

LOOP OBSERVABLES AND THE GAUGE STRUCTURE OF INTERPRETATION

A Preliminary Discussion of Evaluation without an External Frame

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Abstract

An assessment of a social arrangement is made from some interpretive frame, and no frame stands outside the history it assesses. The difficulty this creates is familiar and is usually met either by privileging one frame or by abandoning the possibility of frame-independent content. This paper proposes a third route, taken from the treatment of the same structural problem in gauge theory. Where a quantity defined at a point depends on a choice of local frame, the quantity carried around a closed loop does not, and the loop-carried quantity is the frame-independent content of a system that has no privileged frame anywhere. The paper develops the proposal in three steps. It fixes the signature of an assessment, arguing that the object evaluated is the transformation and never the transformed structure, and that the resulting type is recursive and coalgebraic: an assessment returns a restructured world together with a new assessor. It identifies the equivalence appropriate to such an object as bisimulation, from which it follows that no scalar quotient of the assessment relation exists. And it identifies interpretive frames with gauge, bisimilarity with gauge equivalence, and the frame-independent content with holonomy around closed circuits. Two results follow that the paper verifies numerically. Loop traces are invariant under every change of frame while edge-local quantities are not, and the failure of translations to compose to the identity around a cycle is a measurable anholonomy. The paper states the relation to generative justice in one line: where that programme locates a conserved quantity, this one locates a holonomy, and the two differ in precisely the way a local conservation law differs from a global obstruction.

Keywords. holonomy; gauge invariance; bisimulation; coalgebra; interpretive frame; generative justice

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Use of generative artificial intelligence. Generative AI assistants (Anthropic’s Claude and OpenAI’s ChatGPT) were used extensively in preparing this manuscript: for literature survey, for drafting prose from the author’s specifications, for adversarial review of the argument, for the construction of the figures, for consistency and citation auditing, and for typesetting. Formulations and objections arose in the course of that exchange as well as from the author. Every claim, argument, and citation was reviewed and decided by the author, who is responsible for the content and for any errors. Bibliographic details are to be verified against the primary sources before reliance.

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1 INTRODUCTION

Every assessment of a social arrangement is made from somewhere. The party assessing holds a vocabulary, a set of distinctions, a body of precedent and a sense of what counts as a reason, and these are formed within the history the assessment is about. No frame stands outside that history, and the observation is not a sceptical flourish but a condition of the problem: a frame that stood outside would have to be acquired from somewhere, and there is nowhere else.

The received responses are two, and both are unsatisfactory in the same way. One privileges a frame, whether by appeal to reason, to a procedure, or to a position held to be neutral, and the privilege is always contestable by the argument just given. The other concludes that frame-independent content is unavailable and that assessments are reports of the frame that made them, which delivers a result no one can act on and which is self-undermining when applied to itself.

A third route is available and has been worked out at length for a structurally identical problem. In a gauge theory, a quantity defined at a point depends on a choice of local frame that is arbitrary and unremovable; there is no canonical frame anywhere and no theorem produces one. Nothing follows about the unavailability of physical content. Quantities carried around *closed loops* are invariant under every change of local frame, and the loop-carried quantity is the physical content of a theory that has no privileged frame at any point. The structure of the escape is worth naming precisely, because it is what this paper transposes: the frame-independent content is recovered not by finding a better frame but by identifying which quantities are indifferent to the choice, and those quantities are the ones defined on closed circuits.

The transposition is developed in three steps.

The first fixes what is being evaluated. An assessment applied to a transformed structure evaluates the wrong object, since the same structure may be produced by a transformation that may be performed and by one that may not, and no inspection of the result distinguishes them. The object of evaluation is therefore the transformation, and the assessment is a map from transformations to something. §4 argues that the something includes a further assessor, so that the type is recursive: an assessment returns a restructured world together with the assessor that world now carries. That type is the type of a Mealy machine, and its natural home is coalgebra.

The second fixes the equivalence. For objects of that type, the appropriate notion of sameness is bisimulation: two assessment arrangements are equivalent when they sustain the same observable interactions at every future step. §6 derives the consequence, which is that no scalar quotient of the arrangement exists, so that the familiar demand for a measure of justice is asking for something the object's type excludes.

The third makes the gauge identification. An interpretive frame is a local choice, made at a position in a relational structure, of the vocabulary under which what happens there is described. §7 argues that the redundant part of interpretive difference is gauge, that

the criterion separating the redundant part from the substantive part is bisimilarity, and that the frame-independent content is accordingly the holonomy around closed circuits of translation between positions. §8 verifies the two claims that carry the account: loop traces are invariant under every change of frame, and edge-local quantities are not.

Two consequences are stated as results. The first is negative and is due to the structure of the gauge problem itself: no global choice of interpretive frame is available, and this is not a practical difficulty but an obstruction of the kind Gribov identified, which §9 states in its transposed form. The second concerns the absence of a global measure. §10 shows that an assessment field with non-trivial holonomy admits no potential, so that a locally determinate direction of assessment coexists with the non-existence of any global quantity of which that direction is the gradient. Three properties the companion papers of this series required are thereby obtained together: no global scalar, a determinate local direction, and dependence on the path taken.

The relation to generative justice can be stated in one line and is the paper's principal claim about its own position. Eglash's criterion is a *conserved quantity*: value generated returns to its generator, and the criterion is stated locally, holds without reference to any observer, and requires no loop [Eglash (2016)]. The criterion offered here is a *holonomy*: it is defined only on closed circuits, it is trivial in the absence of curvature, and it detects an obstruction in place of a conservation. The two are compatible and they are not the same kind of thing, and the difference is exactly the difference between a local conservation law and a global obstruction.

The debts and the limits are stated together. The apparatus is borrowed and no part of it is original to this paper; what is offered is an identification of the social structure to which the apparatus applies, and an argument that the identification is forced by the problem in place of chosen for its elegance. §2 records that the paper supplies no criterion of justice, that its worked model is a lattice construction and no claim about any actual society, and that the difficulty about diffeomorphism-invariant observables which motivates the whole apparatus in physics is an open problem there and is not solved here.

The argument proceeds in five parts. Part I fixes the object (§2) and states the prior formulations (§3). Part II fixes the signature (§4), gives the coalgebraic type (§5), and derives the equivalence (§6). Part III makes the gauge identification (§7), verifies the loop observables (§8), and states the obstruction (§9). Part IV treats the absence of a potential (§10) and the formulation without background time (§11). Part V records the positions declined (§12) and the limits of the account (§13). PART I

THE PROBLEM OF EVALUATION WITHOUT AN EXTERNAL FRAME

2 SCOPE OF THE TRANSPOSITION AND RESTRICTIONS PLACED ON IT

This section fixes what the paper does and, more importantly for a paper of this kind, what it does not. It states the object, the four restrictions bounding it, the terms on which formal apparatus is borrowed, and the positions declined in advance. The discipline governing the borrowing is stated at length, since a transposition of physical formalism into social theory fails characteristically by borrowing the vocabulary without the constraints that give it content.

2.1 *The object of the transposition and its four restrictions*

The object is the structure of assessment in a system that contains no position exterior to itself. Four restrictions bound it.

The first is to *structure* and never to content. The paper asks what form an assessment can take given that no frame is privileged, and it supplies no assessment of anything.

The second is to *relational systems with distinguishable positions*. The construction requires that a system be decomposable into positions between which translation occurs, and that translations compose. Systems admitting no such decomposition lie outside it.

The third is to *frame-dependence as the difficulty*. Other difficulties about assessment, concerning power, interest, information and capacity, are real and are treated in the companion papers of this series. They are held fixed here.

The fourth is to *admissibility of a formalism*. The paper argues that a certain apparatus fits a certain problem, and the argument is that the fit is forced by features of the problem. Whether the resulting account is true of any society is a separate question on which this paper offers nothing.

2.2 *The discipline governing the borrowed apparatus*

Four rules are observed throughout, and they are stated so that a reader can check compliance.

No physical claim is made. Nothing in this paper asserts that societies are gauge theories, that social quantities obey any physical law, or that any physical constant, scale or

mechanism applies to social material. The apparatus is mathematical, and gauge theory is where the mathematics was developed.

Every borrowed term is given a social definition before use, and the definition is what the argument uses thereafter. Where a borrowed term carries connotations the social definition does not license, the connotations are excluded explicitly at the point of definition.

Where the physics is unsettled, the paper says so and declines to lean on it. This applies in particular to §11, where the construction of observables invariant under the relevant symmetry is an open problem in the physical case and is not solved by being transposed.

No numerical result about any society is derived. §8 reports computations on an explicitly constructed model, and what they verify is that the model has the properties claimed for it.

2.3 *Results imported from companion work*

Three results are imported and used without re-argument.

From the companion paper on the legibility of contribution, the paper takes the result that the evidence available to an assessor is composed by the assessor's position, which is the substantive reason the frame problem is not merely epistemic.

From the companion paper on the admissibility of representational transformation, the paper takes Condition 6.1 and the test of attainability that accompanies it, and it takes the question that paper leaves open at its close: a route back is a structure over transformations in place of a property of any one of them, and the apparatus adequate to that structure is what this paper supplies.

From the analysis of translation between normative languages in the companion work on multilingual democracy, the paper takes the finding that a residue survives translations each of which is individually faithful. §8 argues that this residue is a holonomy, which converts a reported observation into a measurable quantity.

2.4 *Positions declined in advance*

Four positions are declined and the declinations hold throughout.

The paper asserts *no criterion of justice*. It supplies a form that a criterion must take in a system without a privileged frame, and the companion papers supply the criteria.

The paper asserts *no quantity to be maximised*. §6 shows that the relevant equivalence admits no scalar quotient, and §10 shows that the assessment field admits no potential. A maximisation recommendation would therefore contradict two of the paper's own results.

The paper asserts *no claim that interpretive difference is redundant*. The gauge identification applies to the redundant part of interpretive difference and to no other part, and §7 states the criterion separating them and insists on it. An account treating all interpretive difference as gauge would have made interpretation inert, which contradicts the companion papers and is false besides.

The paper asserts *no model of any actual society*. The construction of §8 is a lattice of positions with group-valued edges, chosen because it is the smallest structure carrying the properties at issue.

3 PRIOR FORMULATIONS OF ASSESSMENT WITHOUT AN EXTERIOR POSITION

This section states the formulations already available for the structure this paper treats, and fixes for each what it settles and what it leaves open. The method is concession first. Two of the formulations own components of the account outright, and they are treated first and at greatest length so that the size of what remains is visible before it is claimed.

3.1 *The phase factor as the intrinsic description in gauge theory*

The transposition this paper attempts has a precise ancestor in the physics it borrows from, and the ancestor states the paper's own thesis for its own domain. Wu and Yang establish that an intrinsic and complete description of a gauge field in a region is given by the non-integrable phase factor, which carries more than the field strength and less than the potential, and that the correct mathematical setting is a connection on a principal fibre bundle [Wu & Yang (1975)]. Their argument proceeds from the Aharonov–Bohm effect, in which a particle is affected in a region where the field strength vanishes while the loop integral of the potential does not [Aharonov & Bohm (1959)]. Wilson's loop variable supplies the observable computed at §8 [Wilson (1974)], and Berry's geometric phase establishes that a system carried around a closed circuit acquires a quantity settled by the geometry of the circuit [Berry (1984)], with the adiabatic assumption removed by Aharonov and Anandan [Aharonov & Anandan (1987)].

The concession is unreserved and it is the largest in this paper. The proposition that the frame-independent content of a system with no privileged local frame resides on closed circuits is not a discovery of this paper. It is a settled result in the physics, stated with a precision the present work does not approach, and the whole of Part III is an attempt to identify a social structure to which it applies.

The differentiation is correspondingly narrow and should be stated plainly. What is offered here is the identification, together with an argument that the identification is forced by the frame problem in place of chosen for its elegance, and together with the criterion at §7 separating redundant interpretive difference from substantive difference. That criterion has no counterpart in the physical case, since there the gauge group is given and the question of which differences are redundant does not arise as a question about the world.

3.2 *The reality of gauge quantities in philosophy of physics*

Whether the loop quantities of a gauge theory should be read as the theory's ontology, and what to say about the local quantities that are not invariant, is a developed question in the philosophy of physics, and Healey's treatment is the standard reference [Healey (2007)].

The question there is what a gauge theory says the world contains, and the available positions range over local, non-local and holonomy interpretations.

The concession is complete. The differentiation is that this paper takes no position on the physical question and borrows the mathematical structure alone, under the disciplines fixed at §2. A reader who holds a local interpretation of gauge theories is not contradicted by anything here, since nothing here concerns what a gauge theory says the physical world contains.

3.3 *Legal power in Hohfeld*

Hohfeld's analysis of jural relations identifies a *power* as the capacity to alter the jural relations of oneself or another [Hohfeld (1913)]. A power is therefore not a claim about how things stand but an operator on the structure of claims, duties, privileges and liabilities, and its exercise leaves a different structure in place.

The concession is complete and the priority is a century old. The signature argued for at §4, in which an assessment maps a structure to a structure, is the signature of a Hohfeldian power, and this paper adds no part of that identification. What §5 adds is the recursive component: the operator returns, together with the restructured relations, the operator that the restructured relations now carry. Hohfeld's analysis is synchronic and states the correlatives obtaining at a moment; the addition here concerns what the exercise of a power does to the powers available afterwards.

3.4 *Autopoietic law in Luhmann*

Luhmann's account of law as an autopoietic system holds that the legal system produces its own elements, applies its own code, and generates the criteria by which its operations are assessed, so that no external position supplies those criteria [Luhmann (2004)]. This is the closest available position to the one taken here and it must be conceded without qualification, since it already contains the central negative claim: there is no exterior from which the system may be assessed, and the search for one is a category error.

The differentiation concerns what follows from the negative claim. Luhmann's system observes and produces, and the account is descriptive: it explains how a system that supplies its own criteria operates and it declines to supply a criterion of its own. The present paper asks a further question, which is whether any content survives the absence of an exterior position, and it answers that a determinate class of quantities does. That answer is unavailable within the systems-theoretic vocabulary because that vocabulary lacks the notion of a quantity carried around a circuit of translations between positions, and supplying it is what the borrowed apparatus is for.

3.5 *Revision constrained by postulates in the AGM tradition*

The theory of belief revision developed by Alchourrón, Gärdenfors and Makinson constrains a revision operator by postulates on the operator itself, in place of by a metric on

the states it produces [Alchourrón et al. (1985)]. The form is exactly the form required here: the question raised by an operator-valued account of assessment is what constrains the operator, and the AGM answer is that the constraints are stated on the operation and are checkable without an external standard.

The concession is full, and the connection to jurisprudence is not incidental, since Alchourrón was a legal theorist and the tradition was developed with normative systems in view. The differentiation is that AGM postulates are stated over a single agent's corpus and presuppose a fixed underlying language. The problem treated here arises where the language itself varies by position, so that the operators at different positions are not operators on a common object, and it is precisely that variation which the gauge identification of §7 is introduced to handle.

3.6 Relational quantum mechanics and the partial observable

Rovelli's relational interpretation holds that the state of a system is defined only relative to another system, and that a description from no position is unavailable [Rovelli (1996)]. The associated notion of a *partial observable*, a quantity to which a number can be attached by a measurement while carrying no prediction on its own, together with the *complete observable* formed as a correlation between partial observables, supplies the technical form of the position [Rovelli (2002)].

The concession is full and the paper depends on it at §11. The differentiation is one of domain and is small: the present account applies the structure to interpretive frames in a social system, where the analogue of a measurement is a translation between positions and the analogue of a correlation is a circuit that returns to its origin. The transfer is stated as a transfer and §2 governs it.

3.7 Output as a changed relation in restorative and transitional justice

Two bodies of practice treat the output of a justice process as a changed relation between parties in place of a verdict about a past act. Restorative practice organises encounters whose product is an altered relation among the participants, and transitional arrangements are constructed to change the terms on which a society proceeds after a conflict.

The concession is that the operator conception is not novel in practice, and practitioners in both traditions have long described their work in terms this paper's signature formalises. The differentiation is that neither tradition asks what content survives the absence of a privileged frame, since both proceed within a frame settled for the occasion by the parties or by an authorising instrument.

3.8 Inquiry that is not blocked, and criticism that is taken up

Two results imported by the companion paper of this series belong here as precedents for the loop conception in particular. Peirce [Peirce (1898)] makes the continued availability

of inquiry constitutive of the standing of a conclusion, and Longino [Longino (1990)] requires the uptake of criticism and not merely the existence of venues for it. Both locate the content of an assessment in what happens across a sequence of exchanges in place of at any one of them, which is the temporal counterpart of the claim that content resides on circuits.

Claim 3.1. (The residue left by prior formulations). *Existing formulations supply the operator signature, the absence of an exterior position, the constraint of an operator by postulates on its operation, the relativity of a description to a position, the treatment of an outcome as a changed relation, and, in the physics, the whole apparatus by which frame-independent content is located on closed circuits. What none supplies is a criterion, applicable where the admissible changes of frame are not given in advance, distinguishing the interpretive differences that are redundant from those that are degrees of freedom of the system. That criterion is the object of Part III, and the results of §8 are the borrowed apparatus applied once it is fixed.*

PART II

ASSESSMENT AS AN OPERATOR

4 THE SIGNATURE OF AN ASSESSMENT

This section fixes what an assessment is applied to and what it returns. Its objective is to establish that the object of evaluation is the transformation and never the transformed structure, and its method is an argument from the indistinguishability of results together with a statement of the resulting signature. The section closes with what the signature excludes.

4.1 The argument from indistinguishability of results

Let S be a relational structure and t a transformation carrying S to $S' = t(S)$. An assessment of the form $J(S')$ evaluates the result. The difficulty is that S' carries no record of how it was reached. The same S' may be produced by a transformation that destroyed a distinction while leaving a route by which the distinction may be restored, and by a transformation that destroyed the same distinction and closed the route. On the account of the companion paper these two differ in admissibility, and no inspection of S' distinguishes them, since what differs is a property of the passage in place of a property of the endpoint.

The argument generalises beyond that particular criterion. Any criterion sensitive to the manner of a change is unavailable to a function of the changed state alone, and criteria of that kind are the norm in the assessment of institutions, where consent, notice, reason and availability of challenge are properties of how a result was arrived at.

4.2 The signature that follows

The object evaluated is therefore t , and the signature is

$$J : \text{Struct} \longrightarrow \text{Struct},$$

so that an assessment is itself a transformation of the structure it assesses. This is the signature of a Hohfeldian power [Hohfeld (1913)], and §3 conceded the priority.

Two features of the signature carry consequences.

An assessment has the same type as the transformations it assesses. It is therefore itself subject to whatever criterion governs transformations, and the criterion applies to it in the ordinary way. This looks like a regress and is self-application: a criterion that could not be applied to its own instrument would require a position exempt from assessment, which is the exterior position §1 denied. A verdict-returning signature, of the form $J : \text{Struct} \rightarrow \{0, 1\}$ or $J : \text{Struct} \rightarrow \mathbb{R}$, avoids the self-application by placing the assessment at a level the structure does not contain, and that level is unavailable.

An assessment changes what it assesses. This is a consequence of the signature and not an additional claim, and it is the formal counterpart of the observation that an authoritative interpretation of a past alters the terms on which subsequent interpretations are made.

4.3 Positions the signature excludes

The signature excludes an assessment that leaves its object untouched, and any account requiring a purely observational assessment is inconsistent with it.

The signature excludes a scalar or binary output as the whole of an assessment. It does not exclude the derivation of scalar summaries from an assessment for particular purposes, and §6 states the precise sense in which such summaries fail to be faithful.

The signature is silent about which transformations are assessments and which are merely transformations. Punishment, reform and conquest all carry structures to structures, and §3 recorded that the constraint distinguishing them is stated on the operation in the manner of the AGM postulates [Alchourrón et al. (1985)] and not derived from the signature.

5 THE RECURSIVE TYPE AND ITS COALGEBRAIC FORM

This section refines the signature. Its objective is to establish that the assessment returned by an assessment is part of what an assessment returns, and that the resulting type is coalgebraic, and its method is to state the refinement, identify the resulting type, and name what the identification delivers. The section closes with the restrictions on the identification.

5.1 The refinement

An assessment made at one time is made by a party whose vocabulary, precedents and standards were formed by assessments made earlier. After an assessment, the standards available for the next assessment differ from those available before it: a precedent has been set, a distinction has been drawn or foreclosed, a category has been extended. The assessing capacity is therefore among the things an assessment transforms.

The refined signature makes this explicit. An assessment carries a structure to a re-structured world together with the assessment that world now carries,

$$J \cong \text{Struct} \longrightarrow (\text{Struct} \times J).$$

5.2 The type so obtained

The refined signature is a recursive type, and it is a familiar one. A machine that consumes an input, emits an output, and yields a successor machine is a *Mealy machine*, and the type above is exactly that of a Mealy machine over the alphabet Struct . In coalgebraic terms it is a coalgebra for the functor $F(X) = \text{Struct} \rightarrow (\text{Struct} \times X)$, and the final coalgebra of that functor is the space of stream transducers, whose elements are behaviours in place of states [Rutten (2000)].

Three things follow from the identification and are used later.

The object is a behaviour. Two assessment arrangements are the same object when they behave the same way indefinitely, and §6 makes this precise.

The object does not terminate. A final verdict would be a state at which the transducer stops emitting, and the type has no such state. This is the formal content of the position that an assessment is always open to a further assessment, and it is a property of the type in place of a stipulation added to it.

The object carries its own context. Whatever an assessment must retain about the history that produced it is carried in the successor machine, so no separate context parameter is required and none should be added. The alternative signature, in which a context C is supplied as an argument, would make the assessment a function evaluable at any context and therefore separable from every history, which is the exterior position again in another notation.

5.3 Restrictions on the identification

The identification is a statement about type and carries no dynamics. Nothing here fixes how an assessment transforms a structure, and the whole content of any particular account lies in that unstated part.

The identification supplies no existence claim. That an object of this type exists for a given social arrangement is an assumption, and it is a substantial one wherever the arrangement's assessing capacity is not stable enough to be treated as a machine at all.

The identification is not an appeal to computation. No claim is made that assessment is computable, that societies compute, or that any implementation is intended. The apparatus is used for its notion of behavioural equivalence, which §6 needs.

6 BISIMULATION AND THE ABSENCE OF A SCALAR QUOTIENT

This section derives the equivalence appropriate to the object fixed at §5, and draws the consequence that motivates the whole account. Its objective is to establish that the demand for a measure of justice asks for something the object's type excludes, and its method is to state the equivalence, state the proposition, and prove it. The section closes with the reading the proposition licenses and the readings it does not.

6.1 The equivalence

For objects individuated by input-output behaviour at a terminating computation, equality of results is the correct equivalence. For objects that do not terminate and are individuated by the interactions they sustain, it is not, and the appropriate notion is *bisimulation*: two systems are bisimilar when every transition of one can be matched by a transition of the other leading to states that are again bisimilar, so that no sequence of observations distinguishes them [Park (1981), Milner (1989)].

Applied here, two assessment arrangements are equivalent when, for every structure presented, they emit restructured worlds that stand in the relation and yield successors that are again equivalent. In the vocabulary of the companion papers, the observations are the acts a person may perform and the answers he receives, so that two arrangements are equivalent when the same acts are available under both and the same answers are owed.

Proposition 6.1. (Absence of a faithful scalar quotient). *Let $\sim$ be bisimilarity on assessment arrangements. (i) Any map μ from arrangements to a totally ordered set that respects $\sim$ identifies arrangements that differ in the acts available under them, whenever two non-bisimilar arrangements receive the same value. (ii) A map that separates all non-bisimilar arrangements cannot be totally ordered unless bisimilarity classes are themselves totally ordered by some independent relation, and no such relation is supplied by the type. (iii) Consequently a scalar measure of an assessment arrangement is either coarser than bisimilarity or presupposes an ordering the type does not contain.*

6.2 Proof of the three clauses

For clause (i), let $A \not\sim B$ and $\mu(A) = \mu(B)$. By the definition of bisimilarity there is a finite sequence of observations distinguishing A from B , so the two differ in an act available or an answer owed at some point, and μ has identified them.

For clause (ii), a map separating all non-bisimilar arrangements is injective on bisimilarity classes, so a total order on its image induces a total order on the classes. The type of §5 supplies no such order: it supplies a set of behaviours and the relation $\sim$, and $\sim$ is

an equivalence and no order. Any total ordering is therefore additional structure imposed from outside the type.

Clause (iii) is the disjunction of the two.

6.3 *The reading the proposition licenses*

The proposition says that the demand for a measure of justice is a demand for additional structure, and that the additional structure has to be argued for separately. It does not say that no such structure can ever be supplied. What it removes is the supposition that a measure is implicit in the object and awaits discovery, and that supposition is what motivates most attempts to construct one.

The result also explains a familiar experience in the assessment of institutions. Arrangements are frequently incomparable in a way that resists resolution, and participants report that a comparison is being forced. On this account the report is accurate: the comparison requires an order the object does not carry, and the party supplying the order is making a substantive choice which the appearance of measurement conceals.

6.4 *Readings the proposition excludes*

The proposition excludes no particular scalar summary. Summaries are useful for particular purposes and the account is not an argument against measurement as such. What it establishes is that a summary is coarser than the object, and therefore that two arrangements receiving the same summary may differ in what a person may do under them.

The proposition establishes nothing about which arrangements are better. It concerns the availability of an ordering and not its content.

The proposition does not show that assessments are incomparable in practice. Parties compare arrangements constantly and act on the comparisons, and they do so by supplying, from their own position, exactly the additional structure clause (ii) identifies.

What that position contributes is the subject of Part III. PART III

THE GAUGE STRUCTURE OF INTERPRETATION

7 INTERPRETIVE FRAMES AND THE CRITERION OF REDUNDANCY

This section makes the identification on which the rest of the paper depends. Its objective is to establish that a determinate part of interpretive difference is redundant in the technical sense, and to fix the criterion separating that part from the rest. Its method is to state the identification, to state the objection that defeats a careless version of it, and to answer the objection with the equivalence derived at §6. The section closes with what the identification excludes.

7.1 *The identification*

Let a relational system be decomposed into *positions*, each carrying a vocabulary in which what happens there is described. A *change of frame* at a position is a relabelling of that vocabulary. A *translation* along a relation between two positions is a map carrying descriptions at the first into descriptions at the second.

The identification is that a change of frame is a gauge transformation. Its two defining features are present. It is local, in that it may be performed at one position without being performed at any other. And the composition of translations must transform consistently under it, so that a change of frame at a position alters every translation incident on that position in a determinate way.

7.2 *The objection that defeats the careless version*

Gauge redundancy requires that the content of the theory be indifferent to the choice. If every difference between interpretive frames were gauge, then interpretation would make no difference to anything, and the companion papers of this series establish the contrary at length: a description is causally efficacious, and two descriptions of one situation may license different acts and produce different histories.

The objection is decisive against the version of the identification that treats all interpretive difference as gauge, and that version is declined at §2. What is required is a criterion partitioning interpretive difference into a redundant part and a substantive part, applicable without appeal to a privileged frame.

7.3 *The criterion, which is the equivalence already derived*

The criterion is bisimilarity. Two frames are *gauge-equivalent* when the assessment arrangements they induce are bisimilar in the sense of §6: the same acts are available under both, and the same answers are owed. Frames differing in vocabulary while licensing the same acts and owing the same answers stand in the relation, and their difference is redundant. Frames licensing different acts do not stand in it, and their difference is a degree of freedom of the system in place of a choice of description.

Three features recommend the criterion.

It is stated in terms already fixed. §6 derived bisimilarity from the type of the object, and no new apparatus is introduced to obtain the partition.

It appeals to no privileged frame. Whether two frames license the same acts is settled by the acts, and the question can be put from any position.

It is falsifiable in the direction that matters. A single act available under one frame and unavailable under the other establishes that the difference is substantive, which prevents the identification from being used to dismiss disagreements as merely verbal.

7.4 Positions the identification excludes

The identification does not say that vocabulary is unimportant. It says that a difference of vocabulary which changes no act is redundant *for the purpose of locating frame-independent content*, and vocabulary matters for many other purposes, several of them treated in the companion papers.

The identification supplies no procedure for deciding, in a given case, whether two frames are bisimilar. Determining this requires knowing the acts available under each, and §13 records the difficulty of establishing that from any position.

The identification is not an argument that social systems have a gauge group. Which changes of frame are admissible in a given system is an empirical question about that system, and the constructions of §8 stipulate a group in place of deriving one.

8 LOOP OBSERVABLES AND THEIR VERIFICATION

This section establishes the paper's central positive result. Its objective is to show that quantities defined on closed circuits are invariant under every change of frame while quantities attached to single translations are not, and that translations each of which loses nothing may fail to compose to the identity around a circuit. Its method is a construction, two propositions, and their numerical verification. The section closes with the reading the results license.

8.1 The construction of the loop and edge quantities

A relational system is a finite graph. Vertices are positions in the sense of §7. A directed edge $e = (u, v)$ carries a translation U_e , an invertible map from the frame at u to the frame at v . Translations are taken in $SU(2)$ throughout, so that every translation is invertible and loses nothing: whatever is expressible at u is expressible at v after translation. A change of frame at a position is a group element g_v acting by

$$U_e \longmapsto g_u U_e g_v^{-1}, \quad e = (u, v).$$

Two quantities are compared. The *edge-local* quantity is $w_e = \text{Re Tr}(U_e)$, attached to a single translation. The *loop* quantity for a closed cycle $c = (e_1, \dots, e_k)$ is $W_c = \text{Re Tr}(U_{e_1} \cdots U_{e_k})$.

The choice of $SU(2)$ is a stipulation and carries no claim. A finite non-abelian group would serve equally, and the results below depend on non-commutativity and on nothing else.

Proposition 8.1. (Invariance of loop observables). *W_c is invariant under every change of frame at every position, and w_e is not. Where every translation is a pure change of frame, so that $U_e = h_u h_v^{-1}$ for some assignment h , every loop quantity takes its trivial value.*

Proposition 8.2. (Anholonomy of lossless translations). *Translations each of which is invertible and loses nothing may fail to compose to the identity around a closed circuit. The failure*

is measurable as the rotation angle of the composite, and it is zero exactly when the assignment of translations is a pure change of frame.

8.2 *Verification of the two propositions*

The verification is `verify/holonomy.py` and is supplied with the paper. The graph has eight positions and twelve relations, and five independent cycles are tracked.

For Proposition 8.1, five hundred independent changes of frame were applied at all eight positions simultaneously. The loop quantities moved by at most 7.2×10^{-16} , with a mean deviation of 3.2×10^{-16} , which is floating-point noise. The edge-local quantities moved by up to 3.77 on a scale whose full range is 4, with a mean deviation of 2.60. The contrast is the result: the same transformation that leaves the loop quantities fixed to machine precision moves the edge-local quantities across most of their range. Where every translation was constructed as a pure change of frame, the holonomy angle over all five cycles was zero identically.

For Proposition 8.2, translations were built as a pure change of frame composed with a distortion of controlled magnitude. At zero distortion the holonomy angle vanishes on every cycle. At a distortion of 0.10 the mean angle is 0.365 radians and every cycle exceeds the threshold of 0.01. At 0.50 the mean is 2.18 radians and the maximum is 3.14, which is the largest angle the group admits. Every translation used was unitary to within 3.2×10^{-16} and therefore invertible and lossless. Table 1 and Figure 1 report the results.

8.3 *The reading the results license*

The first result is the transposition announced at §1. In a system where every position carries its own frame and no frame is privileged, quantities attached to single translations carry no frame-independent content, and quantities carried around closed circuits carry it entirely. An assessment that is to survive change of frame is therefore an assessment of a circuit, and an assessment attached to a single relation is an artefact of the frames at its endpoints.

The second result converts a reported observation into a measurable one. The companion work on multilingual democracy records a residue surviving translations each of which is individually faithful. Proposition 8.2 identifies that residue: the pairwise faithfulness of translations places no constraint whatever on the composite around a circuit, since faithfulness is a property of each map separately and the composite is a property of the assignment. The residue is the holonomy, and it is zero exactly when the assignment is a pure change of frame, which is to say when the parties differ in vocabulary alone.

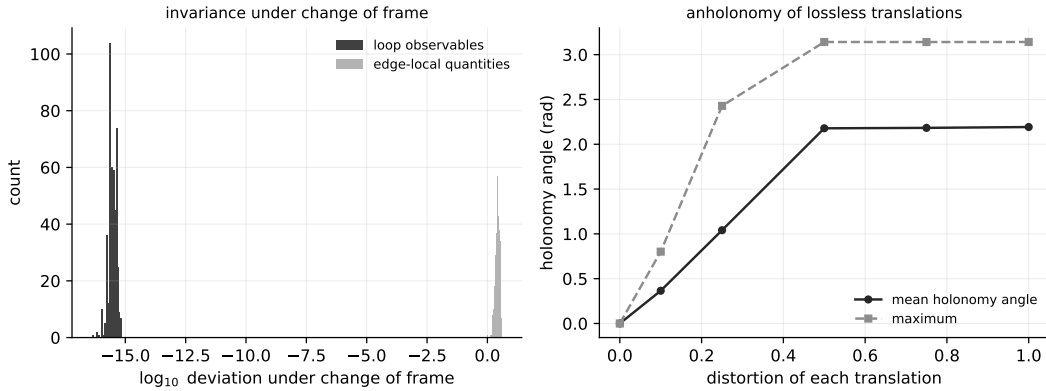
8.4 *Readings the results exclude*

Nothing here shows that any actual system has non-trivial holonomy. The construction stipulates translations and reports what follows; whether a given social arrangement ex-

Table 1: Verification of Propositions 8.1 and 8.2, from `verify/holonomy.py`. Eight positions, twelve relations, five independent cycles. The upper block reports deviation under five hundred independent changes of frame applied at all positions. The lower block reports the holonomy angle as the distortion of each translation is raised, with every translation invertible and lossless throughout.

<i>Deviation under change of frame</i>			
Quantity	maximum	mean	scale
loop observables W_c	7.2×10^{-16}	3.2×10^{-16}	4
edge-local w_e	3.77	2.60	4
<i>Holonomy angle by distortion of each translation</i>			
Distortion	mean angle	maximum	fraction above 0.01
0.00	0.0000	0.0000	0.0000
0.10	0.3652	0.8016	1.0000
0.25	1.0408	2.4286	1.0000
0.50	2.1782	3.1414	1.0000
0.75	2.1830	3.1409	1.0000
1.00	2.1923	3.1415	1.0000

Figure 1: Loop observables against edge-local quantities. The left panel gives the distribution of deviations under five hundred independent changes of frame at all positions; loop observables move by floating-point noise while edge-local quantities move across most of their range. The right panel gives the holonomy angle around closed circuits as the distortion of each individually lossless translation is raised from zero.



hibits the structure is an empirical question the paper does not address.

Nothing here shows that non-trivial holonomy is bad. The results identify an invariant and supply no evaluation of it, and §12 records that supplying one would require the ordering Proposition 6.1 denies.

Nothing here licenses the inference that a large holonomy indicates a large disagreement. The angle is a group-theoretic quantity in a stipulated group, and no interpretation of its magnitude in social terms is offered.

9 THE OBSTRUCTION TO A GLOBAL CHOICE OF FRAME

This section states the negative result that accompanies the positive one. Its objective is to establish that a global choice of interpretive frame is unavailable as a matter of structure, and its method is to state the physical result, state its transposed form, and fix the conditions under which the transposition holds. The section closes with what follows for practice.

9.1 *The result in its original setting*

Fixing a gauge means selecting, for each configuration, one representative from the family related by changes of frame. In an abelian theory this can be done globally and without ambiguity. In a non-abelian theory it cannot: Gribov showed that the standard gauge-fixing conditions admit multiple distinct configurations satisfying the same condition, so that the selection is not unique, and Singer established that the obstruction is topological and that no continuous global gauge fixing exists [Gribov (1978), Singer (1978)].

9.2 *The transposed form*

Fixing an interpretive frame globally means selecting, for the whole system, one vocabulary in which all positions describe what happens. Where the group of admissible changes of frame is non-commutative, and the construction of §8 shows that non-commutativity is what produces holonomy at all, the corresponding statement holds: no global selection is available that is both unambiguous and continuous in the configuration.

Two consequences are worth stating separately.

A global frame may be imposed and cannot be derived. Legal codification, official history and standardised measurement all impose one, and the imposition is a choice made at a position in the system. This is consistent with the imposition being necessary, useful, or the best available course.

The impossibility is structural and not a matter of effort or of goodwill. A party who fails to find a neutral vocabulary has not failed at a task that better parties accomplish.

9.3 *Conditions under which the transposition holds*

The transposition requires two conditions and holds only under them, and stating them is what keeps the section from being an appeal to authority.

The admissible changes of frame must fail to commute. Where they commute, the abelian case obtains, global gauge fixing is available, and nothing in this section applies. Whether admissible reframings commute in a given system is an empirical question.

The configuration space must carry enough structure for the topological argument to have content. Singer's result concerns a specific class of spaces, and the transposed statement is offered here as a structural analogy in place of a derived theorem. §2 governs, and §13 records this as the weakest step in the paper.

9.4 The consequence for practice

The consequence is narrow and is worth stating precisely because a broad version is tempting. It is available, and often necessary, to fix a frame locally in order to compute anything at all, and the results of §8 are unaffected by which local frame is chosen. What is unavailable is the claim that a fixed frame is the neutral one. The practical recommendation that follows is therefore procedural and modest: a party imposing a frame should record that a frame has been imposed, since the content that survives the imposition is defined on circuits and remains computable by anyone who knows what was imposed. PART IV

ASSESSMENT WITHOUT A GLOBAL QUANTITY AND WITHOUT BACKGROUND TIME

10 THE ASSESSMENT FIELD AND THE ABSENCE OF A POTENTIAL

This section draws the consequence of §8 for the form an assessment can take. Its objective is to establish that a locally determinate direction of assessment coexists with the non-existence of any global quantity of which that direction is the difference, and its method is a discrete construction over the same graph, a proposition, and its verification. The section closes with the three properties the result delivers together.

10.1 The construction of the assessment field

Retain the graph of §8 and replace the translations by real numbers. An edge $e = (u, v)$ carries a scalar a_e , read as the assessed change in passing from position u to position v . A *potential* is an assignment φ of a number to each position satisfying $a_e = \varphi(v) - \varphi(u)$ for every edge, so that the assessed change between any two positions is the difference of a single global quantity evaluated at them.

A potential is what a measure of justice would be. It assigns to each position a value, and the assessed change between positions is the difference of the values. The question of whether an assessment field admits a potential is therefore the question of whether the familiar demand for such a measure can be met, restated so that it has a determinate answer.

Proposition 10.1. (Local determinacy without a global quantity). (i) *On a connected graph, a potential exists exactly when the sum of a_e around every cycle vanishes, so that the cycle sums are the obstruction, and the potential is then unique up to an additive constant.* (ii) *Where no potential exists, the assessed change between two positions depends on the path taken between them.* (iii) *The local direction of assessment is nonetheless fully determined at every relation: each a_e has a definite value, and what fails is the existence of a global quantity of which those values are the differences.*

10.2 Verification of the three clauses

The verification is `verify/potential.py`. The graph has eight positions and twelve relations and is connected, so the incidence map has rank seven and the space of assignments admitting no potential has dimension five, which is the number of independent cycles.

Where the assignment is constructed from a potential, the largest cycle sum is 4.4×10^{-16} and the assessed change between two given positions is identical across all twelve distinct paths between them, to within 4.4×10^{-16} . Where the assignment is drawn without that construction, the largest cycle sum is 1.58, the least-squares residual of the equation $a = B\varphi$ is 1.46 in place of zero, and the assessed change between the same two positions spreads by 3.20 across the same twelve paths. Every edge continues to carry a definite value throughout, in the range $[-1.39, 1.16]$, which is clause (iii).

Table 2 and Figure 2 report a sweep in which the assignment is mixed between a component derived from a potential and a component drawn freely. Both the cycle sums and the spread across paths rise monotonically with the weight of the second component, and both vanish exactly at weight zero.

10.3 The three properties obtained together

The result delivers three properties that the companion papers of this series each required separately, and it delivers them from one construction.

There is no global scalar. Proposition 6.1 established that the type of an assessment admits no faithful scalar quotient; Proposition 10.1 establishes the corresponding statement for the field, and the obstruction is the same object that §8 identified as the frame-independent content. The conclusion is not isolated. Arrow's impossibility theorem establishes, for a different construction and under different conditions, that no aggregation of individual orderings into a social ordering satisfies a short list of requirements [Arrow (1951)], and the cyclic phenomena that result are the social-choice counterpart of a non-vanishing cycle sum. The present result is therefore an addition to a family and no novelty in kind.

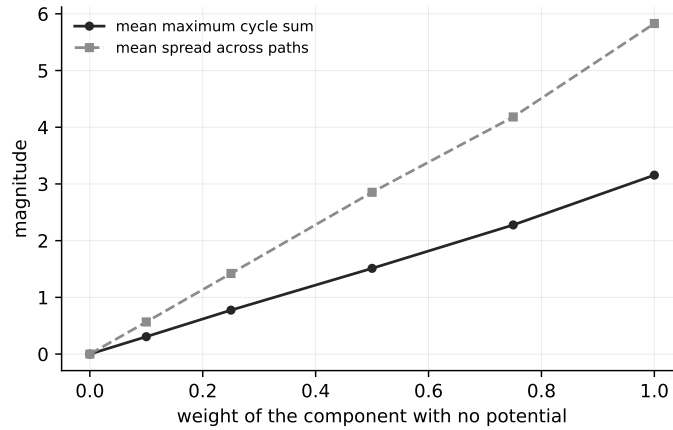
The local direction is determinate. An assessment at a particular relation, between two particular positions, has a definite value and may be acted on. This is what allows an account with no global measure to be usable, and it answers the objection that a theory declining a measure has left its holders unable to act.

The assessment is path-dependent. Which sequence of positions was traversed settles the accumulated assessment, so that the history of a system enters its assessment constitutively in place of evidentially. This is the formal counterpart of the position that an assessment made at one time alters the terms on which later assessments are made, which §4 obtained from the signature.

Table 2: Verification of Proposition 10.1, from `verify/potential.py`. Eight positions, twelve relations, five independent cycles, twelve distinct paths between the two positions compared. The assignment is mixed between a component derived from a potential and a component drawn freely, with the weight of the second given in the first column.

Weight of the free component	mean maximum cycle sum	mean path spread
0.00	0.0000	0.0000
0.10	0.3068	0.5665
0.25	0.7744	1.4216
0.50	1.5115	2.8531
0.75	2.2770	4.1814
1.00	3.1552	5.8290

Figure 2: The obstruction to a potential and its consequence for path dependence. Both the largest cycle sum and the spread of the assessed change across the twelve distinct paths between two positions rise with the weight of the component admitting no potential, and both vanish where the assignment is derived from a potential.



10.4 Readings the result excludes

The result establishes nothing about which assessments are correct. It concerns the availability of a certain form and not the content of any instance of it.

The result does not show that global measures are useless. A measure may be constructed by fixing a reference position and a set of paths, and the construction is often useful; what the result establishes is that such a measure depends on the paths chosen, so that two parties using different paths obtain different values without either making an error.

The result carries no claim that any actual assessment field has non-trivial cycle sums. The construction stipulates a field and reports what follows.

11 THE FORMULATION WITHOUT BACKGROUND TIME

This section removes an assumption the preceding sections carried tacitly. Its objective is to show that the account is better stated without a background time, and that the difficulty

about assessment altering its own object dissolves when it is, and its method is to state the difficulty, apply the relational treatment, and then state the costs. The section closes with the costs, which are substantial.

11.1 *The difficulty carried by the time-indexed formulation*

§4 obtained that an assessment alters what it assesses. Stated against a background time this appears paradoxical: the assessment at time t is made of a structure which the assessment then changes, so the object assessed is not the object that exists afterwards, and the assessment appears to be about something it has destroyed.

The appearance of paradox is produced by the background time and not by the structure. It arises because the formulation privileges a foliation, a division of the history into successive instants, and then asks after the relation between the instants.

11.2 *The relational treatment*

In the relational treatment there is no background time and no privileged foliation. A choice of time is itself a choice of frame, and the analysis of §7 applies to it: it is local, it is not derivable, and quantities depending on it are not frame-independent. What replaces evolution is correlation. A quantity to which a value can be attached by a determinate operation, while carrying no significance on its own, is a *partial observable*; a correlation between partial observables is a *complete observable*, and complete observables are what the theory is about [Rovelli (2002)].

Transposed, an assessment made at a position and a moment is a partial observable. It carries no frame-independent content on its own, which §8 already established for a different reason. The correlation between an assessment and the structure it assesses is a complete observable, and it is defined on the whole history in place of at an instant.

11.3 *Gains delivered by the relational reformulation*

Two gains follow and both are substantial.

The paradox of §11.1 dissolves. There is no instant at which the object assessed is replaced by another, since the history is one object and the assessment and the structure assessed are correlated within it. What looked like an assessment destroying its object is a correlation holding across a history.

The Hume difficulty is removed entirely for this account, in place of merely being avoided. An assessment stated as a condition on a whole history requires no forecast, since it asserts nothing about what will happen, and constrains which histories are admissible. This is the form in which the companion papers' conditions are best stated: the condition on attainability of a route back is a constraint on admissible histories, and the safety component identified there is exactly a set of histories closed under taking finite prefixes.

11.4 *The costs the reformulation imposes*

Two costs must be stated and neither is small.

The demand of a person suffering now becomes harder to express, not easier. Where no foliation is privileged, the present has no distinguished status, and a condition binding now requires a foliation to say what now is. The available answer is relational: the foliation is supplied by the affected party's own trajectory, so that what binds now is what binds at the position of the person concerned along his own history. This is coherent and it is an additional commitment, and an account that took the timeless formulation without making it would have lost the ability to speak to anyone in particular.

The construction of observables invariant under the relevant symmetry is an open problem in the physical case. It has not been solved there and it is not solved by being transposed here. §2 committed the paper to saying so where the physics is unsettled, and this is the principal instance. What the paper takes from the relational treatment is the form of the position and not a worked technique.

11.5 *Readings the reformulation excludes*

The reformulation carries no claim that time is unreal, that social processes are timeless, or that the experience of succession is illusory. It concerns which quantities are frame-independent and holds fixed everything else.

The reformulation does not make interpretation inert. Two assessments leading respectively to different courses are two different histories and no two descriptions of one history, and §7 fixed the criterion that separates these cases. Removing background time converts a causal claim into a constraint claim, and it leaves the substantive content of the claim untouched. PART V

THE BOUNDARIES OF THE ACCOUNT

12 POSITIONS DECLINED AND THE GROUNDS FOR DECLINING THEM

This section argues the four declinations announced at §2. Its objective is to show that each follows from a result established in the body of the paper in place of being adopted for caution, and its method is to derive each from the result that supports it. The section closes with the declination whose ground is different in kind.

12.1 *Grounds for declining a criterion of justice*

The paper supplies the form a criterion must take in a system with no privileged frame, and that form is restrictive: by Proposition 6.1 the criterion admits no faithful scalar expression, by Proposition 8.1 its frame-independent content is defined on circuits, and by Proposition 10.1 it corresponds to no global quantity. A criterion satisfying those constraints is supplied

by the companion papers of this series, and supplying another here would duplicate them without the case material that makes them assessable.

The division is worth stating explicitly, since a reader may reasonably ask what this paper would be for otherwise. The companion papers state conditions and defend them against objections drawn from practice. This paper establishes what kind of object such a condition can be, and its results constrain any condition whatever, including conditions the author of this series would reject.

12.2 Grounds for declining a quantity to be maximised

Two results of this paper exclude a maximand independently, and the exclusion is therefore overdetermined.

Proposition 6.1 establishes that no scalar respecting bisimilarity separates arrangements that differ in the acts available under them. A maximand is such a scalar, so a recommendation to maximise it recommends acting on a quantity that is blind to the distinctions the account exists to track.

Proposition 10.1 establishes that the assessment field admits no potential where cycle sums fail to vanish. A maximand is a potential, so the recommendation presupposes the vanishing of exactly the quantity §8 identified as the frame-independent content of the system. Maximising is available only where there is nothing frame-independent to detect.

12.3 Grounds for declining the treatment of all interpretive difference as gauge

§7 stated the objection and the criterion answering it, and the declination is recorded here because it is the one most likely to be attributed to the paper by a reader who has taken the gauge vocabulary and left the criterion behind.

Interpretive difference is gauge exactly where the arrangements induced are bisimilar. A difference in the acts available or the answers owed is a difference in the system and no difference in its description, and the whole apparatus of Part III applies to the first kind only. An account that dropped the criterion would hold that no interpretation ever makes a difference, which contradicts the companion papers, contradicts §4 of this paper, and is false.

12.4 Grounds for declining any claim about an actual society

This declination differs in kind from the other three, since its ground is the state of the evidence and not a result.

Every construction in this paper stipulates its objects. The group of admissible changes of frame is stipulated at §8 and is not derived from any system; the graph is constructed; the translations are drawn. What the computations establish is that the constructions have the properties claimed for them, which is what a verification of a proposition should establish and is nothing more.

What would be required to say anything about an actual arrangement is an identification, in that arrangement, of the positions, the admissible reframings, and the translations between positions, together with evidence that reframings compose as the construction assumes. §13 records the difficulty of obtaining any of these.

13 LIMITS OF THE ACCOUNT AND DIRECTIONS FOR FURTHER WORK

This section states what remains unsettled. Its objective is to place the account's weaknesses on the record in a form a critic can use, and its method is to order them by severity, beginning with the weakest step in the paper. The section closes with the directions left open and the relation of this paper to its companions.

13.1 The weakest step and its consequences

The transposition of the Gribov and Singer results at §9 is the weakest step in the paper, and §9.3 said so at the point of use. Those results concern a specific class of configuration spaces with structure that the social case has not been shown to possess, and the transposed statement is a structural analogy in place of a derived theorem.

Two consequences follow. The negative claim of §9, that no global choice of frame is available, is the least secure claim in the paper and should be read as a conjecture with a motivation. And the results that do not depend on it are unaffected: Propositions 6.1, 8.1, 8.2 and 10.1 are established independently, and a reader who rejects §9 entirely loses none of them.

13.2 The absence of an observer

The account states which quantities are frame-independent and says nothing about who is positioned to compute them. Computing a loop observable requires knowing the translations around a circuit, which requires occupying, or having reliable reports from, every position on it. The companion paper on the legibility of contribution establishes that reports across positions are systematically composed by the positions they come from, so the material required is exactly the material that paper shows to be difficult to obtain.

This is the same gap the companion papers record, and it is recorded here as shared in place of solved. An account identifying an invariant that no party is positioned to measure has produced a theoretical result and no instrument.

13.3 Limits internal to the constructions

The group is stipulated. Whether the admissible reframings in any social system compose, whether they form a group at all, and whether they fail to commute are three separate empirical questions, and the third carries §9 and part of the significance of §8. Where reframings commute, the abelian case obtains and the account reduces to something much weaker.

The positions are stipulated. A decomposition of a social system into positions between which translation occurs is assumed throughout, and systems admitting no stable decomposition lie outside the account by §2. Whether any actual system admits one is not addressed.

The magnitude of a holonomy is uninterpreted. §8 reports angles in a stipulated group, and no reading of those magnitudes in social terms is offered or available. The account distinguishes zero from non-zero and no more.

13.4 The open problem inherited from the physical case

§11 depends on the relational treatment, and the construction of observables invariant under the relevant symmetry is unsolved in the physical case. The paper takes the form of the position and no technique, and a reader who holds that the physical problem is fatal to the relational programme will hold the same of §11.

13.5 Directions the account leaves open and the relation to the companion papers

Three directions follow.

The first is the identification problem of §13.3: what would have to be established about a social arrangement for the constructions here to apply to it. This is the shortest route from the present results to a claim about anything.

The second is the observer problem of §13.2, which the three papers of this series record in common and none of them answers.

The third is the relation between the invariant identified here and the conditions stated in the companion papers. Those conditions are stated per person and at a position, and §8 establishes that quantities so stated carry no frame-independent content. Whether the conditions can be restated on circuits without losing what makes them conditions on persons is the question this series leaves open, and it is the question on which its coherence depends.

That question has a determinate form and is worth stating as the closing item. Eglash's criterion is a conserved quantity and is stated locally, and the criterion of this series is a holonomy and is stated on circuits. A criterion of the first kind can be applied by a party at a position; a criterion of the second kind cannot. Whether an account of justice can be built on quantities of the second kind, and remain a criterion that a person may invoke on his own behalf, is not settled by anything in these three papers.

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