

Lacanian Psychoanalysis and Relational Field Dynamics: Toward a Dynamical Geometry of Subjective Trajectories

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Abstract

This paper develops a preliminary field-dynamical framework for representing subjective trajectories in Lacanian psychoanalysis. Building on a discrete generative account of psychoanalytic traversal, the present study examines how historically situated subjective configurations may be represented through continuous relational fields, nonlinear evolution, and a state-dependent geometry of possible trajectories. The framework introduces a relational configuration space, field variables, coupled evolution laws, historical and exogenous conditions, and observation maps connecting continuous configurations with fine-grained Lacanian descriptions and coarser Real–Symbolic–Imaginary representations.

The analysis then extends the dynamical model geometrically. A relational metric is introduced to represent how effective distances and available directions of change may depend upon the current configuration and its history. Gauge structure, parallel transport, recurrence, attractors, metastability, bifurcation, criticality, and hysteresis are considered as candidate formal tools for distinguishing different forms of subjective return and transformation. Field histories and coarse-graining are also examined as a bridge between discrete generative trajectories and continuous effective descriptions.

The proposed construction is a formal representation rather than a claim that psychoanalytic subjectivity literally implements a physical field theory or quantum field theory. Classical field theory, dynamical systems, differential geometry, and selected QFT-inspired structures are treated as distinct mathematical resources whose psychoanalytic interpretation requires independent justification. The paper’s contribution lies in organizing these resources into a multi-resolution account of relationally generated subjective trajectories and in preparing a later investigation of emergent relational geometry and the “spacetime of desire.”

Keywords: Lacanian psychoanalysis; relational field dynamics; subjective trajectories; dynamical systems; relational geometry; field theory; gauge structure; coarse-graining; historical dependence; psychoanalytic formalization.

Discussion Paper Note

This paper is a theoretical and exploratory study of mathematical representations for subjective trajectories in Lacanian psychoanalysis. Its purpose is to investigate whether concepts from dynamical systems, field theory, differential geometry, gauge theory, and related mathematical frameworks can provide useful formal descriptions of relationally situated psychoanalytic processes.

The mathematical constructions developed in the paper should be interpreted according to the following methodological qualifications.

1. The relational field is a formal representation.

The field variables introduced in this paper represent selected properties of a relational psychoanalytic model. They are not identified with known physical fields.

A notation such as

$$\Phi^a(x, t)$$

therefore means that the model assigns one or more continuously varying quantities to a formal domain. The coordinate x need not represent physical space, and t need not coincide directly with phenomenological, clinical, or chronological time.

The interpretation of the domain, field components, and observation scale must be stated separately.

2. Field-theoretic language does not imply a quantum ontology of the psyche.

This paper uses mathematical structures that also occur in classical field theory, quantum field theory, gauge theory, and related areas of mathematical physics.

The use of these structures does not establish that subjective experience, desire, the unconscious, or Lacanian structures are quantum physical systems.

In particular, expressions involving fields, actions, potentials, gauge transformations, connections, path ensembles, or effective theories should be read according to the formal definitions supplied in the paper.

A physical interpretation requires independent evidence and is outside the present claim.

3. Classical field theory and QFT-inspired constructions are kept distinct.

Several parts of the paper require only ordinary continuous dynamics, classical fields, or differential geometry.

For example,

$$\partial_t \Phi = F[\Phi]$$

is a dynamical equation and does not by itself introduce quantum field theory.

Similarly,

$$S[\Phi] = \int \mathcal{L}(\Phi, \partial_\mu \Phi, x) d^{d+1}x$$

is an action-based field representation. The existence of an action does not by itself imply quantization.

Where ideas associated with quantum field theory are considered, their additional assumptions will be stated explicitly.

4. The configuration space is model-dependent.

Let

$$\mathcal{M}$$

denote the relational configuration space of the model.

The paper does not assume that $\mathcal{M}$ is a directly observed psychological space or a uniquely determined ontology of subjectivity.

Its coordinates are selected according to the distinctions required by the formal problem.

Different models may therefore use different configuration spaces while describing overlapping psychoanalytic phenomena.

5. A subjective trajectory is distinguished from its observable projection.

A continuous trajectory may be represented as

$$\Gamma : t \mapsto X(t) \in \mathcal{M}.$$

The corresponding Lacanian observation is introduced through a map such as

$$\mathcal{O}_L : \mathcal{M} \longrightarrow \mathcal{Y}_L.$$

A further coarse observation may produce an RSI description:

$$\Pi_{\text{RSI}} : \mathcal{Y}_L \longrightarrow \{R, S, I\}.$$

Consequently,

$$X(t) \mapsto o(t) \mapsto a(t)$$

contains several representational levels.

Equality at a coarse level does not imply equality at a finer level.

For example,

$$a(t_1) = a(t_2) = S$$

does not imply

$$X(t_1) = X(t_2).$$

A return to the same register can therefore occur while the complete relational configuration has changed.

6. The present paper preserves the distinction between symbolic recurrence and dynamical closure.

The previous discrete framework distinguished repeated symbols, repeated Lacanian positions, and repeated generative configurations.

The continuous framework retains an analogous hierarchy.

A return to the same observable category,

$$\mathcal{O}(X(t_1)) = \mathcal{O}(X(t_2)),$$

does not establish state-space closure.

A genuine closed trajectory requires an appropriate condition such as

$$X(t_1) = X(t_2),$$

together with any additional state variables required by the model.

Concepts such as holonomy require still more structure, including a well-defined connection and a genuinely closed path.

7. **The relational metric is an analytical structure.**

The paper considers geometries of the form

$$ds^2 = g_{\mu\nu}(x, \Phi, h) dx^\mu dx^\nu.$$

Such a metric is intended to represent effective relational distance or local accessibility within the selected model.

It is not assumed to be a physical spacetime metric.

The meaning of distance, curvature, neighborhood, and accessibility depends upon the formal interpretation assigned to the coordinates and metric.

A state-dependent or history-dependent metric expresses the hypothesis that the effective geometry of possible change can depend upon the current relational configuration.

8. **Geometric language should be used only where the required mathematical structure has been defined.**

Terms such as

manifold, metric, connection, curvature, holonomy

refer to distinct mathematical structures.

The paper does not use these terms interchangeably.

A metric does not automatically provide a gauge connection.

A recurrent path does not automatically define a closed loop.

A closed loop does not automatically possess a psychoanalytically meaningful holonomy.

Each stronger construction requires the corresponding mathematical and interpretive assumptions.

9. **Gauge transformations concern representational equivalence only after an invariance structure has been specified.**

A transformation

$$\Phi \mapsto g \cdot \Phi, \quad g \in G,$$

can be interpreted as a gauge transformation only when the model defines which transformations belong to G and which observables remain invariant.

For an observable $\mathcal{O}$,

$$\mathcal{O}(g \cdot \Phi) = \mathcal{O}(\Phi)$$

may identify several formal descriptions as equivalent under the selected representation.

The existence of several psychoanalytic descriptions of a phenomenon is not by itself sufficient to establish gauge symmetry.

The relevant transformation group and invariant quantities remain formal problems to be justified.

10. Dynamical concepts retain their mathematical meanings.

The paper considers fixed points, stability, attractors, metastability, bifurcation, criticality, recurrence, and hysteresis.

These terms are not used merely as metaphors.

For example, an attractor claim requires a defined state space, evolution law, and relevant asymptotic behavior.

A bifurcation requires qualitative change in the dynamical structure under variation of a parameter.

Hysteresis requires history-dependent response under an explicitly defined driving or parameter variation.

Where the available model supports only a weaker structural analogy, the paper will state that limitation.

11. Historical dependence does not require that every model be non-Markovian.

A historically situated subject can be represented in several ways.

One model may enlarge the state so that relevant history is encoded in the current configuration,

$$X_t = (x_t, M_t).$$

Another may use an explicit memory kernel,

$$\partial_t \Phi(t) = F[\Phi(t)] + \int_0^t K(t - \tau) \Phi(\tau) d\tau.$$

These are different formal representations of historical dependence.

The paper does not assume in advance that one formulation provides the unique mathematical account of psychoanalytic history.

12. Retroaction is distinguished from reverse physical time evolution.

Lacanian retroaction motivates a model in which later developments can alter the interpretation of earlier events.

This can be represented through a changing valuation,

$$\nu_t \longrightarrow \nu_{t+1},$$

even while the dynamical trajectory remains temporally ordered.

The paper therefore distinguishes

state evolution, historical interpretation, retroactive semantic revision.

Retroaction does not require literal backward propagation through physical time.

13. Variational formulations are candidate representations rather than universal assumptions.

Some relational dynamics may admit a useful action-based representation,

$$\delta S[\Phi] = 0.$$

Other processes may be dissipative, driven, stochastic, history-dependent, or otherwise poorly represented by a simple conservative variational model.

The paper therefore treats the Lagrangian and action formulations as one formal family among several.

The existence of an elegant action principle is not taken as evidence that subjective dynamics are fundamentally conservative.

14. Potential landscapes are effective representations.

A function such as

$$V(\Phi)$$

may be used to represent relative stability, local minima, barriers, or changes in the effective dynamics.

Such a potential is a model-dependent object.

The paper does not assume that desire literally minimizes a scalar energy functional.

If a potential is introduced, its analytical role must follow from the evolution law or effective formulation being used.

15. Path ensembles do not imply quantum superposition.

A family of admissible trajectories can be represented as

$$\Gamma(\Phi_i, \Phi_f) = \{\gamma : \Phi_i \rightarrow \Phi_f\}.$$

A general aggregate may be written

$$K(\Phi_f, \Phi_i) = \int_{\Phi_i}^{\Phi_f} \mathcal{D}\gamma W[\gamma].$$

In the present framework, $W[\gamma]$ is initially a general trajectory weight.

Its interpretation may concern probability, accessibility, frequency, cost, or another explicitly defined quantity.

The specifically quantum expression

$$W[\gamma] = \exp\left(\frac{i}{\hbar} S[\gamma]\right)$$

will not be assumed unless the additional physical and mathematical interpretation required by that form has been justified.

16. Coarse-graining can alter the effective model.

The transition

$$\Phi_{\text{fine}} \longrightarrow \Phi_{\text{eff}}$$

is not assumed to mean deletion of variables while leaving the remaining dynamics unchanged. Coarse-graining can generate effective interactions and modify the form of the resulting dynamics. This observation is relevant to the relation between fine Lacanian descriptions and coarse RSI observations.

The projection

$$\Gamma_L \longrightarrow L_{\text{RSI}}$$

should therefore be distinguished from a full renormalization procedure.

The paper uses renormalization language only when scale-dependent effective dynamics or an explicitly defined coarse-graining transformation warrants it.

17. Symmetry breaking, vacuum structure, and related QFT concepts are treated cautiously.

Concepts such as spontaneous symmetry breaking and structured vacuum states may provide useful formal comparisons for historically selected relational configurations or latent generative structure.

The paper does not identify psychoanalytic subject formation with spontaneous symmetry breaking, nor Lacanian lack with the quantum vacuum.

Any such relation remains an explicitly marked structural analogy unless a stronger mathematical model is constructed.

18. Spin foams and group field theory remain outside the central formal claims of Paper II.

The present paper may motivate the possibility that relational geometry is itself generated rather than fixed in advance.

A later study will examine whether spin-foam and group-field-theoretic structures can provide a formal model of such emergent relational geometry.

Accordingly, Paper II does not claim that subjective spacetime is a spin foam, that Lacanian relations carry physical spin representations, or that a group field theory describes the actual mechanism of desire.

These possibilities belong to a subsequent exploratory stage of the research programme.

19. The model remains a single-subject approximation.

The paper represents one historically situated subject interacting with an external relational environment.

Exogenous conditions may enter through terms such as

$$u(t)$$

or

$$J(x, t).$$

The endogenous dynamics of other subjects are not modeled simultaneously.

A fully coupled architecture would require, for example,

$$X_A(t) \longleftrightarrow X_B(t),$$

or interacting relational fields for several subjects.

Such coupled dynamics belong to later work.

20. **Formal adequacy is distinguished from causal explanation.**

A model can reproduce or organize a trajectory without identifying the complete mechanism that produced the corresponding psychoanalytic phenomenon.

The paper therefore distinguishes

phenomenon,

psychoanalytic description,

formal dynamical representation,

and

causal mechanism.

A successful mathematical representation at one level does not establish an identity among these levels.

21. **The framework is multi-model rather than committed to one unique dynamics.**

Different mathematical models may preserve different properties of the same subjective trajectory.

A finite-dimensional dynamical system may be sufficient for one analytical question.

A spatially or relationally extended field may be necessary for another.

A memory-bearing model may be required when historical dependence changes future accessibility.

A gauge-geometric model may become useful when representational redundancy and parallel transport are part of the research problem.

The existence of a richer model does not make a simpler model analytically inferior when the simpler model preserves the required distinctions.

22. **Formal complexity is introduced only when it performs analytical work.**

The paper therefore adopts the same principle of formal economy as the preceding generative study:

Use the least elaborate formal structure that preserves the distinctions required by the problem, and introduce additional mathematical machinery only when the additional structure changes the analysis.

The use of a field, manifold, metric, gauge connection, path ensemble, or effective action must consequently be justified by the distinction that the construction preserves.

23. **The paper's possible novelty lies in the domain-specific construction rather than in the mathematical tools themselves.**

Dynamical systems, classical field theory, variational methods, differential geometry, gauge theory, coarse-graining, and related mathematical machinery are established fields.

The paper does not claim novelty from these constructions.

Its possible contribution lies in their source-constrained organization around Lacanian subjective trajectories, the relation between discrete and continuous representations, the separation of fine

Lacanian and coarse RSI observables, the treatment of history-dependent relational geometry, and the explicit distinction among formal description, psychoanalytic interpretation, and causal mechanism.

24. All physical and psychoanalytic identifications remain revisable.

The paper is a discussion-stage theoretical construction.

Definitions, mappings, state variables, observation schemes, and geometric structures may require revision as the formal and psychoanalytic analysis develops.

The framework is therefore intended to support explicit reasoning and comparison rather than to present a completed mathematical theory of subjectivity.

Taken together, these qualifications define the intended status of the paper. The study investigates how continuous fields and dynamical geometry can be used to formalize relationally situated Lacanian trajectories while keeping mathematical representation, psychoanalytic interpretation, and physical ontology analytically distinct.

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1 Introduction

This section establishes the problem addressed by the paper and the mathematical status of the proposed relational field model. Its objective is to explain why a continuous dynamical representation is useful for the study of Lacanian subjective trajectories, which distinctions such a representation can preserve beyond a discrete symbolic model, and where the interpretive limits of field-theoretic language lie. The discussion proceeds from the psychoanalytic problem of historically situated traversal, through the relation between discrete and continuous representations, to the possibility of a state-dependent relational geometry. It then states the formal and epistemic commitments of the paper and summarizes the intended contribution.

Lacan's Graph of Desire provides a structured account of relations among the subject, the Other, signification, demand, desire, fantasy, and related mathemes [2, 3, 4]. A formal reconstruction of such relations can represent admissible symbolic itineraries. A further problem arises once the object of analysis is treated as historically evolving rather than only as a sequence of discrete positions. Two observations associated with the same Lacanian position may occur under different internal configurations, different histories, and different conditions of future accessibility.

The present paper investigates this problem through dynamical systems, relational fields, and differential geometry. Concepts from nonlinear dynamics provide established tools for studying trajectories, stability, attractors, bifurcation, recurrence, and hysteresis [8]. Differential geometry provides distinct structures for representing manifolds, metrics, connections, curvature, and parallel transport [5]. Selected constructions associated with field theory and quantum field theory are considered where they provide additional formal resources, while their physical interpretations are kept separate from the psychoanalytic model [7].

The central proposal is modest. A historically situated subjective process may be studied through a continuous relational state or field whose evolution, observation, and effective geometry are represented explicitly. This does not require that subjectivity be a physical field, that the unconscious be a quantum system, or that Lacanian concepts have hidden physical counterparts. The mathematical framework is introduced as a family of representations whose adequacy depends upon the psychoanalytic distinctions they preserve.

1.1 Scope and Research Problem

The present study concerns the trajectory of a single historically situated subject. External persons, institutions, events, and relational conditions may affect the modeled trajectory, but their complete endogenous dynamics are not included. The first objective is therefore to construct a continuous single-subject model before considering the substantially more complicated problem of mutually coupled subjective dynamics.

Let

$$\Omega_R$$

denote a formal relational domain, and let

$$\Phi^a(\xi, t)$$

denote a collection of field variables defined on that domain. The index a distinguishes field components, $\xi \in \Omega_R$ identifies a location or relational coordinate in the selected model, and t orders the evolution.

The notation does not assume that ξ is a coordinate in physical space. Depending upon the construction, Ω_R may represent a relational, structural, or otherwise formally defined domain.

At time t , the complete field configuration is denoted by

$$\Phi_t = \{\Phi^a(\xi, t)\}.$$

A general evolution law can be written schematically as

$$\frac{\partial \Phi^a(\xi, t)}{\partial t} = F^a[\Phi; h_t, u_t](\xi, t),$$

where h_t represents historically relevant information and u_t represents external conditions or forcing. This equation is initially an architectural statement rather than a proposed law of psychoanalytic motion. It identifies the kinds of objects that a continuous model may need to distinguish.

The field configuration is also separated from its psychoanalytic observation. Let

$$\mathcal{O}_L$$

be an observation map from the continuous configuration to a selected Lacanian representation:

$$\Phi_t \xrightarrow{\mathcal{O}_L} o_t.$$

A further coarse observation may produce an RSI classification:

$$o_t \xrightarrow{\Pi_{\text{RSI}}} a_t, \quad a_t \in \{R, S, I\}.$$

The resulting hierarchy is

$$\Phi_t \longrightarrow o_t \longrightarrow a_t.$$

Consequently,

$$a_{t_1} = a_{t_2}$$

does not imply

$$o_{t_1} = o_{t_2},$$

and neither equality implies

$$\Phi_{t_1} = \Phi_{t_2}.$$

A subject may therefore return to the same coarse symbolic description while occupying a different relational configuration.

This distinction motivates the central research problem of the paper:

How can historically situated Lacanian trajectories be represented through continuous relational dynamics in which field configuration, symbolic observation, historical dependence, and effective geometry remain formally distinct, and which mathematical structures are required to preserve the differences relevant to psychoanalytic interpretation?

Several subordinate problems follow from this formulation. A dynamical model must specify what counts as a state, what determines admissible evolution, which aspects of history remain dynamically relevant, how external conditions enter the system, how observable Lacanian descriptions are recovered from continuous configurations, and under which conditions recurrence or apparent return constitutes genuine dynamical closure.

The geometry of the state space creates an additional problem. If relational distance and future accessibility depend upon the current subjective configuration or its history, a fixed flat state space may suppress precisely the distinctions that the model is intended to preserve. The geometry itself may therefore need to become part of the dynamical representation.

1.2 From Generative Trajectories to Relational Fields

A discrete generative representation and a continuous field representation address related aspects of subjective trajectory formation. Their relation should be established without treating either representation as the fundamental description from which the other must follow.

In a discrete model, generation may be represented schematically as

$$C_0 \Rightarrow C_1 \Rightarrow \cdots \Rightarrow C_n,$$

with a corresponding Lacanian itinerary

$$o_0 o_1 \cdots o_n.$$

Such a representation is well suited to questions concerning admissible symbolic transitions, derivational histories, recursive generation, and coarse-grained languages.

A continuous representation instead considers a trajectory

$$\Gamma : t \mapsto \Phi_t$$

through a space of relational configurations.

Discrete observations can be recovered by selecting observation times

$$t_0 < t_1 < \cdots < t_n$$

and applying the Lacanian observation map:

$$o_k = \mathcal{O}_L(\Phi_{t_k}).$$

The symbolic itinerary becomes

$$\mathcal{O}_L(\Phi_{t_0}) \mathcal{O}_L(\Phi_{t_1}) \cdots \mathcal{O}_L(\Phi_{t_n}).$$

Thus a continuous history may possess a discrete observable representation.

The reverse construction is more difficult. A symbolic sequence such as

$$o_0 o_1 \cdots o_n$$

does not determine a unique continuous trajectory between its observations. Many curves may satisfy the same sampled sequence:

$$\Gamma_1 \neq \Gamma_2$$

while

$$\mathcal{O}_L(\Gamma_1(t_k)) = \mathcal{O}_L(\Gamma_2(t_k))$$

for every sampled time t_k .

The relation between discrete and continuous models is therefore generally many-to-many once hidden states, unsampled evolution, and different interpolation laws are admitted.

This observation is important for the status of the present paper. The continuous field model does not supersede a discrete generative grammar. The two representations preserve different kinds of structure.

The discrete representation can preserve exact symbolic order and derivational alternatives.

The continuous representation can preserve local variation, neighborhoods, rates of change, stability, perturbation response, and geometrical relationships among states.

Their common observable projections allow the models to be compared without requiring their internal structures to coincide.

A useful architecture is consequently

$$\begin{array}{ccc}
\text{discrete generative history} & \longrightarrow & \text{Lacanian observation} \\
& \wr & \\
\text{continuous field history} & \longrightarrow & \text{Lacanian observation.}
\end{array}$$

The symbol in the middle indicates correspondence at a selected observational resolution rather than formal equivalence of the generators.

This multi-model position is especially important when history matters. A sequence of repeated symbols may look simple in a discrete language while the underlying continuous state has moved through substantially different regions of configuration space.

Conversely, a complicated continuous trajectory may project to a simple symbolic sequence when most of its variation is irrelevant to the selected psychoanalytic question.

The choice between discrete and continuous representation is therefore task-dependent.

1.3 Dynamical Geometry and the Spacetime of Desire

Continuous dynamics alone do not determine the geometry of the state space. This subsection introduces the stronger possibility explored in the paper: the effective geometry of subjective trajectories may depend upon relational configuration and history.

Let

$$\mathcal{M}_R$$

denote an effective relational state manifold with local coordinates

$$q^\mu.$$

A metric

$$g_{\mu\nu}$$

defines an infinitesimal relational distance

$$ds^2 = g_{\mu\nu}(q) dq^\mu dq^\nu.$$

A fixed metric already introduces distinctions unavailable from a symbolic trajectory alone. Two paths connecting the same observable states can have different lengths, local directions, and neighborhoods.

The present framework allows a stronger possibility:

$$g_{\mu\nu} = g_{\mu\nu}(q, \Phi, h).$$

The effective geometry may then depend upon the current field configuration and historically relevant conditions.

A coordinate displacement

$$dq^\mu$$

can therefore have different relational significance in two configurations:

$$ds_1^2 = g_{\mu\nu}^{(1)} dq^\mu dq^\nu$$

and

$$ds_2^2 = g_{\mu\nu}^{(2)} dq^\mu dq^\nu.$$

This gives mathematical form to a simple but important idea: an apparently similar change need not occupy the same effective distance for two historically different configurations.

The paper uses the phrase *dynamical geometry* for this possibility. The phrase does not imply general relativity or physical spacetime. It refers to a relational geometry whose effective metric, accessibility structure, or connection may depend upon the evolving configuration.

This perspective also changes the interpretation of recurrence.

Suppose

$$q(T) = q(0).$$

Coordinate return alone does not establish complete historical return.

If the effective state contains additional variables,

$$\Phi_T \neq \Phi_0,$$

or if the relevant geometry has changed,

$$g_{\mu\nu}(T) \neq g_{\mu\nu}(0),$$

the later state remains distinguishable from the earlier one.

Where a gauge structure is subsequently justified, an even stronger notion of historical return becomes available. A connection can define parallel transport along a closed path, and its holonomy can record a transformation accumulated around that path. The present paper introduces such machinery only after the required geometric structures have been specified [5].

The expression *spacetime of desire* designates the limiting conceptual problem suggested by this construction. Instead of assuming permanently that desire moves through a fixed background of subjective possibilities, one may ask whether relational history also participates in generating the effective geometry within which later trajectories become possible.

Paper II approaches this problem through state-dependent and history-dependent geometry. A stronger construction in which relational geometry itself emerges from combinatorial histories, spin-foam-like complexes, or group-field-theoretic generation lies beyond the central formal claims of the present study.

This boundary is deliberate. The paper first establishes a defensible continuous relational geometry before considering whether the geometry itself can be generated from more primitive relational events.

1.4 Formal and Epistemic Commitments

The use of mathematical physics in psychoanalytic formalization requires a strict separation among several levels of claim.

The first level is the Lacanian source structure. Concepts such as the divided subject, the Other, demand, desire, fantasy, signification, and the Graph of Desire derive from Lacan's psychoanalytic work [2, 3, 4].

The second level is established mathematics. Dynamical systems, differential equations, field variables, manifolds, metrics, gauge connections, path integrals, and coarse-graining possess meanings independently of their use in this paper [8, 5, 7].

The third level is the present formal reconstruction. Objects such as the relational field

$$\Phi,$$

the relational configuration manifold

$$\mathcal{M}_R,$$

the observation map

$$\mathcal{O}_L,$$

and any proposed relational metric

$$g_{\mu\nu}(q, \Phi, h)$$

belong to the model constructed here.

The fourth level is psychoanalytic interpretation. A mathematical trajectory receives psychoanalytic meaning only through an explicitly stated relation between the formal variables and the psychoanalytic distinctions under analysis.

The fifth level concerns causal mechanism. A successful relational field model does not establish that the corresponding psychological, neurobiological, linguistic, or social process implements that mathematical architecture.

The paper therefore maintains the separation

Lacanian source structure $\longrightarrow$ formal dynamical model $\longrightarrow$ psychoanalytic interpretation,

while treating

causal mechanism

as an additional problem requiring independent evidence.

The same restraint applies specifically to quantum field theory. A path ensemble, effective action, gauge symmetry, or renormalization-like coarse-graining can be mathematically useful without implying quantum subjectivity. A genuinely quantum interpretation would require assumptions that the present paper does not make.

The framework also adopts a principle of formal economy. Additional mathematical structure is introduced only when it preserves a distinction required by the analysis.

A finite-dimensional dynamical system may therefore be preferable when a field adds no relevant information.

A field becomes useful when relationally distributed degrees of freedom matter.

A metric becomes useful when neighborhood and effective distance matter.

A connection becomes useful when transport and representational comparison along paths matter.

A path ensemble becomes useful when alternative histories must be aggregated.

The existence of a mathematically richer formalism does not make it epistemically superior.

Finally, the paper remains a single-subject approximation. External conditions may enter through

$$u_t$$

or a source term such as

$$J(\xi, t),$$

while the internal dynamics of another subject are left outside the present state space. Reciprocal and coupled subjective fields constitute a later problem.

1.5 Intended Contribution and Paper Structure

The intended contribution of this paper lies in the organization of established mathematical resources around a specific problem of psychoanalytic representation.

First, the paper develops a continuous relational field architecture that distinguishes complete field configuration, Lacanian observation, RSI coarse-graining, historical context, and external forcing.

Second, it introduces a dynamical geometry in which effective relational distance and accessibility can depend upon configuration and history. This allows recurrence at the observable level to be separated from genuine state-space return and creates the conditions under which stronger concepts such as parallel transport and holonomy can later be used precisely.

Third, the paper compares several dynamical regimes relevant to subjective trajectory analysis, including stability, attractors, metastability, bifurcation, criticality, recurrence, and hysteresis. These concepts are introduced according to their mathematical requirements rather than as descriptive metaphors.

Fourth, the paper develops an interface between discrete generative trajectories and continuous field histories. The relation permits the same Lacanian observation to arise from several discrete or continuous generators, making representational underdetermination explicit.

Fifth, the paper examines path ensembles, effective descriptions, and coarse-graining as possible bridges toward field-theoretic treatment while maintaining a distinction between general weighted histories and specifically quantum amplitudes.

The remainder of the paper develops these claims in stages. Section 2 identifies the properties of Lacanian structure that a dynamical representation must preserve. Section 3 defines the relational configuration space, field variables, and observation maps. Section 4 introduces evolution, interaction, forcing, memory, and dissipative dynamics. Section 5 considers variational and effective field formulations. Section 6 develops the geometry of subjective trajectories, and Section 7 examines representational invariance, connection, parallel transport, and holonomy.

Section 8 studies the principal dynamical regimes supported by the framework. Section 9 introduces families of field histories and their possible weighting. Section 10 examines scale, effective dynamics, and coarse-graining. The subsequent exploratory treatment of emergent relational geometry marks the boundary between the present model and the later problem of a generated “spacetime of desire.”

Section 12 returns to the semantic relation between mathematical states and Lacanian interpretation. Section 13 compares the continuous model with the discrete generative architecture. Section 14 evaluates the formal scope, epistemic limitations, and unresolved problems of the construction. Section 15 summarizes the resulting account of relational field dynamics and dynamical subjective geometry.

2 Lacanian Structure and the Problem of Dynamics

This section identifies the features of Lacanian structure that motivate a dynamical representation. Its role is to determine what a continuous model must preserve before field variables, evolution equations, or geometric structures are introduced. The analysis proceeds through four steps. It first distinguishes structural positions from trajectories through those positions. It then considers desire, demand, signification, and retroaction as sources of relational and historical dependence. The third subsection examines recurrence and the possibility that repeated symbolic observations correspond to historically different states. The final subsection extracts a set of formal requirements that will guide the construction of the relational configuration space in Section 3.

The purpose of the section is limited. It does not claim that Lacan supplied a dynamical system, differential equation, field theory, or geometric model of subjectivity. The Graph of Desire and the concepts associated with it provide the psychoanalytic source structure [2, 3, 4]. The dynamical language introduced below belongs to the present reconstruction.

2.1 Structural Positions and Trajectories

The Graph of Desire organizes several distinct positions, mathemes, pathways, and relations within a structured diagram [2, 3, 4]. A formal analysis can therefore begin by distinguishing the structural organization of the graph from a trajectory represented through that organization.

Let

$$\mathfrak{G}_L$$

denote a formal reconstruction of the relevant Lacanian structure, and let

$$\mathcal{O}_L$$

denote the set of selected structural objects within that reconstruction.
A symbolic itinerary may be written as

$$\gamma_L = o_0 o_1 \cdots o_n, \quad o_k \in \mathcal{O}_L.$$

Such a sequence records an ordered traversal at a selected level of resolution. It does not yet specify how the subject moves between successive observations, how long a transition takes, how strongly alternative directions are available, or how the subject's history affects the meaning of a later return.

A continuous model introduces an additional level.

Let

$$X(t) \in \mathcal{M}_R$$

denote the complete relational configuration of the selected model at time t , where $\mathcal{M}_R$ is a relational configuration space to be defined in Section 3.

A Lacanian observation is obtained through

$$\mathcal{O}_L : \mathcal{M}_R \longrightarrow \mathcal{Y}_L.$$

Hence

$$o(t) = \mathcal{O}_L(X(t)).$$

Sampling the continuous trajectory at times

$$t_0 < t_1 < \cdots < t_n$$

produces

$$o_0 o_1 \cdots o_n = \mathcal{O}_L(X(t_0)) \mathcal{O}_L(X(t_1)) \cdots \mathcal{O}_L(X(t_n)).$$

The symbolic itinerary can therefore be understood as an observation of a richer trajectory.

This construction immediately creates an important distinction:

$$\text{structural position} \neq \text{complete relational state}.$$

Two times may satisfy

$$\mathcal{O}_L(X(t_1)) = \mathcal{O}_L(X(t_2)) = o$$

while

$$X(t_1) \neq X(t_2).$$

The same Lacanian observation can therefore correspond to several distinct relational configurations.

This is not merely a technical possibility. It is required whenever the current structural description does not contain all information relevant to the subject's subsequent trajectory.

Suppose two histories arrive at the same observed position o :

$$X^{(1)}(t_1) \xrightarrow{\mathcal{O}_L} o,$$

$$X^{(2)}(t_2) \xrightarrow{\mathcal{O}_L} o.$$

If their future trajectories differ,

$$X^{(1)}(t_1 + \Delta t) \neq X^{(2)}(t_2 + \Delta t),$$

then the observation o alone is insufficient to specify the future evolution.

A dynamical model must consequently distinguish between an observable Lacanian position and the state variables that determine local evolution.

This distinction also prevents the Graph of Desire from being interpreted as a literal phase portrait. The diagram provides a psychoanalytic relational structure. A phase space or field configuration space introduced in the present paper is an additional mathematical construction whose coordinates and evolution laws require independent definition.

The relation between the two can therefore be written as

$$\text{relational state} \longrightarrow \text{Lacanian observation},$$

rather than

$$\text{Lacanian node} = \text{complete dynamical state}.$$

The same principle applies to the Real, Symbolic, and Imaginary. An RSI classification describes the state at an even coarser observational resolution. It should therefore be treated as a projection of relational configuration rather than as a complete dynamical coordinate system.

2.2 Desire, Demand, Signification, and Retroaction

A dynamical representation must also preserve the relational character of the Lacanian structures from which it is constructed. The Graph of Desire does not present desire as an isolated scalar property possessed by a self-contained subject. Its organization relates the subject to the Other, signification, demand, fantasy, and the production of meaning [2, 3, 4].

The present model therefore begins from relational dependence.

Let the complete state contain several components,

$$X(t) = (x^1(t), x^2(t), \dots, x^m(t)),$$

where the components represent formally defined dimensions of the relational configuration.

Their evolution should in general be allowed to depend upon one another:

$$\dot{x}^a = F^a(x^1, \dots, x^m; h_t, u_t).$$

The expression is schematic. It does not identify a particular Lacanian concept with one coordinate x^a . Its purpose is to state that the components of the model need not evolve independently.

The dependence is particularly important for any later interpretation of desire.

A simplistic representation might introduce one scalar

$$d(t)$$

and assume that the dynamics of desire are completely described by

$$\dot{d} = F(d).$$

Such a model would already assume that the future of desire is determined by its current scalar value. The relational structure of the Graph of Desire suggests a more cautious architecture:

$$\dot{d} = F(d, X_{\text{rel}}, h_t, u_t),$$

where X_{rel} represents the other relational variables retained by the model.

The point is structural. A formal variable representing an aspect of desire should be permitted to change according to its relation with the rest of the configuration.

Demand introduces a similar issue. If an external or relational event enters the model through

$$u_t,$$

its effect need not be independent of the current state.

A general driven system can therefore have the form

$$\dot{X}(t) = F(X(t), u(t)).$$

Two subjects, or two historical states of the same subject, can receive the same external input while producing different trajectories:

$$u^{(1)}(t) = u^{(2)}(t)$$

but

$$X^{(1)}(t + \Delta t) \neq X^{(2)}(t + \Delta t).$$

The difference arises because response depends upon the relational state in which the input is received.

Signification introduces a further complication. The meaning assigned to an earlier event may depend upon later developments. The previous discrete model represented this possibility through a time-indexed interpretive valuation. The same distinction remains necessary in a continuous model.

Let

$$\gamma_{0:t}$$

denote the relational history available up to time t , and let

$$\nu_t$$

denote the interpretive valuation at that stage.

For an earlier event at time

$$\tau < t,$$

retroactive reinterpretation permits

$$\nu_{t+\Delta t}(X(\tau) \mid \gamma_{0:t+\Delta t}) \neq \nu_t(X(\tau) \mid \gamma_{0:t}).$$

The past state

$$X(\tau)$$

has not been physically replaced. Its position within the interpreted history has changed.

This distinction is essential because it separates three temporal objects:

dynamical time,

historical accumulation,

and

interpretive time.

The dynamical trajectory may remain forward ordered:

$$X(t) \longrightarrow X(t + \Delta t),$$

while interpretation of an earlier segment is revised.

Lacanian retroaction therefore does not require a dynamical equation in which physical time runs backward. It requires a model in which later information can alter the semantic status of earlier events.

Several implementations remain possible.

One possibility is to keep interpretation outside the state dynamics:

$$X(t) \longrightarrow \nu_t.$$

Another is to include historically accumulated variables within the state:

$$X(t) = (x(t), M(t)),$$

where

$$M(t)$$

contains information relevant to later interpretation.

A third possibility is to use an explicit memory functional over the trajectory.

The present paper does not determine in advance which implementation is required. It preserves the distinction so that later mathematical choices can be evaluated according to the information they retain.

2.3 Symbolic Recurrence and Historical Difference

Recurrence provides one of the clearest reasons for distinguishing symbolic observation from complete dynamical state. Its role in the present section is to establish several increasingly strong forms of return before attractors, closed orbits, and holonomy are introduced later.

Let

$$a(t) = \Pi_{\text{RSI}}(\mathcal{O}_L(X(t)))$$

be the RSI observation of the continuous state.

Suppose

$$a(t_1) = a(t_2) = I.$$

This establishes recurrence at the register level.

It does not establish

$$\mathcal{O}_L(X(t_1)) = \mathcal{O}_L(X(t_2)).$$

Distinct fine Lacanian observations may project to the same register.

Even if

$$\mathcal{O}_L(X(t_1)) = \mathcal{O}_L(X(t_2)),$$

the complete relational states may still satisfy

$$X(t_1) \neq X(t_2).$$

The hierarchy can therefore be written as

RSI return,

fine Lacanian return,

relational state return.

These should remain distinct throughout the paper.

A still stronger condition concerns the local dynamical state. If the system is governed by a first-order autonomous equation

$$\dot{X} = F(X),$$

then equality of the complete state at two times has strong implications for subsequent evolution. A historically dependent model may require additional variables. If the effective state is

$$Z(t) = (X(t), M(t)),$$

then

$$X(t_1) = X(t_2)$$

does not constitute complete state return whenever

$$M(t_1) \neq M(t_2).$$

The relevant closure condition becomes

$$Z(t_1) = Z(t_2).$$

Likewise, for a non-Markovian model whose evolution depends upon a segment of past history, equality of instantaneous configurations may remain insufficient.

This motivates a general principle:

The state used to define dynamical recurrence must contain every variable that the selected model requires to determine admissible continuation.

The principle prevents a symbolic return from being promoted too quickly to a claim of dynamical closure.

The distinction can be illustrated by a trajectory whose coarse observations are

$$I \rightarrow R \rightarrow S \rightarrow S \rightarrow I.$$

The first and final observations agree at the RSI level.

Several possibilities remain.

The underlying fine Lacanian states may differ:

$$o_0 \neq o_4.$$

The fine observations may agree while the continuous relational states differ:

$$o_0 = o_4, \quad X(t_0) \neq X(t_4).$$

The relational states may agree while a memory variable differs:

$$X(t_0) = X(t_4), \quad M(t_0) \neq M(t_4).$$

Only after the complete state required by the model has returned can a stronger dynamical notion of closure be considered.

This hierarchy also clarifies the later use of geometry.
Suppose the state trajectory forms a closed curve

$$\gamma : [0, T] \longrightarrow \mathcal{M}_R$$

with

$$\gamma(0) = \gamma(T).$$

The existence of the closed curve still does not by itself define a nontrivial holonomy. A connection must first be specified on the relevant bundle or geometric structure.

Only then can parallel transport around γ produce an object such as

$$U_\gamma = \mathcal{P} \exp \left(\oint_\gamma A \right).$$

This stronger construction is developed only in Section 7.

The present subsection therefore establishes a sequence of increasingly strong notions:

$$\begin{aligned} \text{symbolic recurrence} &\prec \text{fine structural recurrence} \prec \text{state recurrence} \\ &\prec \text{closed dynamical trajectory} \\ &\prec \text{geometric transport around a closed trajectory.} \end{aligned}$$

where the ordering indicates additional required structure rather than greater psychoanalytic importance.

Historical difference can persist at every earlier level.

2.4 Requirements for a Dynamical Representation

The preceding analysis allows the dynamical problem to be stated without yet committing to a particular field equation or geometric construction. This subsection extracts the minimum requirements that the subsequent relational field model should satisfy.

The first requirement is *state-observation separation*.

The model must distinguish

$$X(t)$$

from

$$\mathcal{O}_L(X(t)).$$

A Lacanian observation records selected structural information. It need not contain the complete state required for dynamical evolution.

The second requirement is *multi-resolution observation*.

The architecture should permit

$$X(t) \xrightarrow{\mathcal{O}_L} o(t) \xrightarrow{\Pi_{\text{RSI}}} a(t).$$

Fine and coarse representations can therefore be compared without requiring their state spaces to coincide.

The third requirement is *relational coupling*.

The components of the state should be allowed to interact:

$$\dot{x}^a = F^a(x^1, \dots, x^m).$$

Independent scalar evolution should appear only when justified by the problem.

The fourth requirement is *historical differentiation*.

Two states with the same current observation must be permitted to have different futures when their dynamically relevant histories differ.

This may be achieved through an enlarged state

$$Z(t) = (X(t), M(t)),$$

through an explicit history variable

$$h_t,$$

or through a memory functional.

The fifth requirement is *external responsiveness*.

The single-subject model must permit external conditions to alter the trajectory:

$$\dot{X} = F(X, u).$$

The external input u remains exogenous within the present paper.

The sixth requirement is *nonlinearity where relational dependence requires it*.

A purely linear system,

$$\dot{X} = AX,$$

may be useful as a local approximation. The model should not assume linearity globally when the phenomena under study include multiple stable regimes, thresholds, bifurcations, hysteresis, or state-dependent response.

The seventh requirement is *open-system compatibility*.

The subjective trajectory may exchange influence with an external relational environment. The mathematical framework should therefore permit driven, dissipative, and non-conservative dynamics.

A conservative action principle is one candidate representation rather than a universal starting assumption.

The eighth requirement is *interpretive retroaction*.

Later developments must be allowed to alter the interpretation of earlier states without requiring literal reverse temporal evolution.

The formal architecture should therefore distinguish

$$X(t)$$

from

$$\nu_t.$$

The ninth requirement is *resolution-sensitive recurrence*.

A return at the RSI level should remain distinguishable from return at the fine Lacanian level and from equality of the complete dynamical state.

The tenth requirement is *geometric extensibility*.

The state space must permit the later introduction of geometric structure when effective distance, neighborhood, curvature, or transport becomes analytically relevant.

The construction should therefore allow a progression from

$$\mathcal{M}_R$$

to

$$(\mathcal{M}_R, g)$$

and, where independently justified, to a richer structure containing a connection:

$$(\mathcal{M}_R, g, A).$$

The eleventh requirement is *model pluralism*.

The formal requirements above do not imply one unique evolution law.

A finite-dimensional system may satisfy them in one application.

A field theory may be appropriate when distributed relational degrees of freedom matter.

A stochastic or non-Markovian model may become necessary under different assumptions.

The mathematical form should follow the distinctions required by the analysis.

The twelfth requirement is *epistemic separation*.

For any proposed model,

$$\mathfrak{M},$$

the following objects must remain distinguishable:

Lacanian source structure,

formal dynamical state,

observable psychoanalytic description,

historical interpretation,

and

causal mechanism.

The satisfaction of the formal requirements establishes only the adequacy of a representation relative to a specified analytical problem.

Taken together, these requirements motivate the architecture developed in the next section. Instead of beginning immediately with a particular Lagrangian or differential equation, Section 3 first defines the objects on which any such dynamics would have to act: the relational domain, configuration space, field variables, local and global states, observation maps, and historical and exogenous conditions.

3 Relational Configuration Space

This section defines the state objects on which the subsequent dynamics will act. Its role is to separate the relational domain, field variables, complete configurations, observable Lacanian descriptions, and historically relevant conditions before any specific evolution equation is selected. The construction proceeds from the underlying relational domain to local field variables and global configurations, then introduces observation maps and finally distinguishes endogenous state, accumulated history, and exogenous input. The method remains deliberately general so that finite-dimensional, field-theoretic, memory-bearing, and later geometric models can be compared within one architecture.

The central distinction is between the domain on which relational variables are defined and the space whose points represent complete configurations. Let

$$\Omega_R$$

denote the relational domain and

$$\mathcal{M}_R$$

the corresponding configuration space. A point of $\mathcal{M}_R$ is a complete relational state of the selected model. When the model is field-theoretic, such a point is itself an entire field configuration over Ω_R .

Thus,

$$\xi \in \Omega_R$$

and

$$\Phi_t \in \mathcal{M}_R$$

belong to different mathematical levels.

This distinction will become essential when geometry is introduced. A metric on the relational domain, a metric on configuration space, and a metric on a finite-dimensional reduced state manifold are different constructions and should not be conflated.

3.1 Subjective Configuration Space

The first task is to define what counts as a state of the dynamical model. The configuration should contain enough information to distinguish trajectories whose future admissibility differs, while avoiding an unnecessary commitment to a complete ontology of subjectivity.

Let

$$\mathcal{M}_R$$

denote the set of admissible relational configurations.

A subjective trajectory is then represented as a curve

$$\Gamma : t \mapsto X(t) \in \mathcal{M}_R.$$

The symbol $X(t)$ denotes the complete state required by the selected dynamical description at time t .

In a finite-dimensional model one may write

$$X(t) = (x^1(t), x^2(t), \dots, x^n(t)),$$

with local coordinates

$$q^\mu$$

on an appropriate region of $\mathcal{M}_R$.

Such coordinates are model variables. They are not assumed to correspond one-to-one with Lacanian concepts.

For example, a coordinate

$$x^1$$

should not be identified with “desire” merely because the model requires a first degree of freedom. Any psychoanalytic interpretation of a coordinate requires an explicit semantic argument.

The field-theoretic case requires a richer state space. Let

$$\Phi^a(\xi, t)$$

be a collection of field variables indexed by

$$a = 1, \dots, N.$$

At fixed time t , the entire collection

$$\Phi_t = \{\Phi^a(\xi, t) \mid \xi \in \Omega_R, a = 1, \dots, N\}$$

constitutes one configuration.

The corresponding configuration space can be written schematically as

$$\mathcal{M}_R \subseteq \text{Map}(\Omega_R, \mathcal{V}),$$

where $\mathcal{V}$ is the value space of the fields.

If the fields are real-valued,

$$\mathcal{V} = \mathbb{R}^N.$$

More complicated value spaces may be introduced only when the later dynamics require them.

In the field case, $\mathcal{M}_R$ may therefore be infinite-dimensional. The present section does not assume that this configuration space already carries every structure needed for differential geometry. Smoothness, metric structure, connection, or measure will be added only when a later construction requires them.

A finite-dimensional state can also arise as a reduction of the field configuration.

Let

$$Q^\mu[\Phi]$$

be a collection of functionals of the field. Then

$$q^\mu(t) = Q^\mu[\Phi_t]$$

defines reduced coordinates.

The map

$$\mathcal{Q} : \mathcal{M}_R \longrightarrow \mathcal{M}_{\text{red}}$$

can therefore project the full field configuration onto a lower-dimensional state manifold.

This provides three possible dynamical resolutions:

local relational field,

complete field configuration,

and

reduced dynamical state.

The appropriate level depends upon the problem being studied.

If a finite collection of effective variables captures the relevant dynamics, the reduced system may be sufficient.

If different parts of the relational domain interact and their distribution matters, the field representation retains information that a finite state vector may suppress.

The configuration space should therefore be understood as an analytical construction whose dimensionality and structure depend upon the distinctions that the model is required to preserve.

3.2 Relational Field Variables

The relational field provides a continuously varying representation of properties distributed across the selected relational domain. Its objective is to permit local variation, coupling, propagation, and state-dependent interaction without assigning each psychoanalytic distinction to an isolated scalar variable.

Let

$$\Phi = (\Phi^1, \Phi^2, \dots, \Phi^N)$$

denote an N -component relational field.

Each component is a function

$$\Phi^a : \Omega_R \times T \longrightarrow \mathcal{V}_a,$$

where T is the temporal domain and $\mathcal{V}_a$ is the value space of the component.

For real scalar components,

$$\Phi^a(\xi, t) \in \mathbb{R}.$$

The field index a and relational coordinate ξ perform different roles.

The index a distinguishes kinds of modeled quantity.

The coordinate ξ distinguishes locations within the domain over which those quantities vary.

The relational coordinate should remain interpretation-neutral at the foundational stage.

It may eventually index a continuously parameterized relational domain, positions within a reduced structural manifold, or another formal dimension whose local organization is theoretically justified.

It should not be assumed that

$$\xi$$

is physical position.

Similarly, a field component should not be named directly after a Lacanian concept until the interpretation map has been specified.

A safer initial notation is

$$\Phi^a$$

rather than expressions such as

$$\Phi_{\text{desire}}$$

or

$$\Phi_{\text{symbolic}}.$$

Such labels can be introduced later if the model supplies an operational meaning for them.

The fields may interact locally. A generic evolution can therefore depend upon the full vector

$$\Phi(\xi, t) = (\Phi^1(\xi, t), \dots, \Phi^N(\xi, t)).$$

It may also depend upon derivatives such as

$$\partial_\mu \Phi^a$$

once a differentiable structure has been defined on the relevant domain.

A local field equation might then take the general form

$$\partial_t \Phi^a = F^a(\Phi, \partial_\mu \Phi, \partial_\mu \partial_\nu \Phi, \dots).$$

The present section does not select the function F^a .

Its purpose is to establish that relational evolution may depend on local configuration, interaction among components, and, where justified, variation across the relational domain.

Nonlocal dependence can also be represented.

For example,

$$\partial_t \Phi^a(\xi, t) = F_{\text{loc}}^a[\Phi](\xi, t) + \int_{\Omega_R} K^a_b(\xi, \xi') \Phi^b(\xi', t) d\mu(\xi')$$

allows the state at ξ to depend upon field values elsewhere in the relational domain.

Here

$$K^a_b(\xi, \xi')$$

is an interaction kernel and $d\mu$ a measure defined on the domain.

This construction should be introduced only where relational influence is genuinely nonlocal in the selected formal representation. The existence of social or symbolic relations does not by itself require an integral kernel.

The distinction among local, nonlocal, and globally reduced models therefore remains a modeling choice.

The relational field is useful precisely when the distribution of the state contains information that would disappear from an ordinary state vector.

3.3 Local and Global Configuration

A field representation introduces a distinction between the state at one relational location and the state of the system as a whole. This subsection formalizes that distinction and identifies several intermediate levels of description.

At a point

$$\xi \in \Omega_R,$$

the local field state is

$$\Phi(\xi, t).$$

The complete field configuration is

$$\Phi_t = \Phi(\cdot, t).$$

Thus,

$$\Phi(\xi, t)$$

is a local value, while

$$\Phi_t$$

is a point in configuration space.

Equality of a local value does not imply equality of global configurations.

For two times t_1 and t_2 , it is possible that

$$\Phi(\xi_0, t_1) = \Phi(\xi_0, t_2)$$

while

$$\Phi_{t_1} \neq \Phi_{t_2}.$$

The rest of the relational field may have changed.

This distinction is analogous to the difference established in the generative model between repeated observable symbols and repeated complete configurations.

The field formulation extends that distinction spatially or relationally.

A local region

$$U \subseteq \Omega_R$$

can also define an intermediate state,

$$\Phi_t|_U,$$

which retains more information than a pointwise observation while suppressing the rest of the field.

The resulting hierarchy is

$$\Phi(\xi, t) \prec \Phi_t|_U \prec \Phi_t,$$

where $\prec$ denotes increasing retained relational information.

A global observable can be defined as a functional

$$Q[\Phi_t].$$

Examples of formal functionals include

$$Q[\Phi_t] = \int_{\Omega_R} f(\Phi(\xi, t)) d\mu(\xi)$$

or another explicitly chosen summary.

Such a quantity may function as an effective order parameter when it captures a collective feature relevant to the dynamics.

The paper does not assume that every psychoanalytic observable should be expressed through a single global integral.

Different observations may depend on different regions, components, or functionals of the field.

A family of observables can therefore be written as

$$Q^\alpha[\Phi_t], \quad \alpha = 1, \dots, m.$$

These observables define a reduced description

$$Q[\Phi_t] = (Q^1[\Phi_t], \dots, Q^m[\Phi_t]).$$

Several full configurations can satisfy

$$Q[\Phi_{t_1}] = Q[\Phi_{t_2}]$$

while

$$\Phi_{t_1} \neq \Phi_{t_2}.$$

The reduced variables therefore define equivalence classes of field configurations at the selected observational resolution.

This point will become important for coarse-graining and effective dynamics. A reduced state may evolve according to approximately closed equations under some conditions. In other cases, unresolved degrees of freedom influence the future and appear as memory, noise, or effective interaction terms.

The distinction between local and global configuration therefore prepares the later analysis of both coarse-graining and history dependence.

3.4 Lacanian Observables

The relational field does not possess psychoanalytic meaning through its mathematical form alone. This subsection defines the observation layer through which a field configuration can receive a source-constrained Lacanian description.

Let

$$\mathcal{Y}_L$$

denote the space of fine Lacanian observations relevant to the present analysis.

Its elements may refer to selected positions, structured relations, mathemes, or other objects whose psychoanalytic status has been established from the source material [2, 3, 4].

Define an observation map

$$\mathcal{O}_L : \mathcal{M}_R \longrightarrow \mathcal{Y}_L.$$

For a configuration

$$\Phi_t \in \mathcal{M}_R,$$

the fine observation is

$$o_t = \mathcal{O}_L(\Phi_t).$$

The map $\mathcal{O}_L$ belongs to the present formal reconstruction.

It must therefore specify which properties of Φ_t justify a particular Lacanian observation.

A simple observation rule may use reduced variables:

$$\mathcal{O}_L(\Phi_t) = \mathcal{O}_L(Q^1[\Phi_t], \dots, Q^m[\Phi_t]).$$

A more complex rule may depend upon the spatial or relational organization of the entire configuration.

The observation map need not be injective.

Indeed, the framework generally expects

$$\Phi_t^{(1)} \neq \Phi_t^{(2)}$$

while

$$\mathcal{O}_L(\Phi_t^{(1)}) = \mathcal{O}_L(\Phi_t^{(2)}).$$

The same Lacanian observation may therefore correspond to several field configurations.

This defines a fiber

$$\mathcal{O}_L^{-1}(o) = \{\Phi \in \mathcal{M}_R \mid \mathcal{O}_L(\Phi) = o\}.$$

A Lacanian observation can consequently be treated as a coarse partition of configuration space.

The RSI representation introduces an additional projection,

$$\Pi_{\text{RSI}} : \mathcal{Y}_L \longrightarrow \mathcal{Y}_{\text{RSI}},$$

where, in the simplest discrete case,

$$\mathcal{Y}_{\text{RSI}} = \{R, S, I\}.$$

The complete observation chain is

$$\Phi_t \xrightarrow{\mathcal{O}_L} o_t \xrightarrow{\Pi_{\text{RSI}}} a_t.$$

This immediately produces nested observational equivalence.

If

$$\mathcal{O}_L(\Phi_1) = \mathcal{O}_L(\Phi_2),$$

then the configurations are indistinguishable at the fine Lacanian resolution.

If

$$\Pi_{\text{RSI}}(\mathcal{O}_L(\Phi_1)) = \Pi_{\text{RSI}}(\mathcal{O}_L(\Phi_2)),$$

they are indistinguishable only at the coarser RSI resolution.

The second equivalence can hold while the first fails.

The hierarchy can therefore be written as

$$\text{field configuration} \longrightarrow \text{fine Lacanian observable} \longrightarrow \text{RSI observable}.$$

The observation architecture provides a direct interface with the discrete generative model developed previously.

A discrete trajectory

$$o_0 o_1 \cdots o_n$$

can be obtained by sampling a continuous history:

$$o_k = \mathcal{O}_L(\Phi_{t_k}).$$

The same symbolic word can arise from several continuous trajectories.

Consequently, agreement at the level of symbolic observation does not establish equivalence of the continuous models that generated it.

This distinction will be developed explicitly in Section 13.

3.5 Historical and Exogenous Variables

The final component of the configuration architecture concerns historical dependence and external influence. Its objective is to distinguish information contained within the dynamical state from contextual information and forcing that remain outside the state representation.

Let

$$\Phi_t$$

denote the relational field configuration.

A model may additionally contain an endogenous memory state

$$M_t.$$

The augmented dynamical state is then

$$Z_t = (\Phi_t, M_t).$$

If Z_t contains all information required by the selected deterministic model to determine local continuation, the evolution can be written schematically as

$$\dot{Z}_t = F(Z_t, u_t).$$

The memory variable M_t is part of the formal state.
It should be distinguished from the broader historical context

$$h_t.$$

The object h_t denotes historically relevant information that the analysis may use even when that information is not represented as an endogenous state variable.

Thus,

$$M_t$$

and

$$h_t$$

need not contain identical information.

A modeling decision may later absorb part of h_t into M_t , producing a richer state representation.

Conversely, a reduced model may leave historically relevant information outside the state and represent its influence through explicit history dependence.

A non-Markovian field model can be written schematically as

$$\partial_t \Phi(t) = F[\Phi(t), \mathcal{H}_t],$$

where

$$\mathcal{H}_t = \{\Phi(\tau) \mid 0 \leq \tau \leq t\}$$

denotes the relevant trajectory history.

A memory-kernel representation provides one possible realization:

$$\partial_t \Phi^a(\xi, t) = F^a[\Phi_t](\xi) + \int_0^t K^a_b(t, \tau) \Phi^b(\xi, \tau) d\tau.$$

The existence of such a representation is a formal possibility. The paper does not assume that psychoanalytic history is governed by a particular kernel.

External influence is represented separately.

Let

$$u_t$$

denote a general exogenous variable.

For a field model, an external source may be written as

$$J^a(\xi, t).$$

The evolution can then take the schematic form

$$\partial_t \Phi^a(\xi, t) = F^a[\Phi, h](\xi, t) + J^a(\xi, t).$$

The source term can represent the formal effect of an encounter, utterance, institutional condition, environmental event, or other external perturbation when such an interpretation is justified.

The present model does not include the complete dynamics that generated J^a .

This maintains the single-subject approximation.

A coupled two-subject system would instead require states such as

$$\Phi_A \quad \text{and} \quad \Phi_B$$

with reciprocal dependence,

$$\partial_t \Phi_A = F_A[\Phi_A, \Phi_B],$$

$$\partial_t \Phi_B = F_B[\Phi_B, \Phi_A].$$

Such a system introduces mutual adaptation, feedback, and co-generation of the relational environment. These dynamics are intentionally reserved for a later research series.

The single-subject architecture developed here can therefore be summarized as

$$(\Phi_t, M_t; h_t, u_t) \longrightarrow (\Phi_{t+\Delta t}, M_{t+\Delta t}),$$

followed by observation,

$$\Phi_t \xrightarrow{\mathcal{O}_L} o_t \xrightarrow{\Pi_{\text{RSI}}} a_t.$$

The variables have distinct formal roles:

Φ_t relational field configuration,

M_t endogenous model memory,

h_t historically relevant context,

u_t external input,

o_t fine Lacanian observation,

and

a_t coarse RSI observation.

These distinctions provide the state architecture required by the remainder of the paper.

The next section introduces dynamics on this configuration space. It examines local and coupled evolution laws, interaction terms, external forcing, memory dependence, dissipation, and the conditions under which a field representation adds analytical information beyond a finite-dimensional state model.

4 Relational Field Dynamics

This section develops the dynamical architecture acting on the relational configuration space introduced in Section 3. Its role is to specify how relational field configurations may evolve, interact, respond to external conditions, retain historical dependence, and exhibit irreversible behavior. The objective is to identify a family of mathematically distinguishable dynamical structures before any particular Lagrangian, metric, or psychoanalytic interpretation is imposed. The section proceeds from general evolution laws to coupled field components, nonlinear interactions, external forcing, memory-bearing dynamics, and dissipative processes.

The starting point is a relational field configuration

$$\Phi_t = \Phi(\cdot, t) \in \mathcal{M}_R.$$

A dynamical model assigns an evolution rule to such configurations. In its most general schematic form,

$$\partial_t \Phi = \mathcal{F}[\Phi; h, u],$$

where $\mathcal{F}$ is an evolution operator, h represents historically relevant conditions, and u represents external influence.

This expression should be understood as an architectural template. The formal content of $\mathcal{F}$, the dimensionality of the field, the regularity of the solutions, and the interpretation of its parameters remain model-dependent.

The dynamics developed below therefore describe a space of candidate representations rather than one uniquely specified law of subjective evolution.

4.1 Evolution Laws

The first task is to distinguish several levels at which relational dynamics can be represented. The simplest case is a finite-dimensional reduced state

$$q(t) = (q^1(t), \dots, q^n(t)) \in \mathcal{M}_{\text{red}}.$$

An autonomous first-order system has the form

$$\dot{q}^\mu = F^\mu(q).$$

The corresponding trajectory is determined locally by a vector field

$$F = F^\mu(q) \frac{\partial}{\partial q^\mu}$$

on the reduced state space.

This representation is appropriate when a finite collection of variables contains the distinctions required by the analysis.

A relational field model instead uses

$$\Phi^a(\xi, t), \quad a = 1, \dots, N,$$

and may evolve according to

$$\partial_t \Phi^a(\xi, t) = F^a[\Phi](\xi, t).$$

The functional notation

$$F^a[\Phi]$$

allows the rate of change at one point to depend upon more than the local field value.

A local evolution law may take the form

$$\partial_t \Phi^a = F^a(\Phi, \partial_i \Phi, \partial_i \partial_j \Phi, \dots),$$

where the derivatives refer to coordinates defined on the relational domain Ω_R .

The simplest purely local model would be

$$\partial_t \Phi^a(\xi, t) = f^a(\Phi(\xi, t)).$$

A spatially or relationally coupled field may additionally contain gradient terms, for example

$$\partial_t \Phi^a = f^a(\Phi) + D^a_b \nabla^2 \Phi^b.$$

Here D^a_b is a coupling coefficient and ∇^2 is defined only after the relational domain has been supplied with the geometric structure required to define it.

The appearance of a Laplacian therefore already carries more structure than the abstract evolution equation

$$\partial_t \Phi = \mathcal{F}[\Phi].$$

The paper will maintain this distinction throughout.
A still more general evolution can be written as

$$\partial_t \Phi_t = \mathcal{F}_t[\Phi_t],$$

allowing explicit temporal dependence.

The model may also depend upon historically accumulated or external variables:

$$\partial_t \Phi_t = \mathcal{F}[\Phi_t; h_t, u_t].$$

The selection among these representations should follow the inferential problem.

If the relevant distinction concerns only local stability around an effective state, a finite-dimensional approximation may be sufficient.

If the distribution of relational variables across Ω_R influences the future trajectory, a field representation becomes useful.

If the evolution depends upon previous history, an instantaneous configuration may require augmentation or replacement by a history-bearing state.

The mathematical form therefore follows the information that must remain dynamically available.

4.2 Coupled Field Components

A relational field can contain several interacting components. The purpose of this subsection is to formalize co-evolution without assigning each field component prematurely to a Lacanian concept.

Let

$$\Phi = (\Phi^1, \Phi^2, \dots, \Phi^N).$$

The general coupled evolution is

$$\partial_t \Phi^a = F^a[\Phi^1, \Phi^2, \dots, \Phi^N].$$

The evolution of one component can therefore depend upon the others:

$$\frac{\delta F^a}{\delta \Phi^b} \neq 0$$

for some

$$a \neq b.$$

Such a dependency represents dynamical coupling within the formal model.

For two components,

$$\Phi^1 \quad \text{and} \quad \Phi^2,$$

a simple schematic system is

$$\partial_t \Phi^1 = F^1(\Phi^1, \Phi^2),$$

$$\partial_t \Phi^2 = F^2(\Phi^1, \Phi^2).$$

The two fields may influence one another asymmetrically. Thus,

$$\frac{\partial F^1}{\partial \Phi^2}$$

and

$$\frac{\partial F^2}{\partial \Phi^1}$$

need not coincide.

This matters for relational modeling because mutual dependence does not imply symmetric influence. For N components, a local linear approximation near a configuration Φ^* may be written

$$\partial_t \delta \Phi^a = J^a_b \delta \Phi^b,$$

where

$$\delta \Phi^a = \Phi^a - \Phi^{a*}$$

and

$$J^a_b = \left. \frac{\partial F^a}{\partial \Phi^b} \right|_{\Phi^*}.$$

The matrix or operator J describes the local coupling structure of the linearized model.

This local representation is useful for questions of stability and sensitivity. The nonlinear evolution remains primary when the trajectory moves beyond the neighborhood in which the linearization is valid.

The coupling structure can also depend upon the state:

$$J^a_b = J^a_b(\Phi).$$

In that case, the effective influence among components changes during the trajectory.

A more general relational model may therefore take the form

$$\partial_t \Phi^a = F^a(\Phi; G(\Phi, h)),$$

where G represents an effective coupling structure.

This possibility is conceptually important for the later geometric construction. The relations governing possible change may themselves depend upon the current configuration.

The paper does not yet interpret such a coupling matrix as a metric, connection, or adjacency structure. These mathematical objects have different definitions and will be introduced separately where required.

4.3 Interaction Terms

Coupling among field components can be represented explicitly through interaction terms. The objective of this subsection is to distinguish independent evolution, pairwise coupling, higher-order interaction, and nonlocal relational dependence.

Consider a decomposition

$$\partial_t \Phi^a = F_0^a[\Phi^a] + F_{\text{int}}^a[\Phi].$$

The term

$$F_0^a$$

describes the evolution assigned to component a in the absence of the selected interactions, while

$$F_{\text{int}}^a$$

collects the coupling terms retained by the model.
A simple pairwise nonlinear interaction may contain

$$g^a_{bc}\Phi^b\Phi^c.$$

The corresponding evolution can be written

$$\partial_t\Phi^a = F_0^a[\Phi^a] + g^a_{bc}\Phi^b\Phi^c.$$

Higher-order interactions may contain terms such as

$$\lambda^a_{bcd}\Phi^b\Phi^c\Phi^d.$$

The use of these expressions does not imply that psychoanalytic relations are literally polynomial interactions. Polynomial terms provide one controlled way of representing nonlinear coupling and can be replaced by other functions when the analytical problem requires them.

The interaction strength may also depend upon the relational location:

$$g^a_{bc} = g^a_{bc}(\xi).$$

Or it may depend upon the current state:

$$g^a_{bc} = g^a_{bc}(\Phi, h).$$

The latter case introduces adaptive coupling.

A trajectory can then alter the effective relations governing its subsequent evolution.

This provides a dynamical precursor to the later idea of state-dependent geometry. At this stage, however, g^a_{bc} denotes an interaction coefficient rather than the metric tensor $g_{\mu\nu}$.

The two should remain typographically and conceptually distinct.

Nonlocal interaction can be represented through an integral kernel:

$$F_{\text{int}}^a(\xi, t) = \int_{\Omega_R} K^a_b(\xi, \xi'; \Phi_t) \Phi^b(\xi', t) d\mu(\xi').$$

This expression allows the evolution at ξ to depend upon relational states at other locations ξ' .

The kernel itself may be fixed,

$$K^a_b(\xi, \xi'),$$

or state-dependent,

$$K^a_b(\xi, \xi'; \Phi_t).$$

A state-dependent kernel produces another form of adaptive relational structure.

The significance of such interactions depends on their interpretation. Introducing a nonlocal kernel merely because psychoanalysis concerns relations would add mathematical complexity without explanatory gain.

A nonlocal formulation becomes useful when the selected relational domain contains interactions whose dependence cannot be represented adequately by local derivatives or finite-dimensional coupling.

The same principle applies to higher-order interactions. They should enter the model when pairwise coupling fails to preserve a distinction required by the analysis.

The interaction structure therefore remains subordinate to the principle of formal economy established in Section 1.

4.4 External Forcing

The present paper models one historically situated subject within an open relational environment. External events can therefore modify the trajectory without being generated endogenously by the modeled field.

Let

$$J^a(\xi, t)$$

denote an external source.

A driven field equation has the schematic form

$$\partial_t \Phi^a(\xi, t) = F^a[\Phi](\xi, t) + J^a(\xi, t).$$

The source can be localized,

$$J^a(\xi, t) = J_0^a \delta(\xi - \xi_0) f(t),$$

when the formal domain and measure make such a representation meaningful.

It can instead be distributed across a region:

$$J^a(\xi, t) = J_{\text{ext}}^a(\xi, t).$$

The distinction allows the model to represent external influence with different relational extents.

The effect of an external source depends upon the current state.

Suppose the dynamics are

$$\partial_t \Phi = F(\Phi) + G(\Phi)J.$$

Then the same external input

$$J_1 = J_2$$

can produce different instantaneous responses when

$$G(\Phi_1) \neq G(\Phi_2).$$

The model therefore permits state-dependent susceptibility.

This formal structure is important for psychoanalytic interpretation. An encounter, utterance, demand, institutional condition, or other event need not have one state-independent effect.

Its modeled consequence can depend upon the relational configuration into which it enters.

Historical dependence strengthens this distinction. If the response function takes the form

$$G = G(\Phi_t, h_t),$$

two configurations with the same currently observed Lacanian state can respond differently because their histories differ.

External forcing can also alter parameters rather than enter additively.

Let

$$\lambda(t)$$

be a control parameter. Then

$$\partial_t \Phi = F(\Phi; \lambda(t)).$$

An external condition may therefore change the dynamical regime itself.

This representation will become relevant when bifurcation and hysteresis are considered in Section 8. A transient perturbation can be represented schematically as

$$\lambda(t) = \lambda_0 + \delta\lambda(t),$$

while sustained environmental change may shift the background parameter over a longer interval. The distinction between source forcing and parameter modulation should remain explicit:

$$J(\xi, t)$$

acts as an input to the evolution equation, while

$$\lambda(t)$$

changes the law under which the evolution occurs.

Both can be useful relational representations.

Neither requires the present paper to model the external system that generated the input.

This preserves the single-subject boundary of the model.

4.5 Memory and History Dependence

The current field configuration may fail to contain all information required for future evolution. This subsection develops several formal representations of historical dependence while keeping them conceptually distinct.

The first approach enlarges the state.

Let

$$M(t)$$

be an endogenous memory variable and define

$$Z(t) = (\Phi_t, M(t)).$$

The augmented dynamics can then be written

$$\dot{Z} = \mathcal{F}(Z, u).$$

Equivalently,

$$\partial_t \Phi = F(\Phi, M, u),$$

$$\dot{M} = H(\Phi, M, u).$$

A process that appears history-dependent when observed only through Φ may therefore admit a first-order representation in the enlarged state Z .

This does not imply that every historical dependence can or should be reduced to a finite-dimensional memory variable.

The second approach retains an explicit history functional.

Let

$$\mathcal{H}_t = \{\Phi_\tau \mid 0 \leq \tau \leq t\}.$$

Then

$$\partial_t \Phi_t = F[\Phi_t, \mathcal{H}_t].$$

The future can depend on the trajectory through which the current configuration was reached. The third approach uses a memory kernel:

$$\partial_t \Phi^a(\xi, t) = F^a[\Phi_t](\xi) + \int_0^t K^a_b(t, \tau) \Phi^b(\xi, \tau) d\tau.$$

If the kernel depends only upon the time difference,

$$K^a_b(t, \tau) = K^a_b(t - \tau),$$

the influence of a previous state depends upon how long ago it occurred.

Different kernels encode different forms of persistence.

A rapidly decaying kernel gives recent history greater influence.

A slowly decaying kernel retains a longer formal memory.

An oscillatory or sign-changing kernel can represent more complicated temporal dependence.

These interpretations concern the mathematical model. Their psychoanalytic meaning requires an additional semantic argument.

A fourth possibility is that historical context changes the parameters of the evolution law:

$$\partial_t \Phi = F(\Phi; \theta(h_t)).$$

The state can therefore evolve under an effective dynamical system whose parameters depend upon accumulated history.

This case differs from explicit memory in the trajectory equation. History acts by changing the effective law.

A fifth possibility concerns the geometry developed later in the paper:

$$g_{\mu\nu} = g_{\mu\nu}(q, \Phi, h).$$

Here history modifies the effective geometry within which subsequent trajectories unfold.

These alternatives should remain distinguishable:

memory in the state,

memory in the evolution functional,

history-dependent parameters,

and

history-dependent geometry.

They can produce superficially similar path dependence while making different formal commitments.

The paper therefore avoids using “memory” as one undifferentiated mechanism.

This distinction will also matter for hysteresis. A hysteretic response can arise from several different underlying architectures. Observing hysteresis alone does not determine which memory representation generated it.

4.6 Dissipation and Irreversibility

A subjective trajectory is modeled here as part of an open relational system. The dynamics should therefore permit loss, dissipation, relaxation, and irreversible effective behavior. This subsection introduces these possibilities before the variational formulation of the next section so that the existence of an action principle is not mistaken for a universal assumption of conservative dynamics.

Consider a reduced state $q(t)$ with equation

$$\dot{q} = F(q).$$

Suppose there exists a scalar functional

$$V(q)$$

whose derivative along trajectories satisfies

$$\frac{dV}{dt} = \nabla V \cdot F.$$

If

$$\frac{dV}{dt} < 0$$

within a selected region away from stationary states, V decreases along the corresponding trajectories.

Such a function can provide a formal representation of relaxation toward a stable configuration.

For a field,

$$\mathcal{V}[\Phi]$$

may play an analogous role.

A gradient-flow model has the form

$$\partial_t \Phi = -\mathcal{G} \frac{\delta \mathcal{V}}{\delta \Phi},$$

where $\mathcal{G}$ is an appropriate positive operator or mobility structure.

The evolution then moves according to the gradient structure defined by the model.

This representation can be useful for effective relaxation dynamics.

It should not be interpreted as the general law of desire.

A damped dynamical system provides another example:

$$\ddot{q}^\mu + \gamma^\mu{}_\nu \dot{q}^\nu + F^\mu(q) = 0.$$

The term

$$\gamma^\mu{}_\nu \dot{q}^\nu$$

introduces dissipation.

For a field, an analogous schematic equation is

$$\partial_t^2 \Phi^a + \gamma^a{}_b \partial_t \Phi^b - D^a{}_b \nabla^2 \Phi^b + \frac{\partial V}{\partial \Phi^a} = J^a.$$

This equation combines inertial, dissipative, spatial or relational coupling, effective potential, and external forcing terms.

It is presented as a formal template rather than as a proposed psychoanalytic field equation.

Irreversibility can also arise through coarse-graining.

A fine description may retain variables whose elimination produces an effective reduced equation containing memory or dissipation.

Schematically,

$$\Phi_{\text{fine}} \longrightarrow q_{\text{eff}}$$

may transform a more detailed evolution into

$$\dot{q}_{\text{eff}} = F_{\text{eff}}(q_{\text{eff}}, \mathcal{H}_t).$$

The reduced dynamics can therefore possess effective irreversibility even when the finer representation has a different structure.

This observation will become important in Section 10.

Historical irreversibility should also be distinguished from mathematical non-invertibility.

A subject may acquire a new historical context after an event even when the underlying differential equation is formally reversible.

Conversely, a dissipative model may erase information about earlier states without implying any specific psychoanalytic meaning.

The paper therefore distinguishes

dynamical reversibility,

effective irreversibility,

and

historical non-equivalence.

These three properties can coexist in different combinations.

The relational field architecture developed in this section can now be summarized as

$$\partial_t \Phi^a = F_{\text{local}}^a[\Phi] + F_{\text{int}}^a[\Phi] + F_{\text{mem}}^a[\mathcal{H}_t] + J^a,$$

with the understanding that a particular model may contain only a subset of these terms.

At the augmented-state level, the same architecture may be written

$$\dot{Z} = \mathcal{F}(Z; h, u), \quad Z = (\Phi, M).$$

The central result of this section is therefore a family of continuous relational dynamics capable of representing coupling, nonlinear response, external forcing, historical dependence, and dissipation while preserving the separation between complete field configuration and psychoanalytic observation.

Section 5 now asks when part of this dynamics can be represented through an action, Lagrangian density, effective potential, or related field-theoretic structure, and where such a variational representation ceases to be appropriate.

5 Variational and Effective Field Formulations

This section examines when the relational dynamics developed in Section 4 can be represented through variational and field-theoretic structures. Its role is to distinguish a general evolution law from the stronger assumption that the dynamics arise from an action functional. The section first introduces the action and Lagrangian density, then derives the corresponding Euler–Lagrange field equations, considers interaction potentials and coupled fields, distinguishes conservative from driven and dissipative dynamics, and finally introduces the effective action as a higher-level description. The method is constructive

but cautious: variational formulations are used where they organize the dynamics, while no claim is made that subjective processes fundamentally obey a conservative physical action principle.

The distinction begins with the general evolution law

$$\partial_t \Phi = \mathcal{F}[\Phi; h, u].$$

Such an equation does not require an action.

A variational formulation imposes additional structure by seeking a functional

$$S[\Phi]$$

whose stationary configurations generate the selected equations of motion. This additional structure can be analytically useful because it organizes local dynamics, interactions, symmetries, conserved quantities, and later geometric extensions within one formal object. Its introduction therefore requires justification by the modeling problem rather than by analogy with physics alone.

5.1 Action Functional and Lagrangian Density

Let the relational field be

$$\Phi = (\Phi^1, \dots, \Phi^N)$$

on a domain

$$\Omega_R \times T.$$

When the relational domain has been supplied with the differentiable and measure structures required by the formulation, an action can be introduced as

$$S[\Phi] = \int_T dt \int_{\Omega_R} \mathcal{L}(\Phi, \partial_\mu \Phi, \xi, t) d\mu(\xi),$$

where

$$\mathcal{L}$$

is the Lagrangian density of the selected model.

The index μ refers to coordinates on the domain over which derivatives are defined. These coordinates should not be interpreted automatically as physical spacetime coordinates.

The Lagrangian density may be decomposed schematically as

$$\mathcal{L} = \mathcal{L}_{\text{kin}} - \mathcal{V} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{ext}}.$$

The terms denote, respectively, a dynamical or gradient contribution, an effective potential, interactions among field components, and externally driven contributions where these can be incorporated variationally.

A simple scalar-field template is

$$\mathcal{L} = \frac{1}{2} G_{ab}(\Phi) \partial_\mu \Phi^a \partial^\mu \Phi^b - V(\Phi).$$

Here

$$G_{ab}(\Phi)$$

describes coupling among field components within the kinetic term.

At this stage, G_{ab} should not be identified with the relational state-space metric introduced later. The two objects may eventually be related in a specific model, but their mathematical roles are initially distinct.

For several fields, the potential can contain both independent and interaction contributions:

$$V(\Phi) = \sum_a V_a(\Phi^a) + V_{\text{int}}(\Phi^1, \dots, \Phi^N).$$

A polynomial example is

$$V_{\text{int}} = \frac{1}{2} m_{ab} \Phi^a \Phi^b + \frac{1}{3!} g_{abc} \Phi^a \Phi^b \Phi^c + \frac{1}{4!} \lambda_{abcd} \Phi^a \Phi^b \Phi^c \Phi^d.$$

This expression illustrates increasingly higher-order coupling.
The coefficients

$$m_{ab}, \quad g_{abc}, \quad \lambda_{abcd}$$

have no psychoanalytic meaning until an explicit interpretation is supplied.

The action should therefore be understood as a compact formal description of a candidate relational dynamics rather than as a hidden physical quantity possessed by the subject.

A central advantage of the action formulation is that several aspects of the model are collected into one functional:

$$\text{local evolution,} \quad \text{interaction,} \quad \text{boundary conditions,} \quad \text{symmetry structure.}$$

This economy becomes especially useful once gauge transformations and geometric structure are introduced later in the paper.

5.2 Euler–Lagrange Field Equations

If the dynamics admit a variational formulation, their equations of motion follow from stationary variation of the action.

Let

$$\Phi^a \longrightarrow \Phi^a + \varepsilon \eta^a,$$

where the variation η^a satisfies the selected boundary conditions.

The stationary-action condition is

$$\delta S = 0.$$

For a local Lagrangian density depending upon the fields and their first derivatives, this produces the Euler–Lagrange equations

$$\frac{\partial \mathcal{L}}{\partial \Phi^a} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \Phi^a)} \right) = 0.$$

This construction is standard field theory [7]. Its use here belongs to the present relational reconstruction.

For the simple Lagrangian

$$\mathcal{L} = \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - V(\Phi),$$

the corresponding schematic equation is

$$\partial_\mu \partial^\mu \Phi + \frac{dV}{d\Phi} = 0.$$

The exact form of the differential operator depends upon the geometry and signature assigned to the domain.

The present framework therefore does not assume a Minkowski operator merely because the notation resembles relativistic field theory.

If the relational domain is treated as an ordinary spatial domain evolving in an external time parameter, a more appropriate field equation may instead take the form

$$\partial_t^2 \Phi - c_R^2 \nabla_R^2 \Phi + \frac{\partial V}{\partial \Phi} = 0,$$

where

$$c_R$$

is a model parameter controlling propagation across the relational domain.

The symbol c_R does not represent the physical speed of light.

Likewise, a first-order dissipative field model may be more appropriate than a second-order wave equation:

$$\partial_t \Phi = -\frac{\delta \mathcal{V}}{\delta \Phi}.$$

The selection among these equations is therefore a substantive modeling decision.

The variational formulation is useful only if the resulting equations preserve the distinctions required by the psychoanalytic problem.

A particularly important issue is the relation between field dynamics and observation.

Even when

$$\Phi$$

satisfies an Euler–Lagrange equation, the Lacanian observation remains

$$o_t = \mathcal{O}_L(\Phi_t).$$

The psychoanalytic categories are not generated automatically by the variational calculus.

The field equations govern the formal state.

The observation map supplies the Lacanian readout.

5.3 Interaction Potentials

The potential term provides one way to represent local stability, competition among configurations, and coupling among several field components. Its role is therefore broader than the representation of a single scalar “energy landscape.”

Let

$$V : \mathcal{V} \longrightarrow \mathbb{R}$$

be an effective potential.

Stationary configurations satisfy

$$\frac{\partial V}{\partial \Phi^a} = 0.$$

Their local stability depends upon additional dynamical and second-order properties.

For example, the Hessian

$$H_{ab} = \frac{\partial^2 V}{\partial \Phi^a \partial \Phi^b}$$

describes the local curvature of the potential in field-component space.

A positive-definite Hessian at a stationary point can support a local minimum of the potential.

Such a minimum may contribute to a stable or metastable dynamical regime, depending upon the complete equations of motion.

This distinction matters. A local minimum of V should not be called an attractor unless the corresponding dynamics actually produce attraction.

The potential may possess several minima:

$$\Phi_*^{(1)}, \quad \Phi_*^{(2)}, \quad \dots$$

This provides one formal way to represent multiple locally persistent configurations.

A schematic double-well potential,

$$V(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4, \quad \lambda > 0,$$

contains two symmetry-related minima when

$$\mu^2 > 0.$$

The mathematical structure is familiar from field theory. In the present paper it may serve as a controlled example of multiple effective configurations.

The example should not be interpreted as a claim that psychoanalytic subject formation literally results from spontaneous symmetry breaking.

Its analytical value lies in demonstrating how one dynamical system can contain several locally distinguished regimes and transitions among them.

For several fields, interaction terms can change the location and stability of these regimes.

Consider

$$V(\phi, \psi) = V_\phi(\phi) + V_\psi(\psi) + g\phi^2\psi^2.$$

The effective landscape experienced by ϕ depends upon the value of ψ , and conversely.

This gives a simple formal representation of relational dependence:

effective possibility for one component

depends upon

configuration of another component.

The coupling constant g may also be generalized to a state-dependent quantity,

$$g = g(\Phi, h),$$

although such a generalization changes the model substantially and should be introduced only where required.

A potential may also depend explicitly upon external conditions:

$$V = V(\Phi; \lambda(t)).$$

As

$$\lambda(t)$$

changes, minima may move, disappear, or exchange stability.

This provides a natural interface with bifurcation analysis.

The field can therefore undergo qualitative reorganization because the effective landscape itself changes.

Historical dependence creates another possibility:

$$V = V(\Phi; h_t).$$

Two subjects or two historical stages can then encounter different effective landscapes even when the current observable configuration appears similar.

This is a precursor to the stronger construction developed later in which history modifies the effective geometry itself.

5.4 Conservative and Non-Conservative Dynamics

An action principle gives a powerful representation of conservative and symmetry-structured systems, but the relational model developed here is explicitly open to external forcing, dissipation, memory, and irreversible effective behavior. The variational architecture must therefore be treated as one component of a broader dynamical framework.

A conservative mechanical analogy begins from

$$S[q] = \int L(q, \dot{q}, t) dt.$$

For time-independent systems with appropriate symmetries, conserved quantities can follow from the variational structure.

A relational subjective model need not satisfy these assumptions.

External forcing produces equations such as

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}^\mu} - \frac{\partial L}{\partial q^\mu} = Q_\mu^{\text{ext}},$$

where

$$Q_\mu^{\text{ext}}$$

represents a non-conservative generalized force.

For a field, an external source can yield

$$\frac{\partial \mathcal{L}}{\partial \Phi^a} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \Phi^a)} \right) = J_a.$$

Dissipation may require additional structure.

For a reduced model with a Rayleigh-type dissipation function

$$\mathcal{R}(\dot{q}),$$

one can write schematically

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}^\mu} - \frac{\partial L}{\partial q^\mu} + \frac{\partial \mathcal{R}}{\partial \dot{q}^\mu} = Q_\mu^{\text{ext}}.$$

This extends the conservative variational equation without treating dissipation as part of an ordinary potential.

A field-theoretic analogue can include damping directly in the evolution equation:

$$\partial_t^2 \Phi + \gamma \partial_t \Phi - D \nabla^2 \Phi + \frac{\partial V}{\partial \Phi} = J.$$

The term

$$\gamma \partial_t \Phi$$

breaks the simple conservative form.

Historical dependence creates a stronger challenge.

An equation containing

$$\int_0^t K(t - \tau) \Phi(\tau) d\tau$$

is nonlocal in time.

It may admit generalized variational representations under additional assumptions, but the present paper does not require every such model to be forced into a local action principle.

The same caution applies to stochastic dynamics.

A model such as

$$\partial_t \Phi = F[\Phi] + \eta$$

with noise term η belongs to a different formal class from a simple deterministic Euler–Lagrange system.

The central distinction is therefore:

$$\text{action-generated dynamics} \subset \text{candidate relational dynamics}.$$

The inclusion is conceptual rather than a formal set-theoretic theorem.

It states that the paper permits relational dynamics whose most transparent representation is not a conventional action-based field theory.

This flexibility is important for the later psychoanalytic interpretation. The elegance of a variational formulation should not determine which historical or relational processes the model is allowed to represent.

5.5 Effective Action and Effective Description

The final subsection distinguishes the classical action used to specify a candidate field dynamics from the concept of an effective action. This distinction becomes important when the analysis moves across scales or suppresses unresolved degrees of freedom.

In quantum field theory, the effective action provides a systematic object for encoding quantum-corrected dynamics and is commonly expressed as a functional of an expectation-value field [7]. The present paper does not assume that subjective dynamics possess such a quantum interpretation.

The concept is nevertheless useful because it highlights a more general problem:

the effective dynamics of retained variables

need not have the same form as

the dynamics of the finer description.

Let the fine configuration contain two sets of variables,

$$\Phi \quad \text{and} \quad \chi,$$

where Φ is retained and χ is unresolved at the selected scale.

A fine action may be written

$$S[\Phi, \chi].$$

After eliminating or averaging over the unresolved degrees of freedom by an appropriate procedure, one obtains an effective description for Φ ,

$$S_{\text{eff}}[\Phi].$$

The effective action can contain terms absent from the simplest original description. Schematically,

$$S[\Phi, \chi] \longrightarrow S_{\text{eff}}[\Phi].$$

This provides an important lesson for the relational framework.

A coarse psychoanalytic description should not be expected to consist merely of the fine description with several variables deleted.

The eliminated variables may alter the effective interactions among those that remain.

For example, a fine model may contain

$$S[\Phi_1, \Phi_2, \chi],$$

while the effective model for

$$\Phi_1, \Phi_2$$

contains a new coupling

$$g_{\text{eff}}\Phi_1\Phi_2$$

that summarizes the influence of the unresolved variable χ .

This is one reason to distinguish simple projection from dynamical coarse-graining.

The RSI map

$$\Pi_{\text{RSI}}$$

is primarily an observation map.

An effective-field construction instead asks how the law governing retained variables changes when finer degrees of freedom are suppressed.

The two operations can therefore be represented separately:

$$\Phi_{\text{fine}} \xrightarrow{\text{observation}} a_{\text{RSI}},$$

and

$$S_{\text{fine}} \xrightarrow{\text{coarse-graining}} S_{\text{eff}}.$$

The first changes representational resolution.

The second changes the effective dynamical description.

A later section will examine this distinction in greater detail.

For the present paper, the notation

$$\Gamma[\bar{\Phi}]$$

may be reserved for an effective action when the specifically field-theoretic construction is intended, while

$$S_{\text{eff}}[\Phi]$$

can denote a more general effective action-like functional produced by a specified coarse-graining procedure.

The distinction prevents a general effective model from being presented as a quantum effective action without the mathematical construction that would justify that terminology.

The same caution applies to renormalization.

An effective description at another scale is not yet an RG flow.

A genuine renormalization-group analysis requires a scale parameter, a specified coarse-graining transformation, and evolution of couplings or effective theories under changes of scale.

These additional structures are introduced only in Section 10 where appropriate.

The variational architecture of this section can therefore be summarized as

$$\mathcal{L} \longrightarrow S[\Phi] \longrightarrow \text{Euler--Lagrange dynamics,}$$

with possible extensions to

$$S[\Phi, \chi] \longrightarrow S_{\text{eff}}[\Phi].$$

The first chain organizes a candidate microscopic or fine relational dynamics.

The second expresses the possibility that a lower-resolution description possesses its own effective interactions.

Neither construction determines the psychoanalytic meaning of the fields.

That meaning remains supplied through the observation and interpretation layers established earlier.

The next section turns from evolution laws to geometry. It asks how a relational configuration space can be supplied with notions of distance, neighborhood, local direction, curvature, and state-dependent accessibility, and under which conditions the geometry itself may change along a subjective trajectory.

6 Dynamical Geometry of Subjective Trajectories

This section introduces geometric structure into the relational dynamical framework. Its role is to formalize notions of proximity, direction, accessibility, curvature, and path-dependent change that cannot be recovered from symbolic ordering alone. The section proceeds cautiously because several different spaces have appeared in the preceding analysis. A geometry on the relational domain, a geometry on field-component space, a geometry on a finite-dimensional reduced state manifold, and a geometry on the full field configuration space are mathematically distinct constructions. The present section therefore identifies the carrier of each geometric object before assigning it a psychoanalytic interpretation.

The principal object developed below is an effective state manifold

$$\mathcal{M}_{\text{red}},$$

whose points represent reduced relational configurations. When coordinates

$$q^\mu$$

are defined on this manifold, a trajectory is

$$\gamma : t \longmapsto q(t) \in \mathcal{M}_{\text{red}}.$$

A metric

$$g_{\mu\nu}$$

can then define effective relational distance and local geometry.

The full field configuration space

$$\mathcal{M}_R$$

may also admit a geometric structure, but such a construction is generally infinite-dimensional and requires additional functional-analytic assumptions. The finite-dimensional effective geometry is therefore developed first.

This ordering follows the principle of formal economy: geometric machinery is introduced at the lowest-dimensional level at which it preserves the distinctions required by the analysis.

6.1 Relational State Manifold

A state space becomes a differentiable manifold when the model supplies the local coordinate structure required to compare nearby configurations and define differentiable trajectories. Differential geometry provides the standard mathematical framework for manifolds, metrics, connections, curvature, and related structures [5].

Let

$$\mathcal{M}_{\text{red}}$$

denote an effective relational state manifold of dimension n .

A local coordinate chart assigns

$$q = (q^1, \dots, q^n)$$

to states in an open region

$$U \subseteq \mathcal{M}_{\text{red}}.$$

The coordinates are model variables.

They should not be identified directly with Lacanian concepts merely because they provide a convenient parameterization of the state space.

A continuous subjective trajectory is represented as

$$\gamma(t) = q(t).$$

Its tangent vector is

$$\dot{\gamma}(t) = \dot{q}^\mu(t) \frac{\partial}{\partial q^\mu}.$$

The tangent vector belongs to the tangent space

$$T_{q(t)}\mathcal{M}_{\text{red}}.$$

This provides a formal meaning for a local direction of subjective change.

Two states that are close in coordinate values need not yet be close in a geometric sense. Coordinate differences such as

$$\Delta q^\mu$$

depend upon the chosen parameterization.

A metric is required before coordinate-independent local distances can be defined.

The reduced manifold may be obtained from the full field configuration through a map

$$\mathcal{Q} : \mathcal{M}_R \longrightarrow \mathcal{M}_{\text{red}}.$$

Thus,

$$q^\mu = Q^\mu[\Phi].$$

Different field configurations can project to the same reduced state:

$$\Phi_1 \neq \Phi_2$$

while

$$\mathcal{Q}(\Phi_1) = \mathcal{Q}(\Phi_2).$$

The geometry on $\mathcal{M}_{\text{red}}$ therefore describes the effective dynamics at the selected resolution. It does not automatically encode all distinctions present in the full field configuration space.

This relation parallels the earlier observation hierarchy

$$\Phi \longrightarrow o_L \longrightarrow a_{\text{RSI}}.$$

Different kinds of reduction can therefore coexist:

dynamical reduction

through

$$\mathcal{Q},$$

and

psychoanalytic observation

through

$$o_L.$$

They need not identify the same equivalence classes.

6.2 Relational Metric

A metric introduces a local measure of relational distance on the effective state manifold.

Let

$$g_q : T_q \mathcal{M}_{\text{red}} \times T_q \mathcal{M}_{\text{red}} \longrightarrow \mathbb{R}$$

be a metric at the state q .

In local coordinates,

$$ds^2 = g_{\mu\nu}(q) dq^\mu dq^\nu.$$

The tensor

$$g_{\mu\nu}$$

determines the local geometry of the selected model.

The use of the word *distance* requires care.

The quantity

$$ds$$

does not initially represent physical spatial distance.

It may instead represent an effective difference in relational state, accessibility, deformation cost, sensitivity, or another quantity whose meaning is specified by the model.

For a tangent vector

$$v = v^\mu \frac{\partial}{\partial q^\mu},$$

its squared norm is

$$\|v\|_g^2 = g_{\mu\nu} v^\mu v^\nu.$$

Two coordinate changes of equal magnitude can therefore correspond to different effective relational changes.

Suppose

$$dq_{(1)} = (dq^1, 0, \dots, 0)$$

and

$$dq_{(2)} = (0, dq^2, \dots, 0).$$

Even when

$$|dq^1| = |dq^2|,$$

their metric lengths can differ:

$$g_{11}(dq^1)^2 \neq g_{22}(dq^2)^2.$$

This provides one formal way to represent anisotropy in subjective possibility.

Change along one relational direction may be locally easier, larger, or more significant than an equal coordinate displacement along another.

Off-diagonal terms

$$g_{\mu\nu}, \quad \mu \neq \nu,$$

encode geometric coupling among coordinate directions.

The metric can therefore represent a geometry in which relational dimensions are not independent.

A trajectory has length

$$L[\gamma] = \int_{t_0}^{t_1} \sqrt{g_{\mu\nu} \dot{q}^\mu \dot{q}^\nu} dt$$

for a positive-definite metric.

This length depends upon the complete path rather than only upon its endpoints.

Two trajectories

$$\gamma_1 \quad \text{and} \quad \gamma_2$$

connecting the same states can therefore satisfy

$$L[\gamma_1] \neq L[\gamma_2].$$

The mathematical observation is important for the relational model because identical initial and final observations do not imply identical histories.

The metric provides one possible way to make differences among those histories geometrically explicit.

6.3 Configuration- and History-Dependent Geometry

A fixed metric assumes that the local geometry of state space is independent of the trajectory through which a state was reached. The relational model allows a stronger possibility: the effective geometry itself may depend upon the current field configuration or historically accumulated variables.

A state-dependent metric can be written as

$$g_{\mu\nu} = g_{\mu\nu}(q).$$

This is the ordinary situation on a curved manifold.

The relational framework may additionally define

$$g_{\mu\nu} = g_{\mu\nu}(q; Q[\Phi]),$$

where aspects of the underlying field configuration influence the effective metric.

If two full field states project to the same reduced coordinate,

$$Q(\Phi_1) = Q(\Phi_2) = q,$$

the induced metrics may nevertheless differ:

$$g_{\mu\nu}^{(1)}(q) \neq g_{\mu\nu}^{(2)}(q).$$

The reduced coordinate q is then insufficient to specify the effective geometry.

One solution is to enlarge the state.

Another is to treat the geometry as dependent upon unresolved variables.

Historical dependence introduces a further possibility:

$$g_{\mu\nu} = g_{\mu\nu}(q, h_t).$$

The same coordinate configuration can then possess different local geometry under different histories:

$$q(t_1) = q(t_2)$$

while

$$g_{\mu\nu}(q, h_{t_1}) \neq g_{\mu\nu}(q, h_{t_2}).$$

Consequently, the same infinitesimal displacement

$$dq^\mu$$

has different effective lengths:

$$ds_{t_1}^2 = g_{\mu\nu}(q, h_{t_1}) dq^\mu dq^\nu,$$

$$ds_{t_2}^2 = g_{\mu\nu}(q, h_{t_2}) dq^\mu dq^\nu.$$

This provides a precise formal expression for one of the central hypotheses of the paper:

The effective accessibility and significance of a relational change may depend upon the history through which the current configuration was formed.

The hypothesis should remain distinguishable from ordinary memory in the equation of motion. History can affect dynamics through

$$\dot{q} = F(q, h),$$

while the metric remains fixed.

Alternatively, history can affect geometry through

$$g = g(q, h).$$

A model may also contain both effects.

These are different mathematical hypotheses.

The geometry can likewise depend upon an endogenous memory variable:

$$g_{\mu\nu} = g_{\mu\nu}(q, M).$$

Then the complete state

$$Z = (q, M)$$

determines both the dynamics and the effective geometry.

The paper will refer to such cases as *configuration-dependent* or *history-dependent relational geometry*, according to which variables determine the metric.

6.4 Curvature and Effective Constraint

Once a metric has been defined, the relational state manifold can possess curvature. The purpose of introducing curvature is to describe how local geometric structure varies across the state space, rather than to invoke physical spacetime curvature by analogy.

For a metric-compatible Levi-Civita connection, the Christoffel symbols are

$$\Gamma_{\mu\nu}^{\rho} = \frac{1}{2} g^{\rho\sigma} (\partial_{\mu} g_{\sigma\nu} + \partial_{\nu} g_{\sigma\mu} - \partial_{\sigma} g_{\mu\nu}).$$

The corresponding curvature tensor is

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_{\mu} \Gamma_{\nu\sigma}^{\rho} - \partial_{\nu} \Gamma_{\mu\sigma}^{\rho} + \Gamma_{\mu\lambda}^{\rho} \Gamma_{\nu\sigma}^{\lambda} - \Gamma_{\nu\lambda}^{\rho} \Gamma_{\mu\sigma}^{\lambda}.$$

These are standard geometric constructions [5].

Their psychoanalytic significance does not follow from their mathematical definition.

A flat relational state space has, under the relevant assumptions,

$$R^{\rho}{}_{\sigma\mu\nu} = 0.$$

A curved state space has nontrivial curvature.

The useful conceptual distinction is that curvature can make the relation among nearby directions depend upon location in state space.

This allows the geometry of available change to vary across configurations.

For example, two initially parallel directions need not remain geometrically equivalent under transport through a curved region.

Likewise, a local coordinate approximation that appears simple near one state may fail across a larger region.

Curvature can therefore represent an effective constraint on the geometry of possible trajectories.

The term *constraint* should be interpreted carefully.

Curvature does not function as an external force.

It modifies the geometric structure with respect to which motion, distance, transport, and deviation are defined.

A separate dynamical force may still act on the system.

The distinction becomes clearer by comparing

geometry

with

vector field.

A dynamical law

$$\dot{q}^\mu = F^\mu(q)$$

specifies a local direction of evolution.

A metric

$$g_{\mu\nu}(q)$$

specifies geometric relations among directions.

The two structures are logically independent until the model supplies a relation between them.

A trajectory can therefore be constrained dynamically,

$$F(q),$$

geometrically,

$$g(q),$$

or through both.

This separation will prevent later statements about attractors or bifurcations from being attributed incorrectly to curvature alone.

6.5 Geodesic and Driven Motion

A metric allows the definition of geodesic motion. The objective of this subsection is to distinguish trajectories determined primarily by the geometry from trajectories driven by an independently specified relational dynamics.

For the Levi–Civita connection associated with g , an affinely parameterized geodesic satisfies

$$\frac{d^2 q^\mu}{d\tau^2} + \Gamma_{\alpha\beta}^\mu \frac{dq^\alpha}{d\tau} \frac{dq^\beta}{d\tau} = 0.$$

Such a curve is determined by the connection and initial data.

There is no reason to assume that a subjective trajectory must satisfy this equation.

The relational dynamics developed in Section 4 may instead produce

$$\dot{q}^\mu = F^\mu(q, h, u),$$

or a second-order equation

$$\frac{D^2 q^\mu}{D\tau^2} = f^\mu(q, \dot{q}, h, u),$$

where

$$\frac{D}{D\tau}$$

denotes covariant differentiation along the trajectory.
The right-hand side

$$f^\mu$$

represents a non-geodesic driving term.
The distinction provides several possible model classes.
A purely geodesic model treats the effective geometry as sufficient to determine unforced motion.
A driven geometric model combines

$$g_{\mu\nu}$$

with an independent dynamical field.
A dissipative geometric model additionally includes friction-like or relaxational terms.
A history-dependent geometric model allows either the driving field, the metric, or both to depend upon the accumulated trajectory.
These models should not be collapsed into one equation merely for mathematical elegance.
The concept of geodesic motion can nevertheless serve an analytical purpose. It establishes a reference trajectory associated with the selected geometry.
One may compare an actual modeled trajectory

$$\gamma$$

with a corresponding geodesic

$$\gamma_{\text{geo}}.$$

Deviation between the two can then be attributed, within the model, to forcing, dissipation, memory, or other non-geometric effects.
This decomposition is formal.
It should not be interpreted as a division between a subject's "natural desire" and external interference without an independent psychoanalytic argument.
Geodesic deviation also provides a geometric description of sensitivity between nearby paths.
For a separation vector

$$\eta^\mu$$

between neighboring geodesics, the standard equation is

$$\frac{D^2 \eta^\mu}{D\tau^2} = -R^\mu{}_{\nu\rho\sigma} u^\nu \eta^\rho u^\sigma,$$

up to sign convention, where

$$u^\mu = \frac{dq^\mu}{d\tau}.$$

The equation shows how curvature affects the relative behavior of nearby geodesics.
The present paper does not identify this expression directly with psychological divergence.
Its value is to demonstrate that once a relational geometry is defined, differences among nearby trajectories can have a geometric component in addition to differences in the external driving field.

6.6 Geometry as a Dynamical Object

The preceding subsections treated the metric as dependent on position, configuration, or history. The strongest version of the relational proposal allows the geometric structure itself to evolve.

Let

$$g_{\mu\nu}(t)$$

denote a time-dependent metric.

More generally,

$$g_{\mu\nu} = g_{\mu\nu}(q, t; \Phi, h).$$

The trajectory then evolves within a geometry that changes during the process.

Schematically,

$$\dot{q}^\mu = F^\mu(q, g, h, u),$$

while

$$\partial_t g_{\mu\nu} = H_{\mu\nu}(q, g, \Phi, h, u).$$

This produces coupled dynamics between state and geometry:

$$(q, g) \longrightarrow (q', g').$$

The state affects the effective geometry, and the geometry affects later state evolution.

Such feedback is conceptually stronger than evolution on a fixed manifold.

It represents the hypothesis that relational history can modify the structure of subsequent possibility.

The hypothesis can be written schematically as

$$\text{relational trajectory} \longrightarrow \text{geometric modification} \longrightarrow \text{modified future accessibility}.$$

At this stage, the equation

$$\partial_t g_{\mu\nu} = H_{\mu\nu}$$

is only a general architectural form.

The paper does not adopt Einstein's field equations, Ricci flow, or another established geometric evolution law merely because they provide available examples.

A specific evolution law for the metric requires an interpretation of what the metric represents and evidence for how that quantity should change.

The same caution applies to the topology of the state space.

A changing metric does not imply changing topology.

Topological change is a stronger mathematical claim and is outside the present construction unless explicitly introduced.

The geometry can also be induced from the underlying field.

Suppose the reduced coordinates are

$$q^\mu = Q^\mu[\Phi].$$

A field-dependent metric may be written schematically as

$$g_{\mu\nu} = \mathcal{G}_{\mu\nu}[\Phi].$$

Then

$$\Phi_t \longrightarrow g_{\mu\nu}(t)$$

defines an induced effective geometry.
The complete architecture becomes

$$\Phi_t \xrightarrow{\mathcal{Q}} q(t),$$

$$\Phi_t \xrightarrow{\mathcal{G}} g(t),$$

followed by dynamics depending on both:

$$\dot{q} = F(q, g, h, u).$$

This provides a direct formal expression of the idea that two states with similar reduced coordinates can possess different future accessibility because their unresolved relational field configurations induce different geometries.

A still richer construction would place a metric directly on the full configuration space.
For tangent perturbations

$$\delta\Phi_1, \quad \delta\Phi_2$$

at a configuration Φ , one could define

$$\mathcal{G}_\Phi(\delta\Phi_1, \delta\Phi_2).$$

In a local schematic form,

$$\mathcal{G}_\Phi(\delta\Phi_1, \delta\Phi_2) = \int_{\Omega_R} d\mu(\xi) G_{ab}(\Phi(\xi)) \delta\Phi_1^a(\xi) \delta\Phi_2^b(\xi).$$

This would define geometry directly on a space of fields rather than on a finite-dimensional reduced manifold.

Such an infinite-dimensional construction requires assumptions concerning the admissible function space, regularity, positivity, and analytical properties of the metric.

It is therefore retained as an extension rather than assumed as the default geometry of Paper II.

The hierarchy of possible geometries can now be made explicit:

geometry of the relational domain Ω_R ,

geometry of field-value space,

geometry of the reduced state manifold $\mathcal{M}_{\text{red}}$,

and

geometry of the full configuration space $\mathcal{M}_R$.

These geometries may be related in a particular model, but they are not interchangeable.

The central construction of the present paper is the dynamical geometry of the effective state manifold:

$$(\mathcal{M}_{\text{red}}, g_{\mu\nu}(q, \Phi, h)).$$

This geometry permits local distance, anisotropy, curvature, and state-dependent accessibility to vary across a subjective trajectory.

The construction also prepares the next formal step.

A metric determines one form of geometric structure, but it does not by itself define the representational equivalences that may exist among different descriptions of the same relational state.

Section 7 therefore introduces gauge transformations, invariant observables, connections, parallel transport, and holonomy as additional structures. Their role is to distinguish changes in relational state from changes that belong only to the selected representation and to formalize what may be accumulated when a trajectory genuinely closes.

7 Gauge Structure and Relational Invariance

This section introduces a second geometric layer beyond the metric structure developed in Section 6. Its role is to distinguish changes in the represented relational state from changes that arise only from the choice of representation. The objective is to determine when several formal descriptions may be treated as equivalent, which quantities remain invariant under such transformations, and how relational information can be compared along a trajectory. The section proceeds from representational redundancy to group actions and invariant observables, then introduces gauge orbits, connections, parallel transport, curvature, and finally holonomy around genuinely closed trajectories.

Gauge theory provides an established mathematical framework for describing representational redundancy through group actions, connections, and gauge-invariant quantities [5]. The present paper does not assume in advance that Lacanian psychoanalysis possesses a gauge symmetry. The construction below instead specifies the conditions that would have to be satisfied before gauge-theoretic language becomes appropriate.

The distinction is important because several mathematical structures already appear in the model:

$$\mathcal{M}_{\text{red}}, \quad g_{\mu\nu}, \quad \Phi, \quad \mathcal{O}_L.$$

A metric defines local geometric relations among states. A gauge structure addresses a different problem: several mathematical representatives may encode one relational state or one set of observable properties.

The two structures may interact, but they should not be identified.

7.1 Representational Redundancy

Gauge structure becomes relevant only when the representation contains degrees of freedom whose variation does not change the relational content selected by the model.

Let

$$\mathcal{R}$$

denote a space of formal representatives.

An element

$$r \in \mathcal{R}$$

may contain coordinates, field components, frames, phases, symbolic labels, or other representational information used to describe the relational state.

Suppose two representatives

$$r_1, \quad r_2$$

correspond to the same state at the level relevant to the analysis.

We may then introduce an equivalence relation

$$r_1 \sim_G r_2.$$

The physical analogy would call this gauge equivalence. In the present paper, that terminology is justified only after a transformation structure has been defined.

The existence of several verbal, symbolic, or psychoanalytic descriptions of one phenomenon does not itself establish gauge redundancy.

A stronger condition is required.

There must be a specified family of transformations

$$T : \mathcal{R} \longrightarrow \mathcal{R}$$

such that the model identifies

$$r$$

and

$$T(r)$$

with respect to a defined class of observables.

Let

$$\mathfrak{O} = \{\mathcal{O}_\alpha\}$$

denote those observables.

Representational equivalence requires

$$\mathcal{O}_\alpha(T(r)) = \mathcal{O}_\alpha(r)$$

for every observable $\mathcal{O}_\alpha$ whose invariance defines the selected equivalence.

This formulation makes the dependence on analytical purpose explicit.

Two representations can be equivalent for one family of observables while remaining distinguishable for another.

For example, suppose the analysis retains only the RSI observation

$$\Pi_{\text{RSI}} \circ \mathcal{O}_L.$$

Two fine representations may then satisfy

$$\Pi_{\text{RSI}}(\mathcal{O}_L(r_1)) = \Pi_{\text{RSI}}(\mathcal{O}_L(r_2))$$

while

$$\mathcal{O}_L(r_1) \neq \mathcal{O}_L(r_2).$$

This equality is a coarse observational equivalence.

It should not yet be called gauge equivalence, because the equality arises from projection and does not by itself supply an invertible transformation group relating r_1 and r_2 .

The distinction among

coarse observational equivalence,

coordinate reparameterization,

and

gauge equivalence

is therefore maintained throughout the paper.

7.2 Gauge Transformations

A gauge structure begins when a group acts on the space of formal representatives while leaving the selected relational content invariant.

Let

$$G$$

be a group and let

$$\rho : G \times \mathcal{R} \longrightarrow \mathcal{R}$$

be a group action.

For

$$g \in G,$$

write

$$r \longmapsto g \cdot r.$$

The group law requires

$$e \cdot r = r$$

for the identity $e \in G$, and

$$g_1 \cdot (g_2 \cdot r) = (g_1 g_2) \cdot r.$$

A transformation

$$r \mapsto g \cdot r$$

can be interpreted as gauge-like when the observables selected as relationally meaningful satisfy

$$\mathcal{O}_\alpha(g \cdot r) = \mathcal{O}_\alpha(r).$$

The group G should therefore be derived from a specified invariance problem.

The present paper does not assume that

$$G = U(1), \quad SU(2), \quad SO(n),$$

or another familiar physical gauge group.

Selecting such a group because it is mathematically familiar would introduce a symmetry without explaining what relational transformations it represents.

Instead, the relevant problem is to determine whether the psychoanalytic formalization contains transformations of the form

$$r \longmapsto r'$$

that alter representational variables while preserving the relational quantities under study.

A finite-dimensional state representation may contain a local frame

$$e_a(q)$$

for each state

$$q \in \mathcal{M}_{\text{red}}.$$

Changing that frame through

$$e_a \mapsto g_a^b(q) e_b$$

can leave coordinate-independent geometric content unchanged.

This is one familiar source of gauge-like redundancy.

A relational field representation may similarly contain internal variables

$$\Phi^A$$

transforming as

$$\Phi \mapsto \rho(g)\Phi.$$

If selected observables remain invariant under this transformation, the internal representation contains redundant degrees of freedom with respect to those observables.

The transformation may be global,

$$g = \text{constant},$$

or local,

$$g = g(q).$$

Local transformations create the need for additional geometric machinery because ordinary derivatives of transformed fields generally acquire terms involving derivatives of $g(q)$.

This motivates the introduction of a connection later in the section.

The central methodological point is therefore

a gauge group should encode a justified equivalence of representations

rather than serving as an ornamental analogy to gauge physics.

7.3 Gauge-Invariant Observables

Once a gauge action has been defined, the next task is to identify quantities that depend only upon the equivalence class of a representative.

Let

$$r \in \mathcal{R}$$

and

$$g \in G.$$

An observable

$$\mathcal{I} : \mathcal{R} \longrightarrow \mathbb{R}$$

is gauge invariant when

$$\mathcal{I}(g \cdot r) = \mathcal{I}(r).$$

More generally, the codomain may be another space of observable quantities.

The invariant therefore descends to the quotient of the representation space by the gauge action.

This distinction is potentially important for psychoanalytic formalization.

Suppose several mathematical descriptions differ in variables that depend upon a selected coordinate system, frame, symbolic coding, or parameterization.

If a quantity survives all transformations admitted by the model, it becomes a candidate for a more representation-independent relational observable.

This does not imply that the quantity is ontologically fundamental.

It states only that it is invariant under the particular transformation group chosen by the model.

The hierarchy is therefore

$$\text{representative} \longrightarrow \text{gauge equivalence class} \longrightarrow \text{invariant observable}.$$

The Lacanian observation map may interact with this structure in several ways.

One possibility is that the full Lacanian observation is invariant:

$$\mathcal{O}_L(g \cdot r) = \mathcal{O}_L(r).$$

Another possibility is that only the RSI projection is invariant:

$$\Pi_{\text{RSI}}(\mathcal{O}_L(g \cdot r)) = \Pi_{\text{RSI}}(\mathcal{O}_L(r)).$$

These cases represent different resolutions of invariance.

The second condition is weaker.

A transformation can preserve the coarse RSI observation while altering fine Lacanian structure.

The corresponding gauge interpretation would therefore be valid only if the analysis has explicitly selected the coarse observable as the relevant invariant.

This provides a direct connection between gauge structure and the multi-resolution framework developed earlier.

Invariance is always relative to what the model declares observable.

7.4 Gauge Orbits and Relational Equivalence

The action of the group G partitions the representation space into orbits.

For

$$r \in \mathcal{R},$$

the gauge orbit is

$$[r]_G = G \cdot r = \{g \cdot r \mid g \in G\}.$$

All representatives in this orbit are identified under the selected gauge equivalence.

The quotient space is

$$\mathcal{R}/G.$$

Its points represent gauge-equivalence classes rather than individual representatives.

This construction provides a useful conceptual distinction between

$$r$$

and

$$[r]_G.$$

The first is one description.

The second is the relational content retained after the selected representational redundancy has been removed.

A psychoanalytic application could therefore distinguish

representation-dependent variables

from

orbit-level relational structure.

The latter becomes a candidate for comparison across different symbolic or coordinate representations.

This possibility connects naturally to the broader problem of cross-relational invariance.

Suppose two descriptions

$$r_A$$

and

$$r_B$$

belong to apparently different symbolic systems.

If there exists a transformation structure under which

$$[r_A]_G = [r_B]_G,$$

the model can treat them as alternative representatives of one relational equivalence class.

Such a claim requires considerably more than intuitive similarity.

The transformation group, its action, and the invariant quantities must all be specified.

The orbit structure can also possess stabilizers.

For a representative r , define

$$G_r = \{g \in G \mid g \cdot r = r\}.$$

A nontrivial stabilizer indicates that some transformations leave the representative itself unchanged.

Different states may possess different stabilizer subgroups.

This can lead to distinct symmetry types across the state space.

The present paper does not yet construct a psychoanalytic classification from stabilizer structure, but the distinction becomes relevant if symmetry breaking or changes in relational invariance are studied later.

Gauge orbits should also be distinguished from the observational fibers introduced previously.

The fiber

$$\mathcal{O}_L^{-1}(o)$$

contains all field configurations that yield the same Lacanian observation.

A gauge orbit

$$[r]_G$$

contains all representatives related by the specified group action.

These sets may coincide in a particular model, but there is no general reason for them to do so.

Thus,

$$\boxed{\text{same observation} \not\Rightarrow \text{same gauge orbit}}$$

without additional assumptions.

7.5 Connection and Parallel Transport

A local gauge transformation can vary from one point of the base manifold to another. Comparison of internal representatives at different states then requires a rule for transport.

This is the role of a connection.

Let

$$\pi : P \longrightarrow \mathcal{M}_{\text{red}}$$

be a principal bundle with structure group G , where such a bundle construction is justified.

A point

$$q \in \mathcal{M}_{\text{red}}$$

belongs to the base manifold.

The fiber

$$\pi^{-1}(q)$$

contains the gauge-related internal representatives associated with that base state.

A connection specifies how representatives in neighboring fibers are compared [5].

Locally, a gauge connection may be represented by a Lie-algebra-valued one-form

$$A = A_\mu(q) dq^\mu.$$

If the gauge group has Lie algebra

$$\mathfrak{g},$$

then

$$A_\mu(q) \in \mathfrak{g}.$$

For a field Ψ transforming in a representation of G , the covariant derivative may be written

$$D_\mu \Psi = \partial_\mu \Psi + A_\mu \Psi,$$

with the precise sign and representation convention determined by the model.

The connection compensates for the local variation of gauge representatives.

Parallel transport along a path

$$\gamma : [0, 1] \longrightarrow \mathcal{M}_{\text{red}}$$

then defines a transport operator schematically written as

$$U_\gamma = \mathcal{P} \exp \left(- \int_\gamma A \right),$$

where

$$\mathcal{P}$$

denotes path ordering when required.

This construction should be interpreted carefully in the relational model.

Parallel transport does not automatically represent psychological memory.

It defines how an internal relational quantity is compared along a path once a connection has been specified.

The connection may become psychoanalytically interesting when a representation-dependent internal structure must be transported through changing subjective configurations.

For example, suppose

$$\Psi(t)$$

is an internal relational variable attached to the state

$$q(t).$$

Transport along the subjective trajectory may yield

$$\Psi(t_1) = U_{\gamma(t_0, t_1)} \Psi(t_0).$$

The final variable depends upon the path when the connection has nontrivial curvature.

This gives a formal mechanism through which relational comparison can depend upon the route taken through state space.

That mathematical path dependence should remain distinct from the memory kernels introduced in Section 4.5.

The two may coexist:

dynamical memory

and

geometric transport

are different sources of history dependence.

7.6 Curvature of the Gauge Connection

The connection possesses its own curvature.

For a Lie-algebra-valued connection one-form A , the curvature is

$$F = dA + A \wedge A.$$

In local coordinates,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu].$$

This curvature belongs to the gauge connection.

It should be distinguished from the Riemann curvature

$$R^\rho{}_{\sigma\mu\nu}$$

derived from the metric connection in Section 6.4.

The two curvatures answer different questions.

Riemann curvature concerns the geometry of the state manifold.

Gauge curvature concerns the transport of internal gauge structure over that manifold.

They may interact in a richer theory, but one should not be used as a synonym for the other.

This distinction is particularly important for interdisciplinary interpretation.

A statement that the “relational field is curved” is ambiguous unless the paper specifies whether it refers to

metric curvature,

gauge curvature,

or another geometric structure.

Gauge curvature has a direct relation to path dependence of parallel transport.

When

$$F = 0$$

on an appropriate simply connected region, the connection may be locally flat.

When

$$F \neq 0,$$

transport around sufficiently small loops can produce a nontrivial transformation.

This prepares the final construction of the section.

7.7 Holonomy and Historical Return

Holonomy provides the strongest notion of relational return considered in Paper II. Its role is to formalize how a state may return to the same point of the base manifold while an internally transported relational quantity acquires a nontrivial transformation.

Let

$$\gamma : [0, T] \longrightarrow \mathcal{M}_{\text{red}}$$

be a genuinely closed path satisfying

$$\gamma(0) = \gamma(T).$$

Given a connection A , parallel transport around the loop produces

$$U_\gamma = \mathcal{P} \exp \left(- \oint_\gamma A \right).$$

The resulting transformation belongs, in the appropriate representation, to the holonomy associated with the connection and the loop [5].

If

$$U_\gamma = I,$$

the transported internal object returns unchanged.

If

$$U_\gamma \neq I,$$

the base-state trajectory closes while the transported relational structure does not return trivially.

This gives a precise mathematical distinction among several kinds of return:

RSI return,

fine Lacanian return,

state-space return,

closed path,

and

holonomic return.

A trajectory such as

$$I \rightarrow R \rightarrow S \rightarrow S \rightarrow I$$

establishes only the first of these unless additional structures have been defined.

Even

$$q(T) = q(0)$$

is insufficient for a holonomy claim.

The model also requires

A

and an internal quantity on which the connection acts.

Only then does

U_γ

become meaningful.

The psychoanalytic interest of the construction lies in the possibility of formalizing historical return with retained transformation.

One may return to the same effective relational state coordinate while a transported internal structure reflects the path through which the state was reached.

Schematically,

$$q(T) = q(0)$$

while

$$\Psi(T) = U_\gamma \Psi(0)$$

with

$$U_\gamma \neq I.$$

This provides a mathematically disciplined version of the intuition that “returning to the same place” need not mean returning in the same relational condition.

The formulation should remain narrower than that intuition.

Holonomy describes a transformation produced by parallel transport with respect to a defined connection.

It does not represent every form of psychological history dependence.

A memory-bearing dynamical system may display historical difference without possessing any gauge connection.

A hysteretic system may return to the same external parameter while occupying a different state.

A retroactively reinterpreted history may change semantic valuation without producing a closed geometric path.

Holonomy is therefore one specific geometric form of path dependence among several distinct forms developed in the paper.

Gauge-invariant information can also be extracted from the holonomy. In a matrix representation, for example, a quantity such as

$$\text{Tr}(U_\gamma)$$

may remain invariant under changes of gauge representative under the usual conditions.

Such quantities provide candidate path observables when the corresponding gauge structure has been justified.

The present paper does not yet assign a psychoanalytic meaning to a Wilson loop or related gauge-theoretic observable.

That step would require a specific group, representation, connection, and interpretive map.

The section therefore ends with a limited geometric claim.

If a relational state model admits a justified gauge equivalence and connection, then subjective trajectories can carry path-dependent transport information beyond what is contained in their endpoints:

$$\gamma \mapsto U_\gamma.$$

For closed trajectories,

$$\gamma(0) = \gamma(T),$$

this information can be encoded through holonomy.

The resulting hierarchy is

$$\text{state manifold} \longrightarrow \text{gauge bundle} \longrightarrow \text{connection} \longrightarrow \text{parallel transport} \longrightarrow \text{holonomy}.$$

Each step introduces additional mathematical structure and therefore requires additional justification.

The next section returns from geometry to dynamics. It examines fixed points, local stability, attractors, metastability, recurrence, bifurcation, criticality, and hysteresis, allowing the paper to distinguish several dynamical forms of persistence, transformation, and historical return from the specifically geometric path dependence developed here.

8 Dynamical Regimes of Subjective Fields

This section classifies several qualitative regimes that can arise within the relational dynamics developed in the preceding sections. Its role is to distinguish persistence, recurrence, transition, instability, and history-dependent response at the level of the dynamical system itself. The section proceeds from fixed points and local stability to attractors and metastability, then considers recurrence, bifurcation, criticality, and hysteresis. The method is deliberately structural: each concept is introduced through the mathematical conditions that justify its use before any psychoanalytic interpretation is assigned.

The relevant state may be finite-dimensional,

$$q(t) \in \mathcal{M}_{\text{red}},$$

or field-theoretic,

$$\Phi_t \in \mathcal{M}_R.$$

For exposition, several definitions are first stated for an autonomous finite-dimensional system,

$$\dot{q} = F(q; \lambda),$$

where

$$\lambda$$

denotes one or more parameters. Analogous distinctions can be formulated for fields, nonautonomous systems, and augmented history-bearing states.

The concepts used in this section belong to established nonlinear dynamics [8]. Their application to Lacanian subjective trajectories belongs to the present formal reconstruction.

8.1 Fixed Points and Local Stability

The simplest persistent dynamical regime is a fixed point.

For

$$\dot{q} = F(q),$$

a state

$$q^*$$

is a fixed point when

$$F(q^*) = 0.$$

If the system begins exactly at q^* , it remains there under the selected autonomous dynamics.

The existence of a fixed point should be distinguished from its stability.

Let

$$q(t) = q^* + \delta q(t).$$

Linearization around q^* gives

$$\delta \dot{q} = J(q^*) \delta q,$$

where

$$J^\mu{}_\nu(q^*) = \left. \frac{\partial F^\mu}{\partial q^\nu} \right|_{q^*}$$

is the Jacobian matrix.

The eigenvalues of J provide information about local stability under the usual assumptions.

If all eigenvalues have negative real parts,

$$\operatorname{Re} \lambda_i < 0,$$

the fixed point is locally asymptotically stable.

Perturbations sufficiently close to the state decay:

$$\delta q(t) \longrightarrow 0$$

as

$$t \longrightarrow \infty.$$

If at least one eigenvalue has positive real part,

$$\operatorname{Re} \lambda_i > 0,$$

the fixed point is locally unstable.
A field-theoretic stationary configuration satisfies

$$\partial_t \Phi^* = 0.$$

Linear perturbation

$$\Phi = \Phi^* + \delta\Phi$$

produces an operator equation

$$\partial_t \delta\Phi = \mathcal{J}_{\Phi^*} \delta\Phi,$$

where

$$\mathcal{J}_{\Phi^*}$$

is the linearized evolution operator.

The spectrum of this operator determines local behavior under the selected boundary conditions and function space.

A stable fixed configuration may provide a formal representation of a persistent relational organization.

This interpretation requires caution.

A fixed point in the model does not mean that the person is psychologically unchanging.

It states that the variables retained by the model have reached a stationary configuration at the selected resolution.

Unobserved variables may continue to evolve.

The Lacanian observation may also remain constant over a much larger set than the fixed point itself:

$$\mathcal{O}_L^{-1}(o)$$

may contain many nonstationary configurations.

Consequently,

$$o(t) = o^*$$

for an interval does not imply

$$\dot{q} = 0.$$

Observable persistence and dynamical stationarity remain distinct.

8.2 Attractors and Basins

A fixed point is only one possible persistent regime. A broader concept is an attractor.

Let

$$\varphi_t(q_0)$$

denote the flow generated by the dynamical system from initial state q_0 .

An invariant set

$$\mathcal{A} \subseteq \mathcal{M}_{\text{red}}$$

is an attractor, under the selected definition and assumptions, when nearby trajectories approach $\mathcal{A}$ asymptotically and the set satisfies the required invariance properties.

The corresponding basin of attraction is schematically

$$\mathcal{B}(\mathcal{A}) = \{q_0 \mid \text{dist}(\varphi_t(q_0), \mathcal{A}) \rightarrow 0\}.$$

The basin contains initial configurations whose long-term trajectories approach the attractor. Several attractors may coexist:

$$\mathcal{A}_1, \quad \mathcal{A}_2, \quad \dots$$

Their basins partition part of the state space according to asymptotic behavior. This permits a formal distinction between

current location

and

long-term dynamical organization.

Two nearby states can eventually approach different attractors if they lie on opposite sides of a basin boundary.

Conversely, two initially distant configurations can approach the same attractor.

This structure can become relevant when the observable map is coarse.

Suppose

$$q_1 \in \mathcal{B}(\mathcal{A}_1)$$

and

$$q_2 \in \mathcal{B}(\mathcal{A}_2),$$

while

$$\mathcal{O}_L(q_1) = \mathcal{O}_L(q_2).$$

The two states appear identical at the current Lacanian observational resolution while belonging to different regions of future dynamical accessibility.

The observation therefore fails to determine the asymptotic regime.

The same distinction can arise at the RSI level:

$$\Pi_{\text{RSI}}(\mathcal{O}_L(q_1)) = \Pi_{\text{RSI}}(\mathcal{O}_L(q_2))$$

while their long-term trajectories diverge.

This provides one formal reason for retaining a continuous state beneath a coarse symbolic representation.

An attractor should also be distinguished from a potential minimum.

For gradient dynamics,

$$\dot{q} = -\nabla V(q),$$

a local minimum of V may correspond to a locally attracting state under appropriate conditions.

For general non-gradient systems, the relation between attractors and a scalar potential can be much less direct.

The paper therefore reserves the word *attractor* for a dynamical property rather than using it as a synonym for any apparently stable psychoanalytic pattern.

8.3 Metastability

Many relational configurations may persist for substantial periods without being globally stable. The concept of metastability is useful for describing such regimes.

A metastable configuration or region exhibits long-lived local persistence while remaining capable of transition to another dynamical regime.

Schematically, suppose the effective landscape contains two locally stable regions,

$$\mathcal{A}_1 \quad \text{and} \quad \mathcal{A}_2,$$

separated by an effective barrier.

A trajectory may remain near

$$\mathcal{A}_1$$

for a long interval before a perturbation, parameter change, fluctuation, or slow deformation enables transition toward

$$\mathcal{A}_2.$$

The persistence time can be much longer than the local relaxation time within the metastable region. This creates a separation of time scales:

$$\tau_{\text{local}} \ll \tau_{\text{escape}}.$$

Such separation can matter for subjective trajectory analysis.

A configuration can appear stable at the observational time scale while remaining dynamically capable of reorganization over a longer interval.

The distinction is particularly important when data or interpretation cover only a finite window.

Suppose

$$t_{\text{obs}} \ll \tau_{\text{escape}}.$$

The observer may classify the configuration as effectively stable even though the longer dynamical model places it in a metastable regime.

Metastability therefore depends partly upon scale.

A state can be persistent relative to one observation interval and transient relative to another.

The concept also differs from symbolic recurrence.

A trajectory may move repeatedly among several regions without any of those regions being metastable.

Likewise, a metastable configuration may exhibit little symbolic recurrence before its eventual transition.

Metastability concerns persistence and escape in the dynamical organization of the state space.

8.4 Recurrence

Recurrence was introduced earlier at symbolic and structural resolutions. The present subsection formulates recurrence directly within the continuous dynamical state space.

Let

$$q(t)$$

be a trajectory.

A simple approximate return occurs when

$$d(q(t_2), q(t_1)) < \varepsilon$$

for some

$$t_2 > t_1$$

and tolerance

$$\varepsilon > 0,$$

where d is the distance associated with the selected geometry.
Exact return satisfies

$$q(t_2) = q(t_1).$$

For an autonomous deterministic first-order system whose state representation is complete, exact return has stronger consequences than observational recurrence because the subsequent evolution from the repeated state is determined by the same dynamical law.

The situation changes when the effective state excludes dynamically relevant history.
Suppose

$$q(t_2) = q(t_1)$$

while

$$M(t_2) \neq M(t_1).$$

For the complete augmented state

$$Z(t) = (q(t), M(t)),$$

there has been no exact return:

$$Z(t_2) \neq Z(t_1).$$

Likewise, if the geometry is history-dependent,

$$g(t_2) \neq g(t_1),$$

equality of the coordinate state q may remain insufficient to establish complete recurrence in the enlarged model.

The relevant state for recurrence should therefore be the state that closes the selected dynamics.

This gives the hierarchy

RSI recurrence

↖

fine Lacanian recurrence

↖

reduced-state recurrence

↖

complete-state recurrence.

A closed periodic orbit introduces a stronger condition.

For some period

$$T > 0,$$

a periodic trajectory satisfies

$$q(t + T) = q(t)$$

for all relevant t .

The image of such a solution forms a closed orbit in state space.

This condition should be distinguished from a single accidental equality

$$q(t_1) = q(t_2).$$

Where a gauge connection is also defined, the closed orbit can support the holonomy construction of Section 7.7.

Recurrence therefore forms an interface among symbolic observation, dynamical state, and geometric transport while remaining distinct at each level.

8.5 Bifurcation

A relational system may change qualitatively as a parameter varies. The mathematical concept of bifurcation provides a framework for such changes [8].

Consider

$$\dot{q} = F(q; \lambda),$$

where

$$\lambda$$

is a control parameter.

A bifurcation occurs at a critical parameter value

$$\lambda_c$$

when the qualitative organization of the dynamical system changes as λ passes through or approaches λ_c .

The change may concern the number, stability, or type of invariant sets.

For example, a fixed point may lose stability.

New equilibria may appear.

A stable cycle may emerge.

Basins of attraction may reorganize.

The precise phenomenon depends upon the bifurcation type.

A simple normal form is

$$\dot{x} = \lambda x - x^3.$$

The fixed points satisfy

$$x(\lambda - x^2) = 0.$$

For

$$\lambda < 0,$$

the origin is the only real equilibrium.
For

$$\lambda > 0,$$

additional equilibria appear at

$$x = \pm\sqrt{\lambda}.$$

This equation is a mathematical example of qualitative reorganization.

It is not proposed as an equation of subject formation.

The relevance of bifurcation to the relational framework lies in the possibility that a gradual change in an external or internal parameter can produce a qualitative change in available trajectories.

If

$$\lambda(t)$$

represents an effective relational condition, the system may approach

$$\lambda_c$$

without displaying a proportionally gradual change in its long-term behavior.

Near the bifurcation, local stability can weaken.

The same perturbation may then produce a larger or longer-lived effect.

The observable Lacanian state may remain unchanged while the underlying dynamical regime approaches a transition.

Thus,

$$\mathcal{O}_L(q_{\lambda_1}) = \mathcal{O}_L(q_{\lambda_2})$$

does not imply comparable stability when

$$\lambda_1$$

and

$$\lambda_2$$

lie at different distances from a bifurcation.

This provides a formal distinction between

current observation

and

proximity to qualitative reorganization.

The distinction will be central to the treatment of criticality.

8.6 Criticality

The term *criticality* is used across several mathematical and physical contexts, including phase transitions, bifurcations, critical phenomena, and self-organized critical systems. The present paper adopts a deliberately restricted dynamical meaning.

A relational state is described as approaching a critical regime when the selected dynamical model approaches a parameter or state region at which its qualitative organization changes or its existing stability structure becomes degenerate.

The simplest case is proximity to a bifurcation:

$$\lambda \rightarrow \lambda_c.$$

Suppose a stable fixed point satisfies

$$F(q^*; \lambda) = 0.$$

Its linear stability is governed by eigenvalues

$$\eta_i(\lambda)$$

of the Jacobian.

A loss of stability may occur when

$$\text{Re } \eta_k(\lambda) \rightarrow 0$$

for at least one mode.

The return rate associated with that mode then becomes slow.

Under appropriate conditions, perturbations decay increasingly slowly as the critical parameter is approached.

This provides one mathematically defined sense in which a system can become more sensitive near qualitative reorganization.

The paper does not assume that every subjective crisis is a bifurcation.

Nor does it assume that a subject should be kept near a critical point.

The formal value of criticality lies in distinguishing at least three regimes:

deep within a stable basin,

near a stability boundary or critical region,

and

after qualitative reorganization.

These regimes can have very different implications for local prediction.

Deep within a strongly attracting region, a small perturbation may decay rapidly.

Near loss of stability, the same approximation may become unreliable over a shorter horizon.

After bifurcation, the previous local model may cease to describe the relevant attractor structure.

This motivates a resolution-dependent approach to prediction.

Let

$$q(t)$$

be the current state and let

$$T_q \mathcal{M}_{\text{red}}$$

be its tangent space.

Away from critical reorganization, a local linear approximation

$$\delta \dot{q} = J(q) \delta q$$

may remain useful over a selected neighborhood.

Near a critical region, higher-order terms and alternative future branches can become more important.

A short-horizon local model may still be useful:

$$q(t + \Delta t) \approx q(t) + F(q(t)) \Delta t$$

for sufficiently small

$$\Delta t.$$

The trustworthy horizon, however, can shrink as local stability weakens or nonlinearities dominate. This observation is particularly relevant to historically situated judgment.

A trajectory near qualitative reorganization can require greater epistemic caution than one lying deeply inside a robust dynamical regime.

The concept of criticality therefore concerns the structure of the model's possible evolution.

Its psychoanalytic interpretation remains an additional step.

The present usage also does not imply self-organized criticality.

Such a claim would require a model showing how the dynamics themselves drive the system toward a critical regime without external tuning.

No such mechanism is assumed here.

8.7 Hysteresis and Historical Dependence

Hysteresis provides a dynamical form of history dependence in which the current response depends upon the path through parameter or state space.

Let

$$\lambda$$

be a slowly varying control parameter and let

$$Q$$

be an observable of the system.

A single-valued memoryless response would have the form

$$Q = Q(\lambda).$$

In a hysteretic regime, the observed value can depend additionally upon the history of parameter variation:

$$Q(t) = Q(\lambda(t); \mathcal{H}_t).$$

Thus the same current parameter value can support different states depending upon whether the system arrived from one direction or another.

Suppose

$$\lambda(t_1) = \lambda(t_2) = \lambda_0.$$

Hysteresis permits

$$q(t_1) \neq q(t_2).$$

A familiar dynamical mechanism is multistability combined with parameter variation.

As λ increases, the system may remain on one stable branch until a stability threshold is reached.

During subsequent decrease, the system may remain on another branch until a different threshold is crossed.

The forward and reverse transitions therefore occur at different parameter values.

Schematically,

$$\lambda_{\uparrow} \neq \lambda_{\downarrow}.$$

The resulting loop in an observable plot such as

$$Q \text{ versus } \lambda$$

is a hysteresis loop.

The relational interpretation is potentially important.

Returning an external condition to its previous value does not require the subjective state to return to its previous configuration.

The trajectory through the intervening regime matters.

This is a stronger statement than saying generically that history matters.

Hysteresis has a specific dynamical structure.

It should therefore be distinguished from the other forms of path dependence introduced in this paper.

First, hysteresis differs from an explicit memory kernel.

A system can display hysteresis through multistability and slow parameter variation even when its enlarged dynamical state is Markovian.

Conversely, a memory-kernel model need not display a hysteresis loop.

Second, hysteresis differs from retroactive interpretation.

Retroaction concerns the changing meaning assigned to an earlier event.

Hysteresis concerns the current dynamical state or observable response as a function of the path through the system's parameter history.

Third, hysteresis differs from holonomy.

Holonomy requires a connection and parallel transport around a closed path.

Hysteresis requires a history-dependent dynamical response.

The corresponding structures can coexist, but they should not be identified.

The distinctions can be summarized as

$$\text{memory} \rightarrow \text{past states affect evolution,}$$

$$\text{hysteresis} \rightarrow \text{response depends on the path through a control history,}$$

$$\text{retroaction} \rightarrow \text{later history changes interpretation of earlier history,}$$

and

$$\text{holonomy} \rightarrow \text{parallel transport around a closed path yields a transformation.}$$

This taxonomy is important because the four phenomena can produce superficially similar statements such as “the system remembers its history” while referring to mathematically different structures.

The dynamical regimes developed in this section can now be organized by the kind of structure they describe:

fixed point local stationarity,

attractor asymptotic organization,

metastability long-lived local persistence,

recurrence return within state space,

bifurcation qualitative change of dynamical structure,

criticality proximity to a selected qualitative reorganization,

and

hysteresis path-dependent response.

No one of these concepts supplies a general mathematical translation of psychoanalytic subjectivity.

Their value lies in distinguishing several forms of persistence and change that a symbolic trajectory alone may collapse into the same sequence of observations.

The next section changes the level of analysis again. Instead of following one dynamical trajectory selected by an evolution law, it considers families of admissible field histories between relational configurations. This permits the introduction of trajectory weights and path aggregation while preserving the distinction between a general ensemble of histories and a specifically quantum path integral.

9 Field Histories and Path Ensembles

This section moves from the evolution of one relational configuration to a family of possible histories connecting selected configurations. Its role is to formalize alternative trajectories, assign them general weights, and clarify the relation between discrete derivational sums and continuous history ensembles. The section proceeds from the definition of admissible field histories to trajectory-weight functionals, path aggregation, the relation to the discrete generative model, and the interpretation of the resulting ensemble. The construction remains deliberately more general than a quantum path integral. A specifically quantum amplitude is introduced only as a special mathematical form requiring additional assumptions.

The dynamical sections above consider a trajectory

$$\gamma : t \mapsto \Phi_t$$

generated by a selected evolution law. An ensemble formulation considers a larger object:

$$\Gamma(\Phi_i, \Phi_f),$$

the collection of admissible histories connecting an initial configuration Φ_i with a final configuration Φ_f .

This change of level is conceptually important. A single observed subjective history does not determine that it was the only formally available path between its boundary conditions. Likewise, a symbolic sequence may suppress many continuously distinct histories.

The ensemble formulation therefore asks which histories are admitted, how they differ, and whether some paths should contribute more strongly than others to the selected effective description.

9.1 Families of Admissible Histories

Let

$$\mathcal{M}_R$$

denote the relational configuration space.

A finite-time field history is a map

$$\gamma : [t_i, t_f] \longrightarrow \mathcal{M}_R$$

satisfying

$$\gamma(t_i) = \Phi_i, \quad \gamma(t_f) = \Phi_f.$$

The set of all histories satisfying the selected boundary and regularity conditions is written

$$\Gamma(\Phi_i, \Phi_f).$$

The phrase *all histories* is always relative to a formal specification.

One may require, for example,

$$\gamma \in C^1([t_i, t_f], \mathcal{M}_R),$$

or another function class appropriate to the model.

One may also impose dynamical constraints.

Let

$$\mathcal{C}[\gamma] = 0$$

denote the collection of admissibility conditions.

Then

$$\Gamma_{\text{adm}}(\Phi_i, \Phi_f) = \{\gamma \mid \gamma(t_i) = \Phi_i, \gamma(t_f) = \Phi_f, \mathcal{C}[\gamma] = 0\}.$$

The constraints can express several different commitments.

A strong dynamical condition may require γ to satisfy the evolution equation exactly:

$$\partial_t \Phi = \mathcal{F}[\Phi].$$

A weaker history space may include trajectories that depart from the classical equation while receiving different weights.

A stochastic model may instead define admissibility through a transition law or probability measure over trajectories.

These constructions should remain distinct.

If the dynamical equation and initial condition determine one unique trajectory, then

$$\Gamma_{\text{dyn}}(\Phi_i)$$

may contain only one deterministic history.

A path-ensemble formulation becomes informative when the model permits alternative histories through incomplete boundary information, stochastic evolution, coarse-grained unresolved variables, variational comparison, or another source of multiplicity.

The same distinction applies to reduced state trajectories.

Let

$$q(t) = \mathcal{Q}[\Phi_t].$$

Several field histories can generate the same reduced trajectory:

$$\gamma_1 \neq \gamma_2$$

while

$$\mathcal{Q}[\gamma_1(t)] = \mathcal{Q}[\gamma_2(t)]$$

for all selected times.

Likewise, several reduced trajectories can generate the same Lacanian observation sequence:

$$\mathcal{O}_L(\gamma_1(t_k)) = \mathcal{O}_L(\gamma_2(t_k)).$$

The hierarchy is therefore

field history $\longrightarrow$ reduced dynamical history $\longrightarrow$ Lacanian itinerary $\longrightarrow$ RSI trajectory.

Multiplicity can arise at every level.

This gives the continuous framework the same structural feature encountered in the discrete generative model: one observable history may possess several possible generative realizations.

9.2 Trajectory Weights

Once a family of histories has been defined, an additional structure may assign a numerical or algebraic value to each trajectory.

Let

$$W : \Gamma_{\text{adm}}(\Phi_i, \Phi_f) \longrightarrow \mathbb{K}$$

be a trajectory-weight functional.

The codomain

$$\mathbb{K}$$

depends upon the interpretation.

Possible choices include

$$\mathbb{R}_{\geq 0}, \quad \mathbb{R}, \quad \mathbb{C},$$

or another explicitly defined algebraic structure.

The value

$$W[\gamma]$$

has no psychoanalytic meaning until the weight has been interpreted.

Possible interpretations include

probability density,

relative accessibility,

empirical frequency,

transition cost,

effective salience,

or another quantitatively defined object.

These quantities should not be interchanged merely because each can be represented numerically.

A probabilistic path model requires positivity and normalization appropriate to the selected measure:

$$W[\gamma] \geq 0.$$

A cost functional need not satisfy probability normalization.

A complex amplitude belongs to a still different formal interpretation.

The action introduced in Section 5 supplies one possible source of trajectory weighting.

A general real-valued choice may take the form

$$W[\gamma] = \exp(-\alpha S_{\text{eff}}[\gamma]),$$

where

$$\alpha > 0$$

is a model parameter.

Such an expression favors histories with lower values of the selected effective functional.

Its interpretation depends upon what

$$S_{\text{eff}}$$

means in the model.

It should not be called a quantum amplitude.

A specifically quantum-field-theoretic weighting has the schematic form

$$W_Q[\gamma] = \exp\left(\frac{i}{\hbar} S[\gamma]\right),$$

which leads to interference among complex amplitudes in the usual quantum formulation [7].

The present paper does not assign this physical meaning to subjective histories.

Accordingly, the default notation remains

$$W[\gamma].$$

The quantum form is retained only as a mathematically more specialized possibility whose interpretation would require independent justification.

Trajectory weights may also depend upon history-dependent geometry.

For example,

$$W[\gamma] = \mathcal{W}[\gamma, g[\gamma], \hbar].$$

The weight can then depend upon both the path and the geometry induced along that path.

This creates a stronger form of relational dependence: the history affects the geometry, and the geometry contributes to the evaluation of the history.

Such feedback should be introduced only where the model specifies how $g[\gamma]$ is generated.

9.3 Path Aggregation

A trajectory ensemble becomes analytically useful when the contributions of alternative histories are combined.

A general history aggregate can be written schematically as

$$K(\Phi_f, t_f; \Phi_i, t_i) = \int_{\Gamma_{\text{adm}}(\Phi_i, \Phi_f)} \mathcal{D}\gamma W[\gamma].$$

The notation

$$\mathcal{D}\gamma$$

denotes a formal integration over histories.

Its mathematical status depends upon the model.

In many applications of mathematical physics, path-integral notation requires substantial care concerning measures, regularization, discretization, and limiting procedures. The present paper therefore does not treat $\mathcal{D}\gamma$ as an automatically defined measure on an arbitrary psychoanalytic configuration space.

A more modest finite approximation may begin from a discretized family

$$\{\gamma_1, \dots, \gamma_N\}$$

and define

$$K_N = \sum_{k=1}^N W[\gamma_k].$$

A continuum expression can then be considered only when the trajectory space and limiting procedure have been specified.

The aggregate K also requires interpretation.

If $W[\gamma]$ is a probability density, then integration over histories may produce a transition probability or a related statistical quantity.

If $W[\gamma]$ is a cost-derived weight, K may represent aggregate accessibility according to that cost model.

If $W[\gamma]$ is a complex amplitude, the sum has an interference interpretation characteristic of quantum theory.

The same symbol K should therefore not be assigned one universal meaning.

The present framework uses

$$K_W$$

when useful to emphasize dependence upon the weighting scheme:

$$K_W(\Phi_f, \Phi_i) = \int \mathcal{D}\gamma W[\gamma].$$

The history ensemble can also be conditioned upon intermediate observations.

Suppose

$$\mathcal{O}_L(\gamma(t_k)) = o_k$$

at selected times t_k .

Then define

$$\Gamma(\Phi_i, \Phi_f \mid o_1, \dots, o_m)$$

as the histories consistent with those observations.

A conditional aggregate becomes

$$K_W(\Phi_f, \Phi_i \mid o_1, \dots, o_m) = \int_{\Gamma(\Phi_i, \Phi_f \mid o_1, \dots, o_m)} \mathcal{D}\gamma W[\gamma].$$

This construction is particularly relevant to psychoanalytic observation.

A finite symbolic itinerary may constrain a continuous history without determining it uniquely.

The ensemble then represents the unresolved continuous trajectories compatible with the observed sequence.

9.4 Relation to Discrete Derivations

The path-ensemble construction has a direct formal relation to the derivational architecture of the preceding generative paper.

Let

$$\mathcal{D}(o_i, o_f)$$

denote a set of admissible discrete derivations connecting two formal Lacanian states.

A weighted discrete propagator-like quantity can be written

$$K_D(o_f, o_i) = \sum_{d \in \mathcal{D}(o_i, o_f)} W_D(d).$$

Here

$$W_D(d)$$

is a weight assigned to a derivation.

The continuous analogue is

$$K_C(\Phi_f, \Phi_i) = \int_{\Gamma(\Phi_i, \Phi_f)} \mathcal{D}\gamma W_C[\gamma].$$

The correspondence can therefore be represented schematically as

$$\begin{array}{ccc} \text{discrete derivation} & \longrightarrow & W_D(d) \\ \downarrow & & \downarrow \\ \text{continuous history} & \longrightarrow & W_C[\gamma]. \end{array}$$

The vertical relation requires a discrete-to-continuous bridge.

It is not automatic.

A derivation

$$d = o_0 \Rightarrow o_1 \Rightarrow \dots \Rightarrow o_n$$

may correspond to a family of continuous trajectories satisfying

$$\mathcal{O}_L(\gamma(t_k)) = o_k.$$

Define

$$\Gamma_d = \{\gamma \mid \mathcal{O}_L(\gamma(t_k)) = o_k \text{ for the observations encoded by } d\}.$$

Then

$$d \longrightarrow \Gamma_d$$

is generally a one-to-many relation.

The continuous weight associated with the discrete derivation could therefore be obtained, under a specified aggregation rule, from

$$W_D(d) = \int_{\Gamma_d} \mathcal{D}\gamma W_C[\gamma].$$

This equation is a candidate bridge rather than an established identity.

Its validity depends upon the definitions of

$$\Gamma_d, \quad W_C, \quad W_D,$$

and the history measure.

The reverse relation also need not be unique.

A continuous trajectory may cross several observational boundaries and admit different symbolic segmentations depending upon the sampling rule.

Thus,

$$\gamma \longrightarrow d$$

depends upon the observation and discretization procedure.

The relation between Paper I and Paper II should therefore be understood as *discrete and continuous histories can be related through observation and coarse-graining without being formally identical*.

This point prevents a continuum model from being presented as the uniquely correct completion of the grammar model.

The two architectures preserve different information.

9.5 History Ensembles across Observational Resolutions

The multi-resolution observation architecture can be extended from individual trajectories to entire trajectory ensembles.

Let

$$\Gamma_w = \{\gamma \mid \Pi_{\text{RSI}} \circ \mathcal{O}_L(\gamma) = w\}$$

denote the family of continuous histories that produce a selected RSI trajectory w under the chosen observation protocol.

Likewise, for a fine Lacanian itinerary

$$\gamma_L,$$

define

$$\Gamma_{\gamma_L} = \{\gamma \mid \mathcal{O}_L(\gamma) = \gamma_L\}.$$

These sets satisfy a nested structure.

If

$$\Pi_{\text{RSI}}(\gamma_L) = w,$$

then

$$\Gamma_{\gamma_L} \subseteq \Gamma_w.$$

A coarse trajectory therefore corresponds to a larger class of continuous histories than a fine Lacanian itinerary.

Weights can be aggregated accordingly.

For a fine itinerary,

$$W_L(\gamma_L) = \int_{\Gamma_{\gamma_L}} \mathcal{D}\gamma W_C[\gamma].$$

For the RSI word,

$$W_{\text{RSI}}(w) = \int_{\Gamma_w} \mathcal{D}\gamma W_C[\gamma].$$

Equivalently, when the decomposition is well defined,

$$W_{\text{RSI}}(w) = \sum_{\gamma_L: \Pi_{\text{RSI}}(\gamma_L)=w} W_L(\gamma_L).$$

This reproduces, at the continuous level, the many-to-one architecture of the generative paper.

The same observable word can accumulate contributions from

multiple discrete derivations,

multiple fine Lacanian itineraries,

and

multiple continuous field histories.

These multiplicities belong to different representational levels.

They should therefore be distinguished when probability, accessibility, or another quantitative weight is interpreted.

9.6 Interpretation of the Path Ensemble

The path ensemble introduces substantial expressive power. It also creates a risk of importing specifically quantum interpretations into a model that has not justified them. This subsection therefore states the epistemic status of the construction explicitly.

The weakest interpretation of

$$\Gamma(\Phi_i, \Phi_f)$$

is simply

the family of trajectories admitted by a formal model.

This interpretation requires no quantum assumption.

A weighted ensemble adds

$$W[\gamma].$$

Its meaning depends upon the definition of the weight.

A probabilistic ensemble can represent uncertainty over possible histories.

A statistical ensemble can summarize observed variability.

An accessibility model can rank trajectories according to a defined relational criterion.

An optimization model can compare histories through a cost functional.

A variational model can privilege stationary or extremal histories.

These are mathematically distinct uses of a trajectory ensemble.

The specifically quantum interpretation adds further structure, including complex amplitudes and interference.

For a conventional field-theoretic path integral, the schematic expression

$$K = \int \mathcal{D}\Phi \exp\left(\frac{i}{\hbar} S[\Phi]\right)$$

belongs to quantum theory [7].

The present paper does not infer from this formal resemblance that a subjective trajectory exists in a physical quantum superposition of psychoanalytic histories.

Accordingly,

$$W[\gamma]$$

remains the primary notation of the relational model.

The QFT expression is useful for identifying one mathematically rich special case and for preparing the conceptual transition to Paper III.

The history-ensemble perspective nevertheless produces an important result without any quantum ontology:

A subjective trajectory can be represented relative to a space of alternative admissible histories, and an observed path can be analyzed together with the other trajectories that remain compatible with the selected boundary, dynamical, and observational conditions.

This changes the emphasis from

Which single path occurred?

to

What structure does the space of possible histories possess?

That question becomes especially important near bifurcation, criticality, multistability, and coarse-graining.

Several distinct futures may remain locally accessible even when only one is eventually realized.

The ensemble also provides a natural bridge toward emergent geometry.

If relational geometry depends upon history,

$$g = g[\gamma],$$

then different admissible histories can carry different effective geometries.

The ensemble becomes schematically

$$\{(\gamma, g_\gamma, W[\gamma])\}.$$

This is already more than a collection of paths on one fixed background.

The background structure itself may differ across histories.

Paper II stops short of deriving geometry from a sum over combinatorial histories. That stronger problem belongs to the later spin-foam and group-field-theoretic investigation of the “spacetime of desire.”

The result of the present section can therefore be summarized through the sequence

$$\text{boundary configurations} \longrightarrow \Gamma(\Phi_i, \Phi_f) \longrightarrow W[\gamma] \longrightarrow K_W(\Phi_f, \Phi_i),$$

together with observational projections

$$\gamma \longrightarrow \gamma_L \longrightarrow w_{\text{RSI}}.$$

The architecture permits multiple continuous histories to contribute to the same fine or coarse symbolic trajectory while retaining a strict distinction between general weighted histories and quantum amplitudes.

Section 10 now addresses the complementary problem of scale. Instead of aggregating alternative histories with fixed observational resolution, it asks how the variables, interactions, and effective dynamics change when fine relational degrees of freedom are systematically suppressed.

10 Coarse-Graining, Effective Dynamics, and Renormalization

This section examines how relational dynamics change when distinctions present at a finer level of description are suppressed. Its role is to distinguish three operations that can easily be conflated: observational projection, dynamical reduction, and scale-dependent coarse-graining. The section first defines fine and coarse field variables, then considers elimination of unresolved degrees of freedom, the emergence of effective interactions, and the relation between the continuous field model and RSI observation. It subsequently introduces scale dependence and concludes by identifying the additional structures required before the language of renormalization can be used in a stronger sense.

The central distinction is

$$\text{coarse observation} \neq \text{reduced state} \neq \text{effective dynamics}.$$

A coarse observation can map many configurations to the same symbolic description without specifying how those configurations evolve.

A reduced state retains only selected dynamical variables.

An effective dynamical model additionally specifies how the retained variables evolve after the unresolved variables have been removed, integrated out, averaged over, or otherwise represented indirectly.

The distinction is important for the present programme because the RSI projection introduced earlier is primarily observational:

$$\Phi \xrightarrow{\mathcal{O}_L} o_L \xrightarrow{\Pi_{\text{RSI}}} a_{\text{RSI}}.$$

A genuine dynamical coarse-graining asks a different question:

How does the law governing the retained variables change when finer relational degrees of freedom are suppressed?

10.1 Fine and Coarse Field Variables

Let the fine relational configuration be decomposed schematically as

$$\Phi = (\phi, \chi),$$

where

$$\phi$$

denotes the variables retained at a selected coarse resolution and

$$\chi$$

the variables unresolved at that resolution.

The fine state belongs to

$$\mathcal{M}_{\text{fine}},$$

while the retained variables belong to a reduced space

$$\mathcal{M}_{\text{eff}}.$$

A coarse-graining map may be written

$$\mathcal{C}_\ell : \mathcal{M}_{\text{fine}} \longrightarrow \mathcal{M}_\ell,$$

where

$$\ell$$

denotes the resolution scale.

The corresponding coarse field is

$$\Phi_\ell = \mathcal{C}_\ell[\Phi].$$

The scale parameter ℓ is deliberately abstract.

Depending upon the model, it may represent temporal resolution, relational resolution, structural aggregation, spatial resolution on Ω_R , or another explicitly defined scale.

It should not be introduced as a physical length unless the relational domain supports that interpretation.

A simple averaging operation provides one example.

For a kernel

$$K_\ell(\xi, \xi'),$$

define

$$\Phi_\ell^a(\xi) = \int_{\Omega_R} K_\ell(\xi, \xi') \Phi^a(\xi') d\mu(\xi').$$

When K_ℓ broadens as ℓ increases, the resulting field retains progressively less fine variation.

This is one possible coarse-graining operator.

Other models may use block variables, spectral truncation, basis reduction, projection onto collective coordinates, or probabilistic aggregation.

The choice of coarse-graining map determines which distinctions are removed.

For two fine configurations

$$\Phi_1 \neq \Phi_2,$$

it may happen that

$$\mathcal{C}_\ell[\Phi_1] = \mathcal{C}_\ell[\Phi_2].$$

The coarse description therefore defines an equivalence relation,

$$\Phi_1 \sim_\ell \Phi_2$$

whenever

$$\mathcal{C}_\ell[\Phi_1] = \mathcal{C}_\ell[\Phi_2].$$

The equivalence class

$$[\Phi]_\ell$$

contains configurations indistinguishable at resolution ℓ .

This construction resembles the observational fibers introduced earlier, but the intended role is different.

The fiber

$$\mathcal{O}_L^{-1}(o)$$

collects states with the same psychoanalytic observation.

The class

$$[\Phi]_\ell$$

collects states identified by a specified coarse-graining transformation.

The two partitions may overlap without coinciding.

10.2 Elimination of Unresolved Degrees of Freedom

The suppression of variables becomes dynamically significant when the unresolved degrees of freedom influence the evolution of those that remain.

Suppose the fine dynamics are

$$\dot{\phi} = F_\phi(\phi, \chi),$$

$$\dot{\chi} = F_\chi(\phi, \chi).$$

If χ is removed from the retained description, it does not generally follow that

$$\dot{\phi} = F_\phi(\phi, 0).$$

The unresolved variables can continue to affect the coarse dynamics.

Formally, elimination of χ can produce an effective equation of the form

$$\dot{\phi}(t) = F_{\text{eff}}(\phi(t)) + \int_0^t K(t - \tau) \phi(\tau) d\tau + \eta(t).$$

The three terms can be interpreted, within the formal model, as

effective deterministic dynamics,

memory induced by unresolved variables,

and

effective residual fluctuations.

The appearance of memory or noise under reduction is important for the relational programme.

A model that is Markovian at a fine resolution can appear non-Markovian after dynamically relevant variables are removed.

Thus,

coarse historical dependence

need not imply that the fine dynamics contained an explicit memory kernel.

It may arise from incomplete state representation.
This gives another reason to distinguish

$$M_t$$

as an explicitly modeled memory variable from

effective memory generated by coarse-graining.

The two can produce similar reduced equations while representing different model architectures.
The same issue appears in field dynamics.
Suppose

$$\Phi(\xi, t) = \Phi_{<}(\xi, t) + \Phi_{>}(\xi, t),$$

where the subscripts denote retained and unresolved modes according to a specified scale decomposition.

The fine dynamics may couple these modes:

$$\partial_t \Phi_{<} = F_{<}[\Phi_{<}, \Phi_{>}].$$

Removing $\Phi_{>}$ can alter the effective law governing $\Phi_{<}$.

A coarse field is therefore not generally governed by the fine equation with the fine variables simply omitted.

10.3 Emergent Effective Interactions

One of the most important consequences of coarse-graining is that new effective interactions can appear among the retained variables.

Suppose the fine action has the schematic form

$$S[\phi, \chi] = S_\phi[\phi] + S_\chi[\chi] + S_{\text{int}}[\phi, \chi].$$

A formal elimination of χ can produce

$$S_{\text{eff}}[\phi] = S_\phi[\phi] + \Delta S[\phi].$$

The correction

$$\Delta S[\phi]$$

can contain interactions that were absent from the isolated S_ϕ .

For example, a fine description may contain no direct coupling between ϕ_1 and ϕ_2 ,

$$S_{\text{fine}} = S_1[\phi_1] + S_2[\phi_2] + S_\chi[\chi] + g_1\phi_1\chi + g_2\phi_2\chi.$$

After the unresolved variable χ is eliminated, the effective description may contain a term of the schematic form

$$g_{\text{eff}}\phi_1\phi_2.$$

The retained variables now interact effectively through the degrees of freedom that were removed.
This possibility is conceptually significant for psychoanalytic coarse-graining.

A stable relational pattern observed at a larger scale need not correspond to one primitive variable present at the finer level.

It can emerge from repeated interactions among finer variables.

Likewise, a coarse interaction can be real within the effective model even when it is mediated by processes no longer represented explicitly.

This gives a formal meaning to the distinction

microscopic relation

and

effective relational coupling.

The latter should not be dismissed as unreal merely because it is absent from the finest vocabulary. Its validity depends upon whether it organizes the retained dynamics accurately at the selected scale.

This principle is especially compatible with a relational approach in which the relevant objects of one level may emerge from interactions at another.

The effective model can therefore possess its own vocabulary and interaction structure.

The relation

fine model $\longrightarrow$ effective model

is more than deletion.

It is a transformation of dynamical description.

10.4 RSI as an Effective Observable

The RSI representation provides a useful comparison with dynamical coarse-graining.

Recall the observational chain

$$\Phi_t \xrightarrow{\mathcal{O}_L} o_t \xrightarrow{\Pi_{\text{RSI}}} a_t.$$

The RSI symbol

$$a_t \in \{R, S, I\}$$

is a coarse observable.

Several continuous configurations can satisfy

$$a(\Phi_1) = a(\Phi_2).$$

This equivalence does not determine whether the coarse RSI trajectory itself obeys a closed dynamical equation.

Suppose

$$a_t = S.$$

The future probability of observing I or R may depend upon fine variables hidden inside the fiber

$$a^{-1}(S).$$

A purely RSI-level evolution law,

$$P(a_{t+\Delta t} \mid a_t),$$

may therefore fail to be Markovian.

One may need

$$P(a_{t+\Delta t} \mid a_t, a_{t-1}, \dots),$$

or a latent-state model whose hidden variables retain the information lost by the RSI projection.

This gives a precise connection between Paper I and Paper II.

A coarse symbolic grammar may display history dependence because continuous states with different future accessibility have been collapsed into the same symbol.

Thus,

same RSI word

does not imply

same effective continuous history.

The opposite problem also occurs.

A family of fine trajectories may differ continuously while all distinctions relevant to a selected psychoanalytic question are preserved by the same RSI word.

In that case, the coarse representation may be sufficient.

The adequacy criterion remains task-relative:

$\mathcal{C}_\ell$

or

Π_{RSI}

is appropriate when the distinctions removed by the map do not alter the inference being made.

RSI should therefore be treated as an effective observable resolution rather than as the unique macroscopic dynamics of the relational field.

10.5 Scale Dependence of Relational Description

A relational process can be described at several temporal and structural scales.

For example, the same history may contain

momentary relational events,

short episodes,

recurrent patterns,

and

longer-lived subjective organizations.

The variables useful at one scale need not remain useful at another.

Let

Φ_ℓ

denote the effective state at scale ℓ .

Its dynamics may be written

$$\partial_t \Phi_\ell = F_\ell[\Phi_\ell].$$

The subscript on F_ℓ is important.

The evolution law itself may depend upon scale.

Thus,

$$F_{\ell_1} \neq F_{\ell_2}$$

even when both describe coarse versions of the same underlying system.
The effective couplings can also depend upon scale:

$$g_i = g_i(\ell).$$

A term negligible at one resolution can dominate at another.
Conversely, a fine interaction may average out and disappear from the effective model.
This produces a scale-dependent hierarchy of descriptions:

$$(\Phi_{\ell_1}, F_{\ell_1}) \longrightarrow (\Phi_{\ell_2}, F_{\ell_2}) \longrightarrow (\Phi_{\ell_3}, F_{\ell_3}),$$

with increasing coarse-graining.

There is no requirement that the same psychoanalytic vocabulary be optimal at every level.

For example, a fine Lacanian itinerary may be useful for describing structural transitions over one interval, while an effective relational variable may summarize a longer-lived organization.

The same principle applies to geometry.

A metric

$$g_{\mu\nu}^{(\ell)}$$

defined at one scale may differ from the metric appropriate to another:

$$g_{\mu\nu}^{(\ell_1)} \neq g_{\mu\nu}^{(\ell_2)}.$$

Coarse-graining can therefore modify the effective geometry in addition to the effective dynamics.

This possibility is particularly important for the later idea of emergent relational geometry.

The geometry observed at a larger scale may itself be an effective product of finer relational organization.

10.6 Toward Renormalization of Subjective Description

The language of renormalization becomes appropriate only after a scale-dependent family of effective models has been defined.

In quantum and statistical field theories, renormalization-group methods describe how effective couplings and theories change under transformations of scale [7]. The present relational framework does not assume that subjective dynamics possess a physical RG structure.

It instead asks which additional ingredients would be required before an analogous formal construction becomes meaningful.

Let

$$\mathcal{R}_b$$

denote a coarse-graining transformation associated with scale factor b .

The field transforms as

$$\Phi \longmapsto \Phi' = \mathcal{R}_b[\Phi].$$

The effective couplings transform correspondingly:

$$g_i \longmapsto g'_i.$$

An RG-like map can therefore be written

$$\mathbf{g}' = \mathcal{T}_b(\mathbf{g}),$$

where

$$\mathbf{g} = (g_1, \dots, g_n)$$

denotes the effective coupling vector.

If a continuous scale parameter

$$s = \log b$$

is introduced, one may formally write

$$\frac{dg_i}{ds} = \beta_i(\mathbf{g}),$$

where

$$\beta_i$$

describes the scale dependence of the coupling.

This equation should not be introduced merely because scale is being discussed.

A genuine RG-like construction requires at least:

1. a specified family of scales;
2. a coarse-graining transformation between those scales;
3. a parameterized family of effective models;
4. a rule describing how effective couplings transform;
5. quantities whose behavior under repeated coarse-graining can be compared.

Without these elements, the paper should use the weaker terms *coarse-graining*, *effective description*, or *scale dependence*.

If the required structure is established, several further concepts become available.

A fixed point of the scale transformation satisfies

$$\mathbf{g}^* = \mathcal{T}_b(\mathbf{g}^*).$$

In differential form,

$$\beta_i(\mathbf{g}^*) = 0.$$

Such a fixed point represents scale-invariant effective structure under the selected transformation.

Perturbations around the fixed point can be classified according to whether they grow or decay under repeated coarse-graining.

This provides a formal vocabulary for distinguishing

scale-persistent structure

from

scale-specific detail.

The potential psychoanalytic interest is considerable.

A relational pattern that survives several changes of descriptive scale may represent a more robust effective regularity than one visible only at a particular resolution.

However, persistence across scale should not be treated as evidence of ontological fundamentality. It means only that the selected structure is stable under the defined coarse-graining transformation. A second possibility concerns universality. Distinct fine relational models,

$$\mathfrak{M}_1 \neq \mathfrak{M}_2,$$

could conceivably approach similar effective descriptions under repeated coarse-graining:

$$\mathcal{R}_b^n(\mathfrak{M}_1) \approx \mathcal{R}_b^n(\mathfrak{M}_2)$$

for sufficiently coarse scales.

If such behavior were established, macroscopically similar subjective dynamics could arise from different fine relational histories.

This would provide a formal analogue of universality.

The present paper does not claim that psychoanalytic systems possess such universality classes.

It identifies the question as a possible direction once a genuine scale-transformation structure has been constructed.

A third possibility concerns the geometry itself.

If the effective metric depends upon scale,

$$g_{\mu\nu} = g_{\mu\nu}^{(\ell)},$$

one can ask whether repeated coarse-graining drives the geometry toward a stable effective form. Schematically,

$$g^{(\ell)} \longrightarrow g^{(b\ell)}.$$

This suggests a possible bridge between

coarse-grained relational dynamics

and

emergent relational geometry.

The stronger question is whether a smooth effective geometry may arise from relational structures that are not themselves geometrical at the finest resolution.

That question marks the boundary of the present section and the beginning of the next conceptual layer of the paper.

The distinction developed here can be summarized as

$$\text{observation} \quad \Phi \rightarrow o,$$

$$\text{reduction} \quad \Phi \rightarrow q,$$

$$\text{coarse-graining} \quad \Phi_{\text{fine}} \rightarrow \Phi_{\ell},$$

$$\text{effective dynamics} \quad F_{\text{fine}} \rightarrow F_{\ell},$$

and, only when the required scale structure exists,

renormalization $\mathbf{g} \rightarrow \mathcal{T}_b(\mathbf{g})$.

These operations solve different formal problems.

Their separation is important for the central thesis of Paper II. A subjective trajectory can possess different effective descriptions at different scales, and the geometry and dynamics appropriate to those scales need not be simple restrictions of one fine model.

The next section develops the strongest implication of this observation. Instead of assuming that subjective trajectories move through a geometry given in advance, it asks whether an effective relational geometry can itself emerge from relational events and their histories. The spin-foam and group-field-theoretic possibilities are introduced only as exploratory directions, establishing the conceptual boundary between the present field model and the subsequent study of the “spacetime of desire.”

11 Emergent Relational Geometry and the Spacetime of Desire

The preceding sections have treated subjective dynamics as evolution within a relational configuration space equipped, where required, with effective geometric structure. This section considers a stronger possibility. The geometry that organizes subjective trajectories may itself arise from a more primitive relational organization rather than being supplied in advance.

The purpose of the section is exploratory. It does not construct a spin-foam model of psychoanalysis, assign quantum geometry to desire, or identify subjective states with quantum-gravitational degrees of freedom. Instead, it examines whether several mathematical ideas developed in background-independent approaches to quantum geometry can clarify a more general formal problem: how a network of relational events and their compositions might generate an effective geometry of possible trajectories.

The distinction is important. Earlier sections considered

$$(\mathcal{M}_{\text{red}}, g_{\mu\nu})$$

as an effective geometric state space and allowed

$$g_{\mu\nu} = g_{\mu\nu}(q, \Phi, h)$$

to vary with configuration and history.

The present section asks whether a stronger relation might be considered:

relational events and compositions $\longrightarrow$ effective relational geometry.

Spin foams and group field theories provide established examples in quantum gravity in which combinatorial and algebraic structures participate in the description or generation of quantum geometry [6, 1]. Their physical interpretation belongs to quantum gravity. The present paper uses selected structural features only to formulate a possible future architecture for relational subjective geometry.

11.1 Geometry Beyond a Pre-Given Manifold

The dynamical geometry developed in Section 6 begins with a manifold

$$\mathcal{M}_{\text{red}}$$

and supplements it with a metric

$$g_{\mu\nu}.$$

Even when the metric changes according to

$$\partial_t g_{\mu\nu} = H_{\mu\nu},$$

the differentiable manifold is already available as the carrier of that geometry.

This is a substantial modeling assumption.

It assumes that the possible configurations of the relational system can be organized from the outset through a space possessing sufficient smooth structure for coordinates, tangent vectors, metrics, and differential equations to be defined.

For many effective models, this assumption is entirely appropriate.

A reduced state manifold may provide the most economical representation of the phenomena under analysis.

A different problem arises if the smooth manifold is itself interpreted as an effective large-scale structure.

One may then begin from a more primitive object

$$\mathfrak{R}$$

containing relational events, local compositions, incidence relations, and other discrete or algebraic data.

The effective manifold would arise through a reconstruction or coarse-graining map,

$$\mathcal{E}_\ell : \mathfrak{R} \longrightarrow (\mathcal{M}_\ell, g^{(\ell)}),$$

where ℓ specifies the descriptive scale.

The direction of explanation becomes

$$\mathfrak{R} \longrightarrow \mathcal{M}_\ell \longrightarrow g^{(\ell)} \longrightarrow \text{effective trajectories.}$$

This differs from assigning geometry to an already specified state space.

The primitive relational structure does not need to possess a metric in the same form as the resulting effective manifold.

Likewise, notions such as

$$\text{distance, curvature, dimension,}$$

may acquire their effective meanings only after an appropriate reconstruction has been defined.

This possibility should be distinguished from the ordinary state-dependent metric

$$g_{\mu\nu}(q).$$

In the latter case, geometry varies over a manifold.

In the emergent construction, the manifold-level geometry itself belongs to the effective description.

The distinction may be expressed schematically as

$$\text{dynamical geometry: } \mathcal{M} \rightarrow g(t),$$

and

$$\text{emergent geometry: } \mathfrak{R} \rightarrow (\mathcal{M}, g).$$

A complete theory could contain both:

$$\mathfrak{R}_t \longrightarrow (\mathcal{M}_t, g_t),$$

with the underlying relational structure and its effective geometry evolving together.

The present paper does not provide such a theory.

Its purpose is to identify the mathematical boundary beyond which the configuration-space formulation of Paper II would have to be extended.

11.2 Relational Events and Complexes

A convenient intermediate representation is a combinatorial complex.

In the spin-foam approach to quantum gravity, histories are represented using two-complexes containing vertices, edges, and faces, supplemented by group-theoretic labels and amplitudes [6].

The present construction borrows only the combinatorial architecture.

Define an abstract relational two-complex

$$\mathcal{C}_R = (V, E, F, \partial),$$

where

$$V$$

is a set of vertices,

$$E$$

a set of edges,

$$F$$

a set of faces, and

$$\partial$$

specifies the relevant incidence and boundary relations.

The mathematical object is combinatorial before psychoanalytic meaning is assigned to it.

A provisional relational interpretation may associate vertices with localized relational events,

$$v \in V \mapsto \text{relational event},$$

edges with admissible continuations or relations between events,

$$e \in E \mapsto \text{relational continuation},$$

and faces with extended consistency structures formed through sequences of such relations,

$$f \in F \mapsto \text{extended relational history or constraint}.$$

These assignments belong to the present exploratory reconstruction.

They are not the meanings of vertices, edges, and faces in spin-foam quantum gravity.

The complete complex can then encode a structured relational history,

$$\mathcal{C}_R \mapsto \text{relational history structure}.$$

This representation differs from the continuous history

$$\gamma : [t_i, t_f] \rightarrow \mathcal{M}_R$$

developed in Section 9.

The latter assumes that the configuration space already exists.

The complex representation can be considered prior to that assumption.

Instead of writing

$$\gamma(t) \in \mathcal{M}_R,$$

one begins with a collection of relational events and their compositions.

A smooth trajectory may arise only after coarse-graining.
The relationship can therefore be represented as

$$\mathcal{C}_R \xrightarrow{\mathcal{E}_\ell} \gamma_\ell \subset \mathcal{M}_\ell.$$

Different complexes could conceivably generate similar effective trajectories:

$$\mathcal{C}_R^{(1)} \neq \mathcal{C}_R^{(2)}$$

while

$$\mathcal{E}_\ell \left(\mathcal{C}_R^{(1)} \right) \approx \mathcal{E}_\ell \left(\mathcal{C}_R^{(2)} \right).$$

This would constitute a further level of representational underdetermination.
The hierarchy would then become

relational event complex $\longrightarrow$ continuous effective trajectory $\longrightarrow$ fine Lacanian itinerary $\longrightarrow$ RSI trajectory.

Each arrow removes information.

The construction therefore extends the multi-resolution architecture developed throughout the paper.

11.3 Spin-Foam-Inspired Representation

A spin foam in its established quantum-gravitational setting consists of a labelled two-complex. In commonly used formulations, representation labels are associated with faces and intertwiner data with edges, while amplitudes are assembled from contributions associated with faces, edges, and vertices [6].

Schematically, a fixed two-complex $\mathcal{C}$ supports an amplitude of the form

$$Z_{\mathcal{C}} = \sum_{\{\rho_f, \iota_e\}} \prod_f A_f \prod_e A_e \prod_v A_v,$$

where

$$\rho_f$$

denotes representation data on faces and

$$\iota_e$$

intertwiner data on edges.

This equation belongs to the established spin-foam architecture.

The relational model should initially use a weaker structure.

Let

$$\Lambda_R = \{\lambda_f, \kappa_e, \xi_v\}$$

denote abstract labels associated with the elements of $\mathcal{C}_R$.

A labelled relational complex is

$$(\mathcal{C}_R, \Lambda_R).$$

Its weight can be represented generally as

$$W_R [\mathcal{C}_R, \Lambda_R].$$

For a fixed combinatorial complex, one may define

$$\mathcal{Z}_R(\mathcal{C}_R) = \sum_{\Lambda_R} W_R[\mathcal{C}_R, \Lambda_R],$$

provided that the label space and summation rule have been specified.
The notation is deliberately different from assigning spins

$$j_f$$

to subjective relations.

There is presently no justified identification

$$\text{desire} \longleftrightarrow j,$$

$$\text{subject} \longleftrightarrow f,$$

or

$$\text{Other} \longleftrightarrow \iota.$$

Such assignments would confuse mathematical labels with psychoanalytic concepts.

The relational labels should instead encode whatever local degrees of freedom the future model can define independently.

Examples might include

relational compatibility,

orientation,

transition type,

local constraint class,

or another precisely defined variable.

These examples are possible modeling roles rather than claims about the correct relational ontology.

The total relational history can then depend on both

combinatorics

and

labels.

Two complexes with identical combinatorial structure can therefore remain different when their labels differ:

$$\mathcal{C}_R^{(1)} = \mathcal{C}_R^{(2)}$$

while

$$\Lambda_R^{(1)} \neq \Lambda_R^{(2)}.$$

Conversely, two differently organized complexes might yield similar effective geometry after coarse-graining.

The exploratory state-sum structure therefore permits a distinction among

event organization,

local relational degrees of freedom,

and

effective geometry.

11.4 Labels and Relational Degrees of Freedom

The use of labels becomes substantially stronger when the model possesses an actual symmetry group.

Suppose a group

$$G$$

has been independently justified as acting on the relevant relational variables.

Faces may then, in a spin-foam-inspired specialization, carry irreducible representation labels

$$\rho_f$$

of G .

Each representation is associated with a representation space

$$V_{\rho_f}.$$

Edges meeting several labelled faces may carry invariant maps or intertwiners. Schematically,

$$\iota_e \in \text{Inv} \left(\bigotimes_{f \supset e} V_{\rho_f}^{\epsilon_{ef}} \right),$$

where

$$\epsilon_{ef}$$

encodes the orientation or dualization convention selected by the model.

This structure is meaningful only after G , its action, the relevant representations, and the interpretation of the labels have been defined.

The gauge discussion of Section 7 therefore becomes a prerequisite rather than an optional metaphor.

One cannot move directly from

several relational descriptions

to

representation labels.

The intermediate mathematical structure is required:

defined transformation group $\longrightarrow$ representations $\longrightarrow$ invariant composition.

This point is particularly important because familiar spin-foam notation can create an illusion of explanatory depth.

Writing

$$j_f$$

does not explain what relational property is being represented.
Likewise, selecting

$$SU(2)$$

does not acquire psychoanalytic significance simply because $SU(2)$ is important in loop quantum gravity.

A relational use of representation theory would require a demonstrated symmetry of the relational model itself.

Until that step is completed, the abstract labels

$$\lambda_f, \quad \kappa_e, \quad \xi_v$$

are epistemically preferable.

They preserve the combinatorial architecture without claiming an unsupported group-theoretic ontology.

If a justified group structure is eventually found, the abstract labels can be refined:

$$\lambda_f \longrightarrow \rho_f,$$

$$\kappa_e \longrightarrow \iota_e.$$

This gives a clear criterion for increasing mathematical specificity.

The richer formalism is introduced only when it preserves an independently identified relational invariance.

11.5 From Local Events to Effective Geometry

The central problem of an emergent relational geometry is the construction of a map from discrete relational organization to a smooth or approximately smooth effective description.

Let

$$(\mathcal{C}_R, \Lambda_R)$$

be a labelled relational complex.

At scale ℓ , define schematically

$$\mathcal{E}_\ell : (\mathcal{C}_R, \Lambda_R) \longrightarrow (\mathcal{M}_\ell, g^{(\ell)}, F_\ell).$$

The map produces three conceptually distinct effective structures:

$$\mathcal{M}_\ell$$

as an effective state manifold,

$$g^{(\ell)}$$

as an effective geometry, and

$$F_\ell$$

as an effective dynamical law.

A successful emergence construction would need to explain each of them.

The first problem concerns dimensionality.

A combinatorial complex does not automatically determine the dimension of an effective manifold.

The reconstruction must establish why some number

$$d_{\text{eff}}$$

of effective coordinates becomes sufficient at the selected scale.

The second problem concerns locality.

The discrete relation

$$e = (v_i, v_j)$$

defines combinatorial adjacency.

Metric locality requires substantially more:

$$d_\ell(q_i, q_j).$$

A derivation is needed to explain how adjacency and label information produce effective distance.

The third problem concerns metric structure.

The reconstruction should explain how

$$g_{\mu\nu}^{(\ell)}$$

is inferred from the underlying relational data.

One may write schematically

$$g_{\mu\nu}^{(\ell)} = \mathcal{G}_{\mu\nu}^{(\ell)} [\mathcal{C}_R, \Lambda_R],$$

but this notation merely states the problem.

The functional

$$\mathcal{G}^{(\ell)}$$

must be constructed before the equation has explanatory content.

The fourth problem concerns dynamics.

An event complex encodes relational organization.

A dynamical model additionally requires a rule governing how complexes are generated, modified, or weighted.

One possibility is

$$\mathcal{C}_R^{(n)} \longrightarrow \mathcal{C}_R^{(n+1)}.$$

Another is an ensemble of complete complexes,

$$\{\mathcal{C}_R\},$$

with weights

$$W_R[\mathcal{C}_R].$$

These correspond to different notions of relational history.

The fifth problem concerns scale.

If geometry is emergent, one expects

$$\mathcal{E}_{\ell_1} \neq \mathcal{E}_{\ell_2}$$

in general.

Consequently,

$$g^{(\ell_1)} \neq g^{(\ell_2)}.$$

This connects the present section directly to the coarse-graining framework of Section 10.

The continuum geometry can then be treated as a scale-dependent effective description rather than a primitive background.

The desired chain is

$$(\mathcal{C}_R, \Lambda_R) \longrightarrow \text{coarse relational variables} \longrightarrow (\mathcal{M}_\ell, g^{(\ell)}) \longrightarrow \gamma_\ell.$$

A trajectory

$$\gamma_\ell$$

would therefore appear only after the effective geometry has emerged.

This reverses the logical order of the earlier field-dynamical model, where trajectories were defined on a configuration space from the beginning.

The two models can coexist at different descriptive levels:

event-complex description

at the finer level and

field-geometric description

at the effective level.

This relation would provide a deeper realization of the scale-dependent architecture developed in Section 10.

11.6 Group-Field-Theoretic Perspective

Group field theory provides a particularly relevant structural example because the combinatorial complexes need not be inserted only as fixed background structures. In established GFT constructions, the basic fields are defined on products of group manifolds, while their perturbative Feynman expansions generate combinatorial structures whose amplitudes reproduce spin-foam amplitudes in the relevant models [1].

Schematically, a GFT field may take the form

$$\varphi : G^d \longrightarrow \mathbb{C}.$$

The variable

$$(g_1, \dots, g_d) \in G^d$$

contains group-theoretic data associated with the elementary building block represented by the field. A general GFT-like action can be written schematically as

$$S[\varphi, \bar{\varphi}] = S_{\text{kin}}[\varphi, \bar{\varphi}] + S_{\text{int}}[\varphi, \bar{\varphi}].$$

The interaction term is designed so that elementary building blocks are composed according to a specified combinatorial pattern.

The perturbative expansion then has a schematic form

$$Z = \sum_{\Gamma} \frac{1}{\text{sym}(\Gamma)} \lambda^{N(\Gamma)} \mathcal{A}_{\Gamma},$$

where

$$\Gamma$$

labels generated Feynman diagrams or complexes,

$$N(\Gamma)$$

counts interaction events according to the selected model, and

$$\mathcal{A}_\Gamma$$

is the corresponding amplitude.

The exact construction is model-dependent [1].

The structural feature relevant to the present paper is the sequence

field degrees of freedom $\longrightarrow$ interaction $\longrightarrow$ combinatorial complex $\longrightarrow$ effective geometry.

This suggests a possible extension of relational field dynamics.

Paper II currently uses

$$\Phi(\xi, t)$$

as a field defined over an already selected relational domain.

A later theory might instead seek an elementary relational field

$$\varphi_R$$

whose interaction structure generates the relational complexes on which an effective geometry is reconstructed.

Schematically,

$$S_R[\varphi_R] \longrightarrow \{\mathcal{C}_R\} \longrightarrow (\mathcal{M}_\ell, g^{(\ell)}).$$

This construction is substantially stronger than the relational field theory developed earlier in the paper.

The earlier field

$$\Phi$$

lives on a specified domain.

The hypothetical field

$$\varphi_R$$

would participate in generating the combinatorial structures from which the effective domain and geometry arise.

For this reason, the present paper does not define

$$\varphi_R : G^d \rightarrow \mathbb{C}$$

as a psychoanalytic field.

Such a definition would require answers to several prior questions:

Which group G is justified?

What relational degrees of freedom do its elements encode?

What is the elementary relational building block?

What interaction glues those blocks?

Which observables survive coarse-graining?

How does a smooth effective relational geometry emerge?

Until these questions are answered, group field theory remains a source of formal architecture rather than a completed model of subjective dynamics.

11.7 The Spacetime of Desire

The phrase *spacetime of desire* can now be given a restricted formal meaning.

Within Lacanian psychoanalysis, desire is embedded within a structured relation among subject, Other, demand, signification, fantasy, and related positions rather than functioning as an isolated state variable [2, 3, 4].

The present paper has progressively represented this relational organization through several levels:

Lacanian structural positions,

symbolic trajectories,

continuous relational fields,

dynamical state geometry,

and finally

candidate emergent relational geometry.

At the effective level, define

$$\mathfrak{S}_D^{(\ell)} = \left(\mathcal{M}_\ell, g^{(\ell)}, F_\ell, \mathcal{O}_L \right).$$

Here,

$$\mathcal{M}_\ell$$

specifies the effective space of relational configurations,

$$g^{(\ell)}$$

their effective geometry,

$$F_\ell$$

the dynamics governing possible trajectories, and

$$\mathcal{O}_L$$

the map through which those trajectories acquire a selected Lacanian description.
The notation

$$\mathfrak{S}_D^{(\ell)}$$

will be called the *effective spacetime of desire* at scale ℓ .
The word *spacetime* is used here in a formal relational sense.
It does not identify the structure with physical spacetime.
The effective object combines

configuration,

geometry,

possible evolution,

and

historical accessibility.

Its purpose is to express that desire is modeled through trajectories whose possibilities depend upon a relationally structured environment.

The stronger emergent hypothesis can then be written

$$(\mathcal{C}_R, \Lambda_R) \xrightarrow{\mathcal{E}_\ell} \mathfrak{S}_D^{(\ell)}.$$

Relational events and their organization would generate, at the selected scale, the effective spacetime within which subjective trajectories become representable.

Under this formulation, history performs two roles.

A trajectory evolves within the effective geometry:

$$\gamma_\ell \subset \mathfrak{S}_D^{(\ell)}.$$

At a finer level, relational history may also contribute to the generation of that geometry:

$$\text{history} \longrightarrow \text{effective geometry} \longrightarrow \text{later possible history}.$$

This gives a recursive architecture,

$$\mathfrak{R}_t \longrightarrow g_t \longrightarrow \gamma_{t:t+\Delta t} \longrightarrow \mathfrak{R}_{t+\Delta t},$$

in which generated history modifies the conditions of subsequent generation.

The intuition is stronger than ordinary motion through a fixed phase space.

A phase-space trajectory assumes a space of possibilities and determines a path through it.

The emergent relational proposal allows prior relational generation to participate in determining the later space of effective possibilities.

This idea can be stated without invoking quantum ontology:

generation of trajectory

and

generation of effective trajectory space

become coupled problems.

Spin foams and group field theories are relevant because they provide established mathematical examples in which histories, combinatorial structures, algebraic labels, and geometry are related in ways that differ from ordinary fields propagating on a fixed background [6, 1].

Their use in the present framework remains analogical and architectural.

A future theory would need to derive its own relational group, elementary variables, amplitudes or weights, coarse-graining map, and continuum geometry.

Accordingly, the present section establishes no equation of the form

$$\text{Lacanian desire} = \text{quantum geometry}.$$

It establishes a narrower research direction:

$$\text{relational generation} \longrightarrow \text{combinatorial history} \longrightarrow \text{effective subjective geometry}.$$

The architecture also clarifies the division between the present paper and a subsequent study.

Paper II develops

continuous relational fields

and

dynamical geometry.

A later paper can investigate

generated relational complexes,

spin-foam-inspired sums over labelled histories,

group-field-theoretic generation,

and

emergence of effective relational geometry.

The phrase *spacetime of desire* therefore names the research horizon rather than a completed physical theory.

The remainder of the present paper returns to the interpretive level. Section 12 examines how field configurations, relational forces, historical geometry, gauge-invariant structure, and subjective trajectories can be related to Lacanian psychoanalytic concepts without converting the mathematical architecture into a literal physical ontology.

12 Psychoanalytic Interpretation

The preceding sections developed a relational field framework, a dynamical geometry, gauge-theoretic structures, several dynamical regimes, path ensembles, coarse-graining, and an exploratory account of emergent geometry. The present section returns to the psychoanalytic level and specifies how these formal objects may be interpreted without identifying them directly with Lacanian concepts. Its role is therefore semantic and methodological. The section first relates field configurations to Lacanian positions, then considers desire through the weaker language of relationally structured dynamical tendency. It subsequently examines retroaction and history-dependent geometry, gauge-invariant structure, subjective trajectories within relational spacetime, and finally the boundary between formal representation and psychoanalytic meaning.

Lacan's Graph of Desire organizes relations among the divided subject, the Other, signification, demand, desire, fantasy, and associated mathemes [2, 3, 4]. The mathematical objects developed in this paper are not additional Lacanian concepts. They form an interpretive architecture for representing selected structural and historical distinctions already motivated by the psychoanalytic source material.

The interpretive direction is therefore

$$\text{formal configuration} \longrightarrow \text{psychoanalytic observation} \longrightarrow \text{interpretation},$$

rather than

$$\text{formal configuration} = \text{psychoanalytic object}.$$

The distinction remains central throughout the section.

12.1 Field Configuration and Lacanian Position

The relational field

$$\Phi_t$$

contains the variables retained by the continuous model at time t .

A Lacanian position is obtained through the observation map

$$\mathcal{O}_L : \mathcal{M}_R \longrightarrow \mathcal{Y}_L.$$

Thus,

$$o_t = \mathcal{O}_L(\Phi_t).$$

The interpretation of o_t depends upon the source-constrained vocabulary selected from the Lacanian framework.

The field configuration itself should remain analytically prior to that interpretation.

This prevents statements of the form

$$\Phi^1 = \text{desire}$$

or

$$\Phi^2 = \text{the Symbolic}$$

from being introduced without argument.

A more defensible construction treats Lacanian descriptions as observables of a richer relational configuration.

Several states can therefore satisfy

$$\mathcal{O}_L(\Phi_1) = \mathcal{O}_L(\Phi_2)$$

while

$$\Phi_1 \neq \Phi_2.$$

Psychoanalytically, this permits two historically distinct relational configurations to receive the same structural description at the selected resolution.

The same principle applies to the RSI projection:

$$\Phi_t \xrightarrow{\mathcal{O}_L} o_t \xrightarrow{\Pi_{\text{RSI}}} a_t.$$

An RSI designation therefore constitutes a coarse observation of a configuration.
It should not be treated as a complete state variable.
For example,

$$a(t_1) = a(t_2) = I$$

indicates equality at the chosen register-level observation.
It does not establish

$$o(t_1) = o(t_2),$$

and still less

$$\Phi_{t_1} = \Phi_{t_2}.$$

This distinction gives the continuous model an interpretive advantage over a purely symbolic trajectory.

The same Lacanian position can be embedded in different neighborhoods of possible change.
Two configurations can therefore receive the same psychoanalytic label while having different

local stability,

future accessibility,

historical dependence,

or

effective relational geometry.

The structural position remains meaningful, but it does not exhaust the dynamical state.

12.2 Desire and Relational Dynamical Tendency

The language of dynamics naturally introduces terms such as force, potential, direction, and attraction. Their psychoanalytic interpretation requires particular caution.

Desire in Lacanian psychoanalysis is embedded within relations among demand, the Other, signification, lack, fantasy, and the divided subject [2, 3, 4]. It should therefore not be reduced to a scalar mechanical force acting upon an otherwise self-contained subject.

The present framework uses the weaker notion of a *relational dynamical tendency*.

Let

$$\dot{q}^\mu = F^\mu(q, h, u)$$

describe the effective dynamics.

The vector

$$F(q, h, u)$$

specifies the local direction and rate of modeled change.

Its psychoanalytic interpretation can include the organization of desire only when the variables and observation map justify such a reading.

Thus,

$$F$$

is not identified with desire itself.

It represents the formal dynamical tendency generated by the relational configuration retained by the model.

A useful interpretive chain is

relational configuration $\longrightarrow$ dynamical tendency $\longrightarrow$ trajectory $\longrightarrow$ Lacanian interpretation.

This permits desire to influence the interpretation of a trajectory without requiring a one-to-one mathematical variable called “desire.”

The same caution applies to a potential

$$V(\Phi).$$

A minimum of V does not mean that desire seeks a minimum.

Likewise, an attractor does not mean that the subject consciously desires the attracting state.

An attractor is a property of the formal dynamics.

A potential minimum is a property of a selected effective function.

Desire is a psychoanalytic concept whose relation to either object must be argued separately.

The phrase *relational force* may therefore be used only in a restricted formal sense.

If the equation of motion contains

$$\frac{D^2 q^\mu}{D\tau^2} = f^\mu,$$

then

$$f^\mu$$

is a force-like term within that mathematical model.

Its psychoanalytic interpretation may concern demand, external perturbation, fantasy, symbolic constraint, or another relational contribution only after the corresponding semantic map has been specified.

The methodological advantage of this weaker formulation is that it preserves Lacan’s relational organization while still allowing continuous dynamics to be analyzed.

Desire need not become a Newtonian force for dynamical geometry to be useful.

12.3 Retroaction and History-Dependent Geometry

Retroaction provides one of the strongest points of contact between Lacanian interpretation and the history-dependent structures introduced earlier.

The relevant distinction is between

past event,

present state,

and

present interpretation of the past.

Let an earlier relational state be

$$\Phi_\tau, \quad \tau < t.$$

At time t , its interpretation is represented by

$$\nu_t(\Phi_\tau).$$

Later developments can produce

$$\nu_{t+\Delta t}(\Phi_\tau) \neq \nu_t(\Phi_\tau).$$

The earlier event has not been dynamically replaced.

Its relational meaning within the current history has changed.

This permits retroaction to be represented without reversing the direction of physical or formal time.

The distinction is

$$\Phi_\tau \text{ fixed as an earlier event,}$$

while

$$\nu_t(\Phi_\tau)$$

remains historically revisable.

The dynamical geometry introduces a second form of historical dependence.

Suppose

$$g_{\mu\nu} = g_{\mu\nu}(q, h_t).$$

Then prior history changes the effective geometry of current possibilities.

The consequence is stronger than semantic reinterpretation alone.

Two states satisfying

$$q(t_1) = q(t_2)$$

may possess different effective local geometry:

$$g_{\mu\nu}(q, h_{t_1}) \neq g_{\mu\nu}(q, h_{t_2}).$$

Their later trajectories can therefore differ even though the current reduced coordinates coincide.

The paper should keep these two processes distinct:

retroaction

concerns the changing interpretation of earlier events, while

history-dependent geometry

concerns the changing structure of current and future accessibility.

They can nevertheless interact.

A later event may revise the meaning of an earlier episode,

$$\nu_t \rightarrow \nu_{t+\Delta t},$$

and this reinterpretation may become dynamically relevant if the model allows the updated historical state to modify

F

or

g .

Schematically,

$$\nu_t \longrightarrow h_t \longrightarrow g_t \longrightarrow F_t \longrightarrow \gamma_{t:t+\Delta t}.$$

This equation represents a candidate architecture.

It does not claim that interpretation literally produces metric curvature.

Its purpose is to show how a revised historical organization could enter a formal model of future possibility.

A particularly useful distinction follows.

History can affect the present through at least three separate channels:

semantic valuation,

dynamical state,

and

effective geometry.

These mechanisms may coincide empirically in some cases, but they remain formally distinct hypotheses.

12.4 Symbolic Observation and Gauge-Invariant Structure

The gauge-theoretic construction developed in Section 7 raises an interpretive question: which features of a psychoanalytic description belong to the relational structure being modeled, and which depend upon the particular formal representative?

Suppose

$$r \longmapsto g \cdot r, \quad g \in G,$$

is a justified gauge transformation.

If the Lacanian observation satisfies

$$\mathcal{O}_L(g \cdot r) = \mathcal{O}_L(r),$$

then the observation depends only upon the gauge-equivalence class.

In that case one may write

$$\mathcal{O}_L([r]_G)$$

without reference to a particular representative.

This would provide a strong form of representational invariance.

The paper does not assume that every Lacanian concept is gauge invariant.

A weaker case is

$$\Pi_{\text{RSI}}(\mathcal{O}_L(g \cdot r)) = \Pi_{\text{RSI}}(\mathcal{O}_L(r)).$$

Here only the coarse RSI classification survives the transformation.

Fine psychoanalytic distinctions may still change.

This distinction is important for interpretation.

A quantity may be invariant at one descriptive scale and representation dependent at another.

Accordingly,

invariance

should always be indexed by

transformation group

and

observational resolution.

The psychoanalytic significance of gauge invariance would lie in identifying relations that survive a defined family of representational changes.

Such invariance might become useful when comparing different symbolic descriptions, coordinate systems, or formal encodings of one relational organization.

The conclusion must remain modest.

The existence of several interpretations does not itself establish a gauge symmetry.

Nor does cross-cultural or cross-linguistic similarity establish one.

A genuine gauge interpretation requires

$$G,$$

its action,

$$r \mapsto g \cdot r,$$

and a set of invariant observables.

Only after these structures are defined can the quotient

$$\mathcal{R}/G$$

be treated as a representation-independent relational space.

This makes gauge structure potentially useful for psychoanalytic formalization while preventing “gauge” from becoming a synonym for perspective dependence.

12.5 Subjective Trajectory and Relational Spacetime

The phrase *subjective trajectory* refers in this paper to the modeled history of a relational configuration:

$$\gamma : t \mapsto q(t)$$

or, at the field level,

$$\gamma : t \mapsto \Phi_t.$$

The trajectory receives psychoanalytic meaning through observation and interpretation.

It should not be understood as a literal path of a psychological object moving through physical space.

The dynamical geometry permits a more specific formulation.

Let

$$\mathfrak{S}_D = (\mathcal{M}_{\text{red}}, g, F, \mathcal{O}_L)$$

denote the effective relational spacetime introduced in Section 11.7.

A subjective trajectory is then a path

$$\gamma \subset \mathfrak{S}_D.$$

Its possible evolution depends upon

current relational configuration,

dynamical law,

effective geometry,

history,

and

external conditions.

The resulting architecture can be written

$$(\Phi_t, h_t, u_t) \longrightarrow g_t \longrightarrow F_t \longrightarrow \gamma \longrightarrow \mathcal{O}_L(\gamma).$$

This expresses the strongest continuous claim of Paper II.

A subjective trajectory can be represented as historically situated motion through an effective relational geometry whose local structure may itself depend upon the state and its history.

The phrase *spacetime of desire* names this relational organization.

It should therefore be understood as

the effective geometry of relationally situated subjective possibility,

rather than as physical spacetime occupied by desire.

The geometry provides formal meanings for distinctions such as

near and distant configurations,

locally accessible directions,

constrained regions,

alternative paths,

and

history-dependent return.

These notions can enrich psychoanalytic trajectory analysis without claiming that a clinician directly observes a manifold or metric.

The geometry belongs to the model.

Its psychoanalytic value depends upon whether it preserves distinctions that would otherwise be lost.

The concept also clarifies subjective individuality.

Two subjects may receive similar symbolic descriptions while inhabiting different effective relational geometries:

$$\mathcal{O}_L(q_A) = \mathcal{O}_L(q_B)$$

while

$$g_A \neq g_B.$$

Even for the same subject,

$$\mathcal{O}_L(q_{t_1}) = \mathcal{O}_L(q_{t_2})$$

can coexist with

$$g_{t_1} \neq g_{t_2}.$$

A psychoanalytic category therefore need not determine one universal geometry of continuation. This is one of the principal interpretive consequences of the relational field model.

12.6 Formal Representation and Psychoanalytic Meaning

The final subsection establishes the epistemic boundary of the paper.

The mathematical model contains objects such as

$$\Phi, \quad F, \quad g, \quad A, \quad W[\gamma],$$

together with dynamical concepts such as

attractor,

bifurcation,

criticality,

hysteresis,

and

holonomy.

These objects possess mathematical definitions independently of psychoanalysis.

Lacanian psychoanalysis contains a different vocabulary, including the divided subject, the Other, demand, desire, fantasy, signification, and the registers of the Real, Symbolic, and Imaginary [2, 3, 4].

The task of formalization is to define relations between these vocabularies.

A valid interpretation therefore requires at least three steps.

First, the mathematical object must be defined independently:

$$m \in \mathfrak{M}.$$

Second, the relevant psychoanalytic distinction must be established from the source framework:

$$p \in \mathfrak{P}_L.$$

Third, an interpretation rule must specify why

$$m$$

is useful for representing some aspect of

$$p.$$

This relation can be written

$$\mathcal{I}_P : \mathfrak{M} \longrightarrow \mathfrak{P}_L,$$

with the understanding that $\mathcal{I}_P$ may be partial and many-to-one.
The existence of

$$\mathcal{I}_P(m) = p$$

does not imply

$$m = p.$$

Likewise, successful formal representation does not establish that the subject implements the mathematical mechanism described by m .

The paper therefore distinguishes four layers:

psychoanalytic phenomenon,

Lacanian conceptual description,

formal mathematical representation,

and

causal mechanism.

Relations among these layers require separate arguments.

This framework also permits several mathematical representations of the same psychoanalytic distinction.

For a given psychoanalytic object p , one may have

$$\mathcal{I}_P(m_1) = \mathcal{I}_P(m_2) = p$$

while

$$m_1 \neq m_2.$$

For example, a recurrent psychoanalytic pattern might be represented through

a symbolic grammar,

an attractor,

a metastable field configuration,

or

a history-dependent effective model,

depending upon which aspect of the phenomenon is under study.

These models are not automatically equivalent.

Their adequacy should be assessed according to the distinctions they preserve and the predictions or explanations they support.

This motivates a principle of interpretive economy:

A mathematical structure should receive psychoanalytic interpretation only to the extent required by the distinctions it successfully represents. Additional physical or ontological meaning should not be inferred from formal resemblance alone.

The principle applies especially to the more elaborate constructions of the paper.
 A gauge orbit does not automatically represent the unconscious.
 A curvature tensor does not automatically represent psychic constraint.
 A path integral does not imply quantum desire.
 A spin foam does not represent the subject.
 A group field theory does not become a psychoanalytic theory merely by renaming its variables.
 Their value lies in the formal distinctions they make available.
 The principal interpretive contribution of Paper II can therefore be stated more narrowly.
 Lacanian subjective trajectories may be represented through a multi-resolution dynamical architecture in which

structural observation,

 continuous relational state,

 historical dependence,

 effective geometry,

 and

 alternative histories

remain analytically distinguishable.

The framework thereby permits the same observed Lacanian position to occur under different dynamical and geometric conditions.

It also permits historical transformation to affect later accessibility without requiring symbolic position alone to contain the complete state.

The resulting relation can be summarized as

$$\boxed{\text{Lacanian structure} \longrightarrow \text{relational observation} \longleftarrow \text{field dynamics and geometry}}$$

rather than a direct identification between psychoanalysis and mathematical physics.

The next section makes this pluralism explicit by comparing the continuous field model developed here with the discrete generative architecture of the preceding paper. It examines discretization, continuum reconstruction, shared observables, model non-equivalence, and the conditions under which the two formalisms can be treated as complementary resolutions of the same psychoanalytic problem.

13 Relation to the Generative-Grammar Model

This section relates the continuous field-geometric framework developed in the present paper to the discrete generative architecture developed in the preceding study. Its objective is to identify the mathematical interfaces between discrete derivations and continuous trajectories while preserving the different information carried by the two representations. The analysis proceeds through six steps. It first distinguishes discrete and continuous trajectory descriptions, then examines discretization of field histories and continuum representations compatible with symbolic histories. It subsequently defines the shared observation layer, establishes the non-equivalence of the two model classes, and concludes by characterizing them as complementary formal resolutions.

The comparison begins from two different kinds of history.

A discrete generative model produces a derivation

$$d : C_0 \Rightarrow C_1 \Rightarrow \cdots \Rightarrow C_n$$

and a corresponding observable itinerary

$$\gamma_L^{(d)} = o_0 o_1 \cdots o_n.$$

The continuous model instead represents a history as

$$\gamma_C : [t_i, t_f] \longrightarrow \mathcal{M}_R$$

or, after dynamical reduction,

$$\gamma_{\text{red}} : [t_i, t_f] \longrightarrow \mathcal{M}_{\text{red}}.$$

The two descriptions can be related through observation, sampling, and coarse-graining. They need not share the same state variables, transition structure, or notion of history.

Their common point of comparison is therefore the observable layer rather than an assumed identity of generators.

13.1 Discrete and Continuous Trajectories

The discrete model organizes psychoanalytic histories through configurations, production rules, derivational alternatives, and symbolic observations.

Let

$$\mathcal{C}_D$$

denote its configuration space and let

$$\Rightarrow$$

denote the derivation relation.

A finite derivation has the form

$$C_0 \Rightarrow C_1 \Rightarrow \cdots \Rightarrow C_n.$$

An observation or readout map

$$\rho_D : \mathcal{C}_D \longrightarrow \mathcal{Y}_L$$

produces

$$o_k = \rho_D(C_k).$$

The resulting itinerary is

$$o_0 o_1 \cdots o_n.$$

The continuous model uses a different state architecture.

A field history is

$$\gamma_C(t) = \Phi_t \in \mathcal{M}_R,$$

with Lacanian observation

$$o(t) = \mathcal{O}_L(\Phi_t).$$

The distinction between the two models can therefore be represented as

$$\mathcal{C}_D \xrightarrow{\rho_D} \mathcal{Y}_L$$

and

$$\mathcal{M}_R \xrightarrow{\mathcal{O}_L} \mathcal{Y}_L.$$

The shared codomain

$$\mathcal{Y}_L$$

allows their outputs to be compared.

It does not require

$$\mathcal{C}_D = \mathcal{M}_R.$$

The discrete configuration $\mathcal{C}_k$ may contain derivational information whose continuous analogue is distributed across a segment of field history.

Conversely, the continuous state Φ_t may contain local variation, velocity, geometric neighborhood, or unresolved degrees of freedom absent from the discrete configuration.

The relation is therefore mediated by observation:

$$\begin{array}{ccc} \mathcal{C}_D & \xrightarrow{\rho_D} & \mathcal{Y}_L \\ & & \uparrow \\ \mathcal{M}_R & \xrightarrow{\mathcal{O}_L} & \mathcal{Y}_L. \end{array}$$

Two models can agree at the level of

$$o_0 o_1 \cdots o_n$$

while describing different internal processes.

This observation supplies the basic principle for the remainder of the section:

Agreement of observable psychoanalytic trajectories establishes representational compatibility at the selected resolution. It does not establish equivalence of the underlying generative or dynamical models.

13.2 Discretization of Field Histories

A continuous trajectory can generate a discrete symbolic history through a specified discretization and observation procedure.

Let

$$\gamma_C : [t_i, t_f] \longrightarrow \mathcal{M}_R$$

be a continuous field history.

Choose sampling times

$$t_0 < t_1 < \cdots < t_n.$$

The corresponding sampled states are

$$\Phi_{t_0}, \Phi_{t_1}, \dots, \Phi_{t_n}.$$

Applying the Lacanian observation map gives

$$o_k = \mathcal{O}_L(\Phi_{t_k}).$$

The discretized observable trajectory is therefore

$$\mathcal{D}_{\mathbf{t}}[\gamma_C] = o_0 o_1 \cdots o_n,$$

where

$$\mathbf{t} = (t_0, \dots, t_n)$$

denotes the sampling protocol.

The dependence upon $\mathbf{t}$ is essential.

A different sampling scheme may produce a different discrete itinerary from the same continuous trajectory.

For example, suppose the continuous history passes through three observational regions,

$$A \rightarrow B \rightarrow C,$$

but the sampling times occur only before and after the intermediate passage.

The resulting symbolic trajectory can be

$$A \rightarrow C.$$

The discrete representation has then omitted the intermediate observation B .

Discretization therefore depends upon temporal resolution.

Observation adds a second dependence.

Let

$$\mathcal{O}_L^{(1)}$$

and

$$\mathcal{O}_L^{(2)}$$

be two observation maps with different resolutions.

The same sampled continuous states may yield

$$\mathcal{D}_{\mathbf{t}}^{(1)}[\gamma_C] \neq \mathcal{D}_{\mathbf{t}}^{(2)}[\gamma_C].$$

The construction therefore contains two distinct operations:

sampling in time

and

projection in descriptive resolution.

The RSI sequence requires an additional projection:

$$\mathcal{D}_{\mathbf{t}}^{\text{RSI}}[\gamma_C] = \Pi_{\text{RSI}}(\mathcal{D}_{\mathbf{t}}^L[\gamma_C]).$$

The full chain becomes

$$\gamma_C \longrightarrow \{\Phi_{t_k}\} \longrightarrow \{o_k\} \longrightarrow \{a_k\}.$$

Each transformation can remove information.

Continuous variation between sampling times disappears first.

Fine relational distinctions are then removed by the Lacanian observation map.

Additional distinctions disappear under RSI coarse-graining.

A discrete generative model reconstructed from the final sequence therefore cannot, in general, recover the original field history uniquely.

13.3 Continuum Representation of Symbolic Histories

The reverse direction begins from a discrete symbolic itinerary and asks which continuous trajectories are compatible with it.

Let

$$w_L = o_0 o_1 \cdots o_n$$

be a fine Lacanian itinerary.

Given observation times

$$\mathbf{t} = (t_0, \dots, t_n),$$

define the compatible continuous history set

$$\Gamma_C(w_L; \mathbf{t}) = \{\gamma \mid \mathcal{O}_L(\gamma(t_k)) = o_k \text{ for } k = 0, \dots, n\}.$$

In general,

$$|\Gamma_C(w_L; \mathbf{t})| > 1.$$

A symbolic itinerary therefore corresponds to a family of possible continuous histories.

This family can differ in several respects.

Compatible trajectories may have different lengths,

$$L[\gamma_1] \neq L[\gamma_2],$$

different velocities,

$$\dot{\gamma}_1(t) \neq \dot{\gamma}_2(t),$$

different intermediate states,

$$\gamma_1(t) \neq \gamma_2(t),$$

different histories of external forcing,

$$u_1(t) \neq u_2(t),$$

or different effective geometries,

$$g_1(t) \neq g_2(t).$$

All may nevertheless produce the same sampled symbolic itinerary.

The ambiguity becomes larger after RSI projection.

For

$$w_{\text{RSI}} = a_0 a_1 \cdots a_n,$$

define

$$\Gamma_C(w_{\text{RSI}}) = \{\gamma \mid \Pi_{\text{RSI}} \circ \mathcal{O}_L(\gamma) = w_{\text{RSI}}\},$$

under the selected observation protocol.
Then generally

$$\Gamma_C(w_L) \subseteq \Gamma_C(w_{\text{RSI}})$$

whenever

$$\Pi_{\text{RSI}}(w_L) = w_{\text{RSI}}.$$

The continuum reconstruction problem is therefore underdetermined.
A discrete word supplies constraints on the continuous trajectory rather than a unique interpolation.
The same principle applies at the derivational level.
Suppose several discrete derivations satisfy

$$d_1 \neq d_2$$

while

$$\rho_D(d_1) = \rho_D(d_2) = w_L.$$

Each derivation may correspond to a different compatible family of continuous histories:

$$d_i \mapsto \Gamma_C(d_i).$$

The relation between derivation and continuous path is therefore potentially many-to-many.
This is important for the interpretation of Paper I.

A derivational history should not automatically be treated as a sampled version of one hidden continuous trajectory.

The discrete derivation may preserve formal generative distinctions that do not possess a unique continuum counterpart.

Conversely, a continuous history may preserve geometric distinctions that have no expression in the derivational system.

13.4 Shared Observables

The strongest systematic relation between the two models lies in their shared observation spaces.
Let

$$\mathcal{Y}_L$$

denote the fine Lacanian observation space and

$$\mathcal{Y}_{\text{RSI}}$$

the coarse RSI observation space.
The discrete model contains

$$\rho_D : \mathcal{C}_D \longrightarrow \mathcal{Y}_L,$$

while the continuous model contains

$$\mathcal{O}_L : \mathcal{M}_R \longrightarrow \mathcal{Y}_L.$$

Both can subsequently use

$$\Pi_{\text{RSI}} : \mathcal{Y}_L \longrightarrow \mathcal{Y}_{\text{RSI}}.$$

The architecture can therefore be written

$$\begin{array}{ccccc} \mathcal{C}_D & \xrightarrow{\rho_D} & \mathcal{Y}_L & \xrightarrow{\Pi_{\text{RSI}}} & \mathcal{Y}_{\text{RSI}} \\ & & \uparrow & & \uparrow \\ \mathcal{M}_R & \xrightarrow{\mathcal{O}_L} & \mathcal{Y}_L & \xrightarrow{\Pi_{\text{RSI}}} & \mathcal{Y}_{\text{RSI}}. \end{array}$$

The common observation spaces permit empirical or interpretive comparison.

For example, a discrete derivation and continuous trajectory can be said to agree at the fine observational level when

$$\rho_D(C_k) = \mathcal{O}_L(\Phi_{t_k})$$

for the selected correspondence between derivational stages and observation times.

They can agree only at the RSI level when

$$\Pi_{\text{RSI}}(\rho_D(C_k)) = \Pi_{\text{RSI}}(\mathcal{O}_L(\Phi_{t_k}))$$

while their fine observations differ.

This defines several possible degrees of compatibility:

RSI compatibility,

fine-observation compatibility,

and

trajectory-level compatibility.

The last condition requires substantially more information than agreement of symbolic outputs.

Shared observables also permit model comparison without forcing a common internal ontology.

One model may represent a transition through a production rule,

$$C_i \Rightarrow C_j,$$

while another represents the corresponding observable change through a continuous segment,

$$\gamma_{ij} \subset \mathcal{M}_R.$$

Their comparison can be made through

$$\rho_D(C_i), \quad \rho_D(C_j),$$

and

$$\mathcal{O}_L(\gamma_{ij}(t_i)), \quad \mathcal{O}_L(\gamma_{ij}(t_j)).$$

This is sufficient to establish compatibility at the chosen observational resolution.

No direct identification between

$$C_i$$

and

$$\Phi_{t_i}$$

is required.

13.5 Model Non-Equivalence

The existence of a shared observation layer does not establish formal equivalence between the discrete and continuous models.

Several independent differences prevent such an inference.

The first difference concerns state structure.

A discrete configuration

$$C \in \mathcal{C}_D$$

may contain rule-state, derivational, or control information.

A continuous state

$$\Phi \in \mathcal{M}_R$$

contains continuously parameterized relational variables.

There need not exist a bijection

$$\mathcal{C}_D \leftrightarrow \mathcal{M}_R.$$

The second difference concerns time.

A derivation is ordered through discrete transitions,

$$C_k \Rightarrow C_{k+1}.$$

A continuous trajectory is parameterized through

$$t \in [t_i, t_f].$$

Mapping derivational step number to physical, phenomenological, or formal continuous time requires an additional construction.

The third difference concerns intermediate information.

The discrete transition

$$C_i \Rightarrow C_j$$

may contain no representation of the states traversed between the two configurations.

The continuous model can encode an entire segment

$$\gamma_{ij}(t), \quad t_i \leq t \leq t_j.$$

The fourth difference concerns geometry.

The continuous model can define

$$g_{\mu\nu},$$

trajectory length,

$$L[\gamma],$$

curvature,

$$R^p_{\sigma\mu\nu},$$

and, where justified, parallel transport and holonomy.

These objects have no automatic counterparts in the discrete grammar.

A discrete model can be enriched with graph metrics, transition costs, or other structures, but those additions constitute further modeling choices.

The fifth difference concerns local stability.

The continuous model can analyze

$$J(q),$$

eigenvalues,

$$\operatorname{Re} \lambda_i,$$

attractors, bifurcations, and critical regimes.

A discrete generative system can exhibit its own forms of recurrence, reachability, or long-run behavior, but the mathematical definitions differ.

The sixth difference concerns generative information.

A grammar can distinguish two derivations

$$d_1 \neq d_2$$

that produce the same terminal word.

A continuous model defined only over the terminal observations may fail to retain the derivational distinction.

The seventh difference concerns resolution.

The field model may contain infinitely many degrees of freedom, while the grammar may operate over a finite or countable symbolic configuration structure.

The two models therefore answer different formal questions.

The discrete architecture is especially suited to

admissibility,

derivation,

symbolic ordering,

and

generative alternatives.

The continuous architecture is especially suited to

local variation,

stability,

perturbation,

geometry,

critical transition,

and

history-dependent accessibility.

No general transformation has been established that preserves every relevant structure in both directions.

The paper therefore makes no equivalence claim of the form

$$\mathfrak{M}_D \cong \mathfrak{M}_C.$$

The relation is weaker:

$$\mathfrak{M}_D \sim_{\mathcal{O}} \mathfrak{M}_C$$

when the two models agree under a specified observation protocol $\mathcal{O}$.

The symbol

$$\sim_{\mathcal{O}}$$

denotes observational compatibility within the present paper. It does not denote gauge equivalence or mathematical isomorphism.

13.6 Complementary Formal Resolutions

The relation developed above suggests a multi-resolution architecture rather than a competition between discrete and continuous formalizations.

Let

$$\mathfrak{M}_D$$

denote a discrete generative model and

$$\mathfrak{M}_C$$

a continuous relational field model.

Their usefulness depends upon the distinction under analysis.

When the research problem concerns whether a symbolic transition can be generated, the discrete model may provide the more economical description.

When the problem concerns how close a state lies to loss of stability, the continuous model supplies additional structure.

When derivational history matters, the grammar may distinguish paths that a coarse continuous observation suppresses.

When local geometry or hysteresis matters, the field model can distinguish states that produce the same discrete symbol.

The two models can therefore be organized around a common observational interface:

$$\mathfrak{M}_D \xrightarrow{\rho_D} \mathcal{Y}_L \xrightarrow{\Pi_{\text{RSI}}} \mathcal{Y}_{\text{RSI}},$$

$$\mathfrak{M}_C \xrightarrow{\mathcal{O}_L} \mathcal{Y}_L \xrightarrow{\Pi_{\text{RSI}}} \mathcal{Y}_{\text{RSI}}.$$

This permits one psychoanalytic trajectory to possess several formal descriptions.

At the discrete level,

$$d = C_0 \Rightarrow \cdots \Rightarrow C_n.$$

At the continuous level,

$$\gamma : t \mapsto \Phi_t.$$

At the fine observational level,

$$w_L = o_0 o_1 \cdots o_n.$$

At the coarse level,

$$w_{\text{RSI}} = a_0 a_1 \cdots a_n.$$

The complete architecture is therefore

$$\begin{array}{ccccc} d & \longrightarrow & w_L & \longrightarrow & w_{\text{RSI}} \\ & & \uparrow & & \uparrow \\ \gamma & \longrightarrow & w_L & \longrightarrow & w_{\text{RSI}}. \end{array}$$

The two histories meet at the observation layer while retaining different internal organizations.

This structure also clarifies the role of model choice.

There is no requirement to use the richest available formalism in every case.

A finite-state or grammatical representation may be sufficient for one psychoanalytic distinction.

A finite-dimensional nonlinear system may be sufficient for another.

A relational field may become useful when distributed degrees of freedom matter.

A geometric model becomes useful when distance, direction, and transport matter.

A path ensemble becomes useful when alternative histories must be retained.

The selection principle is therefore

psychoanalytic distinction $\longrightarrow$ required formal information $\longrightarrow$ least elaborate adequate model.

This principle connects the two papers methodologically.

The discrete and continuous models are members of a common family of source-constrained formal reconstructions.

Their relationship is defined through projection, observation, discretization, and coarse-graining rather than through an assumed one-to-one correspondence.

The resulting framework also leaves open the possibility of additional formal resolutions.

A stochastic model may become appropriate when uncertainty is central.

A coupled multi-subject model may become necessary when reciprocal dynamics cannot be treated as external forcing.

An emergent-geometric model may become appropriate when the state space itself should be generated from more primitive relational events.

The present two-model comparison therefore represents one stage in a broader multi-resolution programme.

The principal result of this section can be summarized as follows:

one psychoanalytic trajectory

may admit

multiple formal representations

whose internal structures differ while selected observables remain compatible.

The task is consequently to identify which distinctions each representation preserves, which distinctions it suppresses, and which inferential questions those choices permit.

Section 14 now evaluates the resulting framework as a whole. It distinguishes the established mathematical resources from the present construction, examines the limits of field-theoretic and QFT-inspired language, addresses identifiability and empirical underdetermination, and clarifies the scope of the single-subject approximation.

14 Discussion

This section evaluates the scope and epistemic status of the relational field-dynamical framework developed in the preceding sections. Its objective is to distinguish established mathematical structures from the present psychoanalytic reconstruction, clarify the role of field-theoretic and QFT-inspired formalisms, identify the limits of physical analogy, and examine the problems of identifiability, empirical interpretation, and model scope. The discussion concludes by specifying several open problems that must be resolved before the framework can support stronger theoretical or empirical claims.

The paper has progressively moved through several representational levels:

Lacanian source structure $\longrightarrow$ relational state $\longrightarrow$ field dynamics $\longrightarrow$ effective geometry $\longrightarrow$ history ensembles $\longrightarrow$ c

Each transition introduces additional mathematical structure.

Accordingly, each transition also increases the burden of interpretation.

The principal contribution of the paper lies in organizing these structures into a source-constrained and multi-resolution architecture for subjective trajectories. The mathematics itself is largely established elsewhere. The novelty, where present, lies in the arrangement of those mathematical resources around a particular psychoanalytic representational problem and in the distinctions maintained among observation, state, history, geometry, interpretation, and causal mechanism.

14.1 Established Mathematics and the Present Construction

The mathematical components used throughout the paper belong to several well-established areas.

Nonlinear dynamical systems provide the concepts of fixed points, local stability, attractors, bifurcations, recurrence, and related qualitative regimes [8].

Differential geometry provides manifolds, metrics, connections, curvature, parallel transport, gauge structure, and holonomy [5].

Classical and quantum field theories provide action functionals, Euler–Lagrange field equations, interaction terms, effective actions, and path-integral constructions [7].

Spin-foam and group-field-theoretic research provides established examples of combinatorial histories, representation-labelled complexes, and field-theoretic generation of structures related to quantum geometry [6, 1].

None of these mathematical constructions originates in the present paper.

The paper’s own construction begins when these resources are organized around a relational representation of Lacanian subjective trajectories.

Examples include the relational field

$$\Phi,$$

the observation map

$$\mathcal{O}_L,$$

the RSI projection

$$\Pi_{\text{RSI}},$$

the distinction among relational domain, configuration space, and reduced state manifold,

$$\Omega_R, \quad \mathcal{M}_R, \quad \mathcal{M}_{\text{red}},$$

and the candidate history-dependent metric

$$g_{\mu\nu}(q, \Phi, h).$$

Likewise, the architecture

$$\Phi_t \longrightarrow o_t \longrightarrow a_t$$

belongs to the present formal reconstruction.

The distinction matters because mathematical validity and interpretive adequacy are separate questions.

For example, the equation

$$\frac{\partial \mathcal{L}}{\partial \Phi^a} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \Phi^a)} \right) = 0$$

is an established Euler–Lagrange equation.

Its mathematical validity does not establish that a relational subjective field should obey that equation.

Likewise,

$$R^\rho{}_{\sigma\mu\nu}$$

has an established geometric definition.

Its use in a subjective geometry requires a prior definition of the relevant manifold and metric.

The present framework therefore separates

mathematical availability

from

modeling justification.

A formal structure becomes relevant only when it preserves a distinction required by the psychoanalytic problem.

This principle also constrains claims of novelty.

The introduction of a metric, gauge connection, path integral, or coarse-graining transformation into a psychoanalytic model does not itself constitute a new mathematical theory.

A stronger contribution would require either

a new mathematical construction,

a new derivation,

a new invariant or theorem,

or

a demonstrably useful formal synthesis for a previously unresolved representational problem.

The present paper primarily pursues the last of these possibilities.

14.2 Field Theory and QFT-Inspired Structures

The terminology of field theory requires particular precision because several different formal levels appear in the paper.

The basic relational dynamics,

$$\partial_t \Phi = F[\Phi],$$

can be interpreted entirely within classical field theory or infinite- dimensional dynamical systems. Likewise, the action

$$S[\Phi] = \int \mathcal{L}(\Phi, \partial_\mu \Phi) d\mu$$

does not by itself require quantization.

A field can therefore be useful as a representation of distributed relational degrees of freedom without any quantum interpretation.

The stronger QFT-related structures appear only later.

These include a specifically complex trajectory amplitude,

$$W_Q[\gamma] = \exp\left(\frac{i}{\hbar} S[\gamma]\right),$$

a quantum effective action, and sums over field histories of the conventional QFT type [7].

The paper deliberately keeps these constructions separate from the general weighted history functional

$$W[\gamma].$$

This allows the path-ensemble architecture to remain useful even when the weight represents

probability,

accessibility,

cost,

or another non-quantum quantity.

The same distinction applies to gauge structure.

Gauge invariance does not require quantum mechanics.

Connections, bundles, gauge transformations, and holonomy can be defined in classical geometric settings [5].

The presence of a gauge group in a relational model would therefore establish a representational symmetry before it established anything quantum.

Spin foams and group field theories occupy a still stronger level.

Their established physical meanings belong to quantum-gravitational research [6, 1].

Section 11 therefore uses them only as architectural precedents for a possible sequence

elementary relational variables $\longrightarrow$ interaction complexes $\longrightarrow$ effective geometry.

The paper does not derive such a model.

It identifies the conditions that a later model would need to satisfy.

The resulting hierarchy should remain explicit:

dynamical system

⊂

field-dynamical representation

⊂

variational field formulation

followed, only under additional assumptions, by structures associated with

quantized fields,

spin foams,

or

group field theory.

Increasing mathematical sophistication does not imply increasing empirical adequacy.

The appropriate formal level depends upon the distinctions that the model is required to preserve.

14.3 Limits of Physical Analogy

The framework borrows extensively from mathematical physics. This borrowing creates both analytical possibilities and interpretive risks.

The principal risk is semantic transfer.

A mathematical term can retain its formal definition while acquiring an unsupported psychological interpretation.

For example,

field

in the present model refers to a distributed formal variable.

It does not establish the existence of a physical field in the subject.

Similarly,

potential

does not imply psychological energy,

attractor

does not imply an object of desire,

curvature

does not imply physical spacetime curvature,

and

holonomy

does not provide a generic synonym for memory.

The same caution applies to several stronger analogies.

An effective vacuum state in a field theory should not be identified with Lacanian lack.

Spontaneous symmetry breaking should not be identified directly with subject formation.

Quantum superposition should not be identified with ambiguity or indeterminacy in subjective experience.

A spin foam should not be identified with a person's relational history merely because both involve networks or histories.

These analogies may motivate questions.

They do not constitute derivations.

A useful criterion is whether the imported mathematical structure generates a distinction that could be false, tested, compared, or rejected.

For example, a history-dependent metric makes the specific claim that

$$g_{\mu\nu}(q, h_1) \neq g_{\mu\nu}(q, h_2)$$

can matter for future accessibility.

This is stronger than saying metaphorically that experience "changes the landscape."

Likewise, a hysteresis model predicts a specific type of path dependence:

$$\lambda(t_1) = \lambda(t_2)$$

while

$$q(t_1) \neq q(t_2)$$

because the control histories differ.

Such a model can, at least in principle, be compared with alternatives.

The methodological preference of the paper is therefore for *operational analogy* over lexical analogy.

A mathematical concept becomes informative when its internal relations are preserved in the target model.

A shared word or intuitive resemblance is insufficient.

The physical analogy should also remain reversible in the epistemic sense.

If a particular analogy ceases to clarify the psychoanalytic distinction, the framework should be able to remove it without collapsing the entire model.

This is another reason for maintaining modularity among

field dynamics,

geometry,

gauge structure,

and

history ensembles.

Each component can be evaluated independently.

14.4 Identifiability and Underdetermination

The multi-resolution architecture creates a substantial identifiability problem.

An observed psychoanalytic trajectory generally does not determine one unique underlying formal model.

Suppose the observed sequence is

$$w_L = o_0 o_1 \cdots o_n.$$

Several discrete derivations may generate that sequence:

$$d_1 \neq d_2,$$

while

$$\rho_D(d_1) = \rho_D(d_2) = w_L.$$

Several continuous trajectories may likewise satisfy

$$\mathcal{O}_L(\gamma_1) = \mathcal{O}_L(\gamma_2) = w_L.$$

The underdetermination increases after RSI projection:

$$\Pi_{\text{RSI}} \circ \mathcal{O}_L(\gamma_1) = \Pi_{\text{RSI}} \circ \mathcal{O}_L(\gamma_2).$$

Consequently,

same observation

can be compatible with

different states,

different histories,

different dynamics,

and

different geometries.

The inverse problem

observations $\longrightarrow$ underlying model

is therefore generally many-to-one in the forward direction and underdetermined in the reverse direction.

This problem occurs at several levels.

First, the observation map may be non-injective:

$$\mathcal{O}_L(\Phi_1) = \mathcal{O}_L(\Phi_2).$$

Second, different dynamical laws may produce observationally equivalent trajectories:

$$F_1 \neq F_2$$

while

$$\mathcal{O}_L(\gamma_{F_1}) = \mathcal{O}_L(\gamma_{F_2}).$$

Third, different geometries may permit the same observed sequence:

$$g_1 \neq g_2.$$

Fourth, different memory architectures may produce similar historical dependence. For example, an observed hysteresis-like pattern could arise from

multistability,

explicit memory,

hidden variables,

or another unresolved mechanism.

The observation alone does not identify the cause.

This makes model comparison necessary.

A proposed formalization should therefore be evaluated against alternatives that generate the same observable data.

Useful criteria may include

parsimony,

predictive discrimination,

stability under perturbation,

cross-resolution consistency,

and

interpretive adequacy.

The principle of formal economy acquires particular importance under underdetermination.

When two models explain the same distinctions, the mathematically richer model requires an additional reason for its complexity.

The problem also limits retrospective inference.

An observed trajectory should not be used to reconstruct a unique hidden subjective history unless the identifiability conditions of the model have been established.

This limitation is especially important for clinical or biographical interpretation, where many relational histories may remain compatible with a small number of observations.

14.5 Empirical and Clinical Interpretation

The present paper is primarily a conceptual and formal study. It does not provide an empirically estimated relational field, a clinically validated metric, or a fitted dynamical system for an individual subject.

Accordingly, the formal objects introduced here should not be treated as directly measurable clinical variables.

A future empirical implementation would first require an operational observation layer.

Let

$$y(t)$$

denote empirically available observations.

These might arise from discourse, behavioral sequences, self-report, interaction records, or another defined source.

A measurement model would relate observations to the latent relational state:

$$y(t) = H(\Phi_t) + \epsilon(t),$$

where

$$H$$

is a measurement map and

$$\epsilon$$

represents observational error or unresolved variation.

A psychoanalytic classification would add a second mapping:

$$y(t) \longrightarrow o_t$$

or, depending upon the research design,

$$\Phi_t \longrightarrow o_t.$$

The distinction matters.

If the Lacanian observation is inferred from empirical data, then $\mathcal{O}_L$ is itself an inferential construction and may contain classification uncertainty.

The complete empirical architecture could therefore take the form

$$\Phi_t \longrightarrow y_t \longrightarrow \hat{o}_t \longrightarrow \hat{a}_t.$$

Uncertainty can enter at every arrow.

A clinical application would require additional safeguards.

First, the mathematical model should support clinical judgment rather than replace it automatically.

Second, a fitted attractor or estimated critical regime would remain a model property rather than an intrinsic label attached permanently to a person.

Third, a state-space reconstruction would depend upon the variables selected, the observational window, the sampling procedure, and the assumed model class.

Fourth, any intervention guided by the dynamics would require independent ethical and clinical justification.

The distinction between descriptive and prescriptive use is therefore essential.

The equation

$$\dot{q} = F(q)$$

may describe an estimated trajectory.

It does not by itself specify which trajectory should be encouraged.

Likewise, identifying a state near a bifurcation does not imply that the system should be pushed toward or away from the transition.

Such decisions require values, clinical aims, contextual knowledge, and ethical judgment that are not supplied by the dynamical model.

The present framework is therefore better understood at this stage as a formal language for organizing possible empirical questions.

Examples include whether

apparently repeated symbolic states

possess different continuation distributions,
whether

history improves prediction beyond current observation,

whether

reduced dynamics display multistability or hysteresis,

and whether

different descriptive resolutions preserve the same predictive structure.

These questions can eventually support empirical comparison among models.

The present paper does not claim that such comparisons have already been performed.

14.6 Scope of the Single-Subject Approximation

The entire field-dynamical construction of Paper II is intentionally limited to one modeled subject.

External persons, institutions, utterances, and events enter through variables such as

$$u(t)$$

or

$$J(\xi, t).$$

Their own dynamics are not modeled endogenously.

This approximation makes the present construction tractable.

It also removes an important class of relational phenomena.

Suppose two subjects are represented by

$$\Phi_A$$

and

$$\Phi_B.$$

A coupled system would require

$$\partial_t \Phi_A = F_A[\Phi_A, \Phi_B],$$

$$\partial_t \Phi_B = F_B[\Phi_B, \Phi_A].$$

The trajectory of A changes the relational conditions experienced by B , and the trajectory of B changes those experienced by A .

The interaction can therefore generate feedback:

$$\Phi_A \longrightarrow \Phi_B \longrightarrow \Phi'_A.$$

More generally, the relational field may itself be co-generated:

$$\Phi_{AB} = \mathcal{R}(\Phi_A, \Phi_B).$$

This raises questions that cannot be reduced to external forcing.

The relational geometry of A may depend upon B ,

$$g_A = g_A(\Phi_A, \Phi_B),$$

while the geometry of B simultaneously depends upon A :

$$g_B = g_B(\Phi_B, \Phi_A).$$

The resulting system is a coupled dynamical geometry.

A still richer model may require a shared relational field

$$\Psi_{AB}$$

that cannot be decomposed completely into

$$\Phi_A$$

and

$$\Phi_B.$$

Such models are likely to be necessary for studying reciprocal trust, recognition, conflict, transference-like dynamics, coordination, and other processes whose structure depends upon mutual response.

The present paper deliberately postpones these questions.

Its objective is to establish a sufficiently clear single-subject formal language before adding reciprocal dynamics.

The approximation should therefore be understood as

external relational influence without endogenous modeling of the other's trajectory

rather than as a claim that subjective dynamics are fundamentally single-agent processes.

The distinction is especially important within a relational framework.

Treating another subject as an exogenous input is a methodological simplification.

It is not an ontological commitment.

14.7 Open Problems

The framework leaves several major problems unresolved. These problems define the boundary between the present conceptual construction and a mature relational dynamical theory.

The first problem concerns the definition of the relational domain

$$\Omega_R.$$

The paper permits several interpretations of this domain while avoiding a premature identification with physical space. A stronger theory must specify what its coordinates represent, which transformations preserve its relevant structure, and when a field description adds information beyond a finite-dimensional state model.

The second problem concerns the choice of state variables.

The present field components

$$\Phi^a$$

remain interpretation-neutral.

A future implementation must determine which variables are needed to preserve the relevant Lacanian distinctions and how those variables can be related to observable material.

The third problem concerns the observation map

$$\mathcal{O}_L.$$

The map is conceptually central and presently abstract.

A stronger theory requires explicit criteria for assigning a continuous state or observed history to a fine Lacanian description.

Without those criteria, the relation between formal state and psychoanalytic interpretation remains underspecified.

The fourth problem concerns the metric

$$g_{\mu\nu}.$$

Several possible meanings have been proposed, including effective distance, accessibility, sensitivity, and deformation cost.

A mature model must select one meaning and derive or estimate the metric accordingly.

The fifth problem concerns the relation between dynamics and geometry.

The paper allows

$$\dot{q} = F(q, g, h, u)$$

and

$$\partial_t g = H(q, g, \Phi, h, u),$$

but does not specify a general law coupling F and g .

Different coupling laws can generate qualitatively different subjective geometries.

The sixth problem concerns gauge structure.

A genuine relational gauge theory requires a justified group

$$G,$$

a defined action,

$$r \mapsto g \cdot r,$$

and specified invariant observables.

The present paper provides the architecture for such a construction without identifying a definitive psychoanalytic gauge group.

The seventh problem concerns model identifiability.

One must determine which combinations of

$$F, \quad g, \quad M, \quad \mathcal{O}_L$$

can be inferred from finite observations and which remain observationally equivalent.

The eighth problem concerns scale.

Section 10 distinguished projection, reduction, coarse-graining, and renormalization. A genuine scale-dependent relational theory still requires explicit transformations

$$\mathcal{R}_b$$

and effective flows

$$\frac{dg_i}{ds} = \beta_i(\mathbf{g}).$$

The existence of useful relational fixed points or universality classes remains an open question.

The ninth problem concerns emergent geometry.

Section 11 proposed the exploratory map

$$(\mathcal{C}_R, \Lambda_R) \longrightarrow (\mathcal{M}_\ell, g^{(\ell)}).$$

No reconstruction theorem, continuum limit, or empirically justified relational complex has yet been supplied.

This problem constitutes the central task of the proposed subsequent study on the spacetime of desire.

The tenth problem concerns coupled subjects.

The single-subject approximation must eventually be replaced, where necessary, by dynamics such as

$$\partial_t \Phi_A = F_A[\Phi_A, \Phi_B], \quad \partial_t \Phi_B = F_B[\Phi_B, \Phi_A].$$

Such coupling may generate qualitatively new dynamical regimes that cannot be represented as external forcing.

The eleventh problem concerns stochasticity.

The present paper has mentioned stochastic dynamics only briefly.

A fuller treatment must distinguish

epistemic uncertainty,

unresolved deterministic variables,

and

intrinsic stochastic terms within the selected model.

These possibilities produce different interpretations of trajectory ensembles and effective noise.

The twelfth problem concerns empirical falsifiability.

A formal framework becomes substantially stronger when competing model classes can be distinguished through observations.

Future work should therefore seek claims of the form

$$\mathfrak{M}_1 \Rightarrow P_1(y),$$

$$\mathfrak{M}_2 \Rightarrow P_2(y),$$

with

$$P_1(y) \neq P_2(y)$$

for some observable domain.

Such contrasts would permit the formalism to move beyond retrospective interpretive flexibility.

Taken together, these open problems suggest a staged research programme.

The present paper establishes

relational field dynamics

and

effective dynamical geometry.

A subsequent study can investigate

emergent relational geometry

through complexes, representation structures, spin-foam-inspired histories, and group-field-theoretic generation.

A further series can address

coupled subjective dynamics

in which the relational field is co-generated by several evolving subjects.

Empirical work would constitute another distinct stage.

This staged development preserves the principal methodological commitment of the present study: additional mathematical structure should be introduced only when the previous level cannot preserve a distinction required by the problem.

The discussion therefore returns to the central claim of Paper II.

A continuous field-geometric representation can enrich the analysis of Lacanian subjective trajectories by distinguishing states that appear identical at a symbolic resolution while differing in dynamics, history, stability, geometry, or future accessibility.

The framework remains a formal reconstruction.

Its value depends upon the precision with which these distinctions can be defined, related to psychoanalytic interpretation, and eventually compared with alternative models.

Section 15 summarizes this construction and states the scope of the resulting proposal.

15 Conclusion

This paper has developed a preliminary relational field-dynamical framework for representing historically situated subjective trajectories in Lacanian psychoanalysis. Its central objective has been to extend a discrete generative account of psychoanalytic traversal into a continuous representation capable of preserving distinctions concerning local variation, historical dependence, stability, geometry, and alternative possible histories.

The resulting architecture separates several levels that are easily conflated.

At the dynamical level,

$$\Phi_t \in \mathcal{M}_R$$

represents the complete relational field configuration retained by the model.

At the psychoanalytic observational level,

$$\Phi_t \xrightarrow{\mathcal{O}_L} o_t$$

produces a fine Lacanian description.

At a still coarser resolution,

$$o_t \xrightarrow{\Pi_{\text{RSI}}} a_t$$

produces an RSI observation.

These mappings make explicit that equality of symbolic observations does not require equality of the underlying dynamical states.

Thus,

$$a_{t_1} = a_{t_2}$$

does not imply

$$o_{t_1} = o_{t_2},$$

and

$$o_{t_1} = o_{t_2}$$

does not imply

$$\Phi_{t_1} = \Phi_{t_2}.$$

This distinction supplies the basic motivation for a continuous relational model.

The paper subsequently introduced several classes of dynamical structure.

Relational field equations permit coupled, nonlinear, externally driven, and history-dependent evolution.

Memory can be represented through enlarged states, explicit history functionals, kernels, or history-dependent parameters.

Dissipative and irreversible effective behavior can be incorporated without requiring every relational process to arise from a conservative action principle.

Where a variational description is useful, the framework permits

$$S[\Phi] = \int \mathcal{L}(\Phi, \partial_\mu \Phi) d\mu,$$

while maintaining the distinction between a classical action, a general effective description, and specifically quantum-field-theoretic constructions.

The continuous state space was then supplemented with geometry.

An effective relational state manifold

$$\mathcal{M}_{\text{red}}$$

may carry a metric

$$g_{\mu\nu},$$

allowing relational distance, anisotropy, local direction, and curvature to be represented explicitly.

The stronger possibility

$$g_{\mu\nu} = g_{\mu\nu}(q, \Phi, h)$$

permits the effective geometry itself to depend upon configuration and history.

This yields one of the principal claims explored by the paper:

A repeated psychoanalytic position may occur within a transformed relational geometry, so that symbolic return need not imply return to the same local structure of possibility.

The distinction between dynamical and geometric return was further refined through gauge structure. Where a justified transformation group and connection exist, the framework can distinguish

state recurrence

from

parallel transport around a closed path.

Holonomy then provides one specific geometric form of historical non-equivalence.

The paper has emphasized throughout that holonomy should remain distinct from memory, hysteresis, and retroactive interpretation.

The analysis of dynamical regimes similarly separated several forms of persistence and transformation:

fixed points,

attractors,

metastability,

recurrence,

bifurcation,

criticality,

and

hysteresis.

These concepts describe mathematically different organizations of trajectories and therefore provide more precise alternatives to a generic language of repetition or change.

The introduction of trajectory ensembles extended the analysis from one realized path to a family of admissible histories:

$$\Gamma(\Phi_i, \Phi_f).$$

A general weighting

$$W[\gamma]$$

permits probability, accessibility, cost, or another specified quantity to be associated with alternative histories without presupposing quantum superposition.

The corresponding history aggregate,

$$K_W(\Phi_f, \Phi_i) = \int \mathcal{D}\gamma W[\gamma],$$

was therefore kept conceptually separate from the specifically quantum choice

$$W_Q[\gamma] = \exp\left(\frac{i}{\hbar} S[\gamma]\right).$$

This distinction preserves the usefulness of path-based reasoning without requiring a quantum ontology of subjectivity.

Coarse-graining introduced a further level of analysis.

The paper distinguished

observation,

state reduction,

dynamical coarse-graining,

and

renormalization.

A coarse symbolic observable such as RSI may suppress differences among continuous states without determining the effective dynamics of the retained variables.

Likewise, eliminating fine degrees of freedom may generate memory, fluctuation, or new effective interactions.

A coarse relational description should therefore not be treated as the fine description with variables merely removed.

The relation to the preceding generative-grammar model follows the same multi-resolution principle.

The discrete model and the continuous model can share a Lacanian observation space:

$$\mathcal{C}_D \xrightarrow{\rho_D} \mathcal{Y}_L,$$

$$\mathcal{M}_R \xrightarrow{\mathcal{O}_L} \mathcal{Y}_L.$$

They may therefore be observationally compatible,

$$\mathfrak{M}_D \sim_{\mathcal{O}} \mathfrak{M}_C,$$

without being mathematically equivalent:

$$\mathfrak{M}_D \not\equiv \mathfrak{M}_C.$$

The discrete representation is particularly suited to symbolic generation, derivation, and admissible ordering.

The continuous representation is particularly suited to local dynamics, stability, perturbation, geometry, and path dependence.

Their relation is therefore complementary rather than hierarchical.

The strongest exploratory step of the paper concerns the possibility that relational geometry itself may emerge from more primitive relational organization.

Instead of assuming only

$$\mathcal{M} \longrightarrow g(t),$$

one may eventually consider

$$\mathfrak{R} \longrightarrow (\mathcal{M}, g),$$

where an event-based or combinatorial relational structure contributes to the generation of an effective geometry.

Spin-foam and group-field-theoretic ideas were introduced only as architectural precedents for this stronger problem.

Paper II does not construct a quantum geometry of subjectivity.

It prepares the formal question of how

relational generation $\longrightarrow$ effective subjective geometry

might be investigated in a later study.

The phrase *spacetime of desire* names this research horizon.

At the effective level, it refers to the structured space of relational possibilities within which subjective trajectories unfold:

$$\mathfrak{S}_D^{(\ell)} = \left(\mathcal{M}_\ell, g^{(\ell)}, F_\ell, \mathcal{O}_L \right).$$

The term does not identify desire with physical spacetime.

It denotes a formal geometry of historically conditioned relational possibility.

The paper's principal methodological commitment can therefore be stated in its most general form:

psychoanalytic distinction $\longrightarrow$ required formal information $\longrightarrow$ least elaborate adequate representation.

This principle governs the transition from grammar to fields, from fields to geometry, from geometry to gauge structure, and from individual trajectories to history ensembles.

It also sets a boundary on interpretation.

The paper distinguishes

Lacanian source structure,

mathematical representation,

psychoanalytic interpretation,

and

causal mechanism.

The formal framework does not collapse these levels.

Its intended contribution is instead to show that a historically situated Lacanian trajectory can be represented across several resolutions while preserving distinctions among symbolic position, continuous state, history, stability, geometry, and future accessibility.

The resulting framework remains preliminary.

Its stronger development requires explicit relational variables, an operational observation map, identifiable dynamical laws, a justified relational metric, and, where gauge language is retained, a demonstrable transformation group and invariant structure.

A later study may then address the more demanding problem of emergent relational geometry and the generation of the spacetime of desire.

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