastic systems, the object can be the transition kernel:

$$Q_{\text{kernel}}(\mathcal{G}) = P(x_{t+1} \mid x_t, u_t). \quad (403)$$

Two systems can occupy the same present state while having different transition probabilities.

The object is therefore stronger than state comparison and weaker than complete historical identity.

11.40 Reachable Set

The reachable set is a central object for generative justice.

Let

$$Q_{\text{reach}}(\mathcal{G}; \tau) = \mathcal{R}_\tau(x_t). \quad (404)$$

The reachable set captures futures that remain possible under the selected action and disturbance classes.

This object avoids the claim that one specific counterfactual future would have occurred.

A harm can contract, expand, distort, fragment, or redirect the reachable set.

11.41 Reachable-Set Volume

A coarse object can be the size or volume of the reachable set:

$$Q_{\text{reach-vol}}(\mathcal{G}) = \mu(\mathcal{R}_\tau), \quad (405)$$

where μ is an appropriate measure.

This object can approximate option-space expansion.

Its limitation is severe: two reachable sets can have equal volume and entirely different composition.

Volume should therefore be treated as one attribute of future possibility rather than the future structure itself.

11.42 Reachable-Set Geometry

The geometry or topology of the reachable set can be compared:

$$Q_{\text{reach-geom}}(\mathcal{G}) = \text{Geom}(\mathcal{R}_\tau). \quad (406)$$

This can capture fragmentation, connectivity, distance to desirable regions, or concentration around narrow pathways.

A subject can possess many nominal options while all of them require passage through one dominating institution.

Geometry can therefore reveal structures concealed by simple cardinality.

11.43 Viable Set

The viable set contains states from which legitimate constraints can continue to be satisfied.

Let

$$Q_{\text{viable}}(\mathcal{G}) = \mathcal{V}. \quad (407)$$

This object is relevant where nominal reachability includes futures that are unsafe, rights-violating, economically unsustainable, or practically inaccessible.

Generative assistance can aim to expand viable rather than merely reachable possibility.

11.44 Safe Set

A preventive analysis can compare safe regions:

$$Q_{\text{safe}}(\mathcal{G}) = \mathcal{S}_{\text{safe}}. \quad (408)$$

A system can be evaluated by whether current or future states remain inside the safe set.

This object is especially relevant to prevention and interruption rather than post-harm equivalence.

11.45 Action Set

The set of actions available to the subject can itself be the restorative object:

$$Q_{\text{action}}(\mathcal{G}) = \mathcal{U}_A. \quad (409)$$

Action-set comparison is related to capability comparison but can be formally narrower.

A legal right can exist while the practical action set remains restricted by cost, retaliation, or information.

11.46 Policy Set

For organizations or collective actors, the available policy set can be the object:

$$Q_{\text{policy}}(\mathcal{G}) = \Pi. \quad (410)$$

This can matter where a harm reduces institutional capacity to respond.

A community deprived of resources can retain formal governance authority while losing feasible policy options.

11.47 Dynamical Law

The evolution law itself can be the restorative object.

For continuous dynamics,

$$Q_{\text{dyn}}(\mathcal{G}) = F. \quad (411)$$

For discrete dynamics,

$$Q_{\text{dyn}}(\mathcal{G}) = \Phi. \quad (412)$$

Restoration of the dynamical law is much stronger than restoration of the current state.

It asks whether the process generating future development has been restored or transformed into an equivalent process.

11.48 Parameter Structure

A parameter vector or structure can be selected as the object:

$$Q_{\theta}(\mathcal{G}) = \theta. \quad (413)$$

This is useful where harm changes effective rates, sensitivities, thresholds, or dependencies while preserving the form of the dynamical law.

A workplace can retain the same formal rules while the practical retaliation parameter changes.

11.49 Attractor Set

The attractor family can be a restorative object:

$$Q_{\text{attr}}(\mathcal{G}) = \mathfrak{A}(\Phi). \quad (414)$$

Attractor comparison asks whether the system retains corresponding long-run modes.

Eglash and Garvey's use of attractors in generative justice provides an important precedent for treating such structures as normatively relevant [16].

The object can be compared through number, type, topology, stability, or correspondence.

11.50 Attractor Geometry

The internal geometry of an attractor can matter:

$$Q_{\text{attr-geom}}(\mathcal{G}) = \text{Geom}(\mathcal{A}). \quad (415)$$

Two systems can possess attractors of the same qualitative type while differing in internal structure. Whether the distinction matters depends upon the restorative purpose.

11.51 Basin Structure

The basin of attraction can be a separate object:

$$Q_{\text{basin}}(\mathcal{G}) = \{\mathcal{B}(\mathcal{A}_i)\}_i. \quad (416)$$

A restored system can possess the same attractor but a smaller basin.

This corresponds to reduced resilience.

Basin structure is therefore especially useful for distinguishing apparent recovery from robust recovery.

11.52 Basin Boundary

The distance to a basin boundary can itself be the object.

Let

$$Q_{\text{boundary}}(\mathcal{G}) = d(x_t, \partial\mathcal{B}). \quad (417)$$

A system near a critical boundary can appear observationally normal while remaining highly vulnerable to small disturbance.

This object can therefore capture fragility invisible in current-state comparison.

11.53 Bifurcation Structure

The pattern of qualitative changes under parameter variation can be compared.

Let

$$Q_{\text{bif}}(\mathcal{G}) = \mathfrak{Bif}. \quad (418)$$

This object becomes relevant where harm changes thresholds at which the system moves into another regime.

A subject or institution can therefore be restored under ordinary conditions while becoming more susceptible to future regime change.

11.54 Stability Structure

A stability object can represent local or global stability properties:

$$Q_{\text{stab}}(\mathcal{G}) = \mathfrak{S}_{\text{stab}}. \quad (419)$$

The object can include eigenvalue structure, Lyapunov stability, recovery rate, or another domain-specific measure.

The later dynamical sections will distinguish these alternatives.

11.55 Resilience

Resilience can be selected as a restorative object.

Let

$$Q_{\text{res}}(\mathcal{G}) = \varrho(\mathcal{G}, \Delta_{\text{adm}}). \quad (420)$$

The object is indexed to a disturbance class.

Holling's distinction between stability and resilience supports preserving these concepts separately [24].

A restored system can have equal current output and lower resilience.

11.56 Recovery Time

Recovery time after perturbation can be the object:

$$Q_{\text{rec-time}}(\mathcal{G}, \delta) = T_{\text{rec}}. \quad (421)$$

A person or institution can return to ordinary functioning after both systems experience the same disturbance while one requires much longer recovery.

The difference can matter normatively because greater recovery time can consume other resources and opportunities.

11.57 Perturbation Response

The response function itself can be the object:

$$Q_{\text{resp}}(\mathcal{G}) = [\delta \mapsto Q_{\tau}(\mathcal{G}, \delta)]. \quad (422)$$

This is a strong object for assessing whether restored systems react similarly to future stress.

The later robustness section will formalize several comparison relations over this object.

11.58 Response Distribution

For stochastic systems, the response object can be

$$Q_{\text{resp-dist}}(\mathcal{G}) = [\delta \mapsto P(O_{t+\tau} \mid \delta)]. \quad (423)$$

This object supports statistical indistinguishability under perturbation.

It is stronger than baseline-output comparison and weaker than complete structural identity.

11.59 Causal Graph

A causal structure can itself be the object.

Let

$$Q_{\text{causal}}(\mathcal{G}) = \mathcal{C} = (V_C, E_C). \quad (424)$$

The object identifies causal relations among selected variables.

Two systems can produce similar observational distributions while having different causal graphs.

Causal-structure restoration therefore cannot be inferred from output similarity alone [43].

11.60 Structural Causal Model

A richer object can include structural equations:

$$Q_{\text{SCM}}(\mathcal{G}) = M_{\text{SCM}}. \quad (425)$$

This object captures the mechanism through which variables respond to intervention.

It becomes relevant where justice concerns not only current distribution but the process generating that distribution.

11.61 Interventional Response Structure

An interventional object can be represented as

$$Q_{\text{int}}(\mathcal{G}) = \{P(Y \mid do(u)) : u \in \mathcal{U}_{\text{adm}}\}. \quad (426)$$

This object asks whether the same intervention produces the same response in the restored and reference systems.

It is especially relevant to assistance, policy, treatment, and resource conversion.

11.62 Mediating Structure

A restorative object can concern the mechanisms through which an intervention produces its outcome.

Let

$$Q_{\text{med}}(\mathcal{G}) = \mathcal{M}_{\text{causal}}. \quad (427)$$

This can represent whether a resource produces capability through the same institutional or relational pathway.

Different mediators can produce similar endpoints.

The process can matter independently where the mediating pathway contains domination, dependency, or extraction.

11.63 Conversion Function

The conversion mechanism can be selected directly:

$$Q_{\text{conv}}(\mathcal{G}) = \mathcal{K}. \quad (428)$$

This object asks how resources become capabilities, relations, or generated outputs.

The same monetary compensation can therefore be compared not only by amount but by whether the subject can convert it into useful future conditions.

Conversion is a central bridge between resource-oriented and capability-oriented restoration.

11.64 Conversion Efficiency

A scalar or vector attribute of the conversion process can be defined:

$$Q_{\text{conv-eff}}(\mathcal{G}) = \eta_{\mathcal{K}}. \quad (429)$$

This can measure the amount of capability or usable condition produced by a resource input.

The object should be used cautiously. Efficiency can obscure the quality of the result, burden of conversion, and normative status of the process.

11.65 Conversion Cost

A system can have the same conversion outcome at different cost.

Let

$$Q_{\text{conv-cost}}(\mathcal{G}) = C_{\mathcal{K}}. \quad (430)$$

A person can maintain equal professional output only through much greater effort after harm.

The increased conversion cost can therefore be a separate object of justice.

11.66 Control Structure

The distribution of authority over generative conditions can be compared:

$$Q_{\text{control}}(\mathcal{G}) = \chi. \quad (431)$$

The object can include:

- access authority;
- revocation power;
- ownership;
- decision rights;
- veto power;
- governance participation;
- appeal rights.

This object is central to non-domination and generative justice [44, 15].

11.67 Dependency Structure

Dependency can be represented separately from authority.

Let

$$Q_{\text{dep}}(\mathcal{G}) = D. \quad (432)$$

The object can identify which subjects depend upon which actors or conditions for essential generative functions.

A restored system can reproduce current access while maintaining concentrated dependency.

Dependency therefore provides a useful object for relational and structural justice.

11.68 Decision-Right Distribution

A collective system can be compared through the distribution of decision rights:

$$Q_{\text{decision}}(\mathcal{G}) = \mathbf{d}. \quad (433)$$

This can represent participation in governance, control of reparative funds, community authority, or organizational voice.

A reparations program can transfer resources while leaving decision-right distribution unchanged.

11.69 Value Output

Generated value can be the object:

$$Q_{\text{value}}(\mathcal{G}) = v. \quad (434)$$

The value can be economic, epistemic, social, cultural, ecological, or another selected type.

Value-output equality is a comparatively weak generative criterion because it does not specify who generated the value, who controls it, or how it is distributed.

11.70 Value Vector

A multidimensional value representation can take the form

$$Q_{\text{value-vector}}(\mathcal{G}) = \mathbf{v} = (v_1, \dots, v_k). \quad (435)$$

The vector can preserve heterogeneous value dimensions without immediate aggregation.

Later value theory will determine whether the dimensions are commensurable.

11.71 Value-Generation Function

The process generating value can itself be the object:

$$Q_{\text{gen-fn}}(\mathcal{G}) = \Gamma. \quad (436)$$

This object asks whether similar value emerges through similar generative processes.

It is substantially stronger than output equality.

11.72 Value-Circulation Structure

The circulation map can be selected:

$$Q_{\text{circ}}(\mathcal{G}) = \Lambda. \quad (437)$$

This object identifies where generated value flows, who retains it, who controls it, and whether it becomes part of later generative conditions.

It is a central object for generative justice [15, 17].

11.73 Extraction Rate

A scalar object can summarize extraction:

$$Q_{\text{extract}}(\mathcal{G}) = \epsilon_{\text{ext}}. \quad (438)$$

This can represent the proportion of generated value leaving the generating system.

Equal extraction rates do not imply identical circulation structures.

The scalar is therefore a compressed attribute of a richer object.

11.74 Recursive Retention

A generative system can be compared by how much value returns as future condition:

$$Q_{\text{retain}}(\mathcal{G}) = \rho_{\text{rec}}. \quad (439)$$

This object captures one dimension of self-sustaining generativity.

It can be relevant to community development, education, research, and ecological systems.

11.75 Generative Recursion Structure

A richer object represents the recursive architecture itself:

$$Q_{\text{recursion}}(\mathcal{G}) = (C_t \rightarrow V_{t+1} \rightarrow C_{t+1} \rightarrow V_{t+2} \rightarrow \dots). \quad (440)$$

This object becomes central where harm interrupted a recursive production cycle.

Restoration of one resource does not establish restoration of the recursion.

11.76 Generative Mechanism

The broad generative mechanism introduced in Equation 169 can itself be the primary object:

$$Q_{\text{gen}}(\mathcal{G}) = \Gamma^G. \quad (441)$$

This object includes the production of future states, relations, conditions, and values.

Generative-process equivalence later in the paper will be defined over this object.

11.77 Generative Rate

A compressed generative attribute can represent the rate at which new valued states or conditions are produced:

$$Q_{\text{gen-rate}}(\mathcal{G}) = g_t. \quad (442)$$

The object can be useful for empirical comparison.

Its normative limitation is severe. A higher rate of generation is not necessarily more just.

The object should therefore never be treated as a general maximand.

11.78 Generative Diversity

Generativity can also be compared through diversity of generated possibilities:

$$Q_{\text{gen-div}}(\mathcal{G}) = D_G. \quad (443)$$

A system generating many distinct options can differ normatively from one producing high volume along one path.

Generative diversity can therefore complement generative rate.

11.79 Generative Autonomy

A subject's control over the direction of generation can be selected as the object:

$$Q_{\text{gen-aut}}(\mathcal{G}) = A_G. \quad (444)$$

The object can include authority to choose projects, exit, redirect resources, revise goals, or reject institutional assistance.

It connects generativity with relational autonomy and non-domination.

11.80 Meaning Structure

A meaning structure can be selected as the restorative object.

Let

$$Q_{\text{meaning}}(\mathcal{G}) = \mathcal{M}. \quad (445)$$

The object can include:

- subjective meaning;
- public meaning;
- symbolic classification;
- cultural interpretation;
- moral significance;
- institutional narrative.

Meaning comparison is especially relevant to apology, recognition, reputational repair, memorialization, and historical injustice.

11.81 Recognition State

Recognition can be represented as a separate object:

$$Q_{\text{recognition}}(\mathcal{G}) = R_A^{\text{rec}}. \quad (446)$$

The object can include social standing, acknowledgment, legitimacy, credibility, or membership.

Recognition theory makes clear why material equality alone does not settle this dimension [25, 20].

11.82 Public Narrative

A public narrative can itself be compared:

$$Q_{\text{narrative}}(\mathcal{G}) = N_{\text{pub}}. \quad (447)$$

This can be relevant where the harm involved historical misrepresentation, stigmatization, or public denial.

Narrative restoration does not imply material restoration.

Its inclusion as a separate object prevents symbolic and material response from being conflated.

11.83 Reputation Distribution

Reputation can be represented through the distributed beliefs of other actors:

$$Q_{\text{rep}}(\mathcal{G}) = \rho_A. \quad (448)$$

The object can be a vector, network-weighted distribution, or probabilistic belief structure.

A corrected official record can coexist with an unrepaired reputation object.

11.84 Classification State

Institutional classification can be a distinct object:

$$Q_{\text{class}}(\mathcal{G}) = z. \quad (449)$$

Examples include disciplinary status, eligibility, risk category, credit classification, or professional standing.

Correction of this object is often administratively tractable.

Its downstream effects remain separate.

11.85 Classification Propagation Graph

The distributed propagation of a classification can itself be the object:

$$Q_{\text{class-net}}(\mathcal{G}) = \mathcal{P}_z. \quad (450)$$

This captures which institutions or actors received and used the classification.

A correction can restore the source classification while leaving the propagation graph historically unchanged.

11.86 Provenance Graph

The provenance structure can be the restorative object:

$$Q_{\text{prov}}(\mathcal{G}) = \mathcal{P}. \quad (451)$$

The object records claims, evidence, revisions, corrections, sources, and downstream uses.

It is especially important where justice requires traceability rather than erasure.

11.87 Historical Event Sequence

The historical event sequence can be selected:

$$Q_{\text{hist-seq}}(\mathcal{G}) = h_{0:t}. \quad (452)$$

This is stronger than current-state comparison.

It allows the theory to distinguish worlds that arrive at the same state through different histories.

11.88 Historical Partial Order

Where events are concurrent, the object can be a partial order:

$$Q_{\text{hist-po}}(\mathcal{G}) = (\mathcal{E}, \prec_H). \quad (453)$$

This object preserves causal or temporal ordering without forcing a single linear narrative.

It is useful for distributed institutional histories.

11.89 Historical Provenance

Historical provenance concerns not only what happened but how present objects were generated.

Let

$$Q_{\text{hist-prov}}(\mathcal{G}) = \text{Prov}(x_t). \quad (454)$$

Two equal current states can possess different provenance.

The difference can matter for responsibility, legitimacy, authorship, property, recognition, and historical justice.

11.90 Narrative Continuity

For subjects, narrative continuity can be selected as the restorative object.

Let

$$Q_{\text{narr-cont}}(\mathcal{G}) = N_A^{\text{self}}. \quad (455)$$

This object concerns how the subject understands continuity across harm and restoration.

It resists reduction to behavior or memory count.

The later procedural analysis will examine why maximizing counterfactual similarity can conflict with narrative continuity.

11.91 Memory Structure

Memory can be selected as a comparison object:

$$Q_{\text{memory}}(\mathcal{G}) = M_A. \quad (456)$$

Memory comparison becomes relevant in idealized thought experiments involving memory alteration or perfect reconstruction.

The object is normatively sensitive because memory supports learning, identity, testimony, and responsibility.

11.92 Identity Structure

A theory can attempt to compare identity-relevant features:

$$Q_{\text{id}}(\mathcal{G}) = I_A. \quad (457)$$

The object can include legal identity, narrative identity, social identity, continuity of commitments, or another theory-dependent structure.

There is no theory-neutral universal identity metric.

The object therefore requires unusually explicit philosophical assumptions.

11.93 Agency Structure

Agency can itself be the object:

$$Q_{\text{agency}}(\mathcal{G}) = A_A. \quad (458)$$

Relevant dimensions can include:

- decision authority;
- practical alternatives;
- information;
- freedom from coercion;
- capacity for revision;
- ability to exit;
- control over future direction.

Agency restoration can diverge from resource restoration.

11.94 Revisability

The capacity to revise a trajectory can be selected as an object:

$$Q_{\text{rev}}(\mathcal{G}) = \mathcal{R}_{\square}. \quad (459)$$

A restorative arrangement can provide immediate support while locking the subject into one future. Revisability therefore captures a specifically generative form of autonomy.

11.95 Exit Capacity

Exit capacity can be represented as

$$Q_{\text{exit}}(\mathcal{G}) = E_A. \quad (460)$$

This object concerns whether the subject can leave a relationship, institution, or assistance arrangement without disproportionate generative collapse.

Exit is especially important for non-domination.

11.96 Institutional Trust

Institutional trust can be selected as an object:

$$Q_{\text{trust}}(\mathcal{G}) = T_I. \quad (461)$$

Trust can affect reporting, investment, participation, and long-term cooperation.

A legal victory can restore rights while institutional trust remains low.

11.97 Civic Trust

At collective scale, civic trust can become the object.

Large-scale reparations theory has explicitly connected reparations with civic trust and social solidarity [13].

Civic trust differs from individual trust because it concerns expectations about public institutions and fellow members of the political community.

The object can therefore be relevant to historical and collective repair.

11.98 Social Solidarity

Social solidarity can be selected as another collective object:

$$Q_{\text{solidarity}}(\mathcal{G}) = S_{\text{soc}}. \quad (462)$$

This object concerns willingness to support others, shared institutional commitment, or recognition of common membership.

It should remain analytically distinct from conformity or cultural homogeneity.

11.99 Collective Capacity

A collective can possess capabilities not reducible to individuals.

Let

$$Q_{\text{collective}}(\mathcal{G}) = K_{\text{collective}}. \quad (463)$$

Examples include:

- ability to govern common resources;
- ability to preserve language;
- ability to produce shared knowledge;
- ability to organize politically;
- ability to maintain ecological practices.

Collective restoration can therefore require more than distribution of individual compensation.

11.100 Institutional Capacity

The capacity of an institution to perform a function can be compared:

$$Q_{\text{inst-cap}}(\mathcal{G}) = K_I. \quad (464)$$

A community can regain resources while local institutions remain unable to govern them.

Institutional capacity therefore becomes an important object in transformative and reparative practice.

11.101 Ecological State

An ecological state can be selected:

$$Q_{\text{eco-state}}(\mathcal{G}) = x_{\text{eco}}. \quad (465)$$

The object can include species composition, water quality, biomass, soil condition, or another ecological variable.

Ecological-state restoration is often insufficient where resilience or regime structure has changed.

11.102 Ecological Function

A functional ecological object can be represented as

$$Q_{\text{eco-fn}}(\mathcal{G}) = F_{\text{eco}}. \quad (466)$$

This can concern nutrient cycling, pollination, water regulation, habitat function, or another ecological process.

A restored ecosystem can be functionally similar without historical species identity.

11.103 Ecological Resilience

Ecological resilience can be represented separately:

$$Q_{\text{eco-res}}(\mathcal{G}) = \varrho_{\text{eco}}. \quad (467)$$

This object is directly connected to Holling's resilience framework [24].

The example illustrates why the same taxonomy can apply beyond individual subjects.

11.104 Information State

The information available to an agent or institution can be selected as the object:

$$Q_{\text{info}}(\mathcal{G}) = I_t. \quad (468)$$

Loss of information can reduce generativity even where physical resources remain unchanged.

Restoration can involve access to data, archives, explanation, or records.

11.105 Knowledge Structure

Knowledge can be represented more richly:

$$Q_{\text{knowledge}}(\mathcal{G}) = K_t. \quad (469)$$

The object can include propositional knowledge, practical know-how, distributed knowledge, tacit knowledge, and institutional memory.

A lost research environment can degrade collective knowledge even when individual documents remain.

11.106 Distributed Knowledge

The effective distributed knowledge of a group can be selected:

$$Q_{\text{dist-know}}(\mathcal{G}) = K_t^{\text{eff}}. \quad (470)$$

This object depends upon both individual knowledge and network accessibility.
It is therefore a relational-epistemic object.

11.107 Attention Structure

Attention can function as a scarce generative condition.

Let

$$Q_{\text{attn}}(\mathcal{G}) = A_t^{\text{attn}}. \quad (471)$$

Harm can consume attention through litigation, fear, surveillance, repeated administrative burden, or recovery.

Restoration of resources can leave the attentional burden unchanged.

This object is especially relevant to administrative and procedural justice.

11.108 Time Allocation

Time allocation can be represented as

$$Q_{\text{time}}(\mathcal{G}) = \mathbf{t}_A. \quad (472)$$

The vector can include work, care, recovery, education, administration, and rest.

A person can receive financial compensation while continuing to spend large amounts of time repairing institutional errors.

Time is therefore a distinct restorative object rather than merely a monetary cost.

11.109 Administrative Burden

Administrative burden can be selected as an object:

$$Q_{\text{admin}}(\mathcal{G}) = B_{\text{admin}}. \quad (473)$$

The object can include documentation, repeated explanation, travel, delay, forms, appeals, and cognitive effort required to obtain a remedy.

A restorative system can provide substantial benefit while imposing high administrative burden.

The burden therefore belongs to process as well as outcome analysis.

11.110 Procedural State

The procedural state introduced in Section 9.51 can itself be compared:

$$Q_{\text{proc}}(\mathcal{G}) = p_t. \quad (474)$$

Relevant dimensions can include:

- consent;
- participation;
- notice;
- transparency;
- contestability;
- privacy;
- reversibility;
- proportionality.

This object becomes central when process justice is evaluated independently of restorative outcome.

11.111 Procedural Trajectory

The whole restorative process can be compared:

$$Q_{\text{proc-traj}}(\mathcal{G}) = p_{0:n}. \quad (475)$$

Two cases can end in the same substantive state through radically different procedures. Procedural trajectory therefore captures more than final procedural status.

11.112 Institutional Response Time

Response time can be the object:

$$Q_{\text{latency}}(\mathcal{G}) = \tau_I. \quad (476)$$

Delay can be substantively important where generative deterioration occurs faster than institutional response.

This object therefore belongs both to process and generative outcome.

11.113 Correction Propagation

The degree to which a correction has propagated can be selected:

$$Q_{\text{corr-prop}}(\mathcal{G}) = C_P. \quad (477)$$

This can represent the fraction or structure of institutional nodes updated after a correction. The object is especially relevant to distributed reputational and classification harms.

11.114 Institutional Interface Integrity

The functioning of institutional handoffs can be compared:

$$Q_{\text{interface}}(\mathcal{G}) = I_F. \quad (478)$$

This object can include interoperability, referral success, continuity of benefits, correction transmission, and evidence portability.

An individual remedy can fail because interfaces among institutions fail.

11.115 System Redundancy

Redundancy can be selected as a resilience object:

$$Q_{\text{redundancy}}(\mathcal{G}) = R_D. \quad (479)$$

This can represent the number and independence of alternative pathways to critical conditions.

A restored system can regain current access while retaining a dangerous single point of failure.

11.116 Fragility

Fragility can be compared directly:

$$Q_{\text{frag}}(\mathcal{G}) = F_G. \quad (480)$$

This object can represent sensitivity to shocks, dependence upon one actor, or proximity to failure thresholds.

Fragility comparison is especially useful where ordinary output appears restored.

11.117 Option Value

Future options themselves can possess value even when not exercised.

Let

$$Q_{\text{option}}(\mathcal{G}) = V_{\text{option}}. \quad (481)$$

This object can represent the value of retaining future flexibility.

The concept is relevant to option-preserving governance and subject-directed regeneration.

Its valuation remains theoretically difficult because unexercised options can be heterogeneous and uncertain.

11.118 Opportunity Cost

A restorative object can be the opportunities forgone through the harmful history.

Let

$$Q_{\text{opp-cost}}(\mathcal{G}) = C_{\text{opp}}. \quad (482)$$

This object is counterfactual and should remain sensitive to uncertainty.

It differs from actual resource loss.

11.119 Singular Opportunity

A particular historical opportunity can be selected:

$$Q_{\text{sing-op}}(\mathcal{G}) = o^*. \quad (483)$$

Examples include:

- collaboration with a specific person;
- participation in a unique historical event;
- publication within a particular intellectual moment;
- education during a critical developmental period.

The object is especially resistant to substitution.

It therefore provides a useful test case for the limits of general equivalents.

11.120 Historical Window

The opportunity window itself can be compared:

$$Q_{\text{window}}(\mathcal{G}) = W_{[t_0, t_1]}. \quad (484)$$

The object concerns whether a condition was available during the period in which its generative significance was highest.

Delayed access can restore the condition while failing to restore the window.

11.121 Temporal Location

The temporal location of an event can be an explicit object:

$$Q_{\text{time-pos}}(\mathcal{G}) = t(e). \quad (485)$$

This is important where sequence matters.

The same education, funding, legal status, or publication received at another time can have different effects.

11.122 Event Sequence

A selected sequence of events can be compared:

$$Q_{\text{event-seq}}(\mathcal{G}) = (e_1, \dots, e_n). \quad (486)$$

This object can support process-level historical comparison without requiring complete history.

11.123 Event Dependency Graph

The causal or generative dependencies among events can be selected:

$$Q_{\text{event-dep}}(\mathcal{G}) = G_E. \quad (487)$$

A restored system can reproduce similar events while changing dependency structure.

This difference can matter where autonomy, causation, or generative recursion is at issue.

11.124 Path Dependency Structure

Path dependence itself can be the object:

$$Q_{\text{pathdep}}(\mathcal{G}) = P_D. \quad (488)$$

This can represent which historical states constrain later transitions.

Arthur and Pierson provide important analyses of path dependence and increasing returns in technological and political processes [3, 45].

A harm can increase path dependence by narrowing future transitions.

11.125 Lock-In Structure

Lock-in can be selected as a related object:

$$Q_{\text{lockin}}(\mathcal{G}) = L_I. \quad (489)$$

This can represent the cost or difficulty of leaving a trajectory once entered.

A restorative intervention can improve current state while leaving harmful lock-in intact.

11.126 Switching Cost

Switching cost can be compared:

$$Q_{\text{switch}}(\mathcal{G}) = C_{\text{switch}}. \quad (490)$$

This object is important for exit, re-entry, and career transition.

Equal formal opportunities can coexist with unequal switching costs.

11.127 Adaptive Capacity

The capacity to adapt can itself be restorative:

$$Q_{\text{adapt}}(\mathcal{G}) = A_C. \quad (491)$$

Adaptive capacity differs from current adaptation.

It concerns the ability to respond to future change.

This object is relevant to resilience and long-horizon generativity.

11.128 Learning Capacity

Learning capacity can be represented as

$$Q_{\text{learn}}(\mathcal{G}) = L_C. \quad (492)$$

A subject or institution can recover current performance while having reduced capacity to learn from future experience.

Learning capacity is therefore a distinct generative object.

11.129 Institutional Learning Capacity

At organizational scale,

$$Q_{\text{inst-learn}}(\mathcal{G}) = L_I. \quad (493)$$

can represent the ability to detect error, revise rules, preserve provenance, and prevent recurrence.

A reparative institution that cannot learn can continue generating the same class of harm.

11.130 Correction Capacity

The ability of a system to correct its own errors can be selected:

$$Q_{\text{correct}}(\mathcal{G}) = C_R. \quad (494)$$

This object can include:

- detectability of error;
- appeal;
- record revision;
- propagation of correction;
- institutional responsiveness.

Correction capacity is therefore a generative governance object.

11.131 Reparability

The expected reparability of future harm can itself be compared:

$$Q_{\text{reparability}}(\mathcal{G}) = R_P. \quad (495)$$

A system designed so that errors are easy to reverse differs from one in which small mistakes create permanent consequences.

Reparability is therefore relevant to preventive justice.

11.132 Reversibility

Reversibility can be represented as

$$Q_{\text{revrs}}(\mathcal{G}) = R_V. \quad (496)$$

The object can concern reversibility of institutional action, technical action, or subject choice.

Reversibility should remain distinct from reparability. An intervention can be formally reversible while its historical consequences are not.

11.133 Historical Recoverability

The degree to which a prior historical state can be reconstructed can itself be an object:

$$Q_{\text{hist-rec}}(\mathcal{G}) = H_R. \quad (497)$$

This object can depend upon records, physical preservation, relational continuity, and counterfactual knowledge.

It becomes relevant to the preventive design of archives and provenance systems.

11.134 Epistemic Reparability

A related object concerns whether later actors can know enough to correct an error.

Let

$$Q_{\text{epi-rep}}(\mathcal{G}) = E_R. \quad (498)$$

A system with good provenance, auditable decisions, and revisable records has higher epistemic reparability than a system whose decisions leave no trace.

This object links knowledge governance to justice of return.

11.135 Measurement Structure

The measurement process can itself be compared:

$$Q_{\text{measure}}(\mathcal{G}) = O. \quad (499)$$

Different observation maps can make different harms visible.

A restorative system can therefore improve measurement without yet improving the underlying subject condition.

This distinction matters for evaluation.

11.136 Classification Regime

The rules by which subjects are classified can be the restorative object:

$$Q_{\text{class-reg}}(\mathcal{G}) = Z. \quad (500)$$

A harmful classification can be corrected individually while the regime producing similar errors remains unchanged.

Transformative justice can therefore target the classification regime itself.

11.137 Epistemic Access

The subject's access to information about how they are classified or governed can be compared:

$$Q_{\text{epi-access}}(\mathcal{G}) = A_E. \quad (501)$$

This object can include access to reasons, records, evidence, and appeal.

Epistemic access contributes to contestability and agency.

11.138 Contestability

Contestability can be selected directly:

$$Q_{\text{contest}}(\mathcal{G}) = C_T. \quad (502)$$

A subject can possess a formal right while lacking a usable mechanism to challenge decisions.

Contestability therefore links procedural justice with effective generativity.

11.139 Privacy State

Privacy can become an independent restorative object where harm or repair involves exposure.

Let

$$Q_{\text{privacy}}(\mathcal{G}) = P_R. \quad (503)$$

A public correction can improve recognition while reducing privacy.

The two objects should therefore remain distinct.

11.140 Dignity-Related Condition

Some theories will treat dignity as a separate normative object:

$$Q_{\text{dignity}}(\mathcal{G}) = D_A. \quad (504)$$

Dignity is difficult to formalize without philosophical commitments.

Its inclusion in the taxonomy prevents technically measurable objects from exhausting the field of justice.

11.141 Authorship and Credit

Authorship, attribution, or intellectual credit can be a restorative object:

$$Q_{\text{credit}}(\mathcal{G}) = C_A. \quad (505)$$

This object is relevant to research, art, invention, community knowledge, and generative justice.

A financial payment can coexist with unresolved misattribution.

11.142 Ownership Structure

Ownership of generated assets can be selected:

$$Q_{\text{ownership}}(\mathcal{G}) = O_W. \quad (506)$$

A reparative project can generate substantial value while ownership remains external.

Ownership therefore belongs to the generative architecture.

11.143 Data Control

Data generated through a process can be a distinct object:

$$Q_{\text{data-control}}(\mathcal{G}) = D_C. \quad (507)$$

This is particularly relevant to computational reparations, community research, and digital governance.

Control over reparative data can determine future extraction or self-governance.

11.144 Knowledge Ownership

The rights associated with knowledge generated through repair can also be selected:

$$Q_{\text{know-own}}(\mathcal{G}) = K_O. \quad (508)$$

This object distinguishes the production of knowledge from control over its use.

It is especially relevant where communities generate value through research conducted by external institutions.

11.145 Infrastructure Control

A reparative program can be compared through control of the infrastructure it creates:

$$Q_{\text{infra-control}}(\mathcal{G}) = I_C. \quad (509)$$

A community can receive a facility while another actor controls access, maintenance, data, or future investment.

Infrastructure transfer and infrastructure control are therefore distinct objects.

11.146 Intergenerational Transmission

The mechanism through which conditions pass across generations can be selected:

$$Q_{\text{intergen}}(\mathcal{G}) = T_G. \quad (510)$$

This can include wealth, education, cultural practice, institutional access, ecological condition, and social status.

Historical injustice often operates through such transmission.

Restoration of current resources can leave the intergenerational mechanism unchanged.

11.147 Cultural Transmission

Cultural transmission can be a separate object:

$$Q_{\text{culture}}(\mathcal{G}) = C_T. \quad (511)$$

The object can include language, practices, archives, ritual, knowledge, and intergenerational teaching structures.

A material substitute cannot automatically reproduce this generative process.

11.148 Ecological Transmission

Ecological conditions can also be transmitted across generations.

Let

$$Q_{\text{eco-trans}}(\mathcal{G}) = E_T. \quad (512)$$

This object concerns how current environmental conditions constrain or enable future life and production.

It connects restorative justice with intergenerational justice.

11.149 Collective Historical Memory

Collective memory can be selected:

$$Q_{\text{coll-mem}}(\mathcal{G}) = M_C. \quad (513)$$

A reparative process can alter public memory without altering material distribution.

Collective memory is therefore an independent object.

11.150 Historical Recognizability

The capacity of future actors to know that a harm occurred can itself be restoratively important.

Let

$$Q_{\text{hist-recog}}(\mathcal{G}) = R_H. \quad (514)$$

Destroying every record can increase resemblance to a no-harm world while reducing historical recognizability.

This creates a direct conflict between some forms of historical equivalence and epistemic justice.

11.151 Counterfactual Accessibility

A system's ability to support credible counterfactual reconstruction can be selected as an epistemic object:

$$Q_{\text{cf-access}}(\mathcal{G}) = C_F. \quad (515)$$

This can depend upon data quality, provenance, causal knowledge, and model stability.

It is not itself a restorative outcome for the claimant, but it affects the quality of later restorative judgment.

11.152 Model Robustness

A restorative analysis can compare the robustness of its own models:

$$Q_{\text{model-rob}}(\mathcal{G}) = M_R. \quad (516)$$

This meta-level object becomes important where different plausible models produce different restorative conclusions.

The atlas therefore includes objects about the represented system and objects about the epistemic apparatus representing it.

11.153 Reference Robustness

The stability of a conclusion across plausible references can be selected:

$$Q_{\text{ref-rob}} = R_B. \quad (517)$$

This object measures or classifies how sensitive a restorative judgment is to baseline selection. It is especially useful for policy decisions under reference indeterminacy.

11.154 Value Robustness

A conclusion can also be tested across value theories:

$$Q_{\text{value-rob}} = R_V. \quad (518)$$

An intervention supported by several normative frameworks can possess a different kind of robustness from one justified only under a highly specific theory.

This meta-level object will become relevant in the normative atlas.

11.155 Procedural Robustness

A restorative process can be evaluated by whether procedural legitimacy remains stable under institutional stress:

$$Q_{\text{proc-rob}} = R_P. \quad (519)$$

A procedure that remains fair only when all actors cooperate voluntarily can be fragile.

This object connects robustness with process justice.

11.156 Composite Restorative Object

Many practical cases require a composite object:

$$Q_{\text{comp}}(\mathcal{G}) = (Q_1(\mathcal{G}), Q_2(\mathcal{G}), \dots, Q_n(\mathcal{G})). \quad (520)$$

Equation 520 can combine resources, capabilities, network position, health, recognition, and future reachability.

A composite object should not automatically be reduced to one scalar.

The comparative relation can operate componentwise, lexically, through partial ordering, or through another rule.

11.157 Hierarchical Restorative Object

Some comparison objects have hierarchical structure.

For example,

$$Q_{\text{hier}} = (Q_{\text{resource}}, Q_{\text{capability}}, Q_{\text{relation}}, Q_{\text{generative}}) \quad (521)$$

can represent layers in which resource restoration supports capability, capability supports relation, and relation supports generativity.

The hierarchy should not be interpreted as a universal causal order.

Its usefulness lies in making dependencies explicit.

11.158 Nested Restorative Objects

An object can contain another object.

A network contains relations. A trajectory contains states. A capability structure contains opportunities. A generative mechanism contains conversion and value circulation. A history contains events.

Formally,

$$Q_j \subset Q_i \quad (522)$$

can denote analytical nesting where appropriate.

Nested objects permit restoration at different levels of granularity.

11.159 Cross-Level Restorative Objects

Some objects connect several levels simultaneously.

Examples include:

- resource-to-capability conversion;
- individual-to-network influence;
- institutional rule-to-behavior response;
- community-to-ecological feedback;
- value-output-to-future-condition recursion.

These are cross-level objects because their meaning lies in the mapping between levels.

Such mappings are especially important for generativity.

11.160 Restorative Object Granularity

The same domain can be represented at several granularities.

Reputation can be one scalar, a vector of beliefs, a network distribution, or a historical propagation graph.

Capability can be one indicator, a profile, a set, or a dynamic opportunity system.

A practical theory should therefore record granularity.

Greater granularity can reveal more structure while increasing epistemic and administrative burden.

11.161 Restorative Object Scale

Every object can also be indexed by scale:

$$Q^{(s)}(\mathcal{G}). \quad (523)$$

Individual, household, organizational, community, societal, ecological, and intergenerational objects can differ.

An object restored at one scale can remain unrestored at another.

11.162 Restorative Object Horizon

Some objects are inherently time-indexed.

Reachability, trajectory, resilience, value circulation, and capability development depend upon horizon τ .

The object can therefore be represented as

$$Q_{\tau}(\mathcal{G}). \quad (524)$$

Short-horizon and long-horizon versions can produce different conclusions.

11.163 Restorative Object Observability

Objects differ in observability.

Resources can often be directly recorded. Capabilities are partly inferred. Relations can be partially observed. Counterfactual reachable sets are model-dependent. Meaning can be qualitative. Historical-generative mechanisms can be deeply uncertain.

The atlas should therefore attach an epistemic status to each object.

Let

$$\text{Obs}(Q) \quad (525)$$

denote the observability profile of object Q .

This prevents highly abstract objects from being treated as equally measurable with directly observed resources.

11.164 Restorative Object Controllability

Objects also differ in the extent to which institutions can alter them.

Money can be transferred directly. Legal status can sometimes be corrected. Trust cannot be commanded. Network relationships cannot be recreated unilaterally. Public meaning can be influenced while remaining distributed. Historical identity cannot ordinarily be rewritten.

Let

$$\text{Ctrl}(Q) \quad (526)$$

denote the degree or type of controllability of object Q .

Controllability is distinct from observability.

An institution can observe an object it cannot control and control an object it cannot fully observe.

11.165 Restorative Object Fungibility

Objects differ in substitutability.

Let

$$\text{Fung}(Q) \quad (527)$$

denote the fungibility profile.

Money has high fungibility within its domain. A specific relationship has low fungibility. Some equipment has moderate fungibility. Historical timing can have very low fungibility.

Fungibility strongly affects which comparative relations are plausible.

11.166 Restorative Object Singularity

Singularity is the inverse problem.

Let

$$\text{Sing}(Q) \quad (528)$$

denote the degree to which the object's value depends upon identity, timing, provenance, or unique relation.

High singularity makes substitution weaker as a restorative claim.

11.167 Restorative Object Reparability

Different objects have different expected reparability.

Let

$$\text{Rep}(Q) \quad (529)$$

denote the reparability profile of object Q .

Resource amounts can be highly repairable. Historical windows can be weakly repairable. Network position can be partially repairable. Memory can be technically alterable in hypothetical cases while normatively protected.

Reparability therefore includes practical and normative dimensions.

11.168 Restorative Object Normative Salience

The fact that an object is measurable or restorable does not establish that it matters morally.

Let

$$\text{Sal}_V(Q) \quad (530)$$

denote the normative salience of Q under value theory V .

Different theories can assign different salience to the same object.

This dimension will be developed in the value-theory sections.

11.169 Restorative Object Institutional Fit

Some institutions are better suited to some objects.

Let

$$\text{Fit}(I, Q) \quad (531)$$

denote the institutional fit between institution I and object Q .

Courts fit formal status and legally cognizable loss. Healthcare institutions fit bodily and functional objects. Professional organizations fit credential and network objects. Social protection fits resource floors. Community institutions can fit relational and cultural objects.

Institutional fit is therefore another dimension of the atlas.

11.170 Restorative Object Conflict

Improving one object can worsen another.

For example:

- public correction can improve recognition while reducing privacy;
- reinstatement can restore status while reducing subjective safety;
- surveillance can improve evidence while reducing autonomy;
- strong structural reconstruction can increase fidelity while reducing present agency.

A multi-object theory therefore requires conflict analysis.

Let

$$\Delta Q_i(R) \cdot \Delta Q_j(R) < 0 \quad (532)$$

represent an intervention improving one selected object while worsening another under an appropriate sign convention.

The later normative sections will address how such conflicts should be handled.

11.171 Restorative Object Complementarity

Objects can also be complementary.

Restoration of legal status can increase the value of financial compensation. Health rehabilitation can make professional re-entry usable. Record correction can improve network restoration. Income support can make education feasible.

Complementarity can be represented schematically by

$$V(Q_i, Q_j) > V(Q_i) + V(Q_j) \quad (533)$$

where the expression denotes superadditive interaction under a selected valuation.

This is another reason layered restoration can outperform isolated remedies.

11.172 Restorative Object Dependency

One object can depend upon another.

Let

$$Q_i \rightarrow Q_j \quad (534)$$

denote that restoration of Q_i is a practical prerequisite for effective restoration of Q_j .

Housing stability can precede rehabilitation. Legal status can precede employment. Correction can precede network re-entry.

Dependencies create sequencing requirements.

11.173 Object Dependency Graph

A set of restorative objects can be represented by a dependency graph

$$\mathfrak{D}_Q = (\mathfrak{Q}, E_Q^D) . \quad (535)$$

Equation 535 can guide sequencing of interventions.

The graph is not equivalent to the social causal system. It represents restorative dependencies relevant to practice.

11.174 Primary and Secondary Restorative Objects

A case can distinguish primary and secondary objects.

A primary object is directly affected by the harmful intervention.

A secondary object changes through the subsequent generative history.

For example, wrongful dismissal directly changes employment status and income. Later network position and career opportunity can become secondary objects.

The distinction can help legal attribution without denying normative relevance of secondary effects.

11.175 Direct and Emergent Restorative Objects

Some objects are directly represented at the individual level. Others emerge only at collective or system level.

Income is a direct object. Civic trust is emergent. Network topology is relational. Institutional culture can be collective. Ecological resilience can be systemic.

Emergent objects should not be inferred mechanically from individual variables.

11.176 Material and Non-Material Restorative Objects

The taxonomy also cuts across material and non-material domains.

Material objects include property, income, equipment, infrastructure, and bodily conditions.

Non-material objects include recognition, trust, reputation, legal standing, knowledge, narrative, and opportunity.

The distinction is analytically useful while imperfect. Many non-material objects have material effects and many material objects acquire value through social meaning.

11.177 Observable and Latent Restorative Objects

Objects can also be divided into observable and latent categories.

Let

$$\mathfrak{Q} = \mathfrak{Q}_{\text{obs}} \cup \mathfrak{Q}_{\text{latent}} . \quad (536)$$

Latent objects can include trust, perceived security, or unobserved capability.

Their inclusion requires inference.

The theory should distinguish direct measurement from model-based estimation.

11.178 Static and Dynamic Restorative Objects

A major divide runs between static and dynamic objects.

Static objects include current resources, status, and attributes.

Dynamic objects include trajectories, transition rules, reachable sets, attractors, response functions, and generative mechanisms.

Let

$$\Omega = \Omega_{\text{static}} \cup \Omega_{\text{dynamic}}. \quad (537)$$

The distinction is central to the paper because many traditional restorative practices operate primarily on static objects while generative harm can alter dynamic ones.

11.179 Historical and Ahistorical Restorative Objects

Some objects can be defined without explicit provenance. Others are intrinsically historical.

Money amount can be represented ahistorically. Authorship, provenance, narrative identity, and historical legitimacy cannot.

Let

$$\Omega = \Omega_{\text{ahist}} \cup \Omega_{\text{hist}}. \quad (538)$$

This distinction will later help identify which forms of equivalence can ignore history without losing the relevant value.

11.180 Endogenous and Exogenous Restorative Objects

Some objects lie inside the selected generative system. Others are treated as environmental.

The classification is model-relative.

A labor-market condition can be exogenous to an individual model and endogenous to a societal model.

Reference and scale should therefore accompany object selection.

11.181 Outcome and Process Objects

A final major distinction concerns outcomes and processes.

Outcome objects include state, resource, capability, and value.

Process objects include trajectory, procedure, transition mechanism, value-generation function, and historical sequence.

The distinction is essential because two systems can produce the same outcome through different processes.

The later theory of process justice will build directly upon this separation.

11.182 A Multi-Axis Classification of Restorative Objects

The taxonomy can now be represented through several orthogonal dimensions.

Each object Q can be classified by

$$\Theta(Q) = (s, \tau, o, c, f, h, d, n), \quad (539)$$

where:

- s denotes scale;
- τ denotes temporal horizon;
- o denotes observability;
- c denotes controllability;
- f denotes fungibility or singularity;
- h denotes historical dependence;

- d denotes dynamic or static character;
- n denotes normative salience under a specified theory.

Equation 539 does not require numerical values. The dimensions can be categorical or partially ordered.

The metadata help explain why the same comparative relation can be appropriate for one object and inappropriate for another.

11.183 Summary Taxonomy of Comparison Objects

Table 3 summarizes the principal families of comparison objects developed in this section.

Table 3 is a map rather than a ranking. No object family is universally more fundamental than the others.

11.184 Object Selection as a Normative Act

The choice of comparison object is itself normatively significant.

- Selecting income privileges economic loss.
- Selecting capabilities privileges substantive possibility.
- Selecting relations privileges social structure.
- Selecting recognition privileges social meaning.
- Selecting value circulation privileges generative political economy.
- Selecting history privileges provenance and temporal continuity.

The object chosen for restoration therefore reveals part of the theory of justice even before a comparative relation is selected.

11.185 Object Selection as an Epistemic Act

Object selection also reflects what institutions can know.

- A court can measure some objects more reliably than others.
- A regulator can observe aggregate patterns.
- A clinician can observe functional state.
- A subject can report meaning and usability.
- A network analyst can reconstruct relational structure only where data exist.
- The available object can therefore be partly determined by epistemic infrastructure.
- This dependence creates a risk that the measurable becomes mistaken for the important.

11.186 Object Selection as an Institutional Act

Institutions also define which objects become administratively legible.

- A damages schedule makes money visible.
- A licensing system makes status visible.
- A health assessment makes functional capacity visible.
- A professional database makes credentials visible.
- A reparations commission can make historical recognition visible.
- Institutional categories therefore shape restorative reality.
- This observation connects the object taxonomy to the reflexivity developed in Section 8.

11.187 Object Selection as a Procedural Act

The subject can contest the institution's choice of restorative object.

An employer can define the problem as lost salary while the employee identifies reputational and network loss.

A medical system can define the target as physical function while the patient prioritizes independent living.

A reparations program can emphasize payment while the community prioritizes land or institutional authority.

A legitimate restorative process should therefore permit appropriate contestation of object selection.

11.188 Object Multiplicity

Complex harms are usually multi-object.

Let

$$\Omega_H = \{Q_1, Q_2, \dots, Q_n\} \quad (540)$$

denote the family of objects affected by harm H .

A post-harm response can operate upon a subset

$$\Omega_R \subseteq \Omega_H. \quad (541)$$

Equation 541 permits the theory to describe partial restoration without implying that untreated objects are irrelevant.

11.189 Object Coverage

The coverage of a restorative program can be represented by

$$\text{Cov}(R) = \frac{|\Omega_R|}{|\Omega_H|} \quad (542)$$

where cardinality is meaningful.

Equation 542 is illustrative and should not be interpreted as a general justice metric because objects differ greatly in importance.

Its purpose is to distinguish breadth from depth of restoration.

A program can cover many low-salience objects while missing one critical generative structure.

11.190 Object Criticality

Some restorative objects are more central to future generation than others.

Let

$$\kappa_Q(Q) \quad (543)$$

denote the criticality of object Q within the generative architecture.

Criticality can depend upon:

- number of downstream dependencies;
- substitutability;
- temporal sensitivity;
- control concentration;

- recursive generative effects;
- role in preserving agency.

Object criticality can guide practical prioritization without reducing justice to generative efficiency.

11.191 Object Salience and Object Criticality

Normative salience and generative criticality should remain distinct.

A highly generatively central object can be normatively objectionable.

A normatively important object can have limited downstream generative effect.

Let

$$\text{Sal}_V(Q) \neq \kappa_Q(Q) \quad (544)$$

in general.

Equation 544 prevents generativity from becoming the sole normative criterion.

11.192 Object Replacement

A lost object can be replaced by another object Q' .

Replacement is represented by

$$Q \xrightarrow{\text{sub}} Q'. \quad (545)$$

The existence of substitution does not determine equivalence.

The next section will distinguish identity, isomorphism, functional similarity, simulation, and other relations that can connect Q and Q' .

11.193 Object Translation

Institutions sometimes translate one object into another.

A lost relationship can be translated into monetary compensation.

A health loss can be translated into disability percentage.

A historical injustice can be translated into standardized reparations.

Let

$$T_{ij} : Q_i \longrightarrow Q_j \quad (546)$$

denote such translation.

Object translation can be institutionally necessary.

Its validity depends upon the value theory and the information preserved under the transformation.

11.194 Loss Under Object Translation

Translation can discard structure.

Let

$$L_{ij} = Q_i - T_{ji}(T_{ij}(Q_i)) \quad (547)$$

schematically represent information or value lost when object Q_i is translated into Q_j and reconstructed.

Equation 547 is conceptual rather than algebraic.

It captures why monetary compensation can be useful while remaining non-equivalent to the original relational or historical object.

11.195 Object Conversion and General Equivalents

Money functions institutionally as a common target of object translation.

Many losses are mapped into monetary value:

$$T_{\$} : \mathcal{Q} \longrightarrow \mathbb{R}_{\geq 0}. \quad (548)$$

Equation 548 can support compensation.

The existence of $T_{\$}$ does not prove that $T_{\$}$ is injective, structure-preserving, or normatively complete.

Different losses can map to the same monetary value.

The reverse mapping from money to the original object can be unavailable.

This is the formal foundation for the later political-economic analysis of general equivalents.

11.196 Object Restoration and New Generation

A restorative intervention can fail to reproduce the original object while creating a new object of comparable or greater value.

Let

$$Q_{\text{lost}} \not\approx Q_{\text{new}}, \quad V(Q_{\text{new}}) > V(Q_{\text{lost}}) \quad (549)$$

under a selected value theory.

Equation 549 represents future-directed regeneration.

It demonstrates again why historical equivalence and normative improvement are different dimensions.

11.197 Object-Level Non-Equivalence and Practical Success

A practical response can succeed without restoring equivalence on every object.

A person can lose a singular relationship permanently while gaining strong new resources and capabilities.

A community can lose one historical institution while creating a better governance structure.

A rehabilitation process can fail to reproduce previous bodily state while supporting high present autonomy.

The theory should therefore distinguish:

$$\text{object-level non-equivalence} \quad (550)$$

from

$$\text{overall practical failure.} \quad (551)$$

These are not synonymous.

11.198 Object-Level Equivalence and Overall Injustice

The converse is equally important.

A system can achieve equivalence on the selected object while remaining unjust overall.

Equal income can coexist with domination.

Equal function can be produced through coercive treatment.

Equal network structure can be imposed without consent.

Equal output can coexist with extraction.

The object taxonomy therefore prepares the way for a later theory of multi-dimensional substantive justice.

11.199 Object Families and Comparative Relations

The taxonomy of objects does not determine which mathematical or conceptual relation should connect them.

Entity identity invites equality or identity.

Networks can invite isomorphism, homomorphism, embedding, or similarity.

Transition systems can invite simulation or bisimulation.

Trajectories can invite metric proximity, orbit equivalence, or conjugacy.

Distributions can invite equality, divergence measures, or stochastic dominance.

Reachable sets can invite equality, inclusion, homeomorphism, or approximate Hausdorff similarity.

Causal systems can invite interventional equivalence.

Generative mechanisms can invite simulation, abstraction, or process equivalence.

Historical structures can invite identity, order isomorphism, provenance correspondence, or weaker narrative relations.

The next section therefore moves from the object axis of the atlas to the relation axis.

11.200 The Object Axis of the Atlas

Let

$$\Omega = \{Q_{\text{entity}}, Q_{\text{resource}}, Q_{\text{attribute}}, Q_{\text{status}}, Q_{\text{capability}}, Q_{\text{relation}}, Q_{\text{network}}, Q_{\text{state}}, Q_{\text{trajectory}}, Q_{\text{trace}}, Q_{\text{transition}}, Q_{\text{reach}}, Q_{\text{dynamic}}, Q_{\text{control}}, Q_{\text{generative}}\} \quad (552)$$

denote the object axis of the atlas.

Equation 552 is open-ended.

Its purpose is to ensure that a claim of restoration identifies the object before the comparative relation is chosen.

11.201 Implications for Restorative Theory

The taxonomy developed in this section yields several conclusions.

First, restoration is object-relative.

Second, equality in one object does not imply equality in another.

Third, static and dynamic objects should remain distinct.

Fourth, resources, capabilities, relations, control, and generative processes occupy different levels of analysis.

Fifth, singular and historical objects resist simple substitution.

Sixth, system-level and individual-level objects can diverge.

Seventh, procedural objects are legitimate objects of justice rather than external constraints only.

Eighth, future possibility can be represented through reachable and viable sets without requiring one deterministic counterfactual biography.

Ninth, robust restoration can require comparison of response structures rather than ordinary baseline outputs.

Tenth, value generation and value circulation should be separated.

Eleventh, meaning, recognition, provenance, and history cannot be inferred from material equality.

Twelfth, composite objects can support plural analysis without requiring one scalar index.

Thirteenth, object selection is simultaneously epistemic, institutional, procedural, and normative.

Fourteenth, the choice of object already reveals part of the theory of justice before any equivalence relation has been specified.

11.202 Transition to Comparative Relations

The first axis of the Atlas of Equivalence is now available.

A restorative comparison has an explicit reference system and an explicit comparison object.

The remaining question is:

What relation between the post-restoration object and the reference object should count as sameness, preservation, correspondence, simulation, substitutability, adequacy, or improvement?

This question requires a taxonomy that extends beyond strict mathematical equivalence relations.

Equality, congruence, isomorphism, homomorphism, embedding, homeomorphism, diffeomorphism, topological conjugacy, simulation, mutual simulation, bisimulation, trace equivalence, observational equivalence, statistical indistinguishability, interventional equivalence, metric proximity, inclusion, dominance, refinement, and sufficiency preserve different structures.

Some are symmetric; others are directional. Some are equivalence relations; others are preorders, mappings, or approximate criteria. Some preserve internal structure; others preserve only selected observations. Some compare outcomes; others compare possible responses. Some are stronger than others under specified assumptions; many remain incomparable.

Section 12 therefore develops the second major taxonomy of the atlas: the comparative relations through which restorative objects can be judged equivalent, corresponding, substitutable, simulated, approximately similar, sufficient, or superior.

Table 3: Principal families of objects of restorative comparison

Object family	Illustrative objects	Characteristic use	Characteristic limitation
Entities	Property, equipment, documents, specific assets	Restitution	Identity does not imply generative significance
Resources	Income, capital, housing, access, time	Compensation and distribution	Conversion differs across subjects and histories
Attributes	Health, rank, productivity, centrality	Administrative and empirical comparison	Compresses underlying structure
Formal status	Employment, license, membership, entitlement	Legal and institutional restoration	Formal status can differ from effective access
Capabilities	Opportunity sets, capability profiles	Substantive freedom	Requires specification of dimensions and comparison rule
Relations	Trust, mentorship, authority, recognition	Relational repair	Often singular and historically constituted
Networks	Topology, position, multiplex structure	Professional and collective restoration	Structural similarity can ignore relational meaning
Distributions	Income, health, opportunity, joint distributions	Population and distributive justice	Aggregate equivalence can conceal individual differences
States	Current bodily, economic, professional condition	Endpoint restoration	Does not determine future dynamics
Trajectories	Life course, recovery path, institutional development	Temporal restoration	Counterfactual trajectories can be uncertain
Traces	Observable behavior sequences	Behavioral comparison	Can conceal branching and internal structure
Transition structures	State transitions, transition kernels	Simulation and behavioral potential	Requires model specification
Reachability	Reachable and viable future sets	Future opportunity and generativity	Can be difficult to identify
Dynamics	Transition law, parameters, orbit structure	Process restoration	Formal model can remain incomplete
Attractor structures	Attractors, basins, bifurcations	Long-run dynamics and resilience	Model-dependent and domain-specific
Perturbation response	Response functions, recovery time, robustness	Resilience under future stress	Depends upon selected perturbation class
Causal structures	Causal graph, structural equations, intervention responses	Mechanism and policy comparison	Identification can be incomplete
Conversion structures	Resource-to-capability and capability-to-output maps	Effective use of returned resources	Often partially observed
Control structures	Ownership, authority, dependency, decision rights	Non-domination and generative justice	Can be normatively contested
Value structures	Output, generation, circulation, recursive retention	Political economy and generative justice	Value itself can be multi-dimensional
Meaning structures	Recognition, narrative, stigma, public interpretation	Symbolic and relational repair	Resists simple quantification
Historical structures	Events, provenance, memory, identity, path dependence	Historical and reparative justice	Strong restoration can be structurally unavailable
Procedural objects	Consent, contestability, privacy, process trajectory	Procedural justice	Distinct from substantive outcome
Composite objects	Multi-dimensional restorative profiles	Comprehensive evaluation	Requires rules for conflict and aggregation

12 Comparative Relations of Restoration

This section develops the second major axis of the Atlas of Equivalence: the relations through which two restorative objects can be judged identical, equivalent, structurally corresponding, behaviorally substitutable, mutually simulable, approximately similar, sufficient, or ordered in value. The preceding section identified the object Q being compared. The present section specifies the relation E in a restorative claim of the form introduced in Equation 195.

The principal objective is to prevent the term *equivalence* from being used as though it denoted one universal standard. Mathematics, computer science, dynamical systems, probability theory, causal analysis, and order theory contain many distinct comparative relations. These relations preserve different structures. Equality preserves the represented object itself. Isomorphism preserves structure up to relabeling. Homomorphism preserves specified operations or relations in one direction. Simulation preserves possible behavior asymmetrically. Bisimulation preserves branching behavior through a mutual matching condition. Trace equivalence preserves observable execution sequences while abstracting from some branching structure. Topological conjugacy preserves dynamical organization under an invertible change of coordinates. Statistical indistinguishability preserves behavior relative to a selected statistical test class. Causal and interventional equivalence preserve selected responses to interventions. Dominance, refinement, sufficiency, and preorder relations express directional adequacy without requiring symmetry.

The methodological structure of the section is therefore comparative. Each relation is introduced through five questions: which mathematical structure the relation preserves, whether the relation is symmetric or directional, what a restorative interpretation would mean, which objects from Section 11 can naturally support the relation, and which dimensions remain outside the comparison. The section then examines relations among the relations themselves through refinement, implication, partial ordering, and test-relative strength. It concludes by constructing the relation axis $\mathfrak{E}$ that will be combined with the object axis $\mathfrak{Q}$ in Section 13.

The terminology in this section is deliberately broader than *equivalence relation*. Several important restorative relations, including homomorphism, simulation, set inclusion, dominance, and sufficiency, do not satisfy the mathematical axioms of an equivalence relation. The general term *comparative relation* is therefore used for the full family.

12.1 Comparative Relations as Typed Relations

Let $Q : \mathcal{G} \rightarrow \mathcal{Q}$ be a selected comparison-object projection. A comparative relation over $\mathcal{Q}$ is represented by Equation 553.

$$E_Q \subseteq \mathcal{Q} \times \mathcal{Q}. \quad (553)$$

Equation 553 makes the relation type-dependent. A graph isomorphism acts upon graph objects. Bisimulation acts upon transition systems or state relations. A Wasserstein distance acts upon probability measures under suitable conditions. Topological conjugacy acts upon dynamical systems. Set inclusion acts upon sets. A dominance relation acts upon ordered value or opportunity structures.

The restorative judgment is then given by Equation 554.

$$Q(\mathcal{G}^R) E_Q Q(\mathcal{G}^{\text{ref}}). \quad (554)$$

Equation 554 emphasizes that the meaning of E_Q depends upon Q . The same symbol should not be assumed to preserve the same substantive information across object types.

12.2 Equivalence Relations

A mathematical equivalence relation E on set $\mathcal{Q}$ satisfies reflexivity, symmetry, and transitivity. These conditions are stated in Equations 555 through 557.

$$qEq \quad \text{for every } q \in \mathcal{Q}, \quad (555)$$

$$q_1Eq_2 \implies q_2Eq_1, \quad (556)$$

$$q_1Eq_2 \wedge q_2Eq_3 \implies q_1Eq_3. \quad (557)$$

Equations 555 through 557 imply that $\mathcal{Q}$ is partitioned into equivalence classes. Within a specified formalism, objects in one class are indistinguishable under the structure preserved by E .

This mathematical property has a useful restorative interpretation. A restorative equivalence standard declares certain differences irrelevant to the particular claim being evaluated. The normative importance of that abstraction requires separate justification.

12.3 Exact Identity

The strongest ordinary relation at the represented-object level is exact identity:

$$q_R = q_{\text{ref}}. \quad (558)$$

Equation 558 requires the represented objects to be the same under the formal representation.

For an identifiable asset, exact identity can mean return of the same object. For a scalar resource, it can mean equality of amount. For a state vector, it can mean equality of all included coordinates. For an event record, it can mean identity of the stored representation.

Identity is highly demanding only relative to the selected representation. If q contains income alone, equal income satisfies identity in the income space while leaving health, relations, and history unconstrained.

Identity should therefore be described as *object-relative exactness* rather than complete restoration.

12.4 Numerical Equality

Many restorative practices employ equality of a quantitative attribute:

$$v_R = v_{\text{ref}}. \quad (559)$$

Equation 559 can describe equal income, equal monetary value, equal resource quantity, equal measured function, or another scalar.

This relation is central to compensation because a general equivalent frequently permits losses and transfers to be represented numerically.

Numerical equality preserves the selected quantity. It does not preserve the structure through which the quantity was produced, the meaning of the quantity, or the future consequences associated with it.

12.5 Vector Equality

Where the object is multidimensional, exact equality can require equality in all components:

$$\mathbf{q}_R = \mathbf{q}_{\text{ref}}. \quad (560)$$

Equation 560 is stronger than equality of an aggregated score because it preserves every represented coordinate.

A vector containing income, healthcare access, professional status, and network centrality can therefore support a stronger restorative claim than a single weighted index.

The result remains sensitive to the dimensions selected for inclusion.

12.6 Set Equality

For opportunity, capability, action, reachable, or viable sets, exact equality takes the form

$$S_R = S_{\text{ref}}. \quad (561)$$

Equation 561 preserves membership of every represented option.

Set equality can constitute a strong capability or reachability criterion. It remains silent about the costs of moving among elements, probabilities of reaching them, subjective recognition of the options, and relations among the elements.

12.7 Identity up to Representation

Two real systems can be identical under a selected projection while differing outside it. Let Π be a representation map. Representational identity is given by Equation 562.

$$\Pi(\mathcal{G}_1) = \Pi(\mathcal{G}_2). \quad (562)$$

Equation 562 is weaker than ontological identity of the represented systems.

This distinction is central to restorative language. A legal system can truthfully establish representational identity within the dimensions it controls while remaining unable to claim broader identity.

12.8 Congruence

Congruence expresses equivalence compatible with a specified family of operations or contexts.

Let E be an equivalence relation and let f be an operation defined on the relevant structure. A congruence condition takes the form

$$q_1 E q_2 \implies f(q_1) E f(q_2). \quad (563)$$

Equation 563 is valuable for restoration because it requires equivalence to survive specified future operations.

A salary equality that disappears as soon as ordinary taxation, benefit eligibility, or institutional rules are applied may be weaker than a congruent restoration under those operations.

Congruence therefore begins to connect static equivalence with future compatibility.

12.9 Structural Isomorphism

Isomorphism preserves structure under a bijective relabeling.

For structures

$$\mathcal{S}_1 = (X_1, \Omega_1)$$

and

$$\mathcal{S}_2 = (X_2, \Omega_2),$$

an isomorphism is a bijection $\phi : X_1 \rightarrow X_2$ preserving the relevant operations or relations.

For a binary relation R , the preservation condition can be written as Equation 564.

$$x R_1 y \iff \phi(x) R_2 \phi(y). \quad (564)$$

Equation 564 permits two systems to have different elements while preserving the relational architecture.

A restorative interpretation can therefore say that a replacement environment has the same represented structure as the reference even though its members differ.

For example, a professional network can be graph-isomorphic to the earlier network while containing different collaborators. This is a strong structural claim and a weak historical-identity claim.

12.10 Graph Isomorphism

For graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, graph isomorphism requires a bijection $\phi : V_1 \rightarrow V_2$ such that Equation 565 holds.

$$(u, v) \in E_1 \iff (\phi(u), \phi(v)) \in E_2. \quad (565)$$

Equation 565 can support comparison of organizational, professional, community, or information networks.

Graph isomorphism preserves adjacency structure. It need not preserve the identity of participants, substantive meaning of relations, edge histories, or external context.

A restorative claim of network isomorphism should therefore be distinguished from restoration of singular relationships.

12.11 Typed and Weighted Isomorphism

Social networks frequently contain edge types and weights. A stronger isomorphism can require preservation of labels and weights.

Let ℓ denote edge type and ω denote edge weight. The conditions are stated in Equations 566 and 567.

$$\ell_1(u, v) = \ell_2(\phi(u), \phi(v)), \quad (566)$$

$$\omega_1(u, v) = \omega_2(\phi(u), \phi(v)). \quad (567)$$

Equations 566 and 567 produce a stronger structural correspondence.

The stronger relation can remain normatively weaker than historical identity because participant identity, trust provenance, and meaning can differ.

12.12 Order Isomorphism

Where the object is ordered, an order isomorphism preserves ordering.

For partially ordered sets $(P, \preceq_P)$ and $(Q, \preceq_Q)$, a bijection $\phi : P \rightarrow Q$ is an order isomorphism when Equation 568 holds.

$$x \preceq_P y \iff \phi(x) \preceq_Q \phi(y). \quad (568)$$

Equation 568 can apply to ranked opportunities, institutional authority, precedence relations, or historical partial orders.

An order-isomorphic restoration preserves relative ordering while allowing absolute values to differ.

12.13 Historical-Order Isomorphism

For partially ordered event histories $(\mathcal{E}_1, \prec_1)$ and $(\mathcal{E}_2, \prec_2)$, historical-order isomorphism requires a bijection preserving the event-order relation.

The criterion is stated in Equation 569.

$$e_i \prec_1 e_j \iff \phi(e_i) \prec_2 \phi(e_j). \quad (569)$$

Equation 569 is weaker than identity of event content and stronger than endpoint equivalence.

It can support comparison of processes whose sequence structure matters.

12.14 Homomorphism

Homomorphism preserves specified structure without requiring a bijection.

For algebraic structures with operation $\star$, a homomorphism ϕ satisfies Equation 570.

$$\phi(x \star_1 y) = \phi(x) \star_2 \phi(y). \quad (570)$$

A homomorphism can identify several elements of the source or omit parts of the target. It therefore expresses structure preservation under abstraction or compression.

For restorative comparison, homomorphism is useful when the reference contains distinctions that the restored system represents at a coarser level while preserving selected operations.

Homomorphism is generally directional. Existence of a homomorphism from $\mathcal{S}_1$ to $\mathcal{S}_2$ does not imply existence of a homomorphism in the reverse direction.

12.15 Graph Homomorphism

For graphs, a homomorphism $\phi : V_1 \rightarrow V_2$ preserves adjacency:

$$(u, v) \in E_1 \implies (\phi(u), \phi(v)) \in E_2. \quad (571)$$

Equation 571 is weaker than graph isomorphism.

A restorative interpretation can allow several original relations to map into one replacement relation while preserving specified connectivity.

Such compression can be useful for functional restoration and inadequate for singular relational restoration.

12.16 Epimorphic Compression

A surjective structure-preserving map can represent compression from a richer reference into a coarser restored structure.

Let

$$\phi : Q_{\text{ref}} \twoheadrightarrow Q_R. \quad (572)$$

Equation 572 expresses the possibility that every element of the restored structure has a reference antecedent while several reference elements are identified together.

This relation can model institutional compression of several opportunities, roles, or pathways into fewer replacement categories.

The loss of distinction should be evaluated explicitly.

12.17 Embedding

An embedding preserves the source structure inside the target.

A restorative embedding can be represented by an injective map

$$\iota : Q_{\text{ref}} \hookrightarrow Q_R \quad (573)$$

that preserves the relevant structure.

Equation 573 can express a strong future-oriented criterion: every relevant reference structure remains represented inside the restored system, while the restored system can contain additional structures.

Embedding is therefore particularly suitable for a restoration that permits new possibilities in addition to old ones.

12.18 Reference-Preserving Extension

A simple set-based version of restorative embedding is

$$Q_{\text{ref}} \subseteq Q_R. \quad (574)$$

Equation 574 states that the restored system contains every reference element and possibly more.

For capability and reachable sets, this relation can be normatively attractive. It allows the subject to retain all relevant old options while acquiring new ones.

The relation can become problematic where the additional possibilities impose burdens, domination, or unwanted complexity. Set expansion alone is therefore insufficient as a complete theory of justice.

12.19 Set Inclusion

Set inclusion provides a directional relation:

$$S_1 \subseteq S_2. \quad (575)$$

Equation 575 is reflexive and transitive and is generally asymmetric.

It defines a partial order on sets under ordinary conditions.

For restorative analysis, inclusion is central to capability, reachable-set, action-set, and viable-set comparison.

A restored opportunity set can dominate the reference by inclusion without being equal to it.

12.20 Proper Inclusion

Strict expansion is represented by

$$S_{\text{ref}} \subset S_R. \quad (576)$$

Equation 576 identifies one form of better-than-reference restoration.

The normative interpretation remains dependent upon the quality and burden of the added options.

12.21 Projection Preservation

A comparative relation can require equality after projection into a selected subspace.

Let π be a projection. Projection equivalence is given by Equation 577.

$$\pi(q_R) = \pi(q_{\text{ref}}). \quad (577)$$

Equation 577 is common in institutional practice. A compensation process can preserve monetary value while abstracting from the original object. A functional rehabilitation process can preserve task ability while abstracting from bodily state.

Projection equivalence should always identify the discarded dimensions.

12.22 Quotient Equivalence

A system can be compared after differences deemed irrelevant have been identified through an equivalence relation $\approx$.

The quotient object is

$$Q/\approx. \quad (578)$$

Restoration can then require equality in the quotient:

$$[q_R] = [q_{\text{ref}}]. \quad (579)$$

Equation 579 expresses formally that some distinctions are intentionally ignored.

The ethical significance lies in the choice of $\approx$. Differences treated as irrelevant by one theory can be central to another.

12.23 Functional Equivalence

Functional equivalence compares what objects do rather than their internal structure or identity.

Let F_Q denote a selected functional map. Functional equivalence is defined by Equation 580.

$$F_Q(q_R) = F_Q(q_{\text{ref}}). \quad (580)$$

Equation 580 can describe replacement of a laboratory, assistive device, institutional role, or service by another object that performs the same selected function.

Functional equivalence is highly important for practical repair because exact identity is often impossible.

Its principal limitation lies in function selection. A mentor can provide similar professional advice while differing in intellectual relation, trust, network position, and historical meaning. Functional equivalence is therefore indexed to the function represented by F_Q .

12.24 Multi-Functional Equivalence

Where an object serves several functions, equivalence can require preservation of a vector of functions:

$$\mathbf{F}(q_R) = \mathbf{F}(q_{\text{ref}}). \quad (581)$$

Equation 581 reduces the risk that one visible function substitutes for a richer set of generative roles.

A research institution, for example, can provide funding, legitimacy, collaboration, equipment, intellectual community, and future access simultaneously.

12.25 Role Equivalence

A relational object can be compared through the role it occupies in a wider structure.

Let $\text{Role}(q, \mathcal{S})$ denote the structural role of object q within system $\mathcal{S}$. Role equivalence is stated in Equation 582.

$$\text{Role}(q_R, \mathcal{S}_R) = \text{Role}(q_{\text{ref}}, \mathcal{S}_{\text{ref}}). \quad (582)$$

Equation 582 permits replacement of a particular entity while preserving its place within the wider generative architecture.

The criterion is stronger than isolated functional equivalence where the function depends upon structural position.

12.26 Simulation

Simulation compares possible behavior directionally.

Let $\mathfrak{T}_1 = (S_1, L, \rightarrow_1)$ and $\mathfrak{T}_2 = (S_2, L, \rightarrow_2)$ be labeled transition systems. A relation $\mathcal{S} \subseteq S_1 \times S_2$ is a simulation when, for every $(p, q) \in \mathcal{S}$, each transition from p can be matched by an appropriate transition from q .

The condition is stated in Equation 583.

$$p \xrightarrow{a}_1 p' \implies \exists q' \left[q \xrightarrow{a}_2 q' \wedge (p', q') \in \mathcal{S} \right]. \quad (583)$$

Equation 583 is directional. System 2 simulates System 1 when every relevant behavior of System 1 can be matched by System 2.

Simulation and related semantics occupy an established place in concurrency theory; van Glabbeek's linear-time–branching-time spectrum provides a systematic account of simulation, trace, bisimulation, and related process semantics [66].

For restorative justice, simulation has an especially attractive interpretation:

Every relevant generative action or transition available in the reference system remains reproducible in the restored system.

The restored system can possess additional transitions. Simulation can therefore support restoration through preservation plus expansion.

12.27 Restorative Simulation

Let $\mathfrak{T}_{\text{ref}}$ be the reference transition system and $\mathfrak{T}_R$ the restored transition system.

A restorative simulation relation is written as

$$\mathfrak{T}_{\text{ref}} \preceq_{\text{sim}} \mathfrak{T}_R. \quad (584)$$

Equation 584 means that the restored system can simulate the selected behavior of the reference system.

For capability analysis, this can mean that every relevant reference action remains available after restoration.

For generative analysis, it can mean that every selected generative transition can be matched.

The criterion is stronger than simple set inclusion when branching and transition structure matter.

12.28 Forward Simulation

Forward simulation preserves the ability to match future transitions from related states.

Its restorative interpretation is prospective:

Starting from the restored configuration, future evolution can reproduce every selected reference transition through an appropriate correspondence.

Forward simulation is therefore naturally aligned with future-directed generativity.

It does not require reconstruction of past history.

12.29 Backward Simulation

Some systems admit backward-style simulation conditions that relate predecessor structures or abstract implementation relations.

For restorative theory, backward comparison can become relevant where provenance or reconstructability matters.

The concept should be applied cautiously because social histories can be structurally non-invertible, as discussed in Section 8.22.

12.30 Mutual Simulation

Two systems are mutually simulable when each simulates the other:

$$\mathfrak{T}_1 \preceq_{\text{sim}} \mathfrak{T}_2 \quad \text{and} \quad \mathfrak{T}_2 \preceq_{\text{sim}} \mathfrak{T}_1. \quad (585)$$

Equation 585 is symmetric at the system level.

Mutual simulation can remain weaker than bisimulation because the two simulation relations need not be realized through one common state-matching relation.

This distinction matters for branching behavior.

12.31 Bisimulation

Bisimulation requires transitions to be matched in both directions through one common relation.

Let $\mathcal{B} \subseteq S_1 \times S_2$. A strong bisimulation satisfies the two conditions in Equations 586 and 587.

$$p \xrightarrow{a}_1 p' \implies \exists q' \left[q \xrightarrow{a}_2 q' \wedge (p', q') \in \mathcal{B} \right], \quad (586)$$

$$q \xrightarrow{a}_2 q' \implies \exists p' \left[p \xrightarrow{a}_1 p' \wedge (p', q') \in \mathcal{B} \right]. \quad (587)$$

If such a relation connects the initial states, the systems are bisimilar.

Milner's process-calculus work gives strong bisimulation a central role in behavioral equivalence for communicating systems [38]. Van Glabbeek's spectrum situates bisimulation among several linear-time and branching-time semantics [66].

A restorative interpretation of bisimulation is demanding:

Every relevant transition available to the reference can be matched by the restored system, and every corresponding transition of the restored system can be matched within the reference structure, recursively across future branching.

This standard can approach behavioral indistinguishability at the represented transition level.

12.32 Restorative Bisimulation

For restorative systems, write

$$\mathfrak{T}_R \sim_{\text{bis}} \mathfrak{T}_{\text{ref}}. \quad (588)$$

Equation 588 is stronger than one-way simulation.

It can be appropriate where the restorative objective is preservation of the reference branching structure.

Its normative desirability remains context-dependent. A restored subject can legitimately acquire additional possibilities absent from the reference. Such additional possibilities can break bisimulation while improving generativity.

Bisimulation is therefore formally strong without possessing automatic normative priority.

12.33 Weak Bisimulation

Weak bisimulation abstracts from selected internal or unobservable transitions.

Let τ denote an internal transition. Weak transition notation $\Rightarrow$ permits finite sequences of internal steps around observable actions.

A weak bisimulation can therefore match behavior while ignoring differences classified as internal.

For restoration, weak bisimulation raises an immediate normative question:

Which differences are legitimate to hide as internally irrelevant?

A system can appear behaviorally equivalent after internal suffering, effort, dependency, or surveillance has been abstracted away.

The abstraction boundary must therefore be normatively justified.

12.34 Probabilistic Bisimulation

For stochastic transition systems, bisimulation can require matching probability distributions over equivalence classes rather than deterministic successors.

A generic condition can be written as Equation 589.

$$P_1(C \mid p, a) = P_2(C \mid q, a) \quad \text{for every relevant class } C. \quad (589)$$

Equation 589 can support comparison of uncertain future responses.

The criterion is useful where generative possibilities are probabilistic rather than deterministic.

12.35 Approximate Bisimulation

Exact bisimulation can be excessively strong for empirical systems.

An approximate form permits bounded mismatch:

$$d(\mathcal{O}(p), \mathcal{O}(q)) \leq \varepsilon \quad (590)$$

together with approximate transition matching under an appropriate formalism.

Equation 590 illustrates the role of a tolerance parameter.

Approximate bisimulation is attractive for restoration because social and biological systems rarely admit exact state correspondence.

Its legitimacy depends upon the metric d , tolerance ε , and the dimensions included in $\mathcal{O}$.

12.36 Trace Equivalence

Trace equivalence compares sets of observable execution sequences.

For transition systems $\mathfrak{T}_1$ and $\mathfrak{T}_2$, trace equivalence can be written as

$$\mathcal{L}_{\text{tr}}(\mathfrak{T}_1) = \mathcal{L}_{\text{tr}}(\mathfrak{T}_2). \quad (591)$$

Equation 591 preserves which observable sequences can occur.

Trace semantics generally abstracts from branching structure. Van Glabbeek's linear-time–branching-time spectrum makes the distinction between trace-based, simulation-based, and bisimulation-based semantics explicit [66].

For restoration, trace equivalence can mean:

The restored and reference systems can produce the same relevant observable histories.

Two systems can satisfy this criterion while differing in which choices are available at intermediate points.

12.37 Trace Inclusion

A directional trace relation is

$$\mathcal{L}_{\text{tr}}(\mathfrak{T}_{\text{ref}}) \subseteq \mathcal{L}_{\text{tr}}(\mathfrak{T}_R). \quad (592)$$

Equation 592 means every observable reference history remains possible in the restored system.

The restored system can contain additional traces.

This criterion is naturally compatible with future-option expansion.

12.38 Failure Equivalence

Process semantics can distinguish systems by the actions they can refuse after specified traces.

A failure observation records a trace and a set of unavailable actions.

For restorative analysis, failure semantics can be relevant where equivalent positive behavior hides different inaccessible options.

Two subjects can perform the same observed work while one has lost the ability to refuse certain institutional demands.

The formal apparatus of failure semantics is established within the broader linear-time–branching-time spectrum [66].

12.39 Readiness Equivalence

Readiness semantics compares which actions are available after a trace.

Its restorative interpretation is directly relevant to capability and opportunity.

Two systems can produce the same past traces while differing in the set of actions currently available.

Readiness therefore lies conceptually between trace comparison and richer branching comparison.

12.40 Testing Equivalence

Testing equivalence compares systems according to the outcomes of a specified family of tests.

Let $\mathcal{T}$ be a test class. A generic testing equivalence is represented by Equation 593.

$$T(q_1) = T(q_2) \quad \text{for every } T \in \mathcal{T}. \quad (593)$$

Equation 593 provides a general framework that extends beyond concurrency theory.

For restoration, a test can be an observation, institutional challenge, ordinary environmental change, or selected perturbation.

Testing equivalence makes explicit that indistinguishability is always relative to a test family.

12.41 Observational Equivalence

Observational equivalence requires equality under selected observation maps.

Let $\mathcal{O}$ be a family of admissible observations. Then Equation 594 defines a basic form.

$$O(\mathcal{G}_R) = O(\mathcal{G}_{\text{ref}}) \quad \text{for every } O \in \mathcal{O}. \quad (594)$$

Equation 594 can formalize a common practical judgment that two systems are indistinguishable under the observations relevant to an institution.

Its limits follow directly from the observation family. A workplace can appear restored through salary, attendance, and output while trust or internal effort remains unobserved.

Observational equivalence therefore inherits the ontology of its measurement system.

12.42 Observer-Relative Equivalence

Different observers can possess different observation maps.

For observer j , equivalence can be written as

$$O_j(\mathcal{G}_R) = O_j(\mathcal{G}_{\text{ref}}). \quad (595)$$

Equation 595 can hold for one observer and fail for another.

An employer can observe equivalent output. The subject can experience different usability. A clinician can observe different physiological response. A network analyst can observe different relational position.

The plurality of observers therefore prevents observational equivalence from becoming a universal restorative criterion.

12.43 Contextual Equivalence

A stronger observational criterion compares systems inside a family of contexts.

Let $C[\cdot]$ denote an admissible context. Contextual equivalence can require

$$O(C[\mathcal{G}_1]) = O(C[\mathcal{G}_2]) \quad \text{for every } C \in \mathcal{C}. \quad (596)$$

Equation 596 asks whether the systems remain indistinguishable after being embedded in relevant environments.

For restorative justice, context can include ordinary workplace changes, resource variations, institutional interactions, or social environments.

This relation is stronger than baseline observational equality.

12.44 Behavioral Equivalence

Behavioral equivalence is used here as a broad family rather than one uniquely defined mathematical relation.

A generic behavioral criterion requires correspondence of a selected behavior map $\mathcal{B}$:

$$\mathcal{B}(\mathcal{G}_R) = \mathcal{B}(\mathcal{G}_{\text{ref}}). \quad (597)$$

The map can contain outputs, traces, decisions, responses, or task performance.

Behavioral equivalence is stronger or weaker depending upon the behavior class included.

Its restorative interpretation should therefore specify the behavior space rather than use the term as a free-standing claim.

12.45 Task Equivalence

A practical form of behavioral equivalence concerns ability to perform a specified task set.

Let $\mathcal{T}_{\text{task}}$ denote the relevant tasks. Task equivalence can require

$$\text{Perf}_R(t) = \text{Perf}_{\text{ref}}(t) \quad \text{for every } t \in \mathcal{T}_{\text{task}}. \quad (598)$$

Equation 598 is useful in occupational or rehabilitation contexts.

Equal task performance can conceal differences in effort, stress, autonomy, future resilience, or meaning.

12.46 Performance Equivalence

A weaker criterion can compare performance metrics:

$$P_R = P_{\text{ref}}. \quad (599)$$

Equation 599 is often institutionally tractable.

The simplicity creates a risk of treating measured output as a proxy for the whole generative system.

Performance equality should therefore remain explicitly local.

12.47 Effort-Adjusted Equivalence

Where equal output is produced at unequal cost, the effort required can be included.

Let C_{effort} denote effort or burden. An effort-adjusted relation can require both Equations 600 and 601.

$$P_R = P_{\text{ref}}, \quad (600)$$

$$C_{\text{effort},R} \sim C_{\text{effort},\text{ref}}. \quad (601)$$

The pair of conditions is stronger than output equivalence alone.

This relation is relevant where a subject maintains ordinary performance through exceptional compensatory effort after harm.

12.48 Topological Equivalence

Topological comparison preserves continuity structure while ignoring metric details.

Two topological spaces X and Y are homeomorphic when there exists a bijective continuous map

$$h : X \longrightarrow Y \quad (602)$$

whose inverse is also continuous.

Equation 602 permits deformation without tearing or gluing in the topological sense.

For restorative analysis, homeomorphism can be relevant to reachable sets, attractors, basins, or other geometric structures.

A post-harm possibility space can therefore be topologically equivalent to the reference while differing metrically in distances, costs, and probabilities.

12.49 Homeomorphism of Opportunity Spaces

Suppose reachable sets $\mathcal{R}_R$ and $\mathcal{R}_{\text{ref}}$ are homeomorphic.

The relation can be written as

$$\mathcal{R}_R \cong_{\text{top}} \mathcal{R}_{\text{ref}}. \quad (603)$$

Equation 603 preserves broad topological organization.

It does not preserve transition cost, probability, subjective usability, or historical provenance.

The relation can therefore be valuable for structural analysis and insufficient for capability justice by itself.

12.50 Diffeomorphism

Where smooth structure matters, a diffeomorphism requires a smooth bijection with a smooth inverse.

For smooth manifolds M_1 and M_2 ,

$$\phi : M_1 \longrightarrow M_2 \quad (604)$$

is a diffeomorphism when the relevant smoothness conditions hold.

Equation 604 preserves substantially more geometric structure than homeomorphism.

Diffeomorphic comparison can become relevant where restorative models use continuous state manifolds.

Its philosophical relevance depends upon whether the preserved smooth structure corresponds to anything normatively important.

12.51 Topological Conjugacy

Topological conjugacy compares dynamical systems rather than spaces alone.

Let $f : X \rightarrow X$ and $g : Y \rightarrow Y$ be dynamical maps. They are topologically conjugate when there exists a homeomorphism $h : X \rightarrow Y$ satisfying Equation 605.

$$h \circ f = g \circ h. \quad (605)$$

Equation 605 means that the dynamics correspond under a continuous change of coordinates.

Topological conjugacy is a standard strong relation in dynamical-systems classification; Katok and Hasselblatt treat equivalence, classification, and invariants as foundational issues in modern dynamical-systems theory [27].

For restorative justice, topological conjugacy offers a powerful idealized standard:

The restored and reference systems possess the same qualitative dynamical organization up to an invertible continuous relabeling of states.

The relation can preserve orbit structure while ignoring quantitative timing or metric differences.

12.52 Smooth Conjugacy

A stronger dynamical relation requires the conjugating map to be a diffeomorphism.

Smooth conjugacy preserves smooth dynamical structure beyond topological orbit organization.

For human or social applications, this criterion will often be far stronger than available evidence supports.

Its presence in the atlas remains useful because it marks one end of the structural-strength spectrum.

12.53 Semiconjugacy

Semiconjugacy weakens conjugacy by allowing the comparison map to be non-invertible.

A semiconjugacy satisfies

$$h \circ f = g \circ h \quad (606)$$

without requiring h to possess an inverse.

Equation 606 permits a richer system to project onto a coarser dynamical system.

For restorative comparison, semiconjugacy can represent preservation of a reference dynamic at an abstract level while allowing additional internal structure.

12.54 Orbit Equivalence

Orbit equivalence preserves correspondence among orbits while permitting differences in temporal parameterization.

A generic orbit-equivalence condition requires a bijection h mapping each orbit of one system onto an orbit of the other.

For restoration, this relation can be appropriate where the sequence of qualitative states matters while exact speed of development does not.

A subject can follow a similar professional or rehabilitative path over a different timescale.

Whether timing can be ignored depends upon the value of lost time.

12.55 Time-Reparameterized Equivalence

A stronger statement of timing flexibility can compare trajectories through a monotone time transformation θ .

The relation is represented by Equation 607.

$$\gamma_R(t) = h(\gamma_{\text{ref}}(\theta(t))). \quad (607)$$

Equation 607 can treat delayed but qualitatively similar recovery as equivalent.

The normative acceptability of this abstraction is limited where time itself is a lost good.

12.56 Metric Equivalence

Where the object space has a metric d , exact equality can be relaxed to distance-based comparison.

Exact metric identity remains

$$d(q_R, q_{\text{ref}}) = 0. \quad (608)$$

Approximate metric equivalence uses a tolerance:

$$d(q_R, q_{\text{ref}}) \leq \varepsilon. \quad (609)$$

Equation 609 is among the most general forms of approximate restoration.

Its substantive meaning depends entirely upon the metric and threshold.

12.57 Weighted Metric Equivalence

For vector objects, a weighted distance can be used:

$$d_W(\mathbf{q}_1, \mathbf{q}_2) = \left[\sum_{i=1}^n w_i d_i(q_{1i}, q_{2i})^p \right]^{1/p}. \quad (610)$$

Equation 610 makes normative weighting explicit.

The weights w_i encode judgments concerning relative importance.

Mathematical convenience does not supply those judgments.

12.58 Norm Equivalence

In a normed vector space, approximate equivalence can be written as

$$\|q_R - q_{\text{ref}}\| \leq \varepsilon. \quad (611)$$

Equation 611 is convenient for state vectors and parameter structures.

The norm determines which deviations are treated as large or small.

12.59 Hausdorff Proximity

For reachable, viable, or capability sets embedded in a metric space, the Hausdorff distance provides one possible set comparison.

Let A and B be nonempty sets. The Hausdorff distance is represented in Equation 612.

$$d_H(A, B) = \max \left\{ \sup_{a \in A} \inf_{b \in B} d(a, b), \sup_{b \in B} \inf_{a \in A} d(a, b) \right\}. \quad (612)$$

Approximate set restoration can require

$$d_H(\mathcal{R}_R, \mathcal{R}_{\text{ref}}) \leq \varepsilon. \quad (613)$$

Equation 613 preserves closeness of entire sets rather than only volume.
The result remains sensitive to the metric on the underlying state space.

12.60 Distributional Equality

For probability distributions P_R and P_{ref} , the strongest distributional relation is equality:

$$P_R = P_{\text{ref}}. \quad (614)$$

Equation 614 can apply to state distributions, trajectory distributions, or perturbation responses.

Distributional equality does not imply sample-path identity.

It can therefore preserve population or stochastic structure while allowing different realized histories.

12.61 Equality in Distribution

Random variables X_R and X_{ref} are equal in distribution when

$$P(X_R \leq x) = P(X_{\text{ref}} \leq x) \quad \text{for all relevant } x. \quad (615)$$

Equation 615 can support a statistical restoration claim without requiring the same individual realization.

This criterion can be relevant to population-level reparations or risk restoration.

It can be ethically inadequate where justice concerns individual history.

12.62 Moment Matching

A weaker statistical comparison matches selected moments:

$$\mathbb{E}[X_R^k] = \mathbb{E}[X_{\text{ref}}^k] \quad k = 1, \dots, m. \quad (616)$$

Equation 616 is easier to estimate than full distributional equality.

Different distributions can share the selected moments.

Moment matching should therefore be described as a partial statistical equivalence.

12.63 Statistical Distance

A general probabilistic approximation can require

$$D(P_R, P_{\text{ref}}) \leq \varepsilon, \quad (617)$$

where D is a selected divergence or distance.

Possible choices include total variation distance, Wasserstein distance, Kullback–Leibler divergence under suitable conditions, or another domain-specific measure.

Each captures different features.

The divergence selection should therefore be interpreted as part of the restorative criterion.

12.64 Statistical Indistinguishability

Statistical indistinguishability can be defined relative to a test family.

Let $\mathcal{T}_{\text{stat}}$ denote admissible statistical tests. Two systems are ε -indistinguishable when no selected test distinguishes them beyond the specified advantage.

A generic form is given in Equation 618.

$$\sup_{T \in \mathcal{T}_{\text{stat}}} |P_R(T = 1) - P_{\text{ref}}(T = 1)| \leq \varepsilon. \quad (618)$$

Equation 618 is highly relevant to the paper's perturbation-response thought experiment.

The result is test-relative. A richer test class can reveal differences hidden from a weaker class.

12.65 Baseline Statistical Indistinguishability

A restored and reference system can be statistically indistinguishable under ordinary conditions:

$$D(P_R(O), P_{\text{ref}}(O)) \leq \varepsilon. \quad (619)$$

Equation 619 does not establish indistinguishability under stress or intervention.

This limitation motivates response-conditioned criteria.

12.66 Perturbation-Conditioned Statistical Equivalence

Let Δ_{adm} be an admissible perturbation class. Robust statistical equivalence can require

$$D(P_R(O | \delta), P_{\text{ref}}(O | \delta)) \leq \varepsilon \quad \text{for every } \delta \in \Delta_{\text{adm}}. \quad (620)$$

Equation 620 formalizes the criterion introduced conceptually in Section 1.7.

It is substantially stronger than ordinary statistical similarity.

The perturbation class remains a normative and epistemic choice.

12.67 Interventional Equivalence

Two systems can be compared by their response to interventions.

Let $\mathcal{U}_{\text{adm}}$ denote the admissible intervention class. Interventional equivalence can require

$$P_R(Y | do(u)) = P_{\text{ref}}(Y | do(u)) \quad \text{for every } u \in \mathcal{U}_{\text{adm}}. \quad (621)$$

Equation 621 follows the distinction between observation and intervention emphasized in structural causal analysis [43].

For restoration, the criterion asks whether the same treatment, assistance, policy, resource, or environmental change produces the same selected response.

This relation is especially important where equal current states hide altered conversion mechanisms.

12.68 Partial Interventional Equivalence

A weaker relation requires equivalence only for a subset of interventions:

$$P_R(Y | do(u)) = P_{\text{ref}}(Y | do(u)) \quad u \in \mathcal{U}_0. \quad (622)$$

Equation 622 can be practically appropriate where only certain interventions matter to the institutional purpose.

The scope should be stated explicitly.

12.69 Causal-Graph Isomorphism

Causal structures can be compared structurally.

Let $\mathcal{C}_R = (V_R, E_R)$ and $\mathcal{C}_{\text{ref}} = (V_{\text{ref}}, E_{\text{ref}})$. A causal-graph isomorphism preserves the directed-edge structure under a bijection.

This relation is stronger than equality of observational distributions.

It remains weaker than equality of structural functions or counterfactual responses.

12.70 Structural-Equation Equivalence

A stronger causal relation can compare the structural mechanisms themselves.

For structural equations

$$X_i = f_i(\text{Pa}_i, U_i),$$

equivalence can require corresponding functions to agree after a mapping between variables.

Such a criterion preserves intervention-generating mechanisms.

Its practical identification can be substantially harder than observational or interventional equivalence on a limited action class.

12.71 Counterfactual Equivalence

The strongest causal comparison can require equality of selected counterfactual distributions:

$$P_R(Y_u) = P_{\text{ref}}(Y_u) \quad \text{for every selected } u. \quad (623)$$

Equation 623 concerns potential outcomes under hypothetical interventions.

The epistemic demands are high because counterfactual equivalence depends upon causal structure beyond ordinary observation.

12.72 Conversion Equivalence

A generative system can be compared through its resource-conversion map.

Let $\mathcal{K}_R$ and $\mathcal{K}_{\text{ref}}$ be conversion functions. Exact conversion equivalence is

$$\mathcal{K}_R(c) = \mathcal{K}_{\text{ref}}(c) \quad \text{for every relevant } c. \quad (624)$$

Equation 624 asks whether the same condition produces the same capability or usable output.

This relation directly formalizes the paper's concern that equal compensation can generate unequal future possibilities.

12.73 Approximate Conversion Equivalence

An approximate criterion can require

$$d(\mathcal{K}_R(c), \mathcal{K}_{\text{ref}}(c)) \leq \varepsilon \quad c \in \mathcal{C}_{\text{adm}}. \quad (625)$$

Equation 625 is naturally indexed to a condition class.

A subject need not convert every imaginable resource identically for the restorative process to be adequate.

12.74 Control-Structure Equivalence

Control structures can be compared through equality or isomorphism of authority relations.

Let χ_R and χ_{ref} represent decision rights. A structural criterion can require

$$\chi_R \cong \chi_{\text{ref}}. \quad (626)$$

Equation 626 can preserve access authority, revocation power, appeal, ownership, and governance participation.

Generative justice and non-domination make this object especially important [44, 15].

12.75 Control Dominance

Restoration need not reproduce the earlier control structure where the earlier structure was unjust.

A transformed system can dominate the reference under a selected non-domination criterion:

$$\chi_R \succeq_{\text{ND}} \chi_{\text{ref}}. \quad (627)$$

Equation 627 allows improvement beyond historical equivalence.

This relation illustrates why directional order relations belong in the atlas.

12.76 Value Equivalence

A value-based relation can require equal value under a valuation function V :

$$V(q_R) = V(q_{\text{ref}}). \quad (628)$$

Equation 628 is weaker than object identity whenever different objects possess the same value.

Monetary compensation frequently relies upon such a structure.

The theory of value determines whether this equivalence is meaningful.

12.77 Value-Vector Equivalence

For multidimensional value,

$$\mathbf{V}(q_R) = \mathbf{V}(q_{\text{ref}}). \quad (629)$$

Equation 629 preserves all represented value dimensions.

It remains possible for the underlying objects and histories to differ.

12.78 Value Dominance

A directional relation can require the restored system to be at least as good as the reference under every selected value dimension:

$$\mathbf{V}(q_R) \geq \mathbf{V}(q_{\text{ref}}) \quad (630)$$

componentwise, with strict inequality in at least one dimension for strict dominance.

Equation 630 permits better-than-reference restoration.

The value dimensions must be normatively justified.

12.79 Pareto Dominance

For multi-party restorative systems, Pareto dominance can be defined over individual evaluations V_i .
System R_1 weakly Pareto-dominates R_2 when Equation 631 holds.

$$V_i(R_1) \geq V_i(R_2) \quad \text{for every } i, \quad (631)$$

with strict inequality for at least one subject in the strict form.
Pareto dominance is normatively attractive where available.
It provides no ranking when restorative interests conflict.

12.80 Lexicographic Dominance

Some values can be given lexical priority.

Let V_1 have priority over V_2 . Lexicographic comparison first compares V_1 ; V_2 matters only when V_1 ties.

This can represent rights constraints, minimum safety, or agency requirements that cannot be traded freely against compensation.

Lexicographic relations become important where stronger equivalence in one domain cannot justify violation of a protected process value.

12.81 Preorder

A preorder is reflexive and transitive without requiring antisymmetry or symmetry.

Let

$$q_1 \preceq q_2 \quad (632)$$

denote such a relation.

Preorders are highly useful for restorative comparison because many notions of “at least as capable,” “at least as generative,” or “no worse than” are directional.

Simulation itself induces preorder-like structures in many process settings.

Preorders therefore deserve equal conceptual status with equivalence relations in the atlas.

12.82 Partial Order

A partial order is reflexive, antisymmetric, and transitive.

Restorative systems can be partially ordered when some outcomes are comparable and others are not.

Let

$$R_1 \succeq_R R_2 \quad (633)$$

denote restorative ordering under a selected set of dimensions.

The possibility of incomparability is normatively important because plural values need not admit a complete ranking.

12.83 Total Order

A total order compares every pair of objects.

Many institutional scoring systems implicitly construct total orders through one scalar.

Such ordering is administratively convenient.

Its use should be justified where the underlying values are heterogeneous or incommensurable.

The atlas therefore treats total-order representation as one option rather than the default.

12.84 Dominance Without Equivalence

A restorative outcome can be preferable to the reference without being equivalent to it.

This relation is represented by

$$q_R \succ q_{\text{ref}} \quad \text{and} \quad q_R \not\sim q_{\text{ref}}. \quad (634)$$

Equation 634 is central to transformative justice and subject-directed new trajectories.

It shows why the atlas cannot consist solely of equivalence relations.

12.85 Sufficiency

Sufficiency compares an object with a threshold or acceptable region.

Let $S_{\text{adequate}} \subseteq \mathcal{Q}$. Sufficiency is defined by

$$q_R \in S_{\text{adequate}}. \quad (635)$$

Equation 635 does not require comparison with one specific reference object.

The relation is central to social-protection floors, minimum capability thresholds, safety, and emergency support.

It can coexist with an independent claim for corrective compensation above the threshold.

12.86 Threshold Equivalence

Two objects can be treated as equivalent for an institutional purpose when both satisfy the same threshold class.

Let

$$C_k = \{q : \theta_{k-1} \leq f(q) < \theta_k\}. \quad (636)$$

Objects q_1 and q_2 are threshold-equivalent when they lie in the same class C_k .

This relation can be administratively necessary for eligibility.

It discards differences within the class.

The lost distinctions should therefore be recognized where they matter to individual justice.

12.87 Refinement

One comparative relation refines another when it distinguishes at least as much structure.

Let E_2 refine E_1 when

$$q_1 E_2 q_2 \implies q_1 E_1 q_2. \quad (637)$$

Equation 637 means every pair equivalent under E_2 is also equivalent under E_1 .

The converse need not hold.

Refinement creates a formal notion of comparative strength.

12.88 Strict Refinement

E_2 strictly refines E_1 when Equation 637 holds and there exists at least one pair satisfying E_1 without satisfying E_2 .

This relation allows the atlas to represent families of increasingly discriminating criteria.

Formal refinement should remain distinct from normative superiority.

12.89 Coarsening

A relation E_1 is coarser than E_2 when E_2 refines E_1 .

Coarsening intentionally ignores distinctions.

Institutional practice frequently uses coarse relations to achieve administrability.

The normative question concerns whether the ignored distinctions are relevant to the restorative purpose.

12.90 Relation Implication

A comparative relation can imply another under specified assumptions.

Write

$$E_1 \implies E_2 \quad (638)$$

when satisfaction of E_1 guarantees satisfaction of E_2 for the specified object class.

For example, graph isomorphism implies equality of many graph invariants. Bisimulation implies several weaker behavioral equivalences under common transition-system settings. Topological conjugacy implies a correspondence of qualitative orbit structure.

The assumptions should be stated because implication relations can depend upon the object class.

12.91 Relation Incomparability

Two relations can preserve different structures without either refining the other.

Let

$$E_1 \parallel E_2 \quad (639)$$

denote incomparability in the refinement order.

One relation can preserve topology while another preserves probability. One can preserve network adjacency while another preserves output distribution.

This incomparability is central to the atlas.

It prevents all restorative standards from being arranged on one ladder from weak to strong.

12.92 Equivalence-Class Granularity

An equivalence relation partitions object space into classes.

A finer relation produces smaller equivalence classes.

Let $[q]_E$ denote the equivalence class of q . If E_2 refines E_1 , then

$$[q]_{E_2} \subseteq [q]_{E_1}. \quad (640)$$

Equation 640 gives a geometric interpretation of stronger formal discrimination.

The normative issue concerns how small the acceptable restorative class should be.

12.93 Approximation Relation

An approximation relation allows bounded difference:

$$q_R \approx_\varepsilon q_{\text{ref}}. \quad (641)$$

Equation 641 is shorthand for a criterion defined by a metric, divergence, test family, simulation error, or another tolerance rule.

Approximation is essential for empirical restoration.

The relation should never be treated as self-explanatory because ε requires a substantive justification.

12.94 Asymmetric Approximation

Some approximation measures are directional.

A restoration can require every reference behavior to be approximated by the restored system while permitting additional restored behavior.

A generic directional criterion is

$$d_{\rightarrow}(q_{\text{ref}}, q_R) \leq \varepsilon. \quad (642)$$

Equation 642 is suitable for future-expansion criteria.

12.95 Tolerance Relation

A tolerance relation can treat objects within some threshold as sufficiently similar even when transitivity fails.

For metric d ,

$$q_1 T_{\varepsilon} q_2 \iff d(q_1, q_2) \leq \varepsilon. \quad (643)$$

Equation 643 is reflexive and symmetric under ordinary metric conditions and need not be transitive.

This point matters conceptually. Institutional notions of “close enough” need not form mathematical equivalence classes.

The atlas therefore includes tolerance relations separately from equivalence relations.

12.96 Similarity Relation

A similarity function can assign degree

$$s(q_1, q_2) \in [0, 1]. \quad (644)$$

Equation 644 permits graded restorative comparison.

Similarity can be symmetric or asymmetric depending upon construction.

A high similarity score should not be interpreted as justice without a theory of the features entering s .

12.97 Kernel Similarity

A positive-definite kernel or another representation can compare objects through feature similarity.

This family is useful for high-dimensional empirical objects.

Its use in justice requires strong transparency concerning feature selection, training data, and interpretability.

Formal sophistication does not remove normative model risk.

12.98 Classification Equivalence

Institutions frequently treat objects as equivalent when they receive the same classification:

$$C(q_1) = C(q_2). \quad (645)$$

Equation 645 is administratively powerful.

It can conceal substantial within-class heterogeneity.

Classification equivalence should therefore be distinguished from substantive equivalence.

12.99 Legal Equivalence

Law can declare different objects or statuses equivalent for a legal purpose.

A legal-equivalence relation can be represented as

$$q_1 \sim_{\text{law}} q_2. \quad (646)$$

Equation 646 means the legal system treats the objects as equivalent with respect to a specified rule or consequence.

Legal equivalence can diverge from economic, relational, functional, or historical equivalence.

The divergence is a feature of institutional compression rather than an automatic defect.

12.100 Economic Equivalence

Economic or market equivalence can be defined through equal price or exchange value:

$$p(q_1) = p(q_2). \quad (647)$$

Equation 647 supports substitution through general equivalents.

Its limits are particularly visible for singular relations, rights, public goods, recognition, and historical value.

12.101 Compensatory Equivalence

Compensatory equivalence is an institutional relation asserting that transfer c adequately compensates loss l under a selected valuation rule:

$$l \sim_{\text{comp}} c. \quad (648)$$

Equation 648 should be distinguished from identity or substitution.

A compensatory relation can be justified even where the loss cannot be recreated.

This distinction is essential to the political economy of general equivalents.

12.102 Substitutability

Two objects are substitutable relative to a purpose when one can replace the other without unacceptable loss of the selected function.

Let F denote the purpose. Substitutability can be represented by

$$q_1 \sim_{\text{sub}, F} q_2. \quad (649)$$

The relation is purpose-relative.

A new laboratory can substitute for an old laboratory for some research tasks. A new collaborator can substitute for another for selected technical functions. The same substitution can fail for historical or relational purposes.

12.103 Perfect Substitution

Perfect substitution under purpose F requires equality of all relevant functional consequences:

$$F(q_1, c) = F(q_2, c) \quad \text{for every relevant context } c. \quad (650)$$

Equation 650 is substantially stronger than ordinary market substitution.

Its feasibility decreases as the context family expands.

12.104 Partial Substitution

Partial substitution preserves only a subset of functions or contexts.

Let $\mathcal{C}_0 \subset \mathcal{C}$. Then

$$F(q_1, c) = F(q_2, c) \quad c \in \mathcal{C}_0. \quad (651)$$

Equation 651 is a realistic model for many restorative replacements.

The omitted contexts should be disclosed.

12.105 Interchangeability

Interchangeability requires that two objects can be swapped within a selected system without changing the selected outputs.

For context family $\mathcal{C}$,

$$O(C[q_1]) = O(C[q_2]) \quad C \in \mathcal{C}. \quad (652)$$

Equation 652 resembles contextual equivalence.

The relation is especially useful for resources and institutional components.

Singular social relations frequently fail this standard over sufficiently rich context classes.

12.106 Reachability Equality

For reachable sets,

$$\mathcal{R}_R = \mathcal{R}_{\text{ref}} \quad (653)$$

is the strongest direct set criterion.

Equation 653 preserves which represented futures can be reached.

It does not preserve transition costs, probabilities, timing, or internal pathways.

12.107 Reachability Dominance

A future-expansion criterion is

$$\mathcal{R}_{\text{ref}} \subseteq \mathcal{R}_R. \quad (654)$$

Equation 654 means every reference future remains reachable and the restored system can contain additional futures.

This is one of the most natural formalizations of subject-directed generative expansion.

It requires qualification where some added futures impose risks or unwanted constraints.

12.108 Viability Dominance

For viable sets,

$$\mathcal{V}_{\text{ref}} \subseteq \mathcal{V}_R \quad (655)$$

requires every reference viable state to remain viable after restoration.

Equation 655 is stronger normatively than raw reachability dominance where inadmissible options have been removed.

Its content depends upon the constraints defining viability.

12.109 Attractor Correspondence

Attractor structures can be compared through a mapping

$$\phi_{\mathcal{A}} : \mathfrak{A}_{\text{ref}} \longrightarrow \mathfrak{A}_R. \quad (656)$$

Equation 656 can require one-to-one correspondence of attractors or a weaker relation.

The mapping can preserve attractor type, stability, basin relation, or another selected invariant.

This criterion is especially relevant to the dynamical justice lineage already present in generative-justice work [16].

12.110 Attractor Isomorphism

A stronger criterion can require isomorphism of an attractor graph whose nodes represent attractors and edges represent possible regime transitions.

Let $G_{\mathcal{A},R}$ and $G_{\mathcal{A},\text{ref}}$ be such graphs. The criterion is

$$G_{\mathcal{A},R} \cong G_{\mathcal{A},\text{ref}}. \quad (657)$$

Equation 657 preserves the represented long-run regime architecture.

It can hold while basin sizes differ substantially.

12.111 Basin Equivalence

Basin structures can be compared by equality, homeomorphism, measure, geometry, or another relation.

A strong structural criterion is

$$\mathcal{B}_R \cong_{\text{top}} \mathcal{B}_{\text{ref}}. \quad (658)$$

Equation 658 preserves basin topology.

A weaker criterion can compare basin measure:

$$\mu(\mathcal{B}_R) = \mu(\mathcal{B}_{\text{ref}}). \quad (659)$$

The two criteria preserve different properties.

12.112 Stability Equivalence

Two systems can be equivalent with respect to a selected stability property.

Let Stab be a stability classifier or invariant. The criterion is

$$\text{Stab}(\mathcal{G}_R) = \text{Stab}(\mathcal{G}_{\text{ref}}). \quad (660)$$

Equation 660 can compare Lyapunov stability, structural stability, recovery rate, or another domain-specific property.

The meaning of stability should be specified explicitly.

12.113 Resilience Equivalence

For resilience functional ϱ ,

$$\varrho(\mathcal{G}_R, \Delta_{\text{adm}}) = \varrho(\mathcal{G}_{\text{ref}}, \Delta_{\text{adm}}) \quad (661)$$

defines resilience equivalence.

Equation 661 preserves a summary of disturbance response.

It is weaker than full response-function equivalence.

Holling's work supports the analytical distinction between stability and resilience that motivates separate comparison [24].

12.114 Recovery-Time Equivalence

For perturbation class Δ , systems can be compared by recovery time:

$$T_{\text{rec},R}(\delta) = T_{\text{rec,ref}}(\delta) \quad \delta \in \Delta. \quad (662)$$

Equation 662 can capture hidden fragility despite similar baseline output.
Recovery-time equivalence remains only one component of resilience.

12.115 Bifurcation Equivalence

Two parameterized systems can be compared by their bifurcation structure.

Let $\mathfrak{Bif}(\mathcal{G})$ denote the selected bifurcation object. Then

$$\mathfrak{Bif}(\mathcal{G}_R) \cong \mathfrak{Bif}(\mathcal{G}_{\text{ref}}) \quad (663)$$

requires structural correspondence of regime changes under parameter variation.
This can be important where harm changes the thresholds at which future instability occurs.

12.116 Parameter Equivalence

A weaker dynamical comparison matches selected parameters:

$$\theta_R = \theta_{\text{ref}}. \quad (664)$$

Equation 664 is easy to state and potentially weak as a restorative claim.
Different model structures can assign different meaning to the same parameter value.

12.117 Sensitivity Equivalence

Two systems can be compared through derivatives or finite responses with respect to selected inputs.

A local sensitivity criterion can be written as

$$\frac{\partial O_R}{\partial u} = \frac{\partial O_{\text{ref}}}{\partial u}. \quad (665)$$

Equation 665 captures first-order response similarity.
It can hold locally while large perturbations reveal substantial difference.

12.118 Local Equivalence

A relation can hold only within a neighborhood U around the current state.

Let

$$\mathcal{G}_R \sim_{E,U} \mathcal{G}_{\text{ref}}. \quad (666)$$

Equation 666 expresses local restoration.

A subject can be dynamically equivalent for ordinary small variations while remaining different under large shocks.

The domain of validity should therefore accompany the relation.

12.119 Global Equivalence

A global relation holds across the full represented domain:

$$\mathcal{G}_R \sim_{E, \mathcal{X}} \mathcal{G}_{\text{ref}}. \quad (667)$$

Equation 667 is much stronger than local equivalence.

For social systems, global claims will often be epistemically difficult.

The atlas preserves the distinction to prevent local correspondence from being generalized automatically.

12.120 Finite-Horizon Equivalence

A comparative relation can be restricted to horizon τ :

$$\mathcal{G}_R \sim_E^\tau \mathcal{G}_{\text{ref}}. \quad (668)$$

Equation 668 can hold for one year and fail over ten years.

This is especially relevant to employment, rehabilitation, and ecological restoration.

12.121 Asymptotic Equivalence

Two systems can differ initially and converge over time.

A metric form is

$$\lim_{t \rightarrow \infty} d(x_R(t), x_{\text{ref}}(t)) = 0. \quad (669)$$

Equation 669 treats long-run convergence as the comparison standard.

The criterion can conceal severe intermediate harms.

A justice theory should therefore decide whether path cost matters independently of eventual convergence.

12.122 Long-Run Statistical Equivalence

A stochastic or ergodic system can be compared through long-run distributions:

$$\pi_R = \pi_{\text{ref}}, \quad (670)$$

where π denotes a stationary or long-run distribution where such a distribution exists.

Equal long-run distributions can coexist with different transient dynamics and individual histories.

The criterion is therefore primarily system-level.

12.123 Pathwise Equivalence

A strong stochastic comparison can require sample paths to correspond under a coupling.

Let $X_R(t)$ and $X_{\text{ref}}(t)$ be coupled processes. A pathwise criterion can require

$$P(X_R(t) = X_{\text{ref}}(t) \text{ for all relevant } t) = 1. \quad (671)$$

Equation 671 is extremely strong.

Its role in the atlas is primarily conceptual because ordinary social restoration rarely supplies enough information for such a claim.

12.124 Historical Identity

Historical identity requires equality of the represented history:

$$\mathcal{H}_R = \mathcal{H}_{\text{ref}}. \quad (672)$$

Equation 672 is stronger than state, behavioral, or structural equivalence.

After harm and restoration, strict historical identity with a no-harm history generally fails because the harm and restorative event belong to actual history.

This makes historical identity an extreme criterion.

12.125 Historical Isomorphism

A weaker historical relation can preserve event structure up to relabeling:

$$\mathcal{H}_R \cong \mathcal{H}_{\text{ref}}. \quad (673)$$

Equation 673 can preserve event dependency or order while allowing different event identities.

Such a criterion can be meaningful for formal process comparison and weak for personal historical significance.

12.126 Provenance Equivalence

Provenance structures can be compared through isomorphism or another structure-preserving relation.

Let $\mathcal{P}_R$ and $\mathcal{P}_{\text{ref}}$ be provenance graphs. A provenance-equivalence criterion can require

$$\mathcal{P}_R \cong \mathcal{P}_{\text{ref}}. \quad (674)$$

Equation 674 preserves how claims, evidence, decisions, and outputs derive from earlier elements.

It is especially relevant to authorship, institutional legitimacy, and archival justice.

12.127 Narrative Equivalence

Narrative equivalence is necessarily less formal unless a specific narrative representation is chosen.

A general form can require

$$N(\mathcal{H}_R) \sim_N N(\mathcal{H}_{\text{ref}}), \quad (675)$$

where N maps histories into a narrative structure.

Equation 675 can support philosophical discussion of identity and recognition.

The relation should not be confused with historical event identity.

12.128 Meaning Equivalence

Meaning structures can be compared through

$$\mathcal{M}_R \sim_M \mathcal{M}_{\text{ref}}. \quad (676)$$

Equation 676 requires a theory of how meaning is represented.

In many applications the relation will remain qualitative.

Its inclusion is important because identical resources and functions can possess different social meaning.

12.129 Recognition Equivalence

A recognition-based comparison can require corresponding social standing, acknowledgment, or legitimacy:

$$R_R^{\text{rec}} \sim R_{\text{ref}}^{\text{rec}}. \quad (677)$$

Equation 677 can be satisfied independently of resource equivalence.

Recognition theory provides the normative background for treating this object as distinct [25, 20].

12.130 Trust Equivalence

Trust structures can be compared as relation matrices or distributions:

$$T_R \sim_E T_{\text{ref}}. \quad (678)$$

Equation 678 can represent equality, approximate similarity, or structural correspondence.

Trust is historically generated and difficult to command directly, which limits strong restorative claims.

12.131 Agency Equivalence

Agency can be compared through selected dimensions such as option availability, decision authority, revisability, and exit.

Let $\mathbf{A}$ denote an agency profile. Agency equivalence is

$$\mathbf{A}_R \sim_E \mathbf{A}_{\text{ref}}. \quad (679)$$

Equation 679 is especially relevant where an institution can recreate output through coercive means.

Equal output with unequal agency fails this criterion.

12.132 Revisability Equivalence

A system can be compared through the subject's ability to revise or leave the current trajectory.

Let $\mathcal{R}_{\sqsubseteq}$ denote revisability. A criterion is

$$\mathcal{R}_{\sqsubseteq R} \sim_E \mathcal{R}_{\sqsubseteq \text{ref}}. \quad (680)$$

Equation 680 is useful for assistance programs that otherwise risk creating dependency.

12.133 Exit Equivalence

Exit conditions can be compared by the available paths and associated costs.

Let $\mathcal{E}_A$ denote the exit structure. The relation is

$$\mathcal{E}_{A,R} \sim_E \mathcal{E}_{A,\text{ref}}. \quad (681)$$

Equation 681 can reveal domination hidden by current resource equality.

12.134 Procedural Equivalence

Procedural structures can themselves be compared.

Let P_R and P_{ref} denote procedures. A generic procedural equivalence relation is

$$P_R \sim_E P_{\text{ref}}. \quad (682)$$

The selected object can contain consent, notice, participation, appeal, privacy, and review.

Procedural equivalence should remain distinct from the question whether either procedure is just. Two equally unjust procedures can be equivalent.

12.135 Procedural Dominance

A transformed restorative process can improve upon the reference:

$$P_R \succeq_J P_{\text{ref}}. \quad (683)$$

Equation 683 can express improved participation, privacy, contestability, or non-domination.

This relation is relevant where the historical procedure itself contributed to the harm.

12.136 Generative-Process Equivalence

The generative mechanism can be compared directly.

Let Γ_R^G and Γ_{ref}^G be the restored and reference generative mechanisms.

A generic relation is

$$\Gamma_R^G \sim_E \Gamma_{\text{ref}}^G. \quad (684)$$

Equation 684 is intentionally open. The relation E can be isomorphism, simulation, bisimulation, conjugacy, interventional equivalence, or another structure-preserving relation.

The later dedicated section on generative-process equivalence will examine these options in detail.

12.137 Recursive-Generation Equivalence

Recursive value cycles can be compared over repeated generation.

Let $\Gamma^{(n)}$ denote n rounds of generation. A finite-depth criterion can require

$$\Gamma_R^{(n)} \sim_E \Gamma_{\text{ref}}^{(n)} \quad n \leq N. \quad (685)$$

Equation 685 is stronger than one-step output equivalence.

It asks whether returned conditions continue generating comparable future conditions over repeated cycles.

12.138 Value-Circulation Equivalence

For circulation maps Λ_R and Λ_{ref} ,

$$\Lambda_R \sim_E \Lambda_{\text{ref}} \quad (686)$$

compares how generated value is retained, distributed, extracted, or reinvested.

This relation is central to generative justice.

Equal total output can coexist with radically different circulation structures.

12.139 Value-Flow Isomorphism

A value-flow network can be compared through graph isomorphism.

Let $G_{V,R}$ and $G_{V,\text{ref}}$ represent value-flow graphs. The criterion is

$$G_{V,R} \cong G_{V,\text{ref}}. \quad (687)$$

Equation 687 preserves flow topology while allowing actor relabeling.

Historical and distributive significance can remain different.

12.140 Recursive-Retention Equivalence

A weaker value-cycle comparison can match the proportion of generated value returning as future conditions:

$$\rho_R = \rho_{\text{ref}}. \quad (688)$$

Equation 688 is a scalar summary.

It can conceal differences in who controls the retained value and what forms it takes.

12.141 Historical-Generative Equivalence

The strongest family considered in this paper combines historical and generative structures.

Let

$$Q_{\text{HG}}(\mathcal{G}) = (\mathcal{H}, \Gamma^G, \mathcal{R}, \Lambda, \mathcal{R}_{\text{reach}}). \quad (689)$$

Historical-generative equivalence requires a specified relation over this compound object:

$$Q_{\text{HG}}(\mathcal{G}_R) \sim_{E_{\text{HG}}} Q_{\text{HG}}(\mathcal{G}_{\text{ref}}). \quad (690)$$

Equation 690 is deliberately abstract. Different strong relations can be chosen for the components.

The criterion demonstrates that “complete restoration” is a constructed compound standard rather than a primitive concept.

12.142 Relation Strength and Normative Strength

Formal strength and normative strength should remain separate.

Let E_2 refine E_1 . Then

$$E_2 \Rightarrow E_1 \quad (691)$$

can hold formally.

It does not follow that

$$J(E_2) \geq J(E_1). \quad (692)$$

Equation 692 expresses one of the central claims of the paper.

A stricter equivalence can require invasive intervention, erase valued new development, or preserve an unjust historical structure.

12.143 Stronger Equivalence and Greater Intervention

Stronger restorative equivalence can require stronger intervention.

Let $C_R(E)$ denote the intervention cost or intrusiveness required to attain relation E .

In some domains,

$$E_2 \text{ refines } E_1 \implies C_R(E_2) \geq C_R(E_1) \quad (693)$$

can hold.

Equation 693 is a possible structural relationship rather than a universal theorem.

It becomes normatively important where stronger intervention threatens agency, privacy, identity, or third-party rights.

12.144 Strong Equivalence and Historical Erasure

Historical equivalence can become stronger only by eliminating differences created through the actual history.

A hypothetical intervention that removes memories, records, and adaptations can increase counterfactual similarity.

The same intervention can reduce historical recognizability and agency.

This trade-off shows why historical distance cannot be minimized mechanically.

12.145 Weak Equivalence and Institutional Humility

A weaker relation can sometimes be institutionally appropriate because it matches the institution's legitimate epistemic scope.

A court can establish monetary equivalence for a compensable loss without claiming network equivalence.

A rehabilitation program can establish functional sufficiency without claiming identity of bodily state.

An employer can restore formal status without claiming restoration of trust.

Weakness in formal scope can therefore express institutional humility rather than institutional failure.

12.146 Relation Selection as a Normative Act

Choosing a comparative relation determines which differences count.

Identity treats every represented difference as relevant.

Isomorphism ignores labels while preserving structure.

Homomorphism permits compression.

Simulation requires preservation in one direction.

Bisimulation requires reciprocal matching of branching behavior.

Trace equivalence ignores some branching differences.

Topological conjugacy ignores coordinate representation.

Statistical indistinguishability ignores differences invisible to selected tests.

Sufficiency ignores differences above a threshold.

Dominance treats improvement as acceptable without equivalence.

Each choice therefore contains a substantive stance concerning permissible difference.

12.147 Relation Selection as an Epistemic Act

Relation choice also determines the information required.

Identity can require detailed state information.

Bisimulation requires transition structure.

Topological conjugacy requires a dynamical model.
 Interventional equivalence requires causal knowledge.
 Statistical indistinguishability requires distributions and test classes.
 Sufficiency can require only threshold-relevant information.
 A more demanding relation can therefore exceed the epistemic capacity of the institution.

12.148 Relation Selection as a Practical Act

Relations also differ in implementability.
 Market-value equality is often administratively feasible.
 Network isomorphism can be difficult.
 Bisimulation of a high-dimensional social system can be primarily ideal-theoretical.
 Historical-generative identity can be structurally unavailable.
 Practical feasibility should therefore be recorded separately from conceptual coherence.

12.149 Relation Selection as a Procedural Act

Affected subjects can reasonably contest the relation selected by an institution.
 An institution can regard equal salary as sufficient. The subject can argue that secure authority and professional access matter.
 A rehabilitation provider can regard task performance as sufficient. The subject can identify unsustainable effort or reduced autonomy.
 Procedural justice therefore includes contestability of the comparative standard itself.

12.150 Exact and Approximate Families

Many relations admit exact and approximate versions.
 Let E denote an exact relation and E_ε an approximation. The general structure is

$$E \subseteq E_{\varepsilon_1} \subseteq E_{\varepsilon_2} \quad \text{for } 0 \leq \varepsilon_1 \leq \varepsilon_2. \quad (694)$$

Equation 694 produces a nested family of tolerance standards.
 The appropriate ε depends upon normative salience, measurement error, feasibility, and the institutional purpose.

12.151 Test-Class Monotonicity

Equivalence becomes harder to satisfy as the admissible test class expands.
 Let

$$\mathcal{T}_1 \subseteq \mathcal{T}_2. \quad (695)$$

If systems are indistinguishable under $\mathcal{T}_2$, they are also indistinguishable under $\mathcal{T}_1$ under the same decision criterion.
 This relation is stated schematically in Equation 696.

$$\sim_{\mathcal{T}_2} \implies \sim_{\mathcal{T}_1}. \quad (696)$$

Equation 696 shows that test selection contributes to comparative strength.
 A restoration claim should therefore identify the test class.

12.152 Horizon Monotonicity

A similar relation can arise with temporal horizon.

Equivalence over a long horizon can imply equivalence over a nested shorter horizon, depending upon the relation.

Let $\tau_1 < \tau_2$. A pathwise finite-horizon criterion can satisfy

$$\sim^{\tau_2} \implies \sim^{\tau_1} . \quad (697)$$

Equation 697 does not hold for every relation without additional assumptions.

Its presence illustrates that temporal horizon contributes to the strength of a restorative claim.

12.153 Scale Dependence of Comparative Relations

The same relation can hold at one scale and fail at another.

For example,

$$\mathcal{G}_R^{(\text{organization})} \sim_E \mathcal{G}_{\text{ref}}^{(\text{organization})} \quad (698)$$

can coexist with

$$\mathcal{G}_R^{(\text{individual})} \not\sim_E \mathcal{G}_{\text{ref}}^{(\text{individual})} . \quad (699)$$

Equations 698 and 699 illustrate scale-relative restoration.

Relation metadata should therefore include scale.

12.154 Object Dependence of Comparative Strength

A relation that is strong for one object can be weak for another.

Exact equality of one scalar is weaker in informational content than graph isomorphism of a large network.

The formal symbol alone therefore does not define comparative strength.

Strength is object-relative:

$$\text{Str} (E, Q) . \quad (700)$$

Equation 700 anticipates the object–relation matrix of the next section.

12.155 Composite Comparative Relations

Complex restoration can require several relations simultaneously.

Let the composite relation be

$$E_{\text{comp}} = E_1 \wedge E_2 \wedge \cdots \wedge E_n . \quad (701)$$

Equation 701 can require resource equality, network reachability preservation, and procedural sufficiency simultaneously.

The conjunction is stronger than each component.

It can also become infeasible or normatively excessive.

12.156 Disjunctive Comparative Relations

Some institutional designs permit alternative routes to adequacy.

A criterion can take the form

$$E_{\text{alt}} = E_1 \vee E_2. \quad (702)$$

Equation 702 can represent a program in which either reinstatement or an accepted alternative trajectory satisfies a practical objective.

Disjunctive standards are especially relevant to subject-directed restoration.

12.157 Conditional Comparative Relations

A relation can depend upon contextual conditions:

$$C \implies E_1, \quad \neg C \implies E_2. \quad (703)$$

Equation 703 can represent different restorative standards when literal return is feasible and when it is unavailable.

Conditional relations permit practical flexibility while preserving explicit decision rules.

12.158 Lexically Ordered Comparative Relations

Several relations can be ordered lexically.

For example,

$$E_{\text{rights}} \succ_{\text{lex}} E_{\text{agency}} \succ_{\text{lex}} E_{\text{fidelity}}. \quad (704)$$

Equation 704 is illustrative.

Such ordering can express a theory in which rights constraints are satisfied before counterfactual similarity is optimized.

The atlas records the structure without selecting one universal order.

12.159 Relation Lattices

Some families of relations can be organized through refinement into a lattice or partial lattice structure.

If two relations E_1 and E_2 possess a common refinement and common coarsening, one can ask whether meet and join operations exist.

The prospective meet can represent the strongest relation implied by both:

$$E_1 \wedge E_2. \quad (705)$$

The prospective join can represent the weakest relation refining both under an appropriate construction:

$$E_1 \vee E_2. \quad (706)$$

Whether the full restorative relation space forms a lattice is left open.

The heterogeneity of object classes and directional relations makes a global lattice structure unlikely without additional restrictions.

12.160 Linear-Time and Branching-Time Families

The distinction between linear-time and branching-time semantics is especially useful for restorative analysis.

Linear-time relations compare realized or possible sequences.

Branching-time relations preserve the structure of choices available at intermediate states.

Van Glabbeek's spectrum systematically distinguishes these semantics [66].

The justice interpretation is direct.

Two subjects can have access to the same final outcomes and realized traces while differing in the choices available along the way.

A branching-sensitive criterion can therefore capture agency and option structure omitted by a purely linear-time comparison.

12.161 Branching Structure and Agency

Suppose two systems reach the same endpoint.

In System 1, the subject had several meaningful choices. In System 2, the same path was imposed.

Endpoint equality can hold while branching structure differs.

Let $\text{Branch}(x)$ denote the available transitions at state x . A branching-sensitive comparison can require

$$\text{Branch}_R(x) \sim_E \text{Branch}_{\text{ref}}(x). \quad (707)$$

Equation 707 provides a formal bridge between transition semantics and procedural autonomy.

12.162 Choice-Preserving Equivalence

A specifically normative branching relation can require preservation of a selected set of meaningful choices.

Let $\mathcal{C}(x)$ denote those choices. Then

$$\mathcal{C}_{\text{ref}}(x) \subseteq \mathcal{C}_R(\phi(x)). \quad (708)$$

Equation 708 resembles simulation while restricting attention to normatively relevant actions.

This criterion can be more appropriate than full bisimulation where additional restored options are desirable.

12.163 Freedom-Preserving Simulation

A freedom-preserving simulation can require that every action classified as a valuable option in the reference has a corresponding action in the restored system.

Let $\mathcal{U}_V$ denote the valuable-action subset. Then

$$u \in \mathcal{U}_{V,\text{ref}} \implies \exists u' \in \mathcal{U}_{V,R} \text{ matching } u. \quad (709)$$

Equation 709 shows how a formal simulation relation can be value-filtered.

The value filter must be supplied by normative theory.

12.164 Generativity-Preserving Simulation

Similarly, simulation can be restricted to transitions that generate specified future capacities or value.

Let $\rightarrow_G$ denote generatively relevant transitions. A criterion requires

$$x \xrightarrow{a}_{G,\text{ref}} x' \implies \exists y' \left[y \xrightarrow{a'}_{G,R} y' \wedge (x', y') \in \mathcal{S}_G \right]. \quad (710)$$

Equation 710 will later support the dedicated theory of generative-process equivalence.

12.165 Historical Simulation

A restored system can simulate the consequences of a reference history without sharing the history itself.

Let historical state h_{ref} map to restored state h_R . Historical simulation can require future transitions from the reference history to remain reproducible after restoration.

This criterion is weaker than historical identity and stronger than endpoint equality.

It can be a useful model for future-directed repair after irreversible history.

12.166 Meaning-Preserving Morphism

A relation can preserve selected meaning labels while allowing underlying entities to change.

Let $\mathcal{M}$ assign meanings to objects. A meaning-preserving map ϕ satisfies

$$\mathcal{M}_R(\phi(q)) = \mathcal{M}_{\text{ref}}(q). \quad (711)$$

Equation 711 is conceptual.

It can become relevant to cultural restoration, recognition, ritual, or institutional roles.

Meaning preservation is difficult to establish empirically and can remain a qualitative relation.

12.167 Provenance-Preserving Morphism

A map can preserve provenance relations:

$$q_i \prec_{\text{prov}} q_j \implies \phi(q_i) \prec_{\text{prov}} \phi(q_j). \quad (712)$$

Equation 712 can support comparison of archives, authorship structures, and institutional records.

The criterion preserves derivational order without requiring identity of every record.

12.168 Resource-to-Capability Commutativity

A particularly useful restorative relation can compare conversion diagrams.

Let R_C map reference resources into restored resources and R_K map reference capabilities into restored capabilities.

A commutative preservation condition is

$$R_K \circ \mathcal{K}_{\text{ref}} = \mathcal{K}_R \circ R_C. \quad (713)$$

Equation 713 means that converting then mapping produces the same represented result as mapping then converting.

This criterion is stronger than equality of resource amount because it preserves the structure connecting resources to capabilities.

It provides one formal bridge between capability theory and structural comparison.

12.169 Value-Cycle Commutativity

A similar diagram can compare value generation and circulation.

Let R_C map conditions and R_V map generated value. A preservation criterion for generation is

$$R_V \circ \Gamma_{\text{ref}} = \Gamma_R \circ R_C. \quad (714)$$

Equation 714 means that corresponding conditions generate corresponding value under the restorative mapping.

For circulation, require

$$R_C \circ \Lambda_{\text{ref}} = \Lambda_R \circ R_V. \quad (715)$$

Equation 715 preserves the recursive condition–value cycle.

Together, Equations 714 and 715 define a strong form of structural generative correspondence.

12.170 Commuting Restoration Diagram

The previous relations can be summarized through a commuting diagram structure.

A restorative mapping preserves a generative cycle when the corresponding generation and circulation paths commute.

This criterion is particularly attractive for the later theory of generative-process equivalence because it compares the architecture of generation rather than one output.

The requirement remains representation-relative.

12.171 Relation Families Indexed by Value Theory

The same mathematical relation can receive different normative importance under different value theories.

Let V index the normative interpretation. The relation is written

$$E^V. \quad (716)$$

Equation 716 does not modify the mathematics. It records that the relevance of the preserved structure depends upon V .

Resource equality, capability-set inclusion, non-domination dominance, and generative-cycle preservation illustrate different value-indexed selections.

12.172 Relation Families Indexed by Institutional Purpose

The relation can also be indexed by institutional purpose I :

$$E^I. \quad (717)$$

A court can use compensatory equivalence. A regulator can use safety sufficiency. A rehabilitation service can use functional equivalence. A professional body can use credential identity. A generative-assistance program can use reachability dominance.

Institutional purpose therefore affects legitimate relation choice.

12.173 Relation Families Indexed by Temporal Phase

Different temporal phases can favor different relations.

Let

$$E^{\text{pre}}, \quad E^{\text{ongoing}}, \quad E^{\text{post}} \quad (718)$$

denote phase-specific comparative criteria.

Prevention can focus on safe-set inclusion and robustness. Interruption can focus on reduction of harmful transitions. Post-harm justice can use restitution, compensation, capability, relation, reachability, or generative-process comparisons.

The same case can therefore contain several relation types across time.

12.174 Relation Families Indexed by Epistemic Strength

A relation can be conceptually definable while unsupported by available evidence.

Let

$$K(E, Q) \quad (719)$$

denote the epistemic basis supporting application of relation E to object Q .

A system can support resource equality with high confidence and historical-generative equivalence with very low confidence.

The atlas should preserve this difference.

12.175 Relation Feasibility

Let

$$F(E, Q, I) \quad (720)$$

denote feasibility of evaluating or achieving relation E on object Q for institution I .

Formal definability and institutional feasibility are independent.

This distinction allows idealized relations to remain analytically useful without becoming immediate policy recommendations.

12.176 Relation Cost

Achieving a relation can have resource, informational, temporal, and procedural cost.

Let

$$C(E, Q) \quad (721)$$

denote the cost profile associated with pursuing E for object Q .

A stronger relation can require more monitoring, intervention, or control.

This cost must enter institutional design.

12.177 Relation Intrusiveness

A separate dimension concerns intrusion into the subject or third parties.

Let

$$I_{\text{intr}}(E, Q) \quad (722)$$

denote the intrusiveness required to establish or attain the relation.

Behavioral equivalence can require limited intervention. Internal-state identity can require invasive measurement. Historical identity can require impossible or normatively objectionable intervention.

Intrusiveness is therefore independent of formal strength.

12.178 Relation Revisability

Some restorative relations permit easy later revision.

A monetary transfer leaves many future paths open. A permanent institutional reconstruction can be harder to reverse. Memory modification in an idealized case can have low reversibility.

Let

$$R_{\text{rev}}(E, Q) \quad (723)$$

denote the revisability profile associated with pursuing the relation.

Revisability can become a procedural reason to prefer one adequate relation over another.

12.179 Relation-Induced Dependency

A restorative standard can itself generate dependency.

A program requiring continuous institutional monitoring to maintain equivalence can create a different generative structure from one that becomes self-sustaining.

Let

$$D_{\text{new}}(E, Q) \quad (724)$$

denote dependency generated by pursuing E .

Generative justice has particular reason to attend to this dimension.

12.180 Relation Robustness

A restorative conclusion can be robust across modeling choices.

Let $\mathfrak{M}$ be a plausible model family. Relation robustness can require

$$Q(\mathcal{G}_R) E_M Q(\mathcal{G}_{\text{ref}}) \quad \text{for every } M \in \mathfrak{M}. \quad (725)$$

Equation 725 increases epistemic confidence where the conclusion is model-invariant.

12.181 Reference Robustness of Relations

A relation can also hold across several plausible reference systems:

$$Q(\mathcal{G}_R) EQ(B) \quad \text{for every } B \in \mathfrak{B}. \quad (726)$$

Equation 726 identifies reference-robust restoration.

This criterion can be valuable where baseline disputes remain unresolved.

12.182 Comparative Relation Metadata

Each relation can be described through metadata

$$\Theta(E) = (r, s, t, d, a, o, p, f), \quad (727)$$

where:

- r denotes reflexivity;
- s denotes symmetry;
- t denotes transitivity;
- d denotes directionality;
- a denotes approximation structure;
- o denotes information required;
- p denotes structures preserved;
- f denotes practical feasibility.

Equation 727 permits systematic comparison of relations before normative interpretation is added.

12.183 Summary Classification of Comparative Relations

Table 4 summarizes the principal relation families developed in this section. The table identifies characteristic preservation properties and restorative interpretations without ranking the relations normatively.

Table 4 demonstrates that the relation axis is not a single hierarchy. Several relation families preserve orthogonal structures.

12.184 The Comparative-Relation Axis

Let

$$\mathfrak{E} = \{=, \cong, \text{Hom}, \text{Emb}, \sim_{\text{func}}, \preceq_{\text{sim}}, \sim_{\text{bis}}, \sim_{\text{tr}}, \sim_{\text{obs}}, \sim_{\text{top}}, \sim_{\text{conj}}, \approx_{\varepsilon}, \sim_{\text{dist}}, \sim_{\text{int}}, \subseteq, \supseteq, \text{Suff}, \text{Refine}, \dots\} \quad (728)$$

denote the relation axis of the atlas.

Equation 728 deliberately includes relations that are not mathematical equivalence relations.

The notation functions as a catalogue of possible comparative grammars.

12.185 Formal Strength Spectrum

Within restricted object classes, some relations can be partially ordered by formal strength.

For labeled transition systems, for example, branching-sensitive equivalences typically distinguish more structure than trace equivalence, a distinction systematically developed in the linear-time–branching-time spectrum [66].

Within dynamical systems, smooth conjugacy preserves more differentiable structure than topological conjugacy, while topological conjugacy preserves more dynamical structure than equality of a few observables.

Within probability, equality of distributions is stronger than matching a finite set of moments.

The full atlas contains no universal strength order because relation families operate on heterogeneous objects.

12.186 Normative Non-Monotonicity

Formal refinement does not produce monotonic growth in justice.

Suppose E_2 refines E_1 :

$$E_2 \Rightarrow E_1. \quad (729)$$

A normative theory can nevertheless prefer restoration satisfying only E_1 when achievement of E_2 requires coercion, erasure of valued adaptation, excessive surveillance, or reconstruction of an unjust historical structure.

The normative relation can therefore satisfy

$$J(E_1) > J(E_2) \quad (730)$$

in a particular case.

Equation 730 is one of the most important results of the comparative taxonomy.

12.187 Formal Weakness and Normative Sufficiency

A formally weak relation can be normatively sufficient.

A social-protection program can require only threshold sufficiency.

A court can require monetary compensation according to a legally justified valuation.

A rehabilitation service can require functional adequacy.

A subject-directed assistance program can require viable reachability toward a selected future.
The correct comparative standard therefore depends upon institutional and normative purpose.

12.188 Relation Pluralism

Complex restoration can legitimately employ different relations for different objects.

A single case can require:

- identity for return of an artifact;
- monetary equivalence for income loss;
- sufficiency for current social protection;
- capability inclusion for future options;
- graph correspondence for network re-entry;
- interventional similarity for rehabilitation;
- dominance for a transformed control structure;
- procedural adequacy for the restorative process.

This is *relation pluralism*.

Pluralism becomes incoherent only when the purposes and objects of the relations are concealed or contradictory.

12.189 Relation Bundles

A restorative standard can therefore be represented as a bundle

$$\mathbf{E} = (E_1, E_2, \dots, E_n). \quad (731)$$

Each E_i applies to object Q_i .

The resulting restorative claim is

$$Q_i (\mathcal{G}^R) E_i Q_i (\mathcal{G}_i^{\text{ref}}) \quad i = 1, \dots, n. \quad (732)$$

Equation 732 supplies the formal basis for a multi-object restorative profile.

12.190 Relation Conflict

Two comparative standards can favor different restorative interventions.

Bisimulation with the counterfactual reference can favor reconstruction of old branching structure.

Reachability dominance can favor creation of additional new options.

Historical identity can favor preservation of past relations.

Subject-directed sufficiency can favor departure from those relations.

These conflicts belong to normative theory rather than formal relation theory alone.

The atlas makes the conflict visible before attempting resolution.

12.191 Relation Compatibility

Two relations are compatible for a case when one restorative intervention can satisfy both.

Let $\mathcal{R}(E_i)$ denote the intervention set capable of producing relation E_i . Compatibility requires

$$\mathcal{R}(E_1) \cap \mathcal{R}(E_2) \neq \emptyset. \quad (733)$$

Equation 733 makes clear that formal criteria can be individually coherent and jointly infeasible.

This issue becomes important in multi-object justice.

12.192 Minimal Common Relation

Where several theories demand different relations, one can search for a weaker criterion supported by all.

Let $E_1, \dots, E_n$ be proposed standards. A minimal common relation $E_\cap$ can be defined where appropriate as a relation implied by all:

$$E_i \Rightarrow E_\cap \quad i = 1, \dots, n. \quad (734)$$

Equation 734 can support robust institutional action under normative disagreement.

The existence and uniqueness of such a relation are not guaranteed.

12.193 Maximal Justified Relation

A different problem asks for the strongest relation justified by the available evidence, resources, and normative constraints.

Let $\mathcal{E}_{\text{adm}}$ denote admissible relations. A maximal justified relation can be represented schematically by

$$E^* \in \text{Max}(\mathcal{E}_{\text{adm}}) \quad (735)$$

under the selected refinement ordering.

Equation 735 does not imply that institutions should always maximize formal strength.

The admissible set itself incorporates normative and practical constraints.

12.194 Comparative Relations and Claims of Completion

The comparative taxonomy provides a precise reason that claims of complete restoration require caution.

A system can satisfy

$$q_R = q_{\text{ref}} \quad (736)$$

for one object and fail

$$r_R \sim_E r_{\text{ref}} \quad (737)$$

for another.

Even within one object, it can satisfy a coarse relation and fail a refined relation.

Completion is therefore always indexed to an object family and relation family.

12.195 Relation Metadata for the Atlas

A relation entry in the atlas should record at least:

- formal domain;
- reflexivity;
- symmetry;
- transitivity;
- directionality;
- structures preserved;
- information discarded;
- exact or approximate character;
- test or context dependence;
- temporal dependence;
- scale dependence;
- epistemic requirements;
- practical feasibility;
- intervention burden;
- characteristic normative interpretations;
- characteristic failure modes.

This metadata transforms the atlas from a list of mathematical terms into a comparative analytical instrument.

12.196 Relation Selection Protocol

A restorative inquiry can select a relation through the following conceptual sequence.

First, specify the restorative object Q .

Second, specify the reference B .

Third, identify the structures of Q treated as normatively salient.

Fourth, determine which differences may legitimately be abstracted away.

Fifth, select the weakest relation preserving every salient structure.

Sixth, identify whether stronger relations add normatively useful information.

Seventh, evaluate the epistemic and practical requirements of the relation.

Eighth, evaluate the process required to attain it.

This protocol favors adequacy over maximal formal strength.

12.197 Principle of Comparative Explicitness

The taxonomy supports a general principle:

A claim of restorative equivalence is incomplete unless it specifies the comparative relation under which equivalence, correspondence, adequacy, or improvement is asserted.

This principle complements the earlier requirements to specify the reference and comparison object.

12.198 Principle of Preserved Structure

A second principle concerns interpretation:

The substantive meaning of a restorative relation is determined by the structures that the relation preserves and the structures that it permits to vary.

The principle provides a common language for identity, isomorphism, simulation, bisimulation, statistical indistinguishability, and sufficiency.

12.199 Principle of Normative Non-Monotonicity

A third principle concerns justice:

A formally stronger restorative relation does not, by virtue of formal strength alone, constitute a more just restorative standard.

This principle will become central when the paper turns from the atlas to substantive and procedural justice.

12.200 The Relation Axis and the Object Axis

The first two primary axes of the atlas are now available.

Section 11 constructed the object family

$$\mathfrak{Q} = \{Q_1, Q_2, \dots\}. \quad (738)$$

The present section constructed the relation family

$$\mathfrak{E} = \{E_1, E_2, \dots\}. \quad (739)$$

The formal comparison space is therefore

$$\mathfrak{C} = \mathfrak{Q} \times \mathfrak{E}. \quad (740)$$

Equation 740 contains candidate pairs such as

$$(Q_{\text{network}}, E_{\text{isomorphism}}), \quad (741)$$

$$(Q_{\text{transition}}, E_{\text{bisimulation}}), \quad (742)$$

$$(Q_{\text{reach}}, E_{\text{inclusion}}), \quad (743)$$

$$(Q_{\text{response}}, E_{\text{statistical}}), \quad (744)$$

and

$$(Q_{\text{generative}}, E_{\text{simulation}}). \quad (745)$$

Equations 741 through 745 illustrate that the formal and normative meaning emerges from the pair rather than from either axis alone.

12.201 Implications for the Atlas

Several conclusions follow from the comparative-relation taxonomy.

First, restorative equivalence is plural. No single mathematical relation captures every meaningful sense of return.

Second, equivalence relations form only one part of the comparison space. Directional relations such as simulation, inclusion, dominance, and sufficiency are indispensable for future-oriented justice.

Third, isomorphism preserves structure while permitting identity to change. This makes it useful for replacement and dangerous as a substitute for historical identity.

Fourth, homomorphisms and abstractions make institutional compression formally visible.

Fifth, simulation provides a natural language for preservation of reference capabilities while permitting additional restored possibilities.

Sixth, bisimulation provides a strong branching-sensitive standard while lacking automatic normative priority.

Seventh, trace and observational equivalence are useful where only behavior is legitimately observable, while both can conceal internal or branching differences.

Eighth, topological and dynamical relations provide standards for preserving qualitative organization beyond current-state equality.

Ninth, statistical indistinguishability is always relative to a test class and can conceal differences outside that class.

Tenth, interventional equivalence is stronger than observational similarity for questions concerning how assistance, resources, or policy act upon the system.

Eleventh, approximate relations require explicit metrics and tolerances.

Twelfth, dominance permits a restored system to improve upon the reference without reproducing it.

Thirteenth, sufficiency supports social floors and minimum protection while remaining distinct from corrective equivalence.

Fourteenth, relation refinement supplies a formal language of stronger and weaker standards while normative value remains non-monotonic in formal strength.

Fifteenth, historical and generative relations occupy unusually demanding parts of the atlas because they preserve process, provenance, and recursive generation.

Sixteenth, relation choice is simultaneously mathematical, epistemic, institutional, procedural, and normative.

12.202 Transition to the Object–Relation Space

The taxonomy of comparison objects and the taxonomy of comparative relations have so far been developed separately. Their separation was necessary because neither determines the other.

The same object can be compared under several relations.

A network can be identical, isomorphic, homomorphic, approximately similar, functionally substitutable, or reachability-preserving.

A reachable set can be equal, included, approximately close, homeomorphic, or normatively dominant.

A transition system can be trace-equivalent, mutually simulated, bisimilar, or observationally indistinguishable.

A generative mechanism can be structurally isomorphic, simulable, interventionally equivalent, approximately response-equivalent, or dominant under a selected value theory.

At the same time, many formal object–relation pairs are incoherent or uninformative. Bisimulation is naturally defined for transition structures and requires additional construction before application to a scalar resource. Graph isomorphism is meaningful for network objects and says little about subjective meaning unless meaning is represented within the graph. Distributional equality is appropriate for stochastic objects and insufficient for historical provenance.

Section 13 therefore combines the two axes. It constructs the object–relation space

$$\mathfrak{C} = \mathfrak{Q} \times \mathfrak{C}$$

as an atlas rather than a mere Cartesian list. The analysis classifies object–relation pairs according to formal coherence, structural strength, normative interpretation, epistemic identifiability, technical feasibility, and institutional usefulness. It also begins to examine refinement paths across the atlas and the possibility that restorative standards form several intersecting partial orders rather than one universal hierarchy.

DRAFT

Table 4: Principal families of comparative relations for restorative analysis

Relation family	Characteristic structure	Restorative interpretation	Characteristic limitation
Identity / equality	Exact represented sameness	Same entity, value, state, set, or vector	Strong only relative to selected representation
Congruence	Equivalence preserved under operations	Restoration remains equivalent after specified processes	Depends upon operation family
Isomorphism	Bijective structure preservation	Same relational or structural organization under relabeling	Ignores identity and provenance
Homomorphism	Directional structure preservation	Coarser restorative structure preserves selected operations	Can collapse distinctions
Embedding	Injective preservation inside larger structure	Reference structure remains available within expanded system	Additional structure remains unevaluated
Functional equivalence	Equality of selected function	Replacement performs the same restorative role	Function selection can omit relational value
Simulation	One-way transition matching	Restored system can reproduce reference possibilities	Asymmetric and branching-sensitive only to selected degree
Mutual simulation	Simulation in both directions	Both systems reproduce each other's represented behavior	Can remain weaker than bisimulation
Bisimulation	Recursive two-way branching correspondence	Comparable future branching under matched transitions	Can reject normatively valuable new options
Trace equivalence	Equality of observable execution sequences	Same observable histories remain possible	Can ignore branching structure
Observational equivalence	Indistinguishability under selected observations	Institution cannot distinguish restored and reference systems	Observer and measurement dependent
Contextual / testing equivalence	Indistinguishability across selected contexts or tests	Correspondence remains under a specified test environment	Test family determines strength
Homeomorphism	Topological structure preservation	Possibility or attractor spaces share qualitative topology	Ignores metric and probability
Topological conjugacy	Homeomorphic correspondence of dynamics	Qualitative dynamical organization is preserved	Highly demanding model assumptions
Metric approximation	Bounded distance	Restored object is sufficiently close to reference	Metric and tolerance require justification
Distributional equivalence	Equality or closeness of distributions	Stochastic outcome structure is restored	Can conceal individual histories
Statistical indistinguishability	No selected test reliably distinguishes systems	Responses are empirically indistinguishable under test class	Relative to tests and sample regime
Interventional equivalence	Same responses to selected interventions	Resources, treatment, or policy act similarly after restoration	Requires causal knowledge
Counterfactual equivalence	Same selected potential outcomes	Systems agree under hypothetical interventions	High epistemic burden
Set inclusion	Directional containment	All reference options remain available	Additional options can have mixed value
Dominance	At least as good under selected ordering	Restored system improves upon or matches reference	Depends upon value ordering
Sufficiency	Membership in acceptable region	Adequate threshold has been reached	Can leave corrective loss unresolved
Refinement	One relation preserves all distinctions of another and more	Comparative standards can be ordered by formal strength	Greater formal strength lacks automatic normative priority
Historical correspondence	Identity, order, provenance, or narrative preservation	Historical structure itself is part of restoration	Strong forms can be unattainable or undesirable
Generative-process	Correspondence of value-	Processes of future genera-	Requires rich model and

13 The Object–Relation Space of Restorative Claims

This section combines the two principal taxonomies developed in Sections 11 and 12. Its role is to construct the central comparative atlas of the paper. The comparison object Q identifies the feature of a generative system placed under restorative evaluation. The comparative relation E identifies the form of correspondence, preservation, simulation, approximation, sufficiency, or dominance required between the post-restoration object and the selected reference object. The present section treats the pair (Q, E) as the elementary unit of restorative comparison.

The analysis has five objectives. First, it distinguishes formally coherent object–relation pairs from combinations that require additional construction. Second, it identifies families of restorative claims produced by recurring pairings, including resource equivalence, relational isomorphism, capability inclusion, behavioral bisimulation, dynamical conjugacy, perturbation-response indistinguishability, causal equivalence, and historical-generative correspondence. Third, it introduces metadata for comparing these pairings across epistemic identifiability, institutional feasibility, temporal horizon, scale, normative salience, and procedural burden. Fourth, it examines refinement and implication relations among different points in the atlas. Fifth, it establishes the architecture through which later sections can move from a broad comparative map toward detailed analysis of dynamical, causal, generative, substantive, and procedural forms of restoration.

The central methodological proposition of this section is that a restorative claim acquires determinate meaning only after the reference, comparison object, and comparative relation have all been specified. The formal comparison is therefore represented by Equation 746.

$$Q(\mathcal{G}^R) \ E \ Q(\mathcal{G}^{\text{ref}}). \quad (746)$$

Equation 746 provides the elementary grammar of the atlas. The remaining indices developed earlier, including temporal horizon, test class, approximation tolerance, epistemic basis, system scale, and procedural conditions, enrich this elementary unit without replacing it.

13.1 The Comparative Product Space

Let Ω denote the family of restorative comparison objects and $\mathfrak{E}$ the family of comparative relations. The unrestricted object–relation product space is defined in Equation 747.

$$\mathfrak{C}_0 = \Omega \times \mathfrak{E}. \quad (747)$$

Equation 747 contains every formal pairing of an object and a relation. Many elements of $\mathfrak{C}_0$ are uninformative or undefined without additional structure. Bisimulation, for example, requires a transition system or another object equipped with a suitable transition semantics. Graph isomorphism requires graph structure. Hausdorff distance requires a metric space. Stochastic dominance requires ordered probability distributions. Topological conjugacy requires dynamical maps on topological spaces.

The practically meaningful atlas is therefore a restricted subset $\mathfrak{C} \subseteq \mathfrak{C}_0$, defined in Equation 748.

$$\mathfrak{C} = \{(Q, E) \in \Omega \times \mathfrak{E} : E \text{ is well-defined on the represented structure of } Q\}. \quad (748)$$

Equation 748 establishes formal coherence as the first filter of the atlas.

Formal coherence alone does not establish restorative relevance. A further subset contains pairings that possess a defensible restorative interpretation. Let this set be $\mathfrak{C}_R$, defined in Equation 749.

$$\mathfrak{C}_R = \{(Q, E) \in \mathfrak{C} : (Q, E) \text{ bears a justified relation to a restorative purpose}\}. \quad (749)$$

Equation 749 makes clear that mathematical well-formedness and normative relevance are separate filters.

13.2 Object–Relation Cells

Each element

$$c_{ij} = (Q_i, E_j) \quad (750)$$

of the atlas is called an *object–relation cell*. Equation 750 identifies the unit at which restorative claims are classified.

Examples include:

- $(Q_{\text{resource}}, =)$: exact resource equality;
- $(Q_{\text{network}}, \cong)$: network isomorphism;
- $(Q_{\text{capability}}, \subseteq)$: capability inclusion;
- $(Q_{\text{transition}}, \sim_{\text{bis}})$: bisimulation of transition structure;
- $(Q_{\text{reach}}, d_H \leq \varepsilon)$: approximate reachable-set correspondence;
- $(Q_{\text{response}}, \sim_{\text{stat}})$: statistical indistinguishability of perturbation responses;
- $(Q_{\text{causal}}, \sim_{\text{int}})$: interventional equivalence;
- $(Q_{\text{generative}}, \preceq_{\text{sim}})$: simulation of generative mechanisms;
- $(Q_{\text{history}}, \cong_H)$: historical-order correspondence.

The pair is the minimum adequate unit because neither the object nor the relation independently determines the restorative meaning.

13.3 Cell Metadata

Each object–relation cell can be associated with metadata describing its formal, epistemic, institutional, and normative properties.

Let the metadata function be represented by Equation 751.

$$\Theta(Q, E) = (F, S, T, O, I, C, N, P, R), \quad (751)$$

where:

- F denotes formal coherence;
- S denotes structural strength;
- T denotes temporal scope;
- O denotes observability and epistemic requirements;
- I denotes institutional fit;
- C denotes practical and computational cost;
- N denotes normative salience;
- P denotes procedural implications;
- R denotes expected reparability or attainability.

Equation 751 permits two formally similar cells to differ substantially in institutional usefulness.

Table 5: Illustrative directly defined object–relation cells

Comparison object	Comparative relation	Restorative interpretation
Scalar resource	Equality	The restored quantity equals the reference quantity
Resource vector	Vector equality	Every represented resource dimension equals the reference
Capability set	Set inclusion	Every reference capability remains available after restoration
Network	Graph isomorphism	The represented relational topology is preserved under relabeling
Transition system	Simulation	Every selected reference transition can be matched in the restored system
Transition system	Bisimulation	Branching behavior can be recursively matched in both directions
Reachable set	Set equality	The same represented future states remain reachable
Reachable set	Hausdorff proximity	Future possibility spaces remain within a specified metric tolerance
Probability distribution	Distributional equality	The same stochastic distribution is recovered
Response distribution	Statistical indistinguishability	Selected tests cannot reliably distinguish restorative and reference responses
Causal system	Interventional equivalence	Selected interventions produce corresponding outcome distributions
Dynamical system	Topological conjugacy	Qualitative dynamics are preserved under an invertible continuous state mapping
Historical partial order	Order isomorphism	The represented temporal or provenance order is preserved

13.4 Formal Coherence Classes

The cells of the atlas can be divided into four broad formal-coherence classes.

The first class contains directly defined pairings. Resource equality, graph isomorphism, set inclusion, distributional equality, and transition-system bisimulation belong to this class when their objects carry the required structure.

The second class contains pairings that become coherent after an explicit representation step. A social relation can be compared by graph isomorphism after the relation has been represented as a graph. A capability structure can be compared through Hausdorff distance after the opportunity space has been embedded in a metric space.

The third class contains metaphorically suggestive pairings that require substantial formal development before they become mathematical claims. Statements such as “identity bisimulation” or “narrative conjugacy” can be conceptually interesting while lacking a defined formal object until additional semantics are supplied.

The fourth class contains pairings that are inappropriate under the selected representation. A scalar income variable has no meaningful graph isomorphism unless additional graph structure is first introduced.

A formal atlas should distinguish these classes rather than presenting every term as equally applicable.

13.5 Directly Defined Cells

A directly defined cell requires no additional representational transformation beyond the selected object.

Several important examples are summarized in Table 5.

Table 5 illustrates the diversity of claims that can be described casually as restoration.

13.6 Representation-Mediated Cells

A representation-mediated cell first transforms object Q into another formal structure $M(Q)$, after which relation E becomes applicable.

The structure is represented by Equation 752.

$$Q \xrightarrow{M} M(Q) \xrightarrow{E} M(Q'). \quad (752)$$

Equation 752 makes the mediation explicit.

For example, a professional community can be represented as a multiplex graph, after which graph isomorphism becomes meaningful. A subject's future option space can be embedded in a metric manifold, after which metric or topological relations can be evaluated. A historical narrative can be represented through a partially ordered event graph, after which order isomorphism becomes available.

The restorative conclusion depends upon both the representation map M and the relation E . Representation choice therefore belongs to the epistemic basis of the claim.

13.7 Representation Dependence

Different representations of the same social object can produce different relation judgments.

Let M_1 and M_2 be two representations of object Q . It is possible that Equation 753 holds.

$$M_1(Q_R) E M_1(Q_{\text{ref}}) \quad \text{while} \quad M_2(Q_R) \not E M_2(Q_{\text{ref}}). \quad (753)$$

Equation 753 shows that a restorative equivalence can be representation-relative even when the formal relation is fixed.

A network represented only by adjacency can appear isomorphic while a typed, weighted, historically labeled representation reveals substantial difference.

13.8 Representational Refinement

One representation can refine another by retaining additional structure.

Let M_2 refine M_1 . A coarse equivalence can hold while the refined equivalence fails:

$$M_1(Q_R) E_1 M_1(Q_{\text{ref}}) \quad \text{and} \quad M_2(Q_R) \not E_2 M_2(Q_{\text{ref}}). \quad (754)$$

Equation 754 formalizes a recurring pattern in restorative practice. A claimant can be restored under an institution's coarse representation and remain unrestored under a richer description.

13.9 Entity–Relation Region of the Atlas

For identifiable entities, the most natural relations include identity, replacement, substitutability, functional equivalence, provenance correspondence, and value equivalence.

The region is represented schematically by Equation 755.

$$\mathfrak{C}_{\text{entity}} = \{(Q_{\text{entity}}, =), (Q_{\text{entity}}, \sim_{\text{sub}}), (Q_{\text{entity}}, \sim_{\text{func}}), (Q_{\text{entity}}, \sim_{\text{prov}}), (Q_{\text{entity}}, \sim_V)\}. \quad (755)$$

Equation 755 contains several distinct notions of return.

Return of the identical object is historically stronger than return of an equivalent market substitute. Functional equivalence can be stronger or weaker than market equivalence depending upon the relevant purpose. Provenance equivalence becomes especially important for artifacts, archives, cultural objects, authorship, and property whose history contributes to value.

13.10 Resource–Relation Region

Resource objects naturally support equality, vector equality, inclusion, dominance, sufficiency, metric proximity, and value equivalence.

The region is represented by Equation 756.

$$\mathfrak{C}_{\text{resource}} = \{(Q_{\text{resource}}, =), (Q_{\text{resource}}, \approx_{\varepsilon}), (Q_{\text{resource}}, \succeq), (Q_{\text{resource}}, \text{Suff}), (Q_{\text{resource}}, \sim_V)\}. \quad (756)$$

Equation 756 contains many comparisons used by law and political economy.

Equal monetary compensation belongs to a different cell from resource sufficiency. A social-protection floor can satisfy sufficiency while remaining far below corrective equality. Resource dominance can provide more resources than the reference while leaving conversion structure unchanged.

13.11 Capability–Relation Region

Capability objects naturally support set equality, inclusion, dominance, threshold sufficiency, metric proximity, simulation, and reachable-set relations.

Let the principal region be represented by Equation 757.

$$\mathfrak{C}_{\text{cap}} = \{(Q_{\text{cap}}, =), (Q_{\text{cap}}, \subseteq), (Q_{\text{cap}}, \supseteq), (Q_{\text{cap}}, \text{Suff}), (Q_{\text{cap}}, \preceq_{\text{sim}})\}. \quad (757)$$

Equation 757 reveals several possible interpretations of capability restoration.

Exact capability-set equality reproduces the represented option set. Inclusion preserves every reference capability while permitting additional capabilities. Sufficiency requires an adequate subset or threshold. Simulation becomes relevant when capabilities are represented dynamically through transitions rather than as a static set.

13.12 Capability Inclusion and Subject Freedom

Capability inclusion has a particularly attractive future-oriented interpretation:

$$\mathcal{K}_{\text{ref}} \subseteq \mathcal{K}_R. \quad (758)$$

Equation 758 preserves all represented reference capabilities while allowing new ones.

This relation aligns naturally with subject-directed regeneration. Its normative adequacy depends upon whether the additional options are genuinely usable and whether preserving every reference capability requires unjustified intervention.

13.13 Relation–Relation Region

Individual relations can be compared through identity, type equivalence, functional equivalence, quality similarity, substitution, historical correspondence, and meaning preservation.

The region is represented by Equation 759.

$$\mathfrak{C}_{\text{rel}} = \{(Q_{\text{rel}}, =), (Q_{\text{rel}}, \sim_{\text{type}}), (Q_{\text{rel}}, \sim_{\text{func}}), (Q_{\text{rel}}, \approx_{\text{quality}}), (Q_{\text{rel}}, \sim_{\text{hist}})\}. \quad (759)$$

Equation 759 demonstrates why replacement of a mentor, collaborator, caregiver, or community relation requires specification of the relation preserved.

A replacement can be functionally adequate and historically non-equivalent.

13.14 Network–Relation Region

Networks support one of the richest relation families.

Let

$$\mathfrak{C}_{\text{network}} = \{(Q_{\text{network}}, =), (Q_{\text{network}}, \cong), (Q_{\text{network}}, \text{Hom}), (Q_{\text{network}}, \text{Emb}), (Q_{\text{network}}, \approx), (Q_{\text{network}}, \sim_{\text{reach}})\}. \quad (760)$$

Equation 760 distinguishes several restoration standards.

Network identity requires the same represented nodes and relations. Isomorphism allows node re-labeling while preserving topology. Homomorphism permits structural compression. Embedding preserves the reference structure within a larger restored network. Approximation permits bounded topological or metric difference. Reachability preservation focuses on who remains connected to whom through relevant paths.

These relations can produce different judgments concerning professional and community re-entry.

13.15 Status–Relation Region

Formal statuses naturally support identity, equality, legal equivalence, hierarchical order preservation, and functional equivalence.

A formal reinstatement claim usually occupies the cell

$$(Q_{\text{status}}, =). \quad (761)$$

Equation 761 can be fully satisfied while the institutional-position, network, trust, and capability cells remain far from the reference.

This pattern provides a precise formal description of partial restoration.

13.16 State–Relation Region

State objects support exact equality, metric proximity, norm proximity, distributional equality, classification equivalence, and threshold relations.

The principal region is represented by Equation 762.

$$\mathfrak{C}_{\text{state}} = \{(Q_{\text{state}}, =), (Q_{\text{state}}, \approx_{\varepsilon}), (Q_{\text{state}}, \sim_{\text{dist}}), (Q_{\text{state}}, \text{Suff})\}. \quad (762)$$

Equation 762 contains many conventional concepts of recovery.

The central limitation is temporal. State comparison says little about future dynamics unless additional transition structure is included.

13.17 Trajectory–Relation Region

Trajectories support equality, metric proximity, time-reparameterized equivalence, orbit equivalence, pathwise equivalence, trace correspondence, and distributional comparison.

The region is represented by Equation 763.

$$\mathfrak{C}_{\text{traj}} = \{(Q_{\text{traj}}, =), (Q_{\text{traj}}, \approx), (Q_{\text{traj}}, \sim_{\text{time}}), (Q_{\text{traj}}, \sim_{\text{orbit}}), (Q_{\text{traj}}, \sim_{\text{path}})\}. \quad (763)$$

Equation 763 is central to diachronic justice.

Time-reparameterized equivalence can regard delayed recovery as qualitatively similar. Exact trajectory equality preserves timing. The difference becomes normatively important when time itself is part of the loss.

13.18 Trace–Relation Region

Observable trace objects naturally support equality, trace inclusion, approximate trace distance, and statistical trace comparison.

Let

$$\mathfrak{C}_{\text{trace}} = \{(Q_{\text{trace}}, =), (Q_{\text{trace}}, \subseteq), (Q_{\text{trace}}, \approx), (Q_{\text{trace}}, \sim_{\text{stat}})\}. \quad (764)$$

Equation 764 is especially useful where internal structure remains inaccessible.

Trace-based restoration is weaker than branching-sensitive restoration in many transition-system settings [66].

13.19 Transition-System Region

Transition systems support simulation, mutual simulation, bisimulation, trace-based semantics, readiness, failure, testing equivalence, and abstraction relations.

The region is represented schematically by Equation 765.

$$\mathfrak{C}_{\text{trans}} = \{(Q_{\text{LTS}}, \preceq_{\text{sim}}), (Q_{\text{LTS}}, \sim_{\text{msim}}), (Q_{\text{LTS}}, \sim_{\text{bis}}), (Q_{\text{LTS}}, \sim_{\text{tr}}), (Q_{\text{LTS}}, \sim_{\text{test}})\}. \quad (765)$$

Equation 765 contains a family of relations with different sensitivity to branching structure.

This region is particularly important for future-option and agency analysis.

13.20 Simulation as a Generative-Preservation Cell

One-way simulation has a useful restorative interpretation when the normative goal is preservation of reference possibilities with allowance for new possibilities.

The corresponding cell is

$$(Q_{\text{transition}}, \preceq_{\text{sim}}). \quad (766)$$

Equation 766 can support a criterion under which every normatively relevant reference transition remains reproducible.

This cell is often more compatible with future expansion than strict bisimulation.

13.21 Bisimulation as a Branching-Fidelity Cell

The cell

$$(Q_{\text{transition}}, \sim_{\text{bis}}) \quad (767)$$

requires reciprocal matching of branching behavior.

Equation 767 represents high behavioral fidelity.

Its normative limitation is equally clear. A restored system containing valuable new branches can fail bisimulation because the reference cannot match those new possibilities.

The cell therefore measures branching fidelity rather than generative superiority.

13.22 Reachable-Set Region

Reachable sets support equality, inclusion, proper inclusion, Hausdorff proximity, homeomorphism, measure equality, stochastic comparison, and dominance.

The region is represented by Equation 768.

$$\mathfrak{C}_{\text{reach}} = \{(Q_{\text{reach}}, =), (Q_{\text{reach}}, \subseteq), (Q_{\text{reach}}, \supseteq), (Q_{\text{reach}}, d_H \leq \varepsilon), (Q_{\text{reach}}, \cong_{\text{top}})\}. \quad (768)$$

Equation 768 is among the most important regions for future-directed justice.

Set equality privileges fidelity. Superset inclusion privileges option preservation and expansion. Hausdorff proximity privileges geometric similarity. Homeomorphism privileges qualitative topological organization.

These standards are not interchangeable.

13.23 Viable-Set Region

Viable sets support the same formal family as reachable sets while adding normative or safety constraints to set membership.

Let

$$\mathfrak{C}_{\text{viable}} = \{(Q_{\text{viable}}, =), (Q_{\text{viable}}, \subseteq), (Q_{\text{viable}}, \supseteq), (Q_{\text{viable}}, \approx)\}. \quad (769)$$

Equation 769 is particularly relevant to assistance and capability justice because it excludes futures classified as practically or normatively inadmissible.

13.24 Dynamical-Law Region

Dynamical laws support equality, parameter correspondence, semiconjugacy, topological conjugacy, smooth conjugacy, orbit equivalence, and approximate simulation.

The principal region is represented by Equation 770.

$$\mathfrak{C}_{\text{dyn}} = \{(Q_{\text{dyn}}, =), (Q_{\text{dyn}}, \sim_{\text{semi}}), (Q_{\text{dyn}}, \sim_{\text{conj}}), (Q_{\text{dyn}}, \sim_{\text{orbit}}), (Q_{\text{dyn}}, \approx)\}. \quad (770)$$

Equation 770 moves restorative analysis from present state toward process-generating structure.

Topological conjugacy preserves qualitative dynamical organization [27]. Semiconjugacy permits abstraction. Approximate relations are more realistic for empirical systems.

13.25 Attractor-Relation Region

Attractor structures support identity, set equality, correspondence, graph isomorphism, homeomorphism, metric proximity, and dominance.

Let

$$\mathfrak{C}_{\text{attr}} = \{(Q_{\text{attr}}, =), (Q_{\text{attr}}, \cong), (Q_{\text{attr}}, \cong_{\text{top}}), (Q_{\text{attr}}, \approx)\}. \quad (771)$$

Equation 771 formalizes several possible meanings of “restored attractor structure.”

The mere number of attractors is much weaker than topological correspondence of the attractors and their transition structure.

13.26 Basin-Relation Region

Basin objects support equality, inclusion, measure equality, homeomorphism, boundary distance comparison, and approximate geometric correspondence.

Let

$$\mathfrak{C}_{\text{basin}} = \{(Q_{\text{basin}}, =), (Q_{\text{basin}}, \subseteq), (Q_{\text{basin}}, \cong_{\text{top}}), (Q_{\text{basin}}, \approx)\}. \quad (772)$$

Equation 772 distinguishes restoration of long-run modes from restoration of robustness around those modes.

This region will be developed further in the dedicated dynamical sections.

13.27 Perturbation-Response Region

Response functions support equality, metric proximity, distributional equivalence, statistical indistinguishability, simulation, and dominance.

The region is represented by Equation 773.

$$\mathfrak{C}_{\text{resp}} = \{(Q_{\text{resp}}, =), (Q_{\text{resp}}, \approx_{\varepsilon}), (Q_{\text{resp}}, \sim_{\text{dist}}), (Q_{\text{resp}}, \sim_{\text{stat}})\}. \quad (773)$$

Equation 773 contains the formal version of the question whether restorative and reference systems respond indistinguishably under admissible perturbations.

This cell family is significantly stronger than comparison under ordinary baseline conditions.

13.28 Causal-Relation Region

Causal objects support graph isomorphism, structural-equation equivalence, interventional equivalence, counterfactual equivalence, abstraction, and approximate causal similarity.

Let

$$\mathfrak{C}_{\text{causal}} = \{(Q_{\text{causal}}, \cong), (Q_{\text{SCM}}, \sim_{\text{struct}}), (Q_{\text{int}}, \sim_{\text{int}}), (Q_{\text{cf}}, \sim_{\text{cf}})\}. \quad (774)$$

Equation 774 organizes relations of increasing causal depth.

Observational equality can hold while interventional equivalence fails. Interventional equivalence over a limited action class can hold while complete structural-equation equivalence fails.

The region will be developed in detail in the later causal section [43].

13.29 Conversion-Relation Region

Conversion structures support function equality, approximate function equivalence, commutative structural preservation, interventional equivalence, simulation, and dominance.

The region is represented by Equation 775.

$$\mathfrak{C}_{\text{conv}} = \{(Q_{\text{conv}}, =), (Q_{\text{conv}}, \approx), (Q_{\text{conv}}, \sim_{\text{int}}), (Q_{\text{conv}}, \preceq_{\text{sim}}), (Q_{\text{conv}}, \succeq)\}. \quad (775)$$

Equation 775 is central to the difference between returning resources and restoring their effective generative use.

13.30 Control-Relation Region

Control structures support identity, structural isomorphism, dominance, non-domination ordering, rights sufficiency, and embedding.

Let

$$\mathfrak{C}_{\text{control}} = \{(Q_{\text{control}}, =), (Q_{\text{control}}, \cong), (Q_{\text{control}}, \succeq_{\text{ND}}), (Q_{\text{control}}, \text{Suff}_R)\}. \quad (776)$$

Equation 776 is especially important where the reference control structure was itself unjust.

In such cases, dominance under non-domination or rights criteria can be more normatively appropriate than structural equality.

13.31 Value-Output Region

Value outputs support equality, vector equality, approximate equality, distributional comparison, dominance, and sufficiency.

Let

$$\mathfrak{C}_{\text{value}} = \{(Q_{\text{value}}, =), (Q_{\text{value}}, \approx), (Q_{\text{value}}, \succeq), (Q_{\text{value}}, \text{Suff})\}. \quad (777)$$

Equation 777 contains many familiar welfare and economic comparisons.

Output relations remain insufficient for theories concerned with the process, control, or circulation of value.

13.32 Value-Circulation Region

Value-circulation structures support identity, graph isomorphism, flow equivalence, simulation, proportional correspondence, and dominance.

Let

$$\mathfrak{C}_{\text{circ}} = \{(Q_{\text{circ}}, =), (Q_{\text{circ}}, \cong), (Q_{\text{circ}}, \sim_{\text{flow}}), (Q_{\text{circ}}, \preceq_{\text{sim}}), (Q_{\text{circ}}, \succeq_{GJ})\}. \quad (778)$$

Equation 778 provides one formal location for generative justice's concern with circular value flows and control of production [15, 17].

13.33 Generative-Mechanism Region

Generative mechanisms support equality, homomorphism, simulation, bisimulation, semiconjugacy, interventional equivalence, commutative preservation, and approximation.

The region is represented by Equation 779.

$$\mathfrak{C}_{\text{gen}} = \{(Q_{\text{gen}}, =), (Q_{\text{gen}}, \text{Hom}), (Q_{\text{gen}}, \preceq_{\text{sim}}), (Q_{\text{gen}}, \sim_{\text{bis}}), (Q_{\text{gen}}, \sim_{\text{int}}), (Q_{\text{gen}}, \approx)\}. \quad (779)$$

Equation 779 forms one of the conceptual centers of the paper.

Its detailed treatment is deferred until Section 19, where the relation among conditions, value production, relation formation, and recursive generation can be analyzed directly.

13.34 Meaning-Relation Region

Meaning structures support identity, qualitative correspondence, classification equivalence, narrative equivalence, recognition equivalence, and approximate semantic similarity.

Let

$$\mathfrak{C}_{\text{meaning}} = \{(Q_{\text{meaning}}, =), (Q_{\text{meaning}}, \sim_{\text{qual}}), (Q_{\text{meaning}}, \sim_{\text{narr}}), (Q_{\text{meaning}}, \sim_{\text{rec}})\}. \quad (780)$$

Equation 780 is less mathematically standardized than the graph or transition regions.

Its inclusion is essential because legal or monetary equality can coexist with different social meaning.

13.35 Recognition-Relation Region

Recognition objects support equality, sufficiency, dominance, relational correspondence, and qualitative equivalence.

A transformed post-harm system can achieve superior recognition without reproducing the reference recognition structure.

This makes dominance particularly important where the reference contained misrecognition [25, 20].

13.36 History-Relation Region

Historical structures support identity, event equality, order isomorphism, provenance isomorphism, simulation, narrative correspondence, and partial-order embedding.

Let

$$\mathfrak{C}_{\text{hist}} = \{(Q_{\text{history}}, =), (Q_{\text{history}}, \cong_{\text{ord}}), (Q_{\text{history}}, \cong_{\text{prov}}), (Q_{\text{history}}, \preceq_{\text{sim}}), (Q_{\text{history}}, \sim_{\text{narr}})\}. \quad (781)$$

Equation 781 contains some of the strongest and most epistemically demanding restorative comparisons.

Historical identity with a no-harm reference is generally unavailable once the harm and repair have occurred.

Weaker historical relations can remain meaningful.

13.37 Memory–Relation Region

Memory objects can be compared through identity, content similarity, structural correspondence, narrative coherence, or functional equivalence.

The restorative meaning of such comparisons is normatively sensitive.

A hypothetical process producing memory identity with a counterfactual subject can require erasure of actual historical memory.

The cell can therefore achieve strong formal fidelity and low procedural legitimacy.

13.38 Identity–Relation Region

Identity-related objects support several very different comparative standards:

- numerical identity;
- legal identity;
- continuity;
- narrative correspondence;
- structural similarity of commitments or traits.

The term *identity restoration* is therefore radically ambiguous without a theory of personal or collective identity.

This region should remain explicitly philosophical.

13.39 Agency–Relation Region

Agency objects support equality, dominance, capability inclusion, simulation, sufficiency, and procedural correspondence.

Let

$$\mathfrak{C}_{\text{agency}} = \{(Q_{\text{agency}}, =), (Q_{\text{agency}}, \succeq), (Q_{\text{agency}}, \subseteq), (Q_{\text{agency}}, \text{Suff})\}. \quad (782)$$

Equation 782 is especially important when stronger counterfactual fidelity threatens present subject direction.

13.40 Procedural–Relation Region

Procedural objects support equality, structural isomorphism, threshold sufficiency, dominance, simulation, and rights-based admissibility.

The region is represented by Equation 783.

$$\mathfrak{C}_{\text{proc}} = \{(Q_{\text{proc}}, =), (Q_{\text{proc}}, \cong), (Q_{\text{proc}}, \text{Suff}), (Q_{\text{proc}}, \succeq_J)\}. \quad (783)$$

Equation 783 demonstrates that process itself can be represented in the same atlas while retaining normative independence from substantive outcome.

13.41 Composite-Object Region

Composite objects support componentwise equivalence, lexicographic comparison, Pareto dominance, weighted distance, partial order, and multi-criterion sufficiency.

Let

$$Q_{\text{comp}} = (Q_1, \dots, Q_n). \quad (784)$$

Equation 784 can be compared componentwise:

$$Q_i^R E_i Q_i^{\text{ref}} \quad i = 1, \dots, n. \quad (785)$$

Equation 785 preserves plural structure.

A scalar aggregation should be introduced only where the underlying value theory justifies it.

13.42 Object–Relation Compatibility

The atlas requires a compatibility relation between object and comparison relation.

Let

$$\text{Comp}(Q, E) \in \{0, 1\} \quad (786)$$

denote whether relation E is formally applicable to the current representation of Q .

Equation 786 is binary only at the formal level.

A richer classification can distinguish:

- directly applicable;
- applicable after explicit representation;
- conceptually interpretable;
- presently underformalized;
- formally unsuitable.

This classification is more useful for a philosophical atlas than a simple yes–no test.

13.43 Normative Compatibility

A formally coherent cell can remain normatively irrelevant.

Let

$$\text{NComp}_V(Q, E) \quad (787)$$

denote normative compatibility under value theory V .

Graph isomorphism of a professional network can be formally precise. A theory emphasizing singular relationships can regard it as normatively weak. A structural institutional theory can regard it as highly relevant.

The normative status therefore depends upon the theory of value.

13.44 Epistemic Identifiability of Cells

Each cell has an epistemic identifiability profile.

Let

$$\text{Iden}(Q, E \mid K) \quad (788)$$

denote whether the available epistemic basis K supports determination of the relation.

Resource equality can be highly identifiable. Counterfactual reachability can be weakly identifiable. Historical-generative bisimulation can be largely ideal-theoretical.

The atlas should display these differences explicitly.

13.45 Institutional Feasibility of Cells

A cell can also be evaluated through institutional feasibility:

$$\text{Feas}(I, Q, E) . \quad (789)$$

A court can evaluate monetary equality more readily than network bisimulation. A regulator can evaluate repeated institutional risk. A rehabilitation provider can evaluate functional performance and selected response properties.

Institutional fit is therefore cell-specific.

13.46 Attainability of Cells

Identifiability and attainability should remain distinct.

A system can know that two networks are non-isomorphic while lacking power to reconstruct the reference network.

Conversely, a system can be technically capable of altering a variable while lacking sufficient knowledge of the target.

Let

$$\text{Attain}(Q, E) \quad (790)$$

denote the feasibility of producing the relation through admissible intervention.

This dimension will become central to the idealized-technology sections.

13.47 Procedural Cost of Cells

A cell can require intrusive processes for measurement or attainment.

Let

$$C_P(Q, E) \quad (791)$$

denote the procedural burden associated with the cell.

Resource equality can require financial information. Behavioral equivalence can require observation. Internal-state identity can require invasive measurement. Historical reconstruction can require extensive data about third parties.

A cell's normative desirability should therefore incorporate both the outcome it preserves and the process required to establish or achieve it.

13.48 Cell Strength

Relation strength is object-relative.

Let

$$\text{Str}(Q, E) \quad (792)$$

denote the structural information preserved by cell (Q, E) .

Exact equality of one scalar can preserve less information than approximate bisimulation of a rich transition system.

The atlas therefore avoids ranking relations independently of their objects.

13.49 Cell Refinement

One cell can refine another when it concerns the same or a refined object and preserves strictly more relevant structure.

Let

$$(Q_2, E_2) \preceq_{\text{ref}} (Q_1, E_1) \quad (793)$$

mean that satisfaction of cell (Q_2, E_2) implies satisfaction of cell (Q_1, E_1) under the specified assumptions.

Equation 793 defines a relation on cells rather than on objects alone.

This permits the atlas to contain refinement paths.

13.50 Vertical Refinement

A vertical refinement changes the comparative relation while holding the object fixed.

For transition systems, one possible sequence is

$$\sim_{\text{bis}} \Rightarrow \sim_{\text{sim}} \Rightarrow \sim_{\text{trace}} \quad (794)$$

under suitable definitions and assumptions.

Equation 794 illustrates increasingly coarse behavioral comparison.

The exact implication structure follows the process semantics selected [66].

13.51 Horizontal Refinement

A horizontal refinement changes the object representation while retaining a related comparative relation.

For example, one can move from salary equality to multidimensional resource equality:

$$(Q_{\text{income}}, =) \longrightarrow (Q_{\text{resource-vector}}, =). \quad (795)$$

Equation 795 preserves more dimensions without changing the relation family.

Another example moves from untyped network isomorphism to typed weighted network isomorphism.

13.52 Diagonal Refinement

A diagonal refinement strengthens both object and relation.

One can move from equal current output to perturbation-conditioned statistical response equivalence:

$$(Q_{\text{output}}, =) \longrightarrow (Q_{\text{response}}, \sim_{\text{stat}}). \quad (796)$$

Equation 796 changes both what is compared and how it is compared.

Many important advances in restorative analysis take this form.

13.53 Resource-to-Capability Refinement Path

A characteristic refinement path begins from resources and proceeds toward effective possibility.
A schematic path is given in Equation 797.

$$(Q_{\text{resource}}, =) \longrightarrow (Q_{\text{conversion}}, \sim) \longrightarrow (Q_{\text{capability}}, \subseteq) \longrightarrow (Q_{\text{reach}}, \subseteq). \quad (797)$$

Equation 797 progressively shifts attention from possession toward usable and future possibility.
The sequence is analytical rather than a universal hierarchy.

13.54 Status-to-Relation Refinement Path

A second path begins from formal status:

$$(Q_{\text{status}}, =) \longrightarrow (Q_{\text{position}}, \sim) \longrightarrow (Q_{\text{network}}, \sim) \longrightarrow (Q_{\text{relation}}, \sim). \quad (798)$$

Equation 798 illustrates why reinstatement can be a first layer of restoration rather than the final layer.

13.55 State-to-Dynamics Refinement Path

A third path moves from current state toward future dynamics:

$$(Q_{\text{state}}, =) \longrightarrow (Q_{\text{trajectory}}, \approx) \longrightarrow (Q_{\text{reach}}, \sim) \longrightarrow (Q_{\text{response}}, \sim) \longrightarrow (Q_{\text{dyn}}, \sim_{\text{conj}}). \quad (799)$$

Equation 799 demonstrates increasing attention to future behavior and system structure.

13.56 Output-to-Generativity Refinement Path

A fourth path moves from produced value toward the mechanism of generation:

$$(Q_{\text{value}}, =) \longrightarrow (Q_{\text{circ}}, \sim) \longrightarrow (Q_{\text{recursion}}, \sim) \longrightarrow (Q_{\text{gen}}, \sim). \quad (800)$$

Equation 800 is particularly important for the relation between distributive and generative justice.
Equal output can coexist with different circulation and generative mechanisms.

13.57 Observation-to-Causation Refinement Path

A fifth path moves from observed similarity toward causal similarity:

$$(Q_{\text{output}}, \sim_{\text{obs}}) \longrightarrow (Q_{\text{response}}, \sim_{\text{stat}}) \longrightarrow (Q_{\text{int}}, \sim_{\text{int}}) \longrightarrow (Q_{\text{SCM}}, \sim_{\text{struct}}). \quad (801)$$

Equation 801 progressively increases the causal content of the restorative claim.
The epistemic requirements increase accordingly.

13.58 History Refinement Path

Historical comparison can similarly be refined:

$$(Q_{\text{state}}, =) \longrightarrow (Q_{\text{event-seq}}, \sim) \longrightarrow (Q_{\text{prov}}, \cong) \longrightarrow (Q_{\text{history}}, \cong_H). \quad (802)$$

Equation 802 shifts the restorative object from endpoint toward provenance and historical organization.

The final relation remains substantially weaker than literal identity of actual and counterfactual histories.

13.59 Normative Non-Monotonicity Across Refinement Paths

Every refinement path developed above concerns structural or informational strength. Normative value need not increase monotonically along the path.

Let $c_1 \preceq_{\text{ref}} c_2$ denote that cell c_2 is formally more refined than cell c_1 . A normative evaluation J can satisfy Equation 803.

$$J(c_2) < J(c_1). \quad (803)$$

Equation 803 can arise where achieving c_2 requires coercion, surveillance, memory alteration, third-party burden, or restoration of an unjust historical structure.

The atlas therefore contains refinement relations and normative evaluations as separate layers.

13.60 Object–Relation Partial Orders

Under restricted object families, cells can form partial orders under formal refinement.

Let

$$(\mathcal{C}_S, \preceq_S) \quad (804)$$

denote a subspace of cells equipped with refinement relation $\preceq_S$.

Equation 804 can be meaningful for transition semantics, set relations, statistical approximation, or dynamical equivalence.

The full atlas need not admit one global partial order because object types and relation families remain heterogeneous.

13.61 Local Lattice Structures

Some restricted regions can possess meet and join operations.

A family of partitions induced by equivalence relations on one object space can often be ordered by refinement. A family of capability sets can be ordered by inclusion. A family of tolerance relations can be nested by ε .

These local lattice-like structures can support systematic analysis.

The existence of local structure should not be generalized to the full atlas without proof.

13.62 Atlas Incomparability

Many cells are incomparable in formal strength.

Network isomorphism and equality of health state preserve different objects.

Bisimulation and monetary equivalence answer different questions.

Historical provenance correspondence and perturbation-response indistinguishability preserve orthogonal structures.

The atlas therefore contains genuine incomparability rather than merely missing information.

This property is philosophically important. A plural theory of justice can require several incomparable restorative achievements simultaneously.

13.63 Cross-Cell Implication

Although many cells are incomparable, some cross-cell implications can be established under additional assumptions.

For example, if resource equality holds and conversion functions are identical, capability equality can follow:

$$[r_R = r_{\text{ref}}] \wedge [\mathcal{K}_R = \mathcal{K}_{\text{ref}}] \implies \mathcal{K}_R(r_R) = \mathcal{K}_{\text{ref}}(r_{\text{ref}}). \quad (805)$$

Equation 805 illustrates how a multi-cell conjunction can imply another restorative claim.

Such implications help clarify which assumptions are hidden when institutions infer capability restoration directly from resource return.

13.64 Failure of Cross-Cell Implication

The converse is often invalid.

Resource equality alone does not imply capability equality:

$$r_R = r_{\text{ref}} \not\Rightarrow \mathcal{K}_R(r_R) = \mathcal{K}_{\text{ref}}(r_{\text{ref}}). \quad (806)$$

Equation 806 provides a formal statement of one of the central capability-oriented criticisms of resource-only restoration [55, 50].

Similarly, state equality does not imply dynamical equivalence, and output equality does not imply value-circulation equivalence.

13.65 Cross-Cell Commutativity

The commutative diagrams introduced in Section 12.168 provide another form of cross-cell relation.

A restorative mapping can preserve the relation between two objects when the diagram commutes.

For resource-to-capability conversion, the condition is restated in Equation 807.

$$R_K \circ \mathcal{K}_{\text{ref}} = \mathcal{K}_R \circ R_C. \quad (807)$$

Equation 807 evaluates preservation of the mapping between resource and capability rather than either object alone.

This family of cross-cell relations is especially important for generative systems.

13.66 Generative Commutativity

A generative cycle can be represented through condition and value mappings. Let R_C map conditions and R_V map generated values. Preservation of the generative step requires Equation 808.

$$R_V \circ \Gamma_{\text{ref}} = \Gamma_R \circ R_C. \quad (808)$$

Preservation of circulation requires Equation 809.

$$R_C \circ \Lambda_{\text{ref}} = \Lambda_R \circ R_V. \quad (809)$$

Equations 808 and 809 define a cross-object preservation criterion for recursive generation.

This is stronger than equality of resources or outputs alone.

13.67 Cell Bundles

A practical restorative standard can require a bundle of cells.

Let

$$\mathcal{B}_C = \{(Q_1, E_1), (Q_2, E_2), \dots, (Q_n, E_n)\}. \quad (810)$$

Equation 810 represents a plural restorative specification.

A workplace restoration bundle could include:

- employment-status identity;
- salary equality;

- capability sufficiency;
- non-domination dominance;
- network-access inclusion;
- procedural sufficiency.

No single cell needs to represent the whole restorative judgment.

13.68 Conjunctive Cell Bundles

A conjunctive bundle requires every selected cell to be satisfied:

$$\bigwedge_{i=1}^n \left[Q_i(\mathcal{G}^R) \ E_i \ Q_i(\mathcal{G}_i^{\text{ref}}) \right]. \quad (811)$$

Equation 811 defines a strong multi-dimensional restorative criterion. Its feasibility declines as more demanding cells are added.

13.69 Disjunctive Cell Bundles

A disjunctive bundle permits alternative restorative paths:

$$\bigvee_{i=1}^n \left[Q_i(\mathcal{G}^R) \ E_i \ Q_i(\mathcal{G}_i^{\text{ref}}) \right]. \quad (812)$$

Equation 812 can represent subject-directed choice between reinstatement, transfer, compensated exit, or another legitimate route.

The institutional legitimacy of disjunction depends upon whether the alternatives are genuinely comparable or voluntarily selectable.

13.70 Lexically Ordered Cell Bundles

A bundle can impose lexical priority.

For example, rights and safety can be treated as prior constraints, after which capability or fidelity is considered.

A schematic order is represented by Equation 813.

$$(Q_{\text{rights}}, \text{Suff}) \succ_{\text{lex}} (Q_{\text{agency}}, \text{Suff}) \succ_{\text{lex}} (Q_{\text{fidelity}}, \approx). \quad (813)$$

Equation 813 is illustrative.

The later normative sections will consider which theories support lexical ordering.

13.71 Weighted Cell Bundles

A weighted bundle aggregates cell distances or scores.

Let d_i measure deviation under cell i . An aggregate distance is given in Equation 814.

$$D_{\text{bundle}} = \sum_{i=1}^n w_i d_i. \quad (814)$$

Equation 814 is administratively convenient.

Its legitimacy depends upon the commensurability of dimensions and justification of weights.

The atlas therefore treats weighted aggregation as one normative operation among several.

13.72 Partially Ordered Cell Bundles

Where values remain plural, bundles can be compared through partial order.

Let

$$\mathcal{B}_1 \succeq_{\text{PO}} \mathcal{B}_2 \quad (815)$$

when $\mathcal{B}_1$ is at least as good as $\mathcal{B}_2$ across every selected non-tradeable dimension and strictly better in at least one, subject to the selected ordering rule.

Equation 815 permits incomparability.

This can be preferable to artificial scalarization.

13.73 Restorative Profiles as Atlas Coordinates

A restorative profile can be represented as a vector of cell outcomes:

$$\mathbf{R} = (r_{(Q_1, E_1)}, r_{(Q_2, E_2)}, \dots, r_{(Q_n, E_n)}) . \quad (816)$$

Equation 816 locates a case within several regions of the atlas simultaneously.

A profile can contain Boolean, ordinal, metric, probabilistic, or qualitative entries.

The resulting representation is richer than a single restoration score.

13.74 Atlas Coordinates Beyond Object and Relation

The complete atlas includes additional coordinates beyond (Q, E) .

Let the full restorative coordinate be represented by Equation 817.

$$\alpha = (B, Q, E, \mathcal{T}, \tau, \varepsilon, K, s, P) , \quad (817)$$

where:

- B denotes the reference;
- Q denotes the comparison object;
- E denotes the comparative relation;
- $\mathcal{T}$ denotes tests, interventions, or perturbations;
- τ denotes temporal horizon;
- ε denotes approximation tolerance;
- K denotes epistemic basis;
- s denotes scale and system boundary;
- P denotes relevant procedural conditions.

Equation 817 is the full coordinate system underlying the paper's use of the term *atlas*.

13.75 Equivalent Claims with Different Coordinates

Two institutional statements can use the same ordinary-language phrase while occupying different atlas coordinates.

For example, “employment has been restored” can mean:

- formal status equality;
- salary equality;
- functional work-capacity sufficiency;
- network-access restoration;
- capability-set inclusion;
- behavioral equivalence under ordinary work conditions.

The surface phrase is therefore semantically underdetermined.

Atlas coordinates convert the phrase into a precise claim.

13.76 Different Claims with the Same Cell

The converse also occurs. Two cases can occupy the same object–relation cell while differing in reference, horizon, test class, or value theory.

Two resource-equality claims can use different baselines. Two bisimulation claims can use different transition abstractions. Two statistical-indistinguishability claims can use different perturbation classes.

The cell therefore remains necessary and insufficient for the complete claim.

13.77 Atlas Regions and Institutional Practice

Different institutions tend to occupy different regions of the atlas.

Courts frequently operate in entity, status, resource, causal, and compensatory regions.

Social protection frequently operates in resource-sufficiency and capability-sufficiency regions.

Healthcare and rehabilitation frequently operate in bodily, functional, response, and capability regions.

Organizations frequently operate in status, position, relation, network, and procedural regions.

Regulators frequently operate in transition, risk, robustness, and systemic regions.

Generative-justice analysis places special emphasis on control, value-circulation, recursion, and generative-mechanism regions.

This institutional clustering reflects differences in authority, evidence, and purpose.

13.78 Atlas Regions and Justice Traditions

Justice traditions also privilege different regions.

Corrective justice frequently selects historically indexed loss and responsibility cells.

Resource egalitarianism emphasizes resource-distribution cells.

Capability theory emphasizes capability, conversion, and opportunity cells.

Relational equality emphasizes relation, standing, and institutional-position cells.

Non-domination emphasizes control, dependency, and exit cells.

Recognition theory emphasizes meaning, status, and social-standing cells.

Generative justice emphasizes control, circulation, recursion, and generative mechanisms.

The atlas therefore provides a shared map on which theoretical differences can be represented without reducing one tradition to another.

13.79 Atlas Regions and Temporal Phases

The temporal phases identified earlier also occupy characteristic regions.

Pre-harm governance emphasizes safe sets, resilience, control, redundancy, reparability, and option preservation.

During-harm intervention emphasizes transition rates, harmful-process interruption, evidence state, propagation networks, and interim resource sufficiency.

Post-harm justice emphasizes restitution, compensation, capability, recognition, re-entry, reachable futures, generative mechanisms, and historical provenance.

The atlas is therefore temporal in application even where its basic object–relation cells are timeless abstractions.

13.80 Atlas Regions and Reparability

Different regions possess different typical reparability.

Resource-equality cells can often be highly attainable.

Formal-status cells can be attainable where institutions retain authority.

Network-history cells can be partially attainable.

Singular-relation identity can be unavailable.

Historical identity with a no-harm world can be structurally unavailable after the harm has occurred.

This pattern supports differentiated prevention.

Cells with low expected reparability can justify stronger ex ante safeguards where other normative conditions are satisfied.

13.81 Atlas Regions and Fungibility

Fungibility also varies across regions.

Resources and market-valued goods occupy comparatively fungible regions.

Relations, provenance, historical timing, identity, and meaning occupy less fungible regions.

A general equivalent can therefore move a case from one region to another.

For example, a singular lost relation can be translated into monetary compensation.

The monetary cell can be satisfied while the relational cell remains non-equivalent.

This structure makes the limits of compensation visible without denying its value.

13.82 Atlas Regions and Subject Direction

Subject-directed justice tends to favor relations that preserve or expand viable future possibilities.

Capability inclusion, reachability dominance, action-set expansion, agency sufficiency, and revisability belong naturally to this region.

Strict historical identity belongs less naturally where the subject prefers a new future.

The atlas therefore provides a formal vocabulary for conflicts between counterfactual fidelity and present agency.

13.83 Atlas Regions and Transformative Justice

Transformative justice often occupies dominance, refinement, or structural change cells rather than equivalence cells.

A non-dominating control structure can dominate the historical reference.

A community-controlled reparative institution can differ structurally from the pre-harm institution while being normatively superior.

A transformed value-circulation system can fail historical isomorphism and succeed generative-justice criteria.

This region demonstrates why the title *Atlas of Equivalence* should be understood broadly. The atlas contains equivalence, preservation, dominance, sufficiency, and other comparison relations required to understand return.

13.84 Normative Layer over the Atlas

The formal atlas $\mathfrak{C}_R$ does not itself determine justice.

A normative interpretation map is required:

$$\mathcal{N}_V : \mathfrak{C}_R \longrightarrow \mathfrak{J}_V, \quad (818)$$

where V denotes the relevant value theory.

Equation 818 maps formal restorative claims into normative evaluations.

The same cell can receive different values under different theories.

Capability-set inclusion can be central to one theory and secondary to another. Network isomorphism can be important for institutional structure and insufficient for a theory of singular relational value. Bisimulation can be formally impressive and normatively excessive.

The later value-theory sections will develop this layer systematically.

13.85 Procedural Layer over the Atlas

A separate procedural map evaluates the process required to establish or attain a cell:

$$\mathcal{P}_J : (Q, E, R) \longrightarrow \mathfrak{J}_P. \quad (819)$$

Equation 819 can evaluate consent, participation, privacy, burden, reversibility, contestability, and non-domination.

A cell can be substantively desirable and procedurally impermissible.

This layer prevents the atlas from becoming an optimization system for maximal fidelity.

13.86 Epistemic Layer over the Atlas

The epistemic layer evaluates whether a cell can be identified from available knowledge.

Let

$$\mathcal{K}_A : (Q, E, K) \longrightarrow \mathfrak{C}_K. \quad (820)$$

Equation 820 can classify a claim as directly observed, statistically estimated, model-dependent, partially identified, bounded, speculative, or ideal-theoretical.

This layer is particularly important for counterfactual and historical cells.

13.87 Feasibility Layer over the Atlas

A fourth layer evaluates current or idealized attainability.

Let

$$\mathcal{F}_A : (Q, E, I, T) \longrightarrow \mathfrak{F}, \quad (821)$$

where I denotes institutional capacity and T technological capacity.

Equation 821 distinguishes:

- currently implementable;
- institutionally developable;
- technologically speculative;

Table 6: Illustrative object–relation atlas for restorative claims

Object	Exact / structural	Directional	Behavioral / dynamic	Approximate / statistical	Normative-order form
Resource	Equality	Inclusion	Conversion preservation	Metric proximity	Sufficiency / dominance
Capability	Set equality	Inclusion	Simulation	Set distance	Threshold / dominance
Relation	Identity / type correspondence	Substitution	Role preservation	Quality similarity	Relational sufficiency
Network	Isomorphism	Homomorphism / embedding	Reachability preservation	Graph similarity	Structural dominance
State	Equality	Component inclusion	Finite-horizon evolution	Metric proximity	Threshold
Transition system	Structural identity	Simulation	Bisimulation / trace semantics	Approximate simulation	Choice dominance
Reachable set	Set equality	Inclusion	Reachability simulation	Hausdorff proximity	Option dominance
Dynamics	Law equality	Semiconjugacy	Topological conjugacy	Approximate dynamics	Resilience dominance
Response	Function equality	Response dominance	Perturbation correspondence	Statistical indistinguishability	Robustness sufficiency
Causal system	Structural equality	Causal abstraction	Interventional equivalence	Approximate causal similarity	Policy dominance
Generative mechanism	Structural identity	Homomorphism / simulation	Recursive-process correspondence	Approximate response equivalence	Generative dominance
History	Historical identity	Order embedding	Historical simulation	Narrative similarity	Recognition adequacy
Procedure	Procedural identity	Rights inclusion	Process simulation	Outcome-tolerance relation	Procedural sufficiency / dominance

- ideal-theoretical;
- structurally unattainable under the selected definition.

The feasibility layer is distinct from normative legitimacy.

13.88 The Multi-Layer Atlas

The full atlas can therefore be represented by Equation 822.

$$\mathfrak{A}_R = (\mathfrak{C}_R, \mathcal{N}_V, \mathcal{P}_J, \mathcal{K}_A, \mathcal{F}_A). \quad (822)$$

Equation 822 distinguishes the formal comparison space from its normative, procedural, epistemic, and feasibility interpretations.

This separation is one of the central architectural contributions of the paper.

13.89 A Compact Atlas Matrix

Table 6 provides a compact illustration of the object–relation space. The table is not exhaustive. Its purpose is to show how several important objects admit several distinct comparison standards.

Table 6 demonstrates that the atlas is multi-relational even within one object row.

13.90 Atlas Navigation by Restorative Question

The atlas can be entered through the practical question presented by the case.

A claim concerning returned property begins in the entity and resource regions.

A claim concerning actual freedom to use returned resources moves toward conversion and capability regions.

A claim concerning professional re-entry moves toward status, relation, and network regions.

A claim concerning long-term stability moves toward attractor, basin, and response regions.

A claim concerning whether the same assistance will now produce the same effect moves toward causal and interventional regions.

A claim concerning circular return of value moves toward circulation and generative-mechanism regions.

A claim concerning historical identity moves toward event, provenance, memory, and narrative regions.

The atlas therefore functions as a navigation system for specifying the restorative problem.

13.91 Atlas Navigation by Institutional Function

Institutional function provides another entry point.

A court can begin with legal status, identifiable loss, responsibility, and compensatory equivalence.

A regulator can begin with safe-set, transition, resilience, and risk cells.

A social-protection institution can begin with resource and capability sufficiency.

A rehabilitation provider can begin with function, conversion, response, and subject-directed target cells.

A reparations commission can combine historical, recognition, resource, institutional, and generative cells.

This navigation method reduces pressure on institutions to occupy cells outside their legitimate competence.

13.92 Atlas Navigation by Temporal Phase

Temporal phase supplies a third entry.

Pre-harm questions naturally begin with risk, control, resilience, redundancy, safe-set, and reparability cells.

Ongoing-harm questions begin with transition, propagation, interruption, and interim-resource cells.

Post-harm questions begin with restitution, compensation, capability, recognition, re-entry, and future-generativity cells.

Long-term institutional learning adds provenance, correction-capacity, and systemic-resilience cells.

The temporal architecture and comparative atlas therefore form one integrated framework.

13.93 Atlas Navigation by Value Theory

Value theory supplies a fourth entry.

Utilitarian analysis can privilege welfare and aggregate-value regions.

Resource theories can privilege resource distributions.

Capability theory can privilege capability, conversion, and opportunity regions.

Relational theories can privilege standing and relation structure.

Republican theory can privilege control, dependency, and exit.

Recognition theory can privilege meaning and public-standing regions.

Generative justice can privilege control, value circulation, recursion, and generative mechanisms.

The atlas allows these theories to be compared through the objects and relations they select.

13.94 Atlas Navigation by Failure Mode

The atlas can also diagnose why a restorative intervention appears inadequate.

A resource-equal intervention with poor outcome can suggest conversion failure.

A formally reinstated subject with weak participation can suggest relational or network failure.

A behaviorally normal subject with high fragility can suggest response or basin difference.

A well-compensated community with continuing extraction can suggest circulation or control failure.

A technically successful intervention rejected by the subject can suggest procedural or agency failure.

The atlas therefore supports diagnosis in addition to classification.

13.95 Atlas Navigation by Residual Difference

After one cell has been satisfied, remaining differences can be projected into other objects.

Let cell $c_1 = (Q_1, E_1)$ be satisfied. Residual analysis considers the family

$$\mathfrak{R}_{\text{residual}} = \left\{ Q_j : Q_j(\mathcal{G}^R) \not\models_j Q_j(\mathcal{G}^{\text{ref}}) \right\}. \quad (823)$$

Equation 823 defines the unresolved comparative space.

This procedure prevents one successful dimension from terminating inquiry prematurely.

13.96 Residual Difference and Institutional Handoff

Residual objects can determine institutional handoff.

A court can settle the monetary cell and refer unresolved rehabilitation to health services.

An employer can restore status while professional-network repair requires a professional community.

A social-protection system can stabilize income while historical recognition requires another institution.

The atlas therefore links formal decomposition with the institutional division of labor developed earlier.

13.97 Atlas Minimalism

A practical application should select only cells relevant to the legitimate restorative purpose.

The existence of a large atlas does not justify measuring every object.

Let

$$\mathfrak{C}_{\text{case}} \subset \mathfrak{C}_R \quad (824)$$

denote the case-specific subatlas.

Equation 824 embodies a principle of analytical minimalism.

A small, well-justified subatlas is preferable to indiscriminate high-dimensional measurement.

13.98 Atlas Expansion

The case-specific subatlas can expand when evidence of unresolved dimensions appears.

Let

$$\mathfrak{C}_{t+1} = \mathfrak{C}_t \cup \Delta \mathfrak{C}_{t+1}. \quad (825)$$

Equation 825 represents adaptive enlargement of the comparative inquiry.

This permits a restorative process to remain epistemically humble and revisable.

13.99 Atlas Contraction

Cells can also become irrelevant when the subject's goals, institutional role, or available evidence changes.

A restorative architecture should permit contraction of the active subatlas:

$$\mathfrak{C}_{t+1} = \mathfrak{C}_t \setminus \Delta \mathfrak{C}_{t+1}^-. \quad (826)$$

Equation 826 prevents the analytical framework from becoming a permanent surveillance apparatus.

13.100 Atlas Provenance

The selection and revision of the case-specific atlas should itself possess provenance.

Let

$$\mathfrak{P}_{\mathfrak{C}} = (\mathfrak{C}_0 \rightarrow \mathfrak{C}_1 \rightarrow \cdots \rightarrow \mathfrak{C}_n). \quad (827)$$

Equation 827 records which cells were introduced, removed, or redefined and why.

This is particularly important where the object of justice changes during a long restorative process.

13.101 Atlas Disagreement

Parties can disagree at several levels of the atlas.

They can disagree over the reference.

They can disagree over the comparison object.

They can agree on the object and disagree over the comparative relation.

They can agree on both and disagree over the epistemic evidence.

They can agree formally and disagree over normative salience.

They can agree on the substantive target and disagree over the permissible process.

Identifying the level of disagreement can prevent several debates from being mistaken for factual disputes.

13.102 Object Disagreement

An employer can define a dispute through income and formal status.

The employee can define the same history through trust, network position, and future professional opportunity.

The disagreement concerns Q , not yet E .

The atlas makes this difference explicit.

13.103 Relation Disagreement

Parties can agree that capability is the relevant object and disagree over the standard.

One can require equality with the no-harm capability set.

Another can require sufficiency.

Another can require inclusion of the reference set within an expanded new set.

This disagreement concerns E .

13.104 Reference Disagreement

Parties can also agree on object and relation while disagreeing on the reference.

Both can seek income equality while one uses pre-harm salary and another uses expected no-harm career income.

The numerical dispute is therefore downstream of baseline disagreement.

13.105 Model Disagreement

Two parties can agree on B , Q , and E while representing Q differently.

One network model can include professional collaboration only. Another can include mentorship, reputation, and institutional authority.

The disagreement concerns representation.

This layer belongs to the epistemic architecture of the atlas.

13.106 Normative Disagreement

Parties can agree that one restorative cell is satisfied and disagree about whether the cell is sufficient for justice.

This is a disagreement in the normative layer $\mathcal{N}_V$.

Formal agreement therefore does not imply philosophical agreement.

13.107 Procedural Disagreement

Parties can also agree on the correct substantive cell and disagree about how it may be attained.

One intervention can achieve network correspondence through compulsory relocation or disclosure of sensitive information.

The subject can reject the process while accepting the restorative objective.

This disagreement belongs to the procedural layer.

13.108 Comparative Closure

An institution eventually requires criteria for closing its comparative inquiry.

Comparative closure can occur when:

- every cell within the institution's mandate has been adjudicated;
- remaining cells belong to other institutions;
- evidence is insufficient for stronger claims;
- the subject declines further intervention;
- additional comparison would impose disproportionate burden.

Closure should identify unresolved cells rather than implicitly declaring them satisfied.

13.109 Comparative Reopening

A closed comparison can be reopened when material new evidence or changed conditions justify review.

The reopening rule should be explicit to preserve both revisability and institutional finality.

This principle mirrors the broader treatment of restorative provenance.

13.110 Atlas-Based Restorative Statement

A fully specified restorative statement can take the form represented by Equation 828.

$$\mathcal{R} = [B; Q; E; \mathcal{T}; \tau; \varepsilon; K; s; P]. \quad (828)$$

Equation 828 should be read as:

Relative to reference B , object Q satisfies comparative relation E under test class $\mathcal{T}$, temporal horizon τ , tolerance ε , epistemic basis K , scale s , and procedural conditions P .

This representation is the most compact statement of the comparative framework developed so far.

13.111 Illustrative Workplace Atlas

A workplace case can illustrate how several cells coexist.

Suppose a worker is wrongfully excluded for two years and later reinstated.

The following claims can be separately evaluated:

1. formal employment status equality;
2. salary compensation under a monetary-equivalence rule;
3. professional-capability inclusion;
4. network-position correspondence;
5. trust-state similarity;
6. secure-access dominance under non-domination;
7. perturbation-response similarity under ordinary organizational stress;
8. procedural sufficiency of the re-entry process.

Successful reinstatement satisfies the first cell.

Payment can satisfy the second.

Neither result logically establishes the remaining cells.

The example shows how the atlas decomposes a broad claim of workplace restoration into several independently testable and normatively interpretable claims.

13.112 Illustrative Research Atlas

A researcher deprived of laboratory and institutional access presents another multi-cell case.

Potential cells include:

1. equipment-resource equality;
2. funding-value equivalence;
3. institutional-status identity;
4. research-capability inclusion;
5. collaboration-network similarity;
6. opportunity-set reachability;
7. generative-process simulation;
8. publication-trajectory approximation;
9. subject-directed future sufficiency.

A later grant can exceed the monetary value of the original lost resources while remaining non-equivalent to the historical collaboration network.

A new institution can provide greater capability while producing low historical fidelity.

The atlas permits these judgments to coexist.

13.113 Illustrative Reputational Atlas

Reputational harm creates a distributed comparative problem.

Relevant cells can include:

1. official-record correction identity;
2. classification-propagation update;
3. public-narrative correspondence;
4. reputation-distribution similarity;
5. professional-network reachability;
6. future opportunity dominance;
7. historical-recognizability preservation.

Correction of the official source record can satisfy one cell while downstream reputation remains substantially different.

Erasure of every historical record could improve some forms of no-harm similarity while reducing historical recognizability.

The atlas therefore exposes conflicts among restorative objectives.

13.114 Illustrative Collective Atlas

A collective reparative case can include:

1. property restitution;
2. community-resource dominance;
3. institutional-control transformation;
4. cultural-transmission support;
5. collective-capacity restoration;
6. public-recognition adequacy;
7. generative value-circulation reform;
8. ecological resilience restoration;
9. intergenerational opportunity expansion.

No single equivalence relation can represent all of these objects.

A collective reparations program is therefore naturally a cell bundle.

13.115 Formal Atlas and Practical Modesty

The breadth of the atlas should increase conceptual precision rather than institutional ambition.

A complete map of possible comparison cells does not create a duty to measure or manipulate every object.

The atlas performs three more modest functions:

1. it reveals which claim an institution is actually making;
2. it reveals dimensions left outside that claim;
3. it supplies alternative cells when the selected comparison fails to represent the normative problem.

This role is compatible with institutional minimalism and data minimization.

13.116 The Atlas as a Theory of Comparative Questions

The atlas is therefore best understood as a theory of comparative questions.

For any restorative claim, it asks:

1. Which reference is being used?
2. Which object is being compared?
3. Which relation is being required?
4. Which observations, perturbations, or interventions define the test?
5. Which temporal horizon matters?
6. Which differences are tolerated?
7. Which evidence supports the judgment?
8. At which scale does the claim hold?
9. Which process was used to attain the result?
10. Which value theory makes the selected preservation relevant?

The framework does not assume that every question must receive a precise numerical answer.

Its contribution lies in preventing several distinct questions from being compressed into one undefined assertion of restoration.

13.117 Core Comparative Principle

The first central principle of the atlas can now be stated:

No restorative claim is complete without specification of the reference system, comparison object, and comparative relation.

This principle identifies the minimum comparative structure.

13.118 Indexed Comparative Principle

A stronger principle includes the remaining indices:

The meaning and evidentiary strength of a restorative claim depend upon the reference, comparison object, comparative relation, test class, temporal horizon, approximation tolerance, epistemic basis, scale, and procedural conditions under which the claim is asserted.

This principle supplies the conceptual architecture for the rest of the paper.

13.119 Normative Interpretation Principle

A third principle concerns justice:

A formally coherent restorative comparison acquires normative force only through a theory explaining why the preserved structure matters.

This principle prevents mathematical strength from becoming normative authority by itself.

13.120 Procedural Independence Principle

A fourth principle concerns process:

Satisfaction of a restorative object–relation cell does not independently establish the justice of the process through which the cell was attained.

This principle preserves the distinction between restorative fidelity and procedural legitimacy.

13.121 Atlas Expansion Beyond Equivalence

The title *Atlas of Equivalence* is retained because equivalence provides the central comparative problem. The formal space developed here extends beyond equivalence relations.

The atlas includes:

- equality;
- structural equivalence;
- simulation;
- approximation;
- inclusion;
- dominance;
- sufficiency;
- refinement;
- transformation.

This wider scope is necessary because justice can require preservation, improvement, or adequate future possibility even where return to an equivalent historical state is unavailable or undesirable.

13.122 Implications for Subsequent Formal Analysis

The object–relation space yields several consequences for the sections that follow.

First, temporal horizon requires independent treatment because the same cell can hold instantaneously and fail over longer periods.

Second, state, trajectory, and reachability comparison require separate formal analysis because they occupy different regions of the atlas.

Third, attractor, basin, and stability comparisons require a dedicated dynamical treatment.

Fourth, perturbation-response equivalence requires explicit specification of admissible disturbances and statistical tests.

Fifth, causal and interventional comparison require stronger assumptions than observational equivalence.

Sixth, generative-process equivalence requires comparison across several linked objects, including conditions, conversion, value generation, circulation, and recursive regeneration.

Seventh, value theory must determine which cells carry normative importance.

Eighth, substantive and procedural justice must evaluate different dimensions of the resulting restorative architecture.

13.123 Transition to Temporal Horizons

The first three dimensions of the comparative architecture have now been specified:

$$(B, Q, E). \quad (829)$$

Equation 829 contains the reference, object, and relation.

The next dimension concerns time.

A restored system can satisfy a selected cell at the moment of return and depart from the reference shortly afterward. A subject can display similar behavior for one month and very different long-run resilience. A network can be temporarily reconstructed and later collapse. A resource transfer can create immediate equality and long-run divergence. An ecological system can reproduce current composition while possessing different asymptotic dynamics.

Section 14 therefore examines temporal horizons of restorative comparison. It distinguishes instantaneous, finite-horizon, pathwise, long-run, asymptotic, recurrent, and intergenerational forms of equivalence and shows how the temporal index changes the meaning of otherwise identical object–relation cells.

14 Temporal Horizons of Restorative Comparison

This section develops the temporal dimension of the Atlas of Equivalence. The previous sections specified the reference system B , comparison object Q , and comparative relation E . These three components remain insufficient for many restorative claims because the same object–relation pair can hold at one time horizon and fail at another. Resource equality can be achieved immediately and disappear within months. Functional performance can recover while long-run resilience remains impaired. A network can be reconstructed temporarily and later fragment. An ecological system can regain a familiar state while approaching a different long-run regime. A subject can appear to have returned to a previous level of activity while the future opportunity structure remains altered.

Time therefore enters restorative comparison as an independent analytical coordinate rather than as background metadata. The temporal index specifies *when*, *for how long*, and *over which portion of a trajectory* a restorative relation is asserted.

The section distinguishes several families of temporal comparison:

1. instantaneous comparison;

2. interval and finite-horizon comparison;
3. trajectory and pathwise comparison;
4. cumulative and integrated comparison;
5. rolling and moving-window comparison;
6. recurrent and repeated-event comparison;
7. recovery-time and transient comparison;
8. long-run and asymptotic comparison;
9. lifecycle and developmental comparison;
10. intergenerational comparison;
11. historical-window and sequence-sensitive comparison.

The central claim is that restorative equivalence is generally *horizon-relative*. A system can satisfy the selected relation at t_1 , fail it over $[t_1, t_2]$, recover it asymptotically, and still remain historically non-equivalent to the reference. These are different statements rather than contradictions.

14.1 The Temporal Coordinate of Restorative Claims

Let the basic restorative cell be (B, Q, E) , consisting of reference, comparison object, and comparative relation. A temporally indexed restorative claim is represented by Equation 830.

$$Q(\mathcal{G}^R) \sim_E^\tau Q(\mathcal{G}^{\text{ref}}), \quad (830)$$

where τ denotes the temporal horizon or temporal specification relevant to the comparison.

Equation 830 should be interpreted broadly. The parameter τ can denote a single time, an interval, a collection of times, a stopping time, a moving window, an asymptotic limit, a life stage, or an intergenerational horizon.

A more explicit temporal specification is represented by Equation 831.

$$\mathcal{T}_H = (t_0, t_1, \Delta t, \Omega_t, \vartheta), \quad (831)$$

where:

- t_0 denotes the beginning of the comparison period;
- t_1 denotes its endpoint where finite;
- Δt denotes temporal resolution;
- Ω_t denotes the set or family of times actually evaluated;
- ϑ denotes the temporal aggregation or comparison rule.

Equation 831 makes clear that two studies using the same nominal horizon can remain different if they use different sampling frequencies or aggregation rules.

14.2 Instantaneous Restorative Comparison

The simplest temporal claim concerns one time t . Instantaneous comparison is represented by Equation 832.

$$Q(\mathcal{G}_t^R) \text{ E } Q(\mathcal{G}_t^{\text{ref}}). \quad (832)$$

Equation 832 can be appropriate for objects such as:

- current salary;
- current legal status;
- current account balance;
- current possession of property;
- current measured functional state;
- current access permission;
- current classification in an institutional record.

Instantaneous comparison is institutionally attractive because it is often easy to observe and easy to communicate.

Its limitation is equally clear. The claim establishes correspondence only at the selected moment.

A subject can possess equal salary today while facing weaker future employment security. A network can have the same number of active ties at one measurement time while those ties have lower persistence. A patient can display the same functional performance during a clinical visit while requiring greater effort or recovering more slowly afterward.

Instantaneous restoration should therefore be interpreted as a local temporal claim.

14.3 Instantaneous Equality and Dynamic Divergence

Let two systems satisfy

$$x_R(t_1) = x_{\text{ref}}(t_1). \quad (833)$$

Equation 833 does not imply equality at later times. If the transition rules differ,

$$\Phi_R \neq \Phi_{\text{ref}}, \quad (834)$$

then there can exist $t_2 > t_1$ such that

$$x_R(t_2) \neq x_{\text{ref}}(t_2). \quad (835)$$

Equations 833 through 835 formalize one of the central reasons that current-state equality cannot function as a universal standard of return.

The result is especially important where harm changed conversion structures, expectations, resilience, or transition probabilities.

14.4 Finite-Horizon Restorative Comparison

A finite-horizon claim evaluates correspondence across a bounded interval $[t_0, t_0 + \tau]$.

Let

$$\Gamma_\tau^R = \{Q(\mathcal{G}_t^R) : t \in [t_0, t_0 + \tau]\} \quad (836)$$

and

$$\Gamma_\tau^{\text{ref}} = \{Q(\mathcal{G}_t^{\text{ref}}) : t \in [t_0, t_0 + \tau]\}. \quad (837)$$

The finite-horizon restorative claim is represented by Equation 838.

$$\Gamma_\tau^R E_\tau \Gamma_\tau^{\text{ref}}. \quad (838)$$

Equation 838 can use pointwise equality, path distance, average difference, maximum deviation, trace equivalence, or another interval-level relation.

The choice among these relations changes the substantive meaning of restoration.

14.5 Pointwise Interval Equivalence

The strongest ordinary finite-horizon condition requires the selected relation at every time:

$$Q(\mathcal{G}_t^R) E Q(\mathcal{G}_t^{\text{ref}}) \quad \text{for every } t \in [t_0, t_1]. \quad (839)$$

Equation 839 preserves the relation throughout the interval.

This criterion is stronger than equality of interval averages and stronger than endpoint equality.

For employment, it can require salary equality throughout the period rather than equal annual totals reached through severe interim deprivation.

For capability, it can require continuing access rather than eventual return of access after a long gap.

14.6 Maximum-Deviation Comparison

Where an object space possesses metric d , a finite-horizon comparison can bound the maximum deviation:

$$\sup_{t \in [t_0, t_1]} d(Q(\mathcal{G}_t^R), Q(\mathcal{G}_t^{\text{ref}})) \leq \varepsilon. \quad (840)$$

Equation 840 ensures that deviation never exceeds the specified tolerance during the interval.

This criterion is useful where temporary severe departures matter independently of later averaging.

14.7 Average-Deviation Comparison

A weaker criterion can bound average deviation:

$$\frac{1}{t_1 - t_0} \int_{t_0}^{t_1} d(Q(\mathcal{G}_t^R), Q(\mathcal{G}_t^{\text{ref}})) dt \leq \varepsilon. \quad (841)$$

Equation 841 permits temporary larger deviations if they are compensated by periods of closer correspondence.

The normative legitimacy of such temporal averaging depends upon the object.

Temporary income loss can sometimes be compensated later. Temporary denial of life-saving treatment, political participation, or a unique developmental opportunity can have much weaker temporal substitutability.

Temporal aggregation therefore requires a value theory.

14.8 Cumulative Comparison

Some objects accumulate over time. Total income, total exposure, total burden, care hours, educational time, and cumulative access can be represented through integrals.

Let

$$A_R = \int_{t_0}^{t_1} Q(\mathcal{G}_t^R) dt \quad (842)$$

and

$$A_{\text{ref}} = \int_{t_0}^{t_1} Q(\mathcal{G}_t^{\text{ref}}) dt. \quad (843)$$

A cumulative equality criterion is

$$A_R = A_{\text{ref}}. \quad (844)$$

Equation 844 allows temporal redistribution of the quantity.

This can be meaningful for fungible resources and misleading for temporally singular conditions.

14.9 Cumulative Equality and Temporal Non-Equivalence

Two histories can satisfy equal cumulative quantity while differing greatly in sequence.

Consider one system receiving resource r continuously and another receiving the same total amount after prolonged deprivation.

The cumulative equality

$$\int_{t_0}^{t_1} r_R(t) dt = \int_{t_0}^{t_1} r_{\text{ref}}(t) dt \quad (845)$$

does not imply generative equivalence because earlier availability can produce later resources, relations, and options.

This is a temporal form of non-commutativity.

14.10 Discounted Temporal Comparison

Some economic and policy analyses discount future values. A generic discounted comparison can be written as

$$V_\delta = \int_{t_0}^{t_1} \delta(t) V(Q_t) dt, \quad (846)$$

where $\delta(t)$ is a temporal weighting function.

Equation 846 can be useful for financial valuation and long-horizon policy.

Its normative interpretation requires caution.

Discounting can reflect opportunity cost, uncertainty, impatience, social time preference, mortality, or institutional convention. These rationales are not normatively interchangeable.

For restorative justice, use of discounting should therefore state what temporal weighting represents.

14.11 Temporal Weighting Without Monetary Discounting

Temporal weighting can also apply to non-monetary objects.

Let $w(t)$ represent the significance of a condition at time t :

$$J_T = \int_{t_0}^{t_1} w(t)j(t) dt. \quad (847)$$

Equation 847 can represent developmental sensitivity, critical career stages, ecological seasonal windows, or periods of special dependency.

The weighting function should not be assumed to decline monotonically with time.

Some periods can have greater generative significance than earlier or later periods.

14.12 Historical Windows

A *historical window* is a bounded period during which a condition has a distinctive generative significance.

Let

$$W = [t_a, t_b]. \quad (848)$$

Availability of condition c can be represented by indicator $A_c(t)$. Window-sensitive restoration can require

$$A_{c,R}(t) = A_{c,\text{ref}}(t) \quad t \in W. \quad (849)$$

Equation 849 is stronger than eventual availability of c .

Examples include:

- education during a developmental period;
- participation in an electoral process;
- access to a unique research collaboration;
- timely medical treatment;
- use of land during a seasonal period;
- publication within a specific historical debate.

Once the window closes, later provision of the same nominal condition can have different generative significance.

14.13 Sequence-Sensitive Comparison

Restorative outcomes can depend upon event order.

Let

$$\mathbf{e} = (e_1, e_2, \dots, e_n) \quad (850)$$

denote the sequence of relevant events.

Two histories can contain the same event set and differ in ordering:

$$\{e_1, \dots, e_n\}_R = \{e_1, \dots, e_n\}_{\text{ref}} \quad (851)$$

while

$$\mathbf{e}_R \neq \mathbf{e}_{\text{ref}}. \quad (852)$$

Equations 851 and 852 show why event membership alone is insufficient where sequencing matters.

Path-dependence research in economics and politics provides established examples of sequence-sensitive institutional development [3, 45].

14.14 Temporal Non-Commutativity of Interventions

Intervention order can also matter.

Let R_1 and R_2 be two restorative interventions. In general,

$$R_2 \circ R_1 \neq R_1 \circ R_2. \quad (853)$$

Equation 853 can describe cases such as:

- rehabilitation before or after stable housing;
- reinstatement before or after organizational reform;
- compensation before or after legal-status correction;
- public recognition before or after establishment of factual findings.

The ordering can change both effectiveness and meaning.

14.15 Trajectory Comparison

A trajectory-level restorative claim compares whole paths rather than states.

Let

$$\gamma_R : [t_0, t_1] \longrightarrow \mathcal{X} \quad (854)$$

and

$$\gamma_{\text{ref}} : [t_0, t_1] \longrightarrow \mathcal{X}. \quad (855)$$

A trajectory comparison can use metric

$$D_\Gamma (\gamma_R, \gamma_{\text{ref}}). \quad (856)$$

Equation 856 can represent pointwise difference, integrated distance, maximum deviation, dynamic time warping, or another domain-specific path relation.

Trajectory comparison captures duration and sequence that endpoint comparison loses.

14.16 Pathwise Equality

The strongest trajectory relation is

$$\gamma_R(t) = \gamma_{\text{ref}}(t) \quad \text{for every } t \in [t_0, t_1]. \quad (857)$$

Equation 857 is extremely demanding.

It can be conceptually useful for identifying the strongest possible temporal fidelity.

Actual post-harm histories will generally fail this criterion when compared with a no-harm trajectory because the harm and repair have already occurred.

14.17 Pathwise Approximation

A practical trajectory criterion can require

$$D_{\Gamma}(\gamma_R, \gamma_{\text{ref}}) \leq \varepsilon. \quad (858)$$

Equation 858 permits bounded temporal deviation.

The meaning depends upon the path metric.

A metric insensitive to short severe episodes can be inappropriate for justice where such episodes matter independently.

14.18 Time-Reparameterized Comparison

A subject can follow a similar qualitative trajectory at a different speed.

Let $\theta : [t_0, t_1] \rightarrow [t'_0, t'_1]$ be an increasing time reparameterization. A time-adjusted relation can require

$$\gamma_R(t) \approx \gamma_{\text{ref}}(\theta(t)). \quad (859)$$

Equation 859 can be useful for rehabilitation where recovery follows the same sequence over a longer period.

Its normative legitimacy depends upon whether delay itself constitutes loss.

For age-sensitive, political, ecological, or historical opportunities, time reparameterization can erase precisely the dimension that matters.

14.19 Lagged Restorative Equivalence

A simpler lagged criterion is

$$Q(\mathcal{G}_{t+\ell}^R) \approx Q(\mathcal{G}_t^{\text{ref}}), \quad (860)$$

where ℓ is the recovery lag.

Equation 860 is useful when delayed recovery is expected.

The lag should itself remain visible rather than disappear inside the equivalence statement.

14.20 Recovery-Time Comparison

Let T_{rec} denote the time required for a system to return to an acceptable region after perturbation.

A recovery-time equality criterion is

$$T_{\text{rec},R} = T_{\text{rec},\text{ref}}. \quad (861)$$

An approximate criterion is

$$|T_{\text{rec},R} - T_{\text{rec},\text{ref}}| \leq \varepsilon_T. \quad (862)$$

Equations 861 and 862 provide a simple way to distinguish apparent state recovery from dynamic recovery.

A subject who eventually returns to the same level of performance while requiring substantially longer recovery after ordinary disturbances remains dynamically different.

14.21 Transient Burden

The transient process before recovery can itself be compared.

Let $b(t)$ represent burden over the recovery interval. Cumulative transient burden is

$$B_{\text{trans}} = \int_{t_0}^{T_{\text{rec}}} b(t) dt. \quad (863)$$

Equation 863 captures cases where equal recovery time conceals very different intermediate cost.

This object can include pain, administrative burden, lost income, effort, uncertainty, or another dimension.

14.22 Overshoot and Undershoot

Recovery trajectories can overshoot or undershoot the reference before stabilizing.

Let error be

$$e(t) = Q(\mathcal{G}_t^R) - Q(\mathcal{G}_t^{\text{ref}}). \quad (864)$$

Maximum overshoot can be represented by

$$M_O = \sup_t e(t) \quad (865)$$

for appropriately oriented scalar quantities.

Equation 865 can matter where aggressive intervention creates temporary excesses or new risks before stabilization.

14.23 Stabilization Time

Restoration can also be evaluated by the time required to remain within an acceptable band around the reference.

Let

$$T_{\text{stab}} = \inf \left\{ t : d(Q(\mathcal{G}_s^R), Q(\mathcal{G}_s^{\text{ref}})) \leq \varepsilon \forall s \geq t \right\}. \quad (866)$$

Equation 866 differs from the first return time because a system can temporarily re-enter the acceptable region and later leave it.

This distinction is particularly relevant to recurrent harms and fragile re-entry.

14.24 First-Passage Comparison

A first-passage time records when a system first reaches a target region $\mathcal{A}$:

$$T_{\mathcal{A}} = \inf \{ t \geq t_0 : x_t \in \mathcal{A} \}. \quad (867)$$

Equation 867 can represent time to re-employment, time to regain independent living, time to restore formal status, or time to re-enter a professional network.

First passage is weaker than stable restoration.

14.25 Duration Above a Threshold

Where adequacy is threshold-based, restoration can depend upon the duration for which the condition remains adequate.

Let $I_{\text{adequate}}(t)$ be an indicator of threshold satisfaction. Then

$$D_{\text{adequate}} = \int_{t_0}^{t_1} I_{\text{adequate}}(t) dt \quad (868)$$

measures time spent within the adequate region.

Equation 868 distinguishes continuous security from intermittent access.

14.26 Continuity of Access

Access can be equal in total duration and differ in continuity.

Suppose two systems each provide six months of access. One provides a continuous six-month period; the other alternates access and exclusion.

The generative consequences can differ because interruption can prevent long-horizon projects.

A continuity-sensitive object can therefore include maximal uninterrupted duration:

$$D_{\text{cont}} = \max \{ \text{length of continuous intervals of access} \}. \quad (869)$$

Equation 869 is especially relevant to research, employment, healthcare, education, and housing.

14.27 Temporal Fragmentation

Temporal fragmentation measures how often a condition is interrupted.

Let N_{break} denote the number of interruptions within the horizon.

Two systems can have equal cumulative access and different N_{break} .

Fragmentation can increase administrative cost, reduce planning, disrupt trust, and prevent projects requiring uninterrupted duration.

This dimension should therefore remain separate from total availability.

14.28 Persistence

A restored condition can be evaluated by persistence.

For event A_t representing continued availability, persistence probability over horizon τ can be written as

$$P(A_{t+s} = 1 \forall s \in [0, \tau] \mid A_t = 1). \quad (870)$$

Equation 870 is relevant where current restoration is insecure.

Current access with low persistence differs from secure access.

14.29 Survival of Restorative Conditions

Where loss can recur, a survival function can represent continued restoration:

$$S_R(t) = P(T_{\text{failure}} > t). \quad (871)$$

Equation 871 can model continued employment, continued institutional access, continued system operation, or continued ecological regime persistence.

A reference system can have survival function $S_{\text{ref}}(t)$, permitting distributional comparison.

14.30 Hazard of Restorative Failure

The instantaneous risk of losing the restored condition can be represented by a hazard function $h_R(t)$.

A restorative criterion can require

$$h_R(t) \leq h_{\text{ref}}(t) + \varepsilon_h. \quad (872)$$

Equation 872 can be stronger than current access equality because it compares future vulnerability.

14.31 Repeated-Harm Comparison

Some harms recur.

Let event times be

$$\{t_1, t_2, \dots, t_n\}. \quad (873)$$

A post-harm system can be evaluated by recurrence frequency, intensity, or inter-event time.

A successful intervention can reduce recurrence without reproducing the counterfactual no-harm system.

This is especially relevant to workplace retaliation, recurrent discrimination, repeated service denial, ecological disturbances, and institutional failure.

14.32 Recurrence Interval

Let

$$\Delta_i = t_{i+1} - t_i \quad (874)$$

denote the interval between harmful events.

Increasing recurrence intervals can indicate improvement even where the process has not disappeared completely.

The comparison remains weaker than elimination of recurrence.

14.33 Recurrence Probability

The probability of recurrence within horizon τ can be represented as

$$P_R(H \text{ recurs within } \tau). \quad (875)$$

A guarantees-of-non-repetition framework can therefore be interpreted partly through reduction of recurrence probability, while remaining broader than a statistical criterion [65].

14.34 Repeated Opportunity Structure

Some generative opportunities recur periodically.

Examples include annual hiring cycles, grant competitions, elections, school admissions, seasonal agricultural opportunities, and professional conferences.

Loss of one occurrence can be partly mitigated by later occurrences while remaining historically non-equivalent.

Let

$$\Omega = \{\omega_1, \omega_2, \dots\} \quad (876)$$

denote recurring opportunities.

The substitutability of ω_{i+1} for ω_i depends upon changes in age, network, competition, subject state, and surrounding history.

14.35 Periodic Restorative Comparison

Where systems are periodic, comparison can be made phase-by-phase.

For period T ,

$$Q_R(t + T) \approx Q_R(t) \quad (877)$$

and similarly for the reference.

Periodic equivalence can require preservation of phase structure rather than one-time equality.

This can be relevant to seasonal ecological and economic systems.

14.36 Moving-Window Comparison

A moving-window comparison evaluates recent performance continuously.

Let window length be w . Define

$$M_R(t; w) = \frac{1}{w} \int_{t-w}^t Q(\mathcal{G}_s^R) ds. \quad (878)$$

A reference moving-window statistic $M_{\text{ref}}(t; w)$ can be defined analogously.

Comparison can then require

$$d(M_R(t; w), M_{\text{ref}}(t; w)) \leq \varepsilon \quad (879)$$

for the relevant range of t .

Moving windows are useful where short-term volatility should be smoothed without ignoring long-term drift.

14.37 Rolling Restorative Assessment

A long assistance or rehabilitation program can reassess restorative status at regular intervals:

$$\mathcal{R}_{t_k} = \mathcal{A}(Q_{[t_{k-1}, t_k]}, B, E). \quad (880)$$

Equation 880 supports revisable intervention.

Rolling assessment should remain proportionate. It can become burdensome or surveillant if every dimension of life is continually measured.

14.38 Temporal Resolution

Restorative conclusions can depend upon sampling frequency.

Let observations occur at times

$$t_k = t_0 + k\Delta t. \quad (881)$$

A large Δt can miss short harmful episodes. A small Δt can create unnecessary surveillance and noise.

Temporal resolution is therefore an epistemic and procedural design choice.

14.39 Aliasing of Harm

When a system changes faster than the observation frequency, important variation can disappear from the recorded trajectory.

A subject observed only annually can appear fully restored while experiencing several shorter periods of exclusion during the year.

The restorative analogue of temporal aliasing therefore arises when the measurement schedule is too coarse for the process being evaluated.

The appropriate resolution should reflect the characteristic time scale of the relevant harm.

14.40 Characteristic Time Scale

Let

$$\tau_Q \tag{882}$$

denote the characteristic time scale of object Q .

For income, τ_Q can be monthly or annual. For acute health deterioration, it can be hours or days. For professional-network development, it can be years. For ecological regime change, it can span decades.

A temporal comparison should use a horizon and resolution appropriate to τ_Q .

14.41 Mismatch Between Institutional and Generative Time

Let τ_I denote institutional response time and τ_G the characteristic time scale on which generative loss develops.

The mismatch condition introduced earlier can be restated as

$$\tau_I \gg \tau_G. \tag{883}$$

Equation 883 identifies cases in which a procedurally legitimate institution is too slow to preserve the object being governed.

This is one reason interim mechanisms are necessary.

14.42 Multi-Time-Scale Generative Systems

Generative systems frequently operate at several time scales simultaneously.

A workplace conflict can alter daily stress, monthly output, annual career progression, and decade-scale professional identity.

A representation can decompose the system into components

$$x_t = \left(x_t^{\text{fast}}, x_t^{\text{medium}}, x_t^{\text{slow}} \right). \tag{884}$$

Equation 884 makes temporal heterogeneity explicit.

Restoration can occur rapidly in one component and slowly in another.

14.43 Fast-State and Slow-State Restoration

Suppose salary is restored immediately while trust and network position recover slowly.

Then

$$T_{\text{rec}}^{\text{salary}} \ll T_{\text{rec}}^{\text{trust}} \tag{885}$$

can hold.

Equation 885 shows why a single declaration date of “restoration” can be misleading.

A multi-object system can possess a vector of recovery times:

$$\mathbf{T}_{\text{rec}} = (T_1, \dots, T_n). \quad (886)$$

14.44 Temporal Coupling Across Objects

Recovery in one object can depend upon prior recovery in another.

Let

$$T_j = f(T_i, \mathcal{G}) \quad (887)$$

represent temporal dependence between restorative objects.

Housing stability can precede health rehabilitation. Formal correction can precede professional re-entry. Income support can precede retraining.

Temporal ordering and object dependency therefore interact.

14.45 Temporal Bottlenecks

The slowest critical component can determine system-level recovery.

Let C be the set of critical restorative objects. A simple bottleneck representation is

$$T_{\text{system}} = \max_{i \in C} T_i. \quad (888)$$

Equation 888 is schematic.

Some systems can function partially before every component recovers. The equation nevertheless captures the idea that one unresolved critical condition can delay effective system-level restoration.

14.46 Temporal Complementarity

Two interventions can be complementary only when temporally coordinated.

For interventions R_1 and R_2 , value can depend upon delay Δ :

$$V(R_1, R_2; \Delta). \quad (889)$$

Equation 889 can decline when one intervention arrives too late to support the other.

For example, retraining can be ineffective if income support begins only after the person has already lost housing.

14.47 Temporal Substitution

Some temporal losses can be partially substituted by later gains.

Let a later intervention R_{t_2} compensate for loss at t_1 .

Temporal substitutability can be represented by

$$Q_{\text{loss}}(t_1) \sim_{\text{sub}, \tau} Q_{\text{return}}(t_2). \quad (890)$$

Equation 890 is relation- and object-specific.

Financial loss can be relatively substitutable. A missed election, developmental period, or unique collaboration can be much less substitutable.

14.48 Temporal Fungibility

The degree to which value at one time can substitute for value at another can be represented through a temporal fungibility function

$$F_T(Q, t_1, t_2). \quad (891)$$

Equation 891 can vary by object.

Money can have relatively high temporal fungibility within moderate horizons. Health, trust, cultural transmission, and developmental opportunities can have lower temporal fungibility.

The concept helps explain why delay can alter the adequacy of compensation.

14.49 Temporal Singularity

The opposite case concerns temporally singular opportunities.

Let

$$S_T(Q, W) \quad (892)$$

denote the extent to which value of object Q depends upon historical window W .

High temporal singularity reduces the plausibility of later substitution.

This concept applies to:

- age-sensitive development;
- historical political events;
- unique research encounters;
- finite ecological windows;
- time-specific family relations.

14.50 Temporal Criticality

A period can also be critical because small intervention during that period has large downstream effects.

Let

$$\kappa_T(t) \quad (893)$$

represent temporal criticality.

A high- κ_T interval is one in which preservation or disruption of a condition has unusually large generative consequences.

Temporal criticality can arise through thresholds, developmental stages, network formation, institutional transitions, or path-dependent branching.

14.51 Critical Junctures

Political and historical analysis often uses the concept of critical junctures to describe periods during which choices can have enduring institutional consequences. Pierson's treatment of increasing returns and path dependence is particularly relevant to the importance of timing and sequence [45].

Within the present framework, a critical juncture can be represented as an interval W_c for which changes in transition structure have unusually large long-horizon effects.

Restoration after a critical juncture can therefore require more than return of the immediate pre-juncture state.

14.52 Temporal Depth of Harm

The *temporal depth* of a harm concerns how deeply its consequences have entered subsequent system evolution.

Let H occur at t_0 . A rough temporal-depth measure can depend upon the number of subsequent generative stages influenced by H :

$$D_H = \max \{n : H \rightsquigarrow Q_{t_n}\}, \quad (894)$$

where $\rightsquigarrow$ denotes relevant causal or generative influence.

Equation 894 is conceptual rather than a universal metric.

A deeply propagated harm can require broader restorative analysis than a shallow reversible loss.

14.53 Generational Depth

For intergenerational harms, depth can be measured by the number of generations through which consequences propagate.

Let generation index be g . A transmission process can be represented by

$$\mathcal{G}^{(g+1)} = \Phi^{(g)}(\mathcal{G}^{(g)}). \quad (895)$$

Equation 895 can contain wealth, education, institutional position, ecology, cultural knowledge, and other transmitted conditions.

Historical reparations literature directly confronts the temporal and intergenerational difficulties of responsibility and repair [67, 62, 12].

14.54 Intergenerational Reference

An intergenerational restorative claim can compare generation-indexed systems:

$$Q(\mathcal{G}^{R,(g)}) \ E \ Q(\mathcal{G}^{\text{ref},(g)}) \quad g = 0, \dots, G. \quad (896)$$

Equation 896 extends the horizon beyond the original claimant.

This raises additional normative questions concerning identity, representation, responsibility, and the interests of future persons.

14.55 Intergenerational Resource Comparison

A simple intergenerational object can be resource distribution:

$$R_R^{(g)} \ E \ R_{\text{ref}}^{(g)}. \quad (897)$$

This comparison can be useful and limited.

Intergenerational harm can propagate through institutions and relations even when resource levels converge later.

14.56 Intergenerational Capability Comparison

A capability-oriented intergenerational claim is

$$\mathcal{K}_R^{(g)} \ E \ \mathcal{K}_{\text{ref}}^{(g)}. \quad (898)$$

Equation 898 can capture differences in education, health, mobility, political participation, and future opportunity.

The reference becomes increasingly uncertain as g increases.

14.57 Intergenerational Generative Comparison

A deeper comparison concerns the mechanism transmitting generative conditions:

$$\Gamma_R^{(g)} \sim_E \Gamma_{\text{ref}}^{(g)}. \quad (899)$$

Equation 899 asks whether the process by which one generation produces conditions for the next has been restored or transformed.

This criterion can be more informative than equality of one generation's resources.

14.58 Lifecycle Horizons

Some restorative claims should be located within a human lifecycle rather than calendar time alone.

Let λ denote life stage. A lifecycle-indexed comparison is

$$Q(\mathcal{G}_\lambda^R) \sim_E Q(\mathcal{G}_\lambda^{\text{ref}}). \quad (900)$$

Equation 900 can distinguish childhood, education, early career, caregiving periods, retirement, or other stages.

The same delay can have different significance depending upon life stage.

14.59 Developmental Timing

Capabilities and preferences can develop differently depending upon when conditions become available.

A simple developmental model is

$$x_{\lambda+1} = F_\lambda(x_\lambda, c_\lambda). \quad (901)$$

Equation 901 makes stage dependence explicit.

Provision of c at stage $\lambda + 5$ need not reproduce its effect at stage λ .

This is one reason temporally delayed restitution can have limited developmental equivalence.

14.60 Career Horizons

Professional harms often operate over career stages.

Let

$$\lambda_{\text{career}} \in \{\text{entry, formation, consolidation, senior, transition}\}. \quad (902)$$

Equation 902 is illustrative.

Loss of one year at entry can have different generative consequences from loss of one year after professional consolidation because network formation, credential accumulation, and opportunity structures differ.

Career restoration should therefore avoid assuming temporal homogeneity.

14.61 Institutional Lifecycle Horizons

Organizations and communities also pass through stages.

A newly formed institution can be especially vulnerable to early disruption. A mature institution can possess redundancy unavailable at formation. A declining institution can have limited capacity to convert restored resources into durable future value.

Temporal horizon should therefore be indexed to institutional development where relevant.

14.62 Ecological Horizons

Ecological restoration can require substantially longer horizons than ordinary administrative processes.

A system can regain species counts quickly and remain dynamically altered. Soil, hydrology, succession, and resilience can develop over years or decades.

Holling's resilience framework provides an established reason to distinguish short-term return to observed state from persistence of system structure under disturbance [24].

Ecological restorative claims should therefore specify whether they concern:

- immediate composition;
- medium-term function;
- long-run regime stability;
- resilience under repeated disturbance.

14.63 Political Horizons

Political and legal restoration also operate across distinct horizons.

An election can restore representation immediately. Institutional trust can recover slowly. A constitutional reform can alter governance over decades. Historical reparations can change collective memory across generations.

A single political event can therefore satisfy an instantaneous institutional criterion and remain incomplete under longer relational or historical horizons.

14.64 Temporal Horizon and Reference Choice

Reference choice can change with horizon.

A short-horizon reference can be the pre-harm actual state. A medium-horizon reference can be the expected no-harm trajectory. A long-horizon reference can be a normative or subject-directed future.

Let

$$B = B(\tau). \quad (903)$$

Equation 903 represents horizon-dependent baseline selection.

This dependence should be explicit.

Otherwise an analysis can silently compare the short-term result with one reference and the long-term result with another.

14.65 Temporal Horizon and Object Choice

The comparison object can also change with horizon.

At t_0 , formal status can be the relevant object. Over one year, capability and network position can matter. Over ten years, trajectory, reachable set, and generative mechanism can become more informative.

Let

$$Q = Q(\tau). \quad (904)$$

Equation 904 formalizes this shift.

Temporal analysis can therefore require movement across atlas cells.

14.66 Temporal Horizon and Relation Choice

The comparative relation can similarly depend upon horizon.

Immediate return can use equality. Medium-term analysis can use metric proximity. Long-run analysis can use stability, convergence, or dominance.

Let

$$E = E(\tau). \quad (905)$$

Equation 905 shows that the temporal coordinate can reshape the whole comparative specification.

14.67 Temporal Horizon and Tolerance

Approximation tolerance can also vary with horizon.

Short-term volatility can justify wider temporary tolerance. Long-term institutional stabilization can justify stricter requirements.

Alternatively, uncertainty can increase with horizon and justify wider epistemic tolerance.

Let

$$\varepsilon = \varepsilon(\tau). \quad (906)$$

Equation 906 distinguishes normative tolerance from uncertainty-driven tolerance.

The reason for changing ε should be stated.

14.68 Temporal Horizon and Epistemic Uncertainty

Counterfactual uncertainty generally increases with temporal distance where branching possibilities accumulate.

Let $U(\tau)$ denote uncertainty concerning the reference trajectory. In many path-dependent systems,

$$U(\tau_2) \geq U(\tau_1) \quad \text{for } \tau_2 > \tau_1 \quad (907)$$

can occur, although monotonicity is not universal.

Equation 907 provides one reason long-horizon restoration should often shift from precise trajectory claims toward reachable sets, distributions, bounds, or structural comparison.

14.69 Temporal Horizon and Model Stability

A model valid over a short interval can become unreliable over longer horizons.

Let model error be

$$\epsilon_M(\tau). \quad (908)$$

Long-horizon claims should account for possible growth in $\epsilon_M(\tau)$.

This is particularly important where the intervention changes the dynamics being modeled.

14.70 Temporal Horizon and Reflexivity

Reflexive effects can accumulate through time.

A public classification changes behavior. The changed behavior produces new records. The records alter future decisions.

The number of feedback cycles increases with horizon.

Consequently, a long-horizon restorative model can require explicit treatment of the observer, record, and intervention as parts of the evolving system.

14.71 Temporal Horizon and Open-Ended Generation

Long horizons increase the probability that new states, technologies, relationships, or institutions emerge beyond the original state-space specification.

Let

$$\mathcal{X}_{t+\tau} = \mathcal{X}_t \cup \Delta\mathcal{X}_\tau. \quad (909)$$

Equation 909 makes long-horizon state-space expansion explicit.

A restorative comparison over sufficiently long periods can therefore require model revision rather than extrapolation within a fixed ontology.

14.72 Temporal Granularity and Open-Endedness

Long horizon and coarse temporal resolution are different.

A century-long history observed annually differs from a one-year process observed hourly.

The temporal specification should therefore distinguish:

$$\text{horizon} \neq \text{resolution}. \quad (910)$$

Equation 910 is elementary and important.

Long-horizon analysis can still require fine resolution during critical subperiods.

14.73 Adaptive Temporal Resolution

An efficient restorative architecture can use finer resolution during high-risk periods and coarser resolution during stable periods.

Let

$$\Delta t = \Delta t(t). \quad (911)$$

Equation 911 represents adaptive observation.

This can reduce data burden while preserving sensitivity to critical transitions.

Such monitoring remains subject to privacy and proportionality.

14.74 Event-Triggered Comparison

Temporal evaluation can be triggered by events rather than fixed dates.

Let event set $\mathcal{E}^*$ contain:

- change of supervisor;
- relocation;
- recurrence of symptoms;
- new disciplinary action;
- change of legal status;
- major ecological disturbance.

A restorative reassessment occurs when

$$e_t \in \mathcal{E}^*. \quad (912)$$

Equation 912 aligns institutional attention with changes likely to alter generative structure.

14.75 State-Triggered Comparison

Reassessment can also occur when a state variable crosses a threshold.

Let

$$Q(\mathcal{G}_t) \notin \mathcal{S}_{\text{acceptable}}. \quad (913)$$

Equation 913 can trigger review.

This architecture is useful for safety and resilience.

It should avoid reducing complex subjects to one score where the object is multidimensional.

14.76 Long-Run Restorative Comparison

A long-run comparison focuses on persistent or recurrent behavior after transients have diminished.

A generic long-run object can be represented by

$$Q_{\infty}(\mathcal{G}) = \lim_{T \rightarrow \infty} \mathcal{A}_T [Q(\mathcal{G}_{0:T})], \quad (914)$$

where $\mathcal{A}_T$ is an appropriate long-run aggregation operator.

The limit need not exist for every system.

Long-run comparison can instead use invariant measures, attractors, recurrence properties, or lim-sup/liminf structures.

14.77 Asymptotic State Equivalence

A simple asymptotic criterion is

$$\lim_{t \rightarrow \infty} d(Q(\mathcal{G}_t^R), Q(\mathcal{G}_t^{\text{ref}})) = 0. \quad (915)$$

Equation 915 permits temporary divergence followed by convergence.

It can be appropriate where long-run state matters and path burden is evaluated separately.

14.78 Asymptotic Equivalence and Historical Loss

Asymptotic convergence does not erase earlier loss.

A subject can eventually reach the same income while having lost years of consumption, professional experience, or family time.

The relation

$$\lim_{t \rightarrow \infty} d(Q_R(t), Q_{\text{ref}}(t)) = 0 \quad (916)$$

can therefore coexist with

$$\int_{t_0}^{\infty} d(Q_R(t), Q_{\text{ref}}(t)) dt > 0. \quad (917)$$

Equations 916 and 917 distinguish eventual convergence from historical equivalence.

14.79 Long-Run Distributional Equivalence

For stochastic systems, long-run comparison can concern stationary or invariant distributions.

Let

$$\pi_R \quad (918)$$

and

$$\pi_{\text{ref}} \quad (919)$$

be long-run distributions where meaningful.
A criterion is

$$\pi_R = \pi_{\text{ref}}. \quad (920)$$

Equation 920 can be useful for population or stochastic institutional systems.
It says little about which individual experienced which history.

14.80 Ergodic-Average Comparison

Where appropriate, one can compare time averages:

$$\bar{Q}_R = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T Q(\mathcal{G}_t^R) dt. \quad (921)$$

A corresponding reference average $\bar{Q}_{\text{ref}}$ can be defined.
Equality of long-run averages is substantially weaker than equality of trajectories.
This distinction is essential where distribution over time has independent normative importance.

14.81 Attractor-Horizon Comparison

Long horizons naturally lead to attractor analysis.

Let

$$\omega(x_0) \quad (922)$$

denote the omega-limit set of an initial state under the selected dynamics.
A restorative criterion can compare

$$\omega_R(x_R) \text{ } E \text{ } \omega_{\text{ref}}(x_{\text{ref}}). \quad (923)$$

Equation 923 compares long-run dynamical organization rather than finite-time state.
The detailed attractor and basin analysis appears in later sections.

14.82 Finite-Horizon and Asymptotic Conflict

An intervention can perform well asymptotically and poorly over a finite horizon.

A severe short-term intervention can move the system quickly toward a desirable long-run regime while imposing large immediate burdens.

Conversely, a comfortable short-term policy can preserve present output while allowing long-run fragility to increase.

Temporal justice therefore requires explicit trade-offs between transient and long-run performance.

14.83 Temporal Pareto Comparison

One intervention temporally dominates another where it is at least as good at every relevant time and strictly better at some time.

Let scalar value be V_t . Intervention R_1 temporally dominates R_2 when

$$V_t(R_1) \geq V_t(R_2) \quad \text{for every relevant } t \quad (924)$$

with strict inequality at least once.

Where such dominance exists, temporal trade-off is unnecessary.

In many cases, trajectories cross and no temporal dominance exists.

14.84 Trajectory Crossing

Two restorative policies can satisfy

$$V_{t_a}(R_1) > V_{t_a}(R_2) \quad (925)$$

and

$$V_{t_b}(R_1) < V_{t_b}(R_2) \quad (926)$$

for $t_a < t_b$.

Equations 925 and 926 create a temporal valuation problem.

A theory of justice must determine how present and future interests are related.

14.85 Temporal Lexical Priority

Some theories can assign lexical priority to avoidance of severe immediate harm before long-run optimization.

A schematic ordering is

$$\text{basic safety now} \succ_{\text{lex}} \text{future fidelity}. \quad (927)$$

Equation 927 is illustrative.

Other contexts can prioritize preservation of long-run ecological viability over small short-term gains.

The atlas itself does not select one temporal priority rule.

14.86 Temporal Rights Constraints

Rights can constrain how long a person may be subjected to provisional measures.

A temporary restriction can be permissible for duration τ_0 and become disproportionate when extended.

Let admissibility depend upon duration:

$$R \in \mathcal{A}(\tau). \quad (928)$$

Equation 928 formalizes temporal proportionality.

The same intervention can change normative status through duration alone.

14.87 Temporal Procedural Justice

Procedural justice has temporal structure.

Relevant dimensions include:

- timeliness of notice;
- duration before hearing;
- frequency of review;
- time allowed for response;
- duration of interim measures;
- delay in correction;
- timing of compensation;

- timing of public recognition.

A procedure can contain every formal safeguard and remain unjust through extreme delay. Temporal procedure is therefore an independent object of evaluation.

14.88 Delay to Recognition

Recognition received decades after harm can retain value and differ from timely recognition.

Let

$$T_{\text{recog}} = t_{\text{recognition}} - t_{\text{harm}}. \quad (929)$$

Equation 929 can enter evaluation of historical and institutional repair.

Long delay can affect reputation, trust, and whether the original subject remains able to receive the acknowledgment.

14.89 Delay to Compensation

Compensation can also lose practical value through delay.

Let amount be M and delay τ . Its generative effect can be represented as

$$G_M = G(M, \tau, \mathcal{K}_{t+\tau}), \quad (930)$$

where $\mathcal{K}_{t+\tau}$ is the conversion structure when payment is received.

Equation 930 shows why identical payment at two dates can generate different futures.

14.90 Delay to Correction

A false classification corrected later can have generated downstream uses during the delay.

Let propagation process be $P_z(t)$. Total exposure before correction can be represented by

$$E_z = \int_{t_0}^{t_{\text{corr}}} P_z(t) dt. \quad (931)$$

Equation 931 gives one reason correction speed can matter independently of eventual accuracy.

14.91 Timing of Apology and Satisfaction

Symbolic remedies also have temporal context.

An apology offered voluntarily before litigation can possess a different social meaning from an apology issued after decades of denial.

The material content can be identical while relational interpretation differs.

Time therefore enters meaning and recognition cells as well as material cells.

14.92 Temporal Structure of Trust Repair

Trust often requires repeated successful interaction.

Let

$$T_{t+1} = F_T(T_t, e_{t+1}), \quad (932)$$

where e_{t+1} denotes the next relational event.

Equation 932 illustrates why trust restoration can be temporally irreducible.

An institution can change a rule instantly. The trust consequences of that rule can emerge only through subsequent history.

14.93 Temporal Structure of Network Repair

Networks also rebuild through repeated interaction.

Let

$$\mathcal{R}_{t+1} = F_R(\mathcal{R}_t, E_{t+1}). \quad (933)$$

Equation 933 shows why network restoration can have a natural time scale longer than formal rein-statement.

A policy can facilitate interaction without manufacturing immediate relational identity.

14.94 Temporal Structure of Skill Recovery

Skill can decay during non-use and recover through practice.

Let

$$K_{t+1} = F_K(K_t, p_t), \quad (934)$$

where p_t denotes practice or use.

Equation 934 makes re-entry time-dependent.

Returning access without allowing time for skill recovery can misclassify temporary reduced performance as permanent incapacity.

14.95 Temporal Structure of Reputation Repair

Reputation is distributed across actors and can update at different rates.

For actor j ,

$$\rho_{j,t+1}^A = F_j(\rho_{j,t}^A, z_{t+1}, e_{t+1}), \quad (935)$$

where z_{t+1} can include correction and e_{t+1} subsequent behavior or information.

Equation 935 demonstrates why source-record correction and reputation restoration occupy different horizons.

14.96 Temporal Structure of Value Circulation

Value circulation unfolds recursively.

Let

$$c_t \rightarrow v_{t+1} \rightarrow c_{t+1} \rightarrow v_{t+2}. \quad (936)$$

Equation 936 shows why the timing of return affects later generativity.

A transfer arriving before the next generative cycle can participate in that cycle. The same transfer arriving afterward cannot retroactively enter it.

14.97 Missed Recursive Cycles

Suppose the reference system completes n generative cycles during the period of deprivation while the harmed system completes $m < n$.

The missed-cycle count is

$$N_{\text{miss}} = n - m. \quad (937)$$

Equation 937 is a crude measure.

Its conceptual value lies in making clear that later return of the initial input does not recreate the intermediate recursive stages.

14.98 Temporal Depth of Compensation

Compensation can target different levels of the temporal generative chain.

It can compensate:

- the original lost resource;
- first-order output;
- some subsequent lost earnings;
- transition costs;
- future opportunity effects.

The farther the compensation extends into counterfactual future development, the greater the epistemic uncertainty generally becomes.

A temporal architecture should therefore distinguish levels of compensatory depth.

14.99 Temporal Bounds on Counterfactual Claims

Where long-horizon point estimates are weak, a restorative theory can use time-dependent bounds.

Let

$$Q_{\min}(\tau) \leq Q^{\text{cf}}(\tau) \leq Q_{\max}(\tau). \quad (938)$$

Equation 938 can widen with horizon.

The widening of bounds is itself informative.

It indicates where the theory should shift from precise counterfactual reconstruction toward robust or structural claims.

14.100 Temporal Robustness Across Counterfactual Models

Let $\mathfrak{M}$ be a family of plausible counterfactual models. A time-indexed robust claim can require

$$Q_R(\tau) \ E \ Q_{\text{ref}}^M(\tau) \quad \text{for every } M \in \mathfrak{M}. \quad (939)$$

Equation 939 can hold over short horizons and fail at longer horizons as the models diverge.

This pattern can guide institutional confidence.

14.101 Temporal Reference Robustness

A similar analysis can test several reference systems over time.

Let $\mathfrak{B}$ be the plausible reference family. A restorative policy is temporally reference-robust where

$$Q_R(t) \ E \ Q_B(t) \quad \text{for every } B \in \mathfrak{B} \text{ and relevant } t. \quad (940)$$

Equation 940 is demanding and can identify strong common interventions.

14.102 Temporal Minimum Common Restoration

Where models and references diverge over the long run, one can identify a finite horizon over which common restorative claims remain supported.

Let

$$\tau^* = \sup \{ \tau : \text{the selected claim remains robust across the plausible family} \}. \quad (941)$$

Equation 941 defines a maximum robust horizon.

Beyond τ^* , institutions can continue assistance while reducing the strength of counterfactual claims.

14.103 Temporal Modesty

The concept of a maximum robust horizon supports temporal epistemic humility.

An institution can say:

The evidence supports this restorative comparison for the next two years; longer career effects remain substantially uncertain.

Such a statement can be more accurate than presenting a twenty-year counterfactual forecast with false precision.

14.104 Horizon-Specific Restoration Profiles

The restorative profile introduced earlier can be indexed by horizon:

$$\mathbf{R}(\tau) = (r_1(\tau), r_2(\tau), \dots, r_n(\tau)). \quad (942)$$

Equation 942 can reveal that resource restoration is immediate, capability restoration is medium-term, and relational restoration remains weak over the long term.

A single scalar completion date cannot represent this structure.

14.105 Temporal Restoration Surface

For multiple objects and time, restorative status can be represented as a surface:

$$R : \Omega \times \mathbb{T} \longrightarrow \mathcal{Y}, \quad (943)$$

where $\mathcal{Y}$ contains the restorative status or distance for each object and time.

Equation 943 provides a richer representation than one completion percentage.

The surface can contain recovery, relapse, divergence, and stabilization.

14.106 Temporal Restoration Frontier

Under limited resources, increasing the speed of restoration in one object can reduce the resources available to another.

Let

$$\mathcal{F}_T \quad (944)$$

denote the feasible frontier of recovery-time combinations across objects.

A policy can then face trade-offs between rapid income restoration, slower network rebuilding, health rehabilitation, and institutional reform.

This problem belongs to later distributive and normative analysis.

14.107 Temporal Justice Under Scarcity

Scarcity produces temporal prioritization problems.

When every restorative intervention cannot occur immediately, institutions must decide sequencing.

Possible criteria include:

- urgency;
- irreversibility;
- severity;

- dependency structure;
- expected generative amplification;
- rights;
- vulnerability;
- feasibility.

No one criterion is universally sufficient.

The temporal atlas makes the sequencing problem visible before normative selection.

14.108 Urgency

Let urgency of object Q_i be represented by

$$U_i = U \left(\frac{dL_i}{dt}, R_i, S_i \right), \quad (945)$$

where dL_i/dt represents the rate of additional loss, R_i reparability, and S_i severity.

Equation 945 is schematic.

Urgency becomes high where additional delay rapidly generates large and weakly reparable loss.

14.109 Temporal Priority and Reparability

Where two harms have equal current severity, lower reparability can increase the importance of earlier intervention.

This relation is consistent with the preventive principle developed in Section 5.44.

Temporal prioritization should nevertheless remain constrained by rights and proportionality.

14.110 Temporal Priority and Recursive Amplification

A rapidly self-amplifying harm can justify priority even when the initial loss is modest.

Let growth of loss be

$$\frac{dL}{dt} = g(L, t). \quad (946)$$

Where g creates strong positive feedback, delay increases future burden nonlinearly.

Interruption can therefore have value disproportionate to the current-state difference.

14.111 Temporal Priority and Dependency

An object that enables several later remedies can deserve earlier attention.

If restoration of Q_1 is a prerequisite for $Q_2, Q_3, \dots, Q_k$, then delaying Q_1 delays the whole restorative sequence.

The dependency graph introduced in Section 11.173 therefore has a temporal interpretation.

14.112 Temporal Closure

A temporal restorative inquiry eventually requires closure.

Closure can occur at horizon τ_c even when later consequences remain possible.

Let

$$\text{Close}(Q, E, \tau_c) \quad (947)$$

denote institutional closure of the comparison.

Closure should specify whether it applies to:

- legal liability;
- compensation amount;
- rehabilitation support;
- institutional monitoring;
- historical research.

These functions can have different closure dates.

14.113 Temporal Reopening

A closed comparison can be reopened after a triggering event or new evidence.

Let

$$\text{Reopen}(Q, E \mid D_{\text{new}}) \quad (948)$$

represent such reconsideration.

Reopening protects against premature finality. Unlimited reopening can undermine security.

The balance belongs to institutional design.

14.114 Temporal Finality and Generative Openness

Institutional finality and generative openness should therefore be separated.

A damages judgment can become final. The subject's future remains open. A reparations program can complete payments. Collective historical interpretation can continue. A rehabilitation phase can end. Later social protection can remain available.

Temporal closure should apply to bounded institutional questions rather than to the claim that history itself has ended.

14.115 Temporal Equivalence and Subject Adaptation

The present subject changes during long restorative processes.

Let subject state evolve as

$$A_{t+1} = F_A(A_t, R_t, E_t). \quad (949)$$

Equation 949 means that the target selected at the beginning of restoration can become inappropriate later.

Long-horizon restorative plans therefore require revisability.

14.116 Reference Drift

The reference system can also change.

A subject-directed target can evolve. The labor market can change. A clinical standard can improve. A community's political goals can be revised.

Let

$$B_{t+1} = \mathcal{U} (B_t, D_{t+1}, P_{A,t+1}, V_{t+1}) . \quad (950)$$

Equation 950 formalizes controlled reference revision.

Reference drift should preserve provenance to prevent opportunistic reduction of institutional obligations.

14.117 Target Drift

A distinction should be drawn between legitimate reference revision and institutional target drift.

Target drift occurs when an institution changes the success criterion after failure to meet the original criterion without adequate justification.

A transparent system records:

- the original target;
- the new target;
- the evidence prompting revision;
- the normative reason for revision;
- the consequences for prior claims.

This protects the temporal integrity of restorative evaluation.

14.118 Temporal Reflexivity of Restoration

Restorative interventions alter future temporal dynamics.

A compensation payment can accelerate transition. An apology can alter future trust. A surveillance regime introduced to monitor recovery can change behavior. A re-entry program can create new professional relations.

The post-restoration trajectory is therefore

$$\gamma^{H,R} \quad (951)$$

rather than a delayed copy of γ^{-H} .

The longer the horizon, the more significant these new interactions can become.

14.119 Temporal Non-Closure

The reflexive structure creates temporal non-closure.

At time t_1 , an institution evaluates restoration and produces record Z_{t_1} .

That record influences the system at t_2 .

A later evaluation must therefore evaluate a system containing the earlier evaluation.

The sequence is represented by

$$\mathcal{G}_{t_1} \rightarrow Z_{t_1} \rightarrow \mathcal{G}_{t_2} \rightarrow Z_{t_2} \rightarrow \cdots . \quad (952)$$

Equation 952 connects the temporal atlas with the reflexive non-closure developed in Section 8.35.

14.120 Temporal Comparison of Records

Records themselves can be compared through time.

A valid historical record should distinguish:

- claim time;
- decision time;
- correction time;
- supersession time;
- downstream-use time.

Collapsing these dates can convert a historically contingent classification into an apparently timeless fact.

Temporal provenance is therefore essential to restorative knowledge.

14.121 Temporal Status of Superseded Claims

A claim can be validly recorded as having existed at t_0 and invalid as a current classification at t_1 .

The distinction can be represented by

$$z \in \mathcal{H}_{t_0} \quad \text{and} \quad z \notin \mathcal{A}_{t_1}^{\text{current}}, \quad (953)$$

where $\mathcal{A}_{t_1}^{\text{current}}$ denotes currently active classifications.

Equation 953 allows historical preservation without present operational validity.

14.122 Temporal Granularity of Responsibility

Responsibility can change through the timeline.

One actor initiates harm. Another actor permits continuation. A third actor delays correction. A fourth actor controls post-harm assistance.

The temporal architecture can therefore distinguish responsibility for:

- initiation;
- continuation;
- failure to interrupt;
- delayed correction;
- defective restoration;
- recurrence.

The same history can therefore contain several temporally differentiated forms of responsibility.

14.123 Temporal Granularity of Justice

Justice can also be evaluated stage by stage.

Let

$$J_t = J(\mathcal{G}_t, R_t, P_t) \quad (954)$$

denote the normative evaluation at time t .

A process can become more just over time without making earlier injustice disappear.

Temporal improvement and retrospective evaluation therefore remain distinct.

14.124 Retrospective and Prospective Temporal Evaluation

A retrospective evaluation asks what should have happened earlier.

A prospective evaluation asks what should happen now.

Let

$$J_{\text{retro}} \quad (955)$$

and

$$J_{\text{pros}} \quad (956)$$

denote the two orientations.

They can recommend different actions.

Retrospective justice can identify reinstatement as the appropriate remedy that should have occurred at t_1 . Years later, prospective justice can support a different subject-directed future.

This distinction is central to historically changed subjects.

14.125 Temporal Conflict Between Retrospective and Prospective Justice

Suppose historical correction favors trajectory A , while the present subject now prefers trajectory B .

Then

$$R_{\text{retro}} \neq R_{\text{pros}}. \quad (957)$$

Equation 957 does not imply that one orientation is mistaken.

The retrospective reference can remain relevant to compensation and acknowledgment while the prospective reference governs future assistance.

14.126 Temporal Pluralism

A complex restorative case can therefore require several horizons simultaneously.

Let

$$\mathfrak{T}_R = \{ \tau_{\text{now}}, \tau_{\text{short}}, \tau_{\text{medium}}, \tau_{\text{long}}, \tau_{\text{intergen}} \}. \quad (958)$$

Equation 958 defines a temporal horizon family.

Different objects can be associated with different elements of $\mathfrak{T}_R$.

This is *temporal pluralism*.

14.127 Horizon Bundles

A multi-object restorative claim can assign one horizon to each object:

$$\tau = (\tau_1, \tau_2, \dots, \tau_n). \quad (959)$$

Equation 959 can represent:

- immediate salary correction;
- six-month rehabilitation;
- three-year professional re-entry;
- ten-year career impact;
- intergenerational institutional reform.

The resulting profile is temporally heterogeneous by design.

14.128 Horizon Conflict

Interventions optimized for one horizon can be poor at another.

Let R_1 maximize short-term outcome and R_2 maximize long-run generativity.

It is possible that

$$V_{\text{short}}(R_1) > V_{\text{short}}(R_2) \quad (960)$$

and

$$V_{\text{long}}(R_1) < V_{\text{long}}(R_2). \quad (961)$$

Equations 960 and 961 produce temporal conflict.

The normative theory must determine how horizons are weighted or prioritized.

14.129 Horizon Dominance

Where one intervention performs at least as well across all selected horizons, temporal dominance exists.

Let $\mathfrak{T}$ be the selected horizon family. Intervention R_1 dominates R_2 when

$$V_{\tau}(R_1) \geq V_{\tau}(R_2) \quad \text{for every } \tau \in \mathfrak{T} \quad (962)$$

with strict inequality for at least one τ .

Equation 962 provides one robust decision criterion.

14.130 Horizon Robustness

A restorative conclusion is horizon-robust if it remains valid across a selected range:

$$Q_R \geq Q_{\text{ref}} \quad \text{for every } \tau \in [\tau_{\min}, \tau_{\max}]. \quad (963)$$

Equation 963 is stronger than a point-horizon result.

Horizon robustness is particularly valuable where the institution does not know which future horizon will ultimately matter most.

14.131 Horizon Sensitivity

A restorative claim can also be tested for sensitivity to horizon.

Let outcome score be $R(\tau)$. Horizon sensitivity can be represented by

$$S_{\tau} = \frac{dR(\tau)}{d\tau} \quad (964)$$

where differentiability is meaningful.

A large $|S_{\tau}|$ indicates that the conclusion changes rapidly with horizon.

This suggests caution in statements of completion.

14.132 Temporal Bifurcation of Restorative Judgment

A small change in horizon can sometimes change the classification of a system.

For threshold criterion $R(\tau) \geq r_{\min}$, let τ_c satisfy

$$R(\tau_c) = r_{\min}. \quad (965)$$

The judgment can change across τ_c .

This is a temporal classification threshold rather than a dynamical bifurcation in the strict mathematical sense.

It illustrates why a declared horizon should accompany institutional restorative status.

14.133 Temporal Atlas Coordinate

The full atlas coordinate introduced in Equation 817 can now be refined by replacing the simple horizon parameter with a temporal specification:

$$\mathfrak{a}_T = (B, Q, E, \mathcal{T}, \mathcal{T}_H, \varepsilon, K, s, P), \quad (966)$$

where $\mathcal{T}_H$ is defined in Equation 831.

Equation 966 makes time a structured coordinate of the atlas.

14.134 Temporal Metadata

Every temporally consequential restorative claim should record:

- comparison start time;
- comparison horizon;
- observation resolution;
- relevant historical windows;
- aggregation rule;
- discounting or temporal weights where used;
- recurrence treatment;
- lag allowance;
- closure and review rules;
- long-run assumptions.

This metadata prevents apparently identical claims from concealing incompatible temporal assumptions.

14.135 Illustrative Workplace Horizon Profile

Consider a worker reinstated after wrongful exclusion.

At the moment of reinstatement:

- formal status can satisfy exact equality;
- salary can satisfy numerical equality;
- institutional access can be formally restored.

Over six months:

- work capacity can approach reference levels;
- trust can remain lower;
- network access can remain incomplete.

Over several years:

- career trajectory can converge;
- seniority loss can remain;
- the subject can choose a different professional direction.

The statement “employment has been restored” is therefore strongly horizon-dependent.

14.136 Illustrative Research Horizon Profile

Consider restoration of research access.

Instantaneously, laboratory access can be returned.

Over one project cycle, lost methodological practice can be recovered.

Over several publication cycles, collaborative network effects can change.

Over a decade, the no-harm publication trajectory becomes increasingly counterfactual and difficult to identify.

A sensible comparison can therefore move from resource identity at short horizon toward capability, reachability, and generative-process comparison at longer horizons.

14.137 Illustrative Health Horizon Profile

Health-related restoration similarly changes with horizon.

Immediate comparison can concern bodily state and symptom control.

Medium-term comparison can concern functional capacity and return to activity.

Long-term comparison can concern recurrence, resilience, adaptation, and subject-selected life plans.

The best-feasible reference can also change as the medical prognosis becomes clearer.

14.138 Illustrative Reputational Horizon Profile

A reputational correction can be instantaneous at the source record.

Downstream institutional correction can require weeks or months.

Professional network effects can persist for years.

Historical memory can remain indefinitely.

The appropriate restorative claim should therefore distinguish source-record correction, propagation correction, network repair, and historical recognizability across different horizons.

14.139 Illustrative Collective Horizon Profile

Collective historical repair can involve:

- immediate acknowledgment;
- medium-term compensation;
- institutional reform over several years;
- rebuilding community capacity across decades;
- intergenerational effects on education, wealth, ecology, and cultural transmission.

Large-scale reparations practice already reveals the importance of combining short-term measures with longer goals of recognition, civic trust, social solidarity, and institutional transformation [13, 12].

14.140 Temporal Limits of Restorative Measurement

Longitudinal measurement has practical and normative limits.

Subjects can change location. Institutions can disappear. Measurement definitions can change. Records can become incompatible. Long-term observation can become surveillance.

A restorative theory should therefore avoid assuming that every long-horizon object must be continuously measured.

Sampling, bounded review, subject consent, and institutional handoff can reduce measurement burden.

14.141 Temporal Limits of Counterfactual Precision

Long-horizon counterfactual claims are especially sensitive to contingency.

If several branches are plausible at each generative stage, the set of possible counterfactual histories can expand rapidly.

This does not make long-run justice impossible.

It shifts the appropriate object from a single precise path toward:

- distributions;
- reachable sets;
- robust bounds;
- structural properties;
- present needs;
- subject-directed futures.

Temporal uncertainty therefore changes the grammar of comparison.

14.142 Temporal Limits of Historical Reconstruction

The farther a restorative process is from the original harm, the more likely that:

- relevant actors have changed;
- records are incomplete;
- surrounding institutions have evolved;
- social meanings have changed;
- the subject has formed new commitments;
- third-party reliance interests have developed.

Long delay therefore changes both epistemic and normative structure.

Historical fidelity can become less feasible while present-oriented assistance and transformative governance become more important.

14.143 Temporal Limits of Literal Restitution

Literal restitution becomes weaker where time changes the surrounding system.

Returning land, status, employment, or institutional access decades later can remain important while failing to reproduce the earlier generative environment.

The object can be restituted at t_1 and remain temporally non-equivalent to possession during $[t_0, t_1]$.

This temporal difference should remain visible in institutional language.

14.144 Temporal Limits of Compensation

Compensation can also lose equivalence through delay.

A payment can be increased to reflect lost income or interest.

Some generative effects remain non-fungible because delayed money cannot recreate:

- elapsed developmental stages;
- historical network formation;
- lost political participation;
- missed biological opportunities;
- unique relational events.

Temporal value therefore exceeds financial discounting.

14.145 Temporal Limits of Bisimulation

A restored system can be bisimilar to a reference from time t_1 onward while their histories before t_1 differ.

This distinction can be represented by

$$\mathfrak{T}_{R,t \geq t_1} \sim_{\text{bis}} \mathfrak{T}_{\text{ref},t \geq t_1} \quad (967)$$

and

$$\mathcal{H}_{t < t_1}^R \neq \mathcal{H}_{t < t_1}^{\text{ref}}. \quad (968)$$

Equations 967 and 968 show that future behavioral fidelity does not imply historical identity.

This is a particularly clean formal illustration of the paper's broader argument.

14.146 Temporal Limits of Topological Conjugacy

Similarly, two post-restoration dynamical systems can become topologically conjugate from a selected time onward.

This establishes qualitative dynamical correspondence.

It does not reconstruct earlier trajectories or meanings generated by the history.

Formal dynamical equivalence and historical equivalence therefore occupy different temporal objects.

14.147 Temporal Limits of Statistical Indistinguishability

Statistical indistinguishability can also be horizon-relative.

Two systems can be indistinguishable over one year and distinguishable over ten years.

Let test family depend upon horizon:

$$\mathcal{T}_{\text{stat}} = \mathcal{T}_{\text{stat}}(\tau). \quad (969)$$

Longer observation can reveal slower processes absent from short-term data.

A statistical restoration claim should therefore specify both test family and horizon.

14.148 Temporal Limits of Interventional Equivalence

The same intervention can produce similar immediate response and different long-run response.
A stronger criterion therefore requires

$$P_R(Y_{t+\tau} \mid do(u)) \sim P_{\text{ref}}(Y_{t+\tau} \mid do(u)) \quad (970)$$

for the relevant family of τ .

Equation 970 extends causal comparison into the temporal domain.

14.149 Temporal Limits of Generative-Process Equivalence

A restored generative mechanism can match the reference for one cycle and diverge recursively later.
Let

$$\Gamma_R^{(n)} \sim_E \Gamma_{\text{ref}}^{(n)} \quad (971)$$

denote equivalence through n recursive generations.

Increasing n strengthens the criterion.

One-step generation equality therefore cannot establish long-run recursive equivalence.

14.150 Generative Horizon

The *generative horizon* of a restorative claim is the depth of future recursive generation over which correspondence is asserted.

Let

$$H_G = n \quad (972)$$

for discrete recursive systems, or $H_G = \tau$ for continuous-time systems.

A compensation can restore $H_G = 0$ resource state while leaving larger generative horizons unresolved.

This terminology distinguishes immediate return from deeper generative restoration.

14.151 Generative-Horizon Robustness

A restorative intervention is generatively robust through horizon H_G where

$$\Gamma_R^{(k)} \sim_E \Gamma_{\text{ref}}^{(k)} \quad k = 1, \dots, H_G. \quad (973)$$

Equation 973 creates a graded notion of recursive restoration.

Its epistemic requirements increase with H_G .

14.152 Temporal Horizon and Normative Salience

The normative importance of an object can itself vary with time.

Immediate safety can dominate short-term evaluation. Autonomy can become central after emergency stabilization. Network rebuilding can matter over medium horizons. Intergenerational ecology can dominate long-term policy.

Let

$$\text{Sal}_V(Q, \tau) \quad (974)$$

denote time-indexed normative salience.

Equation 974 makes clear that a fixed multi-object weighting can be inappropriate across all horizons.

14.153 Temporal Horizon and Institutional Competence

Institutions also operate over different horizons.

Organizations can act immediately. Courts can determine responsibility over longer periods. Social-protection systems can maintain continuity. Regulators can alter medium-term structure. Historical institutions can preserve memory over generations.

Let

$$\text{Fit}(I, Q, E, \tau) \quad (975)$$

denote institutional fit including horizon.

An institution can be well suited to one cell over one horizon and poorly suited to the same cell over another.

14.154 Temporal Division of Institutional Labor

A coherent system can therefore distribute temporal functions.

For example:

- emergency organization: hours to days;
- administrative review: days to months;
- litigation: months to years;
- rehabilitation: months to years;
- professional re-entry: years;
- institutional reform: years to decades;
- historical memory: intergenerational.

The durations are illustrative rather than universal.

The point is structural: different institutions govern different temporal segments of justice.

14.155 Temporal Interfaces

Handoffs among institutions require temporal continuity.

A subject should not lose essential support during the interval between institutions.

Let I_i and I_j have service intervals $[t_a, t_b]$ and $[t_c, t_d]$.

A gap exists where

$$t_c > t_b \quad (976)$$

and no bridge support covers (t_b, t_c) .

Equation 976 formalizes one type of institutional handoff failure.

14.156 Temporal Overlap

Institutional intervals can also overlap.

Overlap can provide redundancy and create conflicting obligations.

Let

$$[t_a, t_b] \cap [t_c, t_d] \neq \emptyset. \quad (977)$$

Equation 977 identifies the interval in which coordination is required.

14.157 Temporal Dependency in Institutional Handoffs

One institution can require completion of another process before acting.

Where this dependency creates excessive delay, the architecture can become generatively harmful.

Parallel processing can reduce such temporal dependency where legal and institutional constraints permit.

14.158 Temporal Fairness

Temporal justice is partly distributive.

Different claimants can wait different periods for comparable remedies.

Let waiting time be W_i .

A system can therefore be evaluated through distribution

$$P(W). \quad (978)$$

Large systematic differences in waiting time can produce unequal generative effects even where nominal remedies are equal.

14.159 Queue Position and Generative Consequence

Administrative queues can therefore become substantive.

A person earlier in a queue can preserve housing, employment, or health that a later claimant loses while waiting.

Temporal equality of institutional process can require more than identical formal procedure.

It can require attention to the consequences of waiting.

14.160 Priority Queues

Institutions can prioritize cases by urgency, irreversibility, or vulnerability.

A priority function can be represented as

$$\Pi_i = \Pi(U_i, R_i, S_i, D_i), \quad (979)$$

where U_i denotes urgency, R_i reparability, S_i severity, and D_i dependency.

Equation 979 is illustrative.

Priority rules require procedural justification because they distribute scarce temporal attention.

14.161 Temporal Equality and Temporal Equity

Equal waiting time and equitable waiting time are distinct.

Identical processing time can have unequal consequences for subjects facing different rates of deterioration.

A generative approach can therefore support differentiated urgency while preserving procedural fairness.

The normative theory must determine when unequal timing is justified.

14.162 Intergenerational Temporal Equity

Long-horizon repair also distributes burdens across generations.

Current generations can bear costs of historical reparations. Future generations can receive benefits from institutional reform. Environmental restoration can impose current restraint for future resilience.

The temporal distribution of burden is therefore a distinct justice problem.

The atlas identifies the time structure; later value theory determines the appropriate distribution.

14.163 Intergenerational Discounting and Justice

Discounting future generations raises especially strong normative questions.

A high discount rate can make distant ecological or institutional effects appear small.

A zero discount rate can create other difficulties where future population and uncertainty are large.

The present paper does not adopt one universal intergenerational discount rule.

The relevant contribution is to require explicit temporal weighting rather than allow it to remain hidden.

14.164 Temporal Interpretation of Prevention

Prevention can be represented as preservation of future sets over a horizon.

Let

$$\mathcal{R}_\tau^P \tag{980}$$

denote the reachable set after preventive intervention.

A preventive policy can seek

$$\mathcal{R}_\tau^{\text{safe}} \subseteq \mathcal{R}_\tau^P \tag{981}$$

while avoiding unjustified contraction of legitimate options.

The horizon matters because preventive burdens and benefits can occur at different times.

14.165 Temporal Interpretation of Interruption

Interruption acts upon the short-run propagation rate.

Let harmful-state variable be H_t . A successful interruption can require

$$\frac{dH_t}{dt} \leq 0 \tag{982}$$

under an appropriately defined scalar or ordered harm measure.

Equation 982 is schematic.

The purpose is to stop further generation before complete restoration is possible.

14.166 Temporal Interpretation of Assistance

Assistance is future-oriented.

Let viable target set be $\mathcal{T}_A^{\text{adm}}$.

Assistance can be evaluated by whether there exists a future time $t + \tau$ such that

$$\mathcal{R}_\tau(\mathcal{G}_t^{HA}) \cap \mathcal{T}_A^{\text{adm}} \neq \emptyset. \tag{983}$$

Equation 983 makes the temporal orientation of generative assistance explicit.

14.167 Temporal Interpretation of Transformation

Transformative justice can require transition toward a new structure over a finite reform horizon.

Let normative target region be $\mathcal{N}^*$.

A transformative policy can require

$$\exists \tau \leq \tau_{\max} : \mathcal{G}_{t+\tau}^R \in \mathcal{N}^*. \tag{984}$$

Equation 984 permits gradual transition.

The path remains subject to procedural justice.

14.168 Temporal Interpretation of Guarantees of Non-Repetition

Guarantees of non-repetition possess inherently temporal content.

Their significance lies in future risk reduction rather than replication of the past.

Let recurrence probability over horizon τ be $p_H(\tau)$.

A non-repetition measure can seek

$$p_H^R(\tau) < p_H^{\text{pre-reform}}(\tau). \quad (985)$$

Equation 985 provides one partial formalization of a broader institutional concept recognized in reparations practice [65].

14.169 Temporal Comparison of Institutional Learning

Institutional learning unfolds across cases and time.

Let policy state be Π_t . Learning can be represented by

$$\Pi_{t+1} = L(\Pi_t, D_t), \quad (986)$$

where D_t contains experience from prior cases.

A restorative institution can therefore be evaluated not only by one case outcome but by whether future cases improve.

14.170 Learning Horizon

The relevant horizon for institutional learning can exceed the horizon of the individual claim.

A compensation case can close in one year while institutional prevention is evaluated over ten years.

This difference reinforces the need for function-specific closure.

14.171 Temporal Externalities

Restorative interventions can create benefits or burdens that appear later.

Let

$$\epsilon_{ij}(t + \tau) \quad (987)$$

denote the effect of intervention for subject i on subject j at future horizon τ .

Third-party evaluation should therefore include delayed externalities where material.

14.172 Temporal Spillover

A policy can shift cost across time rather than eliminate it.

For example, intensive short-term compensation without structural reform can reduce immediate burden while allowing future recurrence.

The distinction between cost elimination and temporal displacement should therefore remain explicit.

14.173 Temporal Burden Shifting

Let total burden be B_t . A policy can satisfy

$$B_{t_1}^R < B_{t_1}^{\text{ref}} \quad (988)$$

while

$$B_{t_2}^R > B_{t_2}^{\text{ref}} \quad (989)$$

for $t_2 > t_1$.

Equations 988 and 989 identify temporal burden shifting.

This phenomenon is relevant to fiscal, ecological, health, and organizational restoration.

14.174 Temporal Moral Hazard

A remedy design can alter future incentives.

Where compensation is guaranteed without preventive reform, responsible actors can underinvest in prevention.

Where assistance is highly conditional, recipients can avoid experimentation for fear of losing support.

These effects unfold temporally and belong to the generative consequences of restorative design.

14.175 Temporal Commitment

Some restorative arrangements require credible long-term commitment.

A one-year promise of access can be insufficient for a five-year research project.

Let required project horizon be τ_P and guaranteed access horizon τ_A .

A basic adequacy condition is

$$\tau_A \geq \tau_P. \quad (990)$$

Equation 990 illustrates why duration can be a component of the generative condition itself.

14.176 Credible Commitment

Formal duration and credible duration differ.

A five-year promise that can be arbitrarily withdrawn can have lower generative significance than a secure three-year right.

Temporal security must therefore be combined with control and non-domination objects.

14.177 Temporal Security Profile

Let survival probability of access be

$$S_c(\tau) = P(c \text{ remains available through } \tau). \quad (991)$$

Equation 991 defines one temporal security profile.

The entire function $S_c(\tau)$ can be compared with the reference rather than current binary access.

14.178 Temporal Reachability

Reachability is inherently horizon-dependent.

Let

$$\mathcal{R}_{\tau_1}(x) \subseteq \mathcal{R}_{\tau_2}(x) \quad (992)$$

for $\tau_1 < \tau_2$ where additional time weakly expands reachable states.

This monotonicity need not hold under deadlines, aging, resource depletion, or irreversible windows.

Temporal reachability therefore requires care.

14.179 Deadline-Constrained Reachability

A state can be reachable eventually and unreachable before a relevant deadline.

Define

$$\mathcal{R}_{\leq \tau}(x) \quad (993)$$

as the states reachable within time τ .

A restoration can satisfy unrestricted reachability and fail deadline-constrained reachability.

This distinction is central to time-sensitive opportunities.

14.180 Time-to-Reach Distribution

For target region $\mathcal{A}$, define the time-to-reach random variable

$$T_{\mathcal{A}}. \quad (994)$$

A restored and reference system can possess the same eventual reachability and different distributions of $T_{\mathcal{A}}$.

A stronger restorative criterion can therefore compare

$$P_R(T_{\mathcal{A}} \leq \tau) \sim_E P_{\text{ref}}(T_{\mathcal{A}} \leq \tau). \quad (995)$$

14.181 Temporal Capability

A capability can depend upon the time required to exercise it.

An opportunity that becomes available only after an excessive delay can have reduced practical value.

Let capability k have availability time T_k . A temporal capability profile can be represented by

$$\mathcal{K}_T = \{(k, T_k)\}. \quad (996)$$

Equation 996 refines the ordinary capability set by adding temporal accessibility.

14.182 Temporal Agency

Agency also has temporal structure.

A choice available only after all meaningful alternatives have disappeared does not provide the same agency as timely choice.

Let decision time be t_D and option-expiration time be t_E .

Meaningful temporal agency can require

$$t_D < t_E. \quad (997)$$

Equation 997 is a simple formalization of timely choice.

14.183 Temporal Contestability

A right to appeal can become ineffective when the appeal is decided after the relevant opportunity disappears.

Let

$$T_{\text{appeal}} \quad (998)$$

denote appeal duration and T_{window} the remaining opportunity window.

Effective contestability can require

$$T_{\text{appeal}} < T_{\text{window}}. \quad (999)$$

Equation 999 shows why procedure and timing cannot be separated.

14.184 Temporal Recognition

Recognition can arrive too late to affect some relational consequences while remaining morally meaningful.

The temporal profile of recognition can therefore contain:

- preventive recognition;
- timely acknowledgment;
- delayed acknowledgment;
- posthumous recognition;
- intergenerational recognition.

These forms have different practical and symbolic effects.

14.185 Posthumous Restoration

Some restorative acts occur after the original subject's death.

Literal subject-directed assistance is unavailable. Other objects remain relevant:

- historical record;
- family or descendant interests;
- public recognition;
- property;
- institutional reform;
- prevention of recurrence.

Posthumous cases therefore demonstrate the temporal separability of restorative objects.

14.186 Restoration Across Institutional Generations

Institutions themselves can change so extensively that later bodies inherit responsibility or reparative tasks without institutional identity in a simple sense.

The temporal theory must therefore distinguish:

- continuity of legal personality;
- continuity of function;
- continuity of resources;
- continuity of historical responsibility;
- continuity of governance structure.

This is especially relevant to states, universities, religious institutions, corporations, and long-lived public bodies.

14.187 Temporal Identity of Collective Subjects

Collectives can persist through turnover of individual members.

Let collective at t_0 be C_{t_0} and at t_1 be C_{t_1} .

Questions of intertemporal identity determine whether duties, claims, or recognition can be transmitted.

The paper does not require one metaphysical theory of collective identity. The relevant institutional theory should state which continuity criteria it uses.

14.188 Temporal Attribution

Causal attribution also changes with horizon.

Immediate effects can be strongly attributable to the harmful intervention.

Later outcomes can involve increasing numbers of mediators, independent causes, and choices.

Let attributable fraction or confidence be represented by

$$A_H(\tau). \quad (1000)$$

Equation 1000 need not be monotonic, though attribution often becomes more difficult over long horizons.

The concept explains why legal compensation can use a shorter attribution horizon than philosophical analysis of generative loss.

14.189 Temporal Causal Chains

A harmful intervention can initiate a chain

$$H_{t_0} \rightarrow X_{t_1} \rightarrow X_{t_2} \rightarrow \dots \rightarrow X_{t_n}. \quad (1001)$$

Equation 1001 does not imply deterministic causation.

At each stage, other variables can enter.

Temporal depth and causal complexity therefore increase together.

14.190 Temporal Mediation

Later loss can be mediated through intermediate objects.

For example:

dismissal $\rightarrow$ income loss $\rightarrow$ relocation $\rightarrow$ network loss $\rightarrow$ career change.

The restorative relevance of each stage can differ.

A temporal causal model can identify intervention points at which later propagation could have been interrupted.

14.191 Temporal Counterfactual Branching

At each stage, several counterfactual transitions can be possible.

Let branching factor at stage k be b_k .

The number of possible histories can grow schematically as

$$N_{\text{hist}} \approx \prod_{k=1}^n b_k. \quad (1002)$$

Equation 1002 is illustrative rather than a claim that social possibilities are countable or independent. Its purpose is to show why long-horizon unique-trajectory claims become epistemically fragile.

14.192 Temporal Coarsening of Counterfactual Objects

As horizon increases, comparison can move toward coarser but more robust objects.

A possible progression is

$$\text{specific state} \rightarrow \text{trajectory distribution} \rightarrow \text{reachable set} \rightarrow \text{structural invariant.} \quad (1003)$$

Equation 1003 is not mandatory.

It illustrates an epistemically conservative strategy for long-horizon restorative analysis.

14.193 Temporal Refinement of Observed History

The opposite can occur for actual history.

As additional records become available, the representation of the actual past can become finer.

Thus counterfactual uncertainty can expand with horizon while historical knowledge can improve with archival discovery.

These two epistemic movements should not be conflated.

14.194 Temporal Comparison Under Structural Change

A reference and restored system can operate under different institutional rules at later times.

Let

$$\Phi_{\text{ref},t} \neq \Phi_{R,t}. \quad (1004)$$

The comparison can therefore require time-varying transition systems.

A static equivalence relation can be insufficient.

This is especially relevant to long-term institutional transformation.

14.195 Nonstationary Restorative Systems

Many social systems are nonstationary.

A parameter can evolve:

$$\theta = \theta(t). \quad (1005)$$

A restorative criterion should therefore avoid assuming a stationary distribution or fixed transition law unless justified.

Long-horizon social restoration often requires explicitly nonstationary models.

14.196 Regime-Specific Horizons

A system can pass through different regimes.

Let regime variable be

$$z_t \in \{1, \dots, m\}. \quad (1006)$$

Restorative equivalence can hold within one regime and fail across regime transition.

The relevant horizon should therefore identify whether regime changes are inside the comparison interval.

14.197 Critical Transition Timing

The timing of a critical transition can itself be a comparison object.

Let

$$T_{\text{crit},R} \tag{1007}$$

and

$$T_{\text{crit},\text{ref}} \tag{1008}$$

denote transition times.

A restorative intervention can preserve long-run regime while changing the time at which the transition occurs.

This difference can matter substantially in ecological, economic, health, or institutional systems.

14.198 Temporal Resilience

Resilience has temporal dimensions including:

- time to absorb disturbance;
- time to recover;
- duration before regime shift;
- persistence under repeated shocks;
- recovery between shocks.

Holling's distinction between stability and resilience motivates preserving these dimensions separately [24].

The next dynamical sections will formalize these structures in greater depth.

14.199 Repeated Perturbation Horizons

A system can recover from one shock and fail under repeated shocks.

Let perturbations occur at

$$\delta_1, \delta_2, \dots, \delta_n. \tag{1009}$$

A repeated-perturbation restorative criterion can compare

$$\mathcal{Q}(\mathcal{G}_R; \delta_{1:n}) \ E \ \mathcal{Q}(\mathcal{G}_{\text{ref}}; \delta_{1:n}). \tag{1010}$$

Equation 1010 is stronger than one-shock robustness.

It can reveal cumulative fragility.

14.200 Inter-Shock Recovery

The time between disturbances can be crucial.

Let shock interval be Δ_s and recovery time T_{rec} .

Where

$$\Delta_s < T_{\text{rec}}, \tag{1011}$$

the system experiences the next shock before completing recovery.

A restored system can therefore appear resilient under isolated testing and fragile under realistic repeated exposure.

14.201 Temporal Test Classes

The test class $\mathcal{T}$ introduced in the atlas should therefore include temporal structure.

Let

$$\mathcal{T}_\tau = \{T : T \text{ acts over horizon } \tau\}. \quad (1012)$$

A system can be equivalent under $\mathcal{T}_{\tau_1}$ and distinguishable under $\mathcal{T}_{\tau_2}$.

The strength of the restorative claim grows with the richness of relevant temporal tests.

14.202 Temporal Test-Class Refinement

Let

$$\mathcal{T}_1 \subseteq \mathcal{T}_2. \quad (1013)$$

Equivalence under the richer class $\mathcal{T}_2$ can imply equivalence under $\mathcal{T}_1$.

Longer or more diverse temporal testing therefore creates a form of comparative refinement.

The epistemic and procedural cost also increases.

14.203 Temporal Approximation

A time-varying tolerance can be represented by

$$d(Q_R(t), Q_{\text{ref}}(t)) \leq \varepsilon(t). \quad (1014)$$

Equation 1014 allows larger temporary deviation during transition and stricter long-term requirements.

The shape of $\varepsilon(t)$ requires substantive justification.

14.204 Convergence Envelope

A restorative policy can be required to remain inside a shrinking error envelope:

$$d(Q_R(t), Q_{\text{ref}}(t)) \leq \varepsilon_0 e^{-\lambda t}. \quad (1015)$$

Equation 1015 is an idealized example.

It can represent a planned convergence path.

Human and social restoration need not follow exponential dynamics. The equation illustrates how temporal expectations can be specified formally.

14.205 Non-Monotonic Recovery

Recovery need not be monotonic.

The restorative distance

$$d_t = d(Q_R(t), Q_{\text{ref}}(t)) \quad (1016)$$

can decrease, increase, and decrease again.

A single early measurement can therefore misclassify the process.

Non-monotonicity is common where new environments, stressors, or adaptations emerge.

14.206 Relapse

A relapse occurs when a system enters an acceptable region and later exits it.

Let $t_a < t_b < t_c$ satisfy

$$Q_R(t_a) \notin \mathcal{A}, \quad Q_R(t_b) \in \mathcal{A}, \quad Q_R(t_c) \notin \mathcal{A}. \quad (1017)$$

Equation 1017 distinguishes first return from stable return.

This distinction applies to health, employment, institutional trust, and ecological systems.

14.207 Durable Restoration

A durable restoration remains within the accepted relation for a specified minimum duration τ_D :

$$Q_R(t) \in Q_{\text{ref}}(t) \quad \forall t \in [t_0, t_0 + \tau_D]. \quad (1018)$$

Equation 1018 is stronger than instantaneous restoration and weaker than indefinite equivalence.

Durability requirements should match the object and institutional purpose.

14.208 Sustained Generativity

A future-directed generative criterion can require that a restored system continue generating viable future conditions across multiple cycles.

Let

$$G_t = G(\mathcal{G}_t). \quad (1019)$$

Sustained generativity can require

$$G_t \geq G_{\min} \quad t \in [t_0, t_1]. \quad (1020)$$

Equation 1020 is meaningful only after the generativity measure and normative threshold have been specified.

The paper does not treat generativity itself as a universal scalar maximand.

14.209 Temporal Expansion of Possibility

A subject-directed assistance process can be evaluated through growth of viable future sets:

$$\mathcal{V}_t \subseteq \mathcal{V}_{t+\tau}. \quad (1021)$$

Equation 1021 represents one possible form of future-oriented improvement.

The set can expand while historical fidelity remains low.

14.210 Temporal Contraction of Possibility

A restorative arrangement can also appear adequate initially while gradually reducing future options:

$$\mathcal{V}_{t+\tau} \subset \mathcal{V}_t. \quad (1022)$$

Equation 1022 can reveal dependency created by assistance, declining employability, or unsustainable support.

This is one reason long-horizon analysis matters.

14.211 Restorative Dependency Over Time

Some interventions produce conditions that remain viable only while the restorative institution continuously supplies support.

Let support input be $u_R(t)$. A dependency-sensitive comparison can ask whether

$$\lim_{t \rightarrow \infty} u_R(t) = 0 \quad (1023)$$

while the desired generative conditions remain viable.

Equation 1023 is one possible ideal of self-sustaining restoration.

It is not universally appropriate. Some disabilities, public infrastructures, or social rights legitimately require permanent collective support.

The normative issue is unjust dependency, not support as such.

14.212 Permanent Support and Justice

A long-run requirement for support does not imply restorative failure.

Public education, healthcare, disability support, infrastructure, and social protection can be enduring institutional commitments.

The relevant comparison concerns whether continuing support preserves agency and just relations rather than whether every subject becomes independent of all institutions.

This distinction is important for avoiding an overly individualistic concept of self-sufficiency.

14.213 Temporal Autonomy

Autonomy can be evaluated across time.

A restorative process can provide choice initially and reduce choice later through lock-in.

Let

$$A(t) \quad (1024)$$

denote the relevant autonomy profile.

A temporally adequate process should consider both entry and exit conditions throughout the program.

14.214 Lock-In Horizon

Let T_L denote the time after which leaving a restorative arrangement becomes substantially more costly.

The existence of a short T_L can create pressure for early decisions before the subject has sufficient information.

Revisable assistance should therefore consider the timing of lock-in.

14.215 Reversibility Window

Some interventions remain reversible only for a period.

Let

$$W_R = [t_0, t_R] \quad (1025)$$

denote the interval during which reversal remains feasible.

Subject consent and review can be especially important before t_R .

After the window closes, the intervention can create new irreversibility.

14.216 Temporal Consent

Consent can require renewal where a restorative process changes substantially over time.

Consent at t_0 does not necessarily authorize every future intervention.

Let

$$C_t \in \{0, 1\} \quad (1026)$$

denote valid consent at time t .

A long-term process can require consent at relevant transition points.

14.217 Temporal Participation

Subject participation can also vary through the process.

A plan designed collaboratively at the beginning can become institutionally driven later.

Participation should therefore be treated as a trajectory, not a one-time event.

14.218 Temporal Contestation

Contestability should remain available while significant decisions remain revisable.

A right to contest only after irreversible implementation provides weaker procedural protection than timely contestation.

This is a temporal form of due process.

14.219 Temporal Privacy

Data collected for restorative monitoring can persist longer than the purpose that justified collection.

A privacy-sensitive system should therefore specify retention horizon

$$\tau_{\text{retain}} \quad (1027)$$

Long-term retention should require independent justification.

Restoration should not create permanent surveillance by default.

14.220 Temporal Expiration of Restorative Classifications

Provisional classifications can include expiration times.

Let classification z expire at

$$t_{\text{exp}}(z). \quad (1028)$$

Equation 1028 prevents temporary protective labels from becoming permanent through institutional inertia.

Review can renew a classification where justified.

14.221 Temporal Integrity of Assistance

Assistance should also avoid abrupt termination that recreates vulnerability.

A transition-out period can permit gradual replacement of special support with ordinary institutional conditions.

This is especially important where the subject has reasonably relied upon the support.

14.222 Tapering

Let support intensity be $u(t)$. A tapering policy can satisfy

$$\frac{du}{dt} < 0 \quad (1029)$$

over an appropriate transition period.

Equation 1029 is illustrative.

Abrupt and gradual withdrawal should be evaluated through the subject's conversion and security structures.

14.223 Temporal Integrity of Compensation Plans

Structured compensation over time can differ from lump-sum compensation.

A payment stream

$$M(t) \quad (1030)$$

can provide predictable security while reducing immediate flexibility.

A lump sum provides flexibility and can expose the recipient to other risks.

These alternatives occupy different temporal and agency profiles.

14.224 Temporal Integrity of Collective Reparations

Collective reparations can fail when funding is announced without long-term maintenance.

Infrastructure requires operation, staffing, repair, and governance over time.

The relevant restorative object is therefore often a sustained institutional capacity rather than a one-time transfer.

14.225 Maintenance Horizon

Let reparative infrastructure have required maintenance horizon τ_M .

A sustainable restorative plan should specify resources and governance through that horizon.

One-time construction without maintenance can satisfy an entity-return cell and fail a functional or generative cell.

14.226 Temporal Interpretation of Community Control

Community control also requires persistence.

Temporary consultation differs from durable decision authority.

A control relation can therefore be indexed by duration:

$$\chi = \chi(t). \quad (1031)$$

A reparative institution can begin participatory and later recentralize.

Long-horizon control comparison can reveal this shift.

14.227 Temporal Interpretation of Value Circulation

A one-time return of value differs from sustained circular value flow.

Let recursive retention rate be $\rho(t)$.

Generative-justice evaluation can compare

$$\rho_R(t) \text{ } E \text{ } \rho_{\text{ref}}(t) \quad (1032)$$

over a relevant horizon [15, 17].

This distinction separates reparative transfer from durable generative reform.

14.228 Temporal Interpretation of Non-Domination

Freedom from arbitrary control requires temporal security.

A subject who is free from interference today and remains exposed to arbitrary revocation tomorrow can remain dominated.

A non-domination object should therefore include the future distribution of control, not current interference alone [44].

14.229 Temporal Interpretation of Recognition

Recognition can be episodic or durable.

An institution can issue one apology while continuing practices that reproduce misrecognition.

Long-horizon recognition therefore concerns whether social standing remains embedded in later relations and institutional practice.

14.230 Temporal Interpretation of Capability

Capability should similarly distinguish temporary access from durable substantive freedom.

A one-day opportunity can be formally present and practically insignificant for a long-term project.

Capability horizons should therefore match the activities the capability is supposed to enable [55, 50].

14.231 Temporal Interpretation of Equality

Equality at one time does not imply equality over time.

Let inequality measure be

$$I(t). \quad (1033)$$

A restorative policy can equalize current holdings and allow rapid later divergence.

Dynamic equality therefore concerns the processes generating future distribution as well as the present distribution.

14.232 Temporal Interpretation of Relational Equality

Relational standing can also evolve.

A subject can be publicly reinstated while informal hierarchy re-emerges.

Relational equality therefore requires repeated interaction and institutional structure over time [2, 71].

14.233 Temporal Horizon as a Source of Theoretical Disagreement

Justice theories can appear to disagree when they implicitly use different horizons.

A corrective account can focus on historical loss. A capability account can focus on present and future opportunity. A generative account can focus on recursive long-run circulation. A social-protection account can focus on immediate sufficiency.

Some theoretical disagreement therefore concerns temporal orientation rather than incompatible values alone.

The atlas makes horizon explicit so that these differences can be separated.

14.234 Temporal Horizon as a Source of Institutional Disagreement

Institutions can likewise reach different conclusions because they evaluate different horizons.

An emergency agency asks whether the person is safe tonight.

A court asks what historical loss is compensable.

A rehabilitation service asks what function can be recovered over months.

A pension system considers decades.

Each can issue a valid judgment within its horizon.

Coordination requires preventing one horizon-specific judgment from being mistaken for a global conclusion.

14.235 Temporal Scope Discipline

The section supports a principle of temporal scope discipline:

The duration and temporal structure of a restorative claim should not exceed the duration and temporal structure actually supported by its evidence, comparison object, and institutional purpose.

A one-day measurement cannot establish ten-year resilience without additional model assumptions.

A ten-year statistical trend does not establish the experience of every day within the period.

14.236 Temporal Explicitness Principle

A second principle is:

A restorative claim should specify whether it concerns a moment, interval, trajectory, repeated process, long-run regime, lifecycle stage, or intergenerational horizon.

This prevents the word *restored* from hiding the temporal domain of the claim.

14.237 Temporal Non-Substitutability Principle

A third principle is:

Equality of cumulative quantity across time does not establish equality of historical or generative significance where timing, sequence, continuity, or recursive generation matters.

This principle gives a general form to the difference between delayed return and uninterrupted possession.

14.238 Temporal Non-Monotonicity Principle

A fourth principle is:

Improvement in restorative status at one horizon does not guarantee improvement at every longer or shorter horizon.

This principle follows from crossing trajectories, delayed effects, recurrence, and path dependence.

14.239 Temporal Pluralism Principle

A fifth principle is:

Different restorative objects can legitimately require different temporal horizons within the same case.

This principle supports horizon bundles rather than one universal restoration date.

14.240 Long-Horizon Epistemic Humility Principle

A sixth principle is:

As the restorative horizon extends into increasingly contingent future development, claims should shift toward representations whose epistemic support remains defensible, including bounds, distributions, reachable sets, and robust structural properties.

This principle limits false counterfactual precision.

14.241 Temporal Agency Principle

A seventh principle is:

The present subject should retain meaningful authority to revise future restorative goals as the subject and surrounding world develop over time.

This principle connects temporal comparison with subject-directed regeneration.

14.242 Temporal Layer of the Atlas

The temporal layer can now be represented as a family

$$\mathfrak{T}_A = \{T_{\text{instant}}, T_{\text{interval}}, T_{\text{path}}, T_{\text{recurrent}}, T_{\text{asymptotic}}, T_{\text{lifecycle}}, T_{\text{intergen}}\}. \quad (1034)$$

The complete comparative space becomes

$$\mathfrak{C}_T = \mathfrak{Q} \times \mathfrak{E} \times \mathfrak{T}_A. \quad (1035)$$

Equation 1035 is a three-axis core of the extended atlas.

Reference, test class, tolerance, epistemic basis, scale, and procedure provide additional dimensions.

14.243 Summary of Temporal Comparison Forms

Table 7 summarizes the principal temporal forms developed in this section.

Table 7 demonstrates that the temporal axis contains several distinct objects of comparison rather than one scalar horizon.

14.244 Implications for the Dynamical Analysis

The temporal analysis yields several consequences for the following formal sections.

First, state equality and trajectory equivalence must remain separate.

Second, reachable sets should be indexed by deadlines and horizons.

Third, perturbation response should include recovery duration and repeated disturbance.

Fourth, attractor and basin comparison naturally belongs to long-horizon analysis.

Fifth, short-run observational equivalence can fail under long-run or recurrent testing.

Sixth, long-horizon counterfactual precision is generally weaker than short-horizon historical reconstruction.

Seventh, recursive generative restoration should be indexed by generative depth or horizon.

Eighth, intergenerational repair requires comparison of transmission mechanisms in addition to current distributions.

Ninth, procedural justice contains timing, duration, and review-frequency dimensions.

Tenth, a single date of “complete restoration” is generally inadequate for multi-object generative harms.

Table 7: Principal temporal forms of restorative comparison

Temporal form	Primary object	Restorative interpretation	Characteristic limitation
Instantaneous	State at one time	Current correspondence	Says little about persistence or future dynamics
Finite horizon	Interval of states or outputs	Correspondence over bounded duration	Depends upon horizon and aggregation rule
Pathwise	Entire trajectory	History of evolution is compared	High informational burden
Cumulative	Integrated quantity	Total exposure or value is compared	Can conceal timing and sequence
Moving window	Recent interval statistics	Rolling stability or performance	Can smooth critical short episodes
Recovery time	Time to target region	Speed of recovery	Does not capture burden during recovery
Stabilization time	Time to durable acceptable region	Persistent rather than momentary recovery	Requires definition of acceptable region
Recurrent	Repeated events or shocks	Durability under recurrence	Requires long observation
Asymptotic	Long-run state or distribution	Eventual convergence or regime similarity	Can ignore severe transient injustice
Lifecycle	Stage-indexed condition	Timing relative to human development	Reference is stage-sensitive
Intergenerational	Cross-generation transmission	Long-run historical and structural repair	High epistemic and normative uncertainty
Historical window	Time-specific opportunity	Preservation of temporally singular value	Later substitution can be structurally weak

14.245 Transition to State, Trajectory, and Reachability

The object–relation atlas has now acquired an explicit temporal layer. The next formal task is to examine three closely related and frequently conflated objects in greater depth:

1. present state;
2. trajectory or orbit;
3. reachable future set.

A system can match the reference state and fail trajectory equivalence. It can approximate the reference trajectory while retaining a different future reachable set. It can preserve or expand reachability while following a historically different path.

These distinctions are central to the paper’s account of generativity because future possibility cannot be inferred from present state alone.

Section 15 therefore develops state, trajectory, orbit, trace, and reachable-set comparison systematically. It examines exact and approximate state restoration, path correspondence, time-reparameterized trajectories, reachable-set equality and inclusion, deadline-constrained reachability, and the relation between present restoration and future generative possibility.

15 State, Trajectory, and Reachability in Restorative Comparison

This section develops a detailed comparison among three formal objects that are frequently conflated in restorative reasoning: present state, realized trajectory, and future reachability. Their separation is central to the theory of generativity. A restored system can occupy the same present state as the reference

and retain a different future option structure. It can follow a trajectory similar to the reference and nevertheless possess different branching possibilities. It can recover a large reachable set while having passed through a very different and normatively significant history.

The section therefore examines state restoration, state-subspace restoration, distributional state comparison, trajectory correspondence, orbit comparison, trace comparison, time-reparameterized paths, reachable sets, viable sets, deadline-constrained reachability, transition costs, recognizable opportunity, and the formal relations connecting these objects. Particular attention is given to non-implication results: state equality does not imply trajectory equivalence; trajectory similarity does not imply equality of reachable sets; trace equivalence does not imply equality of branching structure; and equal reachable sets do not imply equal practical accessibility.

The analysis builds on the temporal distinctions developed in Section 14 and prepares the dynamical analysis of attractors, basins, and stability in the following section.

15.1 Three Distinct Comparative Objects

Let x_t denote current state, $\gamma_{[t_0, t_1]}$ a realized trajectory, and $\mathcal{R}_\tau(x_t)$ the reachable set over horizon τ .

The three comparison objects are:

$$Q_{\text{state}}(\mathcal{G}) = x_t, \quad (1036)$$

$$Q_{\text{traj}}(\mathcal{G}) = \gamma_{[t_0, t_1]}, \quad (1037)$$

and

$$Q_{\text{reach}}(\mathcal{G}) = \mathcal{R}_\tau(x_t). \quad (1038)$$

Equations 1036 through 1038 define three distinct projections of the same generative system.

The state answers:

Where is the system now?

The trajectory answers:

How did the system move through the relevant period?

The reachable set answers:

Which future states remain possible from the current configuration?

These questions are logically independent.

15.2 Exact State Restoration

The strongest state-level criterion is exact equality:

$$x_R(t) = x_{\text{ref}}(t). \quad (1039)$$

Equation 1039 can apply to a scalar, vector, or structured state.

For example, a restored employment state can include salary, title, access, workload, and benefits. If every represented coordinate matches the reference, exact state restoration holds relative to that representation.

The criterion remains representation-relative.

A state vector that omits network position, trust, or health cannot support a claim about those omitted objects.

15.3 Partial State Restoration

Often only a subset of state coordinates is restored.

Let I be the index set of restored coordinates. Then

$$x_R^I(t) = x_{\text{ref}}^I(t) \quad (1040)$$

can hold while other coordinates differ.

Equation 1040 provides a formal language for bounded institutional claims.

A court can restore salary and legal status while leaving health and network effects unresolved.

15.4 Approximate State Restoration

Where exact equality is unnecessary or infeasible, approximate restoration can be defined by a metric d_X :

$$d_X(x_R(t), x_{\text{ref}}(t)) \leq \varepsilon_X. \quad (1041)$$

Equation 1041 requires justification of both the metric and tolerance.

A Euclidean metric on heterogeneous social variables can be inappropriate without normalization and normative interpretation.

The distance function therefore belongs to the substantive structure of the claim.

15.5 Weighted State Distance

For a vector state $x = (x_1, \dots, x_n)$, a weighted distance can be written as

$$d_W(x_R, x_{\text{ref}}) = \left[\sum_{i=1}^n w_i |x_{R,i} - x_{\text{ref},i}|^p \right]^{1/p}. \quad (1042)$$

Equation 1042 makes weighting explicit.

The weights w_i are normatively substantive.

A model that assigns low weight to autonomy or recognition can make two systems appear close despite large differences in those dimensions.

15.6 State Equality Under Abstraction

Let α map a rich state into a coarser representation.

Then

$$\alpha(x_R) = \alpha(x_{\text{ref}}) \quad (1043)$$

can hold while

$$x_R \neq x_{\text{ref}}. \quad (1044)$$

Equations 1043 and 1044 formalize institutional compression.

A status record can show identical employment while the richer social state differs.

15.7 State Distribution Restoration

For stochastic or population-level systems, the object can be the state distribution:

$$P_R(X_t) \quad (1045)$$

and

$$P_{\text{ref}}(X_t). \quad (1046)$$

Exact distributional restoration requires

$$P_R(X_t) = P_{\text{ref}}(X_t). \quad (1047)$$

Equation 1047 can be appropriate for population or institutional analysis.
It does not imply equality of individual histories.

15.8 Marginal and Joint State Restoration

Two systems can share all selected marginal distributions and differ in joint structure.

Let X and Y be two dimensions. It can hold that

$$P_R(X) = P_{\text{ref}}(X) \quad (1048)$$

and

$$P_R(Y) = P_{\text{ref}}(Y) \quad (1049)$$

while

$$P_R(X, Y) \neq P_{\text{ref}}(X, Y). \quad (1050)$$

Equations 1048 through 1050 are important for distributional justice.

Income and health can have equal marginals while disadvantage remains concentrated among the same persons in one system.

15.9 State Restoration and Hidden Variables

An observed state can omit latent variables relevant to future dynamics.

Let

$$x_t = (x_t^{\text{obs}}, x_t^{\text{latent}}). \quad (1051)$$

An institution can establish

$$x_{R,t}^{\text{obs}} = x_{\text{ref},t}^{\text{obs}} \quad (1052)$$

while latent components differ.

Equation 1052 supports only observational state restoration.

The later divergence of the systems can reveal the hidden difference.

15.10 State Equality and Transition Non-Equality

Let the systems share state x :

$$x_R = x_{\text{ref}} = x. \quad (1053)$$

Suppose their transition maps differ:

$$\Phi_R \neq \Phi_{\text{ref}}. \quad (1054)$$

Then for some input u ,

$$\Phi_R(x, u) \neq \Phi_{\text{ref}}(x, u). \quad (1055)$$

Equations 1053 through 1055 prove the basic non-implication:

$$\text{state equality} \not\Rightarrow \text{transition equivalence}. \quad (1056)$$

This distinction is foundational for the paper.

15.11 State Equality and Reachability Non-Equality

If transition systems differ, equal states can have different reachable sets:

$$x_R = x_{\text{ref}} \quad (1057)$$

and

$$\mathcal{R}_\tau^R(x_R) \neq \mathcal{R}_\tau^{\text{ref}}(x_{\text{ref}}). \quad (1058)$$

Thus

$$\text{state equality} \not\Rightarrow \text{reachability equality}. \quad (1059)$$

Equation 1059 provides a formal reason why return of a current condition does not guarantee return of future possibility.

15.12 Trajectory as a Historical Object

A trajectory retains temporal order.

For continuous time,

$$\gamma : [t_0, t_1] \longrightarrow \mathcal{X}. \quad (1060)$$

For discrete time,

$$\gamma = (x_{t_0}, x_{t_1}, \dots, x_{t_n}). \quad (1061)$$

Equations 1060 and 1061 preserve more information than the endpoint state.

A trajectory can encode delay, temporary deprivation, instability, recovery, and sequence.

15.13 Exact Trajectory Restoration

Exact trajectory restoration requires

$$\gamma_R(t) = \gamma_{\text{ref}}(t) \quad \forall t \in [t_0, t_1]. \quad (1062)$$

Equation 1062 is a very strong condition.

Where the reference is a no-harm history, the actual repaired trajectory generally cannot satisfy it because the harm and restoration altered the path.

The criterion remains analytically useful as an upper bound on fidelity.

15.14 Trajectory Distance

A more practical comparison uses a path metric.

One example is the L^p trajectory distance

$$D_p(\gamma_R, \gamma_{\text{ref}}) = \left[\int_{t_0}^{t_1} d_X(\gamma_R(t), \gamma_{\text{ref}}(t))^p dt \right]^{1/p}. \quad (1063)$$

Approximate trajectory restoration requires

$$D_p(\gamma_R, \gamma_{\text{ref}}) \leq \varepsilon_\gamma. \quad (1064)$$

Equation 1064 captures overall path similarity.

It can underweight short extreme deviations where p and the temporal resolution are poorly chosen.

15.15 Supremum Trajectory Distance

A stronger path criterion uses maximum deviation:

$$D_\infty(\gamma_R, \gamma_{\text{ref}}) = \sup_{t \in [t_0, t_1]} d_X(\gamma_R(t), \gamma_{\text{ref}}(t)). \quad (1065)$$

Equation 1065 is sensitive to the worst temporal deviation.

This can be normatively appropriate where short severe departures matter.

15.16 Trajectory Comparison with Temporal Weighting

A weighted path distance can assign different significance to different times:

$$D_w(\gamma_R, \gamma_{\text{ref}}) = \int_{t_0}^{t_1} w(t) d_X(\gamma_R(t), \gamma_{\text{ref}}(t)) dt. \quad (1066)$$

Equation 1066 can represent critical periods.

The weighting function $w(t)$ requires explicit normative justification.

15.17 Time-Reparameterized Trajectory Comparison

Two paths can have similar shape and different timing.

Let θ be an increasing reparameterization of time. A time-reparameterized comparison can require

$$\gamma_R(t) \approx h(\gamma_{\text{ref}}(\theta(t))). \quad (1067)$$

Equation 1067 permits delay while preserving qualitative path.

This relation is useful for rehabilitation and dangerous for temporally singular opportunities.

The acceptability of θ depends upon the value of time.

15.18 Trajectory Shape

A comparison can abstract from precise timing and absolute position to preserve trajectory shape.
Let

$$\text{Shape}(\gamma_R) \sim \text{Shape}(\gamma_{\text{ref}}). \quad (1068)$$

Equation 1068 is useful for qualitative dynamical analysis.
Its formal definition depends upon the representation chosen.

15.19 Orbit Comparison

The orbit of state x under dynamics Φ is

$$\mathcal{O}_\Phi(x) = \{\Phi^t(x) : t \geq 0\}. \quad (1069)$$

Orbit comparison ignores some temporal parameterization while preserving the set of states visited under the dynamics.

Orbit equality is

$$\mathcal{O}_{\Phi_R}(x_R) = \mathcal{O}_{\Phi_{\text{ref}}}(x_{\text{ref}}). \quad (1070)$$

This relation is stronger than endpoint equality and weaker than exact time-indexed trajectory equality.

15.20 Orbit Equivalence Under Mapping

An orbit correspondence can use a map h :

$$h(\mathcal{O}_{\Phi_{\text{ref}}}(x)) = \mathcal{O}_{\Phi_R}(h(x)). \quad (1071)$$

Equation 1071 is related to dynamical conjugacy and orbit equivalence.
It becomes relevant where exact state labels differ while qualitative evolution is preserved.

15.21 Trace Comparison

A trace is the observable projection of a trajectory.

Let observation map be O . Then

$$\text{tr}_O(\gamma) = O \circ \gamma. \quad (1072)$$

Trace equality requires

$$\text{tr}_O(\gamma_R) = \text{tr}_O(\gamma_{\text{ref}}). \quad (1073)$$

Equation 1073 can hold while internal state trajectories differ.
This is useful where only observable behavior is legitimately within institutional scope.

15.22 Trace-Language Comparison

For branching systems, compare the sets of possible observable traces:

$$\mathcal{L}_{\text{tr}}(\mathcal{G}_R) \ E \ \mathcal{L}_{\text{tr}}(\mathcal{G}_{\text{ref}}). \quad (1074)$$

Possible relations include equality and inclusion.

Trace-language equality preserves observable sequence possibilities while ignoring some branching structure, consistent with the distinctions in process semantics [66].

15.23 Trace Equality and Branching Difference

Two systems can generate the same trace set while offering different choices at intermediate states.
Therefore,

$$\mathcal{L}_{\text{tr},R} = \mathcal{L}_{\text{tr},\text{ref}} \quad (1075)$$

does not imply

$$\mathfrak{T}_R \sim_{\text{bis}} \mathfrak{T}_{\text{ref}}. \quad (1076)$$

Equations 1075 and 1076 are normatively important where available choice matters independently of realized outcomes.

15.24 Trajectory Equality and Reachability Difference

Two systems can follow the same realized path while retaining different unexercised options.
Let

$$\gamma_R = \gamma_{\text{ref}} \quad (1077)$$

and

$$\mathcal{R}_\tau^R \neq \mathcal{R}_\tau^{\text{ref}}. \quad (1078)$$

Thus

$$\text{trajectory equality} \not\Rightarrow \text{reachability equality}. \quad (1079)$$

Equation 1079 captures a central distinction between realized history and unexercised freedom.

15.25 Reachability Definition

Let x_t be current state, $\mathcal{U}_{\text{adm}}$ admissible actions, and Ξ_{adm} admissible disturbances.
The finite-horizon reachable set is

$$\mathcal{R}_\tau(x_t) = \{x_{t+\tau} : x_{t+\tau} \text{ can be reached from } x_t \text{ under } \mathcal{U}_{\text{adm}}, \Xi_{\text{adm}}\}. \quad (1080)$$

Equation 1080 treats future possibility as a set-valued object.

The definition is indexed to admissible actions and disturbances.

Changing those classes changes the reachable set.

15.26 Reachability Equality

Exact reachable-set restoration requires

$$\mathcal{R}_\tau^R(x_R) = \mathcal{R}_\tau^{\text{ref}}(x_{\text{ref}}). \quad (1081)$$

Equation 1081 is a strong future-oriented criterion.

It requires the same represented set of future states over the specified horizon.

It does not require equal probabilities, costs, or paths to those states.

15.27 Reachability Inclusion

A future-expansion criterion is

$$\mathcal{R}_\tau^{\text{ref}}(x_{\text{ref}}) \subseteq \mathcal{R}_\tau^R(x_R). \quad (1082)$$

Equation 1082 requires every reference future to remain possible while permitting new futures. This relation is especially attractive for subject-directed generative justice. Its normative adequacy depends upon practical usability and the quality of added options.

15.28 Strict Reachability Expansion

Strict future expansion occurs when

$$\mathcal{R}_\tau^{\text{ref}} \subset \mathcal{R}_\tau^R. \quad (1083)$$

Equation 1083 can represent a better-than-reference possibility structure. It does not prove that the restored system is more just overall. New reachable states can include harmful or dominating paths.

15.29 Reachability Contraction

A harmful intervention can contract future possibility:

$$\mathcal{R}_\tau^H \subset \mathcal{R}_\tau^{\text{ref}}. \quad (1084)$$

Equation 1084 provides a precise representation of one form of generative harm. The lost elements need not have been certain future outcomes. Their loss concerns the structure of possible futures.

15.30 Reachability Redirection

A harm can alter the composition of future possibilities without simply shrinking the set.

It can occur that

$$\mathcal{R}_\tau^H \not\subseteq \mathcal{R}_\tau^{\text{ref}} \quad (1085)$$

and

$$\mathcal{R}_\tau^{\text{ref}} \not\subseteq \mathcal{R}_\tau^H. \quad (1086)$$

Equations 1085 and 1086 describe redirection rather than simple contraction. The subject loses some futures and gains others. Normative analysis must evaluate the composition, not only the size, of the sets.

15.31 Reachable-Set Cardinality

For finite or countable sets, one can compare cardinality:

$$|\mathcal{R}_\tau^R| \ E \ |\mathcal{R}_\tau^{\text{ref}}|. \quad (1087)$$

Equal cardinality is a weak criterion. Two option sets can have the same size and entirely different normative value. Cardinality can therefore be a descriptive attribute rather than a sufficient restorative standard.

15.32 Reachable-Set Measure

For measurable sets, compare

$$\mu(\mathcal{R}_\tau^R) \leq \mu(\mathcal{R}_\tau^{\text{ref}}). \quad (1088)$$

Measure preserves more geometric information than cardinality in continuous spaces and remains insensitive to composition.

Equal volume can coexist with loss of uniquely valuable regions.

15.33 Reachable-Set Geometry

A richer object concerns geometry:

$$\text{Geom}(\mathcal{R}_\tau^R) \leq \text{Geom}(\mathcal{R}_\tau^{\text{ref}}). \quad (1089)$$

Relevant features can include:

- connectedness;
- fragmentation;
- distance to desirable regions;
- narrow corridors;
- bottlenecks;
- boundary structure.

Geometry can reveal whether a large option set is practically fragile.

15.34 Hausdorff Reachability Comparison

For metric state space, approximate reachable-set restoration can use Hausdorff distance:

$$d_H(\mathcal{R}_\tau^R, \mathcal{R}_\tau^{\text{ref}}) \leq \varepsilon_R. \quad (1090)$$

Equation 1090 requires every point in each set to lie near some point in the other set.

The criterion is symmetric.

A future-expansion theory can prefer a directional distance where reference options must be approximated but additional restored options are permitted.

15.35 Directed Reachability Distance

Define the directed set distance

$$d_{\rightarrow}(A, B) = \sup_{a \in A} \inf_{b \in B} d(a, b). \quad (1091)$$

A reference-preserving criterion can require

$$d_{\rightarrow}(\mathcal{R}_\tau^{\text{ref}}, \mathcal{R}_\tau^R) \leq \varepsilon. \quad (1092)$$

Equation 1092 permits restored states outside the reference set while requiring every reference future to have a nearby restored counterpart.

This is often more compatible with transformative or generative assistance than symmetric Hausdorff distance.

15.36 Deadline-Constrained Reachability

Define the set of states reachable within deadline τ :

$$\mathcal{R}_{\leq \tau}(x) = \bigcup_{0 \leq s \leq \tau} \mathcal{R}_s(x). \quad (1093)$$

A state can belong to unrestricted reachability and fail deadline-constrained reachability.

This distinction is central to time-sensitive opportunities.

A career outcome reached after retirement age is not equivalent to one available during active career formation merely because the endpoint remains theoretically reachable.

15.37 Time-to-Reach Function

For target state or region A , define

$$T_A(x) = \inf \{t \geq 0 : \Phi^t(x) \in A\}. \quad (1094)$$

Equation 1094 adds temporal accessibility to reachability.

A stronger restorative criterion can require

$$T_{A,R} \leq T_{A,\text{ref}} + \varepsilon_T. \quad (1095)$$

15.38 Reachability Probability

In stochastic systems, future possibility has probability.

For target set A ,

$$p_A(x, \tau) = P(X_{t+\tau} \in A \mid X_t = x). \quad (1096)$$

Two systems can have equal reachable sets and very different $p_A(x, \tau)$.

Thus

$$\text{reachability equality} \not\Rightarrow \text{probability equality}. \quad (1097)$$

15.39 Probabilistic Reachability Restoration

A stronger criterion can require

$$|p_{A,R}(x_R, \tau) - p_{A,\text{ref}}(x_{\text{ref}}, \tau)| \leq \varepsilon_p. \quad (1098)$$

Equation 1098 is useful where available futures differ in likelihood.

A formally available opportunity with near-zero probability can have limited practical significance.

15.40 Transition Cost

Reachability also depends upon cost.

Let a path γ have cost

$$C(\gamma) = \int_{t_0}^{t_1} c(x_t, u_t) dt. \quad (1099)$$

The minimum cost to reach target A is

$$C_A^*(x) = \inf_{\gamma: x \rightarrow A} C(\gamma). \quad (1100)$$

Equation 1100 distinguishes nominal from practical reachability.

Two systems can reach the same target while one requires substantially more money, effort, risk, or time.

15.41 Multi-Dimensional Transition Cost

Transition cost can be vector-valued:

$$\mathbf{C}(\gamma) = (C_{\text{money}}, C_{\text{time}}, C_{\text{effort}}, C_{\text{risk}}, C_{\text{relational}}) . \quad (1101)$$

Equation 1101 avoids forced aggregation of heterogeneous burdens.

A capability can be formally available and practically unusable because one cost dimension is prohibitive.

15.42 Cost-Constrained Reachability

Given budget vector $\mathbf{B}$, define

$$\mathcal{R}_{\tau, \mathbf{B}}(x) = \{y : y \text{ is reachable within } \tau \text{ with } \mathbf{C} \leq \mathbf{B}\} . \quad (1102)$$

Equation 1102 is a stronger practical opportunity object than unconstrained reachability.

It connects directly to capability and effective access.

15.43 Viability

Reachability asks whether a target can be attained. Viability asks whether the system can remain within an admissible region while evolving.

Let $\mathcal{K}_{\text{adm}} \subseteq \mathcal{X}$ be a constraint set.

The viability kernel is

$$\text{Viab}(\mathcal{K}_{\text{adm}}) = \{x_0 \in \mathcal{K}_{\text{adm}} : \exists u(\cdot) \text{ such that } x_t \in \mathcal{K}_{\text{adm}} \text{ over the relevant horizon}\} . \quad (1103)$$

Equation 1103 distinguishes desirable reachable futures from futures that require unacceptable intermediate states.

15.44 Viability Restoration

Exact viability restoration requires

$$\text{Viab}_R = \text{Viab}_{\text{ref}} . \quad (1104)$$

A future-expansion criterion is

$$\text{Viab}_{\text{ref}} \subseteq \text{Viab}_R . \quad (1105)$$

Equation 1105 is especially attractive for subject-directed regeneration because it preserves viable reference futures while allowing new viable futures.

15.45 Reachability and Viability Non-Equivalence

A target can be reachable through an inadmissible path.

Thus

$$y \in \mathcal{R}_\tau(x) \quad (1106)$$

does not imply

$$y \in \text{Viab}(\mathcal{K}_{\text{adm}}). \quad (1107)$$

This distinction matters where a career can be recovered only through unacceptable coercion, health risk, or dependence.

15.46 Recognized Reachability

The subject can fail to recognize some objectively reachable futures.

Let

$$\hat{\mathcal{R}}_\tau^A(x) \subseteq \mathcal{R}_\tau(x) \quad (1108)$$

denote the recognized reachable set.

Equation 1108 depends upon information, expectation, imagination, trust, and social support.

A returned option can remain outside $\hat{\mathcal{R}}_\tau^A$ even after formal access is restored.

15.47 Recognized-Reachability Restoration

A subject-sensitive criterion can require

$$\hat{\mathcal{R}}_{\tau,R}^A \supseteq \hat{\mathcal{R}}_{\tau,\text{ref}}^A. \quad (1109)$$

Equation 1109 is stronger than objective reachability inclusion in one respect and more epistemically difficult in another.

It asks whether the subject can actually perceive and act upon the restored possibilities.

15.48 Reachability and Information

Information can expand recognized reachability without changing objective reachability.

Let information intervention I satisfy

$$\mathcal{R}_\tau^A \text{ unchanged} \quad (1110)$$

while

$$\hat{\mathcal{R}}_\tau^A \subset \hat{\mathcal{R}}_\tau^{A,I}. \quad (1111)$$

This structure explains why advice, mentoring, and information can have generative value even without creating new external options.

15.49 Reachability and Trust

Low trust can make an objectively available path practically unusable.

A subject can decline to enter a pathway because the institution is perceived as unsafe or arbitrary.

Thus trust belongs to the conversion from objective reachability to practical reachability.

A richer model can define

$$\mathcal{R}_\tau^{\text{practical}} = \Psi(\mathcal{R}_\tau, T_A, \mathbf{C}, I_A), \quad (1112)$$

where T_A denotes trust, $\mathbf{C}$ transition costs, and I_A information.

15.50 Reachability and Control

An option depending upon arbitrary permission from another actor has different security from one protected by rights.

Let path p depend upon gatekeeper G . The path remains formally reachable while its secure reachability is lower.

A secure reachable set can therefore be defined as

$$\mathcal{R}_\tau^{\text{sec}} = \{y \in \mathcal{R}_\tau : \text{path to } y \text{ satisfies selected security conditions}\}. \quad (1113)$$

Equation 1113 connects reachability with non-domination [44].

15.51 Reachability and Path Dependency

The current reachable set depends upon prior history where history changes transition structure.

Let

$$\mathcal{R}_\tau = \mathcal{R}_\tau(x_t, h_{0:t}). \quad (1114)$$

Equation 1114 formalizes historical dependence.

Two subjects with identical observed current state can retain different future possibility structures because their histories altered hidden or relational conditions.

15.52 Reachability and Adaptive Preference

Preference affects which reachable states the subject treats as desirable.

Let preferred target set be

$$\mathcal{T}_A \subseteq \mathcal{R}_\tau. \quad (1115)$$

A harmful history can alter $\mathcal{T}_A$.

Restorative theory should distinguish loss of reachability from change in preference over reachable states.

The two can interact without being identical.

15.53 Subject-Directed Reachability

A future-directed assistance criterion can require

$$\mathcal{R}_\tau^R \cap \mathcal{T}_A^{\text{adm}} \neq \emptyset. \quad (1116)$$

A stronger criterion can require several viable routes:

$$|\mathcal{R}_\tau^R \cap \mathcal{T}_A^{\text{adm}}| \geq k \quad (1117)$$

where cardinality is meaningful.

This can preserve genuine choice rather than one institutionally approved route.

15.54 Reachability Redundancy

Two systems can reach the same targets with different numbers of independent paths.

Let $\kappa(x, y)$ denote the number or capacity of independent pathways from x to y .

A restorative criterion can compare

$$\kappa_R(x_R, y_R) \geq \kappa_{\text{ref}}(x_{\text{ref}}, y_{\text{ref}}). \quad (1118)$$

Equation 1118 captures robustness of access.

One path controlled by a single gatekeeper differs from several independent paths.

15.55 Reachability Bottlenecks

A bottleneck is a state, actor, or transition through which many desirable paths must pass.

Let b denote such a bottleneck. A concentration measure can be represented by

$$B_R = \frac{\text{desirable paths passing through } b}{\text{all desirable paths}}. \quad (1119)$$

High B_R can indicate domination or fragility.

Restoration of endpoint reachability can therefore remain inadequate where bottleneck concentration increases.

15.56 Reachability Topology

Topology can distinguish connected and fragmented opportunity spaces.

A connected reachable set can permit gradual transition among futures. A fragmented set can require large jumps or inaccessible intermediate states.

Topological comparison can therefore provide information beyond set size.

A homeomorphism criterion can require

$$\mathcal{R}_\tau^R \cong_{\text{top}} \mathcal{R}_\tau^{\text{ref}}. \quad (1120)$$

15.57 Reachability Components

Let

$$\pi_0(\mathcal{R}_\tau) \quad (1121)$$

denote the set of connected components.

A simple structural comparison can ask whether the number and organization of components correspond.

This can reveal fragmentation after harm.

15.58 Reachability and Distance to Target

The target can remain reachable while becoming more distant.

Let d_{path} be shortest-path or minimum-cost distance.

Compare

$$d_{\text{path}, R}(x_R, A) \quad (1122)$$

with

$$d_{\text{path,ref}}(x_{\text{ref}}, A) . \quad (1123)$$

The difference can represent increased transition burden.

15.59 Reachability and Optionality

Reachable sets can be interpreted as one formalization of optionality.

However, more reachable states do not automatically imply greater freedom.

Additional states can be:

- irrelevant;
- harmful;
- inaccessible in practice;
- dominated by existing options;
- generated through coercive conditions.

Normative interpretation therefore requires a value-filtered reachable set.

15.60 Value-Filtered Reachability

Let V identify states satisfying a selected normative predicate.

Define

$$\mathcal{R}_\tau^V = \{y \in \mathcal{R}_\tau : V(y) = 1\} . \quad (1124)$$

Equation 1124 restricts reachability to normatively admissible or valuable futures.

The choice of V belongs to value theory.

15.61 Dominated Options

Some options add no meaningful freedom because another available option is at least as good under every relevant dimension.

Let option y_1 be dominated by y_2 :

$$y_2 \succeq_V y_1 . \quad (1125)$$

A capability analysis can therefore consider the nondominated frontier rather than raw option count.

15.62 Reachability Frontier

Define the nondominated reachable frontier

$$\mathcal{F}_R = \text{ND}(\mathcal{R}_\tau^V) . \quad (1126)$$

Equation 1126 can provide a more informative object than full reachability where many states are normatively redundant.

15.63 Reachability Frontier Restoration

A restorative criterion can require

$$\mathcal{F}_{\text{ref}} \subseteq \mathcal{F}_R \quad (1127)$$

or approximate frontier correspondence.

This criterion combines future possibility with value filtering.

It remains sensitive to the underlying normative order.

15.64 Trajectory and Reachability Joint Object

Trajectory and reachability can be represented jointly:

$$Q_{\text{TR}}(\mathcal{G}) = (\gamma_{[t_0, t]}, \mathcal{R}_\tau(x_t)). \quad (1128)$$

Equation 1128 captures both realized history and remaining future possibility.

This joint object is particularly useful for historically changed subjects.

The same current option set can carry different meaning depending upon the path through which it was reached.

15.65 State, Trajectory, and Reachability Triple

The three central objects can be combined into

$$Q_{\text{STR}}(\mathcal{G}) = (x_t, \gamma_{[t_0, t]}, \mathcal{R}_\tau(x_t)). \quad (1129)$$

Equation 1129 provides a compact representation of present, past path, and future possibility.

A strong restorative comparison can compare all three components.

The components should not automatically be aggregated into one scalar.

15.66 Three-Way Non-Implication

The distinctions developed above can be summarized by three non-implications:

$$\text{state equivalence} \not\Rightarrow \text{trajectory equivalence}, \quad (1130)$$

$$\text{trajectory equivalence} \not\Rightarrow \text{reachability equivalence}, \quad (1131)$$

$$\text{reachability equivalence} \not\Rightarrow \text{historical equivalence}. \quad (1132)$$

Equations 1130 through 1132 establish the basic logical architecture of this section.

15.67 State and Trajectory Conjunction

A stronger restorative criterion can require both endpoint and path similarity:

$$[x_R(t_1) E_X x_{\text{ref}}(t_1)] \wedge [\gamma_R E_\Gamma \gamma_{\text{ref}}]. \quad (1133)$$

Equation 1133 prevents a matching endpoint from hiding a severely different path.

15.68 State and Reachability Conjunction

Another criterion requires present and future correspondence:

$$[x_R \ E_X \ x_{\text{ref}}] \wedge [\mathcal{R}_\tau^R \ E_R \ \mathcal{R}_\tau^{\text{ref}}]. \quad (1134)$$

Equation 1134 is especially suitable for functional restoration where current state and future opportunity both matter.

15.69 Trajectory and Reachability Conjunction

A historically sensitive future-oriented criterion is

$$[\gamma_R \ E_\Gamma \ \gamma_{\text{ref}}] \wedge [\mathcal{R}_\tau^R \ E_R \ \mathcal{R}_\tau^{\text{ref}}]. \quad (1135)$$

Equation 1135 preserves both path and future possibility.

This can be too demanding where history is irreversible but remains useful as a comparative ideal.

15.70 State–Trajectory–Reachability Bundle

A comprehensive bundle is

$$\mathcal{B}_{\text{STR}} = \{(Q_{\text{state}}, E_X), (Q_{\text{traj}}, E_\Gamma), (Q_{\text{reach}}, E_R)\}. \quad (1136)$$

Equation 1136 can be used in later case analyses.

Different theories can choose different relations for each component.

15.71 Workplace State–Trajectory–Reachability Example

Consider a worker reinstated after prolonged exclusion.

State comparison can ask whether:

- salary is restored;
- title is restored;
- formal access is restored.

Trajectory comparison can ask whether:

- career progression was interrupted;
- professional experience accumulated differently;
- periods of unemployment or stress created additional burden.

Reachability comparison can ask whether:

- promotion remains realistically reachable;
- professional exit remains feasible;
- alternative employers remain accessible;
- future work can occur without dependence upon the same gatekeeper.

The three layers can produce different restorative judgments.

15.72 Research State–Trajectory–Reachability Example

For a researcher:

State restoration can include current funding, affiliation, and laboratory access.

Trajectory restoration can concern publication timing, accumulated collaboration, training, and missed research cycles.

Reachability restoration can concern which future projects, collaborators, institutions, and intellectual directions remain viable.

A later grant can strongly improve current state and reachability while leaving historical trajectory non-equivalent.

This is a central example of future-directed justice after irreversible history.

15.73 Health State–Trajectory–Reachability Example

For health-related loss:

State comparison can concern current bodily and functional measures.

Trajectory comparison can concern duration of pain, hospitalization, treatment, or lost activity.

Reachability comparison can concern future work, mobility, care, family life, and independent living.

A subject can recover present function while retaining a narrower future option set because recurrence risk remains high.

15.74 Reputational State–Trajectory–Reachability Example

For reputational harm:

State comparison can concern the current official classification.

Trajectory comparison can concern the period during which the false classification circulated.

Reachability comparison can concern future employment, collaboration, credit, membership, or public standing.

Correction of the current record therefore satisfies only the state layer.

15.75 Collective State–Trajectory–Reachability Example

For collective historical harm:

State comparison can concern present wealth, land access, institutions, or public recognition.

Trajectory comparison can concern historical dispossession, exclusion, migration, or institutional suppression.

Reachability comparison can concern future collective self-government, cultural transmission, ecological stewardship, education, and economic development.

The three layers are especially important because literal historical trajectory restoration can be impossible while future generativity remains transformable.

15.76 State Restoration as Institutional Compression

Many institutions naturally prefer state comparison because current state is observable and administratively tractable.

This creates a systematic risk:

$$\text{institutional measurability} \Rightarrow \text{preference for state objects.} \quad (1137)$$

Equation 1137 is a sociological tendency rather than a logical law.

The atlas provides a corrective by making trajectory and reachability available as explicit alternative objects.

15.77 Trajectory Comparison and Counterfactual Burden

Trajectory comparison becomes epistemically difficult when the reference path is counterfactual.

The longer the horizon, the more uncertain the exact path.

This suggests a methodological shift:

precise counterfactual trajectory $\rightarrow$ trajectory distribution or bounds $\rightarrow$ reachable-set comparison. (1138)

Equation 1138 expresses an epistemically conservative movement from one predicted path toward a set of possible futures.

15.78 Reachability as Counterfactual Moderation

Reachability has a particular advantage in uncertain historical cases.

Instead of claiming

The subject would have become x^* .

the theory can claim

The harmful intervention removed or substantially altered a region of futures that was previously accessible.

The second claim can require less counterfactual precision.

It therefore fits the paper's modest approach to historical generativity.

15.79 Reachability and Loss of Chance

Loss-of-chance reasoning can be represented through change in probability of reaching a target region.

Let A be a favorable target. The loss of chance can be represented by

$$\Delta p_A = P_{\text{ref}}(A \text{ reached}) - P_H(A \text{ reached}). \quad (1139)$$

Equation 1139 avoids claiming that A would certainly have occurred.

This structure is conceptually compatible with probabilistic approaches to counterfactual harm.

15.80 Reachability and Option Destruction

An intervention can destroy an option even if the subject would never have chosen it.

Let option y satisfy

$$y \in \mathcal{R}_{\text{ref}} \quad (1140)$$

and

$$y \notin \mathcal{R}_H. \quad (1141)$$

The loss concerns freedom or generativity rather than realized outcome.

Whether such loss is compensable or normatively salient depends upon the theory of justice.

15.81 Reachability and Unchosen Options

Capability theory is especially relevant because substantive freedom can matter independently of which functioning is ultimately chosen [55, 39, 50].

The reachable-set framework supplies a dynamical analogue.

A subject's future freedom can contract even where the realized trajectory remains unchanged.

15.82 Trajectory Constraint Without Endpoint Change

A subject can reach the same endpoint through a more constrained path.

Suppose

$$x_R(t_1) = x_{\text{ref}}(t_1). \quad (1142)$$

The restored trajectory can nevertheless contain fewer branching choices or higher costs.

This distinction is especially relevant to coercion and domination.

Endpoint equality therefore cannot substitute for process analysis.

15.83 Path Freedom

A trajectory can be augmented with the branching options available along it.

Define a path-freedom object

$$\mathcal{F}_\gamma = \{\mathcal{U}(x_t) : t \in [t_0, t_1]\}, \quad (1143)$$

where $\mathcal{U}(x_t)$ is the available action set along the path.

Equation 1143 distinguishes a freely chosen path from an externally imposed path with the same states.

15.84 Path-Freedom Restoration

A stronger trajectory criterion can require

$$\mathcal{F}_{\gamma,R} \subseteq \mathcal{F}_{\gamma,\text{ref}}. \quad (1144)$$

Equation 1144 connects branching-time semantics with relational autonomy and procedural justice.

15.85 Trajectory Burden

A path can also be characterized by burden:

$$B_\gamma = \int_{t_0}^{t_1} b(x_t, u_t) dt. \quad (1145)$$

Two trajectories can reach the same endpoint and differ in B_γ .

The burden can include effort, uncertainty, humiliation, administrative cost, risk, or another selected dimension.

15.86 Burden-Adjusted Trajectory Comparison

A burden-adjusted restorative comparison can require both state/path correspondence and burden correspondence:

$$\gamma_R \sim_E \gamma_{\text{ref}} \quad (1146)$$

and

$$B_{\gamma,R} \leq B_{\gamma,\text{ref}} + \varepsilon_B. \quad (1147)$$

This prevents apparent recovery from concealing much higher internal cost.

15.87 Reachability Under Perturbation

Future possibility can change under disturbance.

Let

$$\mathcal{R}_\tau(x; \delta) \quad (1148)$$

denote the reachable set after perturbation δ .

A robust reachability criterion can require

$$\mathcal{R}_\tau^{\text{ref}}(x_{\text{ref}}; \delta) \subseteq \mathcal{R}_\tau^R(x_R; \delta) \quad (1149)$$

for every $\delta \in \Delta_{\text{adm}}$.

Equation 1149 anticipates the dedicated perturbation section.

15.88 Reachability Fragility

A system can possess a large reachable set under nominal conditions and lose most of it after small perturbation.

Define reachability fragility by a change measure

$$F_R(\delta) = d_S(\mathcal{R}_\tau(x), \mathcal{R}_\tau(x; \delta)). \quad (1150)$$

Large $F_R(\delta)$ indicates fragile option structure.

A restored system can therefore match ordinary reachability and remain less robust.

15.89 State-Space Boundary and Reachability

Reachability depends upon the modeled state space.

If the state space omits social meaning, health, or relational variables, the reachable-set comparison cannot represent them.

Thus

$$\mathcal{R}_\tau^{M_1} \neq \mathcal{R}_\tau^{M_2} \quad (1151)$$

can hold for different representations M_1 and M_2 .

Reachability is therefore representation-relative as well as horizon-relative.

15.90 Reachability and Open-Ended State Spaces

If new states can emerge, the reachable set itself can expand through state-space innovation.

Let

$$\mathcal{X}_{t+1} = \mathcal{X}_t \cup \Delta \mathcal{X}_{t+1}. \quad (1152)$$

A restored system can generate futures that were not representable at the reference time.

This makes strict equality with the historical reachable set potentially undesirable.

Future-directed justice can favor generative expansion.

15.91 Reachability and Innovation

Innovation can be represented as creation of new reachable regions:

$$\Delta \mathcal{R}_{\text{new}} = \mathcal{R}_R \setminus \mathcal{R}_{\text{ref}}. \quad (1153)$$

Equation 1153 can represent new careers, institutions, technologies, or relationships generated after harm.

The existence of valuable new regions complicates any theory defining justice solely through restoration of the old reachable set.

15.92 Historical Loss and New Reachability

A subject can simultaneously satisfy:

$$\mathcal{R}_{\text{lost}} = \mathcal{R}_{\text{ref}} \setminus \mathcal{R}_R \neq \emptyset \quad (1154)$$

and

$$\mathcal{R}_{\text{new}} = \mathcal{R}_R \setminus \mathcal{R}_{\text{ref}} \neq \emptyset. \quad (1155)$$

Equations 1154 and 1155 describe mixed historical loss and new generation.

A complete normative theory must evaluate both without pretending they cancel automatically.

15.93 Reachability Balance

A simplistic scalar balance could compare value of lost and new regions:

$$\Delta V_R = V(\mathcal{R}_{\text{new}}) - V(\mathcal{R}_{\text{lost}}). \quad (1156)$$

Equation 1156 can be useful under a specified value theory.

It does not erase historical attribution.

A subject can be better off now and still have a valid claim concerning the earlier unjust deprivation.

15.94 Reachability and Corrective Justice

Corrective justice can use lost reachable regions as evidence of opportunity loss.

The epistemic challenge is that the size and value of $\mathcal{R}_{\text{lost}}$ can be difficult to identify.

The framework therefore favors explicit uncertainty rather than assuming the full counterfactual set is known.

15.95 Reachability and Assistance

Generative assistance can be evaluated through expansion of viable future regions:

$$\mathcal{R}_{\tau}^H \subset \mathcal{R}_{\tau}^{HA}. \quad (1157)$$

Equation 1157 does not require equality with the no-harm reference.

This makes reachability especially suitable for future-directed assistance.

15.96 Reachability and Transformative Justice

Transformative justice can seek a new reachable structure that dominates the historical one under selected values:

$$\mathcal{R}_\tau^R \succeq_V \mathcal{R}_\tau^{\text{ref}}. \quad (1158)$$

Equation 1158 permits structural change beyond historical reconstruction.
The dominance relation depends upon the value ordering over futures.

15.97 Reachability and Non-Domination

A reachable future can be formally available and dependent upon arbitrary power.
A non-domination-filtered reachable set can therefore be defined as

$$\mathcal{R}_\tau^{\text{ND}} = \{y \in \mathcal{R}_\tau : \text{path to } y \text{ satisfies non-domination conditions}\}. \quad (1159)$$

Equation 1159 distinguishes mere access from secure freedom [44].

15.98 Reachability and Relational Equality

Similarly, a future can be reachable only through subordinate or stigmatizing relations.
A relational-equality-filtered reachable set can require acceptable standing along the path.
This shows how relational justice can modify the geometry of opportunity rather than merely add one terminal value.

15.99 Reachability and Recognition

Recognition can alter reachability by changing how other actors respond to the subject.
A public correction can open employment, collaboration, or membership paths.
Thus recognition is not only a symbolic endpoint; it can transform transition structure.
This connects recognition theory with generative dynamics [25, 20].

15.100 Reachability and Value Circulation

Value circulation affects future reachability because retained value becomes new generative condition.
Let recursive retention be ρ . Then

$$\mathcal{R}_\tau = \mathcal{R}_\tau(x_t, \rho, \Lambda). \quad (1160)$$

Equation 1160 links reachability directly to generative justice [15, 17].
Two subjects with equal present resources can have different future opportunity sets because value circulates differently.

15.101 Reachability and Recursive Generation

The reachable set at horizon τ_2 depends upon conditions generated at earlier horizon τ_1 .
Let

$$c_{t+\tau_1} = \Gamma(\mathcal{G}_{t:t+\tau_1}). \quad (1161)$$

Then later reachability is

$$\mathcal{R}_{\tau_2} = \mathcal{R}(c_{t+\tau_1}, x_{t+\tau_1}). \quad (1162)$$

Equations 1161 and 1162 show why early loss can reshape later option structure recursively.

15.102 Generative Reachability

A specifically generative reachable set can contain states that themselves retain the capacity to generate further valuable possibilities.

Define

$$\mathcal{R}_\tau^G = \{y \in \mathcal{R}_\tau : G(y) \geq G_{\min}\}, \quad (1163)$$

where G is a selected generative criterion.

Equation 1163 distinguishes terminal options from options that support further generation.

The criterion G requires normative specification.

15.103 Recursive Reachability Depth

A future state can be evaluated by the depth of subsequent viable generation it supports.

Let

$$D_G(y) = \max \{n : \text{viable generative continuation exists for } n \text{ stages}\}. \quad (1164)$$

Equation 1164 is conceptually useful for distinguishing fragile one-step opportunities from durable generative pathways.

15.104 Reachability Tree

A branching representation can organize future possibilities as a tree or directed acyclic structure where appropriate.

Let

$$\mathfrak{T}_R^{\text{future}} = (V_R, E_R). \quad (1165)$$

Comparison can concern:

- branching factor;
- path depth;
- desirable leaves;
- bottlenecks;
- probabilities;
- transition costs.

This representation makes future generativity more explicit than a flat set.

15.105 Branching-Factor Comparison

Let $b(x)$ denote the number or effective diversity of available next transitions.

A restorative criterion can compare

$$b_R(x_R) \text{ } E \text{ } b_{\text{ref}}(x_{\text{ref}}). \quad (1166)$$

Equal branching factor remains a weak criterion because the content of branches can differ.

15.106 Branching Diversity

A weighted diversity measure can account for qualitatively different options.

Let

$$D_B = D(p_1, \dots, p_k) \quad (1167)$$

denote a diversity functional over branch weights.

This can distinguish one dominant path plus many negligible alternatives from several meaningful alternatives.

The normative interpretation depends upon what constitutes a meaningful branch.

15.107 Reachability and Bisimulation

Bisimilar transition systems often preserve more branching structure than equal reachable sets.

It is possible to have

$$\mathcal{R}_R = \mathcal{R}_{\text{ref}} \quad (1168)$$

while

$$\mathcal{T}_R \not\sim_{\text{bis}} \mathcal{T}_{\text{ref}}. \quad (1169)$$

The systems can reach the same states through different branching architectures.

Thus reachability equality is generally weaker than branching-sensitive equivalence.

15.108 Reachability and Simulation

Simulation can imply a form of reference behavior preservation stronger than raw set inclusion where transitions are labeled and structured.

If the reference system is simulated by the restored system, every selected reference transition can be matched.

This creates a dynamic interpretation of capability preservation.

Simulation therefore occupies a useful middle ground between reachable-set inclusion and bisimulation.

15.109 State, Trajectory, and Reachability Refinement Path

One analytical refinement path is

$$(Q_{\text{state}}, =) \longrightarrow (Q_{\text{traj}}, \approx) \longrightarrow (Q_{\text{reach}}, \subseteq) \longrightarrow (Q_{\text{transition}}, \preceq_{\text{sim}}) \longrightarrow (Q_{\text{transition}}, \sim_{\text{bis}}). \quad (1170)$$

Equation 1170 progressively incorporates more temporal and branching structure.

The sequence is not a universal normative hierarchy.

15.110 Normative Non-Monotonicity Along the Refinement Path

A stronger formal cell can be less normatively attractive.

Bisimulation can reject valuable new options. Exact trajectory replication can erase valued adaptation. State equality can be normatively sufficient for a narrowly bounded legal purpose.

Formal refinement and justice therefore remain independent dimensions.

15.111 Epistemic Burden Along the Refinement Path

Epistemic requirements generally increase as the comparison moves from state toward transition structure.

- Current state can often be directly observed.
- Trajectory requires longitudinal information.
- Reachability requires a model of possible transitions.
- Simulation requires structured transition semantics.
- Bisimulation requires even richer branching information.
- The atlas should therefore record the evidentiary basis at each level.

15.112 Institutional Fit Along the Refinement Path

Different institutions naturally stop at different levels.

- A court can identify current status and some historical trajectory effects.
- A rehabilitation provider can evaluate function and short-horizon reachability.
- A regulator can study transition structures and repeated patterns.
- A philosophical or ideal-theoretical model can examine bisimulation and full reachable-set structure.
- Institutional restraint can therefore justify use of a formally weaker cell.

15.113 State, Trajectory, and Reachability Under Scarcity

Resource scarcity can affect each object differently.

- A system can restore current state cheaply and future reachability only at much greater cost.
- A subject can receive immediate compensation while long-term network reconstruction remains expensive.
- The distributive theory must decide how scarce resources should be allocated across these layers.

15.114 State, Trajectory, and Reachability Under Uncertainty

Uncertainty also differs across the objects.

- Current state can be highly certain.
- Actual past trajectory can be partially documented.
- Counterfactual trajectory can be uncertain.
- Reachable sets can be model-dependent but robust at coarse structural level.
- This asymmetry can make reachability a useful compromise between current-state minimalism and precise counterfactual biography.

15.115 State, Trajectory, and Reachability Under Reflexivity

Restorative assessment can itself alter all three objects.

- A classification changes current state.
- A public judgment changes subsequent trajectory.
- Knowledge of available assistance changes future choices and reachability.
- The comparison process therefore enters the system being compared.
- This is another expression of reflexive non-closure.

15.116 Restorative State Declaration

Institutions should distinguish statements such as:

- The current formal state has been restored.
- from

The historical trajectory has been restored.

and

The future opportunity structure has been restored.

The first can be true while the latter two are false or unknown.
This linguistic distinction is institutionally important.

15.117 Restorative Trajectory Declaration

A trajectory claim should specify:

- interval;
- path representation;
- temporal metric;
- time-reparameterization allowance;
- reference uncertainty.

Without this metadata, the phrase “career trajectory restored” remains too ambiguous.

15.118 Restorative Reachability Declaration

A reachability claim should specify:

- horizon;
- admissible actions;
- admissible disturbances;
- transition costs;
- deadline constraints;
- security conditions;
- value filters;
- epistemic model.

The raw phrase “opportunities have been restored” otherwise obscures major assumptions.

15.119 State–Trajectory–Reachability Metadata

A complete STR comparison can therefore be represented by

$$\Theta_{\text{STR}} = (B, X, D_{\Gamma}, \mathcal{U}_{\text{adm}}, \Xi_{\text{adm}}, \tau, \mathbf{C}, V, K), \quad (1171)$$

where:

- B is the reference;
- X is the state representation;

Table 8: State, trajectory, trace, and reachability in restorative comparison

Object	Preserved information	Characteristic restorative question	Characteristic limitation
State	Current represented configuration	Where is the system now?	Does not determine path or future dynamics
Trajectory	Ordered realized evolution	How did the system reach the current condition?	Does not represent unexercised possibilities
Observable trace	Observed sequence under selected map	Does outward behavior correspond?	Can hide internal and branching differences
Orbit	States visited under dynamics	Is qualitative evolution structurally similar?	Can abstract away timing
Reachable set	Possible future states	Which futures remain possible?	Ignores path costs and probabilities unless enriched
Viable set	Future states maintainable within constraints	Which futures remain sustainably available?	Depends upon admissibility criteria
Recognized reachable set	Possibilities perceived as actionable by subject	Which futures can the subject meaningfully pursue?	Difficult to observe and historically adaptive
Cost-constrained reachability	Futures reachable within resource or burden limits	Which options are practically usable?	Requires multidimensional cost model

- D_Γ is the trajectory relation or distance;
- $\mathcal{U}_{\text{adm}}$ is the admissible action class;
- Ξ_{adm} is the disturbance class;
- τ is the horizon;
- $\mathbf{C}$ is the transition-cost structure;
- V is the value filter;
- K is the epistemic basis.

Equation 1171 provides a compact formal schema for this region of the atlas.

15.120 A Comparative STR Matrix

Table 8 summarizes the main differences among state, trajectory, trace, and reachability objects.

Table 8 shows why no one of these objects can substitute universally for the others.

15.121 State Restoration Principle

The first principle of this section is:

Equality of present represented state establishes only state-relative restoration and should not be interpreted as trajectory, reachability, or dynamic restoration without additional evidence.

15.122 Trajectory Principle

The second principle is:

Endpoint correspondence does not determine the justice of the path through which the endpoint was reached.

This principle gives formal content to the distinction between current result and historical process.

15.123 Reachability Principle

The third principle is:

Future generativity is represented more directly by structures of reachability, viability, and transition than by current state alone.

This principle motivates the paper's emphasis on future possibility.

15.124 Branching Principle

The fourth principle is:

Equality of realized trajectories or trace sets does not establish equality of the branching choices available along those trajectories.

This principle connects formal process semantics with agency.

15.125 Practical-Accessibility Principle

The fifth principle is:

Formal reachability should be distinguished from practical reachability once time, cost, information, trust, security, and control are taken into account.

15.126 Counterfactual-Modesty Principle

The sixth principle is:

Where precise long-horizon counterfactual trajectories are weakly identified, restorative analysis should consider whether distributions, reachable sets, bounds, or structural relations support a more defensible claim.

15.127 Future-Expansion Principle

The seventh principle is:

A just post-harm future can legitimately contain viable possibilities absent from the no-harm reference, so preservation and expansion of valuable reachability can be normatively preferable to exact future-set equality.

15.128 Historical-Residual Principle

The eighth principle is:

Expansion of present and future opportunity does not by itself erase the historical significance of opportunities previously lost.

This principle preserves corrective history alongside future-directed generation.

15.129 Implications for Dynamical Restoration

The distinctions developed in this section lead directly to dynamical restoration.

A system can match current state and differ in attractor structure.

It can possess a similar reachable set and differ in basin geometry.

It can recover ordinary trajectories and remain closer to a critical transition.

It can regain a stable mode while becoming more vulnerable to perturbation.

The next section therefore moves from state, path, and reachability to the long-run organization of dynamics.

15.130 Transition to Attractors, Basins, and Stability

Section 16 develops restorative comparison of attractors, basins, stability, bifurcation structure, regime transitions, criticality, and resilience.

The section asks whether two systems that appear restored at the level of current state or ordinary trajectory can nevertheless occupy different dynamical landscapes, and which comparative relations are appropriate for evaluating those differences.

16 Attractors, Basins, Stability, and Critical Transitions

This section develops the dynamical region of the Atlas of Equivalence. The preceding section distinguished present state, realized trajectory, and future reachability. The present section moves from these objects toward the organization of the dynamical landscape itself: attractors, basins of attraction, stability, resilience, bifurcations, regime transitions, hysteresis, recovery dynamics, and proximity to critical boundaries.

The distinction is important for restorative justice. Two systems can occupy nearly identical present states while possessing different attractors, basin geometries, recovery rates, or critical thresholds. A worker can return to ordinary performance while remaining much more vulnerable to organizational disturbance. An institution can reproduce previous aggregate output while operating closer to a regime of collapse. An ecological system can regain a familiar observable state while possessing reduced resilience and a narrower basin of attraction. Conversely, a transformed post-harm system can possess a different attractor landscape that is normatively preferable to the historical reference.

Dynamical concepts therefore provide comparisons unavailable at the level of state equality alone. They also require substantial methodological restraint. The social systems considered in this paper are not presumed to possess low-dimensional deterministic attractors merely because attractor language is analytically suggestive. Attractors, basins, bifurcations, and stability are formal objects available where a defensible dynamical representation supports them. The framework remains compatible with stochastic, high-dimensional, time-varying, partially observed, and open-ended systems for which weaker dynamical objects are more appropriate.

The section proceeds from attractor and basin definitions through several forms of stability and resilience. It then examines multistability, bifurcation, critical transition, hysteresis, early-warning structure, stochastic regime change, basin deformation, and restoration of dynamical landscapes. The final subsections locate these concepts within generative justice and specify the object–relation cells available for later restorative analysis.

16.1 Dynamical Landscapes as Analytical Representations

Let a continuous-time generative system be represented by Equation 1172.

$$\dot{x} = F(x, u, \xi; \theta), \quad (1172)$$

where x denotes state, u intervention or action, ξ disturbance, and θ relevant parameters. Equation 1172 provides the baseline dynamical representation used in this section.

A discrete-time system can be represented by Equation 1173.

$$x_{t+1} = \Phi(x_t, u_t, \xi_t; \theta_t). \quad (1173)$$

Equation 1173 permits dynamical comparison without assuming differentiability or continuous time.

The formal concepts developed below apply only after the relevant system, variables, time scale, and transition rules have been specified. This condition is especially important in social applications because the effective evolution law can itself change through learning, policy, interpretation, or institutional transformation.

16.2 Invariant Sets

An invariant set is a region preserved by the system's dynamics. For a flow Φ^t , set $A \subseteq \mathcal{X}$ is invariant when Equation 1174 holds.

$$\Phi^t(A) = A \quad \text{for every relevant } t. \quad (1174)$$

Equation 1174 provides a general object from which equilibria, periodic orbits, and more complicated invariant structures can be distinguished.

Invariant-set comparison can be useful even where the stronger language of an attractor is unjustified. A recurring institutional configuration or persistent region of social behavior can sometimes be represented as an invariant or approximately invariant region without assuming convergence from a large surrounding neighborhood.

16.3 Fixed Points

A fixed point x^* of a discrete-time system satisfies Equation 1175.

$$\Phi(x^*) = x^*. \quad (1175)$$

Equation 1175 represents a state preserved by one step of the dynamics.

For the continuous-time system in Equation 1172, an equilibrium satisfies Equation 1176.

$$F(x^*; \theta) = 0. \quad (1176)$$

Equation 1176 defines an equilibrium relative to the selected model and parameter values.

A restorative comparison based only upon equilibrium location remains limited. Two systems can possess the same equilibrium and differ greatly in stability, basin size, convergence rate, or response to perturbation.

16.4 Attractors

An attractor is an invariant set toward which states in an associated neighborhood evolve under the selected dynamics. Because formal definitions vary across dynamical settings, the present paper uses the term at the level appropriate to the model rather than imposing one universal definition.

Let $\mathcal{A}$ be an attracting invariant set. A simplified convergence condition is represented by Equation 1177.

$$d(\Phi^t(x), \mathcal{A}) \longrightarrow 0 \quad \text{as } t \longrightarrow \infty \quad (1177)$$

for states x in an appropriate attracting region. Equation 1177 captures the restorative feature of interest: long-run behavior can be organized around a persistent set even where current state varies.

The attractor itself can be a fixed point, periodic orbit, invariant manifold, more complicated invariant set, or another structure appropriate to the formal system [27].

16.5 Attractor Families

A system can possess several attractors. Let the attractor family be represented by Equation 1178.

$$\mathfrak{A}(\mathcal{G}) = \{\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_m\}. \quad (1178)$$

Equation 1178 makes multistability explicit.

A restorative system can therefore differ from the reference through:

- disappearance of an attractor;
- creation of a new attractor;
- displacement of attractor location;
- alteration of attractor type;
- change in accessibility of an attractor;
- change in the basin associated with an attractor;
- change in transition probability among attractors.

Attractor count alone cannot capture these differences.

16.6 Basins of Attraction

The basin associated with attractor $\mathcal{A}$ contains states whose trajectories approach that attractor under the selected dynamics. A simplified basin definition is given by Equation 1179.

$$\mathcal{B}(\mathcal{A}) = \{x \in \mathcal{X} : d(\Phi^t(x), \mathcal{A}) \longrightarrow 0\}. \quad (1179)$$

Equation 1179 adds a crucial dimension to attractor comparison.

Two systems can possess corresponding attractors while the set of states that return to those attractors after disturbance differs substantially.

Eglash and Garvey's analysis of "basins of attraction for generative justice" provides a direct precedent for connecting attractor-basin structures with generative forms of justice [16]. The present framework treats basin structure as one element within a broader atlas rather than as a complete theory of justice.

16.7 Basin Geometry

Basin geometry concerns the shape and organization of $\mathcal{B}(\mathcal{A})$.

Relevant properties include:

- basin size or measure;
- connectedness;
- distance from current state to basin boundary;
- narrow corridors;
- coexistence with competing basins;
- boundary complexity;
- dependence upon parameter values.

Basin geometry is a stronger restorative object than attractor identity alone because it describes how much disturbance the system can absorb before moving toward another regime.

16.8 Basin Measure

Where a suitable measure μ exists, basin size can be summarized by Equation 1180.

$$M_{\mathcal{B}} = \mu(\mathcal{B}(\mathcal{A})). \quad (1180)$$

Equation 1180 provides a coarse measure of the region associated with the attractor.

Equal basin measure does not imply equal basin geometry. One basin can be broad and compact while another is fragmented or contains narrow vulnerable regions.

16.9 Distance to Basin Boundary

Let $\partial\mathcal{B}$ denote the boundary of the current attractor basin. A simple local robustness quantity is represented by Equation 1181.

$$d_{\partial\mathcal{B}}(x) = \inf_{y \in \partial\mathcal{B}} d(x, y). \quad (1181)$$

Equation 1181 measures distance from current state to the nearest represented basin boundary.

A restored subject can occupy the same observable state as the reference while being much closer to a critical boundary in latent or relational dimensions.

The quantity therefore provides one formal representation of hidden fragility.

16.10 State Restoration and Basin Restoration

Present-state equality and basin correspondence are logically distinct.

It is possible that Equation 1182 holds.

$$x_R = x_{\text{ref}} \quad (1182)$$

while Equation 1183 also holds.

$$\mathcal{B}_R \neq \mathcal{B}_{\text{ref}}. \quad (1183)$$

Together, Equations 1182 and 1183 establish that state restoration does not imply basin restoration.

This distinction is particularly important for post-harm systems whose ordinary performance has returned while vulnerability to future perturbation remains high.

16.11 Attractor Restoration and Basin Non-Equivalence

Corresponding attractors can coexist with different basin structures:

$$\mathcal{A}_R \cong \mathcal{A}_{\text{ref}} \quad \text{and} \quad \mathcal{B}_R \not\cong \mathcal{B}_{\text{ref}}. \quad (1184)$$

Equation 1184 shows why restoration of the long-run mode does not establish restoration of resilience around that mode.

16.12 Local Stability

Stability concerns behavior of trajectories beginning near a reference state or invariant set.

For equilibrium x^* , Lyapunov stability informally requires that sufficiently small initial deviations remain small. A standard form is represented by Equation 1185.

$$\forall \varepsilon > 0 \exists \delta > 0 : d(x_0, x^*) < \delta \implies d(\Phi^t(x_0), x^*) < \varepsilon \quad \forall t \geq 0. \quad (1185)$$

Equation 1185 states a local persistence property rather than a recovery property.

A system can be stable without trajectories converging back rapidly after disturbance.

16.13 Asymptotic Stability

An asymptotically stable equilibrium combines local stability with local convergence.

A simplified convergence condition is represented by Equation 1186.

$$x_0 \in U(x^*) \implies \Phi^t(x_0) \longrightarrow x^*. \quad (1186)$$

Equation 1186 adds return toward the equilibrium from an appropriate neighborhood.

Restorative comparison can therefore distinguish whether a state is merely locally stable or actively attracting.

16.14 Stability Rate

Two asymptotically stable systems can converge at different rates.

A stylized local convergence relation is given by Equation 1187.

$$d(\Phi^t(x), x^*) \leq Ce^{-\lambda t} d(x, x^*), \quad (1187)$$

where $\lambda > 0$ represents a convergence rate under the specified model. Equation 1187 illustrates how recovery speed can become a separate restorative object.

The paper does not assume exponential recovery for social systems. The equation is an idealized example of a rate-sensitive comparison.

16.15 Recovery Rate and Restorative Burden

A slower recovery rate can impose cumulative burden even when eventual state is the same.

Let recovery error be $e(t)$. Cumulative recovery burden is represented by Equation 1188.

$$B_{\text{rec}} = \int_0^T e(t) dt. \quad (1188)$$

Equation 1188 distinguishes eventual convergence from the cost of reaching it.

16.16 Stability and Resilience

Stability and resilience should remain analytically distinct. Holling's classic analysis distinguishes behavior near equilibrium from the capacity of systems to persist through disturbance and remain within a regime [24].

Within the present framework, stability concerns the behavior of trajectories around a state or invariant set. Resilience concerns the system's ability to absorb perturbation, preserve essential organization, remain within an acceptable regime, or recover after disturbance.

A general resilience functional can be represented by Equation 1189.

$$\varrho = \varrho(\mathcal{G}, \Delta_{\text{adm}}, \tau), \quad (1189)$$

where Δ_{adm} is a perturbation class and τ a temporal horizon. Equation 1189 makes resilience explicitly perturbation- and horizon-dependent.

16.17 Engineering and Regime-Oriented Resilience

Two broad resilience questions are useful.

One concerns speed of return toward a selected state after disturbance.

Another concerns how large a disturbance can be absorbed before the system moves into another regime.

These questions can produce different rankings.

A system can return rapidly from small perturbations while possessing a narrow basin and therefore low tolerance for larger shocks. Another system can recover slowly while remaining within the same broad regime under much larger disturbances.

Restorative claims should therefore identify the resilience concept employed.

16.18 Resilience Radius

A simplified resilience radius can be defined as the minimum perturbation magnitude required to move the system outside the selected basin.

Let perturbation operator be P_δ . The radius is represented by Equation 1190.

$$r_{\text{res}} = \inf \{ \|\delta\| : P_\delta(x) \notin \mathcal{B}(\mathcal{A}) \}. \quad (1190)$$

Equation 1190 provides an idealized robustness measure.

The result depends strongly upon the perturbation norm and direction. Social disturbances rarely have a natural single magnitude scale.

16.19 Directional Resilience

Perturbations of equal nominal magnitude can have different effects in different directions.

Let unit direction be v . A directional resilience threshold can be written as Equation 1191.

$$r_{\text{res}}(v) = \inf \{ a \geq 0 : x + av \notin \mathcal{B}(\mathcal{A}) \}. \quad (1191)$$

Equation 1191 shows why one scalar resilience number can be insufficient.

A worker can remain robust to workload variation and highly vulnerable to a specific authority-related cue. An institution can tolerate financial shocks and remain fragile to legitimacy crises.

16.20 Resilience Profile

A richer object is the resilience profile across disturbance types:

$$\varrho = \{ \varrho(\delta) : \delta \in \Delta_{\text{adm}} \}. \quad (1192)$$

Equation 1192 supports multidimensional perturbation comparison.

The dedicated perturbation section will later develop this object further.

16.21 Alternative Stable States

A system can possess more than one locally stable regime under the same or similar parameter conditions.

Let

$$\mathfrak{A}_{\text{stable}} = \{ \mathcal{A}_1, \dots, \mathcal{A}_m \}, \quad m > 1. \quad (1193)$$

Equation 1193 represents multistability.

The existence of alternative attractors is central to critical-transition analysis and is developed extensively in Scheffer's treatment of complex systems [52].

For restorative theory, multistability matters because return of parameters to their previous values can fail to return the system to its previous attractor.

16.22 Multistability and Restorative Ambiguity

Suppose both desirable and undesirable attractors exist.

A resource transfer can move the current state without altering the landscape sufficiently to leave the undesirable basin.

Conversely, a structural reform can change basin geometry without producing an immediate observable state improvement.

The restorative theory must therefore distinguish intervention upon state from intervention upon the dynamical landscape.

16.23 State Intervention

A state intervention changes x while leaving the transition law fixed:

$$x \xrightarrow{R_x} x', \quad F \text{ unchanged.} \quad (1194)$$

Equation 1194 can describe a one-time resource transfer, immediate reinstatement, or another intervention primarily affecting current state.

If x' remains inside the same undesirable basin, long-run dynamics can return the system toward the earlier regime.

16.24 Structural Intervention

A structural intervention modifies the evolution law or its parameters:

$$F \xrightarrow{R_F} F'. \quad (1195)$$

Equation 1195 can alter attractor existence, location, basin geometry, or stability.

Examples can include changes in institutional authority, anti-retaliation structure, funding architecture, governance rights, or value circulation.

Structural intervention is therefore central to transformative restoration.

16.25 Combined State and Structural Intervention

Many restorative processes require both interventions:

$$(x, F) \xrightarrow{R} (x', F'). \quad (1196)$$

Equation 1196 separates immediate correction from alteration of the processes that would otherwise recreate the harmful condition.

16.26 Bifurcation Structure

Let the dynamics depend upon parameter θ :

$$\dot{x} = F(x; \theta). \quad (1197)$$

Equation 1197 permits qualitative dynamical structure to change as θ varies.

A bifurcation occurs when a qualitative property of the dynamical system changes at a critical parameter configuration.

The bifurcation object can include:

- critical parameter values;
- appearance or disappearance of equilibria;

- change of stability;
- emergence of cycles;
- change in basin organization;
- transition among qualitative regimes.

The specific mathematical definition depends upon the model class [27, 52].

16.27 Bifurcation Threshold

Let θ_c denote a critical parameter at which a selected transition occurs.

A restorative system can have threshold $\theta_{c,R}$ and reference threshold $\theta_{c,\text{ref}}$.

Threshold restoration can be represented by Equation 1198.

$$|\theta_{c,R} - \theta_{c,\text{ref}}| \leq \varepsilon_\theta. \quad (1198)$$

Equation 1198 compares the point at which qualitative change occurs.

A restored system can therefore match present state and remain much closer to a critical threshold.

16.28 Critical Transitions

A critical transition is a large qualitative change in system behavior associated with movement across a threshold, loss of stability, or another mechanism of regime change.

Scheffer's work develops critical transitions across ecological and social examples, while Scheffer and colleagues review generic early-warning patterns associated with some classes of transitions [52, 53].

Within the present framework, a critical transition is relevant to restoration when harm shifts the system into another regime or changes its proximity to such a shift.

16.29 Distance to Critical Transition

Let $\mathcal{C}$ denote a represented critical boundary. A proximity measure can be defined by Equation 1199.

$$d_{\text{crit}}(x) = \inf_{y \in \mathcal{C}} d(x, y). \quad (1199)$$

Equation 1199 provides one measure of distance from the current state to the selected critical boundary.

A system can therefore be restored in ordinary observable variables while remaining closer to a regime transition.

16.30 Criticality Profile

Criticality can be multidimensional where several boundaries or transition mechanisms exist.

A profile can be represented by Equation 1200.

$$\mathbf{d}_{\text{crit}} = (d_1, d_2, \dots, d_k). \quad (1200)$$

Equation 1200 prevents one scalar from obscuring different failure modes.

16.31 Critical Slowing Down

Near some bifurcation-induced transitions, recovery from small perturbations can become slower. This phenomenon is commonly described as *critical slowing down* [53].

Let local perturbation amplitude be δx_0 . A simple recovery model is represented by Equation 1201.

$$\delta x(t) \approx \delta x_0 e^{\lambda t}, \quad \lambda < 0. \quad (1201)$$

Equation 1201 implies slower recovery as λ approaches zero from below.

This local form should be treated as model-dependent rather than as a universal diagnostic of social transition.

16.32 Early-Warning Indicators

Scheffer and colleagues identify several statistical patterns that can appear before certain critical transitions, including increasing recovery time, autocorrelation, and variance [53].

Let X_t be an observed time series. A lag-one autocorrelation statistic is represented by Equation 1202.

$$\rho_1 = \text{Corr}(X_t, X_{t-1}). \quad (1202)$$

Equation 1202 provides one possible early-warning observable.

Its increase should not be interpreted as a universal proof that a transition is imminent. Different mechanisms can produce similar patterns, and some critical transitions lack reliable generic warnings.

16.33 Variance as an Early-Warning Observable

A second statistic is local variance:

$$\sigma_X^2 = \text{Var}(X_t). \quad (1203)$$

Equation 1203 can increase under some forms of reduced local stability.

Again, the statistic requires domain-specific interpretation and should be combined with mechanistic understanding where possible [53].

16.34 Early-Warning Signals and Preventive Justice

Early-warning analysis has a natural connection with the pre-harm and during-harm temporal architecture developed earlier.

Where the cost of crossing a critical boundary is large and reparability is low, credible evidence of decreasing resilience can increase the relevance of early intervention.

This implication remains constrained by:

- false-positive risk;
- uncertainty concerning the model;
- cost of intervention;
- rights and autonomy;
- distribution of preventive burdens;
- reversibility of the proposed response.

A statistical warning should therefore trigger proportionate inquiry rather than automatic coercive intervention.

16.35 Hysteresis

Hysteresis describes situations in which the path of return differs from the path through which the transition occurred.

Let parameter θ increase until transition at $\theta_{\uparrow}$, and later decrease. Recovery can occur only after reaching a different threshold $\theta_{\downarrow}$.

Hysteresis is represented schematically by Equation 1204.

$$\theta_{\downarrow} \neq \theta_{\uparrow}. \quad (1204)$$

Equation 1204 shows why restoration of the original parameter value can fail to restore the original regime.

Scheffer's treatment of alternative stable states and critical transitions provides extensive examples of this structure [52].

16.36 Hysteresis and the Justice of Return

Hysteresis provides a particularly clear formal analogue of the paper's generative-return problem.

Suppose condition c deteriorates and the system transitions into another regime. Later restoration of c to its pre-harm value can leave the system in the altered regime.

The result can be represented by Equation 1205.

$$c_R = c_{\text{pre}} \quad \text{while} \quad x_R \in \mathcal{A}_H. \quad (1205)$$

Equation 1205 gives a dynamical form to the principle that condition restitution can fail to restore generativity.

16.37 Path Dependence and Dynamical Return

Hysteresis is one formal mechanism of path dependence. Broader path dependence can arise through learning, network accumulation, increasing returns, and institutional feedback [3, 45].

A present state can therefore depend upon the route through parameter and state space.

Dynamical restoration should consequently identify whether the reference assumes reversible movement through the same path or permits a different return path.

16.38 Alternative Return Path

A restorative intervention can return the system to a desirable regime through a path unavailable in the original history.

Let the harmful path be γ_H and restorative path be γ_R . A successful transformation can satisfy Equation 1206.

$$\gamma_R \neq \gamma_{\text{ref}} \quad \text{and} \quad x_R \in \mathcal{N}_{\text{desirable}}. \quad (1206)$$

Equation 1206 formalizes recovery through a new path.

Historical fidelity remains low while normative outcome can be high.

16.39 Regime Restoration

A weaker dynamical goal than state restoration is return to the same desirable regime.

Let regime map be $Z(x)$. Regime restoration requires Equation 1207.

$$Z(x_R) = Z(x_{\text{ref}}). \quad (1207)$$

Equation 1207 permits internal variation within the regime.

This criterion can be appropriate where exact state identity lacks normative importance.

16.40 Regime Dominance

A transformed system can occupy a regime evaluated as superior to the historical reference.

Let regime ordering be $\succeq_V$. The relation is represented by Equation 1208.

$$Z_R \succeq_V Z_{\text{ref}}. \quad (1208)$$

Equation 1208 supports transformative justice.

A non-dominating institution can be normatively preferable to the pre-harm institution even if their attractor structures differ substantially.

16.41 Normative Classification of Attractors

Attractors are mathematically descriptive objects. Justice requires a separate normative classification.

Let

$$V_A : \mathfrak{A} \longrightarrow \mathfrak{V} \quad (1209)$$

assign a normative interpretation to an attractor. Equation 1209 makes explicit that dynamical stability and normative desirability are different properties.

A stable system can be unjust. An unstable system can be moving away from injustice. A highly resilient extractive institution can be normatively worse than a fragile transitional system moving toward reform.

16.42 Stable Injustice

A harmful or dominating configuration can form a persistent attractor-like regime.

Its stability does not create normative legitimacy.

This distinction is particularly important for governance because interventions designed to maximize stability can entrench unjust structures.

The restorative objective should therefore be defined over *normatively evaluated* dynamical structure rather than stability alone.

16.43 Unstable Justice Transitions

A system undergoing just transformation can temporarily exhibit instability.

Institutional reform can disrupt established routines. Redistribution of authority can create temporary uncertainty. New participatory structures can require learning.

Short-run instability therefore cannot automatically be interpreted as restorative failure.

A temporal and normative theory is required to distinguish transitional instability from deterioration.

16.44 Attractor Fidelity and Normative Improvement

Historical attractor fidelity and normative improvement can diverge.

Let F_A denote similarity to the reference attractor structure and V normative evaluation.

It is possible that Equation 1210 holds.

$$F_A(\mathcal{G}_{R_2}) < F_A(\mathcal{G}_{R_1}) \quad (1210)$$

while Equation 1211 also holds.

$$V(\mathcal{G}_{R_2}) > V(\mathcal{G}_{R_1}). \quad (1211)$$

Together, Equations 1210 and 1211 demonstrate normative non-monotonicity in dynamical fidelity.

16.45 Basin Expansion

For a normatively desirable attractor, a transformed restored system can possess a basin containing the reference basin:

$$\mathcal{B}_{\text{ref}} \subseteq \mathcal{B}_R. \quad (1212)$$

Equation 1212 represents one form of dynamical dominance.

The restored system preserves all represented reference states that converge toward the desirable regime and adds further states with the same long-run destination.

This can be more attractive than exact basin equality.

16.46 Basin Contraction

A restored system can preserve the attractor while possessing a smaller basin:

$$\mathcal{B}_R \subset \mathcal{B}_{\text{ref}}. \quad (1213)$$

Equation 1213 represents reduced robustness.

A subject can therefore appear recovered while requiring a narrower range of future conditions to remain stable.

16.47 Competing Basin Expansion

Restorative analysis should also examine undesirable basins.

Let $\mathcal{B}^-$ denote the basin of an undesirable regime. An intervention can be normatively favorable where Equation 1214 holds.

$$\mathcal{B}_R^- \subset \mathcal{B}_{\text{ref}}^-. \quad (1214)$$

Equation 1214 can matter even where the current state has not yet changed substantially.

This is a structural form of prevention.

16.48 Basin Redistribution

In multistable systems, policy can redistribute the relative size or accessibility of several basins.

Let basin measures be $\mu(\mathcal{B}_1), \dots, \mu(\mathcal{B}_m)$.

A basin profile is represented by Equation 1215.

$$\mathbf{B} = (\mu(\mathcal{B}_1), \dots, \mu(\mathcal{B}_m)). \quad (1215)$$

Equation 1215 provides a coarse summary of the dynamical landscape.

Its normative interpretation requires classification of the attractors.

16.49 Basin Accessibility

Basin size and accessibility are distinct.

A large desirable basin can be difficult to reach from the current state because barriers, costs, or intervention limits constrain transition.

Let minimum intervention cost to enter basin $\mathcal{B}$ be represented by Equation 1216.

$$C_{\text{enter}}(x, \mathcal{B}) = \inf_u C(u) \quad \text{subject to } \Phi^u(x) \in \mathcal{B}. \quad (1216)$$

Equation 1216 connects basin theory with practical restorability.

16.50 Escape from Harmful Basins

Where the current state lies within a harmful basin, assistance can aim to cross its boundary rather than reproduce the original state directly.

Let $\mathcal{B}^-$ be the harmful basin. An escape intervention satisfies Equation 1217.

$$R(x) \notin \mathcal{B}^-. \quad (1217)$$

Equation 1217 describes one future-oriented form of generative assistance.

The target after escape can be a newly selected desirable regime rather than the historical reference.

16.51 Assistance as Basin Transition

Some assistance can be interpreted as enabling a transition from one basin to another.

Examples can include:

- sufficient income to exit an exploitative relation;
- retraining enabling movement into another profession;
- legal status permitting entry into a different institutional field;
- rehabilitation enabling participation in another activity regime.

The basin metaphor becomes useful only where the underlying model supports a persistent dynamical distinction. It should not be applied merely as rhetorical language.

16.52 Control of Basin Geometry

Governance can act upon the landscape rather than one subject.

Changing rules, authority, resource distribution, or institutional interfaces can alter basin structure for many subjects simultaneously.

A parameter intervention is represented by Equation 1218.

$$\theta \xrightarrow{R_\theta} \theta'. \quad (1218)$$

Equation 1218 can move basin boundaries or change attractor existence.

This is a formal analogue of structural governance.

16.53 Governance of Transition Structure

A deeper intervention changes the transition rule itself:

$$\Phi \xrightarrow{R_\Phi} \Phi'. \quad (1219)$$

Equation 1219 can represent institutional redesign that changes how future events translate into states.

This intervention is distinct from moving subjects within an unchanged institutional landscape.

16.54 Restoration of Rules and Restoration of Outcomes

The same outcome can be produced under different rule structures.

A subject can achieve equal income through secure rights or discretionary support.

A community can receive equal resources through self-governance or external administration.

Outcome equality therefore does not establish dynamical-rule equivalence.

The distinction is central to non-domination and generative justice [44, 15].

16.55 Structural Stability

Structural stability concerns whether qualitative dynamical organization remains unchanged under sufficiently small perturbations of the dynamical system itself.

The concept occupies a central place in dynamical-systems theory [27].

For restorative analysis, structural stability can be interpreted as robustness of the represented dynamical organization to small model or parameter changes.

A transformed institution whose desirable behavior depends upon exact fine-tuning can be less robust than one whose qualitative structure persists across a neighborhood of configurations.

16.56 Approximate Structural Stability

Empirical social systems can require a weaker criterion.

Let $\mathcal{N}_\delta(F)$ denote a neighborhood of transition laws around F . An approximate structural criterion can require Equation 1220.

$$F' \in \mathcal{N}_\delta(F_R) \implies \text{Qual}(F') \sim_E \text{Qual}(F_R), \quad (1220)$$

where Qual extracts the qualitative structure relevant to the restorative purpose. Equation 1220 provides a model-relative robustness criterion.

16.57 Parameter Robustness

A simpler criterion evaluates variation in selected parameters.

Let admissible parameter neighborhood be Θ_δ . A robust restorative condition can require Equation 1221.

$$Q(\mathcal{G}_R(\theta)) \sim_E Q(\mathcal{G}_{\text{ref}}(\theta)) \quad \forall \theta \in \Theta_\delta. \quad (1221)$$

Equation 1221 tests restoration across a parameter range rather than at one calibrated point.

16.58 Parameter Drift

Real institutions and environments change through time.

Let parameter evolution satisfy Equation 1222.

$$\dot{\theta} = G(\theta, t). \quad (1222)$$

Equation 1222 makes long-term drift explicit.

A restorative arrangement can be robust under current parameters and fragile under plausible future drift.

16.59 Moving Attractors

When parameters change, attractors themselves can move.

Let

$$\mathcal{A} = \mathcal{A}(\theta(t)). \quad (1223)$$

Equation 1223 shows why a fixed historical attractor can become an inappropriate long-term target. This structure is especially important under technological, institutional, and environmental change.

16.60 Rate-Induced Transition

A system can fail to track a moving stable regime when external conditions change too rapidly, even if the corresponding equilibrium remains locally stable under frozen parameters.

The present paper treats this as a general reminder that transition risk can depend upon the rate of environmental change as well as parameter values.

A restorative policy therefore requires attention to adaptation time and speed of institutional change.

16.61 Time-Varying Basin Structure

When dynamics vary through time, basin structure becomes time-indexed:

$$\mathcal{B} = \mathcal{B}(t). \quad (1224)$$

Equation 1224 limits the usefulness of a static landscape metaphor for rapidly changing social systems.

Restorative comparison can therefore require finite-time or time-dependent basin concepts rather than asymptotic fixed landscapes.

16.62 Finite-Time Stability

An institution can require stability only over a finite operational horizon.

Let horizon be τ . A finite-time stability criterion can require Equation 1225.

$$d(x(t), x^*(t)) \leq \varepsilon \quad t \in [t_0, t_0 + \tau]. \quad (1225)$$

Equation 1225 is more realistic than asymptotic stability for some policy and rehabilitation settings.

16.63 Metastability

A system can remain for long periods within a region that is not permanently stable.

Such metastable configurations can look like restored regimes over ordinary observation horizons.

The distinction becomes important when a system has a long escape time relative to institutional monitoring.

A restorative theory should therefore avoid equating long residence with permanent stability.

16.64 Residence Time

Let $\mathcal{D}$ be a region representing the current regime. Residence time is represented by Equation 1226.

$$T_{\text{res}} = \inf \{t > 0 : X_t \notin \mathcal{D}\}. \quad (1226)$$

Equation 1226 can be stochastic.

A system with equal current state and lower expected residence time can be less dynamically secure.

16.65 Stochastic Dynamics

Noise can induce transitions even where deterministic dynamics remain within a basin.

A stochastic representation can be written as Equation 1227.

$$dX_t = F(X_t; \theta) dt + \Sigma(X_t) dW_t. \quad (1227)$$

Equation 1227 introduces stochastic disturbance through W_t .

The model is illustrative and should be used only where such a representation is appropriate.

16.66 Noise-Induced Transitions

Under stochastic dynamics, a system can leave a deterministic basin because of random fluctuation.

The relevant restorative object can therefore be transition probability rather than deterministic basin membership.

Let $p_{i \rightarrow j}(\tau)$ be the probability of transition from regime i to regime j within horizon τ . A comparison is represented by Equation 1228.

$$p_{i \rightarrow j}^R(\tau) \leq p_{i \rightarrow j}^{\text{ref}}(\tau). \quad (1228)$$

Equation 1228 extends basin comparison to stochastic systems.

16.67 Transition Hazard

A regime-transition hazard can be represented by

$$h_{i \rightarrow j}(t) = \lim_{\Delta t \rightarrow 0} \frac{P(T_{i \rightarrow j} \in [t, t + \Delta t] \mid T_{i \rightarrow j} \geq t)}{\Delta t}. \quad (1229)$$

Equation 1229 provides a time-sensitive object for stochastic regime vulnerability.

A restored system can have equal current state and larger transition hazard.

16.68 Stationary Distribution Across Regimes

Where a stochastic system admits a meaningful stationary distribution over regimes, let

$$\pi = (\pi_1, \dots, \pi_m). \quad (1230)$$

Equation 1230 represents long-run occupation probabilities.

Distributional restoration can compare π_R with π_{ref} .

This system-level relation does not recover individual historical paths.

16.69 Potential-Landscape Representation

Some gradient-like systems permit representation through a potential function $U(x)$ satisfying a relation such as Equation 1231.

$$\dot{x} = -\nabla U(x). \quad (1231)$$

Equation 1231 allows minima of U to correspond to stable equilibria in the simple gradient case.

The familiar “landscape” image of wells and barriers is intuitive in such systems.

The paper does not assume that every social dynamical system admits a global potential function. Many systems are non-gradient, path-dependent, nonstationary, stochastic, or reflexive.

Landscape language should therefore remain formal where its mathematical conditions are satisfied and heuristic elsewhere.

16.70 Barrier Height

Within a defensible potential representation, barrier height can approximate difficulty of transition between regimes.

Let a stable state lie at x_a and saddle or boundary point at x_s . Barrier height is represented by Equation 1232.

$$\Delta U = U(x_s) - U(x_a). \quad (1232)$$

Equation 1232 can provide a formal analogue of transition difficulty.

Its use outside gradient-like systems requires another formal definition rather than literal transfer of the potential metaphor.

16.71 Dynamical Landscape Transformation

A structural restorative intervention can alter:

- attractor location;
- number of attractors;
- basin geometry;
- boundary location;
- transition probabilities;
- recovery rates;
- parameter thresholds.

Such transformation is broader than movement of the current state.
The object of justice can therefore be the architecture of future dynamics.

16.72 Restoration of Attractor Identity

The strongest local attractor criterion is

$$\mathcal{A}_R = \mathcal{A}_{\text{ref}}. \quad (1233)$$

Equation 1233 requires exact identity under the selected representation.
This criterion is generally stronger than necessary where state labels or coordinates differ.

16.73 Attractor Isomorphism

A structural criterion allows a bijection $\phi_{\mathcal{A}}$ among attractors preserving selected properties:

$$\phi_{\mathcal{A}} : \mathfrak{A}_{\text{ref}} \longrightarrow \mathfrak{A}_R. \quad (1234)$$

Equation 1234 can preserve attractor type, transition relations, or normative classification without requiring identical state coordinates.

16.74 Attractor-Graph Comparison

A multistable system can be represented by an attractor-transition graph

$$G_{\mathcal{A}} = (V_{\mathcal{A}}, E_{\mathcal{A}}), \quad (1235)$$

where nodes represent regimes and edges represent possible transitions. Equation 1235 permits graph isomorphism, embedding, simulation, or dominance to compare regime architecture.

This abstraction is useful where detailed continuous dynamics are unavailable but regime transitions can be identified.

16.75 Attractor-Graph Simulation

A restored attractor graph can simulate the reference where every relevant reference regime transition can be matched.

This criterion can preserve future regime options while permitting additional restored regimes.

It is therefore more compatible with transformative expansion than exact attractor-graph isomorphism.

16.76 Basin Equality

Exact basin restoration is represented by Equation 1236.

$$\mathcal{B}_R(\mathcal{A}_R) = \mathcal{B}_{\text{ref}}(\mathcal{A}_{\text{ref}}). \quad (1236)$$

Equation 1236 is a strong robustness criterion.

The basin can also be compared through inclusion, topology, geometry, measure, or boundary distance.

16.77 Basin Inclusion

For a desirable reference attractor, a dominance-style criterion is

$$\mathcal{B}_{\text{ref}} \subseteq \mathcal{B}_R. \quad (1237)$$

Equation 1237 means every state that returned to the desirable reference regime also returns to the corresponding restored regime.

This can be normatively stronger than basin equality where the enlarged region is desirable.

16.78 Basin Topological Equivalence

A basin can be compared through homeomorphism:

$$\mathcal{B}_R \cong_{\text{top}} \mathcal{B}_{\text{ref}}. \quad (1238)$$

Equation 1238 preserves broad topological organization while allowing metric differences.

It says little about how far the current state lies from the boundary.

16.79 Boundary Correspondence

Let $\partial\mathcal{B}_R$ and $\partial\mathcal{B}_{\text{ref}}$ denote basin boundaries.

A metric comparison can require

$$d_H(\partial\mathcal{B}_R, \partial\mathcal{B}_{\text{ref}}) \leq \varepsilon_{\partial}. \quad (1239)$$

Equation 1239 uses Hausdorff distance where a common metric representation is justified.

Boundary correspondence can be more informative than basin measure alone.

16.80 Stability Equivalence

Let Stab denote a selected stability descriptor. Stability equivalence requires

$$\text{Stab}(\mathcal{G}_R) = \text{Stab}(\mathcal{G}_{\text{ref}}). \quad (1240)$$

Equation 1240 is meaningful only after the stability concept has been specified.

Possible descriptors include local Lyapunov stability, convergence rate, structural stability, or finite-time persistence.

16.81 Resilience Equivalence

For resilience functional ϱ , equivalence is represented by Equation 1241.

$$\varrho(\mathcal{G}_R, \Delta_{\text{adm}}, \tau) = \varrho(\mathcal{G}_{\text{ref}}, \Delta_{\text{adm}}, \tau). \quad (1241)$$

Equation 1241 is explicitly indexed to disturbance class and horizon.

A different perturbation class can reverse the restorative judgment.

16.82 Resilience Dominance

A transformed restored system can exceed the reference under a selected resilience criterion:

$$\varrho_R \succeq \varrho_{\text{ref}}. \quad (1242)$$

Equation 1242 supports better-than-reference restoration.

Its normative value depends upon whether the resilient regime is itself just.

16.83 Bifurcation Equivalence

Let $\mathfrak{Bif}(\mathcal{G})$ denote the selected bifurcation structure.

A structural comparison is

$$\mathfrak{Bif}(\mathcal{G}_R) \cong \mathfrak{Bif}(\mathcal{G}_{\text{ref}}). \quad (1243)$$

Equation 1243 is much stronger than current state equality.

It asks whether qualitative regime changes occur under corresponding parameter variation.

16.84 Hysteresis Equivalence

Let the forward and reverse transition thresholds be represented by $(\theta_{\uparrow}, \theta_{\downarrow})$.

A hysteresis-profile comparison can require

$$(\theta_{\uparrow,R}, \theta_{\downarrow,R}) \preceq (\theta_{\uparrow,\text{ref}}, \theta_{\downarrow,\text{ref}}). \quad (1244)$$

Equation 1244 makes return-path asymmetry an explicit restorative object.

16.85 Criticality Equivalence

A criticality profile can also be compared:

$$\mathbf{d}_{\text{crit},R} \preceq \mathbf{d}_{\text{crit},\text{ref}}. \quad (1245)$$

Equation 1245 can identify residual vulnerability even where ordinary performance has recovered.

16.86 Criticality Dominance

For undesirable critical transitions, greater distance from the boundary can be normatively favorable:

$$d_{\text{crit},R} \geq d_{\text{crit},\text{ref}}. \quad (1246)$$

Equation 1246 represents one form of structural improvement.

The direction of the inequality depends upon the meaning of the boundary and the chosen metric.

16.87 Attractor Comparison Under Uncertainty

In empirical systems, attractor structure can remain uncertain.

Let $\mathfrak{M}$ be a family of plausible dynamical models. Model M produces attractor family $\mathfrak{A}^M$.

A robust claim can require

$$\mathfrak{A}_R^M \text{ } E \text{ } \mathfrak{A}_{\text{ref}}^M \quad \forall M \in \mathfrak{M}. \quad (1247)$$

Equation 1247 is stronger epistemically than a result obtained from one fitted model.

16.88 Basin Comparison Under Partial Observability

Basin structure is particularly difficult to infer when the full state is unobserved.

A system can appear to move toward one attractor in observed coordinates while latent variables place it near another boundary.

The restorative theory should therefore distinguish:

- observed basin evidence;
- model-inferred basin structure;
- experimentally perturbed response;
- ideal-theoretical basin structure.

These epistemic statuses should not be conflated.

16.89 Empirical Identification of Attractors

Repeated observation near a region does not independently prove the existence of a mathematical attractor.

Apparent persistence can arise from:

- genuine attraction;
- external forcing;
- slow drift;
- institutional constraint;
- limited observation horizon;
- measurement compression.

Attractor language in empirical social analysis therefore requires model validation and explicit assumptions.

16.90 Empirical Identification of Alternative Stable States

Claims of alternative stable states require particular caution because different observed regimes can arise from different parameter conditions rather than multistability under the same relevant conditions.

Scheffer emphasizes the evidentiary difficulty of establishing alternative basins and regime structure even in ecological systems [52].

Social applications should therefore avoid inferring multistability merely from the existence of several observed social states.

16.91 Formal Model and Social Interpretation

A mathematical attractor does not automatically correspond to a social “preference,” “culture,” or “identity.”

The interpretive mapping must be supplied separately.

Let

$$\mathcal{I}_S : \mathcal{A} \longrightarrow \mathcal{S}_{\text{social}} \quad (1248)$$

map formal attractor $\mathcal{A}$ into a social interpretation. Equation 1248 makes the interpretive step explicit. This protects the formal model from being mistaken for an exhaustive ontology.

16.92 Attractor Reification

A major conceptual risk is reifying an estimated attractor as a natural essence of the subject or community.

A stable behavioral pattern can arise from constraint, exclusion, or institutional reinforcement.

Its persistence does not establish that the pattern reflects the subject’s authentic preference.

The same caution applies to undesirable labels such as “dependency attractor” or “low-productivity equilibrium.”

Normative and causal interpretation must precede institutional action.

16.93 Adaptive Preference and Dynamical Stability

A stable preference structure can develop under prolonged deprivation.

Dynamical stability of the observed preference does not settle whether the underlying opportunity structure is just.

Capability analysis therefore remains necessary alongside dynamical analysis [55, 50].

This is another instance of formal strength lacking automatic normative authority.

16.94 Domination as a Stable Regime

Relations of domination can become dynamically self-sustaining.

A subject adapts to concentrated control. Reduced contestation strengthens the controlling institution. The institution then acquires greater capacity to reproduce the relation.

Such a pattern can be modeled as a reinforcing regime where appropriate.

Republican non-domination supplies the normative reason for rejecting stability as the goal [44].

16.95 Generative Justice and Attractor Structure

Eglash and Garvey explicitly introduced basin-of-attraction language into generative justice, linking self-organizing dynamics and generative value circulation [16].

The present framework extends that intuition in three ways.

First, attractor and basin structure are treated as only one family of restorative objects.

Second, different comparison relations are distinguished explicitly.

Third, normative evaluation is separated from dynamical correspondence.

This extension prevents the attractor metaphor from becoming the complete definition of generative justice.

16.96 Generative Attractors

Where a system repeatedly regenerates conditions supporting further value production, the recursive process can admit a persistent dynamical regime.

Let generative state be g_t . A recursive system can be represented by Equation 1249.

$$g_{t+1} = \Gamma(g_t). \quad (1249)$$

Equation 1249 permits analysis of persistent generative regimes.

A high-output attractor is not automatically a just generative attractor. Control, circulation, agency, and externalities remain separate objects.

16.97 Extractive Attractors

An extractive regime can also be self-sustaining.

Value produced by one group can strengthen institutions that maintain the conditions of extraction.

The resulting system can be dynamically robust and normatively undesirable.

This structure demonstrates why resilience maximization is inadequate as a general justice objective.

16.98 Restorative Attractors

A restorative intervention can seek a regime in which the system itself continues producing conditions of repair or future generativity.

Examples can include:

- durable anti-retaliation structures;
- community-controlled productive infrastructure;
- social protection enabling continued participation;
- professional networks capable of reproducing access without repeated emergency intervention.

The defining feature is recursive maintenance rather than one-time correction.

16.99 Self-Sustaining Restoration

Let external restorative input be $u_R(t)$, and let the desirable region be $\mathcal{D}$.

A self-sustaining restoration can satisfy Equation 1250.

$$u_R(t) \longrightarrow 0 \quad \text{while} \quad x_t \in \mathcal{D} \quad (1250)$$

over the selected horizon. Equation 1250 describes one important form of generative independence.

Permanent legitimate public support remains compatible with justice, so this criterion should not be universalized. Its relevance depends upon whether continued dependence upon the restorative actor is itself problematic.

16.100 Support-Dependent Stable Regimes

Some desirable states require continuing support.

Let external support be $u^* \neq 0$. A supported regime can satisfy Equation 1251.

$$x_t \in \mathcal{D} \quad \text{only while} \quad u_R(t) \in \mathcal{U}^*. \quad (1251)$$

Equation 1251 does not establish injustice.

Healthcare, accessible infrastructure, disability support, education, and social protection can legitimately require durable collective support.

The normative question concerns the quality of the relation through which the support is supplied.

16.101 Attractor Exit as Agency

A normatively desirable dynamical system can require more than stability inside one regime. It can also require the subject to possess viable routes of exit.

Let $\mathcal{A}$ be the current desirable regime and $\mathcal{T}_A$ a subject-selected alternative target.

Agency-sensitive dynamics can require Equation 1252.

$$\mathcal{R}_\tau(x) \cap \mathcal{T}_A \neq \emptyset. \quad (1252)$$

Equation 1252 prevents a highly stable beneficial regime from becoming a form of lock-in.

16.102 Lock-In

Lock-in can be represented through high transition cost or absence of viable paths from the current regime.

Let exit cost be C_{exit} . Severe lock-in can satisfy Equation 1253.

$$C_{\text{exit}} \gg C_{\text{ordinary}}. \quad (1253)$$

Equation 1253 is schematic.

Path-dependence literature provides established accounts of how increasing returns can produce persistent lock-in [3, 45].

A stable restorative arrangement should therefore be evaluated for exit as well as persistence.

16.103 Adaptive Capacity

Adaptive capacity concerns the ability to modify behavior, structure, or policy in response to changing conditions without catastrophic loss of valued function.

Let adaptation operator be $\mathcal{A}_d$. A simple adaptive condition is represented by Equation 1254.

$$\forall \delta \in \Delta_{\text{adm}} \exists a \in \mathcal{A}_d : Q(\mathcal{G}^{a,\delta}) \in \mathcal{S}_{\text{acceptable}}. \quad (1254)$$

Equation 1254 distinguishes adaptation from passive return to one fixed equilibrium.

This can be especially relevant to open-ended social and ecological systems.

16.104 Transformability

Where the existing regime has become undesirable or untenable, justice can require the capacity to create a qualitatively new regime.

Let current regime be Z_1 and normatively preferred new regime be Z_2 . Transformability can be represented by Equation 1255.

$$\exists R : Z_1 \xrightarrow{R} Z_2. \quad (1255)$$

Equation 1255 is structurally different from resilience, which often concerns persistence or recovery of an existing regime.

Transformative justice can therefore value transformability where resilience of the status quo would preserve injustice.

16.105 Resilience, Adaptation, and Transformation

The three concepts correspond to different dynamical objectives:

- resilience concerns persistence or recovery within a valued regime;
- adaptation concerns modification that preserves valued functioning under changed conditions;

- transformation concerns movement into another regime where the previous one is no longer normatively adequate.

The appropriate objective depends upon whether the reference regime itself deserves preservation.

16.106 Critical Transitions and Intervention Timing

The value of intervention can increase sharply as a system approaches a boundary whose crossing is difficult to reverse.

Let intervention cost before transition be C_{pre} and restorative cost after transition be C_{post} .
A low-reparability case can satisfy Equation 1256.

$$C_{\text{post}} \gg C_{\text{pre}}. \quad (1256)$$

Equation 1256 supplies a dynamical reason for greater preventive significance.

The comparison remains incomplete without accounting for false alarms and the costs of preventive intervention.

16.107 Finite-Step Emergency Governance

Near a critical transition, institutions can require shorter-horizon control than their ordinary long-run governance process provides.

Let current state be x_t and critical set be $\mathcal{C}$. A finite-step protective objective can require Equation 1257.

$$x_{t+k} \notin \mathcal{C} \quad k = 1, \dots, N. \quad (1257)$$

Equation 1257 represents a bounded emergency objective: preserve the system from crossing the critical region while fuller evaluation continues.

This objective is distinct from selecting the optimal long-run attractor.

16.108 Criticality and Procedural Restraint

Criticality should not become a universal justification for emergency power.

The estimated distance to a threshold can itself be uncertain. Institutional action can alter the model. False-positive warnings can impose severe burdens.

A criticality-based intervention should therefore remain:

- proportionate;
- reviewable;
- time-limited where appropriate;
- transparent about model uncertainty;
- sensitive to rights and third-party effects.

This requirement links dynamical analysis with procedural justice.

16.109 Critical Slowing Down and Institutional Response

If reliable evidence indicates slower recovery from ordinary disturbances, an institution can interpret this as evidence of declining local resilience rather than waiting for full regime change.

The appropriate response can include additional observation, reduction of stressors, redundancy, or temporary support.

The observation itself does not determine which intervention is justified.

Mechanism, uncertainty, and procedural constraints remain necessary.

16.110 Early Warning and False Precision

An early-warning statistic can be numerically precise while the causal interpretation remains uncertain. For example, an increase in autocorrelation can result from several mechanisms. The paper therefore distinguishes:

$$\text{indicator change} \neq \text{identified transition mechanism.} \quad (1258)$$

Equation 1258 expresses a methodological constraint on dynamical governance.

The same warning applies to social “tipping point” rhetoric unsupported by a specified state space and dynamical model.

16.111 Attractor Metaphor and Formal Attractor

The term *attractor* is widely used metaphorically to describe recurring social patterns.

The present paper distinguishes such metaphorical use from a formal attractor claim.

A formal claim should specify:

- state space;
- time evolution;
- invariant or attracting set;
- convergence or persistence criterion;
- relevant basin;
- evidentiary basis.

Where these elements are unavailable, language such as persistent regime, self-reinforcing configuration, or recurring pattern can be more accurate.

16.112 Basin Metaphor and Formal Basin

The same discipline applies to basin language.

A formal basin requires a specified attractor and dynamics.

A social “basin” used only to mean a vague set of favorable circumstances should not be treated as mathematically established.

The distinction preserves the value of dynamical thinking while avoiding formalistic overstatement.

16.113 Potential Metaphor and Formal Potential

A visual landscape of hills and valleys can be heuristically useful.

A formal potential function requires mathematical conditions that many social systems lack.

The paper therefore uses potential landscapes only where the model justifies them and otherwise speaks more generally of basin, transition, or stability structure.

16.114 Attractor Reconstruction and Subject Agency

A technically powerful institution can attempt to return the subject to a previous behavioral regime.

Such reconstruction can conflict with present agency if the subject now prefers another viable trajectory.

The normative target should therefore distinguish:

$$\mathcal{A}_{cf} \quad (1259)$$

from

$$\mathcal{T}_A^{\text{adm}}, \quad (1260)$$

where Equation 1259 denotes a reconstructed counterfactual regime and Equation 1260 denotes the present subject's admissible target region.

The two need not coincide.

16.115 Attractor Reconstruction and Identity

Restoration of an earlier behavioral attractor can require changing memories, commitments, or relationships formed through the actual history.

The stronger the attractor-level reconstruction becomes, the more important identity and procedural constraints become.

Dynamical fidelity therefore remains subordinate to the broader theory of justice.

16.116 Attractor Reconstruction and Third Parties

A social attractor is often relational.

Reconstructing an earlier relational regime can require third parties to resume roles they no longer choose.

This makes exact social-attractor reconstruction normatively constrained even where technically imaginable.

Network and relational consent therefore limit attainable dynamical equivalence.

16.117 Collective Attractors

Organizations, communities, and institutions can possess persistent collective regimes under suitable formalization.

A collective attractor cannot be inferred merely from stable aggregate output.

The analysis should consider:

- internal relation structure;
- rules;
- distribution;
- information flow;
- external forcing;
- member turnover.

A persistent collective pattern can coexist with severe instability for particular members.

16.118 Aggregate Stability and Individual Instability

An organization can maintain stable aggregate output while repeatedly replacing members who experience harm.

Let organizational output satisfy Equation 1261.

$$Y_R(t) \approx Y_{\text{ref}}(t). \quad (1261)$$

Equation 1261 can coexist with large turnover or individual instability.

System-level stability should therefore not be interpreted as individual-level justice.

16.119 Individual Stability and Systemic Instability

The reverse configuration is also possible.

A particular subject can be successfully stabilized through intensive support while the institution continues producing harms for others.

Individual restoration therefore cannot substitute for structural analysis.

16.120 Multi-Scale Attractor Structure

Attractor structures can exist at several modeled scales.

Let

$$\mathfrak{A}^{(s)} \quad (1262)$$

denote the attractor family at scale s . Equation 1262 permits individual, organizational, community, and societal dynamical regimes to differ.

A restorative intervention can improve one scale and worsen another.

16.121 Cross-Scale Dynamical Coupling

Let individual dynamics depend upon institutional state:

$$\dot{x}_A = F_A(x_A, x_I), \quad (1263)$$

and institutional dynamics depend upon member states:

$$\dot{x}_I = F_I(x_I, x_1, \dots, x_n). \quad (1264)$$

Equations 1263 and 1264 represent coupled generative dynamics.

A restorative intervention at one level can therefore reshape attractor structure at another.

16.122 Attractor Structure and Distribution

A dynamically stable regime can contain unequal distributions.

Let distributional state within attractor $\mathcal{A}$ be $D_{\mathcal{A}}$.

Two systems can possess corresponding attractor structure and different distributions:

$$\mathcal{A}_R \cong \mathcal{A}_{\text{ref}} \quad \text{while} \quad D_{\mathcal{A},R} \neq D_{\mathcal{A},\text{ref}}. \quad (1265)$$

Equation 1265 shows that dynamical equivalence does not imply distributive justice.

16.123 Attractor Structure and Recognition

A social regime can persist through recognition and classification mechanisms.

Changing public meaning can therefore alter transition structure.

Recognition interventions can be dynamically consequential while remaining distinct from material redistribution [25, 20].

This provides one bridge between symbolic and dynamical restoration.

16.124 Attractor Structure and Control

Control relations can stabilize particular regimes.

A system in which one actor possesses discretionary control over critical conditions can produce a persistent dependency regime.

Changing control structure can therefore alter basin and attractor organization.

This connects non-domination and dynamical transformation [44].

16.125 Attractor Structure and Value Circulation

Recursive value circulation can also stabilize regimes.

Let retained value become future generative condition:

$$c_{t+1} = \Lambda(\Gamma(c_t)). \quad (1266)$$

Equation 1266 can produce self-reinforcing generative or extractive dynamics.

The attractor structure therefore depends partly upon who controls generated value and where it circulates [15, 17].

16.126 Attractor Structure and Assistance

Assistance can act at several dynamical levels:

- move the current state;
- reduce transition costs;
- create new reachable paths;
- alter parameters;
- change institutional rules;
- enlarge the basin of a desirable regime.

These interventions should not be treated as equivalent.

A one-time payment primarily moving state differs from a legal reform changing the transition landscape.

16.127 Attractor Structure and Prevention

Prevention can aim to keep the system sufficiently far from an undesirable basin boundary.

Let minimum acceptable distance be $d_{\min}$. A preventive criterion is represented by Equation 1267.

$$d_{\partial B^-}(x_t) \geq d_{\min}. \quad (1267)$$

Equation 1267 is meaningful only where the relevant boundary and metric are sufficiently identified. It illustrates how dynamical information can guide preventive governance.

16.128 Attractor Structure and Interruption

During ongoing harm, interruption can prevent the state from entering an undesirable basin.

Let harmful basin be B^- . A finite-horizon interruptive condition is given by Equation 1268.

$$x_{t+k} \notin B^- \quad k = 1, \dots, N. \quad (1268)$$

Equation 1268 again separates immediate containment from full restoration.

16.129 Attractor Structure and Post-Harm Response

After a regime shift has occurred, post-harm response can require:

- movement of state toward a viable region;
- alteration of basin geometry;
- removal of the harmful attractor;
- creation of an alternative desirable regime;
- reduction of transition cost toward the desirable basin;
- support during the transition.

The appropriate combination depends upon reparability and subject preference.

16.130 Dynamical Repair and Historical Repair

A dynamically restored future can remain historically non-equivalent.

Suppose future dynamics satisfy Equation 1269.

$$\Phi_R \sim_E \Phi_{\text{ref}} \quad \text{for } t \geq t_1. \quad (1269)$$

Equation 1269 can coexist with different past histories.

Dynamical repair therefore addresses future evolution and does not erase prior loss.

16.131 Historical Repair and Dynamical Non-Equivalence

The converse can also occur.

An institution can acknowledge history, compensate loss, and correct records while leaving the future dynamical structure unchanged.

Historical justice and dynamical transformation therefore occupy distinct atlas regions.

16.132 Dynamical Restoration Profile

A dynamical restorative profile can be represented by Equation 1270.

$$\mathbf{D}_R = (r_{\mathcal{A}}, r_{\mathcal{B}}, r_{\text{stab}}, r_{\text{res}}, r_{\text{bif}}, r_{\text{crit}}, r_{\text{hyst}}), \quad (1270)$$

where the entries describe restoration of attractors, basins, stability, resilience, bifurcation structure, criticality, and hysteresis. Equation 1270 preserves multidimensionality rather than forcing one dynamical-restoration score.

16.133 Dynamical Object–Relation Cells

Table 9 summarizes several central cells of the dynamical region of the atlas.

Table 9 demonstrates that dynamical restoration contains several non-equivalent standards.

16.134 Dynamical Refinement Path

A schematic refinement path can move from state toward richer dynamical structure:

$$(Q_{\text{state}}, =) \longrightarrow (Q_{\text{attr}}, \cong) \longrightarrow (Q_{\text{basin}}, \sim) \longrightarrow (Q_{\text{res}}, \sim) \longrightarrow (Q_{\text{bif}}, \cong). \quad (1271)$$

Equation 1271 increases the amount of future dynamical structure included in the comparison.

The sequence is analytical and lacks universal normative priority.

Table 9: Illustrative dynamical object–relation cells for restorative comparison

Object	Comparative relation	Restorative interpretation	Characteristic limitation
Attractor	Identity	The same represented attracting set is restored	Coordinate and history sensitive
Attractor family	Isomorphism	Long-run regimes correspond structurally	Basin geometry can remain different
Attractor graph	Simulation	Reference regime transitions remain reproducible	Allows additional restored regimes
Basin	Equality	The same initial states converge to the corresponding regime	Strong and model-dependent
Basin	Inclusion	The desirable restored basin contains the reference basin	Requires normative classification of the attractor
Basin boundary	Metric proximity	Critical boundaries remain geometrically close	Depends upon metric and representation
Stability	Property equivalence	Selected local stability property is restored	Stability concept must be specified
Recovery rate	Approximate equality	Post-disturbance convergence occurs at comparable speed	Captures only one resilience dimension
Resilience	Profile equivalence	Response across selected perturbations corresponds	Perturbation-class dependent
Bifurcation structure	Structural correspondence	Qualitative regime changes occur under corresponding parameter variation	High modeling burden
Criticality	Distance comparison	Proximity to undesirable transition is comparable	Boundary can be uncertain
Hysteresis	Threshold-profile equivalence	Forward and reverse transition asymmetry is restored	Requires identifiable regime structure

16.135 Dynamical Non-Implication Structure

Several non-implications are central.

State equality does not imply attractor equivalence:

$$x_R = x_{\text{ref}} \not\Rightarrow \mathfrak{A}_R \cong \mathfrak{A}_{\text{ref}}. \quad (1272)$$

Equation 1272 separates current configuration from long-run regime structure.

Attractor correspondence does not imply basin equivalence:

$$\mathfrak{A}_R \cong \mathfrak{A}_{\text{ref}} \not\Rightarrow \mathfrak{B}_R \cong \mathfrak{B}_{\text{ref}}. \quad (1273)$$

Equation 1273 separates regime structure from robustness.

Basin correspondence does not imply equal recovery dynamics:

$$\mathfrak{B}_R \cong \mathfrak{B}_{\text{ref}} \not\Rightarrow T_{\text{rec},R} = T_{\text{rec},\text{ref}}. \quad (1274)$$

Equation 1274 separates basin structure from transient response.

16.136 Dynamical Equivalence and Causal Equivalence

Dynamically similar observed behavior does not establish equality of causal mechanism.

Two systems can possess similar attractors while different causal structures generate them.

The later causal section therefore remains necessary even after detailed dynamical comparison.

16.137 Dynamical Equivalence and Generative-Process Equivalence

Likewise, dynamical equivalence does not establish equality of value generation, control, or circulation.

Two organizations can display similar stable output while one relies upon extractive relations and another upon autonomous collaboration.

Generative-process comparison therefore extends beyond dynamical form.

16.138 Dynamical Equivalence and Procedural Justice

A restored system can be placed into a highly stable desirable state through coercive intervention.

The dynamical result can be strong and the procedure unjust.

This provides another direct example of the independence between restorative fidelity and procedural justice.

16.139 Dynamical Equivalence and Subject Preference

A counterfactual model can identify an attractor corresponding to the subject's likely no-harm career.

The present subject can prefer a new target regime.

Dynamical restoration should therefore distinguish reconstruction of the counterfactual landscape from support for a subject-directed landscape.

16.140 Dynamical Equivalence and Third-Party Effects

Changing basin geometry for one subject can change conditions for others.

Restoring one organizational role can alter another person's role. Returning land can alter current community arrangements. Rebuilding one market structure can shift risk.

A coupled dynamical analysis should therefore evaluate system-wide effects where material.

16.141 Dynamical Comparison Under Model Uncertainty

Dynamical restoration can be especially sensitive to model structure.

Different plausible models can disagree about:

- number of attractors;
- basin boundaries;
- bifurcation type;
- transition threshold;
- resilience;
- expected effect of intervention.

A restorative institution should therefore avoid treating one estimated landscape as uniquely established where several models fit the observations.

16.142 Dynamical Bounds

Where exact basin or threshold identification is unavailable, analysis can use bounds.

For a critical parameter,

$$\theta_{\min} \leq \theta_c \leq \theta_{\max}. \quad (1275)$$

Equation 1275 permits robust intervention planning without false point precision.

The same method can apply to basin measure, resilience radius, or transition probability.

16.143 Robust Dynamical Intervention

Let $\mathfrak{M}$ be the plausible model family. A robust intervention R can require

$$Q(R(\mathcal{G}); M) \in \mathcal{S}_{\text{acceptable}} \quad \forall M \in \mathfrak{M}. \quad (1276)$$

Equation 1276 favors interventions whose success does not depend upon one fragile model. Such robustness can be especially valuable near critical transitions.

16.144 Dynamical Minimalism

The existence of sophisticated dynamical tools should not create a presumption that every restorative case requires attractor estimation.

A simpler object is preferable where it is sufficient.

State correction can be enough for a clerical error. Capability comparison can be more appropriate than bifurcation analysis for many assistance cases. Attractor analysis becomes useful where persistent regimes, multistability, thresholds, or resilience are genuinely relevant.

This is the principle of dynamical minimalism.

16.145 Dynamical Escalation

A case can move toward richer dynamical analysis when simpler restorative objects fail to explain observed behavior.

Examples include:

- repeated relapse after apparent state restoration;
- disproportionate response to small perturbation;
- persistent return to a harmful institutional configuration;
- abrupt regime changes;
- strong path dependence;
- evidence of alternative stable modes.

Dynamical modeling is therefore a diagnostic escalation rather than a universal starting point.

16.146 Workplace Dynamical Example

Consider a workplace in which prolonged bullying or retaliation has occurred.

After formal intervention:

- salary can return;
- formal role can return;
- observable performance can return.

The dynamical questions remain:

- does ordinary disagreement trigger renewed withdrawal?
- does a change of supervisor destabilize participation?
- is the subject's professional path robust to organizational restructuring?
- does the institution itself continue reproducing retaliation?

A state-level declaration of successful reinstatement can therefore coexist with weak dynamical restoration.

16.147 Research-System Dynamical Example

A research environment can possess self-reinforcing regimes.

Strong collaboration generates publications. Publications generate funding. Funding creates further collaboration.

An exclusion can interrupt this cycle and move the researcher into another professional regime.

Later funding can improve current resources without automatically reconstructing the earlier recursive basin.

The appropriate post-harm response can therefore involve creation of a new self-sustaining research regime rather than exact reconstruction of the lost one.

16.148 Reputational Dynamical Example

A false reputational classification can generate a negative feedback structure:

- classification reduces access;
- reduced access lowers visible participation;
- lower participation is interpreted as reduced competence;
- the interpretation reinforces exclusion.

Correcting the original classification changes one state variable.

Breaking the self-reinforcing regime can require network, institutional, and recognition interventions as well.

16.149 Collective Dynamical Example

A community subjected to long-term extraction can enter a persistent regime in which:

- resource loss reduces institutional capacity;
- weak institutional capacity reduces control over generated value;
- reduced control increases future extraction;
- extraction further weakens institutional capacity.

A one-time compensatory payment can move current state substantially.

A generative intervention seeks to alter the recursive dynamics through which the regime reproduces itself.

This provides a direct connection between dynamical structure and generative justice.

16.150 Ecological Dynamical Example

Ecological systems provide established examples of alternative regimes, resilience, hysteresis, and critical transitions [24, 52].

An ecosystem can regain an observable variable while remaining in a different regime or possessing reduced basin resilience.

The ecological case therefore demonstrates why state restoration and dynamical restoration should remain distinct.

Its mathematical structures should not be transferred mechanically into social systems.

16.151 Dynamical Metadata

A dynamical restorative claim should record at least:

- state-space representation;
- temporal scale;
- dynamical law or transition model;
- parameter assumptions;
- attractor definition;
- basin definition;
- perturbation class;
- stability concept;
- resilience concept;
- critical-transition model;
- epistemic uncertainty;
- normative classification of regimes.

This metadata prevents terms such as stability, resilience, tipping point, and attractor from becoming undefined rhetorical labels.

16.152 Attractor Principle

The first principle of this section is:

Restoration of present state does not establish restoration of the long-run dynamical regimes toward which the system evolves.

16.153 Basin Principle

The second principle is:

Correspondence of attractors does not establish correspondence of the basins, boundaries, or perturbation ranges through which those attractors remain accessible and stable.

16.154 Resilience Principle

The third principle is:

Restorative resilience must be indexed to a disturbance class, temporal horizon, and normatively valued regime.

16.155 Hysteresis Principle

The fourth principle is:

Where system dynamics exhibit hysteresis or path dependence, return of an external condition to its earlier value does not generally entail return of the system to its earlier regime.

16.156 Criticality Principle

The fifth principle is:

A system can appear restored under ordinary state variables while remaining closer than the reference to a critical transition.

16.157 Normative Stability Principle

The sixth principle is:

Dynamical stability and resilience possess no automatic positive normative status; the regime whose persistence is being evaluated must itself be normatively assessed.

16.158 Transformability Principle

The seventh principle is:

Where the historical regime is unjust or no longer viable, the appropriate restorative objective can be transformation into a new desirable regime rather than restoration of the previous attractor structure.

16.159 Dynamical Humility Principle

The eighth principle is:

Attractor, basin, bifurcation, and tipping-point language should be used as formal claims only where the represented state space, evolution law, and evidentiary basis support those concepts.

16.160 Dynamical Layer of the Atlas

The dynamical layer of the atlas can now be represented by Equation 1277.

$$\mathcal{D}_A = \{Q_{\text{attr}}, Q_{\text{basin}}, Q_{\text{stab}}, Q_{\text{res}}, Q_{\text{bif}}, Q_{\text{crit}}, Q_{\text{hyst}}\} \times \mathfrak{E}_D, \quad (1277)$$

where $\mathfrak{E}_D$ contains the comparative relations meaningful for dynamical objects. Equation 1277 defines the dynamical region within the larger object–relation atlas.

16.161 Implications for Perturbation and Robustness Analysis

The present section establishes that dynamical restoration requires more than comparison of attractor location.

The response to perturbation determines:

- whether the system remains inside the same basin;
- how far it moves;
- how quickly it recovers;
- whether repeated shocks accumulate;
- whether the perturbation changes the transition law itself;
- whether restoration remains valid outside nominal conditions.

These questions require a dedicated perturbation framework.

16.162 Transition to Perturbation, Robustness, and Recovery Dynamics

Section 17 therefore develops perturbation and robustness comparison in greater depth. It distinguishes perturbation classes, amplitude, direction, duration, frequency, repeated shocks, response operators, recovery time, robustness margins, distributional response, and adversarial or institutional stress tests.

The central question becomes whether a restored and reference system remain comparably generative when they are moved away from the calm baseline under conditions that matter to their future operation.

17 Perturbation, Robustness, and Recovery Dynamics

This section develops the perturbation and robustness region of the Atlas of Equivalence. Section 16 established that present-state equality, attractor correspondence, basin structure, and resilience are distinct restorative objects. The present section asks how those differences become visible when the restored and reference systems are subjected to disturbances, interventions, environmental changes, or ordinary operational stress.

The central methodological move is from baseline comparison to *response comparison*. A restored system and a reference system can appear identical under calm conditions while diverging sharply when exposed to perturbation. The comparison therefore concerns a response operator rather than a single state. This makes robustness a family of indexed claims rather than a binary property.

The section distinguishes perturbation amplitude, direction, duration, frequency, timing, correlation, and source. It then examines one-shot and repeated disturbances, compound shocks, recovery trajectories, recovery time, residual displacement, robustness margins, stress-test classes, adversarial perturbations, stochastic response, model uncertainty, and the procedural limits of testing. The later causal section will then distinguish perturbation from causal intervention more precisely.

17.1 Perturbation as a Comparative Operator

Let δ denote a disturbance applied to a system state, parameter, environment, relation, or input. A perturbation operator is represented by Equation 1278.

$$\mathcal{P}_\delta : \mathcal{G} \longrightarrow \mathcal{G}^\delta. \quad (1278)$$

Equation 1278 is intentionally broad. δ can represent:

- a resource shock;
- change of supervisor;
- unexpected expense;
- organizational conflict;
- policy change;
- health stressor;
- environmental disturbance;
- loss of one network relation;
- increase in workload;
- reputational challenge;
- temporary loss of infrastructure.

The perturbation itself is not assumed to be harmful. It is a test or event through which the response structure of the system becomes visible.

17.2 Perturbation Classes

A restorative claim should specify the class of disturbances under which robustness is evaluated.

Let

$$\Delta_{\text{adm}} = \{\delta_1, \delta_2, \dots\} \quad (1279)$$

denote the admissible perturbation class.

Equation 1279 can include only ordinary foreseeable disturbances or a broader stress-test family.

The scope of Δ_{adm} is normatively and epistemically significant. A system robust to ordinary work-load variation can remain fragile to authority-related conflict. A community robust to moderate economic change can remain vulnerable to ecological disturbance.

Robustness is therefore always class-relative.

17.3 Perturbation Metadata

Each perturbation can be described through metadata

$$\Theta_\delta = (a, v, d, f, s, \tau_\delta, c), \quad (1280)$$

where:

- a denotes amplitude;
- v denotes direction or type;
- d denotes duration;
- f denotes frequency or repetition structure;
- s denotes source;
- τ_δ denotes timing;
- c denotes contextual conditions.

Equation 1280 makes explicit that two shocks of equal amplitude can differ substantially in their effects.

17.4 Perturbation Amplitude

Where a perturbation space carries a norm, disturbance amplitude can be represented by

$$\|\delta\|. \quad (1281)$$

Equation 1281 is useful only where a meaningful metric exists.

For social systems, equal numerical magnitude across different perturbation types is rarely self-explanatory. A small formal change in authority can have a larger generative effect than a larger financial fluctuation.

Amplitude should therefore be interpreted relative to the modeled object.

17.5 Perturbation Direction

Let unit vector v denote perturbation direction and a its amplitude:

$$\delta = av. \quad (1282)$$

Equation 1282 makes anisotropy visible.

A restored system can be robust in one direction and fragile in another.

This motivates directional robustness profiles rather than one scalar robustness index.

17.6 Perturbation Duration

A transient disturbance can differ from a sustained disturbance.

Let

$$\delta(t) = \begin{cases} \delta_0, & t \in [t_a, t_b], \\ 0, & \text{otherwise.} \end{cases} \quad (1283)$$

Equation 1283 defines a disturbance of duration $t_b - t_a$.

A system that tolerates a brief workload increase can fail under prolonged increase of the same magnitude.

Duration is therefore part of the perturbation object.

17.7 Perturbation Timing

The same disturbance can have different effects depending upon when it occurs.

Let

$$\delta_{t_a} \neq_{\text{effect}} \delta_{t_b} \quad (1284)$$

denote a difference in consequence associated with temporal placement.

Equation 1284 is especially relevant near critical windows, recovery periods, developmental stages, or transition phases.

Perturbation analysis therefore interacts directly with the temporal layer of the atlas.

17.8 Single-Shock Response

For one perturbation δ , define the response operator

$$\mathcal{Q}_\tau(\mathcal{G}, \delta) = O(\Phi_\delta^\tau(\mathcal{G})), \quad (1285)$$

where O is the selected observation map.

Equation 1285 provides the basic object for response comparison.

A restored system can then be compared with the reference by

$$\mathcal{Q}_\tau(\mathcal{G}^R, \delta) \ E \ \mathcal{Q}_\tau(\mathcal{G}^{\text{ref}}, \delta). \quad (1286)$$

17.9 Response Trajectory

Rather than compare one post-shock state, the full response trajectory can be used:

$$\gamma_\delta = \left\{ x^\delta(t) : t \in [t_0, t_0 + \tau] \right\}. \quad (1287)$$

Equation 1287 captures peak deviation, recovery, overshoot, oscillation, and residual displacement.

This object is richer than recovery time alone.

17.10 Response Deviation

Let nominal trajectory be $x^0(t)$. Response deviation is

$$e_\delta(t) = d\left(x^\delta(t), x^0(t)\right). \quad (1288)$$

Equation 1288 measures the effect of perturbation relative to the system's own unperturbed behavior.

For restorative comparison, one can then compare $e_{\delta,R}(t)$ with $e_{\delta,\text{ref}}(t)$.

17.11 Peak Response

The maximum deviation under disturbance is

$$M_\delta = \sup_{t \in [t_0, t_0 + \tau]} e_\delta(t). \quad (1289)$$

Equation 1289 provides one robustness measure.

Equal peak response does not imply equal recovery time or equal long-run residual effect.

17.12 Integrated Response Burden

Cumulative disturbance burden can be represented by

$$B_\delta = \int_{t_0}^{t_0 + \tau} e_\delta(t) dt. \quad (1290)$$

Equation 1290 captures both amplitude and duration.

It is analogous to cumulative recovery burden developed in the previous section.

17.13 Recovery Time

Let acceptable region be

$$\mathcal{A}_\varepsilon = \{x : d(x, x^0) \leq \varepsilon\}. \quad (1291)$$

Recovery time is

$$T_{\text{rec}} = \inf \left\{ t \geq t_0 : x^\delta(s) \in \mathcal{A}_\varepsilon \forall s \geq t \right\}. \quad (1292)$$

Equation 1292 uses sustained return rather than first re-entry.

This distinction avoids classifying temporary recovery as stable restoration.

17.14 Residual Displacement

A perturbation can leave persistent residual change.

Define

$$R_\infty = \limsup_{t \rightarrow \infty} d \left(x^\delta(t), x^0(t) \right). \quad (1293)$$

Equation 1293 distinguishes transient response from lasting structural alteration.

A restored system can recover ordinary output and retain larger residual displacement in latent variables.

17.15 Recovery Overshoot

A system can recover beyond the nominal state before settling.

Let scalar response be $y(t)$ with baseline y^0 . Overshoot is

$$O_\delta = \sup_t [y(t) - y^0]. \quad (1294)$$

Equation 1294 can represent compensatory overcorrection.

Overshoot can be beneficial, irrelevant, or harmful depending upon the object.

17.16 Response Symmetry

Responses to positive and negative perturbations can differ.

Let $+\delta$ and $-\delta$ be opposite disturbances. Response symmetry can be evaluated through

$$\mathcal{Q}(\mathcal{G}, +\delta) \sim -\mathcal{Q}(\mathcal{G}, -\delta) \quad (1295)$$

where such a representation is meaningful.

Asymmetry can indicate nonlinearities, thresholds, or path dependence.

17.17 Robustness Margin

A robustness margin can be defined as the largest perturbation magnitude under which a selected condition remains satisfied.

Let criterion be

$$Q(\mathcal{G}^\delta) \in \mathcal{S}_{\text{acceptable}}. \quad (1296)$$

The robustness margin is

$$m_R = \sup \left\{ a : Q(\mathcal{G}^\delta) \in \mathcal{S}_{\text{acceptable}} \forall \|\delta\| \leq a \right\}. \quad (1297)$$

Equation 1297 is a generic robustness measure.

The choice of norm and perturbation family remains substantive.

17.18 Directional Robustness Margin

For perturbation direction v ,

$$m_R(v) = \sup \left\{ a : Q(\mathcal{G}^{av}) \in \mathcal{S}_{\text{acceptable}} \right\}. \quad (1298)$$

Equation 1298 produces a robustness profile over directions.

This can reveal hidden vulnerability concealed by an average robustness score.

17.19 Robustness Surface

The mapping

$$v \mapsto m_R(v) \quad (1299)$$

defines a directional robustness surface.

A restored and reference system can then be compared through the geometry of their robustness surfaces.

This is a stronger criterion than one scalar resilience radius.

17.20 Finite-Horizon Robustness

A system can be robust only over a specified time interval.

Let acceptable condition hold through horizon τ . Define

$$m_R(\tau) = \sup \left\{ a : Q(\mathcal{G}_t^\delta) \in \mathcal{S}_{\text{acceptable}} \forall t \in [0, \tau], \|\delta\| \leq a \right\}. \quad (1300)$$

Equation 1300 makes robustness explicitly horizon-dependent.

A system can be robust for six months and fragile over several years.

17.21 Repeated Perturbations

Real systems often experience repeated shocks.

Let

$$\delta = (\delta_1, \delta_2, \dots, \delta_n) \quad (1301)$$

denote a disturbance sequence.

The response operator becomes

$$\mathcal{Q}(\mathcal{G}, \delta). \quad (1302)$$

Equation 1302 can reveal cumulative vulnerability invisible under isolated testing.

17.22 Inter-Perturbation Interval

Let perturbation times be $t_1, \dots, t_n$. Define

$$\Delta_i = t_{i+1} - t_i. \quad (1303)$$

If

$$\Delta_i < T_{\text{rec},i}, \quad (1304)$$

the next shock arrives before full recovery from the previous one.

Equation 1304 can create cumulative drift even where each individual perturbation is tolerable.

17.23 Cumulative Perturbation Load

A simple cumulative load is

$$L_\delta = \sum_{i=1}^n w_i \|\delta_i\|. \quad (1305)$$

Equation 1305 is only one possible summary.

Sequence, timing, direction, and recovery state can make two equal cumulative loads produce different outcomes.

17.24 Order Effects

Perturbations can be non-commutative.

For two perturbations,

$$\mathcal{P}_{\delta_2} \circ \mathcal{P}_{\delta_1} \neq \mathcal{P}_{\delta_1} \circ \mathcal{P}_{\delta_2}. \quad (1306)$$

Equation 1306 can arise because the first shock alters the state or dynamics through which the second operates.

Sequence therefore belongs to robustness analysis.

17.25 Compound Perturbations

Several disturbances can occur simultaneously.

Let compound shock be

$$\delta^{\text{comp}} = \left(\delta^{(1)}, \delta^{(2)}, \dots, \delta^{(k)} \right). \quad (1307)$$

Equation 1307 can represent simultaneous income loss, health stress, network disruption, and administrative burden.

A system robust to each component separately can fail under the compound event.

17.26 Interaction Among Perturbations

Let response to individual shocks be $\mathcal{Q}(\delta_1)$ and $\mathcal{Q}(\delta_2)$.

Non-additive interaction exists when

$$\mathcal{Q}(\delta_1 + \delta_2) \neq \mathcal{Q}(\delta_1) + \mathcal{Q}(\delta_2). \quad (1308)$$

Equation 1308 makes compound vulnerability explicit.

This is especially important in coupled social and health systems.

17.27 Correlated Perturbations

Perturbations can be statistically dependent.

Let

$$\text{Cov}(\delta_i, \delta_j) \neq 0. \quad (1309)$$

Equation 1309 matters because independence assumptions can underestimate systemic risk.

Employment loss, housing instability, healthcare access, and immigration status can be strongly coupled.

17.28 Common-Cause Perturbations

Several observed shocks can share one underlying cause.

A broad institutional reform, recession, natural disaster, or conflict can simultaneously alter several generative conditions.

Robustness analysis should therefore distinguish multiple independent shocks from one common-cause disturbance.

This distinction becomes especially important for insurance and social protection.

17.29 Systemic Perturbations

A systemic perturbation affects many subsystems simultaneously.

Let

$$\delta^{\text{sys}} : \mathcal{G}_1, \dots, \mathcal{G}_n \longrightarrow \mathcal{G}'_1, \dots, \mathcal{G}'_n. \quad (1310)$$

Equation 1310 captures shocks whose effects cannot be analyzed independently at the individual level.

Collective robustness therefore requires system-scale comparison.

17.30 Localized Perturbations

A localized disturbance affects one node or relation.

For network $G = (V, E)$, removal of node v is represented by

$$G' = G \setminus \{v\}. \quad (1311)$$

Equation 1311 can test dependence upon one actor.

Similarly, edge removal can test robustness to loss of one relation.

17.31 Network Robustness

Network robustness can be assessed through preservation of:

- connectivity;
- giant component;
- shortest paths;
- access to critical nodes;
- redundancy;
- centrality structure.

A restored professional network can appear similar under ordinary operation and collapse after loss of one key gatekeeper.

This reveals residual dependency.

17.32 Node-Removal Robustness

Let $G - v$ denote the network after removal of node v .

A simple robustness criterion is

$$Q(G_R - v) \geq Q(G_{\text{ref}} - v) \quad \forall v \in V_{\text{test}}. \quad (1312)$$

Equation 1312 can compare sensitivity to loss of selected actors.

17.33 Edge-Removal Robustness

Similarly,

$$Q(G_R - e) \geq Q(G_{\text{ref}} - e) \quad (1313)$$

tests dependence upon specific relations.

A system whose restored functioning depends upon one fragile relation can fail this criterion.

17.34 Institutional Stress Tests

A stress test is a deliberately selected perturbation scenario used to evaluate system response.

Let

$$\mathcal{S}_{\text{stress}} = \{\delta^{(1)}, \dots, \delta^{(m)}\} \quad (1314)$$

denote the stress-test suite.

Equation 1314 can contain:

- supervisor replacement;
- temporary budget reduction;
- moderate workload increase;
- ordinary institutional disagreement;
- change of service provider;
- network-node loss;
- temporary health deterioration;
- adverse public attention.

The suite should reflect realistic future conditions rather than dramatic scenarios selected merely to produce failure.

17.35 Stress-Test Equivalence

A restored system can satisfy stress-test equivalence where

$$\mathcal{Q}(\mathcal{G}^R, \delta) \leq \mathcal{Q}(\mathcal{G}^{\text{ref}}, \delta) \quad \forall \delta \in \mathcal{S}_{\text{stress}}. \quad (1315)$$

Equation 1315 is test-suite relative.

Passing a finite suite does not establish universal robustness.

17.36 Stress-Test Coverage

Let Δ_{relevant} denote the broader relevant disturbance class.

Coverage is represented schematically by

$$\text{Cov}(\mathcal{S}_{\text{stress}}) = \frac{\mu(\mathcal{S}_{\text{stress}})}{\mu(\Delta_{\text{relevant}})} \quad (1316)$$

where a meaningful measure exists.

Equation 1316 is conceptual.

Its purpose is to remind the analyst that robustness conclusions depend upon the scope of tested scenarios.

17.37 Adversarial Perturbations

An adversarial perturbation seeks to produce maximal failure within an admissible disturbance budget.

Let disturbance set be

$$\Delta_{\varepsilon} = \{\delta : \|\delta\| \leq \varepsilon\}. \quad (1317)$$

Worst-case response is

$$W_R = \sup_{\delta \in \Delta_{\varepsilon}} L(\mathcal{G}^R, \delta), \quad (1318)$$

where L is a selected loss function.

Equation 1318 gives a robust-control-style interpretation of restorative fragility.

Its practical use in human settings requires strong procedural restraint.

17.38 Adversarial Robustness Comparison

A restored system can be compared with the reference by

$$W_R \leq W_{\text{ref}} + \varepsilon_W. \quad (1319)$$

Equation 1319 asks whether worst-case loss under bounded disturbance is comparable.

This relation is stronger than average-case robustness.

17.39 Ethical Limits of Adversarial Testing

Actual subjects should not be deliberately exposed to harmful perturbations merely to establish formal robustness.

Adversarial analysis can instead use:

- historical data;
- simulation;
- non-harmful proxy tests;
- organizational scenario analysis;
- natural variation;
- conservative bounds.

The distinction between analytical perturbation and imposed real-world harm is essential.

17.40 Counterfactual Stress Testing

A model can evaluate hypothetical disturbance without applying it.

Let

$$\hat{\mathcal{Q}}(\mathcal{G}, \delta) \quad (1320)$$

denote simulated response.

Equation 1320 reduces intervention burden while introducing model uncertainty.

Simulated robustness and observed robustness should therefore be distinguished.

17.41 Observed and Simulated Robustness

Let

$$R_{\text{obs}} \quad (1321)$$

denote robustness supported by actual observed perturbations, and

$$R_{\text{sim}} \quad (1322)$$

robustness estimated through simulation.

These quantities can differ.

A complete claim should report the epistemic basis.

17.42 Natural Experiments in Robustness

Naturally occurring disturbances can provide response evidence without deliberate exposure.

A change of management, market fluctuation, routine illness, or ordinary policy change can reveal whether the restored system remains stable.

The evidentiary interpretation still requires causal caution because natural events are rarely controlled experiments.

17.43 Response Function

The full deterministic response function can be written as

$$\mathcal{R}_Q : \Delta_{\text{adm}} \longrightarrow \mathcal{Y}, \quad (1323)$$

where $\mathcal{Y}$ is the response space.

Equation 1323 treats robustness as a mapping rather than a scalar.

A strong restorative criterion can compare the entire function.

17.44 Response-Function Equality

Exact equality is

$$\mathcal{R}_Q^R(\delta) = \mathcal{R}_Q^{\text{ref}}(\delta) \quad \forall \delta \in \Delta_{\text{adm}}. \quad (1324)$$

Equation 1324 is a demanding robustness criterion.

It is stronger than equal resilience scores.

17.45 Approximate Response-Function Equivalence

A practical criterion is

$$\sup_{\delta \in \Delta_{\text{adm}}} d_Y \left(\mathcal{R}_Q^R(\delta), \mathcal{R}_Q^{\text{ref}}(\delta) \right) \leq \varepsilon_R. \quad (1325)$$

Equation 1325 provides a uniform approximation criterion over the disturbance class.

The result depends upon d_Y , Δ_{adm} , and ε_R .

17.46 Average-Case Response Equivalence

Where perturbations have distribution $P(\delta)$, average response distance can be defined by

$$\mathbb{E}_\delta \left[d_Y \left(\mathcal{R}_Q^R(\delta), \mathcal{R}_Q^{\text{ref}}(\delta) \right) \right] \leq \varepsilon. \quad (1326)$$

Equation 1326 is weaker than worst-case equivalence.

Rare severe perturbations can be hidden by the average.

17.47 Distributional Response

For stochastic systems, response is a distribution:

$$P_G(Y \mid \delta). \quad (1327)$$

A distributional restorative criterion can require

$$D(P_R(Y \mid \delta), P_{\text{ref}}(Y \mid \delta)) \leq \varepsilon \quad (1328)$$

for every relevant δ .

Equation 1328 is stronger than equality of expected response alone.

17.48 Expected-Response Equality

A weaker criterion compares means:

$$\mathbb{E}_R[Y | \delta] = \mathbb{E}_{\text{ref}}[Y | \delta]. \quad (1329)$$

Equation 1329 can hold while response variance or tail risk differs.

17.49 Variance of Response

Compare

$$\text{Var}_R(Y | \delta) \quad (1330)$$

with

$$\text{Var}_{\text{ref}}(Y | \delta). \quad (1331)$$

Higher variance can indicate less predictable response even when expected response is equal.

17.50 Tail-Risk Comparison

For severe outcomes, tail probabilities can matter more than the mean.

Let loss variable be L . Compare

$$P_R(L > \ell | \delta) \quad (1332)$$

with

$$P_{\text{ref}}(L > \ell | \delta). \quad (1333)$$

A restored system can have equal average response and higher severe-failure risk.

This difference can be normatively significant.

17.51 Quantile Response

A quantile-based criterion can compare

$$Q_{\alpha,R}(Y | \delta) \leq Q_{\alpha,\text{ref}}(Y | \delta). \quad (1334)$$

Equation 1334 can be useful where protection against bad outcomes is more important than mean equivalence.

17.52 Stochastic Dominance of Responses

Where an ordering over outcomes exists, one response distribution can dominate another.

For losses, first-order stochastic dominance can be formulated through cumulative distributions.

A transformed restored system can therefore be normatively preferable without being distributionally equal to the reference.

This again shows why dominance belongs alongside equivalence in the atlas.

17.53 Response Correlation Structure

For multi-dimensional response $Y = (Y_1, \dots, Y_k)$, equal marginals can coexist with different dependence structures.

The covariance matrix is

$$\Sigma_Y = \text{Cov}(Y). \quad (1335)$$

Equation 1335 can reveal coupled failures.

A system can therefore be robust in each component separately and fragile when components fail together.

17.54 Coupled Response

For coupled subsystems $i = 1, \dots, n$, response can be represented by

$$\Delta \mathbf{x} = (\Delta x_1, \dots, \Delta x_n). \quad (1336)$$

Equation 1336 supports analysis of burden transfer.

A restorative policy can stabilize one subsystem by shifting disturbance into another.

17.55 Burden Shifting Under Perturbation

Let total response burden be distributed among actors:

$$\mathbf{B}_\delta = (B_1, \dots, B_n). \quad (1337)$$

A system can preserve aggregate performance while increasing burden on one vulnerable actor.

Robustness of the whole therefore does not establish distributive justice.

17.56 Aggregate Robustness and Individual Fragility

Suppose organizational output satisfies

$$Y_R^\delta \approx Y_{\text{ref}}^\delta \quad (1338)$$

while one member's burden satisfies

$$B_{i,R}^\delta \gg B_{i,\text{ref}}^\delta. \quad (1339)$$

Equations 1338 and 1339 demonstrate why robustness must be scale-indexed.

17.57 Robustness and Redundancy

Redundancy can increase robustness by preserving function after one component fails.

Let n_{ind} denote the number of sufficiently independent pathways to a critical condition.

A restored system can therefore improve robustness through

$$n_{\text{ind},R} > n_{\text{ind},\text{ref}}. \quad (1340)$$

Equation 1340 represents one form of better-than-reference restoration.

17.58 Robustness and Diversity

Redundancy based upon identical components can fail under one common-mode shock.

Diversity of pathways can therefore be a distinct robustness object.

A restorative design can compare functional diversity across alternatives.

The value of diversity depends upon whether the alternatives are genuinely independent and usable.

17.59 Robustness and Modularity

Modular systems can contain disturbances within one subsystem.

Let coupled system have adjacency or influence matrix A . Reduced cross-module coupling can limit propagation.

A restorative intervention can therefore improve robustness by changing architecture rather than increasing resource quantity.

This is particularly relevant to distributed governance and information systems.

17.60 Robustness and Decentralization

Decentralization can reduce single-point failure and can also create coordination problems.

Robustness should therefore not be assumed to increase monotonically with decentralization.

The appropriate object can be the distribution of control and redundancy rather than a binary centralized/decentralized classification.

17.61 Robustness and Institutional Interfaces

Institutional interfaces can themselves fail under stress.

A system can function well under ordinary conditions and break when one agency is overloaded or unavailable.

Stress testing should therefore include handoff and referral pathways where they are critical to restoration.

17.62 Robustness and Information Infrastructure

Correction, referral, and evidence systems can be tested under:

- delayed communication;
- missing records;
- inconsistent identifiers;
- partial system outage;
- conflicting institutional decisions.

A robust restorative architecture should degrade gracefully rather than collapse when one information channel fails.

17.63 Graceful Degradation

Let performance under perturbation amplitude a be $P(a)$.

Graceful degradation can be represented by bounded decline:

$$\left| \frac{dP}{da} \right| \leq L \quad (1341)$$

over a selected region, where differentiability is meaningful.

Equation 1341 illustrates a system that loses function gradually rather than catastrophically.

This can be preferable where abrupt failure has severe consequences.

17.64 Cliff Effects

A cliff effect occurs where small additional disturbance produces large loss.

Let threshold a_c satisfy

$$\lim_{\epsilon \downarrow 0} |P(a_c + \epsilon) - P(a_c - \epsilon)| \gg 0. \quad (1342)$$

Equation 1342 is a stylized representation of abrupt failure.

Restored systems near such thresholds can be substantially more fragile than baseline performance suggests.

17.65 Robustness Margin and Criticality

Robustness margin and distance to critical boundary are related but distinct.

A local geometric boundary distance can be small while practical perturbations rarely move in that direction.

Conversely, a larger geometric distance can coexist with highly amplified dynamics.

Both geometry and response should therefore be considered.

17.66 Sensitivity

Local sensitivity of observable Y to parameter or input u is

$$S_u = \frac{\partial Y}{\partial u}. \quad (1343)$$

Equation 1343 is a local property.

Equal local sensitivity does not imply equal response to large perturbations.

17.67 Sensitivity Profile

For input vector $\mathbf{u}$,

$$\mathbf{S} = \nabla_{\mathbf{u}} Y. \quad (1344)$$

Equation 1344 provides a directional local response structure.

A restored and reference system can be compared through their gradients or Jacobians where the model supports such analysis.

17.68 Local Linear Response

Near a reference point,

$$\Delta Y \approx J \Delta u, \quad (1345)$$

where J is the relevant Jacobian.

Equation 1345 is useful for small disturbances.

It becomes unreliable near strong nonlinearities, thresholds, or regime changes.

17.69 Nonlinear Response

A nonlinear response can satisfy

$$\mathcal{R}(a\delta) \neq a\mathcal{R}(\delta). \quad (1346)$$

Equation 1346 means response does not scale proportionally with perturbation magnitude.

This structure can produce disproportionate vulnerability to moderate stress.

17.70 Robustness Across Scales

Perturbation response should be evaluated at the relevant scale.

Let

$$\varrho^{(s)}(\mathcal{G}) \quad (1347)$$

denote robustness at scale s .

An organization can remain robust while one member experiences collapse. A community can remain robust while one ecosystem component fails. A national system can remain robust by transferring burden locally.

Scale should therefore accompany every robustness claim.

17.71 Cross-Scale Robustness

A stronger criterion considers whether robustness at one scale is achieved by destabilizing another.

Let

$$\Delta\varrho^{(s_1)} > 0 \quad (1348)$$

and

$$\Delta\varrho^{(s_2)} < 0. \quad (1349)$$

Equations 1348 and 1349 describe cross-scale trade-off.

Such trade-offs belong to later substantive-justice analysis.

17.72 Robustness Under Model Uncertainty

Let $\mathfrak{M}$ be a plausible model family.

A robust restorative conclusion can require

$$\mathcal{Q}(\mathcal{G}^R, \delta; M) \in \mathcal{Q}(\mathcal{G}^{\text{ref}}, \delta; M) \quad (1350)$$

for all $M \in \mathfrak{M}$ and relevant δ .

Equation 1350 protects against dependence upon one fragile model.

17.73 Robustness Under Parameter Uncertainty

Let uncertain parameter set be Θ .

A robust criterion can require

$$Q(\mathcal{G}^R; \theta, \delta) \in \mathcal{S}_{\text{acceptable}} \quad \forall \theta \in \Theta, \delta \in \Delta. \quad (1351)$$

Equation 1351 is a worst-case form.

Less conservative alternatives can use probability over Θ .

17.74 Probabilistic Robustness

If uncertainty over model or parameter is probabilistic, define success probability

$$p_{\text{rob}} = P \left(Q(\mathcal{G}^{R,\delta}) \in \mathcal{S}_{\text{acceptable}} \right). \quad (1352)$$

Equation 1352 can be compared with the reference.

The probability itself does not determine an acceptable threshold.

17.75 Robustness Confidence

A robustness claim should distinguish estimated robustness from confidence in that estimate.

Let

$$\text{Conf}(\varrho_R \mid K) \quad (1353)$$

denote the epistemic confidence supported by basis K .

A narrow confidence interval around a weakly specified model can still be misleading.

Model uncertainty and statistical uncertainty should therefore remain separate.

17.76 Robustness and Counterfactual Reference Uncertainty

The reference response itself can be counterfactual.

Let

$$\mathfrak{R}_{\text{ref}}^\delta = \{ Q(B_i, \delta) : B_i \in \mathfrak{B} \}. \quad (1354)$$

Equation 1354 represents several plausible reference responses.

The restored system can then be compared against the family rather than one precise counterfactual trajectory.

17.77 Robustness Bounds

Where exact reference response is unavailable, one can use bounds:

$$Q_{\min}(\delta) \leq Q_{\text{ref}}(\delta) \leq Q_{\max}(\delta). \quad (1355)$$

A restored response satisfying the acceptable reference region can support a robust but weaker claim.

This approach preserves action under uncertainty.

17.78 Stress-Test Minimal Common Response

Suppose several models disagree about exact response but agree that one class of intervention remains safe across the tested disturbances.

The corresponding policy can be selected from

$$\mathcal{U}_\cap = \bigcap_{M \in \mathfrak{M}} \mathcal{U}_{\text{safe}}^M. \quad (1356)$$

Equation 1356 defines a model-robust intervention set.

17.79 Robustness and Preventive Governance

Preventive governance can use robustness information to preserve distance from failure.

A policy can seek

$$m_R \geq m_{\min}. \quad (1357)$$

Equation 1357 defines a minimum robustness margin.

The threshold $m_{\min}$ requires normative and institutional justification.

17.80 Robustness and Interruption

During active harm, interruption can reduce the magnitude or duration of perturbation:

$$\delta_H \xrightarrow{I} \delta'_H. \quad (1358)$$

where

$$\|\delta'_H\| < \|\delta_H\| \quad (1359)$$

or its duration is shortened.

The objective is to keep the system inside a viable or desirable region while full repair remains unresolved.

17.81 Robustness and Post-Harm Assistance

Post-harm assistance can improve robustness by:

- increasing redundancy;
- reducing transition costs;
- improving health;
- diversifying income;
- broadening professional networks;
- reducing dependence upon one gatekeeper;
- improving information and contingency planning.

The resulting benefit can remain invisible in baseline-state comparison.

17.82 Robustness and General Equivalents

Money can increase robustness by providing flexible response capacity.

A financial buffer can absorb unexpected shocks without requiring the institution to know the precise future disturbance in advance.

This is another form of epistemic humility through fungibility.

The effect depends upon conversion structure and availability of market substitutes.

17.83 Robustness and Specific Enabling Conditions

Some shocks cannot be absorbed through money alone.

Legal status, healthcare, network access, accessible infrastructure, and institutional rights can function as specific robustness conditions.

A plural restorative architecture therefore combines general buffers with specific protections.

17.84 Robustness and Social Protection

Social protection can be interpreted as a system-level shock absorber.

Income security, healthcare access, and other basic guarantees can prevent one shock from propagating into housing, health, nutrition, or employment collapse.

This interpretation is compatible with the social-protection floor discussed earlier [26].

Its normative justification remains broader than instrumental resilience alone.

17.85 Robustness and Insurance

Insurance pools uncertain shocks across persons and time.

Its contribution to robustness can be represented as reduction in the post-shock loss distribution:

$$L^{\text{insured}} \prec L^{\text{uninsured}} \quad (1360)$$

under an appropriate order.

The relation depends upon coverage, exclusions, timing, and claims procedure.

17.86 Robustness and Non-Domination

A system can be robust only because one powerful actor continuously stabilizes it through discretionary control.

Such robustness can coexist with domination.

The robustness object should therefore be evaluated jointly with control structure [44].

A preferable restored system can possess slightly lower raw robustness and much greater secure agency.

17.87 Robustness and Generative Justice

Generative justice adds another question:

Does the system regenerate the conditions that allow it to remain robust, and who controls that regeneration?

A community can survive one shock through external aid and remain structurally dependent.

Another can use the aid to build locally controlled infrastructure that improves future shock absorption.

The second process differs at the level of recursive generativity and value circulation [15, 17].

17.88 Recursive Robustness

Let robustness at generation n be ϱ_n .

A recursively robust process can satisfy

$$\varrho_{n+1} \geq \varrho_n - \varepsilon \quad (1361)$$

across selected cycles.

Equation 1361 is illustrative.

It asks whether successful response to one disturbance preserves or improves the system's ability to face later disturbances.

17.89 Robustness Depletion

Some responses consume buffers and reduce later resilience.

Let reserve state be b_t . A perturbation response can satisfy

$$b_{t+1} < b_t. \quad (1362)$$

Repeated recovery can therefore hide progressive depletion.

A system can appear to recover repeatedly until the buffer is exhausted.

17.90 Reserve Capacity

Reserve capacity can itself be a restorative object:

$$Q_{\text{reserve}}(\mathcal{G}) = B_{\text{reserve}}. \quad (1363)$$

This can include financial savings, spare staffing, relational redundancy, health reserve, or ecological buffer.

Equal ordinary output with lower reserve capacity indicates weaker robustness.

17.91 Recovery-Induced Strengthening

A perturbation can sometimes produce learning or structural improvement.

Let robustness before and after recovery be

$$Q_{\text{post}} > Q_{\text{pre}}. \quad (1364)$$

Equation 1364 represents strengthening through successful adaptation.

The paper does not treat harm as justified merely because later growth occurs.

The historical wrong and later learning remain normatively distinct.

17.92 Recovery-Induced Fragility

The opposite can occur:

$$Q_{\text{post}} < Q_{\text{pre}}. \quad (1365)$$

Equation 1365 describes recovery that depletes resources or creates brittle compensatory structures.

This is a key reason to evaluate more than return to baseline output.

17.93 Robustness and Adaptation

Adaptation changes the system in response to disturbance.

Let

$$\mathcal{G} \xrightarrow{\delta} \mathcal{G}^\delta \xrightarrow{A} \mathcal{G}^{\delta,A}. \quad (1366)$$

Equation 1366 distinguishes passive robustness from active adaptation.

A restored system can achieve resilience through changed strategy rather than return to its prior state.

17.94 Robustness and Transformation

When the current regime cannot remain viable under realistic disturbance, transformation can be preferable to increasing robustness of the existing regime.

This distinction follows directly from the previous section's separation of resilience and transformability.

A theory of justice should therefore ask whether the object worth protecting is the current regime or a deeper function, right, capability, or generative process.

17.95 Functional Robustness

A system can preserve function while changing internal structure.

Let functional output be F_Q . Functional robustness requires

$$F_Q(\mathcal{G}^\delta) \in \mathcal{S}_F \quad (1367)$$

for all relevant perturbations.

Equation 1367 can be appropriate where internal structure is not normatively central.

17.96 Structural Robustness

Structural robustness requires preservation of selected internal relations under perturbation:

$$Q_{\text{struct}}(\mathcal{G}^\delta) \subseteq Q_{\text{struct}}(\mathcal{G}). \quad (1368)$$

Equation 1368 is stronger than functional robustness.

The relevant structure should be justified by the restorative purpose.

17.97 Relational Robustness

Relational robustness concerns persistence of important social relations under ordinary stress.

A mentoring or collaboration relation can exist only under calm conditions and collapse during disagreement.

A restored relation can therefore be tested by whether trust, communication, and procedural channels remain functional under reasonable perturbation.

Actual harmful provocation should not be required to perform such testing.

17.98 Recognition Robustness

Public recognition can also be fragile.

A subject can regain standing until one new accusation or institutional change reactivates an earlier stigma.

Recognition robustness therefore concerns whether correction has become durable across downstream social systems.

17.99 Reputational Stress Testing

A non-harmful reputational stress test can examine whether:

- corrected records remain current after database synchronization;
- new decision-makers receive the corrected status;
- superseded flags reappear;

- automated processes continue using invalid historical states.

This is primarily a technical and institutional stress test rather than a social provocation of the person.

17.100 Procedural Robustness

A procedure can operate fairly under ordinary conditions and fail under high caseload, urgency, conflict, or staff turnover.

Procedural robustness can therefore be represented as

$$P^\delta \in \mathcal{S}_{\text{just}} \quad \forall \delta \in \Delta_{\text{proc}}. \quad (1369)$$

Equation 1369 asks whether procedural safeguards survive stress.

17.101 Rights Robustness

A right is especially meaningful where it remains effective during adverse conditions.

A reporting right that disappears during organizational crisis provides weaker protection than one institutionally secured across stress.

Rights robustness therefore concerns persistence of practical enforceability.

17.102 Robustness of Contestability

Appeal and contestation mechanisms can be tested for continued accessibility during stress.

If the very condition generating harm also disables the channel of complaint, the institution has low contestability robustness.

This connects procedural architecture with generative resilience.

17.103 Robustness of Exit

Exit can also be stress-dependent.

A person can leave a relation under ordinary conditions and become trapped when income, housing, or legal status deteriorates.

Let exit-cost function be

$$C_{\text{exit}} = C_{\text{exit}}(\delta). \quad (1370)$$

A non-domination-sensitive system should avoid extreme growth of C_{exit} under ordinary adverse conditions.

17.104 Robustness of Subject-Directed Futures

A future selected by the subject can be fragile if it depends upon one narrow support pathway.

Let target set be $\mathcal{T}_A$. A robust target-access criterion can require

$$\mathcal{R}_\tau^{R,\delta} \cap \mathcal{T}_A \neq \emptyset \quad \forall \delta \in \Delta_{\text{adm}}. \quad (1371)$$

Equation 1371 asks whether the chosen future remains reachable under foreseeable disturbance.

17.105 Robust Reachability

Define robust reachable set as

$$\mathcal{R}_\tau^{\text{rob}} = \bigcap_{\delta \in \Delta_{\text{adm}}} \mathcal{R}_\tau(x; \delta). \quad (1372)$$

Equation 1372 contains states reachable under every admissible perturbation.

This is a conservative object.

A less conservative version can require reachability with high probability over the perturbation distribution.

17.106 Robust Viability

Similarly, define robust viability kernel

$$\text{Viab}_{\text{rob}} = \{x : \exists u(\cdot) \text{ maintaining admissibility for all } \delta \in \Delta_{\text{adm}}\}. \quad (1373)$$

Equation 1373 combines future possibility with worst-case admissibility.

Its conservatism must be normatively justified.

17.107 Robustness and Conservatism

Worst-case robustness can become excessively conservative.

Designing against every imaginable disturbance can consume large resources, restrict autonomy, and suppress innovation.

The perturbation class should therefore reflect relevant, plausible, or normatively required disturbances.

Robustness should protect generativity without eliminating it through excessive defensiveness.

17.108 Over-Robustification

A system can become so optimized for resistance that it loses flexibility, adaptation, or openness.

Examples include:

- excessive procedural rigidity;
- over-redundant bureaucracy;
- intrusive monitoring;
- overcapitalization against remote risks;
- suppression of experimentation.

Robustness therefore competes with flexibility and generative openness in some contexts.

17.109 Robustness–Flexibility Trade-Off

Let robustness be ϱ and flexibility be F_{lex} .

An intervention can satisfy

$$\Delta \varrho > 0 \quad (1374)$$

and

$$\Delta F_{\text{lex}} < 0. \quad (1375)$$

Equations 1374 and 1375 create a normative trade-off.
The appropriate balance depends upon the object and risk environment.

17.110 Robustness–Efficiency Trade-Off

Redundancy and reserve capacity can reduce short-run efficiency.

A system optimized for average performance can therefore be fragile.

A restorative theory concerned with generativity should avoid evaluating every buffer as wasted capacity.

At the same time, unlimited redundancy is infeasible.

17.111 Robustness–Privacy Trade-Off

Monitoring can improve detection of impending failure and reduce privacy.

Let robustness gain be $\Delta\varrho$ and privacy loss $\Delta P < 0$.

A technically effective monitoring regime can remain procedurally unjust.

This trade-off becomes especially important in health, employment, and data governance.

17.112 Robustness–Agency Trade-Off

Institutions can increase robustness by restricting choices.

A highly controlled system can be less likely to deviate and more dominating.

Robustness therefore requires evaluation jointly with agency and non-domination.

17.113 Robustness–Historical Fidelity Trade-Off

A transformed system can be more robust than the historical reference while being less similar to it.

Let

$$\varrho_R > \varrho_{\text{ref}} \quad (1376)$$

and

$$F_H(\mathcal{G}_R) < F_H(\mathcal{G}_{\text{ref}}). \quad (1377)$$

Equations 1376 and 1377 can describe transformative improvement.

17.114 Baseline Equivalence and Robustness Non-Equivalence

The central non-implication of this section is

$$Q(\mathcal{G}^R) = Q(\mathcal{G}^{\text{ref}}) \not\Rightarrow \mathcal{R}_Q^R = \mathcal{R}_Q^{\text{ref}}. \quad (1378)$$

Equation 1378 means equal baseline object does not imply equal response function.

This is one of the strongest reasons to include perturbation comparison in the atlas.

17.115 Single-Shock Equivalence and Repeated-Shock Non-Equivalence

Likewise,

$$\mathcal{Q}_R(\delta) = \mathcal{Q}_{\text{ref}}(\delta) \quad (1379)$$

does not imply

$$\mathcal{Q}_R(\delta_1, \dots, \delta_n) = \mathcal{Q}_{\text{ref}}(\delta_1, \dots, \delta_n). \quad (1380)$$

Equations 1379 and 1380 distinguish one-shot robustness from cumulative robustness.

17.116 Average-Case Equivalence and Tail Non-Equivalence

It can hold that

$$\mathbb{E}[Y_R] = \mathbb{E}[Y_{\text{ref}}] \quad (1381)$$

while

$$P_R(Y < y_{\text{crit}}) > P_{\text{ref}}(Y < y_{\text{crit}}). \quad (1382)$$

Equations 1381 and 1382 show why average robustness can conceal catastrophic risk.

17.117 Robustness and Hysteresis

Perturbation can push a system across a threshold after which removing the disturbance does not restore the earlier regime.

Thus response analysis should ask whether

$$\mathcal{P}_{-\delta} \circ \mathcal{P}_{\delta}(\mathcal{G}) = \mathcal{G} \quad (1383)$$

holds.

Failure of Equation 1383 is one form of perturbation-induced irreversibility.

17.118 Perturbation-Induced Rule Change

A disturbance can alter the transition rule itself:

$$\Phi \xrightarrow{\delta} \Phi^{\delta}. \quad (1384)$$

Equation 1384 means the post-shock system is not merely displaced within the old landscape.

This can occur through learning, institutional reform, trauma, technological change, or ecological restructuring.

17.119 Robustness Under Rule Change

A richer robustness object should therefore include both state and rule response:

$$Q_{\text{rob}} = (x^{\delta}, \Phi^{\delta}). \quad (1385)$$

Equation 1385 distinguishes temporary state deviation from structural transformation.

17.120 Recovery to a New Regime

A system can recover function without returning to the old regime.

Let

$$F_Q(\mathcal{G}^{\text{new}}) = F_Q(\mathcal{G}^{\text{ref}}) \quad (1386)$$

while

$$\Phi_{\text{new}} \neq \Phi_{\text{ref}}. \quad (1387)$$

Equations 1386 and 1387 describe adaptive functional recovery.

This can be normatively sufficient or superior where structural identity is unnecessary.

17.121 Robustness and Subjective Experience

External response can appear robust while subjective burden increases.

A subject can maintain performance under stress through greater effort, fear, or exhaustion.

Robustness claims should therefore specify whether internal burden is part of the response object.

This is another reason effort-adjusted comparison matters.

17.122 Effort-Adjusted Robustness

Let performance response be Y and effort cost C_E .

A richer comparison requires

$$Y_R^\delta \sim Y_{\text{ref}}^\delta \quad (1388)$$

and

$$C_{E,R}^\delta \sim C_{E,\text{ref}}^\delta. \quad (1389)$$

Equations 1388 and 1389 prevent hidden compensatory effort from being mistaken for full robustness.

17.123 Recovery Quality

Recovery can therefore be represented by a profile

$$\mathbf{R}_\delta = (M_\delta, T_{\text{rec}}, B_\delta, R_\infty, C_E, \varrho). \quad (1390)$$

Equation 1390 contains peak response, recovery time, cumulative burden, residual displacement, effort cost, and robustness.

This is more informative than one recovery score.

17.124 Perturbation Response as an Atlas Region

The perturbation region of the atlas can be represented as

$$\mathfrak{C}_{\text{pert}} = \{(Q_{\text{response}}, =), (Q_{\text{response}}, \approx_\varepsilon), (Q_{\text{response}}, \sim_{\text{dist}}), (Q_{\text{response}}, \sim_{\text{stat}}), (Q_{\text{robust}}, \succeq)\}. \quad (1391)$$

Equation 1391 contains exact, approximate, distributional, statistical, and dominance-style response comparisons.

17.125 Stress-Test Cell

A stress-test restorative cell is represented by

$$[B; Q_{\text{response}}; E; \mathcal{S}_{\text{stress}}; \tau; \varepsilon; K]. \quad (1392)$$

Equation 1392 records the reference, response object, relation, stress suite, horizon, tolerance, and epistemic basis.

This is a fully indexed robustness claim.

17.126 Robustness Profile

A system can be summarized across several perturbation classes:

$$\varrho = (\varrho_{\text{economic}}, \varrho_{\text{relational}}, \varrho_{\text{institutional}}, \varrho_{\text{health}}, \varrho_{\text{environmental}}). \quad (1393)$$

Equation 1393 avoids collapsing heterogeneous robustness into one scalar.

Table 10: Perturbation dimensions in restorative robustness analysis

Perturbation dimension	di-	Formal descriptor	Restorative significance	Characteristic risk
Amplitude		$\ \delta\ $	Tolerance to disturbance magnitude	Magnitude can be incomparable across types
Direction		v	Reveals anisotropic vulnerability	One scalar robustness score can conceal weakness
Duration		$[t_a, t_b]$	Distinguishes transient and sustained stress	Short tests can overstate durability
Frequency		$\{t_i\}$	Captures repeated disturbance	Single-shock testing can miss cumulative failure
Sequence		$(\delta_1, \dots, \delta_n)$	Captures order effects	Perturbations can be non-commutative
Correlation		$\text{Cov}(\delta_i, \delta_j)$	Captures systemic co-occurrence	Independence assumptions can understate risk
Compound structure		δ^{comp}	Tests coupled vulnerabilities	Separate tests can miss nonlinear interaction
Horizon		τ	Distinguishes short- and long-run robustness	Short horizons can hide slow failure
Source		Institutional, environmental, relational, endogenous	Supports mechanism-sensitive analysis	Same measured shock can have different meaning

17.127 Perturbation Matrix

Table 10 summarizes major perturbation dimensions and their restorative significance.

Table 10 shows that perturbation specification is part of the comparison rather than a secondary methodological detail.

17.128 Response Matrix

Table 11 summarizes several response objects.

Table 11 demonstrates that “robustness” itself is a family of objects.

17.129 Workplace Robustness Example

Consider a worker formally reinstated after prolonged exclusion.

A baseline assessment can show equal salary and ordinary output.

A robustness assessment can examine:

- response to change of supervisor;
- response to ordinary disagreement;
- persistence of anti-retaliation protection;
- ability to retain network access after organizational restructuring;
- practical exit options if conflict recurs.

The system can satisfy state restoration and fail relational or institutional robustness.

Table 11: Response objects for restorative robustness comparison

Response object	Formal form	Restorative interpretation	Characteristic limitation
Peak deviation	M_δ	Largest departure under shock	Ignores duration
Recovery time	T_{rec}	Speed of durable return	Ignores burden before recovery
Integrated burden	B_δ	Total response cost over time	Depends upon burden metric
Residual displacement	R_∞	Persistent post-shock difference	Can miss transient severity
Robustness margin	m_R	Largest tolerated disturbance	Depends upon perturbation norm
Response function	$\delta \mapsto \mathcal{R}_Q(\delta)$	Full disturbance-response mapping	High data and modeling burden
Response distribution	$P(Y \mid \delta)$	Stochastic response structure	Requires distributional identification
Tail risk	$P(L > \ell \mid \delta)$	Probability of severe failure	Threshold sensitive

17.130 Research Robustness Example

For a researcher, perturbations can include:

- temporary funding gap;
- departure of one collaborator;
- equipment failure;
- change of institution;
- rejection of one grant or publication.

A robust research system retains several viable paths after these ordinary events.

A fragile restored system can depend entirely upon one person, one grant, or one institution.

The difference is generatively significant even where current resources are equal.

17.131 Reputational Robustness Example

After correction of a false institutional record, robustness analysis can test whether:

- the correction survives database updates;
- derivative records remain corrected;
- new decision-makers receive the superseding information;
- the old classification is not reactivated automatically.

This form of stress testing concerns information infrastructure rather than deliberate social provocation.

17.132 Collective Robustness Example

A reparative community program can create new infrastructure.

Baseline evaluation can confirm that the infrastructure exists.

Robustness evaluation can ask whether it survives:

- funding variation;
- leadership transition;
- loss of one external partner;
- moderate economic shock;
- political turnover.

A community-controlled architecture with diversified support can be more robust than an externally dependent program of equal current value.

17.133 Ecological Robustness Example

Ecological systems provide established examples of disturbance response, resilience, alternative regimes, and critical transitions [24, 52, 53].

The ecological case demonstrates why ordinary state similarity can be insufficient where disturbance response differs.

The formal structures should not be transferred mechanically to social systems.

17.134 Robustness and Ordinary Life

The strongest practical insight of perturbation analysis does not require extreme stress testing.

Ordinary life itself continually introduces moderate disturbances.

A genuinely usable restoration should therefore survive ordinary variability in:

- people;
- schedules;
- resources;
- workloads;
- policies;
- health;
- relationships.

A condition that functions only under unusually protected circumstances can possess low practical robustness.

17.135 Perturbation Minimalism

Testing should use the least intrusive perturbation evidence sufficient for the restorative purpose.

Historical observations, routine variation, simulation, and benign tests should be preferred over deliberate exposure to substantial harm.

This principle follows the broader institutional minimalism developed earlier.

17.136 Perturbation Proportionality

The burden of testing should be proportionate to the stakes of the robustness claim.

A routine reimbursement does not require elaborate stress testing.

A safety-critical system, fragile ecological restoration, or high-stakes institutional re-entry can justify stronger analysis.

17.137 Perturbation Consent

Where testing affects a person directly, consent can be required depending upon the nature and burden of the test.

Simulation and passive observation can reduce this burden.

Consent does not authorize otherwise unjustified harmful exposure.

17.138 Perturbation Privacy

Robustness monitoring can involve sensitive longitudinal information.

A test architecture should specify:

- what data are required;
- how long they are retained;
- who can access them;
- whether raw data can remain local;
- how the subject can contest interpretation.

Robustness should not become a pretext for indefinite surveillance.

17.139 Perturbation Interpretation

A failed stress test does not automatically identify the cause of failure.

It can reveal:

- altered state;
- weak network redundancy;
- insufficient resources;
- changed dynamics;
- measurement error;
- inappropriate test design.

Response comparison and causal explanation therefore remain distinct.

This motivates the next section.

17.140 Perturbation and Intervention

A perturbation is a disturbance used to observe or characterize system response.

A causal intervention is an operation defined relative to a causal model, typically intended to distinguish changes produced by manipulation of a specified variable from passive association [43].

The two concepts can overlap empirically and should remain conceptually distinct.

A workload increase can be treated as a perturbation for robustness analysis and as an intervention in a causal model.

The next section formalizes this distinction more carefully.

17.141 Robustness Principle

The first principle of this section is:

Baseline restorative equivalence does not establish equivalence of response under disturbance.

17.142 Perturbation-Class Principle

The second principle is:

Every robustness claim is indexed to a perturbation class whose amplitude, direction, duration, frequency, timing, and source should be specified where material.

17.143 Repeated-Shock Principle

The third principle is:

Robustness to one isolated perturbation does not establish robustness to repeated, correlated, or compound disturbances.

17.144 Response-Profile Principle

The fourth principle is:

Peak deviation, recovery time, cumulative burden, residual displacement, and tail risk are distinct response objects and should not be compressed into one robustness claim without justification.

17.145 Scale Principle

The fifth principle is:

System-level robustness does not establish robustness or justice for each subsystem or subject.

17.146 Normative Robustness Principle

The sixth principle is:

Greater robustness lacks automatic normative superiority because robust persistence can preserve domination, extraction, or an unjust regime.

17.147 Testing Restraint Principle

The seventh principle is:

Robustness should be assessed through the least intrusive evidence and test architecture sufficient for the legitimate restorative purpose.

17.148 Future-Usability Principle

The eighth principle is:

A restorative arrangement is more practically meaningful when the returned condition remains usable under the ordinary disturbances expected in the subject's future environment.

17.149 Perturbation Layer of the Atlas

The perturbation layer can be represented by

$$\mathfrak{P}_A = (\Delta_{\text{adm}}, Q_{\text{response}}, E_{\text{response}}, \tau, \varepsilon, K). \quad (1394)$$

Equation 1394 contains the disturbance class, response object, comparative relation, horizon, tolerance, and epistemic basis.

This structure integrates perturbation analysis directly into the indexed atlas.

17.150 Implications for Causal Comparison

Perturbation analysis reveals whether systems react differently. It does not by itself identify why.

Two systems can respond differently because:

- their causal graphs differ;
- their structural functions differ;
- their parameters differ;
- their latent states differ;
- their environments differ;
- their histories changed the meaning of the perturbation.

The distinction between observed response and causal mechanism therefore becomes the next analytical problem.

17.151 Transition to Causal and Interventional Equivalence

Section 18 develops causal and interventional comparison. It distinguishes observational equality from causal structure, compares structural causal models, intervention-response distributions, mediation pathways, counterfactual response, and mechanism invariance, and asks when a restored system is equivalent to the reference in the stronger sense that the same interventions generate corresponding outcomes.

The transition is important because robustness tells us whether the systems behave differently under disturbance, while causal analysis asks which generative mechanisms produce those differences.

18 Causal and Interventional Equivalence

This section develops the causal region of the Atlas of Equivalence. The preceding sections distinguished present state, realized trajectory, reachable future, dynamical regime, and response under perturbation. Those comparisons can reveal substantial differences between a restored system and a reference system. They do not by themselves identify the mechanisms through which those differences are generated. The present section therefore introduces causal structure as a separate restorative object.

The distinction is central to the justice of return. Two systems can display the same current state, generate the same observed output distribution, and respond similarly under a limited family of disturbances while differing in the causal mechanisms through which resources, relations, institutions, and interventions produce later outcomes. A returned salary can generate different professional possibilities because health, legal status, trust, or network access now mediate its effects differently. A reinstated employee can display the same productivity while the causal dependence of that productivity upon one supervisor has changed. A community can recover aggregate income while the mechanisms determining who controls productive assets remain structurally different.

Causal restoration therefore concerns more than resemblance. It asks whether corresponding interventions, mechanisms, dependencies, and counterfactual relations are preserved under the selected representation.

The section proceeds through six analytical levels:

1. observational distributions;
2. causal graphs and dependency structures;
3. structural causal mechanisms;
4. interventional response;
5. mediation and path-specific structure;
6. counterfactual response.

These levels should not be interpreted as a universal linear hierarchy of justice. They represent increasingly rich causal claims under a selected causal model. Their epistemic requirements generally increase as the analysis moves from observed association toward counterfactual structure. Pearl's structural causal framework provides the principal formal language used in this section [43]. The paper treats that framework as an analytical interface rather than an exhaustive ontology of historical or social causation.

18.1 Causal Objects in the Restorative Atlas

Let Q_{causal} denote the projection of a generative system into the causal structure relevant to the restorative inquiry. The causal comparison object is represented by Equation 1395.

$$Q_{\text{causal}} : \mathcal{G} \longrightarrow \mathcal{C}. \quad (1395)$$

Equation 1395 makes causal comparison explicitly representation-relative. The space $\mathcal{C}$ can contain a causal graph, structural causal model, intervention-response family, mediation structure, counterfactual distribution, or another causal object.

A restorative claim in the causal region then takes the form represented by Equation 1396.

$$Q_{\text{causal}}(\mathcal{G}^R) \ E_C \ Q_{\text{causal}}(\mathcal{G}^{\text{ref}}), \quad (1396)$$

where E_C is the selected causal comparative relation. Equation 1396 is the elementary causal cell of the atlas.

18.2 Association and Causation

Observed statistical association and causal dependence occupy different analytical levels.

Let observed variables be X and Y . An observational distribution is represented by Equation 1397.

$$P(Y \mid X = x). \quad (1397)$$

Equation 1397 describes the distribution of Y among observations satisfying $X = x$.

A causal intervention is represented by Equation 1398.

$$P(Y \mid do(X = x)). \quad (1398)$$

Equation 1398 describes the distribution of Y after the causal model is modified so that X is set to x in the specified intervention semantics [43].

The distinction is fundamental because Equation 1399 can hold.

$$P(Y | X = x) \neq P(Y | do(X = x)). \quad (1399)$$

Equation 1399 shows why an observed relationship cannot automatically be interpreted as the effect of changing X .

18.3 Observational Equivalence

Two systems are observationally equivalent relative to selected variable family V_O when their observational distributions correspond under the selected relation. An exact criterion is given by Equation 1400.

$$P_R(V_O) = P_{\text{ref}}(V_O). \quad (1400)$$

Equation 1400 can be a strong empirical achievement.

Its causal content remains limited. Distinct causal structures can induce the same observational distribution.

This permits the central non-implication in Equation 1401.

$$P_R(V_O) = P_{\text{ref}}(V_O) \not\Rightarrow C_R = C_{\text{ref}}. \quad (1401)$$

Equation 1401 establishes that observational restoration does not independently establish causal restoration.

18.4 Observational Equivalence in Restorative Practice

Observational equivalence can arise in ordinary restorative settings.

A worker can have:

- equal salary;
- equal attendance;
- equal output;
- equal formal title;

while future output now depends upon extraordinary personal effort or the continued cooperation of one powerful gatekeeper.

The visible observations can therefore correspond while the causal mechanism supporting them differs.

This is one reason state and performance equality should remain bounded claims.

18.5 Causal Graphs

A directed acyclic graph can represent selected causal dependencies among variables where the assumptions of the causal model support such a representation.

Let the causal graph be represented by Equation 1402.

$$G_C = (V_C, E_C), \quad (1402)$$

where V_C is the variable set and E_C the set of directed causal edges. Equation 1402 provides a structural object for causal comparison.

An edge

$$X \longrightarrow Y \quad (1403)$$

represents a direct causal relation relative to the model's variable set and structural assumptions. Equation 1403 should therefore be interpreted relative to the chosen causal granularity.

18.6 Causal-Graph Equality

The strongest graph-level criterion is exact equality:

$$G_{C,R} = G_{C,\text{ref}}. \quad (1404)$$

Equation 1404 requires the same represented variable nodes and directed-edge structure.

This criterion is sensitive to variable naming and representation.

A structurally equivalent system with different variable labels can fail exact graph equality.

18.7 Causal-Graph Isomorphism

A weaker structural criterion permits relabeling.

Let

$$\phi : V_{C,\text{ref}} \longrightarrow V_{C,R} \quad (1405)$$

be a bijection. Equation 1405 defines the node correspondence.

Causal-graph isomorphism requires Equation 1406.

$$X \longrightarrow Y \text{ in } G_{C,\text{ref}} \iff \phi(X) \longrightarrow \phi(Y) \text{ in } G_{C,R}. \quad (1406)$$

Equation 1406 preserves directed dependency structure while allowing labels to change.

It remains weaker than equality of causal mechanisms.

18.8 Graph Structure and Mechanism

Two systems can possess isomorphic causal graphs and different functional relationships along their edges.

Thus Equation 1407 can hold.

$$G_{C,R} \cong G_{C,\text{ref}} \not\Rightarrow F_R = F_{\text{ref}}. \quad (1407)$$

Equation 1407 separates graph structure from structural mechanism.

The same causal architecture can produce quantitatively or qualitatively different intervention responses.

18.9 Structural Causal Models

A structural causal model can be represented by Equation 1408.

$$M = (U, V, F, P(U)), \quad (1408)$$

where:

- U denotes exogenous variables;
- V denotes endogenous variables;
- F denotes structural functions;
- $P(U)$ denotes the exogenous-variable distribution where probabilistic structure is included.

Equation 1408 follows the structural causal-model tradition [43].

For each endogenous variable V_i , the structural mechanism is represented by Equation 1409.

$$V_i = f_i(\text{Pa}_i, U_i), \quad (1409)$$

where Pa_i denotes the modeled endogenous parents of V_i . Equation 1409 expresses the mechanism through which the variable is generated.

18.10 Structural-Mechanism Equality

The strongest mechanism-level restorative criterion requires corresponding structural functions to agree. After an appropriate variable correspondence ϕ , a criterion is given by Equation 1410.

$$f_{i,R} = \phi \circ f_{i,\text{ref}} \circ \phi^{-1} \quad (1410)$$

where the composition is meaningful for the selected representation. Equation 1410 represents mechanism preservation under relabeling.

This is substantially stronger than causal-graph isomorphism.

18.11 Approximate Mechanism Equivalence

Exact equality of structural mechanisms will often be unrealistic.

A metric or test-based approximation can require Equation 1411.

$$d_F(f_{i,R}, f_{i,\text{ref}}) \leq \varepsilon_F \quad \text{for every selected } i. \quad (1411)$$

Equation 1411 creates a family of mechanism-level approximate restorative claims.

The metric d_F and tolerance ε_F require explicit justification.

18.12 Mechanism Families

A restorative inquiry need not compare every mechanism.

Let

$$F^* = \{f_i : i \in I^*\} \quad (1412)$$

denote the selected mechanism family. Equation 1412 supports scope discipline.

A compensation case can focus upon income-generation mechanisms. A rehabilitation case can focus upon resource-to-function conversion. A governance case can focus upon authority-to-access mechanisms.

18.13 Causal Intervention Families

Let

$$\mathcal{U}_C = \{u_1, u_2, \dots\} \quad (1413)$$

denote the family of causal interventions relevant to the restorative inquiry. Equation 1413 can include:

- resource transfers;
- treatment;
- removal of a gatekeeper;
- legal-status correction;
- provision of information;
- change of institutional rule;
- network introduction;
- change in governance rights.

The intervention family determines the scope of interventional equivalence.

18.14 Interventional Response Maps

For outcome object Y , define the intervention-response map through Equation 1414.

$$\mathcal{I}_Y^M : u \mapsto P_M(Y \mid do(u)). \quad (1414)$$

Equation 1414 maps each admissible intervention to its modeled outcome distribution.

The map itself is a causal object.

18.15 Exact Interventional Equivalence

Two systems are interventionally equivalent over intervention family $\mathcal{U}_C$ and outcome family Y^* when Equation 1415 holds.

$$P_R(Y \mid do(u)) = P_{\text{ref}}(Y \mid do(u)) \quad \forall u \in \mathcal{U}_C, Y \in Y^*. \quad (1415)$$

Equation 1415 is stronger than ordinary observational equivalence.

It means that the corresponding interventions generate the same selected response distributions.

18.16 Partial Interventional Equivalence

A bounded institutional purpose can justify a smaller intervention family $\mathcal{U}_0 \subset \mathcal{U}_C$.

Partial interventional equivalence is represented by Equation 1416.

$$P_R(Y \mid do(u)) = P_{\text{ref}}(Y \mid do(u)) \quad \forall u \in \mathcal{U}_0. \quad (1416)$$

Equation 1416 can be entirely adequate for a narrowly defined restorative task.

The intervention family should remain explicit so that the result is not generalized beyond its scope.

18.17 Approximate Interventional Equivalence

A practical approximation is represented by Equation 1417.

$$D(P_R(Y \mid do(u)), P_{\text{ref}}(Y \mid do(u))) \leq \varepsilon_I \quad \forall u \in \mathcal{U}_0. \quad (1417)$$

Equation 1417 requires a distributional distance D and tolerance ε_I .

The criterion can be especially useful when exact empirical equality is unrealistic.

18.18 Interventional Dominance

A restored system can respond better than the reference under the selected interventions.

Let $\succeq_V$ denote the relevant normative ordering over outcome distributions. Interventional dominance is represented by Equation 1418.

$$P_R(Y \mid do(u)) \succeq_V P_{\text{ref}}(Y \mid do(u)) \quad \forall u \in \mathcal{U}_0. \quad (1418)$$

Equation 1418 supports better-than-reference restoration.

It is especially relevant where the reference mechanism was itself unjust or inefficient.

18.19 Observational Equality and Interventional Difference

The central causal non-implication can be written as Equation 1419.

$$P_R(Y, X) = P_{\text{ref}}(Y, X) \not\equiv P_R(Y \mid do(X = x)) = P_{\text{ref}}(Y \mid do(X = x)). \quad (1419)$$

Equation 1419 demonstrates why matching observed data does not guarantee matching causal response.

This distinction is central to restoration of conversion structures.

18.20 Resource Return and Interventional Difference

Suppose the same resource amount r is returned in both systems:

$$r_R = r_{\text{ref}}. \quad (1420)$$

Equation 1420 establishes resource equality.

If conversion mechanisms differ, the intervention effect can satisfy Equation 1421.

$$P_R(Y \mid do(R = r)) \neq P_{\text{ref}}(Y \mid do(R = r)). \quad (1421)$$

Equation 1421 gives a causal formulation of the difference between resource return and generative restoration.

18.21 Conversion as a Causal Mechanism

The conversion map developed earlier can be interpreted causally where the model supports intervention on the input condition.

Let resource R generate capability K through mechanism f_K . The structural equation is represented by Equation 1422.

$$K = f_K(R, X, H, U_K), \quad (1422)$$

where X denotes relevant present conditions, H historical state, and U_K exogenous factors. Equation 1422 shows how identical resources can generate different capabilities after harm.

18.22 Conversion-Mechanism Restoration

A strong conversion criterion compares the causal response to resource interventions:

$$P_R(K \mid do(R = r)) = P_{\text{ref}}(K \mid do(R = r)) \quad r \in \mathcal{R}_0. \quad (1423)$$

Equation 1423 provides a causal version of conversion equivalence.

This criterion is substantially stronger than equal observed capability at one resource level.

18.23 Mechanism Invariance Across Resource Levels

A stronger criterion can compare responses across a family of resource levels:

$$P_R(K \mid do(R = r)) \sim_E P_{\text{ref}}(K \mid do(R = r)) \quad \forall r \in \mathcal{R}_0. \quad (1424)$$

Equation 1424 evaluates whether the restored and reference conversion systems correspond as functions rather than at one point.

18.24 Treatment-Response Restoration

In health and rehabilitation contexts, intervention A can denote treatment.

A treatment-response restorative claim is represented by Equation 1425.

$$P_R(Y \mid do(A = a)) = P_{\text{ref}}(Y \mid do(A = a)). \quad (1425)$$

Equation 1425 asks whether the same treatment now produces a corresponding outcome.

A changed response can reveal altered physiology, environment, adherence structure, or another causal mechanism.

18.25 Policy-Response Restoration

For governance, intervention A can denote policy.

The corresponding comparison is given by Equation 1426.

$$P_R(Y \mid do(A = a)) \neq P_{\text{ref}}(Y \mid do(A = a)) \quad a \in \mathcal{A}_{\text{policy}}. \quad (1426)$$

Equation 1426 can reveal whether an institution has become structurally different even when its baseline outcomes look similar.

18.26 Intervention Scope

Every interventional comparison requires definition of which variables can legitimately be manipulated.

Let the intervention scope be

$$\text{Scope}_I = (V_I, \mathcal{U}_I), \quad (1427)$$

where V_I is the set of intervention variables and $\mathcal{U}_I$ their admissible values. Equation 1427 prevents a causal model from implying that every modeled variable is a legitimate policy target.

Some variables can be measurable without being morally or practically manipulable.

18.27 Causal Manipulability and Normative Admissibility

The set of causally definable interventions can exceed the set of normatively admissible interventions.

Let

$$\mathcal{U}_{\text{adm}} \subseteq \mathcal{U}_{\text{causal}}. \quad (1428)$$

Equation 1428 separates causal definability from normative permission.

A causal model can define an intervention upon memory, relationship, or identity that restorative justice should reject.

18.28 Intervention Burden

Interventions differ in burden.

Let

$$C_I(u) \quad (1429)$$

denote the cost or burden associated with intervention u . Equation 1429 can include:

- financial cost;
- coercion;
- privacy intrusion;
- administrative burden;
- third-party effects;
- irreversibility.

A causally effective intervention can remain normatively inferior because of its burden.

18.29 Mechanism Preservation and Outcome Preservation

Outcome preservation and mechanism preservation are distinct.

It can hold that

$$P_R(Y) = P_{\text{ref}}(Y) \quad (1430)$$

while

$$f_{Y,R} \neq f_{Y,\text{ref}}. \quad (1431)$$

Equations 1430 and 1431 describe identical observed outcomes produced by different causal processes.

Whether the mechanism difference matters depends upon the normative theory.

18.30 Causal Paths

A causal effect can operate through several paths.

Suppose intervention A affects outcome Y through mediator M :

$$A \longrightarrow M \longrightarrow Y. \quad (1432)$$

Equation 1432 identifies one mediated pathway.

A direct path can also exist:

$$A \longrightarrow Y. \quad (1433)$$

Equation 1433 shows that identical total effects can arise from different combinations of direct and mediated effects.

18.31 Mediation Structure

Let mediation object be represented by

$$Q_{\text{med}}(\mathcal{G}) = \mathcal{M}_C. \quad (1434)$$

Equation 1434 can contain mediators, path structure, and path-specific effects.

Mediation is especially important for generative justice because the path through which value or capability is produced can itself contain domination, extraction, or recognition.

18.32 Mediation Equivalence

A mediation-equivalence criterion can require corresponding mediated structures:

$$\mathcal{M}_{C,R} E_M \mathcal{M}_{C,\text{ref}}. \quad (1435)$$

Equation 1435 is stronger than total-effect equivalence.

The relation E_M can preserve graph structure, selected path effects, or another mediation object.

18.33 Equal Total Effect and Different Mediation

Two systems can satisfy

$$TE_R = TE_{\text{ref}} \quad (1436)$$

while

$$\mathcal{M}_{C,R} \neq \mathcal{M}_{C,\text{ref}}. \quad (1437)$$

Equations 1436 and 1437 are normatively important.

The same outcome can be achieved through autonomous participation in one system and coercive dependency in another.

18.34 Path-Specific Restorative Comparison

A restorative theory can evaluate selected causal paths separately.

Let π denote a causal path or path family. A path-specific comparison is represented by Equation 1438.

$$\text{Eff}_R(\pi) \neq \text{Eff}_{\text{ref}}(\pi). \quad (1438)$$

Equation 1438 can be useful where one pathway is normatively problematic even though the total outcome appears acceptable.

Its identification requires strong causal assumptions.

18.35 Causal Bottlenecks

A causal bottleneck is a variable or mechanism through which many desirable effects must pass.

Let mediator M intercept a large family of causal paths from enabling conditions to future opportunities.

A restorative system can therefore be fragile where one actor controls M .

This causal bottleneck can remain invisible in aggregate outcome data.

18.36 Gatekeeper Dependence

Let G represent gatekeeper approval and Y a valued outcome.

A highly concentrated causal structure can satisfy

$$P(Y \mid \text{do}(G = 0)) \ll P(Y \mid \text{do}(G = 1)). \quad (1439)$$

Equation 1439 represents strong dependence upon gatekeeper approval.

A restored system can achieve ordinary outcomes and remain structurally dominated through this mechanism.

18.37 Causal Redundancy

A more robust structure can contain several independent causal routes to the same valued outcome.

Let path family be

$$\Pi_Y = \{\pi_1, \dots, \pi_k\}. \quad (1440)$$

Equation 1440 represents the set of relevant causal routes to Y .

A system with several viable paths can be less vulnerable to loss of one mediator or institution.

18.38 Mechanism Redundancy

Mechanism redundancy differs from network redundancy.

Two network paths can rely upon the same underlying causal mechanism and fail together.

A causal robustness analysis therefore asks whether alternative paths are mechanistically independent enough to survive common-cause failure.

18.39 Causal Sufficiency

A restorative intervention can be causally sufficient for an outcome without being necessary.

Let u be sufficient where

$$do(u) \implies Y \in \mathcal{S}_Y \quad (1441)$$

under the selected model and conditions.

Equation 1441 can be useful for assistance design.

It does not establish that u is the only legitimate intervention.

18.40 Causal Necessity

An intervention or condition can be modeled as necessary relative to a target and context.

If removing u prevents target Y , a simplified necessity condition is represented by Equation 1442.

$$P(Y \in \mathcal{S}_Y \mid do(u = 0)) = 0. \quad (1442)$$

Equation 1442 is highly context-dependent.

Restorative design should avoid treating model-relative necessity as metaphysical necessity.

18.41 Causal Responsibility and Restorative Object

Causal responsibility and restorative adequacy should remain distinct.

An actor can be causally responsible for loss L while another institution is best positioned to provide assistance.

Conversely, an institution can provide effective assistance without being causally responsible for the original harm.

This distinction follows the earlier institutional separation between liability and assistance.

18.42 Causal Attribution

Let H denote harmful intervention and Y later outcome.

A causal attribution quantity can be represented generically by

$$\Delta_H(Y) = P(Y \mid do(H = 1)) - P(Y \mid do(H = 0)). \quad (1443)$$

Equation 1443 represents one average causal-effect style quantity.

The form is illustrative and should not be treated as the unique representation of legal or moral responsibility.

18.43 Attribution and Individual History

An average causal effect need not determine what happened to one individual.

Population-level causal estimation and individual counterfactual attribution should therefore remain distinct.

A restorative system can use population evidence to support bounded inference without claiming exact reconstruction of one person's lost future.

18.44 Counterfactual Variables

Within a structural causal model, a counterfactual variable Y_x represents the value Y would take under intervention $X = x$.

The notation is given in Equation 1444.

$$Y_x. \quad (1444)$$

Equation 1444 provides the basic object for counterfactual causal comparison [43].

Restorative references based upon no-harm histories frequently depend upon such counterfactual objects, explicitly or implicitly.

18.45 Counterfactual Equivalence

A strong causal restorative relation requires equality of selected counterfactual distributions:

$$P_R(Y_u) = P_{\text{ref}}(Y_u) \quad \forall u \in \mathcal{U}_0. \quad (1445)$$

Equation 1445 is stronger than interventional equality over observed populations where the counterfactual model contains additional cross-world structure.

Its epistemic demands are correspondingly high.

18.46 Counterfactual Distribution Families

Where one counterfactual model is not uniquely identified, define

$$\mathfrak{P}_{\text{cf}} = \{P_M(Y_u) : M \in \mathfrak{M}\}. \quad (1446)$$

Equation 1446 represents the set of counterfactual distributions supported by plausible models.

Restorative analysis can then seek results robust across $\mathfrak{P}_{\text{cf}}$.

18.47 Counterfactual Bounds

Where point identification is unavailable, a counterfactual quantity can be bounded:

$$L_u \leq \mathbb{E}[Y_u] \leq U_u. \quad (1447)$$

Equation 1447 supports bounded restorative claims without invented precision.

Pearl's causal framework explicitly treats identification and bounds as distinct epistemic possibilities [43].

18.48 Counterfactual Restoration and Historical Identity

Counterfactual response equality does not imply historical identity.

It can hold that

$$P_R(Y_u) = P_{\text{ref}}(Y_u) \quad (1448)$$

while

$$\mathcal{H}_R \neq \mathcal{H}_{\text{ref}}. \quad (1449)$$

Equations 1448 and 1449 distinguish causal future equivalence from historical equivalence.

This distinction remains important even under hypothetical perfect causal reconstruction.

18.49 Counterfactual Restoration and Subject Change

A counterfactual model can predict how the earlier subject would have responded without the harm.

The actual present subject has undergone another history.

Restorative justice should therefore distinguish:

$$P(Y_u^{A^C}) \quad (1450)$$

from

$$P(Y_u^{A^H}), \quad (1451)$$

where A^C denotes the counterfactual subject and A^H the historically actual post-harm subject. Equations 1450 and 1451 need not coincide.

This difference is a causal version of the historical-subject problem developed earlier.

18.50 Mechanism Change Through Harm

A harmful process can change structural functions.

Let pre-harm mechanism be f . Harm can produce

$$f \xrightarrow{H} f^H. \quad (1452)$$

Equation 1452 formalizes alteration of the process through which later inputs become outputs.

Examples include:

- reporting now producing fear rather than expected protection;
- money producing less opportunity because health deteriorated;
- professional access producing less collaboration because reputation changed;
- institutional participation producing different risk because authority relations changed.

18.51 Mechanism Restoration

A mechanism-oriented restorative intervention seeks

$$f^H \xrightarrow{R} f^R \quad (1453)$$

such that

$$f^R E_F f^{\text{ref}}. \quad (1454)$$

Equations 1453 and 1454 distinguish mechanism repair from state correction.

18.52 Mechanism Transformation

Where the reference mechanism was itself unjust, the objective can be transformation rather than restoration.

Let

$$f^R \succeq_V f^{\text{ref}} \quad (1455)$$

denote normative dominance of the restored mechanism over the historical reference. Equation 1455 supports transformative causal justice.

18.53 Mechanism Invariance

Causal analysis often relies upon some form of mechanism stability across environments or interventions.

Let environment be e . A mechanism is invariant across selected environments when

$$f_i^{(e_1)} = f_i^{(e_2)} \quad (1456)$$

for the relevant e_1, e_2 . Equation 1456 provides one idealized invariance criterion.

Restorative systems can violate this assumption because the restorative intervention itself changes the mechanism.

18.54 Policy-Induced Mechanism Change

A policy can change the behavioral relationship it was designed from.

Let pre-policy mechanism be f and post-policy mechanism f^R :

$$f \xrightarrow{R} f^R. \quad (1457)$$

Equation 1457 connects the causal framework with the broader lesson of the Lucas critique: policy intervention can alter relationships used for prediction [29].

The present paper uses that insight structurally rather than as a universal econometric theorem.

18.55 Reflexive Causal Mechanisms

A causal model can become socially operative when subjects learn its classification or prediction.

Let record or prediction be

$$Z_t = O(\mathcal{G}_t). \quad (1458)$$

Equation 1458 produces an institutional representation.

If Z_t affects later behavior, then

$$X_{t+1} = F(X_t, Z_t). \quad (1459)$$

Equation 1459 places the representation inside the causal system.

This structure is consistent with the looping and self-fulfilling effects discussed earlier [36, 23].

18.56 Causal Non-Closure

Once the model, classification, or restorative intervention enters the causal system, the original causal model can cease to describe the post-intervention world.

A sequence can be represented by

$$M_t \rightarrow R_t \rightarrow \mathcal{G}_{t+1} \rightarrow M_{t+1}. \quad (1460)$$

Equation 1460 is a causal form of reflexive non-closure.

Restorative causal analysis should therefore be revisable.

18.57 Transport Across Environments

A causal relationship identified in one environment need not transfer unchanged to another.

Let environments be e_1 and e_2 . A transport claim asks whether a causal quantity identified in e_1 can be applied to e_2 .

For restorative justice, the harmed and restored systems can themselves function as different environments.

The assumption

$$P_{e_1}(Y \mid do(X = x)) = P_{e_2}(Y \mid do(X = x)) \quad (1461)$$

should therefore be established or justified rather than assumed. Equation 1461 formalizes the transport problem.

18.58 Historical Transportability

The reference system can belong to an earlier historical environment.

A treatment, salary, qualification, or institutional role can have different effects after technologies, markets, policies, or social meanings change.

A causal restorative claim should therefore ask whether the relevant mechanisms are transportable across historical time.

This is the causal counterpart of the static-world fallacy discussed in Section 10.42.

18.59 Subject-Level Transportability

The same causal effect estimated for a population can differ for the particular subject.

Let effect modifier Z influence response:

$$P(Y \mid do(X = x), Z = z_1) \neq P(Y \mid do(X = x), Z = z_2). \quad (1462)$$

Equation 1462 expresses heterogeneous treatment or intervention response.

A historically changed subject can move from one response stratum to another.

18.60 Heterogeneous Causal Effects

Population-average causal effects can conceal heterogeneous individual or group effects.

Let

$$\tau(z) = \mathbb{E}[Y_1 - Y_0 \mid Z = z]. \quad (1463)$$

Equation 1463 represents a conditional causal effect under a potential-outcome interpretation.

Different $\tau(z)$ values imply that one restorative intervention can be effective for some subjects and ineffective or harmful for others.

18.61 Causal Personalization

Subject-directed assistance can therefore use causal heterogeneity when supported by adequate evidence.

Personalization should remain distinct from deterministic prediction of one person's future.

The epistemic basis can support conditional probabilities and ranges rather than certainty.

18.62 Causal Interaction

Two interventions can interact.

Let interventions be A and B . Non-additive causal interaction is represented schematically by

$$\text{Eff}(A, B) \neq \text{Eff}(A) + \text{Eff}(B). \quad (1464)$$

Equation 1464 is especially relevant to layered restoration.

Money and legal-status correction can produce greater capability jointly than either produces separately.

18.63 Restorative Complementarity

Let interventions R_1 and R_2 be restorative.

Complementarity can be represented by

$$V(Y_{R_1, R_2}) > V(Y_{R_1}) + V(Y_{R_2}) - V(Y_0), \quad (1465)$$

where the expression is meaningful under the selected valuation. Equation 1465 captures positive interaction among restorative components.

This provides a causal interpretation of layered support.

18.64 Restorative Substitution

Interventions can also substitute for one another.

Two different mechanisms can produce similar outcomes.

A financial transfer and direct housing provision can both stabilize housing under some conditions.

Causal substitution should be distinguished from historical equivalence of the interventions.

18.65 Mechanism-Specific Substitution

Suppose interventions R_1 and R_2 produce the same outcome:

$$P(Y \mid do(R_1)) = P(Y \mid do(R_2)). \quad (1466)$$

Equation 1466 establishes outcome substitutability under the specified model.

The two interventions can differ in:

- burden;
- agency;
- privacy;
- durability;
- third-party effects;
- future generativity.

Outcome substitution therefore does not settle procedural or normative choice.

18.66 Mechanism-Sensitive Compensation

Compensation can be evaluated through its causal effect rather than amount alone.

Let payment be M and capability outcome K . A mechanism-sensitive criterion compares

$$P_R(K \mid do(M = m)) \quad (1467)$$

with the corresponding reference response.

Equation 1467 formalizes why the same sum can have different restorative significance across histories.

18.67 Mechanism-Sensitive Re-Entry

Formal re-entry can be causally evaluated through its downstream effects.

Let $A = 1$ denote reinstatement. Outcomes can include network access, security, health, and career opportunity.

The relevant object is the vector represented by Equation 1468.

$$\mathbf{Y}_{\text{reentry}} = (Y_{\text{network}}, Y_{\text{security}}, Y_{\text{health}}, Y_{\text{career}}) . \quad (1468)$$

Equation 1468 permits re-entry to succeed in one causal pathway and fail in another.

18.68 Mechanism-Sensitive Recognition

Recognition can alter material outcomes through changed beliefs and institutional responses.

A public correction can causally affect:

- reputation;
- network access;
- employability;
- institutional trust;
- subject expectation.

Symbolic and material repair can therefore be causally coupled even while remaining analytically distinct.

18.69 Mechanism-Sensitive Institutional Reform

Institutional reform can be evaluated by whether it changes the causal mechanism producing recurrent harm.

Let harmful outcome be H and institutional rule R_I .

A structural reform seeks to change

$$P(H \mid do(R_I = r)) . \quad (1469)$$

Equation 1469 focuses upon recurrence mechanism rather than compensation for prior episodes.

18.70 Causal Restoration and Prevention

Causal models can support prevention by identifying variables whose intervention substantially changes risk.

This does not mean that every effective variable should be manipulated.

Preventive selection remains constrained by:

- rights;
- proportionality;
- causal uncertainty;
- distribution of burdens;
- reversibility;
- institutional competence.

Causal efficacy is therefore one input into preventive justice.

18.71 Causal Restoration and Interruption

During ongoing harm, causal analysis can identify mechanisms maintaining the harmful process.

Let harmful process be H_t , sustained by mediator M_t .

An interruptive intervention can target

$$do(M_t = m^*) \quad (1470)$$

where the intervention is normatively admissible. Equation 1470 illustrates mechanism-targeted interruption.

The objective differs from retrospective attribution of blame.

18.72 Causal Restoration and Assistance

Assistance can be designed around present causal barriers.

Suppose resource R fails to become capability K because mediator M is impaired.

An assistance intervention can act jointly upon R and M .

The causal question is which combination expands viable future possibilities most effectively under the subject's chosen goals.

This orientation requires less historical precision than reconstruction of the entire no-harm causal history.

18.73 Causal Restoration and Transformation

Transformative justice can seek to replace the mechanism that reproduces the harm.

Let harmful mechanism be f_H and transformed mechanism f_T .

The transformation is represented by

$$f_H \xrightarrow{R_T} f_T. \quad (1471)$$

Equation 1471 can change the relation among control, resource flow, classification, and opportunity.

A transformed system can therefore possess lower historical fidelity and higher normative value.

18.74 Causal Equivalence and Non-Domination

Non-domination requires attention to causal dependence upon discretionary power.

Two subjects can receive the same current resource while one receives it through a protected entitlement and another through revocable permission.

The observed outcome is equal.

The causal dependence upon another actor differs.

This makes causal structure directly relevant to republican justice [44].

18.75 Causal Equivalence and Relational Equality

Relational equality similarly concerns how outcomes are generated through social relations.

Equal educational access produced through respectful participation differs from equal access dependent upon degrading submission.

The causal pathway itself can therefore be an object of relational justice [2, 71].

18.76 Causal Equivalence and Recognition

Recognition structures can function as causal mechanisms.

Misrecognition can change:

- institutional decisions;
- subject participation;
- network response;
- distribution of opportunity.

Recognition theory therefore intersects with causal analysis through the mechanisms by which social standing affects later states [25, 20].

18.77 Causal Equivalence and Generative Justice

Generative justice is especially concerned with mechanisms through which value is produced, controlled, circulated, and recursively converted into new generative conditions [15, 17].

A causal representation can therefore distinguish:

$$C_t \longrightarrow V_{t+1} \longrightarrow C_{t+1}. \quad (1472)$$

Equation 1472 represents one recursive generative cycle.

Two systems can have equal V_{t+1} while differing in the causal mechanism determining C_{t+1} .

18.78 Causal Value-Circulation Equivalence

Let intervention u change generated value V . The downstream effect upon future conditions C' can be compared by

$$P_R(C' \mid do(V = v)) \stackrel{E}{=} P_{\text{ref}}(C' \mid do(V = v)). \quad (1473)$$

Equation 1473 gives a causal interpretation of value-circulation equivalence.

The structure can differ even when total generated value is equal.

18.79 Causal Control Structure

Control can also be represented causally by asking which actor's decision variables influence access to critical conditions.

Let actor decision D_j affect access A_i .

A dependence quantity can be represented by

$$\Delta_{j \rightarrow i} = P(A_i \mid do(D_j = 1)) - P(A_i \mid do(D_j = 0)). \quad (1474)$$

Equation 1474 makes discretionary control causally visible.

The normative meaning requires a theory of legitimate authority.

18.80 Causal Dependency Profile

For subject A_i , define dependence on several actors or conditions by

$$\square_i = (\Delta_{1 \rightarrow i}, \dots, \Delta_{m \rightarrow i}). \quad (1475)$$

Equation 1475 can reveal concentration of control.

A restored system can preserve access while increasing concentration.

18.81 Causal Redundancy and Non-Domination

Where several independent mechanisms can sustain access, dependence upon one actor is reduced.

Causal redundancy can therefore support non-domination without requiring complete independence from institutions.

This is an example of how causal and normative layers interact.

18.82 Causal Equivalence and Robustness

Interventional equivalence and perturbation robustness overlap while remaining distinct.

Perturbation analysis asks how a system responds when disturbed.

Causal analysis asks what outcome follows from a specified intervention within a causal model.

A disturbance can be poorly localized causally. An intervention can be defined abstractly without occurring naturally.

The two analyses complement one another.

18.83 Response Equivalence and Causal Non-Equivalence

Two systems can display the same response to one perturbation and possess different causal mechanisms.

Thus

$$Q_R(\delta) = Q_{\text{ref}}(\delta) \quad (1476)$$

does not imply

$$M_R = M_{\text{ref}}. \quad (1477)$$

Equations 1476 and 1477 show why stress testing cannot replace causal analysis.

18.84 Causal Equivalence and Dynamical Non-Equivalence

A selected intervention-response family can correspond while long-run dynamics differ outside the tested intervention class.

Interventional equivalence should therefore be indexed to:

- intervention set;
- outcome set;
- environment;
- horizon.

A broad claim of causal equivalence without these indices is underdetermined.

18.85 Temporal Interventional Equivalence

Causal effects can vary with horizon.

A time-indexed criterion is represented by

$$P_R(Y_{t+\tau} \mid do(u_t = u)) \neq P_{\text{ref}}(Y_{t+\tau} \mid do(u_t = u)). \quad (1478)$$

Equation 1478 distinguishes immediate from delayed causal response.

A treatment, payment, or legal reform can have similar immediate effects and different long-run effects.

18.86 Dynamic Treatment Regimes

Restorative interventions can be sequences conditioned upon evolving states.

Let policy be

$$\pi_t : H_t \longrightarrow A_t, \quad (1479)$$

where H_t denotes observed history available at time t . Equation 1479 represents an adaptive intervention policy.

The restorative comparison can then concern the outcome of policies rather than single interventions.

18.87 Policy Equivalence

Two adaptive policies π_1 and π_2 can be compared through their induced outcome distributions:

$$P_R(Y^\pi) \equiv P_{\text{ref}}(Y^\pi). \quad (1480)$$

Equation 1480 generalizes one-time interventional equivalence to longitudinal governance.

This can be relevant to rehabilitation, assistance, and responsive regulation.

18.88 Policy Adaptation and Reflexivity

A policy can change subject behavior, which changes later treatment assignment, which changes later behavior.

Dynamic restorative policy therefore contains causal feedback.

A simple sequence is represented by

$$A_t \rightarrow X_{t+1} \rightarrow A_{t+1} \rightarrow X_{t+2}. \quad (1481)$$

Equation 1481 shows why static causal estimation can be inadequate for long-term assistance.

18.89 Causal Feedback

Where causal feedback exists, directed acyclic representations can require time expansion or another formalism capable of representing cyclic dynamics.

The relevant model should therefore be chosen according to the temporal and causal structure of the problem.

The paper does not assume that every generative system is adequately represented by one static DAG.

18.90 Causal Cycles and Generative Recursion

Generative systems often contain recursive feedback:

$$C_t \rightarrow V_{t+1} \rightarrow C_{t+1} \rightarrow V_{t+2}. \quad (1482)$$

Equation 1482 is acyclic when unfolded through time and cyclic at the level of repeated functional dependence.

This distinction is useful for value-circulation and institutional-feedback analysis.

18.91 Causal Markov Boundaries and Scope

A causal model defines a variable boundary.

Omitted variables can change interpretation of dependencies.

A model in which supervisor behavior appears to directly determine worker output can change after inclusion of health, network access, or procedural security.

Causal scope discipline therefore parallels the representational discipline of the broader atlas.

18.92 Unobserved Confounding

A latent variable U can influence both exposure X and outcome Y :

$$U \longrightarrow X, \quad U \longrightarrow Y. \quad (1483)$$

Equation 1483 represents a basic confounding structure.

Failure to represent U can make observational association misleading.

Restorative causal claims should therefore report identification assumptions rather than only estimated effects.

18.93 Selection Effects

Observed restored populations can differ from harmed populations because only some subjects remain available for measurement.

For example, those most severely affected can leave the institution entirely.

Observed post-restoration success can therefore be biased by selection.

A causal restorative evaluation should identify the population to which the claim applies.

18.94 Survivorship and Restorative Evaluation

An institution can evaluate only subjects who successfully returned and conclude that re-entry is highly effective.

Those unable or unwilling to return disappear from the observed sample.

This produces a distinctive restorative form of survivorship bias.

Future evaluations should therefore preserve information about exit, non-return, and alternative trajectories where ethically and legally appropriate.

18.95 Measurement Error in Causal Variables

Causal variables can be measured imperfectly.

Formal status can be observed accurately while trust, informal authority, or network access are measured weakly.

Measurement error can therefore distort estimated causal relationships.

The epistemic layer of the atlas should distinguish causal uncertainty from measurement uncertainty.

18.96 Causal Model Uncertainty

Several causal models can remain compatible with available evidence.

Let

$$\mathfrak{M}_C = \{M_1, \dots, M_k\}. \quad (1484)$$

Equation 1484 represents the plausible causal model family.

A restorative claim can be evaluated across $\mathfrak{M}_C$ rather than under one selected model.

18.97 Model-Robust Interventional Equivalence

A robust causal claim can require

$$P_R^M(Y \mid do(u)) \approx P_{\text{ref}}^M(Y \mid do(u)) \quad (1485)$$

for all $M \in \mathfrak{M}_C$ and selected interventions u . Equation 1485 is stronger epistemically than a result depending upon one disputed graph.

18.98 Minimal Common Causal Claims

Where models disagree about deeper structure while agreeing about one causal effect, institutions can act upon the common result.

Let

$$\mathcal{C}_\cap = \bigcap_{M \in \mathfrak{M}_C} \mathcal{C}(M) \quad (1486)$$

schematically denote claims supported across the model family. Equation 1486 is conceptual because causal claims need not literally form sets under every representation.

The idea parallels minimal common restoration developed earlier.

18.99 Partial Identification

A causal effect can be only partially identified.

Let causal estimand θ_C satisfy

$$\theta_C \in \Theta_I \quad (1487)$$

for identified set Θ_I . Equation 1487 avoids forced point estimation.

A restorative decision can sometimes be robust across the entire identified set.

18.100 Sensitivity to Causal Assumptions

Let assumption parameter be η . A causal conclusion can be represented as

$$\theta_C = \theta_C(\eta). \quad (1488)$$

Equation 1488 permits analysis of how the restorative judgment changes when causal assumptions change.

High sensitivity suggests epistemic caution.

18.101 Causal Comparison Under Scarcity

Rich causal identification can be expensive.

Detailed longitudinal data, network data, experiments, and mechanism analysis consume resources.

A practical institution should therefore ask whether a stronger causal claim would materially change the restorative decision.

Causal sophistication is valuable where it improves decision quality and unnecessary where a simpler uncontested claim already supports the legitimate response.

18.102 Causal Minimalism

The principle of causal minimalism can therefore be stated:

Use the weakest causal claim sufficient to justify the legitimate restorative action, while preserving the option to refine the analysis when unresolved causal structure materially affects justice.

This principle reduces epistemic burden and data collection.

18.103 Causal Escalation

Stronger causal analysis becomes useful where:

- equal resources produce unequal outcomes;
- baseline restoration repeatedly fails;
- several plausible mediators compete;
- institutional reform requires mechanism targeting;
- responsibility for downstream effects is disputed;
- one intervention helps some subjects and harms others;
- causal pathways contain potential domination or extraction.

These cases justify movement from observational comparison toward mechanism and intervention analysis.

18.104 Causal Comparison and Legal Proof

Legal causation and structural causal analysis should remain conceptually distinct.

Law can incorporate policy, burden of proof, remoteness, foreseeability, evidentiary presumptions, and normative limits on responsibility.

A causal model can inform legal reasoning without replacing legal doctrine.

The later legal sections will therefore treat causal analysis as one input into responsibility and remedy.

18.105 Causal Comparison and Assistance Without Liability

Assistance can be justified by present causal need even where individual liability for the original harm is unresolved.

Suppose current barrier M causally blocks access to a viable future.

Removing M can be justified through assistance policy independently of a final judgment concerning who caused M .

This again separates responsibility from support.

18.106 Causal Comparison and Third Parties

An intervention can improve the claimant's outcome by imposing causal burdens upon third parties.

Let claimant outcome be Y_A and third-party outcome Y_B .

An intervention can satisfy

$$\Delta Y_A > 0 \quad (1489)$$

while

$$\Delta Y_B < 0. \quad (1490)$$

Equations 1489 and 1490 require multi-party normative evaluation.

Causal effectiveness for one subject therefore cannot define justice alone.

18.107 Causal Externalities

For intervention R_A directed toward subject A , define externality upon B by

$$\epsilon_{A \rightarrow B} = \mathbb{E}[Y_B \mid do(R_A)] - \mathbb{E}[Y_B \mid do(R_A = 0)]. \quad (1491)$$

Equation 1491 makes downstream third-party effects explicit.

The valuation of $\epsilon_{A \rightarrow B}$ belongs to substantive justice.

18.108 Causal Spillovers Across Scale

An individual intervention can alter organization-level dynamics.

An organizational reform can alter community outcomes.

A national compensation policy can alter markets.

Causal comparison should therefore be scale-indexed where material spillovers exist.

18.109 Multi-Scale Causal Models

Let variables be partitioned by scale:

$$V = V^{(1)} \cup V^{(2)} \cup \dots \cup V^{(s)}. \quad (1492)$$

Equation 1492 supports causal relations within and across scales.

A restorative mechanism can be equivalent at one scale and different at another.

18.110 Causal Aggregation

Aggregate variables can conceal heterogeneous micro-level mechanisms.

Let aggregate outcome be

$$Y_{\text{agg}} = A(Y_1, \dots, Y_n). \quad (1493)$$

Equation 1493 can remain unchanged while the individual causal structure changes.

This mirrors the earlier distinction between macro stability and individual justice.

18.111 Causal Decomposition of Generative Harm

Generative harm can be decomposed into causal stages:

$$H \longrightarrow C' \longrightarrow K' \longrightarrow R' \longrightarrow Y'. \quad (1494)$$

Equation 1494 can represent harm changing conditions C' , conversion K' , relations R' , and later outcomes Y' .

The decomposition helps identify where restitution, compensation, assistance, or structural reform can act.

18.112 Causal Decomposition of Restoration

A restorative sequence can similarly be represented as

$$R_1 \longrightarrow C^R \longrightarrow K^R \longrightarrow R^R \longrightarrow Y^R. \quad (1495)$$

Equation 1495 shows that one intervention can act through several mediating stages.

Failure can occur at any stage.

18.113 Causal Failure Localization

Suppose formal status has been restored and later capability remains low.

The atlas can ask whether failure lies in:

- resource level;
- conversion mechanism;
- relation structure;
- information;
- health;
- external environment.

Causal localization can therefore guide targeted assistance without implying complete causal knowledge of the subject's history.

18.114 Causal Overreach

Formal causal models can create an appearance of explanatory completeness.

A model can include every measured variable and still omit:

- unmeasured relations;
- meaning;
- historical contingency;
- newly emerging states;
- reflexive responses to the model itself.

Causal rigor within the model should therefore remain distinct from exhaustive social explanation.

18.115 Causal Reification

A variable such as “trust,” “vulnerability,” or “professional access” can be represented causally without becoming a natural, context-free entity.

Its operationalization depends upon the restorative inquiry.

Causal modeling should therefore preserve provenance of variable construction.

18.116 Causal Equivalence and Meaning

Two mechanisms can be behaviorally and causally similar under selected variables while carrying different social meanings.

A payment made as acknowledgment of responsibility differs relationally from an identical payment made as discretionary charity.

If the modeled outcomes exclude meaning and recognition, causal equivalence can hold while relational equivalence fails.

This is another example of object-relative restoration.

18.117 Causal Equivalence and Historical Provenance

A present causal mechanism can be identical to the reference while its historical provenance differs.

An institution can adopt the same rule after litigation that it would otherwise have adopted voluntarily.

Future causal behavior can correspond while history remains different.

Causal equivalence therefore does not subsume provenance.

18.118 Causal Equivalence and Identity

A hypothetical system could reproduce the same complete modeled causal dynamics using different persons, memories, or historical relations.

Causal-system equivalence is therefore distinct from personal or relational identity.

This distinction becomes especially important in the later ideal-technology thought experiments.

18.119 Causal Equivalence and Procedural Justice

A technically effective intervention can produce the desired causal response through unjust procedure.

Memory alteration, coercive treatment, surveillance, or manipulation of third parties can improve causal fidelity while violating agency or rights.

Causal efficacy therefore belongs to substantive evaluation and remains independent of procedural legitimacy.

18.120 Causal Equivalence and Subject Direction

A causal model can identify the intervention most likely to reproduce the counterfactual career.

The present subject can prefer another future.

Subject-directed assistance can therefore choose an intervention maximizing access to the present target set rather than causal fidelity to the lost trajectory.

This distinction prevents counterfactual paternalism from reappearing in causal form.

18.121 Causal Restoration Profile

A causal restorative profile can be represented by Equation 1496.

$$\mathbf{C}_R = (r_{\text{obs}}, r_{\text{graph}}, r_{\text{mech}}, r_{\text{int}}, r_{\text{med}}, r_{\text{cf}}). \quad (1496)$$

Equation 1496 records observational, graph, mechanism-level, interventional, mediation, and counterfactual correspondence.

The profile avoids collapsing several causal claims into one binary label.

18.122 Causal Object–Relation Cells

Table 12 summarizes several central cells of the causal region of the atlas.

Table 12 demonstrates that causal restoration contains several distinct levels of comparison.

18.123 Causal Refinement Path

A schematic refinement path is represented by Equation 1497.

$$(Q_{\text{obs}}, \sim_{\text{dist}}) \longrightarrow (Q_{\text{graph}}, \cong) \longrightarrow (Q_{\text{int}}, \sim_{\text{int}}) \longrightarrow (Q_{\text{mech}}, \sim_{\text{struct}}) \longrightarrow (Q_{\text{cf}}, \sim_{\text{cf}}). \quad (1497)$$

Equation 1497 increases the causal content of the restorative claim.

The path is analytical rather than a universal normative hierarchy.

Table 12: Illustrative causal object–relation cells in restorative comparison

Causal object	Comparative relation	Restorative interpretation	Characteristic limitation
Observed distribution	Distributional equality	Restored and reference observations correspond	Does not identify causal structure
Causal graph	Graph isomorphism	Directed dependency architecture corresponds	Mechanism functions can differ
Structural mechanism	Function correspondence	Variables are generated through corresponding mechanisms	High identification burden
Intervention response	Distributional equality	Same selected intervention produces corresponding outcomes	Intervention-class dependent
Intervention response	Approximate equivalence	Response distributions remain within tolerance	Metric and tolerance require justification
Mediation structure	Path correspondence	Intervention effects operate through corresponding pathways	Path-specific identification can be difficult
Counterfactual distribution	Distributional equivalence	Selected hypothetical interventions generate corresponding potential outcomes	Strong model dependence
Control mechanism	Causal dependence comparison	Access depends upon corresponding authority structure	Normative legitimacy of authority remains separate
Generative cycle	Mechanism correspondence	Condition–value–future-condition causation is preserved	Requires multi-stage causal representation

18.124 Causal Non-Implication Structure

Several non-implications are central.

Observational equivalence does not imply graph equivalence:

$$P_R(V) = P_{\text{ref}}(V) \not\Rightarrow G_{C,R} \cong G_{C,\text{ref}}. \quad (1498)$$

Equation 1498 separates observed data from causal architecture.

Graph equivalence does not imply mechanism equivalence:

$$G_{C,R} \cong G_{C,\text{ref}} \not\Rightarrow F_R \sim F_{\text{ref}}. \quad (1499)$$

Equation 1499 separates topology from structural functions.

Limited interventional equivalence does not imply global causal equivalence:

$$\sim_{\text{int},\mathcal{U}_0} \not\Rightarrow \sim_{\text{int},\mathcal{U}_C} \quad (1500)$$

when $\mathcal{U}_0 \subset \mathcal{U}_C$. Equation 1500 makes intervention scope part of the claim.

18.125 Causal Strength and Normative Strength

A causally stronger restorative relation does not automatically constitute a more just target.

Exact mechanism reconstruction can preserve an unjust historical institution. A transformed mechanism can produce greater agency and capability while deviating from the reference.

The normative theory must therefore evaluate which causal structures deserve preservation.

18.126 Causal Fidelity and Transformative Improvement

Let F_C measure causal fidelity to the reference and V normative value.

It can hold that

$$F_C(\mathcal{G}_{R_2}) < F_C(\mathcal{G}_{R_1}) \quad (1501)$$

while

$$V(\mathcal{G}_{R_2}) > V(\mathcal{G}_{R_1}). \quad (1502)$$

Equations 1501 and 1502 represent transformative causal improvement.

18.127 Causal Minimalism Principle

The first principle of this section is:

Restorative causal analysis should use the weakest causal claim sufficient for the legitimate institutional purpose, while preserving the possibility of refinement where unresolved mechanism materially affects justice.

18.128 Observational Scope Principle

The second principle is:

Equality of observed distributions establishes observational restoration only within the selected variable and measurement space.

18.129 Interventional Explicitness Principle

The third principle is:

A claim of interventional equivalence should specify the intervention family, outcome family, environment, temporal horizon, and approximation criterion.

18.130 Mechanism Principle

The fourth principle is:

Equivalent outcomes can be generated by causally different mechanisms, and the difference can remain normatively important where autonomy, domination, extraction, recognition, or future generativity depends upon the pathway.

18.131 Counterfactual Modesty Principle

The fifth principle is:

Counterfactual restorative claims should reflect the identification limits and model dependence of the causal structures from which those counterfactuals are generated.

18.132 Causal Transformation Principle

The sixth principle is:

Where the historical causal mechanism is unjust, a just restorative objective can transform the mechanism rather than reproduce it.

18.133 Causal Agency Principle

The seventh principle is:

The intervention that most closely reproduces a counterfactual causal trajectory does not acquire automatic authority over the future preferences of the present historical subject.

18.134 Causal Process Principle

The eighth principle is:

Causal efficacy and procedural legitimacy are independent evaluative dimensions.

18.135 Causal Layer of the Atlas

The causal layer can now be represented by Equation 1503.

$$\mathfrak{C}_A^{\text{causal}} = (Q_{\text{obs}}, Q_{\text{graph}}, Q_{\text{mech}}, Q_{\text{int}}, Q_{\text{med}}, Q_{\text{cf}}) \times \mathfrak{C}_C. \quad (1503)$$

Equation 1503 combines observational, structural, interventional, mediation, and counterfactual objects with their admissible comparative relations.

This layer integrates causal reasoning into the broader Atlas of Equivalence.

18.136 Implications for Generative-Process Equivalence

The causal framework makes it possible to state the next problem precisely.

A generative process is more than one causal edge or one intervention-response relation. It concerns recursive production of:

- future conditions;
- capabilities;
- relations;
- value;
- control;
- further generative possibilities.

Two systems can therefore be interventionally equivalent for one outcome and differ in the recursive process through which future generativity is produced.

The next section moves from causal equivalence of selected mechanisms toward comparison of generative processes as compound, temporally recursive structures.

18.137 Transition to Generative-Process Equivalence

Section 19 develops generative-process equivalence. It combines condition structure, conversion, causal response, relation formation, value generation, value circulation, control, and recursive regeneration into one family of process-level comparisons.

The central question becomes:

Even where the restored and reference systems can produce similar states, outcomes, and intervention responses, do they generate future conditions and value through correspondingly just and self-sustaining processes?

That question marks the transition from causal restoration toward the strongest distinctively generative region of the atlas.

19 Generative-Process Equivalence

This section develops the distinctively generative region of the Atlas of Equivalence. The preceding sections examined state, trajectory, reachability, dynamical structure, perturbation response, and causal mechanisms. Those objects make increasingly strong claims about how restored and reference systems behave. They still permit an important residual difference: two systems can produce similar outcomes through processes that differ in their capacity to reproduce future conditions, distribute control, circulate value, create relations, preserve agency, and support further rounds of generation.

Generative-process comparison therefore asks a stronger and differently structured question:

Does the restored system regenerate future conditions, capacities, relations, and value through a process corresponding in the relevant sense to the reference, or through a normatively preferable process where historical replication is neither possible nor desirable?

This question should not be interpreted as requiring one universal metric of “generativity.” Generativity can concern several objects: production of resources, expansion of capabilities, reproduction of knowledge, creation of social relations, circulation of value, preservation of ecological conditions, institutional learning, or generation of further opportunities. A process can be highly generative in one dimension and normatively objectionable in another. The formal framework therefore treats generative process as a structured family of mappings rather than one scalar variable.

The section has six main objectives. First, it defines a generative process as a recursive mapping from existing conditions into future conditions and values. Second, it decomposes the process into conversion, production, circulation, control, relational, and recursive components. Third, it develops several comparative relations ranging from output equivalence to strong recursive process correspondence. Fourth, it distinguishes generative fidelity from generative dominance and transformation. Fifth, it identifies characteristic failure modes, including extraction, capture, conversion impairment, recursive depletion, dependency, and fragile one-cycle restoration. Sixth, it relates the formal analysis to generative justice while preserving the distinction between descriptive generativity and normative justice [15, 17].

19.1 Generative Processes as Recursive Structures

Let the generative state at time t be represented by

$$g_t = (x_t, c_t, \mathcal{R}_t, \chi_t, v_t, h_t), \quad (1504)$$

where x_t denotes present state, c_t generative conditions, $\mathcal{R}_t$ relational structure, χ_t control structure, v_t generated value, and h_t relevant historical information. Equation 1504 provides a compact process state.

A generative process is represented by

$$g_{t+1} = \Gamma_t^G(g_t, u_t, \xi_t), \quad (1505)$$

where u_t denotes action or intervention and ξ_t environmental influence. Equation 1505 emphasizes that generation produces the conditions of later generation as well as immediate output.

The recursive character appears through repeated application:

$$g_{t+n} = \Gamma_{t+n-1}^G \circ \cdots \circ \Gamma_t^G(g_t). \quad (1506)$$

Equation 1506 distinguishes a generative process from a one-shot production function.

19.2 Immediate Output and Future Condition

A process can generate immediate output y_{t+1} and future condition c_{t+1} :

$$(y_{t+1}, c_{t+1}) = \Gamma_t^G(g_t). \quad (1507)$$

Equation 1507 allows two systems to produce equal immediate output while generating different future conditions.

Thus

$$y_{R,t+1} = y_{\text{ref},t+1} \quad (1508)$$

does not imply

$$c_{R,t+1} = c_{\text{ref},t+1}. \quad (1509)$$

Equations 1508 and 1509 provide the simplest formal statement of generative non-equivalence.

19.3 Condition Regeneration

A generative process reproduces or transforms the conditions under which later activity occurs.

Let

$$C_{t+1} = G_C(C_t, X_t, \mathcal{R}_t, V_{t+1}) \quad (1510)$$

represent regeneration of future condition C_{t+1} .

Equation 1510 makes condition regeneration an independent restorative object.

A system can produce one successful outcome by consuming its remaining resources and thereby reduce C_{t+1} . Another system can produce the same outcome while increasing future capacity.

These processes are output-equivalent and generatively different.

19.4 Condition-Regeneration Equivalence

A basic generative criterion requires

$$G_{C,R}(g_R) \ E_C \ G_{C,\text{ref}}(g_{\text{ref}}). \quad (1511)$$

Equation 1511 compares the future conditions produced by the two processes.

The relation E_C can be equality, inclusion, dominance, approximate similarity, or another condition-appropriate relation.

19.5 Conversion Structure

Existing conditions become capabilities or usable opportunities through a conversion mechanism:

$$K_t = \mathcal{K}_t(C_t, X_t, \mathcal{R}_t, H_t). \quad (1512)$$

Equation 1512 connects the generative framework with the capability distinction between resource possession and substantive freedom [55, 50].

A harmed subject can receive the same condition C_t while possessing a different $\mathcal{K}_t$.

Generative restoration therefore requires attention to the conversion process as well as the returned input.

19.6 Conversion-Process Equivalence

A strong conversion criterion is

$$\mathcal{K}_R \sim_{E_K} \mathcal{K}_{\text{ref}}. \quad (1513)$$

Equation 1513 can mean function equality, approximate response equivalence, interventional equivalence, or commutative structural correspondence.

The selected relation should reflect the institutional purpose.

19.7 Relational Generation

Generative processes also produce and transform relations.

Let

$$\mathcal{R}_{t+1} = G_R (\mathcal{R}_t, X_t, C_t, A_t, E_t), \quad (1514)$$

where A_t represents actions and E_t relevant events. Equation 1514 makes relational formation part of generation.

Research activity can create collaborations. Employment can create reputation and trust. Community institutions can create shared governance relations. Care can create reciprocal dependence.

A restored process can reproduce material output while generating different relations.

19.8 Relational-Generation Equivalence

Relational generation can be compared through

$$G_{R,R} \sim_{E_R} G_{R,\text{ref}}. \quad (1515)$$

Equation 1515 asks whether corresponding activities generate corresponding relational structures.

This criterion is stronger than network-state similarity because it compares the process through which the network develops.

19.9 Knowledge Generation

A generative process can produce knowledge in addition to material value.

Let

$$K_{t+1}^{\text{new}} = G_K (K_t, \mathcal{R}_t, E_t, X_t). \quad (1516)$$

Equation 1516 can represent scientific, professional, cultural, or institutional learning.

A research environment can therefore be restored materially while remaining epistemically less generative if collaboration, experiment, or feedback structures have changed.

19.10 Distributed Knowledge Generation

Where knowledge is distributed across agents,

$$K_{t+1}^{\text{dist}} = G_K^{\text{dist}} (K_{1,t}, \dots, K_{n,t}, \mathcal{R}_t). \quad (1517)$$

Equation 1517 makes network structure part of epistemic generation.

Loss of one relational bridge can therefore alter the process even when individual knowledge stocks remain unchanged.

19.11 Value Generation

Generated value is represented by

$$V_{t+1} = \Gamma_V (X_t, C_t, \mathcal{R}_t, A_t). \quad (1518)$$

Equation 1518 can represent economic, epistemic, social, cultural, ecological, or another value dimension.

The formalism does not assume that all values are commensurable.

19.12 Value-Generation Equivalence

A value-generation criterion can require

$$\Gamma_{V,R} \sim_{E_V} \Gamma_{V,\text{ref}}. \quad (1519)$$

Equation 1519 is stronger than equality of one realized output because it compares the value-producing mechanism over a selected domain.

19.13 Value Circulation

Generated value can be retained, transferred, extracted, distributed, or reinvested.

Let

$$D_{t+1} = \Lambda_t (V_{t+1}, \mathcal{A}, \mathcal{R}_t, \chi_t) \quad (1520)$$

represent the circulation structure. Equation 1520 is central to generative justice because generation and circulation can diverge [15, 17].

Equal production does not imply equal circulation.

19.14 Value-Circulation Equivalence

A circulation criterion is

$$\Lambda_R \sim_{E_\Lambda} \Lambda_{\text{ref}}. \quad (1521)$$

Equation 1521 can compare flow graphs, retention proportions, distributions, control rights, or recursive return to future conditions.

The relation should be specified rather than using “circular value flow” as an undefined metaphor.

19.15 Recursive Return of Value

A portion of generated value can return as future generative condition:

$$C_{t+1} = \Psi (\Lambda(V_{t+1})). \quad (1522)$$

Equation 1522 formalizes one generative cycle.

The cycle becomes

$$C_t \xrightarrow{\Gamma_V} V_{t+1} \xrightarrow{\Lambda} D_{t+1} \xrightarrow{\Psi} C_{t+1}. \quad (1523)$$

Equation 1523 provides a central process object for generative comparison.

19.16 Recursive-Cycle Equivalence

A strong process criterion requires correspondence of the whole cycle.

Let mappings R_C , R_V , and R_D connect reference and restored condition, value, and distribution spaces.

The generation step should satisfy

$$R_V \circ \Gamma_{V,\text{ref}} = \Gamma_{V,R} \circ R_C. \quad (1524)$$

Equation 1524 preserves the relation between conditions and generated value.

The circulation step should satisfy

$$R_D \circ \Lambda_{\text{ref}} = \Lambda_R \circ R_V. \quad (1525)$$

Equation 1525 preserves the relation between generated value and its distribution.

The regeneration step should satisfy

$$R_C \circ \Psi_{\text{ref}} = \Psi_R \circ R_D. \quad (1526)$$

Equation 1526 preserves the relation between distributed value and future conditions.

Together, Equations 1524 through 1526 define a strong structure-preserving generative correspondence.

19.17 Control Structure

Generation and circulation occur under control relations.

Let

$$\chi_t : \mathcal{A} \times \mathcal{C} \longrightarrow \mathcal{P} \quad (1527)$$

assign rights or powers over generative conditions.

Equation 1527 can represent authority to access, allocate, modify, revoke, or govern a condition.

Generative justice gives special significance to whether generators retain appropriate control over the conditions and value of production [15, 17].

19.18 Control-Process Equivalence

A generative process can therefore require

$$\chi_R \sim_{E_\chi} \chi_{\text{ref}}. \quad (1528)$$

Equation 1528 compares the control architecture.

This criterion is independent of output equality.

19.19 Secure Control

Current control can differ from secure control.

Let S_χ denote the security of control rights against arbitrary withdrawal.

A stronger criterion compares

$$(\chi_R, S_{\chi,R}) \sim_E (\chi_{\text{ref}}, S_{\chi,\text{ref}}). \quad (1529)$$

Equation 1529 connects generative justice with non-domination [44].

19.20 Dependency Structure

A system can be generative while concentrating dependency upon one actor.

Let D_t denote dependency structure:

$$D_t = D(\chi_t, \mathcal{R}_t, C_t). \quad (1530)$$

Equation 1530 makes dependency a separate object.

A restored system can reproduce output while increasing dependence.

19.21 Generative Autonomy

Generative autonomy concerns the subject's authority to redirect or revise the generative process.

Let

$$A_G = A_G(\mathcal{U}_A, \chi, \mathcal{R}, \mathcal{R}_{\text{reach}}). \quad (1531)$$

Equation 1531 can include:

- ability to choose projects;
- ability to redirect resources;
- ability to leave a relationship;
- ability to revise goals;
- ability to retain generated value;
- ability to reject restorative intervention.

Generative-process restoration can therefore include autonomy without reducing it to output.

19.22 Generative-Process Object

The full generative-process object can be represented by

$$Q_G(\mathcal{G}) = (\mathcal{K}, G_R, \Gamma_V, \Lambda, \Psi, \chi, D, A_G). \quad (1532)$$

Equation 1532 combines conversion, relational generation, value generation, circulation, condition regeneration, control, dependency, and generative autonomy.

The object is deliberately modular. A practical inquiry can select only the components relevant to the case.

19.23 Exact Generative-Process Identity

The strongest represented criterion is

$$Q_G(\mathcal{G}^R) = Q_G(\mathcal{G}^{\text{ref}}). \quad (1533)$$

Equation 1533 requires identity across every included process component.

This standard will rarely be necessary or empirically attainable.

Its analytical value lies in marking the strongest representation-relative notion of generative-process restoration.

19.24 Structural Generative Isomorphism

A weaker criterion preserves process architecture under relabeling.

Let

$$\Phi_G : Q_{G,\text{ref}} \longrightarrow Q_{G,R} \quad (1534)$$

be a bijection preserving the selected mappings.

Structural generative isomorphism requires the conversion, production, circulation, regeneration, and control relations to commute under Φ_G .

This criterion preserves architecture while allowing the identities of particular resources, actors, or states to change.

19.25 Generative Homomorphism

A generative homomorphism permits some reference distinctions to be compressed while preserving selected process relations.

Let

$$\Phi_G : Q_{G,\text{ref}} \longrightarrow Q_{G,R} \quad (1535)$$

preserve the relevant mappings without requiring bijectivity.

This can represent a restored process that implements the essential generative logic of a richer reference system at a coarser level.

The loss of distinction should be normatively evaluated.

19.26 Generative Embedding

A future-expanding restoration can embed the reference generative process inside a richer restored process:

$$Q_{G,\text{ref}} \hookrightarrow Q_{G,R}. \quad (1536)$$

Equation 1536 is especially attractive where the subject develops new capacities after harm.

The reference process remains available as a substructure while the restored system contains additional generative pathways.

19.27 Generative Simulation

A directional generative criterion can be defined through simulation.

Let states g_{ref} and g_R be related by $\mathcal{S}_G$.

If the reference process can perform a generatively relevant transition

$$g_{\text{ref}} \xrightarrow{a}_G g'_{\text{ref}}, \quad (1537)$$

the restored process should possess a corresponding transition

$$g_R \xrightarrow{a'}_G g'_R \quad (1538)$$

with

$$(g'_{\text{ref}}, g'_R) \in \mathcal{S}_G. \quad (1539)$$

Equations 1537 through 1539 define a generativity-preserving simulation.

This criterion allows the restored system to contain additional generative transitions.

19.28 Generative Bisimulation

A stronger criterion requires reciprocal matching of generative transitions:

$$Q_{G,R} \sim_{\text{bis},G} Q_{G,\text{ref}}. \quad (1540)$$

Equation 1540 is formally strong because both systems must match each other's represented generative branching.

Its normative status is limited. New valuable possibilities in the restored system can cause bisimulation to fail.

Generative bisimulation therefore measures fidelity rather than superiority.

19.29 Generative Trace Equivalence

A weaker process criterion compares sequences of generated states or values.

Let

$$\text{tr}_G(\mathcal{G}) \quad (1541)$$

denote the selected generative trace.

Trace equivalence requires

$$\mathcal{L}_{G,R} = \mathcal{L}_{G,\text{ref}}. \quad (1542)$$

Equation 1542 preserves possible observable generative sequences while potentially ignoring branching differences.

19.30 Interventional Generative Equivalence

A causal-process criterion compares the generative response to interventions.

Let Y_G represent future generative conditions. Then

$$P_R(Y_G \mid \text{do}(u)) \ E \ P_{\text{ref}}(Y_G \mid \text{do}(u)) \quad (1543)$$

for $u \in \mathcal{U}_G$.

Equation 1543 asks whether the same interventions generate corresponding future conditions.

This is stronger than comparing one immediate output.

19.31 Recursive Interventional Equivalence

A stronger criterion follows intervention effects through several generative cycles.

Let $Y_G^{(n)}$ denote the generative state after n cycles. Require

$$P_R(Y_G^{(n)} \mid \text{do}(u)) \ E \ P_{\text{ref}}(Y_G^{(n)} \mid \text{do}(u)) \quad (1544)$$

for $n = 1, \dots, N$.

Equation 1544 makes generative depth explicit.

19.32 Generative Horizon

The number of recursive rounds over which a claim holds is its generative horizon:

$$H_G = N. \quad (1545)$$

Equation 1545 provides a compact index of recursive depth.

A one-cycle process equivalence can fail at larger H_G .

19.33 One-Cycle Restoration

A system can reproduce the first round of output while depleting resources required for the second round.
Let

$$Y_R^{(1)} = Y_{\text{ref}}^{(1)} \quad (1546)$$

and

$$Y_R^{(2)} \neq Y_{\text{ref}}^{(2)}. \quad (1547)$$

Equations 1546 and 1547 demonstrate why one-cycle restoration cannot establish recursive generative restoration.

19.34 Recursive Depletion

A process can maintain output by consuming future capacity.

Let reserve condition be B_t . Recursive depletion is represented by

$$B_{t+1} < B_t \quad (1548)$$

while current output remains high.

This can occur through overwork, ecological depletion, debt, institutional burnout, or loss of social trust.

A high-output process can therefore be generatively unsustainable.

19.35 Recursive Accumulation

The converse process increases future condition:

$$B_{t+1} > B_t. \quad (1549)$$

Equation 1549 represents one form of self-reinforcing generativity.

Its normative value still depends upon how the accumulated condition is distributed and controlled.

19.36 Recursive Retention Rate

Let V_{t+1} be generated value and V_{t+1}^{return} the portion returning as future generative condition.

Define

$$\rho_t = \frac{V_{t+1}^{\text{return}}}{V_{t+1}} \quad (1550)$$

where the ratio is meaningful.

Equation 1550 is a scalar summary of circular retention.

Equal ρ_t does not imply equal control, meaning, or distribution.

19.37 Retention-Rate Equivalence

A weak generative comparison is

$$\rho_R = \rho_{\text{ref}}. \quad (1551)$$

Equation 1551 can be empirically tractable.

It should not be mistaken for complete generative-process equivalence.

19.38 Retention and Control

Value can return to the generating system while control remains external.

Let retained value be V^{return} and controlling actor be $q(V^{\text{return}})$.

A community can receive reinvestment while lacking authority over its use.

Generative justice therefore requires independent representation of circulation and control.

19.39 Extraction

Extraction occurs when generated value leaves the generating system or is controlled externally in ways relevant to the selected normative theory.

Let

$$V_{t+1} = V_{t+1}^{\text{return}} + V_{t+1}^{\text{extract}}. \quad (1552)$$

Equation 1552 decomposes generated value.

An extraction ratio can be represented by

$$\epsilon_t = \frac{V_{t+1}^{\text{extract}}}{V_{t+1}}. \quad (1553)$$

Equation 1553 is a coarse summary.

The normative status of extraction depends upon institutions, consent, distribution, taxation, contractual relations, and other factors.

19.40 Extraction Equivalence

Two systems can be compared through

$$\epsilon_R \ E \ \epsilon_{\text{ref}}. \quad (1554)$$

Equation 1554 is weaker than comparison of the full circulation structure.

Equal extraction ratios can hide different beneficiaries and governance.

19.41 Value Capture

Value capture concerns concentration of control over generated value.

Let actor set controlling generated value be C_V .

A concentration metric can be represented by

$$\kappa_V = \text{Conc}(C_V). \quad (1555)$$

Equation 1555 can reveal whether a process becomes more dependent upon a small number of actors.

The specific concentration measure is application-dependent.

19.42 Generative Capture

Generative capture occurs when an actor controls not only current value but the conditions required for future generation.

Let C_G denote critical generative conditions.

A controlling actor A_j has high generative capture where its decision substantially determines access to many elements of C_G .

This structure is especially relevant to domination.

19.43 Capture and Formal Access

Formal access can remain equal while generative capture increases.

A researcher can retain laboratory access while one actor controls all future funding, publication approval, or recommendation.

The present resource state can therefore look restored while the generative control process has changed substantially.

19.44 Manufactured Vulnerability

A dominating process can progressively remove alternative generative conditions and then render the subject dependent upon the dominant relation.

Let outside-option set be $\mathcal{O}_t$.

Manufactured vulnerability can involve

$$\mathcal{O}_{t+1} \subset \mathcal{O}_t \quad (1556)$$

while dependence upon actor A_j increases.

Equation 1556 captures one structural route by which vulnerability is generated rather than merely discovered.

19.45 Dependency Amplification

Let dependency level be D_t .

A self-reinforcing process can satisfy

$$D_{t+1} = F_D(D_t), \quad F_D(D_t) > D_t \quad (1557)$$

over a relevant region.

Equation 1557 represents recursive dependency growth.

Restoration of current resources can leave this mechanism intact.

19.46 Generative Suppression

A process can suppress generation without removing every resource.

Let potential generative transition set be $\mathcal{T}_G$.

Suppression can be represented by

$$\mathcal{T}_{G,H} \subset \mathcal{T}_{G,\text{ref}}. \quad (1558)$$

Equation 1558 captures reduction of the transitions through which future possibilities are created.

19.47 Generative Obstruction

Obstruction can increase transition costs without removing transitions entirely.

Let path cost be $C(\pi)$. Then

$$C_H(\pi) > C_{\text{ref}}(\pi) \quad (1559)$$

for relevant generative paths.

Equation 1559 distinguishes nominal access from effective generative access.

19.48 Generative Distortion

A process can redirect generation toward goals determined by another actor.

Let subject-selected direction be d_A and realized generative direction be d_G .

A distortion measure can be represented schematically by

$$\Delta_d = d(d_A, d_G). \quad (1560)$$

Equation 1560 makes direction of generation an explicit object.

High productivity can coexist with low generative autonomy.

19.49 Generative Direction

Generativity concerns more than quantity.

A process can generate intensely in a direction the subject rejects.

The theory therefore distinguishes:

$$\text{capacity to generate} \quad (1561)$$

from

$$\text{authority over direction of generation.} \quad (1562)$$

These are separate normative dimensions.

19.50 Generative Diversity

A process can generate several qualitatively distinct future pathways.

Let generated possibility set after one cycle be $\mathcal{P}_{t+1}$.

A diversity functional is

$$D_G = D(\mathcal{P}_{t+1}). \quad (1563)$$

Equation 1563 can distinguish broad generative capacity from high volume along one narrow path.

19.51 Generative Concentration

The converse is concentration of generative pathways.

A process can produce much output while nearly all future value depends upon one channel.

This creates fragility and potential domination.

Generative diversity and robustness therefore interact.

19.52 Generative Redundancy

Redundant generative mechanisms can produce future conditions through several independent routes.

Let

$$\Pi_G = \{\pi_1, \dots, \pi_k\} \quad (1564)$$

denote the set of independent or partially independent generative pathways.

Equation 1564 provides a structural object for robust generativity.

19.53 Generative Robustness

A generative process is robust where future generative capacity remains available under relevant disturbance.

Let perturbation class be Δ . A criterion can require

$$G(\mathcal{G}_{t+\tau}^\delta) \geq G_{\min} \quad \forall \delta \in \Delta. \quad (1565)$$

Equation 1565 connects the present section with the perturbation analysis.
The generativity measure G remains theory-dependent.

19.54 Generative Resilience

Generative resilience concerns recovery of the process of generation rather than one output.

Let $\Gamma_t^{G,\delta}$ be the perturbed mechanism.

A generative-resilience criterion can require

$$\Gamma_{t+\tau}^{G,\delta} \sim_E \Gamma_{t+\tau}^{G,\text{ref}} \quad (1566)$$

after a selected recovery horizon.

This criterion is stronger than output recovery.

19.55 Generative Learning

A generative process can learn from disturbance and change its own mechanism.

Let

$$\Gamma_{t+1}^G = L_G(\Gamma_t^G, E_t), \quad (1567)$$

where E_t denotes relevant experience.

Equation 1567 makes model or institutional learning part of the generative process.

A restorative institution can therefore be evaluated by whether each case improves future capacity for prevention and repair.

19.56 Generative Adaptation

A process can change form while preserving valued generative functions.

Let

$$\Gamma_R^G \neq \Gamma_{\text{ref}}^G \quad (1568)$$

and

$$V_G(\Gamma_R^G) \geq V_G(\Gamma_{\text{ref}}^G). \quad (1569)$$

Equations 1568 and 1569 describe adaptive generative improvement.

This can be preferable to historical replication.

19.57 Generative Transformation

Where the historical process itself contains domination, extraction, or exclusion, justice can require replacement by another process.

Let

$$\Gamma_{\text{ref}}^G \xrightarrow{T} \Gamma_{\text{new}}^G. \quad (1570)$$

Equation 1570 represents transformation of the generative mechanism.
The resulting system can be historically non-equivalent and normatively superior.

19.58 Generative Dominance

A transformed process can dominate the reference under a selected vector of generative values.

Let

$$\mathbf{V}_G = (V_{\text{capability}}, V_{\text{agency}}, V_{\text{control}}, V_{\text{circulation}}, V_{\text{robustness}}) . \quad (1571)$$

Generative dominance can be represented by

$$\mathbf{V}_G (\Gamma_R^G) \succeq \mathbf{V}_G (\Gamma_{\text{ref}}^G) . \quad (1572)$$

Equation 1572 permits improvement without equivalence.

19.59 Generative Sufficiency

Some institutions can require an adequate generative threshold rather than historical equivalence.

Let acceptable process set be $\mathcal{S}_G$.

Generative sufficiency is

$$\Gamma_R^G \in \mathcal{S}_G . \quad (1573)$$

Equation 1573 can be appropriate for social protection or assistance.

It does not settle corrective claims concerning lost historical generativity.

19.60 Generative Recovery and Corrective Justice

A subject can achieve strong present generative recovery while retaining a valid claim for past loss.

Let

$$\Gamma_R^G \succeq \Gamma_{\text{ref}}^G \quad (1574)$$

while

$$L_{\text{hist}} > 0 . \quad (1575)$$

Equations 1574 and 1575 show that future success and historical rectification are separate dimensions.

19.61 Generative Recovery and Assistance

Assistance can prioritize increasing present and future generative capacity without reproducing the exact lost process.

Let harmed process be Γ_H^G . Assistance can seek

$$\mathbf{V}_G (\Gamma_{HA}^G) > \mathbf{V}_G (\Gamma_H^G) \quad (1576)$$

under the selected ordering.

This is a future-directed rather than reference-identical criterion.

19.62 Generative Recovery and Re-Entry

Re-entry can restore access to a process without restoring the process itself.

A researcher can regain institutional membership while the collaborative, epistemic, and value-circulation mechanisms remain different.

A stronger re-entry criterion therefore asks whether the institution again supports viable generative cycles.

19.63 Generative Recovery and Recognition

Recognition can affect generative process by changing how others respond.

A public correction can increase:

- network access;
- credibility;
- willingness of others to collaborate;
- subject willingness to participate.

Recognition therefore enters generation through causal and relational pathways while remaining a distinct value object [25, 20].

19.64 Generative Recovery and Non-Domination

A process can be productive and dominated.

The subject can generate substantial value while another actor retains arbitrary power over:

- access;
- publication;
- income;
- exit;
- generated assets;
- future opportunity.

Generative-process justice therefore requires control and dependency analysis in addition to productivity [44].

19.65 Generative Recovery and Capability

Capability sets can be treated as outputs of generative processes.

The process-level question is not only whether $\mathcal{K}_R$ matches the reference at time t , but whether the system continues producing and preserving substantive freedoms over time.

This turns capability from a static endpoint into a recursive object.

19.66 Capability Regeneration

Let capability set at time t be $\mathcal{K}_t$.

A capability-generating process can be represented by

$$\mathcal{K}_{t+1} = G_{\mathcal{K}} (\mathcal{K}_t, C_t, \mathcal{R}_t, X_t). \quad (1577)$$

Equation 1577 makes capability persistence and expansion part of generativity.

A one-time capability floor can coexist with long-run capability contraction.

19.67 Capability-Process Equivalence

A process criterion can require

$$G_{\mathcal{K},R} \sim_E G_{\mathcal{K},\text{ref}}. \quad (1578)$$

Equation 1578 is stronger than capability-set equality at one time.

19.68 Generative Process and Temporal Windows

Generative mechanisms can be time-sensitive.

A research network can generate opportunity only during a particular professional period.

A family relation can generate care during a developmental period.

An ecological condition can regenerate only within a seasonal or climatic window.

Thus

$$\Gamma^G = \Gamma^G(t) \quad (1579)$$

can hold.

Delayed restoration of the input therefore need not restore the lost process.

19.69 Generative Process and Historical Contingency

Generative processes create contingent events that then become future conditions.

Let one contingent encounter be e_t . It can generate

$$e_t \rightarrow \mathcal{R}_{t+1} \rightarrow C_{t+2} \rightarrow V_{t+3}. \quad (1580)$$

Equation 1580 illustrates why a complete enumeration of lost downstream generation can be epistemically unavailable.

The theory should therefore distinguish identified process structure from speculative historical detail.

19.70 Generative Process and Open-Endedness

A generative process can create new state categories or new kinds of value.

Let

$$\mathcal{X}_{t+1} = \mathcal{X}_t \cup \Delta\mathcal{X}_{t+1}. \quad (1581)$$

Equation 1581 means the future state space is not completely fixed in advance.

Exact process equivalence over indefinitely open-ended systems can therefore be ill-defined without an abstraction rule.

19.71 Generative Novelty

Let

$$N_G(t) = \Delta\mathcal{X}_{t+1} \quad (1582)$$

denote newly generated state categories or possibilities.

A restored process can produce valuable novelty absent from the reference.

This gives another reason to permit embedding, simulation, and dominance in addition to exact equivalence.

19.72 Generative Conservatism

A restorative system can become too focused upon preserving the old process and thereby suppress novelty.

High historical fidelity can reduce generative openness.

A subject forced back into the exact professional, relational, or institutional configuration predicted by the no-harm model can lose newly developed possibilities.

This is a process-level form of counterfactual paternalism.

19.73 Generative Openness

Generative openness concerns capacity to create legitimate new paths.

Let

$$O_G = O(\mathcal{R}_{\text{new}}, A_G, R_{\text{rev}}), \quad (1583)$$

where $\mathcal{R}_{\text{new}}$ denotes newly reachable regions, A_G generative autonomy, and R_{rev} revisability.

Equation 1583 is conceptual.

Its purpose is to distinguish open future formation from mere restoration of known options.

19.74 Generative Closure

A process can be highly productive within one fixed pathway and generate little capacity for revision.

This is generative closure.

Examples can include:

- one-employer dependency;
- one-funder dependency;
- one-gatekeeper academic structure;
- highly optimized but brittle infrastructure;
- assistance available only under one life trajectory.

Generative closure can therefore coexist with high output.

19.75 Generative Revisability

A just generative process can preserve the ability to revise direction.

Let revision action set be $\mathcal{U}_{\text{rev}}$.

A process has practical revisability where

$$\mathcal{U}_{\text{rev}} \neq \emptyset \quad (1584)$$

and revision paths remain viable.

This connects generative process with agency.

19.76 Generative Exit

A system can be self-sustaining and still unjust if leaving it produces catastrophic loss.

Let exit cost be C_{exit} .

A process-sensitive restorative comparison should therefore evaluate

$$C_{\text{exit},R} \leq C_{\text{exit},\text{ref}}. \quad (1585)$$

Equation 1585 makes exit part of generative architecture.

19.77 Generative Process and Security

Generative conditions can exist under different degrees of future security.

Let

$$S_G = S(C, \chi, \mathcal{R}). \quad (1586)$$

Equation 1586 can represent persistence, revocability, and procedural protection.

Secure generativity differs from generativity dependent upon arbitrary permission.

19.78 Generative Process and Trust

Trust can alter whether actors invest in future generative cycles.

Let investment decision be

$$I_t = F_I(T_t, C_t, E_t). \quad (1587)$$

Equation 1587 gives one pathway through which trust changes generativity.

Formal condition return can therefore fail when trust remains low enough to suppress investment.

19.79 Generative Process and Meaning

The social meaning of a condition can affect its generative use.

A payment understood as acknowledgment can generate trust differently from an identical payment understood as discretionary charity.

A title restored through vindication can have a different relational effect from the same title granted as temporary accommodation.

Meaning is therefore capable of entering the generative mechanism.

19.80 Meaning-Sensitive Generative Equivalence

Let meaning state be M . A meaning-sensitive process can be represented as

$$\Gamma^G = \Gamma^G(C, M, \mathcal{R}, X). \quad (1588)$$

Equation 1588 makes meaning part of the generative input.

Two materially equal systems can therefore generate different futures because their relational meanings differ.

19.81 Generative Process and Provenance

How a condition was obtained can affect its later use.

A resource received through one's own recognized work can generate different identity, trust, and relation effects from the same resource received after prolonged denial and litigation.

Provenance can therefore be causally and normatively relevant to generation.

19.82 Provenance-Sensitive Generativity

Let provenance be P_C . The generative mechanism can depend upon it:

$$\Gamma^G = \Gamma^G(C, P_C, H). \quad (1589)$$

Equation 1589 gives a formal location for historical meaning inside process comparison.

19.83 Generative Process and Reflexivity

A representation of generativity can itself change generation.

Suppose institution labels subject A as “successfully restored.” That label can alter:

- eligibility for support;
- expectations of colleagues;
- willingness to report difficulties;
- future institutional monitoring.

The evaluation therefore becomes part of the generative process.

19.84 Reflexive Generative Update

Let evaluation record be

$$Z_t = E_G(\mathcal{G}_t). \quad (1590)$$

Later generation becomes

$$g_{t+1} = \Gamma^G(g_t, Z_t). \quad (1591)$$

Equation 1591 makes process evaluation endogenous.

This limits the possibility of a socially operative and permanently external generative metric.

19.85 Generative Non-Closure

The recursive sequence

$$g_t \rightarrow Z_t \rightarrow g_{t+1} \rightarrow Z_{t+1} \rightarrow \dots \quad (1592)$$

shows that generative evaluation can continually alter the object being evaluated.

Equation 1592 is the process-level form of reflexive non-closure.

A generative-justice institution should therefore be revisable rather than claiming final representation of the system.

19.86 Generative Process and Scale

Generative processes operate at several scales.

Let

$$\Gamma^{G,(s)} \quad (1593)$$

denote the process at scale s .

Individual, organizational, community, societal, and ecological generativity can diverge.

A process that increases organizational productivity can reduce individual generative autonomy.

A national infrastructure can improve aggregate opportunity while extracting local ecological value.

Scale should therefore accompany generative claims.

19.87 Cross-Scale Generation

One scale can generate conditions for another.

Let individual process influence collective condition:

$$C_{t+1}^{\text{collective}} = G_C \left(\Gamma_1^{G,(1)}, \dots, \Gamma_n^{G,(1)} \right). \quad (1594)$$

Collective institutions can then generate individual conditions:

$$C_{i,t+1} = G_i \left(C_t^{\text{collective}} \right). \quad (1595)$$

Equations 1594 and 1595 create a coupled cross-scale generative loop.

19.88 Generative Externalities

One subject's generation can alter another's future conditions.

For intervention or activity A_i ,

$$\epsilon_{ij}^G = \Delta C_j \mid A_i \quad (1596)$$

denotes the generative externality on subject j .

Equation 1596 can be positive or negative.

Generative justice therefore cannot be evaluated only from the generator's perspective.

19.89 Positive Generative Externalities

Research, education, public infrastructure, open knowledge, and ecological stewardship can create future conditions for others.

A restorative program can therefore produce benefits beyond the initial claimant.

These effects are relevant to public finance and collective repair.

19.90 Negative Generative Externalities

One system's self-sustaining generation can depend upon extraction from another.

An institution can maintain a stable productive cycle by consuming unpaid care, ecological capacity, or subordinated labor elsewhere.

A process should therefore not be classified as just merely because it is internally self-sustaining.

19.91 Closed-System Fallacy

A generative process can appear circular only because the model excludes the external source from which value is drawn.

Let modeled system be $\mathcal{G}$ and external system $\mathcal{E}$.

If

$$V_{\mathcal{G}} \text{ is sustained by extraction from } \mathcal{E}, \quad (1597)$$

then internal circularity does not establish global generative justice.

The system boundary is therefore normatively important.

19.92 Boundary-Sensitive Generative Comparison

Let boundary specification be s .

A generative claim should therefore be indexed as

$$\Gamma_R^{G,(s)} \sim_E \Gamma_{\text{ref}}^{G,(s)}. \quad (1598)$$

Equation 1598 prevents a local cycle from being interpreted as globally self-sustaining without examining external flows.

19.93 Generative Process and Ecology

Ecological generativity introduces limits on human production.

A process that reproduces economic value while progressively degrading soil, water, climate, or biodiversity can be economically generative and ecologically depletive.

A plural generative profile should therefore include ecological condition where material.

19.94 Ecological Regeneration

Let ecological condition be E_t .

A sustainable generative process can require

$$E_{t+1} \succeq E_{\min} \quad (1599)$$

or another domain-specific ecological criterion.

Equation 1599 is illustrative rather than a universal ecological metric.

19.95 Generative Process and Intergenerational Transmission

A generative process can produce conditions inherited by later generations.

Let generation index be g . Then

$$C^{(g+1)} = \Gamma_{\text{intergen}} \left(C^{(g)} \right). \quad (1600)$$

Equation 1600 can represent transmission of wealth, knowledge, culture, institutional access, ecological condition, or political capacity.

Intergenerational generative equivalence therefore compares transmission mechanisms as well as current holdings.

19.96 Intergenerational Generative Equivalence

A criterion is

$$\Gamma_{\text{intergen},R} \sim_E \Gamma_{\text{intergen},\text{ref}}. \quad (1601)$$

Equation 1601 is particularly relevant to historical injustice.

Equal present resources can coexist with unequal transmission mechanisms.

19.97 Collective Generative Capacity

A collective can generate conditions unavailable to isolated individuals.

Let

$$G_C^{\text{collective}} = G \left(\mathcal{R}, K^{\text{dist}}, \chi, C \right). \quad (1602)$$

Equation 1602 can include collective governance, distributed knowledge, cultural reproduction, or common-resource management.

Restoration of individual resources therefore need not restore collective generativity.

19.98 Institutional Generative Capacity

An institution can be evaluated by its capacity to produce:

- reliable rights;
- information;
- opportunities;
- learning;
- correction;
- future institutional capacity.

An institution that repeatedly requires emergency repair can possess low generative capacity even when individual cases are eventually resolved.

19.99 Correction as Generative Capacity

A self-correcting system can convert detected error into improved future conditions.

Let correction event be e_C .

Institutional learning can satisfy

$$\Gamma_{t+1}^G = L(\Gamma_t^G, e_C). \quad (1603)$$

Equation 1603 makes correction productive of future governance capacity.

19.100 Failure to Learn

An institution can compensate repeatedly while leaving the harmful generative mechanism unchanged:

$$\Gamma_{t+1}^G = \Gamma_t^G \quad (1604)$$

despite repeated harm events.

Equation 1604 describes a low-learning restorative architecture.

Individual remedies can therefore coexist with systemic generative failure.

19.101 Generative Assistance

Generative assistance aims to expand or repair the process through which the subject can create future possibilities.

Let assistance intervention be A . Then

$$\Gamma_H^G \xrightarrow{A} \Gamma_{HA}^G. \quad (1605)$$

A future-oriented criterion is

$$\mathbf{V}_G(\Gamma_{HA}^G) \succeq \mathbf{V}_G(\Gamma_H^G). \quad (1606)$$

Equations 1605 and 1606 formalize assistance as process improvement.

19.102 Assistance and Historical Non-Equivalence

Assistance can create a new generative mechanism that is not historically equivalent to the no-harm process:

$$\Gamma_{HA}^G \not\sim \Gamma_{\text{ref}}^G. \quad (1607)$$

Equation 1607 does not imply failure.

The new process can better serve the present subject.

19.103 Generative Restoration and General Equivalents

General equivalents such as money can support generative restoration because they permit decentralized future choice.

Let monetary resource be M . The subject can map M into several future conditions:

$$M \xrightarrow{\mathcal{K}_A} \{C_1, \dots, C_n\}. \quad (1608)$$

Equation 1608 illustrates why fungibility has epistemic value: the institution need not know the one true future the subject should pursue.

The value of money remains dependent upon conversion structure.

19.104 Limits of General Equivalents in Generative Processes

Money cannot automatically recreate:

- singular relationships;
- historical windows;
- legal rights;
- public recognition;
- ecological conditions;
- institutional trust.

General equivalents therefore support but do not exhaust generative restoration.

19.105 Specific Enabling Conditions

Some generative processes require specific conditions.

Let

$$C^* = \{c_1^*, \dots, c_k^*\} \quad (1609)$$

denote the specific enabling set.

If one critical element is absent, the process can remain weak despite abundant general resources.

This creates complementarity among restorative inputs.

19.106 Generative Complementarity

Suppose output of a generative process is

$$G = G(M, C^*, R, A), \quad (1610)$$

where M is general resource, C^* specific condition, R relation, and A agency.

Equation 1610 permits strong complementarity.

Increasing M alone can have limited effect when C^* or R is missing.

19.107 Generative Bottlenecks

A bottleneck is a condition whose shortage constrains the whole generative process.

Let bottleneck condition be c_b .

A simple Leontief-like representation is

$$G = \min \{ \alpha_1 c_1, \dots, \alpha_b c_b, \dots, \alpha_n c_n \}. \quad (1611)$$

Equation 1611 is illustrative.

It shows why large compensation in non-binding dimensions can fail to restore generativity when one critical condition remains absent.

19.108 Bottleneck Removal

A small targeted intervention can produce large generative improvement when it removes the binding bottleneck.

This is one reason assistance should not be designed entirely through aggregate resource volume.

The challenge is epistemic: institutions can misidentify the bottleneck.

Subject participation is therefore important.

19.109 Generative Process and Procedural Justice

A generative process can produce desirable outputs through an unjust procedure.

Examples include:

- coerced participation;
- intrusive surveillance;
- compulsory identity reconstruction;
- external control of generated value;
- non-consensual relational engineering.

Process-level generative fidelity therefore does not establish procedural justice.

19.110 Generative Process and Consent

A subject can reject a process that would maximize modeled generativity.

Consent can therefore constrain the admissible process set:

$$\mathfrak{G}_{\text{adm}} = \{ \Gamma^G : \Gamma^G \text{ satisfies relevant consent conditions} \}. \quad (1612)$$

Equation 1612 keeps generativity subordinate to procedural legitimacy.

19.111 Generative Process and Privacy

Optimizing a generative process can require detailed data about relationships, health, finances, and behavior.

A privacy-sensitive system should therefore distinguish:

$$\begin{array}{ll} \text{information useful for optimization} & (1613) \\ \text{from} & \end{array}$$

$$\begin{array}{ll} \text{information legitimately collectible.} & (1614) \end{array}$$

These sets need not coincide.

19.112 Generative Process and Contestability

Subjects should be able to challenge institutional claims about what makes them generative.

A model can classify one career path as high-value while the subject values another.

Contestability therefore includes the right to challenge the generative model, not only the intervention selected from it.

19.113 Generative Process and Revisability

Because subjects and environments change, the chosen generative process should remain revisable where possible.

Let policy mechanism at time t be Γ_t^R .

Revision is

$$\Gamma_t^R \xrightarrow{\mathcal{U}} \Gamma_{t+1}^R. \quad (1615)$$

Equation 1615 makes revision an ordinary feature rather than evidence of failure.

19.114 Generative Process and Institutional Humility

An institution rarely knows every condition supporting a person's future generativity.

The appropriate response can therefore be to provide:

- flexible resources;
- secure rights;
- removal of known barriers;
- information;
- optional specific support;
- mechanisms for revision.

This design preserves subject direction while reducing institutional overconfidence.

19.115 Generative Process and Subject Direction

Let present subject priorities be P_A .

A subject-directed generative process can be represented by

$$\Gamma_A^G = \Gamma^G(P_A, C, \mathcal{R}, \mathcal{U}_A). \quad (1616)$$

Equation 1616 treats the subject's priorities as an input to future generation.

This differs from constructing a process solely to reproduce the counterfactual subject.

19.116 Generative Process and Counterfactual Paternalism

Counterfactual paternalism occurs when

$$\Gamma_{cf}^G \quad (1617)$$

is treated as the uniquely authoritative future process despite present subject preference.

Equation 1617 can remain relevant to historical valuation while lacking automatic authority over future-directed assistance.

19.117 Generative Process and Singular Opportunities

A singular opportunity can generate a unique branch of later conditions.

Let singular event be e^* . Then

$$e^* \rightarrow \Gamma_{e^*}^G \quad (1618)$$

can produce a process unavailable after the historical window closes.

Later resources can create another process Γ_{new}^G without reproducing $\Gamma_{e^*}^G$.

This is a process-level account of singular generative loss.

19.118 Generative Process and Historical Windows

Let opportunity window be W .

The process can be defined only for

$$t \in W. \quad (1619)$$

Once W closes, exact process restoration can become structurally unavailable.

Justice can then move toward compensation, recognition, and new generative support.

19.119 Generative Process and Irreversibility

A generative process can be non-invertible.

Let

$$\Gamma^G(g_1) = \Gamma^G(g_2) \quad (1620)$$

for $g_1 \neq g_2$.

Equation 1620 means the later state does not uniquely identify its prior generative condition.

Process reconstruction can therefore remain underdetermined even under perfect observation of the current state.

19.120 Generative Process and Historical Non-Identity

A restored system can become generatively equivalent from time t_1 onward:

$$\Gamma_{R,t \geq t_1}^G \sim_E \Gamma_{\text{ref},t \geq t_1}^G \quad (1621)$$

while

$$\mathcal{H}_{R,t < t_1} \neq \mathcal{H}_{\text{ref},t < t_1}. \quad (1622)$$

Equations 1621 and 1622 distinguish future process restoration from historical identity.

19.121 Generative Process and Better-Than-Reference Futures

A restored process can outperform the historical reference.

Let

$$\Gamma_R^G \succeq_V \Gamma_{\text{ref}}^G. \quad (1623)$$

Equation 1623 permits a transformed future with stronger agency, resilience, control, or capability.

The historical wrong remains part of the normative record.

19.122 Generative Process and Worse-but-Closer Futures

A process can be structurally closer to the reference and normatively worse.

Let fidelity measure be F_G .

It can hold that

$$F_G(\Gamma_2^G) > F_G(\Gamma_1^G) \quad (1624)$$

while

$$V(\Gamma_2^G) < V(\Gamma_1^G). \quad (1625)$$

Equations 1624 and 1625 give a process-level form of normative non-monotonicity.

19.123 Levels of Generative-Process Comparison

Generative-process comparison can be organized into several levels.

Level 1 compares immediate output.

Level 2 compares future-condition regeneration.

Level 3 compares conversion and relational generation.

Level 4 compares value circulation and control.

Level 5 compares recursive multi-cycle behavior.

Level 6 compares robust generation under perturbation.

Level 7 compares open-ended generative structure and subject-directed revisability.

These levels are analytically useful and should not be treated as a universal normative ladder.

19.124 Level 1: Output Equivalence

The weakest generative claim is

$$V_R = V_{\text{ref}}. \quad (1626)$$

Equation 1626 compares current generated value.

It does not establish generative-process equivalence.

19.125 Level 2: Regeneration Equivalence

The second level requires

$$C_{t+1,R} \ E \ C_{t+1,\text{ref}}. \quad (1627)$$

Equation 1627 compares the future conditions generated by the process.

19.126 Level 3: Conversion and Relation Equivalence

The third level requires correspondence of conversion and relation-generation mechanisms:

$$\mathcal{K}_R \sim_E \mathcal{K}_{\text{ref}} \quad (1628)$$

and

$$G_{R,R} \sim_E G_{R,\text{ref}}. \quad (1629)$$

19.127 Level 4: Circulation and Control Equivalence

The fourth level adds

$$\Lambda_R \sim_E \Lambda_{\text{ref}} \quad (1630)$$

and

$$\chi_R \sim_E \chi_{\text{ref}}. \quad (1631)$$

This level asks who controls generation and where value flows.

19.128 Level 5: Recursive Equivalence

The fifth level requires correspondence across several cycles:

$$(\Gamma_R^G)^n \sim_E (\Gamma_{\text{ref}}^G)^n \quad n = 1, \dots, N. \quad (1632)$$

Equation 1632 makes recursive depth explicit.

19.129 Level 6: Robust Generative Equivalence

The sixth level requires recursive correspondence under perturbation:

$$\Gamma_R^{G,\delta} \sim_E \Gamma_{\text{ref}}^{G,\delta} \quad \forall \delta \in \Delta_{\text{adm}}. \quad (1633)$$

Equation 1633 combines generative and robustness layers.

19.130 Level 7: Open Generative Correspondence

The strongest future-oriented level need not seek exact equivalence.

It can require preservation of valuable reference pathways, subject-directed revision, and admissible new generation:

$$\Gamma_{\text{ref}}^G \hookrightarrow \Gamma_R^G \quad (1634)$$

together with

$$\mathcal{U}_{\text{rev},R} \neq \emptyset. \quad (1635)$$

Equations 1634 and 1635 represent a process that preserves valuable generative structure while permitting new development.

19.131 Generative Restoration Profile

A generative restorative profile can be represented by

$$\mathbf{G}_R = (r_{\text{output}}, r_{\text{condition}}, r_{\text{conversion}}, r_{\text{relation}}, r_{\text{circulation}}, r_{\text{control}}, r_{\text{recursive}}, r_{\text{robust}}, r_{\text{open}}). \quad (1636)$$

Equation 1636 avoids compressing generative restoration into one binary label.

19.132 Generative Object–Relation Cells

Table 13 summarizes several principal generative-process cells.

Table 13 demonstrates that generative-process equivalence is a family of comparisons rather than one formal predicate.

Table 13: Illustrative generative-process object–relation cells

Generative object	Comparative relation	Restorative interpretation	Characteristic limitation
Immediate output	Equality	Current generated value corresponds	Says little about future generation
Future condition	Equality / dominance	The process regenerates corresponding enabling conditions	Condition representation can be incomplete
Conversion mechanism	Functional / interventional equivalence	Returned inputs become usable capability in corresponding ways	High causal-identification burden
Relational generation	Structural correspondence	Activity produces corresponding relational architecture	Relational meaning can remain singular
Value generation	Function correspondence	Conditions generate corresponding value	Value dimensions can be contested
Value circulation	Flow equivalence / dominance	Generated value circulates through corresponding structures	Equal flows can hide unequal control
Control structure	Isomorphism / non-domination dominance	Authority over generative conditions corresponds or improves	Historical control can itself be unjust
Recursive cycle	Commutative structural preservation	Generation, circulation, and regeneration preserve their architecture	Strong modeling requirements
Generative transition system	Simulation / bisimulation	Reference generative transitions remain reproducible	Bisimulation can reject valuable novelty
Robust generative process	Perturbation-conditioned equivalence	Generation remains comparable under relevant disturbance	Perturbation-class dependent
Open generative process	Embedding / dominance	Valuable reference processes remain while new legitimate paths are possible	Requires normative specification of valuable novelty

19.133 Generative Non-Implication Structure

Several non-implications are central.

Output equality does not imply condition-regeneration equality:

$$V_R = V_{\text{ref}} \not\Rightarrow C_{t+1,R} = C_{t+1,\text{ref}}. \quad (1637)$$

Condition equality does not imply conversion equivalence:

$$C_R = C_{\text{ref}} \not\Rightarrow \mathcal{K}_R \sim \mathcal{K}_{\text{ref}}. \quad (1638)$$

Conversion equivalence does not imply circulation equivalence:

$$\mathcal{K}_R \sim \mathcal{K}_{\text{ref}} \not\Rightarrow \Lambda_R \sim \Lambda_{\text{ref}}. \quad (1639)$$

Circulation equivalence does not imply control equivalence:

$$\Lambda_R \sim \Lambda_{\text{ref}} \not\Rightarrow \chi_R \sim \chi_{\text{ref}}. \quad (1640)$$

One-cycle equivalence does not imply recursive equivalence:

$$\Gamma_R^G \sim \Gamma_{\text{ref}}^G \text{ at } n = 1 \not\Rightarrow (\Gamma_R^G)^n \sim (\Gamma_{\text{ref}}^G)^n \quad \forall n. \quad (1641)$$

These non-implications define the internal architecture of the generative region of the atlas.

19.134 Generative Refinement Path

A schematic refinement path is

$$(Q_{\text{output}}, =) \longrightarrow (Q_{\text{condition}}, \sim) \longrightarrow (Q_{\text{conversion}}, \sim) \longrightarrow (Q_{\text{circ}}, \sim) \longrightarrow (Q_{\text{gen}}, \preceq_{\text{sim}}) \longrightarrow (Q_{\text{gen}}, \sim_{\text{bis}}). \quad (1642)$$

Equation 1642 progressively incorporates more process structure.

The sequence does not imply monotonic increase in justice.

19.135 Generative Fidelity and Justice

Let F_G denote fidelity to the reference generative process and J denote normative evaluation.

There is no general implication

$$F_G \uparrow \implies J \uparrow. \quad (1643)$$

Equation 1643 is central.

The reference process can contain domination, inequality, extraction, ecological damage, or constraints the present subject rejects.

19.136 Generative Capacity and Justice

Greater generative capacity is also not automatically more just.

A corporation can become more capable of generating profit while intensifying extraction.

An authoritarian institution can become highly capable of reproducing itself.

A scientific system can generate large output while excluding groups from control or recognition.

Generativity is therefore a descriptive and evaluative object whose normative status depends upon what is generated, for whom, through which relations, and under whose control.

19.137 Normatively Filtered Generativity

Let $\mathcal{N}$ denote normative admissibility conditions.

Define admissible generative transitions by

$$\mathcal{T}_G^{\mathcal{N}} = \{\tau_G : \tau_G \text{ satisfies } \mathcal{N}\}. \quad (1644)$$

Equation 1644 excludes generative transitions that violate the selected rights, relational, procedural, or ecological constraints.

This allows the theory to value generativity without maximizing generation as such.

19.138 Generative Justice as a Normative Layer

Generative justice can therefore be represented as a normative map

$$J_G : Q_G \longrightarrow \mathfrak{J}_G. \quad (1645)$$

Equation 1645 evaluates the generative process according to selected principles concerning control, circulation, autonomy, participation, distribution, and future conditions [15, 17].

The formal process and its justice should remain distinct.

19.139 Generative Equality

One normative question concerns whether subjects possess comparable capacities to enter, influence, and benefit from generative processes.

Let generative capability be G_i .

A crude equality criterion is

$$G_i = G_j. \quad (1646)$$

Equation 1646 is rarely sufficient because generative capabilities are multidimensional and differences can be legitimate.

The later justice sections will examine equality, sufficiency, priority, and relational interpretations separately.

19.140 Generative Sufficiency and Priority

A sufficiency theory can require each subject to possess a minimum generative capacity:

$$G_i \geq G_{\min}. \quad (1647)$$

A priority theory can assign additional weight to improvements for those with lower G_i .

These normative choices are independent of the formal process comparison.

19.141 Generative Relational Equality

A relational theory can focus on whether subjects participate in generative systems as equals rather than as subordinated instruments.

The relevant objects include:

- standing;
- voice;
- control;
- exit;
- recognition;
- dependence.

A system can therefore have unequal resources and still satisfy some forms of relational equality, or equal resources and retain domination [2, 71].

19.142 Generative Non-Domination

A non-domination criterion asks whether subjects remain exposed to arbitrary control over generative conditions.

Let arbitrary-control exposure be D_A .

A transformed process can improve where

$$D_{A,R} < D_{A,\text{ref}}. \quad (1648)$$

Equation 1648 can justify deviation from the historical reference.

19.143 Generative Recognition

Recognition can concern who is acknowledged as a generator of value.

Misrecognition can assign credit to institutions or dominant actors while obscuring the contributions of others.

Generative justice therefore intersects with authorship, attribution, cultural knowledge, and public acknowledgment.

19.144 Generative Attribution

Let generated value be V and contributor set $\mathcal{A}_V$.

An attribution map is

$$\alpha_V : V \longrightarrow \mathcal{P}(\mathcal{A}_V). \quad (1649)$$

Equation 1649 records which actors are recognized as contributors.

Attribution can affect later control and opportunity.

19.145 Attribution Equivalence

A restorative criterion can require

$$\alpha_{V,R} \sim_E \alpha_{V,\text{ref}}. \quad (1650)$$

Equation 1650 can matter where harm involved erasure of authorship or contribution.

19.146 Generative Process and Distribution

Generated value can be distributed unequally even when circulation remains within the system.

Let distribution be

$$D_V = (v_1, \dots, v_n). \quad (1651)$$

Equation 1651 provides a separate object from total value and retention.

Generative justice therefore does not eliminate distributive justice.

19.147 Generative Process and Care

Care work can create and sustain generative conditions while remaining underrepresented in conventional production measures.

A generative model can include care as a condition-producing process:

$$C_{t+1}^{\text{care}} = G_{\text{care}}(A_t, R_t). \quad (1652)$$

Equation 1652 makes visible forms of value production that conventional output measures can omit. The normative treatment of care will be developed later in the value sections.

19.148 Generative Process and Public Goods

Some generative processes create conditions shared by many actors.

Examples include:

- public knowledge;
- infrastructure;

- environmental quality;
- institutional trust;
- legal stability.

The generated value cannot always be assigned to one individual recipient.
Collective and public-good objects therefore belong within generative analysis.

19.149 Generative Process and Commons

A commons can generate value through shared governance and recursive maintenance.
The relevant process includes:

- access;
- contribution;
- rule formation;
- monitoring;
- maintenance;
- renewal.

A reparative intervention that transfers a commons resource without restoring governance capacity can therefore be generatively incomplete.

19.150 Generative Process and Institutional Interfaces

Generation often crosses institutional boundaries.

Education generates credentials recognized by employers. Clinical rehabilitation affects work access.
Legal correction affects professional databases.

Let institutional interface be

$$\mathcal{I}_{ij} : C_i \longrightarrow C_j. \quad (1653)$$

Equation 1653 can itself become a generative bottleneck.

19.151 Interface Failure

A process can function within each institution and fail across interfaces.

A correction issued by one body can fail to reach another. A qualification can be restored and remain unrecognized by downstream systems.

Generative-process analysis therefore extends beyond institutional interiors.

19.152 Generative Process and Portability

A capability can be usable only inside one institution.

Portability concerns whether generated conditions remain valuable after transition to another environment.

Let

$$P_C(c, e_1, e_2) \quad (1654)$$

denote the portability of condition c from environment e_1 to e_2 .

Low portability can create lock-in.

19.153 Generative Process and Exit Portability

A subject can leave an institution more safely when credentials, knowledge, assets, and reputation remain portable.

A restorative program that improves present conditions while reducing portability can therefore increase future dependency.

This again shows why current output is insufficient.

19.154 Generative Process and Resilience

A generative process can be robust because several cycles, relations, and resources compensate for one another.

The resilience of the process can therefore depend upon:

- resource reserves;
- relation diversity;
- institutional redundancy;
- control decentralization;
- learning;
- portability;
- subject adaptability.

Generative resilience is thus a composite object.

19.155 Generative Process and Bifurcation

A recursive generative process can shift into another regime when a condition passes a threshold.

For parameter θ ,

$$\Gamma^G = \Gamma^G(\theta). \quad (1655)$$

A critical change in θ can alter the qualitative form of generation.

This links the present section directly to attractor and bifurcation analysis.

19.156 Generative Process and Hysteresis

If a generative process exhibits hysteresis, restoring the original resource level can fail to restore the original recursive cycle.

For example, prolonged exclusion can destroy a network that funding alone cannot recreate.

Hysteresis therefore provides one dynamical mechanism for persistent generative non-equivalence.

19.157 Generative Process and Causal Mediation

The full process can be decomposed into mediated causal stages:

$$C_t \rightarrow K_t \rightarrow A_t \rightarrow V_{t+1} \rightarrow C_{t+1}. \quad (1656)$$

Equation 1656 makes explicit where restoration can fail.

Equal C_t can coexist with impaired K_t . Equal K_t can coexist with constrained A_t . Equal V_{t+1} can coexist with weak return to C_{t+1} .

19.158 Generative Process and Causal Bottlenecks

A single mediator can become the binding generative bottleneck.

Let bottleneck mediator be M_b .

Intervention upon non-binding inputs can then have weak effect:

$$\frac{\partial G}{\partial C_i} \approx 0 \quad (1657)$$

while

$$\frac{\partial G}{\partial M_b} \gg 0. \quad (1658)$$

Equations 1657 and 1658 are illustrative.

They show why more money can fail when the actual constraint lies elsewhere.

19.159 Generative Process and Institutional Diagnosis

The practical value of the framework lies partly in diagnosis.

If output remains low after resource restoration, an institution can examine:

- conversion;
- relational access;
- recognition;
- control;
- bottlenecks;
- security;
- transition costs.

The purpose is targeted assistance rather than total surveillance of the subject's life.

19.160 Generative-Process Minimalism

A full generative-process model should not be required in every restorative case.

A simple resource loss can be resolved adequately through restitution or compensation.

Process-level analysis becomes useful where:

- returned conditions fail to restore practical use;
- harm propagates recursively;
- value circulation is disputed;
- one actor controls future generation;
- repeated restoration repeatedly fails;
- long-term generativity is the explicit institutional objective.

This is the principle of generative-process minimalism.

19.161 Generative-Process Escalation

A case can escalate from simple object restoration toward process analysis when residual non-equivalence persists.

One escalation path is

resource $\rightarrow$ conversion $\rightarrow$ relation $\rightarrow$ circulation $\rightarrow$ recursive generation. (1659)

Equation 1659 supplies a practical diagnostic sequence.

19.162 Generative-Process Metadata

A process-level restorative claim should record at least:

- system boundary;
- generative conditions;
- conversion structure;
- relation-generation structure;
- value-generation object;
- circulation structure;
- control rights;
- recursive horizon;
- perturbation class;
- subject-directed target;
- normative theory;
- epistemic basis.

These elements prevent the term *generativity* from functioning as an undefined positive label.

19.163 Illustrative Workplace Generative Process

Consider a worker reinstated after exclusion.

A narrow state model asks whether title and salary returned.

A generative-process model asks whether employment again produces:

- stable income;
- skill development;
- professional relationships;
- reputation;
- promotion opportunity;
- portable experience;
- future bargaining power.

The same job can therefore be materially restored and generatively altered.

19.164 Illustrative Research Generative Process

A research environment provides a particularly clear generative cycle:

access → research → knowledge → publication → network/funding → new access. (1660)

Equation 1660 illustrates recursive generation.

Restoring laboratory access at the end of several missed cycles does not automatically recreate the network, reputation, or later opportunities those cycles would have produced.

A new research environment can nevertheless generate a different and equally or more valuable future.

19.165 Illustrative Reputational Generative Process

Reputation can participate in a positive or negative generative loop:

recognition → access → participation → visible contribution → recognition. (1661)

A false classification can reverse this cycle.

Correction of the source record repairs one node in the process. Restoration of the generative loop can require broader relational and institutional intervention.

19.166 Illustrative Community Generative Process

A community-controlled generative cycle can take the form

local production → retained value → community infrastructure → greater productive capacity. (1662)

Equation 1662 closely reflects the concern of generative justice with bottom-up value circulation [15, 17].

A one-time payment can increase resources while leaving the circulation structure unchanged.

19.167 Illustrative Educational Generative Process

Education generates more than credentials.

A cycle can include:

instruction → capability → confidence/network → new learning opportunity. (1663)

Equation 1663 makes educational generativity explicit.

Restoration of a certificate can therefore remain weaker than restoration of the process through which learning creates future learning.

19.168 Illustrative Ecological Generative Process

An ecological system can regenerate soil, habitat, biomass, water quality, and future reproductive capacity.

A restoration that recreates one observable composition while degrading the processes maintaining it can therefore be dynamically and generatively weak.

This case illustrates why process restoration can be more informative than snapshot restoration.

19.169 Illustrative Institutional Generative Process

A just institution can itself generate:

- trust;
- reliable expectations;
- correction capacity;
- public knowledge;
- participation;
- future institutional competence.

An institution that resolves individual cases while degrading these conditions can be restoratively active and institutionally generative-poor.

19.170 Generative-Process Principle

The first principle of this section is:

Restoration of generated output does not establish restoration of the process through which future conditions, capacities, relations, and value are produced.

19.171 Recursive-Generation Principle

The second principle is:

A restorative process should distinguish one-cycle correspondence from correspondence across repeated generative cycles.

19.172 Circulation Principle

The third principle is:

Generation of value and circulation of value are distinct restorative objects; equal production can coexist with unequal retention, extraction, and control.

19.173 Control Principle

The fourth principle is:

A system can be highly generative while remaining unjust where subjects lack secure and legitimate control over critical generative conditions.

19.174 Open-Generation Principle

The fifth principle is:

A just restorative process can preserve valuable reference generativity while permitting new legitimate forms of generation beyond the historical reference.

19.175 Generative Non-Monotonicity Principle

The sixth principle is:

Greater formal fidelity to the reference generative process does not automatically imply greater justice.

19.176 Subject-Direction Principle

The seventh principle is:

The present historical subject should retain meaningful authority over the direction in which restored generative capacity is used.

19.177 Boundary Principle

The eighth principle is:

A process should not be classified as self-sustaining or generatively just without examining the external systems from which its conditions and value are drawn.

19.178 Normative-Filtering Principle

The ninth principle is:

Generativity itself is normatively neutral until the generated objects, relations, distributions, procedures, externalities, and forms of control have been evaluated.

19.179 Generative Layer of the Atlas

The generative layer can now be represented by

$$\mathfrak{G}_A = \{Q_{\text{condition}}, Q_{\text{conversion}}, Q_{\text{rel-gen}}, Q_{\text{value-gen}}, Q_{\text{circ}}, Q_{\text{control}}, Q_{\text{recursion}}, Q_{\text{open}}\} \times \mathfrak{E}_G, \quad (1664)$$

where $\mathfrak{E}_G$ contains equality, structural isomorphism, homomorphism, embedding, simulation, bisimulation, interventional equivalence, approximation, sufficiency, and dominance relations appropriate to these objects.

Equation 1664 defines the generative region of the Atlas of Equivalence.

19.180 Implications for Historical and Reflexive Equivalence

The present section has focused on the recursive process that generates future conditions.

A further question remains.

Even if the restored and reference systems now possess equivalent or superior generative processes, their historical provenance can remain different. The harm occurred. The restorative intervention occurred. Subjects remember those events. Institutions can preserve records of them. The process of declaring generative restoration can itself enter later social dynamics.

The strongest remaining comparative problem therefore concerns the relation between generative-process correspondence and historical or reflexive equivalence.

19.181 Transition to Historical and Reflexive Equivalence

Section 20 develops historical and reflexive comparison. It distinguishes historical identity, event-order correspondence, provenance equivalence, memory and narrative structure, record correction, the restoration event itself, and reflexive non-closure.

The central question becomes:

When the future generative process has been restored or transformed, what remains different because the system arrived there through harm, repair, recognition, memory, and institutional knowledge of its own history?

This is the final major formal comparison before the manuscript turns more directly toward value theory and justice.

20 Historical and Reflexive Equivalence

This section develops the historical and reflexive region of the Atlas of Equivalence. The preceding formal sections have progressively strengthened the object of restorative comparison. They moved from present state to trajectory, reachability, dynamical structure, perturbation response, causal mechanism, and recursive generative process. Even strong correspondence across these dimensions does not eliminate a final source of difference: the restored and reference systems can possess different histories.

The harm occurred in one system and did not occur in the reference system. The subject experienced the harmful interval. Other actors responded to it. Records were created. Relationships changed. Knowledge accumulated. Institutions acted or failed to act. A later restorative intervention added another event. The system can subsequently become behaviorally, causally, dynamically, or generatively similar to the reference while remaining historically distinct.

Historical comparison therefore asks which aspects of the path by which the system arrived at its present configuration are part of the restorative object. Reflexive comparison adds a further complication. Records, judgments, measurements, classifications, apologies, reparations, and declarations of restoration can themselves enter the system and alter subsequent relations and behavior. The act of determining whether restoration has occurred can therefore become one of the causes shaping the post-restoration system.

The section has seven objectives. First, it distinguishes historical identity from state, process, and generative equivalence. Second, it develops several weaker historical relations, including event correspondence, order isomorphism, provenance preservation, and narrative continuity. Third, it examines memory, recognition, archives, and historical recognizability as independent restorative objects. Fourth, it analyzes erasure and reconstruction thought experiments. Fifth, it formalizes the restorative event as a new historical event. Sixth, it develops reflexive non-closure. Seventh, it shows why complete historical equivalence can be unavailable, normatively undesirable, or conceptually unstable even under highly idealized technological capacity.

20.1 Historical Objects in the Restorative Atlas

Let the historical projection of system $\mathcal{G}$ be represented by

$$Q_H(\mathcal{G}) = \mathcal{H}, \quad (1665)$$

where $\mathcal{H}$ denotes the historical structure relevant to the restorative inquiry. Equation 1665 does not assume one universal representation of history.

Depending upon the case, $\mathcal{H}$ can include:

- event sequences;

- partial orders among events;
- causal provenance;
- institutional decisions;
- records and classifications;
- relational histories;
- memories;
- public narratives;
- authorship and attribution;
- historical opportunities;
- prior restorative interventions.

A historical restorative comparison is then represented by

$$Q_H(\mathcal{G}^R) \ E_H \ Q_H(\mathcal{G}^{\text{ref}}), \quad (1666)$$

where E_H is the selected historical comparative relation.

20.2 Historical Identity

The strongest historical relation is exact identity:

$$\mathcal{H}_R = \mathcal{H}_{\text{ref}}. \quad (1667)$$

Equation 1667 requires the represented histories themselves to be identical.

Where $\mathcal{G}^{\text{ref}}$ is a no-harm history and $\mathcal{G}^R$ is an actual harmed-and-restored history, this criterion normally fails by construction.

The actual history contains at least:

$$H, \quad R, \quad (1668)$$

where H denotes the harmful event or process and R the restorative intervention.

The no-harm history contains neither in the same form.

20.3 Historical Identity and Future Equivalence

Historical non-identity can coexist with strong future equivalence.

Suppose

$$\Gamma_{R,t \geq t_1}^G \sim_E \Gamma_{\text{ref},t \geq t_1}^G. \quad (1669)$$

Equation 1669 establishes future generative correspondence from t_1 onward.

It can coexist with

$$\mathcal{H}_{R,[t_0,t_1]} \neq \mathcal{H}_{\text{ref},[t_0,t_1]}. \quad (1670)$$

Together, Equations 1669 and 1670 distinguish restoration of future process from restoration of past history.

20.4 Historical Identity and State Identity

Two histories can terminate in the same represented state:

$$x_R(t_1) = x_{\text{ref}}(t_1) \quad (1671)$$

while

$$\mathcal{H}_R \neq \mathcal{H}_{\text{ref}}. \quad (1672)$$

Equations 1671 and 1672 show why endpoint equality cannot establish historical equivalence.

This is especially important for authorship, legitimacy, responsibility, recognition, and narrative identity.

20.5 Event Sets

A weaker historical object is the set of events that occurred.

Let

$$\mathcal{E}_H = \{e_1, e_2, \dots, e_n\}. \quad (1673)$$

Equation 1673 ignores order and causal dependence.

Two histories can contain corresponding event types while ordering them differently.

Event-set equality is therefore a coarse historical relation.

20.6 Event-Sequence Identity

A stronger object preserves event order:

$$\mathbf{e} = (e_1, e_2, \dots, e_n). \quad (1674)$$

Exact event-sequence identity requires

$$\mathbf{e}_R = \mathbf{e}_{\text{ref}}. \quad (1675)$$

Equation 1675 is stronger than equality of the event set.

For no-harm references, exact sequence identity normally fails once harm and restoration enter the actual sequence.

20.7 Event Correspondence

A weaker relation allows corresponding rather than identical events.

Let

$$\phi_E : \mathcal{E}_{\text{ref}} \longrightarrow \mathcal{E}_R \quad (1676)$$

map reference events to restorative-history events.

Equation 1676 can preserve event type, function, causal role, or another selected property.

This permits two histories to be compared structurally without claiming literal identity.

20.8 Historical Partial Orders

Distributed histories need not possess one canonical total ordering.

Let

$$\mathcal{H}_{PO} = (\mathcal{E}, \prec_H) \quad (1677)$$

represent a partially ordered historical event structure.

Equation 1677 can preserve relations such as:

- temporal precedence;
- causal dependence;
- evidentiary dependence;
- institutional supersession;
- authorship derivation.

This representation avoids forcing concurrent or independent events into one linear narrative.

20.9 Historical-Order Isomorphism

Two historical partial orders are order-isomorphic when there exists a bijection ϕ_E satisfying

$$e_i \prec_{\text{ref}} e_j \iff \phi_E(e_i) \prec_R \phi_E(e_j). \quad (1678)$$

Equation 1678 preserves historical ordering while allowing event identities to differ.

This criterion can be relevant for institutional processes whose procedural sequence matters.

20.10 Historical Embedding

A reference history can be embedded into a richer restored history:

$$\mathcal{H}_{\text{ref}} \hookrightarrow \mathcal{H}_R. \quad (1679)$$

Equation 1679 can represent preservation of a reference process within a history containing additional restorative events.

This relation can be more appropriate than historical identity where later learning and transformation have generated legitimate new events.

20.11 Historical Compression

Institutions often compress rich histories into a small number of legally or administratively relevant events.

Let

$$\alpha_H : \mathcal{H} \longrightarrow \hat{\mathcal{H}} \quad (1680)$$

be a historical abstraction map.

Equation 1680 can transform years of workplace experience into categories such as employment, dismissal, reinstatement, and compensation.

Administrative tractability can require such compression.

The excluded history should not be presumed normatively irrelevant.

20.12 Historical Equivalence under Abstraction

Two histories can become equal after abstraction:

$$\alpha_H(\mathcal{H}_R) = \alpha_H(\mathcal{H}_{\text{ref}}) \quad (1681)$$

while

$$\mathcal{H}_R \neq \mathcal{H}_{\text{ref}}. \quad (1682)$$

Equations 1681 and 1682 provide a historical form of representational equivalence. The abstraction rule determines which differences disappear.

20.13 Provenance

Provenance records how a present object, claim, resource, status, or decision was generated. Let

$$\text{Prov}(q_t) = \mathcal{P}_q \quad (1683)$$

denote the provenance of current object q_t .

Equation 1683 can include:

- originating event;
- evidence;
- authorship;
- institutional decisions;
- transformations;
- corrections;
- transfers;
- dependencies.

Equal present objects can possess different provenance.

20.14 Provenance Identity

Exact provenance identity requires

$$\mathcal{P}_{q,R} = \mathcal{P}_{q,\text{ref}}. \quad (1684)$$

Equation 1684 can matter where value or legitimacy depends upon origin.

Examples include authorship, title, evidence, cultural property, and scientific records.

20.15 Provenance Isomorphism

A weaker criterion can preserve provenance structure under relabeling:

$$\mathcal{P}_{q,R} \cong \mathcal{P}_{q,\text{ref}}. \quad (1685)$$

Equation 1685 can preserve derivational architecture while permitting different source entities. This criterion remains weaker than historical identity.

20.16 Provenance and Legitimacy

The legitimacy of a present state can depend upon how it was produced.

The same resource distribution can result from:

- voluntary exchange;
- coercion;
- inheritance;
- compensation;
- restitution;
- administrative error.

Present-state equality therefore does not determine legitimacy where provenance matters.

20.17 Provenance and Authorship

Authorship is inherently provenance-sensitive.

Two identical texts have different authorship histories if one was copied from the other without attribution.

The current informational object can be identical while the provenance relation differs.

This illustrates why historical structure can carry normative importance independently of present content.

20.18 Provenance and Property

Property claims can similarly depend upon acquisition history.

A present possessor and a historical claimant can invoke different provenance structures.

Historical repair can therefore require more than current possession analysis.

The legal resolution of such claims introduces doctrines and policy considerations beyond the formal provenance object.

20.19 Provenance and Institutions

Institutional decisions also possess provenance.

A disciplinary classification can derive from:

- one report;
- derivative reports;
- automated propagation;
- later reviews;
- a final correction.

Correction of the terminal record can leave the derivational history unchanged.

Provenance-preserving correction therefore differs from historical erasure.

20.20 Record States

Let institutional record at time t be

$$Z_t. \quad (1686)$$

A record can move through states such as:

$$Z^{\text{claim}} \rightarrow Z^{\text{verified}} \rightarrow Z^{\text{corrected}} \rightarrow Z^{\text{superseded}}. \quad (1687)$$

Equation 1687 preserves historical revision rather than replacing every prior state with the latest state.

This distinction is important for both accountability and present accuracy.

20.21 Current Validity and Historical Existence

A historical claim can have existed without remaining valid.

Let z denote an earlier classification.

One can represent

$$z \in \mathcal{H} \quad (1688)$$

and

$$z \notin \mathcal{A}_{\text{current}}, \quad (1689)$$

where $\mathcal{A}_{\text{current}}$ contains presently operative classifications.

Equations 1688 and 1689 permit historical preservation without continuing operational harm.

20.22 Correction Without Erasure

Correction can preserve both the earlier error and the later supersession event.

Let

$$\mathcal{H}_R = (\dots, z_{\text{error}}, e_{\text{correction}}, z_{\text{current}}). \quad (1690)$$

Equation 1690 supports accountability and present accuracy simultaneously.

This architecture can be preferable to deleting every trace of the error.

20.23 Erasure

Historical erasure removes some record, memory, or trace from the represented system.

Let erasure operator be

$$\mathcal{E} : \mathcal{H} \longrightarrow \mathcal{H}^-. \quad (1691)$$

Equation 1691 can increase similarity to a no-harm reference when the reference lacks the harmful event.

The same operation can reduce:

- accountability;
- learning;
- historical recognizability;
- evidence;

- narrative continuity;
- future contestability.

Erasure therefore has ambiguous restorative status.

20.24 Erasure and Counterfactual Fidelity

Suppose historical distance is d_H . An erasure operation can satisfy

$$d_H(\mathcal{E}(\mathcal{H}_R), \mathcal{H}_{\text{ref}}) < d_H(\mathcal{H}_R, \mathcal{H}_{\text{ref}}). \quad (1692)$$

Equation 1692 means erasure increases historical resemblance.
It does not follow that justice increases.

20.25 Historical Recognizability

Historical recognizability concerns whether future actors can identify that an event occurred and understand its relation to later states.

Let

$$R_H(\mathcal{H}) \quad (1693)$$

denote the recognizability profile of the history.

Equation 1693 can depend upon:

- record preservation;
- provenance;
- public access;
- interpretability;
- correction metadata;
- archival integrity.

A perfectly erased history can have low R_H .

20.26 Recognizability and Justice

Historical recognizability can serve several normative functions:

- accountability;
- prevention of recurrence;
- recognition;
- historiography;
- legal review;
- institutional learning.

It can conflict with privacy, rehabilitation, or legitimate rights to limited disclosure.
Historical preservation therefore requires procedural and normative qualification.

20.27 Memory

Memory is another historical object.

Let individual memory structure be

$$M_A(t). \quad (1694)$$

Equation 1694 can include autobiographical memory, learned expectations, emotional associations, and remembered relational events.

Memory should not be reduced automatically to explicit propositional recall.

20.28 Memory Identity

A hypothetical memory-identical restoration would require

$$M_{A,R} = M_{A,\text{ref}}. \quad (1695)$$

Equation 1695 is an extreme counterfactual-fidelity criterion.

Where the actual subject experienced harm and the reference subject did not, achieving Equation 1695 could require deletion or rewriting of actual memory.

This raises major identity and procedural concerns.

20.29 Memory Correspondence

A weaker relation can preserve functional or narrative continuity without exact content identity:

$$M_{A,R} \sim_{E_M} M_{A,\text{ref}}. \quad (1696)$$

Equation 1696 can be meaningful in idealized thought experiments.

Its substantive interpretation depends upon the theory of memory and identity.

20.30 Memory and Learning

Memory can be a condition of learning.

Removing memory of harm can remove:

- knowledge of institutional risk;
- evidence;
- acquired caution;
- moral understanding;
- historical testimony.

A memory-erasure intervention can therefore increase counterfactual similarity and reduce epistemic capacity.

20.31 Memory and Agency

The subject can also value retaining memory as part of self-understanding.

A restorative process that alters memory to reproduce the counterfactual subject can therefore violate present agency.

This is among the clearest examples of conflict between strong restorative fidelity and procedural justice.

20.32 Collective Memory

Collective memory can be represented as a distributed historical object:

$$M_C = M_C(\mathcal{A}, \mathcal{R}, Z, N), \quad (1697)$$

where $\mathcal{A}$ denotes actors, $\mathcal{R}$ relations, Z records, and N public narratives.

Equation 1697 emphasizes that collective memory is not one centralized database.

It emerges through institutions, archives, education, testimony, ritual, and public discourse.

20.33 Collective Memory and Repair

Collective reparations can intentionally change public memory through:

- acknowledgment;
- archives;
- memorials;
- education;
- official correction;
- public testimony.

These interventions alter historical interpretation without changing the past events themselves.

20.34 Narrative Structure

A narrative organizes events into relations of significance.

Let

$$N_A = \mathcal{N}(\mathcal{H}_A) \quad (1698)$$

map a subject's history into a narrative structure.

Equation 1698 does not imply that narrative is a deterministic function of objective events.

Interpretation, culture, language, and later experience enter narrative formation.

Ricoeur's account of selfhood and narrative provides a relevant philosophical background for treating identity as historically mediated [49].

20.35 Narrative Continuity

Narrative continuity concerns whether a subject can integrate earlier and later experience into a coherent enough account of self.

Let

$$C_N(N_A) \quad (1699)$$

denote a qualitative or model-specific continuity relation.

The paper does not assume that greater narrative coherence is always better. Ambivalence, rupture, and revision can be legitimate features of lived history.

20.36 Narrative Equivalence

A historical comparison can ask whether

$$N_{A,R} \sim_{E_N} N_{A,\text{ref}}. \quad (1700)$$

Equation 1700 is weaker than event-history identity and strongly theory-dependent.

Its value lies in making narrative a distinct object rather than assuming it follows from state restoration.

20.37 Narrative Transformation

A restorative process can generate a new legitimate narrative:

$$N_{A,H} \xrightarrow{R} N_{A,R}. \quad (1701)$$

Equation 1701 can represent movement from unrecognized harm toward acknowledged history and future-directed agency.

The new narrative need not approximate the no-harm reference narrative.

20.38 Recognition and Historical Meaning

Recognition can change the social meaning of the same historical event.

An injury privately endured and an injury publicly acknowledged have identical event occurrence and different recognition structures.

Recognition theory therefore gives a reason to distinguish historical fact from social meaning [25, 20].

20.39 Recognition Events

An apology, judgment, memorial, official correction, or public acknowledgment is itself a new historical event.

Let

$$e_{\text{rec}} \quad (1702)$$

denote such an event.

Equation 1702 alters the post-harm history rather than deleting the original event.

Recognition therefore increases historical structure even while repairing social meaning.

20.40 Satisfaction and Historical Addition

Measures commonly associated with satisfaction in reparative practice can similarly add:

- acknowledgment;
- truth finding;
- apology;
- memorialization;
- public correction.

These measures do not recreate the no-harm world. They construct a new historical relation to the wrong [65, 12].

20.41 The Restorative Event

Every restorative intervention is itself an event in the actual history.

Let

$$e_R = \text{Event}(R). \quad (1703)$$

The post-restoration history is then

$$\mathcal{H}_R = \mathcal{H}_H \cup \{e_R\} \cup \mathcal{H}_{\text{post}}. \quad (1704)$$

Equation 1704 formalizes the proposition that repair enters history rather than standing outside it.

20.42 Restoration-Event Paradox

Suppose complete restoration were defined as exact identity with the no-harm history.

The restorative event e_R is absent from the no-harm reference and present in the actual restored history.

Thus

$$e_R \in \mathcal{H}_R \quad \text{and} \quad e_R \notin \mathcal{H}_{\text{ref}}. \quad (1705)$$

Equation 1705 creates a structural obstacle to exact historical identity.

Any restoration that occurs becomes evidence that the histories differ.

20.43 Restoration without Historical Erasure

This paradox suggests another model of successful repair.

Justice can aim for:

$$\text{acknowledged harm} + \text{effective repair} + \text{future generativity} \quad (1706)$$

rather than historical disappearance of the harm.

Equation 1706 is conceptual rather than algebraic.

Its significance lies in treating historical difference as compatible with successful justice.

20.44 Historical Transformation

A restorative process can transform the meaning and consequences of history without making the history identical to the reference.

Let historical interpretation and later consequences be represented by

$$\mathcal{H} \xrightarrow{R} \mathcal{H}'. \quad (1707)$$

Equation 1707 preserves the fact of the earlier event while changing its subsequent relational and institutional place.

This is different from reversal.

20.45 Historical Supersession

A later event can supersede the operational authority of an earlier event.

Let

$$e_1 \prec_{\text{sup}} e_2 \quad (1708)$$

mean e_2 supersedes e_1 for a specified institutional purpose.

The earlier event remains historically real.

This relation is useful for corrected classifications, overturned judgments, and revised institutional records.

20.46 Historical Persistence

Some consequences of an event persist even after formal supersession.

Let effect vector be Y_t . Then

$$e_1 \prec_{\text{sup}} e_2 \quad (1709)$$

can coexist with

$$Y_{t>e_2}^{(e_1)} \neq 0. \quad (1710)$$

Equations 1709 and 1710 distinguish correction of authority from elimination of consequences.

20.47 Historical Residuals

A historical residual is a difference remaining after selected substantive restorative objects have been restored.

Let the selected restorative bundle be $\mathcal{B}_C$.

Residual historical difference is represented by

$$\Delta_H = \mathcal{H}_R \setminus_{E_H} \mathcal{H}_{\text{ref}}, \quad (1711)$$

where $\setminus_{E_H}$ denotes difference relative to the chosen historical relation rather than literal set subtraction.

Equation 1711 provides a conceptual location for memory, lost time, irreversible events, and the fact of repair.

20.48 Residual Historical Value

A historical residual need not be purely negative.

The actual restored history can contain:

- learning;
- solidarity;
- new institutions;
- new relationships;
- public acknowledgment;
- legal precedent.

These later goods do not justify the earlier harm.

They demonstrate that actual history can acquire new value while remaining historically distinct from the no-harm world.

20.49 Better Present and Worse History

It can hold that

$$V(\mathcal{G}_{R,t_1}) > V(\mathcal{G}_{\text{ref},t_1}) \quad (1712)$$

while

$$L_{\text{historical}} > 0. \quad (1713)$$

Equations 1712 and 1713 show that present improvement and historical rectification are independent. A better present does not retroactively make the original harm just.

20.50 Worse Present and Richer Recognition

The reverse can also occur.

A process can achieve truthful historical recognition while present material conditions remain poor. Recognition is therefore one restorative dimension rather than a substitute for material repair.

20.51 Historical Fidelity

Let historical-fidelity functional be

$$F_H(\mathcal{H}_R, \mathcal{H}_{\text{ref}}). \quad (1714)$$

Equation 1714 can measure or classify similarity under a selected historical representation.

Greater F_H should not be assumed to imply greater justice.

The actual historical subject can value some post-harm developments that a counterfactual-replication intervention would remove.

20.52 Historical Fidelity and Present Agency

Suppose intervention R_2 produces greater historical fidelity than R_1 :

$$F_H(R_2) > F_H(R_1). \quad (1715)$$

The present subject can nevertheless prefer R_1 :

$$P_A(R_1) \succ P_A(R_2). \quad (1716)$$

Equations 1715 and 1716 express a conflict between counterfactual fidelity and current agency.

20.53 Historical Fidelity and Identity Transformation

Long histories of harm and adaptation can alter commitments and identity.

A perfect reconstruction of the no-harm path can therefore require replacement of characteristics now integral to the actual subject.

This makes historical fidelity increasingly normatively problematic as the required intervention moves from external conditions into memory, relationships, and self-conception.

20.54 Perfect Replica Thought Experiment

Consider an ideal technology able to construct a subject A^* whose observable behavior, internal represented state, memories, preferences, and future causal dynamics match a modeled no-harm subject.

The replica can satisfy

$$Q_i(A^*) = Q_i(A^C) \quad (1717)$$

for every modeled object Q_i .

Equation 1717 creates extremely strong representational equivalence.

It leaves open whether producing A^* constitutes restoration of the historical subject A^H , replacement of that subject, or creation of another subject satisfying the comparison model.

The question cannot be resolved by formal equivalence alone.

20.55 Memory-Erasure Thought Experiment

Consider an intervention that removes all memories of the harm and restores every modeled external condition.

Let memory-erasure operator be $\mathcal{E}_M$. Suppose

$$M(\mathcal{E}_M(A^H)) = M(A^C). \quad (1718)$$

Equation 1718 increases counterfactual memory fidelity.

The intervention can simultaneously remove testimony, learned knowledge, and the actual subject's chosen relation to the event.

This thought experiment demonstrates the independence of fidelity and justice.

20.56 Record-Erasure Thought Experiment

Suppose every institutional record of harm is deleted.

The resulting archive can resemble the archive of a no-harm world.

Yet the deletion can eliminate:

- evidence;
- accountability;
- correction provenance;
- institutional learning;
- future historical knowledge.

Record erasure can therefore improve one historical-similarity metric while damaging another historical value.

20.57 Third-Party Memory

Even if the subject's memory and records were altered, third parties can retain memory of the event.

Let memories of actors be

$$\{M_1, M_2, \dots, M_n\}. \quad (1719)$$

Equation 1719 makes historical memory relational and distributed.

Exact social historical reconstruction can therefore require intervention upon many persons.

This substantially increases the procedural and ethical cost of strong historical equivalence.

20.58 Distributed Historical State

The social history of one subject is partly stored in:

- other persons' memories;
- institutional records;
- network relations;
- public narratives;
- material traces;
- legal documents.

Let distributed historical state be

$$H^D = (M_{1:n}, Z_{1:m}, \mathcal{R}, N, P). \quad (1720)$$

Equation 1720 shows why history cannot be treated as one subject's internal state alone.

20.59 Distributed Historical Equivalence

A strong distributed criterion would require

$$H_R^D \sim_E H_{\text{ref}}^D. \quad (1721)$$

Equation 1721 can become practically and normatively impossible because it requires extensive control over third-party states.

This is one reason exact social historical equivalence should not be treated as a default restorative ideal.

20.60 Historical Entanglement

A harmful event can become entangled with later events.

Let

$$e_H \rightarrow e_1 \rightarrow e_2 \rightarrow \dots \rightarrow e_n. \quad (1722)$$

Equation 1722 represents downstream historical dependence.

Removing e_H from the representation without revising later events can produce an incoherent history.

20.61 Counterfactual Historical Reconstruction

A complete no-harm historical reconstruction therefore requires generating an alternative sequence

$$\mathcal{H}^{-H} = (e_0, e_1^{-H}, e_2^{-H}, \dots). \quad (1723)$$

rather than subtracting e_H from the actual history.

Equation 1723 formalizes the earlier principle that

$$\mathcal{H}^{-H} \neq \mathcal{H}^H - H. \quad (1724)$$

Path dependence and increasing returns provide established reasons that removal of one earlier event can generate a different later path [3, 45].

20.62 Counterfactual Historical Branching

The no-harm world can contain several plausible histories.

Let

$$\mathfrak{H}^{\neg H} = \{\mathcal{H}^{\neg H,1}, \mathcal{H}^{\neg H,2}, \dots\}. \quad (1725)$$

Equation 1725 represents counterfactual historical plurality.

The restorative reference can therefore be a family rather than one exact lost history.

20.63 Historical Bounds

Where precise reconstruction is unsupported, the theory can specify historical constraints rather than one trajectory.

Let admissible historical family satisfy

$$\mathfrak{H}^{\neg H} \subseteq \mathfrak{H}_{\text{plausible}}. \quad (1726)$$

Equation 1726 supports bounded inference.

A reparative claim can remain legitimate even where the exact no-harm biography cannot be identified.

20.64 Historical Robustness

A restorative conclusion is historically robust where it holds across several plausible counterfactual histories.

Let intervention R satisfy

$$\mathcal{N}(R, \mathcal{H}^{\neg H,i}) \geq N_{\min} \quad \forall \mathcal{H}^{\neg H,i} \in \mathfrak{H}^{\neg H}. \quad (1727)$$

Equation 1727 is schematic.

Its significance lies in favoring responses justified across plausible histories.

20.65 Historical Uncertainty and Present Need

Uncertainty about exact history does not eliminate knowledge of present need.

A subject can require healthcare, income, legal correction, or assistance even when long-run counterfactual history is uncertain.

This distinction prevents historical uncertainty from becoming a reason for institutional paralysis.

20.66 Historical Uncertainty and Recognition

Recognition can also be justified at different epistemic levels.

An institution can acknowledge a proven wrongful act without claiming knowledge of every downstream consequence.

This allows calibrated recognition.

20.67 Historical Uncertainty and Compensation

Compensation can incorporate uncertainty through:

- intervals;
- probabilistic loss;
- standardized schedules;

- evidentiary presumptions;
- partial claims;
- ex gratia support.

The formal atlas does not determine which legal mechanism is appropriate. It clarifies the uncertainty to which the mechanism responds.

20.68 Historical Knowledge and Power

The historical record is produced under unequal conditions.

Some actors possess:

- greater ability to document;
- greater institutional credibility;
- control over archives;
- legal expertise;
- authority to classify.

Historical recognizability is therefore partly governed by power.

The justice of return must attend to which histories remain knowable.

20.69 Archive Structure

Let archive be

$$\mathcal{A}_H = (D, P, M, A), \quad (1728)$$

where D denotes documents, P provenance, M metadata, and A access rules.

Equation 1728 makes the archive an institutional object rather than a passive container.

20.70 Archive Access

A historical record can exist and remain practically inaccessible.

Let

$$A_{\text{hist}}(q) \quad (1729)$$

denote access to historical object q .

Historical justice therefore concerns preservation and accessibility.

Privacy and legitimate confidentiality constrain access.

20.71 Archive Power

Control over archival inclusion, metadata, correction, and access can shape future historical knowledge.

A restorative archive should therefore preserve:

- correction provenance;
- contested status where appropriate;
- source distinction;

- temporal metadata;
- access controls;
- appeal mechanisms.

This architecture reduces the risk that one institutional classification becomes a permanent historical fact through repetition.

20.72 Historical Data Minimization

Preservation does not imply unlimited retention of every personal datum.

A just historical architecture can retain:

what is necessary for accountability and provenance (1730)

while restricting

unnecessary personally invasive detail. (1731)

These objectives can coexist.

20.73 Historical Truth and Representation

The archive is a representation of history, not history itself.

Let

$$\Pi_H : \mathcal{H} \longrightarrow \hat{\mathcal{H}} \quad (1732)$$

denote the mapping from events into the institutional historical representation.

Equation 1732 can omit, classify, compress, or reinterpret events.

Historical-restoration claims should therefore distinguish events from their records.

20.74 Record Equality and Historical Non-Equality

Two record systems can become identical:

$$Z_R = Z_{\text{ref}} \quad (1733)$$

while actual histories remain different:

$$\mathcal{H}_R \neq \mathcal{H}_{\text{ref}}. \quad (1734)$$

Equations 1733 and 1734 show that record restoration is not historical identity.

20.75 Record Difference and Historical Truth

Conversely, a truthful archive can intentionally preserve the difference between the actual and no-harm histories.

The archive can state:

The harmful classification existed, was later found invalid, and was superseded.

This representation is historically less similar to a no-harm archive and more accurate concerning actual history.

20.76 Reflexive Records

Records do more than describe.

A classification can change eligibility. A judgment can alter reputation. A diagnosis can alter treatment. A declaration of successful restoration can terminate support.

Thus

$$Z_t \rightarrow \mathcal{G}_{t+1}. \quad (1735)$$

Equation 1735 makes the record causally endogenous.

20.77 Reflexive Classification

Let classification operator be

$$Z_t = C(\mathcal{G}_t). \quad (1736)$$

If later state depends upon the classification,

$$\mathcal{G}_{t+1} = F(\mathcal{G}_t, Z_t), \quad (1737)$$

then the classification enters the process it classifies.

Equation 1737 gives a generic form of reflexive classification.

Merton's discussion of self-fulfilling prophecy and Hacking's work on looping effects provide important precedents for such feedback between classification and social reality [36, 23].

20.78 Self-Fulfilling Restorative Classification

A positive classification can produce the condition it reports.

If an institution declares that a subject is fully restored, colleagues can resume ordinary interaction, thereby increasing actual restoration.

Thus

$$Z_t^{\text{restored}} \rightarrow R_{t+1}^{\text{actual}}. \quad (1738)$$

Equation 1738 shows that evaluation can have constructive causal effects.

20.79 Self-Defeating Restorative Classification

The same declaration can terminate support too early.

Then

$$Z_t^{\text{restored}} \rightarrow \text{support withdrawal} \rightarrow \text{future deterioration}. \quad (1739)$$

Equation 1739 shows that the classification can undermine the condition it reports.

The direction of reflexive effect is therefore not fixed.

20.80 Classification-Induced Lock-In

A restorative label can also constrain future identity.

A subject classified as permanently vulnerable can remain eligible for support while becoming institutionally trapped in a dependent category.

A subject classified as fully recovered can lose access to needed support.

Reflexive justice therefore concerns the effects of categories as well as their accuracy.

20.81 Measurement Reflexivity

Measurement itself can alter behavior.

Repeated assessment can influence:

- self-presentation;
- institutional decisions;
- subject attention;
- expectations;
- allocation of resources.

A restorative metric is therefore not necessarily observationally neutral.

20.82 Metric-Induced Optimization

Once a metric determines success, actors can optimize toward the metric.

Let reported restorative score be

$$S_R = M(Q_1, \dots, Q_n). \quad (1740)$$

Institutions can then alter behavior to maximize S_R , potentially at the expense of omitted objects.

This is another reason the atlas favors plural profiles over one universal score.

20.83 Goodhart-Type Distortion

A restorative indicator used as a target can lose validity as a measure of the underlying object.

The paper uses this as a general governance caution without relying upon one particular formal version of Goodhart's law.

Examples include:

- optimizing return-to-work rate while ignoring job quality;
- maximizing compensation speed while reducing participation;
- maximizing formal reinstatement while suppressing complaints;
- maximizing short-run output while depleting long-run generativity.

20.84 Evaluation as Intervention

Where evaluation changes the system, it can be represented as an intervention:

$$\mathcal{G}_{t+1} = F(\mathcal{G}_t, E_t), \quad (1741)$$

where E_t is the evaluative act.

Equation 1741 means evaluation belongs inside the causal and generative models.

20.85 Observer Inclusion

A reflexive model can include the observing institution as part of the system:

$$\tilde{\mathcal{G}} = (\mathcal{G}, I_{\text{obs}}). \quad (1742)$$

Equation 1742 broadens the system boundary.

The observer's categories, incentives, and interventions then become endogenous.

20.86 Recursive Observation

The observer can also observe the effects of its earlier observation:

$$\mathcal{G}_t \rightarrow Z_t \rightarrow \mathcal{G}_{t+1} \rightarrow Z_{t+1}. \quad (1743)$$

Equation 1743 generates a recursive observer–system relation.

No one assessment can therefore be assumed to remain permanently external.

20.87 Reflexive Non-Closure

The broader structure can be represented as

$$\mathcal{G}_{t+1} = F(\mathcal{G}_t, \mathcal{O}(\mathcal{G}_t), R_t), \quad (1744)$$

where $\mathcal{O}$ is observation or classification and R_t the institutional response generated partly from that observation.

Equation 1744 defines the paper's *reflexive non-closure* condition at a general level.

The description, decision, and remedy become part of the next state of the system.

20.88 Complete Externality

A fully external observer would require

$$\frac{\partial \mathcal{G}_{t+1}}{\partial \mathcal{O}(\mathcal{G}_t)} = 0. \quad (1745)$$

Equation 1745 expresses non-intervention of the observation in an idealized differentiable setting.

Many institutional restorative systems fail this condition because records and classifications influence later treatment.

20.89 Representational Completeness

Let representation map be

$$\Pi : \mathcal{G} \longrightarrow \hat{\mathcal{G}}. \quad (1746)$$

Complete representation would require preservation of every distinction relevant to the selected inquiry.

In open-ended social systems, such completeness can be unattainable or undefined because future relevance itself can change.

20.90 Completeness, Externality, and Intervention

The paper therefore does not assume that one institutional representation can simultaneously satisfy:

1. exhaustive representational completeness;
2. full externality from the represented system;
3. practical intervention upon that system.

The point is methodological rather than a general impossibility theorem.

In reflexive social systems, richer observation often increases the chance that the representation itself becomes causally consequential.

20.91 Reflexive Measurement Burden

More detailed restoration measurement can itself impose burden.

The subject can be required repeatedly to:

- document harm;
- explain relationships;
- undergo assessment;
- disclose private information;
- demonstrate continuing impairment.

Measurement can therefore reduce the very generativity the process seeks to evaluate.

20.92 Measurement-Induced Harm

Let evaluation burden be B_E . The net restorative effect can be represented schematically by

$$\Delta V_{\text{net}} = \Delta V_{\text{restoration}} - B_E. \quad (1747)$$

Equation 1747 is conceptual.

It emphasizes that epistemic refinement can carry substantive cost.

20.93 Reflexive Minimalism

A reflexively aware institution should therefore collect and operationalize only the historical information needed for the legitimate task.

This principle supports:

- bounded data collection;
- purpose limitation;
- explicit retention rules;
- local processing where appropriate;
- subject access and correction;
- separation of historical preservation from current eligibility.

The aim is sufficient historical knowledge without converting repair into permanent surveillance.

20.94 Historical and Reflexive Scales

Historical structure operates at several scales:

- individual;
- relational;
- organizational;
- community;
- national;

- intergenerational.

Let

$$\mathcal{H}^{(s)} \quad (1748)$$

denote history at scale s .

Individual restoration can alter organizational history, and organizational recognition can alter individual narrative.

Historical equivalence is therefore scale-relative.

20.95 Individual and Institutional History

The subject can experience full personal closure while the institution retains a legitimate reason to preserve the case for future learning.

Conversely, an institution can declare a case closed while the subject's historical process remains open.

Closure is therefore role- and scale-specific.

20.96 Intergenerational Historical Structure

Historical events can affect later generations through:

- resource transmission;
- cultural transmission;
- institutional structure;
- environmental condition;
- public narratives;
- legal status.

The later generations did not experience the original event directly, yet their generative conditions can depend upon it.

Historical justice therefore extends beyond autobiographical memory.

20.97 Intergenerational Provenance

Let present condition $C^{(g)}$ have provenance

$$\text{Prov} \left(C^{(g)} \right) = \mathcal{P}^{(0:g)}. \quad (1749)$$

Equation 1749 records cross-generational production of the current condition.

Equal current holdings can therefore possess different intergenerational histories.

20.98 Historical Responsibility Across Generations

Questions of historical responsibility involve more than causal continuity.

They can include:

- institutional continuity;
- inherited benefit;

- ongoing structure;
- political membership;
- present capacity to repair.

The literature on historical injustice shows the complexity of relating past wrong to present obligation [67, 62].

The formal history object cannot resolve these normative questions by itself.

20.99 Historical Supersession and Continuing Structure

A past claim can weaken under changed circumstances while some structural effects continue.

This is why historical injustice cannot be reduced either to permanent unchanged entitlement or to automatic expiration through time.

The relevant object includes the continuing generative relation between past and present.

20.100 Historical Transformation Across Generations

A reparative process can transform intergenerational transmission without reconstructing the original world.

For example, it can alter:

- educational opportunity;
- asset control;
- ecological inheritance;
- cultural transmission;
- institutional participation.

The appropriate comparison can therefore concern future transmission mechanisms rather than literal return to an ancestral state.

20.101 Historical Equivalence and Collective Identity

Collective identity can persist despite member turnover.

Historical comparison can therefore concern institutions, communities, or political bodies whose current members differ from historical members.

The relevant continuity criterion must be specified rather than assumed.

Possible criteria include:

- legal continuity;
- institutional function;
- cultural continuity;
- territorial continuity;
- governance continuity;
- self-identification.

Different theories can select different criteria.

20.102 Historical Relationality

Historical identity is relational as well as temporal.

A subject's history includes relations with other persons, institutions, and environments.

Restoration of an isolated internal state cannot therefore reproduce a social history unless those relations are included.

This is one reason exact historical reconstruction rapidly becomes a multi-party problem.

20.103 Third-Party Historical Agency

Third parties possess their own histories and present agency.

Reconstructing a prior relationship can require another person to resume a role they no longer choose.

The reference history therefore cannot acquire automatic authority over third-party futures.

This places a principled limit on relational historical fidelity.

20.104 Historical Fidelity and Consent

Where historical reconstruction requires present action from several persons, consent constraints can prevent exact reconstruction.

This is not merely a practical limitation.

It can be a requirement of justice.

A less historically faithful future can therefore be more procedurally legitimate.

20.105 Historical Fidelity and Privacy

Complete reconstruction of distributed history can require extensive access to:

- private correspondence;
- health records;
- social networks;
- third-party memories;
- institutional archives.

Historical fidelity therefore has information costs.

Privacy can impose legitimate limits on historical completeness.

20.106 Historical Fidelity and Dignity

A restorative process can force subjects repeatedly to inhabit or reproduce the harmful history.

Extensive reenactment, disclosure, or proof can undermine dignity.

Historical truth and historical procedure should therefore remain separate objects.

20.107 Historical Fidelity and the Right to Move Forward

The subject can possess a legitimate interest in reducing the present institutional salience of the harmful event.

This does not require denying that the event occurred.

A historical architecture can therefore preserve evidence while limiting the event's continuing control over present identity and eligibility.

20.108 Historical Persistence and Present Non-Operativity

Let event e_H remain in history:

$$e_H \in \mathcal{H}. \quad (1750)$$

At the same time, it can be removed from current decision rules:

$$e_H \notin \mathcal{D}_{\text{current}}. \quad (1751)$$

Equations 1750 and 1751 provide a formal distinction between historical preservation and present operational exclusion.

20.109 Right to Historical Correction

Where a record is false, restoration can require correction.

Correction should distinguish:

- deletion of false current status;
- preservation of correction provenance;
- propagation of the correction;
- protection against reuse of superseded information.

These are separate historical and operational objects.

20.110 Correction Propagation

Let the original record propagate across network $\mathcal{N}_Z$.

A correction function can be represented by

$$C_Z : \mathcal{N}_Z \longrightarrow \{0, 1\}, \quad (1752)$$

where $C_Z(v) = 1$ indicates successful correction at node v .

Equation 1752 makes distributed correction a separate object.

Correcting only the source node can be insufficient.

20.111 Historical Consistency Across Records

Multiple institutional records can contradict one another.

Let

$$Z_1, Z_2, \dots, Z_n \quad (1753)$$

be relevant records.

A consistency relation can require

$$Z_i \sim_{\text{current}} Z_j \quad \forall i, j \quad (1754)$$

for current operative status.

Historical metadata can remain different while current classifications agree.

20.112 Historical Versioning

Versioned records can preserve change over time:

$$Z^{(0)} \rightarrow Z^{(1)} \rightarrow \dots \rightarrow Z^{(n)}. \quad (1755)$$

Equation 1755 allows current truth and historical provenance to coexist.

This architecture is often preferable to silent overwriting.

20.113 Historical Ledger

A historical ledger can record events such as:

- claim;
- evidence;
- decision;
- correction;
- appeal;
- supersession.

The ledger need not be immutable in the sense of preventing lawful correction. Its value lies in preserving transparent provenance among versions.

20.114 Immutability and Justice

Technical immutability can preserve evidence and can preserve harmful error.

A just record system therefore requires a distinction between:

immutability of provenance (1756)

and

immutability of substantive classification. (1757)

The first can support accountability. The second can prevent correction.

20.115 Historical Deletion and Legal Rights

In practical information governance, rights of deletion, correction, archival preservation, and public access can conflict.

The formal framework does not determine their legal resolution.

It clarifies that deletion and correction correspond to different historical operations.

20.116 Historical Narrative and Institutional Narrative

The subject and institution can maintain different narratives of the same event.

Let

$$N_A \neq N_I. \quad (1758)$$

Equation 1758 does not imply that every factual question is indeterminate.

It means interpretation and salience can differ even after some factual issues are resolved.

20.117 Narrative Plurality

A just historical system can sometimes preserve several documented perspectives rather than force one exhaustive narrative.

This is especially relevant where:

- factual evidence is incomplete;
- value interpretations differ;
- several communities experienced the event differently.

Narrative plurality requires safeguards against treating demonstrably false claims as equally supported.

20.118 Historical Fact and Historical Interpretation

The framework distinguishes:

F_H (1759)

for historical factual claims and

I_H (1760)

for interpretation.

The two interact while remaining analytically separable.

A restorative institution can correct a factual record without prescribing one complete meaning of the subject's life.

20.119 Historical Meaning and Future Generativity

Interpretation of the past can alter future action.

A history understood as evidence of permanent incapacity can reduce generativity.

The same history understood as one event within an open future can have another effect.

The institution should avoid imposing a mandatory narrative of resilience, victimhood, or recovery.

20.120 Reflexive Recognition

Recognition can itself change the identity or standing it recognizes.

Public acknowledgment can alter social response. Legal vindication can change self-understanding.

Institutional apology can alter trust.

Recognition is therefore not merely a measurement of standing.

It participates in its production.

20.121 Reflexive Compensation

Compensation can similarly change later claims and expectations.

A payment can:

- increase material security;
- signal acknowledgment;
- close legal proceedings;

- change public interpretation;
- alter future institutional obligations.

Compensation is therefore both transfer and historical event.

20.122 Reflexive Reinstatement

Reinstatement changes more than formal status.

It can signal institutional reversal and alter colleagues' beliefs.

It can also reintroduce the subject into the relational environment associated with the original harm.

The same formal remedy can therefore generate different reflexive effects.

20.123 Reflexive Assistance

Assistance can generate identities such as beneficiary, patient, vulnerable person, or supported worker.

These classifications can enable resources and can create stigma or dependency.

A reflexively aware design should minimize unnecessary identity fixation.

20.124 Reflexive Prevention

Preventive classification can also change the risk it attempts to predict.

Labeling a person or institution high-risk can trigger monitoring, exclusion, or resource withdrawal that changes subsequent behavior.

Preventive models therefore require special caution where classification affects the outcome.

20.125 Reflexive Historical Baselines

The selected reference history can itself influence the restorative process.

If an institution declares one counterfactual biography the official baseline, the subject can be pressured toward that path.

The baseline therefore becomes performative.

Reference selection is consequently part of the reflexive system.

20.126 Baseline-Induced Path Dependence

Let selected reference be B . Institutional assistance can then be conditioned upon conformity to B :

$$A_t = A(x_t, B). \quad (1761)$$

Equation 1761 can make the estimated counterfactual future causally influential.

This creates a new path dependence generated by the restoration system itself.

20.127 Reflexive Counterfactual Paternalism

Counterfactual paternalism becomes reflexive when the modeled no-harm future is used to shape the actual subject until the model appears increasingly accurate.

The institution can then interpret the convergence it produced as evidence that the original model was correct.

This feedback should be distinguished from independent validation.

20.128 Reflexive Model Validation

Let prediction be P_t , intervention based upon that prediction be $R(P_t)$, and later observation Y_{t+1} .
If

$$P_t \rightarrow R(P_t) \rightarrow Y_{t+1}, \quad (1762)$$

then agreement between P_t and Y_{t+1} can partly result from the intervention.

Equation 1762 shows why predictive success does not automatically validate an externally independent model.

20.129 Reflexive Governance

A reflexive governance architecture should therefore support:

- model revision;
- provenance;
- subject contestation;
- separation of description and eligibility where possible;
- bounded data retention;
- periodic review of classification effects.

The purpose is to govern the feedback loop rather than pretend it does not exist.

20.130 Reflexive Revision

Let model or classification rule at time t be M_t .

A reflexive update is

$$M_{t+1} = \mathcal{U}(M_t, D_{t+1}, E_{t+1}), \quad (1763)$$

where D_{t+1} contains new evidence and E_{t+1} observed effects of the earlier model.

Equation 1763 makes institutional learning part of the architecture.

20.131 Reflexive Audit

An audit of a restorative system can ask:

- which classifications affected later outcomes;
- whether observed success was partly model-induced;
- whether omitted subjects disappeared from measurement;
- whether support termination affected measured recovery;
- whether the metric created gaming or strategic behavior.

Such auditing extends evaluation beyond predictive accuracy.

20.132 Historical and Reflexive Equivalence

A combined historical-reflexive object can be represented by

$$Q_{HR}(\mathcal{G}) = (\mathcal{H}, \mathcal{P}, M, N, Z, \mathcal{O}, \mathcal{F}_{\text{reflex}}), \quad (1764)$$

where:

- $\mathcal{H}$ denotes historical event structure;
- $\mathcal{P}$ provenance;
- M memory structure;
- N narrative structure;
- Z records and classifications;
- $\mathcal{O}$ observation/evaluation structure;
- $\mathcal{F}_{\text{reflex}}$ reflexive feedback.

Equation 1764 defines the strongest compound historical object used in the atlas.

20.133 Exact Historical-Reflexive Identity

The strongest represented criterion is

$$Q_{HR}(\mathcal{G}^R) = Q_{HR}(\mathcal{G}^{\text{ref}}). \quad (1765)$$

Equation 1765 would require identity of modeled history, provenance, memory, narratives, records, and reflexive feedback.

This is an extreme formal benchmark.

For actual harmed-and-restored systems, it will generally be structurally unavailable or normatively undesirable.

20.134 Historical-Reflexive Isomorphism

A weaker criterion can preserve the represented architecture under a suitable bijection:

$$Q_{HR,R} \cong Q_{HR,\text{ref}}. \quad (1766)$$

Equation 1766 remains demanding because reflexive feedback can generate additional events absent from the reference.

20.135 Historical-Reflexive Simulation

A directional relation can ask whether the restored system can reproduce reference historical functions without requiring complete identity.

For example, it can preserve:

- evidentiary accountability;
- correction capacity;
- recognition;
- future learning;
- subject agency.

This relation can be more useful than strict historical equivalence for actual repair.

20.136 Historical-Reflexive Sufficiency

A sufficiency criterion can require the history to satisfy a set of minimum conditions:

$$Q_{HR,R} \in \mathcal{S}_{HR}. \quad (1767)$$

Equation 1767 can include:

- truthful current records;
- preserved correction provenance;
- adequate recognition;
- privacy protection;
- future contestability;
- institutional learning.

This can be a more realistic institutional target than historical identity.

20.137 Historical-Reflexive Dominance

A transformed system can be historically less similar and normatively superior.

Let value vector include

$$\mathbf{V}_{HR} = (V_{\text{truth}}, V_{\text{recognition}}, V_{\text{agency}}, V_{\text{privacy}}, V_{\text{learning}}, V_{\text{future}}). \quad (1768)$$

A dominance relation is

$$\mathbf{V}_{HR}(Q_{HR,R}) \succeq \mathbf{V}_{HR}(Q_{HR,\text{ref}}). \quad (1769)$$

Equation 1769 permits a repaired world to contain acknowledgment and learning absent from the hypothetical no-harm world.

20.138 No-Harm World and Repaired World

The no-harm world and repaired world answer different normative questions.

The no-harm world asks:

What would have occurred without the wrong?

The repaired world asks:

Given that the wrong occurred, what history and future can now be made just?

The two need not coincide.

This distinction is central to the justice of return.

20.139 Repaired-World Value

A repaired world can contain goods that exist precisely because repair occurred:

- acknowledgment;
- institutional reform;
- clarified rights;
- historical knowledge;
- stronger safeguards;
- new solidarities.

These goods should not be treated as compensation that morally cancels the original harm. They are features of the actual post-harm historical trajectory.

20.140 Counterfactual Superiority and Historical Wrong

It can even happen that the actual repaired world becomes normatively better on some dimensions than the no-harm reference:

$$V(\mathcal{G}^R) > V(\mathcal{G}^{-H}). \quad (1770)$$

Equation 1770 does not entail that the harm was justified.

The causal path through wrongful harm remains normatively significant.

20.141 Better-than-Before Paradox

The possibility in Equation 1770 creates the better-than-before paradox.

If the subject is now better off than before, has restoration become complete?

The answer depends upon the comparison object.

Present welfare can exceed the reference. Historical loss can remain positive. Recognition can remain incomplete. Procedural wrong can remain unrectified.

The paradox dissolves once restorative dimensions are separated.

20.142 Historical Loss without Present Deficit

A historical claim can therefore persist even when

$$Q(\mathcal{G}_t^R) \succeq Q(\mathcal{G}_t^{\text{ref}}). \quad (1771)$$

Equation 1771 indicates no present deficit under object Q .

It does not imply

$$L_H = 0. \quad (1772)$$

Historical loss and present state are different objects.

20.143 Present Deficit without Historical Wrong

The converse is also possible.

A subject can presently be worse off than a comparator without having suffered the particular historical wrong under investigation.

This distinction prevents restorative justice from becoming a universal theory of every present inequality.

Distributional, social-protection, and corrective institutions retain distinct roles.

20.144 Historical Wrong and Future Direction

A complete restorative architecture can therefore contain two simultaneously valid claims:

1. the past wrong should be acknowledged and rectified according to historical and corrective standards;
2. the present subject should retain authority over the future rather than being forced into counterfactual reconstruction.

The framework treats these claims as complementary rather than contradictory.

20.145 Historical Closure

Institutions sometimes require historical closure.

Closure can mean:

- no further liability proceedings;
- final compensation determination;
- completion of a truth process;
- archival transfer;
- end of monitoring.

Closure should specify which institutional question has ended.

It should not imply that the event ceases to be historical.

20.146 Narrative Closure

Narrative closure is distinct from institutional closure.

A subject can continue revising the meaning of the event after legal proceedings end.

An institution should therefore avoid treating legal finality as authority over personal narrative.

20.147 Historical Reopening

Historical questions can reopen after:

- discovery of new evidence;
- release of archives;
- correction of prior error;
- new legal standards;
- identification of additional affected parties.

Reopening rules must balance historical revisability with institutional stability.

20.148 Versioned Historical Closure

A useful architecture can preserve both finality and provenance.

Version n of an institutional conclusion can be final for ordinary operations while later evidence generates version $n + 1$ through a defined exceptional process.

This is stronger than silent rewriting and more revisable than absolute immutability.

Table 14: Illustrative historical and reflexive object–relation cells

Historical object	Comparative relation	Restorative interpretation	Characteristic limitation
Event sequence	Identity	The same represented events occur in the same order	Usually unavailable after actual harm and repair
Historical partial order	Order isomorphism	Temporal and dependency structure correspond	Event identity and meaning can differ
Provenance graph	Graph isomorphism	Derivational structure corresponds	Present social meaning can remain different
Record state	Current-state equality	Present institutional classification is corrected	Historical consequences can persist
Versioned record	Supersession relation	Earlier error remains historically visible while current authority is removed	Requires governance of access and privacy
Memory	Content or structural correspondence	Modeled memory state corresponds	Strong forms can threaten identity and agency
Narrative	Qualitative correspondence	Historical experience is integrated through a corresponding narrative structure	Narrative representation is theory-dependent
Recognition	Sufficiency / dominance	The wrong receives adequate or improved social acknowledgment	Recognition does not substitute for material repair
Historical recognizability	Threshold / dominance	Future actors retain adequate ability to know the history	Can conflict with privacy
Reflexive feedback	Structural comparison	Evaluation and classification affect the system in corresponding ways	Observer becomes part of the system

20.149 Historical-Reflexive Profile

A historical-reflexive restorative profile can be represented by

$$\mathbf{H}_R = (r_{\text{events}}, r_{\text{order}}, r_{\text{provenance}}, r_{\text{memory}}, r_{\text{narrative}}, r_{\text{recognition}}, r_{\text{records}}, r_{\text{reflexivity}}). \quad (1773)$$

Equation 1773 avoids compressing historical repair into one binary judgment.

20.150 Historical-Reflexive Object–Relation Cells

Table 14 summarizes several principal cells of the historical-reflexive region of the atlas.

Table 14 demonstrates that historical repair contains several distinct objects whose normative relations can conflict.

20.151 Historical Refinement Path

A schematic refinement path can move from current record toward richer historical structure:

$$(Q_{\text{record}}, =) \longrightarrow (Q_{\text{event}}, \sim) \longrightarrow (Q_{\text{prov}}, \cong) \longrightarrow (Q_{\text{history}}, \cong_H) \longrightarrow (Q_{HR}, \sim_{HR}). \quad (1774)$$

Equation 1774 progressively includes more provenance, memory, narrative, and reflexive structure. The sequence does not imply increasing normative desirability.

20.152 Historical Non-Implication Structure

Several non-implications are central.

Record equality does not imply historical identity:

$$Z_R = Z_{\text{ref}} \not\Rightarrow \mathcal{H}_R = \mathcal{H}_{\text{ref}}. \quad (1775)$$

Historical event correspondence does not imply memory equivalence:

$$\mathcal{E}_R \sim \mathcal{E}_{\text{ref}} \not\Rightarrow M_R \sim M_{\text{ref}}. \quad (1776)$$

Memory equivalence does not imply provenance equivalence:

$$M_R \sim M_{\text{ref}} \not\Rightarrow \mathcal{P}_R \sim \mathcal{P}_{\text{ref}}. \quad (1777)$$

Future generative equivalence does not imply historical equivalence:

$$\Gamma_R^G \sim \Gamma_{\text{ref}}^G \not\Rightarrow \mathcal{H}_R \sim \mathcal{H}_{\text{ref}}. \quad (1778)$$

These non-implications complete the formal comparison architecture developed in Sections 15 through 19.

20.153 Historical Fidelity and Normative Non-Monotonicity

Let E_2 preserve more historical structure than E_1 .

It does not follow that

$$J(E_2) \geq J(E_1). \quad (1779)$$

Equation 1779 can fail because stronger historical reconstruction can require:

- memory alteration;
- third-party coercion;
- privacy invasion;
- erasure of later valued development;
- restoration of an unjust historical structure.

Historical equivalence therefore exhibits the same normative non-monotonicity as the broader atlas.

20.154 Historical Minimalism

Not every restorative case requires full historical reconstruction.

A minor administrative error can require:

- correction;
- propagation of correction;
- limited provenance.

A mass historical injustice can require a much richer historical architecture.

The depth of historical inquiry should be proportionate to the function and stakes.

20.155 Historical Escalation

Historical analysis becomes more important where:

- provenance determines entitlement;
- harm altered later generative trajectories;
- public denial continues;
- authorship or attribution is disputed;
- institutional learning requires the record;
- historical meaning itself is part of the injury.

These conditions justify movement toward richer historical cells.

20.156 Reflexive Minimalism

Reflexive modeling should also remain proportionate.

A simple reimbursement process need not model self-fulfilling classification.

Reflexive analysis becomes important where institutional categories materially shape the future state they purport to measure.

This includes:

- eligibility classifications;
- risk scores;
- diagnoses;
- reputational labels;
- restoration-completion declarations.

20.157 Workplace Historical-Reflexive Example

Consider a worker reinstated after wrongful exclusion.

Formal status and salary can be restored.

Historical objects remain:

- the exclusion occurred;
- colleagues remember it;
- records were created;
- later correction becomes part of institutional history;
- re-entry can be interpreted through the earlier event.

A declaration that the worker is “fully restored” can itself affect eligibility for continuing support. The case therefore contains both historical and reflexive residuals.

20.158 Research Historical-Reflexive Example

A researcher can regain strong future generativity while retaining a different publication, collaboration, and institutional history.

A later successful career does not reconstruct:

- missed intellectual encounters;
- earlier authorship opportunities;
- unpublished work;
- lost network formation.

A reparative institution can acknowledge the loss while supporting a different future.
The actual history becomes one of harm, correction, and new generation.

20.159 Reputational Historical-Reflexive Example

A false reputational classification illustrates nearly every object in this section.

The classification:

- becomes a record;
- propagates;
- alters decisions;
- changes other actors' beliefs;
- can alter the subject's expectations;
- later receives correction.

Deleting the original record can improve resemblance to a no-harm archive and destroy correction provenance.

A versioned correction can preserve accountability while preventing continued operational use of the invalid classification.

20.160 Collective Historical-Reflexive Example

Collective historical repair can involve:

- archival preservation;
- public acknowledgment;
- institutional reform;
- resource transfers;
- memorialization;
- educational change.

The goal need not be reproduction of the pre-harm social order.

The repaired collective history can explicitly include the wrong and the public response to it.

20.161 Ideal Technology and Historical Limits

The strongest ideal-technology thought experiment removes many practical constraints.

Suppose an institution possesses:

- perfect historical data;
- perfect causal models;
- unlimited resources;
- exact neural reconstruction;
- exact memory editing;
- perfect relationship simulation;
- complete counterfactual computation.

Even under these assumptions, several questions remain.

Which counterfactual history should be selected where several were possible?

Does exact reconstruction of the modeled counterfactual subject preserve the identity of the actual subject?

Can third parties be compelled to reproduce their counterfactual roles?

Should memories and records be erased?

Should a better post-harm future be replaced by a more historically faithful one?

These are normative and metaphysical questions rather than technological shortages.

20.162 Historical Limits under Perfect Knowledge

Perfect knowledge of the actual history does not create the no-harm history.

Knowledge can reconstruct a model of what might have occurred.

It cannot make the unactualized history into the actual past.

Thus

$$K_{\text{perfect}} \not\Rightarrow \mathcal{H}_R = \mathcal{H}^{\neg H}. \quad (1780)$$

Equation 1780 separates epistemic power from historical identity.

20.163 Historical Limits under Perfect Repair

Suppose every present material and functional consequence can be repaired.

Then

$$Q_i(\mathcal{G}^R) = Q_i(\mathcal{G}^{\text{ref}}) \quad (1781)$$

can hold for every selected present-state object Q_i .

Historical identity can still fail because the route to that state differs.

Perfect repair of consequence is therefore distinct from erasure of history.

20.164 Historical Limits under Time Reversal

A still stronger thought experiment permits literal reversal of physical time.

If the world is returned to a pre-harm configuration and then evolves without the harm, one can approach a literal no-harm history.

The philosophical problem changes.

The actual harmed-and-restored future is then removed rather than repaired.

The identity of subjects who developed within that future becomes part of the problem.

Time reversal therefore does not eliminate the normative issue; it moves it to the status of the actual later history and its subjects.

20.165 World-Replacement Thought Experiment

Suppose one could replace the actual world with an exact no-harm world.

Formally,

$$\mathcal{W}^{H,R} \longrightarrow \mathcal{W}^{-H}. \quad (1782)$$

Equation 1782 achieves maximal counterfactual fidelity by eliminating the actual repaired world.

Whether this is restoration depends upon whether replacing the actual historical subjects with their counterfactual counterparts counts as repairing them.

Formal equivalence cannot settle the question.

20.166 Historical Non-Closure under Ideal Technology

If the subject knows that memory was edited, the knowledge itself becomes a new historical event.

If every memory of editing is erased, records or third parties can preserve it.

If every trace is removed, the intervention has nevertheless occurred within the actual causal history unless the entire history is replaced.

The reflexive problem therefore persists under increasingly strong technological assumptions.

20.167 Historical Humility

A practical restorative institution should therefore make bounded claims.

It can say:

The false record has been corrected and the correction propagated.

It can say:

The claimant's current legal status has been restored.

It should hesitate before saying:

The person's history has been restored.

Historical humility improves precision without weakening the practical value of repair.

20.168 Historical Explicitness Principle

The first principle of this section is:

A restorative claim involving history should specify whether the relevant object is event identity, event order, provenance, memory, narrative, recognition, record state, or another historical structure.

20.169 Historical Non-Identity Principle

The second principle is:

Future state, dynamical, causal, or generative equivalence does not independently establish historical identity.

20.170 Restoration-Event Principle

The third principle is:

A restorative intervention becomes part of the actual history and can therefore increase historical difference from a no-harm reference even while improving justice.

20.171 Provenance Principle

The fourth principle is:

Present identity of an object, status, or outcome does not establish identity of the process through which it was produced.

20.172 Recognizability Principle

The fifth principle is:

Historical recognizability can itself be a value of repair, so increasing similarity to a no-harm world through erasure can reduce justice on other dimensions.

20.173 Reflexive Non-Closure Principle

The sixth principle is:

Where observation, classification, or restorative judgment affects subsequent system evolution, the evaluator belongs to the generative process rather than remaining fully external to it.

20.174 Historical Agency Principle

The seventh principle is:

Counterfactual historical fidelity does not acquire automatic authority over the present subject's memory, identity, relationships, or future direction.

20.175 Historical Pluralism Principle

The eighth principle is:

Correction, recognition, archival preservation, privacy, future generativity, and institutional learning can require different historical representations within the same case.

20.176 Historical-Reflexive Layer of the Atlas

The historical-reflexive layer can now be represented by

$$\mathfrak{H}_A = \{Q_{\text{event}}, Q_{\text{order}}, Q_{\text{prov}}, Q_{\text{memory}}, Q_{\text{narrative}}, Q_{\text{recognition}}, Q_{\text{record}}, Q_{\text{reflex}}\} \times \mathfrak{E}_H, \quad (1783)$$

where $\mathfrak{E}_H$ contains identity, isomorphism, embedding, supersession, approximation, sufficiency, dominance, and other relations appropriate to historical objects.

Equation 1783 completes the main formal comparison regions of the Atlas of Equivalence.

20.177 Completion of the Formal Comparative Architecture

The formal analysis developed from Section 10 through the present section now contains:

- reference systems;
- comparison objects;
- comparative relations;
- object–relation cells;
- temporal horizons;
- state, trajectory, and reachability;
- attractors, basins, and stability;
- perturbation and robustness;
- causal and interventional structure;
- generative-process structure;
- historical and reflexive structure.

These components make increasingly precise statements of restorative correspondence possible. They do not yet answer which correspondence should matter morally.

20.178 Formal Possibility and Normative Selection

The atlas contains many formally coherent restorative criteria.

A subject can be compared through:

- equal resources;
- capability inclusion;
- network isomorphism;
- transition simulation;
- perturbation-response equivalence;
- causal equivalence;
- recursive generative correspondence;
- historical recognition.

No mathematical rule determines which of these objects deserves priority. The choice requires value theory.

20.179 Formal Strength and Value

A formally stronger relation can preserve more structure and remain normatively inferior.

A weaker relation can preserve exactly the structure that justice has reason to protect.

The remaining task is therefore to identify what the theory values before substantive and procedural justice can be evaluated.

20.180 Transition to Value Objects

Section 21 begins the normative part of the manuscript by examining the objects to which value can attach.

The analysis distinguishes value assigned to:

- states;
- resources;
- capabilities;
- relations;
- agency;
- recognition;
- processes;
- histories;
- generative capacity;
- ecological and collective conditions.

It also examines scalar, vector, lexical, threshold, partially ordered, and incommensurable forms of value representation.

The transition is necessary because a theory of restoration requires a theory of comparison, and a theory of comparison requires an account of why the compared objects matter.

21 Value Objects

This section begins the normative part of the manuscript. The preceding sections constructed an extensive comparative architecture for restorative claims. They identified reference systems, comparison objects, comparative relations, temporal horizons, states, trajectories, reachable sets, dynamical structures, perturbation responses, causal mechanisms, generative processes, histories, and reflexive effects. That architecture establishes what can be compared and the forms of correspondence that can be demanded. It does not establish why any particular correspondence should matter morally.

The present section therefore shifts from the structure of comparison to the objects of valuation. Its role is deliberately prior to the comparison of normative theories in Section 22. The question here concerns *what can bear value within restorative analysis*. The subsequent section will examine how welfare theories, resource theories, capability approaches, relational theories, rights-based approaches, recognition theories, non-domination, generative justice, sufficiency, priority, and pluralist frameworks assign normative significance to those objects.

The distinction is important because a restorative object and a value object are related without being identical. Income can be selected as a comparison object while a theory ultimately values the capabilities income enables. Professional status can be compared directly while its normative importance

derives from recognition, security, agency, or opportunity. Network position can be measured while the object ultimately valued is non-domination or access to meaningful relations. Generative capacity can be modeled while the theory rejects maximization of generativity as such.

The section has five principal objectives. First, it develops a taxonomy of value objects relevant to the justice of return. Second, it distinguishes modes of value such as intrinsic, instrumental, constitutive, relational, option, and historical value. Third, it examines scalar, vector, threshold, lexical, partially ordered, set-valued, and qualitative representations of value. Fourth, it develops the problems of pluralism, commensurability, incommensurability, incomparability, aggregation, and translation among heterogeneous values. Fifth, it specifies how value-object selection interacts with historical subjects, present agency, third-party interests, generative processes, institutional competence, and measurement.

The central methodological principle is that a theory of justice should identify its value objects before aggregating, optimizing, compensating, or declaring restoration complete.

21.1 Value Objects and Comparative Objects

Let Q_i denote a comparative object from the Atlas of Equivalence. A value object is an object, relation, process, state, history, or possibility to which normative significance is assigned.

Let the family of value objects be

$$\mathfrak{O}_V = \{O_1^V, O_2^V, \dots, O_n^V\}. \quad (1784)$$

Equation 1784 does not require every comparison object to be valued directly.

A comparative object can function as:

- a value object in its own right;
- an indicator of another value object;
- an instrument producing another value object;
- a constraint upon value realization;
- an epistemic proxy used because the underlying value is difficult to observe.

This distinction prevents institutional measurability from determining moral importance automatically.

21.2 Valuation Maps

Let a valuation map be

$$V : \mathfrak{O}_V \times \mathcal{C}_V \longrightarrow \mathcal{V}, \quad (1785)$$

where $\mathcal{C}_V$ denotes relevant evaluative context and $\mathcal{V}$ the value representation.

Equation 1785 is intentionally general. $\mathcal{V}$ need not be the real numbers. It can be:

- a scalar;
- a vector;
- an ordinal scale;
- a partially ordered set;
- a set of admissible rankings;
- an interval;

- a qualitative category;
- a lexical hierarchy.

The choice of codomain already contains normative assumptions about how values can be compared.

21.3 Value Attribution Structure

A more explicit value attribution can be represented by the tuple

$$\mathbf{v} = (O, M, A, C, \tau, s), \quad (1786)$$

where:

- O denotes the value object;
- M denotes the mode of valuation;
- A denotes the evaluator or normative standpoint;
- C denotes context;
- τ denotes temporal horizon;
- s denotes scale.

Equation 1786 makes value claims indexed rather than context-free.

21.4 States as Value Objects

A state can be valued directly.

Let

$$O_{\text{state}}^V = x_t. \quad (1787)$$

Equation 1787 can represent health, material security, environmental condition, or another presently realized state.

State-oriented valuation is particularly natural where immediate welfare, safety, or physical integrity matters.

Its limitation is temporal. A valuable present state can coexist with fragile future dynamics or unjust provenance.

21.5 Resources as Value Objects

Resources can be treated as value objects:

$$O_{\text{resource}}^V = \mathbf{r}. \quad (1788)$$

Equation 1788 can include income, wealth, housing, infrastructure, time, education, information, and productive assets.

Resource theories can assign direct distributive significance to such holdings, while capability approaches emphasize that the conversion of resources into substantive opportunities differs across persons and circumstances [14, 55, 50].

The value-object distinction allows both possibilities to remain visible.

21.6 Functionings as Value Objects

Actual achievements or functionings can themselves bear value.

Let

$$O_{\text{functioning}}^V = f_A. \quad (1789)$$

Equation 1789 can include being healthy, being educated, participating politically, maintaining relationships, or performing a meaningful activity.

Functionings differ from resources and capabilities because they concern realized states or activities rather than inputs or opportunities.

21.7 Capabilities as Value Objects

A capability set can be valued because it represents substantive freedom to achieve alternative functionings:

$$O_{\text{capability}}^V = \mathcal{K}_A. \quad (1790)$$

Equation 1790 makes opportunity itself a value object.

The capability approach provides a major theoretical precedent for evaluating substantive opportunities rather than resources or realized outcomes alone [55, 39, 50].

This distinction is central to restorative justice because return of a resource can fail to restore the capability that the resource previously supported.

21.8 Opportunity Structure as a Value Object

The full structure of available possibilities can also carry value.

Let

$$O_{\text{opportunity}}^V = \mathcal{R}_\tau^V. \quad (1791)$$

Equation 1791 can represent reachable and viable futures after normative filtering.

The value of opportunity can depend upon:

- diversity;
- quality;
- security;
- accessibility;
- transition cost;
- reversibility;
- independence from domination.

Raw option count is therefore an incomplete representation.

21.9 Agency as a Value Object

Agency concerns the subject's ability to form, revise, and pursue directions of action.

Let

$$O_{\text{agency}}^V = A_A. \quad (1792)$$

Equation 1792 can include choice authority, practical alternatives, information, contestability, and capacity for revision.

Agency can possess value independently of whether the final outcome is identical to the reference.

This is especially important where strong restorative fidelity would require coercive reconstruction of the subject's earlier trajectory.

21.10 Autonomy as a Value Object

Autonomy can be represented as a structured object involving choice among valuable options, control over important decisions, and meaningful independence from coercive direction.

Raz's account of autonomy explicitly connects autonomy with access to an adequate range of valuable options and with value pluralism [48].

Within the present framework,

$$O_{\text{autonomy}}^V = (\mathcal{U}_A, \mathcal{K}_A, A_G, R_{\text{rev}}) \quad (1793)$$

can represent action options, capabilities, generative autonomy, and revisability.

Equation 1793 is illustrative rather than a complete philosophical definition of autonomy.

21.11 Relations as Value Objects

Relations can bear value independently of the resources exchanged through them.

Let

$$O_{\text{relation}}^V = r_{ij}. \quad (1794)$$

Equation 1794 can include friendship, care, trust, mentorship, collaboration, family relations, or civic membership.

Some relations are partly valuable because of what they enable. Others can be valued as forms of association in themselves.

This distinction is important for singular relational losses.

21.12 Relational Standing as a Value Object

A person's standing in relation to others can itself carry normative significance.

Let

$$O_{\text{standing}}^V = S_A^R. \quad (1795)$$

Equation 1795 can represent whether a subject participates as an equal, subordinate, dependent, stigmatized, or dominating actor.

Relational equality theories make such standing central rather than treating justice entirely as distribution of divisible goods [2, 71].

21.13 Recognition as a Value Object

Recognition can be treated as a distinct object:

$$O_{\text{recognition}}^V = R_A^{\text{rec}}. \quad (1796)$$

Equation 1796 can include acknowledgment, social standing, credibility, authorship, membership, and public legitimacy.

Recognition theories provide reasons for distinguishing recognition from material redistribution [25, 20].

The two can interact causally without being normatively reducible to each other.

21.14 Rights as Value Objects

Rights can be treated as normative objects whose protection is not fully captured by aggregate welfare.

Let

$$O_{\text{rights}}^V = \mathcal{R}_{\text{rights}}. \quad (1797)$$

Equation 1797 can include rights to bodily integrity, property, due process, privacy, political participation, or another institutionally defined domain.

Rights can function as:

- direct value objects;
- constraints upon permissible restoration;
- lexical priorities;
- protected minimum conditions.

These roles should be distinguished.

21.15 Legal Status as a Value Object

A legal status can itself matter because it structures entitlements, standing, and institutional recognition.

Let

$$O_{\text{legal-status}}^V = s_L. \quad (1798)$$

Equation 1798 can be valued directly and instrumentally through the opportunities it creates.

This is another example of one object carrying several modes of value.

21.16 Non-Domination as a Value Object

Freedom from arbitrary power can be treated as a relational value object.

Let

$$O_{\text{ND}}^V = D_A^-, \quad (1799)$$

where D_A^- denotes the relevant absence or limitation of arbitrary control.

Republican theory provides a reason to value the structure of dependence even where actual interference is absent [44].

A subject can possess equal resources and remain less free because another actor retains discretionary power over those resources.

21.17 Control as a Value Object

Control over generative conditions can itself carry value.

Let

$$O_{\text{control}}^V = \chi_A. \quad (1800)$$

Equation 1800 can include ownership, governance, decision rights, veto rights, allocation authority, and revocation power.

Control is especially important to generative justice, which emphasizes the relation between production, value circulation, and control over the generative process [15, 17].

21.18 Security as a Value Object

The security of access, status, or relation can itself be valuable.

Let

$$O_{\text{security}}^V = S_A. \quad (1801)$$

Equation 1801 can include stability against arbitrary withdrawal, recurrence risk, persistence, and reliability of institutional protection.

Current access and secure access are therefore different value objects.

21.19 Revisability as a Value Object

Revisability concerns the ability to alter commitments or trajectories without catastrophic loss.

Let

$$O_{\text{revisability}}^V = R_{\text{rev}}. \quad (1802)$$

Equation 1802 is especially relevant to restorative processes extending over long horizons.

A plan can provide high current welfare while reducing future revisability.

21.20 Exit Capacity as a Value Object

Exit capacity can be represented by

$$O_{\text{exit}}^V = E_A. \quad (1803)$$

Equation 1803 concerns whether a subject can leave a relation, institution, profession, or assistance arrangement without severe generative collapse.

Exit can have instrumental and anti-dominating value.

21.21 Knowledge as a Value Object

Knowledge can possess value independently of immediate material output.

Let

$$O_{\text{knowledge}}^V = K. \quad (1804)$$

Equation 1804 can include individual knowledge, distributed knowledge, practical knowledge, historical knowledge, and institutional learning.

Knowledge can be instrumental to later action and constitutive of educational, scientific, or cultural practices.

21.22 Epistemic Access as a Value Object

Access to reasons, evidence, records, and relevant information can itself carry normative significance.

Let

$$O_{\text{epistemic--access}}^V = A_E. \quad (1805)$$

Equation 1805 can support agency, contestability, and participation.

A subject can receive a substantively favorable decision while remaining epistemically excluded from the process.

21.23 Historical Truth as a Value Object

Historical truth or justified historical knowledge can function as a value object in reparative contexts.

Let

$$O_{\text{historical--knowledge}}^V = K_H. \quad (1806)$$

Equation 1806 is distinct from historical fidelity to a no-harm reference.

A truthful record of actual harm can be less similar to the no-harm world and more valuable epistemically.

21.24 Historical Recognizability as a Value Object

The possibility that future actors can know the history can itself matter.

Let

$$O_{\text{recognizability}}^V = R_H. \quad (1807)$$

Equation 1807 can support accountability, institutional learning, and recognition.

Its value must be balanced against privacy and legitimate forgetting.

21.25 Provenance as a Value Object

Provenance can matter independently of the present object.

Let

$$O_{\text{provenance}}^V = \text{Prov}(q). \quad (1808)$$

Equation 1808 becomes important for authorship, legitimacy, cultural property, evidence, and historical responsibility.

Two substantively identical current objects can therefore possess different value because of how they came to exist.

21.26 Narrative Continuity as a Value Object

Narrative continuity can be treated as a value object where continuity of self-understanding matters.

Let

$$O_{\text{narrative}}^V = N_A. \quad (1809)$$

Equation 1809 should not be interpreted as a requirement that every person maintain one coherent life story.

The value can instead concern authority to interpret and revise one's own history.

21.27 Identity Continuity as a Value Object

Some restorative interventions can alter identity-relevant characteristics.

Let

$$O_{\text{identity}}^V = I_A. \quad (1810)$$

Equation 1810 can include continuity of memory, commitment, social relation, or legal personhood according to the theory used.

Identity should remain a distinct value object because maximal counterfactual fidelity can conflict with continuity of the actual historical subject.

21.28 Process as a Value Object

The process through which an outcome is produced can itself matter.

Let

$$O_{\text{process}}^V = P. \quad (1811)$$

Equation 1811 can contain:

- consent;
- participation;
- transparency;
- contestability;
- privacy;
- reversibility;
- impartiality;
- respect.

This object will become central to procedural justice.

21.29 Participation as a Value Object

Participation can be valued because affected persons should possess a role in decisions shaping their lives.

Let

$$O_{\text{participation}}^V = P_A^{\text{part}}. \quad (1812)$$

Equation 1812 can have both intrinsic and instrumental significance.

Participation can improve epistemic quality while also respecting standing.

21.30 Contestability as a Value Object

Contestability can be represented by

$$O_{\text{contest}}^V = C_A^T. \quad (1813)$$

Equation 1813 concerns the ability to challenge classification, evidence, procedure, or remedy.

A system can produce accurate outcomes and remain normatively deficient because those outcomes are unchallengeable.

21.31 Privacy as a Value Object

Privacy can function as a value object and as a constraint on restorative knowledge.

Let

$$O_{\text{privacy}}^V = P_A^R. \quad (1814)$$

Equation 1814 can conflict with historical recognizability, causal identification, or monitoring.

The conflict cannot be resolved through epistemic optimization alone.

21.32 Dignity as a Value Object

Dignity can function as a direct normative object even where it resists simple quantification.

Let

$$O_{\text{dignity}}^V = D_A. \quad (1815)$$

Equation 1815 can be relevant where a highly effective restorative intervention degrades or objectifies the subject.

The absence of a cardinal dignity metric does not make dignity normatively irrelevant.

21.33 Care as a Value Object

Care can be valued both as a relation and as a condition-sustaining process.

Let

$$O_{\text{care}}^V = C_A^{\text{care}}. \quad (1816)$$

Equation 1816 can include attentiveness, responsiveness, dependence, and practical support.

Care is particularly relevant to restorative cases in which formal resource return leaves continuing dependence or vulnerability.

21.34 Trust as a Value Object

Trust can function as a relational and generative value object.

Let

$$O_{\text{trust}}^V = T_A. \quad (1817)$$

Equation 1817 can matter because trust supports cooperation, investment, participation, and institutional legitimacy.

Trust should not be maximized indiscriminately. Critical distrust can be rational and protective.

The appropriate object can therefore be justified trust or trustworthy institutional conditions rather than trust level alone.

21.35 Resilience as a Value Object

Resilience can be normatively relevant where persistence of a valued function or regime matters.

Let

$$O_{\text{resilience}}^V = \varrho. \quad (1818)$$

Equation 1818 must remain indexed to the regime whose persistence is valued.

Resilience of an unjust institution can be normatively negative.

21.36 Robustness as a Value Object

Robustness can also be valued where ordinary future disturbances should not destroy restored conditions.
Let

$$O_{\text{robustness}}^V = m_R \quad (1819)$$

for an appropriately defined robustness margin.

Equation 1819 can have instrumental value for future security while conflicting with flexibility or innovation.

21.37 Generative Capacity as a Value Object

Generative capacity concerns the ability to produce new conditions, opportunities, knowledge, relations, or value.

Let

$$O_{\text{gen-cap}}^V = G_A. \quad (1820)$$

Equation 1820 is distinct from generated output.

The value of generative capacity depends upon what can be generated and under what constraints.

21.38 Generative Process as a Value Object

The process of generation can itself carry normative significance.

Let

$$O_{\text{gen-process}}^V = \Gamma^G. \quad (1821)$$

Equation 1821 permits valuation of:

- autonomy in generation;
- circular value return;
- control;
- non-extraction;
- resilience;
- openness;
- revisability.

This is the principal object through which generative justice enters the normative atlas [15, 17].

21.39 Value Circulation as a Value Object

Value circulation can be normatively important separately from value production.

Let

$$O_{\text{circulation}}^V = \Lambda. \quad (1822)$$

Equation 1822 makes it possible to value how generated benefits return to producers, communities, institutions, or future conditions.

Equal total value and unequal circulation can therefore receive different normative evaluations.

21.40 Collective Capacity as a Value Object

Groups can possess capacities that are not reducible to individual holdings.

Let

$$O_{\text{collective-cap}}^V = K_C. \quad (1823)$$

Equation 1823 can include capacity for self-government, cultural reproduction, ecological stewardship, collective knowledge production, or political organization.

Collective reparations can therefore value collective capacity independently of individual transfers.

21.41 Institutional Capacity as a Value Object

Institutional capacity can be valued where institutions are expected to generate reliable rights, public goods, correction, knowledge, or support.

Let

$$O_{\text{institutional-cap}}^V = K_I. \quad (1824)$$

Equation 1824 can be especially important in guarantees of non-repetition and transformative repair.

21.42 Public Goods as Value Objects

Some goods are shared or non-separable across persons.

Let

$$O_{\text{public}}^V = G_P. \quad (1825)$$

Equation 1825 can include public knowledge, clean environments, trustworthy institutions, infrastructure, and civic peace.

Their value cannot always be reconstructed through summation of individually owned resources.

21.43 Commons as Value Objects

A commons can be valued as a resource and as a governance relation.

Let

$$O_{\text{commons}}^V = (C_{\text{resource}}, \chi_{\text{collective}}, \mathcal{R}_{\text{governance}}). \quad (1826)$$

Equation 1826 distinguishes possession of the resource from the collective structure governing it.

21.44 Ecological Conditions as Value Objects

Ecological conditions can bear value independently of their short-run economic use.

Let

$$O_{\text{ecological}}^V = E. \quad (1827)$$

Equation 1827 can include biodiversity, habitat, water, soil, climate stability, ecological function, and resilience.

The framework leaves open whether these values are anthropocentric, non-anthropocentric, relational, intrinsic, or plural.

That question belongs to value theory.

21.45 Intergenerational Conditions as Value Objects

Conditions inherited by future generations can themselves be valued.

Let

$$O_{\text{intergen}}^V = C^{(g+1)}. \quad (1828)$$

Equation 1828 can include wealth, environment, knowledge, culture, legal institutions, and political capacity.

Intergenerational valuation requires explicit treatment of temporal horizon and future persons.

21.46 Time as a Value Object

Time itself can be lost.

Let

$$O_{\text{time}}^V = T. \quad (1829)$$

Equation 1829 can represent years of professional development, time with family, duration of illness, political participation, or historical opportunity.

Time should not always be translated into money.

The value of a period can depend upon its position in a life or historical sequence.

21.47 Historical Windows as Value Objects

A time-specific opportunity window can itself carry value.

Let

$$O_{\text{window}}^V = W_{[t_0, t_1]}. \quad (1830)$$

Equation 1830 is especially important for temporally singular opportunities.

Later availability of the same external resource does not automatically recreate the value of the earlier window.

21.48 Option Value

An option can possess value even where it is never exercised.

Let opportunity o have option value

$$V_{\text{opt}}(o). \quad (1831)$$

Equation 1831 makes freedom to choose a distinct value object.

This is especially relevant to capability and autonomy-oriented approaches.

21.49 Value of Optionality

A set of options can possess value through flexibility rather than through the sum of each option's realized value.

Let

$$V_{\text{optionality}}(\mathcal{K}) \quad (1832)$$

denote the value of retaining the capability set.

Equation 1832 can depend upon diversity, irreversibility, uncertainty, and future preference change.

21.50 Singular Goods as Value Objects

Some goods derive value from identity, provenance, relation, or timing.

Let singular object be s^* .

Its value can satisfy

$$V(s^*) \neq V(s') \quad (1833)$$

even where s' reproduces many functional attributes of s^* .

Equation 1833 captures the difficulty of replacing specific relationships, artworks, cultural objects, historical opportunities, or authorship.

21.51 Meaning as a Value Object

Meaning can be treated as a value object where the significance of a condition is partly constituted through interpretation.

Let

$$O_{\text{meaning}}^V = M. \quad (1834)$$

Equation 1834 can distinguish a payment as acknowledgment from an identical payment as charity. Material identity does not establish meaning identity.

21.52 Symbolic Value

Objects and actions can possess symbolic value because of what they express within a social relation.

Let

$$V_{\text{sym}}(q, C) \quad (1835)$$

denote context-dependent symbolic value.

Anderson's pluralist account of valuation is particularly relevant to the idea that goods can be valued in different modes and that market valuation does not provide a universal common measure [1].

21.53 Value Modes

The same object can be valued in different ways.

Let mode set be

$$\mathfrak{M}_V = \{m_{\text{intrinsic}}, m_{\text{instrumental}}, m_{\text{constitutive}}, m_{\text{relational}}, m_{\text{symbolic}}, m_{\text{option}}, m_{\text{historical}}\}. \quad (1836)$$

Equation 1836 provides a working taxonomy rather than an exhaustive metaethical classification.

An object can occupy several modes simultaneously.

21.54 Instrumental Value

An object has instrumental value through its contribution to another valued object.

Let O_1 contribute to O_2 :

$$O_1 \longrightarrow O_2. \quad (1837)$$

Equation 1837 can represent money enabling healthcare, education enabling employment, or legal status enabling mobility.

Instrumental valuation does not imply that O_1 lacks other modes of value.

21.55 Constitutive Value

An object can be part of the valued practice or relationship itself.

Participation can be constitutive of democratic self-government. Mutual recognition can be constitutive of an equal social relation. Authorship can be constitutive of the meaning of an intellectual work.

Constitutive value is therefore poorly represented by replacement based solely upon downstream output.

21.56 Relational Value

Relational value depends upon the relation among persons, institutions, objects, or histories.

Let

$$V_R = V_R(A_i, A_j, r_{ij}). \quad (1838)$$

Equation 1838 means the value cannot necessarily be assigned to either actor independently.

This is especially important for trust, friendship, care, mentorship, and equal standing.

21.57 Historical Value

Historical value depends upon provenance or temporal relation.

Let

$$V_H = V_H(q, \text{Prov}(q), t). \quad (1839)$$

Equation 1839 allows an object's value to depend upon who produced it, when, and through which history.

Historical value is central to restitution of singular objects and recognition of lost opportunities.

21.58 Option Value and Reversibility

The value of an option can increase where future preferences are uncertain and choices are irreversible.

Let reversibility be R_{rev} .

A generic relation is

$$V_{\text{opt}} = V_{\text{opt}}(\mathcal{K}, R_{\text{rev}}, U_F), \quad (1840)$$

where U_F denotes uncertainty concerning future preference or environment.

Equation 1840 supports the normative importance of keeping futures open.

21.59 Direct and Indirect Value

A value object can matter directly and through its effects upon other values.

Let direct value be $V_D(O)$ and downstream value be $V_I(O \rightarrow O')$.

A complete valuation can therefore be represented schematically by

$$V_{\text{total}}(O) = V_D(O) + V_I(O \rightarrow O') \quad (1841)$$

where additive representation is appropriate.

Equation 1841 is illustrative and should not be used where the value modes are incommensurable.

21.60 Value Pluralism

Restorative cases typically involve several value objects simultaneously.

Let

$$\mathbf{V} = (V_1, V_2, \dots, V_n). \quad (1842)$$

Equation 1842 represents plural valuation without immediate aggregation.

Value pluralism is especially important where objects such as welfare, agency, recognition, rights, historical truth, and generativity cannot be assumed to be instances of one homogeneous good. Anderson develops a pluralist account of valuation that rejects treatment of heterogeneous goods through one universal market measure, while Raz explicitly connects autonomy with pluralism and incommensurability [1, 48].

21.61 Scalar Value Representation

The simplest representation maps the relevant object into the real numbers:

$$V : \mathfrak{D}_V \longrightarrow \mathbb{R}. \quad (1843)$$

Equation 1843 enables:

- ranking;
- maximization;
- subtraction;
- aggregation;
- compensation calculations.

Its convenience is substantial.

Its validity depends upon whether the relevant value domain is adequately represented by one scale.

21.62 Cardinal Representation

A cardinal representation assigns significance to differences in value:

$$V(A) - V(B). \quad (1844)$$

Equation 1844 can support claims about magnitude of gain or loss.

A cardinal representation requires stronger assumptions than ordinal ranking.

21.63 Ordinal Representation

An ordinal representation requires only ranking:

$$A \succeq B. \quad (1845)$$

Equation 1845 states that A is at least as good as B according to the selected criterion.

The difference between A and B need not have a meaningful numerical magnitude.

21.64 Vector Value Representation

A vector representation preserves several dimensions:

$$\mathbf{V}(O) = (V_1(O), \dots, V_k(O)). \quad (1846)$$

Equation 1846 is well suited to restorative analysis where resources, capabilities, agency, recognition, security, and historical values should remain visible separately.

A vector postpones the aggregation problem rather than solving it.

21.65 Componentwise Dominance

For value vectors, a simple partial order is componentwise dominance:

$$\mathbf{V}(A) \succeq \mathbf{V}(B) \iff V_i(A) \geq V_i(B) \quad \forall i. \quad (1847)$$

Equation 1847 permits confident comparison in cases where one option is at least as good in every represented dimension.

Many realistic cases remain incomparable under this rule.

21.66 Partial Orders

A value relation can therefore be a partial order:

$$(\mathcal{V}, \succeq_V). \quad (1848)$$

Equation 1848 permits some alternatives to remain unranked.

This can be more faithful than forcing complete ordering across heterogeneous values.

21.67 Lexical Representation

Some value theories assign priority to one dimension before another.

Let

$$V_1 \succ_{\text{lex}} V_2 \succ_{\text{lex}} V_3. \quad (1849)$$

Equation 1849 can represent rights, bodily safety, or agency as protected before optimization of secondary benefits.

Lexical ordering should be justified explicitly because it restricts ordinary trade-offs.

21.68 Threshold Representation

A value can be represented through an adequacy threshold.

Let

$$V(O) \geq \theta. \quad (1850)$$

Equation 1850 can represent sufficiency in health, income, capability, safety, or procedural protection.

Above-threshold differences can remain relevant or become secondary depending upon the theory.

21.69 Multi-Threshold Representation

Several value dimensions can each have minimum thresholds:

$$V_i(O) \geq \theta_i \quad i = 1, \dots, k. \quad (1851)$$

Equation 1851 permits non-substitutability among basic dimensions.

Excess income cannot compensate for failure to satisfy a minimum bodily-safety condition if the theory gives each dimension an independent threshold.

21.70 Set-Valued Representation

Valuation under uncertainty or pluralism can produce a set of acceptable evaluations:

$$V(O) \in \mathcal{V}(O). \quad (1852)$$

Equation 1852 can represent unresolved disagreement or incomplete information.

Institutional decisions can then seek robustness across the admissible set.

21.71 Interval-Valued Representation

Where value is uncertain, one can use

$$V(O) \in [V_{\min}(O), V_{\max}(O)]. \quad (1853)$$

Equation 1853 avoids false numerical precision.

This can be especially useful for long-horizon counterfactual loss.

21.72 Qualitative Representation

Some values can be represented through qualitative categories rather than numbers.

Let

$$V(O) \in \{\text{protected, adequate, impaired, severely impaired}\}. \quad (1854)$$

Equation 1854 can be more appropriate where numerical differences lack a defensible interpretation.

Qualitative representation is not equivalent to analytical imprecision.

21.73 Distributional Value Representation

The distribution of value across persons can itself be the object:

$$O_{\text{distribution}}^V = P(V_1, \dots, V_n). \quad (1855)$$

Equation 1855 permits evaluation of inequality, concentration, priority, or group-level effects.

Aggregate total value alone does not determine the distribution.

21.74 Temporal Value Profiles

Value can vary through time:

$$V_t = V(O_t, t). \quad (1856)$$

Equation 1856 permits the same object to have different significance at different historical or developmental periods.

This is essential for singular opportunities and historical windows.

21.75 Value Streams

A temporally extended object can be represented by the value stream

$$\mathbf{V}_{[t_0, t_1]} = \{V_t : t \in [t_0, t_1]\}. \quad (1857)$$

Equation 1857 preserves temporal distribution.

A later gain need not be assumed to compensate for an earlier loss.

21.76 Cumulative Value

Where temporal aggregation is legitimate,

$$V_{\text{cum}} = \int_{t_0}^{t_1} w(t) V_t dt. \quad (1858)$$

Equation 1858 introduces a temporal weighting function $w(t)$.

The validity of this representation depends upon whether the value is temporally fungible.

21.77 Path-Dependent Value

Value can depend upon the sequence through which an outcome is produced.

Let history be γ . Then

$$V = V(x_t, \gamma_{[t_0, t]}). \quad (1859)$$

Equation 1859 permits the same endpoint to possess different value depending upon coercion, delay, recognition, or provenance.

This is central to corrective and historical justice.

21.78 State-Dependent Valuation

The subject's valuation can change with state:

$$V_A = V_A(O; x_A). \quad (1860)$$

Equation 1860 becomes important after harm because preferences, priorities, and needs can change. Present valuation should therefore not be treated as automatically identical to counterfactual valuation.

21.79 Subject-Relative Value

Some objects have different value for different subjects.

Let

$$V_i(O) \neq V_j(O). \quad (1861)$$

Equation 1861 can arise from differences in goals, conversion factors, relationships, culture, or history.

Subject-relative valuation is especially important for fungible compensation and personalized assistance.

21.80 Agent-Neutral Value

Other theories can assign value independently of the particular evaluator.

Let

$$V^N(O) \quad (1862)$$

denote an agent-neutral evaluation.

Restorative justice can combine agent-relative interests with agent-neutral constraints such as rights or third-party welfare.

The framework need not assume that all values belong to one standpoint.

21.81 Relationally Indexed Value

Some values exist only relative to a relation.

Let

$$V(O \mid r_{ij}) \quad (1863)$$

denote value indexed to relation r_{ij} .

A mentor's advice can have distinctive value because of the relationship through which it is offered.

This value cannot necessarily be reproduced by an informationally identical statement from another person.

21.82 Position-Dependent Value

The value of a resource can depend upon the subject's position in a social or institutional structure.

Let

$$V = V(r, \pi_A), \quad (1864)$$

where π_A is relational or institutional position.

Equation 1864 gives another reason equal resource amounts can have unequal significance.

21.83 Contextual Value

An object can have different value in different institutional, cultural, or historical contexts:

$$V(O, C_1) \neq V(O, C_2). \quad (1865)$$

Equation 1865 does not imply that valuation is arbitrary.

It means relevant context forms part of the object of evaluation.

21.84 Conditional Value

The value of one object can depend upon another condition.

Let

$$V(O_1 \mid O_2). \quad (1866)$$

Equation 1866 can represent the value of a grant conditional upon legal eligibility, or the value of a professional network conditional upon health sufficient for participation.

This structure creates complementarity among restorative objects.

21.85 Value Complementarity

Two objects can be more valuable jointly than separately.

Where additive comparison is meaningful, complementarity can be represented by

$$V(O_1, O_2) > V(O_1) + V(O_2). \quad (1867)$$

Equation 1867 explains why layered restoration can have effects greater than the sum of isolated interventions.

The representation is illustrative because not every value domain admits addition.

21.86 Value Substitution

One object can substitute for another relative to a selected value:

$$V(O_1) \approx V(O_2). \quad (1868)$$

Equation 1868 does not imply identity of the objects.

Compensation often relies upon such substitution.

The normative problem concerns which value dimensions are preserved and which are lost.

21.87 Imperfect Substitution

A substitute can preserve one value dimension and fail another.

Let

$$V_1(O_1) = V_1(O_2) \quad (1869)$$

while

$$V_2(O_1) \neq V_2(O_2). \quad (1870)$$

Equations 1869 and 1870 provide a precise representation of partial compensatory equivalence.

21.88 Value Conflict

Restorative interventions can improve one value and worsen another.

Let intervention R satisfy

$$\Delta V_1(R) > 0 \quad (1871)$$

and

$$\Delta V_2(R) < 0. \quad (1872)$$

Equations 1871 and 1872 define a value conflict.

Examples include:

- greater historical transparency and lower privacy;
- stronger robustness and lower flexibility;
- greater counterfactual fidelity and lower present agency;
- greater monitoring accuracy and lower dignity.

21.89 Value Trade-Off

A trade-off rule specifies when loss in one dimension can be accepted for gain in another.

Let

$$T_{12} : \Delta V_1 \mapsto \Delta V_2. \quad (1873)$$

Equation 1873 should not be presumed to exist for every pair of values.

The possibility of trade-off itself is a normative claim.

21.90 Commensurability

Two values are commensurable in the relevant sense where they can be assessed through a common evaluative scale or standard adequate for the decision at hand.

Let common scale be m :

$$m : \{O_1, O_2\} \longrightarrow \mathcal{V}_m. \quad (1874)$$

Equation 1874 supplies a common measure.

The existence of a common measure is a substantive assumption.

21.91 Incommensurability

Values can be incommensurable when no appropriate common measure reduces them without unacceptable loss of normative structure.

The literature on incommensurability and practical reason contains several different analyses of what such incommensurability entails [10, 48, 1].

The present framework therefore avoids defining incommensurability through one universal formal condition.

Its practical implication is more modest: absence of a justified common scale should block unexamined scalar aggregation.

21.92 Incomparability

Incomparability should remain distinct from incommensurability.

Two alternatives can lack a common cardinal measure and remain ordinally comparable.

Conversely, the available reasons can fail to establish that one alternative is better, worse, or equal to another under the selected normative structure.

Represent incomparability by

$$A \parallel_V B. \quad (1875)$$

Equation 1875 permits the atlas to represent a genuine failure of complete ordering rather than forcing a tie.

21.93 Indeterminacy

Value indeterminacy can arise when evidence or normative theory does not resolve the ranking.

Let admissible rankings be

$$\mathfrak{R}_V = \{R_1, \dots, R_k\}. \quad (1876)$$

Equation 1876 distinguishes uncertainty or indeterminacy from true equality.

21.94 Value Equality

Two objects can have equal value under one theory:

$$V(O_1) = V(O_2). \quad (1877)$$

Equation 1877 should not be inferred merely from inability to rank the objects.
Equality, incomparability, and uncertainty are different relations.

21.95 Commensuration

Institutions can create a common scale even where values are heterogeneous.

Let

$$C_M : \mathfrak{D}_V \longrightarrow \mathbb{R} \quad (1878)$$

denote an institutional commensuration map.

Equation 1878 can convert injury, time, opportunity, or environmental harm into monetary values.
Commensuration can be administratively necessary without proving deep moral commensurability.

21.96 Translation Loss

A commensuration map can discard value dimensions.

Let reconstruction from common measure be C_M^{-1} where such a map is defined approximately.

Translation loss can be represented schematically by

$$L_V = O - C_M^{-1}(C_M(O)). \quad (1879)$$

Equation 1879 is conceptual rather than literal algebra for heterogeneous objects.

Its purpose is to identify value lost when a complex object is translated into a general equivalent.

21.97 Money as a General Value Proxy

Money can serve as a highly useful common proxy because of its fungibility.

Let

$$M : \mathfrak{D}_V \longrightarrow \mathbb{R}_{\geq 0}. \quad (1880)$$

Equation 1880 can facilitate compensation, insurance, budgeting, and decentralized subject choice.

The map does not establish that every value object is morally equivalent to its monetary representation.

Anderson's critique of universal market valuation is especially relevant to this distinction [1].

21.98 Fungibility and Epistemic Humility

Fungibility has a distinctive restorative value.

Where the institution does not know which future the subject should pursue, a general resource can preserve decentralized choice.

Let monetary transfer m generate a set of subject-directed conversions:

$$\mathcal{K}_A(m) = \{c_1, \dots, c_n\}. \quad (1881)$$

Equation 1881 illustrates the epistemic advantage of general equivalents.

The advantage weakens where markets cannot provide the relevant good.

21.99 Limits of Fungibility

Low-fungibility values include:

- singular relationships;
- authorship;
- historical opportunities;
- bodily integrity;
- some rights;
- collective self-government;
- cultural heritage.

Compensation can remain appropriate while failing to reproduce the original value mode.

21.100 Value Conversion

One value object can be converted into another.

Let

$$T_{ij}^V : O_i^V \longrightarrow O_j^V. \quad (1882)$$

Equation 1882 can represent money into healthcare, education into capability, recognition into social access, or value into future conditions.

Conversion depends upon history and context.

21.101 Conversion Rate

Where quantitative representation is meaningful, a conversion rate can be defined by

$$\eta_{ij} = \frac{\Delta V_j}{\Delta V_i}. \quad (1883)$$

Equation 1883 is highly context-dependent.

It should not be treated as a universal exchange rate among moral values.

21.102 Conversion Asymmetry

Conversion can be asymmetric.

A resource can generate recognition, while recognition cannot recreate the same resource.

Formally,

$$T_{ij}^V \text{ exists} \not\Rightarrow T_{ji}^V \text{ exists}. \quad (1884)$$

Equation 1884 helps explain irreversibility in value conversion.

21.103 Value Aggregation

Where several values are represented numerically, an aggregate can be formed:

$$V_{\text{agg}} = A(V_1, \dots, V_n). \quad (1885)$$

Equation 1885 leaves the aggregation operator A unspecified.

Possible operators include:

- weighted sums;
- minimum functions;
- lexicographic rules;
- social welfare functions;
- multi-criteria decision rules.

No aggregation rule follows from the existence of the value vector itself.

21.104 Weighted Aggregation

A common form is

$$V_{\text{agg}} = \sum_{i=1}^n w_i V_i. \quad (1886)$$

Equation 1886 requires both commensurability and justified weights.

The apparent precision of the result should not obscure those assumptions.

21.105 Bottleneck Aggregation

Where every dimension must exceed a minimum, a minimum-style aggregation can be used:

$$V_{\text{agg}} = \min_i \left\{ \frac{V_i}{\theta_i} \right\}. \quad (1887)$$

Equation 1887 represents the binding dimension.

This can be useful where one severe deficiency cannot be compensated by excess in another dimension.

21.106 Lexical Aggregation

A lexical structure compares higher-priority dimensions first.

For alternatives A and B , the rule can compare V_1 and consider V_2 only where $V_1(A) = V_1(B)$.

This can represent protected rights or non-negotiable process conditions.

Lexical aggregation reduces ordinary compensability.

21.107 Non-Aggregative Evaluation

A theory can reject a final aggregate entirely.

The restorative result can remain a profile:

$$\mathbf{V}_R = (V_{\text{resource}}, V_{\text{capability}}, V_{\text{agency}}, V_{\text{recognition}}, V_{\text{history}}, V_{\text{process}}). \quad (1888)$$

Equation 1888 allows decision through partial ordering, deliberation, thresholds, or procedural choice.

This can be preferable where value dimensions are heterogeneous.

21.108 Interpersonal Value Comparison

Restorative policy can require comparison across persons.

Let

$$V_i(O_i) \quad (1889)$$

and

$$V_j(O_j) \quad (1890)$$

denote subject-indexed values.

Comparing them requires assumptions concerning scale, needs, capabilities, preferences, or another common structure.

Interpersonal comparability should therefore be explicit rather than hidden in an aggregate score.

21.109 Adaptive Preferences and Valuation

Present preferences can be shaped by deprivation or adaptation.

Let subject preference after harm be

$$P_A^H. \quad (1891)$$

A value theory can decide how much authority to assign P_A^H relative to earlier preferences, informed preferences, capabilities, or objective values.

Capability approaches are especially attentive to the possibility that observed preference satisfaction can understate deprivation [55, 50].

The present framework does not solve this problem through one rule.

21.110 Counterfactual Preferences

A reconstructed no-harm subject can have preference

$$P_A^C. \quad (1892)$$

The actual present subject has

$$P_A^H. \quad (1893)$$

The two can differ:

$$P_A^C \neq P_A^H. \quad (1894)$$

Equation 1894 creates a value conflict between counterfactual fidelity and present agency.

21.111 Subject-Defined Value

Some restorative objects should be partly selected by the affected subject.

Let the subject-selected value set be

$$\mathfrak{D}_V^A \subseteq \mathfrak{D}_V. \quad (1895)$$

Equation 1895 can guide assistance toward the subject's present priorities.

Subject selection remains constrained by third-party rights and broader normative requirements.

21.112 Institution-Defined Value

Institutions also define value objects through their mandates.

A court can value legally cognizable injury. A welfare institution can value minimum material security. A regulator can value safety. A truth commission can value recognition and historical knowledge.

Let

$$\mathfrak{D}_V^I \quad (1896)$$

denote the institutionally relevant value set.

Institutional scope should not be mistaken for the complete value structure of the case.

21.113 Third-Party Value Objects

Restoration can affect persons beyond the claimant.

Let third-party value profile be

$$\mathbf{V}_{-A} = (V_1, \dots, V_m). \quad (1897)$$

Equation 1897 can include rights, welfare, privacy, property, or relational autonomy.

A remedy that maximizes claimant fidelity can therefore remain unjust through third-party burdens.

21.114 Collective and Individual Value Conflict

Collective repair can produce gains at community scale and burdens for some members.

Let

$$\Delta V_C > 0 \quad (1898)$$

while

$$\Delta V_i < 0. \quad (1899)$$

Equations 1898 and 1899 require a theory relating collective and individual claims.

Collective value should not automatically override individual rights.

21.115 Intergenerational Value Conflict

Current and future generations can also have conflicting value profiles.

Let current-generation value be V_0 and future-generation value $V_{1:\infty}$.

A policy can satisfy

$$\Delta V_0 > 0 \quad (1900)$$

while

$$\Delta V_{1:\infty} < 0. \quad (1901)$$

Equations 1900 and 1901 identify an intergenerational value conflict.

21.116 Negative Value Objects

The atlas should represent burdens and disvalues as well as positive goods.

Let

$$\mathfrak{D}_D = \{D_1, \dots, D_m\} \quad (1902)$$

contain pain, domination, stigma, insecurity, administrative burden, ecological depletion, and other negative objects.

Equation 1902 avoids assuming that justice can be represented solely through positive goods.

21.117 Harm as Value Loss

For value object O , harm can be represented by

$$L_V = V(O_{\text{ref}}) - V(O_H) \quad (1903)$$

where scalar subtraction is justified.

Equation 1903 is unavailable where values are incommensurable or only partially ordered.

Alternative representations are then required.

21.118 Vector Loss

For plural value,

$$\mathbf{L}_V = \mathbf{V}(O_{\text{ref}}) - \mathbf{V}(O_H). \quad (1904)$$

Equation 1904 preserves dimensions separately.

Some components can be positive while others are negative.

21.119 Historical Loss and Present Gain

Historical loss and present gain can coexist:

$$L_H > 0 \quad (1905)$$

and

$$V_{\text{present}}(\mathcal{G}^R) > V_{\text{present}}(\mathcal{G}^{\text{ref}}). \quad (1906)$$

Equations 1905 and 1906 show why a better present does not erase the normative status of earlier loss.

21.120 Non-Cancellation Across Value Modes

A gain in one value mode does not automatically cancel a loss in another.

For example:

- greater income need not cancel a rights violation;
- public recognition need not cancel bodily harm;
- future opportunity need not erase historical misattribution;
- high generativity need not cancel domination.

A cancellation rule requires a substantive theory of commensurability.

21.121 Value Floors and Side Constraints

Some value objects can function as constraints rather than objectives to maximize.

Let admissible restorative set be

$$\mathcal{R}_{\text{adm}} = \{R : V_i(R) \geq \theta_i \forall i \in I_C\}. \quad (1907)$$

Equation 1907 requires satisfaction of protected value floors before optimization over remaining dimensions.

This architecture will become important in Section 22.

21.122 Value Objects and Generativity

Generativity is one value object among others rather than the master value of the framework.

Let

$$V_G = V(\Gamma^G). \quad (1908)$$

Equation 1908 can be positive only under normatively relevant conditions.

A process generating domination, extraction, ecological damage, or coercive dependency can be highly generative in a descriptive sense and normatively negative.

21.123 Normatively Filtered Generativity

Let admissible generative process set be

$$\mathfrak{G}^N = \{\Gamma^G : \Gamma^G \text{ satisfies } \mathcal{N}\}. \quad (1909)$$

Equation 1909 makes normative conditions explicit.

The theory can then value generativity inside $\mathfrak{G}^N$ without treating arbitrary generation as good.

21.124 Value Objects and Robustness

Robustness should likewise be attached to a valued object.

Let

$$\varrho(O^V) \quad (1910)$$

denote robustness of value object O^V .

This permits distinct questions:

- robustness of income;
- robustness of agency;
- robustness of relational equality;
- robustness of ecological function.

One system can rank differently across these objects.

21.125 Value Objects and Causal Mechanisms

A causal mechanism can matter because of the values it realizes.

Let mechanism f produce value profile

$$\mathbf{V}(f). \quad (1911)$$

Equation 1911 allows comparison of mechanisms that produce equal outputs through different levels of agency, domination, or burden.

21.126 Value Objects and Historical Provenance

Value can attach to both present result and provenance:

$$\mathbf{V} = (V_{\text{current}}, V_{\text{provenance}}). \quad (1912)$$

Equation 1912 prevents present welfare from exhausting historical evaluation.

21.127 Value Objects and Reflexivity

Institutional declaration of what is valuable can alter behavior.

If funding is awarded according to one restorative value score, actors can optimize toward that score.

Let value metric be M_V . Reflexive response is

$$M_V \rightarrow A \rightarrow Q. \quad (1913)$$

Equation 1913 means a valuation system can reshape the object it measures.

This strengthens the case for plural and revisable value architectures.

21.128 Value Visibility

Institutions tend to privilege values that are administratively visible.

Let visibility function be

$$\text{Vis}(O^V, I). \quad (1914)$$

Equation 1914 can be high for salary and low for trust, narrative continuity, or lost intellectual opportunity.

Low visibility does not imply low normative importance.

21.129 Value Legibility

A value becomes institutionally legible when it can be translated into the categories through which an institution acts.

Let

$$L_I : O^V \longrightarrow \hat{O}_I^V. \quad (1915)$$

Equation 1915 represents institutional translation.

The translation can simplify a complex value into damages, eligibility, severity grade, or administrative status.

Such legibility can enable action while losing normative detail.

21.130 Value Compression

A compressed institutional representation can be written as

$$\hat{\mathbf{V}} = C_V(\mathbf{V}). \quad (1916)$$

Equation 1916 can reduce a high-dimensional value profile to a score or category.
Compression should preserve provenance concerning which dimensions were omitted.

21.131 Metric Capture

Once one value metric controls resources, actors can orient behavior toward it.

A metric can therefore become a site of institutional power.

A restorative theory should ask:

- who defines the metric;
- which values it makes visible;
- which values it suppresses;
- who can contest it;
- how often it can be revised.

This is a normative extension of the reflexivity analyzed in Section 20.

21.132 Value Uncertainty

Uncertainty can concern facts, theory, or valuation.

Let value uncertainty set be

$$\mathfrak{V}_U = \{V_1, \dots, V_k\}. \quad (1917)$$

Equation 1917 can represent several plausible evaluative models.
A restorative decision can be robust if it performs acceptably across them.

21.133 Value-Theory Robustness

Let intervention R be evaluated under theories V_j .

A robust intervention can satisfy

$$R \in \mathcal{A}_{V_j} \quad \forall V_j \in \mathfrak{V}_U, \quad (1918)$$

where $\mathcal{A}_{V_j}$ is the acceptable set under theory V_j .

Equation 1918 supports action under normative plurality without claiming that all theories are equivalent.

21.134 Value Disagreement

Parties can agree on all facts and disagree about what matters.

One party can value restoration of historical status. Another can value present autonomy. Another can prioritize relational repair. Another can prioritize future capability.

Such disagreement belongs to the value layer rather than the factual layer of the atlas.

Identifying this location can prevent normative conflict from being disguised as empirical disagreement.

21.135 Value Disagreement and Procedure

Where value disagreement cannot be eliminated, procedural legitimacy becomes especially important.

Participation, explanation, contestation, and revisability can provide a legitimate way to govern disagreement without assuming a single uncontested value function.

This creates an important connection between plural value and procedural justice.

21.136 Value Objects and Institutional Division of Labor

Different institutions can legitimately protect different value objects.

For example:

- courts can protect rights and legally cognizable interests;
- welfare institutions can protect material sufficiency;
- healthcare can protect bodily and functional values;
- professional institutions can protect credential and access values;
- archives can preserve historical value;
- regulatory institutions can protect safety and systemic robustness.

Institutional pluralism therefore mirrors value pluralism.

21.137 Value Gaps Across Institutions

A value can fall between institutional mandates.

Trust, lost network position, narrative continuity, or future generative opportunity can be important and weakly governed.

Let uncovered value family be

$$\mathfrak{V}_V^{\text{gap}} = \mathfrak{V}_V \setminus \bigcup_I \mathfrak{V}_V^I. \quad (1919)$$

Equation 1919 defines institutional value gaps.

Such gaps can motivate coordination or assistance rather than expansion of one institution's mandate without limit.

21.138 Value Conflict Across Institutions

Institutions can also protect competing values.

An archive can favor preservation. Privacy law can favor deletion. A court can favor finality. A rehabilitation system can favor revisability.

Institutional coordination should therefore identify the underlying value conflict explicitly.

21.139 Value Object Dependency

Some values depend upon others.

Let

$$O_i^V \rightarrow O_j^V \quad (1920)$$

mean that realization of O_i^V contributes materially to realization of O_j^V .

Income security can support autonomy. Recognition can support professional access. Privacy can support agency. Health can support capability.

Dependency does not imply that the upstream object is only instrumentally valuable.

21.140 Value Dependency Graph

A family of value objects can be represented by

$$G_V = (\mathfrak{D}_V, E_V), \quad (1921)$$

where E_V contains dependency relations.

Equation 1921 can help identify value bottlenecks and complementarities.

It should not be interpreted as a complete causal graph unless causal assumptions are supplied.

21.141 Value Bottlenecks

A value object can function as a bottleneck where failure to protect it blocks several other values.

For example:

- legal status can block employment and mobility;
- bodily safety can block participation;
- privacy can be required for safe reporting;
- recognition can be required for professional re-entry.

Bottleneck significance can justify priority without making the value intrinsically superior.

21.142 Value Redundancy

Several objects can support the same downstream value.

A subject can attain economic security through employment, insurance, savings, or social protection. Such redundancy increases resilience.

It also complicates compensatory equivalence because different upstream objects can generate similar downstream value.

21.143 Value Irreversibility

Some value losses cannot be recreated through later provision of the same input.

Let value transformation be

$$O_t^V \xrightarrow{H} O_{t+1}^V. \quad (1922)$$

If no admissible intervention R produces

$$R(O_{t+1}^V) = O_t^V, \quad (1923)$$

then the value is irreversible under the selected object definition.

Irreversibility can concern history, time, relation, identity, or ecology.

21.144 Value Reparability

Reparability can be treated as a property of value objects.

Let

$$\text{Rep}_V(O^V) \quad (1924)$$

denote expected reparability.

A low-reparability value can justify stronger prevention where severity, uncertainty, rights, and proportionality support such action.

21.145 Value Preservation

A restorative intervention can preserve a value even where the underlying object changes.

Let

$$O_1 \rightarrow O_2 \quad (1925)$$

and

$$V(O_1) = V(O_2). \quad (1926)$$

Equation 1926 defines value preservation relative to the selected valuation.

The relation can be useful and normatively insufficient where provenance or identity matters separately.

21.146 Value Transformation

A restorative process can create a different value profile rather than preserve the earlier one.

Let

$$\mathbf{V}_R \neq \mathbf{V}_{\text{ref}}. \quad (1927)$$

Equation 1927 can still be normatively preferable where

$$\mathbf{V}_R \succeq \mathbf{V}_{\text{ref}} \quad (1928)$$

under the selected partial order.

This is the value-theoretic form of transformative restoration.

21.147 Value Creation Through Restoration

Repair can create new values absent from the reference.

Examples include:

- public acknowledgment;
- stronger institutional safeguards;
- new solidarities;
- improved accessibility;
- new forms of agency.

These gains do not retroactively justify the harm.

They belong to the value profile of the repaired history.

21.148 Value Destruction Through Restoration

A restorative intervention can also destroy values.

Examples include:

- privacy lost through public correction;
- agency lost through compulsory reinstatement;
- valued new relationships displaced by historical reconstruction;

- ecological burden created by material replacement.

Restorative interventions therefore require value-impact analysis rather than assuming every movement toward reference equivalence is positive.

21.149 Restorative Value Profile

A case-level value profile can be represented by

$$\mathbf{V}_R = (V_{\text{welfare}}, V_{\text{resource}}, V_{\text{capability}}, V_{\text{agency}}, V_{\text{relation}}, V_{\text{recognition}}, V_{\text{control}}, V_{\text{history}}, V_{\text{process}}, V_{\text{generativity}}, V_{\text{ecology}}) \cdot (1929)$$

Equation 1929 is illustrative and can be expanded or contracted according to the case.

The profile is an evaluative interface rather than a claim that every component can be cardinally measured.

21.150 Value-Object Matrix

Table 15 summarizes major value-object families and the restorative differences they can reveal.

Table 15 is a map of possible normative objects. It does not rank them.

21.151 Value Representation Matrix

Table 16 summarizes major forms of value representation.

Table 16 demonstrates that the form of value representation should follow the value structure rather than administrative convenience alone.

21.152 Value-Object Selection

A restorative inquiry should identify value objects through an explicit selection process.

A practical sequence can include:

1. identify the harm and affected comparison objects;
2. identify the values plausibly carried by those objects;
3. distinguish direct, instrumental, relational, and historical modes of value;
4. identify subject-selected values;
5. identify rights and third-party constraints;
6. identify collective and intergenerational values where material;
7. select an adequate representation for each value;
8. preserve uncertainty or incomparability where justified.

The sequence avoids premature scalarization.

21.153 Minimal Value Set

A practical institution should use the smallest value set sufficient for its legitimate purpose.

Let

$$\mathfrak{V}_V^* \subseteq \mathfrak{V}_V \quad (1930)$$

denote the case-specific value set.

Equation 1930 supports institutional proportionality.

The existence of the full taxonomy does not justify measuring every value in every case.

21.154 Value-Set Expansion

The active value set can expand when residual injustice becomes visible:

$$\mathfrak{V}_{V,t+1}^* = \mathfrak{V}_{V,t}^* \cup \Delta \mathfrak{V}_{V,t+1}. \quad (1931)$$

Equation 1931 allows iterative discovery.

A case initially framed as income loss can expand to include recognition, health, or network opportunity when evidence supports those dimensions.

21.155 Value-Set Contraction

Values can also leave the active institutional set when they cease to be relevant to the institution's task:

$$\mathfrak{V}_{V,t+1}^* = \mathfrak{V}_{V,t}^* \setminus \Delta \mathfrak{V}_{V,t+1}^-. \quad (1932)$$

Equation 1932 prevents endless expansion of institutional monitoring.

21.156 Value Provenance

The selection of value objects should itself possess provenance.

Let

$$\mathcal{P}_V = (\mathfrak{V}_{V,0}^* \rightarrow \mathfrak{V}_{V,1}^* \rightarrow \cdots). \quad (1933)$$

Equation 1933 records why values were added, removed, or redefined.

This supports contestability and guards against hidden target drift.

21.157 Value Objects and Comparative Relations

A value object does not determine the appropriate comparative relation.

Income can be compared through equality, sufficiency, priority, or dominance.

Capability can be compared through inclusion or thresholds.

Agency can be compared through equality, sufficiency, or non-domination dominance.

Historical truth can be compared through accuracy or sufficiency rather than similarity to the no-harm reference.

The normative theory must therefore select both:

$$(O^V, E^V). \quad (1934)$$

Equation 1934 is the value-theoretic analogue of the object–relation cell.

21.158 Value Objects and Baselines

Different value objects can require different baselines.

Income can use expected no-harm income.

Current safety can use an adequate threshold.

Historical recognition can use a truth-sensitive standard.

Agency can use present subject-directed possibility.

Thus

$$B_i = B(O_i^V). \quad (1935)$$

Equation 1935 permits baseline pluralism.

21.159 Value Objects and Temporal Horizons

Value objects also operate over different horizons.

Let

$$\tau_i = \tau(O_i^V). \quad (1936)$$

Equation 1936 can assign:

- immediate horizon to safety;
- medium horizon to rehabilitation;
- long horizon to professional capability;
- intergenerational horizon to ecological and cultural conditions.

One universal horizon would distort this plurality.

21.160 Value Objects and Scale

A value can be evaluated at several scales:

$$O^{V,(s)}. \quad (1937)$$

Equation 1937 distinguishes individual, relational, collective, institutional, societal, and ecological value.

Improvement at one scale does not establish improvement at another.

21.161 Value Objects and Evidence

Different values require different evidence.

Income can be directly documented.

Capability requires inference concerning substantive options.

Recognition can require distributed social evidence.

Narrative value can require first-person interpretation.

Ecological resilience can require long-term modeling.

The evidentiary architecture should therefore be value-specific.

21.162 Value Objects and Measurement Error

Let measured value proxy be

$$\hat{V} = V + \epsilon_V. \quad (1938)$$

Equation 1938 is a simple representation of measurement error.
For complex values, error can be structural rather than merely numerical.
The wrong proxy can measure another object entirely.

21.163 Proxy Validity

Let proxy P represent value object O^V .

Validity can be represented by

$$\text{Val}(P, O^V). \quad (1939)$$

Equation 1939 should be treated as an empirical and conceptual question.
Salary can be a proxy for material security and a weak proxy for professional recognition.

21.164 Proxy Drift

The relation between proxy and value can change after institutional intervention.

Let

$$\text{Val}_t(P, O^V) \neq \text{Val}_{t+1}(P, O^V). \quad (1940)$$

Equation 1940 is especially relevant under reflexive governance.
A metric once correlated with genuine restoration can become gamed.

21.165 Value Object Conflict Matrix

For case-specific value objects $O_1^V, \dots, O_n^V$, define conflict matrix

$$C_{ij} = \text{Conflict}(O_i^V, O_j^V). \quad (1941)$$

Equation 1941 can record whether interventions improving one value tend to burden another.
The matrix can be qualitative.
Its purpose is to make trade-offs explicit before optimization.

21.166 Value Object Complementarity Matrix

Similarly,

$$K_{ij} = \text{Comp}(O_i^V, O_j^V) \quad (1942)$$

can record complementarities.

A high complementarity between housing security and rehabilitation, for example, can justify coordinated intervention.

21.167 Value Architecture

The complete case-specific value architecture can be represented by

$$\mathfrak{V}_R = (\mathfrak{O}_V^*, \mathfrak{M}_V, E_V, G_V, C_V, K_V, \mathcal{P}_V), \quad (1943)$$

where:

- $\mathfrak{O}_V^*$ is the selected value-object family;
- $\mathfrak{M}_V$ contains modes of valuation;
- E_V contains comparative relations;
- G_V contains dependency relations;
- C_V contains value conflicts;
- K_V contains complementarities;
- $\mathcal{P}_V$ contains provenance of value selection.

Equation 1943 is a structured normative interface rather than one value function.

21.168 Value-Object Explicitness Principle

The first principle of this section is:

A restorative theory should identify which states, resources, capabilities, relations, processes, histories, or future possibilities it treats as bearers of normative value before it aggregates or optimizes them.

21.169 Plural Representation Principle

The second principle is:

Heterogeneous value objects should remain represented plurally where no justified common metric or ordering has been established.

21.170 Commensuration Transparency Principle

The third principle is:

Where an institution translates heterogeneous losses into a common measure, the translation rule and the value dimensions it omits should remain explicit.

21.171 Non-Cancellation Principle

The fourth principle is:

A gain in one value dimension should not be presumed to cancel a loss in another without a justified rule of commensurability or trade-off.

21.172 Present-Agency Value Principle

The fifth principle is:

The present subject's agency is a value object independent of counterfactual historical fidelity and should therefore be represented explicitly where restorative design affects future direction.

21.173 Generativity Neutrality Principle

The sixth principle is:

Generative capacity and resilience acquire positive normative significance only relative to what is generated, how it is generated, for whom, under whose control, and with which external effects.

21.174 Historical Value Principle

The seventh principle is:

Present equivalence does not exhaust value where provenance, recognition, historical truth, lost time, or narrative continuity remain normatively significant.

21.175 Institutional Scope Principle

The eighth principle is:

An institution's value set should reflect its legitimate mandate without being mistaken for the complete normative structure of the restorative case.

21.176 Value Revisability Principle

The ninth principle is:

Case-specific value architectures should remain revisable where subjects, evidence, institutions, and future conditions change.

21.177 Value Layer of the Atlas

The value layer can now be represented as

$$\mathfrak{V}_A = (\mathfrak{D}_V, \mathfrak{M}_V, \mathfrak{R}_V), \quad (1944)$$

where:

- $\mathfrak{D}_V$ contains value objects;
- $\mathfrak{M}_V$ contains modes of valuation;
- $\mathfrak{R}_V$ contains admissible forms of value representation and comparison.

Equation 1944 adds an explicitly normative layer to the formal comparison architecture.

21.178 Implications for Value Theories

The taxonomy developed in this section reveals that normative disagreement can arise at several levels. Theories can disagree about:

- which objects possess value;
- whether value is intrinsic, instrumental, constitutive, or relational;
- whether values are commensurable;
- whether aggregation is legitimate;
- whether rights function as objectives or constraints;
- whether equality, sufficiency, priority, or dominance should govern comparison;
- whose standpoint determines value;
- how present preferences relate to adaptive or counterfactual preferences;
- how individual values relate to collective and intergenerational values.

These disagreements cannot be resolved merely by making the formal restorative model more detailed.

21.179 Transition to Value Theories and Comparative Standards

Section 22 therefore compares the principal normative frameworks through the value objects and comparative standards they privilege.

The analysis will examine, among others:

- welfare and consequentialist approaches;
- resource-based equality;
- capability approaches;
- relational equality;
- recognition;
- republican non-domination;
- rights;
- sufficiency;
- priority;
- care-oriented approaches;
- corrective and reparative justice;
- generative justice;
- pluralist and incommensurability-sensitive approaches.

The purpose will not be to select one universal theory prematurely. It will map how different theories populate the Atlas of Equivalence by selecting different value objects, baselines, comparative relations, aggregation rules, and constraints.

Only after that mapping can the manuscript turn systematically to substantive justice and procedural justice in the later sections.

Table 15: Principal families of value objects in restorative analysis

Value-object family	Illustrative objects	Restorative significance	Characteristic risk of reduction
Welfare and state	Health, safety, experienced well-being	Captures current condition	Can ignore freedom and provenance
Resources	Income, wealth, housing, infrastructure	Supports restitution and distribution	Conversion differences can be hidden
Capabilities	Substantive opportunity, viable futures	Captures freedom to achieve	Selection of capability dimensions remains normative
Agency and autonomy	Choice, revision, exit, direction	Protects subject-directed future	Can conflict with historical fidelity
Relations	Trust, care, mentorship, membership	Captures non-fungible social value	Difficult to quantify or substitute
Relational standing	Equality, dependency, domination	Captures structure of social relations	Can be hidden by resource equality
Recognition	Status, acknowledgment, authorship	Captures social meaning and standing	Symbolic repair can be mistaken for complete repair
Rights	Bodily integrity, privacy, due process	Constrains permissible restoration	Difficult to aggregate with ordinary benefits
Control	Ownership, governance, decision rights	Captures authority over generative conditions	Formal control can differ from effective control
Epistemic values	Knowledge, access to reasons, historical truth	Supports learning and contestability	Can conflict with privacy
Historical values	Provenance, recognizability, memory	Preserves relation to actual history	Historical fidelity can constrain present agency
Process values	Consent, participation, contestability	Evaluates how restoration is achieved	Can be omitted by outcome-only models
Generative values	Capacity, circulation, regeneration, openness	Captures production of future possibility	Generativity can sustain unjust processes
Collective values	Public goods, institutional capacity, commons	Captures non-individual conditions	Aggregation can obscure individual burdens
Ecological values	Function, resilience, biodiversity	Extends restoration beyond human distribution	Economic proxies can omit ecological structure
Intergenerational values	Future conditions, transmission capacity	Captures long-horizon consequences	High epistemic and temporal uncertainty

Table 16: Forms of value representation for restorative analysis

Representation	Formal structure	Analytical use	Characteristic limitation
Scalar	$V \in \mathbb{R}$	Optimization and aggregation	Can compress heterogeneous values
Ordinal	$A \succeq B$	Ranking without cardinal differences	Cannot represent magnitude directly
Vector	$(V_1, \dots, V_n)$	Preserves plural dimensions	Requires later conflict rule
Threshold	$V_i \geq \theta_i$	Sufficiency and protected minima	Above-threshold ranking can remain unresolved
Lexical	$V_1 \succ_{\text{lex}} V_2$	Protects priority values	Restricts trade-offs
Partial order	$(\mathcal{V}, \succeq)$	Allows genuine incomparability	Can leave decisions unresolved
Interval	$V \in [V_{\min}, V_{\max}]$	Represents uncertainty	Requires decision rule under overlap
Set-valued	$V \in \mathcal{V}(O)$	Represents plural admissible evaluations	Can be institutionally complex
Qualitative	Categorical evaluation	Suitable for non-cardinal values	Less amenable to arithmetic aggregation
Temporal profile	$V(t)$	Preserves timing and sequence	Requires temporal comparison rule

22 Value Theories and Comparative Standards

This section maps major normative theories onto the comparative architecture developed in the preceding sections. Section 21 identified the objects capable of carrying normative significance. The present section examines how different theories select among those objects, specify baselines, choose comparative relations, authorize or restrict aggregation, distribute priority, and interpret historical loss.

The role of the section is comparative rather than adjudicative. Welfare, resources, capabilities, equality, sufficiency, priority, relational standing, recognition, rights, non-domination, care, corrective repair, reparative justice, and generative justice illuminate different dimensions of restorative problems. Several theories can support the same intervention for different reasons. A single theory can also support different interventions depending upon the object, baseline, temporal horizon, and institutional function.

The section therefore rejects the assumption that normative disagreement always takes the form of several theories ranking the same scalar outcome. Many disagreements arise earlier. The theories select different value objects. They can also differ concerning who counts, which historical reference matters, whether equality or adequacy matters, which losses permit substitution, whether rights constrain aggregation, and whether the process generating an outcome has independent value.

The analysis has six objectives. First, it represents a normative theory as a structured selector over the Atlas of Equivalence. Second, it maps the major normative families onto value objects and comparative standards. Third, it distinguishes distributive standards such as equality, priority, sufficiency, and maximin-type reasoning from broader theories of justice. Fourth, it examines the treatment of history, process, agency, and generativity within each family. Fifth, it identifies recurring complementarities and conflicts among theories. Sixth, it develops a plural normative architecture through which later sections can analyze substantive and procedural justice without prematurely collapsing them into one master value function.

22.1 Normative Theories as Atlas Selectors

Let $\mathcal{T}^N$ denote a normative theory or a sufficiently specified normative framework. Its operation upon the restorative atlas can be represented by the selector in Equation 1945.

$$\mathcal{S}_{\mathcal{T}^N} : \mathcal{A}_R \longrightarrow (\mathfrak{O}_V^*, \mathfrak{B}^*, \mathfrak{E}_V^*, \mathfrak{P}^*, \mathfrak{C}^*), \quad (1945)$$

where:

- $\mathfrak{O}_V^*$ denotes selected value objects;
- $\mathfrak{B}^*$ denotes selected baselines or references;
- $\mathfrak{E}_V^*$ denotes comparative standards;
- $\mathfrak{P}^*$ denotes priority or aggregation rules;
- $\mathfrak{C}^*$ denotes constraints on admissible action.

Equation 1945 makes the central structure explicit: normative theories organize the atlas by selecting and relating its components.

A theory can therefore differ from another theory along several dimensions even where both use the same measured variable.

22.2 Normative Specification

A case-specific normative specification can be represented by Equation 1946.

$$\mathfrak{N} = (\mathcal{T}^N, O^V, B, E^V, A, C, \tau, s), \quad (1946)$$

where A denotes the aggregation or priority architecture and C the normative constraints. Equation 1946 demonstrates that naming a philosophical tradition does not yet fully specify a restorative decision.

Two capability-oriented analyses, for example, can disagree about which capabilities are relevant, which threshold should be used, whether capability equality is required, and how present preferences influence the target.

22.3 Theories and Comparative Standards

Normative theories should be distinguished from comparative standards.

Equality is a comparative standard.

Sufficiency is a family of standards concerned with having enough.

Priority gives additional normative weight to benefits received by those who are worse off.

Pareto dominance is an ordering relation.

A rights theory can prohibit certain transitions independently of their aggregate outcome.

A capability approach identifies a distinctive informational and evaluative space.

Generative justice emphasizes processes of production, circulation, and control.

These categories operate at different conceptual levels.

Treating them as interchangeable “theories of justice” can obscure the actual source of disagreement.

22.4 Consequentialist and Welfarist Approaches

Consequentialist approaches evaluate acts, institutions, or policies through their consequences. Classical utilitarian traditions give a particularly strong role to aggregate utility or welfare [37]. Contemporary consequentialist theories can employ different accounts of welfare and different aggregation structures.

A simple restorative representation is given by Equation 1947.

$$W = A(w_1, w_2, \dots, w_n), \quad (1947)$$

where w_i denotes the relevant welfare of person i , and A is an aggregation rule. Equation 1947 is deliberately general because utilitarian summation is only one possible consequentialist aggregation.

Within restorative analysis, welfare-oriented approaches can value:

- relief from pain;
- psychological well-being;
- material security;
- future expected welfare;
- third-party effects;
- long-run social consequences.

Their principal analytical strength lies in taking consequences seriously beyond formal return. A remedy that restores property and predictably causes larger future harm can be evaluated through those consequences.

22.5 Consequentialist Restorative Comparison

A simple welfare restorative criterion can compare

$$W(\mathcal{G}^R) \quad \text{and} \quad W(\mathcal{G}^{\text{ref}}). \quad (1948)$$

Equation 1948 permits equality, dominance, or maximization standards.

A better-than-reference outcome can be normatively attractive under this family even where historical fidelity is low.

22.6 Consequentialist Strengths for Restorative Analysis

Consequentialist reasoning contributes several important insights to the justice of return.

First, the remedy itself has consequences.

Second, third-party burdens matter.

Third, timing matters because identical resources can produce different future effects.

Fourth, preventive intervention can be justified partly by expected reduction of future harm.

Fifth, restoration can be evaluated beyond symbolic or formal success.

These features make consequentialist analysis particularly useful for policy design and comparison among feasible interventions.

22.7 Consequentialist Compression

A strong aggregation architecture can compress historical, relational, and distributional differences into one outcome measure.

Suppose intervention R_1 gives welfare vector $\mathbf{w}^1$ and R_2 gives $\mathbf{w}^2$. An aggregate rule can satisfy

$$A(\mathbf{w}^1) > A(\mathbf{w}^2) \quad (1949)$$

despite one person suffering a serious loss under R_1 .

Equation 1949 makes the aggregation issue visible.

Rights, distribution, recognition, and provenance can therefore require independent representation.

22.8 Welfare and Adaptive Preference

Subjective welfare measures can also interact with adaptation.

A person deprived of opportunities can report satisfaction after reducing expectations.

This possibility is one reason capability approaches criticize exclusive reliance upon utility, preference satisfaction, or resources as the evaluative space [55, 50].

Restorative analysis should therefore identify whether welfare represents experienced state, preference satisfaction, informed preference, or another conception.

22.9 Resource-Based Equality

Resource-oriented theories identify holdings, means, or resources as an important space of distributive comparison. Dworkin's equality-of-resources project is a major example of an account that distinguishes resource equality from welfare equality [14].

A resource profile can be represented by

$$\mathbf{R}_i = (r_{i1}, r_{i2}, \dots, r_{ik}). \quad (1950)$$

Equation 1950 can contain income, wealth, entitlements, infrastructure, time, or other means.

A restorative resource-equality criterion can require

$$\mathbf{R}_{A,R} = \mathbf{R}_{A,\text{ref}}. \quad (1951)$$

Equation 1951 supplies a clear standard where the relevant loss concerns fungible or identifiable resources.

22.10 Resource Equality and Institutional Feasibility

Resource standards have strong institutional advantages.

Resources can often be:

- documented;
- transferred;
- budgeted;
- compared;
- administered without deep inquiry into subjective life.

This can make resource-oriented repair epistemically modest.

The limitation appears where persons convert identical resources into substantially different substantive opportunities.

22.11 Resource Equality and Conversion

Let capability conversion be $\mathcal{K}_i$. Equal resources satisfy

$$\mathbf{R}_i = \mathbf{R}_j \quad (1952)$$

while capabilities can satisfy

$$\mathcal{K}_i(\mathbf{R}_i) \neq \mathcal{K}_j(\mathbf{R}_j). \quad (1953)$$

Equations 1952 and 1953 capture the central conversion problem.

The same issue arises diachronically when one subject's conversion structure changes after harm.

22.12 Rawlsian Primary Goods and Institutional Justice

Rawls's justice as fairness evaluates the basic structure of society through a theory of free and equal citizens, equal basic liberties, fair equality of opportunity, and a distributive principle concerning social and economic inequalities [47].

Rawlsian primary goods create a resource-like informational space while the theory contains institutional and lexical structure that cannot be reduced to simple resource equality.

For restorative analysis, the framework is important because it highlights:

- basic liberties;
- institutional opportunity;
- social bases of self-respect;
- distribution across persons;
- priority structures among principles.

A Rawlsian-inspired restorative analysis can therefore evaluate the institutional conditions through which resources and opportunity are secured, rather than comparing material holdings alone.

22.13 Basic Liberties and Restorative Constraints

Where a restorative intervention threatens a protected liberty, higher aggregate benefit does not automatically settle the question.

The relevant architecture can be represented by a constraint set

$$\mathcal{R}_{\text{Rawls--adm}} = \{R : R \text{ satisfies the relevant protected institutional conditions}\}. \quad (1954)$$

Equation 1954 illustrates how a normative theory can restrict the remedy space before distributive comparison begins.

22.14 Capability Approaches

Capability approaches shift attention toward substantive freedoms and the opportunities persons possess to achieve valued ways of being and doing [55, 39, 50].

The central value object is represented by

$$O_{\text{cap}}^V = \mathcal{K}_A. \quad (1955)$$

Equation 1955 makes the subject's capability set, rather than resource possession alone, the principal evaluative space.

This approach is particularly well suited to the justice of return because harm can alter conversion factors.

22.15 Capability Restoration

Possible capability-oriented comparative standards include:

$$\mathcal{K}_{A,R} = \mathcal{K}_{A,\text{ref}}, \quad (1956)$$

$$\mathcal{K}_{A,\text{ref}} \subseteq \mathcal{K}_{A,R}, \quad (1957)$$

and

$$\mathcal{K}_{A,R} \in \mathcal{S}_{\text{cap}}. \quad (1958)$$

Equations 1956 through 1958 represent equality, inclusion, and sufficiency versions of capability-oriented repair.

The capability approach itself does not force one of these comparative relations in every context.

22.16 Capabilities and Subject-Directed Futures

Capability inclusion is especially compatible with the right to a different future developed earlier.

A restored subject can possess every normatively relevant reference capability and additional legitimate possibilities.

This allows historical loss to be acknowledged while avoiding forced return to a counterfactual life plan.

22.17 Capabilities and Conversion Repair

Capability theory also provides a natural explanation for specific enabling support.

Where money no longer converts into the earlier opportunity because health, accessibility, legal status, or social structure changed, a capability-oriented response can justify targeted enabling conditions.

The resulting architecture is naturally plural:

$$\text{general resources} + \text{specific conversion support} + \text{subject direction}. \quad (1959)$$

Equation 1959 summarizes one capability-sensitive form of generative assistance.

22.18 Capability Selection

Capability approaches still face a selection problem.

An institution must decide which capabilities are relevant to the restorative task.

A broad list can become intrusive. A narrow list can omit critical dimensions. A fixed list can fail to reflect context.

This makes participation, public reasoning, and institutional scope important even within a capability framework.

22.19 Egalitarian Standards

Egalitarian approaches give normative significance to equality or to objectionable inequalities.

A generic equality criterion for value object O is

$$V_i(O) = V_j(O). \quad (1960)$$

Equation 1960 is incomplete until the equalisandum is specified.

Sen's influential formulation of the "equality of what" problem directly illustrates the need to identify the evaluative space [55].

Equality of resources, welfare, capability, standing, political rights, and generative control are different normative claims.

22.20 Restorative Equality

Restorative equality can concern comparison between the harmed subject and a reference:

$$V(O_{A,R}) = V(O_{A,\text{ref}}). \quad (1961)$$

Equation 1961 differs from interpersonal equality across persons.

A remedy can restore the claimant to a counterfactual level and increase interpersonal inequality.

Corrective and distributive equality can therefore conflict.

22.21 Interpersonal Equality and Corrective Return

Suppose restitution returns a valuable asset to claimant A and leaves A substantially wealthier than B .

Corrective restoration can favor restitution. An independently distributive theory can favor later taxation or redistribution.

Keeping the two normative questions separate avoids forcing one institution to resolve every dimension simultaneously.

22.22 Relational Equality

Relational egalitarian theories focus upon the character of social relations, including hierarchy, oppression, domination, stigmatization, and standing [2, 71].

The central value object can be represented by

$$O_{\text{rel-eq}}^V = S^R, \quad (1962)$$

where S^R denotes relational standing.

A restorative criterion can require

$$S_{A,R}^R \in \mathcal{S}_{\text{equal-standing}}. \quad (1963)$$

Equation 1963 does not require equal holdings in every material dimension.

It asks whether the subject participates in social relations without objectionable subordination.

22.23 Relational Restoration

A person can receive full financial compensation and remain relationally subordinated.

Examples include:

- continued dependence upon the harasser;
- public stigma despite financial payment;
- nominal participation without equal voice;
- restored access conditional upon humiliating submission.

Relational equality therefore captures dimensions omitted by resource-only and outcome-only standards.

22.24 Relational Equality and Generative Conditions

Relational standing can itself affect generation.

Equal voice changes access to information. Non-subordinate participation changes willingness to invest. Recognition changes network response. Freedom from humiliation can expand effective capability.

Relational equality is therefore both a value object and a causal condition of future generativity.

22.25 Recognition Theory

Recognition theories emphasize the normative significance of social standing, identity, respect, and institutionalized patterns of recognition. Honneth's theory of recognition and Fraser's distinction between redistribution and recognition provide important frameworks for understanding harms that are poorly represented as resource loss alone [25, 20].

Let recognition state be

$$R_A^{\text{rec}}. \quad (1964)$$

A restorative recognition criterion can require

$$R_{A,R}^{\text{rec}} \succeq R_{A,\text{min}}^{\text{rec}}. \quad (1965)$$

Equation 1965 illustrates a sufficiency form of recognition repair.

Other contexts can require correction, public acknowledgment, authorship, or restoration of status.

22.26 Recognition and Redistribution

Recognition and redistribution can interact while remaining distinct.

A false professional accusation can reduce income through reputational damage.

Money can compensate some downstream loss.

Public correction can repair some recognition.

Neither remedy automatically accomplishes the other.

A plural restorative architecture can therefore require both.

22.27 Recognition and Historical Repair

Recognition is especially important where the historical wrong involved denial, erasure, humiliation, or false attribution.

A truthful acknowledgment can be valuable even where every material consequence has already been compensated.

This explains why reparative practice often includes satisfaction, apology, truth finding, and public acknowledgment alongside material remedies [65, 12].

22.28 Republican Non-Domination

Republican theories of freedom emphasize independence from arbitrary power. Pettit's account of freedom as non-domination makes the structure of potential interference normatively relevant even where actual interference is absent [44].

Let arbitrary-control exposure be

$$D_A. \quad (1966)$$

A restorative improvement can require

$$D_{A,R} < D_{A,H}. \quad (1967)$$

Equation 1967 evaluates structural dependence rather than only realized outcomes.

22.29 Non-Domination and Secure Access

A subject can possess access today and remain dominated if another actor can withdraw that access arbitrarily.

Non-domination therefore values secure control, rights, exit, and contestation.

This framework is particularly useful for restorative cases in which the same institution or person who caused the harm retains unilateral control over the remedy.

22.30 Non-Domination and Generative Control

Non-domination intersects strongly with generative justice.

A subject can generate value while another actor retains arbitrary authority over:

- access to productive resources;
- allocation of generated value;
- publication or recognition;
- continuation of the relationship;
- exit.

A process can therefore be highly generative and deeply dominating.

22.31 Rights-Based Approaches

Rights-based approaches give special normative status to protected claims, liberties, powers, immunities, or institutional entitlements.

Within the atlas, rights can operate as direct value objects and as constraints upon admissible restorative interventions.

A general rights constraint can be represented by

$$R \in \mathcal{R}_{\text{rights}} \quad (1968)$$

before other values are aggregated. Equation 1968 formalizes a constraint architecture.

Rawls's priority of basic liberties and Scanlon's contractualist emphasis upon justifiability to persons provide two different routes for resisting unrestricted aggregation of individual claims [47, 51].

22.32 Rights and Restorative Fidelity

A highly faithful restorative intervention can violate rights.

Examples include:

- compulsory memory modification;
- forced reconstruction of a personal relation;
- privacy-invasive monitoring;
- seizure from an innocent third party without due process.

Rights can therefore define boundaries around the pursuit of equivalence.

22.33 Contractualist Justifiability

A contractualist perspective asks whether principles governing a restorative practice can be justified to persons affected by them under the relevant moral standard. Scanlon develops contractualism through principles that persons have reason to accept or reject in the domain of what we owe to one another [51].

For restorative analysis, the value object is partly relational:

$$O_{\text{justification}}^V = J_{\text{interpersonal}} \cdot \quad (1969)$$

Equation 1969 highlights the normative importance of justification among persons rather than aggregate outcome alone.

22.34 Contractualism and Third-Party Burdens

A restorative intervention that substantially benefits one claimant and imposes serious burdens upon an innocent third party requires justification to both.

This structure is especially useful for relational restoration, property reallocation, and institutional restructuring.

The claimant's legitimate historical claim remains important without functioning as an unlimited authorization over others.

22.35 Prioritarian Standards

Prioritarian reasoning gives greater moral weight to benefits received by those who are worse off, without making equality itself the sole source of concern. Parfit's distinction between equality and priority is foundational for this family of views [42].

A simple formal representation is

$$W_P = \sum_{i=1}^n g(w_i), \quad (1970)$$

where g is increasing and concave under one familiar representation. Equation 1970 is illustrative rather than a definition of every prioritarian theory.

The concavity gives additional marginal weight to benefits for those at lower levels of the selected good.

22.36 Priority and Restorative Allocation

Prioritarian reasoning becomes important when restorative resources are scarce.

Suppose two claimants have equally valid claims and one faces a much more severe current deprivation.

A prioritarian allocation can favor the intervention generating the larger weighted improvement for the worse-off claimant.

This can conflict with strict proportionality to historical monetary loss.

22.37 Historical Priority and Present Priority

Two forms of being worse off should be distinguished:

- worse historical loss;
- worse present condition.

A claimant can have suffered the larger historical loss and now be materially secure.

Another can have suffered a smaller attributable loss and now face severe deprivation.

A prioritarian restorative theory must specify which temporal object receives priority.

22.38 Sufficiency Standards

Sufficiency approaches assign distinctive importance to ensuring that persons have enough of the relevant goods. Frankfurt's critique of equality as an independent moral ideal is an important antecedent, while Shields develops a systematic account in which sufficiency reasons have special force around the attainment of enough [19, 57].

Let value object be V_i and threshold θ_i .

A sufficiency condition is

$$V_i \geq \theta_i. \quad (1971)$$

Equation 1971 provides a natural standard for social protection, emergency assistance, safety, and minimum capability.

22.39 Sufficiency and Restorative Justice

Sufficiency is especially useful where urgent institutional purpose concerns stabilization.

A person can require:

- adequate housing;
- basic income security;
- minimum healthcare;
- effective legal access;
- minimum professional viability.

The institution can secure those thresholds before a more complex corrective assessment is complete.

22.40 Sufficiency and Corrective Residuals

Reaching a threshold does not eliminate every corrective claim.

Let

$$V_{A,R} \geq \theta \quad (1972)$$

while

$$V_{A,R} < V_{A,\text{ref}}. \quad (1973)$$

Equations 1972 and 1973 describe a subject who has enough and remains below the relevant corrective reference.

Assistance and corrective justice can therefore reach different conclusions.

22.41 Threshold Selection

A sufficientarian framework must identify:

- the good to which the threshold applies;
- the level of enough;
- whether thresholds vary by context;
- what reasons remain above the threshold.

Shields emphasizes that sufficientarianism concerns a shift in the character or weight of reasons around enough rather than a simple claim that everything above the threshold is morally irrelevant [57].

This distinction is useful for restorative design.

22.42 Lexical Sufficiency

A restorative system can assign priority to securing minimum conditions before pursuing fidelity in secondary dimensions.

A schematic order is

$$\text{safety sufficiency} \succ_{\text{lex}} \text{capability sufficiency} \succ_{\text{lex}} \text{historical fidelity}. \quad (1974)$$

Equation 1974 is illustrative.

Such an ordering can be particularly attractive during emergency response.

22.43 Maximin and Worst-Position Standards

Some distributive reasoning gives special attention to the least advantaged position.

A generic maximin rule is represented by

$$R^* \in \arg \max_R \min_i V_i(R). \quad (1975)$$

Equation 1975 is a generic decision rule and should not be identified mechanically with Rawls's difference principle, which is embedded within a broader theory of justice and institutional priority [47].

Worst-position reasoning can be valuable where catastrophic residual vulnerability deserves special attention.

22.44 Maximin and Restorative Risk

A remedy maximizing average benefit can leave one subject exposed to severe failure.

A maximin-sensitive institution can instead protect the lowest outcome.

This can be relevant to:

- basic safety;
- social protection;
- access to essential healthcare;
- catastrophic ecological risk.

Its conservatism can reduce aggregate gains where extreme worst-case focus dominates every other consideration.

22.45 Pareto Standards

A restorative option R_1 Pareto-dominates R_2 where no affected subject is worse off and at least one is better off according to the selected value space.

The criterion is represented by

$$V_i(R_1) \geq V_i(R_2) \quad \forall i, \quad (1976)$$

with strict inequality for at least one person.

Equation 1976 is attractive where available because it avoids interpersonal sacrifice.

Many restorative cases lie outside the Pareto order because repair shifts resources, authority, or burdens.

22.46 Corrective Justice

Corrective justice focuses upon the normative relation created by wrongful loss and responsibility. Aristotelian and modern corrective traditions differ internally, while contemporary accounts such as Weinrib's and Coleman's provide important analyses of corrective structure and responsibility [11, 69].

Within the restorative atlas, corrective justice naturally selects:

- the claimant;
- the responsible party;
- the wrongful transaction or interaction;
- the loss attributable to that relation;

- a corrective baseline.

A stylized corrective relation can be represented by

$$L_{A \leftarrow B} \xrightarrow{R} C_{A \leftarrow B}, \quad (1977)$$

where $L_{A \leftarrow B}$ denotes loss within the relevant bilateral relation and $C_{A \leftarrow B}$ the corrective response. Equation 1977 is schematic and does not define one particular corrective theory.

22.47 Corrective Baselines

Corrective justice often requires some reference representing the claimant's position absent the wrong. Possible baselines include:

- pre-harm holdings;
- the no-wrong counterfactual;
- legally protected entitlement;
- value wrongfully transferred.

The earlier analysis of baselines shows why these references can diverge. A historical corrective theory therefore requires an explicit baseline theory.

22.48 Corrective Justice and Generative Loss

Generative loss complicates corrective valuation because the wrong can alter future conditions recursively.

A narrowly transactional calculation can capture the initial resource and miss later loss of capability, relation, or opportunity.

A broader corrective analysis can include downstream consequences where the relevant responsibility and evidentiary standards support them.

The challenge is to avoid both arbitrary truncation and unlimited speculative counterfactual extension.

22.49 Corrective Justice and Present Improvement

A claimant can become better off than the no-harm reference after the wrong.

Corrective claims concerning the original wrong can remain conceptually distinct from present welfare.

This follows directly from the historical residual analysis in Section 20.

22.50 Restorative Justice Traditions

Restorative justice traditions emphasize harm, responsibility, participation, repair, relationships, and processes of addressing wrongdoing. Zehr and Braithwaite represent influential strands of this literature [72, 7].

Within the atlas, restorative justice can select:

- harm experienced by affected persons;
- relationships;
- accountability;

- voice;
- acknowledgment;
- repair processes.

This orientation is particularly useful for procedural and relational aspects of the justice of return.

22.51 Restorative Justice and Consent

Relational repair should not be interpreted as compulsory reconciliation.

The value object can be acknowledgment or accountability without requiring restoration of a prior interpersonal relationship.

This distinction is crucial where the earlier relationship was unsafe or dominating.

22.52 Moral Repair

Walker develops moral repair as a problem concerning damaged moral relations, responsibility, trust, and normative expectations [68].

This framework is especially relevant where a wrong damages confidence in the shared normative order.

Material compensation can then be necessary and insufficient.

The relevant value objects can include:

- acknowledgment;
- trustworthiness;
- responsibility;
- normative expectation;
- restored moral standing.

22.53 Reparative Justice

Reparative justice in contexts of gross violations and historical wrong often combines several remedy families. International reparations practice recognizes restitution, compensation, rehabilitation, satisfaction, and guarantees of non-repetition [65, 12].

This practice is normatively significant for the present project because it already rejects the idea that one form of return exhausts repair.

Each remedy family selects different value objects.

22.54 Restitution

Restitution privileges identifiable status, property, liberty, employment, citizenship, or another condition capable of return.

Its characteristic comparative relation is often identity or reinstatement.

Restitution is strongest where the object remains identifiable and usable.

Its power declines where the surrounding generative system has changed.

22.55 Compensation

Compensation uses general equivalents to respond to loss.

Its strengths include fungibility, administrability, and subject discretion.

Its limitations arise from:

- singular value;
- low-fungibility relations;
- changing conversion structures;
- historical meaning;
- disputed valuation.

Compensation therefore constitutes one comparative grammar among several.

22.56 Rehabilitation

Rehabilitation naturally selects function, health, capability, and conversion as value objects.

Its standard can be sufficiency, improvement, functional equivalence, or subject-directed capacity.

A rehabilitation theory centered exclusively on prior function can become paternalistic where the subject now values another future.

22.57 Satisfaction

Satisfaction measures such as acknowledgment, apology, truth, and memorialization select recognition, historical truth, and social meaning as value objects.

They demonstrate that repair can concern the relation to the past rather than only present distribution.

22.58 Guarantees of Non-Repetition

Guarantees of non-repetition select future risk, institutional capacity, resilience, rights, and causal mechanism as value objects.

Their orientation is structurally future-directed.

They therefore connect reparative justice with prevention and governance.

22.59 Historical Justice

Historical injustice theories ask how past wrong can generate present claims despite changes in persons, institutions, property, and circumstances. Waldron and Thompson develop influential but different approaches to the persistence, supersession, and intergenerational significance of historical claims [67, 62].

Within the atlas, historical justice naturally emphasizes:

- provenance;
- inherited benefit and burden;
- institutional continuity;
- historical recognizability;
- intergenerational transmission;
- current consequences of historical structure.

Historical justice therefore requires a temporal architecture richer than present-state distribution alone.

22.60 Historical Claims and Supersession

A historical claim can change in normative force as:

- circumstances change;
- third-party reliance develops;
- descendants replace original actors;
- restitution becomes infeasible;
- present structure continues the original injustice.

The atlas can represent these changes without assuming either permanent unchanged entitlement or automatic disappearance through time.

22.61 Care-Oriented Approaches

Care ethics and political theories of care emphasize dependency, vulnerability, responsibility for meeting needs, relational practices, and the social organization of caring labor. Tronto's political theory of care places caring responsibilities at the center of democratic and institutional analysis [63].

Within restorative analysis, a care-oriented framework can select:

- concrete needs;
- dependency relations;
- responsiveness;
- continuity of support;
- burden upon caregivers;
- voice of care receivers;
- institutional distribution of care.

This orientation can reveal dimensions obscured by abstractions centered upon independent resource holders.

22.62 Care and Historical Subjects

Care is particularly relevant where harm changes the subject's dependence structure.

A person who previously required little institutional support can later require continuing assistance.

Justice then concerns quality and distribution of dependence as well as whether the subject eventually becomes independent.

This insight complements the earlier distinction between legitimate support and dominating dependency.

22.63 Care and Responsiveness

A care-oriented restorative process must remain responsive to changing needs.

Let need profile be

$$N_A(t). \quad (1978)$$

Equation 1978 makes care temporally dynamic.

A fixed remedy established at t_0 can become inappropriate as $N_A(t)$ changes.

22.64 Care and Institutional Burden

Care theory also makes visible who performs the restorative work.

A policy can appear generous while transferring unpaid care burdens to family members.

A complete restorative evaluation should therefore include the distribution of care labor across actors.

22.65 Generative Justice

Generative justice emphasizes bottom-up forms of production in which value can circulate back to the communities, actors, and systems generating it. Eglash's work develops generative justice through unalienated value circulation, and later work explicitly connects computational reparations with the capacity to become generative again [15, 17].

The principal value objects include:

- generative conditions;
- productive autonomy;
- control;
- value circulation;
- recursive return of value;
- community or participant capacity.

Generative justice therefore occupies a process-oriented region of the atlas.

22.66 Generative Justice and Return

The present manuscript extends the return problem by asking which comparative relation defines becoming generative again.

Possible standards include:

- return of productive resources;
- restoration of conversion capacity;
- restoration of value circulation;
- restoration of control;
- simulation of the reference generative process;
- dominance through a newly transformed process.

These are different meanings of generative return.

22.67 Generative Justice and Attractors

Eglash and Garvey have already introduced attractor and basin structures into generative justice [16].

The present atlas therefore does not treat dynamical language itself as the novel contribution.

Its additional contribution lies in comparing attractor/basin standards with alternative objects and relations, including capability, causal response, historical provenance, procedure, and recursive process equivalence.

22.68 Generative Justice and Normative Filtering

Generativity itself cannot function as the sole criterion of justice.

An extractive system can be highly capable of reproducing itself.

A dominating organization can generate large amounts of value.

The normative structure therefore requires

$$\Gamma^G \in \mathfrak{G}^N \quad (1979)$$

before generative capacity receives positive evaluative weight. Equation 1979 makes generativity conditional upon the selected normative constraints.

22.69 Generative Justice and Capability

Generative justice and capability theory are naturally complementary.

Capability asks which substantive freedoms a subject possesses.

Generative justice asks how systems produce, control, circulate, and reproduce the conditions underlying those freedoms.

A combined object can therefore be represented by

$$O_{GK}^V = (\mathcal{K}_A, \Gamma^G, \Lambda, \chi) . \quad (1980)$$

Equation 1980 combines substantive opportunity with the process generating it.

22.70 Generative Justice and Non-Domination

Generative justice and non-domination also converge around control.

A community can receive circulating value while one external actor retains arbitrary authority over the productive infrastructure.

Value circulation alone therefore does not settle the justice of control.

The combined standard can require both generative circulation and secure non-dominating governance.

22.71 Generative Justice and Recognition

Generation can be misrecognized.

Actors can create cultural, epistemic, or economic value while institutions attribute the value to another actor.

A generative-reparative framework can therefore include attribution and recognition alongside material circulation.

22.72 Pluralist Value Theories

Pluralist theories resist reduction of every value to one master measure. Anderson's account emphasizes different modes of valuation, Raz connects value pluralism with autonomy and incommensurability, and Chang's edited volume maps major philosophical problems concerning incommensurability and incomparability [1, 48, 10].

Pluralism fits naturally with the restorative atlas because the case can contain simultaneously:

- material value;
- capability;
- recognition;

- rights;
- historical truth;
- agency;
- generative capacity.

The existence of several values does not determine how conflicts among them should be resolved.

22.73 Pluralism without Scalarization

Let the value profile be

$$\mathbf{V} = (V_1, \dots, V_n). \quad (1981)$$

Equation 1981 can remain unaggregated.
Decision can instead proceed through:

- dominance;
- thresholds;
- lexical constraints;
- partial ordering;
- deliberation;
- procedural choice.

This architecture avoids manufacturing one artificial common currency of every value.

22.74 Incommensurability-Sensitive Standards

Where values lack an adequate common metric, the theory can represent

$$R_1 \parallel_V R_2. \quad (1982)$$

Equation 1982 treats incomparability as a possible result.
Institutional decision can still be necessary.

Procedure then acquires greater importance because the substantive theory alone does not produce a complete ordering.

22.75 Contractual and Deliberative Resolution under Pluralism

Where several normatively admissible solutions remain, a legitimate procedure can select among them.

This does not make procedure a substitute for substantive constraints.

The feasible set is first restricted by rights, thresholds, and relevant normative standards.

Participation and deliberation then operate within the admissible region.

22.76 Ecological Value Theories

Ecological restoration raises a further value question concerning whether ecological systems matter only through human welfare or also through other forms of value.

The present manuscript leaves that metaethical issue open.

The atlas can accommodate:

- anthropocentric ecological value;
- relational environmental value;
- intrinsic ecological value;
- intergenerational value;
- resilience and generativity.

This neutrality is deliberate because the formal framework does not require one environmental ethic.

22.77 Ecological Generativity and Resilience

A theory valuing ecological generativity can select:

$$O_E^V = (E_t, \varrho_E, \Gamma_E^G). \quad (1983)$$

Equation 1983 combines ecological state, resilience, and regenerative process.

Holling's resilience framework provides a reason to distinguish persistent system organization from short-run return to one equilibrium [24].

22.78 Intergenerational Value Theories

Restorative policies can affect persons who do not yet exist at the time of the decision.

Parfit's work on future generations demonstrates the profound difficulties that population and identity raise for ordinary person-affecting intuitions [41].

Within the atlas, intergenerational theories can value:

- inherited resources;
- ecological conditions;
- institutions;
- cultural knowledge;
- future capability;
- avoidance of catastrophic risk.

The relevant comparison can involve a distribution across generations rather than restoration for one present subject.

22.79 Intergenerational Transmission Standards

Let generational value profile be

$$\mathbf{V}^G = (V^{(0)}, V^{(1)}, \dots, V^{(n)}). \quad (1984)$$

Equation 1984 can be compared through sufficiency, priority, sustainability, lexical constraints, or another intergenerational standard.

The choice of standard is normative rather than supplied by the temporal model.

22.80 Future Persons and Historical Repair

A reparative intervention can benefit descendants and burden current persons.

Historical responsibility, inherited advantage, institutional continuity, and future well-being can therefore intersect.

The framework treats these as distinct normative objects whose relationships require justification rather than automatic aggregation.

22.81 Value-Theoretic Treatment of Baselines

Different theories select different baselines.

A corrective theory can use the no-wrong counterfactual.

A sufficientarian theory can use an adequacy threshold.

A relational theory can use equal standing.

A non-domination theory can use secure independence from arbitrary control.

A capability theory can use a capability threshold or reference capability set.

A transformative theory can use a normatively superior future state.

Baseline choice is therefore theory-dependent.

22.82 Baseline Pluralism

A single case can legitimately employ several baselines.

Let

$$\mathbf{B} = (B_{\text{corrective}}, B_{\text{sufficiency}}, B_{\text{agency}}, B_{\text{historical}}) . \quad (1985)$$

Equation 1985 permits different normative questions to use different reference structures.

A claimant can be above the sufficiency threshold and below the corrective reference at the same time.

22.83 Comparative-Relation Selection by Theory

Normative theories also select characteristic comparative relations.

A theory can emphasize:

- equality;
- inclusion;
- sufficiency;
- priority;
- dominance;
- lexical protection;
- simulation;
- historical correspondence.

The relation should be derived from the normative object and theory rather than selected because it is mathematically stronger.

22.84 Equality and Dominance

A restored system can be unequal to the reference and normatively dominate it.

Let

$$Q_R \neq Q_{\text{ref}} \quad (1986)$$

and

$$Q_R \succeq_V Q_{\text{ref}}. \quad (1987)$$

Equations 1986 and 1987 provide the formal structure of transformative improvement. This is especially relevant when the reference itself contains injustice.

22.85 Equality and Sufficiency

A subject can be unequal and adequately situated.

Let

$$V_A \neq V_B \quad (1988)$$

while

$$V_A \geq \theta \quad \text{and} \quad V_B \geq \theta. \quad (1989)$$

Equations 1988 and 1989 distinguish egalitarian from sufficientarian evaluation.

22.86 Equality and Priority

Priority can favor improving the worse-off even where the intervention increases inequality.

Parfit's distinction between egalitarian and prioritarian reasoning is valuable precisely because concern for those who are worse off need not derive from the badness of inequality itself [42].

The atlas therefore records equality and priority as separate comparative standards.

22.87 Sufficiency and Priority

Sufficiency and priority can also diverge.

A sufficientarian architecture can assign especially strong weight to moving persons above a threshold.

A prioritarian architecture can continue assigning extra weight to gains for worse-off persons above that threshold.

Hybrid theories are therefore possible.

22.88 Rights and Welfare

Rights can restrict welfare-enhancing interventions.

Let R_1 satisfy

$$W(R_1) > W(R_2) \quad (1990)$$

while R_1 violates a protected right.

A rights-constrained architecture can rank R_2 as the only admissible option before comparing aggregate welfare.

This demonstrates how constraints alter the feasible restorative set.

22.89 Agency and Welfare

An intervention can increase welfare by directing a subject toward a life plan the subject rejects.

Where agency has independent value, the welfare gain does not settle the decision.

This is especially important in idealized counterfactual reconstruction.

22.90 Recognition and Welfare

A private financial settlement can maximize immediate welfare while a subject also values public correction.

Recognition theory explains why the public standing dimension can remain unresolved despite a favorable welfare result.

22.91 Care and Autonomy

Care and autonomy can appear to conflict when protective support limits choice.

A richer architecture asks whether care expands the subject's effective capacity for future agency.

The relation can therefore be complementary rather than inherently opposed.

22.92 Generativity and Welfare

A welfare-improving intervention can reduce long-run generative capacity.

A one-time transfer can raise consumption and leave the underlying productive or institutional process unchanged.

Conversely, an investment in generative infrastructure can impose short-run cost and improve long-run opportunity.

Temporal horizon therefore mediates the relation between welfare and generativity.

22.93 Generativity and Equality

A policy increasing total generativity can distribute the gains unequally.

Let total generative value satisfy

$$G_R > G_{\text{ref}} \quad (1991)$$

while distribution becomes more unequal.

Equation 1991 does not determine justice.

Generative justice therefore requires distributive and control dimensions alongside productive capacity.

22.94 Generativity and Sufficiency

A minimum level of generative capability can be treated as a sufficiency threshold:

$$G_i \geq G_{\text{min}}. \quad (1992)$$

Equation 1992 can support assistance designed to restore self-directed future possibility without demanding equality of all generative capacities.

22.95 Generativity and Non-Domination

Generative capacity under arbitrary control can be structurally unjust.

A combined criterion can require

$$G_i \geq G_{\min} \quad \text{and} \quad D_i \leq D_{\max}. \quad (1993)$$

Equation 1993 combines adequate generative capacity with bounded domination.

This illustrates the usefulness of hybrid normative architectures.

22.96 Corrective Justice and Capability

Corrective justice and capability theory answer different questions.

Corrective justice asks what is owed because of the wrong.

Capability theory asks whether substantive opportunity is adequate or restored.

A response can therefore include:

$$R = R_{\text{corrective}} + R_{\text{capability—support}}, \quad (1994)$$

where Equation 1994 denotes a conceptual bundle rather than arithmetic commensuration.

The components can be delivered by different institutions.

22.97 Corrective Justice and Sufficiency

Emergency support can be justified before corrective liability is determined.

Later corrective compensation can then address attributable historical loss.

This temporal combination prevents the need for assistance from depending upon completion of adversarial responsibility proceedings.

22.98 Corrective Justice and Recognition

A damages award can correct financial loss and leave public misrecognition untouched.

A declaratory judgment or acknowledgment can respond to the recognition dimension.

The two remedies occupy different value cells.

22.99 Reparative Justice and Generative Justice

Reparative justice can return resources, recognize wrong, rehabilitate subjects, and reform institutions.

Generative justice adds a process-level question concerning whether the repaired system can sustain just production and circulation into the future.

A strong reparative architecture can therefore include:

$$\text{repair of loss} + \text{recognition} + \text{future generative conditions}. \quad (1995)$$

Equation 1995 represents a layered normative structure.

22.100 Theory-Specific Treatment of Historical Change

The theories also differ in their treatment of a historically changed subject.

A strict counterfactual corrective approach can privilege the no-harm trajectory.

A capability approach can privilege current substantive freedoms.

A care approach can privilege current need and dependency.

A non-domination approach can privilege secure future independence.

A recognition approach can privilege truthful acknowledgment of the actual history.

A generative approach can privilege the structure through which future possibility is produced.

These orientations can all be relevant simultaneously.

22.101 Theory-Specific Treatment of the Present Subject

The present subject is therefore located differently across theories.

Let the normative weight assigned to present agency under theory $\mathcal{T}$ be

$$\omega_A^{\mathcal{T}}. \quad (1996)$$

Equation 1996 is conceptual.

Its purpose is to show that theories vary in how directly present choice enters their evaluative structure.

A plural restorative framework should make that difference explicit.

22.102 Theory-Specific Treatment of Third Parties

Third-party effects also enter differently.

Consequentialism can include third-party welfare directly.

Rights theories can prohibit some burdens.

Contractualism asks whether principles are justifiable to each affected person.

Relational equality can examine new hierarchies created by repair.

Corrective justice can focus more narrowly upon the claimant-responsible-party relation.

No single treatment follows automatically from the formal atlas.

22.103 Theory-Specific Treatment of Institutions

Normative theories also differ in institutional focus.

Rawlsian justice concerns the basic structure.

Corrective justice can focus upon bilateral wrong.

Care theory emphasizes institutions distributing dependency and caring labor.

Generative justice emphasizes productive and circulatory systems.

Non-domination emphasizes structures of power.

Recognition theory examines institutionalized status.

The object of justice therefore changes with the theory.

22.104 Theory-Specific Treatment of Process

Some theories treat procedure mainly through its consequences.

Others assign process independent significance.

A contractualist or rights-based theory can reject a process despite favorable aggregate outcome.

Care theory can value responsiveness within the process.

Relational equality can value equal standing during decision making.

Generative justice can value control over production itself.

This plurality prepares the distinction between substantive and procedural justice developed later.

22.105 Normative Theory Matrix

Table 17 summarizes the principal theoretical families discussed in this section. The entries identify characteristic tendencies rather than exhaustive definitions.

Table 17 demonstrates that the theories populate different regions of the atlas.

22.106 Comparative-Standard Matrix

Table 18 separates several comparative standards that can be used inside multiple theories.

Table 18 shows that one comparative standard can appear in several broader theories.

Table 17: Value theories and characteristic restorative standards

Normative family	Primary value objects	Characteristic comparison	Typical reference structure	Characteristic restorative concern
Welfare / consequential	Well-being, outcomes, effects	Aggregation / dominance	Feasible alternatives or counterfactual outcome	Net consequences across affected persons
Resource-based	Resources, holdings, means	Equality / compensation	Pre-harm or distributive resource position	Return or redistribution of means
Rawlsian institutional	Liberties, opportunity, primary goods, institutional standing	Lexical protection and distributive standards	Just basic structure	Institutional conditions for free and equal citizens
Capability	Substantive freedoms, viable possibilities	Equality / inclusion / sufficiency	Reference capability set or capability threshold	Effective opportunity after conversion
Egalitarian	Selected equalisandum	Equality	Relevant comparator distribution	Objectionable inequality
Prioritarian	Selected welfare or advantage	Priority-weighted improvement	Distribution of advantage	Greater weight for worse-off persons
Sufficientarian	Basic goods, capabilities, security	Threshold	Adequacy level	Securing enough
Relational equality	Standing, hierarchy, social relation	Equal-standing adequacy	Non-subordinate social relation	Oppression, hierarchy, stigma
Recognition	Status, acknowledgment, authorship	Correction / sufficiency / dominance	Appropriate social recognition	Misrecognition and status injury
Non-domination	Secure independence, control, exit	Domination reduction / secure sufficiency	Non-arbitrary relation	Exposure to discretionary power
Rights / contractualist	Rights, claims, interpersonal justification	Constraint / justifiability	Protected right or justifiable principle	Limits on aggregate sacrifice
Corrective	Wrongful loss, entitlement, responsibility	Return / compensation	No-wrong position or protected entitlement	Rectification of attributable wrong
Restorative / moral repair	Harm, relationships, accountability, trust	Repair / participation / acknowledgment	Damaged normative relationship	Repair of harm and moral relation
Reparative	Restitution, compensation, rehabilitation, recognition, reform	Plural remedy bundle	No-wrong, adequate, and transformed references	Multi-dimensional response to serious wrong
Care	Need, dependency, responsiveness, care relations	Need-sensitive sufficiency / responsiveness	Adequate caring relation	Concrete dependency and caring responsibility
Generative justice	Control, value circulation, regenerative capacity	Process correspondence / dominance	Just generative process	Future production and circulation of value
Pluralist	Heterogeneous value objects	Partial order / thresholds / lexical rules	Multiple value-specific references	Preservation of irreducible value plurality

22.107 Theoretical Complementarity

Several theories can be complementary because they answer different questions.

Consider a worker harmed through prolonged exclusion.

Corrective justice asks what loss resulted from the wrongful exclusion.

Capability theory asks which substantive professional opportunities remain.

Recognition theory asks whether professional standing was repaired.

Non-domination asks whether one actor still controls the worker's future.

Care asks which present dependencies and needs require support.

Generative justice asks whether work now creates future opportunity and control.

No conceptual contradiction follows from asking all of these questions.

22.108 Theoretical Conflict

Conflict arises when theories recommend incompatible actions.

A counterfactual corrective standard can support reinstatement.

Present agency can support exit with resources for a new career.

Recognition can support public vindication.

Privacy can support limited disclosure.

Historical truth can support archival preservation.

These conflicts require substantive and procedural adjudication.

Table 18: Comparative standards used across normative theories

Standard	Formal orientation	Restorative use	Characteristic limitation
Equality	$V_R = V_{\text{ref}}$	Restoring comparable value or status	Depends upon selected equalisandum
Inclusion	$\mathcal{K}_{\text{ref}} \subseteq \mathcal{K}_R$	Preserving reference options while permitting expansion	Additional options can be low-value or burdensome
Sufficiency	$V_R \geq \theta$	Securing adequate conditions	Threshold selection remains normative
Priority	Greater weight to gains for worse-off persons	Scarce restorative resource allocation	Requires specification of worse-off dimension
Dominance	$\mathbf{V}_R \succeq \mathbf{V}_{\text{ref}}$	Transformative improvement	Often yields incompatibility
Lexical priority	Protected dimension considered first	Rights, safety, or process constraints	Can prohibit otherwise attractive trade-offs
Pareto dominance	No one worse off and some one better off	Robust comparison among feasible remedies	Frequently incomplete in redistributive cases
Maximin	Maximize minimum represented position	Protection against severe worst-position outcomes	Can be highly conservative
Compensation	Equivalent value under selected conversion	General-equivalent response to loss	Translation can omit singular values
Historical correspondence	Selected provenance or trajectory preservation	Corrective and historical repair	Can conflict with present agency
Generative dominance	Improved future generative process	Future-directed transformation	Requires normative filtering of generativity

The atlas identifies their location without pretending that classification alone resolves them.

22.109 Hybrid Normative Architectures

A restorative system can combine several theories through layered rules.

One possible architecture is

$$\mathfrak{N}_H = (C_{\text{rights}}, S_{\text{basic}}, J_{\text{corrective}}, K_{\text{capability}}, D_{\text{nondomination}}, G_{\text{generative}}), \quad (1997)$$

where:

- C_{rights} defines admissibility constraints;
- S_{basic} secures minimum sufficiency;
- $J_{\text{corrective}}$ evaluates historical wrong;
- $K_{\text{capability}}$ evaluates substantive future opportunity;
- $D_{\text{nondomination}}$ evaluates arbitrary dependence;
- $G_{\text{generative}}$ evaluates future recursive capacity.

Equation 1997 illustrates theory combination without scalar aggregation.

22.110 Lexically Layered Hybrid Architecture

A hybrid architecture can impose an order such as

$$\text{rights} \succ_{\text{lex}} \text{basic sufficiency} \succ_{\text{lex}} \text{corrective claims} \succ_{\text{lex}} \text{discretionary optimization}. \quad (1998)$$

Equation 1998 is illustrative.

Another theory can use a different order.

The value of the formal representation lies in making the priority structure visible.

22.111 Threshold-Constrained Optimization

Another architecture first secures thresholds and then optimizes within the admissible region.

Let

$$\mathcal{A}^* = \{ R : V_i(R) \geq \theta_i \forall i \in I_{\text{protected}} \}. \quad (1999)$$

Equation 1999 defines the protected set.

The institution can then select

$$R^* \in \arg \max_{R \in \mathcal{A}^*} W(R). \quad (2000)$$

Equation 2000 illustrates how consequentialist optimization can operate inside rights or sufficiency constraints.

22.112 Partial-Order Hybrid Architecture

A non-aggregative architecture can compare remedies through a partial order:

$$R_1 \succeq_{\mathfrak{N}} R_2 \quad (2001)$$

only where R_1 is at least as acceptable across every protected dimension under the selected framework.

Equation 2001 permits unresolved cases.

Procedure can then choose among nondominated alternatives.

22.113 Normative Robustness Across Theories

Where several plausible theories support the same intervention, the result has a form of normative robustness.

Let theory family be

$$\mathfrak{T}^N = \{ \mathcal{T}_1, \dots, \mathcal{T}_k \}. \quad (2002)$$

The common acceptable intervention set is

$$\mathcal{R}_{\cap} = \bigcap_{j=1}^k \mathcal{R}_{\text{adm}}^{\mathcal{T}_j}. \quad (2003)$$

Equation 2003 identifies remedies acceptable across the selected normative family.

Such agreement can support action despite unresolved philosophical disagreement.

22.114 Normative Disagreement Region

The disagreement region is

$$\mathcal{R}_{\Delta} = \bigcup_j \mathcal{R}_{\text{adm}}^{\mathcal{T}_j} \setminus \mathcal{R}_{\cap}. \quad (2004)$$

Equation 2004 contains interventions whose status depends upon the theory selected. These cases require explicit normative justification or legitimate procedural resolution.

22.115 Overlapping Normative Reasons

One intervention can receive several independent justifications.

For example, anti-retaliation protection can:

- increase welfare;
- expand capability;
- reduce domination;
- improve relational equality;
- protect procedural voice;
- preserve future generativity.

This convergence strengthens the case for the intervention without requiring all theories to become one theory.

22.116 Normative Redundancy

Several independent reasons can support the same minimum requirement.

A basic income floor can be defended through welfare, sufficiency, capability, care, or prevention of domination.

Normative redundancy can increase robustness of institutional justification.

The reasons should remain distinguishable because they can diverge in later cases.

22.117 Normative Conflict Graph

The relationships among theories can be represented through a conflict graph:

$$G_N = (\mathfrak{T}^N, E_N), \quad (2005)$$

where an edge indicates recurrent conflict over a selected restorative dimension.

Equation 2005 is an analytical device rather than a claim that philosophical relations are naturally graph-theoretic.

It can help identify where procedural institutions need explicit priority rules.

22.118 Normative Translation

A practical institution often translates one theory's concern into another institutional language.

For example:

- recognition harm becomes damages;
- capability loss becomes accommodation;

- non-domination becomes enforceable procedural protection;
- generative loss becomes funding or infrastructure.

Translation enables institutional action and can lose theoretical content.
The translation should therefore preserve the original normative provenance.

22.119 Normative Translation Loss

Let concern under theory $\mathcal{T}_1$ be C_1 and institutional representation be $\hat{C}_1$.

Translation loss is represented conceptually by

$$L_{\mathcal{T}} = C_1 - \text{Reconstruct}(\hat{C}_1). \quad (2006)$$

Equation 2006 is schematic.

It reminds the institution that monetary or administrative translation can capture only part of the original value claim.

22.120 Theory and Institutional Competence

Some theories require information or authority beyond one institution's competence.

A court can determine legal rights and damages.

It may be poorly situated to optimize an individual's long-term relational network.

A social-protection body can secure basic sufficiency.

It may lack authority to determine corrective liability.

A restorative architecture should therefore distribute normative functions rather than forcing one institution to implement one complete moral theory.

22.121 Theory and Epistemic Burden

Normative theories differ in their epistemic demands.

Resource equality can require relatively observable information.

Capability analysis requires knowledge of conversion and substantive opportunity.

Welfare maximization can require interpersonal welfare estimation.

Historical corrective justice can require counterfactual reconstruction.

Generative justice can require process and circulation analysis.

The strength of a theory's moral insight should therefore be distinguished from the feasibility of implementing it institutionally.

22.122 Theory and Data Burden

A more comprehensive normative model can require more personal data.

The information required to estimate welfare, capability, relation, health, network, and historical trajectory can become extremely intrusive.

Normative richness therefore creates a possible privacy cost.

Data minimization belongs inside the normative design rather than outside it.

22.123 Theory and Strategic Behavior

Institutionalization of a normative standard can change behavior.

If compensation depends upon measured welfare loss, claimants and institutions can orient evidence toward welfare measures.

If success depends upon employment restoration, organizations can optimize formal reinstatement.

If funding depends upon generativity scores, actors can optimize the score.

Theories therefore acquire reflexive effects when converted into metrics.

22.124 Theory and Revisability

The active normative architecture should remain revisable where experience reveals omitted values or harmful incentives.

Let normative specification evolve according to

$$\mathfrak{N}_{t+1} = \mathcal{U}(\mathfrak{N}_t, D_{t+1}, P_{t+1}), \quad (2007)$$

where D_{t+1} denotes new evidence and P_{t+1} evaluative and procedural feedback.

Equation 2007 makes normative institutional learning explicit.

22.125 Theoretical Modesty

A comprehensive atlas does not require one institution to adopt a complete philosophical synthesis.

Institutional theories can remain modest.

A court can state that a legal remedy satisfies a particular corrective standard.

A welfare agency can state that basic sufficiency has been secured.

Neither needs to declare the claimant fully restored in every morally relevant sense.

Theoretical modesty therefore supports more precise institutional language.

22.126 Normative Layering

The combined framework can distinguish at least four normative layers:

1. admissibility constraints;
2. minimum conditions;
3. historical or corrective claims;
4. future-oriented improvement.

A case can therefore be represented by

$$\mathfrak{N}_L = (C, S, J_H, F), \quad (2008)$$

where C denotes constraints, S sufficiency, J_H historical justice, and F future-oriented evaluation.

Equation 2008 is one possible architecture rather than a universal hierarchy.

22.127 Admissibility Layer

The admissibility layer can include:

- basic rights;
- consent;
- privacy;
- third-party protections;
- prohibition of domination.

Interventions outside this region are excluded or subjected to especially strong justification.

22.128 Sufficiency Layer

The sufficiency layer asks whether urgent basic conditions have been secured.

This layer is particularly important before prolonged litigation or counterfactual estimation is complete.

It aligns naturally with assistance.

22.129 Historical-Justice Layer

The historical layer asks what is owed because of the wrong, including:

- restitution;
- compensation;
- recognition;
- correction;
- responsibility.

Its baseline can differ from the sufficiency threshold.

22.130 Future-Oriented Layer

The future layer concerns capability, non-domination, care, resilience, generativity, and subject-directed possibility.

The objective can be improvement rather than historical replication.

This layer becomes increasingly important where exact repair is structurally unavailable.

22.131 Normative Atlas Coordinate

The complete restorative coordinate can now be extended to include normative theory:

$$\mathbf{a}_N = (B, Q, E, \mathcal{T}, \tau, \varepsilon, K, s, P, O^V, \mathcal{T}^N) . \quad (2009)$$

Equation 2009 records reference, object, relation, testing architecture, horizon, tolerance, epistemic basis, scale, procedure, value object, and normative theory.

This is the most explicit form of the atlas developed so far.

22.132 Theory-Selection Principle

The first principle of this section is:

A restorative judgment should identify the normative theory or standard through which the selected comparison acquires moral significance.

22.133 Evaluative-Space Principle

The second principle is:

Disagreement about justice can arise from disagreement over the appropriate evaluative space before disagreement over the ranking of outcomes begins.

22.134 Standard-Theory Distinction Principle

The third principle is:

Equality, sufficiency, priority, dominance, compensation, and lexical protection should be distinguished from broader theories that determine the objects and contexts to which those comparative standards apply.

22.135 Plural-Theory Principle

The fourth principle is:

A single restorative case can legitimately implicate several normative theories where the theories govern different value objects or institutional functions.

22.136 Constraint Principle

The fifth principle is:

Some values can constrain the admissible means of restoration independently of the strength of the resulting outcome equivalence.

22.137 Corrective-Sufficiency Distinction Principle

The sixth principle is:

Securing an adequate present condition does not independently settle what is owed for an attributable historical wrong.

22.138 Future-Transformation Principle

The seventh principle is:

Where the historical reference is unjust, obsolete, or rejected by the present subject, normative improvement can require transformation rather than equivalence with the reference.

22.139 Generative Pluralism Principle

The eighth principle is:

Generativity should be evaluated together with distribution, agency, control, recognition, rights, and external effects rather than treated as a master value.

22.140 Normative Robustness Principle

The ninth principle is:

Where several plausible normative theories converge upon an intervention, that convergence can support action without requiring resolution of every underlying philosophical disagreement.

22.141 Normative Humility Principle

The tenth principle is:

An institution should state the normative scope of its judgment and avoid presenting satisfaction of one theory-specific standard as complete justice across the entire atlas.

22.142 Implications for Substantive Justice

The normative mapping developed in this section shows that substantive justice cannot be represented by one universal requirement of maximum restorative equivalence.

Substantive evaluation must determine:

- which values count;
- whose values count;
- which baselines apply;
- whether equality, sufficiency, priority, correction, or dominance is required;
- whether historical claims survive present improvement;
- whether future transformation can legitimately replace historical reconstruction;
- how third-party and collective values enter;
- how conflicts among value objects are resolved.

These are the tasks of the next stage of the manuscript.

22.143 Transition to Substantive Justice

Section 23 develops substantive justice of return. It asks which restorative outcomes, opportunity structures, distributions, relations, histories, and generative processes can count as just after the value theory and comparative standards have been made explicit.

The section will distinguish:

- corrective adequacy;
- distributive adequacy;
- capability restoration;
- relational equality;
- recognition;
- non-domination;
- sufficiency;
- priority;
- generative justice;
- transformative improvement;
- third-party and collective justice.

The principal question will therefore shift from

Which theory values which restorative object?

to the more integrated problem:

Under a specified plural normative architecture, what substantive conditions must a restorative arrangement satisfy to count as just?

23 Substantive Justice of Return

This section develops the substantive-justice layer of the theory of return. The preceding sections identified the objects of restorative comparison, the relations through which those objects can be compared, the temporal and historical horizons of comparison, and the normative theories that give those comparisons moral significance. The present section integrates those components into a direct question:

What conditions must the post-harm arrangement satisfy for its substantive result to count as just?

The answer cannot be reduced to one requirement of returning the claimant to a single reference state. Harm can alter resources, capabilities, relations, recognition, control, bodily conditions, historical position, and future generativity. Different normative theories can legitimately assign significance to different components of that alteration. At the same time, a theory that requires complete equivalence across every imaginable dimension would make substantive justice practically unattainable and normatively excessive.

The present section therefore treats substantive justice as a structured evaluation of the post-harm arrangement rather than as one universal metric of restorative fidelity. Its analysis distinguishes corrective adequacy, distributive adequacy, sufficiency, capability, relational equality, recognition, non-domination, historical repair, generative justice, and transformative improvement. It also incorporates third-party interests, collective justice, intergenerational effects, irreversibility, uncertainty, and institutional feasibility.

The central proposition is that substantive justice is indexed to a specified normative architecture. A restorative arrangement can be just under one bounded institutional purpose while remaining incomplete under another. The appropriate conclusion is therefore often a qualified judgment such as “correctively adequate,” “capability-sufficient,” or “generatively improved,” rather than the unqualified declaration that justice has been fully restored.

23.1 Substantive Justice as an Indexed Judgment

Let a restorative arrangement be R , a value architecture be $\mathfrak{V}$, and a normative architecture be $\mathfrak{N}$.

Substantive justice can be represented by

$$J_S (R \mid \mathfrak{V}, \mathfrak{N}, B, \tau, K), \quad (2010)$$

where B denotes the relevant reference or references, τ temporal horizon, and K epistemic basis. Equation 2010 makes substantive justice an indexed evaluation rather than a context-free Boolean property.

23.2 Substantive Justice and Restorative Fidelity

Restorative fidelity measures correspondence with a reference.

Substantive justice asks whether the resulting arrangement is normatively acceptable.

Let fidelity be F_R and substantive justice be J_S . In general,

$$F_R \uparrow \not\Rightarrow J_S \uparrow. \quad (2011)$$

Equation 2011 follows from several cases already developed:

- the historical reference can itself be unjust;
- strong fidelity can require coercion;
- new valuable futures can be lost through exact reconstruction;

- third-party rights can be burdened;
- historical identity can conflict with present agency.

23.3 Substantive Justice and Procedural Justice

The present section evaluates the substantive result.

The following section will evaluate the process through which that result is reached.

Let

$$J_S(R) \quad (2012)$$

denote substantive justice and

$$J_P(R) \quad (2013)$$

procedural justice.

The two can diverge:

$$J_S(R) \text{ high} \quad \text{and} \quad J_P(R) \text{ low.} \quad (2014)$$

Equation 2014 can describe a highly effective intervention imposed through coercion, surveillance, or denial of voice.

23.4 Dimensions of Substantive Justice

A general substantive profile can be represented by

$$\mathbf{J}_S = (J_{\text{corr}}, J_{\text{dist}}, J_{\text{suff}}, J_{\text{cap}}, J_{\text{rel}}, J_{\text{rec}}, J_{\text{ND}}, J_{\text{hist}}, J_{\text{gen}}, J_{\text{trans}}), \quad (2015)$$

where the components represent corrective, distributive, sufficientarian, capability, relational, recognition, non-domination, historical, generative, and transformative dimensions.

Equation 2015 is a profile rather than a claim that each dimension can be placed on one cardinal scale.

23.5 Corrective Adequacy

Corrective adequacy concerns whether the response addresses the loss or wrongful alteration attributable to the harm.

Let wrongful loss object be L_H , and corrective response be R_C .

A generic criterion is

$$R_C \sim_{E_C} L_H, \quad (2016)$$

where E_C denotes the appropriate corrective relation.

The relation can be:

- restitution;
- replacement;
- monetary equivalence;
- capability restoration;
- acknowledgment;

- another legally or normatively justified form.

Corrective justice literature provides several competing accounts of the structure of rectification and responsibility [11, 69].

23.6 Corrective Equality and Historical Loss

Where the corrective object is cardinally representable, one can write

$$V(R_C) = V(L_H). \quad (2017)$$

Equation 2017 expresses one compensatory ideal.

Its meaning depends upon the value object selected.

Equal monetary value does not imply equal relational, historical, or capability value.

23.7 Corrective Adequacy under Uncertainty

Where loss cannot be identified precisely, corrective adequacy can be interval- or set-valued.

Let

$$L_H \in [L_{\min}, L_{\max}]. \quad (2018)$$

A corrective response can be judged adequate relative to the justified loss range rather than an invented point estimate.

This is especially important for long-horizon generative losses.

23.8 Corrective Adequacy and Singular Value

Some losses resist complete substitution.

A lost singular relation or historical opportunity can satisfy

$$\text{Rep}_V(O^*) < 1, \quad (2019)$$

where Rep_V represents a conceptual degree of reparability.

Equation 2019 does not imply that compensation is meaningless.

It indicates that corrective adequacy can require acknowledgment of residual non-equivalence.

23.9 Residual Corrective Loss

Let pre-remedy loss be L_H and value recovered through remedy $V(R_C)$.

Where subtraction is meaningful,

$$L_{\text{res}} = L_H - V(R_C). \quad (2020)$$

Equation 2020 represents unresolved loss.

For plural values, the residual should remain vector-valued.

23.10 Corrective Over-Compensation

A restorative arrangement can exceed the corrective reference.

Let

$$V(R_C) > V(L_H). \quad (2021)$$

Equation 2021 raises a separate question from whether the claimant's present life is better than the no-harm reference.

The excess can be justified through:

- uncertainty;
- deterrence;
- institutional error costs;
- distributive policy;
- independent assistance.

These rationales should not be mislabeled as pure corrective equivalence.

23.11 Distributive Adequacy

Distributive justice concerns how relevant goods, burdens, capabilities, or positions are distributed across persons or groups.

Let distribution after restoration be

$$D_R = (V_1, \dots, V_n). \quad (2022)$$

A distributive evaluation is

$$J_{\text{dist}} = J_{\text{dist}}(D_R). \quad (2023)$$

Equation 2023 can employ equality, priority, sufficiency, maximin, or another standard.

23.12 Corrective and Distributive Independence

Corrective and distributive justice can diverge.

A restitution can be correctively justified and distributively unequal.

A redistribution can improve distributive justice and fail to address a specific historical wrong.

This independence is represented by

$$J_{\text{corr}} \neq J_{\text{dist}} \quad (2024)$$

as evaluative functions.

The same intervention can score differently under each.

23.13 Distribution of Restorative Burdens

Substantive justice also concerns who bears the cost of repair.

Let burden vector be

$$\mathbf{B}_R = (B_1, \dots, B_n). \quad (2025)$$

Equation 2025 can include:

- financial cost;
- loss of opportunity;
- changed institutional role;
- tax burden;
- administrative burden;
- relational burden.

A claimant-centered remedy can therefore remain substantively unjust if its burdens are distributed arbitrarily.

23.14 Sufficiency

Sufficiency asks whether a person possesses enough of the relevant value.

Let threshold be θ_i .

A substantive sufficiency criterion is

$$V_i(R) \geq \theta_i \quad \forall i \in I_{\text{basic}}. \quad (2026)$$

Equation 2026 can apply to:

- basic income;
- bodily safety;
- housing;
- healthcare;
- minimum capability;
- access to justice.

Sufficientarian reasoning provides a natural normative basis for such thresholds [19, 57].

23.15 Sufficiency before Corrective Completion

A subject can require basic support while corrective liability remains unresolved.

This yields the temporal structure

$$R_{\text{suff}} \prec_t R_{\text{corr}}, \quad (2027)$$

where $\prec_t$ denotes temporal precedence.

Equation 2027 is not a normative claim that sufficiency is always more important.

It recognizes that urgent support can legitimately precede final corrective determination.

23.16 Sufficiency and Residual Corrective Claims

The subject can satisfy

$$V_R \geq \theta \quad (2028)$$

and still satisfy

$$V_R < V_{\text{cf}}. \quad (2029)$$

Equations 2028 and 2029 demonstrate that sufficiency and corrective completion are independent.

23.17 Priority

Prioritarian evaluation assigns additional weight to improvement for those who are worse off [42].

Let weighted benefit be

$$\Delta W_i^P = w(V_i) \Delta V_i, \quad (2030)$$

where $w(V_i)$ decreases as advantage increases under one familiar representation.

Equation 2030 illustrates how equal nominal benefits can have different normative weight.

23.18 Priority under Restorative Scarcity

Where resources cannot fully satisfy every claim immediately, priority can be sensitive to:

- present deprivation;
- severity;
- irreversibility;
- vulnerability;
- historical loss;
- urgency.

These factors need not point in the same direction.

A complete priority rule should state which dimension determines worse-off status.

23.19 Capability Justice

Capability justice asks whether the person possesses substantive freedom to pursue valued ways of living [55, 39, 50].

Let capability set be $\mathcal{K}_{A,R}$.

A substantive capability criterion can require

$$\mathcal{K}_{A,R} \in \mathcal{S}_{\text{cap}}. \quad (2031)$$

Equation 2031 can represent a threshold, reference set, or dominance relation.

23.20 Capability Equality

A historical reference can support

$$\mathcal{K}_{A,R} = \mathcal{K}_{A,\text{ref}}. \quad (2032)$$

Equation 2032 is a demanding fidelity criterion.

It can become inappropriate where the present subject has developed new goals.

23.21 Capability Inclusion

A future-expanding criterion is

$$\mathcal{K}_{A,\text{ref}} \subseteq \mathcal{K}_{A,R}. \quad (2033)$$

Equation 2033 preserves reference capabilities while allowing new ones.

This relation is attractive where historical restoration and present freedom are both valued.

23.22 Capability Sufficiency

Where exact reference reconstruction is uncertain or excessive, justice can require

$$\mathcal{K}_{A,R} \succeq \mathcal{K}_{\min}. \quad (2034)$$

Equation 2034 represents adequate substantive freedom.

Corrective residuals can remain after this threshold is met.

23.23 Capability and Conversion

Capability justice requires examination of conversion structure.

Equal resources can produce unequal capability:

$$R_A = R_B \quad \text{and} \quad \mathcal{K}_A(R_A) \neq \mathcal{K}_B(R_B). \quad (2035)$$

Equation 2035 explains why substantive justice can require differentiated support.

23.24 Relational Equality

Relational equality evaluates whether persons stand in social relations free from objectionable hierarchy, stigma, oppression, or subordination [2, 71].

Let relational standing be S_A^R .

A substantive criterion is

$$S_{A,R}^R \in \mathcal{S}_{\text{equal—standing}}. \quad (2036)$$

Equation 2036 can be satisfied despite unequal resources.

It can fail despite exact financial compensation.

23.25 Relational Repair

Relational repair can concern:

- removal of stigma;
- restoration of voice;
- equal participation;
- cessation of humiliation;
- secure organizational standing;
- correction of subordinating role structures.

It need not require restoration of a specific personal relationship.

This is especially important where the earlier relationship itself was harmful.

23.26 Relational Non-Replication

A just result can require that a harmful relationship remain discontinued.

Let harmful relation be r_H .

Substantive justice can require

$$r_H \notin \mathcal{R}_R. \quad (2037)$$

Equation 2037 shows why historical relational fidelity can be normatively negative.

23.27 Recognition Justice

Recognition justice concerns status, social meaning, acknowledgment, authorship, and credibility [25, 20].

Let recognition state be R_A^{rec} .

A substantive criterion can require

$$R_{A,R}^{\text{rec}} \succeq R_{A,\min}^{\text{rec}}. \quad (2038)$$

Equation 2038 permits several forms of repair.

23.28 Corrective Recognition

Where the harm consists partly in false classification or denial of contribution, substantive justice can require correction of the recognition object itself.

Examples include:

- correction of authorship;
- restoration of professional status;
- public acknowledgment;
- correction of defamatory records.

The value is not reducible to monetary consequence.

23.29 Recognition and Material Justice

Recognition and material repair are complementary.

A public apology without practical support can be inadequate.

A payment that leaves a false public record intact can also be inadequate.

A plural substantive criterion can therefore require

$$J_{\text{material}} \geq \theta_M \quad \text{and} \quad J_{\text{recognition}} \geq \theta_R. \quad (2039)$$

Equation 2039 gives neither value automatic substitutability.

23.30 Non-Domination

Republican non-domination evaluates exposure to arbitrary power [44].

Let domination exposure be D_A .

A substantive criterion can require

$$D_{A,R} \leq D_{\max}. \quad (2040)$$

Equation 2040 treats secure independence as a separate object from current benefit.

23.31 Actual Interference and Latent Power

A subject can receive every promised benefit while another actor retains the power to withdraw them arbitrarily.

Thus

$$I_A = 0 \quad (2041)$$

does not imply

$$D_A = 0. \quad (2042)$$

Equations 2041 and 2042 distinguish actual interference from structural domination.

23.32 Non-Domination and Restorative Security

A remedy is stronger where the claimant's future does not depend upon continued favor from the actor responsible for the harm.

Possible substantive protections include:

- enforceable rights;
- independent review;
- portable resources;
- alternative pathways;
- exit options;
- anti-retaliation protection.

These protections change the structure of power even where current resources remain constant.

23.33 Rights-Based Substantive Justice

Rights can define substantive constraints on the post-harm arrangement.

Let protected-right set be $\mathcal{R}_P$.

A substantively admissible result can require

$$R \in \mathcal{A}_{\text{rights}}, \quad (2043)$$

where $\mathcal{A}_{\text{rights}}$ contains outcomes consistent with the relevant rights.

Rights can therefore exclude some otherwise high-value restorative states.

23.34 Rights Restoration

Where the harm consists in loss of a right or legal status, substantive justice can require reinstatement of the right itself:

$$\mathcal{R}_{A,R}^P = \mathcal{R}_{A,\text{ref}}^P. \quad (2044)$$

Equation 2044 can be more appropriate than monetary substitution where the right remains exercisable.

23.35 Rights and Compensation

Compensation for a past rights violation and restoration of the right are different remedies.

Where the right continues to matter, substantive justice can require both:

$$R_{\text{rights}} + R_{\text{comp}} \quad (2045)$$

as a conceptual bundle.

Payment alone does not necessarily authorize continued rights deprivation.

23.36 Historical Justice

Historical justice evaluates provenance, historical wrong, inherited structure, and present consequences of past events [67, 62].

Let historical value profile be

$$\mathbf{H}_V = (V_{\text{provenance}}, V_{\text{recognition}}, V_{\text{record}}, V_{\text{historical-loss}}). \quad (2046)$$

Equation 2046 treats history as a substantive object rather than merely evidence for current welfare.

23.37 Historical Truth and Substantive Justice

A substantively just outcome can require truthful correction of the public or institutional record.

Let current record be Z_R and justified record be Z^* .

A basic criterion is

$$Z_R = Z^*. \quad (2047)$$

Equation 2047 can coexist with preservation of provenance showing that an earlier false record existed.

23.38 Historical Recognizability

Substantive historical justice can also value future recognizability of the wrong.

Let recognizability be R_H .

A threshold criterion is

$$R_H \geq R_{H,\min}. \quad (2048)$$

Equation 2048 must remain balanced against privacy and legitimate limits on disclosure.

23.39 Historical Fidelity and Substantive Justice

Historical fidelity is neither necessary nor sufficient for justice.

A repaired world can diverge strongly from the no-harm world and be just.

A perfectly reconstructed no-harm state can be unjust if it requires erasing the actual subject's agency.

Thus

$$F_H \neq J_S. \quad (2049)$$

Equation 2049 is a central result of the manuscript.

23.40 Generative Justice

Generative justice evaluates whether value-generating processes return, circulate, and reproduce conditions in ways that sustain just generativity [15, 17].

Let the generative profile be

$$\mathbf{G} = (G_{\text{condition}}, G_{\text{conversion}}, G_{\text{circulation}}, G_{\text{control}}, G_{\text{recursive}}). \quad (2050)$$

Substantive generative justice concerns the normative evaluation of $\mathbf{G}$, not generative magnitude alone.

23.41 Generative Sufficiency

A minimum generative condition can be written as

$$\mathbf{G}_{A,R} \succeq \mathbf{G}_{\min}. \quad (2051)$$

Equation 2051 can support assistance where exact restoration is impossible.

The threshold should include normative filters concerning control and externalities.

23.42 Generative Restoration

A stronger criterion can require

$$\Gamma_R^G \sim_E \Gamma_{\text{ref}}^G. \quad (2052)$$

Equation 2052 is appropriate only where reference-process fidelity itself carries normative value. It should not be imposed automatically.

23.43 Generative Dominance

A transformed system can be substantively superior where

$$V_G(\Gamma_R^G) \succeq V_G(\Gamma_{\text{ref}}^G). \quad (2053)$$

Equation 2053 formalizes better-than-reference generative justice. This is especially relevant where the historical process was extractive or dominating.

23.44 Generative Justice and Externalities

A self-sustaining process can be unjust if its sustainability depends upon external depletion.

Let internal generative value be G_{int} and external harm be H_{ext} .

A substantive evaluation should include both:

$$J_G = J_G(G_{\text{int}}, H_{\text{ext}}). \quad (2054)$$

Equation 2054 prevents local circularity from being treated as global justice.

23.45 Care-Oriented Substantive Justice

Care-oriented substantive justice evaluates whether concrete needs, dependencies, and caring relations are adequately supported [63].

Let need profile be $N_A(t)$.

A responsive substantive condition can require

$$S_A(t) \succeq N_A(t), \quad (2055)$$

where $S_A(t)$ denotes the relevant support profile.

Equation 2055 is conceptual rather than a cardinal comparison of all needs.

23.46 Care and Dependency

Dependency is not itself injustice.

The substantive question concerns whether dependency is:

- adequately supported;
- non-exploitative;
- non-dominating;
- responsive;
- fairly distributed.

A theory requiring total independence can therefore misclassify legitimate relations of care.

23.47 Care Burden

A restorative policy can shift support work to family or community members.

Let care burden distribution be

$$\mathbf{B}_{\text{care}} = (B_1^C, \dots, B_n^C). \quad (2056)$$

Equation 2056 should be included in substantive evaluation where unpaid support is a significant part of the remedy.

23.48 Agency and Substantive Justice

The post-harm arrangement should preserve meaningful present agency.

Let agency profile be A_R .

A minimum agency condition can require

$$A_R \succeq A_{\min}. \quad (2057)$$

Equation 2057 does not mean every preference must be satisfied.

It means the restorative structure should preserve meaningful authority over future direction.

23.49 Right to a Different Future

A substantively just future can differ from both the harmed trajectory and the no-harm counterfactual trajectory.

Let

$$\gamma_N \neq \gamma_H \quad (2058)$$

and

$$\gamma_N \neq \gamma_{\text{cf}}. \quad (2059)$$

Equations 2058 and 2059 define the structural possibility of a new legitimate future.

This possibility is central to subject-directed regeneration.

23.50 Counterfactual Paternalism

Counterfactual paternalism occurs when the reconstructed no-harm subject is given greater normative authority over the future than the actual present subject.

Let the institution select remedy R_{cf} solely because it maximizes counterfactual fidelity:

$$R_{\text{cf}} \in \arg \max_R F_{\text{cf}}(R). \quad (2060)$$

Equation 2060 can be substantively unjust where the present subject has legitimate alternative goals.

23.51 Transformative Justice

Transformative justice concerns movement toward a normatively improved structure rather than reproduction of the historical reference.

Let current harmful structure be S_H , historical reference S_{ref} , and transformed structure S_T .

A transformative criterion can require

$$S_T \succeq_V S_{\text{ref}} \quad (2061)$$

on the relevant value dimensions.

Equation 2061 allows the post-harm system to become more just than the reference.

23.52 Transformation and Corrective Claims

Transformative improvement does not erase corrective history.

It can hold that

$$J_{\text{trans}} > J_{\text{ref}} \quad (2062)$$

and

$$L_H > 0. \quad (2063)$$

Equations 2062 and 2063 preserve both future improvement and historical responsibility.

23.53 Unjust Baseline

A pre-harm state can itself be unjust.

Let reference satisfy

$$J(B_{\text{pre}}) < J_{\text{min}}. \quad (2064)$$

Equation 2064 shows why exact restoration to the pre-harm state can be substantively inadequate.

The appropriate target can combine correction of the specific harm with improvement beyond the unjust baseline.

23.54 Unjust Baseline Paradox

If the victim's prior condition was unjust, a theory requiring only return to that state can reproduce injustice while claiming complete restoration.

This is the unjust-baseline paradox.

The paradox is resolved by distinguishing:

- correction of the new harm;
- background distributive or relational injustice.

Different institutions can bear responsibility for the two layers.

23.55 Better-than-Before Justice

A remedy can place the claimant in a better substantive position than the historical reference.

This can occur through:

- over-complete capability restoration;
- stronger rights;
- better institutional safeguards;
- improved accessibility;
- greater non-domination.

Better-than-before outcomes can be justified where they arise from independent justice reasons rather than from denial of historical loss.

23.56 Substantive Over-Restoration

More restorative intervention is not always better.

An intervention can become excessive when it:

- crowds out present agency;
- imposes excessive third-party burdens;
- recreates unwanted historical relations;
- overuses surveillance;
- suppresses legitimate new development.

The substantive objective is just adequacy and improvement, not maximal historical replication.

23.57 Third-Party Justice

Restorative action can impose burdens upon persons who did not cause the harm.

Let claimant gain be

$$\Delta V_A > 0 \quad (2065)$$

and third-party burden be

$$\Delta V_B < 0. \quad (2066)$$

Equations 2065 and 2066 create a multi-party substantive problem.

The claimant's valid claim does not automatically determine the permissible burden upon B .

23.58 Innocent Third Parties

An innocent third party can acquire legitimate interests during the period between harm and repair.

Examples include:

- a current employee occupying a position sought in reinstatement;
- a present possessor of disputed property;
- later collaborators in a reconstructed institution;
- descendants whose lives depend upon intervening history.

Historical correction must therefore account for present relational structure.

23.59 Third-Party Rights Constraints

Let the set of remedies compatible with third-party rights be

$$\mathcal{R}_{3P}. \quad (2067)$$

Substantive selection should occur within

$$R^* \in \mathcal{R}_{\text{claimant}} \cap \mathcal{R}_{3P}. \quad (2068)$$

Equation 2068 represents simultaneous respect for the claimant and third parties.

23.60 Collective Justice

Some harms affect collective institutions, culture, territory, knowledge, or self-government.

Let collective value object be O_C^V .

A collective substantive criterion can require

$$V_C(O_{C,R}^V) \succeq V_{C,\min}. \quad (2069)$$

Equation 2069 can concern:

- governance capacity;
- cultural transmission;
- collective resources;
- institutional autonomy;
- public recognition.

23.61 Collective and Individual Non-Reduction

Collective justice should not be reduced to the sum of individual payments.

Conversely, collective repair should not automatically override individual rights.

The substantive architecture therefore requires both

$$J_C \quad (2070)$$

and

$$J_{I,1}, \dots, J_{I,n}. \quad (2071)$$

These dimensions can conflict.

23.62 Intergenerational Justice

Historical and ecological harms can alter conditions transmitted to later generations.

Let generational value profile be

$$\mathbf{V}^G = (V^{(0)}, V^{(1)}, \dots). \quad (2072)$$

Equation 2072 can be evaluated through thresholds, priority, sustainability, or another intergenerational theory.

Parfit's analysis of future persons demonstrates why identity and population questions complicate ordinary person-affecting comparison [41].

23.63 Intergenerational Generative Justice

A future-oriented criterion can evaluate whether each generation preserves adequate conditions for subsequent generation.

Let transmission map be

$$C^{(g+1)} = \Gamma_{\text{intergen}}(C^{(g)}). \quad (2073)$$

A substantive criterion can require

$$C^{(g+1)} \succeq C_{\min}^{(g+1)} \quad (2074)$$

for the relevant sequence of generations.

23.64 Ecological Substantive Justice

Ecological restoration can involve state, function, resilience, and regenerative capacity.

Let ecological profile be

$$\mathbf{E} = (E_{\text{state}}, E_{\text{function}}, E_{\text{resilience}}, E_{\text{generation}}). \quad (2075)$$

A substantively adequate ecological remedy can require more than recreation of one visible state [24].

The normative importance of ecological values can be anthropocentric, relational, intrinsic, or plural.

23.65 Substantive Justice under Irreversibility

Irreversibility changes the form of substantive justice.

Where exact return is unavailable, justice can shift among:

- compensation;
- recognition;
- rehabilitation;
- future opportunity;
- transformation;
- non-repetition.

The impossibility of literal restoration therefore does not imply absence of substantive duties.

23.66 Irreparability and Residual Justice

Let maximum feasible restorative relation leave residual loss L_{irr} .

A complete substantive judgment should acknowledge

$$L_{\text{irr}} > 0 \quad (2076)$$

rather than redefining the achieved remedy as full equivalence.

The residual can itself justify recognition, prevention, or additional support.

23.67 Substantive Justice under Uncertainty

Substantive decisions often occur before every relevant fact is known.

Let plausible world set be Ω .

A robust remedy can satisfy

$$J_S(R, \omega) \geq J_{\min} \quad \forall \omega \in \Omega^*, \quad (2077)$$

for the selected plausible set Ω^* .

Equation 2077 provides a conservative robustness criterion.

Other contexts can use probabilistic evaluation.

23.68 Epistemic Uncertainty and Burden Allocation

Uncertainty does not disappear from substantive justice.

Institutions must decide who bears the risk of error.

Possible allocations include:

- claimant bears uncertainty;
- responsible institution bears uncertainty;
- public insurance socializes uncertainty;
- burden is shared.

The allocation itself is a substantive justice question.

23.69 False-Negative Burden

A false negative denies support where harm was real.

Let its expected burden be

$$C_{\text{FN}}. \quad (2078)$$

Where deprivation is severe and irreparable, C_{FN} can be high.

This can justify provisional assistance under uncertainty.

23.70 False-Positive Burden

A false positive provides remedy where the relevant claim did not hold.

Let its burden be

$$C_{\text{FP}}. \quad (2079)$$

The substantive architecture should compare both error types rather than presuming one should always dominate.

23.71 Substantive Justice and Prevention

Where later substantive restoration is weakly feasible, prevention can acquire greater normative importance.

A conditioned principle is

$$\text{Rep}_V \downarrow \implies W_{\text{prevent}} \uparrow, \quad (2080)$$

other things equal.

Equation 2080 is not an unrestricted authorization for preventive intervention.

Rights, uncertainty, proportionality, and cost continue to constrain action.

23.72 Substantive Justice and Interruption

During ongoing harm, substantive justice can prioritize preventing further generative loss.

Let cumulative harm be $L(t)$.

An interruptive intervention can seek

$$\frac{dL}{dt} \leq 0. \quad (2081)$$

Equation 2081 defines a substantively valuable intermediate result even before liability and complete remedy are settled.

23.73 Substantive Justice and Assistance

Assistance is justified by present and future need rather than only by historical attribution.

Let viable target set be $\mathcal{T}_A^{\text{adm}}$.

A substantive assistance criterion can require

$$\mathcal{R}_\tau(\mathcal{G}^{HA}) \cap \mathcal{T}_A^{\text{adm}} \neq \emptyset. \quad (2082)$$

Equation 2082 makes future possibility the central object.

23.74 Assistance without Corrective Completion

An assistance system can therefore produce substantive improvement even where corrective justice remains unresolved:

$$J_{\text{assist}} > 0 \quad \text{while} \quad J_{\text{corr}} \text{ unresolved}. \quad (2083)$$

Equation 2083 supports the institutional separation developed earlier.

23.75 Substantive Justice and Non-Repetition

Guarantees of non-repetition concern the future structure that allows harm to recur.

Let recurrence probability be $p_H(\tau)$.

A substantive improvement can require

$$p_{H,R}(\tau) < p_{H,\text{pre}}(\tau). \quad (2084)$$

Equation 2084 is one partial formalization of a broader reparative commitment recognized in international reparations practice [65, 12].

23.76 Structural Reform

Where harm is generated by institutional structure, individual compensation can be substantively incomplete.

Let harmful transition law be Φ_H .

Structural reform produces

$$\Phi_H \xrightarrow{RS} \Phi_R. \quad (2085)$$

Equation 2085 changes the process generating future cases.

This is a substantive object distinct from individual remedy.

23.77 Substantive Justice and Institutional Learning

A just institution can be expected to learn from verified harm.

Let institutional policy state be Π_t .

Learning requires

$$\Pi_{t+1} = L(\Pi_t, e_H) \quad (2086)$$

where e_H is the relevant harm event.

Equation 2086 treats improved future capacity as a substantive benefit.

23.78 Failure of Institutional Learning

Repeated payment without mechanism change can produce

$$\Pi_{t+1} = \Pi_t \quad (2087)$$

despite recurrent harm.

Equation 2087 can be substantively deficient under non-repetition and generative standards even where each individual claimant is compensated.

23.79 Substantive Justice under Scarcity

Scarcity prevents simultaneous maximization of every restorative dimension.

Let feasible remedy set be

$$\mathcal{F}_R. \quad (2088)$$

Substantive justice then selects among

$$R \in \mathcal{F}_R \cap \mathcal{A}_N, \quad (2089)$$

where $\mathcal{A}_N$ is the normatively admissible set.

Equation 2089 distinguishes scarcity from normative permissibility.

23.80 Substantive Priority under Scarcity

Priority can be based upon:

- basic need;
- severity;
- irreversibility;
- causal responsibility;
- historical claim strength;
- generative amplification;
- vulnerability.

No universal ordering follows from scarcity itself.

The priority rule must be normatively stated.

23.81 Substantive Justice and Feasibility

An impossible remedy cannot ordinarily function as the sole operational standard of justice.

Let ideal target be R^I and feasible set $\mathcal{F}$.

A practical target can be

$$R^* \in \arg \max_{R \in \mathcal{F}} J_S(R) \quad (2090)$$

subject to rights and other constraints.

Equation 2090 is one best-feasible representation.

The ideal target can remain relevant for criticism and prevention.

23.82 Feasibility and Moral Dilution

Feasibility should not redefine the magnitude of the original wrong.

A weak feasible remedy can be the best available action and remain far from full repair.

Thus

$$R^* = \text{BestFeasible} \quad (2091)$$

does not imply

$$L_{\text{res}} = 0. \quad (2092)$$

Equations 2091 and 2092 preserve normative honesty.

23.83 Ideal and Non-Ideal Justice

An ideal theory can identify a fully just condition.

A non-ideal theory must decide what should be done under history, uncertainty, scarcity, institutional limitations, and conflict.

The justice of return belongs substantially to non-ideal theory because the wrong has already occurred.

Its central object is action within a historically altered world.

23.84 Substantive Justice and Path Dependence

Path dependence means identical current interventions can produce different substantive effects because of prior history [3, 45].

Let intervention be R .

It can satisfy

$$J_S(R \mid H_1) \neq J_S(R \mid H_2). \quad (2093)$$

Equation 2093 shows why substantive adequacy can be history-relative.

23.85 Substantive Justice and Hysteresis

Where a system exhibits hysteresis, returning the external condition to its earlier value can leave the subject in an altered regime.

Thus

$$C_R = C_{\text{pre}} \quad (2094)$$

and

$$J_S(\mathcal{G}_R) < J_S(\mathcal{G}_{\text{ref}}) \quad (2095)$$

can hold simultaneously.

This is the substantive form of the paper's core non-commutativity problem.

23.86 Substantive Justice and Robustness

A remedy can be substantively inadequate if it succeeds only under unusually protected conditions.

Let stress class be Δ_{adm} .

Robust substantive adequacy can require

$$J_S(R, \delta) \geq J_{\min} \quad \forall \delta \in \Delta_{\text{adm}}. \quad (2096)$$

Equation 2096 incorporates ordinary future stress into the justice evaluation.

23.87 Robustness and Over-Protection

Maximal robustness can reduce freedom, flexibility, and innovation.

A protected environment can become substantively unjust if it eliminates meaningful choice.

Robustness therefore remains one value among several.

23.88 Substantive Justice and Causal Structure

Two outcomes can be numerically equal and substantively different because the causal mechanisms differ.

A person receiving income through an enforceable entitlement and a person receiving identical income through revocable favor occupy different structures of power.

Substantive justice can therefore evaluate

$$Q_{\text{mechanism}, R} \quad (2097)$$

as an independent object.

23.89 Substantive Justice and Process-Generating Structure

The mechanism through which future value is generated can itself be substantively unjust even before procedure is considered.

An extractive production structure can produce high welfare while denying control over generated value.

This is a substantive feature of the institutional arrangement.

Procedural justice later asks how decisions about that arrangement are made.

23.90 Substantive Justice and Reflexivity

A declaration of substantive restoration can change the subject's future.

If the declaration terminates assistance, then

$$Z_{\text{restored}} \rightarrow \text{support withdrawal} \rightarrow \Delta J_S < 0. \quad (2098)$$

Equation 2098 shows why substantive evaluation should consider the causal effects of its own classifications.

23.91 Completion Thresholds

Institutions need criteria for declaring their substantive task complete.

Let completion criteria be

$$\mathbf{J}_S^I \succeq \theta_I. \quad (2099)$$

Equation 2099 is institution-specific.

A court can complete its corrective task while social assistance continues.

23.92 Qualified Completion

Institutional language should therefore use qualified completion statements.

Examples include:

- compensation complete;
- formal status restored;
- rehabilitation threshold met;
- recognition remedy completed;
- monitoring period completed.

These statements are more precise than “the person is fully restored.”

23.93 Substantive Residuals

After completion of one institutional task, unresolved substantive objects can remain.

Let

$$\mathfrak{D}_{\text{res}} = \{O_i^V : J_i < \theta_i\}. \quad (2100)$$

Equation 2100 identifies the remaining normative space.

Residuals can trigger referral rather than indefinite expansion of one institution’s mandate.

23.94 Substantive Handoff

Institutional handoff is justified where the unresolved value object belongs to another institution’s competence.

For example:

- court to rehabilitation provider;
- employer to social-protection institution;
- compensation body to archival or recognition institution.

Substantive decomposition therefore supports institutional pluralism.

23.95 Substantive Justice and Institutional Fragmentation

Plural institutions create a risk that every institution satisfies its own bounded standard while the subject remains substantively unsupported overall.

Let institutional outputs be

$$R_1, \dots, R_n. \quad (2101)$$

Even if

$$J_i(R_i) \geq \theta_i \quad \forall i, \quad (2102)$$

the combined system can still fail where interfaces are absent.

Substantive justice therefore requires coordination at the system level.

23.96 Substantive Gap

Define system-level substantive gap as

$$G_S = \mathfrak{D}_V^* \setminus \bigcup_i \mathfrak{D}_V^{I_i, \text{resolved}}. \quad (2103)$$

Equation 2103 identifies important values left unaddressed by the institutional system.

This gap can be a governance problem even where no individual institution has failed its formal mandate.

23.97 Substantive Justice and Scale

Substantive justice should be indexed by scale.

Let

$$J_S^{(s)} \quad (2104)$$

denote justice at scale s .

An intervention can improve individual justice and worsen institutional or ecological justice.

Scale-specific evaluation prevents aggregate success from hiding local burdens.

23.98 Cross-Scale Substantive Conflict

It can hold that

$$\Delta J_S^{(\text{individual})} > 0 \quad (2105)$$

and

$$\Delta J_S^{(\text{collective})} < 0. \quad (2106)$$

Equations 2105 and 2106 require explicit cross-scale adjudication.

23.99 Substantive Justice and Temporal Horizon

The same remedy can be just at one horizon and inadequate at another.

Let

$$J_S = J_S(\tau). \quad (2107)$$

Equation 2107 can vary because:

- current support expires;
- capability grows;
- trust recovers;
- recurrence occurs;
- third-party effects emerge.

A substantive judgment should therefore identify its horizon.

23.100 Short-Horizon and Long-Horizon Justice

Let

$$J_S(\tau_{\text{short}}) > J'_S(\tau_{\text{short}}) \quad (2108)$$

and

$$J_S(\tau_{\text{long}}) < J'_S(\tau_{\text{long}}). \quad (2109)$$

Equations 2108 and 2109 describe a horizon-dependent reversal.

This can occur where immediate compensation competes with structural investment.

23.101 Substantive Justice and Present Preference

Present preference has normative significance and should not be treated as infallible.

Let current preference be P_A^H .

A plural architecture can treat it as one element:

$$J_S = J_S(P_A^H, \mathcal{K}_A, \mathcal{R}_A, \mathfrak{C}). \quad (2110)$$

Equation 2110 combines present preference with capability, rights, and constraints.

23.102 Adaptive Preference and Substantive Justice

Where preference adapts to deprivation, a theory can resist defining justice solely by present satisfaction [55, 50].

The response should nevertheless avoid simply replacing the actual subject's judgment with an institutional model of the life the subject should want.

This tension is a central limit on paternalistic repair.

23.103 Substantive Justice and Objective Goods

Some theories recognize goods whose value does not depend entirely upon current preference.

Examples can include:

- bodily integrity;
- basic education;
- political liberty;
- minimum healthcare.

Where such goods are included, the theory should make their status explicit.

23.104 Substantive Justice and Plural Value

The general substantive architecture can retain a plural value vector

$$\mathbf{V}_S = (V_1, \dots, V_n). \quad (2111)$$

without reducing it to one score.

Substantive justice can then operate through:

- constraints;

- thresholds;
- partial orders;
- dominance;
- justified trade-offs.

23.105 Substantive Justice under Incomparability

Two remedies can remain incomparable:

$$R_1 \parallel_J R_2. \quad (2112)$$

Equation 2112 need not represent analytical failure.

It can accurately reflect plural values lacking a justified complete order [10, 1, 48].

Procedure then becomes especially important for legitimate selection.

23.106 Pareto Improvement

Where one restorative arrangement is at least as good for every affected person and better for some, it possesses a strong comparative advantage:

$$V_i(R_1) \geq V_i(R_2) \quad \forall i. \quad (2113)$$

Equation 2113 is useful where available.

Many meaningful restorative choices remain outside the Pareto relation.

23.107 Lexical Constraints

A plural theory can give lexical priority to selected dimensions.

For example,

$$\text{basic rights} \succ_{\text{lex}} \text{agency threshold} \succ_{\text{lex}} \text{historical fidelity}. \quad (2114)$$

Equation 2114 is illustrative.

The structure prevents large lower-level benefits from overriding protected higher-level values.

23.108 Threshold-Constrained Substantive Optimization

Another architecture defines

$$\mathcal{A}_S = \{R : V_i(R) \geq \theta_i \forall i \in I_P\}, \quad (2115)$$

then selects among $R \in \mathcal{A}_S$ using a secondary criterion.

This allows efficiency or welfare optimization after rights, safety, or sufficiency constraints are met.

23.109 Substantive Dominance

A restorative arrangement substantively dominates another where it is at least as good across all selected non-tradeable dimensions and strictly better in at least one:

$$\mathbf{V}_S(R_1) \succeq \mathbf{V}_S(R_2). \quad (2116)$$

Equation 2116 permits strong choice without full aggregation.

23.110 Minimum Substantive Justice

A minimum substantive standard can be defined by

$$\mathbf{J}_S \succeq \boldsymbol{\theta}_S. \quad (2117)$$

Equation 2117 means every protected dimension meets its required threshold.
This is a plausible institutional completion condition for some assistance systems.

23.111 Strong Substantive Restoration

A stronger criterion can require:

$$\mathbf{J}_S \succeq \mathbf{J}_{S,\text{ref}} \quad (2118)$$

across the selected dimensions.
Equation 2118 permits dominance rather than exact equality.
It can therefore accommodate transformative improvement.

23.112 Complete Substantive Equivalence

An extreme criterion would require

$$\mathbf{V}_{S,R} = \mathbf{V}_{S,\text{ref}} \quad (2119)$$

across every selected substantive value object.
Equation 2119 is analytically useful and rarely a necessary operational target.
It can also be undesirable where the reference contained injustice.

23.113 Substantive Justice Frontier

Where no remedy dominates every other remedy, the nondominated set is

$$\mathcal{F}_J = \text{ND}(\mathcal{F}_R). \quad (2120)$$

Equation 2120 contains substantively efficient trade-off points under the selected value architecture.
A procedure must still select among them.

23.114 Substantive Justice Matrix

Table 19 summarizes the principal substantive dimensions developed in this section.

Table 19 shows why substantive justice cannot be reduced to one equivalence criterion.

23.115 Remedy–Justice Matrix

Table 20 maps common remedy families to the substantive dimensions they most directly address.

Table 20 demonstrates why multi-dimensional harm often requires a remedy bundle.

Table 19: Principal dimensions of substantive justice of return

Dimension	Primary value object	Characteristic reference	Comparative standard	Characteristic residual problem
Corrective	Wrongful loss, entitlement	No-wrong position	Restitution / compensation	Counterfactual uncertainty
Distributive	Resources, welfare, advantage	Population distribution	Equality / priority / maximin	Conflict with bilateral correction
Sufficiency	Basic goods and capabilities	Adequacy threshold	Threshold attainment	Residual loss above threshold
Capability	Substantive opportunity	Reference or adequate capability set	Equality / inclusion / sufficiency	Conversion and selection problems
Relational	Standing and social relation	Equal social relation	Standing adequacy	Difficult relational reconstruction
Recognition	Status, acknowledgment, authorship	Appropriate recognition	Correction / sufficiency	Symbolic and material divergence
Non-domination	Secure control and exit	Non-arbitrary relation	Domination reduction	Latent power can be difficult to observe
Rights	Protected claims and liberties	Rights-respecting condition	Identity / constraint	Rights can conflict
Historical	Provenance, truth, memory	Actual and counterfactual history	Correction / recognition / preservation	Irreversibility
Generative	Future condition and value circulation	Reference or just generative process	Simulation / sufficiency / dominance	Generativity can sustain injustice
Transformative	Institutional structure	Normatively improved future	Dominance	Transformation can create new burdens
Collective	Shared capacity, public goods	Collective viability	Sufficiency / dominance	Individual and collective conflict
Intergenerational	Future inherited conditions	Cross-generational standard	Sufficiency / priority / sustainability	Identity and uncertainty

23.116 Substantive Remedy Bundles

Let substantive remedy bundle be

$$\mathcal{B}_R = \{R_1, \dots, R_m\}. \quad (2121)$$

The bundle can include restitution, compensation, rehabilitation, recognition, assistance, and institutional reform.

A bundle is substantively adequate where its combined value profile satisfies the selected normative architecture.

23.117 Complementary Remedies

Remedies can be complementary.

Let combined value satisfy

$$V(R_1, R_2) > V(R_1) + V(R_2) - V_0 \quad (2122)$$

where addition is meaningful.

Equation 2122 can describe cases where compensation becomes genuinely useful only after legal access is restored.

23.118 Substitutable Remedies

Remedies can also partially substitute for one another.

A person can choose compensated exit rather than reinstatement.

The two remedies can preserve different value objects.

Substantive justice should therefore identify which dimensions are traded and whether the choice is genuinely voluntary.

Table 20: Remedy families and their principal substantive functions

Remedy family	Primary substantive object	Characteristic contribution	Characteristic limitation
Restitution	Property, status, access	Returns identifiable condition	Condition can have altered generative significance
Compensation	Generalized value loss	Provides fungible resource and subject choice	Weak for singular and relational value
Rehabilitation	Function and capability	Repairs conversion and practical use	Can privilege historical function over current goals
Recognition	Status and historical meaning	Corrects misrecognition and acknowledges wrong	Does not substitute for material repair
Re-entry	Institutional position and opportunity	Reopens participation and future pathways	Formal access can remain relationally insecure
Assistance	Present needs and future possibility	Supports viable subject-directed futures	Need not settle responsibility
Structural reform	Institutional mechanism	Changes future harm-generating structure	Can impose transition burdens
Non-repetition measures	Future risk	Reduces recurrence and strengthens institutions	Future-oriented and not equivalent to compensation

23.119 Subject-Selected Remedy Bundles

Where several remedies satisfy protected substantive constraints, the subject can select among them.

Let admissible remedy family be

$$\mathcal{R}_A^{\text{adm}}. \quad (2123)$$

Subject choice can determine

$$R_A^* \in \mathcal{R}_A^{\text{adm}}. \quad (2124)$$

Equation 2124 preserves agency without making subject preference the sole source of justice.

23.120 Right to Refuse Restoration

A substantively justified restoration option need not be compulsory.

The subject can reject reinstatement, relational reconstruction, or another high-fidelity remedy.

Let refusal set be

$$\mathcal{R}_{\text{refuse}}. \quad (2125)$$

A restorative system should preserve refusal where no overriding justified constraint applies.

This is especially important for relational and identity-affecting remedies.

23.121 Substantive Justice and Exit

Exit can be a substantively just remedy where return to the earlier system is undesired or unsafe.

A compensated exit can include:

- financial resources;
- portable credentials;

- legal protection;
- transition support;
- recognition of the historical wrong.

Exit should not be used to displace claimants merely because institutional reform is inconvenient.

23.122 Substantive Justice and Portability

A remedy is stronger when the claimant can carry restored value into another environment.

Let portability be

$$P_R(O, e_1, e_2). \quad (2126)$$

Low portability can trap the claimant in the institution responsible for the harm.

Portability therefore supports both capability and non-domination.

23.123 Substantive Justice and Resilience of the Remedy

A remedy can be substantively fragile.

Let its value under perturbation δ be

$$V_R(\delta). \quad (2127)$$

A resilient remedy can satisfy

$$V_R(\delta) \geq V_{\min} \quad \forall \delta \in \Delta_{\text{ordinary}}. \quad (2128)$$

Equation 2128 captures durability under ordinary future variation.

23.124 Substantive Justice and Repair Duration

A remedy can be adequate only temporarily.

Let

$$J_S(t) \geq J_{\min} \quad t \in [t_0, t_1]. \quad (2129)$$

Equation 2129 does not establish justice after t_1 .

Duration should therefore be part of the substantive claim.

23.125 Durable Substantive Justice

A durable criterion requires

$$J_S(t) \geq J_{\min} \quad t \in [t_0, t_0 + \tau_D]. \quad (2130)$$

Equation 2130 is appropriate where restoration must persist long enough to become practically useful.

23.126 Substantive Justice and Closure

Substantive closure should identify which value objects have been resolved.

Let resolved set be

$$\mathfrak{O}_V^{\text{closed}}. \quad (2131)$$

Closure should not imply that every object outside $\mathfrak{O}_V^{\text{closed}}$ has been restored.

This preserves institutional honesty.

23.127 Substantive Reopening

A substantive judgment can be reopened where:

- the remedy fails;
- recurrence appears;
- new evidence identifies larger loss;
- the subject's condition materially changes;
- an institutional error is discovered.

Reopening protects against premature closure while preserving bounded finality.

23.128 Substantive Justice under Reflexive Reclassification

If the declaration of completion changes eligibility, the completion test should consider its own effect.

Let

$$J_S^+ = J_S(R \mid Z_{\text{complete}}) \quad (2132)$$

denote justice after the completion classification takes effect.

A robust completion criterion should evaluate J_S^+ , not only the state immediately before classification.

23.129 Substantive Justice under Value Conflict

Where value dimensions conflict, substantive justice can require explicit priority rules.

Let

$$\Delta V_1 > 0 \quad \text{and} \quad \Delta V_2 < 0. \quad (2133)$$

Equation 2133 cannot be resolved by the fact of conflict alone.

Possible approaches include:

- lexical priority;
- threshold protection;
- partial-order dominance;
- justified trade-off;
- subject choice among admissible options.

23.130 Substantive Justice under Incommensurability

Where no justified common scale exists, the theory should resist artificial conversion.

A lost relationship and a monetary transfer can remain incomparable in some dimensions even where compensation is institutionally appropriate.

This does not make institutional action impossible.

It limits the meaning of the claim that the loss has been “made equal.”

23.131 Substantive Justice under Moral Uncertainty

Several plausible normative theories can remain available.

Let theory set be

$$\mathfrak{T}_N = \{T_1, \dots, T_k\}. \quad (2134)$$

A normatively robust remedy can satisfy

$$R \in \bigcap_{j=1}^k \mathcal{A}_{T_j}. \quad (2135)$$

Equation 2135 supports action where several theories converge.

23.132 Overlapping Substantive Consensus

Theories can converge on measures such as:

- stopping ongoing harm;
- securing basic needs;
- correcting false records;
- reducing arbitrary dependency;
- preserving future capability.

The reasons differ while the institutional action overlaps.

This convergence can support practical priority.

23.133 Substantive Disagreement Region

Where theories diverge, define

$$\mathcal{D}_J = \bigcup_T \mathcal{A}_T \setminus \bigcap_T \mathcal{A}_T. \quad (2136)$$

Equation 2136 contains remedies whose status depends upon contested normative commitments.

These are especially important cases for procedural legitimacy.

23.134 Substantive Minimalism

A bounded institution should make the narrowest substantive judgment necessary for its task.

For example:

The claimant has received adequate income support for the present eligibility period.

is stronger in precision than

The claimant has been made whole.

Substantive minimalism prevents institutional scope from becoming moral totalization.

23.135 Substantive Escalation

The analysis should expand when bounded remedies repeatedly fail.

A typical escalation can move through

$$\text{resource} \rightarrow \text{capability} \rightarrow \text{relation} \rightarrow \text{control} \rightarrow \text{generative process.} \quad (2137)$$

Equation 2137 identifies progressively deeper possible sources of residual injustice.

23.136 Substantive Justice Statement

A fully specified substantive judgment can be represented by

$$\mathcal{J}_S = [\mathfrak{D}_V^*; \mathbf{B}; \mathbf{E}; \boldsymbol{\theta}; \mathfrak{T}^N; \tau; K; s]. \quad (2138)$$

Equation 2138 records:

- selected value objects;
- baselines;
- comparative relations;
- thresholds;
- normative theories;
- horizons;
- epistemic basis;
- scale.

The representation makes substantive conclusions auditable.

23.137 Substantive Justice Principle

The first principle of this section is:

Substantive justice of return concerns whether the post-harm arrangement adequately realizes the values selected by a justified normative architecture, rather than whether it maximizes formal similarity to one reference state.

23.138 Corrective Residual Principle

The second principle is:

Present sufficiency, welfare, or future success does not independently eliminate a valid residual claim concerning attributable historical loss.

23.139 Sufficiency Principle

The third principle is:

Urgent basic sufficiency can constitute an independent substantive requirement and can justify assistance before complete corrective adjudication.

23.140 Capability Principle

The fourth principle is:

Return of resources should not be treated as complete substantive restoration where the subject's effective capability to use those resources remains materially impaired.

23.141 Relational Principle

The fifth principle is:

Material equivalence does not establish substantive justice where the post-harm arrangement preserves objectionable subordination, stigma, or dependency.

23.142 Recognition Principle

The sixth principle is:

Recognition, historical truth, and social standing can remain substantively unresolved after material loss has been compensated.

23.143 Non-Domination Principle

The seventh principle is:

Secure access and meaningful exit can matter substantively even where actual interference is absent and current resources are equal.

23.144 Generative Principle

The eighth principle is:

A substantively just remedy should consider whether the post-harm arrangement supports just future generativity rather than only a one-time return of current conditions.

23.145 Transformative Principle

The ninth principle is:

Where the historical reference is unjust or rejected by the present subject, substantive justice can require a normatively improved future rather than replication of the earlier system.

23.146 Third-Party Principle

The tenth principle is:

A valid restorative claim does not create unlimited substantive authority to impose serious burdens upon innocent third parties.

23.147 Residual Honesty Principle

The eleventh principle is:

Where irreparable or institutionally unresolved loss remains, the restorative system should acknowledge the residual rather than redefining the achieved remedy as complete equivalence.

23.148 Qualified Completion Principle

The twelfth principle is:

Institutional declarations of completion should identify the substantive object and standard satisfied rather than imply comprehensive restoration across all dimensions of justice.

23.149 Substantive Layer of the Atlas

The substantive layer can now be represented by

$$\mathfrak{S}_A = (\mathfrak{D}_V^*, \mathbf{B}, \mathbf{E}, \mathfrak{T}^N, \boldsymbol{\theta}, \mathfrak{C}_N), \quad (2139)$$

where:

- $\mathfrak{D}_V^*$ contains selected substantive value objects;
- $\mathbf{B}$ contains relevant references;
- $\mathbf{E}$ contains comparative standards;
- $\mathfrak{T}^N$ contains normative theories;
- $\boldsymbol{\theta}$ contains thresholds where applicable;
- $\mathfrak{C}_N$ contains substantive constraints.

Equation 2139 turns the formal comparison atlas into a structured substantive-justice architecture.

23.150 Implications for Procedural Justice

The present section has shown that even a substantively rich theory of return leaves another major problem unresolved.

Who selects the value objects?

Who determines the reference?

Who decides whether the claimant has recovered?

How can the subject contest the model?

What evidence can the institution collect?

Which third parties are heard?

How should uncertainty be governed?

Can the subject refuse the remedy?

How can a remedy be revised?

These questions concern the legitimacy of the process through which substantive justice is pursued.

23.151 Transition to Procedural Justice

Section 24 therefore develops procedural justice of return. It will examine consent, participation, voice, notice, impartiality, reason giving, transparency, explainability, evidence, privacy, contestability, revisability, reversibility, proportionality, temporal fairness, third-party participation, and institutional accountability.

The central question will shift from

Which substantive restorative condition is just?

to

Through which procedures may a restorative institution legitimately identify, select, impose, revise, and declare that condition?

The distinction is essential because even a substantively desirable result can become unjust when produced through an illegitimate restorative process.

24 Procedural Justice of Return

This section develops the procedural-justice layer of the theory of return. The preceding section examined the substantive conditions under which a post-harm arrangement can count as correctively adequate, capability-supporting, recognition-sensitive, non-dominating, generative, or transformative. The present section turns to the institutional path through which such an arrangement is identified, selected, implemented, reviewed, revised, and declared complete.

Procedure possesses independent normative significance. A substantively valuable outcome can be produced through coercion, secrecy, biased fact finding, unreviewable classification, intrusive surveillance, exclusion of affected persons, or arbitrary exercise of institutional power. Conversely, a procedurally careful process can fail to deliver substantively adequate repair. The theory therefore treats substantive and procedural justice as separate and interacting dimensions.

Procedural-justice research provides strong precedent for distinguishing process from outcome. Thibaut and Walker's social-psychological analysis placed process control and dispute procedures at the center of procedural justice, while Tyler's empirical work emphasizes the importance of perceived fairness, respectful treatment, and legitimacy in people's responses to legal authority [61, 64]. Administrative-law scholarship likewise demonstrates that legitimate procedure cannot be designed independently of institutional function. Mashaw's work on administrative due process examines how procedural forms interact with bureaucratic justice and administrative decision making, while Fuller's analysis of adjudication emphasizes participation through reasoned argument and the limits of adjudicative form [34, 22].

The present section extends these insights into restorative governance. Its central concern is the procedure through which an institution decides what has been lost, which reference is relevant, what counts as restoration, whose values matter, what evidence is admissible, which interventions are permissible, and when the institutional task can legitimately end.

The section develops consent, participation, voice, notice, reason giving, impartiality, evidentiary fairness, accessibility, representation, privacy, contestability, revisability, reversibility, proportionality, temporal fairness, third-party participation, institutional competence, review, closure, and reflexive accountability. It concludes by defining a procedural profile that can later be compared directly with substantive restorative fidelity.

24.1 Procedural Justice as an Independent Evaluative Dimension

Let restorative intervention be R . Procedural justice is represented by the indexed evaluation in Equation 2140.

$$J_P(R \mid P, I, K, \tau, s), \quad (2140)$$

where P denotes the procedure used, I the institutional setting, K the epistemic basis, τ the relevant temporal structure, and s the scale of decision. Equation 2140 makes procedural justice explicitly indexed to institutional context rather than treating procedure as one invariant checklist.

Substantive and procedural justice can therefore be represented jointly by Equation 2141.

$$\mathbf{J} = (J_S, J_P). \quad (2141)$$

Equation 2141 allows high substantive adequacy and low procedural justice, high procedural justice and low substantive adequacy, or stronger performance on both dimensions.

24.2 Procedural Objects

Procedure contains several independently relevant objects. Let the procedural object family be defined by Equation 2142.

$$\mathfrak{O}_P = \{O_{\text{notice}}, O_{\text{voice}}, O_{\text{consent}}, O_{\text{evidence}}, O_{\text{reason}}, O_{\text{impartiality}}, O_{\text{privacy}}, O_{\text{contest}}, O_{\text{review}}, O_{\text{revision}}, O_{\text{time}}\}. \quad (2142)$$

Equation 2142 treats procedural justice as a plural architecture. An institution can satisfy one component and fail another.

24.3 Procedural Architecture

A case-specific procedural architecture can be represented by Equation 2143.

$$\mathfrak{P}_R = (A, N, V, E, D, C, Q, R_v, T, F), \quad (2143)$$

where:

- A denotes access;
- N denotes notice;
- V denotes voice and participation;
- E denotes evidentiary structure;
- D denotes decision and reason giving;
- C denotes contestability;
- Q denotes privacy and information governance;
- R_v denotes review and revisability;
- T denotes temporal structure;
- F denotes finality and closure.

Equation 2143 supplies the structural map used throughout the section.

24.4 Access to Procedure

Procedural justice begins before the hearing, review, or restorative conference. A person who cannot enter the procedure cannot exercise later procedural rights.

Let practical access be represented by Equation 2144.

$$A_P = A_P(K, C, L, T, S, R), \quad (2144)$$

where K denotes knowledge of the procedure, C financial and administrative cost, L linguistic accessibility, T time, S safety, and R required resources. Equation 2144 distinguishes formal availability from effective access.

A complaint mechanism can exist formally while remaining inaccessible because of cost, complexity, fear of retaliation, inaccessible language, or unrealistic deadlines.

24.5 Formal and Effective Procedural Access

Let formal eligibility be A_P^F and effective access be A_P^E . The distinction is represented by Equation 2145.

$$A_P^F = 1 \quad \text{and} \quad A_P^E = 0. \quad (2145)$$

Equation 2145 describes a procedurally available mechanism that remains practically unusable.

This distinction parallels the earlier difference between formal resource return and effective generative access.

24.6 Notice

Affected persons require timely and intelligible notice of decisions capable of altering their restorative rights, status, evidence, support, or obligations.

Let notice at time t_N precede the consequential decision at t_D . A minimal temporal condition is represented by Equation 2146.

$$t_N < t_D. \quad (2146)$$

Equation 2146 captures only the temporal minimum. Adequate notice also requires sufficient content and sufficient time for response.

24.7 Informational Adequacy of Notice

Let notice content be N_C and the information required for meaningful response be I_R . Informational adequacy can be represented by Equation 2147.

$$I_R \subseteq N_C. \quad (2147)$$

Equation 2147 means the notice contains the information reasonably necessary to understand and contest the proposed action.

A notice that merely announces an outcome without stating its basis can fail this condition.

24.8 Comprehensibility

Information can be formally disclosed and practically unintelligible.

Let interpretive accessibility be

$$C_N = C(N_C, L_A, K_A), \quad (2148)$$

where L_A and K_A represent the recipient's linguistic and relevant knowledge conditions. Equation 2148 makes comprehensibility an independent object.

Procedural transparency therefore requires more than document availability.

24.9 Voice

Voice concerns the affected person's opportunity to present facts, values, objections, preferences, and alternative interpretations before consequential decisions are fixed.

Let the subject's contribution be V_A and the institution's decision process be D_I . Effective voice requires the relation represented by Equation 2149.

$$V_A \longrightarrow D_I. \quad (2149)$$

Equation 2149 means the contribution enters the decision process. It does not mean the institution must accept every claim.

Thibaut and Walker's work on procedural control provides an important social-psychological foundation for distinguishing participation in the process from control over the final decision [61].

24.10 Voice and Decisional Authority

Procedural voice and final decisional authority should remain distinct.

Let V_A denote voice and D_A final decision control. A procedure can satisfy

$$V_A > 0 \quad \text{and} \quad D_A = 0. \quad (2150)$$

Equation 2150 can describe adjudication in which the claimant presents reasons while an independent institution decides.

Voice can therefore be meaningful without giving every party veto power.

24.11 Token Participation

Participation becomes tokenistic when the subject can speak while the institutional decision is functionally insensitive to the contribution.

Let decision be

$$D_I = F(E_I) \quad (2151)$$

despite formal collection of V_A .

Equation 2151 represents a process in which subject participation has no modeled influence on the decision.

A procedurally credible system should clarify how relevant submissions can affect the outcome or reasoning.

24.12 Participation

Participation extends beyond speaking in a hearing. It can include involvement in:

- defining the harm;
- selecting restorative goals;
- choosing among admissible remedies;
- designing implementation;
- evaluating progress;
- deciding whether revision is needed.

Participation is especially important where restoration concerns future life direction rather than one completed historical transaction.

24.13 Participation Intensity

Different procedural functions can justify different participation levels.

Let participation intensity be

$$P_A \in [0, 1], \quad (2152)$$

where the scale is illustrative rather than a universal metric. Equation 2152 allows low participation in routine mechanical correction and stronger participation in identity-affecting or long-term restorative design.

24.14 Consent

Consent becomes especially important where restoration directly alters the subject's body, memory, relationships, location, profession, or personal future.

Let the set of interventions for which valid consent is required be

$$\mathcal{R}_C \subseteq \mathcal{R}. \quad (2153)$$

Equation 2153 makes consent intervention-specific rather than assuming that every administrative act requires the same form of agreement.

24.15 Consent Conditions

A valid consent architecture can require

$$C_A = C_A(I, K, V, R_v, P), \quad (2154)$$

where I denotes information, K competence, V voluntariness, R_v revocability where appropriate, and P absence of impermissible pressure. Equation 2154 emphasizes that a formal signature alone does not exhaust consent.

24.16 Consent and Power Asymmetry

Consent given under severe dependency can require special scrutiny.

Let dependency on institution I be D_{AI} . Where

$$D_{AI} \gg 0, \quad (2155)$$

the voluntariness of refusal can be limited. Equation 2155 therefore identifies a condition under which institutional power becomes procedurally relevant.

The appropriate response can include independent advice, alternative options, or separation of support from consent to optional interventions.

24.17 Consent and Essential Support

A subject should not ordinarily be forced to consent to unrelated intrusive measures as the price of receiving independently justified essential support.

Let support entitlement be S_E and optional intervention be R_O . Procedural separation can be represented by Equation 2156.

$$S_E \not\Rightarrow C_A(R_O). \quad (2156)$$

Equation 2156 expresses the independence of essential support from consent to a distinct optional intervention.

24.18 Right to Refuse

Where several substantively admissible remedies exist, the affected subject can have a right to refuse a particular remedy.

Let admissible options be

$$\mathcal{R}_A^{\text{adm}} = \{R_1, \dots, R_k\}. \quad (2157)$$

Equation 2157 makes refusal meaningful only where alternative paths remain practically available. A nominal right to refuse can be empty where refusal leads to abandonment of all support.

24.19 Impartiality

Procedural justice requires attention to conflicts of interest and decisional bias.

Let decision-maker D possess interest I_D affected by the outcome. A conflict indicator can be represented by Equation 2158.

$$\text{Conf}(D, I_D, R) > 0. \quad (2158)$$

Equation 2158 identifies a reason for recusal, independent review, or additional procedural safeguards.

Fuller's account of adjudication emphasizes reasoned participation before an arbiter capable of responding to reasons, a structure incompatible with a decision maker whose judgment is wholly inaccessible to reasoned argument [22].

24.20 Institutional Self-Review

Restorative cases frequently require an institution to evaluate harm allegedly caused by itself.

Self-review can provide expertise and speed.

It can also create:

- reputational incentives;
- defensive fact finding;
- information control;
- conflicts between liability and assistance.

A layered architecture can therefore separate initial internal correction from independent escalation.

24.21 Structural Independence

Independence should be evaluated structurally rather than by formal title alone.

Let decision-maker dependence upon interested unit U be

$$D_{DU} = D(F, H, B, A), \quad (2159)$$

where F denotes funding dependence, H hierarchical dependence, B informational dependence, and A appointment dependence. Equation 2159 provides a multidimensional view of institutional independence.

24.22 Reason Giving

A consequential restorative decision should ordinarily be accompanied by reasons sufficient to identify the factual, normative, and institutional basis of the decision.

Let the decision be

$$D = F(E, N, M), \quad (2160)$$

where E denotes evidence, N normative rules, and M institutional mandate. Equation 2160 provides the structure that a reasoned explanation should make sufficiently visible.

Reason giving supports:

- understanding;
- contestation;

- consistency;
- appellate review;
- institutional learning.

24.23 Reason Responsiveness

A process can issue reasons after the fact without allowing submitted reasons to influence the outcome.

A stronger procedural object is reason responsiveness.

Let party submission be r_A and final reason set be R_D . Responsiveness can be represented by

$$r_A \in \mathcal{R}_{\text{relevant}} \implies r_A \text{ is addressed in } R_D. \quad (2161)$$

Equation 2161 requires engagement with material arguments while permitting rejection of them.

24.24 Transparency

Transparency concerns visibility of rules, evidence, decision standards, responsibilities, and review pathways.

Let transparent procedural information be

$$T_P = (R_u, E_v, S_D, A_I, R_w), \quad (2162)$$

where R_u denotes rules, E_v evidence visibility, S_D decision standards, A_I institutional authority, and R_w review mechanisms. Equation 2162 represents transparency as a structured profile.

Transparency should remain compatible with privacy, confidentiality, and protection of third parties.

24.25 Explainability

Explainability concerns whether affected persons can understand how the decision relates to the information and standards used.

A formally transparent mathematical model can remain practically unexplainable.

Procedural explanation should therefore be calibrated to the decision's stakes, complexity, and contestability.

24.26 Evidence as a Procedural Object

Evidence has both epistemic and procedural dimensions.

Let evidence set be

$$\mathcal{E} = \{e_1, \dots, e_n\}. \quad (2163)$$

Equation 2163 becomes procedurally relevant through questions concerning:

- who can submit evidence;
- who can inspect it;
- how reliability is evaluated;
- whether adverse evidence can be answered;
- who bears proof burdens;
- how uncertainty is handled.

24.27 Evidentiary Symmetry

Parties can possess unequal access to evidence.

Let available evidence be

$$\mathcal{E}_A \neq \mathcal{E}_I, \quad (2164)$$

where A denotes the affected subject and I the institution. Equation 2164 identifies evidentiary asymmetry as a procedural object.

An institution that controls the relevant records can bear stronger disclosure duties precisely because formal equality of proof burdens would operate unequally.

24.28 Burden of Proof

Let $\mathcal{B}_P$ allocate proof burdens across propositions and parties.

A simple representation is

$$\mathcal{B}_P : \mathcal{C} \longrightarrow \mathcal{A}, \quad (2165)$$

where $\mathcal{C}$ contains contested claims and $\mathcal{A}$ parties. Equation 2165 shows that proof burdens are part of the institutional architecture rather than neutral background.

The appropriate burden can vary across:

- emergency assistance;
- liability;
- interim measures;
- permanent sanctions;
- historical recognition.

24.29 Standards of Proof

Different decisions can require different epistemic thresholds.

Let evidence strength for claim c be $K(c)$, and required threshold for decision type d be θ_d .

The standard is represented by

$$K(c) \geq \theta_d. \quad (2166)$$

Equation 2166 permits a lower threshold for reversible emergency support and a higher threshold for irreversible adverse action.

This structure connects epistemic uncertainty with proportionality.

24.30 Evidence Preservation

Procedural justice can require preservation of evidence while harm is ongoing.

Let evidence survival probability through time t be

$$P_E(t) = P(e \text{ remains available at } t). \quad (2167)$$

Equation 2167 makes evidence preservation part of future procedural capability.

Delay that predictably destroys evidence can therefore create procedural harm.

24.31 Adverse-Evidence Disclosure

Where a decision relies materially upon adverse evidence, the affected party ordinarily requires a meaningful opportunity to know and answer it, subject to legitimate confidentiality constraints.

Let material adverse evidence be $\mathcal{E}_M^-$. Disclosure adequacy can be represented by

$$\mathcal{E}_M^- \subseteq \mathcal{E}_{\text{disclosed}}^- \cup \mathcal{E}_{\text{protected}}^-, \quad (2168)$$

where protected evidence requires an independent justification and appropriate procedural substitute. Equation 2168 prevents undisclosed material from silently determining the outcome.

24.32 Epistemic Contestability

Affected persons should be able to challenge:

- factual claims;
- data quality;
- model assumptions;
- causal interpretations;
- counterfactual baselines;
- classifications.

This is especially important where restoration depends upon complex models of future generativity.

24.33 Model Contestability

Let institutional model be M_I and alternative model submitted by the affected subject or reviewer be M_A .

A contestable architecture permits

$$M_I \xrightarrow{\text{challenge}} \{M'_I, M_A, \mathfrak{M}\}, \quad (2169)$$

where $\mathfrak{M}$ can contain several plausible models. Equation 2169 prevents one institutional model from becoming epistemically final merely because it was used first.

24.34 Baseline Contestability

The reference system itself should be contestable where its selection materially affects the remedy.

Let initial baseline be B_0 . Review can produce

$$B_0 \xrightarrow{\text{review}} B_1. \quad (2170)$$

Equation 2170 makes baseline choice an explicit decision subject to reasons and revision.

24.35 Value-Object Contestability

Affected persons should also be able to contest omission of relevant value objects.

An institution can frame a case as lost income while the claimant identifies professional recognition or future capability as material.

Let active value set be $\mathfrak{D}_V^*$. Contestation can produce

$$\mathfrak{D}_V^* \xrightarrow{\text{challenge}} \mathfrak{D}_V^* \cup \Delta \mathfrak{D}_V. \quad (2171)$$

Equation 2171 connects procedure directly to the value architecture developed in Section 21.

24.36 Decision Consistency

Procedural fairness can require similar treatment of relevantly similar cases.

Let cases a and b be equivalent under institutional relevance relation $\sim_I$.

Consistency can be represented by

$$a \sim_I b \implies D(a) \sim_D D(b). \quad (2172)$$

Equation 2172 is conditioned upon relevant similarity rather than superficial identity.

This makes the institutional interpretation of similarity itself contestable.

24.37 Consistency and Individualization

Consistency can conflict with sensitivity to special circumstances.

A purely standardized remedy can ignore subject-specific conversion factors.

A purely individualized system can become unpredictable and unequal.

Procedural justice therefore requires a defensible relation between general rules and case-specific reasons.

24.38 General Rules and Exceptions

Let default rule be R_G and exception function be X .

A structured individualized procedure can be represented by

$$D(a) = R_G(a) + X(a), \quad (2173)$$

where the additive notation is conceptual. Equation 2173 represents general treatment plus justified case-specific adjustment.

The reasons for $X(a)$ should be reviewable.

24.39 Impartial Rules and Differential Effects

A formally uniform rule can produce unequal practical access because affected persons possess different resources or capacities.

Procedural justice therefore distinguishes formal symmetry from effective fairness.

Examples include:

- identical filing deadlines for persons with unequal access to records;
- identical hearing formats despite disability;
- identical evidentiary burdens despite institutional control of data.

Reasonable procedural differentiation can therefore support equality rather than violate it.

24.40 Accessibility

A procedure should be usable by the persons it governs.

Accessibility can include:

- language;
- disability accommodation;
- cost;

- digital access;
- geographic access;
- cognitive complexity;
- support for representation.

Procedural accessibility is an enabling condition of voice and contestation.

24.41 Representation and Assistance

Complex proceedings can create asymmetry between institutional repeat players and individual claimants.

Let effective advocacy capacities be A_I and A_C . Severe asymmetry can be represented by

$$A_I \gg A_C. \quad (2174)$$

Equation 2174 can justify access to counsel, advocacy, ombuds services, navigators, or simplified procedure.

Procedural assistance should be distinguished from deciding the substantive claim on the claimant's behalf.

24.42 Respectful Treatment

Procedural justice also concerns how persons are treated within the process.

Tyler's work emphasizes the importance of fairness, dignity, and respectful treatment for legitimacy and acceptance of authority [64].

Respect can include:

- non-humiliating communication;
- acknowledgment of personhood;
- seriousness toward submitted reasons;
- avoidance of stigmatizing assumptions;
- clear communication.

Respect should not be treated merely as a technique for securing compliance.

It can possess independent relational value.

24.43 Procedural Recognition

A person can receive favorable substantive treatment while being procedurally misrecognized.

For example, an institution can provide compensation while describing the claimant as irrational or troublesome.

Procedural recognition therefore concerns standing within the decision process, not only the final remedy.

24.44 Procedural Equality

Procedural equality concerns whether parties possess appropriately comparable opportunities to participate, present evidence, challenge claims, and access review.

Let procedural opportunity vectors be

$$\mathbf{P}_A = (A_A, V_A, E_A, C_A, R_A) \quad (2175)$$

for affected subject A , with an analogous vector for other parties. Equation 2175 provides a multidimensional object for procedural equality.

24.45 Procedural Equality and Power

Identical formal opportunities can be insufficient under extreme power asymmetry.

A restorative process involving an employer and worker, state and individual, or institution and dependent beneficiary can therefore require compensating procedural protections.

Such protections can include:

- independent review;
- assistance;
- anti-retaliation rules;
- disclosure duties;
- adjusted evidentiary burdens.

24.46 Anti-Retaliation

Voice becomes illusory where participation creates a material risk of retaliation.

Let participation be $V_A = 1$, and expected retaliatory loss be L_{ret} .

Effective voice can require

$$\mathbb{E}[L_{\text{ret}} \mid V_A = 1] \leq \theta_{\text{ret}}. \quad (2176)$$

Equation 2176 represents a bounded retaliation risk rather than assuming voice exists whenever a reporting channel exists.

Anti-retaliation protects the institution's epistemic capacity as well as the speaker.

24.47 Confidentiality

Confidentiality can support safe participation.

Excessive secrecy can undermine accountability and response.

A procedural architecture should therefore distinguish:

- claimant privacy;
- witness confidentiality;
- protected deliberation;
- reasons that must be disclosed;
- public accountability.

One undifferentiated rule of secrecy or publicity is rarely adequate.

24.48 Privacy

Restorative assessment can invite collection of extensive personal data because a richer model can improve prediction of future capability or generativity.

Procedural justice places independent limits on such collection.

Let collected data set be $\mathcal{D}_R$ and minimally necessary data set be $\mathcal{D}_{\min}$.

A data-minimization criterion can be represented by

$$\mathcal{D}_R \subseteq \mathcal{D}_{\min} \cup \mathcal{D}_C, \quad (2177)$$

where $\mathcal{D}_C$ contains additional information collected through an independently justified basis such as valid consent. Equation 2177 constrains informational expansion.

24.49 Purpose Limitation

Information collected for one restorative purpose should not automatically be repurposed.

Let purpose of collection be p_0 and later proposed use p_1 .

A purpose-limitation rule requires

$$p_1 \in \mathcal{P}_{\text{compatible}} \quad (2178)$$

or another independent authorization. Equation 2178 separates restorative data from a general institutional surveillance resource.

24.50 Data Retention

Procedural monitoring should include retention limits.

Let

$$\tau_{\text{retain}} = f(p, l, r), \quad (2179)$$

where p denotes purpose, l legal or accountability requirements, and r future review needs. Equation 2179 makes data retention purpose-sensitive.

Permanent retention should not be inferred from the fact that information was once useful.

24.51 Provenance and Privacy

Procedural accountability can require provenance while privacy requires limits on disclosure.

A versioned architecture can therefore preserve

$$\text{Prov}(Z) \quad (2180)$$

while restricting access to sensitive content.

Equation 2180 identifies the object that can remain available for accountability even where substantive details have restricted access.

24.52 Contestability

Contestability concerns the practical ability to challenge a decision, classification, model, remedy, or completion judgment.

Let initial decision be D_0 . A contestable process permits

$$D_0 \xrightarrow{C} D_1, \quad (2181)$$

where C is a defined challenge mechanism. Equation 2181 does not require that every challenge change the outcome.

It requires the possibility of meaningful reconsideration.

24.53 Grounds of Contestation

A restorative decision can be challenged on several grounds:

- factual error;
- omitted evidence;
- procedural irregularity;
- biased decision making;
- model error;
- inappropriate baseline;
- omitted value object;
- disproportionate remedy;
- changed circumstances.

These grounds should be distinguished because they can lead to different forms of review.

24.54 Internal Review

Internal review can provide speed, expertise, and institutional learning.

Its legitimacy depends upon whether the reviewing body possesses sufficient independence from the initial decision.

Let initial decision unit be I_0 and reviewing unit I_1 . A minimum structural condition can require

$$I_1 \neq I_0. \quad (2182)$$

Equation 2182 expresses formal separation, while actual independence requires the broader structural assessment described earlier.

24.55 External Review

External review can provide stronger independence and weaker contextual knowledge.

The appropriate review architecture depends upon:

- stakes;
- reversibility;
- institutional conflict;
- required expertise;
- urgency;
- resource cost.

Mashaw's analysis of administrative due process is particularly relevant to the need to fit procedural form to administrative function rather than treating one judicial model as universally optimal [34].

24.56 Appellate Structure

Multiple levels of review can be represented by

$$D_0 \xrightarrow{R_1} D_1 \xrightarrow{R_2} D_2. \quad (2183)$$

Equation 2183 permits escalation of increasingly serious or contested issues.

Unlimited layers can create delay, cost, and strategic exhaustion.

24.57 Review Intensity

Review can examine:

- legality;
- procedure;
- factual findings;
- proportionality;
- remedy adequacy;
- model assumptions.

A reviewing institution should state its review scope.

Otherwise affected persons can mistake a narrow legality review for a complete reconsideration of substantive restoration.

24.58 Revisability

Revisability concerns the capacity to alter a restorative arrangement after implementation when evidence, conditions, or subject priorities change.

Let restorative plan at time t be R_t . Revisability is represented by

$$R_{t+1} = \mathcal{U} (R_t, D_{t+1}, P_{A,t+1}, E_{t+1}), \quad (2184)$$

where D_{t+1} is new evidence, $P_{A,t+1}$ subject preference, and E_{t+1} observed effect. Equation 2184 makes revision an ordinary feature of long-horizon restoration.

24.59 Revisability and Institutional Learning

Revision can concern the individual plan and the governing institutional rule.

Let institutional policy be Π_t . Policy learning is represented by

$$\Pi_{t+1} = L (\Pi_t, \mathcal{C}_t), \quad (2185)$$

where $\mathcal{C}_t$ contains experience from completed cases. Equation 2185 makes procedural learning part of future justice.

24.60 Reversibility

Reversibility concerns whether an intervention can be withdrawn or altered without creating large new harm.

Let reversal operator be $\mathcal{R}^{-1}$. Approximate reversibility can be represented by

$$d (\mathcal{R}^{-1} (\mathcal{R}(x)), x) \leq \varepsilon. \quad (2186)$$

Equation 2186 is available only where the intervention admits a meaningful reversal operation.

Irreversible interventions require stronger ex ante procedural safeguards.

24.61 Reversibility Gradient

Procedural burden can increase as reversibility decreases.

Let reversibility score be r , with lower values indicating less reversibility. A general design principle can be represented by

$$r \downarrow \implies S_P \uparrow, \quad (2187)$$

where S_P denotes required procedural safeguard intensity. Equation 2187 is a conditioned institutional principle rather than a universal quantitative law.

24.62 Proportionality

Procedural burden should be proportionate to the stakes, uncertainty, and reversibility of the decision.

Let safeguard intensity be

$$S_P = F(H, U, I, R_v), \quad (2188)$$

where H denotes severity of potential harm, U uncertainty, I intrusiveness, and R_v reversibility. Equation 2188 represents proportional procedure.

Routine correction can justify a lighter procedure than memory alteration, forced relocation, or long-term deprivation of support.

24.63 Procedural Minimalism

More procedure is not always more justice.

Excess procedure can impose:

- delay;
- cost;
- repeated disclosure;
- psychological burden;
- strategic advantage for repeat players.

A just procedure should therefore use the least burdensome architecture capable of adequately protecting the relevant interests.

24.64 Procedural Overload

Let procedural protection gain be $G_P(n)$ and burden be $B_P(n)$, where n represents increasing procedural complexity.

Net procedural value is represented by

$$V_P(n) = G_P(n) - B_P(n). \quad (2189)$$

Equation 2189 illustrates how additional procedure can eventually reduce net access or effectiveness. No universal functional form is assumed.

24.65 Procedural Burden

Procedural burden can include:

- time;
- monetary cost;
- documentation;
- repeated testimony;
- emotional effort;
- travel;
- expert fees;
- cognitive complexity.

A procedurally fair system should evaluate the distribution of these burdens.

24.66 Administrative Burden and Restorative Access

Let administrative burden be B_A . Effective access can decline as burden rises:

$$\frac{\partial A_P^E}{\partial B_A} < 0 \quad (2190)$$

where the local relation is meaningful. Equation 2190 illustrates why procedurally complex rights can become practically unavailable.

24.67 Timeliness

A procedurally valid decision can become unjust through excessive delay.

Let decision duration be

$$T_D = t_{\text{decision}} - t_{\text{claim}}. \quad (2191)$$

Equation 2191 makes delay a procedural object.

The acceptable duration depends upon the rate at which substantive harm develops during waiting.

24.68 Time-Sensitive Procedure

Let additional loss rate be dL/dt . A procedure can require faster action where

$$\frac{dL}{dt} \gg 0. \quad (2192)$$

Equation 2192 identifies cases in which ordinary institutional timelines can become substantively destructive.

Interim measures can therefore be procedurally required.

24.69 Interim Measures

Interim measures provide temporary protection while full adjudication remains incomplete.

Let interim intervention be R_I , final decision be D_F , and provisional duration be τ_I .

The architecture is represented by

$$R_I \rightarrow D_F \quad \text{within } \tau_I. \quad (2193)$$

Equation 2193 links rapid protection with later fuller review.

Interim measures should possess review and expiration conditions where their burdens are significant.

24.70 Temporary Measures and Evidentiary Thresholds

Reversible temporary support can justify a lower evidentiary threshold than an irreversible sanction.

Let θ_I be the interim threshold and θ_F final threshold. One possible structure is

$$\theta_I < \theta_F. \quad (2194)$$

Equation 2194 is justified only where the relative error costs and reversibility support such differentiation.

24.71 Temporal Equality

Waiting time can itself be distributed unequally.

Let claimant waiting times be

$$\mathbf{T} = (T_1, \dots, T_n). \quad (2195)$$

Equation 2195 creates a distributive procedural object.

Systematic delay for particular categories of claimants can generate unequal substantive consequences.

24.72 Urgency-Sensitive Timing

Equal waiting time can also be unjust where deterioration rates differ.

Let urgency be U_i . A priority rule can be represented by

$$T_i = F(U_i, S_i, R_i), \quad (2196)$$

where S_i denotes severity and R_i reparability. Equation 2196 permits justified temporal differentiation.

24.73 Hearing and Adjudicative Form

Adjudicative forms are especially useful where a defined dispute can be structured through reasoned claims and responses before a decision maker. Fuller's analysis emphasizes participation through reasoned argument as a central feature of adjudication and also identifies limits where problems become strongly polycentric [22].

This matters for restoration because some issues are well suited to adjudication:

- entitlement;
- historical fact;
- responsibility;

- legal status.

Other issues, such as redesigning a person's long-term life trajectory or balancing highly interconnected institutional relationships, can require broader governance forms.

24.74 Polycentric Restorative Problems

A problem is procedurally polycentric where changing one part materially changes many others.

Let decision variables be $x_1, \dots, x_n$ with cross-effects

$$\frac{\partial x_i}{\partial x_j} \neq 0 \quad (2197)$$

for many $i \neq j$. Equation 2197 gives a schematic representation of polycentric interdependence.

Collective reparations, institutional redesign, and network reconstruction can possess this structure.

24.75 Procedure Matching

The procedure should fit the type of restorative problem.

Let procedure family be $\mathfrak{P}$ and problem object be Q . Institutional fit can be represented by

$$\text{Fit}(P, Q). \quad (2198)$$

Equation 2198 makes procedural form a function of the problem rather than a universal template.

Mashaw's administrative-law analysis likewise supports attention to procedural design within institutional and administrative context [34].

24.76 Adjudication

Adjudication is well suited to bounded disputes involving:

- factual disagreement;
- legal entitlement;
- responsibility;
- interpretation of established rules.

Its strengths include reasoned argument, record creation, independence, and review.

Its limitations become stronger in highly dynamic, polycentric, or future-directed restorative design.

24.77 Administrative Procedure

Administrative procedures can provide:

- expertise;
- continuing monitoring;
- flexible assistance;
- standardized access;
- ongoing revision.

They can also generate bureaucratic opacity, repeat-player asymmetry, and distance from individual experience.

Procedural justice therefore requires attention to the internal design of administrative institutions rather than simply contrasting administration with courts.

24.78 Restorative Dialogue

Restorative dialogue can support acknowledgment, voice, and relational understanding [72, 7].

It should remain conditioned upon:

- safety;
- consent;
- power asymmetry;
- absence of coercive reconciliation;
- availability of alternatives.

Dialogue can be valuable without becoming a mandatory route to substantive repair.

24.79 Mediation

Mediation can support flexible agreement where parties possess sufficient capacity to negotiate.

Its procedural risks include:

- power imbalance;
- confidentiality hiding systemic harm;
- pressure to compromise valid rights;
- weak precedent.

A mediation option should therefore coexist with routes appropriate for rights and public-interest questions.

24.80 Participatory Design

Where the restorative object concerns future institutional structure, participatory design can be more appropriate than bilateral adjudication.

Affected groups can participate in defining:

- goals;
- implementation;
- monitoring;
- revision;
- governance.

The procedure remains subject to representation and minority-protection questions.

24.81 Representation of Collective Interests

Collective cases require rules concerning who may speak for the group.

Let represented group be G and representative be r . Representation adequacy can be expressed by

$$\text{Rep}(r, G) \geq \theta_R. \quad (2199)$$

Equation 2199 is conceptual. Relevant criteria can include mandate, accountability, inclusiveness, and revisability.

No single actor should be presumed to embody every internal group interest.

24.82 Internal Dissent

Collective reparations can contain internal disagreement.

Let group preference set be

$$\mathfrak{P}_G = \{P_1, \dots, P_k\}. \quad (2200)$$

Equation 2200 preserves internal plurality.

A procedurally legitimate collective process should provide some method for minority or dissenting views to remain visible.

24.83 Third-Party Participation

Third parties should participate where the proposed remedy materially alters their rights, obligations, property, or relationships.

Let affected third-party set be

$$\mathcal{A}_{3P} = \{B : |\Delta V_B(R)| \geq \theta_{3P}\}. \quad (2201)$$

Equation 2201 identifies materially affected third parties through a threshold concept.

Participation intensity should be proportionate to the significance of the effect.

24.84 Third-Party Veto

Third-party participation does not automatically imply veto authority.

The relevant procedural power depends upon:

- rights affected;
- causal responsibility;
- severity of burden;
- availability of alternatives;
- institutional mandate.

This distinction prevents participation from becoming obstruction while protecting legitimate interests.

24.85 Public Participation

Some restorative decisions have public consequences extending beyond identifiable parties.

Examples include:

- institutional reform;
- historical memorialization;
- environmental restoration;
- public archive policy.

Public participation can therefore become procedurally relevant.

Its design should avoid allowing majoritarian preferences to erase claimant rights.

24.86 Reasoned Public Justification

Where decisions alter public institutions, reasons can require broader justification than private settlement terms.

A public justification can identify:

- institutional objective;
- evidence;
- affected values;
- trade-offs;
- review mechanisms.

This supports legitimacy and institutional learning.

24.87 Procedural Legitimacy

Procedural legitimacy concerns whether an institution possesses and exercises authority through a process capable of justifying its decisions to those subject to them.

Tyler's empirical work links procedural fairness with legitimacy and compliance [64]. The present framework gives legitimacy a broader normative role that cannot be reduced to producing compliance.

Let legitimacy profile be

$$L_P = L(A, F, V, R, C), \quad (2202)$$

where A denotes lawful or justified authority, F fairness, V voice, R reasons, and C contestability. Equation 2202 is illustrative rather than a complete theory of legitimacy.

24.88 Acceptance and Legitimacy

A party's acceptance of an outcome does not independently establish procedural justice.

Acceptance can arise from:

- agreement;
- resignation;
- fear;
- exhaustion;
- lack of alternatives.

Procedural justice should therefore evaluate institutional conditions rather than infer legitimacy from behavioral compliance alone.

24.89 Procedural Trust

A fair process can contribute to justified trust in institutions.

Trust can also be inappropriate where the institution remains unreliable.

The relevant substantive-procedural object is therefore institutional trustworthiness rather than maximal subjective trust.

24.90 Procedural Predictability

A person should ordinarily be able to anticipate how the restorative procedure operates.

Let rule stability be S_R .

Procedural predictability can require

$$S_R \geq S_{\min} \quad (2203)$$

over the relevant decision horizon. Equation 2203 supports planning and reduces arbitrary surprise. Predictability should coexist with revisability where circumstances materially change.

24.91 Rule Change

Procedural rules can evolve.

Let rule set be Π_t . Revision can be represented by

$$\Pi_t \xrightarrow{\mathcal{U}} \Pi_{t+1}. \quad (2204)$$

Equation 2204 is procedurally legitimate only where the transition itself follows appropriate authority, notice, and non-retroactivity constraints.

24.92 Retroactivity

Changing decision standards after a claim has been filed can alter the claimant's position unexpectedly.

Let rule applicable at claim time be Π_{t_0} and later rule be Π_{t_1} .

Retroactive application is represented by

$$D_{t_1} = F(\Pi_{t_1}, C_{t_0}). \quad (2205)$$

Equation 2205 identifies the structure requiring independent justification.

24.93 Non-Arbitrariness

Procedural justice requires decisions to follow reasons, rules, and evidence rather than uncontrolled discretion.

Let decision variation unexplained by legitimate factors be ϵ_D .

A non-arbitrariness criterion can seek

$$|\epsilon_D| \leq \varepsilon_D. \quad (2206)$$

Equation 2206 is schematic. Procedural non-arbitrariness is broader than statistical consistency. It includes reviewability of the reasons for differential treatment.

24.94 Discretion

Restorative systems often require discretion because individual circumstances differ.

Let discretionary decision space be

$$\mathcal{D}_I = \{d : d_{\min} \preceq d \preceq d_{\max}\}. \quad (2207)$$

Equation 2207 represents bounded discretion.

Procedural justice requires standards governing how that discretion is used, recorded, and reviewed.

24.95 Reasoned Discretion

A discretionary decision should ordinarily identify the reasons that move it within the permitted range.
This preserves flexibility while reducing arbitrary variation.

Reasoned discretion is especially important where fixed rules cannot represent heterogeneous generative loss.

24.96 Procedural Individualization

Individualization can improve substantive fit and increase informational burden.

Let degree of individualization be ι .

The procedural architecture should select ι according to stakes, heterogeneity, and privacy cost rather than maximizing it automatically.

24.97 Standardization

Standardized procedures can improve predictability and access.

They can also compress heterogeneous values.

A hybrid system can use standardized minimum remedies with optional individualized escalation.

This architecture is particularly useful where large claim volumes coexist with a smaller number of complex cases.

24.98 Triage

Triage can direct cases toward different procedural tracks.

Let case characteristics be X_c . Track assignment is represented by

$$P_c = \mathcal{T}(X_c). \quad (2208)$$

Equation 2208 can assign routine correction, assistance, mediation, investigation, or adjudication.

Triage criteria should themselves be transparent and contestable.

24.99 Risk of Procedural Misclassification

Triage can misclassify a complex harm as a routine case or treat a simple claim as requiring excessive procedure.

Let true procedural need be P^* and assigned track P_c .

Misclassification occurs where

$$P_c \neq P^*. \quad (2209)$$

Equation 2209 creates a reason for escalation mechanisms and review.

24.100 Procedural Escalation

A case should move to a richer procedural form when simpler mechanisms cannot adequately protect the values at stake.

One escalation path is represented by

$$\text{informal correction} \rightarrow \text{administrative review} \rightarrow \text{independent review} \rightarrow \text{adjudication}. \quad (2210)$$

Equation 2210 is illustrative.

Different systems can use other sequences.

24.101 Procedural De-Escalation

Procedure can also become lighter when the contested issue is resolved.

De-escalation reduces ongoing burden and prevents institutionalization of the claimant's harmed status.

A system should therefore avoid keeping a person indefinitely inside an intensive restorative procedure after the relevant function has ended.

24.102 Procedural Re-entry

Subjects should be able to re-enter the procedural system when a completed remedy later fails under defined conditions.

Let closure occur at t_c and material failure at $t_f > t_c$.

A reopening mechanism can be represented by

$$C_{t_c} \xrightarrow{e_f} R_{t_f}, \quad (2211)$$

where e_f is a recognized reopening event. Equation 2211 permits bounded finality and future protection.

24.103 Procedural Finality

Finality provides security, limits costs, and allows institutions and parties to move forward.

Let final decision be D_F .

Finality can be represented by

$$D_F \notin \mathcal{R}_{\text{ordinary-review}} \quad (2212)$$

after ordinary review is exhausted. Equation 2212 does not exclude exceptional reopening for new evidence, fraud, or material remedy failure where the institutional design permits it.

24.104 Finality and Revisability

Finality and revisability apply to different objects.

A liability judgment can become final.

A rehabilitation plan can remain revisable.

A historical record can remain correctable.

A future support plan can respond to changed conditions.

The architecture should therefore assign finality separately across procedural objects.

24.105 Procedural Closure

Closure should identify:

- which issue is closed;
- which evidence was considered;
- which remedy was completed;
- which review routes remain;
- which future conditions can trigger reopening.

This prevents institutional closure from being misinterpreted as complete restoration of the subject.

24.106 Completion Declarations

A declaration of completed restoration has reflexive consequences.

Let completion declaration be Z_C . Later state can depend upon it:

$$\mathcal{G}_{t+1} = F(\mathcal{G}_t, Z_C). \quad (2213)$$

Equation 2213 makes completion declarations part of the procedure's causal structure.

A declaration that terminates support should therefore consider the post-declaration state.

24.107 Procedural Reflexivity

Procedures create records, expectations, classifications, and incentives.

Let procedural act be P_t . A reflexive update is represented by

$$\mathcal{G}_{t+1} = F(\mathcal{G}_t, P_t). \quad (2214)$$

Equation 2214 shows why procedure cannot always be treated as a neutral path toward a pre-existing substantive result.

24.108 Procedural Feedback

Subjects and institutions adapt to procedure.

A difficult claims process can discourage future claims. A predictable process can encourage early reporting. An opaque process can shift disputes into public or informal channels.

Procedure therefore helps generate the future epistemic environment of the institution.

24.109 Procedural Learning

Let procedure at time t be P_t .

Institutional learning can be represented by

$$P_{t+1} = L_P(P_t, D_t, E_t), \quad (2215)$$

where D_t contains outcome evidence and E_t procedural experience. Equation 2215 allows procedure to evolve in response to observed failures.

24.110 Procedural Audit

Procedural systems should be auditable at the aggregate level.

Relevant questions include:

- who enters the system;
- who exits before decision;
- how long cases take;
- which evidence is commonly unavailable;
- which groups experience higher denial or appeal rates;
- whether decisions change on review;
- whether support ends prematurely.

Audit should remain compatible with privacy and data minimization.

24.111 Error Correction

A just procedure requires capacity to identify and correct its own errors.

Let error state be D_E , correction mechanism C , and corrected state D_C .

The correction path is represented by

$$D_E \xrightarrow{C} D_C. \quad (2216)$$

Equation 2216 is itself a procedural capability.

A system with low initial error and no correction can be less just over time than a system capable of transparent revision.

24.112 Correction Provenance

Correction should preserve enough provenance to show:

- what was changed;
- when;
- why;
- by whom;
- what downstream systems were updated.

This supports accountability while preventing superseded information from remaining operational.

24.113 Procedural Robustness

A procedure should remain fair under ordinary institutional stress.

Let perturbation class be Δ_P .

Procedural robustness can require

$$J_P(P^\delta) \geq J_{P,\min} \quad \forall \delta \in \Delta_P. \quad (2217)$$

Equation 2217 asks whether due-process features survive workload spikes, staff turnover, emergencies, or system outages.

24.114 Procedural Failure under Stress

A procedure can appear fair under low caseload and collapse under high demand.

Examples include:

- review delays;
- loss of individualized reasons;
- automatic denial;
- inaccessible appeal;
- data errors.

Procedural resilience is therefore part of institutional design.

24.115 Emergency Procedure

Emergency conditions can justify temporary simplification of ordinary procedure.

The justification weakens as the emergency ends or as measures become more burdensome and durable.

Let procedural compression be $C_P(t)$. A restoration condition can require

$$C_P(t) \longrightarrow C_P^{\text{ordinary}} \quad (2218)$$

as emergency conditions recede. Equation 2218 prevents temporary emergency procedure from becoming permanent by inertia.

24.116 Procedural Provisionality

A provisional decision should be visibly provisional.

Let provisional status be Z_P .

The procedure should record

$$Z_P \neq Z_F, \quad (2219)$$

where Z_F denotes final status. Equation 2219 prevents temporary classification from acquiring unwarranted final authority.

24.117 Procedural Non-Domination

Procedure can itself be a site of domination.

An institution can possess arbitrary authority to:

- demand information;
- delay decisions;
- change standards;
- terminate support;
- deny review.

A non-dominating procedure therefore requires bounded discretion, reasons, review, and predictable rights [44].

24.118 Procedural Dependence

Let claimant dependence upon decision-maker D be

$$\Delta_D = |V_A(\text{do}(D = 1)) - V_A(\text{do}(D = 0))|. \quad (2220)$$

Equation 2220 is a schematic causal measure of how much one actor's decision controls the claimant's position.

High dependence can justify stronger procedural checks.

24.119 Procedural Recognition and Equal Standing

A process can embody relational equality when affected persons are treated as participants entitled to reasons rather than objects of administrative management [2, 71].

Procedural equal standing can therefore include:

- voice;
- respectful treatment;
- access to reasons;
- ability to challenge;
- absence of humiliating conditions.

24.120 Procedural Care

Care-oriented procedure emphasizes attentiveness and responsiveness to concrete needs [63].

A procedurally caring institution can adapt communication, timing, and support to the claimant's practical situation while preserving equal normative standing.

Responsiveness should not become discretionary paternalism.

24.121 Procedural Responsiveness

Let subject need state be $N_A(t)$ and procedural support be $S_P(t)$.

A responsive relation can be represented by

$$S_P(t) = F(N_A(t), C_I), \quad (2221)$$

where C_I contains institutional constraints. Equation 2221 permits support to adapt to need without changing the substantive decision standard arbitrarily.

24.122 Procedural Agency

Procedural agency concerns the subject's ability to shape the route through which restoration occurs.

Possible elements include:

- selecting among remedy options;
- determining disclosure preferences;
- choosing representation;
- requesting review;
- withdrawing from optional relational processes.

Procedural agency is especially important where several substantively adequate routes exist.

24.123 Procedural Capability

Formal procedural rights can be treated as capabilities only where they are practically exercisable.

Let formal right set be $\mathcal{R}_P$ and effective procedural capability set be $\mathcal{K}_P$.

The conversion relation is represented by

$$\mathcal{K}_P = \mathcal{C}_P(\mathcal{R}_P, X_A, I). \quad (2222)$$

Equation 2222 connects procedural justice with the capability approach [55, 50].

24.124 Procedural Generativity

A fair procedure can generate future institutional capacity.

It can produce:

- better records;
- improved rules;
- trustworthiness;
- learning;
- safer future reporting;
- new institutional knowledge.

Procedure can therefore possess generative value beyond resolving the current case.

24.125 Extractive Procedure

The opposite structure extracts information or labor from claimants without returning meaningful capacity.

Examples include repeated documentation requirements that primarily protect the institution, or mandatory disclosure generating institutional learning without improving claimant access.

A generative procedural theory should examine where the informational value of participation flows.

24.126 Value Return from Procedure

Let knowledge generated by a case be K_C . A generative procedural system can convert part of that knowledge into future safeguards:

$$K_C \xrightarrow{L_I} P_{t+1}^{\text{improved}}. \quad (2223)$$

Equation 2223 represents institutional learning as return of value generated through the claimant's participation.

24.127 Procedural Distribution

Procedural resources are scarce.

Institutions distribute:

- hearing time;
- investigation;
- expert support;
- legal assistance;
- review capacity.

Procedural justice therefore has a distributive dimension in addition to its case-level dimension.

24.128 Procedural Resource Allocation

Let procedural resource allocation be

$$\mathbf{R}_P = (r_1, \dots, r_n). \quad (2224)$$

Equation 2224 can be evaluated through equality, priority, urgency, or complexity.
Equal allocation is not always fair where cases differ sharply in stakes and difficulty.

24.129 Procedural Priority

Priority rules can allocate greater procedural resources to cases involving:

- severe ongoing harm;
- low reparability;
- high vulnerability;
- major rights implications;
- high evidentiary complexity.

The priority rule should remain transparent and reviewable.

24.130 Procedural Equality and Repeat Players

Institutions can be repeat players while claimants participate once.

Repeat players accumulate:

- procedural knowledge;
- data;
- legal expertise;
- strategic experience.

Procedural design can reduce this asymmetry through navigators, templates, disclosure rules, representation, and independent advice.

24.131 Procedural Complexity

Complexity can protect nuance and create exclusion.

Let complexity be C_P and substantive precision be S_P .

Increasing C_P can initially improve S_P and later reduce effective access.

This creates a procedural design problem rather than a reason to maximize or minimize complexity universally.

24.132 Procedural Compression

High-volume systems can compress procedure through standardized forms, presumptions, or automated triage.

Compression can be legitimate where:

- the issue is routine;
- error is easily reversible;
- escalation remains available;
- the compressed representation fits the claim.

Compression becomes problematic where it suppresses dimensions material to the case.

24.133 Automated Decision Support

Automation can support consistency, information retrieval, and routine calculation.

It creates procedural concerns concerning:

- model opacity;
- data quality;
- contestability;
- proxy discrimination;
- feedback loops;
- overreliance on historical data.

Automated support should therefore preserve identifiable human or institutional responsibility for consequential decisions.

24.134 Automation and Reason Giving

A model output alone does not constitute an adequate reason.

Let automated score be S_M , evidence be E , and governing rule be N .

A reasoned decision should preserve a relation such as

$$D = F(S_M, E, N) \quad (2225)$$

rather than treating S_M as self-justifying. Equation 2225 locates automated inference within a broader accountable decision structure.

24.135 Human Review

Human review is procedurally valuable only where the reviewer possesses actual authority and sufficient information to reconsider the automated result.

Nominal human confirmation of an effectively fixed output provides weak contestability.

24.136 Procedural Record

A consequential restorative decision should ordinarily generate a sufficient record for review.

Let procedural record be

$$\mathcal{Z}_P = (E, S, D, R, V), \quad (2226)$$

where E denotes evidence, S submissions, D decision, R reasons, and V version/provenance information. Equation 2226 supports review and correction.

The record should remain no more intrusive than necessary.

24.137 Record Completeness and Data Minimization

Procedural accountability favors sufficient records.

Privacy favors limited collection.

The appropriate target is therefore task-relative sufficiency:

$$\mathcal{Z}_{\min} \subseteq \mathcal{Z}_P \subseteq \mathcal{Z}_{\max}^{\text{legitimate}}. \quad (2227)$$

Equation 2227 expresses a bounded record architecture.

24.138 Procedural Provenance

The procedural record should preserve how the decision changed over time.

Let

$$D^{(0)} \rightarrow D^{(1)} \rightarrow \dots \rightarrow D^{(n)}. \quad (2228)$$

Equation 2228 allows correction without pretending the earlier decision never existed.

This supports historical accountability and present accuracy.

24.139 Procedural Publicity

Some procedural information should be public even where individual case details remain private.

Examples include:

- governing rules;
- aggregate outcomes;
- review standards;
- institutional responsibilities;
- audit findings.

Publicity can support systemic accountability without exposing unnecessary personal data.

24.140 Procedural Secrecy

Confidentiality can protect claimants and encourage reporting.

It can also hide recurring institutional harm.

A just architecture therefore distinguishes privacy of persons from secrecy of institutional patterns.

Aggregate reporting can sometimes preserve both.

24.141 Procedural Third-Party Privacy

Claimant access to evidence can conflict with third-party privacy.

Procedural solutions can include:

- redaction;
- summaries;
- independent review;
- limited disclosure;
- confidentiality orders.

The procedural substitute should preserve meaningful contestability to the greatest feasible extent.

24.142 Procedural Safety

Participation can expose persons to emotional, relational, or physical risk.

A procedurally safe system can offer:

- remote participation;
- separate sessions;
- representation;
- protected communication;
- choice concerning direct encounter.

Restorative dialogue should never require unsafe confrontation as the price of repair.

24.143 Procedural Dignity

The institution should avoid unnecessary repetition of humiliating or traumatic facts.

Where a fact has already been adequately established, repeated proof demands can constitute procedural burden without corresponding epistemic gain.

A provenance-rich record can reduce this burden by allowing verified information to be reused appropriately.

24.144 Procedural Memory

Institutions should remember prior verified findings sufficiently to prevent claimants from restarting the same proof process repeatedly.

Institutional memory can therefore protect procedural dignity.

It should coexist with correction mechanisms so that erroneous findings do not become permanent.

24.145 Procedural Forgetting

Institutions also require bounded forgetting.

Temporary vulnerability labels, provisional classifications, and irrelevant historical details should not remain perpetually active.

Procedural justice therefore concerns what institutions remember, what they archive, and what they stop using.

24.146 Completion and Data Expiration

Completion can trigger expiration of monitoring or temporary data.

Let monitoring dataset be $\mathcal{D}_M$. After purpose expiration,

$$\mathcal{D}_M \xrightarrow{\text{expiry}} \mathcal{D}_{\text{archive}} \quad \text{or} \quad \emptyset, \quad (2229)$$

according to legitimate retention requirements. Equation 2229 prevents restorative monitoring from becoming permanent by default.

24.147 Procedural Burden on Third Parties

Witnesses and third parties can also face burdens from restorative proceedings.

A just procedure should avoid unnecessary demands while preserving evidence needed for fair determination.

This is another reason procedure should be proportional to the stakes.

24.148 Procedural Burden on Institutions

Institutional capacity matters because procedures that cannot be delivered reliably can create new injustice through delay and inconsistency.

Let available capacity be C_I and procedural demand be D_P .

A capacity condition is represented by

$$D_P \leq C_I. \quad (2230)$$

Equation 2230 is schematic.

Where demand exceeds capacity, the solution can involve additional resources, simplification, prioritization, or institutional division of labor.

24.149 Procedural Institutional Fit

A procedure can be fair in abstraction and badly matched to the institution implementing it.

The fit relation can be represented by

$$\text{Fit}(P, I, Q, \tau). \quad (2231)$$

Equation 2231 includes institutional authority, expertise, speed, independence, and capacity.

24.150 Division of Procedural Labor

Different procedural tasks can be allocated to different institutions:

- emergency protection;
- fact finding;
- liability determination;
- rehabilitation planning;
- historical recognition;
- long-term monitoring.

This division can increase competence and create coordination gaps.

24.151 Procedural Handoffs

A handoff should transfer enough information and authority for the next institution to act without forcing complete repetition.

Let handoff be

$$H_{ij} : \mathcal{Z}_{P,i} \longrightarrow \mathcal{Z}_{P,j}. \quad (2232)$$

Equation 2232 should preserve relevant provenance, current status, and privacy constraints.

24.152 Handoff Gaps

A procedural gap occurs where one institution closes before another acquires responsibility.

Let closure time be t_c^i and receiving time t_o^j .

A gap exists when

$$t_o^j > t_c^i \quad (2233)$$

without bridge support. Equation 2233 makes coordination a procedural-justice issue.

24.153 Procedural Overlap

Overlapping institutions can also issue inconsistent decisions.

Let outputs be D_i and D_j . Conflict exists where

$$D_i \not\sim D_j \quad (2234)$$

on an object requiring shared interpretation. Equation 2234 can justify a conflict resolution or precedence rule.

24.154 Procedural Consistency Across Institutions

Interinstitutional consistency should preserve the distinction between different mandates.

A court's liability judgment and a welfare agency's need assessment need not match because they answer different questions.

Consistency therefore concerns compatible use of shared facts and current status rather than identical conclusions.

24.155 Procedural Justice and Historical Records

Historical correction requires procedural safeguards concerning:

- evidence;
- notice;
- competing claims;
- correction;
- access;
- versioning.

Historical truth cannot be generated legitimately by unilateral institutional rewriting.

24.156 Procedural Justice and Recognition

Recognition can be substantively correct and procedurally defective.

An apology drafted without the affected community's participation can fail to address the meaning of the wrong.

Conversely, participation cannot substitute for accurate recognition.

Substantive and procedural recognition therefore remain distinct.

24.157 Procedural Justice and Capability

A restorative procedure should preserve the claimant's capability to use the remedy.

A process that consumes years, resources, and health can reduce the capability the eventual remedy is intended to restore.

Procedural burden is therefore capable of producing substantive capability loss.

24.158 Procedural Justice and Generativity

Procedure can preserve or damage future generativity.

A process can:

- interrupt work;
- consume financial resources;
- damage relationships;
- create public stigma;
- generate useful institutional learning.

Procedural design should therefore include the generative effects of the process itself.

24.159 Procedural Non-Interference with Future Generation

Let the generative cost of procedure be $C_G(P)$.

A proportionality criterion can require

$$C_G(P) \leq C_{G,\max} \quad (2235)$$

relative to the stakes and epistemic benefit. Equation 2235 prevents procedure from destroying disproportionate amounts of the value it seeks to restore.

24.160 Procedural Justice and Collective Repair

Collective procedures require additional safeguards concerning:

- representation;
- internal dissent;
- transparency of allocation;
- governance of shared resources;
- long-term accountability.

Collective agreement should not be inferred merely from participation of one recognized representative.

24.161 Procedural Justice and Intergenerational Repair

Future persons cannot participate directly.

Intergenerational procedural legitimacy therefore requires institutional representation of future interests through mechanisms such as:

- sustainability constraints;
- fiduciary duties;
- long-term impact analysis;
- protected future funds;
- ecological safeguards.

The appropriate mechanism depends upon the institutional context.

24.162 Procedural Justice and Ecological Repair

Ecological restoration raises the additional problem that affected nonhuman systems do not participate in ordinary deliberation.

Procedural design can therefore require expertise, long-term monitoring, public participation, and precautionary safeguards.

Human representation of ecological interests remains epistemically limited and should be treated with corresponding humility.

24.163 Procedural Justice under Uncertainty

Uncertainty should be governed procedurally rather than hidden.

Let uncertainty set be Ω .

A decision should identify, where material:

- what is known;
- what remains uncertain;
- which assumptions were used;
- who bears residual risk;
- when review will occur.

This makes uncertainty contestable.

24.164 Allocation of Epistemic Risk

Let probability of error be p_e , false-positive burden be C_{FP} , and false-negative burden be C_{FN} .

Expected error burden can be represented by

$$E_C = p_{FP}C_{FP} + p_{FN}C_{FN}. \quad (2236)$$

Equation 2236 is a simplified decision-theoretic representation.

Procedural justice additionally asks who bears each type of error and whether the allocation is acceptable.

24.165 Precautionary Procedure

Where potential harm is severe and weakly reparable, procedure can authorize reversible protective measures before full certainty.

Such measures should include:

- bounded duration;
- periodic review;
- least-restrictive design;
- transparent uncertainty;
- preservation of later contestation.

Precaution therefore changes procedure as well as substantive action.

24.166 Procedural Justice under Moral Disagreement

Parties can agree on facts and disagree about values.

A legitimate procedure should permit those normative disagreements to be identified explicitly.

It should avoid presenting one contested value theory as though it were an empirical conclusion.

24.167 Procedural Resolution of Incomparability

Where two substantively admissible remedies remain incomparable, procedure can provide a legitimate selection mechanism.

Let nondominated remedy set be

$$\mathcal{F}_J = \{R_1, \dots, R_k\}. \quad (2237)$$

Equation 2237 defines the region within which subject choice, deliberation, negotiation, or institutional rule can operate.

Procedure should not manufacture a false scalar ranking merely to avoid acknowledging incomparability.

24.168 Procedural Choice among Admissible Futures

Where several futures satisfy rights and substantive thresholds, the subject's choice can receive strong authority.

Let admissible future set be $\mathcal{F}_A$. Subject-directed selection is

$$f_A^* \in \arg \max_{f \in \mathcal{F}_A} P_A(f), \quad (2238)$$

where P_A represents the subject's preference ordering. Equation 2238 applies only after inadmissible options have been excluded.

24.169 Procedural Paternalism

Paternalistic procedure arises when the institution limits voice or choice on the ground that it knows the subject's good.

Some paternalistic interventions can be defensible under conditions such as incapacity or severe immediate risk.

The burden of justification should increase with:

- intrusiveness;
- irreversibility;
- value disagreement;
- availability of less restrictive alternatives.

24.170 Procedural Counterfactual Paternalism

Counterfactual paternalism has a procedural form when the institution privileges the reconstructed preferences of the no-harm subject over the expressed preferences of the present subject.

Let counterfactual preference be P_A^C and present preference be P_A^H .

Procedural domination can arise where

$$P_A^C \succ_{\text{institution}} P_A^H \quad (2239)$$

without independent justification. Equation 2239 makes the conflict explicit.

24.171 Procedural Right to a Different Future

The right to a different future therefore has a procedural dimension.

The subject should be able to present and select a future trajectory that differs from the counterfactual reference where it remains substantively admissible.

This protects present agency from being displaced by restorative modeling.

24.172 Procedural Justice and Remedy Choice

Remedy design can involve choice among:

- reinstatement;
- compensated exit;
- transfer;
- rehabilitation;
- long-term support;
- public recognition.

A fair procedure should explain available alternatives and their consequences.

24.173 Remedy Choice under Dependency

Choice can be distorted where one option carries catastrophic loss of basic support.

Let utility or viability of refusal be V_{refuse} .

Meaningful remedy choice can require

$$V_{\text{refuse}} \geq V_{\text{min}}. \quad (2240)$$

Equation 2240 formalizes the intuition that choice requires a minimally viable alternative.

24.174 Procedural Justice and Implementation

Procedural justice continues after the remedy is selected.

Implementation requires:

- clear responsibilities;
- milestones;
- access to information;
- monitoring;
- correction of implementation failures.

A fair decision can become procedurally ineffective if implementation is opaque or unaccountable.

24.175 Implementation Provenance

Let implementation events be

$$\mathcal{E}_I = (e_1, \dots, e_n). \quad (2241)$$

Equation 2241 allows progress, delay, and responsibility to be recorded.

This is particularly useful for long restorative plans.

24.176 Milestone Review

Let implementation milestones be $m_1, \dots, m_k$.

Review at milestone m_i can evaluate

$$R_{m_i} = F(D_{m_i}, P_{A,m_i}, J_{S,m_i}). \quad (2242)$$

Equation 2242 combines new evidence, subject preference, and substantive performance.

Milestone review avoids waiting until complete failure becomes visible.

24.177 Procedural Monitoring

Monitoring should be limited to the information necessary to evaluate implementation and safety.

More monitoring can improve detection and reduce privacy.

The appropriate level therefore belongs to the procedural proportionality analysis.

24.178 Subject-Controlled Monitoring

Where several monitoring arrangements provide adequate institutional assurance, the subject can be allowed to select the less intrusive route.

This gives privacy and agency practical weight rather than treating them as abstract values.

24.179 Procedural Failure and Remedy Failure

A remedy can fail because:

- the substantive theory was wrong;
- implementation failed;
- circumstances changed;

- the procedure omitted important evidence;
- the subject's goals changed.

Failure diagnosis should therefore distinguish substantive from procedural causes.

24.180 Procedural Error Taxonomy

Let error family be

$$\mathfrak{E}_P = \{E_{\text{access}}, E_{\text{evidence}}, E_{\text{bias}}, E_{\text{reason}}, E_{\text{delay}}, E_{\text{review}}, E_{\text{implementation}}\}. \quad (2243)$$

Equation 2243 distinguishes several procedural failure modes.
Different errors require different corrections.

24.181 Procedural Repair

A flawed restorative procedure can itself require repair.

Possible responses include:

- rehearing;
- correction of records;
- renewed evidence collection;
- independent review;
- apology;
- compensation for procedural burden.

Thus the process of repair can itself become the object of later repair.

24.182 Recursive Procedural Justice

Let initial restorative process be P_0 , procedural challenge be C_1 , and corrected process be P_1 .

The sequence is represented by

$$P_0 \xrightarrow{C_1} P_1 \xrightarrow{C_2} P_2 \rightarrow \dots. \quad (2244)$$

Equation 2244 reveals a potential regress.

Institutional systems therefore need bounded review and finality while preserving exceptional correction.

24.183 Procedural Non-Closure

Because procedural decisions alter future states and later reviews evaluate those altered states, complete procedural externality is unavailable in many restorative systems.

The sequence can be represented by

$$\mathcal{G}_t \rightarrow P_t \rightarrow \mathcal{G}_{t+1} \rightarrow P_{t+1}. \quad (2245)$$

Equation 2245 is the procedural form of reflexive non-closure developed in Section 20.

Table 21: Principal dimensions of procedural justice of return

Procedural dimension	Primary procedural object	Restorative function	Characteristic failure
Access	Practical entry to procedure	Makes procedural rights usable	Formal procedure remains inaccessible
Notice	Timely intelligible information	Enables preparation and response	Decision arrives before meaningful response
Voice	Ability to present reasons and evidence	Represents affected perspectives	Token participation
Consent	Voluntary authorization	Protects agency for intrusive interventions	Dependency distorts voluntariness
Impartiality	Independent decision structure	Limits interested judgment	Institution judges its own interest
Evidence	Disclosure, proof, provenance	Supports accurate and contestable findings	Institution controls decisive evidence
Reason giving	Explanation of factual and normative basis	Enables understanding and review	Outcome announced without responsive reasons
Accessibility	Language, disability, cost, complexity	Makes participation effective	Equal formal rules create unequal access
Privacy	Data minimization and use limitation	Limits informational intrusion	Restoration becomes surveillance
Contestability	Challenge to facts, models, baselines, values	Permits correction	Initial classification becomes final by default
Review	Independent reconsideration	Corrects error and abuse	Review lacks authority or independence
Revisability	Adaptation to new evidence and conditions	Supports long-horizon restoration	Initial plan becomes rigid
Reversibility	Capacity to undo interventions	Limits irreversible error	High-impact intervention cannot be corrected
Timeliness	Decision and review duration	Prevents process-generated loss	Delay transforms the harm
Implementation	Execution and monitoring	Connects decision to actual remedy	Nominal remedy remains unrealized
Finality	Closure and reopening rules	Provides security and bounded process	Premature closure or endless review

24.184 Procedural Humility

A restorative institution should therefore avoid claiming that its procedure produces an epistemically final representation of the subject.

It can instead state:

This decision is justified for this institutional purpose, on this evidentiary record, under these standards, subject to the specified routes of review.

Such bounded claims improve legitimacy and precision.

24.185 Procedural Justice Profile

A case-level procedural profile can be represented by Equation 2246.

$$\mathbf{J}_P = (J_{\text{access}}, J_{\text{notice}}, J_{\text{voice}}, J_{\text{consent}}, J_{\text{impartiality}}, J_{\text{evidence}}, J_{\text{reason}}, J_{\text{privacy}}, J_{\text{contest}}, J_{\text{review}}, J_{\text{revision}}, J_{\text{time}}, J_{\text{implementation}}) . \quad (2246)$$

Equation 2246 preserves procedural plurality.

One component should not automatically substitute for another.

24.186 Procedural-Justice Matrix

Table 21 summarizes the principal procedural dimensions developed in this section.

Table 21 demonstrates that procedural justice cannot be represented adequately by one variable such as participation or speed.

Table 22: Procedural forms and characteristic restorative functions

Procedural form	Characteristic function	Comparative strength	Recurring constraint
Routine administrative correction	Clear factual or status error	Speed and low burden	Weak for contested complex harms
Administrative review	Eligibility, assistance, implementation	Expertise and continuity	Internal conflict and opacity
Adjudication	Rights, liability, contested facts	Reasoned argument and formal review	Cost, delay, polycentric limits
Mediation	Negotiable multi-interest disputes	Flexibility and party input	Power asymmetry and weak precedent
Restorative dialogue	Acknowledgment and relational repair	Voice and interpersonal understanding	Requires consent and safety
Participatory design	Collective or structural reform	Distributed knowledge and legitimacy	Representation and coordination complexity
Independent audit	Systemic performance and recurring error	Cross-case accountability	Distance from individual context

24.187 Procedure–Function Matrix

Table 22 illustrates the differing strengths and limitations of several procedural forms.

Table 22 supports procedure matching rather than a single universal restorative process.

24.188 Procedural Justice and Substantive Fidelity

A high-fidelity restorative outcome can be reached through an unjust process.

Let restorative fidelity be F_R and procedural justice be J_P . It can hold that

$$F_R \rightarrow 1 \quad \text{while} \quad J_P \rightarrow 0. \quad (2247)$$

Equation 2247 captures the central independence between fidelity and procedural legitimacy.

24.189 Procedural Justice and Substantive Improvement

A procedurally just process can also produce an outcome below the substantive ideal because of uncertainty, scarcity, or limited authority.

Thus

$$J_P \geq J_{P,\min} \quad \text{and} \quad J_S < J_{S,\text{ideal}} \quad (2248)$$

can hold simultaneously. Equation 2248 prevents procedural fairness from being treated as proof of substantive justice.

24.190 Procedural Minimum

A minimum procedural standard can be represented by

$$\mathbf{J}_P \succeq \boldsymbol{\theta}_P. \quad (2249)$$

Equation 2249 requires every protected procedural dimension to satisfy its threshold.

The threshold vector can vary according to the intervention's stakes and institutional form.

24.191 Procedural Dominance

Procedure P_1 procedurally dominates P_2 where

$$\mathbf{J}_P(P_1) \succeq \mathbf{J}_P(P_2) \quad (2250)$$

under the selected partial order. Equation 2250 permits strong comparison without requiring scalar aggregation.

24.192 Procedural Trade-Offs

Many procedural designs involve trade-offs.

Examples include:

- speed and evidentiary depth;
- privacy and transparency;
- standardization and individualization;
- finality and revisability;
- participation and administrative burden.

These trade-offs should be stated directly rather than hidden inside institutional custom.

24.193 Speed and Accuracy

Let decision delay be T_D and epistemic error probability be p_e .

A procedural design can generate relation

$$p_e = f(T_D), \quad (2251)$$

where the form depends upon the process. Equation 2251 represents the possibility that additional time improves evidence while delay generates substantive loss.

The optimal balance cannot be determined from epistemic accuracy alone.

24.194 Privacy and Transparency

Transparency can improve accountability and reduce privacy.

The relevant architecture can distinguish:

- public rules;
- private personal evidence;
- public aggregate statistics;
- protected internal deliberation;
- disclosed decision reasons.

This decomposition reduces the apparent need to choose one extreme.

24.195 Finality and Revisability

Finality protects reliance and closure.

Revisability protects against error and changed circumstances.

A layered system can assign strong finality to adjudicated historical liability and continuing revisability to future-oriented support.

This is preferable to treating the entire restorative relationship as either permanently open or completely closed.

24.196 Standardization and Individualization

Standardization supports predictability and low cost.

Individualization supports substantive fit.

A tiered process can combine:

$$P = P_{\text{standard}} + P_{\text{escalated}} \quad (2252)$$

as a conceptual architecture. Equation 2252 represents standardized access with richer procedure available where material complexity appears.

24.197 Participation and Burden

More participation can increase legitimacy and information while imposing greater time and emotional burden.

Participation should therefore be an available capacity rather than an obligation to repeatedly relive the harm.

Representation and written submissions can sometimes substitute for direct personal participation.

24.198 Procedural Choice

Where several procedures are legitimate, parties can sometimes choose among them.

Let procedural options be

$$\mathcal{P}_A^{\text{adm}} = \{P_1, \dots, P_k\}. \quad (2253)$$

Equation 2253 can include written review, hearing, mediation, or assisted dialogue.

Choice should remain conditioned upon the nature of the legal or public issue.

24.199 Procedural Justice Statement

A fully specified procedural judgment can be represented by

$$\mathcal{J}_P = [I; P; \mathfrak{D}_P; \theta_P; K; \tau; s; \mathcal{R}_v; F], \quad (2254)$$

where I denotes institution, P procedure, $\mathfrak{D}_P$ procedural objects, θ_P thresholds, K epistemic basis, τ horizon, s scale, $\mathcal{R}_v$ review and revision rules, and F finality conditions. Equation 2254 makes procedural claims auditable and bounded.

24.200 Procedural Independence Principle

The first principle of this section is:

Substantive restorative success does not independently establish the justice of the procedure through which that result was selected or produced.

24.201 Effective Access Principle

The second principle is:

Formal availability of a restorative procedure should be distinguished from the affected person's practical capability to use it safely and effectively.

24.202 Voice Principle

The third principle is:

Affected persons should possess a meaningful opportunity to introduce relevant facts, values, preferences, and objections into consequential restorative decisions.

24.203 Reason-Giving Principle

The fourth principle is:

Consequential restorative decisions should identify their material evidentiary, normative, and institutional grounds in a form sufficient for understanding and contestability.

24.204 Evidentiary Fairness Principle

The fifth principle is:

Proof burdens, disclosure duties, and evidentiary standards should account for institutional control of information and the consequences of error rather than assuming that formal symmetry always produces procedural fairness.

24.205 Consent and Refusal Principle

The sixth principle is:

Intrusive, identity-affecting, relational, or future-directing restorative interventions require correspondingly strong attention to consent and meaningful refusal.

24.206 Contestability Principle

The seventh principle is:

Material factual findings, classifications, baselines, models, and completion judgments should remain contestable through procedures proportionate to their effects.

24.207 Revisability Principle

The eighth principle is:

Long-horizon restorative arrangements should permit revision when material evidence, conditions, or legitimate subject priorities change.

24.208 Reversibility Principle

The ninth principle is:

As an intervention becomes less reversible and more intrusive, the required strength of ex ante procedural safeguards generally increases.

24.209 Temporal Fairness Principle

The tenth principle is:

Procedural justice includes the distribution and consequences of time, so extreme delay can constitute injustice even where every formal procedural stage eventually occurs.

24.210 Privacy Principle

The eleventh principle is:

The informational usefulness of personal data for restorative optimization does not independently establish a legitimate basis for collecting, retaining, or repurposing that data.

24.211 Third-Party Participation Principle

The twelfth principle is:

Third parties whose rights or substantial interests are materially affected by a proposed restorative arrangement should receive procedural protection proportionate to the nature of that effect.

24.212 Institutional Fit Principle

The thirteenth principle is:

Procedural form should be matched to the restorative function, institutional competence, and structure of the problem rather than derived from one universal model of adjudication or participation.

24.213 Procedural Minimalism Principle

The fourteenth principle is:

A restorative procedure should use no more informational, temporal, and participatory burden than is reasonably necessary to protect the values at stake.

24.214 Qualified Finality Principle

The fifteenth principle is:

Procedural finality should attach to specified institutional questions while preserving defined routes for exceptional correction, implementation failure, and changed future conditions.

24.215 Reflexive Procedure Principle

The sixteenth principle is:

Where classifications, evaluations, or declarations of completion alter the future system, those effects should enter the procedural assessment rather than being treated as external consequences.

24.216 Procedural Layer of the Atlas

The procedural layer can now be represented by

$$\mathfrak{P}_A = (\mathfrak{O}_P, \mathfrak{E}_P, \mathfrak{I}_P, \mathfrak{R}_P, \mathfrak{T}_P), \quad (2255)$$

where:

- $\mathfrak{O}_P$ contains procedural objects;
- $\mathfrak{E}_P$ contains procedural comparative standards;
- $\mathfrak{I}_P$ contains institutional forms;
- $\mathfrak{R}_P$ contains review, revision, and reopening rules;
- $\mathfrak{T}_P$ contains temporal procedural structure.

Equation 2255 adds procedural justice as an independent layer of the Atlas of Equivalence.

24.217 Implications for Fidelity–Procedure Conflict

The substantive and procedural layers can now be compared directly.

A stronger restorative intervention can produce:

- greater counterfactual similarity;
- greater capability;
- stronger generative fidelity;

while simultaneously requiring:

- greater coercion;
- more surveillance;
- stronger interference with third parties;
- reduced reversibility;
- weaker present agency.

The resulting conflict cannot be resolved by maximizing either restorative fidelity or procedural justice in isolation.

24.218 Transition to Fidelity and Procedure

Section 25 therefore examines direct conflicts between substantive restorative fidelity and procedural justice. It will analyze cases in which stronger equivalence requires increasingly intrusive intervention, including memory reconstruction, relational reconstruction, identity-affecting repair, compulsory reinstatement, extensive data collection, and intervention upon third parties.

The central question becomes:

How far may an institution pursue stronger restorative equivalence when each additional increment of fidelity requires greater interference with consent, agency, privacy, identity, third-party rights, or procedural legitimacy?

That conflict marks the transition from parallel substantive and procedural analysis toward an integrated theory of the limits of restoration.

25 Restorative Fidelity and Procedural Constraint

This section examines conflicts between restorative fidelity and procedural justice. The preceding sections developed these dimensions separately. Section 23 asked which post-harm conditions can count as substantively just. Section 24 asked through which procedures those conditions can legitimately be identified, selected, implemented, revised, and declared complete. The present section studies cases in which the two dimensions point in different directions.

The conflict becomes especially important as restorative technology and institutional capacity increase. Weak forms of restoration can often be achieved through low-intrusion measures such as correction of records, return of property, or payment of compensation. Stronger forms of equivalence can require deeper intervention into relationships, institutions, memory, identity, third-party behavior, information systems, and future life direction. The additional restorative fidelity can therefore carry procedural costs.

The central problem can be stated as follows:

How far may an institution pursue stronger equivalence with a restorative reference when each additional increment of fidelity requires greater interference with agency, consent, privacy, identity, reversibility, or the rights of others?

The section develops this problem through a fidelity–procedure space. It distinguishes weak and strong restorative fidelity, defines procedural cost and procedural admissibility, identifies several recurrent conflict types, analyzes the possibility of Pareto improvements and nondominated restorative options, and develops stopping principles for cases in which further fidelity would require unjustifiable intervention. It also examines the possibility that a lower-fidelity remedy can be more just overall than a higher-fidelity remedy.

25.1 Restorative Fidelity

Let F_R denote restorative fidelity relative to a specified indexed restorative claim.

A general representation is

$$F_R = F(B, Q, E, \mathcal{T}, \tau, \varepsilon, K), \quad (2256)$$

where the arguments retain the meanings developed in the earlier atlas: reference B , comparison object Q , comparative relation E , test class $\mathcal{T}$, horizon τ , tolerance ε , and epistemic basis K .

Equation 2256 does not imply one universal cardinal measure of fidelity. It can represent an ordered family of restorative standards.

25.2 Procedural Cost

Let procedural cost be represented by

$$C_P = C_P(C_{\text{coercion}}, C_{\text{privacy}}, C_{\text{identity}}, C_{\text{third}}, C_{\text{burden}}, C_{\text{irreversibility}}). \quad (2257)$$

Equation 2257 collects several procedural burdens associated with the restorative intervention.

The representation is vector-valued unless a justified aggregation rule is available.

25.3 The Fidelity–Procedure Space

A restorative option R_i can be located in a two-layer space:

$$R_i \mapsto (F_R(R_i), J_P(R_i)). \quad (2258)$$

Equation 2258 shows that restorative fidelity and procedural justice are independent axes.

The most difficult cases arise where

$$F_R(R_2) > F_R(R_1) \quad (2259)$$

and

$$J_P(R_2) < J_P(R_1). \quad (2260)$$

Equations 2259 and 2260 define a fidelity–procedure conflict.

25.4 Procedural Admissibility

Some restorative interventions should be excluded before fidelity is optimized.

Let the procedurally admissible set be

$$\mathcal{R}_P^{\text{adm}} = \{R : \mathbf{J}_P(R) \succeq \boldsymbol{\theta}_P\}. \quad (2261)$$

Equation 2261 requires protected procedural dimensions to satisfy their minimum thresholds.

Restorative choice then occurs within

$$R^* \in \mathcal{R}_P^{\text{adm}}. \quad (2262)$$

Equation 2262 expresses a central principle of the section: higher fidelity outside the procedurally admissible region does not become legitimate merely because it better reproduces the reference.

25.5 Maximum Admissible Fidelity

A constrained restorative ideal can therefore be defined as

$$R^* \in \arg \max_{R \in \mathcal{R}_P^{\text{adm}}} F_R(R). \quad (2263)$$

Equation 2263 identifies the highest fidelity achievable within the accepted procedural constraints.

This is different from unconstrained maximum fidelity.

25.6 Unconstrained Fidelity

Unconstrained fidelity is

$$R^F \in \arg \max_{R \in \mathcal{R}} F_R(R). \quad (2264)$$

If

$$R^F \notin \mathcal{R}_P^{\text{adm}}, \quad (2265)$$

then the most faithful intervention is procedurally inadmissible.

This possibility is central to the theory of return.

25.7 Fidelity Gradients

Restorative fidelity often increases incrementally.

Let intervention intensity be α . Then

$$F_R = F_R(\alpha) \quad (2266)$$

and

$$C_P = C_P(\alpha). \quad (2267)$$

A fidelity gradient is

$$\frac{dF_R}{d\alpha}, \quad (2268)$$

while the procedural-cost gradient is

$$\frac{dC_P}{d\alpha}. \quad (2269)$$

Equations 2268 and 2269 permit comparison of marginal gains in restorative similarity with marginal procedural burden.

25.8 Diminishing Fidelity Returns

Restorative interventions can exhibit diminishing returns.

Large improvements can occur through simple correction, while later increments require increasingly intrusive measures.

A stylized condition is

$$\frac{d^2 F_R}{d\alpha^2} < 0. \quad (2270)$$

Equation 2270 is illustrative.

This structure is especially plausible where early repair addresses major material deficits and later fidelity concerns increasingly singular historical or relational details.

25.9 Increasing Procedural Cost

Procedural burden can increase nonlinearly:

$$\frac{d^2 C_P}{d\alpha^2} > 0. \quad (2271)$$

Equation 2271 can arise where stronger restoration requires deeper surveillance, coercion, or third-party intervention.

Together, Equations 2270 and 2271 create a natural region in which further fidelity becomes increasingly difficult to justify.

25.10 Procedural Stopping Point

Let α^* be the highest intervention level satisfying the procedural constraints.

Then

$$\alpha^* = \sup \left\{ \alpha : R(\alpha) \in \mathcal{R}_P^{\text{adm}} \right\}. \quad (2272)$$

Equation 2272 defines a procedural stopping point.

The remaining fidelity gap is

$$\Delta F_{\text{res}} = F_{\text{max}} - F_R(\alpha^*). \quad (2273)$$

Equation 2273 should be acknowledged rather than erased by redefining the achieved state as complete restoration.

25.11 Consent–Fidelity Conflict

One recurring conflict concerns consent.

Suppose intervention R_2 produces greater fidelity than R_1 :

$$F_R(R_2) > F_R(R_1). \quad (2274)$$

If the subject consents to R_1 and refuses R_2 , the institution faces a choice between historical fidelity and present agency.

Where the intervention directly alters the subject's body, memory, identity, relationships, or future life direction, refusal can have strong procedural weight.

25.12 Consent as a Side Constraint

For interventions requiring valid consent, define

$$\mathcal{R}_C = \{R : C_A(R) = 1\}. \quad (2275)$$

The restorative choice becomes

$$R^* \in \mathcal{R}_P^{\text{adm}} \cap \mathcal{R}_C. \quad (2276)$$

Equation 2276 blocks fidelity gains that require non-consensual intervention where consent is normatively required.

25.13 Compulsory Reinstatement

Reinstatement illustrates the conflict clearly.

Suppose the no-harm reference contains employment at institution I .

Exact positional fidelity can favor return:

$$p_{A,R} = p_{A,\text{ref}}. \quad (2277)$$

The present subject can prefer compensated exit.

Compulsory reinstatement can therefore increase reference fidelity while reducing agency.

A just system should distinguish the right to reinstatement from an obligation to accept reinstatement.

25.14 Relational Reconstruction

Stronger relational fidelity can require reconstruction of relationships.

Let reference relation be r_{AB}^{ref} .

Restorative fidelity can seek

$$r_{AB}^R \sim r_{AB}^{\text{ref}}. \quad (2278)$$

This goal can conflict with the current preferences of A , B , or both.

A relationship cannot ordinarily be justly restored by treating one party's historical claim as authority over another party's present agency.

25.15 Relational Consent

Let consent of both parties be

$$C_A(r) = 1 \quad \text{and} \quad C_B(r) = 1. \quad (2279)$$

Equation 2279 defines one procedural precondition for voluntary relational reconstruction.

Where such consent is absent, other remedies can address the historical loss without recreating the relation.

25.16 Memory Reconstruction

The memory-erasure thought experiment from Section 20 becomes a direct fidelity–procedure conflict.

Suppose intervention R_M makes

$$M_{A,R} = M_{A,cf}. \quad (2280)$$

Equation 2280 maximizes one dimension of counterfactual fidelity.

The intervention can alter:

- actual autobiographical history;
- testimony;
- learned knowledge;
- identity continuity;
- present preferences.

A strong fidelity gain therefore creates equally strong procedural and identity concerns.

25.17 Memory Editing and Consent

If memory editing were technologically possible, valid consent would remain necessary and potentially insufficient.

A subject consenting before the intervention can become a different preference-bearing subject after the intervention.

The procedure therefore raises questions concerning:

- continuity of consent;
- reversibility;
- identity;
- preservation of testimony;
- third-party interests.

Technical possibility does not collapse these normative distinctions.

25.18 Identity Reconstruction

A stronger intervention can attempt to restore the counterfactual subject's preferences, commitments, or personality.

Let actual subject state be I_A^H and counterfactual state be I_A^C .

Maximum identity fidelity seeks

$$I_A^R = I_A^C. \quad (2281)$$

Equation 2281 raises the question whether the actual historical subject is being repaired or replaced. The comparison cannot be resolved by fidelity alone.

25.19 Identity Continuity Constraint

Let acceptable identity-continuity relation be

$$I_A^R \sim_{\text{cont}} I_A^H. \quad (2282)$$

Equation 2282 can operate as a procedural or substantive constraint upon restorative intervention.

A highly faithful intervention violating Equation 2282 can therefore be inadmissible.

25.20 Privacy–Fidelity Conflict

More accurate restoration often requires more information.

Let restorative accuracy or fidelity increase with data:

$$F_R = F_R(\mathcal{D}). \quad (2283)$$

The institution can therefore seek a larger dataset

$$\mathcal{D}_1 \subset \mathcal{D}_2. \quad (2284)$$

while privacy protection decreases.

This creates a structural conflict between epistemic precision and informational self-determination.

25.21 Marginal Value of Information

Let the marginal fidelity gain from data item d be

$$\Delta F(d). \quad (2285)$$

Let privacy burden be

$$C_{\text{priv}}(d). \quad (2286)$$

A data-minimizing restorative procedure should reject low-value high-intrusion information where the additional fidelity does not justify the procedural cost.

25.22 Surveillance-Based Restoration

A highly monitored system can measure recovery with great precision.

Continuous monitoring can also change the subject's behavior and create a new relation of control.

The intervention can therefore increase

$$F_{\text{measured}} \quad (2287)$$

while reducing

$$J_{\text{privacy}} \quad \text{and} \quad J_{\text{ND}}. \quad (2288)$$

Restorative epistemology should therefore remain subordinate to procedural and relational constraints.

25.23 Third-Party Rights–Fidelity Conflict

Strong historical restoration can require intervention upon innocent third parties.

Suppose exact employment restoration requires displacement of current employee B .

Let claimant fidelity gain be

$$\Delta F_A > 0 \quad (2289)$$

and third-party burden be

$$\Delta V_B < 0. \quad (2290)$$

Equations 2289 and 2290 create a multi-party conflict.

The claimant's historical entitlement does not by itself determine the permissibility of the third-party burden.

25.24 Third-Party Substitution

Where exact restoration burdens an innocent third party, alternative restorative paths can seek to preserve claimant value through another mechanism.

Let R_E be exact historical restoration and R_A an alternative remedy.

It can hold that

$$F_R(R_E) > F_R(R_A) \quad (2291)$$

and

$$J_{\text{overall}}(R_A) > J_{\text{overall}}(R_E). \quad (2292)$$

Equations 2291 and 2292 provide a central example of lower-fidelity greater-justice restoration.

25.25 Historical Truth–Privacy Conflict

Historical recognizability can require public preservation of information.

Privacy can justify limits on access.

Let archival publicity level be a . Then historical recognizability can increase with a :

$$\frac{dR_H}{da} > 0, \quad (2293)$$

while privacy can decrease:

$$\frac{dP_A}{da} < 0. \quad (2294)$$

Equations 2293 and 2294 demonstrate a direct value conflict.

Versioning, restricted access, and aggregate publication can sometimes reduce the conflict.

25.26 Recognition–Privacy Conflict

Public recognition can carry greater symbolic value than private recognition.

The subject can nevertheless prefer privacy.

A recognition remedy should therefore distinguish the right to acknowledgment from compulsory publicity.

The subject's historical claim does not automatically create a public-duty exception to privacy.

25.27 Public Apology and Subject Preference

An institution can consider a public apology necessary to acknowledge systemic wrong.

A claimant can prefer a private resolution.

The conflict can sometimes be resolved through forms of public acknowledgment that do not identify the claimant.

The procedural architecture should therefore disaggregate institutional recognition from personal disclosure.

25.28 Transparency–Confidentiality Conflict

Procedural legitimacy can favor transparent reasoning.

Protection of witnesses or private information can favor confidentiality.

The appropriate target is structured transparency rather than maximal disclosure.

Let

$$\mathcal{I}_{\text{public}} \cup \mathcal{I}_{\text{restricted}} = \mathcal{I}_{\text{decision}} \quad (2295)$$

with a justified division between the two sets.

Equation 2295 preserves accountability and confidentiality simultaneously where possible.

25.29 Speed–Participation Conflict

Rapid intervention can preserve generative conditions.

Extensive participation can require time.

Let delay cost be $C_D(t)$ and participation value be $V_P(t)$.

The procedural design must balance

$$\frac{dC_D}{dt} \quad (2296)$$

against

$$\frac{dV_P}{dt}. \quad (2297)$$

No universal optimum exists.

Emergency interim action followed by fuller review can reduce the conflict.

25.30 Evidence–Dignity Conflict

More evidence can increase factual accuracy while requiring repeated testimony or intrusive documentation.

A person should not have to repeatedly prove a verified harm merely because different institutional units fail to share lawful information.

Procedural provenance and verified-record reuse can reduce this conflict.

25.31 Historical Fidelity–Finality Conflict

Historical fidelity can favor reopening whenever new evidence appears.

Finality favors closure.

Let fidelity gain from reopening be

$$\Delta F_H^{\text{reopen}} \quad (2298)$$

and closure value be

$$V_{\text{finality}}. \quad (2299)$$

A justified reopening rule should identify conditions under which $\Delta F_H^{\text{reopen}}$ is sufficiently material to overcome the institutional value of finality.

25.32 Revisability–Reliance Conflict

Procedural revisability can burden persons who rely on settled arrangements.

Let reliance interest be L_R .

Reopening can improve claimant justice and damage legitimate reliance.

A layered architecture can therefore preserve strong finality for some objects and revisability for others.

25.33 Robustness–Agency Conflict

Institutions can increase robustness by reducing the subject's freedom.

For example, a tightly controlled support program can prevent financial instability while limiting choice.

Let robustness gain be

$$\Delta \varrho > 0 \quad (2300)$$

and agency loss be

$$\Delta A < 0. \quad (2301)$$

Equations 2300 and 2301 create a substantive–procedural conflict.

A robust remedy should not become a form of institutional domination.

25.34 Optimization–Contestability Conflict

Highly optimized restorative systems can rely on complex models.

Greater model complexity can improve predictive fit and reduce intelligibility.

Let predictive fidelity be $F_M(c)$ and contestability be $C_M(c)$, where c represents complexity.

It can occur that

$$\frac{dF_M}{dc} > 0 \quad (2302)$$

and

$$\frac{dC_M}{dc} < 0. \quad (2303)$$

Equations 2302 and 2303 demonstrate why predictive accuracy alone cannot determine procedural legitimacy.

25.35 Perfect Prediction and Illegitimate Intervention

Even a perfectly accurate model does not independently authorize every intervention it recommends.
Suppose

$$P(Y | M) = 1 \quad (2304)$$

for a modeled outcome.

Consent, rights, privacy, and third-party interests remain distinct.

Epistemic certainty does not eliminate normative constraints.

25.36 Perfect Causal Knowledge and Procedure

The same conclusion applies to causal knowledge.

Suppose the institution knows exactly which intervention would reproduce the counterfactual result.

That knowledge can answer:

What would produce the reference outcome?

It cannot independently answer:

May the institution impose that intervention?

The second question remains procedural and normative.

25.37 Omnipotence and Procedural Constraint

Unlimited technical capacity does not imply unlimited legitimate authority.

Let technically feasible set be

$$\mathcal{R}_{\text{tech}}. \quad (2305)$$

The legitimate set can satisfy

$$\mathcal{R}_{\text{legit}} \subset \mathcal{R}_{\text{tech}}. \quad (2306)$$

Equation 2306 is fundamental.

Technological expansion can enlarge the conflict between what can be restored and what may be restored.

25.38 Restorative Power

As restorative power increases, institutions acquire greater capacity to alter persons and systems.

Let intervention capacity be P_R .

Procedural safeguards can need to increase with restorative power:

$$P_R \uparrow \implies S_P \uparrow, \quad (2307)$$

other things equal.

Equation 2307 is a normative design principle.

25.39 Restoration as Governance Power

The authority to define what a person should be “restored to” is itself a form of governance power.
It includes power to define:

- the baseline;
- relevant values;
- acceptable identity;
- completion;
- residual impairment.

The justice of return therefore requires governance of the power to restore.

25.40 Baseline Power

Selecting baseline B shapes the direction of institutional intervention.

Let the restorative target be

$$T_R = T(B). \quad (2308)$$

Equation 2308 makes baseline selection practically consequential.

A procedurally just system should therefore make baseline choice visible and contestable.

25.41 Value-Selection Power

Institutions also exercise power by defining which value objects count.

Let selected value set be $\mathfrak{D}_V^I$.

Omitted value is

$$\mathfrak{D}_V^{\text{omit}} = \mathfrak{D}_V^* \setminus \mathfrak{D}_V^I. \quad (2309)$$

Equation 2309 makes value omission an object of procedural review.

25.42 Completion Power

The power to declare restoration complete can terminate:

- support;
- monitoring;
- eligibility;
- institutional responsibility.

Completion therefore requires procedural safeguards proportional to its consequences.

25.43 Subject-Directed Stopping

Where further restoration concerns optional identity, relational, or future-direction interventions, the subject can possess strong authority to stop.

Let subject stopping decision be

$$S_A = 1. \quad (2310)$$

Where no overriding duty applies, the process should permit

$$S_A = 1 \implies \alpha_{\text{next}} = \alpha_{\text{current}}. \quad (2311)$$

Equation 2311 formalizes a right against continued restorative escalation.

25.44 Institutional Stopping

Institutions also require stopping rules to prevent endless pursuit of marginal fidelity.

A stopping rule can consider:

- residual value;
- procedural burden;
- uncertainty;
- third-party effects;
- feasibility;
- subject preference.

The stopping rule should distinguish acknowledged residual loss from abandoned responsibility.

25.45 Marginal Restorative Justification

For an increment ΔR , let fidelity gain be ΔF , substantive gain ΔJ_S , and procedural cost ΔC_P .

Further intervention requires justification of the vector

$$(\Delta F, \Delta J_S, -\Delta C_P). \quad (2312)$$

Equation 2312 preserves the distinct dimensions rather than immediately reducing them to one utility score.

25.46 Dominated Restorative Options

An intervention R_2 is dominated by R_1 where R_1 provides at least as much substantive justice and fidelity with no greater procedural burden.

A simple condition is

$$F_R(R_1) \geq F_R(R_2), \quad (2313)$$

$$J_S(R_1) \geq J_S(R_2), \quad (2314)$$

and

$$C_P(R_1) \leq C_P(R_2), \quad (2315)$$

with at least one strict relation.

Dominated options can be removed without resolving deeper trade-offs.

25.47 Fidelity–Procedure Frontier

The nondominated restorative set is

$$\mathcal{F}_{FP} = \text{ND}(\mathcal{R}). \quad (2316)$$

Equation 2316 contains interventions for which stronger fidelity requires some procedural or substantive sacrifice.

The frontier is a decision space rather than a complete decision rule.

25.48 Procedural Threshold Frontier

Applying procedural thresholds produces

$$\mathcal{F}_{FP}^{\text{adm}} = \mathcal{F}_{FP} \cap \mathcal{R}_P^{\text{adm}}. \quad (2317)$$

Equation 2317 removes options violating protected procedural minima.

Selection among the remaining options can use subject choice, substantive dominance, institutional mandate, or another legitimate rule.

25.49 Subject Choice on the Admissible Frontier

Where several options on $\mathcal{F}_{FP}^{\text{adm}}$ are substantively acceptable, present subject preference can receive strong decision authority.

Let

$$R_A^* \in \arg \max_{R \in \mathcal{F}_{FP}^{\text{adm}}} P_A(R). \quad (2318)$$

Equation 2318 formalizes one form of subject-directed restoration.

25.50 Third-Party Constrained Frontier

Where third-party rights apply, the relevant set becomes

$$\mathcal{F}_{FP}^{3P} = \mathcal{F}_{FP}^{\text{adm}} \cap \mathcal{R}_{3P}. \quad (2319)$$

Equation 2319 prevents subject preference from overriding protected third-party interests.

25.51 Minimum Procedural Loss

For a required substantive target $J_S \geq \theta_S$, the institution can seek the least procedurally burdensome intervention:

$$R^* \in \arg \min_R C_P(R) \quad (2320)$$

subject to

$$J_S(R) \geq \theta_S. \quad (2321)$$

Equations 2320 and 2321 represent a least-intrusive restorative principle.

25.52 Least-Restrictive Restoration

A least-restrictive principle requires the institution to prefer a remedy that achieves the legitimate restorative objective with less interference where the substantive result is materially comparable.

Let two remedies satisfy

$$J_S(R_1) \sim J_S(R_2) \quad (2322)$$

and

$$C_P(R_1) < C_P(R_2). \quad (2323)$$

Then R_1 has a procedural advantage.

25.53 Minimum Necessary Fidelity

Some institutional purposes require only a bounded level of fidelity.

Let required fidelity threshold be $F_{\min}^I$.

The institution can stop once

$$F_R(R) \geq F_{\min}^I \quad (2324)$$

and substantive protected values are satisfied.

Equation 2324 prevents institutions from pursuing unnecessary deeper intervention merely because more fidelity is technically possible.

25.54 No General Duty to Maximize Fidelity

The theory therefore rejects a general principle

$$R^* = \arg \max_R F_R(R). \quad (2325)$$

Equation 2325 is too strong as a universal restorative rule.

Restorative fidelity is one evaluative dimension within a larger justice architecture.

25.55 Fidelity as Evidence Rather than Objective

In some cases, fidelity can function primarily as evidence that the relevant value has been restored.

For example, equal income can be evidence of economic repair.

The ultimate value object can be material security or capability.

This distinction allows institutions to stop pursuing literal equality once the underlying substantive value is adequately restored.

25.56 Reference Fidelity and Value Fidelity

Reference fidelity asks whether the restored system resembles the reference.

Value fidelity asks whether the values the reference instantiated are preserved.

Let

$$F_{\text{ref}} \quad (2326)$$

and

$$F_V. \quad (2327)$$

A transformed remedy can satisfy

$$F_{\text{ref}} \downarrow \quad \text{and} \quad F_V \uparrow . \quad (2328)$$

Equation 2328 captures transformative restoration.

25.57 Functional Fidelity

A remedy can preserve the function served by a lost condition without recreating the condition itself.

Let historical object be q and valued function be $f(q)$.

Functional restoration requires

$$f(q_R) \sim f(q_{\text{ref}}) . \quad (2329)$$

Equation 2329 can permit lower historical fidelity with lower procedural cost.

25.58 Capability Fidelity

Similarly, if the relevant value is capability, the target can be

$$\mathcal{K}_{A,R} \succeq \mathcal{K}_{A,\text{ref}} \quad (2330)$$

without reconstructing the exact historical objects that generated the reference capability.

This can preserve present agency and reduce coercive restoration.

25.59 Generative Fidelity

If the relevant value is future generativity, the target can be

$$\Gamma_R^G \succeq_V \Gamma_{\text{ref}}^G . \quad (2331)$$

Equation 2331 permits a different process to provide equal or better future generative value.

Historical replication is therefore unnecessary where generative dominance is achieved through legitimate means.

25.60 Procedural Superiority of Transformative Restoration

A transformed remedy can be both less historically faithful and procedurally superior.

For example, compensated exit can preserve agency better than compulsory reinstatement.

A new professional network can be easier to build justly than forced restoration of old relationships.

Transformative restoration therefore can improve both future substantive value and procedural legitimacy.

25.61 Fidelity Sacrifice

Sometimes justice requires knowingly accepting a residual fidelity gap.

Let procedural threshold make stronger fidelity inadmissible.

Then

$$F_R(R^*) < F_{\text{max}} \quad (2332)$$

is compatible with a justified restorative decision.

The gap should be acknowledged as residual non-equivalence.

25.62 Residual Recognition

Where stronger restoration is procedurally unavailable, acknowledgment can recognize the remaining loss.

Let residual gap be ΔF_{res} .

Recognition response can be represented by

$$R_{\text{rec}} = R_{\text{rec}}(\Delta F_{\text{res}}). \quad (2333)$$

Equation 2333 does not compensate automatically for the gap.

It prevents the system from pretending the residual disappeared.

25.63 Residual Compensation

Where procedural constraints prevent exact return, compensation can address part of the residual value.

Let

$$C_{\text{res}} = C(\Delta V_{\text{res}}). \quad (2334)$$

Equation 2334 provides one alternative to intrusive reconstruction.

Compensation can remain partial where the lost value is non-fungible.

25.64 Residual Assistance

Future-oriented assistance can also address residual non-equivalence.

Let

$$A_{\text{res}} : \mathcal{G}_R \longrightarrow \mathcal{G}_{R+A}. \quad (2335)$$

Equation 2335 seeks to improve future capability without reconstructing the past.

This is especially appropriate where present agency favors a new trajectory.

25.65 Fidelity and Procedural Rights

Some procedural rights can function as strong constraints against deeper restoration.

Examples include:

- bodily integrity;
- privacy;
- freedom of association;
- right to refuse treatment;
- third-party autonomy.

These rights place parts of the restorative state space outside institutional reach.

25.66 Protected Non-Restoration

A protected non-restoration region contains differences from the reference that the institution may not legitimately eliminate.

Let

$$\Delta_{\text{protected}} = \{\delta : \delta \text{ may not be removed through admissible intervention}\}. \quad (2336)$$

Equation 2336 is conceptually important.

Some remaining non-equivalence is therefore evidence of respect for rights rather than institutional failure.

25.67 Normatively Valuable Difference

Some post-harm differences from the reference can possess positive value.

Let

$$\Delta^+ = Q_R \setminus Q_{\text{ref}} \quad (2337)$$

denote newly generated features.

If

$$V(\Delta^+) > 0, \quad (2338)$$

then eliminating the difference can reduce justice.

This is especially relevant to new relationships, knowledge, capabilities, and institutional reforms created after harm.

25.68 Difference Preservation

A restorative process can therefore intentionally preserve selected differences.

Let the protected difference set be

$$\Delta_{\text{keep}} = \Delta_{\text{protected}} \cup \Delta^+. \quad (2339)$$

Equation 2339 contains differences preserved because eliminating them would violate rights or destroy valued developments.

25.69 Restorative Target after Procedural Filtering

The final target can be defined as a filtered reference:

$$B^* = \text{Filter}(B, \mathcal{N}, P_A, \Delta_{\text{protected}}, \Delta^+), \quad (2340)$$

where $\mathcal{N}$ contains normative constraints and P_A present subject preferences.

Equation 2340 does not imply literal editing of the historical reference.

It represents construction of a legitimate restorative target informed by, yet not identical to, the reference.

25.70 Filtered Fidelity

Fidelity can then be evaluated relative to the filtered target:

$$F_R^* = F(\mathcal{G}_R, B^*). \quad (2341)$$

Equation 2341 can be more normatively meaningful than fidelity to the raw no-harm reference.

25.71 Restorative Target Construction

A legitimate restorative target can therefore be constructed through several steps:

1. identify the historical reference;
2. identify the values instantiated by the reference;
3. identify unjust or obsolete features of the reference;
4. identify present subject priorities;
5. identify third-party constraints;
6. identify protected post-harm developments;
7. construct the admissible target region.

This procedure transforms the reference from a command into an evidentiary and normative input.

25.72 Admissible Target Region

Let the admissible target region be

$$\mathcal{T}_R^{\text{adm}} = \{x : J_S(x) \geq \theta_S, J_P(x) \geq \theta_P, x \text{ respects protected differences}\}. \quad (2342)$$

Equation 2342 provides a general alternative to exact target identity.

The subject and institution can then choose among several legitimate futures.

25.73 Plural Legitimate Futures

A just restorative system need not produce a unique future.

Let

$$\mathcal{T}_R^{\text{adm}} = \{T_1, \dots, T_k\}, \quad (2343)$$

where several targets satisfy the protected conditions.

Equation 2343 gives formal expression to the right to a different future.

25.74 Fidelity within a Target Set

Historical fidelity can remain relevant as a secondary criterion inside the admissible target region:

$$T^* \in \arg \max_{T \in \mathcal{T}_R^{\text{adm}}} F_R(T), \quad (2344)$$

provided the subject or legitimate procedure has not selected another admissible target for independent reasons.

Equation 2344 gives fidelity a bounded role.

25.75 Subject Preference within the Target Set

Where the relevant values permit subject choice,

$$T_A^* \in \arg \max_{T \in \mathcal{T}_R^{\text{adm}}} P_A(T). \quad (2345)$$

Equation 2345 can override Equation 2344 where present agency has priority.

Table 23: Principal conflicts between restorative fidelity and procedural justice

Fidelity target	Stronger restorative intervention	Procedural value at risk	Potential lower-intrusion alternative
Employment trajectory	Compulsory reinstatement	Agency and refusal	Compensated exit with portable support
Prior relationship	Relational reconstruction	Freedom of association and third-party agency	Recognition and alternative network support
Counterfactual memory	Memory editing	Identity, consent, historical truth	Recognition, therapy, future-directed support
Counterfactual identity	Personality or preference reconstruction	Continuity and autonomy	Capability expansion under present preferences
Detailed causal fidelity	Extensive personal monitoring	Privacy and non-domination	Bounded data and robust lower-resolution models
Historical recognizability	Full public disclosure	Privacy and third-party confidentiality	Restricted archive and aggregate acknowledgment
Exact positional restitution	Displacement of innocent third party	Third-party rights and reliance	Alternative position, compensation, or structural remedy
Model accuracy	Highly complex automated decision	Contestability and explainability	Simpler model with review and uncertainty bounds
Long-run robustness	Intensive institutional control	Agency and revisability	Redundancy, portability, and voluntary safeguards

25.76 Fidelity and Procedural Justice Matrix

Table 23 summarizes major conflict families.

Table 23 demonstrates that procedural constraint often redirects the form of restoration rather than ending the duty to repair.

25.77 Restorative Response to Procedural Limits

When stronger fidelity is procedurally unavailable, the institution has several possible responses:

- preserve the procedurally legitimate level of restoration;
- acknowledge residual loss;
- compensate remaining value where appropriate;
- support an alternative future;
- reform structures contributing to recurrence;
- preserve historical recognition.

Procedural limitation therefore changes the restorative strategy rather than automatically eliminating responsibility.

25.78 Procedural Constraint as Justice

A residual difference from the reference can be normatively required.

If eliminating the difference requires coercion or rights violation, restraint can be the just outcome. Thus

$$F_R(R_1) < F_R(R_2) \quad (2346)$$

and

$$J_{\text{overall}}(R_1) > J_{\text{overall}}(R_2) \quad (2347)$$

can hold simultaneously.

Equations 2346 and 2347 provide one of the central results of the paper.

25.79 Justice of Non-Restoration

There are cases in which justice requires that one dimension remain unrestored.

Examples include:

- a harmful relationship;
- an unjust institutional hierarchy;
- a historical dependency;
- a counterfactual preference rejected by the present subject.

Non-restoration in these dimensions can be a positive normative achievement.

25.80 Partial Restoration

Partial restoration should therefore be distinguished from failed restoration.

Let restorative profile be

$$\mathbf{R} = (r_1, \dots, r_n). \quad (2348)$$

Some dimensions can remain deliberately below full fidelity because procedural constraints or normative transformation justify the difference.

25.81 Justified Residual Difference

Let residual difference be Δ_R .

Partition it as

$$\Delta_R = \Delta_{\text{unresolved}} \cup \Delta_{\text{protected}} \cup \Delta_{\text{valuable}}. \quad (2349)$$

Equation 2349 distinguishes:

- unresolved loss;
- difference protected by procedural or rights constraints;
- positively valued new difference.

This partition prevents every residual difference from being labeled injustice.

25.82 Unresolved Residuals

Only

$$\Delta_{\text{unresolved}} \quad (2350)$$

should be treated straightforwardly as remaining restorative deficiency.

Even this category can contain irreparable loss whose appropriate response is recognition rather than further reconstruction.

25.83 Protected Residuals

Protected residuals arise because eliminating them would violate rights, consent, privacy, identity, or third-party justice.

Their persistence can therefore indicate procedural success.

25.84 Valuable Residuals

Valuable residuals arise where actual history created legitimate new capabilities, relations, knowledge, or institutions.

Justice can require their preservation.

25.85 Integrated Justice Profile

An integrated restorative profile can now be represented by

$$\mathbf{J}_R = (F_R, J_S, J_P, \Delta_{\text{unresolved}}, \Delta_{\text{protected}}, \Delta_{\text{valuable}}). \quad (2351)$$

Equation 2351 is more informative than a single completion score.

25.86 Integrated Admissibility

A restorative option is integratively admissible where

$$J_S(R) \geq \theta_S \quad (2352)$$

and

$$J_P(R) \geq \theta_P. \quad (2353)$$

Equations 2352 and 2353 impose dual minimum conditions.

Historical fidelity can then guide selection within the admissible set where it remains normatively relevant.

25.87 Integrated Dominance

Restorative option R_1 integratively dominates R_2 where

$$F_R(R_1) \geq F_R(R_2), \quad (2354)$$

$$J_S(R_1) \geq J_S(R_2), \quad (2355)$$

and

$$J_P(R_1) \geq J_P(R_2), \quad (2356)$$

with at least one strict relation.

Dominated options can be rejected without resolving deeper normative incomparability.

25.88 Integrated Nondominated Set

The integrated restorative frontier is

$$\mathcal{F}_J^* = \text{ND} (F_R, J_S, J_P). \quad (2357)$$

Equation 2357 contains the restorative options remaining after obvious dominance relations are removed.

Procedural institutions must then govern choice among these alternatives.

25.89 Fidelity Priority

There can be contexts in which fidelity deserves unusually strong weight.

Examples include:

- return of a wrongfully taken unique object;
- correction of authorship;
- restoration of a legal status;
- correction of an official record.

In such cases, the protected object itself is highly specific and procedural burden can remain low.

The theory therefore does not generally subordinate fidelity.

It rejects only unconditional fidelity maximization.

25.90 Procedural Priority

Other cases give procedural values stronger priority.

Examples include:

- identity-affecting intervention;
- intimate relationship reconstruction;
- memory intervention;
- intervention upon innocent third parties.

The deeper the restorative intervention reaches into protected personal or relational domains, the stronger the procedural justification generally becomes.

25.91 Domain-Sensitive Priority

The relative weight of fidelity and procedure should therefore depend upon the object.

Let priority parameter be

$$\lambda_Q = \lambda(Q). \quad (2358)$$

Equation 2358 is conceptual.

It means that restoring money, legal status, memory, and relationships should not all use the same fidelity–procedure trade-off.

25.92 Fungibility and Procedural Intrusion

High-fungibility objects often permit low-intrusion substitutes.

Money can compensate value without reconstructing the subject's internal state.

Low-fungibility objects can tempt institutions toward more intrusive reconstruction.

This creates a paradox: the harder a value is to replace, the stronger the temptation to intervene deeply, and the stronger the procedural constraints against doing so can become.

25.93 Singularity Constraint

For a singular object O^* , exact replacement can be unavailable.

The institution should therefore avoid treating increasing intrusion as a guaranteed route to equivalent value.

Singularity can create a substantive residual that no procedurally legitimate intervention can eliminate.

25.94 Historical Distance and Procedural Cost

As historical time passes, restoring the earlier world can require modification of more intervening relations and third-party interests.

Let historical distance be τ_H .

A stylized relation is

$$\frac{dC_P}{d\tau_H} > 0 \quad (2359)$$

in cases where more later history must be displaced to recreate the reference.

Equation 2359 is not universal.

It identifies one structural reason late exact restoration can become more procedurally difficult.

25.95 Historical Distance and Transformative Preference

As time passes, present subject goals can increasingly diverge from the counterfactual trajectory.

The normative importance of subject-directed transformation can therefore grow with historical distance.

This does not automatically reduce corrective claims concerning the earlier harm.

25.96 Irreversibility and Procedural Fidelity

Where harm is irreversible, exact restoration can require artificial reconstruction rather than reversal.

That reconstruction can become increasingly interventionist.

The theory should therefore distinguish:

$$\begin{array}{ll} \text{reversal of reversible loss} & (2360) \\ \text{from} \end{array}$$

$$\text{construction of a reference-like substitute.} \quad (2361)$$

The second operation requires independent procedural justification.

25.97 Procedurally Constrained Best Restoration

The central optimization problem of this section can now be stated as

$$R^* \in \arg \max_{R \in \mathcal{R}_P^{\text{adm}}} J_S(R) \quad (2362)$$

subject to an optional secondary fidelity objective

$$F_R(R) \rightarrow \max. \quad (2363)$$

Equations 2362 and 2363 place substantive justice and procedural admissibility before fidelity maximization.

Different normative theories can adopt different priority structures.

25.98 Fidelity-Constrained Justice

Another institutional design can require a minimum fidelity threshold:

$$F_R(R) \geq F_{\min} \quad (2364)$$

and then maximize justice within that region.

This architecture can be appropriate where fidelity to an entitlement or legal status has independent normative force.

The framework therefore supports several orderings rather than one universal optimization rule.

25.99 Lexical Rights Constraint

A rights-based architecture can impose

$$J_{\text{rights}} \succ_{\text{lex}} F_R. \quad (2365)$$

Equation 2365 means no fidelity gain can justify crossing the specified rights boundary.

This is especially relevant to bodily integrity, privacy, and third-party agency.

25.100 Threshold Procedure Architecture

A threshold architecture can require

$$J_P \geq \theta_P \quad (2366)$$

then evaluate fidelity and substantive justice jointly inside the admissible region.

This permits procedural trade-offs above the minimum threshold while protecting basic safeguards.

25.101 Partial-Order Integration

Where no scalar aggregation is justified, define

$$R_1 \succeq_{FP} R_2 \quad (2367)$$

only where R_1 is at least as good in every protected fidelity, substantive, and procedural dimension.

The result can be a substantial set of incomparable options.

Legitimate procedure then governs selection.

25.102 Institutional Division of Fidelity Tasks

Conflicts can also be reduced by dividing functions.

A court can correct status and award compensation.

A rehabilitation institution can support capability.

An archive can preserve history under privacy safeguards.

A social-protection institution can secure future sufficiency.

No single institution must maximize every dimension of fidelity.

25.103 Procedural Specialization

Different restorative objects can therefore use different procedural forms.

For example:

- property return through adjudication;
- rehabilitation through collaborative planning;

- historical acknowledgment through public process;
- future support through administrative review.

Procedure matching reduces the temptation to use one intrusive method for every dimension.

25.104 Restorative Modularity

A modular restorative architecture separates interventions that can be accepted or rejected independently.
Let remedy bundle be

$$\mathcal{B}_R = \{R_1, \dots, R_n\}. \quad (2368)$$

The subject can accept

$$\mathcal{B}_A \subseteq \mathcal{B}_R \quad (2369)$$

where the modules are genuinely separable.
Modularity can reduce coercive bundling.

25.105 Bundling and Coercion

A procedurally problematic system can condition essential compensation upon acceptance of intrusive optional remedies.

Let essential module be R_E and optional module R_O .
Coercive bundling occurs where

$$R_E \text{ available} \iff C_A(R_O) = 1. \quad (2370)$$

Equation 2370 can undermine meaningful consent.

25.106 Unbundling Principle

Where feasible, substantively independent remedy components should be offered separately.

This allows the subject to receive independently justified support without surrendering procedural control over unrelated interventions.

25.107 Worked Workplace Conflict

Consider a worker excluded through wrongful managerial conduct.

Several restorative options exist:

1. salary compensation only;
2. formal reinstatement;
3. reinstatement plus relational reconstruction;
4. compensated exit with credential correction and transition support.

Option 3 can have the highest historical positional fidelity.

If the worker rejects renewed interaction with the responsible manager, Option 3 can have lower procedural legitimacy than Option 4.

A capability- and non-domination-sensitive architecture can therefore prefer the less historically faithful future.

25.108 Worked Research Conflict

Consider a researcher excluded from a laboratory.

Exact reconstruction can require:

- return to the same laboratory;
- restoration of former collaborations;
- reconstruction of missed projects.

The collaborators now have independent commitments.

The researcher can prefer a new institution.

A just remedy can instead provide funding, credential correction, network support, and recognition of lost opportunity.

The resulting process can have lower relational fidelity and higher present agency.

25.109 Worked Reputational Conflict

Suppose a false reputational record was widely disseminated.

Maximal historical correction can require contacting every person who received the original information.

Such outreach can further publicize the event.

A privacy-sensitive strategy can instead:

- correct active institutional records;
- propagate corrections to known operational databases;
- preserve correction provenance;
- avoid unnecessary new disclosure.

The remedy sacrifices maximal historical informational symmetry in order to protect privacy and reduce new harm.

25.110 Worked Memory Thought Experiment

Suppose perfect memory editing can eliminate all trauma-related memory and recreate the modeled no-harm psychological state.

The intervention maximizes selected psychological fidelity.

The subject prefers to retain the memory and receive support for living with the actual history.

A justice framework respecting identity and agency can favor the lower-fidelity option.

This thought experiment reveals that complete restoration and complete justice can diverge even under ideal technical capacity.

25.111 Worked Collective Conflict

Suppose a historically displaced community can be reconstructed in its former territorial configuration only by displacing current residents.

Exact territorial fidelity can conflict with third-party rights and later historical reliance.

A plural reparative architecture can instead combine:

- land return where feasible;
- compensation;

Table 24: Structural conflicts between restorative fidelity and procedural justice

Restorative dimension	Fidelity pressure	Procedural constraint	Possible resolution form
Position	Exact reinstatement	Present agency	Choice between reinstatement and supported exit
Relation	Recreate earlier relation	Mutual consent	Recognition plus alternative relational support
Memory	Recreate no-harm memory state	Identity and consent	Preserve memory with supportive repair
Historical record Information	Erase evidence of harm Collect more data for precise modeling	Accountability and truth Privacy	Versioned correction Data minimization and uncertainty bounds
Third-party structure	Restore earlier allocation	Third-party rights	Alternative compensation and transformation
Robustness	Increase institutional control	Agency and non-domination	Redundancy and secure rights
Historical publicity	Maximize recognizability	Privacy	Restricted archives and selective public acknowledgment

- governance rights;
- cultural protection;
- public recognition.

The resulting arrangement can be less historically exact and more just across current parties.

25.112 Fidelity–Procedure Conflict Matrix

Table 24 summarizes the principal structural conflicts developed in this section.

Table 24 shows that procedural constraint often changes the form of legitimate restoration.

25.113 Procedural Constraint Principle

The first principle of this section is:

Restorative fidelity should be pursued only within procedures and interventions that satisfy independently justified constraints concerning agency, consent, rights, privacy, reversibility, and third-party justice.

25.114 No-Maximization Principle

The second principle is:

There is no general duty to maximize similarity to the restorative reference once the marginal fidelity gain requires disproportionate procedural intrusion.

25.115 Least-Intrusive Restoration Principle

The third principle is:

Where two remedies achieve materially comparable substantive justice, the remedy with lower coercive, privacy, identity, and third-party burden has a procedural advantage.

25.116 Protected-Difference Principle

The fourth principle is:

Some differences between the restored and reference systems should remain because eliminating them would violate procedural rights or destroy legitimate post-harm developments.

25.117 Residual-Honesty Principle

The fifth principle is:

Where procedural limits prevent stronger restoration, the remaining difference should be acknowledged as residual non-equivalence rather than hidden through an inflated declaration of completeness.

25.118 Present-Subject Principle

The sixth principle is:

The reconstructed counterfactual subject should not automatically possess greater authority over future restorative choices than the actual present historical subject.

25.119 Third-Party Constraint Principle

The seventh principle is:

A claimant's legitimate restorative interest does not create unlimited authority to reconstruct the reference at the expense of innocent third parties.

25.120 Restorative-Power Principle

The eighth principle is:

As institutional capacity to alter persons, relations, and systems increases, the normative importance of procedural constraint increases rather than decreases.

25.121 Transformative Alternative Principle

The ninth principle is:

Where stronger historical fidelity is procedurally objectionable, justice should consider alternative routes that preserve or improve the relevant values through a new legitimate future.

25.122 Filtered-Reference Principle

The tenth principle is:

The operative restorative target should be informed by the historical reference while remaining filtered through present rights, agency, third-party claims, and normatively valuable post-harm developments.

25.123 Integrated Layer of the Atlas

The integrated fidelity–justice layer can be represented by

$$\mathfrak{I}_A = (F_R, \mathfrak{S}_A, \mathfrak{P}_A, \Delta_{\text{unresolved}}, \Delta_{\text{protected}}, \Delta_{\text{valuable}}), \quad (2371)$$

where F_R denotes restorative fidelity, $\mathfrak{S}_A$ substantive justice, $\mathfrak{P}_A$ procedural justice, and the remaining terms distinguish three forms of residual difference.

Equation 2371 provides the conceptual bridge from ordinary restorative analysis to the idealized technology thought experiments that follow.

25.124 Implications for Idealized Restorative Capacity

The present section establishes that stronger technical capacity does not collapse the normative architecture.

Even if future institutions could:

- reconstruct precise counterfactual trajectories;
- simulate complete relational histories;
- edit memory;
- rebuild networks;
- restore bodily states;
- reproduce causal mechanisms;

they would still need to answer:

- whether the subject consents;
- whether third parties consent;
- whether the reference is just;
- whether valued later developments should be preserved;
- whether identity continuity survives;
- whether the information required may legitimately be collected.

Technical solvability therefore leaves normative admissibility unresolved.

25.125 Transition to Idealized Restoration under Expanded Technical Capacity

Section 26 develops this conclusion under deliberately expanded technical assumptions. It examines hypothetical perfect observation, causal knowledge, simulation, bodily repair, memory reconstruction, network reconstruction, and near-unlimited intervention capacity.

The purpose is not technological forecasting. The thought experiments isolate which limits of restoration arise from present technological weakness and which remain because restoration itself is indexed, historical, relational, normatively plural, and procedurally constrained.

The central question becomes:

If nearly every technically remediable difference could be eliminated, which differences would still remain conceptually, historically, or normatively irreducible, and which differences should justice deliberately preserve?

26 Idealized Restoration under Expanded Technical Capacity

This section examines the justice of return under deliberately expanded technical assumptions. The preceding sections showed that contemporary restorative practice is constrained by incomplete observation, uncertain counterfactuals, limited causal identification, institutional fragmentation, resource scarcity, irreversible biological and relational change, and the procedural costs of intervention. Some of those limits can weaken as technology, institutional capacity, scientific knowledge, or available resources improve.

The present section therefore removes many ordinary practical constraints and asks which restorative limits remain.

The analysis considers hypothetical systems capable of:

- near-complete observation of relevant states and histories;
- highly accurate causal modeling;
- counterfactual simulation;
- precise bodily and functional repair;
- memory modification;
- reconstruction of networks and institutional environments;
- large-scale coordination across institutions;
- extensive control over future dynamics;
- abundant restorative resources.

These assumptions are methodological devices rather than technological predictions. Their purpose is to separate five different families of limits:

1. technical limits concerning what can be changed;
2. epistemic limits concerning what can be known;
3. structural limits concerning what follows from the system's dynamics;
4. historical limits concerning the identity of actual and counterfactual histories;
5. normative and procedural limits concerning what may legitimately be changed.

The distinction matters because a limitation that disappears with better technology differs fundamentally from one that persists under perfect observation or unlimited intervention capacity. A lost bodily function can in principle become repairable through future medicine. A no-harm history, by contrast, remains a counterfactual history even if its consequences can be reproduced exactly. Likewise, perfect causal knowledge can identify which intervention would reproduce a reference outcome while leaving unresolved whether that intervention respects identity, agency, privacy, third-party rights, or present values.

The section therefore treats expanded technical capacity as a stress test for the concept of restoration itself.

26.1 Role of the Idealization

The idealization serves three purposes.

First, it separates contingent present-day limitations from deeper conceptual ones.

Second, it tests whether the theory of indexed restoration remains necessary when technological constraints become weak.

Third, it identifies which restorative differences justice could deliberately preserve even if eliminating them became technically possible.

The method can be represented by a sequence of expanding capability sets.

Let

$$\mathcal{U}_0 \subset \mathcal{U}_1 \subset \cdots \subset \mathcal{U}_m \quad (2372)$$

denote increasingly powerful intervention sets. Equation 2372 allows analysis of how the restorative frontier changes as technical capability expands.

26.2 Technical Restoration Frontier

For intervention set $\mathcal{U}_k$, define the technically attainable restorative region by

$$\mathcal{A}_k^T = \{Q(R_u(\mathcal{G}^H)) : u \in \mathcal{U}_k\}. \quad (2373)$$

Equation 2373 contains the comparison objects reachable through the available technology.

As k increases, the attainable region can expand:

$$\mathcal{A}_k^T \subseteq \mathcal{A}_{k+1}^T. \quad (2374)$$

Equation 2374 expresses increasing technical capacity.

The normative admissible region need not expand in the same way.

26.3 Technical and Normative Frontiers

Let the normatively admissible restorative region be

$$\mathcal{A}^N = \{R : J_S(R) \geq \theta_S, J_P(R) \geq \theta_P\}. \quad (2375)$$

Equation 2375 is determined by substantive and procedural constraints.

The practically available just region at capability level k is therefore

$$\mathcal{A}_k^J = \mathcal{A}_k^T \cap \mathcal{A}^N. \quad (2376)$$

Equation 2376 shows that greater technical power expands justice-relevant possibilities only where the new possibilities remain normatively admissible.

26.4 Expanded Observation

Consider first an ideal observation technology capable of measuring every state variable relevant to the chosen restorative representation.

Let the true represented state be

$$x_t = (x_t^1, \dots, x_t^n). \quad (2377)$$

An ideal observation map satisfies

$$O^*(x_t) = x_t. \quad (2378)$$

Equation 2378 removes measurement error within the specified state space.

Several earlier epistemic problems then disappear.

The institution no longer needs to infer latent variables represented within x_t .

It can observe relational position, physiological state, network structure, resource security, or other modeled variables directly.

This improvement is substantial.

It does not resolve the choice of state space.

26.5 Perfect Observation and Representation

Let representation M_1 contain variable family $\mathcal{X}_1$, and richer representation M_2 contain $\mathcal{X}_2$.

It can remain true that

$$\mathcal{X}_1 \subset \mathcal{X}_2 \quad (2379)$$

even where observation within each model is perfect. Equation 2379 shows that perfect measurement does not determine which distinctions should be represented.

A model can perfectly observe salary, health, and network structure while excluding narrative meaning.

A richer model can include meaning while excluding future emergent categories.

Observation solves measurement within a representation.

It does not generate a uniquely correct representation of the social world.

26.6 Expanded Historical Observation

Suppose a hypothetical archive records every event relevant to the restorative inquiry.

Let actual history be

$$\mathcal{H}^H = \{e_t : t \in [t_0, t_1]\}. \quad (2380)$$

Equation 2380 represents a complete actual historical record relative to the modeled event ontology. Such an archive could eliminate many disputes concerning:

- which events occurred;
- event timing;
- authorship;
- institutional actions;
- resource transfers;
- communication histories.

Historical uncertainty concerning the actual path would decline dramatically.

Counterfactual uncertainty can remain.

26.7 Perfect Archive and Counterfactual History

Knowing every event in the actual history does not directly reveal which history would have occurred without intervention H .

The relevant distinction is

$$\mathcal{H}^H \neq \mathcal{H}^{-H}. \quad (2381)$$

Equation 2381 separates complete historical observation from the no-harm counterfactual.

Lewis's possible-worlds treatment of counterfactuals and Pearl's structural causal approach provide different formal traditions for reasoning about such unactualized alternatives [28, 43].

Perfect archival knowledge therefore solves one epistemic problem and leaves a different one intact.

26.8 Perfect Causal Knowledge

Suppose next that the institution possesses the exact causal model relevant to the defined state space.

Let

$$M_C^* = (U, V, F, P(U)) \quad (2382)$$

be the true structural causal model under the selected ontology. Equation 2382 removes uncertainty concerning the causal graph and structural mechanisms within that representation.

The institution can then calculate

$$P(Y \mid do(X = x)) \quad (2383)$$

exactly for interventions supported by the model. Equation 2383 eliminates statistical uncertainty concerning the selected interventional response.

This capability would dramatically improve restorative design.

It would not itself determine which intervention should be chosen.

26.9 Perfect Causal Knowledge and Normativity

Suppose intervention u^* uniquely maximizes similarity to the reference:

$$u^* = \arg \max_u F_R(R_u(\mathcal{G}^H)). \quad (2384)$$

Equation 2384 identifies the fidelity-maximizing intervention under perfect causal knowledge.

Its moral status remains a separate function:

$$J(u^*). \quad (2385)$$

Equation 2385 can be low even where Equation 2384 is known with certainty.

Causal knowledge identifies consequences.

Normative theory evaluates their legitimacy.

26.10 Perfect Counterfactual Simulation

Consider a still stronger capability.

Suppose the institution can compute the exact no-harm counterfactual trajectory:

$$\gamma_{[t_0, t_1]}^{-H}. \quad (2386)$$

Equation 2386 removes uncertainty concerning one specified counterfactual model.

The institution can know exactly:

- which job the subject would have held;
- which relationships would have formed;
- which income would have been earned;
- which health state would have occurred;

- which future opportunities would have become reachable.

This thought experiment removes the most demanding practical problem of counterfactual identification.

Several deeper questions remain.

26.11 Counterfactual Model Selection

Perfect simulation requires a specification of what is being simulated.

If the harmful event is removed, the system can contain stochastic, indeterministic, or branching elements.

Let the no-harm history family be

$$\mathfrak{H}^{-H} = \{\mathcal{H}_1^{-H}, \dots, \mathcal{H}_m^{-H}\}. \quad (2387)$$

Equation 2387 permits several possible no-harm histories.

A perfect simulator can assign exact probabilities:

$$P(\mathcal{H}_i^{-H}) = p_i. \quad (2388)$$

Equation 2388 still leaves a normative question concerning which history should become the restorative reference.

The most probable history, best history, expected history, and subject-preferred history can differ.

26.12 Counterfactual Selection under Determinism

Suppose further that the system is completely deterministic and the exact no-harm trajectory is therefore unique.

The reference-selection problem narrows.

Normative selection remains.

The unique no-harm life is still a life the actual present subject did not live.

The actual subject can now prefer another future.

Uniqueness of the counterfactual therefore does not create normative authority over the present subject.

26.13 Perfect Bodily Repair

Consider a hypothetical medicine capable of restoring every bodily variable to its no-harm counterfactual value.

Let bodily state be b_t .

Perfect bodily restoration satisfies

$$b_{R,t} = b_{cf,t}. \quad (2389)$$

Equation 2389 can eliminate bodily residuals within the selected representation.

This would solve many currently irreparable injuries.

The historical fact that the injury occurred remains different.

The subject can also have formed new relationships, commitments, or meanings during the injured period.

Perfect bodily repair is therefore powerful and partial.

26.14 Perfect Functional Repair

A still more functionally oriented intervention can restore every selected function:

$$F_i(\mathcal{G}^R) = F_i(\mathcal{G}^{\text{cf}}) \quad \forall i \in I_F. \quad (2390)$$

Equation 2390 establishes functional equivalence over the selected family.

It leaves open:

- provenance;
- memory;
- recognition;
- process;
- control;
- third-party history.

Functional completeness therefore remains object-relative.

26.15 Perfect Capability Restoration

Suppose every capability available in the no-harm reference can be recreated:

$$\mathcal{K}_{A,\text{cf}} \subseteq \mathcal{K}_{A,R}. \quad (2391)$$

Equation 2391 preserves every reference capability while permitting new ones.

This can be normatively preferable to exact capability equality because it allows the actual subject to retain valuable later development.

The remaining questions concern whether the capabilities are secure, usable, and subject-directed.

26.16 Perfect Reachability Restoration

Suppose future reachable sets can be matched exactly:

$$\mathcal{R}_\tau^R = \mathcal{R}_\tau^{\text{cf}}. \quad (2392)$$

Equation 2392 establishes equality of represented future possibility over horizon τ .

The actual histories remain different.

The practical meanings of the same options can also differ if memory, provenance, or recognition remain different.

26.17 Perfect Dynamical Restoration

Suppose technology can alter the system until its future transition law matches the reference.

Let

$$\Phi_R = \Phi_{\text{cf}}. \quad (2393)$$

Equation 2393 establishes exact transition-law identity within the modeled state space.

Attractor, basin, perturbation-response, and reachability structures can then also correspond under appropriate conditions.

This still does not establish historical identity.

26.18 Perfect Causal Restoration

A strong causal criterion can require

$$M_{C,R} = M_{C,cf}. \quad (2394)$$

Equation 2394 means the selected causal graph, structural mechanisms, and exogenous distributions correspond exactly.

The restored system then responds to every modeled intervention in the same way as the counterfactual system.

Historical provenance can remain different.

Identity can remain different.

The act of causal reconstruction itself creates new history.

26.19 Perfect Generative-Process Restoration

Suppose conversion, relational generation, value production, circulation, control, and recursive regeneration can all be reconstructed.

Then

$$Q_G(\mathcal{G}^R) = Q_G(\mathcal{G}^{cf}). \quad (2395)$$

Equation 2395 establishes strong generative-process identity under the modeled process representation.

This is substantially stronger than returning resources.

It remains weaker than historical identity.

26.20 Perfect Network Reconstruction

Consider a technology capable of reconstructing the exact relational network that would have existed without the harm.

Let reference network be

$$G_{cf} = (V_{cf}, E_{cf}). \quad (2396)$$

Perfect network restoration would require

$$G_R \cong G_{cf} \quad (2397)$$

or stronger node-level identity where the same persons are required.

Equation 2397 creates a major procedural problem.

Each other person in the network possesses present agency.

The claimant's historical loss does not automatically authorize reconstruction of their counterfactual relationships.

26.21 Network Reconstruction and Third-Party Agency

Let desired reference edge be

$$e_{AB}^{cf} \in E_{cf}. \quad (2398)$$

The actual present person B can decline relation e_{AB} .

Thus

$$C_B(e_{AB}) = 0. \quad (2399)$$

Equation 2399 places the reference edge outside the admissible reconstructed network where mutual consent is required.

Technical reconstructability therefore differs from legitimate reconstructability.

26.22 Perfect Institutional Reconstruction

Suppose an institution can be returned exactly to its counterfactual structural state.

Let institutional architecture be

$$I = (R, A, C, P, N), \quad (2400)$$

where R denotes rules, A authority, C capacities, P procedures, and N networks.

Exact institutional reconstruction satisfies

$$I_R = I_{cf}. \quad (2401)$$

Equation 2401 can still be normatively undesirable where the historical institution itself contained injustice that the counterfactual harm did not cause.

Technical fidelity therefore cannot replace baseline evaluation.

26.23 Perfect Environmental Reconstruction

Suppose ecological technology can recreate every selected physical and biological variable of the counterfactual environment.

Let ecological state be E_t . Exact state restoration is

$$E_{R,t} = E_{cf,t}. \quad (2402)$$

Equation 2402 does not automatically restore:

- historical provenance;
- evolutionary history;
- lost organisms as historically identical individuals;
- cultural relations to the prior environment.

Ecological function can be restored more fully than ecological history.

26.24 Perfect Memory Reconstruction

Consider a hypothetical technology capable of changing all memories associated with the harm.

Let actual memory be M_A^H , counterfactual memory M_A^C , and restored memory M_A^R .

Maximum memory fidelity requires

$$M_A^R = M_A^C. \quad (2403)$$

Equation 2403 eliminates one major historical difference inside the subject's represented internal state.

The intervention creates several residual problems.

26.25 Memory Reconstruction and Testimony

The actual subject can possess knowledge of the harm that would disappear after memory reconstruction.

Let testimony capacity be T_A .

It can occur that

$$T_A^H > T_A^R. \quad (2404)$$

Equation 2404 represents loss of historical testimony through fidelity-enhancing memory intervention.

Historical similarity and epistemic justice can therefore diverge.

26.26 Memory Reconstruction and Identity

Ricoeur's account of selfhood highlights the relation among memory, narrative, and identity [49].

Within the present framework, identity continuity can be represented by

$$I_A^R \sim_{\text{cont}} I_A^H. \quad (2405)$$

Equation 2405 can fail even where Equation 2403 succeeds.

Perfect memory fidelity therefore does not establish restoration of the same historical subject.

26.27 Perfect Preference Reconstruction

Suppose the institution can also reconstruct the preferences the subject would have possessed without the harm.

Let

$$P_A^R = P_A^C. \quad (2406)$$

Equation 2406 appears to solve the conflict between present and counterfactual preferences only by altering the present preference-bearing subject.

The procedure therefore changes the agent whose consent and welfare are being evaluated.

This creates an identity-sensitive regress.

26.28 Preference Reconstruction Regress

Before the intervention, subject A^H can reject preference reconstruction.

After the intervention, reconstructed subject A^R can endorse it.

The institutional question concerns whose preference has authority over whether the transition should occur.

Let

$$P_{A^H}(R_P) < 0 \quad (2407)$$

and

$$P_{A^R}(R_P) > 0. \quad (2408)$$

Equations 2407 and 2408 demonstrate why post-intervention endorsement cannot automatically legitimate an earlier non-consensual identity change.

26.29 Perfect Emotional Reconstruction

A still stronger technology could reproduce every affective disposition of the counterfactual subject.

Let affective state be A_t^E . Exact correspondence is

$$A_{R,t}^E = A_{cf,t}^E. \quad (2409)$$

Equation 2409 can eliminate fear, resentment, grief, or distrust associated with the harm.

Such intervention can remove genuine downstream burdens.

It can also erase reasonable responses to actual history.

Normative evaluation therefore remains necessary even when psychological repair is technically exact.

26.30 Perfect Recognition Reconstruction

Suppose every relevant third party can be induced to hold exactly the counterfactual recognition state.

Let distributed recognition be

$$\mathbf{R}^{\text{rec}} = (R_1^{\text{rec}}, \dots, R_n^{\text{rec}}). \quad (2410)$$

Perfect recognition reconstruction requires

$$\mathbf{R}_R^{\text{rec}} = \mathbf{R}_{cf}^{\text{rec}}. \quad (2411)$$

Equation 2411 can require manipulation of the beliefs or memories of others.

This creates third-party autonomy and epistemic integrity concerns.

26.31 Recognition through Truthful Correction

A less faithful alternative can preserve truthful social knowledge.

Instead of making others believe that harm never occurred, the institution can produce

$$R_i^{\text{rec}} = R_i(H, C_H, R), \quad (2412)$$

where C_H represents correction or acknowledgment.

Equation 2412 creates a recognition state different from the no-harm world and potentially superior in historical truth.

26.32 Perfect Record Reconstruction

Suppose all records can be edited until

$$Z_R = Z_{cf}. \quad (2413)$$

Equation 2413 makes the current archive identical to the no-harm archive under the selected record representation.

This can require deletion of:

- evidence of harm;
- evidence of correction;
- institutional responsibility;
- later legal decisions.

Exact archival fidelity can therefore reduce historical truth.

26.33 Perfect Erasure

Consider the strongest erasure technology.

It removes:

- the subject's memory;
- third-party memory;
- digital records;
- institutional records;
- material traces.

Let distributed trace set be $\mathcal{T}_H$. Perfect erasure yields

$$\mathcal{T}_{H,R} = \emptyset. \quad (2414)$$

Equation 2414 can make the observable restored world extremely similar to the no-harm world.

The actual causal history still contained the erasure operation unless the world history itself is replaced.

26.34 Erasure Event

Let erasure event be e_E .

Then

$$e_E \in \mathcal{H}_R \quad (2415)$$

while

$$e_E \notin \mathcal{H}_{cf}. \quad (2416)$$

Equations 2415 and 2416 show that perfect trace erasure does not create literal historical identity.

It produces a history in which erasure occurred.

26.35 Perfect Replica

Consider a technology capable of producing a perfect replica of the counterfactual subject.

Let replica be A^* . Suppose

$$Q_i(A^*) = Q_i(A^C) \quad \forall i \in I_Q. \quad (2417)$$

Equation 2417 establishes complete represented equivalence over the chosen object family.

The remaining question concerns personal identity.

If A^* is another subject, creation of A^* does not restore A^H .

If A^H is transformed into A^* , identity continuity becomes the central issue.

The atlas cannot resolve that question through structural equivalence alone.

26.36 Replacement and Restoration

The perfect-replica thought experiment forces a distinction between two operations:

$$R : A^H \longrightarrow A^R \quad (2418)$$

and

$$\text{Replace} : A^H \longrightarrow A^*. \quad (2419)$$

Equation 2418 denotes transformation of the historical subject.

Equation 2419 represents replacement by a counterfactual-equivalent subject where identity continuity fails.

The two operations can be observationally indistinguishable to outsiders.

Their normative meanings differ radically.

26.37 World Reconstruction

Extend the thought experiment from one subject to the whole world.

Let actual harmed-and-repaired world be

$$\mathcal{W}^{H,R} \quad (2420)$$

and counterfactual no-harm world be

$$\mathcal{W}^{-H}. \quad (2421)$$

Suppose technology can transform every present variable until

$$Q_i(\mathcal{W}^{H,R}) = Q_i(\mathcal{W}^{-H}) \quad \forall i. \quad (2422)$$

Equation 2422 achieves maximal represented present-world similarity.

The histories still differ unless historical events themselves are replaced.

26.38 World Replacement

Suppose technology instead replaces the actual world with the exact counterfactual world:

$$\mathcal{W}^{H,R} \longrightarrow \mathcal{W}^{-H}. \quad (2423)$$

Equation 2423 maximizes counterfactual fidelity by eliminating the actual post-harm world.

The normative question becomes whether elimination of actual historical persons, relationships, and developments constitutes repair.

The thought experiment therefore transforms restoration into a problem of world replacement.

26.39 Time Reversal

A still stronger idealization permits reversal to a pre-harm time t_0 :

$$\mathcal{W}_{t_1}^H \xrightarrow{\mathcal{T}^{-1}} \mathcal{W}_{t_0}. \quad (2424)$$

After reversal, the harm is prevented and a new trajectory begins.

Equation 2424 can generate a world in which the harm never occurs from the perspective of the restarted trajectory.

The actual later history between t_0 and t_1 is thereby removed.

The question is no longer simply whether harm can be repaired.

The question becomes whether erasing an actual interval of lived history and the persons constituted through it is permissible.

26.40 Temporal Replacement

Time reversal therefore functions as temporal replacement.

Let actual interval be

$$\mathcal{H}_{[t_0, t_1]}^H. \quad (2425)$$

The reversed world replaces it with

$$\mathcal{H}_{[t_0, t_1]}^{-H}. \quad (2426)$$

Equations 2425 and 2426 represent different histories.

Technical time manipulation does not dissolve the question of which history's subjects possess claims.

26.41 Omniscient Restorer

Consider an idealized restorer possessing complete knowledge of:

- actual history;
- every causal mechanism;
- every counterfactual history;
- every present preference;
- every future consequence of every possible intervention.

Let epistemic state be

$$K^* = K_{\text{complete}}. \quad (2427)$$

Equation 2427 eliminates empirical uncertainty within the assumed ontology.

Normative disagreement can remain complete.

26.42 Omniscience and Value Conflict

Suppose the restorer knows with certainty that intervention R_1 maximizes welfare and intervention R_2 better preserves rights.

The conflict is

$$W(R_1) > W(R_2) \quad (2428)$$

and

$$J_{\text{rights}}(R_2) > J_{\text{rights}}(R_1). \quad (2429)$$

Equations 2428 and 2429 show that epistemic perfection does not create a unique normative ranking.

26.43 Omnipotent Restorer

Now suppose the restorer also possesses unlimited technical power.

Let feasible set satisfy

$$\mathcal{R}_{\text{tech}} = \mathcal{R}_{\text{conceivable}}. \quad (2430)$$

Equation 2430 removes scarcity and technical constraints.

The legitimate set can still satisfy

$$\mathcal{R}_{\text{legit}} \subset \mathcal{R}_{\text{tech}}. \quad (2431)$$

Equation 2431 is one of the central results of the idealized analysis.

Power expands possibility.

It does not create moral permission.

26.44 Unlimited Resources

Suppose restorative resources are effectively unlimited.

Then distributive scarcity among restorative claimants disappears.

Several conflicts remain:

- identity;
- consent;
- third-party autonomy;
- historical truth;
- value pluralism;
- reference selection.

Unlimited resources solve trade-offs arising from scarcity.

They do not solve trade-offs among non-fungible normative objects.

26.45 Unlimited Compensation

Suppose unlimited money can be transferred.

Let compensation be

$$M \rightarrow \infty. \quad (2432)$$

Equation 2432 can eliminate material scarcity for the claimant.

It cannot force the existence of:

- a deceased person;
- a historical opportunity window;
- a particular past intellectual encounter;
- a lost ecological history;
- a consensual relationship.

Unlimited general equivalents therefore do not create universal fungibility.

26.46 Unlimited Simulation

Suppose every possible future can be simulated exactly.

Let future policy set be Π . The institution can calculate

$$\mathcal{O}(\pi) \quad \forall \pi \in \Pi, \quad (2433)$$

where $\mathcal{O}(\pi)$ is the complete consequence of policy π .

Equation 2433 removes decision uncertainty.

If the outcomes remain incomparable under the value architecture, perfect simulation still does not produce a unique choice.

26.47 Perfect Prediction and Free Choice

The idealization also exposes a distinction between predictability and agency.

Suppose the subject's future choice can be predicted with certainty:

$$P(A_{t+1} = a^* \mid K^*) = 1. \quad (2434)$$

Equation 2434 does not itself give the institution authority to choose a^* for the subject.

Prediction of agency and exercise of agency remain different normative objects.

26.48 Perfect Preference Knowledge

Suppose the institution knows every present and future preference the subject would have under every intervention.

The institution can identify the complete preference map

$$\mathcal{P}_A : R \longmapsto P_A^R. \quad (2435)$$

Equation 2435 removes uncertainty about preferences.

The institution must still decide:

- which temporal preference has authority;
- how adaptive preferences are treated;
- whether some rights constrain preference satisfaction;
- how third-party interests enter.

Perfect preference knowledge therefore does not create a complete normative theory.

26.49 Perfect Value Knowledge

An even stronger thought experiment assumes that every value object is known perfectly.

Let value vector be

$$\mathbf{V}^* = (V_1, \dots, V_n) \quad (2436)$$

with perfect measurement.

Equation 2436 removes measurement uncertainty.

If the values are only partially ordered, the alternatives can remain incomparable:

$$R_1 \parallel_V R_2. \quad (2437)$$

Equation 2437 shows that incomparability can persist under perfect information.

26.50 Perfect Moral Theory Thought Experiment

Suppose one pushes the idealization further and assumes a uniquely correct and fully known moral theory.
Let this theory be $\mathcal{T}^*$.

The institution can then rank every admissible intervention according to

$$J^*(R). \quad (2438)$$

Equation 2438 removes normative uncertainty by assumption.

Even here, exact historical identity remains distinct from justice.

The morally best repaired world can differ from the no-harm world.

The idealization therefore separates a final conceptual point: justice and historical equivalence are distinct objects even under complete normative knowledge.

26.51 Technical Completeness

The term *complete restoration* can refer to several different forms of completeness.

Technical completeness can be defined relative to intervention set $\mathcal{U}$:

$$C_T = 1 \iff \text{every technically targetable restorative object reaches its target.} \quad (2439)$$

Equation 2439 says nothing about whether the targets are correctly chosen.

26.52 Representational Completeness

Representational completeness would require the model to preserve every distinction relevant to the restorative purpose.

Let representation be Π . A purpose-relative criterion can be written as

$$C_R(\Pi; Q^*) = 1. \quad (2440)$$

Equation 2440 is indexed to a relevance set Q^* .

Open-ended systems make unrestricted representational completeness difficult to define because future relevance can itself change.

26.53 Epistemic Completeness

Epistemic completeness can be defined by

$$C_K = 1 \quad (2441)$$

where all facts, mechanisms, and counterfactuals required by the chosen model are known.

Equation 2441 is an idealization.

It remains conceptually distinct from normative completeness.

26.54 State Completeness

State completeness requires

$$x_R = x_{\text{ref}}. \quad (2442)$$

Equation 2442 is the simplest strong restorative criterion.

It leaves trajectory, dynamics, and history open.

26.55 Functional Completeness

Functional completeness requires

$$F_i(\mathcal{G}_R) = F_i(\mathcal{G}_{\text{ref}}) \quad \forall i \in I_F. \quad (2443)$$

Equation 2443 remains indexed to the selected function family.

26.56 Dynamical Completeness

Dynamical completeness can require

$$\Phi_R \cong \Phi_{\text{ref}} \quad (2444)$$

under the chosen dynamical relation.

Equation 2444 can include attractor, basin, stability, and perturbation-response correspondence.

26.57 Causal Completeness

Causal completeness can require

$$M_{C,R} \sim_{\text{causal}} M_{C,\text{ref}}. \quad (2445)$$

Equation 2445 can be defined through structural or interventional equivalence.

26.58 Generative Completeness

Generative completeness can require

$$Q_G(\mathcal{G}_R) \sim_G Q_G(\mathcal{G}_{\text{ref}}). \quad (2446)$$

Equation 2446 captures recursive process correspondence.

It does not entail historical identity.

26.59 Historical Completeness

Historical completeness in the strongest sense requires

$$\mathcal{H}_R = \mathcal{H}_{\text{ref}}. \quad (2447)$$

Equation 2447 differs categorically from the preceding forms.

Where actual harm and repair occurred, exact historical identity with a no-harm reference generally requires replacement or erasure of the actual history.

26.60 Normative Completeness

Normative completeness can mean that every protected substantive and procedural dimension satisfies its justified standard:

$$\mathbf{J}_S \succeq \boldsymbol{\theta}_S \quad \text{and} \quad \mathbf{J}_P \succeq \boldsymbol{\theta}_P. \quad (2448)$$

Equation 2448 does not require historical identity.

A normatively complete repair can therefore remain historically different from the reference.

26.61 Completeness Vector

The different meanings of completeness can be summarized by

$$\mathbf{C} = (C_T, C_R, C_K, C_S, C_F, C_D, C_C, C_G, C_H, C_N), \quad (2449)$$

where the components denote technical, representational, epistemic, state, functional, dynamical, causal, generative, historical, and normative completeness.

Equation 2449 makes clear why the phrase “complete restoration” is underdetermined without qualification.

26.62 Non-Implications among Completeness Concepts

Several non-implications follow immediately.

Technical completeness does not imply normative completeness:

$$C_T = 1 \not\Rightarrow C_N = 1. \quad (2450)$$

Normative completeness does not imply historical completeness:

$$C_N = 1 \not\Rightarrow C_H = 1. \quad (2451)$$

Causal completeness does not imply historical completeness:

$$C_C = 1 \not\Rightarrow C_H = 1. \quad (2452)$$

Generative completeness does not imply state identity:

$$C_G = 1 \not\Rightarrow C_S = 1. \quad (2453)$$

Equations 2450 through 2453 demonstrate the need for indexed language even under ideal technology.

26.63 Residual Classes under Ideal Technology

Expanded technology can reduce some residuals while leaving others.

Let total residual difference be

$$\Delta_{\text{total}} = \Delta_T \cup \Delta_K \cup \Delta_S \cup \Delta_H \cup \Delta_N \cup \Delta_R, \quad (2454)$$

where:

- Δ_T contains technical residuals;
- Δ_K epistemic residuals;
- Δ_S structural residuals;
- Δ_H historical residuals;
- Δ_N normative residuals;
- Δ_R reflexive residuals.

Equation 2454 provides a taxonomy of remaining differences.

26.64 Technical Residuals

Technical residuals are differences that remain because available intervention technology cannot alter the relevant object.

Examples under present conditions include:

- some bodily losses;
- deceased persons;
- destroyed ecosystems;
- inaccessible historical evidence.

Many technical residuals can shrink with future capacity.

They should therefore be distinguished from principled normative limits.

26.65 Epistemic Residuals

Epistemic residuals remain because the reference, model, causal structure, or value cannot be identified fully.

Under the strongest omniscience idealization, Δ_K can be set to zero by assumption.

The remaining residuals then reveal which problems are deeper than uncertainty.

26.66 Structural Residuals

Structural residuals arise from the form of the system.

Examples include:

- many-to-one dynamics;
- open-ended state spaces;
- stochastic branching;
- emergent variables;
- path dependence.

Some structural residuals can be overcome through world manipulation.

Others reappear as historical or identity problems once the intervention becomes sufficiently strong.

26.67 Historical Residuals

Historical residuals arise because the actual harmed-and-repaired history differs from the no-harm history.

Even where every present represented variable is equal, the relation

$$\mathcal{H}^{H,R} \neq \mathcal{H}^{\neg H} \quad (2455)$$

can remain.

Equation 2455 disappears only by redefining the historical object, erasing the actual history, or replacing the world.

26.68 Normative Residuals

Normative residuals are differences deliberately preserved because eliminating them would violate rights, consent, identity, third-party justice, or another normative constraint.

Let

$$\Delta_N = \Delta_{\text{protected}} \cup \Delta_{\text{valuable}}. \quad (2456)$$

Equation 2456 shows that some non-equivalence is part of justice rather than a failure to achieve it.

26.69 Reflexive Residuals

Reflexive residuals arise because observation and intervention become part of the system.

Let the restorer observe the system through $\mathcal{O}$, select intervention R , and alter the next state:

$$\mathcal{G}_{t+1} = F(\mathcal{G}_t, \mathcal{O}(\mathcal{G}_t), R_t). \quad (2457)$$

Equation 2457 shows that the act of restoration changes the object later being compared.

Hacking's looping effects and Merton's self-fulfilling prophecy provide important social-theoretical precedents for such feedback [23, 36].

26.70 Reflexive Non-Closure under Perfect Observation

Suppose observation is perfect.

The institutional record can still influence later states:

$$Z_t^* = O^*(\mathcal{G}_t) \quad (2458)$$

and

$$\mathcal{G}_{t+1} = F(\mathcal{G}_t, Z_t^*). \quad (2459)$$

Equations 2458 and 2459 show that perfect accuracy does not make the record causally external.

Reflexive non-closure therefore persists after measurement error disappears.

26.71 Reflexive Non-Closure under Perfect Policy

Suppose the institution selects the best policy under its current model.

The policy can alter the behavior generating the model's previously observed relationships.

This structure is analogous to the broader lesson of the Lucas critique that policy changes can alter the behavioral relations used for prediction [29].

The restorative model can therefore require continuous updating even under extremely strong initial knowledge.

26.72 Model Updating under Ideal Capacity

Let model at time t be M_t .

After intervention R_t ,

$$M_{t+1} = \mathcal{U}(M_t, \mathcal{G}_{t+1}). \quad (2460)$$

Equation 2460 makes revision structurally necessary in reflexive systems.

Ideal computational capacity can make revision instantaneous.

It does not make revision unnecessary.

26.73 Open-Ended Ontology

Generative systems can create new categories of state or value.

Let

$$\mathcal{X}_{t+1} = \mathcal{X}_t \cup \Delta\mathcal{X}_{t+1}. \quad (2461)$$

Equation 2461 means tomorrow's relevant state space can contain objects absent from today's ontology.

Perfect knowledge at time t can therefore fail to imply a permanently complete ontology.

This is another form of generative non-closure.

26.74 Perfect Knowledge at One Time

Let

$$K_t = K_t^*. \quad (2462)$$

Equation 2462 denotes complete knowledge of everything represented at time t .

If new value categories emerge at $t + 1$,

$$\mathfrak{D}_{V,t+1} \supset \mathfrak{D}_{V,t}, \quad (2463)$$

then Equation 2462 does not establish permanent completeness.

26.75 Ideal Restorative Digital Twin

Consider a perfect digital twin of the affected system.

Let

$$\mathcal{D}_t \cong \mathcal{G}_t \quad (2464)$$

for every modeled variable and dynamic relation.

Equation 2464 permits exact simulation of candidate remedies.

The twin can improve epistemic decision quality dramatically.

It remains a representation.

Its construction requires selection of:

- system boundaries;
- state variables;
- value objects;
- relevant relations;
- normative constraints.

A perfect twin relative to one representation is therefore compatible with representational contestability.

26.76 Digital Twin and Privacy

A highly complete digital twin can require extraordinarily intimate data.

Let informational fidelity be F_D and privacy cost be C_{priv} .

It can occur that

$$F_D \uparrow \implies C_{\text{priv}} \uparrow . \quad (2465)$$

Equation 2465 means ideal simulation can remain procedurally undesirable.

The institution can rationally prefer a lower-resolution model whose decisions are robust across remaining uncertainty.

26.77 Ideal Simulation and Minimal Necessary Knowledge

Expanded technical capacity therefore strengthens, rather than eliminates, the question of necessity.

Let the minimum information required to select a just intervention be

$$K_{\text{min}}^J . \quad (2466)$$

The institution need not collect

$$K_{\text{all}} \quad (2467)$$

merely because doing so is technically possible.

Data minimization can therefore remain rational under ideal technology.

26.78 Ideal Restoration and Institutional Power

Expanded technical capacity transforms restorative institutions into unusually powerful governance actors.

A system capable of rewriting memory, reconstructing networks, or selecting counterfactual life trajectories exercises power over the constitution of subjects.

Let restorative power be

$$P_R = P(\mathcal{U}_{\text{tech}}, K, A) , \quad (2468)$$

where A denotes institutional authority.

Equation 2468 increases with both technical capacity and informational control.

Procedural constraint therefore becomes more important as restorative power increases.

26.79 Ideal Technology and Non-Domination

A technically benevolent institution can remain dominating if subjects depend upon its discretionary power to define and modify their lives.

Republican non-domination therefore remains relevant under highly capable restorative governance [44].

Secure rights, contestability, distributed authority, and exit remain valuable even if the institution almost always chooses well.

26.80 Perfect Benevolence Thought Experiment

Suppose the restorative institution is perfectly benevolent.

It always seeks the subject's good and never intentionally abuses power.

Arbitrary structural power can still remain if future benevolence is merely an assumption rather than an institutional guarantee.

More importantly, benevolence does not determine which conception of the subject's good is authoritative under value pluralism.

Perfect motives therefore do not eliminate governance questions.

26.81 Perfect Trustworthiness

Suppose the institution is also perfectly trustworthy.

Privacy and consent can still have value beyond protection against misuse.

A person can value authority over intimate information and life direction even when misuse probability is zero.

Thus

$$P(\text{misuse}) = 0 \quad (2469)$$

does not imply

$$V_{\text{privacy}} = 0. \quad (2470)$$

Equations 2469 and 2470 distinguish instrumental from constitutive or autonomy-related privacy value.

26.82 Perfect Accuracy and Contestability

Suppose the institution never makes a factual error.

Contestability can still possess value through equal standing, justification, and protection against future institutional change.

A right to reasons can therefore remain meaningful even where the result is always substantively correct.

This thought experiment shows that some procedural values are independent of error correction.

26.83 Perfect Outcome and Procedural Wrong

Let outcome satisfy

$$J_S(R) = J_{S,\max}. \quad (2471)$$

Suppose the intervention was imposed through severe procedural violation:

$$J_P(R) < \theta_P. \quad (2472)$$

Equations 2471 and 2472 establish that perfect substantive outcome and procedural justice remain distinct even under ideal technology.

26.84 Perfect Procedure and Residual History

The reverse structure also remains possible.

Let

$$J_P(R) = J_{P,\max}. \quad (2473)$$

Historical non-equivalence can persist:

$$\mathcal{H}_R \neq \mathcal{H}_{\text{cf}}. \quad (2474)$$

Equations 2473 and 2474 demonstrate that a perfectly fair process cannot make the harm historically unoccurred.

26.85 Ideal Restoration Frontier

Under expanded technical capacity, define the ideal nondominated set

$$\mathcal{F}_I = \text{ND} (F_R, J_S, J_P, H_{\text{truth}}, A_{\text{present}}). \quad (2475)$$

Equation 2475 preserves restorative fidelity, substantive justice, procedural justice, historical truth, and present agency as separate evaluative dimensions.

Technical power can enlarge $\mathcal{F}_I$.

It need not collapse the frontier to a single point.

26.86 Ideal Pareto Improvements

Expanded technology can create genuine improvements without new trade-offs.

For example, future medicine can restore bodily function with lower procedural burden.

Let intervention R_2 satisfy

$$F_R(R_2) > F_R(R_1), \quad (2476)$$

$$J_S(R_2) \geq J_S(R_1), \quad (2477)$$

and

$$J_P(R_2) \geq J_P(R_1). \quad (2478)$$

Equations 2476 through 2478 describe a genuine technology-enabled Pareto improvement in restorative design.

The theory welcomes such improvements.

Its caution concerns the assumption that every increase in technical power has this structure.

26.87 Ideal Technology and New Trade-Offs

New technology can also create restorative options that previously did not exist.

Memory editing, neural reconstruction, total behavioral simulation, and large-scale relational engineering generate new choices and therefore new normative conflicts.

Technical progress can thus increase the dimensionality of the restorative decision problem.

26.88 Option Expansion

Let available restorative set before technological expansion be $\mathcal{R}_0$ and after expansion be $\mathcal{R}_1$.

Then

$$\mathcal{R}_0 \subset \mathcal{R}_1. \quad (2479)$$

Equation 2479 can increase freedom.

It can also create pressure to use highly intrusive interventions because they are now possible.

A right to decline technical restoration therefore becomes increasingly important.

26.89 Right to Remain Historically Different

A subject can possess a legitimate interest in preserving aspects of the actual post-harm self.

Let difference from the counterfactual subject be

$$\Delta_A = A^H \setminus A^C. \quad (2480)$$

Equation 2480 is conceptual.

Some elements of Δ_A can be:

- unwanted consequences of harm;
- valuable learning;
- new commitments;
- new relationships;
- identity-forming experiences.

Justice should not presume that every element must be removed.

26.90 Right to Retain Post-Harm Development

Let valued post-harm development be Δ_A^+ .

A preservation condition can require

$$\Delta_A^+ \subseteq A^R. \quad (2481)$$

Equation 2481 permits restoration to remove harmful consequences while preserving later values.

This creates a repaired subject different from both the harmed subject and the counterfactual subject.

26.91 Selective Restoration

The preceding argument supports selective restoration.

Let actual difference decompose into

$$\Delta_A = \Delta_A^- \cup \Delta_A^+ \cup \Delta_A^0, \quad (2482)$$

where the components denote harmful, valued, and normatively neutral differences.

Equation 2482 permits the restorative target to reduce Δ_A^- , preserve Δ_A^+ , and leave Δ_A^0 to subject choice where feasible.

26.92 Selective Historical Reconstruction

Historical repair can follow the same logic.

Let historical differences be partitioned as

$$\Delta_H = \Delta_H^{\text{harm}} \cup \Delta_H^{\text{truth}} \cup \Delta_H^{\text{development}}. \quad (2483)$$

Equation 2483 distinguishes harmful consequences, truthful historical provenance, and valuable later development.

Justice can seek to remove harmful consequences while preserving the other categories.

26.93 Repaired World as a Third World

The theory therefore distinguishes three worlds:

$$\mathcal{W}^H, \quad \mathcal{W}^{\neg H}, \quad \mathcal{W}^R. \quad (2484)$$

Equation 2484 denotes the harmed world, no-harm counterfactual world, and repaired actual world. The repaired world need not converge completely to either of the other two.

Its normative objective can be

$$\mathcal{W}^R \in \mathcal{W}_{\text{just}}^{\text{postharm}}. \quad (2485)$$

Equation 2485 defines the target as a just post-harm world rather than a replica of the no-harm world.

26.94 Justice after Irreversible History

This formulation provides a general answer to the strongest restoration problem.

The normative task after irreversible history is not exhausted by asking how closely the actual world can approximate the world in which the wrong never occurred.

It also asks what justice requires among the persons, institutions, memories, relationships, and possibilities that now exist.

The post-harm world therefore has independent normative reality.

26.95 Historical Truth as a Constraint on Ideal Restoration

Where historical truth has independent value, perfect reconstruction of the no-harm appearance can become normatively inferior.

Let historical truth value be V_H .

An intervention can satisfy

$$F_R \uparrow \quad (2486)$$

while

$$V_H \downarrow. \quad (2487)$$

Equations 2486 and 2487 formalize the conflict.

A repaired world can therefore preserve evidence that distinguishes it from the no-harm world.

Table 25: Idealized restorative capacities and residual limits

Idealized capacity	Restorative problem reduced	Residual problem	Primary atlas layer
Perfect observation	Measurement error and latent-state uncertainty	Representation and value selection	Epistemic / object
Perfect archive	Uncertainty concerning actual events	Counterfactual history and privacy	Historical
Perfect causal knowledge	Mechanism and intervention uncertainty	Normative admissibility	Causal / normative
Perfect counterfactual simulation	Uncertainty concerning selected no-harm trajectories	Reference authority and present agency	Reference / procedural
Perfect bodily repair	Physical and functional residuals	History, identity, recognition	State / historical
Perfect network reconstruction	Relational structural loss	Third-party agency and consent	Relational / procedural
Perfect memory reconstruction	Internal memory divergence	Identity, testimony, consent	Historical / procedural
Perfect generative reconstruction	Future process divergence	Historical non-identity and normative filtering	Generative / historical
Perfect world-state reconstruction	Present represented difference	Actual historical difference	Historical
Time reversal	Post-harm causal sequence	Erasure of actual later subjects and history	Identity / historical
Unlimited resources	Scarcity	Non-fungibility and normative conflict	Value / distributive

26.96 Recognition as Historical Addition

Recognition creates another divergence from the no-harm reference.

An apology, judgment, or memorial has no counterpart in the no-harm world.

Let recognition event be e_{rec} .

Then

$$e_{\text{rec}} \in \mathcal{H}_R \quad (2488)$$

and

$$e_{\text{rec}} \notin \mathcal{H}_{\text{cf}}. \quad (2489)$$

Equations 2488 and 2489 show that successful recognition can increase historical difference while increasing justice.

26.97 Institutional Learning as Historical Addition

The same structure applies to institutional reform.

Let learning event produce improved institution I_R^+ .

Then

$$I_R^+ \succeq_J I_{\text{cf}} \quad (2490)$$

can hold.

Equation 2490 means the repaired institution can become more just than the institution in the no-harm reference.

Exact counterfactual reconstruction would erase this improvement.

26.98 Idealized Restoration Matrix

Table 25 summarizes several idealized technical capabilities and the limits that remain after each capability is granted.

Table 25 shows that different idealizations remove different categories of constraint.

Table 26: Residual limits under expanded restorative capacity

Limit family	Characteristic source	Response to technical progress	Residual justice question
Technical	Unavailable intervention	Can substantially shrink	Which previously impossible repairs become feasible?
Epistemic	Unknown state, cause, or counterfactual	Can shrink with observation and modeling	How much knowledge is necessary and legitimate to collect?
Structural	Dynamics, branching, open-endedness	Can partly shrink through stronger control	Does reconstruction preserve the relevant system or replace it?
Historical	Actual occurrence of harm and repair	Persists unless history is erased or replaced	Which historical differences should remain recognized?
Normative	Rights, consent, value conflict	Does not disappear through technical power	Which technically possible interventions remain impermissible?
Reflexive	Observation and intervention enter the system	Can become faster but remains structurally present	How should the institution govern its own feedback effects?

26.99 Residual-Limit Matrix

Table 26 organizes the main residual-limit families.

Table 26 demonstrates why technological optimism and restorative justice answer different questions.

26.100 Technology-Sensitive Reparability

Reparability is historically variable.

Let technology level be k . Then

$$\text{Rep}(O; k) \quad (2491)$$

can increase over time.

An injury classified as practically irreversible today can become highly repairable under later medicine or institutional technology.

Preventive governance should therefore distinguish current reparability from conceptual irreversibility.

26.101 Current and Conceptual Irreparability

Let

$$\text{Rep}_{\text{current}} \quad (2492)$$

and

$$\text{Rep}_{\text{conceptual}} \quad (2493)$$

denote current and conceptual reparability.

A large difference between them indicates a technological limitation.

A small difference at the conceptual level can indicate a deeper historical or normative limit.

26.102 Technological Optimism and Prevention

Expected future repairability should not automatically justify present harm.

Let expected future repair probability be p_R .

Preventive duty can remain strong where:

- future technology is uncertain;
- repair itself is burdensome;
- historical loss remains;
- third-party consequences cannot be reconstructed;
- repair requires intrusive intervention.

Future repair therefore does not convert prevention into an obsolete concern.

26.103 Perfect Reparability and Wrongdoing

Even if every consequence could later be repaired perfectly, the permissibility of causing the initial harm would remain a separate question.

Suppose harm H is followed by perfect substantive repair R^* :

$$J_S(R^*(\mathcal{G}^H)) = J_S(\mathcal{G}^{-H}). \quad (2494)$$

Equation 2494 does not establish that H was permissible.

Wrongdoing, risk imposition, procedure, and historical experience remain separate normative objects.

26.104 Perfect Restoration as Ex Ante License

A dangerous inference would be:

$$\text{Rep}(H) = 1 \implies H \text{ permissible.} \quad (2495)$$

Equation 2495 is rejected.

Reparability can affect the severity and prevention calculus.

It does not independently authorize the initial harm.

26.105 Restorative Moral Hazard

High restorative capacity can create moral hazard if institutions become less careful because later repair is expected to be available.

Let preventive effort be P_E and expected reparability be Rep .

A possible institutional relation is

$$\frac{\partial P_E}{\partial \text{Rep}} < 0. \quad (2496)$$

Equation 2496 identifies a governance risk.

A just system should avoid allowing improved repair capacity to weaken appropriate prevention.

26.106 Restoration Insurance

Expanded technical capacity can support insurance against currently unpredictable restorative needs.

A public restorative fund can provide flexible future resources without requiring certainty concerning the eventual form of repair.

Such architecture preserves the epistemic-humility advantage of general equivalents while leaving space for later specific intervention.

26.107 Restorative Option Value under Technical Change

Where future technologies can create new repair possibilities, preserving future choice can have additional value.

Let available restorative options at future time t be $\mathcal{R}_t$.

Option-preserving policy can seek

$$\mathcal{R}_t \subseteq \mathcal{R}_{t+1}. \quad (2497)$$

Equation 2497 favors interventions that avoid prematurely eliminating future restorative possibilities.

26.108 Reversible Interim Restoration

Under uncertainty concerning future technology, interim remedies can be designed to remain revisable.

Let interim remedy be R_I and future improved remedy be R_F .

A compatibility condition is

$$R_F \circ R_I \text{ remains feasible.} \quad (2498)$$

Equation 2498 makes option preservation part of current restorative design.

26.109 Technological Lock-In

A present intervention can block later superior restoration.

Let

$$\mathcal{R}_{\text{future}}(R_1) \subset \mathcal{R}_{\text{future}}(R_2). \quad (2499)$$

Equation 2499 means remedy R_1 reduces future repair options more than R_2 .

Where present benefits are comparable, the more reversible path can have greater option value.

26.110 Expanded Capacity and Institutional Humility

Greater capacity can tempt institutions toward greater substantive ambition.

The opposite design principle can be preferable:

As technical capacity expands, institutions should become more explicit about the boundary between what they can change and what they are authorized to change.

Institutional humility therefore remains relevant under powerful technology.

26.111 Expanded Capacity and Subject Sovereignty

Where technology can alter memory, preference, identity, or relationships, subject authority over those domains becomes increasingly important.

The right to refuse restoration can become as important as the right to receive it.

This reverses the intuition that better technology simply increases the amount of restoration a benevolent institution should provide.

26.112 Expanded Capacity and Third-Party Sovereignty

The same principle applies to others affected by relational reconstruction.

A perfect technology cannot create consent by technical precision.

Third-party autonomy therefore places durable limits upon exact social reconstruction.

26.113 Expanded Capacity and Public Authority

Some forms of restoration affect public institutions and cannot be governed solely through private consent.

Examples include:

- public historical records;
- legal status;
- collective reparations;
- ecological restoration;
- institutional reform.

Expanded capacity therefore increases the need to clarify the public authority under which restorative intervention occurs.

26.114 Idealized Restorative Constitution

A highly capable restorative regime would require a constitutional architecture governing:

- protected domains beyond ordinary restorative manipulation;
- consent requirements;
- data access;
- baseline selection;
- counterfactual modeling;
- third-party rights;
- review;
- institutional separation of powers.

Technological power therefore transforms restoration into a constitutional governance problem.

26.115 Separation of Restorative Powers

One institution should not automatically control every stage of powerful restoration.

Possible functions include:

- evidence collection;
- reference construction;
- intervention design;
- implementation;

- review;
- archival oversight.

Separating these functions can reduce concentration of restorative power.

26.116 Reference Authority

Under expanded technology, the power to select the reference becomes especially consequential.

Let candidate references be

$$\mathfrak{B} = \{B_1, \dots, B_k\}. \quad (2500)$$

Equation 2500 can include no-harm counterfactual, pre-harm actual state, rights-based reference, capability threshold, and subject-directed transformed future.

No technological advance selects among these references automatically.

26.117 Value Authority

The same applies to value objects.

Let candidate value sets be

$$\mathfrak{V} = \{\mathfrak{V}_V^1, \dots, \mathfrak{V}_V^m\}. \quad (2501)$$

Equation 2501 can generate different restorative targets from the same perfectly known facts.

Value selection therefore remains a governance task.

26.118 Ideal Technical Capacity and Political Economy

Expanded restorative technologies can themselves be unevenly distributed.

Even where the technology is conceptually capable of perfect bodily or causal repair, access can remain unequal.

Let access probability be

$$P(R^* | i). \quad (2502)$$

Equation 2502 can vary by wealth, citizenship, institutional membership, or political power.

The political economy of restorative capacity therefore becomes a separate justice problem.

26.119 Restorative Technology as a New Generative Condition

Restorative technology can itself become a valuable generative condition.

Let access to restorative infrastructure be C_R^T .

Future capability can depend upon it:

$$\mathcal{K}_{t+1} = G(\mathcal{K}_t, C_R^T). \quad (2503)$$

Equation 2503 means unequal access to repair can recursively amplify inequality.

26.120 Restorative Technology Capture

Control over highly powerful repair technologies can create new domination.

Let actor A_j control access to C_R^T .

The dependence profile can become

$$D_i = D(A_j, C_R^T). \quad (2504)$$

Equation 2504 shows that technologies designed to repair vulnerability can generate another vulnerability if control is concentrated.

26.121 Open Restorative Infrastructure

A generative-justice approach can therefore ask whether restorative infrastructure itself is governed through:

- accessible standards;
- distributed knowledge;
- interoperable systems;
- public or community control;
- correction and contestation.

The justice of repair extends to the infrastructure through which repair becomes possible.

26.122 Idealized Restorative Capability Profile

The idealized technical environment can be summarized by

$$\mathbf{T}_R^* = (T_{\text{obs}}, T_{\text{hist}}, T_{\text{causal}}, T_{\text{sim}}, T_{\text{body}}, T_{\text{memory}}, T_{\text{network}}, T_{\text{institution}}, T_{\text{environment}}). \quad (2505)$$

Equation 2505 records observational, historical, causal, simulation, bodily, memory, network, institutional, and environmental capacities.

The profile can be expanded without altering the central conceptual conclusion.

26.123 Idealized Justice Profile

The corresponding justice profile remains

$$\mathbf{J}^* = (J_S, J_P, F_R, V_H, A, D, R_{3P}), \quad (2506)$$

where the components include substantive justice, procedural justice, restorative fidelity, historical value, agency, non-domination, and third-party rights.

Equation 2506 does not collapse as $\mathbf{T}_R^*$ approaches technical perfection.

26.124 Technical Expansion without Normative Collapse

The relationship can therefore be summarized by

$$\mathbf{T}_R \uparrow \not\Rightarrow \dim(\mathbf{J}) \downarrow. \quad (2507)$$

Equation 2507 states that increasing technical capacity does not reduce the normative problem to one dimension.

In some cases it creates more legitimate options and therefore a richer choice problem.

26.125 Idealized Restoration Principle

The first principle of this section is:

Expanded technical capacity can reduce practical and epistemic obstacles to restoration without eliminating the need to specify the reference, object, comparative relation, value theory, and procedure.

26.126 Observation Principle

The second principle is:

Perfect observation within a representation does not independently establish that the representation contains every normatively relevant distinction.

26.127 Counterfactual Principle

The third principle is:

Perfect knowledge of a no-harm counterfactual does not grant that counterfactual automatic authority over the present subject's future.

26.128 Causal-Power Principle

The fourth principle is:

Perfect knowledge of which intervention would reproduce a reference outcome does not independently establish permission to impose that intervention.

26.129 Historical Non-Identity Principle

The fifth principle is:

Exact present, functional, causal, dynamical, or generative equivalence can coexist with historical non-identity between the repaired world and the no-harm world.

26.130 Replacement Principle

The sixth principle is:

As restorative intervention approaches complete counterfactual reconstruction, the distinction between repairing the historical subject and replacing the subject with a counterfactual-equivalent configuration becomes increasingly important.

26.131 Third-Party Agency Principle

The seventh principle is:

Perfect reconstruction of social relations remains constrained by the present agency and rights of other persons whose counterfactual roles the reconstruction would require.

26.132 Historical Truth Principle

The eighth principle is:

Increasing similarity to a no-harm world through erasure can reduce justice where historical truth, accountability, testimony, or institutional learning carry independent value.

26.133 Protected Difference Principle

The ninth principle is:

Some differences from the reference should remain even when they become technically removable because their preservation protects rights, identity, agency, or valuable post-harm development.

26.134 Reflexive Non-Closure Principle

The tenth principle is:

Perfect observation and intervention do not eliminate reflexive non-closure where records, models, classifications, and restorative acts enter the future dynamics of the system.

26.135 Repaired-World Principle

The eleventh principle is:

The proper normative target after an actual wrong can be a just repaired world that differs from both the harmed world and the world in which the wrong never occurred.

26.136 Restorative-Power Constraint Principle

The twelfth principle is:

The expansion of restorative power increases the importance of constitutional, procedural, and non-domination constraints upon who may define and implement restoration.

26.137 Idealized Layer of the Atlas

The idealized layer of the atlas can be represented by

$$\mathfrak{I}_T = (\mathcal{U}_{\text{tech}}, K^*, \mathfrak{B}, \mathfrak{D}_V, \mathcal{A}^N, \Delta_H, \Delta_N, \Delta_R), \quad (2508)$$

where:

- $\mathcal{U}_{\text{tech}}$ is the expanded intervention set;
- K^* is the idealized epistemic state;
- $\mathfrak{B}$ is the family of possible references;
- $\mathfrak{D}_V$ is the value-object architecture;
- $\mathcal{A}^N$ is the normatively admissible region;
- Δ_H contains historical residuals;
- Δ_N contains normatively preserved residuals;
- Δ_R contains reflexive residuals.

Equation 2508 summarizes what remains once ordinary technological limitations have been deliberately weakened.

26.138 Implications for Legal Models

The idealized analysis has direct implications for law.

Law operates under technical scarcity today, yet many of its central questions would survive dramatic increases in restorative capacity.

Legal institutions would still need to decide:

- which interests are legally protected;
- which counterfactual is legally relevant;
- when restitution yields to compensation;
- how innocent third parties are protected;
- which remedies require consent;
- when privacy limits proof or reconstruction;
- how finality and revisability should interact;
- which residual losses receive recognition despite technical non-restoration.

The legal problem is therefore not merely an imperfect technological substitute for perfect restoration.

Law supplies normative, institutional, evidentiary, and procedural structure to the problem of return itself.

26.139 Transition to Legal Models of Return

Section 27 therefore turns from idealized capacity toward legal forms of restoration. It examines how restitution, compensation, damages, specific performance, injunctions, rehabilitation, declaratory relief, public-law remedies, reparations, and guarantees of non-repetition instantiate different object–relation cells within the Atlas of Equivalence.

The central legal question will be:

Which forms of equivalence does law actually attempt to produce, which differences does it convert into general equivalents, which residuals does it recognize without eliminating, and which restorative interventions does it forbid even when stronger factual reconstruction would be possible?

That transition moves the manuscript from the ideal limits of restoration toward the institutional grammar through which contemporary legal systems govern return.

27 Legal Models of Return

This section translates the preceding comparative and normative architecture into legal forms of remedy. Law does not ordinarily speak in the full vocabulary of state equivalence, reachability, basin geometry, causal mechanisms, generative recursion, historical provenance, and value pluralism. It nevertheless operates through a rich remedial grammar that selects particular objects of return, particular baselines, particular substitutes, and particular limits upon what may be restored.

The legal question is therefore broader than whether law “compensates” harm. Different legal remedies implement different comparative relations. Restitution can seek return of an identifiable object or status. Compensatory damages can translate heterogeneous loss into a general equivalent. Specific performance can require actual performance of an obligation rather than monetary substitution. Injunctions can prevent future deterioration or require corrective action. Declaratory relief can alter legal status and

authoritative classification without directly transferring resources. Rehabilitation can repair functional or capability loss. Satisfaction and public acknowledgment can address recognition and historical meaning. Guarantees of non-repetition and structural remedies can alter the mechanisms through which future harms would otherwise be reproduced.

International reparations practice already reflects this plurality by distinguishing restitution, compensation, rehabilitation, satisfaction, and guarantees of non-repetition [65, 12]. Private-law accounts of corrective justice likewise emphasize the relation between wrong, claimant, defendant, and remedy [11, 69]. Administrative and adjudicative theory adds a further insight: remedy cannot be separated completely from institutional form, competence, procedure, and authority [22, 34, 33].

The purpose of this section is neither to state the positive law of one jurisdiction nor to collapse heterogeneous legal traditions into one doctrinal scheme. It develops an analytical map of legal remedy forms and shows how they populate the Atlas of Equivalence. Particular jurisdictions can assign different doctrinal names, availability conditions, standards, and institutional competences to these forms.

The section proceeds through four levels. First, it represents legal remedies as institutionally authorized transformations of legally recognized objects. Second, it examines the major remedial forms and their implicit comparative relations. Third, it analyzes characteristic legal limits including causation, proof, finality, third-party rights, proportionality, feasibility, and institutional competence. Fourth, it asks how legal remedies can move from one-time correction toward protection of future generativity without converting law into unlimited management of persons and social systems.

27.1 Legal Remedy as an Indexed Transformation

Let a legally cognizable claim be represented by the tuple

$$\mathcal{L} = (A, B, W, I_L, Q_L, B_L, P_L), \quad (2509)$$

where A denotes the claimant, B the relevant responding party or public institution, W the legally recognized wrong or triggering condition, I_L the legally protected interest, Q_L the remedial object, B_L the legally selected baseline, and P_L the governing procedural framework. Equation 2509 represents the legal claim as a bounded institutional construction rather than the complete moral history of the parties.

A legal remedy can then be represented as

$$R_L : (\mathcal{G}^H, \mathcal{L}) \longrightarrow \mathcal{G}^L, \quad (2510)$$

where $\mathcal{G}^L$ is the post-remedy system. Equation 2510 makes law one institutional mechanism capable of altering selected dimensions of the harmed system.

27.2 Legal Interests and Restorative Objects

The legally protected interest and the restorative comparison object should remain analytically distinct.

Let the protected interest be I_L , and the remedial object be Q_L . Their relationship is represented by

$$Q_L = \Psi_L(I_L, W, B_L). \quad (2511)$$

Equation 2511 means the form of legal remedy depends upon how the legal system characterizes the protected interest, wrong, and baseline.

The same factual event can therefore generate different remedial objects under different legal causes of action or institutional mandates.

27.3 Legal Baselines

Legal remedies employ several possible baselines.

Let the family of legal baselines be

$$\mathfrak{B}_L = \{B_{\text{pre}}, B_{\text{entitlement}}, B_{\text{no-wrong}}, B_{\text{performance}}, B_{\text{adequacy}}, B_{\text{public}}\}. \quad (2512)$$

Equation 2512 contains pre-wrong state, protected entitlement, no-wrong counterfactual, promised or required performance, adequacy threshold, and public-law reference.

Different remedies can therefore pursue different forms of return even within the same legal dispute.

27.4 Legal Remedy Families

For analytical purposes, the major remedy families can be represented by

$$\mathfrak{R}_L = \{R_{\text{rest}}, R_{\text{dam}}, R_{\text{spec}}, R_{\text{inj}}, R_{\text{decl}}, R_{\text{rehab}}, R_{\text{sat}}, R_{\text{struct}}, R_{\text{NR}}\}, \quad (2513)$$

where the terms denote restitution, damages or compensation, specific performance, injunctive relief, declaratory relief, rehabilitation, satisfaction, structural relief, and guarantees of non-repetition. Equation 2513 is a functional taxonomy rather than a jurisdiction-specific doctrinal classification.

27.5 Restitution

Restitution most directly approximates an identity-like restorative relation where an identifiable object, status, possession, or legally protected position can be returned.

Let legally relevant object be q . A simple restitutionary relation is

$$q_R = q_{\text{entitlement}}. \quad (2514)$$

Equation 2514 expresses restoration of the legally protected object itself.

This can apply analytically to property, legal status, access, possession, employment position, or another identifiable legal condition, depending upon the governing law.

27.6 Object Restitution

Object restitution is strongest where the original object remains available and legally returnable.

Let original object be q^* . Exact restitution requires

$$q_R = q^*. \quad (2515)$$

Equation 2515 has unusually high object fidelity.

Its normative significance can be especially strong where the object is singular, historically meaningful, or difficult to replace.

27.7 Status Restitution

Legal status can also be restored.

Let status before wrongful alteration be s_L^* . Status restitution requires

$$s_{L,R} = s_L^*. \quad (2516)$$

Equation 2516 can apply to a legally recognized position whose value depends upon authoritative institutional classification.

Status restoration can carry material, relational, and recognition effects simultaneously.

27.8 Restitution and Changed Context

Return of the same legal object can have different practical significance after history has changed.

It can hold that

$$q_R = q_{\text{ref}} \quad (2517)$$

and

$$V_G(q_R) \neq V_G(q_{\text{ref}}), \quad (2518)$$

where V_G denotes generative value in context. Equations 2517 and 2518 formalize the limitation of object-level restitution.

The law can restore the object while leaving capability, relation, or trajectory differences unresolved.

27.9 Restitution and Third-Party Structure

Exact return can become difficult where later third-party interests have developed.

Let claimant restorative interest be I_A and third-party interest be I_B . A restitutionary intervention can satisfy

$$\Delta I_A > 0 \quad (2519)$$

while

$$\Delta I_B < 0. \quad (2520)$$

Equations 2519 and 2520 show why restitution can require rules concerning reliance, priority, bona fide third-party interests, or alternative compensation.

The specific legal doctrine varies across fields and jurisdictions.

27.10 Restitution and Fungibility

The stronger the identity-sensitive value of the object, the more attractive specific restitution can become.

Where objects are highly fungible, a substitute can preserve much of the relevant value at lower procedural or administrative cost.

Let substitutability be $\sigma(q)$. A conceptual relation is

$$\sigma(q) \uparrow \implies V_{\text{specific--return}} - V_{\text{substitute}} \downarrow. \quad (2521)$$

Equation 2521 is a stylized value relationship rather than a legal rule.

27.11 Monetary Damages and Compensation

Monetary relief translates legally cognizable loss into a general equivalent.

Let legally assessed loss be L_L . Monetary relief can be represented by

$$M_R = \mathcal{C}_M(L_L), \quad (2522)$$

where $\mathcal{C}_M$ is the legal commensuration function. Equation 2522 makes explicit that a monetary award is a conversion rather than literal reproduction of the lost object.

This legal strategy has exceptional institutional importance because money is fungible, divisible, transferable, administrable, and capable of supporting subject-directed future choice.

27.12 Compensatory Equivalence

A simple compensatory ideal seeks

$$V_M(M_R) \sim_E V_L(L_L), \quad (2523)$$

where V_M and V_L represent the value dimensions being translated. Equation 2523 should not be read as a claim that every loss possesses a natural monetary equivalent.

Law creates an administrable equivalence for a particular remedial purpose.

27.13 Pecuniary Loss

Pecuniary loss is especially amenable to monetary translation where market, contractual, wage, or accounting evidence supplies a common scale.

Let lost financial stream be y_t^L . A time-sensitive representation is

$$L_{\text{pec}} = \sum_{t=t_0}^{t_1} w_t y_t^L, \quad (2524)$$

where w_t contains the legally relevant temporal adjustment. Equation 2524 is illustrative and does not prescribe a particular jurisdiction's damages method.

27.14 Non-Pecuniary Loss

Pain, suffering, dignity, relational harm, loss of reputation, and loss of amenity can resist market-based valuation.

Where law nevertheless awards money, the monetary sum can function as:

- compensation;
- acknowledgment;
- administrable substitution;
- institutional closure.

The legal sum should therefore not automatically be interpreted as proof that the underlying value was fully commensurable with money.

27.15 Monetary Translation Loss

Let original multidimensional loss be

$$\mathbf{L} = (L_1, \dots, L_n). \quad (2525)$$

A monetary award compresses this through

$$M_R = C_M(\mathbf{L}). \quad (2526)$$

Equation 2526 can preserve administrable value while discarding provenance, relational identity, historical timing, or symbolic meaning.

The lost dimensions constitute a legal translation residual.

27.16 General Equivalents and Subject Direction

The fungibility of money can also protect present agency.

The claimant can transform the award into a subject-selected portfolio of conditions:

$$M_R \xrightarrow{\mathcal{K}_A} \{c_1, \dots, c_n\}. \quad (2527)$$

Equation 2527 expresses the epistemic advantage of compensation where the legal institution cannot know the one correct future for the claimant.

This point becomes central to the political economy of general equivalents in Section 28.

27.17 Future Loss

Damages can also seek to compensate future legally recognized consequences.

Let future loss distribution be

$$P(L_{t>t_1}). \quad (2528)$$

Equation 2528 shows that future damages require causal and probabilistic judgments rather than mere observation of completed loss.

The longer the horizon, the greater the potential role of uncertainty, alternative causes, adaptation, and changed environments.

27.18 Loss of Opportunity

A legal system can sometimes recognize loss of an opportunity even when the ultimate outcome was uncertain.

The relevant object is the change in opportunity probability:

$$\Delta p = p_{\text{ref}} - p_H. \quad (2529)$$

Equation 2529 is a natural bridge between legal damages and the reachability analysis developed earlier.

Its doctrinal treatment varies substantially across legal systems and fields.

27.19 Delay and Interest

A monetary remedy paid later is not generally equivalent to the same nominal amount paid earlier.

Let award amount be M and delay be Δt . Temporal adjustment can be represented by

$$M_{\text{adj}} = T_M(M, \Delta t). \quad (2530)$$

Equation 2530 represents the general legal need to account for time without prescribing a particular interest rule.

Delay can also generate non-monetary losses that this adjustment does not capture.

27.20 Liquidated and Standardized Valuation

Legal systems sometimes employ predetermined, scheduled, or standardized values rather than individualized reconstruction.

Let individualized loss be L_i and schedule be S_L . The award is

$$M_i = S_L(c_i), \quad (2531)$$

where c_i denotes the legally relevant claim category. Equation 2531 exchanges precision for administrability, consistency, and speed.

A review or exceptional route can preserve fairness for cases poorly represented by the schedule.

27.21 Specific Performance

Specific performance requires actual performance of an obligation rather than monetary substitution. Analytically, it seeks

$$P_R = P_{\text{owed}}, \quad (2532)$$

where P_{owed} denotes the promised or legally required performance. Equation 2532 gives specific performance considerably greater object fidelity than damages.

Its conceptual importance to the atlas is that law sometimes recognizes money as an inadequate substitute for the legally relevant object.

27.22 Specific Performance and Singularity

Specific performance becomes especially intelligible where the performance or object is difficult to substitute.

Let substitutability be $\sigma(P)$. The comparative advantage of specific performance can increase as

$$\sigma(P) \downarrow. \quad (2533)$$

Equation 2533 is an analytical relation rather than a doctrinal test.

The legal insight is nevertheless important: some interests are protected more adequately through actual performance than through monetary conversion.

27.23 Specific Performance and Personal Agency

Specific relief also has limits.

Compelling performance of personal or relational obligations can implicate:

- autonomy;
- supervision difficulty;
- trust;
- continuing relationships;
- institutional manageability.

The atlas therefore predicts that the legal attractiveness of exact performance declines where fidelity requires continuous coercive governance over personal relations.

27.24 Injunctive Relief

Injunctive relief operates upon conduct rather than merely translating completed loss into money.

Let prohibited or required conduct be a . An injunction can be represented by

$$a_t \in \mathcal{A}_L^{\text{ordered}} \quad t \in [t_0, t_1]. \quad (2534)$$

Equation 2534 makes injunction a constraint upon future trajectories.

This gives injunctive relief a natural connection with prevention, interruption, reachability, and non-repetition.

27.25 Prohibitory Injunctions

A prohibitory injunction constrains a harmful or unlawful action.

Let harmful action set be $\mathcal{A}_H$. The ordered system satisfies

$$a_t \notin \mathcal{A}_H \quad (2535)$$

during the injunction's operative period.

Equation 2535 is naturally preventive or interruptive.

It can preserve generative conditions before full post-harm compensation is determined.

27.26 Mandatory Injunctions

A mandatory injunction requires specified affirmative conduct.

Let required action set be $\mathcal{A}_M$. The legal order requires

$$a_t \in \mathcal{A}_M. \quad (2536)$$

Equation 2536 can support correction, reinstatement, removal of an obstruction, disclosure, or another affirmative remedial act depending upon the legal context.

Mandatory relief can therefore move closer to structural intervention than a simple prohibition.

27.27 Interim Injunctions

Interim relief can preserve the possibility of later effective adjudication.

Let threatened transition be

$$x_t \longrightarrow x_{t+1}^H. \quad (2537)$$

An interim order seeks

$$x_{t+1} \notin \mathcal{X}_{\text{irreparable}}. \quad (2538)$$

Equation 2538 maps naturally onto the earlier theory of interruption and option preservation.

The legal value of interim relief can therefore arise from protecting future remedial possibility rather than resolving the entire dispute immediately.

27.28 Irreparable Harm

The legal concept of inadequacy of monetary relief has an important analogue in the atlas.

Let monetary remedy M leave residual value loss

$$\Delta V_{\text{res}}(M) > \theta_V. \quad (2539)$$

Equation 2539 provides an analytical interpretation of why coercive or specific relief can become preferable where money leaves substantial residual loss.

The exact legal standards for equitable relief remain jurisdiction-specific.

27.29 Injunctions and Generative Conditions

An injunction can protect generativity without itself reconstructing a lost future.

Examples include preventing:

- retaliation;
- destruction of evidence;

- dissipation of property;
- discriminatory exclusion;
- environmental destruction.

The legal order can therefore preserve future possibility while later proceedings determine fuller relief.

27.30 Declaratory Relief

Declaratory relief authoritatively defines legal rights or relations without necessarily ordering payment or conduct.

Let disputed legal status be s_L . Declaration produces

$$s_L \xrightarrow{D_L} s_L^*. \quad (2540)$$

Equation 2540 makes legal classification the direct remedial object.

This can have significant downstream material and relational consequences even without immediate coercive relief.

27.31 Declaratory Relief and Recognition

A declaration can function as legal recognition.

Where the injury includes denial of legal standing, authorship, entitlement, or status, authoritative clarification can possess independent value.

Its significance can exceed the direct economic consequences.

27.32 Declaratory Relief and Prevention

Declaration can also prevent future disputes by stabilizing expectations.

Let uncertainty concerning legal relation be U_L . Declaratory relief can produce

$$U_{L,R} < U_{L,H}. \quad (2541)$$

Equation 2541 shows how a non-coercive legal remedy can alter future institutional behavior.

27.33 Record Correction

Legal and administrative systems can correct records or classifications.

Let erroneous record be Z_E , and corrected record be Z_C . The correction is

$$Z_E \xrightarrow{C} Z_C. \quad (2542)$$

Equation 2542 restores current informational status.

Historical provenance can be preserved separately.

27.34 Correction and Propagation

Where erroneous information propagated across institutions, source correction can be insufficient.

Let record network be $G_Z = (V_Z, E_Z)$. Correction coverage is

$$\kappa_C = \frac{|V_Z^C|}{|V_Z^A|}, \quad (2543)$$

where V_Z^A contains active nodes carrying the relevant record and V_Z^C corrected nodes. Equation 2543 gives a simple distributed measure of correction.

A legally adequate correction can therefore require downstream propagation.

27.35 Vacatur, Reversal, and Supersession

Legal systems can remove or supersede the authority of earlier decisions.

Let earlier decision be D_0 and later authoritative decision D_1 .

A supersession relation is represented by

$$D_0 \prec_{\text{sup}} D_1. \quad (2544)$$

Equation 2544 means the earlier decision remains part of legal history while its operative authority changes.

This structure parallels the historical versioning developed in Section 20.

27.36 Reinstatement

Reinstatement seeks return to a prior legally protected institutional position.

Let position be p . A formal reinstatement criterion is

$$p_R = p_{\text{pre}}. \quad (2545)$$

Equation 2545 is a position-level restoration.

The earlier analysis shows why this can remain weaker than restoration of authority, trust, network position, effective capability, or future career trajectory.

27.37 Effective Reinstatement

A stronger object can be represented by

$$Q_{\text{reentry}} = (p, A, N, S, K), \quad (2546)$$

where A denotes authority, N network access, S security, and K relevant capability.

Equation 2546 shows that formal reinstatement can constitute one component of effective re-entry.

Law can directly control some of these components more readily than others.

27.38 Compensated Exit

Where reinstatement is unwanted, unsafe, or relationally unworkable, law can support an alternative future through monetary, status, or transition remedies.

Let exit package be

$$R_{\text{exit}} = (M, C, Z, S_T), \quad (2547)$$

where M denotes monetary support, C portable credentials or rights, Z record correction, and S_T transition support. Equation 2547 provides a legal analogue of subject-directed generative assistance.

Its legal availability depends upon the relevant field and jurisdiction.

27.39 Rehabilitation

Rehabilitation addresses functional, bodily, psychological, occupational, or social capacities impaired by harm.

Let functioning or capability profile be $\mathbf{K}$. Rehabilitation seeks

$$\mathbf{K}_R \succeq \mathbf{K}_{\min} \quad (2548)$$

or another legally relevant functional standard. Equation 2548 places rehabilitation closer to capability restoration than to simple object return.

International reparations frameworks explicitly recognize rehabilitation as a distinct remedial form [65, 12].

27.40 Rehabilitation and Present Subject

Rehabilitation can become paternalistic if it defines success exclusively through return to prior function.

A subject can now prefer another occupation, environment, or life plan.

A more agency-sensitive legal structure can preserve entitlement to rehabilitation while permitting alternative use of restorative resources where consistent with the governing scheme.

27.41 Satisfaction

Satisfaction addresses dimensions such as acknowledgment, truth, apology, commemoration, or correction of public meaning.

Let recognition object be R_A^{rec} . Satisfaction can seek

$$R_{A,R}^{\text{rec}} \succeq R_{A,\min}^{\text{rec}}. \quad (2549)$$

Equation 2549 treats recognition as a legal or reparative object independent of money.

International reparations practice makes this plurality explicit [65, 12].

27.42 Apology

An apology can alter social meaning without restoring material conditions.

Let institutional acknowledgment state be A_I . An apology can produce

$$A_I^H \xrightarrow{R_{\text{apol}}} A_I^R. \quad (2550)$$

Equation 2550 can have recognition, trust, and historical effects.

Its value depends upon truthfulness, authority, context, and relationship to material repair.

27.43 Legal Acknowledgment and Historical Truth

A legally authoritative acknowledgment adds an event to the actual history.

It therefore increases one form of historical difference from the no-harm reference while potentially increasing justice.

This directly confirms the earlier conclusion that historical fidelity and historical justice can diverge.

27.44 Guarantees of Non-Repetition

Guarantees of non-repetition address the future mechanisms through which similar harms could recur [65, 12].

Let recurrence probability under institutional structure I be

$$p_H(\tau \mid I). \quad (2551)$$

A non-repetition measure seeks

$$p_H(\tau \mid I_R) < p_H(\tau \mid I_H). \quad (2552)$$

Equation 2552 moves law from retrospective correction toward future structural governance.

27.45 Structural Remedies

Structural remedies alter rules, institutions, authority, practices, or infrastructure rather than merely transferring value to one claimant.

Let institutional mechanism be Φ_I . Structural remedy is represented by

$$\Phi_I^H \xrightarrow{R_S} \Phi_I^R. \quad (2553)$$

Equation 2553 changes the mechanism through which future legal states are generated.

This is the legal remedy form closest to causal and generative transformation.

27.46 Structural Remedy Scope

Structural relief can affect persons beyond the original parties.

Let affected system be $\mathcal{A}_S$. The scope of remedy is

$$\text{Scope}(R_S) = \mathcal{A}_S. \quad (2554)$$

Equation 2554 makes the scale of legal intervention an independent object.

As scope expands, institutional competence, democratic legitimacy, third-party participation, and monitoring become more important.

27.47 Structural Remedies and Polycentricity

Structural reform can be polycentric because alteration of one institutional rule affects many actors and functions.

Fuller's analysis of the limits of adjudication is relevant to this institutional problem [22].

A court can identify a legal violation while implementation of structural repair requires administrative expertise, iterative monitoring, or distributed governance.

27.48 Structural Orders and Adaptive Implementation

A structural remedy can require an adaptive plan rather than a one-time command.

Let implementation state be I_t , milestone evidence D_t , and revised plan P_t . An adaptive order can be represented by

$$P_{t+1} = \mathcal{U}(P_t, D_t, I_t). \quad (2555)$$

Equation 2555 captures long-horizon restorative governance.

Such orders create stronger procedural and institutional-competence demands than a one-time monetary judgment.

27.49 Public-Law Remedies

Public-law remedies can address unlawful state action, invalid administrative decisions, rights violations, or institutional failures.

Their restorative objects can include:

- legal status;

- decision validity;
- future exercise of authority;
- access to procedure;
- institutional rules.

The legal system can therefore restore legality without reproducing every counterfactual consequence of the unlawful action.

27.50 Annulment and Remand

One public-law model removes an invalid decision and returns the matter for lawful reconsideration.

Let invalid decision be D_H . Annulment followed by remand can be represented by

$$D_H \xrightarrow{\text{vacate}} \emptyset \xrightarrow{\text{remand}} D_R. \quad (2556)$$

Equation 2556 restores procedural legality while leaving the substantive result of the lawful reconsideration open.

This is a striking example of law restoring a decision process rather than a predetermined outcome.

27.51 Procedural Restoration

Where the original wrong concerns defective procedure, the legal remedy can seek

$$P_R \sim_{E_P} P_{\text{lawful}}. \quad (2557)$$

Equation 2557 compares the new process with the required procedural standard.

The final substantive outcome can legitimately differ from the claimant's preferred result.

27.52 Administrative Correction

Administrative institutions can often correct eligibility, status, benefits, records, or service provision without full judicial proceedings.

This can reduce delay and burden.

Mashaw's work emphasizes that administrative justice should be evaluated through the structure and goals of administrative decision systems rather than by assuming that all legitimate procedure must reproduce a courtroom [33, 34].

Administrative correction therefore occupies an important low-cost region of the legal restorative atlas.

27.53 Administrative Assistance

Some legal institutions provide assistance without determining individual fault.

Let eligibility condition be E_A and support be S_A . Assistance can be represented by

$$E_A \geq \theta_E \implies S_A. \quad (2558)$$

Equation 2558 separates support from corrective liability.

This distinction is important where urgent need exists and causal attribution requires long adjudication.

27.54 Social Protection and Legal Return

Social protection can secure a generative floor even where no particular wrongdoer is legally liable.
The legal object can therefore be

$$C_A \succeq C_{\min} \quad (2559)$$

rather than equivalence with a no-harm counterfactual. Equation 2559 illustrates a sufficiency-based legal model.

This complements rather than replaces corrective law.

27.55 Private-Law and Public-Law Division

A harmed subject can therefore receive different responses from different legal domains.

Private law can address bilateral responsibility.

Public law can constrain institutional authority.

Social law can secure minimum conditions.

Administrative law can correct status and entitlement.

Reparations programs can address collective and historical wrong.

The legal system is therefore normatively and institutionally plural.

27.56 Legal Remedy Bundles

A complex case can require several legal remedies.

Let remedy bundle be

$$\mathcal{B}_L = \{R_{\text{rest}}, R_{\text{dam}}, R_{\text{decl}}, R_{\text{rehab}}, R_{\text{NR}}\}. \quad (2560)$$

Equation 2560 can address object return, residual loss, authoritative status, functional repair, and recurrence.

No one component should be presumed to make the others unnecessary.

27.57 Cumulative Legal Relief

The combined effect of remedies can be represented by

$$R_L^{\text{total}} = R_n \circ \dots \circ R_2 \circ R_1. \quad (2561)$$

Equation 2561 emphasizes that remedy order can matter.

A declaration can precede compensation. An injunction can preserve the status quo before final adjudication. Rehabilitation can continue after a damages judgment.

Legal relief is therefore temporally structured.

27.58 Remedy Sequencing

A general sequence can be

$$R_{\text{interim}} \rightarrow R_{\text{status}} \rightarrow R_{\text{comp}} \rightarrow R_{\text{future}}. \quad (2562)$$

Equation 2562 represents interim protection, status correction, compensation, and future-oriented measures.

The correct sequence varies by case.

27.59 Non-Commutativity of Remedies

Remedy order can be non-commutative:

$$R_2 \circ R_1 \neq R_1 \circ R_2. \quad (2563)$$

Equation 2563 can arise where one remedy changes the conditions under which another operates.

For example, correction of legal status can increase the practical value of later financial compensation.

27.60 Interim and Final Relief

Interim relief and final relief perform different functions.

Let interim relief preserve state within safe region $\mathcal{S}$:

$$x_t \in \mathcal{S} \quad t \in [t_0, t_F]. \quad (2564)$$

Final relief then determines the legally justified post-dispute arrangement.

Equation 2564 illustrates why temporary relief should not be judged by the same equivalence criterion as final remedy.

27.61 Legal Causation

Legal remedies frequently depend upon causal connection between wrong and loss.

Let wrong be W , outcome Y , and legally recognized causal relation $\rightsquigarrow_L$.

The relevant structure is

$$W \rightsquigarrow_L Y. \quad (2565)$$

Equation 2565 should not be equated mechanically with one structural causal-model relation.

Legal causation can incorporate doctrinal judgments concerning scope of liability, attribution, proof, foreseeability, remoteness, and policy.

The formal causal analysis of Section 18 therefore informs but does not replace legal causation.

27.62 Factual Causation and Legal Scope

A causal contribution can be scientifically real and legally outside the scope of compensable responsibility.

Let scientific or factual causal set be

$$\mathcal{C}_F. \quad (2566)$$

Let legally compensable causal set be

$$\mathcal{C}_L \subseteq \mathcal{C}_F. \quad (2567)$$

Equation 2567 represents legal compression of the broader causal field.

The boundary requires legal justification rather than scientific proof alone.

27.63 Remoteness and Recursive Harm

Generative systems complicate legal remoteness because one wrong can initiate long recursive chains:

$$W \rightarrow L_1 \rightarrow L_2 \rightarrow \cdots \rightarrow L_n. \quad (2568)$$

Equation 2568 can extend indefinitely in principle.

Law therefore requires stopping rules concerning which downstream consequences remain attributable for remedial purposes.

This legal necessity should be distinguished from a claim that later losses are causally unreal.

27.64 Legal Truncation

Let full modeled consequence set be $\mathcal{L}_C$, and legally recoverable set be $\mathcal{L}_L$.

Then

$$\mathcal{L}_L \subseteq \mathcal{L}_C. \quad (2569)$$

Equation 2569 expresses a fundamental feature of legal remedy: institutional boundaries can intentionally truncate the causal and historical field.

The omitted residual can remain morally relevant even where it is legally non-recoverable.

27.65 Proof

Legal remedy is bounded by evidence.

Let proposition c concern harm, causation, amount, or entitlement. Legal action requires

$$K(c) \geq \theta_L(c), \quad (2570)$$

where $K(c)$ denotes justified evidentiary support and $\theta_L(c)$ the applicable legal threshold. Equation 2570 connects legal remedy with the procedural architecture developed in Section 24.

Different decisions can legitimately use different proof thresholds.

27.66 Proof and Irreparable Loss

Strict proof requirements can create under-repair where the wrong itself destroyed evidence.

Let evidentiary availability after harm be

$$K_H < K_{\text{pre}}. \quad (2571)$$

Equation 2571 identifies an endogenous evidentiary asymmetry.

Legal presumptions, burden shifting, or evidentiary inference can sometimes respond to such structures.

Their doctrinal availability remains jurisdiction-specific.

27.67 Uncertainty and Standardized Remedies

Where individualized counterfactual reconstruction is costly or unreliable, law can adopt standardized rules.

Let individualized estimate be $\hat{L}_i$, and standardized value be $S(c_i)$.

The institutional trade-off is represented by

$$(\text{Precision, Consistency, Cost}). \quad (2572)$$

Equation 2572 is a multi-dimensional procedural choice.

Standardization can increase equal administration while reducing fit for singular cases.

27.68 Legal Presumptions

A legal presumption can allocate uncertainty by allowing one fact to support another absent sufficient contrary evidence.

Let predicate A trigger presumption concerning B :

$$A \Rightarrow_L B. \quad (2573)$$

Equation 2573 is a procedural rather than causal implication.

Presumptions can reduce impossible evidentiary burdens while creating risk of over-inclusion.

27.69 Legal Fictions and Restorative Administration

Law can also employ simplified constructs that permit decision without claiming complete factual reproduction.

A legally treated equivalence can be represented by

$$Q_1 \equiv_L Q_2. \quad (2574)$$

Equation 2574 denotes equivalence for a specified legal purpose.

It should not be confused with physical, historical, or moral identity.

This distinction is central to the Atlas of Equivalence.

27.70 Legal Equivalence

Legal systems routinely construct purpose-relative equivalences.

Money can be treated as adequate compensation for a legally recognized loss.

A corrected record can be treated as restoring current status.

A declaratory judgment can conclusively establish a legal relation.

These relations are institutionally powerful precisely because law does not need to recreate every dimension of reality in order to govern rights and duties.

27.71 Legal Adequacy

A legal remedy can be adequate for its institutional purpose while leaving broader residual loss.

Let legal remedial standard be θ_L . Adequacy is

$$J_L(R_L) \geq \theta_L. \quad (2575)$$

Equation 2575 is distinct from

$$\Delta_{\text{moral}} = 0. \quad (2576)$$

The first can hold while the second fails.

This is an important reason to use bounded legal conclusions.

27.72 Legal Finality

Law requires finality because disputes, property, institutions, and third parties need stable expectations.

Let final legal decision be D_F .

Finality can be represented by

$$D_F \notin \mathcal{R}_{\text{ordinary}} \quad (2577)$$

after ordinary review mechanisms are exhausted.

Equation 2577 does not imply historical or moral completion.

It terminates a defined legal question.

27.73 Finality and Continuing Assistance

A liability judgment can become final while healthcare, rehabilitation, or social support continues.

This separation is institutionally important.

Legal finality should not force all future-oriented assistance into the temporal structure of adjudication.

27.74 Settlement

Settlement allows parties to resolve a dispute through negotiated agreement within the relevant legal framework.

Let feasible settlement set be

$$S_L = \{R : R \text{ is legally and procedurally admissible}\}. \quad (2578)$$

Equation 2578 can contain remedy bundles unavailable through one standardized judicial order.

Settlement can therefore improve flexibility.

It can also reproduce bargaining asymmetries.

27.75 Settlement and Power Asymmetry

Let bargaining capacities be B_A and B_B . Where

$$B_A \ll B_B, \quad (2579)$$

formal agreement can coexist with severe procedural inequality.

Equation 2579 shows why settlement should not be treated automatically as evidence that the remedy represents equal valuation by the parties.

27.76 Waiver

A claimant can waive some remedial claims.

Waiver becomes procedurally problematic where information, voluntariness, or bargaining power is inadequate.

Let waiver validity be

$$W_A = W(I, C, V, S), \quad (2580)$$

where I denotes information, C capacity, V voluntariness, and S scope clarity. Equation 2580 connects legal closure with procedural justice.

27.77 Release and Future Generative Loss

Broad releases can be difficult where future consequences are uncertain.

A claimant can settle a currently visible loss and later discover generative effects outside the anticipated claim.

A legal system therefore faces a trade-off between finality and unknown future loss.

The relevant allocation of uncertainty should be explicit.

27.78 Continuing Harm

A one-time damages remedy can be poorly matched to continuing harm.

Let harm rate be

$$\frac{dL}{dt} > 0. \quad (2581)$$

Equation 2581 indicates that interruption or injunction can be necessary before final monetary quantification.

Legal remedy therefore has a temporal architecture.

27.79 Recurring Harm

Repeated harm creates another mismatch with one-time remedy.

Let recurrence events be $H_1, \dots, H_n$. The legal system can respond repeatedly to each event or alter the recurrence mechanism.

The structural alternative is represented by

$$\Phi_H \xrightarrow{R_S} \Phi_R, \quad (2582)$$

where Equation 2582 changes the process generating repeated claims.

27.80 Legal Remedy and Prevention

Law can operate before completed harm through injunctions, regulatory duties, protective orders, or other preventive mechanisms.

The relevant object is future risk rather than completed loss.

Let risk be

$$\rho_H = P(H | \mathcal{G}_t). \quad (2583)$$

A preventive legal response seeks to reduce ρ_H subject to evidentiary, rights, and proportionality constraints.

27.81 Legal Remedy and Interruption

During ongoing harm, law can seek cessation.

Let ongoing harmful process be H_t .

Interruption seeks

$$H_{t+1} = 0 \quad (2584)$$

or a legally defined reduction in harmful conduct.

Equation 2584 is analytically prior to compensation for completed effects.

27.82 Legal Remedy and Post-Harm Response

After harm, law can deploy a portfolio including:

- restitution;
- damages;
- rehabilitation;
- declaration;

- satisfaction;
- future safeguards.

The portfolio can therefore correspond to several regions of the atlas rather than one return operation.

27.83 Legal Remedy and Capability

Law can affect capability indirectly through resources, status, accessibility, and enforceable rights.

Let legal condition vector be $\mathbf{L}$. Capability is

$$\mathcal{K}_A = \mathcal{C}(\mathbf{L}, \mathbf{R}, X_A), \quad (2585)$$

where $\mathbf{R}$ denotes other resources and X_A conversion factors. Equation 2585 shows that legal remedy can modify the conversion environment without fully determining capability.

27.84 Legal Remedy and Recognition

Judgments and declarations can change public and institutional recognition.

A legal decision can therefore have symbolic and relational effects in addition to enforceable consequences.

Recognition remains partly outside law because social meaning is distributed across wider institutions and communities.

27.85 Legal Remedy and Non-Domination

Law can reduce domination by converting discretionary dependence into enforceable entitlement.

Let discretionary access be A_D and rights-secured access be A_R .

The transformation is represented by

$$A_D \xrightarrow{L} A_R. \quad (2586)$$

Equation 2586 can preserve the same present resource while changing the power relation substantially.

This gives law a distinctive role in generative security [44].

27.86 Legal Remedy and Exit

Legal remedies can also create portable exit options.

Severance rights, transferable benefits, corrected credentials, or enforceable records can reduce dependence upon the original institution.

The value lies partly in what the claimant can do elsewhere.

27.87 Legal Remedy and Generative Processes

Legal rules help constitute generative conditions.

They structure:

- ownership;
- contract;
- institutional access;
- recognition;

- decision rights;
- value circulation.

A remedy can therefore alter the process through which future value is generated, not merely compensate output already lost.

27.88 Generative Legal Remedy

Let generative process be Γ^G .

A generative legal remedy can seek

$$\Gamma_H^G \xrightarrow{R_L} \Gamma_R^G \quad (2587)$$

with

$$\mathbf{V}_G(\Gamma_R^G) \succeq \mathbf{V}_G(\Gamma_H^G). \quad (2588)$$

Equations 2587 and 2588 define a broad future-generative legal effect.

This should not be interpreted as a claim that every court should manage generativity directly.

27.89 Legal Limits on Generative Governance

Law's capacity to improve generativity is constrained by:

- jurisdiction;
- institutional competence;
- rights;
- proof;
- finality;
- separation of powers;
- third-party interests.

Generative justice can therefore inform remedy design without converting legal institutions into unrestricted planners of future lives.

27.90 Legal Minimalism

A legal institution should generally grant the remedy necessary to vindicate the legally protected interest within its competence.

Let legally necessary remedial set be

$$\mathcal{R}_L^* \subseteq \mathcal{R}_{\text{possible}}. \quad (2589)$$

Equation 2589 represents bounded legal intervention.

Other institutions can address residual health, welfare, social, or generative needs.

27.91 Legal Escalation

Stronger legal remedies become relevant where weaker remedies cannot protect the interest.

A schematic escalation is

$$\text{declaration} \rightarrow \text{damages} \rightarrow \text{injunction} \rightarrow \text{structural relief.} \quad (2590)$$

Equation 2590 is an analytical sequence rather than a doctrinal hierarchy.

The appropriate path depends upon the nature and persistence of the wrong.

27.92 Legal Proportionality

The burden imposed by a remedy should remain proportionate to the protected interest and remedial purpose.

Let claimant remedial gain be G_R and legally relevant burden be B_R .

A proportionality assessment can be represented generically by

$$\mathcal{P}_L = \mathcal{P}(G_R, B_R, A_R), \quad (2591)$$

where A_R denotes availability of less burdensome alternatives. Equation 2591 is a general analytical form rather than a particular jurisdiction's proportionality doctrine.

27.93 Least-Burdensome Effective Remedy

Where two remedies vindicate the legally protected interest comparably, the less burdensome remedy can possess an institutional advantage.

Let

$$J_L(R_1) \sim J_L(R_2) \quad (2592)$$

and

$$B_L(R_1) < B_L(R_2). \quad (2593)$$

Equations 2592 and 2593 formalize the comparison.

This principle connects legal remedy with the procedural constraints developed earlier.

27.94 Enforcement

A judgment without effective implementation can have weak restorative effect.

Let ordered remedy be R_O and implemented remedy be R_I .

Enforcement effectiveness can be represented by

$$E_R = d(R_O, R_I). \quad (2594)$$

Equation 2594 measures the implementation gap under an appropriate distance or comparison relation.

Legal justice therefore extends beyond issuance of the remedy.

27.95 Compliance and Coercion

Some remedies depend upon voluntary compliance; others are backed by coercive enforcement.

The stronger the continuing coercion needed to maintain the remedy, the more the legal order approaches ongoing governance rather than one-time correction.

This transition has implications for institutional competence and procedural review.

27.96 Monitoring

Structural and continuing remedies can require monitoring.

Let compliance state be C_t . Monitoring generates

$$Z_t = O_L(C_t). \quad (2595)$$

Equation 2595 creates information supporting enforcement and revision.

Monitoring also creates privacy, cost, and institutional-duration concerns.

27.97 Sunset and Review

Continuing remedies can include review or expiration conditions.

Let remedy remain operative until τ_R , unless renewed through review:

$$R_L(t) = 0 \quad t > \tau_R \quad (2596)$$

absent a renewal decision.

Equation 2596 provides a mechanism for limiting indefinite legal supervision.

27.98 Adaptive Legal Remedies

Where implementation conditions change, legal remedies can sometimes require adaptive governance.

Let remedy state be R_t . Review produces

$$R_{t+1} = \mathcal{U}(R_t, E_t, C_t), \quad (2597)$$

where E_t is evidence and C_t compliance or context. Equation 2597 increases practical responsiveness.

It also raises concerns about finality and the scope of judicial or administrative authority.

27.99 Legal Reflexivity

Law changes the social system it describes.

A judgment can alter:

- reputation;
- incentives;
- expectations;
- bargaining power;
- institutional policy.

Let judgment be D_L . The post-judgment system is

$$\mathcal{G}_{t+1} = F(\mathcal{G}_t, D_L). \quad (2598)$$

Equation 2598 makes legal decision an endogenous event in the generative system.

27.100 Precedent as Generative Effect

A legal decision can affect later cases beyond the original parties.

Let future legal rule state be Π_{t+1} . Decision D_t can contribute to

$$\Pi_{t+1} = G_L(\Pi_t, D_t). \quad (2599)$$

Equation 2599 makes precedent or institutional learning a generative effect of adjudication.

The exact force of precedent varies across legal systems.

27.101 Settlement and Public Learning

Private settlement can resolve the claimant's case while producing less public legal learning than an authoritative decision.

This creates a potential conflict among:

- claimant privacy;
- efficient resolution;
- public precedent;
- institutional accountability.

The appropriate balance cannot be determined through claimant compensation alone.

27.102 Legal Recognition of Residual Loss

Law can acknowledge that a remedy is incomplete.

Let legal remedy satisfy

$$J_L(R_L) \geq \theta_L \quad (2600)$$

while broader residual loss remains

$$\Delta_{\text{res}} > 0. \quad (2601)$$

Equations 2600 and 2601 allow legal adequacy and moral incompleteness to coexist.

Declaratory or recognition remedies can make that residual visible without expanding liability without limit.

27.103 Legal Non-Compensability

Some losses can be legally acknowledged while remaining only partially compensable or outside the scope of available legal relief.

This distinction should not be expressed as though the unremedied value does not exist.

The atlas therefore distinguishes:

$$\Delta_{\text{legally recoverable}} \quad (2602)$$

from

$$\Delta_{\text{normatively relevant}}. \quad (2603)$$

The two sets can differ.

27.104 Legal Under-Repair

Under-repair occurs where the legally delivered remedy falls below the legally justified remedial standard:

$$J_L(R_L) < \theta_L. \quad (2604)$$

Equation 2604 is a legal failure within the institution's own normative frame.

This should be distinguished from broader moral incompleteness beyond legal competence.

27.105 Legal Overreach

Legal overreach occurs where the institution attempts to govern dimensions beyond its legitimate authority or competence.

Let legal mandate be $\mathcal{M}_L$ and intervention scope $\text{Scope}(R)$.

Overreach can be represented by

$$\text{Scope}(R) \not\subseteq \mathcal{M}_L. \quad (2605)$$

Equation 2605 is especially important for ambitious generative or identity-affecting remedies.

27.106 Legal Remedy and Institutional Competence

Different institutions possess different competencies.

Courts can be strong at:

- authoritative legal classification;
- liability;
- rights;
- enforceable orders.

Administrative agencies can be stronger at:

- expertise;
- ongoing implementation;
- monitoring;
- adaptive support.

Social institutions can be stronger at:

- assistance;
- rehabilitation;
- long-term service delivery.

The legal architecture should therefore exploit division of labor.

27.107 Referral and Legal Handoff

Where residual value lies outside the deciding institution's competence, the response can include referral.

Let unresolved object set be $\mathfrak{O}_{\text{res}}$. A referral map is

$$H_L : \mathfrak{O}_{\text{res}} \longrightarrow \mathfrak{I}_{\text{competent}}. \quad (2606)$$

Equation 2606 avoids both abandonment and institutional overreach.

27.108 Legal Fragmentation

Plural remedies create fragmentation risks.

A claimant can face separate proceedings for:

- compensation;
- employment status;
- benefit eligibility;
- record correction;
- rehabilitation.

Each institution can be legally correct within its mandate while the overall restorative system remains difficult to navigate.

Legal coordination is therefore part of effective justice.

27.109 Procedural Economy

Repeated proof of the same verified fact across proceedings creates avoidable burden.

Where legally permissible, interoperable findings and provenance can reduce duplication.

Such reuse must remain sensitive to differing standards of proof and legal purposes.

A fact established for one institutional purpose should not automatically be treated as conclusive for every other purpose.

27.110 Cross-Institutional Fact Use

Let factual finding be f , originating institution I_1 , and receiving institution I_2 .

Cross-use can be represented by

$$f^{I_1} \xrightarrow{\Gamma_{12}} f^{I_2}, \quad (2607)$$

where Γ_{12} contains rules governing recognition, evidentiary weight, privacy, and contestability.

Equation 2607 allows shared epistemic infrastructure without assuming universal legal equivalence.

27.111 Jurisdictional Variation

Remedial doctrine differs across jurisdictions, legal traditions, causes of action, and institutional levels.

Let jurisdiction be j . Available remedy set is

$$\mathfrak{R}_L^{(j)}. \quad (2608)$$

Equation 2608 prevents the present taxonomy from being mistaken for a statement that every remedy is available everywhere.

The atlas supplies a comparative language in which jurisdictional differences can be studied.

27.112 Comparative Legal Atlas

For jurisdictions $j_1, \dots, j_n$, one can map

$$\mathfrak{A}_L^{(j)} = \{(Q_L, E_L, R_L)\}_j. \quad (2609)$$

Equation 2609 identifies which restorative objects, comparative relations, and remedies each jurisdiction recognizes.

This creates a future comparative-law research program.

27.113 Restitutionary Legal Model

The restitutionary model can be summarized by

$$\mathfrak{M}_{\text{rest}} = (Q_{\text{object/status}}, =, B_{\text{entitlement}}). \quad (2610)$$

Equation 2610 seeks identity or direct return relative to an entitlement baseline.

Its strength is high fidelity.

Its limitations arise from changed context, third-party interests, and irreparability.

27.114 Compensatory Legal Model

The compensatory model is

$$\mathfrak{M}_{\text{comp}} = (Q_{\text{loss}}, \sim_{\text{monetary}}, B_{\text{no-wrong}}). \quad (2611)$$

Equation 2611 translates legally recognized loss into a general equivalent.

Its strength is fungibility.

Its limitation is translation loss.

27.115 Specific-Performance Legal Model

The specific-performance model is

$$\mathfrak{M}_{\text{spec}} = (Q_{\text{performance}}, =, B_{\text{promised}}). \quad (2612)$$

Equation 2612 seeks performance fidelity.

Its strength is preservation of a non-fungible performance interest.

Its limitations include coercive intensity and supervision.

27.116 Injunctive Legal Model

The injunctive model is

$$\mathfrak{M}_{\text{inj}} = (Q_{\text{future conduct}}, \in \mathcal{A}_{\text{ordered}}, B_{\text{lawful}}). \quad (2613)$$

Equation 2613 governs future trajectories.

Its strength lies in prevention and interruption.

Its burdens arise from coercion, monitoring, and effects on third parties.

27.117 Declaratory Legal Model

The declaratory model is

$$\mathfrak{M}_{\text{decl}} = (Q_{\text{legal status}}, =, B_{\text{authoritative}}) . \quad (2614)$$

Equation 2614 restores or clarifies authoritative legal meaning.

Its direct coercive burden can be lower than mandatory relief while its downstream effects can be substantial.

27.118 Rehabilitative Legal Model

The rehabilitative model is

$$\mathfrak{M}_{\text{rehab}} = (Q_{\text{function/capability}}, \succeq_{\text{suff}}, B_{\text{adequacy}}) . \quad (2615)$$

Equation 2615 seeks functional or capability adequacy.

Its central risk is paternalistic fixation upon a historical function the subject no longer chooses.

27.119 Recognition Legal Model

The recognition model is

$$\mathfrak{M}_{\text{rec}} = (Q_{\text{recognition/history}}, \succeq, B_{\text{truthful}}) . \quad (2616)$$

Equation 2616 includes declaration, correction, acknowledgment, and satisfaction.

Its limitation is that symbolic repair cannot substitute automatically for material repair.

27.120 Structural Legal Model

The structural model is

$$\mathfrak{M}_{\text{struct}} = (Q_{\text{institutional mechanism}}, \succeq_V, B_{\text{just}}) . \quad (2617)$$

Equation 2617 seeks transformation of future institutional behavior.

Its challenge lies in institutional competence, polycentricity, implementation, and democratic legitimacy.

27.121 Legal Remedy Matrix

Table 27 maps major legal remedy families to their principal atlas objects and comparative relations.

Table 27 demonstrates that legal remedies instantiate several distinct comparative relations.

27.122 Legal Remedy and Temporal Domain Matrix

Table 28 maps remedy families onto the pre-harm, ongoing-harm, and post-harm temporal architecture.

Table 28 shows that legal remedy is distributed across the harm timeline rather than concentrated in ex post compensation.

27.123 Legal Remedy and Value Matrix

Table 29 illustrates how one remedy can engage several normative values.

Table 29 demonstrates that the legal form of remedy and the moral value it serves should remain analytically distinct.

Table 27: Legal remedy families within the Atlas of Equivalence

Legal remedy	Primary comparison object	Characteristic relation	Principal strength	Characteristic residual
Restitution	Object, status, possession	Identity / return	High object fidelity	Changed context and third-party interests
Compensatory damages	Legally cognizable loss	Monetary equivalence	Fungibility and administrability	Non-fungible value and translation loss
Specific performance	Promised or required performance	Performance identity	Preserves unique performance interest	Coercion and supervision
Prohibitory injunction	Future conduct	Exclusion from prohibited set	Prevention and interruption	Continuing enforcement burden
Mandatory injunction	Future affirmative conduct	Required-action inclusion	Direct correction of continuing wrong	Intrusiveness and institutional supervision
Declaratory relief	Legal status and relation	Authoritative classification	Recognition and expectation stabilization	Limited direct material effect
Rehabilitation	Function and capability	Sufficiency / improvement	Repairs conversion and practical use	Risk of paternalistic target selection
Satisfaction	Recognition and historical meaning	Correction / acknowledgment	Addresses non-material historical injury	Does not replace material repair
Structural relief	Institutional mechanism	Transformation / dominance	Reduces systemic recurrence	Polycentricity and legitimacy
Non-repetition	Future risk	Risk reduction	Future-oriented prevention	Cannot compensate past loss

Table 28: Legal remedy forms across the harm timeline

Temporal domain	Primary legal function	Characteristic remedies	Atlas object
Before harm	Protection and prevention	Preventive injunctions, regulatory duties, safeguards	Risk, viability, protected conditions
During harm	Cessation and preservation	Interim orders, prohibitory and mandatory relief	Trajectory, evidence, continuing loss
After harm	Return and compensation	Restitution, damages, declaration	Object, status, value loss
Post-harm recovery	Function and future opportunity	Rehabilitation, re-entry, assistance	Capability, reachability, conversion
Long-run reform	Non-repetition and institutional transformation	Structural guarantees of non-repetition	Causal mechanism, resilience, generative process

27.124 Legal Remedy Profiles

A legal remedy can be represented by the profile

$$\mathbf{R}_L = (r_{\text{object}}, r_{\text{monetary}}, r_{\text{status}}, r_{\text{capability}}, r_{\text{recognition}}, r_{\text{future}}, r_{\text{structural}}). \quad (2618)$$

Equation 2618 allows the same judgment to be evaluated across several restorative dimensions.

A monetary award can score highly on compensation and weakly on recognition.

A declaration can exhibit the reverse pattern.

27.125 Legal Completeness

A legal remedy is complete only relative to the legal question it resolves.

Let legal claim set be $\mathcal{C}_L^*$. Legal completeness is

$$C_L = 1 \iff \forall c \in \mathcal{C}_L^*, c \text{ has received final legal disposition.} \quad (2619)$$

Equation 2619 does not imply complete substantive, historical, or generative restoration.

This is a critical scope distinction.

Table 29: Legal remedy forms and principal value objects

Remedy form	Direct value object	Potential secondary value	Potential conflict
Restitution	Entitlement and possession	Recognition and control	Third-party reliance
Damages	Compensable value	Agency through fungibility	Commensuration loss
Injunction	Security and lawful conduct	Generative preservation	Freedom and proportionality
Declaration	Status and recognition	Trust and expectation	Publicity and privacy
Rehabilitation	Function and capability	Future generativity	Paternalism
Satisfaction	Recognition and historical truth	Collective memory	Privacy and symbolic substitution
Structural reform	Institutional justice	Non-repetition and collective capability	Institutional competence

27.126 Legal Satisfaction and Moral Residual

It can therefore hold that

$$C_L = 1 \quad (2620)$$

and

$$\Delta_{\text{moral}} > 0. \quad (2621)$$

Equations 2620 and 2621 describe a legally completed case with continuing moral or generative residuals.

A precise legal system should be capable of acknowledging this distinction.

27.127 Legal Remedy and the Atlas Coordinate

The restorative atlas coordinate can now be extended by legal form:

$$\alpha_L = (B, Q, E, \tau, \varepsilon, K, O^V, \mathcal{T}^N, P, R_L, I_L). \quad (2622)$$

Equation 2622 records reference, object, relation, horizon, tolerance, epistemic basis, value object, normative theory, procedure, legal remedy form, and legal institution.

This coordinate allows legal doctrine to be compared with the wider theory without assuming that legal remedy exhausts justice.

27.128 Legal-Remedy Explicitness Principle

The first principle of this section is:

A legal remedy should be understood through the object it acts upon and the comparative relation it attempts to realize rather than through the generic language of “making the claimant whole.”

27.129 Legal-Equivalence Principle

The second principle is:

Legal systems can construct purpose-relative equivalences for institutional decision without thereby establishing physical, historical, relational, or moral identity.

27.130 Restitution Principle

The third principle is:

Direct restitution has its strongest claim where the legally protected object remains identifiable, returnable, and normatively significant in its specific form.

27.131 General-Equivalent Principle

The fourth principle is:

Monetary compensation gains institutional power from fungibility and subject-directed convertibility, while its legal usefulness does not establish complete commensurability of heterogeneous losses.

27.132 Specific-Relief Principle

The fifth principle is:

Specific and coercive relief become especially relevant where monetary substitution leaves substantial residual loss or where continuing conduct threatens future restorative possibility.

27.133 Temporal-Legal Principle

The sixth principle is:

Legal remedy should distinguish prevention, interruption, interim preservation, post-harm correction, rehabilitation, and long-run non-repetition because these functions operate upon different temporal objects.

27.134 Causal-Boundary Principle

The seventh principle is:

Legal responsibility necessarily selects and truncates from a wider causal and historical field, so legal non-recoverability should not automatically be interpreted as causal or moral nonexistence.

27.135 Institutional-Competence Principle

The eighth principle is:

The remedial ambitions of a legal institution should remain proportionate to its authority, expertise, procedural capacity, and ability to manage continuing effects.

27.136 Third-Party Legal Principle

The ninth principle is:

Exact legal return can require modification of later third-party interests, so restorative fidelity must be governed together with present rights and reliance structures.

27.137 Legal Residual Principle

The tenth principle is:

A legally adequate remedy can leave acknowledged substantive, historical, or generative residuals beyond the legal institution's remedial mandate.

27.138 Structural-Learning Principle

The eleventh principle is:

Where verified harms arise recurrently from an institutional mechanism, a legal system concerned only with repeated individual compensation can remain generatively incomplete.

27.139 Bounded-Law Principle

The twelfth principle is:

Law should contribute to just return without claiming exclusive authority over every dimension of the claimant's future generativity, identity, relationships, or moral history.

27.140 Legal Layer of the Atlas

The legal layer can now be represented by

$$\mathfrak{L}_A = (\mathfrak{I}_L, \mathfrak{R}_L, \mathfrak{B}_L, \mathfrak{Q}_L, \mathfrak{E}_L, \mathfrak{P}_L), \quad (2623)$$

where:

- $\mathfrak{I}_L$ contains legally protected interests;
- $\mathfrak{R}_L$ contains remedial forms;
- $\mathfrak{B}_L$ contains legal baselines;
- $\mathfrak{Q}_L$ contains legally operative comparison objects;
- $\mathfrak{E}_L$ contains legal comparative relations;
- $\mathfrak{P}_L$ contains procedural and institutional architectures.

Equation 2623 locates law within the wider Atlas of Equivalence.

27.141 Implications for the Political Economy of General Equivalents

The legal analysis reveals why monetary compensation occupies such a central position in modern remedial systems.

Money can:

- translate heterogeneous claims into an administrable medium;
- decentralize future choice to the claimant;
- support settlement;
- permit institutional closure;
- avoid some intrusive forms of specific reconstruction.

These strengths are substantial.

They also create a political-economic problem. Once money becomes the dominant remedial language, harms that are difficult to monetize can become institutionally less visible, while the capacity to convert money back into generativity depends upon markets, infrastructure, legal status, health, and relational position.

The legal success of the general equivalent therefore does not settle its restorative adequacy.

27.142 Transition to the Political Economy of General Equivalents

Section 28 therefore examines money and other general equivalents as infrastructures of restorative translation. It asks why legal and administrative systems rely so heavily upon fungible equivalents, which epistemic and institutional advantages this reliance creates, how conversion into future generativity depends upon social structure, and where monetization becomes incomplete or distorting.

The central question becomes:

When law converts heterogeneous injury into a general equivalent, what exactly has been returned, what has merely been translated, which values have been compressed, and under which conditions can the equivalent become generative again?

28 Political Economy of General Equivalents

This section examines general equivalents as infrastructures of restorative translation. The preceding legal analysis showed why monetary compensation occupies a central position in contemporary remedial systems. Money can convert heterogeneous claims into an administrable medium, permit decentralized choice, support institutional closure, and avoid some of the coercive difficulties of specific reconstruction. These advantages are substantial. They do not imply that money reproduces every value that has been lost.

The present section therefore analyzes monetary compensation through political economy rather than through valuation alone. The restorative significance of a monetary award depends upon the institutional environment through which nominal money becomes goods, services, security, capabilities, relations, and future generative conditions. Prices, market availability, liquidity, taxation, debt, inflation, bargaining power, legal status, discrimination, infrastructure, and time can all alter the conversion process. Equal nominal amounts can consequently produce unequal restorative effects.

The term *general equivalent* is used here in a broad analytical sense. Marx's analysis of money as the general equivalent within commodity exchange is an important historical source for this vocabulary [32]. Simmel's analysis of money emphasizes its capacity to mediate heterogeneous values and social relations, including its relation to freedom and impersonal exchange [58]. Polanyi's political economy, in turn, provides an important reminder that markets and monetary exchange are institutionally embedded rather than self-sufficient domains [46]. The present argument does not require acceptance of any one complete theory of money, capitalism, or value developed by these authors. Their relevance lies in illuminating the social and institutional conditions under which generalized exchange becomes possible.

The section develops five claims. First, monetary compensation should be understood as a conversion interface rather than as automatic restoration of the underlying value. Second, fungibility creates a genuine epistemic advantage because it allows historically changed subjects to direct resources toward futures institutions cannot fully predict. Third, the capacity to convert money into generativity is socially structured. Fourth, monetization can compress or obscure values whose significance depends upon identity, relation, provenance, or collective governance. Fifth, a just restorative architecture should distinguish monetary return from the broader institutional conditions that make the returned equivalent usable.

28.1 General Equivalents as Translation Infrastructure

Let heterogeneous value loss be represented by

$$\mathbf{L} = (L_1, L_2, \dots, L_n). \quad (2624)$$

Equation 2624 can contain material, functional, relational, temporal, reputational, and generative components.

A general-equivalent system applies a translation map

$$\mathcal{M} : \mathbf{L} \longrightarrow m, \quad (2625)$$

where m denotes an amount denominated in the relevant monetary unit. Equation 2625 does not establish ontological or moral identity between $\mathbf{L}$ and m .

It constructs a transferable institutional representation of loss.

28.2 Translation and Reconversion

The restorative process does not ordinarily end with Equation 2625. The subject must convert the monetary equivalent back into conditions capable of supporting life and future action.

Let the reconversion process be

$$m \xrightarrow{C_{A,E}} \mathbf{C}_A, \quad (2626)$$

where $C_{A,E}$ is the conversion structure associated with subject A in environment E , and $\mathbf{C}_A$ is the resulting condition bundle.

Equation 2626 makes the social environment part of monetary restoration.

The complete monetary pathway is therefore

$$\mathbf{L} \xrightarrow{\mathcal{M}} m \xrightarrow{C_{A,E}} \mathbf{C}_A \xrightarrow{\Gamma_A} \mathcal{K}_A \xrightarrow{\Gamma_G} G_A, \quad (2627)$$

where $\mathcal{K}_A$ denotes substantive capabilities and G_A future generative capacity.

Equation 2627 is the central formal structure of this section.

28.3 Monetary Adequacy

Monetary adequacy should therefore be indexed to the value the award is intended to support.

Let target value object be O^V . Monetary adequacy is represented by

$$\text{Adeq}_M(m; O^V, A, E, \tau). \quad (2628)$$

Equation 2628 shows that the same nominal amount can be adequate for one subject, environment, value object, or temporal horizon and inadequate for another.

28.4 Nominal and Real Monetary Value

A nominal monetary amount is distinct from its purchasing power.

Let nominal award at time t be M_t^N , and relevant price index be P_t . A simple real-value representation is

$$M_t^R = \frac{M_t^N}{P_t}. \quad (2629)$$

Equation 2629 demonstrates why nominal equality across time does not establish real-value equality. Inflation can therefore erode restorative value between injury, judgment, payment, and later use.

28.5 General and Subject-Specific Purchasing Power

A broad consumer-price index can fail to represent the goods relevant to a particular restorative need.

Let subject-relevant basket be

$$\mathcal{B}_A = \{g_1, \dots, g_k\}, \quad (2630)$$

with prices $p_{1,t}, \dots, p_{k,t}$.

A subject-specific purchasing-power index can be represented by

$$P_{A,t} = \sum_{i=1}^k w_{A,i} p_{i,t}, \quad (2631)$$

where $w_{A,i}$ reflects the relevance of each good to the restorative bundle.

Equation 2631 is illustrative.

Its importance lies in showing that the relevant cost structure can differ from the average economy-wide basket.

28.6 Relative-Price Risk

Even stable general inflation can coexist with rapid price increases in the specific goods needed for restoration.

Let restorative good g_i have relative price

$$\pi_{i,t} = \frac{p_{i,t}}{P_t}. \quad (2632)$$

An increase satisfying

$$\pi_{i,t+1} > \pi_{i,t} \quad (2633)$$

reduces the ability of a fixed general monetary award to purchase that restorative condition.

This can be especially important for housing, healthcare, education, legal services, or specialized accessibility support.

28.7 Price-Level Indexation

Where monetary obligations are temporally extended, an institution can index payments to preserve a specified purchasing-power relation.

Let base amount be M_0 . An indexed payment is represented by

$$M_t = M_0 \frac{P_t}{P_0}. \quad (2634)$$

Equation 2634 preserves general purchasing power under the selected index.

It does not guarantee preservation of subject-specific restorative purchasing power.

28.8 Money and Fungibility

The central advantage of money is fungibility.

Let the market-accessible condition set purchasable with m be

$$\mathcal{C}_M(m, E) = \{c : p_E(c) \leq m\}. \quad (2635)$$

Equation 2635 makes a monetary award an option set rather than one predetermined restorative object.

The subject can allocate the same award differently according to present priorities.

28.9 Fungibility as Epistemic Humility

Suppose an institution faces uncertainty over which future the subject should pursue.

Let possible subject-directed bundles be

$$\mathfrak{C}_A = \{\mathbf{c}^1, \mathbf{c}^2, \dots, \mathbf{c}^n\}. \quad (2636)$$

A sufficiently fungible award can preserve access to several members of $\mathfrak{C}_A$ without requiring the institution to choose one.

This provides a form of epistemic humility.

Money can decentralize normative and practical choice back to the historical subject.

28.10 Fungibility and the Right to a Different Future

The same property supports the right to a different future.

A person harmed in one profession can use compensation for:

- return to the profession;
- transition to another profession;
- education;
- relocation;
- health;
- entrepreneurial activity;
- temporary rest.

A specific remedy designed exclusively around the counterfactual career can provide greater reference fidelity and less future choice.

General equivalents can therefore support lower-fidelity, higher-agency restoration.

28.11 Fungibility and Institutional Closure

Money also permits institutions to convert uncertain future responsibilities into a bounded present obligation.

Let future obligation stream be $L_{t:T}$. A capitalized award is

$$M_0 = \mathcal{PV}(L_{t:T}), \quad (2637)$$

where $\mathcal{PV}$ is the institutionally selected present-value operator.

Equation 2637 facilitates legal and administrative closure.

It transfers future conversion risk partly or wholly to the recipient.

28.12 Conversion Risk

Conversion risk concerns uncertainty over whether money will remain capable of producing the intended condition.

Let target condition be c^* . The conversion probability is

$$p_C = P(c^* \in \mathcal{C}_M(m, E_{t+\tau})). \quad (2638)$$

Equation 2638 depends upon future prices, availability, regulation, health, infrastructure, and other environmental conditions.

A lump-sum award can therefore transfer significant future conversion risk to the claimant.

28.13 Market Availability

Money purchases only goods or services that can be accessed through some exchange or provision mechanism.

Let market or service availability indicator be

$$A_M(c, E) \in \{0, 1\}. \quad (2639)$$

Even where

$$m \geq p(c), \quad (2640)$$

condition c remains unavailable through monetary conversion if

$$A_M(c, E) = 0. \quad (2641)$$

Equations 2640 and 2641 distinguish affordability from availability.

28.14 Market Completeness

Let restorative condition family be $\mathfrak{C}^*$.

A fully complete restorative market would satisfy

$$A_M(c, E) = 1 \quad \forall c \in \mathfrak{C}^*. \quad (2642)$$

Equation 2642 is an idealization.

Actual restorative markets are incomplete.

Some goods are legally non-tradable, relationally constituted, historically singular, publicly supplied, technologically unavailable, or dependent upon the consent of another person.

28.15 Non-Market Goods

Important restorative objects can lie partly outside ordinary markets.

Examples include:

- legal status;
- public recognition;
- friendship;
- authorship;
- institutional trust;
- voting rights;
- ecological history.

Money can support conditions surrounding these objects.

It does not purchase the object itself in the same institutional sense.

Polanyi's analysis of the institutional embedding of markets is especially relevant to the limits of treating social life as though every significant condition were reducible to market exchange [46].

28.16 Market Access

Market availability at the system level does not imply market access for the subject.

Let effective market-access set be

$$\mathcal{M}_A = \{m_i : A \text{ can effectively transact in market } m_i\}. \quad (2643)$$

Equation 2643 can depend upon:

- legal status;
- geography;
- disability;
- information;
- digital access;
- discrimination;
- documentation.

Money held by a subject with restricted access can therefore have lower restorative conversion value.

28.17 Legal Status and Monetary Convertibility

Let legal status be s_L . Conversion function can be written as

$$\mathcal{C}_{A,E} = \mathcal{C}(m, s_L, X_A, E). \quad (2644)$$

Equation 2644 makes legal status a multiplier or constraint on monetary usefulness.

An amount sufficient for education can remain ineffective where the subject lacks legal eligibility for enrollment.

28.18 Information and Search Costs

Monetary conversion also requires information.

Let gross award be m and search, advice, and administrative costs be C_S . Effective deployable value is

$$m_{\text{eff}} = m - C_S. \quad (2645)$$

Equation 2645 shows why an administratively complex environment can reduce the practical value of compensation.

Information assistance can therefore increase restorative value without increasing the nominal award.

28.19 Transaction Costs

More generally, let transaction costs be C_T . Net purchasing capacity is

$$m_{\text{net}} = m - C_T. \quad (2646)$$

Transaction costs can include:

- legal fees;
- brokerage;

- relocation costs;
- application charges;
- time;
- documentation.

The same award can therefore create different effective value across institutional environments.

28.20 Liquidity

Money is unusually liquid compared with many specific assets.

Let liquidity of asset a be represented by

$$\lambda(a) = \lambda(T_a, C_a, S_a), \quad (2647)$$

where T_a denotes conversion time, C_a conversion cost, and S_a certainty of realizable value.

Equation 2647 is conceptual.

High liquidity can be valuable after harm because needs and priorities remain uncertain.

28.21 Liquidity and Emergency Generativity

A liquid award can prevent secondary harm by allowing rapid payment for housing, transport, healthcare, or basic needs.

Let emergency expenditure need be e_t . A liquidity constraint exists where

$$m_t^{\text{liquid}} < e_t. \quad (2648)$$

Equation 2648 can generate downstream harm even where the subject possesses illiquid wealth.

Liquidity itself can therefore be a restorative condition.

28.22 Stock and Flow

A lump-sum asset and a stable income flow are distinct economic objects.

Let wealth stock be W_t and income flow be Y_t .

A restorative profile is

$$Q_M = (W_t, Y_t). \quad (2649)$$

Equation 2649 shows why equal present wealth need not reproduce lost long-term income security.

28.23 Lump-Sum and Periodic Compensation

A lump-sum award can be represented by

$$M^L = M_0, \quad (2650)$$

while periodic support is

$$\mathbf{M}^P = (M_1, \dots, M_T). \quad (2651)$$

Equations 2650 and 2651 generate different distributions of liquidity, investment opportunity, risk, and institutional dependence.

Neither form is universally superior.

28.24 Lump-Sum Advantages

Lump-sum compensation can support:

- autonomy;
- major transition expenditure;
- investment;
- institutional closure;
- reduced continuing dependency.

These benefits can be especially important where the claimant seeks a different future.

28.25 Periodic-Payment Advantages

Periodic compensation can support:

- stable income;
- inflation adjustment;
- risk sharing;
- adaptation to changing needs.

Its disadvantages can include continuing institutional dependence and reduced portability. The appropriate form therefore depends upon the value object.

28.26 Annuity and Dependency

A recurring payment can stabilize the subject while preserving a continuing power relation.

Let payment depend upon administrator I :

$$Y_{A,t} = F(D_{I,t}), \quad (2652)$$

where $D_{I,t}$ is the institution's recurring decision.

Equation 2652 creates potential non-domination concerns if continuation is discretionary.

Legally secured entitlement changes the structure significantly.

28.27 Entitlement Security

Let secure payment right be R_Y .

A secured flow can satisfy

$$Y_{A,t} = F(R_Y) \quad (2653)$$

rather than depending upon discretionary favor.

Equation 2653 illustrates how legal form changes the political economy of the same monetary amount [44].

28.28 Debt

Existing debt can absorb compensation before it creates new capability.

Let gross award be M and enforceable debt claims be D . Available restorative amount is

$$M_A = M - D_{\text{paid}}. \quad (2654)$$

Equation 2654 shows why equal awards can create different practical effects across balance sheets.

Debt can therefore function as a prior claim on restorative resources.

28.29 Debt Overhang and Generativity

Where debt remains large after compensation, future generated income can continue flowing toward creditors.

Let generated surplus be V_{t+1} . Subject-retained value is

$$V_{A,t+1}^{\text{ret}} = V_{t+1} - D_{t+1}^{\text{service}}. \quad (2655)$$

Equation 2655 connects debt structure with generative value circulation.

A one-time award can therefore restore a balance-sheet position weakly while leaving recursive extraction unchanged.

28.30 Taxation and Net Compensation

Tax treatment can affect effective award value.

Let gross award be M_G , taxes associated with the award or its use be T_M , and net monetary resource be

$$M_N = M_G - T_M. \quad (2656)$$

Equation 2656 is jurisdiction-dependent in practice.

The general analytical point is that gross legal award and usable economic resource are distinct objects.

28.31 Benefit Interactions

A monetary award can also alter eligibility for other public benefits.

Let existing benefit be B_A and post-award benefit be $B_A(M)$.

Net resource gain is

$$\Delta R_A = M + B_A(M) - B_A(0). \quad (2657)$$

Equation 2657 shows that nominal compensation can partly displace other support.

Legal and welfare coordination can therefore materially affect restorative value.

28.32 Means Testing and Restorative Paradox

Where compensation reduces eligibility for means-tested support, the claimant can receive a large nominal award and a much smaller net improvement.

The issue is especially important when compensation addresses historical loss while the benefits address present sufficiency.

Conflating the two purposes can cause one remedy to cancel another institutionally.

28.33 Wealth Effects

Compensation can alter future choices through wealth effects.

Let choice set before compensation be $\mathcal{A}_0$, and after compensation be $\mathcal{A}_M$.

A useful monetary remedy can satisfy

$$\mathcal{A}_0 \subset \mathcal{A}_M. \quad (2658)$$

Equation 2658 gives a capability-like interpretation of monetary restoration.

The additional options can include exit from dominating relations.

28.34 Reservation Options and Bargaining Power

Money can alter bargaining power by improving the subject's outside option.

Let outside-option value be O_A .

Compensation can generate

$$O_A(M) > O_A(0). \quad (2659)$$

Equation 2659 can reduce dependence upon an employer, family member, landlord, or institution.

Monetary compensation can therefore have a non-domination effect beyond consumption value.

28.35 Bargaining Power and Price

Prices faced by the claimant can also depend upon bargaining structure.

Let effective transaction price be

$$p_A(c) = p(c, b_A, b_S, I), \quad (2660)$$

where b_A and b_S are bargaining positions of buyer and seller and I institutional conditions.

Equation 2660 reminds the analysis that “the market price” can fail to describe every subject's actual conversion cost.

28.36 Market Power

Where a restorative good is supplied by a monopolistic or highly concentrated provider, increased compensation can partly increase the price captured by the provider.

Let supplier price response to subsidy or compensation M be

$$\frac{\partial p}{\partial M} > 0. \quad (2661)$$

Equation 2661 is a stylized possibility.

It shows why increasing purchasing power does not always translate one-for-one into increased real access.

28.37 Compensation Capture

More generally, part of the restorative transfer can be captured by:

- creditors;
- landlords;
- monopolistic suppliers;
- intermediaries;

- administrative charges.

Let claimant-retained restorative value be

$$M_{\text{ret}} = M - M_{\text{captured}}. \quad (2662)$$

Equation 2662 is a political-economic analogue of value-circulation analysis.

28.38 Compensation and Value Circulation

The destination of compensation after payment matters.

Let the allocation vector be

$$\lambda_M = (\lambda_A, \lambda_D, \lambda_H, \lambda_I, \lambda_O), \quad (2663)$$

where the components can represent value retained by the claimant, debt repayment, housing, intermediaries, and other destinations.

Equation 2663 makes post-payment circulation visible.

A legal transfer to the claimant's account is therefore the beginning of one economic process rather than necessarily the end of restoration.

28.39 Generative Return from Compensation

A portion of compensation can become future generative condition.

Let reinvested or converted amount be M_G . Future condition is

$$C_{t+1}^G = \Psi(M_G, E, A). \quad (2664)$$

Equation 2664 can represent education, health, housing stability, equipment, business formation, or other generative uses.

The relationship between money and generativity is therefore mediated rather than automatic.

28.40 Consumption and Generativity

A distinction between consumption and generative use should not be interpreted normatively as though consumption were inferior.

Consumption can itself sustain bodily, psychological, and social conditions required for future generation.

Let consumption C_t contribute to future capacity G_{t+1} :

$$G_{t+1} = F(G_t, C_t). \quad (2665)$$

Equation 2665 shows why rent, food, leisure, or rest can be generative expenditures.

28.41 Time Purchase

Money can effectively purchase discretionary time by reducing the need for immediate labor or administrative effort.

Let available discretionary time be T_A . A transfer can produce

$$T_A(M) > T_A(0). \quad (2666)$$

Equation 2666 gives monetary compensation a temporal generative dimension.

Time can support healing, education, caregiving, research, or reflection.

28.42 Time and Historical Non-Fungibility

Money can create future discretionary time and cannot recreate elapsed time.

Let lost period be $[t_0, t_1]$. Later compensated time $[t_2, t_3]$ can satisfy

$$t_3 - t_2 = t_1 - t_0 \quad (2667)$$

while

$$V([t_2, t_3]) \neq V([t_0, t_1]). \quad (2668)$$

Equations 2667 and 2668 show why time has both fungible and historical dimensions.

28.43 Human Capital and Compensation

A monetary award can fund education, training, or health that increases future earning or productive capacity.

Let human-capability stock be H_A . Investment I_H produces

$$H_{A,t+1} = G_H(H_{A,t}, I_H). \quad (2669)$$

Equation 2669 gives money an indirect future-return role.

This should not reduce the person's value to future earnings.

28.44 Network Capital and Monetary Limits

Money can support participation in environments where relationships form.

It can fund:

- travel;
- conference participation;
- relocation;
- membership;
- collaborative projects.

It cannot guarantee the formation of a particular consensual relationship.

Thus

$$M \rightarrow P(r_{AB}) \quad (2670)$$

can influence the probability of relational formation without purchasing r_{AB} itself.

Equation 2670 illustrates a central limit of monetary substitution.

28.45 Reputation and Monetary Limits

Money can support public communication, legal action, or professional rebuilding.

It cannot by itself authoritatively correct a false institutional record.

A reputational remedy can therefore require

$$M + Z_C + R_{\text{rec}}, \quad (2671)$$

where Z_C denotes record correction and R_{rec} recognition.

Equation 2671 is a conceptual bundle rather than an additive valuation.

28.46 Health and Monetary Limits

Money can purchase healthcare where treatment exists and is accessible.

Let treatment set be $\mathcal{T}_H$. A health-restorative condition requires

$$\mathcal{T}_H \neq \emptyset \quad (2672)$$

and

$$\exists t \in \mathcal{T}_H : p(t) \leq M_A. \quad (2673)$$

Equations 2672 and 2673 distinguish technological availability from affordability.

Unlimited money cannot purchase treatment that does not exist.

28.47 Public Infrastructure

Many conversions require public infrastructure.

Let public condition vector be

$$\mathbf{G}_P = (G_{\text{transport}}, G_{\text{health}}, G_{\text{education}}, G_{\text{digital}}, G_{\text{legal}}). \quad (2674)$$

The conversion function becomes

$$\mathcal{C}_A = \mathcal{C}(M, X_A, \mathbf{G}_P). \quad (2675)$$

Equation 2675 shows why individual compensation can be less effective where common infrastructure is weak.

28.48 Private Compensation and Public Conditions

Two claimants receiving identical money can achieve different restorative results because they live within different institutional environments.

Let

$$M_A = M_B \quad (2676)$$

and

$$\mathbf{G}_P^A \neq \mathbf{G}_P^B. \quad (2677)$$

Then it can follow that

$$\mathcal{K}_A(M_A) \neq \mathcal{K}_B(M_B). \quad (2678)$$

Equations 2676 through 2678 provide a political-economic extension of the capability conversion problem [55, 50].

28.49 Compensation and Social Insurance

Individual compensation and social insurance allocate risk differently.

Compensation seeks to respond to an identified loss.

Insurance pools risk across a population.

Let individual uncertain future loss be L_i . A risk pool transforms

$$\{L_1, \dots, L_n\} \xrightarrow{\text{pool}} \mathbf{P}. \quad (2679)$$

Equation 2679 can support resilience where individual counterfactual loss is difficult to predict.

This does not determine who should bear the ultimate funding burden.

28.50 Insurance and Reparability

Insurance is especially useful for losses that are uncertain in incidence and reasonably substitutable by resources.

Its power is weaker where:

- loss is singular;
- repair technology does not exist;
- harm concerns public recognition;
- responsibility requires acknowledgment.

Insurance therefore complements rather than replaces corrective and reparative institutions.

28.51 Public Compensation Funds

A public fund can provide rapid support where individualized liability is slow or fragmented.

Let claimant eligibility be E_i . Payment rule can be

$$E_i \geq \theta_E \implies M_i. \quad (2680)$$

Equation 2680 can prioritize assistance, standardization, or risk pooling.

Later liability recovery can remain institutionally separate.

28.52 Monetary Floors

A restorative or social-protection system can guarantee a minimum economic condition:

$$M_{A,t} \geq M_{\min,t}. \quad (2681)$$

Equation 2681 is a sufficiency standard rather than a historical-equivalence standard.

It can prevent secondary vulnerability during legal or administrative delay.

28.53 Monetary Floors and Generative Floors

A monetary floor becomes a generative floor only where it can be converted into the relevant basic conditions.

Let basic condition set be $\mathcal{C}_{\text{basic}}$. A stronger standard is

$$\mathcal{C}_{\text{basic}} \subseteq \mathcal{C}_M(M_{\min}, E). \quad (2682)$$

Equation 2682 connects monetary sufficiency to real institutional conditions.

28.54 In-Kind Provision

Specific goods or services can be provided directly rather than through money.

Let in-kind remedy be K_g .

A direct-provision relation is

$$K_g \longrightarrow c^*. \quad (2683)$$

Equation 2683 can reduce conversion risk when the required condition is clear.

It can also reduce choice and create paternalistic allocation.

28.55 Money and In-Kind Bundles

A layered architecture can combine

$$R = M + K_g, \quad (2684)$$

where Equation 2684 denotes a conceptual bundle of fungible resources and specific enabling conditions.

This combination can preserve subject choice while securing non-market or highly specialized needs.

28.56 Vouchers and Restricted Equivalents

Between unrestricted money and direct provision lies the restricted equivalent.

Let voucher V be redeemable over subset $\mathcal{C}_V$:

$$\mathcal{C}_V \subset \mathcal{C}_M. \quad (2685)$$

Equation 2685 reduces fungibility in exchange for greater targeting.

The justice of the restriction depends upon whether the reduced option set is justified by the remedial purpose.

28.57 Restricted Equivalents and Paternalism

A voucher can increase the probability that resources are used for one institutionally selected purpose.

It can also communicate that the claimant is not trusted to direct the remedy.

The value of targeting should therefore be weighed against agency and administrative burden.

28.58 Conditional Cash Transfers

Conditional payment structures link money to behavior or status.

Let payment be

$$M = M_0 \mathbf{1} [C_A = 1], \quad (2686)$$

where C_A denotes satisfaction of the specified condition.

Equation 2686 illustrates how monetary support can become an instrument of behavioral governance.

Where the condition is unrelated to the underlying restorative claim, coercion or domination concerns can arise.

28.59 Unconditional Compensation

Unconditional compensation can better preserve subject control.

Its principal institutional cost is reduced control over how the resources affect the intended restorative object.

This trade-off should be explicit rather than resolved by an assumption that either autonomy or targeting always dominates.

28.60 Currency

Monetary claims are denominated in currencies whose purchasing power and convertibility can vary.

Let award be denominated in currency c_1 , while restorative expenditure occurs in c_2 .

The converted amount is

$$M_t^{c_2} = e_t^{c_1/c_2} M_t^{c_1}, \quad (2687)$$

where $e_t^{c_1/c_2}$ is the applicable exchange rate.

Equation 2687 introduces exchange-rate risk into cross-border or internationally mobile restoration.

28.61 Currency Risk

If the restorative horizon is long, future converted purchasing power is

$$M_{t+\tau}^{c_2} = e_{t+\tau}^{c_1/c_2} M^{c_1}. \quad (2688)$$

Equation 2688 can differ materially from the value expected at settlement time.

Currency denomination is therefore part of restorative design where the claimant's future is cross-border.

28.62 Portability

Money can be more portable than institution-specific benefits.

Let portability of restorative resource r across environments be

$$P(r) = P(r; E_1, E_2). \quad (2689)$$

General money can have high $P(r)$ compared with a non-transferable service or benefit.

This portability can support exit and geographical mobility.

28.63 Financial Infrastructure

Monetary value requires infrastructure for storage, transfer, payment, and conversion.

Let financial access be F_A . Effective monetary capability is

$$M_A^{\text{eff}} = F(M, F_A). \quad (2690)$$

Equation 2690 can be affected by access to banking, payment systems, identity documentation, or financial services.

A monetary award can therefore be formally delivered and practically difficult to use.

28.64 Financial Intermediation

Future-oriented compensation often passes through financial intermediaries.

Investment can preserve or increase value and introduce market risk.

Let invested compensation evolve according to

$$M_{t+1} = M_t (1 + r_{t+1}), \quad (2691)$$

where r_{t+1} is the realized return.

Equation 2691 shows that long-term monetary restoration can transfer financial risk to the claimant.

28.65 Risk-Bearing Capacity

Equal capital awards need not provide equal security where subjects have different ability to bear investment risk.

Let risk-bearing capacity be B_A^R . A restorative portfolio should therefore be evaluated through

$$\text{Risk}(M, B_A^R, \tau). \quad (2692)$$

Equation 2692 can justify different payment structures without implying unequal moral worth.

28.66 Uncertainty Allocation

A lump-sum award often moves several uncertainties from institution to claimant:

- longevity;
- inflation;
- treatment cost;
- investment return;
- future labor-market conditions.

The nominal amount should therefore be interpreted together with the risk allocation it creates.

28.67 Risk Sharing

A continuing or indexed payment can retain some risk within the responsible or public institution.

Let claimant share of risk be ρ_A , and institutional share ρ_I , with

$$\rho_A + \rho_I = 1. \quad (2693)$$

Equation 2693 gives a simple representation of how different compensation architectures distribute uncertainty.

28.68 Monetary Value and Social Meaning

Money also carries social meaning.

Simmel's analysis emphasizes that money transforms relations by mediating exchange through an abstract common form [58]. Anderson's pluralist analysis likewise cautions against treating market valuation as the appropriate mode for every kind of good [1].

A payment can be interpreted as:

- compensation;
- acknowledgment;
- purchase;
- charity;
- settlement;
- institutional exit.

The same amount can therefore possess different relational significance.

28.69 Meaning-Sensitive Compensation

Let monetary amount be m and institutional meaning be s_M .

The restorative value can be represented by

$$V_M = V(m, s_M). \quad (2694)$$

Equation 2694 allows identical nominal transfers to have different recognition effects. This is why payment and acknowledgment can be jointly necessary.

28.70 Payment without Recognition

A payment can improve material conditions while communicating denial of responsibility.

Let material value satisfy

$$\Delta V_{\text{material}} > 0 \quad (2695)$$

while recognition value satisfies

$$\Delta V_{\text{recognition}} \leq 0. \quad (2696)$$

Equations 2695 and 2696 describe one form of partial restoration.

28.71 Recognition without Money

The converse can also occur.

Public acknowledgment can improve recognition while leaving material vulnerability unchanged.

The political economy of repair therefore requires separate treatment of symbolic and material channels.

28.72 Monetization

Monetization occurs when a heterogeneous value object is translated into a monetary representation for decision.

Let value object be O^V . Monetization is

$$\mu : O^V \longrightarrow m. \quad (2697)$$

Equation 2697 can be administratively necessary.

The resulting number should remain identified as a representation for a particular institutional purpose.

28.73 Monetization and Ontological Compression

A monetary representation can compress distinctions among:

- identity;
- relation;
- provenance;
- timing;
- control;
- recognition.

Let original object structure be Q and monetary representation $\mu(Q)$. Compression is represented by

$$Q_1 \neq Q_2 \quad \text{and} \quad \mu(Q_1) = \mu(Q_2). \quad (2698)$$

Equation 2698 shows how distinct losses can become administratively equivalent after monetization.

28.74 Monetary Equivalence Classes

The monetization map induces classes

$$[Q]_M = \{Q' : \mu(Q') = \mu(Q)\}. \quad (2699)$$

Equation 2699 contains objects treated as equivalent by the monetary representation.

Members can remain historically, relationally, or generatively non-equivalent.

This is a particularly clear example of why the Atlas of Equivalence is needed.

28.75 Commensuration Power

The authority to define monetary equivalence is a form of institutional power.

Let valuation institution be I_V . It selects map

$$\mu_{I_V} : O^V \longrightarrow m. \quad (2700)$$

Equation 2700 determines which losses become legible in monetary terms and how large they appear.

The politics of compensation therefore includes the politics of valuation.

28.76 Valuation Expertise

Experts can provide information concerning wages, markets, treatment costs, property values, and expected loss.

Expertise does not independently determine the correct normative object.

A highly precise estimate of future earnings remains an estimate of earnings, not of every lost capability or relationship.

Procedural systems should therefore distinguish valuation expertise from value authority.

28.77 Market Price and Normative Value

Let market price be $p(q)$ and normative restorative value be $V(q)$.

In general,

$$p(q) \neq V(q). \quad (2701)$$

Equation 2701 can arise because market price reflects scarcity, bargaining, institutional rules, income distribution, and effective demand.

It need not measure moral significance.

28.78 Market Price and Replacement Cost

Market price can nevertheless provide useful information where the legally or practically relevant object is replaceable through purchase.

Let replacement object be q' . If

$$q' \sim_E q \quad (2702)$$

for the legally relevant relation, then

$$p(q') \quad (2703)$$

can provide a defensible restorative measure.

The adequacy depends upon the relation E , not upon price alone.

28.79 Willingness to Pay and Distribution

Monetary valuations derived from willingness or ability to pay can depend upon existing wealth.

Let monetary willingness to pay be

$$WTP_A = WTP(V_A, Y_A), \quad (2704)$$

where Y_A denotes available resources.

Equation 2704 shows that monetary expression of preference can depend upon budget constraint.

Observed market valuation should therefore not be treated automatically as an interpersonally neutral measure of welfare.

28.80 Ability to Pay and Restorative Access

The same structure operates after compensation.

If critical restorative good c has price $p(c)$, access requires

$$M_A \geq p(c). \quad (2705)$$

When prices exceed the award, the market formally offers the good and the claimant lacks effective access.

The remedy then fails at the conversion stage rather than at legal transfer.

28.81 Price Endogeneity

Large reparative or compensatory programs can themselves affect prices.

Let aggregate program transfer be M^Σ . Relevant price can satisfy

$$p = p(M^\Sigma, S, D, I), \quad (2706)$$

where S and D denote supply and demand conditions and I institutions.

Equation 2706 shows why scaling an individually adequate remedy to a large population can change its real effects.

28.82 Large-Scale Reparations and Supply Constraints

Suppose a program transfers sufficient nominal funds for housing to many claimants while housing supply is highly constrained.

Aggregate purchasing power can rise faster than available housing.

The result can include price increases and transfer of part of the reparative value to incumbent asset owners.

Collective reparations therefore require supply-side and infrastructure analysis in addition to nominal transfer design.

28.83 Supply-Side Restorative Policy

A restorative program can combine individual purchasing power with expansion of the relevant condition supply.

Let transfer be M and supply intervention be S_c . The combined architecture is

$$R = (M, S_c). \quad (2707)$$

Equation 2707 can improve real access where markets are capacity constrained.

This is one bridge from compensation toward generative assistance.

28.84 Collective Infrastructure and Reparations

Collective reparations can direct resources toward infrastructure rather than individual payments alone.
Let collective infrastructure be G_C . Future individual capabilities can satisfy

$$\mathcal{K}_{i,t+1} = F(M_i, G_C). \quad (2708)$$

Equation 2708 demonstrates complementarity between personal resources and common conditions.
The allocation between individual and collective forms is a normative and governance question.

28.85 Individual Transfer and Collective Control

A community can receive large monetary transfers while lacking governance over the institutions controlling future productive conditions.

Generative justice therefore asks whether compensation changes control as well as resource quantity [15, 17].

Let collective control be χ_C . A stronger restorative profile is

$$Q_C = (M_C, \chi_C, \Gamma_C^G). \quad (2709)$$

Equation 2709 separates money, control, and generative process.

28.86 Capital and Generative Conditions

Money can become capital when deployed through institutional arrangements that generate future value.
Let investment I_t produce

$$V_{t+1} = G_V(I_t, L_t, C_t, E_t), \quad (2710)$$

where L_t can include labor and C_t other productive conditions.

Equation 2710 makes future value dependent upon a relational production structure rather than money alone.

Marx's analysis of capital is relevant precisely because monetary value in a capitalist economy is embedded within relations of production and accumulation, rather than functioning merely as a neutral stock of purchasing power [32].

28.87 Capitalization of Reparative Resources

A restorative transfer can be designed to generate recurring future resources.

Let reparative fund be F_R . Its evolution can be represented by

$$F_{R,t+1} = F_{R,t} + r_t F_{R,t} - C_t, \quad (2711)$$

where r_t is return and C_t disbursement.

Equation 2711 can support long-term institutions or collective capacity.

Investment governance then becomes part of reparative justice.

28.88 Ownership of Reparative Capital

A fund nominally dedicated to an affected community can remain externally controlled.

Let formal beneficiary be B_C and control rights be χ_F .

A politically significant distinction is

$$B_C \neq \chi_F. \quad (2712)$$

Equation 2712 indicates separation between benefit and governance.

Generative justice gives this difference independent significance.

28.89 Value Circulation

A reparative economy can be analyzed through the cycle

$$M_R \rightarrow I \rightarrow V \rightarrow \Lambda(V) \rightarrow C_{t+1}^G. \quad (2713)$$

Equation 2713 represents transfer, investment, generated value, circulation, and future condition.

A program that transfers money once and permits all later value to leave the affected system can have a different generative profile from one that sustains recursive local or subject-controlled capacity.

28.90 Extraction after Compensation

Compensation does not necessarily end extraction.

Let transfer be M_R , and subsequent extracted value be V_{ext} .

Net long-horizon restorative value can be represented by

$$V_{\text{net}} = M_R + V_{\text{new}} - V_{\text{ext}}. \quad (2714)$$

Equation 2714 is illustrative.

Its purpose is to prevent a one-time transfer from obscuring the continuation of the structure that generated the original vulnerability.

28.91 Compensation without Structural Change

An institution can repeatedly compensate harms while maintaining the economic structure that produces them.

Let harmful mechanism be Φ_H . If

$$\Phi_{H,t+1} = \Phi_{H,t} \quad (2715)$$

after compensation, then the remedy corrects individual losses without changing the recurrence mechanism.

Equation 2715 can be legally adequate in one case and structurally incomplete at system scale.

28.92 Compensation and Moral Hazard

Where harmful actors can externalize costs through predictable compensation, remedial payments can become incorporated into the cost of harmful conduct.

Let private gain from activity be G_H and expected liability be $E[L]$.

If

$$G_H > E[L], \quad (2716)$$

the activity can remain privately advantageous.

Equation 2716 is a simple political-economic representation rather than a complete theory of deterrence.

It shows why compensation and prevention can require separate institutional mechanisms.

28.93 General Equivalents and Deterrence

Monetary liability can influence future conduct by changing expected costs.

The restorative function and deterrent function should remain distinct.

A payment equal to claimant loss can be correctively adequate while insufficient to alter the harmful actor's future incentives.

Conversely, a larger deterrent sanction should not be misdescribed as compensatory equivalence.

28.94 General Equivalents and Social Power

Money expands choice and can concentrate power.

Simmel's analysis links money to forms of individual freedom created through impersonal exchange, while also tracing broader transformations of social relations under monetary economies [58].

Within the present framework, monetary holdings can increase:

- exit options;
- bargaining power;
- mobility;
- access to expertise.

They can also create asymmetries where wealth gives some actors disproportionate power to shape markets, institutions, or legal processes.

28.95 Money and Non-Domination

A monetary transfer can reduce domination if it creates a viable outside option.

Let domination exposure before and after transfer be D_A^0 and D_A^M .

A beneficial effect satisfies

$$D_A^M < D_A^0. \quad (2717)$$

Equation 2717 gives compensation a republican dimension [44].

The effect weakens where the claimant must continue depending upon the same actor for access to essential non-market conditions.

28.96 Money and Relational Equality

Resource transfers can improve bargaining position and still leave social standing unchanged.

Let economic position improve:

$$\Delta V_{\text{resource}} > 0 \quad (2718)$$

while relational standing remains

$$S_{A,R}^R = S_{A,H}^R. \quad (2719)$$

Equations 2718 and 2719 show why redistribution and relational equality remain separate objects [2, 71].

28.97 Money and Recognition

Compensation can itself express recognition where it is institutionally framed as response to a wrong.

The same amount framed as charity can communicate another relation.

Recognition theory therefore helps explain why source, reason, and public meaning of payment matter [25, 20].

28.98 Money and Capability

Capability theory provides one of the clearest frameworks for analyzing the limits of equal monetary awards.

Let subjects A and B receive equal compensation:

$$M_A = M_B. \quad (2720)$$

Their resulting capabilities can satisfy

$$\mathcal{K}_A(M_A) \neq \mathcal{K}_B(M_B). \quad (2721)$$

Equations 2720 and 2721 express the conversion problem directly [55, 50].

28.99 Money and Resource Equality

A resource-oriented theory can nevertheless assign direct significance to equal monetary or wealth holdings.

Dworkin's equality-of-resources project provides a prominent example of a theory that does not reduce distributive equality to welfare equality [14].

The present framework preserves that distinction.

Resource equality can be normatively important even where capability outcomes differ.

28.100 Money and Plural Value

A pluralist restorative architecture can therefore treat money as one value carrier among several.

Let restorative value profile be

$$\mathbf{V}_R = (V_M, V_{\text{cap}}, V_{\text{rec}}, V_{\text{relation}}, V_{\text{history}}, V_{\text{gen}}). \quad (2722)$$

Equation 2722 prevents a high monetary score from automatically dominating other dimensions.

Anderson's pluralist account is particularly relevant to the limits of using market valuation as a universal evaluative mode [1].

28.101 Money and Incommensurability

An institution can assign a monetary amount to a loss without establishing that the loss is fully commensurable with money.

Let legal or administrative monetary representation be m .

It can hold that

$$\mu(O^V) = m \quad (2723)$$

and

$$O^V \not\equiv_V m. \quad (2724)$$

Equations 2723 and 2724 distinguish administrative commensuration from complete moral equivalence.

28.102 Singular Value

Singular goods are especially resistant to replacement through a general equivalent.

Let singular good be q^* . A monetary substitute can satisfy

$$V_{\text{functional}}(m) \approx V_{\text{functional}}(q^*) \quad (2725)$$

while

$$V_{\text{historical}}(m) \neq V_{\text{historical}}(q^*). \quad (2726)$$

Equations 2725 and 2726 capture partial compensation for singular loss.

28.103 Money and Historical Opportunity

A missed opportunity can have a value arising from its place in a historical sequence.

Later money can create new opportunity and cannot recreate the event itself.

Let lost historical opportunity be $o_{t_0}^*$ and later purchased opportunity o'_{t_1} .

It can hold that

$$V_{\text{future}}(o'_{t_1}) \geq V_{\text{future}}(o_{t_0}^*) \quad (2727)$$

while

$$o'_{t_1} \neq_H o_{t_0}^*. \quad (2728)$$

Equations 2727 and 2728 show how generous monetary support can create an excellent new future without erasing historical loss.

28.104 Money and Re-Creation of Choice

The most defensible interpretation of many monetary remedies is therefore not that money reproduces the lost object.

It recreates a field of choice.

Let post-compensation future set be $\mathcal{F}_A^M$. A successful future-oriented award can satisfy

$$\mathcal{F}_A^M \succeq \mathcal{F}_A^H. \quad (2729)$$

Equation 2729 evaluates compensation through the future it makes possible rather than through fictional identity with the past.

28.105 Monetary Restitution and Monetary Assistance

The same currency can perform conceptually different functions.

Money can be:

- restitutionary substitution;
- compensatory damages;
- emergency assistance;
- social insurance;
- generative investment.

The legal and normative meaning depends upon why the money is transferred.

A single bank balance does not reveal the institutional provenance of the funds.

28.106 Provenance of Money

Let monetary amount be m and provenance be π_M .

A richer monetary object is

$$Q_M = (m, \pi_M). \quad (2730)$$

Equation 2730 allows a damages award, gift, loan, salary, and welfare benefit of equal nominal amount to remain institutionally different.

Provenance can affect rights, repayment, recognition, and social meaning.

28.107 Debt and Grant

A loan and a grant can supply the same present liquidity.

Their future generative effects differ because the loan creates a repayment claim.

Let grant be M_G and loan be M_L . Present equality can satisfy

$$M_G = M_L \quad (2731)$$

while future net resources satisfy

$$R_{A,t+\tau}^G > R_{A,t+\tau}^L. \quad (2732)$$

Equations 2731 and 2732 show why nominal present resources are insufficient without provenance and obligation structure.

28.108 Conditional Liability and Future Claims

Some compensation systems preserve future claims if specified conditions later materialize.

Let contingent claim be

$$C_F = \mathbf{1}[e_F] M_F, \quad (2733)$$

where e_F is a future qualifying event.

Equation 2733 shares uncertainty between claimant and institution.

It can reduce the need for speculative present capitalization.

28.109 Open and Closed Compensation

A closed compensation award ends future monetary claims.

An open or revisable scheme retains some adaptation to later evidence.

Let future claim set after settlement be

$$C_F^{\text{closed}} = \emptyset \quad (2734)$$

for a fully closed award.

A revisable system has

$$C_F^{\text{open}} \neq \emptyset. \quad (2735)$$

Equations 2734 and 2735 allocate uncertainty differently.

28.110 General Equivalent and Administrative Scale

Money is especially attractive for large systems because it is standardized.

A single unit can be used across heterogeneous claims.

Let administrative state space of loss be $\mathcal{L}$. Monetary commensuration reduces it to

$$\mu(\mathcal{L}) \subseteq \mathbb{R}. \quad (2736)$$

Equation 2736 dramatically simplifies budgeting and adjudication.

The same simplification can suppress differences important to justice.

28.111 Scale and Value Compression

As claim volume increases, pressure for standardized monetary treatment often increases.

Let claim volume be N and representational granularity be G_R .

A possible institutional tendency is

$$\frac{\partial G_R}{\partial N} < 0. \quad (2737)$$

Equation 2737 represents administrative compression under scale.

A tiered system can preserve standardized access with richer review for exceptional cases.

28.112 Budget Constraints

Public and institutional compensation operates under budget constraints.

Let total remedial budget be B_R and awards M_i . Feasibility requires

$$\sum_{i=1}^n M_i \leq B_R. \quad (2738)$$

Equation 2738 introduces distributive justice into restorative design.

Full compensation of one claimant can reduce resources available to others.

28.113 Scarcity and Prioritization

Under scarcity, allocation can use:

- proportionality to loss;
- priority to severe deprivation;
- sufficiency;
- equal shares;
- historical responsibility.

The budget constraint does not determine which rule is just.

It merely makes the conflict unavoidable.

28.114 Fiscal Incidence

Publicly funded restoration distributes burdens through taxation, borrowing, or foregone expenditure.

Let funding vector be

$$\mathbf{F}_R = (F_{\text{tax}}, F_{\text{debt}}, F_{\text{reallocation}}). \quad (2739)$$

Equation 2739 shows that the political economy of restoration includes who finances the remedy.

The claimant-side value of compensation is only one half of the distributive picture.

28.115 Intergenerational Financing

Borrowing can shift part of the cost of current restoration into future public budgets.

Let debt-financed amount be D_R . Future burden stream can be represented by

$$\mathbf{B}^F = (B_{t+1}, \dots, B_{t+T}). \quad (2740)$$

Equation 2740 makes intergenerational incidence visible.

Historical reparations can therefore create complex relations among past wrong, present beneficiaries, and future taxpayers.

28.116 Responsible-Party Funding

Where the responsible actor funds compensation, payment can align financial burden with corrective responsibility.

Let responsible actor be B . Funding relation is

$$B \xrightarrow{M} A. \quad (2741)$$

Equation 2741 can carry recognition and deterrence effects unavailable in an anonymous public transfer.

The responsible actor can also lack sufficient resources.

28.117 Judgment-Proof Harm

If legally responsible actor B lacks assets sufficient for the assessed remedy, then

$$W_B < L_A. \quad (2742)$$

Equation 2742 creates a gap between legal entitlement and realizable compensation.

Insurance, public funds, collective liability, or social protection can respond to different parts of this gap.

28.118 Solvency and Remedy Reality

A legal award without solvent funding has weak practical equivalence.

Let ordered monetary remedy be M_O and collected amount M_C .

Collection ratio is

$$\kappa_M = \frac{M_C}{M_O}. \quad (2743)$$

Equation 2743 makes enforcement and solvency part of effective monetary restoration.

28.119 Collective Liability

Some harms arise from organizations or distributed systems rather than one isolated individual.

Funding can therefore be distributed across a liability set $\mathcal{A}_L$.

Let shares satisfy

$$\sum_{j \in \mathcal{A}_L} \lambda_j = 1. \quad (2744)$$

Equation 2744 is a generic allocation structure.

The legal and moral bases for assigning λ_j can differ.

28.120 Restorative Funds and Collective Risk

Pooling contributions before harm occurs can increase payment reliability.

Let pooled reserve be F_P . Solvency condition can be represented by

$$F_P \geq L_\alpha, \quad (2745)$$

where L_α is a selected loss quantile or reserve requirement.

Equation 2745 illustrates how collective financial infrastructure can stabilize restorative capacity.

28.121 Marketization of Repair

Once restorative needs become predictable, specialized markets can emerge around them.

Examples can include:

- insurance;
- legal services;
- rehabilitation;
- claims management;
- financial products.

Marketization can improve availability and create new extraction opportunities.

The political economy of repair therefore includes the industries built around harm and remedy.

28.122 Intermediary Incentives

Let intermediary revenue be R_I and claimant recovery be R_A .

The intermediary's objective need not satisfy

$$\arg \max R_I = \arg \max R_A. \quad (2746)$$

Equation 2746 makes incentive alignment an institutional design issue.

Competition, professional duties, fee rules, or public provision can alter the relationship.

28.123 Administrative Industry and Harm Persistence

A system can become institutionally adapted to managing recurring harm.

Resources flow toward claims processing rather than removal of the underlying mechanism.

Let administrative expenditure be A_H and preventive structural expenditure be P_H .

Persistent imbalance satisfying

$$A_H \gg P_H \quad (2747)$$

can indicate a system optimized for post-harm management.

Equation 2747 is not itself proof of injustice, since some harms are unavoidable.

It is a diagnostic question.

28.124 General Equivalents and Institutional Incentives

Money can make harms administratively manageable.

This is beneficial for claimants and can reduce institutional pressure for structural change if payment becomes cheaper than prevention.

A complete political-economic analysis therefore compares:

$$C_{\text{comp}} \quad \text{and} \quad C_{\text{prevention}}, \quad (2748)$$

alongside the distribution of those costs and benefits.

Equation 2748 should never be the sole normative criterion because rights and non-compensable harms can constrain the choice.

28.125 Compensation Price of Harm

A dangerous institutional effect arises when compensation is interpreted as a price purchasing permission to inflict the harm.

Let assessed compensation be M_H . The inference

$$M_H = \text{Price (permission to harm)} \quad (2749)$$

is rejected.

Equation 2749 confuses ex post remedial translation with ex ante authorization.

This parallels the rejection of reparability as a license for wrongdoing in Section 26.

28.126 General Equivalents and Ex Ante Governance

The political economy of compensation must therefore be connected to ex ante rights, regulation, and prevention.

Some values should not be treated as freely purchasable through expected liability.

Rights can function as constraints rather than prices.

28.127 Compensation and Institutional Learning

A compensation system can generate information about recurring harm.

Let claims dataset be $\mathcal{D}_C$. Institutional learning can be

$$\Pi_{t+1} = L(\Pi_t, \mathcal{D}_C). \quad (2750)$$

Equation 2750 converts compensation administration into a preventive knowledge system.

Privacy and purpose limitation remain necessary.

28.128 Claims Data and Reflexivity

Once institutions optimize against measured claims, reporting behavior can change.

Let measured claim rate be C_t^M , actual harm rate H_t , and reporting probability p_t^R :

$$C_t^M = p_t^R H_t. \quad (2751)$$

Equation 2751 shows why falling claims do not necessarily imply falling harm.

Procedure, trust, eligibility, and incentives affect the measurement.

28.129 Compensation and Reporting Incentives

A credible compensation system can increase reporting because people expect claims to be taken seriously.

This can temporarily increase recorded harm while improving institutional justice.

Evaluation should therefore avoid interpreting every increase in claims as deterioration.

28.130 Monetary Metrics and Goodhart-Type Effects

If institutional success is measured only through amount paid or percentage of claims settled, actors can optimize those indicators while neglecting:

- capability;
- recognition;
- recurrence;
- claimant burden;
- future generativity.

The monetary metric can therefore become a reflexive governance target.

Plural restorative profiles remain preferable.

28.131 Restorative Purchasing-Power Profile

A richer monetary profile can be represented by

$$\mathbf{M}_R = (M_N, M_R, L_Q, P_A, A_M, C_T, S_E, R_G), \quad (2752)$$

where:

- M_N denotes nominal amount;
- M_R real purchasing power;
- L_Q liquidity;
- P_A portability;
- A_M market access;
- C_T transaction cost;
- S_E security of entitlement;
- R_G generative conversion potential.

Equation 2752 demonstrates why nominal award amount is only one dimension of monetary restoration.

28.132 General-Equivalent Adequacy Profile

The broader adequacy profile can be written as

$$\mathbf{A}_{GE} = (A_{\text{valuation}}, A_{\text{purchasing}}, A_{\text{availability}}, A_{\text{conversion}}, A_{\text{agency}}, A_{\text{recognition}}, A_{\text{generative}}). \quad (2753)$$

Equation 2753 separates valuation adequacy from actual purchasing, conversion, agency, recognition, and generative effects.

28.133 General-Equivalent Failure Modes

A monetary remedy can fail through several distinct mechanisms:

1. valuation failure;
2. inflation or price change;
3. market absence;
4. access restriction;
5. high transaction costs;
6. debt or institutional capture;
7. conversion impairment;
8. non-fungibility of the underlying value;
9. continuing structural extraction.

These failure modes should not be collapsed into the statement that the award was simply “too small.”

28.134 Valuation Failure

Valuation failure occurs where the monetary translation understates the legally or normatively selected value.

Let assessed amount be $\widehat{M}$ and justified monetary representation be M^* .

Failure is

$$\widehat{M} < M^*. \quad (2754)$$

Equation 2754 concerns the translation stage.

28.135 Purchasing-Power Failure

Purchasing-power failure occurs when an initially adequate amount loses real capacity:

$$M_{t+\tau}^R < M_t^R. \quad (2755)$$

Equation 2755 concerns time and prices rather than initial valuation.

28.136 Availability Failure

Availability failure occurs where the target condition cannot be purchased:

$$A_M(c^*, E) = 0. \quad (2756)$$

Equation 2756 cannot be corrected solely by raising the monetary amount if supply is genuinely absent.

28.137 Access Failure

Access failure occurs where the market exists but the subject cannot effectively use it:

$$A_M(c^*, E) = 1 \quad (2757)$$

and

$$A_{M,A}(c^*) = 0. \quad (2758)$$

Equations 2757 and 2758 identify an institutional-access problem.

28.138 Capture Failure

Capture failure occurs where a substantial share of the award flows to actors who do not provide corresponding restorative value.

Let capture share be

$$\chi_M = \frac{M_{\text{captured}}}{M}. \quad (2759)$$

Equation 2759 can diagnose value leakage without assuming every payment to another actor is illegitimate.

28.139 Conversion Failure

Conversion failure occurs where available resources do not become substantive capability.

Let capability gain be

$$\Delta \mathcal{K}_A(M) \approx 0 \quad (2760)$$

despite significant monetary transfer.

Equation 2760 can indicate that the binding constraint lies outside money.

28.140 Non-Fungibility Failure

Non-fungibility failure occurs where the underlying object has value that cannot be recreated through exchange.

Let monetary equivalence hold on one dimension:

$$V_1(M) = V_1(O^*) \quad (2761)$$

and fail on another:

$$V_2(M) \neq V_2(O^*). \quad (2762)$$

Equations 2761 and 2762 define partial monetary adequacy.

28.141 Structural Failure

Structural failure occurs where compensation leaves the harm-producing economic or institutional relation intact.

Let harmful mechanism remain

$$\Phi_R = \Phi_H. \quad (2763)$$

Equation 2763 shows why repeated monetary repair can coexist with recurrent harm.

Table 30: General equivalents in restorative political economy

Property	Restorative advantage	Political-economic dependency	Characteristic limitation
Fungibility	Supports subject-directed use	Availability of convertible goods	Cannot reproduce singular goods
Liquidity	Supports urgent adaptation	Financial access and payment infrastructure	Can be absorbed rapidly by prior claims
Divisibility	Allows calibrated awards	Valuation and administrative rules	Can encourage false precision
Portability	Supports exit and geographic mobility	Currency and financial infrastructure	Exchange-rate and legal-access risk
Standardization	Supports scale and administrative consistency	Institutional commensuration	Compresses heterogeneous values
Purchasing power	Converts nominal award into goods and services	Prices and supply	Inflation and local scarcity
Investment capacity	Can generate future value	Financial markets and risk	Transfers investment risk
Bargaining leverage	Can improve outside options	Institutional and market structure	Weak where critical goods are non-market

Table 31: Failure modes in monetary restoration

Failure site	Mechanism	Illustrative response	Atlas layer
Valuation	Loss translated inadequately	Revaluation or plural remedy	Value
Price	Purchasing power erodes	Indexation	Temporal / economic
Supply	Required good unavailable	Capacity or infrastructure expansion	Institutional / generative
Access	Subject cannot enter available market	Legal, informational, or accessibility support	Capability
Transaction	Conversion costs consume award	Navigation or direct provision	Procedural / economic
Capture	Third parties absorb large share	Debt, fee, or market-structure intervention	Political economy
Conversion	Money fails to create capability	Specific enabling support	Capability / causal
Non-fungibility	Underlying value cannot be purchased	Recognition, restitution, or residual acknowledgment	Historical / relational
Structure	Harm-producing mechanism persists	Structural reform and non-repetition	Causal / generative

28.142 General-Equivalent Matrix

Table 30 summarizes principal strengths and limitations of monetary and related general-equivalent remedies.

Table 30 shows that the value of a general equivalent is relationally dependent upon the surrounding economy.

28.143 Monetary Failure Matrix

Table 31 distinguishes several sites at which a monetary remedy can lose restorative force.

Table 31 makes the location of monetary failure diagnostically explicit.

28.144 General Equivalents and the Atlas of Equivalence

Money occupies several object–relation cells simultaneously.

It can function as:

- numerical equality of monetary resources;
- compensatory equivalence for legally assessed loss;
- sufficient resource threshold;

- instrument for capability inclusion;
- buffer supporting robust reachability;
- investment in future generative process.

These uses should remain distinct.

An award can succeed in one cell and fail in another.

28.145 Monetary Equality

The simplest cell is

$$M_R = M_{\text{ref}}. \quad (2764)$$

Equation 2764 is a resource-level equality claim.

It says nothing by itself about real purchasing power, capability, or historical value.

28.146 Purchasing-Power Equivalence

A stronger relation is

$$PP_R \sim_E PP_{\text{ref}}, \quad (2765)$$

where PP denotes purchasing power over the selected basket.

Equation 2765 is still weaker than capability equivalence.

28.147 Capability Equivalence through Money

A stronger cell is

$$\mathcal{K}_{A,R}(M_R) \sim_E \mathcal{K}_{A,\text{ref}}. \quad (2766)$$

Equation 2766 evaluates the result of monetary conversion rather than the amount itself.

28.148 Generative Equivalence through Money

A still stronger criterion is

$$\Gamma_{A,R}^G(M_R) \sim_E \Gamma_{A,\text{ref}}^G. \quad (2767)$$

Equation 2767 asks whether compensation supports a corresponding future generative process.

This standard will often be too demanding for routine legal compensation.

It is analytically useful for diagnosing residual loss.

28.149 Monetary Dominance

A monetary remedy can also produce a future better than the counterfactual reference on selected dimensions.

Let

$$\mathbf{V}(\mathcal{G}_R^M) \succeq \mathbf{V}(\mathcal{G}_{\text{ref}}). \quad (2768)$$

Equation 2768 can hold where a flexible award enables a new preferred trajectory.

Historical loss can remain positive.

28.150 Restorative Monetary Statement

A fully specified monetary restorative claim can be represented by

$$\mathcal{M}_R^* = [M_N; P_A; \tau; \mathcal{C}_{A,E}; A_M; C_T; \pi_M; O^V; E^V] . \quad (2769)$$

Equation 2769 records nominal amount, relevant price structure, horizon, conversion function, market access, transaction cost, provenance, value object, and comparative relation.

This is substantially more informative than reporting the award amount alone.

28.151 General-Equivalent Principle

The first principle of this section is:

A general equivalent is a powerful infrastructure for restorative translation and decentralized choice, while monetary equality should not be presumed to constitute equality of every value translated through it.

28.152 Fungibility Principle

The second principle is:

Fungibility has positive restorative value because it permits the historically changed subject to redirect resources toward a present future that institutions cannot fully predict.

28.153 Conversion Principle

The third principle is:

The restorative value of money depends upon the conversion structure through which monetary resources become substantive conditions, capabilities, and future generativity.

28.154 Real-Value Principle

The fourth principle is:

Nominal equivalence should be distinguished from real purchasing-power equivalence, especially where payment and restorative use occur at different times.

28.155 Market-Availability Principle

The fifth principle is:

Increasing a monetary award cannot restore a condition that is unavailable, legally inaccessible, technologically nonexistent, or constituted through the consent of another person.

28.156 Infrastructure Principle

The sixth principle is:

Private monetary compensation derives part of its restorative value from public and common infrastructures that enable money to be converted into usable conditions.

28.157 Non-Fungibility Principle

The seventh principle is:

Money can compensate for some dimensions of a singular loss while leaving identity, provenance, relational, and historical dimensions non-equivalent.

28.158 Commensuration Principle

The eighth principle is:

Institutional monetization creates a purpose-relative common measure and should not be interpreted automatically as proof of complete moral commensurability.

28.159 Capture Principle

The ninth principle is:

The real restorative value of a transfer depends partly upon how much of the transferred value remains available to the subject and how much is captured by prior claims, intermediaries, or constrained markets.

28.160 Value-Circulation Principle

The tenth principle is:

Evaluation of a monetary remedy should extend beyond the initial transfer where later circulation determines whether the award regenerates future conditions or returns primarily to external actors.

28.161 Structural-Residual Principle

The eleventh principle is:

Compensation can be individually valuable and legally adequate while leaving the institutional mechanism that produces recurrent harm unchanged.

28.162 No-Purchase-of-Permission Principle

The twelfth principle is:

An ex post monetary equivalent for harm should not be interpreted as an ex ante price purchasing permission to impose that harm.

28.163 Political-Economic Layer of the Atlas

The political-economic layer of general equivalents can be represented by

$$\mathfrak{M}_A = (M, P, A_M, C_T, F_A, D, \Lambda_M, \mathcal{C}_{A,E}, \Gamma_G), \quad (2770)$$

where:

- M denotes monetary amount and payment structure;
- P relevant prices and purchasing power;

- A_M market availability and access;
- C_T transaction costs;
- F_A financial infrastructure;
- D debt and prior claims;
- Λ_M post-transfer circulation;
- $C_{A,E}$ subject-environment conversion;
- Γ_G generative effect.

Equation 2770 connects legal monetary remedy with the wider social and institutional system through which money acquires restorative force.

28.164 Implications for Generative Assistance

The analysis in this section suggests a limit to compensation-centered restoration.

Where the principal problem is insufficient nominal resources, money can be an excellent remedy.

Where the principal problem lies in conversion, access, health, information, specific infrastructure, legal status, or relational position, increasing the amount alone can yield weak marginal restorative value.

The practical problem therefore shifts from valuation of past loss toward construction of present and future enabling conditions.

This does not imply replacing cash with paternalistic service provision.

The stronger architecture combines:

$$\text{general resources} + \text{specific enabling conditions} + \text{subject-directed choice.} \quad (2771)$$

Equation 2771 is the conceptual bridge between the political economy of compensation and generative assistance.

28.165 Transition to Generative Assistance

Section 29 develops generative assistance as a distinct institutional function. It examines support directed toward restoring or expanding the subject's ability to generate future possibilities where restitution or monetary compensation alone is insufficient.

The analysis will distinguish assistance from corrective liability and from paternalistic optimization. It will examine flexible resources, specific enabling conditions, conversion support, navigation, re-entry, portability, safe experimentation, revisability, time, and subject-directed future planning.

The central question becomes:

When a general equivalent has been returned but the subject's ability to convert that equivalent into a viable and self-directed future remains impaired, what forms of assistance can legitimately repair the conversion and generative environment without taking control of the future away from the subject?

29 Generative Assistance

This section develops generative assistance as a distinct institutional function within the justice of return. The preceding sections showed that restitution, compensation, legal correction, and monetary transfer can leave substantial residual differences where the subject's capacity to convert returned resources into viable future possibilities has been altered. Generative assistance addresses this residual problem.

Its central object is the subject's practical ability to generate, select, enter, sustain, revise, and leave future trajectories. Assistance can therefore concern resources, information, health, institutional access, networks, credentials, transportation, time, administrative navigation, safe experimentation, and other conditions affecting practical future formation.

Generative assistance differs analytically from several neighboring functions. Corrective justice identifies what is owed because of a wrong. Social protection can secure minimum conditions independently of individual responsibility. Rehabilitation can restore bodily or functional capacities. Compensation provides general resources. Generative assistance can draw upon all of these instruments while focusing specifically upon the conversion of present conditions into self-directed future possibility.

The section develops six propositions. First, assistance should be evaluated through the future possibilities it enables rather than through expenditure alone. Second, a layered architecture combining general resources and specific enabling conditions is often preferable to exclusive reliance upon either cash or direct provision. Third, subject direction is central because the historically actual subject can legitimately pursue a future different from the no-harm counterfactual. Fourth, assistance should remain revisable because both the subject and the surrounding environment evolve. Fifth, support can be long-lasting without becoming dominating where rights, portability, contestability, and exit are preserved. Sixth, successful assistance should expand independent generativity where feasible while avoiding an institutional demand that every subject become economically or socially self-sufficient.

Capability theory provides an important foundation for distinguishing resources from substantive opportunities [55, 39, 50]. Care theory helps illuminate legitimate dependence and responsiveness [63]. Republican non-domination contributes a framework for evaluating discretionary dependency [44]. Generative justice contributes the process-level concern with circulation, control, and the capacity to become generative again [15, 17]. The present framework integrates these insights within an assistance architecture centered upon conversion, future option structure, and subject-directed regeneration.

29.1 Generative Assistance as an Institutional Function

Let the harmed system be $\mathcal{G}^H$, and let assistance intervention be A_G . Generative assistance produces the transition represented by Equation 2772.

$$A_G : \mathcal{G}^H \longrightarrow \mathcal{G}^{HA}. \quad (2772)$$

Equation 2772 defines assistance through its effect upon the post-harm system.

The principal target is future generative capacity rather than exact historical replication.

Let the generative profile be $\mathbf{G}$. A minimal improvement criterion is given by Equation 2773.

$$\mathbf{G}(\mathcal{G}^{HA}) \succeq \mathbf{G}(\mathcal{G}^H). \quad (2773)$$

Equation 2773 permits assistance to improve future capability without requiring equivalence with the counterfactual reference.

29.2 Assistance and Corrective Liability

Corrective liability and assistance can be institutionally separated.

Let liability status be L , and assistance eligibility be E_A . A system can satisfy the relation in Equation 2774.

$$E_A = 1 \quad \text{while} \quad L \in \{0, 1, \text{unresolved}\}. \quad (2774)$$

Equation 2774 means support can be available before or independently of final responsibility adjudication.

This distinction is especially important where delay itself threatens health, housing, professional access, evidence, or future capability.

29.3 Assistance and Social Protection

Social protection can secure a minimum condition floor.

Let minimum protected condition vector be $\mathbf{C}_{\min}$. A social-protection criterion is represented by Equation 2775.

$$\mathbf{C}_A \succeq \mathbf{C}_{\min}. \quad (2775)$$

Equation 2775 can include basic income, housing, healthcare, or access to essential services.

Generative assistance begins from such foundations and asks what additional conditions enable the subject to form and pursue viable futures.

ILO Recommendation No. 202 provides an established institutional example of social-protection floors as a distinct governance function [26].

29.4 Assistance and Rehabilitation

Rehabilitation commonly focuses upon restoration or improvement of functional capacity.

Generative assistance can incorporate rehabilitation and extend beyond it.

Let functional capacity be F_A , and future generative capacity be G_A . The relationship can be represented by Equation 2776.

$$G_A = G(F_A, C_A, R_A, K_A, E_A), \quad (2776)$$

where C_A denotes enabling conditions, R_A relations, K_A knowledge, and E_A environment.

Equation 2776 shows why functional recovery can be necessary and still insufficient for restoration of future possibility.

29.5 Assistance and Compensation

Compensation can provide the principal resource input into assistance.

Let monetary resource be M_A . The resulting capability depends upon the conversion structure:

$$\mathcal{K}_A = \mathcal{C}_{A,E}(M_A, \mathbf{C}_A^*), \quad (2777)$$

where $\mathbf{C}_A^*$ denotes relevant specific enabling conditions. Equation 2777 formalizes the layered architecture developed in Section 28.

An assistance system therefore need not choose between cash and specific support as mutually exclusive models.

29.6 Core Assistance Architecture

A general generative-assistance architecture can be represented by

$$A_G = (M, C^*, N, R, T, X, P, V), \quad (2778)$$

where:

- M denotes flexible general resources;
- C^* denotes specific enabling conditions;
- N denotes navigation and information;
- R denotes relational and institutional access;
- T denotes time and temporal support;

- X denotes experimentation and transition support;
- P denotes portability and exit;
- V denotes review and revisability.

Equation 2778 provides the principal institutional decomposition used throughout this section.

29.7 General Resources

General resources preserve choice.

Let subject-directed conversion set generated by resource amount M be

$$\mathcal{C}_A(M) = \{c : c \text{ is practically obtainable by } A \text{ using } M\}. \quad (2779)$$

Equation 2779 shows why flexible resources can support multiple legitimate futures.

General resources are especially valuable when:

- future preferences remain uncertain;
- the subject possesses superior local knowledge;
- several restorative paths are substantively admissible;
- institutional specification would create unnecessary paternalism.

29.8 Specific Enabling Conditions

Some conditions cannot be recreated efficiently through unrestricted money.

Let specific enabling set be

$$\mathbf{C}^* = (c_1^*, \dots, c_k^*). \quad (2780)$$

Equation 2780 can include:

- legal status;
- accessibility accommodation;
- credential correction;
- healthcare;
- specialized equipment;
- institutional access;
- protected communication channels.

These conditions can function as bottlenecks.

29.9 Assistance Bottlenecks

Let overall generative potential depend upon several conditions through the illustrative structure in Equation 2781.

$$G_A = \min \{\alpha_1 c_1, \dots, \alpha_k c_k\}. \quad (2781)$$

Equation 2781 illustrates a case in which one missing enabling condition constrains the full process. Increasing non-binding resources can then have weak marginal effect.

29.10 Bottleneck Diagnosis

Assistance requires diagnosis of the currently binding constraint.

Let candidate constraint set be

$$\mathfrak{B}_A = \{b_1, \dots, b_m\}. \quad (2782)$$

Equation 2782 can contain financial, legal, health, informational, relational, temporal, or institutional constraints.

The institution should avoid assuming that the most visible deficit is the binding one.

29.11 Subject Knowledge in Bottleneck Diagnosis

The affected person often possesses information unavailable to the institution.

Let institutional estimate of the binding constraint be $\hat{b}_I$, and subject estimate be $\hat{b}_A$.

They can satisfy

$$\hat{b}_I \neq \hat{b}_A. \quad (2783)$$

Equation 2783 creates a reason for participatory diagnosis rather than automatic institutional substitution of judgment.

29.12 Diagnostic Humility

The institution can remain uncertain among several plausible bottlenecks.

Let probability distribution over candidate constraints be

$$P(b_j | K). \quad (2784)$$

Equation 2784 permits staged assistance that gathers information while preserving the subject's options.

This can be preferable to a large irreversible intervention based upon one uncertain diagnosis.

29.13 Navigation

Institutional environments can be difficult to use even where services formally exist.

Let service set be

$$\mathcal{S} = \{s_1, \dots, s_n\}. \quad (2785)$$

The subject's recognized and usable subset is

$$\mathcal{S}_A^U \subseteq \mathcal{S}. \quad (2786)$$

Equation 2786 makes informational and administrative usability explicit.

Navigation assistance can expand $\mathcal{S}_A^U$ without creating new services.

29.14 Navigation as Conversion Support

Let navigation intervention be N_A . Its effect can satisfy

$$\mathcal{S}_A^U(N_A) \supset \mathcal{S}_A^U(0). \quad (2787)$$

Equation 2787 treats navigation as a conversion intervention.

This can include:

- information;
- application support;
- appointment coordination;
- document preparation;
- referral.

29.15 Single Entry

A fragmented system can require repeated entry into different institutions.

A single-entry architecture uses an initial interface I_0 that routes the subject toward relevant functions.

Let routing map be

$$\rho : I_0 \longrightarrow \{I_1, \dots, I_n\}. \quad (2788)$$

Equation 2788 can reduce navigation burden while preserving institutional specialization.

29.16 Case Coordination

Where assistance spans several institutions, a coordination function can manage dependencies.

Let intervention set be

$$\mathcal{A}_C = \{A_1, \dots, A_n\}. \quad (2789)$$

Coordination order is represented by

$$A_i \prec A_j \quad (2790)$$

where A_i must occur before A_j becomes effective.

Equation 2790 makes sequencing explicit.

29.17 Coordination without Totalization

Case coordination can become intrusive if one coordinator acquires access to every aspect of the subject's life.

A bounded architecture should therefore restrict coordination to information required for the relevant intervention dependencies.

Coordination is an interface function rather than a mandate to construct a complete life model.

29.18 Re-Entry

Re-entry concerns renewed practical access to institutions, practices, or communities from which the subject has been displaced.

Let target institutional domain be I . Formal access is

$$A_I^F = 1. \quad (2791)$$

Effective re-entry additionally requires

$$A_I^E = 1. \quad (2792)$$

Equations 2791 and 2792 distinguish formal reinstatement from usable participation.

29.19 Re-Entry Conditions

Effective re-entry can depend upon:

- credentials;
- network access;
- information;
- accessibility;
- safety;
- trust;
- absence of retaliation;
- practical role availability.

A legal restoration of status can therefore be one component of generative re-entry.

29.20 Network Re-Entry

Network damage can persist after formal reinstatement.

Let professional or social network be $G = (V, E)$, and subject node be A .

A network re-entry objective can seek

$$\deg_R(A) \geq d_{\min} \quad (2793)$$

or another context-appropriate structural criterion.

Equation 2793 is illustrative.

The relevant object can instead be access to bridging ties, multiplex relations, or specific functional roles.

29.21 Network Assistance and Consent

Network assistance should create opportunities for relation formation rather than compel particular relationships.

Let offered relational opportunity set be

$$\mathcal{R}_A^O. \quad (2794)$$

Actual relationship set remains

$$\mathcal{R}_A^{\text{actual}} \subseteq \mathcal{R}_A^O \quad (2795)$$

through voluntary participation.

Equation 2795 respects the agency of both the subject and potential relational partners.

29.22 Credential Portability

Credentials can restore access only where other institutions recognize them.

Let credential be c , and recognition function in environment E_j be

$$R_j(c) \in \{0, 1\}. \quad (2796)$$

Portability across environment family $\mathfrak{E}$ can be represented by

$$P_C(c) = \sum_j w_j R_j(c). \quad (2797)$$

Equation 2797 shows why correction of one institutional record can be insufficient where downstream recognition remains fragmented.

29.23 Resource Portability

A restorative resource is more autonomy-supporting where it remains usable after institutional exit.

Let resource r have portability

$$P_R(r) = P(r; E_1, \dots, E_n). \quad (2798)$$

Equation 2798 can distinguish portable cash, credentials, and benefits from institution-specific support.

Portability reduces lock-in.

29.24 Exit

Assistance should preserve meaningful exit where feasible.

Let exit cost be

$$C_{\text{exit}} = C(M, R, L, S), \quad (2799)$$

where the terms can represent monetary, relational, legal, and service losses.

Equation 2799 makes assistance dependency measurable in principle.

A low-exit-cost support architecture can provide continuing help without producing domination.

29.25 Support Dependency

Long-term assistance can create legitimate dependence.

Let subject outcome depend upon support S :

$$Y_A = F(S, X_A). \quad (2800)$$

Equation 2800 does not itself indicate injustice.

The normative question concerns the conditions of dependence.

29.26 Legitimate Dependence

Dependence can be compatible with justice where support is:

- secure;
- rights-based;
- non-stigmatizing;

- responsive;
- contestable;
- portable where feasible.

Care theory is especially important here because it resists treating all dependence as a failure of autonomy [63].

29.27 Dominating Dependence

Dependence becomes problematic where continued access is subject to arbitrary control.

Let continuation decision be D_I . High domination exposure exists where

$$P(S_{t+1} = 0 \mid do(D_I = 0)) \gg 0. \quad (2801)$$

Equation 2801 represents strong dependence upon discretionary institutional approval.

Converting discretionary support into enforceable entitlement can reduce this problem [44].

29.28 Support Security

Let support-security profile be

$$\mathbf{S}_A = (S_{\text{duration}}, S_{\text{legal}}, S_{\text{predictability}}, S_{\text{review}}). \quad (2802)$$

Equation 2802 distinguishes amount of support from security of support.

A smaller secure entitlement can sometimes provide greater practical agency than a larger discretionary benefit.

29.29 Time as Assistance

Time can itself be a restorative resource.

Let discretionary recovery time be T_A .

Assistance can increase

$$T_A^R > T_A^H. \quad (2803)$$

Equation 2803 can be achieved through income support, leave, reduced administrative burden, flexible deadlines, or temporary workload adjustment.

Time can support health, learning, care, reflection, and transition.

29.30 Temporal Pressure

A formally generous assistance package can fail if the subject must use it under unrealistic deadlines.

Let action deadline be d , and minimum practical preparation time be $T_{\min}$.

A temporal-access condition is

$$d - t_0 \geq T_{\min}. \quad (2804)$$

Equation 2804 shows why time is part of effective assistance.

29.31 Waiting as a Cost

Delay before assistance can generate secondary losses.

Let assistance begin at t_A , while harm begins at t_H .

Cumulative waiting burden is

$$B_W = \int_{t_H}^{t_A} \ell(t) dt, \quad (2805)$$

where $\ell(t)$ is the instantaneous burden rate.

Equation 2805 demonstrates why rapid provisional support can be justified even before the final support plan is known.

29.32 Interim Assistance

Interim assistance can preserve viability during diagnosis and adjudication.

Let viable set be $\mathcal{V}$. Interim support A_I seeks

$$x_t^{A_I} \in \mathcal{V} \quad t \in [t_0, t_F]. \quad (2806)$$

Equation 2806 treats temporary assistance as preservation of future options.

29.33 Safe Experimentation

After harm, the subject may be uncertain about which new trajectory is viable.

Assistance can support reversible experimentation among several possibilities.

Let candidate trajectory set be

$$\mathfrak{T}_A = \{T_1, \dots, T_k\}. \quad (2807)$$

A safe experiment with T_i should preserve the ability to return or redirect where feasible.

29.34 Experimental Reversibility

Let experiment X_i move state from x_0 to x_i .

Reversibility is represented by

$$d(R_i^{-1}(x_i), x_0) \leq \varepsilon_X. \quad (2808)$$

Equation 2808 allows exploration with bounded lock-in.

This is particularly useful where the institution lacks confidence concerning the subject's future preferences.

29.35 Experimentation Budget

Safe experimentation can require a dedicated resource buffer.

Let experimentation budget be B_X , and experiment costs be c_i .

A basic feasibility condition is

$$\sum_{i \in I_X} c_i \leq B_X. \quad (2809)$$

Equation 2809 permits exploration without threatening basic subsistence.

29.36 Failure-Tolerant Assistance

A generative system should not treat every unsuccessful experiment as evidence that the subject's assistance should end.

Let trial outcome be Y_i . A failure-tolerant design permits

$$Y_i < Y_{\text{target}} \quad (2810)$$

without automatically setting

$$S_{i+1} = 0. \quad (2811)$$

Equations 2810 and 2811 distinguish learning from disqualification.

29.37 Learning through Assistance

Assistance can itself generate information about the subject's future conversion structure.

Let belief concerning effective intervention be $P_t(A_j)$.

After experience E_t ,

$$P_{t+1}(A_j) = \mathcal{U}(P_t(A_j), E_t). \quad (2812)$$

Equation 2812 gives assistance an adaptive epistemic function.

The information should primarily serve the subject's legitimate support interests rather than unlimited institutional profiling.

29.38 Revisable Assistance Plans

Let assistance plan at time t be A_t .

A revisable architecture satisfies

$$A_{t+1} = \mathcal{U}(A_t, P_{A,t+1}, E_{t+1}, C_{t+1}), \quad (2813)$$

where $P_{A,t+1}$ denotes present subject priorities, E_{t+1} observed effects, and C_{t+1} changed circumstances.

Equation 2813 makes plan revision a normal feature of longitudinal support.

29.39 Revision Triggers

Revision can be triggered by:

- failure of the current intervention;
- new health information;
- change in subject preference;
- change in market or institutional environment;
- appearance of new opportunity;
- newly identified side effects.

Explicit triggers reduce both rigidity and arbitrary institutional change.

29.40 Subject-Initiated Revision

The affected subject should ordinarily possess a route to request modification.

Let revision request be r_A . A responsive system provides

$$r_A \longrightarrow \text{Review}(A_t). \quad (2814)$$

Equation 2814 does not guarantee approval.

It guarantees procedural access to reconsideration.

29.41 Institution-Initiated Revision

Institutions can also identify that an assistance plan has become ineffective or harmful.

Institution-initiated revision should remain transparent and contestable, especially where it reduces support.

This prevents revisability from becoming one-sided administrative discretion.

29.42 Revisability and Security

Excessive revisability can undermine security.

Let support state be S_t . If institutional modification probability is too high,

$$P(S_{t+1} \neq S_t) \gg 0, \quad (2815)$$

the subject can be unable to plan.

Equation 2815 shows why revisions should occur through predictable rules rather than constant discretionary change.

29.43 Adaptive Stability

A desirable assistance architecture combines stability of core entitlements with flexibility of implementation.

Let core entitlement be C_E , and adjustable support vector be $\mathbf{A}_t$.

The structure is

$$C_E = \text{constant} \quad \text{while} \quad \mathbf{A}_{t+1} = \mathcal{U}(\mathbf{A}_t). \quad (2816)$$

Equation 2816 separates security from rigidity.

29.44 Subject-Directed Goals

Generative assistance should normally operate relative to a subject-selected goal region rather than one institutionally predetermined life path.

Let goal set be

$$\mathcal{G}_A^* = \{g_1, \dots, g_k\}. \quad (2817)$$

Equation 2817 can evolve over time.

Assistance then asks which conditions make at least one valued trajectory toward $\mathcal{G}_A^*$ viable.

29.45 Goal Feasibility

Let reachable set under assistance plan A be $\mathcal{R}_\tau^A$.

A minimal feasibility criterion is

$$\mathcal{R}_\tau^A \cap \mathcal{G}_A^* \neq \emptyset. \quad (2818)$$

Equation 2818 means at least one subject-valued goal is practically reachable.

This is a stronger assistance criterion than mere receipt of resources.

29.46 Goal Plurality

A high-quality assistance system can preserve several viable goals.

Let viable goal subset be

$$\mathcal{G}_A^V = \mathcal{G}_A^* \cap \mathcal{R}_\tau^A. \quad (2819)$$

Equation 2819 permits assessment of how much practical future plurality the assistance preserves.

A plan offering exactly one institutionally preferred path can be effective and low in generative autonomy.

29.47 Goal Revision

The subject can revise goals as new experience develops.

Let

$$\mathcal{G}_{A,t+1}^* = \mathcal{G}(\mathcal{G}_{A,t}^*, E_{A,t}). \quad (2820)$$

Equation 2820 makes preference development part of the assistance process.

A restorative plan should avoid treating an early goal statement as permanently binding.

29.48 Adaptive Preference and Assistance

Present preferences can reflect adaptation to deprivation.

Generative assistance should therefore preserve two commitments:

1. respect the actual subject's present agency;
2. preserve or expand opportunities that make later revision possible.

Capability theory is particularly valuable here because it allows attention to opportunities alongside chosen functionings [55, 50].

29.49 Preference Expansion

Assistance can expose the subject to information and possibilities that expand the domain of informed preference.

Let initially recognized options be $\mathcal{O}_A^0$, and later recognized options be $\mathcal{O}_A^1$.

A learning-oriented intervention can satisfy

$$\mathcal{O}_A^0 \subset \mathcal{O}_A^1. \quad (2821)$$

Equation 2821 expands decision space without dictating the chosen option.

29.50 Information without Direction

Providing information should be distinguished from prescribing a life plan.

An assistance institution can provide:

- realistic option descriptions;
- likely costs;
- eligibility information;
- transition requirements;
- uncertainty.

The subject retains authority to select among admissible futures.

29.51 Advisory Power

Advice can nevertheless become influential because the institution controls resources.

Let recommended option be g_I , while subject preference is g_A .

Where receipt of support depends upon accepting g_I ,

$$S_A = 1 \iff g_A = g_I, \quad (2822)$$

the advisory relation becomes coercive.

Equation 2822 identifies a form of paternalistic assistance.

29.52 Bounded Paternalism

Some restrictions can be justified where:

- immediate severe harm is likely;
- decision capacity is materially impaired;
- third-party rights are implicated;
- public funds have a narrowly defined lawful purpose.

The intervention should remain proportional, reviewable, and as reversible as possible.

Paternalistic authority should not be inferred merely from the fact that support is publicly funded.

29.53 Assistance and Care

Care-oriented assistance emphasizes responsiveness to concrete dependency and need.

Let care-support profile be $C_A(t)$, and need profile be $N_A(t)$.

A responsiveness relation can be represented by

$$C_A(t) = F_C(N_A(t)). \quad (2823)$$

Equation 2823 permits support intensity and type to change with the subject's actual situation.

Care does not require the disappearance of dependence.

29.54 Assistance and Autonomy

Autonomy-supporting assistance expands the subject's capacity to direct future action.

Let autonomy profile be

$$\mathbf{A}_U = (O, E, R, C), \quad (2824)$$

where O denotes option structure, E exit, R revisability, and C control.

Equation 2824 makes autonomy multidimensional.

A support program can increase material resources and reduce one of these dimensions.

29.55 Assistance and Non-Domination

A non-dominating assistance system should reduce exposure to discretionary control.

Let institutional domination exposure be D_I .

A desirable relation is

$$D_I(A_G) \leq D_{\max}. \quad (2825)$$

Equation 2825 can be supported through clear entitlements, review rights, predictable duration, portability, and independent appeal [44].

29.56 Assistance and Recognition

Assistance can become stigmatizing if it communicates incapacity or inferiority.

Let material support value be V_M , and recognition effect be V_R .

A poorly designed program can satisfy

$$\Delta V_M > 0 \quad (2826)$$

while

$$\Delta V_R < 0. \quad (2827)$$

Equations 2826 and 2827 show why assistance design affects relational standing.

29.57 Non-Stigmatizing Assistance

Non-stigmatizing design can include:

- rights-based eligibility;
- ordinary service interfaces;
- privacy;
- avoidance of humiliating proof;
- subject control over disclosure.

The goal is to provide support without institutionalizing a subordinate identity.

29.58 Assistance and Historical Recognition

Assistance can coexist with acknowledgment that the subject should not have needed the support absent the historical harm.

This distinction matters.

Otherwise the subject can be reframed as a naturally dependent beneficiary rather than a person whose generative conditions were altered.

Historical provenance therefore protects against depoliticization of assistance.

29.59 Assistance and Corrective Provenance

Let support amount be S , and provenance be π_S .

A richer support object is

$$Q_S = (S, \pi_S). \quad (2828)$$

Equation 2828 allows identical material support to retain different institutional meanings, such as entitlement, rehabilitation, reparation, insurance, or discretionary aid.

29.60 Assistance as a Bridge

Assistance can function as a bridge between harmed and self-sustaining generative processes.

Let subject generative process under support be $\Gamma_{A,S}^G$, and independent process be $\Gamma_{A,I}^G$.

A transitional objective can be represented by

$$\Gamma_{A,S}^G \xrightarrow{\tau} \Gamma_{A,I}^G. \quad (2829)$$

Equation 2829 is appropriate only where independent generation is feasible and normatively desirable.

It should not become a universal requirement.

29.61 Permanent Assistance

Some subjects or collective systems can require continuing support.

Let required support horizon satisfy

$$\tau_S \rightarrow \infty. \quad (2830)$$

Equation 2830 does not imply restorative failure.

A permanent support relation can be just where it remains secure, non-dominating, responsive, and compatible with agency.

29.62 Support Dependence and Generativity

A supported system can remain highly generative.

Let external support be S and generated value be G .

It can hold that

$$S > 0 \quad (2831)$$

and

$$G \gg 0. \quad (2832)$$

Equations 2831 and 2832 reject the assumption that self-financing independence is the sole form of successful generativity.

29.63 Self-Sustaining Generativity

Where self-sustaining generation is a relevant goal, define external support requirement S_t .

A self-sustaining process can satisfy

$$S_t \longrightarrow 0 \quad (2833)$$

while maintaining

$$G_t \geq G_{\min}. \quad (2834)$$

Equations 2833 and 2834 define one form of successful transition.

The criterion is inappropriate where continued support is itself a just social condition.

29.64 Assistance Intensity

Support intensity can be represented by

$$I_A(t) = I(M_t, C_t^*, N_t, R_t, T_t). \quad (2835)$$

Equation 2835 combines several support dimensions.

A program can adapt $I_A(t)$ over time rather than using one fixed package.

29.65 Tapering

Where support can safely decrease, gradual tapering can reduce cliff effects.

Let support be S_t . A taper satisfies

$$S_{t+1} = S_t - \Delta S_t, \quad (2836)$$

with ΔS_t constrained by observed generative capacity.

Equation 2836 permits gradual transition.

29.66 Benefit Cliffs

A benefit cliff occurs where a small increase in independent resources causes a large loss of support.

Let independent resource be Y . A cliff exists where

$$\left| \frac{dS}{dY} \right| \gg 1 \quad (2837)$$

over a narrow region.

Equation 2837 can discourage experimentation, employment, or transition.

Smoother tapering can preserve incentives and security.

29.67 Transition Security

A subject may avoid a new opportunity if failure would eliminate the previous support structure.

Let trial state be T and fallback support be S_F .

Transition security requires

$$S_F > 0 \quad (2838)$$

for a defined period following unsuccessful transition.

Equation 2838 can make experimentation practically viable.

29.68 Fallback Rights

Fallback support should be clearly defined rather than discretionary.

Let conditions for reactivation be $\mathcal{C}_F$. A fallback rule is

$$c \in \mathcal{C}_F \implies S_F. \quad (2839)$$

Equation 2839 creates predictable risk sharing between the subject and institution.

29.69 Safe Failure

Generative assistance should permit some failures without catastrophic loss.

Let experiment outcome be Y_X , and subsistence or capability floor be $V_{\min}$.

Safe failure requires

$$Y_X < Y_{\text{target}} \implies V_A \geq V_{\min}. \quad (2840)$$

Equation 2840 converts experimentation from existential risk into bounded learning.

29.70 Redundancy

A robust assistance architecture can preserve more than one route to essential conditions.

Let support channels be

$$\mathcal{S}_R = \{s_1, \dots, s_k\}. \quad (2841)$$

Equation 2841 reduces dependence upon one administrative interface.

Redundancy becomes particularly important for health, income, and legal access.

29.71 Assistance Resilience

Let disturbance class be Δ_A .

A robust assistance system satisfies

$$J_S(A_G^\delta) \geq J_{\min} \quad \forall \delta \in \Delta_A. \quad (2842)$$

Equation 2842 means ordinary disruptions do not immediately destroy essential support.

29.72 Administrative Resilience

Support systems themselves can fail through staff absence, budget delay, software outage, or organizational restructuring.

Institutional resilience therefore belongs to the assistance object.

A claimant should not bear catastrophic loss because one administrative node temporarily fails.

29.73 Information Portability

The subject can benefit from carrying verified information between institutions.

Let verified information package be K_A^V . Portability is represented by

$$K_A^V \xrightarrow{\Gamma_{ij}} K_{A,j}^V, \quad (2843)$$

where Γ_{ij} preserves legal meaning, provenance, and privacy.

Equation 2843 can reduce repeated proof burden.

29.74 Data Minimization

Generative assistance can tempt institutions to collect extensive longitudinal data.

Let collected data be $\mathcal{D}_A$, and minimum necessary data be $\mathcal{D}_{\min}$.

A minimization principle requires

$$\mathcal{D}_A \subseteq \mathcal{D}_{\min} \cup \mathcal{D}_{\text{consented}}. \quad (2844)$$

Equation 2844 limits optimization through surveillance.

29.75 Outcome Monitoring

Assistance should monitor whether the selected support is producing useful conditions.

Let outcome vector be

$$\mathbf{Y}_A = (Y_{\text{capability}}, Y_{\text{security}}, Y_{\text{access}}, Y_{\text{agency}}, Y_{\text{generativity}}). \quad (2845)$$

Equation 2845 provides a plural monitoring object.

Monitoring should remain proportional to the program's purpose.

29.76 Subject-Reported Usability

Institutional provision can be formally complete and practically unusable.

Let provided support be S , and subject-reported usability be $U_A(S)$.

A basic criterion is

$$U_A(S) \geq U_{\min}. \quad (2846)$$

Equation 2846 gives first-person evidence a defined role without making it the sole evaluative standard.

29.77 Objective and Subjective Usability

Usability can be evaluated through both objective and subject-relative indicators.

Let

$$U = (U_{\text{obs}}, U_A). \quad (2847)$$

Equation 2847 can reveal disagreement between measured availability and lived usability.

Such disagreement should trigger inquiry rather than automatic privileging of one side.

29.78 Assistance Success

Assistance success should be defined relative to its declared purpose.

A generic success criterion is

$$\mathbf{G}_{A,t+\tau} \succeq \boldsymbol{\theta}_G \quad (2848)$$

over the specified horizon.

Equation 2848 can include security, capability, agency, and future possibility.

Success need not imply equivalence with the no-harm trajectory.

29.79 Assistance Failure

Assistance can fail because:

- the wrong bottleneck was targeted;
- support intensity was insufficient;
- timing was poor;
- support was inaccessible;
- market or institutional conditions changed;
- the subject's goal changed;
- support created new dependency or burden.

Failure diagnosis should therefore precede withdrawal of assistance.

29.80 Assistance Failure Localization

Let assistance chain be

$$M \rightarrow C \rightarrow K \rightarrow A \rightarrow G. \quad (2849)$$

Equation 2849 represents resources, conditions, capabilities, action, and generativity.

Failure can occur at any link.

The intervention should target the failing link rather than assume insufficient motivation or effort.

29.81 Effort and Structural Constraint

Observed outcome can depend upon both subject effort E_A and structural conditions C_S :

$$Y_A = F(E_A, C_S). \quad (2850)$$

Equation 2850 prevents low outcome from being interpreted automatically as low effort.

The institution should distinguish controllable and structural factors.

29.82 Administrative Attribution

Assistance systems can generate moralized classifications such as “compliant,” “motivated,” or “independent.”

These classifications can obscure structural barriers.

A generative framework should prefer descriptions of observable constraints, actions, and outcomes over broad character judgments where possible.

29.83 Assistance and Dignity

Repeated demonstration of need can itself be burdensome.

Where a stable condition has already been verified, the system should avoid unnecessary repeated proof.

Institutional memory can therefore protect dignity.

Correction mechanisms should remain available if the verified condition changes.

29.84 Assistance and Privacy

Longitudinal support can expose intimate information across multiple life domains.

A privacy-respecting architecture should separate:

- eligibility data;
- service-delivery data;
- outcome monitoring;
- research or policy evaluation.

Access to one category should not imply unrestricted access to all categories.

29.85 Assistance and Contestability

Subjects should be able to challenge:

- eligibility decisions;
- benefit levels;
- termination;
- mandatory conditions;
- data use;
- institutional models of success.

Contestability prevents the support system from defining the subject's future unilaterally.

29.86 Assistance and Institutional Learning

Aggregate assistance experience can reveal recurring structural bottlenecks.

Let case evidence be $\mathcal{D}_A^{1:n}$. Institutional learning can produce

$$\Pi_{t+1} = L(\Pi_t, \mathcal{D}_A^{1:n}). \quad (2851)$$

Equation 2851 allows individual support to generate preventive institutional knowledge.

Privacy and aggregation constraints remain necessary.

29.87 Repeated Assistance Demand as Structural Evidence

If similar assistance needs recur across many cases, the problem can lie in the institutional environment rather than isolated individual deficits.

Let repeated bottleneck b occur with frequency

$$f(b) \geq \theta_b. \quad (2852)$$

Equation 2852 can trigger review of the structural condition generating the demand.

Assistance can thereby inform preventive governance.

29.88 From Assistance to Structural Reform

Where assistance repeatedly compensates for the same institutional barrier, structural modification can become more efficient and just.

Let per-case support cost be C_A , recurrence count be n , and structural reform cost be C_S .

A simple economic comparison is

$$nC_A > C_S. \quad (2853)$$

Equation 2853 can support investigation of structural reform.

Cost is only one consideration. Rights, distribution, and institutional competence also matter.

29.89 Assistance and Prevention

Assistance can prevent secondary harms even where it cannot reverse the original loss.

Let original loss be L_0 , and secondary loss without assistance be L_1 .

Preventive assistance seeks

$$L_1(A_G) < L_1(0). \quad (2854)$$

Equation 2854 gives assistance an independent preventive function.

29.90 Secondary Vulnerability

An initial harm can create vulnerability to later losses.

For example:

- income loss can produce housing loss;
- reputational loss can produce network exclusion;
- health impairment can produce employment instability.

Generative assistance should therefore target propagation as well as the initial deficit.

29.91 Manufactured Vulnerability and Assistance

Where vulnerability was created through domination or exclusion, assistance should avoid treating the resulting dependency as a natural property of the subject.

Let vulnerability be

$$V_A = V(H, R, C). \quad (2855)$$

Equation 2855 makes vulnerability partly relational and historical.

Assistance can then target the structure maintaining the vulnerability.

29.92 Assistance and Unequal Exchange

A subject can receive support while continuing to participate in a relation that extracts disproportionate value.

Let support be S and extraction be E_X .

Net generative condition can be represented by

$$G_{\text{net}} = G(S) - E_X. \quad (2856)$$

Equation 2856 shows why assistance can have weak effect where extractive structure remains unchanged.

29.93 Anti-Extractive Assistance

Assistance can reduce extraction through:

- portable resources;
- independent income;
- legal advice;
- alternative networks;
- secure exit;
- direct control over productive assets.

These interventions alter bargaining and control structures rather than only consumption.

29.94 Assistance and Collective Generativity

Generative assistance can be directed toward groups and communities.

Let collective generative capacity be

$$G_C = G(R_C, K_C, \chi_C, I_C), \quad (2857)$$

where R_C denotes relations, K_C distributed knowledge, χ_C control, and I_C infrastructure.

Equation 2857 shows why collective assistance can require governance and infrastructure rather than individual transfers alone.

29.95 Collective Assistance

Collective assistance can include:

- shared infrastructure;
- community-controlled funds;
- training institutions;
- public archives;
- ecological restoration;
- governance capacity.

The internal distribution of benefits and control remains a separate justice question.

29.96 Community Control

Let assistance asset be A_C , and governance rights be χ_C .

A community-control criterion can require

$$\chi_C \succeq \chi_{\min}. \quad (2858)$$

Equation 2858 distinguishes assistance provided to a community from assistance governed by the community.

Generative justice gives this distinction particular significance [15, 17].

29.97 Distributed Knowledge

Collective assistance can benefit from distributed local knowledge.

Let community knowledge be

$$K_C^D = (K_1, \dots, K_n, \mathcal{R}_K). \quad (2859)$$

Equation 2859 includes individual knowledge and the relations through which it is combined.

Externally designed assistance can fail where it ignores this distributed epistemic structure.

29.98 Assistance and Public Goods

Some support is most efficiently and justly supplied as public or common infrastructure.

Let public generative condition be G_P . Individual capability can satisfy

$$\mathcal{K}_A = F(M_A, G_P). \quad (2860)$$

Equation 2860 shows the complementarity between individual resources and shared infrastructure.

29.99 Assistance and Intergenerational Generativity

Assistance can alter transmission to later generations.

Let current intervention be $A_G^{(0)}$. Future condition satisfies

$$C^{(1)} = \Gamma_{\text{intergen}}(C^{(0)}, A_G^{(0)}). \quad (2861)$$

Equation 2861 is relevant to education, health, housing, environmental conditions, and collective capacity.

Current support can therefore possess long-horizon public value.

29.100 Assistance and Ecological Conditions

Generative assistance should avoid creating private capability through destruction of shared ecological conditions.

Let subject gain be ΔG_A , and ecological loss be ΔE .

It can occur that

$$\Delta G_A > 0 \quad (2862)$$

while

$$\Delta E < 0. \quad (2863)$$

Equations 2862 and 2863 require multi-scale evaluation.

29.101 Assistance and Labor

Assistance systems often assume paid employment as the primary marker of successful recovery.

This can be too narrow.

Generative activity can include:

- caregiving;
- education;
- research;

- civic activity;
- artistic production;
- community work.

Employment can be one generative pathway without becoming the universal criterion of successful assistance.

29.102 Employment-Centered Assistance

Where employment is the subject's chosen goal, assistance can target:

- skills;
- credentials;
- workplace accommodation;
- network access;
- transportation;
- transitional income.

Success should include job security, usability, and future opportunity rather than placement alone.

29.103 Placement and Generativity

Let employment placement indicator be E_P .

It can satisfy

$$E_P = 1 \quad (2864)$$

while future generativity remains low:

$$G_A < G_{\min}. \quad (2865)$$

Equations 2864 and 2865 show why placement rate is an insufficient assistance metric.

29.104 Research Assistance

A research context provides a clear example of generative assistance.

Relevant conditions can include:

- funding;
- institutional affiliation;
- data access;
- laboratory access;
- mentorship;
- publication infrastructure;
- conference participation;
- credential portability.

A one-time monetary award can be useful and insufficient if key institutional interfaces remain closed.

29.105 Research Generative Cycle

Let the research cycle be represented by

$$A \rightarrow R \rightarrow K \rightarrow P \rightarrow N \rightarrow A', \quad (2866)$$

where A denotes access, R research activity, K knowledge, P publication, N network formation, and A' future access.

Equation 2866 shows how assistance at one stage can restore later recursive opportunity.

29.106 Workplace Assistance

A workplace case can require:

- income continuity;
- anti-retaliation protection;
- role clarification;
- alternative reporting channel;
- training or transition support;
- compensated exit.

The appropriate bundle depends upon whether the subject seeks return, transfer, or departure.

29.107 Health-Related Assistance

Health-related generative assistance can combine treatment with practical conversion conditions such as:

- transportation;
- housing stability;
- work accommodation;
- information;
- financial support.

The goal is practical capability rather than treatment consumption alone.

29.108 Reputational Assistance

Reputational harm can require assistance beyond source-record correction.

Possible support includes:

- propagation of correction;
- credential verification;
- public clarification;
- professional re-entry support;
- new network access.

Assistance should avoid unnecessary republication of the harmful allegation.

29.109 Migration and Cross-Border Assistance

Generative conditions can span jurisdictions.

A subject can need:

- portable funds;
- recognized credentials;
- lawful status;
- language access;
- cross-border records.

Assistance therefore requires interface design where future trajectories cross institutional boundaries.

29.110 Portability across Jurisdictions

Let condition c be recognized in jurisdiction j through $R_j(c)$.

Cross-jurisdiction portability is

$$P_J(c) = \sum_j w_j R_j(c). \quad (2867)$$

Equation 2867 provides an abstract measure of whether the restored condition travels with the subject.

29.111 Assistance and General Equivalents

The political-economic analysis of Section 28 supports a central assistance principle:

$$A_G = M + C^* + N + P_A, \quad (2868)$$

where the addition denotes a conceptual bundle of flexible resources, specific conditions, navigation, and subject direction.

Equation 2868 should not be read as requiring every component in every case.

It specifies the main classes from which a case-specific architecture can be constructed.

29.112 Minimal Generative Assistance

A minimal assistance package is the smallest intervention sufficient to make a subject-valued viable future available.

Let assistance package be A . A minimum criterion is

$$A^* \in \arg \min_A C(A) \quad (2869)$$

subject to

$$\mathcal{R}_\tau^A \cap \mathcal{G}_A^* \neq \emptyset. \quad (2870)$$

Equations 2869 and 2870 formalize a minimal effective assistance principle.

Cost $C(A)$ can include institutional and subject burden rather than financial cost alone.

29.113 Expanded Generative Assistance

Where resources permit, assistance can aim beyond one viable path toward a diverse and secure option set.

Let target number or diversity of viable futures be measured by $D(\mathcal{R}_\tau^A)$.

An expanded criterion can require

$$D(\mathcal{R}_\tau^A \cap \mathcal{G}_A^*) \geq D_{\min}. \quad (2871)$$

Equation 2871 values future plurality.

29.114 Assistance and Opportunity Security

An opportunity can be formally reachable and practically fragile.

Let opportunity o survive ordinary disturbances with probability

$$P_S(o) = P(o \text{ remains reachable under } \Delta_{\text{ordinary}}). \quad (2872)$$

Equation 2872 provides one security criterion.

Assistance can therefore seek durable rather than momentary access.

29.115 Assistance Robustness

A generative-assistance robustness profile can be represented by

$$\mathbf{R}_A^G = (R_{\text{income}}, R_{\text{access}}, R_{\text{health}}, R_{\text{network}}, R_{\text{exit}}, R_{\text{revision}}). \quad (2873)$$

Equation 2873 makes fragility across support dimensions visible.

29.116 Assistance Metrics

Institutional evaluation should avoid one universal success indicator.

Useful metrics can include:

- viable future-set expansion;
- security of basic conditions;
- subject-reported usability;
- portability;
- exit capacity;
- support dependence;
- generative diversity;
- recurrence of crisis.

The metric family should follow the assistance purpose.

29.117 Assistance Outcome Profile

A case-level assistance profile can be represented by

$$\mathbf{A}_R = (a_{\text{resource}}, a_{\text{conversion}}, a_{\text{navigation}}, a_{\text{reentry}}, a_{\text{portability}}, a_{\text{agency}}, a_{\text{security}}, a_{\text{generativity}}). \quad (2874)$$

Equation 2874 avoids treating amount spent as the sole measure of assistance.

Table 32: Principal components of generative assistance

Assistance component	Primary object	Generative function	Characteristic risk
Flexible resources	Money and transferable resources	Supports subject-selected conversion	Market and conversion failure
Specific enabling conditions	Legal, health, accessibility, institutional conditions	Removes binding bottlenecks	Paternalistic misdiagnosis
Navigation	Information and administrative access	Makes existing services usable	Over-centralized case management
Re-entry	Institutional position and access	Reopens generative participation	Formal access without relational usability
Network support	Relational opportunity	Creates paths to collaboration and recognition	Coercive relational engineering
Time support	Discretionary and recovery time	Enables transition, learning, and healing	Administrative delay can consume the benefit
Safe experimentation	Reversible trials	Supports discovery of viable futures	Failure can be misread as disqualification
Portability	Transferable resources and status	Supports exit and cross-institutional mobility	Recognition gaps across institutions
Fallback support	Security after failed transition	Makes experimentation safe	Can become unpredictable if discretionary
Revisability	Adaptive support plan	Responds to changing goals and conditions	Excessive instability

29.118 Assistance Matrix

Table 32 summarizes the principal assistance functions developed in this section.

Table 32 demonstrates that generative assistance is an architecture of conversion and future possibility rather than a single benefit.

29.119 Assistance Failure Matrix

Table 33 identifies recurring failure modes.

Table 33 shows that insufficient money is only one possible explanation for unsuccessful assistance.

29.120 Assistance Eligibility

Eligibility rules should correspond to the institutional purpose.

Let assistance need score or qualification relation be N_A . Eligibility can be represented by

$$N_A \in \mathcal{N}_{\text{eligible}}. \quad (2875)$$

Equation 2875 need not require final proof of fault where the assistance institution is need-based.

29.121 Eligibility and Stigma

Eligibility categories can create identities such as “victim,” “disabled,” “dependent,” or “failed.”

These categories can enable support and constrain future recognition.

A just design should minimize identity fixation where a narrower functional classification is sufficient.

29.122 Eligibility and Dynamic Need

Need can change.

Let eligibility state be E_t . A dynamic system permits

$$E_{t+1} = F_E(E_t, N_{t+1}, C_{t+1}). \quad (2876)$$

Equation 2876 supports entry, continuation, modification, and exit based upon relevant change.

Table 33: Failure modes in generative assistance

Failure mode	Underlying mechanism	Potential response	Atlas layer
Resource insufficiency	Support volume too low	Increase flexible resources	Resource
Conversion failure	Resources cannot become capabilities	Specific enabling support	Capability
Navigation failure	Services exist but remain unusable	Information and coordination	Epistemic / procedural
Re-entry failure	Formal access lacks practical usability	Network, role, or safety support	Relational
Portability failure	Support disappears after transition	Portable entitlement or credential	Control / reachability
Dependency failure	Support depends on discretionary control	Rights, review, exit	Non-domination
Temporal failure	Support arrives after critical window	Interim and rapid assistance	Temporal
Rigidity failure	Plan cannot adapt	Periodic review and revision	Procedural
Extraction failure	Support is absorbed by continuing harmful structure	Anti-extractive and structural intervention	Political economy
Metric failure	Success proxy distorts assistance	Plural outcome profile	Reflexive

29.123 Eligibility Review

Review should avoid requiring complete re-proving of stable historical facts.

Let historical fact set be $\mathcal{H}_V$, and current variable need set be $\mathcal{N}_t$.

A bounded review can reuse

$$\mathcal{H}_V \quad (2877)$$

and reassess only

$$\mathcal{N}_t. \quad (2878)$$

This reduces repeated administrative burden.

29.124 Assistance Termination

Termination is a consequential institutional decision.

Let support terminate at t_T .

A responsible termination assessment should consider the post-termination state

$$\mathcal{G}_{t_T+\tau}^{-S} \quad (2879)$$

rather than only the state immediately before withdrawal.

Equation 2879 incorporates the reflexive effect of termination.

29.125 Premature Termination

A subject can appear stable precisely because support is active.

Let supported state satisfy

$$V_A^S \geq V_{\min} \quad (2880)$$

while unsupported state satisfies

$$V_A^{-S} < V_{\min}. \quad (2881)$$

Equations 2880 and 2881 show why present adequacy cannot by itself justify termination.

29.126 Support-Independence Test

Where transition away from support is a legitimate goal, independence can be tested gradually.

Let support intensity decline from S to $S - \delta$. A safe test requires

$$J_S \left(\mathcal{G}^{S-\delta} \right) \geq \theta_S. \quad (2882)$$

Equation 2882 permits cautious tapering rather than abrupt withdrawal.

29.127 Long-Term Follow-Up

Long-term follow-up can identify delayed failure.

It should be proportionate and should not become permanent surveillance.

A light-touch follow-up can preserve:

- re-entry into support;
- updated information;
- institutional learning.

Participation can become optional once the institutional need for active monitoring ends.

29.128 Assistance Closure

Closure should specify which support function has ended.

Let completed assistance object set be

$$\mathfrak{D}_A^C. \quad (2883)$$

Equation 2883 can include completed navigation, completed transition funding, or completed rehabilitation.

Closure should not imply that every historical loss has been repaired.

29.129 Reopening

A closed assistance case can reopen under defined conditions.

Let reopening event set be

$$\mathcal{E}_R = \{e_{\text{failure}}, e_{\text{recurrence}}, e_{\text{new evidence}}, e_{\text{transition collapse}}\}. \quad (2884)$$

Equation 2884 preserves bounded finality while recognizing long-horizon uncertainty.

29.130 Generative-Assistance Frontier

Assistance options can trade agency, cost, security, and generativity.

Let option set be $\mathcal{A}$. The nondominated assistance frontier is

$$\mathcal{F}_A = \text{ND} (G, A, S, -C), \quad (2885)$$

where G denotes generativity, A agency, S security, and C burden or cost.

Equation 2885 defines a plural decision space.

29.131 Subject Choice on the Assistance Frontier

Where several assistance plans satisfy protected thresholds, subject choice can select among the nondominated options:

$$A^* \in \arg \max_{A \in \mathcal{F}_A^{\text{adm}}} P_A(A). \quad (2886)$$

Equation 2886 gives present agency a direct role after normative constraints are satisfied.

29.132 Assistance Minimalism

Generative assistance should avoid constructing an unnecessarily comprehensive model of the person's life.

A bounded principle is:

Provide the least intrusive combination of resources, enabling conditions, and institutional support sufficient to make substantively acceptable subject-directed futures practically available.

This protects privacy and agency while preserving effectiveness.

29.133 Assistance Escalation

Support can escalate when simpler interventions fail.

A schematic path is represented by

cash $\rightarrow$ navigation $\rightarrow$ specific enabling support $\rightarrow$ coordinated assistance $\rightarrow$ structural intervention. (2887)

Equation 2887 is diagnostic rather than a universal sequence.

The relevant step depends upon the identified bottleneck.

29.134 Assistance De-Escalation

Support can also become lighter as the subject's viable future set becomes more secure.

De-escalation should preserve fallback mechanisms where sudden withdrawal would recreate vulnerability.

The goal is reduced unnecessary institutional involvement rather than symbolic independence.

29.135 Generative-Assistance Statement

A fully specified assistance claim can be represented by

$$\mathcal{A}_G^* = [\mathcal{G}_A^*; \mathcal{R}_\tau^A; M; C^*; N; P; S; V; K; \tau], \quad (2888)$$

where:

- $\mathcal{G}_A^*$ denotes subject-valued goals;
- $\mathcal{R}_\tau^A$ reachable futures under assistance;
- M general resources;
- C^* specific enabling conditions;
- N navigation;

- P portability;
- S support security;
- V revisability;
- K epistemic basis;
- τ horizon.

Equation 2888 makes assistance claims explicit and auditable.

29.136 Future-Possibility Principle

The first principle of this section is:

Generative assistance should be evaluated through the viable and usable future possibilities it creates rather than through the amount of resources delivered alone.

29.137 Layered-Support Principle

The second principle is:

Where conversion depends upon both fungible resources and specific enabling conditions, assistance should permit layered support rather than treating cash and direct provision as mutually exclusive models.

29.138 Subject-Direction Principle

The third principle is:

The historically actual subject should possess meaningful authority over which substantively admissible future the assistance is used to pursue.

29.139 Conversion Principle

The fourth principle is:

Failure of a monetary or resource transfer should trigger examination of conversion conditions before the institution concludes that greater effort or motivation is required from the subject.

29.140 Navigation Principle

The fifth principle is:

Institutional resources that are formally available and practically difficult to discover or access remain incomplete generative conditions.

29.141 Re-Entry Principle

The sixth principle is:

Formal reinstatement should be distinguished from effective re-entry into the relational, informational, and institutional structures through which future opportunity is generated.

29.142 Safe-Experimentation Principle

The seventh principle is:

Where the subject's future trajectory is uncertain, assistance should support reversible experimentation and preserve a viable fallback where feasible.

29.143 Portability Principle

The eighth principle is:

Restorative resources and rights should be portable across institutional transitions where unnecessary lock-in would otherwise constrain exit and future agency.

29.144 Legitimate-Dependence Principle

The ninth principle is:

Continuing support does not constitute restorative failure merely because the subject remains dependent upon it; the relevant questions concern security, control, dignity, responsiveness, and the possibility of domination.

29.145 Revisability Principle

The tenth principle is:

Long-horizon assistance should remain revisable as evidence, circumstances, and the subject's legitimate priorities change, while core entitlements remain sufficiently stable for planning.

29.146 Non-Stigmatization Principle

The eleventh principle is:

Assistance should avoid converting temporary or historically produced vulnerability into a permanent subordinating identity where a narrower functional classification is sufficient.

29.147 Structural-Learning Principle

The twelfth principle is:

Repeated appearance of the same assistance bottleneck across cases should trigger investigation of the institutional structure generating that bottleneck.

29.148 Generative-Assistance Layer of the Atlas

The generative-assistance layer can now be represented by

$$\mathfrak{A}_G = (M, C^*, N, R, T, X, P, S, V, \mathcal{G}_A^*), \quad (2889)$$

where the components denote flexible resources, specific enabling conditions, navigation, relational access, time, experimentation, portability, support security, revisability, and subject-directed goals.

Equation 2889 locates assistance as the bridge between post-harm remedy and future generativity.

29.149 Implications for Preventive Governance

Generative assistance reveals recurrent patterns of lost or impaired enabling conditions. Those patterns can inform prevention.

If many subjects repeatedly require the same post-harm intervention, the underlying institution can possess a preventable structural weakness.

Assistance data can therefore identify:

- fragile interfaces;
- recurring bottlenecks;
- single points of dependency;
- delayed intervention;
- low-portability conditions;
- recurring conversion failure.

The next step is to ask how governance can protect generative conditions before they are lost.

29.150 Transition to Preventive Governance

Section 30 develops preventive governance of generative conditions. It examines critical-condition mapping, early warning, redundancy, anti-retaliation, option preservation, intervention thresholds, precaution, reversible safeguards, evidence preservation, distributed monitoring, and protection against recursive propagation of harm.

The central question becomes:

If post-harm assistance can identify the conditions whose loss repeatedly constrains future generativity, how should institutions protect those conditions before the loss becomes difficult, costly, or impossible to repair?

This transition moves the manuscript from the governance of post-harm regeneration toward the governance of reparability itself.

30 Preventive Governance of Generative Conditions

This section develops preventive governance as the temporal counterpart to generative assistance. Section 29 examined how institutions can restore or expand future possibility after generative conditions have already been impaired. The present section asks how institutions can identify, protect, buffer, and preserve those conditions before their loss produces difficult, costly, or historically irreversible consequences.

Preventive governance differs from a general policy of minimizing every possible risk. Generative systems require experimentation, variation, movement, relationship formation, institutional discretion, and exposure to uncertainty. A system designed to eliminate all disturbance can suppress the very generativity it seeks to protect. Prevention therefore concerns preservation of valuable future possibility under uncertainty while retaining agency, revisability, proportionality, and room for legitimate change.

The central object of prevention is consequently neither one fixed state nor one predetermined future. It is the structure of conditions through which subjects and systems remain capable of generating admissible future trajectories.

The section develops six major propositions. First, preventive governance should identify critical generative conditions and distinguish them from ordinary resources whose loss is readily reversible.

Second, expected reparability, propagation potential, temporal singularity, and dependence structure should inform the urgency of protection. Third, prevention should preserve option structure through redundancy, buffers, portability, and reversible safeguards. Fourth, early warning should support staged intervention rather than automatic punishment. Fifth, reporting, evidence preservation, and distributed epistemic infrastructure are themselves preventive conditions. Sixth, preventive power requires limits because excessive monitoring, paternalistic intervention, and risk classification can create new forms of domination and generative suppression.

The analysis draws upon several bodies of existing work without reducing preventive governance to any single one of them. Resilience theory distinguishes persistence and recovery of systems under disturbance [24]. Research on critical transitions and early-warning signals demonstrates how changes in system dynamics can precede regime shifts [52, 53]. Republican non-domination contributes a reason to protect subjects against discretionary control exercised in the name of safety [44]. Generative justice adds attention to the conditions through which value and future productive capacity remain under just forms of circulation and control [15, 17]. The framework developed here connects these concerns through an explicitly restorative and preventive architecture.

30.1 Preventive Governance as Preservation of Future Possibility

Let the system at time t be $\mathcal{G}_t$, and let its admissible future set over horizon τ be

$$\mathcal{F}_{t,\tau}^{\text{adm}} = \mathcal{R}_\tau(\mathcal{G}_t) \cap \mathcal{N}, \quad (2890)$$

where $\mathcal{R}_\tau$ denotes the reachable set and $\mathcal{N}$ the normatively admissible region. Equation 2890 makes future possibility itself a preventive object.

A preventive intervention P is valuable where it preserves or improves this set:

$$\mathcal{F}_{t+\tau}^{\text{adm}}(P) \succeq \mathcal{F}_{t+\tau}^{\text{adm}}(0). \quad (2891)$$

Equation 2891 permits prevention to be evaluated through inclusion, viability, diversity, security, or another future-appropriate relation.

30.2 Prevention and Static Preservation

Preventive governance should not be interpreted as preservation of every current state variable.

A changing system can remain healthy and generative.

Let current state be x_t and later admissible state be x_{t+1} . Legitimate change can satisfy

$$x_{t+1} \neq x_t \quad (2892)$$

while

$$\mathcal{F}_{t+1,\tau}^{\text{adm}} \succeq \mathcal{F}_{t,\tau}^{\text{adm}}. \quad (2893)$$

Equations 2892 and 2893 distinguish preservation of generativity from preservation of stasis.

30.3 Critical Generative Conditions

Let the generative condition family be

$$\mathcal{C} = \{c_1, \dots, c_n\}. \quad (2894)$$

A condition is preventively critical where its loss produces a large contraction of valuable future possibility, severe propagation, or weakly reparable harm.

Let removal of condition c_i produce future set $\mathcal{F}^{-i}$. A simple criticality measure is

$$K_i = D_F(\mathcal{F}, \mathcal{F}^{-i}), \quad (2895)$$

where D_F is an appropriate distance, dominance, or loss relation over future sets.

Equation 2895 makes criticality a consequence of lost future structure rather than the nominal size of the condition.

30.4 Criticality Profile

Criticality is multidimensional.

Let

$$\mathbf{K}_i = (K_i^R, K_i^P, K_i^S, K_i^T, K_i^D, K_i^C), \quad (2896)$$

where the components denote reachability impact, propagation potential, singularity, temporal sensitivity, dependency, and collective impact.

Equation 2896 avoids compressing criticality into one universal risk score.

30.5 Reparability

Let expected reparability of condition c_i after loss be

$$\rho_i = \text{Rep}(c_i). \quad (2897)$$

Equation 2897 can reflect technical, institutional, economic, relational, and historical reparability.

A condition that can be replaced quickly and cheaply requires a different preventive response from one whose loss destroys a singular historical window.

30.6 Reparability-Weighted Prevention

A general preventive tendency can be represented by

$$\rho_i \downarrow \implies W_i^P \uparrow, \quad (2898)$$

other relevant factors held constant, where W_i^P is the normative weight assigned to preventing loss of c_i .

Equation 2898 is a qualified principle. Low reparability alone does not authorize intrusive prevention. Severity, probability, rights, uncertainty, and intervention burden remain relevant.

30.7 Singularity

A condition can be critical because it is singular.

Let substitutability be σ_i . Low substitutability satisfies

$$\sigma_i \rightarrow 0. \quad (2899)$$

Equation 2899 can describe a unique relationship, historical opportunity, ecological condition, authorship event, or time-specific institutional window.

Preventive protection can be especially important where no later equivalent can be produced through money or ordinary replacement.

30.8 Temporal Windows

Let condition c_i possess a critical temporal window

$$W_i = [t_i^0, t_i^1]. \quad (2900)$$

If intervention occurs after t_i^1 , effective reparability can fall sharply.

A time-sensitive reparability function can be written as

$$\rho_i = \rho_i(t). \quad (2901)$$

Equation 2901 explains why speed can itself be a component of preventive justice.

30.9 Propagation Potential

Initial loss can generate downstream losses.

Let the propagation graph be

$$G_H = (V_H, E_H), \quad (2902)$$

where nodes are affected conditions and edges represent pathways of influence.

For condition c_i , propagation potential can be represented by

$$P_i = \sum_j w_{ij} \Pr(c_j \text{ impaired} \mid c_i \text{ impaired}). \quad (2903)$$

Equation 2903 is illustrative.

The central point is that a small initial impairment can justify rapid attention where it initiates large recursive effects.

30.10 Recursive Amplification

Let loss evolve through

$$L_{t+1} = F_L(L_t). \quad (2904)$$

Where

$$F_L(L_t) > L_t \quad (2905)$$

over the relevant range, harm is recursively amplifying.

Equations 2904 and 2905 identify situations where early interruption can have disproportionately high value.

30.11 Dependency Structure

A generative system can depend strongly upon a small number of nodes or actors.

Let dependency matrix be

$$D = [d_{ij}], \quad (2906)$$

where d_{ij} represents the dependence of condition or actor i upon j .

A concentrated dependency structure can produce single points of failure.

30.12 Single Points of Failure

Let node j be a critical dependency for many functions.

A simple concentration index is

$$S_j = \sum_i d_{ij}. \quad (2907)$$

Equation 2907 provides a diagnostic indicator of potential systemic fragility.

Examples can include:

- one gatekeeper controlling professional access;
- one database controlling downstream classification;
- one supplier controlling essential infrastructure;
- one administrator controlling continuing support.

30.13 Redundancy

Redundancy creates alternative pathways where one condition fails.

Let the set of alternative paths to valued target g be

$$\Pi_g = \{\pi_1, \dots, \pi_k\}. \quad (2908)$$

Equation 2908 makes redundancy a reachability property.

A system with several partially independent paths can preserve future possibility after loss of one path.

30.14 Redundancy and Efficiency

Redundancy can impose cost.

Let redundancy level be r , operating cost $C(r)$, and robustness $\varrho(r)$.

The trade-off can be represented by

$$\frac{dC}{dr} > 0 \quad \text{and} \quad \frac{d\varrho}{dr} > 0 \quad (2909)$$

over a relevant region.

Equation 2909 shows why preventive governance cannot maximize redundancy without limit.

The relevant question concerns the value of resilience relative to cost, complexity, and agency.

30.15 Diversity

Diversity can reduce common-mode failure where different pathways respond differently to disturbance.

Let pathway responses be $R_{\pi_i}(\delta)$.

Diversity has preventive value where

$$R_{\pi_i}(\delta) \neq R_{\pi_j}(\delta) \quad (2910)$$

for relevant perturbations.

Equation 2910 means one disturbance is less likely to destroy every pathway simultaneously.

30.16 Modularity

Modularity limits propagation across subsystems.

Let system consist of modules $\mathcal{M}_1, \dots, \mathcal{M}_k$.

Weak intermodule coupling can reduce propagation:

$$\left| \frac{\partial x_i}{\partial x_j} \right| \ll 1 \quad (2911)$$

for selected cross-module pairs.

Equation 2911 gives a local formal interpretation of modular isolation.

Excessive modularity can inhibit coordination and shared learning.

30.17 Buffers

Buffers absorb temporary disturbance before critical conditions collapse.

Let resource reserve be B_t , disturbance burden be δ_t , and critical minimum be $B_{\min}$.

A basic buffer condition is

$$B_t - \delta_t \geq B_{\min}. \quad (2912)$$

Equation 2912 can apply to financial reserves, staffing, time, storage, network redundancy, or ecological capacity.

30.18 Buffer Depletion

Repeated shocks can consume reserves.

Let

$$B_{t+1} = B_t - \delta_t + r_t, \quad (2913)$$

where r_t is replenishment.

Equation 2913 makes replenishment rate part of preventive resilience.

A system can survive isolated shocks and remain vulnerable to repeated ones.

30.19 Minimum Reserve

A preventive system can establish a reserve threshold

$$B_t \geq B_{\text{safe}}. \quad (2914)$$

Equation 2914 can trigger replenishment before the system reaches a critical boundary.

The appropriate threshold depends upon uncertainty, shock distribution, and cost of reserve maintenance.

30.20 Option Preservation

Prevention can preserve options even where no immediate harm is visible.

Let future option set before intervention be $\mathcal{O}_t$.

A preventive decision should avoid unnecessary contraction:

$$\mathcal{O}_{t+1}^P \supseteq \mathcal{O}_{t+1}^0. \quad (2915)$$

Equation 2915 gives option value an explicit governance role.

This is especially important under uncertainty concerning future preferences and technologies.

30.21 Reversibility

Preventive interventions should remain reversible where uncertainty is high.

Let intervention be P , with reversal operator P^{-1} .

Approximate reversibility can satisfy

$$d(P^{-1}(P(x)), x) \leq \varepsilon_P. \quad (2916)$$

Equation 2916 permits protective action while limiting lock-in.

30.22 Intervention Reversibility and Evidence Threshold

Reversible measures can justify lower evidentiary thresholds than irreversible ones.

Let required evidence for intervention P be $\theta_K(P)$.

A general relation can be

$$\text{Rev}(P) \downarrow \implies \theta_K(P) \uparrow. \quad (2917)$$

Equation 2917 is a proportionality principle, not a universal numerical law.

30.23 Precaution

Precaution becomes relevant where potential harm is serious, uncertainty is material, and later repair can be weak.

Let estimated probability of harm be p_H , severity S_H , reparability ρ_H , and intervention burden C_P .

A schematic preventive decision object is

$$\Pi_P = (p_H, S_H, \rho_H, C_P). \quad (2918)$$

Equation 2918 makes precaution multidimensional.

Low probability alone should not decide the issue where severity and irreparability are extreme.

30.24 Precaution and Proportionality

Precaution should remain proportional.

Let protective intensity be α_P .

A legitimate intervention should consider whether

$$\alpha_P \leq \alpha_{\max}(S_H, p_H, \rho_H, K), \quad (2919)$$

where K denotes epistemic support.

Equation 2919 prevents uncertainty from functioning as unlimited authority for restrictive action.

30.25 Least-Restrictive Prevention

Where several interventions reduce risk comparably, the less intrusive measure has a procedural advantage.

Let

$$\Delta\rho_H(P_1) \sim \Delta\rho_H(P_2) \quad (2920)$$

and

$$C_P(P_1) < C_P(P_2). \quad (2921)$$

Equations 2920 and 2921 support selection of P_1 where other relevant considerations are comparable.

30.26 Early Warning

Preventive governance requires mechanisms for detecting movement toward dangerous regions before critical loss occurs.

Let warning indicator vector be

$$\mathbf{W}_t = (W_t^S, W_t^T, W_t^N, W_t^R, W_t^E), \quad (2922)$$

where the components represent state, trend, network, relational, and event-based signals. Equation 2922 allows multiple forms of evidence to contribute to preventive assessment.

30.27 State-Based Warning

A state-based warning occurs when a variable approaches a critical boundary.

Let critical threshold be x_c .

Warning can be triggered where

$$d(x_t, x_c) \leq \varepsilon_W. \quad (2923)$$

Equation 2923 is appropriate where the relevant boundary is known.

30.28 Trend-Based Warning

A system can remain far from a known threshold and deteriorate rapidly.

Let indicator be z_t .

A trend warning can use

$$\frac{dz_t}{dt} < -\theta_z. \quad (2924)$$

Equation 2924 captures deterioration rate rather than state level alone.

30.29 Critical-Transition Warning

Systems approaching critical transitions can exhibit changes in recovery and variance. The ecological literature on early-warning signals provides an important precedent for studying such dynamical indicators [53].

Let recovery rate from perturbation be λ_t . A slowing recovery can be represented by

$$|\lambda_t| \downarrow. \quad (2925)$$

Equation 2925 can function as one warning signal where the underlying dynamical assumptions are empirically justified.

No single indicator should be treated as universal evidence of an approaching social crisis.

30.30 Network Warning

Network deterioration can precede visible outcome loss.

Let network connectivity or bridging capacity be B_t^N .

A warning can be generated by

$$B_{t+1}^N < B_t^N - \theta_B. \quad (2926)$$

Equation 2926 can be relevant to organizational, professional, infrastructure, or community networks.

30.31 Event-Based Warning

Some events should trigger review even before measurable deterioration appears.

Let trigger-event family be

$$\mathcal{E}_W = \{e_1, \dots, e_k\}. \quad (2927)$$

Equation 2927 can contain retaliation reports, unexpected access revocation, data-integrity failures, or abrupt resource withdrawal.

Event-based warning is especially useful where the event itself threatens a critical generative condition.

30.32 Warning Aggregation

Multiple warnings can be combined without forcing a single scalar score.

Let warning profile be $\mathbf{W}_t$.

A partial-order trigger can require

$$\mathbf{W}_t \succeq \boldsymbol{\theta}_W. \quad (2928)$$

Equation 2928 permits several independent thresholds.

This can be more transparent than one opaque composite risk number.

30.33 Warning Confidence

A warning should include uncertainty.

Let warning confidence be

$$K_W = P(H \mid \mathbf{W}_t, M), \quad (2929)$$

where M denotes the warning model.

Equation 2929 makes the warning conditional upon a model rather than presenting it as direct knowledge of future harm.

30.34 False Positives

Preventive systems can intervene where severe harm would not have occurred.

Let false-positive probability be

$$p_{\text{FP}} = P(P = 1, H = 0). \quad (2930)$$

Equation 2930 matters because preventive intervention can itself burden subjects and third parties.

30.35 False Negatives

A false negative occurs when the system fails to intervene before harm.

Let

$$p_{\text{FN}} = P(P = 0, H = 1). \quad (2931)$$

Equation 2931 can carry especially large cost where harm is irreversible or recursively amplifying. Preventive threshold design therefore depends upon asymmetric error costs.

30.36 Error-Cost Profile

Let false-positive burden be C_{FP} and false-negative burden C_{FN} .
The preventive error profile is

$$C_E = (p_{FP}C_{FP}, p_{FN}C_{FN}). \quad (2932)$$

Equation 2932 preserves both error types rather than reducing them immediately to one expected-cost scalar.

30.37 Staged Intervention

Warnings should ordinarily trigger staged intervention.
Let intervention levels be

$$\mathcal{P} = \{P_0, P_1, \dots, P_m\}, \quad (2933)$$

with increasing intensity.

Equation 2933 permits a progression from observation and consultation toward stronger protective action as evidence and risk increase.

30.38 Escalation Thresholds

Let warning state be W_t . Escalation from P_i to P_{i+1} occurs when

$$W_t \in \mathcal{E}_{i+1}, \quad (2934)$$

where $\mathcal{E}_{i+1}$ is the specified escalation region.

Equation 2934 makes intervention escalation rule-governed rather than purely discretionary.

30.39 De-Escalation Thresholds

Protective measures should also possess de-escalation conditions.
Let

$$W_t \in \mathcal{D}_{i-1} \quad (2935)$$

trigger movement from P_i to P_{i-1} .

Equation 2935 prevents preventive interventions from remaining permanently active after the triggering conditions disappear.

30.40 Hysteresis in Intervention Rules

Using separate escalation and de-escalation thresholds can reduce rapid switching.
Let escalation threshold be θ_E and de-escalation threshold θ_D , with

$$\theta_D < \theta_E. \quad (2936)$$

Equation 2936 creates a hysteresis band.

This can stabilize governance where indicators fluctuate near one threshold.

30.41 Finite-Step Emergency Simulation

In high-risk contexts, institutions can simulate a bounded number of future steps rather than attempting complete long-run prediction.

Let transition model be F , and horizon be h .

A finite-step forecast is

$$x_{t+h} = F^h(x_t, u_{t:t+h-1}, \xi_{t:t+h-1}). \quad (2937)$$

Equation 2937 supports near-term intervention without claiming complete future knowledge.

30.42 Emergency Horizon

Let the longest horizon over which predictions remain sufficiently robust be h^* .

Define

$$h^* = \sup \{h : K_h \geq K_{\min}\}, \quad (2938)$$

where K_h denotes confidence or robustness at horizon h .

Equation 2938 prevents emergency models from being extended beyond their epistemic competence.

30.43 Temporary Protective Measures

Where risk is serious and evidence remains incomplete, reversible temporary measures can preserve the system while fuller review occurs.

Let measure P_T operate until t_R .

The measure satisfies

$$P_T(t) = 0 \quad t > t_R \quad (2939)$$

unless affirmatively renewed.

Equation 2939 creates a default sunset.

30.44 Review of Temporary Measures

Temporary measures should generate evidence for review.

Let new evidence accumulated during the temporary period be ΔK .

The later decision can be represented by

$$P_{t_R} = \mathcal{U}(P_T, K_0 + \Delta K). \quad (2940)$$

Equation 2940 converts provisional protection into an information-generating stage.

30.45 Anti-Retaliation

Retaliation can suppress the information required for prevention.

Let reporting action be $R_A = 1$, and expected retaliatory loss be L_{ret} .

Effective reporting capacity requires

$$\mathbb{E}[L_{\text{ret}} \mid R_A = 1] \leq \theta_{\text{ret}}. \quad (2941)$$

Equation 2941 makes anti-retaliation a precondition of preventive epistemology.

30.46 Safe Reporting Channels

A system can maintain several reporting routes:

$$\mathcal{R}_{\text{report}} = \{r_1, \dots, r_k\}. \quad (2942)$$

Equation 2942 reduces dependence upon one gatekeeper.

Channels can include internal, independent, anonymous, professional, or external routes depending upon context.

30.47 Reporting Redundancy

If one reporting channel is controlled by the actor generating the risk, redundancy becomes especially important.

Let availability of channel r_i under disturbance δ be $A_i(\delta)$.

Reporting robustness requires

$$\sum_i A_i(\delta) \geq 1 \quad \forall \delta \in \Delta_{\text{report}}. \quad (2943)$$

Equation 2943 requires at least one usable channel under the selected disturbance class.

30.48 Evidence Preservation

Evidence can disappear while institutions delay action.

Let evidence availability at time t be E_t .

A simple decay model is

$$E_{t+1} = \delta_E E_t, \quad 0 \leq \delta_E \leq 1. \quad (2944)$$

Equation 2944 illustrates why delay can reduce future epistemic recoverability.

Preservation measures can therefore be justified before substantive interpretation is complete.

30.49 Preservation without Interpretive Freezing

Evidence preservation should be distinguished from freezing one interpretation.

Let raw evidentiary object be e , and interpretations be $I_1(e), \dots, I_k(e)$.

Preservation should seek

$$e_{t+1} = e_t \quad (2945)$$

without requiring

$$I_1(e) = I_2(e). \quad (2946)$$

Equations 2945 and 2946 support evidentiary integrity and later contestability simultaneously.

30.50 Provenance Preservation

Evidence should retain provenance.

Let evidentiary provenance be

$$\text{Prov}(e) = (o, t, a, m), \quad (2947)$$

where o denotes origin, t time, a actor or source, and m subsequent modification history.

Equation 2947 supports later verification, correction, and accountability.

30.51 Distributed Evidence

Critical evidence can be distributed across actors and institutions.

Let evidence set be

$$\mathcal{E} = \bigcup_{j=1}^n \mathcal{E}_j. \quad (2948)$$

Equation 2948 means no single institution necessarily possesses a complete view.

Preventive governance can therefore require interoperable evidentiary interfaces.

30.52 Shared Epistemic Infrastructure

Multi-party governance can share epistemic infrastructure without centralizing every decision.

Let shared infrastructure be

$$\mathcal{I}_E = (D, M, S, P), \quad (2949)$$

where D denotes distributed data interfaces, M shared measurement standards, S security, and P provenance.

Equation 2949 can support coordination while preserving decentralized control over raw data and decisions.

30.53 Shared Evidence and Distributed Decision

Let institution i possess decision function

$$D_i = F_i(\mathcal{I}_E, K_i, N_i), \quad (2950)$$

where K_i denotes local knowledge and N_i the institution's normative mandate.

Equation 2950 allows shared evidence with heterogeneous decision rules.

Common epistemic infrastructure therefore does not imply centralized governance.

30.54 Local Processing

Where raw data are sensitive, institutions can process information locally and share only the transformed output required for joint governance.

Let raw local data be D_i , local transformation T_i , and shared output S_i :

$$S_i = T_i(D_i). \quad (2951)$$

Equation 2951 reduces unnecessary global exposure of raw personal data.

30.55 Minimum Necessary Sharing

Let shared information be $\mathcal{D}_S$, and minimum information needed for the common preventive function be $\mathcal{D}_{\min}$.

A minimization criterion is

$$\mathcal{D}_S \subseteq \mathcal{D}_{\min} \cup \mathcal{D}_{\text{authorized}}. \quad (2952)$$

Equation 2952 limits common infrastructure from becoming general surveillance.

30.56 Polycentric Monitoring

Several actors can monitor different parts of a system.

Let monitoring nodes be $M_1, \dots, M_k$, with observation sets $\mathcal{O}_1, \dots, \mathcal{O}_k$.

The combined monitoring field is

$$\mathcal{O}_P = \bigcup_{i=1}^k \mathcal{O}_i. \quad (2953)$$

Equation 2953 permits specialized observation without requiring one omniscient authority.

30.57 Monitoring Independence

Monitoring can fail where the actor being monitored controls the observation channel.

Let monitor M depend upon monitored actor A through D_{MA} .

High dependence satisfies

$$D_{MA} \gg 0. \quad (2954)$$

Equation 2954 identifies a reason for independent or redundant monitoring.

30.58 Monitoring Burden

Preventive monitoring can itself impose cost and alter behavior.

Let monitoring intensity be m , epistemic benefit $K(m)$, and burden $B(m)$.

Net preventive value can be represented by

$$V_M(m) = K(m) - B(m). \quad (2955)$$

Equation 2955 illustrates why maximum monitoring is not automatically optimal.

30.59 Surveillance Risk

Continuous preventive observation can create:

- privacy loss;
- behavioral conformity;
- chilling effects;
- dependency upon institutional approval;
- classification feedback.

Preventive epistemology therefore requires privacy and non-domination constraints.

30.60 Classification Feedback

Risk classification can alter the future being predicted.

Let classification be

$$Z_t = C(\mathcal{G}_t). \quad (2956)$$

If later state is

$$\mathcal{G}_{t+1} = F(\mathcal{G}_t, Z_t), \quad (2957)$$

then the classification is reflexive.

Equations 2956 and 2957 show why preventive models should be evaluated partly through the effects of their own labels.

30.61 Preventive Stigmatization

A high-risk label can reduce opportunity even when intended as protection.

Let risk label be Z_H . It can produce

$$\mathcal{R}_\tau(Z_H) \subset \mathcal{R}_\tau(Z_0), \quad (2958)$$

where Z_0 denotes absence of the label.

Equation 2958 identifies a preventive intervention that itself contracts future possibility.

30.62 Risk Classification and Contestability

Persons affected by consequential risk classifications should possess a route to challenge:

- data;
- classification criteria;
- current applicability;
- downstream use.

Contestability prevents preventive classifications from becoming permanent identity assignments.

30.63 Expiration of Preventive Labels

Let preventive classification Z_P expire after period τ_Z unless renewed.

The expiration rule is

$$Z_P(t) = 0 \quad t > t_0 + \tau_Z \quad (2959)$$

absent a new justified determination.

Equation 2959 reduces historical lock-in from temporary risk assessments.

30.64 Critical Infrastructure

Some generative conditions are infrastructural.

Let infrastructure family be

$$\mathcal{I}_G = \{I_{\text{legal}}, I_{\text{financial}}, I_{\text{health}}, I_{\text{information}}, I_{\text{transport}}, I_{\text{ecological}}\}. \quad (2960)$$

Equation 2960 includes systems whose failure affects many subjects simultaneously.

Preventive governance at this scale concerns resilience and shared capacity as well as individual protection.

30.65 Infrastructure Dependency

Let individual capability be

$$\mathcal{K}_i = F(R_i, \mathcal{I}_G). \quad (2961)$$

Equation 2961 shows how common infrastructure conditions private generativity.

Individual compensation after systemic failure can therefore be less effective than infrastructure protection.

30.66 Infrastructure Redundancy

Let infrastructure function f be deliverable through channels $I_1, \dots, I_k$.

A redundancy condition is

$$\sum_{j=1}^k A_j \geq r_{\min}, \quad (2962)$$

where A_j indicates channel availability and $r_{\min}$ is the required minimum number of available pathways.

Equation 2962 can be adapted to context.

30.67 Common-Mode Failure

Nominally separate channels can share one hidden dependency.

Let channels I_1 and I_2 both depend upon component c :

$$I_1 \leftarrow c \rightarrow I_2. \quad (2963)$$

Equation 2963 shows why numerical redundancy alone does not guarantee resilience.

Preventive analysis should identify shared dependencies.

30.68 Institutional Portability

Portability protects subjects when one institution fails.

Let entitlement or credential q be recognized across institutions through

$$R_j(q) = 1 \quad j \in J_q. \quad (2964)$$

Equation 2964 reduces dependency upon one institutional node.

Portability can therefore function as preventive non-domination.

30.69 Preventive Social Protection

Social-protection floors can buffer shocks before they cascade into severe generative loss [26].

Let floor be $C_{\min}$. A preventive social-protection condition is

$$C_{A,t} \geq C_{\min} \quad \forall t \in [t_0, t_1]. \quad (2965)$$

Equation 2965 converts social protection into a continuity safeguard.

30.70 Shock Absorption

Let external shock be δ . Without protection, subject state becomes x^δ . With protection P_S ,

$$d(x^{\delta, P_S}, x^0) < d(x^\delta, x^0). \quad (2966)$$

Equation 2966 formalizes the buffering role of social protection.

30.71 Preventive Exit

Exit can prevent escalation where a relation becomes harmful.

Let current relation be r , and exit cost be C_{exit} .

Preventive governance can seek

$$C_{\text{exit}} \leq C_{\text{max}}. \quad (2967)$$

Equation 2967 makes viable exit part of risk governance.

A subject trapped in one institution can be vulnerable even before overt harm occurs.

30.72 Preventive Portability

Portable credentials, benefits, records, and resources reduce the cost of leaving a deteriorating system.

Portability therefore serves prevention by preserving alternative pathways before harm becomes severe.

30.73 Preventive Non-Domination

Prevention should also protect against concentration of arbitrary power.

Let domination exposure be D_A .

A preventive governance objective can require

$$D_A \leq D_{\text{max}}. \quad (2968)$$

Equation 2968 can justify independent review, portable rights, divided authority, or alternative channels [44].

30.74 Gatekeeper Risk

A gatekeeper controls access to a valuable condition.

Let actor g control condition set $\mathcal{C}_g$.

Gatekeeper concentration can be represented by

$$K_g = \frac{|\mathcal{C}_g|}{|\mathcal{C}|}. \quad (2969)$$

Equation 2969 can indicate vulnerability where one actor controls many critical conditions.

The metric is illustrative rather than universally sufficient.

30.75 Gatekeeper Redundancy

Preventive alternatives can include:

- multiple approvers;
- appeal;
- independent access routes;
- automatic entitlement;
- portable credentials.

These mechanisms reduce the generative significance of one adverse gatekeeper decision.

30.76 Preventive Governance of Reputation

Reputational information can propagate rapidly before factual correction.

Let classification propagate through network G_Z .

Propagation at time t can be represented by

$$Z_{t+1} = \mathcal{P}_Z(Z_t, G_Z). \quad (2970)$$

Equation 2970 creates a preventive case for provenance, confidence labels, appeal, and correction propagation.

30.77 Provisional Classification

Unverified information should remain visibly provisional where the distinction matters.

Let provisional classification be Z^P , and verified classification Z^V .

The system should preserve

$$Z^P \neq Z^V. \quad (2971)$$

Equation 2971 prevents temporary allegation from becoming operationally indistinguishable from established fact.

30.78 Correction Propagation

A correction should travel at least as far as the operationally relevant erroneous record.

Let erroneous dissemination set be V_E , and corrected set V_C .

A strong correction condition is

$$V_E^{\text{active}} \subseteq V_C. \quad (2972)$$

Equation 2972 protects against future harm from stale classifications.

30.79 Preventive Governance of Employment and Professional Access

Professional systems can preserve generativity through:

- anti-retaliation;
- portable credentials;
- multiple supervisors or reviewers;
- independent grievance channels;
- transparent access rules.

The preventive objective is to avoid one conflict producing irreversible professional exclusion.

30.80 Professional Redundancy

Let career-generating access depend upon evaluators $e_1, \dots, e_k$.

A multi-evaluator system reduces the control of one evaluator where

$$\text{Influence}(e_i) < 1 \quad (2973)$$

for each single evaluator.

Equation 2973 is a conceptual condition against absolute gatekeeping.

30.81 Preventive Governance of Research Systems

Research generativity can be protected through:

- independent authorship records;
- portable data and provenance;
- funding continuity mechanisms;
- independent misconduct review;
- multiple collaboration pathways.

The aim is to prevent one institutional conflict from destroying an entire research trajectory.

30.82 Authorship Provenance

Let contribution events be $e_1, \dots, e_n$.

Authorship provenance can be represented by

$$\mathcal{P}_A = (\mathcal{E}, \prec, \alpha), \quad (2974)$$

where $\prec$ orders contribution events and α records attribution.

Equation 2974 can preserve evidence before recognition disputes arise.

30.83 Preventive Governance of Health-Related Generativity

Health conditions can become generative bottlenecks long before complete functional loss occurs.

Preventive support can include:

- early accommodation;
- time flexibility;
- continuity of treatment;
- income protection;
- preservation of professional status.

Such measures can reduce secondary social and economic harm without making medical treatment itself the sole object.

30.84 Preventive Governance of Ecological Systems

Ecological systems illustrate especially clearly why prevention can dominate later reconstruction.

Alternative stable regimes, thresholds, and hysteresis can make return after a critical transition difficult [24, 52].

Let ecological state parameter be x , and critical threshold be x_c .

A preventive safety margin is

$$m_E = d(x, x_c). \quad (2975)$$

Equation 2975 makes distance to critical transition a preventive object.

30.85 Ecological Uncertainty

Ecological dynamics are often only partially known.

A precautionary system can therefore protect a safety region

$$\mathcal{S}_E \subset \mathcal{X}_E \quad (2976)$$

that remains robust across several plausible models.

The objective is resilience under uncertainty rather than exact optimization against one forecast.

30.86 Preventive Governance of Collective Capacity

Communities and institutions can lose generative capacity gradually.

Preventive indicators can include:

- loss of distributed knowledge;
- concentration of decision authority;
- collapse of participation;
- infrastructure deterioration;
- loss of intergenerational transmission.

The preventive object is collective capacity rather than the welfare of one individual alone.

30.87 Intergenerational Prevention

Some losses become visible only across generations.

Let transmitted condition be

$$C^{(g+1)} = \Gamma \left(C^{(g)} \right). \quad (2977)$$

Equation 2977 can represent ecological, cultural, institutional, or economic transmission.

Preventive governance can seek

$$C^{(g+1)} \succeq C_{\min}^{(g+1)}. \quad (2978)$$

Equation 2978 makes future inherited conditions part of present governance.

30.88 Institutional Memory

Prevention depends upon institutional capacity to remember prior failures.

Let case history archive be $\mathcal{H}_I$.

Learning function is

$$\Pi_{t+1} = L \left(\Pi_t, \mathcal{H}_I \right). \quad (2979)$$

Equation 2979 allows historical cases to alter future policy.

A system that repeatedly forgets verified harms recreates preventable vulnerability.

30.89 Institutional Forgetting

Institutional memory must remain selective.

Permanent retention of every individual detail can create privacy and stigma.

The preventive objective is therefore to preserve:

- structural lessons;
- relevant provenance;
- validated risk mechanisms;

while limiting unnecessary continued use of personal classifications.

30.90 Learning without Permanent Labeling

Let structural lesson be L_S , and personal identifier be I_A .

A privacy-sensitive learning transformation can seek

$$(I_A, e_H) \xrightarrow{T} L_S \quad (2980)$$

while restricting future circulation of I_A .

Equation 2980 separates institutional learning from permanent personal labeling.

30.91 Preventive Auditing

Audits can examine whether harmful patterns recur across cases.

Let case indicators be $z_{i,t}$.

Aggregate pattern is

$$Z_t^\Sigma = \mathcal{A}(z_{1,t}, \dots, z_{n,t}). \quad (2981)$$

Equation 2981 can reveal patterns invisible within isolated case review.

Aggregation should remain sensitive to heterogeneous case meanings.

30.92 Audit Independence

Preventive audits should possess enough independence to identify failures that the operating unit has incentives to minimize.

Independence can be institutional, informational, financial, or professional.

No single formal separation guarantees effective independence.

30.93 Preventive Governance and Incentives

Actors respond to incentives created by governance.

Let behavior be

$$a_i = F(R_i, C_i, S_i), \quad (2982)$$

where R_i denotes rewards, C_i costs, and S_i social or institutional signals.

Equation 2982 reminds preventive governance that formal rules alter the strategic environment.

30.94 Compensation and Prevention Incentives

If harmful actors expect to internalize only a small fraction of downstream loss, compensation alone can have weak preventive effect.

Let private benefit of risky behavior be B_H , expected private remedial cost C_H , and externalized loss L_X .

Risk can remain attractive where

$$B_H > C_H \quad (2983)$$

despite

$$L_X \gg 0. \quad (2984)$$

Equations 2983 and 2984 identify a political-economic prevention problem.

30.95 Preventive Alignment

A preventive institution can alter incentives so that actors bear more of the expected consequences of risky conduct.

Let internalized cost be C_H^I .

Alignment improves where

$$C_H^I \uparrow \quad (2985)$$

toward the relevant social cost.

This is only one preventive strategy. Rights-based prohibitions can remain necessary where harm should not be treated as purchasable risk.

30.96 Preventive Governance and Rights

Rights can constrain preventive policy in two directions.

They can justify protection against threatened interference.

They can also limit intrusive surveillance or control imposed in the name of prevention.

Let permissible preventive set be

$$\mathcal{P}_{\text{rights}} = \{P : P \text{ satisfies protected rights}\}. \quad (2986)$$

Equation 2986 means prevention operates inside a rights-constrained region.

30.97 Preventive Paternalism

Preventive intervention can become paternalistic where institutions constrain a person's choices because they predict future harm to that person.

The justification burden should increase with:

- intervention intensity;
- irreversibility;
- uncertainty;
- disagreement about value;
- availability of less restrictive alternatives.

The presence of a predictive model does not settle the normative question.

30.98 Subject-Directed Prevention

Where several risk-reduction measures are substantively adequate, the subject can choose among them.

Let admissible prevention set be $\mathcal{P}_A^{\text{adm}}$.

Subject-directed selection is

$$P_A^* \in \arg \max_{P \in \mathcal{P}_A^{\text{adm}}} U_A(P), \quad (2987)$$

where U_A denotes the subject's ordering over admissible options.

Equation 2987 preserves agency within the protected region.

30.99 Preventive Governance and Collective Risk

Some risks cannot be managed through individual choice because one actor's decision affects many others.

Let action a_i generate external risk

$$R_{ij} = R(a_i, j). \quad (2988)$$

Equation 2988 creates a legitimate role for collective governance.

The appropriate intervention still depends upon severity, probability, distribution, and rights.

30.100 Systemic Risk

Systemic risk arises where local failures propagate through a network.

Let failure probability vector be $\mathbf{p}$, and propagation matrix be A .

A stylized systemic exposure is

$$\mathbf{r} = A\mathbf{p}. \quad (2989)$$

Equation 2989 makes network structure part of risk.

Prevention can therefore target connectivity, redundancy, modularity, or buffers rather than only local failure probability.

30.101 Correlation Risk

Multiple safeguards can fail together because they share exposure.

Let failure indicators be X_i and X_j .

High correlation satisfies

$$\text{Corr}(X_i, X_j) \rightarrow 1. \quad (2990)$$

Equation 2990 reduces the real value of nominal redundancy.

Preventive diversification should therefore examine independence as well as quantity.

30.102 Scenario Analysis

Where probability estimates are weak, governance can examine several plausible scenarios.

Let scenario family be

$$\Omega = \{\omega_1, \dots, \omega_m\}. \quad (2991)$$

A robust preventive intervention can satisfy

$$J(P, \omega_i) \geq J_{\min} \quad \forall \omega_i \in \Omega^*, \quad (2992)$$

where Ω^* is the selected plausible set.

Equation 2992 supports decision under model uncertainty without pretending to know one probability distribution exactly.

30.103 Stress Testing

Preventive institutions can test how systems respond under adverse but plausible conditions.

Let stress suite be

$$\Delta_{\text{stress}} = \{\delta_1, \dots, \delta_k\}. \quad (2993)$$

A preventive robustness criterion is

$$Q(\mathcal{G}^{\delta_i}) \in \mathcal{S} \quad \forall \delta_i \in \Delta_{\text{stress}}. \quad (2994)$$

Equation 2994 links preventive governance with the perturbation architecture developed in Section 17.

30.104 Ethical Limits of Stress Testing

Testing should not create the harmful event it is intended to prevent where safer alternatives exist.

Preferred methods can include:

- simulation;
- historical data;
- benign proxy perturbations;
- sandbox environments.

This preserves the principle that epistemic gain alone does not authorize harmful experimentation on persons.

30.105 Preventive Governance and Model Uncertainty

Let plausible model family be

$$\mathfrak{M} = \{M_1, \dots, M_k\}. \quad (2995)$$

A preventive action robust across models lies in

$$\mathcal{P}_{\cap} = \bigcap_{i=1}^k \mathcal{P}_{\text{safe}}^{M_i}. \quad (2996)$$

Equation 2996 identifies a common-safe policy region.

Such robust policies can be preferable where model disagreement is large.

30.106 Model Revision

Preventive models should update after new evidence and interventions.

Let

$$M_{t+1} = \mathcal{U}(M_t, D_{t+1}, E_{t+1}), \quad (2997)$$

where D_{t+1} denotes new data and E_{t+1} intervention outcomes.

Equation 2997 makes preventive governance adaptive.

30.107 Open-Ended Risk

Some future harms cannot be fully enumerated in advance because the system generates new states, technologies, relationships, and values.

Let future hazard set satisfy

$$\mathcal{H}_{t+1} \supset \mathcal{H}_t. \quad (2998)$$

Equation 2998 makes permanent hazard completeness unavailable.

Preventive governance therefore requires adaptive capacity in addition to identified-risk control.

30.108 Adaptive Capacity

Let institutional adaptive capacity be

$$A_I = A(K, R, V, D), \quad (2999)$$

where K denotes learning capacity, R resources, V revisability, and D distributed decision capacity. Equation 2999 treats adaptability as a preventive condition.

30.109 Transformability

Some systems should be capable of leaving an obsolete regime rather than resisting every disturbance.

Let current regime be S_0 and improved regime S_1 .

Transformability requires a viable transition

$$S_0 \rightsquigarrow S_1 \quad (3000)$$

where S_1 is normatively preferable.

Equation 3000 distinguishes resilience from rigid preservation of an unjust or obsolete regime.

30.110 Over-Prevention

Prevention becomes excessive where efforts to suppress risk substantially reduce legitimate generativity.

Let prevented harm value be $V_H(P)$, and generativity loss caused by prevention be $L_G(P)$.

A preventive intervention can become counterproductive where

$$L_G(P) > V_H(P) \quad (3001)$$

under the relevant normative representation.

Equation 3001 is illustrative and should not be used to monetize non-commensurable rights automatically.

30.111 Innovation Suppression

Highly risk-averse governance can reduce experimentation and novelty.

Let innovation or legitimate new-state generation be N_G .

It can satisfy

$$\frac{dN_G}{d\alpha_P} < 0 \quad (3002)$$

as preventive intensity α_P increases beyond some region.

Equation 3002 gives generative openness a role in preventive design.

30.112 Preventive Lock-In

A preventive measure can create institutional lock-in.

Let intervention P narrow future policy set from $\mathcal{U}_0$ to $\mathcal{U}_P$:

$$\mathcal{U}_P \subset \mathcal{U}_0. \quad (3003)$$

Equation 3003 creates a reason to value reversible and modular safeguards.

30.113 Preventive Capture

Actors can capture preventive institutions and use risk language to protect their own power.

Let preventive authority over actor A be $P_I(A)$.

Capture occurs where that authority is redirected toward private advantage rather than the public or protected purpose.

Preventive legitimacy therefore requires review, transparency, and contestability.

30.114 Risk Discourse and Power

Classifying a group, person, or practice as risky can justify exclusion before harm has occurred.

The burden of evidence and the possibility of appeal therefore become especially important.

Preventive governance should focus on conditions and mechanisms where feasible rather than essentializing persons as risks.

30.115 Mechanism-Based Prevention

Where causal evidence identifies a harmful mechanism, governance can target the mechanism rather than broad categories of persons.

Let causal mechanism be M_H .

Preventive intervention seeks

$$M_H \xrightarrow{P} M_S, \quad (3004)$$

where M_S is a safer mechanism.

Equation 3004 can reduce discriminatory or overbroad intervention where the causal structure is sufficiently understood.

30.116 Outcome-Based Prevention

Where mechanism knowledge is weak, governance can use observable outcome thresholds.

Let harm indicator be Y_t .

Intervention can trigger where

$$Y_t \geq \theta_Y. \quad (3005)$$

Equation 3005 is administratively simple and can react late where the first visible outcome already represents substantial harm.

30.117 Hybrid Prevention

A hybrid architecture can combine state indicators, causal evidence, event triggers, and subject reports.

Let hybrid warning function be

$$W_t = F_W(S_t, C_t, E_t, R_t), \quad (3006)$$

where S_t denotes state indicators, C_t causal indicators, E_t events, and R_t reports.
Equation 3006 reduces dependence upon one epistemic channel.

30.118 Preventive Resource Allocation

Prevention competes for finite institutional resources.

Let preventive budget be B_P , and intervention costs be c_i .

Feasibility requires

$$\sum_i c_i \leq B_P. \quad (3007)$$

Equation 3007 creates a priority problem.

The budget constraint does not determine which risks deserve protection.

30.119 Preventive Priority

Priority can be informed by:

- severity;
- reparability;
- propagation;
- vulnerability;
- singularity;
- number of affected persons;
- rights implications.

A plural priority profile can be represented by

$$\mathbf{P}_i = (S_i, 1 - \rho_i, P_i, V_i, N_i, R_i). \quad (3008)$$

Equation 3008 preserves several dimensions before aggregation.

30.120 Preventive Triage

Cases can be assigned to preventive tracks according to their priority profile.

Let triage function be

$$T_P : \mathbf{P}_i \longrightarrow \{L_0, \dots, L_m\}. \quad (3009)$$

Equation 3009 maps cases to intervention intensity levels.

Triage should remain reviewable because misclassification can itself create harm.

Table 34: Principal functions of preventive governance

Preventive function	Primary object	Governance contribution	Characteristic risk
Critical-condition mapping	Generative dependencies	Identifies high-consequence conditions	False confidence in model completeness
Redundancy	Alternative pathways	Reduces single-point failure	Cost and coordination burden
Buffers	Reserve capacity	Absorbs temporary shocks	Idle-resource pressure
Option preservation	Future reachable set	Maintains flexibility under uncertainty	Can delay useful commitment
Early warning	Dynamic and event indicators	Enables earlier intervention	False positives and stigma
Staged intervention	Risk-sensitive response	Matches intensity to evidence and severity	Escalation can become sticky
Anti-retaliation	Reporting capability	Protects preventive information flow	Enforcement difficulty
Evidence preservation	Future epistemic access	Protects later adjudication and learning	Privacy and retention burden
Distributed monitoring	Multi-source information	Reduces dependence on one observer	Coordination and data-governance complexity
Portability and exit	Alternative institutional paths	Reduces lock-in and domination	Can reduce institution-specific investment
Structural reform	Harm-generating mechanism	Reduces recurrence	Institutional scope and legitimacy

Table 35: Illustrative warning and intervention architecture

Warning level	Illustrative evidence state	Illustrative response	Procedural characteristic
Baseline	No material warning	Routine monitoring	Low intrusion
Elevated	Weak deterioration or isolated report	Consultation, evidence preservation	Reversible
Serious	Converging indicators or repeated events	Temporary safeguards, independent review	Time-limited and contestable
Critical	High likelihood of severe or weakly repairable harm	Strong interruption or protective action	Heightened review and reasons
Post-critical	Immediate danger reduced	De-escalation, learning, structural review	Sunset and reassessment

30.121 Preventive Justice Frontier

Preventive options can trade risk reduction against agency, privacy, cost, and generative openness.

Let preventive option P have profile

$$\mathbf{V}_P = (-\rho_H, A, P_v, G, -C), \quad (3010)$$

where $-\rho_H$ denotes risk reduction, A agency, P_v privacy, G generativity, and C cost.

The nondominated preventive frontier is

$$\mathcal{F}_P = \text{ND}(\mathbf{V}_P). \quad (3011)$$

Equation 3011 creates a structured decision space without forcing one scalar optimization.

30.122 Preventive Governance Matrix

Table 34 summarizes the major preventive functions developed in this section.

Table 34 shows that prevention is a portfolio of structural, epistemic, and procedural functions.

30.123 Warning and Intervention Matrix

Table 35 illustrates how warning strength can correspond to progressively stronger interventions.

Table 35 represents a staged structure rather than a universal risk protocol.

Table 36: Failure modes in preventive governance

Failure mode	Mechanism	Potential response	Atlas layer
Late detection	Indicators appear after critical window	Earlier distributed warning	Temporal
Single-point failure	One actor or node controls critical condition	Redundancy and portability	Network / control
Evidence loss	Delay destroys future proof	Early preservation and provenance	Epistemic
Reporting suppression	Retaliation or dependency	Independent protected channels	Procedural / non-domination
Over-prevention	Risk control suppresses legitimate generativity	Least-restrictive staged intervention	Normative
Classification lock-in	Temporary risk label becomes permanent	Expiration and contestability	Reflexive / historical
Monitoring capture	Observer depends upon monitored actor	Independent or plural monitoring	Institutional
Model failure	Single risk model misrepresents system	Model plurality and revision	Epistemic
Common-mode failure	Nominally redundant safeguards share dependency	Diverse pathways	Robustness
Structural recurrence	Individual incidents treated without mechanism change	Structural reform	Causal / generative

30.124 Preventive Failure Matrix

Table 36 summarizes recurring preventive failure modes.

Table 36 emphasizes that prevention can fail through both insufficient and excessive intervention.

30.125 Preventive Governance Statement

A fully specified preventive claim can be represented by

$$\mathcal{P}^* = [\mathcal{C}_{\text{crit}}; \mathbf{K}; \boldsymbol{\rho}; \mathbf{W}; \mathcal{P}_{\text{stage}}; \mathcal{E}; \mathcal{D}; \mathcal{R}_v; \tau; K], \quad (3012)$$

where:

- $\mathcal{C}_{\text{crit}}$ contains critical conditions;
- $\mathbf{K}$ contains criticality profiles;
- $\boldsymbol{\rho}$ contains reparability estimates;
- $\mathbf{W}$ contains warning indicators;
- $\mathcal{P}_{\text{stage}}$ contains staged interventions;
- $\mathcal{E}$ contains evidence infrastructure;
- $\mathcal{D}$ contains distributed monitoring structure;
- $\mathcal{R}_v$ contains review and de-escalation rules;
- τ denotes horizon;
- K denotes epistemic basis.

Equation 3012 makes preventive governance auditable and revisable.

30.126 Critical-Condition Principle

The first principle of this section is:

Preventive governance should prioritize conditions whose loss substantially contracts valuable future possibility, propagates recursively, or is weakly reparable.

30.127 Reparability Principle

The second principle is:

Lower expected reparability increases the relevance of prevention and early interruption while leaving rights, uncertainty, proportionality, and intervention burden independently relevant.

30.128 Option-Preservation Principle

The third principle is:

Under uncertainty concerning future preferences and system development, preventive governance should preserve valuable options where feasible rather than prematurely fixing one future.

30.129 Redundancy Principle

The fourth principle is:

Critical generative functions should avoid unnecessary dependence upon a single actor, channel, infrastructure node, or evidentiary source where redundant pathways are feasible.

30.130 Early-Warning Principle

The fifth principle is:

Early-warning information should support proportionate staged intervention and should not be treated as conclusive proof of future harm.

30.131 Reversible-Protection Principle

The sixth principle is:

Where evidence remains incomplete, reversible and time-limited safeguards have a procedural advantage over irreversible interventions capable of producing comparable immediate protection.

30.132 Anti-Retaliation Principle

The seventh principle is:

Preventive epistemology requires protected reporting conditions because a system cannot reliably govern emerging harm when those closest to the process face serious retaliation for making it visible.

30.133 Evidence-Preservation Principle

The eighth principle is:

Preservation of evidence can justifiably precede final interpretation where delay would materially reduce future recognizability, reviewability, or accountability.

30.134 Distributed-Epistemic Principle

The ninth principle is:

Multi-party preventive governance can share standards, evidence interfaces, and provenance while preserving distributed collection, local processing, and institution-specific decision authority.

30.135 Non-Domination Principle

The tenth principle is:

Risk prevention should reduce avoidable vulnerability to arbitrary power rather than creating new discretionary dependencies through monitoring or protective control.

30.136 Anti-Stigmatization Principle

The eleventh principle is:

Preventive classification should target conditions, mechanisms, and time-bounded risk states where feasible rather than converting uncertain future risk into permanent identity.

30.137 Adaptive-Governance Principle

The twelfth principle is:

Because generative systems and their relevant state spaces evolve, preventive governance should maintain capacity for model revision, policy revision, de-escalation, and transformation.

30.138 Anti-Overprevention Principle

The thirteenth principle is:

Prevention should protect generativity without suppressing legitimate experimentation, diversity, agency, and transformation through excessive risk control.

30.139 Structural-Learning Principle

The fourteenth principle is:

Recurring post-harm assistance and compensation needs should be treated as evidence for possible upstream structural weakness rather than as isolated downstream administrative events.

30.140 Preventive Layer of the Atlas

The preventive layer of the atlas can be represented by

$$\mathfrak{P}_G = (\mathcal{C}_{\text{crit}}, \mathbf{K}, \boldsymbol{\rho}, \mathbf{W}, \mathcal{B}, \mathcal{R}, \mathcal{E}, \mathcal{D}, \mathcal{I}, \mathcal{A}), \quad (3013)$$

where:

- $\mathcal{C}_{\text{crit}}$ contains critical generative conditions;
- $\mathbf{K}$ contains criticality profiles;
- $\boldsymbol{\rho}$ contains reparability structures;
- $\mathbf{W}$ contains warning systems;
- $\mathcal{B}$ contains buffers and redundancy;
- $\mathcal{R}$ contains reversible protective measures;
- $\mathcal{E}$ contains evidence-preservation architecture;
- $\mathcal{D}$ contains distributed monitoring;
- $\mathcal{I}$ contains intervention and escalation rules;
- $\mathcal{A}$ contains adaptive learning mechanisms.

Equation 3013 completes the manuscript's prevention–interruption–restoration–assistance continuum.

30.141 Completion of the Temporal Governance Architecture

The analysis from Sections 5 through 30 now supports four distinct temporal functions:

1. preservation of critical generative conditions before severe loss;
2. interruption of harmful processes while they are unfolding;
3. post-harm correction, restitution, compensation, and recognition;
4. assistance directed toward renewed or transformed future generativity.

These functions can be represented by

$$\mathfrak{T}_J = (P, I, R, A), \quad (3014)$$

where P denotes prevention, I interruption, R post-harm remedy, and A generative assistance. Equation 3014 demonstrates that the justice of return extends temporally beyond ex post return.

30.142 From General Architecture to Applied Comparison

The manuscript has now developed the principal analytical and normative components required for applied comparison:

- reference systems;
- comparison objects;
- comparative relations;
- temporal horizons;
- dynamical, causal, generative, historical, and reflexive objects;
- value theories;
- substantive and procedural justice;
- legal remedies;
- general equivalents;
- generative assistance;
- preventive governance.

The remaining task is to show how these components behave when several are applied simultaneously to concrete classes of harm.

30.143 Transition to Applied Comparative Analyses

Section 31 therefore develops a set of applied comparative analyses across workplace exclusion, research and academic opportunity, reputational injury, health-related generative loss, collective and historical injustice, ecological restoration, and other representative cases.

The purpose will not be to produce jurisdiction-specific legal advice or to claim empirical validation of every formal model. The applied analyses will instead demonstrate how different choices of reference, object, relation, horizon, value theory, procedure, and institutional form generate different meanings of successful return.

The central practical question becomes:

When the full atlas is brought to a concrete case, which forms of restoration are actually achieved, which values remain unresolved, which differences are justifiably preserved, and which institutional functions should act before, during, and after the harm?

31 Applied Comparative Analyses

This section applies the analytical architecture developed in the preceding sections to several stylized classes of restorative problems. The purpose is to demonstrate how the Atlas of Equivalence changes the interpretation of familiar claims such as reinstatement, compensation, correction, rehabilitation, reparation, and ecological restoration when the relevant reference, comparison object, comparative relation, temporal horizon, value theory, procedure, and institutional function are specified separately.

The analyses are illustrative rather than jurisdiction-specific. They do not claim that every legal remedy discussed is available in every legal system, that every formal model can presently be estimated empirically, or that one remedial architecture should govern every case belonging to the same broad category. Their role is methodological. Each case is used to show how apparently simple statements

such as “the job was restored,” “the record was corrected,” or “the environment was rehabilitated” can conceal several distinct comparative claims.

The section uses a common sequence. Each case identifies the relevant harm, candidate references, principal comparison objects, appropriate comparative relations, temporal structure, normative values, institutional responses, residual differences, and preventive implications. The same event can therefore produce several valid but non-equivalent restorative assessments.

The analyses also test the manuscript’s broader claim that restoration should rarely be represented by one Boolean variable. In complex cases, successful return often consists of a profile containing some restored dimensions, some transformed dimensions, some unresolved residuals, and some differences that justice should deliberately preserve.

31.1 Common Applied Analysis Architecture

Let an applied restorative case be represented by the tuple in Equation 3015.

$$\mathfrak{C}_A = (H, \mathbf{B}, \mathbf{Q}, \mathbf{E}, \mathcal{T}, \tau, \mathbf{V}, \mathbf{J}, \mathbf{R}, \Delta), \quad (3015)$$

where:

- H denotes the harm or harmful process;
- $\mathbf{B}$ contains candidate reference systems;
- $\mathbf{Q}$ contains selected comparison objects;
- $\mathbf{E}$ contains the corresponding comparative relations;
- $\mathcal{T}$ contains tests and observations;
- τ contains relevant temporal horizons;
- $\mathbf{V}$ contains value objects and normative theories;
- $\mathbf{J}$ contains substantive and procedural justice requirements;
- $\mathbf{R}$ contains institutional responses;
- Δ contains residual differences.

Equation 3015 is not intended to create one universal decision algorithm. It is a disciplined way to prevent different restorative questions from being silently collapsed into one.

31.2 Applied Comparison Cells

Within each case, a restorative claim can be expressed through a comparison cell

$$c_i = (B_i, Q_i, E_i, \tau_i). \quad (3016)$$

Equation 3016 identifies the reference, object, comparative relation, and horizon relevant to one claim.

Several cells can be simultaneously valid.

For example, a workplace case can contain:

- salary equality relative to a no-wrong income baseline;
- status identity relative to formal employment status;

- capability sufficiency relative to present professional opportunity;
- network inclusion relative to professional re-entry;
- non-domination relative to future managerial control.

No one of these cells determines the others.

31.3 Applied Residual Classification

Residual difference will be classified using the partition developed earlier:

$$\Delta = \Delta_{\text{unresolved}} \cup \Delta_{\text{protected}} \cup \Delta_{\text{valuable}}. \quad (3017)$$

Equation 3017 distinguishes unresolved loss, differences preserved because stronger reconstruction would violate normative constraints, and positively valued developments generated after the harm.

This classification is especially important in the applied cases because it prevents all divergence from the counterfactual reference from being interpreted as remedial failure.

31.4 Workplace Exclusion and Institutional Re-Entry

Consider a worker who experiences sustained exclusion, harassment, retaliatory conduct, or wrongful removal from a professional role. The stylized case assumes that the worker later receives some combination of reinstatement, financial compensation, record correction, or transfer.

A simple legal or administrative description can state that the worker has been “returned to work.”

The atlas reveals several different possible meanings of this statement.

31.5 Workplace Reference Systems

Candidate references include:

- the worker’s formal status immediately before exclusion;
- the no-exclusion counterfactual career trajectory;
- an adequate professional-capability threshold;
- a non-dominating employment relationship;
- a new subject-selected professional future.

These references can be represented by

$$\mathbf{B}_W = (B_{\text{pre}}, B_{\text{cf}}, B_{\text{cap}}, B_{\text{ND}}, B_{\text{new}}). \quad (3018)$$

Equation 3018 shows why a workplace case does not possess one self-evident baseline.

31.6 Workplace Comparison Objects

Relevant objects can include

$$\mathbf{Q}_W = (Q_{\text{salary}}, Q_{\text{status}}, Q_{\text{authority}}, Q_{\text{network}}, Q_{\text{trust}}, Q_{\text{capability}}, Q_{\text{security}}, Q_{\text{trajectory}}). \quad (3019)$$

Equation 3019 distinguishes current material return from future professional generativity.

A worker can therefore recover salary and title while remaining outside the informal network through which meaningful work, information, recommendation, and promotion are generated.

31.7 Formal Reinstatement

Formal reinstatement can satisfy

$$Q_{\text{status}}(\mathcal{G}^R) = Q_{\text{status}}(\mathcal{G}^{\text{pre}}). \quad (3020)$$

Equation 3020 is a strong legal-status result.

It does not imply

$$Q_{\text{network}}(\mathcal{G}^R) = Q_{\text{network}}(\mathcal{G}^{\text{cf}}). \quad (3021)$$

Equation 3021 can fail because colleagues, projects, supervisors, and institutional expectations have changed during the period of exclusion.

31.8 Workplace Trajectory Loss

Suppose the no-harm professional trajectory is γ_{cf} and the realized trajectory is γ_H .

A later return to the same formal position can satisfy endpoint equality:

$$x_R(t_1) = x_{\text{cf}}(t_1) \quad (3022)$$

while

$$\gamma_R \neq \gamma_{\text{cf}}. \quad (3023)$$

Equations 3022 and 3023 make missed professional development, reputation, project experience, and network formation visible.

31.9 Workplace Reachability

A stronger restorative question asks whether future professional options are again practically reachable.

Let the worker's professional reachable set be $\mathcal{R}_{W,\tau}$.

An inclusion-oriented criterion is

$$\mathcal{R}_{W,\tau}^{\text{ref}} \subseteq \mathcal{R}_{W,\tau}^R. \quad (3024)$$

Equation 3024 allows the restored worker to pursue a future different from the exact historical counterfactual while preserving equivalent or greater substantive professional possibility.

31.10 Workplace Non-Domination

Suppose reinstatement returns the worker to the authority of the same manager who possesses discretionary power over assignments, evaluations, and continued access.

Formal status can be restored while domination exposure remains high.

Let managerial domination exposure be D_W . A non-dominating restoration requires

$$D_W^R \leq D_{W,\text{max}}. \quad (3025)$$

Equation 3025 can justify independent review, alternative supervision, portable credentials, or meaningful exit [44].

31.11 Workplace Remedy Bundle

A richer workplace remedy can be represented by

$$R_W = (R_{\text{status}}, R_{\text{money}}, R_{\text{record}}, R_{\text{reentry}}, R_{\text{security}}, R_{\text{exit}}). \quad (3026)$$

Equation 3026 separates formal correction, compensation, record repair, effective re-entry, secure future conditions, and exit.

Not every case requires every component.

31.12 Workplace Subject Direction

The worker can prefer a new professional future instead of returning to the earlier workplace.

Let the admissible future set be

$$\mathcal{F}_W^{\text{adm}} = \{f_{\text{return}}, f_{\text{transfer}}, f_{\text{exit}}\}. \quad (3027)$$

Where all three options satisfy protected substantive thresholds, subject choice can legitimately determine the future path.

A compensated exit can therefore be less historically faithful and more just than compulsory reinstatement.

31.13 Workplace Prevention

Preventive workplace governance can focus upon:

- independent reporting channels;
- anti-retaliation protection;
- portable records and credentials;
- multiple evaluation routes;
- early separation of conflicting parties where necessary;
- preservation of evidence;
- review of concentrated gatekeeper power.

The preventive object is not merely absence of complaints.

It is preservation of secure professional generativity.

31.14 Workplace Residuals

After substantial repair, unresolved residuals can include:

- missed projects;
- lost time;
- altered relationships;
- historical memory of exclusion.

Protected residuals can include the worker's decision not to recreate prior relationships.

Valuable residuals can include new professional directions discovered during or after the harmful period.

The case therefore demonstrates the full residual partition in Equation 3017.

31.15 Research and Academic Opportunity

Research and academic opportunity provide an especially clear case of temporally singular and recursively generative loss.

A researcher excluded from a laboratory, project, dataset, publication process, conference, grant, or mentorship network can lose far more than the immediate resource.

The loss can alter the future sequence through which later intellectual and professional opportunities are generated.

31.16 Research Generative Structure

A simplified academic-generative cycle can be represented by

$$A \rightarrow R \rightarrow K \rightarrow O \rightarrow N \rightarrow A', \quad (3028)$$

where A denotes access, R research activity, K knowledge, O output, N network formation, and A' later access.

Equation 3028 shows why loss of one early condition can propagate recursively.

31.17 Research Reference Systems

Candidate references include:

- access immediately before exclusion;
- the no-exclusion research trajectory;
- an adequate independent research-capability set;
- a new subject-selected research trajectory.

The no-harm trajectory can be especially uncertain because research depends upon contingent discovery, collaboration, publication, and funding.

31.18 Research Opportunity Windows

Suppose a research opportunity exists only during interval

$$W_R = [t_0, t_1]. \quad (3029)$$

If access is restored at $t_2 > t_1$, the same nominal resource can be returned without recreating the opportunity.

Thus

$$Q_{\text{resource}}^R = Q_{\text{resource}}^{\text{pre}} \quad (3030)$$

can coexist with

$$Q_{\text{opportunity}}^R \neq Q_{\text{opportunity}}^{\text{cf}}. \quad (3031)$$

Equations 3030 and 3031 capture the singularity of historical academic windows.

31.19 Authorship

Authorship demonstrates the importance of provenance.

Let work q remain textually identical before and after correction.

A correct attribution can require

$$\text{Prov}_R(q) = \text{Prov}^*(q). \quad (3032)$$

Equation 3032 is not reducible to monetary value.

Recognition and historical provenance are the principal restorative objects.

31.20 Research Networks

Academic opportunity often depends upon network position.

Let collaboration network be $G_R = (V, E)$.

A researcher can regain institutional affiliation while remaining outside important relational pathways.

A useful comparison can therefore concern structural or functional access rather than exact restoration of every former edge.

31.21 Research Capability

Let research-capability set be $\mathcal{K}_R$.

A future-oriented restorative criterion can require

$$\mathcal{K}_R^{\text{post}} \succeq \mathcal{K}_R^{\text{min}}. \quad (3033)$$

Equation 3033 can be satisfied through a new institution, new collaborators, new funding, or new infrastructure.

This illustrates capability restoration without historical replication [55, 50].

31.22 Research Remedy Bundle

An applied bundle can include

$$R_{\text{research}} = (M, Z_C, C_P, F, N, A_D), \quad (3034)$$

where M denotes financial support, Z_C record or attribution correction, C_P portable credentials, F future funding, N network access, and A_D data or research access.

Equation 3034 allows the remedy to support a new trajectory rather than attempt literal recreation of every lost event.

31.23 Research Prevention

Preventive academic governance can include:

- contribution provenance;
- portable authorship records;
- independent misconduct review;
- data-access provenance;
- multiple mentorship channels;
- funding continuity during dispute where appropriate;

- anti-retaliation.

The objective is to reduce the ability of one gatekeeper or one dispute to destroy an entire intellectual trajectory.

31.24 Research Residuals

Even an excellent later career can leave:

- missed collaboration;
- lost historical priority;
- unproduced research;
- unavailable earlier intellectual development.

These residuals can be historically real without requiring a conclusion that future generativity remains permanently impaired.

31.25 Reputational Injury and Distributed Correction

Reputational injury provides a particularly strong example of relational, informational, historical, and reflexive harm.

Suppose an erroneous classification, allegation, or institutional judgment propagates through several records or social nodes.

The harm is not located solely in the source statement.

It resides partly in the distributed downstream beliefs and decisions created by that statement.

31.26 Reputational Propagation

Let reputational state propagate through network G_Z .

A simplified propagation process is

$$Z_{t+1} = \mathcal{P}(Z_t, G_Z). \quad (3035)$$

Equation 3035 makes reputation a distributed state.

Correction of one source node does not automatically reverse all downstream effects.

31.27 Source Correction

Let erroneous source record be Z_E and corrected source record Z_C .

Source correction satisfies

$$Z_E \xrightarrow{C} Z_C. \quad (3036)$$

Equation 3036 can establish current institutional accuracy.

It does not establish downstream recognition equivalence.

31.28 Correction Propagation

Let V_E be the set of operational nodes that received the erroneous classification and V_C the set receiving the correction.

Correction coverage is

$$\kappa_C = \frac{|V_C|}{|V_E|}. \quad (3037)$$

Equation 3037 provides one coarse distributed measure.

It should be refined where nodes differ substantially in institutional power or future impact.

31.29 Operational and Historical Records

A corrected system can preserve the historical existence of the error while removing its current authority.

Let

$$Z_E \in \mathcal{H} \quad (3038)$$

and

$$Z_E \notin \mathcal{D}_{\text{current}}. \quad (3039)$$

Equations 3038 and 3039 permit accountability and current accuracy simultaneously.

31.30 Erasure and Privacy

Deleting every trace of the original allegation can reduce future stigma.

It can also eliminate evidence of institutional error and correction.

The appropriate remedy can therefore involve:

- operational removal;
- protected historical provenance;
- limited access;
- correction propagation.

The strongest counterfactual fidelity is not automatically the most just record architecture.

31.31 Reputational Capability

Reputation matters partly because it conditions future access.

Let professional capability after correction be $\mathcal{K}_A^{Z_C}$.

A stronger practical criterion than source correction is

$$\mathcal{K}_A^{Z_C} \succeq \mathcal{K}_{A,\min}. \quad (3040)$$

Equation 3040 asks whether corrected information has become usable in future institutional decisions.

31.32 Reputational Remedy Bundle

A reputational remedy can include

$$R_{\text{rep}} = (Z_C, P_C, D_L, M, N_R), \quad (3041)$$

where Z_C denotes source correction, P_C correction propagation, D_L authoritative declaration, M compensation where relevant, and N_R re-entry or recognition support.

Equation 3041 shows why reputational repair is rarely one-dimensional.

31.33 Reputational Prevention

Preventive safeguards can include:

- distinction between allegation and verified finding;
- provenance;
- limited distribution of provisional information;
- correction channels;
- expiry of temporary classifications;
- downstream correction requirements.

The central preventive object is future recognizability without unjustified permanent stigmatization.

31.34 Health-Related Generative Loss

Health-related harm demonstrates the interaction among bodily state, functional capacity, capability, time, care, employment, and social participation.

The applied analysis should distinguish restoration of bodily state from restoration of practical life possibility.

31.35 Health Comparison Objects

Relevant objects can include

$$\mathbf{Q}_H = (Q_{\text{body}}, Q_{\text{function}}, Q_{\text{capability}}, Q_{\text{time}}, Q_{\text{care}}, Q_{\text{income}}, Q_{\text{agency}}). \quad (3042)$$

Equation 3042 makes clear that medical repair and restorative justice overlap without being identical.

31.36 Bodily Restoration

Suppose treatment restores the modeled bodily state:

$$b_R = b_{\text{ref}}. \quad (3043)$$

Equation 3043 can represent very strong medical success.

The subject can still have lost time, employment, relationships, savings, or confidence during the period of illness.

31.37 Functional Restoration

A functional criterion can require

$$F_R \succeq F_{\min}. \quad (3044)$$

Equation 3044 can be appropriate for rehabilitation.

Capability can remain below its prior level where the social environment no longer permits effective use of the restored function.

31.38 Health Capability

Let capability depend upon

$$\mathcal{K}_A = G(b_A, F_A, M_A, E_A, S_A), \quad (3045)$$

where M_A denotes material resources, E_A environment, and S_A support.

Equation 3045 demonstrates why equal medical outcomes can yield unequal substantive futures.

31.39 Care

Continuing care can be necessary after maximal feasible recovery.

A support-dependent life should not automatically be classified as failed restoration.

The relevant questions concern:

- adequacy;
- security;
- dignity;
- autonomy;
- distribution of caring burden.

Care theory is particularly useful for this analysis [63].

31.40 Time Loss

Medical recovery can restore future function without restoring elapsed time.

Let illness period be $[t_0, t_1]$.

A later period of equal duration satisfies

$$t_3 - t_2 = t_1 - t_0 \quad (3046)$$

while

$$V([t_2, t_3]) \neq V([t_0, t_1]). \quad (3047)$$

Equations 3046 and 3047 identify a historical residual that no later medical intervention necessarily eliminates.

31.41 Health Remedy Bundle

A health-related restorative bundle can contain

$$R_H = (T_M, R_{\text{rehab}}, M, A_W, C, S_L), \quad (3048)$$

where T_M denotes treatment, R_{rehab} rehabilitation, M material resources, A_W workplace or educational accommodation, C care, and S_L livelihood support.

Equation 3048 represents a layered rather than purely medical response.

31.42 Health Subject Direction

The no-harm counterfactual can point toward return to a prior occupation.

The actual subject can now prefer another path.

A capability-sensitive remedy can therefore preserve:

$$\mathcal{K}_{A,\text{ref}} \subseteq \mathcal{K}_{A,R} \quad (3049)$$

while allowing the person to choose another functioning.

This reflects the distinction between capability and compulsory return.

31.43 Health Prevention

Preventive governance can include:

- early accommodation;
- continuity of treatment;
- income protection;
- preservation of status;
- protection against health-related exclusion;
- timely information.

The objective is to prevent a health impairment from recursively producing housing, employment, network, or financial collapse.

31.44 Collective and Historical Injustice

Collective and historical injustice places strong pressure on individualized models of restoration. The relevant objects can include land, public institutions, cultural transmission, political standing, collective memory, infrastructure, and intergenerational generativity.

International reparations practice already recognizes plural remedial forms including restitution, compensation, rehabilitation, satisfaction, and guarantees of non-repetition [65, 12].

The atlas adds a structured method for asking which object each remedy addresses.

31.45 Collective Reference Systems

Possible references include:

- a pre-injustice collective condition;
- a no-wrong counterfactual trajectory;

- present equal citizenship or standing;
- current collective capability sufficiency;
- a transformed future under community governance.

These references can diverge sharply after long historical periods.

31.46 Collective Objects

A collective restorative object family can be represented by

$$\mathbf{Q}_C = (Q_{\text{land}}, Q_{\text{resources}}, Q_{\text{governance}}, Q_{\text{culture}}, Q_{\text{memory}}, Q_{\text{infrastructure}}, Q_{\text{generation}}) . \quad (3050)$$

Equation 3050 prevents collective repair from being reduced to individual monetary distribution.

31.47 Collective Resource Return

Resource restitution can satisfy

$$R_C^{\text{post}} = R_C^{\text{entitled}} \quad (3051)$$

for a specified collective resource.

The value of the resource can depend heavily upon control, infrastructure, and governance.

Formal title alone can therefore be incomplete.

31.48 Collective Control

Let collective governance rights be χ_C .

A control-sensitive criterion is

$$\chi_C^R \succeq \chi_{C,\text{min}} . \quad (3052)$$

Equation 3052 distinguishes receipt of resources from authority over their future use.

This distinction is central to generative justice [15, 17].

31.49 Collective Recognition

A public acknowledgment can alter historical standing without directly changing material distribution.

Collective recognition can include:

- official acknowledgment;
- correction of public narratives;
- archival access;
- memorialization;
- educational change.

Recognition should not become a substitute for material or institutional repair.

31.50 Intergenerational Transmission

Let collective condition be transmitted through

$$C^{(g+1)} = \Gamma_C \left(C^{(g)} \right). \quad (3053)$$

Equation 3053 can include language, knowledge, resources, governance capacity, and environmental conditions.

A reparative intervention can therefore be evaluated through its effect upon future transmission.

31.51 Third-Party Structure

Long historical periods generate third-party reliance and new populations.

Exact historical reconstruction can therefore impose large burdens upon persons who did not participate in the original wrong.

The applied problem becomes one of balancing:

- historical claim;
- present rights;
- collective control;
- feasible transformation.

This is precisely the kind of case in which lower historical fidelity can be more just.

31.52 Collective Remedy Bundle

A plural reparative bundle can be represented by

$$R_C = (R_{\text{resource}}, M, \chi_C, R_{\text{rec}}, I_C, R_{\text{NR}}), \quad (3054)$$

where the terms denote resource return, monetary transfer, governance rights, recognition, collective infrastructure, and non-repetition.

Equation 3054 directly reflects the plural remedy architecture of reparative justice.

31.53 Collective Residuals

No collective remedy can necessarily restore:

- lives already lived under the injustice;
- lost cultural knowledge;
- exact historical demography;
- extinct ecological or social relations.

Residual recognition therefore becomes particularly important in long-horizon historical justice [67, 62].

31.54 Collective Prevention

Guarantees of non-repetition can target the institutional mechanisms through which collective injustice was reproduced.

The appropriate object is therefore not merely memory of the past but causal and generative structure of future governance.

31.55 Ecological Restoration

Ecological restoration provides a case in which state, function, resilience, history, and future generativity can diverge especially clearly.

The restored appearance of an ecosystem does not necessarily imply restoration of ecological process, biodiversity, resilience, or historical provenance.

31.56 Ecological Reference Systems

Possible references include:

- pre-disturbance ecological state;
- modeled no-disturbance trajectory;
- functional ecological threshold;
- resilience target;
- transformed ecosystem adapted to changed climatic or social conditions.

The appropriate reference can therefore change with irreversible environmental conditions.

31.57 Ecological Objects

Let ecological object profile be

$$\mathbf{Q}_E = (Q_{\text{state}}, Q_{\text{species}}, Q_{\text{function}}, Q_{\text{resilience}}, Q_{\text{basin}}, Q_{\text{history}}). \quad (3055)$$

Equation 3055 makes ecological restoration multidimensional.

31.58 Ecological State Restoration

A visible-state restoration can satisfy

$$x_E^R \approx x_E^{\text{ref}}. \quad (3056)$$

Equation 3056 can concern vegetation, water quality, population abundance, or another measured state.

The system can remain dynamically fragile.

31.59 Ecological Functional Restoration

A functional comparison asks whether selected ecological functions satisfy

$$F_{E,i}^R \sim F_{E,i}^{\text{ref}} \quad \forall i \in I_F. \quad (3057)$$

Equation 3057 can remain compatible with different species composition or historical pathway.

31.60 Ecological Resilience

Holling's distinction between resilience and simple return to equilibrium is especially relevant here [24].

Let resilience be ϱ_E .

A restored ecosystem can satisfy

$$x_E^R \approx x_E^{\text{ref}} \quad (3058)$$

while

$$\varrho_E^R < \varrho_E^{\text{ref}}. \quad (3059)$$

Equations 3058 and 3059 describe visually successful and dynamically fragile restoration.

31.61 Ecological Basin Structure

Where alternative stable regimes exist, a system can remain near a desired state while its basin has contracted.

Let basin be $\mathcal{B}_E$.

A stronger restorative criterion can require

$$\mathcal{B}_E^R \supseteq \mathcal{B}_E^{\text{ref}}. \quad (3060)$$

Equation 3060 is directly connected to the resilience and critical-transition analysis developed earlier [52].

31.62 Ecological Historical Identity

Even perfect recreation of species and functions cannot reproduce the actual evolutionary or ecological history that was lost.

A recreated forest and the historical forest can therefore satisfy

$$Q_{\text{state}}^R = Q_{\text{state}}^{\text{ref}} \quad (3061)$$

and

$$Q_{\text{history}}^R \neq Q_{\text{history}}^{\text{ref}}. \quad (3062)$$

Equations 3061 and 3062 parallel the manuscript's broader distinction between state and history.

31.63 Ecological Transformative Restoration

Climate, land use, or other background conditions can make the historical ecological state unsustainable.

A transformed ecological target can therefore satisfy

$$\mathbf{V}_E(E_R) \supseteq \mathbf{V}_E(E_{\text{historical}}) \quad (3063)$$

without reproducing the historical state.

Equation 3063 provides an ecological example of better-than-reference restoration.

31.64 Ecological Prevention

Ecological systems illustrate the importance of prevention where critical transitions exhibit hysteresis or low reversibility [52, 53].

Preventive governance can emphasize:

- safety margins;
- early warning;
- biodiversity;
- redundancy;
- distributed monitoring;
- precaution.

The cost of early intervention can be much lower than the cost of attempting to reverse a regime shift.

31.65 Administrative Classification and Automated Decision

A further applied case concerns administrative or automated classifications that affect eligibility, reputation, access, or risk status.

This case is useful because it connects evidence, causality, procedure, reflexivity, correction, and future generativity.

31.66 Administrative Classification Objects

Let classification be Z_A .

Relevant comparison objects include

$$\mathbf{Q}_A = (Q_{\text{classification}}, Q_{\text{record}}, Q_{\text{eligibility}}, Q_{\text{decision}}, Q_{\text{reputation}}, Q_{\text{future access}}). \quad (3064)$$

Equation 3064 makes the distinction between the classification itself and its downstream consequences explicit.

31.67 Classification Correction

Suppose classification Z_E is erroneous.

Direct correction is

$$Z_E \xrightarrow{C} Z_C. \quad (3065)$$

Equation 3065 can restore the current record.

Downstream decisions can remain affected if the earlier classification was copied or used before correction.

31.68 Decision Propagation

Let downstream decision set be

$$\mathcal{D}_Z = \{D_1, \dots, D_n\}. \quad (3066)$$

A complete administrative correction should ask which D_i depended materially upon Z_E .

The relevant causal object is therefore larger than the source database.

31.69 Automated Classification and Reflexivity

A risk score can change institutional treatment, which then changes later data.

Let score be

$$S_t = M(X_t), \quad (3067)$$

and next state be

$$X_{t+1} = F(X_t, S_t). \quad (3068)$$

Equations 3067 and 3068 make the classification causally endogenous.

A later model trained on X_{t+1} can therefore learn partly from the effects of its own earlier classifications.

31.70 Administrative Procedural Justice

The restorative response should therefore include:

- notice;
- access to material reasons;
- correction;
- contestability;
- review;
- downstream propagation of correction.

A high predictive accuracy rate cannot substitute for these procedural conditions.

31.71 Administrative Non-Domination

Where a person depends upon one opaque classification system for essential access, the system can create domination even without frequent errors.

Portability, multiple review channels, and enforceable correction rights can therefore have independent justice value.

31.72 Administrative Prevention

Preventive architecture can include:

- provenance;
- versioning;
- expiration of provisional labels;
- audit of feedback effects;
- separation of raw data and operational classification;
- correction propagation.

The objective is to prevent one early classification from becoming an irreversible institutional identity.

31.73 Cross-Border Status and Portable Generativity

Cross-border mobility provides another useful case because resources, rights, credentials, and institutional recognition do not necessarily travel together.

A person can possess substantial financial resources and remain unable to convert them into opportunity because legal or institutional status differs across jurisdictions.

31.74 Cross-Border Comparison Objects

Relevant objects include

$$\mathbf{Q}_X = (Q_{\text{status}}, Q_{\text{money}}, Q_{\text{credential}}, Q_{\text{mobility}}, Q_{\text{service}}, Q_{\text{network}}). \quad (3069)$$

Equation 3069 highlights the interaction between general equivalents and non-monetary institutional conditions.

31.75 Portability Failure

Suppose credential c is valid in jurisdiction j_1 and unrecognized in j_2 :

$$R_{j_1}(c) = 1 \quad (3070)$$

and

$$R_{j_2}(c) = 0. \quad (3071)$$

Equations 3070 and 3071 demonstrate that possession of the credential and practical professional capability are different objects.

31.76 Cross-Border General Equivalents

Money can travel more easily than legal status or credential recognition.

This creates a case in which fungibility solves part of the restorative problem and leaves institutional conversion unresolved.

The appropriate assistance can therefore combine portable money with legal, credential, language, and institutional interfaces.

31.77 Cross-Border Generative Assistance

A cross-border support bundle can include

$$A_X = (M, S_L, C_P, N, I_A), \quad (3072)$$

where M denotes portable resources, S_L legal-status support, C_P credential portability, N navigation, and I_A institutional access.

Equation 3072 demonstrates the interaction between political economy and generative assistance.

31.78 Public Infrastructure Failure

A final applied class concerns failure of public or shared infrastructure.

Examples can include transportation, healthcare access, information systems, public records, environmental systems, or social-protection interfaces.

The central feature is that individual compensation can be insufficient where the common condition itself remains impaired.

31.79 Infrastructure Comparison Objects

Let infrastructure object profile be

$$\mathbf{Q}_I = (Q_{\text{availability}}, Q_{\text{capacity}}, Q_{\text{reliability}}, Q_{\text{access}}, Q_{\text{redundancy}}, Q_{\text{resilience}}). \quad (3073)$$

Equation 3073 distinguishes infrastructure state from user-level access and robustness.

31.80 Infrastructure Compensation

Individual users can receive money after infrastructure failure.

Let payment be M_i .

If the critical service remains unavailable,

$$A_M(c^*) = 0, \quad (3074)$$

then increasing M_i alone cannot reproduce the lost condition.

Equation 3074 is a direct applied example of the market-availability limit developed in Section 28.

31.81 Infrastructure Restoration

A stronger remedy targets the infrastructure:

$$I_H \xrightarrow{R_I} I_R. \quad (3075)$$

Equation 3075 can improve many users' capabilities simultaneously.

The relevant restorative scale is collective.

31.82 Infrastructure Resilience

Restoration to ordinary operation can remain incomplete if the system retains the same single points of failure.

Let reliability be R_I and resilience be ϱ_I .

A transformed repair can seek

$$R_I^R \geq R_I^{\text{pre}} \quad (3076)$$

and

$$\varrho_I^R > \varrho_I^{\text{pre}}. \quad (3077)$$

Equations 3076 and 3077 illustrate better-than-before structural restoration.

31.83 Infrastructure Prevention

Preventive design can include:

- redundancy;
- distributed monitoring;
- common standards;
- reserve capacity;
- local fallback;
- interoperability.

Table 37: Cross-case application of the restorative atlas

Case	Primary objects	Strong restorative relation	Characteristic residual	Central institutional challenge
Workplace exclusion	Status, network, capability, control	Effective re-entry / reachability	Lost trajectory and trust	Return without renewed domination
Research opportunity	Access, authorship, network, opportunity	Capability / generative restoration	Singular missed intellectual windows	Future support without forced historical replication
Reputational injury	Record, recognition, distributed belief	Correction plus propagation	Historical memory and downstream effects	Truth, privacy, and distributed correction
Health-related loss	Body, function, capability, time	Functional and capability adequacy	Lost time and altered life trajectory	Care without paternalistic reconstruction
Collective injustice	Resources, governance, culture, history	Plural reparative bundle	Intergenerational and historical loss	Historical claims under present third-party structure
Ecological harm	State, function, resilience, basin	Functional / resilience restoration	Lost ecological history	Restoration under nonlinear dynamics
Administrative classification	Record, status, eligibility, future access	Correction plus causal propagation	Reflexive effects of prior classification	Contestability and correction across systems
Cross-border loss	Status, money, credentials, mobility	Portable capability	Institutional conversion gaps	Interoperability across jurisdictions
Infrastructure failure	Availability, capacity, reliability, resilience	Structural restoration	Service interruption and downstream losses	Collective repair beyond individual compensation

Table 38: Temporal governance functions across applied cases

Case	Prevention	Interruption	Post-harm response	Generative assistance
Workplace	Anti-retaliation and gatekeeper redundancy	Temporary separation and continuity	Reinstatement, compensation, correction	Transition, re-entry, portable support
Research	Provenance and multiple access routes	Preserve project and authorship evidence	Correction and compensation	Funding, network, institutional re-entry
Reputation	Provisional labeling and restricted dissemination	Freeze harmful propagation	Correction and declaration	Professional re-entry
Health	Early accommodation and continuity	Prevent secondary livelihood collapse	Treatment and compensation	Care, time, work or educational support
Collective	Institutional safeguards	Stop continuing structural harm	Plural reparations	Governance and collective capacity
Ecological	Safety margins and early warning	Emergency containment	Rehabilitation and restoration	Long-term ecological management
Administrative	Versioning and model audit	Suspend consequential erroneous use	Correction and review	Restore eligibility and access
Infrastructure	Redundancy and buffers	Emergency service continuity	Repair and compensation	Alternative access and resilient redesign

The applied case demonstrates why prevention, repair, and infrastructure investment belong to one temporal governance architecture.

31.84 Cross-Case Comparison

The cases differ substantially in object, temporality, and institutional form.

Their common structure can be seen through the matrix in Table 37.

Table 37 shows that the strongest relevant restorative relation varies substantially across domains.

31.85 Cross-Case Temporal Architecture

Table 38 compares preventive, interruptive, post-harm, and assistance functions across several cases.

Table 38 illustrates why temporal function is as important as remedial form.

31.86 Cross-Case Residual Structure

Several recurring residual types appear across the cases.

They include:

- lost time;

- historically singular opportunity;
- altered relationships;
- distributed reputational memory;
- irreversible ecological history;
- new third-party interests;
- valued post-harm development.

These residuals should not be forced into one category of uncompensated loss.

Some are actionable. Some are compensable. Some require recognition. Some are normatively protected from elimination.

31.87 Cross-Case Role of General Equivalents

Money performs differently across the cases.

In workplace and health cases, it can secure income continuity and time.

In research cases, it can fund alternative institutional pathways.

In reputational cases, it has weaker ability to change public meaning.

In collective cases, it can fund infrastructure while remaining insufficient for governance or recognition.

In ecological cases, monetary resources can finance restoration but cannot purchase historical identity.

The applied comparison therefore confirms the political-economic conclusion that the strength of general equivalents depends upon the conversion environment.

31.88 Cross-Case Role of Capabilities

Capability analysis provides a particularly portable applied standard.

Across all cases, the relevant question can be stated as whether the subject or collective possesses meaningful substantive opportunity after the intervention [55, 39, 50].

The content of the capability set differs:

- professional options in workplace cases;
- intellectual and institutional options in research;
- social and professional access in reputation;
- life activity in health;
- collective self-government in historical injustice.

Capability theory therefore supplies a common grammar while leaving domain content plural.

31.89 Cross-Case Role of Recognition

Recognition also recurs across domains.

A formally correct resource transfer can fail where:

- the worker remains stigmatized;
- the researcher remains misattributed;

- the reputational record remains false;
- the collective wrong remains publicly denied.

Recognition is therefore neither merely symbolic nor universally substitutive. Its practical importance depends upon the harm.

31.90 Cross-Case Role of Non-Domination

Non-domination provides another recurring standard.

A restored subject can remain dependent upon:

- the same manager;
- the same academic gatekeeper;
- an opaque classification system;
- discretionary assistance.

Secure rights, portability, independent review, and exit therefore recur as cross-domain restorative mechanisms [44].

31.91 Cross-Case Role of Generative Justice

Generative justice asks whether value and future capacity circulate in ways that permit continued generation [15, 17].

Across the cases, this shifts attention from:

- compensation to future professional capacity;
- research funding to intellectual recursion;
- record correction to restored institutional access;
- collective payment to community-controlled infrastructure.

The applied cases therefore confirm that generative justice is best understood as one process-oriented normative layer rather than a substitute for all other justice theories.

31.92 Cross-Case Procedural Constraints

Several procedural constraints recur:

- subject participation;
- third-party rights;
- privacy;
- contestability;
- revisability;
- proportionality.

Their relative weight increases as the intervention moves from external resources toward identity, relationships, or distributed social structures.

31.93 Cross-Case Institutional Division of Labor

The applied cases also support institutional pluralism.

Courts or adjudicative institutions can determine rights and legally cognizable loss.

Administrative institutions can correct status and coordinate services.

Social-protection systems can secure material floors.

Professional institutions can support re-entry.

Archives and truth-oriented institutions can preserve historical recognition.

Community institutions can govern collective generative conditions.

No single institution is naturally competent across the entire atlas.

31.94 Cross-Case Handoff Problem

Institutional plurality creates the risk that unresolved objects fall between mandates.

Let institution I_i resolve object set $\mathfrak{D}_i$.

The uncovered set is

$$\mathfrak{D}_{\text{gap}} = \mathfrak{D}_V^* \setminus \bigcup_i \mathfrak{D}_i. \quad (3078)$$

Equation 3078 identifies a system-level restorative gap.

Applied analysis should therefore include handoff architecture rather than evaluating institutions only in isolation.

31.95 Applied Restorative Profiles

For each case c , define restorative profile

$$\mathbf{R}^{(c)} = (r_{\text{state}}, r_{\text{resource}}, r_{\text{capability}}, r_{\text{relation}}, r_{\text{recognition}}, r_{\text{history}}, r_{\text{generativity}}, r_{\text{procedure}}). \quad (3079)$$

Equation 3079 permits cross-case comparison without assuming that the numerical value of each component is directly commensurable.

31.96 Applied Completion Statements

The case analyses show why institutional completion should be scoped.

Appropriate statements include:

Formal employment status has been restored.

The erroneous institutional classification has been corrected and propagated to the identified operational systems.

The rehabilitation threshold has been met for the specified functional domain.

The ecological system has regained the selected functional and resilience thresholds over the monitored horizon.

These statements are more informative than a general declaration that the affected system has been made whole.

31.97 Applied Residual Statements

The same precision should apply to residuals.

A residual statement can identify:

- what remains different;
- whether the difference is unresolved, protected, or valued;
- whether another institution bears responsibility;
- whether future review remains available.

This is especially important where complete restoration is unavailable.

31.98 Applied Robustness Analysis

Each applied case should also ask whether the restored condition survives ordinary future stress.

Let case-specific disturbance class be $\Delta_{\text{ordinary}}^{(c)}$.

A robust applied criterion is

$$J_S \left(R^{(c)}, \delta \right) \geq \theta_S \quad \forall \delta \in \Delta_{\text{ordinary}}^{(c)}. \quad (3080)$$

Equation 3080 prevents calm-baseline success from being mistaken for durable restoration.

31.99 Applied Counterfactual Modesty

Several cases demonstrate severe uncertainty concerning the exact no-harm trajectory.

Research, reputation, and collective historical cases are especially sensitive to contingency.

Where exact trajectory identification is weak, applied analysis should prefer:

- ranges;
- probability distributions;
- capability sets;
- reachability bounds;
- historical acknowledgment.

This preserves corrective reasoning without manufacturing a fictitious exact biography.

31.100 Applied Normative Pluralism

The same remedy can be justified by different theories.

Portable compensation can support:

- welfare;
- capability;
- non-domination;
- subject-directed generativity.

Record correction can support:

- rights;
- recognition;
- capability;
- historical truth.

Normative convergence can therefore strengthen practical justification without requiring theoretical unification.

31.101 Applied Value Conflict

The cases also produce recurring conflicts:

- historical fidelity and present agency;
- transparency and privacy;
- collective repair and third-party rights;
- resilience and flexibility;
- speed and participation.

The applied value of the atlas lies partly in locating these conflicts before a decision is made.

31.102 Applied Transformation

Across several cases, the most just outcome is a transformed future.

The worker may enter a new institution.

The researcher may develop a new field of inquiry.

The harmed collective may create a new governance structure.

The ecosystem may require a new resilient regime under changed environmental conditions.

A transformed future should therefore be represented by

$$T_N \neq T_H \quad \text{and} \quad T_N \neq T_{cf}, \quad (3081)$$

while satisfying

$$J(T_N) \succeq J_{\min}. \quad (3082)$$

Equations 3081 and 3082 provide the applied expression of the right to a different future.

31.103 Applied Atlas Workflow

A practical use of the atlas can follow the sequence below:

1. identify the harmful event or process;
2. identify candidate references;
3. decompose the affected objects;
4. select comparative relations appropriate to each object;
5. specify temporal horizons;

6. identify value theories and constraints;
7. distinguish substantive and procedural justice;
8. map institutional competencies;
9. select remedy, assistance, and preventive functions;
10. identify residual differences;
11. establish review and closure conditions.

This workflow does not automate normative judgment.
It disciplines the structure of that judgment.

31.104 Applied Comparison Principle

The first principle of this section is:

Concrete restorative disputes should be decomposed into explicit reference–object–relation cells before claims of successful return are made.

31.105 Case-Specificity Principle

The second principle is:

The same remedy form can realize different restorative values across cases because its effect depends upon the subject, institutional environment, conversion structure, and historical context.

31.106 Multi-Object Principle

The third principle is:

Complex harms typically affect several objects simultaneously, so restoration of one legally or administratively visible object should not be interpreted automatically as restoration of the full generative system.

31.107 Effective-Access Principle

The fourth principle is:

Formal status, nominal resources, or corrected records should be distinguished from effective capability to use the restored condition in future practice.

31.108 Historical-Residual Principle

The fifth principle is:

Successful future restoration can coexist with historical residual loss, especially where time, singular opportunities, relationships, or ecological history cannot be recreated.

31.109 Transformative-Future Principle

The sixth principle is:

Where exact historical return is unavailable, unwanted, or normatively inferior, the applied objective can become construction of a new just future rather than approximation of the no-harm trajectory.

31.110 Institutional-Handoff Principle

The seventh principle is:

A bounded institutional remedy should identify residual objects requiring handoff rather than either abandoning them or expanding one institution's mandate without limit.

31.111 Durability Principle

The eighth principle is:

Applied restorative success should be tested under ordinary future disturbance where the practical value of the remedy depends upon continuing usability.

31.112 Residual-Classification Principle

The ninth principle is:

Differences remaining after intervention should be classified as unresolved, normatively protected, or positively valued rather than treated as one undifferentiated restoration gap.

31.113 Applied-Humility Principle

The tenth principle is:

Applied restorative claims should remain no broader than the evidence, institutional competence, comparison object, horizon, and normative standard actually support.

31.114 Applied Layer of the Atlas

The applied layer can now be represented by

$$\mathcal{A}_{\text{applied}} = (\mathcal{C}_A, \mathbf{R}, \mathcal{I}, \Delta, \mathcal{R}_v), \quad (3083)$$

where:

- $\mathcal{C}_A$ contains the case-specific comparison architecture;
- $\mathbf{R}$ contains remedy, assistance, and preventive responses;
- $\mathcal{I}$ contains responsible institutions;
- Δ contains classified residuals;
- $\mathcal{R}_v$ contains review and revision mechanisms.

Equation 3083 converts the general theory into a case-comparison interface.

31.115 Limits Revealed by Application

The applied analyses reveal several limits that remain important for the theory.

First, high-dimensional modeling can improve conceptual clarity while increasing measurement and institutional burden.

Second, some comparison objects remain difficult to observe empirically.

Third, references can remain contested.

Fourth, value pluralism can generate genuine incomparability.

Fifth, institutional competence places practical limits on the depth of restorative assessment.

Sixth, the atlas itself can become excessively complex if every conceivable object is activated in every case.

These problems do not invalidate the framework.

They define important limits and open research questions.

31.116 Transition to Open Problems

Section 32 therefore turns from application toward unresolved theoretical, formal, empirical, legal, institutional, and technological questions.

The open problems include:

- construction of defensible reference families under uncertainty;
- empirical measurement of relational and generative objects;
- identification of appropriate comparative relations across scales;
- governance of incomplete and reflexive models;
- normative aggregation under value pluralism;
- empirical estimation of reparability;
- institutional implementation of cross-domain restorative profiles;
- design of privacy-preserving shared epistemic infrastructure;
- comparative study of legal remedy architectures;
- evaluation of long-horizon generative assistance and prevention.

The central research question becomes:

Which parts of the Atlas of Equivalence can be operationalized reliably in specific domains, which parts should remain qualitative or deliberative, and how can institutions use the framework without converting a theory of restorative precision into an apparatus of excessive measurement and control?

32 Open Problems in Restorative Comparison and Generative Justice

The preceding sections developed a broad comparative, normative, legal, and institutional architecture for analyzing justice after harm. The framework distinguishes references, comparison objects, comparative relations, temporal horizons, causal and generative structure, historical provenance, value objects, substantive justice, procedural justice, legal remedies, general equivalents, generative assistance, preventive governance, and applied case analysis.

The resulting architecture is intentionally incomplete.

Its purpose is to make restorative claims more explicit, not to suggest that all relevant objects can already be measured, all counterfactuals can be identified, or all normative disagreements can be resolved through formal modeling. Several of the most important questions exposed by the framework remain open at the levels of theory, mathematics, empirical research, law, institutional design, technology, and political philosophy.

The role of this section is therefore to identify a research agenda rather than to close the theory. The open problems are organized according to the part of the restorative architecture in which the uncertainty arises. This structure matters because different unresolved questions require different methods. Some are formal questions. Some require empirical study. Some are problems of normative theory. Some concern institutional feasibility. Others may remain resistant to complete formalization by their nature.

The central methodological principle is that unresolved complexity should remain visible. A restorative theory becomes weaker when it converts uncertainty, incomparability, or incomplete observability into artificial precision merely for the sake of producing one determinate score.

32.1 Reference Construction under Counterfactual Uncertainty

The first major problem concerns construction of the restorative reference.

A no-harm counterfactual is often neither directly observed nor uniquely identified. Let the admissible reference family be

$$\mathfrak{B} = \{B_1, B_2, \dots, B_n\}. \quad (3084)$$

Equation 3084 can contain pre-harm actual states, counterfactual trajectories, capability thresholds, rights-based references, statistical comparators, and transformed future targets.

The open problem is how to construct $\mathfrak{B}$ without either selecting one speculative reference prematurely or allowing so many references that the comparison loses practical force.

32.2 Reference-Family Reduction

A practical institution may need to reduce $\mathfrak{B}$ to a smaller set

$$\mathfrak{B}^* \subseteq \mathfrak{B}. \quad (3085)$$

The reduction rule should account for:

- empirical plausibility;
- normative relevance;
- subject perspective;
- institutional competence;
- robustness across model assumptions.

A major open question concerns which reduction rules can be justified without silently embedding one contested theory of justice.

32.3 Reference Robustness

Restorative conclusions should ideally remain stable across plausible references.

Let conclusion under reference B_i be $J(R \mid B_i)$.

A robust conclusion satisfies

$$J(R \mid B_i) \succeq J_{\min} \quad \forall B_i \in \mathfrak{B}^*. \quad (3086)$$

Equation 3086 provides one possible definition of reference robustness.

An open problem concerns how large the plausible reference family should be and how disagreement among references should affect remedy design.

32.4 Reference Drift

The appropriate restorative target can change over time.

Let reference at time t be B_t .

It can evolve according to

$$B_{t+1} = \mathcal{U}_B(B_t, E_{t+1}, V_{t+1}, P_{A,t+1}), \quad (3087)$$

where E_{t+1} denotes new evidence, V_{t+1} changed value conditions, and $P_{A,t+1}$ present subject priorities.

The open question is when reference updating constitutes justified adaptation and when it becomes target drift that weakens the original corrective claim.

32.5 Reference Authority

The framework also leaves open who possesses legitimate authority to select the reference.

Possible actors include:

- the affected subject;
- courts;
- administrative institutions;
- experts;
- affected communities;
- plural deliberative bodies.

No single actor has automatic epistemic and normative supremacy across all cases.

The governance of reference selection therefore remains a major institutional problem.

32.6 Comparison-Object Selection

The second broad problem concerns selection of comparison objects.

The atlas intentionally contains many possible objects:

$$\Omega = \{Q_1, \dots, Q_m\}. \quad (3088)$$

Equation 3088 can include resources, states, capabilities, relations, networks, trajectories, causal mechanisms, generative processes, histories, recognition, and procedural objects.

Activating every object in every case would be unmanageable.

The open problem is how to identify a minimal sufficient object set.

32.7 Minimal Sufficient Restorative Representation

Let case-specific object set be $\Omega^* \subseteq \Omega$.

A desirable representation should satisfy two competing criteria:

$$\text{Coverage}(\Omega^*) \geq \theta_C \quad (3089)$$

and

$$\text{Burden}(\Omega^*) \leq \theta_B. \quad (3090)$$

Equations 3089 and 3090 define a conceptual minimal-sufficiency problem.

The challenge is to determine which omitted objects create unacceptable normative loss.

32.8 Latent Value Objects

Some important restorative values can remain invisible until the remedy fails.

A case initially represented through income and formal status can later reveal trust, network position, or portability as binding objects.

Let currently represented object set be Ω_t . Discovery can produce

$$\Omega_{t+1} = \Omega_t \cup \Delta\Omega_{t+1}. \quad (3091)$$

Equation 3091 makes ontology expansion part of restorative analysis.

A central open question concerns how institutions can remain open to object discovery without making every case indefinitely revisable.

32.9 Comparative-Relation Selection

The choice of comparative relation creates another unresolved problem.

Let admissible relation family for object Q be

$$\mathfrak{E}(Q) = \{E_1, \dots, E_k\}. \quad (3092)$$

Equation 3092 can include identity, inclusion, simulation, bisimulation, dominance, sufficiency, approximation, or historical correspondence.

A stronger mathematical relation is not automatically a more appropriate normative relation.

The theory therefore requires principled methods for relation selection.

32.10 Relation Refinement

Suppose a case begins with coarse relation E_0 and later evidence motivates a stronger relation E_1 .

Refinement can be represented by

$$E_1 \preceq_{\text{refine}} E_0. \quad (3093)$$

Equation 3093 means satisfaction of E_1 preserves at least the structure preserved by E_0 under the relevant formal ordering.

The open question is when refinement is worth its additional epistemic, measurement, and procedural cost.

32.11 Incomparable Comparative Relations

Many relations do not lie on one scale.

Causal equivalence, historical equivalence, capability inclusion, and recognition sufficiency can be mutually incomparable.

A future formal research problem is to identify useful local partial orders among relation families without imposing an artificial global hierarchy.

32.12 Formal Semantics of Restorative Claims

The manuscript has introduced a general indexed relation

$$\mathcal{G}^R \sim_{E, Q, \mathcal{T}, \tau, \varepsilon, K} \mathcal{G}^{\text{ref}}. \quad (3094)$$

Equation 3094 provides a compact comparative grammar.

A formal open problem is to develop a rigorous semantics for a useful subset of such claims.

This includes specifying:

- admissible system classes;
- observation algebras;
- intervention languages;
- approximation structures;
- compositional properties;
- cross-scale mappings.

The goal would be a formal toolkit rather than one universal restorative logic.

32.13 Restorative Modal Logic

Reachability, possibility, necessity, intervention, and historical branching suggest a possible modal representation.

Let

$$\Diamond_R \phi \quad (3095)$$

mean that proposition ϕ is reachable through an admissible restorative intervention, and let

$$\Box_R \phi \quad (3096)$$

mean that ϕ holds across the selected restorative policy family.

A future research program could study which modal structures adequately capture restorative possibility without reducing historical and normative structure to ordinary transition logic.

32.14 Restorative Type Systems

Another formal direction is the development of typed restorative objects.

For example, resource, relation, history, and capability objects should not necessarily admit the same comparison operators.

Let object type be

$$\tau_Q : Q \longrightarrow \mathcal{T}_Q. \quad (3097)$$

A type system could restrict invalid comparisons before normative evaluation.

This could prevent errors such as applying numerical equality to objects whose meaning is relational or historical.

32.15 Composition of Restorative Claims

Complex remedies combine several restorative cells.

Let cells be $c_1, \dots, c_n$. Their composition can be represented by

$$\mathcal{C}_R = c_1 \otimes \dots \otimes c_n. \quad (3098)$$

Equation 3098 leaves the composition operator undefined.

A major formal problem is determining when restorative cells compose conjunctively, lexically, conditionally, or through partial orders.

32.16 Cross-Scale Composition

Individual and collective restorative conditions can interact.

Let individual profiles be J_i and collective condition be J_C .

There is no general rule implying

$$J_C = F(J_1, \dots, J_n). \quad (3099)$$

Equation 3099 leaves open how individual and collective justice should be composed where collective goods and governance have emergent properties.

32.17 Temporal Aggregation

Restorative value can vary through time.

Let value stream be $V(t)$.

A common aggregate would be

$$V_\tau = \int_0^\tau w(t)V(t) dt. \quad (3100)$$

Equation 3100 requires a weighting function $w(t)$.

The open question is which value domains admit such temporal aggregation and which contain historically non-substitutable periods.

32.18 Discounting

Discounting future value is especially controversial where restorative claims concern delayed justice, intergenerational harm, or ecological loss.

Let discount function be $d(t)$.

A discounted value is

$$V_D = \int_0^\tau d(t)V(t) dt. \quad (3101)$$

Equation 3101 is mathematically simple and normatively demanding.

Different discount rules can substantially change the apparent value of long-run restorative or preventive interventions.

32.19 Temporal Non-Substitutability

A central open problem is identifying when value at one time can substitute for value at another.

Let temporal substitutability relation be

$$V_{t_1} \sim_{\text{temp}} V_{t_2}. \quad (3102)$$

For income, some degree of intertemporal substitution can exist.

For childhood, historical opportunity, authorship priority, or ecological transition windows, substitution can be much weaker.

A general theory of temporal substitutability remains underdeveloped.

32.20 Empirical Measurement of Generativity

One of the most important empirical challenges concerns generativity itself.

The concept should not be reduced to output volume.

A plausible generativity profile can include

$$\mathbf{G} = (G_{\text{option}}, G_{\text{conversion}}, G_{\text{recursion}}, G_{\text{diversity}}, G_{\text{autonomy}}, G_{\text{retention}}). \quad (3103)$$

Equation 3103 separates option generation, conversion, recursive reproduction, diversity, autonomy, and retained value.

The open empirical question is which indicators can measure these dimensions in specific domains without turning generativity into a vague or all-encompassing construct.

32.21 Domain-Specific Generativity Metrics

Generativity in employment, research, ecological systems, education, and communities need not share one metric.

Let domain d possess generativity representation

$$\mathbf{G}^{(d)}. \quad (3104)$$

The research problem is to identify the invariant conceptual core and the domain-specific indicators.

A successful theory should permit domain variation without rendering the concept unfalsifiable.

32.22 Measurement of Capability Sets

Capability approaches provide a compelling evaluative space and face difficult measurement questions.

The actual capability set is rarely observed directly.

Let observed functioning and resources be Y , and latent capability set be $\mathcal{K}$.

Empirical work must infer

$$P(\mathcal{K} \mid Y, X, E), \quad (3105)$$

where X contains subject conversion factors and E environment.

The open problem is how much capability inference can be justified without constructing an excessively intrusive model of the subject.

32.23 Measurement of Relational Objects

Trust, standing, domination, recognition, and network usability are difficult to observe directly.

Potential indicators include:

- reported experience;
- observed interaction;
- network structure;
- institutional decisions;
- perturbation response.

No single indicator fully captures the relational object.

The research challenge concerns triangulation across several evidence types.

32.24 Measurement of Historical Recognizability

Historical recognizability was introduced as the future ability to know that an event occurred and understand its relation to later states.

Let recognizability be

$$R_H = R(P, A, I, C), \quad (3106)$$

where P denotes preservation, A access, I interpretability, and C correction provenance.

The open problem is whether this can be operationalized meaningfully across archives, law, public memory, and digital systems.

32.25 Measurement Burden

Greater measurement depth can create higher burden.

Let measurement depth be m , information gain $K(m)$, and subject burden $B(m)$.

A design problem is represented by

$$\max_m [K(m) - B(m)]. \quad (3107)$$

Equation 3107 is illustrative and should not imply that privacy and dignity are always monetizable costs.

The open question is how to determine a sufficient rather than maximal measurement architecture.

32.26 Proxy Validity

Institutions often use proxies because core value objects are difficult to measure.

Let proxy be P and target value object V^* .

Proxy validity can vary across contexts:

$$\text{Val}(P, V^* \mid E_1) \neq \text{Val}(P, V^* \mid E_2). \quad (3108)$$

Equation 3108 creates a research problem concerning transportability of restorative metrics.

32.27 Metric Drift

A metric can lose validity after institutions optimize against it.

Let proxy validity at time t be v_t .

Reflexive drift can satisfy

$$v_{t+1} < v_t. \quad (3109)$$

Equation 3109 makes metric revalidation an ongoing institutional obligation.

32.28 Causal Identification of Generative Harm

Generative harm often propagates through long causal chains.

Let

$$H \rightarrow C_1 \rightarrow C_2 \rightarrow \cdots \rightarrow G_t. \quad (3110)$$

Equation 3110 represents harm moving through conditions before affecting later generativity.

The empirical challenge is identifying which links are causally supported and which remain speculative.

Pearl's structural causal framework provides tools for distinguishing observational and interventional claims, but restorative applications raise additional problems of history, interference, and changing mechanisms [43].

32.29 Dynamic Causal Models

Many restorative systems contain feedback and changing causal structure.

A time-indexed structural model can be represented by

$$X_{t+1} = F_t(X_t, U_t). \quad (3111)$$

Equation 3111 allows mechanisms to evolve.

An open problem is how to integrate such dynamic models with the legal and normative need for intelligible causal attribution.

32.30 Interference

One person's intervention can affect others' outcomes.

Let outcome for person i be

$$Y_i = Y_i(A_1, \dots, A_n). \quad (3112)$$

Equation 3112 violates simple no-interference assumptions.

Workplace, network, collective, and ecological restoration often possess this structure.

The open question is how to estimate restorative causal effects in systems with substantial interference.

32.31 Mechanism Change after Harm

Harm can change the causal mechanism itself.

Let pre-harm mechanism be F_0 and post-harm mechanism be F_H .

It can hold that

$$F_0 \neq F_H. \quad (3113)$$

Equation 3113 means counterfactual estimation based upon post-harm data can fail to recover the earlier mechanism.

This is especially important for trust, health, organizations, and ecological systems.

32.32 Causal Contribution and Legal Responsibility

Scientific causal contribution and legal responsibility remain different objects.

Let factual causal contribution be C_F and legal responsibility be R_L .

A translation relation can be represented by

$$R_L = \Psi_L(C_F, N_L, P_L), \quad (3114)$$

where N_L contains legal norms and P_L policy or institutional considerations.

The open comparative-law problem is how different legal systems perform this translation.

32.33 Reparability Estimation

The framework repeatedly relies upon reparability.

Empirically, reparability is difficult to estimate.

Let reparability of object Q under intervention set $\mathcal{U}$ be

$$\rho_Q = P(Q_R \sim_E Q_{\text{ref}} \mid \mathcal{U}, K, \tau). \quad (3115)$$

Equation 3115 provides one possible probabilistic interpretation.

The open research problem concerns estimation of ρ_Q when interventions are heterogeneous, histories differ, and the relevant equivalence relation is non-scalar.

32.34 Reparability as a Vector

A scalar reparability measure can conceal major differences.

A richer representation is

$$\boldsymbol{\rho} = (\rho_{\text{state}}, \rho_{\text{function}}, \rho_{\text{capability}}, \rho_{\text{history}}, \rho_{\text{generativity}}). \quad (3116)$$

Equation 3116 can distinguish high state reparability from low historical reparability.

The problem is how to estimate each component empirically.

32.35 Reparability and Prevention Thresholds

Preventive governance requires rules linking reparability to intervention.

Let preventive intensity be P_I .

One possible relation is

$$P_I = F(1 - \rho, S, U, C), \quad (3117)$$

where S is severity, U uncertainty, and C intervention cost.

The open normative problem is how to calibrate such functions without allowing low reparability to justify excessive preventive control.

32.36 Critical-Transition Estimation in Social Systems

Critical-transition theory provides useful conceptual tools and raises a major empirical problem when transferred from ecological or physical systems to social institutions.

Early-warning indicators can depend heavily upon model assumptions [53].

The open problem is identifying when a social process is genuinely governed by a dynamical structure in which critical slowing down, basin boundaries, or bifurcation concepts have empirical validity.

Formal analogy alone is insufficient.

32.37 Attractor Identification

Observed persistence does not prove the existence of one attractor in the formal dynamical sense.

Let observed stable pattern be S_O .

The inference

$$S_O \implies A_{\text{formal}} \quad (3118)$$

requires model justification.

Equation 3118 should therefore be treated as a hypothesis rather than a default interpretation.

32.38 State-Space Reconstruction

Many social systems do not come with a natural state space.

Researchers must decide which variables constitute state.

Let candidate state representation be

$$x_t = \Phi(y_{1,t}, \dots, y_{n,t}). \quad (3119)$$

Equation 3119 creates a major modeling choice.

A future research program should study how restorative conclusions vary under alternative state-space constructions.

32.39 Reachability Estimation

Reachability is normatively attractive because it captures future possibility.

It is empirically difficult because many actions have never been attempted.

Let estimated reachable set be

$$\hat{\mathcal{R}}_\tau. \quad (3120)$$

The open problem is how to infer $\hat{\mathcal{R}}_\tau$ from limited behavior without confusing unobserved action with impossible action.

32.40 Recognized and Objective Reachability

The subject can fail to recognize objectively available options.

Let objective reachable set be $\mathcal{R}_\tau$ and recognized set be $\hat{\mathcal{R}}_{A,\tau}$.

Their difference is

$$\Delta\mathcal{R}_A = \mathcal{R}_\tau \setminus \hat{\mathcal{R}}_{A,\tau}. \quad (3121)$$

Equation 3121 can be reduced through information, trust, and navigation.

Empirically distinguishing objective absence from epistemic non-recognition remains difficult.

32.41 Value Pluralism and Decision

The framework permits value vectors, partial orders, lexical structures, and incomparability.

The institutional challenge is how to make decisions where

$$R_1 \parallel_V R_2. \quad (3122)$$

Equation 3122 represents genuine incomparability.

The open problem is to design legitimate procedures for decision without manufacturing one artificial scalar ranking [1, 48, 10].

32.42 Value Aggregation under Pluralism

Where some aggregation is institutionally necessary, the choice of aggregation rule becomes central.

Let value vector be $\mathbf{V}$ and aggregation operator A :

$$V_{\text{agg}} = A(\mathbf{V}). \quad (3123)$$

Equation 3123 can conceal:

- weighting assumptions;
- interpersonal comparisons;
- threshold violations;
- lexical values;
- incommensurable objects.

An open research direction concerns transparent multi-criteria methods that preserve these distinctions.

32.43 Lexical Priority

Rights, bodily integrity, privacy, or basic sufficiency can receive lexical priority.

Let

$$V_1 \succ_{\text{lex}} V_2. \quad (3124)$$

Equation 3124 restricts ordinary trade-offs.

The open normative problem is determining which values deserve lexical protection and under which institutional contexts.

32.44 Threshold Selection

Sufficientarian and rights-based approaches depend upon thresholds.

Let threshold be θ .

The unresolved question concerns the derivation of

$$\theta = \Theta(V, C, I), \quad (3125)$$

where V is value theory, C context, and I institutional purpose.

Thresholds that appear technically neutral can embed substantial normative judgments.

32.45 Interpersonal Comparison

Many remedial and preventive institutions must compare burdens and benefits across persons.

Let subject values be V_i and V_j .

Interpersonal comparison requires some mapping

$$\Gamma_{ij} : V_i \longrightarrow V_j. \quad (3126)$$

Equation 3126 is highly theory-dependent.

The problem becomes acute in allocation under scarcity and third-party conflict.

32.46 Present Preference and Adaptive Preference

The framework gives substantial authority to the present subject and recognizes adaptive preference concerns.

The unresolved question is how to balance:

$$P_A^H, \quad P_A^C, \quad \mathcal{K}_A, \quad V_{\text{objective}}. \quad (3127)$$

Equation 3127 represents present preference, counterfactual preference, capability, and possible objective-value claims.

No general weighting rule has been established.

32.47 Subject Authority under Identity Change

The problem becomes sharper where harm changes identity or preference deeply.

Which temporal version of the subject should have authority over restorative choice?

A purely historical answer risks counterfactual paternalism.

A purely present-preference answer can ignore severe adaptive deprivation.

A future theory requires a principled account of diachronic agency.

32.48 Collective Value and Internal Dissent

Collective restoration introduces another unresolved question.

Let community preferences be

$$\mathfrak{P}_C = \{P_1, \dots, P_n\}. \quad (3128)$$

Equation 3128 can contain deep internal disagreement.

The open problem is how collective rights, community governance, and individual dissent should interact in reparative decisions.

32.49 Representation of Future Persons

Intergenerational justice requires representation of persons who cannot participate.

Let future-value estimate be

$$V_F = \mathbb{E}[V(G_{t+\tau})]. \quad (3129)$$

Equation 3129 depends upon uncertain models and contested values.

The institutional problem is how future interests should enter current restorative and preventive governance without granting experts unchecked authority.

32.50 Legal Comparative Research

The legal atlas creates a substantial comparative-law research program.

For jurisdiction j , define legal remedy architecture

$$\mathfrak{L}^{(j)} = \left(\mathfrak{B}_L^{(j)}, \mathfrak{Q}_L^{(j)}, \mathfrak{E}_L^{(j)}, \mathfrak{R}_L^{(j)} \right). \quad (3130)$$

Equation 3130 allows systematic comparison of:

- which losses are legally recognized;
- which remedies are available;
- how counterfactuals are constructed;
- which values are monetized;
- how third-party interests are protected;
- when structural relief is permitted.

This comparative program remains largely open.

32.51 Legal Treatment of Generative Loss

A central legal research question concerns whether existing categories of damages and equitable relief adequately capture generative loss.

Potentially relevant objects include:

- lost opportunity;
- impaired future earning capacity;
- professional-network loss;
- lost control over future value generation;
- delayed capability formation.

The challenge is to recognize genuine downstream loss without expanding legal liability into unlimited speculative counterfactual chains.

32.52 Legal Stopping Rules

Generative loss can propagate recursively.

Law requires stopping rules.

Let causal loss chain be

$$W \rightarrow L_1 \rightarrow L_2 \rightarrow \cdots \rightarrow L_n. \quad (3131)$$

Equation 3131 creates a research question concerning the proper boundary between legally recoverable and broader morally relevant loss.

Comparative study of remoteness, foreseeability, scope of liability, and loss of chance could illuminate this boundary.

32.53 Legal Recognition of Residual Loss

Another question concerns whether legal systems need richer ways to acknowledge residual loss even where additional coercive relief is unavailable.

Possible institutional forms include:

- declarations;
- findings;
- public acknowledgment;
- preserved legal provenance.

This could allow legal closure without implying metaphysical or moral completeness.

32.54 Generative Assistance Evaluation

Generative assistance requires empirical evaluation beyond expenditure.

Let assistance outcome profile be

$$\mathbf{A} = (A_{\text{cap}}, A_{\text{security}}, A_{\text{agency}}, A_{\text{portability}}, A_{\text{generativity}}) . \quad (3132)$$

Equation 3132 raises the empirical question of how each dimension should be operationalized and over which horizon.

Longitudinal studies are especially important.

32.55 Causal Evaluation of Assistance

Assistance programs are difficult to evaluate because participants can differ systematically from non-participants, and the programs themselves can adapt over time.

A future research agenda should distinguish:

- immediate service use;
- capability change;
- future reachability;
- robustness;
- support dependence.

The relevant outcome is not one-time employment or income alone.

32.56 Permanent Support Evaluation

A theory of generative assistance also needs better criteria for judging permanent support.

Continued dependence can be compatible with justice.

The unresolved empirical and normative question is how to distinguish:

$$\begin{array}{l} D_{\text{legitimate}} \\ \text{from} \end{array} \quad (3133)$$

$$D_{\text{dominating}} . \quad (3134)$$

The distinction likely requires security, rights, exit, dignity, and control rather than duration alone.

32.57 Prevention Evaluation

Successful prevention creates a familiar empirical problem: the prevented event is unobserved.

Let observed harm after intervention be $H(P)$, and counterfactual harm without intervention be $H(0)$.

Preventive effect is

$$\Delta H = H(0) - H(P). \quad (3135)$$

Equation 3135 depends upon counterfactual inference.

This makes preventive success intrinsically harder to observe than completed harm.

32.58 Preventive False-Positive Evaluation

Preventive systems can also create substantial burden in cases where no harm would have occurred.

Evaluation therefore requires both

$$C_{\text{FN}} \quad (3136)$$

and

$$C_{\text{FP}}. \quad (3137)$$

The open problem is how to compare these burdens when one concerns unrealized future harm and the other concerns actual intervention.

32.59 Early-Warning Validation

Early-warning indicators must be validated prospectively.

An indicator that appears retrospectively before known crises can fail in new cases.

Let warning score be W_t .

Validation requires estimating

$$P(H_{t+\tau} \mid W_t) \quad (3138)$$

across out-of-sample contexts.

The research challenge is especially severe where institutions adapt in response to the warning, thereby changing the event probability.

32.60 Privacy-Preserving Epistemic Infrastructure

The manuscript repeatedly proposes distributed evidence, local processing, and shared standards.

A major technological and governance research program concerns how to implement these principles.

Let institution i hold private dataset D_i .

A shared query can be represented by

$$S = F(T_1(D_1), \dots, T_n(D_n)), \quad (3139)$$

where T_i exposes only the transformation necessary for the common function.

The open problem is how to design such infrastructures while preserving auditability, correction, consent, and provenance.

32.61 Distributed Provenance

A distributed restorative system needs provenance that can cross institutions without centralizing every raw record.

Let provenance fragments be $\mathcal{P}_1, \dots, \mathcal{P}_n$.

The challenge is to construct

$$\mathcal{P}^* = \text{Merge}(\mathcal{P}_1, \dots, \mathcal{P}_n) \quad (3140)$$

while preserving authenticity, access control, correction, and contextual meaning.

This problem sits at the intersection of law, security, information systems, and institutional governance.

32.62 Correction Propagation Infrastructure

Distributed erroneous records require distributed correction.

Let erroneous propagation graph be G_E and correction graph G_C .

An ideal condition is

$$G_C \succeq G_E^{\text{operational}}. \quad (3141)$$

Equation 3141 means correction reaches every operational node materially affected by the original record.

The open technological problem is how to support this without creating a permanent centralized identity ledger.

32.63 Automated Restorative Decision Support

Advanced computational systems can support:

- counterfactual modeling;
- resource estimation;
- causal analysis;
- case triage;
- remedy comparison.

Their use creates major governance questions.

A model can improve prediction and reduce explainability, contestability, or privacy.

The appropriate role of automation should therefore be studied as an institutional-design problem rather than a purely technical optimization problem.

32.64 Model Authority

One unresolved question concerns the authority assigned to model outputs.

Let model recommendation be R_M and institutional decision be R_I .

A high-deference architecture satisfies

$$P(R_I = R_M) \rightarrow 1. \quad (3142)$$

Equation 3142 can become problematic even when model accuracy is high if review is merely nominal.

Future research should examine meaningful human and institutional control over restorative models.

32.65 Model Contestability

A technically sophisticated restorative model should remain challengeable.

Contestability requires access to enough information to question:

- variables;
- data;
- assumptions;
- reference selection;
- value weights;
- uncertainty.

The open problem is how to provide meaningful contestability without requiring every affected person to become a technical expert.

32.66 Reflexive Model Governance

Models change behavior.

Let model output at time t be $M_t(X_t)$, and behavior after exposure be

$$X_{t+1} = F(X_t, M_t(X_t)). \quad (3143)$$

Equation 3143 means model performance can deteriorate because its own use changes the system.

A future restorative infrastructure therefore requires reflexive model audits.

32.67 Reflexive Non-Closure

The concept of reflexive non-closure remains underdeveloped formally.

The general structure is

$$\mathcal{G}_{t+1} = F(\mathcal{G}_t, \mathcal{O}_t, R_t), \quad (3144)$$

where observation $\mathcal{O}_t$ and restoration R_t become part of the next system state.

A major formal problem is to determine which kinds of fixed points, equilibria, or invariant restorative judgments are possible in such systems.

32.68 Reflexive Stability

Suppose evaluative rule J_t influences the system, and later evaluation uses the changed system.

A reflexively stable architecture would satisfy

$$J_{t+1} \approx J_t \quad (3145)$$

after the effects of its own use are incorporated.

Equation 3145 suggests a formal research direction for stable governance under endogenous measurement.

32.69 Open-Ended Ontology

Generative systems can create new kinds of relevant objects.

Let ontology at time t be $\mathcal{O}_t$.

Open-endedness is represented by

$$\mathcal{O}_{t+1} = \mathcal{O}_t \cup \Delta\mathcal{O}_{t+1}. \quad (3146)$$

Equation 3146 creates a challenge for any fixed restorative taxonomy.

The open problem is how an institutional system can revise its ontology without losing consistency and legal predictability.

32.70 Restorative Ontology Governance

Ontology expansion itself requires governance.

New categories can:

- reveal previously invisible harm;
- create new eligibility;
- create new classifications;
- redistribute institutional authority.

The politics of category creation therefore belongs within restorative governance.

32.71 Institutional Complexity

The manuscript advocates institutional pluralism.

Pluralism creates coordination problems.

Let institution set be

$$\mathcal{I} = \{I_1, \dots, I_n\}. \quad (3147)$$

Each institution controls object subset $\mathfrak{Q}_{I_i}$.

The system-level gap is

$$\mathfrak{Q}_{\text{gap}} = \mathfrak{Q}^* \setminus \bigcup_i \mathfrak{Q}_{I_i}. \quad (3148)$$

Equation 3148 identifies unresolved objects.

A major institutional research question concerns how to reduce these gaps without constructing one totalizing restorative bureaucracy.

32.72 Institutional Overlap

Institutions can also overlap.

Let shared object domain be

$$\mathfrak{Q}_{ij} = \mathfrak{Q}_{I_i} \cap \mathfrak{Q}_{I_j}. \quad (3149)$$

Equation 3149 creates potential duplication, contradictory decisions, and forum selection.

The open problem is designing precedence, coordination, and review rules.

32.73 Institutional Handoffs

Restorative cases frequently cross institutional boundaries.

Let handoff from I_i to I_j be

$$H_{ij} : \mathcal{S}_i \longrightarrow \mathcal{S}_j. \quad (3150)$$

Equation 3150 should preserve relevant status, evidence, provenance, and privacy constraints.

The practical design of these interfaces remains an important research area.

32.74 Institutional Latency

Delay can transform reparability.

Let institutional response time be T_I , and critical window length be T_C .

A high-risk condition is

$$T_I > T_C. \quad (3151)$$

Equation 3151 indicates that the institution can be procedurally correct and temporally incapable of effective restoration.

Future institutional research should therefore measure response latency together with outcome quality.

32.75 Institutional Learning

The theory assumes institutions can learn from repeated cases.

Empirically, learning can be weak.

Let lesson extraction be L_t , and policy update be

$$\Pi_{t+1} = U(\Pi_t, L_t). \quad (3152)$$

Equation 3152 raises the question of which governance arrangements actually cause lessons to propagate into institutional practice.

32.76 Correction Propagation in Institutions

Institutional learning also requires correction propagation.

An error corrected in one unit can remain active elsewhere.

The open design problem concerns how versioned institutional knowledge can be updated reliably without erasing historical provenance.

32.77 Organizational Incentives

Institutions can possess incentives to minimize visible harm, deny liability, or close cases rapidly.

Let institutional objective be

$$U_I = U_I(C, R, L, P), \quad (3153)$$

where C denotes cost, R reputation, L liability, and P public performance.

Equation 3153 need not align with claimant restoration.

Future research should examine incentive-compatible restorative governance.

32.78 Restorative Moral Hazard

High capacity for compensation or repair can reduce prevention.

Let preventive effort be e_P , and expected reparability be ρ .

A possible relation is

$$\frac{\partial e_P}{\partial \rho} < 0. \quad (3154)$$

Equation 3154 identifies a moral-hazard hypothesis requiring empirical study.

Institutions should not treat reparability as permission to tolerate avoidable harm.

32.79 Political Economy of Restorative Industries

Claims management, legal services, insurance, rehabilitation, consulting, and technology can form industries around harm and repair.

An open political-economic question concerns whether these industries generate incentives toward:

- efficient repair;
- administrative expansion;
- recurring dependency;
- prevention;
- commodification of harm.

The institutional ecology of restoration deserves study in its own right.

32.80 General-Equivalent Capture

Section 28 identified the possibility that compensation is partly captured by creditors, intermediaries, or constrained markets.

A future empirical program should estimate the retained-value ratio

$$\eta_M = \frac{M_{\text{retained}}}{M_{\text{gross}}}. \quad (3155)$$

Equation 3155 could be linked to capability and generative outcomes.

32.81 Supply Effects of Large Reparative Programs

Large transfers can alter local prices and supply.

Let aggregate reparative transfer be M^Σ , and restorative-good price be

$$p = p(M^\Sigma, S). \quad (3156)$$

Equation 3156 makes program scale endogenous to real restorative value.

Future reparations research should therefore combine transfer analysis with supply, infrastructure, and ownership structure.

32.82 Generative Circulation Measurement

Generative justice gives value circulation a central role.

Empirically, this requires tracing where generated value flows.

Let value-flow network be

$$G_V = (N_V, E_V, w_V), \quad (3157)$$

where edge weights represent transfers or retained value.

A major research direction concerns measurement of circularity, extraction, retention, and control without reducing all value to money [15, 17].

32.83 Control Measurement

Formal ownership and effective control can differ.

Let control profile be

$$\mathbf{C} = (C_{\text{decision}}, C_{\text{exit}}, C_{\text{allocation}}, C_{\text{information}}, C_{\text{revision}}). \quad (3158)$$

Equation 3158 suggests a multidimensional empirical approach to control.

How these dimensions should be weighted remains open.

32.84 Cross-Cultural Value Translation

Restorative concepts can carry different meanings across cultures and legal traditions.

Recognition, autonomy, care, property, community, dignity, and historical repair need not map identically.

Let value translation between contexts C_1 and C_2 be

$$T_{12}^V : V^{C_1} \longrightarrow V^{C_2}. \quad (3159)$$

Equation 3159 can be lossy.

A major open problem is how to preserve plural normative meaning in cross-jurisdictional and international restorative institutions.

32.85 Plural Legal Ontologies

Different legal systems can classify the same harm through different categories.

A property-based system, dignity-based system, capability-oriented policy, and collective-rights framework can produce different remedial objects.

Comparative law should therefore study ontological translation as well as doctrinal outcome.

32.86 Scale Translation

Restorative insights can fail when moved across scales.

A successful individual remedy does not automatically become a successful collective remedy.

Let scale transformation be

$$T_s : Q^{(s_1)} \longrightarrow Q^{(s_2)}. \quad (3160)$$

Equation 3160 can lose emergent collective structure.

The open problem is determining which restorative relations are scale-stable.

32.87 Micro–Macro Non-Equivalence

Equal aggregate outcome can coexist with unequal individual trajectories.

Let aggregate state satisfy

$$Q_{\text{macro}}^R = Q_{\text{macro}}^{\text{ref}} \quad (3161)$$

while

$$Q_i^R \neq Q_i^{\text{ref}} \quad (3162)$$

for many individuals.

Equations 3161 and 3162 show why aggregate restoration does not establish justice at the micro level.

32.88 Robustness across Scale

A restorative intervention can be robust at organizational scale and fragile at individual scale.

Future empirical work should therefore test

$$Q^{(s_1)} \neq Q^{(s_2)}. \quad (3163)$$

Equation 3163 makes scale-indexed robustness explicit.

32.89 Ecological Value and Nonhuman Generativity

The ecological applications raise an unresolved normative question concerning whose generativity matters.

The framework can represent ecological processes and resilience.

It does not settle whether nonhuman systems possess intrinsic, relational, or instrumental value.

A future extension could examine the relation among:

$$V_{\text{human}}, \quad V_{\text{ecological}}, \quad G_{\text{ecological}}. \quad (3164)$$

Equation 3164 leaves the underlying environmental ethic open.

32.90 Restoration under Climate Change

Climate change makes historical ecological baselines increasingly difficult to treat as stable targets.

The no-harm historical ecosystem can become nonviable under altered future conditions.

This creates a particularly important empirical and normative research area for transformative ecological restoration.

32.91 Digital and Informational Harms

Digital systems create new restorative problems involving:

- persistent records;
- replicated data;
- algorithmic classification;
- automated propagation;
- cross-platform identity.

The ease of copying information makes exact deletion difficult, while broad deletion can conflict with provenance and accountability.

The historical-reflexive architecture is especially relevant to this domain.

32.92 Information Deletion and Provenance

A major technological and legal problem concerns how to support both correction and historical accountability.

Let active record be Z_A , archived provenance be Z_P , and public access rule be A_Z .

A future architecture can seek

$$Z_A = Z_{\text{correct}} \quad (3165)$$

while preserving

$$Z_P = \text{Prov}(Z_A). \quad (3166)$$

Equations 3165 and 3166 require careful privacy and access governance.

32.93 Identity Reconstruction Technologies

Future technologies capable of altering memory, cognition, or identity would make several thought experiments in Section 26 practical rather than hypothetical.

A corresponding research agenda would require:

- consent theory;
- diachronic identity;
- continuity of legal personhood;
- evidentiary preservation;
- rights against restorative modification.

The theory developed here suggests that technical capability would increase, rather than eliminate, the need for procedural constraint.

32.94 Restorative Constitutionalism

Highly capable restorative institutions could exercise extraordinary power over memory, classification, identity, resources, and future trajectories.

A long-run research problem is therefore the constitutional governance of restorative power.

Possible institutional questions include:

- separation of restorative functions;
- protected non-intervention domains;
- judicial or independent review;
- data governance;
- limits on counterfactual reconstruction;
- rights to refuse restoration.

This could become a distinct field of restorative constitutionalism.

32.95 Right to Remain Different

The manuscript repeatedly identifies differences that justice can deliberately preserve.

A future theory should clarify whether there exists a more general right to remain historically different from one's modeled no-harm self.

Let counterfactual difference be

$$\Delta_A = A^R \setminus A^C. \quad (3167)$$

The normative question concerns which components of Δ_A fall within the subject's protected authority.

32.96 Residual Justice

A major theoretical problem concerns justice where full repair is unavailable.

Let residual loss be

$$\Delta_{\text{unresolved}} > 0. \quad (3168)$$

What is owed after all feasible and legitimate restorative interventions have been exhausted?

Possible responses include:

- acknowledgment;
- continuing compensation;
- assistance;
- memorialization;
- prevention of recurrence.

A systematic theory of residual justice remains to be developed.

32.97 Justice after Irreparable Loss

Irreparable loss challenges the language of restoration itself.

Where the relevant object cannot be recreated, the normative task shifts from equivalence toward the construction of a just relation to the residual.

This transition deserves deeper philosophical analysis.

32.98 Historical Truth and Forgiveness

Another open problem concerns the relationship among historical truth, recognition, forgiveness, closure, and justice.

Forgiveness can transform the moral relation without changing the historical fact.

It should not be treated as a mandatory condition of restoration.

The framework presently leaves the theory of forgiveness largely outside its scope.

32.99 Reconciliation

Reconciliation likewise deserves separate treatment.

A repaired legal or material relation can exist without interpersonal or political reconciliation.

A future theory could distinguish:

$$R_{\text{legal}}, \quad R_{\text{material}}, \quad R_{\text{relational}}, \quad R_{\text{reconciliatory}}. \quad (3169)$$

Equation 3169 would prevent reconciliation from becoming an unexamined endpoint imposed upon affected persons.

32.100 Restorative Success and Subjective Closure

Institutional completion and subjective closure can diverge.

Let institutional closure be C_I and subjective closure be C_A .

It can hold that

$$C_I = 1 \quad (3170)$$

while

$$C_A = 0. \quad (3171)$$

Equations 3170 and 3171 show why institutions should not define personal emotional completion.

The relationship between institutional justice and subjective closure remains an important interdisciplinary research question.

32.101 Restorative Success without Forgiveness

A further conceptual question is whether complete institutional repair requires any change in the subject's attitude toward the wrongdoer.

The present framework suggests that it does not.

A subject can receive adequate remedy, possess secure future capability, and continue to condemn the historical wrong.

This distinction deserves more explicit philosophical development.

32.102 Restorative Success Metrics

Institutions require operational metrics.

A future research program should distinguish at least:

$$\mathbf{S}_R = (S_{\text{legal}}, S_{\text{material}}, S_{\text{capability}}, S_{\text{relational}}, S_{\text{historical}}, S_{\text{generative}}, S_{\text{procedural}}). \quad (3172)$$

Equation 3172 preserves multiple success dimensions.

The open empirical problem is how to measure them efficiently and comparably without transforming the profile into an excessively burdensome bureaucracy.

32.103 Benchmark Datasets

Formal and computational development would benefit from benchmark cases.

Such datasets could contain:

- simulated dynamic systems;
- anonymized administrative trajectories;

- legal remedy histories;
- network disruptions;
- ecological restoration cases.

The ethical design of these datasets is itself nontrivial because restorative cases frequently contain sensitive historical information.

32.104 Synthetic Restorative Systems

Synthetic environments can help evaluate formal models without experimenting on real harmed subjects.

Let synthetic system be

$$\tilde{\mathcal{G}} = (\tilde{X}, \tilde{\Phi}, \tilde{H}). \quad (3173)$$

Equation 3173 permits controlled variation of harm, repair, history, and intervention.

Such environments could test whether proposed restorative metrics recover known ground truth.

32.105 Simulation and External Validity

Simulation can establish formal behavior without establishing social validity.

Let model result be R_M and empirical domain result be R_E .

External validity concerns

$$R_M \sim_{\text{transport}} R_E. \quad (3174)$$

Equation 3174 should therefore be tested rather than presumed.

32.106 Longitudinal Empirical Research

The theory is fundamentally temporal.

Cross-sectional data can miss:

- delayed effects;
- recovery trajectories;
- recurrence;
- support dependence;
- institutional learning.

Longitudinal research is therefore especially important.

Let outcome trajectory be

$$\mathbf{Y}_{0:T} = (Y_0, \dots, Y_T). \quad (3175)$$

Equation 3175 provides a richer empirical object than one endpoint.

32.107 Natural Experiments

Institutional reforms, changes in remedy rules, or phased policy implementation can generate opportunities for causal study.

Such designs can help estimate the effects of:

- correction propagation;
- portable benefits;
- independent review;
- staged prevention;
- assistance programs.

The framework can therefore generate empirically testable research questions despite its philosophical scope.

32.108 Qualitative and Ethnographic Research

Several core objects are difficult to understand through administrative data alone.

Qualitative and ethnographic methods can illuminate:

- relational meaning;
- institutional usability;
- subject interpretation;
- hidden gatekeeping;
- adaptive preference;
- perceived security.

A comprehensive empirical program should therefore combine formal modeling with interpretive methods rather than forcing every object into numerical form.

32.109 Participatory Research

Affected persons can contribute to the design of restorative metrics and research questions.

Participatory research can help identify omitted value objects.

It also raises questions concerning representation, burden, compensation for participation, and conflict among affected persons.

The methodology therefore requires its own procedural safeguards.

32.110 Cross-Disciplinary Translation

The framework draws upon jurisprudence, political philosophy, economics, dynamical systems, causal inference, resilience, administrative law, and restorative practice.

A continuing research problem is translation across these fields.

Terms such as:

- equivalence;
- restoration;

- resilience;
- causation;
- capability;
- recognition;
- generativity;

carry different meanings across disciplines.

Conceptual precision is therefore necessary to prevent false synthesis.

32.111 Formal Metaphor and Formal Model

The manuscript repeatedly uses mathematical structures.

A critical methodological problem is distinguishing formal models from formal metaphors.

Let representation be M .

A formal model requires at least:

$$M \rightarrow \mathcal{P}_{\text{testable}}, \quad (3176)$$

where $\mathcal{P}_{\text{testable}}$ contains implications that can in principle be evaluated against data or logically derived within the formal system.

A metaphor can remain philosophically useful without satisfying this condition.

The distinction should remain explicit.

32.112 Ontological Modesty

The formal architecture should not be treated as an exhaustive ontology of persons, communities, histories, or ecosystems.

The manuscript deliberately treats many categories as analytical interfaces.

A future research program should continue to test whether each formalization clarifies rather than reifies its object.

32.113 Normative Modesty

The framework likewise does not provide one final theory of justice.

Its normative contribution lies partly in making disagreements explicit.

Future development should preserve the distinction among:

- comparison;
- value;
- substantive justice;
- procedural justice.

Collapsing these layers would remove much of the theory's analytical advantage.

32.114 Institutional Modesty

No institution should be presumed competent to operationalize the entire atlas.
The research agenda should therefore investigate modular implementation.
Let institutional module set be

$$\mathfrak{M}_I = \{M_1, \dots, M_k\}. \quad (3177)$$

Each module can govern a bounded object family while sharing carefully designed interfaces.
This approach can preserve depth without creating a total restorative state.

32.115 Restorative Minimalism

A major practical research question concerns the smallest architecture that improves restorative reasoning materially.

The full atlas is conceptually useful and operationally expensive.
Future work should therefore identify minimal profiles for different institutional domains.
A small-claims compensation process may require only resource, temporal, and procedural cells.
A complex historical reparations program can require a substantially richer profile.

32.116 Domain-Specific Atlases

One promising direction is construction of domain-specific atlases:

$$\mathfrak{A}^{(d)} \subseteq \mathfrak{A}_R. \quad (3178)$$

Equation 3178 can define smaller restorative grammars for:

- employment;
- health;
- research;
- reputation;
- ecological restoration;
- collective reparations.

These atlases could be empirically refined without requiring one universal operational system.

32.117 Domain-Specific Validation

Each domain-specific atlas should be validated against:

- actual institutional cases;
- expert judgment;
- subject experience;
- legal practice;
- empirical outcomes.

The framework should be revised where the taxonomy fails to distinguish practically important cases.

32.118 Restorative Profiles and Decision Support

A future decision-support system could present a restorative profile without producing a single automated answer.

Let profile be

$$\mathbf{R} = (R_1, \dots, R_n). \quad (3179)$$

The system could identify:

- strong dimensions;
- residual dimensions;
- uncertainty;
- conflicts;
- institutionally unavailable remedies.

This could improve deliberation while preserving human and institutional judgment.

32.119 Restorative Dashboards and Metric Risk

Operational dashboards are attractive and potentially dangerous.

Once indicators become visible and consequential, institutions can optimize them.

A future research program should therefore study governance of restorative dashboards, including:

- metric provenance;
- metric expiry;
- qualitative annotations;
- omitted-value warnings;
- anti-gaming audits.

32.120 Open Problems in Historical Identity

The strongest philosophical questions concern historical identity.

Even if every present state and future dynamic can be reproduced, the actual history differs from the no-harm history.

The remaining questions include:

- which historical differences matter morally;
- whether provenance can carry independent value;
- whether subject identity depends upon actual history;
- when historical difference should be preserved.

These questions cannot be reduced to engineering.

32.121 Open Problems in Personal Identity

Ideal restoration thought experiments raise the question of whether transforming a person into a counterfactual-equivalent state preserves the same person.

Let historical subject be A^H , and reconstructed subject A^R .

The theory currently requires only a continuity relation

$$A^R \sim_{\text{cont}} A^H. \quad (3180)$$

Equation 3180 leaves the metaphysics of personal identity open.

A deeper treatment would require engagement with philosophy of mind and personal identity.

32.122 Open Problems in Collective Identity

Collective identity raises analogous questions across generations.

Which present community is the relevant bearer of a historical claim?

Possible continuity criteria include:

- institutional continuity;
- cultural continuity;
- self-identification;
- territory;
- legal succession.

No universal criterion has been established.

32.123 Open Problems in Ecological Identity

Ecological restoration raises a parallel issue.

A recreated ecosystem can match composition and function while possessing a different history.

The meaning of ecological identity under reconstruction therefore deserves further philosophical and ecological analysis.

32.124 Restorative Justice and Uncertainty about Justice

The final broad theoretical problem is moral uncertainty itself.

Let plausible normative theories be

$$\mathfrak{T} = \{T_1, \dots, T_n\}. \quad (3181)$$

A robust restorative intervention can lie in

$$\mathcal{R}^* = \bigcap_{i=1}^n \mathcal{R}_{\text{acceptable}}^{T_i}. \quad (3182)$$

Equation 3182 identifies an overlapping normative-consensus region.

When this region is empty, procedure must govern deeper normative disagreement.

Table 39: Principal open research problems

Research domain	Core unresolved problem	Promising method	Principal risk
Reference construction	Plural counterfactual and normative baselines	Robust reference families	False counterfactual precision
Formal comparison	Object–relation composition	Typed formal semantics and partial orders	Overformalization
Generativity	Domain-valid measurement	Longitudinal mixed-method studies	Vagueness or metric capture
Capability	Latent opportunity estimation	Resource–conversion–functioning models	Intrusive inference
Causality	Dynamic feedback and interference	Longitudinal causal models	Mechanism misspecification
Reparability	Object-specific recovery probability	Intervention and longitudinal studies	False confidence
Value pluralism	Decision under incomparability	Thresholds, partial orders, deliberation	Artificial scalarization
Law	Translation from causal loss to legal remedy	Comparative remedial analysis	Jurisdictional overgeneralization
Assistance	Future-generative outcome evaluation	Longitudinal program evaluation	Employment-centered proxies
Prevention	Validation of early-warning systems	Prospective monitoring and stress testing	Over-prevention
Epistemic infrastructure	Distributed evidence with privacy	Local processing and provenance systems	Surveillance centralization
Reflexivity	Models changing modeled behavior	Recursive model audits	Metric drift
Collective repair	Representation of internal plurality	Participatory and institutional analysis	Majoritarian capture
Ecological restoration	Transformative reference under environmental change	Resilience and scenario analysis	Historical-baseline fixation

32.125 Normative Sensitivity Analysis

Restorative recommendations should therefore be tested across plausible normative specifications.

Let recommendation under theory T_i be R_i^* .

Sensitivity can be represented by

$$S_N = \text{Var}(R_1^*, \dots, R_n^*), \quad (3183)$$

where the variance symbol is conceptual and can be replaced by an appropriate distance or disagreement measure.

High normative sensitivity should reduce institutional confidence in claims of uniquely correct restoration.

32.126 Research Priority Matrix

Table 39 summarizes several major open research areas.

Table 39 emphasizes that no single methodology can resolve the framework’s open questions.

32.127 Operationalization Priority

The first practical research priority should not be operationalization of the entire atlas.

A more defensible strategy is progressive operationalization.

Let initial domain-specific model be

$$\mathfrak{A}_0^{(d)} \subset \mathfrak{A}^{(d)}. \quad (3184)$$

Empirical use can then generate

$$\mathfrak{A}_{t+1}^{(d)} = \mathcal{U}(\mathfrak{A}_t^{(d)}, E_{t+1}). \quad (3185)$$

Equation 3185 treats the atlas itself as revisable.

32.128 Theory as a Revisable Interface

The framework should therefore be understood as a revisable interface among disciplines and institutional practices.

Its categories are useful where they reveal distinctions that matter.

They should be modified where empirical research or philosophical argument shows that the distinctions are inadequate.

This is consistent with the manuscript's broader commitment to generativity and revisability.

32.129 Open-Problem Principle

The first principle of this section is:

The Atlas of Equivalence should be treated as a research architecture whose categories remain open to empirical, formal, legal, and normative revision.

32.130 Operational-Modesty Principle

The second principle is:

Only those restorative objects and relations that can be justified conceptually and supported adequately by evidence should be operationalized in a given institutional domain.

32.131 Uncertainty-Preservation Principle

The third principle is:

Where reference, causal structure, value, or future trajectory remains uncertain, the uncertainty should be represented directly rather than converted into a spurious point estimate.

32.132 Qualitative-Retention Principle

The fourth principle is:

Restorative objects that resist defensible numerical representation should remain available to qualitative, legal, historical, or deliberative analysis rather than being excluded merely because they are difficult to measure.

32.133 Formalization-Discipline Principle

The fifth principle is:

Mathematical language should be used where it clarifies structure, permits derivation, or supports empirical testing, and should not convert philosophical analogy into unsupported formal authority.

32.134 Domain-Validation Principle

The sixth principle is:

Restorative constructs should be validated within particular domains before they are assumed to transfer across legal, organizational, ecological, or cultural contexts.

32.135 Reflexive-Governance Principle

The seventh principle is:

Metrics, models, classifications, and restorative interventions should be periodically evaluated for the effects they create in the systems they measure and govern.

32.136 Institutional-Restraint Principle

The eighth principle is:

Conceptual comprehensiveness of the theory should not be interpreted as a mandate for comprehensive institutional surveillance, intervention, or centralized control.

32.137 Plural-Method Principle

The ninth principle is:

Formal modeling, quantitative measurement, legal analysis, qualitative research, historical inquiry, and participatory methods should be treated as complementary sources of restorative knowledge rather than competitors for one universal method.

32.138 Residual-Research Principle

The tenth principle is:

The study of what remains after the best feasible and legitimate repair should be treated as a substantive research domain rather than as the leftover category of restoration.

32.139 Open Research Architecture

The research agenda can be summarized by

$$\mathfrak{R}_O = (R_{\text{formal}}, R_{\text{empirical}}, R_{\text{normative}}, R_{\text{legal}}, R_{\text{institutional}}, R_{\text{technical}}, R_{\text{historical}}) . \quad (3186)$$

Equation 3186 identifies formal, empirical, normative, legal, institutional, technical, and historical research streams.

Their interaction is likely to be more productive than attempting to solve the problem of return within one discipline.

32.140 Transition to Limitations and Scope

The existence of this large research agenda also clarifies the limitations of the present manuscript.

The theory has developed a comparative and normative grammar.

It has not empirically validated every object in the atlas.

It has not produced one general algorithm for restorative decisions.

It does not establish that every social system should be modeled dynamically.

It does not claim that every loss can be quantified.

It does not resolve every dispute among theories of justice.

Section 33 therefore turns explicitly to the limits of the present contribution.

The next section distinguishes:

- conceptual scope;
- formal scope;

- empirical scope;
- normative scope;
- legal scope;
- institutional scope;
- technological scope.

Its purpose is to prevent the breadth of the Atlas of Equivalence from being misread as a claim of universal explanatory or prescriptive completeness.

33 Limitations and Scope

This section states the boundaries of the present contribution. The manuscript has developed a broad architecture for analyzing generativity, harm, return, reparability, value, legal remedy, assistance, prevention, and the comparison of restored and reference systems. The breadth of that architecture creates a particular interpretive risk: a reader can mistake conceptual inclusiveness for a claim that the theory provides a complete ontology of social reality, a universal mathematical model, an empirically validated measurement system, a final theory of justice, or a decision algorithm suitable for direct institutional deployment.

The manuscript makes none of those claims.

Its central contribution is more limited and more specific. It proposes that restorative claims become clearer when the analyst specifies what is being compared, against which reference, through which comparative relation, over what time horizon, under which epistemic assumptions, and according to which value and justice standards. It further argues that return of an external condition does not generally establish restoration of the historical, relational, capability, causal, or generative structure in which that condition once operated.

The Atlas of Equivalence is therefore an analytical grammar. It organizes questions that are often conflated. It does not determine every answer to those questions.

The limitations developed below concern conceptual scope, ontological commitment, formalization, empirical validation, causal inference, normative theory, legal doctrine, institutional design, measurement burden, technological assumptions, cultural translation, temporal reach, scale, and the framework's own reflexive effects.

33.1 Scope of the Central Claim

The most general claim of the manuscript can be expressed as

$$\text{Rest} \left(\mathcal{G}^R, \mathcal{G}^{\text{ref}} \right) \quad (3187)$$

being underdetermined until the comparison is indexed.

A more explicit form is

$$\text{Rest} \left(\mathcal{G}^R, \mathcal{G}^{\text{ref}}; Q, E, \mathcal{T}, \tau, \varepsilon, K \right). \quad (3188)$$

Equation 3188 is the manuscript's principal analytical commitment.

The theory does not require that every restorative institution implement the full tuple in a formal computational system.

It requires only that strong claims of restoration should remain sensitive to the dimensions they implicitly compare.

33.2 Scope of Generativity

The manuscript uses *generativity* to describe the capacity of subjects, relations, institutions, or systems to produce, reproduce, diversify, and revise future possibilities.

This concept is intentionally broader than productivity.

It can include:

- capability formation;
- value creation;
- relational development;
- learning;
- institutional reproduction;
- ecological regeneration;
- option generation.

The concept does not imply that all generation is normatively good.

A system can generate domination, extraction, exclusion, or environmental depletion.

Accordingly,

$$G > 0 \not\Rightarrow J > 0. \quad (3189)$$

Equation 3189 is a basic scope condition.

Generativity becomes normatively relevant only after the generated objects, distribution, control, externalities, and procedural conditions are evaluated.

33.3 Generativity as an Analytical Interface

The manuscript does not claim that generativity is a natural kind with one universally measurable essence.

In different domains, the relevant structure can differ substantially.

Research generativity can concern access, knowledge production, recognition, network formation, and subsequent access.

Ecological generativity can concern regeneration, resilience, reproduction, and adaptive capacity.

Institutional generativity can concern learning, rule revision, distributed knowledge, and capacity formation.

The shared concept is therefore an analytical interface rather than a claim of domain identity.

33.4 Ontological Modesty

The framework does not require a fixed ontology of persons, institutions, relations, or social facts.

The manuscript has used categories such as state, event, relation, process, attribute, capability, history, and generative condition because they provide useful analytical distinctions.

These categories are not presented as exhaustive first-level constituents of reality.

Let formal object family be

$$\Omega = \{Q_1, \dots, Q_n\}. \quad (3190)$$

Equation 3190 should be read as a modeling vocabulary.

It does not imply

$$\text{Reality} = \bigcup_{i=1}^n Q_i. \quad (3191)$$

Equation 3191 rejects ontological exhaustiveness.

33.5 Relation as a Methodological Device

The manuscript gives substantial importance to relational and historical constitution.

It does not require the metaphysical thesis that relation is the ultimate substance of reality.

Relation functions here as a methodological device for analyzing how value, identity, capability, control, and meaning depend upon wider systems.

The framework is therefore compatible with several deeper ontological positions.

33.6 Subject Ontology

The theory does not provide a final account of what a subject is.

Its applied arguments require only that some systems possess normatively relevant continuity, agency, interests, capabilities, or recognized standing.

This leaves open questions concerning:

- metaphysical personal identity;
- collective subjecthood;
- artificial agents;
- nonhuman subjectivity;
- distributed agency.

The theory can analyze restorative relations before those deeper ontological questions are fully resolved.

33.7 Formal Scope

The mathematical expressions used throughout the manuscript have several different roles.

Some specify genuine formal objects, such as sets, mappings, causal models, graphs, trajectories, partial orders, or reachability relations.

Others provide structural notation intended to discipline conceptual reasoning.

The manuscript does not claim that every displayed expression constitutes a validated mathematical model of an empirical system.

A formal expression should therefore be interpreted according to its role.

33.8 Formal Model and Conceptual Formalization

Let $\mathcal{M}_F$ denote a formal model with explicit semantics and $\mathcal{M}_C$ a conceptual formalization.

The distinction is

$$\mathcal{M}_F \neq \mathcal{M}_C. \quad (3192)$$

Equation 3192 matters because a conceptual tuple can clarify dimensions of a problem without supporting quantitative prediction.

Future domain-specific work would be required to convert selected parts of the framework into empirically estimable formal models.

33.9 No Universal State Space

The manuscript frequently uses state-space language.

It does not claim that every social, legal, historical, or moral problem possesses one naturally given finite-dimensional state space.

Let one researcher choose state representation

$$x_t = \Phi_1(\mathcal{G}_t), \quad (3193)$$

while another chooses

$$z_t = \Phi_2(\mathcal{G}_t). \quad (3194)$$

Equations 3193 and 3194 can yield different dynamical descriptions.

The state representation itself therefore requires justification.

33.10 Limits of Dynamical Analogy

Concepts such as attractor, basin, bifurcation, hysteresis, resilience, and critical transition are powerful when the domain supports the relevant mathematical assumptions.

They should not be transferred into social theory merely because the vocabulary is intuitively suggestive.

Observed persistence is not sufficient evidence of an attractor.

Abrupt political or organizational change is not sufficient evidence of a formal bifurcation.

The manuscript therefore treats dynamical systems as a set of analytical tools whose empirical applicability must be demonstrated case by case.

33.11 No Universal Metric

The framework deliberately rejects the assumption that one metric can represent all restorative distance.

Let d_i denote a metric appropriate to object Q_i .

In general,

$$d_i \neq d_j. \quad (3195)$$

Equation 3195 expresses metric pluralism.

Historical provenance, monetary loss, network topology, bodily function, and recognition need not admit the same distance function.

Some domains may admit no defensible cardinal metric at all.

33.12 Partial Orders and Incomparability

The atlas allows partial ordering and incomparability.

This increases conceptual honesty and reduces algorithmic convenience.

Where

$$R_1 \parallel R_2, \quad (3196)$$

the framework does not always produce a unique remedy.

Equation 3196 is a limitation if the framework is expected to function as a complete decision algorithm.

It is an intentional feature if the goal is to preserve genuine normative plurality.

33.13 No Universal Optimization Function

The manuscript does not propose a universal objective function

$$R^* = \arg \max_R U(R) \quad (3197)$$

for all restorative decisions.

Equation 3197 would require a justified common scale across rights, welfare, recognition, history, capability, generativity, procedure, and third-party interests.

No such scale has been established.

33.14 Empirical Scope

The manuscript is primarily theoretical and synthetic.

It has not conducted original large-sample empirical validation of the full framework.

The applied cases in Section 31 are analytical demonstrations rather than empirical tests demonstrating causal effectiveness of a particular remedy architecture.

Accordingly, the theory should not be read as evidence that every proposed intervention produces positive real-world outcomes.

33.15 No General Effect-Size Claims

The framework does not establish numerical effect sizes such as

$$\Delta G = \beta A_G \quad (3198)$$

for generative assistance, prevention, compensation, or structural reform.

Equation 3198 would require domain-specific empirical estimation.

The manuscript instead identifies variables and relations that future empirical work can investigate.

33.16 Measurement Limits

Several central objects remain difficult to measure.

These include:

- capability sets;
- counterfactual opportunity;
- trust;
- relational standing;
- domination;
- historical recognizability;
- future generativity.

The framework therefore should not be operationalized by assigning arbitrary numbers merely because institutional software requires numerical inputs.

33.17 Latent Variables

Many restorative objects are latent.

Let observed variables be Y , and latent restorative object be Q^* .

Inference requires

$$P(Q^* | Y). \quad (3199)$$

Equation 3199 depends upon modeling assumptions.

The uncertainty in those assumptions should therefore propagate into restorative conclusions.

33.18 Self-Report Limits

First-person reports provide indispensable evidence concerning usability, preference, trust, meaning, pain, and perceived security.

They can also be affected by:

- memory;
- adaptation;
- strategic context;
- uncertainty;
- changing interpretation.

The framework therefore does not treat either self-report or external measurement as epistemically sufficient by itself.

Triangulation is often preferable.

33.19 Administrative-Data Limits

Administrative data are generated for institutional purposes.

They can omit:

- people who never entered the system;
- informal harms;
- failed applications;
- invisible relational conditions;
- unrecognized value objects.

Administrative visibility should therefore not be confused with complete social visibility.

33.20 Selection Effects

Observed restorative outcomes can be strongly affected by selection.

Let participants in assistance program be $A = 1$.

The observed quantity

$$\mathbb{E}[Y | A = 1] \quad (3200)$$

does not by itself identify the program's causal effect.

Equation 3200 can reflect prior differences in need, motivation, access, or eligibility.

Causal conclusions therefore require appropriate empirical design.

33.21 Survivorship

Restorative systems can observe those who remain engaged and lose information about persons who exit because the system is inaccessible or harmful.

Let observed population be O and original affected population be A .

Where

$$O \subset A, \quad (3201)$$

success rates based only upon O can be misleading.

Equation 3201 is particularly relevant to burdensome complaint, rehabilitation, and assistance systems.

33.22 Causal Scope

The manuscript uses causal language extensively while distinguishing causal models from legal, moral, and historical judgments.

A scientifically plausible causal relation does not independently establish:

- legal liability;
- moral blame;
- required compensation;
- appropriate remedy.

Those conclusions require additional normative and institutional premises.

33.23 No Complete Counterfactual Identification

Outside the idealized thought experiments of Section 26, no claim is made that actual counterfactual life trajectories can generally be identified exactly.

Let counterfactual family be

$$\mathfrak{H}^{-H} = \{H_1, \dots, H_n\}. \quad (3202)$$

The practical task can therefore involve distributions, ranges, bounds, or qualitative possibilities rather than one reconstructed biography.

33.24 No-Harm Counterfactual as Analytical Reference

The no-harm counterfactual is analytically useful because it makes loss visible.

It should not automatically be treated as the uniquely authoritative future.

This distinction is especially important after long periods of historical change.

33.25 Mechanism Instability

The causal mechanisms generating outcomes can change after harm, intervention, or institutional reform.

Let mechanism at time t be F_t .

In general,

$$F_{t+1} \neq F_t. \quad (3203)$$

Equation 3203 limits transport of causal estimates across time.

It also weakens simple projections of long-run generative loss.

33.26 Interference and Network Dependence

Many applied domains violate assumptions that each person's outcome depends only upon that person's treatment.

Let outcome for person i be

$$Y_i = Y_i(A_1, \dots, A_n). \quad (3204)$$

Equation 3204 is especially relevant to workplaces, professional networks, collective reparations, and ecological systems.

This complicates both causal estimation and remedy evaluation.

33.27 Normative Scope

The manuscript does not defend one complete moral theory.

It draws upon several traditions because different restorative problems involve different normative objects.

These include:

- welfare;
- resources;
- capabilities;
- relational equality;
- recognition;
- rights;
- non-domination;
- care;
- corrective justice;
- reparative justice;
- generative justice.

The framework maps their implications.

It does not prove that they can be reconciled into one coherent master theory.

33.28 Normative Underdetermination

The formal atlas can identify that two remedies differ on several dimensions.

It cannot determine their moral ranking until a normative theory is supplied.

Let comparison data be D_R .

Then

$$D_R \not\Rightarrow J(R) \quad (3205)$$

without additional normative premises.

Equation 3205 is a central limitation.

Formal comparison does not generate normativity by itself.

33.29 Value Selection

The framework cannot determine mechanically which value objects should matter in every context.

Selecting capability, recognition, historical truth, privacy, or generativity as a value object is itself a normative act.

Institutional implementation therefore requires explicit value governance.

33.30 No Complete Theory of Rights

Rights appear throughout the manuscript as side constraints, protected objects, and institutional entitlements.

The manuscript does not provide a complete theory of:

- the foundation of rights;
- conflicts among rights;
- rights hierarchy;
- rights of future persons;
- rights of nonhuman systems.

Those questions remain dependent upon broader legal and philosophical theories.

33.31 No Complete Theory of Care

Care is used to illuminate responsiveness, dependency, and support.

The manuscript does not attempt a complete ethics or political theory of care.

Questions concerning gendered labor, family structure, social reproduction, and the institutional distribution of care deserve fuller treatment than the present manuscript provides.

33.32 No Complete Theory of Recognition

Recognition is treated as a distinct value object.

The manuscript does not resolve all disputes concerning:

- identity-based recognition;
- status recognition;
- cultural recognition;
- recognition and redistribution.

It uses recognition theory primarily to demonstrate that material equivalence can coexist with unresolved social standing.

33.33 No Complete Theory of Generative Justice

The manuscript draws upon generative justice and extends the analysis toward restoration, reparability, equivalence, assistance, and prevention.

It does not claim to exhaust generative justice as developed in the existing literature.

In particular, the manuscript's Atlas of Equivalence should not be read as a replacement for work on circular value flows, community production, indigenous knowledge, computational systems, or anti-alienation.

Its contribution is a complementary comparative grammar.

33.34 Legal Scope

Section 27 develops a functional taxonomy of legal remedies.

It does not constitute legal advice.

It does not state the complete positive law of any jurisdiction.

It does not imply that remedies such as restitution, specific performance, structural relief, declaratory relief, or rehabilitation are equally available across legal systems.

33.35 Jurisdictional Variation

Legal systems differ concerning:

- causes of action;
- standing;
- causation;
- proof;
- remedies;
- limitation periods;
- institutional jurisdiction.

Any applied legal use of the framework therefore requires jurisdiction-specific analysis.

33.36 Legal Vocabulary and Analytical Vocabulary

Terms used analytically in this manuscript do not necessarily correspond to formal doctrinal categories.

For example, *generative loss* can describe a theoretically relevant form of downstream impairment without constituting an independently recognized legal head of damage.

The distinction should remain explicit.

33.37 No Automatic Expansion of Liability

The theory's recognition of long causal and generative chains should not be read as an argument that legal liability should extend indefinitely.

Let causal consequence set be $\mathcal{L}_C$ and legally recoverable set $\mathcal{L}_L$.

The theory permits

$$\mathcal{L}_L \subset \mathcal{L}_C. \quad (3206)$$

Equation 3206 can be justified by institutional considerations including proof, remoteness, finality, and third-party reliance.

33.38 Institutional Scope

The manuscript argues for institutional pluralism.

This introduces practical complexity.

A system with courts, administrative agencies, social-protection institutions, professional bodies, archives, healthcare providers, and community organizations can distribute competence effectively and create coordination failure.

The theory does not provide one complete institutional constitution for such systems.

33.39 No Universal Institutional Form

No single institution is proposed as the universal restorer.

The appropriate architecture depends upon:

- object;
- temporal function;
- authority;
- expertise;
- procedural requirements;
- scale.

A centralized restorative agency can solve coordination problems and create concentration of power.

A decentralized system can preserve specialization and create fragmentation.

The manuscript does not resolve this trade-off generally.

33.40 Administrative Capacity

Many proposed practices require:

- trained staff;
- interoperable records;
- review capacity;
- timely response;
- data governance.

Institutions with low administrative capacity can be unable to implement high-dimensional restorative assessment reliably.

A simpler architecture can then be more just than an ambitious system that operates inconsistently.

33.41 Institutional Cost

The framework itself can create administrative cost.

Let analytical depth be d and institutional cost be $C_I(d)$.

A plausible relationship is

$$\frac{dC_I}{dd} > 0. \quad (3207)$$

Equation 3207 shows why maximum analytical completeness is not automatically desirable.

Institutional design should seek sufficient rather than maximal resolution.

33.42 Bureaucratic Expansion

A framework intended to protect subjects can become a basis for expanding bureaucratic observation.

A system could attempt to measure:

- networks;
- trust;
- health;
- preference;
- future options;
- generativity.

Such comprehensiveness can itself become intrusive.

The manuscript therefore rejects an implementation principle of “measure everything that the theory can describe.”

33.43 Surveillance Risk

Let restorative information collected be $\mathcal{D}_R$.

More data can improve modeling:

$$K(\mathcal{D}_R) \uparrow \quad (3208)$$

and increase privacy burden:

$$C_{\text{priv}}(\mathcal{D}_R) \uparrow . \quad (3209)$$

Equations 3208 and 3209 define an implementation trade-off.

The framework’s epistemic ambition should remain subordinate to privacy and institutional legitimacy.

33.44 Classification Risk

Operationalizing the atlas can create new labels.

For example, subjects can be classified according to:

- reparability;
- vulnerability;
- generativity;
- resilience;
- risk.

These labels can affect future treatment.

The model can therefore create the very relational structures it is intended to observe.

33.45 Metric Capture

Once a restorative indicator becomes an institutional target, actors can optimize the indicator.

Let metric be M , and target value be V .

It can occur that

$$M \uparrow \tag{3210}$$

while

$$V \nmid. \tag{3211}$$

Equations 3210 and 3211 capture a general metric-capture risk.

Restorative metrics therefore require recurring validity review.

33.46 Technological Scope

The idealized technology thought experiments in Section 26 are deliberately unrealistic.

They include hypothetical perfect observation, causal knowledge, memory intervention, network reconstruction, and world-level control.

These thought experiments are methodological.

They are not forecasts.

33.47 No Prediction of Future Technology

The manuscript does not predict that:

- memory can be rewritten perfectly;
- personal identity can be reconstructed;
- counterfactual histories can be simulated exactly;
- ecosystems can be recreated completely.

The examples isolate conceptual limits by asking what would remain if ordinary technical limits disappeared.

33.48 Technology and Institutional Reality

Existing institutions operate under far weaker conditions.

They face:

- missing data;
- model uncertainty;
- limited budgets;
- fragmented authority;
- strategic behavior.

The idealized section should therefore not be used to justify present-day high-intrusion interventions on the assumption that future technical precision will solve their risks.

33.49 Artificial Intelligence

Computational systems can assist with classification, causal modeling, simulation, record comparison, or decision support.

The manuscript does not argue that artificial intelligence should independently determine restorative justice.

Such systems inherit:

- data limitations;
- reference-selection problems;
- value disagreement;
- model uncertainty;
- procedural constraints.

The central problems of justice therefore survive improvements in prediction.

33.50 No Technical Solution to Normative Pluralism

Suppose a computational system predicts every consequence perfectly.

If two value theories rank the consequences differently, the normative conflict remains.

Thus

$$K = K^* \not\Rightarrow J = J^* \quad (3212)$$

where K^* denotes ideal knowledge and J^* a uniquely justified decision.

Equation 3212 rejects technological solutionism.

33.51 Cultural Scope

The framework draws heavily upon traditions developed in modern jurisprudence, political philosophy, economics, systems theory, and contemporary governance.

Its concepts can travel imperfectly across cultures.

Terms such as autonomy, capability, recognition, property, subject, and restoration can carry different normative structures in different intellectual and cultural traditions.

33.52 No Universal Cultural Translation

Let concept in context C_1 be V^{C_1} .

Translation to C_2 is

$$T_{12}(V^{C_1}) = V^{C_2} + \epsilon_{12}. \quad (3213)$$

Equation 3213 represents possible semantic and normative translation loss.

The manuscript has not developed a complete comparative cultural theory of restoration.

33.53 Relational Traditions beyond the Present Literature

The relational and process-oriented character of the framework has possible resonances with intellectual traditions beyond the Western philosophical literature used most heavily in the formal sections.

The present manuscript does not claim to have developed those comparisons with sufficient historical or philological depth.

Such connections should be pursued through dedicated scholarship rather than brief analogical citation.

33.54 Translation into International Institutions

International institutions operate across legal and cultural systems.

A common restorative vocabulary can support coordination and also flatten local meanings.

The atlas can function as a translation interface only where local value structures remain visible.

It should not become an instrument for imposing one universal institutional ontology.

33.55 Temporal Scope

The manuscript emphasizes history and temporal horizon.

It nevertheless cannot resolve every temporal problem.

Several unresolved issues remain:

- how far backward corrective responsibility extends;
- how far forward future consequences should count;
- how future uncertainty should be discounted;
- when claims lose institutional priority.

These questions depend partly upon the normative and legal domain.

33.56 Historical Distance

As time between harm and remedy increases, the number of intervening events, actors, and legitimate reliance interests tends to grow.

The framework recognizes this complexity and does not provide one universal formula for adjusting claims by historical distance.

33.57 Intergenerational Scope

The framework allows intergenerational value and transmission.

It does not provide a complete population ethics.

Questions concerning:

- future persons;
- nonidentity;
- population size;
- discounting;
- obligations among generations

remain largely outside the developed theory.

33.58 Scale Scope

The framework applies conceptually from individual cases to collective and ecological systems.

This breadth creates a danger of assuming scale invariance.

A relation useful at one scale can fail at another.

Let restorative rule at scale s_1 be $R^{(s_1)}$.

In general,

$$T_{s_1 \rightarrow s_2} \left(R^{(s_1)} \right) \neq R^{(s_2)}. \quad (3214)$$

Equation 3214 rejects automatic transfer across scales.

33.59 Individual and Collective Non-Reduction

Collective repair cannot always be reduced to the sum of individual remedies.

Individual claims likewise should not disappear inside collective improvement.

The framework recognizes both levels and does not provide one complete theory for resolving all conflicts between them.

33.60 Ecological Scope

Ecological applications are illustrative extensions.

The manuscript does not provide a complete theory of environmental ethics, ecological modeling, conservation science, or restoration ecology.

Concepts such as resilience, basin structure, and regime shift should be applied with domain expertise.

33.61 Nonhuman Moral Standing

The framework can represent ecological value.

It does not determine whether that value is:

- intrinsic;
- instrumental;
- relational;
- rights-based.

The moral standing of animals, species, ecosystems, and future ecological systems remains outside the manuscript's central claims.

33.62 Scope of Prevention

Preventive governance occupies a significant part of the theory.

This creates a risk of conceptual expansion from justice after harm into general risk governance.

The manuscript therefore limits preventive analysis to conditions whose preservation is meaningfully connected to serious restorative problems, reparability, rights, or generative vulnerability.

It does not propose that all social risk become a restorative-justice problem.

33.63 Preventive Uncertainty

Preventive models necessarily act before the full event is observed.

False positives and false negatives therefore remain unavoidable.

The framework provides principles for staged, reversible, and contestable intervention.

It does not eliminate the underlying uncertainty.

33.64 Over-Prevention

A system can preserve safety by suppressing experimentation, agency, innovation, or dissent.

The manuscript therefore rejects prevention as an absolute maximand.

Preventive success should remain constrained by generative openness.

33.65 Scope of Assistance

Generative assistance is introduced as a distinct institutional function.

The concept can become too broad if every condition contributing to a person's future is treated as an assistance obligation.

The manuscript therefore does not claim that institutions have a general duty to maximize every person's generativity.

33.66 Bounded Assistance

Assistance obligations require a normative basis such as:

- corrective responsibility;
- social protection;
- legal entitlement;
- care obligations;
- public policy mandate.

The theory helps design assistance once such a basis exists.

It does not derive unlimited assistance obligations from generativity alone.

33.67 No Universal Self-Sufficiency Standard

The manuscript rejects economic self-sufficiency as a universal definition of successful restoration.

Some just lives involve continuing dependency, public support, care, or shared infrastructure.

Generative assistance therefore evaluates control, security, capability, and agency rather than treating disappearance of support as the sole success metric.

33.68 Scope of Historical Truth

Historical truth is treated as a possible independent value.

The manuscript does not claim that every historical detail should remain publicly accessible forever.

Historical recognizability can conflict with:

- privacy;
- dignity;
- data minimization;
- legitimate closure.

The appropriate archival architecture is context-dependent.

33.69 No Absolute Archivalism

The framework therefore does not endorse the principle

$$\text{Preserve}(e) = 1 \quad \forall e. \quad (3215)$$

Equation 3215 would create severe privacy and governance problems.

Preservation should remain purpose-sensitive.

33.70 Scope of Reflexive Non-Closure

Reflexive non-closure is proposed as a methodological principle rather than a formal impossibility theorem.

The manuscript does not prove that every socially operative representation must alter the system it represents.

It argues more modestly that in many institutional contexts, records, classifications, evaluations, and remedies enter the causal environment and therefore should not automatically be modeled as external observations.

33.71 No Universal Non-Closure Theorem

The framework should not be read as asserting a theorem that completeness, externality, and non-intervention are logically incompatible in every possible system.

The claim is methodological and domain-sensitive.

Where observation can genuinely remain external, the reflexive complication can be absent.

33.72 Scope of the Atlas

The Atlas of Equivalence is intentionally expansive.

It functions best as a map of possible questions.

It is not intended to become a mandatory checklist in which every cell must be filled for every case.

Let full atlas be $\mathfrak{A}$, and case-specific active atlas be

$$\mathfrak{A}^* \subset \mathfrak{A}. \quad (3216)$$

Equation 3216 is central to practical use.

Only the relevant subset should be activated.

33.73 Atlas Complexity

The number of possible object–relation combinations can become very large.

Let number of objects be n_Q and comparative relations n_E .

The unconstrained cell count is

$$N_C = n_Q n_E. \quad (3217)$$

Equation 3217 illustrates the combinatorial growth.

Most cells will be incoherent or irrelevant in any specific domain.

Future work therefore needs pruning, typing, and domain-specific atlases.

33.74 No Exhaustive Enumeration Claim

The object and relation taxonomies are broad and remain open.

The manuscript does not claim that every possible restorative object or comparative relation has been enumerated.

New domains can require new categories.

33.75 Atlas as a Living Taxonomy

Let the atlas at version v be $\mathfrak{A}^{(v)}$.

Revision can produce

$$\mathfrak{A}^{(v+1)} = \mathcal{U} \left(\mathfrak{A}^{(v)}, E, C, D \right), \quad (3218)$$

where *E* denotes empirical findings, *C* conceptual critique, and *D* domain-specific development. Equation 3218 treats the taxonomy itself as revisable.

33.76 Scope of Applied Cases

The cases in Section 31 were selected because they illuminate different structures.

They are not intended to represent the full universe of restorative problems.

The manuscript gives relatively limited treatment to:

- criminal sentencing;
- family law;
- war and mass displacement;
- transitional justice in detail;
- indigenous legal orders;
- animal justice;
- disability theory as an independent field;
- medical ethics;
- technology-specific harms.

Each could support a separate domain-specific study.

33.77 No Case-Specific Legal or Policy Conclusions

The stylized applied cases are not recommendations concerning any actual person, institution, or legal dispute.

They illustrate analytical distinctions.

Empirical and legal facts would be necessary before the framework could support a specific real-world recommendation.

33.78 Epistemic Position of the Analyst

The analyst is not outside the system in every case.

Selection of:

- variables;
- references;
- values;
- evidence;
- thresholds

can reflect the analyst's institutional and conceptual position.

The framework therefore requires reflexivity concerning its own use.

33.79 Observer Dependence

Let analyst a and analyst b construct representations

$$\Pi_a(\mathcal{G}) \tag{3219}$$

and

$$\Pi_b(\mathcal{G}). \tag{3220}$$

The two need not be identical.

This does not imply that all interpretations are equally valid.

It means the representation should expose its standpoint and justification.

33.80 Subject Participation Limits

The manuscript gives substantial importance to subject voice and direction.

Participation also has limits.

Affected persons can:

- disagree internally;
- lack information;
- experience decision fatigue;
- prefer limited participation;
- face conflicts with third-party rights.

Subject participation should therefore be meaningful and non-coercive rather than maximal by default.

33.81 No Subject Sovereignty over Third Parties

Present agency does not imply unlimited authority to impose restorative burdens upon others.

The subject can possess strong authority over personal future direction while third-party rights constrain relational reconstruction and resource allocation.

33.82 Third-Party Scope

The manuscript treats third-party justice as a major constraint.

It does not provide a complete theory for determining when a person becomes a relevant third party.

Material effect, rights, reliance, causal responsibility, and institutional position can all matter.

Future work should clarify these thresholds.

33.83 Scope of Better-than-Before Restoration

The theory permits transformative outcomes superior to the historical reference.

It does not imply that every restorative institution should maximize improvement beyond the reference.

A court deciding a narrow corrective claim can legitimately stop at a jurisdictionally appropriate remedy even where broader social improvement would be desirable.

Transformative justice and bounded institutional responsibility should remain distinct.

33.84 Scope of Residual Justice

The manuscript repeatedly identifies residual loss.

It does not yet provide a complete normative theory of what is owed for every irreparable residual.

Recognition, compensation, memorialization, continuing assistance, and prevention are possible responses.

Their appropriate combination remains open.

33.85 Irreparability

The theory does not claim that any current classification of a loss as irreparable is permanent.

Technical and institutional change can alter reparability.

The more durable distinction is between current inability to repair and historical non-identity.

33.86 Historical Non-Identity

The claim that a harmed-and-repaired history differs from a no-harm history is conceptually strong and normatively incomplete.

Historical difference does not automatically imply an additional compensable claim.

Its normative significance depends upon value theory, institutional context, and the nature of the lost history.

33.87 No Metaphysical Claim of Irreversibility

The manuscript does not attempt to prove that physical time reversal, world reconstruction, or other speculative interventions are metaphysically impossible.

Its argument is narrower.

Even if technically possible, such interventions would create additional questions concerning identity, consent, replacement, and justice.

33.88 Language of Restoration

Terms such as *restore*, *repair*, *return*, *reparation*, *rehabilitation*, and *restitution* have different technical meanings across disciplines.

The manuscript uses *restoration* as the broad umbrella concept and preserves narrower legal or institutional meanings where relevant.

Readers should therefore attend to the indexed object and relation rather than assume that the ordinary-language word itself determines the formal claim.

33.89 Scope of Equivalence

The title's *Atlas of Equivalence* uses equivalence broadly.

Many relations in the atlas are not mathematical equivalence relations because they can lack reflexivity, symmetry, or transitivity.

Examples include:

- dominance;
- sufficiency;
- simulation;
- inclusion;
- approximation.

The technically broader term throughout the manuscript is *comparative relations*.

33.90 No Global Equivalence Hierarchy

The manuscript does not provide one global ordering

$$E_1 < E_2 < \dots < E_n \quad (3221)$$

from weak to strong restoration.

Equation 3221 would misrepresent relations that preserve different structures.

Local refinement orders can exist.

A universal hierarchy generally does not follow.

33.91 Limits of Stronger-Equivalence Intuition

A stronger comparative relation can preserve more structure and impose greater normative or procedural costs.

Thus

$$E_2 \succ_{\text{formal}} E_1 \quad (3222)$$

does not imply

$$J(E_2) > J(E_1). \quad (3223)$$

Equations 3222 and 3223 summarize one of the framework's central limitations on mathematical intuition.

33.92 Scope of Monetary Analysis

Section 28 treats money as a general equivalent and restorative conversion interface.

It does not provide a complete theory of:

- money creation;
- monetary policy;
- financial markets;
- price theory;
- macroeconomic distribution.

The political-economic analysis is limited to the restorative role of fungibility, purchasing power, access, conversion, capture, and circulation.

33.93 No Anti-Monetary Claim

The manuscript does not argue that monetary compensation is inherently inferior to specific restoration.

Money can possess exceptional restorative advantages through:

- fungibility;
- liquidity;
- portability;
- subject direction.

The argument concerns the scope of monetary equivalence, not rejection of money as a remedy.

33.94 No Anti-Restitution Claim

Likewise, the framework does not argue against restitution.

Where an identifiable object or legal status remains returnable and meaningful, direct restitution can possess very high restorative value.

The caution concerns the inference from object return to complete restoration of the entire generative system.

33.95 No Anti-Prevention Claim

The framework does not use concerns about paternalism to reject prevention.

Prevention can be especially important where harm is severe, recursively amplifying, or weakly reparable.

The limitation is that prevention must itself satisfy procedural and normative constraints.

33.96 No Anti-Formalization Claim

The manuscript's repeated warnings about formal overreach should not be read as a rejection of mathematics.

Formalization is valuable where it:

- reveals assumptions;
- separates objects;
- clarifies non-implications;
- supports empirical testing;
- identifies structural trade-offs.

The limitation concerns unjustified formal authority.

33.97 No Claim of Grand Theory

The manuscript does not propose a final unified theory of justice, law, social systems, or human flourishing.

Its ambition is architectural.

It proposes a way to organize restorative comparison across several theoretical traditions and practical domains.

The architecture should remain open to revision.

33.98 Boundary between Framework and Doctrine

The manuscript is best understood as a framework that can host domain-specific doctrine.

For example:

- law determines legally cognizable interests and remedies;
- medicine determines clinically meaningful functional measures;
- ecology determines valid resilience models;
- affected communities contribute local value and historical knowledge.

The atlas organizes these contributions.

It does not replace them.

33.99 Boundary between Framework and Empirical Science

The framework generates hypotheses such as:

- portability can reduce domination;
- early interruption can reduce recursive loss;
- formal reinstatement can fail to restore network access;
- conversion support can improve capability beyond equal cash transfer.

These remain empirical propositions where applied to real populations.

The manuscript does not treat conceptual plausibility as empirical confirmation.

33.100 Boundary between Framework and Moral Judgment

The atlas can expose that a remedy improves capability, reduces historical fidelity, and burdens a third party.

The final normative judgment still requires reasons concerning how those values should be weighted or constrained.

The framework therefore structures judgment without replacing moral judgment.

33.101 Boundary between Framework and Political Decision

Collective restorative policies often require legitimately authorized political choice.

Technical experts can provide:

- causal estimates;
- simulations;
- cost analysis;
- risk information.

They cannot derive legitimate political authority from expertise alone.

The institutional process remains part of justice.

33.102 Practical Use under Bounded Rationality

Real decision makers possess limited time, information, and cognitive capacity.

The full theory should therefore be used selectively.

Let case complexity be C and available analytical capacity be A .

Where

$$C > A, \tag{3224}$$

the institution requires simplification.

Equation 3224 does not justify arbitrary simplification.

It creates a need for transparent domain-specific heuristics.

33.103 Heuristics

A practical heuristic can ask:

1. what was lost;
2. what reference matters;
3. what has actually been returned;
4. what future remains possible;
5. what residual remains;
6. what procedure and institution are appropriate.

This six-question structure can capture much of the framework's practical value without implementing the entire atlas.

33.104 Domain-Specific Simplification

Different domains can activate different core objects.

For example:

- employment can prioritize status, capability, network, and control;
- health can prioritize function, capability, care, and time;
- reputation can prioritize record, recognition, propagation, and privacy;
- ecology can prioritize state, function, resilience, and history.

Such simplification is compatible with the framework's purpose.

33.105 Risk of Framework Reification

Once a taxonomy becomes institutionalized, its categories can appear natural and complete.

This would contradict the framework's methodological intent.

The atlas should therefore preserve the possibility that an affected person, institution, or future researcher can say:

The relevant restorative object is missing from the current model.

That possibility is part of the framework's generative openness.

33.106 Risk of Institutional Self-Validation

An institution can define restoration through metrics it already performs well.

Let institutional metric set be $\mathcal{M}_I$.

If success is defined exclusively by

$$J_I = F(\mathcal{M}_I), \tag{3225}$$

the institution can systematically omit values outside its measurement architecture.

Equation 3225 motivates external review and subject participation.

33.107 Scope of Conclusions

The conclusions that follow in Section 34 should therefore be read at the appropriate level.

The manuscript supports the following broad conclusions:

1. restorative return is comparison-relative;
2. external condition return does not generally imply restoration of generative significance;
3. history and path dependence matter;
4. value theory and procedure are necessary parts of restoration;
5. money is powerful and partial;
6. assistance and prevention complement ex post remedy;
7. complete historical equivalence is distinct from justice.

The manuscript does not support the stronger claims that:

1. every loss can be formally modeled;
2. every restorative object can be measured;
3. one algorithm can select the just remedy;
4. one institution should implement the whole framework;
5. exact counterfactual reconstruction is generally possible;
6. generativity is the sole or supreme value.

33.108 Conceptual-Scope Principle

The first limitation principle is:

The framework is a grammar for specifying restorative comparison and should not be interpreted as a complete ontology of persons, societies, institutions, or ecosystems.

33.109 Formal-Scope Principle

The second principle is:

Formal notation should be interpreted according to its analytical role, and only domain-validated components should be treated as empirical mathematical models.

33.110 Empirical-Scope Principle

The third principle is:

The manuscript proposes testable structures and distinctions while leaving domain-specific effect sizes, causal estimates, and operational validity to empirical research.

33.111 Normative-Scope Principle

The fourth principle is:

The Atlas of Equivalence does not generate a unique theory of justice and should remain compatible with principled disagreement among several normative frameworks.

33.112 Legal-Scope Principle

The fifth principle is:

The legal analysis provides a comparative remedial grammar and should not be treated as a statement of jurisdiction-specific positive law or legal advice.

33.113 Institutional-Scope Principle

The sixth principle is:

Conceptual comprehensiveness does not imply that one institution should collect, measure, govern, or repair every dimension represented in the theory.

33.114 Technological-Scope Principle

The seventh principle is:

Idealized restorative technologies are thought experiments used to isolate conceptual limits and should not be interpreted as forecasts or present policy recommendations.

33.115 Cultural-Scope Principle

The eighth principle is:

Restorative concepts require contextual and cross-cultural translation, and the framework should not be used to erase locally grounded normative vocabularies.

33.116 Scale-Scope Principle

The ninth principle is:

Restorative objects and relations should be re-evaluated when moving across individual, institutional, collective, ecological, and intergenerational scales.

33.117 Anti-Surveillance Principle

The tenth principle is:

The availability of a high-dimensional restorative model does not create a general authorization to collect the personal data required to estimate every dimension of that model.

33.118 Anti-Reification Principle

The eleventh principle is:

The categories of the atlas should remain revisable analytical interfaces and should not be treated as fixed natural kinds merely because they become institutionalized.

33.119 Residual-Modesty Principle

The twelfth principle is:

Where the theory cannot determine a unique restorative judgment, the remaining uncertainty or incomparability should be acknowledged rather than concealed by artificial precision.

Table 40: Scope and principal limitations of the present framework

Domain	Present contribution	Excluded stronger claim	Required future work
Conceptual	Indexed restorative grammar	Complete ontology of social reality	Domain-specific conceptual refinement
Formal	Structured notation and selected formal relations	Universal mathematical model	Typed semantics and empirical validation
Empirical	Testable hypotheses and applied structures	Validated effect sizes across domains	Longitudinal and mixed-method research
Causal	Distinction among observational, causal, and counterfactual objects	Exact reconstruction of actual life counterfactuals	Domain-specific causal identification
Normative	Plural mapping of value theories	Final unified theory of justice	Substantive normative argument
Legal	Comparative remedy architecture	Jurisdiction-independent legal doctrine	Comparative and doctrinal legal research
Institutional	Division-of-labor principles	Universal restorative agency	Implementation and governance studies
Technological	Idealized limit analysis	Prediction of future restorative technology	Engineering and ethics research
Cultural	Portable analytical vocabulary	Lossless universal translation	Comparative intellectual and institutional study
Ecological	Restorative and resilience interfaces	Complete environmental ethics or ecology	Domain-specialist ecological research

Table 41: Supported claims and excluded stronger interpretations

Supported claim	Excluded stronger interpretation
Restoration depends upon the selected object and comparison relation.	There exists one objectively unique metric of total restoration.
Returning a condition can fail to restore its earlier generative significance.	Every returned condition necessarily has lower value after harm.
Historical and relational structure can matter normatively.	Historical fidelity always dominates present agency.
Money is a powerful general equivalent with conversion limits.	Monetary compensation is intrinsically inadequate.
Generative assistance can complement corrective remedy.	Institutions have unlimited duties to maximize individual generativity.
Low reparability can increase the importance of prevention.	Any weakly reparable risk justifies intrusive preventive control.
Formal models can clarify restorative structure.	Every social restorative problem should be mathematically modeled.
Plural normative theories select different atlas cells.	Every normative disagreement can be reconciled within one master theory.

33.120 Scope Matrix

Table 40 summarizes the principal boundaries of the manuscript.

Table 40 makes explicit that the manuscript's contribution is architectural across each domain.

33.121 Claim Matrix

Table 41 distinguishes representative claims supported by the manuscript from stronger claims that fall outside its scope.

Table 41 is intended to reduce predictable overinterpretation of the framework.

33.122 Relationship between Limitations and Contribution

The limitations identified in this section do not merely weaken the theory.

Several are direct consequences of the manuscript's central commitments.

Value pluralism creates incomparability because the theory refuses artificial scalarization.

Institutional pluralism creates coordination problems because the theory refuses to treat one institution as universally competent.

Ontological openness creates model uncertainty because the theory refuses to treat its taxonomy as exhaustive.

Procedural constraint limits maximal restorative fidelity because the theory refuses to subordinate agency and rights to reference similarity.

These limitations therefore define the kind of theory being proposed.

33.123 Deliberate Incompleteness

The architecture is deliberately incomplete in three senses.

First, its taxonomy remains revisable.

Second, its normative layer remains plural.

Third, its operationalization remains domain-specific.

This deliberate incompleteness is preferable to a superficially complete theory that obtains determinacy by hiding contested assumptions.

33.124 The Boundary of Formalization

The manuscript therefore proposes a practical boundary:

Formalize until formalization clarifies the object, assumptions, comparison, or trade-off.

Stop where additional formal precision would exceed the conceptual or empirical precision of the underlying claim.

This boundary is especially important for history, identity, meaning, and recognition.

33.125 The Boundary of Institutional Knowledge

An institution should know enough to perform its legitimate restorative function.

It need not know everything that could theoretically improve prediction.

Let sufficient institutional knowledge be K_I^* .

The desired condition is

$$K_I \succeq K_I^* \quad (3226)$$

without presuming

$$K_I = K_{\text{total}}. \quad (3227)$$

Equations 3226 and 3227 express epistemic sufficiency rather than total informational ambition.

33.126 The Boundary of Restoration

The deepest limitation is the boundary between restoring an actual historical subject and constructing another world.

As intervention becomes stronger, the project can move from:

$$\text{repair} \rightarrow \text{reconstruction} \rightarrow \text{replacement}. \quad (3228)$$

Equation 3228 is conceptual.

The manuscript argues that justice does not require crossing this sequence as far as technology permits.

The appropriate stopping point depends upon agency, identity, history, rights, third-party interests, and value.

33.127 Transition to Conclusion

With these limitations made explicit, the manuscript can return to its central problem without the burden of claiming a universal solution.

The theory began from a simple but consequential observation: after a harmful interruption, returning a resource, status, opportunity, or external condition does not necessarily recreate the generative system in which that condition once had its earlier meaning.

The subsequent analysis expanded this observation into a comparative architecture of references, objects, relations, histories, values, remedies, assistance, and prevention.

Section 34 draws these strands together.

The conclusion will emphasize three final points.

First, return is not one operation. It is an indexed family of comparative and normative claims.

Second, justice after harm can require a new legitimate future rather than a simulation of the world in which the harm never occurred.

Third, the most important practical question can therefore shift from

Has the lost thing been returned?

toward

Which conditions, relations, capabilities, histories, and generative possibilities remain, and what form of just future can now be built from them?

34 Conclusion

This manuscript began from a comparatively simple restorative problem.

A subject can lose a resource, status, opportunity, relation, bodily capacity, institutional position, or other condition through harm. An institution can later return something bearing the same external description. The job can be reinstated. The money can be repaid. The record can be corrected. The resource can be restored. The legal status can be reissued.

Yet the returned condition can enter a system that has already changed.

Trust can have altered. Networks can have reorganized. Health can have changed. Opportunity windows can have closed. New dependencies can have formed. The subject can have developed new preferences, relations, commitments, knowledge, and vulnerabilities. Institutions and third parties can also have changed.

The central conclusion of the manuscript is that a claim of restoration is therefore incomplete until the analyst specifies what has been restored, against which reference, through which comparative relation, over what temporal horizon, under which epistemic assumptions, and according to which theory of value and justice.

The justice of return is consequently neither a single act of replacement nor a single metric of compensation.

It is a structured family of comparative and normative problems.

34.1 Return as an Indexed Relation

The formal expression developed throughout the manuscript can be summarized by

$$\text{Rest} \left(\mathcal{G}^R, \mathcal{G}^{\text{ref}}; Q, E, \mathcal{T}, \tau, \varepsilon, K \right). \quad (3229)$$

Equation 3229 states that restoration is indexed by:

- a restored system $\mathcal{G}^R$;

- a reference system $\mathcal{G}^{\text{ref}}$;
- a comparison object Q ;
- a comparative relation E ;
- a test or observation family $\mathcal{T}$;
- a temporal horizon τ ;
- a tolerance ε ;
- an epistemic basis K .

This indexing is the conceptual core of the Atlas of Equivalence.

It explains why several claims that appear contradictory can all be true.

A person can be monetarily restored and capability-impaired.

A worker can be formally reinstated and relationally excluded.

A record can be corrected and historically different.

An ecosystem can recover visible state and remain dynamically fragile.

A community can receive resources and remain without control.

The apparent contradiction disappears once the comparison object and relation are specified.

34.2 From Return of Conditions to Return of Generativity

The manuscript has distinguished return of a condition from restoration of the generative system in which the condition once operated.

Let external condition be c . It can satisfy

$$c_R = c_{\text{pre}} \quad (3230)$$

while the wider generative system satisfies

$$\mathcal{G}^R \neq \mathcal{G}^{\text{ref}}. \quad (3231)$$

Equations 3230 and 3231 capture the manuscript's originating problem.

The value of a condition is often generated through its relations to other conditions, institutions, persons, temporal windows, and future pathways.

The same nominal resource can therefore acquire different practical and normative significance after harm.

34.3 Temporal Non-Commutativity

The manuscript formalized this historical asymmetry through a non-commutativity relation.

Let harm occur at t_0 , altered evolution continue until t_1 , and restoration occur at t_1 .

A generic structure is

$$R_{t_1} \circ \Phi_{t_1, t_0}^H \circ H_{t_0} \neq \Phi_{t_1, t_0}^{\text{ref}}. \quad (3232)$$

Equation 3232 states that return after historical evolution does not generally reproduce the world in which deprivation never occurred.

This is the basis for distinguishing:

- condition restitution;
- capability restoration;

- trajectory restoration;
- historical restoration;
- generative restoration.

The distinction is especially important for time-sensitive and recursively generative losses.

34.4 State, Trajectory, and Reachability

The manuscript has shown that restorative comparison can occur at several dynamical levels.

State comparison asks whether the current condition corresponds.

Trajectory comparison asks whether the historical or future path corresponds.

Reachability asks which futures remain possible.

These objects can be represented by

$$(x_t, \gamma_{[t_0, t_1]}, \mathcal{R}_\tau(x_t)) . \quad (3233)$$

Equation 3233 demonstrates why endpoint equality can coexist with substantial restorative loss.

A person can arrive at the same salary and possess a substantially different future option structure.

A restored ecosystem can occupy a similar current state and remain in a different basin of attraction.

A corrected legal status can coexist with a different institutional trajectory.

34.5 Dynamics and Restorative Structure

The dynamical analysis has also introduced attractors, basins, resilience, hysteresis, bifurcation, perturbation response, and recovery time as possible restorative objects.

These tools are useful where the domain supports the relevant assumptions.

They reveal a further distinction between appearance and structural stability.

A system can look restored under calm conditions and remain vulnerable to ordinary future perturbation.

Durable restoration therefore sometimes requires more than state similarity.

34.6 Causal Restoration

Causal analysis adds another layer.

Two systems can produce the same observed outcome through different mechanisms.

Let observational equality satisfy

$$P_R(Y) = P_{\text{ref}}(Y) \quad (3234)$$

while intervention responses satisfy

$$P_R(Y \mid \text{do}(X = x)) \neq P_{\text{ref}}(Y \mid \text{do}(X = x)) . \quad (3235)$$

Equations 3234 and 3235 demonstrate why observational similarity can fail to establish causal restoration.

This distinction is important where resources are returned while the mechanisms converting those resources into opportunity remain altered.

34.7 Generative-Process Restoration

The generative-process analysis extended the comparison from causal effect to recursive production.

A generative cycle can be represented as

$$C_t \rightarrow V_{t+1} \rightarrow \Lambda \rightarrow C_{t+1}, \quad (3236)$$

where conditions generate value, value circulates, and part of that value returns to future generative conditions.

Restorative justice can therefore ask whether:

- conversion has been restored;
- control has been restored;
- value circulation has been restored;
- recursive generation has been restored.

This perspective is especially useful where one-time transfer leaves the future process unchanged.

34.8 Historical and Reflexive Difference

History itself remains an object of restorative comparison.

The repaired world and the no-harm world can be written as

$$\mathcal{H}^{H,R} \neq \mathcal{H}^{-H}. \quad (3237)$$

Equation 3237 remains true even when selected present states, functions, causal mechanisms, or capabilities have been restored exactly.

The distinction becomes stronger because repair itself enters the actual history.

Apology, compensation, judgment, rehabilitation, and institutional reform become new events.

The repaired history is therefore neither the harmed history without repair nor the no-harm history.

It is a third historical trajectory.

34.9 The Repaired World

The manuscript has consequently distinguished three worlds:

$$\mathcal{W}^H, \quad \mathcal{W}^{-H}, \quad \mathcal{W}^R. \quad (3238)$$

Equation 3238 denotes the harmed world, the no-harm counterfactual world, and the repaired actual world.

The principal normative target after harm need not be

$$\mathcal{W}^R = \mathcal{W}^{-H}. \quad (3239)$$

A more defensible target can be

$$\mathcal{W}^R \in \mathcal{W}_{\text{just}}^{\text{postharm}}. \quad (3240)$$

Equation 3240 represents the set of just futures available after the actual history has occurred.

This is one of the manuscript's most important conclusions.

34.10 Value Objects

Restorative comparison acquires normative significance only through value.

The manuscript has identified a plural family of value objects including:

- welfare;
- resources;
- capability;
- agency;
- autonomy;
- relational standing;
- recognition;
- rights;
- non-domination;
- history;
- provenance;
- care;
- trust;
- resilience;
- generativity;
- ecological and intergenerational conditions.

No single value was elevated into a universal master value.

This pluralism is deliberate.

34.11 Value Theory and Comparative Standard

The manuscript has also separated broad normative theories from comparative standards.

Equality, sufficiency, priority, dominance, lexical protection, compensation, and inclusion are different comparative structures.

Welfare theories, capability approaches, relational equality, recognition, rights theories, corrective justice, non-domination, care, reparative justice, and generative justice provide different reasons for selecting and ordering those structures.

The resulting conclusion is:

Restoration requires a theory of comparison, and a theory of comparison requires a theory of value.

The converse is equally important:

A theory of value does not determine the legitimacy of every process through which the valued result can be produced.

This leads directly to the distinction between substantive and procedural justice.

34.12 Substantive Justice

Substantive justice of return was represented as a plural profile:

$$\mathbf{J}_S = (J_{\text{corr}}, J_{\text{dist}}, J_{\text{suff}}, J_{\text{cap}}, J_{\text{rel}}, J_{\text{rec}}, J_{\text{ND}}, J_{\text{hist}}, J_{\text{gen}}, J_{\text{trans}}) . \quad (3241)$$

Equation 3241 reflects corrective, distributive, sufficientarian, capability, relational, recognition, non-domination, historical, generative, and transformative dimensions.

A restorative system can therefore succeed in one dimension and remain deficient in another.

34.13 Procedural Justice

Procedural justice was developed independently through objects such as:

- access;
- notice;
- voice;
- consent;
- impartiality;
- evidence;
- reason giving;
- privacy;
- contestability;
- review;
- revisability;
- timeliness.

A procedurally legitimate restoration should therefore be evaluated through more than its outcome.

The same substantive result can be reached through different procedures with different justice properties.

34.14 Fidelity and Procedure

One of the manuscript's central results is the possibility that stronger restorative fidelity reduces procedural justice.

Let two interventions satisfy

$$F_R(R_2) > F_R(R_1) \quad (3242)$$

and

$$J_P(R_2) < J_P(R_1). \quad (3243)$$

Equations 3242 and 3243 show why maximizing resemblance to a reference is not a sufficient theory of restorative justice.

This problem becomes particularly acute for:

- memory reconstruction;
- compulsory reinstatement;
- relational reconstruction;
- extensive surveillance;
- intervention upon innocent third parties.

34.15 Protected Difference

The framework therefore distinguishes several forms of residual difference.

Let

$$\Delta_R = \Delta_{\text{unresolved}} \cup \Delta_{\text{protected}} \cup \Delta_{\text{valuable}}. \quad (3244)$$

Equation 3244 separates:

- unresolved restorative loss;
- differences protected because removing them would violate rights or agency;
- differences possessing positive value after harm.

This partition changes the meaning of incomplete historical fidelity.

Some differences should disappear.

Some cannot disappear.

Some should remain.

34.16 The Right to a Different Future

The actual present subject should not be governed entirely by the modeled preferences or trajectory of the counterfactual no-harm subject.

Let harmed trajectory be γ_H , counterfactual trajectory γ_C , and legitimate new trajectory γ_N .

The framework permits

$$\gamma_N \neq \gamma_H \quad (3245)$$

and

$$\gamma_N \neq \gamma_C. \quad (3246)$$

Equations 3245 and 3246 express the right to a different future.

A just remedy can therefore help construct a new trajectory rather than forcing the subject back toward a life that exists only as a model of what might have been.

34.17 Legal Models of Return

Law already contains several distinct models of return.

The manuscript has interpreted:

- restitution as object or status return;
- damages as monetary translation;
- specific performance as performance fidelity;

- injunctions as control of future trajectories;
- declarations as authoritative status correction;
- rehabilitation as functional or capability repair;
- satisfaction as recognition and historical response;
- structural remedies as mechanism transformation.

This plurality is important.

It shows that law does not operate through one undifferentiated concept of making a claimant whole.

34.18 Legal Equivalence

Legal systems construct purpose-relative equivalences.

Let

$$Q_1 \equiv_L Q_2 \quad (3247)$$

mean that Q_1 and Q_2 are treated as equivalent for a specified legal purpose.

Equation 3247 need not imply historical, functional, relational, or moral identity.

This distinction allows law to remain institutionally workable without claiming metaphysical completeness.

34.19 General Equivalents

Money occupies a particularly important place within this architecture.

Its strengths include:

- fungibility;
- liquidity;
- portability;
- divisibility;
- subject-directed use.

Its restorative power is therefore real.

Its limits are also real.

The full monetary pathway was represented as

$$\mathbf{L} \rightarrow \mathbf{M} \rightarrow \mathbf{C} \rightarrow \mathcal{K} \rightarrow \mathbf{G}. \quad (3248)$$

Equation 3248 shows that monetary transfer is one stage within a broader conversion process.

Prices, access, debt, institutions, health, infrastructure, and social structure can all alter the later stages.

34.20 Fungibility as Epistemic Humility

One important positive conclusion concerns fungibility.

Where the institution cannot know which future the changed subject should pursue, general resources allow the subject to redirect value toward a chosen future.

This gives money and other flexible resources a form of epistemic humility.

The institution supplies capacity without fully specifying the subject's life plan.

The limitation appears where the needed condition is unavailable through exchange, structurally inaccessible, relationally constituted, or historically singular.

34.21 Generative Assistance

Where money and restitution leave conversion impaired, generative assistance becomes relevant.

Its central architecture was represented by

$$A_G = (M, C^*, N, R, T, X, P, V), \quad (3249)$$

where the terms denote flexible resources, specific enabling conditions, navigation, relational access, time, experimentation, portability, and revisability.

Generative assistance is therefore neither a simple cash transfer nor a complete institutional plan for the subject's future.

Its function is to make viable futures practically reachable while preserving subject direction.

34.22 Assistance and Legitimate Dependence

The manuscript has also rejected a narrow definition of successful assistance as the disappearance of all support.

A subject can remain dependent upon:

- public infrastructure;
- social protection;
- care;
- accommodation;
- shared resources.

Such dependence can be just where it is secure, non-dominating, dignified, and compatible with agency.

The relevant contrast is therefore not simply dependence and independence.

It is the difference among secure interdependence, supported agency, and dominating dependence.

34.23 Preventive Governance

The temporal analysis has moved further upstream.

If some losses are weakly reparable, prevention and early interruption become more important.

Preventive governance therefore focuses upon:

- critical generative conditions;
- reparability;
- early warning;
- redundancy;
- buffers;
- portability;
- evidence preservation;
- anti-retaliation;
- staged intervention.

The qualified principle is

$$\text{Rep}(Q) \downarrow \implies W_{\text{prevention}} \uparrow, \quad (3250)$$

other things equal.

Equation 3250 remains constrained by rights, uncertainty, proportionality, and intervention burden.

34.24 Prevention without Stasis

Preventive governance should preserve generativity rather than freeze one social state.

A living system requires change.

The preventive objective is therefore closer to

$$\mathcal{F}_{t+\tau}^{\text{adm}} \text{ remains viable} \quad (3251)$$

than to

$$x_{t+\tau} = x_t. \quad (3252)$$

Equations 3251 and 3252 distinguish preservation of future possibility from preservation of stasis.

34.25 The Full Temporal Architecture

The manuscript now supports four temporally distinct institutional functions:

$$\mathfrak{T}_J = (P, I, R, A), \quad (3253)$$

where:

- P denotes prevention;
- I interruption;
- R post-harm remedy;
- A generative assistance.

Equation 3253 is a central practical conclusion.

Justice after harm cannot be understood solely as an ex post remedial function.

34.26 Institutional Division of Labor

No institution is naturally suited to every component of this architecture.

Courts can determine rights and liability.

Administrative institutions can correct status and coordinate services.

Social-protection institutions can secure minimum conditions.

Healthcare institutions can support function.

Archives can preserve historical provenance.

Community institutions can govern collective resources.

The result is institutional pluralism.

Its corresponding risk is fragmentation.

34.27 Institutional Handoff

A restorative system should therefore identify unresolved objects and transfer them to competent institutions where possible.

Let unresolved set be

$$\mathcal{O}_{\text{res}} = \{O_i : J_i < \theta_i\}. \quad (3254)$$

Equation 3254 provides the basis for referral rather than either abandonment or unlimited institutional expansion.

34.28 Reflexive Non-Closure

One of the manuscript's deeper methodological conclusions concerns reflexivity.

Records, classifications, models, judgments, and restorative interventions can alter the systems they describe.

A generic structure is

$$\mathcal{G}_{t+1} = F(\mathcal{G}_t, O(\mathcal{G}_t), R_t). \quad (3255)$$

Equation 3255 means observation and intervention can become endogenous.

The theory has called this *reflexive non-closure of generative systems*.

The implication is methodological.

Restorative models and classifications should remain revisable because their own use can change the system later being evaluated.

34.29 Ideal Technology and Persistent Normativity

The idealized thought experiments showed that technical progress does not collapse the normative architecture.

Even perfect observation, causal knowledge, simulation, bodily repair, memory intervention, or network reconstruction would leave questions concerning:

- reference authority;
- identity;
- consent;
- third-party agency;
- historical truth;
- present subject preference.

The distinction can be stated as

$$\mathcal{R}_{\text{legitimate}} \subseteq \mathcal{R}_{\text{technically possible}}. \quad (3256)$$

Equation 3256 becomes more important as restorative power increases.

34.30 Complete Restoration

The phrase *complete restoration* was shown to contain several possible meanings.

The manuscript distinguished:

- technical completeness;
- state completeness;
- functional completeness;
- causal completeness;
- dynamical completeness;
- generative completeness;
- historical completeness;
- normative completeness.

These forms do not imply one another.

A normatively complete repair can remain historically different.

A technically complete intervention can remain procedurally unjust.

A causally reconstructed system can remain a different history.

This conclusion places a strong limit on undifferentiated claims of being made whole.

34.31 Applied Cases

The applied analyses showed how the framework behaves across distinct domains.

Workplace exclusion demonstrated the difference between formal reinstatement and effective professional re-entry.

Research opportunity demonstrated recursive and temporally singular loss.

Reputational injury demonstrated distributed correction and historical provenance.

Health-related harm demonstrated the difference between bodily repair and life capability.

Collective injustice demonstrated the interaction among resource, governance, recognition, history, and third-party structure.

Ecological restoration demonstrated the difference among state, function, resilience, basin structure, and ecological history.

Administrative classification demonstrated reflexive informational harm.

Cross-border mobility demonstrated institutional conversion and portability.

Infrastructure failure demonstrated why collective conditions can dominate individual compensation.

The cases therefore support one general applied conclusion:

The appropriate meaning of restoration depends upon the object and system in which the loss occurred.

34.32 The Atlas of Equivalence

The Atlas of Equivalence has been constructed as a multidimensional map rather than a hierarchy.

Its basic comparison space is

$$\mathfrak{C} = \mathfrak{Q} \times \mathfrak{E}, \quad (3257)$$

where $\mathfrak{Q}$ contains comparison objects and $\mathfrak{E}$ comparative relations.

The complete architecture adds references, time, epistemic basis, value, procedure, legal form, assistance, and prevention.

Its purpose is therefore diagnostic.

It asks which comparison an institution is actually making and which relevant comparisons remain outside its claim.

34.33 Atlas as Grammar rather than Score

The atlas should not be converted automatically into one scalar index of restoration.

Its principal contribution lies in preserving distinctions.

A single score would risk recreating the very compression the framework was designed to expose.

The atlas is therefore closer to a grammar than to a ranking table.

It tells the analyst which questions can be asked, how they differ, and which inferences fail across them.

34.34 No Global Hierarchy of Equivalence

A central formal conclusion is that there is no general global hierarchy in which every comparative relation can be ordered from weak to strong.

Identity, bisimulation, capability inclusion, causal equivalence, historical correspondence, and recognition sufficiency preserve different structures.

A relation can be formally stronger along one axis and normatively less appropriate in a particular case.

Therefore,

$$E_2 \succ_{\text{formal}} E_1 \not\Rightarrow J(E_2) > J(E_1). \quad (3258)$$

Equation 3258 summarizes the manuscript's critique of stronger-equivalence intuition.

34.35 Residual Honesty

Where full repair is unavailable, institutions should state the residual.

A bounded restorative statement is stronger than a false declaration of completeness.

Let achieved remedy satisfy

$$J_L \geq \theta_L \quad (3259)$$

while residual loss remains

$$\Delta_{\text{unresolved}} > 0. \quad (3260)$$

Equations 3259 and 3260 can coexist without contradiction.

This form of institutional honesty is especially important for irreparable historical loss.

34.36 Qualified Completion

The manuscript therefore recommends qualified completion statements.

An institution can say:

The legal status has been restored.

The monetary compensation has been paid.

The rehabilitation threshold has been met.

The current official record has been corrected.

Such statements are more precise than declaring the affected person or system fully restored.

34.37 Contribution to Justice Theory

The manuscript's contribution to justice theory is modest in one sense and broad in another.

It does not replace existing theories of corrective justice, distributive justice, capability, recognition, non-domination, care, reparative justice, or generative justice.

It provides a comparative architecture through which their different objects and standards can be related.

The key distinction is between:

1. what is compared;
2. how it is compared;
3. why that comparison matters;
4. whether the resulting state is substantively just;
5. whether the process is procedurally just.

This separation can reduce conceptual conflict among theories that are often answering different questions.

34.38 Contribution to Legal Theory

For legal theory, the framework provides a grammar for understanding why different remedies correspond to different forms of return.

It also clarifies why legal adequacy need not imply complete historical or moral repair.

Law can remain bounded and institutionally honest by specifying the interest, object, relation, and remedy it has addressed.

34.39 Contribution to Political Economy

For political economy, the framework emphasizes that money is a powerful restorative interface whose real value depends upon conversion structures, markets, public infrastructure, debt, control, and value circulation.

Compensation therefore belongs inside a broader social system.

The same nominal award can generate radically different futures depending upon that environment.

34.40 Contribution to Governance

For governance, the framework integrates prevention, interruption, remedy, and assistance.

This temporal architecture moves analysis away from the assumption that justice begins only after a completed harm.

It also moves prevention away from the assumption that risk reduction should be maximized without regard to agency or generative openness.

34.41 Contribution to Formal Analysis

For formal analysis, the manuscript proposes a modular vocabulary of states, trajectories, reachable sets, causal mechanisms, generative processes, histories, partial orders, and comparative relations.

The formalism is intended to clarify assumptions and non-implications.

It is not offered as a universal mathematical reduction of justice.

34.42 Contribution to Future Research

The principal research agenda includes:

- domain-specific restorative atlases;
- empirical measurement of generativity and capability;
- reparability estimation;
- comparative legal analysis;
- longitudinal assistance evaluation;
- privacy-preserving epistemic infrastructure;
- reflexive model governance;
- theories of residual justice.

The breadth of these questions confirms that the present manuscript is an architecture for further inquiry rather than a final closed system.

34.43 A Compact Restorative Architecture

The entire framework can be compressed into the layered structure

$$\mathfrak{R}^* = (B, Q, E, \tau, K, V, J_S, J_P, I, R, A, P, \Delta), \quad (3261)$$

where:

- B denotes reference;
- Q comparison object;
- E comparative relation;
- τ temporal horizon;
- K epistemic basis;
- V value architecture;
- J_S substantive justice;
- J_P procedural justice;
- I institutional allocation;
- R post-harm remedy;
- A generative assistance;
- P prevention;
- Δ residual difference.

Equation 3261 is the manuscript's final integrative representation.

34.44 Restorative Sequence

A practical restorative inquiry can therefore proceed through the sequence

harm → reference → objects → relations → values → justice → response → residual. (3262)

Equation 3262 should be understood as an analytical sequence.

Real institutions can move iteratively among the stages.

34.45 Final Principles

The manuscript can be summarized through the following principles.

1. A claim of restoration should specify the object, reference, comparative relation, temporal horizon, and epistemic basis.
2. Return of an external condition does not generally establish return of its earlier generative significance.
3. State, trajectory, reachability, causal structure, generative process, and history are distinct restorative objects.
4. Historical evolution can make restoration temporally non-commutative.
5. Value theory is necessary to determine which forms of equivalence matter.
6. Stronger formal equivalence does not generally imply greater justice.
7. Substantive and procedural justice should remain analytically distinct.
8. Present subject agency can justify a new future different from both the harmed and no-harm trajectories.
9. Money is a powerful general equivalent whose restorative force depends upon conversion and institutional context.
10. Generative assistance should combine flexible resources, specific enabling conditions, and subject direction where appropriate.
11. Low reparability can increase the importance of prevention and early interruption.
12. Some residual differences represent unresolved injustice, some are protected by rights and agency, and some possess positive value.
13. Institutional completion should remain bounded to the object and mandate actually resolved.
14. The repaired world should be judged as an actual historical world, rather than only by its resemblance to a world in which the wrong never occurred.

34.46 The Justice of Return

The phrase *justice of return* can now be given a more precise meaning.

It is the justice governing:

- what is returned;
- what return can mean after historical change;

- which equivalences matter;
- which differences remain;
- which new futures become legitimate.

The central question therefore moves beyond whether an institution can restore an earlier object.

It asks whether the post-harm world contains the conditions through which subjects, communities, and systems can continue in ways that are substantively just, procedurally legitimate, historically truthful, and generatively viable.

34.47 Final Perspective

A theory of return begins naturally with the past.

Something was taken, destroyed, interrupted, distorted, or made inaccessible.

The demand for justice therefore turns backward toward the lost condition.

Yet every actual remedy occurs in the present and acts upon the future.

This temporal structure creates the distinctive difficulty of restoration.

Justice must recognize what was lost without treating the past as the only legitimate model of what should come next.

It must acknowledge the no-harm counterfactual without replacing the actual historical subject with a hypothetical one.

It must value fidelity without converting similarity into the sole criterion of justice.

It must preserve history without making injury the permanent identity of the person or community.

It must provide resources without assuming that resources alone regenerate capability.

It must assist without taking ownership of the future.

It must prevent serious and weakly repairable harm without transforming prevention into permanent control.

The resulting conception of restoration is therefore neither simple reversal nor simple compensation.

It is a governance problem concerning history, relation, value, agency, generativity, and the conditions of future possibility.

The deepest practical implication can be stated concisely:

After harm, justice should ask not only how much of the lost world can be recreated, but also which differences should remain, which capabilities should be rebuilt, which relations should be transformed, and which new futures should be made possible.

The justice of return is therefore ultimately a justice of historical continuation.

Its task is to respond faithfully to loss while leaving room for a future that need not be identical to the past in order to be just.

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