

Linguistic Competence Reproduction

A Preliminary Study of Participant Formation and Support Renewal

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Abstract

Competent current language use can coexist with a weakening capacity to form further competent participants. This paper investigates the historically organized relations connecting participation, learning, correction and enabling support in the reproduction of selected linguistic competences. Comparison of cognitive, communicative, socialization and participatory-learning accounts informs a Generative Relational interpretation of participant formation. The philosophical argument distinguishes supported contribution from supplied answers, learning from disclosure of prior capacity, and changed targets from lost competence. History-bearing configurations retain these differences across changing participants and practices. A restricted nonlinear three-field model investigates mutually enabling operational activities and establishes conditional coexistence of inactive and positive stable regimes. A partial-history operator construction distinguishes executable formation from repeated boundary descriptions. A finite selection procedure identifies continuing boundary families, with explicit assumptions for lifting their paths to concrete formative histories; robustness to every admitted choice is a stronger requirement. A developed fictional multilingual inquiry demonstrates how the distinctions change warranted findings. The authority argument supports accountable instructional asymmetry and accessible contestation without assigning automatic continuity or repayment obligations. The results are bounded philosophical arguments and mathematical consequences within several possible models, rather than a general acquisition theory or calibrated law of linguistic competence.

Keywords: linguistic competence; participant formation; language socialization; generative return; non-linear fields; history operators

Scope Note

This paper studies the formation of further competent participants in a selected linguistic practice. Its central object includes the learning relations and supports through which that formation can recur. Current fluency, an archive of expressions, an effective lesson and a renewable arrangement for participant formation are different achievements. The analysis keeps their connections visible without requiring every competent speaker to become a teacher.

The Generative Relational framework is used as a revisable metalanguage for historical, philosophical, empirical and mathematical inquiry. Its inherited accounts of formative capacity, generative return, history-bearing configurations and changing operations already anticipate much of the problem. Established linguistic and anthropological research supplies independent explanations and objections. Translation into a common vocabulary does not settle disagreements about cognition, social relations, evidence or authority.

The mathematical constructions are selected representations. A historical architecture retains the formation of participants and their relations. A nonlinear three-field model examines specified mutually enabling activities. An operator construction tests the execution and renewal of formative histories. These models do not measure meaning, establish a complete acquisition theory or prove that any community follows the displayed dynamics. In particular, the field's stable positive regime is not a certificate of competent or legitimate practice.

Published learning studies and a separately stipulated learning-circle example constrain the interpretation. This paper provides no new empirical dataset. References to the author's supplied manuscripts identify the versions consulted without implying publication or independent validation. The source packet records actual reading scopes, computational checks and remaining obligations. The draft is intended for substantive criticism and author review.

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Notices

Status. This discussion draft presents an evolving inquiry. The definitions, translations, formal constructions, empirical interpretations, and conclusions remain open to criticism and author review. It proposes no universal competence ranking, compulsory language policy or empirically calibrated learning law.

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Statement On The Use Of Language Models. The author selected the research direction, supplied prior manuscripts and a reference exposition, and requested consistency with the Generative Relational research programme. AI-assisted research, source comparison, drafting, formal checking, and document preparation contributed substantially to this draft. These contributions include proposed arguments and interpretations that remain subject to the author's review. Preparation of a checked draft does not establish the author's approval of every claim. Authorship credit remains with the human author; the use of these tools is disclosed to support critical assessment of the manuscript and its limitations.

1 Participant Renewal

Linguistic competence reproduction concerns the relations through which a practice forms further participants capable of understanding, contributing and responding to correction in selected contexts. The immediate philosophical difficulty is a gap between successful present activity and the renewal of its participants. A group can communicate fluently, maintain an extensive archive and produce new expressions while newcomers lack the opportunities needed to acquire the relevant capacities. Conversely, newcomers can acquire a useful range of capacities while the original teaching arrangement becomes unavailable to those who follow them. The paper investigates the structure of these differences and the conditions under which a formative success contributes to further formation.

The scope is deliberately intermediate. The object exceeds a single learner's test result because opportunities, correction, access and support help determine what a result means and whether it can recur. It remains narrower than the reproduction of an entire language or culture. A selected practice can involve overlapping languages, changing participants and disputed standards. Its competence target might concern interpreting a particular class of descriptions, explaining a distinction, repairing misunderstandings or contributing to a recurring deliberation. The inquiry does not require a decision about where one whole language ends.

1.1 A Formative Discontinuity

Consider a stipulated multilingual neighborhood learning circle organized around descriptions used in a repair workshop. The workshop supplies a setting for language practice, rather than mechanical instruction in this paper. Experienced participants can recognize a request, explain why two descriptions differ and negotiate an unfamiliar case. A newcomer initially repeats a demonstrated phrase. Over time, the newcomer learns to choose among expressions, explain a choice, request clarification and identify an inappropriate application. Those developments concern a specified linguistic profile. They can occur without the newcomer being able to organize the next learning group, prepare suitable contrasts or diagnose another learner's misunderstanding.

Suppose that the original facilitator then leaves. Some former learners continue using the expressions successfully. Yet the next group receives only recordings, fragmented notes and occasional answers from people who lack time for sustained interaction. The situation does not establish that recordings are ineffective or that only face-to-face mentorship supports learning. It establishes a question: which relations that made the earlier formation possible have successors now? The answer depends on what later participants can actually do with the materials, their available interlocutors and their opportunities to revise an interpretation. The same stock of experienced users can support different answers.

This example also gives a positive meaning to reproduction. A later group need not repeat the former group's vocabulary exactly. It can develop another explanation, divide instructional tasks differently or use a new medium. The reproductive claim concerns a declared capacity and its formative relations across that change. Historical continuity identifies how the successor formation arose; functional comparison specifies what it retains; evaluation asks whether the retained and changed features deserve support. Each requires its own grounds.

1.2 The Analytical Contribution

The Generative Relational preservation taxonomy already distinguishes a generative capacity, the conditions of its execution and the historical comparison supporting a preservation claim [10]. The current paper uses that inheritance to examine participant formation. Its proposed contribution is a bounded architecture connecting three questions: what competence is being formed; what historical relations enable formation; and which consequences of a formative episode help make another episode possible. Reproductive capacity therefore belongs to an arrangement with differentiated participants and supports, even when important learning changes occur within an individual.

The account does not obtain a social explanation by denying cognitive processes. A learner's perception, memory, attention and inferential organization remain relevant. The claim is that their development and evidential interpretation leave additional questions about access to examples, uptake of contributions, correctable mistakes and renewed opportunities. Equally, an attractive institutional description does not establish that anyone learned. Keeping the individual and relational questions jointly accountable is more informative than assigning either an unqualified explanatory priority.

The paper's mathematics makes selected dependencies inspectable. A history-bearing representation distinguishes a new participant from a replacement label on an unchanged state. A nonlinear field model examines how several operational activities can jointly sustain one another while small isolated improvements decay. A regional potential balance and an exact open-input example show how continuing local activity can depend on external support even while the whole model dissipates its potential. A history algebra distinguishes a repeated boundary pattern from a concretely executable renewal process. A further finite construction selects the greatest boundary family admitting a continuing choice from a declared repertoire. Its historical lifting conditions state when that choice carries concrete participant formation, while a stronger comparison tests robustness to every admitted choice. These are conditional mathematical achievements, not proofs of linguistic meaning or guarantees of practical control.

The philosophical development gives those calculations a more specific object. It distinguishes a learner's contribution within ordinary supports from an answer supplied on the learner's behalf; acquisition from selection or disclosure of a prior capacity; and a revised learning target from failure on an unchanged target. The developed inquiry returns different findings when these histories differ, even where the first observed answer is the same. The authority argument then explains why useful instructional asymmetry requires task-related reasons and accessible contestation. Together these developments make a positive account of reproductive capacity possible without treating every dependence as a defect or every continuity as desirable.

1.3 Method And Evidential Standing

The inquiry combines conceptual reconstruction, comparison of primary literature, conditional mathematical analysis and developed examples. Established research provides several explanations of learning and transmission with different objects. The paper preserves their differences rather than placing them on a progression culminating in a relational framework. The studies discussed later provide evidence for particular learning tasks and experimental arrangements. They do not directly test the field law or the renewal algebra.

The formal argument adopts an explicit hierarchy of commitments. History-bearing configurations express a methodological preference inherited from prior work. A labelled two-complex is one available representation. A finite active-site field and a countable history Hilbert space are stronger, independent choices for particular investigations. Their fixed carriers do not represent the whole future of a linguistic practice. A changed participant or repertoire requires the typed historical extension and comparison data developed before those specializations. The power formalism supplies this distinction between general historical dynamics and selected field or algebraic realizations [9].

1.4 Argument Sequence

Section 2 reconstructs the literature interfaces. The next two sections develop reproductive capacity and historical participant formation. Section 5 specifies the historical and operational objects needed by the mathematics. Sections 6 and 7 introduce a nonlinear model and establish its conditional regimes. Section 8 addresses executable renewal across changing participants. Sections 9 and 10 examine evidence and authority before Section 11 returns to the learning-circle example. Section 12 states what the combined investigation establishes and what remains open.

2 Linguistic Learning Accounts

Linguistic competence reproduction intersects several established inquiries whose explanatory objects require careful separation. Grammatical knowledge, contextually appropriate participation, directed learning, and transmission across successive learners can each be investigated without thereby explaining renewal of an entire formative arrangement. This section compares primary contributions to identify what the present investigation inherits and what its selected question still requires. The comparison is problem-oriented. It preserves disagreements about the constitution of learning and the interpretation of evidence, while developing a modest place for a historical investigation of further participant formation. The sources supply neither a universal developmental sequence nor a progression culminating in Generative Relational theory.

2.1 Competence Attribution

Chomsky's distinction between competence and performance separates knowledge of a language from its actual use. The opening of *Aspects of the Theory of Syntax* specifies an idealized speaker and a homogeneous community; actual performance involves additional interacting factors. The proposed grammar characterizes the speaker's knowledge, while its assignment of structural descriptions supplies no algorithm for the practical production or perception of utterances [1, pp. 3–9]. This distinction matters methodologically. A change in observed success need not be a change in the underlying object being attributed. Moreover, explaining why a practice has ceased to form newcomers concerns a different explanandum from describing an acquired grammar. The present inquiry does not infer the inadequacy of grammatical investigation merely from that difference of scope.

Hymes broadens competence to include knowledge and ability for use. His account distinguishes formal possibility, feasibility, contextual appropriateness, and actual occurrence; those dimensions need not coincide. He also treats communicative events as having properties arising through the interaction of participants rather than simply reflecting standardized individual competence [11, pp. 276–284]. The relevance is substantial: contextual differentiation and interactional organization already belong to a well-developed predecessor problem. Calling competence relational cannot itself establish a contribution. Hymes's distinctions also prevent a convenient observed regularity from silently standing for everything a participant can understand or do.

For the present argument, these accounts motivate an explicit attribution question. An investigator should specify the linguistic activity, the relevant variations, the permitted assistance, and the conditions under which an apparent success could be accidental. Consider a newcomer who correctly repeats a phrase after a prompt. The episode provides evidence of one achievement. Whether the newcomer can select that phrase for a different occasion, recognize an inappropriate application, or help another participant acquire its use requires further inquiry. These questions need not form a mandatory ladder, and their answers may depend on different supports. They are distinguishable because the activities demand different discriminations.

These contrasts motivate a declared competence profile whose criteria remain answerable to the selected practice and evidence. Section 3 develops that specification, including the risks of criteria that are either too weak or unnecessarily demanding.

2.2 Socialization Histories

Ochs and Schieffelin place novice formation within a lifespan account of language socialization. Their theoretical chapter treats novices as active participants, recognizes changing relations of expertise, and considers communicative arrangements, material environments, differentiated access, and power. It also connects participant formation with the transformation of languages and communities [15, pp. 1–17]. Historical conditions and participant renewal are therefore inherited concerns. The present paper should be assessed by the precision of its selected distinctions and consequences, rather than by a claim to have discovered the social constitution of linguistic learning.

The remaining task can be stated through an analytic comparison. Suppose two arrangements currently sustain competent exchanges among their established members. One also supplies occasions on which unfamiliar participants can attempt relevant contributions and encounter interpretable responses. The other has lost those occasions, although its members continue their established exchanges. Their present outputs may be similar while the available histories of further participation differ. This is a proposed discriminating case, not a report about the prevalence of either arrangement. Investigating it requires attention to who can enter, what participation makes accessible, and which forms of correction a newcomer can actually use.

Such a comparison must also allow formation to change its conditions. A newcomer may introduce an unfamiliar distinction, reveal a failure in an inherited example, or become a source of instruction for an established participant. A model that predetermines a permanent class of experts and a permanent class of recipients would exclude those possibilities by construction. Conversely, permitting role changes does not establish that every change is successful learning. Historical openness increases the explanatory obligation: one must say which achievement continues, which changes, and what warrants the description of competence in the revised practice.

2.3 Participation Conditions

Lave and Wenger describe legitimate peripheral participation as an analytical perspective on learning, rather than a teaching technique. Their initial chapter connects participation to access, resource control, and relations of power; it also resists a single center or a linear scale of acquisition. Participation can enable growing involvement or hold someone in a restricted position [14, pp. 29–43]. These commitments directly resist a picture in which learning consists simply in moving a packet of knowledge between otherwise unchanged individuals. They also make it inappropriate to announce access conditions as an addition wholly absent from prior learning theory.

The distinction between access and effective formation nevertheless needs local specification. An institution may formally permit a newcomer to observe an exchange while withholding the records or contextual distinctions needed to understand it. Alternatively, a novice may acquire considerable ability without receiving the status that established members associate with that ability. In the first stipulated case, permission does not establish a usable formative opportunity. In the second, withheld recognition does not by itself establish incapacity. An adequate investigation must retain both possibilities instead of using institutional admission as the definition of learning.

Reproductive analysis also asks what becomes available after an initial formation succeeds. A participant could acquire a selected competence through resources that are subsequently exhausted or closed. Whether another participant can then follow a formative history is a further question about the resulting arrangement. The distinction is compatible with the importance of distributed participation: the first learner need not personally become a teacher. Further formation may depend on a different division of activity among peers, materials, experienced participants, and supporting organizations. Its success requires a specified continuation, rather than a presumption that every acquired competence carries its own renewal conditions.

2.4 Pedagogical Generalization

Csibra and Gergely propose natural pedagogy as a communicative basis for acquiring generalizable knowledge, including culturally important information whose rationale may be difficult to recover by observation alone. Their account connects ostensive address, referential expectations, and an interpretive orientation toward generalization, while distinguishing the proposal from a restriction to formal schooling [2, pp. 148–152]. Here it is used as an explanatory hypothesis concerning a formative interaction. Its evolutionary and cross-cultural claims are not required premises of the present paper, and the review is not treated as independent verification of every experiment it discusses.

The present use of this hypothesis concerns the recipient's interpretation of a communicative episode. Our further question concerns the arrangement that makes another such episode possible. Even if an

episode successfully supports generalization, an additional account is needed of access to suitable contributors, examples and occasions. This is a proposed difference of explanatory target, not a refutation of natural pedagogy. The distinction between participating competently and helping another participant learn is developed in Section 3.

Assistance is also relationally specific. An explanation can presuppose distinctions its intended recipient has not acquired. A correction can be accurate yet unusable because its timing, form, or practical implications are inaccessible. Repetition can stabilize a response without making the relevant contrast available. Accordingly, a model of formative operations needs more than a label denoting instruction: it must state the conditions under which a recipient can take part in the operation. Where those conditions are unknown, the model identifies an evidential obligation. Assigning a learning transition by definition would conceal precisely the dependence the model is meant to investigate.

2.5 Transmission Success

Kirby, Cornish, and Smith investigate artificial-language transmission through successive adult learners. Their two experimental arrangements show that increased transmissibility can accompany different structural outcomes, depending on the training constraints. They explicitly distinguish adaptation in learnability and structure from improvement in communicative functionality [12, pp. 10682–10686]. The case supplies a disciplined reason to specify what reproduction preserves. Greater stability in a selected output comparison does not establish preservation of every distinction that might matter to a practice.

The philosophical extension proposed here concerns a change of explanatory target. A transmission experiment can appropriately supply the next participant and the conditions of training in order to investigate what happens across the resulting sequence. An inquiry into formative renewal asks whether a selected arrangement can maintain or rebuild such conditions. Its question is not answered merely by making the sequence longer. One must examine the operations that keep further participation possible, including their domains, supports, and consequences for later activity. This extension respects the experiment's purpose while identifying an additional target.

The distinction also disciplines the use of simulations. Repeatedly applying a learning map demonstrates properties of that map under its specified conditions. If the supply of learners, interpretable tasks, corrective responses, or admissible transitions is guaranteed by the simulation, their reproduction has been assumed rather than derived. A more informative model can represent selected dependencies explicitly and compare what follows when different enabling relations persist. Even then, a conditional result concerns the modeled architecture; evidence is needed before it becomes an explanation of an actual linguistic practice.

2.6 Public Formative Materials

Goodwin's analysis of professional vision examines how coding, selective attention and graphic practices organize professional objects of knowledge. In the archaeological training material, a color chart participates in a historically developed practice of classification; applying its categories still requires work and can produce disagreement. A completed code can conceal uncertainty and the interaction through which it was selected [3]. This source supports a bounded point: preserving an instructional artifact and preserving the competence required for its situated use are different achievements. It does not establish that every linguistic distinction depends on an analogous chart or profession.

The question for the present paper is how to describe the formation of a competent contribution when public materials partly constitute its content. A simple input model may treat a diagram as information delivered to an already individuated learner. A historical account can also ask how learning changes what the participant recognizes as relevant in that diagram and how the diagram itself was organized to make a distinction available. These questions supplement one another when their respective objects are retained. The diagram's history does not explain the learner's every response; a successful response does not exhaust the diagram's institutional or formative role. Their connection needs an account of actual use.

Goodwin's later study analyzes action assembled through heterogeneous semiotic resources and successive transformations of public materials. Its discussion of a speaker with severe lexical restriction shows how other participants' talk, prosody and embodied conduct can contribute to consequential participation. Its apprenticeship discussion also examines how visible correction can organize both an activity and the formation of its competent participants [4, pp. 8–11, 19–21]. These observations challenge an attribution rule that counts only unsupported individual verbal production. They do not, by themselves, certify the renewability of a particular support arrangement.

The present analysis adds a specific reproductive question to that challenge. Suppose an arrangement supports a contribution through a familiar partner's interpretive work. A successor partner may need to learn that work, including how to recognize an attempted contribution and distinguish endorsement from accommodation. Reproducing the supported competence can therefore require formation on more than one side of the interaction. The learner's development and the supporter's development need not be symmetric or interchangeable. Their coordination is precisely what an account centered solely on the final utterance leaves unspecified.

That argument is a proposed extension, not a further reported finding from Goodwin's cases. It identifies a question an actual longitudinal inquiry could examine. Which contributions remain available when a familiar partner leaves? What must the successor learn, and what changes during that learning? A practice may answer through planned preparation, informal adaptation or a reorganization of the task. Each possibility has a different historical structure, even if all are summarized institutionally as replacement of a support role. The distinction becomes important when a reproduction claim rests on the continuity of that role.

2.7 Participation Structures

Rogoff and colleagues contrast learning organized through attentive participation in shared activity with specialized instructional arrangements. Their stated contrast is multidimensional and explicitly does not divide learning into two exhaustive kinds. Their account includes observation and listening oriented toward participation, with attention to the roles of experienced contributors and learners [16, pp. 175–178]. The present paper uses that proposed organization of learning rather than treating the review as fresh verification of every study it cites. Its relevance is to the selection of a formative unit and the danger of making the teacher–learner lesson the only admissible unit.

A practice can distribute formative contributions across an activity that has another immediate purpose. Experienced members may explain an unusual choice while performing ordinary work; a newcomer can observe alternatives, anticipate a later contribution and eventually participate. Under such a description, the absence of a formal lesson is not the absence of a learning relation. Equally, the newcomer's physical presence does not establish attentive access, intelligibility or acquisition. The reproductive inquiry needs to identify which part of the activity makes further participation possible and what evidence supports that role.

This problem alters the interpretation of the formal channels developed later. Responsive guidance cannot be equated with a number of explicit corrections if the chosen practice supplies its informative differences through demonstration, observation or ordinary responses. Support renewal may include protecting an occasion for legitimate observation rather than preparing a new formal teacher. A model whose variables were measured only by scheduled lessons could miss the formative arrangement even if its equations were internally correct. Operationalization therefore begins with the practice and question; the convenient availability of a data series does not decide what the field represents.

There is also a substantive limit to the proposed three-channel model. A practice may contain a formative route that succeeds without the particular triadic interaction assumed by that model. In such a case a missing modeled channel would not establish absent reproduction. The appropriate response would be to reconsider the interaction law or choose another representation. The model investigates one mechanism, and the literature supplies reasons to preserve alternatives. This is a stronger methodological commitment than calling an equation illustrative while interpreting all empirical departures as defects of the practice.

2.8 Competing Explanatory Commitments

The accounts just considered cannot be synthesized merely by increasing the number of objects in a diagram. They disagree about what requires explanation and what constitutes the relevant achievement. An account of grammatical knowledge can treat a particular performance as the joint effect of knowledge and additional factors. An interactional account can investigate a supported act whose organization crosses participants and materials. A socialization account can trace the history through which participation and its significance develop. Each can be adequate for a bounded question while leaving a further question open. The reproductive inquiry must therefore declare which attributions it adopts, rather than presenting its breadth as a refutation of narrower investigations.

One consequence concerns explanatory division of labor. If a learner's response changes, the present framework can distinguish a changed personal discrimination, a new contribution from a partner, a transformed task or a different criterion of assessment. It cannot decide which occurred from the response alone. Historical and interactional evidence may discriminate among these possibilities; an independently supported cognitive account may explain part of the change. A Generative Relational description is useful when it keeps the relevant contributions and comparisons connected. It gains no additional support simply by declaring them all relational.

A second consequence concerns disagreement about competence itself. A practice-specific supported achievement and an attribution of transferable knowledge can coincide, but neither description should silently replace the other. Transfer has a specified range; supported participation has a specified contribution structure. The paper's competence profiles make these commitments explicit without claiming to settle the general ontology of linguistic knowledge. The following section develops the positive attribution rules and their limits, so that later formal representations inherit a substantive target rather than supplying one by notation.

2.9 Reproductive Explananda

The preceding comparisons leave three connected targets for the present investigation: a selected competence exercised by current participants, a formative history through which further participants acquire a relevant profile, and a continuation that carries or rebuilds conditions for another such history. Their separation is analytical. The same encounter may contribute to all three, and an explanation may fail if it isolates them despite constitutive dependence. The proposed distinctions become useful when they expose a concrete missing transition, conflicting demand, or unearned inference.

The literature also constrains the paper's ambition. A relational framework cannot resolve disagreements over mental representation, social participation, or generic knowledge by giving them a common diagram. It can make selected commitments explicit and investigate their conditional consequences. Nor does successful formation settle the legitimacy of its criteria, the distribution of its burdens, or a duty to continue. Those questions concern the standing of the formative practice and remain open to reasons that an account of reproduction alone does not supply. The subsequent philosophical and formal investigations therefore begin with declared targets and supports, preserving the difference between an operative learning history, a justified competence attribution, and a defensible practice of renewal.

3 Reproductive Capacity

The reproductive capacity of a linguistic practice concerns its ability to form further competent participants through a historically organized arrangement. The arrangement can include current users, learners, examples, material supports and opportunities for response. This section specifies the achievement being attributed to it. The argument distinguishes a competence profile from a successful performance, participant formation from competent current use, and renewal of a formative arrangement from the continued presence of its original providers. These distinctions establish the object that the subsequent historical and formal accounts must preserve.

3.1 Competence Profiles

A competence claim needs an object and a scope. Someone can understand a request, produce an appropriate description, recognize a consequential ambiguity, or help repair a misunderstanding within a particular practice. Those capacities can develop unevenly. A person who responds appropriately to a distinction may still find it difficult to formulate, while someone who repeats its canonical wording may miss a relevant difference between cases. The present inquiry therefore uses a *competence profile*: an explicitly selected family of interpretive, productive and participatory capacities, together with the contexts and supports relative to which their availability is claimed. The profile is an analytical specification, not a numerical rank of speakers or a complete definition of linguistic knowledge.

Consider a stipulated multilingual repair circle. Participants are learning to distinguish a completed removal from a condition they call “return-ready,” which also requires that relevant information for later return remain available. The expression continues the illustration used in the preceding convention-inquiry [6]. Only the linguistic organization of the example matters here; the example supplies no technical procedure or judgment of mechanical safety. A selected profile might concern identifying a missing record, requesting clarification and explaining why the description should remain provisional. Correctly echoing the expression after a facilitator supplies it establishes a narrower achievement than independently recognizing the missing-record case.

Independence is itself relative to a specified intervention. The participant may use a shared diagram, consult a colleague or compare earlier descriptions. Removing those resources can change the activity being assessed. An inquiry that seeks competence within this supported practice should ask whether the participant makes the relevant distinctions through its ordinary resources. An inquiry that seeks competence after a particular facilitator departs asks a further question. Neither target gains priority simply by requiring fewer supports. A person can possess substantial competence while lacking an opportunity to exercise it; conversely, an arrangement can produce a successful answer without establishing every capacity attributed to that person.

The selected variation is therefore substantive. Variation should disclose a difference that matters to the competence claim: a new interlocutor, an unfamiliar example, an omitted cue or a relevant correction. It need not demand success in arbitrary contexts. Nor must competence invariably take the form of an explicit statement of reasons. Discrimination can be displayed through an appropriate question, a changed action, a gesture or a request for help. Where reflective explanation is part of the target, it needs its own evidence. This makes the profile demanding enough to exclude mere token repetition without identifying competent participation with complete verbal mastery.

3.2 Supported Competence Attribution

An attribution to a participant and an attribution to an arrangement need not compete for exclusive possession of the same achievement. They answer different questions about its organization. To attribute a competence to a participant is to identify a contribution that the participant can make across a specified range of supported occasions. To attribute a formative capacity to an arrangement is to identify relations through which contributions of that kind can develop. A successful group response can warrant the second description only when the relevant developmental relation is present. Conversely, the fact that a response depends on others does not erase the participant’s contribution. The inquiry must locate that contribution within the dependence.

Suppose a learner uses a bilingual comparison sheet when deciding whether a description leaves a return condition unresolved. The sheet supplies vocabulary and contrasts. The learner identifies which comparison matters, notices that a new case fits neither example, and asks for clarification. Attributing this discrimination to the learner is compatible with treating the sheet as a constitutive resource of the supported task. The attribution would change if a facilitator silently selected every relevant comparison and the learner merely read the highlighted answer. The words might be identical, but the learner’s contribution to selecting the distinction would differ. What matters is the organization of contribution, not a count of how many people or objects participate in the final response.

This analysis gives a reason for testing variation, while also limiting what variation may legitimately demand. A changed example can reveal whether the participant selects the relevant contrast. Withdrawing an ordinary accessibility resource can instead destroy the practice whose competence was being assessed. A defensible test varies a feature that the competence claim represents as nonessential, or openly states that it is investigating a stronger target. It does not infer a deficit from failure after arbitrarily removing a constitutive support. The distinction can itself be disputed. A diagram may initially supply the very comparison being learned and later become a convenient reminder. A formation history should allow the role of the same artifact to change.

The opposite danger is to make the support specification so permissive that the attribution loses content. If any successful response counts, including a response entirely supplied by someone else, the test cannot distinguish the learner's emerging discrimination from the arrangement's ability to answer on the learner's behalf. The remedy is an articulated contribution requirement. For one task the learner must select a relevant example; for another, identify a missing distinction or appropriately seek assistance. These are different requirements and should not be combined into an unrestricted demand for autonomy. Appropriate help-seeking can itself be a competence when the task requires recognizing a limit and engaging a suitable source.

A competence profile consequently has two boundaries: the range of occasions and the contribution expected within those occasions. A person might reliably identify uncertainty while needing assistance to formulate a public explanation. That is a determinate achievement. Its description becomes misleading only when it is substituted for a different claim, such as being able to explain the distinction to a novice. Recognizing partial achievements can therefore increase precision while avoiding both indiscriminate success attribution and an ideal of the completely self-sufficient speaker.

The boundary between personal and distributed competence remains revisable through this inquiry. Different decompositions can be adequate for different questions, but they incur testable commitments. A replacement-partner claim requires evidence across appropriate partners. A facilitator-departure claim requires an account of what replaces the facilitator's actual work. A claim about one stable partnership can remain warranted even if both stronger claims fail. The paper preserves these distinctions because reproduction frequently changes partners and supports while leaving a broad institutional role name unchanged. A nominal successor is not yet an adequate successor arrangement.

3.3 Further Participant Formation

Reproductive capacity introduces a second object. A practice can presently support the selected profile among experienced participants while lacking a route through which another person can acquire it. Familiar members may anticipate one another's questions, recognize old examples and rely on comparisons that remain inaccessible to newcomers. Their performance can be competent. The failure concerns the relation between that achievement and further formation, rather than the reality of their existing understanding.

The distinction has a simple logical basis. A present-use claim concerns what specified participants can do under current conditions. A formation claim concerns a history through which a participant acquires or develops the target capacities. The first description leaves that history unspecified. To infer the second from the first, an additional account is needed of entry, engagement, response, retained learning and subsequent use. The fact that teaching sometimes takes place through participation makes these activities potentially connected; it does not make their connection true by definition. A first visible success can also disclose a capacity already possessed, so that event alone does not establish its acquisition.

In the repair circle, experienced members might complete every session efficiently while newcomers merely carry messages. A different organization might let newcomers compare descriptions, try an application and encounter responses that help them distinguish an error from an unfamiliar acceptable use. Both organizations could produce similar immediate task outputs. Only the second stipulation includes a formative route appropriate to the selected profile. Whether that route succeeds remains an empirical question: access and instruction establish an opportunity, while achieved formation requires evidence of changed capacities across the relevant occasions.

The learner need not reproduce the facilitator's entire profile. Complementary roles can support a practice, and new participants can contribute capacities that earlier participants lacked. Reproduction concerns specified continuities of competent participation, with allowable variation made explicit. An inquiry must consequently say which differences are acceptable transformations and which would defeat its particular target. Exact copying makes the target unnecessarily rigid; describing every historical descendant as successful reproduction makes it uninformative.

3.4 Formative Causation

A learning history supplies more than a temporal sequence ending in success. It should identify how the participant's contribution changed and which relations plausibly contributed to that change. Otherwise selection and formation can be confused. A circle that admits only participants already able to use its central distinctions can display increasingly fluent sessions while having formed none of the relevant competence. A different circle may show more hesitation precisely because it admits novices and makes their questions consequential. Neither fluency nor hesitation identifies the formative process without the history of admission, assistance and changed performance.

The positive causal question can be organized around contrasts that the practice itself makes intelligible. Suppose a learner initially treats two descriptions as equivalent, encounters a case in which their consequences differ, receives an explanation, then distinguishes further cases. The sequence supports a formation hypothesis because the later discrimination is connected to a previously unresolved difference. Its force increases if the learner can identify a further relevant instance, correct an earlier application or explain why an apparently similar case falls outside the distinction. A single retrospective statement that the explanation was helpful supplies a different, narrower kind of evidence. Testimony can matter without carrying every part of the causal claim by itself.

Alternative explanations remain substantive possibilities. The learner may have understood earlier but lacked an opportunity to display the understanding. The later task may contain an unnoticed cue. Repeated exposure may have mattered more than the particular correction. A newly accessible translation may have disclosed existing knowledge while also enabling later learning. These possibilities need not be exclusive. The researcher can identify a compound history in which disclosure, access and formation contribute at different stages. A useful account specifies their respective roles instead of assigning the entire change to the last visible intervention.

The distinction between an enabling condition and a formative contribution is similarly relative to the question. Making a meeting accessible can enable instruction without determining what is learned. If exclusion had previously prevented participation, the access change can nevertheless be causally decisive for that person's formation. Calling it merely background would obscure the actual contrast. Equally, an accessible room does not explain the content of a distinction that was learned there. Causal importance and semantic specification perform different explanatory work. Their separation permits a historical explanation to retain both institutional access and situated interpretation without treating either as a complete cause.

Formation can also operate through sequences whose contributions are distributed. An example supplies a contrast, a peer notices the learner's question, a later encounter makes its significance clear, and a record permits the earlier claim to be reconsidered. The successful episode may occur after the original teacher has left. The relevant unit is then an organized history of contributions, not a single intervention and immediate outcome. This supports the history-bearing construction used later: it preserves the relations needed to distinguish delayed formation from an unsupported claim that the most recent event created the competence.

A recursive formative explanation makes one further claim. It says that what participants acquire also changes how subsequent learning becomes possible. Evidence for a learner's new discrimination does not yet establish that recursion. One must trace a further effect: a revised example becomes available to others, an earlier exclusion is removed, or a participant becomes able to notice and respond to another's difficulty. The effect may fail to occur even after substantial learning. Generative return therefore remains an additional historical relation to investigate, rather than a favorable name for every successful

educational outcome.

3.5 Distributed Pedagogical Capacity

Competent use and pedagogical capacity can also differ within one person. A fluent participant can have difficulty selecting examples, recognizing a learner's misunderstanding or changing an explanation. A less fluent participant may be especially capable of arranging comparisons that help others. Neither possibility establishes a universal separation between users and teachers. It establishes that an attribution of one capacity needs an argument before it can support an attribution of the other.

The formative arrangement need not locate pedagogy in one teacher. One member can supply examples, another can interpret a learner's question, and a documented comparison can preserve an earlier correction. Learners can also help one another notice a difference. These contributions become a pedagogical capacity of the arrangement only through relations that make them jointly usable. A room containing knowledgeable people, records and willing learners may fail if there is no suitable occasion for engagement or if the records presuppose distinctions the learners are trying to acquire. The relevant unit is the organized formation relation, with individual capacities retained where they explain its operation.

This account leaves room for directed instruction, observation, imitation, peer interaction and incidental learning. It imposes no requirement that every formative episode include overt correction. A quiet adjustment can matter, and repeated unproblematic use can also contribute. When a stronger claim includes the capacity to recognize and address a mistake, however, the inquiry needs evidence of a suitable possibility of response. Absence of recorded correction cannot alone establish either perfect mastery or an absence of learning.

A recurrent danger is to treat a newcomer's difficulty as evidence that the person lacks an underlying ability when the arrangement has withheld a relevant opportunity. The reverse inference is equally weak: the existence of a supportive arrangement does not establish that the target has been learned. Separating capacity, access, enacted participation and achieved formation allows both possibilities to be investigated. The separation preserves responsibility for the arrangement without converting every unsuccessful performance into an institutional failure.

3.6 Transformative Reproduction

Reproduction raises an apparent tension. A successor must retain enough for a continuation claim to be informative, yet a linguistic practice capable of further expression cannot be exhausted by copying earlier performances. The tension is resolved locally by specifying the relation that continues. If the target concerns the ability to distinguish a completed action from an unresolved future condition, a successor can express that distinction through different words and examples. If the target instead concerns command of a particular historical register, those changes may be material. The declaration of a target is not a judgment that either aim deserves universal priority.

Even a target initially stated with care can change through learning. A newcomer may discover that the accepted examples combine two different kinds of uncertainty. The practice then faces several possibilities. It can retain the older profile for a restricted task, divide it into two profiles, or replace it with a revised distinction. Each possibility supports a different continuation claim. Saying simply that competence was reproduced would hide the decision; saying that any changed profile ends the practice would hide the historical work through which the revision became intelligible.

The appropriate comparison records both retained commitments and changes of target. Suppose an earlier profile required asking whether the return record was complete. A later profile distinguishes completeness from the recipient's ability to interpret the record. Someone who learned the earlier profile may possess one component of the later competence. That partial relation can be stated positively. It neither licenses full equivalence nor makes the earlier learning irrelevant. A learner who introduces the second distinction also changes the task for later participants, so reproduction becomes a history of revised formative possibilities rather than a sequence of identical outputs.

Table 1: Distinct Reproductive Claims. The comparison concerns a declared competence profile and formative arrangement, rather than an unrestricted ranking of speakers or communities.

Claim	Selected Achievement	Further Question
Present competent use	Current participants realize the selected profile.	Can another participant acquire it through available relations?
Participant formation	A further participant acquires the selected profile.	Can the arrangement continue or renew its formative supports?
Support renewal	Required roles and resources remain accessible or receive adequate replacements.	Do the renewed relations actually support another formation history?
Repeated reproduction	Further formation occurs across specified changes of participants and supports.	Which future changes remain outside the established scope?

This account needs a constraint against retrospectively protecting every outcome as successful transformation. A proposed change should identify the problem it addresses, the previous distinctions it retains or abandons, the consequences of that choice and the evidence of subsequent understanding. Participants may reasonably disagree about whether the new profile serves their purposes. The descriptive inquiry should retain that disagreement. Historical descent can establish how the revision arose while leaving open whether it improves the practice or warrants replacing the previous target.

Nor should the criterion of success belong exclusively to the established facilitator. The facilitator's expertise may explain why an objection initially misdescribes the task. The learner's experience may disclose an exclusion that the established account overlooks. Assessing these possibilities requires attention to the content and consequences of their reasons. It cannot be settled by treating either position as automatically authoritative. The later normative discussion therefore examines how a practice can coordinate instruction while keeping this comparison practically possible.

The formal account will preserve the same asymmetry. A correspondence can transport selected task distinctions across a changed carrier without mapping every old task onto every new one. A missing correspondence is a limit on the comparison, not a proof that nothing was learned. A transported task can be preserved while additional tasks are newly formed. This gives transformation a precise place inside a bounded reproduction inquiry without turning the paper into a theory of every possible semantic innovation.

3.7 Renewable Formation Arrangements

A further participant can acquire the target profile while the arrangement that formed that participant becomes less able to continue. A facilitator may supply extensive unpaid preparation for one cohort, use an irreplaceable collection of examples, or bear all interpretive work. Formation can succeed for that cohort even if those supports are subsequently unavailable. The achievement is substantial, but a claim of continuing reproduction needs additional premises about the renewal or substitution of its supports.

Conversely, renewal need not reproduce the original allocation of work. New participants might maintain examples, make questions accessible through another language, create further occasions for practice, or help another learner compare cases. A separate institution might provide continuing support. The relevant continuity can be achieved through changed relations. It therefore matters which role must remain available, who can occupy it and whether the proposed replacement is practically accessible.

Table 1 distinguishes the claims that are easily compressed into a declaration that a practice is being reproduced. Each row identifies an achievement and the additional question it leaves open. The rows are not a universal stage sequence: support can be prepared before a learner arrives, and an experienced learner can simultaneously assist others.

The strongest row still concerns a bounded history or a conditional capacity. Finite success does not guarantee indefinite renewal. A later change can remove a required relation, alter the target or reveal a burden the earlier arrangement could absorb only temporarily. The philosophical issue is the adequacy of the stated continuation claim, not the discovery of an entirely self-sufficient linguistic system.

3.8 Reproductive Success Criteria

The resulting account assigns reproductive capacity to an arrangement relative to a competence profile, a class of prospective participants, selected formation conditions and a declared continuation scope. A warranted judgment should identify what can be learned, through which relations, with what remaining dependencies and across which changes. Those qualifications make the claim open to counterevidence. They also permit an explanation of partial success when one capacity becomes available while another remains absent or uncertain.

The framework inherits the prior Generative Relational treatment of competence reproduction and teachable methods [5, pp. 41–44]. Its specifically linguistic task is to identify the interpretive and practical discriminations whose formation connects current use with further participation. It does not attribute novelty to the general discovery that learning depends on social relations. Established learning accounts already investigate those relations; the present distinctions organize a narrower question about their continuation and transformation.

Finally, reproduction is initially descriptive. A practice can reproduce misleading classifications, exclusive roles or burdensome expectations. Failure to reproduce every inherited feature can sometimes be welcome, and a participant can reasonably decline a teaching role. Evaluating those possibilities requires reasons about the selected practice and the persons affected. The formation account provides information relevant to that judgment; the occurrence of further competent performance does not supply the judgment by itself.

4 Historical Participant Formation

Participant formation concerns a transformation of situated capacities and relations across a history. A Generative Relational account must explain what that historical description contributes beyond a list of learner attributes and instructional inputs. This section develops the contribution through manifestation, historically extended participants, changing formative interfaces and generative return. Its method is conceptual reconstruction grounded in prior manuscripts. The aim is to specify the organization a formal representation should retain while leaving learning mechanisms open to independent investigation.

4.1 Relational Manifestation

A displayed answer is an occurrence within a particular encounter. Its interpretation as an exercise of competence requires a further account of what the participant can distinguish and do. Its interpretation as evidence of reproductive capacity requires a further account of how such competence can be formed and renewed. The three claims concern connected objects, but one observed event can leave the latter two substantially underdetermined. GR's distinction between manifestational and relational reality supplies an inherited vocabulary for keeping those inferences explicit [7, pp. 17–19].

The repair-circle illustration makes the difference consequential. Suppose two newcomers both request a missing record. In one stipulated history, a facilitator quietly identifies each omission immediately before the request. In the other, the newcomer has learned which omission matters, recognizes it in a further example and knows how to reopen an earlier description. The requests can be verbally identical. Their sustaining relations differ in a way relevant to an assessment of what would happen when that cue disappears. The first participant may nevertheless possess other substantial capacities, including an ability to coordinate appropriately with the facilitator. The error would be to enlarge that achievement into a different competence claim without evidence.

Relational reality adds an explanatory question: does an organization acquire enough historical stability and efficacy to condition later encounters? A formed practice can make certain questions intelligible, preserve usable examples and sustain expectations about who will respond. Its effects can persist despite changes in the words used on a particular occasion. Such efficacy supports treating the arrangement as an analytical object, while its specific linguistic interpretation still depends on the distinctions and conse-

quences established by the inquiry. A persistent allocation of roles does not become adequate pedagogy merely because it is persistent.

This distinction also leaves causal explanation and normative assessment separate. An authoritative correction can determine what a learner says without being well supported. A justified objection can receive little uptake. A formation history should retain both the grounds offered and the organization through which they become consequential. Neither the frequency of an utterance nor the strength of its institutional support settles its correctness. Prior normative-formation work explicitly separates formative influence from validity and investigates how earlier influence changes later conditions of participation [5, pp. 7–9, 11–14].

4.2 Historically Extended Participants

The participant in this account is a historically extended being, whose present possibilities are partly constituted through earlier encounters, embodied practices, material relations and social positions. Treating that participant as a GR spacetime object preserves situated occurrence and historical organization. It does not require a smooth manifold, a universal clock or the reduction of personal agency to a trajectory fixed in advance. Different dimensions of a person's history matter to different explanatory questions; a bounded inquiry retains those relevant to the selected formation claim.

This matters even where a compact state description predicts the next response. Two learners can have similar current response profiles while possessing different relations to the formative arrangement. One may know how to seek another explanation, retain access to earlier examples and be recognized as someone whose correction deserves attention. Another may produce the same answers while depending on a single intermediary. If the question concerns continued learning after that intermediary departs, the omitted history and its continuing relations can become explanatory. The purpose of historical extension is to retain such dependencies, rather than to demand a complete biography for every linguistic judgment.

The history of the arrangement also includes its relations to several participants. Learning need not align with one institutional timetable. An earlier question can receive an answer much later; a privately noticed contrast can enter shared discussion through another learner; a record made for one encounter can become useful elsewhere. A historically situated representation can retain these partly ordered connections without placing all participants on a common developmental ladder. It can also distinguish historical occurrence from present interpretation. An earlier correction remains part of the record when participants subsequently judge it mistaken.

A labelled two-complex is one possible representation of this organized history. Events, continuations and declared composition relations can receive different incidence roles. The modelling interpretation must specify those roles; a person does not automatically become a vertex, and a face does not automatically represent agreement. A participant's involvement may occupy a historically extended subconfiguration. The earlier language papers adopt this object–representation distinction [6, Section 4].

Further spin-foam structure requires additional representation labels, compatibility data and weights with an identified analytical purpose. Introducing that structure would not by itself establish that the represented learning has occurred. Prior work separates substrate combinatorics, accessible observations and the bridge to linguistic or developmental interpretation [8, pp. 5–6]. The present philosophical claim is therefore conditional: a history-bearing representation is useful where it retains relations required by an explanation that a more compressed description omits. Its adequacy remains accountable to the explanatory task.

4.3 Changing Formation Interfaces

A formative interface is the organized relation through which attempts, responses, examples and opportunities become mutually usable. The term does not presume a technical device or a sharp boundary between an individual and a social exterior. A shared diagram can contribute to a learner's present discrimination, while the skill of using that diagram is itself historically formed. At another scale, the diagram, learner and practice of correction can be represented as one coupled configuration. The distinction

between capacity and condition remains useful when its boundary is stated.

Formation can transform this interface. Suppose a newcomer asks whether “return-ready” covers a case that experienced members had habitually handled without comment. The resulting comparison could clarify an existing distinction, disclose that the inherited example was misleading, or produce a new way of recording uncertainty. The newcomer then contributes to the conditions of later learning. The event is not adequately described as the receipt of an unchanged item when it also changes which examples are available and how later questions receive a response.

The stronger interpretation still needs evidence. A new explanation can make an old distinction more accessible without changing its practical content. A learner’s alternative wording can express an unchanged understanding. An institution can announce participation while retaining sole control over the consequences of every question. The relevant investigation traces whether the contribution modifies the examples, responses, opportunities or criteria that later participants actually encounter. Recursive description identifies a possible mechanism; it does not establish that the mechanism operates in every teaching encounter.

This recursive organization is inherited within GR. Normative Formation already treats a formed distinction as a later source and describes transformation among conceptual, institutional, material and interpretive contributions [5, pp. 11–14]. The present application asks how a learner can become a contributor to a specifically linguistic formative arrangement. It retains the learner’s agency without assuming that every contribution is novel, successful or entitled to adoption. An objection may be mistaken. A proposed simplification may erase a distinction the activity requires. The interface becomes analytically richer when it can represent those differences, rather than treating participation as an undifferentiated beneficial input.

Changed interfaces also complicate a claim of preservation. New participants can make the same selected competence teachable through different examples or roles. They can also alter what competence is being sought. In the former case, a continuation claim needs a supported comparison between differently realized capacities. In the latter, the investigation should identify the revised target. Historical descent alone leaves this distinction open. Reproduction consequently permits transformation while remaining answerable to the content of its declared continuity claim.

4.4 Participant Agency Within Formative Histories

A history-bearing description must retain what participants contribute to formation without treating their actions as the execution of a complete programme already stored in the arrangement. An explanation, refusal or question can alter the practical situation to which the next action responds. The teacher and learner can consequently change together, although the changes need not be equal or equally voluntary. A representation that fixes the learner as the sole changing component would miss this possibility even if it included a long sequence of interactions.

Consider the difference between asking for an example and challenging the classification that makes the example relevant. The first request can be answered within the existing teaching repertoire. The second can require participants to reconsider what they are trying to teach. An experienced facilitator may initially hear the challenge as a failure to understand, while the learner may have identified a real ambiguity. Subsequent inquiry can vindicate either interpretation or disclose a third. The historical record should preserve the question, the response and the later reasoning without making the final assessment the meaning of the earlier event by definition. That separation lets formation include correction of the formative arrangement itself.

This possibility is stronger than a rule saying that learners can select among predefined responses. Selection can exercise meaningful agency, but it does not exhaust the ways a participant can change a practice. A new contrast can require a new task, alter the domain of an existing operation or make a previously unavailable comparison intelligible. The inherited field formalism permits changing operation families and set-valued descendant relations [9, pp. 55–56]. In this application those formal possibilities need a record of the linguistic work that motivates the new operation. Merely enlarging a symbol set supplies no account of how the new contrast came to matter.

Agency is also compatible with refusing the role that an arrangement makes available. A learner may become competent at a task and decline an invitation to teach it. The refusal changes which reproductive histories are practically available without negating the earlier formation. A model can represent the new operation domain, but the participant's reasons remain part of its interpretation rather than an error term. This point becomes especially important when a mathematical recurrence requires fresh participants at every stage. Such a requirement is a declared conditional resource of the proof, not an entitlement to actual people's continued participation.

The same restraint applies to institutional agency. A programme can change admission rules, teaching materials or the status of a correction. Its capacity to do so can help explain subsequent formation. Whether its action is justified is a further question, including whose alternatives become unavailable. A coherent historical model can represent a coercively effective programme as well as a defensible one. Its descriptive generality is useful only if the paper continues to distinguish those evaluations. The present inquiry therefore retains reasons, standing and consequences alongside the operational records where they matter to the declared claim.

4.5 Historical Relevance Selection

Treating a participant as historically extended does not require storing every event of the person's life. The problem is to justify the selection of history relative to a question. Three relations can make an earlier event relevant: it can have contributed to the present capacity, remain operative through a retained support, or provide evidence for a continuity claim. These roles can coexist, but none automatically implies the others. A forgotten explanation may have contributed to learning; a retained record may remain useful even if its original author no longer participates; a certificate may document an episode while failing to support any present formation operation.

This distinction supplies a reason for retaining some apparently redundant details. If two arrangements have the same number of competent participants, their histories can still differ in whether several people learned to prepare examples or one person continues to supply every example privately. The difference becomes relevant when that person plans to leave. Retaining it allows an inquiry to assess the actual succession claim. Recording unrelated biographical detail would add volume without increasing that explanatory capacity. Historical comprehensiveness is therefore relative to the dependencies and comparisons at issue.

There is an objection that an adequately enriched current state could retain all these relevant dependencies without being called a history-bearing object. The objection is well founded at the level of predictive representation. Nothing here proves that one notation is uniquely adequate. The historical description contributes when it preserves provenance, the order of changes, or the distinction between retained occurrence and present interpretation that the alternative state representation discards. If the enriched state already retains those relations, the accounts may differ mainly in organization. The stronger claim of an irreducible collective or historical substance does not follow. The research obligation is to show what a chosen compression loses, rather than to reject compression in advance.

4.6 Generative Return

The formation relation can leave participants differently able to contribute to its continuation. One learner might become able to maintain examples, another to help interpret a difficult question, and another to recognize when the existing arrangement needs outside assistance. These capacities can renew the practice without making each person a self-sufficient teacher. The relevant return concerns how the activity of formation changes the participants' later possibilities and the support organization that they can help sustain.

Prior GR work already names *generative return* as a provisional evaluative consideration. Its examples include making expert methods teachable, preserving reasons for later interpreters and allowing initial educational dependence to contribute to increasing intellectual independence [5, pp. 40–43]. The concept therefore belongs to the inherited foundation. Applying it here requires a linguistic account of

what becomes available: a comparison, an interpretive skill, access to a role, a usable correction practice or the ability to help form another participant. It should not become a general quantity that ranks all these achievements on one scale.

Dependence itself needs a differentiated assessment. Continuing reliance on colleagues, shared records or specialized expertise can be constitutive of competent participation. An aim of eliminating those relations would misdescribe the practice. The more specific question concerns whether a participant gains suitable capacities to use, question, supplement or help renew them. A facilitator can exercise warranted leadership while making the grounds of correction accessible. A decentralized arrangement can leave newcomers dependent on tacit gatekeeping. Organizational form alone decides neither case.

Nor does successful reproduction establish an unlimited obligation to continue it. Learners may wish to use a capacity without becoming mentors; facilitators may have commitments that limit their availability. A renewable arrangement can distribute support across institutions and participants without assigning each speaker a duty to perpetuate the practice. Assessment must consider the purposes of the selected competence, actual undertakings, burdens and possible alternatives. The present paper identifies consequences relevant to those reasons while leaving a comprehensive theory of educational or preservation duties outside its scope.

These distinctions constrain the formal investigations that follow. A field of selected activities can illuminate their mutual support without becoming a scalar measure of linguistic competence. A historical operation can change the participants, supports or realizable operation families. An algebra can distinguish an executable correction from a merely recorded permission, while its composition rules alone establish neither learning nor legitimate authority. The constructive task is to connect these representations through declared interpretive maps and limited claims. Their value lies in making the conditions of reproductive continuation examinable without erasing the historical agency and normative questions that motivated the inquiry.

5 Formative History Architecture

A formative history represents how particular participants, examples and supports become related over time. The purpose of this section is to specify that object before choosing numerical or algebraic specializations. The construction first identifies a represented historical carrier, then defines competence tests and formative operations, and finally distinguishes completed formation from renewable formative opportunity. Its claims concern the selected representation; the existence and interpretation of its labels require independent inquiry.

5.1 Represented Historical Carriers

Let X denote a selected history-bearing relational formation. Its scope includes participants, linguistic encounters, retained examples, correction episodes and support arrangements relevant to the question. A representation of X may use a labelled two-complex K with vertices V , edges E , faces F , attachment maps and labels. Vertices can identify event positions, edges selected relations between them, and faces declared interaction episodes with their boundary relations. Faces acquire their meaning through a specification of those attachments; an arbitrary triangle drawn between three agents supplies no such specification. An event precedence relation is additional data, not an automatic consequence of the undirected incidence structure.

An active configuration ω consists of the retained history K , an active interface B , participant-position records Π , and the selected operation and assessment data $\mathcal{N}$. Equation 1 states this bookkeeping convention.

$$\omega = (K, B, \Pi, \mathcal{N}) \in \Omega. \quad (1)$$

The class Ω in Equation 1 permits different complexes and participant families. It is an investigator's class of descriptions, not a single fixed social state space assumed to govern every possible history. The

active interface records who can presently interact, which materials are usable, and which requests for correction or support can receive a response. It need not coincide with the topological boundary of K .

A participant is identified through a historical position and its continuations. Two positions carrying the same role name need not identify the same person or the same competences. Conversely, the same person can occupy different roles across episodes. Retaining these distinctions avoids interpreting participant renewal as a mere permutation of timeless points. The selected two-complex does not itself establish a physical spacetime ontology. A spin-foam realization would add representation labels, amplitudes and further boundary data with a justified role; none is required for the argument here.

5.2 Competence Profiles And Assessment

Choose a participant position p and a finite or explicitly delimited family Q of context-sensitive tasks. Tasks may concern discriminating two uses, producing an appropriate contribution, explaining an application or repairing a misunderstanding. Each $q \in Q$ is represented by a partial assessment $q_p : \Omega \rightarrow \{0, 1\}$. A value one records satisfaction of the declared test, zero records failure, and undefinedness records lack of applicability or insufficient assessment information. The associated recorded profile is given in Equation 2.

$$C_Q(p, \omega) = \{q \in Q : q_p(\omega) \text{ is defined and } q_p(\omega) = 1\}. \quad (2)$$

Equation 2 is a profile of stipulated assessments, not the definition of competence as a metaphysical property. It retains passing tests only; the partial assessments must also be kept when distinguishing a failed test from an undefined one. A substantive attribution can require evidence across occasions, transfer within a specified range, responsiveness to reasons and an account of residual support. A single positive record need not supply that evidence. The tests themselves can misdescribe the practice or privilege an unjustified standard.

A formation target specifies a profile Q , a novice position p , contexts of exercise and allowed support conditions. Support is named positively: a dictionary, interpreter, partner or accessible interface may be part of competent activity. Removing every support would change the object. When an inquiry seeks reduced dependence on a particular facilitator, it must identify that dependence and the replacement supports permitted. Teaching capability requires different tasks, such as selecting instructive contrasts or responding to another learner's error. It cannot be inferred by relabelling the same profile.

5.3 Situated Formative Operations

Let β be an operation schema, such as offering an example, arranging a practice opportunity, challenging a correction or preparing a successor facilitator. Its realization at ω is a partial transformation selected by the schema. Equation 3 separates selection from execution.

$$F_\beta : \Omega \rightarrow \text{PMap}(\Omega), \quad T_\beta^{[\omega]} = F_\beta(\omega), \quad \omega' = T_\beta^{[\omega]}(\omega). \quad (3)$$

Here $\text{PMap}(\Omega)$ denotes the selected class of typed partial transformations on Ω . The final expression in Equation 3 exists only when both the schema and its selected transformation are applicable. Concrete applicability includes the conditions represented at the chosen resolution. It does not prove that participants know how to select the operation, consent to it or possess unrecorded material resources. The preservation taxonomy already separates these levels of availability [10, Section 7].

A learning operation can alter the participant record, available examples and future operation domains together. For example, a newcomer's successful challenge can create a new contrast and make later correction depend on it. The result is stronger than increasing a weight on an unchanged list of responses. Where operation families change, an old schema needs a declared descendant correspondence to a new family; there may be several descendants or none. This is an application of the inherited changing-operation architecture [9, Section 9], not a new general definition of generativity.

The realized history retains intermediate configurations as in Equation 4.

$$h = (\omega_0, \beta_1, \omega_1, \dots, \beta_n, \omega_n), \quad \omega_i = T_{\beta_i}^{[\omega_{i-1}]}(\omega_{i-1}). \quad (4)$$

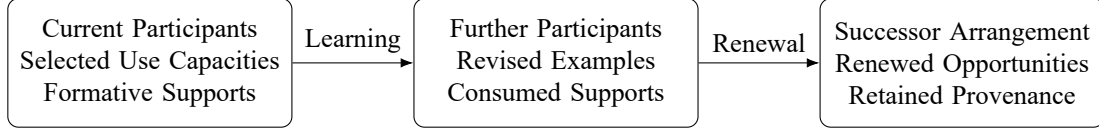


Figure 1: Selected Formative History. A successful learning episode and renewal of the relations supporting another episode are separately represented. Neither arrow requires every learner to become a teacher.

Equation 4 distinguishes the participants who learned, the contributions they made and the supports consumed from a final competence report. The composite endpoint transformation intentionally forgets some of this information. Equal endpoints can therefore leave different histories of exclusion, dependence or formative contribution.

5.4 Historical Anchors And Renewed Boundaries

In an anchored representation, a retained historical subcomplex K_- embeds into later complexes through $\iota_i : K_- \hookrightarrow K_i$. A continuation $f_{ij} : K_i \rightarrow K_j$ preserves incidence and the historical-record labels declared fixed by the representation, and preserves its anchor when Equation 5 holds.

$$f_{ij} \circ \iota_i = \iota_j. \quad (5)$$

Equation 5 is inherited from the power formalism's anchored-extension regime [9, Section 8]. It permits new participants and operation domains outside the retained anchor. It does not freeze the present interpretation of earlier events. A later participant can challenge an old correction while the record of that correction remains intact.

Figure 1 distinguishes three successive formation boundaries. Arrows mean realized extensions with preserved provenance; they do not mean that every event has a single cause or follows one universal clock.

5.5 Competence Target Correspondence

Historical preservation of an event record does not settle which competence target a later arrangement continues. The target may be expressed through new words, accessible materials or a divided set of tasks. The comparison therefore needs a relation between task descriptions in addition to the anchor maps. The following restricted construction makes that burden explicit without defining learning as invariant assessment.

Suppose Q and Q' are source and successor task families, and let $Q_0 \subseteq Q$ be the tasks for which the inquiry supplies a correspondence. Choose an injective map $\theta : Q_0 \rightarrow Q'$. Injectivity represents a comparison that keeps the selected source tasks distinguishable; a many-to-one translation would support a weaker target. Let $A \subseteq \Omega$ be the family of source configurations where a partial comparison $\Phi : A \rightarrow \Omega'$ and participant correspondence $p \mapsto p'$ are defined. Here Ω' is a declared successor description class. The stipulated assessment-compatibility condition is Equation 6.

$$q'_{\theta(q),p'}(\Phi\omega) = q_{q,p}(\omega) \quad (q \in Q_0, \omega \in A), \quad C_{\theta(Q_0)}(p', \Phi\omega) = \theta(C_{Q_0}(p, \omega)). \quad (6)$$

The notation $q_{q,p}$ in Equation 6 names the partial assessment for task q and participant p ; $q'_{\theta(q),p'}$ names its successor. Equality of partial assessments includes agreement of definedness. The second identity follows from the first: a corresponding task belongs to the successor passing profile exactly when its source assessment is defined and equal to one.

This elementary transport is a comparison between already described capacities, not a law requiring a novice's assessments to remain unchanged during learning. It might compare one participant's supported competence before and after an interface replacement, or corresponding achievements of different cohorts. Applying it to a novice before and after instruction would normally be inappropriate

because learning changes the relevant assessment profile. The map Φ and the actual formative operation must therefore retain their separate types and interpretations.

The construction gives three informative outcomes. Agreement on Q_0 supports the retained comparison, while tasks in $Q \setminus Q_0$ remain outside it. A new task in $Q' \setminus \theta(Q_0)$ can register an additional achievement without being mislabeled as a recovered old one. Failure of definedness leaves the comparison unresolved rather than recording a failed competence. These outcomes follow from the specified task carriers; their linguistic warrant depends on whether θ really preserves the relevant distinction and support conditions.

For the learning-circle illustration, the source task may ask whether a record contains all specified information. A successor arrangement can retain that task and add a task concerning whether another participant can interpret the information. Matching success on the first task says nothing about success on the second. Conversely, a source participant's failure on the added task need not show that the earlier task was never learned. The correspondence makes the claim more precise by retaining both continuity and enlargement.

The most demanding part of this comparison is usually interpretive rather than algebraic. Identical task wording can acquire a different role when the recipient, purpose or consequence changes. Different wording can preserve a relevant distinction. The empirical inquiry must therefore explain the examples, allowed assistance, error cases and uptake through which the task correspondence is defended. Equation 6 records the resulting commitment and its consequence; it does not manufacture that correspondence from a matching string.

The distinction between comparison and execution is inherited from the field-power account's separation of interface transport and substantive dynamical action [9, pp. 33–35]. The preservation taxonomy additionally shows why separately retained capacity and support can fail to retain their joint operation [10, Section 8]. A successor can possess a suitable explanation and an available learner while lacking an interpretable occasion on which the two become related. In such a case task comparison and resource inventory can both succeed while the formative word remains undefined. The operator domain must retain the missing coupling.

The third boundary in Figure 1 is the distinctive object of the reproductive inquiry. It can retain a role pattern with different people, materials and legitimate standards. A repeated role pattern alone does not establish that concrete operations realizing it remain available. The field model next investigates one possible support mechanism; the algebra later returns to this execution gap.

6 Coupled Reproduction Fields

A field model can investigate how activities supporting participant formation coexist, reinforce one another and respond to a loss of support. Its role here is conditional: the model supplies a substantive dynamical law for selected activities, while historical and empirical analysis must justify their identification. This section defines the carriers and interaction, establishes global forward evolution, and specifies how regional descriptions retain their interfaces. Section 7 then examines the resulting regimes.

6.1 Activity Carriers

Select a finite active-site family $I = \{1, \dots, m\}$ from the historical representation. A site is an analytically distinguished learning setting or activity interface. It need not be a geographical point or an individual. Symmetric weights $w_{ij} = w_{ji} \geq 0$, with $w_{ii} = 0$, specify exchange between sites. They are model data requiring an interpretation such as shared practice opportunities or available facilitation. The graph Laplacian L is defined by Equation 7.

$$(Lr)_i = \sum_{j=1}^m w_{ij}(r_i - r_j), \quad r \in \mathbb{R}^m. \quad (7)$$

Equation 7 uses the active interface graph, not every edge in the historical complex. Old events need not diffuse. The graph is a deliberately restricted part of the more general architecture.

Three nonnegative fields $x, y, z \in \mathbb{R}_{\geq 0}^m$ describe amplitudes of selected operational activities: supported participation opportunities, responsive guidance or repair, and renewal of enabling support. An application would first identify countable or otherwise observable activities and a time window, then defend a normalization of their intensities. For example, the participation channel could record usable scheduled encounters rather than enrollment; the guidance channel could record opportunities to resolve a specified class of misunderstanding; the renewal channel could record support work that creates later opportunities. These examples specify possible measurement obligations. No such dataset or calibration is supplied here.

The fields are neither individual competence scores nor quantities of meaning. Guidance can include tacit demonstration, responsive pacing and correction embedded in activity. A zero value of y means absence of the selected modeled channel, not the universal absence of learning. Separate normalization permits dimensionless variables; it does not make the activities substantively identical. The symmetric coefficients used below are a tractable specialization, not an empirical assertion that the three channels share losses or costs.

6.2 An Explicit Nonlinear Interaction

Let $a, c > 0$ describe loss and large-amplitude saturation in the selected units, $g \geq 0$ describe reinforcing interaction, and $D \geq 0$ describe interface exchange. A chosen potential functional is given in Equation 8.

$$\mathcal{E}(x, y, z) = \frac{D}{2} \sum_{r \in \{x, y, z\}} r^T L r + \sum_{i=1}^m \left[\frac{a}{2} (x_i^2 + y_i^2 + z_i^2) - g x_i y_i z_i + \frac{c}{4} (x_i^4 + y_i^4 + z_i^4) \right]. \quad (8)$$

The quadratic graph term in Equation 8 penalizes differences across the declared exchange interfaces. The cubic interaction represents mutual reinforcement: sustained activity in two channels can help reconstitute the third. The quartic terms bound that reinforcement. The functional is an analytical potential, not welfare, instructional quality, thermodynamic energy or a total social resource budget.

Assume Euclidean gradient evolution with a common time scale. Differentiating Equation 8 gives the law in Equation 9.

$$\begin{aligned} \dot{x}_i &= -D(Lx)_i - ax_i + gy_i z_i - cx_i^3, \\ \dot{y}_i &= -D(Ly)_i - ay_i + gx_i z_i - cy_i^3, \\ \dot{z}_i &= -D(Lz)_i - az_i + gx_i y_i - cz_i^3. \end{aligned} \quad (9)$$

Equation 9 is a modeling assumption with a transparent derivation from the proposed potential. It is not derived from language socialization research or from the existence of a two-complex. Delays, asymmetric influence, heterogeneous rates, external inputs and changing interfaces can all require another law. Selecting a gradient law excludes sustained cycles in the potential and substantially restricts the possibilities. Its analytical purpose is to expose one cooperative renewal mechanism in detail.

The triadic potential has a useful distinction from a rule requiring all three channels to be positive before anything can happen. When $z_i = 0$ and $x_i, y_i > 0$, the local contribution to $\dot{z}_i$ is positive. Two retained activities can therefore regenerate a missing third within this model. One isolated positive channel at an isolated site cannot do so. This dependence is a chosen mechanism, not a general criterion of linguistic reproduction.

6.3 Forward Evolution

Proposition 6.1 establishes that the selected law has a coherent forward evolution. The result licenses the later calculations; it does not validate their empirical interpretation.

Proposition 6.1 (Positive Global Evolution). *For a finite symmetric nonnegative interface graph and $a, c > 0$, $g, D \geq 0$, each nonnegative initial condition of Equation 9 has a unique nonnegative solution for all $t \geq 0$. Its potential satisfies Equation 10.*

$$\frac{d\mathcal{E}}{dt} = - \sum_{i=1}^m (\dot{x}_i^2 + \dot{y}_i^2 + \dot{z}_i^2) \leq 0. \quad (10)$$

Proof. The polynomial vector field is locally Lipschitz, giving local existence and uniqueness. At a boundary coordinate $x_i = 0$ with all coordinates nonnegative, its derivative is $D \sum_j w_{ij} x_j + g y_i z_i \geq 0$. The same argument applies to y_i, z_i , so the positive orthant is forward invariant. The gradient identity gives Equation 10 by the chain rule.

For $R_i = x_i^2 + y_i^2 + z_i^2$, the arithmetic–geometric mean and Cauchy inequalities give $|x_i y_i z_i| \leq R_i^{3/2}/(3\sqrt{3})$ and $x_i^4 + y_i^4 + z_i^4 \geq R_i^2/3$. The graph term is nonnegative. Consequently $\mathcal{E}$ is bounded below by the sum of $aR_i/2 - gR_i^{3/2}/(3\sqrt{3}) + cR_i^2/12$. The positive quartic term makes this bound coercive on the finite-dimensional carrier. A nonincreasing potential confines the solution to a bounded closed sublevel set. Local existence therefore extends to every finite future time. $\square$

Equation 10 establishes an invariant bound and a direction of potential change, not monotone improvement in learning. A solution can decrease $\mathcal{E}$ while every represented activity decays. Even persistent positive activity could implement a poor curriculum or unwarranted correction regime. Those possibilities are outside the potential’s definition and remain open to philosophical and empirical evaluation.

6.4 Regional Interfaces

The field-realization protocol inherited from the power manuscript requires compatibility of carriers, restrictions and dynamics [9, Sections 5–6]. A simple restriction of the numerical values to a site subset does not generally supply an autonomous local law. Let $U \subset I$, and let $U^c = I \setminus U$. The actual restricted x dynamics contains the boundary contribution in Equation 11.

$$\dot{x}_U = -DL_{UU}x_U + b_U^x - ax_U + g(y_U \odot z_U) - cx_U^{\odot 3}, \quad b_U^x = -DL_{UU^c}x_{U^c}. \quad (11)$$

Here $\odot$ denotes componentwise multiplication, and matrix subscripts indicate the corresponding rows and columns. Analogous expressions hold for y, z . Equation 11 retains both the full-graph diagonal block and the time-dependent exterior input. Replacing L_{UU} by the induced-subgraph Laplacian and setting the input to zero changes the system. It can erase the very dependency whose reproduction is at issue.

A regional record therefore includes internal values, the declared boundary trace and a common normalization. Two overlapping records must agree on shared coordinates and the total exchange contribution induced by the same global history. Their raw boundary inputs need not coincide: an edge can be internal to one region and external to the other. Restricting both descriptions to the same subregion and recomputing its boundary trace gives a common comparison. This gives an explicit finite consistency requirement. It establishes neither a unique reconstruction of an unknown exterior nor a general sheaf-gluing theorem for language. Its practical implication is specific: continued local activity can depend on support outside the selected observation region.

6.5 Regional Potential Balance

The regional distinction has a further mathematical consequence. The potential decrease proved for the full graph does not automatically become a potential decrease for a region treated as an open subsystem. This matters when sustained local activity is used as evidence that a selected setting renews its own

formative support. The following calculation specifies the missing exchange term within the chosen finite model.

For $U \subseteq I$, let L_U^{int} be the Laplacian containing only edges with both endpoints in U . Define $\mathcal{E}_U$ by the same formula as Equation 8, using this internal Laplacian and summing the local potential only over $i \in U$. Write $\mathbf{r}_U = (x_U, y_U, z_U)$ for the three restricted fields together. For each channel $r \in \{x, y, z\}$ and $i \in U$, put $q_i^r = D \sum_{j \notin U} w_{ij}(r_j - r_i)$, and collect these terms as $\mathbf{q}_U$. The full dynamics then gives Equation 12.

$$\dot{\mathbf{r}}_U = -\nabla \mathcal{E}_U + \mathbf{q}_U, \quad \frac{d\mathcal{E}_U}{dt} = -\|\dot{\mathbf{r}}_U\|^2 + \langle \mathbf{q}_U, \dot{\mathbf{r}}_U \rangle. \quad (12)$$

Proposition 6.2 (Open Regional Balance). *For a solution of Equation 9, its restriction to U satisfies Equation 12. Its internal potential does not increase whenever the exchange pairing does not exceed $\|\dot{\mathbf{r}}_U\|^2$, but the sign of its rate of change is not determined by the global dissipation statement alone.*

Proof. Split each Laplacian sum into neighbors inside and outside U . The internal terms and the local polynomial equal $-\nabla \mathcal{E}_U$; the remaining terms equal $\mathbf{q}_U$. The chain rule gives $d\mathcal{E}_U/dt = \langle \nabla \mathcal{E}_U, \dot{\mathbf{r}}_U \rangle$. Substituting $\nabla \mathcal{E}_U = \mathbf{q}_U - \dot{\mathbf{r}}_U$ proves the second identity. The stated sufficient inequality follows directly. The exchange pairing has no fixed sign in these assumptions. $\square$

A concrete global solution can exhibit the regional increase. Take two sites with $D = w_{12} = a = c = 1$ and $g = 3$. Initially set all three channels to 0.1 at the selected site and 0.3 at the other. The selected site's intrinsic drift is -0.071 and its incoming exchange is 0.2, giving a local derivative of 0.129 in each channel. Its internal potential therefore has derivative $3(0.071)(0.129) = 0.027477 > 0$. The other site's derivative is -0.257 in each channel, so the full potential has derivative $-3(0.129^2 + 0.257^2) = -0.24807 < 0$. These exact initial derivatives concern an autonomous two-site model. They establish the claimed difference of signs without assuming a constant exterior or assigning the potential an empirical meaning.

Proposition 6.2 is an elementary consequence of the adopted gradient law. Its point here is the interpretive restriction it makes exact. The exchange pairing is not a measure of educational labor, economic subsidy or thermodynamic work. It records how the declared boundary input enters this regional potential balance. An application that associates the boundary with shared guidance or access must independently establish that correspondence. The calculation then prevents an investigator from assigning the full-system Lyapunov property to a region after deleting its incoming relations.

6.6 Externally Supported Activity

A one-site open example makes the attribution issue concrete. Suppose all three local coordinates equal ρ , all connected exterior coordinates equal a prescribed trace $e(t)$, and the total boundary exchange weight is $d = D \sum_{j \notin U} w_{ij} > 0$. The equal-channel sector remains invariant. The local law and the constant input that sustains a prescribed positive level ρ_0 are given in Equation 13.

$$\dot{\rho} = -a\rho + g\rho^2 - c\rho^3 + d(e - \rho), \quad e_0 = \rho_0 + \frac{a\rho_0 - g\rho_0^2 + c\rho_0^3}{d}. \quad (13)$$

The second expression is obtained by setting the first to zero. It is an admissible nonnegative boundary trace only where $e_0 \geq 0$; this is an explicit condition, not guaranteed for every target level. The construction prescribes an open input. It does not assert that this trace is generated autonomously by some unspecified exterior system.

Choose the illustrative dimensionless coefficients $a = c = d = 1$, $g = 3$ and $\rho_0 = 0.1$. Equation 13 gives $e_0 = 0.171$, which holds the local activities constant. If the site is then decoupled, its law becomes $\dot{\rho} = \rho(-1 + 3\rho - \rho^2)$. The smaller positive stationary level is $(3 - \sqrt{5})/2$, exceeding 0.1. The derivative is strictly negative between zero and that level, so the decoupled solution starting at 0.1 decreases to zero. To justify the limit, monotonicity and nonnegativity first give a limit; continuity of the autonomous vector

field requires that limit to be stationary, and zero is the only such point in the attained interval. Thus the sustained open trajectory and the decaying isolated trajectory follow from the same local interaction.

Decoupling differs from retaining the edge while setting the exterior trace to zero. The latter still contributes $-d\rho$ and represents another system. This distinction is often substantively consequential: removing an exchange relation and leaving a relation whose other endpoint no longer supplies the former activity can have different effects. The model establishes the distinction conditionally; determining which change corresponds to a facilitator's withdrawal requires the actual history.

The example supports a positive attribution discipline. Persistent local activity can be accurately described as externally supported activity when the external contribution is part of the declared arrangement. A claim of participant reproduction additionally requires the competence and formation witnesses developed elsewhere in this paper. Persistence should not be reported as evidence of autonomy from that contribution. Extending the region to include the contributor can reveal another support history; it does not automatically terminate the inquiry at a perfectly closed social whole. One selects the region appropriate to the claim and makes its retained dependencies explicit. That is the concrete work performed by a history-bearing regional description in this example.

6.7 Historical Extension

The fixed graph describes a selected interval and activity organization. New participants, settings or task types can require a new graph and new variables. An extension map must then state which old sites are retained, which new sites are introduced and how new activity receives an initial value. A zero coordinate in a universal preallocated vector is not evidence that a later participant or practice was already historically available.

The historical operation of Section 5 can generate a new active carrier. The field law then governs a chosen sector on that carrier. Neither step replaces the other. Field persistence examines an already articulated activity architecture; participant formation can change that architecture and its future operations. This division preserves the contribution of field theory without asking a polynomial on one fixed carrier to generate every new meaning.

7 Reproduction Field Regimes

The nonlinear law permits a precise distinction between the existence of a sustaining regime and the capacity to reach it from a selected condition. This section derives the homogeneous equilibria, checks their stability against all finite field perturbations, compares their potentials and examines interface effects. The results remain conditional on the explicit law and normalization; no empirical critical value for language reproduction is inferred.

7.1 Homogeneous Equilibria

Spatially uniform, channel-symmetric configurations have $x_i = y_i = z_i = s$ for every site. The graph term vanishes, and the full law restricts to Equation 14.

$$\dot{s} = -as + gs^2 - cs^3 = -s(a - gs + cs^2). \quad (14)$$

Equation 14 is a restricted invariant sector of the three-field dynamics, not a replacement of competence by a scalar. The reduction is legitimate because equal coordinates have equal derivatives under the specified symmetric law. Write $\Delta = g^2 - 4ac$. When $\Delta > 0$, the two positive constant amplitudes are stated in Equation 15.

$$s_- = \frac{g - \sqrt{\Delta}}{2c}, \quad s_+ = \frac{g + \sqrt{\Delta}}{2c}. \quad (15)$$

Equation 15 specifies two equilibria in addition to zero. Both are positive because $a, c > 0$. When $\Delta < 0$ there is no positive homogeneous equilibrium; when $\Delta = 0$ the two roots coincide.

For an isolated site, these are all the nonnegative equilibria. If one coordinate of an equilibrium is zero, its equation requires the product of the other two to vanish when $g > 0$; their loss terms then make all coordinates zero. For a positive equilibrium, multiplying the three respective equations by x, y, z gives $ax^2 + cx^4 = ay^2 + cy^4 = az^2 + cz^4 = gxyz$. The function $ar^2 + cr^4$ is strictly increasing for $r > 0$, so $x = y = z$. This argument is a single-site classification. Coupled sites can also admit nonuniform equilibria; the homogeneous calculation does not classify them.

7.2 Full Field Stability

Let $\lambda_k \geq 0$ be an eigenvalue of the symmetric graph Laplacian. At a positive homogeneous equilibrium s , the channel Jacobian has diagonal $-a - 3cs^2$ and off-diagonal entries gs . Its eigenvectors split into the common-channel direction $(1, 1, 1)$ and its two-dimensional orthogonal complement. The full field eigenvalues are given in Equation 16.

$$\begin{aligned}\mu_{\parallel,k} &= s(g - 2cs) - D\lambda_k, \\ \mu_{\perp,k} &= -2gs - 2cs^2 - D\lambda_k \quad \text{with channel multiplicity two.}\end{aligned}\tag{16}$$

Equation 16 uses $a = gs - cs^2$ to simplify the Jacobian. It checks perturbations that change channel balance and spatial distribution, not merely perturbations along the reduced scalar sector.

Proposition 7.1 (Coexisting Stable Regimes). *For $\Delta > 0$, the zero field and the homogeneous s_+ field are locally asymptotically stable in the full finite system. The homogeneous s_- field is unstable. At $\Delta = 0$, the positive constant equilibrium is not locally asymptotically stable, despite having no positive linearized eigenvalue.*

Proof. At zero, the linearization is three copies of $-aI - DL$, with strictly negative eigenvalues. At s_+ , Equation 15 gives $g - 2cs_+ = -\sqrt{\Delta} < 0$; all eigenvalues in Equation 16 are negative. At s_- , the spatially constant mode has $\lambda = 0$ and common-channel eigenvalue $s_- \sqrt{\Delta} > 0$, proving instability. Standard local estimates for a smooth finite-dimensional vector field with a strictly stable linearization give local asymptotic stability in the first two cases.

At $\Delta = 0$, write $s_* = g/(2c) > 0$. The homogeneous equation becomes $\dot{s} = -cs(s - s_*)^2$. A positive initial amplitude just below s_* decreases toward zero; one just above decreases toward s_* . Arbitrarily nearby lower states therefore leave a neighborhood of s_* , so the equilibrium is not locally asymptotically stable. $\square$

Proposition 7.1 distinguishes an existing sustaining configuration from the conditions of its initiation. In the homogeneous sector, an initial value $0 < s(0) < s_-$ approaches zero, while $s(0) > s_-$ approaches s_+ . Equality at s_- remains at the unstable equilibrium. The signs of the cubic between its roots establish these statements directly. The number s_- is a basin boundary only within this sector; it is not a universal threshold for arbitrary three-field states or regions.

The result has a bounded interpretation. Loss of sufficiently much coordinated activity can place the same modeled arrangement in a basin from which its unforced dynamics does not restore the positive regime. Conversely, persistence of an established arrangement does not prove that a small isolated pilot will grow. The difference arises from the chosen reinforcing nonlinearity and loss terms. The mathematics does not determine which real intervention would supply the relevant activities, whether anyone wishes to participate or whether the competence target is justified.

7.3 Potential Selection

Local stability leaves a further question about the potential's global minima. The distinction matters because an equilibrium can be locally stable without having the lowest potential. Neither property is a social evaluation. Proposition 7.2 provides the exact comparison for the nonnegative finite carrier.

Proposition 7.2 (Homogeneous Potential Minima). *On a connected graph with $D > 0$, the global minimum of Equation 8 is the zero field when $g^2 < 9ac/2$, and the homogeneous s_+ field when $g^2 > 9ac/2$. At $g^2 = 9ac/2$, both are global minima. If $D = 0$, sites can choose independently among the global minimizers of the one-site potential; for a disconnected graph the same qualification applies componentwise.*

Proof. At one site fix $R = x^2 + y^2 + z^2$. Nonnegative coordinates satisfy $xyz \leq (R/3)^{3/2}$ and $x^4 + y^4 + z^4 \geq R^2/3$, with simultaneous equality at $x = y = z = \sqrt{R/3}$. Replacing a triple by this equal triple therefore cannot increase its local potential. A global minimum of the one-site potential is either zero or an equal positive triple. The preceding equilibrium analysis identifies the latter candidate as s_+ ; s_- is unstable and cannot be a minimum.

At a positive stationary triple the local potential simplifies as in Equation 17.

$$V(s, s, s) = \frac{s^3}{4}(2g - 3cs). \quad (17)$$

Equation 17 is zero at $s = 2g/(3c)$. Combining this with $a = gs - cs^2$ gives $g^2 = 9ac/2$. For s_+ its value is positive below this parameter value and negative above it. When $\Delta < 0$, zero is the only stationary candidate; the double-root case also has positive nonzero potential. This establishes the single-site minimum throughout the parameter range.

The graph contribution is nonnegative, so the total potential is bounded below by m times the one-site global minimum. A homogeneous choice attains that bound. Equality requires each site to attain its one-site global minimum and each positive-weight edge to join identical triples. Connectivity gives the claimed homogeneous alternatives; absent edges or $D = 0$ give the stated qualifications. $\square$

Proposition 7.2 leaves a parameter interval $4ac < g^2 < 9ac/2$ where the positive constant regime is locally stable although zero has lower potential. Selecting a regime by minimizing $\mathcal{E}$ would therefore misrepresent some stable histories of the actual dynamical law. Nor does lowest potential mean the most desirable practice: the model has no welfare, rights or competence-quality variable. The calculation clarifies a mathematical distinction often obscured by undifferentiated appeals to stability.

7.4 Regional Coupling And Finite Perturbations

Interface exchange can help a depleted site receive activity from another site, but it can also draw activity away from a sustaining concentration. The sign of its practical effect depends on the initial distribution and the nonlinear local dynamics. There is no general monotonicity claim that more connection improves reproduction. Equation 11 identifies the contribution that must be examined for each region.

Figure 2 shows reproducible illustrative calculations with $a = c = 1$. The homogeneous curves use $g = 3$ and initial amplitudes on opposite sides of s_- . The two-site calculations begin with a small active concentration and compare weak with stronger exchange. These are mathematical trajectories, not fitted learning outcomes.

Strict stability is not dependent on exact coefficient equality. For a fixed finite graph and $\Delta > 0$, the positive homogeneous equilibrium has an invertible negative-definite Jacobian. The implicit function theorem therefore continues it to a nearby equilibrium under sufficiently small smooth coefficient perturbations. Continuity retains positivity and eigenvalues with negative real parts. The zero equilibrium also remains stable for small changes of positive loss parameters when no forcing is introduced. This proves a local robustness statement for the mechanism, not the same explicit thresholds for heterogeneous models or robustness to arbitrary changes of learning architecture.

The formal checks compare numerical trajectories with the analytically derived roots and Jacobian, check positivity and potential decline, and verify the regional boundary identity. They include asymmetric initial fields and an unequal-coefficient example. Numerical agreement supplements the proofs; it does not establish their philosophical interpretation. The next section examines a separate issue left open by every positive field trajectory: whether identifiable formative operations can renew an actual sequence of participants and supports.

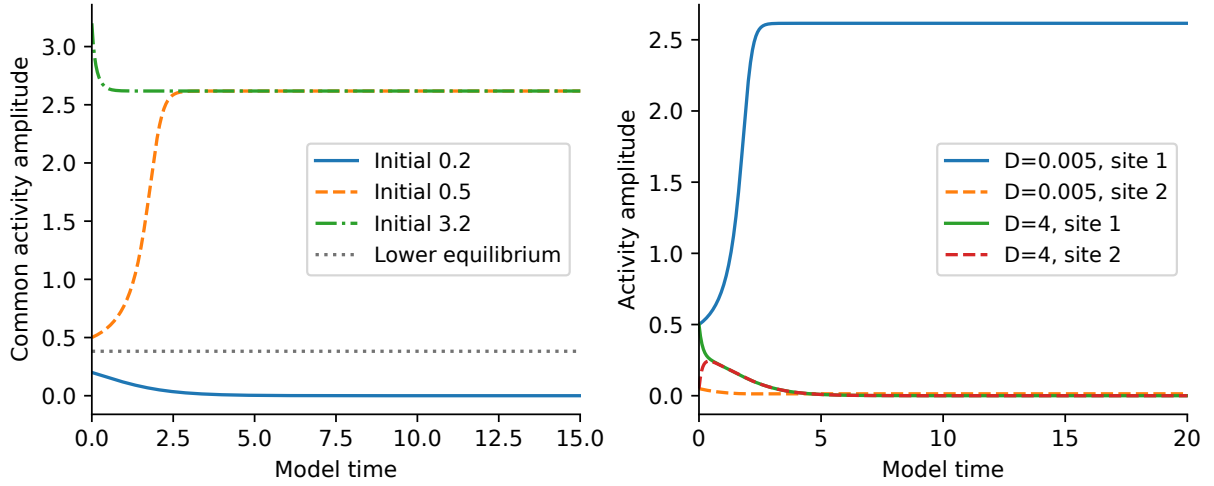


Figure 2: Illustrative Reproduction Field Regimes. The left panel displays homogeneous activity trajectories on opposite sides of the lower equilibrium. The right panel displays selected two-site trajectories under two exchange strengths. The amplitudes concern the specified model channels, not measured competence. Parameters and numerical checks are retained in the source packet.

8 Formative Operation Algebras

The operation algebra addresses a question distinct from positive field activity: whether a sequence that forms a participant leaves an executable arrangement for another sequence. A finite field can remain active while the relevant operation is unavailable, and a successful endpoint can conceal exhausted support. This section inherits a partial-history representation, adds a renewal criterion with explicit execution conditions, and tests it against a finite resource example. Its algebraic claims concern declared formative operations rather than an intrinsic algebra of meaning.

8.1 Historical Operation Representation

Choose a countable family Ω_0 of symbolic configurations of the type in Equation 1. The continuous field states are not all included in this countable family; symbolic descriptions can instead record selected regimes or externally supplied summaries. Select a countable operation-label family. Each β denotes a determined partial map $T_\beta : D_\beta \subseteq \Omega_0 \rightarrow \Omega_0$ after its schema selection has been specified. A realized branch of a nondeterministic schema needs its own label. Unresolved branching would require a different transition representation.

Let $\mathcal{H}$ be the set of finite admissible histories, including empty histories at starting configurations, and use $\mathcal{H} = \ell^2(\mathcal{H})$ with orthonormal basis e_h . This is a mathematical space of history records, not a claim about quantum cognition. For $\text{end}(h)$ the final configuration, define the append operator by Equation 18.

$$L_\beta e_h = \begin{cases} e_{h \cdot \beta}, & \text{end}(h) \in D_\beta, \\ 0, & \text{end}(h) \notin D_\beta. \end{cases} \quad (18)$$

The appended history in Equation 18 includes its new configuration. Distinct applicable histories remain distinct after appending, because the complete prefix is retained. Thus the operator extends by continuity with norm at most one, and is a partial isometry. If P_{D_β} projects onto applicable histories and $P_{\text{ran}\beta}$ onto appended histories, Equation 19 follows.

$$L_\beta^* L_\beta = P_{D_\beta}, \quad L_\beta L_\beta^* = P_{\text{ran}\beta}. \quad (19)$$

To verify Equation 19, apply each side to a basis history. The adjoint removes a final applicable β step and annihilates other histories. Extending equality from finite combinations by boundedness proves the

operator identities. This construction and its domain distinction are inherited from the previous local-convention investigation [6, Section 7]. Erasing a recorded suffix by an adjoint is not an enacted reversal of learning, harm or resource use.

For any bounded readout $q : \mathcal{H} \rightarrow \mathbb{C}$, let $M_q e_h = q(h) e_h$. The unital norm-closed algebra generated by these readouts, the L_β and their adjoints is a concrete operator algebra closed under adjoints. Its full mathematical closure contains sums and limits without an interpretation as available actions. A formative claim must specify its interpreted generators and their domains rather than treating all algebra elements as executable.

8.2 Executable Renewal

Fix a nonempty set $S \subseteq \mathcal{H}$ of histories with a declared formative boundary: an available participant-forming role arrangement, its supports and a suitable prospective participant. Membership in S is a substantive specification. It must include the dependencies whose continued availability is being claimed. It need not impose identical people or unchanged linguistic content.

Consider a fixed operation word $w = \beta_1 \cdots \beta_k$, with $k \geq 1$, whose concrete schemas can introduce fresh participant labels. Its operator $W = L_{\beta_k} \cdots L_{\beta_1}$ acts in chronological order from right to left. Let D_w be the set of histories where every intermediate step is executable. On that domain the complete prefix is retained, so W remains a partial isometry. Let P_S and P_{D_w} denote the corresponding history projections.

Proposition 8.1 (Executable Boundary Renewal). *The word w is executable at every history in S and returns every such history to S if and only if Equation 20 holds.*

$$P_{D_w} P_S = P_S, \quad (I_{\mathcal{H}} - P_S) W P_S = 0. \quad (20)$$

Proof. For a basis vector e_h with $h \in S$, the first identity says exactly that the word executes completely. Its image is then the basis vector of the resulting history. The second identity says that this history belongs to S . Conversely these two facts on every selected basis vector give both identities on finite combinations and, by boundedness, on their closure. $\square$

Proposition 8.1 deliberately includes executability. A zero operator would satisfy the second identity while accomplishing nothing. Equally, executing a word once does not establish either identity on its later outputs. The result specializes an inherited domain-and-preservation criterion to a formative boundary. Its value is the explicit obligation to identify what must be renewed, not novelty of the operator identity itself.

Proposition 8.2 (Iterated Formative Extension). *Suppose Equation 20 holds. Suppose also that each execution of w introduces at least one participant label absent from its initial history, establishes the declared competence witness for that participant, and preserves the chosen historical anchor. Then every $h \in S$ admits arbitrarily many consecutive executions of w , each retaining the earlier history and a fresh formation witness.*

Proof. The execution condition gives the first extension. The return condition places its result in S , so the same execution condition gives the next. Induction yields a defined prefix for every finite number of cycles. At each step the freshness assumption excludes earlier participant labels and the witness assumption supplies the stated formation record. Composition of the anchor-preserving continuation maps retains the same anchor at every finite stage. The nested finite histories define an infinite symbolic sequence, although only the arbitrary finite prefixes are needed for the claim. $\square$

Proposition 8.2 states a conditional extension result, not a prediction of indefinite social reproduction. A bounded time horizon, exhausted supplies, absent prospective participants or a changed permission regime can invalidate its hypotheses. It neither guarantees an acceptable duration for each cycle

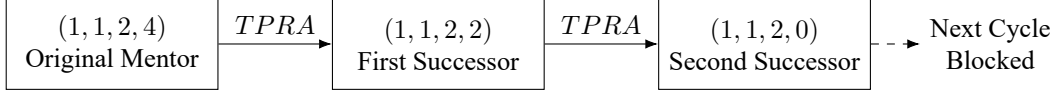


Figure 3: Repeated Boundary Counts With Exhausted Support. The active counts repeat while the retained reserve decreases. New participant identities remain in the history even when the role counts coincide.

nor supplies a participant's policy for selecting among competing cycles. The history witnesses also require independent interpretation as learning. Declaring a fresh label competent inside a model does not establish that a person acquired the capacity.

8.3 A Finite Support Counterexample

A small example shows why a repeated coarse boundary is weaker than Proposition 8.2. Record four nonnegative integers (u, m, b, q) : currently active users, users additionally prepared for the selected mentoring role, usable support units and a finite reserve. Assume $m \leq u$. The counts are a projection of historically distinct people, with fresh learner labels retained in the full history. The four partial operations in Equation 21 are stipulated bookkeeping. For the concrete lift of the displayed cycle, T selects the next unused participant label, P prepares that newcomer, and A records the predecessor's departure. The count projection alone does not supply those historical identifications.

$$\begin{aligned}
 T : (u, m, b, q) &\mapsto (u + 1, m, b - 1, q), & m \geq 1, & & b \geq 1, \\
 P : (u, m, b, q) &\mapsto (u, m + 1, b - 1, q), & u > m \geq 1, & & b \geq 1, \\
 R : (u, m, b, q) &\mapsto (u, m, b + 2, q - 2), & m \geq 2, & & q \geq 2, \\
 A : (u, m, b, q) &\mapsto (u - 1, m - 1, b, q), & m \geq 2. & &
 \end{aligned} \tag{21}$$

In Equation 21, T forms a user, P prepares a user for the selected mentoring tasks, R makes two reserve units usable, and A records the former mentor's departure. The prerequisites for R represent a chosen support arrangement, not a general requirement for two mentors. Support units are explicitly counted resources of the example, not quantities of competence or generosity. Real participants can divide the roles differently.

Starting at $(1, 1, 2, q)$ with $q \geq 2$, the word $TPRA$ gives Equation 22.

$$(A \circ R \circ P \circ T)(1, 1, 2, q) = (1, 1, 2, q - 2). \tag{22}$$

Equation 22 returns the first three coordinates while consuming two reserve units. From $q = 4$, two cycles succeed and the third fails at R . The coarse signature (u, m, b) has repeated at both successful boundaries. The full history and reserve record show why further renewal is unavailable. In general there are exactly $\lfloor q/2 \rfloor$ complete cycles under these rules: each complete cycle consumes two units, and the explicit sequence executes whenever at least two remain. On the third attempted cycle from $q = 4$, formation and preparation can still occur before R fails. The failure concerns complete support renewal, not the absence of any new learning.

Figure 3 displays the comparison. Its arrows mean completed words with newly formed participants and old-mentor departure. The final blocked arrow is a domain failure, not a negative competence score.

Noncommutativity in this example has an explicit domain consequence. At $(1, 1, 2, 4)$, P is unavailable, whereas $P \circ T$ is executable. Thus preparation requires the selected prior formation in this stipulated arrangement. The conclusion does not follow merely from different history words; it follows from the stated operation domains. A different arrangement could prepare several competencies concurrently and require different maps.

8.4 Liftability And Continued Support

A coarse model can certify concrete recurrence of a role signature only if its transitions lift to executable concrete histories. The example lacks that property once the reserve is exhausted. Adding a support-renewal operation could restore it, but its resources, time, consent and effects must enter the model. Treating the reserve as automatically replenished would move the dependency outside the description without explaining it.

This is where algebra and historical interpretation meet. Reproduction can preserve an organizational role while changing its occupants, yet its continuation depends on more than a role isomorphism. Some dependency must be supplied again, replaced or explicitly treated as a bounded external condition. The appropriate conclusion may therefore be finite-horizon reproduction rather than indefinite closure. That is a substantive positive achievement when its horizon and support conditions are stated.

8.5 Renewable Boundary Selection

Proposition 8.2 assumes a set of histories and a fixed word that returns every history to that set. A practical arrangement may instead have several ways to prepare another formation episode. One route uses a familiar facilitator; another prepares a peer contribution; a third obtains outside support. A constructive question remains even inside a completely specified model: which boundaries permit continued formation through some available succession of such routes? The following finite construction answers that question under explicit historical lifting assumptions. It provides a way to find a suitable boundary family rather than assuming one has already been supplied.

Let B be a finite set of boundary descriptions and let $r : \mathcal{H}_0 \rightarrow B$ be a surjection from a selected family of concrete histories $\mathcal{H}_0 \subseteq \mathcal{H}$. Each boundary retains the dependencies relevant to the modeled continuation, including support and the selected competence target. Its finiteness is a restricted description, not finiteness of the histories or possible participants. Choose a finite family $\mathcal{W}$ of nonempty operation words. A word represents a complete participant-formation episode with a specified support update. The assertion that it introduces a fresh competent participant is a separate witness condition on its concrete execution. It cannot be inferred from the word's name or the return of its boundary counts.

For each $w \in \mathcal{W}$, choose a partial boundary map t_w and retain the actual partial history map T_w . The decisive assumptions are expressed in Equation 23.

$$t_w : D_w^B \subseteq B \longrightarrow B, \quad r^{-1}(D_w^B) \subseteq \text{Dom}(T_w), \quad r(T_w h) = t_w(r(h)). \quad (23)$$

In Equation 23, the final identity holds for every $h \in r^{-1}(D_w^B)$, and $T_w h$ is required to remain in $\mathcal{H}_0$. These are uniform lifting requirements across each retained boundary class. They permit a conservative declared domain: some concrete executable episodes may be omitted. Such omission restricts the completeness of the analysis without invalidating its sufficient continuations. Conversely, a boundary edge supported by just one favorable representative would not satisfy the displayed requirement.

The reserve example explains the importance of this choice. If r retains only (u, m, b) , the same boundary class contains both well supplied and exhausted histories. The complete word $TPRA$ fails to lift at the latter. Adding a cyclic arrow to that coarse graph therefore cannot establish renewal. One must retain the reserve, restrict the admitted history family through a warranted external-support assumption, or abandon that edge. The finite construction begins only after these alternatives have been resolved inside the selected model.

Let $C \subseteq B$ be the boundary states satisfying the declared candidate requirements for continuation. Membership in C does not yet certify renewability; that is the question the subsequent construction tests. These requirements need not mean that every earlier participant remains or that the next task is word-for-word identical. They specify the comparison actually claimed. Starting with $S_0 = C$, define the successive retained sets by Equation 24.

$$S_{n+1} = \{b \in S_n : \text{there exists } w \in \mathcal{W} \text{ with } b \in D_w^B \text{ and } t_w(b) \in S_n\}. \quad (24)$$

Equation 24 removes boundaries whose available complete episodes all leave the currently retained family. The recursion is a finite controlled-invariance calculation, not a new general theory of learning. Its philosophical role is to reveal an additional distinction: an arrangement can have enough support for one next episode while lacking a route that leaves support for another.

Proposition 8.3 (Finite Renewable Boundary Kernel). *The decreasing sequence in Equation 24 becomes constant after at most $|C|$ strict removals. Its final set S_* is the greatest subset $A \subseteq C$ such that every $b \in A$ admits a word w with $b \in D_w^B$ and $t_w(b) \in A$. From each boundary in S_* , a boundary-dependent choice of words gives arbitrarily long finite sequences remaining in C .*

Proof. The defining membership condition gives $S_{n+1} \subseteq S_n$. Every strict inclusion removes at least one point of the finite set C , so there are at most $|C|$ such inclusions. If two consecutive sets agree, applying the same defining operation again leaves that set unchanged; call it S_* . Its defining equality supplies a permitted successor in S_* for every point in S_* . Because the set is finite, select one such word at each point and apply the selected choices successively. Induction gives a sequence of every finite length inside S_* .

For maximality, suppose A has the stated property. Clearly $A \subseteq S_0$. If $A \subseteq S_n$, the successor promised for each $b \in A$ also lies in S_n , so $b \in S_{n+1}$. Induction gives $A \subseteq S_n$ at every stage and hence $A \subseteq S_*$. This proves greatestness. $\square$

Proposition 8.3 is a positive result inside the declared boundary model. Its words can vary across successive episodes; it does not require one canonical teaching sequence. A boundary removed late in the calculation may still support several successful cohorts. Removal means that the selected indefinitely repeatable continuation claim fails, not that those finite achievements did not occur. A finite horizon may itself be the appropriate target of an actual programme.

8.6 Concrete Historical Continuation

The passage from the boundary calculation to participant formation requires the lifting data in Equation 23. Assume each lifted word also supplies the fresh-participant witness and anchor continuation required by Proposition 8.2. Select a word $\pi(b)$ for each $b \in S_*$ as in the proof above. The function π is a mathematical selection on a finite boundary set. It does not establish that a real participant knows the boundary, consents to the operation or can act on the selection.

Proposition 8.4 (Lifted Renewal Selection). *Under the uniform lifting and witness assumptions, every history $h \in \mathcal{H}_0$ with $r(h) \in S_*$ admits arbitrarily many consecutive finite renewal episodes selected by π . Every episode retains the historical anchor and adds the stipulated fresh participant and formation witness.*

Proof. Let $b = r(h)$. The selected word $\pi(b)$ is defined at b and has its boundary output in S_* . Uniform lifting makes the concrete word executable at h , sends its output back into $\mathcal{H}_0$, and identifies that output's boundary with the selected successor. Its additional witness assumptions establish participant freshness, formation and anchor continuation. The same argument applies to the resulting history. Induction constructs any prescribed finite number of episodes; composition of the anchor maps preserves the retained anchor throughout. $\square$

The proof of Proposition 8.4 identifies exactly where a coarse recurrence becomes informative: its edges must remain concretely available after the histories they help generate. If the abstraction discards a declining resource or a changing task, uniform lifting can fail. If the task cannot be interpreted as learning, the stipulated witness has no empirical warrant. These are different failures, and neither is repaired by a longer simulated sequence.

The selection also has an algebraic formulation that retains its local conditions. Write $\hat{S} = r^{-1}(S_*)$ and partition it into sets $H_w = \{h \in \hat{S} : \pi(r(h)) = w\}$. Let P_{H_w} and $P_{\hat{S}}$ denote the corresponding

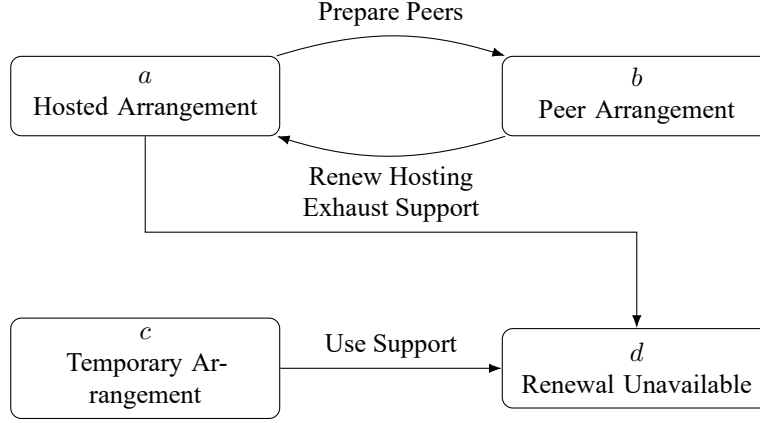


Figure 4: A Constructed Renewable Boundary Graph. Each arrow denotes an assumed complete episode with a concrete historical lift. The cycle supports selected continuation; the alternative exit prevents a claim that every admitted choice sustains renewal.

history-basis projections, and let W_w be the fixed-word operator already defined. Each selected piece satisfies Equation 25.

$$P_{D_w} P_{H_w} = P_{H_w}, \quad (I_{\mathcal{H}} - P_{\hat{S}}) W_w P_{H_w} = 0 \quad (w \in \mathcal{W}). \quad (25)$$

Equation 25 applies the earlier execution and return identities to the histories on which a word is actually selected. This restriction matters: a word need not work everywhere in $\hat{S}$ if another word supplies the next episode elsewhere. The formulation therefore represents differentiated support strategies without identifying every algebraic combination with an executable action.

One could form the bounded finite sum $\sum_{w \in \mathcal{W}} W_w P_{H_w}$, but it need not be a partial isometry merely because the source projections are disjoint. Different word lengths can send histories of different lengths to the same final history. Additional unique parsing or cycle-marker assumptions would be needed for that stronger claim. Keeping the separate identities in Equation 25 avoids silently importing the fixed-word injectivity argument into a different construction.

8.7 Choice-Sensitive Continuation

A four-boundary example distinguishes availability of a sustaining choice from reliability under every admitted choice. Let $B = C = \{a, b, c, d\}$. A prepared hosted arrangement a can either pass to a peer arrangement b or select a route ending at d . The peer arrangement can return to a . A temporary arrangement c has a single successor d , and d has no complete renewal word. The descriptions are stipulated boundary types with their lifting requirements assumed, not empirical types of language communities. Figure 4 gives the complete transition graph.

Equation 24 first removes d , then c , and retains $S_* = \{a, b\}$. Starting at c allows one episode without continued renewal. Starting at a permits the sustaining cycle, but also permits a choice that ends it. The graph does not identify which choice ought to be made. A participant may have sufficient reasons to end the hosted arrangement, and the descriptive kernel cannot turn a possible cycle into an obligation to remain within it.

A stronger robustness claim would require every admitted successor to remain inside the retained set. Its recursion must also exclude states with no available operation, as in Equation 26.

$$\begin{aligned} A(b) &= \{w : b \in D_w^B\}, & R_0 &= C, \\ R_{n+1} &= \{b \in R_n : A(b) \neq \emptyset \text{ and } t_w(b) \in R_n \text{ for every } w \in A(b)\}. \end{aligned} \quad (26)$$

For Figure 4, Equation 26 removes d , then a and c , then b , leaving the empty set. This result concerns the stated repertoire. A justified change to which operations are available changes the robustness question; silently discarding the inconvenient exit would change the model.

The contrast has an interpretation more precise than calling a formation stable or unstable. Some arrangements retain an executable continuation choice in the declared model while remaining vulnerable to specific actions. Others can tolerate a declared class of disruptions. A claim about the second requires more than the existence of the first. Where disruptions have several possible outcomes, one can refine the construction to require a chosen action whose every admitted outcome remains viable. That stronger model would need its own outcome relation and is not assumed by the deterministic example.

Finite branching is material when one infers a continuing path from success at every finite horizon. Consider a root that can choose any positive integer n , after which only n episodes remain. Every finite horizon can be met by choosing a sufficiently large n , yet no single choice begins an infinite continuation. This countably branching counterexample prevents a horizon-by-horizon success claim from being substituted for one indefinitely repeatable arrangement. The finite kernel uses one stable choice structure and explicit lifts; it does not make that substitution. Actual social claims remain bounded by changing participants, resources and purposes.

8.8 Field Interpretation Boundaries

The field and algebra should remain connected without being identified. A field trajectory may motivate a hypothesis about support availability. The operator domain must still specify how that activity makes a particular operation executable. Conversely, an algebraic renewal cycle says nothing about its attraction from arbitrary initial activity profiles. A bridge between them would require measured or otherwise justified realization data. The present paper provides two complementary investigations and states that remaining obligation explicitly.

9 Learning Evidence

Evidence of participant formation must connect a recorded change to the capacity and support conditions under investigation. The preceding models identify several possible failures of that connection, but they do not determine what a real learner acquired. This section examines two primary experimental programmes and then derives methodological questions for a reproductive inquiry. The experiments are retained at their own scale; the proposed extensions are explicitly the present paper's analysis.

9.1 Situated Phonetic Learning

Kuhl, Tsao and Liu studied English-learning infants beginning at approximately nine months. Twelve short sessions supplied live Mandarin exposure or live English control exposure; another experiment supplied recorded Mandarin in audio-only or audiovisual form. The live Mandarin condition improved discrimination of a selected Mandarin phonetic contrast relative to the English control. The recorded conditions did not show the corresponding improvement. The analyzed groups contained 10 and 11 infants in the first experiment and 15 and 13 in the second. Attention also differed between live and recorded conditions. The authors discuss possible contributions of social and referential cues; the study does not isolate one universal interaction mechanism [13].

For the present inquiry, the relevant inference is limited: comparable linguistic materials need not yield equivalent learning opportunities under changed delivery conditions. The experiment neither establishes general fluency nor tests whether the learners can subsequently help form other participants. It does not show that recordings, technological mediation or solitary learning are generally ineffective.

The philosophical lesson concerns specification of a replacement. A preserved record can retain selected acoustic information while changing the learner's relation to its presentation, timing and uptake. Whether that change matters depends on the target. A replacement suitable for later vocabulary review might be unsuitable for a different formative task. The adequacy of the replacement therefore cannot be inferred solely from a description of its stored content. It needs a comparison of the activity it enables for the specified participants.

This argument leaves several explanatory alternatives open. Differences in attention, contingency, perceptual organization or motivation can contribute to a learning difference. Describing their joint setting as relational does not distinguish them. A Generative Relational reconstruction earns value by retaining those alternatives, identifying their interfaces and guiding a discriminating inquiry. It adds little if it merely renames all unexplained variance as relational reality. The field's three channels are not estimated from this experiment, and their symmetric interaction is not an explanation of its particular effect.

9.2 Experimental Transmission Chains

Kirby, Cornish and Smith passed learned artificial-language outputs to later adult learners in experimental chains. In one experiment, increasing learnability could accompany underspecified expression. A second experiment filtered repeated labels in the training material and produced more compositional structure. The authors explicitly distinguish increased learnability and structure from increased communicative functionality [12]. These findings constrain the inference from repeated transmission to retention of a selected repertoire. The supporting-information correction is recorded in the source ledger; no corrected supporting table is used here.

The present paper's further question concerns the experimental boundary. A chain of learner outputs can be an appropriate object for investigating changes of form. A reproductive inquiry also asks how the next learner, training opportunity and evaluative setting become available. When those conditions are supplied by a research design, they need not be generated by the preceding learner's success. This is a difference of explanatory target, not a defect in the experiment. Requiring every experiment to explain its entire infrastructure would make bounded research impossible.

The comparison instead supports a disciplined extension. First specify the feature transmitted and the conditions held fixed. Then enlarge the target only when the inquiry concerns the reproduction of one of those conditions. The finite reserve example in Figure 3 performs that enlargement mathematically: the role boundary repeats until a previously hidden support variable prevents another cycle. The experiment does not provide evidence for that particular reserve law. It shows why the target of an output-transmission claim should remain visible when the analysis turns to an institutional or community process.

9.3 Assessment Dependence

An assessment can itself supply instruction. A request to explain a choice may disclose a contrast, while an examiner's hesitation may cue a repair. The resulting successful response has a history. An inquiry should state whether it tests prior capacity, learning during the encounter or supported joint performance. Treating these outcomes as distinguishable does not make one automatically inferior. A pedagogical interaction may aim precisely at learning during assessment; a historical attribution asks another question.

The same consideration applies to competence profiles. If a participant can act successfully only with a specified partner, the investigator must decide whether that supported relation belongs to the target or masks a narrower claim about independent activity. A claim of independence from one teacher can remain compatible with dictionaries, peers and accessible technology. The relevant contrast is between declared support regimes, not between relational dependence and a fictitious absence of dependence.

Table 2 organizes several evidential claims. Its rows are separate targets rather than mandatory stages. A target can be relevant even when the later rows are outside the inquiry.

9.4 Reproductive Inference

The strongest local evidence for reproduction would join several kinds of record: a justified competence target, histories of participant formation, changes in formative assistance and evidence that the relevant supports remain available to successors. Even that evidence has a horizon. Two successful cohorts

Target	Relevant Observation	Further Obligation
Task performance	A situated response meets a declared criterion	Establish the criterion's applicability and record assistance
Selected competence	Performance, interpretation and repair across specified occasions	Address cue dependence and the justified range of transfer
Participant formation	A documented learning history establishes the selected profile	Distinguish prior capability, selection and learning during assessment
Formative assistance	A participant helps another acquire the selected capacity	Identify the learner's change and the remaining support
Renewed opportunity	A successor arrangement supports another formation episode	Account for access, resources, support renewal and changed participants

Table 2: Distinct Evidential Targets. The distinctions prevent a result at one target from silently supplying evidence for another. They do not rank every practice on a universal scale.

support an inference about those histories and perhaps a carefully defended expectation, not indefinite continuation.

The field and algebra suggest different additional observations. To assess the field mechanism, one needs time-sensitive evidence about the selected activities, their losses, mutual effects and interface exchanges. To assess the algebraic renewal claim, one needs the applicability of concrete operations after earlier steps, including resources and permissions that coarse role counts omit. A single summary success rate is unlikely to answer both questions. The required evidence remains proportionate to the specific model claim rather than an indiscriminate demand to record everything.

10 Formative Authority

Participant formation involves judgments about adequate performance, useful correction and acceptable continuation. This section examines the authority attached to those judgments. Its purpose is to distinguish the practical coordination needed for learning from an unrestricted entitlement to define competence or require reproduction. The argument develops reasons for bounded instructional authority, then considers contestation, variation and the distribution of formative work.

10.1 Task Authority

A learning arrangement needs enough coordination for examples and corrections to have determinate purposes. If participants agree to learn a selected way of distinguishing requests, a facilitator may be warranted in identifying a response that fails that purpose. The warrant arises from the task, relevant competence and the facilitator's accountable role. It does not follow from fluency alone. Someone who speaks effectively can misunderstand a learner's difficulty, offer misleading explanations or claim authority outside the domain of their knowledge.

This bounded authority can be positively specified. The facilitator should make the selected task and support conditions intelligible, relate corrections to their practical point, and permit a suitable route for questioning misapplication. The exact route depends on the setting: a novice may first use examples and demonstrations rather than an explicit metalinguistic argument. Requiring advanced public argument before allowing a correction to be challenged would make formal access depend on the very competence still being acquired.

The prior normative-formation manuscript already distinguishes formative efficacy, standing and contestability, and discusses the reproduction of competence as part of generative return [5, Sections 13–14]. The current application asks how those inherited distinctions operate within a learning history. A practice can expand a learner's ability to challenge a correction; it can also make future participation conditional on displaying the teacher's preferred interpretation. Both are formative effects. Their difference requires an argument about the purposes and authority of the arrangement.

10.2 Competence Criteria

A competence criterion can be well suited to one purpose and unsuitable for another. Accurate interpretation of a selected request may be necessary for a particular shared task, while a preferred accent may be irrelevant. Treating both as a single fluency standard can convert an instructional choice into a broader exclusion. The analysis therefore asks what each criterion contributes to the declared activity and how exceptions are handled when they become relevant.

The possibility of contestation does not require abandoning all standards whenever disagreement occurs. A practice can retain a provisional criterion while testing an objection and preserving evidence needed for later revision. The important distinction is between justified task coordination and the closure of meaningful opportunities to question its application. A perfectly stable correction regime can fail the latter test. Neither the field's positive equilibrium nor an invariant operator projection measures that failure without additional normative and evidential structure.

Learner variation can also be reproductively significant. A new participant may preserve a useful distinction through another expression or introduce a contrast the existing repertoire handles poorly. Calling every such change an error would make reproduction equivalent to copying before the continuity question has been argued. Calling every change generative progress would make the opposite mistake. A defensible judgment identifies the retained task, the proposed difference, its consequences for others and the process through which the comparison can be revised.

10.3 Accountable Instructional Asymmetry

The preceding account permits an asymmetry of instructional roles. Someone who has repeatedly distinguished a class of cases can initially direct a novice's attention more effectively than the novice can direct their own. Requiring symmetric authority at every moment would prevent some of the assistance through which that competence can develop. The relevant question is what warrants this local asymmetry and what prevents it from becoming an unrestricted power over the learner's standing.

Three premises support a limited answer. First, the participants have a reason to pursue the specified activity, whether through voluntary agreement or another independently justified educational relationship. Second, the instructor's interventions have a defensible connection to the capacity being developed. Third, the learner remains someone whose difficulties, objections and changing purposes can bear on the arrangement. These premises support a defeasible warrant for directed correction as an accountable contribution to formation. They do not justify a correction merely because its author occupies the instructor's position. Where agreement is absent or institutionally constrained, the first premise requires a substantive justification; the present paper cannot supply it by calling the encounter formative.

Consider a learner who applies "return-ready" to a case for which the return record exists but is inaccessible to the next participant. The facilitator can point to the access condition and explain why the application fails the agreed purpose. This is more than an announcement of preference: it connects the correction to a consequence the parties can examine. Now suppose the learner responds that an accessible oral explanation would serve the same purpose. The response can challenge the proposed means while preserving the task. An instructor who treats the challenge as incompetence would assume the exclusivity of the written format instead of defending it. The appropriate next step is to examine whether the alternative actually supports the relevant access, including its duration and later availability.

That example gives contestation a constructive role. A challenge can reveal an overnarrow criterion, a missing support or a mistaken interpretation of what the learner already understands. It can also fail: the oral explanation might reach only the present pair while the declared task requires access for a later participant. Accountability permits a reasoned rejection in that case. It requires a route through which the difference can be demonstrated, rather than a guarantee that every objection changes the standard. The learner's standing requires appropriate consideration of their difficulties, responses and objections; it does not make every proposed alternative adequate.

The form of that route must fit the stage of formation. A novice may indicate the same distinction by sorting examples, rejecting a suggested answer or showing a counterexample in another language. Requiring

a polished explanation in the target idiom would confound the capacity at issue with an advanced means of defending its attribution. This does not entail treating every silence as dissent or every incorrect answer as a hidden insight. It means that the inference from performance to understanding should remain sensitive to available means of expression. Responsive scaffolding can improve both the assessment and the learner's future ability to question it.

10.4 Criteria Revision

Changes in a competence standard should be examined at the same resolution. Suppose participants add a requirement to explain the return conditions to someone who did not attend the original session. The change may improve the practice, but it enlarges the task. Earlier learners who cannot yet perform it have not thereby lost the narrower capacity previously demonstrated. Equation 6 supplies a comparison for the retained tests; the added test requires its own formative and evidential history. This separation protects an accurate account of learning while allowing the practice to become more demanding.

There are also revisions that correct an earlier misdescription. If the declared target always included access by a later participant, but an assessment ignored that condition, a revised assessment need not constitute a new target. It can reveal that the earlier success claim was inadequately supported. Distinguishing these cases requires the history of purposes, instructions and uses. The written wording of a test cannot settle the matter alone. Conversely, a retrospective assertion that the broader criterion was always intended does not establish that participants had a reasonable opportunity to learn or contest it.

The normative stakes therefore depend partly on the comparison claimed. Introducing a new requirement can be justified while imposing a duty to provide additional learning opportunities or a transition period, given appropriate institutional commitments. Correcting an invalid assessment can be justified while requiring a remedy for people who reasonably relied on its result. These are conditional reasons, not universal prescriptions derived from the field potential. Their force comes from the purposes, undertakings and effects of the actual relationship.

An efficient formation process can obscure this distinction. If a new cohort quickly learns the revised task, the improvement may make it tempting to describe the previous cohort as deficient in general. Yet the new cohort's success establishes neither the fairness of a retrospective judgment nor the availability of the same support to earlier participants. A historical representation contributes by keeping these formative conditions separate. It cannot determine their fairness without the normative premises just identified. This is a specific division of work between the GR representation and the argument about authority.

10.5 Participation And Exit

Successful reproduction supplies no automatic duty to continue a language or become a teacher. Participants can have competing commitments, prefer another language, limit their involvement or leave. A collective interest in continuity can provide a reason to offer support, preserve alternatives or negotiate responsibilities. The strength and allocation of any obligation need a further argument. The existence of a possible successor role does not assign that role to the person whose learning made it possible.

Exit and shared dependencies can nevertheless interact. A facilitator's withdrawal may affect people who reasonably relied on a promised course. A learner's decision not to teach later has a different history of commitment. Treating both as failures of generative return would erase that difference. The appropriate inquiry identifies actual undertakings, foreseeable effects and feasible alternatives. This paper does not derive a general withdrawal rule; a later practical investigation can examine particular responsibilities and handover arrangements.

These distinctions matter especially when reproduction is discussed across generations. Future participants are not simply inputs into a preservation process. Their emerging capacities can expand the range of futures they may endorse, including transformed or multilingual participation. Preserving an opportunity to learn can therefore be valuable without establishing a right to dictate its eventual use. The

analytical target should keep that possibility visible rather than defining successful reproduction by a predetermined allegiance.

10.6 Formative Burdens

Learning and support renewal consume time and attention as well as material resources. The relevant burden can fall on a small group whose work disappears inside aggregate counts of successful learners. The finite reserve example offers a structural caution: a repeated active boundary can draw down an omitted support. In a real setting the omitted term might be unpaid preparation, restricted access to space or the sustained availability of a particular person. Identifying it requires evidence; no person should be represented as a disposable reserve.

A generative-return argument asks what a practice gives back to the conditions of further formation. Return can take several forms: preparing examples, making a room accessible, contributing time, enabling an alternative site or helping someone challenge an exclusion. It need not require each beneficiary to repay the same service. A narrow repayment model could impose the heaviest burden on those whose participation is already difficult. A more adequate account specifies the collective arrangement and its justified distribution of responsibilities.

The collective character of renewal supplies a further reason to distinguish benefit from indebtedness. Suppose a learner benefits from accessible materials prepared by several people. That history can explain why continued provision matters and may support an invitation to contribute. It does not identify a determinate creditor, a unique form of repayment or a fixed amount of labor owed. Those additional relations need agreement or another defended principle. Otherwise the very distribution that enabled access can be reconstructed after the event as an open-ended claim over the beneficiary.

A defensible allocation can instead begin with the work that needs doing, the commitments actually made and the participants' feasible contributions. One person might maintain examples; another might secure access to a venue; another might contribute no continuing labor while facing circumstances that make such labor impracticable. The arrangement can still reproduce a competence if its distributed support is renewed. The algebraic lifting assumptions require executable support; they do not require that every learner provide it or that the same people carry each cycle. Keeping those claims separate prevents a mathematical renewal condition from becoming an unargued norm of personal repayment.

The resulting normative position remains deliberately incomplete. The paper supports a distinction between sustaining a formative opportunity and coercively reproducing a preferred identity. It explains why competence and efficacy are insufficient grounds for unrestricted authority. It does not provide a universal allocation rule for educational labor, funding or language preservation. Those questions need more particular commitments and evidence than the present models contain.

11 Reproduction Inquiry

The stipulated multilingual learning circle can now be examined with the distinctions developed throughout the paper. This section constructs a bounded inquiry rather than reporting an actual programme. It specifies the target, develops competing explanations of apparent success, traces support renewal and examines the contribution of the formal models. The purpose is to show what an investigation would need to learn before claiming that the arrangement reproduces a linguistic competence.

11.1 A Selected Learning Target

Suppose the circle uses the provisional expression "return-ready" to distinguish a completed removal from a state in which the relevant return conditions have been documented and understood. The expression is a stipulated object of linguistic inquiry, not an instruction for performing physical work. Participants already possess substantial linguistic capacities in different languages. Their learning target

Uncertainty	Proposed Observation	Inference Boundary
Cue-dependent answers	Compare responses to changed examples and explicitly record assistance	Transfer applies only to the sampled contrast and support regime
Prior competence	Retain a baseline encounter and the intervening learning history	Baseline failure can itself reflect unfamiliar assessment conditions
Formative assistance	Observe a later novice's changing understanding during a supported episode	The first learner's fluency alone supplies no teaching result
Support renewal	Trace who supplies time, examples, access and correction for the next group	Repeated role counts can hide dependence on the original facilitator
Changed standards	Record the reasons and uptake of proposed criterion revisions	Agreement does not by itself establish legitimate authority

Table 3: A Stipulated Learning-Circle Inquiry. Each observation addresses a specific uncertainty while preserving the further conditions needed for its interpretation.

concerns this local distinction, its appropriate application and its explanation to others. The inquiry is therefore narrower than childhood acquisition or general command of any named language.

The selected profile includes recognizing the contrast in examples, explaining why a borderline case is unclear, asking for missing information and correcting a misleading description. Several expressions can realize the distinction if participants have a warranted interpretation bridge. A learner may use a translation aid or consult a record when those supports are part of the declared task. The profile records those conditions so that a later claim of unassisted use cannot silently inherit evidence from a differently supported exercise.

A separate formative profile concerns helping another newcomer acquire the distinction. It includes selecting contrasting examples, noticing when the learner is merely echoing a cue, making room for clarification and adjusting an explanation. No single person must perform all these tasks. One participant can prepare accessible records, another can facilitate comparisons and another can provide a responsive discussion setting. Reproductive capacity concerns their arrangement and its continuation, not a requirement of universal pedagogical self-sufficiency.

11.2 Rival Histories Of Success

Imagine that most newcomers give the expected answer at the end of a session. At least four histories can produce that manifestation. Some may have learned the selected distinction. Others may recognize the facilitator's cues without discriminating the underlying cases. Some may have entered with relevant competence that the assessment merely reveals. Others may construct a correct answer jointly with a partner. The last history can be a successful supported performance rather than an error in the programme; its interpretation depends on the target.

The inquiry should therefore vary or record features that distinguish these histories. A later encounter with a changed example can test the claimed range of application. An explanation request can disclose which contrast the learner uses, while its own teaching effects should be recorded. A change of partner can examine a particular dependence, without assuming that partner-independent performance is always the goal. Observation before instruction helps distinguish prior competence from formation during the episode. None of these methods guarantees transparent access to understanding; together they can improve a bounded attribution.

Table 3 links each uncertainty to a proposed observation and an inference boundary. The rows are designs for discussion, not an empirically validated assessment instrument.

11.3 A Worked Attribution Sequence

The proposed observations can be developed into a single constructed history. Call three newcomers Mira, Tomas and Sora. These names designate fictional participants, and every observation in this subsection is stipulated for analytical comparison. The history is not a disguised empirical report or an estimate of a programme's likely effectiveness. It supplies sufficiently different outcomes to make the earlier distinctions answerable to a case.

At the initial encounter, Mira already distinguishes physical completion from the availability of return conditions in her existing language. She does not recognize the circle's expression. Tomas recognizes the expression from a notice but treats it as equivalent to "finished." Sora understands the proposed contrast when the record is read aloud, but cannot access the format used in the written exercise. If all three initially fail a written label-selection test, that common result conceals three different relations to the target. Mira needs a bridge to a local expression; Tomas needs to reorganize an application distinction; Sora needs an accessible means of participation. All three may learn something in the session, but the same claim about the acquisition of a previously absent distinction would be unwarranted.

During the first exercise, the facilitator places every successful example on a green card and every unsuccessful example on a gray card. All three eventually select the expected answer. The exercise can introduce useful examples, yet its final responses cannot distinguish attention to the relevant conditions from attention to card color. A second encounter uses the same accessible presentation but distributes colors independently of the answers. Mira gives a correct explanation through her translation bridge. Tomas follows the colors and misclassifies two stipulated contrasts. Sora asks for an absent access detail and declines to label that case settled. The changed encounter revises the apparent equivalence of the first results.

Mira's contribution supports the narrower finding that a prior contrast has become usable through a new local expression. It does not establish that the session created the underlying distinction. Tomas's performance defeats the claim of transfer across the sampled color variation at that stage; it does not show a general inability to learn. Sora's request supplies positive evidence of the selected uncertainty response, provided asking for the missing detail is itself part of the declared competence profile. The inquiry has therefore learned something affirmative about each participant without reducing their trajectories to one passing proportion.

Suppose Tomas then works through contrasting cases with Mira. The pair explicitly discusses a case whose physical work is complete while the next participant lacks access to its return record. At a later encounter Tomas classifies a changed example and explains the access condition without the color cue. The observed sequence now supports an attribution of learning across these episodes. Its support is local: it concerns the demonstrated contrast, the available translation bridge and the sampled delay. It does not isolate Mira's contribution from the previous facilitator's work or show competence across every possible return condition. The formative history supplies a plausible dependency account while leaving that stronger causal decomposition open.

Finally, suppose Sora responds effectively with an audio record in both encounters but struggles when that record is removed. If audio access was part of the target regime, this finding identifies a support dependence within the successful attribution. Reclassifying it as failure would change the task. If independent memory was the declared target, the same evidence would support a more limited success claim. The difference cannot be settled by the investigator's preference for autonomous performance. It is resolved through the purpose and publicly specified conditions of the competence under investigation.

This sequence demonstrates why historical detail earns its place. The final answer alone underdetermines what changed; the baseline and intervening encounters distinguish expression learning, application revision and access support. The representation need not retain every fact about the participants. It must retain enough of this trajectory to support the comparison being claimed. If a compact state description explicitly records those distinctions, it can be an adequate alternative representation. The GR objection concerns their unexplained loss, not the use of compact descriptions itself.

11.4 The Successor Episode

Suppose the first group retains the competence profile, but only the original facilitator knows how to prepare new contrasts. The group has achieved participant formation and preserved a dependency. It has not yet shown renewal of that part of the formative arrangement. This assessment is more precise than calling the whole programme a success or a failure. It also identifies a constructive next step: the participants can jointly develop usable examples and examine whether later facilitators can adapt them.

If that preparation occurs, the next group may learn through a different division of work. The historical object changes: new participants enter, examples acquire new uses and some corrections are revised. A history-bearing representation can retain those changes without requiring a fixed participant set. The reproductive comparison then names which competence and support relations have successors. It need not claim that the two groups are identical or that every prior convention remains intact.

Consider a further possibility. The original facilitator formally withdraws but continues answering messages, repairing materials and resolving difficult cases. A coarse report records a successful handover. A fuller history shows that the apparent successor arrangement still depends on the same person's unrecorded work. The dependency might be acceptable and mutually agreed. What fails is an unsupported claim that the corresponding support has been reproduced elsewhere. Accurate description permits a more useful discussion of its duration, alternatives and burdens.

11.5 A Worked Support Handover

Continue the constructed history by admitting a fourth fictional participant, Jin. Tomas can now apply the selected distinction to familiar and changed cases. He initially helps Jin by giving the answer whenever Jin hesitates. Jin's successful repetitions therefore establish an assisted output, while leaving the learning target unresolved. Mira suggests retaining the contrast and asking Jin to identify which access condition is missing. Sora provides an audio version that Jin can revisit. After these adjustments Jin responds to a further example and supplies a relevant clarification request. In this stipulated history the formative effect belongs to an arrangement of contributions. Tomas's initial fluency did not alone establish formative capacity, and the later success need not belong exclusively to the person who asked the final question.

The support history is equally specific. Before the original facilitator leaves, the group stores the examples and their reasons for correction. That transfer preserves an artifact. It does not yet establish that someone can select new contrasts, repair a misleading example or host the next encounter. Suppose the next session uses a new case that makes the original answer key ambiguous. Tomas notices the ambiguity, Mira checks the proposed distinction with the group, and Sora updates the accessible record after the revision is agreed. These stipulated events provide evidence that some support functions have successors. The artifact is now part of an active interpretive and corrective arrangement, rather than its existence being treated as sufficient evidence of that arrangement.

Now examine two possible continuations. In the first, the original facilitator still receives every difficult question privately and supplies the decisive correction. The visible group has acquired hosting and routine guidance functions while retaining an external dependence for unfamiliar cases. In the second, the group resolves the selected new cases through the declared comparison procedure, and the original facilitator's messages are limited to matters outside the stated target. Withdrawal then supports a stronger finding about renewal of the selected functions. It still does not show that the group can handle every future difficulty. The comparison must state which functions and challenges were actually retained.

This handover gives the historical anchor a concrete role. The successor arrangement inherits examples and correction records whose meaning depends on earlier encounters, yet new participants alter who can interpret and revise them. Preserving the records alone would satisfy a weaker documentary claim. Preserving an executable route through their use, contestation and repair supports the relevant formative claim. The distinction is not an abstract preference for process over objects: it identifies the operations whose continued availability separates the two stipulated histories.

11.6 Target Revision During Reproduction

Suppose the group next proposes that “return-ready” also require a record usable by a participant joining remotely. An accessible record for those present does not necessarily meet that condition. The original task profile and its support regime should therefore remain identifiable while the new requirement is investigated. Tomas may retain the previous competence and need further formation for remote communication. Sora’s experience with accessible media may become a formative contribution, rather than a reason to treat her earlier support dependence as an inferior version of the same capacity. These are possible consequences of the stipulated change, not general claims about any group defined by an access need.

There are three possible findings. The new requirement might expose an earlier failure to apply a target that already included remote access. It might add a genuinely new task. Or the parties might disagree about which description is warranted because the earlier purpose was ambiguous. The inquiry can preserve that disagreement while still reporting success on the common retained tests. An injective task correspondence of the form in Equation 6 identifies the restricted comparison; it does not decide the contested extension. A claimed loss of competence is warranted only after the comparison and the evidence have been specified at the relevant level.

This example also limits a tempting account of reproductive success. If the next group handles remote access better, the practice has gained a capacity relative to that target. It does not follow that every earlier form has been preserved or that the change benefits everyone equally. Some participants may lose access if remote participation becomes the only admitted route. The present analysis can identify the altered support conditions and the people affected. A judgment about the acceptability of that reorganization requires the additional reasons discussed in Section 10.

11.7 Model Correspondences

The three-field model can represent a selected pattern in this inquiry only after operational channels have been identified. Usable encounter opportunities might inform x ; responsive guidance might inform y ; work creating later opportunities might inform z . These are candidate bridges, not measured equalities. The common coefficients and triadic interaction would require additional justification. A setting where support is externally guaranteed or learning occurs through a different channel may need another model entirely.

The model nevertheless sharpens a question. If an established circle continues through coordinated activities, does a small separated attempt have access to the same sustaining relations? Proposition 7.1 shows why existence of a positive regime is insufficient to answer that question under one explicit non-linear law. Figure 2 also shows why stronger exchange can alter the fate of a localized activity pattern. Neither result licenses a recommendation to isolate a community or manufacture a crisis. The substantive channels and effects must be established first.

The algebra addresses a different correspondence. A complete formative word could include admission to a practice opportunity, acquisition of the selected profile, preparation of part of the future support, and transfer of an agreed role. Its actual domain must include available time, materials and willing participants. The finite reserve example asks whether repeated observable roles conceal a consumed dependency. The operator criterion then requires the concrete output to satisfy the next cycle’s domain, including that dependency. This is the specific inferential gain over merely drawing a cycle in a diagram.

11.8 Alternative Continuation Choices

The finite kernel in Figure 4 allows the handover to be examined beyond one repeated script. Interpret a as an arrangement with current hosting support and an opportunity to prepare peers; interpret b as one whose prepared participants can undertake the selected renewal of hosting. Each edge still has to include a complete formation episode and its support update, with the concrete witness assumptions of

Proposition 8.4. Preparing peers is not by itself equivalent to introducing a newly competent participant, so the short edge names cannot discharge those assumptions.

The modeled a – b continuation offers a choice of alternating complete episodes. A competing route from a to d completes a present episode while exhausting the retained support. A temporary arrangement c can likewise reach d . The elimination procedure first identifies the missing continuation at d , then the dependence of c on it. Its retained family is $\{a, b\}$. In the constructed inquiry this says exactly which declared boundary conditions admit a continuing selection. It does not prove that every workshop has these four states or that the verbal descriptions provide a sufficient empirical abstraction.

The distinction between an available selection and robust continuation is substantive. If the support-exhausting route remains admitted at a , the all-choice construction eventually retains no state. The group could therefore have an executable renewal route without every admitted decision preserving it. One response would be to make the dependency intelligible and negotiate a way to maintain the route. Another would be to provide support that changes the terminal boundary. A third would be to complete the present episode and knowingly end the programme. The theorem orders none of these choices morally; it clarifies their consequences within the declared transition structure.

This is also where stronger empirical claims would need additional evidence. The selector assumes that its chosen operation can be identified and executed at the relevant boundary. A group that cannot distinguish a from c cannot necessarily act on the mathematical selection. A sponsor who can withdraw an unmodeled support can invalidate a uniform lift. A participant who declines a proposed role can change an operation's actual domain. These observations indicate how to refine the history and inquiry; they do not turn an existence theorem into a practical guarantee. The positive result is the identification of a sufficient continuing family under declared assumptions, together with explicit points where those assumptions can be investigated and challenged.

11.9 Revisable Findings

A responsible inquiry could report several positive findings without claiming indefinite reproduction: participants demonstrated a specified profile under named supports, with newly acquired and previously available capacities distinguished; some participants developed selected formative capabilities; the next cohort had an executable opportunity; and a limited support horizon was secured. It could also report unresolved questions about the transfer range, future resources or contested standards. That mixed account is more informative than a single vitality score because its components answer different questions and permit different forms of correction.

The proposed inquiry remains a philosophical and methodological design. Its implementation would need collaboration with actual participants, locally appropriate tasks, proportionate evidence and a defensible interpretation of authority. The present paper supplies an architecture for making those obligations visible. It does not substitute a completed formal model for the work of learning what a community is trying to continue and how its members wish to participate.

12 Reproduction Research Boundaries

Linguistic competence reproduction has been treated as a historically organized capacity to form further participants while renewing selected formative relations. The resulting account distinguishes competent current use, acquired participant capacity, formative assistance and renewed opportunity. These achievements can support one another without forming a universal developmental ladder. The distinctions preserve the role of individual learning while making its relational supports and historical consequences available for inquiry.

The philosophical argument develops a positive conception of reproduction that permits transformation. Successor participants can retain a useful capacity through changed expressions, tools and divisions of work. Historical witnesses identify their relation to earlier formations; context-sensitive comparisons identify what continues; normative judgment addresses the authority and value of that continuation. No

single one of these tasks silently supplies the others. The analysis also keeps access and exit visible, so a capacity to reproduce a practice does not become an obligation imposed on every learner. Supported competence can be attributable when a participant makes the required contribution within declared ordinary supports. A formative claim additionally needs grounds for identifying learning rather than the disclosure of a prior capacity. Those grounds can be distributed across a history of partners and materials; they need not identify one instructor as the unique source of the change.

The authority argument supplies a related constructive result. Instructional asymmetry can be warranted by a justified task, relevant competence and accountable intervention, while remaining open to challenges expressed in forms available to the learner. A changed target requires a new comparison and, where the relevant commitments support it, additional formative opportunity. The fact that support renewal benefits a practice does not assign identical repayment duties to its learners. These distinctions make continued formation assessable without converting its existence into an unrestricted reason to reproduce a preferred identity.

The formal investigations establish definite but restricted consequences. The three-field law has a unique forward evolution on its nonnegative cone and a decreasing potential. Under specified parameters, an inactive regime and a positive regime coexist as stable solutions. Full field stability, global potential minimization and initiation from a selected condition remain distinct. Regional dynamics requires its actual boundary input. Its internal potential can initially increase while the global potential decreases; an exact supported-site example also distinguishes continued open activity from decay after decoupling. These results identify a dependency that a claim of local autonomous renewal would have to address. The history algebra separately gives executable boundary-renewal conditions and an inductive formation result. A finite reserve example shows why recurrent active counts can conceal a bounded continuation horizon. A constructive finite refinement then identifies the greatest boundary family admitting a choice of continuing episodes. Uniform lifting, fresh participant witnesses and historical anchors connect that boundary calculation to arbitrarily long finite concrete continuations under the assumptions. Requiring every admitted choice to preserve continuation can give a different answer, including an empty family where a selected cycle exists. Neither result establishes participants' ability or obligation to choose the modeled route.

These constructions remain several possible models. The polynomial potential is not a fitted learning law, its amplitudes are not competence quantities, and its minimum is not a normative ideal. The symbolic operation algebra represents specified histories and tests; it does not establish semantic adequacy or practical access merely by admitting a map. The chosen fixed sectors leave genuine historical and representational change to explicitly typed extensions. An eventual bridge between field activity and operation domains requires further substantive evidence.

Established linguistic, anthropological and learning research already supplies substantial accounts of socialization, participation, instruction and transmission. Prior Generative Relational manuscripts already develop formative capacity, generative return, historical anchors and changing operations. The present inquiry claims a bounded elaboration of participant renewal and its conditional formal dependencies within that inheritance. It does not establish a new general theory of language acquisition or an exhaustive account of cultural continuity. The worked inquiry illustrates that elaboration by changing its conclusions across baseline assessment, cue variation, supported learning, participant handover and target revision. Its fictional observations establish no empirical finding; they demonstrate which distinctions do inferential work and what actual evidence would have to discriminate.

Further work can test selected competence profiles, compare actual support arrangements and investigate whether the proposed nonlinear mechanism fits any identifiable setting. Other work can examine dormancy, recovery, evidence, authority and practical succession with their own targets. Those inquiries should retain the present paper's central discipline: specify whose capacity, which formative relation, what historical continuation and which evidential or normative claim are actually being made.

References

- [1] Noam Chomsky. *Aspects of the Theory of Syntax*. The MIT Press, Cambridge, MA, 1965.
- [2] Gergely Csibra and György Gergely. Natural pedagogy. *Trends in Cognitive Sciences*, 13(4):148–153, 2009.
- [3] Charles Goodwin. Professional vision. *American Anthropologist*, 96(3):606–633, 1994.
- [4] Charles Goodwin. The co-operative, transformative organization of human action and knowledge. *Journal of Pragmatics*, 46(1):8–23, 2013.
- [5] Wanhong Huang. A Preliminary Discussion of Normative Formation under Asymmetric Generative Capacity. Author-supplied working manuscript, consulted October 2026.
- [6] Wanhong Huang. Local Linguistic Convention Formation: A Preliminary Study of Shared-Use Availability. Author-supplied discussion draft, consulted October 2026.
- [7] Wanhong Huang. The Relational Reality of Artificial Intelligence under Ontological Uncertainty. Author-supplied working manuscript, consulted October 2026.
- [8] Wanhong Huang. Topology or Metaphor? Substrate Combinatorics, Emergent Invariants, and Formal Obligations for Models of Language and Subject Development. Author-supplied working manuscript, consulted October 2026.
- [9] Wanhong Huang. A preliminary field-theoretic formalism of power in Generative Relational theory. Author-supplied draft, 1 October 2026, 94 pages, 2026.
- [10] Wanhong Huang. A preliminary taxonomy of preservation in Generative Relational theory. Author-supplied Round 4 source package, 103-page manuscript, 2026.
- [11] Dell H. Hymes. On communicative competence. In John B. Pride and Janet Holmes, editors, *Sociolinguistics: Selected Readings*, pages 269–293. Penguin, Harmondsworth, 1972.
- [12] Simon Kirby, Hannah Cornish, and Kenny Smith. Cumulative cultural evolution in the laboratory: An experimental approach to the origins of structure in human language. *Proceedings of the National Academy of Sciences*, 105(31):10681–10686, 2008.
- [13] Patricia K. Kuhl, Feng-Ming Tsao, and Huei-Mei Liu. Foreign-language experience in infancy: Effects of short-term exposure and social interaction on phonetic learning. *Proceedings of the National Academy of Sciences*, 100(15):9096–9101, 2003.
- [14] Jean Lave and Etienne Wenger. *Situated Learning: Legitimate Peripheral Participation*. Cambridge University Press, Cambridge, 1991.
- [15] Elinor Ochs and Bambi B. Schieffelin. The theory of language socialization. In Alessandro Duranti, Elinor Ochs, and Bambi B. Schieffelin, editors, *The Handbook of Language Socialization*, pages 1–22. Wiley-Blackwell, 2011.
- [16] Barbara Rogoff, Ruth Paradise, Rebeca Mejía Arauz, Maricela Correa-Chávez, and Cathy Angelillo. Firsthand learning through intent participation. *Annual Review of Psychology*, 54:175–203, 2003.