

Historical Trust Formation in Repeated Interaction: A Preliminary Dynamical Formalism

Wanhong HUANG
huangwanhong@serendip.ngo

Abstract

This paper develops a preliminary formal architecture for trust formation over repeated interaction histories. Trust is treated as a declared relational state whose carrier may be scalar, vector-valued, graph-valued, latent, probabilistic, or otherwise structured. Rather than proposing another universal or parametric trust-update equation, the paper asks a prior representational question. The central problem is historical compression: when do the represented present trust state and current interaction record contain enough information for future trust evolution, and when must a model retain additional history, latent structure, or conditional uncertainty? The paper defines typed trust states and pre-update interaction records, gives deterministic and stochastic factorization criteria for state sufficiency, and develops diagnostics for explicit memory, hysteretic repair, basin dependence, power-conditioned feasibility, process composition, and observational uncertainty. One recurring structural consequence is that invertibility of an interaction-trajectory descriptor does not entail invertibility of the induced trust process. Repair may satisfy a declared target without restoring the pre-breach state, and state-level process composition itself requires feasibility and congruence conditions beyond one-step update factorization. The framework therefore separates trajectory algebra from trust-process semantics and representational adequacy from causal, normative, and ontological interpretation. An empirical bridge specification links the formal diagnostics to measurement, event coding, support, approximation, and uncertainty requirements. The contribution is methodological and architectural rather than a claim of novelty for Markov sufficiency, stochastic kernels, basin concepts, partial transformations, or recursive trust updating individually: it supplies conditions for locating where historically relevant information resides, distinguishing several mechanisms that can produce similar observations, and stating which stronger empirical claims require additional evidence.

Keywords: trust dynamics; historical compression; repeated interaction; hysteretic repair; process semantics

Discussion Paper Note

This manuscript is the fifth paper in a cumulative research series on trajectory-sensitive social description, historical reconstruction, processual normativity, and interaction-history representation. Paper I developed braid theory as one lower-level descriptor of realized trajectories rather than a generative ontology. Paper II inserted historical evidence and set-valued reconstruction between realized history and trajectory description. Paper III developed a processual-justice architecture in which a declared normative evaluation determines which historical distinctions must survive compression. Paper IV developed a meta-level formalism for normatively structured interaction histories, partial composition, continuation equivalence, and relational-state sufficiency [9, 7, 10, 8].

The present paper changes the research object. It asks how trust itself may form, persist, deteriorate, and recover through repeated interaction. Trust is not treated as another normative label attached to the interaction history. It is represented as a potentially history-dependent relational state whose evolution may preserve memory that is absent from a topological or normative summary of the same interaction sequence.

The formal objective is deliberately pluralist. Trust may be modeled differently across empirical and theoretical domains, and the paper does not assume a universal scalar measure, a universal Markov property, or a unique mechanism of repair. The proposed machinery is intended to state clearly which assumptions are required by particular dynamical representations and where irreversible or non-Markovian behavior requires moving beyond group-valued trajectory description.

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Notices

Status. This manuscript is a preprint and discussion paper presenting a complete preliminary formal construction of historical trust dynamics in repeated interaction. It is intended for public scholarly discussion and may be revised as empirical applications, comparative tests, and the wider trajectory-sensitive interaction-history research programme develop. No universal trust law is asserted.

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Statement on the use of language models. OpenAI’s ChatGPT was used for exploratory dialogue, formal reconstruction, source discovery followed by source verification, argumentative criticism, and drafting in $\text{\LaTeX}$. The author selected the research questions, directed and approved the theoretical commitments and epistemic status of the claims, verified the cited sources, and bears sole responsibility for the manuscript, including its definitions, formal constructions, arguments, conclusions, and errors. Verification records for cited external sources are maintained in the accompanying source ledger.

1 Introduction

Trust in repeated interaction is often intelligible only through history. Reliance after a long sequence of dependable conduct differs from reliance formed before substantial interaction; an otherwise similar breach can have different consequences early and late in a relationship; and a successful apology, compensation, or subsequent performance can repair a relation without returning it to its pre-breach condition. The analytical object of interest is therefore a relational state generated through an ordered interaction history, together with the question of which parts of that history remain relevant to later trust evolution.

Existing trust research supports this historical problem while leaving the substantive ontology open. Cross-disciplinary work emphasizes vulnerability, risk, and interdependence [21]; organizational models distinguish trust from perceived trustworthiness, risk taking, and behavioral consequences [16]; and trust and distrust may require more than a single scalar axis [13]. Temporal evidence also motivates explicit attention to sequence. Initial trust can depend strongly on early relationship conditions [18]; early breaches may have more persistent effects than later breaches in some repeated-interaction settings [14]; and repair after deception can remain incomplete despite promises, apologies, and later trustworthy conduct [22]. These findings justify modeling timing, sequence, violation type, and prior history as candidate dynamical variables without establishing a universal law of trust hysteresis.

The paper also enters an established literature on dynamic trust modeling. Human–automation and human–robot research has represented trust with recursive difference equations, cumulative-history terms, Bayesian updating, probabilistic states, and related state-space constructions, and recent review work surveys a broad family of such mathematical models [6, 4, 20]. The present paper therefore does not claim novelty for treating trust as time-varying, for using recursive or probabilistic updates, or for asking whether a present state summarizes prior interaction. Its question is one level more architectural: given a declared trust representation, which historical distinctions are retained, which are lost, and what additional mathematical structure is required when the chosen representation fails?

A parallel caution applies to the language of path dependence. Historical sociology and political science have long emphasized that sequence, timing, contingency, self-reinforcement, and difficult reversibility require more precise treatment than the generic statement that “history matters” [15, 19]. The present formalism does not use *path dependence* as an umbrella label for every historical effect. It separates at least five possibilities that can otherwise be conflated: insufficient present-state coding, explicit retained memory, basin dependence under a fixed continuation rule, restricted repair reachability, and differences in full-history-conditioned stochastic laws. This narrower diagnostic use is intended to complement rather than replace substantive path-dependence theories.

The paper asks a narrower formal question: what dynamical structure is required when trust formation depends on repeated interaction history? It extends the interaction-history architecture developed in the preceding paper, which distinguishes histories, typed interaction information, partial composition, continuation structure, and relational-state representations [8]. Trust adds a state-specific requirement. A history representation adequate for a normative or descriptive inquiry may still compress distinctions relevant to expectations, vulnerability, memory, or future reliance. Conversely, histories that remain distinct for another inquiry may converge to the same represented trust state. Trust therefore requires a declared state space, equivalence structure, event interface, and prediction target of its own.

Let $\mathcal{T}_c$ denote a trust-state space declared for context c . The carrier need not be scalar, symmetric, continuous, or directly observable. A minimal local update has the form stated in Equation 1.

$$T_{t+1} = F_c(T_t, e_t, z_t), \quad (1)$$

where $T_t \in \mathcal{T}_c$, e_t is the represented interaction occurrence, and z_t collects independently declared contextual or role information. Equation 1 is a factorization claim: it is appropriate only when the selected present state and input record are sufficient for the chosen next-state target. A history-sensitive alternative is stated in Equation 2.

$$T_t = \Phi_c(h_{0:t}, T_0, C_{0:t}), \quad (2)$$

where $h_{0:t}$ is the interaction-history prefix and $C_{0:t}$ is its contextual history. The central compression problem is to determine when the information in Equation 2 can be represented adequately by a smaller state-record interface of the form in Equation 1. Failure of that compression may call for a richer current state, a finer record, explicit memory, latent structure, or a stochastic target; the failure alone does not identify which mechanism is appropriate.

A second problem concerns reversibility. Trajectory languages such as braid groups can contain algebraic inverses, while trust repair can remain historically marked. If a breach process b induces K_b and a later repair process r induces K_r , the formalism must be able to represent the non-restoring case in Equation 3.

$$K_r(K_b(T)) \neq T. \quad (3)$$

Equation 3 specifies a possible relation between repair and restoration rather than an empirical universal. The same separation applies to a trajectory descriptor with an inverse. A topologically or syntactically inverse process p^{-1} need not induce a semantic inverse on trust, as expressed in Equation 4.

$$K_{p^{-1}} \circ K_p \neq \text{id}_{\mathcal{T}_c}. \quad (4)$$

The paper therefore treats trajectory composition and trust-process composition as related but distinct structures. Depending on the declared representation, trust evolution may require partial transformations, relations, stochastic kernels, path categories, or explicit history dependence rather than a group action by automorphisms.

Three further issues follow from this separation. First, early interactions may alter later regime membership under a declared dynamics; basin dependence provides one formal language for such timing effects, while stronger claims about primacy or criticality require additional evidence [18, 14]. Second, institutional position, role obligations, material dependence, unequal exit options, and other power-relevant conditions can restrict feasible interaction, observation, repair, and continuation paths. This emphasis is compatible with relational power-dependence traditions in which alternatives and dependence matter to the power relation [3]; the paper nevertheless leaves substantive theories of power to the application domain. It represents those constraints separately from the trust state rather than treating power as an unexplained scalar modifier. Third, residual variation after historical conditioning may reflect unresolved history, latent heterogeneity, observation error, or genuinely non-degenerate conditional transition laws. A stochastic extension must therefore preserve mechanism-identification discipline rather than use probability as a generic label for unexplained deterministic defects.

The formalism also keeps trust distinct from neighboring constructs. Trustworthiness concerns properties or judgments attributed to the party trusted; reliance and cooperation concern behavior; distrust may be a distinct relational attitude; and justice, legitimacy, or welfare introduce evaluative criteria of a different type. These objects may interact, but observable cooperation can persist under dependence or coercion, and trust can change without an immediate behavioral response. Keeping the types separate is necessary for interpreting both deterministic and stochastic results.

Within the broader series, Paper I supplied conditional trajectory descriptors, Paper II addressed history reconstruction under evidential uncertainty, Paper III formalized theory-relative historical compression for normative evaluation, and Paper IV developed typed interaction histories, partial composition, continuation equivalence, and generic relational-state sufficiency [9, 7, 10, 8]. The present paper specializes that architecture to historical trust formation. The mathematical ingredients used in that specialization—factorization, equivalence classes, partial maps, relations, stochastic kernels, and basin terminology—are not presented as new mathematical primitives. The contribution lies in their typed assembly into a trust-specific diagnostic architecture and in the explicit separation of claims that are often allowed to travel together: state sufficiency from memory, repair from restoration, feasibility from response, realized variation from law-level variation, and prediction from causal or normative interpretation. Its main tasks are to define admissible trust-state and interaction-record representations; test when history can be compressed into them; characterize explicit memory, repair hysteresis, basin dependence, power-conditioned feasibility, and process composition; extend the framework to conditional probability

laws; and state the empirical operationalization, identification, and scope conditions under which those constructions are informative.

The contribution is methodological and formal. It does not select one universal substantive theory of trust. Instead, it provides criteria for stating what historical information a model retains, detecting when a chosen compression loses transition-relevant distinctions, identifying which additional structures are needed for stronger process claims, and separating representational adequacy from causal, normative, and ontological interpretation. Sections 2–5 establish the state, record, and factorization architecture. Sections 6–8 develop memory, repair, and basin dependence. Sections 9–11 add constrained feasibility, process semantics, and stochastic evolution. Section 12 supplies empirical operationalization and states the interpretive limits of the formalism, Section 13 synthesizes the resulting research programme, and Appendix A provides finite diagnostic constructions without interrupting the main argumentative sequence.

2 Conceptual Architecture of Historical Trust

This section establishes the conceptual interface used by the later dynamics. Its objective is to specify what counts as a represented *trust state*, which neighboring constructs remain separately typed, and which contextual assumptions must be declared before a transition law is meaningful. Because psychological, organizational, institutional, computational, and philosophical accounts operationalize trust differently, the section adopts a pluralist modeling architecture informed by the distinctions among vulnerability, interdependence, perceived trustworthiness, risk taking, and behavioral consequences [21, 16].

2.1 Analytical Role of a Trust State

Let c denote a declared empirical or theoretical context. A *trust state space* is a modeling object

$$\mathfrak{T}_c, \tag{5}$$

whose elements represent the trust-relevant relational condition retained by the analysis at a given historical position. Equation 5 does not require $\mathfrak{T}_c$ to be numerical, one-dimensional, continuous, symmetric, or directly observable. Depending on the domain, an element of $\mathfrak{T}_c$ may be scalar, vector-valued, graph-valued, set-valued, probabilistic, latent, or otherwise structured.

The term *state* is used here in a representational rather than ontological sense. A state is whatever retained object the declared model uses to summarize trust-relevant historical information at a specified resolution. Whether such a summary is sufficient for subsequent trust evolution is a later question, not part of Definition 2.2. Carrier pluralism, however, does not make the framework construct-free. The same mathematical carrier could represent confidence, dependence, reputation, legitimacy, affect, or another relational variable; calling it a trust state therefore requires a substantive anchor supplied by the application.

Research Obligation 2.1 (Substantive anchoring of the trust carrier). An application using $\mathfrak{T}_c$ as a trust-state carrier should declare the trustor, trustee, and relevant domain when those distinctions matter; state the substantive interpretation that warrants the label *trust*; and identify the observational, elicitation, or theoretical bridge by which represented states are connected to that interpretation. Mathematical adequacy of a state-transition model alone does not establish that the modeled variable is trust rather than a neighboring construct.

Obligation 2.1 is intentionally compatible with more than one substantive theory. It rules out only an unanchored relabeling of a generic state variable as trust. The empirical bridge in Section 12 later sharpens the measurement and identification requirements when the state is estimated from data.

Definition 2.2 (Trust-state representation). For a declared context c and an admissible interaction-history domain Γ_c^{tr} , a trust-state representation is a map

$$\Theta_c^{\text{tr}} : \Gamma_c^{\text{tr}} \longrightarrow \mathfrak{T}_c. \quad (6)$$

The value $\Theta_c^{\text{tr}}(h)$ is the represented trust state after history h at the resolution adopted by the model. Equation 6 records this history-to-state map.

Definition 2.2 intentionally allows different histories to map to the same represented state. Such compression may be justified, or it may discard memory needed for future trust evolution. Sections 5 and 6 test that distinction explicitly rather than building it into the definition.

2.2 Directed and Domain-Typed Trust Relations

Many trust judgments are directed: a trustor places trust in a trustee. They are also often domain-sensitive: confidence regarding one task, role, claim, resource, or vulnerability need not extend to another. To retain these distinctions without requiring a universal decomposition, let

$$\mathcal{A}_c \quad (7)$$

be the declared set of participant bearers relevant to context c , as recorded in Equation 7, and let

$$\mathcal{D}_c^{\text{tr}} \quad (8)$$

be the declared set of trust domains used by the inquiry. Equation 8 records that domain carrier. A *trust coordinate* is then indexed by a triple

$$(x, y, d) \in \mathcal{A}_c \times \mathcal{A}_c \times \mathcal{D}_c^{\text{tr}}, \quad x \neq y, \quad (9)$$

where x is the trustor, y the trustee, and d the declared domain.

Equation 9 does not imply that every trust model decomposes into independent coordinates. It identifies the minimum typing information that a model must retain when direction and domain matter. In particular, neither symmetry nor cross-domain invariance is presumed:

$$T_{x \rightarrow y}^d \not\equiv T_{y \rightarrow x}^d, \quad T_{x \rightarrow y}^{d_1} \not\equiv T_{x \rightarrow y}^{d_2}. \quad (10)$$

Equation 10 is a modeling caution, not an empirical claim that the corresponding states must differ in every case. A particular domain may justify equality, symmetry, or aggregation, but those properties require separate assumptions.

A relation can also involve collective, institutional, or role-bearing participants rather than only natural persons. The participant carrier should therefore be declared by the substantive inquiry. If an office, organization, team, or other collective is treated as a trust-bearing or trust-target object, the model must state what historical continuity licenses that treatment. This preserves the bearer/role/office distinction developed in the preceding interaction-history framework rather than silently converting institutional roles into person-level trust variables [8].

2.3 Trust, Trustworthiness, Reliance, Cooperation, and Distrust

The dynamical formalism requires several neighboring concepts to remain distinct because they occupy different places in a causal or representational architecture. Organizational trust models distinguish perceived trustworthiness from trust and distinguish both from risk-taking behavior [16]. Cross-disciplinary work likewise treats trust as connected to vulnerability and interdependence rather than as identical to an observed cooperative act [21]. For the present paper, these distinctions are represented by typed objects rather than by a single vocabulary-dependent definition.

Let

$$W_c(h; x, y, d) \in \mathfrak{W}_c \quad (11)$$

denote whatever trustworthiness-relevant assessment the declared model attributes to trustee y from the position of trustor x in domain d after history h . The codomain $\mathfrak{W}_c$ may encode ability-, benevolence-, integrity-, reliability-, competence-, or other domain-specific considerations, but Equation 11 does not require any fixed universal list.

Let

$$R_c(h; x, y, d) \in \mathfrak{R}_c^{\text{act}} \quad (12)$$

represent an observed or reconstructed reliance-related action. Equation 12 is intentionally action-typed rather than state-typed. Let

$$C_c(h; x, y, d) \in \mathfrak{C}_c^{\text{coop}} \quad (13)$$

represent a cooperation-related record when cooperation is analytically relevant. The formalism does not identify the values in Equations 11–13 with the trust state $\Theta_c^{\text{tr}}(h)$. High assessed trustworthiness may fail to produce reliance under severe external risk; reliance may occur under coercion, lack of alternatives, contract, or strategic calculation; and observable cooperation can arise without the relational attitude represented by the chosen trust model.

Distrust requires an additional caution. Some theories model distrust as the low end of a single trust continuum, whereas other work argues that trust and distrust may coexist because the underlying expectations are multidimensional [13]. The present framework therefore allows, but does not require, separate trust and distrust coordinates. If a domain treats them separately, the state space may contain a product or coupled structure such as

$$\mathfrak{T}_c \subseteq \mathfrak{T}_c^+ \times \mathfrak{T}_c^- \times \mathfrak{Z}_c, \quad (14)$$

where $\mathfrak{T}_c^+$ carries trust-relevant components, $\mathfrak{T}_c^-$ carries distrust-relevant components, and $\mathfrak{Z}_c$ collects any additional relational variables retained by the model. Equation 14 is illustrative rather than canonical.

The analytical separation can be summarized by

$$\text{trust state} \not\equiv \text{trustworthiness assessment} \not\equiv \text{reliance action} \not\equiv \text{cooperation record}. \quad (15)$$

Equation 15 should be read as a typing rule for the model, not as a denial of causal or evidential relations among the quantities.

2.4 Vulnerability, Risk, and Exposure Conditions

Trust becomes analytically important in situations where another party’s conduct matters to the trustor under some form of vulnerability or interdependence. Both cross-disciplinary and organizational accounts emphasize this structure [21, 16]. The present framework records it through an exposure condition rather than building a particular psychological definition into the state space.

For a trust coordinate (x, y, d) and history h , let

$$E_c(h; x, y, d) \in \mathfrak{E}_c \quad (16)$$

represent the declared exposure configuration. Depending on the substantive domain, it may encode stakes, dependence, available alternatives, reversibility, monitoring possibilities, institutional protections, or the potential consequences of trustee action. Equation 16 therefore provides context for interpreting both trust-related states and reliance behavior.

The model should not infer trust merely from exposure or risk. A participant can be exposed without trusting, and can avoid exposure despite substantial trust because the opportunity for reliance never arises. Nor should the model infer the magnitude of trust from the magnitude of material risk without an explicit substantive argument. Formally, no monotone relation

$$E_c(h; x, y, d) \mapsto T_{x \rightarrow y}^d \quad (17)$$

is assumed by default. Equation 17 leaves open domain-specific relations among exposure, trust, and action.

Table 1: Conceptual boundaries for the trust-dynamics formalism.

Object	Retained analytical role	Default inference excluded
Trust state	History-conditioned relational condition retained by the model	Scalarity, symmetry, direct observability
Trustworthiness	Assessment or property attributed to the trustee under a declared account	Identity with trust state or reliance
Reliance	Action exposing the trustor to trustee-dependent consequences	Proof of trust
Cooperation	Coordinated or mutually adjusted behavior	Proof of trust or justice
Distrust	Negative or guarded relational component when separately modeled	Mere arithmetic complement of trust
Exposure / risk	Stakes, dependence, alternatives, and vulnerability conditions	Monotone determination of trust magnitude
Initial condition	Pre-history or boundary condition for subsequent dynamics	Neutral or universally identical starting state

2.5 Historical Trust and Initial Conditions

The adjective *historical* in this paper does not mean that every past event must remain explicitly represented. It means that the present trust state is allowed to depend on an ordered interaction history and that the adequacy of any compression of that history must be demonstrated rather than presumed. For a history prefix $h_{0:t}$, the most permissive representation introduced in the Introduction is

$$T_t = \Phi_c(h_{0:t}, T_0, C_{0:t}), \quad (18)$$

where T_0 is an initial trust condition and $C_{0:t}$ collects declared contextual conditions. Equation 18 allows two relations exposed to similar recent events to differ because their earlier histories or initial conditions differ.

Initial trust deserves separate representation because a newly formed relation need not begin from a neutral or zero state. Prior category beliefs, institutional assurances, reputational information, roles, or other pre-interaction conditions may contribute to the starting representation. Research on initial trust in organizational relationships supplies one established motivation for treating T_0 as a substantive object rather than as an arbitrary numerical constant [18]. The present paper does not adopt a single theory of how initial trust is generated; it requires only that the assumptions determining T_0 be declared when they affect later dynamics.

Historical dependence also does not imply that older information must matter forever. A domain may justify forgetting, discounting, saturation, or aggregation. The later formal problem is to determine whether the resulting compressed state retains enough information for the intended predictions or explanations. Thus the contrast is not between “memory” and “no memory” in the abstract, but among different equivalence relations on histories induced by different state representations.

2.6 Conceptual Packet for the Dynamical Formalism

The preceding distinctions can be collected into a context-specific modeling packet

$$\mathfrak{M}_c^{\text{tr}} = (\Gamma_c^{\text{tr}}, \mathcal{A}_c, \mathcal{D}_c^{\text{tr}}, \mathcal{T}_c, \mathfrak{W}_c, \mathfrak{R}_c^{\text{act}}, \mathfrak{C}_c^{\text{coop}}, \mathfrak{E}_c). \quad (19)$$

Equation 19 does not assert that every application needs every component or that the displayed tuple is ontologically fundamental. It is a bookkeeping device for separating the objects that later equations might otherwise conflate.

Table 1 records the corresponding formalization boundaries.

Equation 19 therefore serves as a declaration interface: an application should state which components it represents and how they are related before specifying trust dynamics. A numerical score is one possible specialization, while Section 3 develops the broader typed state carriers used below.

3 Typed Relational Trust States

This section specifies the internal structure available to a trust-state representation before any update law is imposed. Its objective is to construct carriers that can preserve direction, domain specificity, multidimensionality, participant structure, and partial observability without presuming a scalar geometry or universal metric. The method is representational: typed carriers induce explicit notions of state equivalence, readout resolution, and informational refinement that later sections use to state dynamical sufficiency.

3.1 Directed Trust Coordinates and Typed State Carriers

Let $\mathcal{A}_c$ denote the ambient participant-bearing set declared for context c , and let $\mathcal{D}_c^{\text{tr}}$ denote the set of trust domains introduced in Section 2. A directed trust coordinate is a triple

$$q = (x, y, d) \in \mathcal{Q}_c^{\text{tr}} \subseteq \{(x, y, d) \in \mathcal{A}_c \times \mathcal{A}_c \times \mathcal{D}_c^{\text{tr}} : x \neq y\}, \quad (20)$$

where inclusion in $\mathcal{Q}_c^{\text{tr}}$ indicates that the domain declares the corresponding trust relation meaningful. Equation 20 permits a model to exclude pairs or domains for which a trust coordinate would be substantively inappropriate.

A fixed coordinate set need not be active throughout every history. For $h \in \Gamma_c^{\text{tr}}$, let

$$\mathcal{Q}_c(h) \subseteq \mathcal{Q}_c^{\text{tr}} \quad (21)$$

be the coordinates active or interpretable after h . Equation 21 permits entry, exit, role-dependent availability, or domain-specific activation without identifying participant persistence with trust-coordinate persistence.

For each $q \in \mathcal{Q}_c^{\text{tr}}$, let $\mathfrak{T}_{c,q}$ be a declared local carrier for the trust-relevant state associated with q . The carriers need not be mutually isomorphic. A scalar confidence level, a vector of differentiated expectations, a probability distribution, an interval, or a finite qualitative state may therefore be used at one coordinate without forcing the same representation at another.

The most permissive typed state carrier used in this paper is the partial-section space

$$\mathfrak{T}_c^{\text{sec}} := \left\{ \tau : \text{Dom}(\tau) \longrightarrow \coprod_{q \in \mathcal{Q}_c^{\text{tr}}} \mathfrak{T}_{c,q} : \text{Dom}(\tau) \subseteq \mathcal{Q}_c^{\text{tr}}, \tau(q) \in \mathfrak{T}_{c,q} \right\}, \quad (22)$$

where the disjoint union keeps the local types visible. A concrete trust-state space is then any declared subset

$$\mathfrak{T}_c \subseteq \mathfrak{T}_c^{\text{sec}} \quad (23)$$

whose additional constraints encode the domain's admissible joint states. Equations 22–23 provide an ambient representation only; they do not assert that social trust literally consists of mathematical sections.

The history-to-state representation introduced in Equation 6 can now be required to respect active typing:

$$\text{Dom}(\Theta_c^{\text{tr}}(h)) = \mathcal{Q}_c(h) \quad \text{for all } h \in \Gamma_c^{\text{tr}} \quad (24)$$

whenever the application treats active-coordinate support itself as part of the represented state. If support is analytically irrelevant or separately encoded, Equation 24 may be weakened accordingly.

3.2 State-Space Coupling across Relations and Domains

Directed trust coordinates can interact. Trust in one person or domain may constrain, enable, or co-vary with trust elsewhere, while other applications may reasonably approximate coordinates as independent. The state carrier should therefore make coupling an explicit modeling choice rather than building independence into a Cartesian product by default.

For $q \in \mathcal{Q}_c^{\text{tr}}$, define the coordinate projection on states containing q by

$$\pi_q : \mathfrak{T}_c(q) \longrightarrow \mathfrak{T}_{c,q}, \quad \pi_q(\tau) = \tau(q), \quad (25)$$

where

$$\mathfrak{T}_c(q) := \{\tau \in \mathfrak{T}_c : q \in \text{Dom}(\tau)\}. \quad (26)$$

Equations 25–26 keep the single-coordinate projection well typed when support varies across states.

For two coordinates q and q' , let

$$\mathfrak{T}_c(q, q') := \{\tau \in \mathfrak{T}_c : q, q' \in \text{Dom}(\tau)\} \quad (27)$$

Equation 27 defines the states on which both coordinates coexist. On that domain, define the paired projection

$$\pi_{q,q'} : \mathfrak{T}_c(q, q') \longrightarrow \mathfrak{T}_{c,q} \times \mathfrak{T}_{c,q'}, \quad \pi_{q,q'}(\tau) = (\tau(q), \tau(q')). \quad (28)$$

For coordinates with $\mathfrak{T}_c(q, q') \neq \emptyset$, call q and q' *state-space decoupled* relative to $\mathfrak{T}_c$ when

$$\pi_{q,q'}(\mathfrak{T}_c(q, q')) = \pi_q(\mathfrak{T}_c(q, q')) \times \pi_{q'}(\mathfrak{T}_c(q, q')). \quad (29)$$

Failure of Equation 29 indicates a static admissibility coupling: some coordinate combinations allowed separately do not occur jointly in the declared state space.

Definition 3.1 (State-space coupling). For $q, q' \in \mathcal{Q}_c^{\text{tr}}$, write

$$q \bowtie_c^{\text{st}} q' \quad (30)$$

when Equation 29 fails. Equation 30 records this coupling relation.

Equation 28 supplies the joint readout used in Equation 29. Definition 3.1 captures only coupling encoded in the admissible state carrier. It does not yet assert that the *dynamics* of q and q' are coupled. Dynamic coupling belongs to the update analysis of Section 5. Conversely, absence of state-space coupling does not guarantee dynamical independence.

The distinction matters for multidomain trust. A model may permit all combinations of professional and personal trust states while still specifying updates in one domain that affect the other. Another model may encode structural constraints directly in $\mathfrak{T}_c$, for example because certain combinations of trust and distrust are ruled out by the chosen measurement or latent-state model. The present formalism leaves both possibilities available.

3.3 Geometries, Orders, and Readout Structures

A state carrier becomes geometrically informative only after additional structure is declared. Nothing in Equations 22–23 supplies a canonical metric, linear structure, topology, total order, or probability model. Those structures may be useful, but each answers a different analytical question.

Let Λ_c index a declared family of state readouts. For each $\lambda \in \Lambda_c$, let

$$\varphi_{c,\lambda} : \mathfrak{T}_c \rightharpoonup \mathfrak{V}_{c,\lambda} \quad (31)$$

be a partial map to a readout space $\mathfrak{V}_{c,\lambda}$. Equation 31 can represent a scalar summary, a qualitative classification, a network statistic, a coordinate-specific estimate, or any other declared feature. Partiality permits a readout to be meaningful only on a subset of trust states.

Table 2: Illustrative trust-state geometries and the assumptions they add.

State architecture	Typical analytical use	Additional assumptions requiring justification
Scalar	Compact ordering or regression target	One-dimensional comparability; chosen scale meaning
Vector-valued	Multiple trust-relevant dimensions	Coordinate semantics; aggregation or comparison rule
Directed network	Simultaneous pairwise/domain-typed relations	Node/edge persistence; network semantics
Partial typed section	Variable participants, domains, or heterogeneous coordinate carriers	Support interpretation; coordinate correspondence
Probability distribution / latent state	Uncertain or indirectly observed trust state	Observation model; probabilistic semantics
Set-valued state	Partial identification or interval-type uncertainty	Set interpretation; update rule for sets

When comparative ordering is substantively justified, a readout space may carry a preorder

$$\preceq_{c,\lambda} \subseteq \mathfrak{V}_{c,\lambda} \times \mathfrak{V}_{c,\lambda}. \quad (32)$$

Equation 32 need not be total. Two states may therefore be incomparable with respect to a declared trust feature, especially when one relation combines greater positive trust on one dimension with greater distrust or vulnerability on another.

Similarly, a metric

$$d_c^{\text{tr}} : \mathfrak{T}_c \times \mathfrak{T}_c \longrightarrow \mathbb{R}_{\geq 0} \quad (33)$$

may be introduced only when the application can justify what the resulting distance means. Equation 33 is therefore optional. Later language concerning “nearby states,” “basins,” or “critical regions” will require either such a metric, a topology, an order-theoretic neighborhood structure, or another explicitly declared replacement.

Table 2 summarizes several admissible specializations of $\mathfrak{T}_c$ without treating any one as canonical.

The table records representational choices rather than a hierarchy of sophistication. A scalar state can be adequate for a narrowly defined empirical task, while a high-dimensional state can be analytically excessive if the intended inference requires only a coarse distinction.

3.4 Partial Observability and Observation Equivalence

Trust states are frequently inferred from reports, behavior, institutional records, or experimental choices rather than directly observed. The formalism therefore distinguishes the represented trust state from its observation layer. Let

$$O_{c,\mu} : \mathfrak{T}_c \rightharpoonup \mathfrak{V}_{c,\mu} \quad (34)$$

be an observation map for channel μ in a declared index set $\mathcal{M}_c$. The channel may correspond to survey responses, reliance behavior, cooperation records, third-party reports, physiological measurements, or another admissible observation mechanism. Field research has operationalized distinct affect- and cognition-based interpersonal trust dimensions [17], while experimental economics has used investment decisions to study trust and reciprocity [1]. Those examples illustrate different observation channels rather than interchangeable measurements. In particular, behavioral transfers can combine trust with other-regarding preferences unless the design supplies additional identification structure [2]. Equation 34 therefore does not assert that any one observable is trust itself.

For a family $M \subseteq \mathcal{M}_c$ whose channels are jointly defined at states T and T' , define observation equivalence by

$$T \sim_{O_M} T' \iff O_{c,\mu}(T) = O_{c,\mu}(T') \text{ for every } \mu \in M. \quad (35)$$

Equation 35 separates equality of the latent or represented trust state from observational indistinguishability. Unless the combined observation family is injective on the domain of interest,

$$T \sim_{O_M} T' \not\Rightarrow T = T'. \quad (36)$$

Equation 36 is particularly important when cooperation or reliance is used as an empirical proxy: identical behavior may be compatible with different trust states because incentives, coercion, alternatives, or exposure conditions differ.

The same observation structure induces an equivalence relation on histories through the state representation. For histories $h, g \in \Gamma_c^{\text{tr}}$ on which the relevant maps are defined, let

$$h \sim_{O_M \circ \Theta} g \iff \Theta_c^{\text{tr}}(h) \sim_{O_M} \Theta_c^{\text{tr}}(g). \quad (37)$$

Equation 37 makes explicit that observational compression can be coarser than the trust-state compression itself.

3.5 State Equivalence and Informational Refinement

Every trust-state representation identifies histories that receive the same represented state. Define

$$h \sim_{\Theta} g \iff \Theta_c^{\text{tr}}(h) = \Theta_c^{\text{tr}}(g). \quad (38)$$

The relation in Equation 38 is a representational equivalence on history. It does not imply that h and g are historically identical, that they have the same moral status, or that they will necessarily generate the same future trust dynamics. The last question is deferred to the state-sufficiency tests of Sections 5–6.

Different trust-state representations can retain different amounts of historical information. Let

$$\Theta_1 : \Gamma \rightarrow \mathfrak{T}_1, \quad \Theta_2 : \Gamma \rightarrow \mathfrak{T}_2 \quad (39)$$

be two representations of the same history domain. Write

$$\Theta_1 \succeq_{\text{info}} \Theta_2 \quad (40)$$

when the information preorder in Equation 40 holds, meaning that Θ_2 factors through Θ_1 on their realized images: there exists

$$r_{21} : \text{Im}(\Theta_1) \longrightarrow \text{Im}(\Theta_2) \quad \text{such that} \quad \Theta_2 = r_{21} \circ \Theta_1. \quad (41)$$

Equation 41 means that every distinction retained by Θ_2 can be recovered from Θ_1 , although Θ_1 may retain additional distinctions. The relation $\succeq_{\text{info}}$ is generally a preorder on raw representation maps rather than a partial order, because two differently coded representations can factor through each other. Quotienting by mutual factorability yields the corresponding partial order on informational-equivalence classes.

Proposition 3.2 (Factorization and equivalence refinement). *For maps Θ_1 and Θ_2 in Equation 39, the following are equivalent:*

1. $\Theta_1 \succeq_{\text{info}} \Theta_2$;
2. the induced history equivalences satisfy

$$\sim_{\Theta_1} \subseteq \sim_{\Theta_2}. \quad (42)$$

Proof. If Equation 41 holds and $\Theta_1(h) = \Theta_1(g)$, applying r_{21} gives $\Theta_2(h) = \Theta_2(g)$, proving Equation 42. Conversely, assume Equation 42. Define r_{21} on $\text{Im}(\Theta_1)$ by $r_{21}(\Theta_1(h)) := \Theta_2(h)$. If $\Theta_1(h) = \Theta_1(g)$, then $h \sim_{\Theta_1} g$, hence $h \sim_{\Theta_2} g$ and $\Theta_2(h) = \Theta_2(g)$. The definition is therefore well defined and yields Equation 41. $\square$

Proposition 3.2 gives the paper a representation-independent language for comparing trust-state models. A high-dimensional state is not automatically better; it is merely more informative when it induces a finer equivalence relation. Whether the additional information is useful depends on the downstream task.

3.6 State Identity, Feature Identity, and Analytical Resolution

Exact state equality may be unnecessarily strong for many inquiries. For a readout $\varphi_{c,\lambda}$ from Equation 31, define feature equivalence on the domain of that readout by

$$T \sim_{\varphi_\lambda} T' \iff \varphi_{c,\lambda}(T) = \varphi_{c,\lambda}(T'). \quad (43)$$

Equation 43 allows a model to preserve, for example, the same vulnerability-sensitive trust class or the same domain-specific ordering while leaving other internal components unresolved.

For a family $L \subseteq \Lambda_c$, define the joint readout

$$\varphi_{c,L}(T) := (\varphi_{c,\lambda}(T))_{\lambda \in L} \quad (44)$$

whenever all selected readouts are defined. Equation 44 collects the selected feature resolution into one map. The resulting equivalence

$$T \sim_{\varphi_L} T' \iff \varphi_{c,L}(T) = \varphi_{c,L}(T') \quad (45)$$

in Equation 45 provides a declared analytical resolution coarser than exact state identity whenever $\varphi_{c,L}$ is non-injective.

The same logic applies to histories through composition with Θ_c^{tr} . Thus “same trust” requires a declared resolution: exact state identity, selected-feature equality, observational equivalence, or another task-specific equivalence. Sections 4–5 use those distinctions to define transition inputs and test dynamical sufficiency.

4 Trust-Relevant Interaction Records

This section defines the interaction information permitted to enter a trust transition. Its role is pre-dynamical: a record represents independently identifiable features of an occurrence, while the trust effect of that record remains a separate object for Section 5. The section therefore specifies record schemas, directional and domain projections, historical non-erasure, and refinement relations without encoding an outcome into the event label itself.

This separation prevents a simple circularity. Labels such as “trust increasing,” “trust destroying,” or “successfully reparative” already presuppose the transition that a dynamical model is meant to explain. By contrast, whether a commitment was made, whether a later performance satisfies a declared fulfilment rule, whether an explanation was offered, or whether a subsequent action satisfies a declared evidential criterion can in principle be identified independently of the resulting trust state. The formalism therefore treats trust-relevant records as inputs whose dynamical significance is not built into their names.

4.1 Event Occurrences and Trust-Relevance Support

Let $\mathcal{E}_c^*$ be a declared ambient carrier of event types or event tokens for context c . An event-decomposition map assigns to each trust history a finite or locally finite set of event carriers:

$$\mathcal{E}_c : \Gamma_c^{\text{tr}} \longrightarrow 2^{\mathcal{E}_c^*}, \quad h \longmapsto \mathcal{E}_c(h). \quad (46)$$

Equation 46 is representation-relative. An event carrier may correspond to an action, communication, institutional decision, observed outcome, disclosure, omission, or another analytically justified unit. The paper does not assume a unique natural event decomposition.

Define the occurrence domain

$$\mathcal{O}_c := \{(h, e) : h \in \Gamma_c^{\text{tr}}, e \in \mathcal{E}_c(h)\}. \quad (47)$$

Equation 47 supplies the common domain on which event-level record maps will be defined. For each occurrence, let the trust-relevance support be a map

$$\text{Aff}_c : \mathcal{O}_c \longrightarrow 2^{\mathcal{Q}_c^{\text{tr}}}, \quad \text{Aff}_c(h, e) \subseteq \mathcal{Q}_c(h). \quad (48)$$

Membership in Equation 48 means only that an occurrence is admissibly treated as potentially relevant to a directed/domain-specific trust coordinate. It does not imply a nonzero trust-state change.

A single occurrence may be relevant to several trust coordinates:

$$|\text{Aff}_c(h, e)| > 1 \quad (49)$$

may therefore hold. Equation 49 permits one occurrence to be relevant across multiple domains, to several implicated actors, or to separately tracked person-, role-, office-, or collective-level relations. No common direction or magnitude of later trust change is implied.

4.2 Pre-Update Record Schemas

Let $\mathcal{Z}_c^{\text{rec}}$ be a declared carrier of trust-relevant interaction records. A local record extractor is a partial map

$$\text{Rec}_c : \mathcal{O}_c \dashrightarrow \mathcal{Z}_c^{\text{rec}}. \quad (50)$$

Equation 50 permits the record of an occurrence to depend on independently identifiable historical context, including role, institutional, temporal, evidential, or exposure conditions. It may not be defined by consulting the trust transition that the record will later help explain.

Research Obligation 4.1 (Pre-update provenance). For every component of $\text{Rec}_c(h, e)$ used as an input to a trust-transition model, the component’s carrier, interpretation, and identification rule should be specifiable without using the resulting next trust state, the sign of the resulting trust change, or the eventual success of a later repair as an input to its definition.

Obligation 4.1 permits retrospective coding when the coding rule relies on independently available evidence. It excludes labels whose operational meaning is simply “whatever later lowered trust” or “whatever later restored trust.”

A useful heterogeneous record schema is

$$\text{Rec}_c(h, e) = (I_e, R_e, D_e, K_e, P_e, M_e, X_e, V_e, U_e), \quad (51)$$

where the displayed components are optional typed fields rather than mandatory coordinates of every application. In Equation 51, I_e records participant incidence and source/target structure; R_e records relevant role, office, or institutional-capacity information; D_e records the trust domain or domains; K_e records declared commitments or undertakings; P_e records observed performance or outcome information; M_e records messages, disclosures, explanations, promises, apologies, or other communicative acts; X_e records exposure conditions such as dependence, stakes, alternatives, monitoring, or reversibility; V_e records evidential or verification status; and U_e collects additional domain-specific context.

The tuple in Equation 51 is bookkeeping rather than ontology. A narrow application may need only a few fields, while another may replace them with a different typed family. For a selected feature family $S \subseteq \{I, R, D, K, P, M, X, V, U\}$, let pr_S be the corresponding projection and define

$$\text{Rec}_{c,S} := \text{pr}_S \circ \text{Rec}_c. \quad (52)$$

Equation 52 makes record resolution explicit. Adding fields retains more information, but greater input resolution is not automatically more useful.

4.3 Directed and Domain-Specific Record Projections

The same occurrence can carry different trust relevance for different directed coordinates. For $q \in \text{Aff}_c(h, e)$, let

$$\text{pr}_{c,q}^{\text{rec}} : \mathfrak{Z}_c^{\text{rec}} \rightarrow \mathfrak{Z}_{c,q}^{\text{rec}} \quad (53)$$

be a declared coordinate-relative projection. Equation 53 declares the coordinate-specific projection itself. The resulting record is

$$\text{rec}_{c,q}(h, e) := \text{pr}_{c,q}^{\text{rec}}(\text{Rec}_c(h, e)). \quad (54)$$

Equation 54 allows one shared occurrence to yield different directed or domain-specific input records. Reciprocal coordinates (x, y, d) and (y, x, d) are not required to receive identical projections, and the same occurrence may be represented differently across domains d_1 and d_2 .

Trust-relevance support and the state-space coupling relation from Section 3 answer different questions. In particular,

$$q, q' \in \text{Aff}_c(h, e) \not\Rightarrow q \bowtie_c^{\text{st}} q' \quad (55)$$

and

$$q \bowtie_c^{\text{st}} q' \not\Rightarrow \exists e \in \mathcal{E}_c(h) \text{ such that } q, q' \in \text{Aff}_c(h, e). \quad (56)$$

Equations 55–56 separate static admissibility coupling among trust-state coordinates from the event-level support on which dynamic coupling may later be defined.

4.4 Commitment, Performance, and Breach Candidates

A breach should not be defined merely as an occurrence that lowers trust. The formalism instead permits a breach candidate to be derived from independently specified commitment and performance records.

Let $\mathfrak{K}_c$ be a commitment carrier and $\mathfrak{P}_c$ a performance carrier. Define partial extraction maps

$$\text{K}_c : \mathcal{O}_c \rightarrow \mathfrak{K}_c, \quad \text{P}_c : \mathcal{O}_c \rightarrow \mathfrak{P}_c. \quad (57)$$

Equation 57 records the relevant commitment and performance information without yet pairing them. For each history h , let

$$\mathcal{L}_c^{\text{KP}}(h) \subseteq \mathcal{E}_c(h) \times \mathcal{E}_c(h) \quad (58)$$

be a declared linking relation. Equation 58 supplies the admissible commitment–performance pairings whose pairs (e, e') identify a performance occurrence e' as relevant to a commitment occurrence e under the chosen temporal, causal, or institutional rules.

Let

$$\Vdash_c^{\text{ful}} \subseteq \mathfrak{K}_c \times \mathfrak{P}_c \quad (59)$$

be an independently specified fulfilment relation. Equation 59 defines the criterion against which a linked performance is evaluated. The breach-candidate set is then

$$\mathcal{B}_c(h) := \left\{ (e, e') \in \mathcal{L}_c^{\text{KP}}(h) : \text{K}_c(h, e) \not\Vdash_c^{\text{ful}} \text{P}_c(h, e') \right\}, \quad (60)$$

with the understanding that the required extraction maps must be defined on the linked occurrences. Equation 60 identifies a mismatch relative to a declared commitment/performance rule. Membership does not imply that trust actually falls, that the mismatch was culpable, that the commitment was legitimate, or that every affected party interprets the occurrence as betrayal.

The separation is useful when non-fulfilment arises from incapacity, force majeure, conflicting obligations, ambiguity, disputed evidence, or coercion. Those distinctions can be retained in role, exposure, evidential, or contextual fields without redefining breach by the trust effect that follows.

4.5 Communication, Evidence, and Deception-Relevant Records

Communication can affect trust through channels distinct from observed performance. Experimental repair studies have found different trust responses to apology and denial depending on the type and later evidential status of an alleged violation [11], while deception can alter subsequent repair trajectories even when later trustworthy conduct is observed [22]. These findings motivate retaining a promise, explanation, disclosure, apology, denial, and later verification as separately typed record features when the application requires them. The record schema therefore keeps communicative content and evidential status separate.

Let

$$\text{Msg}_c : \mathcal{O}_c \rightarrow \mathfrak{M}_c^{\text{msg}}, \quad \text{Ver}_c : \mathcal{O}_c \rightarrow \mathfrak{V}_c^{\text{ev}} \quad (61)$$

be the declared message-content and verification maps. Equation 61 distinguishes what was represented or asserted from the evidential status later attached to that content.

When an application has an independently justified consistency relation

$$\models_c^{\text{cons}} \subseteq \mathfrak{M}_c^{\text{msg}} \times \mathfrak{V}_c^{\text{ev}}, \quad (62)$$

a message/evidence pair can be classified as inconsistent when the relation in Equation 62 fails. Intentional deception requires additional evidence concerning agency, knowledge, or intent whenever the substantive account treats those properties as necessary. The formalism therefore does not identify inconsistency with deception by default.

This distinction allows an inaccurate statement, a mistaken forecast, a knowingly false statement, and a strategically incomplete disclosure to receive different pre-update records even when their observed downstream outcomes coincide. Section 5 can then test whether those retained distinctions matter dynamically.

4.6 Repair Attempts and Historical Non-Erasure

The same provenance discipline applies to repair. A later occurrence should not be labeled “successful repair” merely because trust eventually rises. For a breach candidate $b \in \mathcal{B}_c(h)$, let

$$\text{Resp}_c(h, b) \subseteq \mathcal{E}_c(h) \quad (63)$$

be the set of occurrences linked to b by a declared response rule. The set in Equation 63 may include apologies, explanations, compensation, renewed commitments, institutional remedies, verified corrective conduct, or other independently identifiable responses.

Let

$$\mathcal{A}_c^{\text{rep}}(h) \subseteq \{(b, r) : b \in \mathcal{B}_c(h), r \in \text{Resp}_c(h, b)\} \quad (64)$$

be a declared repair-attempt set. Membership in Equation 64 means only that response occurrence r satisfies the application’s independently specified criteria for attempting to address breach candidate b . It does not imply restoration, improvement, forgiveness, or erasure of the breach history.

Claims of successful repair therefore belong to the dynamical layer. Restoration requires comparison of pre-breach and post-response trust states; improvement requires a declared trust feature and an appropriate preorder; hysteretic repair requires still stronger path-sensitive structure. Section 7 will formalize those distinctions after the update and memory machinery has been established.

4.7 Record Equivalence and Input Resolution

Two interaction occurrences can receive the same selected record even when they belong to different histories. For a selected feature family S , define

$$(h, e) \sim_{\text{Rec}_S} (g, f) \iff \text{Rec}_{c,S}(h, e) = \text{Rec}_{c,S}(g, f), \quad (65)$$

whenever both selected records are defined. Equation 65 formalizes “the same represented interaction occurrence” at a declared input resolution. No equality of trust effect follows from this definition; whether equal records have equal effects after different prior histories is precisely one of the factorization questions of Section 5.

Record representations can also be ordered by informational refinement. Let

$$\text{Rec}_1 : \mathcal{O} \rightarrow \mathfrak{Z}_1, \quad \text{Rec}_2 : \mathcal{O} \rightarrow \mathfrak{Z}_2 \quad (66)$$

be total record maps on a common occurrence domain $\mathcal{O}$. Write

$$\text{Rec}_1 \succeq_{\text{rec}} \text{Rec}_2 \quad (67)$$

Equation 67 names this record-information preorder. It holds when there exists a map

$$s_{21} : \text{Im}(\text{Rec}_1) \longrightarrow \text{Im}(\text{Rec}_2) \quad \text{such that} \quad \text{Rec}_2 = s_{21} \circ \text{Rec}_1. \quad (68)$$

Equation 68 means that every distinction retained by Rec_2 can be recovered from Rec_1 . As with trust-state refinement, mutual factorability can hold between differently coded record maps, so $\succeq_{\text{rec}}$ is a pre-order unless representations are identified up to informational equivalence.

Proposition 4.2 (Record factorization and equivalence refinement). *For the maps in Equation 66,*

$$\text{Rec}_1 \succeq_{\text{rec}} \text{Rec}_2 \iff \sim_{\text{Rec}_1} \subseteq \sim_{\text{Rec}_2}. \quad (69)$$

Proof. If Equation 68 holds, equality under Rec_1 implies equality under Rec_2 . Conversely, assume the equivalence refinement in Equation 69. Define $s_{21}(\text{Rec}_1(\xi)) := \text{Rec}_2(\xi)$ for $\xi \in \mathcal{O}$. Refinement guarantees that this definition is independent of the chosen representative, yielding Equation 68. $\square$

Proposition 4.2 supplies the input-resolution order used in Section 5, where $\Theta_c^{\text{tr}}(h)$ and $\text{Rec}_c(h, e)$ are tested jointly for next-state sufficiency.

5 Trust-State Update Factorization

This section asks when the represented current trust state and pre-update interaction record are sufficient for the represented next trust state. Its diagnostic method is exact factorization: the section defines the corresponding defect set and then separates whole-state, feature-specific, and coordinate-local prediction. A failed factorization establishes insufficiency of the selected interface; locating the missing information among state refinement, record refinement, explicit memory, latent structure, or another model class requires further analysis.

5.1 Admissible One-Step Extensions

A trust-history domain need not be closed under arbitrary concatenation, and the event carrier used to describe a new occurrence need not exist in the pre-extension history. The update problem is therefore based on a declared one-step successor relation rather than on an assumed total composition law. Let

$$\rightsquigarrow_c \subseteq \Gamma_c^{\text{tr}} \times \mathcal{E}_c^* \times \Gamma_c^{\text{tr}} \quad (70)$$

be an admissible successor relation. Equation 70 is read

$$h \xrightarrow[e]{c} h^+ \quad (71)$$

when $(h, e, h^+) \in \rightsquigarrow_c$, the post-extension history contains the newly represented occurrence $e \in \mathcal{E}_c(h^+)$, and h^+ is admissibly treated as a one-step extension of h at the declared analytical resolution. Equation 71 does not require that the history domain possess a globally associative composition operation.

For a selected record family S , define the update domain

$$\mathcal{U}_{c,S} := \{(h, e, h^+) \in \sim_c : \text{Rec}_{c,S}(h^+, e) \text{ is defined}\}. \quad (72)$$

The record in Equation 72 is evaluated on the extended history because Section 4 permits independently identifiable contextual, role, evidential, and exposure information to enter an occurrence record. This convention does not allow the record extractor to consult the resulting trust-state change; Obligation 4.1 remains in force.

For $u = (h, e, h^+) \in \mathcal{U}_{c,S}$, define its represented pre-state, selected event record, and represented post-state by

$$\tau^-(u) := \Theta_c^{\text{tr}}(h), \quad z_S(u) := \text{Rec}_{c,S}(h^+, e), \quad \tau^+(u) := \Theta_c^{\text{tr}}(h^+). \quad (73)$$

Equation 73 keeps the three roles separate: τ^- is the state representation before the occurrence, z_S is the pre-update representation of the occurrence, and τ^+ is the state representation after the admissible extension.

5.2 State–Record Factorization

The update input signature combines the two pre-update objects from Equation 73:

$$\text{In}_{c,S} : \mathcal{U}_{c,S} \longrightarrow \mathfrak{T}_c \times \mathfrak{Z}_{c,S}^{\text{rec}}, \quad \text{In}_{c,S}(u) = (\tau^-(u), z_S(u)), \quad (74)$$

where $\mathfrak{Z}_{c,S}^{\text{rec}}$ denotes the realized carrier of the selected record representation. The next-state target is

$$\text{Next}_{c,S} : \mathcal{U}_{c,S} \longrightarrow \mathfrak{T}_c, \quad \text{Next}_{c,S}(u) = \tau^+(u). \quad (75)$$

Equations 74–75 turn the Markov-style question into a factorization problem on realized update occurrences.

Definition 5.1 (Exact update sufficiency). The pair $(\Theta_c^{\text{tr}}, \text{Rec}_{c,S})$ is *exactly update-sufficient* on $\mathcal{U}_{c,S}$ when there exists a map

$$F_{c,S} : \text{Im}(\text{In}_{c,S}) \longrightarrow \mathfrak{T}_c \quad (76)$$

Equation 76 declares the candidate update operator on realized state–record inputs. The required factorization is

$$\text{Next}_{c,S} = F_{c,S} \circ \text{In}_{c,S}. \quad (77)$$

Definition 5.1 gives precise content to the provisional update equation in the Introduction. Equation 77 says that once the represented current state and selected event record are fixed, no further distinction among admissible prior histories is required to recover the represented next state. The condition is relative to the declared state representation, record resolution, successor relation, and history domain; it is not a claim that social trust is intrinsically Markovian.

The input and output maps induce equivalence relations on $\mathcal{U}_{c,S}$:

$$u \sim_{\text{In}_S} v \iff \text{In}_{c,S}(u) = \text{In}_{c,S}(v), \quad (78)$$

$$u \sim_{\text{Next}} v \iff \text{Next}_{c,S}(u) = \text{Next}_{c,S}(v). \quad (79)$$

Equations 78–79 distinguish equality of represented inputs from equality of represented trust effects.

Proposition 5.2 (Update factorization criterion). *The following statements are equivalent:*

1. $(\Theta_c^{\text{tr}}, \text{Rec}_{c,S})$ is exactly update-sufficient on $\mathcal{U}_{c,S}$;

2.

$$\sim_{\text{In}_S} \subseteq \sim_{\text{Next}}; \quad (80)$$

3. whenever $u, v \in \mathcal{U}_{c,S}$ satisfy

$$\tau^-(u) = \tau^-(v), \quad z_S(u) = z_S(v), \quad (81)$$

they also satisfy $\tau^+(u) = \tau^+(v)$.

Equation 81 is the explicit equal-input condition used in Statement 3.

Proof. If Equation 77 holds and $u \sim_{\text{In}_S} v$, applying $F_{c,S}$ to the common input gives $\text{Next}_{c,S}(u) = \text{Next}_{c,S}(v)$, proving Equation 80. The equivalence of Statements 2 and 3 is the expansion of the two induced equivalence relations. Conversely, assume Statement 3. Define $F_{c,S}$ on $\text{Im}(\text{In}_{c,S})$ by Equation 82.

$$F_{c,S}(\text{In}_{c,S}(u)) := \text{Next}_{c,S}(u). \quad (82)$$

Statement 3 guarantees that this value is independent of the representative u , so the map is well defined and satisfies Equation 77. $\square$

Proposition 5.2 is purely representational. It identifies when the selected input signature is sufficient for a declared next-state representation; it does not establish a causal mechanism or determine how $F_{c,S}$ should be estimated empirically.

5.3 Update Defects and Missing Dynamical Information

When factorization fails, the failure can be represented explicitly. Define the exact update-defect set

$$\Delta_{c,S}^{\text{upd}} := \{(u, v) \in \mathcal{U}_{c,S}^2 : \text{In}_{c,S}(u) = \text{In}_{c,S}(v), \text{Next}_{c,S}(u) \neq \text{Next}_{c,S}(v)\}. \quad (83)$$

Equation 83 contains pairs of admissible one-step extensions that are indistinguishable at the selected state-record input resolution but lead to different represented next states.

Corollary 5.3 (Defect criterion). *Exact update sufficiency holds if and only if*

$$\Delta_{c,S}^{\text{upd}} = \emptyset. \quad (84)$$

Proof. Equation 84 is exactly Statement 3 of Proposition 5.2 written as the absence of counterexamples. $\square$

Corollary 5.3 turns exact update sufficiency into an explicit emptiness test. A nonempty defect has a limited but useful interpretation. It establishes that the selected state representation and record map are dynamically insufficient for exact next-state recovery on the declared update domain. It does not by itself reveal where the missing information belongs. At least four repairs are logically possible: enrich the represented current state; enrich the event record; retain an explicit history or memory variable; or replace the deterministic next-state target with a stochastic or set-valued object when the residual variation is not resolved by the available information. Sections 6 and 11 examine the latter two possibilities in more detail.

This distinction prevents an immediate inference from

$$\Delta_{c,S}^{\text{upd}} \neq \emptyset \quad (85)$$

to the claim that trust has irreducible long memory. Equation 85 may instead diagnose an impoverished state representation, an under-specified event record, latent heterogeneity, or unresolved observational uncertainty.

5.4 Feature-Specific Update Sufficiency

Exact recovery of the complete represented next state can be stronger than a particular inquiry requires. Let $\varphi_{c,\lambda}$ be a state readout from Equation 31. Define the readout-compatible update domain

$$\mathcal{U}_{c,S}^\lambda := \{u \in \mathcal{U}_{c,S} : \varphi_{c,\lambda}(\tau^+(u)) \text{ is defined}\} \quad (86)$$

and the feature target

$$\text{Next}_{c,S}^\lambda(u) := \varphi_{c,\lambda}(\tau^+(u)). \quad (87)$$

Equations 86–87 restrict the prediction problem to the declared downstream feature.

Definition 5.4 (Feature-update sufficiency). The state–record signature is λ -*update-sufficient* when there exists

$$F_{c,S}^\lambda : \text{Im}(\text{In}_{c,S}|_{\mathcal{U}_{c,S}^\lambda}) \longrightarrow \mathfrak{V}_{c,\lambda} \quad (88)$$

Equation 88 declares the feature-level update operator. The required factorization is

$$\text{Next}_{c,S}^\lambda = F_{c,S}^\lambda \circ \text{In}_{c,S}|_{\mathcal{U}_{c,S}^\lambda}. \quad (89)$$

Definition 5.4 permits a state representation to be sufficient for one forecasting or analytical task while remaining insufficient for complete next-state recovery. Its defect set is

$$\Delta_{c,S}^{\text{upd},\lambda} := \{(u, v) \in (\mathcal{U}_{c,S}^\lambda)^2 : \text{In}_{c,S}(u) = \text{In}_{c,S}(v), \text{Next}_{c,S}^\lambda(u) \neq \text{Next}_{c,S}^\lambda(v)\}. \quad (90)$$

Equation 90 records the pairs that remain indistinguishable at the selected input resolution but differ on the chosen next-state feature. By the same factorization argument as Proposition 5.2, Equation 89 holds if and only if $\Delta_{c,S}^{\text{upd},\lambda} = \emptyset$.

Exact update sufficiency implies feature-update sufficiency for every readout defined on the realized next states:

$$\Delta_{c,S}^{\text{upd}} = \emptyset \implies \Delta_{c,S}^{\text{upd},\lambda} = \emptyset. \quad (91)$$

The converse of Equation 91 need not hold. Two next states may differ while agreeing on the feature relevant to the declared inquiry.

5.5 Coordinate-Local Prediction and Representational Dynamic Coupling

The typed state carrier permits a more local question: can the next value of one directed trust coordinate be predicted from that coordinate’s present value and its coordinate-relative event record, or must additional state coordinates be retained?

Fix $q \in \mathcal{Q}_c^{\text{tr}}$. Let

$$\mathcal{U}_{c,q} := \left\{ u = (h, e, h^+) \in \mathcal{U}_{c,S} \mid \begin{array}{l} q \in \text{Dom}(\tau^-(u)) \cap \text{Dom}(\tau^+(u)), \\ q \in \text{Aff}_c(h^+, e), \text{rec}_{c,q}(h^+, e) \text{ defined} \end{array} \right\}. \quad (92)$$

Equation 92 restricts attention to transitions on which the coordinate exists before and after the occurrence and has a coordinate-relative input record.

The coordinate-local input and output maps are

$$\text{In}_{c,q}^{\text{loc}}(h, e, h^+) := (\pi_q(\tau^-), \text{rec}_{c,q}(h^+, e)), \quad (93)$$

$$\text{Next}_{c,q}(h, e, h^+) := \pi_q(\tau^+). \quad (94)$$

Equations 93–94 ask whether the local coordinate carries enough state information for its own represented update.

Call q *locally update-sufficient* on $\mathcal{U}_{c,q}$ when $\text{Next}_{c,q}$ factors through $\text{In}_{c,q}^{\text{loc}}$. Failure can be represented by

$$\Delta_{c,q}^{\text{loc}} := \left\{ (u, v) \in \mathcal{U}_{c,q}^2 : \text{In}_{c,q}^{\text{loc}}(u) = \text{In}_{c,q}^{\text{loc}}(v), \text{Next}_{c,q}(u) \neq \text{Next}_{c,q}(v) \right\}. \quad (95)$$

Equation 95 is a representation-relative witness that the update of q cannot be recovered from the chosen local state and local record alone.

To test whether additional current-state coordinates repair the defect, let $R \subseteq \mathcal{Q}_c^{\text{tr}}$ be a finite set with $q \in R$, and define

$$\mathfrak{T}_c(R) := \{ \tau \in \mathfrak{T}_c : R \subseteq \text{Dom}(\tau) \}, \quad \pi_R(\tau) := \tau|_R. \quad (96)$$

Equation 96 supplies the typed multicoordinate projection on states containing R . On the subdomain $\mathcal{U}_{c,q;R} \subseteq \mathcal{U}_{c,q}$ whose pre-states lie in $\mathfrak{T}_c(R)$, define

$$\text{In}_{c,q|R}(u) := (\pi_R(\tau^-(u)), \text{rec}_{c,q}(h^+, e)). \quad (97)$$

Equation 97 allows a coordinate update to depend on a declared neighborhood of present trust relations without forcing the entire global state into every local transition.

Definition 5.5 (Representational dynamic support). A set $R \ni q$ is *q-update-sufficient* on $\mathcal{U}_{c,q;R}$ when $\text{Next}_{c,q}$ factors through $\text{In}_{c,q|R}$. An inclusion-minimal such R , when one exists in the declared candidate family, is called a *minimal representational dynamic support* for q .

Definition 5.5 gives a conservative meaning to dynamic coupling. If $\{q\}$ is not update-sufficient but a larger R is, the chosen representation requires information outside q to recover its next value. This is not yet a causal claim that every coordinate in $R \setminus \{q\}$ physically influences q ; causal attribution requires additional intervention, identification, or substantive assumptions.

The distinction from state-space coupling is immediate. It is possible that

$$q \not\bowtie_c^{\text{st}} q' \quad (98)$$

Equation 98 is compatible with a minimal q -update support containing q' , because static admissibility and update dependence concern different structures. Conversely, state-space coupling in Equation 30 does not by itself show that q' is needed to predict the next value of q .

5.6 Input Refinement and the Location of Missing Information

A failed update factorization can sometimes be repaired by retaining more pre-update information. Let

$$l_1 : \mathcal{U} \rightarrow \mathfrak{X}_1, \quad l_2 : \mathcal{U} \rightarrow \mathfrak{X}_2 \quad (99)$$

Equation 99 declares two input signatures on a common update domain. Write

$$l_1 \succeq_{\text{upd}} l_2 \quad (100)$$

when there exists

$$s_{21} : \text{Im}(l_1) \rightarrow \text{Im}(l_2) \quad \text{such that} \quad l_2 = s_{21} \circ l_1. \quad (101)$$

Equations 100–101 define an information preorder on realized update inputs.

Proposition 5.6 (Monotonicity under input refinement). *Let $Y : \mathcal{U} \rightarrow \mathfrak{Y}$ be any declared next-step target. If Y factors through l_2 and Equation 101 holds, then Y also factors through l_1 .*

Proof. If $Y = f_2 \circ l_2$ and $l_2 = s_{21} \circ l_1$, then Equation 102 holds.

$$Y = (f_2 \circ s_{21}) \circ l_1. \quad (102)$$

Thus $f_1 := f_2 \circ s_{21}$ supplies the required factorization through l_1 . $\square$

Proposition 5.6 formalizes a simple but useful discipline: retaining additional pre-update information cannot destroy an already available factorization for the same target, although it may make the representation unnecessarily detailed. The converse fails in general. A refined input may repair a defect that remains present at a coarser resolution.

Proposition 5.6 leaves the location of missing information open. It may lie in the present state, the event record, retained history, latent structure, or the choice of deterministic target. Section 6 isolates the explicit-history case without presuming that every defect is memory.

6 Historical Memory in Trust Evolution

This section analyzes the historical distinctions that remain when the state–record factorization of Section 5 is insufficient. Memory is treated as a representation of retained historical information rather than as a primitive psychological substance. The section tests candidate memory maps by defect resolution, orders them by informational refinement, develops recursive and finite-horizon forms, and uses a full-history control to distinguish compression failure from residual structure that history summaries alone cannot resolve.

The central discipline is that a nonempty update-defect set is not yet evidence of irreducible long memory. A candidate memory representation earns a dynamical role only when the distinctions it preserves are relevant to the selected next-state target. Conversely, a history summary can be substantively meaningful while remaining unnecessary for a particular prediction task. Memory in the present section therefore means *retained historical information at a declared representational resolution*; it need not coincide with conscious recollection, institutional recordkeeping, or any specific cognitive mechanism.

6.1 Memory Representations and Historical Equivalence

Let $\mathfrak{M}_c$ be a declared carrier of historical-memory representations for context c . A memory representation is a map

$$M_c : \Gamma_c^{\text{tr}} \longrightarrow \mathfrak{M}_c. \quad (103)$$

Equation 103 may encode, for example, an ordered record tail, a breach count, the recency of selected event types, a role-conditioned interaction summary, a sequence motif, or another history-derived object whose construction is independently specified. The map may also return a designated boundary value for histories that are shorter than the nominal memory window. The definition does not require that the represented participants consciously store or retrieve the encoded information.

Every memory map induces an equivalence relation on the history domain:

$$h \sim_M g \iff M_c(h) = M_c(g). \quad (104)$$

Equation 104 identifies histories that the selected memory representation treats as indistinguishable. Combining the trust-state representation with memory yields the joint map

$$J_c^{\Theta, M} : \Gamma_c^{\text{tr}} \longrightarrow \mathfrak{T}_c \times \mathfrak{M}_c, \quad J_c^{\Theta, M}(h) = (\Theta_c^{\text{tr}}(h), M_c(h)). \quad (105)$$

Equation 105 records the joint historical information retained by the represented trust state and the selected memory summary. The induced joint equivalence satisfies

$$h \sim_{\Theta, M} g \iff h \sim_{\Theta} g \text{ and } h \sim_M g. \quad (106)$$

Equation 106 makes explicit that memory can refine the historical distinctions retained by the current trust state.

A memory representation may nevertheless add no information beyond that state.

Definition 6.1 (State-redundant memory). A memory map M_c is *state-redundant* relative to Θ_c^{tr} when there exists a map

$$r_M : \text{Im}(\Theta_c^{\text{tr}}) \longrightarrow \mathfrak{M}_c \quad (107)$$

Equation 107 declares the readout through which the memory value would be recoverable from the represented current state. State redundancy requires

$$M_c = r_M \circ \Theta_c^{\text{tr}}. \quad (108)$$

Definition 6.1 is representation-relative. A breach marker stored inside an enriched trust state may be redundant under one state architecture and genuinely additional under a coarser architecture. The distinction concerns recoverable information, not whether the historical feature is substantively important.

6.2 Memory-Augmented Update Factorization

For an update occurrence $u = (h, e, h^+) \in \mathcal{U}_{c,S}$, define the pre-update memory value by

$$m_M^-(u) := M_c(h). \quad (109)$$

Equation 109 is evaluated on the pre-extension history. It therefore preserves the causal and evidential direction established for the state–record interface: the memory input may use information available from the prior history, but it may not be defined by consulting the resulting post-update trust state.

The memory-augmented input signature is

$$\text{In}_{c,S}^M(u) := (\tau^-(u), z_S(u), m_M^-(u)). \quad (110)$$

The selected memory resolves the exact next-state problem when there exists a map

$$F_{c,S}^M : \text{Im}(\text{In}_{c,S}^M) \longrightarrow \mathfrak{T}_c \quad (111)$$

Equation 111 declares the candidate deterministic trust-update map on realized augmented inputs. The required factorization is

$$\text{Next}_{c,S} = F_{c,S}^M \circ \text{In}_{c,S}^M. \quad (112)$$

Equation 112 is the memory-augmented analogue of Equation 77.

Define the residual update-defect set after memory augmentation by

$$\Delta_{c,S}^{\text{upd}|M} := \left\{ (u, v) \in \mathcal{U}_{c,S}^2 : \text{In}_{c,S}^M(u) = \text{In}_{c,S}^M(v), \text{Next}_{c,S}(u) \neq \text{Next}_{c,S}(v) \right\}. \quad (113)$$

The memory-assisted factorization in Equation 112 holds if and only if

$$\Delta_{c,S}^{\text{upd}|M} = \emptyset. \quad (114)$$

Equation 114 follows by the same factorization argument used in Proposition 5.2.

Definition 6.2 (Dynamically resolving memory at a fixed interface). A memory map M_c is *dynamically resolving* for the fixed pair $(\Theta_c^{\text{tr}}, \text{Rec}_{c,S})$ when

$$\Delta_{c,S}^{\text{upd}} \neq \emptyset \quad \text{and} \quad \Delta_{c,S}^{\text{upd}|M} = \emptyset. \quad (115)$$

Definition 6.2 gives a deliberately local meaning to the claim that memory is required. It establishes that historical information carried by M_c repairs a defect at the selected state–record resolution. It does not establish that no alternative enrichment of the trust state or event record could accomplish the same task.

Memory augmentation can only refine the input partition.

Proposition 6.3 (Defect monotonicity under memory augmentation). *For any memory map M_c ,*

$$\Delta_{c,S}^{\text{upd|M}} \subseteq \Delta_{c,S}^{\text{upd}}. \quad (116)$$

Proof. If $(u, v) \in \Delta_{c,S}^{\text{upd|M}}$, then Equation 110 gives equality of the pre-state and selected record as components of the equal augmented inputs. The next states differ by Equation 113. Hence (u, v) also satisfies Equation 83, proving Equation 116. $\square$

The converse inclusion need not hold. A useful memory representation removes precisely those defect pairs that it distinguishes while leaving unresolved pairs whose relevant historical difference remains compressed.

Proposition 6.4 (State-redundant memory cannot repair an update defect). *If M_c is state-redundant relative to Θ_c^{tr} , then*

$$\Delta_{c,S}^{\text{upd|M}} = \Delta_{c,S}^{\text{upd}}. \quad (117)$$

Proof. Proposition 6.3 gives one inclusion. For the reverse inclusion, let $(u, v) \in \Delta_{c,S}^{\text{upd}}$. Then $\tau^-(u) = \tau^-(v)$ and $z_S(u) = z_S(v)$. By Equation 108, equality of the pre-states implies $m_M^-(u) = m_M^-(v)$. Thus the augmented inputs are equal while the next states differ, so $(u, v) \in \Delta_{c,S}^{\text{upd|M}}$. $\square$

Equation 117 therefore shows that a state-redundant historical label leaves the exact defect set unchanged. Proposition 6.4 prevents a merely relabeled state variable from being presented as evidence for historical memory. At the fixed interface, a memory term has explanatory or predictive significance only if it retains distinctions that the current state does not already determine.

6.3 Full-History Control and the Source of Residual Defects

A maximal diagnostic within the declared history domain is obtained by retaining the pre-update history itself. Let

$$H_c : \Gamma_c^{\text{tr}} \longrightarrow \Gamma_c^{\text{tr}}, \quad H_c(h) = h. \quad (118)$$

Equation 118 is not proposed as a practical model. It is an upper-information control against which compressed memory summaries can be compared.

The corresponding input signature is

$$\text{In}_{c,S}^{\text{hist}}(u) := (\tau^-(u), z_S(u), h(u)), \quad u = (h(u), e, h^+), \quad (119)$$

where the notation in Equation 119 merely names the pre-history component of the update tuple. Its residual defect set is

$$\Delta_{c,S}^{\text{upd|hist}} := \left\{ (u, v) \in \mathcal{U}_{c,S}^2 : \text{In}_{c,S}^{\text{hist}}(u) = \text{In}_{c,S}^{\text{hist}}(v), \text{Next}_{c,S}(u) \neq \text{Next}_{c,S}(v) \right\}. \quad (120)$$

Any memory map in Equation 103 is a compression of the full-history identity map. This yields a useful impossibility result.

Proposition 6.5 (Full-history residual criterion). *For every memory map M_c on Γ_c^{tr} ,*

$$\Delta_{c,S}^{\text{upd|hist}} \subseteq \Delta_{c,S}^{\text{upd|M}}. \quad (121)$$

Consequently, if

$$\Delta_{c,S}^{\text{upd|hist}} \neq \emptyset, \quad (122)$$

then no memory summary depending only on the pre-update history can restore deterministic exact update sufficiency while the selected current-state and event-record representations remain fixed.

Proof. If two update occurrences have equal full-history inputs, then they share the same pre-history h . Equation 103 therefore assigns them the same memory value for every M_c . Equality of the state and record components is already included in the full-history input equality. A pair in Equation 120 is thus also a pair in Equation 113, proving Equation 121. $\square$

Equation 122 is the decisive negative diagnostic for history-only deterministic repair at the fixed state–record interface. Proposition 6.5 sharply limits what can be inferred from failed deterministic prediction. A nonempty full-history residual can arise when the selected event record merges occurrences that have different effects, when the successor relation represents several admissible continuations under the same pre-history and record, when latent variables are omitted, when the represented post-state is only partially identified, or when genuinely stochastic variation is part of the intended model. Section 11 develops stochastic representations for such cases. The full-history control therefore separates *history compression* from other sources of residual uncertainty without deciding among those other sources by itself.

6.4 Information Order and Minimal Resolving Memory

Different memory summaries can retain different amounts of historical information. Let

$$M_1 : \Gamma_c^{\text{tr}} \rightarrow \mathfrak{M}_1, \quad M_2 : \Gamma_c^{\text{tr}} \rightarrow \mathfrak{M}_2 \quad (123)$$

Equation 123 places two candidate memories on a common history domain so that their retained distinctions can be compared. Write

$$M_1 \succeq_{\text{mem}} M_2 \quad (124)$$

when there exists a map

$$r_{21}^{\text{mem}} : \text{Im}(M_1) \longrightarrow \text{Im}(M_2) \quad \text{such that} \quad M_2 = r_{21}^{\text{mem}} \circ M_1. \quad (125)$$

Equations 124–125 define the same information-refinement preorder used for trust states and update signatures, now restricted to historical summaries.

Proposition 6.6 (Residual-defect monotonicity under memory refinement). *If $M_1 \succeq_{\text{mem}} M_2$, then*

$$\Delta_{c,S}^{\text{upd}|M_1} \subseteq \Delta_{c,S}^{\text{upd}|M_2}. \quad (126)$$

Proof. Equality of M_1 -values implies equality of M_2 -values by Equation 125. Therefore every pair that remains indistinguishable under the more informative memory M_1 also remains indistinguishable under M_2 . If its next states differ, the pair belongs to both residual-defect sets, yielding Equation 126. $\square$

Because mutual factorability can hold between differently coded memory maps, $\succeq_{\text{mem}}$ is generally a preorder. Write $M_1 \succ_{\text{mem}} M_2$ when $M_1 \succeq_{\text{mem}} M_2$ while $M_2 \not\succeq_{\text{mem}} M_1$. Let $\mathcal{C}_c^{\text{mem}}$ be a declared candidate family of substantively admissible memory maps. A member $M_\star \in \mathcal{C}_c^{\text{mem}}$ is a *minimal resolving memory* when it satisfies Equation 114 and there is no resolving $M \in \mathcal{C}_c^{\text{mem}}$ with $M_\star \succ_{\text{mem}} M$. Minimality is therefore relative to the declared candidate family and the selected next-state target, modulo informationally equivalent recodings.

A minimal resolving memory need not be unique. Two incomparable summaries can each retain enough information for exact prediction while encoding different historical features. The formalism should therefore avoid interpreting minimality as discovery of a unique natural memory variable unless the substantive domain supplies additional identification conditions.

6.5 Recursive Memory and Augmented-State Closure

Explicit memory is often useful because a history summary can itself be updated as new interactions occur. Define the post-update memory on $\mathcal{U}_{c,S}$ by

$$m_M^+(u) := M_c(h^+) \quad \text{for } u = (h, e, h^+). \quad (127)$$

Equation 127 evaluates the same memory representation after the admissible history extension. The memory representation is *recursively closed* on $\mathcal{U}_{c,S}$ when there exists

$$G_{c,S}^M : \text{Im}(\text{In}_{c,S}^M) \longrightarrow \mathfrak{M}_c \quad (128)$$

Equation 128 declares the candidate recursion on realized augmented inputs. Recursive closure requires

$$m_M^+ = G_{c,S}^M \circ \text{In}_{c,S}^M. \quad (129)$$

Equation 129 requires the next memory value to be recoverable from the same represented pre-state, current record, and current memory used by the trust update. Applications may impose a stronger recursion of the form $m^+ = G(m^-, z_S)$ when the current trust state is unnecessary for updating the memory summary.

Recursive closure matters because it identifies when the explicit-memory model can be rewritten as an ordinary first-order dynamics on an augmented state. Define

$$\hat{\Theta}_c^{\text{tr},M}(h) := (\Theta_c^{\text{tr}}(h), M_c(h)) \quad (130)$$

with realized carrier

$$\hat{\mathfrak{T}}_c^M := \text{Im}(\hat{\Theta}_c^{\text{tr},M}) \subseteq \mathfrak{T}_c \times \mathfrak{M}_c. \quad (131)$$

Equation 131 identifies the realized augmented-state carrier rather than assuming the full Cartesian product is reachable. The corresponding realized augmented update domain is

$$\hat{\mathcal{D}}_{c,S}^M := \left\{ ((\tau, m), z) \in \hat{\mathfrak{T}}_c^M \times \mathfrak{Z}_{c,S}^{\text{rec}} : (\tau, z, m) \in \text{Im}(\text{In}_{c,S}^M) \right\}. \quad (132)$$

Equation 132 prevents the augmented recursion from being read as a total map on state–record combinations that never occur in the declared update domain.

Proposition 6.7 (Augmented-state closure). *Suppose Equation 112 holds and M_c satisfies the recursion in Equation 129. Then the augmented state in Equation 130 admits an exact one-step update factorization on the realized domain in Equation 132 through its current value and the selected record.*

Proof. Define the augmented update map on the realized domain by

$$\hat{F}_{c,S}^M : \hat{\mathcal{D}}_{c,S}^M \longrightarrow \hat{\mathfrak{T}}_c^M, \quad \hat{F}_{c,S}^M((\tau, m), z) := \left(F_{c,S}^M(\tau, z, m), G_{c,S}^M(\tau, z, m) \right). \quad (133)$$

Equation 133 combines the trust-state and memory recursions into one augmented transition. Equation 112 gives the first component of the post-update pair, while Equation 129 gives the second. Hence for every $u = (h, e, h^+) \in \mathcal{U}_{c,S}$,

$$\hat{\Theta}_c^{\text{tr},M}(h^+) = \hat{F}_{c,S}^M \left(\hat{\Theta}_c^{\text{tr},M}(h), z_S(u) \right). \quad (134)$$

Equation 134 is the required exact update factorization for the augmented state. $\square$

Proposition 6.7 clarifies the relation between memory and state. Once a sufficient memory variable is recursively closed, it can be incorporated into a richer state representation. The distinction between “state” and “explicit memory” is therefore partly a modeling boundary. The substantive question is which historical distinctions the representation preserves and how interpretable that preservation remains, rather than whether a variable is placed syntactically inside or outside the state tuple.

The proposition also explains why a successful Markovization does not establish historical insignificance. An augmented state can be first-order precisely because it carries forward information generated by earlier interactions. Markovianity at the augmented resolution can therefore coexist with strong path dependence relative to a coarser state representation.

6.6 Finite-Horizon Record Memory

Some applications can order the trust-relevant records contained in a history and ask whether only a bounded recent portion is dynamically necessary. Let

$$\text{Trace}_{c,S} : \Gamma_c^{\text{tr}} \longrightarrow (\mathfrak{Z}_{c,S}^{\text{rec}})^{<\omega} \quad (135)$$

be a declared finite ordered record-trace map, where $(\mathfrak{Z}_{c,S}^{\text{rec}})^{<\omega}$ denotes the set of finite record sequences. Equation 135 is available only when the application has justified an order and a rule for selecting the historical records represented in the trace. It need not coincide with the complete event history.

Let $\mathbb{N}_0 := \{0, 1, 2, \dots\}$. For $k \in \mathbb{N}_0$, let

$$\text{Tail}_k : (\mathfrak{Z}_{c,S}^{\text{rec}})^{<\omega} \longrightarrow \mathfrak{W}_{c,S}^{(k)} \quad (136)$$

Equation 136 declares the finite-window extraction operator. It returns the final $\min\{k, n\}$ records of a sequence of length n , with the empty tail used at $k = 0$. Define the k -record memory by

$$\mathbf{M}_{c,S}^{(k)} := \text{Tail}_k \circ \text{Trace}_{c,S}. \quad (137)$$

Equation 137 retains order inside the selected window. Count-based, exponentially weighted, motif-based, or domain-specific summaries can be introduced as further compressions of this tail when justified.

Let

$$\Delta_{c,S}^{(k)} := \Delta_{c,S}^{\text{upd}|\mathbf{M}_{c,S}^{(k)}}. \quad (138)$$

Equation 138 names the residual deterministic defect at memory horizon k . Because $\mathbf{M}_{c,S}^{(k+1)}$ determines $\mathbf{M}_{c,S}^{(k)}$, Proposition 6.6 gives

$$\Delta_{c,S}^{(k+1)} \subseteq \Delta_{c,S}^{(k)}. \quad (139)$$

Equation 139 formalizes the intuitive fact that lengthening the retained record tail cannot create a deterministic defect that was already absent at a shorter window.

When the set is nonempty, define the minimal sufficient record horizon by

$$k_{c,S}^* := \min \left\{ k \in \mathbb{N}_0 : \Delta_{c,S}^{(k)} = \emptyset \right\}. \quad (140)$$

Equation 140 is meaningful only relative to the declared trace, current-state representation, event-record resolution, update domain, and exact next-state target. A finite $k_{c,S}^*$ shows that the retained record tail of that length is sufficient at the chosen representation. It does not imply that older interactions are socially irrelevant; their effects may already have been compressed into $\Theta_c^{\text{tr}}(h)$ or into the retained tail itself.

If no finite k satisfies Equation 140, the justified conclusion is narrower than a claim of intrinsically infinite social memory. The result shows only that no finite tail in the selected record-trace family resolves the update problem across the examined history domain. A different state representation, a different historical summary, a richer current event record, or a stochastic model may still provide an adequate description.

6.7 Diagnostic Interpretation of Memory Findings

The preceding constructions support a graded interpretation rather than a binary declaration that a system either “has memory” or “has no memory.” Table 3 summarizes the main deterministic cases.

The table also clarifies the evidential burden for claims about long memory. A pair of superficially similar trajectories with different next trust states does not establish memory unless similarity is defined at the selected state–record resolution and the relevant historical distinction is independently represented. A fitted memory parameter likewise does not establish a unique underlying mechanism when an enriched state, latent heterogeneity, or a changed event interpretation could generate the same observations.

Table 3: Diagnostic patterns for deterministic update defects under historical-memory refinement.

Base defect Δ^{upd}	Candidate-memory defect $\Delta^{\text{upd M}}$	Full-history defect $\Delta^{\text{upd hist}}$	Warranted interpretation
Empty	—	—	The selected state–record interface is already sufficient for the exact deterministic target. Additional explicit memory is unnecessary for that target.
Nonempty	Empty	Empty	The candidate memory resolves distinctions compressed by the selected state–record interface. Memory is dynamically resolving at that fixed interface.
Nonempty	Nonempty	Empty	The candidate memory remains too coarse, while richer pre-history information can in principle resolve the deterministic update problem.
Nonempty	Nonempty	Nonempty	Compression of prior history is not sufficient to explain the residual defect. Event resolution, successor ambiguity, latent structure, partial identification, or stochastic modeling requires examination.

Three reporting levels are therefore useful. First, *state–record insufficiency* is established by Equation 85. Second, *memory resolution at a fixed interface* is established by Equation 115. Third, a stronger claim that explicit memory is required across a model family needs an audit showing that the admissible memory-free state and record refinements in that family fail while at least one admissible historical summary succeeds. The third claim is model-family relative and should not be inferred from the second without that additional comparison.

Historical memory is therefore a sufficiency question at a declared resolution. A state can already encode history, an explicit memory can resolve omitted distinctions, recursive memory can be absorbed into an augmented state, and a full-history residual can show that historical compression alone is insufficient. Section 7 adds the stronger trajectory comparison needed for hysteresis.

7 Hysteretic Trust Repair

This section formalizes breach and repair when a later response can improve a relation without retracing its earlier trust trajectory. The objective is to distinguish repair attempts, target satisfaction, restoration, hysteretic nonclosure, matched-path hysteresis, and repair-family irreversibility. The method is trajectory-comparative: breach–response episodes are typed independently of their outcomes, while restoration and hysteresis are evaluated only after the relevant state resolution and external return condition have been declared.

The empirical trust-repair literature motivates the need for these distinctions without fixing their universal form. Repeated trustworthy conduct, promises, and apologies can support recovery after some violations, while deception can leave more persistent deficits; the timing of a breach can also alter later trust trajectories [22, 14]. Review work likewise treats trust repair as a heterogeneous family of short- and longer-term processes and notes that recovery need not erase every consequence of a violation [12]. The present paper therefore does not claim novelty for the substantive idea of incomplete repair. Its narrower contribution is to distinguish target satisfaction, exact or feature-level restoration, returned-condition nonclosure, matched-path hysteresis, and repair-family reachability as formally different claims. These

findings justify a formalism capable of representing incomplete restoration and timing sensitivity. They do not establish that every breach produces hysteresis or that every residual difference is irreversible.

7.1 Breach–Response Episodes in Continuation Structure

The one-step successor relation in Equation 70 supplies the local temporal architecture. First define its event-forgetting projection on histories by

$$h \rightsquigarrow_c g \iff \exists e \in \mathcal{E}_c^* \text{ such that } (h, e, g) \in \rightsquigarrow_c. \quad (141)$$

Equation 141 records one admissible historical step while suppressing the event label after reachability has been established. Let

$$\preceq_c^{\rightsquigarrow} := (\rightsquigarrow_c)^* \quad (142)$$

denote the reflexive–transitive closure of that projected relation. Equation 142 therefore records continuation reachability rather than a total temporal order. Thus $h \preceq_c^{\rightsquigarrow} g$ means that $g = h$ or that a finite chain of admissible one-step extensions leads from h to g . The construction introduces no global composition law beyond the successor structure already declared in Section 5.

A breach–response episode requires three historical positions and independently typed event information. Let

$$\mathcal{R}_c^{\text{ep}} \subseteq \Gamma_c^{\text{tr}} \times \Gamma_c^{\text{tr}} \times \Gamma_c^{\text{tr}} \times \left(\bigcup_{h \in \Gamma_c^{\text{tr}}} \mathcal{B}_c(h) \right) \times \mathcal{E}_c^* \quad (143)$$

be a declared set of tuples. Equation 143 declares the ambient episode carrier. Write

$$\rho = (h_\rho^0, h_\rho^b, h_\rho^r, b_\rho, r_\rho) \quad (144)$$

that satisfy the following conditions: $h_\rho^0 \preceq_c^{\rightsquigarrow} h_\rho^b \preceq_c^{\rightsquigarrow} h_\rho^r$; the breach candidate b_ρ is represented in $\mathcal{B}_c(h_\rho^b)$; and the pair (b_ρ, r_ρ) belongs to the repair-attempt relation $\mathcal{A}_c^{\text{rep}}(h_\rho^r)$ from Equation 64. Equation 144 records the selected baseline, post-breach, and post-response positions of one analytically declared episode.

Membership in $\mathcal{R}_c^{\text{ep}}$ is outcome-independent. In particular, the episode selector may use event timing, commitment/performance links, response links, institutional roles, and other pre-declared historical criteria, but it does not consult whether trust later rises, whether the baseline state is restored, or whether the response is judged successful. This preserves the pre-update provenance discipline of Obligation 4.1 at the multi-step episode level.

For $\rho \in \mathcal{R}_c^{\text{ep}}$, define the three represented trust states by

$$T_\rho^0 := \Theta_c^{\text{tr}}(h_\rho^0), \quad T_\rho^b := \Theta_c^{\text{tr}}(h_\rho^b), \quad T_\rho^r := \Theta_c^{\text{tr}}(h_\rho^r). \quad (145)$$

Equation 145 separates the baseline, breach-position, and post-response states without assigning a normative or dynamical verdict to any of them.

7.2 External Return Conditions

Hysteresis requires more than a difference between T_ρ^0 and T_ρ^r . A post-response state may differ because the surrounding relationship, task, exposure, role, or institutional condition also changed. The formalism therefore introduces a separately declared non-trust configuration map

$$\Xi_c : \Gamma_c^{\text{tr}} \longrightarrow \mathfrak{X}_c^{\text{ext}}, \quad (146)$$

where $\mathfrak{X}_c^{\text{ext}}$ carries those external, control-like, or relational-environment features that the application wishes to hold fixed when testing return. Equation 146 may include role configuration, contractual status, task domain, exposure conditions, available safeguards, or other independently represented circumstances. It excludes the trust state itself and any field defined by the post-response trust outcome.

The map in Equation 146 induces external-configuration equivalence

$$h \sim_{\Xi} g \iff \Xi_c(h) = \Xi_c(g). \quad (147)$$

Equation 147 states equality only at the declared control resolution. A richer application may replace Ξ_c by a more informative configuration map when distinctions discarded by the current map materially affect the repair comparison.

Definition 7.1 (Return-qualified repair episode). A breach–response episode $\rho \in \mathcal{R}_c^{\text{ep}}$ is *return-qualified* relative to Ξ_c when

$$h_{\rho}^0 \sim_{\Xi} h_{\rho}^r. \quad (148)$$

Definition 7.1 does not say that the entire world has returned to an earlier state. It says only that the selected non-trust configuration has returned to the same represented value. Time itself may be included in Ξ_c when calendar position, delay, or elapsed duration is substantively relevant; omitting those variables asserts that they are being treated as irrelevant at the chosen comparison resolution.

7.3 Restoration at Declared Analytical Resolutions

Repair can be evaluated at several resolutions already available in the state architecture. Exact restoration is the strongest state-level criterion considered here.

Definition 7.2 (Exact trust-state restoration). A repair episode $\rho \in \mathcal{R}_c^{\text{ep}}$ achieves *exact trust-state restoration* when

$$T_{\rho}^r = T_{\rho}^0. \quad (149)$$

Definition 7.2, through Equation 149, refers to equality in the declared state carrier $\mathfrak{T}_c$. It is therefore representation-relative. An enriched state that explicitly retains a breach marker may fail exact restoration even when a coarser state representation returns to its earlier value.

A particular inquiry may require only restoration of selected trust features. For a joint readout $\varphi_{c,L}$ from Equation 44, define *L-feature restoration* by

$$\varphi_{c,L}(T_{\rho}^r) = \varphi_{c,L}(T_{\rho}^0), \quad (150)$$

whenever both readouts are defined. Equation 150 is weaker than Equation 149 whenever $\varphi_{c,L}$ is non-injective. Likewise, for a jointly defined observation family $M \subseteq \mathcal{M}_c$, *observational restoration* is

$$T_{\rho}^r \sim_{O_M} T_{\rho}^0. \quad (151)$$

Equation 151 records return only at the selected observation channels and therefore need not identify the underlying represented state.

A repair criterion can also be target-based rather than restoration-based. Extract from each episode the pre-response specification

$$p_{\rho}^{\text{rep}} := (h_{\rho}^0, h_{\rho}^b, b_{\rho}), \quad (152)$$

and let $\mathcal{P}_c^{\text{rep}}$ be the set of realized specifications in Equation 152. A declared repair-goal rule is then

$$\mathcal{A}_{c,\alpha}^{\text{goal}} : \mathcal{P}_c^{\text{rep}} \longrightarrow 2^{\mathfrak{T}_c}, \quad p_{\rho}^{\text{rep}} \longmapsto \mathcal{A}_{c,\alpha}^{\text{goal}}(p_{\rho}^{\text{rep}}). \quad (153)$$

Equation 153 permits criterion α to encode a domain-specific threshold, safety condition, working-relation region, or other substantively justified target. Because the rule is indexed by the pre-response specification rather than by the realized post-response state, its target can be declared before the repair outcome is evaluated. A repair episode satisfies criterion α when

$$T_{\rho}^r \in \mathcal{A}_{c,\alpha}^{\text{goal}}(p_{\rho}^{\text{rep}}). \quad (154)$$

Equation 154 makes explicit that successful repair under a declared criterion need not mean restoration of the pre-breach state.

Definition 7.3 (Historically marked criterion-satisfying repair). A repair episode ρ is *historically marked at state resolution* under criterion α when Equation 154 holds while

$$T_\rho^r \neq T_\rho^0. \quad (155)$$

Definition 7.3 formalizes one sense in which repair can be successful without historical erasure. The post-response relation lies inside the declared repair target while retaining a state-level distinction from the baseline. That distinction may correspond to caution, revised vulnerability, retained suspicion, altered expectations, or another represented feature, but the formalism does not assign a universal substantive interpretation to it.

The retained distinction can be made feature-specific when two readout families play different roles. Suppose L_{goal} contains the features used to judge restoration for the task at hand and L_{mark} contains features intended to detect a historical residual. A selectively restored and marked episode satisfies

$$\varphi_{c, L_{\text{goal}}}(T_\rho^r) = \varphi_{c, L_{\text{goal}}}(T_\rho^0), \quad (156)$$

$$\varphi_{c, L_{\text{mark}}}(T_\rho^r) \neq \varphi_{c, L_{\text{mark}}}(T_\rho^0). \quad (157)$$

Equations 156–157 provide a non-scalar representation of partial recovery: the features used for the declared repair goal may return while another represented component continues to distinguish the post-response state from the baseline.

7.4 Hysteretic Nonclosure under Returned Conditions

A return-qualified episode provides the minimum comparison structure needed to discuss a hysteretic loop at the selected external resolution. The term *nonclosure* is used because the post-response trajectory returns to the same represented external configuration while the trust state may fail to return to its starting point.

Definition 7.4 (Exact hysteretic repair witness). A repair episode $\rho \in \mathcal{R}_c^{\text{ep}}$ is an *exact hysteretic repair witness* relative to Ξ_c when it is return-qualified and

$$T_\rho^r \neq T_\rho^0. \quad (158)$$

Equation 158 should be interpreted at the declared state resolution. It does not establish that the same relation could never return to T_ρ^0 under a different continuation. It also does not identify the source of the residual difference. Memory, changed latent variables, imperfect control matching, or stochastic variation can each matter in applications; the present section isolates the deterministic trajectory relation that later empirical work would need to identify.

Feature-level hysteretic nonclosure is defined analogously. For a readout family L , a return-qualified episode is an L -hysteretic witness when

$$\varphi_{c, L}(T_\rho^r) \neq \varphi_{c, L}(T_\rho^0). \quad (159)$$

Equation 159 is useful when exact latent-state identity is neither observable nor required by the research question.

Exact nonclosure does not automatically appear in every available readout. Let $\Phi_c = \{\varphi_{c, \lambda}\}_{\lambda \in \Lambda_c}$ be the declared readout family. Call Φ_c *point-separating* on a subset $D \subseteq \mathfrak{T}_c$ when

$$T, T' \in D, \quad T \neq T' \implies \exists \lambda \in \Lambda_c \text{ such that } \varphi_{c, \lambda}(T) \neq \varphi_{c, \lambda}(T'). \quad (160)$$

Equation 160 states the informational condition under which state differences can be detected by at least one declared feature channel.

Proposition 7.5 (Readout witness for exact hysteretic nonclosure). *Let ρ be an exact hysteretic repair witness, and suppose Φ_c is point-separating on a set containing T_ρ^0 and T_ρ^r . Then at least one $\lambda \in \Lambda_c$ satisfies*

$$\varphi_{c,\lambda}(T_\rho^r) \neq \varphi_{c,\lambda}(T_\rho^0). \quad (161)$$

Proof. Definition 7.4 gives $T_\rho^r \neq T_\rho^0$. Applying the point-separation property in Equation 160 to that pair yields a readout satisfying Equation 161. $\square$

Proposition 7.5 is deliberately conditional. If the declared readouts are not point-separating, state-level hysteretic nonclosure may remain observationally or operationally hidden. Conversely, observed feature nonclosure establishes nonclosure only at the selected feature resolution unless the observation/readout map is sufficiently identifying.

7.5 Matched-Path Hysteresis

A single return-qualified episode can exhibit nonclosure, but stronger evidence for path dependence compares two admissible return routes with matched boundary conditions. Let $\rho, \sigma \in \mathcal{R}_c^{\text{ep}}$ be two return-qualified episodes. Their external return conditions are matched when

$$\Xi_c(h_\rho^0) = \Xi_c(h_\rho^r) = \Xi_c(h_\sigma^0) = \Xi_c(h_\sigma^r). \quad (162)$$

They are *state-start-matched* when, in addition,

$$T_\rho^0 = T_\sigma^0. \quad (163)$$

Equations 162–163 establish common represented return conditions while allowing the intervening histories to differ.

Definition 7.6 (Matched-path witness). A pair (ρ, σ) satisfying Equations 162–163 is a *matched-path hysteresis witness* at exact state resolution when

$$T_\rho^r \neq T_\sigma^r. \quad (164)$$

Definition 7.6 captures route-sensitive endpoint dependence. The initial trust state and the represented external endpoints agree, yet the terminal trust states differ after different admissible trajectories. This condition is stronger than showing that one repair episode failed to restore its baseline because it uses another matched trajectory as a comparator. The comparison remains representation-relative: omitted control variables can mimic path dependence if Ξ_c is too coarse.

A feature-level matched-path witness replaces Equation 164 with

$$\varphi_{c,L}(T_\rho^r) \neq \varphi_{c,L}(T_\sigma^r). \quad (165)$$

Equation 165 supports a narrower claim that route differences persist in the selected trust feature even when full state identity remains unidentified.

The matched-path construction also clarifies why event order can matter without requiring every historical detail to remain explicit. Two paths may contain similar local event types while differing in their ordering, timing, breach position, or intervening memory state. When an augmented state from Section 6 records the relevant distinction, the two paths can be represented by first-order dynamics on that augmented state while still ending at different trust states under matched external conditions.

7.6 Irreversibility within Repair-Family Reachability

Hysteretic nonclosure under one response sequence is weaker than irreversibility. To state the stronger property, the admissible family of repair continuations must be declared. Fix an episode ρ through its post-breach history h_ρ^b and breach candidate b_ρ . Let

$$\mathcal{C}_c^{\text{rep}}(h_\rho^b, b_\rho) \subseteq \left\{ g \in \Gamma_c^{\text{tr}} : h_\rho^b \preccurlyeq_c^{\sim} g \right\} \quad (166)$$

be a declared family of continuations that satisfy the application's repair-response admissibility rules for b_ρ . Equation 166 may restrict participants, available institutional remedies, duration, resource bounds, or permissible response types; those restrictions are part of the scope of any subsequent irreversibility claim.

The corresponding reachable trust-state set is

$$\text{Reach}_c^{\text{rep}}(h_\rho^b, b_\rho) := \left\{ \Theta_c^{\text{tr}}(g) : g \in \mathcal{C}_c^{\text{rep}}(h_\rho^b, b_\rho) \right\}. \quad (167)$$

Equation 167 records what states can be reached within the declared repair family, without asserting that the family contains every physically, socially, or institutionally conceivable future.

Definition 7.7 (Repair-family exact irreversibility). The pre-breach baseline of episode ρ is *exactly unreachable within the declared repair family* when

$$T_\rho^0 \notin \text{Reach}_c^{\text{rep}}(h_\rho^b, b_\rho). \quad (168)$$

Definition 7.7 supplies a scoped notion of irreversibility. It is stronger than Equation 158, because a single post-response endpoint can fail to equal the baseline even when another admissible continuation would restore it.

Proposition 7.8 (Protocol nonrestoration does not establish repair-family irreversibility). *For a repair episode ρ , Equation 155 does not imply Equation 168 unless additional assumptions exclude every other admissible continuation reaching T_ρ^0 .*

Proof. Equation 155 concerns only the realized endpoint T_ρ^r . Equation 168 quantifies over the full set in Equation 167. A reachable-state set may contain both T_ρ^r and T_ρ^0 . Therefore nonidentity of the realized endpoint with the baseline leaves baseline reachability undecided. $\square$

Proposition 7.8 therefore limits irreversibility claims to the continuation family actually quantified over. Feature-level irreversibility can be defined more weakly when only a selected readout matters. The baseline feature is unreachable within the declared repair family when

$$\varphi_{c,L}(T_\rho^0) \notin \left\{ \varphi_{c,L}(T) : T \in \text{Reach}_c^{\text{rep}}(h_\rho^b, b_\rho), \varphi_{c,L}(T) \text{ defined} \right\}. \quad (169)$$

Equation 169 can hold or fail independently of exact state reachability when $\varphi_{c,L}$ is non-injective.

7.7 Quantifying Residual Displacement under Additional Geometry

The preceding definitions require only equality and reachability. Some applications also justify a metric or another distance-like structure on the relevant state or feature space. When the metric d_c^{tr} from Equation 33 is substantively meaningful, define the exact-state residual displacement of a return-qualified episode ρ by

$$H_c^{\text{st}}(\rho) := d_c^{\text{tr}}(T_\rho^r, T_\rho^0). \quad (170)$$

Equation 170 supplies a magnitude only under the additional metric assumptions already identified in Section 3. It is not a universal scalar measure of hysteresis.

Table 4: Diagnostic distinctions among repair, restoration, hysteresis, and irreversibility.

Finding	Formal condition	Warranted interpretation
Repair attempt	$(b_\rho, r_\rho) \in \mathcal{A}_c^{\text{rep}}(h_\rho^r)$	The response meets independently declared attempt criteria; no trust outcome follows by definition.
Criterion-satisfying repair	Equation 154	The post-response state lies in the declared repair target. Exact restoration need not follow.
Exact restoration	Equation 149	The represented post-response trust state equals the represented baseline state.
Hysteretic nonclosure	Equations 148 and 158	The selected external configuration returns while the represented trust state does not.
Matched-path hysteresis	Equations 162–164	Matched boundary conditions with different intervening trajectories lead to different represented endpoints.
Repair-family irreversibility	Equation 168	Exact baseline restoration is unreachable within the explicitly declared admissible repair family.

If the realized joint-readout image $\text{Im}(\varphi_{c,L})$ carries a justified metric $d_{c,L}^{\text{feat}}$, define the feature residual by

$$H_{c,L}^{\text{feat}}(\rho) := d_{c,L}^{\text{feat}}(\varphi_{c,L}(T_\rho^r), \varphi_{c,L}(T_\rho^0)). \quad (171)$$

Equation 171 permits empirical work to estimate residual displacement at a declared resolution while preserving the distinction between state-level and feature-level recovery.

The metric quantities in Equations 170–171 should not be interpreted as proof of a particular mechanism. A large residual can arise from a highly persistent historical mark, a coarse external return map, a changed latent condition, or another omitted distinction. The diagnostic architecture of Sections 5–6 remains necessary for locating the source of the residual.

7.8 Hysteresis after Augmented-State Markovization

Historical memory and hysteresis answer different questions. Section 6 asks which historical information is needed to close the next-state update. The present section asks whether trajectories close under a declared return or matched-path comparison. A process can therefore be Markovian on an augmented state and still display hysteretic nonclosure in the trust component or in the augmented state itself.

Suppose a recursively closed memory M_c yields the augmented state $\hat{\Theta}_c^{\text{tr},M}$ and exact augmented update in Proposition 6.7. For a repair episode ρ , write

$$\hat{T}_\rho^0 := \hat{\Theta}_c^{\text{tr},M}(h_\rho^0), \quad \hat{T}_\rho^r := \hat{\Theta}_c^{\text{tr},M}(h_\rho^r). \quad (172)$$

Equation 172 makes the retained memory component explicit at the two comparison points.

A return-qualified episode can satisfy

$$\Xi_c(h_\rho^0) = \Xi_c(h_\rho^r) \quad \text{while} \quad \hat{T}_\rho^0 \neq \hat{T}_\rho^r. \quad (173)$$

Equation 173 is fully compatible with first-order augmented dynamics. The earlier history matters because its effects are carried by the current augmented state. Markovization therefore concerns update closure; hysteresis concerns trajectory return properties. Conflating the two would incorrectly treat a successful state augmentation as evidence that history has ceased to matter.

Table 4 summarizes the principal distinctions established in this section.

The hierarchy in Table 4 is important for claim discipline. A repair attempt is an event-level classification; a repair criterion evaluates an outcome; restoration compares states; hysteresis compares trajectories under returned or matched external conditions; and irreversibility quantifies over an admissible continuation family. Each stronger claim therefore requires additional structure beyond the preceding one.

The section therefore separates successful repair from formal cancellation: target satisfaction can coexist with residual displacement, and matched external return can coexist with different terminal trust states. Section 8 studies one dynamical mechanism through which earlier interaction can make such later differences persistent.

8 Early Trust Formation under Basin Dependence

This section studies a conditional dynamical mechanism through which early interactions can remain consequential: distinct early branches may enter different attraction basins under a common later continuation rule. Its objective is to state the additional structures required for basin analysis, define basin-splitting and timing-sensitive witnesses, and distinguish local basin fragility from stronger claims about criticality or general primacy. The method specializes the earlier update architecture only after a dynamically closed state, continuation rule, and topology have been declared.

Research on initial trust motivates treating the pre-interaction condition as a substantive object rather than a neutral numerical origin [18]. Experimental evidence that an early trust breach can have more persistent consequences than a later breach likewise motivates explicit timing comparisons [14]. These sources support asking whether early history can alter later dynamical possibilities. They do not establish that trust relations generally have multiple attractors, sharply separated basins, or a universal primacy effect.

8.1 Dynamical Prerequisites of Basin Analysis

A basin is defined relative to a dynamics. A trust-state carrier by itself therefore does not determine basin structure. The present construction begins by selecting a state representation for which one-step evolution is closed at the resolution used in the basin analysis.

Let $\mathfrak{X}_c$ denote a declared dynamically closed carrier and let

$$\Psi_c : \Gamma_c^{\text{tr}} \longrightarrow \mathfrak{X}_c \quad (174)$$

be its history representation. Equation 174 may instantiate Ψ_c as the trust-state representation Θ_c^{tr} when exact state–record factorization holds, or as the augmented representation $\widehat{\Theta}_c^{\text{tr},M}$ from Equation 130 when explicit recursively closed memory is required. Other state enrichments are admissible when their construction and empirical interpretation are independently justified.

For a selected record family S , let

$$F_{c,S}^{\mathfrak{X}} : \mathcal{D}_{c,S}^{\mathfrak{X}} \longrightarrow \mathfrak{X}_c, \quad \mathcal{D}_{c,S}^{\mathfrak{X}} \subseteq \mathfrak{X}_c \times \mathfrak{Z}_{c,S}^{\text{rec}} \quad (175)$$

be a declared exact update map on the realized state–record inputs relevant to the specialization. Equation 175 is inherited from Equation 76 when $\mathfrak{X}_c = \mathfrak{T}_c$, and from Equation 133 when $\mathfrak{X}_c = \widehat{\mathfrak{T}}_c^M$. The notation therefore packages previously established update structures rather than introducing a new substantive theory of trust.

Standard basin language becomes available only after the post-window continuation rule is also declared. Equation 176 defines that rule. Let

$$\pi_{c,S} : \mathfrak{X}_{c,S}^{\pi} \longrightarrow \mathfrak{Z}_{c,S}^{\text{rec}} \quad (176)$$

be a time-homogeneous continuation rule on a subset $\mathfrak{X}_{c,S}^{\pi} \subseteq \mathfrak{X}_c$ satisfying

$$(x, \pi_{c,S}(x)) \in \mathcal{D}_{c,S}^{\mathfrak{X}} \quad \text{and} \quad F_{c,S}^{\mathfrak{X}}(x, \pi_{c,S}(x)) \in \mathfrak{X}_{c,S}^{\pi} \quad (177)$$

for every $x \in \mathfrak{X}_{c,S}^{\pi}$. Equation 177 makes the chosen post-window domain forward closed under the declared state–record update map. This is a mathematical closure condition at the selected state resolution; it does not yet guarantee that the chosen record is socially feasible from every history compressed into the

same state. When representative histories admit different feasible labels, a state-only continuation rule can be mathematically evaluable while lacking a common history-level implementation. Definition 9.7 later states the corresponding uniform-feasibility condition for a selected policy.

The induced autonomous map is

$$\Phi_{c,S}^\pi(x) := F_{c,S}^\pi(x, \pi_{c,S}(x)), \quad \Phi_{c,S}^\pi : \mathfrak{X}_{c,S}^\pi \longrightarrow \mathfrak{X}_{c,S}^\pi. \quad (178)$$

Equation 178 is a policy-conditioned or regime-conditioned dynamical specialization. It can represent a stabilized interaction protocol, an institutional environment, a repeated behavioral rule, or another fixed post-window continuation scheme. It does not assert that naturally occurring trust dynamics are globally autonomous. Time-varying, stochastic, strategic, or history-dependent continuation rules require a richer construction; the stochastic case is deferred to Section 11.

For $n \in \mathbb{N}_0$, define the iterates recursively by

$$(\Phi_{c,S}^\pi)^0 := \text{id}_{\mathfrak{X}_{c,S}^\pi}, \quad (\Phi_{c,S}^\pi)^{n+1} := \Phi_{c,S}^\pi \circ (\Phi_{c,S}^\pi)^n. \quad (179)$$

In Equation 179, $\text{id}_{\mathfrak{X}_{c,S}^\pi}$ denotes the identity map on the declared post-window state domain. The equation supplies the common later dynamics used throughout the basin comparisons below.

Assumption 8.1 (Declared neighborhood structure). The carrier $\mathfrak{X}_{c,S}^\pi$ is equipped with a topology $\mathcal{O}_{c,S}^\pi$, or with another neighborhood structure from which the topological notions used below are explicitly derived.

Assumption 8.1 supplies structure that is absent from the bare trust-state carrier. A metric specialization is permitted when a substantively meaningful metric has been justified, as already required by Equation 33. Basin conclusions are therefore relative to the selected state resolution, continuation rule, and neighborhood structure.

8.2 Attraction-Basin Regime Membership

The basin construction uses attracting sets rather than presuming a particular technical definition of a dynamical attractor. Let $A \subseteq \mathfrak{X}_{c,S}^\pi$ be nonempty. A state $x \in \mathfrak{X}_{c,S}^\pi$ is said to approach A under $\Phi_{c,S}^\pi$ when

$$\forall U \in \mathcal{O}_{c,S}^\pi \text{ with } A \subseteq U, \quad \exists N \in \mathbb{N}_0 \quad \forall n \geq N, \quad (\Phi_{c,S}^\pi)^n(x) \in U. \quad (180)$$

Equation 180 defines set-wise asymptotic approach through the declared topology. It does not require a scalar notion of “trust level.”

Definition 8.2 (Attracting set and basin). A nonempty closed set $A \subseteq \mathfrak{X}_{c,S}^\pi$ is an *attracting set for the declared specialization* when it is forward invariant,

$$\Phi_{c,S}^\pi(A) \subseteq A, \quad (181)$$

and there exists an open neighborhood $V \in \mathcal{O}_{c,S}^\pi$ with $A \subseteq V$ such that every $x \in V$ satisfies Equation 180. Its basin is

$$\mathcal{B}_{c,S}^\pi(A) := \{x \in \mathfrak{X}_{c,S}^\pi : x \text{ satisfies Equation 180}\}. \quad (182)$$

Equations 181 and 182 respectively state the invariance condition and the resulting basin. Definition 8.2 deliberately uses a modest attracting-set notion. Compactness, minimality, recurrence, smoothness, or stability in a stronger dynamical-systems sense can be added where an application supports them. The term “high-trust attractor,” for example, would require a separately justified readout connecting the attracting set to the substantive interpretation of high trust.

Proposition 8.3 (Forward invariance of basin membership). *If $x \in \mathcal{B}_{c,S}^\pi(A)$, then*

$$(\Phi_{c,S}^\pi)^m(x) \in \mathcal{B}_{c,S}^\pi(A) \quad (183)$$

for every $m \in \mathbb{N}_0$.

Proof. Fix $m \in \mathbb{N}_0$ and an open neighborhood U of A . Because $x \in \mathcal{B}_{c,S}^\pi(A)$, Equation 180 supplies N such that $(\Phi_{c,S}^\pi)^n(x) \in U$ for all $n \geq N$. For the state $y = (\Phi_{c,S}^\pi)^m(x)$, Equation 179 gives $(\Phi_{c,S}^\pi)^n(y) = (\Phi_{c,S}^\pi)^{n+m}(x)$. Taking $n \geq \max\{0, N - m\}$ places the latter state in U . Thus y satisfies Equation 180. $\square$

Equation 183 states the persistence result in Proposition 8.3. The proposition supplies the persistence property needed for early-formation analysis. Once a state lies in a basin, continued application of the same declared post-window dynamics preserves that basin membership, even though the state may continue to change within the basin.

Let J_c index a declared family of attracting sets $\mathcal{A}_{c,S}^\pi = \{A_{c,j} : j \in J_c\}$. Define the basin-membership profile of a state by

$$\beta_{c,S}^\pi(x) := \{j \in J_c : x \in \mathcal{B}_{c,S}^\pi(A_{c,j})\}. \quad (184)$$

Equation 184 remains set-valued because the formalism does not assume that the declared attracting sets or their basins are mutually exclusive. A regime classification becomes single-valued only after the application establishes an appropriate disjointness or selection rule.

8.3 Early-Formation Basin Splitting

Basin dependence concerns the relation between an early interaction path and a later common dynamics. The early path itself need not be generated by $\Phi_{c,S}^\pi$; it can contain interventions, breaches, role changes, or other interactions that precede the common post-window regime.

A length- k formation path is a finite successor chain

$$\eta = (h_0, e_1, h_1, e_2, h_2, \dots, e_k, h_k) \quad (185)$$

with $h_{i-1} \xrightarrow[e_i]{c} h_i$ for every $i \in \{1, \dots, k\}$. Equation 185 derives the early-window object directly from the successor relation in Equation 71. For each stage, write

$$x_i(\eta) := \Psi_c(h_i), \quad z_i(\eta) := \text{Rec}_{c,S}(h_i, e_i) \quad (186)$$

whenever the selected record is defined. Equation 186 separates the represented state reached at a stage from the record of the occurrence that produced that stage.

The ordered record trace of the formation path is

$$\mathbf{z}_{c,S}(\eta) := (z_1(\eta), \dots, z_k(\eta)). \quad (187)$$

Equation 187 is path-local and should not be confused with the more general history trace in Equation 135.

Two length- k formation paths η and η' form a *common-root early comparison* when

$$h_0(\eta) = h_0(\eta') \quad (188)$$

and both endpoints $x_k(\eta)$ and $x_k(\eta')$ lie in $\mathfrak{X}_{c,S}^\pi$. Equation 188 is stronger than matching only the represented initial trust state. It removes differences in the modeled pre-window history at the comparison root; applications that can support only coarser matching should state that weaker design explicitly.

Definition 8.4 (Early basin-splitting witness). A common-root pair (η, η') is an *early basin-splitting witness* for attracting sets $A_{c,j}$ and $A_{c,\ell}$ when

$$x_k(\eta) \in \mathcal{B}_{c,S}^\pi(A_{c,j}), \quad x_k(\eta') \in \mathcal{B}_{c,S}^\pi(A_{c,\ell}), \quad \mathcal{B}_{c,S}^\pi(A_{c,j}) \cap \mathcal{B}_{c,S}^\pi(A_{c,\ell}) = \emptyset. \quad (189)$$

Definition 8.4 gives a precise basin-dependence statement. Two admissible early branches begin from the same modeled history and, after a fixed number of early interactions, enter disjoint basins under the same subsequent continuation rule. The definition does not say that the two early paths are equally likely, normatively equivalent, or observationally indistinguishable.

Proposition 8.5 (Persistence of an early basin split). *If (η, η') satisfies Equation 189, then for every $m \in \mathbb{N}_0$,*

$$(\Phi_{c,S}^\pi)^m(x_k(\eta)) \in \mathcal{B}_{c,S}^\pi(A_{c,j}), \quad (\Phi_{c,S}^\pi)^m(x_k(\eta')) \in \mathcal{B}_{c,S}^\pi(A_{c,\ell}). \quad (190)$$

Proof. Apply Proposition 8.3 separately to the two endpoint states in Equation 189. $\square$

Equation 190 states the two persistent basin memberships. Proposition 8.5 formalizes one mechanism through which an early difference can remain consequential under later similar interaction rules. The later trajectories need not remain far apart in every readout, yet their declared basin memberships cannot merge while the same $\Phi_{c,S}^\pi$ governs both paths and the basins in Equation 189 remain disjoint.

For a pair of common-root formation paths, define the first represented basin-profile divergence, when it exists, by

$$k_{\text{split}}(\eta, \eta') := \min \{i \in \{0, \dots, k\} : \beta_{c,S}^\pi(x_i(\eta)) \neq \beta_{c,S}^\pi(x_i(\eta'))\}. \quad (191)$$

Equation 191 locates the earliest represented stage at which the selected future-regime classification differs. It is a model-relative diagnostic index, not a claim that the corresponding interaction is the unique social cause of the later divergence.

8.4 Timing-Sensitive Basin Membership

An early basin split demonstrates sensitivity to an early branch. A stronger timing claim requires holding more of the interaction content fixed. The ordered traces in Equation 187 permit one such comparison.

For two length- k record traces $\mathbf{z} = (z_1, \dots, z_k)$ and $\mathbf{z}' = (z'_1, \dots, z'_k)$, define permutation equivalence by

$$\mathbf{z} \sim_{\text{perm}} \mathbf{z}' \iff \exists \sigma \in \mathfrak{S}_k \text{ such that } z'_i = z_{\sigma(i)} \text{ for every } i. \quad (192)$$

Here $\mathfrak{S}_k$ denotes the symmetric group on the k record positions. Equation 192 preserves the multiset of represented records while allowing their order to differ. Equality here is equality at the selected record resolution; unrecorded contextual differences remain a possible alternative explanation in empirical applications.

Definition 8.6 (Order-sensitive basin witness). A common-root pair (η, η') is an *order-sensitive basin witness* when

$$\mathbf{z}_{c,S}(\eta) \sim_{\text{perm}} \mathbf{z}_{c,S}(\eta') \quad \text{and} \quad \beta_{c,S}^\pi(x_k(\eta)) \neq \beta_{c,S}^\pi(x_k(\eta')). \quad (193)$$

Definition 8.6 shows that the same represented record content, differently ordered, can be associated with different later attraction profiles in the declared model. It is a sequence-sensitivity result. A substantive primacy claim requires an additional interpretation connecting record positions to an empirical notion of early influence.

A narrower relocation comparison can hold all but two record positions fixed. Let $\sigma_{ij} \in \mathfrak{S}_k$ denote the transposition exchanging positions i and j . Two traces satisfy a single-transposition timing contrast when

$$\mathbf{z}_{c,S}(\eta') = \sigma_{ij} \cdot \mathbf{z}_{c,S}(\eta) \quad (194)$$

for declared positions $1 \leq i < j \leq k$. Equation 194 supplies a particularly transparent comparison when the focal record at one position is moved later while one comparison record is moved earlier.

If Equation 194 holds and the terminal basin profiles differ, the model exhibits timing sensitivity to that relocation at the selected record and state resolutions. The conclusion remains narrower than a general primacy law. Establishing that early placement is systematically more influential than later placement requires repeated contrasts, a declared effect measure, and an empirical or probabilistic design capable of comparing effect magnitudes rather than one basin-switching witness.

8.5 Basin-Boundary Robustness Margins

Basin membership can be robust or fragile to small perturbations only after a meaningful neighborhood or metric structure has been declared. Under Assumption 8.1, the topological boundary of a basin is

$$\partial \mathcal{B}_{c,S}^\pi(A) := \overline{\mathcal{B}_{c,S}^\pi(A)} \cap \overline{\mathfrak{X}_{c,S}^\pi \setminus \mathcal{B}_{c,S}^\pi(A)}. \quad (195)$$

The overline in Equation 195 denotes closure in $\mathcal{O}_{c,S}^\pi$. The equation identifies states whose every sufficiently local topological neighborhood is sensitive to basin membership distinctions.

Suppose additionally that $\mathfrak{X}_{c,S}^\pi$ carries a substantively justified metric $d_{c,S}^\mathfrak{X}$. For $x \in \mathcal{B}_{c,S}^\pi(A)$, define the basin robustness margin by

$$r_{c,S}^\pi(x; A) := \inf \left\{ d_{c,S}^\mathfrak{X}(x, y) : y \in \mathfrak{X}_{c,S}^\pi \setminus \mathcal{B}_{c,S}^\pi(A) \right\}, \quad (196)$$

with $r_{c,S}^\pi(x; A) = +\infty$ when the complement is empty. Equation 196 measures the size of the largest guaranteed local perturbation scale available from the metric and basin geometry. In particular, if

$$0 < \varepsilon < r_{c,S}^\pi(x; A) \quad \text{and} \quad d_{c,S}^\mathfrak{X}(x, y) < \varepsilon, \quad (197)$$

then $y \in \mathcal{B}_{c,S}^\pi(A)$. Equation 197 follows directly from the infimum definition and gives the margin its robustness interpretation.

A small or vanishing margin indicates sensitivity at the selected state geometry. It does not by itself establish dynamical criticality. Criticality may refer to bifurcation under parameter variation, divergent susceptibility, slowing down, scale-free behavior, or another domain-specific criterion; those claims require additional structures beyond basin proximity.

Research Obligation 8.7 (Criticality claim discipline). Any application that describes a trust state or regime as *critical* or *near-critical* should declare the parameterized dynamical family, the mathematical criterion for criticality, the observable or inferential link to the substantive trust claim, and the evidence supporting proximity to the critical set. Basin-boundary proximity alone is insufficient.

Research Obligation 8.7 preserves the distinction between a useful geometric sensitivity diagnostic and a stronger dynamical-systems claim.

8.6 Basin-Constrained Repair Reachability

Basin dependence can constrain later repair under a fixed continuation dynamics, but the scope of that constraint should remain explicit. Let $A_{c,j}$ and $A_{c,\ell}$ have disjoint basins under $\Phi_{c,S}^\pi$. If an early or breach-related episode moves the represented state from the first basin into the second, Proposition 8.3 implies that subsequent iterations of the same $\Phi_{c,S}^\pi$ cannot return the generic dynamical state to the first basin.

Formally, if

$$x^0 \in \mathcal{B}_{c,S}^\pi(A_{c,j}), \quad x^b \in \mathcal{B}_{c,S}^\pi(A_{c,\ell}), \quad \mathcal{B}_{c,S}^\pi(A_{c,j}) \cap \mathcal{B}_{c,S}^\pi(A_{c,\ell}) = \emptyset, \quad (198)$$

then

$$(\Phi_{c,S}^\pi)^n(x^b) \neq x^0 \quad (199)$$

for every $n \in \mathbb{N}_0$. Equation 199 follows because the left-hand state remains in the second basin by Proposition 8.3, while x^0 belongs to the disjoint first basin.

Equation 199 is a fixed-policy reachability result on the generic carrier $\mathfrak{X}_c$. It does not automatically establish exact trust-state irreversibility as defined in Section 7. When $\mathfrak{X}_c$ is augmented, two different augmented states can share the same trust component. Moreover, a repair intervention may change the continuation rule itself. A repair-family irreversibility claim therefore still requires the quantification in Equation 168 or an explicitly related continuation family.

This distinction is substantively important. Basin structure can explain why ordinary continuation under an unchanged interaction regime fails to restore a prior relation, while institutional intervention, changed incentives, altered roles, or a deliberately different repair protocol may move the process across the earlier basin boundary. Basin dependence therefore constrains a declared dynamics; it does not create an ontology of socially permanent trust regimes.

Table 5: Diagnostic distinctions for early-formation and basin-dependence claims.

Finding	Formal requirement	Warranted interpretation
Different early endpoints	Common root with $x_k(\eta) \neq x_k(\eta')$	Early branches generate different represented states; no long-run regime claim follows.
Different basin profiles	Equation 189 or unequal profiles in Equation 184	Early branches alter attraction-region membership under the same declared post-window dynamics.
Order-sensitive basin membership	Equation 193	The ordering of the same represented record content can affect later basin membership at the selected resolution.
Single-relocation timing sensitivity	Equation 194 plus different terminal basin profiles	Relocating a selected record within an otherwise single-transposition comparison changes later regime membership in the model.
Small basin robustness margin	Equation 196 yields a small value	Basin membership is locally fragile in the declared metric; criticality does not follow.
Fixed-policy nonreturn	Equations 198–199	The unchanged post-window rule cannot return the generic state to the baseline across disjoint invariant basins.

8.7 Diagnostic Taxonomy of Basin Findings

The basin formalism supports graded conclusions. Table 5 separates several findings that can otherwise be conflated under the language of “early trust” or “path dependence.”

The distinctions in Table 5 also clarify the status of a primacy claim. Basin splitting can provide a mechanism through which an early difference remains consequential because later common dynamics preserve basin membership. Order-sensitive or relocation comparisons can show that timing matters within the model. A general statement that early interactions are more influential than later interactions requires additional comparative evidence about effect magnitude, prevalence, and boundary conditions.

Basin dependence is conditional on the declared closed state, continuation rule, topology, and attracting sets. Under those assumptions, early branches can alter later regime membership; institutional and material constraints can in turn restrict which branches or basin-crossing interventions are feasible. Section 9 makes that feasibility interface explicit.

9 Power-Constrained Trust Dynamics

This section introduces power-relevant conditions as constraints on historical trust dynamics rather than as a scalar component of trust. It separates two channels: *constraint selection*, which changes the feasible interaction, observation, repair, or continuation set, and *constraint responsiveness*, in which an independently identified constraint profile carries additional transition-relevant information. Set-valued feasibility and factorization tests extend the existing successor, record, repair, and basin structures while leaving substantive theories of power to the application domain.

The scope is deliberately restrained. Classical power-dependence analysis already emphasizes that power is relational and that dependence turns partly on the value and availability of alternatives [3]. Role authority, resource control, material dependence, sanction capacity, unequal exit options, institutional permissions, and control over evidence can therefore be power-relevant in particular applications, but the formalism does not infer power merely from any difference between two feasible sets. A substantive application must identify which constraint is being represented and why it warrants a power interpretation. This distinction is especially important for trust because reliance or compliance may occur under coercion, lack of alternatives, contractual duty, or strategic necessity and therefore cannot be equated with the represented trust state; Section 2 already separates those objects.

9.1 Power-Relevant Constraint Profiles and Feasible Successors

The successor relation in Equation 70 records the interaction extensions admitted by the declared history model. Power-relevant constraints can be represented as an additional restriction on that baseline process space. For a history $h \in \Gamma_c^{\text{tr}}$, define its baseline one-step successor set by Equation 200.

$$\text{Succ}_c(h) := \left\{ (e, h^+) \in \mathcal{E}_c^* \times \Gamma_c^{\text{tr}} : h \xrightarrow[e]{c} h^+ \right\}. \quad (200)$$

Equation 200 contains only extensions already admissible under the underlying interaction-history representation. The present section therefore does not use power language to enlarge the process space by fiat.

Let $\mathcal{R}_c^{\text{pc}}$ be a declared carrier of *power-relevant constraint profiles*, and let Equation 201 define the profile associated with each history.

$$\text{PC}_c : \Gamma_c^{\text{tr}} \longrightarrow \mathcal{R}_c^{\text{pc}}. \quad (201)$$

An element of $\mathcal{R}_c^{\text{pc}}$ may retain role permissions, institutional authority, resource dependencies, sanction exposure, exit availability, monitoring access, or other independently specified constraints relevant to the application. Equation 201 does not impose an ordering, metric, scalar magnitude, or universal decomposition on those profiles. In particular, $\text{PC}_c(h)$ is a constraint descriptor rather than a general measure of how much power an actor possesses.

Research Obligation 9.1 (Constraint-profile provenance). Every component of $\text{PC}_c(h)$ used to restrict or explain a trust transition should have an identification rule that does not depend on the sign or magnitude of the resulting trust-state change. A power interpretation additionally requires a substantive account of the relevant role, authority, dependence, resource, sanction, exit, or institutional mechanism.

Obligation 9.1 is the constraint analogue of the pre-update provenance rule in Obligation 4.1. It prevents a circular coding practice in which a condition is called powerful merely because it is followed by a large trust effect.

Equation 202 defines the feasible-successor correspondence for a history and candidate constraint profile.

$$\text{Feas}_c : \Gamma_c^{\text{tr}} \times \mathcal{R}_c^{\text{pc}} \longrightarrow 2^{\mathcal{E}_c^* \times \Gamma_c^{\text{tr}}}, \quad \text{Feas}_c(h, k) \subseteq \text{Succ}_c(h). \quad (202)$$

Equation 202 allows a constraint profile to remove otherwise represented continuations while preserving the baseline typing supplied by Equation 200. The realized power-constrained successor relation is defined in Equation 203.

$$h \xrightarrow[e]{c, \text{pc}} h^+ \iff (e, h^+) \in \text{Feas}_c(h, \text{PC}_c(h)). \quad (203)$$

Equation 203 is therefore a subrelation of the baseline successor relation. It can represent, for example, a role rule that makes one response unavailable, a dependence relation that removes a credible exit, or an institutional authority structure that permits one class of intervention while excluding another. Whether such a restriction is normatively justified, coercive, legitimate, or unjust remains a separate question.

For a selected record family S , the corresponding realized constrained update domain is

$$\mathcal{U}_{c,S}^{\text{pc}} := \left\{ (h, e, h^+) \in \mathcal{U}_{c,S} : h \xrightarrow[e]{c, \text{pc}} h^+ \right\}. \quad (204)$$

Equation 204 changes the domain on which trust updates are realized; it does not yet change the update law itself.

Proposition 9.2 (Exact update sufficiency under feasible-domain restriction). *If $(\Theta_c^{\text{tr}}, \text{Rec}_{c,S})$ is exactly update-sufficient on $\mathcal{U}_{c,S}$, then its restriction to $\mathcal{U}_{c,S}^{\text{pc}}$ is exactly update-sufficient.*

Proof. Exact update sufficiency on $\mathcal{U}_{c,S}$ provides the factorization in Equation 77. Restricting both $\text{In}_{c,S}$ and $\text{Next}_{c,S}$ to the subset in Equation 204 preserves the same equality with the same update map on the realized restricted image. $\square$

Proposition 9.2 separates selection from transition specification. Removing feasible paths can alter which trust trajectories occur without creating a new deterministic update defect on paths that remain. A claim that power changes the trust effect of an otherwise matched occurrence therefore requires more than feasible-set restriction alone.

9.2 Constraint-Responsive Trust Updates

A power-relevant condition can also carry transition information. To test that possibility without building it into the event label, let $u = (h, e, h^+) \in \mathcal{U}_{c,S}^{\text{pc}}$ and define its pre-update constraint descriptor by Equation 205.

$$\kappa^-(u) := \text{PC}_c(h). \quad (205)$$

Equation 205 uses only the pre-extension history. The descriptor can overlap with role or exposure fields already retained in $z_S(u)$; whether it adds information is therefore an empirical and representational question rather than an assumption.

The constraint-augmented input signature is defined in Equation 206.

$$\text{In}_{c,S}^{\text{pc}}(u) := (\tau^-(u), z_S(u), \kappa^-(u)). \quad (206)$$

Equation 206 is a refinement of the state–record input in Equation 74. Its residual defect set is

$$\Delta_{c,S}^{\text{upd|pc}} := \left\{ (u, v) \in (\mathcal{U}_{c,S}^{\text{pc}})^2 : \text{In}_{c,S}^{\text{pc}}(u) = \text{In}_{c,S}^{\text{pc}}(v), \tau^+(u) \neq \tau^+(v) \right\}. \quad (207)$$

Equation 207 asks whether the represented next state is determined once the selected state, event record, and independently identified constraint profile are jointly fixed.

Because Equation 206 refines the original input signature, the refinement logic of Proposition 5.6 yields Equation 208 on the constrained update domain.

$$\Delta_{c,S}^{\text{upd|pc}} \subseteq \Delta_{c,S}^{\text{upd}} \cap (\mathcal{U}_{c,S}^{\text{pc}})^2. \quad (208)$$

Equation 208 states only that adding information cannot create an equal-input/different-output pair that was absent at the coarser input resolution.

Definition 9.3 (Constraint-resolving update interface). The power-relevant descriptor PC_c is *constraint-resolving at the selected update interface* when

$$\Delta_{c,S}^{\text{upd}} \cap (\mathcal{U}_{c,S}^{\text{pc}})^2 \neq \emptyset, \quad \Delta_{c,S}^{\text{upd|pc}} = \emptyset. \quad (209)$$

Equation 209 gives the operative condition in Definition 9.3. It means that some update ambiguity present at the selected state–record resolution disappears when the pre-update constraint profile is retained. The finding supports a claim of representational relevance for that descriptor. It does not by itself establish that the constraint profile causally produces the trust difference, because the descriptor may proxy for omitted variables or jointly generated historical structure.

A narrower diagnostic isolates pairs in which the nominal state–record interface is matched while the constraint descriptor and next state differ. Define the constraint-sensitive witness set by Equation 210.

$$\mathcal{W}_{c,S}^{\text{pc}} := \left\{ (u, v) \in (\mathcal{U}_{c,S}^{\text{pc}})^2 : \tau^-(u) = \tau^-(v), z_S(u) = z_S(v), \kappa^-(u) \neq \kappa^-(v), \tau^+(u) \neq \tau^+(v) \right\}. \quad (210)$$

A nonempty set in Equation 210 shows that the selected state–record representation does not absorb every transition-relevant distinction correlated with the declared constraint profile. The interpretation remains conditional on Obligation 9.1 and on the adequacy of the matching variables.

Research Obligation 9.4 (Causal restraint for constraint-sensitive transitions). A nonempty witness set in Equation 210, or successful defect resolution in Equation 209, should be described as a representational or predictive dependence unless the application supplies a separate causal design or substantive identification argument.

Obligation 9.4 preserves the distinction between a constraint-aware dynamical representation and a causal theory of power effects.

9.3 Visibility Constraints and Evidential Coarsening

Power-relevant structure can matter even when it does not alter the underlying occurrence. Institutional control over records, unequal monitoring access, selective disclosure, or asymmetric verification can change which event distinctions are available to an observer or analyst. The formal issue is therefore one of record resolution.

Fix an observation regime ω and let Equation 211 define a visibility map on the selected record carrier.

$$\text{Vis}_{c,S}^\omega : \mathfrak{Z}_{c,S}^{\text{rec}} \longrightarrow \mathfrak{Y}_{c,S}^\omega. \quad (211)$$

Equation 211 may be injective or many-to-one. When a particular institution or power relation determines which ω is available, that determination belongs to the substantive application; the mathematical map alone does not establish a power relation.

For $u \in \mathcal{U}_{c,S}^{\text{pc}}$, define the visible record and visible update input by Equation 212.

$$\tilde{z}_S^\omega(u) := \text{Vis}_{c,S}^\omega(z_S(u)), \quad \tilde{\text{In}}_{c,S}^\omega(u) := (\tau^-(u), \tilde{z}_S^\omega(u)). \quad (212)$$

The corresponding visible-input defect set is

$$\tilde{\Delta}_{c,S}^{\text{upd},\omega} := \left\{ (u, v) \in (\mathcal{U}_{c,S}^{\text{pc}})^2 : \tilde{\text{In}}_{c,S}^\omega(u) = \tilde{\text{In}}_{c,S}^\omega(v), \tau^+(u) \neq \tau^+(v) \right\}. \quad (213)$$

Equation 213 records ambiguity created or preserved at the observer's accessible record resolution.

Proposition 9.5 (Visibility-coarsening defect monotonicity). *For a fixed observation regime ω ,*

$$\Delta_{c,S}^{\text{upd}} \cap (\mathcal{U}_{c,S}^{\text{pc}})^2 \subseteq \tilde{\Delta}_{c,S}^{\text{upd},\omega}. \quad (214)$$

Proof. If (u, v) lies in the left-hand side of Equation 214, then $\tau^-(u) = \tau^-(v)$, $z_S(u) = z_S(v)$, and $\tau^+(u) \neq \tau^+(v)$. Equality of the full selected records implies equality after applying the common map $\text{Vis}_{c,S}^\omega$. Hence the visible inputs in Equation 212 are equal while the represented next states differ, placing (u, v) in the right-hand defect set. $\square$

Proposition 9.5 does not say that visibility loss necessarily creates a defect. It states that coarsening cannot repair an update ambiguity already present at a finer record resolution. A many-to-one visibility map can additionally merge records whose trust effects differ, producing a nonempty visible defect even when the selected full-record interface is exact. In an empirical application, apparent historical unpredictability may therefore arise from evidence access rather than from long memory alone.

9.4 Constraint-Restricted Repair Reachability

Section 7 defined irreversibility relative to a declared repair family. Power-relevant constraints can shrink that family by making some remedies, exits, disclosures, institutional appeals, or compensatory actions unavailable. To express that restriction, let $\preceq_c^{\text{pc}}$ denote the reflexive-transitive closure of the realized constrained successor relation in Equation 203. Equation 215 defines the constrained repair family for an episode ρ .

$$\mathcal{C}_c^{\text{rep,pc}}(h_\rho^b, b_\rho) := \mathcal{C}_c^{\text{rep}}(h_\rho^b, b_\rho) \cap \left\{ g \in \Gamma_c^{\text{tr}} : h_\rho^b \preceq_c^{\text{pc}} g \right\}. \quad (215)$$

Equation 215 retains repair continuations reachable through the declared constrained process space.

The corresponding trust-state reachability set is

$$\text{Reach}_c^{\text{rep,pc}}(h_\rho^b, b_\rho) := \left\{ \Theta_c^{\text{tr}}(g) : g \in \mathcal{C}_c^{\text{rep,pc}}(h_\rho^b, b_\rho) \right\}. \quad (216)$$

Equation 216 is a constrained counterpart of Equation 167.

Proposition 9.6 (Reachability contraction under power constraints). *The constrained repair reachability set satisfies*

$$\text{Reach}_c^{\text{rep}, \text{pc}}(h_\rho^b, b_\rho) \subseteq \text{Reach}_c^{\text{rep}}(h_\rho^b, b_\rho). \quad (217)$$

Consequently, exact unreachability of T_ρ^0 in the broader repair family implies exact unreachability in the constrained family, whereas the converse implication does not follow without additional assumptions.

Proof. Equation 215 is a subset of Equation 166. Applying the common history-to-trust map Θ_c^{tr} yields Equation 217. If the baseline is absent from the larger reachable set, it is absent from every subset. Its absence from the constrained subset leaves open whether an excluded but otherwise admissible repair continuation reaches it. $\square$

Proposition 9.6 sharpens the scope discipline for irreversibility. A relation may be effectively unable to restore a baseline under current institutional or resource constraints even though restoration remains reachable in a broader process family. Calling that finding socially or metaphysically irreversible would exceed the quantified continuation space.

A useful comparison between two constraint profiles $k_1, k_2 \in \mathcal{R}_c^{\text{pc}}$ can therefore focus on the histories they make reachable rather than on a scalar ranking of “power.” If one independently justified constraint regime removes every repair path to a declared target that remains reachable under another, the formal result concerns asymmetric repair opportunity. The normative evaluation of that asymmetry requires additional criteria.

9.5 Constraint-Compatible Continuation Rules and Basin Analysis

The basin construction in Section 8 assumes a state-based continuation rule $\pi_{c,S}$. A power-constrained application should verify that the records prescribed by that rule are feasible for every history compressed into the represented state. Otherwise the autonomous map can silently use a continuation that some histories represented by the same state cannot realize.

For $h \in \Gamma_c^{\text{tr}}$, define the feasible selected-record set by Equation 218.

$$\mathfrak{Z}_{c,S}^{\text{pc}}(h) := \{ \text{Rec}_{c,S}(h^+, e) : (e, h^+) \in \text{Feas}_c(h, \text{PC}_c(h)), \text{Rec}_{c,S}(h^+, e) \text{ defined} \}. \quad (218)$$

Equation 218 records which selected inputs can actually be generated from a particular history under the declared constraint profile.

Let $\Psi_c : \Gamma_c^{\text{tr}} \rightarrow \mathfrak{X}_c$ be the dynamically closed representation used for basin analysis in Equation 174. For a represented state $x \in \text{Im}(\Psi_c)$, define the uniformly feasible record set by Equation 219.

$$\mathfrak{Z}_{c,S}^{\text{unif}, \text{pc}}(x) := \bigcap_{\substack{h \in \Gamma_c^{\text{tr}} \\ \Psi_c(h) = x}} \mathfrak{Z}_{c,S}^{\text{pc}}(h). \quad (219)$$

Equation 219 takes an intersection because a state-only policy should prescribe a record that remains feasible for every history the state representation treats as equivalent. An empty intersection indicates that the selected state has compressed distinctions relevant to feasible continuation.

Definition 9.7 (Constraint-compatible state-based continuation rule). A post-window rule $\pi_{c,S} : \mathfrak{X}_{c,S}^\pi \rightarrow \mathfrak{Z}_{c,S}^{\text{rec}}$ is *constraint-compatible at the state resolution Ψ_c* when

$$\pi_{c,S}(x) \in \mathfrak{Z}_{c,S}^{\text{unif}, \text{pc}}(x) \quad (220)$$

for every $x \in \mathfrak{X}_{c,S}^\pi \cap \text{Im}(\Psi_c)$.

Equation 220 states the condition in Definition 9.7. If it fails, the induced autonomous map in Equation 178 may remain mathematically well defined while lacking a common feasible implementation across histories collapsed into the same state. The appropriate response is application-dependent: refine

Table 6: Diagnostic distinctions for power-constrained trust dynamics.

Finding	Formal requirement	Warranted interpretation
Feasible-path restriction	Equations 202–204	The declared constraint profile removes some baseline continuations; no change in the trust-update law follows by itself.
Constraint-resolving input	Equation 209	Retaining the pre-update constraint profile resolves ambiguity at the selected state–record interface; causal power attribution requires further identification.
Constraint-sensitive pair	Nonempty set in Equation 210	Matched state–record inputs can have different represented outcomes across different constraint profiles at the selected resolution.
Visibility coarsening	Equations 211–214	Restricted evidence access can preserve or enlarge update ambiguity; long memory need not be the source of the apparent unpredictability.
Repair-family contraction	Equation 217	Current constraints reduce represented repair reachability; constrained nonreturn remains weaker than unreachability in a broader family.
State-policy incompatibility	Failure of Equation 220	A state-only continuation rule prescribes records unavailable to at least one history represented by that state; the dynamical state or policy representation requires refinement.

the state, retain the relevant constraint profile as part of an augmented state, restrict the policy domain, or use a history-dependent or set-valued continuation model. The condition is policy-specific: it requires only the record actually selected by $\pi_{c,S}$ to be uniformly feasible. By contrast, state-invariant record feasibility in Equation 230 requires equality of the entire locally realizable record set and is therefore the stronger condition needed for a label-complete state-level process quotient.

Research Obligation 9.8 (Feasibility audit for power-conditioned basin claims). A basin, nonreturn, or basin-crossing claim interpreted under power or institutional constraints should verify Equation 220, or explicitly replace the state-only continuation rule with a richer representation that retains the distinctions governing feasibility.

Obligation 9.8 prevents a model from describing a basin-crossing intervention as available merely because the state-space geometry contains a path. Social reachability depends on the admissible process family as well as on the geometry of the state representation.

9.6 Diagnostic Taxonomy of Power-Constrained Findings

The formal constructions support several distinct conclusions. Table 6 separates those conclusions so that feasible-set asymmetry, transition dependence, evidence loss, and repair restriction are not collapsed into a single undifferentiated notion of power.

Table 6 also clarifies what the section does not establish. None of the listed findings supplies a universal ordering of actors by power, a normative judgment about legitimacy, or a claim that constrained behavior reveals the underlying trust state. The formalism instead locates where independently identified power-relevant conditions enter a historical trust model: the feasible process space, the information retained by the update interface, the evidence available to observers, the repair family, and the continuation rules used in longer-run dynamics.

Power constraints therefore act on reachability, information, and feasible continuation without being absorbed into trust itself. A nominal trajectory or inverse descriptor may exist even when the corresponding social continuation fails Obligation 9.8. Section 10 carries that partiality into the compositional process representation.

10 Process-Algebraic Representation of Trust Dynamics

This section derives a compositional process representation from the successor, feasibility, update, memory, and repair structures already established. It first constructs a labeled history path category, then states the congruence required for compression to trust states, separates deterministic partial-map semantics from relation-valued semantics, and finally locates repair, power constraints, and trajectory descriptors within that architecture. The term *process-algebraic* refers here to typed identity, sequential composition, partial transformations, relations, and generated process families rather than to a claim that one established process-algebra formalism is uniquely appropriate for trust.

10.1 Labeled Trust-Process Steps

The successor relation in Equation 70 records admissible history extension, whereas the selected record map in Section 4 supplies the trust-relevant label attached to that extension. Combining the two produces a labeled transition relation on histories. For a selected record family S , define

$$h \xrightarrow[z]{c,S} h^+ \quad (221)$$

when there exists $e \in \mathcal{E}_c^\star$ such that $(h, e, h^+) \in \mathcal{U}_{c,S}$ and $\text{Rec}_{c,S}(h^+, e) = z$. Equation 221 forgets the event token after its selected record has been formed while retaining the admissible history endpoints.

For each history $h \in \Gamma_c^{\text{tr}}$, define the locally realizable record set by

$$\mathfrak{Z}_{c,S}^{\text{loc}}(h) := \left\{ z \in \mathfrak{Z}_{c,S}^{\text{rec}} : \exists h^+ \in \Gamma_c^{\text{tr}} \text{ such that } h \xrightarrow[z]{c,S} h^+ \right\}. \quad (222)$$

Equation 222 is a history-level feasibility object. Two histories with the same represented trust state can therefore have different locally realizable record sets even when every record that is realizable from both histories produces the same represented next state.

A finite realized process path of length $n \in \mathbb{N}$ is a sequence

$$\eta = (h_0, z_1, h_1, z_2, \dots, z_n, h_n) \quad (223)$$

such that $h_{i-1} \xrightarrow[z_i]{c,S} h_i$ for every $i \in \{1, \dots, n\}$. Equation 223 preserves the ordered record labels together with the history endpoints that make their sequential realization admissible. For every $h \in \Gamma_c^{\text{tr}}$, let $\mathbf{1}_h$ denote the length-zero path at h .

The source, target, and record trace of a finite process path are defined by

$$\text{src}(\eta) := h_0, \quad \text{tgt}(\eta) := h_n, \quad \text{tr}_S(\eta) := (z_1, \dots, z_n). \quad (224)$$

Equation 224 keeps endpoint information separate from the visible trace. Distinct realized paths can share the same trace and the same represented trust endpoints while differing in histories, event tokens, latent variables, institutional conditions, or other retained structure.

10.2 The History Process Category

Sequential composition is available when the target history of one realized path is exactly the source history of another. If $\eta = (h_0, z_1, h_1, \dots, z_n, h_n)$ and $\xi = (h_n, y_1, k_1, \dots, y_m, k_m)$ are realized paths, define their concatenation by

$$\xi \diamond \eta := (h_0, z_1, h_1, \dots, z_n, h_n, y_1, k_1, \dots, y_m, k_m). \quad (225)$$

Equation 225 is defined only for endpoint-compatible pairs. Its record trace satisfies

$$\text{tr}_S(\xi \diamond \eta) = \text{tr}_S(\eta) \parallel \text{tr}_S(\xi), \quad (226)$$

where $\parallel$ denotes concatenation of finite ordered record sequences. Equation 226 fixes the temporal order convention: the labels of η occur before the labels of ξ .

Proposition 10.1 (History-path category). *Let the objects be histories in Γ_c^{tr} , and let the morphisms $h \rightarrow g$ be finite realized paths from h to g , including the length-zero paths. With composition $\diamond$ from Equation 225, these objects and morphisms form a category, denoted $\mathcal{P}_{c,S}^{\text{hist}}$.*

Proof. Endpoint-compatible path concatenation is associative because adjoining three finite step sequences yields the same ordered sequence under either parenthesization. For each history h , the zero-length path $\mathbf{1}_h$ leaves every composable path unchanged on the corresponding side. Thus associativity and identity laws hold on the declared path domain. $\square$

Proposition 10.1 requires no assumption that the original history carrier admits arbitrary concatenation. The category is generated by the declared successor edges themselves. In this sense, the compositional structure is recovered from admissible local extension rather than imposed on Γ_c^{tr} as a total binary operation.

The history-to-trust map Θ_c^{tr} assigns a represented state to every object of $\mathcal{P}_{c,S}^{\text{hist}}$. It does not automatically define a category whose objects are trust states, because histories mapped to the same state can possess different labeled futures. The next subsection states the additional condition required for a state-level quotient.

10.3 Transition Congruence for Trust-State Compression

Define history equivalence at the represented trust-state resolution by

$$h \approx_{\Theta} h' \iff \Theta_c^{\text{tr}}(h) = \Theta_c^{\text{tr}}(h'). \quad (227)$$

Equation 227 is the object-level compression already implicit in the trust-state representation. A compositional quotient requires this equivalence to respect labeled successors in both directions.

Definition 10.2 (Deterministic trust-process transition congruence). The equivalence $\approx_{\Theta}$ is a *deterministic trust-process transition congruence* for record family S when, for every $h \approx_{\Theta} h'$ and every $z \in \mathfrak{Z}_{c,S}^{\text{rec}}$, the following two conditions hold. First,

$$z \in \mathfrak{Z}_{c,S}^{\text{loc}}(h) \iff z \in \mathfrak{Z}_{c,S}^{\text{loc}}(h'). \quad (228)$$

Second, whenever $h \xrightarrow[z]{c,S} g$ and $h' \xrightarrow[z]{c,S} g'$,

$$g \approx_{\Theta} g'. \quad (229)$$

Equations 228–229 require histories compressed into the same trust state to admit the same next record labels and require every realized successor under a common label to enter the same represented trust-state class. Because the definition also applies when $h = h'$, multiple history-level successors carrying the same label may remain, but they cannot disagree at the represented trust-state resolution. The condition therefore contains both a feasibility component and a deterministic represented-outcome component.

The feasibility component can be isolated through the local record sets in Equation 222. Call record feasibility *state-invariant* when

$$h \approx_{\Theta} h' \implies \mathfrak{Z}_{c,S}^{\text{loc}}(h) = \mathfrak{Z}_{c,S}^{\text{loc}}(h'). \quad (230)$$

Equation 230 is stronger than the exact update-factorization criterion from Section 5: factorization compares outcomes only when matching state–record inputs are already realized, whereas state-invariant feasibility requires the matching record to be realizable from every history represented by that state.

For each record label $z \in \mathfrak{Z}_{c,S}^{\text{rec}}$, define its represented feasibility domain by

$$D_{c,S}(z) := \left\{ T \in \text{Im}(\Theta_c^{\text{tr}}) : \exists h \in \Gamma_c^{\text{tr}} \text{ with } \Theta_c^{\text{tr}}(h) = T \text{ and } z \in \mathfrak{Z}_{c,S}^{\text{loc}}(h) \right\}. \quad (231)$$

Equation 231 is defined independently of deterministic outcome factorization. Under state-invariant feasibility, membership in $D_{c,S}(z)$ is equivalent to feasibility of z from every history represented by T ; without that condition it records only existential feasibility at the compressed state resolution.

Proposition 10.3 (Factorization and feasibility criterion for process congruence). *Suppose the selected record family S is fixed. Then $\approx_\Theta$ is a deterministic trust-process transition congruence if and only if both of the following conditions hold:*

1. *exact trust-state update sufficiency holds on $\mathcal{U}_{c,S}$ as in Definition 5.1;*
2. *record feasibility is state-invariant as in Equation 230.*

Proof. Assume exact update sufficiency and state-invariant record feasibility. Equation 230 gives Equation 228. If $h \approx_\Theta h'$ and both $h \xrightarrow[z]{c,S} g$ and $h' \xrightarrow[z]{c,S} g'$ are realized, the two transitions have the same represented pre-state and selected record. Exact update sufficiency therefore gives $\Theta_c^{\text{tr}}(g) = \Theta_c^{\text{tr}}(g')$, which is Equation 229.

Conversely, Equation 228 is exactly state-invariant record feasibility. To prove exact update sufficiency, take any two realized update triples with equal represented pre-states and equal selected records. Their pre-histories satisfy $h \approx_\Theta h'$, and their post-histories are labeled successors under the same z . Equation 229 therefore makes the represented post-states equal, which is the factorization criterion in Proposition 5.2. $\square$

Proposition 10.3 separates two reasons why a trust-state quotient can fail. A nonempty update defect from Equation 83 is an *outcome inconsistency*: the same represented state and record can lead to different represented next states. Failure of Equation 230 is a *feasibility inconsistency*: histories represented by the same trust state do not admit the same next record labels. Either failure prevents the represented trust state from supporting a complete state-only process calculus at the chosen resolution. The criterion is representation-generic. If a richer closed representation is used, such as the augmented state in Equation 130, the same quotient test applies after replacing Θ_c^{tr} by that representation and re-establishing both represented-outcome sufficiency and label-feasibility invariance. Augmented one-step closure alone therefore does not imply process congruence.

Research Obligation 10.4 (State-level process quotient audit). A claim that trust dynamics can be represented compositionally on the trust-state carrier alone should verify both exact update sufficiency and state-invariant record feasibility, or state explicitly which richer state, memory, constraint, or history representation replaces them.

Obligation 10.4 extends the one-step sufficiency audit of Section 5. It prevents deterministic state-transition notation from concealing histories that share the same trust state yet possess different feasible futures.

10.4 State-Congruent Partial Transformation Semantics

Suppose the conditions of Proposition 10.3 hold. Use the feasibility domain $D_{c,S}(z)$ from Equation 231. State-invariant feasibility gives the domain a representative-independent interpretation, and exact update sufficiency defines a partial state transformation

$$F_{c,S}^z : D_{c,S}(z) \longrightarrow \text{Im}(\Theta_c^{\text{tr}}), \quad F_{c,S}^z(T) := \Theta_c^{\text{tr}}(h^+), \quad (232)$$

where $h \xrightarrow[z]{c,S} h^+$ and $\Theta_c^{\text{tr}}(h) = T$. Proposition 10.3 makes the right-hand side of Equation 232 independent of the selected representative history and successor.

For a finite record word $w = (z_1, \dots, z_n)$, define its partial transformation recursively by

$$F_{c,S}^w := F_{c,S}^{z_n} \circ \dots \circ F_{c,S}^{z_1}, \quad (233)$$

with domain consisting of those initial states for which every successive partial map in Equation 233 is defined. For the empty word ϵ , define

$$F_{c,S}^\epsilon := \text{id}_{\text{Im}(\Theta_c^{\text{tr}})}. \quad (234)$$

If w is followed by v , the corresponding partial transformations satisfy

$$F_{c,S}^{w\|v} = F_{c,S}^v \circ F_{c,S}^w \quad (235)$$

on the natural domain of the composite. Equations 233–235 convert sequential record traces into partial endomorphisms of the represented trust-state carrier while preserving the temporal concatenation convention.

Let

$$\mathfrak{M}_{c,S}^{\text{tr}} := \{F_{c,S}^w : w \in (\mathfrak{Z}_{c,S}^{\text{rec}})^{<\omega}\} \quad (236)$$

where transformations agreeing as partial functions are identified. Equation 235 shows that the set in Equation 236 is closed under partial-function composition, while Equation 234 supplies the identity. It is therefore a generated partial-transformation monoid on the represented state carrier, with domains carrying the admissibility information.

Proposition 10.5 (Trace semantics under process congruence). *Let η be a realized path with trace $w = \text{tr}_S(\eta)$. Under trust-process transition congruence,*

$$F_{c,S}^w(\Theta_c^{\text{tr}}(\text{src}(\eta))) = \Theta_c^{\text{tr}}(\text{tgt}(\eta)). \quad (237)$$

Proof. Apply Equation 232 successively along the labeled steps of η . Each intermediate represented state is the image of the corresponding intermediate history. The recursive composition in Equation 233 therefore yields the represented terminal state. $\square$

Equation 237 states the endpoint identity established by Proposition 10.5. The proposition gives a genuine state-level compositional semantics only under the congruence conditions. It is stronger than writing a formal composite of one-step update symbols: the represented record word must remain realizable through the compressed state sequence.

10.5 Relation-Valued Semantics when State Compression Is Incomplete

When exact update sufficiency or state-invariant feasibility fails, a deterministic partial map $F_{c,S}^z$ is not justified at the selected state resolution. A relation-valued representation remains available without forcing hidden distinctions into a single output. For each record z , define the realized atomic transition relation

$$\mathcal{R}_{c,S}(z) := \left\{ (T, T^+) \in \mathfrak{T}_c \times \mathfrak{T}_c : \exists h, h^+ \in \Gamma_c^{\text{tr}} \text{ such that } h \xrightarrow[z]{c,S} h^+, \Theta_c^{\text{tr}}(h) = T, \Theta_c^{\text{tr}}(h^+) = T^+ \right\}. \quad (238)$$

Equation 238 represents every observed or modeled state transition associated with label z without asserting functional uniqueness.

Relational composition gives an abstract state-level sequential semantics. For $w = (z_1, \dots, z_n)$, define

$$\mathcal{R}_{c,S}^\circ(w) := \mathcal{R}_{c,S}(z_n) \circ \dots \circ \mathcal{R}_{c,S}(z_1), \quad (239)$$

where $\circ$ is ordinary relation composition. For the empty word, define

$$\mathcal{R}_{c,S}^\circ(\epsilon) := \text{Id}_{\text{Im}(\Theta_c^{\text{tr}})}. \quad (240)$$

Equations 239–240 require only matching intermediate represented trust states.

The history-realizable relation for the same word is narrower. Define

$$\mathcal{R}_{c,S}^{\text{real}}(w) := \left\{ (\Theta_c^{\text{tr}}(\text{src}(\eta)), \Theta_c^{\text{tr}}(\text{tgt}(\eta))) : \eta \in \text{Mor}(\mathcal{P}_{c,S}^{\text{hist}}, \text{tr}_S(\eta) = w) \right\}. \quad (241)$$

Equation 241 requires one compatible history path to realize the entire record sequence. Because the morphisms of $\mathcal{P}_{c,S}^{\text{hist}}$ include the zero-length paths, it also gives $\mathcal{R}_{c,S}^{\text{real}}(\epsilon) = \text{Id}_{\text{Im}(\Theta_c^{\text{tr}})}$.

Proposition 10.6 (Abstract composition can over-approximate realizable process semantics). *For every finite record word w ,*

$$\mathcal{R}_{c,S}^{\text{real}}(w) \subseteq \mathcal{R}_{c,S}^{\circ}(w). \quad (242)$$

The inclusion in Equation 242 can be strict when represented intermediate state equality permits relation composition across histories whose next-step possibilities are incompatible.

Proof. A realized history path supplies, at every adjacent pair of represented states, a member of the corresponding atomic relation in Equation 238. Therefore its endpoints belong to the relational composite, giving the inclusion. Strictness is possible when two atomic transitions share an intermediate represented state but no single intermediate history supports both adjacent steps; relation composition sees only the represented state equality and can concatenate them abstractly. $\square$

Proposition 10.6 identifies a second compositional hazard beyond update nonuniqueness. Even relation-valued state semantics can generate spurious multi-step possibilities when state compression erases constraints on which transitions can follow which histories. History-level path semantics remains the conservative reference representation in that case.

Under transition congruence, the over-approximation gap closes exactly. For a partial function F , write $\text{Gr}(F) := \{(x, F(x)) : x \in \text{Dom}(F)\}$ for its graph.

Proposition 10.7 (Relational exactness under transition congruence). *If Definition 10.2 holds, then for every finite record word w ,*

$$\mathcal{R}_{c,S}^{\text{real}}(w) = \mathcal{R}_{c,S}^{\circ}(w) = \text{Gr}(F_{c,S}^w). \quad (243)$$

Proof. For a one-label word, transition congruence makes $\mathcal{R}_{c,S}(z)$ the graph of the well-defined partial map $F_{c,S}^z$. The empty-word equality follows from Equations 234, 240, and 241. For a longer word, suppose an abstract relational composite reaches an intermediate represented state. State-invariant feasibility allows the next label to be realized from every history representing that state, and outcome congruence places every such successor in the unique represented state prescribed by the corresponding partial map. Induction therefore lifts the entire abstractly composable word to a realized history path and identifies its endpoint with $F_{c,S}^w$. Proposition 10.6 supplies the reverse inclusion for the realized relation. $\square$

Proposition 10.7, summarized by Equation 243, makes precise the sense in which the partial-map construction is the deterministic specialization of relation-valued semantics: transition congruence removes both outcome ambiguity and history-incompatible abstract compositions at the represented state resolution.

10.6 Algebraic Cancellation in Breach-Repair Processes

A process representation allows repair to be compared with algebraic inversion without presupposing that the two coincide. Suppose a breach-relevant record word w_b and a repair-response word w_r induce partial transformations through Equation 233. The composite repair-after-breach transformation is

$$F_{c,S}^{w_b \parallel w_r} = F_{c,S}^{w_r} \circ F_{c,S}^{w_b}, \quad (244)$$

using the word-order convention of Equation 226. Equation 244 is defined only on states for which the breach path and subsequent repair path are both admissible.

Exact algebraic cancellation on a domain $D \subseteq \text{Dom}(F_{c,S}^{w_b \parallel w_r})$ would require

$$F_{c,S}^{w_b \parallel w_r} \Big|_D = \text{id}_D. \quad (245)$$

Equation 245 is stronger than successful repair under a target criterion and corresponds to exact return of every represented state in the declared domain D .

Define the cancellation-defect set by

$$\mathfrak{C}_{c,S}(w_b, w_r; D) := \left\{ T \in D : F_{c,S}^{w_b \parallel w_r}(T) \neq T \right\}. \quad (246)$$

Equation 246 localizes failure of the repair process to act as a left-cancelling continuation for the breach process on the selected domain.

Proposition 10.8 (Hysteretic witness as a local cancellation defect). *Let ρ be a return-qualified repair episode whose breach and repair segments admit record words w_b and w_r under a process-congruent representation. Let T_ρ^0 be the pre-breach trust state. If ρ is an exact hysteretic repair witness in Definition 7.4, then*

$$T_\rho^0 \in \mathfrak{C}_{c,S}(w_b, w_r; \{T_\rho^0\}). \quad (247)$$

Proof. Process congruence and Proposition 10.5 identify the post-repair trust state with $F_{c,S}^{w_b \parallel w_r}(T_\rho^0)$. Exact hysteretic nonclosure gives $T_\rho^r \neq T_\rho^0$ through Equation 158. Substituting into Equation 246 yields Equation 247. $\square$

Proposition 10.8 gives an algebraic reading of one hysteretic episode without identifying repair with an inverse operation in general. A repair word can satisfy the target condition in Equation 154 while the cancellation-defect set remains nonempty. Conversely, exact cancellation on one state or one restricted domain does not establish a global inverse: the breach transformation may be many-to-one, may have a smaller image than its domain, or may fail to admit the proposed repair word elsewhere.

For relation-valued semantics, exact cancellation is naturally expressed by comparison with the identity relation. If $\mathcal{R}_b$ and $\mathcal{R}_r$ are breach and repair relations, a strong exact-return condition on D would require

$$(\mathcal{R}_r \circ \mathcal{R}_b)|_D = \text{Id}_D. \quad (248)$$

Equation 248 can fail because some starting states have no repair continuation, because multiple post-repair states remain possible, or because the available repair trajectories return to historically marked states rather than the original state.

10.7 Power-Constrained Process-System Restriction

The power-relevant feasibility interface of Section 9 can be represented directly at the process level. Let $\xrightarrow[c,S,\text{pc}]{z}$ denote the labeled one-step relation obtained by restricting Equation 221 to the realized power-constrained successor relation in Equation 203. Finite paths built only from those constrained steps form a category

$$\mathcal{P}_{c,S}^{\text{pc}} \subseteq \mathcal{P}_{c,S}^{\text{hist}}, \quad (249)$$

with the same history objects and a restricted collection of morphisms. Equation 249 is a wide subcategory because every identity path remains available while some nonidentity paths can be removed by feasibility restrictions.

The inclusion in Equation 249 gives a process-level explanation of the repair-reachability contraction in Proposition 9.6. A power-constrained repair family selects morphisms from a smaller category, so the set of reachable trust targets can contract even when the trust-update semantics assigned to every retained step is unchanged.

A state-level quotient of $\mathcal{P}_{c,S}^{\text{pc}}$ requires a constrained analogue of transition congruence. In particular, histories represented by the same state must admit compatible constrained labeled futures. The uniformly feasible record intersection in Equation 219 is one state-based diagnostic for this requirement. If the intersection is empty, a common state-only process action under the selected constraint regime is unavailable at that resolution.

Research Obligation 10.9 (Constraint-sensitive process representation). When a process-algebraic trust model is interpreted under power or institutional constraints, the model should state whether constraints restrict the history process category, alter the state-transition semantics, alter record visibility, or change more than one of these structures. These mechanisms should not be collapsed into a single algebraic parameter without an independently justified reduction.

Obligation 10.9 preserves the channel distinctions established in Section 9: process availability, transition information, evidential visibility, and repair reachability can vary independently.

10.8 Semantic Scope of Trajectory-Descriptor Invertibility

The broader paper series permits braid-like, topological, or other trajectory descriptors for interaction histories. A descriptor can carry a stronger algebraic structure than the induced trust dynamics. To state that interface abstractly, let G_c^{traj} be a group of trajectory descriptors and let

$$B_c : \text{Mor}(\mathcal{P}_{c,S}^{\text{hist}}) \rightarrow G_c^{\text{traj}} \quad (250)$$

be a partial descriptor map. Whenever both sides are defined for composable paths, suppose the descriptor respects path composition through

$$B_c(\xi \diamond \eta) = B_c(\xi) B_c(\eta). \quad (251)$$

Equation 251 makes the descriptor multiplicative at the trajectory-description layer. Because G_c^{traj} is a group, every descriptor value $B_c(\eta)$ has an algebraic inverse $B_c(\eta)^{-1}$.

That inverse has no automatic trust-dynamical interpretation. Three additional properties would be required to treat it as a trust-restoring inverse: an admissible history path carrying the inverse descriptor must exist, its record semantics must be defined at the selected trust-state resolution, and the resulting trust transformation must cancel the original trust transformation on the declared domain. Formally, if $\bar{\eta}$ is a candidate inverse-descriptor path with

$$B_c(\bar{\eta}) = B_c(\eta)^{-1}, \quad (252)$$

trust-state inversion would additionally require a condition such as

$$F_{c,S}^{\text{tr}_S(\bar{\eta})} \circ F_{c,S}^{\text{tr}_S(\eta)} = \text{id} \quad (253)$$

on a declared common domain. Equation 253 is independent of the group identity represented in Equation 252.

Proposition 10.10 (Descriptor invertibility does not imply trust-process invertibility). *The existence of $B_c(\eta)^{-1}$ in G_c^{traj} does not, from Equations 250–251 alone, imply the existence of an admissible path $\bar{\eta}$ satisfying Equation 252, nor does it imply Equation 253 when such a path exists.*

Proof. Group inversion is an operation internal to G_c^{traj} . The partial descriptor map in Equation 250 need not be surjective onto inverse elements by admissible paths, and Equations 250–251 impose no requirement that trust-state semantics be bijective or that a path carrying an inverse descriptor cancel the original partial transformation. The two implications therefore require additional assumptions absent from the stated structure. $\square$

Proposition 10.10 formalizes the distinction motivating the paper. A braid-like interaction description may be invertible while trust evolution is dissipative, many-to-one, history-sensitive, partially defined, or constrained by unavailable repair paths. The appropriate trust semantics can consequently live in a partial-transformation monoid or a relation category even when the trajectory descriptor lives in a group.

Table 7: Diagnostic distinctions for process-algebraic representations of trust dynamics.

Representation or finding	Formal requirement	Warranted interpretation
History path category	Proposition 10.1	Admissible trust-relevant interaction steps can be sequentially composed when their history endpoints match; no state compression is assumed.
Exact one-step state update	Definition 5.1	Matching represented state and record determine a matching represented next state on realized transitions; equal states may still have different feasible next labels.
State-level process quotient	Definition 10.2 or Proposition 10.3	Represented trust states support compatible labeled futures and can serve as compositional process states at the selected resolution.
Partial transformation monoid	Equations 232–236	Record words act compositionally as deterministic partial transformations on represented trust states.
Relation-valued semantics	Equations 238–241	Nonunique or incompletely compressed transitions are retained without forcing deterministic state updates; abstract relational composition may over-approximate realizable paths.
Cancellation defect	Equation 246	A declared repair-after-breach process fails exact algebraic return on the selected state domain; successful target-based repair may still occur.
Power-constrained process family	Equation 249	Institutional or resource constraints remove admissible process morphisms and can contract reachability without changing retained transition semantics.
Inverse trajectory descriptor	Equations 252–253	Topological or braid-like inversion is distinct from trust-state inversion and requires separate realizability and semantic-cancellation conditions.

10.9 Diagnostic Taxonomy of Process Representations

The process architecture supports several levels of representation whose assumptions should remain visible. Table 7 summarizes the main distinctions.

Table 7 shows why a single algebraic object is insufficient across all model resolutions. The history path category is the conservative compositional baseline. A trust-state partial-transformation monoid becomes available only after a transition congruence has been justified. Relation-valued state semantics preserves nonuniqueness when that quotient is unavailable, but Proposition 10.6 warns that purely state-level relational composition can still admit sequences that no single history realizes.

The process architecture retains identity and sequential composition while making invertibility, cancellation, unrestricted reachability, and state-only composition conditional. Section 11 extends the same architecture from partial maps and relations to conditional probability kernels.

11 Stochastic Trust Evolution under Historical Uncertainty

This section extends the deterministic architecture to conditional uncertainty while preserving the distinction among unresolved history, latent heterogeneity, observation error, and stochastic transition structure. It takes a full-history-conditioned kernel as the reference object, defines distributional factorization and memory diagnostics, separates latent and observation layers, and develops partial stochastic process semantics and probabilistic repair. Probability is therefore introduced only with an explicit law or empirical construction, not as a generic replacement for a failed deterministic map.

11.1 Measurable Trust States and Historical Reference Kernels

A stochastic update model requires measurable structure in addition to the trust-state carrier introduced in Section 3. Fix a context c and trust-state selector S . Equip the represented trust-state space $\mathfrak{T}_c$ with a declared sigma-algebra Σ_c^{tr} , and write

$$\mathcal{P}_c^{\text{tr}} := \mathcal{P}(\mathfrak{T}_c, \Sigma_c^{\text{tr}}) \quad (254)$$

for the set of probability measures on that measurable space. Equation 254 adds no substantive probability law; it only declares the codomain in which such laws can be represented.

The full-history input from Equation 119 has realized image

$$\mathfrak{X}_{c,S}^{\text{hist}} := \text{Im}(\text{In}_{c,S}^{\text{hist}}). \quad (255)$$

Equation 255 names the realized conditioning domain used by the stochastic extension. Equip $\mathfrak{X}_{c,S}^{\text{hist}}$ with a declared sigma-algebra $\Sigma_{c,S}^{\text{hist}}$. A historical reference kernel is then a Markov kernel

$$\mathbb{K}_{c,S}^{\text{hist}} : \mathfrak{X}_{c,S}^{\text{hist}} \times \Sigma_c^{\text{tr}} \longrightarrow [0, 1], \quad (256)$$

where $A \mapsto \mathbb{K}_{c,S}^{\text{hist}}(x, A)$ is a probability measure for every $x \in \mathfrak{X}_{c,S}^{\text{hist}}$ and $x \mapsto \mathbb{K}_{c,S}^{\text{hist}}(x, A)$ is measurable for every $A \in \Sigma_c^{\text{tr}}$. Equation 256 is interpreted as the selected model's conditional law of the next represented trust state given the retained pre-history, current represented state, and selected current record.

Research Obligation 11.1 (Probability provenance). A stochastic trust model should state what licenses the numerical probabilities in its kernel: repeated-data estimation, a substantive random mechanism, Bayesian predictive uncertainty under a declared prior and likelihood, calibrated expert assessment, or another explicit construction. Multiplicity of admissible successors alone supplies a possibility set and does not determine probability weights.

The requirement in Obligation 11.1 limits stochastic interpretation to cases with an explicit probability basis. The successor relation in Section 5 can identify several admissible continuations while remaining silent about their relative probability.

When the full-history residual defect in Equation 120 is empty, equality of realized full-history inputs implies equality of represented next states. The same fiber argument used in Proposition 5.2 therefore defines a unique set-theoretic next-state map on the realized historical-input image. If that map is measurable, write it as

$$F_{c,S}^{\text{hist}} : \mathfrak{X}_{c,S}^{\text{hist}} \longrightarrow \mathfrak{T}_c. \quad (257)$$

Equation 257 is thus a measurable deterministic benchmark licensed by full-history outcome sufficiency, not an additional primitive. Its Dirac kernel is

$$\mathbb{K}_{c,S}^{\delta}(x, A) := \mathbf{1}_A(F_{c,S}^{\text{hist}}(x)). \quad (258)$$

Equation 258 shows that deterministic updating is a special case of the stochastic representation rather than a competing formal language.

11.2 Distributional Update Sufficiency

The state-record input from Equation 74 is a coarsening of the full-history input. Define the canonical projection on realized historical inputs by

$$\pi_{c,S}^{\text{sr}} : \mathfrak{X}_{c,S}^{\text{hist}} \longrightarrow \text{Im}(\text{In}_{c,S}), \quad \pi_{c,S}^{\text{sr}}(\text{In}_{c,S}^{\text{hist}}(u)) := \text{In}_{c,S}(u). \quad (259)$$

Equation 259 is well defined because the full-history input contains the current state and record components used by $\text{In}_{c,S}$. It is surjective onto the realized state-record image. Equip that image with the quotient sigma-algebra

$$\Sigma_{c,S}^{\text{sr}} := \left\{ B \subseteq \text{Im}(\text{In}_{c,S}) : (\pi_{c,S}^{\text{sr}})^{-1}(B) \in \Sigma_{c,S}^{\text{hist}} \right\}. \quad (260)$$

Equation 260 is the final measurable structure induced by the historical-input projection. It makes $\pi_{c,S}^{\text{sr}}$ measurable and ensures that any measurable scalar function on the historical-input space that is constant on projection fibers descends measurably to the realized state–record image.

Definition 11.2 (Stochastic update sufficiency). The selected state–record representation is *stochastically update-sufficient* relative to $\mathbb{K}_{c,S}^{\text{hist}}$ when there exists a Markov kernel

$$\mathbb{K}_{c,S}^{\text{sr}} : \text{Im}(\text{In}_{c,S}) \times \Sigma_c^{\text{tr}} \longrightarrow [0, 1] \quad (261)$$

such that

$$\mathbb{K}_{c,S}^{\text{hist}}(x, A) = \mathbb{K}_{c,S}^{\text{sr}}(\pi_{c,S}^{\text{sr}}(x), A) \quad (262)$$

for every $x \in \mathcal{X}_{c,S}^{\text{hist}}$ and every $A \in \Sigma_c^{\text{tr}}$.

Equation 261 defines the candidate quotient kernel, while Equation 262 states the required law-level factorization. Definition 11.2 is the distributional analogue of exact update sufficiency in Definition 5.1. Equation 262 permits different realized next trust states after the same represented input while requiring their conditional law to be the same across histories compressed into that input. The Markovian claim is therefore resolution-relative and distributional.

The corresponding fiber defect is

$$\Delta_{c,S}^{\text{ker}} := \left\{ (x, y) \in (\mathcal{X}_{c,S}^{\text{hist}})^2 : \pi_{c,S}^{\text{sr}}(x) = \pi_{c,S}^{\text{sr}}(y), \mathbb{K}_{c,S}^{\text{hist}}(x, \cdot) \neq \mathbb{K}_{c,S}^{\text{hist}}(y, \cdot) \right\}. \quad (263)$$

Equation 263 records distributional history dependence at the selected state–record resolution.

Proposition 11.3 (Fiber criterion for distributional sufficiency). *Under the quotient measurable structure in Equation 260, Definition 11.2 holds if and only if*

$$\Delta_{c,S}^{\text{ker}} = \emptyset. \quad (264)$$

Proof. Under Equation 262, two historical inputs with the same $\pi_{c,S}^{\text{sr}}$ value receive the same probability measure, so Equation 264 follows. Conversely, assume Equation 264. Define the state–record probability measure at a realized input z to be $\mathbb{K}_{c,S}^{\text{hist}}(x, \cdot)$ for any x satisfying $\pi_{c,S}^{\text{sr}}(x) = z$. Fiber equality makes the assignment independent of the representative, while countable additivity and normalization are inherited from the historical kernel. For each $A \in \Sigma_c^{\text{tr}}$, the scalar map $x \mapsto \mathbb{K}_{c,S}^{\text{hist}}(x, A)$ is measurable and constant on fibers. Its descended map on the quotient is measurable by Equation 260. The induced assignment is therefore a Markov kernel and satisfies Equation 262. $\square$

Proposition 11.3 separates equality of conditional laws from the choice of conditioning resolution while making the quotient measurability explicit. A nonempty set in Equation 263 means that explicit historical information remains predictive of the next-state distribution at the selected resolution. It does not establish why that historical information matters.

The deterministic and stochastic defect concepts are related but non-equivalent. Under a Dirac historical kernel of the form in Equation 258, equality of conditional laws is equivalent to equality of deterministic next states. With non-Dirac kernels, a deterministic update defect can coexist with an empty kernel defect because different realized outcomes may be draws from the same conditional law.

Corollary 11.4 (Deterministic embedding). *Suppose the historical reference kernel has the Dirac form in Equation 258. Under the quotient sigma-algebra in Equation 260, stochastic update sufficiency is equivalent to the existence of a measurable map on the realized state–record image through which $F_{c,S}^{\text{hist}}$ factors via $\pi_{c,S}^{\text{sr}}$.*

Proof. Two Dirac probability measures on $\mathcal{T}_c$ are equal exactly when their support points are equal. Hence the kernel defect is empty exactly when $F_{c,S}^{\text{hist}}$ is constant on every fiber of $\pi_{c,S}^{\text{sr}}$. Because $F_{c,S}^{\text{hist}}$ is measurable and the quotient sigma-algebra is final with respect to $\pi_{c,S}^{\text{sr}}$, the induced state–record map is measurable. Proposition 11.3 then gives the equivalence. $\square$

Corollary 11.4 confirms that the stochastic extension preserves the earlier deterministic criterion rather than redefining it.

11.3 Historical Memory at the Level of Conditional Laws

A memory summary can also be evaluated by how much distributional history dependence it resolves. Let $M_c : \Gamma_c^{\text{tr}} \rightarrow \mathfrak{M}_c$ be a memory map of the form introduced in Equation 103. Define the memory-augmented projection

$$\pi_{c,S}^M(\text{In}_{c,S}^{\text{hist}}(u)) := (\text{In}_{c,S}(u), M_c(h(u))), \quad (265)$$

where $h(u)$ is the pre-update history component named in Equation 119. Equation 265 keeps the current state–record input distinct from the retained historical summary.

The memory-conditioned kernel defect is

$$\Delta_{c,S}^{\text{ker}|M} := \left\{ (x, y) : \pi_{c,S}^M(x) = \pi_{c,S}^M(y), \mathbb{K}_{c,S}^{\text{hist}}(x, \cdot) \neq \mathbb{K}_{c,S}^{\text{hist}}(y, \cdot) \right\}. \quad (266)$$

Equation 266 asks whether the selected memory removes distributional differences that remain after conditioning only on the represented state and current record.

Proposition 11.5 (Distributional defect monotonicity under memory refinement). *If $M_1 \succeq_{\text{mem}} M_2$ in the sense of Equation 124, then*

$$\Delta_{c,S}^{\text{ker}|M_1} \subseteq \Delta_{c,S}^{\text{ker}|M_2}. \quad (267)$$

Proof. Equality of the more informative M_1 values implies equality of the M_2 values by Equation 125. Hence every pair left indistinguishable by $\pi_{c,S}^{M_1}$ is also left indistinguishable by $\pi_{c,S}^{M_2}$. If the associated historical conditional laws differ, the pair belongs to both defect sets, proving Equation 267. $\square$

Proposition 11.5 is the law-level analogue of Proposition 6.6. It supports a conservative empirical strategy: a candidate historical summary earns a role when it removes predictive distributional differences that a coarser representation leaves unresolved. A fitted memory coefficient is insufficient by itself because the same predictive gain can sometimes be reproduced by a richer present state, a richer event record, a latent variable, or a different observation model.

A full-history-conditioned kernel differs conceptually from the deterministic full-history control in Section 6. Once $\mathbb{K}_{c,S}^{\text{hist}}$ has been specified, conditioning on the same historical input trivially identifies the same law. Residual variation inside that law is therefore not a further history-compression defect. The substantive question becomes whether that within-law variation represents a modeled random mechanism or an unresolved mixture over variables omitted from the conditioning set.

11.4 Latent-State Mixtures and Mechanism Non-Identification

A coarse stochastic kernel can arise from heterogeneous latent states even when the transition becomes deterministic after latent refinement. Let $(\mathfrak{L}_c, \Sigma_c^{\text{lat}})$ be a declared latent-state space, and equip $\mathfrak{X}_{c,S}^{\text{hist}} \times \mathfrak{L}_c$ with the product sigma-algebra $\Sigma_{c,S}^{\text{hist}} \otimes \Sigma_c^{\text{lat}}$. Introduce a latent conditional kernel

$$\mathbb{Q}_{c,S}^{\text{lat}} : \mathfrak{X}_{c,S}^{\text{hist}} \times \Sigma_c^{\text{lat}} \longrightarrow [0, 1] \quad (268)$$

and an outcome kernel conditional on both historical input and latent state,

$$\mathbb{K}_{c,S}^{\text{lat}} : (\mathfrak{X}_{c,S}^{\text{hist}} \times \mathfrak{L}_c) \times \Sigma_c^{\text{tr}} \longrightarrow [0, 1]. \quad (269)$$

Equations 268–269 keep latent uncertainty and conditional transition uncertainty in separate kernels. The observed historical kernel is compatible with this latent representation when

$$\mathbb{K}_{c,S}^{\text{hist}}(x, A) = \int_{\mathfrak{L}_c} \mathbb{K}_{c,S}^{\text{lat}}((x, \ell), A) \mathbb{Q}_{c,S}^{\text{lat}}(x, d\ell). \quad (270)$$

Equation 270 separates uncertainty about the latent relational condition from uncertainty in the transition conditional on that condition.

A deterministic latent completion is the special case in which there exists a measurable map

$$G_{c,S}^{\text{lat}} : \mathfrak{X}_{c,S}^{\text{hist}} \times \mathfrak{L}_c \longrightarrow \mathfrak{T}_c \quad (271)$$

with

$$\mathbb{K}_{c,S}^{\text{lat}}((x, \ell), A) = \mathbf{1}_A(G_{c,S}^{\text{lat}}(x, \ell)). \quad (272)$$

Equations 271–272 formalize one way in which apparently stochastic trust evolution can result from unresolved heterogeneity.

Proposition 11.6 (Coarse-kernel mechanism non-identification). *A historical kernel $\mathbb{K}_{c,S}^{\text{hist}}$ does not, without additional restrictions, identify whether its within-input variation arises from a non-degenerate transition conditional on a fixed latent state or from a mixture over latent states with deterministic conditional transitions.*

Proof. One representation uses a singleton latent space and sets the conditional outcome kernel equal to $\mathbb{K}_{c,S}^{\text{hist}}$, leaving the transition uncertainty inside the outcome kernel. A second representation takes $\mathfrak{L}_c = \mathfrak{T}_c$, sets $\mathbb{Q}_{c,S}^{\text{lat}}(x, \cdot) = \mathbb{K}_{c,S}^{\text{hist}}(x, \cdot)$, and defines $G_{c,S}^{\text{lat}}(x, \ell) = \ell$. Equation 270 then reproduces the same historical kernel while Equation 272 is deterministic conditional on ℓ . The two representations therefore induce the same coarse kernel with different allocations of uncertainty. $\square$

Proposition 11.6 is a representational non-identification result, not a substantive claim that the next trust state is literally a pre-existing latent cause. The second construction in the proof is deliberately generic. Its purpose is to show that the coarse predictive law alone cannot establish metaphysical or causal irreducibility of randomness. Substantive latent variables require independent interpretation, measurement, intervention, or structural restrictions.

Research Obligation 11.7 (Latent-state interpretability). When a latent state is used to explain stochastic trust transitions, the model should specify what distinctions the latent variable represents, how those distinctions differ from the represented trust state and historical memory, and which observations or interventions could in principle discriminate the latent account from alternative stochastic decompositions.

Obligation 11.7 preserves the distinction among three representational locations for missing information: current relational state, retained history, and latent structure.

11.5 Transition Uncertainty and Observation Uncertainty

A stochastic transition model should also remain distinct from uncertainty in observing the trust state. Let $(\mathfrak{Y}_c, \Sigma_c^{\text{obs}})$ be a measurable observation space and let

$$\mathbb{O}_c : \mathfrak{T}_c \times \Sigma_c^{\text{obs}} \longrightarrow [0, 1] \quad (273)$$

be an observation kernel. Equation 273 generalizes the deterministic readout maps of Section 3 by allowing noisy, partial, or probabilistic measurement.

Given a historical transition kernel, the predictive law of a next observation is

$$\mathbb{P}_{c,S}^{\text{obs}}(x, B) := \int_{\mathfrak{T}_c} \mathbb{O}_c(\tau, B) \mathbb{K}_{c,S}^{\text{hist}}(x, d\tau), \quad B \in \Sigma_c^{\text{obs}}. \quad (274)$$

Equation 274 composes transition uncertainty with observation uncertainty.

The separation matters empirically. Different latent trust transitions can generate the same observed distribution through a coarse observation kernel, and a deterministic latent transition can yield noisy observations. Conversely, highly reliable observation does not imply that the trust transition itself is deterministic. Identification therefore depends jointly on the transition model, observation model, and data design.

Research Obligation 11.8 (Uncertainty-layer separation). Empirical applications should state whether uncertainty is assigned to trust-state evolution, state observation, latent-state inference, event-record classification, or several of these layers. A single residual-error term should not be interpreted simultaneously as evidence for each mechanism.

Obligation 11.8 prevents observational noise from being mistaken for historical or transition stochasticity.

11.6 Partial Stochastic Process Semantics

Section 10 derived deterministic partial transformations only after histories compressed into the same state were shown to admit compatible labeled futures. The stochastic extension requires an analogous quotient condition at the level of transition laws. Let z denote a record label and let $D_{c,S}(z) \subseteq \mathfrak{T}_c$ be the state feasibility domain from Equation 231; under state-invariant feasibility it is uniformly feasible across representative histories.

Assumption 11.9 (Measurable record-feasibility domains). For every record label used in stochastic process composition, $D_{c,S}(z) \in \Sigma_c^{\text{tr}}$.

Assumption 11.9 is needed because later word composition evaluates probabilities of entering the next label’s feasibility domain. Equip each $D_{c,S}(z)$ with the trace sigma-algebra inherited from Σ_c^{tr} . A state-level stochastic action for z is a partial kernel

$$\mathbb{K}_{c,S}^z : D_{c,S}(z) \times \Sigma_c^{\text{tr}} \longrightarrow [0, 1]. \quad (275)$$

Equation 275 is warranted only when histories represented by the same state and carrying the same feasible label induce the same next-state law at the selected resolution.

Proposition 11.10 (Stochastic state-process quotient). *Suppose state-invariant record feasibility in Equation 230 holds and the state–record representation is stochastically update-sufficient in the sense of Definition 11.2. Then every realized record label z induces a well-defined partial kernel on the domain $D_{c,S}(z)$ by*

$$\mathbb{K}_{c,S}^z(T, A) := \mathbb{K}_{c,S}^{\text{sr}}((T, z), A). \quad (276)$$

Proof. State-invariant feasibility makes membership in $D_{c,S}(z)$ independent of the representative history. Stochastic update sufficiency makes the next-state law depend on a realized historical input only through its represented state–record pair. Consequently, histories with the same represented state T and feasible label z induce the common measure in Equation 276. This defines a probability measure for every $T \in D_{c,S}(z)$, and measurability follows from the declared state–record kernel restricted to the partial domain. $\square$

Proposition 11.10 is the stochastic analogue of the deterministic quotient criterion in Section 10. It requires both common label feasibility and common transition laws across histories compressed into the same represented state.

For finite record words, stochastic composition must also respect future-label feasibility. Define the word domains recursively by

$$\mathcal{D}_{c,S}^\varepsilon := \text{Im}(\Theta_c^{\text{tr}}), \quad \mathcal{D}_{c,S}^z := D_{c,S}(z), \quad \mathcal{D}_{c,S}^{w||z} := \{\tau \in \mathcal{D}_{c,S}^w : \mathbb{K}_{c,S}^w(\tau, D_{c,S}(z)) = 1\}. \quad (277)$$

Equation 277 makes almost-sure feasibility part of the domain of the longer word rather than an untyped side condition. Set

$$\mathbb{K}_{c,S}^\varepsilon(\tau, A) := \mathbf{1}_A(\tau) \quad (278)$$

for the empty word. For a word w followed by a label z , define

$$\mathbb{K}_{c,S}^{w||z}(\tau, A) := \int_{D_{c,S}(z)} \mathbb{K}_{c,S}^z(\tau', A) \mathbb{K}_{c,S}^w(\tau, d\tau'), \quad \tau \in \mathcal{D}_{c,S}^w. \quad (279)$$

The restricted integration domain in Equation 279 ensures that the partial kernel is evaluated only where the final record label is defined.

Proposition 11.11 (Closure of measurable stochastic word semantics). *Under Assumption 11.9 and Proposition 11.10, every finite record word w has a measurable domain $\mathcal{D}_{c,S}^w \in \Sigma_c^{\text{tr}}$, and Equation 279 recursively defines a Markov kernel from the trace measurable space on $\mathcal{D}_{c,S}^w$ to $(\mathfrak{T}_c, \Sigma_c^{\text{tr}})$.*

Proof. The empty-word domain is measurable and its kernel is the identity Dirac kernel. Atomic domains are measurable by Assumption 11.9, and Proposition 11.10 supplies the corresponding atomic kernels. Assume the claim holds for w . Because $D_{c,S}(z)$ is measurable, the map $\tau \mapsto \mathbb{K}_{c,S}^w(\tau, D_{c,S}(z))$ is measurable on $\mathcal{D}_{c,S}^w$; therefore the unit-level set in Equation 277 is measurable. For each $A \in \Sigma_c^{\text{tr}}$, extend $\tau' \mapsto \mathbb{K}_{c,S}^z(\tau', A)$ by zero outside $D_{c,S}(z)$. The extension is measurable, so integration against the kernel $\mathbb{K}_{c,S}^w$ yields a measurable function of the starting state. On $\mathcal{D}_{c,S}^{w||z}$ the preceding kernel assigns probability one to $D_{c,S}(z)$; normalization and countable additivity are therefore inherited from the atomic probability kernel. This completes the induction. $\square$

Proposition 11.11 gives the stochastic word calculus an explicit partial domain. A word is undefined at starting states for which the model assigns positive probability to an intermediate state where the next prescribed record is infeasible.

Proposition 11.12 (Dirac embedding of partial process maps). *Under Assumption 11.9, suppose the deterministic partial transformations $F_{c,S}^z$ from Equation 232 are measurable on their trace-measurable domains. Define*

$$\mathbb{K}_{c,S}^z(\tau, A) := \mathbf{1}_A(F_{c,S}^z(\tau)) \quad (280)$$

for $\tau \in D_{c,S}(z)$. Then, wherever the deterministic word map $F_{c,S}^w$ is defined, its starting state belongs to $\mathcal{D}_{c,S}^w$ and the recursively composed kernel satisfies

$$\mathbb{K}_{c,S}^w(\tau, A) = \mathbf{1}_A(F_{c,S}^w(\tau)) . \quad (281)$$

Proof. For the empty word, the deterministic identity is defined on $\text{Im}(\Theta_c^{\text{tr}}) = \mathcal{D}_{c,S}^\varepsilon$, and Equation 278 gives the claimed Dirac law. The one-label case is Equation 280. Assume the claim holds for a word w , and suppose $F_{c,S}^{w||z}$ is defined at τ . Then $F_{c,S}^w(\tau) \in D_{c,S}(z)$. By the induction hypothesis, $\mathbb{K}_{c,S}^w(\tau, \cdot)$ is the Dirac measure at $F_{c,S}^w(\tau)$, so $\mathbb{K}_{c,S}^w(\tau, D_{c,S}(z)) = 1$. Equation 277 therefore gives $\tau \in \mathcal{D}_{c,S}^{w||z}$. Substituting the Dirac induction hypothesis into Equation 279 evaluates the integral at $F_{c,S}^w(\tau)$ and yields the Dirac measure at $F_{c,S}^z(F_{c,S}^w(\tau)) = F_{c,S}^{w||z}(\tau)$. $\square$

Equation 281, proved in Proposition 11.12, shows that stochastic process semantics extends the partial-transformation monoid without replacing it. The stochastic layer is needed only when a non-degenerate conditional law is substantively warranted.

If the unit-probability condition defining $\mathcal{D}_{c,S}^{w||z}$ in Equation 277 fails, several richer constructions are possible, including a subprobability kernel that records mass lost to infeasibility or a total kernel on an enlarged state space containing an explicit unavailable or terminated state. Each construction changes the semantics and should therefore be declared rather than silently normalizing away infeasible probability mass.

11.7 Probabilistic Repair and Distributional Hysteresis

The repair framework of Section 7 distinguishes acceptable repair from exact restoration. Stochastic dynamics preserve that distinction. Let $\mathfrak{A}_c^{\text{rep}}(\tau_0) \in \Sigma_c^{\text{tr}}$ be a declared acceptable-repair target set for a pre-breach state τ_0 . For a feasible breach–repair word $w^{\text{br,rep}}$, define the target-attainment probability by

$$p_{c,S}^{\text{rep}}(\tau_0, w^{\text{br,rep}}) := \mathbb{K}_{c,S}^{w^{\text{br,rep}}}(\tau_0, \mathfrak{A}_c^{\text{rep}}(\tau_0)) . \quad (282)$$

Equation 282 quantifies the chance of reaching the declared repair target under the selected process model. It equals a restoration probability only when the target set itself encodes the declared restoration criterion.

A law-level analogue of matched-path hysteresis compares terminal conditional laws after externally matched return protocols. Let $w^{(1)}$ and $w^{(2)}$ be two feasible process words that satisfy the same declared external return condition from the same represented starting state τ_0 . Their terminal laws are

$$\mu_{c,S}^{(i)}(A) := \mathbb{K}_{c,S}^{w^{(i)}}(\tau_0, A), \quad i \in \{1, 2\}. \quad (283)$$

Distributional hysteresis at that resolution is present when

$$\mu_{c,S}^{(1)} \neq \mu_{c,S}^{(2)}. \quad (284)$$

Equation 284 is weaker than asserting that every realized pair of terminal states differs and stronger than observing one discrepant realized pair. It states that the matched path histories induce different terminal trust distributions under the selected stochastic model.

Research Obligation 11.13 (Probabilistic repair calibration). A probability of repair, restoration, or distributional hysteresis should be interpreted only relative to the declared target set, process policy, conditioning variables, and probability provenance. A high target-attainment probability does not establish exact restoration, and a single unsuccessful realization does not establish a low repair probability.

Obligation 11.13 keeps event-level outcomes distinct from population- or model-level probability statements.

11.8 Diagnostic Taxonomy of Historical Uncertainty

The stochastic extension distinguishes several mechanisms that can generate apparently unpredictable trust evolution. Table 8 summarizes their formal signatures and the conclusions that each signature can support.

Table 8 provides the interpretive boundary needed for the paper’s later examples. The stochastic formalism is most informative when it narrows the location of uncertainty rather than merely replacing a failed deterministic map with a probability distribution. Historical memory, latent-state heterogeneity, observation error, process infeasibility, and model-internal transition uncertainty remain distinct representational mechanisms even when they generate similar observed variability.

The stochastic extension preserves the deterministic architecture as a Dirac special case while adding law-level factorization, latent-mixture ambiguity, observation uncertainty, and target-relative probabilistic repair. Appendix A compares these mechanisms in finite constructions.

Table 8: Diagnostic distinctions for stochastic trust evolution under historical uncertainty.

Finding or representation	Formal diagnostic	Warranted interpretation
Deterministic update defect	Equation 83	The selected state–record input does not determine one represented next state; the source of the missing information remains open.
Distributional history dependence	Equation 263	Histories compressed into the same state–record input induce different next-state laws under the selected historical kernel.
Stochastic state–record sufficiency	Definition 11.2	Explicit prior history adds no further predictive information for the next-state law after the represented state and record are fixed.
Memory-resolved law dependence	Equation 266 together with Proposition 11.5	A declared historical summary resolves some distributional distinctions left by a coarser representation.
Latent-state mixture	Equation 270	The coarse kernel can be represented as averaging over latent states; causal or ontological interpretation requires independent support.
Observation uncertainty	Equations 273–274	Noise or partial observability can enter after trust-state evolution and should not automatically be attributed to the transition itself.
Partial stochastic process	Equations 275–277	Record-labeled stochastic transitions compose only where future labels remain feasible with probability one under the selected semantics.
Probabilistic repair	Equation 282	Repair becomes a target-attainment probability under a declared process model; acceptable repair remains distinct from exact restoration.
Distributional hysteresis	Equations 283–284	Externally matched paths induce different terminal trust laws; one discrepant realized outcome is insufficient for this conclusion.

12 Scope Boundaries of the Trust-Dynamics Formalism

This section states the interpretation boundaries of the preceding formal results. Its objective is to separate representation-relative adequacy from empirical operationalization and identification, causal explanation, cross-context transport, normative evaluation, aggregation, approximation, and ontological interpretation. Each boundary is expressed as an additional condition or bridge that must be supplied before a stronger claim can be drawn from the formal diagnostics.

12.1 Representation-Relative Validity of Dynamical Claims

A trust-dynamics result in this paper is indexed by modeling choices rather than by the word “trust” alone. For a declared context c , collect the principal deterministic ingredients used by the analysis into the specification

$$\mathbf{B}_{c,S} := (\Gamma_c^{\text{tr}}, \Theta_c^{\text{tr}}, \succ_c^{\text{tr}}, S, \mathfrak{T}_c, \mathfrak{Z}_{c,S}), \quad (285)$$

where the history domain, trust-state representation, admissible-successor relation, selected record map, trust-state carrier, and selected-record carrier have the meanings established in Sections 2–5. Equation 285 is a bookkeeping object rather than an additional ontology. Memory maps, constraint profiles, continuation rules, metrics, sigma-algebras, stochastic kernels, observation kernels, and repair families are appended when a claim requires them.

A statement such as exact update sufficiency, finite-memory sufficiency, basin membership, repair nonreturn, process congruence, or distributional hysteresis is therefore a predicate of an explicitly en-

riched specification. Write this schematically as

$$D(\mathbf{B}_{c,S}; \mathbf{a}) = 1, \quad (286)$$

where D denotes a declared diagnostic and $\mathbf{a}$ collects the additional structures required by that diagnostic. Equation 286 means that the diagnostic holds for the stated representation and assumptions. It does not assert that the same predicate is invariant under arbitrary changes of state resolution, event typing, history domain, or admissible process family.

The dependence on representation is substantive. Proposition 5.6 shows that refining update inputs can remove defects; Proposition 6.6 establishes the analogous monotonicity for historical-memory representations; and Proposition 9.5 shows that evidential coarsening can preserve or create apparent ambiguity. Consequently, a statement that “history matters” at one resolution may become a state-sufficiency statement after a richer state representation is introduced, while a statement that an update is deterministic at one selected domain may cease to be adequate after the domain is expanded to include previously omitted cases.

Remark 12.1 (Representation-relative historical dependence). Within the present framework, historical dependence is established relative to a declared compression of histories. A nonempty update-defect or kernel-defect set shows that the chosen compressed representation is insufficient for the corresponding deterministic or distributional task. Additional argument is required before that insufficiency is interpreted as a distinct psychological memory mechanism, an ontological property of trust, or a universal feature of social relations.

Remark 12.1 preserves the distinction developed in Section 6: explicit memory is one possible representation of omitted historical information, not the only possible one. The worked construction in Section A.2 makes the same point by showing that one bit of history can be represented either as a memory variable or as part of a refined present state.

12.2 Empirical Identification and Observational Equivalence

Formal well-definedness does not imply empirical identification. Section 3 already allows the represented trust state to be partially observed, and Section 11 separates transition uncertainty from observation uncertainty. The present boundary can be stated at the model level.

Let $\mathfrak{D}_c$ be an observable-data space with sigma-algebra Σ_c^{obs} . For a candidate trust-dynamics model m in a declared model class $\mathfrak{C}_c^{\text{mod}}$, let

$$\mathbb{P}_{c,m}^{\text{obs}} \in \mathcal{P}(\mathfrak{D}_c, \Sigma_c^{\text{obs}}) \quad (287)$$

denote the probability law for the observables generated by that model under the declared sampling and observation design. Equation 287 makes the observation design part of the identification problem rather than treating the latent trust model as directly observed. Two models are observationally equivalent for that design when Equation 288 holds.

$$m \sim_c^{\text{obs}} m' \iff \mathbb{P}_{c,m}^{\text{obs}} = \mathbb{P}_{c,m'}^{\text{obs}}. \quad (288)$$

Equation 288 is design-relative. A richer measurement protocol, longer observation window, or intervention may split an equivalence class that remains indistinguishable under a poorer design.

For a structural property $\Pi : \mathfrak{C}_c^{\text{mod}} \rightarrow \mathfrak{V}_{\Pi}$, empirical identification from the declared observables requires constancy of Π on every relevant observational-equivalence class, as stated in Equation 289.

$$m \sim_c^{\text{obs}} m' \implies \Pi(m) = \Pi(m'). \quad (289)$$

Equation 289 supplies a minimal identification criterion for properties such as a memory horizon, latent-state interpretation, transition kernel, repair probability, or basin assignment.

Proposition 12.2 (Observational non-identification criterion). *If there exist $m, m' \in \mathfrak{C}_c^{\text{mod}}$ such that $m \sim_c^{\text{obs}} m'$ while $\Pi(m) \neq \Pi(m')$, then Π is not identified by the declared observational design on any model class containing both m and m' .*

Proposition 12.2 follows directly from Equation 289. Its value is diagnostic rather than algebraically difficult. The latent-mixture result in Proposition 11.6 is one concrete instance: a coarse predictive kernel can be compatible with different underlying mechanisms. Likewise, Section A.8 shows that a non-degenerate observed distribution can be produced by a deterministic trust transition followed by an imperfect observation channel.

The implication for empirical work is that fitting a model does not by itself identify the mechanism represented by the fitted parameters. A trust score, transition matrix, estimated memory coefficient, or inferred latent class obtains its substantive meaning from the measurement construction, model restrictions, and identifying assumptions. Where multiple models remain observationally equivalent, the present framework supports reporting that equivalence rather than selecting one mechanism by formal convenience alone.

12.3 Empirical Operationalization of Dynamical Diagnostics

This subsection supplies an empirical bridge between the formal diagnostics and observable research designs. Its role is to specify the minimum measurement, sampling, and comparison objects required before a formal predicate is treated as empirically testable. The bridge remains pluralist: interpersonal trust has been measured through field instruments that distinguish affective and cognitive components [17], while experimental work has used investment decisions as behavioral indicators of trust and reciprocity [1]. Behavioral indicators require their own identification argument. Cox’s triadic design, for example, shows that transfers in a single trust-game design can combine trust or reciprocity with altruism or inequality aversion [2]. The present framework therefore treats survey scales, behavioral choices, latent-variable scores, institutional indicators, and other measurements as candidate observation channels rather than interchangeable observations of one universal trust variable.

Definition 12.3 (Empirical bridge specification). An empirical bridge specification for a trust-dynamics diagnostic declares: (i) the relation or unit whose trust state is represented; (ii) the observation schedule or event-time rule; (iii) the measurement model linking observed indicators to the trust-state representation; (iv) the rule that encodes pre-update interaction information into the selected record; (v) the sampling, assignment, or intervention regime that determines which histories are observed; (vi) the comparison or loss criterion used to operationalize exact or approximate equality; and (vii) the uncertainty model used for measurement error, sampling variation, or latent-state estimation.

Definition 12.3 turns empirical implementation into part of the model specification. The measurement model is especially important when the represented trust state is multidimensional or latent. A self-report scale, an investment decision, an observed delegation action, and an inferred latent state can each be informative under an appropriate theory of measurement, while equality among those observables does not follow from the shared label “trust.” Longitudinal applications also need to establish that changes in the measurement instrument or response process have not created apparent trust dynamics. Cross-group and longitudinal measurement-invariance analysis provides one established family of tests for this problem [23].

The event-record interface requires comparable discipline. The selected record $z_S(u)$ is defined from information available before the successor trust state is used to classify the transition. Empirical coding should preserve that temporal direction. A breach indicator, repair type, constraint profile, or interaction category that is constructed retrospectively from the observed trust outcome can mechanically create or erase apparent update defects. Where the record is coded from human annotation, language models, administrative data, or behavioral logs, reliability and uncertainty in that coding belong to the observation layer rather than being silently absorbed into the trust transition.

Table 9: Empirical operationalization of selected trust-dynamics diagnostics.

Formal diagnostic	Minimum empirical object	Candidate design or estimand	Principal identification risk
Update sufficiency	Repeated or approximately matched state–record inputs with successor-state measurements	Estimate conditional successor variation within state–record fibers; compare against a declared tolerance or predictive loss	Sparse support, measurement error, and unmeasured state coordinates can mimic a nonempty defect
Historical-memory contribution	Pre-update history summaries measured in addition to the current state and record	Compare held-out prediction or conditional-law fit with and without declared history summaries, preserving temporal ordering	Omitted present state, leakage from future information, and latent heterogeneity can be mislabeled as memory
Repair and hysteresis	Prospective breach/repair sequences with a declared return condition and pre-specified repair target	Experimental or longitudinal comparison of matched return histories; estimate restoration or target-attainment probabilities	Repair selection, changing external conditions, and outcome-defined repair labels can invalidate the matched-path interpretation
Basin dependence	Repeated trajectories under a common continuation rule with sufficient follow-up to characterize regimes	Estimate a common transition law, propagate uncertainty, and test whether early histories robustly enter different attracting regions	Unobserved regime changes, policy heterogeneity, and finite follow-up can create apparent basin separation
Power-conditioned feasibility	Observed feasible-action/support sets and independently measured constraint profiles	Estimate support or opportunity sets before comparing transition responses; assess overlap where causal contrasts are intended	Structural non-overlap and post-treatment conditioning can make matched comparisons undefined or causally misleading
Stochastic transition law	Repeated conditional observations or a likelihood for successor states together with an observation model	Estimate conditional kernels and compare richer conditioning sets or latent-state alternatives	Latent mixtures, omitted variables, and observation noise can reproduce the same coarse kernel
Cross-context transport	Context-specific measurement models, state/record semantics, and overlap on a transport domain	Test measurement comparability, then evaluate state/record transport and dynamic compatibility on supported cases	Measurement non-invariance or semantic mismatch can make equal numerical scores incomparable

Table 9 maps the principal formal diagnostics to empirical objects that can support an application. The designs are examples rather than universal prescriptions. Trust-repair experiments already illustrate why the details of the intervention matter: apology and denial can have different consequences across competence- and integrity-related allegations [11]; later trustworthy conduct can restore trust after some forms of untrustworthy behavior while deception can leave persistent deficits [22]; and the timing of a breach can alter subsequent restoration trajectories in repeated interaction [14]. Those findings motivate prospective repair and timing designs while leaving the present formal criteria domain-relative.

Table 9 also separates prediction from mechanism identification. A useful empirical sequence is

to test whether the coarse representation predicts adequately, then ask whether a richer representation materially improves prediction, and only then evaluate which substantive mechanism can identify that improvement. For longitudinal data, held-out evaluation should preserve relationship and temporal structure so that information from a later part of the same trajectory does not leak into the representation being tested. For sparse continuous states, exact repeated fibers may be unavailable; the approximate criterion of Section 12.9 or another declared local-comparison rule is then required.

Research Obligation 12.4 (Operationalization disclosure). An empirical application should report the bridge in Definition 12.3, the support on which each diagnostic is evaluated, the approximation or loss criterion used in place of exact equality when applicable, and the alternative mechanisms that remain observationally equivalent under the chosen design.

Obligation 12.4 is methodological rather than prescriptive about one statistical estimator. The same formal diagnostic can be studied with controlled experiments, longitudinal field data, Bayesian state-space models, nonparametric transition estimates, qualitative event reconstruction combined with formal coding, or mixed methods when those approaches supply the required bridge. The relevant standard is correspondence between the empirical design and the formal claim being evaluated.

12.4 Predictive Dependence and Causal Interpretation

The update diagnostics in this paper are primarily representational and predictive. They compare histories, state–record signatures, constraint profiles, and conditional laws. Such comparisons can reveal that one representation carries additional information about subsequent trust evolution. A causal claim requires a counterfactual or intervention interpretation whose identification conditions exceed an observed conditional association [5]. The present diagnostics therefore establish what would happen under a changed feature only when the corresponding intervention and identifying assumptions are supplied explicitly.

Let $\mathcal{I}_c$ be a declared family of admissible interventions. A causal extension requires an intervention-indexed family of transition laws or kernels, written schematically as

$$\{\mathbb{K}_c^{\text{do}(i)} : i \in \mathcal{I}_c\}, \quad (290)$$

together with assumptions connecting the observed regime to the intervention regimes. Equation 290 does not prescribe a particular causal framework. It records the extra object required before an observational contrast is interpreted as an intervention effect.

For example, the constraint-sensitive witness set of Section 9.2 may show that matched represented state–record inputs with different independently specified constraint profiles have different next states. That is evidence of residual predictive dependence at the chosen resolution. A causal interpretation additionally requires a defensible intervention, natural experiment, structural model, longitudinal identification strategy, or comparable design that addresses selection, confounding, and post-treatment conditioning as appropriate to the application. Obligation 9.4 therefore remains binding even when the formal witness is exact.

Remark 12.5 (Causal restraint for historical variables). Historical variables often encode consequences of earlier processes as well as candidate causes of later transitions. An intervention on a historical summary, breach label, institutional constraint, or memory variable may therefore require a model of how that intervention can be realized without silently changing other parts of the history. The formalism does not supply such a model automatically.

Remark 12.5 is especially important for path-dependent analyses. A pair of histories may differ in order, timing, breach, or repair while also differing in many downstream consequences generated by those events. The matched-path constructions of Sections 7 and 8 define precise representational comparisons. Empirical causal claims about those comparisons require a domain-specific design capable of supporting the relevant counterfactual interpretation.

12.5 Cross-Context Transport and Measurement Comparability

A trust model calibrated in one context does not become comparable with another model merely because both use a numerical scale or the same verbal label. The trust-state carrier $\mathfrak{T}_c$, record carrier $\mathfrak{Z}_{c,S}$, active coordinate set, observation channels, and exposure conditions may differ across contexts. Measurement-invariance analysis provides one established method for testing whether a measurement structure is comparable across groups or time points [23]; semantic and dynamical comparability require further structure. Cross-context transport therefore requires declared maps between the relevant objects.

Consider contexts c and c' . Let

$$\psi_T : \mathfrak{T}_c \rightarrow \mathfrak{T}_{c'}, \quad \psi_Z : \mathfrak{Z}_{c,S} \rightarrow \mathfrak{Z}_{c',S'} \quad (291)$$

be partial transport maps for represented trust states and selected interaction records. Equation 291 allows transport to be undefined where the two contexts lack a defensible counterpart.

Suppose deterministic state–record update maps $F_{c,S}$ and $F_{c',S'}$ are available on declared realized domains. Dynamic transport compatibility on a domain $D_{c \rightarrow c'}$ requires the commutation condition in Equation 292.

$$\psi_T(F_{c,S}(\tau, z)) = F_{c',S'}(\psi_T(\tau), \psi_Z(z)) \quad \text{for } (\tau, z) \in D_{c \rightarrow c'}. \quad (292)$$

Equation 292 is a semiconjugacy-type compatibility requirement for the selected dynamics. It is stronger than numerical rescaling alone because it asks the translated update structure to be preserved on the declared transport domain.

For stochastic models, suppose the state–record kernels $\mathbb{K}_{c,S}^{\text{sr}}$ and $\mathbb{K}_{c',S'}^{\text{sr}}$ of Equation 261 are available. An analogous transport condition compares their pushforward probability laws. If ψ_T is measurable on the relevant domain, compatibility requires Equation 293 for all measurable $A \subseteq \mathfrak{T}_{c'}$ under consideration.

$$(\psi_T)_\# \mathbb{K}_{c,S}^{\text{sr}}((\tau, z), \cdot)(A) = \mathbb{K}_{c',S'}^{\text{sr}}((\psi_T(\tau), \psi_Z(z)), A). \quad (293)$$

Equation 293 states equality of transported conditional laws; it does not assert that such a transport exists in a given empirical domain.

Definition 12.6 (Declared trust-dynamics transport). A cross-context transport from (c, S) to (c', S') is a pair of partial maps of the form in Equation 291, together with an explicitly stated transport domain and the semantic justification for matching the transported states and records. The transport is dynamically compatible when the applicable deterministic condition in Equation 292, the stochastic condition in Equation 293, or another declared structure-preservation criterion is satisfied.

Definition 12.6 makes comparability an object of analysis. Without such a bridge, equality of raw numbers, latent-class labels, or qualitative categories across contexts is not enough to establish equality of represented trust states. The same caution applies to longitudinal transport when institutions, participant roles, measurement instruments, or relevant trust domains change over time.

12.6 Normative Evaluation and the Direction of Trust Change

The dynamical formalism describes represented trust evolution and its admissible processes. It does not supply a welfare ordering over trust states. Trust can facilitate cooperation and reliance in some settings, while misplaced trust can expose participants to exploitation, manipulation, unsafe delegation, or institutional capture. Conversely, reduced trust can be a rational or ethically appropriate response to evidence of unreliability, injustice, or abuse. The direction of a trust-state change therefore carries no default normative sign in the present framework.

If an application requires normative evaluation, let

$$\mathcal{N}_c : \mathfrak{T}_c \times \mathfrak{E}_c \times \mathfrak{J}_c \rightarrow \mathfrak{V}_c^{\text{norm}} \quad (294)$$

represent a separately declared evaluative map, where $\mathfrak{E}_c$ carries exposure conditions and $\mathfrak{J}_c$ denotes whatever justice-, legitimacy-, rights-, welfare-, or other normatively relevant information the application chooses to retain. Equation 294 is intentionally external to the trust-state update law.

No order-preservation assumption of the form in Equation 295 is imposed by the trust formalism.

$$\tau \preceq_c^{\text{tr}} \tau' \not\Rightarrow \mathcal{N}_c(\tau, e, j) \preceq_c^{\text{norm}} \mathcal{N}_c(\tau', e, j). \quad (295)$$

Equation 295 states a modeling boundary: even where a trust preorder $\preceq_c^{\text{tr}}$ has been declared, a higher trust state need not be normatively preferable without an additional substantive argument linking trust to the evaluative criterion.

This separation also applies to repair. A target-satisfying repair process can satisfy a declared trust criterion while leaving questions of apology, responsibility, compensation, procedural justice, rights, legitimacy, or institutional reform unresolved. Exact restoration can likewise be normatively undesirable if the pre-breach relation itself was exploitative or unjust. The trust-repair targets of Section 7 therefore remain trust-dynamical targets rather than complete theories of social repair.

Remark 12.7 (Trust dynamics and neighboring normative concepts). The formalism permits empirical or theoretical relations among trust, justice, trustworthiness, cooperation, legitimacy, and institutional performance, while preserving their distinct types. A result about one type does not transfer to another without an explicit linking model.

Remark 12.7 extends the typing rule in Equation 15 from neighboring behavioral concepts to normative interpretation. It also preserves the division of labor across the present paper series: interaction-history and justice-evaluation structures may provide inputs to trust dynamics, but trust dynamics do not subsume those normative analyses.

12.7 Open Systems, Changing Participants, and Structural Regime Change

Many constructions in the paper fix a context, active participant set, trust-coordinate carrier, and admissible history domain for the interval being modeled. That local closure is analytically useful, but social systems can change their own modeling substrate. Participants enter or leave; organizations merge or dissolve; legal roles change; communication channels disappear; a trustee can be replaced by a collective; or a relation can move into a domain in which the prior trust coordinate is no longer meaningful.

One way to represent such change is to index the trust-state carrier by a regime variable ρ_t and use a disjoint-union carrier of the form in Equation 296.

$$\mathfrak{T}^{\text{open}} := \coprod_{\rho \in \mathcal{R}} \mathfrak{T}_{\rho}, \quad (296)$$

where $\mathcal{R}$ is a declared set of representational regimes. Equation 296 permits the admissible state type itself to change. A transition between components then requires an additional regime-transition rule rather than an ordinary endomorphism on one fixed carrier.

The need for Equation 296 marks a boundary of the simpler fixed-carrier results. For example, a basin defined under one continuation rule and one state space need not remain meaningful after a regime change that alters the carrier or feasible process family. Likewise, a finite-memory horizon established before an institutional transformation need not remain sufficient afterward. The formalism can be extended to such cases, but the extension requires the changing structure to be represented explicitly.

The same point applies to identity continuity. If an empirical application treats “the same” institution, organization, AI system, or collective as persisting across substantial internal replacement, the criterion of identity belongs to the model specification. The trust-dynamics equations do not decide that criterion.

12.8 Aggregation, Population Inference, and Relational Heterogeneity

Typed trust states can carry many directed participant–domain coordinates. A population summary or institutional score is therefore an additional compression, not a primitive consequence of the local state representation. Let

$$A_c : \mathfrak{T}_c \longrightarrow \mathfrak{G}_c \quad (297)$$

be a declared aggregation map into a group-, population-, organizational-, or system-level summary carrier $\mathfrak{G}_c$. Equation 297 may average, pool, project, cluster, or otherwise summarize heterogeneous local trust coordinates.

Aggregation is dynamically sufficient for a selected update task only when equal aggregated states cannot conceal distinctions that matter to the aggregated successor. On the realized update domain, a necessary fiber condition is stated in Equation 298.

$$A_c(\tau^-(u)) = A_c(\tau^-(v)), \quad z_S(u) = z_S(v) \implies A_c(\tau^+(u)) = A_c(\tau^+(v)). \quad (298)$$

Equation 298 is the aggregation analogue of the update-factorization criterion. Its failure means that the aggregate summary discards distinctions needed for the selected aggregate dynamics.

Proposition 12.8 (Aggregate update criterion). *On a realized deterministic update domain, an aggregate state–record update map $F_{c,S}^A : \text{Im}(A_c \circ \tau^-, z_S) \rightarrow \mathfrak{G}_c$ with $A_c(\tau^+(u)) = F_{c,S}^A(A_c(\tau^-(u)), z_S(u))$ exists if and only if Equation 298 holds for every pair of realized update occurrences.*

Proposition 12.8 is the same quotient logic used in Proposition 5.2, now applied after aggregation. It prevents group-level trust dynamics from being inferred by averaging local trust coordinates and then assuming that the average is dynamically closed. Population inference requires additional sampling assumptions as well: a dynamically adequate aggregate for one observed population need not transport to another population with different role structures, exposure distributions, or interaction opportunities.

12.9 Exact Formal Criteria and Approximate Empirical Application

The core results use exact equality because exact relations make the logical structure transparent. Empirical data typically contain measurement error, finite-sample uncertainty, continuous variation, and model misspecification. Applying the formalism empirically therefore requires an approximation rule rather than replacing approximate equality with exact equality by convention.

Suppose the represented trust-state space carries a metric d_c^{tr} of the type allowed in Section 3.3. For a realized state–record input x in the image of the deterministic update signature, define its output fiber by

$$\mathcal{Y}_{c,S}(x) := \{ \tau^+(u) : \text{In}_{c,S}^{\text{upd}}(u) = x \}. \quad (299)$$

Equation 299 collects the represented next states associated with one repeated input signature. Its metric diameter is

$$\text{diam}_c^{\text{tr}} \mathcal{Y}_{c,S}(x) := \sup_{y, y' \in \mathcal{Y}_{c,S}(x)} d_c^{\text{tr}}(y, y'). \quad (300)$$

when the supremum is defined. Equation 300 is zero for every realized input exactly when the deterministic fiber criterion holds in a metric space.

For a declared tolerance $\varepsilon \geq 0$, define an ε -sufficient update representation by Equation 301.

$$\sup_x \text{diam}_c^{\text{tr}} \mathcal{Y}_{c,S}(x) \leq \varepsilon. \quad (301)$$

Equation 301 is one possible approximation criterion rather than a universal empirical standard. A scientifically defensible tolerance must be tied to measurement resolution, substantive effect size, predictive loss, decision consequences, or another declared criterion. In stochastic models, analogous approximation can be formulated with a declared distance or divergence between conditional probability laws.

Remark 12.9 (Approximation does not remove identification requirements). Replacing exact equality by a tolerance can make a representation practically adequate for a declared task, while leaving mechanism identification unresolved. Approximate predictive sufficiency and causal or ontological identification remain distinct claims.

Remark 12.9 prevents empirical convenience from collapsing the boundaries established earlier in this section. A model may predict within an acceptable tolerance while remaining one member of a large observational-equivalence class.

12.10 Trajectory Descriptors, Process Semantics, and Ontological Restraint

The process-algebraic results of Section 10 and the braid interface developed across the series have a limited semantic role. A trajectory descriptor can encode order, crossing, interaction pattern, or another structural feature of a history. Its algebraic operations belong to the descriptor unless a separate semantics maps those operations into admissible social processes and represented trust transitions.

The distinction can be written schematically as the pair of maps in Equation 302.

$$\Gamma_c^{\text{tr}} \xrightarrow{B_c} \mathfrak{B}_c, \quad \Gamma_c^{\text{tr}} \xrightarrow{\Theta_c^{\text{tr}}} \mathfrak{T}_c. \quad (302)$$

Equation 302 supplies two representations of histories with different analytical roles. A group inverse, equivalence relation, or composition law in $\mathfrak{B}_c$ transfers to trust dynamics only when an additional structure-preserving relation between B_c and the trust-process semantics has been established. Section 10 showed explicitly why an inverse trajectory descriptor need not provide an admissible inverse social process or a trust-restoring inverse transformation.

The same restraint applies to the language of state, field, memory, basin, attractor, and stochasticity. These terms name mathematical or model-theoretic structures under declared assumptions. Their successful use does not establish that social reality is ultimately composed of those mathematical objects. The paper therefore retains the ontological suspension already built into the broader relational programme: formal structures are assessed by their analytical adequacy, explanatory usefulness, empirical support, and revisability rather than by an inference from mathematical convenience to ultimate ontology.

12.11 Boundary Matrix for Stronger Interpretive Claims

Table 10 summarizes the principal interpretive boundaries. Each row pairs a result that the current formalism can support with the additional structure required for a stronger claim.

Table 10 turns the limitations into research obligations rather than generic disclaimers. The formalism is strongest when it makes visible which inferential step has already been established and which step remains external to the current model.

12.12 Scope of the Present Contribution

The present paper consequently makes a bounded architectural and diagnostic contribution. It supplies a typed language for historical trust states and interaction records, deterministic and stochastic sufficiency tests, and explicit diagnostics for retained history, repair, basin dependence, power-conditioned feasibility, process composition, and uncertainty. These components organize established ideas rather than claiming ownership of them: dynamic trust research already includes recursive, cumulative-history, Bayesian, probabilistic, and other state-based models [6, 4, 20]; path-dependence scholarship has long disciplined claims about sequence, contingency, self-reinforcement, and difficult reversibility [15, 19]; and trust-repair scholarship provides substantive theories and evidence concerning violation and recovery [12]. Factorization, quotienting, partial transformations, stochastic kernels, relations, and basins are likewise established mathematical objects.

The claimed contribution is the typed assembly and the failure tests between layers: when state-record compression is insufficient; when retained history resolves that defect; when repair succeeds

Table 10: Scope boundaries and additional requirements for stronger trust-dynamics claims.

Formal result	Supported interpretation	Additional requirement for a stronger claim
Coherent state dynamics on a carrier labeled “trust”	The declared state variable admits the stated mathematical representation	Substantive anchoring under Obligation 2.1, plus an observational or theoretical bridge before interpreting the state as trust
Nonempty update or kernel defect	Chosen compressed representation is dynamically insufficient on the declared domain	Evidence distinguishing explicit memory, omitted state, latent heterogeneity, measurement error, or another mechanism
Memory-assisted factorization	Added historical summary resolves a declared defect	Evidence that the summary corresponds to a substantively distinct memory mechanism rather than an alternative state encoding
Constraint-sensitive witness	Constraint descriptor improves or changes prediction at matched represented inputs	Intervention design or causal model supporting an effect of the constraint itself
Basin split under a common rule	Early histories occupy different basins for one declared continuation dynamics	Evidence that the specialization is empirically adequate and remains stable under relevant regime changes
Restricted-family nonreturn	Restoration is unreachable within the declared process family	Justification that the process family exhausts substantively relevant repair interventions before interpreting nonreturn as irreversibility
Non-degenerate stochastic kernel	Conditional uncertainty remains under the declared conditioning representation	Identification argument separating transition randomness from latent mixtures, omitted variables, and observation uncertainty
Successful state-level process quotient	Selected state and record representation supports compositional process semantics on the realized domain	Domain expansion tests before treating the quotient as globally adequate
Cross-context numerical similarity	Two contexts use values that appear comparable under a chosen coding	Measurement invariance where measured scales are compared, plus semantic state/record transport and, where needed, dynamic compatibility of the form in Definition 12.6
Increase in represented trust	Trust state moves upward under a declared trust preorder	Separate normative argument before inferring welfare, justice, legitimacy, or institutional desirability
Braid or trajectory inverse	Descriptor admits an algebraic inverse	Admissible inverse social path and semantic cancellation before inferring trust restoration

without restoration; when feasible-process restrictions differ from response effects; when a state quotient supports compositional process semantics; and when stochastic residuals remain non-identifying among latent mixtures, observation error, unresolved history, and non-degenerate conditional laws.

The framework does not determine a universal definition or metric of trust, a universal memory law, a domain-independent repair standard, or a general causal law of trust formation. Nor does it establish that trust should be maximized, restoration is always desirable, stochastic residuals are irreducible, basin boundaries are critical points, or braid-theoretic inverses represent social reversibility. Each stronger conclusion requires the corresponding substantive, empirical, causal, normative, or mathematical bridge identified above. In this sense the boundary is constructive: the formalism tests whether a declared historical compression is adequate for a task and identifies what additional bridge a stronger interpretation

would require. Section 13 synthesizes that division of labor.

13 Conclusion

This paper developed a preliminary formal architecture for historical trust formation in repeated interaction. Its organizing problem is representational: a model must decide which parts of interaction history are retained in the represented trust state, which are encoded in the current interaction record, which remain as explicit memory or latent structure, and which uncertainty is represented probabilistically. The resulting framework does not select one substantive theory of trust. It supplies typed conditions for asking when a declared representation is dynamically adequate and for locating what kind of additional structure is required when it is not.

13.1 Historical Compression and Relational State Adequacy

The first principal result is that historical dependence can be analyzed as a compression problem. Sections 2–5 separated the trust-state carrier from the pre-update interaction record and defined exact factorization as the criterion for deterministic next-state sufficiency. When histories that share the same represented state and record can still produce different represented successors, the interface has compressed a dynamically relevant distinction. That diagnosis is intentionally weaker than a mechanism claim: the missing distinction may belong in a richer state, a finer record, an explicit memory variable, latent structure, or another target representation.

Section 6 refined that diagnosis by treating memory as retained historical information at a declared resolution. A candidate memory representation earns a dynamical role when it resolves a defect that persists at the selected state–record interface. Information refinement, recursive memory, finite-horizon tests, and the full-history control then distinguish several cases that ordinary references to “path dependence” tend to merge. In particular, residual variation after full-history conditioning cannot be attributed to historical compression alone.

This structure yields a representation-relative notion of Markovian adequacy. A compact state can be sufficient in one task and insufficient in another; feature-level prediction can close even when exact state prediction does not; and an augmented state can absorb explicit memory without erasing the historical origin of its components. The relevant question is therefore not whether trust is intrinsically Markovian or non-Markovian, but which representation is sufficient for a declared inferential target.

13.2 Repair, Feasibility, and Non-Invertible Process Structure

The second principal result concerns repair and composition. Section 7 distinguished a repair attempt from target satisfaction, restoration, hysteretic nonclosure, matched-path hysteresis, and repair-family irreversibility. A relation can satisfy an acceptable repair criterion while remaining distinguishable from its pre-breach state. Likewise, nonreturn under one repair protocol or one constrained continuation family is weaker than unreachability under every admissible intervention.

Sections 8 and 9 added two sources of path restriction. Basin dependence can make early differences persist under a common later dynamics, while power-relevant conditions can restrict feasible successors, visible evidence, repair opportunities, and continuation rules. Both results are conditional on declared structures. A basin split under one continuation policy does not establish permanent social irreversibility, and constrained nonreturn does not establish nonreturn in a broader repair family.

Section 10 then made the algebraic consequence explicit. The conservative compositional object is the labeled history path system. Compression to a trust-state process calculus requires more than one-step outcome factorization: histories represented by the same trust state must also support compatible labeled futures. When that transition congruence holds, record labels induce deterministic partial transformations and generated process families. When it fails, relation-valued or history-level semantics preserve the unresolved distinctions.

This process architecture explains why trajectory invertibility and trust-process invertibility are different claims. A braid-like or other trajectory descriptor can possess an algebraic inverse while the inverse social path is infeasible or while the induced trust transformation fails to restore the original state. Repair can therefore be meaningful without being group-theoretic cancellation. The paper's non-group language is a consequence of partiality, reachability, and historical compression rather than an assertion that every trust process is irreversible.

13.3 Uncertainty, Identification, and Interpretive Discipline

The third result is an identification discipline for stochastic and empirical interpretations. Section 11 took the full-history-conditioned probability law as the stochastic reference object and asked whether that law factors through a coarser state-record interface. This separates variation among realized outcomes from variation in the conditional laws that generate them. Two histories can realize different successors while sharing the same conditional law, just as equal represented states and records can conceal different history-conditioned laws.

The stochastic extension also shows why a fitted kernel does not identify its own mechanism. The same coarse non-degenerate law can arise directly as a conditional transition kernel or as a mixture over deterministic latent transitions. A separate observation kernel can generate noisy measured trust even when the latent represented transition is deterministic. Unresolved history, latent heterogeneity, observation error, and non-degenerate transition uncertainty therefore remain distinct model classes unless additional evidence identifies among them.

Section 12 generalizes that restraint and adds an empirical bridge specification. A formal diagnostic becomes an empirical claim only after the relation or unit, observation schedule, trust measurement, pre-update record coding, sampling or assignment regime, approximation criterion, and uncertainty model have been declared. Predictive dependence requires an intervention or causal model before receiving a causal interpretation. Cross-context comparison requires explicit transport between state and record representations rather than numerical similarity alone. Aggregate trust statistics introduce another compression whose autonomous dynamics require their own fiber conditions. Normative evaluation likewise remains separately typed: an increase in represented trust does not by itself imply greater welfare, justice, legitimacy, or desirability. Finally, state, memory, basin, attractor, and stochastic-kernel language are representational commitments rather than proofs about ultimate social ontology.

These boundaries are part of the framework's positive contribution. They specify what additional bridge is required for a stronger claim and thereby prevent a formal diagnostic from carrying more empirical or normative content than its assumptions support.

13.4 Empirical and Comparative Research Priorities

The completed architecture suggests four connected research priorities. They follow from the paper's diagnostic contribution: locating where historical information resides, determining which process claim has actually been established, and stating what additional bridge is required before moving from mathematical adequacy to mechanism, causation, normativity, or ontology. The first is empirical comparison of alternative historical compressions. Longitudinal relational data can be used to compare state, record, and memory representations by deterministic or distributional defect structure, predictive performance, stability across samples or contexts, and representational parsimony. Such work should pre-specify the observation channel, event coding, support, approximation rule, and uncertainty treatment so that an apparent defect is not created by measurement drift or retrospective outcome coding. It can then test whether a finite horizon is adequate, whether a persistent historical summary adds information, and whether apparent memory disappears after refining the present state or event record.

The second priority is direct study of repair and constrained reachability. Experimental and observational designs can vary apology, compensation, verification, institutional change, third-party mediation, time, renewed interaction, and available exit or appeal. Multi-feature trust measurement would then permit separate tests of target satisfaction, exact restoration, matched-path hysteresis, and repair-family

reachability. Power-conditioned analyses can additionally ask whether superficially similar acts differ because the feasible alternatives surrounding them differ.

The third priority is mechanism identification under uncertainty. Repeated measurement, richer covariates, latent-state models, explicit observation models, and manipulated information can test whether residual transition variation is better represented as unresolved history, latent heterogeneity, measurement uncertainty, or a stable non-degenerate conditional law. Reporting the conditioning information and observation process is essential for interpreting residual stochasticity.

The fourth priority is transport across domains and changing relational systems. Interpersonal, organizational, institutional, intergroup, and heterogeneous human–AI relations may each support trust-like states without sharing the same carrier or event semantics. Comparative work should therefore establish which structural features can be transported and which require context-specific typing. Open-system extensions are also needed when participants, offices, institutions, legal regimes, or admissible interaction types change during the modeled process rather than remaining fixed.

The paper’s overall conclusion is methodological. Repeated interaction can generate relational states whose relevant history may sometimes be compressed into a compact current representation and may sometimes require explicit memory, constrained reachability, latent structure, non-invertible process semantics, or stochastic laws. Which structure is appropriate is a question to be demonstrated for the task and domain rather than assumed in advance. Historical trust formation can then be studied as a disciplined relation among interaction trajectories, represented relational states, feasible processes, and conditional uncertainty, with causal inference and normative evaluation retained as additional and explicitly declared layers.

A Formal Diagnostic Examples

This appendix tests the paper’s diagnostics on finite constructions with fully declared mechanisms. The examples are comparative: similar terminal differences or non-degenerate outcome laws are generated by memory, unequal record feasibility, basin splitting, constrained repair, common stochastic laws, latent mixtures, observation error, and distributional hysteresis. Their purpose is to show which formal tests discriminate these mechanisms and which interpretations remain underidentified; they are not empirical models of a particular social domain.

A.1 Finite Construction Conventions

Each example fixes its own finite trust-state carrier and realized record set. The context c and record selector S are suppressed locally when they play no role in the calculation. A symbol such as L , M , or H denotes a distinct represented trust state only within the example in which it is declared; the symbols do not assert a universal low–medium–high scale. Dirac probability measures use the notation δ_τ from Section 11.

The examples are diagnostic constructions. Their purpose is to evaluate implications of the formal definitions under transparent assumptions. A mechanism that appears in one construction should therefore be read as a possibility within the framework rather than as an empirical claim about trust generally.

A.2 One-Bit Historical Memory for a Coarse Update Defect

Consider the finite trust-state carrier $\mathfrak{T}^{(1)} = \{L, H\}$ and a single selected record s . Let two admissible update occurrences $u_0 = (h_0, e_0, h_0^+)$ and $u_1 = (h_1, e_1, h_1^+)$ satisfy the common represented input and divergent represented outputs in Equation 303.

$$\tau^-(u_0) = \tau^-(u_1) = L, \quad z_S(u_0) = z_S(u_1) = s, \quad \tau^+(u_0) = H, \quad \tau^+(u_1) = L. \quad (303)$$

Equation 303 places (u_0, u_1) in the update-defect set of Equation 83. At the coarse state–record resolution, the next represented state is therefore not a function of (L, s) .

Now define a one-bit historical summary on the two pre-histories by Equation 304.

$$M^{(1)}(h_0) = 0, \quad M^{(1)}(h_1) = 1. \quad (304)$$

For the realized augmented inputs, define the memory-assisted update map by Equation 305.

$$F_{\text{mem}}^{(1)}(L, s, m) = \begin{cases} H, & m = 0, \\ L, & m = 1. \end{cases} \quad (305)$$

Equations 304–305 recover both post-states in Equation 303, so the residual memory defect is empty on this finite realized domain. The example instantiates Definition 6.2. It also shows why the word “memory” remains representation-relative: the same bit could instead be incorporated into a refined present-state representation $(\tau, M^{(1)})$. The formal result is resolution of the coarse update defect, not an ontological proof that trust possesses a separate memory substance.

A.3 State Compression with Unequal Record Feasibility

Exact update factorization is weaker than a complete state-level process quotient. Let two histories g_a and g_b satisfy $\Theta^{\text{tr}}(g_a) = \Theta^{\text{tr}}(g_b) = L$. Declare their locally realizable record sets by Equation 306.

$$\mathfrak{J}^{\text{loc}}(g_a) = \{s, r\}, \quad \mathfrak{J}^{\text{loc}}(g_b) = \{s\}. \quad (306)$$

Suppose both s -labeled successors have represented post-state H , while the sole r -labeled successor from g_a has represented post-state L . The realized state–record update therefore has the well-defined values in Equation 307.

$$F^{(2)}(L, s) = H, \quad F^{(2)}(L, r) = L. \quad (307)$$

Equation 307 is exactly update-sufficient on the realized update domain: every duplicated input (L, s) has the same represented outcome, while (L, r) occurs only from g_a . The factorization criterion in Proposition 5.2 is therefore satisfied.

The state-invariant feasibility condition nevertheless fails because Equation 306 assigns different feasible label sets to histories represented by the same trust state. Consequently, Proposition 10.3 does not permit a complete deterministic trust-state process quotient. The example isolates a feasibility inconsistency that outcome factorization alone cannot detect.

A.4 Early Record Order with Basin Splitting

A finite example can also separate early order effects from later common dynamics. Let the represented state carrier be $\mathfrak{X}^{(3)} = \{x_0, a, b\}$ with the discrete topology. Define two early partial record maps E^p and E^q on the realized inputs by Equation 308.

$$E^p(x_0) = a, \quad E^q(a) = a, \quad E^q(x_0) = b, \quad E^p(b) = b. \quad (308)$$

Under the temporal word convention of Equation 233, the two permutations of the same record multiset have the endpoints in Equation 309.

$$E^q \circ E^p(x_0) = a, \quad E^p \circ E^q(x_0) = b. \quad (309)$$

After the early window, impose the common continuation rule $\Phi^{(3)}$ in Equation 310.

$$\Phi^{(3)}(a) = a, \quad \Phi^{(3)}(b) = b, \quad \Phi^{(3)}(x_0) = a. \quad (310)$$

With the discrete topology, $\{a\}$ and $\{b\}$ are attracting sets for Equation 310; their basins contain a and b , respectively. Equation 309 therefore supplies an order-sensitive basin witness in the sense of Definition 8.6. Because both endpoints subsequently evolve under the same $\Phi^{(3)}$, the split persists by Proposition 8.5. The construction establishes order-sensitive basin membership under the declared specialization. It does not, by itself, establish a general primacy effect or empirical criticality.

A.5 Constraint-Limited Repair Reachability

Consider a breached trust state L and a pre-breach restoration target H . At the unconstrained process level, let two records be feasible from L : a repair record r and a maintenance record m . Their represented transitions are defined by Equation 311.

$$F^r(L) = H, \quad F^m(L) = L. \quad (311)$$

Suppose an independently specified constraint profile $q^\dagger$ removes r from the feasible record set while retaining m . The baseline and constrained feasible sets are then those in Equation 312.

$$\mathfrak{Z}^{\text{base}}(L) = \{r, m\}, \quad \mathfrak{Z}^{q^\dagger}(L) = \{m\}. \quad (312)$$

Under the baseline repair family, H is reachable from L in one step by r . Under the constrained family generated only by m , every finite continuation remains at L . The reachability contrast is stated in Equation 313.

$$H \in \mathcal{R}^{\text{base}}(L), \quad H \notin \mathcal{R}^{q^\dagger}(L). \quad (313)$$

Equation 313 is a concrete instance of the contraction result in Proposition 9.6. Nonreturn under $q^\dagger$ is therefore compatible with broader reversibility. A substantive power interpretation of $q^\dagger$ would still require the independent provenance demanded by Obligation 9.1.

A.6 Common Conditional Law with Divergent Realizations

Observed outcome variation can occur without distributional history dependence. Let the trust-state carrier be $\mathfrak{T}^{(5)} = \{L, H\}$ with the discrete sigma-algebra. Consider two full-history inputs x_1 and x_2 that project to the same represented state–record input, as specified in Equation 314.

$$\pi^{\text{sr}}(x_1) = \pi^{\text{sr}}(x_2) = (L, s). \quad (314)$$

Assign both historical inputs the same conditional next-state law in Equation 315.

$$\mathbb{K}^{\text{hist}}(x_1, \cdot) = \mathbb{K}^{\text{hist}}(x_2, \cdot) = \frac{1}{2}\delta_L + \frac{1}{2}\delta_H. \quad (315)$$

A realized dataset can contain one next state H after x_1 and one next state L after x_2 . Such realized outcomes disagree pointwise, so a deterministic map fitted directly to the two occurrences would exhibit a deterministic update defect. Equation 315, however, makes (x_1, x_2) absent from the kernel-defect set in Equation 263. The state–record representation is stochastically update-sufficient for this pair. The example therefore separates within-law realization variation from history-dependent conditional laws.

A.7 Coarse-Kernel Ambiguity from Latent Mixtures

A non-degenerate coarse kernel can admit structurally different representations. For the realized input $x = (L, s)$, let the coarse next-state law be Equation 316.

$$\mathbb{K}^{\text{sr}}(x, \cdot) = 0.7\delta_H + 0.3\delta_L. \quad (316)$$

Equation 316 can be used directly as a non-degenerate conditional transition kernel. It also admits a deterministic latent-mixture representation. Let the latent carrier be $\Lambda = \{\lambda_H, \lambda_L\}$ with weights and deterministic latent-conditioned outcomes given by Equation 317.

$$Q(\lambda_H | x) = 0.7, \quad Q(\lambda_L | x) = 0.3, \quad G(x, \lambda_H) = H, \quad G(x, \lambda_L) = L. \quad (317)$$

Integrating the Dirac laws at the two values of G against Q reproduces Equation 316. This is the finite numerical instance of Proposition 11.6. The predictive coarse kernel is identified by the construction; the interpretation as within-condition transition uncertainty or uncertainty over deterministic latent states is not identified without additional restrictions.

A.8 Observation Error after a Deterministic Trust Transition

Observed stochasticity can also enter after a deterministic trust transition. Retain the trust states $\{L, H\}$ and suppose the true next trust state after input $x = (L, s)$ is deterministically H , as in Equation 318.

$$\mathbb{K}^{\text{sr}}(x, \cdot) = \delta_H. \quad (318)$$

Let an observed indicator $Y \in \{y_L, y_H\}$ be generated by an observation kernel whose values at the true state H are given by Equation 319.

$$\mathbb{O}(H, \{y_H\}) = 0.8, \quad \mathbb{O}(H, \{y_L\}) = 0.2. \quad (319)$$

Combining Equations 318–319 through the predictive observation law of Equation 274 yields Equation 320.

$$\Pr(Y = y_H \mid x) = 0.8, \quad \Pr(Y = y_L \mid x) = 0.2. \quad (320)$$

The non-degenerate observed distribution in Equation 320 is generated entirely by the observation layer in this construction. It therefore supplies no basis for inferring a non-degenerate trust-transition kernel. The example makes the transition–measurement separation in Obligation 11.8 operational.

A.9 Probabilistic Repair with Distributional Hysteresis

The final example combines target-relative repair with matched-path distributional hysteresis. Let $\mathfrak{T}^{(7)} = \{L, M, H\}$ with starting state H . Consider a control return word w^{ctl} and a breach–repair return word $w^{\text{br,rep}}$. Assume both satisfy the same declared external return condition from Section 7. Their terminal laws are specified in Equation 321.

$$\mathbb{K}^{w^{\text{ctl}}}(H, \cdot) = \delta_H, \quad \mathbb{K}^{w^{\text{br,rep}}}(H, \cdot) = 0.6 \delta_H + 0.4 \delta_M. \quad (321)$$

Declare the acceptable repair target and exact restoration target by Equation 322.

$$\mathfrak{A}^{\text{acc}}(H) = \{M, H\}, \quad \mathfrak{A}^{\text{rest}}(H) = \{H\}. \quad (322)$$

The target-attainment probabilities calculated from Equation 282 are then those in Equation 323.

$$p^{\text{acc}}(H, w^{\text{br,rep}}) = 1, \quad p^{\text{rest}}(H, w^{\text{br,rep}}) = 0.6. \quad (323)$$

Equation 323 shows that certain attainment of an acceptable repair target can coexist with uncertain exact restoration. Moreover, the two terminal laws in Equation 321 are unequal. Under the stipulated matched external return condition, Equation 284 therefore diagnoses distributional hysteresis. A single realization that happens to terminate at H after $w^{\text{br,rep}}$ would remain compatible with that law-level hysteresis because the comparison concerns terminal distributions rather than one realized pair.

A.10 Comparative Diagnostic Matrix

The finite constructions demonstrate that similar observational patterns can correspond to different formal mechanisms. Table 11 summarizes the discriminating test supplied by each example.

Table 11 compares the resulting diagnoses. Across the examples, terminal differences, repeated-outcome variability, and observed nonreturn remain underdetermined until the relevant state, record, history, feasible process family, probability provenance, and observation interface are declared. Section 12 states the corresponding interpretation limits.

Table 11: Diagnostic interpretation of the formal diagnostic examples.

Construction	Decisive formal feature	Supported interpretation
One-bit memory	Equations 303–305	A coarse deterministic update defect is resolved by one declared historical distinction; the distinction may be represented as memory or absorbed into a refined state.
Unequal record feasibility	Equations 306–307	Exact outcome factorization can hold while a complete state-level process quotient fails because feasible labels differ across compressed histories.
Early record order	Equations 308–310	Two permutations of the same early record multiset enter different basins under one later continuation rule.
Constraint-limited repair	Equations 312–313	Restoration can be unreachable under a declared constraint while remaining reachable in the broader repair family.
Common stochastic law	Equations 314–315	Divergent realized outcomes can arise without distributional history dependence when their conditional laws coincide.
Latent mixture	Equations 316–317	One coarse predictive kernel can support both a direct non-degenerate transition representation and a deterministic latent-mixture representation.
Observation error	Equations 318–320	Non-degenerate observations can arise after a deterministic trust transition through a separate measurement channel.
Probabilistic repair	Equations 321–323	Certain acceptable repair can coexist with uncertain exact restoration and with distributional hysteresis under matched external return conditions.

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