

Knowledge Pressing in Evolving Relational Spacetime: Inference and Translation of Generative Representations An Exploratory Formal Investigation

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Abstract

The reconstruction of generative mechanisms from historical observations connects established research in system identification, program induction, causal modeling, and historical interpretation. Evolving relational systems additionally raise questions about how generative representations change, how epistemic and practical grammars become intelligible across heterogeneous systems, and how epistemic activity may participate in their subsequent development. This discussion paper proposes *knowledge pressing* as a provisional framework for examining these coupled processes within an evolving relational-spacetime reference model. Candidate generative representations may include grammars, dynamical equations, action functionals, programs, probabilistic mechanisms, and institutionally constituted rules. The proposed investigation addresses historical inference, partial translation, admissible relational continuation, and the constitution of practical agency, while separating model-relative future possibilities from claims about metaphysical freedom. Observational underdetermination, epistemic situatedness, translation loss, and normative authority remain explicit limitations. The present version develops the Introduction, research foundations, knowledge pressing, typed generative models, evidence-indexed inference, historical model evolution, contextual translation witnesses, and model-indexed whole-spacetime continuation. Finite examples illustrate limitations of trace matching, changes in cellular topology, and conditions of action-dependent accessibility. The agency section develops historically situated practical and collective action, a restricted monotonicity proposition, and philosophical distinctions concerning free will. The epistemic-limits section establishes conditional results for query-relative identifiability and model-supported continuations, and gives an intervention-based counterexample. Controlled synthetic computational investigations illustrate grammar changes, inverse dynamics, structural translation, branching loss, and action-conditioned reachability; these do not constitute real-world validation. A proposed federated epistemic infrastructure specifies versioned historical records, model and inference provenance, contextual translation certificates, continuation artifacts, and procedures for participation, access control, and contestation; it remains an unimplemented design. The discussion assesses representational plurality, translation, continuation, and agency, while the conclusion consolidates the paper's conditional results, empirical boundaries, and open research agenda. Appendix A specifies notation and typed interfaces. Appendix B develops relative cellular continuation, homology changes under attachments, and independently checked finite examples. Appendix C specifies restricted relational rewrite grammars, rule evolution, endpoint ambiguity, and translation obligations. Appendix D examines inverse heat dynamics, sampling non-identifiability, variational non-uniqueness, and qualified continuous-discrete correspondences. Appendix E develops translation obligations, branching-sensitive counterexamples, compositional correspondence, approximate commutation, and historically versioned witnesses. Appendix F establishes additional identifiability counterexamples concerning finite observation horizons, temporal sampling, causal interventions, and changes in epistemic observation. Appendix G consolidates reproducibility contracts, an executable verification registry, environment records, regeneration tolerances, and limits of computational inference.

Keywords: knowledge pressing; relational spacetime; historical inference; generative representation; epistemic translation; generative continuation; agency.

Draft status. Version 2.2, 21 October 2026. Sections 1–14 and Appendices A–G drafted; comprehensive editorial and independent scientific review remain planned. Reproducible synthetic experiments and a conceptual infrastructure design are included.

DRAFT

Discussion Paper Note

This independent discussion paper investigates knowledge pressing in evolving relational spacetime, with emphasis on inference, translation, and the historically situated continuation of generative representations. Knowledge pressing is introduced as a provisional analytical expression for connected processes of epistemic reconstruction, cross-system interpretation, revision, and participation in further relational development. It is intended to cover prospective generativity as well as established history, without separating past and future into independent domains.

The work considers heterogeneous candidate representations, including rewrite grammars, differential equations, action functionals, stochastic models, programs, and normative or practical rule systems. The framework adopts ontological suspension: relational spacetime is a posited reference structure; recorded histories and inferred generative models are partial epistemic constructions. Predictive adequacy, self-understanding, and normative legitimacy are evaluated separately.

Version 0.7 contains complete first drafts of Sections 1–6, including typed generative model interfaces, a conditional framework for inverse inference, and a treatment of historically evolving generative and epistemic grammars with elementary ambiguity examples. Sections on inter-system translation, continuation, agency, computational experiments, and infrastructure remain planned. No original empirical findings, validated forecasting system, or resolved metaphysical account of free will is claimed. The earlier two-complex future-creation study informs the continuation terminology while remaining a distinct manuscript.

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Applications of inferred generative representations should distinguish observations, model fit, causal interpretation, epistemic practices, and normative authority. Translations involving persons, communities, and institutions should respect consent, allow correction and contestation, and examine representational losses and differences in power.

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Notices

Publication status. Independent preliminary discussion paper, version 0.4, 20 October 2026. Sections 1–4 are drafted with verified references. Subsequent technical formalization and computational sections remain pending. The work has not been peer reviewed.

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AI-assisted research and preparation. OpenAI’s ChatGPT contributed to research dialogue, literature discovery and verification, initial drafting, and LaTeX preparation. The human author directs the research objectives and assumes responsibility for accuracy, source use, interpretation, and dissemination. ChatGPT is not identified as an author.

Research status and independence. The manuscript reports no original human-participant research, institution-specific findings, or validated applications. It expresses an independent preliminary research programme without implied institutional endorsement.

1 Introduction

Knowledge pressing in evolving relational spacetime concerns the epistemic activity through which historically situated systems encounter evidence, infer generative structures, translate among heterogeneous representations, and participate in the formation of subsequent relational possibilities. The term *knowledge pressing* is used provisionally for this coupled field of activity. It identifies a process in which knowledge is continually produced, examined, revised, and brought into interaction with practices that may alter what can subsequently occur. The object of inquiry is an evolving relational configuration together with the historically constituted conditions under which that configuration becomes intelligible and can be acted upon. The investigation treats historical constitution and prospective continuation as mutually implicated aspects of relational development. This paper proposes a preliminary formal language for several components of that problem, concentrating on generative representation, historical inference, inter-system translation, and continuation.

The motivation is partly epistemic and partly practical. Scientific measurements, archival records, computational traces, situated testimony, and institutional documentation preserve distinct aspects of events and their connections. Such records can support reconstruction of how a system’s present organization arose. They can also constrain conjectures about the mechanisms responsible for its transformations and about possible continuations under changed conditions. Yet a historical record is produced through particular instruments, categories, selections, and institutional practices. Its coverage depends on resources and access, while its interpretation depends on the capacities and interests of the observer. Even an extensive shared record would remain a mediated source of evidence. The prospect of increasingly comprehensive, computationally navigable historical records therefore raises an inference problem: which generative representations are compatible with the accessible trajectories, and what further observations or interventions would discriminate between rival candidates?

Relevant scientific methods already address important parts of that question. Computational mechanics has investigated the inference of predictive structure from temporal observations [14]. Sparse identification of nonlinear dynamics has shown how governing equations can be recovered within appropriately restricted libraries of candidate terms [9]. Related work has developed data-driven discovery of partial differential equations from spatiotemporal measurements [36]. Probabilistic program induction has illustrated how compositional generative programs can explain and generalize observed exemplars [21]. Causal representation learning addresses the recovery of higher-level variables and relations capable of supporting transfer and explanatory analysis [40]. These research traditions offer established tools and cautions. Their success within particular model classes does not by itself license a general claim that the historical constitution of an arbitrary system can be uniquely reconstructed from its outputs.

The candidate generative representation may differ profoundly across domains. A grammar may specify admissible productions and rewriting operations; a differential equation may relate continuously varying states and their derivatives; an action functional may characterize stationary histories relative to a variational structure; a probabilistic kernel may represent state-conditioned distributions of subsequent events; a program may encode operations and control flow. Institutional and practical systems may also be investigated through historically constituted rules, permissions, obligations, exceptions, and interpretive practices. Such normative descriptions have commitments and validation conditions of their own. The present research uses *generative representation* as an umbrella term for candidate accounts of how structures, processes, or practices may arise and change, while requiring each representation to specify its domain, observational interface, assumptions, and conditions of adequacy. No single mathematical grammar is presumed to be adequate for all cases.

A second difficulty arises when the generative representation itself evolves. A fixed-parameter model may become inadequate because its parameters drift, its operative relations change, new operations become available, or the observer’s available conceptual language changes. These possibilities have different interpretations. An observed shift may reflect transformation in the system, a revised measurement process, or a newly acquired ability to distinguish previously unrecognized structure. An inquiry into evolving relational spacetime must therefore keep changes in the modeled system analytically distinct

from changes in the means of observing and describing it. In the social and institutional cases, learning and description may also enter the system's subsequent development: participants can react to classifications, alter conventions, and reorganize relations through deliberate action. The epistemic process is historically situated within the domain whose continuation it seeks to understand.

The distinction is especially relevant to the proposed use of relational spacetime. In the present paper, relational spacetime is a *posited reference structure* for investigating the historical formation of events, couplings, and potentially higher-order relational primitives. A two-complex, with event-like vertices and coupling-like edges, provides one possible finite representation; the semantics of faces and higher-dimensional decorations require explicit specification in each application. Entities and subjects can be modeled as historically emergent, relationally sustained structures, rather than identified axiomatically with individual vertices. The geometry, topology, and dynamical description of a relational system are epistemic choices to be assessed against evidence and purpose. Topological information may distinguish patterns of connection, incidence, loops, or structural transformation, but topological similarity does not imply identity of causal mechanisms, meanings, or norms. Ontological suspension permits the mathematical investigation to proceed without treating a selected representation as exhaustive access to reality.

The inference problem is accompanied by a problem of translation. Distinct relational systems may use different observables, generative rules, epistemic standards, or practical grammars. A system whose interactions are modeled by a set of dynamical equations may be compared with one represented through a rewrite grammar, provided a suitable interface and a relevant criterion of correspondence can be identified. In social or institutional settings, translation also involves the historically situated categories through which participants recognize actions, reasons, authority, and consequences. Existing scholarship has long treated the difficulty of translation and the status of meaning as substantive philosophical problems; Quine's account of translation is a notable instance [33]. This paper therefore investigates *partial and context-indexed correspondences* between generative representations, including the relations, transitions, or consequences they may preserve and the distinctions they may lose. Translation may be asymmetric, many-valued, incomplete, and historically revisable. Its adequacy is evaluated relative to stated epistemic and practical purposes.

An additional possibility emerges when translation involves interaction. Learning another system's grammar can change the observer's own vocabulary, investigative capacities, and available practices. A negotiated intermediary language or a shared operational rule may alter the future behavior of both participating systems. Under such circumstances, translation is among the processes that contribute to further relational formation. Its consequences may include newly accessible collaborations, changing institutional powers, or the disappearance of previously available possibilities. Generative translation should consequently be studied both for representational adequacy and for the effects of epistemically mediated interaction. This coupling introduces questions of consent, correction, interpretive authority, and the asymmetrical ability of actors to have their own categories recognized. An inferentially successful representation need not confer authority to intervene in another system or decide the legitimacy of its norms.

Continuation designates a further mathematical object of interest. A reconstructed history and a candidate family of generative models can be used to investigate compatible prospective relational structures. One convenient finite formulation takes an observed or reconstructed historical complex and asks for admissible extensions, indexed by the model, epistemic context, and selected intervention conditions. Such continuations may attach new events and relations, modify available operational rules, or transform the topology of the represented complex. An extension can be understood as part of a whole historical-and-prospective structure rather than as a future fragment detached from its conditions of emergence. The continuation family expresses possibilities *within* a particular epistemic and generative description. Its members may differ from physically realizable alternatives, and practical access to an alternative may require resources, permissions, coordination, and conditions absent from the abstract model. Importantly, the generative rules determining admissibility may themselves become objects of historical transformation; the inquiry then concerns changes in the family of continuations as well as differences among its members.

The evolving character of continuation also introduces a question of agency. A historically constituted subject may acquire capacities for interpretation, deliberation, coordination, and intervention that alter the relational conditions of later action. In some situations, an agent's activity changes the operative rules or practical accessibility of future paths, rather than merely selecting among options specified by an unchanged model. Research in enactive approaches to agency has examined individuality, normativity, and interactional asymmetry as relevant to identifying an agent [6]. Philosophical discussions of free will further distinguish alternative possibilities, sourcehood, control, and moral responsibility [29]. The present framework proposes to connect model-relative continuation and historically situated agency while preserving distinctions among epistemic uncertainty, ontological indeterminacy, practically accessible action, and normative responsibility. A branching mathematical model by itself settles none of the longstanding metaphysical controversies surrounding free will. Nevertheless, an investigation of how epistemic capacities change the conditions for action can clarify a limited and substantive part of the problem.

Several limits of the inference project are already apparent. Observationally equivalent models may attribute different generative mechanisms to the same trajectory. Under-determination is a recognized issue in the philosophy of science, including the possibility that available evidence supports competing accounts under different background assumptions [42]. Further observation, interaction, or controlled intervention may reduce some ambiguities, while other distinctions remain outside the available tests. In addition, a model that accurately predicts observable behavior may fail to represent the internal categories through which a community interprets its practices. Likewise, a structural mapping can preserve selected transformations without preserving semantic or normative commitments. A responsible formalism should expose these different kinds of adequacy and their limitations rather than aggregate them prematurely into a universal measure of understanding.

The present inquiry has four connected objectives. The first is to specify historical relational observations and heterogeneous families of generative representations as distinct, evidence-indexed analytical objects. The second is to articulate an inverse problem for historically evolving generative mechanisms, together with model plurality, uncertainty, and differences between system changes and epistemic revision. The third is to examine the translation of generative representations through partial correspondences that can be evaluated on observable and prospective consequences. The fourth is to investigate how the inference and translation process can participate in continuation, including changes in accessible possibilities, the emergence of shared grammars, and the constitution of practical agency. The framework's integrative ambition concerns the articulation of relationships among these problems; it offers no claim to have originated the constituent methods or the philosophical questions themselves.

An accompanying computational programme is proposed to make the investigation more discriminating. Controlled relational rewrite systems could supply known production rules against which learned candidate grammars are tested. Differential or variational examples could examine reconstruction under incomplete observation and compare equivalent or non-identifiable models. Paired generative systems could permit an explicit comparison between output alignment and translation that preserves selected operations under previously unseen conditions. Further experiments could introduce time-varying rules, topology-changing continuations, or action-conditioned interactions. Such examples will be synthetic and conditional upon specified assumptions. They can illuminate the formalism and its failure cases, while the applicability of the approach to actual scientific, institutional, or social systems requires separate empirical and interpretive work.

The paper builds on a companion exploratory treatment of topological epistemology and two-complex continuation, particularly its distinction between a posited reference domain, an observer's epistemic construction, and represented future extensions. The present investigation shifts the principal emphasis toward the discovery of heterogeneous generative representations, their historically evolving grammars, translation between systems, and their interaction with continuation and agency. This division of labor permits the two inquiries to remain complementary. Formal constructions originating in topology, dynamics, grammar induction, causal modeling, philosophy of translation, and theories of agency are treated as existing intellectual resources and will be attributed accordingly.

The paper proceeds in the following order. Section 2 reviews the disciplinary foundations and their boundaries. Section 3 articulates knowledge pressing, evolving relational spacetime, and epistemic observation. Section 4 introduces heterogeneous generative representations; Section 5 formulates the historical inference problem; and Section 6 examines changes in generative and epistemic grammars. Section 7 develops cross-system translation. Section 8 treats generative continuation and changes in future relational possibilities, followed by the discussion of historically constituted agency and free will in Section 9. Section 10 addresses identifiability, interpretive plurality, and epistemic limitations. Section 11 presents proposed computational models and comparative tests. Section 12 considers implementation as an epistemic infrastructure; Section 13 develops the discussion and open research questions; and Section 14 concludes. The scope remains exploratory: definitions, formal claims, illustrative tests, and philosophical interpretations are to be assessed under their stated assumptions and through subsequent research.

2 Research Background and Conceptual Foundations

This section identifies the disciplinary foundations required to investigate knowledge pressing in evolving relational spacetime. Its purpose is to establish the inferential achievements and limitations of existing methods before developing a common vocabulary for generative representations, historically changing mechanisms, inter-system translation, and continuation. The discussion proceeds from inverse dynamical problems and variational modeling to grammatical and causal inference, then to historically situated interpretation and agency. Table 1 summarizes the distinct inferential objects and boundaries at issue. The comparison is methodological: the various traditions address related problems under different assumptions about observations, admissible explanations, interventions, and validation. Their intersection motivates a research programme; it does not yet establish a single general theory of generative understanding.

2.1 Inverse Problems and Dynamical-System Identification

In dynamical-system identification, the task is to reconstruct a model of a system’s evolution from measurements of its behavior. The observations may be time series, state transitions, inputs and outputs, or experimental responses. Classical inverse problems already make the relation between observable trajectories and posited dynamics explicit: an observation process restricts the information available about the generating mechanism, and additional assumptions determine what can reasonably be inferred. Even with abundant data, inference depends on the choice of state variables, the sampling design, the observational noise model, and the family of equations under consideration.

Modern data-driven approaches illustrate how substantial parts of this task can be supported computationally. Bongard and Lipson examined automated symbolic modeling of coupled nonlinear dynamical systems from time-series observations [8]. Schmidt and Lipson investigated the automated extraction of analytical relationships from experimental data [39]. Sparse Identification of Nonlinear Dynamics (SINDy) provides a particularly clear example of the role played by a model-class assumption: candidate derivatives are expressed using a library of possible functions, and sparsity-promoting regression is used to select a comparatively small number of terms [9]. This approach can recover informative differential equations when the chosen variables, library, and observations are adequate. Its success does not imply an unrestricted recovery procedure for arbitrarily constituted relational systems.

The distinction between model discovery and mechanism discovery is fundamental to the present investigation. A differential equation may describe the evolution of measured variables while omitting unmeasured interactions that are causally relevant. An observationally successful representation may also depend on a particular spatial or temporal scale. When the observational interface changes, the reconstructed model may change even if the underlying physical dynamics remain stable. In addition, different models can be consistent with the same finite observational record. These are familiar limitations of inverse reasoning, and they motivate an evidence-indexed family of candidate generative

representations rather than an automatic identification of a uniquely recovered law.

The relevance to relational spacetime is direct but conditional. If events, couplings, or relational observables admit an appropriate dynamic representation, techniques of system identification may constrain how those observables develop. A relational system with changing participants, variable topology, or historically constituted interpretive rules requires additional representation choices. The question for later sections is how to preserve the specificity of successful inverse methods while permitting heterogeneous model families to participate in a broader inferential process.

2.2 Differential Equation Discovery and Spatiotemporal Structure

Partial differential equations describe changes in fields distributed over space and time, frequently in relation to local derivatives, boundary conditions, and conservation or transport mechanisms. The data-driven discovery of PDEs extends the inverse problem to spatiotemporal measurements. Rudy and colleagues developed a sparse-regression procedure for selecting candidate PDE terms from observed data [36]. This work demonstrates that equation discovery can concern spatially structured temporal evolution as well as finite-dimensional time series. It also clarifies the significance of the measurement regime: spatial resolution, temporal discretization, derivative estimation, noise, and boundary information affect which terms can be supported.

Several distinctions matter when such methods are situated within a relational-spacetime framework. A spatial coordinate used to index a measured field need not coincide with a fundamental ontological coordinate. A PDE reconstructed over a fixed domain may cease to be adequate if the relevant domain, boundary conditions, or relational topology changes. Conversely, changes in a recorded field may result from an altered observation procedure rather than an alteration of the generating dynamics. These possibilities require separate hypotheses and evidence. They become especially salient when a system is represented by a changing network or cell complex instead of a predetermined continuous manifold.

Topological information can nonetheless be an important source of constraints. Components, cycles, incidence relations, and changes in attachment may restrict admissible reconstructions even when the available numerical measurements are sparse. Such restrictions do not automatically determine a dynamical law. Their value depends on a well-specified connection between the recorded relational structure and the observables to which the proposed equation applies. Accordingly, the later formalism treats topology as a possible component of historical evidence and model admissibility, with the particular meaning of each higher-dimensional primitive specified locally.

The term *spacetime* in the present title therefore denotes a posited evolving relational reference domain for the inquiry. It does not assert that all scientific systems share a common physical spacetime description, nor that a partial differential equation and a discrete rewrite grammar are interchangeable. Their potential correspondences must be established through explicit observables, boundary conditions, transformation rules, and domains of validity.

2.3 Variational and Action-Based Representations

Variational formulations offer another influential representation of generative dynamics. In suitable physical systems, an action functional determines stationarity conditions, and these conditions yield equations of motion. Inferring an action or Lagrangian from observed trajectories may be useful when variational structure, symmetries, or conservation properties provide meaningful constraints. Recent computational work has explored learning Lagrangian representations using parameterized models; Lagrangian Neural Networks are one example [12]. The connection with knowledge pressing concerns the possibility of reconstructing a compact generative principle from temporally distributed consequences under specific physical assumptions.

A simple example exhibits an important limit of inverse recovery. For generalized coordinates q , a sufficiently smooth function $F(q, t)$, and a Lagrangian L , consider the transformed Lagrangian in

Equation 1.

$$L'(q, \dot{q}, t) = L(q, \dot{q}, t) + \frac{d}{dt}F(q, t). \quad (1)$$

Under fixed-endpoint variations and the usual differentiability assumptions, the added term changes the action only by the endpoint contribution $F(q(t_1), t_1) - F(q(t_0), t_0)$. The Euler–Lagrange equations are consequently unchanged. Equation 1 gives an elementary instance in which distinct analytical expressions generate the same local equations of motion. The permissible transformation is mathematically well characterized in this setting; analogous claims about other model families would require their own derivations.

The example matters because a recovered generative representation may be determined only up to a specified equivalence relation. Even an extensive trajectory record can be consistent with multiple symbolic expressions that have identical tested consequences. A responsible inverse framework should therefore retain the type of equivalence at issue, such as gauge freedom, observational indistinguishability, change of coordinates, or restricted-domain approximation, rather than treating all representational differences as evidence of different mechanisms.

Variational principles also have a carefully bounded role outside physics. In some domains, objective functions and optimization constraints may provide useful accounts of choices or transformations. In institutions and human communities, however, inferred optimization criteria need not coincide with participants’ reasons, legitimate obligations, or shared normative standards. An action functional is an instructive member of a heterogeneous representation family; it is not proposed here as a universal explanatory language for social agency.

2.4 Grammar Induction, Program Learning, and Structural Rewriting

Grammatical inference seeks rules governing the production or recognition of structured sequences. Gold’s investigation of language identification in the limit established a classical framework for examining what a learner can identify from specified forms of evidence over time [19]. Angluin’s work on learning regular sets through membership queries and counterexamples demonstrates the additional possibilities opened by structured interaction with an informative teacher [4]. Such results are conditional on the target language class, access to a defined kind of oracle, and the adopted success criterion. Their importance here lies in making the epistemic resources for rule inference explicit.

Program induction extends the generative perspective from linguistic sequences to structured concepts and procedures. Lake, Salakhutdinov, and Tenenbaum demonstrated probabilistic program induction for concept learning under a specific representational scheme [21]. In a related but distinct line, computational mechanics investigates structural characteristics of processes through their observed histories and predictive organization [14]. These traditions motivate the study of candidate mechanisms that can generate more than a memorized collection of observations. The inferred objects nevertheless vary in type: a regular-language recognizer, a probabilistic drawing program, and a predictive state model provide different explanatory and operational capacities.

For relational systems, the analogous representation may be a graph or higher-dimensional rewrite scheme. One rule might introduce a new coupling between previously unconnected events; another might transform a local incidence configuration or attach a higher-dimensional face. In the two-complex interpretation used in the companion inquiry, vertices correspond to events and edges to couplings; the interpretation of faces depends on the specified construction. An entity or subject may emerge through a historically constituted substructure rather than being stipulated as a primitive vertex. This vocabulary permits illustrations of topology-changing continuation without asserting that every such change is governed by a uniquely identifiable rewrite grammar.

Grammatical inference also introduces a central distinction between passive and interactive epistemic access. A record of positive examples may support a different collection of candidate grammars than a setting in which counterexamples or targeted membership queries are available. In a physical experiment, the corresponding questions concern interventions and additional measurements; in social interpretation,

they may concern dialogue, observation of practice, or requests for explanation. The legitimacy, feasibility, and risks of these interactions vary by domain. The later treatment of translation therefore allows epistemic participation while retaining explicit constraints on what kinds of inquiry are permissible.

2.5 Causal Representation and Historically Changing Mechanisms

Causal inference asks how observed dependencies relate to mechanisms, interventions, and counterfactual consequences. The distinction matters because temporal succession or predictive association does not itself establish a causal account. Peters, Janzing, and Schölkopf present a systematic treatment of causal models and inference using observational and interventional information [32]. Schölkopf and collaborators identify causal representation learning as a further challenge: the variables suitable for causal analysis may themselves need to be learned from lower-level observations [40]. These works provide relevant conceptual resources for separating recorded events, inferred causal variables, and the assumptions required to interpret an intervention.

Historical generative inference adds a question about the stability of those assumptions over time. A mechanism can change through parameter drift, altered constraints, changing interaction structures, or a modification of the effective variables through which its behavior is represented. Bayesian online change-point detection offers one established way to examine abrupt variations in generative parameters under a specified probabilistic model [1]. Detecting a statistical change, however, does not on its own discriminate among a physical rule change, a change in measurement, a new population composition, or an altered conceptual vocabulary. Such distinctions call for independent evidence and for models that describe the observational interface as well as the putative generative process.

The historical constitution of the observer is consequently important. What counts as a state variable, valid measurement, meaningful category, or admissible intervention may be revised through scientific discovery or institutional change. An improved representation of a persistent mechanism can appear in the historical record as a change of scientific theory. Conversely, a genuinely evolving mechanism may be concealed by a representation that averages over distinct regimes. An epistemic model should remain open to both possibilities and avoid conflating its own revisions with changes in the system being studied.

For the present research programme, causal representation offers a way to ask what candidate models might imply for action-conditioned continuations. Such implications remain indexed to the assumptions of the causal model, observational coverage, and permitted interventions. When actions alter the rules of subsequent interaction, model revision and continuation inference become coupled problems. The later formal sections will distinguish model-admissible possibilities from physically realizable developments and the options practically accessible to particular subjects.

2.6 Hermeneutics, Incommensurability, and Epistemic Translation

Historical understanding and translation have long been studied outside mathematical inference. Philosophical hermeneutics gives particular attention to the historical situatedness of understanding, the formation of interpretive horizons, and the role of dialogue. Gadamer's account, as surveyed by Malpas, treats interpretation as embedded in historically mediated practices rather than as the mechanical decoding of fixed meanings [24]. Such accounts are pertinent when the objects of inquiry include institutional practices, human reasons, legal traditions, and the ways communities constitute their own categories. They should be engaged as substantive intellectual predecessors rather than retroactively reconstructed as instances of a contemporary algorithm.

Quine's analysis of translation draws attention to the possible indeterminacy of interpretation under available behavioral evidence [33]. The literature on incommensurability in science further distinguishes failures of direct translation from the possibility of comparison or understanding across historically different conceptual taxonomies [30]. In the later development of Kuhn's position, learning to operate within another scientific lexicon is a different achievement from replacing each of its terms with an equivalent

in one's original language. This distinction resonates with the paper's interest in inferring another system's generative grammar, but the resonance is conceptual; it supplies neither an identification theorem nor a guaranteed method for constructing a cross-system mapping.

The term *translation* is therefore used in a deliberately plural and context-indexed manner. In a mathematical setting, translation might preserve selected operations, observables, boundary conditions, or predicted transitions. In an institutional setting, it might support an intelligible account of procedures, permissions, legitimate authorities, and experienced consequences. In interpersonal dialogue, the process may modify the interpretive capacities of the participants. Each setting provides different criteria of adequacy, and practical success in one does not establish semantic completeness in another.

Power and recognition also matter. An institution may classify another community's practices according to its own available categories while leaving important distinctions unexpressed. Greater predictive accuracy does not confer interpretive authority, and a shared historical record does not guarantee equal participation in defining its meaning. For this reason, the later translation formalism will represent partial correspondence, evidence, disagreement, and identifiable remainders as central outcomes. It also permits a translation process to contribute to new, jointly developed practices, with the legitimacy of those practices evaluated in their relevant contexts.

2.7 Generative Continuation, Agency, and Free Will

The preceding traditions concern reconstruction and interpretation, but generative models also invite investigation of subsequent relational development. An inferred differential equation can support conditional trajectories; a grammar can generate structures that have not yet appeared in a record; a causal model can organize intervention-dependent outcomes; and a practical rule system can delimit institutionally available actions. These are different senses of a *possible continuation*. Their formal relationship must be specified rather than inferred from a common vocabulary of futures.

Continuation is especially significant when generative capacities evolve. A scientific discovery may create new feasible interventions, while a legal change can transform the institutional availability of a previously conceivable action. New coordination practices may permit kinds of cooperation that existing participants could neither anticipate nor implement. In such cases, future development is partly conditioned by transformations of the processes that define or enable later options. The historical record and the set of candidate continuations therefore belong to one evolving inquiry. The companion two-complex study offers a provisional formal treatment of reconstructed whole-spacetime extensions; the present paper will examine how heterogeneous generative representations and their translation may inform that programme, without simply repeating its earlier constructions.

The connection to agency is philosophical as well as technical. Barandiaran, Di Paolo, and Rohde examine agency in terms that include individuality, interactional asymmetry, and normativity [6]. Their account illustrates why observable action alone may provide an incomplete description of agential organization. A relational investigation can ask how subjects and their capacities emerge through histories of dependence, coordination, conflict, and self-regulation, and how those capacities constrain or enlarge the continuations accessible to them.

Questions of free will require further care. The philosophical literature distinguishes alternative possibilities, the nature of control, self-determination, and moral responsibility [29]. Multiple branches in a computational model may express uncertainty, model-relative alternatives, or action-conditioned paths. Those branches do not independently establish the metaphysical openness of the future or the freedom of an agent. Likewise, a deterministic generative equation alone does not settle the compatibility of freedom and determinism. The investigation can examine how knowledge changes practical agency while treating the larger metaphysical controversy as an open philosophical boundary.

This separation is not merely terminological. A subject may correctly infer that an action is physically conceivable while lacking the resources, permission, bodily capacity, or collective support needed to realize it. A model may identify a predictable behavior without representing the subject's reasons for acting. In a historically constituted relational setting, freedom as practical capability and freedom as a

Table 1: Selected disciplinary foundations and the inferential boundaries relevant to knowledge pressing.

Research tradition	Typical epistemic object	Key boundary for the present investigation
System identification and PDE discovery	Dynamical equations conditioned on measurements and model libraries	Partial observability, model adequacy, changes of domain or variables
Variational inference	Action or Lagrangian representation under physical assumptions	Equivalence of representations, boundary conditions and variational scope
Grammar and program learning	Structural generation rules or executable procedures	Identifiability under specified data and query access
Causal representation learning	Variables, mechanisms and intervention consequences	Assumed interventions, latent variables and changing mechanisms
Hermeneutics and translation	Historically situated meaning, interpretive practices and conceptual correspondence	Contextual understanding, incommensurability and normative authority
Agency and continuation	Capacities for action and admissible prospective developments	Model-relative possibility, practical accessibility and metaphysical claims

metaphysical property of action require different analyses. Their possible connections warrant investigation, but claims about one should not be imported as proofs of the other.

2.8 Research Position and Methodological Boundaries

The reviewed traditions supply distinct forms of generative inference, translation, and continuation. Table 1 summarizes their principal objects, sources of epistemic constraint, and the additional questions they pose for the present investigation. The table is an analytical orientation, not a hierarchy of methodological merit or a claim that every tradition can be represented faithfully by the same formal operator.

The proposed *knowledge pressing* vocabulary has a specific and limited role: it groups a set of questions concerning the historically situated acquisition, revision, translation, and practical use of generative representations. It is not presented as a standard term in the surveyed fields. This grouping may prove useful if a common research design can expose interactions that separate disciplinary accounts leave implicit, especially between changing model classes, translation across systems, and action-conditioned continuation. Its value will depend on precise definitions and discriminating examples, rather than the breadth of its terminology.

The paper’s principal methodological commitment is to preserve distinguishable explanatory levels. A posited relational reference domain provides a convenient way to formulate events, couplings, and higher-order structures. The historical record supplies observer-indexed evidence about such structures. Candidate generative models organize possible mechanisms or observable consequences. A system’s internally constituted grammar concerns its actual practices of recognition and action. Translation identifies conditional correspondences between representations. Continuation concerns the constraints and possibilities of further relational development. Agency introduces questions of control, authorization, participation, and normativity. These levels can interact without being identified as the same object.

This position also gives a concrete meaning to the paper’s exploratory character. Sections 3–10 will propose definitions and examine their logical consequences under stated assumptions. The computational programme in Section 11 will require explicit generative testbeds, baselines, and failure cases. No results of those prospective tests are asserted in the present background section. The ensuing investigation begins with the historical and epistemic objects needed to formulate these questions precisely, carrying forward the established achievements and the irreducible limits of the traditions surveyed here.

3 Knowledge Pressing in Evolving Relational Spacetime

This section establishes the reference structures, observational distinctions, and provisional epistemic vocabulary used in the formal investigation. Its objective is to characterize knowledge pressing as an historically constituted activity within relational development, with attention to both the reconstruction of generative processes and participation in their possible continuation. The construction proceeds through a posited relational-spacetime reference, observer-indexed records, historically formed epistemic capacities, and a conditional account of pressing and continuation. Finite two-complexes provide a tractable structural example. Their use is a choice of mathematical representation, while differential, grammatical, probabilistic, and institutional descriptions remain available for systems whose phenomena require different resources.

3.1 Relational-Spacetime Reference and Historical Structure

A relational-spacetime reference supplies a common formal location for asking how events are connected, how interactions develop, and how an observer's history is embedded in the domain being investigated. For the finite cellular illustration, a reference history is a family of decorated two-dimensional cell complexes indexed by a partially ordered set of observation-independent historical indices. Vertices stand for events, edges for couplings among events, and faces for higher-order primitives whose interpretation depends on the application. In particular, a vertex does not denote an individual person or a self-sufficient agent. Subjects and institutions, when represented, have historically distributed support across parts of the relational structure. Likewise, face incidence is a combinatorial property; the semantic or physical interpretation of a face remains a separate modeling decision.

Definition 3.1 (Finite relational reference history). Let $(T, \preceq)$ be a partially ordered index set. A *finite relational reference history* over T consists of a family of finite two-dimensional CW complexes $K_{\leq t}$, cellular embeddings $\iota_{s,t} : K_{\leq s} \hookrightarrow K_{\leq t}$ for $s \preceq t$, and contextual decorations λ_t . For comparable indices, the embeddings satisfy the coherence relations in Equation 2.

$$\iota_{t,t} = \text{id}_{K_{\leq t}}, \quad \iota_{r,t} = \iota_{s,t} \circ \iota_{r,s} \quad (r \preceq s \preceq t). \quad (2)$$

The complexes record accumulated incidence, while the decorations may include activity status, attributed states, interval information, admissible relations, or application-specific measurements. The family is denoted by $\mathcal{R} = (T, \preceq, K, \iota, \lambda)$.

The embeddings of Definition 3.1 encode accumulation within a formal history, not monotonic growth of a currently active network. A coupling that ceases to operate may remain represented as a past event or relation; its cessation can be expressed through a later decoration. The active relational configuration at t can consequently differ in topology from its earlier configuration even when the cumulative complex embeds into later complexes. For an application in which removal or revision of cellular incidence itself is essential, one can replace embeddings by more general history morphisms, spans, or event-sourced replacements. Section 3 uses the embedding case to keep observational and continuation issues explicit without asserting that every relational history possesses this particular structure.

The order $\preceq$ is an index of historical accessibility or precedence specified by the model. It need not be total, metrically calibrated, or identical to physical causation. Edge orientation, where supplied, can indicate a convention for incidence or coupling and by itself does not establish a causal relation. A PDE-based representation would additionally require spaces, differential operators, boundary data, and regularity conditions; an action-based representation would require an appropriate class of fields, paths, or configurations. The relational reference is therefore a flexible carrier of questions about constitution and development, rather than a universal equation governing disparate domains.

Ontological suspension governs the interpretation of $\mathcal{R}$. A posited reference may be useful for reasoning about histories that precede an observer's access, but its existence as a mathematical object does not establish that reality is exhaustively cellular or that the complete historical domain is available for

inspection. The reference is an explicitly assumed domain against which epistemic constructions can be compared. In a concrete study it must be replaced, restricted, or enriched according to the available evidence and justified modeling assumptions.

3.2 Historically Constituted Epistemic Systems

Epistemic access takes place through systems whose capabilities arise within histories. Instruments, languages, disciplinary standards, memory, institutional authorizations, and learning procedures affect what an observer can register and what it can infer. The observer may itself be constituted through interacting biological, social, technical, and normative processes. The formal analysis consequently uses an observer index a and does not identify an epistemic system with a primitive vertex of $K_{\leq t}$.

For a given observer a at t , an epistemic configuration is provisionally described by Equation 3.

$$\mathcal{E}_{a,t} = (\mathcal{L}_{a,t}, \mathcal{O}_{a,t}, \mathcal{I}_{a,t}, \mathcal{Q}_{a,t}, \mathcal{A}_{a,t}, \mathcal{N}_{a,t}). \quad (3)$$

Here $\mathcal{L}_{a,t}$ is the collection of representational languages or categories available to the observer; $\mathcal{O}_{a,t}$ denotes observation procedures; $\mathcal{I}_{a,t}$ denotes inferential procedures; $\mathcal{Q}_{a,t}$ denotes procedures of inquiry, criticism, and revision; $\mathcal{A}_{a,t}$ describes available modes of action or participation; and $\mathcal{N}_{a,t}$ specifies operative norms, resource constraints, and authorization conditions. These components are analytical roles, not independent modules established for every actual subject. In particular, the observer's own practical grammar may be internally contested or incompletely articulable in $\mathcal{L}_{a,t}$.

A historically constituted epistemic system can be modeled as having a support $S_{a,t}$ distributed across the relational reference, together with the configuration $\mathcal{E}_{a,t}$. The support need not be a disjoint subcomplex and need not uniquely individuate a subject. Its role is to make dependence on relational history expressible: changes in instruments, education, available sources, or institutional position can change the capacities represented by Equation 3. Likewise, the epistemic system can alter its own conditions of access by developing new questions, measurements, or practices. These are possibilities for joint historical analysis, not a determination of metaphysical personhood.

The historical constitution of an observer and its epistemic self-description must also be distinguished. An observer may reconstruct parts of the processes that constituted its capacities without possessing an exhaustive record of those processes. An AI system, a scientific community, and an individual investigator can all be considered under appropriately specified observational and interpretive models, yet the adequacy of each model must be evaluated in its own domain. Predictive success concerning an agent's outward behavior alone does not establish knowledge of that agent's lived meaning, self-understanding, or normative authority. Such distinctions will become important for inter-system translation.

3.3 Epistemic Records, Provenance, and Observational Mediation

A historical reference and an accessible historical record play different roles. Let $D_{a,\leq t}$ denote the observations, reports, and representations available to epistemic system a by t . The production of $D_{a,\leq t}$ depends on accessible events, instruments, selection practices, memory, recording technologies, and interpretive categories. A schematic observation relation is given in Equation 4.

$$D_{a,\leq t} \sim \mathcal{O}_{a,t}(\mathcal{R}_{\leq t}; \mathcal{E}_{a,t}, \zeta_{a,t}). \quad (4)$$

The notation $\sim$ allows the observation mechanism to be stochastic, incomplete, or set-valued, with $\zeta_{a,t}$ collecting unmodeled disturbances and selection factors. In a fully specified probabilistic model this notation would require a likelihood or probability kernel; in a qualitative study it may stand for a documented interpretive process. Equation 4 is a schema for indexing the dependence of records on observation, rather than a claim that a unique observable map is available.

A practical recorded item can retain several distinctions. A minimal evidence entry d_i is defined for bookkeeping in Equation 5.

$$d_i = (c_i, \tau_i^{\text{target}}, \tau_i^{\text{record}}, s_i, p_i, m_i), \quad (5)$$

Table 2: Analytical levels in the preliminary knowledge-pressing framework. The table distinguishes a posited relational reference from records, models, practices, and candidate continuations; these levels need not correspond to independent physical components.

Level	Analytical role	Principal limitation or question
Relational reference	Supplies a posited domain of events, couplings, and higher-order relations	Its ontological adequacy is assumed only conditionally
Epistemic record	Retains accessible observations, testimony, transformations, and provenance	Coverage, selection, source access, and interpretation
Candidate generative model	Explains or reproduces aspects of recorded development	Identifiability, adequacy, model-class dependence
Internal and practical grammar	Organizes a system’s recognized meanings, norms, and admissible actions	Authority, contestation, situated self-understanding
Candidate continuation	Represents possible extensions and transformations under stated assumptions	Modal interpretation, interventions, accessibility, realization

where c_i is the recorded content or referent, τ_i^{target} denotes the time or temporal interval to which the content refers, τ_i^{record} denotes when it was entered or made accessible, s_i identifies the source where such identification is lawful, p_i records provenance or transformations of the source, and m_i states the relevant observation method and qualifications. The tuple in Equation 5 is an illustrative schema, not a mandatory storage format. It helps distinguish the time of a reported event from the time of recording, and a primary observation from an interpretation circulated through several intermediaries.

Provenance does not certify truth on its own. Different observers may produce records about the same episode that are partially overlapping, differently categorized, contradictory, or inaccessible to one another. Privacy and institutional authorization may also make deliberate non-disclosure appropriate. A global repository, even if widely adopted, would consequently produce a distributed evidential environment rather than an unmediated copy of relational spacetime. Existing discussions of scientific underdetermination reinforce the distinction between evidence, model choice, and the breadth of conclusions licensed by observations [42].

The layers relevant to the present framework are distinguished in Table 2. They are related through practices of observation, inference, and intervention, while remaining analytically distinguishable.

3.4 Knowledge Pressing as a Historically Situated Process

Knowledge pressing is provisionally used here to name epistemic activity through which evidence is encountered, generative representations are proposed and revised, correspondences across heterogeneous systems are investigated, and representations become involved in subsequent inquiry and action. The phrase is a working theoretical designation. Its utility depends on whether it helps separate and connect identifiable processes beyond vocabulary already supplied by archival research, historiography, system identification, scientific modeling, and interpretation. A practical instance may involve a scientist reconstructing dynamics, a mediator learning another institution’s operating grammar, or a community revising its understanding of a changing ecological system.

Let $\mathfrak{M}_{a,t}$ be a family of candidate generative representations entertained by observer a at t . A local pressing step may be represented as a conditional epistemic update, as in Equation 6.

$$P_{a,t}^c : (D_{a,\leq t}, \mathcal{E}_{a,t}, \mathfrak{M}_{a,t}) \rightrightarrows (\mathfrak{M}'_{a,t}, \mathcal{E}'_{a,t}, \mathfrak{U}_{a,t}^c), \quad (6)$$

where c specifies the inquiry context and $\mathfrak{U}_{a,t}^c$ is a family of proposed questions, translations, and admissible epistemic or practical interventions. The multivalued arrow permits uncertainty and competing

updates. An output $\mathfrak{M}'_{a,t}$ can remain a heterogeneous set of grammars, differential or variational models, programs, or socially situated rule descriptions. Equation 6 supplies an organizational interface for the later formal developments; it neither specifies a universal learning algorithm nor guarantees that a candidate representation is true.

The components of pressing may occur together or in varying orders. Observation may be designed in response to an emerging model; a proposed translation may expose missing evidence; dialogue with another system may alter the categories used in the original inference. The historical formation of the epistemic operator is consequently part of the inquiry. Its capacities, admissibility rules, and objectives can be revised through learning, institutional change, and new technical resources. Research on predictive structure inference and grammatical learnability provides established precedents for examining model recovery under stated observational regimes [14, 19]. The present inquiry seeks to keep the broader coupling between inferential, interpretive, and participatory processes visible while developing domain-specific models where warranted.

An epistemic update can remain internal to an observer's records and practices. It can also have external consequences when a translated interpretation changes communication, an experiment changes the observed system, or a collective decision reorganizes available relations. For this reason, knowledge pressing is considered within a coupled evolution of reference and epistemic configurations. One may write the schematic relation in Equation 7.

$$(\mathcal{R}_{\leq t}, \mathcal{E}_{a,t}) \xrightarrow{F_{a,t}^{c,u}} (\mathcal{R}_{\leq t'}, \mathcal{E}_{a,t'}), \quad t \preceq t', \quad (7)$$

where u indexes admissible actions or interactions and $F_{a,t}^{c,u}$ denotes a context-dependent relation of possible joint developments. Equation 7 is not proposed as a universal dynamical law. When no causal interaction with the investigated system occurs, the epistemic component may change while the represented external process evolves independently. When interaction occurs, the effects require their own empirical or formal justification. Future pressing may also revise the historical interpretation of the earlier process, even when the earlier occurrence is taken as fixed.

3.5 Historical Constitution and Relational Continuation

Historical constitution and future continuation belong to one evolving relational process, while an observer usually knows the process through asymmetric access to records. An observer may hold evidence about earlier events, encounter presently active relations, and construct candidate continuations conditioned on its generative models and available actions. Such a temporal distinction serves the epistemic problem and does not commit the framework to an ontological separation between past and future. The meaning of “future” here is indexed to the standpoint and temporal structure of the model.

For the cellular example, an extension retains a coherent embedding of the reconstructed historical complex into a larger complex containing additional cells or new decorations. The basic admissible-continuation object is stated in Definition 3.2.

Definition 3.2 (Model-indexed relational continuation). Given an observer-indexed reconstruction $\hat{K}_{a,\leq t}$, a candidate generative model $M \in \mathfrak{M}_{a,t}$, and a context c , a *model-indexed continuation* is a tuple $(K^+, j, \lambda^+, M, c)$ in which K^+ is a candidate extended complex and $j : \hat{K}_{a,\leq t} \hookrightarrow K^+$ is a cellular embedding compatible with the stipulated historical incidence and with the admissibility conditions of M in context c . The modeled family of these tuples is denoted $\text{Cont}_{a,t}(M; c)$.

The embedding in Definition 3.2 is a formal compatibility condition with the reconstructed history. It need not imply that later active configurations preserve earlier relations, since λ^+ may mark transformations or cessation. If a generative mechanism changes during continuation, the tuple may be extended by a sequence of models $(M_s)_{s \succeq t}$ with explicit transition conditions. The continuation is then a *whole-spacetime* construction whose future components remain attached to a marked epistemic history, following the design of the preceding two-complex investigation. In particular, a continuation model describes

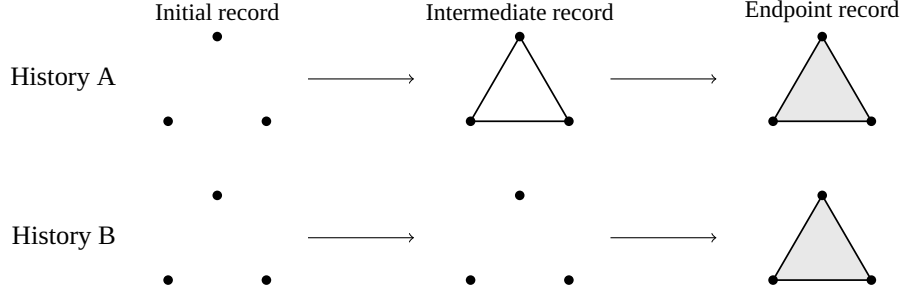


Figure 1: Two constructed histories with indistinguishable endpoint incidence and different intermediate structures. Black points indicate events, edges indicate couplings, and pale fill indicates the attached higher-order primitive f . Cell incidence is a modeling convention; the figure does not assign actors to vertices or infer a causal arrow from the drawing.

a family of admissible extensions; it does not identify a realized future solely from the topology of the historical complex.

Several modalities must be treated separately. An observer may be able to describe a continuation without having an adequate model supporting it. A model may admit a continuation that is physically unrealistic under unmodeled constraints. A physically realizable development may remain inaccessible to a particular individual because of insufficient resources, coercion, or institutional limitations. Finally, the eventual realized development is an historical occurrence, with its own subsequent record. Comparisons among these classes require a shared domain and explicit criteria; an unconditional nesting of epistemically conceivable, model-admissible, physically realizable, and practically accessible continuations would be unjustified.

Knowledge pressing may transform the conditions that shape these possible continuations. Inference can reveal previously unrecognized options, translations can enable coordination across institutional grammars, and new practices can alter existing constraints. The effects may also close possibilities, concentrate authority, or introduce dependencies. The investigation therefore asks both which future structures a specified model permits and how the processes establishing those permissions or capacities may themselves evolve. The formal study of these transformations is developed in the later continuation section; the present subsection establishes the modal and historical distinctions on which it will depend.

3.6 An Illustrative Relational-History Ambiguity

A small cellular example makes visible how historical information constrains generative interpretation. Consider three event vertices v_0, v_1, v_2 . At a later observed endpoint, edges e_{01}, e_{12}, e_{20} and a two-cell f form a filled triangular disk whose face attaches along the cycle. Two hypothetical histories can yield that same terminal complex. In history A, the three edges have appeared by an intermediate index, while the face is attached only later. In history B, the vertices remain unconnected at the intermediate index and the edges and face appear together at the endpoint. Figure 1 gives a schematic comparison.

At the intermediate index the first homology differs, under the specified cell structures, as stated in Equation 8.

$$H_1(K_{\text{mid}}^A; \mathbb{Z}) \cong \mathbb{Z}, \quad H_1(K_{\text{mid}}^B; \mathbb{Z}) = 0, \quad K_{\text{end}}^A \cong K_{\text{end}}^B. \quad (8)$$

History A has an unfilled cycle at the intermediate index; history B has only isolated vertices there. The shared endpoint is a filled disk, with vanishing first homology. Consequently, endpoint incidence alone is insufficient to distinguish these two specified construction histories, whereas an accurate intermediate observation could discriminate between them. Equation 8 illustrates a restricted identifiability issue. Even the fully observed sequence might admit different generating algorithms, intentions, or causal descriptions. Historical resolution improves the available constraints without automatically identifying a unique generative law.

The same example indicates why epistemic missingness should not be inferred from topological holes. The unfilled intermediate cycle in history A is an actual feature of the posited cellular structure. A missing report about whether the face was present would instead be a limitation of $D_{a,\leq t}$. Only a justified observation or inference model can connect those two claims. The distinction is essential when topology is used to support explanations of scientific data, archived processes, or social relations.

3.7 Epistemic Plurality, Translation, and Normative Scope

A shared relational reference does not produce a shared interpretive grammar. Epistemic systems can differ in the kinds of observations they recognize, the models they regard as adequate, the norms governing valid claims, and the forms of action authorized by their institutions. The same event sequence may be interpreted through differential dynamics, a generative grammar, a procedural account, or a participant's historically developed vocabulary. Some correspondences may be established through common observations and interaction. Others may remain partial, context-dependent, or contested. Established problems of scientific incommensurability and interpretation provide a relevant background for this research direction [30, 24].

For two systems a and b , the preliminary object of translation is a family of context-indexed relations among candidate representations and practices, supported by recorded encounters between the systems. It may include a common observable, a relation between state transitions, a partial mapping of production rules, or an explanation of the institutional conditions under which a concept operates. This preliminary conception does not posit a privileged metalanguage that settles the meanings of the participating systems. Later formal sections will specify how candidate correspondences can be evaluated for selected properties, including the preservation of relevant generative consequences and the identification of remainders.

A hypothetical care-planning example illustrates the difference between shared events and shared meanings. A clinical team may use physiological stability to support a proposed discharge; a family may evaluate the available resources and obligations for daily care; and an administrative service may consider eligibility and procedural authorization. A shared historical record may help each party reconstruct relevant developments, while the three grammars continue to assign different practical implications to "readiness." Translation may support a jointly revisable care arrangement, but a predictive model of one party's actions does not confer authority to define that party's obligations. The example is illustrative; the effectiveness of any such intervention requires evidence from actual practice.

The formation of a bridging representation can itself alter subsequent relations. A new shared description may make a joint activity feasible, introduce new obligations, or redistribute interpretive authority. These changes belong to the continuation of the relational system and must be investigated in their material and normative context. In particular, a translation that improves coordination for one institution may diminish participation or agency for another. Knowledge pressing therefore includes attention to who can challenge a record, correct a model, authorize a practical translation, and contest the consequences of its use.

The conceptual structure established in this section distinguishes a posited relational history, observer-indexed evidence, candidate generative representations, internally constituted grammars, and model-conditioned continuations. These objects form the starting vocabulary for the technical sections on generative model families, inference, evolving mechanisms, translation, continuation, and agency. Their separation also states a methodological commitment: explanatory adequacy, semantic intelligibility, practical capacity, and normative legitimacy each require appropriate grounds for assessment. The working concept of knowledge pressing is useful to the extent that it helps investigate how these distinct yet interacting dimensions participate in historically evolving relational spacetime.

4 Generative Representations of Relational Systems

This section specifies the representational objects on which historical inference and inter-system translation may operate. Its objective is to compare generative grammars, differential dynamics, variational principles, stochastic models, executable programs, and practical or normative grammars while retaining the distinctions among their semantics. A minimal typed interface first makes the models comparable at the level of admissible histories, observations, and candidate continuations. The subsequent subsections develop representative constructions, including a cellular attachment example, and then examine hybrid descriptions, evidential adequacy, and equivalence relative to a selected question. The formal interface is an analytical proposal for the present discussion paper. It is neither a universal syntax for all knowledge nor evidence that every historically constituted practice admits a complete mathematical generator.

4.1 Candidate Representations and Typed Model Interfaces

A *generative representation* is taken here to be an explicitly delimited description of how selected configurations, events, relations, or practices can be produced or transformed under specified conditions. The expression includes models of physical and computational processes and formal descriptions of institutionally regulated practices. Its adequacy is relative to the modeled phenomena. A generative representation may specify a differential law, a production relation, a probability distribution over trajectories, or a set of action-enabling and action-restricting conditions. These descriptions carry different meanings; their shared role in the investigation is to constrain accounts of possible histories or continuations.

Definition 4.1 (Typed generative representation). A *typed generative representation* is a descriptor M of the form specified in Equation 9.

$$M = (\tau_M, \mathcal{C}_M, \mathcal{U}_M, \mathcal{H}_M, \text{Gen}_M, \text{Obs}_M, \text{Adm}_M, \text{Int}_M). \quad (9)$$

Here τ_M identifies the model family and its mathematical type; $\mathcal{C}_M$ is a class of configurations; $\mathcal{U}_M$ contains inputs, environmental conditions, or intervention contexts; and $\mathcal{H}_M$ is a class of model histories or trajectories. The generator Gen_M assigns well-typed generative descriptions to configurations and contexts, which may be trajectories, sets of traces, evolution operators, or probability laws. The observation interface Obs_M relates model histories to evidential records; Adm_M states domain restrictions and admissibility conditions; and Int_M records the interpretation, units, or context in which the model's objects are used. The latter component may be partial or contested.

The type marker τ_M is substantive. A PDE model requires functional spaces and boundary conditions, whereas a probabilistic kernel requires measurable spaces; a rewrite system needs typed matching conditions and a rule application relation. Accordingly, Equation 9 is a *family of interfaces*, not a claim that Gen_M has the same codomain for each model. In deterministic cases, the generator can yield a single trajectory conditional on initial data; in nondeterministic cases it may yield a set of trajectories; in stochastic cases it yields a law over trajectories. These outputs can be compared only after an appropriate observation or interpretation interface has been specified. Similarly, Int_M cannot be treated as an algorithm that settles the meaning of a community's practices independently of its participants.

The distinction between a postulated reference structure, an observer-indexed record, and a candidate explanatory representation in Section 3 is retained. The reference history $\mathcal{R}$ supplies an optional domain of comparison; the record $D_{a, \leq t}$ supplies what an observer has actually registered; and M is one possible device for explaining selected features of those records. The same record may admit distinct models of different types, and a model may be informative for a limited class of questions without describing the entire system. A type-aware registry for inference would consequently maintain separate evidence links, domains, assumptions, and revision histories for each M .

Table 3 summarizes what the major model families generate and what additional information is required before their outputs can be interpreted as statements about a relational history. The columns deliberately distinguish the generated object from its epistemic conditions of use.

Table 3: Representative generative model families and their epistemic interfaces. The descriptions are examples of admissible model types rather than exhaustive definitions of the corresponding disciplines.

Family	Generative output	Required interpretation or evidence
Rewrite grammar	Derived strings, structures, or cell-complex transitions	Rule matching, admissibility, event and cell semantics
ODE/PDE	State or field trajectories	Variables, units, domain, initial and boundary data
Action principle	Stationary configurations or histories	Fields, variations, boundary terms, physical interpretation
Stochastic model	Probability law over transitions or histories	Sample space, observation likelihood, conditioning context
Program or automaton	Computed outputs and execution traces	Types, execution semantics, environment, termination or resource limits
Practical or normative grammar	Conditions for authorized or interpretable acts	Institutions, exceptions, participants' interpretations and actual compliance

4.2 Generative and Relational Rewrite Grammars

Grammatical representations describe construction through admissible production rules. For a conventional formal grammar, a starting symbol and a set of productions induce a derivation relation on strings or trees. A relational rewrite grammar substitutes a typed relational object for the string and rules governing local configurations for string substitutions. An elementary schematic rule is expressed in Equation 10.

$$(K, \lambda) \xrightarrow[\mu]{\rho} (K', \lambda'), \quad \rho : L \rightsquigarrow R. \quad (10)$$

The rule ρ names a local replacement or attachment, while μ denotes an admissible match of its left-hand pattern L in the current complex K . The decorations λ and λ' may register labels, active status, or temporal information. The semantics of the rule must say which incidences can be preserved, which cells can be introduced, and whether the transformation records deletion or merely deactivation. A sequence of such applications constitutes a derived model history, subject to the specified matching restrictions. Grammar induction and identification of languages from presented information have long-established formal precedents [19, 4]; learning graph or cellular rewrite rules constitutes a related but technically distinct task.

A finite cellular example makes the distinction between a production and its observation concrete. Consider vertices v_0, v_1, v_2 with oriented edges e_{01}, e_{12}, e_{02} . A rule can attach a two-cell f to the existing triangular cycle. Under the usual cellular orientation convention, its boundary is given in Equation 11.

$$\partial_2 f = e_{01} + e_{12} - e_{02}, \quad \partial_1 \partial_2 f = 0. \quad (11)$$

Attaching f may eliminate the homology class of that cycle while keeping the one-skeleton's edges present. In a model where f is a physically or socially meaningful primitive, its addition may also have consequences that ordinary one-dimensional network analysis cannot express. The exact interpretation of f remains domain-dependent: a cellular face is neither automatically an actor nor automatically a causal mechanism. Equation 11 is a finite combinatorial example, with no claim that the incidence relation alone reconstructs the history or motivation of the attachment.

Rewrite grammars are useful for representing possible *changes to the admissible form of a system*. An institutional arrangement may acquire a new procedure; a computational architecture may gain a new operation; a relational structure may permit a new class of cellular attachment. In such uses the reconstructed grammar may itself change, and the distinction between an observed rule change and a change in an investigator's categorization must remain open. Section 3 has already illustrated how distinct trajectories can lead to observationally similar endpoints. Rule discovery therefore requires histories rich enough to constrain the derivation process, together with an explicit account of unobserved intermediate steps.

4.3 Dynamical Systems and Partial Differential Equations

Differential descriptions are appropriate when the phenomenon under study admits meaningful state variables, differentiability assumptions, and continuous temporal or spatial evolution. A controlled finite-dimensional model may take the form in Equation 12.

$$\dot{x}(t) = f_{\theta}(x(t), u(t), t), \quad x(t_0) = x_0. \quad (12)$$

Here x is the selected state representation, u denotes externally specified inputs or interventions, and θ collects model parameters. The mathematical task of recovering f_{θ} from selected measurements depends on the observation map, excitation or variation in the data, noise, and admissible function class. Sparse model discovery provides a widely studied method under appropriate assumptions [9]. A recovered vector field describes selected dynamical relations in a chosen coordinate system; its interpretation as a fundamental law requires separate physical justification.

For spatially distributed phenomena, a model may act on fields $q(t, \xi)$ over a domain Ω . Equation 13 gives a general schematic form.

$$\partial_t q(t, \xi) = F_{\theta}(q, \nabla q, \Delta q, u; t, \xi), \quad \xi \in \Omega. \quad (13)$$

Its meaning depends on boundary and initial conditions, regularity, observability, and the definition of the differential operators. PDE discovery from spatiotemporal data has an established methodological literature [36]. A representation in the form of Equation 13 can describe field evolution and transport within a fixed domain, while the creation or deletion of cells, changing incidence relations, and historical redefinition of permissible operations may call for a coupled model. Moving boundaries, free-boundary problems, hybrid PDE systems, and topology-changing geometric formulations offer different possible choices; none is assumed to provide an unrestricted account of social or epistemic change.

Relational spacetime can enter differential modeling through several routes: relationally indexed state variables, coupling-dependent operators, fields defined on graphs or complexes, or changing interaction structures. These choices are mathematically distinct. For example, a graph Laplacian on a fixed vertex set requires an adjacency or incidence operator, whereas an evolving cellular complex requires the operators themselves to be indexed by the history. A model that adequately predicts a selected observable may nonetheless omit the constitution of the observational category, a regulatory constraint, or the reasons through which participants interpret their actions. Such omissions matter when differential descriptions are proposed as inputs to epistemic translation.

4.4 Action Functionals and Variational Representations

A variational representation specifies admissible histories or fields through an action and a stationarity condition. On a path space $\mathcal{Q}$ with prescribed endpoint conditions, one illustrative form is stated in Equation 14.

$$S[q] = \int_{t_0}^{t_1} L(q(t), \dot{q}(t), t) dt, \quad \delta S[q] = 0. \quad (14)$$

For a sufficiently regular Lagrangian and the usual fixed-endpoint variations, stationary paths satisfy the Euler–Lagrange equations. The action representation can encode symmetries and conservation properties, subject to the assumptions of the model. Learned Lagrangian parameterizations supply a concrete example of computational inference in this family [12]. In a field theory the action generally depends on fields and their derivatives over a spacetime domain, with corresponding boundary, constraint, and gauge conditions.

The inference of an action from observed histories presents an additional kind of underdetermination. The Lagrangian can change by a total time derivative without changing the Euler–Lagrange equations under the usual boundary assumptions. Scaling by a nonzero constant also preserves the stationary paths in the elementary unconstrained setting. Section 2 discusses this issue through a worked example. As a consequence, the data may identify a family of variational descriptions rather than a unique expression

for S . Inferences about physical interpretation, conserved quantities, or admissible continuation should respect the equivalence relation relevant to the inquiry.

Action-based models are especially valuable for our investigation as a *different representation of generative regularity*. They show that a generative account need not be an executable local production rule: global constraints on histories can sometimes define the candidate trajectories. The use of an action also makes the distinction between model-admissible histories and actually realized histories explicit. The mathematical concept of a stationary path is not, by itself, a theory of deliberate action or normative permission. Later discussion of agency will use separate structures for capacities, reasons, and practical constraints.

4.5 Stochastic Generative Models

Stochastic representations assign probability measures or transition kernels to selected states and histories. For a discrete-time Markov model, a transition kernel P_θ supplies the conditional law in Equation 15.

$$x_{n+1} \mid (x_n, u_n, c_n) \sim P_\theta(\cdot \mid x_n, u_n, c_n). \quad (15)$$

The conditioning variable c_n can record relevant contextual features, although its inclusion does not automatically make the Markov assumption appropriate. Given an initial law and the required measurability conditions, the kernel can define distributions over trajectories. Hidden-state and non-Markovian models require additional structure. Probabilities inferred from sparse or selectively recorded histories also depend on observation, sampling, and missingness mechanisms; the same empirical frequencies may support different causal accounts.

A small example exhibits the differing commitments of a grammar and a stochastic model. Let z_n denote position on the integers and let a rewrite grammar permit increments $+1$ and -1 at each step. Adding the kernel in Equation 16 assigns probabilities to the two options.

$$P(z_{n+1} = z_n + 1 \mid z_n) = P(z_{n+1} = z_n - 1 \mid z_n) = \frac{1}{2}. \quad (16)$$

The rewrite grammar alone describes admissible steps; Equation 16 adds a probabilistic commitment. In this model, the conditional mean increment vanishes, but a mean-field summary cannot recover an individual path or all the relations among its possible continuations. This elementary example will recur as a benchmark motivation: correspondence at the level of one summary statistic is weaker than correspondence of trajectory laws, and correspondence of trajectory laws remains weaker than equality of underlying causal mechanisms.

Stochastic models can also assign probabilities to changes in model parameters or structural configurations. Their utility for a historically changing grammar depends on how the change process is represented and whether observational evidence can discriminate parameter drift from changes in model type. Uncertainty about the model should additionally be distinguished from randomness posited within the model. The former is epistemic and may respond to new evidence; the latter is a characteristic of a candidate generative description. Neither alone establishes ontological indeterminacy or a conclusion about free will.

4.6 Programs, Automata, and Computational Generators

Computational representations characterize generation through typed procedures, state machines, evaluation rules, or executable programs. A simple transition program has a configuration space, input language, and operational semantics assigning successor configurations. More elaborate programs can contain procedures for composing structures, changing internal state, or calling other generators. Probabilistic program induction has shown how concept learning can be framed through inferred generative programs [21]. In the present analysis a program can be a candidate representation of observed structural production, provided that its outputs and execution traces have a justified relation to the source system.

A program can reproduce a finite dataset through memorization while providing little information about the mechanisms responsible for its production. The distinction between reproduction and structural inference becomes clear when the program is evaluated on interventions or longer continuations absent from the training history. Resource limits, termination conventions, randomness, and interaction with an environment are part of the relevant semantics. A claim that two programs are equivalent consequently requires specification of the compared observations, permitted inputs, and computational context. If programs dynamically rewrite their own instructions, the language used to describe the rewriting and the scope of admissible self-modification must also be specified.

Automata and grammars offer finite combinatorial systems for studying identification and translation under controlled conditions. Such systems are attractive for the later controlled computational investigations: rule recovery can be checked against a known generator, while observation loss and time-limited samples can be varied independently. They are also limited examples. Success on a finite-state benchmark does not establish recovery of a physical action principle or understanding of the meanings attached to institutional practices. Cross-family generalization will therefore require separate tests.

4.7 Institutional, Normative, and Practical Grammars

Historically constituted institutions and communities use categories, procedures, norms, and practices that determine how actions are described, evaluated, permitted, or contested. These are plausible objects for a broad inquiry into generative representations, provided that the role of a normative grammar is stated precisely. It can specify conditions under which a practice counts as institutionally valid, who has authority to act, which exceptions apply, and what kinds of reasons are recognized. Its relation to actual behavior is mediated by interpretation, compliance, resistance, and enforcement. A descriptive predictive model of conduct and a rule of institutional validity therefore address different questions.

For an illustrative defeasible rule system $\mathcal{N}$, let ϕ denote a specified condition, a an actor, and α a candidate act. Equation 17 records a model-relative inference of an obligation, with χ denoting a relevant exception condition.

$$(\mathcal{N}, \phi, \neg\chi) \vdash_d \text{Obl}(a, \alpha). \quad (17)$$

The defeasible consequence sign $\vdash_d$ indicates that the conclusion depends on the procedural logic selected for $\mathcal{N}$. The expression is an analytical illustration of one way to formalize aspects of a normative practice. It does not mean that observers possess the authority to impose that formalization on participants, or that the represented rule uniquely determines actual conduct. Different parties may dispute the applicability of ϕ , the recognition of χ , or the source of the rule's authority. The record of those disputes forms part of the relevant relational history.

Consider a hypothetical discharge-planning process. A clinical account of physiological stability, a family account of available care capacities, and an institutional account of eligibility can refer to the same patient history while encoding different practical predicates. A model of medical risk may yield predicted outcomes; an administrative grammar may yield permissible procedures; an account of family obligations may require interpretive dialogue. Productive translation would examine correspondences and differences across these grammars with the involvement of relevant participants. Mathematical consistency of an administrative rule cannot serve by itself as a test of ethical justice or consent.

The normative family also raises a problem for the generative terminology. A rule specifying what *ought* to occur does not directly generate what *does* occur. In a practical system, norms can condition available actions, induce sanctions, sustain expectations, or become objects of conflict; these effects require empirical and interpretive evidence. The proposed typed interface therefore allows Gen_M to describe institutional possibilities and recognized operations, while the empirical observation relation registers actual histories separately. This distinction is essential if an inter-system translation is to preserve meaning without reducing obligations to behavioral regularities.

4.8 Hybrid, Multiscale, and Topology-Changing Representations

Real investigations often require several model families at once. A transport process may combine a PDE describing concentration with discrete events that create or remove connections; a scientific research practice may combine dynamical measurement models, software workflows, institutional procedures, and historically changing epistemic criteria. A hybrid representation may therefore consist of typed component models $(M_i)_{i \in I}$ and explicitly specified coupling relations. A coupling between model i and model j is represented schematically by Equation 18.

$$R_{ij} \subseteq \mathcal{Z}_i \times \mathcal{Z}_j \times \mathcal{C}_{ij}, \quad (18)$$

where $\mathcal{Z}_i$ and $\mathcal{Z}_j$ are selected interface quantities or predicates, and $\mathcal{C}_{ij}$ specifies the correspondence context. Depending on the domain, R_{ij} may be a deterministic operator, a stochastic coupling, a partial semantic alignment, or an empirically documented practice of mutual coordination. Equation 18 gives a place to record such a relation without supposing that the quantities share units, time scales, or normative standing.

A topology-changing relational model particularly needs to separate the cumulative record from the active configuration. The embeddings of Definition 3.1 preserve an accumulated formal history even when decorations register deactivated relations. A rule on the active configuration may add a new face, alter a coupling, or deactivate an edge, while the event-sourced record retains evidence of the transition. A differential generator can describe continuous change between such discrete updates. In a finite example, the resulting object can be a sequence of typed cellular complexes with fields or weights on their cells. For large physical systems the continuum limit, numerical discretization, and relation between cellular observables and physical fields would require their own analysis.

A multiscale representation can map a fine description to a coarser observable, yet a satisfactory coarse model need not permit recovery of the fine generative mechanism. The same issue applies when a system's own epistemic grammar is represented through an external analyst's simplified categories. Hybrid modeling accordingly increases expressive range and also increases the number of interfaces requiring justification. The interdisciplinary usefulness of a model should be assessed through explicit questions, well-defined correspondences, and recorded losses, rather than inferred from the mere coexistence of several mathematical languages.

4.9 Representational Adequacy and Model-Relative Equivalence

Generative models are candidates for knowledge pressing insofar as they help reconstruct and critically examine selected histories and possible continuations. Their evaluation requires specifying the questions, observational conditions, and domains for which adequacy is claimed. Useful criteria include agreement with observed trajectories, robustness to reasonable observational perturbations, conditional predictive or reconstructive performance, consistency with known physical constraints, and fidelity to relevant participant interpretations. These criteria need not be aggregated into a single numerical score, particularly when competing normative commitments are involved.

For a chosen class of queries $\mathcal{Q}$ and contextual conditions $\mathcal{C}$, one possible exact behavioral equivalence is defined in Equation 19.

$$M \equiv_{\mathcal{Q}, \mathcal{C}} N \iff \text{Ans}_M(q, c) = \text{Ans}_N(q, c) \text{ for each } (q, c) \in \mathcal{Q} \times \mathcal{C}. \quad (19)$$

Here Ans_M is meaningful only where the query has an agreed interface and answer semantics for model M ; therefore Equation 19 presupposes comparable outputs. A tolerance relation may replace exact equality in empirical applications, with the metric and error tolerance specified locally. Equivalence for a restricted collection of observed queries does not imply equivalence for unobserved interventions, later historical periods, institutional meanings, or counterfactual continuations. The equivalence relation is tied to the inquiry and can change when new questions become relevant.

Proposition 4.2 (Finite-horizon observational indistinguishability). *For any positive integer L , there exist two distinct deterministic generators over the alphabet $\{a, b\}$ whose output prefixes of every length at most L coincide, although their outputs differ at a later step.*

Proof. Let M_0 emit a at every step. Let M_1 emit a for its first L steps and emit b at step $L + 1$, with arbitrary fixed behavior thereafter. The observed prefixes of lengths at most L coincide, whereas the length- $(L + 1)$ prefixes differ. The generators are distinct by construction. $\square$

Proposition 4.2 states an elementary, deliberately limited result. It is an illustrative construction rather than a new theorem of system identification. Its purpose is to make the inference limitation explicit: a finite history can support agreement among multiple candidate generators even when their possible continuations diverge. Stronger identification conclusions require restrictions on model classes, observational mechanisms, interventions, or other prior knowledge. Existing formal learning results likewise depend on the information provided to the learner and the class of admissible languages [19, 4].

A representation may also have several distinct forms of adequacy. A model can be *reconstructively adequate* for recorded observations, *generatively adequate* for specified out-of-sample transformations, *interpretively adequate* relative to informed participants and evidence, and *practically adequate* for a delimited intervention. These are working evaluation categories whose exact measurement or deliberative procedures must be developed in later sections. In particular, a predictive representation of a community’s behavior may remain inappropriate as an authoritative rendering of that community’s own grammar.

The model families considered in this section establish the objects required by the subsequent investigation. Section 3 supplies a reference of historical relational formation; the current section supplies typed candidates for expressing generative regularities and practices. Inference must then determine which candidate representations the available evidence supports, historical analysis must account for changes in the models and in their domains of validity, and translation must investigate justified correspondences between model families. Continuation adds an additional requirement: the representations must be assessed for how they constrain or illuminate accessible future structures, including situations where the rules governing those structures themselves change. The explicit preservation of type, interpretation, observational scope, and uncertainty is the principal methodological commitment carried forward.

5 Inference of Generative Representations

This section formulates the recovery of candidate generative representations from historically situated evidence as an inverse problem. Its objective is to give the typed model interface of Section 4 an explicit inferential role while preserving the differences among dynamical equations, rewrite grammars, action functionals, programs, and practical rules. The argument proceeds from an observation-indexed inverse relation to candidate admissibility, evidential scoring, topological constraints, identifiability, inquiry, model-conditioned continuation, and revision. A finite-dimensional construction establishes a restricted non-identifiability result and clarifies why additional measurements may sometimes be more informative than a longer record. The mathematical devices developed here are proposed methodological tools; they do not amount to a general identification theorem for heterogeneous relational systems.

5.1 Historical Observation and the Inverse Generative Problem

A history can be approached from two different directions. A forward representation begins with a model and admissible initial or contextual conditions, then generates a family of histories or possible observations. Historical inference begins with available records and asks which representations could have contributed to their formation. This distinction is familiar in inverse problems and system identification, where the parameters or form of a dynamical law may be inferred from measured trajectories [9, 36, 8]. Its application to heterogeneous relational histories additionally requires an explicit account of who acquired the record, through which observation procedure, and under which interpretation of the recorded variables.

Let $D_{a,\leq t}$ denote the available records of epistemic system a , in the sense of Section 3.3. Let q identify a particular inferential question, for example the dynamics of a measured concentration, the rule of a cellular attachment, or the conditions under which an institutional act is recognized. The inferential context c includes the accessible observation instruments, admissible external inputs, temporal resolution, units, and background restrictions. A heterogeneous inverse problem is provisionally represented by the correspondence in Equation 20.

$$\text{Inv}_{a,t}^{q,c} : D_{a,\leq t} \Rightarrow \mathfrak{M}_{a,t}^{q,c} \subseteq \coprod_{\tau \in \mathbf{T}} \mathfrak{M}_{\tau}, \quad (20)$$

where $\mathbf{T}$ is an index of admitted model types and $\mathfrak{M}_{\tau}$ is a type-specific class of representations. The disjoint union retains the type of each candidate. The multivalued arrow emphasizes that several models may remain compatible with the same evidence. The superscripts register the question and context, and do not imply that a unique algorithm can recover the corresponding set. In particular, the codomain may be restricted by the observer's historically available representational languages $\mathcal{L}_{a,t}$ in Equation 3.

A forward model must be connected to the evidence through a well-typed observation interface. For a deterministic model, this may consist of a trajectory solution followed by an instrument response. For a stochastic model it may be a likelihood on recorded values. A rewrite grammar may generate a set of derivation traces, with a projection that hides unrecorded production steps. A practical grammar may constrain which acts are deemed admissible by identified participants, while the actual occurrence of an act requires separate evidence. The observation interface in Definition 4.1 therefore carries an essential part of the inverse problem: different observation procedures can make different aspects of the same modeled history identifiable.

Inference may be formulated without a fully specified probability law. An archive of dated legal decisions, for example, may support a qualitative comparison of institutional interpretations and exceptions where a numerical likelihood would require unjustified assumptions. Conversely, an instrument with calibrated error characteristics can support a probabilistic comparison. A coherent methodology should state which evidential relation is being used, what counts as disagreement, and which uncertainties can be quantified. The use of one common notation for candidate models does not turn interpretive disagreement into measurement error.

5.2 Evidence-Indexed Candidate Spaces and Admissibility

A model candidate must first satisfy structural and interpretive conditions independent of how well it fits recorded outputs. For a PDE, those conditions may include its spatial domain, initial and boundary conditions, regularity assumptions, and units. For a cellular grammar, they may include types of cells, legal matches, preservation of incidence, and allowable attachments or deactivations. For an action functional, the field space, boundary terms, variation class, and relevant symmetry or gauge convention are part of the candidate specification. For a normative rule system, the relevant jurisdiction, institutional authority, historical period, exceptions, and available participant accounts constrain the interpretation.

Definition 5.1 (Evidence-compatible candidate). Fix an epistemic configuration $\mathcal{E}_{a,t}$, a question q , context c , and historical evidence $D_{a,\leq t}$. A typed representation M is an *evidence-compatible candidate* at tolerance ε when it satisfies its own admissibility conditions and there exists a well-typed evidential comparison $\text{Comp}_{q,c}$ such that Equation 21 holds.

$$M \in \mathfrak{M}_{a,t}^{q,c}(\varepsilon) \iff \text{Adm}_M(c) = \text{true}, \quad \text{Comp}_{q,c}(D_{a,\leq t}, M) \leq \varepsilon. \quad (21)$$

The comparison is defined only for the specified question and evidence type. For non-metric evidence, the comparison may be a preorder or a documented compatibility judgment, with an appropriately redefined threshold condition.

Definition 5.1 is procedural. It specifies which assumptions a particular investigation must supply; it does not declare that all model families possess one natural shared discrepancy measure. In a finite

grammar-learning example, $\text{Comp}_{q,c}$ could count violations of observed transitions and known prohibitions. In a measured ODE, it could be a residual weighted by instrument uncertainty. In a documentary reconstruction, compatibility may depend on source criticism and whether an interpretation can accommodate dated contrary cases. Numerical comparison across these settings would require an additional and justified calibration of evidential standards.

The inferential domain itself is historically situated. A representational family unavailable to an observer at time t may become available at a later time through scientific learning, contact with another community, or the development of new conceptual tools. Thus, the change of $\mathfrak{M}_{a,t}^{q,c}$ over time can reflect epistemic enlargement, a change in historical evidence, or a genuine change in the modeled system. These possibilities should remain distinguished in any later analysis of evolving generative rules. A contemporary analyst can construct retrospective models of events, yet such models are not necessarily concepts that the participants of those events possessed.

5.3 Evidential Comparison and Model Search

Within a model family that supports quantitative prediction, a candidate may be assessed through its data likelihood or a specified predictive loss. In the familiar Gaussian observation case, a model predicts a vector $\hat{y}_M$ and the observations y have a known positive-definite error covariance Σ . Equation 22 gives one possible discrepancy; the log determinant term can be omitted for comparisons with fixed Σ .

$$\mathcal{L}_{\text{obs}}(M; D, c) = \frac{1}{2}(y - \hat{y}_M)^\top \Sigma^{-1}(y - \hat{y}_M) + \frac{1}{2} \log \det \Sigma. \quad (22)$$

Equation 22 presupposes a defined vector observation, a meaningful covariance model, and predictions obtained under comparable inputs and observation procedures. It is unsuitable as a generic measure of interpretive agreement or institutional legitimacy. In grammar induction, a candidate can instead be evaluated against positive and negative observations or queries, with learnability conditional on the information supplied; foundational results in grammatical identification make this dependence explicit [19, 4].

One can also compare model complexity, structural constraints, held-out observations, and the plausibility of required auxiliary assumptions. For a model family with a meaningful common scale, Equation 23 illustrates a penalized search objective.

$$\hat{M} \in \arg \min_{M \in \mathfrak{M}_{a,t}^{q,c}(\varepsilon)} \{ \mathcal{L}_{\text{obs}}(M; D, c) + \lambda \mathcal{R}(M) \}. \quad (23)$$

Here $\mathcal{R}$ is a specified regularizer and λ sets its relative weight. In sparse equation discovery, regularization can favor compact expressions that explain trajectories, subject to a chosen feature library and noise model [9]. A low score is evidence of adequacy under the selected loss and sample, not evidence of a uniquely true mechanism. Moreover, regularization can suppress mechanisms missing from the feature library and reward a simple model whose assumptions are materially incomplete.

Model comparison becomes especially delicate across types. A grammar describing admissible cellular transformations, a PDE modeling continuous fields, and an institutional rule describing permission may address different explananda even when their outputs concern the same episode. A justified comparison requires a common question and an explicitly specified observational projection. When this condition is absent, a structured collection of complementary representations is more defensible than selecting a winner. Such pluralism also matters for historically evolving systems, because a description suitable for one temporal or spatial scale may be inadequate for another.

5.4 Relational Topology as Inferential Evidence

The relational reference permits structural evidence beyond state vectors or text sequences. In the finite two-complex setting, observations may include incidences of vertices, edges, and faces, changes in the active subcomplex, or persistence of cycles across successive recorded indices. A cellular model that

Table 4: Inferential roles of historical evidence. Each role requires a specified observation and interpretation interface; the same record may support several roles under different assumptions.

Evidence	Potential constraint	Principal ambiguity
Observed outputs	State trajectories, measurements, recorded acts	Latent mechanisms and observational noise
Structural histories	Incidence, order, attachments, relational cycles	Hidden operations and nonunique production rules
Situated interpretation	Reasons, institutional rules, actor accounts, source context	Contestability, change of categories, and authority

generates a candidate history must reproduce the structural features relevant to the question. Betti numbers, boundary operators, or diagrammatic summaries can contribute such features when their underlying cellular interpretation is appropriate. These summaries are invariants of a specified structure, and they may conceal the differences among distinct production paths.

For a filtered family of finite complexes $K_{\leq s}$, let $\beta_k(K_{\leq s})$ denote the rank of its k th homology group over a fixed field. Equation 24 records a possible observable signature at finitely many indices.

$$\Phi_{\text{top}}(K) = (\beta_0(K_{\leq s_j}), \beta_1(K_{\leq s_j}), \beta_2(K_{\leq s_j}))_{j=1}^m. \quad (24)$$

The field coefficients, filtration, and selected observation indices must be fixed before interpreting Equation 24. A cellular attachment can destroy an unfilled one-cycle while introducing a two-cell, yet distinct histories may share exactly the same signature. Homology is therefore one source of constraints on generative models, not a complete reconstruction of causality, sequence, meaning, or agency. The intermediate-history ambiguity in Section 3 provides a simple example of information gained through temporally resolved structure rather than endpoint topology alone.

A practical inference procedure could jointly compare numerical observations, relational incidences, and documentary provenance when these arise from the same study. Such integration requires explicit relationships between evidence types. A topological hole in the modeled complex, for example, does not by itself represent a gap in evidence, a missed social relationship, or an absence of normative authority. Those possibilities concern different levels of representation. Likewise, attaching a face to a formal two-complex does not imply an identified physical interaction or a recognized institutional act without additional domain-specific evidence.

Table 4 distinguishes three roles that evidence can play in constraining candidate representations. The separation helps prevent a successful description of observable structure from being treated as recovery of the system’s full generative constitution.

5.5 Observational Equivalence and Model Uncertainty

Historical inference should retain unresolved distinctions among candidates when the available observations do not separate them. For stochastic models, exact observational equivalence over an experiment class $\mathcal{Q}$ means equality of the induced observational laws for each admissible experiment. For deterministic models it means equality of observed outputs under the selected conditions. An index $\mathcal{Q}$ is essential: equivalence can disappear when a new variable is measured, an input changes, or a longer interval is observed. Within a fixed family and observation interface, Equation 25 defines a model-relative relation.

$$M \sim_{\mathcal{Q}} M' \iff P_M(D \mid q) = P_{M'}(D \mid q) \text{ for every } q \in \mathcal{Q}. \quad (25)$$

Equation 25 assumes that the observational laws share a measurable output space for each q . Its extension to qualitative histories would require an appropriate notion of indistinguishability of supported judgments, rather than an unqualified probability model. Observational equivalence should also be distinguished from parameter uncertainty produced by limited samples. Structural identifiability studies the

first issue under idealized information; practical identifiability concerns the constraints supplied by finite and noisy measurements, a distinction developed explicitly in partially observed dynamical models [34].

Proposition 5.2 (Product-parameter non-identifiability). *Consider the deterministic system $\dot{x}(t) = abx(t)$ with fixed observed initial state $x(0) = x_0 \neq 0$, where $a, b > 0$. Suppose only $x(t)$ is observed, with arbitrary temporal resolution on an interval, and a and b are otherwise unrestricted positive parameters. Then distinct pairs (a, b) are observationally equivalent for this experiment whenever they have the same product; the parameters are not separately identifiable from these observations.*

Proof. The solution is $x(t) = x_0 \exp(abt)$. Given any $r > 0$, the transformed parameters $(ra, b/r)$ have the same positive product as (a, b) and therefore generate the same trajectory for every observed t . Choosing $r \neq 1$ gives a distinct admissible pair. Hence the observation map is not injective in the parameter pair. $\square$

The construction in Proposition 5.2 is elementary and used solely as an analytical illustration. More numerous observations of the same state variable under the unchanged model do not resolve this symmetry, even in the ideal noise-free limit. A new measurement that directly constrains a , a physical restriction fixing one parameter, or a controlled intervention that modifies the dependence of the observable on the parameters may break the equivalence. The appropriate choice requires knowledge of the model’s physical or practical meaning; parameters cannot be separated merely by naming them differently.

In a Bayesian specification, uncertainty can be represented by a probability distribution over models and parameters, conditional on declared priors and likelihoods. In a set-valued specification, it can be represented by a version space of candidates consistent with the record. Both have limitations. Priors can materially shape conclusions under weak evidence; a version space depends on the admitted model families and compatibility criteria. Moreover, models from different families may be observationally equivalent for a task yet support different interpretations or counterfactuals outside that task. The investigation should consequently preserve the evidence, assumptions, question, and unresolved alternatives along with any selected model.

5.6 Additional Evidence and Permissible Interventions

Additional inquiry becomes important when candidate representations agree on current records but disagree about observations or continuations under newly accessible conditions. The relevant inquiry may involve a new sensor, increased temporal resolution, a documentary search, a structured question to knowledgeable participants, or an ethically and institutionally permissible experiment. Classical active grammar learning uses queries and counterexamples to restrict a candidate language class [4]; Bayesian experimental design studies choice of observations or experiments under a specified decision criterion [11]. These approaches illustrate ways in which an epistemic system can shape its evidence, subject to limits of access and legitimacy.

In a probabilistic model space with posterior distribution $\pi(M \mid D)$, one possible inquiry criterion is the expected information gain in Equation 26.

$$\text{EIG}(q \mid D) = \mathbb{E}_{Y \sim p(\cdot \mid D, q)} [D_{\text{KL}}(\pi(\cdot \mid D, Y, q) \parallel \pi(\cdot \mid D))] . \quad (26)$$

Equation 26 requires a probabilistic predictive distribution, a well-defined posterior, and an admissible family of inquiries q . Its use does not justify acting solely to maximize information. Information gain must be evaluated alongside anticipated harm, privacy, research ethics, participant consent, institutional authorization, cost, and the possibility that observation or intervention changes the object being studied. In social settings, the inquiry may alter trust, incentives, categories, or power relations, so a model that treats the experiment as exogenous can itself be inadequate.

A further distinction concerns causal identification. Two models may fit passive observational data while predicting different outcomes under intervention. Identification of such effects demands assumptions about the causal structure, interventions, confounding, and the stability of relevant mechanisms

[32, 40]. Generative correspondence alone supplies no permission to interpret every relational edge as causal or every fitted action rule as the reasons governing a person’s conduct. Where interventions are unavailable, the investigator may still delimit competing explanations, undertake sensitivity analysis, or register questions requiring independent evidence.

5.7 Generative Possibilities and Qualified Continuation

Inference connects with continuation when candidate models are used to investigate configurations and relations beyond the observed interval. The distinction between prediction and continuation matters in the present project. Prediction estimates selected future observables within an assumed model and observation regime. Generative continuation may additionally ask which new relational structures, interactions, or representational grammars become admissible when the system develops or a participant acts. A candidate model that performs well on historical reconstruction may fail to describe such transformations, especially after a change in mechanisms or in the epistemic capacities of the observer.

For a fixed candidate M , recorded history $D_{a,\leq t}$, and permissible context u , consider a class of model-admissible whole-spacetime extensions as in Equation 27.

$$\mathfrak{C}_M(D_{a,\leq t}; u) = \{(K^+, j) : j : K_{\leq t} \hookrightarrow K^+, (K^+, j) \models \text{Adm}_M(D_{a,\leq t}, u)\}. \quad (27)$$

In Equation 27, $K_{\leq t}$ is a selected reconstructed complex compatible with the available record, j preserves the designated historical inclusion, and $\models$ denotes satisfaction of the specified continuation constraints. For a model whose natural histories are differential trajectories or programs, K^+ may be replaced by a suitable typed extension; the displayed form applies to the finite cellular reference. When several historical reconstructions remain compatible with evidence, the inference problem should consider their associated families of extensions rather than silently treating one reconstruction as observed fact.

Model-admissibility should be kept separate from physical realizability, practical accessibility, and normative authorization. For example, a rewrite grammar may generate a structure that would require unavailable resources, and an institutional rule might permit a legal action that a particular participant cannot safely undertake. Distinct candidate models can also imply different continuation families. The corresponding epistemic possibility set can be described as their indexed family or a qualified union, but the union is a statement of model-relative uncertainty. It provides no proof that all represented branches are ontologically open under identical conditions, nor that a subject’s capacity to choose among them amounts to metaphysical free will.

The generative rules themselves may change through later interactions. In that case Equation 27 represents only the continuations licensed by a specified model of change. A historical account of the transformation of that model’s grammar, or of the observer’s representational vocabulary, will require further objects and assumptions. These are the focus of Section 6 on historical evolution and Section 8 on relational continuation, as laid out in the manuscript plan. Inference serves continuation by clarifying conditions, candidate mechanisms, and unresolved alternatives; it may also become part of the causal or interpretive context of later developments.

5.8 Epistemic Revision and the Scope of Inference

An evolving record supports an iterative inferential process. New observations may remove candidates, change their evidential standing, demand a new observation model, or motivate a previously unavailable representational family. The resulting revision is indexed by the observer’s epistemic configuration. It should also preserve earlier versions for audit, since changing a record’s interpretation or a model’s assumptions is itself a historical event relevant to knowledge pressing. A minimal versioned inferential state can be recorded as a tuple $\mathcal{J}_{a,t} = (D_{a,\leq t}, \mathfrak{M}_{a,t}^{q,c}, \mathcal{E}_{a,t}, \Pi_{a,t})$, where $\Pi_{a,t}$ stores the provenance of modeling decisions and revisions.

For a probabilistic specification with fixed model class and likelihood, the familiar posterior update

is given in Equation 28.

$$\pi_{t+1}(M) = \frac{p(D_{t+1} \mid M, D_{\leq t}) \pi_t(M)}{\int_{\mathfrak{M}} p(D_{t+1} \mid M', D_{\leq t}) \pi_t(dM')}. \quad (28)$$

Equation 28 assumes the denominator is positive and finite, a compatible conditional likelihood, and a prior distribution on a fixed measurable candidate space $\mathfrak{M}$. A historically changing model class may instead require an expanded state space, explicit model transitions, or reassessment of previous inferences. A change in the posterior does not by itself determine whether the underlying system changed, the observational conditions changed, or the evidence simply made an existing ambiguity visible.

The epistemic purpose of inference is thus broader than finding one compact formula. An adequate record of inquiry makes clear which generative descriptions have been considered, which evidence and assumptions support them, where explanations remain observationally indistinguishable, and how possible continuations depend on the models selected. For a physical process, some of these judgments can be disciplined through experiments and quantitative prediction. For normative and practical systems, sources of authority, participant interpretation, and contestation provide additional conditions that prediction alone cannot satisfy. A unified workflow can retain these differences without claiming a unified ground of validity.

Within the argument of this paper, historical inference provides a set of conditional generative descriptions that can be compared and revised. Their historical evolution will be studied in Section 6; the interpretation and translation of their heterogeneous forms will be studied in Section 7. Subsequent continuation and agency questions require further distinctions between an observer's knowledge of possibilities and the ways in which subjects participate in changing the conditions of possibility themselves. These connections motivate the broader idea of knowledge pressing while leaving the mathematical and ethical limits of inference explicit.

6 Historical Evolution of Generative Representations

The inference framework of Section 5 associates historical evidence with candidate generative representations at a particular epistemic and temporal position. The present section examines how those representations, their admissible operations, and the capacities through which they are inferred may develop over time. Its objective is to distinguish evolution of the investigated system from evolution of the observer's reconstruction, and to identify the additional structure required when even the language of a generative model can change. The discussion treats parameter variation, changes of generative rules, transformations of representational categories, epistemic revision, endogenous feedback, and the identification of historical transitions. Elementary counterexamples specify limits on what observational change can establish. These constructions are methodological proposals; a temporal change in a fitted model is treated as evidence to investigate, with its possible causes kept open.

6.1 Parameter Evolution and Structural Transformation

A generative representation can change at more than one level. Within a fixed model family, a numerical parameter can vary while the governing equation, allowable states, and operations remain unchanged. A structural transition can alter the interaction pattern, introduce a new cell or relation type, change the available production rules, or replace the state variables themselves. A third possibility is the alteration of the observation process: new instruments or a revised institutional vocabulary can make a previously unrecorded structure accessible while the investigated system follows the same underlying development. A fourth concerns the analyst's model selection, which may change as additional evidence enters a historical record. The differences have consequences for the interpretation of novelty and continuation.

For a type-preserving dynamical model, a convenient local expression is given in Equation 29.

$$x_{s+1} = F_{\tau}(x_s, u_s; \theta_s), \quad \theta_{s+1} = U_{\tau}(\theta_s, x_s, u_s, \eta_s), \quad (29)$$

where τ specifies a fixed model type, $x_s \in X_\tau$, u_s denotes admitted inputs or interactions, and η_s collects specified exogenous effects. The function U_τ is a hypothetical rule of parameter evolution. Equation 29 can represent gradual variation, external driving, or adaptation within one expressible family. It does not automatically account for the appearance of a new kind of event, a new face-attachment operation, or an unanticipated conceptual distinction.

A change of structural model is expressed more naturally through a sequence of typed representations and transition relations. The schematic construction in Equation 30 permits the type at a later position to differ from the earlier type.

$$(M_s, \tau_s, x_s) \xrightarrow{\Gamma_{s,s'}} (M_{s'}, \tau_{s'}, x_{s'}), \quad \Gamma_{s,s'} \subseteq (\mathfrak{M}_{\tau_s} \times X_{\tau_s}) \times (\mathfrak{M}_{\tau_{s'}} \times X_{\tau_{s'}}), \quad (30)$$

where $\Gamma_{s,s'}$ records whatever model-transition and state-comparison conditions the local application can justify. Its use does not postulate a canonical transformation between unrelated model languages. It may be a total map in a tightly controlled simulation, a partial relation between different scientific formalisms, or a qualitative, evidence-supported account of changes in a practice. Some transitions can admit a stable identification of older variables; others require a new observational interface. The same notation can preserve a documented discontinuity without claiming a unique mechanism for it.

A further distinction concerns the timescales at which change is investigated. Physical dynamics, biological learning, institutional reform, and historical shifts in interpretive vocabulary may develop on overlapping but differently resolved timescales. A coarse observation schedule can present gradual structural change as a sudden event, while a fine-resolution archive can reveal a sequence of precursor interactions. A report of a change point should therefore include sampling resolution, admissible model families, and the temporal support of its evidence. Existing approaches to change detection and concept drift provide established techniques for data streams and changing statistical relations [7, 16, 1]. Their results concern particular observation and modeling assumptions and provide methodological precedents rather than a general solution to ontological reconstruction.

6.2 Changes in Generative Rules and Relational Topology

A relational system may develop by repeating an available transformation, by changing the conditions under which a transformation is permitted, or by acquiring a further transformation. These alternatives are distinct even when their endpoint complexes are isomorphic. A cellular rewrite grammar can make their difference explicit because it separately represents the admissibility of an operation and its effect on the relational structure. The typed grammar interface in Section 4.2 supplies the elementary setting.

Consider a finite complex with vertices, edges, and possibly faces. During an initial modeled interval, grammar G^- permits vertex and edge attachment and permits the formation of a closed one-cycle. An augmented grammar G^+ additionally admits a two-cell attachment whenever a specified admissible cycle is present. Its new operation is described in Equation 31.

$$G^+ = G^- \cup \{r_f\}, \quad r_f : (K, c) \mapsto K \cup_c f, \quad \partial_2 f = c, \quad \partial_1 c = 0, \quad (31)$$

where c is a valid attaching cycle and ∂_i denotes the cellular boundary operator. For this constructed example, Equation 31 preserves the chain-complex identity $\partial_1 \partial_2 f = 0$. If c initially represents a non-trivial homology class, the attachment may change H_1 and H_2 as the complex develops; the exact effect depends on the wider complex and attaching map. Its primary role here is to exhibit a change in the available production rule, with identifiable consequences for admissible continuation structures.

Proposition 6.1 (Typed exclusion under a fixed grammar). *Let G be a finite rewrite grammar whose admissible productions preserve a type alphabet A , and let every initial state have cells typed in A . Under a sound observation procedure that correctly identifies cell types, a verified observed cell of type $\sigma \notin A$ cannot be generated by a derivation using only G and those initial states.*

Proof. The initial states have types in A . By hypothesis, each admissible production maps a state with types in A to a state with types in A . Induction on the number of productions shows that every derivable

state retains only types in A . Sound observation of a cell with type $\sigma \notin A$ is incompatible with such a derivation. $\square$

Proposition 6.1 establishes a restricted exclusion result, with its force supplied by the closed alphabet and the soundness assumption. In empirical settings, an unfamiliar cell type could arise from a revised classification, a previously unavailable measurement, an incorrect initial model, or an actual transformation outside the stipulated grammar. A researcher who introduces a new type to explain an observation has revised a representation; the change in the underlying system remains a separate matter for inquiry. Further, a sufficiently expressive fixed universal grammar could encode the same sequence through a hidden state or parameter switch. The finite example identifies what is excluded by a specified model, rather than proving an intrinsic distinction between system-level evolution and hidden implementation.

For evolving relational spacetime, the order and embedding of these attachments are consequential. A historical record of active cells at one time does not necessarily reveal which rule was available before the cell appeared. Even a complete incidence record may omit the institutional authorization, energetic constraints, or latent state that made an operation admissible. Reconstruction of a rule change therefore calls for observations of prohibitions, trials, attempted interventions, rejected operations, or explicit alterations of governing procedures, in addition to the topology of successful endpoints. This point becomes central when the represented grammar is a social or normative practice whose admissibility is also contested by participants.

6.3 Evolution of Representational Languages

Generative mechanisms are represented through historically available languages. The language $\mathcal{L}_{a,t}$ of the epistemic configuration in Equation 3 contains categories that make some distinctions intelligible, and its available operators make some inferences expressible. A change of language may consist in refining a previous category, adding a concept, reinterpreting a term, or altering which statements count as well-formed explanations. These changes may occur without a corresponding change in the investigated system, yet they may also influence the system when the categories enter scientific practice, institutional decision-making, or social interaction.

To represent a language change, let $\text{Expr}(\mathcal{L}_{a,s})$ be a collection of expressible descriptions at s . A situated re-expression relation is introduced in Equation 32.

$$\Phi_{a;s,s'} \subseteq \text{Expr}(\mathcal{L}_{a,s}) \times \text{Expr}(\mathcal{L}_{a,s'}), \quad (32)$$

where a pair in $\Phi_{a;s,s'}$ expresses a justified local correspondence, not semantic identity. The relation may be partial, many-to-many, and dependent on background practices and the questions being asked. It can register the survival of an older distinction, a qualified reinterpretation, or the absence of a satisfactory expression in the later language. Equation 32 is a historical re-description interface within an epistemic system; The subsequent section on heterogeneous translation will investigate the more demanding problem of correspondences between heterogeneous systems.

An episode of scientific conceptual change can provide a familiar analogy. Two theoretical vocabularies can describe overlapping observations while dividing phenomena into different categories. The study of incommensurability and historically situated interpretation cautions against treating conceptual overlap as a single lossless dictionary [30, 24]. The analogy also has limits: changes in a formal type signature can sometimes be recorded exactly, while social meanings and practical purposes may be disputed or enacted in ways that escape a finite set of definitions. Historical translation should therefore preserve conditions of use, disagreements, and the agents authorized to interpret particular expressions.

The evolution of language also bears on the inference operation itself. A model family available at s' can describe aspects of an earlier history that were inexpressible at s . Such retrospective reconstruction is useful, although it should be distinguished from reconstructing how observers at s themselves understood the history. Treating a later explanatory category as the earlier participants' operative concept introduces an anachronism. An adequate evidence record can hold both the retrospective model and a historically situated account of the concepts participants used, with attribution and temporal scope.

6.4 Observer Revision and Epistemic Co-evolution

An epistemic system can modify its instruments, training, evidence standards, objectives, and practical capacities while investigating a relational system. Section 3.2 represented these capacities by $\mathcal{E}_{a,t}$; the inference relation in Equation 20 is indexed by the corresponding historical conditions. A sequence of reconstructions can consequently change because new evidence is obtained, because an existing record is reinterpreted, or because the observer's language now admits different explanations. These are forms of epistemic evolution even where the investigated process has remained stable.

A useful accounting device separates the observed target process from the observation and reconstruction procedures. For a selected quantitative model, Equation 33 provides one schematic decomposition.

$$\begin{aligned} X_{s+1} &= F_{M_s}(X_s, u_s, \eta_s), \\ D_{a,s} &= O_{a,s}(X_s, \zeta_s), \\ \mathfrak{M}_{a,s} &= \mathfrak{l}_{a,s}(D_{a,\leq s}; \mathcal{E}_{a,s}), \end{aligned} \tag{33}$$

where M_s is a hypothesized model of the investigated process and $\mathfrak{M}_{a,s}$ is the observer's collection of inferred candidates. Distinct symbols are used because a candidate set inferred by an observer should not be conflated with a model used to represent the target. The dependencies in Equation 33 are schematic and conditional: an observation relation need not be a deterministic function, and social evidence can include narrated, negotiated, or institutionally mediated interpretations beyond a finite data vector.

Proposition 6.2 (Observational ambiguity of a recorded transition). *Fix integers $T \geq n \geq 1$ and a binary record $y_s = 0$ for $s < n$ and $y_s = 1$ for $n \leq s \leq T$. There exist two deterministic models reproducing the record exactly: one with a change in the target's state-transition rule and a time-invariant observation map, and another with a time-invariant target state and transition rule but a time-varying observation map.*

Proof. In the first model, let $x_s = 0$ for $s < n$ and $x_s = 1$ thereafter. Define $x_{s+1} = 0$ for $s < n - 1$, $x_n = 1$ at $s = n - 1$, and $x_{s+1} = x_s$ for $s \geq n$. The observation map is $O_s(x) = x$ for every s . In the second model, let $x_s = 0$ and $x_{s+1} = x_s$ for every s , but set $O_s(x) = x$ for $s < n$ and $O_s(x) = x + 1$ for $s \geq n$. Both models yield precisely the stipulated y_s at each recorded time. Their explanations locate the temporal change in different parts of the observation process. $\square$

Proposition 6.2 is an elementary constructed counterexample, closely related in spirit to the underdetermination discussed in Section 5.5. Its point is specific: a change in recorded output does not identify whether the target's generative rule changed or the observation interface changed. More abundant observations of that same output channel can retain the ambiguity. An independently calibrated instrument, an additional observation channel, or an admissible intervention may help distinguish the explanations when the chosen models predict different responses to those additional inquiries. Such evidence also has historically situated acquisition conditions and costs. Similar challenges arise when an institution changes reporting categories while its underlying practices remain stable, or when an institution genuinely changes its practices without altering the terminology used in records.

Because the observer may also participate in the target's development, the decomposition of a target and an observer is sometimes only a useful analytical partition. A scientific measurement can perturb the investigated process; a public classification can change the way institutional actors respond; and feedback from translated representations can modify interaction norms. The epistemic system has a material and relational history and may belong to the very spacetime whose development it seeks to understand. This is compatible with the coupled historical interpretation of knowledge pressing in Section 3.4, while leaving a model of the specific coupling to be justified case by case.

6.5 Endogenous Rule Transformation and Generative Capacity

Historically evolving systems may alter their admissible operations through their own interactions. A learned skill, a negotiated convention, a newly available instrument, or a modification of institutional

procedures can change which future acts are possible for particular participants. The source of a change, however, requires careful attribution. Exogenous disturbances, changes to recording instruments, decisions by identifiable agents, and endogenous feedback can yield superficially similar records. A theoretical description of generative capacity should therefore specify its modeled boundary, the interactions admitted within that boundary, and the contextual conditions under which a capacity is exercised.

For a modeled evolving grammar, one can describe a history-dependent rule-update relation as in Equation 34.

$$G_{s'} \in \mathcal{U}_{s,s'}(G_s, K_{\leq s}, \mathcal{E}_{a,s}, c_{s,s'}), \quad s' \succ s, \quad (34)$$

where $\mathcal{U}_{s,s'}$ gives candidate next grammars consistent with stipulated mechanisms, interaction records, and contextual restrictions. The expression is generally set-valued because several transformations may be admissible and the account may be incomplete. The presence of the epistemic configuration $\mathcal{E}_{a,s}$ is relevant only when the epistemic system participates in the investigated process; for a target whose dynamics are independent of the observer, this argument can be omitted. Even when $\mathcal{U}_{s,s'}$ is specified computationally, the ability to generate a new rule is relative to the meta-rules allowed by the chosen implementation. An empirically open-ended generative process should not be equated uncritically with a finite rule-update program.

An evolving collection of grammars may change the corresponding model-indexed continuation family. Let $\text{Cont}_{a,s}(G_s; c)$ retain the meaning established in Definition 3.2. The induced continuation family with admissible grammar updates is provisionally expressed in Equation 35.

$$\text{Cont}_{a,s}^{\mathcal{U}}(G_s; c) = \bigcup_{(G_r)_{r \succeq s} \in \text{Traj}_{\mathcal{U}}(G_s; c)} \text{Cont}_{a,s}((G_r)_{r \succeq s}; c), \quad (35)$$

where $\text{Traj}_{\mathcal{U}}$ denotes the admitted sequences of rules and the final continuation term is understood in the extended, rule-sequence sense already described after Definition 3.2. Equation 35 is a union of *model-admissible* possibilities, not a measure of physical realization, a prediction of actual outcomes, or a moral ranking. A change in its members may represent new kinds of accessible structures, the closure of formerly admitted transformations, or both. The available actions of an actual subject impose additional constraints that are deferred to the agency analysis.

A model that appears to enlarge future possibilities may also amplify the power of one participant to close alternatives for others. For institutional and normative grammars, distinctions between an authorized rule change and a contested imposition are essential. A procedure that is computationally representable as a rule update acquires neither legitimacy nor justice from that fact. The histories of who can initiate changes, whose objections are recorded, and which participants bear the costs belong to the evidence required for a situated evaluation.

6.6 Historical Change Detection and Model-Class Discovery

The foregoing distinctions generate a practical methodological task: identify the smallest class of historical changes that available evidence can support and specify what further evidence would discriminate among competing accounts. Change-point analysis, concept-drift adaptation, and sequential inquiry already provide relevant techniques under declared statistical assumptions [7, 1, 16]. Their application to the broader class of generative representations demands explicit interfaces for state types, temporal windows, evidence provenance, and model adequacy.

One investigation can compare candidate histories under different restrictions: a stationary model with fixed observations; a fixed model family with varying parameters; a grammar or model class with changing operations; and a stable target mechanism observed through a changing interface. Table 5 summarizes the principal alternatives and the kinds of additional evidence they invite.

For a quantitative sequence observed in comparable conditions, a structured model-selection calculation can be useful. Equation 36 defines an illustrative penalized criterion over segmented time ranges,

Table 5: Distinct explanations for temporal differences in recorded relational trajectories. The evidence column gives possible discriminating inquiries rather than universally decisive tests.

Interpretive account	Candidate change	Potential additional evidence
Parameter variation	Coefficients within a stable model family	Calibrated repeat measurements, controlled inputs, parameter sensitivity
Structural change	Rule set, state types, interaction topology	Derivation traces, records of failed operations, independent structural observations
Observation change	Sensor, archive selection, coding scheme	Instrument calibration, parallel channels, source-version records
Epistemic change	Inference method, vocabulary, standards of justification	Dated model versions, training records, contemporaneous descriptions
Interaction-induced change	Feedback among agents, practices, and generative constraints	Intervention histories, competing participant accounts, subsequent behavior

with each local model evaluated under its own well-typed observation interface.

$$\hat{\mathcal{H}} \in \arg \min_{\mathcal{H} \in \mathfrak{H}_c} \left\{ \sum_{j=1}^{J(\mathcal{H})} \mathcal{L}_j(D_{I_j}, M_j, O_j) + \lambda J(\mathcal{H}) + \rho \mathcal{R}(\mathcal{H}) \right\}, \quad (36)$$

where $\mathcal{H}$ is a hypothesized segmentation and model history, I_j are its segments, M_j and O_j are local generator and observation descriptions, $\mathcal{L}_j$ are justified local discrepancies, and $\mathcal{R}$ penalizes stipulated complexity or unsupported transitions. The scale and interpretation of these penalties must be specified before empirical comparisons. A lower value under Equation 36 is a conditional preference within the admitted class $\mathfrak{H}_c$. It is neither evidence of a unique historical mechanism nor a substitute for qualitative source criticism when meanings, authorizations, or practical grammars are under investigation.

Model-class discovery raises a further challenge. If a research procedure searches only for parameter changes, it may miss a newly relevant state variable or a transition to a different family of representations. Conversely, a search space that permits arbitrarily complex changing grammars may fit sparse data while offering little out-of-sample explanatory value. A defensible protocol therefore includes a declaration of candidate languages, records of unsuccessful models, retrospective checks using earlier evidence, new measurements where permissible, and reports of residual incompatibilities. The appropriate balance among expressive power, stability, complexity, and observational fit depends on the inferential question and on available resources. The inquiry may remain open even after a useful model has been adopted for local action.

6.7 Continuity, Reconfiguration, and Epistemic Scope

A final issue concerns the meaning of historical continuity when the representational type changes. Earlier states and later states may lack a common coordinate system or a canonical identification of their variables. Continuity may nonetheless be investigated through shared evidence, persistence of selected relational supports, locally preserved invariants, overlapping practices, or documented paths of translation. For cellular histories, compatible inclusions and higher-order attaching maps may retain parts of the earlier complex while allowing later configurations to transform. For interpretive histories, continuity can be recorded through communities of practice, inherited usages, legal precedents, and explicit disputes about the meaning of inherited rules. Each criterion is local to the epistemic purposes of the inquiry.

The same point applies to an account of generative novelty. A structure is novel relative to a specified grammar if the earlier admissible derivations cannot produce it, as in Proposition 6.1. It may be epistemically novel when an observer gains a category or recognition procedure capable of identifying it. Its appearance may be locally unprecedented in the available historical archive even where the archive is incomplete. These are different attributions with different evidential requirements. A robust knowledge-

pressing infrastructure would preserve the distinction in its record rather than attach one undifferentiated novelty label to the event.

Historical evolution of generative representations therefore gives knowledge pressing two intertwined but analytically distinguishable tasks. One is the investigation of mechanisms and the conditions under which their generative capacities develop. The other is the historical reconstruction and revision of the epistemic practices through which these mechanisms become intelligible. Their coupling may contribute to the future development of the investigated system, including through interpretation and inter-system contact. Subsequent formal analysis of translation will examine when such contacts support partial correspondences between heterogeneous generative languages; the continuation and agency sections will investigate the possible relational futures and historically constituted capacities to act that emerge under specified conditions. The present section establishes the need to model change at the levels of dynamics, grammar, observation, and epistemic constitution before making claims about how knowledge participates in future creation.

7 Translation across Heterogeneous Generative Systems

The preceding sections specify generative representations as typed models, relate candidate models to historically situated evidence, and distinguish mechanism evolution from revision of an observer's account. The present section investigates correspondences between representations inferred within different relational systems. Its objective is to clarify what a translation can preserve, how its adequacy can be evaluated, and how differences in representational type and historical constitution limit the resulting understanding. The discussion introduces context-indexed translation interfaces, distinguishes structural and dynamic correspondences, gives elementary conditions for preserving selected generative consequences, and considers translation loss, asymmetric relations, composition, and historical revision. The formal constructions concern well-defined classes of models and observations. Their extension to culturally or institutionally constituted practices requires evidence of situated use and participation beyond mathematical matching.

7.1 Heterogeneous Relational Systems and Translation Contexts

Let A and B denote relational systems investigated through observation-indexed histories $D_{a,A,\leq t}$ and $D_{b,B,\leq s}$. The observers a and b may belong to the systems concerned, be external investigators, or occupy overlapping positions. At a particular epistemic stage they associate those records with families of admissible typed models $\mathfrak{M}_A$ and $\mathfrak{M}_B$, as in the inverse correspondence of Equation 20. A translation inquiry concerns selected members of these families, their interpretation, and the historical evidence connecting the systems. The two systems may be distinct communities of practice, scientific descriptions that use different mathematical languages, or models of an interacting physical and computational process. Each setting demands its own interpretation of a valid correspondence.

A *translation context* c specifies the question for which mutual intelligibility is sought, the observations or operations exposed by each system, the relevant historical periods, and who may evaluate or contest the proposed relationship. Context is essential because the same pair of models can admit distinct comparisons. A hydrodynamic differential equation and a cellular simulator can be compared through specified field measurements, whereas a community's interpretation of a warning may require records of local practices, explicit consultation, and attention to institutional authority. In the first case, comparison can sometimes be expressed through numerical error. In the second, adequacy involves situated meaning and practical consequences whose evaluations may remain disputed.

The research programme treats inter-system translation as the construction of a *partial, revisable relation* among selected components of generative descriptions and their historically situated interpretations. It does not presuppose a universal translation language that fully contains each system's categories. This restraint has precedents in philosophical accounts of translation, interpretation, and incommensurability [33, 24, 30]. Related formal traditions provide more constrained tools for comparing processes and dif-

fering logics, including behavioral equivalence in concurrency theory and the institution framework for logical systems [27, 17, 18]. The present framework draws on these traditions as examples of rigorously delimited comparisons, without claiming that their constructions automatically establish translations of social meaning.

7.2 Partial Observation Interfaces and Translation Witnesses

Two representations can be compared when the inquiry supplies some common evidential or operational interface. The word “common” indicates an explicitly constructed comparison setting, not an observer-independent metalanguage. A suitable interface may register measurements with compatible units, traces of named actions, selected incidences of a cell complex, or documented instances of a practice. It may consist of partial maps because certain source events have no counterpart within the present receiving language. A relation is appropriate when several candidate correspondences remain possible.

Definition 7.1 (Contextual generative translation witness). Fix a translation question and context c , typed models M_A and M_B , and admissible history spaces $\mathcal{H}_A$ and $\mathcal{H}_B$. A *contextual translation witness* is the data described in Equation 37.

$$\Theta_{AB}^c = (Z_A^c, Z_B^c, \alpha_A^c, \alpha_B^c, R_c, \Pi_c), \quad \alpha_i^c : \mathcal{H}_i \rightarrow Z_i^c, \quad R_c \subseteq Z_A^c \times Z_B^c, \quad (37)$$

where each α_i^c selects an admitted, possibly partial, observation or interpreted feature of histories from system i , R_c records the candidate correspondence between comparable features, and Π_c records its evidential basis, scope, participating interpreters, known exclusions, and version history. The witness induces the history relation in Equation 38 on its domain of definition.

$$\mathcal{R}_{AB}^c = \{(h_A, h_B) \in \text{dom}(\alpha_A^c) \times \text{dom}(\alpha_B^c) : (\alpha_A^c(h_A), \alpha_B^c(h_B)) \in R_c\}. \quad (38)$$

Definition 7.1 imposes a limited common *comparison interface*, while allowing each source model to retain its own state types, generators, and interpretive conventions. The maps α_i^c need not be embeddings; they may omit temporal, geometric, causal, or normative distinctions. The relation R_c need not be a function or an equivalence relation. A translator may record several readings of one source feature, or identify no warranted reading in a particular target vocabulary. A witness may also be intentionally one-sided: one institution may require an interpretation of another’s procedural rule for a narrow operational purpose, without implying a reciprocal translation of its own rules.

The evidential component Π_c matters because plausible-looking structural mappings can have radically different warrants. One witness may be derived from a calibrated physical instrument; another from a researcher-constructed analogy; a third from negotiated descriptions provided by participants. These sources of warrant should remain visible. The inference methods of Section 5 can propose candidate witnesses, but observational fit alone does not establish their meaning or authority. A revision may replace the interface, narrow the domain of admissibility, or preserve alternative mappings as unresolved.

7.3 Structural, Dynamical, and Semantic Correspondences

Table 6 distinguishes several translation targets and clarifies which evidence can establish adequacy for each one.

A structural correspondence concerns selected features of configurations or histories: event adjacency, incidence, order, cell attachment, connectivity, conserved quantities, or other observables justified by the local question. A dynamic correspondence concerns transformations or conditional continuations. A semantic correspondence concerns the interpretation of the modeled quantities or practices in their respective systems. These dimensions interact, but success in one supplies only restricted evidence for another.

For relational cell complexes, a strong structural map would preserve typed incidence and the boundary maps. When a comparison is made using cellular chain groups, the familiar chain-map condition is

Table 6: Levels of contextual translation between generative representations. Preservation requirements are local to a declared question and model class.

Translation target	Candidate preservation requirement	Principal evidential limitation
Observed outputs	Matched values or records under specified instruments	Other generators may fit the same record
Structural relations	Incidence, boundary, order, or selected invariants	Structure may lack common semantics
Generative transitions	Selected source steps admit related target steps	Trace agreement can hide branching differences
Probabilistic dynamics	Compatible laws on a specified observable space	Sampling, conditioning, and model misspecification
Epistemic and practical meaning	Contextual intelligibility checked with relevant interpreters	Interpretive plurality and unequal authority
Continuation	Selected admissible extensions retain warranted counterparts	Rule changes and action constraints may invalidate the bridge

expressed by Equation 39.

$$f_{n-1} \circ \partial_n^A = \partial_n^B \circ f_n, \quad f_n : C_n(K_A) \longrightarrow C_n(K_B). \quad (39)$$

When Equation 39 holds for a family of suitable linear maps, the maps are compatible with cellular boundaries and induce homology maps. Such maps capture a selected algebraic aspect of topology; they need not preserve event semantics, the sequence of cellular constructions, or the practical meaning of a face. In particular, two complexes can share homological information while supporting different histories or production rules. More restrictive correspondences may require typed attaching histories, orientations, temporal incidence, or designated higher-dimensional primitives.

A dynamical correspondence requires an interpretation of transitions. A discrete rewrite grammar, a smooth flow, and a stochastic transition kernel generate different mathematical objects. For two discrete models a state relation and an action correspondence can sometimes be examined directly. For a continuous system and its numerical discretization, a relation may instead be established at chosen observation times and spatial resolutions. A stochastic model may require a comparison of induced probability measures rather than a pointwise correspondence of trajectories. This multiplicity is consistent with the typed interface of Definition 4.1: the appropriate translation test depends on the model family and the question being asked.

Semantic correspondence asks whether an inferred mapping conveys the relevant concepts and practical implications to participants. A scientific quantity often comes with a unit, measurement procedure, and scope of interpretation. A legal or administrative category can also depend on precedent, jurisdiction, procedural exceptions, and disputed authority. Even an exact structural mapping between rule systems may fail to preserve what an action means to the people concerned. Philosophical work on historically situated interpretation and incommensurable taxonomies identifies reasons to treat such translation as an interpretive inquiry [24, 30]. The admissible meaning of a correspondence may therefore need dialogical validation and a record of objections alongside its formal specification.

7.4 Generative Consequence Preservation

Translation of completed records and translation of generative capacities are distinct achievements. Two models may assign compatible features to all histories already observed while permitting different future transformations. A generative comparison consequently asks whether a source operation, continuation, or possible response has a corresponding target operation within an explicitly specified domain. The answer depends on the permitted action vocabulary, observation resolution, temporal horizon, and chosen direction of comparison.

For finite labeled transition models, let S_A and S_B be state sets, L_A and L_B action labels, and $\rightarrow_A$ and $\rightarrow_B$ labeled step relations. Let $Q \subseteq L_A \times L_B$ specify an admissible pairing of actions and $\rho \subseteq S_A \times S_B$

relate states. A bounded *forward generative correspondence* is required to match source transitions by target transitions under Equation 40.

$$(x, y) \in \rho, \quad x \xrightarrow{\ell}_A x' \implies \exists m, y' [y \xrightarrow{m}_B y', (\ell, m) \in Q, (x', y') \in \rho]. \quad (40)$$

Equation 40 is a simulation-type condition, familiar in process semantics [27, 17]. It is meaningful only for an identified set of admitted states and steps and a correspondence of their observables. A two-way condition, with the directions exchanged, imposes a stronger comparison. Some translation purposes require only one-way preservation because the receiving system has additional capabilities or must represent only selected source actions. Translation at a fixed finite horizon may also use a bounded matching relation, permitting the translation to expire or be reevaluated as the systems develop.

Proposition 7.2 (Finite trace preservation under a forward correspondence). *Suppose initial states $x_0 \in S_A$ and $y_0 \in S_B$ satisfy $(x_0, y_0) \in \rho$, and the step condition of Equation 40 holds along all source paths of length at most n from x_0 and their corresponding target states. Then each source action trace of length at most n has a target trace of the same length whose labels are related componentwise by Q . This conclusion concerns the specified action-trace projection; state observations require an additional matching condition.*

Proof. At length zero the paired initial states provide the empty trace. Suppose a source trace of length $k < n$ has a corresponding target trace ending at states related by ρ . For a source step extending that trace, Equation 40 supplies a target step with a Q -related label and successor states again related by ρ . Induction on the trace length completes the construction. $\square$

Proposition 7.2 is an elementary consequence of simulation, included to identify the exact strength of one preservation claim. It establishes neither equality of trace sets nor equivalence of branching structures. Inference from historical records may yield several candidate state relations ρ , and their scope can differ as additional behaviors are observed. Models may also agree at one observation resolution while diverging when the query becomes more detailed.

A finite counterexample distinguishes observed traces from generative branching. Model A permits a single a -transition to a state at which either b or c is possible. Model B permits two distinct a -transitions, leading separately to a state permitting only b and a state permitting only c . The sets of completed action traces are identical, namely $\{ab, ac\}$, but after observing a the models represent different availability structures. A state of A after its a -step has both b and c options, while each a -successor in B has only one. Thus a strong two-way step correspondence matching the labels and all possibilities cannot relate the initial states. The example is a standard kind of distinction between trace and branching semantics, developed systematically in process algebra [17]. It also illustrates why historically recorded outputs can leave a source system's response capacities incompletely identified.

7.5 Translation between Different Mathematical Languages

Some translations connect models whose mathematical objects have different types. A continuous differential equation and an executable difference scheme supply a useful controlled example. Consider the one-dimensional heat equation on a specified bounded interval, with initial data and consistent boundary conditions. Its model and a simple explicit finite-difference implementation are recorded in Equation 41.

$$\partial_t u = \kappa \partial_{xx} u, \quad u_i^{k+1} = u_i^k + \mu(u_{i-1}^k - 2u_i^k + u_{i+1}^k), \quad \mu = \frac{\kappa \Delta t}{(\Delta x)^2}. \quad (41)$$

The differential model describes a continuum field, while the numerical system is a finite set of local updates on a discrete lattice. A translation witness may associate a continuum solution with samples at lattice points and compare these with simulated values. Under suitable smoothness, consistent boundary treatment, and the familiar stability restriction $0 < \mu \leq 1/2$ for this explicit scheme, refinement can support approximation of the corresponding continuum evolution. The preservation claim concerns selected sampled trajectories under those conditions; it is not an equivalence of the complete differential

and computational grammars. A coarse numerical model may have different admissible states, exhibit discretization artifacts, or fail when its stability assumptions are violated.

A second example concerns the translation of higher-order relational quantities. For a finite cell complex K , a real one-cochain α assigns a value to each oriented edge. Its coboundary $d\alpha$ is a two-cochain. Equation 42 expresses the duality between cochain and chain boundaries for an oriented two-chain f .

$$\langle d\alpha, f \rangle = \langle \alpha, \partial_2 f \rangle. \quad (42)$$

Equation 42 supports an exact algebraic relation within the chosen cellular complex. When a continuum geometry and discretization map are additionally specified, this identity can help construct comparisons with circulation or flux calculations. It does not assign a universal physical, social, or semantic interpretation to the value on an edge or face. The event and coupling semantics of relational spacetime remain distinct from the roles of numerical edge and face cochains used for an illustrative measurement. A correspondence between discretized and continuum descriptions earns its interpretation through the particular observables and approximation properties selected for the inquiry.

These examples suggest a general discipline for translating generative models: specify the interfaces, state what mathematical object is being preserved, describe the scale and conditions of comparison, and retain the mismatch between the remaining model structures. The same discipline is applicable to other pairings, including programs and rewrite grammars or stochastic and deterministic models, provided their domains and observation processes are well defined.

7.6 Translation Loss, Incomparability, and Interpretive Asymmetry

Translation loss arises when a correspondence omits or merges distinctions that matter in one system. It may involve a missing variable, an unavailable action, an interpretive category without a suitable counterpart, or a relation preserved only after temporal or spatial aggregation. Some losses can be measured within a specified numerical comparison; others require descriptions of the affected practices and the perspectives from which those losses matter. Treating the existence of a relation R_c as evidence of complete understanding would conceal these differences.

One simple formal case is the collapse of distinguishable source features under a deterministic mapping. Let a context-specific projection $f : Z_A^c \rightarrow Z_B^c$ be defined for the features under investigation. If two distinct source features have the same target image, downstream deterministic translation cannot recover their original distinction from that image alone.

Proposition 7.3 (Persistent information collapse in composed deterministic translation). *Let $f : Z_A^c \rightarrow Z_B^c$ and $g : Z_B^c \rightarrow Z_C^c$ be functions. If $z_1 \neq z_2$ but $f(z_1) = f(z_2)$, then $(g \circ f)(z_1) = (g \circ f)(z_2)$. Consequently, $g \circ f$ admits no left inverse on a domain containing both z_1 and z_2 .*

Proof. Equality of the images under f remains equality after applying g . A left inverse would map the common image to both z_1 and z_2 , contradicting their distinctness. $\square$

Proposition 7.3 is deliberately elementary. Its relevance concerns translation design: a feature discarded at an intermediate interface may remain inaccessible to later translators unless fresh evidence or a separate channel is introduced. A many-to-many relation with contextual information can avoid some premature collapse, while simultaneously introducing ambiguity. The formal loss illustrated here is only one type. Epistemic and normative losses may be recognized differently by participants, particularly when the translated category bears obligations, permissions, or social recognition.

Incomparability is also context-relative. Two grammars may fail to admit a useful correspondence concerning the question c , while admitting one for another restricted question c' . A map that identifies comparable measurement outcomes need not translate agents' intentions. A system can possess an action or concept for which another system has no established counterpart, even if the systems can communicate about that absence. Documenting this failure is a positive epistemic result: it specifies a boundary of understanding and can motivate a new inquiry, a more adequate intermediary representation, or a revision of the institutional process that requires the translation.

Asymmetry is expected in partial translation. A system with a richer measurement vocabulary may convey a coarse quantity to a system using an aggregate variable; returning from that aggregate to its components requires additional evidence or assumptions. Normative asymmetry can be more consequential: one party may possess greater authority to declare equivalence, choose a reference vocabulary, or demand that another party accommodate its classifications. A documented translation should identify who controls its context, who validates it, and whose interpretations are represented as provisional. Neither formal symmetry nor successful prediction confers interpretive or normative legitimacy.

7.7 Compositional Translation and Intermediary Representations

An intermediate system B may facilitate communication between systems A and C . In a deliberately simplified setting, contextual correspondences can be treated as relations with compatible middle types. The relational composition is given in Equation 43.

$$R_{AC}^c \supseteq R_{BC}^c \circ R_{AB}^c, \quad (x, z) \in R_{BC}^c \circ R_{AB}^c \iff \exists y [(x, y) \in R_{AB}^c \wedge (y, z) \in R_{BC}^c]. \quad (43)$$

The inclusion in Equation 43 is a *proposed acceptance condition* for a composite witness, when the contexts and types are compatible; it is not an automatic relation among separately inferred translations. In applications, context changes, conflicting meanings, or incompatible authority conditions may prevent such an acceptance. Even when relational composition is well typed, it can create additional ambiguous pairs and may fail to preserve the stronger operational properties associated with either component witness.

One can therefore distinguish formal composability, evidential support, and interpretive acceptance. A translation pipeline may be formally composable because its intermediate variables agree, yet provide little evidence that the final relation conveys the original system's generative capacities. Where conditional predictions, probabilities, or uncertainties are propagated across bridges, dependence assumptions and provenance must be made explicit. A single numerical confidence score for a composite translation is generally ungrounded without a validated uncertainty model.

An intermediary representation need not be selected in advance. Repeated inter-system interaction may motivate the development of a new language, shared measurement protocol, or procedural practice that provides useful observables unavailable in either initial interface. This is an instance of possible *generative translation*: the process of seeking correspondence participates in changing the capacities through which later correspondences become accessible. For an institutional setting, creation of a bridging practice raises questions about whose terms are adopted and what obligations the new procedure establishes. A shared grammar can increase mutual intelligibility while simultaneously introducing new exclusions or shifting the balance of interpretive authority.

7.8 Historically Revised Translation and Relational Continuation

Translations are situated within evolving epistemic and generative histories. The models $M_{A,t}$ and $M_{B,t}$ may change, previously hidden transitions may be observed, and the relation selected for comparison may become inadequate. A versioned translation history can record which correspondences remain supported across those changes. Equation 44 describes a provisional update relation for witness families.

$$\Theta_{AB}^{c,t'} \in \text{Rev}_{AB}^c(\Theta_{AB}^{c,t}, D_{A,\leq t'}, D_{B,\leq t'}, H_{AB,\leq t'}, \mathcal{E}_{a,t'}, \mathcal{E}_{b,t'}), \quad t \preceq t'. \quad (44)$$

Here $H_{AB,\leq t'}$ denotes available evidence of interaction between the systems, while $\mathcal{E}_{a,t'}$ and $\mathcal{E}_{b,t'}$ register the relevant epistemic configurations. Equation 44 describes a class of permissible revisions rather than an independently validated algorithm. In contexts without interaction histories, admissible inferences may be substantially weaker. The record of failed correspondences and disagreements should be retained because it can inform later inquiry and the interpretation of subsequent relational changes.

Translation also relates to model-indexed continuation. A witness grounded exclusively in histories up to t may match the available outputs while fail to transfer some of the continuations generated after t .

The generative-preservation condition of Equation 40 offers a restricted way to examine this difference for discrete transition models. In a richer setting, the inquiry may compare selected observables on admissible whole-spacetime extensions, using the continuation objects of Definition 3.2. Their relation remains conditioned on the chosen models, their observation interfaces, and the availability of compatible extension contexts. A failed extension test can prompt revision of the source model, target model, or proposed translation.

In settings where participants act on their interpretations, translation can have further consequences. A translated scientific forecast may enable a local intervention; an institutional dialogue may generate a new procedure; a shared concept may alter what participants can recognize or contest. Those changes become part of subsequent histories and may transform both the admissible grammars and the practical capacities of participating subjects. The possibility of such feedback follows from the research setting, while its concrete presence and effects demand independent evidence. Whether a continuation becomes more intelligible, more accessible, or normatively preferable remains a situated question. The following section will formalize model-indexed relational continuation in greater detail and examine how evolving generative grammars condition possible future structures.

8 Generative Continuation and Possible Relational Futures

Knowledge pressing concerns the historical constitution of generative systems and the further relational developments in which epistemic activities may participate. This section specifies a restricted formal account of such developments. Its objective is to connect model-indexed inference and cross-system translation to prospective relational configurations while preserving the distinction between what an observer can describe, what a model permits, what practical intervention can reach, and what subsequently occurs. The discussion first elaborates whole-spacetime extensions of reconstructed histories, then develops model-indexed continuation families, topological transformations, evolving rules, action-conditioned accessibility, translation-mediated possibilities, and retrospective incorporation of realized developments into new histories. Elementary cellular examples and a conditional set-theoretic proposition provide precise, limited demonstrations. Questions concerning metaphysical openness, deliberation, and free will are reserved for the subsequent agency section.

8.1 Whole-Spacetime Continuation and Historical Embedding

The preceding reference model considers a relational history through a cell complex and temporal, causal, or interpretive decorations (Definition 3.1). At an epistemic stage t , an observer a holds a reconstruction $\hat{K}_{a,\leq t}$ whose evidence may be incomplete, contested, or revised. A prospective continuation uses this reconstructed history as a marked part of a more extensive relational structure. In this restricted cellular setting, the candidate object is a complex K^+ with an embedding of the reconstructed prefix. The embedding marks historical compatibility, whereas the additional cells and decorations describe what the candidate adds. The resulting representation treats history and its prospective extension as portions of one possible whole-spacetime construction. Temporal indexing serves the situated inference problem and does not posit two independently existing ontological domains called past and future.

A literal embedding carries a subtle modeling requirement. A complex representing only presently active relations may delete an edge or a norm when the relation ceases. A history-preserving complex, by contrast, can keep the record of the edge and mark its period of activity with a temporal annotation. We adopt the latter interpretation for the present formal development: past cells remain in the history representation, while their activity or meaning may change. When retrospective evidence leads to a new interpretation of an earlier record, the observer's *reconstruction* can change, although that change should not be confused with a modification of the posited historical occurrence itself. The same distinction is relevant to historical archives and institutional records, where preservation of evidence does not guarantee an unchanging account of what that evidence means.

The continuation object introduced in Definition 3.2 is sharpened here by recording the provenance of its historical prefix and the rules governing future additions. Equation 45 gives the descriptive tuple for a candidate in a specified inquiry.

$$\mathbf{C}_{a,t}^{M,c} = (K^+, j, \lambda^+, \gamma, \Pi), \quad j : \widehat{K}_{a,\leq t} \hookrightarrow K^+. \quad (45)$$

Here M is a typed generative representation, c is a declared context, γ is a derivation or admissibility witness when the model supplies one, and Π records the source of the reconstruction, its uncertainty, and relevant revision history. In a continuous model, γ might specify an initial-boundary-value solution and its regularity assumptions; in a cellular grammar it might be a sequence of typed rewrite applications; and in an institutional model it might record authorized transitions under a documented procedural grammar. The tuple does not assert that a common mechanical semantics exists for these cases. The common interface merely asks how the continuation is constructed, what evidence it inherits, and which conditions support its interpretation.

Composition of compatible historical embeddings is mathematically straightforward, although it matters for the architecture of successive inquiries. If $j_{01} : K_0 \hookrightarrow K_1$ and $j_{12} : K_1 \hookrightarrow K_2$ preserve the stipulated cell structures, then $j_{12} \circ j_{01}$ embeds K_0 into K_2 . The intermediate continuation remains recoverable as a marked stage when its inclusion and decorations are retained. It is therefore possible to preserve a chain of epistemic and representational updates without assuming that an individual endpoint determines the preceding derivation. The historical ambiguity illustrated in Section 3 continues to apply whenever the intermediary attachments or their provenance have been lost.

8.2 Model-Admissible Continuations and Epistemic Modalities

The continuation family $\text{Cont}_{a,t}(M; c)$ collects the model-indexed extensions of a reconstruction in Definition 3.2. Different models compatible with the same recorded history can supply different admissible families. Consequently, an inference procedure that preserves several candidate models also preserves a structured uncertainty over future possibilities. Its output may be a collection of extensions, a probability law over them, a constrained reachable set, or a qualitative account of possible practices, depending on the type of the generative model. A probability law can be interpreted only with its measurable sample space and conditioning assumptions; an unweighted set of branches has no inherent probability measure.

For a declared collection of historically compatible models $\mathfrak{M}_{a,t}(D, c)$, the broad model-supported family can be written as Equation 46.

$$\mathfrak{C}_{a,t}^{\text{model}}(D, c) = \bigcup_{M \in \mathfrak{M}_{a,t}(D, c)} \text{Cont}_{a,t}(M; c). \quad (46)$$

Equation 46 uses a tagged union when continuation objects belong to different types; a comparison of genuinely heterogeneous elements additionally requires a declared observation or translation interface. The union gives a conservative inventory of extensions admitted by at least one currently retained model. It can be useful for exploratory inquiry, but its size has little intrinsic meaning across different model languages. Counting synthetic branches and describing the conditions under which a person can access an institutional opportunity are distinct tasks.

Four modalities should be distinguished. An *epistemically formulable* continuation belongs to an observer's currently expressible repertoire. A *model-admissible* continuation is licensed under a particular specification of generative constraints. A *physically realizable* continuation satisfies the relevant physical conditions, insofar as these can be established. A *practically accessible* continuation can be pursued by a specified subject or collectivity under available resources, opportunities, restrictions, and action capacities. A realized continuation is subsequently established through historical evidence. Membership among these classes depends on their separate criteria, and a proposed nesting between them requires an explicit comparison domain. For example, an impoverished family's conception of a future educational path need not be empty even where its present opportunities to obtain that path are severely constrained.

The familiar formal distinction between reachable states and paths satisfying additional constraints supplies one technical precedent. Timed automata analyze temporal behavior within explicitly modeled clock conditions [3]; viability theory studies evolution subject to stated constraints, including feasible controlled trajectories [5]. These results do not themselves settle which futures a social system can imagine or which arrangements would be legitimate. They show why a continuation model should state its dynamics, admissibility restrictions, and temporal resolution rather than attach a single undifferentiated label of possibility to a predicted outcome.

A related inferential distinction concerns *possibility under uncertainty*. When the observer retains competing models M_1 and M_2 , an extension supported by M_1 alone may be included in the exploratory union even though its status under M_2 is excluded or undefined. Reporting the provenance of support allows readers to distinguish a robustly shared continuation, a disputed extension, and one that was generated only under an especially speculative model. For bounded problems, this can support robust-control or sensitivity analyses. For interpretive systems, the corresponding evidence may include conflicting accounts and the different practical assumptions under which each account is intelligible.

8.3 Historical Evidence and Prospective Revision

Continuations are constrained by available evidence, yet the relation between evidence and possible futures is mediated by model selection. For a fixed historical reconstruction, context, and model semantics, eliminating unsupported models can restrict the broad union of admissible extensions. The following elementary result specifies that conditional claim.

Proposition 8.1 (Restriction under fixed continuation semantics). *Let $\mathfrak{M}_2 \subseteq \mathfrak{M}_1$ be two collections of typed models defined on a common tagged continuation universe. Assume a fixed reconstructed prefix, context, and continuation relation $\text{Cont}_{a,t}(M; c)$ for each M in either collection. Then the unions of their model-admissible continuations satisfy Equation 47.*

$$\bigcup_{M \in \mathfrak{M}_2} \text{Cont}_{a,t}(M; c) \subseteq \bigcup_{M \in \mathfrak{M}_1} \text{Cont}_{a,t}(M; c). \quad (47)$$

Proof. Every model indexed in the left union is also indexed in the right union. An extension admitted by such a model therefore belongs to the right union. $\square$

Proposition 8.1 is a set-theoretic consequence of its fixed-semantics assumptions, rather than a substantive discovery about historical dynamics. Its purpose is to isolate a common ambiguity: an observer may report a reduction of projected futures because evidence has excluded some candidate models, while the investigated system's own generative capacities have not changed. The result does not apply automatically if the historical prefix is revised, a new representation type is introduced, the observation interface changes, or the rule-evolution model is updated. Each of those changes can alter what the expressions on the two sides even designate.

Evidence-conditioned revision may also reveal futures that an earlier epistemic language could not describe. Suppose an observer initially records only ordinary edges, then acquires a meaningful account of a two-cell attachment in a social or physical model. The observer may now distinguish a future in which a cycle is filled by an admissible higher-order relation from one in which the cycle persists. This epistemic enrichment does not by itself establish the historical appearance of a new generative law. The additional possibility may have belonged to the investigated mechanism all along. Conversely, a documented rule reform may change the mechanism itself without being represented promptly in the observer's archive. Temporal alignment between observations, model versions, and institutional or physical events is therefore central to a responsible account of prospective revision.

8.4 Topological Transformation in Cellular Continuations

Higher-dimensional relational representations are useful when future generation can change cycles, boundaries, and the possible attachments of cells. The present examples employ finite two-dimensional

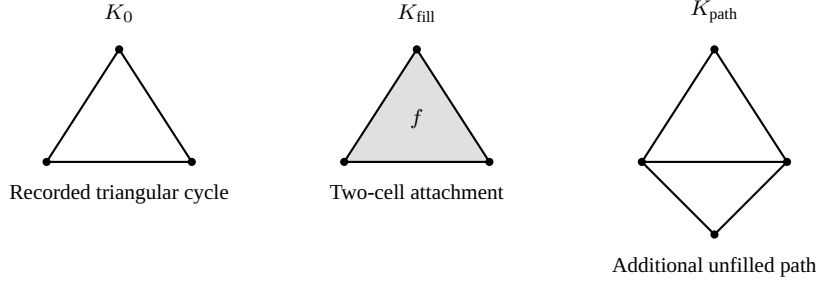


Figure 2: Two distinct cellular continuations of a recorded triangular cycle. The center branch attaches a two-cell and fills the original cycle; the right branch adds a two-edge path through a new vertex while preserving the original one-skeleton. The diagram illustrates model-relative structural options, without assigning probabilities or normative rankings.

cell complexes with explicitly stipulated semantics. They do not assume that a homological feature is an agent, value, normative standing, or causal force. The topological quantities summarize selected structural relations that may have application-specific interpretations.

Consider a reconstructed historical one-skeleton K_0 consisting of three vertices and a closed triangle of three edges. With coefficients in $\mathbb{F}_2$, its first homology has dimension one. Two elementary continuations are available within a grammar allowing either a two-cell attachment or a new path joining two existing vertices through a fresh vertex. Attaching a face f along the triangular cycle fills that cycle, giving first Betti number zero. Attaching a two-edge path v_0-w-v_2 without a face creates an additional independent cycle in the connected one-dimensional complex and gives first Betti number two. Figure 2 displays these extensions, and Equation 48 records their homological signatures.

$$\beta_1(K_0) = 1, \quad \beta_1(K_{\text{fill}}) = 0, \quad \beta_1(K_{\text{path}}) = 2. \quad (48)$$

The calculations in Equation 48 follow from the corresponding cellular chain complexes over $\mathbb{F}_2$. For K_{fill} , the one nonzero independent two-cell boundary kills the original one-cycle. For K_{path} , four vertices and five edges form a connected graph with cycle rank $5-4+1 = 2$. A change in the value of β_1 indicates a change in selected topology under these constructions; it says nothing by itself about whether a new institutional arrangement creates justice, or whether an individual's choices become more free. Those evaluations require other data and interpretive commitments.

This finite example also isolates the importance of *rule availability*. If the working grammar permits only face attachment, the new-path branch is excluded. If it permits both operations, the two alternatives become model-admissible. If a later rule update permits detachment, deactivation, or a three-dimensional relation, the representational setting requires further specification. The distinction between newly available operations and newly observed operations is retained even when the same visual result can be generated by different derivations.

8.5 Evolution of Generative Rules and Prospective Horizons

A fixed representation M_t may supply useful short-horizon continuations while providing an inadequate account of future transformations of the generating grammar itself. The historical-evolution framework of Section 6 therefore admits sequences of models or rule sets coupled through typed update relations. Extending Equation 35, one can consider the family of whole-spacetime continuations associated with admitted rule histories. A time-indexed horizon h supplies a bounded version of the construction in Equation 49.

$$\mathfrak{C}_{a,t}^{U,h} = \bigcup_{\mathbf{M} \in \text{Traj}_U^h(\mathfrak{M}_{a,t}; c)} \text{Cont}_{a,t}^{\leq h}(\mathbf{M}; c), \quad (49)$$

where Traj_U^h is a collection of admissible model histories with at most h future transition stages, and $\text{Cont}_{a,t}^{\leq h}$ denotes their tagged whole-spacetime extensions over that horizon. A finite bound makes the

model comparison operational for suitably computable generators, though a finite search may still be intractable. Equation 49 remains conditional on the update mechanism U . A learned rule-update process is itself a candidate representation, with observational and identifiability limitations inherited from Section 5.

Changes to generative grammar can produce different effects. A new operation may admit a type of configuration that had previously been unreachable under the stipulated rules. Removal of a permission may close an earlier path. An interaction may alter which agents can invoke an operation, even when the formal rule remains in a common registry. These situations suggest distinguishing the *syntax of available transformations*, their *operational availability* in a specific context, and the *historically realized enactment* of a transition. A formal extension space indexed only by abstract production rules can overstate practical agency where permissions, material prerequisites, and power differ among participants.

At longer horizons, uncertainty may grow because a model permits many trajectories or because its model class ceases to describe relevant transformations. Those are separate difficulties. Counting model-admissible branches is meaningful only after specifying a finite search grammar and equivalence criterion; continuous models often admit uncountably many trajectories, and different discretizations can change computational branch counts. An epistemic infrastructure should therefore present a horizon together with its model version, approximations, unmodeled mechanisms, and the status of the resulting possibilities. A prospective view can remain useful even when it supplies only qualitative comparisons or restrictions on feasible paths.

8.6 Action-Conditioned and Practically Accessible Continuations

The continuation construction can incorporate actions without treating an isolated subject as a primitive cell of relational spacetime. A subject may be reconstructed as a historically constituted subconfiguration with capacities, dependencies, and access to specific interactions. For an operational inquiry we can represent the subject at time t through a context-indexed interface $\mathcal{A}_{a,t}^{s,c}$ of actions recognized as available to subject s according to available evidence. The superscript records that availability is reported by an epistemic system and may require correction from the subject and affected participants. This interface may encode bodily ability, technical instruments, legal authorizations, social cooperation, economic resources, and exposure to coercion. Its content is historically variable.

An *action-conditioned continuation* is a model-admissible extension accompanied by an action or intervention history whose compatibility with the extension is specified. Equation 50 defines a restricted accessible family for an agent-indexed inquiry.

$$\mathfrak{C}_{a,t}^{\text{acc}}(s; M, c) = \{ \mathbf{C} \in \text{Cont}_{a,t}(M; c) : \exists \mathbf{u} \in \text{Path}(\mathcal{A}_{a,t}^{s,c}) \text{ Realize}_{M,c}(\mathbf{u}, \mathbf{C}) \}. \quad (50)$$

Here Path contains admissible action sequences and $\text{Realize}_{M,c}$ is a *modeled* relation of action and continuation, including relevant environmental and cooperative assumptions. The notation does not assert that the subject can guarantee a resulting future. In stochastic or interactive settings, the relation may capture positive support under a stated law, positive feasibility under a control condition, or an institutionally valid attempt. Those interpretations should be separated in any concrete application. Within the fixed model and context of Equation 50, the accessible family is a subset of the model-admissible family by construction; a stronger claim about the true range of the subject's practical freedom would require empirical and normative arguments.

A person may participate in changing their action interface through learning, new relationships, assistance, or collective organizing. A translation between professional systems may reveal an existing procedure, while a negotiated agreement can generate a new one. Conversely, surveillance, legal restrictions, economic precarity, or unequal bargaining power can close practical possibilities. The role of knowledge pressing here is conditional: epistemic reconstruction can influence action only through historically situated relations of attention, interpretation, material access, authorization, and cooperation. A richer representation of future options may improve an observer's understanding while leaving a subject's capacity to pursue those options unchanged.

In control-theoretic applications, action-conditioned feasibility admits well-developed formal methods; viability theory provides a precise setting for investigating constraints on state evolution [5]. In human and institutional settings, reduction of freedom to a reachability calculation would disregard dimensions such as coercion, recognition, and the authority to formulate one's own ends. Work on institutional legibility has drawn attention to the dangers of treating an administratively simplified model as a complete description of local practical knowledge [41]. Equation 50 should therefore be read as a local epistemic instrument for investigating constrained opportunities, alongside qualitative and participatory evidence. Its relation to philosophical theories of free will is examined in the subsequent agency section.

8.7 Translation-Mediated Continuation and Co-Evolution

A translation witness Θ_{AB}^c , as defined in Section 7, supports comparison between selected histories or generative features of systems A and B . For possible futures, the relevant issue is whether the correspondence survives admissible extensions and whether the interaction enabled by translation changes either system's subsequent generative conditions. The first question concerns *preservation*; the second concerns *co-evolution*. These should be investigated separately, since a structurally faithful correspondence may have little practical effect, and an institutionally consequential translation may remain partial or semantically disputed.

For compatible whole-spacetime extensions $\mathbf{C}_A$ and $\mathbf{C}_B$, the context can specify prospective observation interfaces $\alpha_{A,+}^c$ and $\alpha_{B,+}^c$. If their outputs are related by the witness relation R_c , they constitute a candidate continuation correspondence. The condition is recorded in Equation 51.

$$(\mathbf{C}_A, \mathbf{C}_B) \in \mathcal{R}_{AB,+}^c \iff (\alpha_{A,+}^c(\mathbf{C}_A), \alpha_{B,+}^c(\mathbf{C}_B)) \in R_c, \quad (51)$$

provided both observation maps are defined. Equation 51 states compatibility with a selected prospective interface, not equivalence of complete futures. If a translation is warranted only for histories already observed, extending α_i^c to prospective objects requires fresh assumptions and later tests. The forward-step preservation condition of Section 7 gives a stronger criterion for a restricted class of transition systems, but its adequacy still depends on the observed variables, labels, and branching semantics selected by the inquiry.

Consider a community organization and an emergency-management agency. A translation between local flood indicators and an agency's warning classifications may initially connect observations in the two systems. Interaction can then lead to a new consultation protocol, a revised threshold policy, or the incorporation of a previously neglected local observation. In such a case, the translation process participates in forming new relational connections and possibly new institutional rules. A more elaborate global infrastructure might document the records, model revisions, translation witnesses, and the emergence of the revised practice. The present example is hypothetical and should not be understood as an empirical claim about particular disaster-response institutions.

The possibility of translation-mediated generativity also creates a risk of asymmetric imposition. A system with institutional authority may define the shared vocabulary and reject interpretations that its interface cannot express. Formal alignment can then appear successful while obscuring a loss of meaningful agency for less powerful participants. Evidence about consultation, refusals, permissions, and subsequent harms should be preserved alongside a technically adequate correspondence. The epistemic objective is intelligibility and revision under declared constraints, while the normative evaluation of an interaction remains subject to the affected participants and the institutional procedures through which their claims can be considered.

8.8 Realization, Irreversibility, and Historical Feedback

At a later historical stage t' , some prospective descriptions can be compared with recorded developments. The comparison requires a new epistemic record $D_{a,\leq t'}$, which may itself be incomplete and subject to interpretation. It should be possible to identify which model version generated a prediction,

which assumptions selected the continuation, what interventions were actually undertaken, and where the observed evolution diverged from the candidate. This allows continuation models to inform retrospective learning without treating an earlier forecast as a definitive account of what had been physically possible.

A prospective model's failure may indicate a mistaken generative rule, an unmodeled event, an incorrect action assumption, an unstable translation witness, or a later change in the operative grammar. A model's apparent success may reflect a coarse observation interface under which several distinct mechanisms remain indistinguishable. Thus, the same inferential discipline developed for historical reconstruction applies prospectively and retrospectively. The continuation record can be interpreted as a versioned sequence of evidence-linked hypotheses and tests, with explicit treatment of missed possibilities, obsolete rules, and decisions that altered the subsequent data-generating process.

Irreversibility complicates this cycle. A physical intervention may alter a habitat, a medical decision may create a new dependency, and an institutional commitment may constrain future participation. Some changes can be reversed in a formal model, whereas restoration of the earlier social or ecological situation may remain practically infeasible. A continuation framework should record domain-specific notions of reversibility, the costs and authority of reversal, and the persistence of consequences. A topological operation that is invertible on abstract complexes need not be reversible at the level of affected people or material systems.

The cumulative significance of generative continuation is therefore twofold. First, it develops a typed, evidence-conditioned representation of how historical relational configurations may be extended under explicit generative and action constraints. Second, it identifies ways in which epistemic activities, including inquiry and translation, can participate in transforming the rules and conditions of subsequent development. Within this limited formal framework, knowledge pressing concerns the co-evolution of relational structures, inferential capacities, and possible continuations. It leaves open the metaphysical status of alternatives and the contested question of which relational conditions constitute freedom. Those questions motivate the forthcoming analysis of historically constituted agency, deliberation, and free will.

9 Agency, Free Will, and Epistemic Participation

The purpose of this section is to examine the relation between historically constituted subjects and the continuation of relational spacetime. Sections 3 and 8 introduced epistemic systems with distributed historical support and model-indexed possible continuations. The present analysis asks how a subject can deliberate, intervene, revise its capacities, and participate in modifying the conditions of further generation. Agency is approached through a restricted, context-indexed account of practical capacities and responsive conduct. Free will is then treated as a distinct philosophical question concerning control, alternatives, sourcehood, and responsibility. The discussion develops operational definitions, a conditional proposition, two examples, and a comparison of different modal and normative interpretations. It draws on philosophy of action, autonomy, compatibilism, relational agency, and the capability approach while leaving contested metaphysical and moral premises explicit [38, 29, 26, 10, 35].

9.1 Historical Constitution of Relational Subjects

A relational account of historical constitution begins with the observation that an individual subject depends on biological organization, developmental processes, language, social relations, institutional arrangements, and available practices. In the illustrative cellular model of Section 3, a subject is consequently represented by distributed historical support and relations rather than by a privileged vertex. Vertices denote events; edges denote couplings; faces denote application-dependent higher-order primitives. Attributed subjecthood requires additional criteria for bodily, experiential, institutional, or deliberative continuity. These criteria are specific to the inquiry and may themselves be disputed.

For a subject s and an observer a , a provisional *subject description* may be indexed by a historical stage t and a context c . Equation 52 records the analytical roles of its components.

$$\text{Subj}_{a,t}^c(s) = (S_{a,t}^s, \mathcal{E}_{a,t}^s, \mathcal{A}_{a,t}^{s,c}, \text{Delib}_{a,t}^{s,c}, \Pi_{a,t}^{s,c}). \quad (52)$$

Here $S_{a,t}^s$ is the inferred historical support; $\mathcal{E}_{a,t}^s$ describes situated epistemic resources; $\mathcal{A}_{a,t}^{s,c}$ is an action interface; $\text{Delib}_{a,t}^{s,c}$ refers to accessible descriptions of deliberation, reasons, or practical orientation; and $\Pi_{a,t}^{s,c}$ records provenance, uncertainty, and the standpoint from which the description is constructed. Equation 52 is deliberately observer-indexed. The subject's own self-description, third-party behavioral observation, and an institution's administrative classification may differ. None acquires final authority simply by being formalized.

The account avoids a reduction of subjecthood to a selected trajectory pattern. A network can have identifiable persistence and feedback without possessing the phenomenology, practical standpoint, or forms of accountability commonly associated with a person. Conversely, the absence of an articulable self-description in a dataset does not negate the standing of an affected person. Philosophical accounts of agency differ over whether reasons, intentions, self-reflection, and causal initiation are essential to action; relational and enactive accounts further emphasize organization and histories of interaction [38, 6]. The formal description is therefore an epistemic instrument for studying specified aspects of agency, with no presumption that it settles personhood.

9.2 Epistemic Capacities and Practical Action

A subject may recognize a possible action yet lack the means to perform it. Conversely, familiar practices can be executable without an explicit conceptual model. For knowledge pressing, the difference between conceivability, admissibility, and practical access is essential. Section 8.6 introduced an action-conditioned continuation family using an admissible path through $\mathcal{A}_{a,t}^{s,c}$. The present discussion refines that interface by separating an observer's record of opportunities from the subject's actual ability to use them.

For a fixed model M , context c , and epistemic record $D_{a,\leq t}$, let $\text{Act}_{a,t}^{s,c}$ be the described set of locally considered actions. A relation $\text{Feas}_{M,c}(s, u; K)$ states that the proposed action u is feasible at reconstructed configuration K under stated assumptions about bodily capacities, resources, permissions, collaborators, and environmental conditions. Equation 53 gives the resulting model-relative action set.

$$\mathcal{F}_{a,t}^s(M, c) = \{u \in \text{Act}_{a,t}^{s,c} : \text{Feas}_{M,c}(s, u; \hat{K}_{a,\leq t})\}. \quad (53)$$

The predicate in Equation 53 is a place for domain-specific evidence and evaluation, not a generally computable test for what a person can do. It is especially sensitive to inaccessible information and institutional bias. A patient may be legally eligible for an appointment yet lack affordable transport; a researcher may know a method but lack the equipment; a member of a community may understand a proposed emergency measure but possess no authority to implement it. Formal eligibility, subjective recognition, and effective access occupy different explanatory levels.

The capability approach offers an established account of substantive opportunity, attending to personal, social, and environmental conditions that influence the conversion of resources into achievable functionings [35]. The present framework borrows the methodological importance of conversion conditions without equating a model-generated continuation set with an adequate measure of a person's well-being or freedom. A complete assessment may require testimony, institutional histories, material indicators, and contestable evaluations of what counts as an opportunity.

9.3 Action-Conditioned Futures and Alternative Possibilities

A continuation model makes possible developments expressible relative to a declared model and history. It does not follow that multiple model branches are physically available under a fixed complete state of the world. Equation 54 refines the continuation family by making the assumed action policy explicit.

$$\mathfrak{C}_{a,t}^{s,\pi}(M, c) = \{\mathbf{C} \in \text{Cont}_{a,t}(M; c) : \text{Cons}_{M,c}(\mathbf{C}, \pi, \mathcal{F}_{a,t}^s)\}. \quad (54)$$

Here π is a conditional plan or policy defined on a specified class of perceived states and histories, while $\text{Cons}_{M,c}$ records modeled consistency among that policy, feasible action sequences, and the continuation. It may include stochastic environmental responses, other participants' actions, and transitions of the subject's own epistemic configuration. Thus, different values of π can yield different modeled continuations without the claim that a particular subject could have enacted both policies under exactly the same total circumstances.

At least four kinds of possibility must be kept distinct. *Representational possibility* means that an epistemic language can describe an extension. *Model admissibility* means that a particular model permits it under the assumptions in force. *Practical accessibility* refers to an agent's effective opportunities under relevant material and social conditions. *Metaphysical alternative possibility* concerns what could occur holding fixed whichever facts and laws a philosophical interpretation treats as relevant. The latter notion cannot be read directly from the first three. A branching model may express ignorance, stochastic variation, response to distinct interventions, or rules that actually admit multiple transitions. Clarifying its branching semantics is a prerequisite for interpreting it.

The free-will literature distinguishes the requirement of alternative possibilities from accounts emphasizing the source and quality of control over action [29, 26]. A deterministic representation might provide a useful explanation of reasons-responsive agency under a compatibilist account, while an incompatibilist may dispute its adequacy for sourcehood or the ability to do otherwise. The present framework assumes neither philosophical resolution. Its contribution here is a disciplined representation of distinct questions and of the evidence needed for the narrower empirical claims.

9.4 Deliberation, Reasons, and Responsive Control

An epistemic system can form candidate representations of its circumstances, consider action alternatives, receive objections, and revise its plans. Such activities can enter the generative dynamics of a subject's behavior. Yet a model that predicts outward actions may be silent about the subject's own understanding, reasons, and capacity to endorse or refuse proposed ends. The philosophy of action offers multiple accounts of intention and acting for reasons, and theories of autonomy examine whether deliberation is appropriately the subject's own [38, 10].

For a restricted inquiry, a *reason-responsive policy model* specifies a collection of perceived circumstances Z , a collection of considerations R , and a policy family $\{\pi_r : r \in R\}$. The operational question concerns sensitivity of action to changes in identified considerations under fixed background constraints. Equation 55 describes a simple observational contrast.

$$\Delta_{a,t}^{s,c}(r, r'; z) = d_A(\pi_r(z), \pi_{r'}(z)), \quad z \in Z, \quad (55)$$

where d_A is a stipulated distance or dissimilarity on policy outputs. The contrast in Equation 55 provides an operational sensitivity probe, not a necessary or sufficient condition for free action. A responsive policy could have been installed through coercion or manipulation. The observed consideration r might be misidentified, and small output differences may conceal profound changes in the subject's reasons. Some forms of autonomy concern an agent's capacity to question and revise the standards by which considerations count as reasons, a process that a fixed input-output contrast alone cannot capture.

Historical constitution complicates the interpretation of deliberation constructively. Education, conversation, and translation can enlarge the subject's vocabulary of available reasons. Trust and recognition can change whose testimony is considered credible. Injury, fear, or institutional pressure can restrict the conditions under which deliberation occurs. A process of knowledge pressing may therefore alter the informational, affective, and normative conditions of subsequent decisions. The causal and ethical assessment of such interventions demands separate investigation. Providing information, restructuring a choice environment, and manipulating another subject's deliberation may all change policy outputs while involving very different relations of authority and responsibility.

9.5 Self-Modification and Evolving Capacities for Agency

Agency is historically dynamic. A subject's learning can change the policies it considers, its ability to formulate goals, its practical competences, and the social relationships through which an action becomes possible. Changes can also be involuntary: illness, loss of infrastructure, institutional exclusion, or conflict can alter capacity. The question of self-modification is therefore situated within processes of mutual conditioning rather than as an isolated act of self-creation.

Let $\mathcal{G}_{a,t}^s$ denote an attributed practical grammar, consisting of available action descriptions, admissible combinations, and context-specific constraints. A modeled update can depend on interaction, deliberation, and unobserved historical influences, as expressed in Equation 56.

$$\mathcal{G}_{a,t+1}^s \in \mathcal{U}_{a,t}^{s,c}(\mathcal{G}_{a,t}^s, D_{a,\leq t+1}, \pi_t, \eta_t). \quad (56)$$

The update relation in Equation 56 admits a family of possible grammar changes, and η_t stands for influences omitted from the explicitly modeled record. Its form can range from learning a new motor procedure to negotiating a new institutional right. These cases differ in mechanism, authority, and normative significance. The update is not postulated as a universal self-determination law; it represents a class of questions about how epistemic and practical capacities evolve together.

A change in $\mathcal{G}_{a,t}^s$ can alter the continuation family even when the material environment is initially held fixed. For example, learning to interpret another profession's procedural grammar can make an already authorized procedure intelligible and usable. A collective agreement can go further by generating a previously unavailable procedure. Other changes may close off formerly available actions. The historical formation of practical agency is consequently associated with transformations in the structure of accessible paths, including their constraints, dependences, and potential irreversibility.

There is an important limitation. An expanded grammar of conceivable actions does not establish an expanded set of feasible or worthwhile actions. The epistemic system must investigate the conditions that translate competence into effective opportunity. Nor does the mere possibility of changing one's own capacities settle whether the source of that change is free under any particular metaphysical or normative account.

9.6 Relational Power, Coercion, and Accessible Futures

Power is relevant to generative continuation insofar as subjects and institutions can influence the geometry, availability, and evolution of other participants' action conditions. For this paper, the term *power* denotes a family of situated capacities to affect relational configurations, permissions, resources, and the conditions of subsequent generation. It is not introduced as an empirically established universal scalar field. Physical field metaphors may be useful for exploring distributed influence, but their applicability to normative institutions requires additional definitions and tests.

Consider two subjects who face the same publicly described procedure. Their practical opportunities may differ because one can access necessary resources, obtain permission, or safely refuse, while the other cannot. Their observed behavioral histories could nevertheless be identical over the available record. Differences in access, dependence, and coercion may remain invisible if a model records only completed actions. This is one reason to preserve action interfaces and contextual evidence rather than reconstruct agency solely from successful transitions.

A useful restricted implication concerns the monotonicity of modeled opportunity under an explicitly fixed background. Define $\text{Acc}_{M,c}(\mathcal{F})$ as the family of continuation objects associated with action sequences assembled from a feasible action set $\mathcal{F}$, using the same continuation model and the same consistency relation throughout. The conditions of the following proposition deliberately exclude strategic responses or collateral changes in other agents' capabilities.

Proposition 9.1 (Monotonicity of modeled access under fixed conditions). *Suppose $\mathcal{F}_1 \subseteq \mathcal{F}_2$, every sequence admissible under $\mathcal{F}_1$ remains admissible under $\mathcal{F}_2$, and the model, contextual constraints, and action–continuation consistency relation are identical for the two comparisons. Then $\text{Acc}_{M,c}(\mathcal{F}_1) \subseteq \text{Acc}_{M,c}(\mathcal{F}_2)$.*

Proof. Any continuation witnessed by a sequence admissible under $\mathcal{F}_1$ has the same witness under $\mathcal{F}_2$, since admissible sequences are nested and the consistency relation is unchanged. Hence every member of the first accessible family belongs to the second. $\square$

Proposition 9.1 is an elementary consequence of set inclusion. It identifies the assumptions needed for a narrow statement about modeled opportunity; it proves no claim about increasing autonomy, moral standing, or overall freedom. In actual relational systems, giving one actor greater powers can restrict another’s opportunities, and adding choices can modify incentives, surveillance, burdens, and coercion. Such changes violate the fixed-background assumptions. Comparing the consequences of institutional interventions therefore requires modeling the responses of affected parties and the authority through which new possibilities become available.

9.7 Collective Agency and Inter-System Participation

Many continuations depend on coordinated activities. A person may recognize a generative possibility and still require consent, expertise, infrastructure, or joint commitment to realize it. Institutions can create capacities unavailable to isolated participants, while coordination can also establish unequal dependencies. A relational account accordingly allows collective practices to contribute to the generation and revision of possibilities without assuming that a collective subject is always reducible to the sum of independent preferences.

A minimal model considers a family of participants S with individual action interfaces and an interaction protocol Γ_c . For a joint history $\mathbf{u}_S$, the protocol records feasible cooperation and the conditions under which a collective intervention is treated as compatible with a continuation. Equation 57 expresses a context-indexed collective accessibility family.

$$\mathcal{C}_{a,t}^{S,\Gamma_c}(M) = \{\mathbf{C} \in \text{Cont}_{a,t}(M; c) : \exists \mathbf{u}_S \in \text{JointPath}_{\Gamma_c}(S), \text{Cons}_{M,c}(\mathbf{C}, \mathbf{u}_S)\}. \quad (57)$$

Equation 57 captures a formal dependence on coordination rules. A collective continuation can become feasible through shared resources and negotiated authorizations; another can be blocked when a required participant withholds consent. These are context-dependent relations, and the label “collective” conveys no automatic legitimacy. Representation of a community’s agreement must distinguish elected representatives, consent procedures, actual participation, dissent, and the people bearing consequences. The governance of coordination is part of the historical relational structure under investigation.

Translation between heterogeneous practical grammars can modify Γ_c . A scientific warning, an administrative protocol, and a community’s locally recognized signs may acquire a jointly negotiated correspondence, thereby altering how they coordinate protective action. The translation need not eliminate semantic disagreement. It may create a working interface that allows cooperation while preserving incompatible interpretations. Such an interface can itself become an object of subsequent criticism and reform. The case connects the translation witnesses of Section 7 to continuation while preserving the participants’ distinct epistemic and normative standpoints.

9.8 Free Will, Metaphysical Openness, and Self-Determination

The philosophical problem of free will exceeds the empirical question of whether subjects can act and modify practical constraints. The classic debate concerns, among other issues, whether causal determinism is compatible with freedom, whether freedom requires alternative possibilities, whether an agent must be the relevant source of an action, and how responsibility should be understood [29, 26]. The framework developed here helps make distinct explanatory claims explicit. It provides no general theorem about the truth or falsity of compatibilism, libertarianism, or hard incompatibilism.

For clarity, the formal possibilities developed in the preceding sections may be interpreted through three independent questions. The first is *epistemic*: which continuations remain consistent with evidence and an observer’s models? The second is *dynamical*: which continuations are permitted by a particular physical or social model under stated state and intervention assumptions? The third is *agential and normative*: which processes qualify as action attributable to a subject, and what kind of control or sourcehood

Table 7: Distinct inquiries concerning relational continuation and freedom. A formal model can address selected empirical relations while leaving metaphysical and normative criteria open.

Inquiry	Relevant question	Adequate basis for assessment
Epistemic openness	Which extensions are consistent with available evidence?	Observation model, uncertainty, provenance, competing candidates
Dynamical admissibility	Which extensions satisfy assumed laws or rules?	Domain-specific model, conditions, derivations and tests
Practical opportunity	Which actions can a situated subject effectively attempt?	Resources, ability, authorization, cooperation, lived evidence
Autonomy and control	Under what conditions does action express self-government?	Deliberative and relational histories, coercion and manipulation, philosophical account
Metaphysical freedom	Could the subject have acted otherwise in the relevant sense?	Explicit metaphysical premises and argument beyond model branching
Normative responsibility	Who may be held answerable, and on which grounds?	Context-specific ethical and institutional justification

is required for freedom or responsibility? A model may answer aspects of the first two without providing an account of the third.

Table 7 records different questions and the kinds of evidence or argument required for them. Its separations are methodological; the philosophical theories connecting these questions remain contested.

Consider a deterministic model that describes a subject's deliberation as dependent on inherited biological and social conditions. This description is compatible with the idea that the subject's deliberation has genuine causal effects within the system; whether those effects amount to free action depends on an account of freedom. Now consider a stochastic model with several supported future paths. The multiplicity of paths is likewise insufficient on its own to establish self-government or answerability. Random variation and autonomous control are different concepts. These contrasts help prevent the topology of a model's continuation space from being promoted into a conclusion about free will.

The relation between self-determination and historical constitution raises a further difficulty. Agents may revise their commitments and the practical grammars through which they deliberate, yet even those revisions occur under inherited conditions. Some theories emphasize reflective endorsement or responsiveness to reasons; others impose historical conditions meant to distinguish autonomous formation from manipulation [10, 26]. Our formalism can record parts of those histories and support comparison among explanations. It cannot replace participant testimony, legal standing, and moral reasoning with a universal numerical freedom score.

9.9 Epistemic Participation, Normative Plurality, and Future Creation

Knowledge pressing can be causally implicated in further relational generation when inference or translation modifies the actions people can recognize, the interfaces through which they cooperate, or the rules governing permissible interventions. A research finding may alter an institution's practice; a new translation may reveal a previously inaccessible procedure; critical historical understanding may allow a community to contest a prevailing account of its circumstances. Such activities are historically conditioned and their generative consequences may include both expanded and diminished possibilities.

A hypothetical disaster-response case illustrates the full argument. A community's practical grammar may connect unusual river signs with household decisions, while a technical agency models rainfall, catchment dynamics, and probabilistic flood risk. A translation interface can connect selected observations without requiring either grammar to be absorbed into the other. Joint inquiry may reveal a previously absent evacuation arrangement or a conflict over authorized decisions. The resulting collective protocol may change the relations of trust, responsibility, and effective access for future emergencies.

The historical record can capture which interpretation was proposed, by whom, how it was contested, which decisions followed, and how later outcomes altered the protocol. This is a possible trajectory of epistemic participation, not evidence that a specific system of knowledge pressing will produce agreement or improved welfare.

Power and justice remain integral to evaluating these processes. A formally accurate explanation can be used to manipulate, exclude, or govern a less powerful subject without meaningful participation. An apparent expansion of collective options can coincide with losses for particular persons or communities. Likewise, an institutional classifier can assign responsibility while overlooking coercion, disability, or asymmetrical access. A practical infrastructure should preserve the grounds on which inferences and normative judgments are made, and make contestation possible for affected participants. It should avoid the premise that one epistemic system enjoys a privileged, globally authoritative view of all others.

The resulting account of agency has a limited but substantive role. It identifies historically distributed subjects and institutions as possible participants in the transformation of generative conditions; it connects inference and translation to action under specified constraints; and it requires the assessment of freedom and responsibility to acknowledge both material possibilities and contested normativity. Knowledge pressing, in this sense, concerns the co-development of relational structures, generative representations, epistemic capacities, and practical agencies. Its continuation-oriented interpretation does not require a metaphysically open future, and its mathematical constructions establish no general verdict on free will. The forthcoming section investigates the broader limits of identifiability, translation, and modal claims underlying this account.

10 Formal Properties, Identifiability, and Epistemic Limits

The formal framework developed in Sections 5–9 permits an epistemic system to retain candidate generators, examine partial translations, and investigate admissible continuations of an evolving relational history. The present section establishes limits on the conclusions that these operations warrant. Its objectives are to distinguish identifiable properties from identifiable representations, specify how observation and intervention constrain inquiry, examine information lost through structural summaries and translation interfaces, and delimit the interpretation of future possibilities and normative judgments. The method is conditional: precisely defined observation classes support elementary propositions and counterexamples, whereas claims about situated understanding and authority require additional interpretive and institutional evidence. The results are modest consequences of established mathematical ideas. Their role is to expose the assumptions that subsequent computational experiments must make visible.

10.1 Observational Equivalence Relative to an Inquiry

A claim of generative recovery presupposes an observation procedure and a class of questions that the recovered representation is expected to answer. Let $\mathfrak{M}_c$ be a specified family of typed generative models in context c . Let $\mathcal{Q}$ be a collection of admissible observation protocols, where each $q \in \mathcal{Q}$ specifies a measurement space $\mathcal{O}_q$, an observation horizon, and any allowed inputs. When a model M supplies a probability law on $\mathcal{O}_q$, denote it by P_M^q . A deterministic generator can be represented by a point-mass law, provided its observation map and input are specified. Equation 58 defines the family of observational laws available under this inquiry.

$$\Omega_{\mathcal{Q},c}(M) = (P_M^q)_{q \in \mathcal{Q}}, \quad P_M^q \in \mathcal{P}(\mathcal{O}_q). \quad (58)$$

Equation 58 is a typed signature rather than a universal numerical descriptor: distinct protocols may have distinct measurable spaces, and the family exists only when the selected model provides the specified predictions. For documentary or interpretive inquiry, an alternative signature might collect warranted judgments, recognized practices, or responses to contextually legitimate questions. Such signatures require separately justified interpretation rules. A common notation for signatures does not imply a common metric across these inquiries.

Two models are observationally equivalent for $\mathcal{Q}$ when their signatures agree, consistent with Equation 25. A refinement of instruments or an enlargement of the query class can split an equivalence class; a coarser projection can merge classes. Neither operation necessarily changes the generative mechanisms under investigation. This dependence on observational design underlies the established distinction between structural identifiability under ideal data and practical identifiability under finite, noisy data [34]. With historically evolving mechanisms, the temporal range of $\mathcal{Q}$ is especially consequential: a generator that remains stable during the sampled interval may change later, while another may remain stable throughout, without the recorded prefix separating them.

10.2 Query-Relative Identifiability and the Evidential Fiber

An epistemic inquiry often seeks a particular property of a system, such as a conserved quantity, a selected continuation constraint, or the presence of an admissible action, rather than its complete generative description. Let $\psi : \mathfrak{M}_c \rightarrow Y$ be a well-defined property or answer map taking values in a specified set Y . Fix an observational signature o that is realized by at least one candidate model. The associated *evidential fiber* is given in Equation 59.

$$\mathfrak{F}_{\mathcal{Q},c}(o) = \{M \in \mathfrak{M}_c : \Omega_{\mathcal{Q},c}(M) = o\}. \quad (59)$$

An observable record determines ψ within this model class precisely when its value is unchanged across the fiber. The following proposition states that elementary criterion in a form useful for assessing candidate outputs of knowledge pressing.

Proposition 10.1 (Query-relative identifiability on an observational fiber). *Fix $\mathcal{Q}$, c , $\mathfrak{M}_c$ and a nonempty fiber $\mathfrak{F}_{\mathcal{Q},c}(o)$. A property $\psi : \mathfrak{M}_c \rightarrow Y$ can be assigned a unique value from the signature o , uniformly across models in the fiber, if and only if ψ is constant on $\mathfrak{F}_{\mathcal{Q},c}(o)$.*

Proof. If the property has a unique value determined by o , each model consistent with o must have that value, so ψ is constant on the fiber. Conversely, if ψ is constant on the fiber, assigning this common value to o is unambiguous relative to the admitted model family. The statement does not assert that the assigned value is true of an unobserved system outside that family. $\square$

Proposition 10.1 separates the ambitions of *model identification* and *question identification*. Even where two parameterizations or mechanisms cannot be distinguished, they may agree on a scientifically important answer. Conversely, an excellent fit to observed trajectories may leave a targeted intervention response unresolved. In this sense, the question being pressed matters as much as the class of representations that an inference engine can search.

A finite-data analogue requires an evidential compatibility set, such as the family from Definition 5.1, rather than the idealized exact fiber of Equation 59. With uncertainty tolerance ε , one may report the set of supported property values specified in Equation 60.

$$\Psi_{\mathcal{Q},c}(D; \varepsilon) = \{\psi(M) : M \in \mathfrak{M}_{a,t}^{q,c}(\varepsilon)\}. \quad (60)$$

Equation 60 requires that ψ be defined on the retained candidates. If several values remain, the output can represent a qualified range or explicit alternatives. Such an answer is informative even when no single explanatory model has been selected. For qualitative normative inquiries, the same set notation may organize attributed positions, but the values are not interchangeable with objective scientific parameters.

10.3 Interventions, Counterfactual Responses, and Observation Mechanisms

Historical records commonly arise under a narrow range of conditions. They therefore constrain how a system behaved in that range more directly than how it would respond to a new intervention. Causal analysis distinguishes observed associations from questions about intervention, requiring additional assumptions and often new forms of evidence [32, 11]. The following example uses deterministic differential

equations to make the distinction explicit without assigning causal meaning to observational equivalence alone.

Suppose the state x begins at zero, the measured output is $y = x$, and an external input u is controlled. Consider two candidate models M_+ and M_- distinguished by $\sigma \in \{+1, -1\}$. Their dynamics are given in Equation 61.

$$\dot{x}(t) = \sigma u(t), \quad x(0) = 0, \quad y(t) = x(t), \quad \sigma \in \{+1, -1\}. \quad (61)$$

Under the historical protocol $u(t) = 0$, both models generate the same complete output $y(t) = 0$, even with continuous and noise-free observation. Under a new nonzero integrable input, their output paths are those in Equation 62.

$$y_\sigma^u(t) = \sigma \int_0^t u(s) ds. \quad (62)$$

For an input with nonzero integral at a chosen observation time, Equation 62 separates the candidates. The example is elementary, but its consequence matters for translation: two systems that agree on a historical record can support incompatible predictions for an action-conditioned continuation. An interface calibrated exclusively on the original record can therefore misrepresent the receiving system's response to a previously unobserved practice or intervention.

Intervention is itself situated. A proposed experiment may be technically impossible, unsafe, unlawful, coercive, or epistemically inappropriate. In social and institutional inquiries, asking a participant to perform an unfamiliar act may change their understanding of the situation. The effect of the inquiry is then part of the investigated relational development. The response also depends on whether instruments, classifications, and observation protocols remain stable. Changes in recorded data may be due to a new mechanism, a new mode of observing the mechanism, or both, as analyzed in Section 6. Historical documentation and procedural accountability are therefore part of identifying what an intervention actually changed.

10.4 Structural Invariants and the Incompleteness of Topological Evidence

The topological structure of a relational model can reveal constraints that are poorly represented by a simple sequence of measured outputs. Incidence and boundary information, connected components, cycles, attachments, and changing homology can distinguish important classes of construction. Their usefulness depends on the fidelity of the chosen relational representation and the historical resolution of the record. The same topological invariant can occur in many distinct relational structures and trajectories.

For instance, a triangular cycle and a quadrilateral cycle, treated as one-dimensional cell complexes without filled faces, both have one connected component and one independent one-cycle. Their first Betti number is one, although their cell counts and incidence structures differ. Attaching a two-cell along either cycle eliminates its first homology class over an appropriate coefficient field, while the events and operations by which the attachment arose may remain different. A coarse topological signature such as Equation 24 is therefore unable to recover a unique cellular grammar. Even a chain map satisfying Equation 39 records compatibility of boundaries without necessarily preserving historical ordering, causal interpretation, or the semantics of the represented primitives.

This example exposes distinct senses of structural adequacy. Homology can be sufficient for a question about the existence of a specified cycle. It can be insufficient for a question about the sequence of attachments, which agents participated in a practice, or which actions remain permitted. More complete incidence records, temporally indexed attachments, decorations, and corroborating source histories can improve reconstruction. Yet these additions still leave the observation mechanism and interpretive conventions to be specified. In particular, higher-dimensional formalisms should be justified by their explanatory or inferential contribution rather than their expressive generality alone.

10.5 Translation Adequacy under Generative Continuation

Section 7 introduces contextual translation witnesses, differentiates observed-output matching from preservation of generative transitions, and shows that trace agreement can conceal branching differences. Continuation introduces a further difficulty: a translation judged adequate over one historical interval may cease to preserve the relevant behavior after the participating systems change their grammars. The source and receiving models, the context of comparison, and the evidential warrant for their correspondence may evolve together.

Consider two finite generators whose observed action histories up to a horizon T are identical. Generator A retains a rule admitting action b after a shared prefix a , and generator B retains the same rule during the observed period. After T , an endogenous rule update in B removes b and admits only c , whereas A retains b . A translation witness that matches the earlier a - and b -transitions is valid for its sampled horizon under a suitably bounded interpretation. An unqualified extension of that witness to the later rule configuration would make an unsupported claim. This remains true even if the former translation was perfect for the behaviors it originally compared.

A usable translation record should accordingly specify at least four indices: a pair of model versions, the temporal interval of warranted comparison, a context of admissible observations and interventions, and an evidential provenance. For a translation relation $\mathcal{R}_{AB}^{c,t}$ and a chosen continuation query q , a prospective preservation test is meaningful only for paths admitted within the selected domains. An appropriate condition can be written as Equation 63.

$$\forall (h_A, h_B) \in \mathcal{R}_{AB}^{c,t}, \forall h'_A \in \text{Next}_A^q(h_A) : \exists h'_B \in \text{Next}_B^q(h_B) \text{ with } (h'_A, h'_B) \in \mathcal{R}_{AB}^{c,t}. \quad (63)$$

Equation 63 is a restricted forward-simulation condition over histories for which the relation is defined, with the chosen query q specifying comparable transitions. It does not guarantee preservation of all possible actions, probabilities, normative reasons, or future changes to the rules. If the receiving model changes, a new witness or new validation is required. Reverse preservation may be needed when mutual ability to perform actions is the research question. The distinction between trace and branching semantics, and their respective preservation standards, is well established in process theory [17, 27].

For practical and normative grammars, failure of a translation may have several origins: a genuinely absent target practice, a difference in social recognition, a temporary lack of shared evidence, a contested interpretation, or an asymmetric institutional power relation. Mathematical non-existence of a specified formal map provides evidence only within the chosen type signatures and interfaces. It cannot establish that the participating communities lack any possible route to mutual intelligibility. New intermediary concepts and negotiated practices can change the interface itself, which is one of the generative processes discussed in Section 9.

10.6 Model-Relative Modal Claims and the Limits of Future Inference

A continuation model assigns possibilities relative to assumptions about structure, rules, action, and admissibility. Historical inference may support several candidate models, each with its own family of continuations. The broad union of Equation 46 reports what at least one retained model admits; it need not indicate what every candidate jointly supports. For a fixed comparison universe $\mathcal{U}$ and a nonempty family $\mathfrak{M}$, two useful bounds are given in Equation 64.

$$\mathfrak{C}^{\text{shared}}(\mathfrak{M}) = \bigcap_{M \in \mathfrak{M}} \text{Cont}(M; c), \quad \mathfrak{C}^{\text{supported}}(\mathfrak{M}) = \bigcup_{M \in \mathfrak{M}} \text{Cont}(M; c). \quad (64)$$

The intersections and unions in Equation 64 presuppose that the continuation families have been mapped into the same meaningful comparison universe with shared prefix, temporal scope, and typing. When heterogeneous models lack that common interface, the responsible representation is an indexed family with explicit partial correspondences. Even under a common interface, the shared family may be empty because the admitted models disagree, rather than because no physical continuation exists.

Table 8: Forms of evidential and practical warrant for outputs of knowledge pressing. Each assessment remains indexed to the declared question, system, and participating evaluators.

Inquiry output	Possible basis for validation	Limitation requiring disclosure
Trajectory reconstruction	Primary sources, timestamped records, provenance, cross-source consistency	Missing or selective records; observer dependence
Generative mechanism	Fit, out-of-sample tests, interventions, structural restrictions	Non-identifiability and model-class dependence
Translation witness	Specified preservation tests, target-system consultation, version history	Partial coverage, loss, changed grammars
Prospective continuation	Model assumptions, constrained reachability, scenario comparison	Epistemic versus physical or practical possibility
Normative assessment	Articulated reasons, affected-party participation, review and appeal	Plural standards and unequal interpretive authority

Proposition 10.2 (Bounds under restriction of candidate models). *Assume nonempty candidate families $\mathfrak{M}_2 \subseteq \mathfrak{M}_1$ in a fixed continuation universe, with an unchanged context, historical prefix, and continuation semantics for each retained model. Then the shared-continuation family for $\mathfrak{M}_1$ is contained in that for $\mathfrak{M}_2$, while the supported-continuation family for $\mathfrak{M}_2$ is contained in that for $\mathfrak{M}_1$.*

Proof. A continuation belonging to every $\text{Cont}(M; c)$ for $M \in \mathfrak{M}_1$ belongs to every continuation family indexed by the subset $\mathfrak{M}_2$. Similarly, a continuation belonging to a family indexed by some $M \in \mathfrak{M}_2$ also belongs to one indexed by $\mathfrak{M}_1$. Both inclusions follow directly from the definitions in Equation 64. $\square$

Proposition 10.2 complements the restriction result of Proposition 8.1. Eliminating incompatible models narrows the exploratory union while potentially enlarging the set of continuations shared by every surviving model. Neither tendency is a claim about the number of physically open futures. Furthermore, neither tendency is assured across time if the system itself evolves, the observer changes representational language, or the comparison domain is altered. A past-to-future dichotomy would conceal these dependencies; the present framework instead treats each inferential update as situated within the evolving process it attempts to describe.

The difference between model-relative multiplicity and free will remains decisive. Several represented continuations may arise from incomplete knowledge, stochastic dynamics, distinct possible actions, or different rules of historical evolution. Those sources of multiplicity require separate interpretation. The agency discussion in Section 9 therefore distinguishes practical opportunity, autonomy, sourcehood, and normative responsibility from the mere presence of formal branching.

10.7 Situated Interpretation, Power, and Normative Assessment

Scientific observations, interpretive claims, and normative judgments require different forms of warrant. A model may accurately predict when an institution invokes a rule while leaving open whether the rule is justified. A historically faithful reconstruction can describe why two communities understand an event differently without authorizing one interpretation as the uniquely legitimate account. Inter-system translation can make a practice more intelligible and can also amplify the interpretive power of the institution controlling the translation interface. These possibilities should inform the design of evaluation procedures and public epistemic infrastructures.

Table 8 identifies kinds of warrants associated with several outputs of knowledge pressing. The entries describe possible evidence and review procedures, not a universal ranking of forms of knowledge.

The warrant categories in Table 8 should not be collapsed into a single scalar notion of accuracy. Computational validation can establish that an implemented rewrite system satisfies specified structural constraints. It cannot thereby establish that a vulnerable community has consented to the institutional use of its history. Interpretation of community practices may benefit from explicit feedback, yet agreement

by a powerful spokesperson does not necessarily represent dissenting members. Likewise, an automated model can accurately identify a historical pattern while facilitating surveillance or inappropriate prediction of individuals' behavior. Ethical assessment requires attention to which records were collected, with what authority, whose participation is possible, and how challenge or withdrawal can operate.

The philosophical literature on understanding and scientific incommensurability already identifies limits to neutral translation between categories and conceptual schemes [24, 30]. Knowledge pressing does not dissolve those limits by positing a richly recorded relational spacetime. It proposes additional means of investigating historically constituted practices, with interpretive claims remaining attributable to particular perspectives. A translation that preserves a selected formal invariant can be mathematically sound and still normatively contested; a negotiated interpretive bridge can be socially valuable while resisting a unique formalization. The difference should be preserved in the architecture and reporting of any resulting system.

10.8 Methodological Conditions for Subsequent Investigation

The preceding analysis yields methodological conditions for the computational investigations proposed for the next section, which remain to be undertaken. These conditions are proposed as evaluation requirements rather than as established features of a universal inference engine.

First, each benchmark should declare the generative model family, observation mechanism, temporal scope, and exact question being tested. A grammar-recovery experiment may score the recovery of rules, the prediction of held-out structures, and the detection of rule changes separately. A dynamical-law experiment may distinguish parameter fit, structural identifiability, and intervention response. An inter-system translation experiment should specify whether it assesses observed traces, branching behavior, probabilistic responses, or situated meaning. The performance measures appropriate to one task cannot automatically be carried into another.

Second, controlled experiments should include deliberately indistinguishable models and representations with insufficient observation. Synthetic studies have the advantage that their known generators permit targeted tests of recovery and failure. That advantage is also their principal external-validity limitation: the generator was selected by the experimenter, and its semantics may be far simpler than those of a scientific or institutional system. A method that succeeds on an artificial grammar has not thereby learned another society's practical grammar. Domain evidence and participation would still be necessary for claims about real social systems.

Third, prospective evaluation should vary the continuation horizon and the possibility of grammar revision. A translation learned under one model version should be tested again following a rule change. Claims of future accessibility should incorporate modeled resources and action constraints and disclose which practical barriers were omitted. Where action itself changes the investigated system, observations gathered after intervention should be attributed to the resulting coupled process.

Fourth, reported conclusions should be calibrated to their evidential status. An exactly proved inclusion is a property of the stated sets and mappings. A simulated success is an empirical result within the synthetic environment, conditional on implementation and experimental design. A historical interpretation is a situated claim supported by sources and methods of interpretation. A normative recommendation requires reasons concerning authority, participation, and consequences. Responsible epistemic infrastructures need to preserve these distinctions through provenance, versioning, and avenues for contestation.

Taken together, these limitations make the research programme more precise. Inference need not identify a unique mechanism to yield a determinate answer to a specified question; translation can be useful while preserving selected consequences only; and continuation can be investigated as an evolving family of model- and action-conditioned structures without asserting metaphysical openness. The next section develops reproducible examples capable of testing these restricted claims and exhibiting their failure modes. Knowledge pressing is thereby treated as an historically situated practice of inquiry and participation whose mathematical, empirical, interpretive, and normative warrants remain distinct.

11 Illustrative Models and Computational Investigations

The preceding sections develop inferential, translational, and continuation interfaces without presupposing a unique mathematical language for generative systems. The present section investigates a small set of executable cases in which the generating mechanism, observation process, and translation target are explicitly specified. Its objective is to exhibit the information that different observation protocols preserve or discard, quantify selected effects of measurement corruption, and test finite correspondences between representations. Five controlled constructions are considered: an evolving graph grammar, inverse heat dynamics and variational ambiguity, translation between transition representations, a trace-equivalent branching counterexample, and action-conditioned reachability. The experiments use synthetic data, stipulated model classes, and transparent baselines. They are methodological illustrations, rather than empirical tests of social understanding or a general-purpose system for knowledge pressing.

11.1 Experimental Design and Reproducibility

Every numerical experiment has a known data-generating process and a stated observation interface. This distinction is necessary because a successful result relative to the specified generator family may conceal ambiguities in a wider family. The software, fixed random seed, numerical settings, and machine-readable outputs are supplied in `experiments/run_all.py` and `experiments/results.json`. The script uses NumPy, SciPy, SymPy for one symbolic identity, and Matplotlib. Running it from the manuscript directory regenerates the data summaries and the two vector figures used in this section. The seed is 20261021. The code makes no network requests, and it contains assertions for key structural and algebraic claims.

The numerical experiments distinguish direct access to a generated record, corrupted observational features, and exact access to a model’s transition operator. These are different evidential privileges. The evaluated metrics are a change-point error in event indices, parameter error and future field-prediction error, a residual for a cross-representation correspondence, and the cardinality of action-feasible reachable sets. The results are conditional on the chosen observation domains and do not supply measures of semantic or normative translation. Research on sparse dynamical discovery, grammatical inference, and structural identifiability provides established precedents for these controlled questions [9, 36, 4, 34].

Table 9 indicates how the cases relate to the formal distinctions of Sections 7–10. None is intended to estimate the performance of existing museum practice, journalism, professional mediation, or real-world institutional inference. The finite models facilitate inspection of successes and failures without assigning them unwarranted social interpretations.

11.2 Relational Grammar Recovery and a Historical Rule Change

A connected graph is initialized with two vertices and a single edge. For the first 32 updates, the generator appends a new vertex joined to an existing vertex by one edge. For the remaining 28 updates, it connects two already connected nonadjacent vertices by a new edge. A deterministic lexicographic rule selects eligible new edges. This produces a simple historical change between a tree-attachment grammar and a cycle-creating grammar. At each event index k , the graph has V_k vertices and E_k edges. For a connected finite graph its first Betti number equals $E_k - V_k + 1$. The signatures of the two update families appear in Equation 65.

$$(\Delta V_k, \Delta E_k, \Delta \beta_{1,k}) = \begin{cases} (1, 1, 0), & 1 \leq k \leq 32, \\ (0, 1, 1), & 33 \leq k \leq 60. \end{cases} \quad (65)$$

The differences in Equation 65 give a deliberately transparent rule-classification signal. From the exact chronological record, the change point is recoverable without a sophisticated inference engine. The purpose of the numerical test is to expose the additional role of record quality. Independent corruption flips the inferred binary event label with probability q . A single-regime Bernoulli model is compared

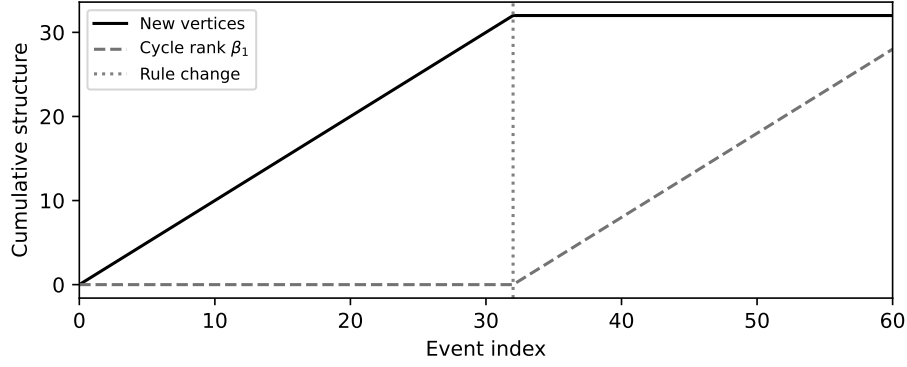


Figure 3: One exactly observed synthetic history with a grammar change after event 32. Tree attachments increase the vertex count while preserving the cycle rank; subsequent chord insertions increase the first Betti number. The plot reports known structural properties of the generated graph, rather than inferred latent mechanisms.

Table 9: Controlled computational cases and the information tested. Each result concerns the stated synthetic model and observation regime.

Case	Observed or supplied data	Tested question
Graph grammar	Event-indexed changes in vertices and independent cycles	Recovery of a transition between two production rules under corrupted event labels
Heat dynamics and action	Noisy periodic fields; symbolic variational expressions	Sensitivity of inverse coefficient estimates and ambiguity of action representation
Matrix translation	Two exactly known, relabeled transition operators	Existence and specificity of a label correspondence preserving iterated dynamics
Branching semantics	Finite labeled transition systems and their trace sets	Difference between output-trace equality and branching equivalence
Action-conditioned continuation	A finite directed network, capacities, and budget	Effects of learned access rules and a later rule revision on reachable states

with a two-regime model by searching all candidate change points between events 6 and 54. Each segment probability receives a $1/2$ pseudo-count in its fitted likelihood; the two-regime model is selected when twice the maximized log-likelihood improvement exceeds $2 \log 60$. This is an explicit complexity-penalized decision rule, related to standard change-detection approaches [7, 16].

Figure 3 shows the change in structural signatures. The program performs 200 independent corruption replicates at each of three values of q . With $q = 0$, the mean absolute change-point error is 0 events, and the two-regime specification is selected in all replicates. At $q = 0.15$, the mean absolute error is 1.035 events, 90.5% of estimates lie within three events of the true change, and two regimes are selected in all replicates. At $q = 0.35$, the corresponding values are 6.620 events, 51.0%, and 51.5%. These are simulation frequencies under a stipulated corruption mechanism; they are neither confidence levels for the underlying system nor evidence that a new discovery method is superior to existing alternatives.

The demonstration illustrates the difference between recovering a change under known event classes and inferring the classes themselves. A real grammar-discovery task may lack event segmentation, topological completeness, and a known two-family grammar. Observers might record changing connectivity while failing to recognize newly admissible rules, or might change the measurement procedure while the source rules remain stable. Consequently, the strong performance under complete synthetic observation should be read alongside the observational ambiguity developed in Section 10.

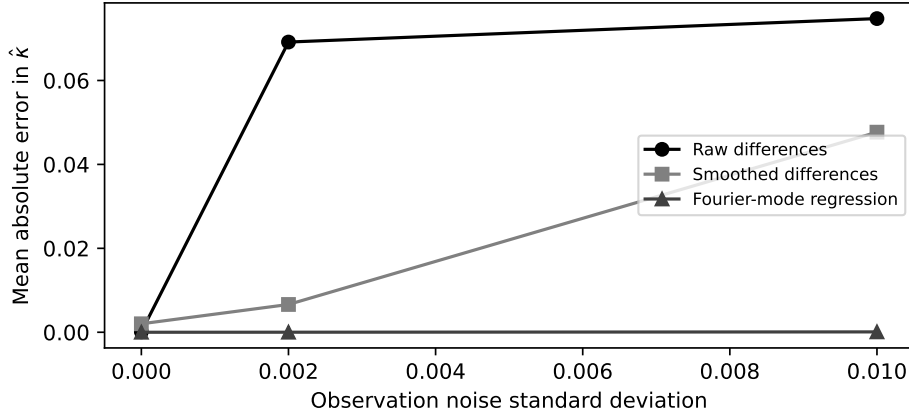


Figure 4: Parameter-estimation sensitivity in the synthetic heat-equation example. Each point is the mean absolute diffusivity error from 40 independently corrupted fields. The strong Fourier result depends on a correctly specified periodic and low-dimensional spectral model.

11.3 Differential Inference, Observation Sensitivity, and Variational Ambiguity

The second case concerns inverse estimation of a diffusion coefficient from observations of a periodic field. The stipulated generator solves the one-dimensional heat equation on a unit circle with diffusivity $\kappa = 0.075$. The initial condition contains two specified Fourier modes. The exact field is expressed in Equation 66.

$$\begin{aligned} \partial_t u &= \kappa \partial_{xx} u, & \kappa &= 0.075, \\ u(x, t) &= e^{-4\pi^2 \kappa t} \sin(2\pi x) + 0.4e^{-16\pi^2 \kappa t} \cos(4\pi x). \end{aligned} \quad (66)$$

Equation 66 permits controlled observations on 96 equally spaced grid points and 151 time points over $0 \leq t \leq 1.2$. Independent Gaussian noise of standard deviation σ is added to the field. Three deliberately different inverse estimators operate within the known heat-equation model family: ordinary least squares applied to central-difference approximations of u_t and u_{xx} ; the same estimator after fixed Gaussian smoothing; and log-amplitude regression on the first spatial Fourier mode for $t \leq 0.9$. The last method is afforded a strong prior advantage: the analyst knows that periodic Fourier modes form the relevant representation, and the low-frequency mode is known to be excited.

The local-difference coefficient estimator is stated in Equation 67, with sums over interior observed times and all spatial grid points.

$$\hat{\kappa}_{\text{local}} = \frac{\sum_{j,i} \hat{u}_{xx}(x_i, t_j) \hat{u}_t(x_i, t_j)}{\sum_{j,i} \hat{u}_{xx}(x_i, t_j)^2}. \quad (67)$$

Noise in the second spatial derivative of Equation 67 is amplified by the inverse square of grid spacing. In 40 synthetic replicates with $\sigma = 0.002$, the mean absolute coefficient error is approximately 0.06916 for raw local differences, 0.00662 after smoothing, and 0.0000112 for the specified Fourier estimator. At $\sigma = 0.01$, these mean absolute errors increase to approximately 0.07475, 0.04768, and 0.0000867, respectively. The tests use the same analytic two-mode generator in all cases, and the result does not establish that the Fourier method will succeed when the boundary conditions, active modes, and model family are unknown.

Figure 4 presents the effect of noise. A limited future-continuation test starts from the clean analytic field at $t = 0.6$ and advances each fitted spectral model to $t = 1.0$. At $\sigma = 0.01$, mean root-mean-square field errors on this held-out interval are approximately 0.05472, 0.02949, and 0.0000412 for raw, smoothed, and Fourier-based estimates. Because the clean initial field and correct modal structure are supplied, this is a conditional parameter-to-continuation test. It is not a full forecast from a noisy, independently inferred initial state.

Action-based inference has a related but distinct representational ambiguity. Consider $L_0 = (\dot{x}^2 - \omega^2 x^2)/2$ and $L_1 = L_0 + d(\lambda x^2/2)/dt$, with constant λ . Their Euler–Lagrange expressions are identical. The symbolic verification in the accompanying script simplifies their difference to zero. This tests a gauge-like freedom of the variational description, and it shows why recovering equations of motion does not uniquely recover an action representation. The computation establishes the algebraic identity for the stated class; it is not an empirical inference of a Lagrangian from trajectories.

11.4 Translation between Distinct Transition Representations

A controlled translation case represents the same finite stochastic dynamics in two formally distinct ways: a directed weighted transition relation and a row-stochastic matrix using different labels for its five states. The first representation associates source states with weighted successors; the second applies a hidden permutation to the row and column indices. A source-to-target state correspondence is represented by a permutation matrix C . The required one-step intertwining condition is Equation 68.

$$P_A C = C P_B, \quad r(C) = \|P_A C - C P_B\|_F. \quad (68)$$

The matrices are supplied exactly, and the search enumerates the $5! = 120$ possible bijective label correspondences. The true correspondence has residual zero in Equation 68; the next-lowest residual is approximately 0.90554, and an unaligned identity mapping has residual approximately 1.12250. Since the best correspondence commutes with the transition operators, its corresponding eight-step residual is below 1.7×10^{-16} in floating-point computation. This controlled positive example demonstrates a valid typed translation witness and a check of iterated generative behavior. It does not show how to discover analogous mappings when the state spaces are unknown, the correspondence is nonbijective, or the semantics of the states depend on disputed social or normative categories.

11.5 Trace Agreement and Branching-Structure Loss

The fourth case uses two finite labeled transition systems whose action traces through termination are identical. The first contains one a -successor with both b and c available; the second has two different a -successors, one admitting b and the other c . For both systems the set of nonempty action prefixes through two steps is $\{a, ab, ac\}$. The accompanying code enumerates these sets and uses iterative relation elimination to test strong bisimilarity. The initial states fail the bisimulation test, in accordance with the distinction described in Section 7 and established process semantics [27, 17].

This is a structural counterexample, rather than a noisy prediction benchmark. Trace equality preserves the possibilities expressible as action sequences under the selected observation protocol. It omits information concerning which possibilities are jointly available after a particular transition. A knowledge-pushing system that stores only completed event strings may therefore lose information relevant to practical choices, negotiation, or prospective intervention. Additional state and branching observations can repair this deficiency in the constructed case, although more complex environments may have further observational ambiguities.

11.6 Action-Conditioned Continuation and Rule Revision

The final example uses a directed network with seven states numbered 0 through 6 and a cumulative action budget of 4. Each edge has an action cost and may require a specified practical skill. The initial network is fixed, the observer begins in state 0, and the set of reachable states is found using a shortest-cost search. The query is restricted to model-admissible, resource-feasible continuation. Its results are reported in Table 10. Without the translation-related skill, states $\{0, 1, 2, 3\}$ are reachable. Adding that skill, while holding the material budget and graph fixed, makes states $\{0, 1, 2, 3, 4, 5\}$ accessible. A subsequent admissible rule revision adds a low-cost edge and makes state 6 accessible as well.

The successive reachability changes demonstrate two kinds of contribution to continuation: an epistemically mediated capacity to operate under an already available rule, followed by a transformation of

Table 10: Selected reproducible numerical results. Errors and success fractions are conditional on synthetic generators, supplied model families, and the specified observation processes.

Metric and experimental condition	Computed result
Graph change-point MAE, $q = 0, 0.15, 0.35$	0.000, 1.035, 6.620 events
Change point within ± 3 events, same q values	100.0%, 90.5%, 51.0%
Heat diffusivity MAE, $\sigma = 0.01$ (raw/smoothed/Fourier)	0.07475 / 0.04768 / 0.0000867
Heat continuation RMSE, $\sigma = 0.01$ (same order)	0.05472 / 0.02949 / 0.0000412
Best / second matrix correspondence residual	0.00000 / 0.90554
Two transition systems: traces / bisimulation	Identical / fails
Budget-4 reachable state count: base / skill / revision	4 / 6 / 7

the generative rule set. The example does not evaluate whether the new possibilities are valuable or just. Its subject is a mathematical state and a declared action interface, not a model of metaphysical free will. In particular, the count of reachable states does not measure autonomy, responsibility, or subjective experience. The continuation model also presumes that the relevant resources and permissions are represented correctly, an assumption that would require independent inquiry in a real institution.

Table 10 collects the results most relevant to the paper’s distinctions. The results concern different questions and permit no common performance score. Informative observations can distinguish generators conflated by coarser records. Valid formal translation can preserve iterated behavior under its assumptions. Changed knowledge or rules can alter accessible model-relative continuations. Inferred structure, interpretive legitimacy, and free will require different forms of evidence.

11.7 Limitations, Failure Cases, and Reproduction Conditions

Several restrictions delimit the experiments. The graph generator uses two easily distinguished update types, with no hidden semantic categories, and its synthetic corruption process is independent across events. The change-point procedure knows that a single possible structural transition separates two stationary Bernoulli regimes. The heat example begins with an exact solution of a familiar equation and supplies the Fourier method with a correct mode family and periodic geometry. The matrix experiment observes the entire transition operator, so the state-permutation search is considerably easier than translation between partially observed systems. The branching test is a finite exact semantics check; the accessibility example stipulates the complete action graph, costs, skills, and budget. None measures the effect of an AI agent, inferential performance in realistic historical archives, or the validity of translating a community’s internal practical grammar.

The computational package contains the executable source, a JSON record of results, fixed seed, and vector figures. Regeneration requires Python and the stated open-source numerical packages, but no external data. Small floating-point discrepancies may occur across software and hardware environments. Corruption replicate summaries depend on the fixed random-number stream. The accompanying example of Lagrangian ambiguity requires SymPy; if unavailable, the script records that the symbolic check was not performed. The verified numerical output supplied with this manuscript records a successful check.

These controlled cases motivate further investigation using learned and audited observation interfaces, wider grammar families, varied sampling regimes, and action-conditioned generalization beyond the fitted horizon. A stronger empirical framework would distinguish changes in the observer’s measurement procedure from changes in the investigated system, and would examine translation with appropriately situated domain participants. Such work requires independent evaluation. The present section supplies reproducible cases of restricted inference and translation, alongside failures resulting from reduced historical or generative information.

12 Epistemic Infrastructure for Knowledge Pressing

The formal and computational investigations above concern relations among incomplete historical evidence, candidate generative representations, contextual translations, and model-indexed continuations. The present section examines the institutional and computational conditions under which these objects could be recorded, interrogated, and revised across multiple organizations. Its objective is to specify a plausible *infrastructure contract*: the minimum information that a system would need to preserve in order for the distinctions developed in Sections 3–10 to remain visible during practical use. The proposal combines versioned evidence, model and inference records, translation witnesses, continuation artifacts, and mechanisms for contestation. It is a research architecture rather than an implemented worldwide system. Although expanded computational capacity and machine learning may lower some costs of reconstruction, automated generation of plausible descriptions also creates new opportunities for evidential error, interpretive homogenization, and concentration of authority.

A hypothetical global infrastructure should be distinguished from a global repository administered by a single body. The primary design considered here is *federated*: participating institutions retain stewardship of their records and substantive procedures while exchanging narrowly specified metadata and approved evidential objects. Federation is an architectural possibility rather than a guarantee of equitable control. Its adequacy depends on legal arrangements, resources, technical implementations, and effective opportunities for affected participants to contest representations. In particular, an interoperable account of an event should not be taken to establish an interoperable interpretation of its meaning.

12.1 Architecture Scope and Epistemic Contracts

A system for knowledge pressing would mediate between at least four operations: recording situated observations, inferring generative representations, translating selected descriptions or processes, and investigating possible continuations. Each operation should expose its dependencies and conditions of applicability. The architecture therefore treats an *epistemic artifact* as a traceable research or practical object with a declared provenance and status, rather than as a claim whose inclusion establishes its truth. Typical artifacts include a dated sensor measurement, an interview transcript with access restrictions, an inferred rewrite rule, a calibrated dynamical model, a translation proposal between organizational concepts, and a scenario generated under a specified intervention.

Figure 5 depicts a modular organization. The record layer supports evidence retrieval without choosing a single theory of causation; the inference layer admits different model families; translation relates selected interfaces; continuation explores model-relative prospective structures. Participation and governance apply across these activities. The arrows indicate permitted information flows rather than a necessary chronological order: translations can motivate further observations, simulations can expose inadequacies in previous models, and participant objections can change the admissible evidence and inquiry context.

The underlying design principles have substantial antecedents. The FAIR principles emphasize findability, accessibility, interoperability, and reuse of data and associated workflows [43]. The W3C PROV-O recommendation offers a vocabulary for provenance relationships involving entities, activities, and agents [22]. DCAT 3 addresses the description and federation of data catalogs, including dataset series and version information [2]. These facilities solve important parts of interoperability. The additional concern developed here is the preservation of *epistemic distinctions* among observation, reconstruction, translation, and prospective inference. It should be implemented, where feasible, through extensions and profiles of existing standards rather than a competing universal schema.

12.2 Historical Records, Relational Structure, and Provenance

The historical layer records assertions about observed events, their relationships, and their histories of documentation. An event may be represented by a vertex of a reconstructed relational complex, a coupling by an edge, and a higher-order primitive by an appropriately typed face, consistent with Section 3.

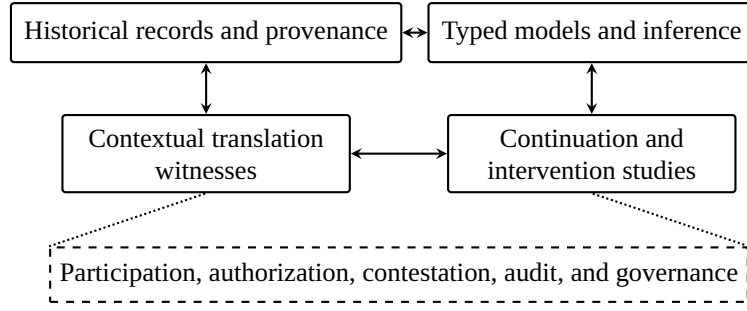


Figure 5: Proposed modular organization of epistemic infrastructure. The components exchange evidence-indexed artifacts with declared domains and versions. Governance and situated participation apply across the system. The arrangement is a conceptual architecture, without an assumption that all components are controlled by one organization.

Such representation presupposes choices about observational scale and the semantics of the cell types. An organization may instead hold measurements as time series or practical histories as annotated documents. The infrastructure should permit connections among these artifacts without requiring immediate conversion into a common two-complex.

A basic record should separate at least *event time* from *recording time*. The former describes when a purported occurrence took place; the latter records when the relevant assertion or observation entered a particular repository. A third time may be required for publication, correction, or authorized use. Maintaining these distinctions helps an inquiry reconstruct what information was available to an observer at a historical decision point. A later correction should be linked to, and normally distinguishable from, the earlier assertion. This supports faithful reconstruction of an epistemic history even when the currently preferred account changes.

Definition 12.1 (Versioned epistemic record). A *versioned epistemic record* at repository r is the structured object in Equation 69, interpreted relative to a declared schema S_r .

$$E_r = (\iota, v, q, \tau_e, \tau_r, s, p, \sigma, \lambda), \quad (69)$$

Here ι identifies a stable referent or assertion lineage, v identifies a particular version, q is the typed content or claim, τ_e and τ_r are event and recording-time attributes, s indicates the source or acquisition activity, p links derivations and evidential dependencies, σ states epistemic qualifications and contestations, and λ specifies access, use, and retention conditions. Components may be partial, with missingness explicitly represented. The object is a record of an assertion or observation; its construction does not certify that q holds in the investigated system.

Definition 12.1 is deliberately less expressive than the full relational account in Section 3. Its purpose is operational separation of content, witness, and stewardship. A provenance link may connect the assertion that an edge exists with a measurement activity, a textual source, a coding decision, or a later model inference. Corroborating sources need not be epistemically independent. The system should therefore record source relationships where these are known and avoid treating repeated copies of one report as independent confirmations. Likewise, a provenance graph provides a history of recorded operations, but a path through it is not in itself evidence of physical or social causation.

Immutable versions and explicit supersession links are suitable for scientific reconstruction, but permanence should be conditional on the type of record. Consent, legal duties, personal safety, and restrictions on the retention of sensitive testimony may justify deletion or limited retention. In such cases the system may preserve a minimal permitted trace of a correction or withdrawal while removing protected content, subject to local governance and law. The proposed federation should distinguish *metadata discoverability* from authorization to access raw material; publishing the existence of a record can itself be sensitive. The RO-Crate specification provides an existing way to package related research objects and machine-readable metadata [13], although it does not by itself resolve these institutional obligations.

12.3 Typed Model Registry and Inferential Workflows

The model layer associates versioned evidence with heterogeneous generative representations, including rewrite grammars, ODEs, PDEs, action functionals, stochastic models, programs, and descriptions of practical rules. A registry entry should record the model’s type, semantics where formally specified, training or estimation domain, admissible transformations, required inputs, observation interface, dependencies, fit diagnostics, and known failure cases. An action functional, for example, might require a configuration space, admissible paths, boundary conditions, and an interpretation of variational equivalence; a graph grammar instead requires typed production rules and their matching semantics. A shared registry contract therefore need not impose a single internal model language.

The inverse correspondence of Section 5 associates evidence with multiple possible representations. An operational inference run should record which evidence snapshot was supplied, which candidate family and priors were admitted, which algorithm or expert procedure was used, and which comparisons were executed. The output is a versioned *candidate family* together with its evidential standing, not automatically a single endorsed mechanism. Failed fits, inconclusive comparisons, and rejected candidates can be important for subsequent inquiry when retention is warranted and affordable.

A minimal run signature is given in Equation 70.

$$\text{Run}_{a,t}^c : (\mathcal{E}^v, \mathfrak{M}^{\text{ad}}, \kappa, \rho) \mapsto (\mathfrak{M}^{\text{out}}, \mathcal{W}, \mathcal{L}), \quad (70)$$

where $\mathcal{E}^v$ is a versioned evidential snapshot, $\mathfrak{M}^{\text{ad}}$ an admitted model family, κ the inferential procedure and configuration, and ρ the declared resources and permissions. The output $\mathfrak{M}^{\text{out}}$ is the retained model family; $\mathcal{W}$ contains diagnostic or evidential warrants, and $\mathcal{L}$ contains limitations and unresolved alternatives. Equation 70 is an interface specification, not a theorem claiming algorithmic recoverability.

A viable implementation would allow inference engines to be added incrementally. A sparse-regression method may operate on a particular class of differentiable measurements; a grammar learner may require candidate production types; a historical interpreter may return annotated, contestable accounts rather than a statistical posterior. The interface should make differences in evidential modality visible. Cross-domain comparisons are legitimate only when a shared test has been specified and independently examined. The numerical examples of Section 11 show that a method benefiting from supplied structural information may perform well within its stipulated family while offering little evidence about success under unfamiliar grammar classes.

12.4 Translation Services and Evidence-Carrying Correspondences

The translation layer stores and evaluates contextual witnesses of the kind defined in Definition 7.1. An operational translation request specifies the source and receiving systems, the versions of their candidate representations, a particular question, and the observations or interventions available for checking correspondence. A scientific request could investigate whether a finite-volume simulator preserves selected predictions of a continuum PDE over a bounded space-time domain. A legal or administrative request might instead ask how two institutions recognize authority or document an emergency warning, using interviews, procedural records, and authorized professional review as the relevant evidence.

A translation artifact should report *what was related, on which domain, by which interpretive or computational operation, and which consequences were preserved or lost*. Table 11 gives a concise account of the proposed artifact types. A correspondence may be partial and one-directional; mismatches and unresolved terms are legitimate outputs. Automated translators may suggest alignments, but a high score on text similarity or a matching trace need not validate the generative or normative mapping. The branching counterexample in Section 11 illustrates how equal observed traces can coexist with different transition possibilities.

At the interface level, a translation witness could be accompanied by a certificate of the form in Equation 71.

$$\text{Cert}_{AB}^c = (\Theta_{AB}^c, \mathcal{E}_{AB}^v, Q_c, V_c, F_c, A_c), \quad (71)$$

Table 11: Principal artifact classes and the distinctions required for their responsible reuse in a proposed federated infrastructure.

Artifact	Minimum traceable content	Principal limitation to communicate
Historical record	Assertion, source, times, version, access rules	Recorded assertion versus underlying event
Inference run	Evidence snapshot, model class, method, candidate family	Fit versus mechanism identification
Translation witness	Source and target versions, context, relation, tests	Selected preservation versus full intelligibility
Continuation study	Initial history, model/rule histories, interventions, horizon	Model possibility versus realized future
Contest or revision	Target artifact, reason, competent reviewer, disposition	Institutional decision versus universal truth

where Θ_{AB}^c is the contextual witness, $\mathcal{E}_{AB}^v$ the versioned source and interaction evidence, Q_c the restricted preservation queries, V_c the tests and situated checks actually conducted, F_c known failures or translation losses, and A_c the competent authorizations or dissenting assessments. Equation 71 specifies an auditable record of support, not a certificate of universal semantic equivalence. It could support the future comparison of competing interpretations by exposing the questions that each proposed translation was designed to answer.

Translation may also participate in continuing historical formation. Participating systems can introduce intermediary vocabularies, negotiate new procedures, or revise their own models after encountering the other’s perspective. Such co-evolution requires versioning on both sides of the bridge. A previously adequate witness can become obsolete as a grammar changes; the infrastructure should expose dependency changes and permit context-specific revalidation, rather than silently treating an earlier relation as indefinitely valid.

12.5 Continuation Studies and Decision-Relevant Possibilities

A continuation service takes versioned historical evidence, one or more candidate generators, and declared intervention or rule-evolution assumptions to construct *conditional* future relational configurations. Its output may be an ensemble of simulated trajectories, a bounded reachability structure, a family of extended two-complexes, or an interpretable account of available actions. As developed in Section 8, the appropriate object may be an extension that retains the historical region as part of a larger whole-spacetime structure. The service should therefore preserve the relation between the inherited history and its generated extensions, rather than detach predicted futures from the evidential and relational conditions used in their construction.

The service should distinguish a future that is representable in the observer’s language from one admitted by a selected model, accessible under specified actions, or physically realizable under constraints external to that model. Its evidential obligations increase when such outputs enter consequential decisions. In an emergency warning system, for example, a model may describe flood exposure, while local evacuation feasibility depends on available transportation, housing, care obligations, and the trustworthiness of communication. Translation between forecast probabilities, administrative thresholds, and local practical grammars can alter decisions; changes in resources or warning policies can then change the available continuations. These claims describe a possible workflow, not a validated intervention or evidence that one shared scenario is sufficient for all participants.

A candidate continuation artifact can be recorded through the dependency relation in Equation 72.

$$C^v = \text{Cont}(H^{vH}, M^{vM}, U^{vU}, \pi^{v\pi}, c, h), \quad (72)$$

where H^{vH} is the admitted historical snapshot, M^{vM} the chosen model, U^{vU} a specified family of rule changes, $\pi^{v\pi}$ the action or intervention policy, c the interpretive context, and h the prospective horizon.

The notation records computational and conceptual dependencies without asserting that Cont is a universal function. Revisions to any dependency should trigger reconsideration of affected outputs. In a probabilistic implementation, model and scenario uncertainty should be reported separately when the available evidence supports that distinction. In an interpretive implementation, the system may produce several reasoned narratives accompanied by explicit disagreements rather than a probability distribution.

The continuation service should avoid implicit normative optimization. A larger set of reachable configurations can include harms as well as benefits. A statistical preference score assigned by an algorithm is different from a moral judgment or a legitimate public decision. The agency analysis in Section 9 further cautions that formal reachability does not establish free will. Decision support should therefore state whose goals, constraints, or evaluative criteria have been admitted, and permit alternative accounts to be inspected without silently translating one party's priorities into the authoritative objective.

12.6 Participation, Interpretive Authority, and Contestation

Knowledge pressing involves people who are sources of evidence, objects of representation, interpreters, and agents affected by decisions. These positions can overlap but entail different rights and responsibilities. A subject represented in an historical record may wish to correct a factual mistake, dispute an interpretation, withhold sensitive testimony, or object to the use of a model in an institutional decision. A professional interpreter may identify a translation failure that is invisible to output-level tests. A local community may recognize practical constraints omitted by an otherwise sophisticated dynamical model. The infrastructure should distinguish these forms of intervention and preserve their consequences in the relevant artifact histories.

An operational contestation mechanism would provide a route to identify an affected artifact, specify the type of challenge, document qualified responses, and publish or communicate the disposition under applicable access rules. Multiple legitimate interpretations may persist after review. A record may become *contested* while remaining useful evidence of what was reported at a particular time; a translation witness may be restricted to a smaller domain without being deleted; a deployed decision rule may need suspension even if its scientific input remains valid. Rights of reply, procedural fairness, and opportunities to appeal must be grounded in actual institutional authority and legal obligations. An algorithm cannot supply those obligations simply by recording a disagreement.

Participation also has costs. Repeated review requests can impose uncompensated labor on communities and experts, especially when an organization externalizes its uncertainty onto affected people. A credible governance arrangement would allocate time and resources for consent, review, translation, and response, and would identify situations in which the burden of justification belongs to the institution proposing to use the model. Historical knowledge is unevenly distributed, and the people most capable of challenging a representation may have the least available time or institutional power. These constraints bear directly on the reliability and legitimacy of an apparently inclusive epistemic process.

12.7 Federated Governance, Security, and Institutional Limits

The worldwide scope of the motivating thought experiment does not justify unrestricted central collection of human histories. Broad interoperability creates opportunities for inference that cross the purposes for which data were originally gathered. Linkage across innocuous records may reveal sensitive facts, and a detailed relational history may enable surveillance even when individual files have been de-identified. The infrastructure should therefore support purpose limitation, selective disclosure, local stewardship, revocable access where technically and legally possible, and retention policies that distinguish scientific reproducibility from the continued exposure of personal information. The appropriate controls vary among public meteorological measurements, anonymized research simulations, clinical data, and testimony concerning political violence.

Minimum governance functions include authentication and authorization; contextual consent and lawful basis checks; segregation of public metadata from restricted content; provenance-preserving correction and deletion procedures; audit logging; review of model use in consequential decisions; and

Table 12: Illustrative evaluation dimensions for a pilot epistemic infrastructure. These are proposed tests and do not represent achieved system performance.

Dimension	Possible operational test	Important qualification
Historical reconstruction	Recover a dated decision’s available evidence and earlier versions	Historical availability versus later correction
Model inference	Recover known mechanisms or valid equivalence classes under controlled missingness	In-family performance versus open-world inference
Translation	Test preservation of selected interventions, transitions, or situated interpretations	Scope of query, participant disagreement
Continuation	Reproduce scenario outputs and stated model assumptions	Model admissibility versus physical feasibility
Epistemic revision	Audit propagation of corrections through dependent artifacts	Recomputability and permitted retention
Accessibility and cost	Compare staff effort, comprehension, and error rates with an existing workflow	Unequal skills, infrastructure, and review burdens
Governance and recourse	Test access limits, explanation requests, challenges, and remedies	Institutional legitimacy and legal specificity

access to remedial processes. Computed outputs may need sensitivity reviews because even a derived graph or inferred generative model can leak information about individuals and organizations. Security controls should address corrupted evidence, source spoofing, model supply-chain compromise, adversarial instructions in documents, and false provenance claims. Cryptographic integrity can establish that a particular artifact has not changed since a recorded commitment; it cannot establish the accuracy of the assertion contained within the artifact.

For AI-supported components, the voluntary NIST AI Risk Management Framework provides a useful existing approach to governance, mapping, measurement, and management of risks across the AI life cycle [28]. Such guidance helps organize responsibilities but should not be mistaken for a universal legal or ethical authorization. The present paper’s concern is broader than AI and includes scholarly, human, institutional, and computational forms of inference. James Scott’s analysis of administrative simplification is also a relevant caution: representations that facilitate central legibility can erase locally consequential forms of knowledge [41]. A federated design can mitigate certain concentration risks, but may still reproduce unequal bargaining power, resource disparities, and asymmetrical control over schemas or translation standards.

12.8 Incremental Implementation and Evaluation Criteria

An initial implementation should begin with a bounded domain in which generative rules and observational procedures are sufficiently inspectable to establish meaningful benchmarks. One candidate is a scientific workflow using controlled rewrite grammars and dynamical models. Versioned observations, reproducible inference runs, contextual translation witnesses, and continuation simulations could be packaged as research objects and compared with conventional archival and modeling workflows. The experiments of Section 11 provide initial synthetic fixtures, but do not constitute an implementation or evaluation of the proposed infrastructure. The next engineering step would be to implement an end-to-end path with controlled access, artifact versioning, dependency tracking, and independent reproduction of both successful and failed inquiries.

The evaluation should distinguish technical performance from epistemic and institutional adequacy. Table 12 proposes a non-exhaustive evaluation matrix. Metrics should be chosen locally and compared with appropriate baselines, such as an existing laboratory data repository, established archival practice, ordinary professional translation, or a current clinical or emergency-coordination process. An infrastructural innovation earns its cost only where it addresses a demonstrable bottleneck with acceptable new burdens.

Deployment should proceed through a modest sequence: evidence and provenance profiles; one or

two inspectable model families; reproducible inference and translation runs; scenario-dependency tracking; and an evaluated participation and governance protocol. Each stage should preserve backwards compatibility where practical while allowing obsolete representations and interfaces to be revised. Successful dissemination may involve interfaces adapted to different readers and modalities, but such interfaces should expose evidence and permit the user to choose how to investigate the record. Multimodal communication is an accessibility option, not a substitute for epistemic verification.

The proposal thus identifies a possible infrastructural consequence of the formal investigation. The core technical contribution would be the reliable handling of *evidence-bearing, versioned generative and translation artifacts* whose conditions remain inspectable as systems evolve. The potential societal value depends on whether this handling improves actual inquiry and cooperation at reasonable cost, while preserving appropriate rights and opportunities to contest representations. A worldwide network could support shared warning and scientific understanding, yet it would remain a federation of situated epistemic practices with persistent disagreements and varying authority. In this sense, knowledge pressing is an ongoing practice of reconstruction, translation, continuation, and revision; an infrastructure can support those activities without claiming to close their interpretive or normative questions.

13 Discussion and Open Questions

The preceding sections have developed knowledge pressing as an exploratory account of historically situated epistemic activity within evolving relational spacetime. The account connects the inference of candidate generative representations, the contextual translation of heterogeneous models and practices, and the investigation of possible relational continuations. This section assesses the reach of those connections and identifies the conditions under which they could contribute to further theoretical or practical inquiry. The discussion first considers representational plurality and inter-system translation, then examines continuation, agency, and empirical adequacy. It concludes with specific open questions and an assessment of the present manuscript's evidential standing. The analysis treats formal constructions, executed synthetic experiments, historical interpretation, and institutional design as different modes of inquiry whose conclusions require different warrants.

13.1 Knowledge Pressing as Situated Generative Inquiry

The notion of knowledge pressing introduced in Section 3 refers provisionally to epistemic processes by which historically constituted observers encounter relational evidence, develop and revise representations, interpret other systems, and participate in further developments. The term designates a research object and organizing perspective. It makes no claim to name a historically unprecedented epistemic practice. Scientific model discovery, archival reconstruction, comparative interpretation, deliberation, and cross-cultural mediation supply established practices whose relevance extends well beyond the present formulation. One potential value of the framework is to locate their relationships and differences within a common research agenda while preserving the specificity of each practice.

The definition places two dependencies in view. First, epistemic access is historically constituted. Observers inherit instruments, categories, languages, resource constraints, institutional responsibilities, and prior records. Their representations are consequently indexed to both the investigated system and the conditions of observation. Second, epistemic activity may itself enter the developing relational process. A measurement can trigger an intervention; a translation can establish a new procedure; the publication of a forecast can modify the behavior that the forecast sought to anticipate. The corresponding continuation is consequently a history of coupled observation and participation when the particular case supports such coupling. Neither dependency entails that an observer possesses a complete ontology of the system.

The significance of historical spacetime is thus methodological as well as representational. A final configuration of a network or two-complex may omit the formation order, failed transformations, abandoned possibilities, and intermediate relations through which the configuration arose. Yet even a detailed trajectory does not uniquely disclose its generative mechanism, as the counterexamples in Sec-

tions 5 and 10 establish under their respective assumptions. Recording trajectories expands available evidence and enables additional questions. Inferential adequacy continues to depend on the observation operator, model family, query, and permitted interventions.

A worldwide system of interoperable historical records could, in principle, make evidence available across disciplinary and institutional boundaries. Such availability would be valuable even if the additional epistemic services proposed here proved infeasible. Its existence would not yield a single universal account of meaning. Historical recording itself depends on decisions concerning what counts as an event, which couplings merit representation, which participants may speak for a practice, and which material may ethically be retained or disclosed. A shared infrastructure should make these selections more inspectable while allowing consequential differences in perspective to remain explicit.

13.2 Representational Plurality and Scale-Dependent Adequacy

Section 4 characterizes candidate generators through typed interfaces in order to compare their evidential roles. The collection includes rewrite rules, continuous dynamics, action functionals, stochastic models, computational programs, and institutional grammars. This plurality is substantive. Each formalism has its own state objects, admissible transformations, observational semantics, and conditions for interpretation. A PDE may describe concentration transport at a specified scale; a rewrite grammar may specify admissible attachments of cells; a practical grammar may express permissions or obligations recognized by an institution. The existence of an abstract interface does not establish a semantics-preserving translation between these descriptions.

Scale is part of the problem. A continuum equation may arise as a limiting approximation of a discrete model, while discontinuous changes in graph connectivity can become invisible under a chosen coarse-graining. Conversely, a representation defined at a fine scale may be computationally expensive and epistemically unnecessary for a decision concerning an aggregate outcome. In social systems, a behavioral regularity established over many interactions may coexist with different local reasons for action. A predictive abstraction and an interpretive account can therefore have distinct and legitimate domains of adequacy. Scientific discovery methods such as sparse dynamical identification and program induction exemplify the value of constrained model classes for recoverable inference [9, 36, 21]; they do not establish a general solution to cross-domain representation choice.

The selection of representational scales should be tied to explicit queries. For a coastal warning system, an aggregate fluid model may support the prediction of inundation while local historical and institutional records help explain which households can act on a warning. Converting the latter question into a continuous field without an adequate empirical and interpretive justification may conceal the very constraints of interest. Equally, forcing meteorological dynamics into a social narrative without preserving quantitative evidence would impair physical prediction. A useful infrastructure would permit different models to interact through declared observations, tasks, and compatibility tests, while retaining their original qualifications.

This requirement extends to relational topology. Topological invariants can reveal connectedness, cycles, and certain forms of structural change. Their informational value depends on how cells are typed and how observations relate to the investigated processes. Identical homology groups do not imply identical histories, transition relations, causal organization, or meanings; Section 3 supplies a finite illustration of the historical issue. Future work on two-complex and higher-order representations should specify the semantics of higher-dimensional primitives before attributing a particular physical or social mechanism to a topological feature.

13.3 Generative Translation and the Limits of Common Interfaces

The translation correspondence developed in Section 7 addresses an intermediary level between matching descriptions and claiming equivalence of entire systems. A contextual witness can state which observables, transformations, or consequences are preserved, over which domain, and relative to which

evidence and version of a model. Such witnesses make room for unequal or partial coverage. The direction of translation matters: a vocabulary developed to communicate physical forecast uncertainty to an administrative institution may preserve decision-relevant probability thresholds while leaving out the physical variables required for independent scientific reconstruction.

A revealing distinction concerns *trace agreement* and *branching agreement*. The same series of observable outputs may be generated by two systems with different unobserved alternatives. Consequently, translating an observed history can succeed under one test while misrepresenting the consequences of a prospective intervention. The branching example developed formally in Section 7 and revisited computationally in Section 11 exhibits this limitation. Comparable distinctions have long been investigated through behavioral equivalences and process semantics [27, 17]. The present discussion uses these antecedents to motivate context-specific tests of generative preservation, without claiming a new general equivalence relation.

Translation between mathematical representations is further distinguished from translation between historically constituted practical grammars. The latter may involve reasons, obligations, legitimacy, identity, and disagreement about the very terms of description. The philosophical discussions of interpretation and incommensurability surveyed in Section 2 caution against treating a mapping between terms as a neutral recovery of meanings [33, 24, 30]. A mathematical certificate that a designated invariant is preserved has a different warrant from the claim that members of a community recognize a translated practice as appropriately described. Both may be valuable. Neither should be used to silently replace the other.

An especially important possibility is the development of a new intermediary grammar through interaction. Such a grammar may be created through scientific collaboration, legal accommodation, interpretation, or negotiated procedural reform. Its formation changes the future interactions available to participating systems. This possibility supports the view that translation is sometimes generative. It also introduces an epistemic risk: a dominant institution may promote its own categories as a purportedly neutral intermediary language. Historical evidence about the emergence of a shared interface, opportunities for correction, and the treatment of rejected interpretations become relevant to assessing the resulting translation.

13.4 Continuation, Modality, and the Evolution of Possibility

The continuation formalism in Section 8 treats a candidate extension as a whole relational configuration containing its inherited historical part. A temporal distinction between recorded and unrecorded developments remains useful for locating evidence; the mathematical treatment avoids assigning past and future to independent ontological domains. An inferred generative representation constrains possible extensions according to specified rules. The production of new evidence, introduction of an action, or modification of a generative rule can change which extensions are represented and which are practically accessible. The history of pressing itself may thereby become part of the continuing configuration.

Modal claims nevertheless require explicit indices. *Epistemically conceivable* continuations depend on an observer's representational capacities; *model-admissible* continuations depend on specified dynamics and constraints; *practically accessible* continuations depend additionally on material, legal, social, and agential conditions. A claim about *physical realizability* would require domain-specific evidence beyond consistency with a convenient mathematical generator. The distinction is consequential because new knowledge can enlarge an observer's represented repertoire without changing the underlying dynamics, while invention or collective institutional action may alter practical access or the rules governing later possibilities.

The elementary bounds for retained candidate models in Section 10 are useful only under their fixed-universe assumptions. Removing an unsupported model changes the epistemically represented continuation family. By itself, it says nothing about a change in the physical world's possibilities. Similarly, a new institutional permission can create a practically accessible path without establishing ontological indeterminism. These differences prevent an analysis of future creation from conflating learning, alteration

of resources, change of rules, and metaphysical openness.

Continuation can also produce epistemic loss. Irreversible actions may eliminate alternative procedures, destroy evidence, displace participants, or transform the institutions through which earlier claims were interpretable. A model that scores future options only by their number would omit these features. A more informative analysis would consider the kinds of possibilities preserved or lost, whose capacities are affected, the time horizon of assessment, and the legitimacy of the evaluative procedure. Such assessment involves explanatory and normative judgments in addition to formal reachability. Viability-theoretic approaches already connect admissible trajectories to constraints over time [5]; the distinct challenge here is to examine coupled changes in the models, interpreters, and practical constraints used to identify those trajectories.

13.5 Agency, Freedom, and Relational Conditions of Action

Section 9 describes a subject through historically constituted capacities for interpreting reasons, deliberating, and acting within practical and relational constraints. Subjects in this treatment are emergent and context-dependent, rather than primitive event vertices. This formal stance makes a substantive question available: how can the actions of subjects participate in transforming the relationships and generative conditions through which their present capacities arose? Learning, cooperation, institutional reform, technical invention, and the formation of new practices offer distinct mechanisms that could be investigated empirically.

The resulting account has close connections to longstanding philosophical and social-scientific treatments of agency, autonomy, and substantive opportunity [6, 38, 10, 35]. It remains compatible with more than one position in the free-will debate. A model of reason-responsive deliberation may help characterize aspects of practical control, while leaving unsettled whether an actor could have done otherwise under exactly the same complete conditions. Conversely, a stochastic branching model can represent multiple outcomes while leaving the significance of authorship and control undetermined. Discussions of compatibilism and alternative possibilities continue to address these differences [29, 26].

There is a further normative risk in treating historically constituted agency as merely a function of what a model predicts. Successful behavioral prediction may be achieved through observation of constrained options, unequal resources, or systematic coercion. Predictive success does not establish that the resulting behavior expresses meaningful consent or self-determination. The availability of a nominal action in a formal transition system is therefore distinct from the conditions under which a person can understand, initiate, refuse, and sustain that action. The present account situates agency in an evolving relational environment while leaving the standard of legitimate agency to explicit philosophical and practical examination.

Collective agency raises additional complications. An association may develop capabilities that no single participant possesses in isolation; it may also internalize exclusions and unequal decision power. A translation interface that helps two organizations cooperate may expand their collective range of action while restricting opportunities for individuals whose views are absent from the adopted grammar. Accordingly, a formal analysis of access should be supplemented by questions about distribution, institutional responsibility, and voice. These are not residual issues to be resolved after successful system identification. They affect what histories are recorded, what models are accepted, which translations are authorized, and which future possibilities are pursued.

13.6 Interpretation of the Computational and Infrastructure Results

The computational studies in Section 11 examine limited, controlled cases in which the generating mechanism or intended correspondence is available to the experimenter. A graph-growth example tests the recovery of an introduced rule transition under complete and corrupted records. The heat-equation study illustrates how noise and representational assumptions alter coefficient recovery. The variational example shows the non-uniqueness of a Lagrangian description under the relevant transformation. Finite transition systems make translation and branching differences exactly inspectable, and an action-conditioned

reachability example demonstrates the dependence of modeled access on capacities and rules. These studies establish properties of the chosen synthetic examples and implementations. They do not demonstrate a domain-general inference engine, real-world interpretive adequacy, or a deployed knowledge-preserving infrastructure.

The experiments nevertheless help specify evaluation requirements. Inference tasks should report the exact unknowns to be recovered, the observation mechanism, the model-class restrictions, and sensitivity to noise or missingness. Translation tasks should specify the relevant preservation target: outputs, transition structure, probability laws, situated meanings, or intervention responses. Continuation tasks should declare their modal scope and distinguish the observer’s uncertainty from the represented system’s constraints. A comparison between an automated method and professional human interpretation would additionally require explicit task definitions, ethically acquired data, and independent expert review. The promise of artificial intelligence lies partly in extending feasible inference and comparison, but the conditions under which it improves understanding remain an empirical question.

The proposed federation in Section 12 responds to these distinctions by keeping observations, inference runs, translation witnesses, continuation studies, and contestations as traceable but separate artifacts. Established standards for data stewardship and provenance provide important foundations [22, 2, 43]. Nevertheless, a repository can preserve the documented origin of an interpretation while leaving its truth, fairness, and practical consequences unresolved. A translation certificate can report passed tests and identified losses, while remaining silent on aspects that were neither observed nor tested. A continuation study can record assumptions about accessible futures, while still omitting important constraints on what affected participants can actually do.

Three levels of evaluation consequently remain distinct. *Artifact-level evaluation* examines correct storage, execution, provenance, and reproducibility. *Epistemic evaluation* examines whether an inference or translation supports the claims for which it is used. *Institutional evaluation* examines accessibility, participation, safety, cost, sustainability, and the allocation of interpretive authority. Robust performance at one level does not entail satisfactory performance at the others. The next phase of research should test these levels separately and examine their interaction in bounded applications before contemplating wider deployment.

13.7 Open Problems for Generative and Epistemic Research

The framework leaves several questions whose answers could change its definitions, technical direction, and practical justification. The following agenda identifies objects of investigation rather than claiming that each problem requires an entirely new mathematical theory.

Joint identification of mechanisms and observation processes. An apparent change in recorded relational dynamics may reflect a change in the investigated mechanism, the observation protocol, or the epistemic categories applied to the record. Under what observational diversity, intervention conditions, or invariance assumptions can these explanations be separated? Controlled change-point experiments can reveal sensitivity to missingness, while historical cases require attention to changes in recording institutions and instruments. An especially difficult extension concerns cases in which observers alter their own observation categories during the period under investigation.

Comparisons between heterogeneous generative languages. A typed interface can compare specified observations or transformations. Which classes of PDE models, grammars, stochastic processes, programs, or normative systems admit useful partial correspondences without a privileged semantic metalanguage? A research programme could classify correspondence failures by missing observables, incompatible state meanings, unequal temporal resolutions, or dependence on untranslatable practical norms. Existing process semantics and categorical approaches provide possible technical resources, but their applicability to socially situated interpretation requires separate demonstration [18, 17].

Historical inference of topology-changing rules. Relational histories may involve changes in cell incidence, higher-order attachments, and the available operations for transforming them. Which observations distinguish rule changes from re-descriptions of a fixed structure? How do evidential gaps affect reconstruction of births or closures of cycles? A useful benchmark would vary record completeness, latent intermediate states, cellular typing, and allowable rule families while asking whether the inferred generator predicts held-out topological transformations.

Model-relative and physically meaningful future possibilities. A mathematically consistent continuation is a consequence of specified model constraints. What additional empirical warrant is needed for a claim about physical realizability or intervention-responsive possibility? Work on constrained dynamical systems may help define viable trajectories in particular domains [5]. For historically evolving social systems, further difficulties arise because participants may respond to the publication of the very scenarios being evaluated. The representation of future alternatives therefore has to remain sensitive to reflexive feedback and changing interpretive conditions.

Translation-mediated creation of shared grammars. Repeated translation can result in negotiated concepts, new procedures, or changed institutions. What evidence would distinguish the formation of a new shared grammar from temporary behavioral accommodation or the imposition of one system's categories upon another? A promising study would record interaction histories, disputes over equivalence, revisions to agreed procedures, and consequences for future actions. The assessment should recognize disagreement among participants as potentially informative evidence rather than treating consensus as the only measure of understanding.

Practical agency and the conditions of freedom. How should a formal continuation model represent understanding, resources, coercion, reasons, institutional authorization, and opportunities for refusal? Which attributes can be measured without reducing the subject's self-understanding to externally imposed behavioral categories? A further philosophical question concerns the relation between historically conditioned self-transformation and freedom as sourcehood or alternative possibility. The present formalism supplies an interface for these investigations; it offers neither a general metric of freedom nor a resolution of the metaphysical controversy.

Infrastructure design and distributed epistemic authority. Can a federated system support effective search and reproducible inference while respecting local stewardship, withdrawal, confidentiality, and the right to challenge an interpretation? A practical pilot should examine the cost of provenance maintenance, error correction, expert validation, version migration, and governance. It should also establish whether the proposed services improve an existing workflow relative to less elaborate professional and technical arrangements. The existence of an interesting theoretical representation alone does not establish that a new infrastructure is required.

13.8 Contribution Status and Research Boundaries

The manuscript's principal contribution is a connected exploratory formulation of historical inference, heterogeneous generative translation, model-indexed continuation, and epistemic participation. The value of such a formulation depends on the precision of its distinctions and its ability to motivate productive comparisons with established work. Its components have antecedents in system identification, program learning, causal inference, formal language theory, process semantics, hermeneutics, viability theory, and philosophical accounts of agency. The current discussion does not establish priority over these traditions or prove that they can be unified under a single complete model.

Table 13 summarizes the status of claims made in the manuscript and the additional work required to support stronger conclusions. The table is designed as a safeguard against treating a conditional proof, illustrative simulation, and infrastructural proposal as interchangeable achievements.

Table 13: Epistemic standing of representative contributions and the evidence needed for further claims.

Object	Current standing	Additional warrant required
Typed generative interfaces	Proposed formal organization of heterogeneous models	Domain-specific semantics and demonstrated comparative utility
Query-relative identifiability	Conditional propositions within declared model families	Evidence for applicability of assumptions to investigated systems
Contextual translation witnesses	Proposed partial correspondences; finite examples and failures	Cross-domain tests and interpretation by relevant participants
Generative continuation	Model-indexed whole-spacetime extensions and limited reachability results	Validated models, intervention evidence, and modal qualification
Agency and free will	Historically situated conceptual and practical analysis	Normative and philosophical argument beyond formal branching
Federated epistemic infrastructure	Unimplemented architecture referencing existing standards	Pilot studies, governance evaluation, and measured practical benefits

As indicated in Table 13, the paper can support relatively narrow results concerning properties of defined sets, maps, and experimental systems, while the general empirical and interpretive claims remain open. Its broadest proposal is methodological: the epistemic reconstruction of historical relational development, the translation of selected generative representations, and the exploration of future continuations should be examined together whenever their actual causal and practical couplings warrant it. Continued work will require formal restrictions, better benchmarks, situated interpretation, and accountable institutional trials. The conclusion consolidates this limited but connected research position.

14 Conclusion

This paper has examined *knowledge pressing* as a provisional account of epistemic activity within evolving relational spacetime. Its objective has been to connect three research problems that acquire different meanings when considered together: inferring generative representations from historically situated evidence, translating those representations across heterogeneous relational systems, and investigating the continuations in which inquiry and translation may themselves participate. The preceding analysis develops a common set of questions and limited interfaces for these problems, together with conditional formal results, controlled computational illustrations, and an institutional design proposal. The resulting position concerns the conditions of epistemic access, the historical development of generative capacities, and the consequences of employing knowledge in an evolving relational environment.

The starting point is a distinction between a posited relational reference structure, the history available to an epistemic system, and the representations that the system can construct from that history. A two-complex or other relational-spacetime model supplies one possible formal reference for events, couplings, and higher-order primitives. The recorded trajectory contains the effects of historically constituted instruments, classifications, sources, and access conditions. The candidate generative representation is then an inferential accomplishment relative to specified observations and questions. This layered treatment permits a common analysis of historical formation and prospective continuation while preserving ontological suspension. In particular, a sufficiently detailed record may improve the reconstruction of a history without certifying that one particular generative mechanism has been uniquely recovered.

The investigation of generative representations consequently proceeds through methodological plurality. Grammars, differential and partial differential equations, variational principles, stochastic transition systems, computational programs, and institutional rules express different kinds of admissible transformation. Their adequacy depends on the investigated process, the observational interface, the intended query, and the available grounds for validation. The typed interfaces introduced in Section 4 organize these differences without treating the formalisms as interchangeable descriptions of a single underlying language. Where a system's own categories and practical norms matter, an externally predictive model and a historically situated interpretation have distinct evidential requirements. This distinction is

particularly consequential for inquiries concerning persons, institutions, and social relations.

Historical inference is formulated in Section 5 through evidence-indexed candidate models and explicitly declared assumptions. The relevant achievement may be the identification of a restricted property, the exclusion of incompatible explanations, or the design of an additional inquiry capable of discriminating among remaining candidates. The conditional results of Section 10 clarify that a property can be identifiable within an observational equivalence class even when its members are generatively distinct. Conversely, models that fit an observed history may diverge under subsequent interventions. This places a useful boundary on prospective claims: an admissible continuation derived from a model is qualified by the evidence and assumptions supporting that model, together with the actions and constraints used to define the continuation.

The historical evolution of generative systems introduces a further difficulty. Parameters, structural rules, available operations, representational vocabularies, and observation mechanisms may change on different timescales. Section 6 separates these transformations and shows, in elementary constructions, why a changing record may leave source dynamics and observer revision observationally entangled. Inference therefore requires attention to the constitution and possible revision of its own inferential resources. An epistemic system can acquire a new grammar for describing a process, even when evidence is insufficient to conclude that the process's underlying rules changed. Equally, a generative system can develop new operations that are difficult to express using the observer's inherited language. The investigation of this co-evolution remains open to both mathematical and interpretive methods.

Translation extends the inferential problem to relations between systems. Section 7 proposes contextual witnesses for partial correspondence among observed quantities, transitions, structures, and selected generative consequences. A translation may be useful because it preserves the response to a specified intervention or supports intelligible interaction within a defined context. It may also lose branching information, obscure relevant differences, or fail after one system's grammar changes. The distinction between matching traces and preserving branching structure is demonstrated formally and computationally. A successful translation thus carries qualifications concerning domain, direction, available evidence, preserved properties, and known losses. Where translation concerns legal categories, normative practices, or the self-understanding of participants, those qualifications also involve situated interpretation and contestable authority. Computational correspondence alone supplies a limited form of warrant.

Generative continuation provides the temporal and modal setting in which these epistemic activities become consequential. Section 8 treats a future relational structure as a possible extension of an inherited history, subject to specified generative rules and embedding conditions. The analysis distinguishes continuations conceivable to an observer, admitted by a model, considered physically realizable, and accessible through particular actions, resources, and relationships. These distinctions permit prospective inquiry while maintaining continuity between inherited and emerging structure. Knowledge pressing may change the epistemic description of a continuation, enable actions previously unavailable to a participant, or enter a historical process in which the operative rules themselves change. The resulting account concerns an evolving whole, in which what becomes a future realization subsequently enters the historical conditions of further inference and action.

The role of agency sharpens that account. Section 9 examines subjects as historically constituted within relational arrangements that enable and constrain their capacities. Deliberation, interpretation, learning, cooperation, and institutional modification may affect practically accessible continuations. The elementary accessibility proposition concerns a fixed model and specified conditions; its scope does not extend automatically to autonomy, responsibility, or moral freedom. The philosophical question of free will accordingly remains substantive. Epistemic indeterminacy, physically possible alternatives, opportunities for action, reasons-responsive control, and normative responsibility call for different analyses. Within the present inquiry, an especially productive question is how subjects can participate in revising the relational conditions and practical grammars through which their capacities to act are formed, and how those revisions alter the positions of other participants.

The computational studies in Section 11 give these distinctions finite, reproducible form. Controlled changes in a graph-generation rule become harder to detect as historical observations are corrupted.

Diffusion-coefficient estimation varies markedly with the estimator and supplied representational knowledge. Distinct variational descriptions can produce the same Euler–Lagrange dynamics. A known correspondence between finite stochastic representations preserves iterated transitions, while trace-equivalent transition systems can differ in branching structure. Finally, acquiring a stipulated skill and changing an available rule produce different increments in a model’s resource-feasible reachability. The scripts, numerical outputs, and figures permit examination of these claims under their stated assumptions. These cases establish neither a general system for discovering arbitrary generative laws nor validated translation of lived or institutional meaning; their contribution is to expose particular inferential opportunities and failure modes in inspectable settings.

The proposed infrastructure in Section 12 translates the methodological agenda into a possible architecture of versioned historical records, provenance-bearing inference runs, typed model repositories, contextual translation witnesses, and dependency-tracked continuation studies. Its federated design also incorporates access restrictions, opportunities for correction, and the ability to contest representations and their uses. Such controls are part of the epistemic specification, since decisions about observation, categorization, disclosure, and validation affect whose histories and practices can become intelligible to others. A practically successful implementation would require evidence of benefits over existing scientific, archival, educational, journalistic, and institutional practices. Its performance would also depend on maintenance costs, privacy risks, institutional incentives, unequal resources, and the distribution of authority among participating systems.

The paper’s contribution is therefore primarily an organized formal and methodological investigation. It combines historically indexed inverse inquiry, heterogeneous generative representations, partial translation, changing model languages, and whole-spacetime continuation within one provisional vocabulary. Its propositions give restricted consequences of declared mathematical assumptions; its experiments test constructed mechanisms with known reference conditions; and its infrastructure component supplies a proposal for evaluation. The term *knowledge pressing* does not establish a new disciplinary practice by itself. Its potential analytical usefulness depends on whether the combined framework clarifies problems, supports discriminating inquiry, or leads to improvements that existing methods do not already provide at comparable cost and with appropriate safeguards.

A research programme following this investigation should develop more demanding benchmarks for representation inference under changing observation mechanisms, clarify translation between formalisms with different state and event ontologies, and study the durability of correspondence under intervention and endogenous rule evolution. Further work on topological continuation should specify which higher-dimensional features are identifiable from partial histories and which future claims remain model-relative. The study of epistemic participation would benefit from situated empirical partnerships in which participants can correct an inferred grammar and contest proposed translations. Investigations of practical agency should test how learning and institutional changes affect genuinely available action while leaving moral and metaphysical questions to appropriate philosophical and normative argument. Incremental infrastructure pilots can then examine whether provenance, model revision, and translation certificates improve comprehension, scientific inquiry, accountability, or cooperative practice.

Taken together, these directions locate knowledge pressing within the historical generation of both understanding and future possibilities. Inference makes selected generative relations available for scrutiny; translation may render aspects of heterogeneous systems mutually intelligible; continuation specifies how such understanding can be related to transformations that remain open under declared conditions. Their coupling is historically contingent and requires case-sensitive evidence. The resulting research perspective treats the development of knowledge and the development of relational spacetime as potentially interacting processes, with epistemic limitations, plurality of representations, and the practical conditions of agency remaining central to its further elaboration.

A Notation, Types, and Structural Conventions

This appendix consolidates the mathematical vocabulary used in Sections 3–12 and specifies type and scope conventions needed to compare their constructions. Its purpose is to facilitate verification and future implementation of the examples, rather than to supply an overarching mathematical language into which all physical, computational, and socially constituted systems must fit. A notation table first identifies the objects that recur throughout the manuscript. Subsequent subsections distinguish their domains, examine how observational equivalence depends on the question posed, and state conditions for comparing translations and continuations. The appendix retains the ontological suspension described in Section 3: a mathematical reference complex is a posited modeling resource, and the observation of its purported properties remains historically situated.

Table A.1 identifies the principal symbols used below and the scope of their interpretation.

Table A.1: Working notation for the relational reference, epistemic system, generative representation, translation, and continuation. The indicated types are relative to the investigation and do not imply a universal common semantics.

Symbol	Role	Type or condition of use
$T, \preceq$	Historical index and precedence	Partially ordered set; the order is not automatically physical causation
$K_{\leq t, \iota_{s,t}}$	Cumulative reference complex and embedding	Finite CW 2-complex; coherent cellular embeddings when $s \preceq t$
$\mathcal{R}, \lambda_t$	Posited reference history and decorations	Structured family of complexes, incidence and contextual attributes
$a, \mathcal{E}_{a,t}$	Observer and epistemic configuration	Observer index; historically situated observation, inference, inquiry, action and norms
$D_{a,\leq t}$	Available historical evidence	Typed, potentially incomplete and versioned epistemic records
M, τ_M	Candidate model and its type	Typed descriptor with generative, observational and interpretive interfaces
$\mathfrak{M}_{a,t}^{q,c}$	Candidate model family	Evidence- and question-indexed subset of a disjoint union of model types
q, c	Inquiry and context	Selected explanandum; instruments, units, intervention and interpretive constraints
Θ_{AB}^c	Translation witness	Partial observation interfaces, correspondence relation and evidential qualifications
$\mathfrak{C}_t^{M,c}$	Model-admissible continuations	Whole-history extensions admitted by model M under context c
$A_{a,t}$	Available action modes	Situated capacities and permissions, distinct from model-consistent branches

A.1 Symbols and Domains of Use

The table distinguishes mathematical types from the analytical roles assigned to them. A symbol indicates a family of objects rather than a universal datatype. For example, the history space of a probabilistic process carries measurable structure, while a space of cellular histories may consist of finite derivations with combinatorial incidence data. The formal requirements of each case remain those of the particular model type under investigation.

Indexing by t does not imply that the observer has simultaneous access to everything assigned that index in the posited history. In particular, $D_{a,\leq t}$ is an epistemic record available to observer a by t , whereas $K_{\leq t}$ belongs to the posited reference in Definition 3.1. An investigator at a later index may reconstruct an earlier episode using evidence that was unavailable to the original participants. Such retrospective knowledge should be recorded separately from the epistemic resources held by those participants.

A.2 Cellular Reference Histories and Active Configurations

Definition 3.1 uses coherent cellular embeddings to represent a cumulative finite reference history. Over a fixed coefficient field $\mathbb{F}$, the chain groups $C_k(K_{\leq t}; \mathbb{F})$ are free vector spaces generated by oriented k -cells. Their boundary maps and the chain-complex condition are stated in Equation A.1.

$$\partial_{k,t} : C_k(K_{\leq t}; \mathbb{F}) \longrightarrow C_{k-1}(K_{\leq t}; \mathbb{F}), \quad \partial_{k-1,t} \circ \partial_{k,t} = 0. \quad (\text{A.1})$$

For $k = 2$, the vanishing composite in Equation A.1 expresses the familiar fact that the cellular boundary of a face is a cycle. This is a combinatorial consistency property, not a claim that physical or social transformations are completely specified by cellular incidence. If a cell embedding $\iota_{s,t}$ induces a chain map $\iota_{s,t\#}$, its compatibility with the boundary maps has the form in Equation A.2.

$$\partial_{k,t} \circ \iota_{s,t\#} = \iota_{s,t\#} \circ \partial_{k,s}. \quad (\text{A.2})$$

The coherence of the embeddings in Equation 2 and the chain-map relation in Equation A.2 have distinct purposes. The former ensures consistency across historical indices, while the latter concerns the induced algebraic representation of incidence. In a non-embedding representation of historical revisions, corresponding properties would need to be specified for the selected morphisms or spans rather than assumed.

An accumulated complex and an *active configuration* must also be distinguished. Let λ_t include an activity map on recorded cells. An active configuration may be formed by selecting a subcomplex $K_t^{\text{act}} \subseteq K_{\leq t}$ when the selected cells are closed under taking boundaries. Where activity status does not satisfy this closure property, the active object should instead be represented by a typed incidence structure or a separate derived complex. This avoids treating the withdrawal of a practical coupling as literal deletion of its historical occurrence.

The Betti numbers $\beta_k(K)$ depend on the chosen coefficient field and the precise complex whose homology is computed. Equality of Betti numbers provides limited information: it may fail to distinguish non-isomorphic complexes and, more fundamentally for this paper, their different generating histories. The examples in Sections 3, 5, and 8 make different uses of endpoint, intermediate, and continuation topologies. Subsequent computations must state which of these objects supplies the reported invariants.

A.3 Typed Generators and Observation Interfaces

The model descriptor of Definition 4.1 identifies roles shared by heterogeneous representations. It does not determine a common mathematical codomain for the generator. A useful supplementary convention is to assign a type-specific semantic carrier $\mathcal{G}_{\tau_M}$ and require that generation be well typed in the sense of Equation A.3.

$$\text{Gen}_M : \mathcal{C}_M \times \mathcal{U}_M \rightharpoonup \mathcal{G}_{\tau_M}, \quad \text{Real}_M : \mathcal{G}_{\tau_M} \rightrightarrows \mathcal{H}_M. \quad (\text{A.3})$$

The optional realization relation Real_M in Equation A.3 specifies which histories satisfy or instantiate the generated object. It may be the identity for a generator producing trajectories, the solution relation for a differential equation or variational condition, a derivation semantics for a grammar, or a support relation for a probability law. This is an *appendix-level convention* for clarifying the earlier descriptor rather than a replacement of its meaning. In particular, it does not assert that every model has a computable realization operator or that the semantic carriers of distinct model families can be identified.

A model's observation interface can similarly be deterministic, stochastic, relational, or interpretive. For a numerical investigation with a measurable observation space $\mathcal{Y}_{q,c}$, it may be a function from trajectories to data or a probability kernel. A textual or normative inquiry may instead specify admissible documented interpretations and their sources. The pair of relations in Equation A.4 illustrates the separation of model realization from comparison with evidence.

$$\text{Obs}_M^{q,c} : \mathcal{H}_M \rightrightarrows \mathcal{Y}_{q,c}, \quad \text{Comp}_{q,c} : \mathcal{Y}_{q,c} \times \mathcal{D}_{q,c} \rightharpoonup \mathcal{W}_{q,c}. \quad (\text{A.4})$$

Here $\mathcal{D}_{q,c}$ collects the admitted evidence and $\mathcal{W}_{q,c}$ is the type of an evidential comparison result. A numerical loss may take values in nonnegative reals, while a qualitative comparison may produce a documented compatibility relation. The numerical inequality in Definition 5.1 is one special case of this general convention. It is important not to transfer a numerical comparison unaltered to a dispute about the legitimacy or interpretation of a social practice.

For statistical inference, additional objects such as a sigma-algebra, sampling design, error model, and conditioning information may be required. For variational inference, boundary conditions and permitted variations belong to admissibility. For normative or institutional description, interpretive authority and the possible divergence between an official rule and actual conduct belong to the context and evidence. These requirements belong to the types themselves, rather than becoming interchangeable merely because all models are registered using one data structure.

A.4 Evidence Fibers and Query-Relative Properties

A fixed question and observation context can support a deterministic signature map $\sigma_{q,c} : \mathfrak{M}^{q,c} \rightarrow \mathcal{Y}_{q,c}$, when every candidate possesses a comparable output on the selected observation protocol. For an observed signature y , the corresponding evidential fiber is defined in Equation A.5.

$$\mathfrak{F}_{q,c}(y) = \{M \in \mathfrak{M}^{q,c} : \sigma_{q,c}(M) = y\}. \quad (\text{A.5})$$

As explained in Section 10, a query-valued property Q may be identifiable on the fiber of Equation A.5 even when the full model is not. This requires Q to be constant on that fiber. In a noisy or partially observed setting, equality can be replaced by a stated statistical or compatibility criterion, but the resulting notion of identifiability will then inherit its assumptions. If the observations vary with an evolving measurement system, fibers at different indices should not be compared without accounting for those observational changes.

One elementary illustration is a parameterized scalar evolution with an observed product $\theta = ab$. Measurements that identify θ can still leave the individual values of a and b underdetermined. The property $Q(a, b) = ab$ is identifiable in the stipulated setting, whereas $Q(a, b) = a$ need not be. The distinction concerns the selected observational experiment; additional inputs or interventions could supply constraints unavailable in the original record. This illustration is consistent with the product-parameter example of Section 5 and demonstrates why the recovery of one feature should not be reported as recovery of a unique mechanism.

A.5 Translation Witnesses and the Domain of Composition

The translation object in Definition 7.1 consists of partial interfaces α_A^c and α_B^c , a relation R_c between selected feature spaces, and an evidential qualification Π_c . A translation witness has an inferential status as well as a mathematical type. Its comparison may be meaningful for a restricted class of histories while carrying little evidence about unobserved responses, incompatible categories, or subsequent changes of grammar.

When translation is tested against generative transitions, the source and target systems must expose a compatible test interface. Let S_A, S_B be state descriptions appropriate to one query, let F_A^u, F_B^v be transition maps or relations for matched admissible inputs, and let $r : S_A \rightarrow S_B$ be a partial state correspondence. A strong, explicitly conditional preservation requirement can be written as Equation A.6.

$$r(F_A^u(x)) = F_B^v(r(x)), \quad x \in \text{dom}(r), \quad (\text{A.6})$$

provided $F_A^u(x)$ and the other displayed expressions are defined, $F_A^u(x)$ remains in the domain of r , and the matched inputs u, v are specified. For relation-valued transitions, Equation A.6 must be reformulated as forward simulation, backward simulation, or another selected relation-preservation condition. The equation is a test for a particularly strong correspondence on a delimited domain, not the definition of meaningful translation for social systems.

Suppose partial functions r_{AB} and r_{BC} are proposed in successive translation contexts. Their composite is defined on the restricted domain stated in Equation A.7.

$$\text{dom}(r_{BC} \circ r_{AB}) = \{x \in \text{dom}(r_{AB}) : r_{AB}(x) \in \text{dom}(r_{BC})\}. \quad (\text{A.7})$$

Equation A.7 identifies a mathematical precondition for compositional mapping. It does not settle whether the contexts, semantic interpretations, authorized participants, and retained evidence can likewise be composed. The failure of these additional conditions can invalidate a practical or normative translation even if a formal function composition exists. Moreover, a non-injective map may discard distinctions that subsequent translation cannot recover. The branching counterexamples in Sections 7 and 10 illustrate this issue in controlled model classes.

A.6 Whole-History Continuation and Modal Scope

A continuation in this manuscript is an extension of a specified historical object, rather than a future-only object detached from its conditions of formation. In the embedding-based cellular case, the relevant datum is a pair (K^+, j) with $j : K_{\leq t} \hookrightarrow K^+$, subject to declared compatibility constraints. The collection of model-admissible pairs depends on the model, the time of inquiry, and the continuation context. Equation A.8 makes these indices explicit.

$$\mathfrak{C}_t^{M,c}(K_{\leq t}) \subseteq \{(K^+, j) : j : K_{\leq t} \hookrightarrow K^+ \text{ is a compatible cellular embedding}\}. \quad (\text{A.8})$$

The inclusion in Equation A.8 pertains to the cellular example. A PDE continuation may involve a forward extension of a field solution with initial and boundary data, while a rule-governed practical process may involve admissible subsequent actions. For systems whose recorded reference changes by replacement rather than embedding, continuation requires the corresponding history morphisms. The term *whole-history* signals retention of the constituting history in the model of possible development, without imposing an embedding requirement on every other mathematical formalism.

Let $\mathfrak{M}_t$ be a nonempty evidence-compatible model family evaluated under the *same* context and continuation semantics. The distinction between model-supported and shared continuations can be expressed as the union and intersection in Equation A.9.

$$\mathfrak{C}_t^{\text{supported}} = \bigcup_{M \in \mathfrak{M}_t} \mathfrak{C}_t^{M,c}, \quad \mathfrak{C}_t^{\text{shared}} = \bigcap_{M \in \mathfrak{M}_t} \mathfrak{C}_t^{M,c}. \quad (\text{A.9})$$

These are set-theoretic epistemic summaries. The first records what at least one retained model permits; the second records what each retained model permits within the stipulated comparison. Their physical realizability is a separate question. Their practical accessibility also depends on resource constraints, viable interventions, permissions, and relations of power. As demonstrated in Section 8, eliminating some candidate models narrows the union and can enlarge the intersection, under fixed semantics. If learning also changes the available language, observation protocol, continuation conditions, or model family, this monotonicity need not apply to the entire historical sequence of inquiries.

The agency analysis in Section 9 further distinguishes a model-supported branch from a subject's capacity to deliberate about, select, or realize it. A simulation with several trajectories establishes a property of that simulation; it does not determine whether the investigated world is ontologically indeterministic or whether a subject possesses metaphysical freedom. This limitation matters equally for physical systems and for social settings in which participants can revise their practices through interaction.

A.7 Consistency Checks for Subsequent Formal Investigations

The conventions above suggest a practical checking procedure when designing new examples or implementing an epistemic infrastructure. A proposed mathematical result should identify the question and its observation type, list admissibility conditions for each model family, state whether the claimed correspondence concerns traces, branching, responses to interventions, or situated meanings, and report the

domain over which a continuation is compared. A model’s output should remain distinguishable from its observation, an observation from its interpretation, and an interpretation from any normative warrant claimed on its behalf.

This separation also prevents a particular type of inference error. If a novel translation interface improves the performance of an observer, the result may arise from added evidence, an altered model class, a more informative query, a new intervention, or an actual transformation of the investigated system. Each explanation is empirically and conceptually distinct. The observational ambiguity discussed in Section 6 provides an explicit finite example. Tracking versioned provenance of evidence and translation witnesses, as proposed in Section 12, supports later investigation of these alternatives without deciding among them in advance.

Finally, the conventions are open to extension. Higher-dimensional cell complexes, smooth or singular spaces, categorical structures, probabilistic semantics, and historically constituted practical languages may require new objects that do not reduce to the finite cellular case. Their relationship to the present paper can be established through domain-specific interpretations and partial correspondences. The aim of this appendix has been to make these interfaces auditable while keeping the generative and epistemic plurality of the research programme intact.

B Two-Complex Histories and Admissible Extensions

This appendix develops finite cellular examples underlying the historical and continuation constructions in Sections 3 and 8. Its objective is to make the attachment data, boundary operators, relative homology, and derivation histories independently checkable. The treatment remains deliberately limited to finite CW complexes of dimension at most two over a declared coefficient field. These complexes provide an inspectable representation of certain relational structures; neither cellular incidence nor a homology class, by itself, determines physical causation, social meaning, or normative significance. The exposition first specifies cellular history data and relative chains, then proves a restricted attachment result, computes alternative extensions of a triangular cycle, examines changes of generative grammar, and formulates consistency checks for historical reconstruction.

B.1 Cumulative Histories, Boundary Maps, and Decorations

Let $K_0 \hookrightarrow K_1 \hookrightarrow \dots \hookrightarrow K_m$ be a finite filtration of two-dimensional CW complexes, with oriented cells and cellular inclusion maps. This is a linear-indexed instance of the partially ordered reference structure in Definition 3.1. The complex K_i accumulates cells admitted by the reference construction through stage i ; it need not describe the network of relations currently active at that stage. An activity function or a similarly typed decoration can retain the historical occurrence of a coupling after its practical activity has ended. For the present examples, vertices denote events and edges denote couplings. Two-cells denote formal higher-order relational primitives with specified attaching maps; their application-level interpretation remains open.

Fix a field $\mathbb{F}$, such as $\mathbb{Q}$ or $\mathbb{F}_2$. Cellular boundary matrices are determined by oriented incidence and attaching degrees. For a chosen ordering of the cells, their consistency and the resulting homology are summarized in Equation B.1.

$$\partial_1 \partial_2 = 0, \quad H_k(K; \mathbb{F}) = \ker \partial_k / \text{im } \partial_{k+1}, \quad \beta_k(K) = \dim_{\mathbb{F}} H_k(K; \mathbb{F}). \quad (\text{B.1})$$

Equation B.1 is a property of the cellular chain complex. A numerical implementation should check the vanishing matrix product and compute ranks over the declared field. Rank calculations over floating-point approximations may require tolerances and can be inappropriate for integer incidence data where exact arithmetic is available.

To make the meaning of a historical extension precise, the cellular inclusion should be accompanied by a stage marker, an attachment or update witness, and the provenance of the observer’s reconstruction. These additional records separate a claimed structural change from evidence that supports the claim. A

change in a reconstruction may result from new observations or reclassification even when the posited reference has not changed. Similarly, a system's active graph can change by deactivating a relation while its cumulative historical complex preserves the earlier edge. For later interpretation, the provenance of a cell, its interval of activity, and the assertion that it represents a particular relation should be recoverable as distinct pieces of information.

B.2 Relative Cellular Chains of a Continuation

Let $j : K \hookrightarrow L$ be a cellular inclusion, identified with a subcomplex inclusion for the remainder of this appendix. The pair (L, K) records an entire extended complex together with its inherited historical portion. Relative cellular chains isolate the added part of that pair while retaining the effect of the inherited boundary. Their definition and boundary construction are stated in Equation B.2.

$$C_k(L, K; \mathbb{F}) = C_k(L; \mathbb{F}) / C_k(K; \mathbb{F}), \quad \bar{\partial}_k([c]) = [\partial_k c]. \quad (\text{B.2})$$

The quotient in Equation B.2 is well defined because the boundary of a cellular chain in the subcomplex K is again a chain in K . Its homology, $H_k(L, K; \mathbb{F})$, describes a property of the *pair*; it does not coincide in general with the new absolute homology of L . The distinction is particularly consequential when a new face fills an old cycle.

A continuation is therefore recorded as a marked pair (L, j) together with an optional derivation sequence and contextual qualifications, consistent with Equation 45. Two continuations can have isomorphic endpoint complexes while carrying distinct inclusions, stage histories, or admissibility witnesses. Even when their pairs are isomorphic as undecorated cellular objects, their event timings, practices, causal claims, and epistemic provenance may differ. The formalism represents these distinctions when the relevant decorations and derivation witnesses are retained.

For a pair of finite complexes, the long exact homology sequence relates the inherited, extended, and relative structures. The segment relevant to two-cell attachment is shown in Equation B.3.

$$H_2(K) \longrightarrow H_2(L) \longrightarrow H_2(L, K) \xrightarrow{\delta} H_1(K) \longrightarrow H_1(L) \longrightarrow H_1(L, K). \quad (\text{B.3})$$

All groups in Equation B.3 use coefficients in $\mathbb{F}$. The connecting homomorphism δ conveys a key fact: a new relative two-dimensional class can be associated with the elimination of an existing absolute one-dimensional cycle. Thus an increase in relative H_2 need not imply the creation of an absolute two-dimensional homology class.

B.3 Homology Change under a Single Two-Cell Attachment

Consider a connected finite CW complex K of dimension at most two, and attach one oriented two-cell f along a cellular map $\varphi : S^1 \rightarrow K$. Write $L = K \cup_{\varphi} e^2$. The attaching loop determines a homology class $[\varphi] \in H_1(K; \mathbb{F})$, using the induced map on first homology. The following elementary proposition makes the alternative effects explicit.

Proposition B.1 (Single-face attachment over a field). *For $L = K \cup_{\varphi} e^2$ as above, suppose K is connected and the coefficient ring is the field $\mathbb{F}$. Then $H_1(L; \mathbb{F})$ is naturally isomorphic to $H_1(K; \mathbb{F}) / \langle [\varphi] \rangle$. If $[\varphi]$ is nonzero, $\beta_1(L) = \beta_1(K) - 1$ and $\beta_2(L) = \beta_2(K)$. If $[\varphi]$ vanishes, $\beta_1(L) = \beta_1(K)$ and $\beta_2(L) = \beta_2(K) + 1$. In either case, $\beta_0(L) = \beta_0(K)$.*

Proof. The relative complex (L, K) has precisely one relative two-cell and no relative cells in dimensions zero or one. Consequently, $H_2(L, K; \mathbb{F}) \cong \mathbb{F}$ and $H_1(L, K; \mathbb{F}) = 0$. The relevant exact sequence extends Equation B.3 to an exact segment beginning with $0 \rightarrow H_2(K) \rightarrow H_2(L)$ because $H_3(L, K) = 0$. Under the connecting map, a generator of $H_2(L, K)$ is sent to the attaching class $[\varphi]$ in $H_1(K)$. Exactness therefore identifies $H_1(L)$ with the quotient by the span of $[\varphi]$. If this class is nonzero, the connecting map is injective; $H_2(K) \rightarrow H_2(L)$ is then an isomorphism, while the dimension of H_1

Table B.1: Cell counts, Euler characteristic, and absolute homology over a field for the finite triangular continuation examples. The coefficient-field choice does not alter these particular rank values.

Complex	#V	#E	#F	χ	β_0	β_1	β_2
K	3	3	0	0	1	1	0
L_{fill}	3	3	1	1	1	0	0
L_{path}	4	5	0	-1	1	2	0
L_{both}	4	5	1	0	1	1	0

drops by one. If the class vanishes, the connecting map is zero; exactness gives a short exact sequence $0 \rightarrow H_2(K) \rightarrow H_2(L) \rightarrow \mathbb{F} \rightarrow 0$. This sequence splits as a sequence of vector spaces, yielding one additional dimension in H_2 . Attaching a two-cell changes neither the zero-cells nor path-connectedness, so β_0 is unchanged. $\square$

Proposition B.1 is a standard consequence of cellular relative homology. Its purpose here is to delimit the meaning of *generative continuation*: the topological effect of a newly admitted primitive depends on how its boundary relates to the inherited history. The same operation type, attachment of one face, can eliminate a cycle or create a higher-dimensional homology class. In applications, identifying the attaching map and its evidential basis is therefore at least as important as recording that a face was added.

For a more general two-cell attaching map of degree d to a one-cell circle, the cellular boundary of the face is multiplication by d . Over a field, the result depends on the image of d in that field. For example, over $\mathbb{Q}$, every nonzero d kills the rational first-homology generator; over $\mathbb{F}_2$, an even degree induces the zero boundary map. This coefficient dependence underscores why topological summaries must state the chosen field and attaching conventions. A nontrivial attaching degree need not be represented by an ordinary simplicial face; it is naturally accommodated by CW attachment.

B.4 Two Extensions of a Triangular Historical Cycle

The following finite construction elaborates the illustrative continuation of Section 8. Let K be the one-dimensional boundary of a triangle, with vertices v_0, v_1, v_2 and oriented edges e_{01}, e_{12}, e_{20} . There is no two-cell in K . Its fundamental cellular cycle z and Euler characteristic are specified in Equation B.4.

$$z = e_{01} + e_{12} + e_{20}, \quad \partial_1 z = 0, \quad \chi(K) = 3 - 3 = 0. \quad (\text{B.4})$$

Over a field, K is connected with $\beta_1(K) = 1$ and $\beta_2(K) = 0$. We compare two expansions whose records retain the same inherited triangle.

The first expansion L_{fill} attaches an oriented face f with $\partial_2 f = z$. By Proposition B.1, the old cycle becomes a boundary. Hence $\beta_1(L_{\text{fill}}) = 0$ and $\beta_2(L_{\text{fill}}) = 0$. The relative pair (L_{fill}, K) nevertheless has $H_2(L_{\text{fill}}, K) \cong \mathbb{F}$. The relative connecting map sends its generator to $[z]$ and is an isomorphism in this example.

For the pair (L_{fill}, K) , the relative two-cell is a cycle because its full boundary lies in K , although the same boundary is nonzero in $C_1(K)$. This is the concrete reason that relative H_2 records the attachment while the absolute H_1 class disappears.

The second expansion L_{path} adds a new vertex v_3 and edges e_{03}, e_{31} , forming a second path from v_0 to v_1 without attaching a face. The new cycle $z' = e_{03} + e_{31} - e_{01}$ is independent from the inherited triangular cycle. Thus L_{path} has four vertices, five edges, and no faces, with $\beta_1(L_{\text{path}}) = 2$ and $\chi(L_{\text{path}}) = -1$. For the second pair, the relative chain groups are $C_1(L_{\text{path}}, K) \cong \mathbb{F}^2$ and $C_0(L_{\text{path}}, K) \cong \mathbb{F}$, with relative boundary row $(1, -1)$ under compatible orientations. Thus $H_1(L_{\text{path}}, K) \cong \mathbb{F}$ and $H_2(L_{\text{path}}, K) = 0$. The complexes and their relevant cellular structures are displayed in Figure B.1.

Table B.1 reports the independently checkable Betti numbers. Both branches share the inherited complex, but their topological effects diverge under the stated attachment rules.

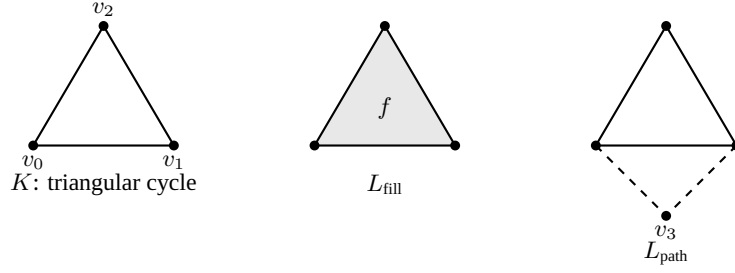


Figure B.1: Two admissible cellular extensions of a common historical triangle. The shaded face is an added two-cell; dashed edges form an added coupling path through a new event. The diagrams encode a selected cellular structure and do not, by themselves, establish the interpretation or desirability of an intervention.

The last row of Table B.1 describes a further complex L_{both} obtained by carrying out both operations. It can be reached by attaching the face before adding the path or by adding the path before attaching the face. The final complex is identical under the specified identification of cells, and its first Betti number is one. Its intermediate history is nevertheless different: the face-first derivation passes through a complex with $\beta_1 = 0$, while the path-first derivation passes through one with $\beta_1 = 2$. The order of operations is therefore information that an endpoint complex and its final Betti numbers do not preserve.

B.5 Temporal Filtrations and Path-Dependent Reconstruction

An indexed series of inclusions provides more information than a single endpoint. For a chosen k , each inclusion $K_i \hookrightarrow K_j$ induces a map in homology, giving a sequence of vector spaces and linear maps. Equation B.5 records this restricted persistence structure.

$$H_k(K_0; \mathbb{F}) \longrightarrow H_k(K_1; \mathbb{F}) \longrightarrow \cdots \longrightarrow H_k(K_m; \mathbb{F}). \quad (\text{B.5})$$

A one-dimensional homology class may appear, persist through some attachments, and become trivial after a face attachment. In the face-first construction above, the original triangular class is killed before a second independent cycle appears; in the path-first construction, two independent cycles coexist before the triangle is filled. A computation based on Equation B.5 therefore distinguishes these histories even though the final Betti numbers agree. Persistent homology provides mathematical summaries of these birth and death relations under a specified filtration; a numerical birth index is not itself an ontological date unless the filtration indices and their relationship to recorded events are established.

For an epistemic application, a filtration is inferred from evidence and may be contested. A historical archive might specify only a partial precedence order, possess uncertain event dates, or record an edge after the interaction occurred. More than one linear ordering can then be compatible with the same surviving evidence. A persistent summary from one chosen linear extension of the partial order should be accompanied by the ordering assumptions that produced it. Where those assumptions change, the relevant persistence diagram or interval summary may change. This issue is distinct from computational instability due to measurement noise: it concerns the constitution and dating of the historical record.

The chain maps of Equation A.2 ensure that cellular inclusions induce maps in homology. The existence of these maps supplies a mathematical framework for comparison, yet causal interpretation still requires an additional account. A homology class changing at an attachment stage indicates a topological event in the selected representation. Inferring that a specific collective action *caused* a social practice to change requires independent observational, counterfactual, and interpretive evidence. The historical and epistemic qualifications developed in Section 5 remain necessary.

B.6 Grammar-Conditioned Admissibility of Attachments

A pair (L, K) describes a structural possibility within the selected cellular category. Whether that extension belongs to a system's available continuations depends on its generative grammar, material and institutional constraints, and the observer's evidence about them. Let $\mathcal{G}$ denote a typed set of permitted attachment rules, where each rule has applicability conditions and a specified effect on the cellular and decorated structure. A finite derivation witness can be represented by Equation B.6.

$$K = K^{(0)} \xRightarrow{r_1} K^{(1)} \xRightarrow{r_2} \dots \xRightarrow{r_n} K^{(n)} = L, \quad r_i \in \mathcal{G}_i. \quad (\text{B.6})$$

The stage-indexed grammar $\mathcal{G}_i$ in Equation B.6 permits historical rule evolution. Under a fixed grammar, $\mathcal{G}_i = \mathcal{G}$ for the entire sequence. In an evolving grammar, the availability of a rule at stage i may depend on earlier interactions, permissions, learning, or a separately specified grammar-update operation. A cellular attachment that is mathematically definable may remain inadmissible under the grammar operating at a particular stage.

For example, suppose the initial grammar admits only the attachment of a face along an existing admissible cycle. Then L_{fill} may belong to the model-admissible continuation family while L_{path} is excluded. A later grammar update that admits the formation of a new two-edge path can make the second branch model-admissible. Whether a particular subject can implement that rule is a further question concerning resources, authority, and action feasibility. Thus grammatical admissibility, practical accessibility, and realized occurrence remain distinct, in line with Section 9.

The derivation witness can include the rule version, the affected cells, preconditions, and the justification for each step. Two sequences yielding the same endpoint can then remain distinguishable. If a reconstruction lacks such data, its inference should retain the ambiguity rather than impute a preferred derivation from the endpoint alone. The historical rule changes studied in Section 6 are one instance of this problem; a grammar that generates observed configurations may be insufficient to identify the historical sequence of rule updates without additional evidence.

B.7 Relative Evidence, Reconstruction, and Formal Scope

The examples establish several distinct uses of cellular topology. Absolute homology describes the state of a specified complex, relative homology describes a marked extension, and maps between homology groups summarize parts of the attachment history. A derivation witness records an additional temporal and operational structure. None of these layers alone supplies an exhaustive generative explanation. The distinction can be made operational in the epistemic infrastructure of Section 12: an artifact describing a continuation should identify the inherited reconstruction, the attachment data, the model and grammar versions, the field used for homology, the evidence supporting the inferred cells, and the procedure by which numerical results were produced.

A cellular representation also depends on a chosen resolution. One observer may encode a complex interaction by one edge; another may represent it through many intervening events and couplings. Subdivision can preserve a topological realization while changing the combinatorial number of cells. Inferred higher-order faces may represent different concepts in different inquiries. Therefore, even a matching homology sequence across two epistemic systems establishes limited structural correspondence. The stronger questions of translation, preserved interventions, and practical meaning require the contextual witnesses of Section 7.

Finally, the inclusion model used here represents accumulation of a posited history. Deletion of an active coupling can be recorded by a change in decorations while retaining its historical cell. If an application's mathematical state genuinely replaces its cellular structure, a different class of morphisms, correspondences, or event-sourced reconstructions is appropriate. A general theory of arbitrary topology-changing relational spacetime is beyond these finite examples. Their contribution is to show, under explicit assumptions, how attachment operations, homological calculations, and historically indexed generative rules can be made precise and independently audited while leaving interpretation and future realization open to further inquiry.

C Generative and Relational Rewrite Grammars

This appendix develops a finite, typed account of relational rewrite grammars for the inference and continuation problems of Sections 5, 6, and 7. Its objective is to specify rule applicability, cellular well-formedness, derivation histories, changes of grammar, and the evidence needed to distinguish alternative generative accounts. The construction first introduces typed cellular states and productions, then gives a restricted operational semantics, analyzes a two-complex derivation example, and closes with identifiability and translation conditions. The apparatus is deliberately local. Established algebraic graph transformation supplies a broader and more developed family of approaches, particularly double-pushout (DPO) graph rewriting and its categorical extensions [15, 20]. The present treatment uses a simpler inclusion-based cellular fragment where assumptions and consequences can be inspected directly. It makes no claim that arbitrary decorated CW complexes automatically possess the adhesive-category properties needed for unrestricted DPO reasoning.

C.1 Typed Cellular States and Persistent Historical Structure

Fix a finite vocabulary of cell types $\Theta = (\Theta_0, \Theta_1, \Theta_2)$, along with an incidence specification indicating which boundary configurations are admissible for each type. A *typed cellular state* is a finite oriented two-dimensional CW complex K with a cell-type function $\tau_k : K_k \rightarrow \Theta_k$ and a decoration assignment α . The type system describes admissible signatures of cellular relations; it does not determine what the cells mean in an application. Consistent with Section 3, zero-cells represent events, one-cells couplings, and two-cells selected higher-order primitives, each subject to the chosen interpretation.

Two different temporal views are useful. The cumulative complex K_i retains cells admitted by stage i . An activity decoration $a_i(c)$ can indicate whether a previously recorded coupling or higher-order relation remains active at that stage. The distinction is consequential when an institution ends a practice: an active relation may cease without deleting the historical record of its former existence. For the basic grammar below, a cell once incorporated into K_i remains in the cumulative structure. State changes that remove cells from an active model are handled by updates to a_i or require a separate replacement semantics. The latter is discussed after the finite construction.

A well-typed extension must be a cellular inclusion, respect inherited types, and preserve boundary incidence. With $j_i : K_i \hookrightarrow K_{i+1}$, the chain compatibility requirement is recorded in Equation C.1.

$$\partial_k^{K_{i+1}} (j_i)_{\#,k} = (j_i)_{\#,k-1} \partial_k^{K_i}, \quad k \in \{1, 2\}. \quad (\text{C.1})$$

Equation C.1 is the inherited chain-map requirement from Appendix B. Checking it together with $\partial_1 \partial_2 = 0$ rules out certain malformed incidence matrices, while additional type and attaching-map checks still remain necessary. In particular, a numerical boundary matrix by itself is insufficient to specify a CW complex's attaching map or the epistemic interpretation of a face.

C.2 Productions, Matches, and Application Conditions

Definition C.1 (Restricted cellular production). A restricted cellular production r comprises a finite typed pattern L_r , a finite typed extension R_r containing L_r as a cellular subcomplex, a guard g_r , and a provenance schema π_r . A match $m : L_r \hookrightarrow K$ is an injective type-preserving cellular map. The guard $g_r(m, K, h, c)$ evaluates the applicability of the rule relative to a recorded history h and context c . If the guard holds and the requested attachment is compatible with the ambient cellular boundary and types, the production yields an extension $K \xrightarrow{r, m} K'$, recording an inclusion $j : K \hookrightarrow K'$ and the witness π_r .

This fragment models *additive* transformations. Its inclusion $L_r \hookrightarrow R_r$ is more restrictive than the standard DPO specification $L \leftarrow I \rightarrow R$, which allows deletion and replacement subject to gluing conditions. The distinction is important: in DPO graph rewriting a valid pushout complement imposes

identification and dangling restrictions, whereas our simple additive construction retains the matched pattern and its incident structure [15]. The local construction is specified in Equation C.2.

$$r = (L_r \hookrightarrow R_r, g_r, \pi_r), \quad (m, h, c) \in \text{Adm}_r(K) \implies K \xrightarrow{r, m, h, c} K'. \quad (\text{C.2})$$

Equation C.2 leaves the realization of K' abstract until a concrete type and attaching convention are supplied. In the small finite examples below, realization takes the union of K with explicitly specified fresh cells of $R_r \setminus L_r$, identifying the copy of L_r with $m(L_r)$ and checking the oriented attaching data. When a fresh face is attached along an existing closed cycle, the complete attaching word or cellular map is part of the rule witness. A positive guard may require the presence of a specific cycle, while a negative guard can prohibit duplication of an already recorded face. Guards involving permissions, institutional legitimacy, or resource constraints require a separate source of interpretation and cannot be verified solely from cellular incidence.

A *derivation witness* records the source and target state identifiers, rule identifier and version, match, guard evaluations, affected cell identifiers, cell orientation and attaching data, and evidence links. A witness is stronger than an observed pair of endpoints because it supplies an operational hypothesis for their relationship. It remains an epistemic artifact: the existence of a derivation in a mathematical grammar does not prove that this derivation actually occurred in a physical or social system.

C.3 Finite Derivations and Historical Grammar Versions

Let $\mathcal{G}_i$ be the set of rule versions admissible at stage i . A history of structural changes is represented as a sequence of witnessed steps and admissibility checks. Equation C.3 states the finite form, with both cellular and grammar states evolving.

$$(K_i, \mathcal{G}_i, h_i) \xrightarrow{r_i, m_i} (K_{i+1}, \mathcal{G}_{i+1}, h_{i+1}), \quad r_i \in \mathcal{G}_i. \quad (\text{C.3})$$

The transition in Equation C.3 does not make grammar change automatic. A distinct versioned update u_i , when admissible, may transform $\mathcal{G}_i$ into $\mathcal{G}_{i+1}$. Updates can add a production, retire one, change a guard, refine a type, or change an observer's coding language. These have different epistemic meanings. A system may acquire a new practice; an observer may discover a previously unrecognized practice; an institution may revise the permission to exercise a practice. Each interpretation requires evidence beyond the fact that the encoded rule set has changed.

For analytical purposes, one can specify an update kernel U without treating it as a universal historical law. Its role is stated in Equation C.4.

$$\mathcal{G}_{i+1} \in U(\mathcal{G}_i, h_i, u_i, c_i), \quad h_{i+1} = h_i \parallel (r_i, m_i, u_i, \pi_i). \quad (\text{C.4})$$

Here $\parallel$ denotes append-only concatenation of records, and π_i contains the provenance of the inferred step. The update mechanism in Equation C.4 can itself be represented by another generative model. Work on higher-order graph grammars provides an existing formal precedent for transformations of rules, rather than only transformations of host graphs [23]. No privileged final level of grammar is assumed in the present exploratory model; finite inquiry selects a level appropriate to its evidence and questions.

C.4 An Elementary Two-Complex Production System

The two-complexes from Appendix B permit a complete finite demonstration. Begin with the boundary cycle K on vertices v_0, v_1, v_2 , oriented edges e_{01}, e_{12}, e_{20} , and no two-cell. Define the *fill rule* r_F to attach a new oriented face f with boundary $e_{01} + e_{12} + e_{20}$, provided that this face has not already been attached. Define the *path rule* r_P to introduce a fresh vertex w and two fresh oriented edges joining v_0 to w and w to v_1 , provided the specified matched vertices are distinct. The rules here apply at the named interface; the construction fixes those matched cells for a reproducible example.

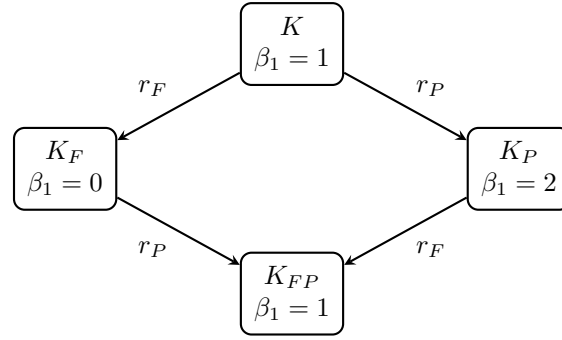


Figure C.1: Finite derivation diamond for two specified additive cellular rules. Both rule orders yield the same final decorated complex when fresh identifiers and the stated endpoint matching are consistently identified. Intermediate first-homology ranks differ, so an endpoint record loses information present in a witnessed derivation history.

For the initial triangle, r_F changes the first Betti number from one to zero, while r_P changes it from one to two. The first intermediate state therefore reveals which production occurred, assuming that its topology is recorded faithfully at that stage. Figure C.1 records the two derivation orders and their intermediate homological differences.

The example has a restricted kind of independence: neither rule changes the guard or the attaching interface of the other, and their fresh-cell sets are disjoint. Therefore the two compositions generate isomorphic endpoint complexes with the same cell labels under the chosen identification. This is a constructed instance of commutation, not an unrestricted Church–Rosser property for relational grammars. In particular, changing a guard to require an unfilled triangle would disable r_P after r_F , breaking the derivation diamond. Similarly, rule updates or resource constraints can introduce dependencies even where the underlying cellular attachments are compatible.

Proposition C.2 (Endpoint ambiguity for commuting additive productions). *Let K be a finite typed cellular state, and let r and s be additive productions with admissible matches such that (i) each production remains applicable after the other, (ii) their newly added cells are disjoint and identified consistently in the final state, and (iii) their inherited attaching interfaces are preserved under both steps. Suppose an observation records only the final typed cellular state up to isomorphism and suppresses operational timestamps and derivation witnesses. Then the two production orders have identical final observations. The observation cannot uniquely determine which production occurred first.*

Proof. Under assumptions (i)–(iii), either order attaches the same two collections of fresh cells to the same inherited interfaces. Their union with K determines isomorphic typed cellular structures, including the specified boundaries and decorations. The endpoint observation is invariant under this isomorphism. Both histories therefore belong to its observational fiber. Since their first steps are different, that fiber does not identify the order of the productions. $\square$

Proposition C.2 is conditional and elementary. It does not assert that two rules commute whenever their final Betti numbers agree. Nor does it require that the complete rule names be ontologically real. It identifies a limit of reconstructing ordered generation from an endpoint record, even when the endpoint cellular data are complete relative to the chosen representation.

C.5 Inference from Partial Historical Evidence

Suppose an observer has a record D with uncertain event order, incomplete attachments, and perhaps only a subset of typed cell states. For a candidate grammar $\mathcal{G}$, define a family of derivation witnesses that satisfy its guards, project to the observations, and obey the declared timing constraints. Equation C.5

gives the finite set-valued form.

$$\text{Der}(\mathcal{G}, D) = \{h : h \text{ is } \mathcal{G}\text{-admissible, } \text{Obs}(h) \sim_D D\}. \quad (\text{C.5})$$

The compatibility relation $\sim_D$ in Equation C.5 allows missing timestamps or incompletely specified observation channels. Where observations include noise, one can replace the hard relation by a likelihood or discrepancy model. The result is a set of candidate histories, rather than a presumed recovery of a unique operational past. Candidate grammars may be distinguished by additional records of intermediate states, traces of rule application, negative observations about forbidden operations, or admissible experiments. As argued in Section 5, inference also depends on the scope and credibility of observation mechanisms.

For the diamond in Figure C.1, an endpoint-only record retains two compatible rule orders. If the intermediate first Betti number is reliably observed and is either zero or two, those two candidate histories are separated under the assumed production system. This is a case where an additional summary measurement is informative. Conversely, if a sensor records only the number of vertices in the final complex, both production orders remain indistinguishable. Such results are query-relative: observing an order-sensitive property can identify a feature of the history without identifying every detail of its generating grammar.

A related ambiguity concerns the *grammar itself*. When a new structural pattern first appears in the archive, at least three candidate accounts may fit it: a new rule was introduced, a pre-existing rule was exercised for the first time, or a new observer began recognizing an already occurring pattern. The states predicted by these accounts can overlap. Distinguishing them requires evidence about the permissions, learning processes, conceptual instruments, and observational changes relevant to the inquiry. Treating these as separate hidden variables is preferable to attributing the observed shift automatically to evolving ontology.

C.6 Translation of Rule Systems and Generative Consequences

A comparison of grammars $\mathcal{G}_A$ and $\mathcal{G}_B$ may align their typed cellular states through partial interfaces and compare admissible productions through a context-dependent rule correspondence. Let F be a partial structure map, and let ρ relate source rules to target rule sequences. A forward preservation obligation is expressed in Equation C.6.

$$K \xrightarrow[\mathcal{G}_A]{r} K' \implies F(K) \xrightarrow[\mathcal{G}_B]{\rho(r)} F(K'), \quad \text{when all terms are defined and admissible in context } c. \quad (\text{C.6})$$

Equation C.6 is a *candidate validation condition*, not an assumed theorem. It permits one source production to correspond to a finite sequence in the target grammar; it does not require reversible translation or exact preservation of all observables. Partial maps must specify their domain, the contextual guard assumptions, and the properties intended to be preserved. As Section 7 develops, matching endpoints under one known experiment provides weaker evidence than validating how interventions and branching possibilities correspond across contexts.

An elementary example is a fine-grained path-attachment rule mapped to a coarser operation that directly introduces a single edge between two existing vertices. Both can connect the same endpoints, while the fine representation records an intervening event and may permit additional future couplings at that vertex. A translation that erases the internal vertex can preserve selected reachability predicates but fail to preserve future branching, specific event histories, or resource constraints. Thus even a correctly tested rule alignment can lose generative information. The intended queries and losses belong in the translation witness, consistent with the infrastructure design of Section 12.

Normative and institutional grammars add a distinct interpretive burden. A formal guard labeled “authorized” has no self-contained account of authority. Its meaning may depend on recognized procedures, contested jurisdiction, historical precedent, and participants’ situated interpretations. An algorithm can test an encoded guard; establishing whether the encoding adequately represents a living practice requires

appropriate domain inquiry and accountable participation. Translation across institutional grammars consequently combines formal evidence with interpretive and normative assessment, without presuming a global arbiter.

C.7 Reproducibility, Scope, and Continuation

The finite diamond construction is accompanied by a standalone exact checker, `experiments/check_k_appendix_c.py`. It enumerates both rule orders, constructs oriented boundary matrices over the rationals, verifies $\partial_1 \partial_2 = 0$, and compares absolute Betti numbers at each stage. It records the endpoint observation signatures, sequence-sensitive signatures, and the availability of each rule under two declared grammar versions. The results are written to `experiments/appendix_c_results.json`. These checks establish correctness of the finite example under its explicit representation; they do not validate general grammar discovery from empirical data.

For future use, a grammar-conditioned continuation study should retain the inherited historical complex, rule versions, applicable matches, guard outcomes, the sequence of inferred transformations, and the provenance of the representation. A model may admit a structural continuation that lies beyond the practical power of any available subject, while an epistemic update can change which continuations an observer can describe. A historically constituted agent may learn to implement a new production, or a community may establish a new cooperative rule. These possibilities connect the local grammar model to Section 9, while leaving questions of autonomy, material conditions, coercion, and free will for philosophical and empirical investigation. The present appendix therefore supplies a restricted, auditable grammar interface for relational continuation, with its limitations visible wherever the mathematical representation meets historically situated practice.

D Differential and Variational Inverse Problems

This appendix specifies several differential, variational, and discrete examples underlying Sections 2, 4, 5, and 7. The objective is to examine what observations constrain about a generative law, which transformations preserve selected dynamical consequences, and why an inferred equation cannot automatically be equated with a unique action principle or a complete relational history. We first distinguish observation interfaces from evolution laws, then construct an explicit inverse problem for the heat equation, examine the ambiguity of Lagrangian recovery, establish a sampling-based non-identifiability result, and derive a discrete variational representation of a harmonic oscillator. The final construction relates a PDE to a conservative cellular update, while identifying properties lost in discretization. These are familiar mathematical objects used as controlled witnesses of the paper’s claims. Neither a universal physics of relational spacetime nor a general theorem of cross-system epistemic translation is proposed here.

D.1 Observation Maps and Differential Inference

Let u be a state field on a spatial region Ω over a time interval I , with dynamics governed by a model $M = (\mathcal{U}, \mathcal{D}, \mathcal{B}, \theta)$. Here $\mathcal{U}$ is the state space, $\mathcal{D}$ a differential evolution operator, $\mathcal{B}$ the initial and boundary conditions, and θ its parameters. For a time-and-space sampling operator $\mathcal{O}_\Sigma$ on a finite observation set Σ , the evidence comprises observed values and their error descriptions, as in Equation D.1.

$$D_\Sigma = \mathcal{O}_\Sigma u + \varepsilon, \quad D_\theta[u] = 0, \quad \mathcal{B}[u] = 0. \quad (\text{D.1})$$

Even when $\varepsilon = 0$, the inverse mapping from D_Σ to M may have multiple solutions. The observational operator and the assumed parameterization therefore belong to the inferential claim. A measurement record of a field is not itself a measurement of all of its spatial derivatives or of its response to every permissible intervention. Sparse discovery algorithms estimate differential generators from data under restrictions on the candidate library, accessible variables, and noise processes [9, 36]; a successful regression is not a general identification theorem.

A transparent case is the one-dimensional heat equation on a periodic spatial interval $\mathbb{T} = [0, 2\pi)$:

$$\partial_t u = \kappa \partial_{xx} u, \quad u(x, 0) = A \cos(kx), \quad \kappa > 0, \quad k \in \mathbb{N}. \quad (\text{D.2})$$

Its classical solution $u(x, t) = A e^{-\kappa k^2 t} \cos(kx)$ permits inference of κ from two observed nonzero mode amplitudes $a(t_1)$ and $a(t_2)$ when k , the time calibration, and the heat-equation model class are known:

$$\hat{\kappa} = -\frac{\log(|a(t_2)|/|a(t_1)|)}{k^2(t_2 - t_1)}, \quad t_2 > t_1, \quad a(t_i) \neq 0. \quad (\text{D.3})$$

Equation D.3 is exact in this idealized setting. It also illustrates dependence on the epistemic grammar: recognizing a periodic Fourier mode, correctly specifying its wavenumber, and treating the process as autonomous diffusion do significant inferential work. If an unknown forcing f is admitted in $u_t = \kappa u_{xx} + f$, each candidate value of κ may be paired with $f = u_t - \kappa u_{xx}$. An observed smooth field alone then fails to identify κ without restrictions or external evidence concerning the forcing. A similar ambiguity arises when observation times have an unknown scale. This is a conditional mathematical limitation, not evidence that the underlying physical process possesses multiple actual diffusion coefficients.

D.2 Action Functionals and Variational Equivalence

The inverse variational problem asks whether observed dynamical equations can be generated by an action and, if so, which actions remain compatible with them. This problem has an established literature concerning existence, uniqueness, and Helmholtz-type conditions [37]. We use a finite-dimensional, regular example to expose one basic non-uniqueness mechanism.

For a coordinate $q(t)$ on $[t_0, t_1]$, consider the Lagrangian and action

$$L_0(q, \dot{q}, t) = \frac{m}{2} \dot{q}^2 - \frac{k}{2} q^2, \quad S_0[q] = \int_{t_0}^{t_1} L_0 dt, \quad m, k > 0. \quad (\text{D.4})$$

With variations fixing $q(t_0)$ and $q(t_1)$, its Euler–Lagrange equation is $m\ddot{q} + kq = 0$. Let $F(q, t)$ be a smooth scalar function on a neighborhood of the relevant paths, and define

$$L_F = L_0 + \frac{d}{dt} F(q, t) = L_0 + F_q(q, t) \dot{q} + F_t(q, t). \quad (\text{D.5})$$

Proposition D.1 (Boundary-term invariance under fixed endpoints). *If $F \in C^2$ and the admissible variations fix the endpoint coordinates, $S_F[q] = \int L_F dt$ and $S_0[q]$ possess the same first variation with respect to q and hence the same interior Euler–Lagrange equation.*

Proof. Integrating the total derivative gives $S_F[q] = S_0[q] + F(q(t_1), t_1) - F(q(t_0), t_0)$. The additional endpoint contribution has zero first variation under the stipulated endpoint condition. Consequently $\delta S_F = \delta S_0$, which proves the claim. Equivalently, the Euler–Lagrange operator annihilates dF/dt by equality of mixed derivatives. $\square$

The choice $F(q, t) = \gamma t q^2/2$, for arbitrary constant γ , produces a family of visibly different Lagrangian expressions with identical interior equations. Nonzero constant rescaling $L \mapsto cL$ also preserves the *solution set* of the Euler–Lagrange equations, although it rescales their left-hand sides and should be distinguished from the exact equality in Proposition D.1. Thus equality of observed paths is still weaker than equality of differential operators, and equality of differential operators is weaker than equality of local Lagrangian formulas. Boundary conditions, momentum definitions, external coupling, and global topology can make apparently harmless representational transformations consequential outside their specified comparison domain. A total derivative local in one coordinate chart is not automatically a globally defined boundary term on every configuration space.

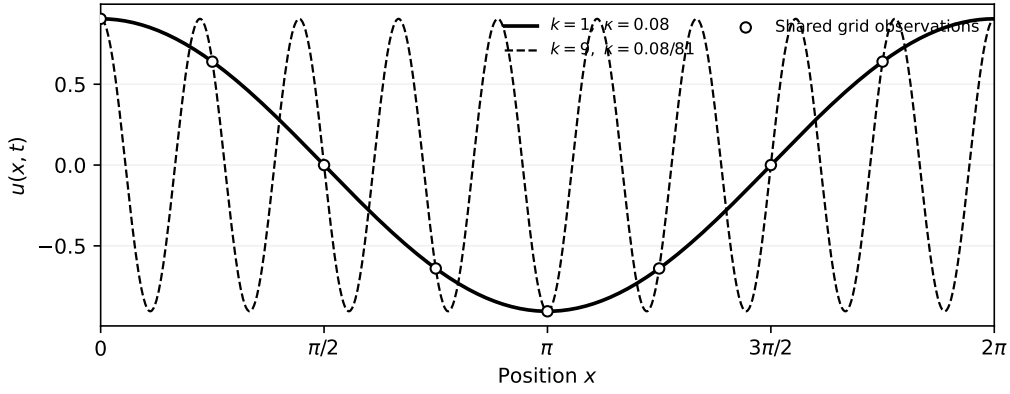


Figure D.1: Two continuous periodic fields and identical eight-point spatial samples at a fixed time. The higher-wavenumber candidate has an adjusted diffusivity producing the same recorded time series at every sampled location. The plot illustrates the restricted observation map, not an ambiguity remaining after unrestricted spatial observation.

D.3 A Sampling Ambiguity for an Inferred PDE

The next counterexample isolates a distinct source of underdetermination arising from spatial observations. It does not appeal to measurement noise, insufficient numerical precision, or an unspecified learning algorithm.

Proposition D.2 (Joint wavenumber–diffusivity ambiguity under grid sampling). *Fix an integer $N \geq 3$, an integer $k \geq 1$, and $\kappa > 0$. On $\mathbb{T}$, set $k' = k + N$ and $\kappa' = \kappa k^2 / (k')^2$. The two periodic heat-equation solutions*

$$u(x, t) = e^{-\kappa k^2 t} \cos(kx), \quad v(x, t) = e^{-\kappa' (k')^2 t} \cos(k'x) \quad (\text{D.6})$$

coincide for all $t \geq 0$ at every grid point $x_j = 2\pi j / N$ but represent different continuous fields and different diffusion coefficients.

Proof. The definitions give $\kappa' (k')^2 = \kappa k^2$. At a sample point, $k'x_j = kx_j + 2\pi j$, so the cosine factors agree. Hence all grid-point time series coincide exactly. Since $k' \neq k$, the two cosine fields differ at some locations between the samples, and $\kappa' \neq \kappa$ for $k \geq 1$. Each field separately solves its stated heat equation, establishing the claimed observational ambiguity. $\square$

Proposition D.2 depends on an observation map that retains only the fixed spatial grid and on a model family that does not already fix the wavenumber. Additional spatial samples, a bandwidth restriction, known initial field, or independent evidence about κ can remove this particular ambiguity. The observation itself cannot distinguish its candidates, so any claim of uniquely recovered physics must state which additional restrictions were imposed. Figure D.1 visualizes one such pair, with $N = 8$, $k = 1$, $k' = 9$, and $\kappa' = \kappa / 81$. The equality holds over the entire observation history, even though their off-grid continuations and spatial responses differ.

D.4 A Discrete Variational Translation of the Oscillator

A continuous Lagrangian and a discrete-time Lagrangian can encode related generative dynamics without being definitionally identical. Discrete variational mechanics supplies an established theory for constructing numerical dynamics from a discretized action [25]. Fix the time step $h > 0$ and sampled configurations $q_j \approx q(jh)$. Define the symmetric discrete Lagrangian and discrete action by

$$L_d(q_j, q_{j+1}; h) = \frac{m}{2h} (q_{j+1} - q_j)^2 - \frac{hk}{4} (q_j^2 + q_{j+1}^2), \quad S_d = \sum_{j=0}^{J-1} L_d(q_j, q_{j+1}; h). \quad (\text{D.7})$$

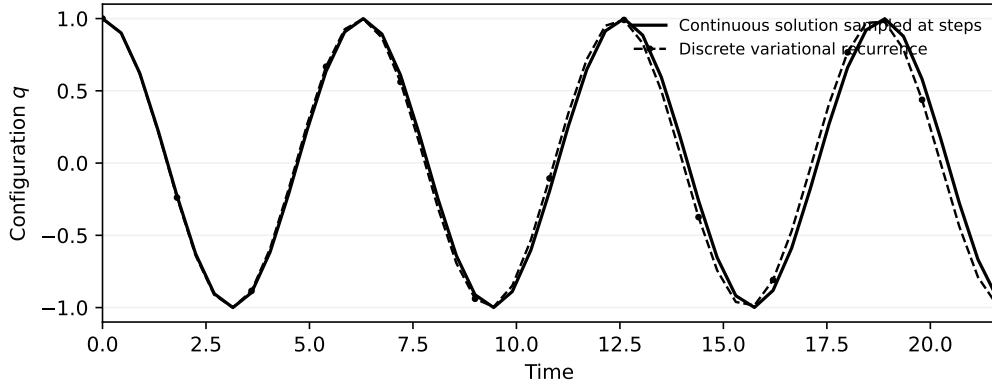


Figure D.2: A harmonic oscillator trajectory and a corresponding discrete variational recurrence at finite step size ($\omega = 1$, $h = 0.45$). The first two discrete values equal the sampled continuous solution, after which numerical phase drift develops. The construction verifies conditional approximation rather than exact trajectory translation.

With q_0, q_J held fixed, the vanishing discrete first variation yields the interior condition

$$D_2 L_d(q_{j-1}, q_j; h) + D_1 L_d(q_j, q_{j+1}; h) = 0.$$

Rearranging gives

$$\frac{m}{h^2}(q_{j+1} - 2q_j + q_{j-1}) + kq_j = 0. \quad (\text{D.8})$$

The recurrence is a discrete generative grammar for allowed triples of sampled configurations. It supplies a deterministic transition on pairs (q_{j-1}, q_j) , while the continuous oscillator supplies a flow on $(q, \dot{q})$. The translation therefore requires specifying a relation between different state types, including a velocity or momentum reconstruction, and need not preserve exact solutions at a nonzero step size.

Proposition D.3 (Consistency and conditional stability of the discrete oscillator). *For a smooth continuous solution of $m\ddot{q} + kq = 0$, substitution of its samples into Equation D.8 produces a residual of order $O(h^2)$ as $h \rightarrow 0$ at fixed times. Setting $\omega^2 = k/m$, the corresponding homogeneous recurrence has bounded solutions for all initial data when $0 < h\omega < 2$. The sampled continuous and discrete solution families are generally unequal for any fixed nonzero h .*

Proof. Taylor expansion around t_j gives $(q(t_j + h) - 2q(t_j) + q(t_j - h))/h^2 = \ddot{q}(t_j) + O(h^2)$ for $q \in C^4$, proving the consistency claim. The recurrence has characteristic polynomial $z^2 - (2 - h^2\omega^2)z + 1$. For $0 < h\omega < 2$, its roots are distinct complex conjugates of modulus one, so every solution is bounded. At $h\omega = 2$ the repeated root $z = -1$ admits linearly growing solutions for some initial data; larger steps yield a root outside the unit circle. Finally, substituting $q(t) = \cos(\omega t)$ into the recurrence gives the nonzero coefficient $\omega^2 - 4h^{-2}\sin^2(\omega h/2)$ for generic $h > 0$ and t_j . $\square$

Proposition D.3 expresses a conditional relationship between a continuous generator and a discrete generator. The approximation is justified by a consistency statement and appropriate initial-data matching, not by an assertion of exact semantic identity. In particular, an action-based inference engine supplied only with discrete samples might recover a discrete Lagrangian that is adequate for the sampled recurrence while leaving its continuous limit, original physical variables, or endpoint conventions underdetermined. Figure D.2 illustrates the finite-step deviation even when the discrete initial sample is matched to the first two continuous samples.

D.5 A Conservative Cellular Approximation of Diffusion

For the periodic heat equation in Equation D.2, a nearest-neighbor grid with spatial step Δx and time step Δt admits the explicit update

$$u_j^{n+1} = (1 - 2r)u_j^n + ru_{j-1}^n + ru_{j+1}^n, \quad r = \frac{\kappa \Delta t}{(\Delta x)^2}, \quad (\text{D.9})$$

Table D.1: Selected differential, variational, and discrete correspondences.

Pair or interface	Supported comparison	Unsecured property
Heat field / modal observation	Parameter recovery with specified Fourier mode and timing	Unique model under unknown forcing or spatial aliasing
Lagrangians L_0, L_F	Same interior Euler–Lagrange equation for fixed endpoints	Identical action values, boundary data, or global interpretation
Continuous / discrete oscillator	$O(h^2)$ consistency and conditional stability	Equality of finite-step trajectories and unrestricted control responses
Heat PDE / cellular update	Conservation, positivity, and restricted numerical stability	Exact continuum evolution or a uniquely recovered PDE

with periodic site indices. This equation can be read as a local weighted rewrite on a fixed one-dimensional cellulation, or as the update of a graph field whose edges encode permitted neighbor coupling. Its weights and incidence have specified numerical meanings, unlike the higher-order primitives of a generic relational two-complex. This is a chosen discretization interface, not evidence that every relational system has a physical diffusion interpretation.

For $0 \leq r \leq 1/2$ the update is a convex combination and preserves nonnegativity. Summing Equation D.9 over the periodic grid gives $\sum_j u_j^{n+1} = \sum_j u_j^n$, so total grid mass is conserved exactly. For a spatial Fourier mode $e^{i\xi j}$, the amplification factor is $1 - 4r \sin^2(\xi/2)$, whose magnitude is at most one for $0 \leq r \leq 1/2$. These statements establish restricted positivity, conservation, and linear stability properties of the numerical generator. They do not imply exact agreement with the continuum heat flow, preservation of every continuum observable, or faithful recovery of the original physical mechanism from the discretized outputs.

Moreover, although the harmonic oscillator has the regular mechanical action in Equation D.4, the dissipative heat equation should not be treated as if it automatically possessed an analogous regular action depending solely on the diffusion field. Extensions using auxiliary variables or other variational constructions require additional modeling choices. A typed translation interface must retain such choices as part of the represented model rather than silently transferring mechanical variational semantics to a dissipative update.

D.6 Translation Obligations and Reproducibility

The constructions in this appendix can be compared through explicit, limited obligations. Table D.1 distinguishes agreement at observed samples from consistency of operators, variational equivalence, and preservation of selected future responses. Each row states a relation to be justified, not an equivalence available automatically for any two models.

An executable symbolic and numerical verifier is provided in `experiments/check_appendix_d.py`; it writes machine-readable results to `experiments/appendix_d_results.json`. Its checks cover Euler–Lagrange gauge invariance, discrete variational derivatives, the Fourier aliasing construction, grid-mass conservation, and numerical agreement of the oscillator recurrence with the derived formula. The illustrations are generated from deterministic analytic or finite-difference models and are not empirical measurements. The two propositions invoke hypotheses that permit exact proof, while the numerical checks are independent consistency tests rather than their logical justification.

Taken together, the appendix supplies concrete instances of a heterogeneous generative interface. A continuous law, a discrete update, and an action principle can become mutually informative under specified observation maps, approximations, and boundary conditions. Their agreement is local to the supported queries and conditions. This is the appropriate mathematical reading of translation for the limited examples considered here: historically obtained evidence can constrain what may be inferred and compared, while the future consequences of a model remain dependent on its admissible interventions, its representational grammar, and the evidence available to revise it.

E Cross-Representation Correspondences

This appendix specifies limited mathematical obligations for a translation between generative representations and examines their behavior under continuation, composition, approximation, and historical revision. Its role is to refine the contextual translation witnesses of Section 7 using operational examples with independently testable properties. First, finite labeled transition systems make trace preservation and branching preservation distinguishable. Second, an explicitly typed composition result identifies when compatible translation witnesses can be chained. Third, an elementary error estimate supplies a bounded comparison for approximate dynamical translation. Finally, versioned witnesses illustrate how learning or rule evolution can invalidate a formerly adequate correspondence. These constructions draw on established simulation, bisimulation, and model-checking traditions [27, 17]; their contribution here is a disciplined interface to the paper’s historically indexed epistemic and generative questions, rather than a new general theory of process equivalence.

E.1 Typed Operational Interfaces and Evidence Scope

Fix two representation versions M_A^v and M_B^w , a comparison context c , and a finite observation horizon n . For a discrete operational illustration, associate to each version a labeled transition structure $\mathcal{L}_i = (S_i, \Sigma_i, \rightarrow_i, s_i^0, O_i)$. Here S_i is a state set, Σ_i an action or production alphabet, $\rightarrow_i \subseteq S_i \times \Sigma_i \times S_i$ a transition relation, s_i^0 a designated initial state, and O_i an observation interface. States may encode generated cellular complexes, program configurations, or selected variables of an otherwise richer model. Such encodings depend on the stated abstraction. In particular, a state in S_i need not be an event vertex of the relational-spacetime reference structure.

A *contextual operational witness* specifies a partial relation $R \subseteq S_A \times S_B$, an admissible label relation $Q \subseteq \Sigma_A \times \Sigma_B$, selected domains $D_i \subseteq S_i$, and provenance Π . An initial pair and a designated query must be recorded before a preservation statement is made. Equation E.1 states the one-step forward obligation on its declared domain.

$$(x, y) \in R, x \xrightarrow{a}_A x', x \in D_A \implies \exists b, y' [(a, b) \in Q, y \xrightarrow{b}_B y', (x', y') \in R], \quad (\text{E.1})$$

where the target transition is required to be admissible in the selected context. For a partial interface, Equation E.1 concerns only source steps within the explicitly declared domain; the domain should be closed under the steps being tested if the statement is intended to extend to paths. Matching the observations $O_A(x)$ and $O_B(y)$ requires a separate relation or numerical comparison of their values. A forward witness provides a translation of selected source possibilities into target possibilities. A reciprocal obligation, checked with the directions exchanged and the same pair relation, provides a stronger branching-sensitive criterion.

The provenance Π records the versions of both model descriptions, the evidential basis for the relation, the allowed queries, the rule applicability assumptions, the comparison horizon, and any failures. A witness inferred from two matching observational histories has a different warrant from one established by an exhaustive transition test. In normative or interpretive settings, the meanings of states and actions require additional participant-based and institutional assessment; the formal relation by itself establishes only the stated operational property.

E.2 Trace Agreement and Branching-Sensitive Translation

For a transition system $\mathcal{L}$ and starting state s , write $\text{Tr}_{\leq n}(\mathcal{L}, s)$ for the action words realizable by paths of at most n steps, including the empty word. Trace comparison is a natural first test when the archive records action sequences but does not directly preserve the intermediate branching state. However, an

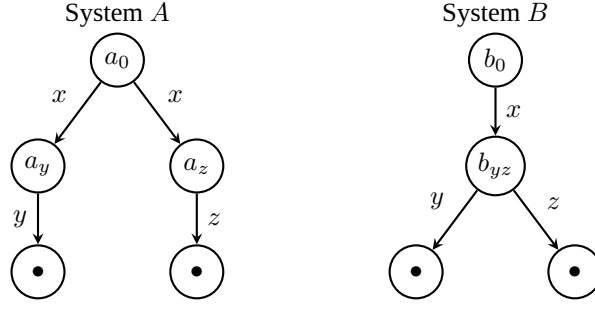


Figure E.1: Two finite generators with identical action-trace languages and different branching structures. System A resolves the choice at its x -transition, while system B retains both outgoing options after x . The displayed models support a mathematical statement about operational structure and make no direct claim about the metaphysical openness of a person's choices.

agreement of trace languages can omit distinctions relevant to choice, control, and future action. Equation E.2 records a particular trace-level adequacy claim.

$$\text{Tr}_{\leq n}(\mathcal{L}_A, s_A^0) = \text{Tr}_{\leq n}(\mathcal{L}_B, s_B^0) \quad \text{under a declared common action vocabulary.} \quad (\text{E.2})$$

Consider a source system A with two x -steps from a_0 to a_y and a_z . Only action y is available at a_y , and only action z at a_z . In a target system B , the sole x -step from b_0 reaches b_{yz} , where both y and z are available. The terminal states have no outgoing transitions. Both systems possess exactly the finite trace language $\{\varepsilon, x, xy, xz\}$, so Equation E.2 holds for any $n \geq 2$. Their post- x choice structures differ: one makes a branch during the x -transition, the other retains a subsequent local choice. These differences can matter for the possible intervention or deliberation that follows observing x .

Proposition E.1 (Trace equivalence without branching equivalence). *For the finite systems A and B just described, the initial states have equal action-trace languages, but there is no strong bisimulation relating a_0 and b_0 while preserving the labels x, y, z .*

Proof. Both systems generate precisely the empty trace and the words x , xy , and xz . Suppose a strong bisimulation R related a_0 and b_0 . Both transitions $a_0 \xrightarrow{x} a_y$ and $a_0 \xrightarrow{x} a_z$ must be matched by the sole x -transition from b_0 , so both (a_y, b_{yz}) and (a_z, b_{yz}) must belong to R . The state b_{yz} admits a z -transition whereas a_y does not, contradicting the reverse matching obligation for (a_y, b_{yz}) . Thus no such relation exists. $\square$

Proposition E.1 illustrates an elementary distinction established in process semantics, rather than a newly discovered impossibility. The two systems have matching completed traces while differing in the placement of a nondeterministic choice. A historical record containing only the action words would leave that branching feature unobserved. Additional transition-level evidence, or a permitted experiment in which the observer can intervene between the x -step and the next action, could discriminate the descriptions. Conversely, a restricted translation query concerning only the eventual occurrence of xy or xz may require no stronger witness than trace agreement. A diagrammatic comparison is provided in Figure E.1.

E.3 Compositional Correspondence and Intermediate Interfaces

Translation can pass through an intermediary model when its signatures and contexts are compatible. Fix three transition systems A, B, C with action alphabets $\Sigma_A, \Sigma_B, \Sigma_C$. To keep the result precise, let $q_{AB} : \Sigma_A \rightarrow \Sigma_B$ and $q_{BC} : \Sigma_B \rightarrow \Sigma_C$ be total label maps on the admitted transitions, and let $R_{AB} \subseteq S_A \times S_B$ and $R_{BC} \subseteq S_B \times S_C$ be forward simulations on step-closed domains. Their relational composition and action-map composition are defined in Equation E.3.

$$R_{AC} = R_{BC} \circ R_{AB} = \{(a, c) : \exists b ((a, b) \in R_{AB} \wedge (b, c) \in R_{BC})\}, \quad q_{AC} = q_{BC} \circ q_{AB}. \quad (\text{E.3})$$

Proposition E.2 (Composition of compatible forward simulations). Assume that R_{AB} is a forward simulation from A to B with label map q_{AB} and that R_{BC} is a forward simulation from B to C with label map q_{BC} . Suppose the chosen domains contain all intermediate states reached by the matching transitions. Then R_{AC} in Equation E.3 is a forward simulation with label map q_{AC} on the composed domain.

Proof. Take $(a, c) \in R_{AC}$ and an admitted step $a \xrightarrow{u}_A a'$. Choose b witnessing the relational composition. The first forward simulation supplies $b \xrightarrow{q_{AB}(u)}_B b'$ with $(a', b') \in R_{AB}$. The second supplies $c \xrightarrow{q_{BC}(q_{AB}(u))}_C c'$ with $(b', c') \in R_{BC}$. Hence $(a', c') \in R_{AC}$, and the label of the target step is $q_{AC}(u)$. $\square$

The result in Proposition E.2 is the familiar elementary composition property of simulations under compatible label translations. Its conditions matter. A purported bridge R_{BC} established only for some states of B may not cover the states used in R_{AB} ; incompatible observation units, label interpretations, authority constraints, and admissibility guards may also invalidate a composite contextual witness. A formally defined relational product can be computed even when no meaningful operational or epistemic translation is supported. In a grammar-to-PDE-to-institutional chain, representation-specific interfaces and justified query translations are therefore part of the data, rather than consequences of composition alone.

A second concern is loss under many-to-one intermediate coding. Suppose a deterministic interface f identifies two source features. Any further deterministic interface g continues to identify their images, as established in Proposition 7.3. Restoring the distinction requires independent evidence, richer intermediate data, or a revised context. Composition can also amplify uncertainty: when two bridges are supported only probabilistically, their uncertainty may be dependent because they draw on the same historical archive. The synthetic tests below use exact finite relations and thereby make none of the assumptions needed for a calibrated probabilistic chain.

E.4 Approximate Translation of Dynamical Representations

Discrete representations of continuous models frequently preserve selected trajectories only approximately. Fix normed state spaces X_A, X_B , deterministic maps $F_A : X_A \rightarrow X_A$ and $F_B : X_B \rightarrow X_B$, and a comparison interface $f : X_A \rightarrow X_B$. Let $\Omega \subseteq X_A$ be a forward-invariant domain for F_A , and assume F_B is L -Lipschitz on a region containing both the translated and target trajectories. The pointwise commutation defect is bounded in Equation E.4.

$$\sup_{x \in \Omega} \|f(F_A(x)) - F_B(f(x))\| \leq \varepsilon, \quad \|F_B(y) - F_B(z)\| \leq L\|y - z\|. \quad (\text{E.4})$$

The first inequality measures a selected one-step mismatch. It is an empirical or analytical assumption in an application, not a property secured by an arbitrary map f .

Proposition E.3 (Finite-horizon propagation of translation error). Let $x_{k+1} = F_A(x_k)$ and $y_{k+1} = F_B(y_k)$, with $x_k \in \Omega$ for $0 \leq k < n$. Under Equation E.4, define $e_k = \|f(x_k) - y_k\|$. Then Equation E.5 holds for the chosen trajectories.

$$e_n \leq L^n e_0 + \varepsilon \sum_{j=0}^{n-1} L^j, \quad n \geq 1. \quad (\text{E.5})$$

Proof. By the triangle inequality and Equation E.4, $e_{k+1} \leq \|f(F_A(x_k)) - F_B(f(x_k))\| + \|F_B(f(x_k)) - F_B(y_k)\| \leq \varepsilon + L e_k$. Induction yields Equation E.5. $\square$

For $0 \leq L < 1$, the accumulated defect is bounded by $\varepsilon/(1 - L)$ as n increases. For $L > 1$, a small local defect may grow across a long continuation. The proposition is an elementary error estimate and applies only within the stated state domains and Lipschitz conditions. It provides no universal criterion

for semantic interpretation or normative translation. In particular, replacing a continuous generator by an approximation introduces numerical and observational conventions that must be tracked as part of the witness; Appendix D.5 examines selected conservation and stability properties for a diffusion update.

An analytic check uses $F_A(x) = 0.85x + 0.12$, $F_B(y) = 0.85y + 0.10$, and $f(x) = x$. The defect is 0.02 and $L = 0.85$. Identical initial values yield an observed error equal to the bound in Equation E.5 at each step. The example confirms the calculation for one controlled linear system; the number 0.02 measures a known parameter mismatch, not an estimate of uncertainty derived from empirical data. If the target grammar changes to $F_B^{\text{new}}(y) = 0.75y + 0.10$, the original L , defect, and validation domain must be reconsidered. The previously recorded certificate should remain historically accessible while its present operational acceptance is reassessed.

E.5 Historical Evolution of Translation Witnesses

A generative representation can change through parameter drift, alteration of admissible operations, revised observations, or reinterpretation of its categories. A useful versioned translation record therefore includes a pair of model identifiers, their temporal applicability, the epistemic agents responsible for the interpretation, and the tests performed on the corresponding rules. Let $\Theta_{AB}^{v,w,c}$ denote an accepted witness between versions v and w . Its continued use after model changes requires a fresh domain-specific validity check. Equation E.6 makes the dependency explicit.

$$\text{Valid}(\Theta_{AB}^{v,w,c}; M_A^{v'}, M_B^{w'}, c') \in \{\text{supported, refuted, undetermined}\}. \quad (\text{E.6})$$

The three-valued assessment in Equation E.6 is a possible reporting convention for a defined battery of tests. It is not a universal truth predicate. A formerly accurate bridge may fail if a new action is introduced, if a grammar's guards are revised, or if a new community interpretation changes the practical force of an institutional term. A bridge may also remain valid for a restricted sublanguage even when the full representation has evolved. Revisions can preserve both the old certificate and its counterexample, thereby documenting the historical process of translation rather than silently overwriting earlier judgments.

A version change may belong to the investigated system or to the epistemic observer. Altered data recording, a newly available sensor, or a different categorization of actions can change observed traces while the underlying practice remains stable. The observational and inferential limitations discussed in Sections 6 and 10 therefore apply equally to inferred translations. Structural match, predictive adequacy, participant recognition, and normative authorization remain distinct forms of warrant. A new correspondence produced through interaction can itself affect future action by making formerly unavailable communication or cooperation practical; this is a possible generative continuation, whose presence requires evidence about the actual participating systems.

E.6 Finite Verification Protocol and Research Scope

The reproducible examples of this appendix are implemented in `experiments/check_appendix_e.py`, with machine-readable output in `experiments/appendix_e_results.json`. The script enumerates the finite trace languages of Proposition E.1, computes the greatest strong bisimulation by iterative elimination of inconsistent state pairs, and verifies forward simulations and their compositions. It also evaluates the linear dynamical error bound from Proposition E.3 and records a change in the local translation defect under a versioned target map. An independent algebraic proof, rather than a finite computation, establishes each proposition's general claim.

Table E.1 summarizes the hierarchy of obligations illustrated in Appendix E. Their distinct strengths matter for knowledge pressing because a proposed correspondence can be adequate for one historical or future-directed query while being insufficient for another.

These limited formal results illuminate a boundary between translating outputs, translating operational possibilities, and learning another system's historically situated generative grammar. A translation

Table E.1: Restricted translation obligations and their remaining epistemic limitations.

Test	Established in the declared domain	Left open
Trace agreement	Equality of selected observable action words	Branching and intervention structure
Forward simulation	Matching of supported source continuations	Extra target behaviors and reciprocal preservation
Bisimulation	Reciprocal single-step matching of specified actions	Full interpretive, causal, or normative equivalence
Composed simulation	Preservation across compatible typed interfaces	Independently warranted semantic acceptance
Approximate commutation	Bounded finite-horizon deviation under stated constants	Bounds outside the domain or after untested rule change
Versioned assessment	Explicit test outcome and provenance for a model pair	Unique or globally authoritative understanding

may be useful without being exhaustive. It may be exact for a finite transition abstraction, approximate for numerical dynamics, or contested for a social practice. The inferential record should specify which kind of adequacy has been established, which possibilities have been suppressed by an observation interface, and how continued interaction may change the relevant grammars. Such disciplined plurality is compatible with knowledge pressing as a process that participates in relational continuation while remaining epistemically revisable and institutionally accountable.

F Identifiability Counterexamples and Technical Verification

This appendix examines identification limits that arise when an epistemic agent infers generative representations from a historically situated observation record. It complements the conditional results in Section 10 and the translation-specific tests in Appendix E. Its emphasis is on the *scope of available inquiries*: a record may identify some properties while leaving other properties, causal responses, model versions, and prospective continuations undetermined. The counterexamples are constructed from finite-state generators, sampled continuous dynamics, simple structural causal models, and changing observation interfaces. Their mathematical ingredients are established in system identification, experimental design, and causal inference [34, 11, 31]. They serve as explicit restrictions on claims made about knowledge pressing, rather than original impossibility theorems about arbitrary relational systems.

F.1 Query-Indexed Observational Signatures

A historical record obtains its inferential force from an observation protocol, a time horizon, an admissible model family, and an inquiry of interest. Let $\mathfrak{M}$ be a declared family of generative models, each with an observation process, and let $\mathcal{Q}$ be a collection of admissible experimental or observational regimes. A regime q specifies what is measured, the times at which measurements are taken, and whether selected interventions are available. For model M and regime q , denote the induced law of the observed history through time n by $P_M^{q,n}$. The *query-indexed observational signature* and its equivalence class are

$$S_{\mathcal{Q},n}(M) = (P_M^{q,n})_{q \in \mathcal{Q}}, \quad [M]_{\mathcal{Q},n} = \{N \in \mathfrak{M} : S_{\mathcal{Q},n}(N) = S_{\mathcal{Q},n}(M)\}. \quad (\text{F.1})$$

For deterministic models, $P_M^{q,n}$ may be a point mass on the output sequence, and signature equality reduces to equality of these sequences. This construction is model-class relative. Its equality is exact, whereas finite empirical data normally support only statistical comparison with explicit uncertainty.

A model query $\psi : \mathfrak{M} \rightarrow \mathcal{V}$ is identifiable relative to $(\mathcal{Q}, n)$ at M precisely when it is constant on $[M]_{\mathcal{Q},n}$. This is the fiber criterion discussed in Proposition 10.1; specifying the query is essential. For example, observing $y(t) = \theta^2 t$ under the restricted family $\theta \in \{-2, 2\}$ identifies θ^2 but not the sign of θ . A successful inference of the former does not justify reporting the latter. Similarly, even when candidate models have different internal state labels, their disagreement may be irrelevant to the particular

observable or intervention being investigated. The legitimate unit of a claim is thus an identified *property* under a declared protocol, rather than necessarily an entire generative representation.

F.2 Arbitrarily Long but Finite Historical Agreement

Even an error-free historical record of considerable length cannot in general determine the full continuation behavior of an unrestricted model family. The following finite construction isolates the missing assumption without invoking noisy observations or approximate inference.

Proposition F.1 (Finite-horizon agreement with divergent next observations). *For every integer $n \geq 0$, there exist two finite deterministic one-action transition systems whose observed histories agree at each time $0, 1, \dots, n$, but whose observations differ at time $n + 1$.*

Proof. For each system take the states $\{0, 1, \dots, n + 1\}$, initial state 0, and the sole action a . Define $i \xrightarrow{a} i + 1$ for $i < n + 1$, and let $n + 1$ be absorbing under a . The first observation function assigns 0 to every state. The second observation function assigns 0 to $0, \dots, n$ and 1 to $n + 1$. The unique paths visit state k at time k through $n + 1$, so their first $n + 1$ observations agree and the observations at time $n + 1$ disagree. Both structures are finite and deterministic, establishing the claim. $\square$

The construction varies the model with n ; it does not assert that any fixed, finitely specified process will remain unidentified indefinitely. Known bounds on state counts, transition structure, observability, or allowed grammars may restrict such ambiguities. The example shows what a finite historical record alone cannot establish without those restrictions. It is especially relevant when an inferred practical grammar gains a new action only after the recorded period. A claim about future continuity must therefore specify which aspects of the evolving generative rule space are being held fixed and which are permitted to change.

F.3 Continuous-Time Aliasing under Historical Sampling

A second ambiguity arises when the observation interface samples a continuous dynamical process at regular time intervals. Consider a planar oscillator represented as a trajectory on the unit circle, with angular frequency $\omega > 0$ and known initial phase:

$$x_\omega(t) = (\cos(\omega t), \sin(\omega t)), \quad t_k = k\Delta, \quad \omega' = \omega + \frac{2\pi m}{\Delta}, \quad m \in \mathbb{Z} \setminus \{0\}, \quad (\text{F.2})$$

where only positive ω' are admitted.

Proposition F.2 (Temporal sampling aliases distinct generators). *For positive frequencies ω and ω' related by Equation F.2, the sampled observations of x_ω and $x_{\omega'}$ agree at every $t_k = k\Delta$. If $\omega' \neq \omega$, the continuous trajectories disagree at some off-grid times.*

Proof. At a sampled time the phase difference is $(\omega' - \omega)k\Delta = 2\pi mk$, so both sine and cosine agree. If the continuous paths agreed for all t in a neighborhood of the origin, differentiating their sine coordinates at $t = 0$ would give $\omega = \omega'$, contradicting $m \neq 0$. $\square$

Unlike Proposition F.1, this ambiguity can persist for an arbitrarily long *sampled* history under a fixed observation schedule. It is not the same as the spatial Fourier aliasing of Appendix D, although the sampling mechanism is analogous. A known frequency band narrower than the relevant aliasing scale, an independently calibrated velocity, or additional noncommensurate observation times can distinguish the candidates. Figure F.1 depicts the different paths between identical recorded sample locations. This is a statement about sampled dynamics, not about the fundamental nature of time or ontological spacetime.

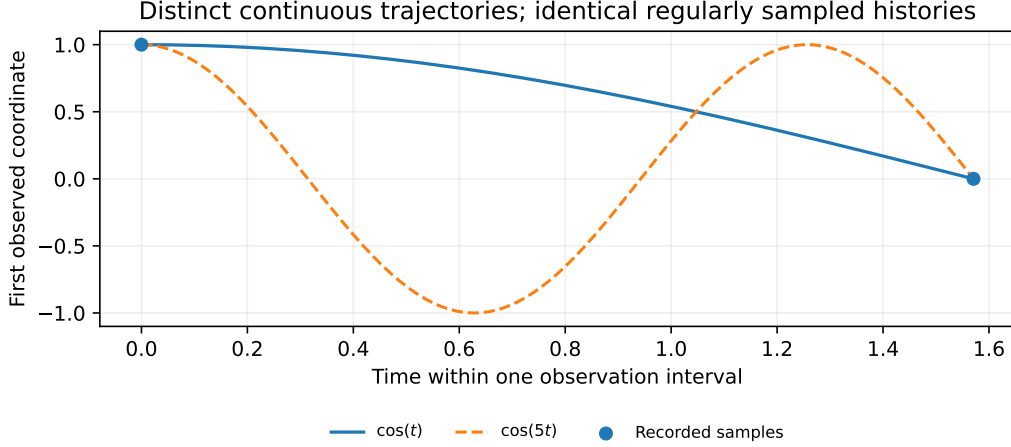


Figure F.1: Two continuous trajectories with $\omega = 1$ and $\omega' = 5$ sampled at $\Delta = \pi/2$. Both models agree at the recorded sample times, but their intermediate planar positions differ. Lines show the first observation interval; markers indicate shared sampled positions. The visual discrepancy vanishes at all integer sampling times by construction.

F.4 Observational Agreement and Distinct Intervention Responses

Historical observations can agree while interventions yield different results. Let $U \sim \text{Bernoulli}(1/2)$ and consider two acyclic structural causal models, with the same observable variables X and Y :

$$\begin{aligned} M_A : X &:= U, & Y &:= X, \\ M_B : X &:= U, & Y &:= U. \end{aligned} \quad P_{M_A}(X = x, Y = y) = P_{M_B}(X = x, Y = y). \quad (\text{F.3})$$

The shared observational law assigns probability $1/2$ to each of $(X, Y) = (0, 0)$ and $(1, 1)$. The models nevertheless encode different causal roles for X : in M_A it is a parent of Y , whereas in M_B it only shares an exogenous cause with Y .

Proposition F.3 (Observational equality does not identify this intervention query). *For the two models in Equation F.3, the observational distributions of (X, Y) are identical, but*

$$P_{M_A}(Y = 1 \mid \text{do}(X = 0)) = 0, \quad P_{M_B}(Y = 1 \mid \text{do}(X = 0)) = \frac{1}{2}. \quad (\text{F.4})$$

Proof. Before intervention, both models imply $X = Y = U$, proving equality of the observational laws. In the structural intervention $\text{do}(X = 0)$, replace the structural assignment to X by the constant 0 while leaving the other assignments intact. Model M_A then gives $Y = X = 0$ surely. In M_B , the equation $Y = U$ is unchanged, so $Y = 1$ with probability $1/2$. $\square$

The intervention rule is part of the stipulated structural causal semantics [31, 32]; it is not inferred from the shared joint distribution. The example thus identifies a limitation and a potential remedy. New evidence from a valid intervention, an externally justified causal restriction, or another identified source of information may distinguish the mechanisms. Yet social and clinical interventions are often constrained by ethics, safety, law, feasibility, and participant agency. The capacity to imagine an intervention mathematically does not establish permission to perform it. Even an ethically permitted intervention may change the participating institutions, subjects, or observation interface, requiring its results to be interpreted within the resulting history.

F.5 Ambiguity between Generative Change and Epistemic Change

A historical change in recorded observations need not originate in the investigated system. For any designated change time $\tau \geq 1$, define a deterministic binary observed series $y_t = 0$ for $t < \tau$ and

$y_t = 1$ for $t \geq \tau$. It admits at least two elementary reconstructions. In a *generator-change* explanation, a latent state z_t switches from 0 to 1 at τ while the observation map remains $O(z) = z$. In an *observer-change* explanation, the latent state remains $z_t = 0$ while a time-dependent observation map changes from $O_t(z) = z$ to $O_t(z) = 1 - z$ at τ . The two accounts satisfy

$$y_t = O(z_t) = O_t(0) = \mathbf{1}_{\{t \geq \tau\}}, \quad t = 0, 1, 2, \dots \quad (\text{F.5})$$

The equality concerns the chosen observation channel. It is not an assertion that the two systems share all internal causal processes. A separately calibrated sensor, an independently recorded change in classification practice, or an invariant measurement can discriminate these constructions. Without such additional evidence, a change-point detector applied only to y_t can identify when the record changes but cannot establish whether the change reflects a generative transition, an altered recording instrument, or a revised interpretive convention. This distinction matters in journalistic archives, scientific instrumentation, institutional reporting, and AI-assisted historical reconstruction. The observer has a history, and its changing epistemic grammar is itself relevant evidence.

F.6 Identification of Continuation and Practical Possibility

The inferential examples above bear directly on model-indexed future continuation. Fix a historical record D , a candidate model family $\mathfrak{M}(D)$, and a query specifying an observational regime, an action or intervention policy π , and a future horizon h . Let $\mathcal{C}_h(M, \pi)$ denote the set of modeled continuation traces of length h for model M , under the declared action semantics. Then define the model-supported and shared continuation sets by

$$\mathcal{C}_h^\exists(D, \pi) = \bigcup_{M \in \mathfrak{M}(D)} \mathcal{C}_h(M, \pi), \quad \mathcal{C}_h^\forall(D, \pi) = \bigcap_{M \in \mathfrak{M}(D)} \mathcal{C}_h(M, \pi). \quad (\text{F.6})$$

As in Section 8, these sets express two distinct epistemic warrants. The union enumerates possibilities supported by at least one retained model; the intersection reports continuation traces admitted by every retained model under the same comparison interface and fixed policy semantics. They need not be exhaustive of physically realizable futures, and membership in either set does not confer a moral or political entitlement to pursue an action.

For the binary causal models above, the common observational history supports the same observed joint law, while the intervention $\text{do}(X = 0)$ yields distinct distributions for Y . Consequently, the intervention-conditioned future query is not identified by the observational record alone. For the finite-horizon construction, the next observable differs even without a policy change. When a grammar evolves, a new action may appear after the recorded period, so even the common *type* of the admissible action space may become uncertain. In that case, defining the intersection in Equation F.6 requires a declared translation or embedding of continuation traces into a common comparison language. Such a bridge is not automatically provided by a global record repository.

A practical agent may further face feasibility constraints due to resources, care obligations, coercion, or collective decision procedures. These belong to action-conditioned accessibility and cannot be read directly from the mathematical membership relation in Equation F.6. Likewise, the presence of two model-supported branches establishes neither ontological indeterminism nor free will. Our framework can examine how revised knowledge and new inter-system translations change *represented* and *practically accessible* options; claims about metaphysical freedom and normative authority require further assumptions and argument.

F.7 Reproducibility, Discriminating Evidence, and Claim Boundaries

The independent verification program `experiments/check_appendix_f.py` constructs the deterministic horizon models, checks temporal-sampling equality and off-grid divergence, exactly enumerates the binary structural causal models, and compares the two drift explanations. Its results are written to

Table F.1: Illustrative sources of non-identifiability and conditional discriminating evidence.

Ambiguity	Identical evidence under restriction	Potential additional warrant
Finite-horizon generator	Observed prefixes through time n	Longer history; verified bounds on generator class
Temporal aliasing	Exact regularly sampled states	Off-grid samples; trusted frequency range
Causal structure	Shared joint law $P(X, Y)$	Valid intervention; defended causal assumptions
Observer versus generator drift	Same recorded change-point series	Calibrated independent observation channel
Modal continuation	Common past, distinct modeled futures	Admissible policy probes; qualified model constraints

experiments/appendix_f_results.json; it also generates Figure F.1. The program checks the particular constructions, while the proofs establish the stated claims for their restricted model families. A successful numerical test of a counterexample does not demonstrate a universal impossibility result for every representational class.

Table F.1 collects potential distinguishing evidence and highlights its corresponding assumptions. The table does not propose that the listed interventions are uniformly available, safe, or authoritative.

A global record and epistemic infrastructure can enlarge accessible evidence and support increasingly sophisticated inference. The examples in this appendix show why even well-maintained records must preserve their observation conditions, model assumptions, and uncertainty. Inference can identify meaningful query-specific properties without uniquely recovering a complete generative law; translation can preserve selected consequences without transporting all future possibilities; and continuation can remain underdetermined even when the observed history is accurately reconstructed. These are constraints to be represented and investigated within knowledge pressing, rather than grounds for abandoning generative inquiry. The epistemic benefit lies partly in learning what is justified, what remains unresolved, and which additional, permissible forms of interaction could improve understanding.

G Computational Reproducibility and Verification

This appendix specifies the reproducibility contract for the controlled investigations in Section 11 and Appendices B–F. Its objective is to make the relation between executable assumptions, generated observations, verification procedures, and manuscript claims inspectable. The account separates exact mathematical checks, floating-point demonstrations, and interpretations of constructed models. It also documents the dependencies and limitations of the experiments, so that an independent reader can regenerate the reported artifacts and assess which claims the executions support. The experiments concern deliberately bounded mathematical systems. Their successful execution provides neither empirical confirmation of a worldwide epistemic infrastructure nor validation of general semantic translation between heterogeneous societies.

G.1 Verification Scope and Evidential Categories

Reproduction begins with a distinction among four evidential operations. A *formal argument* establishes a conditional mathematical statement within a declared domain, while an *exact computational check* verifies a finite instance through symbolic algebra, integer or rational arithmetic, or exhaustive enumeration. A *numerical check* evaluates a specified computation with recorded approximations and tolerances. An *empirical validation* would test a representation against independently acquired observations, credible uncertainty models, and appropriate domain expertise. The present executable suite principally comprises the middle two categories. Proposition proofs in the main manuscript and mathematical appendices remain arguments to be assessed independently of the programs implementing their

illustrative examples.

A verification record may be represented by the typed collection in Equation G.1.

$$\mathcal{V} = (S, \Theta, D, P, E, O, C, \Pi), \quad (\text{G.1})$$

where S identifies the versioned source files, Θ the stipulated mathematical assumptions and model parameters, D the input observations or generated datasets, P the executable procedure, E the software and execution environment, O the resulting observations or derived artifacts, C the assertions and comparison conventions, and Π the provenance binding these elements. These are roles in a research record, not a proposed universal ontology for scientific evidence. The present suite records synthetic input-generating procedures within P and Θ , because external datasets are absent. The elements of Equation G.1 are distributed among the scripts, JSON outputs, figures, this appendix, and the consolidated audit report.

The operative claim attached to a passing check is deliberately narrow: for the recorded environment and specified inputs, a program completed and its declared assertions held. A proof may establish a stronger conditional statement when its assumptions and reasoning are correct. Neither kind of verification entails that the chosen generative representation adequately describes an unobserved real-world system. That inferential step depends on independent evidence and the application-specific validity conditions developed in Sections 5 and 10.

G.2 Source Organization and Artifact Dependencies

The source archive retains an editable XeLaTeX master document, its individual mathematical sections and appendices, scholarly references with a source-verification ledger, and a dedicated `experiments/` directory. The primary synthetic investigation is executed by `experiments/run_all.py`. Appendix-specific programs are named `check_appendix_b.py` through `check_appendix_f.py`. Each program writes a machine-readable JSON record. Procedures producing illustrations also write vector PDF figures into `figures/`. The consolidated runner, `experiments/check_reproducibility.py`, invokes the six programs, checks a selected invariant registry, compares their regenerated JSON records with the archived versions, and writes `experiments/reproducibility_report.json`.

The artifact dependency relation has a directed interpretation: a figure depends on an executable, its parameters, and the software environment; a numerical statement in the manuscript depends on specified entries in an output record; and the strength of the resulting inference depends additionally on whether the construction's assumptions are justified for the investigated system. Source hashes support identification of executable artifacts and output files, while their agreement alone cannot authenticate the mathematical or social interpretation. Changes to the simulation rules, observation maps, or translation criteria create a new experimental configuration, even when filenames remain unchanged. A comparable change to a generative system's observed grammar can therefore require a new translation or continuation validation, as discussed in Section 7 and Appendix E.

G.3 Execution Environment and Reconstruction Procedure

All six computational checks use local synthetic data and require no data download or remote service. The Python suite uses NumPy, SciPy, Matplotlib, and SymPy, while the document build uses XeLaTeX and Biber. Executions described here used the invocation commands shown below, from the manuscript directory. The first command generates the individual numerical and exact-check records and constructs the consolidated audit report; the second builds the typeset manuscript and bibliography.

```
python experiments/check_reproducibility.py
make
```

The consolidated runner sets a noninteractive Matplotlib backend and a fixed Python hash seed for each subprocess. The primary stochastic example additionally uses an explicit NumPy generator seed of 20261021. Platform and package versions are serialized into the audit JSON record and are preferable to

Table G.1: Reproducible synthetic investigations, evidential mode, and the principal assertions checked by the consolidated runner. The registry links executable cases to limited inferential questions.

Suite	Representation and evidential mode	Illustrative validation target
Main, Section 11	Graph rule changes, periodic heat field, finite stochastic transition systems, accessibility; simulation and selected symbolic check	Fixed change-point conditions, parameter estimates under stated noise, exact relabeling, trace-branching distinction, reachability counts
Appendix B	Oriented cellular boundary matrices; exact rational ranks	Boundary consistency, absolute and relative Betti numbers, different intermediate filtrations
Appendix C	Typed cellular productions; exact finite incidence	Endpoint equality and historical rule-order ambiguity
Appendix D	Heat fields, variational oscillator, periodic diffusion; symbolic and numerical	Gauge equality, sampling aliases, discrete recurrence, restricted mass and positivity properties
Appendix E	Labeled transition systems and affine maps; finite enumeration and numerical bounds	Trace agreement without bisimulation, forward-simulation composition, error accumulation, witness revision
Appendix F	Finite automata, sampled oscillators, structural causal models; constructive and numerical	Finite-horizon divergence, temporal aliasing, intervention dependence, observer-generator ambiguity

a timeless claim that one package combination guarantees identical floating-point results. Execution in a new environment should be evaluated with the stated numerical tolerances and error diagnostics. The report retains a SHA-256 digest for each executable, generated JSON result, and archived experiment figure at the end of the run. Because typesetting and graphics backends can embed implementation-dependent data in PDFs, equal binary hashes of vector figures are not a general requirement for scientific agreement.

The requirements for independent regeneration differ among checks. Symbolic expressions and rational-matrix ranks are intended for exact verification. Synthetic time-series simulations use numerical differentiation or regression and consequently require documented floating-point tolerances. The finite transition-system checks test labeled traces and explicitly enumerated candidate relations. Conventions for orientation, state labels, initial conditions, and allowed transitions remain part of the experimental definition. Replacing them without documentation changes the problem being verified.

G.4 Experimental Registry and Independent Assertions

Table G.1 summarizes the six executable families and the scientific question to which each is restricted. A script can implement several logically different checks. In particular, the main suite reports performance of selected estimators under noise, whereas the subsequent appendix checkers give finite counterexamples and exact identities under more explicitly controlled conditions.

The consolidated suite checks twenty-one explicit invariants spanning these categories, in addition to the checks embedded within each script. They include the Betti-number sequences of Appendix B, the Lagrangian-gauge and sampling checks of Appendix D, the finite branching distinction of Appendix E, and the intervention counterexample of Appendix F. The audit runner performs its selected comparisons against regenerated machine-readable results. It does not constitute a second implementation of all six computational algorithms. Agreement among a script, its assertions, and a manuscript description may still reflect a shared modeling error, a mistaken definition, or an omitted relevant alternative. Independent mathematical scrutiny remains necessary.

Table G.2: Outcome of the consolidated local reproducibility audit for the manuscript’s synthetic cases. Counts refer to registered executable suites and assertions, rather than empirical sample sizes or generality of the proposed theory.

Audit dimension	Result and interpretation
Execution coverage	6 of 6 suites completed without a subprocess failure
Invariant registry	21 of 21 declared checks satisfied
Archived JSON comparison	6 of 6 regenerated files structurally compatible under Equation G.2
Source and artifact lineage	SHA-256 digests recorded for executable scripts, generated JSON records, and figure PDFs
External empirical data	None; all observed histories are stipulated or synthetically generated
Generalization status	Experimental validity is limited to the specified cases and model assumptions

G.5 Exact, Numerical, and Statistical Comparison Conventions

Different comparison notions are used deliberately. Integer cell counts, combinatorial incidence, finite reachable sets, and rational symbolic expressions admit exact equality tests under the stated representation. Lagrangian equivalence is checked through simplification of the Euler–Lagrange expressions with the fixed-endpoint convention specified in Appendix D. Matrix intertwinings, sampled field coincidences, recurrence residuals, and finite-horizon error estimates are evaluated numerically, with magnitudes and expected discrepancies retained in the corresponding output records. The regeneration comparison for floating-point JSON entries uses the condition in Equation G.2.

$$|x - y| \leq \varepsilon_{\text{abs}} + \varepsilon_{\text{rel}} \max(|x|, |y|), \quad \varepsilon_{\text{abs}} = 10^{-10}, \quad \varepsilon_{\text{rel}} = 10^{-9}. \quad (\text{G.2})$$

The implementation uses Python’s `math.isclose`, which corresponds to the comparison in Equation G.2 for finite values. JSON keys, sequence lengths, data types, Boolean assertions, and strings are compared exactly. The tolerances are audit choices for repeat execution of these numerical examples; they are neither uncertainty intervals for physical parameters nor thresholds for acceptable translation among social systems. Individual mathematical checks also enforce problem-specific limits, such as near-zero sample discrepancy alongside deliberately nonzero differences at unsampled locations.

The original main experiment includes repeated random corruption of graph events and noisy samples of a periodic heat field. Its reported estimates are conditional on the specified random seed, observation grid, synthetic noise distributions, candidate rule-change family, and regression operators. The results are neither confidence statements for an external population nor a performance benchmark for unrestricted scientific discovery. The finite examples in Appendices B–F mostly concern exact identities, explicit counterexamples, or known-model numerical consistency. Distinguishing these modes prevents the attractive precision of an output file from obscuring the epistemic role of the underlying construction.

G.6 Consolidated Audit and Regeneration Results

The consolidated runner was executed for this manuscript version with all six programs available. Each subprocess terminated successfully, each of the twenty-one registered invariants passed, and every regenerated JSON result agreed with the corresponding archived record under the explicit structural and numerical comparison rules. Table G.2 records the scope of this execution. The complete environment, script digests, output digests, and subprocess diagnostics are stored in the source archive’s `reproducibility_report.json`; the hashes allow comparison against future revisions. The present execution confirms internal procedural repeatability for the installed environment, while leaving independent algorithmic and empirical validation as separate tasks.

The outcomes in Table G.2 are evidence that an independent researcher can attempt the same computations using the supplied source archive. They do not identify an underlying ontological relational spacetime. In particular, the script verifying two observationally equivalent models with divergent interventions checks the exact construction of Appendix F; it does not supply evidence that a specified

real-world system possesses either causal structure. Likewise, a computed simulation relation certifies selected operational correspondences inside the finite model, rather than the meaning, institutional authorization, or normative legitimacy of translation in an external practice.

G.7 Failure Modes, Revision Triggers, and Epistemic Accountability

Reproducibility records require versioning because the subject of investigation and the epistemic apparatus may evolve. A change in a source generator, its state space, an observation operator, a model-selection restriction, a translation witness, or an action-feasibility constraint can alter the supported claims without changing the surface appearance of recorded outputs. When these components change, the relevant assertions and interpretations should be rerun or re-examined. The history of revision should preserve superseded results and their evidential context where ethical and institutional conditions allow. The proposed infrastructure of Section 12 would treat such events as inspectable revisions of an evidential claim, rather than silent replacements of a supposedly timeless model.

Three classes of failure deserve separate treatment. An *execution failure* means that the program could not complete in the specified environment. A *verification failure* means that an asserted mathematical or numerical property did not hold for the current inputs and conventions. An *epistemic failure* means that an internally successful construction is used to support a stronger claim than its assumptions or evidence warrant. The third can persist even when the first two are absent. Provenance, hashes, and repeatable test runs mitigate ambiguity about what was computed, while scrutiny of sampling decisions, representational alternatives, consent, and domain-specific meaning remains necessary for interpreting what was learned.

The experimental suite was not designed to test arbitrary new model families, large-scale AI systems, culturally situated normative grammars, or deployed public infrastructure. It also does not establish that generative possibility sets identify metaphysical freedom, that different communities must accept a proposed translation, or that epistemic improvement produces a desirable historical continuation. These questions require further domain knowledge and plural evaluation practices. The provenance design is thus compatible with revisable claims, contestation, and the separation of scientific reproducibility from institutional legitimacy.

G.8 Research Reuse and Release Conditions

A reader wishing to reuse the experiments should preserve the model definitions, initial conditions, observation schedules, seed choices, state and action labels, comparison tolerances, and software environment in a new execution record. The existing programs may be modified to test alternative hypotheses, but a modified experiment should be assigned a distinguishable version and reported together with any newly obtained negative results. Independent checks that use different algorithms or data sources are particularly valuable, because they can expose assumptions shared by the present scripts and proofs. A more substantial validation program would add heterogeneous real-world records, domain-expert interpretation, uncertainty calibration, and lawful mechanisms for participant correction and disagreement.

This appendix completes the planned computational documentation for the exploratory manuscript. The formal results, synthetic demonstrations, and infrastructure proposal are intended to remain individually inspectable and revisable. Their relationship exemplifies one narrow aspect of knowledge pressing: the preservation of an epistemic trajectory connecting situated evidence, generative reconstruction, translation, and qualified continuation. Further scholarly review, independent verification, and empirical studies are required before claiming that the proposed framework yields practically reliable cross-system understanding.

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