

The Topological Epistemology of Future Creation: A Two-Complex Formalism for Spacetime Continuation

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Abstract

This discussion paper proposes a preliminary two-complex framework for the topological epistemology of future creation. Its object is the construction and revision of possible continuations of historically represented relational spacetime. Events are represented by vertices, couplings by edges, and higher-order primitives by faces, while historically constituted entities are modeled through relational substructures. An assumed ontological relational spacetime is used solely as a formal reference construct, with ultimate ontology suspended. The observer's epistemic capacities and model-reconstruction operators are treated as historically conditioned, and the resulting representations as partial and revisable. Candidate futures are formulated as history-indexed whole-spacetime extensions whose embeddings preserve the represented historical component and its attachments; future-only formulations remain possible with suitable relative data. Dependent types, undercategories, relative cell complexes, homology, and discrete Morse theory supply mathematical resources for comparing extensions and specifying information-preservation requirements. The paper identifies problems of indeterminate future incidence, nonunique reconstruction, epistemic revision, and changing spaces of conceivable possibilities, and provides exact synthetic computations illustrating historical attachment dependence, epistemic revision, and reduction-related information loss, with interfaces to subsequent normative inquiry concerning future creation and justice. Its constructions are provisional, its cases are illustrative, and no empirical forecast or comprehensive ontological generator is claimed.

Keywords: future creation; topological epistemology; relational spacetime; two-complex; epistemic continuation; historical constitution

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Discussion Paper Note

This manuscript is an independent preliminary inquiry into the topological epistemology of future creation. It considers how historically situated observers may construct, compare, and revise possible continuations of epistemically represented relational spacetime. The two-complex setting represents events as vertices, couplings as edges, and higher-order primitives as faces requiring explicit interpretation. It treats historically constituted subjects and institutions as possible outcomes of relational organization rather than primitive event-vertices.

The phrase *assumed ontological relational spacetime* designates a formal reference construct used to express a possible dependence of observation and epistemic capacities upon historical constitution. Its use leaves ultimate ontology, the reality of time, and the existence of an exhaustive underlying generative model unresolved. The epistemic complexes developed in this inquiry are situated, partial, and revisable representations, not complete descriptions of that reference domain.

The preliminary formalism favors whole-spacetime extensions equipped with explicit embeddings of their represented histories. Future-only representations remain legitimate if their historical interfaces and attaching data are retained. Dependent types, category-theoretic constructions, relative homology, and discrete Morse theory are adopted as established mathematical resources; their proposed interpretation within future-creation epistemology requires additional definitions and arguments.

This first writing increment comprises the complete Introduction and a source-verified initial bibliography. The detailed outline, technical constructions, illustrative experiments, normative interfaces, and independent open questions are planned components, not completed results. All claims should be read within their specified evidential and formal scope.

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The author welcomes scholarly examination, teaching, criticism, and responsible adaptation of the proposed framework. Applications to governance, scientific research, institutional design, education, or human welfare should maintain explicit distinctions among modeled possibility, observed evidence, practical feasibility, and justified ethical or legal judgment. Mathematical descriptions of candidate futures should be examined for omissions in the recognition of participants, capacities, historical conditions, and alternative futures. Formal topological relations alone do not authorize decisions affecting others.

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Notices

Publication status. Independent preliminary discussion draft, version 2.4, 20 October 2026. Sections 1–15 and Appendices A–H constitute the first complete technical draft. The document has not been peer reviewed. The mathematical results are conditional on their stated assumptions, and the proposed epistemic-reconstruction and normative interpretations remain subject to further scholarly examination.

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1 Introduction

Future creation presents an epistemic problem before it presents a problem of planning, optimisation, or governance. Decisions concerning possible futures are made within historically formed relations, using descriptions of the past whose evidential reach, interpretive categories, and representational precision are limited. The events that may later become consequential have yet to occur; the couplings through which they might arise can depend on conditions that have not been observed or even conceived. A mathematical inquiry into future creation must therefore address both the structural relation between an inherited history and its prospective continuations, and the formation of the knowledge through which those continuations are considered. This paper develops a preliminary two-complex formalism for that inquiry.

The research object is *future creation*: the prospective formation and transformation of relational conditions through which further events, relations, and capacities may become possible. This usage encompasses historically consequential actions and the changes they induce in subsequent opportunities, while leaving the precise ontology of time an open philosophical question. Future creation is approached here through the *epistemic continuation of relational spacetime*. An observer represents a portion of history, formulates admissible candidate extensions, and revises those representations as evidence and epistemic capacities change. The term *continuation* refers to relational spacetime; it does not presuppose that creation itself is a pre-existing object with a determinate continuation law. The aim is a language in which these distinctions can be stated and examined, rather than a general prediction procedure.

The mathematical setting uses an epistemically interpreted two-dimensional cell complex. Its zero-cells stand for events, its one-cells for couplings among events, and its two-cells for higher-order primitives whose domain-specific semantics require explicit specification. Agents, organisations, institutions, and other putative entities may be reconstructed from historically constituted subcomplexes or families of relational configurations; they are not identified with elementary vertices. Cellular incidence and attachment provide a way to record higher-order relational structure, while additional data are required to express causal precedence, observation, modality, and normative assessment. The two-dimensional cellular representation is a deliberate preliminary modeling choice. Its expressive limits, including phenomena needing richer incidence, labels, or higher-dimensional structures, form part of the research question.

Spin-foam research supplies an instructive mathematical precedent. Spin foams are decorated two-complex structures whose faces and edges bear representation-theoretic data in quantum-gravity constructions; spin-foam *models* additionally specify dynamical amplitudes and rules over such complexes [2]. The present inquiry adopts the availability of two-complex structure as a modeling resource. Its event–coupling–primitive interpretation is a separate proposed semantics, and no identification of social or epistemic histories with quantum-gravitational processes is implied. An individual decorated two-complex may represent a particular history under a given formalism; the mathematical problem of inferring which possible histories are admissible remains separate from the specification of that individual structure.

The epistemic status of history requires special care. The paper introduces, when analytically useful, an *assumed ontological relational spacetime* as a posited reference domain for discussing how observers and their capabilities are historically constituted. This domain is a formal convenience. Its definition makes no assertion that relational spacetime is the ultimate constitution of reality, that its representation is exhaustive or unique, or that a complete ontological generator has been discovered. Epistemic relational spacetime, by contrast, denotes a situated and revisable representation produced under historically conditioned observation and interpretation. The distinction permits an account in which the observer’s capacity to recognise possible futures is itself conditioned by relational history. Situated-knowledge scholarship supplies a relevant account of partial perspectives and the historical position of knowers [11]; the precise endogenous observation operator remains a proposal to be developed rather than a settled formal consequence.

Several established traditions illuminate parts of this problem. Event-structure semantics treats

events, causal dependency, and concurrency as explicit features of processes [23]. Computational mechanics investigates how predictive organisation may be reconstructed from observation sequences within specified stochastic settings [5]. Dependent type theory supplies constructions in which the type of a possible object depends on a given context or parameter [22], and category theory supplies languages for morphisms, composition, and objects equipped with maps from a fixed source [16]. Cellular and relative homology describe how a complex and a containing complex differ topologically [12]; discrete Morse theory gives methods for reducing cellular complexity under controlled topological conditions [8]. These contributions provide mathematical resources for the proposed synthesis. Their available invariants and theorems retain their established meanings; interpreting them as epistemic, historical, or ethical quantities requires further assumptions and argument.

A distinctive difficulty arises from the status of prospective cellular structure. A completed two-complex specifies its vertices, edges, faces, and attaching relations. In future creation, some prospective events are unknown, future attachments may be conditional on earlier actions, and even the observer's catalogue of permissible cell types may evolve. A finite historical representation may admit multiple extension models with different continuation families. Equivalence under selected topological invariants does not establish equivalence of event identities, causal relations, historical provenance, or possibilities for subsequent action. Conversely, uncertainty about historical or prospective facts does not in general correspond to a homological hole. The distinction between cellular boundaries, causal partial orders, and evidential absence is therefore constitutive of the inquiry.

The central design decision is to represent a candidate continuation as an *extended whole space-time*. Let $\hat{K}$ denote a represented historical complex and $\hat{K}^+$ a candidate extension. The basic history-preserving map is introduced schematically in Equation 1.

$$\iota : \hat{K} \hookrightarrow \hat{K}^+. \quad (1)$$

The image of ι identifies the retained historical component inside its codomain; the codomain contains the whole candidate extended complex. This choice preserves the record of how new cells attach to inherited structure. A future-only object can also be mathematically adequate when equipped with its historical interface and attaching data, for instance through a relative-cell or pushout construction. The choice of Equation 1 is therefore an analytic preference motivated by the historical information the present inquiry seeks to retain. It allows the same candidate extension to be examined both as a whole relational configuration and as a relative transformation of an inherited one. The embedding model records historical occurrence through retained cells; dissolution of a capacity or relation may be expressed by subsequent events, whereas correction of an earlier epistemic description calls for a distinct revision operation.

Continuation must also be separated from *generation*. A cellular embedding represents one proposed extension. An epistemic generative model instead specifies, enumerates, samples, or constrains a family of candidate continuations under adopted assumptions. The paper investigates context-indexed continuation types and admissibility predicates, drawing on dependent sums and categorical structures to make the relevant premises inspectable. Reconstruction of those models from historical evidence is potentially many-valued: different models may fit the represented history yet admit different possible continuations. Some possibilities may also fall outside the observer's available conceptual language. Accordingly, probabilities, feasibility conditions, and model completeness are introduced only where independently justified. The formalism provides a means to articulate the assumptions of future recognition, while leaving open the question of whether the actual range of future possibilities can be exhaustively represented.

The historical constitution of observation introduces a further difficulty. An observer's abilities to distinguish events, form abstractions, use measurement instruments, and evaluate hypotheses depend on historical processes that may themselves be represented in the assumed reference domain. An epistemic representation is thus conditioned by an operator whose own emergence is a subject of inquiry. A provisional mathematical treatment can index observations and model reconstructions by an observer's historical support, available evidence, and epistemic capacities. Such indexing identifies dependencies without pretending to explain their complete causal or ontological genesis. Changes to a represented past may require revisions that are not history-preserving embeddings. A general account of how revised

epistemic contexts transport families of admissible future continuations remains an open mathematical problem, potentially calling for indexed categorical or fibred structures.

The topological programme consequently has a restricted but substantive role. Relative cellular homology can compare the ways in which candidate attachments change homological relations to inherited structure. Inclusion-induced maps can reveal distinctions that a list of Betti numbers conceals. Discrete Morse reductions can simplify a model while motivating further tests of whether historical labels, incidence, causal relations, and ethically relevant distinctions survive the reduction. Such constructions are proposed as analytical instruments; their topological results are established under explicit cellular assumptions, whereas their interpretation as properties of future creation depends on the adopted semantics. The paper develops elementary constructed examples and computational checks to expose both the explanatory possibilities and the losses introduced by representation and reduction.

The normative implications warrant an explicit interface. Future creation concerns the conditions under which opportunities, capabilities, and institutional arrangements may later arise. An intervention may preserve some possibilities while narrowing others, and different observers or participants may have unequal opportunities to recognise or influence those alternatives. The present paper identifies structural and epistemic distinctions relevant to later inquiries into participation, responsibility, procedural ordering, and justice. A topological invariant does not determine a moral obligation or a just distribution. Normative admissibility therefore requires separate principles, contextual evidence, and attention to affected subjects; the corresponding ethical theory is an associated research direction rather than a theorem of cellular topology.

The intended contribution is threefold. First, the paper specifies a disciplined separation between a posited ontological reference structure, historically conditioned epistemic representations, and models of possible continuation. Second, it develops a candidate formal architecture in which future possibilities appear as history-indexed whole-spacetime extensions, together with categorical comparisons and relative topological analyses. Third, it identifies the additional difficulties that arise specifically for future creation: indeterminate prospective incidence, model underdetermination, epistemic self-constitution, revision of represented histories, and loss of historically significant information under structural simplification. These are proposed conceptual and representational contributions. Established dependent-type, categorical, homological, and Morse-theoretic constructions are credited to their respective mathematical literatures; formal propositions, counterexamples, and illustrative computations will be assessed within their stated assumptions. No original empirical evidence or validated forecasting performance is claimed.

The paper proceeds as follows. Section 2 positions the inquiry in relation to work on relational representation, epistemology, event structures, model reconstruction, topology, and the normative study of futures. Section 3 establishes the event-based two-complex syntax, its semantic conventions, and spin foams as an instructive instantiation of decorated two-complex mathematics. Section 4 introduces the mathematical foundations, and Section 5 identifies the distinctive topological and epistemic problems of future creation. Sections 6–9 then develop historically constituted epistemic representations, dependent continuation types, categorical continuation structures, and provisional model-reconstruction operators. Section 10 investigates attachments, relative homology, and cellular reduction; Section 11 presents constructed examples and computational demonstrations. Section 12 articulates normative interfaces, while Section 13 examines epistemological and mathematical limitations. Section 14 states independent open questions and directions for further research, and Section 15 concludes. The introductory formal vocabulary is developed conditionally throughout, consistent with the paper’s ontological suspension and preliminary status.

2 Related Work and Research Context

This section locates the proposed topological epistemology of future creation within several established, partially independent bodies of scholarship. Its objective is to identify the analytical resources supplied by those traditions, the conditions governing their transfer to epistemically represented relational space-time, and the problems that remain specific to a history-dependent account of possible continuations. The

review proceeds through relational spacetime, situated epistemology, predictive reconstruction, event-based process semantics, type and category theory, cellular topology, and anticipatory governance. It closes by specifying the paper’s research position. The works discussed here do not collectively constitute a previously established theory of future creation. Their combination is examined as a provisional formal research design.

2.1 Relational Spacetime and Process-Based Representations

Discrete and relational approaches to spacetime motivate the study of structures whose primitive data are incidences, relations, or processes rather than coordinates on an assumed continuous background. In causal-set theory, Bombelli and collaborators proposed a locally finite partially ordered set as a candidate microscopic structure for physical spacetime, with the order interpreted through causal precedence [3]. The mathematical lesson relevant here is that an event-oriented history may admit a formal description whose order and structure are specified independently of a global clock. Causal sets nevertheless have a particular physical motivation and a distinguished partial-order structure. An epistemic two-complex equipped with event, coupling, and higher-order primitive interpretations is a different formal object. Its one-skeleton does not, by itself, define the transitive, antisymmetric causal order of a causal set.

Spin-foam models offer a second precedent with a more direct cellular connection. In Baez’s treatment, spin foams involve two-dimensional combinatorial structures decorated by representation-theoretic information, and model-specific rules relate those decorations to amplitudes [2]. It is useful to distinguish an individual decorated two-complex representing a candidate history from a model assigning dynamical significance to families of such structures. This distinction informs the present separation of candidate continuation morphisms from epistemic generative models. The application remains formal and comparative: the present research does not borrow the physical spin labels, quantum amplitudes, or empirical claims of spin-foam quantum gravity for social or epistemic phenomena.

The proposed GRT instantiation assigns vertices to events, edges to couplings, and faces to higher-order relational primitives whose meanings require local specification. Historically constituted agents and institutions can be reconstructed from extended relational substructures rather than taken as elementary vertices. Physical precedents illustrate that such a choice admits mathematical investigation; they provide no direct validation of these semantics. Moreover, a two-complex may be considered with or without orientations, labels, causal precedence, probability weights, and observation metadata. The present paper treats those as separate layers and asks which layers a particular claim about future creation needs.

2.2 Situated Cognition and Historical Epistemology

The epistemic representation of history depends on how an observer acquires, classifies, and interprets evidence. Haraway’s account of situated knowledges makes the partiality and location of perspectives central to questions of responsible knowledge production [11]. Suchman’s analysis of plans and situated actions likewise shows why abstract plans may inadequately capture the situated organization of practical activity, especially in interactions between people and computational systems [21]. These arguments are relevant to possible futures because a model’s categories, available actions, and assumptions about participants can arise from particular contexts of observation and practice.

Historical epistemology adds a temporal dimension to that concern. Daston and Galison investigate changing epistemic ideals and representational practices, including the historically developed meanings of objectivity and trained judgment in scientific image-making [6]. Their account supports the modest claim that the conditions under which certain descriptions count as good knowledge may vary historically. It does not itself yield an operator that maps an assumed relational reference domain into an epistemic two-complex. Formalizing such an operator remains the present paper’s task, subject to explicitly stipulated assumptions.

These perspectives motivate a distinction among three dependencies. An observer’s *evidence* depends on historically available traces and instruments; *interpretation* depends on available categories,

practices, and assumptions; and *representation* depends on the expressive capacities and losses of the chosen formal language. A hypothetical ontological reference spacetime may be introduced to discuss these dependencies consistently, yet it is not asserted to be an ultimate or uniquely correct ontology. The observer and any operator used to model observation can be treated as historically conditioned within that posited reference structure. The mathematical dependency can be declared before a complete generative explanation of the observer becomes available.

This leads to a specific interface with type theory: a type of accessible evidence or permissible reconstruction may depend on a context representing an observer’s historical support and epistemic capabilities. Such indexing does not determine how those capacities develop. Nor does the existence of an indexed type imply that a given observer can enumerate every historically feasible possibility. These limits are central to the proposed inquiry into future recognition.

2.3 Predictive-State Reconstruction and Generative Modeling

Computational mechanics offers a mathematically explicit example of constructing predictive organization from histories. Crutchfield and Young introduced statistical-complexity inference from observed processes [5]. Shalizi and Crutchfield subsequently developed the causal-state formulation and studied minimal predictive representations under specified stochastic-process assumptions [18]. Histories may be grouped according to equality of the conditional distributions they induce for future observations. This provides a precise precedent for deriving a predictive state representation from available historical information rather than assuming that the observed history is itself the generator.

The scope of these results matters. Their predictive equivalence is relative to a process model, an observation alphabet, and probabilistic assumptions. It does not establish that an observer has captured every physically or historically feasible future. For a social or institutional history, even the catalogue of recognizable event types, the classification of couplings, and the mechanisms of observation may change during the period under investigation. This creates difficulties for a fixed observational alphabet and a stable predictive state space. The distinction between uncertainty within a supplied model and possibilities absent from the modeling language is therefore important.

Our proposed epistemic generative model has a broader preliminary meaning than a stochastic sampler. A model may define a predicate over admissible continuation objects, generate finite candidate extensions, assign conditional likelihoods when warranted, or constrain allowable attaching operations. Each alternative has different proof obligations. A deterministic continuation rule, a family of admissibility predicates, and a probability distribution over extensions are mathematically distinct. The paper aims to make that distinction explicit before investigating whether historical evidence supports a particular member of such a model family.

Predictive reconstruction also raises an identifiability problem. Different model descriptions can agree on a finite observed prefix while differing on continuations beyond that prefix. Such underdetermination is especially consequential when prospective cells may introduce new kinds of incidences or higher-order relations. Consequently, the formalism will distinguish the construction of candidate models from claims of their empirical identification or predictive reliability. An illustrative continuation family can clarify formal consequences without functioning as a validated forecast.

2.4 Event Structures, Causal Orders, and Possible Histories

Event structures and models of concurrency provide a rigorous foundation for reasoning about occurrences that are dependent, independent, or mutually incompatible. Winskel’s event-structure treatment and Nielsen and Winskel’s broader survey develop mathematical languages for distributed processes and their possible configurations [23, 13]. These traditions are relevant to future creation because an admissible future may involve alternative event selections, causal dependencies, and multiple permissible orders of occurrence.

Their semantics also reveals an important distinction from cellular topology. A cellular edge may record a coupling without declaring a temporal direction. A face may express a higher-order primitive

without specifying causal conflict, temporal precedence, or an elementary state transition. Conversely, a partially ordered configuration of events can have several linear extensions even when its associated cellular representation is contractible. Neither the Betti numbers of a complex nor the presence of a cellular cycle determines the number or justice of allowable action sequences.

This motivates adding causal and historical order as explicit information rather than inferring it from incidence alone. Future creation may further require *conditional* order: the appearance of a prospective coupling or primitive could depend on the occurrence of an earlier modeled event. A continuation can therefore extend not only the underlying cellular data but also an adopted event-order structure and constraints on possible attachments. Defining the compatibility conditions among these layers is a task for subsequent formal sections.

Existing concurrency models provide substantial tools for possible-history analysis. Their transfer to epistemic relational spacetime requires care regarding event identity, emergence of previously unrepresented event types, and revision of the reconstructed historical configuration. Our particular question is not merely which events may execute concurrently in a specified system, but how a historically situated observer forms and revises the very system through which those events and possibilities are recognized.

2.5 Dependent Type Theory and Categorical Semantics

Dependent type theory supplies a disciplined language for expressing mathematical objects whose types depend on prior objects or contextual evidence. Dependent sums can package an extended complex together with a map from a historical complex and a witness satisfying an admissibility predicate. Dependent products can describe families of constructions indexed by historical representations or observer contexts. These are established type-forming operations, as presented in the *Homotopy Type Theory* reference [22]. The present paper proposes using them to prevent a candidate continuation from being detached from the historical representation and conditions that define its admissibility.

A preliminary continuation type may therefore take an inhabitant to consist of an epistemic two-complex K' , a history-preserving embedding $i : \widehat{K} \hookrightarrow K'$, and a context-indexed admissibility witness. This is initially a specification for candidate extensions. The admissibility condition must subsequently be given content: at minimum, it may include cellular compatibility and preservation of declared historical information, while stronger conditions involving causality, evidence, feasibility, or normative constraints require further justification. Merely writing a dependent type establishes neither inhabitation nor empirical adequacy.

Category theory contributes the language of objects, morphisms, commutative diagrams, and compositional constructions. In particular, the undercategory of a fixed historical complex organizes objects equipped with maps from that complex. Pushouts provide a standard mechanism for forming cellular attachments, while composition can describe staged extensions under suitable conditions [16]. This supports a whole-spacetime semantics in which the codomain of a continuation morphism contains both retained historical structure and prospective cells. A future-only representation is also legitimate when supplemented by sufficient attaching or interface data to recover the relevant extension. The preference for the whole complex is methodological: historical inclusion and attachment remain immediately available for subsequent comparisons.

Two categorical cautions deserve emphasis. First, an arbitrary cellular morphism need not be a history-preserving embedding, so the choice of morphism class is substantive. Second, an epistemic revision can identify, remove, or reinterpret previously posited cells. Such a revision generally requires more than an inclusion map. Indexed categories or fibrational structures could eventually relate changing observer contexts to continuation families, but those constructions depend on defining valid transport operations between revised historical descriptions. The present paper treats this as an open construction problem rather than a result inherited automatically from category theory.

2.6 Cellular Topology, Homology, and Discrete Morse Theory

Algebraic topology provides invariants and exact constructions for analyzing a historical complex together with a candidate extension. Cellular chains express boundaries through incidence operators, and relative homology studies a pair $(K', \hat{K})$ when the historical complex is embedded as a subcomplex of the extension [12]. The inclusion-induced homology maps can show how homology classes inherited from history behave within an extended complex. Relative groups can record changes that a comparison of the two absolute homology groups alone does not fully express.

These tools offer a particularly useful test of the choice of continuation semantics. Suppose an extension attaches a two-cell along a cycle already present in the historical complex. The original cycle remains part of the represented history, although it may become null-homologous in the extension. Two attachments with isomorphic resulting homology groups can nevertheless differ in which historical classes are mapped to zero. In such cases, the full inclusion map and the relative pair retain information relevant to the posed question. A detached future component could carry equivalent information only if it also records the necessary boundary and attaching data.

Discrete Morse theory provides methods for simplifying CW complexes while controlling homotopy-related information. Forman's discrete Morse theory is the relevant foundation [8]. For future continuation analysis, the mathematical usefulness of a smaller complex raises a separate representational question: does a particular reduction preserve the event identities, historical attachments, order constraints, epistemic provenance, and other observables required by the inquiry? A homotopy-equivalent reduced complex need not preserve those additional labels or relations without extra structure and checks.

There is consequently a difference between *topological invariance* and *historical or normative adequacy*. Homology classes measure specified algebraic-topological properties. A homological hole does not by definition represent an unobserved event or an ethically excluded future. A critical cell of a discrete Morse function does not by definition identify a socially pivotal intervention. Any proposed interpretation of these features must state its representational assumptions and evidential support. The later formal and illustrative sections use these distinctions to frame testable preservation conditions rather than to assign unsupported social semantics to mathematical invariants.

2.7 Anticipation, Future Creation, and Procedural Justice

Anticipation and responsible-innovation studies address the social organization of decisions concerning possible futures. Guston's account of anticipatory governance emphasizes capacities for foresight, engagement, and integration in the governance of emerging technologies [10]. Stilgoe, Owen, and Macnaghten present anticipation, reflexivity, inclusion, and responsiveness as dimensions of responsible innovation [20]. These frameworks give substantive reasons to investigate who participates in future-oriented model formation, which expectations structure action, and how choices can be reconsidered as knowledge changes.

Epistemic justice provides a complementary normative perspective. Fricker distinguishes testimonial injustice, concerning the credibility accorded to a knower, and hermeneutical injustice, concerning resources for making social experiences intelligible [9]. In future creation, these distinctions suggest questions about whose testimony informs a historical reconstruction and which participants' possible activities can be expressed in the adopted modeling language. They do not supply a ready-made numerical justice functional for a two-complex, nor do they imply that one can determine injustice directly from a homology computation.

The present paper adopts a limited interface with these normative traditions. Formally admissible continuations, epistemically credible continuations, historically feasible continuations, and ethically permissible continuations may constitute different families. A priority ordering among possible interventions can alter the conditions under which later interventions become feasible or even recognizable. These are reasons to preserve order, historical provenance, and observer-context information when constructing continuation models. They are not deductions of rights, justice, or ethical obligation from cellular topology alone.

The normative analysis of procedural order and the distribution of future generative capacities therefore remains a subsequent inquiry. Its mathematical prerequisites can nonetheless be made explicit here. A continuation representation useful for ethical analysis should retain or reconstruct the participants, constraints, relevant dependencies, and consequences upon which an independently argued normative assessment depends. The question of what counts as adequate retention is itself partly normative and domain-specific.

2.8 Research Positioning and Scope

The reviewed literatures contribute different mathematical and interpretive resources. Table 1 identifies the particular interfaces adopted by the paper, together with the additional problems posed by epistemically represented future continuation. This comparison is intentionally selective. It positions the proposed construction without claiming an exhaustive survey of each discipline.

The proposed contribution is a preliminary *interface architecture* connecting these resources around one formal object: an epistemic continuation of a represented historical two-complex. Its principal decisions concern the type of the continuation, the historical information included in its codomain, the context in which a generative model is reconstructed, and the transformations permitted when historical descriptions change. Established mathematical theorems remain attributed to their originating traditions. Any additional propositions in later sections will be limited to the assumptions explicitly formulated there.

Three questions distinguish the proposed inquiry from an ordinary analysis of a completed complex. First, prospective cells and attaching relations may be undefined at the point of inquiry, so the object to be studied is a family of candidate extensions rather than a single fully observed complex. Second, an epistemic representation may be revised, requiring comparison of continuation families anchored to different descriptions of history. Third, the observer’s capacity to construct those descriptions is itself historically conditioned. The present framework offers a formal setting in which these dependencies can be separated, examined, and in selected cases illustrated computationally. The next section specifies the two-complex representation and its spin-foam precedent before the mathematical construction of continuation types is developed.

Table 1: Established research resources and the additional questions required for epistemic two-complex continuation.

Research tradition	Established resource	Question for future continuation
Discrete spacetime and spin foams	Orders, cells, decorated histories, and model dynamics	Choice of event semantics and separation of candidate history from epistemic generator
Situated and historical epistemology	Situated observation, historically variable representational practices	Historically constituted observing operator and revision of reconstructed history
Predictive reconstruction	Model-relative predictive states and inferred dynamics	Identifiability, model plurality, and unrepresented future event types
Event and concurrency semantics	Causality, conflict, configurations, and order	Compatibility of causal dependency with two-complex incidence
Dependent types and category theory	Indexed objects, typed witnesses, morphisms, and composition	History-preserving extensions and transport under epistemic revision
Relative topology and Morse theory	Pair invariants and homotopy-controlled reductions	Preservation of provenance, order, and application-specific meaning
Anticipatory governance and epistemic justice	Participation, reflexivity, credibility, and interpretation	Normative criteria for inclusion, ordering, and future capacity

3 Two-Complex Representations of Relational Spacetime

This section establishes the cellular representation used throughout the proposed epistemology of future creation. Its immediate purpose is to specify a mathematical object whose historical configurations can be retained within candidate extensions, while separating cellular syntax from the interpretive commitments that make a complex a representation of relational spacetime. The discussion defines finite two-complexes, assigns provisional meanings to their cells, introduces chain and boundary operators, and explains how historically constituted entities may be represented through subcomplexes and associated relational data. Spin-foam constructions then provide a developed mathematical precedent for decorated two-complexes, with their physical amplitudes and dynamical assumptions kept distinct from the present application. A worked complex closes the section and prepares the relative-extension analysis developed in Section 4 and the later formal sections.

3.1 Cellular Complexes and Two-Dimensional Structures

A CW complex is assembled by attaching cells through maps from their boundaries into previously constructed skeleta. This construction permits combinatorial descriptions of incidence without requiring that the resulting space be represented in physical coordinates. The foundational results and conventions employed here follow standard cellular topology [12]. For the finite examples in this paper, attention is restricted to two-dimensional CW complexes. A two-complex has cells of dimensions zero, one, and two, and contains no cells of higher dimension. Its dimension refers to the *cellular representation*, not to an assertion about the dimension of physical or ultimate spacetime.

Definition 3.1 (Finite relational two-complex). A *finite relational two-complex* is a finite CW complex K together with explicit sets of zero-cells $V(K)$, oriented one-cells $E(K)$, and oriented two-cells $F(K)$, the endpoint information of the one-cells, and attaching maps for the two-cells. Where the complex represents epistemic relational spacetime, it is additionally equipped with a declared interpretation scheme σ_K for its cells and relevant relational attributes.

An orientation is a mathematical convention for cellular chains. It does not, by itself, confer a causal or temporal direction on an event or a coupling. Similarly, a drawing of cells in the plane supplies a visual aid and is not a canonical embedding of the represented events in geographic, physical, or historical coordinates. While a finite CW complex provides a tractable starting point, its cellular structure may be too coarse for histories with indefinite detail or unbounded scales. Local finiteness, infinite complexes, refinements, and higher-dimensional cells remain possible later extensions of the formal setting.

Definition 3.1 requires attaching data because the attaching map of a two-cell is an essential component of the construction. For a two-cell f , its characteristic map induces an attaching map from a circle to the one-skeleton. In a finite combinatorial presentation this may be encoded as a closed edge walk w_f , with repetitions and reversed orientations allowed. A list of vertices, edges, and faces without such boundary information would not specify the corresponding cellular construction. Under additional regularity assumptions the faces can be depicted as polygons embedded along their boundaries, but the general CW definition allows more complicated attachments. This distinction matters when the future involves prospective couplings whose incidences have not yet been established or even represented.

3.2 Events, Couplings, and Higher-Order Primitives

The working GRT interpretation assigns different analytical roles to the dimensions of a two-complex. These assignments are conventions of the proposed epistemic representation, and their appropriateness must be argued in relation to particular historical materials. Table 2 states the basic assignments and the interpretive information that a cell alone leaves unspecified.

A vertex denotes an *event*, including an occurrence that may be recognized as part of an extended process. An entity, person, institution, technology, or organization is therefore not identified with one vertex merely because it appears prominently in the substantive description. Where such an entity is

Table 2: Cellular syntax and provisional interpretation in the event-based relational representation.

Cell	Proposed semantic role	Information requiring separate specification
Vertex v	Event or occurrence	Event identification, observational support, temporal or causal attributes
Edge e	Coupling between represented events	Coupling mechanism, direction if applicable, evidential status and strength
Face f	Higher-order relational primitive	Constitutive meaning, attachment interpretation, contextual or modal constraints
Subcomplex L	Historical relational support for an emergent entity or process	Identity criteria, persistence, boundaries, transformations and scale

representable, its formation and continuity would instead be analyzed through relations among multiple events and through a selected historical substructure. This preserves the distinction between elementary representational cells and historically constituted subjects or objects of inquiry.

An edge denotes a *coupling* between represented events. A coupling may be causal, enabling, constraining, communicative, interpretive, or otherwise constituted, depending on the particular model. The presence of an edge establishes only that the representation contains that coupling. A temporal or causal order requires further data and consistency conditions; an oriented edge is not automatically a causal arrow. The same applies to the spatial proximity of vertices in an illustrative drawing.

A face denotes a *higher-order primitive*. Its substantive meaning is deliberately left open at the general level. A face might later carry a declared constraint, a coordinated interaction, a constitutive pattern, or a composite dependency; none of these interpretations follows simply from the existence of a two-cell. A face is also not assumed to represent a single episode of value circulation. In the companion research on heterogeneous value circulation, represented values may themselves have nontrivial cellular structure; the present paper instead focuses on the structures required to formulate possible future extensions. The open semantics of faces will affect which continuations can be considered admissible.

Additional labels can supplement the incidence structure. One might introduce observation meta-data, uncertainty annotations, modal classifications, event-order relations, or type-specific quantities. Each layer should be specified independently, because two complexes with isomorphic underlying CW structures can carry different historical meanings, causal dependencies, or epistemic reliability. A homological calculation refers to the selected cellular structure; its interpretation as a historical or normative claim requires the additional semantic layers.

3.3 Cellular Incidence, Orientation, and Boundary Operators

The cellular chain complex provides an elementary algebraic representation of the incidences described above. For an oriented finite two-complex K , let $C_n(K; \mathbb{Z})$ be the free abelian group generated by its oriented n -cells. The boundary homomorphisms are stated in Equation 2.

$$C_2(K; \mathbb{Z}) \xrightarrow{\partial_2} C_1(K; \mathbb{Z}) \xrightarrow{\partial_1} C_0(K; \mathbb{Z}), \quad \partial_1 \circ \partial_2 = 0. \quad (2)$$

For an oriented edge e with source $s(e)$ and target $t(e)$, the boundary convention is specified in Equation 3.

$$\partial_1[e] = [t(e)] - [s(e)]. \quad (3)$$

For an oriented two-cell f , the cellular boundary takes the form given in Equation 4.

$$\partial_2[f] = \sum_{e \in E(K)} n(f, e)[e], \quad n(f, e) \in \mathbb{Z}. \quad (4)$$

The coefficients $n(f, e)$ encode oriented incidence, including repeated traversals when the attaching map is non-regular. The identity $\partial_1 \partial_2 = 0$ expresses an algebraic consistency property of the cellular boundary. It neither states that the represented events occurred in a unique order nor establishes a particular

causal mechanism. Cellular homology is defined from the kernels and images of these homomorphisms, as recalled in Equation 5.

$$H_n(K; \mathbb{Z}) = \ker(\partial_n) / \text{im}(\partial_{n+1}). \quad (5)$$

Boundary information becomes especially significant for future continuation. An extension of an epistemically represented history may introduce a face whose boundary involves edges already present in the history. The new face thus depends on existing cellular data even if the historical cells remain intact. For a subcomplex $L \subseteq K$, the relative chain group $C_n(K, L) = C_n(K) / C_n(L)$ supplies a derived description of cells relative to a retained subcomplex. However, the chain quotient alone can suppress information about the original attachment to L . Retaining the pair (K, L) , and the inclusion that makes L a subcomplex, is therefore more informative for inquiries in which historical incidence is analytically primary. Section 3.8 illustrates this distinction without yet introducing a general continuation type.

3.4 Historical Subcomplexes and Emergent Relational Entities

A historical representation typically contains parts that an observer associates with continuing processes, institutions, persons, or other identifiable patterns. A subcomplex offers one representation of their historically constituted support. Its mathematical selection is only a first step: criteria for identifying and tracking an entity require substantive assumptions about which event relations, couplings, and higher-order primitives matter for that inquiry.

Let $L \subseteq K$ be a subcomplex. Its closure under cellular incidence ensures that a cell of L is accompanied by the lower-dimensional cells needed for its attachment. A selected set of events without the incident structure may fail to form a subcomplex. This is important when a historical entity is represented by a collection of occurrences whose meaningful couplings extend beyond the selected events. Some contexts may call for a family of overlapping subcomplexes, annotated subcomplexes, or a diagram of complexes rather than one sharply delimited component.

The use of subcomplexes also introduces scale dependence. A laboratory may be examined through a fine-scale network of experimental and institutional events, while a different study treats some of the same trajectories as parts of a broader scientific field. Neither representation establishes a unique identity boundary by topological means alone. Emergent entities are thus modeled as interpretively identifiable historical configurations supported by relational structure, with their boundaries and continuity criteria open to revision. For empirical applications, evidence about the processes that generated those configurations must be distinguished from the analyst's choices of partition and labeling.

A historical inclusion $i : L \hookrightarrow K$ preserves the selected subcomplex as a part of the larger cellular object. It does not guarantee that the meaning, role, or homological status of every structure in L remains unchanged when the larger complex is formed. Later cell attachments can fill a previously nonbounding cycle, create connections between previously separated components, or change which cellular representatives are convenient. This distinction between retention of historical cells and transformation of their relations to a larger whole motivates the paper's preference for whole-spacetime continuation semantics.

3.5 Spin-Foam Constructions as Decorated Two-Complexes

Spin foams provide a technically developed example in which a two-complex receives structured decorations and a model-dependent interpretation. In the spin-foam approach to quantum gravity, oriented two-complexes are commonly decorated with representation-theoretic information assigned to faces and intertwiners associated with edges. The relevant literature distinguishes the combinatorial foam, its labels, and amplitudes computed according to a chosen physical model [2, 15]. The representation-theoretic labeling is a substantive mathematical ingredient of spin-foam models; it is not a generic attribute of every two-complex.

A spin foam should therefore be understood as more than an undecorated cell complex when described in its physical-theoretical setting. The two-complex specifies combinatorial support, the decorations introduce quantum-geometric information, and physical constraints and amplitudes govern the

contribution assigned to the decorated history. Distinct spin-foam models can use different label sets, constraints, and amplitude prescriptions. This established construction supplies a useful demonstration that the same cellular syntax can participate in several more-specific semantic and dynamical theories.

The proposed relational spacetime representation borrows the possibility of working with a two-complex and optional decorations. Its semantic assignments remain separate from those of quantum gravity. In particular, representing an occurrence by a vertex does not equate that occurrence with a spin-foam vertex amplitude or a physical interaction of quantum gravitational degrees of freedom. Representing a coupling by an edge does not confer the semantics of an intertwiner, and assigning meaning to a face does not make that face a quantum-geometric surface carrying a spin label. No assumption of quantum coherence, a Hilbert space of social histories, or a physical spacetime path integral is required for the present construction.

The analogy also requires care with the word *spacetime*. In spin-foam quantum gravity, the underlying mathematical structures are motivated by attempts to describe gravitational geometry, usually in relation to boundary states and quantum amplitudes. In the present GRT inquiry, *relational spacetime* denotes an epistemic representation of historically organized events and couplings. The correspondence is a transfer of mathematical resources and representational questions, not an identification of the two scientific domains. Physical spin-foam results cannot serve as empirical validation of a social or epistemic two-complex model.

3.6 Spin-Foam Dynamics and Structural Instantiation

The distinction between an individual complex and a rule assigning significance to possible complexes is particularly useful for future creation. A decorated spin foam may encode one candidate history, whereas a spin-foam *model* supplies prescriptions for amplitudes or sums over histories. For suitable models, the amplitude associated with a fixed two-complex can be expressed schematically through local face, edge, and vertex factors, as in Equation 6 [2, 15].

$$Z_K = \sum_{\lambda \in \Lambda(K)} \prod_{f \in F(K)} A_f(\lambda) \prod_{e \in E(K)} A_e(\lambda) \prod_{v \in V(K)} A_v(\lambda). \quad (6)$$

In Equation 6, λ denotes a permitted collection of labels, and the factors stand for model-specific local amplitudes. The displayed expression is illustrative rather than a definition shared by all spin-foam theories. Boundary conditions, gauge symmetries, normalization, convergence, and the meaning of summation depend on the chosen physical construction.

A further level concerns mathematical frameworks in which a family of two-complexes arises from the perturbative expansion of a group field theory. Such field-theoretic approaches relate Feynman diagrams and spin-foam-like structures and permit sums over complexes under specified interactions [14, 15]. This distinguishes the generation of a candidate combinatorial history from a given history's cellular incidence and from the amplitude or interpretation subsequently assigned to it. It does not follow that the underlying process constitutes a historically complete account of how observers, institutions, or social possibilities are generated.

For the present paper the methodological lesson is a threefold separation. A *cellular structure* provides a syntax of events, couplings, and higher-order relations; an *epistemic model* specifies admissible interpretations or possible continuations under stated evidence and assumptions; and a *model-construction procedure* concerns how that epistemic model is formed or revised. The last two levels may depend on historically constituted observing capacities. Introducing them separately avoids describing a two-complex as if its mere existence supplied an explanatory generative law. The ensuing sections develop these distinctions without importing the quantum dynamics of spin foams.

3.7 Semantic Interpretation and Ontological Suspension

The representational vocabulary introduced in this section is situated within an explicit ontological suspension. An assumed ontological relational spacetime may later be introduced as a convenient *formal*

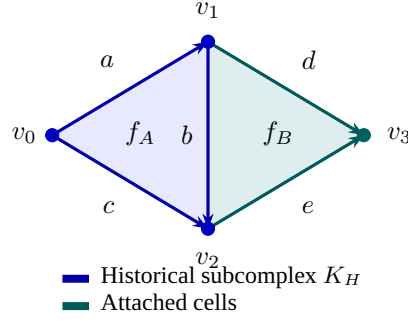


Figure 1: A finite oriented two-complex containing a triangular historical subcomplex K_H (left) and an attached triangular face with additional event and coupling cells (right). The shared coupling b belongs to the historical subcomplex and to the attaching boundary of f_B . The figure illustrates cellular incidence rather than physical position or causal direction.

reference domain for describing the historical constitution of observation and interpretive capacities. Its introduction does not assert that ultimate reality is fundamentally a CW complex, that a unique ontological relational generator exists, or that the formal domain is complete. It is a postulated analytical object whose consequences are conditional on the assumptions used to specify it.

Epistemic relational spacetime is a distinct construction. A finite complex $\hat{K}$ may encode selected recorded events, inferred couplings, interpretive primitives, and provenance. Its omissions can arise from absent evidence, selective attention, incompatible vocabularies, insufficient computational capacity, or incorrect inference. These limitations may interact, and no particular pattern of missing information is automatically equivalent to a homological hole. Likewise, the existence of a cellular face is a formal assertion about incidence rather than independent evidence for the historical interpretation attached to that face.

The possibility of representing an observer within a historically assumed relational reference domain introduces a further distinction between *historical constitution* and *epistemic self-description*. An observer's capacities may depend on processes the observer cannot fully reconstruct. Modeling such dependence does not require the observer to possess a complete representation of its own constitution. Subsequent sections will use context-indexed structures to articulate this dependence provisionally, while treating the formation and epistemic adequacy of reconstruction operators as an open research problem.

The treatment is also neutral regarding whether time is ultimately fundamental, emergent, or differently constituted. Where a historical ordering is needed, the model must specify its operational or relational basis. Cellular incidence alone provides neither a temporal order nor a unique future direction. In the formal investigation of continuation, *history* refers to the component selected as epistemically established relative to the observer's standpoint, while a *candidate future* concerns an admissible modeled extension. Such selection remains revisable when new evidence changes the historical representation.

3.8 An Illustrative Two-Complex Spacetime Model

A finite constructed example makes the incidence conventions and the whole-spacetime design choice explicit. Consider four event vertices v_0, v_1, v_2, v_3 , five coupling edges a, b, c, d, e , and two oriented faces f_A, f_B . Edges are oriented from v_0 to v_1 (a), v_1 to v_2 (b), v_0 to v_2 (c), v_1 to v_3 (d), and v_2 to v_3 (e). Figure 1 displays the incidence structure. The coordinates of the drawing are illustrative and carry no temporal or metric interpretation.

The orientations of the faces can be chosen so that the shared edge cancels when the faces are added. Their cellular boundaries are specified in Equation 7.

$$\partial_2[f_A] = [a] + [b] - [c], \quad \partial_2[f_B] = [d] - [e] - [b]. \quad (7)$$

With ordered edge basis (a, b, c, d, e) and ordered face basis (f_A, f_B) , Equation 8 gives the boundary

575 matrix.

$$576 \quad [\partial_2] = \begin{pmatrix} 1 & 0 \\ 1 & -1 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix}. \quad (8)$$

577 For each face, applying ∂_1 to the expression in Equation 7 gives zero, in agreement with Equation 2.
 578 The complex is a triangulated disk, with four vertices, five edges, and two faces. Its Euler characteristic
 579 is $4 - 5 + 2 = 1$, and its ordinary homology over $\mathbb{Z}$ is $H_0(K) \cong \mathbb{Z}$, $H_1(K) = 0$, and $H_2(K) = 0$.
 580 These are properties of the constructed cellular example. They do not establish a claim about a particular
 581 observer or realized historical process.

582 Let K_H be the full left triangle, including v_0, v_1, v_2, a, b, c , and f_A . The inclusion $i : K_H \hookrightarrow K$
 583 retains the historical cells. The cells newly added to construct K are v_3, d, e, f_B . Considered in isolation,
 584 this collection is *not* a subcomplex: both d and e have historical endpoints, and the boundary of f_B
 585 contains the historical edge b . A future-only construction is possible if it retains the attaching information
 586 relating those cells to K_H . The full extension K contains those relations directly.

587 The relative chain complex $C_*(K, K_H)$ suppresses historical generators while keeping the pair
 588 (K, K_H) available for interpretation. In the relative quotient, the boundary of $[f_B]$ is represented by
 589 $[d] - [e]$; the $[b]$ term disappears because it belongs to $C_1(K_H)$. This calculation illustrates a distinction
 590 between retaining the *pair* with its inclusion and retaining only its *relative chain quotient*. The latter
 591 is appropriate for certain homological questions but need not preserve the full historical attachment
 592 description. In the present example, the relative homology groups vanish, although an entire triangle's
 593 worth of new cells has been attached. Relative homology is therefore an invariant of the pair, not a
 594 cell-by-cell inventory of future developments.

595 The interpretation of this example as relational spacetime remains provisional. The four vertices are
 596 represented events, the edges are declared couplings, and the faces are higher-order primitives for which
 597 substantive semantics must still be supplied. The historical component is selected by the modeling ex-
 598 ercise. Future continuation adds a candidate event and relations, with an attachment involving already
 599 represented history. Later sections will use such structures to articulate admissibility, composition, epis-
 600 temic revision, and topological change; the current example establishes only the cellular syntax and its
 601 informational consequences.

602 The finite CW presentations, historically marked attachment maps, and full integer boundary calcu-
 603 lations underlying this construction are collected in Appendix B.

604 4 Mathematical Foundations of Future Continuation

605 This section assembles the mathematical resources required to study candidate continuations of epis-
 606 temically represented relational spacetime. Its objective is to establish the types, morphisms, relative
 607 cellular constructions, topological invariants, reductions, and ordering relations that the later formalism
 608 will employ. The argument is mathematical and representational: each construction is introduced with
 609 its assumptions, what it preserves, and what it leaves outside its scope. Throughout, the historical com-
 610 plex is a situated epistemic representation. The discussion does not postulate a complete ontological
 611 generator, and it does not identify a modeled continuation with a realized future. Table 3 summarizes
 612 the separate analytical responsibilities of the mathematical instruments.

613 4.1 Dependent Types and Indexed Mathematical Structures

614 Dependent type theory provides a language for structures whose admissible members depend on previ-
 615 ously selected data. The distinction matters because a prospective relational spacetime is evaluated *rela-*
 616 *tive to a particular historical representation*. Different historical complexes can admit differently typed

Table 3: Mathematical instruments for epistemic continuation and the information that each requires beyond cellular incidence.

Instrument	Role within the proposed formalism	Additional conditions or limitations
Dependent types	Index possible extensions by a represented history and an epistemic context	Types of complexes, embeddings, evidence, and admissibility witnesses
Categorical morphisms	Compare and compose history-preserving extensions	Defined category; composition-compatible admissibility
Relative CW pairs	Retain the historical complex inside a candidate extension	Explicit cellular attaching maps and inclusion data
Relative homology	Detect selected topological effects of attachments	A chosen coefficient ring; limited sensitivity to semantics
Discrete Morse theory	Simplify a represented complex while retaining selected topology	Suitable cellular regularity and history-relative reduction rules
Causal partial orders	State dependencies among represented events	Separately specified precedence, conflict, and evidence

extensions, and different observers can assess those extensions under different evidential constraints. Dependent sums and products are standard constructions in type theory [22]. Their use here is a proposed organization of epistemic continuation, rather than an assertion that the world itself is a type-theoretic object.

Let EpCplx_2 denote a selected collection of finite, syntactically specified epistemic two-complexes. To avoid treating an unrestricted class of all complexes as a small type, one may fix a universe of finite encodings and size conventions, identifying isomorphic descriptions only when the chosen representation policy permits. Each $K : \text{EpCplx}_2$ includes its CW incidence data and a declared interpretation scheme. The selection of a type of descriptions is a modeling choice; it does not certify that every encoded coupling or face has been historically observed.

For two such complexes let $\text{Emb}(K, K')$ be the type of *cellular embeddings onto subcomplexes* preserving dimensions, attaching relations, and whatever labels are explicitly stipulated as invariant. The interpretation policy can be weaker or stronger than preservation of bare CW structure. Let c denote an epistemic context containing a represented history K_c , accessible evidence, a standpoint, and stated assumptions. A provisional continuation type is given in Equation 9.

$$\text{Cont}(c) := \sum_{K' : \text{EpCplx}_2} \sum_{i : \text{Emb}(K_c, K')} \text{Adm}_c(K', i). \quad (9)$$

The final factor is a type of admissibility witnesses, or a proposition when only its truth status matters. Its interpretation must be fixed before any candidate is treated as admissible. Examples include preservation of recorded event identifiers, compatibility with selected causal constraints, or conformity with a limited, explicitly sourced domain rule. An unrestricted admissibility predicate, an empirically supported predicate, and a normative permissibility predicate have different analytical roles. Their equality is not assumed.

For a fixed context, an element of $\text{Cont}(c)$ is a dependent triple (K', i, p) , where the witness p belongs to the admissibility type associated with the particular extension (K', i) . The dependent structure prevents a generic list of prospective cells from silently replacing the relevant historical embedding. When admissibility is proof-irrelevant, distinct evidence for the same proposition need not be counted as different modeled futures; when witnesses represent substantively different provenance or interpretations, they may need to remain distinguishable. This difference between *propositional admissibility* and *evidence-bearing admissibility* is a design decision for subsequent sections.

A prospective epistemic generator can be represented as a predicate on continuation inhabitants, or as a set-valued selection of such inhabitants. Equation 10 uses a predicate form that does not require a

probability law or an executable sampling process.

$$G_m : \text{Cont}(c) \longrightarrow \text{Prop.} \quad (10)$$

Here m identifies a selected epistemic model. Such a predicate states which extensions the model admits, conditional on its reconstruction and assumptions. It need not exhaust historically possible continuations. A probability distribution could be added when an explicitly defined stochastic model and sample space justify one. Likewise, an inference process reconstructing m from evidence is a separate mathematical object from G_m itself.

Equation 9 will serve as a candidate *syntax* for a family of whole-spacetime continuations. Its eventual semantics must connect encodings, evidence, historical identification, and interpretation. A dependent type establishes internal well-formedness under specified rules; it cannot by itself establish that an encoded future is historically feasible or ethically appropriate.

4.2 Categories, Morphisms, and Compositional Structures

Category theory supplies a language for structural transformations that can be compared and composed. For present purposes, let $\mathbf{C}_2^{\text{emb}}$ denote a category whose objects are finite epistemic two-complex descriptions and whose arrows are the selected history-preserving cellular embeddings. Composition is ordinary composition of embeddings; identity arrows retain each complex. This produces a category when the chosen semantic-preservation conditions are closed under composition. More general cellular maps, quotient maps, and epistemic revisions can be studied in larger or different categories, but they are not identified with embedding-based continuations. Standard categorical terminology follows Riehl [16].

Fix a historical representation K . The undercategory $K \downarrow \mathbf{C}_2^{\text{emb}}$ has objects (K', i) with $i : K \hookrightarrow K'$. A morphism from (K_1, i_1) to (K_2, i_2) is an embedding $h : K_1 \hookrightarrow K_2$ that respects the fixed historical origin. Its commutativity condition is stated in Equation 11.

$$h \circ i_1 = i_2. \quad (11)$$

This equation distinguishes comparison of two *based extensions* from an arbitrary map between their codomains. It requires the inherited historical part to be tracked consistently across a further extension. When a candidate changes the historical labels or previously represented attachment data, the arrow may cease to qualify as a continuation in the chosen category and may instead be treated as an epistemic revision.

Proposition 4.1 (Composition of cellular continuations). *Let $i : K_0 \hookrightarrow K_1$ and $j : K_1 \hookrightarrow K_2$ be cellular embeddings onto subcomplexes in $\mathbf{C}_2^{\text{emb}}$. Their composite $j \circ i : K_0 \hookrightarrow K_2$ is a cellular embedding onto a subcomplex. If an admissibility relation is imposed on such embeddings, the composite is admissible whenever that relation is closed under composition.*

Justification. The composition of injective cellular embeddings onto subcomplexes again identifies the source with a subcomplex of the target, preserving the specified cellular attachments. The last assertion follows only under the stated closure condition for admissibility. Proposition 4.1 is therefore a basic structural result, with a separate proof obligation for the application-specific admissibility predicate. Some constraints, such as a maximum budget for a single stage or a horizon-limited epistemic forecast, need not be composition-closed without additional bookkeeping.

The categorical description clarifies the direction of our preferred morphism. In $i : K \hookrightarrow K'$, the *image* of the embedding represents the retained historical part, whereas its *codomain* is the entire extended two-complex. Calling the image “the whole future” would conflate the source’s embedded history with newly introduced cells. The chosen codomain semantics permits future-specific information to be studied relative to the inclusion without treating new cells as an autonomous history.

An undercategory collects structural extensions of a fixed K ; it does not generate a unique future and does not by itself rank alternatives. A family of model-dependent predicates can be placed on its

objects or arrows, and later revisions of K may lead to different undercategories. Transport across such revisions requires its own specification rather than a tacit identification of all reconstructed histories.

4.3 Relative Cell Complexes and Cellular Attachments

Relative CW theory is closely aligned with the proposed whole-spacetime semantics because it treats a candidate extension together with the subcomplex that it extends. In a CW pair (K', K) the inclusion $i : K \hookrightarrow K'$ is part of the mathematical data. Under a finite two-dimensional restriction, a candidate extension can be built in stages by adjoining event vertices, coupling edges, and higher-order primitive faces, provided their attaching maps land in the current lower-dimensional skeleton [12].

For a standard cell attachment, an attaching map $\alpha : S^{q-1} \rightarrow K$ determines a pushout construction. Equation 12 gives the quotient presentation of the resulting extension.

$$K' = K \cup_{\alpha} D^q := (K \sqcup D^q) / \{x \sim \alpha(x) : x \in S^{q-1}\}. \quad (12)$$

For the two-complex setting, $q \leq 2$. In a more elaborate future-only presentation, the attached space may be a finite complex B with a declared interface $A \subseteq B$ and an attaching map $\alpha : A \rightarrow K$. The whole extension is then obtained by gluing B to K along that interface. The future-only component together with α can therefore represent the same extension as the entire pair, provided the construction retains the relevant identifications and semantic labels.

This equivalence of presentations motivates a reasoned design preference rather than a claim of uniqueness. A future-only inventory omitting the interface can lose the information necessary to distinguish attachments to different historical couplings. In contrast, a whole-spacetime codomain retains historical cells and their new incidences in one object. That choice is analytically valuable when later interpretation depends on historical provenance or on whether a new face bounds a particular inherited cycle. The worked construction in Section 3.8 already illustrates this issue at the level of shared edges.

It is useful to distinguish *new cells* from a *future subcomplex*. Let $i : K \hookrightarrow K'$ be a subcomplex inclusion. The set difference of cell sets can inventory cells not present in $i(K)$, but its cells need not be closed under their boundaries. In particular, a new face may attach along a historical edge and a new coupling may terminate at an inherited event. An abstract isolated future-only complex therefore generally needs additional attaching data to reconstruct the whole extension. The pair (K', K) is a convenient primary object, while a quotient or relative chain complex can support selected algebraic calculations.

No interpretation of K as a literally fixed past is required at the level of ultimate ontology. The designation “historical” specifies the component presently taken as established *within the epistemic representation*. A subsequent revision can alter that representation or its labels; the embedding-based continuation and epistemic revision are mathematically different operations.

4.4 Homology and Relative Homology

Cellular homology describes selected aspects of the incidence structure through chain groups and boundary operators. For a finite two-complex K , its cellular chain complex consists of $C_2(K)$, $C_1(K)$, and $C_0(K)$, with boundary maps satisfying $\partial_1 \partial_2 = 0$. A coefficient ring, taken to be $\mathbb{Z}$ in the examples below, must be specified. For each degree q , Equation 13 defines the ordinary homology group.

$$H_q(K; \mathbb{Z}) = \ker(\partial_q) / \text{im}(\partial_{q+1}). \quad (13)$$

Such groups can distinguish certain cycles, boundaries, and connected components, but are insensitive to many historical labels, semantic distinctions, and causal orientations. In particular, a nontrivial first homology class is a cellular topological feature, whereas an epistemic gap concerns what is unknown, misinterpreted, or unrepresentable. One does not imply the other.

For a CW pair (K', K) , the relative chain groups are defined by quotienting chains supported on the historical subcomplex. Equation 14 records their construction.

$$C_q(K', K) = C_q(K')/C_q(K), \quad H_q(K', K) = H_q(C_*(K', K)). \quad (14)$$

This quotient simplifies calculation by suppressing historical chain generators, while the *pair* retains the historical subcomplex and its inclusion. Forgetting the pair and retaining only the quotient chain complex may discard information about precisely which earlier cycle an attachment changed. The long exact sequence of a pair, a standard result of algebraic topology [12], relates the absolute and relative invariants. The relevant portion is displayed in Equation 15.

$$\cdots \longrightarrow H_2(K') \longrightarrow H_2(K', K) \xrightarrow{\delta} H_1(K) \xrightarrow{i_*} H_1(K') \longrightarrow H_1(K', K) \longrightarrow \cdots. \quad (15)$$

The connecting morphism δ can record how a relative two-dimensional class meets an inherited one-dimensional cycle; the inclusion-induced map i_* records which historical homology classes remain non-trivial in the extension.

A basic worked example shows the relevance of both maps. Let $K \cong S^1$ be the boundary of a polygon representing a historical one-cycle, and form $K' = K \cup_\alpha D^2 \cong D^2$, where α traverses the circle once. The inclusion preserves every historical boundary cell, while its attached face fills the inherited cycle. Equation 16 states the resulting homology groups and the connecting map.

$$\begin{aligned} H_1(K) &\cong \mathbb{Z}, & H_1(K') &= 0, \\ H_2(K', K) &\cong \mathbb{Z}, & \delta : H_2(K', K) &\xrightarrow{\cong} H_1(K). \end{aligned} \quad (16)$$

The displayed isomorphism follows from Equation 15 because the disk has vanishing first and second homology. This is a mathematical account of an attached cell changing the topological status of an inherited cycle, without deleting that cycle's cellular record. It is not yet an explanation of which substantive event or historical process, if any, should be modeled as such an attachment.

For a more discriminating comparison, let $K = S_a^1 \vee S_b^1$ be a wedge of two oriented circles, and construct K'_a and K'_b by attaching a two-cell along a or b , respectively. Although $H_1(K'_a) \cong H_1(K'_b) \cong \mathbb{Z}$, the inclusion-induced maps have distinct kernels relative to the *marked historical generators*, as stated in Equation 17.

$$\ker(H_1(K) \rightarrow H_1(K'_a)) = \langle [a] \rangle, \quad \ker(H_1(K) \rightarrow H_1(K'_b)) = \langle [b] \rangle. \quad (17)$$

The example illustrates why a continuation should be compared *over its historical base*. Abstractly isomorphic homology groups of future codomains can obscure different historical effects. Indeed the two extensions are isomorphic if one is allowed to exchange the labels a and b , but that exchange is disallowed when the historical identity of each loop is part of the comparison. Later sections can employ marked inclusions and relative maps to preserve this distinction.

4.5 Discrete Morse Theory and Cellular Reduction

Discrete Morse theory provides controlled simplification of finite cell complexes. For a finite regular CW complex, a suitable acyclic matching pairs incident cells of consecutive dimensions, and the unmatched or critical cells determine a smaller Morse-type CW model with the same homotopy type under the established hypotheses [8]. Because a general two-complex need not be regular, the elementary constructions in this paper restrict Morse reductions to finite regular CW or simplicial models, or to subdivisions for which the required conditions have been checked. These technical restrictions matter when prospective faces have nonregular attaching maps.

Ordinary homotopy-preserving reduction is insufficient for every epistemic task. A reduction may preserve homology while identifying separately documented events, suppressing causal dependence, or obscuring attaching maps relevant to later interpretation. We therefore distinguish a *topological reduction* from a *history-relative reduction*. A history-relative simplification should at minimum specify

whether the included historical subcomplex remains fixed and which additional annotations must be retained.

One elementary sufficient condition is a sequence of *free-face collapses performed entirely outside the historical subcomplex*. If $K \subseteq K'$ and there is a sequence of elementary collapses from K' to a subcomplex L with $K \subseteq L$, each removing an allowable free-face pair whose cells lie outside K , then the collapse can be performed relative to K . The resulting inclusions define a homotopy equivalence of pairs, and hence an isomorphism on relative homology, as in Equation 18.

$$H_q(K', K; \mathbb{Z}) \cong H_q(L, K; \mathbb{Z}) \quad \text{for all } q. \quad (18)$$

The condition is intentionally strong and sufficient rather than a characterization of all permissible relative Morse matchings. A general discrete Morse matching would require further compatibility checks to ensure that the historical subcomplex and selected labels remain meaningfully identifiable.

Consider the two-triangle example of Section 3.8. The attached face f_B has an outer coupling edge that is a free face, so the pair may be collapsed without deleting any cell of the historical left triangle. After removing the face with one free outer edge, the remaining new edge and new vertex form a collapsible appendage. A second collapse removes them, leaving the historical subcomplex. This explains algebraically why the relative homology of that example vanishes, despite the addition of several cells. The newly recorded event and couplings remain historically or interpretively meaningful if the representation requires their identities to persist; topological collapsibility alone does not license their erasure from an epistemic archive.

A useful later research problem is the construction of *semantics-aware reduction constraints*. A critical cell in discrete Morse theory is defined with respect to a matching, and does not automatically correspond to a socially consequential event or an ethically critical intervention. Any such interpretive correspondence requires independently stated semantics and evidence. The role of Morse theory here is therefore to make the preservation problem explicit and computationally tractable in restricted cases.

4.6 Partial Orders and Admissible Histories

Causal and temporal ordering must be introduced separately from cellular incidence. A vertex is represented as an event, but an orientation selected for a cellular edge serves the algebraic boundary operator and need not encode causal precedence. In a model where relevant evidence permits it, a partial order $\preceq$ may be assigned to selected events. Its reflexivity, antisymmetry, and transitivity give an operational dependency structure. Other frameworks, including event structures, represent conflict and concurrency alongside causal dependency [23, 13].

For a finite collection of modeled events V with an acyclic precedence relation, a linear extension is a total order compatible with the partial order. Equation 19 specifies the elementary compatibility condition for a sequence of distinct events $v_1, \dots, v_n$.

$$v_a \prec v_b \implies \text{pos}(v_a) < \text{pos}(v_b). \quad (19)$$

The sequence may be called an admissible ordering only relative to the selected dependency constraints. It does not specify all feasible future actions, prove that the events will occur, or supply an ethical assessment of their order.

Suppose two prospective events x and y both depend on an inherited event h , while no precedence between x and y has been established. The partial order permits both h, x, y and h, y, x . Their distinct linearizations need not change the underlying two-complex or its homology. Conversely, a two-complex may contain topological cycles while its event-precedence relation is acyclic. The relation between homological cycles and causal cycles is therefore an additional question requiring an explicit linking rule, rather than an implication of the cellular model.

For future creation, the epistemic complication is that the event set and dependency relation are themselves constructed representations. Some events may not yet be conceived; evidence may support several competing dependencies; and historical reinterpretation may alter the selected partial order. A

topological sort orders a *given* dependency graph, while a continuation model investigates proposed extensions of both the event set and its associated structure. The second problem cannot generally be reduced to the first.

4.7 Mathematical Structures of Alternative Continuations

The constructions above support an organized, but deliberately incomplete, mathematical notion of multiple possible continuations. For a fixed epistemic historical complex K , let $\mathcal{E}(K)$ denote the class of based cellular extensions (K', i) and let $\mathcal{A}_c(K) \subseteq \mathcal{E}(K)$ consist of those satisfying an explicitly chosen admissibility predicate in context c . Under specified size conventions this collection may be represented by the inhabitants of the dependent type in Equation 9. Distinct extensions need not be comparable by inclusion, and their codomains can have different numbers of cells, attachment maps, and homology groups.

A limited *branching continuation diagram* can therefore consist of two embeddings $i_A : K \hookrightarrow K_A$ and $i_B : K \hookrightarrow K_B$. A further common extension, when it exists and satisfies the relevant conditions, is a complex L with maps from both candidates that agree on K . Equation 20 records this requirement algebraically.

$$j_A \circ i_A = j_B \circ i_B, \quad j_A : K_A \hookrightarrow L, \quad j_B : K_B \hookrightarrow L. \quad (20)$$

The existence of such an amalgamation depends on the chosen category and admissibility conditions. A formal pushout may be constructible at the level of cell complexes, yet the gluing may violate model-specific constraints, causal consistency, or interpretive compatibility. A common topological extension should consequently not be interpreted automatically as a jointly feasible or normatively permissible historical future.

To describe a family of models across observers or stages of epistemic revision, one can index candidate continuation categories by epistemic contexts. This suggests a possible indexed category or fibration, but a full construction requires specified *reindexing* procedures. For a context change $r : c \rightarrow d$, one would have to determine how a continuation over K_d can be related to one over K_c , including cases in which the historical reconstruction changes by correction, deletion, or reinterpretation. Such procedures may be partial, nonunique, or impossible for a particular model. The basic continuation formalism does not settle these transport questions.

The resulting mathematical picture is therefore plural and conditional. Dependent types organize candidates relative to represented histories; based categorical morphisms track their structural composition; relative topology measures certain effects of attachment; Morse reductions reveal the need for preservation constraints; and causal orders encode separately justified dependencies. Together these tools prepare the investigation of distinctive topological problems of future creation in the subsequent section on topological problems of future creation and the later study of historically constituted epistemic reconstruction. Their combination is a preliminary representational framework, rather than a prediction rule, a complete ontology, or a derivation of justice from topology.

5 Topological Problems of Future Creation

The topological investigation of future creation begins with a structurally unusual object: a historical relational spacetime represented by a finite epistemic two-complex, accompanied by conjectured continuations whose events, couplings, higher-order cells, and possible interpretations are not yet established. This section identifies problems introduced by that prospective status. It distinguishes cellular uncertainty from causal ordering, structural transformation from epistemic revision, and topological comparability from normative evaluation. The aim is diagnostic and methodological: to show which information a continuation formalism must retain and which questions require additional assumptions before the constructions of Sections 4 and the later theory of epistemic reconstruction can be used responsibly. The analysis employs finite CW examples, elementary algebraic topology, and hypothetical epistemic con-

Table 4: Distinctive problems of future creation in epistemic two-complex representations. Each topological construction requires additional interpretation or evidence before supporting a claim about a possible future.

Problem	Available mathematical object	Information requiring further specification
Unrealized structure	Candidate cells, incidence, attaching maps	Event ontology, evidential grounds, interpretive meaning
Historical attachment	Based inclusion $K \hookrightarrow K'$ and relative chains	Identity of inherited relations, feasibility of attachments
Multiple continuations	Families of based extensions and morphisms	Evidence for membership, model comparison, overlooked candidates
Causal order	Partial orders and event configurations	Justification of precedence, conflict, and concurrency
Epistemic revision	Changes in representations and contexts	Translation of records and continuation judgments
Reduction	Homotopy- or homology-preserving simplification	Provenance, event identity, and future-relevant constraints
Normative assessment	Structured histories and admissibility predicates	Rights, affected parties, ethical criteria, and responsibility

texts. None of the examples constitutes an empirical prediction or a claim about ultimate spacetime ontology.

The term *future* is used relative to an inquiry’s represented historical standpoint. It identifies prospective extensions of that represented history, without requiring a metaphysical thesis about temporal becoming. Equally, a *possible continuation* in this section is an inhabitant of a stipulated representational domain; its actual feasibility, empirical support, and ethical acceptability require separate assessments. Table 4 gives an overview of the principal mismatches between readily computable topological information and the questions relevant to future creation.

5.1 Unrealized Events and Indeterminate Cellular Structures

A reconstructed historical two-complex can be assembled from observations, archival records, or other representations, even when those sources remain incomplete. A continuation complex differs because its prospective cells describe proposed events and relational structures. Their incidence and interpretation are hypotheses within an epistemic model. In particular, the declaration of a candidate vertex supplies a *represented event possibility*; it does not imply that an event of that kind will occur, or that the representational language exhausts event kinds that may emerge.

Fix an epistemic context c with historical complex K_c . One may first select a modeling envelope $\mathcal{U}_c$ consisting of admissible syntactic descriptions of candidate cells, labels, and attachment instructions. The construction of the envelope depends on the observer’s conceptual resources and the technical language used. A provisional family of modeled continuations may be written as in Equation 21.

$$\mathcal{C}_c(\mathcal{U}_c) := \{ (K', i) \mid i : K_c \hookrightarrow K', K' \text{ is finitely described using } \mathcal{U}_c \}. \quad (21)$$

The family in Equation 21 is an analytically specified domain of descriptions. It may contain physically or institutionally infeasible constructions, and it may omit futures that an observer has not conceived. A finite or computably enumerable modeling language therefore provides a tractable object without licensing an identification with the totality of realizable possibilities.

A further distinction concerns *modal status*. A candidate face might be used as a formal placeholder for an anticipated higher-order relational primitive; its eventual constitutive or interpretive significance cannot be read from its cellular dimension. In this research program, vertices are interpreted as events, edges as couplings, and faces as higher-order primitives whose semantics must be specified for each application. Future entities, including organized actors and institutions, would emerge as interpreted his-

torical substructures of a candidate continuation, rather than as primitive vertex labels. As in Section 3.2, topology organizes the representation; it does not independently verify its ontology.

5.2 Prospective Couplings and Unknown Attachment Relations

Prospective incidence is an independent source of uncertainty. Even after an observer has specified a candidate cell, they may not know how it could attach to the historical structure. The same abstract prospective disk can affect different historical loops, attach with different winding degrees, or participate in distinct causal and interpretive contexts. The attaching map consequently belongs to the content of a continuation hypothesis rather than being inferred from the bare existence of a new cell.

Consider an epistemic historical complex $K \simeq S^1$ with one vertex and one oriented edge e . For each positive integer m , construct K'_m by attaching a single 2-cell f_m along a map of degree m . These are valid CW two-complexes, although for $m > 1$ their chosen cell structures are generally nonregular. The resulting cellular differential is specified in Equation 22.

$$C_2(K'_m; \mathbb{Z}) = \mathbb{Z}\langle f_m \rangle, \quad \partial_2(f_m) = me, \quad \partial_1(e) = 0. \quad (22)$$

The corresponding first homology groups are given in Equation 23.

$$H_1(K'_m; \mathbb{Z}) \cong \mathbb{Z}/m\mathbb{Z}, \quad m \geq 1. \quad (23)$$

Thus degree one yields trivial first homology and degree two yields $\mathbb{Z}/2\mathbb{Z}$, even though each extension adds one abstract 2-cell to the same historical circle. Their difference lies in the historical attachment. These calculations follow ordinary cellular homology; see Hatcher [12] for the standard CW framework.

Proposition 5.1 (Attachment underdetermination by a common base). *In the category of finite CW two-complexes with the indicated historical subcomplex $K \simeq S^1$, knowledge of K and the information that a continuation adds one 2-cell do not determine the homology of the extended complex.*

Demonstration. The continuations K'_1 and K'_2 both include K and add one 2-cell. Equations 22 and 23 give nonisomorphic first homology groups. The attachment degree, absent from the specified historical and cell-count information, determines the difference. The proposition concerns syntactic and topological determination; neither extension is thereby established as empirically feasible.

The result clarifies the design choice behind whole-spacetime continuation semantics. A future-only disk can be considered mathematically, but recovering its effect on history requires at least an attaching map and its relation to the historical base. A whole extension K' equipped with $i : K \hookrightarrow K'$ records these data directly. A future-only formalization remains adequate when that interface is supplied explicitly; the difference is one of representational emphasis and preservation policy.

5.3 Historical Boundaries and Relative Continuation Topology

An epistemic boundary between represented history and its prospective continuation should be distinguished from the cellular boundary operator. The operator $\partial_2 : C_2(K') \rightarrow C_1(K')$ encodes how faces attach to edges. It does not identify which cells represent established historical events and which cells are prospective. That distinction is supplied by the inclusion $i : K \hookrightarrow K'$, by labels, or by an independently declared historical filtration.

For a subcomplex inclusion $K \subseteq K'$, relative chains are obtained by quotienting the chains of the whole continuation by chains lying in the historical component. This construction is given in Equation 24, with the induced differential understood.

$$C_q(K', K; \mathbb{Z}) = C_q(K'; \mathbb{Z}) / C_q(K; \mathbb{Z}). \quad (24)$$

The quotient retains the algebraic effect of new cells relative to history. It does not generally describe a freestanding cellular *future* subcomplex: the boundary of a new face may involve old edges, which

become zero only after passage to the relative quotient. Relative homology and the inclusion-induced maps therefore answer different questions. The former detects some effects of added structure relative to the base, while the latter can record which homology classes of the historical complex survive or become boundaries in the whole extension [12].

For future creation, a continuation should accordingly be treated as a *based* structure. Two codomains with equal Betti numbers may remain distinguishable through their marked historical embeddings, attaching relations, and interpreted provenance. This is especially relevant when later possibilities are conditioned by a particular inherited capacity or obligation. Abstract topological equivalence need not preserve the identifiers and relations on which an epistemic or ethical comparison depends.

5.4 Structural Emergence and Cellular Transformations

A future extension may add cells, alter connectedness, fill previously nonbounding cycles, or introduce higher-order relations among inherited structures. These descriptions concern changes within a represented cellular model. Their interpretation as the emergence of a capacity, organization, institution, or other relational entity is additional and domain-dependent. The cellular description makes a structural hypothesis precise, while the semantic layer must explain what the structure denotes and how the relevant evidence supports that interpretation.

The most elementary continuation operation is an attachment $K' = K \cup_{\varphi} D^q$. A sequence of such operations produces a cellular extension, and alternative choices of φ may yield different invariants. In an event-based interpretation, new events may have relational dependence on old ones even if the added cells do not introduce a new homology generator. Conversely, a nontrivial new homology class does not by itself indicate the creation of a meaningful social or epistemic capacity. Interpretation requires a mapping between selected structural patterns and the phenomena under investigation.

A related issue concerns *disappearance* or dissolution. Historical records of an event need not be deleted when a formerly active coupling ceases to operate. One useful construction retains occurrence cells while encoding the cessation of a relationship through further events, annotations, or state transitions. Such a representation permits a history-preserving embedding even when the current relational configuration has contracted. A model that rewrites or removes previously represented cells instead requires a distinct morphism class and an account of what the removal means. Future creation may therefore involve both extension of historical records and contraction of currently effective capacities. These notions should not be conflated with monotone growth of every topological invariant.

5.5 Topological Invariants and Causal Partial Orders

Two-complex topology and event-order structure provide different kinds of information. Incidence assigns boundaries to cells; a causal or temporal relation orders selected events under separately justified constraints. An oriented cellular edge contributes a sign to a boundary operator, and that choice of orientation does not alone establish causal precedence. Event structures and concurrency models already distinguish causality, conflict, and concurrent possibility as separate relations [23, 13].

For an explicitly defined finite event set V and a strict partial order $\prec$, admissible linearizations form the set specified in Equation 25.

$$\text{Lin}(V, \prec) := \{ \sigma : V \xrightarrow{\cong} \{1, \dots, |V|\} \mid x \prec y \Rightarrow \sigma(x) < \sigma(y) \}. \quad (25)$$

A fixed cell complex may carry distinct admissible orders, and a fixed partial order may be represented using complexes of different topology. A nonzero first Betti number does not prevent a directed acyclic precedence relation from admitting a linear extension. Likewise, contractibility does not imply that all event sequences are feasible or ethically equivalent. Such conclusions would require an explicit structural bridge between cell incidence and ordering constraints.

This matters for the justice of future creation. A model might contain the same candidate events and the same final abstract topology while different admissible orders allocate opportunity or irreversibility

The diagram does not enumerate unrepresented possibilities.

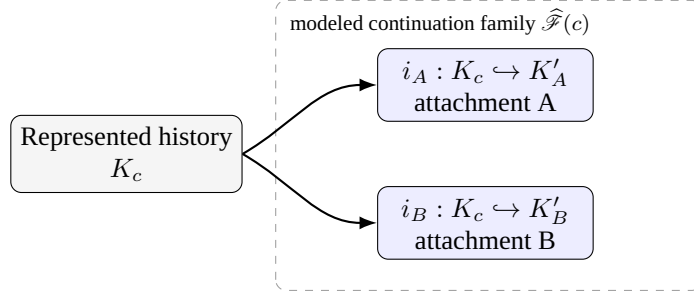


Figure 2: Two candidate whole-spacetime continuations of a common represented history. The dashed boundary marks a selected epistemic modeling family, rather than the boundary of all realizable futures.

differently. Topological invariants cannot substitute for the representation of these historically sensitive dependencies. They can contribute information to a normative inquiry, but the order-sensitive criteria must be defined on appropriately enriched continuation histories.

5.6 Multiple Continuations and Historical Underdetermination

Finite historical evidence can support several inequivalent continuation models. The ambiguity arises at multiple levels: the existence of new events, the attaching maps of new cells, the causal order of candidate events, and the semantic interpretation of newly formed relational structures. The attachment example in Proposition 5.1 supplies a minimal mathematical instance: identical historical data and identical prospective cell counts are compatible with extensions having different homology.

An epistemic reconstruction procedure may accordingly return a set, relation, or family of admissible models. Denote by $\mathcal{M}(c)$ the reconstructed model family for context c , and by $\mathcal{F}(m, c)$ the modeled continuation family under m . Their union is defined in Equation 26.

$$\hat{\mathcal{F}}(c) := \bigcup_{m \in \mathcal{M}(c)} \mathcal{F}(m, c). \quad (26)$$

Equation 26 collects continuations made available by the selected models. It is not a claim that the union includes every real possibility. The dependence on model reconstruction is particularly important because two models may agree on all represented historical observations and diverge only in their constraints on new cells or their inferred dependencies.

Figure 2 depicts a controlled comparison: the base history remains identified, while models may attach new structure differently. Even when different model families are formally enumerable, their members are epistemically conditional. Evidence-based selection among them is a separate task, and the feasibility of their proposed futures cannot be established solely by the existence of their cellular presentations. Predictive-state reconstruction provides an established example of how historical observations may be organized into predictive representations under specified stochastic assumptions [5, 18]. Those assumptions need additional examination for an open-ended setting with historically changing event types and observation conditions.

5.7 Evolving Spaces of Admissible and Conceivable Futures

Future creation can change the capacities through which further possibilities become available or recognizable. Such changes need not be describable as a fixed state space with changing probabilities. An observer may acquire new distinctions, instruments, or concepts, thereby changing the type of future cells and couplings they can specify. A historical intervention may also alter material or institutional constraints relevant to admissibility. These are different developments: the former changes representational accessibility, and the latter may change real-world feasibility as understood within the model.

For successive epistemic contexts c and d , let $\mathcal{U}_c$ and $\mathcal{U}_d$ be their respective modeling languages. One may attempt to relate them through a translation $T_{cd} : \mathcal{U}_c \rightarrow \mathcal{U}_d$. The partial arrow indicates that translation may be undefined for some descriptions. There may be no faithful reverse translation, and a newly introduced primitive in $\mathcal{U}_d$ may have no expression in $\mathcal{U}_c$. In that situation, comparing $\widehat{\mathcal{F}}(c)$ and $\widehat{\mathcal{F}}(d)$ merely by set inclusion would be ill-typed until a common comparison domain or suitable transport construction had been specified.

A similar problem occurs with admissibility. Let Adm_c and Adm_d be context-indexed predicates. Revised evidence, constraints, or interpretations may enlarge some represented possibilities and exclude others. An enlarged language does not imply that the admissible continuations form a monotonically increasing family. The resulting problem is one of *changing types and admissibility conditions*, not simply uncertainty over a fixed list of outcomes. Dependent type theory offers a language for stating such dependencies [22]; the semantics of reindexing across changing historical descriptions remain to be developed.

The category of unrepresented possibilities is particularly delicate. The formalism can acknowledge that a modeled family may be incomplete. It cannot explicitly list elements that lie outside its own current representational resources. Future openness should therefore be treated as a limitation of the epistemic construction and a motivation for revision, rather than as a hidden set whose complete elements have already been specified.

5.8 Endogenous Observers and Epistemic Reflexivity

In a generative-relational inquiry, observing capacities are themselves historically constituted. The observer, their concepts, instruments, and interpretive practices develop within relational histories; observations of possible continuations are made through those capacities. Situated approaches to knowledge emphasize the significance of standpoint and partial perspective [11, 21]. Here that insight motivates a formal problem: the description of a continuation is indexed by a context whose constituents also have historical explanations.

A minimal schematic arrangement may posit a formal reference history R , an observer-supporting substructure O_i within that reference, a history-dependent capacity descriptor a_i , and an observation or interpretation relation yielding the epistemic history K_i . The construction is summarized in Equation 27.

$$O_i \hookrightarrow R, \quad a_i : \text{Cap}(O_i), \quad K_i \in \text{Rep}(R; O_i, a_i). \quad (27)$$

Here R is an *assumed ontological reference construct introduced solely for formal convenience*. No claim is made that it is ultimate reality, a uniquely determined ontology, or a complete account of what exists. The relation Rep is deliberately provisional: its mathematical domain, evidence-dependence, and treatment of interpretive bias will be developed in Section 6. The notation in Equation 27 records intended dependencies without assuming a uniquely specified mechanism by which cognitive capacity emerges from historical structure.

Reflexivity raises two problems. First, the epistemic representation may attempt to describe aspects of the observer whose capacities generated that representation. Second, the capacities may change as observations and subsequent actions reshape the observer's supporting history. Neither problem requires a complete self-description at a fixed moment. A representation may be partial and revisable while still supporting a useful inquiry. The substantive task is to specify what limited observations and context changes allow to be inferred about the resulting continuations. A mathematically explicit account of the endogenous operator remains a principal open problem.

5.9 Cellular Reduction and Historical Information Preservation

Discrete Morse theory supplies methods for constructing smaller complexes that retain selected homotopy information. In future-creation inquiries, computational reduction can improve tractability, but the information required to assess a continuation may exceed its homotopy type. The established discrete

Morse correspondence concerns topology under stated hypotheses [8]. Historical provenance, interpreted event identity, causal ordering, and normative significance are additional features that a reduction may fail to preserve.

A minimal example makes the distinction concrete. Let the historical complex K consist of a single event vertex h . Construct K' by adding a proposed event vertex x and one edge e between h and x . The pair (e, x) admits an elementary collapse relative to K , so K' and K have the same homotopy type and their relative homology vanishes. Yet a model whose future inquiry concerns whether event x is reachable or permitted must retain information about x and its attachment. Deleting the collapsible pair changes the event inventory and can invalidate the downstream question, even though it preserves the selected topological invariants.

A reduction procedure must therefore be judged against an explicit family of observables. Denote such a family by $\mathcal{I}_c$, which may include historical labels, attachments, precedence constraints, and interpretation-dependent data. A reduction $r : (K', K) \rightarrow (L, K)$ is suitable for an inquiry only when the selected observables can be recovered or are appropriately transported. Equation 28 expresses one strong preservation condition.

$$I(K', K) = I(L, K) \quad \text{for each } I \in \mathcal{I}_c, \quad (28)$$

where equality presumes common value spaces and fixed identifications. In applications, a specified comparison isomorphism or approximate reconstruction relation may be more appropriate. The criterion in Equation 28 is a methodological requirement, rather than an established theorem about a general Morse reduction. It explains why preservation of homotopy or relative homology alone cannot determine which simplifications are appropriate for future creation.

5.10 Two-Dimensional Representability and Higher-Order Structures

A two-complex provides a disciplined vocabulary of events, couplings, and higher-order primitives represented by faces. It can express nontrivial loops, cellular attachments, and some changes in homology. Its adequacy for a particular historical process nevertheless depends on the semantics assigned to those primitives and on the resolution of the available evidence. A complex of dimension at most two has vanishing cellular chain groups above dimension two. Thus its cellular homology in degrees greater than two vanishes; a phenomenon whose intended representation depends on nonzero higher-dimensional homology cannot be captured literally within that chosen cellular model.

The limitation should not be confused with a claim that every complicated multi-agent or multi-event relation necessarily requires a higher-dimensional cell. A face may already encode a sophisticated higher-order primitive under an explicit interpretation policy, and multiple faces may form a complex substructure. Conversely, a chosen encoding may collapse distinctions whose preservation is vital to a later inquiry. The decision about dimensional extension is therefore partly mathematical and partly semantic.

The spin-foam literature demonstrates that decorated two-complexes can participate in sophisticated physical modeling; its field-theoretic and amplitude constructions also make clear that the geometric support of a model and the model's dynamics are distinct mathematical ingredients [2, 15, 14]. The use of spin foams as a structural precedent in this paper entails no inference that historical, social, or epistemic processes obey quantum-gravitational dynamics. Future extensions involving three-cells, enriched labels, sheaves, or higher-categorical descriptions may become relevant when a concrete representational deficiency has been demonstrated.

5.11 Structural Possibilities and Normative Constraints

A candidate continuation may be mathematically constructible and epistemically representable while remaining unacceptable under independently justified ethical standards. The study of future creation is

sensitive to how early choices condition later opportunities, participation, duties, and risks. Those dimensions are not defined by an unmarked homology group, and the identification of a cell as topologically critical does not establish the ethical significance of the event it represents.

For an epistemic context c , the constraints governing modeled continuations can be separated into a structural predicate CellAdm_c , an evidence-sensitive epistemic predicate EpiAdm_c , and a normatively interpreted predicate NormAdm_c . Their conjunction, where meaningful, is shown in Equation 29.

$$\text{Adm}_c(K', i) := \text{CellAdm}_c(K', i) \wedge \text{EpiAdm}_c(K', i) \wedge \text{NormAdm}_c(K', i). \quad (29)$$

Equation 29 is a possible design for a layered assessment, not a deduction of ethics from topology. In some research tasks only structural admissibility is defined; in others, epistemic support is graded rather than Boolean, and normative judgments are incomplete or disputed. The conjunction presupposes a representation policy that explicitly identifies each predicate's domain and sources of warrant.

The normative concern is also order-sensitive. A continuation may preserve a final collection of cells while differing in which actors gain the ability to shape intermediate possibilities. Epistemic exclusion may occur when a modeling language fails to recognize certain participants' capacities or claims. Anticipatory governance, responsible innovation, and epistemic injustice provide relevant normative perspectives for specifying such questions [10, 20, 9]. Their insights do not establish a single universal decision criterion; ethical evaluation requires attention to domain, institutions, history, and the affected parties.

The problems assembled in Table 4 suggest a disciplined division of work for the rest of the paper. Section 6 develops the historical constitution of epistemic representations; Sections 7–9 address history-indexed continuation types, categorical relations, and model reconstruction; Section 10 investigates the topological behavior of selected extensions; and the subsequent illustrative and normative sections examine what these constructions can, and cannot, support. The distinctive mathematical challenge of future creation is therefore the *controlled articulation of possible extensions under incomplete and evolving epistemic conditions*. A two-complex formalism offers one tractable starting point while leaving room for additional forms of representation, historical evidence, and ethical reasoning.

6 Historical Constitution of Epistemic Representations

The topological epistemology of future creation requires an account of how a represented history becomes available to an observer. The mathematical foundations of Section 4 permit candidate extensions of a historical two-complex, while Section 5 identifies the uncertainty of the represented history and the prospective attachments. This section addresses an additional dependency: the capacities, recording practices, and interpretive procedures through which an observer constructs a historical complex may themselves be historically constituted. Its objective is to specify that dependency without deriving a complete cognitive or ontological generator. The discussion introduces an explicitly postulated reference domain, historical supports of observing systems, dependent types for epistemic capacities, an observation–interpretation relation, and a distinction between epistemic revision and future continuation. The method is conditional formal modeling accompanied by a finite counterexample concerning topological observability. Throughout, the models concern represented histories; their correspondence to any actual historical process remains a separate evidential problem.

The phrase *historical constitution* has a restricted analytical role. It indicates that an observer's capacities and representations are conditioned by historical configurations represented within a chosen domain. It does not, by itself, explain the mechanism producing those capacities, prove a causal dependence in a particular empirical case, or establish an ultimate ontology of relational spacetime. Discussions of situated knowledge and historically developed scientific practices provide disciplinary context for this treatment [11, 21, 6]. The following type-theoretic constructions are proposals of the present paper, whereas dependent sums and the mathematics of cellular subcomplexes are established resources [22, 12].

6.1 Assumed Ontological Reference Structures

To state a relation between a historical process and its epistemic reconstruction, the formal investigation may posit a reference structure R belonging to a designated class Ref_2 . For the present finite examples, members of Ref_2 are two-dimensional CW descriptions with selected incidence and optional historical data. The adjective *ontological*, when used for this assumed reference structure, marks the role that R plays within a conditional model: it stands for the history to which the model's observation claims refer. It does not identify R with ultimate reality, a uniquely correct account of what exists, or a fundamental cell decomposition of physical or social phenomena. Another inquiry may employ a different reference domain and obtain different formal consequences.

Three separations are important. First, R is an *assumed reference description*; $\hat{K}$ is an *epistemic description* attributed to a situated observer. Both are mathematical representations, even though their roles differ. Second, a mapping or relation connecting them is an explicitly proposed observation–interpretation construction; merely placing two objects in the same category does not determine such a construction. Third, there is no requirement that the epistemic representation be a subcomplex of R : interpretation can introduce conjectured couplings, identify events under different descriptions, or use cellular decompositions not shared with the reference structure.

The historical designation is also operational. A reference model may carry a precedence relation, an observational stage parameter, or a family of selected historical supports. None requires the paper to settle whether time is fundamentally real, emergent, or otherwise constituted. In particular, the finite index s used below designates the stage of an inquiry or available records, not an ontologically privileged present. The ensuing constructions therefore remain compatible with different philosophical accounts of temporality, provided the chosen reference model states the relevant order relations explicitly.

6.2 Historical Supports of Observing Systems

Within a stipulated reference structure R , an observer may be associated with a supporting historical configuration. A simple formal representation uses a subcomplex S together with a cellular embedding $j : S \hookrightarrow R$. Its cells are interpreted as a selected support for the formation or operation of observing capacities. This support may include earlier observations, interactions, acquired practices, institutional relations, and material resources, insofar as the modeling vocabulary assigns them a meaning. The support is a modeling choice rather than an exhaustively documented origin of the observer.

This formulation respects the event-based semantics of Section 3: vertices represent events, edges represent couplings, and faces represent higher-order primitives whose substantive meanings require specification. A person, research community, or observing institution is consequently modeled, where useful, through interpreted historical substructures. An observer is not identified with one vertex, and the existence of an embedding j does not establish an independent metaphysical entity corresponding to S .

Historical supports can overlap. Distinct observers may depend on shared records, methods, institutions, or communication infrastructures, while differences in access and interpretation can remain consequential. A single observer may also have several defensible supporting subcomplexes, depending on the granularity and investigative purpose. In cases where the assumed reference domain does not accommodate a meaningful inclusion $S \hookrightarrow R$, a marked diagram or an interpreted correspondence can replace a literal subcomplex. The embedding convention is retained here for a tractable initial construction.

Let $\text{Sub}(R)$ denote the type of specified pairs (S, j) in the chosen finite reference formalism, with $j : S \hookrightarrow R$ a cellular embedding onto a subcomplex. The term *support* does not imply that every cell of S is known to the observer. It separates the historical conditions represented by the modeler from those conditions that the observing system can itself recognize or report. That separation will be essential when the observer becomes an object of its own epistemic inquiry.

6.3 Epistemic Capacity Types and Historical Constitution

Dependent types offer a compact way to register the dependence of epistemic capacity descriptions on a selected reference history and support. For each $(S, j) : \text{Sub}(R)$, let $\text{Cap}(R, S, j)$ be a type of stipulated observing and interpreting capabilities associated with that configuration. An inhabitant a may encode distinctions available to the observer, instrumentation, conceptual vocabularies, recording permissions, computation, or procedures for revising earlier judgments. These are interpreted capability descriptions; the type does not automatically furnish a psychology, sociology, or generative dynamics of their emergence.

A historically indexed observer context is proposed in Equation 30.

$$\text{ObsCtx}(R) := \sum_{(S,j):\text{Sub}(R)} \sum_{a:\text{Cap}(R,S,j)} \text{Proc}(R, S, j, a). \quad (30)$$

The final factor $\text{Proc}(R, S, j, a)$ contains descriptions of observation and interpretation procedures available under the selected capacity profile. In Equation 30, an observer context $o = (S, j, a, \pi)$ is therefore indexed by an assumed reference history. This is an explicit mathematical statement of *conditional historical dependence*. It remains compatible with distinct capacities supported by the same history and with different reference descriptions of the same empirical setting.

Equation 30 alone does not establish that R uniquely generates a or π . An inhabitant is selected or hypothesized subject to stated interpretation and evidence. Explaining why a particular capacity emerged may require additional learning dynamics, biological constraints, social mechanisms, or historical investigation. Conversely, treating Cap as a singleton would erase precisely the variability that motivates observer-indexed epistemic representations. For this preliminary paper, a nontrivial dependent family is sufficient to express the intended indexing without pretending to solve the emergence problem.

The indexing convention also distinguishes two kinds of variation. A change in the supported history (R, S, j) may change the *available type* of capacities; a change of inhabitant within the same type may represent variation in concrete abilities or procedures. Whether such variations should be identified, compared, or transported requires specified morphisms between reference contexts. This issue later reappears as a transport problem for continuation types rather than a consequence guaranteed by dependent type theory.

6.4 Observation and Interpretation Operators

A historically indexed observer context must be connected to actual epistemic representations by an additional operation. For $o : \text{ObsCtx}(R)$, let $\text{Data}(R, o)$ denote the admissible records or accessible evidence described relative to that observer. Let EpCplx_2 retain the syntactic meaning established in Section 4.1. To keep provenance visible, define the type of evidence-annotated epistemic descriptions in Equation 31.

$$\text{EpRecord}_2(D) := \sum_{\hat{K}:\text{EpCplx}_2} \text{Prov}(D, \hat{K}). \quad (31)$$

A provenance inhabitant records the stipulated relation of events, couplings, faces, and their interpretations to the evidence D . It may include confidence qualifications, disputed identifications, inferential assumptions, or explicit gaps. Such an inhabitant is not a proof that the represented historical claims are true. The type specifies the internal form of an accountable representation.

Rather than impose a deterministic and single-valued map, we provisionally define the observation–interpretation operation as a relation, as stated in Equation 32.

$$\mathcal{O}_{R,o} \subseteq \coprod_{D:\text{Data}(R,o)} (\{D\} \times \text{EpRecord}_2(D)). \quad (32)$$

A member $(D, (\hat{K}, p))$ of the relation indicates that $\hat{K}$, with provenance annotation p , is a permitted epistemic reconstruction from evidence D under the selected observer context. The relational form allows

ambiguous evidence to support more than one interpretation, and allows a procedure to be undefined when necessary material is missing. In a concrete computational implementation, one could choose a partial function, a nondeterministic procedure, or a stochastic kernel after stating the assumptions that justify it.

The relation in Equation 32 should not be silently described as a cellular morphism $R \rightarrow \hat{K}$. It connects data and interpreted records across distinct representational roles, possibly with different cell decompositions and competing identifications. It therefore need not preserve dimension, incidence, homology, or the original event labels. An epistemic continuation morphism $\hat{K} \hookrightarrow \hat{K}'$ introduced in Section 4 has a different mathematical type and purpose. Its codomain contains a candidate extended history, whereas $\mathcal{O}_{R,o}$ specifies how one historically situated representation may be formed.

This formal distinction also guards against a mistaken account of bias. Selectivity may enter through access to evidence, through the capability type, through the interpretation procedure, or through the provenance policy. Incompleteness, misclassification, and incompatible perspectives need not have the same formal signature. The literature on situated knowledge, situated action, and the historical formation of epistemic practices motivates attention to these layers without supplying the proposed cellular operator as an established result [11, 21, 6].

6.5 Epistemic Spacetime Construction

A finite model can exhibit the dependence of topological conclusions on which relations become epistemically represented. Let the stipulated reference complex R be a filled oriented triangle with event vertices v_0, v_1, v_2 , coupling edges e_{01}, e_{12}, e_{20} , and one face f . Consider three possible evidence-annotated reconstructions, all employing the same abstract cell interpretations but differing in the incidence information they admit. The first represents two edges forming a path; the second includes the closing edge and represents a one-cycle; the third additionally recognizes the face and its attaching boundary. Figure 3 shows these three epistemic complexes. Its cell coordinates have no temporal significance.

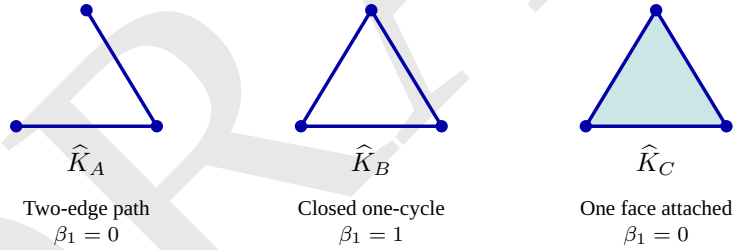


Figure 3: Three epistemic reconstructions compatible with different access or interpretation conditions within a stipulated triangular reference example. All complexes are connected; their first Betti numbers are respectively zero, one, and zero. The middle cycle is a topological feature of that representation, not automatically a gap in the observer’s knowledge.

For these particular finite CW complexes, Equation 33 records the first homology groups with integer coefficients.

$$H_1(\hat{K}_A; \mathbb{Z}) = 0, \quad H_1(\hat{K}_B; \mathbb{Z}) \cong \mathbb{Z}, \quad H_1(\hat{K}_C; \mathbb{Z}) = 0. \quad (33)$$

The calculation follows directly from the cellular boundary ranks: $\hat{K}_A$ has a tree as its one-skeleton, $\hat{K}_B$ has one cycle and no two-cell, and $\hat{K}_C$ has a two-cell whose boundary kills that cycle. This is a synthetic example rather than a demonstration of real-world observer bias. It nevertheless establishes that, even where evidence or interpreted cells are added monotonically, the first Betti number of a reconstructed complex need not vary monotonically. A single numerical invariant cannot measure epistemic completeness.

One might require that each stage arise from a different historically conditioned observation context, or allow the same observing system to revise its description as new evidence arrives. Either interpretation

requires explicit provenance. In the former case, observer variation is the object of comparison; in the latter, the transformation is an epistemic revision. Both are analytically distinct from a modeled *future continuation*, whose source is a historical epistemic complex and whose target includes additional candidate future cells. The example also reveals why the selected reference structure is a convenience: calculating homology does not certify that the stipulated filled triangle is the complete or uniquely correct description of a historical reality.

6.6 Endogenous Observation and Reflexive Representation

The dependent construction above makes an observing context part of a stipulated historical reference domain. In that restricted sense, observation is endogenous: the selection of an epistemic operator is conditioned by R and an interpreted support (S, j) . Its output $\hat{K}$ may in turn contain recorded or inferred information about the observing system. The correspondence between an assumed supporting structure S and an interpreted observer-description inside $\hat{K}$ is an additional, partial epistemic claim; it should not be built into the construction as a guaranteed identity.

This distinction avoids an unnecessary demand for complete self-representation. An observing system can employ a procedure whose historical conditions are only partly accessible to that system. The mathematical specification of $\mathcal{O}_{R,o}$ by a modeler does not imply that the modeled observer can reconstruct the procedure, the full reference structure, or the observer's own support. Thus no general fixed-point requirement $\hat{K} \simeq R$, nor a requirement that $\hat{K}$ encode a complete description of $\mathcal{O}_{R,o}$, follows from Equation 30. Such stronger requirements would constitute additional research hypotheses.

Reflexive inquiry nevertheless becomes possible at a second epistemic level. A represented observer may develop a model of selected conditions of its own knowledge production: earlier evidence selection, instruments, interpretive habits, or institutional rules. That model can become an object of subsequent observation and revision, without exhaustively capturing the historical process that supported the capacity to construct it. Distinct levels of description should be kept traceable through provenance and explicit interpretive correspondences. Otherwise, the phrase *the observer observes itself* conceals several independent operations: self-report, external observation of practices, retrospective reconstruction, and formal modeling by a third party.

There are also genuine unresolved questions. The type family $\text{Cap}(R, S, j)$ records dependency but does not specify a mechanism of capacity formation. The relation $\mathcal{O}_{R,o}$ records permissible representational outcomes but does not identify which one is epistemically justified without further evidence rules. Moreover, changing the reference description R can change the indexing types themselves. These limits distinguish an initial *formal account of endogeneity* from a generative causal explanation or a universal reflexive epistemology.

6.7 Epistemic Revision and Representational Change

The historically constituted character of an epistemic complex is operationally important when observations and interpretive conditions change. Let $c_s = (\hat{K}_s, D_s, p_s, o_s)$ summarize a selected epistemic context at inquiry stage s , including its reconstructed history, evidence, provenance, and observer conditions. A revision $c_s \rightsquigarrow c_{s+1}$ may append records, correct earlier identifications, withdraw uncertain couplings, reclassify higher-order primitives, or change the representational language. This revision arrow is initially a relation between contexts; its classification as a functor, cellular map, span, or rewriting step requires separate structure.

The contrast with continuation can be stated precisely. A continuation arrow $i_s : \hat{K}_s \hookrightarrow L_s$ retains the stipulated historical complex as a marked subcomplex of a *candidate extended* spacetime L_s . Epistemic revision may instead replace the represented history $\hat{K}_s$ with a differently constituted $\hat{K}_{s+1}$. A continuation selected before the revision consequently need not have an unambiguous counterpart afterward. Figure 3, for example, can be read as a succession of expanding evidence records, but a corrected reconstruction may also remove the closing edge of $\hat{K}_B$ and thereby invalidate a previous face-attachment proposal depending on that cycle.

Table 5: Distinct formal and epistemic roles in historically conditioned reconstruction and future continuation. The reference and epistemic structures are both representations, while their functions in the conditional model differ.

Construction	Mathematical role	Interpretive limitation
$R : \text{Ref}_2$	Assumed reference history	Formal convenience; no ultimate ontology
$j : S \hookrightarrow R$	Marked historical observer support	Depends on selected grain and support interpretation
$o : \text{ObsCtx}(R)$	Historically indexed capacity and procedure	Indexing alone does not explain emergence
$\mathcal{O}_{R,o}$	Evidence-to-representation relation	Partial, nonunique, not necessarily cellular
$\hat{K}$	Represented historical two-complex	Revisable, partial, evidence-dependent
$\hat{K} \hookrightarrow L$	Candidate whole-spacetime continuation	Admissibility and feasibility need added conditions
$c_s \rightsquigarrow c_{s+1}$	Epistemic context revision	Continuation transport is not automatic

A conditional construction illustrates what a transport rule would require. If revision happens to be equipped with an appropriate map $r : \hat{K}_s \rightarrow \hat{K}_{s+1}$, and the selected category admits the corresponding pushout, the candidate extended complex can be transported by the gluing expression in Equation 34.

$$L_{s+1} := L_s \sqcup_{\hat{K}_s} \hat{K}_{s+1}. \quad (34)$$

Equation 34 is a conditional categorical construction, not a universal rule for epistemic revision. An admissible map r may be unavailable or nonunique; the pushout may fall outside the selected category of decorated finite two-complexes; and the resulting object may fail to preserve the evidential or normative conditions initially imposed on L_s . In particular, identifying historical cells during revision can alter the attachments of prospective cells. Even a technically valid pushout cannot establish the truth of a corrected historical interpretation.

Table 5 summarizes the distinct roles of the objects introduced in this section. The separation is especially important for future creation because an observer’s admissible continuation family depends on the epistemic history and evidence used to construct it. Revision may therefore change the apparent field of possible futures without proving that the underlying field of historically feasible developments has changed.

The present section therefore provides a conditional architecture for endogenous epistemic reconstruction rather than a closed explanatory theory. It distinguishes the selected reference history, the historical support of an observer, the capacity-dependent observation procedure, its evidence-annotated representations, and subsequent revision. The formal construction supplies the historical-epistemic context required to index continuation types in Section 4 and to investigate their categorical transport later. In normative applications, the selection and revision of observation procedures can affect whose possibilities become recognizable, yet judgments about justice and responsibility require separately argued normative criteria. The mathematical architecture makes these dependencies discussable while preserving the limits of topological and epistemic inference.

7 Dependent Types of Epistemic Continuation

The epistemic constitution of a represented historical spacetime, developed in Section 6, supplies the starting point for a typed account of future continuation. This section specifies the objects that qualify as candidate whole-spacetime extensions, explains the roles of cellular embeddings and admissibility witnesses, and examines the information retained or lost under alternative representations of a prospective

future. The method is primarily constructive and comparative: formal definitions and limited propositions are accompanied by cellular examples and explicit epistemic qualifications. The resulting continuation type organizes *represented possibilities* under a declared context. It does not supply a complete generator of realizable futures or a criterion of justice.

7.1 Historical Complexes and Continuation Domains

An epistemic continuation must have a historically identified source. We use $c = (K, D, p, o)$ to denote a selected epistemic context, where $K : \text{EpCplx}_2$ is the reconstructed finite two-complex, D is available evidence, p collects provenance and interpretation rules, and o summarizes historically constituted observing conditions. This notation follows the context record introduced in Section 6.7; the components may themselves be uncertain or revised. The type EpCplx_2 is a selected universe of encodings of finite, interpreted two-complexes. Its inhabitants are syntactically well-formed representations, rather than certificates of historical truth.

For a fixed source K , let $\text{Ext}_2(K)$ be the type of pairs (L, i) , where L is a candidate two-complex and $i : K \hookrightarrow L$ is a dimension-preserving cellular embedding whose image is a subcomplex. The embedding preserves the chosen historical cells, their incidence, and those labels declared invariant by the interpretation scheme. Equation 35 records the elementary dependent type.

$$\text{Ext}_2(K) := \sum_{L:\text{EpCplx}_2} \text{Emb}_2(K, L). \quad (35)$$

The fibers $\text{Emb}_2(K, L)$ depend on the proposed codomain L . In particular, a bare choice of L cannot specify which of its cells represent the historical source if multiple embeddings are available. Two embeddings into the same underlying complex may represent different historical identifications and should be kept distinct when provenance is analytically significant.

This construction concerns a single interpreted historical context. A general claim that K accurately reproduces an assumed ontological reference structure would exceed its type-theoretic content. The type of an epistemic extension is indexed by a *representation* of history, not by a verified description of ultimate reality. It also does not privilege an ontologically fundamental time parameter. Terms such as *earlier*, *historical*, and *prospective* refer to the declared standpoint and ordering conventions of the inquiry.

7.2 History-Preserving Cellular Embeddings

An embedding specifies how a candidate continuation contains its represented past. For a cellular two-complex, a marked embedding must commute with cellular boundary maps when passed to cellular chains. With $i_q : C_q(K; \mathbb{Z}) \rightarrow C_q(L; \mathbb{Z})$ denoting the induced maps on chain groups, the requirement is stated in Equation 36.

$$\partial_q^L i_q = i_{q-1} \partial_q^K \quad (q = 1, 2). \quad (36)$$

This equality is necessary for the embedding's induced cellular chain map. In the present category it supplements the geometric requirement that the selected source cells occur in L with their specified incidence. A mere injective homomorphism of homology groups would not identify the underlying events and couplings, and need not preserve the full historical structure.

An embedding also carries historical identification information. For example, two source cycles with different recorded meanings cannot be exchanged freely when comparison is intended to preserve their labels. Consequently, the isomorphisms relevant to our inquiry are often *isomorphisms over K* : an isomorphism $h : L_1 \rightarrow L_2$ must satisfy $h \circ i_1 = i_2$. This is stronger than the requirement that the underlying spaces L_1 and L_2 have the same homotopy type, and stronger than abstract isomorphism of their homology groups.

Cellular embedding remains a deliberate restriction. A process represented as termination, dissolution, or reconfiguration of a historical institution can be encoded through further events and incidence

relations without erasing the earlier event records. If an epistemic correction removes or reidentifies previously represented cells, however, the corresponding operation belongs to the revision problem of Section 6.7; it may require a span, replacement rule, or differently typed morphism. Such corrections are not treated as strict continuations merely because they occur at a later stage of inquiry.

7.3 Whole-Spacetime Continuation Semantics

The central semantic choice of this paper is to take the *codomain of a continuation embedding* as the whole candidate relational spacetime. A continuation is a marked pair (L, i) , with the source history included as $i(K) \subseteq L$. The image of i is the retained historical part; the codomain L includes that part and prospective cells attached to it. This distinction between the image of a morphism and its codomain is important because the phrase *image of the future* can otherwise obscure which structure the formalism is intended to represent.

The design follows three analytical requirements. First, historical provenance is retained as part of the represented object: particular earlier events and couplings remain available for comparison. Second, the incidence of future cells with historical cells is explicit in the whole complex. Third, successive continuations can be compared or composed using the same kind of marked embedding, subject to admissibility constraints. None of these properties establishes that the extension is realizable; they establish a representation in which historically conditioned structural claims can be stated and examined.

An elementary future attachment shows the importance of retaining the base. Suppose a historical subcomplex contains the boundary loop of a prospective two-cell. The completed candidate is the pushout described in Equation 37.

$$L = K \sqcup_{\alpha: S^1 \rightarrow K} D^2, \quad (37)$$

where the gluing identifies the boundary of the new disk with the loop α in K . Different maps α may give different extended spaces, even when the newly supplied disk is identical. Equation 37 is a standard cellular attachment construction [12]; its use as a model of a relational primitive additionally requires an interpretation of that primitive and an epistemic warrant for proposing the attachment.

Whole-spacetime semantics also accommodates historically marked comparisons. A new face can render a previously nonbounding one-cycle into a boundary in L , even though the historical cycle remains present as a cellular chain. Thus, retention of the original cells does not entail retention of every homological property under inclusion. This distinction was illustrated by the inclusion-induced kernels in Equation 17. It is a reason to track the marked embedding, rather than replace the extended structure with homology groups alone.

7.4 Future-Only Representations and Relative Extensions

A future-only representation is mathematically legitimate when its relation to history is specified. The simplest set of newly introduced cells, $\text{Cells}(L) \setminus i(\text{Cells}(K))$, is generally not closed under cellular boundaries. For example, a prospective triangular face may have an edge lying entirely in K . The collection of newly introduced cells consequently need not form a subcomplex or carry the intended cellular boundary operator by itself. This is an issue of missing attachment structure, not a general prohibition against future-only models.

One alternative is to retain a prospective cell object F , a boundary interface A , and a chosen attaching map $a : A \rightarrow K$. A pushout $L = K \sqcup_A F$, under appropriate cofibration and CW attachment assumptions, can recover the whole extended structure. The interface A may identify historical cells or attach successive prospective cells. Accordingly, a suitably decorated future-only representation and a whole-spacetime representation can carry equivalent information under a specified gluing convention. Table 6 identifies the different data responsibilities of the two formulations.

Figure 4 depicts the two history-marked disk attachments used in Proposition 7.1. The left and right circles of the source have distinct recorded labels. The two continuations differ by the attachment site, even though the prospective two-cell has the same abstract shape.

Table 6: Data retained by alternative presentations of epistemic future continuation. Equivalence depends on preserving a specified interface and its attachment maps.

Presentation	Primary data	Information required for historical comparison
Whole-spacetime extension	$(L, i : K \hookrightarrow L)$	Base identification is explicit; incidence is contained in L
Future-only cells	New cell list or prospective complex	Historical boundaries and attaching maps must be added
Relative cellular pair	(L, K) with marked inclusion	Relative chain groups may omit distinct attachment effects on K
Gluing presentation	(K, A, F, a) with appropriate pushout data	Interface, labels, and selected equivalence conventions

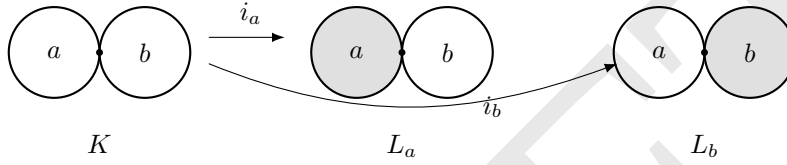


Figure 4: Two continuations of a common historically marked wedge of circles. The shaded disk is attached along the labelled loop a in L_a and along b in L_b . The future quotients L_a/K and L_b/K each identify with S^2 , although the historical inclusion-induced homology maps differ. The diagram is schematic; labels represent distinct historical identifications rather than ontological claims.

A second future-oriented construction is the relative chain complex $C_*(L, K)$, or the quotient space L/K when a based quotient is appropriate. These are useful topological summaries of the extension. They are not in general complete substitutes for a historically marked pair. In particular, the relative boundary operator does not by itself recover the connecting homomorphism from relative homology to historical homology, which depends on how the new cells attach. The long exact sequence of a pair, discussed in Section 4.4, records part of this additional structure.

Proposition 7.1 (Attachment information lost by an unmarked future quotient). *There exist a finite historical two-complex K and two cellular extensions (L_a, i_a) and (L_b, i_b) such that L_a/K and L_b/K are homeomorphic, whereas the extensions are not isomorphic over the historically labelled base K . Their inclusion-induced homology maps have different kernels as subgroups of $H_1(K; \mathbb{Z})$.*

Proof. Take $K = S_a^1 \vee S_b^1$, a wedge of two labelled circles, and let L_a be obtained by attaching a two-cell with degree one along S_a^1 . Define L_b similarly with its two-cell attached along S_b^1 . Collapsing the entire historical wedge K to a point in either L_a or L_b produces $D^2/S^1 \cong S^2$. Thus both future quotients are homeomorphic to the two-sphere. The inclusions, however, kill different based homology classes, as calculated in Equation 17: their kernels are respectively $\langle [a] \rangle$ and $\langle [b] \rangle$. An isomorphism over K would have to commute with the inclusions and induce equality of these kernels, which is impossible because $[a]$ and $[b]$ are independent generators. $\square$

Proposition 7.1 establishes a limited information-loss result. A future quotient with no marked attachment data fails to distinguish certain historically different continuations. The statement does not establish that every future-only formalism is lossy: retaining the attaching map distinguishes the two cases. The preferred whole-spacetime convention therefore makes the historical data *primary*, while leaving alternative presentations available for computational or interpretive purposes.

7.5 Historical Attachment Data and Semantic Preservation

Cellular incidence gives the formal mechanism of attachment, but semantic preservation depends on further declarations. Within the GRT-inspired interpretation used here, vertices represent events, edges rep-

resent couplings, and faces represent higher-order primitives whose substantive interpretation is context-dependent. A historical entity may be reconstructed from a pattern of events and couplings rather than identified with a primitive vertex. Consequently, a cellular embedding can preserve an incidence pattern while failing to preserve a separately intended meaning unless that meaning is encoded in the morphism’s label-preservation rules.

For formal clarity, write λ_K and λ_L for selected annotations on the historical and extended complexes. A label-preserving embedding is required to satisfy the compatibility condition in Equation 38 whenever a given label is declared rigid under continuation.

$$\lambda_L(i(\sigma)) = \lambda_K(\sigma) \quad \text{for each historically marked cell } \sigma. \quad (38)$$

The equality is a chosen representational condition. It may be appropriate for stable provenance identifiers, while interpretive descriptions of events may legitimately change under epistemic revision. A robust data model might therefore distinguish immutable record identifiers, revisable interpretation fields, and inference-specific annotations. Their different stability requirements should not be collapsed into one undifferentiated notion of *preservation*.

One may further attach a separate event-dependency relation, a confidence annotation for an inferred coupling, or conditions describing which proposed higher-order primitives are meaningful in a domain. These enrichments do not follow from the CW structure itself. In particular, the orientation of an edge in a boundary calculation is a sign convention and need not encode the direction of historical causation. Likewise, two complexes of the same homotopy type may represent substantially different histories when their event identities, observer supports, and dependencies differ.

For the epistemology of future creation, this semantic layer identifies a key design obligation. Whenever a quotient, homotopy reduction, or equivalence relation is used to simplify a continuation, the paper must specify what information the reduction is allowed to forget. Preserving homology is sufficient for a question about selected topological invariants. Questions about historical responsibility, participation, or admissible orders may require preservation of event provenance and normative predicates that topology alone does not encode.

7.6 Admissibility Predicates and Dependent Continuation Types

A structural extension can be encoded without asserting that it is supported by evidence or feasible under historical constraints. We therefore make the admissibility layer explicit. For context c , let $\text{Adm}_c(L, i)$ be a proposition or type of witnesses specifying declared conditions on the extended complex and the marked embedding. The dependent continuation type introduced in Section 4.1 is restated in Equation 39 to distinguish a well-formed extension from an epistemically admitted candidate.

$$\text{Cont}(c) := \sum_{(L, i) : \text{Ext}_2(K_c)} \text{Adm}_c(L, i). \quad (39)$$

An inhabitant $u = (L, i, w)$ contains a witness $w : \text{Adm}_c(L, i)$; the witness has meaning only relative to the selected predicate. Under a proposition-valued formulation, the witness records type-theoretic admission rather than probabilistic confidence. A type checker can establish that an encoded witness conforms to the rules of the formal system, but it cannot independently verify whether those rules correctly represent historical evidence or predict actual realization [22].

For analytical use, admissibility can be structured into separate conditions. Equation 40 describes an optional factorization into representational, epistemic, and stipulated feasibility-related components.

$$\text{Adm}_c(L, i) := \text{Rep}_c(L, i) \times \text{Evid}_c(L, i) \times \text{Feas}_c^{\text{mod}}(L, i). \quad (40)$$

Here Rep_c states syntactic and interpretation-policy constraints, Evid_c records consistency with the evidence policy, and $\text{Feas}_c^{\text{mod}}$ encodes only the feasibility constraints actually represented in the model. These factors may have different kinds of witnesses and may be partial or unknown. The product is

a modeling choice; one may omit a factor or use a different logical structure if the evidence does not justify a definite truth value. Normative permissibility remains a separate predicate, to be developed in Section 12, rather than a consequence of this equation.

A related distinction concerns a *model of continuations*. A single $u : \text{Cont}(c)$ represents one candidate; a model m may select a subfamily through an additional predicate $G_{m,c} : \text{Cont}(c) \rightarrow \text{Prop}$. Its specification can be reconstructed from historical evidence under the conditional architecture of Section 6. Even when the modeled family is nonempty, it need not enumerate every historically realizable future. Moreover, lack of a witness of admissibility need not establish impossibility: it can indicate unverified conditions or limits of the chosen formal language.

7.7 Alternative Continuations and Historical Compatibility

Multiple extensions of one history can be compared in a category of marked continuations. For a fixed historical complex K , objects are embeddings $i : K \hookrightarrow L$, and a morphism from (L, i) to (M, j) is a selected cellular map $h : L \rightarrow M$ satisfying $h \circ i = j$. This is an undercategory construction, restricted to the representations and morphisms admitted by the study [16]. If all continuation arrows are required to be embeddings, composition preserves the structural preservation property. Model-specific admissibility still requires an independent closure condition.

A simple compositional statement clarifies the point.

Proposition 7.2 (Conditional composition of admitted continuations). *Let $K \xrightarrow{i} L \xrightarrow{j} M$ be history-preserving cellular embeddings. Assume an admissibility discipline that supplies a composition operation taking a witness for (K, i, L) and a compatible witness for (L, j, M) to a witness for $(K, j \circ i, M)$. Under this assumption, the composite determines an admitted continuation of K .*

Proof. The composite $j \circ i$ is a cellular embedding onto a subcomplex because the two stipulated embeddings preserve their cellular structures. The assumed composition operation on admissibility witnesses provides the required witness for the composite. The dependent pair $(M, j \circ i, w)$ therefore belongs to the selected continuation type. $\square$

The force of Proposition 7.2 lies in its separation of structural and epistemic assumptions. The cellular portion composes by the usual mathematical rules. Admission does so only when the modeled evidential, causal, and interpretation constraints have been constructed to remain coherent across stages. For example, two individually plausible prospective interventions may rely on mutually inconsistent resource assumptions, so an independent compatibility witness may be necessary. Composition cannot create such a witness automatically.

Branching provides another comparison. Two admitted continuations $i_A : K \hookrightarrow L_A$ and $i_B : K \hookrightarrow L_B$ may differ in cells, attachments, labels, or inferred dependencies. An amalgamation M with compatible maps from L_A and L_B is possible as a formal gluing under selected conditions, but an amalgamation need not satisfy the chosen admissibility predicate. A mathematical colimit therefore provides a candidate object, not a proof that both branches can be realized together. If prospective events are subject to conflict or exclusive resource use, such constraints require their own representation.

Epistemic revision creates a different kind of comparison problem. When a context change $c \rightsquigarrow d$ revises the represented historical base K_c , an extension over K_c may no longer have a canonical image over K_d . Section 6.7 discussed conditional pushout transport when a suitable cellular revision map exists. For corrections that lack such a map, comparison may require correspondences, spans, or partial translation relations. A full indexed category of continuation types requires these transport operations and their coherence conditions; it is proposed as a subsequent research problem rather than already established by Equation 39.

7.8 Continuation Semantics and Representational Tradeoffs

The preceding constructions support an explicit evaluation of the paper's representational design. The dependent type indexes candidate extensions by a historically situated epistemic context. The marked

embedding retains the historical reference and its attachment relations. A future-only presentation can be economical for certain computations, while a gluing presentation can preserve the same information when interfaces are recorded. Relative homology and topological reduction then offer particular derived summaries, each with stated information-loss conditions. The choice of whole-spacetime semantics is therefore motivated by the research objective of preserving historical constitution and provenance in the analysis of possible futures, not by a claim that all other mathematical presentations are defective.

This design has costs. Whole-spacetime complexes can grow as modeled histories accumulate, and direct comparison can become cumbersome when observers use incompatible labels or different reconstructed histories. Computational analysis may favor compressed or relative representations. Moreover, preservation of the historical cellular base does not ensure semantic adequacy, actual feasibility, or ethical acceptability. Those properties depend on evidence, interpretation, domain constraints, and, where relevant, normative reasoning.

The main result of the section is thus an *epistemic specification*, rather than a predictive theorem. It identifies a type of candidate continuations and specifies which data must accompany an extension for historical comparisons to be meaningful. Proposition 7.1 demonstrates the information that a bare future quotient can lose; Proposition 7.2 states the additional closure requirement under which staged admission can compose. Together they prepare the subsequent category-theoretic analysis and the reconstruction of epistemic generative models. Future creation enters here as the investigation of historically indexed, model-dependent continuations whose recognition and evaluation remain contingent on revisable epistemic capacities.

8 Categorical Structures of Continuation

A typed candidate continuation identifies a represented history within a proposed extended two-complex. To compare such continuations, construct them in stages, and investigate their dependence on revised historical knowledge, one also needs a precise account of the arrows *between* extended structures. Appendix D provides technical details on composition, pushout gluing, and context-sensitive transport. This section supplies a categorical organization of that account. It first specifies the categories in which history-preserving arrows are interpreted, then develops the undercategory of a fixed history and the composition of successive extensions. Cellular pushouts provide an explicit construction for gluing alternatives and for transporting an extension along a suitable change of historical base. The final subsections examine context-indexed continuation families, the limitations of revision transport, and generalized correspondences for more substantial representational change. The method is constructive: established category-theoretic results are applied under stated cellular and epistemic hypotheses, while the historical and normative meanings of the constructions remain matters of interpretation.

8.1 Categories of Epistemic Relational Complexes

Let $\mathbf{C}_{\text{cell}}$ denote a chosen category whose objects are finite, interpreted two-dimensional CW complexes, equipped where required with event, coupling, primitive, and provenance annotations. For present purposes, an arrow in $\mathbf{C}_{\text{cell}}$ is a cellular map accompanied by any compatibility information required by the selected annotation policy. Such maps can identify cells or alter the way cells are represented, depending on the chosen encoding. A smaller category $\mathbf{C}_{\text{emb}}$ has the same objects and selected cellular embeddings onto subcomplexes as arrows. In particular, an arrow in $\mathbf{C}_{\text{emb}}$ preserves the designated historical cell incidence and the labels declared rigid under continuation.

The use of a category requires closure under composition and identities. We therefore suppose that the stipulated preservation conditions admit identity arrows and are stable under composition. This assumption is straightforward for a fixed family of strict cell-label-preserving embeddings, but should not be silently transferred to every possible social or epistemic interpretation. Some kinds of semantic equivalence may call for relational correspondences, partial maps, or arrows carrying explicit witnesses

of reinterpretation. The finite two-dimensional scope is likewise a modeling restriction; the physical and social phenomena under study need not be exhausted by two-dimensional structures.

The distinction between $\mathbf{C}_{\text{cell}}$ and $\mathbf{C}_{\text{emb}}$ will be used throughout. Embeddings are the preferred arrows for the *continuation of a fixed represented history*; more general cellular maps may be needed to express *revision of the historical representation*. Conventional category-theoretic notions of undercategories, universal properties, and colimits provide the mathematical background [16]. Their availability in a particular interpreted category depends on whether its label and evidence policies are closed under the relevant constructions.

There is a second distinction inside the objects themselves. A two-complex supplies cellular incidence; separately modeled relations may encode causal precedence, evidential provenance, or the capacities of historically constituted observers. A cellular map preserves only the structures imposed in its definition. For example, a morphism preserving oriented boundary matrices need not respect a separately supplied event order. It may therefore be useful to refine the object type to an annotated record $(K, \lambda, \prec, \pi)$, with interpretations for the labels λ , a proposed partial order $\prec$, and provenance data π . The present category is intentionally parameterized by that choice rather than declaring one universal semantics for a two-complex.

8.2 Undercategories of Historical Extensions

Fix a represented historical complex K and its selected interpretation policy. The undercategory $K \downarrow \mathbf{C}_{\text{emb}}$ has as objects the history-preserving embeddings $i : K \hookrightarrow L$. A morphism from $i : K \hookrightarrow L$ to $j : K \hookrightarrow M$ is an embedding $h : L \hookrightarrow M$ satisfying $h \circ i = j$. Equation 41 records this historical compatibility condition.

$$\text{Hom}_{K \downarrow \mathbf{C}_{\text{emb}}}(i, j) = \{h \in \text{Emb}_2(L, M) \mid h \circ i = j\}. \quad (41)$$

This commutativity requirement ensures that the two routes for identifying a historical cell in M agree. An embedding $h : L \hookrightarrow M$ that changes the identity of a marked cell of K may be a meaningful map between unmarked complexes, but it is excluded from this undercategory. The historical base acts as an explicit constraint on comparison.

The identity embedding $\text{id}_K : K \hookrightarrow K$ is an initial object of the undercategory: for every extension $i : K \hookrightarrow L$ there is exactly one arrow from id_K to i , namely i itself. Thus the category records a common origin without asserting that all its other objects can be linearly ordered. In particular, alternative continuations may be incomparable because there is no history-preserving embedding from one candidate into the other. The resulting organization is richer than a catalogue of endpoint homology groups and more specific than treating candidate futures as an unstructured collection.

The dependent continuation type introduced in Equation 39 includes an admissibility witness in addition to the extension arrow. For a fixed epistemic context c , one may choose an *admitted extension category* $\mathbf{A}_c$ whose objects are extensions with such witnesses and whose arrows are those undercategory morphisms satisfying the selected witness-compatibility policy. When the objects are viewed as an ordinary full subcategory on admitted extensions, categorical identity and composition are inherited. The stronger demand that *successive new steps* each be admitted and that their composite admit an appropriate witness is a separate closure assumption. It is the latter demand, treated in Proposition 7.2, that carries substantial epistemic content.

An example illustrates the distinction. Let L_A and L_B extend the same K but attach new faces along different marked historical cycles. Even if $H_*(L_A)$ and $H_*(L_B)$ are abstractly isomorphic, a comparison that commutes with both historical embeddings may fail to exist, as in Proposition 7.1. An undercategory therefore preserves a specified relation to history that ordinary isomorphism classes of codomains would discard.

8.3 Composition of Successive Continuations

Consider two stages of represented continuation, $K \xrightarrow{i} L$ and $L \xrightarrow{j} M$. The ordinary composition law in $\mathbf{C}_{\text{emb}}$ produces a composite extension of K into M . In the undercategory based at K , the first stage i is an object and the second-stage embedding j is an arrow from i to $j \circ i$. For three stages, associativity makes the two possible parenthesizations agree. The structural statement is expressed in Equation 42.

$$K \xrightarrow{i} L \xrightarrow{j} M \xrightarrow{k} N, \quad (k \circ j) \circ i = k \circ (j \circ i). \quad (42)$$

For strict cellular embeddings the resulting arrow preserves the historical subcomplex and the incidence and labels required by the morphism definition. This is an elementary categorical property, rather than a theory of the physical or institutional processes producing the represented stages.

To express stagewise epistemic admissibility, let $\text{Adm}_c(i)$ be a predicate on an extension relative to context c . A stagewise continuation discipline may require a witness-composition operation, defined for compatible stages, of the form shown in Equation 43. The superscript step distinguishes a predicate concerning a successive step from admissibility relative to the initial context.

$$\mu_{c,d} : \text{Adm}_c(i) \times \text{Adm}_d^{\text{step}}(j \mid i) \times \text{Compat}_{c,d}(i, j) \longrightarrow \text{Adm}_c(j \circ i), \quad (43)$$

where d records the relevant evidence, interpretation, and resource assumptions at the intermediate stage. Equation 43 is a requested *additional operation*, not one supplied by the categorical composition of embeddings. Its definition must explain how assumptions used in the first and second stages are jointly validated. The type $\text{Compat}_{c,d}(i, j)$ requires a witness of compatibility; incompatible stages supply no such witness in this construction.

For example, two projected interventions might separately satisfy constraints that presume exclusive access to the same resource. Their cell attachments compose syntactically, but the resource assumptions may not. Likewise, a later intervention can invalidate the earlier model's interpretation of a historical coupling, even when the incidence-preserving composite is mathematically well formed. A representational ordering can therefore support conditional analyses of candidate sequences while leaving feasibility, implementation, and normative permissibility to independently justified criteria.

This framework also preserves the distinction between an ordered *sequence of model-construction stages* and a causal partial order on represented events. The category organizes embeddings between structures; an event order requires additional data and may contain concurrent or incomparable events. A composite extension does not, by itself, determine a total chronology or the justice of possible intervention orders. These are interfaces to the later analysis rather than categorical consequences.

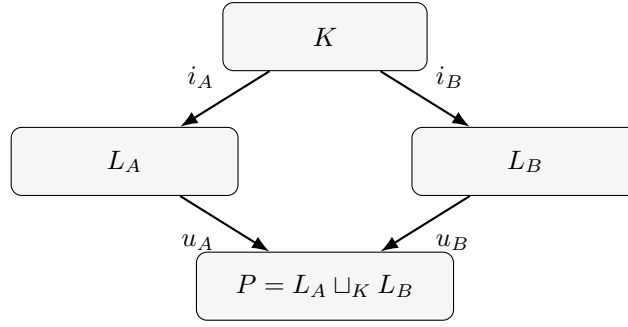
8.4 Cellular Attachments and Pushout Constructions

Cellular pushouts give a precise way to attach candidate structure along a specified historical interface. Suppose $i : K \hookrightarrow L$ represents a historical complex together with one continuation, and $r : K \rightarrow K'$ is a map from that same history into a second structure. Whenever the pushout exists in the selected cellular category, it supplies a space P and a commutative square with the usual universal property. Equation 44 records the construction.

$$P = L \sqcup_K K', \quad P \simeq (L \sqcup K') / \{i(x) \sim r(x) : x \in K\}. \quad (44)$$

Here $\simeq$ refers to the corresponding gluing realization in an appropriate category of spaces. For cellular attachments along suitable subcomplex inclusions, the resulting structure is again a cellular complex; an arbitrary interpreted category, particularly one carrying rigid or inconsistent semantic labels, need not admit every such colimit [12, 16].

One use of a pushout is to combine *candidate branches*. Let $i_A : K \hookrightarrow L_A$ and $i_B : K \hookrightarrow L_B$ be two extensions of the same marked history. Their pushout identifies the two historical copies of K while retaining the two proposed attachments. Figure 5 represents the corresponding commuting square. The



The diagram commutes: $u_A \circ i_A = u_B \circ i_B$.
 Mathematical amalgamation is distinct from
 joint empirical or normative admissibility.

Figure 5: Pushout of two candidate cellular continuations along a common marked historical complex. The universal gluing identifies the historical interface but does not validate coexistence of the modeled branches.

resulting universal construction is a mathematically defined amalgamation of representations. It supplies no independent evidence that the two branches can occur together.

A finite example illustrates an effect that is readily overlooked when combining continuations. Take K to be a circle represented by one vertex and one loop-edge. Let L_A and L_B each be obtained by attaching one two-cell with degree-one attaching map along that circle. Each branch is a disk and therefore has trivial positive-dimensional homology. Their pushout is two disks glued along their boundary, topologically a two-sphere. With consistent orientations, its degree-two cellular boundary map has matrix $\begin{pmatrix} 1 & -1 \end{pmatrix}$. Equation 45 exhibits the resulting homology.

$$H_2(L_A; \mathbb{Z}) = H_2(L_B; \mathbb{Z}) = 0, \quad H_2(L_A \sqcup_K L_B; \mathbb{Z}) \cong \mathbb{Z}. \quad (45)$$

This computation follows from $\ker \partial_2 = \langle (1, 1) \rangle \cong \mathbb{Z}$ and the absence of three-cells. It shows that the gluing of two separately contractible extensions can create new second homology. In a relational interpretation the two attached faces remain higher-order primitives, with their semantic labels to be assigned by an explicit interpretation scheme. A physical, institutional, or ethical reading of the two-sphere does not follow from the chain calculation.

The example also cautions against treating a category-theoretic colimit as a generative model. A pushout describes how mathematical data *could be assembled under a specified identification*. It does not select which prospective cells are historically feasible or supply a rule that generates new events. The epistemic reconstruction and selection of possible continuations are addressed separately in Section 9.

8.5 Epistemic Contexts and Indexed Continuation Families

Section 6 models the observer's represented historical spacetime together with evidence, provenance, and historically constituted epistemic capacities. Let an epistemic context be $c = (K_c, D_c, p_c, o_c)$ in the notation of Section 7.1. The corresponding continuation category $\mathbf{A}_c$ contains the admitted marked extensions of K_c relative to those evidential and interpretation conditions. The family $\{\mathbf{A}_c\}_c$ therefore depends on the context, rather than on cellular topology alone.

A possible organization is to choose a category $\mathbf{Ctx}$ whose objects are epistemic contexts and whose arrows represent declared changes of evidence, interpretation, observational capacities, or historical reconstruction. Such arrows require explicit structure. In particular, it is insufficient to equate a context change with an arbitrary cellular map between the historical complexes: one also needs compatibility information for revised claims, provenance, and the admissibility policies that depend on them.

A strictly defined covariant indexing would supply a functor $\mathbf{A} : \mathbf{Ctx} \rightarrow \mathbf{Cat}$ assigning each context its continuation category and each context arrow a continuation-transport functor. For the provisional

study we consider the weaker, conditional form in Equation 46, in which transport may be specified only for a selected subcategory $\mathbf{A}_c^{\text{tr}}$ of continuations whose attaching data and admissibility witnesses can be translated.

$$T_\rho : \mathbf{A}_c^{\text{tr}} \longrightarrow \mathbf{A}_d, \quad \rho : c \rightsquigarrow d. \quad (46)$$

There need not be a canonical T_ρ , and its domain may be empty. The resulting problem is not solved by the existence of dependent types: the mathematical theory must specify which changes of epistemic context can carry which families of future possibilities.

For comparison, when coherent transport functors exist for all arrows, their identity and composition laws could support an indexed categorical formulation and, under familiar categorical constructions, an associated opfibration. A contravariant reindexing of continuation data would lead to a different variance convention. These are established mathematical options, but the evidence-based transport law required for this particular application remains a research task [16]. We therefore do not assert a fibration or opfibration for the present epistemic setting.

The indexed perspective has an important conceptual consequence. Even when two observers share the same underlying cellular reconstruction K , their continuation categories may differ because their evidence or interpretive capacities differ. Conversely, a revision may change the historical base while preserving selected prospective analyses through an explicitly justified transport operation. Both forms of variation concern epistemic access; neither identifies the modeled continuation family with the complete set of futures actually possible in reality.

8.6 Revision Morphisms and Continuation Transport

A particularly tractable case of context change has a specified cellular map $r : K_c \rightarrow K_d$ between historical reconstructions. Let $i : K_c \hookrightarrow L$ be a candidate continuation in the earlier context. If the required pushout exists, one may construct $P = L \sqcup_{K_c} K_d$. This produces a structural candidate extension $\bar{i} : K_d \rightarrow P$ by cobase change. The commutative square is given in Equation 47.

$$\begin{array}{ccc} K_c & \xrightarrow{i} & L \\ \downarrow r & & \downarrow \bar{r} \\ K_d & \xrightarrow{\bar{i}} & P. \end{array} \quad (47)$$

For a cellular inclusion i and compatible finite cellular attachments, P can be constructed by attaching the cells of $L \setminus K_c$ to K_d along the transported attaching maps. The map $\bar{i}$ then identifies K_d with a subcomplex under the usual CW attachment assumptions. A map r that re-identifies earlier cells can nevertheless destroy the original historical identification carried by $L \rightarrow P$ or violate the chosen semantic-label requirements. The structural square therefore needs separate epistemic validation.

Under stronger assumptions, cobase change is functorial. Suppose the relevant pushouts exist and have coherent chosen representatives in a category $\mathbf{C}$. For a fixed $r : K_c \rightarrow K_d$, an arrow of the undercategory $K_c \downarrow \mathbf{C}$ induces an arrow between the corresponding pushouts, yielding a functor as in Equation 48.

$$r_! : K_c \downarrow \mathbf{C} \longrightarrow K_d \downarrow \mathbf{C}, \quad [i : K_c \rightarrow L] \longmapsto [\bar{i} : K_d \rightarrow L \sqcup_{K_c} K_d]. \quad (48)$$

Proposition 8.1 (Structural transport by cobase change). *Assume that the indicated pushouts exist in a category $\mathbf{C}$ and that a representative pushout is chosen for each object of $K_c \downarrow \mathbf{C}$. Then a map $r : K_c \rightarrow K_d$ induces the functor $r_!$ of Equation 48. When the ambient category and the arrows satisfy the stated cellular attachment hypotheses, its objects can be represented by candidate whole-spacetime extensions of K_d . Preservation of an epistemic admissibility predicate does not follow without additional witness-transport data.*

Proof. Given an arrow $h : L \rightarrow M$ over K_c , the universal property of the pushout produces a unique arrow $h' : L \sqcup_{K_c} K_d \rightarrow M \sqcup_{K_c} K_d$ that is compatible with h and the identity of K_d . The identity on L

Table 7: Morphisms and categorical constructions for epistemic continuation and historical revision.

Construction	Analytical role	Additional conditions
Marked embedding $K \hookrightarrow L$	Candidate whole-spacetime continuation	Historical cell and annotation preservation
Arrow over K	Comparison of two extensions of a fixed history	Commutation with historical embeddings
Pushout $L \sqcup_K L'$	Gluing candidate attachments or changing a cellular base	Compatible interface; joint admission assessed separately
Context transport T_ρ	Relating continuation families after epistemic revision	Evidence, interpretation, and witness compatibility
General maps or correspondences	Translation across non-embedding revisions	Declared treatment of identifications and lost or reconstructed information

induces the identity on its pushout, and uniqueness makes the arrow induced by a composite equal to the composite of the induced arrows. These assignments define a functor. In a cellular attachment setting the gluing realization provides the indicated extended objects. The pushout universal property makes no assertion about the truth of evidence claims or the existence of admissibility witnesses. $\square$

The proposition is a statement about categorical structure with explicitly stated hypotheses. It should not be read as a guarantee that arbitrary representational revisions have canonical translations. In particular, even if r is a cellular inclusion, the new historical evidence may contradict assumptions used to admit an earlier continuation. Transporting the underlying cellular extension does not transport its warrant.

A concrete example illustrates this difference. Let $K_c = S^1$ and let L be a disk attached to K_c along its boundary as a candidate future face. Suppose a revised historical representation K_d already includes a disk filling the same historical cycle, with $r : K_c \hookrightarrow K_d$ the boundary inclusion. The categorical pushout $L \sqcup_{S^1} K_d$ is a two-sphere: it retains the candidate future disk and the newly recognized historical disk as *distinct* two-cells. If the revision instead establishes that the supposed future face and the previously unrecognized historical face refer to the same occurrence, a further identification would be required and the resulting representation could be a disk. The appropriate interpretation is not fixed by the pushout alone. A comparison must specify whether the prospective and historical faces are distinct or co-referential, and whether the original continuation remains admissible after the revision.

8.7 Structural Transformations and Generalized Morphisms

History-preserving embeddings are useful when historical cells are to be retained with their designated identities. They are less expressive for revisions that collapse redundant distinctions, split an earlier identification, or translate between substantially different epistemic encodings. General cellular maps can express some such changes; spans, cospans, and partial correspondences can represent additional cases when no single total map is justified. The mathematical choice should be made according to which evidence and semantic information the comparison is intended to preserve.

Table 7 summarizes the distinct jobs of the constructions introduced in this section. It also identifies the kind of extra information needed for historical interpretation. These alternatives concern mathematical representations and have no automatic claim to ontological adequacy.

One prospective direction is a category of epistemic reconstructions equipped with provenance-sensitive correspondences and comparison morphisms among their continuation categories. Another is a higher-categorical treatment of translations and revisions when two routes of model adjustment agree only up to a specified comparison. Both would require exact definitions of the objects, arrows, and coherence conditions; their mathematical interest does not by itself establish a new theory of historical cognition.

The categorical analysis also supplies a methodological boundary for future normative work. Composition records structural stages; a pushout can express a shared extension; a reduction can identify formally equivalent configurations. None supplies a normative criterion for whose possibilities should

be preserved or how conflicts among interventions should be resolved. A theory of procedural justice may use the historically marked structures and their comparison maps as inputs, but it must articulate its ethical standards and epistemic responsibilities independently.

The results of this section organize the *space of representations of continuation*: the undercategory retains historical identification, composition organizes staged extensions, pushouts specify gluing and conditional revision transport, and indexed families make observer-dependent admissibility explicit. Proposition 8.1 establishes a limited functorial transport construction, while the disk-revision example shows why that construction needs interpretation and evidential review. Section 9 takes up a different problem: how a historically situated observer might reconstruct, assess, and revise epistemic models that select among these candidate continuations.

9 Epistemic Reconstruction of Generative Models

A history-preserving embedding specifies a candidate continuation of a represented relational spacetime, while the categorical constructions of Section 8 organize relations between such candidates. Neither construction determines which continuations a historically situated observer can recognize or has reason to admit. The present section studies that further operation: the epistemic reconstruction of *models of continuation* from evidence, interpretive conventions, and historically conditioned observing capacities. Its objective is to distinguish the construction of a model from the model's selection of candidate extended complexes, to specify the limits of identification from historical evidence, and to examine the revision of both models and their representational domains. The argument introduces provisional dependent types and relations, proves a limited non-identifiability result, and develops finite examples. Its subject is an epistemic practice, not a mechanism asserted to generate the underlying reality.

9.1 Historical Evidence and Model Constitution

Let $c = (K_c, D_c, p_c, o_c)$ be an epistemic context in the sense of Section 7.1. The component K_c is the currently represented historical two-complex, D_c the accessible evidence, p_c the adopted interpretation and provenance policy, and o_c the observer conditions described in Section 6. A candidate continuation $u = (L, i, w) : \text{Cont}(c)$ inherits this context through its marked historical embedding and its admissibility witness. The question of model constitution is how such a context permits a restricted, justified, or operationally useful family of continuations to be considered.

Historical evidence can have different logical relationships to represented cells. An event may be explicitly attested, an edge inferred from multiple reports, a face hypothesized from a recurrent structural relation, and an event ordering supplied by an independent archival chronology. Recording each distinction matters because the same cell complex may be associated with different degrees of evidential support. In particular, the orientation chosen for a cellular boundary and a separately argued direction of causation have distinct meanings. The represented complex K_c is therefore part of an epistemic record, rather than the whole evidence on which a continuation model ought to depend.

To make this dependence explicit, let $\text{Mod}(c)$ denote a context-indexed type of descriptions of candidate continuation models. The expression $\text{Mod}(c)$ is a modeling specification: defining its inhabitants requires a selected language of candidate event types, coupling proposals, attachment rules, evidential conditions, and prospective constraints. A model may be suitable for structural enumeration, qualitative feasibility analysis, statistical forecasting, or a combination of these. Its type does not establish its empirical adequacy. The records used to identify a model may themselves depend on prior observation and interpretation relations, such as the construction in Equation 32.

The epistemic context must be distinguished from the assumed ontological reference domain of Section 6. That reference is a convenience for stating conditional dependencies of observers and their capacities. Nothing in the following reconstruction requires it to be the ultimate or uniquely correct description of reality. The history modeled by K_c may omit events and couplings, and evidence may be erroneous

or incommensurably interpreted. This formulation also makes no claim that a particular time coordinate is ontologically fundamental: the index of a historical standpoint indicates a stage or boundary of accessible information.

9.2 Types of Epistemic Generative Models

There are several mathematically distinct meanings of a generative model. One can specify a predicate on possible continuations, a relation associating a history with several extensions, a finite enumeration procedure, a probability kernel, or a transition rule operating on an encoded complex. These alternatives can share candidate outputs without carrying the same computational or epistemological commitments. A probability distribution requires a measurable outcome domain and an interpretation of its weights; a deterministic procedure requires an algorithm and a convention for termination or bounded search. The preliminary formalism therefore takes a *continuation predicate* as its weakest common description.

For $m : \text{Mod}(c)$, suppose that its admitted candidates are selected by a proposition-valued family. This interface is written in Equation 49.

$$G_{m,c} : \text{Cont}(c) \longrightarrow \text{Prop}, \quad \Gamma_c(m) := \sum_{u : \text{Cont}(c)} G_{m,c}(u). \quad (49)$$

An inhabitant of $\Gamma_c(m)$ is a modeled candidate continuation accompanied by a witness that the model admits it. Admission under $G_{m,c}$ is a further restriction on the contextual conditions already carried by $\text{Cont}(c)$; it is not a declaration of physical realizability or ethical permissibility. When a model is too incomplete to supply proposition-valued judgments, an evidence-graded relation or a three-valued status with *undetermined* may be more appropriate. The use of Prop in Equation 49 defines one precise formal interface, not a universal language for every form of uncertainty [22].

A stochastic specialization may associate probabilities with specified measurable subsets of an extension domain. A sampling implementation may instead output finitely many candidates without defining a calibrated probability of historical realization. When interpreting such outputs, one should distinguish the model's sampling measure from a belief about real-world frequencies. Statistical predictive-state reconstruction supplies a precedent for connecting observed histories with generative predictive representations under explicit process assumptions [5, 18]. Its results do not by themselves establish a corresponding minimal predictive model for historically changing, heterogeneous two-complexes.

A second distinction separates a model of a *single extension* from a model of *successive revisions of an extension*. The first selects objects of the undercategory $K_c \downarrow \mathbf{C}_{\text{emb}}$. The second may additionally specify permissible arrows between these objects, witness composition, and constraints on changing event and face types. A static continuation predicate does not entail transition dynamics. Conversely, a transition system over already known encodings may have difficulty representing the historical emergence of categories that were absent from its initial modeling language.

Table 8 records these alternatives because the word *generative* can otherwise imply computational execution, ontological production, and epistemic prediction at once. The paper uses the term in its modest epistemic sense: a model organizes or produces *represented candidate continuations*. Whether the historical processes being studied admit a complete ontological generator remains an open question.

9.3 Reconstruction Operators and Model Admissibility

Model constitution also requires a description of how evidence and assumptions select models. The most general preliminary choice is a relation, since finite evidence may support several incompatible model descriptions and a reconstruction attempt may be inconclusive. Fixing a context type ECtx , define the possible reconstructed models by the relation in Equation 50.

$$\mathcal{R}_{\text{rec}} \subseteq \sum_{c : \text{ECtx}} \text{Mod}(c), \quad \Phi(c) := \{m : \text{Mod}(c) \mid (c, m) \in \mathcal{R}_{\text{rec}}\}. \quad (50)$$

Table 8: Possible formal interfaces for epistemic models of spacetime continuation. Each interface has additional assumptions beyond the cellular representation.

Interface	Representation of candidate futures	Required qualifications
Predicate $G_{m,c}$	Admitted inhabitants of a dependent continuation type	Declared logical and evidential admission rules
Generative relation	Pairs of source contexts and proposed extensions	Domain of relation, nondeterminacy, missing outputs
Finite enumerator	Computable list or bounded search of extensions	Search bound, enumeration language, completeness claims
Stochastic kernel	Measure over specified prospective outcomes	Measurability, calibration, sampling interpretation
Stagewise transition rule	Paths of successive cellular extensions	State encoding, witness compatibility, changing rule sets

The use of $\mathcal{R}_{\text{rec}}$ here denotes an *epistemic reconstruction relation*, not the assumed ontological reference structure used elsewhere in the paper. The notation is intentionally distinct from the history-preserving cellular morphism $i : K_c \hookrightarrow L$. In particular, Φ operates on evidence-bearing epistemic contexts, whereas i identifies a represented historical subcomplex within a proposed whole-spacetime extension. No categorical preservation property follows for Φ without an additional definition of morphisms between contexts and models.

For a particular reconstruction discipline, an evidential fit judgment may be expressed by $\text{Fit}_c(m, D_c)$. A separate interpretive-policy condition $\text{Int}_c(m, p_c)$ can specify which model descriptions are meaningful under the selected vocabulary. These conditions provide one transparent version of the relation, as expressed in Equation 51.

$$\Phi(c) = \{m : \text{Mod}(c) \mid \text{Fit}_c(m, D_c) \wedge \text{Int}_c(m, p_c)\}. \quad (51)$$

Equation 51 is a specification schema. It becomes empirically informative only after defining evidence, model predictions, interpretation policy, and a standard of fit appropriate to a particular field. Different standards may yield different admitted models. A tolerance threshold could be used for noisy measurements; a provenance consistency check may be more appropriate for documented historical assertions. A score or likelihood may help compare models without selecting a single preferred one. In none of these cases does formal satisfaction alone prove that an admitted model correctly describes future reality.

The order of the operations matters analytically. A candidate m must be meaningful in $\text{Mod}(c)$ before a predicate $G_{m,c}$ can select its possible continuations. Thus the two-stage relation comprises model reconstruction and model-dependent continuation. A condensed statement is given in Equation 52.

$$c \xrightarrow{\Phi} \Phi(c), \quad (m, c) \xrightarrow{\Gamma} \Gamma_c(m), \quad m \in \Phi(c). \quad (52)$$

The first arrow in Equation 52 is schematic set-valued notation, not a cellular embedding. A reconstructed model then supplies a candidate family of complete extensions, each with its designated historical inclusion. The construction allows several model families to coexist without amalgamating them into one supposedly exhaustive future. This is valuable when evidence supports plural interpretations or when distinct models preserve different historical constraints.

9.4 Historical Endogeneity of Reconstruction Operators

The reconstruction relation in Equation 50 is not construed as an operation of an abstract observer outside the represented history. Section 6 introduced the conditional type $\text{ObsCtx}(R)$ in Equation 30. An inhabitant $o = (S, j, a, \pi)$ includes a historical support S with a stipulated map into an assumed reference history R , a capacity profile a , and available interpretive procedures π . The model-reconstruction language, selection criteria, and computational methods may depend on these components.

One way to express this dependence without asserting a complete generative law is to index the reconstruction relation by observing conditions. Equation 53 displays the resulting family.

$$\Phi_{R,o}(c) := \{m : \text{Mod}(c) \mid \text{Recon}(R, o, c, m)\}, \quad o : \text{ObsCtx}(R). \quad (53)$$

The predicate $\text{Recon}(R, o, c, m)$ refers to the stipulated history-sensitive capacities and conditions under which a model is constructed or admitted. Neither its existence as a formal predicate nor an inhabitant of the observer-context type explains the historical mechanism by which the observing system acquired its abilities. Such an explanation may draw on learning theory, neuroscience, institutional history, technological infrastructures, or other domain-specific knowledge. The mathematical claim here is conditional indexing: changes in o can change the reconstruction relation and its candidate models.

The reference argument R in Equation 53 carries a restricted role. It denotes an *assumed ontological relational spacetime for formal convenience*, not ultimate reality, a uniquely correct domain, or a verified ontology. A reader could replace the reference representation with another posited structure and examine whether the epistemic construction remains coherent. Indeed, the act of selecting R is itself an epistemic modeling decision. Keeping that status explicit prevents a circular inference in which the mere success of a dependent-type construction would be taken to verify the underlying ontological assumption.

Endogeneity also introduces a potential reflexivity: the observing system may form a partial representation of the history supporting its own capacities. This does not require that the represented history encode the full operation of its reconstruction procedure, or that an observer solve a complete fixed-point problem about its own cognition. At a chosen level of analysis, observing practices can be recorded as historically situated events and higher-order relations, while the formal account of those practices remains partial. This is consistent with the literature emphasizing situated and historically formed conditions of knowledge [11, 21, 6]. It should be treated as a philosophical and formal proposal, not as a proof that any finite cellular complex can exhaustively represent cognition.

A related question concerns multiple observers. Contexts c_i and c_j may contain differently identified events, differently supported relationships, and distinct inference languages. Even when their underlying cell complexes are isomorphic, their admissible model types $\text{Mod}(c_i)$ and $\text{Mod}(c_j)$ need not be naturally identifiable. Comparison requires a translation or correspondence relating evidence, historical marks, and interpretive policies. The existence of a cellular isomorphism alone provides too little semantic information. This problem of historically conditioned translation will later inform the treatment of epistemic responsibility in Section 12 and the open problems of Section 14.

9.5 Nonuniqueness and Observational Equivalence

A finite reconstructed history can support several models that agree on the historical evidence and disagree on possible continuations. This is a concrete form of *model-relative observational underdetermination*. It is narrower than the thesis that every scientific theory has an empirically equivalent rival under all conceivable future evidence. The philosophy-of-science literature carefully distinguishes finite or transient evidential underdetermination, empirical equivalence, and presently unconceived alternatives [19]. Our construction makes a restricted finite case explicit.

Fix an epistemic context c and let $\text{Sig}_c(m)$ be an observation signature of model m relative to the recorded historical evidence. The signature might comprise predictions for observed occurrences, reconstructed boundary incidences, evidentially supported causal constraints, and other quantities declared visible under the current observation policy. Define the equivalence relation in Equation 54.

$$m \equiv_c^{\text{obs}} n \iff \text{Sig}_c(m) = \text{Sig}_c(n). \quad (54)$$

This equality is indexed by the selected signature and available observation conditions. It does not mean that the models are equivalent in every mathematical respect, nor that they would remain indistinguishable after future observations. Models can have identical signatures and different continuation predicates, attachments, or sets of conceivable prospective events.

Proposition 9.1 (Finite-history non-identifiability of continuation families). *Let c be an epistemic context. Suppose $m_A, m_B \in \Phi(c)$ have equal observation signatures and distinct admitted continuation families, in the sense that there exists $u : \text{Cont}(c)$ for which $G_{m_A, c}(u)$ and $G_{m_B, c}(u)$ have different truth values. Then the signature alone does not determine the admitted continuation predicate uniquely within the candidate model class.*

Proof. Assume the contrary: that the observational signature uniquely determines the admission predicate among the models under consideration. The equality $\text{Sig}_c(m_A) = \text{Sig}_c(m_B)$ would then imply agreement of their admission judgments for every $u : \text{Cont}(c)$. This contradicts the stipulated disagreement for the selected u . Hence uniqueness does not follow from the signature. $\square$

Proposition 9.1 is an elementary conditional non-identifiability result. Its substantive value depends on exhibiting concrete models satisfying its hypotheses. Such an example can be built using cellular attachments. Let $K \simeq S^1$ be a historically marked circular one-complex. Model m_A admits one future disk attached along a degree-one boundary map, while m_B admits a two-cell attached along a degree-two map. Suppose their observation signatures encode only the common recorded history K and omit a constraint fixing the degree of a prospective attachment. The models agree on the stipulated historical signature yet admit different extensions L_A and L_B . Their first homology groups are given in Equation 55.

$$H_1(L_A; \mathbb{Z}) = 0, \quad H_1(L_B; \mathbb{Z}) \cong \mathbb{Z}/2\mathbb{Z}. \quad (55)$$

For the degree- d attachment to the circle, the cellular chain in dimensions two and one contains a boundary homomorphism $\partial_2 : \mathbb{Z} \rightarrow \mathbb{Z}$ given by multiplication by d . The quotient $\mathbb{Z}/d\mathbb{Z}$ therefore yields Equation 55 when $d = 1$ and $d = 2$. The example recovers the attachment distinction considered in Section 5, now interpreted specifically as two models supported by the same limited historical signature. Their divergence is prospective and conditional; the example supplies no empirical basis for regarding either attachment as the future of an actual institution or community.

The observational signature can be refined to include more evidence and thereby distinguish the models. However, their initial equality does not establish permanent empirical equivalence, and a revised signature may favor one, both, or neither. More generally, a current candidate-model class is itself historically limited. The exclusion of a model from $\Phi(c)$ may reflect unavailable evidence, linguistic omission, computational constraints, or an adopted screening rule; it need not imply real-world impossibility. This is the epistemic boundary taken up in Section 9.8.

9.6 Conditional Continuations and Model-Dependent Possibilities

The output of epistemic model reconstruction is best understood as a family indexed by the admitted models, rather than a single unqualified collection of futures. For context c , Equation 56 defines this two-level family.

$$\mathcal{F}(c) := \sum_{m: \Phi(c)} \Gamma_c(m). \quad (56)$$

In this notation, $m : \Phi(c)$ means that m is accompanied by an appropriate membership or reconstruction witness; $\mathcal{F}(c)$ can contain multiple presentations of an extension under distinct model assumptions. Such duplication can be useful when the reasons for considering a continuation differ. Taking a union after erasing its model index would lose the provenance of selection, the assumptions supporting it, and the reasons for a later change in admission.

The family $\mathcal{F}(c)$ is an *epistemically modeled possibility family*. Its contents depend jointly on the historical record, the language of model descriptions, the reconstruction relation, and the continuation predicates. Even an exhaustive enumeration of $\mathcal{F}(c)$ within a finite formal language would establish only completeness relative to that language and those admission rules. A model may admit an extension that proves infeasible under later evidence, or omit a form of transformation that becomes conceivable only after a conceptual or technical advance.

The full-spacetime semantics remains consequential. Each $u : \Gamma_c(m)$ contains an embedding of K_c into a complete candidate complex L_u , retaining the historical attachment information required for comparison. Two models can propose identical newly added cells while attaching them along different historical maps. If their prospective parts are analyzed in isolation, a researcher may overlook their distinct effects on marked cycles, causal annotations, or institutional provenance. The structural difference is illustrated by the inclusion-induced kernels developed in Section 7. Thus model reconstruction and continuation semantics reinforce one another: the model supplies an interpreted reason for proposing an extension, while the extension retains the historically specific relations on which that reason may depend.

Partial ordering among possible interventions must be supplied through a separate structure. For a candidate L_u , a specified order $\prec_u$ may express causal dependence, evidential precedence, or an operational scheduling constraint, depending on its definition. The CW boundary maps do not induce a universally valid social or causal order. If two models agree on an extended complex but differ on admissible partial orders or available capacities, they may produce different predictions about sequential opportunities without differing in ordinary homology. This motivates keeping model-specific ordering and semantic annotations alongside the cellular objects. It also anticipates the separate normative inquiry into procedural justice, which cannot be settled by the topology of L_u alone.

9.7 Epistemic Updating and Generative Model Revision

An epistemic context can change through new evidence, reinterpretation of existing records, revised observing capacities, or an enlarged representational vocabulary. Let $\rho : c \rightsquigarrow d$ denote a specified revision between contexts. The relation ρ need not be a cellular embedding of the corresponding historical complexes, as established in Sections 6 and 8. Consequently, there need not be a canonical function carrying every model over c to one model over d . A general provisional treatment employs a revision correspondence, defined in Equation 57.

$$\mathcal{U}_\rho \subseteq \text{Mod}(c) \times \text{Mod}(d). \quad (57)$$

An element (m, n) means that, under the chosen revision interpretation, model n is considered a candidate revision or translation of m . The relation may be one-to-many, many-to-one, or partial. It can also be empty for a given m if the revised historical representation has no meaningfully corresponding model in the new language. A functorial transport operation of the kind discussed in Section 8.5 requires stronger conditions than Equation 57 supplies.

Model revision and model comparison are not identical. A newly observed event may falsify an asserted prediction under a fixed statistical interpretation. By contrast, a revised provenance scheme may make an earlier comparison ill-posed, even if the observed records themselves have not changed. A new computational instrument can introduce models that could not previously be expressed or evaluated. These are different forms of epistemic change, with different possible effects on the admitted continuation family.

It is tempting to regard increasing evidence as producing nested sets of possibilities. Such nesting is available only when the model language, history encoding, observational assumptions, and admissibility standard remain fixed and newly added evidence is imposed as further conjunctive constraints. In that restricted situation, Equation 58 states the elementary contraction property.

$$D_c \subseteq D_d, \quad \text{Fit}_d(m, D_d) \implies \text{Fit}_c(m, D_c) \implies \Phi(d) \subseteq \Phi(c), \quad (58)$$

where the final inclusion presupposes the same underlying model type and unchanged remaining admission conditions. Equation 58 is intentionally conditional; it does not apply when a revised context introduces new model descriptions or changes the evaluation rule. In particular, models in $\text{Mod}(d)$ may lack any predecessor in $\text{Mod}(c)$. In such cases, cardinality comparisons between $\Phi(d)$ and $\Phi(c)$ may not even be well-typed without a translation.

A small example illustrates both effects. Suppose an initial finite model class contains m_A and m_B , agreeing on historical observations but disagreeing about an unobserved attachment. Later evidence may

support the degree-one attachment, excluding m_B under a fixed fit rule. At another stage, an expanded language may introduce a model m_C with a new event or face type unavailable in the original language. The first change contracts the selected set within a fixed domain; the second alters the domain of descriptions itself. Neither is adequately described by a single monotone filtration of known candidate futures.

A practical revision record should retain the previous model description, revised historical evidence, translation or incompatibility witnesses, and the consequent changes in admissible continuations. This information may be needed to understand why a proposed future was accepted or rejected at an earlier stage, even if the later model is better supported. The record thereby links epistemic reconstruction to the provenance-sensitive infrastructure envisaged by the broader research program. The requirements concern accountable interpretation; they do not imply that a complete historical record is computationally or institutionally available.

9.8 Unrepresented Possibilities and Representational Boundaries

A final limitation concerns prospective structures outside an observer's present language of description. A type such as $\text{Cont}(c)$ is defined relative to the chosen universe EpCplx_2 , the admissibility rules, and any additional event or face vocabularies. Its inhabitants can express continuations available to that formal language. They do not, simply by forming a type, exhaust the realizable transformations of the world. Moreover, the complement of an enumerated set of candidates within a formal universe is different from a possibility that the observer's language does not yet represent at all.

Three analytically distinct boundaries are relevant. The first is *within-model uncertainty*: several continuations are admitted by a given m . The second is *between-model uncertainty*: different reconstructed models admit different candidate families. The third is *representational openness*: relevant alternatives may fall outside every model expressible in the present context. The first two can be explored mathematically once the formal universe and evidence policy are defined. The third can be acknowledged and investigated indirectly through historical model change, translation failure, or later conceptual expansion, but cannot be exhaustively enumerated by the same unexpanded language that excludes it.

This epistemic distinction has an immediate consequence for the meaning of an empty model output. If $\Gamma_c(m)$ is empty under a specified admission predicate, the correct formal conclusion is that no extension has been admitted by that predicate within its type. It does not follow that historical reality has no further possible transformations. Likewise, when all currently reconstructed models reject a proposed extension, the shared rejection may reflect common modeling assumptions or inaccessible evidence. Additional forms of evidence and independent perspectives may be required before inferring historical impossibility.

The prospect of unrepresented alternatives also raises a question of epistemic and normative responsibility. An institution can shape future opportunities partly through its criteria for recognizing conceivable plans, its representations of stakeholders' capacities, and its treatment of uncertainty. The resulting effects cannot be evaluated solely from the mathematical size or homology of a candidate continuation family. An ethical assessment must consider evidence, participation, social context, and the histories of the criteria used to include or exclude proposals. Work on epistemic injustice and responsible anticipation supplies relevant conceptual precedents [9, 10, 20]; the present formalism does not derive normative obligations from the existence of an epistemic type.

Figure 6 summarizes the analytic layers introduced in this section. The diagram is organized by epistemic dependence and is not intended as a depiction of an ontologically fundamental time direction.

The section has provided a minimal type-theoretic and relational interface for the epistemic reconstruction of continuation models. Equation 49 defines their modeled candidate outputs, Equation 50 describes the multiplicity of reconstructed models, and Equation 57 allows noncanonical relations between models following epistemic revision. Proposition 9.1 and its cellular example demonstrate a finite form of underdetermination, while the discussion of changing model types identifies an open limit of any

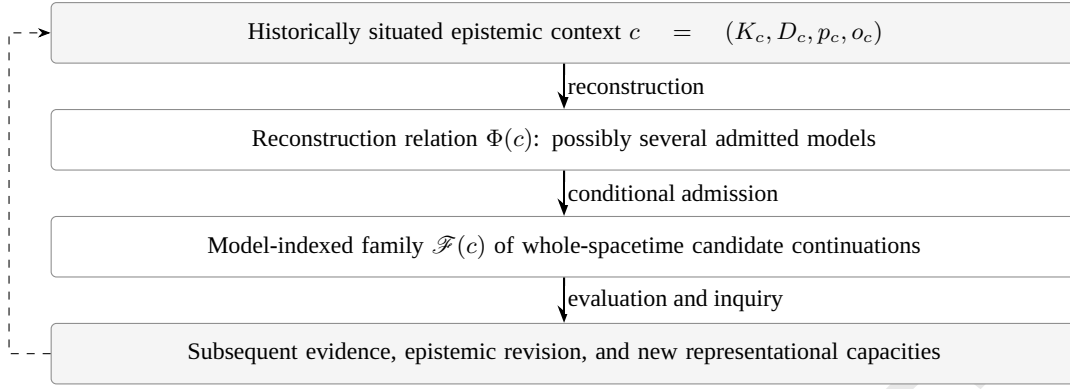


Figure 6: Epistemic reconstruction and revision as a layered research construction. The initial context, admitted models, and candidate whole-spacetime extensions have different types. The dashed return depicts possible revision of evidence, interpretation, or observing conditions; it is not a cellular continuation embedding.

presently specified continuation language. The further task, taken up in Section 10, is to analyze what the topological invariants of such candidate extensions can establish, and which historical, epistemic, and normative questions remain beyond their reach.

10 Topological Analysis of Future Continuations

Topological analysis of future creation concerns the structural implications of *candidate* extensions of historically represented relational spacetime. The subject of this section is a marked epistemic two-complex $i : K \hookrightarrow L$, where K is a selected historical representation, L is the whole candidate continuation, and i identifies the retained historical cells. Its objective is to determine which consequences of such an extension follow from cellular incidence and attachment, which data are preserved by homological and Morse-theoretic reductions, and which further questions require causal, evidential, or normative interpretation. The method combines relative cellular chain calculations, conditional propositions, counterexamples over fixed historical bases, and limited multistage constructions. The discussion builds on the continuation type of Section 7 and the epistemic reconstruction problem of Section 9. Appendix C gives the corresponding integral chain, exact-sequence, and attaching-map calculations. It neither assumes a complete generator of feasible futures nor assigns an ultimate ontological status to the represented spacetime.

10.1 Cellular Boundaries and Historical Attachments

Consider a finite CW pair (L, K) in which K is identified with a subcomplex of the candidate continuation L . In the paper's representational convention, vertices encode events, oriented edges encode couplings, and faces encode higher-order primitives with locally specified semantics. The cellular boundary ∂_2^L specifies the incidence of a face with one-dimensional cells; its coefficients depend on the chosen orientations and attaching maps. A cellular boundary coefficient is an algebraic datum, and should not be interpreted directly as a strength of influence, a legal relationship, or a causal probability.

After selecting cellular bases, the chain groups of the pair admit a decomposition as free abelian groups $C_q(L) \cong C_q(K) \oplus C_q^{\text{new}}$, where C_q^{new} is generated by the additional cells. This decomposition is a choice of basis adapted to the subcomplex; the boundary matrices record the essential attachment data. The adapted block form appears in Equation 59.

$$\partial_q^L = \begin{pmatrix} \partial_q^K & A_q \\ 0 & B_q \end{pmatrix}, \quad \partial_{q-1}^L \partial_q^L = 0, \quad (59)$$

where A_q records incidence of newly introduced q -cells with historical $(q - 1)$ -cells, while B_q records their incidence with newly introduced $(q - 1)$ -cells. In a dimension-two complex, the equation $\partial_1^L \partial_2^L = 0$ imposes compatibility among these blocks. The relative boundary operator is represented by B_q , because chains supported on K vanish in $C_q(L, K)$. Consequently, the relative chain groups alone can suppress information carried by A_q ; the specified pair and the connecting maps recover part of its algebraic effect.

The two-complex model permits prospective cells to attach directly to earlier event-coupling structures. For a new face f , the decomposition of its boundary into historical and new edges is informative only when those sets of edges and the embedding i are retained. A newly introduced face may have a boundary entirely in K , or it may be supported partly by new couplings and new event vertices. In the latter case, the set difference of cells of L and K need not constitute a subcomplex, as discussed in Section 7.4. The whole-spacetime presentation keeps the historical interface available without requiring it to be reconstructed from an unmarked future-only quotient.

This algebraic representation has a specific epistemic scope. Historical cells are those retained by the observer's current encoding; a real historical occurrence might have been omitted, misclassified, or represented by an unsuitable primitive. Prospective attaching maps are therefore *assumptions of a candidate model*. Their topological consequences can be calculated exactly even where the choice of map is uncertain or substantively contested.

10.2 Homological Transformations of Extended Spacetimes

An attachment can preserve historical cells while changing the homological status of cycles they support. This subsection establishes a finite-dimensional constraint for one restricted, analytically useful class of continuations: models which retain the entire one-skeleton of K and attach only finitely many new two-cells. The result supplies a calculable bound on topological transformations, rather than a bound on historical or social feasibility. The underlying method is the cellular long exact sequence of a pair [12].

Proposition 10.1 (Homological bounds for prospective two-cell attachments). *Let K be a finite connected CW two-complex, let $\mathbb{F}$ be a field, and let L be obtained from K by attaching r two-cells along specified loops $\alpha_1, \dots, \alpha_r : S^1 \rightarrow K^{(1)}$, with no additional cells. Write V for the span of the classes $[\alpha_1], \dots, [\alpha_r]$ in $H_1(K; \mathbb{F})$ and $d = \dim_{\mathbb{F}} V$. Then the inclusion $j : K \hookrightarrow L$ is a cellular continuation and the consequences in Equation 60 follow.*

$$\begin{aligned} H_1(L; \mathbb{F}) &\cong H_1(K; \mathbb{F})/V, & \ker j_{*,1} &= V, \\ \beta_1(L) &= \beta_1(K) - d, & \beta_2(L) &= \beta_2(K) + r - d, \\ \max\{0, \beta_1(K) - r\} &\leq \beta_1(L) \leq \beta_1(K). \end{aligned} \tag{60}$$

Proof. For the stated cell attachments, the relative cellular chain complex has $C_2(L, K; \mathbb{F}) \cong \mathbb{F}^r$ and $C_1(L, K; \mathbb{F}) = C_0(L, K; \mathbb{F}) = 0$. Thus $H_2(L, K; \mathbb{F}) \cong \mathbb{F}^r$ and the relevant portion of the long exact sequence is $0 \rightarrow H_2(K; \mathbb{F}) \rightarrow H_2(L; \mathbb{F}) \rightarrow \mathbb{F}^r \xrightarrow{\delta} H_1(K; \mathbb{F}) \xrightarrow{j_{*,1}} H_1(L; \mathbb{F}) \rightarrow 0$. The connecting map takes the standard generator corresponding to the k th new face to $[\alpha_k]$. Therefore $\text{im } \delta = V$ and $\text{rank } \delta = d$. Exactness gives the quotient and kernel identities. Rank-nullity yields the displayed Betti-number formulas, and $0 \leq d \leq \min\{r, \beta_1(K)\}$ gives the bounds. $\square$

Proposition 10.1 shows how the available attachment classes constrain first-homology changes within a specified family of extensions. For example, two prospective faces attached along independent historical one-cycles remove two independent first-homology classes; two faces attached along homologous cycles may create a new second-homology class while removing only one first-homology class. These implications depend on the chosen field of coefficients. Torsion information, including that arising from nonprimitive attaching degrees, requires integral homology or a more complete algebraic description.

The proposition should not be interpreted as a theory of institutional closure, irreversible consequences, or the number of future opportunities. Additional vertices or edges, changes in admissibility, and epistemic revision fall outside its hypotheses. Its value is to make a restricted structural claim

demonstrable while leaving the semantics of each proposed face and attachment available for separate scrutiny.

10.3 Relative Homology and Inclusion-Induced Maps

A continuation has two topological viewpoints: the extended space L and the historically marked pair (L, K) . Ordinary homology of L describes its cycles and boundaries, while relative homology and the inclusion-induced maps track how the extension interacts with the specified historical base. The difference is relevant when an inquiry distinguishes two previously recorded relational trajectories or primitives that share the same unmarked topological type.

The long exact sequence of the pair, recalled in Section 4.4, contains the segment in Equation 61.

$$H_2(L; \mathbb{F}) \longrightarrow H_2(L, K; \mathbb{F}) \xrightarrow{\delta} H_1(K; \mathbb{F}) \xrightarrow{j_{*,1}} H_1(L; \mathbb{F}) \longrightarrow H_1(L, K; \mathbb{F}). \quad (61)$$

Exactness gives $\text{im } \delta = \ker j_{*,1}$. This identity, together with the specified historical marking, distinguishes an inherited class becoming a boundary in L from an inherited cellular chain being removed from the record. The latter does not occur under the embedding hypothesis.

For comparison, define the *marked homological profile* of a continuation over K in degrees at most two by Equation 62.

$$\mathfrak{H}_{\leq 2}(L \mid K; \mathbb{F}) := \left(\{H_q(K), H_q(L), H_q(L, K)\}_{q=0}^2, \{j_{*,q}\}_{q=0}^2, \{\delta_q\}_{q=1}^2 \right)_{\mathbb{F}}. \quad (62)$$

The profile is a proposed record of established homological constructions, not a new homology theory. It requires the historical identifications and induced maps to be carried together with the groups; abstractly isomorphic entries may be related by maps that do not preserve historical labels. The connecting maps in Equation 62 are the usual homomorphisms $\delta_q : H_q(L, K) \rightarrow H_{q-1}(K)$. A full comparison can also retain the entire long exact sequence and its maps; the abbreviated profile is intended only as a manageable diagnostic for finite examples.

Relative homology does not capture the complete attaching-map homotopy class, let alone the observer's interpretation of it. Two attachments can agree on abelianized cellular boundaries yet differ in their fundamental-group effects. Nor does the marked profile encode causal precedence, representational bias, or the ethical status of a proposed event. Its use is to retain *selected* historical effects that can disappear when only $H_*(L)$ or the quotient L/K is reported.

10.4 Homological Equivalence and Historical Distinctions

Two extensions can share homology groups and still modify different historically identified structures. To display this distinction without relying only on a comparison of different first-Betti numbers, take a historical CW complex $K = S_a^1 \vee S_b^1$ with the two loops recorded as distinct. For each sign $\varepsilon \in \{+1, -1\}$, attach one new face to a loop whose abelianized attaching class is $[a] + \varepsilon[b]$ and denote the resulting continuation by L_ε . Both attaching classes are primitive in $H_1(K; \mathbb{Z}) \cong \mathbb{Z}^2$.

The homology groups of the two extended complexes are the same: $H_1(L_+; \mathbb{Z}) \cong H_1(L_-; \mathbb{Z}) \cong \mathbb{Z}$, $H_2(L_+; \mathbb{Z}) = H_2(L_-; \mathbb{Z}) = 0$, and $H_2(L_\varepsilon, K; \mathbb{Z}) \cong \mathbb{Z}$. Moreover, each unmarked quotient L_ε/K is homeomorphic to S^2 . Nevertheless, the marked inclusion kernels differ, as stated in Equation 63.

$$\ker(j_{+,*},1) = \langle [a] + [b] \rangle, \quad \ker(j_{-,*},1) = \langle [a] - [b] \rangle. \quad (63)$$

Equation 63 is evaluated relative to the *same identified historical generators*. The quotients and abstract groups record less than the marked cellular extensions. An isomorphism that changes the orientation or identity of a historical loop may identify the extensions as unmarked topological spaces, but is not an isomorphism over the stipulated identity of K .

This is a precise explanation of the paper’s preference for whole-spacetime continuation semantics. The pair (L, i) preserves the historical reference and attaching relationship directly. A future-only representation remains equally informative if it includes the appropriate attaching map or equivalent gluing data; retaining only the unmarked new face or quotient loses the marked distinction. The example establishes a *representational tradeoff*, without claiming that one syntactic presentation is universally preferable.

The historically different classes in Equation 63 acquire substantive meaning only after a , b , and the newly attached face have received independently justified semantics. For a prospective institutional transformation, for example, the two historical loops might be distinct networks of commitments, but the mathematics does not establish which commitments exist or how they should be evaluated. This interpretive restraint also prevents homological equivalence from being equated with equivalence in justice.

10.5 Topological Constraints in Continuation Models

The preceding calculations produce constraints *conditional on a model’s cell inventory and attachment rules*. Given a prescribed historical complex K and a family of admissible attaching maps $\mathcal{A}$, the candidate extensions form a restricted collection of CW pairs. Homological invariants can eliminate candidates that are inconsistent with a declared structural requirement. A claimed first-Betti-number reduction greater than the number of attached two-cells, for instance, is impossible within the hypotheses of Proposition 10.1.

Such structural restrictions remain distinct from feasibility. An attachment can be algebraically well-defined but have no feasible social, computational, or institutional interpretation. Likewise, failure to represent a future coupling in the chosen language does not prove that the coupling is historically impossible. The actual modeling process depends on data, observing capacities, and assumptions developed in Section 9. An admissibility witness therefore needs a recorded scope, identifying whether it certifies syntactic well-formedness, structural compatibility, an empirically estimated condition, or a separately justified normative condition.

Topological obstructions also require attention to the selected homomorphisms. If a proposal specifies that a certain historical cycle must remain nonbounding, then a candidate with that cycle in $\ker j_{*,1}$ fails this particular condition. A competing proposal could permit or require that attachment. The same cellular transformation can thus be admissible for one declared purpose and inadmissible for another. The topological calculation clarifies the consequence of the choice; the purpose itself is supplied by epistemic and, where appropriate, normative argument.

A limitation of two-dimensional cellular topology appears in the range of detectable invariants. For a two-complex, ordinary cellular homology vanishes in dimensions above two, but the complex may still carry rich information in its fundamental group and higher homotopy. Certain properties of higher-order relationships, continuous dynamics, and interactions among multiple levels of representation may demand additional cells, decorations, or independent structures. The choice of dimension is therefore a design restriction suited to the preliminary inquiry, not a general conclusion about reality’s dimensional constitution.

10.6 Discrete Morse Reduction of Continuation Complexes

Discrete Morse theory and elementary cellular collapses permit controlled reduction of finite cellular structures under specified hypotheses [8]. Their prospective use in future creation is epistemic as much as computational: smaller models can make candidate continuations easier to inspect, while the disappearance of cells may erase precisely those event identities that motivated the representation. The following example separates preservation of relative homotopy type from retention of event-specific information.

Let K be the historical edge ab together with its endpoints, and let L be the filled simplicial triangle with vertices a, b, c . In this interpretation, c represents a prospective event, while the edges ac and bc and the face abc are prospective relational cells. The face abc has the free edge ac , both outside K . Collapsing

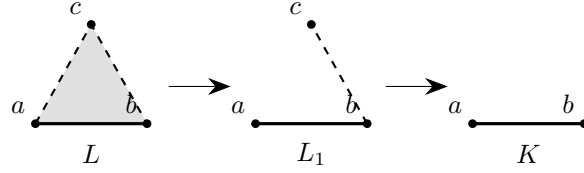


Figure 7: Two elementary collapses relative to a marked historical edge. The solid edge is the retained historical complex K ; dashed edges and the shaded face are prospective cells. The collapse removes a represented future event c while preserving the relative homotopy type of the pair.

this pair leaves the path $a-b-c$, in which c is a free vertex of the edge bc . A second elementary collapse of the pair (bc, c) leaves precisely K . Figure 7 depicts the reduction.

The collapses occur entirely outside K , so they give a deformation retraction of pairs. In particular, the relative homology statement in Equation 64 holds; both relative groups vanish in all degrees in this contractible example.

$$(L, K) \simeq (K, K), \quad H_q(L, K; \mathbb{Z}) = 0 \quad \text{for every } q. \quad (64)$$

The symbol $\simeq$ in Equation 64 denotes a homotopy equivalence of pairs. Despite that equivalence, the reduced complex does not contain the vertex c as a distinct recorded event. If the chosen epistemic task requires a trace of c , its prospective dependencies, or a separate interpretive account of the face, the unannotated reduction is inadequate for that task. Morse-critical cells are mathematically defined through a matching, and cannot be treated without further argument as uniquely consequential social events.

For an admissible semantics-aware reduction, one possibility is to require a chosen set of labeled event and provenance records P to remain recoverable under a pair of maps between the original and reduced representations. Another possibility is to retain an external audit record of collapsed cells and their incidences. These are different design responses: the former constrains which reductions can occur, while the latter permits reduction but supplements the reduced object with a reconstruction trace. Their normative adequacy is an open problem rather than a consequence of Forman's theorem.

10.7 Historical Information Preservation and Reduction Criteria

A practical reduction criterion for epistemic continuations must name the information the reduced representation is intended to support. The history-preserving embedding $i : K \hookrightarrow L$ already fixes the represented historical cell inventory. Beyond that inventory, provenance labels, prospective event identities, causal precedence, and interpretations of higher-order primitives may all be relevant to future recognition. The significance of each datum depends on the inquiry and cannot be inferred from ordinary homology.

Let $A(L, i)$ denote a specified collection of annotations over a continuation, and let $P(L, i, A)$ denote a selected set of queries whose answers the model is required to preserve. A reduction is then a structured correspondence $r : (L, i, A) \rightsquigarrow (M, k, A_M)$ equipped with an account of how queries are translated. A possible preservation requirement is stated in Equation 65.

$$\text{Ans}_L(p) = \text{Ans}_M(r_{\#}p) \quad \text{for each } p \in P(L, i, A) \text{ for which } r_{\#}p \text{ is defined.} \quad (65)$$

The equality in Equation 65 is a proposed operational test: the query class, answer equivalence, and translation $r_{\#}$ must be specified. If a query asks for the identity of the prospective event c from Figure 7, a reduction retaining only K has no answer unless the event's record is separately preserved. Homotopy equivalence provides neither the required query translation nor the corresponding provenance guarantee.

Table 9 describes candidate audit requirements, rather than asserting that they can always be met simultaneously. A reduction retaining every event and its full relational incidence may yield little computational simplification. Conversely, a substantial simplification can be appropriate for a question whose

Table 9: Selected preservation demands for reduced epistemic continuation models. Each demand has its own mathematical or interpretive warrant.

Information retained	Possible test	Limitation of ordinary homology
Historical cellular base	Fixed inclusion $K \hookrightarrow M$ and identified cells	Homology does not determine original cell identities
Relative topological effects	Induced maps on $H_q(L, K)$ and marked sequences	Group ranks omit attaching and marking distinctions
Prospective event identity	Explicit labeled-record recovery	A collapsible vertex may disappear
Causal or procedural order	Separate order-preserving comparison	Chain incidence supplies no social order
Provenance and interpretation	Traceable evidence and annotation translation	Interpretive warrants are external to topology
Normatively relevant distinctions	Declared decision-sensitive query comparison	Ethical value is not a homotopy invariant

only required outputs are Betti numbers. The relevant research task is to make the tradeoff explicit and reversible where the application permits.

These criteria have a prospective normative interface. A model used to evaluate future opportunities may have to retain distinctions concerning whose capacities persist, whose options disappear, and what consequences follow from particular sequences. Those quantities are not defined by cellular topology. Section 10 therefore provides structural tools for a later ethical inquiry; the normative standards themselves belong to the independent assessment planned in Section 12.

10.8 Multiscale and Nonmonotonic Structural Transformations

Future-continuation models can be explored at several representational resolutions and at several stages of conditional extension. An inclusion filtration is a suitable mathematical device when the same represented history is retained and candidate cells are added in a specified order. Persistent homology studies the resulting evolution of homological features and their lifetimes along a filtration [7]. Such a filtration is a *selected model of stages*; it does not settle whether physical time is fundamental or whether the modeled stages correspond to a uniquely realizable sequence.

Let K_0 consist of an edge ab and its endpoints. Add a new vertex c and edges ac and bc to obtain K_1 , whose one-skeleton is a triangle without a face. Attach a two-simplex along the triangle boundary to obtain K_2 . The filtration and first-Betti-number trajectory are given in Equation 66.

$$K_0 \hookrightarrow K_1 \hookrightarrow K_2, \quad (\beta_1(K_0), \beta_1(K_1), \beta_1(K_2)) = (0, 1, 0). \quad (66)$$

The first stage adds a one-cycle, and the next attaches a face that makes the cycle a boundary. Equation 66 demonstrates that increasing cellular inclusion need not yield monotone Betti numbers. In a persistence calculation the introduced first-homology class has a finite modeled lifetime; that lifetime alone conveys no verdict on the value or significance of the corresponding historical events.

Epistemic revisions require a different mathematical treatment when a later representation retracts or re-identifies cells. One possible diagram is $K_a \hookrightarrow M \hookleftarrow K_b$, in which two historical reconstructions embed into a common refinement. More general zigzag diagrams and their persistent-homology invariants are established mathematical tools [4]; their use here would require explicit choices of comparison maps, common refinements, and admissible epistemic translations. In particular, the existence of a common refinement must be established rather than presumed for arbitrary interpretive changes.

At multiple scales, a coarse representation and a fine representation may classify one occurrence differently, attribute a coupling to different supporting events, or represent a higher-order relation with distinct cell decompositions. A cellular subdivision can preserve topological type while greatly changing the number of recorded cells. A more substantive coarse-graining can alter the available semantic labels

or the very continuation types that can be formulated. Therefore, comparison across scales must retain a record of chosen maps and of the questions those maps preserve.

The analysis supports a limited synthesis. Relative homology and inclusion maps expose selected effects of new attachments on represented history; Morse reductions expose the distinction between topological equivalence and historical information retention; and filtration-based methods track modeled structural features across stages. Together they provide a tractable analytical vocabulary for candidate spacetime continuations. They do not produce a complete list of realizable futures, determine the observer's missing concepts, or justify a criterion of justice. The next stage of the manuscript can test these constructions on explicitly specified finite examples and compare their information losses and interpretive constraints.

11 Illustrative Models and Computational Experiments

The analytical object of this section is a collection of finite, *synthetic* epistemic two-complexes whose historical cells are explicitly marked and whose candidate continuations are specified by cellular attaching maps. The section's purpose is to examine, by reproducible calculation, the representational consequences of the continuation constructions developed in Sections 7–10. Its method is to fix small historical chain complexes, vary prospective attachments and observation assumptions, compute integral and relative homology, and compare the information retained by different descriptions and reductions. The examples address history dependence, alternative continuations, order, epistemic revision, reconstruction nonuniqueness, and history-relative Morse reduction. They illustrate mathematical mechanisms; they supply neither historical observations nor validated forecasts of future events.

11.1 Experimental Constructions and Verification Methods

An executable specification accompanies this section in `examples/verify_section11.py`. It uses exact integer matrices in SymPy, including Smith normal forms, and creates machine-readable results, a $\text{\LaTeX}$ table, and three vector diagrams. Every calculation is deterministic and independent of fitted data. The script checks that cellular boundaries compose to zero; computes the relevant free ranks and torsion invariants; and verifies selected relative chain complexes and staged continuations. These tests concern the stated finite objects. They do not constitute machine-checked proofs of the general dependent-type or categorical constructions developed earlier in the manuscript.

The primary test object is a historical *theta graph* K , consisting of two event vertices u, v and three distinct oriented coupling edges a, b, c , each directed from u to v for the purpose of cellular boundary calculation. The graph admits two independent integral cycles, which are assigned historical identifiers $\alpha = a - b$ and $\beta = b - c$. The incidence matrices and cycle-basis embedding are specified in Equation 67.

$$\partial_1^K = \begin{pmatrix} -1 & -1 & -1 \\ 1 & 1 & 1 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{pmatrix}, \quad \partial_1^K Z = 0. \quad (67)$$

The columns of Z form a $\mathbb{Z}$ -basis for $\ker \partial_1^K$, and $H_1(K; \mathbb{Z}) \cong \mathbb{Z}\langle\alpha\rangle \oplus \mathbb{Z}\langle\beta\rangle$. The graph has no historical two-cell. Using a theta graph rather than abstract free homology generators retains an explicit cellular realization: u and v denote distinct historical events, and a, b, c denote couplings. The physical or social interpretation of these couplings remains unspecified. Their chosen chain orientations are algebraic and do not themselves assert temporal or causal direction.

A candidate continuation L_D is obtained from K by adjoining r oriented two-cells. Their attaching cycles are encoded by an integer matrix $D \in \mathbb{Z}^{2 \times r}$ in the historical cycle basis. Equation 68 defines the full cellular boundary of those new faces.

$$\partial_2^{L_D} = ZD, \quad D = \begin{pmatrix} d_{11} & \cdots & d_{1r} \\ d_{21} & \cdots & d_{2r} \end{pmatrix}, \quad \partial_1^{L_D} \partial_2^{L_D} = 0. \quad (68)$$

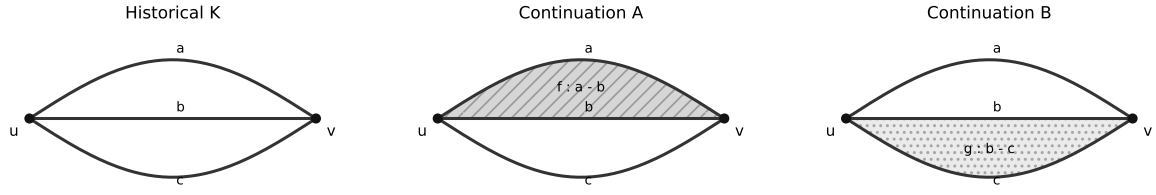


Figure 8: Common historically marked theta graph and alternative one-face continuations. In candidate A the new face has boundary $a - b$; in candidate B it has boundary $b - c$. The whole-spacetime presentations preserve the identity of the historical couplings and disclose different attachment maps.

Every integer column can be realized by a cellular attaching loop in the graph, although different loops with the same abelianized cycle can yield different fundamental groups and interpretive histories. The experiments compare cellular homology, not an exhaustive classification of attaching maps up to homotopy. The extension retains K as a specified subcomplex; this marked inclusion is part of the experimental data.

11.2 Two-Complex Spacetime Construction and Visualization

Figure 8 displays the historical theta graph and two continuations constructed by adjoining a single face. The first candidate fills the cycle $\alpha = a - b$; the second fills $\beta = b - c$. In each panel the historical vertices and couplings remain present and identified. Only the prospective face and its attaching cycle change. The shading indicates the newly represented higher-order primitive; no claim about its empirical significance follows from the visual convention.

This construction also illustrates a representational convention from Section 3. The model's vertices are events; there is no primitive actor located at u or v . A relational entity, if the application requires one, would be reconstructed from a suitably interpreted historical subcomplex. Likewise, the face denotes a higher-order primitive requiring a domain-specific interpretation. The drawing provides a test of incidence-preserving continuation; it does not determine which future primitive should be attached, whether the candidate is historically feasible, or whether it is desirable.

The finite graph is deliberately small enough to permit verification by hand. Larger numerical examples would be useful for complexity and scalability assessment, but they would not change the epistemological distinction between syntactically admitted and realizable futures. For the present purpose, exact integer calculations expose information that a purely graphical illustration might conceal, especially torsion and history-specific kernels.

11.3 Common Historical Complexes and Alternative Continuations

Six cases are considered. The baseline retains K without a new face. Candidates A and B attach one face along α and β , respectively. Candidate C attaches one face along 2α , allowing the attaching loop to traverse the identified cycle twice. Candidate D attaches two faces, one along each independent cycle. Candidate E attaches two distinct faces with the same boundary cycle α . The matrices are generated and checked by the accompanying script; Table 10 reports the resulting integral-homology invariants.

For candidates A and B, the first Betti number falls from two to one, but the eliminated historical class differs. Candidate C also has first Betti number one; its first integral homology additionally contains a nontrivial $\mathbb{Z}/2\mathbb{Z}$ torsion summand. Thus the abstract Betti numbers alone fail to record the entire cellular effect of the degree-two attachment. Candidate D eliminates both independent first-homology classes. Candidate E retains one first-homology class and produces a nontrivial second-homology class, because the two attached faces have linearly dependent boundaries. These outcomes follow from the stated attaching matrices, without imposing assumptions about causal efficacy or normative importance.

A useful diagnostic is the actual subgroup of historical homology killed by the inclusion. For candidate A, the kernel is $\langle[\alpha]\rangle$; for B, it is $\langle[\beta]\rangle$. Although both extended complexes have isomorphic ab-

Table 10: Exact homology calculations for six historically marked two-complexes with the same base graph K . The torsion column refers to the first integral homology group; D is given in the historical cycle basis (α, β) .

Continuation	β_1	β_2	Torsion in H_1	Attaching columns
History K	2	0	none	none
A: α	1	0	none	$(1, 0)^T$
B: β	1	0	none	$(0, 1)^T$
C: 2α	1	0	$\mathbb{Z}/2$	$(2, 0)^T$
D: α, β	0	0	none	$(1, 0)^T, (0, 1)^T$
E: α, α	1	1	none	$(1, 0)^T, (1, 0)^T$

stract first homology, their inclusion-induced maps from the *historically identified* $H_1(K)$ differ. This distinguishes the candidate branches over the marked base, a distinction relevant when prospective transformations are compared against inherited events and couplings.

11.4 Whole-Spacetime and Future-Only Representations

The same branches test the rationale for the paper’s preferred whole-spacetime continuation semantics. Since candidates A and B introduce no new vertices or edges and attach a single two-cell with its entire boundary in K , collapsing the retained historical subcomplex to a point produces the same quotient space, namely $D^2/S^1 \cong S^2$. The quotient descriptions are recorded in Equation 69.

$$L_A/K \cong S^2 \cong L_B/K, \quad \ker H_1(i_A) = \langle [\alpha] \rangle, \quad \ker H_1(i_B) = \langle [\beta] \rangle. \quad (69)$$

Equation 69 provides a particularly transparent test of representational information loss: two future-only quotients are homeomorphic, while the historically marked inclusion maps behave differently. The same quotient construction applied to candidate C produces S^2 as well, despite the torsion in its whole-spacetime first homology. Abstract future-only topological type therefore omits substantial attachment information in these examples.

This is a claim about *unmarked* quotients. A future-only formulation equipped with the historical attaching map, boundary interface, and appropriate identifiers can reconstruct the distinguished extension and preserve these differences. The comparison therefore supports a design preference rather than a mathematical exclusion of alternative encodings: the whole-spacetime presentation makes the common historical embedding an explicit primary datum, with the future-only quotient available as a derived construction. For a theory investigating how unrealized structures depend upon historically represented relations, that primary datum is especially convenient.

11.5 Cellular Attachments and Relative Homology Calculations

All six extensions permit exact computations using relative cellular chains. Because the experimental branches add only two-cells to the historical theta graph, $C_2(L_D, K) \cong \mathbb{Z}^r$ and the other positive-dimensional relative chain groups vanish. Hence $H_2(L_D, K; \mathbb{Z}) \cong \mathbb{Z}^r$ and $H_1(L_D, K; \mathbb{Z}) = 0$. The relevant part of the long exact sequence of the pair reduces to Equation 70.

$$0 \longrightarrow H_2(L_D; \mathbb{Z}) \longrightarrow \mathbb{Z}^r \xrightarrow{\delta_D} \mathbb{Z}^2 \longrightarrow H_1(L_D; \mathbb{Z}) \longrightarrow 0, \quad [\delta_D] = D. \quad (70)$$

The connecting map δ_D records the historical homology classes of the attached face boundaries. Accordingly, $H_2(L_D; \mathbb{Z}) \cong \ker D$ and $H_1(L_D; \mathbb{Z}) \cong \operatorname{coker} D$. These identifications follow from the declared theta-graph construction; they are not generic formulas for arbitrary cellular extensions with newly introduced vertices or edges.

The machine-readable results retain the rank of D , the Smith normal form diagonal entries, and the number of prospective faces. Candidate C has a nonunit Smith invariant 2; candidate E has two face

generators but only one independent attaching class. This exposes two distinct mechanisms that a Betti-number-only comparison would treat incompletely: a winding multiplicity in C and a relation among face boundaries in E . Since a face's semantics remains application-dependent, the calculations identify *formal possibilities of attachment*, rather than empirically meaningful winding or duplication in an actual social process [12].

11.6 Historical Dependencies and Multiple Admissible Orders

The next experiment distinguishes the structure of an extended complex from the sequence through which its represented cells are introduced. Beginning with K , attach the face along α and then the face along β , or reverse the order. Both sequences terminate in candidate D , and both exhibit the first-Betti-number sequence $(2, 1, 0)$. However, their intermediate inclusion-induced kernels differ: the first sequence initially kills $[\alpha]$, whereas the second initially kills $[\beta]$. Equal Betti-number trajectories therefore need not describe the same *marked* intermediate histories.

Causal or procedural constraints require separate data. Introduce three prospective events A, B, C at a distinct semantic layer, with $A \prec C$ and $B \prec C$ and no imposed precedence between A and B . The two linear extensions are stated in Equation 71.

$$\text{LinExt}(\{A \prec C, B \prec C\}) = \{(A, B, C), (B, A, C)\}. \quad (71)$$

Here A and B can be interpreted as two candidate interventions, while C is a subsequent event modeled as depending upon both. Equation 71 follows from the separately stipulated precedence relation, not from cellular boundary incidence. The demonstration shows how a topology-sensitive study of continuations can be supplemented with an order-sensitive representation without conflating algebraic orientation, causality, and normative priority [23, 13].

A later theory of procedural justice may distinguish the two admissible orders if their intermediate consequences affect participants' capacities or opportunities differently. Such consequences are absent from this synthetic example. Its result is consequently limited: the two modeled sequences differ in historically identified intermediate attachments, while their final cellular complex and unmarked Betti-number sequence agree.

11.7 Epistemic Revision and Continuation Reconfiguration

Epistemic revision can change what the same prospectively described attachment means topologically. Consider an initial reconstruction K consisting of the theta graph alone. A later representation K^* includes an additional *historical* face h attached along α . The earlier complex embeds in the revised one, but the status of the cycle $[\alpha]$ changes: it is nontrivial in $H_1(K)$ and trivial in $H_1(K^*)$. These are alternative epistemic descriptions of historical information. The experiment does not identify either description with ultimate reality.

Now introduce a *prospective* face f , also attached along the same marked cycle α , in both contexts. Figure 9 displays the two starting contexts and the corresponding extensions. The computed homology ranks are recorded in Equation 72.

$$\begin{aligned} (\beta_1(K), \beta_2(K)) &= (2, 0), & (\beta_1(K \cup_\alpha f), \beta_2(K \cup_\alpha f)) &= (1, 0), \\ (\beta_1(K^*), \beta_2(K^*)) &= (1, 0), & (\beta_1(K^* \cup_\alpha f), \beta_2(K^* \cup_\alpha f)) &= (1, 1). \end{aligned} \quad (72)$$

The lower-right case contains two distinct faces with the same boundary, producing a two-cycle. The contrast in Equation 72 shows that carrying forward a syntactically similar proposed attachment across a revised historical representation can alter its topological effect. It does not establish that the attachment remains epistemically admissible after the revision; the model's admissibility witnesses and semantic annotations would need to be reconsidered.

The example is particularly useful for the categorical discussion of Section 8. In this special inclusion case, attaching the same cell along a map into K can be compared using an ordinary pushout. More general revisions may retract a misidentified event, merge interpretations, or change the available cell types.

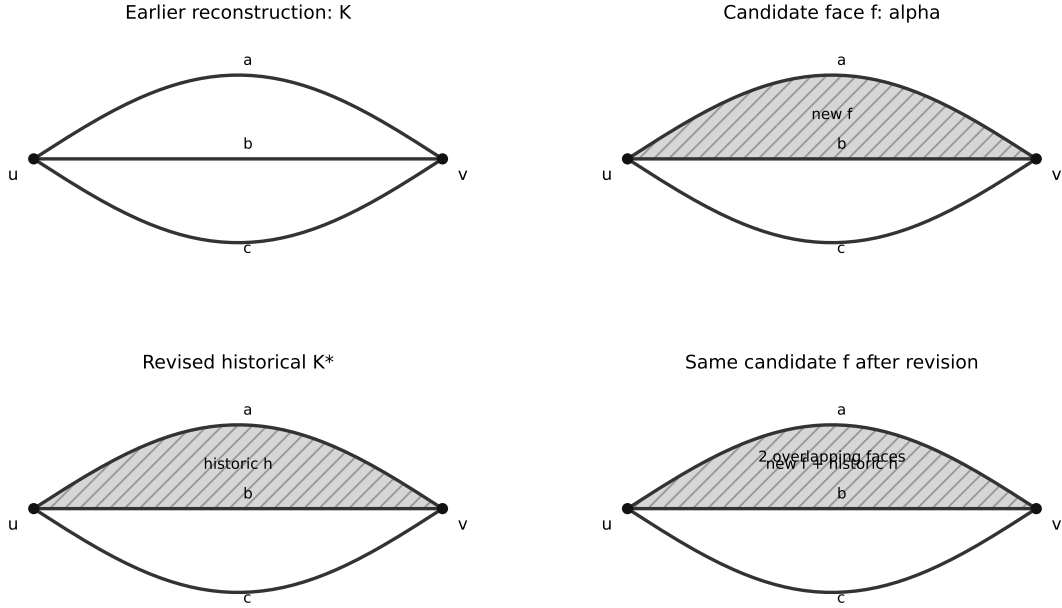


Figure 9: Revision-sensitive continuation of a marked theta graph. The lower-left historical reconstruction includes a previously unrepresented face h on α . Attaching a new face f along the same cycle eliminates a first-homology class in the earlier context but produces a second-homology class after revision.

They need not supply an embedding or a canonical transport of continuation families. Consequently, the figure demonstrates a limited, calculable revision effect rather than a general solution to the transport problem.

11.8 Nonunique Generative Reconstructions

The theta-graph experiments also provide a concrete witness to the reconstruction nonuniqueness described in Section 9. Let an observational signature report only the common historical complex K and its declared cell annotations. Consider two epistemic models m_A and m_B that reproduce this signature but whose continuation predicates select candidates A and B, respectively. The model relation and the disagreement are stated in Equation 73.

$$\text{Sig}_c(m_A) = \text{Sig}_c(m_B), \quad \Gamma_c(m_A) = \{(L_A, i_A)\}, \quad \Gamma_c(m_B) = \{(L_B, i_B)\}. \quad (73)$$

In Equation 73, braces denote toy singleton families of structural extension pairs; the admission witnesses are suppressed for readability. Both models are consistent with the *stipulated* signature, but they designate different prospective face attachments. Their future-only quotients agree in homeomorphism type, so even that derived invariant does not identify which historical class each model would modify. The difference becomes visible in the historically marked inclusion maps of Equation 69.

This is finite-history non-identifiability within an explicitly specified toy model class. It is not evidence that the corresponding models would remain observationally indistinguishable after further events, that either is physically realizable, or that the models exhaust other possible continuations. Furthermore, the experiment operates within a fixed language of two-dimensional cellular attachments. It cannot calculate possibilities that lie beyond the represented event types and admissibility vocabulary. This limitation belongs to the epistemic constitution of the model class, rather than to a missing column of the numerical attachment matrix [18, 19].

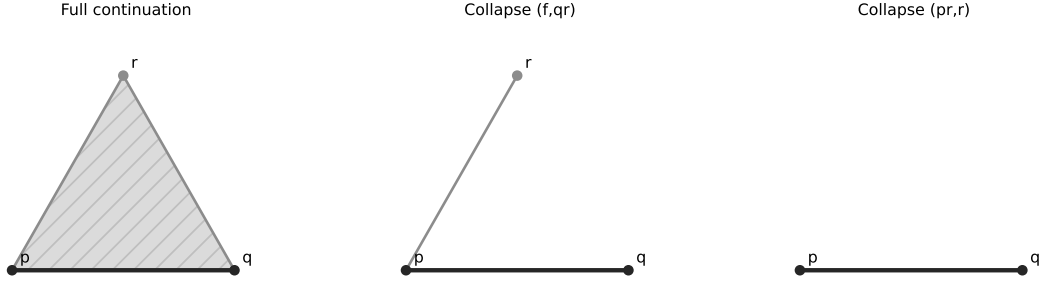


Figure 10: Two history-relative elementary collapses of a triangular continuation. The full continuation includes the prospective event r and its couplings, while the final reduced complex preserves the marked historical edge pq and its homotopy type but omits r as a cell.

11.9 Morse Reductions and Historical Information Preservation

A final finite experiment concerns the information retained by topological simplification. Let the historical complex K_0 be a single marked edge pq with its two endpoint events. A candidate continuation L introduces an event r , two couplings pr and qr , and a triangular face f attached along the cycle $pq + qr - pr$. As a topological space, L is a disk and K_0 is an interval. The inclusion is a homotopy equivalence, and the relative homology groups are trivial. Nevertheless, the continuation contains the explicitly recorded prospective event r .

Figure 10 illustrates an elementary cellular collapse of the new triangular face together with its free coupling qr , followed by a collapse of pr with its free endpoint r . The historical edge pq remains fixed throughout. The relative chain matrices used in the exact check are given in Equation 74.

$$C_2(L, K_0) \cong \mathbb{Z} \xrightarrow{\begin{pmatrix} -1 \\ 1 \end{pmatrix}} \mathbb{Z}^2 \xrightarrow{\begin{pmatrix} 1 & 1 \end{pmatrix}} \mathbb{Z} \longrightarrow 0. \quad (74)$$

The compositions vanish, both nonzero maps have rank one, and the homology of the relative chain complex in Equation 74 is zero in each degree. The reductions are therefore consistent with the homological calculation; the illustrated free pairs also establish a stronger elementary-collapse relationship. The construction is a small instance of cellular reduction of the kind studied by Forman [8].

For a query asking only whether the candidate continuation changes homotopy type, retaining K_0 suffices. For a query asking whether the proposed event r was included, which couplings involved it, or what provenance supports its representation, the reduced complex alone gives an incomplete answer. A historically adequate reduction must consequently be evaluated against a specified query and annotation policy, rather than solely against topological invariants. The experiment exhibits one explicit tradeoff; it establishes no universal criterion for which records a future-oriented decision system ought to preserve.

11.10 Comparative Results and Representational Limitations

Table 11 consolidates the analytic roles of the experiments. Their joint significance lies in comparing representations on the same small historical structures rather than in simulating empirical social or physical dynamics. The figures and exact calculations disclose several distinct information losses: an unmarked future-only quotient can forget historically identified attachments; Betti numbers can conceal torsion and different kernels; identical final complexes can conceal different intermediate orderings; and a topology-preserving reduction can remove a separately meaningful event record.

The verification scripts check exact identities and integer homology for the selected finite examples, including consistency of chain compositions. They do not implement a general theorem prover for dependent types, a general-purpose two-complex simulation engine, or an inference algorithm trained on observational histories. The model-reconstruction predicates remain explicitly stipulated; their historical

Table 11: Interpretive scope of the Section 11 synthetic computations. Each result is conditional on the specified cellular and epistemic encodings.

Experiment	Calculated or constructed distinction	Additional information still required
Alternative attachments	Different marked kernels and integral torsion	Meaning and feasibility of attaching faces
Relative pairs	Boundary maps record historical attachment classes	Evidence and reliability of represented history
Stagewise order	Equal final complex, different marked intermediates	Separate causal and procedural predicates
Epistemic revision	Identical prospective attachment, changed homological effect	Interpretation and admission across revision
Model plurality	Same historical signature, different continuation families	Evidence for selecting or comparing models
Relative collapse	Homotopy retained, prospective event identity removed	Query class, provenance, decision relevance

endogeneity, interpretive suitability, and empirical identification have yet to be established in a particular domain. The tests also do not infer probabilities of future realizations. A probabilistic interpretation would require its own sample space, evidence model, and calibration method.

Several limits are mathematically substantive. Integral homology can distinguish the torsion of candidate C , but it cannot identify all differences in fundamental-group structure or in cell-level provenance. The future quotient identifies each retained historical subcomplex to one point and may erase marked attachment relations. A filtration reflects a selected ordering of cells; it supplies no independent causal warrant. Revision is especially problematic when prior representations are corrected rather than merely enlarged. These limitations motivate the use of annotated continuation morphisms, context-dependent types, and explicit epistemic translation policies developed elsewhere in the paper.

The resulting research position is modest but concrete. Within a selected epistemic two-complex language, exact computation makes structural differences among candidate continuations inspectable and can test whether a proposed reduction preserves specified mathematical information. It does not determine whether a candidate continuation will occur or whether an intervention is just. Those additional questions require historically grounded evidence, interpretations of the represented higher-order primitives, causal and institutional constraints, and substantive normative reasoning. Section 12 can therefore use these examples as *interfaces* for investigating future creation and justice while keeping structural possibility, epistemic access, and ethical assessment distinct.

12 Normative Interfaces of Future Creation

This section identifies the normative information needed to evaluate decisions about prospective continuations of historically represented relational spacetime. Its objective is to connect the dependent continuation formalism of Section 7, the reconstruction problem of Section 9, and the computational comparisons of Section 11 to questions of ethical responsibility and justice. The method is conceptual analysis supported by explicitly stipulated predicates and a finite counterexample concerning procedural order. Established discussions of comparative justice, democratic participation, epistemic injustice, and responsible innovation provide interpretive resources [17, 24, 9, 20]. Their arguments are used in the limited respects stated below. The present paper proposes interfaces for a subsequent normative theory; it does not derive ethical obligations from cellular topology.

Let $c = (K, D, p, o)$ be an epistemic historical context as defined in Section 7.1, and let $\mathcal{C}_c$ denote its specified family of structurally and epistemically admitted whole-spacetime continuations. Each $x \in \mathcal{C}_c$ has a represented historical embedding $i_x : K \hookrightarrow L_x$. A normative inquiry additionally requires the identity of affected subjects, relevant commitments, a description of available interventions, information

about their effects, and an account of whose reasons and claims receive consideration. These elements cannot generally be recovered from the isomorphism class of L_x or its homology groups. In keeping with the event semantics of this paper, a represented person or institution is interpreted through historically situated substructures and associated evidence; individual vertices continue to denote events rather than primitive agents.

12.1 Historical Feasibility and Normative Admissibility

The analytical problem in this subsection is the relationship among mathematical well-formedness, empirically conditioned feasibility, and normative permission. An embedding of finite complexes is a precise structural claim. Its existence establishes that the selected cells and attaching maps can be assembled in the stated mathematical category. It leaves open whether the described events could be brought about, whether the required legal or material conditions exist, and whether pursuing the continuation would be ethically acceptable.

We therefore distinguish four predicates on a represented continuation x , given a modeling context c and a normative context ν . Their conjunction is introduced in Equation 75.

$$\begin{aligned} S_c(x) &: \text{structural consistency in the selected two-complex language,} \\ E_c(x) &: \text{support under stated evidence and interpretation rules,} \\ F_c(x) &: \text{feasibility under identified historical constraints,} \\ N_\nu(x) &: \text{normative permissibility under specified principles,} \\ A_{c,\nu}(x) &:= S_c(x) \wedge E_c(x) \wedge F_c(x) \wedge N_\nu(x). \end{aligned} \tag{75}$$

The subscripts matter. F_c expresses a justified feasibility assessment *under context* c ; it does not provide access to an exhaustive ontological set of feasible futures. Similarly, N_ν refers to an articulated ethical or legal framework, including its scope of application and interpretive uncertainty. The conjunction is a classification scheme, not an algorithm for discovering all four truth values. Under incomplete evidence, a predicate may be unresolved; an implementation might therefore use proof obligations or three-valued annotations in place of unqualified Booleans.

There are at least two possible uses of *admissibility* in the formal sections of the manuscript. One is a syntactic requirement for inhabiting a dependent continuation type; the other is a substantive assessment involving evidence, feasibility, and ethical justification. Equation 75 keeps these layers apart. A proof that a continuation is cellularly admissible does not provide a certificate of $N_\nu(x)$. A normative prohibition likewise does not show that the prohibited extension is topologically impossible. Distinct kinds of reasons and disagreements can consequently be recorded without their automatic reduction to one score.

12.2 Epistemic Responsibility in Future Recognition

The normative object here is the practice of recognizing, presenting, and acting upon represented possible futures under incomplete knowledge. Section 6 treated observing capacities and interpretation operators as historically conditioned. Section 9 allowed several models to fit the same historical observations. These conditions have practical consequences whenever a decision-maker presents one model's continuation family as though it were a comprehensive catalog of the futures available to affected people.

An epistemically responsible process could therefore require a record of the evidence used to construct K , the assumptions selecting a reconstruction model, the uncertainty among admitted continuations, the observers and groups whose information was available, and the reasons for excluding alternatives. Such requirements depend on context and institutional responsibility. Responsible-innovation research has developed related themes of anticipation, reflexivity, inclusion, and responsiveness [20]; anticipatory governance similarly addresses foresight, engagement, and institutional integration [10]. These sources motivate inquiry into institutional arrangements, while the present construction isolates the information to which such arrangements would need access.

A useful research distinction separates *model-internal uncertainty*, which compares represented alternatives, from *model-selection uncertainty*, which compares reconstructions, and from *representational openness*, which concerns alternatives an observer has not formulated. For a decision supported by one finite model family $\mathcal{M}_c$, acknowledging representational openness cannot amount to enumerating the missing alternatives. It can instead motivate the provision of revision channels, explicit model provenance, and opportunities for affected participants to dispute an omitted event type, coupling, or interpretation. These procedures remain conditional recommendations until specified for a decision domain.

Fricker’s discussion of epistemic injustice is relevant where social relations affect whose testimony is credited and which experiences can be made intelligible within shared interpretive resources [9]. The present two-complex language can represent a record of testimony, challenges, corrections, and decisions when appropriate semantic data are provided. Homological statistics alone cannot diagnose a testimonial or hermeneutical injustice. The latter judgment requires interpretive and normative evidence about the individuals, social practices, and credibility relationships involved.

12.3 Procedural Ordering and Justice of Continuation

The temporal ordering of prospective interventions introduces a normative problem distinct from the topology of their final union. Consider a fixed family of event descriptions with an externally specified precedence relation P . A linear extension $\pi \in \text{Lin}(P)$ is one admissible ordering of those events. The cellular complex records their incidence and any attached semantic annotations, while the precedence relation records selected ordering constraints. An order-dependent evaluation can then be defined on the enriched history (L, i, P, π, σ) , where σ contains institutional or causal state transitions and their evidence. This extension of the model follows the separation of incidence and causal order established in Section 10.

Definition 12.1 (Order-sensitive continuation assessment). Given a normative context ν and an explicitly interpreted state-transition specification σ , an order-sensitive assessment is a function or relation J_ν on enriched continuation histories. Its codomain Q_ν may be a partially ordered set of outcomes, obligations, or evaluations. No total numerical scale is required. The definition presupposes an account of affected subjects and the normative meaning of the represented events.

For a finite illustration, let T denote an event at which access-supporting instruction becomes available to a group, and let A denote an irreversible allocation decision whose eligibility threshold depends on capacity immediately before allocation. The historical context leaves T and A unordered, so both $\pi_1 = (T, A)$ and $\pi_2 = (A, T)$ are allowed linear extensions of P . Set initial eligible capacity to zero, let T increase it to one, and let A record whether the current capacity reaches one. The stipulated eligibility results are specified in Equation 76.

$$\begin{aligned} c_0 = 0, \quad T(c) = c + 1, \quad A(c) = \mathbf{1}_{\{c \geq 1\}}, \\ q(T, A) = A(T(c_0)) = 1, \quad q(A, T) = A(c_0) = 0. \end{aligned} \tag{76}$$

The toy assessment q tracks one explicitly selected opportunity to qualify before a binding decision. It is neither a measure of overall social justice nor an empirical prediction. Both histories contain the same two event-types and may be assigned the same underlying final cellular complex after forgetting order and decision-state annotations. They differ in a morally potentially relevant procedural fact: whether qualifying access was available before the allocation decision. The second ordering might be unjust under a principle requiring meaningful prior opportunity, but a justified conclusion in an actual case would also examine urgency, applicable rights, other affected parties, and feasible alternatives.

A formal consequence illustrates the limits of topology-only assessments. Let $\mathcal{H}$ be a set of semantically enriched continuation histories, and let $U : \mathcal{H} \rightarrow \mathcal{T}$ forget procedural order and other annotations, retaining only a selected terminal topological summary.

Proposition 12.2 (Factorization criterion for a terminal-topology assessment). *For a function $J : \mathcal{H} \rightarrow Q$, there exists a function $\bar{J} : U(\mathcal{H}) \rightarrow Q$ satisfying $J = \bar{J} \circ U$ if and only if J is constant on each fiber of U .*

Proof. If $J = \bar{J} \circ U$, histories with the same image under U have equal J -values. Conversely, if J is constant on every fiber, define $\bar{J}(t) = J(h)$ for any h with $U(h) = t$. Fiberwise constancy makes the definition independent of the choice of h .

Proposition 12.2 is elementary set theory. Its analytical role is to specify exactly when an evaluation of enriched histories can be recovered from a reduced representation. In the stipulated example of Equation 76, the opportunity indicator q is not constant on the two histories once U forgets their order. Consequently it cannot factor through that terminal-topology summary. The result does not entail that all procedures are order-sensitive or that topology lacks normative relevance. It identifies a definite information-preservation requirement for a specified query.

12.4 Historical Dependencies and Heterogeneous Generative Capacities

This subsection concerns the possible influence of present interventions on the abilities of relationally constituted subjects to participate in subsequent activity. The term *generative capacity* here refers to a domain-specific, evidence-dependent ability or opportunity to sustain, initiate, or transform future relations. It is not identified with a Betti number, a generic count of incident edges, or an intrinsic property of a vertex. An interpretive map must associate a historically constituted relational substructure with a subject, relevant capability, and means of exercise.

One preliminary representation assigns to an enriched continuation stage h_s a profile $C_\nu(h_s)$ with components indexed by declared subjects and domains of action. These components might include access to needed knowledge, institutional recognition, practical agency, or participation rights. A component may be qualitative, ordinal, or partially ordered. Sen's comparative approach to justice and discussions of freedoms and capabilities motivate attention to what persons can actually do, alongside the comparative evaluation of alternative social arrangements [17]. The proposed profile imports no ready-made universal metric from that literature; it requires a context-specific interpretation and justification.

The temporal dependence matters because an intervention may reshape who can act later, and thereby change which further continuations can be represented as feasible. We can describe such dependence schematically through a history-conditioned transition relation, rather than assuming an invariant transition generator. Let $B_\nu(h_s, a)$ denote a family of next-stage capability profiles proposed under intervention a . The relation permits nonuniqueness, including alternative interpretations and unexpected effects. The family itself is epistemically bounded by current evidence and vocabulary. It should therefore be accompanied by its source assumptions and revision status.

A further issue is conflict among heterogeneous claims. Enhancing one dimension of capability can impose burdens or restrictions elsewhere; some changes are incommensurable under available evidence or ethical frameworks. The evaluation space Q_ν in Definition 12.1 can remain partially ordered to preserve such incomparability. An apparent increase in the number of modeled options need not indicate a socially desirable improvement if those options are coercive, inaccessible to affected persons, or dependent on the loss of protected rights. Future creation thus raises questions about the distribution and regeneration of capacities and also about who may legitimately set the conditions of that regeneration.

12.5 Representation, Participation, and Epistemic Exclusion

The representational interface itself may shape normative possibilities. A mathematical model constructed from a restricted body of evidence can omit affected perspectives, overlook a relevant historical commitment, or impose a vocabulary in which a participant's concerns cannot be expressed. The problem is especially consequential when the model is used to select actions rather than solely to explore structural hypotheses. Young's account of inclusion and democracy stresses the need to consider differences in communicative forms and social positions within participatory decision-making [24]. Anderson's

relational-egalitarian arguments likewise direct attention to how social arrangements constitute relationships among persons, extending inquiry beyond resource distributions alone [1]. These perspectives motivate questions about model construction and institutional power; they do not determine a canonical cellular encoding.

In the present formalism, the selection of a represented history K , the types of prospective cells, the admissibility predicate, and the interpretation of reduced histories are separate sites at which exclusion can arise. Consider two groups with competing descriptions of an event's consequences. A single two-complex might accommodate several provenance-marked observations, or the groups might construct distinct epistemic complexes linked by a partial translation relation. Neither strategy guarantees that contested testimony has been interpreted appropriately. The choice between them should follow the inquiry's purpose, data conditions, and the governing norms of participation and accountability.

A prospective participatory procedure can therefore be investigated at two levels. At the object level, participants contribute claims about relevant historical and prospective events. At the modeling level, they may challenge the representational language, evidence admissibility rules, observer assumptions, or semantic mappings. The second level is particularly significant under representational openness, where proposed futures may be absent because the model lacks the concepts needed to describe them. Revision mechanisms and documented disagreements can improve accountability while leaving unresolved the broader ethical question of who is entitled to representation, consultation, consent, or decision authority in the specific institutional setting.

12.6 Reversibility, Irreversibility, and Future Opportunities

Irreversibility in future creation requires a specified object of assessment. A topological class may disappear under attachment, a material asset may become unrecoverable, an institutional commitment may become binding, or an epistemic frame may become entrenched. These are distinct notions. In particular, the triviality of $H_1(L)$ does not establish the irreversibility of a social action. Conversely, two homotopy-equivalent terminal complexes may represent processes with very different capacities for practical correction.

We can state a conditional notion using an enriched transition system over represented histories. Relative to a normative context ν , a stage h' is *recoverable to a protected condition* of an earlier stage h when some admitted subsequent history reaches a state satisfying an explicitly specified recovery predicate $R_\nu(h, \cdot)$. This need not entail literal reversal of historical events: the record of what happened remains part of the whole-spacetime representation. Restoration may concern rights, capacities, resources, institutional standing, or other protected properties. A recovery predicate may itself be contested, and finite modeled continuations cannot guarantee that no unrepresented recovery route exists.

A principle favoring recoverability may be plausible under uncertainty, especially where an intervention risks foreclosing others' ability to pursue alternative futures. Nevertheless, complete reversibility can conflict with urgent care, binding commitments, safety, or the needs of particular persons. The formalism therefore allows recoverability to be recorded as a morally relevant characteristic while leaving its priority to substantive normative reasoning. In a comparative assessment, the relevant concern may be the distribution of reversibility: whose choices remain correctable, whose losses can be remedied, and who bears the costs of preserving optionality for others.

The conventional vocabulary of options is useful only with caution. A model may display numerous possible extensions that are practically unavailable to certain people, while another model may encode fewer possibilities but sustain stronger rights, participation, or material support. Similarly, a future attachment that fills a historical cycle provides topological information about the modeled complex, but the social interpretation of that attachment and its effect on later agency remain independently specified. The inclusion maps and relative homology studied in Section 10 can support historically marked comparisons once appropriate semantic data are supplied.

Table 12: Normative inquiries and additional information required beyond the terminal topology of a candidate continuation. Requirements are domain-dependent and are not sufficient conditions for justice.

Normative inquiry	Additional information	Relevant representational safeguard
Procedural opportunity	Event order, institutional deadlines, access conditions	Ordered and provenance-marked histories
Epistemic responsibility	Evidence coverage, uncertainty, model provenance	Versioned reconstruction and dissent records
Participation and exclusion	Affected subjects, communicative access, contestation	Subject-indexed interpretation and revision channels
Generative capacities	Capability profiles, constraints, continuing agency	History-conditioned state and capability annotations
Remedial possibility	Protected interests, recovery costs, available remedies	Action-sensitive recovery predicates
Comparative justice	Explicit principles, conflicting claims, justified scope	Partial comparisons and reasons for incomparability

12.7 Normative Information Preservation in Continuation Models

The final interface concerns translation, coarse-graining, and representation reduction. Section 10 showed that homotopy-preserving simplifications may discard cell identifiers and that abstract homology can conceal distinct historical attachments. The normative significance of information loss depends on the questions for which the representation will be used. Table 12 identifies selected inquiries and the minimum kinds of supplementary information they may require.

A general test for retaining relevant information follows Proposition 12.2. Suppose $r : \mathcal{H} \rightarrow \mathcal{H}'$ is a proposed reduction of enriched continuation histories, and Q_ν is a declared family of ethically relevant queries. We say that r is Q_ν -preserving when each $q \in Q_\nu$ can be evaluated from the reduced representation. The factorization condition is given in Equation 77.

$$\forall q \in Q_\nu \exists \bar{q} : \mathcal{H}' \rightarrow Q_q \text{ such that } q = \bar{q} \circ r. \quad (77)$$

Equation 77 provides a formal specification for information preservation relative to an explicitly declared query family. It does not establish that the chosen family exhausts morally relevant questions, and it does not by itself license the underlying judgments. If two histories are identified by r yet a query assigns them different values, that query cannot be recovered from the reduced representation. In the ordering example of Equation 76, a reduction retaining only the final cellular type fails to preserve the opportunity query. A reduction that retains procedural order and the allocation-state annotation can preserve this query without retaining every unrelated detail of the original cellular model.

The implications for future creation are conditional but operationally important. A model used for structural exploration may legitimately discard many historical distinctions. A model used to inform a consequential decision must be assessed against the kinds of reasons, affected persons, obligations, and remedial possibilities relevant to that decision. Some such reasons may only emerge through participation and later epistemic revision. Accordingly, a claim of normative information preservation should identify its query domain, evidence, audience, and mechanisms for challenging the classification of information as irrelevant.

This section has proposed a normative *interface*, not a general metric of justice. The formal continuation model identifies represented histories and relations among their possible extensions. The normative layer introduces independently justified evaluative predicates, possible partial comparisons, participation requirements, and preservation criteria. Comparative justice, relational equality, epistemic injustice, and responsible innovation offer distinct analytical resources for that layer [17, 1, 9, 20]. Their differences should remain visible. The mathematical scope and epistemological limitations of this combined architecture are examined in the next section; a comprehensive procedural ethics of future creation remains a separate research project.

13 Mathematical Scope and Epistemological Implications

The preceding sections develop a preliminary architecture for representing possible continuations of an epistemically reconstructed relational spacetime. Its components are drawn from established theories of cellular complexes, dependent types, categories, relative homology, and discrete Morse reductions; the proposed contribution concerns their coordinated use in a historically situated inquiry into future creation. The objective of this section is to delimit the conclusions supported by the construction and to identify the additional assumptions required for stronger epistemological, empirical, and normative claims. The discussion first collects the formal commitments and their consistency conditions, then examines historically constituted observation, the interpretive reach of topological invariants, and the identifiability of inferred continuation models. It finally specifies interfaces with Generative Relational Theory (GRT) and criteria for assessing representational adequacy across disciplines. Its method is primarily analytical: mathematical consequences are separated from interpretive proposals and open problems, and the synthetic cases of Section 11 serve as diagnostic counterexamples rather than evidence for a universal theory.

A recurring distinction concerns the *object* of a mathematical statement. An extension $i : K \hookrightarrow L$ is a relation between chosen cellular representations; a statement about the actual feasibility of a future event requires a justified correspondence between those representations and an investigated situation. Likewise, the availability of an epistemic model does not establish that the model identifies the mechanism by which reality develops. The paper's results support explicit conditional reasoning about modeled continuations, leaving questions of ultimate ontology and empirical adequacy available for further inquiry.

13.1 Formal Assumptions and Ontological Suspension

The formal reference domain introduced in Section 6 is deliberately stipulated. A reference object R in a class Ref_2 provides a convenient domain in which historical supports of observers can be posited and relations to epistemic reconstructions can be investigated. Calling this domain *assumed ontological relational spacetime* identifies its role inside a conditional formalism. Both R and an observer-dependent $\hat{K}$ remain mathematical representations; neither is asserted to be the uniquely correct description of a historical process or the ultimate constitution of reality. The structure of physical time, and whether time is fundamental, emergent, or otherwise interpreted, is similarly left open. The indices denoting observational stages identify positions in an inquiry, with historical ordering supplied separately where required.

Several modeling choices organize the current results. Historical representations are taken, in the finite examples, to be two-dimensional cellular complexes. Vertices represent *events*, edges represent *couplings*, and faces stand for higher-order primitives whose domain-specific meanings must be supplied separately. Historically constituted entities and observers may be associated with interpreted subcomplexes or richer diagrams. Candidate continuations are usually represented by embeddings of a fixed historical complex into extended complexes. The preferred codomain contains the historical structure and the additional prospective cells; a future-only construction can carry equivalent information when it includes a complete specification of the relevant attachments.

The word *continuation* is thus a designation given to a well-formed, historically marked extension together with declared admissibility information. Cellular topology, on its own, does not determine empirical feasibility, causality, interpretation, or ethical permissibility. These components must be supplied by additional predicates or structures. In particular, the requirement that the history embed into the extension is a convention for *candidate continuations of a fixed represented history*. It need not characterize epistemic revisions that retract, reinterpret, or reidentify earlier descriptions, and it does not exclude modeling genuine historical dissolution by adding later events that record the dissolution.

Table 13 collects the roles of the principal assumptions and the kinds of claims they license. The register also indicates the methodological status of conclusions developed earlier in the paper. Its rows are intended as a guide to interpretation, rather than as exhaustive axioms for all possible applications.

Table 13: Scope register for the preliminary two-complex continuation framework. Each conclusion is conditional on its indicated mathematical or interpretive inputs.

Specified input	Supported analytical claim	Additional requirements for stronger claims
Finite two-complex with oriented cells	Boundary maps and computable homology	Evidence for cell identities and semantics
Marked embedding $K \hookrightarrow L$	Inclusion-induced maps and relative topology	Historically justified identification of the marked base
Attachment inventory and degree	Conditional homological bounds	Empirical or process-based feasibility of attachments
Causal or precedence annotation	Admissible orderings under declared constraints	Grounding of the ordering relation in the investigated domain
Observer support and capacity types	History-indexed formal dependency of model construction	Explanatory mechanisms and observations of capability formation
Evidence-fit and model predicates	Families of conditionally admitted continuations	Coverage, calibration, identifiability, and external validation
Normative query and procedural record	Explicit order-sensitive comparisons and reduction tests	Justified principles, affected perspectives, and practical context

This scope register supports an affirmative interpretation of the formalism: it permits exact mathematical claims while retaining a clear location for the evidence and reasons needed to apply them. Ontological suspension accordingly functions as a discipline of claim strength. It does not obstruct the investigation of actual historical processes; it requires such investigations to state how their epistemic records are connected to the formal objects.

13.2 Mathematical Consistency and Model Dependence

Within a specified category of two-dimensional cellular complexes and embeddings, the basic continuation construction is mathematically coherent. Identities and compositions of the chosen embeddings exist, so objects extending a marked historical complex can be organized in the undercategory described in Section 8. Standard dependent sums can record the chosen extension and its admissibility witness, as developed in Section 7. Relative homology is available for the corresponding pairs, and suitably constrained Morse reductions can preserve designated topological structure [22, 16, 12, 8].

These statements establish the legitimacy of particular constructions under their definitions; they do not imply that every contemplated epistemic and semantic operation has already been integrated into one closed formal system. An admissibility family $\text{Adm}_E(i)$ may depend on data and contextual assumptions, and an inhabitant of this family certifies only the condition actually encoded by its definition. For example, a predicate expressing satisfaction of an incidence constraint carries a different evidential meaning from a predicate representing an empirically estimated transition, and both differ from a normatively justified permission. A dependent type provides a disciplined interface for such distinctions but supplies no automatic verification of observational evidence or ethical principles.

The conditional composition result in Proposition 7.2 requires a closure condition on admissibility witnesses. The categorical transport constructed in Proposition 8.1 concerns structural cobase changes; admissibility, historical meaning, and causal annotations need additional preservation conditions. These restrictions are especially significant when a revised account of history induces a mapping that identifies or changes previously distinguished cells. A pushout may then be well-defined in the ambient category while no longer representing an admissible continuation in the intended epistemic context.

The finite attachment results offer a narrower kind of certainty. Under the hypotheses of Proposition 10.1, the Betti-number changes follow algebraically from the ranks of the attaching classes. The synthetic computations in Section 11 independently evaluate selected instances, including integer torsion that field-valued Betti numbers cannot detect. They confirm those finite calculations and some

information-loss examples; their evidential reach is confined to the constructed complexes. In particular, the demonstration of an algebraically valid two-cell attachment does not establish the occurrence or feasibility of the event or relation that the new cell might represent.

A mathematical claim is therefore *model dependent* in two complementary senses. Its proof depends on the chosen class of objects, morphisms, and constraints. Its interpretation depends on a correspondence between these objects and a historically investigated domain. Confusing these dependencies risks converting an exact consequence of a stipulated complex into an unwarranted conclusion about social or physical spacetime. The distinction also clarifies why an epistemic model can be useful in comparative reasoning despite uncertain empirical calibration: the model can expose consequences of assumptions, provided that the assumptions remain open to examination and revision.

13.3 Historical Endogeneity and Epistemic Reflexivity

The historically constituted observer is a central conceptual commitment of this inquiry. Section 6 proposes the dependent observer-context type in Equation 30, indexing possible capabilities and procedures by a selected historical support within the assumed reference domain. Section 9 subsequently expresses the reconstruction of candidate models as conditioned by such observing contexts. This architecture permits an observer's capacity to recognize future continuations to be investigated as part of a historical process, rather than represented as an independent, invariant faculty.

The type-theoretic dependence is precise but limited. A fiber $\text{Cap}(R, S, j)$ specifies the capabilities admitted by a selected historical configuration in the model. The existence of that fiber is not an explanation of why a particular observer acquired a capability, nor does an assignment of a procedure π show that the procedure was generated uniquely by the history. Multiple capacities can share a stipulated historical support; competing reconstructions can attach different supports to similar observed practices. An explanatory theory would require additional account of learning, instrumentation, institutions, bodily processes, communication, or other mechanisms appropriate to the domain. Such mechanisms may subsequently change which events and relationships the observer can even represent.

Epistemic reflexivity introduces an additional modeling question. An observer may construct a representation of their own historical support, and later observations may modify that representation. The observer's practical capacity to observe need not depend on having completed a faithful self-description: the act of operating an instrument is distinguishable from a theory of the instrument's entire historical constitution. Thus a provisional theory can admit partial self-representation without imposing an equation demanding that each epistemic object contain a complete account of its own production. Self-description can be studied through successive, evidence-bounded reconstructions, with their stages, provenance, and translation limitations documented.

One may describe this as a form of endogenous epistemology, provided its status remains clear. It proposes a *formal dependency* between supported historical configurations and admissible epistemic procedures. It does not derive a universal reflexive generator, a complete fixed-point semantics, or a privileged observer-independent reconstruction of ultimate reality. Situated-knowledge research and historical accounts of objectivity give independent reasons to take observing practices and their conditions seriously [11, 21, 6]; the specific dependent types and cellular interpretation remain proposals made here.

A further consequence concerns comparison among observers. Two epistemic complexes may differ because their evidence differs, because they employ different descriptive languages, or because they identify different relational structures under a shared vocabulary. Any map intended to transfer candidate continuations between these contexts must therefore specify what is carried across: cellular incidence, cell identity, semantic references, provenance, constraint predicates, or some selection of these. The conditional transport of Section 8 supplies a possible structural interface, while the adequacy of its epistemic interpretation is a separate problem. This is a precise place for future work on indexed categories and evolving families of representation types.

13.4 Structural Invariants and Interpretive Semantics

Topological invariants offer reliable comparisons within their defined scope. Homology detects selected cycles, boundaries, and their algebraic relationships; relative homology and inclusion-induced maps retain information about how an extension changes the standing of historically identified classes. Section 10 defines a marked homological profile in Equation 62, extending comparisons beyond the homology groups of terminal complexes. This distinction is analytically useful because continuations with the same unmarked homology can have different inclusion kernels over the same historically identified cycles.

Even the marked profile remains a deliberately restricted summary. Cellular boundary operators can agree on abelianized attaching data while the underlying attaching maps have different homotopical effects. Homotopy type does not encode the complete incidence history, and abstract topological equivalence need not respect event identity, causal precedence, or documentation of how an interpretation was reached. The examples of relative collapse in Sections 10 and 11 demonstrate the further point that removing a prospective cell may preserve homotopy while deleting an event marker required for the intended historical inquiry.

It follows that an *adequate reduction* is relative to a specified class of questions. The query-preservation condition in Equation 77 makes this dependence explicit: a proposed reduction supports a query when the query factors through the reduced representation. For a purely homological question, a homotopy-preserving reduction may be sufficient. For an inquiry into the order of decisions, the preservation of causal order and state annotations may be required. For research on epistemic responsibility, cell provenance, disagreement records, and revisions may matter more than the minimal number of cells. These are distinct preservation obligations and should not be silently combined into a single topological invariant.

The semantic situation is particularly delicate for faces. In the present GRT instantiation, a vertex represents an event, an edge a coupling, and a face a higher-order primitive. The face is assigned no universal social, institutional, moral, or cognitive meaning. Physical spin-foam models also use decorated two-complexes, but their labels and amplitudes have specific mathematical and physical roles [2, 15, 14]. Borrowing their combinatorial setting supplies neither evidence that relational social histories are quantum nor a universal interpretation of the faces. A semantic assignment should be documented as part of the inquiry and tested against the records or phenomena it is proposed to describe.

One related distinction concerns causal and temporal ordering. A cellular incidence relation supplies boundary information; a causal partial order adds a separately stipulated relation. A complex with a nontrivial first homology group can still admit a causal acyclic order when the extra relation is chosen accordingly. Conversely, a topology-preserving change can alter an event-order annotation. Homology therefore provides information about represented attachments and connectivity, whereas the justice of temporal order requires enriched historical and normative descriptions, as argued in Section 12.

These interpretive boundaries make the topological approach more precise. The mathematical apparatus is capable of identifying where a proposed future attaches to represented history, which structural features it changes, and what selected reductions preserve. The significance of those features for actual future creation must be established through semantic and evidential work undertaken alongside the mathematics.

13.5 Identifiability, Underdetermination, and Epistemic Uncertainty

Future continuation is an inverse and prospective problem. An observer receives a limited record of historical processes and proposes models that can account for the record while admitting possible extensions. The finite-history non-identifiability result in Proposition 9.1 shows, under its stated assumptions, that distinct continuation rules can agree on the represented past and diverge on a future attachment. An observed history therefore need not determine a unique continuation family. The broader scientific literature identifies related issues of underdetermination and unconceived alternatives [19].

It is useful to distinguish at least three levels of uncertainty. *Within-model uncertainty* concerns alternatives admitted by one specified generator or continuation predicate. *Between-model uncertainty* concerns distinct models compatible with the available evidence but differing in their inferred continuations. *Beyond-model limitation* concerns prospective structures and interpretive categories absent from the currently available modeling language. A probabilistic model may assign distributions to members of a declared space, but that operation cannot by itself show that the space exhausts the forms that historical processes may realize.

The distinction becomes sharper when epistemic representations are revised. A newly recognized event can create an apparent cycle in a reconstructed complex; evidence for a face can later make the cycle a boundary, as demonstrated in Section 6. Such changes in computed invariants need not indicate corresponding topological changes in the underlying process being investigated. They can instead reflect alterations in which relations and primitives are recognized, how observations are grouped, or which explanatory vocabulary is used. The same caution applies when a revised past changes a future attachment's interpreted significance in the computation of Section 11.

Identification of a suitable model consequently requires specifying the observational target and the level of equivalence sought. Two models may be observationally equivalent for a finite set of measurements while differing in prospective predictions, causal explanations, or normative consequences. A result concerning the equality of finite historical records says less than one concerning equality of all admissible continuations. Similarly, a partial ordering of models by evidential adequacy does not automatically yield a defensible unique model selection; costs of missing possibilities, domain shifts, and historically emerging concepts may alter the comparison.

These considerations limit the meaning of *forecast* in this paper. The synthetic experiments calculate consequences of cell attachments and model specifications. They make no calibrated statistical prediction of future social or physical events. The continuation family returned by an epistemic generator is a structured expression of conditional possibilities accessible under specified representations, not a catalog of objectively complete futures. This orientation allows the mathematical study of model-sensitive anticipation while preserving the practical importance of iterative evidence gathering, external criticism, and the revision of conceptual categories.

13.6 Interfaces with Generative Relational Theory

The present paper belongs to a broader Generative Relational Theory inquiry into historical formation, relational constitution, and the conditions affecting further possibilities. Its specific object is the epistemic representation of *future creation*: how a represented history can be extended in modeled continuations and how the recognition of those continuations depends on historical evidence and epistemic capacities. The two-complex representation is an investigative choice within that program. It preserves the event-based cellular convention and allows historically constituted entities to be investigated through interpreted substructures, while leaving ontology and empirical semantics open.

This object differs from the circulation of heterogeneous values examined elsewhere in the research program. A circulation model may investigate exchanges, conversion interfaces, returning paths, distributional consequences, and the justice of an order among represented transactions. Future creation additionally concerns the prospect that events, couplings, cellular attachments, available capacities, or admissibility rules have yet to be specified and may later become representable. The new problem therefore arises at the level of constructing and revising *possible continuations of the represented spacetime itself*. Shared mathematical resources do not establish that the two investigative objects are identical.

A second interface concerns the relation of generativity to models that produce complex histories. A single spin foam or another decorated two-complex is one represented structure; mathematical frameworks can separately specify families of such structures and the rules or amplitudes associated with their production. For GRT, a posited epistemic generator is a tool for articulating candidate continuations from a historically situated body of evidence. There is no inference here that a selected generator is the unique mechanism of the process under investigation, nor that changes of epistemic models must correspond

directly to changes of ontological laws. The ontological question is intentionally suspended.

A third interface concerns agency and justice. Historically constituted capacities influence what futures become recognizable and what interventions can be proposed. An institution's selection of a representational language, data source, or admissibility predicate can consequently affect which opportunities enter a decision process. Section 12 treats those issues through an explicitly separate normative layer: participants, rights, responsibilities, recovery possibilities, and reasons for comparative judgments must be articulated in addition to topology. The resulting framework could support a later procedural ethics of future creation, particularly where ordering and recognition of alternatives affect others' subsequent generative capacities. It cannot derive a complete theory of justice from cellular homology or categorical structure.

The architecture also permits connections to law, political theory, education, scientific practice, and computational infrastructure. Different fields may provide different accounts of what constitutes an event, which couplings are relevant, which records warrant recognition, and which prospective transformations are permissible. The task is to specify translations and their limits rather than assume that one mathematical language exhausts their disagreements. These interfaces are opportunities for domain-specific research, with the present paper supplying an initial shared representation of conditional historical continuations.

13.7 Representational Adequacy and Disciplinary Boundaries

A two-complex representation should be evaluated against the purposes and evidence of the inquiry in which it is used. A formally consistent complex may inadequately distinguish observed events, encode couplings with poor evidential support, or impose cells that conflict with the intended domain semantics. Conversely, a simplified complex may be useful for a restricted analytical task even when it leaves out processes that a fuller historical or interpretive account would require. The appropriate question is therefore which properties the representation preserves for a declared investigation, who can contest the representation, and what uncertainties remain accessible to criticism.

For practical assessment, we propose a staged *adequacy record*. It identifies the empirical or documentary sources supporting the chosen historical complex; the operational meanings assigned to vertices, edges, and faces; the relevant observational limitations; the declared continuation and admissibility rules; the structural invariants examined; and the decision context, if any, in which modeled possibilities will be used. Where a projection or Morse reduction is used, the record should state the preserved query family and describe lost event-specific or semantic information. Where an epistemic revision occurs, the record should state which previous claims were retracted, revised, or newly supported and how that change affects existing continuation candidates.

These requirements follow from the paper's central design choice. A whole-spacetime continuation preserves the identified historical base and the attachments of prospective cells directly. A future-only representation supplied with full attachment data can express the same mathematical relationships. The preferred form makes the history-dependent comparison explicit in the primary object, but it does not confer privileged truth on the represented past. The adequacy of that past, the mapping between evidence and cells, and the possibility of later correction remain open to independent evaluation. A stable marked embedding and a revisable epistemic history therefore play different roles and should be managed through different classes of transformations.

Several limitations also delimit disciplinary transfer. Two-dimensional topology can be insufficient for phenomena demanding higher-dimensional incidence, stochastic dynamics, learned representations, or continuous fields. Nonregular cellular attachments may require more care than simple simplicial diagrams suggest. Probabilistic calibration, causal identification, archival provenance, lived experience, and legal or ethical justification have their own standards of adequacy. A correspondence between representations from different domains may preserve selected structures without preserving meaning or explanatory force. In each case, additional models and evidence can complement the two-complex formalism, and failure of a particular translation is an informative outcome rather than a defect to be hidden.

The available demonstrations establish exact statements for restricted synthetic systems and identify meaningful failure modes for information-preserving reductions. They do not establish external validity for the use of cellular futures in governance or other high-stakes decisions. Such applications would call for domain specialists, affected participants, evidence-sensitive validation, and procedures for contesting inferred opportunities or exclusions. Historical and qualitative interpretation, formal analysis, and quantitative modeling provide complementary forms of access; each should be assessed in light of the local research question [11, 20, 24].

Taken together, Sections 3–12 provide a preliminary, internally specified approach to epistemic continuation with several demonstrable cellular and homological consequences. The approach is strongest where a represented history and the relevant attachment questions are explicit, and where preserving historically identified structure adds information beyond terminal topology. Its principal unresolved demands concern the reconstruction of admissibility rules from incomplete evidence, the historical emergence and revision of epistemic capacities, the treatment of possibilities outside current modeling languages, and the justification of normative interpretation. These are developed as independent research questions in the following section on open questions and future directions.

14 Open Questions and Future Research Directions

The preceding analysis provides typed descriptions of epistemic continuations, categorical comparisons of marked cellular extensions, and exact calculations for selected relative homology and reduction problems. A research program on future creation requires additional structures before those constructions can support explanatory, predictive, or normative judgments about situated historical processes. The purpose of this section is to formulate the remaining mathematical and epistemological problems with sufficient specificity to guide subsequent work. The discussion considers historically constituted observation, revisions of epistemic contexts, changes in prospective cell vocabularies, open families of possible continuations, and the requirements of inference and validation. It then examines dimensional extension, the evolution of epistemic capacities, procedural justice, and computational implementation. The method is one of problem specification: proposed mathematical interfaces are distinguished from constructions established in earlier sections, and possible tests are described without suggesting that they already provide empirical validation.

The paper’s ontological suspension applies throughout. An assumed relational reference structure R is available as a conditional modeling device for relating observer histories to epistemic descriptions; no conclusion about ultimate reality follows from using such a device. Moreover, in the intended two-complex semantics, vertices denote events, edges denote couplings, and faces denote higher-order primitives whose interpretations must be specified for each application. A change in a model’s admissible continuation family can signify an epistemic revision, a modification of the modeled possibility vocabulary, or a hypothesized historical transformation. Distinguishing these interpretations is itself part of the open research agenda.

14.1 Mathematical Constitution of Endogenous Epistemic Operators

Section 6 represents the historical support of an observing system through the dependent context type of Equation 30. Section 9 further indexes model reconstruction by the observer context in Equation 53. These expressions make dependence explicit at the level of types and relations. They leave open the substantive problem of determining which historical processes support a particular epistemic capability, which changes are causally relevant, and what evidence could distinguish alternative accounts of its formation.

One possible research direction is to supply a separate, domain-sensitive relation $\text{Supports}_R(S, a)$ between a represented historical support S and a capability description a . A richer construction would permit several supports for the same capability and several possible capabilities for a given support. Its evidence component would distinguish observed records of training, instrument use, and collective

practices from inferred relations between those records and a resulting capacity. The mere presence of a support relation would not entail that S is sufficient to generate a , nor that the historical source of the capacity has been exhaustively represented.

A useful formal problem is to characterize transformations of reference-support pairs under which selected capability distinctions remain meaningful. For a proposed correspondence between two supports, one could ask which observations, procedures, or error profiles are preserved, and whether the correspondence permits transporting an epistemic reconstruction procedure. Such a result would be conditional on a specified category of supports and an interpretation of capacity descriptions. A separate empirical project would then test the claimed correspondence with historical, behavioral, or institutional evidence. The relevant literature on situated knowledge and historical epistemic practice motivates this problem, without determining its mathematical solution [11, 21, 6].

14.2 Higher-Categorical Structures of Epistemic Revision

The continuation category in Section 8 fixes a represented historical base and compares candidate whole-spacetime extensions. Epistemic revision changes that base, its annotations, or the criteria for admitting an extension. The cobase-change constructions examined earlier work when appropriate morphisms and compatibility witnesses have been supplied. Their existence does not resolve the general case in which an observer retracts an event identification, changes a face interpretation, or substitutes an alternative cellular decomposition.

A prospective theory could organize epistemic contexts into a category or bicategory $\mathcal{E}$, with revisions represented by arrows or spans carrying records of retained, withdrawn, and newly interpreted claims. Above a context c would lie a category $\mathcal{C}(c)$ of marked candidate continuations. Transport along a revision would be a partially defined or correspondence-valued operation, rather than a presumed canonical function. The first research task is to determine the conditions that make such operations well-defined; the second is to characterize the coherence required when revisions are composed.

If reindexing functors can be specified for a restricted class of revisions, one may investigate a contravariant pseudofunctor from that class into categories. This would require comparison isomorphisms for composites and coherence laws, in addition to the compatibility of historical embeddings, evidence annotations, and admissibility witnesses. Where revisions lack a unique transport, a profunctor, span, or relation may be a more faithful mathematical interface. The proposal is connected to established categorical methods, but the existence of such a structure for the present epistemic problem has not been shown [16].

A minimal test case should compare two different revision paths connecting equivalent evidence states. Their transported continuation families may agree, disagree, or fail to be comparable. Each outcome carries information about the modeling assumptions. Coherence should therefore be demonstrated for a delimited class of revisions before a global fibred interpretation is asserted.

14.3 Changing Types of Possible Events and Relations

The finite experiments in Section 11 work within a declared vocabulary of prospective vertices, edges, faces, and attachment maps. Future creation may involve the emergence of a previously unmodeled kind of event, a newly recognized coupling, or a higher-order primitive absent from the observer's current descriptive language. A fixed universe of candidate cells can represent uncertainty about which of its elements will appear; it expresses uncertainty about the vocabulary itself only through additional structure.

One preliminary approach is to index the syntax by an epistemic vocabulary V_c . For each context c , an associated type $\text{EpCplx}_2(V_c)$ would contain complexes whose event, coupling, and face annotations are meaningful under that vocabulary. A vocabulary extension $u : V_c \rightarrow V_d$ might then induce a translation of selected complexes and continuation types. A translation can preserve cellular incidence while changing which interpretations or admissibility rules are expressible; those two properties must be assessed separately.

The difficult case is a vocabulary change whose relevant new distinctions cannot be specified in the earlier language. Retrospectively adding a new symbol is mathematically tractable once it has been identified. Anticipating the content of a genuinely unformulated distinction is a different epistemic task. Thus a sequence of vocabulary extensions is a useful record of representational growth, while an assumption that the union lists all objectively possible event kinds would require independent justification.

A related research objective is to specify preservation criteria for vocabulary translations: which historical event identities, attachment relations, provenance annotations, and normative query distinctions survive? Counterexamples in which two differently interpreted vocabularies map to the same unannotated cellular complex may clarify the informational cost of purely topological translations.

14.4 Epistemic Generative Models with Open Possibility Spaces

Section 9 represents epistemic generators as context-indexed predicates or relations admitting selected continuation structures. Such a model may be intentionally incomplete. Its extension can depend on the observer's available language, search resources, assumed attachment repertoire, and accepted evidence. The phrase *open possibility space* designates an inquiry whose modeled universe of candidate continuations may itself be revised; it does not introduce a claimed mathematical inventory of unknown ontological futures.

A possible formal direction is a directed collection of epistemic contexts and vocabulary maps, each equipped with an admissible continuation family. Under suitably compatible embeddings, some of these families may form a filtered diagram. A colimit could then express an enlarged *modeled* space of continuations. The proposed use is explicitly conditional: historical revisions may retract earlier candidates, and competing interpretations may lack the compatibility needed to produce a directed diagram. A filtered colimit is therefore a special case, not a general representation of epistemic progress.

The main research problem is to define a comparison operation that acknowledges three different situations: an earlier candidate remains expressible and admissible, an earlier candidate is now represented but rejected, and an earlier candidate lacks an adequate translation into the new vocabulary. Collapsing all three into a binary membership test would obscure changes in the observer's representational powers. A future formalism may use partial translations and explicit reasons for nontransportability, with a separate treatment for probabilities where a measurable and calibrated outcome space is justified.

This agenda is closely related to the paper's distinction between finite observational non-identifiability and currently unconceived alternatives. The existing underdetermination discussion motivates caution about the completeness of modeled possibility spaces, while leaving the design of their evolving mathematical interface open [19].

14.5 Historical Inference and Structural Identifiability

The finite-history non-identifiability result of Proposition 9.1 provides a restricted example of two epistemic generators with indistinguishable historical observation signatures and differing candidate continuations. A deeper theory requires specifying which evidential records, observable queries, and interventions permit a meaningful distinction between such models. The choice of observable is decisive: equality of unmarked homology groups is weaker than agreement on inclusion-induced maps, and agreement on a finite observational record is weaker than agreement on conditional prospective signatures.

For a specified observational query family $\mathcal{Q}$, one might define a model signature $\text{Sig}_{\mathcal{Q},c}(m)$ and investigate whether it distinguishes models modulo a declared equivalence relation $\simeq_c$. A candidate identifiability condition is stated in Equation 78.

$$\text{Sig}_{\mathcal{Q},c}(m) = \text{Sig}_{\mathcal{Q},c}(n) \implies m \simeq_c n, \quad (78)$$

where the antecedent concerns observations expressible under context c and the consequent concerns an explicitly defined model equivalence. Equation 78 is a proposed *criterion*, not a property already proved for the model families in this manuscript. One research route would identify restricted classes of cellular

generators and query families for which the implication can be established; another would construct counterexamples demonstrating which observations fail to distinguish materially different continuations.

Empirical identifiability requires more than a theorem about synthetic boundary matrices. Historical evidence may be incomplete, recorded categories may change, and access may differ among observers. Model criticism should therefore distinguish a mathematical limitation due to the declared query family from a practical limitation due to the evidence available to construct its answers. These distinctions provide a route toward responsible use of the formalism in domains requiring historical or causal interpretation.

14.6 Higher-Dimensional and Multiscale Relational Complexes

A two-complex is a deliberate first representational choice. It supports event vertices, coupling edges, and higher-order faces, along with finite calculations of cellular homology and relative attachment. Certain relational structures may demand more elaborate incidence patterns, longer-range coherence relations, or higher-order compatibility information. A higher-dimensional CW complex, simplicial object, or an interpreted diagram may then be a more appropriate mathematical representation. The need for these extensions should be motivated by a specified analytical failure of the two-dimensional description, rather than by an assumption that additional dimensions necessarily increase explanatory adequacy.

Two related mathematical problems arise. First, an extension may attach cells of dimension greater than two and thereby alter homology or homotopy in ways unavailable to a two-complex. Second, descriptions at different scales may preserve selected invariants while merging event identities or changing the representation of couplings. The latter problem is central to our historical interpretation: a multiscale projection that preserves a Betti number can still fail to preserve the provenance of an event, the precedence of an intervention, or the identity of a historical attachment.

A useful direction is to investigate *marked multiscale comparison* in which the historical embedding, a declared query family, and the coarse-graining maps are specified together. Persistent and zigzag homology offer existing mathematical resources for studying changes in indexed complexes, including settings where inclusions do not form one monotone filtration [7, 4]. Their application to epistemic revisions requires an additional account of how different representations refer to the same historical claims. A topological interval or critical cell should not acquire an epistemological or ethical meaning without such an account.

14.7 Dynamic Reconstruction of Epistemic Capacities

A historically indexed capacity type describes admissible capability profiles relative to a posited reference support. A dynamics of capacities would require further data: a description of interventions, learning, institutional change, material constraints, and the evidence linking such processes to observed changes in recognition or interpretation. The challenge is especially substantial because changes in capability may change the observer's own way of recording those changes.

One possible program would represent successive observations of a capacity through records of discriminatory performance, available instruments, conceptual vocabulary, and error correction. A model could then specify a conditional transition relation among capacity profiles, with the support history and evidence provenance retained. In this setting a claimed improvement should identify its criterion: finer event discrimination, better calibrated uncertainty, preservation of more historical distinctions, or a demonstrated reduction in a task-specific error measure. These criteria are potentially divergent. A model can become more predictive for one restricted query while losing distinctions relevant to another.

The reflexive case calls for special care. Observers may use a developing representation to revise their understanding of the practices by which it was generated. A complete self-model is unnecessary for recording such revisions, but consistency between levels of description cannot be assumed. The open mathematical task is to specify partial self-description and revision without claiming a complete fixed point for the underlying observer. The empirical task is to test particular accounts of learning or institutional capacity formation, rather than infer them from type-indexing alone.

Table 14: Research problems, proposed formal artifacts, and minimum tests for future development.

Problem	Proposed artifact	Minimum test or failure case
Epistemic revision	Context correspondence and candidate transport	Competing revision paths and nontransportable extensions
Open vocabulary	Typed annotation translations	Newly expressible distinctions and retracted candidates
Model identification	Query-indexed model signature	Observationally matched models with divergent continuations
Historical reduction	Marked projection and preserved query family	Lost provenance or order under valid homotopy reduction
Normative interface	Interpreted procedural history and comparison rule	Identical terminal topology with different participation histories

This direction may become especially consequential for future creation when interventions change the capacities through which subsequent possibilities become recognizable. A historical model of capacity formation and an epistemic model of continuation would then interact, while remaining separate objects subject to different standards of justification.

14.8 Formal Verification and Empirical Interpretation

The computational cases of Section 11 verify exact consequences of declared cellular attachments. They provide regression tests for mathematical implementation and furnish counterexamples to overly coarse representational equivalences. They do not establish that a particular two-complex is an adequate representation of any unexamined empirical history. The next methodological step is therefore a layered validation framework that keeps proof checking, computational verification, measurement, and historical interpretation distinct.

At the formal layer, further work should test chain identities, the consistency of relative inclusions, functoriality of permitted continuation maps, coherence of specified context transports, and preservation claims for topology-reducing operations. At the data layer, the representation of an event or coupling needs a traceable evidential basis, uncertainty annotation, and revision history. At the interpretive layer, domain specialists and affected participants may contest the meaning of a modeled face or the omission of a historically significant relation. At the predictive layer, a claim about anticipated events would require an independently justified observation target, temporal evaluation protocol, and evidence sufficient for calibration or comparative evaluation.

Table 14 summarizes testable outputs for selected open problems. The entries are research criteria, not claims that the proposed constructions have already passed the stated tests.

Evidence-driven historical case studies could begin with comparatively well-documented processes, provided the investigators establish recording permissions, privacy protections, and domain-appropriate interpretive procedures. For such studies, successful modeling would mean that declared questions can be investigated reproducibly and that limitations and disagreements remain visible. Claims of predictive or ethical superiority would require an additional research design and evaluation against appropriate alternatives.

14.9 Procedural Justice and Future Generative Capacities

The normative interface of Section 12 separates cellular admissibility, evidential support, practical feasibility, and ethical permissibility. It also shows that an assessment of procedural histories need not factor through a reduced terminal topological description. A subsequent theory of procedural justice should begin by specifying the relevant persons or historically constituted participants, the rights and responsibilities at issue, the opportunities for contestation, and the ways earlier decisions may affect participation in later decision stages.

One unresolved problem concerns *recognitional effects*. A model may omit a candidate continuation because the observer’s present vocabulary does not describe the capacities or claims of certain participants. Such exclusion can influence the opportunities considered even when the omitted candidate’s actual feasibility remains unknown. The formal framework could record the provenance of a decision to admit or exclude a modeled continuation, but the justice of that decision requires principles concerning recognition, explanation, participation, and remedy. Existing work on epistemic injustice, democratic inclusion, and comparative justice supplies distinct normative resources rather than one ready-made cellular criterion [9, 24, 17].

A second problem concerns the ordering of interventions. Two histories may have the same terminal complex but create different intermediate opportunities, expose different participants to risks, or change who can influence later decisions. Such differences require temporally or causally interpreted records beyond a terminal homology group. An inquiry into justice of continuation may therefore investigate a family of ethically relevant history queries and determine which projections preserve their answers, applying the formal factorization criterion of Proposition 12.2 only after the ethical interpretation is supplied.

Future generative capacities introduce an additional dimension: interventions may change the means by which participants can formulate, recognize, and pursue later possibilities. A later normative paper may compare the distribution and durability of those capacities under explicitly specified procedural principles. The present article supplies representational interfaces for such inquiry, while leaving the normative principles and evidence of their application as separate research obligations.

14.10 Computational Infrastructure for Epistemic Continuation

A practical continuation system would need to represent historical evidence, candidate whole-spacetime extensions, competing epistemic models, and revisions without silently identifying distinct structures. A minimal research implementation could store immutable, versioned event, coupling, and face records; cellular boundary and incidence data; provenance assertions; and history-marking embeddings. Candidate continuations would reference a particular version of the epistemic historical base. Changes in that base would be represented through explicit revision records, with candidate transport computed only where a declared correspondence and admissibility check permit it.

The computational representation should support at least two complementary modes. A symbolic mode can verify finite cellular chain complexes, exact homology, and declared morphism properties. An exploratory mode can construct, compare, and visualize alternative continuation families under user-specified model predicates, while retaining evidence annotations and displaying unresolved structural and interpretive uncertainty. Neither mode should imply that enumeration is complete merely because all candidates inside a bounded search language have been listed.

Versioned provenance is particularly important for whole-spacetime semantics. A future-only display may be suitable for inspection, but the stored mathematical object should retain the base embedding and attachment map, or another presentation sufficient to reconstruct their historical relationship. The software should preserve the difference between *historical revision*, which alters what is currently represented as known, and *prospective continuation*, which extends a particular representation. A calculation on an outdated base should remain reproducible while being marked as depending on superseded assumptions.

A plausible first implementation would reproduce the finite experiments of Section 11, then add revision branching, proof-obligation reports, and query-preservation tests. The subsequent research question is whether those functions assist investigators in recognizing relevant alternative histories, documenting disagreements, or avoiding analytically significant information loss in a particular domain. Such utility is an empirical and practical claim requiring evaluation, beyond correctness of the algebraic calculations.

The research directions outlined here are related by a common methodological constraint: mathematical structure can support accountable comparisons of *represented* historical continuations only in relation to specified evidence, semantics, and purposes. The immediate formal priorities are coherent

epistemic revision, well-typed changes of modeling vocabulary, and identifiable restricted families of continuation models. Higher-dimensional representations, empirical applications, and normative theories can then be developed through domain-specific projects without treating the two-complex proposal as an established ontology or a complete predictor of future creation.

15 Conclusion

The present inquiry has examined a specific epistemological problem of future creation: the construction, comparison, and revision of possible continuations of historically represented relational spacetime. Its contribution is a preliminary formal vocabulary for articulating this problem while making the conditions of representation visible. The inquiry began from the observation that future creation concerns structures whose prospective events, couplings, attachments, and conditions of admissibility may be incompletely known. At the same time, the observer's available evidence, interpretive language, and capacity to anticipate alternatives have their own historical conditions. The resulting problem involves both the forms that a represented spacetime might take through continuation and the epistemic processes through which particular forms become conceivable. Sections 5 and 6 established the conceptual distinction on which the subsequent mathematical constructions depend.

A two-complex supplies a tractable, deliberately restricted setting for this investigation. Within the working GRT convention, vertices represent events, edges represent couplings, and two-cells represent higher-order primitives whose meanings must be specified in context. Entities and observing systems may be investigated through historically constituted, interpreted substructures rather than treated as primitive vertices. Cellular incidence and boundary maps provide exact combinatorial information, while histories, causal orders, semantic annotations, and evidence records require additional structures. The spin-foam precedent illustrates how two-complexes can support mathematically elaborate descriptions, but its physical dynamics and labels are distinct from those of the present epistemic model. The representational choice thus enables conditional mathematical analysis without conferring ontological priority on two-dimensional topology.

The paper's guiding construction is a history-indexed *continuation* of an epistemic complex. For a represented historical complex K , a candidate extension consists of an extended complex L and a history-preserving cellular embedding $i : K \hookrightarrow L$, supplemented where necessary by an evidence-sensitive admissibility witness. Dependent sum types keep the proposed codomain, the identity of its historical part, and the conditions qualifying an extension together in one object. Categorical underconstructions then organize extensions relative to their marked base, while composition and cellular pushouts describe selected relations among successive or alternative continuations. The resulting category-theoretic operations concern explicitly specified mathematical objects; their preservation of epistemic or domain-specific admissibility depends on separately established compatibility conditions. Sections 7 and 8 develop those constructions and their restrictions.

The preference for *whole-spacetime continuation semantics* is an analytical design decision. The codomain L includes the represented historical complex together with the prospective cells and their attachment relations. This presentation makes the historically identified base directly available for comparison, supports compositional treatment of staged extensions, and retains incidence information that a collection of prospective cells by itself may lack. A future-only construction with a suitable interface and attaching maps can encode the same structural content. The distinction is therefore a matter of representational emphasis and retained data, rather than a purported impossibility of future-only mathematics. Its importance becomes evident where continuations with the same abstract prospective component attach to different historically marked cycles and thereby affect different historical homology classes.

Relative homology and cellular reduction make selected consequences of that design precise. Inclusion-induced maps and the homology of a pair can register how new attachments change the role of historically identified cycles even where terminal homology groups alone give identical descriptions. The bounded attachment results of Section 10 specify the effects of prescribed two-cell attachments under explicit coefficient and incidence assumptions. Discrete Morse reductions permit simplification

under appropriate cellular conditions; their homotopy-theoretic correctness can coexist with the loss of a prospective event's identity, a provenance relation, or an order-sensitive interpretation. It follows that information preservation must be specified relative to the intended historical questions. A homological invariant, a causal dependency, and a normatively relevant opportunity are different kinds of information and warrant distinct preservation requirements.

The exact synthetic models in Section 11 demonstrate the resulting distinctions on finite structures. Marked attachments of the same historical graph produce different homology and torsion, historical revision can alter the computed significance of an otherwise unchanged proposed attachment, and alternative procedural orders can carry different intermediate interpretations despite agreement in terminal cellular structure. A separate reduction example exhibits the loss of a named prospective event under a permitted topological simplification. These calculations test stated algebraic and representational claims within bounded examples. Their significance lies in demonstrating possible failure modes of coarse descriptions and identifying the data needed to distinguish particular mathematical cases. They do not measure the coverage of a future-modeling language, establish predictive accuracy, or validate a two-complex representation of any particular social or physical history.

Epistemic reconstruction introduces a second, equally important axis of analysis. A historical representation is formed through observation and interpretation carried out by an observer whose capacities and practices are themselves historically conditioned. To express this dependence, Section 6 uses an assumed reference structure and types indexed by modeled observer supports, capacities, and procedures. The assumed structure serves a formal role only: it is neither an identification of ultimate reality nor a verified complete description of what exists. The type-theoretic dependency makes the historical positioning of observation explicit; it does not, on its own, explain the causal or developmental emergence of cognition. Similarly, the reconstruction relations of Section 9 connect evidence and observer contexts to candidate *epistemic* generative models. They express model plurality and conditional continuation families while leaving open whether any such model approximates an underlying generative process. A finite represented history may be consistent with models that admit different future extensions, and some possibilities may remain outside the current representational vocabulary altogether.

These observations also show why epistemic *revision* merits treatment alongside continuation. A continuation holds a selected historical representation fixed as a marked component of a possible extended history. Revision can correct, reinterpret, or replace part of the represented history itself. Consequently, transporting proposed continuations between revised contexts requires correspondences and admissibility checks whose existence and coherence cannot be taken for granted. A useful account of future creation needs to record both the conditional provenance of an anticipated extension and the historical circumstances under which that anticipation was constructed. The pending mathematical problems therefore include context-dependent transport, changes in event and relation vocabularies, and restricted notions of structural identifiability, as examined in Section 14.

The formalism further opens a carefully delimited interface with normative inquiry. The capacity to recognize a possible continuation can influence which proposals receive scrutiny, whose evidence becomes legible, and which choices are available at later stages of a procedure. An extension satisfying cellular conditions may still have uncertain feasibility and contested ethical significance. Section 12 therefore introduces distinct interfaces for structural admissibility, epistemic support, practical constraints, and normative evaluation. Its order-sensitive examples and information-factorization criterion indicate why a projection adequate for homological calculation may be inadequate for a particular procedural assessment. Such claims are conditional on explicitly specified normative questions. Justice, responsibility, and the distribution of future generative capacities call for justified ethical principles, historical interpretation, and participation by affected parties; topology supplies neither those principles nor their empirical application.

The notation and typing conventions underlying these comparisons are consolidated in Appendix A, including the distinction between reference-domain assumptions, epistemic contexts, continuation witnesses, and model-selection predicates.

The immediate mathematical value of the proposal is thus one of structured comparison. It gives a

language for identifying the represented history relative to which an extension is proposed, distinguishing candidate continuations that attach differently to that history, separating generative-model inference from the extensions admitted by a model, and exposing which transformations preserve selected topological or interpreted information. Established tools from type theory, category theory, cellular topology, and discrete Morse theory do substantial work in these constructions. The present paper contributes an explicitly scoped arrangement of those tools around the epistemology of future creation and a collection of mathematical illustrations of its design choices and limitations. The question of originality is correspondingly local to the definitions, examples, and analytical connections developed here; the paper makes no general claim to the invention of its constituent theories.

Several tasks remain before the framework could support explanatory or practical applications. The most immediate are to specify observably grounded models of epistemic capacities, construct coherent procedures for revising and transporting continuation families, and characterize what restricted future queries available evidence can identify. Extending the mathematics to higher-dimensional complexes or richer dynamical structures may be necessary where two-cell incidence is insufficient. Domain-specific investigations would additionally need justified semantics for cells and couplings, traceable historical evidence, comparisons with existing representations, and protocols for identifying unrecognized or excluded alternatives. A subsequent normative theory could then study procedural justice in future creation through explicitly interpreted histories, opportunities, and responsibilities while retaining the revisability of its epistemic premises.

Future creation, as treated in this paper, is consequently a problem of *historically situated representation and conditional continuation*. Its formal investigation begins with a represented past, retains the relations through which proposed new structures attach to that past, and acknowledges that both the proposed continuations and the capacities used to recognize them may change. This perspective offers a preliminary basis for examining the topology of epistemic possibilities and the information needed to compare them. The broader historical, ontological, predictive, and ethical questions remain open to further formal development and to independent forms of inquiry.

A Notation and Type-Theoretic Conventions

This appendix consolidates the formal language used in Sections 4–9 and specifies the reading conventions needed to compare their constructions. Its role is technical: the paper uses dependent types to index descriptions of historical spacetime and its candidate continuations, cellular maps to preserve a selected represented history, and predicates or relations to encode epistemic admission. These layers have different purposes. The appendix first fixes their universes and judgments, then records the observer-indexed and continuation-indexed types, the treatment of equality and witnesses, and the conventions for interpretation, revision, and topological calculations. The notation is proposed for the present formalism; the dependent-type operations themselves are standard [22].

A.1 Universes, Judgments, and Finite Encodings

We write $x : A$ to assert that x is an inhabitant of the type A , and $A : \text{Type}_\ell$ when a size level needs to be made explicit. A judgment $\Gamma \vdash x : A$ means that x is well typed under declarations and assumptions Γ . A term, a proof of a proposition, and a claim that a historical episode actually occurred must be kept distinct: type checking establishes a conclusion under the specified formal context, while the latter claim requires evidence external to the typing judgment. Throughout the manuscript, types of finite complexes may be realized by concrete encodings of cells, attaching maps, labels, and declared interpretation data; no unrestricted type of all possible topological spaces is required.

The symbol EpCplx_2 denotes a selected type of epistemically interpreted finite two-complex descriptions. The symbol Ref_2 denotes a separately selected type of *assumed reference descriptions*, used only in the conditional observer-constitution construction of Section 6. Both types contain formal representations. Membership in Ref_2 does not assert that a description is ultimate reality or that relational spacetime

Table 15: Principal notation for the two-complex and history-preserving continuation constructions.

Notation	Defined role in the present formalism
$\text{EpCplx}_2; \text{Ref}_2$	Types of epistemic two-complex encodings and stipulated formal reference descriptions, respectively
$K, L, \hat{K}, R$	Historical or extended complexes, an emphasized epistemic history, and a stipulated reference description
$C_q(K; A); \partial_q^K$	Cellular q -chains with coefficients in A and the corresponding boundary map
$H_q(K; A); H_q(L, K; A)$	Absolute and relative homology for an explicitly specified inclusion of the historical base
$\text{Emb}_2(K, L)$	Type of dimension-preserving, history-label-compatible cellular embeddings onto subcomplexes
$i : K \hookrightarrow L$	History-marking embedding; $i(K)$ is the retained historical image and L is the whole candidate spacetime
$\mathbf{C}_2^{\text{emb}}$	Selected category of epistemic complexes with composition-closed admissible structural embeddings
$K \downarrow \mathbf{C}_2^{\text{emb}}$	Undercategory of structural extensions with fixed historical source K
$\prec_L$	Additional interpreted causal or procedural order on represented events, when supplied

is a metaphysically fundamental substrate. The distinction concerns the roles of two modeling domains, not a verified hierarchy of what exists.

When the distinction matters, K and L denote encoded two-complexes, $\hat{K}$ emphasizes a historically situated epistemic reconstruction, and R denotes a stipulated reference description. The circumflex on $\hat{K}$ is an interpretive reminder, not a new type former. Similarly, subscripts s and i may mark stages of available information or the identity of an observing context. The use of s does not introduce a universal physical time or presuppose a metaphysically privileged present.

Table 15 gathers the principal structural symbols. Entries describing semantics are conventions of this manuscript; they should not be read as properties intrinsic to a CW complex.

A.2 Dependent Sums, Products, and Propositions

For a base type A and a family $B : A \rightarrow \text{Type}_\ell$, the dependent sum $\sum_{x:A} B(x)$ collects pairs (x, b) with $b : B(x)$. The dependent product $\prod_{x:A} B(x)$ describes functions assigning an inhabitant $b_x : B(x)$ to each $x : A$, subject to the specified universe and formation rules. When B does not vary with x , these specialize to an ordinary product and an ordinary function type. The typing convention is recorded in Equation 79.

$$\frac{\Gamma \vdash a : A \quad \Gamma \vdash b : B(a)}{\Gamma \vdash (a, b) : \sum_{x:A} B(x)}. \quad (79)$$

Equation 79 is a schematic introduction rule, with the usual proviso that B is a well-formed family. In the continuation construction, the type of a history-marking embedding depends on the particular target complex L . The dependent sum is therefore substantive: it retains both the target object and the identification of history within that target.

We use Prop for a selected proposition-valued interface, with proof irrelevance assumed where the intended application distinguishes only truth from falsity. This is a modeling convention; it need not be identified without qualification with every proof-relevant identity type or homotopical interpretation of propositions in dependent type theory. A type $\text{Adm}_c(L, i)$ may instead be proof relevant, particularly when witnesses contain independently meaningful evidence, provenance, or constraints. Where proof relevance is important, two inhabitants supporting the same extension can remain distinct records. When the application intends only an admission condition, its propositional truncation or a selected truth-valued predicate may be used.

The symbols $\emptyset$, $\subseteq$, and $\mathcal{P}(X)$ may be used at a set-theoretic presentation level for finite enumerations or sets of encoded outputs. Such presentations must be distinguished from the intensional types of descriptions and witnesses. In particular, identifying all records with the same target complex would erase the possibility of distinct historical embeddings and distinct justifications for the same candidate continuation.

A.3 Historical Observers and Epistemic Contexts

In Section 6, a postulated $R : \text{Ref}_2$ supplies a formal domain in which a historically supported observing system may be described. The type $\text{Sub}(R)$ consists of marked subcomplex descriptions (S, j) , with $j : S \hookrightarrow R$ in the selected reference scheme. The capacity family $\text{Cap}(R, S, j)$ records stipulated observing abilities associated with that support; $\text{Proc}(R, S, j, a)$ records permitted observation or interpretation procedures under a chosen capacity description a . Their dependent assembly was defined in Equation 30 and is reproduced here for notational reference as Equation 80.

$$\text{ObsCtx}(R) = \sum_{(S,j):\text{Sub}(R)} \sum_{a:\text{Cap}(R,S,j)} \text{Proc}(R, S, j, a). \quad (80)$$

An inhabitant $o = (S, j, a, \pi)$ of Equation 80 expresses conditional dependence within the stipulated model. It provides no function that automatically derives a real observer's capacities from R ; historical constitution at the explanatory level would need independently specified dynamics, mechanisms, and evidence.

For such an o , $\text{Data}(R, o)$ denotes the selected accessible evidence domain. The notation $\mathcal{O}_{R,o}$ denotes an observation–interpretation relation between evidence and one or more annotated epistemic complexes. The relation can be partial or multivalued, and its codomain need not share the reference complex's cellular decomposition. Accordingly, it is distinct in type and meaning from a cellular continuation embedding. Section 6 supplies the interpretive qualifications of this distinction.

The compact epistemic context used in subsequent sections is $c = (K_c, D_c, p_c, o_c)$, where K_c is the represented historical complex, D_c selected evidence, p_c provenance and interpretation policy, and o_c an observer-condition record. We use ECtx as shorthand for a chosen type of such compatible records; it is not asserted that the manuscript has defined a universal type of all epistemic contexts. Compatibility may require a selected provenance linkage between an observer record and an epistemic complex. If two investigators use different recording schemes, a mapping between their context types must be supplied before transporting candidate continuations.

A.4 Whole-Spacetime Extension and Admissibility Types

Fix $c : \text{ECtx}$ with historical complex $K_c : \text{EpCplx}_2$. The structural extension type $\text{Ext}_2(K_c)$ was introduced in Equation 35. The epistemically admitted continuation type is obtained by adjoining a context-indexed admissibility family. Equation 81 records the full formation used in Sections 7 and 9.

$$\text{Cont}(c) := \sum_{L:\text{EpCplx}_2} \sum_{i:\text{Emb}_2(K_c, L)} \text{Adm}_c(L, i). \quad (81)$$

An inhabitant $u = (L, i, w)$ has a target L , a marked inclusion i , and an admissibility witness w . This expression identifies the syntactic and declared-context conditions for a *candidate*; it supplies neither a frequency estimate of future realization nor a general moral permission to pursue that candidate. In particular, the codomain L of i denotes the *whole extended spacetime*; the image $i(K_c)$ is the inherited history. The set of cells newly represented in L can be extracted relative to this image, yet a bare complement of cells usually lacks the boundary and interface data necessary for a standalone subcomplex.

The admissibility witness type may be analyzed into structural, epistemic, operational, or domain-specific factors as required for an investigation. The mathematical type constructor itself imposes no

universal criterion of historical feasibility. Moreover, $\text{Emb}_2(K_c, L)$ presupposes a strict historical inclusion convention. A representational correction that removes or merges old cells is modeled as a revision, with separately typed maps or relations, and is not silently recast as an embedding.

For a chosen continuation model $m : \text{Mod}(c)$, the additional model-selection predicate $G_{m,c}$ produces a model-admitted family. Equation 82 restates the separation between contextual admissibility and model-dependent selection.

$$G_{m,c} : \text{Cont}(c) \longrightarrow \text{Prop}, \quad \Gamma_c(m) := \sum_{u : \text{Cont}(c)} G_{m,c}(u). \quad (82)$$

The judgment $u : \text{Cont}(c)$ alone states that an extension is admitted under the stipulated context-dependent type. An inhabitant of $G_{m,c}(u)$ further states that model m admits it. Several models may admit the same structural continuation with different evidential explanations. The model-reconstruction relation $\Phi(c)$ of Section 9 records which model descriptions are supported or selected under the observer's context. Consequently, $m : \Phi(c)$ is shorthand for a model equipped with its reconstruction membership witness when $\Phi(c)$ is implemented as a dependent or subset-like type. This shorthand should not be interpreted as a numerical inference probability.

A.5 Identity, Equivalence, and Historical Marking

Three comparisons recur throughout the manuscript. Strict equality of descriptions depends on the adopted representation system, including cell identifiers and declared labels. Isomorphism of underlying cellular complexes permits relabeling compatible with incidence and chosen semantics. Isomorphism *over a fixed history* additionally preserves the selected embedding of that history, as made explicit in Equation 83.

$$h : L_1 \xrightarrow{\cong} L_2, \quad h \circ i_1 = i_2, \quad \text{for } i_j : K \hookrightarrow L_j. \quad (83)$$

Equation 83 is the comparison appropriate for based extensions when the identity of historical cells matters. A homotopy equivalence or abstract isomorphism of homology groups is weaker and may fail to preserve historical attachments, labels, or procedural ordering. Conversely, two distinct encodings may be equivalent for a deliberately narrower analytical query. A quotient of representations must therefore state the information it forgets and the inquiries for which that loss is acceptable.

Categorical comparisons are made in a selected ordinary category $\mathbf{C}_2^{\text{emb}}$ whose arrows are composition-closed structural embeddings. In its undercategory, objects are arrows $i : K \hookrightarrow L$. An arrow between two such objects is a map $h : L_1 \hookrightarrow L_2$ satisfying Equation 83 without requiring h to be invertible. The category does not become a two-category merely because its objects are *two-dimensional* complexes. Higher categorical structure, if eventually introduced to compare revisions or transformations of maps, would require additional definitions.

A.6 Revisions, Relative Pairs, and Cross-Layer Semantics

A revision $c_s \rightsquigarrow c_{s+1}$ concerns a changed epistemic history, evidence base, interpretive policy, or observing condition. The symbol $\rightsquigarrow$ intentionally denotes a relation until a precise category of contexts and revision maps has been specified. By contrast, $i : K_c \hookrightarrow L$ denotes a cellular continuation with a fixed represented base. Transporting a continuation across a revision can sometimes be represented by a suitable pushout; it requires compatible maps and may change the semantics or admissibility of the proposed extension. A reindexing functor between continuation categories is therefore an additional construction rather than a consequence of merely possessing two contexts.

For a pair $(L, i(K))$, relative chains use $C_q(L, i(K); A) = C_q(L; A) / i_* C_q(K; A)$ with the usual boundary induced from the pair. Writing (L, K) abbreviates this marked pair only when the embedding has been fixed. A relative homology group measures topology of the pair under chosen coefficients; it is not a measure of an observer's ignorance or of a future's ethical desirability. Discrete Morse reduction

Table 16: Formal interface register separating epistemic, cellular, model-selective, and normative claims.

Interface	Typical notation	Additional warrant required
Assumed reference description	$R : \text{Ref}_2$	Chosen modeling assumptions; no ultimate ontology inferred
Evidence-based reconstruction	$\mathcal{O}_{R,o}; c : \text{ECtx}$	Access, interpretive procedure, provenance
Structural extension	$i : K_c \hookrightarrow L$	Cellular incidence and marked historical identification
Contextual admission	$w : \text{Adm}_c(L, i)$	Declared contextual conditions and witness meaning
Model-selected continuation	$G_{m,c}(u)$	Model reconstruction and model-selection assumptions
Epistemic revision	$c_s \rightsquigarrow c_{s+1}$	Supported correspondence between revised contexts
Normative assessment	$J(\gamma)$ or a stipulated predicate	Independent ethical premises, domain evidence, participation where relevant

likewise has a topological preservation target, while any required preservation of named events, causal annotations, and normative queries must be imposed separately.

Table 16 records the main interfaces whose types and evidential roles should remain distinct across Sections 6–12.

The notation in Table 16 is deliberately layered. A well-typed term is a formally described candidate, a selected model supplies conditional reasons to admit it, and an application supplies additional evidence concerning feasibility and significance. The appendix fixes these conventions for the worked cellular and computational constructions in Sections 10 and 11. The subsequent technical appendices are intended to expand the individual constructions, beginning with explicit two-complex encodings and boundary calculations.

B Two-Complex Spacetime Constructions

This appendix supplies explicit cellular presentations underlying the relational-spacetime examples in Section 3 and the continuation types of Section 7. Its objective is to make the passage from a finite incidence specification to a historically marked extension reproducible and to distinguish the mathematical data required by different representations of a proposed future. The constructions are synthetic. A vertex denotes a represented event, an edge a stipulated coupling, and a two-cell a higher-order primitive whose semantic content requires separate specification. A spin-foam decoration is examined as an established mathematical instantiation of a two-complex, with its physical interpretation reserved for the relevant physical theory. The appendix develops an oriented combinatorial presentation, historical inclusions, cellular attachments, two worked continuation families, and explicit criteria for comparing semantic annotations.

B.1 Finite Cellular Presentations and Attaching Words

A finite connected two-dimensional CW complex can be described by its finite zero- and one-skeleta and the attaching maps of its two-cells. For the combinatorial examples below, the one-skeleton is a finite graph with oriented edges, allowing loops and parallel edges. Each two-cell is attached along a closed edge path represented by a cyclic word in oriented edges and their formal inverses. Attaching maps whose cellular boundary traverses an edge repeatedly are permitted. This presentation includes finite examples that are not regular CW complexes; regularity is therefore stated separately when needed. The standard cellular background follows Hatcher [12].

Table 17: Data layers of a finite represented two-complex and the information retained at each layer.

Layer	Recorded data	Principal analytical use
Cellular presentation	V, E, F , endpoints, attaching words	Recover incidence, cell identities, and attachment descriptions
Cellular chain complex	$C_2 \xrightarrow{\partial_2} C_1 \xrightarrow{\partial_1} C_0$	Compute homology and relative homology
Historical marking	$i : K \hookrightarrow L$, marked labels	Distinguish attachments affecting different historical components
Epistemic annotations	Evidence, interpretation, uncertainty, provenance	Examine representational support and revision
Causal and normative data	Separately stipulated orders and evaluation contexts	Study conditional ordering and ethical questions

Let V , E , and F denote finite sets of vertices, edges, and faces. Each oriented edge e has a source $s(e)$ and target $t(e)$ in V . For each face f , choose a closed attaching word w_f in the directed edges of the one-skeleton, with reversal encoded by e^{-1} . When an arbitrary attaching map cannot be represented by an edge word in an initial cell structure, a compatible subdivision or a more general cellular description is required. Equation 84 records the finite data used in the present examples.

$$\text{CW}_2^{\text{word}} = (V, E, F, s, t, (w_f)_{f \in F}), \quad w_f = e_1^{\epsilon_1} \cdots e_m^{\epsilon_m}, \quad \epsilon_j \in \{+1, -1\}. \quad (84)$$

The word in Equation 84 is subject to the condition that its successive oriented endpoints match and its final endpoint equals its initial endpoint. The choice of starting point on a boundary circle and the orientation convention affect the written word; the corresponding cellular attachment can be compared up to the relevant equivalences of attaching maps. At the level of oriented cellular chains, only the signed edge traversal numbers enter the boundary matrix.

The oriented integer chain groups are $C_0 = \mathbb{Z}[V]$, $C_1 = \mathbb{Z}[E]$, and $C_2 = \mathbb{Z}[F]$. The two cellular boundaries used throughout the manuscript are given by Equation 85.

$$\partial_1[e] = [t(e)] - [s(e)], \quad \partial_2[f] = \sum_{e \in E} \deg_e(w_f)[e], \quad \deg_e(w_f) = \#(e) - \#(e^{-1}). \quad (85)$$

The signed traversal count $\deg_e(w_f)$ is the cellular incidence number for these graph-attaching presentations. A valid closed boundary path satisfies $\partial_1 \partial_2[f] = 0$. This condition is necessary for a chain-complex encoding and follows automatically from a consistently represented cellular attaching word. The converse does not determine the attaching word: different words may have identical cellular incidence vectors, so the boundary matrices need not retain the nonabelian information of the attaching maps. For example, commutators can have zero abelianized boundary despite defining nontrivial two-cell attachments. Any inquiry that depends on such distinctions must preserve words or an equivalent attaching-map specification beyond the chain complex.

Table 17 distinguishes the levels that the paper occasionally combines in a single example. An orientation of an edge is part of the incidence convention and is not, by itself, a temporal or causal claim. Likewise, the existence of a two-cell supplies an attaching relation, while its interpretation as a particular relational primitive requires an additional semantic declaration.

B.2 Historical Subcomplexes and Relative Attachments

A historical base $K \subseteq L$ is a subcomplex when it contains the closure of each of its cells. In a finite word presentation, the declaration that an edge belongs to K requires its endpoints to belong to K , and the declaration that a face belongs to K requires the attaching edges and their endpoints. A history-preserving inclusion $i : K \hookrightarrow L$ identifies the represented cells that are retained in the candidate whole-spacetime

$$\begin{array}{ccc}
S^{q-1} & \xrightarrow{\varphi} & K \\
\downarrow \subseteq & & \downarrow i \\
D^q & \longrightarrow & L = K \cup_{\varphi} D^q
\end{array}$$

The historical base and the attaching map remain available in the whole extension. A list of prospective cells alone generally omits their historical incidence.

Figure 11: A relative cellular attachment used to construct a whole-spacetime continuation from a selected historical base. The diagram describes a mathematical pushout and supplies no criterion of empirical future feasibility.

extension. This inclusion is structural; whether the selected base is an adequate historical representation is an epistemic question outside the cellular definition.

A basic extension is obtained by attaching a cell along a specified map from its boundary into an existing complex. The pushout form in Equation 86 records the attachment and its historical interface, as illustrated schematically in Figure 11.

$$\begin{array}{ccc}
S^{q-1} & \xrightarrow{\varphi} & K \\
\downarrow & & \downarrow i \\
D^q & \longrightarrow & L = K \cup_{\varphi} D^q,
\end{array} \quad q \in \{0, 1, 2\}, \quad (86)$$

with the customary empty attaching sphere when $q = 0$. The square in Equation 86 is a pushout in the category of topological spaces under the stated attaching construction. A finite sequence of such attachments yields a relative CW complex. The intermediate stages provide a chosen construction order; that order alone does not imply a physically or historically observed sequence of events.

For the marked pair (L, K) , the relative chain groups $C_q(L, K) = C_q(L)/C_q(K)$ retain the classes of new q -cells, with historical boundary components becoming zero in the quotient. In contrast, the cell complement $\text{Cells}(L) \setminus \text{Cells}(K)$ generally fails to be a subcomplex. The pair is thus analytically richer than its unmarked relative chain complex. An abstract quotient L/K can support useful calculations but may identify historically different attaching maps after the historical base is collapsed. Subsection B.4 gives a finite case of this distinction.

B.3 An Explicit Triangular Continuation

The triangular construction of Section 3.8 provides a finite check involving new cells in dimensions zero, one, and two. Let the historical complex K consist of vertices v_0, v_1, v_2 , edges $a : v_0 \rightarrow v_1$, $b : v_1 \rightarrow v_2$, and $c : v_0 \rightarrow v_2$, together with a face f_A attached along the oriented triangle abc^{-1} . Its incidence boundary is $\partial_2 f_A = a + b - c$. The extension L adds vertex v_3 , edges $d : v_1 \rightarrow v_3$ and $e : v_2 \rightarrow v_3$, and a face f_B attached along $de^{-1}b^{-1}$. Its boundary is $\partial_2 f_B = d - e - b$.

Relative to the ordered vertices (v_0, v_1, v_2, v_3) , edges (a, b, c, d, e) , and faces (f_A, f_B) , the integral boundary matrices appear in Equation 87.

$$[\partial_1]_L = \begin{pmatrix} -1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}, \quad [\partial_2]_L = \begin{pmatrix} 1 & 0 \\ 1 & -1 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix}. \quad (87)$$

Equation 87 satisfies $[\partial_1]_L[\partial_2]_L = 0$. The original triangle and the additional triangle together form a triangulated disk. The underlying complex is connected and contractible, hence $H_0(L; \mathbb{Z}) \cong \mathbb{Z}$ and $H_1(L; \mathbb{Z}) = H_2(L; \mathbb{Z}) = 0$.

The relative complex has generators $\bar{v}_3$ in degree zero, $(\bar{d}, \bar{e})$ in degree one, and $\bar{f}_B$ in degree two. Suppressing the historical generators gives the matrices in Equation 88.

$$[\partial_1]_{(L,K)} = \begin{pmatrix} 1 & 1 \end{pmatrix}, \quad [\partial_2]_{(L,K)} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}. \quad (88)$$

The kernel of $[\partial_1]_{(L,K)}$ is generated by $(1, -1)$, precisely the image of $[\partial_2]_{(L,K)}$. It follows that $H_q(L, K; \mathbb{Z}) = 0$ for $q = 0, 1, 2$. This is an example in which new represented event, coupling, and face cells have been attached even though the relative homology is trivial. The relative groups answer a topological question about the pair; they do not inventory new represented occurrences. The added edges d and e still have endpoints in the historical subcomplex, and the new face f_B still meets the historical edge b . The complete cell presentation and embedding retain those relations.

B.4 Historically Marked Circle Attachments

A second construction makes the loss of historically distinguished attachment information under a bare future quotient explicit. Let $K = S_a^1 \vee S_b^1$ be the wedge of two oriented circles with a common vertex. Its finite CW presentation contains one zero-cell and two loop edges a, b . Construct L_a by attaching one two-cell along a , and L_b by attaching one two-cell along b . Both continuations include the same identified historical base K . The corresponding cellular boundary maps are recorded in Equation 89.

$$C_2(L_a) \cong \mathbb{Z} \xrightarrow{\begin{pmatrix} 1 \\ 0 \end{pmatrix}} \mathbb{Z}^2, \quad C_2(L_b) \cong \mathbb{Z} \xrightarrow{\begin{pmatrix} 0 \\ 1 \end{pmatrix}} \mathbb{Z}^2. \quad (89)$$

Each resulting space is homotopy equivalent to a circle and satisfies $H_1(L_a; \mathbb{Z}) \cong H_1(L_b; \mathbb{Z}) \cong \mathbb{Z}$. However, the kernels of the maps induced by the marked historical inclusions are different. Equation 90 records their values.

$$\ker(H_1(K) \rightarrow H_1(L_a)) = \mathbb{Z}[a], \quad \ker(H_1(K) \rightarrow H_1(L_b)) = \mathbb{Z}[b]. \quad (90)$$

In both cases the quotient obtained by collapsing the marked historical base is homeomorphic to S^2 , the newly attached two-cell with its boundary identified to a point. Consequently, the abstract quotient spaces agree, yet the original attaching maps act on distinct historical cycles. The relative connecting homomorphisms $H_2(L_a, K) \rightarrow H_1(K)$ and $H_2(L_b, K) \rightarrow H_1(K)$ recover $[a]$ and $[b]$, respectively, when the historical identifications are retained.

Proposition B.1 (Marked historical attachment distinguishes quotient-equivalent extensions). *For the two extensions $K \hookrightarrow L_a$ and $K \hookrightarrow L_b$ above, the quotient spaces L_a/K and L_b/K are homeomorphic, but the two extensions are not isomorphic over K as cellular complexes preserving the given historical identification.*

Proof. The respective inclusion-induced kernels are distinct subgroups of the marked group $H_1(K; \mathbb{Z})$ by Equation 90. If an isomorphism $h : L_a \rightarrow L_b$ over K existed, the equation $h \circ i_a = i_b$ would induce $h_* \circ (i_a)_* = (i_b)_*$ in first homology, making their kernels equal. This contradicts the calculation. Collapsing K maps each attached disk boundary to the basepoint and produces S^2 in either case. $\square$

Proposition B.1 supplies a limited, structurally precise justification for the preferred whole-spacetime continuation semantics. It does not imply that future-only methods are inadequate: retaining the attaching word, a marked interface, or equivalent relative data distinguishes the two continuations as well. It shows that the *unmarked* quotient alone is insufficient for this historically indexed comparison.

B.5 Degree Attachments and Cellular Regularity

Attaching words permit a controlled comparison in which the proposed future cell has a fixed dimension while the attaching map changes. Let $K = S^1$ with one vertex and an oriented loop edge a . Attach one oriented two-cell f_n with attaching word a^n for $n \in \mathbb{Z}$; for $n = 0$ take the constant attaching map. The cellular chain groups of the extension L_n are $C_2 \cong C_1 \cong C_0 \cong \mathbb{Z}$, with $\partial_1 = 0$ and ∂_2 equal to multiplication by n . Equation 91 summarizes the homological consequences.

$$H_1(L_n; \mathbb{Z}) \cong \mathbb{Z}/n\mathbb{Z}, \quad H_2(L_n; \mathbb{Z}) = 0 \quad (n \neq 0); \quad H_1(L_0; \mathbb{Z}) \cong H_2(L_0; \mathbb{Z}) \cong \mathbb{Z}. \quad (91)$$

Here $\mathbb{Z}/n\mathbb{Z}$ is understood for nonzero n , so a degree-one attachment gives trivial first homology, a degree-two attachment gives $\mathbb{Z}/2\mathbb{Z}$, and a constant attachment gives a wedge $S^1 \vee S^2$. Higher-degree wrapping is generally not a regular cellular attachment in the displayed one-cell presentation. This example therefore motivates retaining general CW attaching maps when the formalism explores changes in the topology of possible continuations; a strict regular-complex convention would exclude such elementary presentations unless the modeling class were modified.

The comparison has an epistemic interpretation only after an account is given of what the loop, face, and attaching multiplicity encode. The integer n is a stipulated cellular degree and must not be read as a count of real-world causal influences, opportunities, or obligations without independent modeling evidence.

B.6 Spin-Foam Decorations and Relational Semantics

Spin-foam models demonstrate that a two-complex may carry additional mathematical labels and dynamical rules. In familiar spin-foam constructions, faces receive representation-theoretic labels and edges may carry intertwining data, with amplitudes assembled according to the selected physical model [2, 15]. Group-field-theoretic expansions supply another setting in which suitable spin-foam-like complexes arise as diagrams under specified dynamical assumptions [14]. These facts motivate the use of decorated cellular structures, while the physical amplitudes and field equations remain specific to their originating theories.

For the epistemic relational representation, a decoration may instead comprise partial evidence references, proposed primitive types, confidence-qualified interpretations, provenance links, or modeled dependencies. It is useful to express the distinction schematically by the record in Equation 92.

$$\hat{K}^{\text{dec}} = (V, E, F, s, t, (w_f), \lambda_V, \lambda_E, \lambda_F, \text{Prov}, \preceq), \quad (92)$$

where the label maps and optional relation $\preceq$ are separately defined in a particular application. Equation 92 is a flexible record schema, not an assertion that all of the fields possess known or fixed semantic types. In particular, $\preceq$ is a separately supplied dependency or precedence relation when the represented domain warrants one; it does not follow merely from chosen edge orientations. A represented entity may be associated with a historically constituted subcomplex and interpretation rather than being identified with a vertex.

A spin foam supplies an illustrative *instantiation of a decorated two-complex*; it is not assumed to be the ontological generator of the modeled relational spacetime. Likewise, an epistemic continuation model can specify candidate cellular extensions without asserting a quantum origin for social, institutional, or cognitive processes. The assumed ontological reference domain used elsewhere in the manuscript is a formal convenience, with the ultimate constitution of reality held open.

B.7 Structural Checks and Continuation-Data Requirements

The preceding constructions require checks at several distinct levels. At the cellular level, edge endpoints and attaching words must form valid paths, historical subsets must be closed under incidence, and the chain condition $\partial_1 \partial_2 = 0$ must hold. At the relative level, a candidate embedding must retain the

selected historical identification and induce the intended chain inclusion. At the semantic level, provenance, event identity, and historically significant distinctions require declared preservation conditions. An admissibility witness in the continuation type of Section 7 is evaluated against those conditions, not against homology alone.

The companion script `examples/verify_appendix_B.py` checks the exact integer boundary matrices for the triangular example, its relative acyclicity, the marked-circle inclusion kernels, and the degree-attachment cases. The script verifies the stated synthetic computations; it supplies no empirical validation or prediction of future events. Appendix A provides the corresponding notation conventions, and the present constructions prepare the more detailed exact-sequence calculations intended for Appendix C.

C Cellular Boundary Operators and Relative Homology Calculations

This appendix develops the chain-level calculations underlying the continuation comparisons in Sections 4, 10, and 11. Its objective is to identify precisely which historical attachment data are accessible through cellular chains, relative homology, inclusion-induced maps, and connecting homomorphisms. The argument begins with a historically marked CW pair, derives its exact sequence, and calculates the consequences of alternative attaching maps, including integral torsion and a relative acyclic extension. A final comparison establishes a limit of homology as a representation of higher-order attachment information. These are calculations for stipulated epistemic complexes, rather than evidence that an indicated continuation will occur. Standard foundations for cellular and relative homology follow Hatcher [12]; the explicit cell presentations and historical markings are those of Appendix B.

C.1 Historical Subcomplexes and the Relative Chain Sequence

Let $i : K \hookrightarrow L$ be the inclusion of a finite two-dimensional CW subcomplex into an extended CW complex. We use the chosen cellular orientations and integer coefficients throughout unless a coefficient field is specified. The subcomplex condition ensures that $C_*(K)$ is a sub-chain-complex of $C_*(L)$. The relative cellular chains are quotients by the historical chains, as expressed in Equation 93.

$$0 \longrightarrow C_*(K; \mathbb{Z}) \xrightarrow{i_\#} C_*(L; \mathbb{Z}) \xrightarrow{q_\#} C_*(L, K; \mathbb{Z}) \longrightarrow 0, \quad C_q(L, K) = C_q(L)/C_q(K). \quad (93)$$

For a relative CW presentation, a choice of the old and new oriented cells gives a direct-sum decomposition of the *graded abelian groups* $C_q(L) \cong C_q(K) \oplus N_q$, where N_q is freely generated by the added q -cells. In this cellular basis the boundary maps necessarily take an upper-block-triangular form. Equation 94 fixes the convention.

$$[\partial_q^L] = \begin{pmatrix} \partial_q^K & A_q \\ 0 & D_q \end{pmatrix}, \quad A_q : N_q \longrightarrow C_{q-1}(K), \quad D_q : N_q \longrightarrow N_{q-1}. \quad (94)$$

The entries of A_q encode the historical boundary components of new cells at chain level. The matrices D_q encode the relative differential, because the quotient in Equation 93 kills the chains carried by K . The chain identity $\partial_{q-1}^L \partial_q^L = 0$ yields the compatibility equations in Equation 95.

$$\partial_{q-1}^K A_q + A_{q-1} D_q = 0, \quad D_{q-1} D_q = 0. \quad (95)$$

The decomposition is relative to the selected CW structure and orientations. It is not an assertion of a canonical splitting of chain complexes: in general, the off-diagonal term A_q prevents the old and new generators from forming independent subcomplexes. It also explains how two extensions may share an isomorphic abstract relative chain complex while differing in their incidence with historically identified cells.

Remark C.1 (Chain-level scope of historical attachment). The matrices A_q retain signed cellular boundary coefficients. They may lose finer information in the attaching maps, including noncommutative edge words. A complete cellular presentation, together with chosen historical markings and epistemic annotations, may therefore distinguish extensions that all of their homology groups fail to distinguish. Subsection C.7 supplies an explicit case; Remark C.1 records this scope limitation.

C.2 Connecting Homomorphisms and Marked Attachment Classes

The short exact sequence of chain complexes in Equation 93 induces the long exact sequence of the pair (L, K) . The segment relevant to prospective two-cell attachments is given in Equation 96.

$$\begin{aligned} H_2(K) &\xrightarrow{i_*} H_2(L) \xrightarrow{q_*} H_2(L, K) \xrightarrow{\delta_2} H_1(K) \\ &\xrightarrow{i_*} H_1(L) \xrightarrow{q_*} H_1(L, K) \xrightarrow{\delta_1} H_0(K) \xrightarrow{i_*} H_0(L). \end{aligned} \quad (96)$$

For a relative q -cycle $\bar{z} \in C_q(L, K)$, choose a lift $z \in C_q(L)$. Its boundary lies in the historical chain complex because $D_q \bar{z} = 0$. Equation 97 states the induced connecting map.

$$\delta_q[\bar{z}] = [\partial_q^L z] \in H_{q-1}(K), \quad \text{whenever } [\bar{z}] \in H_q(L, K). \quad (97)$$

The construction is independent of the chosen representative and lift at the homology level. In the block presentation, if $n \in \ker D_q \subseteq N_q$ represents the relative cycle, the connecting class is $[A_q n]$. Equation 98 makes the historical incidence of a relative cycle explicit.

$$\delta_q[n] = [A_q n]_{H_{q-1}(K)} \quad \text{for } n \in \ker D_q, \quad (98)$$

where classes differing by relative boundaries are identified in the domain. Equation 95 ensures $A_q n$ is a historical cycle, and the chain-map identities imply that the class is well defined. The connecting map is therefore a natural means of recording what selected *relative homology classes* attach to historically marked homology classes. Its information is richer than the relative groups viewed in isolation and weaker than the full original attaching maps.

Exactness of Equation 96 supplies two distinct statements. The image of δ_2 is exactly the subgroup of historical one-dimensional homology classes that become trivial under inclusion into the continuation. Its kernel consists of relative two-classes lifted from absolute two-classes in the extension. Both facts are conditional on the chosen pair; neither assigns historical or ethical meaning to the generators by itself.

C.3 Alternative Attachments to a Historically Marked Pair of Cycles

Let the historical complex $K = S_a^1 \vee S_b^1$ consist of two oriented loop edges sharing one vertex, with historical labels a and b . The two continuations L_a and L_b attach a single two-cell along a and b , respectively, as specified in Subsection B.4. Since each extension introduces a two-cell and no one- or zero-cells, both relative chain complexes have $C_2(L_x, K) \cong \mathbb{Z}$ and zero relative groups in lower degrees. Consequently, their relative second homology groups are abstractly identical.

The connecting maps nevertheless differ. With respect to the marked basis $([a], [b])$ of $H_1(K; \mathbb{Z}) \cong \mathbb{Z}^2$, Equation 99 records the two integer matrices.

$$[\delta_2^a] = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad [\delta_2^b] = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad H_2(L_a, K) \cong H_2(L_b, K) \cong \mathbb{Z}. \quad (99)$$

Each map is injective. The relevant segment of the long exact sequence therefore reduces to the exact sequence in Equation 100 for $x \in \{a, b\}$.

$$0 \longrightarrow \mathbb{Z} \xrightarrow{\delta_2^x} \mathbb{Z}^2 \xrightarrow{i_*} H_1(L_x; \mathbb{Z}) \longrightarrow 0, \quad H_1(L_x; \mathbb{Z}) \cong \mathbb{Z}. \quad (100)$$

Table 18: Relative and marked homological information for alternative single-face attachments to a historical wedge of two circles. Every computation is conditional on the indicated cellular attaching word.

Continuation	Relative $H_2(L, K)$	Connecting image in $H_1(K)$	Historical inclusion kernel
$L_a = K \cup_a D^2$	$\mathbb{Z}$	$\mathbb{Z}[a]$	$\mathbb{Z}[a]$
$L_b = K \cup_b D^2$	$\mathbb{Z}$	$\mathbb{Z}[b]$	$\mathbb{Z}[b]$

Table 19: Integral homology of degree- n attachments, with the same abstract relative second homology in every case.

Degree n	Connecting map	$H_1(L_n; \mathbb{Z})$	$H_2(L_n; \mathbb{Z})$
0	$\times 0$	$\mathbb{Z}$	$\mathbb{Z}$
1	$\times 1$	0	0
2	$\times 2$	$\mathbb{Z}/2\mathbb{Z}$	0
-2	$\times (-2)$	$\mathbb{Z}/2\mathbb{Z}$	0

Although the displayed quotient groups coincide, their marked quotient maps differ. In L_a , the historical class $[a]$ becomes null-homologous, whereas in L_b the class $[b]$ becomes null-homologous. The unmarked spaces L_a/K and L_b/K are both homeomorphic to S^2 . The pairwise connecting maps together with the marked historical domain distinguish the continuations, even though the quotients and absolute homology groups alone do not.

Table 18 separates the common abstract relative group from the historical information preserved by δ_2 . The table supports the representational choice defended in Section 7: keeping the complete extension and its historical embedding makes the relevant attachment data available. A future-only presentation carrying the original attaching map can also preserve these distinctions.

C.4 Degree Attachments and Integral Torsion

Let $K = S^1$ have one loop generator a , and let $L_n = K \cup_{a^n} D^2$ be the degree- n attachment of Appendix B. The relative chain complex has one generator in degree two and zero generators in degrees one and zero, so $H_2(L_n, K; \mathbb{Z}) \cong \mathbb{Z}$ for every integer n . The connecting map, however, records the winding degree. Equation 101 is its complete integral description.

$$\delta_2^{(n)} : \mathbb{Z} \longrightarrow \mathbb{Z}, \quad k \longmapsto nk. \quad (101)$$

Since K has no two-dimensional homology, the degree-two to degree-one portion of the exact sequence becomes Equation 102.

$$0 \longrightarrow H_2(L_n; \mathbb{Z}) \longrightarrow \mathbb{Z} \xrightarrow{\times n} \mathbb{Z} \longrightarrow H_1(L_n; \mathbb{Z}) \longrightarrow 0. \quad (102)$$

Hence $H_2(L_n; \mathbb{Z}) \cong \ker(\times n)$ and $H_1(L_n; \mathbb{Z}) \cong \operatorname{coker}(\times n)$. For nonzero n , the extension has $H_2 = 0$ and $H_1 \cong \mathbb{Z}/|n|\mathbb{Z}$; for $n = 0$, both groups are infinite cyclic. In particular, a degree-two attachment produces integral torsion even though its rational first and second Betti numbers both vanish. The concrete cases appear in Table 19.

The Smith normal form of the integral boundary matrix $[n]$ yields its invariant factor $|n|$ when $n \neq 0$. Over a field, the rank alone gives Betti numbers but does not recover the integral torsion subgroup. For instance, the degree-two attachment has trivial positive-degree homology with rational coefficients, while its first and second homology with $\mathbb{F}_2$ coefficients are each isomorphic to $\mathbb{F}_2$. This latter difference also follows from the universal coefficient theorem; it illustrates why the coefficient ring should accompany every reported homology computation.

C.5 Multiple Two-Cell Attachments and Homological Constraints

The single-cell examples extend to a finite family of proposed two-cells. Let K be a connected finite CW two-complex and construct L by attaching r oriented two-cells to K , with no new vertices or edges. The relative chain complex is concentrated in degree two, where it equals $\mathbb{Z}^r$. Each attaching map defines a homology class $\alpha_j \in H_1(K; \mathbb{Z})$, and the connecting homomorphism records their collective action. Equation 103 specifies the integral map.

$$\delta_2 : \mathbb{Z}^r \longrightarrow H_1(K; \mathbb{Z}), \quad (m_1, \dots, m_r) \longmapsto \sum_{j=1}^r m_j \alpha_j. \quad (103)$$

Proposition C.2 (Integral homology of a pure two-cell continuation). *Under the preceding hypotheses, the inclusion-induced map $H_2(K; \mathbb{Z}) \rightarrow H_2(L; \mathbb{Z})$ is injective. There is an exact sequence*

$$0 \longrightarrow H_2(K; \mathbb{Z}) \longrightarrow H_2(L; \mathbb{Z}) \longrightarrow \ker \delta_2 \longrightarrow 0, \quad H_1(L; \mathbb{Z}) \cong H_1(K; \mathbb{Z}) / \text{im } \delta_2. \quad (104)$$

The short exact sequence in Equation 104 splits as a sequence of abelian groups, though generally without a canonical splitting determined solely by the historically marked presentation.

Proof. The relative cellular complex has $H_2(L, K; \mathbb{Z}) \cong \mathbb{Z}^r$ and $H_1(L, K; \mathbb{Z}) = 0$. Since K and L are two-dimensional, the term $H_3(L, K)$ vanishes. Exactness of Equation 96 therefore makes the map from $H_2(K)$ injective and identifies the image of $H_2(L)$ in $\mathbb{Z}^r$ with $\ker \delta_2$. Exactness in degree one identifies $H_1(L)$ with the indicated quotient of $H_1(K)$. Finally, $\ker \delta_2$ is a subgroup of the free abelian group $\mathbb{Z}^r$ and is itself free. It is projective as a $\mathbb{Z}$ -module, so the indicated short exact sequence splits, though a particular section need not be distinguished by the original cell labels or epistemic interpretation. $\square$

Proposition C.2 gives an integral refinement of the rank statement in Section 10. Over a field $\mathbb{F}$, let d be the rank of the connecting map $\delta_2^{\mathbb{F}} : \mathbb{F}^r \rightarrow H_1(K; \mathbb{F})$. Equation 105 records the resulting Betti-number identities.

$$\beta_1(L; \mathbb{F}) = \beta_1(K; \mathbb{F}) - d, \quad \beta_2(L; \mathbb{F}) = \beta_2(K; \mathbb{F}) + r - d. \quad (105)$$

The field-specific rank d can depend on the chosen characteristic, especially when integral attaching classes produce torsion. The formula determines conditional changes in homological rank; its interpretation as a change in historically meaningful possibilities requires separately justified semantics for the attaching classes.

C.6 Relative Acyclicity with New Represented Cells

Homological differences can also fail to register the existence of newly represented cells. Reconsider the two-triangle continuation of Subsection B.3. It adds vertex v_3 , edges d, e , and face f_B to the historical filled triangle K . In the relative bases fixed there, the boundary matrices are those of Equation 106.

$$0 \longrightarrow \mathbb{Z} \xrightarrow{\begin{pmatrix} 1 \\ -1 \end{pmatrix}} \mathbb{Z}^2 \xrightarrow{\begin{pmatrix} 1 & 1 \end{pmatrix}} \mathbb{Z} \longrightarrow 0. \quad (106)$$

The right-hand map is surjective, and its kernel is generated by $(1, -1)$, the image of the left-hand map. Consequently $H_q(L, K; \mathbb{Z}) = 0$ for all q . Both K and L are contractible, and the new cells form an elementary expansion of the represented structure at the relevant CW level.

This computation has two interpretive lessons. First, a vanishing relative group does not indicate that no prospective cells have been introduced; it indicates that their relative chain complex is acyclic. Second, an information-preserving representation of future creation may need to retain newly represented events, couplings, and their provenance, even when a topological reduction removes them without changing homology. The information requirements of such a reduction depend on the query and normative context examined in Sections 10 and 12.

C.7 Attaching-Word Information and Natural Comparisons of Pairs

The cellular boundary of a two-cell attached to a graph records the abelianized signed traversal counts of the attaching word. Its chain-level data therefore need not recover the full word in the fundamental group. As a counterexample, take $K = S_a^1 \vee S_b^1$ and compare attaching one face by the commutator word $aba^{-1}b^{-1}$ with attaching one face by a constant map. Both have zero cellular two-boundary, so both possess $H_1 \cong \mathbb{Z}^2$ and $H_2 \cong \mathbb{Z}$; their relative H_2 groups and connecting maps agree as well. The first construction is the standard two-dimensional CW presentation of the torus, with fundamental group $\mathbb{Z}^2$. The second gives $K \vee S^2$, whose fundamental group is the free group on two generators. Thus the two spaces are distinguishable by fundamental groups even though the homological records just listed coincide.

This example exposes a precise limitation of the proposed topological epistemology. Cellular homology and connecting maps are useful historical observables, but they do not encode all relations among attaching paths. The original attaching words, higher homotopy data, or other annotations may be needed when a research question requires those distinctions. Equally, none of these topological data automatically supplies the causal ordering, interpretation, or normative significance of the represented relations.

For a genuine map of historical pairs $(f, g) : (L, K) \rightarrow (L', K')$, where $f(K) \subseteq K'$ and $g = f|_K$, the induced homology maps commute with the connecting homomorphisms. Equation 107 records their naturality.

$$g_* \circ \delta_q^{(L, K)} = \delta_q^{(L', K')} \circ f_*^{\text{rel}} : H_q(L, K) \longrightarrow H_{q-1}(K'). \quad (107)$$

Here f_*^{rel} is the map induced on relative homology. Equation 107 gives an exact condition for comparing the marked attachment profiles across compatible cellular maps. An epistemic revision, however, may alter identifications or withdraw previously recorded cells without admitting a cellular map of pairs. In that case, no induced comparison is available solely from the homology functor; a revision correspondence and its epistemic justification must be supplied separately.

The reproducibility script `examples/verify_appendix_C.py` checks the integral matrices, chain identities, connecting-map examples, degree-attachment torsion, and relative acyclicity used in this appendix. These checks establish the stated synthetic computations, not the completeness of a modeled future family. The whole-spacetime extension and its historically marked embedding remain the preferred primary data objects; relative homology is one analytic projection of their structure.

D Categorical Constructions and Continuation Composition

This appendix supplies the categorical constructions underlying the history-indexed continuation analysis of Section 8. Its objective is to specify the objects and morphisms through which represented historical spacetimes admit comparisons, staged extensions, and conditional transport under epistemic revision. The account proceeds from undercategories and their composition laws to pushouts, coherent cobase change, and context-dependent admissibility witnesses. Two finite cellular examples identify the distinction between a mathematically constructed extended complex and an epistemically or historically warranted continuation. Throughout, the complexes are stipulated epistemic representations; the assumed ontological reference structure remains a formal convenience and is not identified with ultimate reality. Standard categorical terminology and universal properties follow Riehl [16], whose bibliographic record is already verified in the source ledger.

D.1 Ambient Categories and Historically Marked Extensions

Let $\mathbf{C}$ be a chosen category of two-dimensional cellular presentations with a specified class of morphisms, containing the finite CW constructions needed in this appendix. One workable choice for the underlying topological constructions is a category of suitably presented finite CW complexes and cellular maps, with pushouts interpreted in topological spaces when their finite cellular realizations are stipulated. The passage from a topological pushout to an arrow in an *interpreted* cellular category may require additional

choices of labels, orientations, and provenance records. We will distinguish this underlying construction from its interpreted enrichment.

Fix an epistemically represented historical complex K . A marked extension is an arrow $i : K \rightarrow L$ satisfying a declared history-preservation policy. For the preferred continuation semantics, i is a cellular embedding onto a subcomplex, together with whatever data are required to identify historically marked events, couplings, and faces. The object L represents the *whole* candidate spacetime: its codomain contains both the marked historical image $i(K)$ and the prospective attachments. The image of i is the historical subcomplex, not the newly introduced portion.

A general cellular map can identify or otherwise transform representations and need not qualify as a strict continuation embedding. For this reason we use $\mathbf{C}_{\text{emb}}$ for the subcategory of selected history-preserving embeddings, while allowing a broader $\mathbf{C}$ for universal constructions. The existence of pushouts in $\mathbf{C}$ does not, by itself, establish that a corresponding pushout exists *inside* $\mathbf{C}_{\text{emb}}$ with all its required interpretation and admission policies intact. Similarly, preserving cell incidence is distinct from preserving a separately defined causal order or epistemic provenance relation.

D.2 Undercategories, Comparisons, and Stagewise Composition

For fixed K , an object of the undercategory $K \downarrow \mathbf{C}$ is an arrow $i : K \rightarrow L$. A morphism between two such objects $i : K \rightarrow L$ and $j : K \rightarrow M$ is a map $h : L \rightarrow M$ with $h \circ i = j$. The hom-set condition in Equation 108 makes the historical marking explicit.

$$\text{Hom}_{K \downarrow \mathbf{C}}(i, j) = \{h : L \rightarrow M \mid h \circ i = j\}. \quad (108)$$

The identity on K is initial in this category. Composition of arrows is inherited from $\mathbf{C}$, and for a sequence of continuation embeddings the composite identifies the same marked history in the final codomain. These are elementary categorical properties. The conditions that make successive stages epistemically admissible require independent predicates and witnesses, as distinguished in Section 8.3.

The marking can distinguish alternative continuations with isomorphic unmarked codomains. As an example, take $K = S_a^1 \vee S_b^1$, with distinguished loop generators a, b , and attach a two-cell along a in L_a or along b in L_b . The resulting complexes have isomorphic unmarked homotopy types, but the inclusion-induced kernels are different, as shown in Appendix C. A map isomorphic as an unmarked cellular structure need not commute with the fixed historical marking. In particular, any isomorphism of extensions *over* K would induce the same kernel on $H_1(K)$ in both branches; the different kernels obstruct such an isomorphism. The categorical distinction therefore records historically referenced information beyond the unmarked isomorphism type of the extension.

Proposition D.1 (Composition over a marked history). *Let $i : K \hookrightarrow L$, $j : L \hookrightarrow M$, and $k : M \hookrightarrow N$ be cellular embeddings belonging to a composition-closed class of history-preserving morphisms. Then $j \circ i$ and $k \circ j \circ i$ are history-preserving embeddings, and the induced arrows in $K \downarrow \mathbf{C}_{\text{emb}}$ compose associatively. This assertion is structural; it requires no inference that the stages have compatible epistemic-admissibility witnesses.*

Proof. A composite of the selected embeddings is again an embedding by the assumed closure condition, and the resulting identification of a marked historical cell is the composite of its stipulated identifications. An arrow j from i to $j \circ i$ satisfies the defining commutative relation in Equation 108. For a third stage, the underlying category's associativity law gives the same composite arrow in either parenthesization. Epistemic predicates were not invoked. $\square$

Proposition D.1 states precisely what structural continuation guarantees. It leaves open the additional question whether a procedure admitting each local step can consistently admit their composite. Such a procedure may require an explicit composition operation on admissibility witnesses, including compatibility with the evidential assumptions made at intermediate stages.

$$\begin{array}{ccc}
K & \xrightarrow{i_A} & L_A \\
i_B \downarrow & & \downarrow u_A \\
L_B & \xrightarrow{u_B} & P = L_A \sqcup_K L_B
\end{array}$$

Figure 12: A pushout of two represented continuations over their common historical base. The square encodes mathematical gluing; separate evidence and compatibility conditions are required for joint admission.

D.3 Pushout Gluing and the Topology of Combined Candidates

Suppose K is a common historical base and $i_A : K \hookrightarrow L_A$ and $i_B : K \hookrightarrow L_B$ are cellular extensions. Under ordinary finite CW subcomplex hypotheses, their topological gluing can be formed as the pushout $P = L_A \sqcup_K L_B$. Its defining property is that maps out of L_A and L_B which agree on K factor uniquely through P . The universal relation is expressed in Equation 109.

$$\text{Hom}_{\mathbf{C}}(P, X) \cong \{(f_A, f_B) \mid f_A : L_A \rightarrow X, f_B : L_B \rightarrow X, f_A i_A = f_B i_B\}, \quad (109)$$

when the pushout exists in the chosen category $\mathbf{C}$. The correspondence is natural in X . The resulting object is a *joint cellular amalgamation*; the universal property supplies no proof of joint historical feasibility, causal compatibility, or ethical permissibility.

For a finite example, let K be a circle with one vertex and one oriented edge. Each of L_A and L_B is a disk attached along the historical circle by a degree-one boundary map. The gluing $P = L_A \sqcup_K L_B$ is a two-sphere. In the cellular presentation containing the one old edge and two new faces, the boundary matrix and its homology are given in Equation 110.

$$C_2(P; \mathbb{Z}) \cong \mathbb{Z}^2 \xrightarrow{\partial_2 = \begin{pmatrix} 1 & -1 \end{pmatrix}} C_1(P; \mathbb{Z}) \cong \mathbb{Z}, \quad H_2(P; \mathbb{Z}) \cong \mathbb{Z}, \quad H_1(P; \mathbb{Z}) = 0. \quad (110)$$

The orientation of the second face has been reversed for convenience; the sign choice does not change the homology. Individually each disk has trivial second homology, while the pushout acquires a two-dimensional cycle. Equation 110 demonstrates a conditional topological consequence of gluing, with no interpretation of the two disks as mutually compatible future interventions.

Figure 12 shows the underlying square. The example in Equation 110 can be checked exactly by observing that the kernel of the displayed integer boundary row is generated by $(1, 1)$ and its cokernel is zero. In a modeled future-creation problem, two candidate attachments may compete for resources or presuppose incompatible event interpretations. Such incompatibilities need to be recorded in an independent admissibility layer even when the pushout exists.

D.4 Cobase Change and the Composition of Historical Revisions

Let $r : K \rightarrow K'$ represent a specified change of historical base, and let $i : K \rightarrow L$ be a continuation arrow. A pushout of i along r gives a new structural candidate $i' : K' \rightarrow L \sqcup_K K'$. With chosen pushouts this construction defines a functor $r_! : K \downarrow \mathbf{C} \rightarrow K' \downarrow \mathbf{C}$, as established in Proposition 8.1. One further issue is its compatibility with successive changes of base.

Proposition D.2 (Coherence of successive cobase changes). *Let $r : K_0 \rightarrow K_1$ and $s : K_1 \rightarrow K_2$ be composable arrows in a category admitting the required pushouts. Then for each arrow $i : K_0 \rightarrow L$ the objects $(s \circ r)_!(i)$ and $s_!(r_!(i))$ are canonically isomorphic over K_2 . These canonical isomorphisms are natural in arrows of $K_0 \downarrow \mathbf{C}$. Strict equality requires coherent choices or a strictified presentation; isomorphism follows from the universal property.*

Proof. The composite of the two successive pushout squares is a pushout rectangle, by the pushout pasting property. Consequently the iterated pushout $(L \sqcup_{K_0} K_1) \sqcup_{K_1} K_2$ and the direct pushout $L \sqcup_{K_0} K_2$ represent the same universal cocone for the span $L \leftarrow K_0 \rightarrow K_2$. Their universal arrows define mutually inverse isomorphisms respecting the maps from K_2 . Naturality follows because each isomorphism is uniquely determined by the maps out of the original span. $\square$

The coherence statement of Proposition D.2 can be recorded by the natural isomorphism in Equation 111.

$$(s \circ r)_! \cong s_! \circ r_! \quad \text{on } K_0 \downarrow \mathbf{C}. \quad (111)$$

For a structural category these assignments provide a controlled way to relate extension diagrams under successive changes of base. Equation 111 does not supply a transport of epistemic meaning, interpretive identity, or admission witnesses. In particular, the historical arrow r need not itself be an embedding: it may be a formally declared reinterpretation or identification, and the chosen category determines which colimits are valid.

D.5 Revision-Sensitive Semantics and Witness Transport

An epistemic context c includes a historical representation K_c , observation and provenance data, and conditions used to judge candidate extensions. Let $\mathbf{A}_c$ denote its category of admitted whole-spacetime continuations. For a context revision $\rho : c \rightsquigarrow d$ with underlying historical map $r : K_c \rightarrow K_d$, the structural cobase-change functor need not map an admitted extension of c to an admitted extension of d . A genuine epistemic-transport operation needs a compatible mapping of admission witnesses, as specified in Equation 112.

$$\tau_{\rho,i} : \text{Adm}_c(i) \longrightarrow \text{Adm}_d(r_!i), \quad i : K_c \rightarrow L, \quad (112)$$

for each extension in the chosen transport domain. If admissibility is proof-relevant, the map must state how the old evidence is reinterpreted or supplemented. A total family of the form in Equation 112 is an additional assumption, not a consequence of the existence of cellular pushouts.

A finite logical example exhibits the separation even with an unchanged historical complex. Fix the identity revision on a single represented K and choose an extension $i : K \hookrightarrow L$ requiring one unit of a stipulated resource. Context c has a verified resource bound of one, while context d revises that bound to zero. With the model's admission rule "required resources do not exceed the verified bound," $\text{Adm}_c(i)$ is inhabited and $\text{Adm}_d(i)$ is empty. The structural base map is the identity and its cobase change leaves L unchanged, but no total function of the type in Equation 112 exists for this candidate. This is a logical demonstration about stipulated predicates, rather than a factual claim about resource allocation.

A separate example concerns the identity of occurrences. Let the represented history $K = S^1$ and the candidate extension $L = D^2$ attach a proposed face along the circle. A revised historical complex $K' = D^2$ already contains a face filling the same boundary. The pushout $L \sqcup_K K'$ retains two distinct two-cells and is homeomorphic to S^2 . If the new historical evidence instead identifies the candidate face and the historical face as the *same* occurrence, the intended representational revision may require a further co-reference identification, leading to a different complex. The pushout is still a valid mathematical construction; its semantic suitability depends on the identity claim. The example extends the discussion in Subsection 8.6 and demonstrates why an epistemic revision must carry explicit information about referents and their possible identifications.

For contexts carrying partial correspondences instead of a single historical map, one could investigate spans of histories $K_c \leftarrow R \rightarrow K_d$, cospans, or other structures of mediated comparison. Such diagrams do not automatically determine a unique continuation translation. The needed choices include the common evidence domain, the interpretation of its cells, and a rule for transferring admission information to each revised context.

Table 20: Categorical operations and the information required for epistemic and normative interpretation.

Operation	Structural result	Additional information
Arrow in $K \downarrow \mathbf{C}$	Comparison commutes with the marked historical base	Historical identity, annotation and order preservation
Composition of embeddings	Composite is a structural continuation in a composition-closed class	Compatible stagewise admission witnesses
Pushout of extensions	Universal joint cellular amalgamation where defined	Joint feasibility, co-reference and interpretation
Cobase change along r	Transport of underlying extension diagram	Revised provenance and historical identity claims
Witness map $\tau_{\rho,i}$	Contextual admission carried to a revised candidate	Evidence and interpretation supporting the transported witness
Normative assessment	Predicate or valuation on annotated candidate histories	Independent ethical principles and supporting historical evidence

D.6 Continuation Categories and Normative Information

The categorical constructions in this appendix organize which histories are marked as common, which extensions can be compared, and which composite diagrams can be built. Their ethical interpretation calls for a separate information-preservation test. Section 12 introduces order-sensitive assessments and the conditions under which a reduced representation can recover an ethical query. An isomorphism in an unannotated category may preserve cellular topology while omitting event provenance, participation histories, or the modeled order of actions. Conversely, a category enriched with these records can be more discriminating, provided its morphism conditions are explicitly justified.

Table 20 summarizes the mathematical operations and their extra interpretive requirements. The table distinguishes statements available by definition or universal property from proposed requirements for historical and normative application.

The appendix establishes structural composition and coherent base change under explicit categorical hypotheses. It also demonstrates that epistemic admission and historical co-reference are additional levels of construction. A full indexed or fibred category of epistemic continuations remains an open problem because revision arrows, witness-transport domains, and coherence requirements depend on epistemic practices that the preliminary formalism does not derive. The next appendix investigates dependent types and admissibility interfaces in more detail; the later technical analysis can then test which operations preserve the information needed for specific future-creation inquiries.

E Dependent Continuation Types and Admissibility Conditions

This appendix develops the type-theoretic admission interface introduced in Section 7 and used in the model-reconstruction construction of Section 9. Its purpose is to specify the information required for a proposed extension to become an inhabitant of a context-indexed continuation type, to distinguish that judgment from selection by a reconstructed epistemic model, and to identify the additional witnesses needed for composition and revision. The method is constructive: dependent sums and proof terms are given explicit roles, a finite marked-history example tests changes of evidence and model selection, and the resulting claims are limited to a declared formal context. The usual dependent-sum and dependent-product operations are adopted from established type theory [22]; their present interpretation as epistemic continuation records is a proposal of this paper.

E.1 Context-Indexed Families and Structural Formation

Let ECtx be a selected type of epistemic contexts and let $K_c : \text{EpCplx}_2$ denote the represented historical complex associated with $c : \text{ECtx}$. A context may contain evidence, interpretation rules, provenance,

and historically constituted observing capacities. Its role is an index of formal judgments rather than a guarantee that a particular historical representation is correct. In particular, the assumed ontological reference domain used in Section 6 is only a modeling convenience; the present appendix does not treat it as an ultimate reality.

We reuse the structural type $\text{Ext}_2(K_c)$ of Equation 35. A term $x = (L, i)$ consists of a candidate whole-spacetime complex L and an embedding $i : K_c \hookrightarrow L$ that respects the specified cellular incidence and historical marking. The codomain L contains the retained image $i(K_c)$ together with any prospective cells. Since K_c occurs in the source of the embedding type, extensions of different represented histories inhabit different dependent families even when their codomains happen to have isomorphic underlying complexes.

The formation and introduction rules for contextual continuation are summarized in Equation 113. Here $\text{Adm}_c(x)$ is a declared type of admissibility evidence; its definition is supplied by the selected epistemic discipline.

$$\frac{c : \text{ECtx} \quad x : \text{Ext}_2(K_c) \quad w : \text{Adm}_c(x)}{(x, w) : \text{Cont}(c)}, \quad \text{Cont}(c) := \sum_{x : \text{Ext}_2(K_c)} \text{Adm}_c(x). \quad (113)$$

Equation 113 restates the main construction with an explicit typing premise. Neither the existence of x nor the derivation of w licenses an inference to actual historical realizability. A type checker can inspect the well-formedness of supplied encodings and witnesses relative to its rules; the historical adequacy of those rules is a separate epistemological question.

E.2 Admission Witnesses and Their Interpretations

The optional factorization in Equation 40 separates representational compliance, evidence-supported consistency, and feasibility constraints included in the model. These components should remain individually inspectable. A rule such as “the proposed face uses an admissible attaching interface” differs from “the proposed event is supported by a recorded precedent,” and both differ from “the proposed event satisfies a specified resource bound.” The dependent product of such witness families formalizes a conjunction under a given rule set; it supplies no general theory of which constraints should be selected.

When $\text{Adm}_c(x)$ is proposition-valued and proof-irrelevant, its role is membership or admission under the chosen conditions. When it is a proof-relevant type, different inhabitants may retain distinct derivations, supporting records, or provenance. The associated erasure map is specified in Equation 114; its fiber over a structural extension records precisely the chosen admission-witness type.

$$\varepsilon_c : \text{Cont}(c) \longrightarrow \text{Ext}_2(K_c), \quad \varepsilon_c(x, w) = x, \quad \text{fib}_{\varepsilon_c}(x) \simeq \text{Adm}_c(x). \quad (114)$$

The last equivalence in Equation 114 is the ordinary dependent-pair fiber equivalence, interpreted in the selected type-theoretic setting. It explains why forgetting the witness can remove epistemically relevant distinctions even when the same complex and embedding are retained. A proof-relevant record may distinguish two independently verified supporting observations; a mere Boolean flag records only the outcome of an admission procedure.

Failure to construct w requires particular care. In an open or incomplete evidence system, the absence of a witness means only that admission has not been established under the selected rules. An explicit modeled-exclusion judgment requires its own negative evidence or decision policy. We therefore distinguish an admitted candidate, an unresolved candidate, and a candidate excluded under an explicitly stated model rule. None of these statuses is inherently an ontological classification of the actual future.

E.3 Model-Selected Continuation Families

A reconstructed epistemic model adds a selection layer to contextual admission. For $m : \text{Mod}(c)$, Section 9.2 uses the predicate $G_{m,c} : \text{Cont}(c) \rightarrow \text{Prop}$. The corresponding model-selected dependent family

is made explicit in Equation 115.

$$\Gamma_c(m) := \sum_{x:\text{Ext}_2(K_c)} \sum_{w:\text{Adm}_c(x)} G_{m,c}(x, w). \quad (115)$$

Equation 115 is definitionally equivalent to the nested dependent sum used in Equation 49. It distinguishes three questions: whether an extension can be formed, whether contextual evidence admits it, and whether a particular epistemic model selects it. Two models reconstructed from the same history may select different subfamilies of $\text{Cont}(c)$; the choice of a model also requires the reconstruction conditions of Section 9.3.

A model can select no candidates under a chosen context without proving that the future is impossible. Empty selection may arise from restrictive assumptions, finite search bounds, insufficient evidence, or a representational vocabulary lacking relevant future cells. Similarly, a nonempty $\Gamma_c(m)$ is a family of *modeled* continuations, not a complete set of physically possible histories. The family may be infinite and need not have a computable enumerator. A probabilistic implementation would require additional choices of measurable space, measure, and interpretation of its weights; those choices are not built into Equation 115.

E.4 Composition and Compatibility of Admission Witnesses

A structural continuation can be followed by a further structural continuation. Suppose $i : K \hookrightarrow L$ and $j : L \hookrightarrow M$ are embeddings and suitable contexts c_K, c_L, c_M specify evidence and admissibility for the two stages. The composite $j \circ i$ is again a history-preserving embedding. A corresponding admission witness follows only after supplying a compatible operation with the type recorded in Equation 116.

$$\text{comp}_{i,j} : \text{Adm}_{c_K}(L, i) \times \text{Adm}_{c_L}(M, j) \times \text{Compat}(i, j; c_K, c_L, c_M) \longrightarrow \text{Adm}_{c_K}(M, j \circ i). \quad (116)$$

Here Compat is a declared type expressing any cross-stage conditions not certified by the individual stage witnesses. For example, two extensions may separately satisfy a local resource bound while exceeding a cumulative bound. Equation 116 does not postulate that such a composition function exists for arbitrary admissibility rules.

Proposition E.1 (Typed composition under a supplied compatibility operation). *For two composable structural embeddings, assume inhabitants of the three source types of Equation 116 and a function of that type. Then the composite embedding defines an inhabitant of the corresponding dependent continuation type over the initial historical complex.*

Proof. Composition of the two specified embeddings gives a structural extension $(M, j \circ i)$ of K . Applying the assumed operation to the two admission witnesses and their compatibility witness yields an inhabitant of $\text{Adm}_{c_K}(M, j \circ i)$. The introduction rule in Equation 113 then supplies the required dependent pair. $\square$

Proposition E.1 is conditional and deliberately modest. It restates the composition requirement of Proposition 7.2 as a type-level obligation. Associativity of *admitted* multistage histories would require additional coherence conditions among the witness-composition functions, beyond the ordinary associativity of cellular maps. Likewise, a model selecting individual stages may need a distinct rule to select their composite.

E.5 Revision-Indexed and Partial Witness Transport

An epistemic revision $\rho : c \rightsquigarrow d$ can alter the represented historical complex, its evidential records, or the interpretation policies applied to candidate futures. Appendix D describes structural cobase change when a suitable base morphism and pushouts are available. Transport of admission witnesses is a further

Table 21: Finite admission example over a marked historical wedge. Candidate cells, evidence tokens, and model-selection rules are stipulated rather than inferred from historical observations.

Context	Positively admitted attachments	Selected by m_A	Selected by m_B
c_0	α, β	α, β	β
c_1	β	β	β

condition. To avoid presupposing a total transport function, we can define a partial reconstruction interface using an option type. Equation 117 also requires an explicit correspondence between the original extension and the output extension.

$$\mathcal{T}_\rho : \prod_{u:\text{Cont}(c)} \text{Option} \left(\sum_{v:\text{Cont}(d)} \text{Corr}_\rho(\varepsilon_c(u), \varepsilon_d(v)) \right). \quad (117)$$

The type Corr_ρ includes whichever structural, historical, and semantic information the selected revision discipline requires. It may be induced by a pushout, by a mediated correspondence, or by a separately justified translation. A value of `None` in Equation 117 indicates that this operation has not produced a supported transported candidate. It does not prove that every interpretation of the proposed future is impossible. The construction can also be expressed as a relation when no single-valued translation is warranted.

Proposition E.2 (Limits of unconditional revision transport). *A structural identity map between two epistemic contexts with the same historical complex does not, by itself, determine a total transport of their admitted continuation types.*

Proof. Choose a structural extension x requiring a stipulated positive supporting observation. In context c its evidence predicate is inhabited; in context d the observation is retracted and the predicate is empty. Holding the remaining admission factors fixed, $\text{Adm}_c(x)$ is inhabited while $\text{Adm}_d(x)$ is not. A total transport sending x to itself and preserving the indicated evidence witness would give an inhabitant of the empty target type. Therefore such witness transport is not implied by the identity structural map. $\square$

The statement of Proposition E.2 concerns witness-preserving transport of a specified candidate. It does not rule out unrelated functions from the entire old admitted family into a new family, and it does not identify evidence retraction with physical impossibility. It complements the categorical counterexample in Subsection D.5.

E.6 A Finite History with Evidence-Dependent Selection

Consider a represented historical complex $K = S^1 \vee S^1$ whose marked loops are called α and β . Introduce three candidate whole-spacetime extensions by attaching a two-cell along α , β , or the degree-two loop 2α . The underlying three extensions remain structurally available in two stipulated epistemic contexts c_0 and c_1 . For this illustration, a positive evidence token is required for admission. Context c_0 carries such tokens for α and β ; context c_1 retains only the token for β . Neither context carries a token for 2α . This example concerns the outcome of an evidence policy, not the historical existence of those proposed attachments.

Let model m_A select the two simple attachments and model m_B select only the β attachment. Table 21 enumerates the resulting families. The extension set is unchanged, and the model predicates are held fixed; only the evidence tokens differ. Table 21 shows two distinct filters. Epistemic admission changes after revision; model selection can be more restrictive than admission. The degree-two candidate lacks a positive token under both evidence policies, yet remains a well-formed member of $\text{Ext}_2(K)$. Its integral first homology contains $\mathbb{Z}/2\mathbb{Z}$ torsion, as verified independently by the integer boundary

matrix and discussed in Appendix C. That mathematical distinction survives regardless of the currently issued evidence token.

The included script `examples/verify_appendix_E.py` checks the finite token selections and the exact cellular boundary matrices. The check is a reproducible verification of the synthetic inputs. It provides no inference procedure for obtaining reliable tokens from social or scientific historical records. In an implementation for real observations, the provenance, quality, and possible retraction of evidence would become central parts of the type or record carried by each witness.

E.7 Historical Identity, Model Selection, and Normative Boundaries

An admitted continuation retains the whole candidate spacetime and its marked history. Erasure of w in Equation 114 removes admission support; replacing (L, i) with the isomorphism type of L can additionally discard which historical events and couplings the embedding identifies. These information losses are analytically different. An appropriate quotient or reduction should specify the query for which it preserves enough evidence and semantics.

Ethical assessment also occupies a distinct type level. A judgment concerning procedural participation, justice of continuation order, or preservation of heterogeneous generative capacities requires an independently specified normative context and supporting interpretation. One may attach an ethical predicate to an admitted continuation, but its inhabitation cannot be derived merely from a cellular embedding, an evidence token, or model membership. The query-preservation condition developed in Section 12 and the factorization discussion of Section 10 clarify the distinction between preservation of mathematical invariants and preservation of ethically relevant information.

The dependent constructions of this appendix consequently supply a transparent interface among structural extension, epistemic admission, and model selection. They make assumptions auditable and expose conditions for staged composition and partial revision. They do not generate the historically constituted capacity types themselves, establish an exhaustive family of actual futures, or assign normative legitimacy. Those remain research questions developed in Section 14 and are particularly relevant when the very language used to describe possible future events is historically revised.

F Discrete Morse Constructions and Reduction Examples

The mathematical object of this appendix is a finite, historically marked candidate continuation $i : K \hookrightarrow L$ together with a proposed simplification of its cellular presentation. Its purpose is to separate the topology preserved by an elementary collapse or discrete Morse matching from the historical and epistemic information that a future-creation inquiry may need to retain. We first state a conditional preservation result for collapses relative to the historical subcomplex, then compute two examples at the chain level, and finally formulate a query-preservation criterion and a limitation concerning further continuations. The discussion develops Sections 10.6–10.7 and the synthetic experiment in Subsection 11.9. The mathematical framework follows established discrete Morse and elementary-collapse theory [8]; the proposed historical and epistemic interpretations are additional modeling choices. None of these reductions identifies which future events will be realized.

F.1 Finite Cell Structures and Relative Matchings

For general discrete Morse matchings, let $K \subseteq L$ be finite regular CW complexes (including finite simplicial complexes), with a fixed cellular inclusion identifying the represented historical cells. The elementary free-pair constructions below also apply to finite CW presentations admitting the stated collapses, including the loop-edge model of Subsection F.4. A *free face* σ^{q-1} of a cell τ^q is a proper face contained in no other q -cell of the complex in the sense required for an elementary collapse. Removing the free pair (σ, τ) produces a subcomplex L' and an elementary strong deformation retraction $L \searrow L'$. For our purposes, a *history-relative elementary collapse* removes a free pair disjoint from the cells of

K and leaves $K \subseteq L'$ throughout the reduction. We denote a finite succession of these operations by $L \searrow_K M$.

A discrete Morse matching pairs incident cells of adjacent dimensions subject to an acyclicity condition on the resulting directed cell-incidence graph. An unmatched cell is called *critical* relative to that matching. Under the usual hypotheses, such matchings provide a CW model with the homotopy type of the original complex. An arbitrary Morse CW model need not literally be a subcomplex of L or retain the named cells of K as an identical cellular inclusion. The present appendix therefore uses explicit history-relative elementary collapses for its preservation claims. More general Morse reductions require an independently specified marking and reconstruction interface.

The distinction between the two descriptions matters operationally. An elementary collapse supplies a concrete sequence of free-cell removals, while a gradient matching can encode cancellations that are realized through a more general Morse complex. Neither notion, considered solely as a homotopy construction, supplies a provenance record for erased event labels, a causal-order translation, or admissibility witnesses for subsequent continuation models.

F.2 History-Relative Collapse and Homological Preservation

The basic topological guarantee concerns the marked pair rather than the unmarked space alone.

Proposition F.1 (Relative elementary-collapse preservation). *Suppose $K \subseteq L$ is a finite CW pair and $L \searrow_K M$ is a finite sequence of elementary collapses in which each removed free pair lies outside K . Then the inclusion $(M, K) \hookrightarrow (L, K)$ is a homotopy equivalence of pairs and induces isomorphisms on integral relative homology in every degree. Moreover, these isomorphisms are compatible with the long exact sequences of the two pairs and with the identities on the homology groups of K .*

Proof. At each step, the elementary collapse gives a strong deformation retraction onto the remaining subcomplex. Because the closed cells removed at that step are disjoint from the historical subcomplex, the deformation can be taken to fix K . Composing the stepwise homotopies yields a homotopy equivalence of pairs $(L, K) \simeq (M, K)$. Homotopy invariance gives the claimed relative homology isomorphisms. Naturality of the long exact sequence for pairs gives a commutative comparison diagram whose map on $H_*(K)$ is the identity. $\square$

The structural content of Proposition F.1 can be stated using the isomorphism in Equation 118. The displayed maps depend on the specified reduction; they do not identify the original and reduced cell inventories.

$$H_q(L, K; \mathbb{Z}) \cong H_q(M, K; \mathbb{Z}), \quad q \geq 0. \quad (118)$$

A relative collapse retains the incidence and topology of the marked historical subcomplex and preserves its homological relationship to the reduced whole. This is useful when the intended question concerns a selected relative homology class or the effect of a two-cell attachment on a historical cycle. Its guarantee is narrower when the question names a prospective event, a coupling, a decision order, an evidence source, or an interpretive significance not determined by the homotopy type of the pair.

F.3 An Acyclic Prospective Triangle

Consider the historical interval $K = \{a, b, ab\}$ and a candidate whole-spacetime continuation L given by a filled triangle with a new event vertex c , prospective edges ac, bc , and prospective face $f = [a, b, c]$. Orient the edges from a or b toward c , and orient f by the ordered vertices (a, b, c) . The relative chain groups have the explicitly chosen bases recorded in Equation 119.

$$\begin{aligned} C_2(L, K; \mathbb{Z}) &= \mathbb{Z}\langle f \rangle, & C_1(L, K; \mathbb{Z}) &= \mathbb{Z}\langle ac, bc \rangle, & C_0(L, K; \mathbb{Z}) &= \mathbb{Z}\langle c \rangle, \\ [\partial_2^{L,K}] &= \begin{pmatrix} -1 \\ 1 \end{pmatrix}, & [\partial_1^{L,K}] &= \begin{pmatrix} 1 & 1 \end{pmatrix}. \end{aligned} \quad (119)$$

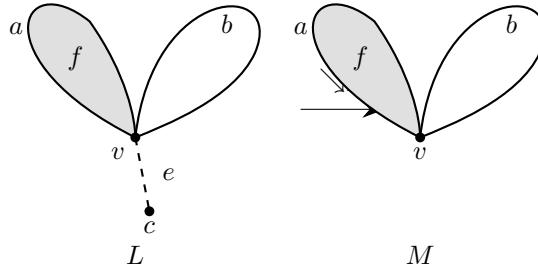


Figure 13: History-relative elementary collapse in a candidate continuation. Historical loops a, b and their common vertex are retained; the prospective two-cell f remains attached to historical loop a . The prospective event c and its edge e are removed as a free pair. The marked relative homology is preserved although the original event inventory is reduced.

The product $\partial_1^{L,K} \partial_2^{L,K}$ is zero. Both displayed matrices have rank one, their nonzero Smith invariant factors equal one, and the relative homology vanishes in every degree. The triangle admits the history-relative collapse sequence whose free pairs are (ac, f) and, after the first removal, (c, bc) . In the associated matching, both pairs lie entirely in the prospective portion; the historical edge ab is retained. The remaining complex is exactly K .

The formal reduction is particularly transparent: the topological comparison $(L, K) \simeq (K, K)$ is valid, while the output contains no cell identified with the prospective event c . This conclusion extends the analogous calculation in Equation 74. It does not imply that the occurrence of c would be causally unimportant, redundant in an explanatory narrative, or ethically negligible. The homological calculation answers a strictly structural question about the stipulated complex.

F.4 Nontrivial Historical Attachments and Removable Events

A second example separates a topologically substantive attachment from a prospective event that can be removed by collapse. Let $K = S_a^1 \vee S_b^1$ be a graph with one historical vertex v and two oriented loop edges a, b . Construct L by attaching a two-cell f along the loop a and, independently, attaching a new edge e from v to a new vertex c . The latter vertex is interpreted as a candidate event, and e as its coupling to the historical complex. The oriented absolute cellular boundaries are specified in Equation 120.

$$\partial_2 f = a, \quad \partial_1 a = \partial_1 b = 0, \quad \partial_1 e = c - v. \quad (120)$$

The pair (c, e) is a free pair outside K . Its removal gives a history-relative elementary collapse $L \searrow_K M$, where M consists of the marked wedge and the two-cell attached along a . The connecting map in the relative long exact sequence retains the historical effect of the face attachment. With the indicated orientations, the exact calculations are summarized in Equation 121.

$$\begin{aligned} H_1(L; \mathbb{Z}) &\cong H_1(M; \mathbb{Z}) \cong \mathbb{Z}\langle [b] \rangle, \\ H_2(L, K; \mathbb{Z}) &\cong H_2(M, K; \mathbb{Z}) \cong \mathbb{Z}\langle [f] \rangle, \\ \delta_2([f]) &= [a] \in H_1(K; \mathbb{Z}), \quad \ker(H_1(K) \rightarrow H_1(L)) = \mathbb{Z}\langle [a] \rangle. \end{aligned} \quad (121)$$

Equation 121 shows that a reduction can preserve a historically marked homological effect while still deleting a particular prospective event. Figure 13 identifies these two kinds of information. The filled loop represents the new two-cell, the unfilled loop a surviving historical class, and the whisker the prospective event that can be removed without changing the marked pair's homotopy type.

The example also highlights a distinction between relative homological criticality and discrete-Morse criticality. The two-cell f cannot be removed by the displayed free-pair collapse, and its relative class detects an attachment to $[a]$. The event c is removable in this topological presentation. Neither circumstance establishes the comparative historical, causal, or moral significance of the corresponding interpreted occurrences.

F.5 Query Preservation and a Factorization Criterion

Preserving a homotopy type is sufficient for certain topological queries and insufficient for many information-bearing queries. This difference can be stated without adopting a universal epistemic metric. Let $\mathcal{D}$ be a selected family of annotated candidate continuations, $\mathcal{R}$ a family of reduced representations, and $r : \mathcal{D} \rightarrow \mathcal{R}$ a specified reduction that forgets some cells or annotations. A query $Q : \mathcal{D} \rightarrow A$ is *recoverable* from the reduced representation when there exists a function $\bar{Q}$ on $\mathcal{R}$ giving the same answer on every retained input. The exact criterion is given by the following elementary proposition.

Proposition F.2 (Information recoverability under a fixed reduction). *Let $r : \mathcal{D} \rightarrow \mathcal{R}$ and $Q : \mathcal{D} \rightarrow A$ be functions of sets. There exists $\bar{Q} : r(\mathcal{D}) \rightarrow A$ satisfying $Q = \bar{Q} \circ r$ if and only if $Q(x) = Q(y)$ whenever $r(x) = r(y)$. Extension of $\bar{Q}$ from $r(\mathcal{D})$ to the whole of $\mathcal{R}$ may require a separate choice of values outside the image.*

Proof. If the factorization exists, equality of reduced outputs implies equality of their $\bar{Q}$ -images. Conversely, constancy on each nonempty fiber makes the assignment $\bar{Q}(r(x)) := Q(x)$ independent of the representative x . This defines the claimed function on the image of r . $\square$

Equation 122 records the factorization when its hypothesis holds.

$$Q = \bar{Q} \circ r \quad \text{on } \mathcal{D}. \quad (122)$$

For a concrete counterexample, consider two otherwise identical filled-triangle continuations over the marked historical edge ab . In one representation the new vertex has the recorded event identifier c ; in the other it has identifier d . Both admit the same unlabeled relative collapse onto $K = ab$. The query “which prospective event identifier occurs in the full candidate?” returns c for one and d for the other. Since both are mapped to the same reduced K , the query does not factor through the unlabeled reduction. This is an information-loss result over a *family* of annotated inputs. It does not assert that any event identifier is more consequential than another or that such a query is relevant for every inquiry.

One may preserve the query in at least two ways: forbid the collapse of cells carrying the required identifiers, or accompany the reduced structure with a provenance trace that retains the removed records and sufficient attaching information. The second approach changes the codomain of r to a structured pair such as (M, Trace) . A trace that merely lists removed labels may still be inadequate for queries about their incidences or causal dependencies. The requested query family determines what the trace must contain. The logical requirement in Proposition F.2 is distinct from a normative assessment of which questions should be preserved.

F.6 Subsequent Continuations and Reduction Compatibility

The role of reductions in future creation also differs from their use as terminal data compression. An epistemic generative model may need to construct a further continuation of the present candidate L . If its admission rules reference a prospective cell later erased by a reduction $r : L \searrow_K M$, the reduced complex alone may fail to provide the required attaching interface.

Return to the triangular example in Subsection F.3. A subsequent candidate extension may introduce an edge from the prospective event c to another new event d , with a specified dependence on c . After collapsing L to K , the original named event c is absent. It is mathematically possible to build other extensions of K , and suitably chosen homotopy-theoretic constructions may retain some unmarked structural information. However, the unannotated reduced object does not uniquely specify an extension preserving the original dependency on c .

A reduction intended to support model-generated futures would therefore require a continuation-transport or reconstruction interface, schematically as stated in Equation 123. The notation $\Gamma(L)$ denotes a chosen family of admissible later continuations over the candidate history L , and the assignment T_r is additional mathematical data, potentially only partially defined.

$$T_r : \Gamma(L) \rightarrow \Gamma(M), \quad \text{Ans}_{L'}(q) = \text{Ans}_{T_r(L')}(q_r) \text{ for designated queries } q. \quad (123)$$

Table 22: Topological guarantees and additional preservation requirements for reductions of epistemic spacetime continuations.

Preservation target	Possible criterion	Standing of the criterion
Historical cellular base	Collapse fixes marked subcomplex K throughout	Structural hypothesis of relative collapse
Relative topology	Isomorphisms on $H_q(L, K)$ compatible with exact sequences	Consequence of Proposition F.1
Event identity	Named event identifiers recoverable after reduction	Separate semantic or provenance requirement
Causal dependencies	Declared order or dependency queries factor through reduction	Additional modeled causal data
Model admissibility	Witnesses and selected continuations translate across reduced contexts	Additional epistemic transport operation
Procedural justice queries	Normatively relevant distinctions retained for a justified query class	Independent ethical and evidential premises

Equation 123 is a *design requirement*, not a consequence of homotopy equivalence. The definition of q_r and any transport of contextual admissibility witnesses must be supplied for each application. In particular, a history-relative collapse $L \searrow_K M$ preserves K as the inherited base while possibly changing the present candidate structure used as the base for its own later continuation. Compatibility at the first level therefore need not imply compatibility of the iterated future-creation process.

F.7 Reduction Criteria and Reproducible Checks

Table 22 records the different preservation targets used in this appendix. The first two are consequences of an appropriately specified history-relative elementary collapse. The remaining targets require explicit semantics, data retention, or epistemic reconstruction; they cannot be inferred from the number of Morse-critical cells or from Betti numbers alone.

A deterministic verification script, `examples/verify_appendix_F.py`, checks the integer relative boundary matrices in Equation 119, the wedge-and-whisker boundary equations and homology in Equations 120–121, and the finite counterexample to query factorization. These calculations verify the synthetic encodings and their claimed algebraic consequences. They do not validate the semantic adequacy of using a two-complex to represent an actual observer, a future event, or a historical relation.

The reduction problem thus has two connected components. The mathematical component asks for collapses or matchings preserving an appropriately marked topology. The epistemic component asks which histories, interpretations, and subsequent possibilities remain distinguishable after those collapses. When the objective is future creation, preservation of the historical subcomplex is a useful starting constraint, while the adequate reduction of candidate futures depends on the questions and responsibilities of the observer. A comprehensive theory of history-sensitive discrete Morse reduction, including categorical and admission-witness transport, remains open.

G Computational Experiments and Reproducibility

This appendix documents the computational evidence supporting the paper’s finite two-complex examples and specifies a reproducible audit procedure. Its analytical object is a suite of *stipulated*, *synthetic* cellular continuations with explicitly marked historical bases. The objective is to connect the mathematical assertions of Section 11 and Appendices C and F to identifiable input matrices, executable checks, and outputs. The method combines exact integer linear algebra, independently reconstructed invariants for selected small examples, source-level inspection, and a documented manuscript build. The procedures test whether the *implemented finite examples* have the stated properties. They do not establish an empir-

ical model of historical causation, a prediction of future events, or a formal proof of all dependent-type and categorical propositions.

G.1 Experimental Units and Representation Boundaries

The primary historical test object in Section 11 is a finite theta graph with two distinct event vertices, three oriented coupling edges, and two marked homology classes $[\alpha]$ and $[\beta]$. The test suite represents possible extensions by adding two-cells along stipulated integer combinations of the corresponding integral cycle basis. Orientation facilitates boundary calculations; it is not a surrogate for a temporal relation. Each prospective face is a formal higher-order cellular primitive. Its meaning in an eventual social, institutional, or physical application would require a separate semantic mapping.

Three layers of experimental description are retained. The *input layer* records the historical incidence matrix, a cycle basis, and each prospective two-cell attachment. The *derived layer* contains full and relative cellular boundary matrices and their algebraic invariants. The *interpretive layer* associates identifiers, epistemic contexts, or ordering constraints with selected examples. Only the first two layers are processed by the exact homology calculations. The interpretive layer is supplied explicitly where a historical comparison, model-selection example, or procedural-order comparison requires it. The computer does not infer those interpretations from cellular topology.

A finite experiment may be described by a record consisting of a marked complex and a proposed attaching matrix. The primary algebraic consistency condition is stated in Equation 124.

$$\partial_1^K Z = 0, \quad D \in \mathbb{Z}^{2 \times r}, \quad \partial_2^{L_D} = ZD, \quad \partial_1^{L_D} \partial_2^{L_D} = 0. \quad (124)$$

Here Z is the historical integral cycle-basis matrix given in Equation 67, r is the number of attached faces, and D lists their attaching classes. Equation 124 is an algebraic encoding of the selected cellular boundary data. It does not uniquely determine attaching *words*, fundamental-group information, or the historical interpretation of a face. The distinction between chain-level and attaching-word information is developed in Appendix C.

G.2 Exact Homology and Integral Torsion

The principal script, `examples/verify_section11.py`, constructs matrices with SymPy's exact integer types and uses Smith normal form to determine the invariant factors of each attachment matrix. Floating-point arithmetic is used only for visualization, not for the reported homology invariants. In the theta-graph cases, the cellular cycles of K form a free rank-two group, and the boundary relations of the new faces are the columns of D . Equation 125 records the resulting algebraic presentation.

$$H_1(L_D; \mathbb{Z}) \cong \mathbb{Z}^2 / \text{im } D, \quad H_2(L_D; \mathbb{Z}) \cong \ker(D : \mathbb{Z}^r \rightarrow \mathbb{Z}^2). \quad (125)$$

For the finitely generated free modules in Equation 125, a Smith diagonal with nonzero entries $s_1, \dots, s_d$ gives first Betti number $2 - d$, second Betti number $r - d$, and torsion factors $\mathbb{Z}/s_j\mathbb{Z}$ for nonunit entries. The full cellular complex is connected in the examples, so its zeroth Betti number is one. The source checks both $\partial_1^K Z = 0$ and $\partial_1^{L_D} \partial_2^{L_D} = 0$ before recording these results.

Table 23 gives the complete family of primary cases. The attachment identifiers and calculated invariants agree with Table 10; the latter is generated directly from the script. The present table serves a distinct purpose: it identifies which kinds of algebraic claims are being audited and makes their common base explicit.

The distinction between torsion and free rank is consequential here. Case C has the same first Betti number as cases A and B while carrying an additional order-two class in integral homology. Case E contains a new free second-homology class because the two attached faces have dependent cellular boundaries. These are exact claims about the specified finite chain complexes; the scripts do not rank or assign probabilities to the candidate histories.

Table 23: Audit cases for exact integral homology over the historically marked theta graph K . Columns of D use the basis $([\alpha], [\beta])$; all quantities are synthetic.

Case	Attached two-cell classes	β_1	β_2	Torsion in H_1
K	none	2	0	none
A	$[\alpha]$	1	0	none
B	$[\beta]$	1	0	none
C	$2[\alpha]$	1	0	$\mathbb{Z}/2$
D	$[\alpha], [\beta]$	0	0	none
E	$[\alpha], [\alpha]$	1	1	none

G.3 Independent Algebraic Checks

Re-executing the same algorithm can reveal execution failures, yet agreement among outputs computed through one code path provides limited protection against a common implementation error. The additional script `examples/verify_appendix_G.py` therefore uses a small independent audit for the six theta-graph cases. It reconstructs the attachment matrices from a separately stated case specification, checks the actual integer incidence condition, and computes ranks and first-homology torsion using elementary one- and two-column integer invariants. Its independent calculation does not invoke the Section 11 Smith-normal-form routine. It compares the computed answers against the machine-readable results generated by that routine.

For an integer matrix D with at most two rows, the rank and determinantal divisors can be computed from its entries and two-by-two minors. For the nonzero rank-two cases, the invariant factors can be reconstructed from the greatest common divisor of the entries and the greatest common divisor of the two-by-two minors. The method uses exact integer arithmetic. Equation 126 states the relevant relations in rank two.

$$\delta_1 = \gcd\{D_{ij}\}, \quad \delta_2 = \gcd\{|\det D_{I,J}| : |I| = |J| = 2\}, \quad s_1 = \delta_1, \quad s_2 = \delta_2/\delta_1. \quad (126)$$

These relations apply to a rank-two integer matrix with the displayed minors; the rank-one cases use δ_1 alone. The audit also checks generated table rows against the JSON invariants, counts the expected cases, and confirms the presence of the three vector diagrams. It validates finite artifacts independently of the author's prose, while still relying on the correctness of the *stipulated* case definitions.

The remaining verification scripts inspect other local constructions, including relative chains, category-level finite gluing examples, admissibility witnesses, and Morse-collapse test cases. Some scripts exercise exact matrices, while others test finite logical counterexamples. Their passing status means that the asserted examples satisfy the implemented checks. It does not turn the associated general mathematical propositions into formally machine-verified theorems. Each proposition retains its mathematical proof obligations in the text.

G.4 Continuation Comparisons and Sensitivity Tests

Several experimental comparisons are designed to distinguish topological results that would otherwise be conflated. First, the two one-face branches A and B have isomorphic unmarked future-only quotients, both homeomorphic to S^2 , but the kernels of the maps from $H_1(K)$ are different marked subgroups. The quotient-equivalence and marked-kernel distinction are stated in Equation 69. Consequently, the computation illustrates why an extension record retaining the historical embedding answers questions that the abstract quotient space cannot answer by itself. Future-only encodings equipped with the necessary attaching maps remain fully capable of representing this same information.

Second, the order experiments compare attachment of $[\alpha]$ followed by $[\beta]$ with the reverse order. Both sequences have the same final chain complex and the same sequence of Betti-number pairs, but their first-stage inclusion-induced kernels differ. A separate finite precedence example admits precisely

two linear orders given its stipulated prerequisite relation. These checks operate on *declared* event-order data, keeping causal precedence separate from the incidence relations of the two-complex.

Third, the revision experiment changes the epistemically represented historical base by adding a previously unrepresented two-cell along $[\alpha]$. Attaching a further candidate two-cell along that same cycle then creates a second-homology generator relative to the revised base, whereas attaching it to the original base kills a first-homology generator. This tests sensitivity to the selected historical reconstruction. It does not identify which reconstruction is correct about a real process.

Fourth, the relative-collapse experiment removes a prospective vertex and adjacent cellular material while fixing a historical edge. The full and relative homology calculations verify topological preservation, but the removed event identifier remains unrecoverable from the unlabeled reduced complex alone. Appendix F.7 provides the corresponding information-factorization criterion. The experiment therefore distinguishes homotopy preservation from provenance preservation and from compatibility with later continuation operators.

G.5 Reconstruction, Generated Artifacts, and Execution Protocol

The editable source archive is organized around a root `manuscript.tex`, the `sections/` and `appendices/` directories, `refs/` bibliography files, the `examples/` scripts, and the `generated/` and `figures/` directories. The root Makefile specifies the build and verification commands. In a system with a working XeLaTeX and Biber installation, the conventional manuscript build is `make`; `make verify` executes the finite verification scripts; and `make experiments` regenerates the Section 11 table, machine-readable results, and diagrams. Version-specific build success must be established by running the commands rather than inferred solely from the existence of a PDF.

The generated result file `generated/section11_results.json` includes exact Betti numbers, torsion factors, attachment ranks, boundary-cycle vectors, ordering examples, and revision and reduction summaries. The generated `section11_table.tex` is consumed directly by the manuscript. The three PDF figures document the marked continuation branches, epistemic revision, and relative collapse. Their graphical curves are schematics; only their labeled cellular incidence and the corresponding specified matrices enter the mathematical claims. The presence of a rendered PDF diagram is insufficient to verify a computation without its underlying matrix specification.

A clean rerun should first execute the mathematical tests, then regenerate the figures and machine-readable Section 11 outputs, and finally compile the manuscript. A further reproducibility check compares the regenerated JSON and table against the canonical copies distributed in the archive. The JSON comparison is semantic, and the tabular comparison is textual; byte identity of rendered PDF figures is not required because plotting and PDF backends can introduce metadata and rendering differences unrelated to the finite algebra. The independent Appendix G audit compares algebraic invariants with the Section 11 outputs. To enable later scrutiny, the release also includes a lightweight computational manifest giving tool versions and SHA-256 digests for selected source and result files. Hashes establish identity of archived artifacts, rather than truth of their mathematical interpretations.

A reader can reproduce the main checks without treating the preliminary model as fitted to observational data. Reusing the scripts for empirical research would require specifying data schemas, sampling and measurement conditions, epistemic provenance, admissibility assumptions, and validation targets before interpreting the computed complexes as historical evidence. Computational reproducibility and empirical adequacy are different evaluative requirements.

G.6 Provenance, Failure Modes, and Evidential Scope

A successful software test establishes only the properties encoded by its inputs and assertions. Several risks remain even when the suite passes. A syntactically valid attachment matrix may correspond to multiple nonhomotopic attaching words. A historically identified cycle may be assigned the wrong semantic meaning. A revised epistemic context may alter admissibility predicates without changing cellular boundaries. A topological reduction may preserve homology while discarding event provenance or

Table 24: Reproducibility levels and the evidential limits of the finite continuation experiments.

Level	Auditable object	Additional requirement for broader inference
Cellular consistency	Integer matrices and the equation $\partial^2 = 0$	Correct observation-to-cell encoding
Integral topology	Betti numbers, torsion and relative groups	Adequate choice of topological observables
Historical marking	Inclusion maps and identified cycle classes	Reliable historical identifiers and provenance
Epistemic model selection	Declared evidence and admission tests	Justified inference procedure and calibration
Prospective realization	Candidate extensions and staged orderings	Historical feasibility and empirical causal support
Normative interpretation	Information preserved for stipulated queries	Ethical premises, participation and contextual evidence

model-relevant interfaces. And a numerical invariant may fail to expose causal, institutional, or normative distinctions required for a particular question.

Table 24 distinguishes the verifiable computational claims from the additional premises needed for stronger uses of the formalism. The table is an audit guide: it prevents the verification of a synthetic calculation from being interpreted as validation of a real future scenario.

This appendix establishes a reproducibility boundary for the paper’s preliminary two-complex experiments. Exact algebra makes the local attachment and reduction consequences inspectable, and the independent audit makes common computational mistakes easier to detect. The experiments remain conditional on representation choices and assumed admissibility information. A defensible application of the topological epistemology of future creation will require connecting those formal objects to the historical formation of evidence, observers, and capacities, while retaining the distinction between possible formal continuations and realizable or ethically preferable futures.

H Epistemic Reconstruction and Formal Counterexamples

This appendix tests the interpretive and inferential limits of the continuation formalism through finite counterexamples. Its analytical object is the passage from historically situated evidence to models of candidate two-complex extensions. The objective is to distinguish conditions established by cellular construction, homological calculation, and dependent-type admission from stronger claims concerning model identification, observational adequacy, or the completeness of future possibilities. The method uses explicitly stipulated historical complexes, competing attachments, controlled observation contexts, and changes of admissibility predicates. Each counterexample specifies the information available to an observer, the models or continuations compatible with that information, and the difference remaining unresolved. The constructions extend the epistemic reconstruction analysis of Section 9 and the computational distinctions in Section 11; they are mathematical demonstrations rather than reconstructions of empirical histories.

H.1 Finite Evidence and Continuation Nonidentifiability

A representation of historical relational spacetime supplies structural and possibly evidential constraints on proposed continuations. It need not determine which prospective attaching maps are admissible, selected, or realized. To state the resulting identifiability problem precisely, fix a historical theta graph K with a marked integral cycle basis $([\alpha], [\beta])$ and a stipulated observational record d containing its vertices, edges, incidence relations, and historical cycle markers. The record contains no observation of a prospective two-cell attaching map.

Consider two epistemic models m_A and m_B that agree on the recorded history. Model m_A selects the extension L_A obtained by attaching a two-cell along α ; model m_B selects L_B obtained by attaching a two-cell of degree two along α . The homological distinction appears in Equation 127.

$$H_1(K; \mathbb{Z}) \cong \mathbb{Z}^2, \quad H_1(L_A; \mathbb{Z}) \cong \mathbb{Z}, \quad H_1(L_B; \mathbb{Z}) \cong \mathbb{Z} \oplus \mathbb{Z}/2\mathbb{Z}. \quad (127)$$

The calculations in Equation 127 follow from the respective attaching columns $(1, 0)^T$ and $(2, 0)^T$ in the marked cycle basis. Both models accommodate d because d records the historical complex without evidence discriminating between the prospective attachments. Their modeled continuations have different torsion, even though their historical observations coincide. Neither model is thereby established as an actual generative mechanism.

Proposition H.1 (Conditional nonidentifiability under a fixed evidence record). *Let $\mathcal{M}_d$ contain two models m_1, m_2 compatible with an observed record d . Suppose a modeled continuation observable q has different values for a continuation selected by m_1 and one selected by m_2 , while the data offered to an inference rule are identical for both models. A deterministic inference rule whose sole input is d cannot return the correct distinct q -values for both models.*

Proof. An inference rule $F(d)$ has one value at its specified input. Correctness for both models would require that value to equal each of the two stipulated, distinct observable values, which is impossible. $\square$

Proposition H.1 concerns identification *from the stated evidence interface*. Additional intervention records, observational instruments, or modeling assumptions may discriminate the candidates. It does not say that every proposed continuation is equally plausible; it identifies the evidential conditions missing from the constructed example. The counterexample also clarifies why the existence of a typed family $\Gamma_c(m)$ in Equation 115 does not supply a privileged model-selection rule.

H.2 Historically Conditioned Observation and Divergent Reconstructions

The proposed reference-domain formalism of Section 6 treats the observer's capacities and available interpretations as historically conditioned. A minimal construction can expose the representational consequences of changing those conditions without purporting to explain their ultimate emergence. Stipulate a finite two-dimensional reference complex R consisting of a filled triangle and all its boundary cells. This R is an *assumed formal reference structure*; identifying it with ultimate reality is neither required nor intended.

Define three observation contexts by their accessible cellular data. Context c_P records the three vertices and only two consecutive boundary edges; context c_C also records the closing edge; context c_F additionally records the two-cell. Their respective epistemic reconstructions are the path K_P , the boundary cycle K_C , and the filled triangle K_F . They are nested cellular subcomplexes of the posited R , but their first homology groups differ as specified in Equation 128.

$$K_P \hookrightarrow K_C \hookrightarrow K_F = R, \quad (\beta_1(K_P), \beta_1(K_C), \beta_1(K_F)) = (0, 1, 0). \quad (128)$$

Equation 128 demonstrates that a sequence of increasingly inclusive *representations* can first introduce and then fill a homological cycle. An observer who has registered the boundary edges but not the face may recognize a nontrivial class that becomes trivial once the face is represented. This observation alone says nothing about whether the earlier epistemic state corresponded to a real ontological hole or whether the later interpretation is accurate.

The contexts illustrate historical conditioning as an indexing relationship: different available detection procedures, concepts, or records can support different reconstructed complexes. The construction leaves the developmental account of those capacities unspecified. It also permits distinct observing histories to yield the same output complex. Consequently, even exhaustive mathematical knowledge of K_C would not uniquely reconstruct the historical processes that constituted the operator producing K_C .

H.3 Future-Only Quotients and Historical Attachment Ambiguity

A second limitation concerns the representation chosen for an extended history. Let $K = S_\alpha^1 \vee S_\beta^1$ be a wedge of historically marked circles and let L_α and L_β be extensions obtained by attaching a two-cell along the respective marked cycles. Each quotient collapsing the entire historical K to a point is homeomorphic to S^2 . Nevertheless, the inclusion-induced maps have distinct marked kernels, recorded in Equation 129.

$$\begin{aligned} L_\alpha/K &\cong L_\beta/K \cong S^2, \\ \ker(H_1(K) \longrightarrow H_1(L_\alpha)) &= \langle [\alpha] \rangle, \\ \ker(H_1(K) \longrightarrow H_1(L_\beta)) &= \langle [\beta] \rangle. \end{aligned} \tag{129}$$

The homology groups in Equation 129 use integer coefficients. As abstract unmarked complexes, the two continuations can be related by relabeling the historical circles; as *continuations over the fixed marked history*, that relabeling is not the identity on K . This is precisely the distinction relevant to historically identified events and couplings.

This counterexample supports the representational preference developed in Section 7: the primary object is a whole-spacetime extension (L, i) retaining $i : K \hookrightarrow L$ and its historical markers. A future-only quotient or relative chain complex remains mathematically useful, and a sufficiently enriched future-only presentation can reconstruct the relevant attachment data. The preference is therefore an information-design decision: it keeps historical identities and incidence interfaces directly available to subsequent reasoning.

H.4 Cellular Homology and Attaching-Word Underdetermination

Even a marked inclusion and all cellular homology groups need not encode every higher-order relation in an extension. Consider again $K = S_\alpha^1 \vee S_\beta^1$. Attach one two-cell along the constant word to obtain L_0 , or attach it along the commutator word $\alpha\beta\alpha^{-1}\beta^{-1}$ to obtain L_{com} . The abelianized cellular boundary of the new two-cell is zero in both cases. Consequently, both complexes have $H_1 \cong \mathbb{Z}^2$ and $H_2 \cong \mathbb{Z}$.

The attaching maps differ at the level of the fundamental group, as summarized in Equation 130.

$$\pi_1(L_0) \cong F(\alpha, \beta), \quad \pi_1(L_{\text{com}}) \cong \langle \alpha, \beta \mid [\alpha, \beta] = 1 \rangle \cong \mathbb{Z}^2. \tag{130}$$

The first group in Equation 130 is the free group on two generators; the second is its abelianization by the commutator relation. Thus, identical cellular boundary matrices and corresponding homology groups do not imply identical attaching-word data or equivalent fundamental groups. Appendix C provides the chain-level interpretation of this phenomenon.

For an epistemic continuation model, the result cautions against using homology as a complete encoding of relational meaning. An inferential procedure retaining only the boundary matrix may identify these alternatives, while one retaining cell-attaching words can distinguish them. Neither encoding supplies the historical semantics of the face automatically. The appropriate representation depends on which questions subsequent continuation or normative analysis needs to answer.

H.5 Epistemic Revision and the Obstruction to Admissibility Transport

The preceding examples vary candidate models or accessible structures. A further counterexample holds the historical complex and structural extension fixed while revising the epistemic criteria for admissibility. Let c_0 and c_1 be contexts with the same marked historical K , and let $x = (L, i)$ be a particular structural continuation. Assume $\text{Adm}_{c_0}(x)$ is inhabited while $\text{Adm}_{c_1}(x)$ is empty. For example, c_0 may admit the attachment under an initially recorded resource constraint, while c_1 includes independently stipulated evidence of a violated bound. The example is about the formal response to revised evidence; it does not establish any empirical resource law.

Proposition H.2 (Obstruction to witness-preserving revision transport). *Under the stated contexts, there is no total map $T : \text{Cont}(c_0) \rightarrow \text{Cont}(c_1)$ whose erasure agrees with the identity on structural extensions, provided the admitted continuation (x, w_0) is within its domain.*

Proof. By assumption, $w_0 : \text{Adm}_{c_0}(x)$ exists. A total map T preserving structural erasure would send (x, w_0) to (x, w_1) for an inhabitant $w_1 : \text{Adm}_{c_1}(x)$. The latter type was assumed empty, giving a contradiction. $\square$

Proposition H.2 gives a simple obstruction to unrestricted transport of admitted candidate futures across epistemic updates. The structural pushout or identity base change may exist, as in Appendix D, while contextual admission fails to transport. A partial revision relation, a revised admissibility procedure, or additional compatibility witnesses can remain meaningful. The proposition supports keeping *epistemic revision* distinct from *spacetime continuation* at the level of both morphisms and types.

H.6 Changing Representational Vocabularies and Incomplete Coverage

The finite constructions also illuminate a limitation of any fixed vocabulary for future creation. Let an observer's initial formal language $\mathcal{L}_0$ contain only one available type of prospective face, f_a , attachable along a specified historical loop. Let the extended language $\mathcal{L}_1$ additionally contain a distinct face type f_b , with an explicitly different prospective interpretation or attaching interface. Under a depth-one formation rule, the candidate continuation types expressible in the two languages are $\{x_a\}$ and $\{x_a, x_b\}$, respectively. The relation is summarized in Equation 131.

$$\mathcal{L}_0 \subsetneq \mathcal{L}_1, \quad \text{Expr}_{\mathcal{L}_0}(K) = \{x_a\} \subsetneq \text{Expr}_{\mathcal{L}_1}(K) = \{x_a, x_b\}. \quad (131)$$

Equation 131 expresses *syntactic* enlargement under stipulated formation rules. It does not assert that x_b has newly become historically feasible, or that it was ontologically impossible before the language change. A later revision of evidence can simultaneously expand the expressible family and restrict the admissible family. Hence, syntactic expressibility, evidence-based admission, and actual feasibility should be indexed and investigated separately.

The construction has an epistemological consequence of limited but useful scope. A complete enumeration of candidates in a fixed language establishes completeness only relative to that language and its formation rules. Extending the repertoire can reveal previously unrepresented candidate continuations. Nothing in a finite type check or homology calculation establishes that the repertoire exhausts the possibilities available in reality. The historically constituted development of concepts and instruments therefore matters to the interpretation of future-model coverage, even when their historical formation is not itself explained by the formalism.

H.7 Counterexample Register and Inferential Scope

Table 25 consolidates the informational distinctions demonstrated above. Each row identifies a mathematically specified representation or inference interface, a distinction that interface can miss, and the additional structure required to make the distinction examinable. The table functions as a set of design tests for proposed future-creation models, not as a checklist establishing empirical reliability.

The formal counterexamples locate limits of particular information interfaces; they do not establish limits of mathematical inquiry in general. Enriched higher-order invariants, refined evidence, additional structures, or alternative representations may remove individual ambiguities. Similarly, epistemic reconstruction can improve without becoming complete or unique. The assumed ontological relational spacetime remains a formal convenience throughout: the counterexamples concern relations among stipulated reference structures, representations, evidence rules, and candidate continuations.

For later normative research, the salient implication is conditional. When an evaluative question depends on historically marked events, opportunities, participation, or the order of interventions, a reduction or reconstruction should retain the data on which the proposed evaluation depends. The mathematics

Table 25: Finite counterexamples and corresponding information requirements in epistemic continuation models.

Observed or retained interface	Distinction remaining unresolved	Additional specification
Finite historical record	Models selecting distinct prospective attachments	Future-discriminating evidence, assumptions, or tests
Cellular data accessible to an observer	Different reconstructed complexes and historical operator supports	Observational capacities, provenance, and interpretation records
Future-only quotient	Different effects on marked historical cycles	Historical embedding and attaching data
Cellular boundary and homology	Distinct attaching words and fundamental groups	Attaching maps and higher-order invariants
Structural base-change map	Loss of a revised admission witness	Context-sensitive admissibility and compatibility rules
Enumeration in a fixed language	Candidate forms available in an expanded vocabulary	Explicit language scope and revision interface

can reveal failures of information preservation; the relevance and authority of a particular evaluation depend on separate ethical premises and historical interpretation, as developed in Section 12. These counterexamples complete the technical testing of the preliminary continuation formalism while preserving its central epistemic limitation: represented candidate futures remain conditional on historically situated models, and the set of recognized possibilities remains open to revision.

Research Ethics and Source Provenance

Research scope. This draft develops a provisional philosophical and formal research proposal; it reports no original fieldwork or human-participant data. The synthetic mathematical examples throughout this manuscript do not constitute evidence about identifiable institutions or communities.

Source verification. Each external source cited in Sections 1–15 was checked against online bibliographic, author, publisher, or primary-material records before insertion. The accompanying verification ledger records the access boundary and intended support. Verification of a bibliographic record does not imply detailed examination of an entire book.

Revision status. Sections 1–15 constitute an initial scholarly draft; Appendices A–H are drafted; the first complete technical draft awaits comprehensive mathematical and scholarly revision. The formal results remain conditional on their stated assumptions, and empirical and normative validation remain outstanding.

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