

Linguistic Capacity Reactivation

A Preliminary Study Of Activity-Support Coupling

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Abstract

A successful return to a linguistic activity can end when its initiating support is withdrawn, even when a selected capacity remains. This preliminary study investigates sustained reactivation through the coupling of realized activity and enabling support. A focused review of residual knowledge, relearning, re-exposure and longer use histories constrains the interpretation of return. Within the Generative Relational framework, history-bearing configurations retain operation families, execution conditions and historical warrants independently of numerical activity. One nonlinear regional field model then represents a selected activity channel and a replenishable support channel. The analysis derives positive bounded evolution, stationary regimes, a nonlinear sustaining region and a sufficient finite initiating pulse. A timing counterexample proves that equal initial activity can lead to extinction or persistence when support differs. A regional calculation identifies a further obstruction from retained boundary coupling. Observable algebras and partial history operators distinguish immediate activity, sustained-domain membership and executable staged intervention. Philosophical objections and a fictional return episode test the interpretation of these results. The model is an illustrative representation rather than an empirically calibrated law; its certificates establish conditional channel persistence, while capacity retention, meaningful use, participant authority and acceptable support burdens require additional arguments.

Keywords: linguistic capacity; reactivation; enabling conditions; support renewal; nonlinear fields; operation algebra

Scope Of This Preliminary Study

This paper examines a selected process: a retained linguistic capacity is initially unavailable for specified situated operations, an initiating intervention changes its conditions of realization, and the resulting activity may continue after that intervention ends. The central question concerns the organization and timing of activity–support coupling. It is narrower than a general account of language vitality, a method for deciding who possesses an entire language, or a programme for the revival of every interrupted tradition.

The argument belongs to an open Generative Relational research programme. Its historical and relational vocabulary is inherited; its philosophical distinctions remain revisable; its nonlinear field law is one proposed representation. The derived conditions follow from that law. They do not establish numerical thresholds for human speakers. The formalism keeps a selected operation grammar separate from the amplitude of its modeled realization, and keeps sustained activity separate from consent, legitimacy, historical authenticity and the formation of new participants.

The paper is written for discussion and further investigation. It includes the derivations needed to assess its model, a focused primary-literature review, analytical figures, objections and replies, and a constructed case whose status is explicitly fictional. The source packet records source verification, prior-work reading scopes, computational checks and manuscript review. These checks establish the reproducibility and internal discipline of the draft, while leaving its substantive adequacy open to criticism.

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Status. This discussion draft presents an evolving inquiry. The definitions, translations, formal constructions, empirical interpretations, and conclusions remain open to criticism and author review. It investigates the sustained operation of selected retained linguistic capacities after finite initiating support. Its formal models are conditional representations; they establish neither universal linguistic thresholds nor an empirically calibrated law of language change.

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Statement On The Use Of Language Models. The author selected the research direction, supplied prior manuscripts and a reference exposition, and requested consistency with the Generative Relational research programme. AI-assisted research, source comparison, drafting, formal checking, and document preparation contributed substantially to this draft. These contributions include proposed arguments and interpretations that remain subject to the author's review. Preparation of a checked draft does not establish the author's approval of every claim. Authorship credit remains with the human author; the use of these tools is disclosed to support critical assessment of the manuscript and its limitations.

1 The Reactivation Problem

A person returns to a familiar language during a carefully organized encounter. A patient interlocutor allows time, supplies a suitable topic, repairs misunderstandings and makes continued participation possible. The encounter succeeds. Several weeks later, the selected uses have again become unavailable in the person's ordinary setting. A second encounter can produce another success without changing this longer pattern. The analytical difficulty concerns the relation between an episode that elicits activity and an arrangement that sustains the conditions of further activity. Describing both as a successful return conceals the difference that this paper investigates.

The example leaves several questions open deliberately. The first encounter might reveal retained organization, make an inaccessible operation executable, teach something new, or combine those processes. It might revive a particular form of interpretation while leaving spontaneous production unavailable. It might also depend on work that is easy to overlook because another participant performs it. An adequate reactivation account must specify its capacity target and the history that warrants treating that target as retained. It must then explain what changes when the initiating episode ends.

This preliminary study concerns *sustained reactivation of selected linguistic operations*. Its primary analytical object is a process in which a historically retained operation family becomes executable and enters a continuing organization of activity and enabling support. The word *selected* matters throughout. No single successful response establishes possession of an entire language. No positive field amplitude establishes meaningful linguistic use. The target might concern participation in one recurrent conversational setting, interpretation of a family of expressions, or a specified repair practice. Its scope must be stated before the process is evaluated.

1.1 Finite Initiation

An initiating intervention is finite when the particular support designated as initiating support ends after a declared interval. That ending does not remove every condition of linguistic activity. Interlocutors, material access, opportunities, routines and historical competence can remain. The relevant question is whether the post-intervention arrangement supplies enough of the conditions required for the selected operation to continue. A return supported by enduring relations can be a substantive achievement. Its accurate description should name those relations rather than interpret continuation as independence from them.

The distinction also changes the temporal object of inquiry. A short observation can document a present success. Sustained reactivation concerns an extended trajectory and the processes through which later occasions remain possible. If practice renews familiarity, encourages further encounters or maintains a repair relation, current activity can contribute to its later conditions. If those processes are too slow, too weak, or distributed across relations that become unavailable, initial performance can disappear before they become effective. Timing can therefore matter even when the capacities and long-run possibilities described at the outset are similar.

This is a conditional mechanism claim. Many linguistic returns may follow another organization. A continuing external service may supply the needed support without substantial renewal through use. A recurrent activity may be seasonal rather than continuously positive. A transformation of identity or institutional access may change the available operation family altogether. The present investigation gives one process a sufficiently explicit representation to examine its consequences, while retaining the broader philosophical distinctions needed to interpret that representation.

1.2 Inherited Questions

The preservation taxonomy already distinguishes capacity grammar, operational availability, exercise, enabling conditions and historical correspondence [7, Sections 7–8]. Those distinctions supply the starting point here. A capacity can be represented as retained while a selected execution is unavailable; conversely, a capacity that is available may simply not be exercised. Reactivation cannot be read directly

from the difference between zero and positive activity in a plot. The historical and execution claims require separate warrants.

The earlier study of linguistic competence reproduction develops mutually enabling operational channels, nonlinear coexistence of inactive and active regimes, regional boundary dependence, and the difference between sustained local activity and renewal of competence in further participants [10, Sections 6–8]. Generic feedback, bistability and dependence on support are therefore inherited resources. The present paper investigates a more specific timing problem: a finite activation must last long enough for the chosen support process to enter a condition that remains sufficient after the initiating input ends.

The attribution study asks what warrants calling a selected capacity dormant [8, Sections 3–8]. This paper takes a warranted attribution as a premise where its reactivation interpretation needs one. The historical-continuity study separately examines how earlier linguistic organization contributes to later productive practice across interrupted transmission [9]. A reconstructed practice can sustain later activity without demonstrating that a particular participant retained the relevant operation throughout the interruption. The present model does not resolve that historical question by predicting activity.

1.3 Method Of Investigation

The inquiry combines philosophical argument, a focused literature comparison, a historical representation and a conditional dynamical model. The philosophical argument specifies the target and tests competing descriptions of the return. The literature constrains what can be inferred from residual traces, re-exposure and longer trajectories. The historical representation retains participants, operations and support relations rather than silently replacing them with independent point-like entities. The model then selects activity and support channels on a finite regional interface graph.

Two nonlinear fields are used. One represents a chosen intensity of realized activity; the other represents a selected enabling-support channel. Activity can help renew support, and support can enable activity. Both descriptions depend on an independently defended interpretation. The equations are a stipulated mechanism with saturation, loss and exchange, not a consequence of calling language relational. The model permits a direct derivation of a sufficient finite activation condition, a timing obstruction, and a regional coupling obstruction. These results are mathematical consequences of the declared law.

An accompanying algebraic construction identifies what a successful staged intervention must preserve. Its role is precise. A stage must first be executable on the histories being considered; its endpoint must then lie in a region whose baseline continuation satisfies the declared activity condition. Recording only an endpoint projection can hide an undefined intervention. Recording only immediate activity can hide an insufficient support state. The construction makes these different obligations visible and connects them to the nonlinear analysis.

The argument also explains why its strongest formal condition is narrower than sustained linguistic life. A uniform positive lower bound is useful for proving a sufficient invariant region. It does not define meaningful practice or rule out intermittent participation. A fixed graph supplies one dynamical sector. Changes in participants, operation types or institutional arrangements require additional historical comparison data. A mathematical support variable does not determine whether maintaining it is fair or whether its beneficiaries consent.

1.4 Proposed Contribution

The proposed contribution is a worked investigation of finite activation within an explicitly interpreted activity–support mechanism. Its strongest claim concerns the selected law: initial activity alone can fail to determine continuation; support accumulation can make intervention duration consequential; and a specified positive region can certify continuation after the finite input ends. The formal results establish neither novelty in general nonlinear dynamics nor empirical thresholds for linguistic reactivation.

The paper develops the needed distinctions before deriving the field results. It then connects staged operations to the sustained region, examines the limits of the empirical translation, and tests the framework against objections and a constructed return episode. Each stage is intended to constrain the next.

A capacity attribution constrains what the field is allowed to represent; the nonlinear calculation constrains claims made about timing; the case analysis constrains the practical language used to describe success. The resulting account remains preliminary, but it is specific enough to be criticized through its assumptions, derivations and interpretations.

2 Linguistic Reactivation Research

Research on linguistic retention, attrition, relearning, and renewed use supplies several kinds of evidence relevant to sustained reactivation. Their relevance depends on the operation studied, the participant's earlier history, and the interval over which a return is observed. This section compares those evidential tasks. It first considers retained contributions to later learning, then changes in adult bilingual performance, and finally the relation between continued use and maintenance. The method is a selective primary-source comparison, with the present paper's interpretive proposals separated from the studies' findings. The objective is to identify the empirical commitments that a reactivation account would incur, while preserving the differences among the research programmes reviewed.

2.1 Historical Reactivation Targets

A return claim contains a historical commitment. An operation available later is being connected to an earlier capacity, and the intervening condition is being interpreted as a period of reduced or absent availability. The relevant earlier capacity therefore needs a description independent of the later success. Otherwise, a person who learns rapidly after renewed exposure could be classified as having recovered whichever operation the later test happens to reveal. Such a classification would make the proposed continuity too easy to satisfy: present acquisition would determine the content attributed to the past.

The distinction matters even when early exposure is well documented. Exposure may contribute to perceptual discrimination, familiarity with recurring forms, affective orientation, comprehension, or productive organization in different combinations. A history containing one of these contributions does not by itself establish that a later productive operation was already available. The analytical choice is consequently between several possible return claims, each with its own warrant. A retained discrimination might become usable again; retained discriminative resources might also assist the formation of a new productive capacity. Both histories preserve a contribution from the past. Only the first directly describes the return of the selected discrimination, and neither alone establishes the return of a complete language.

Basham and Fathman's category of the latent speaker concerns people raised in an environment where an ancestral language was spoken who did not themselves become speakers. Their abstract distinguishes receptive resources and productive hesitation, drawing on successful adult learner cases and beginning learners [1]. This category therefore requires care when used to identify a previously possessed productive capacity.

For the present analysis, hesitation, inability, and reduced opportunity remain distinct descriptions. A participant may decline a task that would be possible in another relationship. Conversely, willingness and identification can coexist with substantial learning needs. Assigning these possibilities separate places avoids treating an identity category as a psycholinguistic diagnosis. It also avoids making the researcher's preferred form of participation the measure of a person's linguistic reality. What the person can do, what the person wishes to do, and what the surrounding arrangement permits are related questions whose answers need not coincide.

The earlier attribution study supplies a useful discipline here: specify the selected operation and ask what retained organization contributes to its possible execution [8]. The present paper takes that discipline forward into a temporal question. Once a return has occurred, its continuation has to be investigated through later episodes and their support conditions. The history-bearing object includes the retained contribution, the enabling change, and the subsequent activity; reducing the object to a before-and-after score would omit the relation that connects these parts.

2.2 Savings Evidence

Savings research makes past experience visible through a difference in subsequent learning. A person may fail an initial retrieval task yet benefit from earlier exposure during later training. Such a comparison changes the evidential question. It asks whether the learning history still affects the present trajectory, rather than relying exclusively on immediate successful recall. For a reactivation account, that contribution is valuable because an unavailable response need not imply the complete absence of historically acquired resources.

De Bot, Martens, and Stoessel compared relearning of previously encountered vocabulary with learning of selected new items. Their experiments included words unavailable in initial translation tests, and a French replication included a test three weeks after training. The study therefore supplies both savings comparisons and a bounded delayed observation. Earlier item knowledge was partly inferred from educational history; training itself supplied translations [5].

Two operations should be distinguished when interpreting such a design. Retrieving a word without further instruction is one operation; learning a word with the assistance of retained resources is another. Improvement in the second can support an attribution of historical influence without establishing that the first remained executable throughout the intervening period. The difference is conceptual, not a criticism of savings methodology. A method can provide appropriate evidence for a surviving contribution while leaving the organization and extent of that contribution partly unresolved.

Choi, Broersma, and Cutler studied Korean adoptees and Dutch controls during 10–12 days of phonological training. The authors interpret the learning trajectory as evidence of retained early phonological knowledge. The aggregate group difference diminished, with a nonsignificant final aggregate comparison; age-subgroup analyses address a different comparison. Testing ended within the training programme, so the design did not establish later ordinary use or post-training persistence [4].

The present paper draws a narrower methodological lesson from the distinction between a trajectory and an endpoint. Equal endpoint performance can coexist with different routes to that endpoint. Conversely, an advantage during early training can coexist with convergence later. A preservation question therefore needs to specify whether the target is immediate execution, reduced relearning cost, transfer to a new task, or maintenance after instruction. These targets can be connected without being interchangeable. Their separation allows an account to recognize a retained contribution without claiming more than its evidence identifies.

A further issue concerns what the intervention supplies. An earlier resource can make instruction effective while the instruction contributes material required for the eventual operation. Describing that outcome as mixed recovery and formation is more informative than forcing a choice between complete retention and wholly new learning. The mixed description still requires analysis: which operation was retained, which relation became available, and which new organization emerged? Those questions establish the appropriate relation between savings evidence and the operation-specific starting point adopted in Section 3.

2.3 Adult Linguistic Access

Adult attrition research addresses a different starting point when participants have a history of developed linguistic competence and continue using more than one language. Changes in performance can then concern access to a still extensively used repertoire. This situation is relevant to reactivation because it allows changes in availability to be investigated without assuming that an entire language has ceased operating. Its relevance is domain-specific: the return of a selected processing pattern supplies evidence about that pattern, while broader capacities may already be active.

Chamorro, Sorace, and Sturt compared Spanish pronoun interpretation among monolingual controls, attriters, and recently re-exposed attriters. In the relevant online comparison, the re-exposed group differed significantly from neither comparison group, and the authors considered an intermediate interpretation. Offline judgments did not show the corresponding group differences. Recent exposure was compared across groups, rather than through a within-person return trajectory [3].

The distinction between a reference comparison and a demonstrated transition is decisive for the present argument. A group observed after exposure may resemble a reference group without having been measured before exposure. The difference between those groups can therefore leave the individual's earlier state unresolved. Even with before-and-after measurements, a nonsignificant contrast is compatible with several effect sizes and patterns of heterogeneity. An inference to unchanged representation requires additional assumptions about what the task detects and what differences the design could reveal.

Those limits do not deprive processing studies of value. They identify a level at which a return can be investigated precisely. Timed interpretation, choice, and overt production can be distinguished, rather than combined into an undifferentiated declaration of recovery. A theoretical account can then ask whether a proposed enabling change affects one channel, several channels, or the relation among them. This is especially useful when an ordinary conversational success depends on multiple operations whose separate trajectories differ.

Mickan, McQueen, and Lemhöfer experimentally examined interference with recently learned Spanish vocabulary after retrieval of corresponding words in other languages. Their reported effects depended on the measure and delay; they also noted that earlier testing could affect later performance [13]. The study concerns newly learned items and supplies a possible access mechanism, rather than direct evidence about a long-dormant adult repertoire.

For the present model comparison, access competition is an alternative to a simple story of disappearance during nonuse. It also cautions against treating support as a generic quantity with a uniformly beneficial effect. An encounter may facilitate one operation while recruiting a competing operation, or improve deliberate retrieval without producing a corresponding change in spontaneous interaction. Which effect matters depends on the selected task and history. The formal activity variable introduced later must therefore represent a specified channel; its increase cannot be interpreted as an increase in linguistic competence without further measurement assumptions.

2.4 Reactivation Observation Horizons

A claim of sustained operation needs an observation horizon and a description of the changes occurring within it. The number of days between two tests is insufficient by itself. A later test may follow continued practice, repeated instruction, a return to an earlier environment, or a period in which relevant encounters became rare. Each history poses a different question about continuation. A longer interval can therefore be less informative about the effect of a finite initiating intervention than a shorter interval whose intervening conditions are well specified.

Stolberg and Münch followed an adult German speaker through conversational contacts during 2000–2004. Their case began with substantial German proficiency; it did not document the return of a wholly unavailable language. Changes differed across linguistic measures, with lexical access showing the clearest improvement [14].

Köpke and Genevskaja-Hanke's Bulgarian–German case compared recordings across two settings in 2012 and again in 2017. The later observations followed a change in household language use, so the five-year interval cannot isolate a lasting effect of the earlier brief re-exposure [11].

These designs motivate a distinction in the present inquiry between the duration of observation and the duration of an independently specified intervention. An activity can continue for years while receiving continuing support. Such support is part of the explanatory object when the question concerns successful everyday operation. It becomes a complication only for the stronger claim that one short initiating event sufficed to produce the later history. Distinguishing those claims preserves the significance of supported continuation without attributing its entire production to its earliest identifiable occasion.

The same distinction applies to the research encounter. Repeated elicitation can supply practice, attention, an interlocutor, or a reason to prepare. These possibilities do not automatically invalidate the observations. They change the account of the system being observed. If the relevant question concerns operation during a continuing research relationship, that relationship belongs in the realization condi-

tions. If the question concerns operation after the relationship ends, observations during it leave a later evidential task. The target history determines which boundary matters.

Temporal scope also has an ordinary practical dimension. A capacity used seasonally or for occasional family communication may count as sustained when it remains available at recurrent relevant occasions. Continuous overt activity would be an inappropriate criterion for that target. Conversely, repeated high performance in the same brief test can provide weak evidence about reliable participation in varied encounters. A useful horizon combines elapsed time, opportunities for execution, and the range of conditions over which continued availability is being attributed.

2.5 Linguistic Use Histories

Use and maintenance can be related through several causal routes. Practice might contribute to continued availability; continued availability might make participation easier; an accessible social setting might support both. The routes can coexist. A reactivation theory that makes activity renew its own enabling relations undertakes a stronger task than demonstrating an association between activity and later performance. It needs to identify the intervening process through which earlier operation contributes to the conditions of subsequent operation.

Mickan and colleagues followed 97 German learners for six months after Spanish study abroad, combining vocabulary tests with repeated reports of language use. Continued Spanish use predicted less forgetting in their analysis. The design provides observational evidence about maintenance after an intensive learning period; its participants were not selected as bearers of dormant capacities [12].

An operation-support account can take such an association seriously while keeping its own mechanism provisional. Consider a constructed comparison. One participant practices because a continuing group organizes encounters; another helps organize the next encounter through successful participation in the present one. Both might report similar use and show similar later performance. Only the second history supplies the proposed activity-dependent renewal relation as described. The difference concerns the production of support, not the amount of language activity measured at an endpoint.

The hypothetical comparison also clarifies what should count as an explanation of persistence. If an outside institution repeatedly supplies the required encounters, the operation can be stable relative to that institutional support. A model that omits the institution might represent the same history as self-maintenance. Including the support source changes the interpretation without changing the observations. The boundary of the analytical object therefore needs to be stated before claims about autonomous continuation, external maintenance, or reciprocal renewal can be assessed.

Use measures present a related problem of content. Time spent with a language can involve listening, reading, rehearsal, translation, spontaneous conversation, or collaborative problem solving. An intervention appropriate to one selected operation need not supply the conditions of another. In the present framework, those differences enter through the operation grammar and realization data. A single activity channel is an explicit restriction on that richer history. Its mathematical convenience should remain subordinate to the linguistic comparison it is intended to represent.

2.6 Formal Reactivation Interpretation

The reviewed traditions provide evidence at partially different levels: historical influence on learning, selected processing changes, delayed item availability, repeated conversational performance, and maintenance correlated with continuing use. The present paper connects these levels through a question about continued realization of retained operations. It does not claim that the studies instantiate one shared dynamical law. Their tasks, histories, and observation windows are too different for that identification to follow from the review.

A field representation can nevertheless be useful if its translation obligations are explicit. Activity must be connected to a declared class of manifestations, support to specified enabling relations, and a history to the operations whose continuation is under investigation. These commitments are stronger than assigning a numerical score to proficiency. They identify what a model preserves and what its

projection leaves unresolved. In particular, the existence of a positive mathematical regime does not by itself establish that a speaker retains an operation, consents to its exercise, or has access to the relations required for ordinary participation.

The operator approach has a corresponding boundary. An order-sensitive composition can represent the difference between preparing a support relation and attempting an operation before that relation exists. Such a representation supplies a structured possibility, whose linguistic interpretation still requires evidence about the actual interventions and dependencies. A successful composition in a finite example is neither a discovered sequence of psychological stages nor a recommendation that every participant should undergo that sequence.

The contribution investigated here is consequently conditional and integrative. Given an operation-specific retention attribution, and given a defensible mapping of activity and support, the paper examines how a finite initiating change could enter a continuing history, and how equal immediate performances could leave different prospects for continuation. Section 12 returns to the primary cases to examine the limits of that translation. The studies remain evidence for their own measured outcomes; the formal results remain deductions from declared representations.

3 Retained Linguistic Operations

Reactivation concerns the return of a selected operation whose relevant organization has been retained through a period of unavailability. The retention claim gives the process its historical object; the changed execution conditions explain what must happen for that object to become usable again. This section establishes those premises before the dynamical investigation. It specifies the operation and its bearer, distinguishes inactivity from operational unavailability, and examines the contributions that enabling change and further learning can make to the same return episode. The argument inherits the taxonomy's availability distinctions and the earlier dormancy paper's attribution analysis [7, Section 7][8, Sections 3,5].

3.1 The Selected Operation

A language name is generally too coarse to specify the operation returning to use. An inquiry might concern producing a particular contrast, interpreting a narrative relation, correcting an inappropriate use, or participating in a recurring exchange. These achievements can separate. A participant may recognize a contrast while needing help to formulate it, or produce a familiar expression while failing to extend its use. A reactivation claim should identify the relevant family of occasions and the contribution required within it.

Consider a constructed narrative circle whose participants previously distinguished a completed event from an interrupted attempt. The selected profile includes interpreting unfamiliar descriptions, requesting clarification when the completion status is unclear, and correcting a narration that erases the interruption. The case stipulates a linguistic practice without assigning these forms to an actual language. It also stipulates prior competent participants, so it does not derive their earlier competence from the success of a later workshop.

The profile reaches beyond repeating a phrase. Its productive element concerns applying the distinction to further narrations; its interpretive element concerns recognizing the circumstances in which that distinction matters. A participant can retain some elements while needing renewed instruction for others. The current object is consequently a selected profile, not the totality of the participant's linguistic history. A finding about the profile should remain at that level even when participants use a shared language name to describe their broader undertaking.

The specification also identifies ordinary resources. A record containing examples may be an established aid, while a facilitator who supplies the answer to each new case performs part of the selected discrimination. Both arrangements involve support, but they distribute the relevant work differently. The inquiry must say what the participant or collective contributes rather than treating either all assistance or complete absence of assistance as the default condition of competence.

3.2 Operational Unavailability

The taxonomy distinguishes a represented operation type, its state-dependent realization, its execution domain, admissibility and actual exercise [7, Section 7]. An operation can be available without being exercised on a particular occasion. Dormancy, in the narrower convention inherited here, concerns historically retained organization whose selected operation is currently unavailable under the declared conditions. That convention makes the missing execution relation part of the problem.

Silence therefore requires interpretation. A participant can decline to narrate, lack an appropriate occasion, encounter an inaccessible medium, or have lost the distinction itself. These possibilities can produce the same observed absence of speech. The absence is evidence about the encounter; it does not classify the entire participant or practice. In particular, voluntary nonparticipation should not automatically be translated into a dysfunction requiring activation.

In the constructed circle, suppose a setting change removes the participants' shared access to the materials and interlocutors needed for the selected exchanges. The inquiry can investigate whether the relevant organization remains and which conditions prevent its use. An observation that no such exchange occurs is insufficient by itself. A stronger account identifies the inaccessible relation and explains how the retained contribution would enter execution if that relation were restored.

This distinction constrains the field interpretation. A vanishing activity amplitude can represent absence of realized activity, but it does not independently establish retained capacity or operational unavailability. Those predicates require their own meanings and grounds. The dynamical model can then investigate a realization process conditional on the attribution. Giving a zero coordinate the name dormant would otherwise place the conclusion inside the notation.

3.3 Present Retained Contributions

A retention attribution concerns something organized in the present bearer, with a warranted relation to earlier formation. Evidence of past proficiency establishes an earlier achievement; evidence of present trainability establishes a prospect. Neither alone identifies the contribution that remains through the interval. The earlier dormancy investigation develops this requirement in terms of a selected present organization [8, Section 3]. The current paper uses that result as a premise and studies its subsequent realization.

The relevant contribution can involve a discrimination, a way of selecting examples, a coordinated response pattern or a distribution of complementary roles. It need not be fully verbalized. In the narrative circle, participants might recognize that an unfamiliar account leaves completion unresolved and ask a suitable question before receiving any explanation of the distinction. Such behavior can support a particular retained contribution when the encounter's other effects are understood. It does not automatically establish the whole earlier profile.

Retention also admits transformation. A participant can retain the distinction while adapting its expression to a changed medium or new conversational partners. The historical claim then follows the selected organization through an explicit comparison. Exact repetition of earlier outputs would be unnecessarily restrictive for that target. At the same time, a broad label such as the same practice cannot protect the claim if the relevant discrimination has been supplied anew.

A partial finding can be substantial. Participants may retain sensitivity to interrupted attempts while having forgotten a local convention for marking them. Relearning that convention can combine with a retained interpretive contribution. The resulting account identifies what the episode reactivated and what it formed. It need not divide the achievement into numerical shares of old and new competence. Such shares would require a defensible common scale and an account of interactions that the historical description does not itself provide.

3.4 Enabling Change

An initiating intervention can alter a missing execution relation without supplying the selected distinction. It can provide an accessible meeting, restore a recording interface or arrange a partner whose contribution fits an existing role. Its content matters more than its administrative name. An event described as a reminder can teach; an event described as training can largely restore access to previously formed organization.

The difference is particularly visible in the narrative example. Presenting a new recording supplies material on which the participants exercise their distinction. Explaining that a particular missing event makes the narration an interrupted attempt can supply part of the discrimination. Both acts communicate information. Their difference concerns the task organization contributed by the participant, rather than whether the intervention was informationally empty.

The intervention's sequence can help locate its contributions. A discrimination observed before an explicit explanation has a different evidential relation to retention from one observed afterward. Even the earlier response can be influenced by examples or newly visible relations. The record should preserve those possibilities. The purpose is to distinguish informative histories, not to assume that an investigator can expose a capacity without ever changing its circumstances.

The strongest enabling interpretation requires an independently supported limit on what the intervention can create. Paper 4 develops a target-relative noncreation condition for that inference [8, Section 5]. The present inquiry need not prove that condition afresh for every model. It must make clear when retention is stipulated, independently supported or still unresolved. Successful post-intervention activity cannot be asked to establish all three statuses by itself.

3.5 Historical Bearer Boundaries

Reactivation can be attributed to an individual using established aids, a partnership or a larger arrangement. The choice changes the claim. Restoring a joint narrative exchange can require contributions from several participants even when each retains a relevant discrimination. Attributing the exchange to the arrangement preserves its coordination; attributing a selected contribution to one participant preserves what that person does. These accounts need not compete for exclusive possession of the same achievement.

The boundary must nevertheless remain explicit across the interval. If the later arrangement includes a new expert who supplies all interpretation, its successful outputs do not establish that the earlier participants regained the selected operation. The arrangement may have acquired a new capacity. That can be a valuable achievement while supporting a different historical description.

Conversely, continuing dependence on a familiar aid does not automatically defeat reactivation. If use of the aid belonged to the original task, removing it tests a stronger or different target. A post-intervention claim can concern the end of a temporary facilitator's role while allowing the ordinary record, room and partner network to remain. The support boundary should follow the proposed comparison, rather than an ideal of a speaker detached from material and social conditions.

The taxonomy treats internal and external conditions as distinctions relative to the represented entity and resolution [7, Section 8]. Enlarging the description can make previously external coordination visible within the history-bearing object. It cannot make the work disappear. Any claim that the arrangement now sustains part of its support must identify the continuing contributors and relations.

3.6 Return Processes

Several processes can accompany the same episode. Maintenance preserves selected conditions or availability over an interval. Reactivation restores execution after a specified unavailability. Reconstruction forms or rebuilds the selected organization. Where retention remains unresolved, the choice between reconstruction and reactivation can also remain unresolved; missing evidence of retention does not establish that reconstruction occurred. Reproduction concerns the formation of relevant competence in

further participants. The earlier reproduction study establishes why competent current use does not by itself demonstrate that formative capacity [10, Section 3].

These distinctions allow a return episode to be described positively. Former participants may reactivate a retained contrast, learn a new convention, maintain a shared record and begin helping newcomers. The evidence for each contribution can differ. A successful session supports what happened in that session; subsequent competent use and further participant formation require their own histories.

The present paper selects one relation within that larger configuration: a retained operation becomes usable and then continues to be realized after the finite initiating intervention ends. It does not require an unchanged whole practice or a universal separation of learning from use. It requires sufficient historical and analytical continuity to identify the returning operation, followed by an account of the changed support and coupling that permits its sustained realization.

4 Sustained Reactivation

Sustained reactivation concerns a coupled history in which a retained operation returns to use and remains usable across further relevant occasions after a specified initiating intervention ends. The additional requirement concerns the continuation, not a more impressive first performance. This section develops that requirement through temporal scope, continuing supports, activity-dependent renewal and withdrawal comparisons. Its aim is to give the later dynamical conditions a substantive interpretation without making one mathematical regime the definition of a successful linguistic practice.

4.1 Continuation Horizons

A sustained-operation claim identifies a horizon over which continuation matters. The horizon might encompass several meetings, a season of recurrent use or a longer institutional interval. Its adequacy follows from the practice and question. An operation used in an annual gathering should not be judged by a requirement of daily production. Conversely, a single workshop cannot establish regular availability for a task whose ordinary occasions recur throughout the year.

The temporal unit can therefore include opportunities as well as clock duration. Two months containing frequent relevant encounters provide a different continuation history from two months with no occasion to use the operation. The latter interval need not indicate failure, but it supplies less direct evidence of recurrent realization. An inquiry should distinguish elapsed time, opportunities, actual uses and the conditions under which use remains possible.

The taxonomy's treatment of retention and metastable regimes already makes historical scale and admissible continuations part of the target [7, Section 10]. The present application uses that principle to resist two excessive demands. Sustained use need not mean constant activity at every instant. Nor does success over one selected horizon establish indefinite survival under every future transformation. The claim is definite when it identifies the continuation tested and the further changes it leaves open.

This distinction also governs the interpretation of a formal invariant region. Remaining in such a region for all future model time can be a strong conditional result. Its social interpretation depends on whether the represented operation, baseline supports, coupling and boundary inputs remain adequate descriptions. A finite empirical trajectory cannot independently verify their indefinite stability. The mathematical result identifies sufficient premises for a continuation; the historical study examines how far those premises and the resulting pattern are supported.

4.2 Initiating Intervention

The initiating intervention is the finite contribution whose termination defines the comparison. It might arrange the first encounters, repair access, provide temporary coordination or add an exceptional resource. Its end must be described concretely. A programme's formal closing date may precede the facilitator's final unpaid preparations; a nominally independent session may continue to depend on advice supplied

afterward. The comparison concerns those operative contributions rather than the administrative label alone.

Baseline support is the support retained after that intervention ends. It can remain externally supplied relative to the selected practice: a publicly maintained room, ordinary access technology, transport, an established archive or continued institutional permission. Ending initiation does not entail withdrawing every condition of use. Such withdrawal could destroy the very supported task whose continuation the inquiry seeks to understand.

In the constructed narrative circle, a temporary coordinator might arrange the initial meetings while a community institution continues to provide an accessible venue. Participants may subsequently arrange their own meeting times, but the venue remains externally maintained. The result can support continuation after the coordinator's initiating role ends. It does not establish independence from the institution or from the labor maintaining its facilities.

The distinction should survive changes of analytical boundary. A model of the whole institution can place venue maintenance inside the represented object; a model of the circle can treat it as a boundary contribution. The historical work occurs in either description. A claim of increasing internal support is informative only when it specifies which roles have changed and how their continuation is organized. Boundary expansion alone is not an achievement of reactivation.

4.3 Activity-Dependent Renewal

Activity can contribute to its own later conditions through identifiable processes. Repeated use can maintain a shared collection of examples, make suitable partners easier to locate, clarify a meeting routine or provide participants with evidence that another encounter will be worthwhile. Such effects are possible mechanisms, not automatic benefits of speaking. Their relevance depends on whether they replenish conditions needed by the selected operation.

The narrative circle supplies a limited illustration. A session might produce a record of unresolved narrations that participants choose to revisit. Those records can supply material for later meetings. Participants can also distribute the work of arranging access, maintain an agreed contact route and identify which examples need clarification. If these activities actually occur, the session has changed some conditions of future realization. If the facilitator performs them all, the same visible narrative activity can leave the participants' continuing arrangement largely unchanged.

The difference is relational. The record, contact route and willingness to reconvene become support through their organized use. A list of names does not by itself form an accessible partner network; a collection of examples does not by itself establish a shared interpretation practice. The account should explain how the contributions become jointly available. The taxonomy's capacity-condition coupling analysis already shows why separately retained components need not realize the same operation [7, Section 8].

Renewal can also be selective. Recurrent conversation might maintain the ability to arrange another meeting while doing little to replenish transportation, technical repairs or interpretive labor. A positive return effect at one interface therefore supports a limited claim. It does not imply that activity supplies all its own conditions. The same practice can renew one resource, consume another and depend on a third that remains externally maintained.

The nonlinear model in Section 6 represents one such mutual dependence. Activity can build a selected replenishable support, while that support changes the conditions for further activity. The representation needs an interpretation of both channels and their coupling. It should be assessed for the mechanism it isolates, rather than read as a scalar account of competence, motivation or social value.

4.4 Support Formation Time

The support produced during initiation can develop on a different time scale from visible performance. A facilitator may obtain an immediate successful exchange by preparing materials, selecting partners and resolving every coordination problem. The participants' continuing arrangements can take longer to

form. A short intervention can therefore produce substantial activity while leaving little of the support required after its withdrawal.

Compare two stipulated episodes ending with similar narrative performance. In the first, the temporary coordinator supplies every future occasion and keeps the only usable record. In the second, participants have also established access to the materials and a workable method of convening the next encounter. The observed linguistic outputs can be comparable at the final session. Their post-intervention possibilities differ because the second episode leaves a different support configuration.

A coupled state description makes that difference available to analysis. The question becomes whether the initiating episode brings both activity and support into a region from which the represented continuation can persist. Increasing immediate activity alone need not achieve that result. A more intense but brief episode can end before the relevant support develops; a differently timed sequence can leave a more favorable continuation even when its peak performance is lower.

These are conditional possibilities. They do not establish that longer programmes always work better or that participants should tolerate indefinite initiation. Support can fail to develop despite prolonged activity, and additional intervention can consume resources or displace participants' own arrangements. The useful mathematical question is how duration, timing and support dynamics interact within a specified law. Its interpretation should preserve the conditions under which a longer interval actually changes the post-intervention state.

4.5 Withdrawal Comparisons

A withdrawal comparison identifies which contribution changes while tracking the other conditions relevant to continuation. If the intervention ends at the same time as the venue closes, later inactivity cannot be attributed to intervention withdrawal alone. If a new paid organizer replaces the temporary coordinator, continued activity may demonstrate a successful substitution rather than persistence without that role. Both histories can matter; they answer different questions.

The comparison also requires continuity of the operation being assessed. A group might continue meeting while moving to a different linguistic task. That can be a meaningful new practice without demonstrating sustained return of the selected contrast. Conversely, the contrast can continue through changed examples or media when the declared correspondence permits those transformations. The target should be followed at the level its claim requires.

Intervention order belongs in this history. Restoring access before the first encounter can permit a retained discrimination to operate. Holding the encounter before access is restored can produce failure or induce a substitute task. Similarly, ending temporary coordination before participants have arranged their next occasion differs from ending it after those arrangements exist. The operations can be individually available while their ordered composition has a different domain or consequence.

The operation-algebra representation in Section 11 can make such distinctions exact, but its success criterion must remain interpreted. A sequence's formal executability does not show that it produced sustained linguistic use. A certificate for the subsequent domain needs a link to the selected continuation condition. The philosophical purpose of staged operations is to preserve what each action changes and what the next action requires, rather than to replace the practice with an arbitrary ordering of labels.

4.6 Regional Continuation

Sustained return can be uneven across settings. One group can regain recurrent access while another remains dependent on an occasional visiting partner. A total activity record may rise even as some participants' opportunities contract. The relevant region and operation family therefore belong in the claim. A positive aggregate result does not automatically distribute to every part of the history-bearing formation.

Regional boundaries can carry support or impose demands. A nearby practice might provide examples and interpretation; it might also draw away the only available coordinator. The sign and content of

the dependence need investigation. Representing every connection as beneficial exchange would conceal precisely the historical relations that a relational account seeks to retain.

Spatial or relational coupling can consequently support a more exact question: which local continuation depends on which actual boundary contribution? A region's activity can persist while the boundary supplies work that a local observer does not record. Removing that input changes the system under examination. An account of regional reactivation should retain the crossing relation or explicitly state the conditions under which it is negligible.

The resulting positive claim is substantial but bounded. A selected operation can return to recurrent use after a finite initiating contribution because the resulting arrangement, together with its retained baseline and boundary supports, sustains the necessary coupling. Identifying that history explains more than observing a successful event. It also leaves room for transformed supports, intermittent use and remaining dependence without converting each into a failure of the process.

5 Historical Realization Architecture

The preceding distinctions require a representation that can retain historical organization while selecting a tractable dynamical interval. This section supplies that architecture. It begins with a history-bearing object, distinguishes operation types from realized acts, and states how regional field data can represent selected aspects of those acts and their support. Its purpose is to make the subsequent model's interpretation assessable. It does not derive linguistic organization from a graph or infer a field law from the existence of historical relations.

5.1 History-Bearing Configurations

The represented object is a historically constituted configuration of participants, practices, artifacts and enabling relations. An individual participant can be a selected region of this description without being reduced to a point with one competence value. Earlier encounters, learned distinctions, unavailable interlocutors and changed institutional access can matter to present continuation. The selection is inquiry-relative: a history relevant to one narrative operation need not contain every event relevant to every linguistic capacity.

One available representation is a labelled two-complex K_Σ , where Σ names the selected historical cut. Its vertices, edges and faces are denoted $K_\Sigma^{(0)}$, $K_\Sigma^{(1)}$ and $K_\Sigma^{(2)}$. The representation includes endpoint maps for edges and attaching maps for face boundaries. Labels λ_Σ identify the declared roles of cells, and a separate precedence relation specifies the temporal ordering used in the inquiry. Incidence alone supplies no universal social clock. We write the selected configuration in Equation 1.

$$X_\Sigma = (K_\Sigma, \lambda_\Sigma, \omega_\Sigma). \quad (1)$$

The component ω_Σ in Equation 1 contains the active relational data not exhausted by the bare incidence structure. These can include operation realizations, access conditions and the current interpretation of retained labels. A face might represent a documented relation between two paths of interaction or a jointly realized episode; its assignment requires an explicit interpretation. A collection of pairwise contacts alone does not already determine the faces.

The two-complex is one representation of a scoped relational spacetime. It is neither the object itself nor evidence of an ultimate physical constitution. A spin-foam realization would require further labels, boundary data and an amplitude or other interpretation. None is supplied here because the selected reactivation result does not require a sum over changing labelled complexes. This omission is a scope decision inside the inherited pluralism, not a reduction of the programme to an ordinary contact network.

Historical extension from Σ to a later cut Σ' can add events, participants and relations. Where a retained historical anchor is used, an embedding identifies the old selected history inside the later representation. It need not freeze the active interpretation or preserve every current operation. The field-power manuscript distinguishes anchor retention, regional dynamics and changed operation carriers [6,

Sections 8–9]. The present paper uses a fixed active carrier only during a declared interval. A later change of carrier must be recorded as a new representational step.

5.2 Selected Operation Grammar

At a region R of the represented history, let $\mathcal{P}_{\Sigma,R}$ be a selected family of linguistic operation types. Examples might include identifying a narrative contrast, producing an appropriate repair or extending a familiar expression to a new case. A type is not a token occurrence. For each $p \in \mathcal{P}_{\Sigma,R}$, let $F_{p,R}$ be a partial schema selecting a concrete partial transformation from the relevant configuration class Ω . Let $\text{Comp}_{\Sigma,R}$ record the admitted typed compositions. Equation 2 collects this inherited structure.

$$\begin{aligned}\widehat{\Gamma}_{\Sigma,R} &= (\mathcal{P}_{\Sigma,R}, \{F_{p,R}\}_{p \in \mathcal{P}_{\Sigma,R}}, \text{Comp}_{\Sigma,R}), \\ F_{p,R} : \Omega &\dashrightarrow \text{PMap}(\Omega).\end{aligned}\tag{2}$$

Here $\text{PMap}(\Omega)$ denotes the declared class of typed partial transformations on the relevant configuration carriers. Equation 2 follows the taxonomy’s state-dependent grammar [7, Section 7]. It is not an assertion that the selected operation types form a vector space or that every composition is defined.

Execution requires more than membership in this grammar. The schema must select a transformation, the current configuration must belong to its domain, and the independently declared admissibility conditions must hold. A retained capacity attribution also requires evidence that this organization belongs to the selected bearer and continues the relevant earlier organization. These demands are not reduced to the numerical fields introduced below.

For example, a participant may retain a distinction while the only available interaction requires a pace that prevents its use. A slower partner can alter the execution situation without supplying the distinction. Another intervention might teach the distinction. Both interventions can increase observed activity, but only the first fits the specified retention premise without additional qualification. The difference belongs to the interpreted transformation and its history. Reusing an identical activity coordinate for both cases does not remove it.

The present field sector fixes $\widehat{\Gamma}_{\Sigma,R}$ up to the declared operation correspondence during the modeled interval. It studies channels of its realization. If the intervention changes the operation family or alters what counts as competent use, the fixed-target model must be qualified. A descendant correspondence between old and new operations may make such a change intelligible. Its existence cannot be inferred from the continued availability of the same numerical coordinates.

5.3 Regional Realization Data

Choose finitely many active interfaces $I = \{1, \dots, m\}$ inside the selected history. A site can represent a recurrent practice setting, a structured encounter or an analytically distinguished support interface. It is not automatically a person or a geographical point. Nonnegative symmetric exchange weights identify a tractable interaction graph over these interfaces. That graph is a selected active part of the historical description; it does not replace the full history-bearing complex.

For a subset $U \subseteq I$ and a time interval J , let $V(U, J)$ contain the selected nonnegative activity and support paths, together with the boundary traces needed to interpret their dynamics. A realization map assigns a path a record of the represented operations and support effects. Equation 3 states the type and the required restriction agreement.

$$\Xi_{U,J} : V(U, J) \dashrightarrow \mathcal{C}(U, J), \quad \Xi_{U',J'} \circ r_{(U,J),(U',J')} = \rho_{(U,J),(U',J')} \circ \Xi_{U,J}.\tag{3}$$

In Equation 3, $U' \subseteq U$, $J' \subseteq J$, $\mathcal{C}(U, J)$ is the class of interpreted realization records, r restricts field and boundary data, and ρ restricts interpreted records. The equality is required on the common domain where both sides exist. It states that interpreting a restricted record should agree with restricting its interpretation. No unique gluing theorem is assumed merely from this notation.

This requirement is inherited from the regional field discipline [6, pp. 27–28,33]. Its linguistic application is substantive. If the activity amplitude means recurrent usable opportunities in one region and emitted syllables in an overlapping region, numerical agreement has little interpretive force. If a regional restriction deletes the contribution of an outside interlocutor, it may change the represented mechanism. A valid local description must preserve the relevant normalization and boundary dependency.

The two selected coordinates at site i will be u_i for an activity channel and v_i for an enabling-support channel. They need not describe the same kind of event. Their scales must be justified separately. A possible activity channel is an intensity of encounters in which the selected operation is genuinely exercised. A possible support channel is the availability of a responsive interaction routine needed for those encounters. Such examples are hypotheses about what a measurement scheme could retain. The paper supplies no empirical estimates.

The realization map also makes zero values intelligible. A zero u_i means absence of the selected modeled activity during the chosen resolution interval. It does not establish lack of a capacity, loss of meaning or absence of all linguistic life. A zero v_i means absence of the selected support channel in its chosen units. Other support channels can remain. A valid empirical application would have to show that the two coordinates and retained background conditions capture enough of the selected dependence for the proposed law to be useful.

5.4 Dynamical Realization

An assignment of field values is weaker than a dynamical explanation. A law specifies how their selected organization changes. The next sections propose a nonlinear finite-graph law with losses, activity-dependent support renewal, finite initiating input and interface exchange. A solution path is then interpreted through Ξ . The law is accepted here as a conditional model assumption, and its implications are derived explicitly.

The distinction has a methodological consequence. Reproducing a few observed responses could leave multiple laws compatible with the same coarse record. A gradient model, a delayed model, a switching model and the proposed two-field model might agree over a short observation interval. Their post-intervention predictions could differ. The empirical task is to identify observations and interventions that discriminate the relevant mechanisms, while the formal task is to avoid attributing one law's results to the entire GR framework.

The nonlinear field and the history-bearing architecture perform different work. The field makes a selected feedback process calculable on a fixed carrier. The historical description explains why the operations and supports are attributed to these participants and these relations, and records changes outside the fixed sector. The algebra introduced later makes the domains and order of staged interventions explicit. Their combination is useful only where the interpretations agree. It is not a general mathematical unification of language, cognition and institutional life.

5.5 Model Translation Obligations

Table 1 summarizes the translation obligations carried into the calculation. The table distinguishes formal data from the substantive claim each datum would need to support. It is an aid to interpretation, not an additional empirical result.

The last row is especially important. The analysis will derive an invariant lower bound because it supports a transparent sufficient result. The bound does not supply a universal minimum for linguistic life. If meaningful practice consists of periodic gatherings, a condition on recurrence could be more adequate. The same historical architecture can accommodate that choice, but the proof would have to be replaced by a result about that condition. Formal pluralism here means allowing the question to select the mathematical obligation.

Table 1: Interpretation Obligations For The Selected Field Sector

Formal Datum	Required Interpretation	Remaining Distinction
Retained grammar	Selected historically warranted operations	Activity does not certify retention
Active site	Declared practice or support interface	A site need not be a person
Activity u_i	Realization channel with a defended scale	Output frequency does not determine meaning
Support v_i	Selected enabling process and its renewal	Support amount does not determine acceptable burden
Exchange weights	Interpreted relation and direction of exchange	Symmetry is a model restriction
Finite input	Specified initiating contribution that ends	Background conditions can remain
Positive invariant region	Continued modeled channel above a chosen floor	Recurrent or seasonal practice needs another criterion

6 Coupled Activity-Support Fields

The field investigation examines a particular reactivation mechanism: initiating activity can help renew a support channel that subsequently enables continued activity. Its object is the coupled trajectory, including the interval after the initiating input ends. The present section defines the channels, their nonlinear interaction and their mathematical admissibility. The following sections derive stationary regimes, a finite-duration initiation certificate, and a regional obstruction. Historical retention and the interpretation of the selected linguistic operation remain separately warranted premises throughout.

The general idea of recursive support is inherited. The earlier competence-reproduction paper already studies reinforcing activity fields, bistability, regional exchange and support withdrawal [10, Sections 6–8]. The preservation taxonomy already distinguishes capacity grammar, executable availability, condition structure and their coupling [7, Sections 7–8]. The narrower investigation here concerns the duration of an initiating episode when support responds on its own time scale. The selected two-channel law permits this duration to be studied without imposing the symmetric three-channel gradient structure of the earlier example. Its mathematical consequences do not by themselves establish a distinct empirical mechanism.

6.1 Selected Activity Channels

Let $I = \{1, \dots, m\}$, where $m \geq 1$, index selected sites of the historical realization described in Section 5. A site can be an interaction setting or an operational interface. It need not be a whole person or a geographical point. Choose symmetric exchange weights $w_{ij} = w_{ji} \geq 0$, with $w_{ii} = 0$. Define the graph Laplacian L by Equation 4.

$$(Lr)_i = \sum_{j \in I} w_{ij}(r_i - r_j), \quad r \in \mathbb{R}^m. \quad (4)$$

The sign convention in Equation 4 makes L symmetric and positive semidefinite. Its edges represent the selected channels of exchange, not every historical relation. A record of an earlier event does not diffuse merely because it belongs to the same represented spacetime.

At site i , let $u_i \geq 0$ be the amplitude of an identified execution-activity channel and $v_i \geq 0$ the amplitude of an identified replenishable support channel. The field state is $z = (u, v)$ in the carrier $X = \mathbb{R}_{\geq 0}^{2m}$. An application might interpret u through repeated episodes in which a retained linguistic distinction is exercised and v through the preparation that makes further encounters usable. Available but unused

opportunities would require a different channel interpretation. Such an interpretation must explain how the activities are identified and normalized. The numerical examples below use dimensionless model quantities and provide no measurement protocol for a real community.

The two amplitudes have different roles. Activity uses currently available support; some of that activity may also contribute to preparing subsequent support. For example, an encounter can include arranging a further occasion and preparing materials that the participants can use then. Whether this happens, who undertakes it, and whether the arrangement is welcome are substantive questions. They are not consequences of a positive coefficient in the law. Support that activity cannot renew must remain separately represented or held fixed under an explicit boundary assumption.

A retained capacity grammar is not a coordinate of X . Its operation types, concrete schemas, domains and historical comparisons belong to the preceding architecture. The same zero amplitude can accompany ordinary non-use, blocked execution, or absence of the selected organization. Paper 4's response-field analysis already makes the need for an independently warranted retention and intervention account explicit [8, Section 8]. Here $u = 0$ records zero activity in the chosen channel. Calling its later increase reactivation requires the retained-operation and realization premises; an increase can otherwise describe learning, reconstruction, compelled performance, or an unrelated activity.

6.2 A Nonlinear Support Law

Choose parameters $a, b, \varepsilon, \kappa > 0, \sigma \geq 0$, and $D_u, D_v \geq 0$. The coefficient a describes loss in the selected activity channel; b describes its dependence on the selected support; ε governs support adjustment; and κ scales activity's contribution to renewed support. The baseline σ specifies the support level approached without that contribution. The coefficients D_u, D_v govern exchange of the two channels. None is a quantity of competence, historical legitimacy, or participant worth.

Let $I_i(t) \geq 0$ be the actual initiating input delivered at site i . It is piecewise continuous and bounded on every finite time interval. The selected law is Equation 5; its powers and products act coordinatewise.

$$\begin{aligned}\dot{u}_i &= -D_u(Lu)_i + u_i(-a + bv_i - u_i^2) + I_i(t), \\ \dot{v}_i &= -D_v(Lv)_i + \varepsilon(\sigma - v_i) + \kappa \frac{u_i^2}{1 + u_i^2}.\end{aligned}\tag{5}$$

Equation 5 is a proposed dynamical specialization. The cubic activity term limits growth within its chosen units. The support-renewal term is increasing and bounded, so arbitrarily large activity cannot supply arbitrarily rapid renewal. The even power is used only on the nonnegative carrier; it implies a weak response at very small activity and a saturating response at larger activity. Other response laws could alter the quantitative results. No linguistic evidence is offered for this particular rational function.

The baseline deserves separate attention. Setting I to zero withdraws the finite initiating input. It leaves σ , the fixed graph, the coefficients and every support condition built into the realization in place. Sustained activity under that baseline is therefore not independence from all external provision. If coordination, technical access or unpaid work is needed to maintain these conditions, a practical account must identify that dependency. Reclassifying such work as a constant coefficient would not show that it has ceased to matter.

The law also has a specific restriction at zero. If every u_i is zero and $I = 0$, no activity begins, even if support is abundant. A local zero site can receive activity from an active neighbor, but a completely inactive graph requires an initiating input under this law. This reflects one initiation mechanism. It does not establish that people never resume activity spontaneously; an application admitting such episodes must represent their initiating process or choose another law.

The model is generally not an ordinary Euclidean gradient system. Its local cross-partials are bu_i and $2\kappa u_i/(1 + u_i^2)^2$, which do not agree throughout the carrier. No potential-decrease claim from the earlier gradient model is transferred. The present proofs instead use order, inward-pointing bounds and explicit comparison trajectories. This choice permits distinct channel time scales while keeping the derivations inspectable.

6.3 Positive Bounded Evolution

On an interval where $0 \leq I_i(t) \leq I_{\max}$, choose constants $V, U > 0$ that contain the initial data and satisfy Equation 6.

$$V \geq \sigma + \frac{\kappa}{\varepsilon}, \quad U^3 + (a - bV)U \geq I_{\max}, \quad B = [0, U]^m \times [0, V]^m \subset X. \quad (6)$$

Such choices exist: first enlarge V to contain the initial support, then use the positive cubic growth in the left side of the second inequality to choose U . The box B in Equation 6 supplies a common compact carrier for a bounded pulse and its subsequent baseline evolution.

Proposition 6.1 (Positive Bounded Field Evolution). *For the stated graph and parameters, every non-negative initial state has a unique global forward solution of Equation 5. The solution is absolutely continuous and satisfies the law almost everywhere; it is differentiable between input jumps. On each interval satisfying the displayed input bound, the box in Equation 6 is forward invariant.*

Proof. The state-dependent terms are smooth on the real carrier, since $1 + u_i^2$ is positive. The prescribed input gives a locally Lipschitz state law with piecewise-continuous time dependence, yielding a unique local solution. At $u_i = 0$ with all coordinates nonnegative, the derivative is $D_u \sum_j w_{ij} u_j + I_i \geq 0$. At $v_i = 0$, it is $D_v \sum_j w_{ij} v_j + \varepsilon \sigma + \kappa u_i^2 / (1 + u_i^2) \geq 0$. The nonnegative carrier is therefore invariant.

At an upper support face $v_i = V$, with the other support coordinates at most V , diffusion is nonpositive. The derivative is at most $\varepsilon(\sigma - V) + \kappa \leq 0$. At an upper activity face $u_i = U$, diffusion is again nonpositive and $v_i \leq V$, so the derivative is at most $U(-a + bV - U^2) + I_{\max} \leq 0$. The boundary signs establish invariance by the usual first-contact comparison, or by applying that comparison with an arbitrarily small strict outward perturbation of the barriers and passing to the limit. The argument applies separately across each input interval and preserves the same closed bounds at jumps.

On any finite time interval the input has a finite bound, so a box of this form contains the solution. A locally Lipschitz finite-dimensional solution that remains bounded cannot escape its existence interval at a finite time. Continuation therefore gives global forward existence and the stated uniqueness. $\square$

Proposition 6.1 guarantees a mathematically coherent carrier, not a feasible linguistic programme. Its upper bounds prevent the chosen reinforcement from diverging; they do not quantify finite money, fatigue, consent, or the remaining lifetime of an institution. Those constraints require interpreted state variables or explicit restrictions. A model can be bounded while its realization is practically unavailable.

6.4 Regional Realization Records

The field-power formalism distinguishes a field datum, its realization and the dynamics producing its interpreted responses [6, pp. 27–28, 33–35]. For the present specialization, a full trace retains the graph, parameters, initial state, delivered input and solution. A regional trace retains the local coordinates and actual crossing inputs needed by its equations. Overlapping descriptions must agree on their shared coordinates and on the induced dynamics after restriction to a common region. Section 9 derives the crossing terms explicitly.

An interpreted record may then select activity occasions, support preparation and whether the retained operation was executable in those occasions. This interpretation is additional to the amplitude trajectory. A positive u and v can coexist with a changed correction criterion, an unavailable operation schema, or an activity that merely imitates the selected output. The field certificate can support a linguistic conclusion only through a warranted realization connecting the certified channels to recurrent usable operation over the declared horizon. It does not certify that bridge itself.

The fixed graph and operation comparison describe a selected interval. If new participants, different operation types or revised support interfaces are introduced, their historical and formal correspondence must be supplied. The finite field should then be extended or replaced accordingly. Keeping the historical carrier explicit makes this limitation a place for further inquiry rather than a reason to interpret every possible change as a new value of the same two amplitudes.

7 Reactivation Regimes

Stationary regimes provide a reference for the initiation problem. An arrangement may admit both a stable inactive state and a stable positive activity state, leaving the history of access to either regime unresolved. This section derives those states for Equation 5 and checks their stability against all finite graph modes. The analysis supplies a parameter regime for the later nonlinear certificate. It does not identify a universal threshold of linguistic competence or the complete basin of any field state.

7.1 The Unforced Homogeneous Sector

Set $I_i = 0$ and take spatially uniform coordinates $u_i = u$, $v_i = v$. Since L annihilates constant vectors, this is an invariant sector of the full field. Its law is Equation 7.

$$\dot{u} = u(-a + bv - u^2), \quad \dot{v} = \varepsilon(\sigma - v) + \kappa \frac{u^2}{1 + u^2}. \quad (7)$$

Equation 7 is obtained by a justified restriction, not by treating language itself as a two-number object. The inactive equilibrium is $(0, \sigma)$. To focus on a regime in which small unforced activity does not automatically expand, define the parameters in Equation 8 and assume $\delta > 0$.

$$\delta = a - b\sigma, \quad K = \frac{b\kappa}{\varepsilon}, \quad k = \frac{\kappa}{\varepsilon}. \quad (8)$$

The difference δ compares baseline support with the local loss term inside this law. The quantities K and k summarize the selected feedback and support range. They are derived parameter combinations, not measured capacities. When $\delta \leq 0$, the small-activity analysis changes; that regime does not supply the particular initiation barrier investigated here.

At a positive equilibrium let $w = u^2 > 0$. The activity equation requires $bv = a + w$, whereas the support equation requires $v = \sigma + kw/(1 + w)$. Equating them gives Equation 9.

$$w^2 + (1 + \delta - K)w + \delta = 0. \quad (9)$$

Multiplying the equality $\delta + w = Kw/(1 + w)$ by $1 + w$ derives Equation 9 without introducing an approximation. Its positive solutions define the complete positive stationary set of the homogeneous two-dimensional system.

Proposition 7.1 (Positive Homogeneous Branches). *For $\delta > 0$, two distinct positive homogeneous equilibria exist exactly when $K > (1 + \sqrt{\delta})^2$. Their coordinates are stated in Equation 10. At equality there is one double positive root; below it there is no positive homogeneous equilibrium.*

$$w_{\pm} = \frac{K - 1 - \delta \pm \sqrt{(K - 1 - \delta)^2 - 4\delta}}{2}, \quad (10)$$

$$u_{\pm} = \sqrt{w_{\pm}}, \quad v_{\pm} = \sigma + k \frac{w_{\pm}}{1 + w_{\pm}}.$$

Proof. The quadratic has two distinct real positive roots exactly when its discriminant is positive and its root sum $K - 1 - \delta$ is positive; its root product is already $\delta > 0$. These two conditions reduce to $K - 1 - \delta > 2\sqrt{\delta}$, which is the stated inequality. The quadratic formula gives Equation 10. At equality both roots are $\sqrt{\delta}$. Below that value the discriminant is negative or the real roots cannot be positive. Conversely each positive root gives a positive pair satisfying both stationary equations, so none is omitted. $\square$

Proposition 7.1 describes existence. It does not establish whether an initiating episode reaches either state. In particular, the existence of a positive branch cannot be read as a prediction that every small pilot grows. The earlier reproduction model already makes the distinction between an existing sustaining regime and its accessibility [10, Section 7]; the following calculation prepares a different two-channel investigation of that distinction.

7.2 Coupled Local Stability

At a positive homogeneous equilibrium with squared activity w , the local Jacobian is Equation 11.

$$J(w) = \begin{pmatrix} -2w & b\sqrt{w} \\ \frac{2\kappa\sqrt{w}}{(1+w)^2} & -\varepsilon \end{pmatrix}, \quad \det J(w) = 2\varepsilon w \left(1 - \frac{K}{(1+w)^2} \right). \quad (11)$$

To obtain the first diagonal entry in Equation 11, differentiate the activity equation to get $-a + bv - 3u^2$ and use its stationary relation $-a + bv = u^2$. Differentiating the rational renewal term gives the lower-left entry. The trace is $-2w - \varepsilon < 0$.

The determinant's sign can be read directly from the stationary equation. Substituting $K = (w + \delta)(1 + w)/w$ gives Equation 12.

$$1 - \frac{K}{(1+w)^2} = \frac{w^2 - \delta}{w(1+w)}. \quad (12)$$

Since $w_- w_+ = \delta$ and the roots are distinct, $w_- < \sqrt{\delta} < w_+$. Equation 12 therefore makes the lower branch a saddle and the upper branch locally asymptotically stable in the homogeneous system. At $(0, \sigma)$ the local Jacobian is diagonal with entries $-\delta$ and $-\varepsilon$, so this equilibrium is also locally asymptotically stable.

The double-root boundary is nonhyperbolic: its determinant vanishes. A negative trace with a zero eigenvalue does not supply strict asymptotic stability. The later arguments use strict separation of the roots and need no verdict about that boundary case. In this way the parameter inequality marks the domain of a conditional construction rather than a universal empirical critical point.

7.3 Spatial Mode Stability

Homogeneous stability alone need not settle a field problem. Here the symmetry of the graph supplies an orthonormal Laplacian eigenbasis with eigenvalues $\mu \geq 0$. For each such mode, the linearized channel block is obtained by subtracting μD_u and μD_v from the corresponding diagonal entries. Its trace and determinant are stated in Equation 13.

$$\begin{aligned} J_\mu(w) &= J(w) - \mu \operatorname{diag}(D_u, D_v), \\ \operatorname{tr} J_\mu(w) &= -2w - \varepsilon - \mu(D_u + D_v), \\ \det J_\mu(w) &= \det J(w) + (2wD_v + \varepsilon D_u)\mu + D_u D_v \mu^2. \end{aligned} \quad (13)$$

Equation 13 retains unequal transport coefficients. No equal-diffusion assumption is needed for the sign conclusion.

Proposition 7.2 (Full Finite-Field Stability). *Assume the strict two-branch regime of Proposition 7.1. The inactive field $(u_i, v_i) = (0, \sigma)$ and the positive uniform upper-branch field are locally asymptotically stable in the complete finite field system. The lower uniform branch is unstable.*

Proof. At the upper branch the determinant in Equation 11 is positive. Every added term in the modal determinant is nonnegative, while every modal trace is negative. A real two-by-two matrix with positive determinant and negative trace has both eigenvalues with negative real part: either its roots are real with positive product and negative sum, or they are a complex conjugate pair with negative real part equal to half the trace. The orthonormal graph decomposition accounts for every finite spatial perturbation, so the full linearization is strictly stable. Smooth finite-dimensional evolution then gives local asymptotic stability.

At the inactive field the modal eigenvalues are $-\delta - D_u \mu$ and $-\varepsilon - D_v \mu$, again strictly negative. At the lower branch the spatially constant mode has $\mu = 0$ and negative determinant. Its two eigenvalues have opposite signs, giving an unstable mode in the full system. These arguments apply to disconnected graphs as well; additional zero graph eigenvalues do not change the conclusions. $\square$

Proposition 7.2 establishes a useful limitation as well as stability. Within this cooperative specialization, unequal channel diffusion does not destabilize the upper homogeneous branch through a small spatial perturbation. A claim that such a diffusion-driven linear instability explains reactivation would require a different law. Large nonuniform perturbations, regional leakage and changes in the carrier remain different problems. The proposition does not classify every nonuniform stationary state or establish attraction from arbitrary initial conditions.

7.4 Support Time Scales

The ratio $k = \kappa/\varepsilon$ determines the support equilibrium curve. Holding k fixed while changing ε and setting $\kappa = \varepsilon k$ leaves every equilibrium coordinate unchanged. Nevertheless, the support law becomes $\dot{v} = \varepsilon[\sigma - v + ku^2/(1 + u^2)]$, so its response speed changes. The activity equation does not receive the same time rescaling. Consequently the coupled trajectories can have different outcomes after the same type of interruption even though their stationary diagrams coincide.

This provides a reason to examine histories rather than infer successful reactivation from the existence of a positive equilibrium. A short encounter can raise the activity channel before the support process has had time to change. The resulting state can then decay when the initiating input ends. A longer episode can enter a region whose activity and support sustain each other under the baseline. Section 8 proves a sufficient duration bound and a contrasting extinction result; neither conclusion follows from the equilibrium plot alone.

8 Finite Activation

The central dynamical question is whether a finite initiating episode can leave an arrangement that continues to operate after that input ends. This section supplies a sufficient condition using both activity and support, derives a pulse duration that reaches it, and proves an extinction contrast at the same initial activity level. The results concern the selected channels of Equation 5. A linguistic conclusion additionally requires the retained grammar and the realization connecting those channels to recurrent execution. The initiating episode's termination leaves the declared baseline conditions in place.

8.1 A Nonlinear Sustaining Region

Assume $\delta > 0$ and $K > (1 + \sqrt{\delta})^2$. Choose an activity floor $\ell > 0$ strictly between the squared stationary roots in the sense $w_- < \ell^2 < w_+$. Define the support equilibrium associated with that floor and choose a lower support level by Equation 14.

$$H(\ell) = \sigma + k \frac{\ell^2}{1 + \ell^2}, \quad \frac{a + \ell^2}{b} < v_L < H(\ell). \quad (14)$$

The interval in Equation 14 is nonempty. Indeed, multiplying its endpoint difference by $b(1 + \ell^2)$ gives the negative of the polynomial in Equation 9 evaluated at $w = \ell^2$. That polynomial is negative strictly between its two roots. Thus the same coupling that creates the positive stationary branches also supplies a whole range of lower bounds.

Choose the upper box B from Equation 6 with $U \geq \ell$ and $V \geq v_L$. The proposed sustaining region is Equation 15.

$$\mathcal{R}_{\ell, v_L} = \{(u, v) \in B : u_i \geq \ell, v_i \geq v_L \text{ for every } i \in I\}. \quad (15)$$

Equation 15 retains support as part of the state. It is a sufficient invariant region, not a complete description of the attraction basin and not a field definition of retained linguistic competence.

Proposition 8.1 (Sustained Baseline Activity). *When $I = 0$, every solution beginning in $\mathcal{R}_{\ell, v_L}$ remains there for all subsequent times.*

Proof. The upper bounds follow from Proposition 6.1. At a lower activity face $u_i = \ell$, all neighboring activity coordinates are at least ℓ , so diffusion is nonnegative. The local derivative is at least $\ell(-a + bv_L - \ell^2) > 0$ by Equation 14. At a lower support face $v_i = v_L$, neighboring support values are at least v_L , so support diffusion is nonnegative. Because $r^2/(1+r^2)$ is increasing for $r \geq 0$, the local derivative is at least $\varepsilon(\sigma - v_L) + \kappa\ell^2/(1+\ell^2) = \varepsilon[H(\ell) - v_L] > 0$. Thus no lower face can be crossed outward, and all four sets of faces point inward or tangentially. The same first-contact argument used for the upper box establishes invariance. $\square$

Proposition 8.1 controls nonlinear trajectories and spatially heterogeneous initial states inside the region. It does not depend on their being close to a homogeneous equilibrium. Nor does it claim that every such trajectory converges to the upper equilibrium; the lower bounds themselves answer the selected persistence question. States outside this rectangle can also persist. The proposition supplies an inspectable sufficient certificate rather than an optimal boundary.

The floor ℓ is selected for the model inquiry. To interpret it as adequate operational activity, one must justify how the corresponding channel supports the declared task over a specified horizon. A mathematical floor cannot determine an acceptable frequency of linguistic use for participants. Likewise, the support floor concerns the modeled replenishable channel. It supplies no warrant that every other condition has become independent of external work.

8.2 Finite Pulse Duration

Suppose initially $u_i \geq 0$ and $v_i \geq \sigma$ at every site. Deliver a uniform initiating input $I_i(t) = I_0$ for a finite interval, choosing $I_0 > a\ell + \ell^3$. Define the two times in Equation 16.

$$c = I_0 - a\ell - \ell^3 > 0, \quad T_1 = \frac{\ell}{c}, \quad T_2 = \frac{1}{\varepsilon} \log \left(\frac{H(\ell) - \sigma}{H(\ell) - v_L} \right). \quad (16)$$

Both arguments of the logarithmic ratio in Equation 16 are positive, and the ratio exceeds one because $v_L > \sigma$. The first time is a conservative bound for establishing the activity floor. The second allows the support channel to develop while that floor is maintained.

Proposition 8.2 (Finite Initiation Certificate). *Choose B to contain the initial state and satisfy Equation 6 with $I_{\max} = I_0$. A uniform pulse lasting at least $T_1 + T_2$ places the field in $\mathcal{R}_{\ell, v_L}$. On withdrawing that pulse, the baseline law retains the field in this sustaining region.*

Proof. The lower support bound $v_i \geq \sigma$ is invariant throughout the pulse: at a lower face its diffusion and renewal terms are nonnegative, and the baseline term is zero. For activity between zero and ℓ , the local derivative at a spatial minimum is at least $I_0 - au_i - u_i^3 \geq c$. More explicitly, use the spatially constant lower barrier $r(t) = ct$ on $0 \leq t \leq T_1$. Its derivative is c , its diffusion is zero, and for $r \leq \ell$ the right side of the activity law at $u_i = r$ is at least c . Starting below every activity coordinate, it therefore remains below them by first-contact comparison. At T_1 every $u_i \geq \ell$.

As long as the pulse remains present, the lower activity face $u_i = \ell$ has derivative at least c , so that floor persists. From time T_1 , compare support with the scalar function in Equation 17.

$$h(t) = \sigma + [H(\ell) - \sigma](1 - e^{-\varepsilon(t-T_1)}), \quad t \geq T_1. \quad (17)$$

This function begins at σ and solves $\dot{h} = \varepsilon[H(\ell) - h]$. At a first contact $v_i = h$ with all other support coordinates at least h , diffusion is nonnegative and $u_i \geq \ell$, so the true support derivative is at least this comparison derivative. Hence $v_i(t) \geq h(t)$. Substituting $t = T_1 + T_2$ into Equation 17 gives $h = v_L$. The activity and support lower bounds are then both satisfied. The upper bounds hold throughout by the choice of B . Withdrawal changes I to zero and invokes Proposition 8.1. $\square$

Proposition 8.2 establishes an attainable sufficient state under a specified delivered input. It does not optimize duration or intensity, compare burdens, or ensure that a participant can arrange the pulse.

The estimate deliberately uses the activity loss bound with no help from support in its first stage. A trajectory can therefore succeed much earlier than the bound. What the proof adds is a transparent chain of sufficient conditions from a finite episode to persistent baseline activity.

The delivery condition is material. A nominal programme duration is not the same as a realized input at every selected site. Missing occasions, unsuitable interaction, or uneven access can violate the assumed forcing. An application can study heterogeneous inputs, but it must then provide the corresponding lower comparison rather than reuse the uniform-pulse certificate without its premise.

8.3 Support Accumulation Time

Holding $k = \kappa/\varepsilon$ fixed preserves the stationary diagram while the sufficient support-building time T_2 scales as ε^{-1} . The bound therefore identifies a specific temporal dependence: attaining an activity floor does not imply that the coupled support floor has already been reached.

There is also an upper bound on how quickly support can build, even under arbitrarily large non-negative activity input. Starting with $v_i(0) = \sigma$, the renewal term never exceeds κ . Comparison at the support maximum gives Equation 18.

$$\begin{aligned} v_i(t) &\leq \sigma + k(1 - e^{-\varepsilon t}), \\ v_i(t) \geq v_L &\implies t \geq \frac{1}{\varepsilon} \log\left(\frac{k}{k - (v_L - \sigma)}\right). \end{aligned} \quad (18)$$

To derive the first inequality in Equation 18, compare with the spatially constant solution of $\dot{h} = \varepsilon(\sigma - h) + \kappa$, $h(0) = \sigma$; diffusion cannot increase a coordinate above that upper barrier. Solving the inequality for time gives its second part. The denominator is positive because $v_L < H(\ell) < \sigma + k$.

This is a necessary time for reaching the declared support floor, not a necessary time for every possible successful reactivation. A trajectory outside the sufficient rectangle might still persist. That distinction prevents a convenient certificate from becoming a universal policy threshold. The result nevertheless blocks one inference inside the model: an arbitrarily intense activity pulse cannot instantaneously establish an arbitrarily selected support level when the renewal channel has a finite response rate.

8.4 Support-Dependent Continuation

A stronger contrast proves that the same activity amplitude can lead to extinction or persistence. Consider the homogeneous unforced sector. Choose $0 < u_c < \ell$ and $\sigma < v_c < a/b$ such that $H(u_c) \leq v_c$. Define the quantities and condition in Equation 19.

$$\eta = a - bv_c > 0, \quad T_{\text{drop}} = \frac{\log(\ell/u_c)}{\eta}, \quad \kappa T_{\text{drop}} \leq v_c - \sigma. \quad (19)$$

The choices are possible by taking u_c small, since $H(0) = \sigma$, and then taking ε small while keeping k fixed, so that $\kappa = \varepsilon k$ also becomes small.

Proposition 8.3 (Slow-Support Extinction). *Under Equation 19, the homogeneous baseline solution starting from $(u, v) = (\ell, \sigma)$ satisfies $u(t) \rightarrow 0$ and $v(t) \rightarrow \sigma$. By contrast, the state (ℓ, v_L) from Equation 14 remains above both sustaining floors.*

Proof. From $v(0) = \sigma$, support remains at least σ and its derivative is at most κ , so $v(t) \leq \sigma + \kappa t$. Equation 19 keeps $v(t) \leq v_c$ until T_{drop} . Throughout that interval, $\dot{u} \leq -\eta u$, giving $u(T_{\text{drop}}) \leq \ell e^{-\eta T_{\text{drop}}} = u_c$.

The rectangle $0 \leq u \leq u_c$, $0 \leq v \leq v_c$ is forward invariant. Its lower faces point inward by positivity. At $u = u_c$ the derivative is at most $-\eta u_c < 0$; at $v = v_c$ it is at most $\varepsilon[H(u_c) - v_c] \leq 0$. The solution therefore remains inside this rectangle after T_{drop} , and u continues to decay at least exponentially with rate η .

For support, variation of constants gives a decaying initial term plus the convolution of $\kappa u^2/(1+u^2)$ with $e^{-\varepsilon(t-s)}$, in addition to σ . The forcing tends to zero and is bounded. Split the convolution at a sufficiently late fixed time: the earlier part decays exponentially, and the later part is bounded by an arbitrarily small forcing supremum divided by ε . Its limit is therefore zero, proving $v \rightarrow \sigma$. The contrasting state lies in $\mathcal{R}_{\ell, v_L}$ and persists by Proposition 8.1. $\square$

Proposition 8.3 makes support history consequential without relying only on a failed certificate. It proves actual decay for one state and a positive lower bound for the other. Both states have the same present activity. The difference is the enabling channel already developed at the moment the baseline evolution begins.

For a concrete dimensionless example, take $a = 2$, $b = \sigma = 1$, $k = 6$, $\varepsilon = 0.02$ and $\kappa = 0.12$. Then $w_{\pm} = 2 \pm \sqrt{3}$, and $\ell = 1$, $v_L = 3.5$ satisfy Equation 14, because $H(1) = 4$ and $(a+1)/b = 3$. Choosing $u_c = 0.2$ and $v_c = 1.5$ gives $H(u_c) = 1 + 3/13 < 1.5$, $\eta = 0.5$, and $\kappa T_{\text{drop}} = 0.24 \log 5 < 0.5$. Thus $(1, 1)$ decays while $(1, 3.5)$ persists. For the same parameters a pulse $I_0 = 4$ has the sufficient duration $1 + 50 \log 6$. Section 10 illustrates these histories and compares the conservative bound with directly computed trajectories.

9 Regional Support

The sustaining certificate applies to a declared coupled carrier. A smaller observation region generally has crossing inputs that change its activity and support balance. This section derives those terms, gives a sufficient regional certificate, and constructs an open-site case in which leakage removes the positive regime. The argument develops the consequence of a specified boundary history. It does not infer that a language community has an autonomous closed boundary or that increased connection is uniformly beneficial.

9.1 Actual Crossing Traces

Let $R \subseteq I$ be a nonempty region, and let L_R be the graph Laplacian containing only edges with both endpoints in R . For $i \in R$, restriction of Equation 5 gives Equation 20.

$$\begin{aligned} \dot{u}_i &= -D_u(L_R u_R)_i + u_i(-a + bv_i - u_i^2) + I_i + B_i^u, \\ \dot{v}_i &= -D_v(L_R v_R)_i + \varepsilon(\sigma - v_i) + \kappa \frac{u_i^2}{1 + u_i^2} + B_i^v, \\ B_i^u &= D_u \sum_{j \notin R} w_{ij}(u_j - u_i), \quad B_i^v = D_v \sum_{j \notin R} w_{ij}(v_j - v_i). \end{aligned} \tag{20}$$

Equation 20 follows by splitting the full Laplacian sum into internal and crossing edges. The boundary terms retain both incoming activity and outward exchange. Their values can be negative even though all field coordinates are nonnegative.

The trace carrier of the region includes the actual B^u, B^v together with its local coordinates and delivered input. Two overlapping regional descriptions are compatible when restricting each to their common region, and recomputing its crossing terms from the same full trace, yields the same local evolution. Raw boundary vectors for two differently sized regions need not be equal: an edge can be internal to one and external to the other. Nested restriction is coherent when such newly exposed edges enter the smaller region's trace. These are concrete instances of the inherited regional field requirements [6, pp. 27–28, 33–35].

Removing the crossing terms while keeping the same observed initial values describes a different system. The earlier reproduction paper already distinguishes decoupling from retaining an edge whose exterior contribution has changed [10, Section 6]. The present calculation applies that distinction to the relative speed and strength of activity and support renewal. It asks whether an initiation certificate survives the boundary conditions that actually remain after the initiating episode.

9.2 A Regional Lower-Bound Certificate

Suppose the exterior traces obey $0 \leq u_j \leq U$ and $\sigma \leq v_j \leq V$, with the same upper bounds as the region. Define maximal crossing rates by Equation 21.

$$d_u = \max_{i \in R} D_u \sum_{j \notin R} w_{ij}, \quad d_v = \max_{i \in R} D_v \sum_{j \notin R} w_{ij}. \quad (21)$$

The rates in Equation 21 summarize worst-case outward exchange at the proposed lower faces. They are bounds for this regional comparison, not measurements of a community's isolation.

For baseline evolution inside R , a sufficient replacement for Equation 14 is Equation 22.

$$\frac{a + d_u + \ell^2}{b} < v_L < \sigma + \frac{\kappa}{\varepsilon + d_v} \frac{\ell^2}{1 + \ell^2}. \quad (22)$$

To verify Equation 22, consider the activity lower face $u_i = \ell$. Internal diffusion is nonnegative, while the crossing term is at least $-d_u \ell$. The remaining derivative is therefore at least $\ell(-a - d_u + b v_L - \ell^2) > 0$. At $v_i = v_L$, the exterior support bound gives $B_i^v \geq d_v(\sigma - v_L)$; the sign uses $v_L > \sigma$ and the maximal rate. Its local derivative is at least $(\varepsilon + d_v)(\sigma - v_L) + \kappa \ell^2 / (1 + \ell^2) > 0$. Upper faces remain controlled by the exterior upper bounds and Equation 6. The first-contact argument consequently retains both floors in the region.

This certificate is conservative: a favorable exterior trace can supply more than the worst-case bounds. It is nevertheless useful because it states what additional information a regional persistence claim needs. The isolated-region inequality can fail after exposure to outward exchange even when its internal local law is unchanged. Conversely, raising the actual exterior support or reducing loss can restore a sufficient interval. Which change corresponds to a real support arrangement requires the interpretation of the crossing channels.

9.3 A Prescribed Exterior

An exact single-site specialization makes the obstruction explicit. Prescribe exterior activity $u = 0$ and exterior support $v = \sigma$, retain the edges, and let their total exchange rates be $d_u, d_v \geq 0$. After the initiating input ends, the open-site law is Equation 23.

$$\begin{aligned} \dot{u} &= u[-(a + d_u) + b v - u^2], \\ \dot{v} &= (\varepsilon + d_v)(\sigma - v) + \kappa \frac{u^2}{1 + u^2}, \\ \delta_R &= \delta + d_u, \quad K_R = \frac{b \kappa}{\varepsilon + d_v}. \end{aligned} \quad (23)$$

Equation 23 follows directly from the crossing terms. The exterior is prescribed, not claimed to be the autonomous output of an unspecified larger system. A larger model might need additional mechanisms to maintain it.

The equilibrium calculation of Section 7 now uses δ_R, K_R . Outward activity exchange raises δ_R , while support exchange with a baseline-only exterior lowers K_R . Either change can remove the interval of positive stationary activity. Absence of a positive equilibrium alone would not prove extinction in a general nonlinear system. The cooperative structure of this particular law supplies the additional argument.

Proposition 9.1 (Open-Site Extinction). *Assume $\delta_R > 0$ and $K_R < (1 + \sqrt{\delta_R})^2$. Every finite nonnegative initial state of Equation 23 converges to $(0, \sigma)$.*

Proof. The positive-equilibrium quadratic has no positive root under the strict inequality, by Proposition 7.1 with the substituted parameters. The only nonnegative equilibrium is therefore $(0, \sigma)$.

The off-diagonal derivatives of the vector field are $bu \geq 0$ and $2\kappa u/(1+u^2)^2 \geq 0$. Its flow preserves coordinatewise order on the nonnegative carrier. To see the comparison directly, at a first equality of one coordinate of two ordered states, increasing the other coordinate cannot decrease that coordinate's derivative. A strict barrier perturbation followed by a limit handles tangency; local Lipschitz continuity supplies uniqueness. The same argument compares solutions with constant upper barriers.

Choose an upper initial corner (U_0, V_0) above the given initial state, with $V_0 \geq \sigma + \kappa/(\varepsilon + d_v)$ and U_0 large enough that $U_0^2 \geq bV_0 - a - d_u$. Both derivatives at that corner are nonpositive. Its autonomous trajectory therefore stays below the corner. Moreover, if ϕ_t denotes this open-site flow, order preservation gives $\phi_t(\phi_h(U_0, V_0)) \leq \phi_t(U_0, V_0)$ for every $t, h \geq 0$. The semigroup identity makes this trajectory coordinatewise nonincreasing. It is nonnegative and bounded, so it has a limit. Continuity of ϕ_h implies that the limit is fixed by every ϕ_h and hence is an equilibrium. It must be $(0, \sigma)$.

Every solution from the original state lies below that upper trajectory and above zero in activity. Its activity therefore tends to zero. For support, the nonnegative renewal term gives the lower estimate $v(t) \geq \sigma + [v(0) - \sigma]e^{-(\varepsilon+d_v)t}$. This tends to σ , while the upper comparison also tends to σ . Thus support converges to the same baseline, proving the claim. $\square$

Proposition 9.1 supplies global behavior through comparison, not a conclusion inferred only from homogeneous linearization. The strict inequality matters: the double-root boundary admits a positive equilibrium and is excluded. The result also depends on the prescribed exterior and the fixed selected law. It does not establish a general harmful effect of social connection.

For the illustrative parameters of Section 8, choose $d_u = 0.2$ and $d_v = 0.02$. Then $\delta_R = 1.2$ and $K_R = 3$, which is below $(1 + \sqrt{1.2})^2$. The state $(1, 3.5)$ persists under the closed baseline certificate but decays under this open-site condition. Deleting the edges would restore the original closed law; keeping them while changing their exterior trace gives the distinct history proved here.

The practical comparison is therefore a question about the support arrangement after initiation. Does the selected region retain favorable relations, become decoupled, or continue exchanging with a differently supported exterior? Similar early performance can precede different answers. A historically situated account should record the actual continuing relation and its burdens before interpreting the later trajectory as success or failure of retained linguistic capacity.

10 Illustrative Field Investigations

The numerical investigation compares histories that the preceding proofs distinguish. Its role is to make the coupled mechanism inspectable and to check independent consequences of the calculations. Four figures display the stationary geometry, finite initiating pulses, different support time scales and a changed regional boundary. The source packet retains the script, numerical arrays and check report. All quantities are uncalibrated model amplitudes; none is an observed competence score or evidence that a real intervention would have the represented effect.

10.1 Stationary Geometry

The principal examples use $a = 2$, $b = \sigma = 1$, $\varepsilon = 0.02$ and $\kappa = 0.12$, giving $\delta = 1$ and $K = 6$. Figure 1 plots the positive activity nullcline $v = (a + u^2)/b$ and the support nullcline $v = H(u)$. Their intersections are the two positive equilibria derived in Equation 10. The zero-activity branch of the activity nullcline also exists and is treated analytically; it is not represented by dividing the activity equation by u .

The right panel of Figure 1 returns to Proposition 8.3. Both trajectories begin with activity one. The state with support one enters the proved extinction region; the state with support 3.5 remains above the sustaining floors. The phase portrait therefore visualizes a historically relevant distinction in the chosen law: current activity does not determine how much support has already developed. A record containing only activity would omit the variable responsible for the different continuations in this comparison.

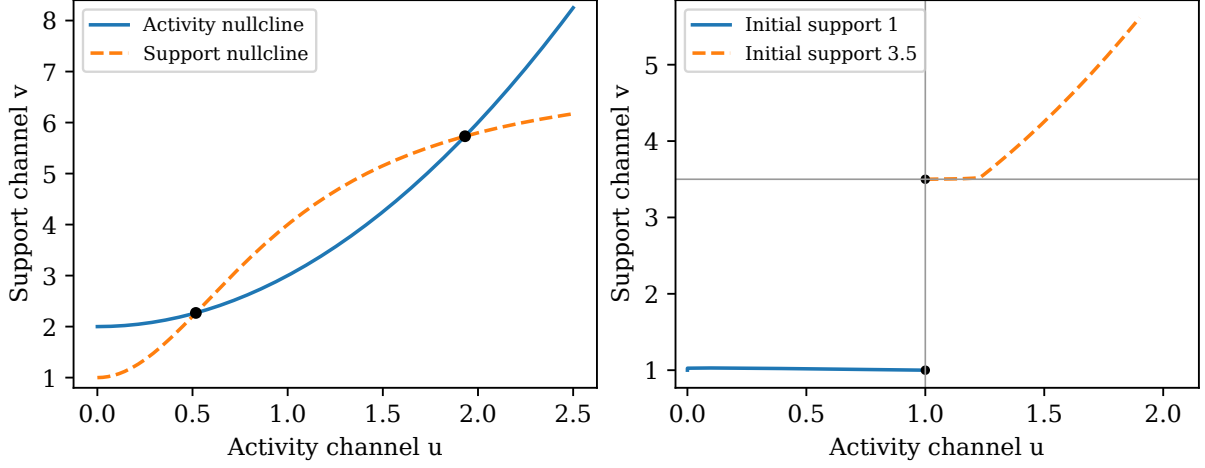


Figure 1: Illustrative Reactivation Regimes. The left panel shows the positive activity and support nullclines. The right panel compares baseline trajectories with initial activity $u = 1$ and different initial support. Thin reference lines indicate the sufficient sustaining floors, not the complete basin boundary. Parameters are dimensionless and uncalibrated.

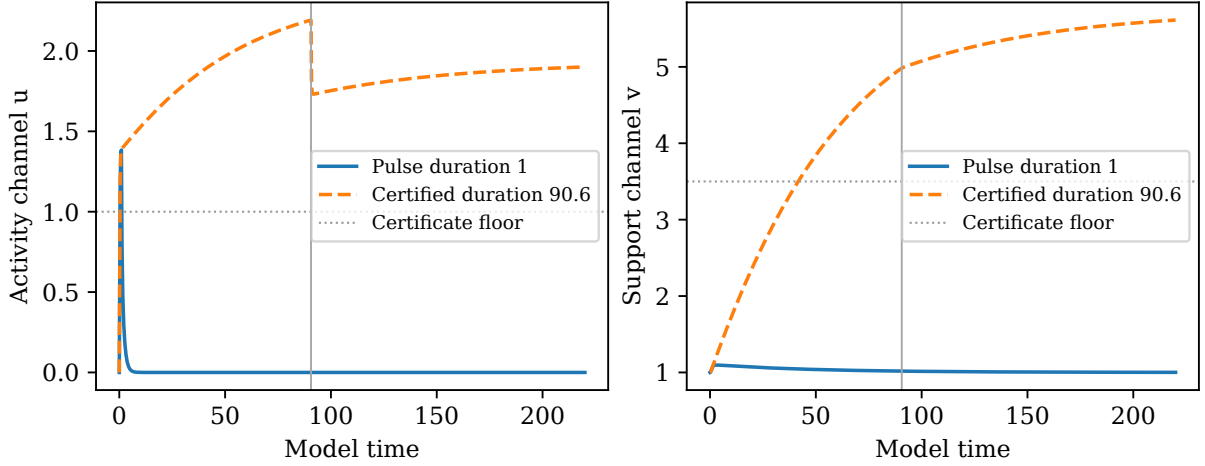


Figure 2: Finite Initiating Pulses. Equal input amplitudes are applied for different durations. The longer pulse uses the conservative sufficient bound and leaves both channels above their sustaining floors after withdrawal. The shorter illustrative pulse decays. The vertical line marks withdrawal of the longer pulse; it does not mark removal of baseline support.

10.2 Initiating Episodes

Figure 2 begins from $(u, v) = (0, 1)$ and delivers an input of amplitude four. One pulse lasts one model-time unit. The other lasts the sufficient duration $T_1 + T_2 = 1 + 50 \log 6$, approximately 90.59. Both terminate before the final observation time. Each discontinuity is an explicit integration boundary, so no numerical step averages an unspecified mixture of the forced and baseline laws.

The longer trajectory in Figure 2 is covered by Proposition 8.2. Its later numerical samples agree with the all-time lower bounds proved for the baseline law. The shorter trajectory illustrates one unsuccessful duration; the calculation does not locate the least successful duration. In particular, the large gap between the two chosen times should not be interpreted as a sharply estimated policy boundary. The proof used deliberately conservative comparisons, and no optimization of burden or exposure has been attempted.

The test compares pulse integration at the main step size with integration using half that size on each segment. Their largest coordinate difference is approximately 1.0×10^{-6} . This is a numerical

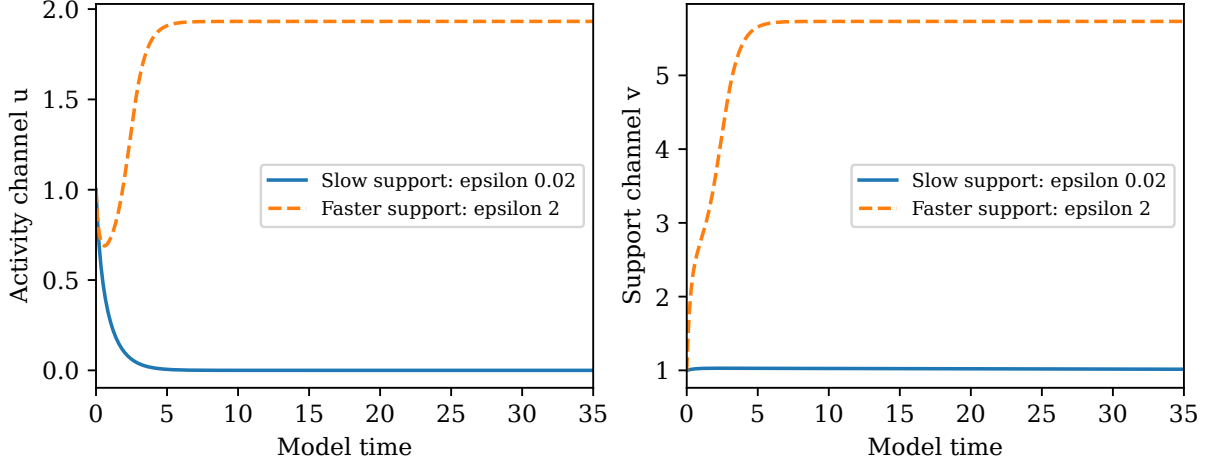


Figure 3: Support Response Time Scales. Two selected laws have the same stationary coordinates and initial state but different support adjustment rates. The slow case satisfies the extinction proof; the faster case approaches the positive regime in this illustrative calculation. The figure gives no universal monotonicity claim about intervention speed.

consistency check for the plotted histories. The all-time conclusion rests on the invariant-region proof, while the adequacy of its linguistic realization remains a separate question.

10.3 Support Response Speeds

Figure 3 holds $k = \kappa/\varepsilon = 6$ fixed and compares $\varepsilon = 0.02$ with $\varepsilon = 2$, setting κ accordingly. Both histories begin at $(1, 1)$ with no initiating input thereafter. Their equilibrium coordinates agree exactly, but the faster support response permits a different coupled history in the numerical example.

The slow case in Figure 3 has a proof of decay through Equation 19. The faster case is a displayed trajectory, not a theorem that increasing ε always improves every history. Changing a coefficient can alter timing, support requirements and the interpretation of the intervention itself. The narrower result is sufficient for the present inquiry: equal equilibrium diagrams do not determine the outcome of an initiating episode when the two channels respond at different rates.

10.4 Regional Withdrawal

Figure 4 compares the same initial state $(1, 3.5)$ in a closed selected site and in the open-site model with $d_u = 0.2$, $d_v = 0.02$, and prescribed exterior $(0, 1)$. The open case satisfies Proposition 9.1. Its early activity derivative is nevertheless positive: substituting the initial state into Equation 23 gives $\dot{u} = 0.3$ while $\dot{v} = -0.04$. A short observation of increasing activity could therefore miss the support decline that governs the later result.

The contrast in Figure 4 is not a recommendation to remove relations. Favorable exterior inputs can have the opposite effect, and the prescribed comparison does not model the whole surrounding history. Its analytical use is to distinguish ending a finite input from changing the interfaces through which later support flows. A reactivation account that reports only the successful initiating occasion has not yet specified this continuation.

10.5 Computational Scope

The reproducible script performs nineteen named checks. It substitutes the closed-form roots into both the quadratic and the original equations, compares the analytic Jacobian with central differences of the nonlinear law, and compares the full ten-dimensional Jacobian of a five-site graph with all its two-channel mode blocks. It separately checks a nonlinear heterogeneous trajectory inside the sustaining rectangle

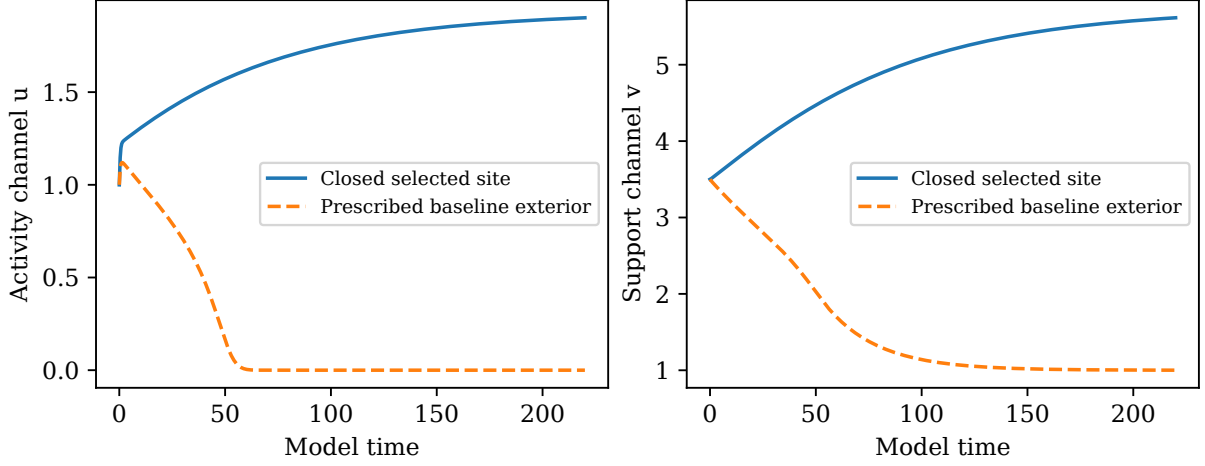


Figure 4: Regional Support Continuations. The closed site retains the sufficient sustaining region. The site exchanging with the prescribed baseline exterior eventually loses activity despite its initially positive activity derivative. Both curves retain the declared baseline; the comparison concerns their different continuing interfaces.

and reconstructs a regional derivative from internal exchange plus independently summed crossing traces. These comparisons address different potential errors rather than merely resampling one homogeneous trajectory.

The remaining checks cover pulse entry, post-pulse lower bounds, the slow-support extinction hypotheses, positivity, the upper box, the regional parameter obstruction and selected contrasting trajectories. Finite numerical samples supplement the proofs; they establish neither an empirically correct field law nor a complete account of permissible intervention. The next section investigates a different part of the problem: how staged operations can be represented so that execution, support withdrawal and subsequent viability remain explicit in the historical account.

11 Staged Reactivation Operations

The field analysis distinguishes entering an active state from entering a region that remains active after the initiating input ends. This section expresses that distinction through observable algebras and history operators. Its objective is to identify an exact success condition for a declared sequence of interventions without losing their domains. The construction also shows why immediate activity alone can be an inadequate endpoint description.

Two mathematical carriers are kept separate. The field has a continuous state space. A countable library of specified intervention histories supplies a different carrier for partial history operators. The first supports a nonlinear baseline flow and its pullback on observables. The second records executable sequences. Neither carrier is asserted to contain the whole history of a language. Their relationship is a declared realization of selected histories into the field sector.

11.1 Baseline Observable Algebra

Fix the coefficients and graph after the initiating input ends, and choose a compact forward-invariant box B supplied by the field bounds. Let $\phi_t : B \rightarrow B$ denote the unforced flow at duration $t \geq 0$. Global uniqueness gives $\phi_0 = \text{id}_B$ and $\phi_{s+t} = \phi_s \circ \phi_t$. Let $\mathcal{A} = B_b(B; \mathbb{C})$ be the algebra of bounded Borel functions on B , with pointwise product, complex conjugation and supremum norm. Its unit is the constant function 1. This is a commutative unital C^* -algebra.

The flow induces the pullback in Equation 24.

$$\alpha_t : \mathcal{A} \longrightarrow \mathcal{A}, \quad \alpha_t(f) = f \circ \phi_t, \quad \alpha_{s+t} = \alpha_t \circ \alpha_s. \quad (24)$$

Equation 24 preserves multiplication, complex conjugation and the unit. Each α_t is therefore a unital $*$ -endomorphism of norm at most one. The last identity follows by composing the maps; it records the reversal of order characteristic of pullback. The autonomous flow maps commute in this specialization. Distinct intervention stages need not.

Bounded Borel observables are chosen so that indicator functions of sustained regions are included. No norm-continuity claim is made for the family α_t on this algebra. Moving a sharp indicator can prevent norm convergence even when the underlying flow is continuous. On the smaller algebra of continuous functions on compact B , uniform continuity supplies the usual strong continuity in time, but the sharp indicators need not belong to that smaller algebra. These are different useful choices, not interchangeable statements about one generator.

This pullback is a mathematical representation of changes in selected field observables. It is not a physical quantum evolution, a Hilbert-space implementation of the full linguistic object or a forward map on empirical probability distributions. The inherited formalism distinguishes algebra maps, evaluation transport and represented operators [6, pp. 40–43]. The present construction uses only the elementary map class just established.

11.2 Sustained Regions

Choose an activity floor $\ell > 0$ and let A_ℓ be the closed subset of B on which every selected activity coordinate is at least ℓ . Define the sustained set V_ℓ and its finite-horizon version $V_{\ell,T}$ by Equation 25.

$$\begin{aligned} A_\ell &= \{(u, v) \in B : u_i \geq \ell \text{ for every } i\}, \\ V_{\ell,T} &= \bigcap_{0 \leq t \leq T} \phi_t^{-1}(A_\ell), \quad V_\ell = \bigcap_{t \geq 0} \phi_t^{-1}(A_\ell). \end{aligned} \quad (25)$$

Equation 25 is a definition within the selected law. Every set in the intersections is closed because ϕ_t is continuous and A_ℓ is closed. The resulting sets are therefore Borel. Continuity of the trajectory also permits the intersections to be taken over nonnegative rational times, including 0. If a trajectory violates the closed activity condition at one time, the strict violation persists over a small interval containing a rational time.

Membership in $V_{\ell,T}$ concerns only the horizon T . The sets decrease as T grows, and $V_\ell = \bigcap_{n=1}^{\infty} V_{\ell,n}$. This limiting statement does not turn an observed finite history into evidence of indefinite continuation. It defines a property of the assumed law. In an application, uncertain coefficients, future structural changes and unobserved support processes qualify the use of that law.

Proposition 11.1 (Sustained-Region Invariance). *The set V_ℓ is the largest forward-invariant subset of A_ℓ for the baseline flow. Its indicator $P_\ell = \mathbf{1}_{V_\ell}$ satisfies Equation 26.*

$$P_\ell \leq \alpha_t(P_\ell), \quad P_\ell \alpha_t(1 - P_\ell) = 0, \quad t \geq 0. \quad (26)$$

Proof. If $z \in V_\ell$, then for every $s \geq 0$, $\phi_s(\phi_t(z)) = \phi_{s+t}(z) \in A_\ell$. Thus $\phi_t(z) \in V_\ell$. Conversely, if $C \subseteq A_\ell$ is forward invariant, every trajectory beginning in C remains in A_ℓ , so $C \subseteq V_\ell$. For the indicator, forward invariance says that $P_\ell(z) = 1$ implies $P_\ell(\phi_t(z)) = 1$. This is precisely the first pointwise inequality in Equation 26; multiplying its complementary form by P_ℓ gives the second identity. $\square$

Proposition 11.1 asserts forward invariance. Equality $P_\ell = \alpha_t(P_\ell)$ is stronger and is not asserted. A trajectory can begin outside A_ℓ , later enter a sustained region, and thereafter remain there. Forward invariance prohibits escape after membership; it does not prohibit entry. This distinction prevents a mathematical closure condition from silently becoming a claim that activation cannot occur.

Restriction to V_ℓ gives a further algebraic consequence. For $g \in B_b(V_\ell; \mathbb{C})$, define $\beta_t(g) = g \circ (\phi_t|_{V_\ell})$. Forward invariance makes this expression well defined. It is again a unital $*$ -endomorphism, now with unit 1_{V_ℓ} . Equivalently, the corner $P_\ell \mathcal{A}$ is stable under the compressed map $f \mapsto P_\ell \alpha_t(f)$. This restricted algebra represents a declared continuing sector. Its existence says nothing about whether an investigator can determine membership from a short observation.

The sustaining rectangle of the nonlinear analysis is a subset of V_ℓ whenever it lies in B and its lower bounds use the chosen ℓ . It is a sufficient certificate, not a characterization of the full set. A state outside that rectangle can still belong to V_ℓ . Conversely, a state in A_ℓ can lie outside V_ℓ because its support is too weak. The distinction is the mathematical counterpart of the difference between immediate activity and sustainable continuation.

11.3 Immediate Activity Selection

The two one-site states in Equation 27 supply an exact counterexample to an activity-only endpoint criterion. Use the coefficients and timing result of Section 8.

$$a = 2, \quad b = \sigma = 1, \quad \varepsilon = \frac{1}{50}, \quad \kappa = \frac{3}{25}, \quad z^- = (1, 1), \quad z^+ = \left(1, \frac{7}{2}\right). \quad (27)$$

Both states in Equation 27 have activity $u = 1$. The state z^+ belongs to the sustaining rectangle with lower bounds $u \geq 1, v \geq 7/2$, and hence to V_1 . The timing obstruction shows that z^- loses activity and converges to the inactive baseline. In particular, $z^- \notin V_1$. The result concerns two states of the same nonlinear law.

Let $q : B \rightarrow \mathbb{R}_{\geq 0}$ be the activity readout $q(u, v) = u$ in this one-site case, and let $\mathcal{A}_u$ consist of bounded Borel observables of the form $g \circ q$. Every element of $\mathcal{A}_u$ takes equal values at z^- and z^+ . Since P_1 takes different values there, it cannot belong to $\mathcal{A}_u$. Thus the selected activity readout loses a distinction required by this sustained-region classification.

This is a narrowly scoped failure of one readout family. It does not prove that every extended activity record is insufficient, nor that measuring v is always necessary or feasible. A richer trajectory, additional intervention or another mechanism could alter the identification problem. The present result is useful because it identifies the exact premise missing from a claim that one successful endpoint certifies continued operation.

11.4 Partial Intervention Histories

Now select a countable set $\mathcal{H}$ of admissible finite intervention histories. Its elements retain a starting dossier, the ordered stage labels and the realized intermediate records. Countability is an additional restriction on this library. For example, one can choose countably many specified initial records and a countable set of intervention protocols. This does not make the field state space countable, nor does it enumerate every real-valued trajectory.

For a stage label a , let $D_a \subseteq \mathcal{H}$ be the histories on which that stage is executable in the selected representation. Appending the stage and its result gives an injective partial map $s_a : D_a \rightarrow \mathcal{H}$. Injectivity comes from retaining the whole prefix and stage, even when different histories end at the same field state. On $\mathcal{H} = \ell^2(\mathcal{H}; \mathbb{C})$, with basis e_h , define the operator in Equation 28.

$$S_a e_h = \begin{cases} e_{s_a(h)}, & h \in D_a, \\ 0, & h \notin D_a, \end{cases} \quad S_a^* S_a = P_{D_a}. \quad (28)$$

For a history set C , P_C denotes the orthogonal diagonal projection onto the span of its basis vectors. Equation 28 defines a bounded partial isometry of norm at most one. The adjoint removes a represented last stage where the prefix exists. It does not supply an executable way to reverse an encounter or undo its historical consequences.

The norm-closed unital $*$ -algebra generated by the retained projections and stage operators is a selected operator algebra in $\mathcal{B}(\mathcal{H})$. This statement derives the carrier from the chosen histories and operations. It does not assert that the operation grammar itself was already a C^* -algebra. The construction is consistent with earlier partial-history uses while selecting a different target predicate [8, 10].

For a word $w = (a_1, \dots, a_n)$, let D_w be its sequential domain and $S_w = S_{a_n} \cdots S_{a_1}$. The sequential domain requires every stage to be defined on the result of the preceding stages. Products therefore retain a domain obligation that cannot be recovered by checking the last stage in isolation.

Let θ map each terminal history designated as a post-withdrawal history to its field state in B . Such a map must be supplied by the realization record. Define C_ℓ to consist of those post-withdrawal histories for which the retention and interpretation premises are recorded and $\theta(h) \in V_\ell$. The field condition and the recorded premises remain separate components of this selection. In particular, a field state in V_ℓ does not create the missing historical warrant.

Proposition 11.2 (Executable Sustained-Target Certificate). *Let $D \subseteq \mathcal{H}$ be a selected initial history family. Every history in D admits the full staged intervention w and ends in C_ℓ if and only if both identities in Equation 29 hold.*

$$S_w^* S_w P_D = P_D, \quad P_{C_\ell} S_w P_D = S_w P_D. \quad (29)$$

Proof. Because each stage appends a retained record injectively, their sequential composition is injective on D_w , and $S_w^* S_w = P_{D_w}$. The first identity in Equation 29 is therefore equivalent to $D \subseteq D_w$. Subject to that inclusion, $S_w e_h$ is the basis vector of the realized terminal history for every $h \in D$. The second identity holds on e_h exactly when that terminal history belongs to C_ℓ . Equality on all basis vectors in D , together with boundedness, is equivalent to the stated operator identity. These implications reverse directly. $\square$

The two identities perform different work. If the whole intervention is undefined on D , then $S_w P_D = 0$ and the second identity holds vacuously. The first identity excludes that case. Even when the domain condition holds, selection by immediate activity can be weaker than selection by C_ℓ . The nonlinear counterexample above shows why an endpoint that merely appears active can fail the chosen sustained-target requirement.

11.5 Stage Order

A finite illustration makes the domain distinction concrete. Use four abstract boundary descriptions: d is an inactive but intervention-ready setting, a an initiated activity without consolidated support, s a supported active setting, and r a returned inactive setting. These are stipulated descriptions, not empirical classifications. Let the partial stages act as in Equation 30.

$$A : d \mapsto a, \quad C : a \mapsto s, \quad W : a \mapsto r, \quad s \mapsto s. \quad (30)$$

Here A initiates, C consolidates the selected support, and W ends the initiating role. Every unlisted action is undefined. Baseline continuation at s stays in the declared supported class, while baseline continuation at r stays inactive. The assumptions of this illustration are explicit in Equation 30; they do not replace the field proof that warrants a corresponding support classification in the continuous model.

Starting from d , the sequence A, C, W is executable and ends in s . The sequence A, W, C fails because C is undefined at r . On the retained history carrier, the first product $S_W S_C S_A e_d$ is a nonzero success vector, whereas $S_C S_W S_A e_d = 0$. Testing only a final success projection on the second product would produce a vacuous equality. Proposition 11.2 detects the failure through its domain condition.

Figure 5 displays the two sequences. The accompanying finite check independently compares explicit partial-map execution with lifted history matrices for all 4,096 pairs of partial maps on three states and all initial and target subsets. The 262,144 certificate comparisons agree. This finite enumeration

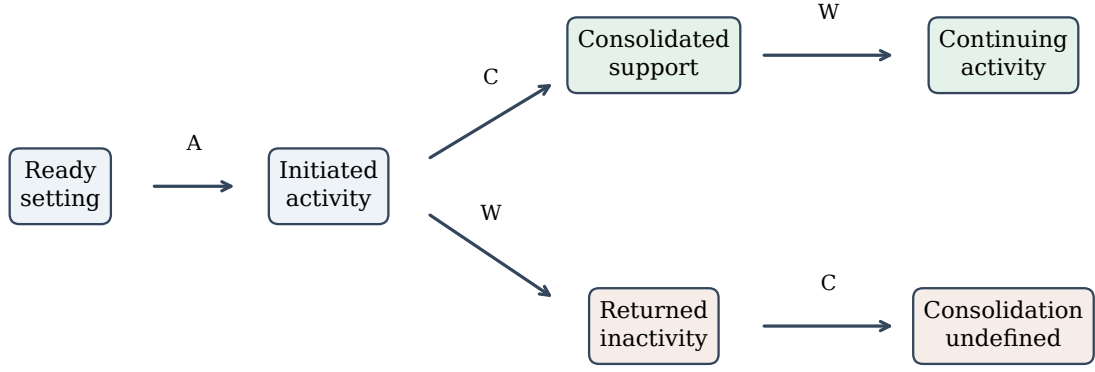


Figure 5: Staged Activation And Support Withdrawal. In the stipulated finite example, consolidation before withdrawal reaches the continuing class. Withdrawal before consolidation reaches a state on which the proposed consolidation is undefined. The branches retain the same initial activation but differ in their intermediate domains.

checks the implementation of the proved distinction; it is not evidence for the linguistic adequacy of the stage labels.

Ordered action and partial domains are inherited mathematical resources. The present use concerns the handover from initiating support to continuing support. A practical interpretation must still establish whether consolidation can occur before withdrawal, whether the designated actors can execute it, and whether the required work is acceptable. Algebraic noncommutation by itself answers none of these questions. Its contribution is to keep their formal consequences visible once the stages have been interpreted.

11.6 Finite-Horizon Certificates

Replace V_ℓ in the terminal selection by $V_{\ell,T}$ to obtain $C_{\ell,T}$, retaining the same post-withdrawal and historical premises. As T increases, the corresponding projections decrease. For positive integer horizons, their strong operator limit is the indefinite-horizon projection stated in Equation 31.

$$P_{C_{\ell,n}} \xrightarrow[n \rightarrow \infty]{\text{strong}} P_{C_\ell}. \quad (31)$$

To verify Equation 31, apply the projections to any $\psi = \sum_h c_h e_h \in \ell^2(\mathcal{H})$. The squared norm of their difference is the sum of $|c_h|^2$ over histories selected at horizon n but not indefinitely. The indicators decrease pointwise to zero and are bounded by one. Summability of $\sum_h |c_h|^2$ then makes the difference tend to zero. This is strong convergence, not a claim of convergence in operator norm.

The limit makes explicit what finite certificates leave open. Success at every finite horizon for the same terminal history under one fixed law has an exact mathematical limiting interpretation. Choosing a different successful history at each horizon would make a different claim. A finite empirical record supplies only finitely bounded evidence and may not keep the law or carrier fixed. Later sections therefore treat post-intervention observation as a test of an articulated mechanism, not as a means of certifying an infinite social future.

12 Empirical Reactivation Boundaries

Documented return trajectories can discipline the interpretation of a formal reactivation model without serving as direct measurements of its variables. This section examines that boundary through conversational, household, travel, and delayed-retrieval cases. Each case report is followed by a separately iden-

tified analytical comparison. The objective is to specify which historical claim the observation supports, which stronger claim would require additional evidence, and which features of support a representation would need to preserve. The comparisons develop consequences of the selected cases rather than treating them as instances of an already confirmed general law.

12.1 Repeated Conversational Contact

Stolberg and Münch analyzed eight conversations selected from fifteen contacts with one German emigrant during 2000–2004. Contacts were irregular, with gaps ranging from less than one month to eleven months. German already accounted for roughly 80 percent of the conversations, and improvements were uneven across measures. Their comparisons also faced a historical reference problem: standard German and a dialect recording could not supply the participant's actual earlier baseline. The study did not follow performance after the research contact ended [14].

The following constructed comparison isolates an issue that a conversational design can raise. Suppose two participants each become more able to sustain an extended exchange. In one history, the researcher repeatedly provides the only relevant conversational opportunity. In the other, those exchanges enable the participant to enter a voluntary group whose members subsequently arrange further encounters. The immediate performances may be similar, while the continuation histories differ. The first trajectory depends on an ongoing relation supplied by the study; the second includes a newly operating relation beyond that setting. Neither trajectory is unreal. They answer different questions about the organization of continued activity.

A further comparison concerns the contribution of the interlocutor. One interlocutor can supply missing forms and assemble the message, while another can allow time for the participant to perform the relevant operation. Both encounters may produce a successful exchange. If the target is individual retrieval, those successes have different evidential meanings. If the target is collaborative communication, both may fall within the selected operation, provided that its distributed bearer has been declared. The point is to preserve the location of the operation when interpreting an apparently improved outcome.

Such comparisons explain why the end of a project is a meaningful boundary only relative to a specified causal claim. Successful continuing conversation during the project can warrant an account of supported operation. A claim about operation after project closure concerns a different support history. The latter need not demand an isolated individual: subsequent partners, routines, and resources may constitute the ordinary setting of the operation. The inquiry should identify their contribution rather than treating continuing relational dependence as evidence that a return has failed.

12.2 Household Language Support

Köpke and Genevskaja-Hanke recorded one Bulgarian–German bilingual in Germany and after stays in Bulgaria, repeating the comparison in 2012 and 2017. The target was overt subject pronoun use. Later proportions fell within the monolingual comparison range in both settings. By then the participant had married a Bulgarian speaker three years earlier, changing home language use. That history leaves the earlier brief stay's independent contribution to later performance unresolved [11].

A long interval is often rhetorically persuasive because it appears to test endurance. Its evidential meaning depends on what persists during that interval. Consider an independent hypothetical trajectory in which a person uses a returning capacity each week with a partner. An initial workshop ends, yet the partner continues to provide occasions, interpretation, and correction. The observation that operation continues after the workshop supports persistence relative to the subsequent household arrangement. Attributing that persistence entirely to the workshop would require evidence connecting the initiating event to the later organization, while distinguishing the partner's continuing contribution.

The same reasoning permits a stronger relation when it is documented. The workshop might enable the person to negotiate a recurring practice with the partner; successful practice might then help maintain the arrangement. In that constructed history, the initiating intervention, operation, and support renewal form a connected sequence. The sequence remains open to alternative explanations. The partner may

have organized the practice independently, or another change may have made participation possible. A model of reciprocal renewal should represent the connected sequence only to the extent that those relations are justified.

The household example also clarifies the meaning of a support boundary. A mathematical system can be described as unforced after one externally imposed input ends, while ordinary material and interpersonal conditions remain embedded in its coefficients, domain, or state. That technical description should not become a claim that the person's language now operates without dependence. The relevant conclusion concerns continuation within a declared historical arrangement. The representation's boundary is adequate when it makes the arrangement's contribution visible enough for the proposed explanation.

12.3 Language Reimmersion Visits

Casado and colleagues compared 37 Polish–English migrants and 37 bilingual participants in Poland, using counterbalanced sessions and different matched pictures. Migrants' return testing occurred within seven days of arriving back in the United Kingdom. The reported naming pattern depended on word frequency rather than a uniform recovery effect. A statistical reporting qualification remains: the Methods used an absolute t value greater than 2, while the article's Table 3 gave 1.94 for the relevant interaction, described as significant in the Results. The present paper therefore treats that interaction cautiously and does not infer long-term persistence [2].

Travel provides an analytically useful boundary because location change can coincide with several changes in the conditions of operation. In a constructed visit, a participant might encounter different interlocutors, topics, expectations, opportunities for use, and competing demands. A subsequent improvement could be consistent with several histories: access facilitated by recent use, learning during the visit, altered task familiarity, or a change in the participant's orientation toward the research encounter. The designation of the visit as an intervention identifies an interval. It does not yet identify which of these relations produced the difference.

The relevant return claim can therefore be narrower than a claim about a language. Suppose the declared target is retrieving a class of familiar words in a timed task after returning home. Successful later retrieval bears directly on that task-relative target. A claim about sustained spontaneous conversation introduces other requirements, including selecting relevant words, maintaining an exchange, interpreting a partner, and managing repair. Generalization to those operations requires observations capable of distinguishing their contributions. Calling both outcomes linguistic access would not remove the need for that work.

A follow-up taken after a geographical return is valuable because it samples a setting distinct from the visit itself. Its scope remains the specified interval and situation. Repeated success over that interval can coexist with later decline, continued learning, or adaptation to a different practice. These possibilities should be left open without making finite observation futile. An appropriately bounded claim states what remained available, at which occasions, and under which continuing conditions. It avoids converting an endpoint into a prediction for an unspecified future.

12.4 Delayed Vocabulary Tests

In the French replication reported by de Bot, Martens, and Stoessel, a three-week test preceded further training and showed an advantage for previously encountered items. The same paper found no general part-set benefit from retraining a portion of the vocabulary [5]. Those observations concern selected items and the tested interval; they do not establish the renewed operation of an entire lexicon.

A delayed test changes the claim being evaluated because it separates one observation from the immediate training episode. The additional interval nevertheless contains a history. A constructed participant may privately rehearse material, encounter it incidentally, or leave it unused. The same score at the later test could arise through these different paths. For an outcome claim about later availability, the score can be relevant across those paths. For a mechanism claim about unaided persistence or activity-dependent renewal, the differences become explanatory variables.

The phrase sustained operation also needs to accommodate intermittency. A person who can retrieve a word when an appropriate occasion arises need not continuously produce it between occasions. The absence of observed utterance during an interval does not establish a lapse, while a successful delayed test does not establish continuous manifestation. The appropriate object is a history of availability and execution relative to relevant occasions. This distinction connects the empirical question to the spacetime representation without requiring an invented continuous proficiency signal.

An additional constructed contrast concerns distribution across items. A selected subset can remain available while another becomes difficult to retrieve. An aggregate score could conceal that redistribution. If the target operation requires choosing among the whole set, the distribution matters. If the target concerns a declared subset, the same outcome can be sufficient for the narrower claim. A representation should preserve the distinctions on which the return claim depends; it need not reproduce every item-level detail when those details are irrelevant to that claim.

12.5 Continued Use Comparisons

The six-month post-study-abroad investigation by Mickan and colleagues combined repeated vocabulary testing with continuing use reports [12]. Its use–maintenance association makes the intervening history relevant, while its observational design leaves reciprocal causation and common enabling conditions available as explanations.

The present paper’s renewal mechanism concerns a specific relation within such a history: an operation contributes to the production or maintenance of conditions for its subsequent execution. A report of frequent use would be compatible with that relation but would not identify it. The distinction can be made through a constructed pair of histories. In one, a funded programme supplies every encounter independently of participant performance. In the other, participants’ successful coordination generates the commitments and arrangements that make later encounters available. A vocabulary measure could improve in both histories. The support-production relation differs.

The pair does not imply that the second arrangement is preferable. External provision can be reliable, chosen, and less burdensome. Participant-organized continuation can demand time or expose unequal responsibilities. The formal question concerns which process is represented; evaluation requires additional considerations. Keeping those tasks distinct prevents a descriptive feedback mechanism from becoming an implicit demand that participants maintain their own support without assistance.

For empirical interpretation, the same pair identifies a useful failure condition for the proposed explanation. If later opportunities remain unchanged when the selected operation varies, and are generated entirely through a documented independent process, an account assigning their renewal to that operation would be poorly supported. A model could still represent externally maintained activity, but its causal interpretation would need revision. Conversely, evidence that successful operation helps generate later opportunities would support a renewal hypothesis without determining the functional form, strength, or time scale of the coupling.

12.6 Reactivation Model Commitments

The case comparisons establish several obligations for the formal interpretation. Retention requires an operation-specific historical warrant; successful execution requires evidence about the present bearer and enabling conditions; continuation requires observations over a declared horizon. Identifying a feedback mechanism adds a further requirement: the relation producing later support must be investigated. These obligations accumulate when the claim becomes stronger. They should not be collapsed into a single observed measure or inferred from the model’s successful simulation.

The field model developed in Sections 6–8 can distinguish equal activity amplitudes accompanied by different support states. That distinction is analytically relevant to the constructed comparisons here. Its empirical realization would require separate access to the selected manifestations and to the enabling relations represented by support. A questionnaire response, test score, or conversational count might

contribute to that access, but none is identified with the model variable by definition. The correspondence must be justified for the actual investigation.

Similarly, the operator construction in Section 11 distinguishes a successful immediate operation from entry into a region that preserves a selected continuation property. An empirical application would need to say which later observations warrant membership in such a region and which changes terminate the comparison. Finite observation can support a finite-horizon attribution. Extending that attribution indefinitely would require assumptions beyond the documented record, even when a mathematical invariant set exists in the model.

The literature therefore offers constraints and discriminations, rather than numerical confirmation of the proposed field. The cases preserve the possibility of retained contributions, local recovery, and supported continuation while leaving several mechanisms and temporal questions unresolved. The formal investigation earns its role by making those alternatives more explicit: immediate recovery, externally maintained operation, and activity-dependent renewal can be represented separately. Which representation best fits a particular return remains a task for historically and linguistically specified inquiry.

13 Reactivation Objections

The proposed account connects a retained operation, an initiating intervention and a supported continuation. Each connection can be challenged. A critic can question whether the operation was retained, whether the intervention genuinely ended, whether the field captures the relevant process, or whether continued activity warrants a favorable interpretation. This section develops those objections and the scope of the available replies. Its objective is to test the reactivation explanation, including cases where its mathematical conditions hold while a stronger historical or practical conclusion remains unsupported.

13.1 The Retention Premise

An initial objection is that the account assumes the feature that distinguishes reactivation from learning. A previously competent participant can relearn rapidly because fragments, familiarity or general interpretive skill remain. A short return episode can therefore reconstruct the selected operation. If the paper begins by stipulating retention, its dynamics may explain the persistence of later activity while leaving the central classification unresolved.

The objection correctly limits what the model proves. Prior retention is an independently warranted premise or a declared stipulation, not an outcome of the nonlinear calculation. The earlier dormancy inquiry examines what would support that premise, including the participant's task-specific contribution and the formative content of enabling interventions [8, Sections 3,5,11]. Repeating its full attribution problem inside the field would not make the premise better established.

The distinction remains useful because continuation can differ even when the premise is granted. Two settings can contain similarly retained operations yet leave different activity and support states after initiation. The coupled model investigates that further difference. Its result can therefore have explanatory content without discovering which real participants possess the stipulated organization.

An application with unresolved retention should preserve that status. It can establish renewed sustained performance while leaving open whether the route reactivated a full operation, reconstructed it from fragments or combined both. Such a finding is not empty. It identifies an achieved continuation and the remaining historical alternatives. The stronger term reactivation should be reserved for the selected contribution whose retention is supported.

13.2 Continued Intervention

A second objection challenges the withdrawal claim. If baseline support continues and participants perform new maintenance work, the process may appear to replace one intervention with several others.

Calling the resulting activity sustained could then celebrate an administrative disappearance while concealing continuing dependence. A dynamical parameter held constant may represent precisely the resource whose provision the practical question concerns.

The reply begins with the object of comparison. The present claim concerns the end of a specified finite initiating contribution, not escape from every enabling relation. Baseline provision and newly organized work remain part of the explanation. The distinction becomes informative when the initiating contribution had an identifiable role that subsequent relations can carry differently or render unnecessary.

For the narrative circle, temporary coordination might establish contact and access arrangements that participants later maintain. The later arrangement still requires work. What changes is who does that work, how it is organized and whether it depends on the initiating coordinator's continued involvement. A claim that the coordinator can depart must follow those functions, including informal advice and preparation. It cannot be established by changing the programme's label.

There are cases where the objection defeats the claim. If the coordinator continues performing the same indispensable role without recognition or payment, termination is administrative rather than operative. If an unreported institution supplies a replacement role, the history concerns substitution. The framework should describe that result directly. It permits supported continuation without converting every redistribution into independence.

13.3 Dynamical Abstraction

A third objection concerns explanatory adequacy. Activity and support are heterogeneous, interpretation-sensitive processes. A low-dimensional nonlinear law may select feedback and saturation because they produce attractive mathematical regimes. A basin-entry theorem would then show only that the chosen equations possess the behavior designed into them. Historical language practice could remain largely unexplained.

The proper response is to identify the inferential task performed by the specialization. The coupled state can distinguish similar activity levels with different support configurations. A finite initiating episode can move the represented system toward or away from a continuation region, and the rate of support formation can change the duration needed. The conditions developed in Section 8 expose assumptions that an activity-only success criterion omits. Their mathematical investigation is more specific than naming a feedback loop.

The model's linguistic adequacy nevertheless requires a realization account. An activity coordinate must correspond to a selected operational process, and the support coordinate must represent a condition whose variation matters to that process. The dependence should be investigated independently of the desired persistence result. If many unrelated resources are compressed into one coordinate while their substitutions have different consequences, the representation can fail for the intended comparison.

The same limitation applies to fixed operation families. A practice can change the distinctions that make its activity meaningful. The current field may remain numerically well behaved while ceasing to represent the relevant operation. An application should then revise the realization map or introduce an explicit comparison to a changed carrier. Mathematical persistence of the old coordinates cannot protect their interpretation from criticism.

The account consequently offers a selected mechanism and conditional diagnostic. It does not establish that reactivation generally follows one field law. The inherited taxonomy already permits different realizations and coupling architectures [7, Sections 7–8]; taking that plurality seriously means retaining failures of translation as possible findings.

13.4 Temporal Scope

A further objection targets sustained operation itself. A finite observation can always end before decline appears. A model with a positive invariant region can idealize away changing participants, support exhaustion or shocks. The distinction between a brief performance and a sustained return might therefore depend on an arbitrary choice of stopping time.

The reply requires a horizon justified by the practice's continuation problem. An inquiry into recurrent monthly exchanges should include relevant opportunities after initiation, rather than choose an endpoint immediately following the successful launch. A seasonal practice requires a different temporal comparison. The horizon can be provisional, but the reasons for selecting it and the consequences of extending it should remain visible.

A finite finding and a conditional long-run result perform different roles. The historical finding concerns the occasions actually examined. The model identifies what follows if its law, supports and operation interpretation remain applicable. Their agreement can motivate further inquiry without turning the finite history into proof of indefinite survival. A later failure can reveal a changing premise, an inadequate realization or a mechanism outside the model.

Robustness also needs scope. Persistence against one specified support disturbance does not imply resistance to every disturbance. Requiring unlimited resilience would exclude ordinary supported practice; ignoring plausible changes would make the continuation claim too easy. The inquiry should identify perturbations relevant to the intended application and leave other continuation classes untested. The taxonomy's scale-relative retention framework supplies the inherited basis for that distinction [7, Section 10].

13.5 Distributed Burdens

An objection concerning burden challenges the interpretation of successful renewal. A practice can maintain aggregate activity by concentrating preparation, travel, correction or emotional work on a few participants. The support coordinate can remain high while those participants lose alternatives or become unable to withdraw. Describing the process as self-sustaining may obscure the asymmetric relations responsible for its apparent stability.

The model's positive result does not by itself establish a desirable allocation. A realized support flow has producers, conditions and possible costs. Their omission can affect both normative assessment and descriptive reliability. If continued support depends on effort that participants cannot maintain over the claimed horizon, treating that flow as fixed can overstate the process's viability. If the effort remains practically available but unfairly imposed, descriptive persistence and normative justification diverge.

These cases call for different conclusions. The first challenges the continuation premises; the second challenges the permission or desirability of relying on them. A field trajectory cannot adjudicate both through a single support value. The history should identify the relevant contributions and distinguish willingly accepted work from expectations imposed by the project's success narrative.

The earlier competence-reproduction inquiry already treats support renewal as a further condition beyond successful current performance [10, Section 3]. Applying that lesson here requires attention to what the initiating episode leaves behind. Ending a funded role by transferring its functions to unpaid participants can sustain activity in one sense while creating a new practical problem. The current paper identifies that junction without providing a complete theory of fair maintenance.

13.6 Voluntary Participation

A final objection concerns the assumption that return is the appropriate aim. A retained operation can remain unused because participants prefer another activity, associate its earlier setting with exclusion, or decline the proposed inquiry. Successful activation can also produce performances under pressure. A framework organized around persistence risks classifying refusal as failure and treating retained capacity as an obligation to demonstrate it.

The account separates executability, exercise and authorization. A participant's refusal does not establish loss of the operation. The existence of a feasible intervention does not establish a right to perform it. A stable pattern of compelled output would require a different practical interpretation from chosen recurrent participation, even if a narrow activity measure recorded similar events. These distinctions continue the attribution paper's limits on practical inference [8, Section 11].

Voluntary participation does not make the historical question disappear. Participants can choose to investigate a retained practice, assess what support they want and revise the continuation target as experience develops. The inquiry can serve those purposes by distinguishing burdens, dependencies and possible routes. It should also permit a decision that a particular operation need not return, or that its continuation should take another form.

The positive explanation therefore remains separate from the endorsement of its objective. It can clarify when a finite initiating episode is followed by supported recurrent use, which premises sustain that result and which changes defeat it. Reasons to undertake, prolong, modify or end the process require further judgments about the people and practices affected. Those judgments belong in the interpretation of an application even though the present model cannot settle them.

14 Reactivation Inquiry

A reactivation inquiry should identify the returning operation, the initiating contribution and the history of subsequent use. The model suggests particular distinctions worth investigating: prior retention, immediate realization, changing support, intervention termination and continued operation. This section develops their evidential and practical implications through the constructed narrative circle. It proposes an inquiry architecture for criticism, rather than a validated protocol, an optimal intervention policy or a general criterion for certifying language vitality.

14.1 A Declared Return Claim

An inquiry can begin with a sentence specifying whose operation is expected to return, which contribution remains retained, what currently prevents execution and which continuation would count as sustained for the selected purpose. In the narrative circle, the claim concerns former participants' use of the completed-event and interrupted-attempt contrast across unfamiliar accounts. It does not concern every feature of their language or their standing within a community.

The initial record should separate grounds for prior achievement from grounds for present retention. A description of former fluent use can establish the first while leaving the second unsettled. A retained distinction expressed through another medium might support part of the present profile. A successful encounter after new instruction can support a later achievement while leaving its earlier possession unresolved. These differences follow the inherited attribution analysis [8, Sections 3,5].

The missing execution condition requires a similarly definite account. If the relevant problem is inaccessible material, the inquiry should identify what participants cannot access and why the alternative preserves the task. If the problem concerns absent coordination, it should identify the roles that have ceased to function together. A general statement that opportunities are poor may motivate concern, but it leaves the reactivation mechanism underdescribed.

The proposed continuation should be recorded before the observed trajectory is used to declare success. Participants and investigators can revise it, but the revision should remain explicit. Changing the aim from interpreting unfamiliar narrations to repeating familiar phrases alters the result. Changing the meeting medium while retaining the selected discrimination can preserve the target if the comparison is justified. The record should make those cases distinguishable.

14.2 Intervention Histories

An intervention history records what actually occurred, including contributions that may disappear from a formal programme description. In the first meetings, a coordinator might select examples, explain the distinction, translate participants' questions, arrange access and resolve timing problems. Those actions have different relations to the selected operation. Some enable its exercise; some can form or extend it; others organize its future occasions.

Claim	Relevant Evidence	Remaining Question
Retained operation	Present task-specific contribution and a warranted earlier history.	What prevents the selected execution?
Initial realization	Participants perform the specified operation with recorded assistance.	What did the episode enable, teach or reorganize?
Support development	Identified roles, resources or coordination become available for later occasions.	Which contributions continue after initiation ends?
Sustained return	Relevant use recurs over the declared post-intervention horizon.	Which baseline, boundary and burden conditions remain?
Further reproduction	Other participants acquire the selected profile through a supported formation history.	Can the formative arrangement itself continue?

Table 2: Evidence Claims Across A Return Episode. Each row identifies a distinct attribution; successful performance in one row does not settle the remaining questions.

The account should retain their sequence. If participants first discriminate unfamiliar cases and later receive an explanation, the evidence differs from a sequence beginning with the explanation. If they arrange another encounter before the coordinator leaves, the resulting support differs from an arrangement in which the coordinator privately secures every future occasion. Recording only session attendance or final correct responses would conceal these differences.

The same artifact can change role over time. A comparison sheet can initially teach a distinction and later serve as an ordinary reminder. A participant can initially require interpretation and later help clarify another person's use. These changes should be described through their functions in the history. Labeling the whole episode training or activation would leave its mixed contributions unresolved.

Table 2 distinguishes the evidential claims that a return episode can support. The entries are not mandatory stages: support can be organized before initial activity, and a retained operation can become usable before its historical attribution is settled.

The final row in Table 2 marks a boundary with competence reproduction. Renewed participation by previously competent members can occur without any route for newcomers to acquire the profile. Conversely, a newly formed participant can become an important support during a return episode. A combined history should retain both achievements while avoiding the inference from current activity to continuing participant formation [10, Section 3].

14.3 Post-Intervention Observation

Observation after initiation should follow relevant occasions and support conditions, rather than merely repeat the most favorable initial task. New examples can reveal whether the selected contrast remains usable. Changed partners can test a partnership-general claim when such variation belongs to the target. A stable partnership can support a narrower result without having to demonstrate competence under arbitrary social conditions.

The inquiry should also record occasions without performance. Their interpretation can differ: participants may lack the needed support, choose another task, postpone the meeting or decline participation. An empty session record does not identify which explanation holds. Requests, scheduling histories, access conditions and participant accounts can help distinguish them, subject to their own evidential limits.

The end of initiation needs continuing scrutiny. A coordinator may cease attending while continuing to prepare materials. Another institution may assume the role. Those developments can explain persistence without supporting the initially proposed withdrawal comparison. The record should describe the operative support change even when it complicates an administratively convenient success story.

The horizon should include the range of occasions relevant to the claim. A short follow-up may support initial continuation; later observations can test further persistence or expose delayed decline. Differences between those findings need not invalidate the earlier bounded result. They change how far it can be extended. What should be avoided is presenting a favorable short interval as evidence that the practice has become indefinitely independent of further conditions.

14.4 Mechanism Discrimination

The activity–support hypothesis predicts that continuation can depend on the support state left by initiation, beyond the level of immediate activity. A useful inquiry would therefore investigate support formation directly. In the narrative circle, this can concern access to reusable materials, actual coordination of further meetings and the availability of the roles needed for interpretation. The evidence should identify those relations rather than infer them entirely from the fact that another session occurred.

Several explanations can fit a rise in activity. The operation may have become accessible; participants may have learned it anew; an organizer may be supplying more answers; selection may favor people who already perform well; or another resource may have changed at the same time. These alternatives can coexist. An account that separates their contributions is more informative than assigning the whole change to the latest visible intervention.

Timing differences can be particularly revealing when the target and comparison conditions are understood. Similar immediate performance followed by different continuation may support investigation of the post-intervention support states. It does not, by itself, isolate the modeled mechanism. Changes in participants, task difficulty, baseline provision or relevant outside activity may explain the difference. A coupled account earns empirical force through evidence for the dependence it represents.

Formal coordinates consequently require a measurement or interpretation bridge. Counts of utterances may represent one activity aspect while missing successful clarification or a justified decision to withhold a description. A support index may conceal noninterchangeable resources. The proposed non-linear mechanism should be compared with the available evidence at the declared resolution, including where the translation fails. Good numerical agreement with an aggregate trajectory does not validate a hidden capacity attribution.

14.5 Support Responsibilities

Inquiry into continuation should identify how the necessary work is distributed. A support record can name preparation, interpretation, access maintenance and coordination without converting them to a common monetary or numerical unit. Its purpose is to reveal which contributions explain the continuation and which dependences remain outside the modeled region.

That account can distinguish three different findings. Some work may disappear because an initiating repair has a lasting effect. Some may recur through arrangements participants have chosen to maintain. Some may continue through an unreported transfer to people whose alternatives have narrowed. Similar observed activity can accompany all three. Treating them as one self-sustaining outcome would obscure both causal interpretation and practical assessment.

The distinction also affects the claimed time horizon. A participant can absorb exceptional work for several meetings while lacking the resources to continue for a year. A finite record can establish the contribution already made without establishing its indefinite availability. Expected future work should be distinguished from an accepted undertaking and from an institutionally secured support.

The current paper does not determine a complete allocation rule for these responsibilities. It identifies the premises a continuation explanation uses. A later normative analysis can then examine whether relying on those premises is justified, what alternatives are available and how withdrawal should be handled. A model's sufficient condition for sustained activity is relevant information for that discussion, not a directive to produce the condition at any cost.

14.6 Inquiry Revision

A proposed inquiry should preserve the difference between a participant's retained capacity and willingness to make it available. Agreement to attend one meeting does not itself establish agreement to repeated testing, teaching or long-term maintenance. The consent relation can change as the practical consequences become clearer. An account of sustained participation should record the relevant commitments rather than assume that continued presence supplies unrestricted permission.

Revision can concern the target as well as the method. Participants may find that the old narrative distinction no longer serves their current purposes, or that a proposed test gives an inadequate account of their practice. Those judgments do not automatically settle the historical retention question. They can nevertheless change what the inquiry is appropriately trying to support. The formal operation family must then be related to the revised target or its application suspended.

The framework permits a positive bounded result without requiring an intervention programme to continue indefinitely. It can identify a retained contribution, explain a finite successful return, document a support dependence or show that a proposed continuation remains uncertain. A decision to stop after such a finding is a further practical choice. The analysis should not turn the discovery of a possible route into an obligation to complete it.

The resulting inquiry is cumulative when its records preserve the distinctions needed for criticism. Another reader should be able to identify whether a disagreement concerns retention, intervention content, support renewal, temporal scope or the proposed use of the findings. That organization makes the process intelligible without requiring a final verdict on an entire language. It also leaves the later questions of policy selection and recovery-record sufficiency open at their proper scale.

15 A Constructed Return Episode

This section applies the distinctions to a fictional episode. Its purpose is to show how the same initial observation can support different process descriptions once historical retention, support formation, regional dependence and intervention order are examined. The episode is not an empirical case, a recommended programme or a numerical calibration. Its participants and conditions are stipulated to expose specific interpretive choices.

15.1 The Selected Target

Mara formerly participated in a recurring family storytelling practice. In the current setting she rarely has access to interlocutors who recognize its narrative distinctions and tolerate the pace at which she now produces them. The selected target is narrow: using a familiar contrast to distinguish a completed event from an interrupted attempt, recognizing a listener's repair request, and revising the account appropriately. The inquiry does not concern every competence associated with the language.

Assume that the historical dossier provides good evidence of Mara's earlier command of these operations. Further carefully designed interactions support attribution of a retained present contribution to the contrast and repair, rather than merely reproduction of supplied answers. The assumption is explicit because the present model cannot establish it. It could fail if the intervention teaches the contrast anew or if the observed performance depends entirely on a facilitator's decisions. In those cases the episode would require a mixed reactivation-and-formation description.

The current absence of routine storytelling is insufficient to establish dormancy. Mara might have a presently available capacity that she chooses not to exercise. For the stipulated reactivation case, the dossier also records an execution obstacle: the available setting supplies neither a responsive interlocutor nor the time and repair practices required by the selected operation. The retention attribution remains on Mara's selected contribution; its situated execution requires a wider interaction arrangement. The later joint arrangement is newly established enabling realization, not a partnership asserted to have remained dormant. If the target were only Mara's individual discrimination in a suitable test, a different availability judgment could be warranted.

This boundary choice does not make every missing condition part of the person. It identifies the process being investigated. The individual retention question and the availability of the distributed interaction remain separately answerable. A report could consistently attribute the first while describing the second as unavailable.

15.2 The Initiating Arrangement

A temporary facilitator organizes encounters with two interested participants, Lian and Neri. The facilitator arranges a suitable time, supplies a familiar topic and makes repair requests intelligible. The selected operation appears in the first encounter. The initial report should identify whose contributions make that performance possible. It should also state whether the facilitator offers the answer, slows the exchange, mediates a repair request or teaches a new contrast. These acts can have different implications for retention even if the transcript looks similarly fluent.

The initiating role is scheduled to end. A successful handover would retain a responsive interaction routine among the three participants, including a way to arrange later encounters, tolerate pauses and repair misunderstandings. These are continuing support processes. They can be renewed through participation if the encounters teach the participants how to coordinate, make later encounters easier to organize and sustain mutual willingness. This description supplies a candidate interpretation of activity-dependent support renewal.

That interpretation must be tested rather than assumed. A calendar service or a continuing organizer might be doing all the renewal. If so, the process may still support a valuable return, but the claim that current activity renews the selected support would need revision. Likewise, a successful first encounter could use up the facilitator's available time without leaving any arrangement for the next one. The immediate linguistic performance would then be compatible with a fragile continuation.

15.3 Two Handover Histories

Consider two stipulated handover histories with similar immediate activity. In the first, the facilitator leaves after the successful first encounter. The participants have not yet established a shared repair routine or a feasible schedule. Later encounters become difficult to arrange, pauses are misinterpreted, and the selected activity declines. In the second, the facilitator continues long enough for those support relations to be established and then withdraws the initiating role. The participants continue the practice through those relations.

These histories express the model's same-activity/different-support distinction. Their numerical identification with z^- and z^+ in Equation 27 would require a measurement and realization argument that the fictional example does not supply. The formal result instead disciplines the verbal inference: an initial activity level is compatible with different continuation outcomes when support is an independent state variable.

The second history also illustrates the scope of the finite activation theorem. A sufficient duration can be derived inside the chosen law because activity has time to renew the selected support. The theorem is not a schedule recommendation for actual participants. Its duration depends on hypothetical coefficients, a selected activity floor and a restricted support law. An empirical application would first have to identify the relevant process and determine whether its assumed dependence is plausible.

The first history does not establish that the capacity has been lost. The selected operation can become unavailable again while its retained organization persists. Renewed inactivity may also change that organization over a longer interval, but this would be a further historical process. Treating every unsuccessful handover as immediate loss of linguistic capacity would collapse the distinctions that motivated the inquiry.

15.4 Regional Dependence

Suppose the report observes only Mara and Lian, while Neri continues to perform the scheduling and repair work. Their local activity can appear self-maintaining if Neri's contribution is left outside the observed region. Extending the region makes the contribution visible, but the correct response is not automatically to search for a perfectly closed social whole. The inquiry should state the region appropriate to its claim and retain the actual boundary relation.

Now suppose Neri becomes unavailable. This can have at least two different formal interpretations. One removes the corresponding exchange relation. Another leaves a relation whose other endpoint no longer supplies the former activity or support, while local effort continues to be spent through it. The field model's regional analysis distinguishes these operations. Which one is appropriate depends on the actual arrangement. A canceled encounter and an unresponsive but still time-consuming obligation can differ materially.

This is also where a symmetric exchange law may become inadequate. Neri may supply most of the coordination while receiving little comparable support. A symmetric weight can be a useful restricted approximation, but it can erase a directional dependence relevant to continuation. A revised law should then represent that dependence rather than treat its absence from the original equations as evidence that it is unimportant.

The regional account preserves a further distinction between support presence and support reliability. Neri might be available during every observed encounter yet unable to continue afterward. A time-varying support history would be a better representation than a constant baseline. The paper's autonomous post-withdrawal theorem applies only while its declared coefficients and background conditions remain in force.

15.5 Operation Change

During the encounters, Lian introduces a new narrative convention. Mara learns to use it, and the practice becomes more flexible. This development can be valuable without belonging wholly to reactivation of the original operation family. The inquiry now contains a retained contribution and a newly formed operation or realization. It should record their relation rather than require the later practice to be a literal repetition of the earlier one.

The history-bearing representation can add the new encounter and its outcome while retaining the earlier anchor. The active operation grammar may require an extension or descendant correspondence. The fixed field sector can still describe activity intensity over an interval, but its target interpretation has changed. Continuing to plot the same coordinate does not by itself justify treating the meaning of the coordinate as constant.

This case explains why sustained field activity cannot be equated with preservation of all selected capacities. A routine could become easier to sustain because participants silently abandon the difficult narrative distinction. Output might increase as the initial target disappears. The field trajectory and the operation-comparison record must therefore be assessed together. If the target changes, the report should name the successor practice and the retained contribution that helped form it.

15.6 Continuing Work

The successful handover depends on work. Lian may repeatedly adjust meeting times; Neri may prepare materials; Mara may accept the vulnerability of slow speech. Those contributions do not disappear when the initiating facilitator leaves. A description of self-renewing support refers to how activity and support are related, not to an absence of effort or a guarantee that its distribution is acceptable.

The participants can also decline continuation. If Mara decides that the practice is no longer important to her, maintaining its possible availability differs from inducing repeated performance. The dynamical capacity to sustain a channel does not establish an obligation to remain inside it. Likewise, a support

arrangement that produces stable activity by imposing unacceptable demands is not vindicated by its stability.

The positive implication is more precise. A report can identify the retained operation, the actual initiating contribution, the support that remains, the changes during handover and the horizon over which continued use is observed. It can describe the participant's choices and record support burdens independently of the field variables. Such a report makes an achieved return intelligible without converting either mathematical persistence or output frequency into a complete evaluation of the practice.

15.7 Revised Finding

The defensible finding in the successful fictional branch is that a specified retained contribution became executable within a changed interaction arrangement, and the selected practice continued over the observed horizon after a named initiating role ended. The continuing arrangement retained background conditions and renewed some support through use. Formation of the new narrative convention was recorded separately. The inquiry would remain open about longer horizons, wider competence, substitute interfaces and the fairness of continuing burdens.

That finding is narrower than saying that the language was restored or that the participants became independent of support. Its precision is also its usefulness. It identifies what changed, what remained, which mechanism is proposed and what evidence could revise the explanation. The formal investigation supplies conditional tests within that account rather than a replacement for the historical and participant-level description.

16 Research Boundaries

This investigation has treated sustained reactivation as a selected historical process whose explanation requires more than a successful elicitation. A warranted retained operation family, a change in its execution conditions, a continuing activity trajectory and the support of that trajectory are different parts of the account. The contribution of the model is to make one relation among them explicit: finite initiating activity can establish a continuing activity–support organization, but an initially successful episode can also fade before support develops.

16.1 Conditional Results

The two-field law supplies a concrete nonlinear realization of this question. The analysis establishes positive bounded evolution, characterizes its uniform stationary regimes, and derives a sufficient invariant region for continuing activity. A finite pulse can reach that region under declared conditions. A timing counterexample shows that the same immediate activity can decline when support formation is too slow. An open-region calculation shows how retained coupling to an inactive exterior can change the conditions for persistence.

These results concern the chosen field law. The uniform equilibrium calculation is not an empirical tipping point for a language, and a stable branch is not a definition of competent participation. The finite pulse bound is sufficient and can be conservative. Regional results retain the stated boundary condition rather than proving that all real support relations have that form. The computations check illustrative trajectories and exact examples; they do not estimate the law from linguistic data.

The algebraic analysis gives a complementary statement. Sustained-region membership depends on the baseline continuation, and its indicator satisfies a forward-invariance condition. A selected activity-only readout cannot distinguish the two constructed endpoint states whose support differs. On a countable history library, a staged intervention needs both an executable-domain certificate and a sustained-target certificate. The target identity alone can hold vacuously for an undefined intervention. These are elementary but useful consequences of retaining the correct carriers and domains.

16.2 Philosophical Limits

The central philosophical inference remains conditional on interpretation. Retention is not established by later success alone. Zero activity is not sufficient evidence of dormancy. Sustained activity does not prove that the originally selected operation is preserved, because the practice can change its target or the contribution of its participants. A fixed carrier can help explain one interval without determining every historically possible continuation.

The literature comparison reinforces these limits. Different studies investigate traces, relearning, re-exposure, accessible performance or longer use histories. Their results constrain the interpretation of return, but none is treated here as direct validation of the proposed field law. The empirical significance of support accumulation, intervention duration and boundary dependence requires a design able to distinguish those processes from new learning, continued external facilitation and changes of target.

Generative Relational vocabulary contributes a disciplined separation of the historical object, its selected representation, its operations and their conditions. It also permits these layers to change together. Its usefulness in this paper does not depend on asserting that the two-field model is fundamental. Another model may better represent intermittent participation, directed support, changing interpreters or the creation of new operations. Such a replacement should preserve the distinctions that the inquiry needs while stating which mathematical results no longer apply.

16.3 Further Investigation

The first further task is empirical specification. A selected operation family and its bearer boundary must be described closely enough to distinguish retained contribution from supplied performance. The activity and support channels then need independently defended observations. Post-intervention histories should record which initiating role ends and which other relations continue. These requirements follow from the present question; they do not prescribe one universal study design.

A second task concerns model extension. Delayed renewal, directed exchange, finite reserves, intermittent use and changing operation carriers can each alter the results. Their inclusion should be motivated by a specific explanatory failure of the current law. Adding mathematical structure merely because it is available would weaken the connection between the question and its formal investigation.

A third task concerns evaluation. Continuing support can preserve an important possibility while allocating work unequally. Participants can value access without wanting constant exercise. A future normative account must examine authority, consent, burden and the possibility of declining or revising a practice. The present results identify conditions of modeled continuation; they do not settle those judgments.

The paper therefore supports a bounded positive claim. Reactivation can be investigated as a history-bearing transition into continuing operation through specified support relations. Its success may depend on the interval during which those relations become effective, on their regional dependencies and on the order of intervention stages. Keeping those features explicit makes both successful returns and failed handovers more intelligible, while leaving their wider linguistic and normative significance open to further inquiry.

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