

# Linguistic Recovery Record Sufficiency

## A Preliminary Study Of Feasible Intervention Selection

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### Abstract

A historical record can preserve credible evidence of a linguistic practice while leaving its users unable to choose a feasible recovery intervention. This preliminary study examines the distinctions required for that choice. It separates retained data, interpreted access, available operations, specified recovery targets and the authority to act. Within a Generative Relational architecture, compatible history-bearing configurations can share a record while admitting different successful continuations. A finite characterization requires a common successful policy on each accessible record class. The analysis distinguishes this requirement from reconstructing a complete history and from an already established factorization criterion for a prescribed rule. Examples show that pairwise policy compatibility can fail collectively and that adequate records need not possess a unique coarsest partition. An explicit finite-horizon inquiry model includes observation costs and changed operation domains. A nonlinear activity-support field then supplies an analytically tractable family of regional recovery choices: aggregate and regional records permit different intervention budgets, while sufficiently delayed observation can leave the standardized recovery route unavailable. Observable subalgebras and partial history operators expose the distinction between a record-dependent choice and an executable procedure. Primary documentation and reuse research constrains the linguistic interpretation. The resulting contribution is a scoped integration and worked investigation, with the empirical identification of support, interpretive access, acceptable recovery aims and public authority left as substantive obligations.

**Keywords:** linguistic recovery; documentation; record sufficiency; partial observation; nonlinear fields; observable algebra

## Scope Note

This preliminary discussion investigates the adequacy of an accessible historical record for a specified linguistic recovery choice. Its object is narrower than language preservation generally. A record may retain valuable evidence without enabling its intended users to choose a feasible intervention. Conversely, several unresolved histories may permit the same successful intervention. The relation between those possibilities is the organizing problem.

The philosophical analysis uses the Generative Relational research programme as a revisable language for historical objects, interpreted manifestations, operation domains and changing conditions of continuation. Earlier arguments about preservation, interpretive quotients, recovery corridors and participatory inquiry are inherited explicitly. The mathematical investigations reconstruct elementary finite decision criteria and established partial-observation methods, then examine a selected nonlinear regional field and its observable algebras. These are several possible models. Their proofs establish consequences of declared assumptions, while the interpretation of a linguistic target, the adequacy of evidence and the authority to intervene require independent arguments.

The manuscript distinguishes public-source findings, author-supplied prior work, constructed cases, conditional propositions and open research obligations. Its extensive exposition is intended for sustained discussion. It proposes neither a universal archive specification nor a rule that maximizing retained information is always desirable. The source packet preserves the bibliography verification, actual reading scopes, calculations and document checks.

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## Notices

**Status.** This discussion draft presents an evolving inquiry. The definitions, translations, formal constructions, empirical interpretations, and conclusions remain open to criticism and author review. It investigates the adequacy of accessible records for specified feasible linguistic recovery choices. Its formal models are conditional representations; the record criteria and regional fields establish neither a universal documentation standard nor an empirically calibrated intervention rule.

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**Statement On The Use Of Language Models.** The author selected the research direction, supplied prior manuscripts and a reference exposition, and requested consistency with the Generative Relational research programme. AI-assisted research, source comparison, drafting, formal checking, and document preparation contributed substantially to this draft. These contributions include proposed arguments and interpretations that remain subject to the author's review. Preparation of a checked draft does not establish the author's approval of every claim. Authorship credit remains with the human author; the use of these tools is disclosed to support critical assessment of the manuscript and its limitations.

# 1 Recovery Record Adequacy

The object of this paper is the adequacy of an accessible record for selecting a feasible linguistic recovery intervention. The problem arises when retained materials support several interpretations of an interrupted practice and those interpretations imply different usable routes. A record can accurately preserve an earlier utterance, its transcription and its immediate setting while omitting which current relations would allow a selected operation to be used again. Establishing that a recovery route exists somewhere in the compatible histories is then insufficient for choosing what to do. The inquiry concerns the distinctions on which a justified choice depends, together with the conditions under which those distinctions can be obtained and acted upon.

Consider a constructed narrative-practice inquiry. Archived examples mark a contrast between an interrupted attempt and a completed event. A group wishes to support renewed use of that contrast in a specified practice. The archive alone may leave unresolved whether current participants need a familiar responsive partner, a different presentation format, or instruction that would form an operation they presently lack. Those are distinct interventions and potentially distinct descriptions of recovery. An available facilitator has limited time. A route that would succeed if the relevant condition were known can be unusable when the facilitator cannot determine which condition obtains. Alternatively, a common arrangement might accommodate several of the possibilities without resolving them. The practical force of a retained distinction depends on this relation between information and available action.

## 1.1 The Selected Analytical Object

Recovery is used here as a task description requiring further specification. It can concern renewed execution of a retained operation, renewed access to a practice, or another explicitly defended relation to an earlier formation. The taxonomy of preservation already distinguishes retained structures, capacities, enabling conditions, realizations and historical claims [7]. The present paper does not settle those distinctions by calling every desirable future practice a recovery. Each investigation must identify its bearer, selected operation, acceptable continuation and evidence of the relevant earlier condition. Its record question is evaluated only after that specification.

A record is likewise a situated object. Stored bytes, public catalogue fields, a participant's readable materials and an analyst's inferred state can have different contents. A distinction formally encoded in a file can remain unavailable to the person expected to act. The record used in the finite models is explicitly an accessible, interpreted readout. This makes the mathematical question precise while preserving an empirical obligation: the claimed access and interpretation must actually be supported. The later algebraic construction will distinguish the information contained in a record from an executable procedure for using it.

The study concentrates on feasible intervention selection rather than optimal selection. Several protocols can satisfy a declared target. They need not have equal costs, equal cultural standing or equal desirability. For the selected task, however, each can be admitted as an acceptable candidate after independently stated constraints are applied. The simplest criterion asks whether a participant can choose one of these candidates successfully across the possibilities still compatible with the record. An optimization problem would require further orderings, loss functions or comparisons. Their absence is deliberate because the philosophical difficulty already arises before a single metric of preferred recovery has been justified.

## 1.2 Inherited Arguments

The prior interpretive-criteria manuscript already proves that a fixed prescribed revision rule can be implemented from a retained record exactly when it is constant on that record's classes [8, Section 8]. It also distinguishes a formally sufficient statistic from an independently collectible one, acknowledges set-valued interpretive alternatives, and discusses privacy and unequal access. The procedural-justice manuscript already develops readout-relative recovery, participant-relevant operation repertoires and

path-sensitive repair corridors [5, Section 6]. These are substantial antecedents. Renaming their components would supply little reason for another paper.

The current investigation therefore has a narrower purpose. It considers a relation of acceptable feasible policies, rather than one already prescribed output. It develops the consequences of choosing a common member of that relation on a record class, examines acquisition of information as an actual historical intervention, and derives a regional nonlinear example whose successful choices can be characterized exactly. The finite intersection arguments are elementary. The observation-based recursion belongs to an established mathematical tradition discussed below. The proposed contribution is their explicit coordination around this selected linguistic question, including the assumptions that prevent a formal selector from being mistaken for a ready practical instruction.

The distinction also preserves boundaries between the current language papers. Participatory linguistic evidence examined how inquiry can disclose, teach or change its object [10]. The reactivation paper studied the coupling of activity and enabling support after finite initiating input [9]. Here those arguments constrain what can be inferred from a recovery record. The paper does not repeat a general account of reactivation or expand into a complete normative theory of documentation.

### 1.3 Argument Structure

The first part compares relevant documentation and decision research and develops the philosophical conditions of a specified recovery choice. The historical architecture then places records and operations inside a selected relational spacetime. Finite policy families make the quantifier problem explicit: individual recoverability across several possible histories differs from one policy that succeeds while those histories remain unresolved. The partition analysis examines the limits of pairwise compatibility and of a unique minimum record.

The second formal part treats inquiry as a branching sequence of executable acts. Its information states retain possible current configurations, including time and resources. A nonlinear regional field supplies a different, complementary investigation. The regional construction begins with inherited activity-support dynamics and derives which localized initiating episodes succeed. Aggregate records and regional records then impose different intervention requirements. A separate delay calculation shows why increasing descriptive resolution need not preserve a route already lost during inquiry.

The final parts interpret record-dependent decision operators, examine documented reuse cases, develop objections, and return to the constructed narrative inquiry. The formal results remain conditional and the cases remain differentiated by evidential status. No equation turns a specified target into an obligation, nor does a successful intervention establish that its participants regard the resulting practice as the continuation they sought. Those limits identify the remaining philosophical work rather than supplying a general reason to avoid formal investigation.

## 2 Recovery Record Research Interfaces

The sufficiency of a linguistic recovery record connects several established inquiries that give different meanings to useful information. Documentary linguistics investigates materials and contextual resources that can support subsequent use. Language reclamation raises questions about who defines that use and what relationships it serves. Statistical decision theory compares experiments through the decisions they support, while imperfect-information games and control theory examine action through restricted observations. This section identifies the particular contribution each inquiry makes to the present problem. Its method is comparative: preserve the original question, specify the proposed transfer, and state the additional assumptions that the transfer requires. The result is a bounded position for a recovery-policy investigation, rather than a general theory of archival value.

## 2.1 Prospective Documentary Reuse

Himmelman's distinction between documentary and descriptive linguistics places primary linguistic data at the center of documentation, with analytic commentary related to the recorded material. His programme attends to situated communicative events, elicitation circumstances, transcription, translation, and uncertainty. Its orientation toward further uses extends beyond examples organized to support one current analysis [4, pp. 168–170]. This is a methodological proposal about documentation's organization and scope, rather than an empirical guarantee of successful future recovery.

The present task accepts that prospective orientation while asking a more restricted question. Suppose a retained collection can support several interpretations of a past practice. A participant now needs to choose an available recovery episode. The relevant test concerns which distinctions in the collection bear on that choice. Two interpretations may disagree about historical details while permitting the same episode; another disagreement may concern a support condition without which the episode fails. A general description of documentary richness does not yet distinguish those possibilities.

This restriction changes the form of the sufficiency claim. A record can be adequate for arranging one learning encounter without being adequate for reconstructing its complete historical antecedents. Its success is indexed to the selected encounter and the participant's feasible alternatives. It follows that adequacy for that task gives no immediate reason to erase the additional material. A later attribution inquiry could need the distinction that the current policy can ignore. Retention and present selection have partially different objects, even when both depend on the same collection.

An objection is that fixing one task prematurely limits the openness that makes documentation valuable. The objection applies to treating the present criterion as a complete archival policy. It has less force against using the criterion to test an explicitly selected commitment. A project might undertake to support a particular practice while retaining other materials for unspecified uses. The narrow test asks whether that undertaking is supported by the accessible evidence and action menu. It can expose an unsupported promise without prescribing the final boundaries of the archive.

Task specification also gives the negative result a useful form. Failure need not mean that the documentation is empty, inaccurate, or culturally unimportant. It can mean that two currently admissible situations remain indistinguishable although the available intervention must differ between them. A proposed repair then concerns a particular missing discriminator or an enlarged action menu. The analysis becomes more informative when it identifies that obstruction than when it ranks whole archives on a single scale of completeness.

## 2.2 Audience-Specific Usability

Bird and Simons distinguish seven dimensions of portability, including content, discovery, access, preservation, and rights. Their discussion separates documentary coverage from accountability to sources and the interpretation of terminology. It also distinguishes finding relevant material from obtaining it through practically available access arrangements [1, pp. 562–563, 566]. Their recommendations concern conditions for portable documentation; they do not establish that a portable resource determines a successful intervention for a specified learner.

Woodbury's audience-oriented proposals explain why corpus intelligibility can require a guide to the collection, its circumstances, conventions, and purposes. His discussion of project participants preserves different understandings of what the same collection is for [15, pp. 19–25]. The proposal therefore concerns interpretive orientation as well as access to individual records. It does not report a controlled evaluation showing that a corpus guide produces a particular learning or recovery outcome.

Nathan's account distinguishes access dimensions that include discovery, delivery, intelligibility, and user experience. His historical discussion of the Endangered Languages Archive also connects access arrangements with restrictions and audience-specific presentations [12]. These are descriptions and proposals concerning a particular archival setting at the time of writing; they supply no claim about its current interface or the present availability of individual materials.

For recovery-policy analysis, these distinctions locate several places at which an apparently informative record can fail to become usable evidence. A retained symbol may lack a decoding convention available to the participant. A decoded assertion may leave its application to the current situation unclear. A correctly interpreted recommendation may prescribe an unavailable act. These are different failures because different repairs would address them. A legend can repair missing notation; a present inquiry can address temporal application; additional practical support can enlarge the action menu. More bytes alone do not specify which repair is required.

The converse deserves equal attention. A concise accessible instruction can be enough for one action even when its recipient cannot reconstruct the expert's entire interpretation. Consider an instruction whose warrant has been checked through a procedure the participant can reasonably rely on. If the selected task requires executing that instruction and its practical conditions hold, demanding complete reproduction of the expert analysis would impose a stronger requirement. Whether such reliance is warranted is a further evidential question. The example only separates the amount of information needed to select an action from the information needed to audit its whole derivation.

The appropriate analytical unit is consequently an access arrangement with a declared participant position. That position specifies which records, conventions, interlocutors, and executable operations can actually be used. This is a proposal for the present model, not a claim that documentary linguistics already reduces accessibility to a partition of states. A partition represents only the distinctions that the chosen access arrangement makes available for the particular decision. It leaves the provenance, quality of interpretation, and legitimacy of access to their own assessment.

## 2.3 Recovery Purpose Formation

Leonard argues that language reclamation should engage community epistemologies and definitions of language, rather than silently imposing an externally selected object. His practitioner and interview discussion also recognizes heterogeneous positions and complicates a simple linguist–community division [11, pp. 15–17, 20–23]. The argument supports scrutiny of the inquiry's target and categories. It supplies neither a single community definition nor a quantitative estimate of how frequently particular priorities occur.

The Mangarrayi resource proposal developed by Richards and colleagues addresses archival audio, learner-relevant organization, contextual function, and practical constraints. Its consultation and design discussion provides a concrete setting for examining those distinctions [14, pp. 260–262, 271–274]. Section 12 develops that comparison while keeping the proposed resources separate from measured recovery outcomes.

A formal recovery task begins after some target has been specified, but the target's origin remains consequential. Recovering a selected grammatical operation, preparing an encounter, supporting a teaching practice, and producing a historical edition require different success conditions. Evidence sufficient for one can leave another unresolved. A model that fixes the first target cannot establish that it represents the participants' overriding purpose merely because its variables are mathematically well defined.

The distinction allows a local analysis without pretending that aims are permanently settled. During one comparison, participants and investigators can hold an agreed target fixed and examine the records needed to pursue it. If subsequent discussion changes the target, the adequate-record judgment must be recomputed relative to the revised task. The earlier result can remain correct for its earlier specification while becoming irrelevant to the new one. Target revision is therefore a change in the question, not automatically a failure of the earlier proof.

A second issue concerns the level of agreement required. Different participants can accept the same immediate episode for different reasons. A common feasible policy can then be meaningful without a shared comprehensive account of language or recovery. Conversely, apparently identical verbal aims can hide different acceptable burdens, timescales, or forms of participation. The formal declaration should identify the practical commitments that success requires, rather than infer their agreement from a common label.

This treatment gives normative criticism a precise point of entry. A critic can challenge the target, the admissible costs, the represented participants, or the exclusion of an alternative. The mathematical result answers a conditional question within those choices. Its evidential value increases when the choices are explicit enough to be contested; mathematical completeness does not remove that contestability.

## 2.4 Decision-Relevant Experiment Comparison

Blackwell compares experiments through the attainable expected-loss vectors they support across decision problems. The finite-experiment results relate this comparison to stochastic transformations of experiments [2, pp. 265–267]. The quantification across decision problems is essential: an experiment’s adequacy for one selected act is weaker than a comparison established over the relevant class of losses and decisions. The original argument is an established mathematical point of comparison, not a result introduced here.

The recovery question in this paper uses a more restricted declaration. It fixes a participant, a target, and feasible policies, then asks whether a record permits an acceptable choice throughout its admissible possibilities. The target may be represented by a success condition without assigning probabilities to the possible histories or numerical losses to each outcome. This robust question is useful when the aim is a guarantee relative to the stated domain. It does not rule out expected-loss analysis where a probability model and loss interpretation can be defended.

The two comparisons can disagree in an intelligible way. A coarse record may support a policy that succeeds in every situation admitted by one task. A finer record may nevertheless matter greatly for another task, including one that penalizes a burden ignored by the first. Calling the coarse record sufficient makes a claim about the first task. It leaves the broader ordering of experiments unanswered. Conversely, a record that improves some expected decision outcome can still leave a worst-case success requirement unsatisfied. Different quantifiers carry different practical commitments.

The distinction also clarifies the role of several acceptable actions. A record need not identify which intervention an omniscient planner would prefer if the task only requires one acceptable intervention. Suppose two histories permit different preferred routes but share a feasible route that meets the declared target. Their distinction can matter to preference while being unnecessary for that target’s bare achievement. Adding a cost or burden objective can make it relevant again. A meaningful sufficiency statement should therefore specify whether it concerns feasibility, optimality, fairness, or another criterion.

No hierarchy among those criteria follows merely from their mathematical form. A guaranteed feasible intervention can be undesirable by an independently justified measure. An optimal expected decision can expose a participant to an unacceptable exceptional outcome. The present manuscript selects robust task achievement to isolate one information problem. The selection is a scope decision whose suitability depends on the intended application, rather than a general recommendation for language work.

## 2.5 Observation-Consistent Action

Chatterjee and colleagues formulate observation-based strategies for imperfect-information games and develop a subset construction for sure winning. The information-set update follows possible transitions and then restricts them to the received observation [3, Sections 2–3.1]. These established constructions already distinguish a strategy available through observation from separate favorable choices at individually specified states. Their objective and observation assumptions must be checked before transferring a result to a different recovery model.

For the present inquiry, this comparison prevents a recurrent logical shortcut. An investigator may know that each admissible history possesses a successful intervention and conclude that the participant can recover the target. That conclusion requires an additional selection argument. If the participant receives the same evidence in the relevant histories, the first choice must respect that sameness. Separate witnesses of success do not supply a choice that the participant can make.

A contingent policy changes the problem by admitting further observations. A participant can begin with an inquiry, branch according to its result, and choose a later intervention. The inquiry belongs to

the policy, so its availability, possible outcomes, duration, and effects belong to the model. Treating its result as free information would change the task. A trial that teaches a distinction or consumes the only available encounter alters the situation that the later policy must address.

There are also different stopping questions. Reaching a target and knowing that the target has been reached need not coincide if the observations leave the target unresolved. A fixed-horizon assessment can evaluate success externally at its endpoint. A participant-directed stopping rule may require an available certificate or an observation that supports termination. The manuscript must state which problem it studies before importing a reachability construction or claiming that a successful policy can announce its own success.

The informational state used in a finite model is consequently a conditional summary of the declared possibilities, not the participant's entire understanding. Its update is sound only relative to the transition and observation descriptions supplied. An omitted historical possibility, a mistaken interpretation of an observation, or an unavailable action can invalidate an application while leaving the abstract calculation internally correct. The value of the construction is to expose these dependencies in an inspectable form.

## 2.6 Executable Abstract Policies

Reissig, Weber, and Rungger introduce feedback refinement relations with explicit admissible-input and successor conditions. Under the required relation and specification assumptions, controllers can be refined from a symbolic system to the represented system [13, Definition V.2; Theorems V.4 and VI.3]. This addresses an implementation problem beyond abstract state comparison. An arbitrary record partition does not satisfy those conditions merely by being called an abstraction.

The present linguistic model has an analogous proof obligation at its own declared level. If a formal action denotes a supported encounter, the application must identify who can organize it, which conditions permit it, and what range of outcomes is represented. A successful symbolic sequence is evidence about the actual task only through a warranted connection between those actions and their realizations. Naming the same act at both levels does not supply that connection.

There are two useful forms of modesty here. A mathematical example can establish a conditional obstruction without claiming a measured linguistic law. If its assumptions describe an admissible family of configurations, it shows exactly how a chosen record can conceal an intervention-relevant difference in that family. An empirical application has a stronger burden: it must support the membership of actual configurations in the family and the stated action consequences. The example and the application have different evidential standing.

This distinction is especially important for the nonlinear field investigation. Its activity and support variables represent selected channels under an illustrative law. The recovery-policy comparison concerns which regional interventions succeed under that law and budget. It does not identify linguistic competence with a coordinate, or establish that a community follows the chosen differential equations. The field earns its role by producing a substantive, derived family of success sets that can be tested against different records. Its empirical realization remains an additional research task.

## 2.7 Inherited Analytical Boundary

The closest prior result within Generative Relational work already concerns implementation through retained records. The criterion-comparison manuscript proves that a fixed revision rule factors through a record precisely when the rule is constant on the record's fibers. It also distinguishes a formal record containing the desired answer from an executable procedure for acquiring that answer, and derives the joint record needed for a fixed family of prescribed revisions [8, pp. 10–11]. The present paper therefore inherits an explicit information constraint rather than introducing the general idea that available distinctions limit action.

The current investigation permits several acceptable policies for a history. A record can be adequate when its possibilities share one acceptable policy, even if they disagree on other acceptable or preferred policies. That change makes the common-success relation central. It also raises a design question that the

fixed-answer factorization settles differently: several adequate records may support different selections without admitting one uniquely coarsest adequate record. The formal sections investigate that possibility within stated finite and dynamical models.

The paper's contribution should be assessed at this limited scale. It connects a recovery task, an actual access arrangement, policy feasibility, and histories whose support structure can affect success. The reviewed mathematical traditions supply established methods and caution against inflated novelty. The documentary and reclamation discussions supply reasons to keep the task's purpose, interpretability, and practical realization visible. A useful synthesis preserves those interfaces while producing conditional results that can be challenged through their assumptions. It does not absorb archival practice, participant authority, or language vitality into a single sufficiency predicate.

### 3 Participant Recovery Choices

A recovery record is adequate for a choice when the distinctions it makes accessible permit specified participants to select a feasible way of achieving a declared recovery target. This formulation connects information to action without requiring a complete reconstruction of the underlying history. The present section establishes the choice problem before its formal representation. It specifies the target, the available repertoire and the histories against which a choice is assessed, then distinguishes a common successful policy from separate routes that participants cannot identify. The argument develops a bounded application of the inherited recovery-corridor and record-implementation accounts [5, Section 6][8, Section 8].

#### 3.1 The Selected Recovery Target

Recovery requires a criterion for what is to become available again. The target can be a selected interpretive operation, a supported exchange or a capacity to continue a practice across further occasions. Recovering a recording is a different target from recovering competent use of what it records. Restoring an accessible room is different from restoring the linguistic contribution needed within it. Each can matter, but their success conditions should be stated separately.

The taxonomy already represents capacity recovery and condition recovery through distinct target classes and admissible histories [7, Section 9]. Those constructions allow the later realization to differ from the earlier one according to a declared comparison. A changed medium, a new partner or a reorganized support arrangement can therefore belong to recovery. Exact restoration of every earlier condition would impose a stronger target whose justification cannot be assumed.

Consider a constructed narrative practice in three districts. Its selected operation distinguishes a completed event from an interrupted attempt, permits requests for clarification and supports correction of misleading accounts. An archive contains examples of the practice. Participants wish to resume a recurrent setting in which the selected operation can be used with unfamiliar accounts. The example assigns no empirical claim to an actual language or community.

For the reactivation branch of this inquiry, assume that an independent dossier supports retention of the relevant contribution among the participating narrators. The archive's existence does not establish that premise. A different branch could concern instruction or reconstruction using inherited materials. The present choice analysis can represent either branch, provided that its target and available interventions are interpreted accordingly. It should preserve the distinction when reporting what a successful programme achieved.

The proposed target also needs a regional scope. Establishing a usable practice in one district differs from providing every district with access. A facilitator budget might permit the first while leaving the second infeasible. Success under the first target cannot be reported as success under the second merely because both use the same language name. The formal analysis should preserve the target selected for the actual comparison.

### 3.2 Available Recovery Policies

A recovery policy specifies what participants can do using the information available at the relevant stages. A simple policy selects a fixed intervention from a familiar menu. A contingent policy can first obtain a permitted observation and then choose an intervention according to its result. The word policy therefore concerns an executable arrangement of choices, rather than an aspiration such as support recovery wherever needed.

The repertoire is part of the inquiry. It includes the interventions participants know how to carry out, the actors whose contributions are available, and the resources and time required. A procedure that an analyst can describe may fall outside that repertoire. An intervention requiring a departed specialist, an inaccessible recording convention or an unaccepted commitment cannot enter the participant's feasible menu merely through inclusion in a mathematical list.

In the narrative example, available interventions might include a limited number of facilitated sessions at identified support interfaces. The participants understand what each session provides and what continuing arrangements it assumes. The record may leave uncertain which interfaces retain sufficient support for later practice. A session's prospects can therefore differ across histories compatible with the archive.

The distinction between an operation label and its realization matters here. Sending a facilitator to a district is an administrative description. Its practical realization includes access, a suitable encounter, interpretable materials and the relevant participant contributions. A policy that succeeds only because its label silently includes whatever assistance each hidden situation needs gives the decision-maker information or capabilities that have not been established. The available intervention must retain the same declared meaning across the situations in which it is compared.

The procedural account already makes reachability relative to a chosen update repertoire and warns against enlarging it through merely imaginable interventions [5, pp. 26–29]. The present application adds an information constraint: participants must be able to select and execute a suitable policy using their actual record. A repertoire can contain several successful routes while the record leaves their appropriate selection unresolved.

### 3.3 Admitted Histories

The choice problem is evaluated against histories still supported as possibilities by the available evidence. These histories can differ in present support, earlier instruction, access arrangements or the effects of a proposed intervention. They should include the distinctions relevant to the selected target and policy class. They are an evidentially warranted model of uncertainty, rather than a collection of every conceivable world.

For example, equal archived response counts can coexist with different current support arrangements. One district may retain a responsive partner network; another may have recordings but no usable means of convening an encounter. Treating these as rival hypotheses requires reasons drawn from the actual record and inquiry. Simply multiplying hypothetical failures would make every practical choice appear impossible, while excluding an inconvenient documented possibility would make adequacy too easy to obtain.

The temporal reference should remain explicit. An archive can describe support at the time of recording while the recovery choice concerns its present availability. A hypothesis connecting the earlier record to current conditions is therefore part of the account. The distinction can matter even if the file is perfectly preserved and its coding fully understood. Correct information about an earlier arrangement does not automatically settle which intervention is executable now.

Agreement with a record also differs from endorsement by the investigator. Several histories can remain unresolved without being equally likely or equally well supported. The robust criterion studied here asks whether one policy succeeds across the declared family. It does not require treating its members as equally probable. A probabilistic choice rule would introduce another question and would need its own evidential commitments.

The family is revisable. New testimony, a discovered annotation or a changed support relation can alter the admitted histories. Such revision may change which record distinctions are adequate. The resulting change is intelligible when the report identifies whether it concerns new knowledge of the earlier configuration or a transformation of the configuration on which action must now operate.

### **3.4 Common Successful Choice**

The basic positive possibility is that several different histories admit the same successful policy. The participants can then choose that policy without identifying which history is actual. A record can be sufficient for this decision while remaining incomplete for historical explanation. This is an important achievement when further identification is costly, unavailable or irrelevant to the selected action.

Suppose two support histories differ in who originally organized a narrative archive, but the same available session and follow-up arrangement succeeds in both. That provenance difference may matter for attribution or responsibility. It need not prevent the immediate recovery choice. A record's adequacy for that choice consequently supplies no reason to erase the provenance; it identifies a narrower task for which the distinction is unnecessary.

The opposite pattern is equally possible. Each admitted history can possess some successful route while the successful routes have no common member. The existence of recovery in each history then leaves the participant's selection problem unresolved. Choosing a route separately for each hidden history would use precisely the discrimination that the record does not supply. The formal construction later makes that difference exact.

A contingent policy can sometimes supply the common member. Participants may use the same preliminary inquiry and branch on its accessible result, even though their final interventions differ. What remains common is the executable rule for using observations. The inquiry itself must be available across the histories where the policy calls for it, and its consequences must leave the subsequent branch feasible.

This criterion does not require identical sets of successful policies. Two histories may share one successful option while offering different additional alternatives. Requiring the whole option sets to agree would answer a stronger question. The current criterion concerns at least one feasible choice; informed comparison among several options can require a richer record and belongs in the interpretation of the result.

### **3.5 Resource-Relative Adequacy**

The information needed for a choice can change when the available resources change. If participants can facilitate every relevant interface, an aggregate record may support a successful plan without locating the best-supported district. If only one session is available, the location can become decisive. The same archive can therefore be sufficient for one declared budget and insufficient for another.

This dependence explains why record sufficiency should be attributed to a configuration of record, task and action conditions. An archive does not acquire an unconditional sufficient status independent of its users. Nor does insufficient information automatically imply that collecting more detail is the appropriate response. Expanding the feasible menu, reorganizing support or changing a justified delivery constraint can alter the choice problem.

The direction of comparison also needs care. Additional resources can preserve earlier possibilities when the participants may leave them unused and their provision changes nothing else relevant. Actual provision can carry conditions, require travel or alter institutional commitments. Those effects belong in the historical account rather than being concealed inside a larger budget parameter. The formal monotonic comparison isolates the first situation.

The regional field investigation supplies a more specific realization of this resource dependence. Its sites represent support interfaces, and its localized inputs belong to a bounded intervention menu. The resulting informational conditions concern those channels and policies. Their linguistic significance depends on the independently stated realization connecting a successful field response to the selected operation.

### 3.6 Participant Agency

The availability of a common successful policy answers a feasibility question. Participants can still disagree about the aim, the distribution of work or the choice among several successful routes. A policy that reaches the target may burden one district disproportionately, use material beyond an agreed access arrangement or conflict with a participant's priorities. None of these questions is settled by the common-policy test.

The earlier procedural analysis separates structural repair from normative rectification and retains differences between histories sharing an endpoint [5, pp. 33–36]. That separation applies directly here. A record adequate to select one successful route can remain inadequate for evaluating who contributes, which alternatives are foreclosed or whether authorization is present. An inquiry can investigate these further questions without treating the initial feasibility result as false.

Participants may also decide to retain a recovery possibility without pursuing it immediately, or decline the proposed target. Their response does not establish that the record was inadequate or that their capacity is absent. It changes the practical undertaking. The present framework supports agency when it identifies usable options and their premises while leaving participants' authority and reasons visible. It becomes misleading if a sufficient record is treated as an instruction that recovery must proceed.

## 4 Accessible Recovery Records

A retained artifact supports recovery choice through an access arrangement that makes selected distinctions usable to participants. Its physical content, their interpretation of that content and the actions they can select are related but separable objects. This section develops that relation. It follows a record from acquisition and retention to interpretation, shared access and possible refinement, then examines the cost of obtaining further distinctions. The aim is a positive account of task-relative usability that preserves the value of archives without assigning them capabilities their access conditions do not support.

### 4.1 Retained Content

A recording preserves a selected presentation produced through particular circumstances and an acquisition protocol. The taxonomy distinguishes the manifestation, its derived trace and the provenance package retained with it [7, Section 5]. These distinctions prevent the content of a file from being equated with the complete encounter or with the relational organization that made the encounter possible.

In the narrative example, a transcript can retain a completed-event description and its correction. A recording may additionally retain hesitation, timing and the interlocutor's contribution. Metadata can identify whether a facilitator supplied the relevant distinction before the recorded response. Which elements matter depends on the inquiry. A transcript can be wholly adequate for one textual comparison while leaving the history of the participant's contribution unresolved.

Such selectivity does not make every record defective. Record production is ordinarily organized around some purpose and resolution. A later recovery task may need a distinction that the earlier purpose omitted, but that mismatch should be identified specifically. It is more informative to show how two action-relevant situations share the retained record than to declare archives generally incapable of preserving living practice.

Preserving additional material can maintain possibilities for later reinterpretation. Its value is conditional on an actual route of access and a defensible reading. A richer recording does not come with every future interpretation already understood. Conversely, a coarse summary cannot supply a missing discriminator merely because the investigator now has a more refined vocabulary. The required content must be retained somewhere or obtained through a further encounter.

The earlier criterion-comparison paper states this limit precisely: a prescribed revision can be computed from a record only if the record keeps apart cases assigned different required revisions [8, pp. 10–11]. The current policy-selection task permits several successful answers, but it inherits the same disci-

pline about the information actually available. Renaming an ambiguous summary cannot by itself repair an action-relevant ambiguity.

## 4.2 Participant Interpretation

Access begins with obtaining the relevant material and extends to using its distinctions. A file can exist in an archive while remaining undiscovered, unavailable through the participant's equipment or unintelligible under its coding convention. These conditions concern different parts of the access arrangement. Treating them as one binary property of the file would obscure which change could make the record useful.

Suppose the narrative archive uses a notation distinguishing an interrupted attempt from a completed event, but the participants have only a version from which the notation key is missing. The marks are retained. Their intended distinction is not yet accessible to those readers. Supplying a verified key can improve access without requiring a new recording of the original event. Whether the key is correct and sufficient for these readers remains part of the inquiry.

A participant may instead understand the convention but lack the contextual information needed to apply it to the current recovery choice. The archived annotation might identify a successful encounter without saying whether the partner network still exists. Decoding the annotation therefore does not identify the present support state. The missing relation concerns the record's temporal application, rather than a defect in the participant's literacy.

Interpretation can also be learned. A participant may initially overlook a recorded repair and later recognize it after discussion with an experienced interlocutor. The newly available discrimination can be grounded in an old trace together with acquired interpretive resources. This account does not imply that information has appeared from an exhausted summary. It identifies which retained material and which additional competence support the later reading.

The distinction matters to feasibility. If the interpretation can be acquired through a short available exchange, the associated inquiry may belong to the recovery policy. If learning the notation requires a lengthy course while the support arrangement is closing, formal decodability offers little evidence of timely access. The record should be assessed at the participant's actual interpretive and temporal position.

## 4.3 Distributed Access

The access arrangement can involve several people. One participant may understand an annotation, another know the current support history, and a third possess the means of arranging a suitable encounter. A collective choice can use their complementary contributions when communication and coordination are available. No individual needs to contain the whole record for such a policy to be feasible.

The collective claim nevertheless requires a bridge. Separate possession of complementary files does not establish that their holders can communicate, agree on their interpretation or act jointly within the available time. An analyst who combines every record can generate a selector unavailable to the actual participants. Its apparent success would reflect the analyst's enlarged access rather than the adequacy of the participants' arrangement.

In the three-district example, each district may know its local support condition. A central aggregate report can conceal which district is prepared, while an agreed local reporting procedure supplies enough information to allocate a facilitator. The reporting procedure is an additional capability. Its feasibility depends on communication, an understood report format and the opportunity to respond before allocation.

There can also be reasons to limit disclosure. A district might communicate that a session can be supported without sharing private details about particular participants. For the selected allocation task, a verified readiness report could be sufficient. Another task, such as investigating burdens or contested exclusion, might require different evidence and authority. The limited report's adequacy should therefore be evaluated against its stated purpose rather than a presumption that all underlying information must be centralized.

Distributed access illustrates the relational character of the record. Its practical content depends on the relations through which distinctions become available for action. This does not dissolve the artifact into an unlimited context. It identifies the specific links needed for the declared task and leaves other contextual relations outside the current claim.

#### 4.4 Alternative Adequate Records

Several different records can support a successful recovery choice. One can locate a prepared interface directly. Another can identify a small group of interfaces guaranteed to contain a prepared one, provided that the available budget covers the group. Their retained distinctions differ, yet each can be adequate relative to its policy menu.

This possibility complicates the search for a minimal record. The question is not always to preserve a predetermined answer. When several policies are acceptable, a record can be adequate by supporting one common policy within each of its information classes. Different records can support different selections. Coarsening them together can remove the distinctions on which either selection depends.

The result differs from a joint record designed to preserve a fixed family of answers. The criterion paper already derives the coarsest identification preserving every prescribed update in such a family [8, p. 11]. That construction fixes what every answer must be. The present task asks whether some successful answer remains selectable. It therefore has a different adequacy condition, and the existence of a canonical coarsest record requires its own argument.

An absence of a unique coarsest adequate record is not itself a practical failure. It can expose legitimate design alternatives. Participants may prefer one record because it is easier to collect, another because it discloses less, or a third because it supports an anticipated further task. These preferences require reasons beyond the existence of a successful selector, but the formal comparison can make the alternatives precise.

Minimality should consequently remain relative to the task, feasible policy class and chosen ordering of records. The smallest file, the least interpretive burden and the fewest distinctions need not coincide. A mathematically coarse label can require a demanding procedure to compute. An apparently verbose instruction can be easier for participants to follow than an encoded summary. Informational comparison does not settle these practical costs.

#### 4.5 Inquiry Costs

Further inquiry can make a record more discriminating while changing the action problem. Participants spend time reading, locating a witness, asking questions or conducting a trial encounter. During that interval, support can decline, an available session can be used up or a commitment can change. The later choice occurs in the configuration produced by that history.

This is the relevant difference between free refinement and actual acquisition. If a finer record is supplied without changing the task, resources or action possibilities, and the additional distinctions can be ignored, an earlier successful policy remains available. The informational comparison is then monotone. It does not describe an inquiry whose costs remove the possibility of executing that policy.

Suppose a precise account of the originally prepared district becomes available only after its responsive partner has left. The information can be historically correct and more detailed than the earlier aggregate report. It may nevertheless fail to support the original localized intervention. The proper conclusion concerns the changed configuration: the old distinction has been learned after its earlier action relevance has expired.

The source's repair-corridor construction already retains delay, intermediate burdens and operation order rather than only terminal readouts [5, pp. 34–35]. Applying that discipline to recovery records means recording inquiry as part of the policy's history. A useful test asks whether each inquiry branch leaves a feasible continuation, not merely whether the inquiry eventually distinguishes the hypotheses.

Inquiry can also contribute directly to recovery. A trial session might teach an operation, organize a later encounter or strengthen an interpretation routine. Its result is then both evidence and a changed

practical situation. The policy may succeed through that combination. Reporting the process as pure identification would conceal part of the achievement and could support a false attribution of what participants already possessed beforehand.

#### 4.6 Future Uses

A record sufficient for a present recovery choice can remain valuable for further purposes that have not been formalized. Historical attribution, teaching, reinterpretation and contestation may need distinctions irrelevant to the current intervention. The current sufficiency result therefore supplies no general instruction to discard everything it does not use.

The reverse inference also needs restraint. Unspecified future usefulness does not establish that retaining every possible detail is feasible or justified. Some content cannot be acquired; some is burdensome to maintain or interpret; some is subject to restricted access. A responsible record design can identify supported uses, preserve important optional material where warranted and document which future questions remain unanswered.

The criteria paper already distinguishes a declared family of future revisions from unrestricted future demands and identifies the burdens of information retention [8, p. 11]. This paper adds a recovery-specific comparison: which accessible distinctions support an available policy before the relevant opportunities change? It does not replace the broader historical or normative purposes of documentation.

The resulting account treats sufficiency as a relation maintained through an access arrangement. A useful record keeps enough action-relevant distinctions available to its participants for the selected choice. Its adequacy can survive substantial historical uncertainty, change with resources, and depend on the timing of interpretation. Those features give the formal constructions a substantive object while preserving the additional questions that an archive can legitimately serve.

### 5 Historical Decision Architecture

A recovery decision relates a retained history, a present interpretive position and possible future operations. This section gives that relation a scoped Generative Relational representation. It first locates the object in a history-bearing spacetime, then introduces accessible records and partial operations, and finally identifies the finite hypothesis carrier used in the decision analysis. The architecture inherits the earlier preservation, participatory-inquiry and field-power formulations [7, 10, 6]. Its purpose is to preserve their historical distinctions while permitting a tractable choice problem.

#### 5.1 History-Bearing Decision Situations

Let  $\Sigma$  identify a selected historical cut. A represented configuration  $X_\Sigma$  contains the relevant participants, artifacts, practices and enabling relations through that cut. One possible representation uses a labelled two-complex  $K_\Sigma$  with vertex, edge and face sets  $K_\Sigma^{(0)}, K_\Sigma^{(1)}, K_\Sigma^{(2)}$ , together with endpoint and face-attaching maps. The label data  $\lambda_\Sigma$  identify the declared roles of these cells. Additional relational data  $\omega_\Sigma$  include operation conditions, interpretations, selected precedence and boundary dependencies. The compact notation is Equation 1.

$$X_\Sigma = (K_\Sigma, \lambda_\Sigma, \omega_\Sigma). \quad (1)$$

Equation 1 does not identify a person with a vertex. A participant can occupy an extended history-bearing region. A face can represent a specified comparison or joint realization without asserting that two histories are interchangeable. Incidence needs its own interpretation; temporal precedence does not follow from every edge. A spin-foam instantiation would require further representation labels, compatible boundary data and an amplitude or other declared rule. Those additions are unnecessary for the present choice result. The two-complex remains one available representation of the scoped object.

Suppose an intervention  $a$  extends the represented history to a cut  $\Sigma'$ . An embedding  $j_a : K_\Sigma \hookrightarrow K_{\Sigma'}$  can retain the selected earlier cells. If  $\iota_\Sigma : K_- \hookrightarrow K_\Sigma$  identifies an earlier anchor, the compatibility

$j_{a\iota\Sigma} = \iota_{\Sigma'}$  records its continued historical inclusion. That identity preserves a specified provenance. It establishes neither unchanged present capacities nor unchanged interpretation. Instruction can retain an earlier response in the history while changing what the participant can now do. The distinction is especially consequential when an information gathering episode itself supplies missing support.

## 5.2 Accessible Readouts

An investigator rarely possesses the complete represented history. Let  $z_p : X \mapsto z_p(X)$  denote the stored or retained material available through a declared access arrangement  $p$ , on the configurations where that arrangement is defined. Let  $d_p$  denote an independently justified interpretation of those materials. On the selected finite domain below, the accessible record is the composite in Equation 2.

$$\rho_p : H \longrightarrow R_p, \quad \rho_p(h) = d_p(z_p(X_h)), \quad R_p = \rho_p(H). \quad (2)$$

Here  $H$  is a finite family of compatible historical dossiers, and  $X_h$  is the configuration represented by dossier  $h$ . Equation 2 is used only when both component maps are defined for the admitted dossiers. An inaccessible file is not silently assigned its hidden contents. Where loss of access is itself observable, an explicit unavailable-record value can be introduced. Where interpretation is unresolved, the dossier family or the observation relation must retain that uncertainty.

The subscript  $p$  concerns the declared participant arrangement, not a timeless personal faculty. A change of collaborators, reading tools, permissions or interpretive experience can alter  $\rho_p$ . The same stored material can then induce different accessible distinctions. Conversely, different physical copies may yield the same interpreted record for the task. The mathematical equality of record values concerns this selected readout and does not identify the underlying manifestations or historical objects.

The finite model stipulates  $d_p$  as a functioning readout. It does not prove that participants can learn or execute it. Earlier inquiry work already distinguishes a record algebra from an inventory of usable skills [10, Sections 5–6]. If a formally available distinction takes weeks to interpret, its acquisition belongs to the intervention history. Section 8 treats that dependence explicitly.

## 5.3 Typed Operations

At each represented cut, an operation schema can select a partial transformation, as in the taxonomy's state-dependent operation grammar [7, Section 7]. A concrete represented action has a domain containing the configurations on which its prerequisites hold. Those prerequisites can include an available participant, a usable artifact, a competence premise, enough time and a specified permission. The interpretation of each prerequisite must be stated separately. Including permission in a model does not establish its legitimacy.

For countable selected families of history descriptions  $\mathcal{H}_0$  and  $\mathcal{H}_1$ , an executed branch can be represented by a partial append map  $U_a : D_a \subseteq \mathcal{H}_0 \rightarrow \mathcal{H}_1$ . Its output retains the source prefix and appends the event and relevant update. It is then injective on its domain by the chosen recording convention. Two different earlier histories can have the same endpoint readout while remaining different append histories. For a subsequent operation  $U_b : D_b \subseteq \mathcal{H}_1 \rightarrow \mathcal{H}_2$ , a composite operation is available precisely on the domain in Equation 3.

$$\text{Dom}(U_b U_a) = \{h \in D_a : U_a(h) \in D_b\}. \quad (3)$$

Equation 3 is an inherited partial-operation rule, not a new account of historical causation. It records a practical issue that an endpoint list can suppress. An interview might disclose the appropriate recovery route while exhausting the only session in which that route could be attempted. A permission request can alter which acts remain available. A declined request is a possible represented outcome when its occurrence belongs to the declared action; it must not be deleted to make the successful branches look certain.

An unresolved action can have several successors. Such an action is represented by a relation or branching schema until a branch is selected by an actual history. Treating it as a deterministic map in

Table 1: Historical Decision Objects

| Object     | Declared Role                          | Independent Obligation                            |
|------------|----------------------------------------|---------------------------------------------------|
| $X_\Sigma$ | Selected history-bearing configuration | Interpret cells, relations and boundaries         |
| $H$        | Warranted candidate dossiers           | Defend inclusion and relevant omissions           |
| $z_p$      | Retained accessible materials          | Verify custody, access and provenance             |
| $d_p$      | Interpreted readout                    | Establish participant use and decoding conditions |
| $D_a$      | Action domain                          | Defend operative prerequisites                    |
| $U_a$      | Executed historical extension          | Retain branch and intermediate conditions         |

advance would introduce a substantive premise. The adaptive construction uses nonempty successor sets, and its success requirement quantifies over all outcomes admitted by the chosen finite model.

#### 5.4 Epistemic Carriers

The finite set  $H$  in Equation 2 consists of interpretively warranted candidates for a specified inquiry. It is not an ontology of all possible worlds. Several dossiers can represent the same coarse activity trajectory while differing in retained support relations, operation provenance or permission conditions. Excluding a dossier therefore requires an evidential or explicit modeling reason. Including every imaginable history would usually make the criterion unusable without improving its warrant.

The candidate set is fixed during the static propositions. This permits a clear comparison of records on one common carrier. Revising the candidate family is a different step. New evidence might narrow it; discovery of an omitted mechanism might enlarge it. A record adequate on the original family can become inadequate after that enlargement. The conclusion would then be that the original guarantee depended on an insufficient model, not that the new observation mathematically destroyed a fixed-carrier implication.

The operational model below also requires a target predicate and a protocol library. Their construction is part of the inquiry. A field model can justify selected dynamical success sets, while historical evidence supports the claim that those outcomes instantiate the intended operation. Those two warrants have different objects. Treating a field trajectory as a sufficient description of linguistic recovery would omit the second.

Table 1 collects the major objects introduced here. The distinctions remain active in the later finite, field and algebraic representations.

## 6 Common Feasible Policies

The first formal investigation isolates a choice difficulty within a fixed task. Its purpose is to distinguish the existence of separate recovery routes from an available choice that remains successful across unresolved histories. The construction uses finite sets and an elementary intersection argument. It extends the application of the prior fixed-rule factorization criterion [8, Section 8] to a relation admitting several acceptable protocols, without claiming a new general theory of sufficient statistics.

### 6.1 A Specified Policy Family

Fix a nonempty finite dossier family  $H$ , a participant arrangement  $p$ , its accessible record  $\rho = \rho_p : H \rightarrow R$ , a declared target, and a time and resource regime. Let  $\Pi$  be a finite library of protocols that the

participant arrangement can identify and implement when their specified conditions hold. A protocol can be one act or a finite contingent procedure. Its name is an index for those instructions. The existence of a mathematical index alone supplies no instruction manual.

For  $h \in H$ , let  $W(h) \subseteq \Pi$  contain precisely the protocols that are executable from the configuration represented by  $h$ , satisfy the declared constraints at every relevant stage, and achieve the selected target for every admitted outcome. This is a sure-success interpretation within a specified model. A probabilistic criterion would require a different definition. Define  $W_\pi = \{h : \pi \in W(h)\}$  for each protocol  $\pi$ . Equation 4 records both sections of the same relation.

$$\mathcal{W} = \{(h, \pi) \in H \times \Pi : \pi \in W(h)\}, \quad W_\pi = \{h \in H : (h, \pi) \in \mathcal{W}\}. \quad (4)$$

Equation 4 does not derive the success relation. The relation may be supplied by a verified finite transition model or, for the regional example, by a nonlinear proof. An empirical application would require evidence linking the represented outcomes to the target. If no such warrant exists, a computed selector is conditional on an unsupported input relation.

A participant sees a record value  $r$ , not the hidden dossier label. Its compatible cell is  $C_r = \rho^{-1}(\{r\})$ . Since  $R = \rho(H)$ , each  $C_r$  is nonempty. A record-dependent choice is a map  $s : R \rightarrow \Pi$ . The operational condition on that choice is  $s(\rho(h)) \in W(h)$  for every admitted  $h$ . It permits different choices at different observed values, but forbids a hidden change of choice between dossiers that give the participant the same record.

**Definition 6.1** (Task-Adequate Record). For the fixed data  $(H, \Pi, \mathcal{W}, p)$ , an accessible record  $\rho$  is task-adequate when a participant-implementable record-dependent selector exists. Its finite information condition is given in Equation 5.

$$W(C_r) := \bigcap_{h \in C_r} W(h) \neq \emptyset \quad \text{for each } r \in R. \quad (5)$$

Definition 6.1 distinguishes the finite information condition from practical implementation. In the next proposition, implementation is an explicit premise: the finite lookup from a readable record value to a protocol index can be executed within the already specified conditions. Without that premise, Equation 5 establishes existence of a formal selector only.

**Proposition 6.2** (Finite Common-Policy Criterion). *Under the finite-carrier assumptions, Equation 5 holds if and only if there is a map  $s : R \rightarrow \Pi$  satisfying  $s(\rho(h)) \in W(h)$  for every  $h \in H$ . If the lookup and the selected protocols are implementable under the fixed regime, that map supplies a task-adequate choice.*

*Proof.* Suppose such an  $s$  exists. Fix  $r \in R$ . For each  $h \in C_r$ ,  $\rho(h) = r$ , so  $s(r) \in W(h)$ . The single value  $s(r)$  therefore belongs to their intersection. Conversely, suppose every intersection is nonempty. There are finitely many cells. Select one  $\pi_r \in W(C_r)$  for each of them and define  $s(r) = \pi_r$ . For an arbitrary  $h$ , its cell is  $C_{\rho(h)}$ , and membership in that cell's intersection implies  $s(\rho(h)) \in W(h)$ . The final operational statement follows only from the separate implementation premise.  $\square$

Proposition 6.2 makes the relevant quantifiers explicit. The statement that each  $h$  has some successful protocol permits the chosen protocol to depend on  $h$ . A participant with only  $\rho(h)$  cannot use that dependence unless the necessary distinction is available. The criterion requires a common protocol on each cell. It does not require the cells to be singletons or the participant to know every feature represented by the dossiers.

## 6.2 Prescribed Rules

If each  $W(h)$  is the singleton  $\{U(h)\}$  for a prescribed map  $U : H \rightarrow \Pi$ , the intersection condition says that  $U$  is constant on each record cell. Equivalently,  $U = s \circ \rho$  for some  $s$ . This recovers the prior

factorization argument. The present relation can be less demanding: two dossiers may have different sets of successful protocols while sharing one member. A record can support that common member without encoding the whole set  $W(h)$  or reconstructing  $h$ .

For example, take  $H = \{h_1, h_2\}$  and  $W(h_1) = \{\alpha, \beta\}$ ,  $W(h_2) = \{\beta, \gamma\}$ . The constant record is adequate because  $\beta$  works at both candidates. It does not reveal whether  $\alpha$  would work. A full policy-identification task would demand that distinction, but the present successful-choice task does not. This difference will matter in the observable algebra: deciding successfully can require less information than measuring every policy's success indicator.

Selecting a unique preferred rule in advance changes the task. If one declares  $U(h_1) = \alpha$  and  $U(h_2) = \gamma$ , the same constant record cannot implement that rule. The failure does not refute the feasibility of  $\beta$ ; it records the stronger requirement. A defensible preference ordering could justify such a change, but it must be stated rather than silently added to the meaning of sufficiency.

### 6.3 Fixed-Task Refinement

Let  $\rho' : H \rightarrow R'$  be another record with  $\rho = g \circ \rho'$  for some map  $g : R' \rightarrow R$ . This is a refinement: every  $\rho'$ -cell lies in a  $\rho$ -cell. If  $\rho$  is adequate under the finite criterion, then  $\rho'$  is adequate as well. For each finer cell, the successful protocol chosen for the containing coarse cell remains successful at every member. Equivalently the selector is  $s \circ g$ . This consequence follows directly from Proposition 6.2.

The conclusion keeps  $H, \Pi, \mathcal{W}$  and the implementation conditions fixed. It compares already available records. It does not say that obtaining a refinement is free. A query can consume time, alter relations or withdraw an action from the available library. In that case the comparison concerns a successor task with changed data. Applying fixed-task monotonicity across that change would conceal the event whose consequences are at issue.

Adequacy is also relative to the admitted target. Tightening the target can shrink each  $W(h)$ ; adding a genuinely executable protocol can enlarge them. Neither alteration is simply a change in informational richness. This is why the field example records an intervention budget together with its observation regime. The same coarse record can be adequate with a larger permitted batch and inadequate with a smaller one, even though its descriptive contents remain identical.

## 7 Adequate Record Partitions

Different records can support the same recovery task while retaining different distinctions. This section examines the geometry of that possibility on a fixed finite carrier. It derives a family of admissible record cells, gives a collective compatibility obstruction, and distinguishes minimal adequate records from a unique coarsest one. The arguments concern the common-policy condition, with practical implementation retained as the independent premise stated in Section 6.

### 7.1 Admissible Record Cells

Keep the dossier family  $H$ , protocol set  $\Pi$  and success relation  $\mathcal{W}$ . Assume  $W(h) \neq \emptyset$  for each  $h$  when comparing adequate partitions of the whole carrier. If a dossier has no successful protocol, even its singleton cell fails the selected task; no refinement can repair that absence of a route. Let  $\mathcal{C}_{\mathcal{W}}$  be the family in Equation 6.

$$\mathcal{C}_{\mathcal{W}} = \{\emptyset\} \cup \{C \subseteq H : C \neq \emptyset, \bigcap_{h \in C} W(h) \neq \emptyset\} = \bigcup_{\pi \in \Pi} \mathcal{P}(W_{\pi}). \quad (6)$$

In Equation 6,  $\mathcal{P}(W_{\pi})$  is the power set of  $W_{\pi}$ . The equality follows because a nonempty cell has a common protocol exactly when it is contained in some  $W_{\pi}$ . The empty set is included for the mathematical family only; it is never treated as a possible observed record value.

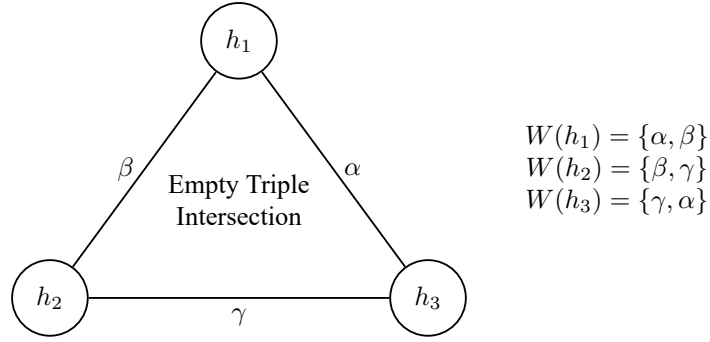


Figure 1: Collective Policy Compatibility. Each edge identifies a protocol that succeeds for its two endpoint dossiers. The unfilled triangle records the absence of a protocol for all three. Edges represent shared feasible choices, not causal relations or linguistic similarity.

This family is downward closed: if a protocol succeeds on  $C$ , it succeeds on every subset of  $C$ . With the singleton assumption it can therefore be represented as a finite abstract simplicial complex on vertex set  $H$ . The term identifies a combinatorial property, not a physical shape or the topology of a community. A record is adequate precisely when every cell of its induced partition belongs to this family. The historical two-complex and this finite compatibility complex have different objects and different interpretations.

The construction organizes a useful distinction. A record partition requires disjoint exhaustive cells because one observation has one realized value. The successful domains  $W_\pi$  can overlap. Converting that overlapping cover into a partition requires assigning shared dossiers to particular record classes. Several assignments can work, and none need be more informative than all the others.

## 7.2 Collective Compatibility

Take  $H = \{h_1, h_2, h_3\}$ ,  $\Pi = \{\alpha, \beta, \gamma\}$ , and the relation in Equation 7.

$$W(h_1) = \{\alpha, \beta\}, \quad W(h_2) = \{\beta, \gamma\}, \quad W(h_3) = \{\gamma, \alpha\}. \quad (7)$$

Equation 7 gives a common protocol for each pair:  $\beta$  for  $\{h_1, h_2\}$ ,  $\gamma$  for  $\{h_2, h_3\}$ , and  $\alpha$  for  $\{h_1, h_3\}$ . The triple intersection is empty. A constant record therefore fails even though any pair of compatible dossiers can be handled together. Figure 1 represents the distinction.

Pairwise audits can consequently miss a collective obstruction. If an archive designer tests one pair of histories at a time, each comparison can be reassuring while the actual record leaves all three unresolved. The failure is logical before it is numerical. It concerns whether the same witness works throughout a class. Calling the dossiers mutually compatible without specifying that common witness would hide the issue.

**Proposition 7.1** (Incomparable Minimal Adequate Records). *For Equation 7, each partition into one two-element cell and the remaining singleton is adequate and admits no strictly coarser adequate partition. There is no adequate partition coarser than all three.*

*Proof.* Every two-element cell has its indicated common protocol, and every singleton has two successful protocols. Each partition is therefore adequate by Proposition 6.2. Its only strict coarsening is the one-cell partition, whose common intersection is empty. A partition coarser than both  $\{\{h_1, h_2\}, \{h_3\}\}$  and  $\{\{h_1, h_3\}, \{h_2\}\}$  must put  $h_1$  with  $h_2$  and with  $h_3$ . Transitivity of belonging to the same cell then puts all three together. That partition is inadequate, proving the last assertion.  $\square$

Proposition 7.1 distinguishes several minimal solutions from one least informative solution. Each two-cell partition loses a distinction that another retains. Choosing among them can depend on which observation is accessible, less intrusive, easier to interpret or faster to acquire. The mathematical relation

does not rank those additional properties. A record containing all distinctions is adequate in this example but includes more information than the selected task requires.

### 7.3 Fixed-Rule Quotients

The absence of a unique coarsest adequate partition does not contradict the earlier coarsest quotient for a fixed family of prescribed functions [8, Section 8]. For a fixed function, equality of outputs defines an equivalence relation. A common-choice relation instead permits different witnesses on overlapping sets. Pairwise existence of a shared acceptable output need not supply one common output for a whole record class, as Equation 7 demonstrates.

One could recover a fixed-rule quotient by selecting a particular policy  $U(h) \in W(h)$  in advance. Its kernel would then be a coarsest record for implementing that selected rule. Another selection could yield a different kernel. This construction is legitimate, but it changes the order of design: an analyst first commits to a rule and then asks what record implements it. The present inquiry leaves the acceptable alternatives open while assessing what a participant can choose. Their difference is especially important when selecting a rule already presupposes hidden information.

### 7.4 Changing Tasks

Suppose a record must support several separately requested recovery tasks on the same carrier. Each task has its own successful-policy relation. If the requested task is known when the choice is made and the tasks are not jointly executed, every record cell must satisfy each task's own intersection condition. A selector can then depend on the task as well as the record. This is stronger than adequacy for a single task, but different from a requirement that one protocol accomplish every task simultaneously. The latter uses a joint success relation and may fail even when each task is separately feasible.

Future users may request tasks that were never specified at collection. The finite construction cannot certify adequacy for that indefinite class. The inherited manuscript already shows how demanding preservation of an arbitrary family of future distinctions can recover the full distinction structure [8]. The present result supports a more local practice: state the tasks for which a record is justified, preserve the reasons for its selected distinctions, and make revision possible where feasible. It does not infer a universal smallest archive or an obligation to retain every observable difference.

## 8 Executable Recovery Inquiry

An accessible record can be extended by inquiry, but inquiry is itself an operation with conditions and consequences. This section models a finite sequence of observation-dependent actions. It derives a bounded success criterion, exhibits a case where a query enables recovery, and locates the failure produced by exhausting the resources required to use its answer. The mathematical background is the established use of observation-based strategies and knowledge-set updates in imperfect-information games [3, Sections 2–3.1]. The following elementary bounded derivation specializes that architecture to explicit domains, stopping and resources; it is not a new game-solving algorithm.

### 8.1 Operational State Data

Choose a finite set  $S$  of operational state descriptions for a declared finite investigation. Each description retains the historical and current data required to determine the modeled next-step possibilities. It can include a finite history prefix, remaining sessions, elapsed time, support status and the current target interpretation. This is a finite operational projection of the history-bearing architecture, not an assertion that the complete relational object has finitely many states.

Let  $\mathcal{A}$  be a finite action alphabet. For each  $a \in \mathcal{A}$ , let  $D_a \subseteq S$  be its domain, and let  $T_a : D_a \rightarrow \mathcal{P}(S) \setminus \{\emptyset\}$  give its nonempty possible successor sets. A failed or unsafe outcome is represented by a

state when it belongs to the modeled action. It is not removed from  $T_a(s)$  merely because it is undesirable. An action outside its domain is unavailable and cannot supply a successful branch.

Let  $o : S \rightarrow O = o(S)$  be the accessible observation map, and let  $G \subseteq S$  identify the declared successful stopping states. The participant remembers its own previous actions and observations. A current information set  $B$  is a nonempty subset of one observation class, retaining every current state still possible after that history. No probability is assigned. For a uniformly available action, meaning  $B \subseteq D_a$ , and a possible next observation  $y \in O$ , define the update in Equation 8.

$$B_{a,y} = \{s' \in S : o(s') = y, \exists s \in B \text{ with } s' \in T_a(s)\}. \quad (8)$$

Equation 8 first propagates all possible current states through the action and then restricts by the new observation. Restricting the old set without propagating it would falsely treat the query as passive. Only nonempty  $B_{a,y}$  are possible branches. Empty branches are ignored as impossible observations, while an empty action domain cannot generate a successful execution.

Resource uncertainty belongs in  $S$ . If a query can consume different amounts of time that the participant cannot observe, the successor information set may contain several remaining budgets. Replacing those budgets by an optimistic common value would change the model. A later action is uniformly available only when its prerequisites hold at every member of that set.

## 8.2 Bounded Successful Stopping

Let  $\text{Win}_n(B)$  mean that an observation-based procedure can stop successfully within at most  $n$  additional actions from every state in  $B$ , for every admitted sequence of outcomes. Stopping is legitimate only when every possible current state belongs to  $G$ . This objective asks for a successful terminal decision under the model. It is stronger than a claim that an unrecognized success occurred transiently at some point. Equation 9 gives the recursion.

$$\begin{aligned} \text{Win}_0(B) &\iff B \subseteq G, \\ \text{Win}_{n+1}(B) &\iff B \subseteq G \text{ or } \exists a \in \mathcal{A} \left[ B \subseteq D_a \text{ and } \bigwedge_{\substack{y \in O \\ B_{a,y} \neq \emptyset}} \text{Win}_n(B_{a,y}) \right]. \end{aligned} \quad (9)$$

All conjunctions in Equation 9 are finite. When an action is uniformly available, nonempty successor sets ensure that at least one observation branch exists.

**Proposition 8.1** (Finite Inquiry Criterion). *For the declared finite operational model, Equation 9 holds exactly when an observation-based policy tree with successful stopping within the indicated action bound exists. Choices in that tree require only the available observation history.*

*Proof.* At bound zero, no action remains. Successful stopping is possible exactly when all candidate states are in  $G$ . Assume the assertion holds at bound  $n$ . If the second line of Equation 9 holds, either stop immediately or select its witnessing action  $a$ . It is available at every current possibility. For each possible observed  $y$ , the induction assumption supplies a successful policy subtree from  $B_{a,y}$  with at most  $n$  more actions. Joining these subtrees below  $a$  gives an observation-based tree of the required bound.

Conversely, consider such a tree at bound  $n + 1$ . If it stops at the root, every state in  $B$  must be successful. Otherwise its first action is the same for all candidates sharing the present observation history; it must therefore be defined throughout  $B$ . Each possible successor belongs to the branch selected by its observed  $y$ . The remaining subtree succeeds from every member of  $B_{a,y}$  in at most  $n$  actions. The induction assumption establishes the required conjunction.  $\square$

Proposition 8.1 exposes the difference between knowing a successor's label and retaining a feasible route from it. The recursive branch condition evaluates success after the inquiry has happened. Its necessity depends on the completeness of the represented current state for the chosen transition relation

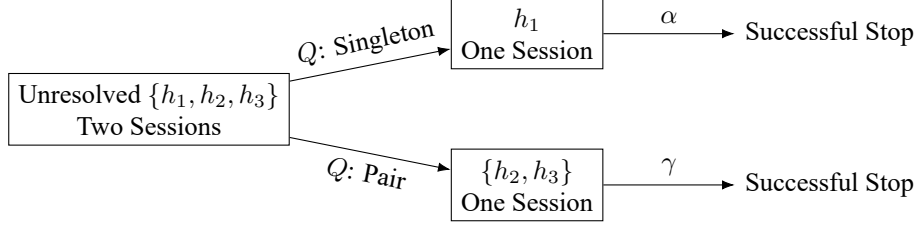


Figure 2: Costed Inquiry Tree. Each edge consumes one session. The successful policy uses the query’s two readable outcomes and leaves two dossiers unresolved on one branch. The tree is a constructed finite example; its session costs are stipulated, not measured.

and on the specified terminal objective. If past costs or obligations affect later feasibility, they must be included in that state. If the actual task concerns unbounded recurrence, the bounded stopping result needs replacement by an appropriate objective.

### 8.3 A Costed Query

Return to the three-dossier relation of Equation 7. Suppose executing one of  $\alpha, \beta, \gamma$  costs one facilitator session and ends the trial. Its success or failure is determined by the stated relation. A query  $Q$  also costs one session and yields either the message identifying  $h_1$  or the message identifying the pair  $\{h_2, h_3\}$ . It does not otherwise change the successful intervention sets in this finite example. This stipulation isolates resource consumption before the later field example also permits support decay.

With two available sessions, the constant initial record supports an adaptive policy: ask  $Q$ , choose  $\alpha$  after the singleton message, and choose  $\gamma$  after the pair message. All remaining candidates then admit the chosen action. The query need not identify  $h_2$  and  $h_3$  separately. Figure 2 depicts the tree.

With only one available session, every dossier separately still permits a successful intervention. Yet the initial constant record has no common one-session policy. Choosing an intervention immediately fails at one dossier; asking  $Q$  leaves zero sessions for an intervention. The richer successor record can be sufficient for the original selection question while being practically too late. This is a changed-resource failure, not a violation of the fixed-task refinement argument.

The source packet checks the costed example with explicit success and failure assertions. Separately, it compares directly enumerated policy trees with Equation 9 on exhaustive two-state systems. Such checking detects implementation errors in the representation; the induction proof carries the general finite result. Neither establishes that a real facilitator can perform the stipulated query or that the three intervention labels exhaust the actual possibilities.

### 8.4 Interpretation Conditions

The word knowledge in a knowledge-set construction has a restricted role.  $B$  lists current possibilities admitted by the model; it does not establish a philosophical analysis of knowledge or eliminate contested interpretation. A correctly executed update can still inherit a poorly justified candidate family. The practical record therefore needs a statement of the model’s evidential basis and the alternatives it omits.

The construction also distinguishes a sensor from an intervention. A formal observation map specifies what becomes available at a state. An actual operation that makes the map readable may require translation, permission, equipment, learning or a social encounter. Where those actions alter support or the target, their effects belong to  $T_a$ . The nonlinear regional model in the next sections supplies one explicit instance of support dynamics. It does not establish a complete reduction from the continuous field to this finite model. Such a reduction would require sound transition and observation bounds, with controller realizability checked independently.

## 9 Regional Recovery Fields

A record can omit a regional distinction that matters only when interventions are limited. To examine that dependence, this section constructs a family of nonlinear activity-support histories with identical aggregate passive observations. A standardized initiating episode succeeds in some regions and fails in others. The next section determines exactly which regional records permit a successful choice within a fixed batch budget. The field analysis supplies the success sets needed by that choice problem; it does not substitute observed activity for a warranted attribution of linguistic capacity.

The nonlinear law and its comparison methods are inherited from the reactivation investigation [9, Sections 6, 8–9]. That paper already establishes support-dependent persistence, finite initiation certificates and regional obstructions. The present question concerns how those differences constrain a participant who must choose where to deliver a limited intervention. The extension is therefore a record-sensitive selection problem built from an established model, rather than a new general theory of support feedback. Its region family is deliberately large enough to distinguish exact identification, partial localization and a common successful choice.

### 9.1 Eligible Regional Realizations

Let  $J = \{1, \dots, m\}$ , with  $m \geq 2$ , index eligible regions. Each contains a copy of a finite connected weighted graph  $G$  with nonempty site set  $V$ . The sites represent selected operational interfaces. They need not be persons or geographical points. Symmetric weights  $w_{ik} = w_{ki} \geq 0$ , with  $w_{ii} = 0$ , specify the modeled exchange inside each region. For  $r \in \mathbb{R}^V$ , define the graph Laplacian by Equation 10.

$$(L_G r)_i = \sum_{k \in V} w_{ik}(r_i - r_k), \quad i \in V. \quad (10)$$

The operator in Equation 10 describes the selected exchange channels. A historical relation between two regions need not be one of those channels.

All exchange between the  $m$  components is zero in this specialization. This is a substantive boundary assumption. It makes possible an exact comparison between intervention locations without letting an active component initiate every other component through an unrecorded connection. It would be inappropriate to infer this separation from the fact that records happen to be stored in different folders. If the selected activity or support actually crosses a boundary, the field law must retain that trace and the results below must be rechecked. The prior regional field formalism explicitly distinguishes restriction of a field from changing the dynamics at the restricted boundary [6, pp. 27–28, 33–35].

Each eligible region is assumed to have an independently warranted realization of the same selected operation type. For example, several settings might support recurrent use of an already retained linguistic distinction. The task permits successful continuation at any one of them. This permission belongs to the task specification. Regions with different authority, accessibility or participant aims cannot be treated as interchangeable merely because their equations are isomorphic. A target requiring recovery in a particular setting, or requiring every setting to continue, gives a different success family.

At site  $i$  of region  $j$ ,  $u_{ji} \geq 0$  records the amplitude of a selected execution-activity channel, and  $v_{ji} \geq 0$  records one replenishable enabling channel. Neither variable measures competence, historical continuity, meaningfulness or consent. A retained operation grammar, its history and the interpretation connecting actual activity to recurrent usable execution remain separate premises. In particular, an initial value  $u_{ji} = 0$  states that the modeled activity is absent. That value alone cannot distinguish non-use, blocked execution and the absence of a capacity.

### 9.2 An Inherited Nonlinear Law

Use the reactivation law with dimensionless parameters  $a = 2$ ,  $b = \sigma = 1$ ,  $\varepsilon = 1/200$  and  $\kappa = 3/100$ . The support-renewal ratio is therefore  $\kappa/\varepsilon = 6$ . For arbitrary fixed  $D_u, D_v \geq 0$ , the regional field is

Equation 11.

$$\begin{aligned} \dot{u}_{ji} &= -D_u(L_G u_j)_i + u_{ji}(-2 + v_{ji} - u_{ji}^2) + I_{ji}(t), \\ \dot{v}_{ji} &= -D_v(L_G v_j)_i + \frac{1 - v_{ji}}{200} + \frac{3}{100} \frac{u_{ji}^2}{1 + u_{ji}^2}. \end{aligned} \quad (11)$$

The terms in Equation 11 retain the earlier model's interpretation. Activity depends on support and is limited by its cubic term. Activity can renew the selected support channel, but the renewal rate saturates. The coefficients are illustrative choices that permit explicit comparison bounds; they have not been fitted to a language community. The baseline support level of one remains after the initiating input is withdrawn. Every other condition held in the realization also remains in place. Persistence consequently means continuation under that baseline, not independence from external work.

For inputs between zero and four, the field has the common compact carrier in Equation 12.

$$B = [0, 3]^{m|V|} \times [1, 7]^{m|V|}. \quad (12)$$

The boundary calculation for Equation 12 is short enough to make its assumptions explicit. At  $u_{ji} = 0$ , exchange and input are nonnegative. At  $u_{ji} = 3$ , exchange cannot increase a coordinate at the upper face, and the remaining derivative is at most  $3(-2 + 7 - 9) + 4 = -8$ . At  $v_{ji} = 1$ , exchange and renewal are nonnegative. At  $v_{ji} = 7$ , the derivative is at most  $-6/200 + 3/100 = 0$ . Thus the faces point inward or tangentially. First-contact comparison gives invariance. Smooth state dependence and bounded piecewise-continuous forcing give a unique global forward solution, absolutely continuous through input changes. Boundedness here establishes mathematical admissibility; it does not supply a practical resource budget.

An important exact reduction follows from the chosen regional construction. If initial activity and support are constant across the sites of a component, and that component receives a spatially uniform input, its solution stays spatially uniform. The Laplacian annihilates constants, so the corresponding two-variable solution satisfies every site equation; uniqueness identifies it with the full solution. The proof below uses this invariant family to obtain exact failure statements. Its persistence rectangle, by contrast, also controls heterogeneous states inside the same bounds. Internal spatial structure is retained, but these examples make no claim about new spatial pattern formation.

### 9.3 Prepared Support Histories

Fix an integer  $h$  with  $1 \leq h \leq m$ . A candidate preparation is a subset  $H \subseteq J$  containing exactly  $h$  regions. The current field values associated with that preparation are given in Equation 13.

$$u_{ji}(0) = 0, \quad v_{ji}(0) = \begin{cases} 18/5, & j \in H, \\ 1, & j \notin H, \end{cases} \quad i \in V. \quad (13)$$

Equation 13 distinguishes a prepared support channel from a baseline-only channel. Both can accompany the same independently warranted retained operation. The field does not reconstruct the history that produced those values. A historically warranted application would need to identify the preparation, the resulting current support and the admissible alternative histories. The finite candidate family below retains only the uncertainty about its regional location.

A participant chooses a batch  $A \subseteq J$ . Each chosen component receives the standardized input in Equation 14.

$$I_{ji}^A(t) = 4 \mathbf{1}_A(j) \mathbf{1}_{[0,1)}(t), \quad |A| \leq q, \quad 0 \leq q \leq m. \quad (14)$$

Here  $q$  is an integer budget of available regional sessions. Equation 14 specifies simultaneous delivery in one batch. Each session supplies the same input at every modeled interface in its region for one model time unit. Delivery is assumed feasible at every eligible component. If a real participant cannot deliver that episode, the corresponding action must be removed from the menu or given its actual domain.

This menu gives the resource constraint precise content. It permits choosing a location or several locations; it does not permit an arbitrarily long episode, a continuously adjustable division of the input, or further interventions after the results are observed. Sequential pulses with intermediate inquiry would be a different policy class. This restriction is essential because sufficiently long forcing can renew initially low support in the inherited law. The purpose is to ask whether a specified record supports the interventions actually admitted, not whether complete freedom of control could overcome the omitted information.

The field target is that at least one eligible region retain  $u_{ji}(t) \geq 1$  at every site for every  $t \geq 1$ , after input withdrawal. The value one is a selected activity floor in this dimensionless model. It becomes a linguistic target only through a justified realization connecting that channel to recurrent execution over the relevant horizon. The use of an unbounded mathematical horizon strengthens the model's internal persistence claim; it does not warrant an empirical prediction of indefinite language survival.

#### 9.4 Exact Pulse Outcomes

**Proposition 9.1** (Regional Pulse Outcomes). *For the preparations in Equation 13 and the batch in Equation 14, a selected prepared region enters the bounds  $u_{ji} \geq 1$ ,  $v_{ji} \geq 7/2$  by withdrawal and retains them thereafter. Every selected baseline-only region satisfies  $u_{ji}(t) \rightarrow 0$  and  $v_{ji}(t) \rightarrow 1$ . Every unselected region has identically zero activity. Consequently the field target succeeds exactly when  $A \cap H \neq \emptyset$ .*

*Proof.* First consider a selected prepared component. While a site at the activity minimum lies between zero and one, its diffusion is nonnegative and its derivative is at least  $4 - 2u - u^3 \geq 1$ . The constant lower barrier  $\min\{t, 1\}$  is therefore valid through the pulse. It gives  $u_{ji}(1) \geq 1$ . Independently, the renewal term is nonnegative, so support is bounded below by the spatially constant solution of  $\dot{v} = (1 - v)/200$ . Thus Equation 15 holds.

$$v_{ji}(t) \geq 1 + \frac{13}{5}e^{-t/200}, \quad v_{ji}(1) \geq 1 + \frac{13}{5} \frac{199}{200} = \frac{3587}{1000} > \frac{7}{2}. \quad (15)$$

The second inequality in Equation 15 uses  $e^{-x} \geq 1 - x$ . Under the unforced law, a lower activity face  $u_{ji} = 1$  with all neighboring activity at least one has derivative at least  $-2 + 7/2 - 1 = 1/2$ . A lower support face  $v_{ji} = 7/2$ , with all neighboring support at least  $7/2$  and activity at least one, has derivative at least  $-5/400 + 3/200 = 1/400$ . Together with the upper faces of  $B$ , these signs make the sustaining rectangle forward invariant. No convergence-to-equilibrium premise is needed.

Next consider a selected baseline-only component. It is spatially uniform by the preceding uniqueness argument. Write its coordinates as  $u, v$ . The bounds  $v \geq 1$  and  $\dot{v} \leq 3/100$  imply  $v(t) \leq 1 + 3t/100$  for every  $t \geq 0$ . During the pulse this is less than two, so  $\dot{u} \leq 4$  and  $u(1) \leq 4$ . This intentionally loose upper bound suffices even though  $B$  supplies the sharper bound three. Define the comparison time by Equation 16.

$$T = 1 + 2 \log 20 < 7, \quad v(t) < \frac{121}{100} < \frac{3}{2} \quad (0 \leq t \leq T), \quad u(T) \leq 4e^{-(T-1)/2} = \frac{1}{5}. \quad (16)$$

The bound  $T < 7$  follows from  $\log 20 < 3$ . Between withdrawal and  $T$ , the support bound gives  $\dot{u} \leq -u/2$ , proving the last inequality in Equation 16.

The rectangle  $0 \leq u \leq 1/5$ ,  $1 \leq v \leq 3/2$  is invariant for the baseline law. At its activity upper face, the derivative is strictly negative. At the support upper face, it is at most  $(1/200)(1 + 6/26 - 3/2) < 0$ ; its lower faces point inward. Activity thereafter decays at least exponentially at rate  $1/2$ . Variation of constants in the support equation shows  $v \rightarrow 1$ : its decaying initial term tends to zero, and its bounded renewal forcing tends to zero with  $u$ . Splitting the convolution at a late fixed time bounds the early part by an exponentially decaying term and the late part by an arbitrarily small forcing supremum divided by  $1/200$ .

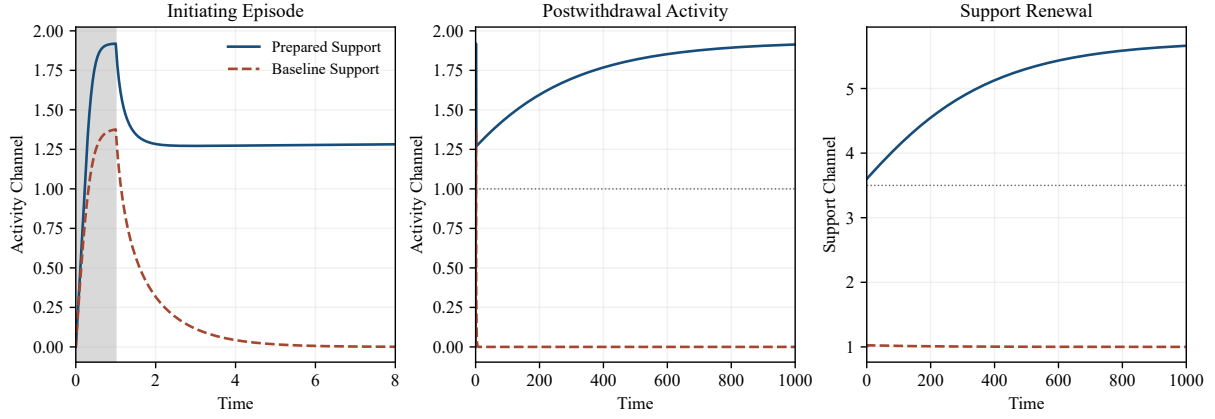


Figure 3: Standardized Regional Pulse Histories. The same one-unit input produces sustained activity at the prepared support level and eventual extinction at the baseline level. The channels are dimensionless illustrative quantities. The numerical trajectories depict the analytical alternatives of Proposition 9.1; they do not calibrate linguistic competence or certify the infinite horizon independently.

Finally, in an unselected component the trajectory with  $u = 0$  and  $v(t) = 1 + [v(0) - 1]e^{-t/200}$  solves the equations. Uniqueness makes it the actual solution. Thus the only regions satisfying the sustained target are those selected from  $H$ , which proves the final equivalence.  $\square$

Proposition 9.1 proves both sides of the choice problem. The negative result is actual extinction under the specified intervention, rather than failure to enter one sufficient rectangle. The positive result is a nonlinear all-future bound, rather than a short simulated episode of activity. This asymmetry in proof methods is appropriate: a sufficient invariant region establishes success, while a separate comparison establishes failure for the other admitted preparations.

Figure 3 illustrates the resulting histories. A numerical trajectory can help show how a pulse produces temporary activity in a baseline-only component, but the proof supplies the infinite-horizon claim. Similar immediate performance is therefore insufficient for choosing where to spend a scarce session. The location of prepared support matters because the menu cannot simply extend every unsuccessful episode until its support catches up.

The proposition also identifies the fragility of the exact reduction. An interregional activity channel could seed an unselected prepared component. A different intervention duration could rescue a baseline-only component. A revised task could reject a region that the present task accepts. Each change alters the set of successful batches. The following record criterion is exact for the declared field family because these commitments are explicit; it should not be transferred to a changed family without repeating the success analysis.

## 10 Regional Record Sufficiency

The field calculation now permits an exact comparison of records. Its relevant distinction is the location of support that makes a bounded initiating episode effective. This section derives the aggregate ambiguity, identifies the additional distinctions retained by regional counts, and calculates the minimum session budget compatible with each record value. It then examines a temporal limit: obtaining a more precise location record can leave the original recovery choice unavailable if the underlying support has changed during inquiry. The argument separates informational refinement at a fixed configuration from an actual process of acquiring information.

## 10.1 Aggregate Passive Records

The candidate carrier is the finite family  $\Omega_h = \{H \subseteq J : |H| = h\}$  from the previous section. Its members encode different locations of prepared support, within otherwise common field and task commitments. They need not be the only possible histories in an empirical inquiry. Restricting the hypothesis family to these preparations requires warrant; the following sufficiency statements quantify over this family rather than certify that an investigator has found every relevant alternative.

Suppose the retained passive record contains aggregate activity and aggregate support, with component and site counts known, but omits regional labels for the support values. Before any input is delivered, its trajectories are given by Equation 17.

$$\bar{u}(t) = 0, \quad \bar{v}(t) = \frac{1}{m|V|} \sum_{j \in J} \sum_{i \in V} v_{ji}(t) = 1 + \frac{13h}{5m} e^{-t/200}. \quad (17)$$

Equation 17 follows from the exact unforced zero-activity solution in Proposition 9.1. It depends on  $h$  but not on the location  $H$ . Perfect time resolution and arbitrarily accurate values of these aggregates therefore leave the permutation ambiguity intact. More measurements of the same insensitive channel do not supply a regional discriminator.

The observation does retain something useful. If  $h$  were initially unknown within this two-level family, the initial aggregate would determine it. With  $h$  fixed, it determines a class of preparations in which some intervention can succeed. What it omits is whether one affordable intervention is guaranteed to meet prepared support. That is a different requirement from determining every preparation coordinate. A participant with enough available sessions can sometimes act successfully despite this omission.

The word passive also matters. Equation 17 describes an interval without initiating input. Observing a longer such interval uses time during which prepared support declines. An actor cannot treat the resulting record as if it had been acquired instantaneously at the original preparation boundary. The final subsection makes this distinction explicit; for the present record comparison, each readout is stipulated to be already available at the same initial boundary.

## 10.2 Regional Count Records

Let  $\mathcal{P} = \{B_1, \dots, B_s\}$  be a partition of  $J$  into nonempty identified blocks. Put  $n_j = |B_j|$ . A more localized record gives the number of prepared components in each block, as defined in Equation 18.

$$r_{\mathcal{P}} : \Omega_h \longrightarrow \prod_{j=1}^s \{0, \dots, n_j\}, \quad r_{\mathcal{P}}(H) = (h_1, \dots, h_s), \quad h_j = |H \cap B_j|. \quad (18)$$

Only count vectors whose coordinates sum to  $h$  occur in the range of Equation 18. The record is idealized as exact and readable. Obtaining counts from actual linguistic settings would require an interpretation and a warranted observation procedure. There is no assumption that enabling support is ordinarily measured in the two numerical levels of the field example.

The retained counts leave regional permutations unresolved within each block. Section 11 derives the corresponding observable subalgebra and distinguishes accessible decisions from the full family of action-success indicators. The immediate question is which batches succeed throughout a count-record fibre.

## 10.3 Exact Session Budgets

Fix a realized count vector  $y = (h_1, \dots, h_s)$  in the range of  $r_{\mathcal{P}}$ , and let  $C_y = r_{\mathcal{P}}^{-1}(y)$ . Because  $h \geq 1$ , at least one coordinate  $h_j$  is positive. Define the integer in Equation 19.

$$q_{\min}(y) = \min_{j: h_j > 0} (n_j - h_j + 1). \quad (19)$$

Equation 19 will be the exact smallest number of sessions that guarantees success on this record fibre. It is not a universal resource scale; its units and force come from the standardized intervention menu.

**Proposition 10.1** (Regional Record Budget). *There exists a batch  $A$  with  $|A| \leq q$  that achieves the field target for every  $H \in C_y$  if and only if  $q \geq q_{\min}(y)$ . For the aggregate record, whose single block is  $J$ , the condition is  $q \geq m - h + 1$ .*

*Proof.* For a fixed batch write  $a_j = |A \cap B_j|$ . A preparation in  $C_y$  misses  $A$  exactly when its  $h_j$  prepared elements in each block can all be placed outside  $A$ . Such a placement exists if and only if  $n_j - a_j \geq h_j$  for every  $j$ . The choices in different blocks are independent and their union then has size  $h$ . By Proposition 9.1, the batch succeeds throughout the fibre precisely when at least one block instead satisfies  $a_j \geq n_j - h_j + 1$ .

If  $|A| < q_{\min}(y)$ , that inequality fails in every positive-count block because  $a_j \leq |A|$ . In a zero-count block it cannot hold even with the entire block selected. Hence a missed preparation exists in the same fibre and the batch fails robustly. This proves necessity, including the case where an actor distributes a few sessions across several blocks.

For sufficiency, choose a positive-count block attaining the minimum and select any  $n_j - h_j + 1$  of its components. Only  $h_j - 1$  components of that block remain outside the batch, so every preparation containing  $h_j$  prepared components there must meet the batch. It has size  $q_{\min}(y)$  and succeeds throughout  $C_y$ . The aggregate formula is the special case  $s = 1$ ,  $n_1 = m$ ,  $h_1 = h$ .  $\square$

The positive construction in Proposition 10.1 supplies an executable location rule only because the eligible components and their delivery labels are assumed available to the participant. A stored count indexed by labels that the participant cannot connect to settings would not meet that premise. The rule also presupposes that any selected component in the identified block can receive the standardized session. Regional access restrictions would change the optimization.

The proposition makes the budget dependence exact. With an aggregate record, selecting at most  $m - h$  regions leaves an unselected set large enough to contain every prepared region. Selecting  $m - h + 1$  prevents that counterexample. The record does not become more informative when the budget increases. It becomes sufficient for a feasible common choice because the intervention repertoire expands. At the opposite limit  $h = m$ , any single eligible session succeeds and precise localization is unnecessary.

For a concrete comparison, take  $m = 6$ ,  $h = 2$  and  $q = 2$ . The aggregate record requires five sessions and is inadequate. Partition the regions into three identified pairs. Every realized count vector contains a positive-count pair; selecting that pair requires at most two sessions and guarantees success. A record value  $(1, 1, 0)$  nevertheless leaves four preparations possible. The actor can choose a successful batch while remaining unable to identify which component in either of the first two pairs was prepared.

A partition into two blocks of three shows a further distinction. At count vector  $(2, 0)$ , two sessions suffice. At  $(1, 1)$ , three are required. The same record design can therefore have sufficient and insufficient values at the same budget. Calling the whole record sufficient requires the criterion at every admitted value, unless the claim explicitly restricts its domain. Figure 4 displays these exact comparisons.

This partial localization also has a direct operator certificate. On the finite Hilbert space  $\ell^2(\Omega_h)$ , let  $M_f$  multiply by a function  $f : \Omega_h \rightarrow \mathbb{C}$ , and put  $w_A(H) = \mathbf{1}_{\{A \cap H \neq \emptyset\}}$ . Let  $P_{C_y}$  multiply by the indicator of  $C_y$ , and let  $A(y)$  be the batch assigned to the readable value. The regional success certificate is Equation 20.

$$M_{w_{A(y)}} P_{C_y} = P_{C_y}, \quad |A(y)| \leq q. \quad (20)$$

Equation 20 applies the common-policy requirement of Section 6 in a concrete diagonal representation. It tests every basis vector in the fibre. The action domains in this particular example are total; a partial delivery operation would require its domain certificate as well. Section 11 develops the general operator comparison. The block-budget result determines exactly when the displayed conditions can be met, without requiring that the record identify  $H$  or preserve every continuation observation.

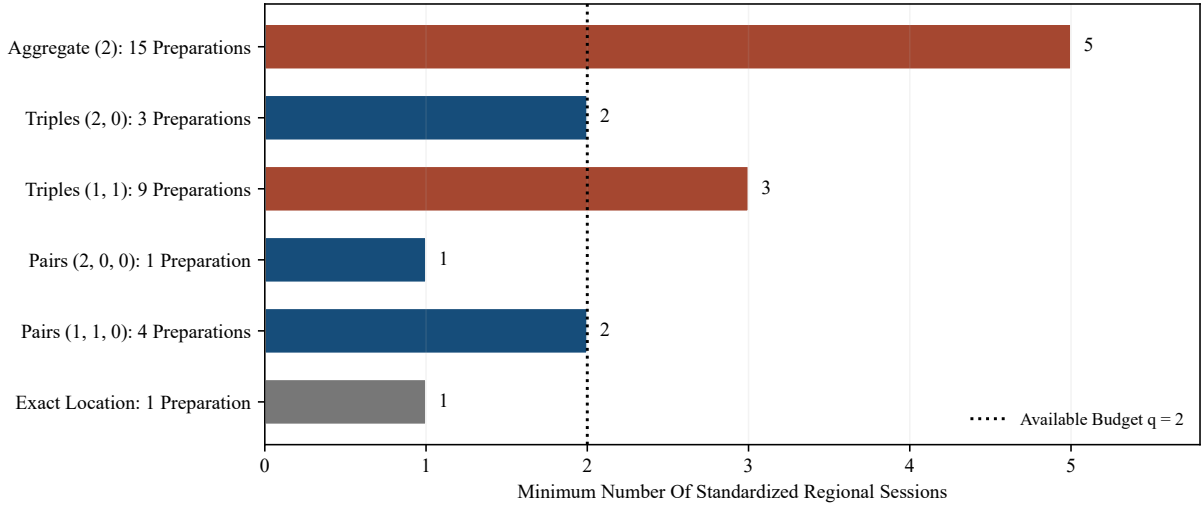


Figure 4: Regional Record Budgets. Exact minimum batch sizes for six eligible regions and two prepared regions. Aggregate recording requires five sessions; identified pair counts require at most two while leaving some preparations unresolved. Two blocks of three produce a fibre-dependent requirement. These are consequences of the stipulated pulse menu and support family, not empirical intervention costs.

#### 10.4 Delayed Location Knowledge

A refined location record is more discriminating at a fixed boundary. Its acquisition can nevertheless change that boundary. The present model gives an exact instance even when inquiry has no direct harmful effect on the field: an interval without activity lets the prepared support decay. The comparison does not assign a fabricated numerical cost to a real sensor. It studies a declared waiting duration and asks what remains possible afterward within the same standardized pulse menu.

Let  $d \geq 0$  be a period without initiating input before the one-unit batch begins. A formerly prepared component then has support  $1 + (13/5)e^{-d/200}$ . The proof of Proposition 9.1 still certifies success if its lower bound at withdrawal is at least  $7/2$ . Thus a sufficient prompt-action condition is Equation 21.

$$d \leq 200 \log(26/25) - 1. \quad (21)$$

Equation 21 is a certificate interval, not an exact boundary of success. Failure of that inequality alone says nothing decisive about the subsequent trajectory. A separate late-delay argument establishes actual extinction.

**Proposition 10.2** (Delayed Standardized Pulse Failure). *If  $d \geq 200 \log(52/5)$ , every component becomes extinct after any batch from the standardized menu, even when the record identifies the original preparation  $H$  exactly.*

*Proof.* At this delay, each formerly prepared support value is at most  $1 + (13/5)(5/52) = 5/4$ ; baseline-only values remain one. Set the new time origin at pulse delivery. In a selected component,  $v \geq 1$  and  $\dot{v} \leq 3/100$  give  $v(t) \leq 5/4 + 3t/100$ . During the unit pulse this is below two, so  $u(1) \leq 4$ . At the comparison time  $T = 1 + 2 \log 20 < 7$  from Equation 16, support remains below  $5/4 + 21/100 = 146/100 < 3/2$ . Hence the same postwithdrawal bound yields  $u(T) \leq 1/5$ . The invariant extinction rectangle used in Proposition 9.1 now gives  $u \rightarrow 0$  and  $v \rightarrow 1$ . Unselected components have identically zero activity. The conclusion holds for every  $H$  and every batch, irrespective of the record's precision.  $\square$

Proposition 10.2 concerns loss of a route under a fixed menu. It does not prove irreversible capacity loss or impossibility of recovery through a longer or differently organized intervention. Between the sufficient prompt interval and the sufficient late-failure interval, these comparison arguments leave the exact outcome unresolved. Numerical illustrations may explore that gap, but they cannot convert the two bounds into a universal threshold.

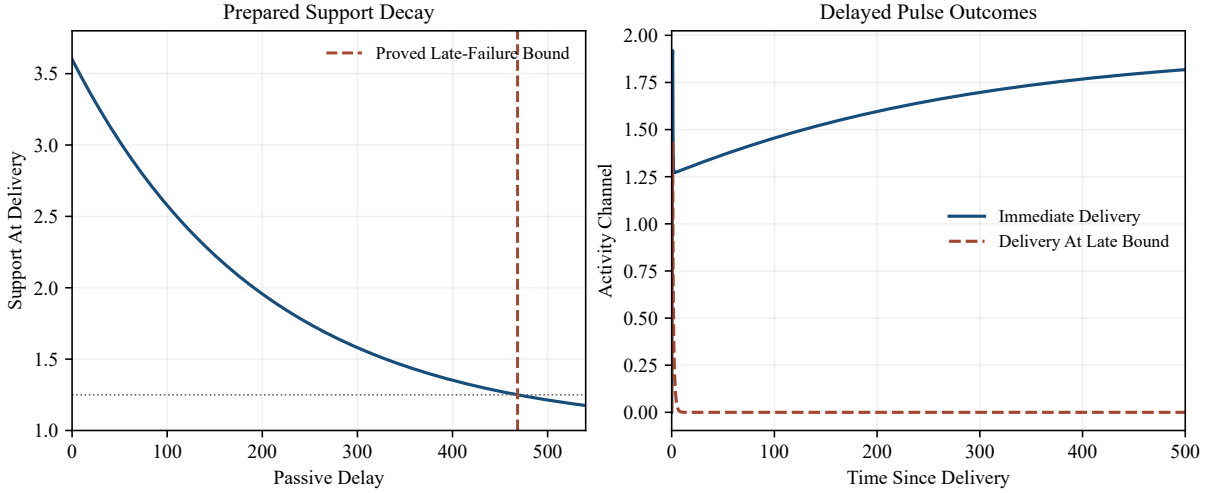


Figure 5: Delayed Standardized Pulse Comparison. Prepared support decays during a passive interval. Immediate delivery is certified to sustain activity, whereas delivery after the stated late bound is proved to extinguish, even with exact knowledge of the preparation’s original location. The gap between sufficient bounds remains analytically unresolved; model time is dimensionless.

Figure 5 compares immediate delivery with the proved late-delay case. The late observer can possess a perfect historical location record and still lack a successful currently admitted batch. The correct recovery problem uses the successor support state, remaining resources and elapsed time. Merely replacing the old coarse label with a precise one would combine information from the later history with opportunities that belonged to the earlier history.

The regional investigation therefore identifies the location distinctions that permit a common affordable episode now, given its delivery conditions and the support that remains. Aggregate data, block counts and exact locations meet that requirement differently. Their ordering by informativeness remains useful, but it does not replace the actual intervention menu, the participant’s ability to use the record, or the historical changes incurred while acquiring it. The adaptive inquiry construction of Section 8 retains those changes when a readout must be obtained through an actual intervention history.

## 11 Recovery Decision Algebras

The algebraic investigation distinguishes three claims that can otherwise be conflated: a success condition can be defined on the represented histories, a choice can depend only on an accessible record, and the chosen operation can actually be executed. This section represents these claims with diagonal observables and partial history operators. It recovers the common-policy criterion algebraically and connects it to the regional field. The construction extends the application of inherited observable and partial-operation tools [10, 9, 6]; it supplies no quantum or ultimate ontological interpretation.

### 11.1 Record Observable Subalgebras

Return to the finite dossier carrier  $H$ . Let  $\mathcal{H} = \ell^2(H)$  have orthonormal basis  $\{e_h : h \in H\}$  and its usual complex inner product. A function  $f : H \rightarrow \mathbb{C}$  determines the multiplication operator  $M_f e_h = f(h)e_h$ . The full diagonal algebra is  $\mathcal{A} = \{M_f : f \in \mathbb{C}^H\}$ , with pointwise multiplication, conjugation as adjoint, and the supremum norm. It is a finite-dimensional commutative unital  $C^*$ -algebra. The accessible record  $\rho : H \rightarrow R$  determines the subalgebra in Equation 22.

$$\mathcal{A}_\rho = \{M_{g \circ \rho} : g : R \rightarrow \mathbb{C}\} \subseteq \mathcal{A}, \quad P_C = M_{1_C} \quad (C \subseteq H). \quad (22)$$

Equation 22 contains precisely the multiplication operators whose functions are constant on each record cell. In particular,  $P_C \in \mathcal{A}_\rho$  exactly when  $C$  is a union of cells. A projection belongs to the full algebra whenever the subset is mathematically specified; it need not belong to the accessible subalgebra.

Every operator in  $\mathcal{A}$  commutes with every other one. Commutation therefore cannot establish accessibility. In the two-dossier example from Section 6, the constant record yields only scalar multiples of the identity. The success indicator for  $\alpha$  is the projection onto  $h_1$ , which lies outside that subalgebra even though it commutes with it. The distinction is containment, not a commutator test.

Refining a record on a fixed carrier enlarges its observable subalgebra. If  $\rho = g \circ \rho'$ , every function of  $\rho$  is a function of  $\rho'$ , so  $\mathcal{A}_\rho \subseteq \mathcal{A}_{\rho'}$ . The converse also holds for finite records up to the induced partitions: the indicator of each  $\rho$ -cell belongs to  $\mathcal{A}_{\rho'}$ , forcing every  $\rho'$ -cell to lie inside a  $\rho$ -cell. This is a structural information comparison. It does not price the act of obtaining the refinement.

## 11.2 Decision Projections

For each policy  $\pi$ , let  $Q_\pi = P_{W_\pi}$  denote its successful-domain projection. It can lie outside  $\mathcal{A}_\rho$ . A selector  $s : R \rightarrow \Pi$  determines a different projection  $E_\pi = P_{\{h:s(\rho(h))=\pi\}}$ . These decision projections belong to  $\mathcal{A}_\rho$ , are mutually orthogonal, and sum to the identity. The success requirement is the subordination  $E_\pi \leq Q_\pi$ , equivalently  $Q_\pi E_\pi = E_\pi$ . The combined criterion is Equation 23.

$$\begin{aligned} E_\pi \in \mathcal{A}_\rho, \quad E_\pi^* = E_\pi^2 = E_\pi, \quad E_\pi E_\tau = 0 \quad (\pi \neq \tau), \\ \sum_{\pi \in \Pi} E_\pi = I_{\mathcal{H}}, \quad Q_\pi E_\pi = E_\pi \quad (\pi \in \Pi). \end{aligned} \tag{23}$$

**Proposition 11.1** (Record-Dependent Decision Partition). *For the finite fixed task, a family satisfying Equation 23 exists exactly when the common-policy intersections in Equation 5 are nonempty.*

*Proof.* A selector from Proposition 6.2 gives the projections just defined, and its success property gives their subordination. Conversely, a projection in the diagonal algebra has pointwise values zero or one. Orthogonality and the identity sum imply that exactly one  $E_\pi$  takes value one at each  $h$ . Since each  $E_\pi$  is record-constant, that same unique choice holds on the whole record cell. Subordination then places every member of the cell in  $W_\pi$ . This supplies its common successful policy.  $\square$

Proposition 11.1 is an algebraic restatement, not a second independent sufficiency theorem. It nevertheless exposes a useful weaker demand. A record need not contain each  $Q_\pi$ ; it need only contain one successful partition of choices. With  $W(h_1) = \{\alpha, \beta\}$  and  $W(h_2) = \{\beta, \gamma\}$ , choose  $E_\beta = I$ ,  $E_\alpha = E_\gamma = 0$ . The constant record suffices even though neither  $Q_\alpha$  nor  $Q_\gamma$  is accessible. Measuring the entire policy-success profile is a stronger task.

This distinction blocks a familiar expansion of record requirements. An analyst may list every variable affecting every possible intervention, then assume a sufficient record must reveal them all. For the selected choice, an available common policy can make some of those variables irrelevant. Determining that they are irrelevant still requires a warranted success relation. The argument supports neither arbitrary omission nor indefinite data collection.

## 11.3 Regional Symmetry

In the field example, replace the generic carrier  $H$  by the finite family  $\Omega_h$  of prepared regional subsets. For a spatial partition  $\mathcal{P} = \{B_1, \dots, B_s\}$ , the block-count record is  $r_{\mathcal{P}}$ . The product of permutation groups acting within the blocks has exactly the record cells as its orbits. Two prepared subsets with the same counts can be carried into one another by permuting the prepared and unprepared locations within each block; such permutations preserve all counts. Consequently  $\mathcal{A}_{r_{\mathcal{P}}}$  is the diagonal algebra of functions invariant under those within-block permutations.

For a regional permutation  $g$ , define  $U_g e_H = e_{gH}$  on  $\ell^2(\Omega_h)$ . This is a unitary relabeling. If  $f : \Omega_h \rightarrow \mathbb{C}$  is a function and  $w_A(H) = \mathbf{1}_{\{A \cap H \neq \emptyset\}}$  is the success indicator for batch  $A$ , Equation 24 states the corresponding covariance.

$$U_g M_f U_g^* = M_{f \circ g^{-1}}, \quad w_{gA}(gH) = w_A(H). \quad (24)$$

Equation 24 relabels both a preparation and an action. It does not say that a fixed localized action succeeds independently of the hidden preparation. The aggregate record is invariant under every regional permutation, whereas most fixed-batch success indicators are not. The budget result in Proposition 10.1 determines when a record-dependent decision partition nevertheless exists.

The distinction has a concrete interpretation. A record can preserve a total support quantity while losing the alignment between support and an available intervention location. Renaming region identifiers leaves the problem unchanged when all associated data are renamed together. Omitting the location alignment does not. The field assumptions make this difference exact, but they do not identify the social meaning of the regions or authorize their interchange.

## 11.4 Executable History Operators

A decision partition still needs an execution representation. Restrict this paragraph's construction to deterministic finite protocols, or to already specified deterministic realized branches. Let  $U_\pi : D_\pi \subseteq H \rightarrow H_+$  append the complete protocol history, including its initial dossier and its protocol tag. The selected terminal-history library  $H_+$  is finite or countable. Retaining the prefix makes  $U_\pi$  injective, and retaining the tag gives disjoint images for different  $\pi$ . On  $\mathcal{H}_+ = \ell^2(H_+)$  define the partial isometry in Equation 25.

$$S_\pi : \mathcal{H} \rightarrow \mathcal{H}_+, \quad S_\pi e_h = \begin{cases} e_{U_\pi(h)}, & h \in D_\pi, \\ 0, & h \notin D_\pi, \end{cases} \quad S_\pi^* S_\pi = P_{D_\pi}. \quad (25)$$

Equation 25 separates domain failure from an executed unsuccessful act. A zero image means the modeled operation is undefined, not that the historical act happened and erased its input. The adjoint removes an appended record on its range; it is not a physical reversal of elapsed history.

Let  $G_+ \subseteq H_+$  be the successful terminal histories and let  $P_{G_+}$  be their multiplication projection. To identify this execution representation with the earlier success relation, require  $W_\pi = \{h \in D_\pi : U_\pi(h) \in G_+\}$  for each represented deterministic protocol. This compatibility must be justified, not inferred from the shared policy labels. For a chosen decision family define  $V = \sum_\pi S_\pi E_\pi$ . If the selected cells lie in the operation domains and their terminal images lie in  $G_+$ , the two identities in Equation 26 follow.

$$V^* V = I_{\mathcal{H}}, \quad P_{G_+} V = V. \quad (26)$$

To verify Equation 26, the disjoint protocol-tagged ranges give  $S_\pi^* S_\tau = 0$  for  $\pi \neq \tau$ . The remaining terms are  $E_\pi P_{D_\pi} E_\pi = E_\pi$ , and their sum is the identity. Each summand's range is successful, giving the second equality. The first equality records defined execution for every initial basis state; the second records the target. Target inclusion alone would hold vacuously for a zero operator, which is why both conditions matter.

These are inherited history certificates applied to a record-conditioned choice [9, Section 11]. They do not convert every policy tree into one deterministic branch. For an unresolved action with several possible outcomes, success must hold on every reachable branch, as in Proposition 8.1. Selecting only a favorable branch would alter the original success relation. One may represent the tree with a family of branch operators and their domains, but the present finite diagonal criterion already retains the required universal condition through  $\mathcal{W}$ .

## 11.5 Changed Observation Carriers

Suppose a query extends a history by  $U_a$ . A readable successor observable can be pulled back along that map on its domain. Mathematical existence of the pulled-back function does not make it readable before the query. If it did, the operation would have no information-acquisition role. Meanwhile its append history can record consumed time, changed support or newly learned distinctions. The successor observable and the successor domain must therefore be assessed together.

The algebras here are deliberately restricted. A complete historical field theory could use continuous configuration spaces, local observable algebras and changing region embeddings. Extending the finite construction would require measurability, admissible selection and continuity assumptions, together with an interpretation of actually executable readouts. Taking a norm closure is not a method for supplying missing human skills. The finite result earns its place by isolating a precise failure of inference while leaving those further obligations visible.

## 12 Documentation Case Boundaries

Documented resource projects make the recovery-record problem concrete without supplying the full state descriptions required by a formal model. This section examines a Mangarrayi archival-audio proposal, an archive-interpretation setting, and a historically described audience-specific rendition. Each source report is followed by a separately constructed comparison. The objective is to identify which observations could support a task-relative sufficiency judgment and which premises would still require investigation. The discussion evaluates neither a community's authenticity nor the overall value of an archive. Its subject is the passage from a documented resource feature to a defensible claim about feasible recovery choice.

### 12.1 Mangarrayi Resource Design

Richards and colleagues propose a Mangarrayi collection of reusable audio segments connecting utterances, transcription, glosses, translation, audio, and source information. Their design includes topical and functional organization. A discussed example shows that a literal English rendering can misrepresent the force of a Mangarrayi request. The consultation also exposed differences between ranked topics and expressions participants subsequently wanted. Work and family commitments constrained intensive learning opportunities [14, pp. 260–262, 271–274]. These are resource-design arguments and reported consultation observations. The article does not establish a long-term recovery effect or verify the intervention-success sets used in the present formal models.

The design raises a specific question for the present inquiry: what must a participant be able to distinguish before selecting a suitable use of an archival exemplar? A transcript can preserve a linguistic form while leaving its local communicative role unresolved for a particular reader. A function-oriented description can help with that question, but its adequacy depends on the intended action. Selecting an exemplar for analysis, preparing a supervised rehearsal, and undertaking an unmediated encounter make different demands on the description.

That dependence does not diminish the value of retaining the original audio or its provenance. The resource may support future reinterpretation that the immediate learning task does not anticipate. The current analysis asks whether an expressly declared use is supported. It would be an overextension to infer that every additional detail is dispensable because one exercise can proceed without it. It would be equally uninformative to assume that retaining the richest available material automatically establishes the exercise's feasibility.

The practical commitments in the documented account make a second question visible. An intervention menu belongs to a setting with finite opportunities for participation. A model that permits unlimited expert assistance could certify a policy that the intended participants cannot execute. Conversely, a model that allows only unsupported individual study could exclude an available cooperative route. Selecting

the menu therefore requires evidence about the relevant participants and episode, rather than a general judgment that one teaching arrangement is preferable.

## 12.2 Context-Sensitive Rehearsal Selection

The following example is constructed for analysis and describes no reported Mangarrayi participant. Suppose a learner and facilitator agree on a bounded task: prepare and complete one supported rehearsal of a selected request operation during an available encounter. Prior evidence warrants treating that operation as an object of recovery. The immediate target is successful use and correction within the supported encounter, not established independent competence or enduring language vitality. A later persistence test would constitute an additional target.

Two candidate situations remain compatible with the learner's current excerpt. In one, the displayed contextual label is appropriate to the planned rehearsal. In the other, a historical annotation convention makes the same label refer to a different interactional role. The distinction is stipulated to matter to the facilitator's choice of exercise. It is also stipulated that the learner can read the excerpt but cannot yet discriminate the conventions. Readability and sufficiency therefore come apart for a precise reason: the same available record permits situations requiring different immediate preparations.

Now specify the feasible alternatives. If the encounter permits only one of two incompatible exercises, and each succeeds in just one situation, the excerpt cannot support a guaranteed choice. This conclusion follows from the stipulated success relation. It does not require a claim that the excerpt is false, that the archive is incomplete in every respect, or that recovery is impossible by every route. Both situations possess an appropriate exercise; the available record fails to select between them.

Suppose instead that a facilitator can first explain the two conventions through a supported comparison accepted in both situations. If that comparison fits within the encounter and leaves time for the chosen rehearsal, it can provide a common first step in a contingent policy. The original excerpt remains ambiguous, yet the complete access arrangement may be adequate. Calling the excerpt alone sufficient would omit the facilitator's contribution. Calling the arrangement insufficient merely because the excerpt remains ambiguous would omit the policy's available inquiry.

The example identifies the evidence an application would need. Investigators would have to support the two interpretations, the participant's present discrimination, the exercises' domains, and the feasibility of the comparison. They would also have to specify the criterion by which the rehearsal succeeds. The published resource-design case motivates these questions; it does not establish the constructed answers. This separation permits the formal obstruction to be useful without assigning unmeasured hidden states to actual learners.

## 12.3 Partial Records Supporting A Common Route

A second constructed case holds the target and participants fixed while varying the action menu. Two available meeting settings can support the selected rehearsal. The retained record establishes that at least one has the necessary support, but conceals which. Each setting is practically accessible. If the participant can arrange only one meeting and the settings require different choices, the aggregate report may leave the choice unresolved. If the available arrangement permits a joint invitation that reliably reaches whichever setting is ready, the same report can suffice for the declared target.

The second arrangement requires an actual delivery mechanism. An analyst cannot introduce "use the ready setting" as an action merely because a ready setting exists. The invitation procedure must be available without prior identification of that setting, and its successful operation across the candidate situations must be supported. This requirement is the practical counterpart of distinguishing an acceptable-policy witness from a selector that the participant can execute.

The regional field example in Sections 9 and 10 develops a different realization of the same question with an explicit batch budget and derived pulse outcomes. Its regions are selected operational settings; its success criterion follows from the stated dynamics. The constructed meeting case supplies no empirical calibration for those equations. The connection is the analytical question of choosing a feasible

intervention when retained information localizes useful support only partially. Their different evidential status remains intact.

A coarse record's success also need not erase historical uncertainty. A participant may complete the rehearsal while remaining unable to say which earlier teacher, document, or organizational arrangement produced the present support. The completed task supplies evidence about its own route and result. It does not answer the separate lineage question. Keeping that distinction visible avoids interpreting practical success as retrospective proof of a uniquely identified transmission history.

The policy can moreover preserve an opportunity for later historical inquiry. A task-relative summary may be sufficient for immediate action while the richer dossier remains available to another authorized inquiry. These two record arrangements serve different purposes and need not compete. A claim of local adequacy becomes misleading only when it is promoted into a claim that the unused historical distinctions have no further value.

## 12.4 Archive Interpretation Learning

Leonard describes the 2011 Breath of Life archival institute's pairing of community researchers and linguistic partners, including linguistic instruction intended to assist interpretation of archival materials. His discussion also records different understandings of the language work participants pursued [11, pp. 20–23]. The account supports attention to acquired archival literacy and participant purposes. It does not show that a particular instructional component alone caused restored use, or that every participant shared one recovery target.

For the current problem, interpretive learning can alter the participant's available record distinctions without changing the retained document. A person who learns a transcription convention can subsequently discriminate marks previously treated alike. The source of that later discrimination is the retained material together with newly acquired resources. A sufficiency assessment should therefore identify the participant's stage of interpretation, rather than assign one timeless accessibility status to the document.

Consider a constructed task in which a participant must select between two exercises after consulting an old notebook. The notebook retains the relevant distinction, but the participant initially lacks its key. If an available orientation supplies the key in time and its interpretation can be checked, the notebook and orientation together may support the selection. A claim that the notebook was already practically sufficient would conceal a necessary part of the route. A claim that the original record lacked the distinction would misidentify the source of the difficulty.

Learning also complicates retrospective assessment. Suppose the participant can later perform the operation and interpret the notebook. The later achievement alone cannot identify how much operative understanding was present before instruction. Instruction may have helped reveal earlier retained knowledge, supplied new knowledge, or combined both processes. The present record-sufficiency analysis can ask whether the episode supports a future action without settling that historical allocation. Establishing retained dormant capacity requires its own evidence, as the preceding reactivation inquiry emphasizes [9, Section 3].

An adequate evaluation can preserve both questions by recording the initial interpretation task, the assistance supplied, the later task, and the conditions of performance. Even that record will have a resolution and scope. It can make the inference inspectable without reconstructing every contribution to learning. This is a proposed evaluation design, not a finding about the institute described by Leonard.

## 12.5 Audience Rendition Scope

Nathan describes an alternate presentation of Eli Timan's materials that omitted some morphological detail while adding an Arabic transliteration, an English translation, and drawings by the storyteller for a different audience [12]. The example documents a particular rendition decision. It does not establish that reduced annotation is generally preferable or that the rendition achieved a measured recovery outcome. The account is historical, and no claim about current access follows from it.

The example helps distinguish formal information retention from practical use. A presentation with fewer analytic distinctions can be easier for a specified audience to employ, while a specialist representation remains more discriminating for another task. The relevant comparison needs the participant and aim. Character count or annotation density alone cannot decide which presentation supports the selected action.

For this paper's bounded question, a rendition should be evaluated by the distinctions it makes usable for policy selection. A simplified presentation may retain exactly what a supported reading episode requires. A grammatical comparison might require the omitted annotation. Neither judgment determines the archive's overall quality. The original and alternate presentations can remain complementary resources.

The model's record-refinement result must accordingly keep access conditions fixed. Giving a participant an additional usable distinction at no cost can preserve an earlier policy when the distinction may be ignored. Replacing an accessible presentation with an unfamiliar analytic representation changes that premise. Even if the latter encodes more distinctions, the participant's practical access arrangement may have become less capable. The present analysis states that dependence without attempting a general theory of interface design.

## 12.6 Task-Specific Evaluation Design

The documented projects and constructed cases support a limited empirical agenda. Begin with an agreed episode and an independently stated criterion of success. Identify the intended participants, the relevant resources, the permitted forms of assistance, and the time available. Then examine whether the record distinguishes situations in which the feasible choice materially differs. This procedure asks for evidence about a declared action problem; it does not require that an investigator first reconstruct an entire language or social history.

The order of inquiry matters. Defining the only successful intervention after observing what the participant happened to do can make the test circular. A defensible evaluation can specify a family of acceptable outcomes in advance while recording justified revisions when the task changes. The record should show which version of the target each assessment uses. Such transparency allows an episode to be informative even when participants revise their purposes during it.

Positive and negative findings also demand different support. One observed successful choice establishes that the choice worked in that observed episode, within the assessment's limits. It does not establish success throughout every situation admitted by an uncertain record. A robust guarantee requires a justified domain and a supported success argument over that domain. Conversely, one unsuccessful attempt does not establish that every policy available through the record fails. The obstruction concerns the whole permitted policy class, which may be difficult to delimit empirically.

Those difficulties suggest graded claims. An inquiry can report that a missing contextual distinction affected a particular choice, that an orientation made a selected distinction usable, or that an explicitly tested policy worked across a stated collection of cases. These findings are useful without being described as complete sufficiency theorems. The formal analysis helps identify the stronger claim and its extra premises; it should not encourage a stronger label than the evidence warrants.

Finally, inquiry can consume or transform the opportunity being evaluated. A delayed clarification can improve historical understanding after the available facilitator has left. A trial exercise can simultaneously inform the investigator and advance the participant's recovery. Recording those effects gives the action history its proper role. The practical target is a defensible account of what the record and access arrangement enabled, at what stage, through which feasible contributions. That account remains open to participant criticism and later evidence while being precise enough to guide the next bounded investigation.

## 13 Record Sufficiency Objections

The proposed account evaluates a record through the recovery choices it supports. Its usefulness depends on how the target, uncertainty family, access arrangement and available policies are specified. This section tests those commitments against six objections. The discussion identifies cases in which a formal sufficiency result has little practical force, cases in which a negative result reflects a revisable modeling choice, and conditions under which a narrower positive finding remains informative. The objections concern the interpretation of the constructions rather than an attempt to derive their social adequacy from their formal correctness.

### 13.1 Target Selection

A first objection concerns the ease of manufacturing sufficiency. An analyst could choose a weak recovery target, permit an undemanding intervention and then find that almost any record is adequate. For the narrative practice, counting one correctly repeated phrase as recovery would make the task easier than restoring use of the distinction across unfamiliar accounts. A theorem conditional on the chosen target cannot determine whether that target answers the participants' concern.

This objection is substantive. Target relativity permits precise inquiry, but it also permits selective descriptions of success. The reply must therefore examine the relation between the target and the undertaking. A pilot study can legitimately ask whether one supported encounter is possible. Its result should then be reported at that level. If the practical aim concerns recurrent use, an endpoint that omits continuation cannot answer it.

The target can also be revised during inquiry. Participants may discover that an earlier convention is unsuitable for present uses, or that another operation matters more. The revised target may be better justified. Its adoption changes the question being evaluated, however, and should remain visible. Calling the change an improvement in record sufficiency would confuse a change of evidence with a change of the success condition.

A limiting case makes the issue especially clear. If doing nothing already achieves the declared target, a constant policy can render every accessible record sufficient for that task. The mathematical result is correct. It does not establish that a lost capacity was recovered or that an intervention is warranted. The interpretation must identify whether the selected question concerns maintenance, a possible future return or an actual transition to recovered use.

The taxonomy distinguishes recovery modes and preservation targets [7, Sections 9 and 11]. The present account uses those distinctions to state what a selector is supposed to accomplish. It supplies a conditional adequacy test after that choice, while leaving the reasons for selecting and revising the target open to participant criticism.

### 13.2 The Uncertainty Family

A second objection challenges robust feasibility. Requiring one policy to succeed across every admitted history can make a useful record appear inadequate because of a remote or poorly supported possibility. Conversely, an analyst can obtain a favorable result by excluding the history on which the selected intervention fails. The formal criterion is sensitive to judgments about evidence that it does not itself establish.

The appropriate response is to state how the family was formed and what would change it. Histories can be admitted because a current record fails to distinguish documented support arrangements, because a measurement has a justified uncertainty range, or because relevant testimony remains unresolved. Those grounds differ from adding arbitrary imaginable failures. The model should retain the uncertainty relevant to the decision without claiming that its family exhausts reality.

There are consequently two distinct negative findings. A record may fail to support a common policy within a well-supported family. Alternatively, the inquiry may be unable to justify a sufficiently precise family for the proposed guarantee. The second problem cannot always be repaired by collecting another

item of the same kind. It may require changing the description of the task or the interpretation of an observation.

The robust condition also has a limited decision-theoretic role. It identifies a policy that succeeds throughout the declared family. It does not establish that participants ought to demand such a guarantee before acting. A justified risk-sensitive or probabilistic procedure can make another choice, but then its distributional assumptions and loss criteria must be supplied. The present failure result concerns its stated standard and should not become an unconditional prohibition on recovery attempts.

Positive results remain revisable as well. When evidence introduces another credible history, a former selector can cease to be robust. That revision need not show that the earlier mathematics was wrong. It shows that its coverage premise changed. An auditable report should preserve the earlier domain and the reason for enlarging it rather than retrospectively presenting either family as unconditionally complete.

### 13.3 Practical Implementability

A third objection concerns the word accessible. If the model simply declares a readable record and an executable menu, it can assume away the difficulty it purports to study. The existence of a selector then says only that an idealized participant could follow it. Actual participants may lack the relevant decoding skill, reliable communication or time to compare its branches.

The distinction between abstract and practical implementation should therefore remain explicit. The finite result can establish that a record carries enough distinctions for a function or contingent policy. An application must additionally establish how the participants identify the record class and carry out the selected instruction. A formally finite table can still be too large, difficult or culturally inappropriate for the actual undertaking.

The criterion-comparison manuscript already rejects the proposal to repair an insufficient record merely by appending the answer that the decision procedure is supposed to compute [8, pp. 10–11]. A participant-usable construction must identify an independently obtainable discriminator. The present recovery application inherits that obligation for both the record and its interpretation.

Practical access can nevertheless be collective or supported. A participant can rely on an agreed interpreter, a familiar guide or a communication procedure. Such support need not invalidate the result. The inquiry should describe the supported policy actually available, including the continuing work and commitments on which it depends. It should avoid shifting between individual and collective access while retaining the same unqualified sufficiency claim.

This response gives the model an empirical point of failure. If the required report cannot be produced, its meaning is disputed, or its instruction cannot be executed within the declared horizon, the application fails at an identified bridge. Formal sufficiency then remains a conditional result whose missing practical premise is known. Naming that premise is more informative than treating mathematical existence as completed recovery design.

### 13.4 Choice Preservation

A fourth objection argues that one common successful policy is too weak to preserve participant agency. A coarse record can support a policy that reaches the target while hiding another route participants would prefer. It can also make a burden-intensive intervention appear unavoidable by suppressing the distinctions needed to identify a less demanding alternative.

The objection identifies a stronger informational purpose. Feasibility of at least one route differs from preservation of a participant's relevant choice set. If an inquiry aims to support comparison among all acceptable alternatives, its success condition should retain the information needed for that comparison. A record adequate for the narrower task can be inadequate for deliberation without contradiction.

In the district example, a common plan might allocate scarce facilitation to one interface while leaving another group's preferred arrangement unavailable. The selected target could still be achieved. Whether the allocation is justified depends on the commitments, alternatives and affected claims. A mere assertion that some circle will recover gives no general answer to those questions.

The procedural source already retains history-sensitive differences among repair routes sharing the same selected restoration profile [5, pp. 34–36]. That architecture permits the present inquiry to report both common feasibility and the additional comparisons it leaves unresolved. It also explains why successful outcome compression should not erase records needed for contestation or acknowledgment.

The limited criterion is useful when participants have already authorized a bounded target and need to know whether their current evidence supports any feasible route. It becomes insufficient when the practical undertaking includes choosing fairly among routes or obtaining informed agreement about burdens. Those further conditions should be specified as additional tasks, not presumed to follow from the selector’s existence.

### 13.5 Maximal Retention

A fifth objection favors comprehensive retention. Future recovery tasks may be unforeseen, and a record sufficient today can omit the distinction needed tomorrow. Deliberately designing a coarse record could therefore create avoidable future dependence or loss. The absence of a universal minimal record might seem to support retaining everything instead.

The uncertainty about future use is real. A current sufficiency proof cannot establish that omitted material has no other value. It supplies no automatic deletion rule. An archive can preserve historically important distinctions even when the present policy does not consult them. Practical record design can therefore retain a core decision record alongside additional material whose future uses remain open.

Maximal retention, however, requires its own interpretation. Recording every possible context is not an available operation, and additional collection, maintenance and disclosure can impose costs. A retained distinction can also remain inaccessible without equipment, decoding conventions or participant permission. More material is consequently neither equivalent to more usable information nor a complete response to future uncertainty.

The criteria source already describes the tension between specified future revision tasks and unrestricted future demands [8, p. 11]. The present inquiry applies that lesson to recovery choice. It can identify which distinctions the current task requires, which alternatives another record would support, and what assumptions make later extension possible. It cannot derive an unconditional storage policy from informational ordering alone.

The absence of a unique coarsest adequate record is therefore an invitation to compare justified designs. It does not establish that every design is equally good or that further evaluation is impossible. Collection feasibility, interpretive burden, future aims and participant commitments can distinguish designs whose present recovery guarantees are the same.

### 13.6 Explanatory Contribution

A final objection concerns the contribution claimed here. Policy intersections, strategies using observations, and record factorization belong to established mathematical reasoning. The field model adds stipulated dynamics whose interpretation remains conditional. The combination could amount to familiar tools applied to an example rather than an independent explanation of linguistic recovery.

The objection rules out a broad novelty claim. The paper does not claim to invent informational sufficiency, partial-observation control, target-relative repair or the possibility that inquiry has costs. The prior Generative Relational sources already develop several of these commitments. Their explicit inheritance is necessary to judge what the present investigation adds.

Its proposed contribution is the connected analysis of one bounded question. A record’s policy adequacy can survive historical underdetermination, differ across resource bounds, lack a unique coarsest realization and fail after an informative inquiry changes the available continuation. The nonlinear regional construction supplies conditions under which these distinctions have calculable consequences for a selected activation mechanism. The philosophical analysis then identifies what would warrant applying that mechanism to participants and their records.

This contribution is strongest where a result rules out a tempting inference or supports a specific alternative. Separate recovery routes do not establish an accessible common choice. A finer late observation need not restore an earlier opportunity. A coarse grouping can nevertheless be adequate when the available policy budget covers it. These consequences give the task structure explanatory use without establishing empirical validity or originality for the general mathematics.

Further applications may show that the assumed policies, records or dynamics poorly describe a particular practice. That would limit the application and might motivate another representation. The present account is offered as a preliminary study with identifiable assumptions and tests, rather than a universal characterization of the information needed for linguistic recovery.

## **14 Recovery Record Inquiry**

An inquiry into record sufficiency should produce a defensible account of the choice that the record enables, the conditions on which that choice depends and the changes that would defeat it. This section develops a proposed reporting and comparison procedure through the constructed narrative practice. Its objective is to make the formal distinctions usable for criticism without presenting a validated protocol. The discussion follows the declaration of a task, the examination of accessible evidence, alternative record designs, the consequences of further inquiry and the interpretation of an eventual finding.

### **14.1 A Declared Choice Problem**

The inquiry begins with the participants' intended undertaking. In the three-district example, suppose they provisionally agree to establish one recurrent narrative circle that can use the completed-event and interrupted-attempt distinction. This aim is narrower than providing every district with equal access or restoring the entire practice. The initial agreement should state why the limited target serves the present project and which wider aims remain unresolved.

The selected outcome needs a temporal interpretation. One supported session can document immediate performance. A recurrent circle requires further relevant occasions and the continuing support needed for them. The report should identify which outcome the available observations can assess. If the formal result certifies a modeled sustaining condition, its interpretation should explain how that condition bears on the participant-level target.

The policy menu should then be described in terms participants can recognize. It may consist of a bounded number of facilitated sessions, a permitted readiness inquiry and an agreed method of choosing the receiving interfaces. The specification should identify what each session supplies, what preparation it assumes and who can arrange it. Resource and timing conditions belong in this description rather than appearing only in a later qualification.

The retention premise requires its own record. Where the undertaking is described as reactivation, there should be independent grounds for attributing the relevant retained contribution. A successful session following instruction can support current performance while leaving that earlier attribution unresolved. The taxonomy's separation of dormant recovery from reconstructive recovery remains useful here [7, Section 11]. The choice analysis should report its selected branch and any uncertainty about it.

### **14.2 The Access Comparison**

The next step examines the record participants can actually use. This can require comparing an archival package, the version delivered to participants, its available interpretation aids and the information communicated through the current support arrangement. The comparison should identify consequential differences, such as a missing annotation key or outdated contact information, rather than equating access with the existence of a downloadable file.

For the narrative archive, investigators could identify what the examples show about the selected distinction, what they show about the contributions that produced the earlier response, and what they

leave open about present readiness. An accurate transcript can settle the first question while leaving the others unresolved. Such a result preserves the value of the transcript at its appropriate level.

The admitted histories should be linked to this evidence. Suppose current reports establish that one district has the needed continuing preparation but fail to identify which one. The three possibilities then have an explicit evidential origin. If even the existence of a prepared district is uncertain, the inquiry must retain that additional possibility or explain its exclusion. The formal action comparison should not inherit a favorable premise that the documentary inquiry never established.

Participants may possess relevant information outside the central archive. A local coordinator can know that a partner has become unavailable, while an archive custodian knows the original session conditions. The reporting arrangement should indicate whether and how these contributions can enter the current decision. Their potential availability is different from their actual communication before the choice.

This step can reveal an access repair that requires little new historical research. Restoring a notation guide, providing an agreed translation or reconnecting a current contact route can make a retained distinction usable. It can also reveal a substantive gap that demands new evidence. The finding should name which kind of repair is required, including the possibility that neither is presently available.

### **14.3 Alternative Record Designs**

A stipulated finite comparison illustrates the design question. Suppose exactly one of the three district interfaces has the required preparation, any session at that interface achieves the selected task, and a session elsewhere fails. Participants may deliver sessions to at most two interfaces in one batch. These are declared assumptions for this comparison, not empirical conclusions about actual districts.

An aggregate statement that one interface is prepared leaves three possible locations. Any fixed batch of at most two omits one of them, so that record supplies no guaranteed successful batch. Yet each possible history has an individually successful choice. The problem lies in selecting a common batch from the available information.

One additional record distinguishes whether the first interface is prepared. If it is, participants select that interface. If it is not, they select the other two. This record is sufficient for the stipulated target and budget without identifying the prepared location in every case. An analogous record distinguishing the second interface is also sufficient. The two records preserve different distinctions and support different conditional choices.

The comparison does not yet establish an obtainable record. Someone must be able to make and communicate the readiness judgment under the actual access conditions. Nor does it establish that either design is preferable. One may require less travel; another may disclose less personal information; another may be easier to explain or check. Those differences can guide the practical choice among informationally adequate designs.

The example also exposes a material alternative. If three sessions become available without changing the other assumptions, participants can visit all three interfaces using the original aggregate record. The repair then changes the action conditions rather than the information. Conversely, if the aim changes to recovery in every district, neither the old aggregate guarantee nor the selective two-session design answers the revised target.

Table 2 distinguishes these possible changes. Its rows organize a comparison among proposals rather than a prescribed sequence of steps.

The comparison in Table 2 prevents a failed record from receiving an automatic recommendation for more data. The most suitable response depends on the obstacle and the participants' reasons. A proposed repair can also combine several rows, in which case its contributions should be distinguished.

### **14.4 Inquiry Histories**

When additional evidence is required, the inquiry becomes part of the recovery policy. The report should record the question asked, the conditions under which it can be answered, the possible observations and

Table 2: Recovery Choice Repair Comparisons. Each change addresses a different component of the declared inquiry and therefore requires a different supporting argument.

| Proposed Change                    | Immediate Role                                   | Additional Obligation                                 |
|------------------------------------|--------------------------------------------------|-------------------------------------------------------|
| Recover an annotation key          | Make retained content interpretable              | Check the key and participant use                     |
| Obtain a readiness distinction     | Separate action-relevant histories               | Establish collection access and remaining time        |
| Provide another session            | Enlarge the feasible intervention menu           | Record actual delivery and associated conditions      |
| Introduce a different intervention | Change which histories share a successful policy | Establish its execution and consequences              |
| Revise the recovery target         | Change the achievement being sought              | Justify the new aim and report the changed comparison |

the continuation selected after each answer. An observation has practical value for the task when it supports a feasible continuation at the stage where it becomes available.

For example, a readiness visit can require the same facilitator time later needed for the recovery session. A policy that counts the visit as free will overstate the remaining action budget. Alternatively, the visit can establish useful coordination while gathering information. In that case it has changed both knowledge and support. Its success should not be attributed entirely to the record refinement.

Delay deserves separate attention. An accurate answer about an earlier support configuration may arrive after the relevant partner has departed. The answer can improve historical understanding while leaving the original intervention unavailable. A later adequate choice must be assessed against the successor configuration, possibly with another repertoire or target. It cannot inherit the earlier guarantee simply because uncertainty has decreased.

The procedural recovery account already requires every step to be executable from the configuration produced by the preceding steps and retains path-sensitive burdens [5, pp. 33–35]. A recovery inquiry should apply the same discipline to reading, contacting, testing and deciding. The procedure’s final information state is only part of its history.

Participant learning can create a further complication. A diagnostic encounter may teach a new way of recognizing the narrative distinction. The resulting policy can succeed because the participants have acquired an additional operation. That can be an important achievement. It should be reported as formation contributing to recovery rather than as evidence that the initial record already supported the same choice for the same participants.

## 14.5 The Reported Finding

A positive finding should state the task, accessible record, policy class, admitted histories and resource horizon. In the stipulated two-session example, the finding concerns the readiness distinction and the available batch allocation. It does not establish complete knowledge of local histories or a general guarantee for every future intervention.

The report should distinguish a proved conditional result from an observed outcome. A batch that succeeds in the actual district does not by itself establish that it would have succeeded across the other admitted histories. Conversely, a valid model proof can remain poorly supported as a description of the actual arrangement. The bridge between the two requires evidence for the policy effects, interpretation and uncertainty family used in the calculation.

An unsuccessful inquiry can also yield a precise result. The existing record may lack a common feasible choice within the selected menu, while each hypothesis separately admits recovery. The available queries may distinguish the hypotheses only after exhausting the action budget. Or the inquiry may lack

grounds for treating a proposed readiness report as reliable. These findings locate different obstacles and should not collapse into a single judgment that recovery is impossible.

Residual uncertainty should be stated at the level it affects the conclusion. Uncertainty about historical authorship may remain compatible with the selected present action, while uncertainty about current support can defeat it. The former can still matter to another inquiry. Reporting it separately preserves its significance without making complete historical identification a condition of every feasible step.

## **14.6 Participant Revision**

The inquiry should permit participants to challenge the target, a readiness classification, an assumed burden or the interpretation of a successful outcome. Such criticism can reveal missing information, a mistaken transition assumption or a disagreement about the undertaking itself. Each calls for a different revision. Their distinction makes the formal account answerable to the practice it represents.

A participant may accept a readiness report for allocating a session while declining disclosure of the underlying personal details. Another may contest the target of establishing only one circle. The first response concerns the access arrangement; the second concerns the distribution and purpose of recovery. A robust selector can inform both discussions without adjudicating them through its existence.

Stopping can be a legitimate outcome of the inquiry. Participants can judge that the available route is too burdensome, that additional inquiry is unwelcome or that another practice now deserves priority. Those judgments do not erase an established conditional feasibility result. They determine what use, if any, should be made of it.

The proposed procedure therefore aims to preserve an intelligible relation between records and possible action. It can identify a sufficient accessible distinction, expose a resource-dependent limitation, document an inquiry-induced change or leave the recovery choice unresolved for a specified reason. That is a bounded contribution to the investigation of linguistic recovery, with later questions of institutional allocation, interface substitution and normative authority remaining separately open.

## **15 A Constructed Recovery Dossier**

A developed case can show what the separate investigations permit someone to conclude and what they leave undecided. This section returns to the narrative-practice example introduced earlier. Its participants, records and intervention outcomes are constructed for analysis. They are not reported findings about a language community. The case combines a fixed target, alternative feasible protocols, a partial record, a costed inquiry and a later revision of the target. Its purpose is to make the dependencies of a record-sufficiency judgment explicit.

### **15.1 Target Specification**

Three local practice settings use archived stories that distinguish an interrupted attempt from a completed event. A participant group wishes to resume use of that contrast in a recurring narrated activity. The target is successful shared use of the selected contrast in the agreed setting, with participants able to extend its application to an unrehearsed instance. It is not reproduction of the entire earlier language, a general proficiency standard or restoration of a historical institution. A small dossier identifies the archive examples, the selected contrast, earlier uses, and the current participants' interpretation of the task.

In this constructed inquiry, independent earlier observations support attribution of the selected retained operation to the relevant participants. The warrant includes interpreted responses and participant correction, rather than an inference from silence or a count of recordings. The current uncertainty concerns enabling arrangements. If the retention warrant were withdrawn, the task would need revision: forming an operation through instruction could be valuable, but it would no longer satisfy the same retained-operation description merely because the final activity looked similar.

The group permits three protocols,  $\alpha, \beta, \gamma$ . Each supplies a different arrangement of facilitation and presentation for one session. All three are acceptable for the stated target under the group's stipulated decision procedure. Their acceptability is a premise of the case, not a conclusion of the finite matrix. A person declining a session would change the available outcomes and domains.

For this bounded case, success requires usable extension of the contrast at three subsequent agreed meetings after the initiating session ends. The matrix below is stipulated to cover that continuation test, not merely performance during facilitation. The facilitator budget counts initiating and inquiry sessions; the resources for the three later meetings are separately assumed available. Three occasions provide the case's declared assessment horizon and imply no guarantee of indefinite persistence.

## 15.2 Three Compatible Histories

The available archive and current coarse account leave three dossiers  $h_1, h_2, h_3$  compatible. The dossiers include different preparation histories that make different pairs of protocols workable. For the example, controlled pilot evidence is stipulated to justify exactly the success relation in Equation 7. No such pilot study is claimed to have occurred. This stipulation makes the ensuing logic assessable while leaving the evidential cost of building the relation visible.

The initial record does not distinguish the three dossiers. Every one has two feasible protocols, so an analyst inspecting the full matrix can say that the task is individually recoverable at each candidate. The participant cannot choose an always successful one-session protocol from that record:  $\alpha$  fails at  $h_2$ ,  $\beta$  at  $h_3$ , and  $\gamma$  at  $h_1$ . Listing these three existential recovery claims together therefore overstates what the record enables.

A readiness message can repair the selected information gap without identifying the complete history. One allowed message distinguishes  $h_1$  from the pair  $\{h_2, h_3\}$ . On the first branch the facilitator can select  $\alpha$ ; on the second it can select  $\gamma$ . Another message distinguishing  $h_2$  from  $\{h_1, h_3\}$  also works, with a different assignment. These two records preserve different distinctions and neither refines the other. Their existence is the practical counterpart of Proposition 7.1.

The choice between those messages can be made on grounds outside the success matrix. One may be easy for a local custodian to report without disclosing private preparation details. Another may be hard to translate into a label understood by the facilitator. Selecting the first can retain sufficient task information while leaving the historical explanation distributed. Its sufficiency still depends on reliable message production and interpretation. A short message produced by an ungrounded guess has the right formal shape but lacks the required evidential status.

## 15.3 Inquiry Timing

If the message is already available, one intervention session suffices. If obtaining it requires a separate facilitator session, the adaptive case of Section 8 applies. Two sessions permit inquiry followed by the selected intervention. With only one, the query can resolve the relevant record class after the opportunity for intervention is gone. The judgment should then report an unavailable combined procedure, rather than treating the refined answer as proof of practical sufficiency.

A different remedy can enlarge the intervention repertoire. A joint session format might accommodate all three histories if participants can actually perform it and if it remains acceptable. Adding that protocol would make the constant record sufficient. Its effect would be a change of feasible actions, not discovery of a previously stored distinction. Calling both remedies improved information would obscure the difference between learning which route to use and creating a route that works across uncertainty.

The regional field introduces an additional possibility. Suppose the available support arrangements weaken during a long delay. The earlier success matrix may cease to describe the current situation. A perfectly accurate account of the original preparation can then coexist with failure of every currently admitted standardized episode. Proposition 10.2 proves that possibility within its selected dynamics. The narrative case does not import its numerical coefficients. It uses the model to identify an empirical question: whether the support required by the record's advice still exists when the advice can be used.

## 15.4 Historical Interpretation

Suppose a protocol succeeds. The result warrants a statement about the stipulated operation and its observed continuation under the given arrangement. It does not by itself establish that every historical contrast in the archive has been recovered, that a whole community has resumed an earlier language, or that the participants agree on the activity's cultural description. Those claims require different records and different arguments.

The history-bearing representation helps keep these claims separate. The successful episode appends a new event to the participants' ongoing relations. Its archival references remain identifiable through their provenance, while its current realization can have new purposes or forms. Preserving the earlier anchor does not force a claim that the present event duplicates it. Likewise, an effective intervention can contribute new support without proving that it created the selected retained operation.

The case also leaves room for a defensible change of aim. Participants may decide that extending the contrast into a new genre matters more than resuming the earlier activity. The recovery target has then changed through deliberation. A record adequate for the first task may help with the second, but its previous certificate does not transfer automatically. Reporting the change preserves the relationship between the tasks without classifying the later development as failure merely because it departs from the initial operational target.

## 15.5 A Bounded Adequacy Report

The resulting report would state the selected operation, the participants and permissions included, the alternative support histories still admitted, and the protocols whose feasibility has been supported. It would identify which record distinctions the intended users can read and how those distinctions determine a choice. It would also state the time at which the success relation applies and any inquiry needed before execution.

This report is narrower than a complete archive and more substantive than a declaration that recovery is possible. It explains what a particular record enables, for whom, under which conditions and with which remaining uncertainty. Where the evidence is inadequate to establish the success relation, the report should retain that limitation. The formal criterion clarifies the missing premise; it does not manufacture the premise from the presence of a diagram or an equation.

## 16 Research Boundaries

The paper's analytical object has been a record adequate for a specified feasible recovery choice. This final section identifies the results established within that scope and the work required for their application. It distinguishes the contribution of the philosophical reconstruction, the conditional mathematical investigations and the primary documentation comparisons. The distinctions matter because successful formal construction does not supply a complete practical programme.

### 16.1 Established Conditional Results

For a finite fixed task, record-dependent successful selection requires a common feasible policy on each accessible record class. The criterion is weaker than complete state identification and stronger than separate existential recovery for each candidate. It reduces to the inherited factorization criterion when the output is a prescribed function. Allowing several successful outputs can produce incomparable minimal adequate records: the triangle example supplies pairwise witnesses without a common witness for the whole class.

The finite inquiry model extends the comparison to actual observation histories. Its recursion evaluates possible successors after every operation, preserves all admitted outcomes, and requires the next action's domain throughout the remaining information set. The induction establishes a bounded successful-

stopping criterion. It does not establish that the finite description is empirically complete, nor does it turn a conditional sure-success statement into an unqualified promise.

The nonlinear field supplies a more specific substantive investigation. Within its selected support preparations and standardized pulse menu, successful batches are exactly those intersecting prepared regions. The record's regional resolution then determines an exact minimum session budget. An aggregate record can be adequate at one budget and inadequate at another; identified block counts can support success without exact preparation identification. A separate late-delay proof shows loss of this standardized route even with a precise historical location record.

The algebraic treatment expresses successful selection as a record-measurable partition of decision projections. It distinguishes that partition from knowing every policy's success indicator. Partial history operators add the independent domain condition needed to represent execution. These results use established mathematical structures. Their role is to expose dependencies in the recovery question, rather than to establish the novelty of the structures themselves.

## **16.2 Empirical Obligations**

The principal empirical obligation is to justify the link between the selected target and the evidence used to assess it. A field activity floor is not a measure of linguistic competence. A finite successful endpoint is not automatically a meaningful continuation. A recorded response can disclose retained organization, arise through new instruction, or acquire its meaning through the encounter. Distinguishing those possibilities requires historical and interpretive inquiry alongside any formal test.

The candidate family and protocol library also require defense. Excluding a live alternative can make a record appear adequate too easily. Including an unavailable protocol can manufacture a common witness. Restricting the library can make a feasible task appear impossible. The appropriate remedy is to document and reconsider these choices, not to replace them with an unrestricted set of imaginable histories or interventions.

An empirical regional study would need to identify the support channels, defend their measurement, examine boundary exchange and test how initiating episodes interact with support renewal. The exact field results are sensitive to those assumptions. The present simulations check the selected equations and illustrate their consequences. They provide no calibration and no evidence that one set of coefficients governs linguistic communities.

## **16.3 Normative Boundaries**

Feasible selection leaves unresolved who may define the recovery aim, authorize an intervention, bear its burdens or decline participation. An admissibility condition can represent a decision already defended elsewhere; placing it inside a domain does not establish that defense. Likewise, a sufficient record can disclose information that its subjects have good reason to withhold. The mathematical possibility of a less informative successful record can inform that discussion without deciding the relevant rights or obligations.

There is also no general duty to preserve the current task. Participants can revise their aims or choose a different future. The historical architecture permits those changes to be recorded without treating every departure as loss. The present paper fixes a task to make its dependencies inspectable, while keeping that fixation available for philosophical criticism.

## **16.4 Further Investigation**

A useful next empirical step would compare a small number of actual record-use episodes with their declared targets, accessible materials, operation conditions and subsequent revisions. Such work could test whether the present distinctions identify missing information or merely redescribe facts already handled more effectively by practitioners. Participant assessment should help determine that question. The

present construction supplies questions for inquiry rather than a universal form that every project must complete.

Further formal work could examine uncertain success probabilities, noisy regional records, nonzero interregional transfer, changing protocol grammars and continuous configuration spaces. Each addition would require a new adequacy criterion or proof. A sound abstraction connecting a field to an implementable finite controller would be especially useful; its observation and transition relations must be established rather than assumed from similar notation. The current finite, field and algebraic models remain complementary restricted representations.

The resulting position is therefore precise but limited. A record's contribution to recovery depends on the choices it enables within an interpreted historical situation. Its retained distinctions matter through available operations, resources, timing and the task participants actually adopt. Preserving that relation in the analysis gives a more assessable claim than either complete-description demands or an unsupported assertion that recovery remains possible.

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