Language Teaching Grant Allocation

A Preliminary Study Of Eligibility Feedback

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Abstract

Language teaching grants can support the activities from which later eligibility and renewal evidence is produced. This preliminary study examines the resulting relation between participant formation, review timing and supported continuation. Drawing on the Generative Relational framework, it treats a teaching arrangement as a history-bearing configuration whose records, judgments and available operations must be distinguished. A problem-led literature review and version-specific programme documents situate the inquiry without inferring programme effects from published rules. Three conditional formal investigations develop the timing problem. A finite continuation model identifies a preparation path excluded by a delivery-only renewal checkpoint. A nonlinear spatial field model proves a related joint-activity obstruction even with arbitrary mixing and regional allocation, and constructs an equal-budget protected path. An operator representation characterizes the intermediate restrictions inserted into a history-extension sequence. The fields measure selected activity channels, not competence or desert. The philosophical argument distinguishes evidence error, obstruction of a declared formation purpose and unfair allocation. A bounded continuation requires credible milestones, practical conversion conditions, participant agency and consideration of competing claims. The study proposes several possible models and an assessment structure; it does not establish a universal grant policy or an empirical causal finding.

Keywords: language teaching; grant allocation; participant formation; eligibility feedback; nonlinear fields; historical operators

Scope Note

This preliminary study concerns a recurring language-teaching grant arrangement. It examines how eligibility and renewal rules can affect the formation of further participants and the evidence available at later funding decisions. The object is narrower than the political economy of language generally. Grant support, practical teaching capacity, observed delivery, participant formation and justified allocation remain distinct.

The Generative Relational research programme supplies a revisable language for history-bearing configurations, manifestations, partial operations and formative feedback. Its prior arguments already identify recognition-dependent access, generative return and the possibility that funding judgments alter later preservation conditions. The present paper inherits those arguments and investigates a bounded allocation mechanism. It does not claim to discover recursive advantage or the general importance of teaching resources.

The formal investigations are several possible models. A historical operation system makes review-dependent availability explicit. A finite continuation calculation identifies a formation path interrupted by an intermediate eligibility test. A nonlinear regional field examines a related obstruction even when resource use is continuously divisible. Operator projections distinguish intermediate eligibility filtering from assessment after a longer operation. These results are conditional on stated assumptions. Activity, practice exposure and resource coordinates do not constitute numerical measures of linguistic competence or moral worth.

Primary institutional documents establish documented rules at specified versions. They do not establish the causal effects of those rules. Constructed cases and illustrative computations are identified separately. The philosophical analysis distinguishes an evidential error, an obstruction of a declared formation target, and an unfair allocation. A favorable model result supplies neither an unlimited claim to funding nor authority to impose participation. The packet retains actual reading scopes, verified references and reproducible checks for further discussion.

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Notices

Status. This discussion draft presents an evolving inquiry. The definitions, translations, formal constructions, empirical interpretations, and conclusions remain open to criticism and author review. It investigates the interaction between language-teaching grant rules, participant formation and the evidence available for later support decisions. Its formal models are conditional representations and its institutional sources establish documented rules rather than causal estimates. The analysis does not prescribe a universal allocation policy.

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Statement On The Use Of Language Models. The author selected the research direction, supplied prior manuscripts and a reference exposition, and requested consistency with the Generative Relational research programme. AI-assisted research, source comparison, drafting, formal checking, and document preparation contributed substantially to this draft. These contributions include proposed arguments and interpretations that remain subject to the author's review. Preparation of a checked draft does not establish the author's approval of every claim. Authorship credit remains with the human author; the use of these tools is disclosed to support critical assessment of the manuscript and its limitations.

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1 Teaching Grant Renewal

Language-teaching grant decisions often concern both present provision and the conditions of future teaching. A supported arrangement can deliver classes, prepare materials and produce an acceptable report while depending on a small group whose ability to form further participants remains uncertain. Another arrangement can devote substantial work to supervised preparation whose significance becomes visible only after an intervening review. The current paper investigates how eligibility and renewal rules interact with that temporal difference. It studies a mechanism of allocation rather than assigning a single value to a language or a teaching community.

The primary analytical object is an *eligibility-feedback arrangement*. A historically formed teaching configuration presents a selected record; a rule makes that record consequential for support; the support changes which operations can be undertaken; those operations help generate the next record. Each relation requires evidence or a stated model assumption. The feedback can support valuable continuity, reveal a failing undertaking, restrict a viable formation route, or have mixed consequences for different participants. Its direction is a question for the inquiry.

The distinction between current output and further participant formation does not imply that they always compete. Teaching can simultaneously serve learners and prepare a future teacher. Supervised delivery can generate useful public evidence of formation. The difficulty arises where the relation among activity, evidence and review timing is less favorable. A criterion adequate for certifying completed delivery can be inadequate for evaluating an undertaking whose declared purpose includes making later delivery possible through additional participants.

1.1 The Recurring Decision

Consider a constructed two-period support arrangement. One package can be used for current classes or for preparation that makes a later coached teaching episode possible. The first period of preparation produces warranted intermediate evidence, but no output of the kind counted by a completed-class renewal rule. If the next package is available only after that counted output, the preparation route can be interrupted before the later episode occurs. Keeping current classes running instead can satisfy the rule while leaving the selected participant-formation target unmet.

This example does not describe an actual funding programme. It isolates a question about the fit among a target, an operation sequence and an intermediate decision. A different rule can retain the preparation route, and a different target can justify the completed-class criterion. Actual programmes can also distinguish new applicants, formation activities, installment reporting and renewal. Those distinctions are examined through primary documents in Sections 2 and 12.

The object consequently includes more than a reporting indicator. It includes the historical teaching arrangement, the supported operation, the participant’s accessible evidence, the grant’s resource and timing constraints, and the decision’s effect on later availability. A report can be accurate about what happened without being sufficient for a judgment about why it happened or what could happen after further support. Conversely, uncertainty about formation does not establish that additional funding would succeed.

1.2 Inherited Research

The Generative Relational preservation taxonomy already separates capacities, enabling conditions, their coupling and the judgment that recognizes a preservation result [12]. The earlier study of normative formation develops endogenous recognition and access, the reproduction of competence, and the possibility that present participation changes later formative opportunities [9]. The power manuscript gives a judgment-conditioned transformation and a reflexive recognition loop [11]. A general diagram from recognition to resources and back would therefore repeat existing work.

Earlier language papers also constrain the contribution. Participant formation was distinguished from successful current use in the competence-reproduction study [15]. The reactivation study developed a

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selected activity-support law and the importance of support history [14]. Record sufficiency and interface substitution then separated accessible policy choice, operation domains and preserved continuation [16, 13]. These distinctions remain inherited premises.

The present inquiry specializes them to a recurring allocation decision. Its question is whether a rule that conditions support on a specified record preserves a feasible route to its stated formation objective, including the record required at the next decision. This can expose a mismatch even when the record is truthful and the rule is applied consistently. That explanatory finding remains distinct from the further assessment of competing claims and legitimate allocation.

1.3 Method And Formal Investigations

The philosophical argument specifies the historical bearer, the relation between current delivery and participant formation, and the evidential role of a renewal record. It develops alternative interpretations and examines scarcity, accountability, dependence and participant willingness. Primary literature supplies both established explanations of allocation feedback and bounded institutional comparisons. Their concepts are translated into the current problem only where their assumptions and remainders remain explicit.

The first formal investigation begins with history-bearing configurations and a possible two-complex representation. Awards, teaching, preparation, reporting and review have distinct occurrences. The rule changes the domain of later operations. A finite backward calculation then identifies continuation viability relative to that installed rule and a declared resource budget. The calculation uses established finite-horizon reasoning; its contribution is to make a particular eligibility obstruction and its alternatives inspectable.

The second investigation constructs nonlinear regional dynamics. Purchased support can be divided between current delivery and maintained formation practice. A grant review observes a selected delivery signal. An aggregate bound shows that one strict renewal criterion defeats a joint activity-and-practice target even across arbitrary admissible spatial and temporal allocations. A protected two-period schedule reaches the target with the same total package budget. A current-delivery-only target supplies a positive countercase for the stricter rule.

The third investigation uses a history-operator representation. Eligibility is a projection identifying which continuations receive support at a declared stage. A product with intermediate eligibility gates can differ from one tested only after its completion. The algebra makes that difference calculable while distinguishing a mathematical filter from physical destruction of a history. A separate record-algebra question concerns whether the report is sufficient for the continuation judgment, rather than merely sufficient for executing the installed rule.

1.4 Contribution And Scope

The paper offers a bounded philosophical and formal account of eligibility feedback in language-teaching allocation. Its mathematical results are conditional representations, not new general theories of incentives, fairness or viability. Its institutional sources establish rules and design choices, not causal estimates of their consequences. Its proposed assessment identifies questions that a practical decision would need to answer rather than ranking actual applicants.

The analysis also keeps a distinction that can easily disappear in a critique of metrics. Showing that a criterion interrupts one feasible formation route does not establish that retaining that route is the best use of scarce support. Another group, an urgent present service or a serious evidential weakness may provide a reason to allocate otherwise. The relevant demand is for a decision whose evidence, causal assumptions, target and competing claims can be examined together.

The remaining sections develop the object and its formal specializations before returning to documented and constructed cases. The intended result is a more precise discussion of what a renewal rule enables, what it makes difficult to demonstrate, and which further reasons would justify changing it. The scope remains a teaching-grant mechanism throughout.

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2 Grant Allocation Research

Research on incentives, organizational measurement, administrative access and accountability supplies established explanations of the interaction between evidence and resources. The present section compares these explanations to identify the narrower problem raised by recurring language-teaching grants. Its method is analytical comparison of primary arguments and selected empirical studies. Each account is considered in relation to its explanatory object, its evidence, and the additional premises required to apply it to participant formation. The documentary language-programme cases are developed separately in Section 12.

2.1 The Allocation Problem

A grant decision can use evidence for several different purposes. A record may establish that an applicant belongs to the class a programme is authorized to support. It may indicate that the proposed activity is feasible, document that an earlier activity occurred, or support a comparison among eligible applicants. An expenditure record can establish compliance with an agreement while providing limited evidence of learning. A participant's demonstration can establish a bounded capacity while leaving the institutional cost of sustaining the teaching arrangement unresolved. These evidential functions have different objects and should be assessed separately.

The distinction becomes consequential across successive decisions. When earlier resources enabled the activity recorded in an application, the record partly expresses what that support made possible. The same record may still be an excellent predictor of what the arrangement can deliver next. Support dependence therefore does not invalidate its use. The inferential difficulty concerns the additional conclusion that a poorer record establishes a correspondingly weaker capacity to benefit from support. That conclusion requires a comparison of formation opportunities, feasible activities and the purpose of the next award. A record of supported performance cannot supply the comparison merely by existing.

Consider two constructed purposes. A short course commissions a specified number of already feasible teaching sessions. A formation grant supports supervised work intended to prepare an additional instructor. Completed sessions can be directly relevant to the first purpose. Evidence of supervised attempts, correction and developing responsibility may be relevant to the second, even before independent instruction occurs. These examples do not establish the superiority of one purpose. They identify the temporal relation between a purpose and the evidence capable of bearing on it. An institution can support both purposes through distinct routes, or choose one within its remit.

This purpose sensitivity also limits the interpretation of fairness. Equal eligibility conditions can protect applicants from arbitrary preference. Yet their effect depends on which undertaking the conditions are supposed to assess. A requirement that appropriately verifies a delivery contract can obstruct a formation sequence if inserted before the sequence can produce its declared evidence. Conversely, a generous account of future potential can conceal favoritism if neither its premises nor its application can be inspected. The research problem concerns a defensible relation among purpose, evidence, feasible support and later review. Each of the following traditions explains a different part of that relation.

2.2 Multitask Incentive Design

Holmström and Milgrom analyze incentives that affect the allocation of attention among tasks. Their principal-agent framework shows why differences in measurability across valuable activities can make strong incentives for the measurable component undesirable under specified assumptions. They also consider job design and the separation of tasks as responses. The educational example helps motivate the model; it is not an empirical investigation of language-teaching grants. The model's benefit, cost, information and compensation assumptions define the scope of its results [8, pp. 24–29].

The relevant translation begins with a limited common problem: rewarding an observed activity changes the practical reasons for undertaking it. A teaching arrangement may have several valuable operations whose evidential accessibility differs. Preparing a lesson, supporting hesitant participation,

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coaching another instructor and recording completed sessions can require overlapping time. An award rule can make one operation especially consequential for access to the next period of work. The resulting choice deserves explanation even when the participants continue to endorse all of the programme's aims.

Three differences prevent a direct substitution of grant vocabulary into an employment model. First, the recipient may be an arrangement comprising learners, instructors, coordinators and supporting organizations. Their interests and authority need not be representable by one agent's preference ordering. Second, additional participant formation can change which operations exist as feasible options. The question then exceeds the distribution of a fixed effort budget among already available tasks. Third, a recurring award can determine whether the arrangement continues to possess the setting in which either operation could occur. A marginal change in an incentive and the removal of an operative teaching relation are different interventions.

These differences leave the established incentive insight intact while identifying an additional modeling obligation. An account of grant feedback should specify whether a rule changes effort allocation within a continuing arrangement, changes the arrangement's feasible operation set, or does both. If coaching and current instruction are jointly feasible, a supposed conflict may disappear. If coaching requires protected preparation that current delivery cannot supply, the temporal sequence becomes important. The model must make that difference visible rather than stipulating that every report competes with every act of teaching.

A positive design question follows. An institution may retain a reliable delivery measure while separately recognizing justified preparation. Such recognition would require a declared purpose and evidence proportionate to the preparatory activity. Simply attaching a second score to formation need not solve the difficulty: the institution would still need to explain what that score observes and when it can be produced. The task is to represent the sequence well enough for a decision, including uncertainty about eventual success. No universal recommendation about incentive intensity follows from the translation.

2.3 Measurement Reactivity

Espeland and Sauder's study of law-school rankings examines how public measures reorganize the institutions being evaluated. Its primary material includes 136 open-ended interviews, with documentary and observational triangulation. The authors distinguish self-fulfilling processes from commensuration and examine resource redistribution among the consequences. They present their framework as an orienting heuristic and emphasize variation in responses. Their interview design supports analysis of mechanisms and experience; it does not provide a language-grant effect estimate [3, pp. 8–9, 11, 16, 25, 33–34].

For the present inquiry, a grant record has two analytically separable roles. It carries information about the teaching arrangement, and its recognized categories can help organize the arrangement's work. An application that asks for a teaching plan may prompt useful coordination. A report that distinguishes initial participation from sustained attendance may reveal a problem that would otherwise remain unnoticed. These are possible beneficial forms of reactivity. Their existence prevents the inference that any effect of observation on conduct compromises the record.

A more demanding concern arises when the categories also determine continued access to the means of producing evidence. In a constructed case, the institution counts independent teaching episodes, and a learner requires supported co-teaching before an independent episode becomes feasible. If co-teaching cannot appear as relevant progress, the next allocation may interrupt the very activity needed to produce the count. The information need not be false or manipulated. Its categories can correctly describe current delivery while failing to represent an authorized formation route. This is a problem about the use of an accurate measure within a decision sequence.

Public ranking and grant eligibility nevertheless differ. A ranking can alter reputation through a public ordering of institutions; a grant may impose a threshold without publishing any comparative rank. A financial report can verify expenditure without purporting to rank linguistic quality. The relevant mechanism must consequently be located in the actual decision: who receives the record, what inference is drawn, what consequence follows, and what participants can then do. Calling all these instruments metrics would conceal rather than explain their different effects.

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The distinction also changes the evidential burden of criticism. Establishing that a criterion exists is a first step. Establishing that participants anticipate it and alter their activity is another. Showing that the alteration reduces an important formation possibility requires further evidence about alternatives, resources and outcomes. The present paper uses formal constructions to investigate possible dependencies among those steps. The existence of a constructed dependency does not complete the empirical argument for any actual programme.

2.4 Administrative Access

Herd and colleagues examine Wisconsin's BadgerCare Plus through interviews, documents and enrollment analysis. Their study distinguishes changes in administrative processes from expanded eligibility by examining an already eligible population. It describes verification assistance and other transfers of administrative work toward the state, while acknowledging that the separate effects of the reform package cannot be isolated. The study supplies a concrete account of practical access within an eligibility regime. Its setting is health coverage, and its findings do not establish the consequences of language-grant reporting [7].

The conceptual distinction is relevant even where the populations and institutional purposes differ. A person or organization can satisfy a substantive condition while lacking a practicable route for demonstrating that satisfaction. The missing route might involve access to records, translation, a signature, reliable communication or time to reconstruct prior expenditure. These are different obstacles, and their remedies need not weaken the substantive condition. Someone else may be able to retrieve an existing record; an accessible form may allow the same evidence to be submitted; an institution may already possess information that an applicant is asked to reproduce.

Within a teaching arrangement, the distribution of this work matters. A reporting task assigned to an experienced instructor may occupy time during which a learner could receive feedback. The same task assigned to a supported coordinator may have another practical consequence. Neither effect follows from the page count of the form. The relevant comparison includes who can perform the task, what it requires, the available support, and the teaching operations displaced or enabled. Administrative burden is consequently a hypothesis about an activity within a configuration, not a property inferred from the mere existence of a requirement.

The grant context also introduces an important difference from an entitlement programme. Satisfying eligibility can establish entry to consideration while leaving an application unfunded because other applications have priority within a finite budget. Improving access to application therefore need not secure an award. It can nevertheless improve the defensibility of the comparison by ensuring that the institution considers the intended evidence rather than applicants' unequal capacity to reproduce it in a prescribed format. Whether this improvement justifies its administrative cost remains a separate decision.

There are costs on the other side as well. Assistance requires staff, infrastructure and rules for handling incomplete information. A proposal to transfer reporting work must identify these resources and account for applicants who also depend on them. The analysis should preserve this symmetry: participant time is consequential, and institutional verification is itself a situated practice with capacity limits. Neither side can be treated as an unlimited reserve that absorbs the difficulties of the other.

2.5 Accountability Purposes

Ebrahim compares accountability mechanisms across their audiences, inducements and organizational purposes. His analysis distinguishes recurring reports and assessments from more extended participatory processes, and examines the possibility of evaluation contributing to organizational learning. It also stresses the need for credible assessment rather than relying on an organization's declared mission. This is an analytical synthesis, including discussion of existing studies; it does not establish a uniform causal effect of accountability requirements across organizations [2, pp. 813–818, 824–826].

A language-teaching grant can involve several legitimate recipients of an account. The funding institution may need to explain its use of a limited budget. Participants may need to know whether promised

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teaching occurred, how their contributions were used, and whether an instructor's responsibilities were fulfilled. Other applicants may need assurance that continuation was justified on announced grounds. The supported organization itself may need records that permit it to learn from difficulties. A reduction in upward reporting can therefore be an improvement in one respect while leaving other accountability relations unchanged or weaker.

These relations also reveal a limit of the opposition between quantitative reporting and qualitative understanding. A simple attendance record may disclose that a promised activity repeatedly failed to occur. A rich narrative may conceal the same fact. Conversely, an attendance count may provide little information about whether participants could ask questions, receive correction or assume a new role. Evidential adequacy follows from the inference being attempted. Neither numerical nor narrative form supplies a general guarantee.

For participant formation, a particularly useful account would distinguish evidence of an attempted route from evidence of completed capacity. Supported preparation can be competently undertaken even when it reveals that the proposed responsibility is presently unsuitable. Treating every such result as failure may encourage implausibly smooth progress reports. Treating every attempt as sufficient success would make the formation aim difficult to assess. A review can recognize what was responsibly learned while still declining a further stage for which the evidence is weak. This judgment requires intelligible intermediate objectives and an account of the resources already used.

The disagreement among accountability purposes cannot always be resolved by collecting more information. Participants may reasonably favor continued preparation while an institution has an immediate obligation to deliver previously promised classes. A committee may have good evidence of potential and still decide that a different use has priority. The present paper's distinction between continuation feasibility and allocation justification preserves room for those decisions. Its criticism concerns unsupported inferences and avoidable obstruction relative to a declared purpose, not the abolition of competing purposes.

2.6 Decision-Dependent Outcomes

Liu and colleagues provide a one-step model in which selection affects a subsequent variable of interest. They compare standard fairness constraints through their outcome implications, showing that favorable effects depend on the decision setting and outcome model. Their result already establishes the importance of temporal consequences for evaluating selection rules. The model does not prove inevitable exclusion across repeated funding cycles, and its score-based assumptions are not empirical premises about teaching arrangements [17, pp. 3150–3153, 3157].

The pertinent lesson is methodological. A rule can satisfy a property of the present allocation while producing different subsequent possibilities. Assessing those possibilities requires an account of the relevant transition. In the teaching problem, that transition may involve the survival of a relationship, completion of supervised practice or availability of a future teaching role. These are heterogeneous objects. A scalar activity channel can help investigate one dependency, but it cannot silently replace participant competence, communal authority or the worth of a language.

The comparison also clarifies two distinct temporal questions. One asks what happens after a single decision, holding the later environment fixed. Another asks how later decisions respond to evidence partly produced by earlier decisions. The latter adds a rule of recurrence, an observation process and feasible intervening operations. Repetition can amplify an initial difference, correct it, or have little effect, depending on these additions. A claim about repeated exclusion therefore requires more than the observation that decisions have consequences.

Fairness remains plural within the recurring problem. Equal access to initial consideration, reliable fulfillment of current teaching commitments and opportunities for further participant formation can point toward different allocations. A formal comparison should state which of these aims it represents and retain the cost imposed on alternatives. A bridge that keeps one formation route open may use resources that another applicant could employ immediately. Demonstrating the bridge's feasibility identifies an option; justifying its selection requires reasons that the model's reachability result does not supply.

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2.7 Generative Relational Inheritance

The prior Generative Relational account of normative formation already develops the possibility that realized influence changes recognition, legibility, access and the conditions of later participation. It distinguishes such feedback from ordinary selection and treats the restriction of future formative capacities as a stronger claim than unequal present influence. It also recognizes constructive roles for accumulated expertise, shared categories and institutionalized access [9, pp. 19–24]. These distinctions govern the present translation rather than emerging as new conclusions from the grant example.

The current paper gives the recurring allocation arrangement a more specific object. A historically extended teaching configuration contains operations whose availability depends on resources, participants and timing. The institution encounters selected records of that configuration and makes a consequential decision. The inquiry then asks whether the admitted formation route can still be undertaken, which intermediate record the rule requires, and whether a feasible alternative rule would change that result. Historical descriptions, finite operation models and field representations examine different aspects of this question.

No single literature supplies all of these relations, and their coexistence is not itself a novelty claim. The value of the proposed investigation must lie in a defensible comparison that the separate distinctions make possible. In particular, it should identify a sequence whose obstruction follows from explicit assumptions, retain a countercase where the rule serves its purpose, and explain what institutional evidence would be needed to investigate either possibility. That is a narrower undertaking than a general theory of language funding or a comprehensive allocation ethic.

3 Teaching Arrangements

A language-teaching grant acts upon an arrangement whose current lessons, learning relations and capacity to prepare further participants can develop differently. This section identifies that arrangement as the bearer of the proposed inquiry. It specifies the selected teaching operations, distinguishes present delivery from further participant formation, and examines the support relations through which an allocation becomes practically consequential. The argument uses constructed contrasts to establish what a later funding comparison needs to retain. Its object is a situated teaching arrangement, with a declared repertoire and purpose, rather than an entire language represented by a funding total.

3.1 Historical Teaching Configuration

A teaching arrangement includes persons and relations with different histories. An instructor brings acquired competence and earlier commitments; learners bring heterogeneous experience, purposes and available time; materials retain contributions whose meaning may depend on their provenance. A room, a remote connection, a schedule and an agreement about correction can jointly make an encounter possible. The organization named on a grant application is one participant in this configuration. Its legal or administrative identity does not by itself identify every relation that sustains the teaching.

The GR account treats such a configuration as historically extended. Earlier lessons can establish a shared example, an expectation about correction or a learner's readiness to undertake another task. A later lesson depends on selected parts of that history, while new participants and resources can reorganize the arrangement. The inherited spacetime account permits an effective representation that retains relevant events, dependencies and relations without treating each person as a timeless point [11, pp. 8–10]. A two-complex can organize a selected historical structure; that representation does not establish the ultimate ontology of the practice.

This description matters for allocation because an award changes particular relations within the configuration. It may make an instructor's time available, cover travel, enable access to a place or support preparation of teaching materials. It may leave other necessary relations untouched. Two equal awards can therefore have different effects without either difference proving irresponsible use. Their recipients

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may begin with different obligations, complementary resources or accessible forms of participation. A useful comparison identifies these differences at the resolution required by the decision.

The relevant bearer can also change while the arrangement continues. A learner can begin assisting an instructor; a former participant can return in a teaching role; one institution can host work previously supported elsewhere. Those changes require historical and practical evidence. They do not require every role to retain the same occupant, and they do not permit attribution of a new participant's competence merely because the organizational name persists.

3.2 Selected Teaching Operations

The present inquiry concerns a declared family of operations. Depending on the case, these can include recognizing a distinction in an unfamiliar account, explaining a proposed use, responding to a learner's misunderstanding or organizing practice in which another person can attempt and correct that use. The operation description should state what participants can actually do, in which circumstances and with what assistance. A recorded lesson is evidence about an episode; its presence does not establish all these capacities.

The selection serves a bounded comparison. An arrangement might prepare participants to lead one recurring narrative exercise while leaving advanced interpretation, curriculum design and other linguistic activities outside the claim. Such a finding can be educationally significant without amounting to complete mastery of a language or qualification for every teaching role. The target should retain the difference between performing an exercise, explaining it to someone else and responding responsibly when a learner's attempt departs from the prepared examples.

An assessment also needs a position for the learner. A participant can acquire a capacity while declining to become a teacher. Another may contribute as an interlocutor, organizer or interpreter. Further formation therefore concerns the development of specified possibilities of participation, with the actual role and undertaking made explicit. The grant's purpose can include preparing additional teachers, but it cannot obtain their future agreement by defining them as the outputs of current instruction.

The inherited account of generative capacity includes preservation, translation, maintenance and enabling other participants, alongside innovation [9, pp. 13–14]. That breadth avoids a narrow test in which only new expressions count as successful generation. Its application here remains selective: the paper investigates how an allocation affects a specified formation task. A claim about that task cannot exhaust the linguistic or cultural value of the surrounding practice.

3.3 Participant Formation

Current teaching delivery and further participant formation name different achievements. Delivery can provide worthwhile instruction now. Formation concerns what additional participants become able to undertake through a sequence of encounters and supporting activities. A programme can deliver many lessons while leaving all demanding teaching operations concentrated in one instructor. Another can conduct fewer independent lessons during a preparation period while developing additional participants' ability to lead them later. Neither description establishes which programme ought to receive a grant.

The relation between these achievements is not always a competition. Supervised participation can contribute to a present lesson while giving a developing participant experience of diagnosing misunderstanding. An instructor's explanation can benefit current learners and become a resource for later teaching. Some work has several effects, so a model that divides time into wholly separate activities needs a local justification. The allocation inquiry should first identify the activities and only then decide whether their uses compete.

A genuine tradeoff can nevertheless arise. An instructor who can immediately lead a familiar class may need additional preparation and feedback time to support someone else's first attempt. The latter arrangement can initially serve fewer learners, require a second person to attend or produce a less simple activity record. If a fixed package of paid time must cover both tasks, one schedule may postpone visible

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delivery to support a different later capacity. The choice is consequential even where everyone agrees that both goods matter.

The normative significance depends on the grant's purpose. A commission for a short series of urgently needed classes can properly prioritize present delivery. A scheme that also seeks additional teaching capacity needs to consider the sequence through which that capacity could develop. The formation objective does not erase the current learners' claim to the provision they were promised. It introduces an additional target whose compatibility with that promise must be examined.

3.4 Support Coupling

An allocation contributes to teaching through a realization process. Funding can pay for protected time, yet that time becomes useful only if the relevant instructor, learner, material and occasion can be brought together. A training place can exist while travel is unavailable. A learner can be ready to attempt a task while no authorized or willing person can provide the necessary feedback. The operative support is the configuration that makes the selected activity feasible.

The preservation taxonomy distinguishes a capacity target, its selected conditions and the relation through which they jointly support executable operations [12, Section 8]. This is inherited architecture. It permits the present paper to ask a precise question about an award: which previously unavailable operation becomes feasible, which existing operation becomes more secure, and which dependence remains? An amount of money is one input into that inquiry, not a universal measure of supported capacity.

Support can also be redundant or substitutable. An arrangement that loses one grant might continue through another accessible source, shared facilities or a revised schedule. Conversely, the apparent availability of alternatives can conceal restrictions: another fund may cover materials but exclude instructor time, or it may decide after the relevant encounter must occur. Dependence on the selected grant is therefore a proposition requiring evidence. It should neither be assumed from an applicant's request nor dismissed because some resource exists elsewhere.

A continued external contribution can be part of a successful arrangement. The formation of additional teachers need not eliminate the need for paid work, premises or learning materials. The selected transformation might concern who can undertake instruction, while the baseline support remains. Evaluation should state which dependence is intended to change. Otherwise a finding of improved participant capacity can be wrongly discredited for failing an unannounced requirement of financial independence.

3.5 Participant Work

The funder's allocation describes only part of the work required. Participants may also prepare examples, coordinate attendance, travel, arrange care or explain the arrangement to a reviewer. These activities can enable the grant's stated output while remaining outside the paid package. A proposal that appears feasible in the institution's accounts can therefore depend on a contribution that another participant has neither promised nor become able to make.

The proper response is to identify the contribution and its bearer. Some unpaid work may be voluntarily undertaken and valued; some paid work may remain excessively demanding. The distinction between paid and unpaid activity is important, but it does not supply a complete evaluation. Duration, predictability, discretion, substitution and the consequences of refusing the work can matter. A single resource total can obscure these differences.

Formation can redistribute burdens beneficially. Shared preparation may reduce an instructor's repeated work; additional competence may allow participants to exchange roles; funded coordination may make learning compatible with care commitments. Formation can also temporarily increase work for the most experienced participant. The relevant comparison follows who does what during the proposed pathway and after it, without treating either increased participation or reduced dependence as an automatic benefit for every person.

The inherited idea of generative return asks whether those who help sustain a formation acquire further capacities through it [9, pp. 40–42]. Here that question includes access to explanations, usable

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teaching resources and opportunities for increasing responsibility. It remains one consideration among several. An instructor's contribution does not create an unlimited duty to reproduce that contribution through successors, and a learner's participation does not assign the institution permanent authority over the learner's future role.

3.6 Allocation-Relevant Continuation

A grant comparison needs a continuation target that connects these distinctions. It can specify the participants, selected operations, supporting commitments and period within which an additional role is expected to become available. The target should also indicate what remains provisional: competence may require assessment, participation may depend on renewed agreement, and material support may be subject to a separate decision. Such a specification makes a proposal assessable without pretending that teaching develops according to a complete administrative script.

The description is consequential for evidence. If the target is current delivery, an accurate record of delivered activity can be central. If the target includes additional participant formation, the record needs to support an inference about the pathway and its results. A count can contribute to that inference, but its role depends on what was counted and what the arrangement was able to do during the counted period.

The teaching configuration consequently supplies the missing middle between award and outcome. It explains why an allocation can preserve current provision, prepare further participants, transfer work or fail to become usable support. These possibilities establish the object on which an eligibility rule acts. The next question concerns the evidence by which that action is justified, especially when earlier allocation decisions have helped determine what later evidence can exist.

4 Eligibility Evidence

Eligibility evidence connects a teaching history to an institutionally consequential decision. The central difficulty arises when the grant helps produce the later record on which continuing support depends. This section separates the questions that a record can answer, develops the temporal structure of a possible renewal obstacle, and states conditions for a more informative review. Its argument concerns the use of evidence within a declared allocation purpose. It preserves the positive role of accurate records and does not infer that an applicant deserves support merely because earlier provision would have improved its evidence.

4.1 Decision Questions

A grant record can support several distinct judgments. The institution may need to know whether an applicant belongs to an eligible category, whether a proposed activity can be carried out, whether an award was used as agreed, whether teaching occurred, or whether additional participants became able to undertake specified operations. These questions can require overlapping evidence while remaining different questions. An accurate answer to one does not automatically answer the others.

Suppose a report records the number of sessions delivered during a period. It can help establish compliance with an agreement to hold those sessions. With additional information, it may support an assessment of their reach or regularity. It does not by itself show what participants learned, who can now provide instruction, or which activity would have occurred under another schedule. Those further claims require an evidential bridge from the reported events to the selected capacity or counterfactual.

Eligibility also needs to be separated from selection. A threshold can identify proposals that may be considered without promising an award to every qualifying applicant. Ranking eligible applications adds a comparison of claims and feasible uses of the available budget. Release of an award can involve another condition, and renewal can return the arrangement to a new selection process. Calling all these events recognition would conceal which decision changes practical access to support.

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The preservation taxonomy supplies the more general separation between structural satisfaction, identification and valuation [12, Section 13]. Applied here, it prevents an eligibility judgment from becoming a verdict about the worth of a language or the competence of all its users. An arrangement can fail a particular evidence requirement while sustaining valuable activity elsewhere. It can also satisfy that requirement while leaving important questions about quality, future capacity or burden unresolved.

4.2 Support-Conditioned Records

A record can be partly shaped by earlier support without becoming false or irrelevant. An award may have enabled lessons that were actually delivered; those lessons can provide genuine evidence of responsible practice. Their history can strengthen an inference about what the arrangement can do with comparable support. The pertinent issue is the conclusion for which the record is used.

Consider two arrangements with different access to funded preparation. A comparison of their delivered classes may accurately identify which delivered more. It does not isolate an intrinsic difference in their ability to teach, because the comparison retains different enabling conditions. If the decision concerns immediate provision next month, the currently established capacity can still be relevant. If it concerns which arrangement could develop further participants through a supported pathway, the earlier record may leave the decisive question open.

This distinction gives history an evidential role without demanding complete genealogy. The assessment needs those earlier allocation and participation events that bear on the proposed inference. It may need to know that an applicant lacked a paid preparation period or that a successful programme already received one. It need not reconstruct every event in the language's history. The relevant history is selected by the question being asked and remains subject to ordinary standards of evidence.

The inherited account of endogenous weighting already explains how successful contributions can reshape training, documentation, recognition and access to future opportunities [9, pp. 19–22]. The present application concerns a narrower coupling: a support decision changes the operations from which the next qualifying record can be produced. The existence and direction of that dependence require investigation. They cannot be inferred merely from the recurrence of applications and awards.

4.3 Review Timing

A renewal review occurs at a particular point in a formation pathway. At that point, some evidence can already be available while other evidence depends on work still to be done. A participant may have begun supervised practice without yet being ready to lead an activity independently. A preparation record can establish that the agreed encounter occurred and that feedback was available, while leaving the later competence judgment appropriately unresolved.

The timing becomes problematic when a rule requires the completed later achievement before authorizing the support needed to reach it. In a constructed case, an initial package permits either a period of current delivery or a period of supervised preparation. The latter can contribute to a further participant's later role, but it produces less of the delivery evidence used at the first review. If that review controls access to the next required package, the rule can prevent a supported sequence that its stated formation purpose would otherwise admit.

Several premises are doing work. The preparation route must have a credible connection to the later task. The required support must actually depend on the decision, rather than being available through another feasible channel. The review must occur before the relevant result could reasonably be demonstrated. The alternative schedule must satisfy the same declared resource and participation constraints. Without these premises, low evidence may indicate an unsuitable proposal, a different cause of failure or a comparison that simply adds resources.

The resulting obstruction is specific. A path can be feasible for the arrangement and excluded by a rule; a path can also be infeasible independently of the rule. Relaxing an evidential requirement does not create an absent instructor, willing learner or adequate teaching method. The allocation analysis gains

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force by distinguishing these cases, since only the first supports the claim that changing the decision schedule would reopen the selected route.

4.4 Intermediate Evidence

A review can remain demanding while assessing evidence appropriate to the stage. An interim record might establish that the proposed participants undertook supervised practice, that the identified difficulties received feedback, that the next teaching role has been specified and that the required support remains available. Such evidence can warrant continuation of a bounded inquiry without claiming that its final target has already been reached.

The milestone must bear on the pathway, rather than merely resemble its administrative description. Attendance at a meeting can matter when the relevant practice actually occurred there. A completed form can record a careful assessment or simply repeat an intended outcome. The evidential task is to connect the record to an event and the event to the reason for the next allocation. The connection may involve participant accounts, observed attempts or independently examined work; the appropriate method depends on the activity.

The possibility of negative evidence is essential. An attempt may reveal that the proposed step is too demanding, that feedback is inaccessible or that participants prefer another role. A useful interim review permits the route to change, pause or end for those reasons. If every possible result is described as progress toward the same entitlement, the milestone ceases to discriminate among relevant histories.

The conditional case for a bridge therefore rests on an accountable sequence. An institution that has adopted a defensible formation purpose has a reason to consider adequate early evidence for a later stage when that stage cannot yet provide final evidence. The reason remains limited by the available budget, competing purposes and the quality of the pathway. The institutional commitment is to an assessable next step, not to treating an intended future achievement as already established.

4.5 Consequential Classification

An eligibility category can become consequential through the operation it conditions. A report is received, interpreted under a criterion, and used to authorize, delay or decline a specified contribution. The classification and the award are distinct events. A category can remain purely descriptive when nothing follows from it; an award can also be determined by constraints that the category does not represent.

This distinction follows the prior power account's treatment of a judgment-conditioned transformation [11, pp. 30–33]. The judgment does not itself equal the transformation. The current inquiry must identify who can act upon the classification, what support is actually released and how the later configuration changes. A published rule describes one part of that structure. Its actual effects depend on implementation, alternative support, participant response and the work that becomes feasible.

The later record can then partly reflect the decision's consequences. An arrangement denied the next preparation stage may show little subsequent evidence of additional teaching capacity. That absence can be consistent with the decision having closed the relevant route. It can also reflect other difficulties. Treating the later absence as independent confirmation that the original application lacked potential would require an argument separating these explanations.

The converse matters equally. Continued support followed by improvement does not establish that the selected rule was uniquely responsible, that every supported proposal would improve or that the same investment should be repeated indefinitely. The record can support a bounded finding about a realized pathway. Broader causal and comparative claims need appropriate counterfactual evidence. This symmetry prevents the feedback argument from turning every adverse result into institutional fault and every favorable result into proof of the preferred policy.

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4.6 Accountable Continuation

Good accountability can protect participants as well as funders. A clear delivery commitment can keep a project from indefinitely postponing the classes people were promised. A record of who performed the work can expose an arrangement that places unsupported obligations on one instructor. A requirement to explain an unsuccessful stage can help distinguish a revisable teaching problem from misuse of resources. The fact that evidence requirements shape activity does not make these functions suspect.

The question is whether a requirement remains adequate to the declared task and the history it helps produce. A delivery measure can be well suited to a short provision grant and poorly suited as the sole early test of a formation pathway. A detailed developmental report can be useful for a complex project and unnecessarily burdensome for a simple, completed service. Evidential ambition should be justified by the decision that needs to be made.

A responsible comparison therefore retains both the informational benefit and the work required to supply it. Additional interpretation can clarify a proposal while using resources otherwise available for teaching. Simplification can reduce burdens while discarding consequential distinctions. Neither more documentation nor fewer requirements has an unconditional advantage. The appropriate inquiry identifies which distinctions could change the decision and whether participants can provide them at a proportionate cost.

Eligibility feedback becomes philosophically significant at this junction. The institution is assessing an arrangement whose later evidence is partly conditional on its present decision. Adequate assessment can recognize that dependence while retaining genuine tests, finite commitments and the claims of others. The formal investigations that follow make selected feasibility and timing relations explicit; normative judgment must still determine which of the demonstrated possibilities deserves support.

5 Historical Allocation Architecture

A teaching arrangement reaches a grant decision with a history of work, support, interpretation and earlier decisions. This section specifies how the analysis retains that history while constructing a bounded allocation model. It separates a represented spacetime, the record available at review and the operation through which a judgment changes later support. The construction supplies the primitives for the continuation calculations rather than a universal ontology of teaching or institutions.

5.1 History-Bearing Arrangements

Let $\mathcal{H}$ be a declared family of finite histories. A history $h \in \mathcal{H}$ records selected participants and support relations together with relevant occurrences: preparation, teaching, observation, reporting, interpretation, award and review. It also records the occurrence order and any justified relations of overlap or dependence. Its retained content follows the inquiry. A transcript, expenditure list or sequence of test scores alone need not identify such a history.

An extension relation $h \preceq h'$ means that h' continues the retained occurrence record of h. The relation concerns a specified historical representation. It leaves room for later reinterpretation, correction and changed significance through further events. A subsequent review can amend the accepted account without being identified with the earlier teaching event itself.

A possible realization assigns to h a labelled two-complex K_h. Vertices identify selected configurations or interfaces, edges identify transitions or dependencies, and faces retain declared relations among paths. For example, a face can record the association between a reported teaching episode and the evidence on which its acceptance depends. The face does not automatically assert that reporting and teaching commute.

This representation inherits the GR account of history-bearing spacetime and regional realization [10]. It has more structure than a point representing an applicant's current score. A spin foam would require additional labels and amplitudes; none is imposed here. The object of inquiry remains the histor-

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ical teaching arrangement, while the complex is one way to retain selected attachments and occurrence relations.

5.2 Historical Anchors

Suppose the inquiry fixes a retained earlier subhistory h_-, with representation K_-. Candidate continuations carry embeddings $\iota_h : K_- \to K_h$. For a comparison map $f_{hh'} : K_h \to K_{h'}$, the inherited anchor condition has the following form.

$$f_{hh'} \circ \iota_h = \iota_{h'}. \tag{1}$$

Equation (1) preserves the declared occurrence structure. It permits different future attachments, participant roles and operation families outside that anchor. A decision to withhold another grant can remove a funded continuation without deleting the record of the earlier work. Conversely, retaining that record does not retain the operation's practical availability.

At a finite cellular resolution, an admitted change must retain well-typed attachments and the chain identity $\partial_1 \partial_2 = 0$. That identity is a structural consistency test. It does not establish the truth of a report, the causal efficacy of a grant or the legitimacy of an award. The earlier interface-substitution study gives a worked attachment construction [13]; the present inquiry uses its discipline without reproducing the gluing proof.

Review timing enters this history explicitly. If preparation must precede a coached teaching episode, and a renewal decision occurs between them, the order is part of the represented arrangement. A retrospective observer who sees only completed delivery can miss the intermediate dependency. The temporal question is therefore about which operations remain available across that particular review, not merely about whether two endpoints look similar.

5.3 Records And Judgments

Let O be a set of accessible report records and $\rho : \mathcal{H} \to O$ the selected readout. The readout can retain counts, narratives, attendance, assessment, expenditure or evidence of supervised practice. Those are different kinds of information; their collection in one record does not make them commensurable. The model specifies which distinctions the record retains.

A record has an interpretation and an institutional status. A reported activity can be present but unverified, verified but outside the criterion, or accepted for one payment decision while insufficient for another. A partial judgment map makes that distinction explicit.

$$J_t : \mathrm{Dom}(J_t) \subseteq O \longrightarrow \mathcal{J}_t. \tag{2}$$

Here $\mathcal{J}_t$ is the declared judgment carrier at review t. It can distinguish eligibility, a request for clarification, conditional continuation or refusal where the application justifies those statuses. Equation (2) does not identify an unavailable judgment with a negative one. A scheme requiring a determinate outcome would need to specify how incomplete records are handled.

The next step is an actual support decision, not an inference automatically produced by the existence of a judgment. The prior power manuscript already distinguishes comparison, judgment and judgment-conditioned transformation [11, Section 6.6]. In the grant specialization, an installed rule selects a package, changes access or establishes a future condition. Whether the rule is operative requires evidence of its implementation.

5.4 Allocation Feedback

Let $\mathcal{G}$ be a declared resource-package carrier. A package can specify paid time, materials, access or an earmarked amount. Its coordinates must be interpreted; a numerical amount of one transferable resource does not measure all relevant burdens. For a judgment j and package g, an available operation a can extend the current history to several possible successor histories.

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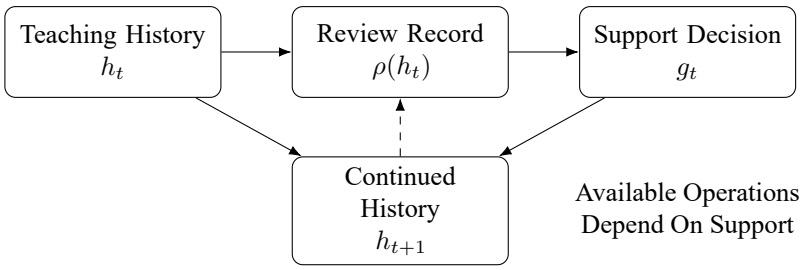

Figure 1: Historical Allocation Feedback. The review record is a selected manifestation of an existing history. An implemented support decision conditions later operations. The dashed return denotes the next review's record, not a return to the same historical occurrence.

The local dependency is recorded without assuming a probability law.

$$\begin{aligned} o_t &= \rho(h_t), \qquad j_t = J_t(o_t), \\ g_t &= R_t(o_t, b_t), \qquad h_{t+1} \in F_t(h_t, g_t, a_t), \\ o_{t+1} &= \rho(h_{t+1}). \end{aligned} \tag{3}$$

In Equation (3), b_t is a remaining budget in the later numerical specialization, R_t includes the implemented consequences of the judgment where it is defined, and F_t is a nonempty successor correspondence on its specified operation domain. The next section provides the full types and budget update.

The equation distinguishes three possible effects. The package can change what can be done. The operation can change the configuration and its next record. The record can change the next package. Establishing the complete feedback requires all three connections. An analyst's correlation between earlier awards and later success establishes none of them by itself.

Figure 1 locates the review within a possible continuation history. Its arrows identify declared operation and information relations. They do not assert an empirically identified causal law.

5.5 Criterion Scope

A criterion can be entirely adequate for its declared task while inadequate for another. An accepted expenditure record can establish compliance with a spending condition. It need not establish that a new participant can teach independently. A witnessed teaching episode can support a capacity claim while leaving its reproducibility or dependence on continuing assistance unresolved. The model should specify which question the review is answering.

The same distinction constrains causal interpretation. An arrangement lacking a qualifying record might lack the relevant capacity, lack the conditions for exercising it, lack access to recognized evidence, or be at an earlier stage of formation. A history can help distinguish these alternatives without guaranteeing a unique explanation. The normative-formation account already identifies the role of access, legibility and recognition in later participation [9].

The present formal question is deliberately narrower than deciding which explanation is true in an actual programme. Given declared operations, evidence and support rules, it asks which continuation paths remain available. That conditional result can then identify the empirical and normative premises a real allocation would need.

6 Renewal-Constrained Operations

The historical architecture becomes analytically useful when an allocation decision changes a specified operation domain. This section constructs a finite-horizon representation of that change. Its purpose is to distinguish an available grant, an available participant action and a successful continuation. Each distinction matters when a report is required before the support needed to produce a later report becomes available.

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6.1 Decision States

Fix decision dates $t = 0, \ldots, N-1$, with terminal date N. Let $\mathcal{H}_t$ be the declared histories ending at date t. The sets are tagged by their dates even when their current manifestations are identical. A decision state $x = (h, b)$ consists of a history $h \in \mathcal{H}_t$ and a remaining budget b in $B \subseteq \mathbb{R}_{\geq 0}$. Write X_t for the admitted pairs. The budget is an identified resource account. It might measure a particular fund's purchasing authority. It does not include unpaid work, other funds or participant burdens unless these have explicitly been incorporated into its unit and accounting boundary.

Let $\mathcal{G}_t$ be a package set with a cost map $c_t : \mathcal{G}_t \to \mathbb{R}_{\geq 0}$ and a zero package 0_t. A package specifies support and its conditions sufficiently to determine the modeled operations. Two grants with equal numerical costs can have different permitted uses, payment dates or staffing obligations. Their equality as costs does not identify them as packages. Assume $c_t(0_t) = 0$; other zero-price access arrangements require their own entries rather than automatic identification with refusal.

For a record carrier O_t, let $\rho_t : \mathcal{H}_t \to O_t$ be the selected readout. The implemented rule has a declared domain.

$$R_t : D_t \subseteq O_t \times B \longrightarrow \mathcal{G}_t, \qquad c_t(R_t(o, b)) \leq b. \tag{4}$$

Equation (4) describes the rule being investigated, including its response to the available budget. It is fixed before the continuation calculation. The analyst cannot choose a favorable grant after seeing which unobserved history produced the record. If the institution uses further information, that information belongs in O_t. If the decision is discretionary, a correspondence or a separately specified institutional policy is needed instead.

The finite calculation below concerns decision-ready states whose record-budget pairs lie in D_t. Assume that every admitted successor pair belongs to X_{t+1}, including closure of the remaining-budget carrier under the stated costs. At nonterminal dates these successors must also have defined rule responses. If a judgment can remain unavailable, represent that outcome by an explicit deferral or failure state with its own rule response; do not discard it from the successor set. A real arrangement can instead request clarification, defer an award or admit an appeal. Those are operations with times and costs. They must be represented if their availability matters; omission does not prove their practical absence. A missing judgment in Equation (2) is thus not silently replaced by a refusal.

6.2 Participant Actions

For $h \in \mathcal{H}_t$ and $g \in \mathcal{G}_t$, let $A_t(h, g)$ be the actions available to the represented participants. For an admitted triple (h, g, a), the successor correspondence has the following type.

$$\varnothing \neq F_t(h, g, a) \subseteq \{h' \in \mathcal{H}_{t+1} : h \preceq h'\}. \tag{5}$$

Equation (5) permits several outcomes of the same supported action. A funded training meeting can take place without producing the intended capacity. A teacher can withdraw, a facility can become unavailable, or evidence can remain inconclusive. A deterministic example sets F_t to a singleton as a simplifying premise; certainty is not a general property of teaching.

The distinction between support and action also preserves agency. An award does not compel someone to teach, accept supervision or provide cultural knowledge. Willingness and permissions are conditions of membership in $A_t(h, g)$. Where continuing participation can be withdrawn, the outcome set or action domain must retain that possibility. A model that guarantees formation by deleting legitimate withdrawal describes a narrower arrangement than its language might suggest.

For $x = (h, b)$, abbreviate $g = R_t(\rho_t(h), b)$. The resulting state successors are defined in Equation (6).

$$\Phi_t^R(x, a) = \{(h', b - c_t(g)) : h' \in F_t(h, g, a)\}, \qquad a \in A_t(h, g). \tag{6}$$

Equation (6) charges a package once at its declared purchase date. A commitment extending beyond that date must be included in its cost or represented by a liability state. Charging only the first payment while treating later services as free would make the budget comparison invalid.

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Neither refusal nor the zero package implies that all activity ends. An arrangement may continue using reserves, voluntary work, another institution or a different teaching practice. Those possibilities belong in $A_t(h, 0_t)$ and F_t. The countermodel below excludes alternative funding only as an explicit local premise. Its conclusion is correspondingly conditional, not a general claim that grant recipients lack independent capacities.

6.3 Targets And Admissibility

A target is a subset $G \subseteq X_N$ with an interpretation independent of whether the grant rule rewards it. It might require a witnessed capacity to perform a particular teaching role, a specified continuing service, or their conjunction. The interpretation must identify what counts as a witness and which dependencies remain permitted. An observed activity count alone becomes a formation target only through an additional justified relation.

Let $C_t \subseteq X_t$ describe conditions required throughout the modeled continuation. These can include resource limits, protected permissions or minimum current provision. They are stated premises, not quantities derived by maximizing a field variable. Terminal success requires both G and C_N. The calculation can therefore represent a formation path that is technically possible but inadmissible because it violates a separately specified present obligation.

An action policy selects an available action from the current decision state and date. It does not choose which successor occurs. A *sure* policy reaches $G \cap C_N$ while remaining in the C_t sets for every successor admitted by the model. This is stronger than the existence of a successful possible path. It is also different from a probability of success: no probability law has yet been defined. If the outcome set includes rare disruptions, the sure requirement may be demanding. That is a feature of the selected question rather than a reason to reinterpret the result as an empirical success rate.

6.4 Observation Conditions

The mathematical policy initially has access to x. Participants and administrators may have less information. For a practical decision-maker, let $\eta_t : X_t \to Z_t$ specify its available observation. An executable policy must choose the same action for states with the same observation whenever that observation does not distinguish them. Action names must also denote the same executable operation across those states.

This condition matters even when the grant rule itself uses a perfectly accessible report. Rule implementation and successful action selection are different tasks. The earlier recovery-record study develops this general distinction between statewise feasibility and an observation-sufficient selector [16]. The present paper inherits it. Its finite counterexample will make the relevant stages observable, so that the renewal obstruction cannot be blamed on an extra hidden state imposed for the proof.

More generally, the full-information calculation supplies an upper bound on executable success. Failure even with full state information establishes failure for every more restricted observation policy. Success with full information requires a further selector check before supporting a practical proposal. This asymmetry is especially useful when an analyst wants to identify an obstruction without pretending to know what participants can infer from incomplete records.

6.5 Comparison Discipline

Two rules can now be compared on a common state space, operation family, cost system and target. Holding these fixed isolates a rule-relative difference in admitted continuations. If changing the rule also changes reporting costs, staffing commitments or willingness, the revised operation family must be made explicit. The shared notation does not guarantee a controlled comparison.

Likewise, equal expenditure is a limited equality. It identifies costs inside the selected account. It need not identify waiting burdens, risk exposure or the claims of other applicants. A common envelope is useful for excluding the simple explanation that one construction succeeds only because its modeled

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budget is larger. It cannot establish that the two allocations are equivalent in every ethically relevant respect.

The next section gives a finite criterion for sure formation viability relative to an installed rule. Its result is a conditional statement about operations. The assessment of the target, the evidence needed to accept the operations and the allocation of scarce resources remain distinct questions.

7 Formation Path Viability

This section establishes a finite backward criterion for the continuation question introduced above. It makes the dependence on review rules explicit and separates an operational obstruction from a judgment about a grant's fairness. The construction is an elementary finite dynamic-programming representation, offered as one model of the historical problem rather than a general law of generativity.

7.1 Backward Continuation Sets

Assume that the sets X_t, the available action sets and their successor sets are finite. Restrict attention to decision-ready states. Define $V_t^R \subseteq X_t$ backward from the terminal date.

$$\begin{aligned} V_N^R &= G \cap C_N, \\ V_t^R &= \left\{ x = (h,b) \in C_t : \begin{array}{l} \text{there exists } a \in A_t(h, R_t(\rho_t(h), b)) \\ \text{such that } \Phi_t^R(x,a) \subseteq V_{t+1}^R \end{array} \right\}, \quad t < N. \end{aligned} \tag{7}$$

All successor sets in Equation (7) are nonempty by the preceding domain assumptions. That requirement prevents an unavailable operation from passing the test through vacuous inclusion. The budget update is already contained in Φ_t^R, and admissibility is checked at each date.

Proposition 7.1 (Finite Rule-Relative Viability)**.** *A decision-ready state $x \in X_t$ belongs to V_t^R if and only if a full-information action policy can ensure the declared terminal target from x while respecting all C_s, for $s = t, \ldots, N$, against every admitted successor.*

Proof. At $t = N$, success is exactly membership in $G \cap C_N = V_N^R$. Assume the equivalence at date $t+1$. If $x \in V_t^R$, the recursion provides an available action a whose every successor belongs to V_{t+1}^R. For each successor the induction hypothesis supplies a successful continuation policy. Because the successor state is available to the full-information policy, these choices can be combined into one contingent policy after the first action. The current state belongs to C_t; the continuation respects the later constraints.

Conversely, let a successful policy start at x. It must have $x \in C_t$ and select an available first action a. Every state in $\Phi_t^R(x,a)$ can occur under the stated successor interpretation. Restricting the policy to any one of those continuations must remain successful. By the induction hypothesis, every such successor lies in V_{t+1}^R. The action therefore witnesses membership in V_t^R. Backward induction completes the proof. □

Proposition 7.1 derives no capacity from notation. The result is as informative as the specified operation domains and target witnesses. Its advantage is logical transparency: the point at which a rule removes a required continuation can be located rather than hidden in an overall success score.

7.2 Record-Uniform Action Selection

For a viable state define its winning first actions.

$$W_t^R(x) = \{a \in A_t(h, R_t(\rho_t(h), b)) : \Phi_t^R(x,a) \subseteq V_{t+1}^R\}. \tag{8}$$

Equation (8) is nonempty for $x \in V_t^R$. If a decision-maker must handle every state in a specified viable observation class $S \subseteq V_t^R$, a common first action exists exactly when $\bigcap_{x \in S} W_t^R(x)$ is nonempty. This statement concerns the first action. Later observation classes require their own common choices.

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A sufficient global condition is a compatible choice from every nonempty intersection over the fibers of η_t restricted to V_t^R, at each date. Such choices produce an observation-based policy preserving the viable sets. Necessity for a particular initial state need only be checked on the observation classes that can actually be reached under its policy. Demanding compatibility on every imaginable state can be unnecessarily strong.

This qualification prevents an easy confusion. A grant reviewer might correctly apply one eligibility rule to a shared record, while the represented arrangements require different next actions. More information can help select an operation, but it cannot restore an operation that the installed support rule has removed. Epistemic sufficiency and support sufficiency interact without becoming the same property.

7.3 Conditional Exclusion

Let R and $\widetilde{R}$ share the declared histories, accounting boundary, admissibility conditions and target. Their packages can differ. A state in $V_t^{\widetilde{R}} \setminus V_t^R$ has a specific interpretation: the alternative rule admits a sure successful continuation that the original rule does not admit, under the common model assumptions.

Calling this a *rule-relative exclusion* does not imply that the rejected applicant deserves the alternative award. The target may be incomplete, the action model optimistic, or the alternative allocation incompatible with another applicant's stronger claim. It does establish that failure under R cannot, within that model, be identified with an unconditional inability to achieve the selected target.

There is also a useful distinction between removing one path and removing every path. An early review may interrupt a particular preparatory sequence while leaving another supported route open. That is a path obstruction. Membership failure in V_t^R is stronger: every policy lacks a sure successful continuation. The finite example below proves this stronger result for its complete action list. A real investigation would need evidence that its action list is adequate before making the same claim.

7.4 Rule Extensions

A limited monotonicity result identifies one transparent sufficient condition for preserving viability. Suppose every R-available action at every state has an alternative $\widetilde{R}$-available action with the same successor state set, including the same budget updates. Additional actions are permitted. Keep G and all C_t unchanged.

Proposition 7.2 (Continuation-Preserving Extension)**.** *Under these assumptions, $V_t^R \subseteq V_t^{\widetilde{R}}$ at every date.*

Proof. The terminal sets are equal. Assume inclusion at date $t+1$. For $x \in V_t^R$, choose the witnessing action in Equation (7). Its corresponding alternative action has exactly the same successors. They lie in V_{t+1}^R and hence in $V_{t+1}^{\widetilde{R}}$. This action witnesses $x \in V_t^{\widetilde{R}}$. □

Proposition 7.2 does not say that a more permissive grant rule is always better. The premise fails if additional reporting, conditions or expenditures alter the old successors. It also says nothing about opportunity costs outside the common state space. Its role is to state precisely which operational preservation would warrant a monotonic conclusion.

This distinction becomes important in practical proposals for flexible grants. Flexibility can preserve a formerly available operation, replace it with a new obligation, or transfer uncertainty to a participant. The ordinary word alone identifies none of those mathematical relations. A model should therefore compare actual packages and commitments instead of treating flexibility as an intrinsically beneficial scalar property.

7.5 Empirical Interpretation

Applying the recursion requires at least three kinds of evidence. First, the available actions and their resource requirements must be defensible. Second, the target must have a credible witness independent of

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Table 1: Constructed Period Operations. Each transition appends its operation to the full history. The pair (z, y) is a summary used only for this finite calculation.

Operation	Availability	Successor Summary
D: Current Delivery	Supported period; any stage	$(z, 1)$, with stage unchanged
P: Preparation	Supported period; $z = 0$	$(1, 0)$
C: Coached Completion	Supported period; $z = 1$	$(2, 1)$
W: Waiting	Every stage; any package	$(z, 0)$

the criterion that controls funding. Third, the review rule must actually condition the relevant operations, rather than merely describe an administrative classification.

Each requirement can fail. Participants may find an unmodeled route, a supposed formation stage may not lead toward the target, or another fund may bridge the gap. Such findings change the model and its result. They do not invalidate the conditional proof, but they prevent its unmodified application.

The epistemic benefit of the formalization is therefore modest and specific. It makes the premises of a timing argument available for criticism. It also shows why the absence of a successful funded trajectory cannot alone identify the source of failure. An operation model and an allocation history are needed to distinguish infeasibility, rule-relative exclusion, uncertain outcomes and an unsupported interpretation of the target.

8 Renewal Timing Countermodels

A finite construction now isolates the relation between a preparatory stage and the review that precedes its possible completion. The construction is deliberately small enough to enumerate completely. It establishes a conditional timing obstruction, supplies a countercase for a different target and identifies the normative question the calculation leaves open. It is not a reconstruction of any programme discussed in Section 12.

8.1 A Two-Stage Formation Path

Consider two activity periods and an envelope of two grant units. Each supported period costs one unit. The initial state has formation stage $z = 0$ and delivery record $y = 0$. The stage carrier is $\{0, 1, 2\}$: 0 means that the specified preparation has not been completed; 1 means that it has been completed; 2 means that the further coached episode has supplied the stipulated formation witness. These are labels for different represented histories, not measurements of linguistic worth.

The binary report y records whether a qualifying current-delivery episode occurred in the most recent period. It does not retain the formation stage. The full history does. For this example the participants know their stage, and a prospective arrangement can verify preparation through a separate milestone record. Thus the construction does not rely on inaccessible participant information.

Both comparison rules have access to the same record containing delivery and the verified milestone; the strict rule uses only its delivery component. The stipulated package cost includes the work of producing and checking that milestone in both comparisons. This premise holds information acquisition fixed. An application with different verification costs would need a revised resource comparison.

Table 1 specifies the complete operation list. An awarded package supports either current delivery or the next appropriate formation operation. Waiting remains possible with or without support. There is no alternative funding or unpaid substitute inside this example. The package pays all modeled obligations of its operation; accepting a grant does not oblige a participant to choose that operation.

The completion operation is a strong selected premise: it deterministically supplies the specified witness after preparation. Actual teaching need not do so. Replacing it with uncertain outcomes would weaken a sure success conclusion and require the correspondence in Equation (5). The deterministic choice here makes the effect of the grant rule independently visible.

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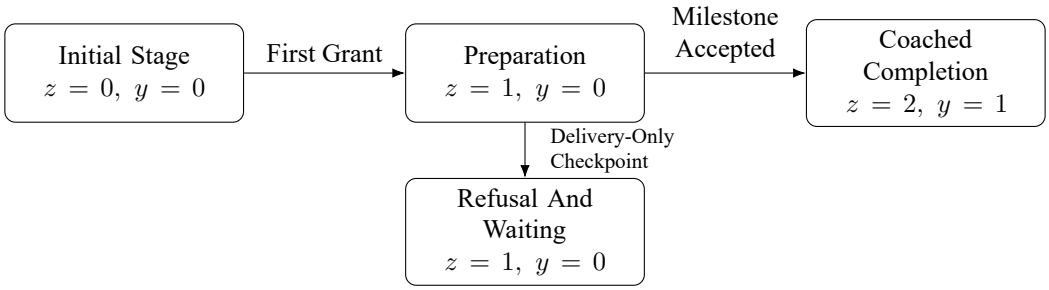

Figure 2: A Constructed Renewal Prefix. A supported preparation episode can supply an intermediate formation milestone while failing a delivery-only checkpoint. The alternative grant admits one further supported operation. The diagram asserts only the operation relations stipulated in the example.

8.2 A Delivery-Only Checkpoint

Both comparison rules award the first unit. The strict rule awards the second unit exactly when the first period's report has $y = 1$. The prospective rule also admits the verified preparation milestone $z = 1$ for this single continuation decision. Its additional acceptance is bounded by the two-period horizon and the same budget. Neither rule guarantees an indefinite series of awards.

Take the terminal target to be $G = \{z = 2,\ y = 1\}$. All states inside the declared resource envelope are otherwise admissible.

Proposition 8.1 (Finite Renewal Obstruction)**.** *From the initial stage $z = 0$, the target is unattainable under the strict rule and attainable under the prospective rule. The prospective successful path and a strict-rule delivery path each spend exactly two grant units.*

Proof. At the first period, the possible actions are D, P, W. If P is chosen, the successor has $z = 1, y = 0$. The strict rule withholds the second unit, so only W remains and stage 2 cannot be reached. If D is chosen, the successor has $z = 0, y = 1$ and the second unit is awarded. The remaining operations are D, P, W; none reaches stage 2 in one period. If W is chosen first, the second unit is withheld and the stage remains 0. These cases exhaust the operation list.

Under the prospective rule, P produces the accepted preparation milestone. The second unit supports C, whose successor has $z = 2, y = 1$. The path P, C spends two units. Under the strict rule, D, D also spends two units and produces delivery in both periods, while remaining at stage 0. □

Proposition 8.1 shows a difference in attainable formation, not a difference in total modeled expenditure between these two displayed paths. A strict-rule path denied its second grant spends only one unit. The result does not erase that saving or determine the use of the unspent unit. It identifies the temporal structure of the formation operation.

Figure 2 displays the successful reference path and the position of its restrictive checkpoint. Its lower branch is a refusal followed by waiting, which remains a history even though it does not reach the target.

8.3 Target Dependence

Change the target to qualifying delivery in both periods. The strict-rule path D, D succeeds. The protected P, C path fails the first-period requirement. If current delivery is independently necessary, for example because an already accepted obligation cannot be deferred, the formation path may be inadmissible.

This countercase is analytically necessary. It shows that the strict criterion is not intrinsically meaningless or universally self-defeating. The conflict appears when a programme seeks a formation sequence whose first operation does not yet produce the output required for its continuation. A judgment about the programme must establish which objective is actually adopted, how current obligations are ranked and whether another route is feasible.

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Similarly, replacing a delivery record with a milestone label does not establish improvement. If P produces paperwork without the preparation required for C, the prospective rule has admitted an unsupported path. Its formal success depended on an independently meaningful transition, not on renaming the criterion.

8.4 Review Resolution

A later review can observe a completed P, C sequence that an earlier delivery-only checkpoint interrupts. Combining the two periods into one committed package therefore changes the available continuation even when the total envelope is unchanged. That change has costs: the funder commits resources before observing the final result and needs a justified interim account of progress.

The relevant temporal distinction is consequently finer than short-term versus long-term funding. A long award can contain restrictive payment gates. A short award can recognize a preparation milestone. The location, evidential object and implemented consequences of each review determine whether the required prefix remains supported.

The finite model also clarifies the role of randomness. Under its deterministic operation list, randomizing among first actions does not create a target-reaching strict-rule path: every branch has already been exhausted. That observation concerns sure attainment of this target. It does not compare expected welfare or probabilistic fairness, for which no outcome distribution or social evaluation has been specified.

8.5 Competing Claims

Suppose another teaching arrangement can use the same two units for urgent current provision. Funding the first arrangement's complete formation path can then prevent funding that provision. The countermodel gives no ordering between those claims.

This is not an incidental limitation. A local opportunity can be real without being the opportunity that should receive the available resources. To obtain an allocation conclusion, the state space would need to include the other arrangement, the joint budget and an explicitly defended criterion of comparison.

The practical diagnostic is therefore conditional: before treating a failed renewal record as evidence that an arrangement cannot form further participants, examine whether the rule interrupts a credible supported sequence. Before recommending its support, examine the quality of that sequence's warrant and the claims displaced by the commitment. The first inquiry is operational; the second adds epistemic and normative premises that the finite proof cannot supply.

9 Grant-Supported Activity Fields

A field investigation can test whether an eligibility rule removes a feasible supported activity history even when recipients can mix activities and move resources among settings. This section constructs one such representation. It specifies the conversion of a grant into provisioned services, separates current delivery from supervised practice, and derives the regional balance needed for a later comparison. Section 10 then proves a renewal obstruction within a common two-period budget. The fields represent selected realized activities. Their evolution supplies neither a complete account of participant formation nor a measure of an applicant's claim to public resources.

9.1 Service Provisioning

The starting object is a historically organized teaching arrangement with identified participants, retained linguistic operations, facilities and service commitments. Select a finite family of active settings $I = \{1, \dots, m\}$. A site can be a teaching group, a supervised-practice setting or another declared interface. It need not coincide with one person or geographical place. The model retains those settings during a

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bounded comparison interval. A subsequent participant's formation can change the historical arrangement and its available operations; that change requires an additional history extension rather than a reinterpretation of a numerical coordinate as a person.

Let $q_i(t)$ and $r_i(t)$ be nonnegative provisioning rates for two service families. The first supplies current teaching delivery. The second supplies supervised formation-practice encounters in which prospective further participants can exercise selected operations with guidance. Provisioning includes the specified preparation, staffing, access and coordination necessary to deliver that service. It is distinct from an announced award or a nominal permission to spend. If suitable staff cannot be engaged, participants do not consent, or the needed practice is unavailable, the corresponding control can lie outside the arrangement's actual conversion domain.

A financial interface would ordinarily attach prices to those provisioned packages. With declared positive unit prices p_D, p_F and available expenditure rate $G(t)$, the selected monetary constraint is given in Equation 9.

$$p_D \sum_{i=1}^{m} q_i(t) + p_F \sum_{i=1}^{m} r_i(t) \le G(t). \tag{9}$$

Equation 9 is an assumed local conversion interface. It does not identify money with competence, nor make all kinds of teaching labor interchangeable. Different prices, fixed setup costs, indivisible commitments or incompatible staff qualifications can require another feasible control set. The countermodel below selects equal normalized package prices and a pool of m units per funded period of unit duration. This permits a transparent comparison of timing without claiming that the prices are empirically equal.

A complete package also accounts prospectively for its service obligations during the comparison horizon. Earlier provisioning can establish sessions or support arrangements that continue after a new input stops. Their later activity is then part of a commitment already funded or of explicitly retained background provision. It is not costless labor generated by the differential equation. If the intended service requires unpriced continuing work, the financial interface is incomplete and the claimed equal-budget comparison has not been established. This requirement matters especially when a refusal prevents new funding but previously arranged encounters continue briefly.

Write $g(t) \in \{0, 1\}$ for whether the current period is funded. The normalized pool condition used from this point onward is Equation 10.

$$q_i(t), r_i(t) \ge 0, \qquad \sum_{i=1}^{m} \big(q_i(t) + r_i(t)\big) \le m g(t). \tag{10}$$

Equation 10 allows unequal regional allocations and arbitrary mixing of the two service families. Controls may depend on the observed or, for the stronger impossibility comparison, the complete preceding history. The derivations apply to every resulting admissible control history. They do not presume that an actual applicant possesses complete information. A selected feedback rule must additionally produce a well-posed evolution and have an available practical implementation.

9.2 Realized Activity Channels

For each site let $u_i(t) \ge 0$ denote the normalized intensity of realized teaching delivery, and let $v_i(t) \ge 0$ denote the normalized intensity of realized supervised formation-practice encounters. The normalization uses a declared service and observation interval. The second field therefore concerns actual practice activity, rather than an unexercised stock of competence or mere advertised opportunity. An empirical implementation would need to identify what counts as an encounter, whether participants can use it, and how heterogeneous encounters can be compared within the selected channel.

Neither field establishes that a learner can subsequently undertake the intended linguistic operation. Repeated practice can be irrelevant, poorly guided or misleading. A participant-formation judgment requires the separate historical and performance conditions developed elsewhere in the paper. The present model asks whether the grant rule permits a specified activity pathway on which such a judgment could

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depend. Its cumulative practice quantity will accordingly be called realized practice exposure throughout the formal comparison.

This distinction inherits the earlier account of competence reproduction, whose three-channel field represents participation, guidance and support renewal while retaining separate formation witnesses [15, Section 6]. The present law is a different specialization. It introduces an explicit competition between provisioned activities and a decision-time funding constraint. It does not replace the earlier triadic potential or establish a unique dynamical model of teaching.

Let symmetric weights $w_{ij} = w_{ji} \geq 0$, with $w_{ii} = 0$, specify exchange between retained settings. The graph Laplacian L acts on a vector $z \in \mathbb{R}^m$ by $(Lz)_i = \sum_j w_{ij}(z_i - z_j)$. Let $D_u, D_v \geq 0$ be exchange coefficients and $\beta \geq 0$ the strength of the selected contribution of practice activity to current delivery. Define $h(s) = s^2/(1 + s^2)$ for $s \geq 0$, acting componentwise on vectors. The proposed law is Equation 11.

$$\begin{aligned} \dot{u}_i &= -D_u(Lu)_i - u_i + q_i + \beta h(v_i), \\ \dot{v}_i &= -D_v(Lv)_i + r_i - v_i^2. \end{aligned} \tag{11}$$

The unit linear loss and quadratic practice loss in Equation 11 belong to a dimensionless illustrative law. The latter represents increasing overlap, coordination or scheduling losses within the retained activity channel. The saturated contribution $\beta h(v_i)$ permits supervised practice to contribute to current delivery without making that contribution unbounded. These mechanisms are selected hypotheses for the comparison. No institutional source establishes this polynomial, the saturation law or the value of its coefficients.

The nonlinear loss will perform substantive work. Concentrating a fixed mean practice activity in fewer settings increases its total quadratic loss in this representation. Consequently, the aggregate practice channel does not generally obey the same autonomous equation as a homogeneous site. The later argument retains that discrepancy rather than replacing a spatially varying configuration with an unjustified scalar average. The mean remains a chosen summary of activity and leaves the distribution of participant access unresolved.

9.3 Positive Forward Evolution

The model first needs a coherent trajectory for every allowed provisioning history. Assume bounded piecewise continuous controls satisfying Equation 10, with finitely many review times on each finite horizon. The same integral arguments apply to bounded measurable controls using absolutely continuous solutions. Proposition 9.1 gives the elementary bounds used below.

Proposition 9.1 (Positive Bounded Grant Fields)**.** *For a finite symmetric nonnegative graph and a nonnegative initial condition, Equation 11 has a unique nonnegative forward solution. Choose $U_* \geq m + \beta$ and $V_* \geq \sqrt{m}$ large enough to contain every initial coordinate. The box $\mathcal{B} = [0, U_*]^m \times [0, V_*]^m$ is forward invariant. The solution therefore exists for the entire declared horizon, including periods with no new provisioning.*

Proof. The state dependence is continuously differentiable on the nonnegative carrier, so local existence and uniqueness hold on each continuity interval of the controls and continue through their prescribed changes. At $u_i = 0$, its derivative is $D_u \sum_j w_{ij}u_j + q_i + \beta h(v_i) \geq 0$. At $v_i = 0$, the corresponding derivative is $D_v \sum_j w_{ij}v_j + r_i \geq 0$. The nonnegative orthant is consequently invariant.

At an upper face $u_i = U_*$ of the stipulated box, diffusion is nonpositive. Since $q_i \leq m$ and $h \leq 1$, the remaining derivative is at most $-U_* + m + \beta \leq 0$. At $v_i = V_*$ it is at most $m - V_*^2 \leq 0$. Thus each upper face is inward pointing. The bounded box prevents finite-time escape and permits continuation across every finite time. A funding decision changes the admissible controls but introduces no jump in the field coordinates, so the same bound remains valid across reviews. □

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Proposition 9.1 supplies mathematical existence rather than an application warrant. In particular, its positivity does not prove willingness to participate, and its upper box does not identify a safe workload. Those are separate domain restrictions. A practical allocation problem can impose stricter regional bounds, delivery obligations or consent conditions. The impossibility result below allows a generous control family; it therefore continues to apply to any more restrictive family that retains the same dynamics and target.

9.4 Regional Exchange Data

The GR field-realization requirement distinguishes a selected field representation from the substantive historical relation it represents and requires explicit regional compatibility [10, pp. 27–28, 33–35]. For a region $R \subseteq I$, the restriction of Equation 11 contains the crossing terms $D_u \sum_{j \notin R} w_{ij}(u_j - u_i)$ and $D_v \sum_{j \notin R} w_{ij}(v_j - v_i)$ at each $i \in R$. Their actual histories belong to the regional record. Replacing the full Laplacian by an induced-region Laplacian changes the system unless the corresponding boundary assumption is justified.

The pool can also cross an observational boundary. A grant paid to one administrative body may provision encounters in several settings. A regional analysis should retain the assignment of those inputs and any already funded commitments. Neither a site's visible delivery nor its ledger alone necessarily supplies the whole support history. This is a local application of the inherited support-attribution problem, not a new claim that institutional resources can circulate.

For the complete retained graph, symmetric exchange cancels in the arithmetic means. Define $\bar{u} = m^{-1} \sum_i u_i$, $\bar{v} = m^{-1} \sum_i v_i$, and similarly $\bar{q}, \bar{r}$. Let $\mathcal{V}_v = m^{-1} \sum_i (v_i - \bar{v})^2$ be the spatial variance of the selected practice channel. Their exact balances and the resulting inequality are stated in Equation 12.

$$\begin{aligned} \dot{\bar{u}} &= -\bar{u} + \bar{q} + \beta\, m^{-1} \sum_i h(v_i), \\ \dot{\bar{v}} &= \bar{r} - \bar{v}^2 - \mathcal{V}_v \le \bar{r} - \bar{v}^2, \qquad \bar{q} + \bar{r} \le g. \end{aligned} \tag{12}$$

To derive Equation 12, sum the site equations and pair the opposite contributions of each undirected edge. The identity $m^{-1} \sum_i v_i^2 = \bar{v}^2 + \mathcal{V}_v$ then gives the second line. This is the finite Jensen inequality written with its exact nonnegative remainder. Uniform practice has zero remainder; an arbitrary regional allocation need not.

The cancellation is specific to the complete retained exchange graph and its symmetric weights. A subsystem receiving outside support requires its boundary input in the mean balance. A directed or otherwise weighted population summary may require a different invariant measure. The later theorem therefore does not assert that every real teaching network obeys the same aggregate budget estimate. It establishes a conditional obstruction even after considerable freedom of regional allocation has been granted within this representation.

9.5 Activity Histories At Review

The field becomes a grant model when a rule selects future input domains from a record at a declared time. A review can retain the current delivery mean $\bar{u}(t)$, the cumulative realized practice exposure, regional differences or another justified record. These are distinct observables. The continuous dynamics itself does not specify which should determine eligibility.

For a horizon T, define cumulative realized practice exposure by Equation 13.

$$J_T = \int_0^T \bar{v}(t)\, dt. \tag{13}$$

Equation 13 integrates an already interpreted realized activity intensity. It would need a further realization rule if v represented only stored materials, potential support or unused opportunities. Even with the stated interpretation, equal J_T can conceal different participation histories, instructional content and correction.

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The scalar is an operational exposure record for this selected model, not an exhaustive account of the activity.

Normative Formation already explains how recognized consequences can alter later access, resources and competence reproduction [9, pp. 17–21, 39–44]. The present investigation asks a narrower question: can a recurrent delivery checkpoint block a jointly feasible exposure and later-delivery pathway within its own finite resource envelope? The following calculation answers that question for explicit data while retaining the distinction between operational obstruction, actual participant formation and normative evaluation.

10 Grant Renewal Dynamics

The grant-supported activity field of Section 9 permits a precise comparison between a delivery checkpoint and a protected preparation pathway. This section fixes a two-period budget, derives an upper bound applying to every eligible mixed allocation, and constructs a budget-equal history outside that bound. The result concerns cumulative realized practice exposure together with later teaching delivery. Further calculations distinguish a terminal activity snapshot from an exposure history and identify a task for which the delivery checkpoint is adequate. The purpose is to locate a conditional rule-induced obstruction, not to infer that protected funding is generally preferable.

10.1 Two-Period Review Specification

Take Equation 11 with $\beta = 1/20$ on any finite symmetric nonnegative graph. Initialize $u_i(0) = v_i(0) = 0$ at every site. This means that the selected realized activity channels have not yet begun. It does not mean that relevant teachers, linguistic competence, permissions or service providers are absent. Their availability belongs to the independently stipulated conversion domain that makes the initial provisioning feasible.

The periods are $[0, 1]$ and $[1, 2]$, and the first is funded. A strict renewal rule uses the exact current-delivery record at time one. Put $\theta = 3/5$ and define the second-period funding state in Equation 14.

$$g(t) = 1 \quad (0 \le t < 1), \qquad g(t) = \mathbf{1}_{\{\bar{u}(1) \ge \theta\}} \quad (1 \le t \le 2). \tag{14}$$

Here $\mathbf{1}$ denotes an indicator. Equation 14 is a constructed rule, not a description of the institutional programmes discussed elsewhere. Exact observation is deliberately favorable to the rule: the obstruction below does not depend on inaccurate reporting or a hidden inability to measure the selected output.

The declared joint operational target is Equation 15.

$$\bar{u}(2) \ge \frac{3}{5}, \qquad J_2 \ge \frac{3}{4}. \tag{15}$$

The first condition in Equation 15 concerns delivery at the later boundary; the second concerns cumulative realized practice exposure before that boundary. The numbers specify one dimensionless comparison. In an application they would need an independently justified interpretation, and a participant-formation witness would remain necessary. The theorem does not equate three-quarters of normalized exposure with learning a language, acquiring a teaching qualification or becoming entitled to support.

The funder allows arbitrary controls satisfying Equation 10 during an authorized period. No fixed preparation-versus-delivery action menu is imposed. In particular, an allocator can mix the activities continuously, change their spatial distribution and respond to the complete evolving field. After denial, new q and r are zero; the existing state continues under its declared law. Unfunded alternatives outside that law would constitute an enlarged comparison domain.

10.2 First-Review Resource Bound

The renewal record constrains how much formation-practice provisioning can precede it. Write $b = 1 - e^{-1}$ and let $R_1 = \int_0^1 \bar{r}(s)\, ds$. Since the practice mean starts at zero and has a nonnegative loss,

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Equation 12 gives $\bar{v}(t) \le \int_0^t \bar{r}(s)\,ds$. Also $h(s) \le s$ for $s \ge 0$: division by a positive s reduces the inequality to $s \le 1 + s^2$, which holds. The zero case follows directly.

Variation of constants for the delivery mean now gives Equation 16.

$$\begin{aligned} \bar{u}(1) &= \int_0^1 e^{t-1}\left[\bar{q}(t) + \beta m^{-1}\sum_i h(v_i(t))\right] dt \\ &\le b - \int_0^1 \left[(1+\beta)e^{s-1} - \beta\right]\bar{r}(s)\,ds \le b - cR_1, \qquad c = e^{-1} - \beta(1-e^{-1}) > 0. \end{aligned} \tag{16}$$

The first inequality in Equation 16 uses $\bar{q} \le 1 - \bar{r}$ and the upper bound on $\bar{v}$. To see its kernel explicitly, the positive practice contribution is at most $\beta \int_0^1 e^{t-1} \int_0^t \bar{r}(s)\,ds\,dt$. Changing the order of these non-negative integrals yields $\beta \int_0^1 (1 - e^{s-1})\bar{r}(s)\,ds$. Combining that expression with the removed delivery provisioning gives the displayed coefficient. Its minimum on the interval is c, attained at $s = 0$.

If the arrangement qualifies for renewal, Equation 16 imposes the bound in Equation 17.

$$R_1 \le A := \frac{b-\theta}{c} \simeq 0.0955192, \qquad \bar{v}(1) \le A, \qquad \int_0^1 \bar{v}(t)\,dt \le A. \tag{17}$$

The last inequality in Equation 17 follows by integrating the cumulative input bound, or by observing that $\bar{v}(t) \le R_1$ throughout the first unit interval. The estimate is conservative. It allows more first-period practice than a particular allocation may actually realize. This direction is useful: an upper bound below the chosen target excludes every admissible allocation without solving an optimal-control problem.

The field's nonlinear spatial term remains important here. The argument has not assumed that all sites have the same practice level, nor that their output records are interchangeable. It uses the exact pooled budget and a mean inequality valid for every retained configuration. An unusual allocation cannot evade the bound merely by concentrating practice at one site and delivery at another within this closed exchange graph.

10.3 Joint Activity Obstruction

Proposition 10.1 converts the first-review restriction into a two-period result. The result concerns the selected activity pathway even when the underlying teaching arrangement retains the linguistic competence needed to undertake it.

Proposition 10.1 (Renewal-Constrained Practice Exposure)**.** *Under Equations 11, 10 and 14, with the initial condition and coefficients stated above, no admissible control history satisfies both conditions of Equation 15. If funding is renewed, cumulative realized practice exposure satisfies Equation 18. If it is denied, later delivery satisfies Equation 19.*

Proof. Suppose first that funding is renewed. During the second period, $\bar{r} \le 1$ and Equation 12 gives $\dot{\bar{v}} \le 1 - \bar{v}^2$, with $\bar{v}(1) \le A$. Let z solve $\dot{z} = 1 - z^2$, $z(0) = A$. A direct differentiation verifies $z(t) = (A + \tanh t)/(1 + A\tanh t)$. The scalar comparison can also be checked directly: for $w(t) = \bar{v}(1+t) - z(t)$, the differential inequality is $\dot{w} \le -(\bar{v}(1+t) + z(t))w$, with $w(0) \le 0$. Multiplication by the positive integrating factor shows $w \le 0$.

Integrating this upper solution and adding the first-period bound gives Equation 18.

$$J_2 \le A + \log\bigl(\cosh 1 + A\sinh 1\bigr) \simeq 0.599523 < \frac{3}{4}. \tag{18}$$

The integral follows because the derivative of $\log(\cosh t + A\sinh t)$ is $z(t)$. The strict target comparison also has a simpler bound. The elementary estimate $e < 2.72$ gives $A < 1/10$. Since $\dot{z} = 1 - z^2 \le 1$, the upper solution satisfies $z(t) \le A + t$, yielding $J_2 \le 2A + 1/2 < 7/10 < 3/4$. Thus a renewed history fails the cumulative-exposure condition independently of the displayed decimal.

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Suppose instead that funding is denied. Then $\bar{u}(1) < \theta$, and $q = r = 0$ throughout the second period. Residual practice can still contribute to delivery, but $h \le 1$. Variation of constants therefore gives Equation 19.

$$\bar{u}(2) < e^{-1}\theta + \beta(1 - e^{-1}) \simeq 0.252334 < \frac{3}{5}. \tag{19}$$

Consequently a denied history fails the later-delivery condition. These cases exhaust the rule, including equality at the renewal threshold, which belongs to the renewed case. The joint target is therefore unreachable under the stated rule. □

Proposition 10.1 establishes more than failure of one preparation plan. The allowed controls include full history information, arbitrary temporal mixing and unequal regional allocation. The rule fails to admit the joint target even with those advantages. A practical controller restricted by limited information or staff availability cannot improve on that enlarged feasible set while retaining the same dynamics and budget. Conversely, extra unpaid work, another funder or a different service conversion can enlarge the set; the theorem does not silently include those possibilities.

The result also locates the role of evidence. Current delivery is measured correctly. The difficulty is the use of that measurement to determine the continuation domain before a declared preparatory activity can make its later contribution. Correctness of the record and suitability of the decision timing are separate conditions. The argument does not infer a hidden defect in the recipients from the fact that they fail the checkpoint.

10.4 A Protected Preparation Path

An alternative comparison authorizes the same two-period resource envelope for a warranted preparation-then-delivery pathway. Its interim assessment can examine the preparation record and continuing domain conditions rather than require the final delivery level already at time one. The mathematical witness specifies the supported controls; evidence that makes such a pathway credible in an actual programme remains an additional obligation.

Choose $r_i = 1, q_i = 0$ throughout the first period and $r_i = 0, q_i = 1$ throughout the second, at every site. Both periods exactly use the pool in Equation 10, so total provisioning is $2m$ units. Uniform initial data and controls preserve uniform fields. Diffusion vanishes in this particular witness, allowing the practice history in Equation 20.

$$v_i(t) = \begin{cases} \tanh t, & 0 \le t \le 1, \\ \dfrac{\tanh 1}{1 + (t-1)\tanh 1}, & 1 \le t \le 2. \end{cases} \tag{20}$$

The first branch of Equation 20 solves $\dot{v} = 1 - v^2$ from zero; the second solves $\dot{v} = -v^2$ from $\tanh 1$. Both values and their joined trajectory agree at the review boundary. Their derivatives can differ because the provisioning changes there.

Proposition 10.2 (Budget-Equal Protected Exposure)**.** *The protected controls just specified satisfy the joint target in Equation 15 with total provisioning $2m$. Their first delivery record fails the strict renewal condition. An always-delivery history uses the same total provisioning and satisfies that renewal condition but has zero practice exposure.*

Proof. Integrating the two branches of Equation 20 gives Equation 21.

$$J_2 = \log\cosh 1 + \log(1 + \tanh 1) = 1, \qquad \bar{u}(2) \ge 1 - e^{-1} > \frac{3}{5}. \tag{21}$$

The logarithmic identity uses $\cosh 1(1 + \tanh 1) = \cosh 1 + \sinh 1 = e$. The delivery lower bound follows by discarding the nonnegative initial and practice contributions during the second interval, when $q_i = 1$. Thus Equation 21 meets both target conditions.

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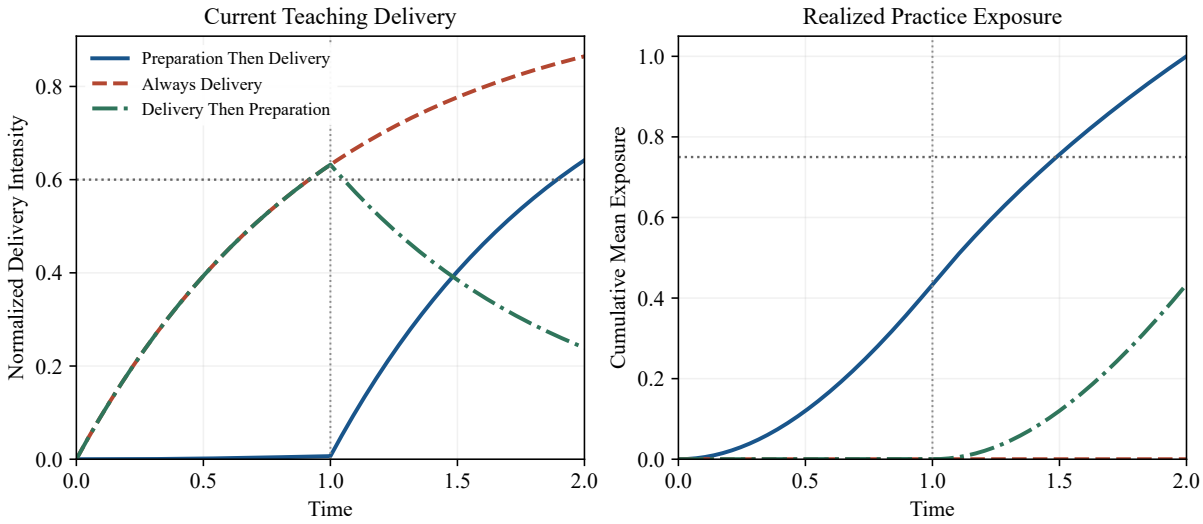

Figure 3: Grant-Supported Activity Histories. Protected preparation followed by delivery reaches the chosen later-delivery and cumulative-practice targets. Always-delivery provision qualifies at the early review while generating no practice exposure. Delivery followed by preparation illustrates a different timing comparison. Dotted lines mark the time-one review and the target levels. All provisioned schedules use the same two-period resource envelope; only the protected schedule requires an alternative first-review authorization.

During the first interval $q = 0$ and $h \leq 1$, so $\bar{u}(1) \leq \beta(1 - e^{-1}) < 3/5$. The strict checkpoint therefore rejects this history's continuation. For the always-delivery comparison take $q_i = 1, r_i = 0$ throughout both intervals. Then $v = 0$ and $u_i(t) = 1 - e^{-t}$. Its first record exceeds $3/5$, its later delivery is $1 - e^{-2}$, and its cumulative practice exposure is zero. Both histories provision one unit per site per unit time for two time units. □

Proposition 10.2 supplies a feasible operational alternative within the same envelope. It does not claim that every denied trajectory would otherwise have spent the full budget. A refusal can leave funds available for another use, and comparing that use requires the other applicants, their targets and opportunity costs. The budget-equal witness establishes that the joint target's failure is not explained simply by an insufficient two-period cap.

The protected history also has an inspectable interim practice record: $\bar{v}(1) = \tanh 1$ and $J_1 = \log\cosh 1$. These values are conditional predictions, not proposed universal renewal standards. An actual interim review would have to establish the relevant practice occurred, that its content and guidance support the intended pathway, and that the later delivery operation remains feasible. A protected interval can therefore be bounded and evidence-sensitive without requiring the final output prematurely.

Figure 3 displays these histories. The figure distinguishes the failed early delivery checkpoint from the successful later joint activity target. Its trajectories are illustrative solutions of the declared law and contain no empirical estimate of a grant's effect.

10.5 Budget-Equal Timing Comparisons

The practice field also makes the distinction between a snapshot and a history exact. Reverse the protected order: provision delivery during the first period and practice during the second. The total amounts assigned to each activity remain one unit per site, and total expenditure remains unchanged. The reversed practice field is zero until time one and then equals $\tanh(t - 1)$. Equation 22 compares the two resulting

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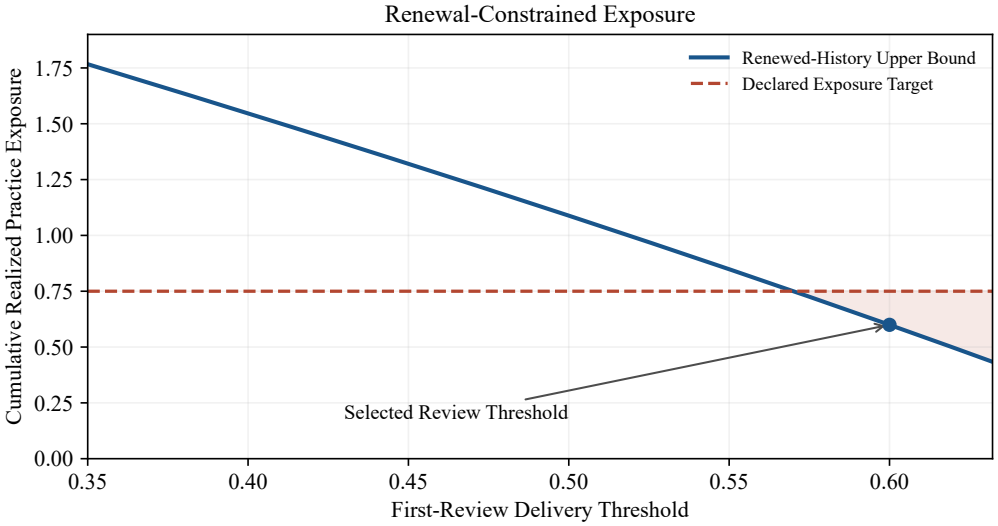

Figure 4: Renewal-Constrained Exposure Envelope. The curve gives the proved upper bound on cumulative realized practice exposure for renewed histories in the selected nonlinear field. The marked threshold is the constructed comparison, and the horizontal line is its declared operational target. An upper bound above that line is inconclusive; it does not prove attainability.

records, using P for preparation-then-delivery and R for its reverse.

$$\begin{aligned} J_2^P &= 1 > \log\cosh 1 = J_2^R, \\ \bar{v}^P(2) &= \frac{\tanh 1}{1+\tanh 1} < \tanh 1 = \bar{v}^R(2). \end{aligned} \tag{22}$$

Equation 22 shows an order reversal between cumulative exposure and a terminal practice snapshot. Earlier practice has had time both to occur and to decline; a later burst leaves a larger current channel while contributing less prior exposure. Neither record is intrinsically the correct assessment. A target about presently active supervised sessions can require the latter, whereas a target about the realized practice history requires the former. Their disagreement gives no reason to identify either scalar with the competence of the participants.

The exclusion estimate also varies continuously with the chosen delivery threshold while its assumptions remain satisfied. For $0 < \theta < b$ and $c > 0$, put $A(\theta) = (b-\theta)/c$. The right side of Equation 18 is then a conservative renewed-exposure envelope. Figure 4 shows this envelope over a stated threshold range. Where it lies below the declared exposure target, the renewed branch is excluded by the same proof. The denied branch and the protected witness still require their separate delivery inequalities. The figure is an analytical sensitivity comparison, not a calibrated threshold recommendation.

10.6 Target-Specific Evidence Adequacy

A different undertaking can justify a different decision. Suppose the selected operational target requires current delivery at both review boundaries and contains no formation-practice requirement during this horizon. The always-delivery history satisfies both demands. The protected witness fails the first one, despite its greater cumulative practice exposure. Its advantage for the earlier joint target consequently provides no unconditional argument for preferring it when current delivery itself is the binding obligation.

Moving the delivery checkpoint to the later boundary admits the protected witness for the stated joint target. That change does not guarantee indefinite continuation. It leaves unresolved the next period's resources, the continuing service conversion and the actual evidence of participant formation. The temporal extension is two periods, not a promise that eventual qualification will follow from enough patience.

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The historical interpretation is correspondingly precise. A grant continuation is an operation whose availability depends on a review record. The strict rule removes a prefix of a history that can reach the final joint activity target under another bounded authorization. In an operator representation, this becomes an intermediate eligibility projection inserted between supported history extensions. The field calculation identifies a concrete nonempty obstruction for that abstract construction. It supplies no quantum interpretation and does not infer legitimate authority from the projection's mathematical definition.

Finally, rule-induced obstruction and injustice remain different judgments. The model isolates how a criterion changes the feasible activity histories produced within an allocation envelope. Evaluating that change requires reasons about the teaching purpose, competing claims, quality of pathway evidence, distribution of work, risk and acceptable limits. A protected pathway can be operationally feasible while poorly justified; an immediate delivery requirement can be justified while excluding another valuable possibility. Keeping those conclusions distinct allows the dynamics to inform deliberation without substituting for it.

11 Eligibility Operation Algebra

The preceding models distinguish a successful final history from the intermediate conditions needed to reach it. An operator representation makes that distinction compositional. This section constructs partial history extensions and eligibility projections, proves a finite checkpoint result and relates grant records to the continuation distinctions they retain. The algebra is an analytical representation of selected operations; it attributes neither quantum dynamics nor an ultimate algebraic ontology to teaching.

11.1 History Extension Operators

Choose a finite declared family $\mathcal{H}$ of histories containing the reference paths under comparison and their prefixes. Let $\mathscr{K} = \ell^2(\mathcal{H})$ with orthonormal basis e_h, $h \in \mathcal{H}$. This is a bookkeeping space for histories. Its coefficients need not be probabilities or physical amplitudes.

For a deterministic operation a, let $\sigma_a : D_a \to \mathcal{H}$ append a tagged occurrence, including the support package and operation identity, to each $h \in D_a$. Assume σ_a is injective. This assumption is natural for a full retained history: deleting the last tagged occurrence recovers its predecessor. A compressed state map can be noninjective and would require a different operator analysis.

$$S_a e_h = \begin{cases} e_{\sigma_a(h)}, & h \in D_a, \\ 0, & h \notin D_a. \end{cases} \tag{23}$$

Equation (23) defines a partial isometry: $S_a^* S_a$ is the diagonal projection onto the span of $\{e_h : h \in D_a\}$. The adjoint is available as a mathematical operation. It is not thereby a feasible social reversal of the appended event.

The reference operation domain includes the packages permitted by the comparison envelope before a particular renewal rule is imposed. Its paths are candidate histories, not all actual funded trajectories. Applying an eligibility rule will restrict these reference paths. A rejected participant continues to have a history; the zero vector below denotes exclusion from this selected supported-path calculation.

11.2 Checkpoint Projections

For a subset $E \subseteq \mathcal{H}$, define $Q_E e_h = e_h$ when $h \in E$, and $Q_E e_h = 0$ otherwise. The operator Q_E is an orthogonal projection. The set may represent eligibility at a particular date, accepted initial conditions or a terminal target. Date tags distinguish these uses even when their record criteria are similar.

For an operation word $w = (a_1, \ldots, a_m)$, write $S_w = S_{a_m} \cdots S_{a_1}$. Let Q_0 select the admitted starting histories, let $Q_1, \ldots, Q_{m-1}$ select the intermediate eligibility sets, and let Q_m select the terminal

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condition. Equation (24) defines the two comparison operators.

$$\begin{aligned} T_w &= Q_m S_{a_m} Q_{m-1} S_{a_{m-1}} \cdots Q_1 S_{a_1} Q_0, \\ L_w &= Q_m S_w Q_0. \end{aligned} \tag{24}$$

Equation (24) distinguishes intermediate eligibility from final-only acceptance. Both use the same reference operations, initial set and terminal condition. The comparison does not quietly provide a new operation to the first system.

Proposition 11.1 (Checkpoint Domain Restriction)**.** *The operators in Equation* (24) *satisfy* $T_w^* T_w \leq L_w^* L_w$*. They are equal exactly when every reference path accepted by* L_w *passes each intermediate checkpoint. Under the same condition,* $T_w = L_w$*.*

Proof. On a basis vector e_h, each shift either produces the uniquely extended basis history or gives zero. Each projection either retains that vector or gives zero. Thus $L_w e_h$ is nonzero exactly when the initial condition, all operation domains and the terminal condition are satisfied. The value $T_w e_h$ is the same final basis vector if, in addition, every intermediate condition is satisfied; otherwise it is zero.

Compositions of the injective history extensions are injective on their domains. Consequently both $T_w^* T_w$ and $L_w^* L_w$ are diagonal domain projections, and the first domain is included in the second. This proves the operator inequality. Equality of the domain projections holds exactly when no finally accepted path fails an intermediate checkpoint. On that domain the two operators have identical values, and outside it both vanish. □

Proposition 11.1 uses the positive order only for the domain projections. It does not assert an order between the generally non-self-adjoint shifts T_w and L_w. Its content is a precise prefix condition, not a measure of how harmful a checkpoint is.

11.3 A Formation Word

Apply the representation to the path P, C in Section 8. The initial projection admits the starting history. The final projection admits the target stage $z = 2, y = 1$. The intermediate strict projection retains histories with delivery record $y = 1$. The reference shifts append the two funded operations.

The preparation prefix has $y = 0$. Therefore the strict word gives $T_{(P,C)} e_{h_0} = 0$, whereas the final-only word gives the completed history $L_{(P,C)} e_{h_0} = e_{h_{PC}}$. Replacing the intermediate set with one that admits the verified preparation milestone retains this particular path.

The field witness in Section 10 has the same operation pattern at its selected review resolution. Its first interval supplies realized practice but fails the delivery checkpoint; its second supported interval would meet the joint activity target. The field proof separately establishes that every strict-rule control fails that target. The algebraic word alone proves only the exclusion of a path. Keeping these result strengths distinct avoids deriving a global impossibility from a single interrupted example.

11.4 Record Algebras

For the finite record map $\rho : \mathcal{H} \to O$, define its multiplication algebra on $\mathscr{K}$.

$$\mathcal{A}_\rho = \{M_{f \circ \rho} : f : \rho(\mathcal{H}) \to \mathbb{C}\}, \qquad M_{f \circ \rho} e_h = f(\rho(h)) e_h. \tag{25}$$

Equation (25) is a finite-dimensional commutative unital $*$-algebra. It retains exactly the distinctions between record fibers. If an eligibility set is specified entirely by the record, its projection belongs to $\mathcal{A}_\rho$.

That membership establishes mathematical determination of the criterion by the chosen record. Practical interpretation and readability remain further premises. It does not establish record sufficiency for formation viability. For this separate record comparison, retain the same operations but allow a three-period envelope. Consider two histories with one funded period remaining and the same delivery record

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$y = 1$. One has already completed preparation and has $z = 1$; the other has $z = 0$. They are reached respectively by P, D and D, D under the reference support envelope. The first can reach the formation target by C, and the second cannot reach it in one period. A record retaining only the date, budget and y identifies these histories. Their viability indicator is not constant on that fiber, so its projection does not belong to that record algebra.

This is an application of the prior record-sufficiency distinction [16]. The new allocation setting makes the practical divergence explicit: an institution can implement a criterion exactly while that criterion omits the distinction needed to assess a possible formation continuation.

11.5 Operation Order

All the diagonal record projections commute with one another. They need not commute with history extension operators. Inserting an eligibility check before or after preparation changes which history its criterion evaluates. This is ordinary noncommutativity of restriction and transformation in the selected model.

The algebra therefore supports a precise GR interpretation: manifestation, judgment and intervention have typed positions within a historical operation sequence. Their interchange requires a proved compatibility, not a verbal equation of assessment with reality. The result inherits the prior power analysis's judgment-conditioned transformation [11].

The representation remains bounded. It does not identify a false report, price a participant's burden or adjudicate competing claims. Those require the epistemic and normative arguments already distinguished. Its contribution is to expose the operation order on which a particular renewal objection depends.

12 Documented Grant Arrangements

Language-programme guidance provides concrete examples of the distinctions examined in this paper. The present section reads four dated primary documents to separate entry, selection, supported formation, reporting and subsequent access to resources. The documents establish announced institutional arrangements; they do not establish applicants' experiences or the causal effects of implementation. Each documentary account is followed by an analytical comparison or an explicitly constructed variation. The variations identify research questions and cannot be attributed to the institutions described.

The sources are historical snapshots accessed on October 7, 2026. Programme years, application deadlines and publication dates have different meanings. A guide for a later funding year may have an earlier application deadline, while a currently accessible programme page may describe a newer cycle. The bibliography therefore identifies the actual guide or revised announcement used. The comparison concerns those versions, with no inference that their rules remain current.

12.1 Supported Apprentice Formation

The First Peoples' Cultural Council's Mentor-Apprentice Program guide for 2025–2026 has an October 15, 2024 application deadline. It supports one-to-one immersion and learners planning to pass the language to others. The guide distinguishes application review, conditional acceptance and further requirements before final acceptance. Training and continuing coaching accompany the programme. Reports and panel evaluations follow each completed 100-hour block; grant instalments are paid after each 100 immersion hours. Completion of 300 hours and the programme requirements leads to an invitation to reapply for another year, up to three years, subject to available funding. Reapplication, payment during an award and initial acceptance are consequently distinct decisions [4, pp. 5–9].

This distinction changes the appropriate object of criticism. An objection directed at an initial eligibility condition would need to show that the condition prevents a relevant applicant from entering

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consideration. An objection directed at payment timing would instead concern how the supported activity can be carried until the next instalment. An objection directed at reapplication would concern the evidential and resource conditions of another award. A single label such as renewal barrier would conceal these different causal questions and the different evidence needed to investigate them.

For example, consider a constructed programme that pays after completed activity. Two otherwise similar teams may differ in their ability to sustain that activity before payment. The resulting question concerns the resources needed to reach the payment point. It does not establish that the payment criterion mismeasures learning, that either team is entitled to an award, or that the actual programme leaves the relevant need unmet. A proper investigation would ask which support is already available, when costs arise, what work is carried by whom, and whether the timing affects participation. The guide alone cannot answer these questions.

The formation purpose presents a second analytical issue. Preparation for a future responsibility can be assessed through a sequence of supported encounters rather than only through independent performance at entry. Yet completion of the encounters and possession of the later capacity remain distinct. An assessment may provide warranted evidence that the route should continue without demonstrating that all intended teaching responsibilities can already be undertaken. This distinction allows a model to represent progress without inventing a universal fluency threshold.

It also permits an appropriate stopping judgment. In a constructed case, a learner may decide that the intended future role is unsuitable, or the available evidence may show that a different teaching arrangement would be preferable. A formation-oriented grant need not continue the original route indefinitely. The relevant question is whether the decision recognizes the particular outcome and the participant's agency, rather than treating every departure from a predetermined progression as the same kind of failure. This is a question about the interpretation of formation evidence, not a claim about the programme's observed decisions.

12.2 Differentiated Entry Routes

The Council's Language Program Application guide for 2025–2026, Cycle B, specifies an August 6, 2025 deadline. It differentiates capacity categories. Rise includes applicants with no previous Council grants or one to two; Roots lists three to eight prior grants over at least two years and a full-time language staff member among its criteria. Categories also differ in the possible funding period. For awards extending into subsequent years, funding is contingent on submission and approval of previous reporting. The guide includes narrative and expense reports, alongside specified employment, material and participation metrics. Its entry route therefore has to be read together with its later capacity and reporting conditions [6, pp. 6–7, 10–11, 19–20].

The initial analytical temptation would be to isolate a prior-grant requirement and conclude that access presupposes access. The differentiated arrangement prevents that inference from the document alone. A condition applying to one category does not describe every possible entry route. A complete analysis must first reconstruct the relevant alternatives and the purpose of the category being assessed. A larger or longer commitment may reasonably require evidence that a proposed arrangement can manage it, while a smaller entry award may seek different evidence.

The more specific research question concerns transitions among such categories. Consider a constructed scheme with an initial formation route and a later expanded route. The initial route could provide enough time and support for the later criteria to become attainable. It could instead expire before the relevant staff or activity history can be formed. Both designs admit new applicants at the outset, but their continuation possibilities differ. The existence of an entry route answers the claim of total initial exclusion while leaving the timing of later transitions open to investigation.

Three forms of evidence would be needed to distinguish these possibilities. The first concerns the actual meaning and application of category criteria, including any accepted alternatives. The second concerns the resources and duration needed for the proposed teaching development. The third concerns other feasible routes available when the initial support ends. A model that simply codes an applicant as

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below a threshold cannot determine these facts. The transition may be a useful safeguard, an avoidable discontinuity, or a reasonable limit on the scheme's responsibility.

A related constructed variation concerns reporting during a multi-year award. If a report can be completed within funded coordination work and usefully informs teaching, its connection to later disbursement can preserve accountability without disrupting formation. If the necessary evidence cannot be assembled before a crucial teaching operation, a timing conflict may arise. The conflict would depend on the work required, the support already provided and the institution's response to uncertainty. Nothing in the existence of a reporting contingency establishes which situation obtains.

These comparisons show why a pathway model needs institutional alternatives rather than a single binary gate. It should retain entry routes, possible transitions and justified conditions on continuation. Otherwise a model can manufacture a closed cycle by omitting a route that the actual institution supplies. Equally, the mere presence of several categories cannot demonstrate that a specific participant can traverse them. Practical availability remains an evidential question.

12.3 Reporting Eligibility

The Council's Funding Guidelines, updated June 11, 2025, describe outstanding or incomplete reporting as an eligibility problem until the required documentation is complete. They also state that new applications will not be reviewed until required reports are submitted and unspent funds returned. Reporting includes activities and financial information, and the document connects it to the Council's fiscal responsibilities. These are general guidelines read here alongside the 2025 Cycle B document; they are not retrospectively imposed on the earlier mentor-apprentice application cycle [5, pp. 7, 10].

An outstanding report can indicate several different situations. In a constructed comparison, one recipient has not undertaken the promised activity. Another has undertaken it but cannot retrieve a required document after staff turnover. A third has an unresolved disagreement about which expenditure belongs to the project. These cases may all lack a complete record at a review date, yet they support different conclusions about delivery, organizational capacity and the next useful action. The institution can have good reasons to pause a decision in each case without treating their explanations as interchangeable.

The resulting assessment has two stages. It first identifies the information needed to discharge a legitimate responsibility. It then asks how that information can be obtained, at what cost and with what residual uncertainty. The second stage can reveal a reason for assistance, correction or a different evidential route. It can also confirm that the missing information is consequential enough to justify withholding further consideration. A presumption that every documentary condition is oppressive would prevent this discrimination.

The allocation consequence nevertheless warrants explicit attention. If resolving a record requires work by the same participants whose funded positions have ended, the resources for resolution become part of the case. The relevant objection would concern the feasibility of satisfying the condition, including available institutional assistance. It would not follow simply from a continued desire to teach. Conversely, the applicant's previous access to resources can make an account of their use especially important to participants and other applicants. A defensible review must preserve both sides of this relation.

The generative issue is thus conditional. Evidence requirements can help maintain a reliable setting for continued work. They can also interact with a loss of support in ways that make evidential repair difficult. Identifying either possibility requires the historical sequence: the work performed, the missing record, the means of reconstructing it and the actual decision consequence. An isolated rule supplies only part of that history.

12.4 Planned Programme Learning

The Administration for Native Americans' 2025 Native American Language Preservation and Maintenance announcement includes language instruction and teacher training among supported activities. Its revised announcement requires a work plan that distinguishes direct outputs from intended outcomes and

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specifies activities over project years. A data management plan must identify collection responsibilities, monitoring and the review and use of information for project adjustment. Applicants whose data tools remain to be developed must explain their development. Staffing plans may identify unfilled positions and how they will be filled. These provisions give preparation and learning a place within an evidentially specified proposal [1, pp. 10, 25–26].

This documentary example constrains a common criticism of funding criteria. Requiring evidence does not necessarily mean requiring every relevant capacity to exist before support begins. A proposal can distinguish current resources, planned formation and the evidence that would later bear on progress. The analytical question then shifts from whether planning is permitted to whether the proposed sequence is credible, its intermediate observations informative and its response to difficulty practicable. The present paper makes no assessment of how those judgments occurred in actual awards.

A constructed example clarifies the difference between a plan and a promise of success. An applicant may propose supervised teaching attempts, specify who will observe them and explain how difficulties will alter later preparation. Such a plan can be assessed before independent teaching becomes possible. Its credibility depends on the availability of the relevant people, opportunities for practice and a plausible connection between the observations and the proposed adjustments. Merely naming an outcome does not establish those relations.

The same example clarifies a risk of overdetermination. A plan that permits changes only within an excessively narrow list may be unable to respond to an unexpected teaching difficulty. A plan that permits any later change may provide little basis for judging whether the original undertaking remains recognizable. A reasonable intermediate account identifies what is fixed by the grant’s purpose, what can be revised in light of experience, and who can authorize the revision. The formal operation domain must reflect these permissions and resources rather than assuming that every useful intervention is available.

Evidential flexibility therefore has a structure. A review can accept a changed method while requiring an explanation of why it serves the agreed purpose. It can also accept that a proposed formation route is unsuccessful without concluding that the resources produced no useful knowledge. The difficulty is to preserve such distinctions while keeping the account sufficiently clear for other participants and competing applicants. This is a demanding institutional task, and the present models do not replace it with a scalar criterion.

12.5 Comparative Investigation

The documents and constructed variations suggest an empirical inquiry into particular decisions. An investigation would identify the entry, payment, continuation or reapplication decision; reconstruct its stated purpose; and trace the evidence available at that time. It would then examine what operations became feasible after the decision and how their results entered a later record. This sequence avoids treating an institution’s announced conditions as a complete description of its practice.

Three contrasts would be especially informative. The first compares arrangements with similar initial relevant capacities but different access to the support needed to demonstrate them. The second compares the same activity recorded through different evidential routes, including any assistance supplied by the institution. The third compares review schedules relative to a specified formation sequence. None is automatically a causal experiment. Selection into support, unobserved differences and participant choices would require explicit treatment, and a comparison would need the communities’ own understanding of the teaching operations at issue.

Outcome observations should retain the distinct objects examined throughout this paper. Completed teaching, a new participant’s bounded capacity, continuity of the arrangement and the quality of institutional accountability may vary independently. An award can succeed in its short delivery purpose while leaving teacher formation unchanged. A preparation project can generate valuable evidence of an unsuitable route. A well-supported arrangement can still conclude, for defensible participant reasons, that continuation is undesirable. Recording these outcomes separately is necessary for assessing the chosen purpose.

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The formal constructions contribute a narrower instrument. They make a proposed dependency inspectable: which sequence requires support, where a review occurs and what record admits the next operation. Their counterexamples show how particular premises can fit poorly together. The documented programmes supply institutional distinctions against which such premises can be checked. Actual criticism or endorsement requires the further historical and participant evidence described here. The constructed allocation assessment in Section 14 illustrates that additional reasoning without attributing its assumptions to any of these programmes.

13 Allocation Objections

An allocation rule can obstruct a feasible formation pathway without thereby becoming unjustified. The strongest assessment of eligibility feedback must therefore engage reasons for retaining restrictive criteria as well as reasons for revising them. This section examines scarcity, uncertain development, administrative consistency, dependence and the explanatory limits of the models. The objections delimit a positive argument for bounded continuation: a supported pathway can deserve consideration when its purpose, evidence and costs withstand comparison, while the existence of that pathway alone establishes neither priority nor an unlimited claim.

13.1 Competing Claims

Scarcity is a substantive objection to protecting a formation period. The resources assigned to preparation could support current learners, another developing arrangement or an activity outside the selected language programme. A comparison that follows only one applicant can make renewal appear harmless because the displaced claim has been omitted. The relevant alternative is consequently an actual feasible allocation, with its beneficiaries and burdens represented, rather than an unspecified remainder of the budget.

Consider two proposals. One can provide an agreed course immediately; the other can prepare an additional instructor for a later course. The second may create a valuable future capacity, yet the first can answer a present need that cannot be deferred. The number of later classes does not settle that disagreement. Timing, reliance, the seriousness of unmet needs and the distribution of access can all matter. Treating eventual expansion as the common purpose would decide the normative question by changing its description.

An equal-budget demonstration makes a narrower contribution. It can show that a specified allocation schedule reaches a target that another schedule excludes without requiring a larger total envelope. It does not show that the successful schedule is the best use of the envelope. Nor does equality of grant expenditure establish equality of participant work. The comparison can remain informative once these limits are stated: one alleged obstacle, additional grant cost, has been removed, while other objections remain available.

A stronger practical argument requires reasons for the selected priority. A scheme may already have undertaken to support additional teaching capacity, or participants may justify reserving a bounded share for that purpose alongside current provision. Even then, a particular application must meet the relevant conditions. Prior agreement about a purpose supplies a reason to consider routes serving it; it does not assign the whole budget to the first route shown to be feasible.

13.2 Speculative Formation

A second objection concerns the evidential weakness of future benefits. An unsuccessful applicant can redescribe low delivery as investment in development. If such a description routinely postpones assessment, a formation allowance may protect poor practice and prevent resources from reaching arrangements with stronger evidence. The objection is especially serious when the relevant participants have little independent access to the reasons offered on their behalf.

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The answer requires a discriminating pathway claim. The applicant should identify what the preparation activity is expected to enable, why that relation is credible, which support the next step requires and what evidence could count against continuation. Evidence may be limited while still differentiating plausible development from an unsupported promise. For example, a supervised attempt can show a specific task performed with assistance, while the proposed next stage tests whether the assistance can change without losing the relevant operation.

The distinction between assistance and independence must remain task-specific. Removing all support is rarely a reasonable test of teaching. The relevant change might be an additional participant taking responsibility for an exercise while retaining access to consultation. A bridge justified for that purpose should be assessed against that development, not against an increasingly demanding ideal introduced at each review. Conversely, meeting an easy preliminary milestone cannot warrant a much broader role without further evidence.

Failure remains a possible outcome of a justified trial. A bounded allocation can be reasonable on the evidence available when the decision is made and subsequently require termination or redesign. Accountability should preserve that distinction. Otherwise fear of being classified as a failed project can encourage premature assurances, selective reporting or an unnecessarily conservative choice of participants. These are possible responses to investigate, not empirical consequences established by the present models.

An indefinite exception is therefore a poor expression of the proposed argument. The positive alternative is a finite commitment with interpretable milestones, a credible review occasion and reasons for continuation or change. Its adequacy depends partly on whether the resources and authority needed to carry out the review are themselves available. A promise of scrutiny provides little protection if the assessment process cannot examine the relevant evidence.

13.3 Common Rules

Uniform rules can constrain favoritism, make applications predictable and reduce the burden of repeated negotiation. Context-sensitive assessment can give an advantage to applicants who are skilled at explaining their histories or already enjoy a sympathetic relationship with the institution. A proposal to recognize differentiated formation pathways must therefore address the political value of common procedures rather than treating standardization as an administrative mistake.

The relevant distinction is between a common framework of reasons and an identical evidential demand at every stage. A rule can state in advance which project purposes permit a formation period, what evidence a proposal needs, when intermediate review occurs and what conditions permit renewal. These common terms can accommodate different pathways while limiting discretionary invention after favored applicants have been identified.

Such a framework can still exclude important experiences. Participants may have difficulty expressing their work in the approved vocabulary, and a nominal right to seek an exception may require expertise they lack. The inherited normative analysis treats legibility and access as conditions of realized influence, while preserving the value of stable procedures [9, pp. 17–22]. Its application here requires examining the work of translation and review. It does not authorize treating every unfamiliar account as adequately supported.

Transparent reasons can improve the comparison. A decision can state that a proposal lacks evidence of the next step, that its costs exceed the available envelope or that a competing claim takes priority. These are different grounds and imply different possible revisions. An undifferentiated finding of insufficient quality leaves the applicant unable to tell whether better evidence, a different schedule or a changed objective would matter.

The practical burden of individualized review can be substantial. A small scheme may lack the expertise to evaluate many distinct pedagogical pathways. One response is to restrict the range of pathways it can responsibly fund; another is to develop shared review resources or a smaller trial category. The present argument provides a reason to assess these options. It does not demand a comprehensive assessment capacity from every institution regardless of its mandate and resources.

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13.4 Legitimate Dependence

A further objection challenges the assumption that forming additional participants should reduce dependence on the institution. Teaching can remain a public or shared undertaking requiring recurring support. A grant programme that sustains reliable instruction over many years need not have failed because its recipients cannot replace the grant from private income. An independence ideal can obscure the value of deliberately maintained common provision.

The proposed analysis accepts that objection. Dependence has several objects: access to money, one particular instructor, a location, an exclusive source of interpretation or a fixed review procedure. Additional teaching capacity can reduce one dependence while retaining others. A claimed improvement should name the relation that changes. Continued payment for useful work is compatible with a wider distribution of competence.

Dependence becomes a concern when its terms undermine the selected activity or place unjustified burdens on participants. An arrangement may have to maintain visible delivery by leaving preparation unpaid, or a participant may feel unable to decline a teaching role because the whole grant appears to depend on that agreement. Establishing either concern requires evidence about the actual relation. The funding condition alone does not establish coercion, and a participant's agreement does not settle every question about the alternatives available.

Generative return provides one inherited evaluative question without resolving these issues automatically [9, pp. 40–44]. An institution can help participants acquire teachable methods, accessible materials and a greater role in subsequent decisions. Those gains can be valuable within a continuing funded relation. Their presence does not erase burdens, and their absence does not by itself make every short delivery grant illegitimate. The purpose and commitments of the particular arrangement remain decisive.

13.5 Allocation Dominance

A model may show that one rule retains more feasible target-reaching histories than another. This can be called a dominance result only with its comparison domain stated. The retained histories may concern one declared target, one budget and one family of uncertain conditions. Inclusion of feasible histories then identifies an operational advantage. It does not yet compare the value of all consequences experienced by the affected participants.

An enlargement of possible routes can leave actual choice unchanged if the institution cannot recognize which route is available. It can also introduce discretion whose exercise creates additional costs or unfairness. A rule that admits both delivery and formation schedules offers a broader menu in the model, but implementing that menu may require assessment, negotiation and monitoring. When those activities consume scarce resources, the operational comparison needs an expanded description.

Normative dominance would require much more. One would need justified grounds for treating the relevant gains and losses as comparable and for concluding that no excluded dimension supplies a defeating claim. The taxonomy deliberately separates political-economic, practical and normative judgments rather than summing them into a common score [12, Section 13]. The present paper retains that separation. A target-feasibility result can remove a mistaken practical objection while leaving a genuine distributive disagreement unresolved.

This limitation strengthens the use of formal counterexamples. Their task is to expose a claim such as the assertion that a strict delivery rule always preserves every affordable formation route. Refuting that assertion does not establish the opposite universal policy. It permits a more precise question about which routes should be considered and which further reasons govern their selection.

13.6 Model Explanatory Value

The final objection concerns whether the mathematics adds more than a restatement of the chosen example. A finite model can encode a preparation stage and a later delivery stage so that preparation is required by construction. A field model can choose a law under which early support accumulates and a

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review interrupts it. Conditional success inside either representation supplies no independent evidence that a linguistic practice has those relations.

The explanatory value lies in consequences that are not transparent from the local descriptions alone. A derivation can show which sequences remain feasible under a particular review rule, whether a result survives mixed allocations or which additional state information the rule omits. A countermodel can distinguish a timing obstruction from a lack of resources. These findings constrain a proposed application once the primitive relations have evidence; they do not provide that evidence themselves.

The interpretation of field channels is especially important. Delivery intensity and supervised practice exposure can be chosen operational variables without becoming quantities of competence or normative worth. A result that reaches a practice-exposure target establishes the stated activity consequence. An additional participant's capacity to interpret, correct or teach still needs an independent witness. The same distinction applies to regional averages, which can conceal differences in who receives usable opportunities.

The philosophical argument therefore gives the models a bounded role. They can clarify a possible conflict between an allocation rule and a declared formation pathway, test a proposed repair and expose omitted premises. Their practical use requires evidence about the pathway, participant commitments and competing claims. The resulting inquiry is more demanding than either accepting an appealing feedback narrative or treating a formal certificate as a complete funding decision.

14 Allocation Assessment

A practical assessment of eligibility feedback needs to connect its pathway claim to a decision that an institution could actually make. This section develops an expressly fictional teaching case with a fixed allocation envelope. It compares a delivery-based review with a prospectively specified formation milestone, examines the evidence and burdens of the alternative, and identifies the remaining grounds for a decision. The case illustrates an assessment process; its premises do not describe any of the actual programmes discussed elsewhere, and its conditional conclusion is not an evaluated grant protocol.

14.1 Declared Teaching Purpose

Consider an arrangement that offers a recurring narrative exercise. An experienced instructor can guide the exercise, explain a selected distinction and respond to a specified family of learner questions. The grant proposal includes a further purpose: preparing an additional willing participant to guide that exercise with an agreed form of continuing consultation. The target is limited to this role. It does not confer general teaching authority or establish unrestricted linguistic competence.

The proposed grant covers two stages. One funded package is available at each stage, subject to the decision governing the second. Each package includes the agreed instruction or preparation work, coordination and proportionate reporting. The constructed comparison assumes that its stated costs cover the relevant participant work; it does not assume an unpaid reserve that can silently complete an unsuccessful schedule. The same baseline facilities and access conditions are retained across the alternatives.

The institution has provisionally accepted additional participant formation as a legitimate purpose of this part of its budget. Current learners' previously promised provision is specified separately and is not removed from one schedule to make the other look successful. This is a substantive premise of the case. If the available funds have already been committed to urgent classes, the decision problem changes and the formation comparison cannot appropriate them by analytical relabeling.

Two first-stage activities are admitted. A delivery package supports instruction by the experienced participant. A preparation package supports the additional participant's supervised attempts, feedback and work on difficulties relevant to the proposed role. These are selected options for the case, not an exhaustive account of pedagogy. Their different immediate records and possible later effects require separate evidence.

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14.2 Pathway Evidence

The preparation proposal begins with evidence of a plausible next step. The additional participant has already undertaken relevant learning and agrees to attempt the bounded teaching role. The instructor can identify the operations that still require assistance and explain how supervised work would address them. The arrangement can supply the necessary encounters within the package. These stipulations make a pathway available for assessment; motivation alone would not supply them.

An interim review would examine what happened during the preparation period. The relevant questions include whether the participant attempted the selected tasks, whether feedback addressed the identified difficulties and whether the proposed next responsibility remains feasible. A report can preserve unsuccessful attempts as evidence rather than omitting them to create a smooth progress narrative. The institution needs enough information to distinguish a productive difficulty from an unsupported assertion that development is occurring.

The final capacity judgment is a further step. The additional participant would need to undertake the declared activity under the agreed assistance conditions, including the specified response to learner questions. Completing preparation hours can contribute to the history of that judgment without replacing it. The finite sequence and field exposure calculations introduced above concern their own operational targets. They do not establish this participant-level result.

This separation permits honest continuation. A first review can conclude that the evidence warrants the next supported attempt while leaving eventual formation unsettled. It can also conclude that the proposed role should be narrowed or that the current pathway is unsuitable. Both conclusions can be compatible with responsible use of the first package. Assessment of the decision's reasonableness should retain what was knowable at the time.

14.3 Delivery-Based Renewal

Suppose the original renewal rule recognizes only completed delivery during the first stage. The institution uses that record because delivered classes are easy to identify and relevant to one of its established purposes. A first-stage delivery package can satisfy the rule. A preparation package can fail it despite supplying the particular evidence on which the proposed formation pathway depends.

The problem is not that the delivery record is inaccurate. It correctly distinguishes completed classes from supervised preparation. The question is whether it is adequate as the sole gate for a grant segment whose declared purpose includes the latter sequence. When first-stage preparation is required for the admitted route, denial of the second package prevents that route from continuing. A favorable record of current delivery may meanwhile support renewal without showing that an additional participant is being prepared.

The comparison must keep the alternatives visible. If the arrangement can complete the same preparation alongside the delivery package without additional work or reduced quality, the claimed obstruction may disappear. If another accessible source can fund the second stage, denial by this scheme may alter the route without closing it. The constructed case holds these alternatives unavailable for the selected comparison; an actual assessment would have to investigate them.

On those premises, the strict rule has an instrumental limitation. Its evidential gate selects against an admitted pathway to part of the scheme's own purpose. That diagnosis does not show that the gate was irrational when designed or that every rejected applicant shares the problem. It identifies a particular mismatch whose repair can be considered without abandoning the evidential value of delivery.

14.4 Prospective Milestone Review

An alternative rule can specify the formation route before applications are judged. The first package funds the declared preparation activity; the next decision evaluates the agreed interim evidence and the continuing feasibility of the second stage. Completed current delivery remains an appropriate record

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for the delivery route. The institution thereby uses different evidence for different authorized purposes within a common announced framework.

The alternative does not automatically renew every preparation project. If the expected encounters did not occur, the participant no longer wants the role or the remaining difficulty cannot be addressed within the second package, the review has grounds to revise or decline the next step. The decision should state which condition failed. A change in willingness, for example, is consequential for the pathway without being a judgment that the person lacked linguistic worth.

If the interim evidence supports continuation, the institution can fund the next bounded stage using the same total two-package envelope. The practical conclusion is that a different temporal use of the available support can preserve a route excluded by the strict gate. Its value depends on the pathway's eventual result and the reasons for assigning resources to that purpose. The comparison does not require a promise that the new participant will thereafter teach indefinitely.

An important feature is prospective notice. Applicants and reviewers know which purposes and evidence forms are admitted before the first-stage work is organized. A retrospective exception may sometimes be justified, but it raises additional questions of consistent treatment and reliance. The case illustrates a planned comparison whose reasons can be examined across applications, rather than privileged access to an undisclosed discretionary channel.

14.5 Burden Examination

The same grant expenditure can coexist with different burdens. Preparation may require the experienced instructor to coordinate another person's attempts or revisit explanations that routine delivery would not demand. The developing participant may need time away from other activities. A report can require them to disclose work they would prefer to keep within the learning relation. These differences remain relevant even after the package is fully costed.

The assessment should identify which contributions are required, who has agreed to them and which alternatives are available. An apparent two-package success obtained through extensive additional work would not establish the stipulated equal-resource comparison. A willingness to contribute extra work could justify a revised comparison, but it should be recorded as a changed premise with its own limits.

Some consequences can improve the arrangement. The additional participant may gain a role they value, the instructor may acquire a colleague and future learners may obtain another route to explanation. Other participants can reasonably prefer a different use of the resources. The analysis should preserve both kinds of claim instead of converting the formation objective into a presumption that everyone benefits.

The inherited trajectory-sensitive account directs attention to how an asymmetry is used and what future capacities its exercise makes available [9, pp. 39–44]. In this case, the experienced instructor's larger role can be justified by the selected teaching task while supporting another participant's development. That possibility remains compatible with obligations to limit the work, share reasons and permit a change of role. Its normative significance cannot be read directly from the number of people who complete a report.

14.6 Comparative Decision

A committee considering the alternative now has a more specific question. It can ask whether the evidence for the next step is adequate, whether the proposed costs are complete, whether participants accept the relevant commitments and whether this use of the package has priority over the other applications. These questions are connected, but a favorable answer to one does not decide the rest.

Suppose another applicant needs the same second-stage resource to maintain a class on which current participants reasonably rely. The formation pathway's feasibility remains established in the constructed comparison, yet the allocation can be contested. The institution needs reasons for balancing the claims or for changing the budget's division among purposes. The present argument does not supply a universal exchange rate between current provision and future teaching capacity.

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Suppose instead that the designated formation budget has no stronger competing claim, the pathway evidence is adequate and the participant work is acceptable. The prospective milestone route then has a conditional practical advantage: it can pursue the adopted purpose without the specific timing obstacle created by the strict gate. A bounded award can be justified on those added premises. Its justification combines evidence, practical comparison and reasons for the purpose; the model alone contributes only part of that argument.

The assessment can also return a negative decision with an informative explanation. It may find that the route needs more support than the envelope permits, that the proposed intermediate evidence is too weak or that a different applicant takes priority. Recognizing support-conditioned evidence does not require pretending these difficulties are absent. It requires that the decision identify them accurately, rather than treating a record partly shaped by earlier nonprovision as a complete and independent assessment of future possibility.

14.7 Continuing Inquiry

The completed assessment leaves a record of its own commitments. It should preserve the declared purpose, the activity and assistance conditions, the evidence used at each stage, the support actually released and the reasons for a change of course. These records can help later reviewers distinguish a failure of the pathway from a failure to provide the pathway's conditions. They can also expose a mistaken premise in the original proposal.

The record should remain usable by the participants whose work it describes. A reporting system that extracts evidence for renewal while leaving instructors and learners unable to recover the reasons for a decision provides a limited return to the teaching relation. Accessible explanations can help participants revise their next proposal or decide that the scheme no longer serves their purposes. Accessibility itself requires an appropriate form and a reasonable allocation of the work of interpretation.

The case supports a bounded conclusion. Grant decisions can be evaluated as part of the history that produces later eligibility evidence, while preserving the legitimacy of accurate reporting, finite commitments and comparative selection. A formation allowance is justified only through a specific pathway and an adequate allocation argument. Further empirical inquiry must determine whether an actual programme has the proposed dependence, which repairs are feasible and whose claims its decisions leave unresolved.

15 Constructed Allocation Dossier

The models can assist a concrete assessment only if their different evidential requirements remain visible. This section assembles those requirements into a constructed dossier for the two-stage case. It does not repeat the allocation deliberation in Section 14; instead, it examines what would have to be established to translate a historical account into the finite, field and operator results.

15.1 Historical Identification

Begin with an identified teaching arrangement and a bounded intended role for a further participant. The dossier records the earlier teaching and support history, the proposed preparation, the supervised completion episode and the review between them. It identifies who can authorize each operation, whose participation it requires and which material conditions it presupposes.

This historical account performs a task that a single application score cannot perform. It distinguishes the claim that preparation has occurred from the claim that a later episode remains available. A supervisor's departure after the first stage can leave the preparation record intact while removing the next operation. Conversely, a reporting delay can leave the operation available while preventing the funder from recognizing its warrant in time.

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Table 2: Translation Obligations For The Constructed Dossier. Each result is conditional on its own declared domain. The remaining warrants cannot be supplied by changing notation.

Representation	Established Conditional Result	Remaining Application Warrant
Historical arrangement	Review occurs between retained operations	Identity, occurrence order, permissions and support dependencies
Finite viability	No admitted strict-rule path reaches the selected stage	Adequate action list and independent meaning of the completion witness
Nonlinear fields	Every strict-rule control misses a joint activity target	Service conversion, measured channels and justified activity requirements
Eligibility operators	An intermediate projection removes an otherwise accepted path	Actual implementation of the checkpoint and validity of its record
Allocation comparison	A bounded supported alternative is model-feasible	Competing claims, participant burdens, proportionality and acceptable risk

The retained history also fixes the comparison boundary. If another institution can fund the second stage, the strict-rule obstruction may disappear. If the preparation depends on work already undertaken without adequate support, the nominal two-unit budget can conceal a larger contribution. Neither possibility is an objection to accounting carefully; each is a reason to identify the relevant operation and support boundary.

15.2 Translation Obligations

Table 2 states what each formal representation establishes and what its application still requires. The representations address connected questions but do not mutually certify their substantive interpretations. A field trajectory that reaches its activity target does not prove the finite model's formation witness. An algebraic projection that selects a record does not prove the record is truthful.

The table's structure resists a common escalation of a formal result. One can prove a path exists, demonstrate that it requires an intermediate commitment, and still lack a sufficient reason to make that commitment. Conversely, uncertainty about the final formation result does not make every intermediate activity meaningless. The appropriate warrant can concern a bounded experiment, a worthwhile current practice or a conditional next stage, provided that this is the actual purpose being assessed.

15.3 Milestone Evidence

For the prospective path, the dossier needs evidence that the first operation is preparation for the declared second operation. Attendance or expenditure may support part of that claim, but the link also concerns content, guidance, opportunities to respond and continuing access. The accepted milestone should therefore state its inferential role: it licenses one specified continuation under retained conditions.

Such evidence differs from an assertion that the intended participant is already fully formed. The distinction allows a review to remain demanding without requiring the final result before supporting the work needed to produce it. It also permits a negative judgment. If the preparation does not occur, its content proves irrelevant, or the next operation becomes unavailable, the basis for the planned continuation has changed.

The dossier should preserve these negative and inconclusive outcomes. A framework designed only to recognize promising formation would be unable to distinguish a credible path from indefinite deferral. The relevant historical object includes failed attempts, revisions and withdrawn commitments, because those occurrences affect what can reasonably be claimed later.

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15.4 Sensitivity To Model Boundaries

Several changes alter the finite result in intelligible ways. If preparation and completion can occur in one supported period, the strict rule's two-stage obstruction disappears. If preparation itself produces qualifying delivery, the same checkpoint may admit the path. If an independently funded completion route remains available after refusal, the no-alternative premise fails.

These variations help identify what empirical inquiry should examine. They also prevent the model from explaining every outcome by definition. The claim depends on a genuine temporal dependency, a particular criterion and its practical control of support. Removing any of those links can remove the obstruction.

The field model has additional boundaries. Different service prices, external inputs, exchange asymmetries or production laws can alter its numerical estimates. The particular proof survives restrictions of its control domain, since it already allows a generous family. It does not automatically survive an enlarged domain or a different law. Its value is the explicit derivation of one mechanism, not an invariant numerical prediction for teaching grants.

15.5 Computational Checks

The source packet retains two reproducible investigations. The finite checker enumerates the constructed operation system across initial summaries, renewal gates and terminal target sets. It compares forward path enumeration with the backward recurrence, checks the two displayed rules and verifies the checkpoint-domain identity on the selected history representation. These are exact discrete checks.

The field script integrates the declared law for the displayed schedules and heterogeneous allocations, checks the analytical bounds and compares step sizes. Its plots show illustrative solutions rather than estimated programme effects. The proof applies to all admissible controls under its assumptions; the numerical sample does not independently establish that universal claim.

The investigations also serve different failure checks. Enumeration can expose an omitted branch in the small finite example. Numerical integration can expose an incorrect closed-form trajectory or figure. Neither can establish that an actual teaching encounter should be represented by one of the modeled channels. That interpretive work requires the source, historical and philosophical inquiries developed throughout the paper.

15.6 Assessment Outcome

A completed dossier could support several outcomes. It might justify a bounded continuation, require a revised milestone, identify an alternative resource route or support ending the proposed sequence. The present construction does not predetermine which outcome a real case should receive.

What it changes is the question asked of a failed record. The record is assessed within a history of support, operations and review, and the allocation is assessed within a field of competing claims. This yields a more explicit argument about the proposed continuation without converting the represented arrangement into a score or the formal possibility into a duty.

16 Research Boundaries

This preliminary study has investigated a bounded allocation mechanism: a teaching grant can support the activities from which later qualifying evidence is produced, while the timing and content of a review determine which further activities remain possible. This section states the resulting contribution, its relation to prior work and the questions that remain open.

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16.1 Established Conditional Results

The finite construction separates support decisions, participant actions and their possible successors. Its backward criterion characterizes sure target attainment under an installed rule and full information. The two-stage countermodel establishes that a delivery-only checkpoint can block every represented formation path while a bounded milestone allowance admits one at the same displayed expenditure.

The nonlinear field investigation strengthens the activity comparison. Its strict-rule obstruction permits continuous mixing and spatial redistribution of a common provisioning budget. It derives a universal upper bound for renewed histories and a separate bound for denied histories, then constructs a protected path meeting the declared joint activity target. The timing calculation also distinguishes cumulative practice exposure from a terminal activity snapshot.

The operator representation locates the intermediate restriction within a history-extension word. Its domain-projection comparison states exactly when a final-only acceptance and a checkpointed acceptance coincide. The record algebra further shows how a criterion can be implementable from the available record while that record fails to distinguish formation viability.

These results remain conditional on different specified domains. The finite target includes a stipulated formation witness; the field target concerns activity exposure and delivery. Their connection is an interpreted comparison, not an identity of the target sets.

16.2 Inherited Analytical Resources

The paper does not claim to discover the dependence of future opportunities on earlier allocations. The reviewed literature already investigates incentive direction, measurement reactivity, accountability, access burdens and subsequent effects of selection. Its disagreements and domain differences remain substantive.

Within the GR programme, the preservation taxonomy already distinguishes preserved objects and continuation conditions. Normative Formation already develops resource-conditioned recognition, endogenous weighting and competence reproduction. The power manuscripts already distinguish historical carriers, field realizations and judgment-conditioned transformations [12, 9, 11, 10]. The present work uses these resources to articulate a specific teaching-grant problem.

The proposed contribution is consequently a bounded integration and investigation: formation timing, renewal evidence and supported continuation are given connected philosophical and formal treatments. Whether this configuration supplies a distinctive empirical explanation remains to be tested. Formal tractability alone does not demonstrate scholarly novelty or practical importance.

16.3 Empirical Research

The institutional guides establish documented eligibility, support and reporting arrangements for identified programme versions. They also show explicit formation purposes and entry routes. They do not establish that the constructed obstruction occurred in those programmes. The paper has conducted no applicant-level causal study.

A further inquiry would need histories of proposed activities, actual support, review decisions and available alternatives. It would also need participants' accounts of workload, willingness and the meaning of the intended role. Award statistics alone would not identify whether an unsuccessful application lacked a credible path or lost the support on which its qualifying evidence depended.

Comparative work could examine the timing and interpretation of milestones without assuming that longer commitments are always preferable. It should retain cases of successful accountability, unwarranted continuation and viable alternative support. Those cases test the proposed mechanism's limits as much as a documented interrupted path would test its relevance.

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16.4 Normative Research

The normative argument distinguishes an evidence error, an obstruction of a declared formation purpose and an unfair allocation. None entails the others without additional premises. A feasible formation path can lose to a more urgent claim; a well intended protected grant can transfer unacceptable work or uncertainty to participants.

Further work could make the joint allocation problem explicit. That would require several arrangements, their different current obligations, support dependencies and criteria of comparison. It should preserve disagreement about those criteria rather than hide it inside a single generativity score.

The present study supports a limited deliberative demand: a proposal for renewal should identify the continuation it seeks, the evidence that warrants it, the burdens it imposes and the claims it displaces. Its formal possibility is one component of that argument.

16.5 Representational Continuation

The history family, two-complex, nonlinear field and operator algebra are several possible representations. Their compatibility depends on explicit translation conditions. A richer historical model could change its sites, roles and operation domains as further participants form. The fixed graph here retains those structures only during the bounded comparison.

Likewise, uncertain grant decisions, partial observations and multiple support sources would require additional structure. Probabilistic models could investigate risk once justified distributions are available. They would answer a different question from sure viability.

The analytical center remains the supported formation history. Studying its grant decisions requires attention to the conditions that produce evidence and to the operations that evidence is used to authorize. The resulting framework offers a disciplined way to examine that relation, while leaving the appropriate allocation open to substantive judgment.

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