

A Grammar of Power

Repertoires and the Reproduction of Asymmetry

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Abstract

Asymmetries of capacity arise continually in social life, and most of them dissolve. A few do not. Accounts that measure power by the resources an actor commands cannot say which will do which, and are embarrassed by the cases in which a thin administration outlasts an overwhelming military advantage, or a body with no coercive capacity outlives every regime that tolerates it. This paper locates the difference in what the exercise of an asymmetry does to the conditions of its own exercise: some acts consume the preconditions that made them possible, and others manufacture them. Power, on the account offered here, is the asymmetry that survives both its own exercise and the continual production of difference against which it must hold. The paper first surveys the formal languages available for describing social dynamics and finds that each locates variability at one level while fixing the level above it, so that none makes the grammar of a system into a moving object; operator algebra, which comes nearest to the requirement, is assessed and set aside for reasons given in detail. A structure of typed operational repertoires and the transitions they induce upon themselves is then constructed, and the mechanism restated within it. Two consequences follow. The Nietzschean thesis that power consists in its own maintenance is reconstructed without the will that carried it: self-maintenance is treated as constitutive of the relation rather than as something any arrangement pursues, and the rarity of maintenance, which Nietzsche read as weakness, is explained instead by the continual production of the difference against which any arrangement must hold. And generativity is shown not to reduce to the number of possibilities a configuration admits: constitutional prohibition contracts the field of available action and remains generative, while the collapse of norms expands it and does not. What distinguishes the two is whose possibilities are enlarged, and whether what is generated returns to the relation that generated it. The argument for this is given by counterexample rather than by a general impossibility result, and the paper is explicit about which of its claims are established and which are proposed.

Keywords: power; asymmetry; relational generation; repertoire; self-reproduction; generativity; formal modelling of social dynamics.

A note on the standing of this paper. This is a theoretical paper, and its central section states a mechanism in ordinary prose before any formal apparatus is introduced. The order is deliberate. A reader who declines the formal construction offered later should still be able to accept or reject the mechanism on its own terms, and the construction is offered as a way of stating the mechanism precisely rather than as its ground. The paper also declines a formalism it has been at pains to describe. Operator algebra is surveyed, credited, and then set aside, and the reasons are given at length, the temptation to borrow its authority being real and the borrowing unable to survive scrutiny. Objection, correction, and counter-evidence are welcome at huangwanhong@serendip.ngo.

1 INTRODUCTION

Power has been understood relationally for some decades now. It is not a substance an actor holds in reserve, on this understanding, but something that exists only in the relation between parties and only in its exercise; it is dispersed rather than concentrated at a centre, productive of subjects and categories rather than merely repressive of them, and immanent to the practices through which social life is conducted rather than imposed upon them from outside.¹ The turn was salutary and is not in question here. What is in question is that the relational account has not been given a formal language adequate to its own ontology, at least in the treatments that have shaped the field. Where power is modelled quantitatively it is generally modelled as a quantity after all, whether a resource endowment, a network centrality, or a bargaining weight, which returns in the formalism precisely what the relational turn removed in the theory. The gap is not a failure of rigour on either side. It reflects the absence of a mathematical language in which a relation, rather than a magnitude, can be the object that moves.

The difficulty this paper takes up can be seen without any formalism at all, in cases where asymmetries of very different magnitude meet very different fates. Consider first a state that has achieved overwhelming military superiority over its neighbours and its own population, and a colonial administration that governs a territory of many millions with a few thousand officials and a garrison too small to hold any province against a determined rising. By any measure of resources commanded, the first asymmetry is the greater by a wide margin. Yet military supremacy of this kind has repeatedly proved among the most perishable of advantages, exhausting the fiscal base that sustains it and provoking the coalitions that end it, while thin colonial administrations have in several instances persisted for the better part of a century. Consider next a firm holding a dominant share of its market, and a professional body that certifies practitioners and can compel no one to do anything. The first commands capital, distribution, and the capacity to price rivals out; the second commands only the recognition of a credential. Dominant market shares have proved unstable on a timescale of decades. Certifying bodies have in several cases survived the regimes, constitutions, and economic orders under which they were founded.

An account that measures power by resources predicts the opposite in each pair. The natural repair is to say that resources are not everything and that legitimacy, institutionalisation, or embeddedness must be added to the account. This is true and insufficient. It names further quantities to be added to the ledger without saying why the ledger is the wrong instrument, and it leaves untouched the feature the cases actually share, which is not the magnitude of

¹The formulations here follow Foucault (1978) and Foucault (1977) most closely. Arendt (1970) arrives at a relational account by a different route, locating power in concerted action rather than in any party's capacity to compel, and Emerson (1962) makes the relational commitment operational through dependence, a line developed in the network literature surveyed by Wasserman and Faust (1994). The contrast with the older capacity-based formulations of Weber (1978) and Dahl (1957) is what the relational turn consists in; Lukes (2005) maps the resulting debates.

anything but a relation between an act and its own preconditions. Overwhelming military superiority is exercised by campaigning, and campaigning consumes the fiscal and demographic base from which superiority was built; the exercise of the advantage subtracts from the conditions of the advantage. A credential is exercised by being required, and each occasion on which it is required confirms that the distinction it marks is a real one; the exercise of the advantage adds to the conditions of the advantage. The two cases differ not in how much is held but in the sign of what the holding does to itself when used.

This suggests the problem the paper takes as its own. Asymmetries of capacity arise continually wherever relations are produced, and the overwhelming majority of them dissolve without leaving institutional trace. A small minority do not dissolve. If what distinguishes the minority is that the exercise of the asymmetry reproduces rather than consumes the conditions of its exercise, then power is not a magnitude to be measured but a property of the relation between an operation and the field of operations that remains available after it. Stating that property precisely requires a language in which the field of available operations is itself the thing that changes, and this is what the available formal languages do not supply. Differential and stochastic models fix a state space and move a point within it. Hybrid models add jumps but paste the jump rule on from outside. Agent-based models let a topology rewire while fixing the space of relations that may be instantiated. Game-theoretic models represent rule-following exactly and receive the rules as exogenous input. Rewriting systems generate genuinely, and generate syntax rather than relations among parties who bear differing capacities. Operator algebra comes nearest of all, taking composable operations as primitive and deriving the trajectory from them rather than the reverse, and it is fixed in its own way: its generators and relations are given in advance, as a differential model's coordinates are given in advance. Each language locates variability at one level and holds fixed the level above. None makes the grammar itself the moving object.

The account developed here does. Its central construction is an operational repertoire, understood as a typed structure of partially composable operations, together with the transitions that operations induce upon the repertoire from which they were drawn. An asymmetry, on this construction, is a power relation when it is returned both by the transition its own exercise induces and by composition with the transitions the field supplies meanwhile. The first is closure and states the constitutive claim; the second is maintenance and states what closure must withstand. Power is maintenance, and the distinction between the two is what allows the account to record an arrangement that reproduces itself perfectly and is undone by what arises elsewhere.

Three consequences are drawn, and it is worth stating them here rather than reserving them. The first concerns a thesis that descends from Nietzsche and recurs, stripped of its original modality, in systems theory and elsewhere: that power consists in its own maintenance. The thesis is right and the modality is the difficulty. Nietzsche has power extending

and discharging itself, with persistence following as a consequence; later formulations retain the self-maintenance and drop the will, but say little about why maintenance succeeds when it does. The reading offered here is that self-maintenance is constitutive rather than purposive. An asymmetry exists only in exercise, and exercise alters the conditions of exercise, so the relation between an asymmetry and its own continuation is definitional rather than causal, and power is the case in which that relation closes. What the present framework adds is an account of why the closure is rare. Because relation is continually produced, ordered difference arises constantly and cheaply, and the field does not stand still beneath any arrangement. Maintenance is therefore work performed against a productive background rather than persistence in a static one, and asymmetry is insufficient for power because asymmetry is abundant.

The second consequence is a negative result, and it is the one the paper would most want defended. It is tempting to identify generativity with the enlargement of possibility and degeneration with its restriction, so that a configuration is generative in proportion to the options it admits. The identification fails in both directions. A constitutional prohibition removes operations from the field and is among the most generative arrangements known; the collapse of shared norms enlarges the field taken as a whole, since restraints that formerly held cease to hold, and is degenerative. What the second case conceals, and Section 8 sets out, is that the enlargement accrues to those already positioned to act while the capacities of everyone else contract. Generativity therefore reduces neither to the number of available options, nor to entropy, nor to freedom in any sense that counts. What separates the two is whose possibilities are enlarged, and whether what the arrangement generates returns to the relation that generated it or is diverted to an end the relation did not require.

The third is a schema. Durable asymmetries, the paper argues, are assembled from four operations working across distinct registers: a differentiation that produces variation, a selection that ranks the variation so that one side's difference confers what the other's does not, a normalisation that renders the ranking unremarkable, and a reproduction that transmits it. The registers are distinct in kind, being severally a distribution, an ordering, a meaning, and an inheritance, and the composite crosses between them rather than remaining within any one. The schema is offered as general rather than illustrative, and the argument for the necessity of each register is given case by case. Its most useful consequence concerns intervention: normalisation is the only register with no material substrate, and an asymmetry deprived of it does not cease but must be continually re-imposed, which is a different and more expensive thing than being reproduced.

The paper proceeds as follows. Section 2 surveys the available formal languages along a common axis, that of what each takes as primitive and what it thereby holds fixed, and derives from the survey the requirements any adequate language must meet. Section 3 assesses operator algebra against those requirements, finds that it meets them and cannot serve nonetheless, and gives the four obstructions in detail. Section 4 states the mechanism philosophically and

without formal apparatus. Section 5 constructs the formal object, and Section 6 restates the mechanism within it. Section 7 applies three distinctions borrowed from operator algebra to the constructed object, which is where they can do work they could not have done earlier. Section 8 sets out the refutation of the counting criterion. Section 9 generalises to the schema of durable asymmetry. Section 10 states what the framework forgoes and what it leaves open.

2 SURVEY OF EXISTING FORMAL LANGUAGES

2.1 *Criteria of Comparison*

The languages surveyed here were built for different purposes and are not in competition. Comparing them requires an axis on which each can be placed without distortion. The axis used throughout is the pair consisting of the language's primitive object and its standing commitment: what the formalism takes as given, and what it thereby holds fixed. Every formalism must hold something still in order to let something else move, so the presence of a commitment is not itself a defect. What matters is where the commitment falls relative to the phenomenon at hand.

A useful diagnostic runs through the whole survey. In each canonical form below, some symbols carry a time index and others do not. Those that carry one are what the language permits to move; those that do not are the commitment. Reading the subscripts is sufficient to locate each language on the axis.

2.2 *Differential and Stochastic Languages*

Ordinary and partial differential equations, together with their stochastic and delayed variants, take a located point as primitive and evolve it under a law of motion. The canonical form is

$$\dot{x}(t) = f(x(t)), \quad x(t) \in M, \quad (1)$$

with the state space M and the vector field f both given in advance. Only x carries the index. Within that commitment these languages are powerful: they express continuous quantitative change, yield equilibria and their stability, and supply rates, which no other language on this list provides with comparable ease.

Composition lies outside what (1) can express. A trajectory records the positions a system has occupied, not the operations performed upon it or the order of their performance, and the two are not interchangeable. Let Φ_a and Φ_b denote the time-one flow maps of two distinct vector fields, representing two social operations. The inequality

$$\Phi_a \circ \Phi_b \neq \Phi_b \circ \Phi_a \quad (2)$$

does hold in general, so order-dependence is present. The inequality (2) holds, however, as a consequence of vector fields the modeller supplied, not as a structural feature the language represents. Nothing in (1) distinguishes an operation from any other displacement, and admissibility is not a structural feature of the language. One may of course restrict the domain of a flow map and thereby forbid a composition, and models routinely do so; the restriction is a stipulation added to (1) rather than something the form expresses, and it does not change as the system evolves. The problem of Section 1 is stated in exactly these terms. An act that consumes its own preconditions is an act after which certain further acts are unavailable, which is a claim about admissibility rather than about position.

A natural response brings the rules inside the state space. If an institutional arrangement governs which acts are available, one may write $M' = M \times R$, with R a space of arrangements, and evolve the pair. This is always possible and it does not help. The enlarged space M' has no natural coordinates on the R factor, an arrangement not being a quantity; the dimension of M' is not fixed, since arrangements bring distinctions into existence and retire others; and the enlargement discards the compositional structure that motivated it, leaving a point moving through a space whose axes cannot be named. The manoeuvre secures a formal representation at the cost of the explanation the representation was to supply.

2.3 *Hybrid and Piecewise Languages*

Hybrid systems extend the differential languages by admitting discontinuous transitions (van der Schaft and Schumacher, 2000). Leaky integrate-and-fire models, hybrid automata, impact mechanics and piecewise-smooth systems all take the form of a tuple

$$H = (M, f, G, R), \tag{3}$$

in which the state flows according to $\dot{x} = f(x)$ while $x \notin G$, and upon reaching the guard set G is assigned a new value by the reset map $R : G \rightarrow M$. The extension is genuine and handles a large class of phenomena that pure differential models handle badly.

Its commitment is easily missed. Of the four components of (3), none carries a time index. The reset map R is stipulated alongside the flow rather than generated by it, so a hybrid model contains a part the mathematics governs and a part the modeller supplies, and the supplied part does not itself change. The state space is likewise fixed across the jump: R maps into M , so the trajectory arrives somewhere else within the same space rather than somewhere the space did not previously contain. It is worth contrasting the alternative treatment in which no discontinuity appears at all, the steep but smooth conductance dynamics of Hodgkin and Huxley (1952) being the canonical instance; there the jump is an artefact of timescale rather than a feature of the model. Hybrid languages capture interruption without capturing structural change. For the present problem this is the wrong extension. The difficulty is not that social

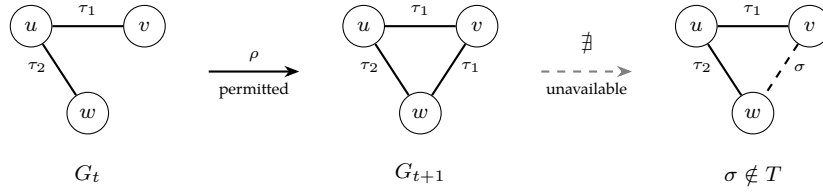


Figure 1: The standing commitment of network languages. The rewiring rule ρ may add an edge of a type already in T , shown at centre. A relation σ of a kind not in T , shown at right, cannot be brought into existence, because T carries no time index in (4).

change is discontinuous, which hybrid models would accommodate, but that the exercise of an asymmetry alters which exercises remain possible, which is a change in R rather than a jump under it.

2.4 Agent-Based and Network Languages

Agent-based and network models take the agent and the edge as primitive and permit the topology to change (Wasserman and Faust, 1994). Writing V for the set of agents and T for the set of relation types the model can represent, a configuration at time t is a graph

$$G_t = (V, E_t), \quad E_t \subseteq V \times V \times T, \quad (4)$$

evolved by a rewiring rule $\rho : G_t \mapsto G_{t+1}$. This is the first language on the list in which something structural moves, and it earns its place for that reason. Models of this kind capture emergent macro-patterns arising from local interaction with a fidelity the aggregate languages cannot match.

The subscript pattern in (4), shown in Figure 1, locates the commitment precisely. The edge set E_t carries an index and the type set T does not, and neither does ρ . A network model may therefore create an edge (u, v, τ) that did not previously exist, but τ must already belong to T : the new edge instantiates a relation the model already knew how to represent. A relation of a kind not in T cannot come into existence.

This distinction is not pedantic in the present context. When a certifying body is founded, the change is not that new links appear among existing nodes. A relation which previously had no representation, being certified by and being disqualified under, becomes available, and with it facts that were formerly not false but unformulable. The requirement is that T carry the index rather than E , and no network formalism supplies that.

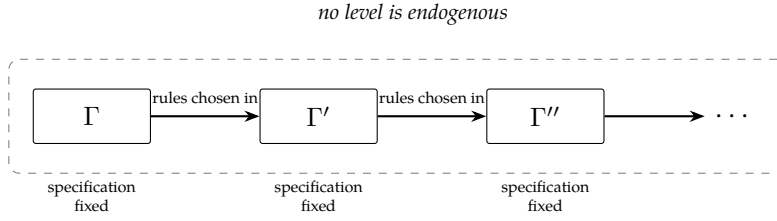


Figure 2: The meta-game regress. Each level treats the specification of the level below as an object of choice while fixing its own, so the commitment is relocated at every step and discharged at none.

2.5 Game-Theoretic and Mechanism-Design Languages

Games take the strategy set and the payoff function as primitive (Osborne and Rubinstein, 1994):

$$\Gamma = (N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N}). \quad (5)$$

These are the languages closest in spirit to the present problem, since they alone represent parties who anticipate one another's responses when choosing what to do. Rule-governed transformation is available elsewhere on this list; strategic anticipation is not. Extensive-form games represent sequence, so a limited compositional structure is available, and mechanism design goes further by treating the rules as objects of choice rather than as fixed background.

The commitment is the specification (5) itself, which enters as exogenous input. Within the intended scope this is not a defect, a mechanism designer being external to the mechanism by construction, but it locates rule-change outside the system rather than within it. The standard extension models rule-change as a meta-game

$$\Gamma' = (N, \{\Sigma_i\}_{i \in N}, \{u'_i\}_{i \in N}), \quad \Sigma_i \subseteq \{\Gamma_1, \Gamma_2, \dots\}, \quad (6)$$

whose players choose among institutional arrangements.

The manoeuvre relocates the commitment without discharging it, as Figure 2 records: the specification in (6) is itself fixed, and iterating produces Γ'' with the same property. No level of the regress is itself endogenous. The present problem requires that the alteration of the available operations be performed by those same operations, which the separation between player and designer forbids.

2.6 Rewriting Systems and Process Calculi

Rewriting systems take the production rule as primitive and are the nearest relatives of what this paper proposes. Formal grammars, term and graph rewriting (Rozenberg, 1997), Petri

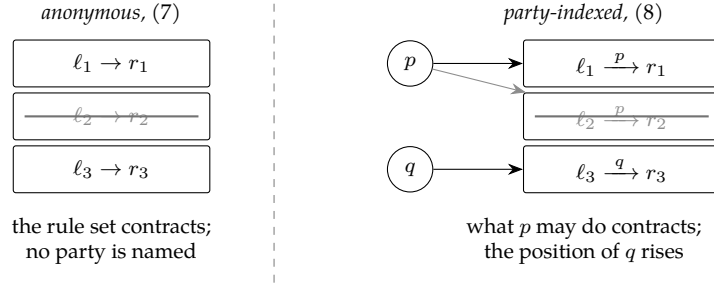


Figure 3: Deletion of a production under the two presentations. On the left the rule set contracts and the question of whose capacity was reduced has no formulation. On the right the same deletion is attributed, and the enlargement of every other party’s relative position becomes stateable.

nets (Reisig, 2013) and process calculi are all specified by a set of productions

$$\mathcal{R} = \{ \ell_k \rightarrow r_k \}_{k \in K}, \quad (7)$$

under which a configuration c transforms to c' whenever some ℓ_k matches a subterm of c . These languages generate rather than evolve: a configuration is a structured object rather than a point, and a rule transforms it into another. They carry a native notion of well-formedness, so not every sequence of transformations is admissible, and composition is therefore partial rather than total. All three of the requirements derived below are satisfied by rewriting systems, and any account of the present kind owes an explanation of why it does not simply use them.

The explanation lies in the anonymity of (7). A grammar rewrites strings, a graph rewriting system rewrites graphs, a process calculus rewrites process terms, and in each case the object transformed is syntactic while the rules apply wherever a pattern matches. Take the phenomenon of a constitutional prohibition, which removes an operation, say the suspension of an election, from what may be done. A graph rewriting system represents this as the deletion of a production from $\mathcal{R}$, and the representation is faithful as far as it goes. It leaves unstated the fact that makes the prohibition interesting: the operation was removed from one party while the possibilities of every other party were thereby enlarged. The rule set contracted, but a rule set belongs to no one, and the question of whose capacity was reduced and whose extended has no formulation in a language whose rules are anonymous.

Further encoding does not relieve this. One may attribute rules to parties by annotation, replacing (7) with

$$\mathcal{R}^* = \{ \ell_k \xrightarrow{p_k} r_k \}_{k \in K}, \quad p_k \in P, \quad (8)$$

where P is a set of parties and p_k names the party permitted to fire the production. The annotation is available and it is exactly the point. What (8) adds is a structure in which operations

are indexed by those who bear them, and that structure, rather than the rewriting, is where the content of the account lies. A system so annotated is on the way to becoming the object constructed in Section 5, and the difference between them is which structure is treated as primary. The account here treats party-indexed availability as the primary object and rewriting as what it undergoes. Nothing prevents a rewriting-theoretic presentation of the same content; the claim is that the content does not reside in the rewriting.

2.7 Operator-Algebraic Languages

Operator algebras take the composable operation as primitive (Bratteli and Robinson, 1987). A C^* -algebra may be specified by generators and relations,

$$\mathcal{A} = \langle G \mid R \rangle, \quad (9)$$

and a dynamics upon it is a one-parameter family of automorphisms

$$\alpha_t \in \text{Aut}(\mathcal{A}), \quad \alpha_s \circ \alpha_t = \alpha_{s+t}. \quad (10)$$

A trajectory, where one is wanted, is derived from the algebra rather than the reverse. The formalism is coordinate-free in the sense that it does not require the modeller to name variables, and non-commutativity is native, so the order-dependence that differential languages receive as input is here a structural feature of the object. These languages further distinguish, with a precision no other formalism on this list matches, the symmetries of a dynamics from those of the observables and from those of a state.

The commitment is visible in (9) and (10) taken together. The index sits on α , not on G or R . Passing from one representation to another, as the standard construction does by associating a representation to each state, leaves $\langle G \mid R \rangle$ untouched: it exhibits a single algebra realised in different ways, which is variability of realisation rather than of the algebra. Operator algebra fixes generators and relations in the way a differential model fixes coordinates, and both are dynamics upon a fixed structure. Because operator algebra comes nearest of any established language to what is required here, and because it is the language most likely to be reached for by anyone approaching this problem, the reasons it cannot serve are set out separately and at length in Section 3.

2.8 Derivation of the Requirements

Table 1 records the survey and Figure 4 displays its common shape. The pattern is consistent across the six entries: each language admits variability at one level and holds fixed the level immediately above. Differential models move a point and fix the space; hybrid models move a point discontinuously and fix the reset map; network models move an edge set and fix the type

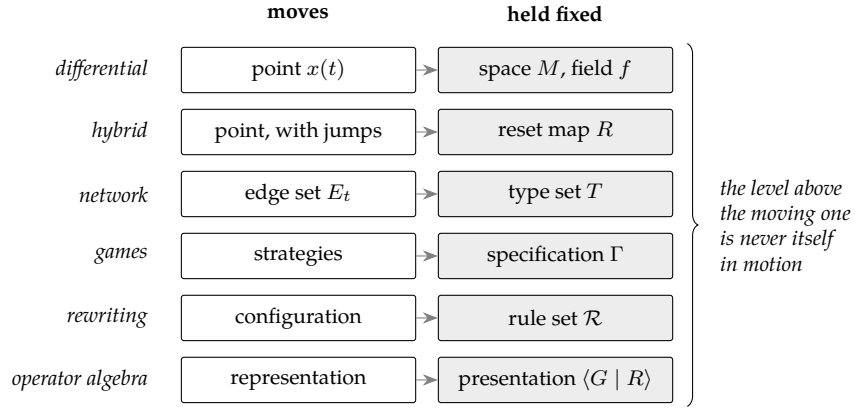


Figure 4: The common pattern of the survey. In every language the commitment sits one level above whatever is permitted to vary, so the grammar of the system is at no point the object that moves.

Language	Primitive	Held fixed	Unavailable for the present problem
Differential, stochastic	Point $x(t) \in M$	M and f	Composition; admissibility cannot be posed
Hybrid, piece-wise	Point with reset R	M, G, R	Reset stipulated, not generated
Agent-based, network	Edge in $V \times V \times T$	Type set T ; rule ρ	New instances only; T carries no index
Game-theoretic, mechanism design	Strategy set, payoff	Specification Γ	Endogenous rule change; the meta-game regress
Rewriting, process calculi	Production $\ell \rightarrow r$	Rule set $\mathcal{R}$; syntactic objects	Anonymous rules; whose capacity changed is unstateable
Operator-algebraic	Composable operation	Generators G , relations R	A fixed presentation is a fixed grammar

Table 1: Formal languages by primitive object and standing commitment. The final column states what each language cannot express about the reproduction of asymmetry, not a general verdict on its adequacy.

set; games move strategies and fix the specification; rewriting systems move configurations and fix the rule set, whose rules bear no parties; operator algebras move representations and fix the presentation. In no case does the grammar of the system, meaning what may be done, by whom, and in what combinations, become the object that moves.

Three requirements follow. The first is that the primitive be a composable operation rather than a located point, the phenomenon being stated in terms of what may follow what. The

second is that composition be order-sensitive natively rather than by stipulation, since the sequence in which operations are performed determines what remains available. The third is that composition be partial, since not every operation may follow every other, and the boundary between the admissible and the inadmissible is itself subject to change.

A fourth requirement is implied by the survey and met by no entry in it, and it is the one that motivates the construction of Section 5. Operations must be borne by parties. An operation available to one party and not to another is the elementary form of the asymmetry under study, and a language whose operations belong to no one cannot state the difference between an arrangement enlarging what one party may do and one enlarging what another may do. The contrast between (7) and (8) is the contrast at issue, and this fourth requirement is what separates the present account from rewriting systems, which satisfy the first three.

3 ASSESSMENT OF THE OPERATOR-ALGEBRAIC CANDIDATE

3.1 *Satisfaction of the Requirements*

Of the languages surveyed, operator algebra alone satisfies the first three requirements without amendment. Its primitive is a composable operation. Composition is order-sensitive natively, non-commutativity being the structural feature from which the subject takes its interest rather than a property added to it. And a C^* -algebra generated by a specified set of relations admits partiality in the sense that matters here: which products are non-zero, which operations annihilate one another, and which compositions are constrained is determined by the relations rather than stipulated case by case.

The temptation to proceed on this basis is considerable, and the reasons are worth stating plainly rather than dismissing. The precedent is the algebraic reformulation of quantum field theory due to Haag and Kastler (1964), in which the algebra of observables rather than any particular Hilbert space is taken as the primary object, and inequivalent representations are read as distinct phases rather than as competing descriptions. Operator algebra carries a developed theory of representation, so that one structure may be realised in many ways; a theory of inequivalence, so that realisations may be distinguished as belonging to genuinely separate sectors; a theory of subalgebras and the maps that project onto them; and a precise separation of the symmetries belonging to a dynamics, to the observables, and to a state. Each of these has an evident bearing on the questions of this paper, and Section 7 returns to three of them and puts them to use. What follows here is not an argument that operator algebra has nothing to offer the study of power. It is an argument that its axioms cannot carry the object this paper requires, and that the difference between borrowing a question and adopting a formalism is one the account must observe rather than blur.

Four obstructions are given. They are independent, and any one of them would suffice.

3.2 *Obstruction from the Involution*

A C^* -algebra is equipped with an involution, a map $a \mapsto a^*$ satisfying $(a^*)^* = a$ and reversing the order of products, $(ab)^* = b^*a^*$. The involution is not a decoration attached to an algebra that would otherwise be complete without it. It is what makes the notions of self-adjointness, positivity, and state available, and these in turn are what connect the algebra to anything that could be called an observable or a probability.

The involution admits no plausible social reading. Let a denote the suppression of a publication. The involution is not the inverse: suppression cannot be undone, and the algebra in any case does not require a to be invertible. Nor is it the adjoint in the sense of some inner product on social states, since no such product has been defined and defining one would import precisely the quantitative structure the relational account was meant to avoid. Nor is it a reversal of the parties, exchanging who suppresses and who is suppressed, since that operation does not satisfy the required identities and in most cases does not exist at all. The candidates are exhausted without a plausible reading having been found.

The point generalises beyond a single unfortunate example. A great many social operations are irreversible in a strong sense: they alter what is available afterward in ways that no subsequent operation restores. An operation of this kind has no natural partner under an order-reversing involution because the structure that would pair it with another has been dissolved by its own performance. One may of course impose an involution formally, declaring $a^* := a$ for every generator and verifying that the axioms hold. This yields a well-defined algebra and no interpretation: the resulting self-adjointness of every element would assert, if it asserted anything, that every social operation is an observable, which is false in an obvious way and useless in a subtle one.

3.3 *Obstruction from the C^* -Identity*

Suppose the involution problem were somehow discharged. There remains the identity from which the subject takes its name, $\|a^*a\| = \|a\|^2$, and it is not one axiom among several. Its consequence is that the norm on a C^* -algebra is determined by the algebraic structure (Bratteli and Robinson, 1987, Ch. 2). An algebra admits at most one norm satisfying it, so the metric and the algebraic content are not independent pieces of data but two aspects of a single object. Everything rigid in the theory descends from this. The uniqueness of the norm gives the automatic continuity of homomorphisms; that gives the isometry of injective homomorphisms; and the structure theorems that make the subject powerful follow in turn.

For a social repertoire of operations no such norm is available. The quantity $\|a\|$ would have to measure the magnitude of a suppression or the size of an act of certification, and no candidate suggests itself that is not either arbitrary or a smuggled-in resource quantity. A resource quantity is what the account set out to displace. Nor would an arbitrary choice

help, since the identity constrains the norm rather than merely accompanying it: a norm not satisfying it yields a Banach $*$ -algebra, and the structure theory does not apply to Banach $*$ -algebras.

The consequence deserves to be stated without softening. An account that calls itself C^* -algebraic while supplying neither a norm nor the identity has borrowed the name without the structure the name denotes. It possesses an associative algebra with a formal involution, which is a considerably weaker object than the one whose reputation it has invoked, and the results that depend on the norm, which is to say the greater part of the structure theory, are unavailable to it.

3.4 *Obstruction from Typing*

An algebra is a structure with one object: every element may be multiplied by every other, and the product is again an element of the same algebra. Partial vanishing is expressible, but the ambient totality is not optional, composition being defined throughout whatever its value.

The social composites this paper is concerned with are not of that form. The sequence by which a durable asymmetry is assembled, treated at length in Section 9, consists of a differentiation that produces variation among parties, a selection that ranks the variation, a normalisation that renders the ranking unremarkable, and a reproduction that transmits it. The first takes a distribution and yields a distribution with greater variance. The second takes variance and yields an ordering. The third takes an ordering and yields a meaning, namely the ordering as it is understood, which is an object of a different kind from the ordering itself. The fourth takes a meaning together with an ordering and yields their transmission across a generation. These operations have distinct domains and distinct codomains, and their composite is well-formed only in one direction. The reverse composites are not merely uninteresting; most of them are undefined, there being no sense in which a normalisation may be applied to an undifferentiated population.

A structure whose operations carry domains and codomains of different kinds, in which composition is defined exactly when the codomain of one matches the domain of the next, is a category and is not an algebra (Mac Lane, 1998). An algebra is the degenerate case in which every domain and codomain coincide. Forcing the social composite into that case requires one of two moves. The first discards the typing, and with it the distinction between a distribution and a meaning that made the composite interesting. The second reintroduces the typing through auxiliary projections, which reconstructs the categorical structure inside the algebra while retaining the advantages of neither.

3.5 *Obstruction from Reversibility*

The three obstructions above are technical. The fourth is not, and it would remain even if the others were somehow answered.

The symmetries of an operator algebra are its automorphisms, and a dynamics is a group of them. The group structure is essential rather than conventional: a time evolution in this setting is a family of maps each of which has an inverse, and the framework was built for physical systems where this is appropriate. Irreversibility enters the theory, when it enters at all, as a derived phenomenon: a semigroup of completely positive maps describing an open system, or a modular flow associated with a state. Both are constructed atop a reversible foundation rather than replacing it.

The primitive of the present account is generation, and generation does not invert. A distinction brought into existence is not removed by any subsequent operation, though it may be superseded; an institution once founded leaves a residue in what remains formulable even after its dissolution; and the whole interest of the mechanism described in Section 4 lies in the asymmetry between an exercise that consumes its conditions and one that manufactures them, which is a statement about a direction that cannot be run backward. To build such an account on a foundation whose basic maps are invertible is to place the phenomenon in the derived layer and the idealisation in the primitive layer, which inverts the order the subject matter requires.

3.6 *Grounds for Constructing a New Object*

The requirements derived in Section 2 survive. The candidate does not. Operator algebra fixes generators and relations in the same way, and for the same kind of reason, that a differential model fixes coordinates: both are languages for dynamics upon a fixed structure, and the object of this paper is the dynamics of the structure.

What is needed, accordingly, is an object meeting the three requirements of Section 2 together with the fourth, that operations be borne by parties, and admitting irreversible transformation of the operational field as its basic move rather than as a derived special case. Section 5 builds it. Before that, and deliberately before any formal apparatus is introduced, Section 4 states in ordinary language the mechanism the construction is meant to capture. The order matters. A reader who declines the construction should still be in a position to accept or reject the mechanism, and a mechanism stated only in the vocabulary of its own formalism cannot be assessed independently of it.

4 THE RELATIONAL MECHANISM OF POWER

This section states the mechanism the remainder of the paper formalises. It does so without formal apparatus and without the vocabulary introduced in Section 5, and the omission is

deliberate. A mechanism stated only in the terms of its own formalism cannot be assessed independently of that formalism, and a reader who finds the construction of the following section unpersuasive should still be in a position to accept or reject what is claimed here. The argument proceeds in seven steps, of which the second and the sixth are where most accounts of power move too quickly.

4.1 From Change to Difference

The account begins from a commitment about what social reality consists in. Relations are not a static web upon which events occur; they are continually produced, and their production is what social life is. On this view change is not something that befalls an otherwise stable arrangement but the ordinary condition of the arrangement, which persists only insofar as it is reproduced.

A consequence follows immediately. Where relations are continually produced, they are not produced identically. Production under varying circumstance yields variation, so difference arises wherever relation is generated, and it arises constantly rather than exceptionally. Two parties to a relation come to differ in what they have done, in what has been done to them, in what they are positioned to do next. This much requires no argument beyond the commitment itself, and it is the starting point rather than a result.

4.2 From Difference to Asymmetry

The step from difference to asymmetry is where accounts of power most often move too quickly, and the distinction matters for what follows.

Two parties differing is not yet an asymmetry between them. A difference is symmetric in the relevant sense: each party has something the other lacks, and nothing in the bare fact of difference determines which lack is consequential. For an asymmetry to obtain, the difference must be ordered, so that what one party has confers a capacity that what the other has does not. The ordering is a further fact and not a restatement of the first.

Nothing in the production of difference supplies the ordering. Change generates variation; it does not rank the variation it generates. The ranking is imposed by whatever selects among differences: a market that prices one skill and not another, a court that recognises one claim and not another, a professional body that certifies one training and not another. This selection is an operation in its own right, distinct from the differentiation that preceded it, and it is treated as such in the schema of Section 9.

The consequence is that an account deriving power directly from change is incomplete. It has passed over the step at which difference becomes consequential, and that step is where a good deal of what is interesting about power resides. Where an asymmetry is contested, the contest is very often not over the difference but over the ordering: not whether two parties

differ, which is rarely in doubt, but whether the difference should confer what it presently confers.

4.3 Exercise and Its Own Conditions

An ordered difference makes certain things possible for one party that are not possible for the other. This is what it is for an asymmetry to exist rather than merely to be recorded.

The exercise of such a possibility has two effects rather than one. The first is the effect intended: something is done, and the situation of the parties is altered accordingly. The second effect is the one this paper is concerned with. The alteration includes what is thereafter available, so the exercise of an asymmetry is also an intervention in the conditions of its own exercise. Every act performed in virtue of an asymmetry adjusts the standing of the asymmetry that permitted it.

The adjustment carries no fixed sign. An act may leave the conditions of its own repetition intact, or diminished, or enhanced, and which of these obtains is a fact about the act and its setting rather than about the magnitude of the advantage that licensed it. It is this variability of sign, rather than any variation in size, that the account takes as the locus of the phenomenon.

4.4 The Ordinary Case of Self-Consumption

Most asymmetries do not survive their own exercise. This is worth stating as the default rather than as an interesting exception, because the reverse assumption is what makes power appear more puzzling than it is.

A military advantage is exercised by campaigning, and campaigning consumes the fiscal capacity, the demographic base, and the willingness that underwrote the advantage. A seizure of authority is exercised by suspending the procedures under which authority was previously conferred, and the suspension destroys the source from which the seizure drew whatever recognition it had. A monopoly is exercised by pricing, and pricing at monopoly levels invites the entry that ends the monopoly. In each case the act is effective and the advantage is real; what fails is the reproduction of the conditions that made the act available.

Asymmetries of this kind arise constantly and leave little trace. They are not failed power, or weak power, or power in an early stage. They are asymmetries that were exercised and thereby dissolved, which is the ordinary fate of an ordered difference in a system where relations are continually produced.

4.5 Persistence as the Mark of Power

A minority of asymmetries behave otherwise. Their exercise reproduces the conditions of their exercise, so that the advantage stands after use as it stood before, and often more securely.

A credential is exercised by being required. Each occasion on which it is required confirms that the distinction it marks is a real one, and the confirmation is what a credential consists in. A bureaucratic procedure is exercised by being followed, and each following establishes the procedure more firmly as what is done. A norm is exercised by being invoked, and invocation is how a norm acquires the standing that makes invocation effective. In these cases the act does not draw down a stock; it deposits.

Power, on the account offered here, is the asymmetry that survives its own exercise. Survival is the mark rather than the substance: what makes an arrangement a power relation is the structural condition described in the following subsection, and persistence is how that condition shows itself. The formulation is deliberately narrow. It does not measure how much a party can compel, or how large the advantage is, or how many others are subject to it. It asks a single question about the relation between an act and what remains available afterward, and it counts as power only what answers that question in one particular way.

Two things follow that recommend the definition. It classifies the cases of Section 1 correctly, assigning the thin administration and the certifying body to power and the overwhelming military advantage and the dominant market share to the ordinary case, which resource accounts cannot do. And it does not require an arrangement to be intended, designed, or understood by anyone in order to count. What reproduces itself reproduces itself whether or not anybody planned that it should.

4.6 Reconstruction of the Self-Maintenance Thesis

A formulation recurs across otherwise unrelated accounts: that power consists in its own maintenance, that what power is about, beneath whatever else it is about, is its going on being power. The reading given to it here differs from the readings usually offered, and the differences are worth setting out, since the thesis is older and better defended than the account of this paper.

The formulation originates with Nietzsche (2002), and it is commonly attributed to him in a form he explicitly rejected. Nietzsche does not hold that life or power aims at self-preservation. He holds the reverse: that a living thing seeks above all to discharge its strength, that self-preservation is among the indirect and frequent consequences of this rather than its object, and that treating preservation as the cardinal drive is to indulge a superfluous teleological principle, a charge he lays against the conatus of Spinoza. The formulation of the thesis is therefore not preservation but discharge and extension, with persistence following as a by-product. The account developed here departs from this and the departure should be recorded rather than concealed.

Two later positions retain the self-maintenance without the will. Luhmann (1979) treats power as a symbolically generalised medium of communication rather than as a capacity held by anyone, and Luhmann (1995) develops the autopoietic account on which social systems

reproduce themselves from their own elements, so that a legal or political order sustains itself through recursive operation rather than through any external warrant. Foucault (1978) and Foucault (1977) make power immanent to the relations it obtains in and existent only in exercise, and add a requirement of some importance: a power relation requires that the party subject to it be maintained throughout as one who acts, since a party reduced to no capacity at all is no longer a party to a relation but an object of force.

The reading offered here is that self-maintenance is constitutive rather than purposive. An asymmetry is not a state that happens to endure; it exists only in exercise, and exercise is what alters the conditions of exercise, so the relation between an asymmetry and its own continuation is not causal but definitional. What is called power is the case in which that relation closes. This preserves what Nietzsche saw, that the continuation of power is not incidental to power, while declining the modality in which he saw it. No aim is attributed to any arrangement, and nothing is said to seek anything.

A structural precedent for a self-maintaining process without an aim exists outside the study of power. In Lacan (2006), desire is constituted by a lack that demand cannot meet, so that desire sustains itself precisely by remaining unsatisfied and its apparent object functions as a vehicle for its continuation rather than as a terminus. The parallel holds in three respects. Both are constituted by a process rather than by a substrate; both would end in their own satisfaction, which is why Foucault's requirement that the subordinate party remain one who acts has a counterpart in the analytic setting; and in both, an appearance of purposiveness arises from the structure with no aim present. That the same form appears in a domain unconnected with social asymmetry is some evidence that it belongs to self-constituting processes generally rather than to power in particular.

The disanalogy is where the present account owes work that the analytic one does not. Desire's self-maintenance is structurally guaranteed, demand being unmeetable, so no explanation is required of why desire persists. Power's self-maintenance is neither guaranteed nor common: the overwhelming majority of ordered differences dissolve, and an account on which power maintains itself owes an explanation of why maintenance is so rarely achieved. Nietzsche has an answer available here and it is one this account rejects. On his view the failure to persist is weakness, an inadequate discharge of strength; the strong have no need to concern themselves with preservation and it is the fearful who pursue it. The cases of Section 1 do not support that reading. An advantage may be exercised vigorously, effectively, and without any deficiency of will, and consume its own conditions precisely through the vigour of its exercise. Self-consumption is not a defect of the party but a property of the relation between an act and what it leaves available.

The explanation offered instead is the generative commitment with which Section 4 began, and it is the contribution of the present framework to a thesis several traditions have reached independently. Because relation is continually produced, difference is continually produced

with it, and ordered difference arises constantly and cheaply. The field does not stand still beneath an asymmetry: new difference is generated, new orderings become available, and the arrangement that was consequential yesterday is unsettled by what has been produced since. Nietzsche saw the same background and read it as conquest, holding that whatever exists is repeatedly reinterpreted to new ends and redirected by some power superior to it, with each subduing obscuring the meaning that went before (Nietzsche, 1998). The present account keeps the restlessness and drops the hierarchy: what unsettles an arrangement need not be a superior power, and is more often the ordinary productivity of relation itself. Maintenance is therefore work performed against a productive background rather than persistence in a static one, and its rarity follows from the abundance of what it must be maintained against. Neither the systems-theoretic nor the analytic account supplies this, the first because autopoietic reproduction is the definition rather than the achievement, the second because desire faces no competition from what it does not desire.

Two consequences follow for what may be claimed. The first is that asymmetry is not sufficient for power, and the insufficiency is not an independent stipulation but a consequence of generativity: were asymmetry scarce it might have sufficed, and it is scarce nowhere. The second is that the observed population of power relations is the maintained population, so that any description of an arrangement is a description of what has so far been maintained. This is a real epistemic constraint and it is not the whole of the account. The constitutive claim concerns what power is; the observation concerns what is available to be studied.

4.7 Generation and Degeneration

A last distinction is needed, and it is stated here qualitatively.

That an asymmetry reproduces itself says nothing about how the reproduction is achieved. Two arrangements may both survive their own exercise and differ entirely in what the survival costs the parties to them. A certifying body persists by making a distinction that others find worth having, and the persistence is purchased by enlarging what those others can do: certified practitioners can undertake what they could not undertake before, and those who deal with them can rely on what they could not previously rely on. A system of exclusion may equally persist, and its persistence is purchased by contracting what the excluded can do, each exercise removing a further possibility from those it acts upon.

Both are power under the account given above, and the account is not thereby defective. What separates them is not whether the asymmetry survives but at whose expense. Where the survival of an asymmetry is sustained by enlarging the possibilities of those it acts upon, the arrangement is generative; where it is purchased by contracting them, the arrangement is degenerative. The two are values of a single condition rather than two conditions, which is why the definition of power does not require amendment to accommodate them.

The distinction is easy to state and easy to state wrongly. It is tempting to make it a matter

of how many possibilities the arrangement admits in total, so that generative arrangements are the permissive ones and degenerative arrangements the restrictive ones. Section 8 shows that this fails in both directions, and that the criterion cannot be recovered by counting anything.

5 CONSTRUCTION OF THE FORMAL OBJECT

The object constructed here is intended to satisfy the four requirements of Section 2 and to state the mechanism of Section 4 precisely. It is assembled in stages: first the collection of available operations, then the typing that makes composition partial, then the transitions that operations induce upon the collection, and finally the presentation that holds the two kinds of movement together.

Two things should be said at the outset. The construction is not a new branch of mathematics and does not claim to be. Its ingredients are standard, and the contribution lies in which structure is treated as primary rather than in any technical novelty. Nor does the construction yield a predictive apparatus; what it yields is a vocabulary in which the distinctions of Section 4 can be stated without ambiguity, together with the conditions under which those distinctions are well defined.

5.1 Repertoires

Let P be a set of parties. A *repertoire* is a collection of operations, each of which is borne by a party and each of which acts upon the relational field the parties inhabit. Writing $\mathcal{A}$ for a repertoire and $a \in \mathcal{A}$ for an operation, every operation carries an attribution

$$\pi : \mathcal{A} \longrightarrow P, \quad (11)$$

so that $\pi(a)$ names the party for whom a is available. The attribution map is what discharges the fourth requirement, and it is the feature that distinguishes a repertoire from a rule set in the sense of (7). The fibre

$$\mathcal{A}_p = \pi^{-1}(p) \subseteq \mathcal{A} \quad (12)$$

collects what is available to the party p , and it is at the level of these fibres, rather than at the level of $\mathcal{A}$ as a whole, that the distinctions of Section 4 are stated.

An asymmetry between two parties is now expressible directly. Where $\mathcal{A}_p$ and $\mathcal{A}_q$ differ, the parties differ in what is available to them, which is the difference of Section 4. The ordering that converts difference into asymmetry is not supplied by (12) and must be added; this is taken up in Section 6, and its absence here is deliberate, the point of Section 4 having been that the ordering is a separate fact.

5.2 Typed Partial Composition

Operations act upon relational facts of different kinds. A differentiation acts upon a distribution and yields a distribution; a selection acts upon a distribution and yields an ordering; a normalisation acts upon an ordering and yields a meaning. To record this, let $\mathcal{O}$ be a collection of *registers*, each register being a kind of relational fact, and equip every operation with a domain and a codomain,

$$a : X \longrightarrow Y, \quad X, Y \in \mathcal{O}. \quad (13)$$

Composition is then defined exactly when the types agree:

$$b \circ a \text{ is defined } \iff \text{cod}(a) = \text{dom}(b). \quad (14)$$

This discharges the third requirement without stipulation. Partiality is not a constraint imposed upon a total operation but a consequence of the operations having types at all, and the inadmissible compositions are inadmissible because they are not well formed rather than because a rule forbids them. Order-sensitivity follows in the same way: where both $b \circ a$ and $a \circ b$ are defined, they are in general distinct, and where only one is defined the asymmetry of order is absolute rather than a matter of degree.

A repertoire equipped with (13) and (14) is a category whose objects are registers and whose arrows are operations, together with the attribution (11). It will be convenient to write $\mathcal{A}$ for this whole structure and to call it a repertoire without further qualification.

5.3 Induced Transitions

The mechanism of Section 4 turns on the second effect of an exercise: that performing an operation alters what is thereafter available. This is recorded by letting each operation induce a transition of the repertoire itself.

Write $\mathfrak{R}$ for the collection of repertoires over the party set P and register collection $\mathcal{O}$. Each operation $a \in \mathcal{A}$ determines a map

$$\Theta_a : \mathcal{A} \longrightarrow \mathcal{A}', \quad (15)$$

where $\mathcal{A}' \in \mathfrak{R}$ is the repertoire in force after a has been performed. The map Θ_a may add operations, remove them, or reattribute them from one party to another, and these are the three ways in which the exercise of an asymmetry alters the conditions of its own exercise.

Not every transition is induced by an operation an asymmetry licenses. Because relation is continually produced, the repertoire is also carried forward by what happens independently of any given arrangement: parties enter and leave, new difference arises among them, and new orderings become available by which difference may be made consequential. Write

$$\Delta : \mathcal{A} \longrightarrow \mathcal{A}^\Delta \quad (16)$$

for a transition of this second kind, and call it a *background transition*. The distinction between (15) and (16) is not one of mechanism, both being transitions of the same structure, but of provenance: the first is induced by the exercise of an asymmetry and the second is not. Background transitions may enlarge the register collection, admit operations no party previously held, or supply orderings under which differences that were inconsequential become consequential. They are the formal counterpart of the productive background described in Section 4, and Section 6 shows that the distinction between an asymmetry that merely closes upon itself and one that is maintained turns on them.

Three further features of (15) deserve emphasis, since they are what the languages of Section 2 could not supply.

The transition is induced by the operation rather than stipulated alongside it. There is no external rule playing the part of the reset map in (3); Θ is determined by the operation, and an operation is not fully specified until its induced transition is given.

The target $\mathcal{A}'$ need not have the same registers as $\mathcal{A}$. A transition may enlarge $\mathcal{O}$, which is how a relational fact that was previously not merely false but unformulable comes into existence, and it is what the fixed type set T of (4) forbids.

The attribution is what changes. Since Θ_a acts on a structure carrying (11), its effect is stated fibrewise: Θ_a may contract $\mathcal{A}_p$ while enlarging $\mathcal{A}_q$, and this is a well-formed description of what a prohibition does. The corresponding description is unavailable in (7), as Figure 3 recorded.

5.4 The Double-Category Presentation

Two kinds of movement have now been introduced. Operations compose with one another within a repertoire, and operations induce transitions between repertoires. These are movements of different kinds and they are not interchangeable, but neither are they independent: an operation performed inside a repertoire is the very thing that carries the repertoire to its successor.

The structure holding both is a double category, in the sense introduced by Ehresmann (1963). Its objects are registers. Its horizontal arrows are operations, composing according to (14). Its vertical arrows are repertoire transitions, composing by performance in sequence. Its cells are squares

$$\begin{array}{ccc} X & \xrightarrow{a} & Y \\ \downarrow \theta & & \downarrow \theta' \\ X' & \xrightarrow{a'} & Y' \end{array} \quad (17)$$

recording that the operation a , performed under the transition θ , appears as a' in the successor repertoire.

The commuting square is the point of the presentation rather than a technical convenience.

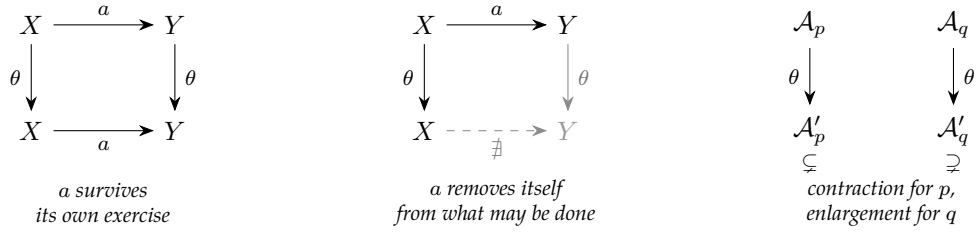


Figure 5: Cells of the double category. On the left an operation whose induced transition leaves it available, which is the formal counterpart of persistence. At centre an operation absent from the successor repertoire, which is self-consumption. On the right the fibrewise effect of a transition, which the attribution map (11) makes stateable and (7) does not.

An operation that rewrites the conditions of its own availability is a single object in this setting, not two coupled stories about the same event, because (17) states the horizontal fact and the vertical fact as one cell. The failure modes of Section 4 become properties of such cells. Where a' does not exist, the operation has removed itself from what may be done, which is self-consumption. Where a' exists and is again a , the operation has left its own availability intact.

5.5 Irreversibility of the Vertical Arrows

A final stipulation completes the construction, and it is the one that separates this object from the operator-algebraic candidate assessed in Section 3.

The vertical arrows are not required to be invertible, and in the intended reading they are typically not. A transition $\theta : \mathcal{A} \rightarrow \mathcal{A}'$ has no assumed partner $\theta^{-1} : \mathcal{A}' \rightarrow \mathcal{A}$. This is not an omission to be repaired in a fuller treatment. Generation does not invert: a register brought into existence is not removed by any subsequent transition, though it may cease to be inhabited; an attribution once contracted is not restored by running anything backward; and the whole content of Section 4 lies in a directional difference between exercises that deposit and exercises that draw down.

The vertical arrows accordingly form a category and not a groupoid, and the dynamics of the object is not a group action. Where the automorphism family (10) placed reversibility in the primitive layer and irreversibility in the derived one, the present construction places irreversibility in the primitive layer. Reversible transitions are the special case, recovered when θ happens to be invertible, and nothing in the framework privileges them.

Two properties follow that are worth recording, since they are used later. A transition may fail to be surjective on operations, which is how possibilities are lost, and it may fail to be injective on registers, which is how distinctions collapse. Section 7 takes up the first of these under the heading of institutional filtering, and the second is the formal counterpart of the collapse of distinction familiar from accounts of relational degradation.

6 FORMALIZATION OF THE MECHANISM

The mechanism of Section 4 is now stated in the vocabulary of Section 5. The order of the two sections was chosen so that this one adds precision rather than content, and the reader should be able to check that nothing new is claimed here beyond what was claimed there.

6.1 Ordering as Added Structure

Section 5 supplies difference between parties without supplying asymmetry. Where the fibres $\mathcal{A}_p$ and $\mathcal{A}_q$ of (12) fail to coincide, the parties differ in what is available to them, and nothing in the construction determines which of the two is thereby advantaged. This corresponds exactly to the step of Section 4 at which difference is not yet asymmetry, and the correspondence is deliberate: an object that delivered asymmetry directly from difference would have built in the step the account holds to be separate.

The ordering is added as follows. Let $\preceq$ be a partial order on the fibres of a repertoire, and write

$$\alpha = (p, q, \preceq), \quad \mathcal{A}_q \prec \mathcal{A}_p, \quad (18)$$

for the assertion that p stands above q under it. The relation $\preceq$ is not derived from set inclusion, and this matters. That $\mathcal{A}_q \subsetneq \mathcal{A}_p$ neither implies nor is implied by $\mathcal{A}_q \prec \mathcal{A}_p$: a party may have fewer operations available and stand higher, as a sovereign bound by few procedures and empowered by them illustrates, and a party may have more operations available and stand lower, as anyone whose many permitted actions are all inconsequential illustrates. Counting operations does not order fibres, and the independence of $\preceq$ from cardinality recorded here is what Section 8 generalises.

What supplies $\preceq$ is the selection of Section 4: a market, a court, a certifying body, some operation that ranks differences and thereby makes them consequential. In the schema of Section 9 this is the second register, and its status as a distinct operation rather than a derived quantity is the formal counterpart of the philosophical point that change generates variation without ranking it.

6.2 Closure Under Self-Induced Transition

An asymmetry α in a repertoire $\mathcal{A}$ licenses a set of operations, namely those available to p and not to q under the ordering:

$$L(\alpha) = \{ a \in \mathcal{A}_p : a \notin \mathcal{A}_q \}. \quad (19)$$

Each such operation induces a transition by (15), and the composite of the licensed operations performed in sequence induces

$$\Theta_{L(\alpha)} : \mathcal{A} \longrightarrow \mathcal{A}', \quad (20)$$

carrying the repertoire to its successor. The asymmetry α has an image α' in the target of (20) whenever the parties, the fibres, and the ordering all survive the transition.

Definition. *An asymmetry α is closed when the transition induced by the operations it licenses returns it:*

$$\Theta_{L(\alpha)}(\alpha) = \alpha.$$

Closure is the formal counterpart of the constitutive claim of Section 4: the exercise of the asymmetry is also an intervention in the conditions of that exercise, and in the closed case the intervention returns what it acted upon. Three points about the form of the condition are worth making explicit, since each answers an objection that would otherwise stand.

The condition is on the asymmetry and not on the repertoire. A repertoire may be reproduced entire without any asymmetry being reproduced, since a repertoire admits fixed points in abundance whether or not any asymmetry obtains within it. The stable egalitarian arrangement is therefore excluded by the condition rather than by a clause appended to it. Two grounds of exclusion should be distinguished. Where all fibres coincide, no difference obtains and α has no subject. Where fibres differ but $\preceq$ ranks none of them above another, difference obtains and α is undefined for want of an ordering. Both cases fall outside the condition, and the second is the one that matters, since egalitarian arrangements are rarely uniform and are characterised by the absence of consequential ranking rather than by the absence of difference.

The condition quantifies over the licensed set (19) rather than over all operations. An arrangement is not disqualified because some unrelated operation would disturb it; what is asked is whether the exercise of the advantage sustains the advantage. This is what makes the condition a statement about self-reference rather than about stability in general.

The condition is silent on magnitude. No cardinality, measure, or weight appears in it. Two asymmetries may differ by any amount in what they permit and stand alike under it, which is the intended consequence: the account holds that the durability of an asymmetry is not a function of its size, and a condition mentioning size would have contradicted that at the outset.

6.3 Maintenance Under Background Transition

Closure is not yet sufficient for power, and the reason is the one Section 4 gave. An asymmetry that returns itself under its own exercise has been tested against one thing only, namely itself. The field meanwhile does not stand still. Because relation is continually produced, background transitions of the form (16) carry the repertoire forward independently: parties enter, difference accumulates, and orderings become available under which what was inconsequential becomes consequential. An asymmetry closed under (20) may nonetheless fail to survive what the field has done in the interval.

Definition. *An asymmetry α is maintained when it is returned by its own induced transition composed with the background transitions the field supplies, that is, when*

$$(\Theta_{L(\alpha)} \circ \Delta)(\alpha) = \alpha$$

for the background transitions Δ admissible in the setting under consideration. Power is maintenance rather than closure.

The strengthening is what the generative commitment requires, and three consequences follow from it.

The distinction between closure and maintenance separates two cases that the earlier condition could not tell apart. An asymmetry may be closed and unmaintained, which is the position of an arrangement that reproduces itself perfectly under its own exercise and is undone by what arises elsewhere. The military advantage of Section 1 is of this kind: campaigning may reproduce the command structure that campaigning requires, and the coalition that forms in response was produced by no operation the advantage licensed. Nothing internal to the arrangement failed. What defeated it was the field's own productivity, and an account with only the closure condition would have to record the case as inexplicable.

Maintenance is a modal condition where closure is a factual one. Closure asks what one transition does; maintenance asks what would happen under the perturbations the setting admits. This is the formal difference between an arrangement that has persisted and one that is disposed to persist, and it is why the account can speak of a tendency without attributing an aim: a disposition is not a purpose, and a modal claim about behaviour under perturbation carries no teleology whatever.

The quantifier over Δ is where empirical content enters, and its scope must be specified rather than left universal. Nothing is maintained against every conceivable perturbation, and an asymmetry required to survive arbitrary background transition would be an asymmetry no arrangement satisfies. What is at issue is the class of background transitions a given setting actually supplies, and this is a matter of fact about the setting rather than something the framework determines. Section 10 records the consequence: the account states the condition and does not decide, for any actual arrangement, which perturbations it must withstand.

6.4 The Two Modes of Failure

The complementary cases are the ones Section 4 identified as ordinary, and the strengthened condition distinguishes two of them where the closure condition alone distinguished one.

An asymmetry is *self-consuming* when its own licensed operations induce a transition under which it fails to return, either because those operations are no longer available,

$$L(\alpha) \not\subseteq \mathcal{A}', \tag{21}$$

or because the ordering (18) does not survive, the parties having ceased to stand in the relation that made the operations consequential. Closure fails, and it fails from within. The centre panel of Figure 5 depicts the case (21). The seizure of authority that suspends the procedures conferring authority is of this kind, and so is the advantage whose exercise exhausts the base that produced it.

An asymmetry is *unmaintained* when it is closed but not maintained: its own exercise returns it, and it does not survive composition with what the field supplies meanwhile. Nothing internal to the arrangement has gone wrong. What has happened is that new difference has arisen, or a new ordering has become available, or parties have entered who stand outside the relation, and the arrangement that reproduced itself perfectly a moment ago no longer finds the conditions it reproduced.

The second mode is the one the account most wants to record, and it bears directly on the reading of Section 4. On a view that treats failure to persist as inadequacy in the party holding the advantage, the unmaintained case must be assimilated to the self-consuming one and read as a defect of exercise. The distinction drawn here denies that assimilation. An arrangement may be exercised with complete competence, return itself under every operation it licenses, and be undone by a productivity that no operation of its own occasioned. Weakness is not the explanation, and vigour would not have been the remedy.

A further observation follows, and it is epistemic rather than constitutive. Under repeated exercise and repeated background transition, asymmetries failing either condition are absent from the repertoires that succeed, while those meeting both remain. Any description of an arrangement made at a later stage is therefore a description of a repertoire in which the failed asymmetries no longer figure, and the population available for study is the maintained population. This explains something about the evidence rather than about the phenomenon: what power is, is stated by the condition of Section 6; what we are in a position to observe, is stated here. Conflating the two would make the account trivial, since it would reduce the claim that power maintains itself to the observation that what remains has remained.

The account does not predict which asymmetries in a given repertoire will prove to be maintained, and Section 10 takes up the reasons. What it supplies is a condition under which the question is well posed, together with the finding that the answer is not read off the size of the advantage.

6.5 The Generative and Degenerative Parameter

The definition given above is satisfied by arrangements that differ in what their persistence costs the parties subject to them, and Section 4 distinguished these qualitatively. The distinction is now stated fibrewise.

Let $\alpha = (p, q, \preceq)$ be a power relation, and let $\Theta = \Theta_{L(\alpha)}$ be the transition it induces. The

relevant comparison is between the fibre of the subordinate party before and after:

$$\mathcal{A}_q \text{ against } \mathcal{A}'_q = \Theta(\mathcal{A})_q. \quad (22)$$

Definition. *A power relation α is degenerative when its maintenance is purchased by contraction of the subordinate fibre, and generative when maintenance is sustained while that fibre is enlarged.*

The right-hand panel of Figure 5 depicts the fibrewise comparison, and the attribution map (11) is what makes (22) a well-formed expression. In a formalism whose operations belong to no one, as in (7), the comparison could not be written down, and this is the point at which the fourth requirement of Section 2 earns its place.

Two features of the definition deserve comment.

Generation and degeneration are values of one condition rather than two conditions. Both presuppose maintenance, and they differ in the fibrewise sign of the transition that sustains it. This is why the definition of a power relation required no amendment to accommodate them, and why an arrangement cannot be generative without being a power relation in the first place. A transient enlargement of the subordinate fibre, not sustained by any self-reproducing asymmetry, is not generative on this account; it is simply an event.

The comparison in (22) is fibrewise and not aggregate. What is compared is $\mathcal{A}_q$ with $\mathcal{A}'_q$, not $\mathcal{A}$ with $\mathcal{A}'$, and the two comparisons can run in opposite directions. A transition may contract the repertoire as a whole while enlarging the subordinate fibre, and the definition attends to the second and disregards the first. Section 8 shows that this is not a technicality but the substance of the matter, since the aggregate comparison yields the wrong verdict in both directions.

7 APPLICATION OF OPERATOR-ALGEBRAIC DISTINCTIONS

Section 3 set aside operator algebra as a foundation. It does not follow that the subject has nothing to contribute, and this section takes up three of its distinctions and applies them to the object constructed in Section 5. The placement is deliberate. Had these appeared alongside the assessment, they would have been analogies with nothing underneath, since the object they are applied to did not then exist. Applied to repertoires and their transitions, they yield distinctions that the fixed-point definition alone does not supply.

The borrowings are of questions rather than of axioms, and the difference should be kept in view. Nothing below asserts that a repertoire is an algebra, that a transition is an automorphism, or that any theorem of operator algebra transfers. What is claimed is that three questions the subject has learned to ask are worth asking here, and that asking them produces results.

7.1 Individuation of Regimes

Operator algebra distinguishes representations of a single algebra that cannot be connected by any transformation preserving the observables (Haag and Kastler, 1964). Such representations belong to separate superselection sectors, and the distinction is structural: no continuous path, and no automorphism, carries one to the other. The physical reading is that the two describe genuinely different phases of the same system rather than different states within one phase.

The corresponding question for repertoires is when two arrangements should count as different regimes rather than as one regime in different conditions. The maintenance condition of Section 6 does not answer this. It tells us which asymmetries persist within an arrangement and says nothing about the individuation of arrangements themselves, so a large change and a change of kind are on its account indistinguishable.

The borrowed distinction supplies a criterion. Call two repertoires *connected* when some sequence of transitions carries one to the other while preserving the register structure, and *separated* when no such sequence exists. Separation is not a matter of distance. Two arrangements may differ in almost every attribution and remain connected, a sequence of transitions carrying the one to the other; and two arrangements may differ in a single register and be separated, no transition preserving the structure being available to bridge them.

This yields a reading of regime change that the magnitude languages of Section 2 cannot state. A constitutional order and its successor are not related as two points along a continuum of arrangements, differing by more than usual. Where the successor lies in a separate sector, the transition between them was not the accumulation of ordinary transitions and cannot be described as such, whatever the sequence of events that in fact produced it. The criterion also disposes of a familiar ambiguity: two states sharing a constitutional text may be separated, if no transition preserving the register structure connects them, and the shared text is then a fact about documents rather than about arrangements.

The criterion is offered as a distinction rather than as a test. Determining whether a bridging sequence exists is not something the framework makes tractable, and Section 10 returns to this.

7.2 Institutional Filtering

A conditional expectation in operator algebra is a projection from an algebra onto a subalgebra, compatible with the multiplication in the sense that elements of the subalgebra pass through it unchanged (Takesaki, 1972). Its interpretation is that of a coarse description: what the larger algebra records is retained only insofar as the smaller one can express it. Where the construction is available, an index measures the size of the discrepancy (Jones, 1983), quantifying how much of the larger structure the smaller one fails to see.

The corresponding structure for repertoires arises whenever an institution acts upon a relational field through a vocabulary of its own. A court recognises claims of certain kinds; a bureaucracy processes cases under categories it maintains; a certifying body registers distinctions its own procedures can express. In each case the institution operates not upon the field as it stands but upon its own reading of it, and the reading is a subrepertoire.

Let $\mathcal{B} \subseteq \mathcal{A}$ be such a subrepertoire, and write

$$E : \mathcal{A} \longrightarrow \mathcal{B} \tag{23}$$

for the assignment carrying each operation to whatever the institution recognises it as. The requirement corresponding to compatibility is that operations already belonging to $\mathcal{B}$ pass unchanged under (23), so an institution reads its own categories correctly and reads everything else through them.

Two consequences follow, and the second is the one worth having.

An institution is thereby blind in a specific and describable way. What E discards is not noise but relational fact: distinctions present in $\mathcal{A}$ that $\mathcal{B}$ cannot express are, from the institution's standpoint, absent. This is the non-surjectivity noted at the end of Section 5, and it gives a formal shape to a familiar observation about administration without recourse to metaphor.

The blindness admits comparison. Where an index analogue is available, arrangements may be ordered by how much of the field their institutions fail to register, and the ordering is not the same as the ordering by how much they do. An institution with a large vocabulary may discard more than one with a small vocabulary, if what it discards is what mattered. The account does not supply such an index in general, and constructing one for a class of cases would be a contribution this paper does not make.

The bearing on what follows is direct. Normalisation, the third register of Section 9, is precisely the operation by which a contingent ordering is rendered as a category an institution can recognise. In the present vocabulary, normalisation moves an ordering from $\mathcal{A}$ into the subrepertoire $\mathcal{B}$ through which the institution acts, after which the ordering is no longer something the institution imposes but something it merely reads.

7.3 *Levels of Symmetry*

The third borrowing is the most useful and the least technical. Operator algebra distinguishes the symmetries of a dynamics, the symmetries of the observables, and the symmetries of a state, and holds them apart as three questions rather than one. The corresponding levels here are the transition, the repertoire, and the fibrewise distribution.

Consider an arrangement in which every party has the same operations available. This is

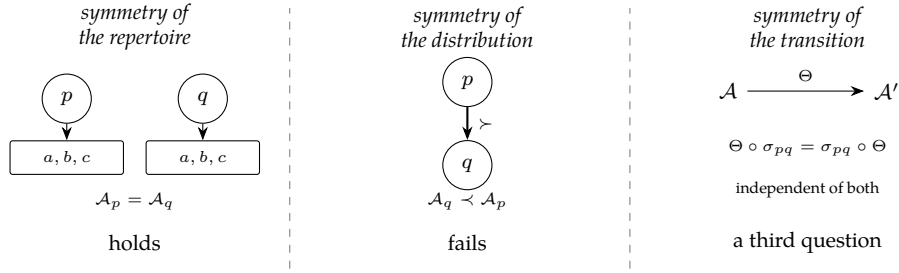


Figure 6: The three levels held apart. Formal equality asserts the first panel and is compatible with the failure of the second, which is the shape of the distinction between formal and substantive equality. The third panel is a further condition that discussions recognising only the first two have no place for.

symmetry at the level of the repertoire: for parties p and q ,

$$\mathcal{A}_p = \mathcal{A}_q. \quad (24)$$

Consider next an arrangement in which the ordering (18) places no party above another, so that no asymmetry obtains. This is symmetry at the level of the distribution, and it is a different condition. And consider finally an arrangement whose transitions treat parties alike, so that Θ commutes with the exchange of p and q . This is symmetry at the level of the transition, and it is a third condition again.

The three are independent, as Figure 6 sets out, and the independence is the result. Equality before the law asserts (24): the same operations are available to all, and this is what a formal guarantee of equal treatment guarantees. Substantive inequality is the failure of symmetry at the level of the distribution, and it is entirely compatible with (24) holding exactly. An arrangement may therefore be symmetric in its repertoire and asymmetric in its distribution without contradiction, which is the formal shape of the long-standing observation that formal and substantive equality are distinct. The gap between the two is what the second and third dimensions of power in Lukes (2005) are addressed to.

What the distinction adds to that observation is a third term usually omitted. Reform that alters what may be done, as against reform that alters who stands where, is a change at the level of the transition, and it is neither of the first two. An arrangement may be reformed in this third sense without either the repertoire or the distribution changing at the moment of reform, the alteration consisting in what subsequent exercises will do. Discussions that recognise only formal and substantive equality have no place for this, and are obliged to classify such reforms as one or the other, generally as merely formal.

Table 2 records the three borrowings and what each contributes.

<i>Distinction</i>	<i>Operator-algebraic content</i>	<i>Applied to repertoires</i>
Superselection	Representations unconnectable by observable-preserving transformation	Regimes separated when no transition preserving register structure bridges them; regime change is not accumulated ordinary change
Conditional expectation	Projection onto a subalgebra, with an index measuring the discrepancy	An institution acts through its own subrepertoire; what it cannot express is absent from its standpoint
Levels of symmetry	Symmetries of dynamics, observables, and state held apart	Symmetry of the transition, the repertoire, and the distribution; formal equality is (24), substantive equality is not

Table 2: Three distinctions borrowed from operator algebra and applied to the object of Section 5. The borrowings are of questions rather than of axioms; no theorem of operator algebra is claimed to transfer.

8 REFUTATION OF THE COUNTING CRITERION

The distinction between generative and degenerative power relations was given in Section 6 as a fibrewise condition. A simpler criterion suggests itself, and it is held widely enough to deserve refutation rather than dismissal. This section states it, shows that it fails in both directions, and identifies what the failure reveals about the criterion that replaces it.

8.1 The Counting Intuition

The intuition is that generativity consists in the enlargement of possibility. An arrangement that admits more is more generative than one that admits less, so that generativity may be measured by the size of what is available and degeneration by its diminution. Written in the vocabulary of Section 5, the proposal is that a transition Θ is generative when

$$|\Theta(\mathcal{A})| > |\mathcal{A}| \quad (25)$$

and degenerative when the inequality runs the other way.

The proposal has evident attractions. It is simple, it requires no attribution map, it agrees with ordinary usage in a wide range of cases, and it connects generativity to notions with established formal treatments, whether the cardinality of an option set, the entropy of a distribution over available actions, or the size of a feasible region. Nothing in what follows disputes that these are well-defined quantities. What is disputed is that any of them tracks the distinction the account requires.

Variants of (25) weighting operations by importance, or counting only operations that are

in some sense significant, do not evade what follows. The counterexamples turn on which party holds the operations rather than on how the operations are counted, and a weighted count remains a count over the repertoire as a whole.

8.2 Counterexamples in Both Directions

Contraction that generates. A constitutional prohibition removes operations from a repertoire. Where a constitution forbids the suspension of elections, the detention of persons without process, or the expropriation of property without compensation, the operations so forbidden are no longer available, and the repertoire is smaller after the prohibition than before. By (25) the transition is degenerative.

The verdict is wrong, and it is wrong in a way that indicates where the criterion has gone astray. What the prohibition removes is drawn almost entirely from one fibre. The operations forbidden are those available to the party in a position to suspend, detain, or expropriate, and their removal is what permits the other parties to act at all in ways that were previously subject to arbitrary reversal. Contracts become worth entering, since they cannot be voided at discretion; associations become worth forming, since they cannot be dissolved at will; and each of these is an enlargement of $\mathcal{A}_q$ effected by a contraction of $\mathcal{A}_p$. The aggregate may fall while the subordinate fibre rises, and the second is what the account attends to.

Expansion that degrades. Consider the dissolution of shared norms, the condition Merton (1938) analyses under the name of anomie, in which what was formerly not done becomes possible. Operations previously excluded, whether by convention, by expectation, or by the anticipated response of others, are now available. The repertoire is larger after the dissolution than before, and by (25) the transition is generative.

This verdict is wrong in the mirror image of the first case. The operations that become available are available in principle to all and in practice to those positioned to use them, so the enlargement accrues to the fibre of whoever can act with least constraint. For the party whose position depended on the dissolved expectations, what was available has contracted sharply, since operations whose effectiveness rested on others' compliance with a norm are no longer effective operations at all. The aggregate rises while the subordinate fibre falls.

Figure 7 sets the two cases side by side. The left panel shows what the counting criterion registers, which is the aggregate; the remaining panels show the fibrewise decomposition that the criterion discards. The aggregate trajectories run in opposite directions from the fibrewise ones in both cases.

The two counterexamples run in opposite directions, which forecloses the natural repair. Were the criterion wrong only about prohibition, one might reverse it and hold that contraction generates. The second case rules this out. No monotone function of $|\mathcal{A}|$ can agree with the account on both cases, since the account assigns opposite verdicts to two transitions whose

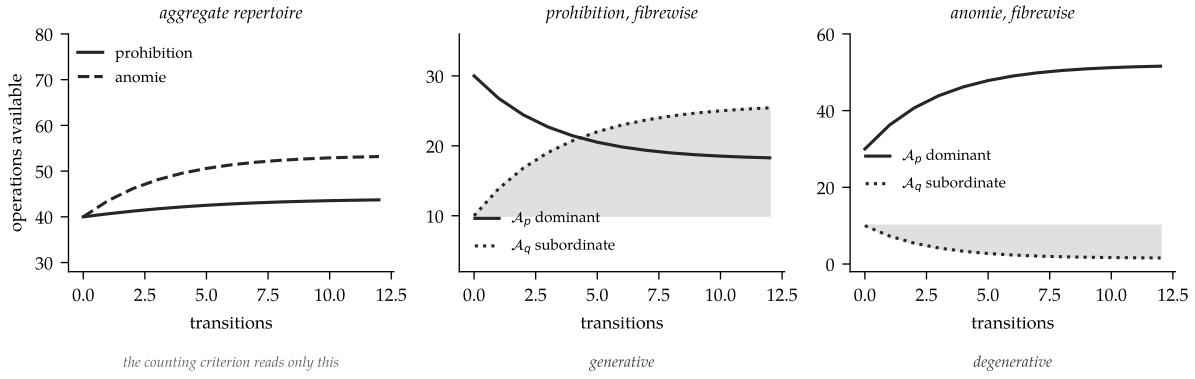


Figure 7: The counting criterion against the fibrewise one. The left panel plots the aggregate size of the repertoire under the two transitions, which is the whole of what (25) reads. The centre and right panels decompose the same transitions by fibre. Prohibition contracts the aggregate while enlarging the subordinate fibre; the dissolution of norms enlarges the aggregate while contracting it. The illustrated trajectories are constructed rather than measured, and no claim is made that any actual arrangement follows them.

aggregate behaviour is itself opposite, and a monotone function of the aggregate must order them as the aggregate orders them. The failure is not of calibration but of the quantity chosen.

8.3 Reformulation of the Criterion

What the counterexamples share is that the aggregate discards the attribution. The quantity $|\mathcal{A}|$ is computed over the repertoire as a whole and is therefore insensitive to which fibre gained and which lost, and it is precisely that information the distinction requires. This is why the criterion of Section 6 was stated fibrewise, and why the attribution map (11) was built into the object rather than added to it.

The reformulation may now be stated as the moral of the two cases. Generativity is not a property of how much an arrangement permits. It is a property of the relation between what an arrangement permits its dominant party and what it thereby permits the others. A quantity insensitive to the attribution cannot express that relation, since the two counterexamples agree in aggregate behaviour where the account separates them and differ in fibrewise behaviour where the account joins them. The consequence for the notions mentioned above is direct: option count, entropy over available actions, and the size of a feasible region are all aggregate quantities and all fail for the same reason.

A related consequence bears on freedom as ordinarily discussed. If freedom is taken to consist in the number of available options, then the second counterexample is a gain in freedom and the first a loss, which reverses the assessment that almost anyone would make of the two cases. The account does not offer a theory of freedom, and nothing here settles what freedom

consists in. It does show that whatever freedom is, it is not the aggregate quantity in (25), since that quantity delivers the wrong verdict on the two cases considered here.

8.4 *Relation to the Value Loop*

A further characterisation follows, and it connects the present result to the account of generation given in Section 4.

Where a power relation is degenerative, what the arrangement generates returns to the party that generated it. The subordinate fibre is contracted, the dominant fibre is thereby secured, and the value produced by the exercise accrues to the position from which the exercise was made. The loop closes upon itself, and the arrangement is sustained by what it takes from those subject to it.

Where a power relation is generative, what the arrangement generates does not return in this way. The certifying body's persistence is purchased by the enlargement of what certified practitioners may do, and the value generated accrues to the relation rather than to either party alone. The loop remains open, in the sense that what is produced is not recovered by the producer.

The two characterisations agree on the cases considered here, and no argument is offered that they agree in general; the loop formulation is a gloss on the fibrewise criterion rather than an independent test, and establishing an equivalence would require a treatment of value that this paper does not attempt. Its usefulness is that it connects this result to the treatment of appropriation elsewhere in the programme to which this paper belongs, where the question of whether generated value is left with the relation that generated it recurs in settings having nothing to do with power.

9 GENERALIZATION TO A SCHEMA OF DURABLE ASYMMETRY

The account has so far given a condition under which an asymmetry persists without saying how such asymmetries are assembled. This section proposes that they are assembled in a common way, and offers the proposal as a general schema rather than as an illustration of the preceding sections.

9.1 *The Four Registers*

A durable asymmetry, on the proposal, is the composite of four operations.

Differentiation takes a population and yields variation within it. Parties come to differ in training, in holdings, in position, in what they have done. This is the step of Section 4 at which relation is produced and, being produced under varying circumstance, produces difference.

Selection takes variation and yields an ordering. Some differences are made consequential and others are not, and the operation that ranks them is distinct from the operation that pro-

duced them. Section 6 recorded this as the addition of $\preceq$, which the construction does not supply on its own.

Normalisation takes an ordering and yields a meaning: the ordering as it is understood, which is to say the ordering rendered unremarkable. What was a contingent ranking becomes the way things are, and the parties to it cease to experience it as something imposed. In the vocabulary of Section 7, normalisation is what moves an ordering into the subrepertoire through which an institution reads the field, after which the institution does not impose the ordering but merely recognises it.

Reproduction takes a normalised ordering and yields its transmission. What holds among the present parties comes to hold among their successors, whether through inheritance, through training, through the persistence of the institutions that recognise it, or through the simple continuation of the practices that constitute it.

Writing the four as arrows,

$$X \xrightarrow{D} V \xrightarrow{S} O \xrightarrow{N} M \xrightarrow{R} X, \quad (26)$$

the composite $R \circ N \circ S \circ D$ carries a population to a population in which the same ordering obtains, which is the closure condition of Section 6 exhibited as a cycle. What the composite must further withstand is the background productivity of the field, and Section 9 argues that one register in particular is what makes it able to.

9.2 *Typing Across Registers*

The four operations do not compose within a single register, and this is why (26) was written with distinct objects. A distribution, an ordering, a meaning, and an inheritance are relational facts of different kinds, and the arrows between them have correspondingly different domains and codomains.

The typing is not decoration. It is the reason the composite could not be written as a product in an algebra, as Section 3 recorded, and it is what makes the reverse composites undefined rather than merely uninteresting: a normalisation cannot be applied to an undifferentiated population, there being no ordering for it to act upon. The partiality of composition established in (14) is therefore doing work here rather than sitting idle.

It follows that a durable asymmetry is not a single kind of thing occurring four times. It is a passage across four kinds of relational fact, and an account that treats power as one substance moving through a system will not have the resources to state it.

9.3 *Argument from Necessity*

The proposal is that all four registers are required, and the argument is given register by register. In each case the question is what becomes of an asymmetry when the register is absent.

Without differentiation there is nothing to order. This case is degenerate and is recorded for completeness.

Without selection there is difference and no asymmetry, which is the step of Section 4 at which most accounts move too quickly. Parties differ, and no difference confers anything. Such arrangements are common and unremarkable, which is precisely the point: variation is ubiquitous and only some of it is made to matter.

Without normalisation an asymmetry may still be held in place, but only by continual re-imposition. The ordering does not stand on its own, so each occasion of its operation requires that it be asserted afresh against parties who experience it as imposed. Such arrangements exist and are recognisable, and what characterises them is expense: surveillance, enforcement, the maintenance of a coercive apparatus, and the periodic demonstration that the ordering still holds. The asymmetry does not cease, but it ceases to reproduce itself and must instead be reproduced by someone.

Without reproduction the asymmetry does not survive the parties to it. An ordering that is differentiated, selected, and normalised, but not transmitted, holds for a generation and lapses. This is the case of the charismatic arrangement that does not outlive its founder, the routinisation problem Weber (1978) identifies, and its familiarity is evidence for the necessity of the fourth register rather than against it.

9.4 *Limits of the Argument*

Sufficiency is not claimed, and the reasons should be stated rather than left to inference.

The four registers may be present and the asymmetry fail to persist for causes lying outside the composite: exogenous shock, the arrival of parties not subject to the ordering, the exhaustion of a material base on which the whole arrangement depended. The schema states what a durable asymmetry is assembled from, not what guarantees that an assembly of those parts will hold.

Nor does the argument from necessity establish that the four registers are the right decomposition rather than one decomposition among several. A different partition of the same passage, into three registers or five, might do the same work. What recommends this one is that each register corresponds to a distinct kind of relational fact and that the necessity argument runs separately for each, but neither consideration excludes alternatives.

9.5 *Instantiation Across Domains*

The schema is offered as general, and the case for generality rests on instances.

In credentialism, analysed at length by Bourdieu and Passeron (1990), differentiation is the variation in training and aptitude; selection is the examination that ranks it; normalisation is the treatment of the credential as evidence of what it certifies rather than as a convention

about who may practise; reproduction is the transmission through institutions that require the credential and schools that supply it.

In colonial administration, differentiation is the difference in armament, organisation, and disease exposure; selection is the conversion of that difference into a claim of authority; normalisation is the elaboration of a doctrine under which the authority is proper rather than merely effective; reproduction is the administrative apparatus, the schooling of a local intermediary class, and the legal forms that outlast the personnel.

In scientific prestige, differentiation is the variation in what is discovered; selection is peer judgement; normalisation is the treatment of eminence as tracking merit rather than as constituting it; reproduction is the training of students by the eminent and the transmission of position through appointment.

In market position, differentiation is the variation in efficiency and holdings; selection is competition; normalisation is the treatment of the resulting distribution as an outcome of desert or of natural process; reproduction is the reinvestment of returns and the inheritance of capital.

The instances differ in almost every respect that matters descriptively, and in the four registers they agree. That agreement is what the generality of the schema amounts to.

9.6 Identification of the Load-Bearing Register

The registers are not equally exposed, and the asymmetry among them has a consequence for intervention.

Differentiation cannot be removed. Parties differ, and an arrangement that prevented them from differing would have to prevent relation from being produced at all. Selection can be altered but not abolished, since any arrangement in which some differences are consequential involves a selection, and the question is which one rather than whether. Reproduction can be interrupted, and much redistributive policy consists in interrupting it, but interruption must be repeated in each generation and the interruption is itself expensive.

Normalisation is different in kind, and the distinction of Section 6 says why. Closure is achieved by the composite as a whole; maintenance is what the composite must further withstand, and normalisation is the register through which it withstands it. An ordering taken as unremarkable is not defended against each new difference the field produces, because new difference is read through the ordering rather than tested against it. This is the same operation Section 7 described as institutional filtering: once an ordering has passed into the subrepertoire through which an institution acts, background transitions arrive already interpreted. Alone among the four registers, normalisation has no material substrate. Differentiation is grounded in the actual variation among parties, selection in the institutions that rank them, reproduction in the mechanisms of transmission. Each of these three can be pointed to independently of what anyone believes about it. Normalisation cannot: it consists in the

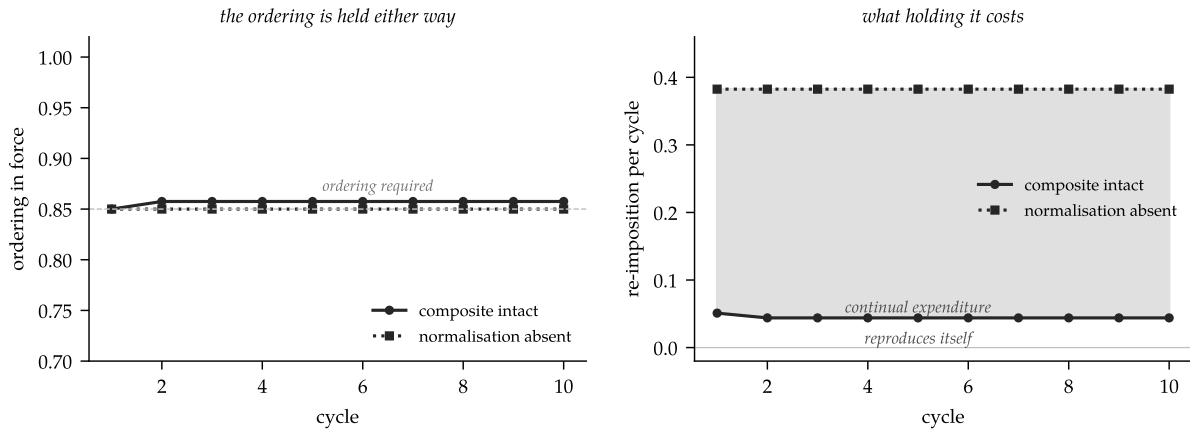


Figure 8: The effect of removing normalisation, under a constructed model. The left panel shows that the ordering may be held at the required level in either case. The right panel shows what holding it costs: with the composite intact the required re-imposition falls toward nothing, since the ordering stands of itself; with normalisation absent the ordering decays between cycles and the cost of restoring it does not fall. The model is illustrative and its parameters are stipulated; no claim is made that any actual arrangement follows these trajectories.

understanding under which the ordering is taken as unremarkable, and an understanding is sustained by nothing except its continuing to be held. The asymmetry among the registers claimed here is this one, and it is offered as an observation about the instances considered above rather than as a theorem about registers in general.

The consequence is illustrated in Figure 8. Where normalisation is removed, closure may still be achieved but maintenance is not: the composite returns to its source under its own operation and no longer withstands what the field supplies, so each new difference must be met as a difference rather than absorbed as a confirmation. The ordering may still be maintained, and frequently is, but its maintenance now requires that it be asserted rather than assumed, and the assertion must be repeated indefinitely. What was reproduced becomes something that must be continually re-imposed, and the difference between these is the difference between an arrangement that costs nothing to hold and one that costs something every cycle.

This identifies the register at which intervention is cheapest, and it does so without recommending any intervention. The finding is structural: an asymmetry deprived of its normalisation is not thereby ended, but it is converted from something that reproduces itself into something that must be reproduced by an identifiable party at a recurring cost, and arrangements of the second kind are considerably easier to end than arrangements of the first. This is also why contests over durable asymmetries are so often contests over description rather than over distribution, a fact that accounts treating power as a quantity have difficulty explaining and that follows directly here.

10 SCOPE AND DIRECTIONS

An account of this kind is best judged by what it declines to claim as much as by what it asserts, and several sections above deferred a limit to this one. They are collected here, together with the work the account leaves open.

10.1 *Limits of Recoverability*

It is tempting to read the framework as offering an inverse problem: given an observed history of a social arrangement, recover the repertoire and the transitions that generated it. The temptation should be resisted, and the reason is not that the problem is difficult.

The problem is underdetermined in a way that no quantity of data relieves. Any observed sequence of arrangements is compatible with many generating repertoires, since the operations available to a party are not observed directly but only through the occasions on which they are exercised, and an operation that is available and never exercised leaves at best indirect evidence of its availability. Two arrangements identical in every observed respect may differ in what was possible but not done, and the difference is precisely what the account holds to be decisive. The situation is worse than the corresponding one for differential models, where at least the state is observed even if the vector field is not.

What follows is that positing a repertoire is a theoretical commitment rather than an inference. The account does not claim otherwise, and the claim it does make is a different one: a posited generating set is more falsifiable than a posited quantity, because relations among the generators forbid certain sequences outright, and a single observed occurrence of a forbidden sequence refutes the posit. A quantitative model accommodates an anomalous observation by adjusting a parameter. A generative posit that forbids what is observed to occur has been refuted. This is a claim about the form of the two kinds of theory rather than a result, and it is taken up below as a direction rather than a finding.

10.2 *Capacities Relinquished by the Account*

Three things available in the languages of Section 2 are not available here, and the account is worse than they are in each respect.

There are no quantitative dynamics. The framework supports no differential equation, yields no trajectory, and permits no calculation of how fast anything happens. Where a question concerns rates, the differential languages are the appropriate instrument and this one is not.

There is no spectral theory and nothing corresponding to it. The structure theorems that make operator algebra powerful were purchased by the axioms Section 3 found unavailable, and declining the axioms means declining the theorems. Nothing here decomposes an ar-

rangement into components, identifies invariants, or classifies arrangements up to any equivalence that a theorem certifies.

There is no prediction. The account states a condition under which an asymmetry persists and does not determine which asymmetries in a given arrangement satisfy it. It is a framework for describing and distinguishing rather than for forecasting, and its usefulness stands or falls on whether the distinctions it draws are the right ones.

A further limit arises from the strengthened condition of Section 6. Maintenance is defined against the background transitions a setting admits, and the framework does not determine that class. Which perturbations an actual arrangement must withstand is a question about the setting, to be settled by whoever knows the setting, and an account that fixed the class in advance would be legislating rather than describing. The condition is therefore well posed only relative to a specification the framework does not supply.

To these should be added the two limits recorded earlier. The criterion of Section 7 for separating regimes is a distinction and not a test, since determining whether a bridging sequence of transitions exists is not made tractable by anything in the construction. And the index analogue that would measure institutional blindness was described but not constructed, so the comparison it would license is at present unavailable.

10.3 Proposal of the Generating-Set Inversion

The direction most worth pursuing is the one identified above and not there developed.

Explanation in the differential languages runs from a law of motion to a trajectory: the vector field is posited and the observed history is what it produces. The inversion proposed here runs from a generating repertoire to the admissible compositions and thence to the possible histories, so that what is posited is not how the arrangement moves but what may be done within it and in what combinations.

The two are not equivalent in explanatory content. A vector field is compatible with any history it produces and forbids nothing that it does not produce; a generating set with relations among its generators forbids compositions outright, and the forbidden compositions are commitments the theory can be held to. The proposal is that this makes the second form of theory more falsifiable in a specific sense, and the proposal has not been tested. What would test it is a case in which relations among posited generators yield a prohibition that survives confrontation with the record, and no such case is offered here. Until one is, the inversion is a direction rather than a result, and the paper marks it as such.

10.4 The Unresolved Coupling of Fast and Slow Change

The framework's principal unresolved difficulty concerns the relation between two kinds of change it holds apart.

Within a repertoire, ordinary social dynamics proceed: parties act, distributions shift, the events that occupy most social description occur. Between repertoires, the operational field is itself transformed. The double category of Section 5 presents these as horizontal and vertical movement respectively, and the commuting square records that an operation performed within a repertoire is what carries the repertoire to its successor. That is a statement of compatibility rather than a theory of the coupling.

What is missing is an account of the timescales. Ordinary dynamics are fast and structural transition is slow, and the two are not merely different in rate but different in kind, so the standard techniques for separating fast and slow variables do not apply. A transition is not the accumulation of many small horizontal movements; it is a change in what horizontal movement consists in. How a great many exercises within a repertoire combine into a transition of it, and under what conditions they fail to, is not answered by anything in this paper.

The question matters because the cases of Section 1 turn on it. An asymmetry that consumes its own preconditions does so through the accumulated effect of ordinary exercise, and the account states the outcome without describing the accumulation. Supplying that description would convert several of the paper's distinctions from classifications into mechanisms, and it is the work this paper most wants done.

10.5 Standing of the Account

The account is offered as a conceptual framework with a formal presentation, and the two should be assessed separately.

The mechanism of Section 4 stands or falls on its own. It claims that asymmetry arises from ordered difference rather than from difference alone, that exercise alters the conditions of exercise, that most asymmetries consume themselves, and that what remains is what reproduced its own conditions. A reader may accept all of this and reject the construction that follows.

The construction of Section 5 is offered as a way of stating that mechanism without ambiguity, and its claim is modest. It is assembled from standard ingredients and contributes no technical novelty; what it contributes is a decision about which structure is primary, namely the party-indexed availability of operations, with rewriting and composition as what that structure undergoes. A reader may accept the construction and dispute the schema of Section 9, which is a further and more exposed proposal.

The result the account would most want defended is the negative one of Section 8. Generativity is not the enlargement of possibility, and a quantity computed over an arrangement as a whole does not distinguish the cases the account must distinguish. That result requires none of the formal apparatus, survives the rejection of everything else in the paper, and is stated as an invitation to counterexample.

A note on method. The formal apparatus of this paper is deliberately thin, and the thinness is a judgement rather than an omission. Two of the paper's results, the reconstruction of the self-maintenance thesis as a selection effect and the refutation of the counting criterion, require no apparatus at all and are stated in Sections 4 and 8 in terms a reader may assess without accepting any construction. The construction of Section 5 is introduced to state the remaining distinctions unambiguously, and it goes no further than that use requires. A more elaborate presentation was available and was declined: the categorical machinery admits considerable development, and developing it in advance of results that would need it would have borrowed a rigour the argument has not earned. Where the account has reached for an established formalism and found it unsuitable, as with operator algebra in Section 3, the reasons have been given at length rather than passed over, on the view that a declined borrowing should be visible in the record.

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