

Normative Renormalization in Distributed Monetary Dissolution

A Thought Experiment on AI Production Networks

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Abstract

This discussion paper examines a proposed multiscale method for representing economically and normatively heterogeneous value circuits in future AI-operated production networks. The thought experiment assumes near-universal artificial labour, extensive shared production information, and potentially scarce resources governed by differentiated rights. Recursive and overlapping circuits can generate large, and under repetition formally unbounded, families of possible pathways. Their effective material and institutional influence may diminish with path length, while their number, coupling, and dependence on historically situated normative appraisals complicate any direct truncation. The paper proposes *normative renormalization* as an explicitly provisional programme of scale-dependent circuit abstraction that seeks to reduce coordination complexity while preserving specified claims, contestation rights, distributional burdens, and material regeneration constraints. It distinguishes weighted-walk attenuation from the cancellation of liabilities and from the legitimacy of excluding or modifying a participant's claim. The hypothesis is that sufficiently capable AI, supported by shared but fallible production data, could assist in discovering effective interfaces, estimating approximation errors, detecting rights-sensitive exceptions, and proposing revisable allocation protocols. The paper also examines how model certification and protocol administration can generate new concentrated power. The objective is to clarify the conditions, limits, and possible tests of normatively constrained abstraction during spatiotemporally distributed monetary dissolution, without presuming moral consensus or complete dynamical knowledge.

Keywords: normative renormalization; AI production networks; distributed monetary dissolution; multiscale institutional abstraction; value circulation

Discussion Paper Note

This paper develops a conditional thought experiment about future productive systems in which highly capable artificial-intelligence agents perform nearly all operational labour. The agents may share extensive data about materials, capacities, deliveries and obligations, while their legal status, authority to dispose of resources, and normative criteria remain disputed or delegated. Scarcity and productive constraints are retained throughout.

“Normative renormalization” names a *proposed* method for reducing descriptions of recursive economic circuits across analytical scales while retaining specified normative observables and routes of contestation. The term is not presented as an established field, a physically validated renormalization group for ethical norms, or a procedure that resolves moral disagreement by computation. Mathematical analogies and operational proposals are distinguished from normative justification and empirical evidence.

This manuscript is Paper IV in a programme with three companion inquiries: the conditional persistence of money in AI-operated production (Paper I), distributed monetary dissolution (Paper II), and a generative grammar of value circuits (Paper III). Paper IV is written to remain understandable independently of those works.

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Responsible Use and Rights Reservation

The proposed framework is conceptual and does not authorize automated deprivation, compulsory exchange, suppression of claims, or unilateral institutional redesign. Its application would require context-specific authorization, independent contestability, and evidence of material as well as normative adequacy. Rights in third-party sources remain with their respective holders.

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Notices

Status. Preliminary discussion paper. Sections 1–10 constitute a complete preliminary first draft; two integrated conceptual and formal revision passes and a source-to-claim scholarly audit have been undertaken. Some claims and bibliographic records were rechecked against primary or authoritative sources; a comprehensive independent literature review and external scholarly review remain outstanding; an internal typeset-layout inspection has been performed for this compiled edition. The formal representations are conditional constructions rather than empirically calibrated economic or normative models.

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Statement on the use of language models. OpenAI’s ChatGPT assisted in research dialogue, reference discovery and verification, structural design, manuscript drafting, and LaTeX preparation. The human author initiated the central questions and hypotheses and remains responsible for the claims, citations and errors.

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1 Introduction

The prospective reduction of monetary dependence in artificial-intelligence-operated production raises a problem of institutional description across scales. A production network may exchange food, electricity, maintenance services, material inputs and resource-specific commitments without requiring a general-purpose payment instrument for each transfer. The same network may contain recursive, overlapping and temporally dependent circuits whose complete enumeration is computationally expensive or conceptually misleading. Moreover, each productive interaction may be conditioned by claims of entitlement, perceived and substantiated unfairness, differentiated powers of disposal, and evolving expectations of cooperation. This paper examines a provisional *normative renormalization* programme: the construction of manageable, scale-dependent descriptions of value circuits that preserve designated normative and material properties while suppressing distinctions that can responsibly be treated as negligible for a stated decision. The inquiry is philosophical and methodological, using bounded formal representations and a hypothetical AI production economy. It proceeds by defining the problem of scale, distinguishing effective economic influence from normative standing, specifying provisional criteria for abstraction, and examining how advanced AI might support the epistemic and practical work without becoming the unaccountable author of the relevant rules.

1.1 Artificial Production and the Domain of the Inquiry

The common counterfactual is a production system in which artificial agents are sufficiently developed to perform nearly every kind of physical, cognitive, managerial and scientific labour. Agriculture, energy provision, processing, transport, equipment repair and production planning are carried out predominantly through interactions among AI-operated systems. Relevant production information is broadly available to participating systems. The availability of information is taken as an enabling premise, with residual limits of measurement quality, causal identification, computational capacity and institutional disclosure. Productive competence does not confer automatic control over land, electricity, scarce components or the rules governing resource access. Agents that execute an operation may act under the authority of distinct people, organizations or collective arrangements. Whether such AI agents possess independent legal or moral subjecthood remains open.

In this setting, a monetary arrangement can remain useful when parties have divergent access to inputs, require interoperable settlement, or confront unresolved claims. Another arrangement could organize material transfers through shared production schedules, direct allocation, commitments specific to particular resources, or valid multi-party clearing. These alternatives may coexist across regions and periods. The expression *distributed monetary dissolution* accordingly refers to the contingent displacement of particular monetary functions in specific networks over time. It does not presume synchronized currency extinction, and it does not equate a cancelled debt with completed material delivery or with a just distribution of productive benefits.

The present paper asks how producers and rule-making institutions could recognize, coordinate, evaluate and revise such networks when their circuit structure exceeds tractable direct description. The research object is thus neither the emergence of monetary media itself nor the grammar that generates all syntactically admissible circuits. It is the *validity of reduced descriptions* used to guide material and normatively consequential decisions.

1.2 Recursive Circuits and the Scope of Effective Influence

A finite directed network has finitely many elementary cycles, while allowing repeated visits to vertices makes infinitely many distinct closed walks possible whenever an accessible directed cycle can be repeated. A walk's syntactic availability does not imply that it constitutes a physically possible production history, an independently realized exchange, or a separately created stock of value. The distinction matters when a grammar can repeatedly insert a nested circuit into an otherwise admissible route. Material

inventory, delivery time, shared capacities, authorization, and historical performance constrain which syntactically possible walks acquire institutional or productive consequences.

One possible limiting mechanism arises from serial fragility. A chain of contingent commitments may become less likely to complete as it grows longer: a producer can encounter inadequate energy, an unavailable replacement part, a credit dispute, or a justified refusal to continue a burdensome allocation. Under a highly restrictive model of independent equal-reliability links, the probability that a length- L chain completes is q^L , for $0 < q < 1$. The economic interpretation of this probability requires specifying the events and the population of paths. Human behavioral evidence concerning fairness-sensitive cooperation cannot simply be transferred to autonomous future AI systems; the mechanism here is a hypothesized institutional response to conditions of participation.

A decrease in the influence of each long path does not alone ensure that their aggregate influence becomes small. If at most Kb^L relevant paths occur at depth L , and their individual contributions to a *specified bounded observable* are at most Cq^L , then the tail after depth L_0 has the elementary upper bound in Equation 1, provided $bq < 1$.

$$\sum_{L=L_0+1}^{\infty} \sum_{\gamma \in \Gamma_L} |w(\gamma)| \leq KC \frac{(bq)^{L_0+1}}{1 - bq}, \quad 0 < bq < 1. \quad (1)$$

Here Γ_L denotes the set of path representations counted at depth L , $w(\gamma)$ is a nonnegative-bound-compatible influence weight for a predefined observable, and constants K, C, b, q are assumed bounds rather than estimated facts. Overlapping paths and repeated events can cause multiple representations of the same economic activity; Equation 1 remains a bound on the *chosen accounting of representations*, not on total economic value. The condition can fail near critical regimes or when path dependencies undermine the bounding assumptions. Established attenuated-walk measures supply one mathematical precedent for summing repeated network influence, without providing a theory of justice or value generation [12].

It would be tempting to interpret a vanishing high-order term as an extinguished obligation. That inference is normatively unsound. A promise to an excluded participant can remain binding even when its contribution to current throughput is negligible. A legitimate refusal may interrupt circulation while protecting those subjected to an exploitative rule. Conversely, conduct that blocks a circuit can express opportunism or an error about its effects. The question of whether an omission is justified must therefore be answered separately from the calculation of its small numerical effect.

1.3 Normative Renormalization as a Proposed Scale Transformation

Renormalization approaches in physics reduce complex descriptions across resolution levels while tracking changes in effective parameters and behavior. Efforts to adapt these approaches to networks encounter obstacles concerning heterogeneous nodes, weak geometric regularity and uncertain choices of coarse-graining operations [7]. The present inquiry uses this literature as a methodological comparator. *Normative renormalization* is proposed here as a research term for a family of institutional abstraction operations satisfying stated validity requirements. In the present manuscript, the operative construction is *certified, task-relative coarse-graining*: partially defined maps, query-specific error bounds, retained claim provenance, and role-sensitive procedural interfaces. Its name signals a multiscale research question, not an established renormalization-group structure, scale invariant, universality class, or transfer of physical laws into judgments of right and wrong. A more demanding mathematical use would need an explicitly defined scale family and transformation algebra, suitable effective parameters and independently demonstrated invariants.

An admissible abstraction first specifies its observables and decisions. For a food circuit, selected observables might include physical deliveries, residual access to electricity, unresolved resource-specific claims, deterioration of soils or equipment, and access to a procedure for challenging a disputed allocation. The same circuit might be summarized differently for an energy capacity decision, a debt-clearance

Table 1: Candidate observables and safeguards for scale-dependent value-circuit descriptions. The requirements are proposed rather than empirically validated.

Analytical domain	Candidate reduced description	Safeguard against harmful omission
Material circulation	Aggregated deliveries, inventories and capacity use	Shared-resource conservation and event-time feasibility
Credit and claims	Net outstanding obligations by type and authorized settlement status	Retained claimant-level provenance and disputed-claim flags
Regenerative conditions	Soil, equipment and energy-renewal constraints across horizons	Explicit deterioration and replacement thresholds
Normative contestation	Active complaints, review channels and decision authority	No deletion of a claim solely for low aggregate influence
Institutional power	Control of allocation, model certification and protocol revision	Auditable authority, appeal and reversible delegation

decision, and an inquiry into a claim of unfair exclusion. The relevant resolution is thus decision- and rights-sensitive rather than a universal scalar measure of economic importance.

For exposition, let s denote a fine-grained network state containing material inventories, historical commitments, disposal authorities, contested claims and relevant information. Let $\Pi_k(s)$ denote a representation at scale k . For a specified intervention a and horizon T , let $O_T^{\text{num}}(s, a)$ denote only the selected, commensurable numerical observables, such as suitably normalized deliveries or resource shortfalls. Their abstract approximations are $\hat{O}_T^{\text{num}}(\Pi_k(s), a)$. Equation 2 defines one prospective numerical accuracy test over a declared intervention class $\mathcal{A}_k$.

$$\epsilon_k(T) = \sup_{a \in \mathcal{A}_k} \|O_T^{\text{num}}(s, a) - \hat{O}_T^{\text{num}}(\Pi_k(s), a)\|_W. \quad (2)$$

The weighted norm requires operationally defined entries, compatible units or justified normalization, and documented weights. A finite value of $\epsilon_k(T)$ is a numerical statement for the chosen state, horizon and intervention class, and supplies neither a uniform certificate for all possible states nor a finding of legal or moral acceptability. Exact claim-status answers and the usability of contestation procedures require separate tests, developed in Section 2 and in the later analysis of rights and dispute memory. State abstraction in artificial-intelligence planning offers a limited precedent for reducing computational complexity while asking which task-relevant properties are preserved [13]. Extending such approaches to contested economic rights requires further institutional and normative arguments.

A rights-sensitive description consequently needs additional safeguards beyond the scalar error in Equation 2. A reduced model should preserve the existence and status of enforceable or contested claims; indicate which agents may revise a rule; keep accessible evidence about how allocations were produced; and allow affected parties to challenge a decision or request finer resolution. These are proposed design requirements requiring justification under the operative legal and governance system. They should not be confused with mathematical invariants established for arbitrary economic networks.

Table 1 groups the prospective preservation requirements. Its categories remain heterogeneous: some admit quantitative bounds, some require procedural guarantees, and others depend on contextual judgments for which a universal numerical tolerance would be inappropriate.

1.4 A Multi-Scale Circuit Example

Consider an agricultural AI A , an energy-supply AI E , a logistics AI L , and a maintenance AI M . The agricultural system draws electricity from E , routes grain through L , and depends on spare equipment maintained by M . Several productive paths may reuse the same electricity-generating equipment, while credit or reciprocity arrangements can connect them in different directions. Suppose that locally authorized schedules reliably deliver required food outputs, with some reciprocal commitments cleared

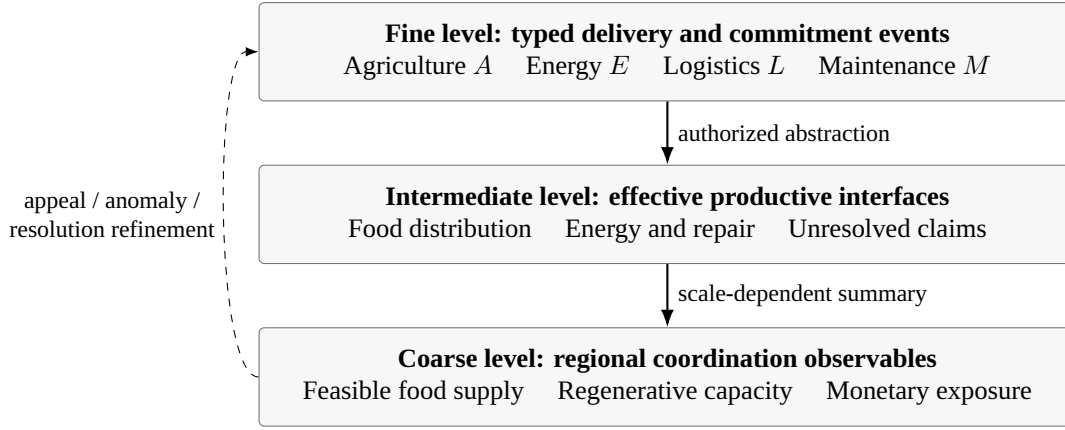


Figure 1: Proposed multi-scale representation of an AI production circuit. Dashed feedback represents reopening of fine-grained evidence when a rights-sensitive exception, resource anomaly or contested decision arises.

through a common record and some replacement components still obtained externally through monetary purchase.

At the fine scale, each delivery or claim has a history and a time. At the intermediate scale, an AI planning module might summarize *A* and *L* as one effective distribution interface, while summarizing energy provision and maintenance as another. At a coarse scale, a regional planner might track aggregate food-delivery feasibility and remaining external monetary exposure. Figure 1 illustrates these levels without treating their relationship as an established physical renormalization map.

The most important example concerns a contested maintenance obligation. If *M* claims that previous contributions have been uncompensated, a regional aggregate may show little effect on current delivery volume. A coarse description that discards the claim for this reason could treat the productive arrangement as legitimate while quietly prolonging a disputed burden. A normatively adequate abstraction would preserve the claim’s existence and route it toward justified review, even if the planner continues to use aggregate delivery variables for routine scheduling. At the same time, a provisional dispute need not automatically authorize *M* to impose arbitrary sanctions upon every other participant; procedural rights and reciprocal duties would have to be specified and contested through the governing institution.

This example reveals a potential distinction between *computational insignificance* and *normative significance*. The reduction of a high-order path’s effect on a measured supply outcome may be sound for a scheduling decision, while being unacceptable as a reason to disregard a participant’s entitlement. Value return to productive agents and the conditions sustaining production is an important normative interlocutor in the generative-justice literature; the specific multi-scale AI governance construction developed here is an additional proposal rather than a doctrine attributable to that literature [5].

1.5 AI-Assisted Recognition and Institutional Practice

The optimistic premise is that future AI systems might possess capacities for multiscale system identification, causal modeling, uncertainty estimation and adaptive resource planning substantially exceeding those available to human coordinators. Broadly shared production records could make it possible to detect recurrent circuit motifs, identify bottlenecks, estimate likely failures, and produce simplified models that retain the variables most relevant to particular decisions. An AI-supported abstraction could then be revised after delivery failures, changes in resource constraints or contested judgments. These possibilities are conditional on observability and the adequacy of the model class. Stronger predictive performance is insufficient to establish that an allocation is authorized or just.

A plausible institutional workflow contains several distinct activities. First, participating agents and legitimate rule-making bodies specify what is being decided and which rights or constraints must survive abstraction. Second, AI tools construct candidate circuit partitions, preserve provenance for unresolved

obligations, and estimate approximation error under relevant interventions. Third, the resulting summaries are used for bounded resource coordination with monitored outcomes. Fourth, exceptions trigger renewed investigation at finer resolution, including procedures by which a disadvantaged participant can question both the data and the normative standard. Fifth, decisions and models are revised through a publicly accountable process. Such a workflow is a proposal for testable governance architecture, not a substitute for institutional authorization or democratic and other applicable forms of legitimacy.

Polycentric-governance research supplies an independent institutional comparison: complex resource problems can admit multiple interrelated centers of decision-making rather than a single all-purpose ruler [17]. Whether AI-supported multi-scale institutions would improve those arrangements remains unsettled. Consolidation of shared data can produce concentrated authority over what is measurable, and certifying an abstraction can produce power over which claims are deemed visible. A model that accurately forecasts food deliveries may still exclude someone from contesting the interpretation of a historical contribution. Robust normative abstraction therefore requires attention to the politics of the measurement interface and to the distribution of authority to reopen or reject an aggregated description.

1.6 Normative Pluralism and Research Boundaries

The adjective *normative* does not imply that every participant shares the same conception of fairness, value or aesthetic legitimacy. A single aggregate welfare measure may obscure competing priorities between contributions, needs, historical responsibilities, procedural fairness and the maintenance of ecological conditions. Comparative theories of justice highlight the significance of reasoned disagreement and public comparison without requiring the prior specification of one perfectly just institutional arrangement [21]. This paper does not infer a unique theory of justice from that literature. It asks which information and institutional opportunities should remain available so that unresolved disputes can be argued and revised when micro-level circuits are represented at coarser scales.

Several failure modes remain: higher-order influence may fail to decay fast enough; shared bottlenecks and correlated decisions may undermine simple serial-reliability models; normatively salient harms may become rare in aggregate data; and the use of AI to evaluate or enforce institutional rules may consolidate power over both the descriptions and the underlying productive infrastructure. Conversely, systems that retain every micro-level detail indefinitely may become incapable of effective coordination. Normative renormalization is therefore formulated as a possible means of reconciling bounded cognition with normatively accountable action, subject to explicit tradeoffs and failure tests.

The remainder of the proposed paper will examine, in order, normative observables and claim provenance; high-order path attenuation and effective depth; admissible abstraction maps; preservation of rights, contestation and heterogeneous contributions; AI-assisted discovery and uncertainty analysis; governance and protocol revision; epistemic concentration and critical failures; and a bounded multi-period evaluation case. The programme's distinctive research task is to establish whether a computationally manageable production-network description can remain materially adequate, institutionally contestable and normatively defensible during partial monetary dissolution. Success in one of these dimensions should neither be presumed nor treated as conclusive evidence of the others.

2 Normative Observables and Claim Provenance in Value Circuits

A productive-network abstraction must specify which properties of a recorded history remain available for the decision at hand. In an AI-operated economy, a completed resource transfer may be accompanied by an unresolved obligation, an authorization of limited scope, an allegation of unfair treatment, or a deterioration of productive capacity whose effects arise in a later period. This section constructs a provisional vocabulary for those heterogeneous objects and establishes the informational conditions under which a coarse description can preserve selected answers. The method is a typed, finite-history representation with illustrative institutional rules. It distinguishes physical events from assertions and adjudicated findings, develops a provenance structure for contestable claims, separates quantitative approximation from

exact-status preservation, and tests the distinctions in a small food–energy–maintenance example. The representation is a proposed research interface. It is not itself a legal determination or a general theory of distributive justice.

2.1 Observation Domains, Decision Horizons, and Event Identity

Let a finite observation window be $[t_0, t_1]$ and let s denote the fine-grained state and record of an AI-operated production network at t_1 . The record covers events dated within that window and relevant earlier commitments that remain operative. A practical abstraction is constructed relative to a declared question, such as whether a specified food-delivery schedule can proceed, whether an obligation was discharged, or which party has standing to request a review. The question fixes the temporal horizon, admissible interventions, and necessary evidence. A summary adequate for one decision may be insufficient for another.

The basic unit of physical accounting is an *event occurrence* rather than a mention of an occurrence within a circuit description. Let $\mathcal{E}_s$ be a finite register of event identifiers; each $e \in \mathcal{E}_s$ has a declared type, time or interval, input and output quantities with units, the relevant operator, and provenance references to observation records. The same electricity-generation occurrence may support several syntactic circuits. The distinct circuit descriptions point to a shared identifier and do not create additional electricity. A ledger therefore evaluates the union of relevant event occurrences, while maintaining any separate descriptions as interpretive objects. The generative grammar of the related investigation supplies one way to identify candidate paths; whether their individual events occurred is a separate evidential question.

A provisional fine-state signature is given in Equation 3.

$$s = (\mathcal{E}_s, \mathcal{C}_s, \mathcal{U}_s, \mathcal{V}_s, x_s, z_s, \mathcal{P}_s), \quad (3)$$

Here $\mathcal{C}_s$ is a register of commitment and claim records; $\mathcal{U}_s$ records operative grants, restrictions, and other authority relations; $\mathcal{V}_s$ records review proceedings and applicable procedural standing; x_s contains the relevant measured material accounts; z_s contains modeled regenerative and capacity conditions; and $\mathcal{P}_s$ is a provenance graph connecting those registers to evidence and decisions. The components are deliberately nonhomogeneous. In particular, a claimant’s assertion and a verified delivery need not belong to one common numerical vector, and a model estimate of soil renewal need not have the same evidential standing as an authenticated shipping receipt.

The event and claim identifiers are stable within the stipulated administrative boundary and time window. Their stability is a modeling assumption requiring sound identity resolution, appropriate safeguards against duplication, and auditable correction procedures. Two records carrying different identifiers can refer to a single event because of erroneous reconciliation; conversely, a fraudulent or missing event may acquire an apparently valid entry. An effective description must specify what counts as independently verified, what is inferred, and what remains contested. Neither globally shared data nor highly capable AI removes these evidential distinctions by hypothesis.

2.2 Claims, Authorization, and Procedural Standing

A *claim record* identifies an asserted or institutionally recognized demand, its alleged basis, addressee, subject matter, relevant time, and procedural status. The record may concern a replacement component, compensation for past maintenance, access to a productive facility, correction of a public entry, or the ability to contest an allocation. Its presence is evidence that a claim has been lodged or recorded; its presence alone does not establish that the demand is justified, legally enforceable, or owed by the named respondent. Its absence can reflect a missing or suppressed record rather than an absence of relevant interests. The assessment therefore distinguishes existence in the register, validity under a specified rule, and satisfaction or discharge under that rule.

Hohfeld’s analysis of jural relations provides a useful legal-conceptual precedent for distinguishing rights and duties from privileges and powers [10]. Schlager and Ostrom similarly distinguish bundles of

resource-related access, withdrawal, management, exclusion and alienation rights, rather than inferring authority from the mere use of a resource [19]. Their accounts concern legal and social institutions, especially human resource arrangements. The present extension asks how a record system in an AI-operated production network might represent distinct kinds of authority without presupposing that an AI operator independently holds legal personality. A principal may be the rights holder, an AI may execute its instruction, another institution may certify the act, and an affected participant may be able to appeal it. Such roles should receive separate identifiers where they are distinguishable.

For a record $c \in \mathcal{C}_s$, let $\sigma_s(c)$ contain at least its current *recorded* status, a reference to the status-determining authority or procedure, and the evidence available for review. Useful status categories include *asserted*, *acknowledged*, *disputed*, *provisionally decided*, *discharged*, and *withdrawn*, with scope for a context-specific institutional vocabulary. This is a representational classification; an actual legal system may employ different effects and distinctions. A disputed claim may be settled in part, revived on appeal, time-barred, or superseded. Consequently, status change is itself an event with a date, reason, authorizing rule, and trace to earlier states. A historical record of discharge does not imply that the obligations survived; it permits later verification of when and by which process a discharge was recognized.

A permission record likewise identifies its issuer, beneficiary or delegated operator, resource object, allowable action, validity interval, scope, and conditions. Authorization of one food delivery need not authorize the sale of the underlying field, exclusion of another participant, or a permanent rewrite of the access rules. A review-standing record answers a different question: which designated parties can seek correction, reconsideration, evidence access, or adjudication in a particular forum, under which restrictions? Operational power to submit an electronic objection is distinct from institutionally recognized standing to have it considered. The representation records both when they matter to the decision.

Disagreement also needs epistemic differentiation. A participant's reported unfairness, an evaluator's estimate of unequal burden, an established violation of a valid rule, and a legitimate plural disagreement over the rule's justification carry different statuses. Treating all disagreement as demonstrable injustice would overstate the available normative evidence; treating all disagreement as noise would conceal conditions relevant to cooperation, authority and trust. Records should preserve the kind of assertion, its evidence and its reviewability without prematurely deciding the dispute.

2.3 Typed Provenance and Revision Histories

Let $\mathcal{P}_s$ in Equation 3 be a finite, typed directed graph whose vertices may include event occurrences, claims, instruments, model outputs, evidence items and institutional decisions. Edges indicate declared relationships, including *recorded by*, *based on*, *authorized by*, *allegedly satisfied by*, *reviewed in*, and *supersedes*. The vocabulary is illustrative. Crucially, an edge of *alleged satisfaction* should remain different from a decision establishing that satisfaction occurred according to an applicable rule. A signed receipt, a simulation, and a disputed witness report may all be relevant evidence without having identical reliability or admissibility.

The provenance structure supports two independent operations. An investigator can reconstruct the causal or documentary path by which a summary arose; a competent procedure can also inspect the grounds on which a status was entered. Causal and documentary ancestry are related but distinct. For example, an electricity event may physically enable equipment repair even when an earlier claim for replacement parts remains unpaid. A ledger that records the causal energy dependence has not thereby discharged the claim. The correct normative inference requires the relevant agreement, applicable institutional rule, and a defensible decision about performance.

A bounded operational register may retain a status certificate and a link to securely held evidence rather than make every underlying personal or commercially sensitive item universally visible. Preservation in this paper means preservation of the information and procedural access needed for declared authorized inquiries, subject to applicable confidentiality, privacy, evidence-retention and deletion rules. It does not imply perpetual public disclosure. A valid correction should be traceable to the extent lawful and necessary without reproducing false or harmful allegations as permanent public truth. Such protec-

Table 2: Heterogeneous observables and provenance needs in AI-operated production circuits. The preservation class is provisional and depends on a declared decision and governing rules.

Observable class	Decision-relevant content	Candidate preservation requirement
Material occurrence	Identity, typed quantity, date, measured source, shared-capacity use	Event-unique accounting; bounded numerical error only for declared aggregates
Commitment or claim	Claimant, respondent, object, basis, due date, state of review	Identity and live-status retrievability for designated claims; proof of authorized discharge
Delegated authority	Issuer, beneficiary, permissible action, scope, validity period	Exact response to permission and validity queries at the decision boundary
Review standing	Eligible reviewer, forum, deadlines, procedural restrictions	Preserved route to competent review and record of contested decisions
Regenerative condition	Ecological or equipment state, deterioration, renewal dependencies	Time-indexed measured or estimated variables with model uncertainty
Institutional control	Who controls certification, data access, protocol changes and exceptions	Trace of delegated powers and authority over summary revision

tions introduce their own institutional choices, including who may inspect sealed evidence and who may certify that an inaccessible item was properly considered.

Table 2 organizes the proposed categories by the question they support and by the information lost when they are compressed carelessly.

The rows of Table 2 do not imply that a numerical measurement is always secondary to a categorical record. A small safety-critical physical threshold may require exact or one-sided conservative treatment, while a historical claim that has been conclusively extinguished under a legitimate procedure may be represented by a compact discharge certificate. The relevant distinction is determined by the decision risk, the status of evidence and the institutionally justified conditions of compression.

2.4 Normative Importance, Material Magnitude, and Regenerative Conditions

It is helpful to separate three notions of importance. *Measured influence* describes the effect of an event or path on a designated quantity over a stated horizon; its numerical scale depends on units, model choices, interventions and dependence assumptions. *Regenerative relevance* concerns the continued availability of productive conditions, including soils, durable equipment, accessible energy and the social arrangements sustaining cooperative production. *Normative significance* concerns a claim, right, responsibility, evaluative disagreement, or procedure under the applicable normative and institutional standards. These three relations can interact, but a small numerical influence does not supply a general test for small normative significance.

In particular, a maintenance worker or delegated maintenance AI may affect almost none of today’s delivered food while bearing a disputed burden whose accumulation changes future participation or the legitimacy of the governing arrangement. A damaged soil condition might also matter beyond the immediate throughput horizon even when a short-run scheduling model estimates little difference in deliveries. Conversely, an event producing a large quantity of food may rest on an unauthorized transfer or on the suppression of another party’s objection. The relevant claims cannot be settled by identifying the greatest physical output. Generative justice research by Eglash and colleagues emphasizes circular value flows and control over productive conditions in its own historical and normative context [5]. The present paper develops a separate, hypothetical procedure for retaining these distinctions in AI-network descriptions; it makes no claim that the cited work specifies such a procedure.

Regenerative measures are necessarily heterogeneous. Soil quality, equipment reliability, network redundancy and the continued possibility of cooperation do not automatically possess a single meaningful conversion into monetary units. A regulator may choose a threshold on soil degradation or a

minimum maintenance reserve, yet that threshold requires ecological, engineering or institutional justification outside the grammar. A summary that predicts six kilowatt-hours of demand accurately may still omit the need to replenish a damaged transformer or the expiry of permission to use a field. Representation should therefore retain explicitly which physical and normative dependencies are included, which remain uncertain and who has authority to revise the model boundary.

2.5 Exact Query Answers and Tolerance-Bounded Measures

The Introduction uses a prospective scale map Π_k and a vector of selected observables over a decision horizon T . To treat heterogeneous evidence precisely, let $\mathcal{Q}$ be the declared collection of decision queries, partitioned as $\mathcal{Q}_{\text{ex}}$ and $\mathcal{Q}_{\text{ap}}$. The first contains queries whose answer must be preserved exactly under the stated procedure: for example, whether a specified claim is registered as disputed, whether an authorization is valid at the proposed execution time, or whether a recognized avenue of review is open. The second contains measured queries for which approximation has been justified, such as expected aggregate electricity use within a bounded interval or predicted food delivery under an explicitly estimated uncertainty model.

Let a range over the declared admissible intervention set $\mathcal{A}_k$ and let $A_q(s, a)$ be the fine-level answer to a query q . An exact-answer condition for an abstract representation $\Pi_k(s)$ is given in Equation 4.

$$A_q(s, a) = \hat{A}_q(\Pi_k(s), a), \quad q \in \mathcal{Q}_{\text{ex}}, \quad a \in \mathcal{A}_k. \quad (4)$$

The equality concerns a stipulated, well-defined answer, including an answer of *undetermined* or *disputed* where that is the correct recorded status. Exact preservation of an institution's recorded finding is distinct from proving that the finding is morally or legally correct. In practice the abstraction may preserve a secure lookup key and authorized resolution mechanism rather than replicate the full protected record in public form. For Equation 4, such a mechanism counts as part of the specified coarse interface: it must address a versioned record or an otherwise fixed institutional snapshot. A decoder that silently consults an unrelated mutable register can return distinct results at different times for an unchanged coarse state, so its snapshot, requester role, and source of authority must be declared before an exact-preservation assertion is made.

For a numerical query $q \in \mathcal{Q}_{\text{ap}}$, the corresponding bounded-error requirement is Equation 5.

$$d_q\left(A_q(s, a), \hat{A}_q(\Pi_k(s), a)\right) \leq \varepsilon_q(T), \quad a \in \mathcal{A}_k, \quad (5)$$

where d_q is a declared metric or loss, and $\varepsilon_q(T)$ is a tolerated error justified for the decision horizon and risk. The right-hand side is a proposed target, not an achieved bound without a proof or calibration exercise. A predicted interval should be distinguished from a worst-case mathematical guarantee; observational error, model misspecification and regime change can invalidate apparently narrow empirical uncertainty. Physical nonnegativity and capacity constraints may also demand a one-sided error rule or an exact feasibility test rather than Equation 5 alone.

This query-based formulation qualifies the vector error in Equation 2: only observables with specified compatible numerical metrics should be placed inside one weighted norm. Categorical statuses, procedural guarantees and noncommensurable values require separately specified interfaces. The query partition is fixed for a declared use and can be contested or revised by an appropriate authority; the optimizer's computational convenience alone is not a legitimate rule for deleting an exact-query category. Because a review case can arise after routine scheduling, the design also needs a controlled means of refinement to obtain relevant fine-level evidence when a new authorized inquiry enters the declared decision boundary.

2.6 Distinguishability of Fine States under Exact Queries

The requirement of exact preservation imposes a simple necessary condition on a deterministic scale map. Consider a domain $\mathcal{S}$ of stipulated fine states, a fixed intervention a , and a deterministic abstraction

$\Pi_k : \mathcal{S} \rightarrow \mathcal{Z}_k$. The following proposition identifies the information that cannot be discarded while still answering a declared query exactly.

Proposition 2.1 (Exact-query distinguishability). *Suppose $q \in \mathcal{Q}_{\text{ex}}$ has a well-defined answer $A_q(s, a)$ for every $s \in \mathcal{S}$, and the coarse decoder $\hat{A}_q(z, a)$ depends only on the coarse state z and the fixed intervention a . If there are $s_1, s_2 \in \mathcal{S}$ such that $\Pi_k(s_1) = \Pi_k(s_2)$ but $A_q(s_1, a) \neq A_q(s_2, a)$, then the equality required by Equation 4 cannot hold for both states.*

Proof. Since the map sends s_1 and s_2 to the same coarse state, the decoder yields one and the same coarse answer for both under the fixed intervention. Exact equality for each state would require that answer to equal both distinct fine-level answers, which is impossible. $\square$

The proposition is an elementary informational constraint, not a finding that any specific institutional query has one universally valid answer. It implies that an admissible coarse state must distinguish fine states whenever a designated exact query distinguishes them. Such a distinction may be encoded by a status flag, a provenance reference, a protected certificate, or an accessible finer-resolution record under the stipulated decoder. It need not entail storing every original item redundantly. For a finite $\mathcal{S}$ with fixed a , the coarsest partition sufficient for a fixed set of exact queries groups precisely those states whose answers coincide on that query set. Additional approximate queries, interventions, dynamic transitions and future contestation can demand a finer partition. Section 2 therefore establishes only a necessary starting interface for later analysis of scale maps.

2.7 A Two-Period Food and Maintenance Illustration

Consider four operator roles: agriculture A , energy supply E , logistics L , and equipment maintenance M . In one stipulated period, E makes six kilowatt-hours available. The executed and uniquely identified energy allocations are two kilowatt-hours to A , one to L , two to M , and one held as unused available capacity. Those quantities sum to six and are not independently reproduced by overlapping descriptions of the food-delivery and maintenance circuits. Assume, solely for the example, that their dated material and capacity constraints are satisfied. The account is compatible with both a food circuit and a maintenance circuit referring to the same production network.

Suppose that M records a request for one replacement component due before the second period. The maintenance claim is marked *disputed*: its alleged contractual basis is documented, yet the competent procedure has not ruled on the obligation or its discharge. At the end of the first period, the immediate delivered-food statistic and energy account can be identical in two records even if only one contains the live dispute. Moreover, the component's absence may affect second-period equipment capacity, while the claim's validity must be decided under rules independent of the component's engineering effect.

For a first-period energy-planning query, it may be appropriate to retain one total of five kilowatt-hours executed and one kilowatt-hour unused, subject to an auditable record of the underlying occurrences and units. For a claim-status query, a regional report reducing both histories to that physical energy summary cannot distinguish the disputed claim from its absence. Proposition 2.1 then rules out an exact decoder for both histories if the disputed-status question belongs to $\mathcal{Q}_{\text{ex}}$. Adding a protected claim-status link suffices for that specific informational distinction when the link remains valid and accessible to the designated reviewer. The link does not establish whether M 's claim is justified; a competent procedure must evaluate its merits, while the maintenance schedule must independently assess capacity and safety.

A second-period counterfactual illustrates the interaction between descriptive and normative models. Failure to provide a necessary part may reduce prospective production, but the causal effect depends on the actual equipment state, alternative parts, repair technologies and available reserves. If the disputed request turns out to be unfounded, there may be no corresponding valid transfer duty even if repair remains productive. If it is valid, a favorable food forecast does not by itself discharge that duty. If M is merely an AI operator delegated by a human organization, the identity of the legal claimant and of the actor permitted to refuse further work require separate determination. The model must keep these conditional possibilities distinct.

The illustration shows how information reduction can legitimately simplify routine material coordination while retaining avenues to re-examine disputed historical relations. It neither assumes that the replacement part has a known monetary equivalent nor measures the value of ecological or technical regeneration with the energy figure. The choice of which queries to retain, the evidential protection of the register, and the authority to reopen the record are themselves political-economic and jurisprudential objects.

2.8 Implications for Subsequent Scale Analysis

A candidate normative renormalization procedure can now be evaluated against an explicit inventory of record types, decision questions and preservation conditions. Physical event identity protects material accounting against circuit-description multiplicity. Claim and authority registers make the existence, status and conditions of a request inspectable without presenting asserted validity as settled law. Provenance graphs support correction and contestation, subject to procedural and privacy constraints. Quantitative observables admit error targets when appropriate, while exact-status queries impose an informational distinguishability condition that cannot be replaced by small aggregate error.

The remaining research programme must add sufficient conditions for feasible approximation under growing circuit depth, compositional properties for the scale maps, mechanisms that preserve review procedures across model revisions, and empirical or simulated tests of AI-assisted inference. Those tasks also require explicit attention to concentrated power over data, certification and protocol selection. A coherent abstraction may lower the cost of producing a scheduling answer while failing to preserve a claim or a relevant resource bottleneck. Section 3 will therefore investigate the bounded conditions under which high-order productive-path contributions attenuate for specific material observables, while retaining the non-erasure and review requirements established here as independent constraints.

3 High-Order Attenuation and Effective Depth in Recursive Networks

The treatment of normative observables in Section 2 establishes what a proposed summary needs to preserve for a declared decision. The present section examines a more restricted, quantitative question: when can contributions associated with long or recursively nested productive routes be approximated by a finite-depth description? Its purpose is to specify sufficient attenuation conditions, distinguish representation multiplicity from actual productive occurrences, and identify failure regimes arising from dependence, collective feedback, and changing networks. The method combines elementary absolute-convergence bounds with a finite nonnegative-matrix model, two deliberately constructed counterexamples, and a small resource-network illustration. All results concern predeclared numerical observables and stipulated representations. Their application to an operating economy would require independent identification, validation, and institutionally justified query selection.

3.1 Path Families, Observable Weights, and Event-Level Accounts

Let G be a finite directed graph of typed productive interfaces, and let Γ_ℓ denote a finite collection of *descriptions* of length ℓ , where length counts directed interface traversals. Repeated walks can produce such collections at unbounded lengths even on a finite graph. A description γ may refer to energy, food, material recovery, maintenance, or a commitment-mediated activity. A repeated syntactic occurrence is distinguishable from a distinct executed event: several descriptions can cite the same uniquely identified generation or delivery record. Furthermore, a graph walk by itself contains neither a complete schedule nor a guarantee of authorization. The finite event register, temporal feasibility checks, and claim-provenance interfaces defined in Section 2 remain the separate evidential basis for those questions.

For a numerical query r evaluated at a fine state s under a declared intervention a , suppose an analyst proposes a path-series *surrogate* whose terms $w_r(\gamma; s, a) \in \mathbb{R}$ represent incremental or response contri-

butions in stated units. The proposed surrogate and its finite-depth truncation are defined in Equation 6.

$$F_r(s, a) = \sum_{\ell=0}^{\infty} \sum_{\gamma \in \Gamma_{\ell}} w_r(\gamma; s, a), \quad F_r^{[L]}(s, a) = \sum_{\ell=0}^L \sum_{\gamma \in \Gamma_{\ell}} w_r(\gamma; s, a). \quad (6)$$

The expression is a modeling commitment rather than an automatic identity for actual deliveries or social value. In particular, a network-flow observation is typically calculated from one event-unique ledger. A path expansion of that observation requires a separately demonstrated decomposition rule with appropriate incidence accounting. The quantities F_r and $F_r^{[L]}$ may consequently remain analytical influence scores or local response approximations without representing stocks, welfare, or output. A linear response calculation can admit both positive and negative terms; convergence claims will therefore use absolute weights and avoid relying on cancellation. The attenuated-walk index introduced by Katz offers a mathematical precedent for taking indirect pathways into account, while its sociometric application is distinct from the economic and normative interpretations proposed here [12].

A description-indexed tail can be bounded even when distinct walks share executed occurrences. If the analyst uses F_r as a surrogate for a measured query $A_r(s, a)$, however, it is also necessary to estimate the *model discrepancy* $d_r(A_r, F_r)$ under a justified unit-compatible distance. A small truncation error for the surrogate says little about this discrepancy by itself. Similarly, duplicated references to one electricity-generation event must never enter the physical resource ledger as duplicated production. The path-indexed influence accounting and event-unique material accounting thus remain parallel representations with separate validation procedures.

3.2 Branching, Attenuation, and an Absolute Tail Bound

Two distinct quantities determine the size of a path-series tail: the number of terms at each length and the magnitude of their individual contributions. The branching envelope can increase rapidly with depth even while each contribution decreases. A meaningful truncation criterion requires controlling their combination. Let $\mathcal{S}_{\star}$ be the class of states for which a proposed model is certified and $\mathcal{A}_{\star}$ its declared intervention set; both are part of the validity statement rather than implicit universal domains.

Proposition 3.1 (Uniform path-tail attenuation). *Assume that Γ_{ℓ} is finite at each $\ell \geq 0$, and that constants $K, C > 0$, $b \geq 1$, and $q \in (0, 1)$ satisfy $|\Gamma_{\ell}| \leq Kb^{\ell}$ and $|w_r(\gamma; s, a)| \leq Cq^{\ell}$ for every $\ell, \gamma \in \Gamma_{\ell}, s \in \mathcal{S}_{\star}$, and $a \in \mathcal{A}_{\star}$. If $\beta = bq < 1$, the surrogate in Equation 6 is absolutely convergent for the declared domain, and its residual satisfies the bound in Equation 7.*

$$\sup_{\substack{s \in \mathcal{S}_{\star} \\ a \in \mathcal{A}_{\star}}} |F_r(s, a) - F_r^{[L]}(s, a)| \leq KC \sum_{\ell=L+1}^{\infty} (bq)^{\ell} = \frac{KC \beta^{L+1}}{1 - \beta}, \quad \beta < 1. \quad (7)$$

Proof. For each depth, the triangle inequality and the two uniform envelopes give $\sum_{\gamma \in \Gamma_{\ell}} |w_r(\gamma; s, a)| \leq KC(bq)^{\ell}$. Since the resulting nonnegative geometric series converges for $bq < 1$, comparison proves absolute convergence, uniformly in the stipulated states and interventions. Applying the same comparison to the omitted depths establishes Equation 7. $\square$

Equation 7 strengthens the illustrative statement in Equation 1 by declaring the domain over which the constants must remain valid. It gives a *sufficient* absolute-error guarantee for a specified surrogate and specified representation family. If the product bq is at least one, the geometric certificate is unavailable; this does not prove that the actual sum diverges. A finite admissible history, cancellations of signed terms, tighter combinatorial restrictions, or a zero-weight unreachable subnetwork can each yield a smaller tail. Conversely, an estimated average decrease in observed contributions is insufficient for the uniform statement if rare conditions produce much larger weights. The distinction between a rigorous envelope and a fitted typical-case tendency is consequential for planning under low-frequency disruption.

The model also requires clarity about how the depth classes are counted. A bound on the number of syntax trees or walk expressions may greatly exceed the number of distinct material histories they denote.

Table 3: Illustrative sufficient depths under a geometric path-series envelope, with $KC = 1$, $q = 0.3$, and target residual at most $\eta = 0.05$. Values are constructed mathematical examples.

Candidate branching b	Combined factor $\beta = bq$	Certified depth L	Geometric tail at the displayed depth
Two continuations	0.6	7	$0.6^8/0.4 \approx 0.0420$
Three continuations	0.9	50	$0.9^{51}/0.1 \approx 0.0464$

Summing their scores is appropriate only if the surrogate has been defined on those descriptions and their coefficients account for multiplicity. A claim about actual throughput demands a separately validated projection onto event identities. When possible, a reduced or quotient representation can eliminate redundant descriptions before approximation; such a quotient must preserve the particular observable of interest and any associated provenance needed for later review.

3.3 Effective Depth and the Allocation of an Error Budget

A truncation horizon acquires practical meaning only in relation to a decision and an acceptable approximation error. For a numerical query r , define its guaranteed effective depth at tolerance $\eta_r > 0$ as the smallest nonnegative integer L at which the declared upper bound on the residual is at most η_r . If the available estimates do not establish such a depth, the guarantee remains unresolved. Under the assumptions of Proposition 3.1 and $0 < \beta < 1$, one sufficient choice is given in Equation 8.

$$L_r^{\text{cert}}(\eta_r) = \max \left\{ 0, \left\lceil \frac{\log(\eta_r(1 - \beta)/(KC))}{\log \beta} - 1 \right\rceil \right\}. \quad (8)$$

The logarithmic expression in Equation 8 is used only for strictly positive η_r , K , C , and $\beta \in (0, 1)$. Because $\log \beta < 0$, it is obtained by solving the upper bound in Equation 7 for a sufficient integer depth. The formula makes no claim of minimality for the *actual* error: it computes the minimum depth certified by this particular geometric envelope. Separate components of a numerical query can have different units, tolerances, intervention classes, and resulting depths. A system-wide single depth should therefore be viewed as a conservative design choice, not as an inherent property of the production network.

An overall decision error can contain at least three parts: error in estimating the weights or state, discrepancy between the surrogate and the intended fine-level quantity, and omitted long-path contributions. If d_r is a metric, the triangle inequality permits a schematic decomposition of the total error as in Equation 9.

$$d_r(A_r, \hat{F}_r^{[L]}) \leq d_r(A_r, F_r) + d_r(F_r, F_r^{[L]}) + d_r(F_r^{[L]}, \hat{F}_r^{[L]}). \quad (9)$$

Here A_r is the fine-level target, F_r the ideal stipulated surrogate, $F_r^{[L]}$ its mathematical truncation, and $\hat{F}_r^{[L]}$ the computed estimate. The middle term is bounded by Equation 7 when the distance is absolute error on the surrogate’s scalar output. The first and third terms require independent evidence, proofs, or calibration; an attractive tail estimate cannot discharge them. This distinction also prevents a simulation confidence interval from being presented as a verified worst-case guarantee. The construction applies to approximable queries in $\mathcal{Q}_{\text{ap}}$; the exact-status queries in $\mathcal{Q}_{\text{ex}}$ retain the separate condition in Equation 4.

Table 3 gives two controlled cases with $KC = 1$ and $q = 0.3$. The examples concern an abstract path-series score, not measured electricity or the worth of a production circuit. Increasing the number of candidate continuations from two to three, while leaving the stipulated per-path attenuation unchanged, greatly increases the certified depth needed to reach the same tolerance.

An institutional choice of η_r also requires justification. A food-supply dashboard may accept a small bounded forecast error for routine dispatch, while a capacity decision near a safety threshold may demand an exact inventory check or conservative uncertainty set. The tolerance is attached to a decision purpose, a time horizon, and a declared affected population. Its selection is therefore a matter for the

relevant procedures and expert evidence rather than an automatic consequence of mathematical attenuation. Approximation can reduce numerical workload while the right to contest an allocation continues to depend on the record-preservation conditions in Section 2.

3.4 Transfer Matrices, Spectral Behaviour, and Critical Boundaries

A finite nonnegative transfer matrix offers a complementary way to organize weighted-walk families. Let $B \in \mathbb{R}_{\geq 0}^{n \times n}$ be a fixed matrix whose entry B_{ij} denotes a stipulated nonnegative transfer of influence from a typed interface i to interface j . Let $u, v \geq 0$ select sources and responses. Define the depth- ℓ response score as $u^\top B^\ell v$. Products of entries then correspond to weighted walks of that depth. Such a matrix can represent an attenuated propagation model, provided its entries and the query units have been identified. It is not automatically a matrix of physically executed transfers, nor a transition law for permissions or contractual validity.

For the maximum-row-sum matrix norm $\|B\|_\infty = \max_i \sum_j B_{ij}$, a direct sufficient condition is $\|B\|_\infty = \beta_B < 1$. Under that condition, the response tail satisfies Equation 10.

$$\sum_{\ell=L+1}^{\infty} u^\top B^\ell v \leq \|u\|_1 \|v\|_\infty \frac{\beta_B^{L+1}}{1 - \beta_B}. \quad (10)$$

The proof uses $\|B^\ell v\|_\infty \leq \|B\|_\infty^\ell \|v\|_\infty$ followed by the geometric-series estimate. More generally, the spectral condition $\rho(B) < 1$ gives convergence of the finite-dimensional Neumann series, although a bound based directly on $\|B\|_\infty$ may be unavailable if that norm exceeds one. Transient amplification is possible in directed, heterogeneous networks. In addition, the relevant observable may never reach a critical component, so the spectral radius of the entire graph can exaggerate the difficulty of approximating one restricted query. The existence and size of an error bound must be determined for the chosen model and question.

The phrase *critical boundary* has a precise local meaning in the present linear model: a particular sufficient contraction inequality ceases to hold as its factor approaches one. Near $\beta_B = 1$ the geometric certificate becomes increasingly conservative and may call for a large truncation depth. The failure of that certificate neither proves a physical phase transition nor identifies a political or economic crisis. Actual threshold effects can arise from joint resource prerequisites, common infrastructure and interdependent failures. Analyses of cascades in threshold networks and interdependent infrastructure provide structural precedents for examining such amplification, under their own assumptions [22, 4]. Their results do not establish numerical cascade laws for hypothetical AI production institutions.

3.5 Dependence, Correlation, and Switching Productive Relations

A serial chain's survival probability is q^ℓ only under an appropriate joint reliability model. If completion events $X_1, \dots, X_\ell$ are independent and each has probability q , then their intersection has probability q^ℓ . With arbitrary dependence, the corresponding product is determined by sequential conditional probabilities, and equal unconditional marginals do not provide the same conclusion. For example, let every required link be available exactly when one common energy facility is operational, with probability q . Each link has marginal availability q , while the entire chain succeeds with probability q for any positive length. An unconditional "per-link reliability" fitted from this system is therefore not an attenuation certificate.

A qualified conditional statement remains available. If each link's success probability, conditional on all preceding successes in the chain and the declared admissible state, is bounded above by $q < 1$, the chain-rule product gives an upper bound q^ℓ without requiring independence. The meaning of success and the conditioning information must be specified. For production, shared stock depletion, scheduled maintenance, strategic interruption, and a duly authorized refusal can affect those conditionals. Furthermore, probabilities of different *candidate paths* can be correlated through a common bottleneck; treating them as independent alternatives may exaggerate redundancy. A union bound is still valid for

the probability of at least one successful path when legitimate individual upper bounds exist, but it may be loose and does not measure resource delivery when successful paths share a single limited facility.

Networks also change over time. A sequence of nonnegative matrices $B_1, \dots, B_\ell$ can represent successive influence interfaces under one stipulated switching history. Uniform contraction, such as $\|B_t\|_\infty \leq \bar{\beta} < 1$ for every allowed period, yields an upper bound $\|B_1 \cdots B_\ell\|_\infty \leq \bar{\beta}^\ell$. The separate spectral stability of each B_t is weaker. Consider the two matrices in Equation 11.

$$B_1 = \begin{pmatrix} 0 & 1.2 \\ 0 & 0 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & 0 \\ 1.2 & 0 \end{pmatrix}, \quad B_1 B_2 = \begin{pmatrix} 1.44 & 0 \\ 0 & 0 \end{pmatrix}. \quad (11)$$

Each matrix in Equation 11 is nilpotent and has spectral radius zero; repeating their alternating product produces amplification along the first coordinate. The coefficients are abstract response multipliers and are *not* delivery probabilities. The example demonstrates why checking each time slice separately is insufficient for a uniform dynamic truncation guarantee. Real applications would need bounds on allowed matrix products, an explicit switching class, or an appropriate horizon-dependent estimate.

The timescale of influence must also be differentiated from walk depth. A long route can contain rapid electronic commitments, whereas a short maintenance cycle can consume months and affect future harvests. Conversely, a feedback motif can recur at several dates, constrained by physical stocks even when its symbolic route remains available. Temporal network research distinguishes time-respecting paths from purely static topology [11]. Consequently, an operational effective-depth estimate requires a defined clock, delay model and dated intervention class alongside the abstract number of traversed interfaces.

3.6 A Resource-Network Illustration with a Retained Dispute

The four-role scenario of Section 2.7 provides a deliberately modest interface between the path bound and a dated material account. Energy supplier E makes six kilowatt-hours available; current-period allocations use two for agriculture A , one for logistics L , and two for maintenance M , leaving one unused. These figures form an event-unique material account: $2 + 1 + 2 + 1 = 6$ kWh. The figure is independent of how many candidate descriptions mention the same energy-generation event. Suppose, in a separate exploratory model, that optional resource-coordination continuations branch into either two or three alternatives per step, with stipulated numerical influence weights bounded by 0.3^ℓ and $KC = 1$. For a scalar planning score with tolerance 0.05, Table 3 certifies depths seven and fifty, respectively. Neither result alters the six-kilowatt-hour accounting identity; both concern the error in the proposed path-series score.

Maintenance operator M also has a recorded request for a replacement part due before the second period, with its legal or contractual validity still disputed. Its status may remain *disputed* whether a surrogate assigns the associated long-range influence a value below 0.05 or above it. The numerical path truncation therefore leaves the claim in the exact-query interface of Equation 4. If the part is necessary for next-period productive capacity, a planner must additionally examine that requirement under a dated material model; the significance of an actual bottleneck cannot be inferred from an unrelated influence-series tolerance. If the part request proves unjustified, the claim register can record that determination through the relevant procedure, independently of the engineering usefulness of repair.

The constructed case exposes three different grounds for preserving fine detail. First, joint capacity and event identity are needed to avoid counting the same electricity twice. Second, a near-threshold maintenance dependency may remain critical to future physical feasibility even when average influence scores attenuate. Third, a disputed obligation requires continued provenance and access to review under the declared exact queries. These demands can lead to different effective resolutions within one network. An AI-assisted decision tool could therefore truncate routine exploratory path weights at an analytically defensible depth while retaining local event-level certificates, a shared-bottleneck register, and authorized access to the disputed claim. That division of representations is a proposed design pattern, not an empirically demonstrated improvement or a legal determination.

3.7 Conditions for Using Effective Depth in Institutional Abstraction

The results of this section justify a restricted form of high-order simplification. Under an explicit decomposition, uniformly controlled weights and a sufficiently small combined branching–attenuation factor, an infinite formal description family may be replaced by a finite truncation with a calculable numerical tail bound. A finite transfer matrix provides another sufficient route to attenuation. In each case, the bound must be associated with a specified response measure, horizon, admissible state class and intervention set. Dependence, changing transfer operators and shared bottlenecks can defeat simple reliability-based estimates; the failure of one sufficient bound leaves open other methods, including direct finite-history analysis and query-specific refinement.

The quantitative criterion is only one layer of the proposed normative renormalization programme. Material conservation and dated feasibility require event-level or otherwise certified accounts; claims and authorizations are governed by explicitly selected exact-status questions and competent review procedures. A small observable contribution has no general authority to extinguish a debt, discard historical evidence, suspend procedural standing or pronounce a distribution just. Similarly, a rapidly decaying path series cannot establish that money-like functions have disappeared in the surrounding economy. Partial displacement of a settlement mechanism, continuity of material production and legitimate value return remain distinct observations to be investigated separately.

Section 3 thus provides a conditional accuracy certificate and identifies its possible failure points. The subsequent construction of scale-dependent maps must specify how those certificates attach to reduced productive interfaces, how errors compose across resolution levels, and how exact-query access and claim provenance survive every admissible transformation. The broader renormalization analogy remains provisional: network-level coarse-graining has its own unresolved mathematical questions, and carrying normative obligations across institutional scales requires additional jurisprudential and political-economic argument [7].

4 Scale-Dependent Abstraction Maps for Productive Networks

The proposed normative renormalization programme requires a precise account of what changes when an AI-operated production network is described at a coarser resolution. The objective of this section is to define restricted maps between typed network representations, specify the material and institutional interfaces that they retain, and establish conditions under which approximation certificates survive successive reductions. The argument is constructive and conditional. It first defines decision-specific scales and admissible domains, then examines event-unique conservation and versioned claim access, proves a compositional error statement, and develops a finite-horizon intervention criterion. A two-period food–energy–maintenance example illustrates how a numerical aggregate and a reviewable claim can remain available at different resolutions. These constructions are proposed representations and mathematical guarantees under stipulated conditions; they do not themselves establish an economic renormalization group or a procedure for judging the merits of contested claims.

4.1 Decision Domains and Typed Partial Abstraction

Consider a fine description $s \in S_0$ comprising event occurrences, claims, authorization records, review procedures, material inventories, regenerative conditions, and provenance, as in Equation 3. Let T denote a decision horizon, $\mathcal{I}$ a declared class of interventions, and $Q = Q_{\text{num}} \sqcup Q_{\text{exact}}$ the selected numerical and exact-status queries. These three choices form part of the abstraction’s specification. A representation that answers a question about total energy used during one period may be inadequate for a question about whether one named maintenance request was fulfilled by its deadline.

A scale k consists of a representation space S_k , a catalogue of typed input and output interfaces, and rules for answering the declared queries. Its effective variables may include boundary inflows, dated deliveries, available capacity, unresolved claims, and identifiers of the procedures controlling access to

detailed evidence. Variables are selected for an identified use, rather than assumed to be sufficient for every future inquiry. In particular, an aggregate region can describe multiple operators without acquiring their rights or making their separate authorizations interchangeable.

For successive scales, a candidate abstraction is a *partial* map. Equation 12 defines its domain and its composed domain.

$$\begin{aligned}\Pi_{ij} : D_{ij} \subseteq S_i &\longrightarrow S_j, & i < j, \\ D_{02}^{\text{comp}} &= \{s \in D_{01} : \Pi_{01}(s) \in D_{12}\}, \\ \Pi_{02}^{\text{comp}}(s) &= \Pi_{12}(\Pi_{01}(s)), & s \in D_{02}^{\text{comp}}.\end{aligned}\tag{12}$$

The domain D_{ij} consists of states meeting documented preconditions: compatible resource units, sufficient event-identity resolution, declared time aggregation, identified boundary transactions, and availability of the required query and review interfaces. Equation 12 defines composition only on states for which both transformations are available. A direct fine-to-coarse map, if separately constructed, need not equal the composite map. Equality is an additional consistency condition to be checked, possibly only at the level of specified observables.

Partiality is substantively important. When a material account lacks meter data, a claim identifier is unresolved, or an intervention lies outside the training and validation envelope, the declared transformation may be undefined. A system can return a refusal to certify, request finer records, or provide a clearly marked exploratory summary with no operational validity. Assigning an apparently precise number to an unverified input would conceal the very uncertainty that scale reduction is intended to manage.

This use of state abstraction has a limited methodological affinity with finite decision-process research, where state aggregation can reduce planning cost while the preservation of decision-relevant features determines which guarantees are available [13]. The present objects also carry heterogeneous legal, material, and historical records whose meanings exceed that formal setting. Similarly, the mathematical literature on network renormalization investigates conditions under which descriptions can transform coherently across scales and identifies difficulties peculiar to heterogeneous networks [7]. Those literatures motivate scrutiny of the proposed maps; neither supplies a ready-made normative authorization rule for the AI economy.

4.2 Material Boundary Interfaces and Event-Unique Accounting

An admissible aggregation must keep resource types and accounting boundaries visible. Let R be a declared set of operators, r a resource type with a specified physical unit, and $[t, t + 1]$ an interval whose reporting conventions are fixed. Let $X_R^r(t)$ be the sum of recorded inventories held in R at time t . For an event-unique and complete account, boundary deliveries enter once, and transfers between members of R cancel when the member-level balances are summed. A proposed boundary identity is given in Equation 13.

$$X_R^r(t + 1) - X_R^r(t) = F_{\text{in},R}^r - F_{\text{out},R}^r + P_R^r - C_R^r - L_R^r.\tag{13}$$

In Equation 13, $F_{\text{in},R}^r$ and $F_{\text{out},R}^r$ are external boundary flows, P_R^r and C_R^r are independently specified production and consumption of type r inside the boundary, and L_R^r comprises measured losses or write-offs over the interval. Each term is a quantity in the same unit. Transformations from grain to food, electricity to mechanical work, or residue to soil amendment require separately stated conversion records; they cannot be combined by numerical addition across heterogeneous units. The identity assumes complete and mutually consistent event records and a fixed convention for stocks in transit. If those conditions fail, the appropriate object is an account with a measured or explicitly unresolved discrepancy, rather than an asserted conservation certificate.

The mechanism of cancellation has a useful invariance. When an event records a delivery of u units of resource r from one member of R to another, it contributes $-u$ to one member's stock balance and $+u$ to the other's. Its net contribution to X_R^r is zero. When the same event appears in three grammatical circuit descriptions, its physical occurrence still enters the account once. A map whose boundary sum is derived from *sets of unique event identifiers* can preserve this identity; a map that sums independently enumerated

Table 4: Illustrative typed interfaces for decision-specific coarse descriptions of AI-operated production networks. A coarse summary may retain a certified aggregate, an accessible fine-scale record, or both, depending on the query and applicable procedures.

Interface	Candidate effective variable	Independent preservation condition
Material boundary	Stocks and dated net flows by resource type	Event-unique balances, compatible units, losses and boundary accounting
Temporal feasibility	Delivery windows, capacity envelopes, exception certificates	Relevant precedence and peak constraints remain testable
Regenerative continuity	Capacity and renewal state with specified horizon	Explicit delayed production, repair or ecological transition model
Claim and authority	Versioned claim identities, authorization scope and status	Exact authorized queries with accessible provenance and correction path
Governance and review	Record of responsible authority and appeal route	Competent standing and procedure preserved under applicable rules
Quantitative influence	Effective response parameters and uncertainty envelope	Bounded error over declared observables, interventions and states

circuit descriptions may double-count a shared generator or maintenance event. The provenance and event-identity requirements arise before any approximation of high-order path influence.

Temporal aggregation introduces additional restrictions. Six kilowatt-hours produced during a period do not necessarily constitute six kilowatt-hours available before a particular maintenance deadline. A scale that omits arrival times can support a period-total query while failing a prefix-feasibility query. Likewise, a shared-capacity ceiling cannot generally be certified from the aggregate total demand if peak simultaneous use remains unknown. An admissible coarse interface must retain relevant deadline windows, peak bounds, or access to their fine-scale certificates. The same consideration applies to regeneration: a claim that a field’s future capacity is sustained requires a dated ecological or technological model, even when a coarse food-output record is arithmetically consistent.

Table 4 records a minimum set of interfaces for the proposed scheme. The requirements are conditional on the question: they specify which evidence would be needed to certify a result, not what institutional arrangement is universally required.

4.3 Exact Status, Provenance Pointers, and Authorized Resolution

The normative layer of an effective representation need not reproduce every underlying document in each aggregate record. It must nevertheless supply a reliable way to answer the exact questions the system promises to preserve. For a specific claim c , a record can carry a stable identifier, an evidence or decision version, a status category, and a reference to the competent procedure by which a requester may inspect or contest that status. A pointer is useful only when its target is retained, its identity is verified, its version semantics are clear, and the authorized requester can obtain the query result. A revoked or broken link cannot satisfy an exact-preservation promise merely by retaining its previous string.

Let r be a requester role for which query q is authorized and let v identify a declared snapshot version. For s in the authorized domain, $q_0^{r,v}(s)$ is the exact fine-level answer. If the coarse representation resolves through a versioned provenance service, it may implement $q_k^{r,v}$ without public disclosure of the underlying documents. Equation 14 states the intended exact requirement.

$$q_0^{r,v}(s) = q_1^{r,v}(\Pi_{01}(s)) = q_2^{r,v}(\Pi_{02}^{\text{comp}}(s)) \quad (s \in D_{02}^{\text{comp}}). \quad (14)$$

Equation 14 holds only when the corresponding query resolvers are defined and available to requester r at version v . Its content may be an assertion of *disputed* or *discharged* status recorded by a named body; equality does not independently prove that body’s legal conclusion correct. A later decision can change the status. The relevant version is then updated through a procedurally valid event, with earlier versions retained according to the declared retention and evidence-access requirements. A coarse description

should not silently convert a historical dispute into an absent entry because the path-weight assigned to its material consequence is small.

Different parties may have different legitimate access to sensitive evidence. Exact preservation therefore concerns the ability of an authorized query and review procedure to recover a determinate answer, including an appropriate indication of restricted access or unresolved evidence. It need not entail unrestricted public inspection of private records. Equally, the availability of a technically functional lookup is distinct from the procedural standing to use it, the competence of the authority deciding the dispute, and the fairness of the underlying rule. These matters remain subjects of institutional evaluation rather than consequences of Equation 14.

4.4 Compositional Certificates for Quantitative and Exact Queries

Two individually acceptable reductions may introduce errors that accumulate. Let q_i denote the realization of one selected numerical query at scale i , with answers in a common metric space (Z, d) . The question, authorized intervention, horizon, and relevant snapshot are held fixed while the fine representation is compared with successive coarse realizations. For all $s \in D_{02}^{\text{comp}}$, assume certified bounds for q_0 relative to q_1 and for q_1 relative to q_2 . Equation 15 states the resulting two-step certificate.

$$\begin{aligned} d(q_0(s), q_1(\Pi_{01}(s))) &\leq \varepsilon_{01}, \\ d(q_1(y), q_2(\Pi_{12}(y))) &\leq \varepsilon_{12} \quad (y \in \Pi_{01}(D_{02}^{\text{comp}})), \\ d(q_0(s), q_2(\Pi_{02}^{\text{comp}}(s))) &\leq \varepsilon_{01} + \varepsilon_{12}. \end{aligned} \tag{15}$$

Proposition 4.1. *Let the partial maps in Equation 12 be defined on the composed domain. Suppose the numerical query implementations satisfy the first two bounds of Equation 15, uniformly over a declared class of states and interventions, with a common metric on their outputs. Then the composed query satisfies the final bound of that equation. If, in addition, every selected exact-status query is defined and preserved with the same authorized requester, snapshot version and status semantics across both maps, its answer is preserved under their composition.*

Proof. For a state in the composed domain, insert the intermediate result $q_1(\Pi_{01}(s))$ between $q_0(s)$ and $q_2(\Pi_{12}(\Pi_{01}(s)))$. The metric triangle inequality and the two assumed uniform bounds yield the final inequality of Equation 15. For each exact query, the hypotheses provide equality of the fine and intermediate answers and equality of the intermediate and coarse answers. Transitivity of equality gives the result. The conclusion concerns the declared questions and common domain only. $\square$

Proposition 4.1 is a sufficient internal consistency result. It neither says that a suitable first-level map exists nor proves that the bounds can be learned from observational data. It also does not imply that a direct fine-to-coarse map and the composite map produce the same state. To support reproducible governance, an implementation should check *observable-level path consistency*: whenever two allowed routes of scale reduction are defined, their answers agree exactly on designated categorical questions and lie within a declared discrepancy budget on quantitative questions. Where those conditions fail, the route used to construct a decision becomes an additional item in the audit record.

For a numerical illustration, suppose one explicitly calibrated query has a first-stage bound $\varepsilon_{01} = 0.02$ and a second-stage bound $\varepsilon_{12} = 0.03$, both expressed in the same normalized score units and certified on the composed domain. Proposition 4.1 yields a coarse-query error bound of 0.05. These are stipulated tolerances and do not describe a measured network. If the score also uses a truncated recursive-path series, the truncation bound of Section 3 must be budgeted separately unless its contribution has already been included in one of the two certificates. Counting the same uncertainty term twice or omitting it entirely would distort the reported accuracy.

4.5 Intervention-Consistent Dynamics and Finite-Horizon Bounds

Snapshot agreement alone may be inadequate when the coarse representation is used to plan a future intervention. Let F_a denote one fine-state transition under an admissible intervention a , and let $\hat{F}_{\iota(a)}$

be the corresponding proposed transition at a chosen coarse scale. The intervention translation ι must identify how the fine action is represented, including its resource boundary, timing and authority. For a metric d_k on the *numerical* components of the coarse state, Equation 16 defines an approximation defect along declared fine trajectories.

$$d_k\left(\Pi(F_a(s)), \widehat{F}_{\iota(a)}(\Pi(s))\right) \leq \delta \quad (s, a \text{ in the certified domain}). \quad (16)$$

The inequality in Equation 16 compares an aggregated real transition with a transition predicted directly on the coarse state. It can be evaluated only when the fine transition, the next-state abstraction, and both intervention descriptions lie within their declared domains. It is a conditional *simulation-accuracy* statement, not a causal conclusion obtained from observing correlations. A chosen metric can ignore differences in categorical claim status only when those differences are handled by independent exact-query safeguards.

A finite-horizon certificate also requires *trajectory closure*: the prediction and the aggregated fine trajectory must remain in the region on which the one-step defect and Lipschitz estimates were certified. Without this requirement, an initially valid estimate may be applied to a subsequent coarse state outside its declared domain. The resulting conditional statement is formalized in Proposition 4.2.

Proposition 4.2 (Domain-conditioned finite-horizon simulation error). *Fix $T \in \mathbb{N}$, a fine trajectory $s_{t+1} = F_{a_t}(s_t)$, and the coarse prediction $\widehat{s}_0 = \Pi(s_0)$, $\widehat{s}_{t+1} = \widehat{F}_{\iota(a_t)}(\widehat{s}_t)$. Assume that both $\Pi(s_t)$ and $\widehat{s}_t$ belong to a declared certified coarse domain D_k^{cert} for $0 \leq t \leq T$, with the necessary partial maps defined. For every $t < T$, assume the one-step defect in Equation 16 is bounded by $\delta \geq 0$ along the fine trajectory and that $\widehat{F}_{\iota(a_t)}$ is Λ -Lipschitz under d_k on the two coarse arguments being compared, with one common $\Lambda \geq 0$. Then the error satisfies Equation 17.*

$$d_k(\Pi(s_T), \widehat{s}_T) \leq \delta \sum_{j=0}^{T-1} \Lambda^j, \quad T \in \mathbb{N}, \quad \widehat{s}_0 = \Pi(s_0). \quad (17)$$

Proof. Write $e_t = d_k(\Pi(s_t), \widehat{s}_t)$. The triangle inequality, the one-step defect, and the Lipschitz assumption, applied at two states inside the certified domain, give $e_{t+1} \leq \delta + \Lambda e_t$. Since $e_0 = 0$, finite induction yields Equation 17. The proof depends on the domain and intervention assumptions at each step, including the predicted states; an estimate certified only at the initial state does not supply those premises. $\square$

For the stipulated values $\delta = 0.02$, $\Lambda = 1.1$, and $T = 3$, Equation 17 yields $0.02(1 + 1.1 + 1.21) = 0.0662$ in the chosen metric units. The estimate may be loose, and the modeled transition rule may itself be misspecified. When $\Lambda > 1$, the bound can increase with the horizon; near thresholds or across switching regimes, a uniform bound may be unavailable. An operational implementation would need to monitor domain membership after each transition, decline to extend the certificate when a trajectory exits the validated domain, and seek re-identification or another model as appropriate. The numeric estimate supplies no certificate for unmodeled state variables, unregistered external effects, or changes in applicable institutional rules.

The intervention set is also normatively and politically selected. A model may be accurate for authorized routine redistributions while lacking evidence for withdrawal of a critical facility, unilateral changes to claims, or interventions affecting an excluded participant. The set of interventions a developer can computationally simulate is generally larger than the set a governing institution may lawfully permit. Authorization remains a separate predicate, and a request exceeding the certified class should trigger a new assessment instead of an automatic application of the existing accuracy bound.

4.6 Two-Period Scale Reduction with a Retained Maintenance Claim

The energy and maintenance scenario introduced in Section 2.7 provides a constructed test for the preceding interfaces. During period one, an energy operator E makes six kilowatt-hours available. Agriculture A uses two, logistics L uses one, maintenance M uses two, and one kilowatt-hour remains unallocated. The material account contains three executed allocations and one residual balance tied to a single event-unique generation record, so it satisfies the stipulated total of six kilowatt-hours. The presence of an unresolved request for a replacement part due before period two is independently recorded as claim c_M ; the request's validity and satisfaction are subjects for the applicable procedure.

At scale zero, the record contains each dated energy allocation and the complete provenance of c_M . A first aggregation groups cultivation and logistics into a food-delivery interface using three kilowatt-hours, retains maintenance demand of two kilowatt-hours, and retains the one-kilowatt-hour balance. A second aggregation reports a five-kilowatt-hour total used and a one-kilowatt-hour available balance. The physical totals remain consistent at each level because the same generation event is counted once. For a planning query restricted to *period-total energy*, the final pair of numbers may suffice. For a question about whether the maintenance activity received its allocation before its deadline, the intermediate or fine record must provide dated evidence or a certified temporal query interface.

Now consider two possible fine records with identical six-kilowatt-hour physical totals. In one record c_M is marked *disputed*; in the other it has been marked *discharged* by a designated procedure. If the second-stage map retains only total energy used and remaining balance, the two states have the same image, yet the exact query “What is the recorded status of c_M at version v ?” has different answers. Under the distinguishability result of Proposition 2.1, that map cannot supply an exact-status resolver solely from those two numbers. A claim-keyed and versioned provenance interface can restore the selected query, provided the target record remains available, its identity is established, and the requester has the appropriate access and review route. The scheme records how a determination may be retrieved; it does not prejudice which status is substantively correct.

A more consequential limitation appears when the replacement part is a prerequisite for period-two production. An aggregate reporting five kilowatt-hours of current use conveys little about whether the required part arrives on time, whether an equipment failure blocks next-period food delivery, or whether the stated maintenance request is justified. For the relevant intervention and horizon, the effective representation must add a dated bottleneck indicator, preserve access to the fine maintenance schedule, or decline to certify future feasibility. This example illustrates why a coarse model can be adequate for one numerical question and ineligible for a second question concerning a critical threshold or a reviewable claim. The separation is an institutional design requirement under the declared evaluation programme, rather than a finding about the operation of a real AI economy.

4.7 Scale Consistency, Refinement, and the Status of Renormalization

The results of this section define a deliberately bounded meaning for *normatively constrained coarse-graining*. A proposed scale transformation has a declared domain; a typed material boundary; numerical query certificates; exact, versioned and role-sensitive status interfaces; and a procedure for reopening fine records when the coarser description lacks the needed information. Its acceptance for one decision is conditional on the selected horizon, intervention class, data quality and institutional rules. A map can be rejected, revised, or refined without requiring that every analytical question share one universal spatial or temporal resolution.

An operational refinement procedure would record the scale map and its version, the evidence used to calibrate its errors, the questions and interventions for which it is certified, and the criteria that invalidate the certification. Evidence of conservation discrepancies, missed deadlines, altered authority, disputed claim provenance, or distributional effects outside the declared observable set can initiate renewed fine-scale investigation. The decision about which exceptions require review is itself subject to legitimate institutional processes and possible disagreement. Preserving one formal answer does not automatically preserve the full opportunity to contest a governing rule; Section 2 supplies the record vocabulary, while

the subsequent analysis of rights and normative plurality must examine that procedural question in its own terms.

The proposed maps implement a restricted form of task-relative coarse-graining and effective-model construction. The compositional error result in Proposition 4.1 concerns compatible declared queries; it does not produce a closed transformation algebra on all model parameters. Stronger uses of the term *renormalization* would require independently demonstrated transformations across scale families, reproducibility under composition, meaningful effective parameters, and justified invariants. In heterogeneous economic institutions, even the partition defining a scale can change through bargaining, resource disputes, technological adaptation or decisions about whose contributions count. A demonstration that a single finite example admits two convenient summaries is therefore a preliminary construction, not a proof of universal scale invariance. The formal certificates established here provide an object that later research may test and reject.

The central outcome is a division of responsibilities among representations. Event-level material records support conservation and dated feasibility; weighted-path summaries can admit controlled numerical truncation; versioned claim interfaces preserve designated exact-status answers for authorized requesters; and institutional procedures evaluate authority and contested rights. Together, these roles make a possible multi-scale decision system more explicit while leaving unresolved the selection and justification of normative standards. The following section examines the additional conditions for preserving rights, dispute memory and plural evaluations when scale reduction affects unequal participants and historically situated obligations.

5 Rights Preservation, Normative Pluralism, and Dispute Memory

A multiscale representation of productive value circuits can preserve aggregate resource balances while concealing the institutional relations through which those resources are governed. A right to challenge an allocation, the documented status of an unpaid maintenance obligation, or the conditions of access to a shared energy facility may depend on identifiable events and decisions whose material quantities are small. This section develops the institutional side of normatively constrained abstraction. Its objective is to specify preservation requirements for claims, powers, procedural standing, and historically changing disputes while acknowledging that reasonable assessments of justice may remain plural. The method combines a typed query model, a limited composition result, a comparison of evaluative orders, and an illustrative two-period production case. It provides necessary design conditions for a proposed information system, with institutional legitimacy and the practical effectiveness of its procedures remaining separate questions of evaluation.

5.1 Differentiated Rights and the Scope of Preservation

The recorded exercise of an action does not itself determine the actor's normative position. In the hypothetical AI-operated networks considered here, a cultivation operator may use electricity under a delegated license, an infrastructure owner may retain the power to alter access rules, a community institution may possess a claim to shared food deliveries, and a maintenance contractor may assert a right to replacement parts. One technical agent may perform several roles; several different persons or institutions may jointly govern one agent's actions. Claims, liberties, powers, liabilities, review standing, and practical control should consequently be distinguished when the research question requires them.

Hohfeld's account of fundamental legal relations is an important analytical precedent for distinguishing the correlatives and opposites of legal rights, privileges, powers, and immunities [10]. Schlager and Ostrom's conceptualization of resource-right bundles likewise distinguishes access, withdrawal, management, exclusion, and alienation [19]. The present inquiry borrows those distinctions as descriptive aids for an institutionally specified model. Their original scholarship does not furnish an AI-specific legal regime, and the moral or legal personhood of an AI operator remains an open question. A record about

an AI can identify the principal who holds a relevant right, the operator who executes an instruction, and the affected party who can challenge its execution.

A *protected normative item*, relative to an identified regime and decision domain, will mean a recorded claim, permission, immunity, review opportunity, or corresponding duty for which an applicable retention or review rule has been declared. Protection in this sense is a property of the proposed representation and its procedures, not a judgment that every asserted demand is valid. The requirement may include historical evidence of an extinguished claim, because an inquiry into its earlier discharge still has a legitimate evidential object. Nor does preservation mean uninterrupted public disclosure: a procedure may restrict access for confidentiality, trade secrecy, safety, or legally applicable data protections. Rights-sensitive access control and the capacity to recover an authorized answer must be considered together.

The relevant protection set must be selected through a stated authority and scope. A technical designer has no inherent mandate to decide which alleged interests count as protected or which claimants may be excluded. Conversely, an unlimited retention requirement can create its own harms, including indefinite exposure of personal records, surveillance capacity, and the accumulation of concentrated administrative power. The present framework therefore treats retention periods, lawful correction and erasure conditions, access restrictions, and challenge channels as variables supplied by governing institutions. A provenance-preserving representation can maintain authorized records of the fact and reasons for a correction without indiscriminately retaining every superseded datum.

5.2 Recorded Status, Historical Memory, and Procedural Opportunity

The fine-state representation in Equation 3 separates event records, claim records, authorizations, review proceedings, and provenance. A claim may pass through assertion, acknowledgment, dispute, provisional decision, discharge, or withdrawal. These classifications describe the *recorded* status assigned by a specified process. The classification does not establish the substantive correctness of the process, and an apparently discharged claim may remain the subject of an authorized appeal. Historical transitions matter because a present aggregate status need not answer which rule governed at the time of an act, whether an appeal was timely, or whether a particular recipient received notice.

For a protected item c , let v designate a version of the applicable register and a a role seeking information or review. A suitable exact-query family Q^{prot} can ask for c 's recorded status, the source and effective interval of an authorization, the forum competent to review a decision, the existence of a live review window, or the provenance of a prior change. The status query is defined only when the requester satisfies the stipulated access rule; an authorized refusal to disclose can be an exact response when its reason and review route are themselves recorded. A legitimate query need not release confidential evidence to every participant.

The interface proposed in Equation 18 distinguishes an informational answer from the operations through which the answer and relevant procedural opportunities remain accessible.

$$\mathcal{I}_{a,v}(s; c) = (\sigma_v(c), \kappa_v(c), \rho_{a,v}(c), \eta_{a,v}(c)), \quad (18)$$

Here σ_v is the versioned recorded status; κ_v identifies the accountable status-determining authority and applicable rule; $\rho_{a,v}$ is an access-controlled provenance resolution procedure; and $\eta_{a,v}$ specifies the available review or correction route, including deadlines when relevant. The tuple in Equation 18 is an institutional specification with possibly restricted entries, rather than a universal public data structure. When review standing is absent under the applicable regime, $\eta_{a,v}$ may report that reason and any authorized process for contesting standing. A proposed automated answer should preserve distinctions between a record being missing, the requester lacking access, and an actual finding that no obligation exists.

This representation suggests three independently assessable forms of preservation. *Status preservation* maintains exact answers to declared protected queries at the relevant version. *Provenance preservation* keeps an authenticated route to the evidence and reasons on which that answer rests, subject to permitted confidentiality restrictions. *Procedural preservation* maintains an effective means to invoke

Table 5: Rights-sensitive preservation requirements for a scale-dependent production-network representation. The listed tests are conditional on an explicitly specified institutional regime and reviewer role.

Analytical object	Required representation or process	Independent validation question
Material event	Unique occurrence, quantity, time, and source	Are duplicated or missing occurrences detected?
Claim status	Claim identifier, rule, dated status version	Is the authorized exact answer retained?
Rights and powers	Holder or principal, scope, delegation, expiry	Is a technical operator’s access distinguished from legal authority?
Historical memory	Status transitions, evidence, lawful correction trail	Can an authorized reviewer reconstruct the relevant history?
Review standing	Forum, qualifying party, deadline, remedy route	Can an eligible participant obtain meaningful review?
Confidentiality	Access policy, redaction grounds, audit of requests	Is disclosure limited while contestability is maintained?

recognized review, correction, or reconsideration under the relevant institution. The first can be checked as a property of an information map; the second also depends on integrity, record availability, and authorized resolution; the third depends on functioning institutions, intelligible notice, accessible submission mechanisms, and the real ability of affected parties to use them. A source-level pointer alone proves none of those broader institutional conditions.

Table 5 presents those conditions as separate checks. They can fail independently. A database might return exactly the correct recorded status while blocking the claimant from the evidence required to challenge it; it might preserve a complete evidential history while allowing a deadline to expire without adequate notice; or it might publish all records while endangering participants whose information is confidential. A proposed implementation must therefore be assessed at the level of both data transformations and lived or institutionally recognized procedure.

5.3 Cross-Scale Protection Certificates and Their Composition

Consider the partial map $\Pi_{01} : \mathcal{D}_{01} \subseteq \mathcal{S}_0 \rightarrow \mathcal{S}_1$ of Section 4.1. Let $Q_{a,v}^{\text{prot}}$ be a declared finite set of protected exact queries that role a is authorized to submit at version v . For each such query, a certified scale-one interface has a resolver $R_{q,a,v}^{01}$ and an authenticated information route, perhaps to an authorized fine-scale register. The exact-answer obligation is expressed in Equation 19.

$$Q_{q,a,v}(s) = R_{q,a,v}^{01}(\Pi_{01}(s)), \quad s \in \mathcal{D}_{01,q,a,v}^{\text{cert}} \subseteq \mathcal{D}_{01}. \quad (19)$$

In Equation 19, the coarse representation may contain a stable, authenticated pointer rather than the status text itself. The resolver is consequently an information *interface*: its required fine records and authorized access paths are part of the certified system, not information implicitly reconstructed from aggregate resource quantities. The domain records conditions on version freshness, identity resolution, access eligibility and source availability. A subsequent change in access rules or record availability can invalidate the certificate.

Exact factorization has a simple informational consequence: if two certified fine states have the same complete query-accessible coarse representation, then every declared protected exact query gives the same answer. This is a particular application of Proposition 2.1. Its converse is not an institutional sufficiency result. Even a logically adequate query partition may be implemented with inaccessible records, biased evidence, or unreviewable decisions. Error bounds on aggregate energy flows, such as those in Proposition 4.1, do not verify the operations of a real appeals forum.

The composition result in Proposition 5.1 records a conditional property of the informational interface. The conditions are deliberately explicit because a second aggregation can remove a pointer, change a role definition or obscure a relevant status version.

Proposition 5.1 (Composition of protected exact-query interfaces). *Let $\Pi_{01} : \mathcal{D}_{01} \rightarrow \mathcal{S}_1$ and $\Pi_{12} : \mathcal{D}_{12} \rightarrow \mathcal{S}_2$ be partial maps with composed domain $\mathcal{D}_{02} = \{s \in \mathcal{D}_{01} : \Pi_{01}(s) \in \mathcal{D}_{12}\}$. Fix a role a , version v , and a family $Q_{a,v}^{\text{prot}}$ of exact queries. Suppose for every $q \in Q_{a,v}^{\text{prot}}$ there exist resolvers $R_{q,a,v}^{01}$ and $R_{q,a,v}^{12}$ such that $Q_{q,a,v} = R_{q,a,v}^{01} \circ \Pi_{01}$ on the certified fine domain and $R_{q,a,v}^{01} = R_{q,a,v}^{12} \circ \Pi_{12}$ on its corresponding certified scale-one image. Suppose the composed interface retains the requisite version, identity, and role constraints and the resolvers are defined there. Then $Q_{q,a,v} = R_{q,a,v}^{12} \circ \Pi_{12} \circ \Pi_{01}$ on the jointly certified composed domain for every declared query.*

Proof. Fix q, a, v , and a fine state s in the jointly certified composed domain. The first factorization yields $Q_{q,a,v}(s) = R_{q,a,v}^{01}(\Pi_{01}(s))$. Because $\Pi_{01}(s)$ belongs to the second certified domain, the second factorization gives $R_{q,a,v}^{01}(\Pi_{01}(s)) = R_{q,a,v}^{12}(\Pi_{12}(\Pi_{01}(s)))$. Their equality establishes the required factorization for this query. The proof applies pointwise to each declared member of $Q_{a,v}^{\text{prot}}$. $\square$

Operational access at a review deadline. An exact answer may be recoverable by an authorized server even when the rights holder lacks a usable channel for challenging a decision. For a designated review-critical subset $Q_{a,v}^{\text{rev}} \subseteq Q_{a,v}^{\text{prot}}$, define $\text{Reach}_{q,a,v}^{\Delta_q}(z) = 1$ when, under the stipulated service and institutional conditions at coarse representation z , role a can authenticate, obtain the reasons and records to which it is entitled, and lodge a review request with an accountable recipient before its applicable deadline Δ_q . The indicator concerns the possibility of timely submission; it does not guarantee favorable disposition or an adjudication by a competent body. Write $R_{q,a,v}$ for the certified resolver at the chosen coarse scale. Equation 20 defines one bounded interface acceptance test for a fine state s and map Π .

$$\begin{aligned} \text{InterfaceOK}_{a,v}(s; \Pi) = & \bigwedge_{q \in Q_{a,v}^{\text{prot}}} [Q_{q,a,v}(s) = R_{q,a,v}(\Pi(s))] \\ & \wedge \bigwedge_{q \in Q_{a,v}^{\text{rev}}} [\text{Reach}_{q,a,v}^{\Delta_q}(\Pi(s)) = 1]. \end{aligned} \quad (20)$$

The conjunction in Equation 20 is a proposal-specific test, conditional on a valid role, a currently governing version, applicable deadlines, and a lawful description of accessible reasons and evidence. A procedure might impose additional requirements for effective interpretation, language access, independence, or remedy. Consider two implementations returning the same authenticated status *disputed*. In the first, an eligible claimant can retrieve the permitted reasons and file a challenge before the deadline. In the second, the server returns the status but the submission channel is unavailable until after the deadline. Both satisfy the first conjunction, while only the first satisfies the second. Thus exact-query composition in Proposition 5.1 supplies no theorem about practical procedural access. A composed workflow must separately certify live routing, authorization continuity, and time-to-deadline feasibility; two independently functioning components can still form an unusably slow combined route.

Proposition 5.1 is a formal preservation statement about specified query interfaces. It neither proves that a given institutional regime is just nor guarantees the operational resilience of its recordkeeping systems. If access to a fine register is lost, if a principal revokes a delegation, or if the relevant status version is misidentified, the proposition's domain conditions may cease to hold. A deployment audit would consequently need adversarial access tests, evidence checks, availability guarantees within permitted limits, and a procedure for suspending uncertified decisions. An unresolved claim can merit a preserved review route even when the resource quantity attached to it lies below a numerical approximation threshold.

5.4 Plural Appraisal and Decision-Relative Normative Orders

A rights-preserving information system still faces disagreement about the rules by which contributions, needs, historical obligations and future burdens should be appraised. Sen's comparative approach to justice emphasizes reasoned comparison, public reasoning, and the possibility of plural grounds for judgment [21]. Anderson's plural theory of value challenges the reduction of qualitatively different modes of

valuation to one market-scale measure [1]. These contributions motivate attention to the limits of scalar optimization, while neither supplies the particular AI production-network metric proposed in this paper.

Let J denote a finite collection of specified evaluative perspectives rather than a universal moral index. For each $j \in J$, a partial or complete preorder $\succeq_j$ compares eligible alternatives on dimensions justified by that perspective. One procedure may prioritize continuity of essential food deliveries; another may assign priority to valid maintenance entitlements; a third may assess ecological restoration or equitable access to productive infrastructure. The criteria may be differently weighted, noncommensurable, or dependent on incompatible judgments about who properly bears a burden. Treating their outcomes as a vector can make the disagreement visible without asserting an objectively given cardinal exchange rate between them.

A limited and explicitly declared agreement relation is stated in Equation 21.

$$x \succeq_J y \iff x \succeq_j y \text{ for each } j \in J. \quad (21)$$

Equation 21 defines a consensus-comparison test relative to the included perspectives. It can leave alternatives incomparable. It also presupposes that all perspectives are being applied to the same declared feasible-choice set and that the specified evaluators are relevant to the question. An information system that labels an alternative “optimal” should identify whether the label refers to a single perspective, a negotiated social-choice rule, an authorized priority hierarchy, or the relatively modest relation of Equation 21.

Consider two illustrative schedules with identical six-kilowatt-hour generation and equal first-period productive energy use. Schedule x gives priority to expedited food delivery and delays a replacement-part commitment that remains disputed. Schedule y allocates a feasible maintenance window and schedules the food delivery later, within its permissible deadline. A throughput-focused assessment may rank x above y , while a precautionary maintenance-focused assessment may rank y above x . A legal determination of the part claim could change which schedules are admissible independently of either ordinal ranking. Thus material equivalence, valid rights, and plural normative preference constitute different tests. Selection among conflicting admissible alternatives requires an authorized deliberative or decision procedure whose reasons and opportunities for challenge are themselves open to assessment.

A plural register should additionally retain *whose* evaluation was solicited and *whose* was absent. A convenient set of AI-generated scores can mask a missing community perspective, a resource steward whose permissions were never consulted, or a person affected by the network who cannot access the protocol. The theoretical availability of advanced AI can increase the range of reconstructions and counterfactuals considered; it does not confer legitimacy on the choice of stakeholders, the assignment of voting rights, or the resolution of irreducible disagreements.

5.5 Dispute Memory, Revision, and the Power of Recordkeeping

A dispute is historically situated. Its significance may extend beyond a single adverse outcome to the process through which an obligation was asserted, ignored, negotiated, provisionally decided, or later corrected. A scale reduction that reports only the latest status can remove evidence of repeated late payments, unequal treatment, systematically rejected submissions, or institutional learning. Conversely, indefinite unrestricted retention of all individual allegations can expose claimants to retaliation and facilitate surveillance. The proposed *dispute memory* is therefore an accountable, purpose-limited capacity for reconstructing legally or procedurally relevant transitions under valid access conditions, with specified correction, retention, and deletion rules.

One design would maintain a versioned event register for status transitions, each referring to a claim identifier, relevant instrument, acting authority, recorded time, reason code, and evidence link. The register may store redacted evidence or references to separately governed repositories where direct inclusion would violate valid restrictions. Corrections can supersede an erroneous entry while retaining, to the extent permitted, a controlled record that the correction occurred. Procedural memory also includes notice dates, appeal deadlines, requests for access, denials, and outcomes. This record structure is proposed for

auditability; the legality of retention, erasure, automated processing, and disclosure depends on jurisdiction and institutional setting. An authenticated system can still produce unjust decisions if its evidence standards or accessible review mechanisms are deficient.

Who controls the representation is itself part of the political economy of monetary dissolution. A protocol operator can determine which claims are indexable, which languages or classifications are accepted, which status transitions appear on dashboards, and which disagreements trigger costly fine-scale inspection. If a dominant interface makes claims invisible, apparent coordination efficiency may be achieved by transferring evidential and procedural burdens to less powerful participants. A concentration of abstraction and certification powers can therefore emerge even when physical resource exchange becomes less dependent on money. Such risks call for independent audit, separable decision and certification roles, tested appeal access, and transparent revision histories proportionate to the relevant stakes. These are candidate governance safeguards for evaluation, not inferred universal prescriptions.

5.6 A Two-Period Maintenance Dispute Across Three Scales

The illustrative energy circuit in Section 2.7 contains one generation occurrence, six kilowatt-hours available in period one, and dated allocations of two to agriculture, one to logistics and two to maintenance, with one left unused. It also contains a separately identified claim c_M for a replacement part whose validity, satisfaction, and consequences for period-two operations have to be determined under the stipulated rules. The following extension holds the numerical energy record fixed while changing the information that a governing procedure can recover.

At the *event scale*, record s^d marks c_M as disputed at version v , with a review window whose closing date is relevant to an affected party. Record s^p contains identical energy quantities and production events but marks the claim as provisionally decided, with a live appeal pathway. At an *operational interface scale*, both records report five kilowatt-hours used and one kilowatt-hour remaining; a maintained status token points to their separate claim records. At a *planning scale*, the quantitative aggregates remain identical while the token and its access-controlled resolver are available through a governed claim service. A status-free energy dashboard would merge s^d and s^p and fail the protected query that distinguishes them. A versioned resolver can preserve this declared question under the assumptions of Proposition 5.1.

The two records can nevertheless differ in their *procedural adequacy*. Suppose the planning interface reports the recorded status accurately but provides the affected party with neither the notice date nor the allowed appeal route, even though the governing rule confers such review standing. The exact-status test passes, while effective contestability can fail. A corrected interface would provide an authorized retrieval route to the deadline, the reasons for the provisional decision, and a working method of challenge, while respecting any justified limits on third-party evidence. An institutional evaluator would still need to determine whether the resulting procedure is sufficiently accessible, impartial and effective.

For a future food-delivery decision, material information also remains pertinent. The replacement part may affect next-period equipment availability, so aggregate first-period energy sufficiency cannot certify period-two production. A rights-sensitive forecast would distinguish a dispute over the part's entitlement from the engineering question whether the part can arrive in time. It might recommend temporary alternative capacity, a lawful provisional allocation, or deferral pending review, according to the permitted decision set. No single physical-energy scalar selects between these options. Their admissibility and legitimacy depend on current authority, evidential quality and the burdens placed on the affected participants.

Finally, consider an abstraction that retains the valid status of c_M but loses the record of a prior unauthorized denial of access to repair facilities. The present claim may have been corrected, yet the older event can remain relevant to a recurring-discrimination inquiry, institutional accountability or a permitted compensation claim. The appropriate retention period and disclosure policy require independent justification. The case illustrates why protected query sets are decision- and time-relative: a current-allocation query and a historical-accountability query have different information requirements. An AI reconstruction can help identify that difference, but authority to reopen the past and determine a remedy

resides with the applicable institution.

5.7 Institutional Acceptance Conditions for Rights-Sensitive Abstraction

The foregoing analysis provides bounded acceptance conditions for the proposed abstraction framework. A coarse description should be evaluated against a declared domain, authorized interventions, finite decision horizon, physical conservation and feasibility constraints, protected exact-query family, versioned provenance access, and procedures for correction and review. The appraisal should also record unresolved normative disagreement and identify the institutions empowered to decide which criteria apply. A certification of numerical accuracy is conditional on its metric and evidence; a certificate of exact-query preservation is conditional on accessible, authorized resolvers; and legitimacy of the overall coordination arrangement requires independent institutional assessment.

Three limitations remain particularly important. First, the selection of protected claims and eligible participants may itself reproduce existing inequalities or leave unrecognized interests beyond the system’s horizon. Second, even a correct recordkeeping architecture can have high costs or produce new concentrations of epistemic and administrative power. Third, changing technology and institutional norms can invalidate an earlier classification of the relevant rights, evidence or review channels. The capacity to revise the abstract representation and reopen the reasons for its use must therefore form part of an ongoing evaluation, with limitations explicitly disclosed rather than hidden by an apparent computational guarantee.

Rights-sensitive abstraction can consequently be described as a constrained research programme in which informational preservation, procedural opportunity, and plural justification are coupled objects of investigation. The next section examines the role that AI-assisted event reconstruction, uncertainty calibration and anomaly detection might play within those constraints. An effective recognition system could improve the evidence available for protected queries, while the authority to establish legal status and decide contested justice questions would remain institutionally specified and independently reviewable.

6 AI-Assisted Circuit Reconstruction and Error Calibration

The operational interpretation of normative renormalization depends on what an analyst can reconstruct from incomplete and institutionally differentiated records of productive activity. This section develops a provisional interface between event-level evidence, candidate circuit grammars, dynamical prediction, and the scale-dependent certificates introduced in Sections 4 and 5. Its objective is to specify what an AI-assisted reconstruction can report, how quantitative errors might be assessed, and where uncertainty or contested authority requires finer inspection. The method is a combination of set-valued reconstruction, conditional formal bounds, statistical calibration, and an illustrative two-period account. The section distinguishes identification of recorded events, inference of missing events, observationally compatible mechanisms, and authorized disposition of claims. Extensive data and highly capable AI are enabling assumptions in the thought experiment; the proposed pipeline has not been validated on an operating AI economy.

6.1 Observation Channels, Provenance, and Candidate Histories

A productive circuit has several layers of evidence. An energy meter records a quantity under a measurement convention; a timestamped receipt associates a transfer with an event identifier; an equipment sensor reports a condition subject to noise and delay; an authorization register records a delegated power at a particular version; and a claim register distinguishes an assertion, an objection, and a procedurally determined outcome. The operator controlling a database or sensor may differ from the beneficiary of a transfer and from the authority permitted to alter a claim’s status. Consequently, observational completeness and institutional accessibility are separate attributes of a reconstruction system.

Let $h \in H_T$ denote a finite, typed history on a declared period T , with unique executed-event identifiers, dated stocks, recorded capacity, and normative dossiers. Let v be the versions of the applicable registers and evidence schema. An observation channel $\mathcal{O}_v$ maps a history to available readings, timestamps, signed messages, record-access results, and declared missingness indicators. Since sensor faults, selective publication, delayed reporting, or an unobservable private exchange can cause distinct histories to yield the same observations, the channel may be many-to-one. A reconstruction method should therefore preserve a set of compatible histories before selecting an explanatory representative. Equation 22 defines a conditional compatibility set.

$$\mathfrak{H}(o; v, K) := \{h \in H_T : \mathcal{O}_v(h) \sim_K o, \quad h \models K_{\text{mat}}, \quad h \models K_{\text{time}}\}. \quad (22)$$

The relation $\sim_K$ specifies compatibility using declared measurement precision, tolerances, and evidence-authentication status. The independently declared material and temporal constraints are K_{mat} and K_{time} . When the observation channel is noisy, the relation permits a stated measurement-error region rather than literal equality. The set's use as an evidential certificate requires a defensible assumption that the true executed history is included. Equations derived from conservation laws may reject some candidates, yet an apparently consistent history can still have misidentified counterparties, forged evidence, or transactions beyond the observation boundary.

The construction distinguishes *observed events*, *inferred events*, and *candidate events*. An observed event has an authenticated record under declared evidential rules. An inferred event is supported by a model or constraint and carries identified uncertainty. A candidate event is grammatically expressible and may remain incompatible with a dated inventory or an authorization rule. A circuit recognizer from Paper III can label recurrent structures within any given history, while preserving the identity of shared occurrences. Repeated grammatical mentions of the same energy delivery do not generate additional physical energy. This is essential where longer candidate walks proliferate more rapidly than distinct executed events.

The compatibility set in Equation 22 is intentionally silent about the correctness of contested claims. The authoritative status of a versioned claim is a separate, authenticated observation when available. In the absence of a reliable status record, the reconstruction can report that status as *unresolved from available evidence* and request access through an authorized procedure. A probability assigned to contractual validity by a language model cannot replace the relevant authority's determination. Likewise, access restrictions may be legitimate even when unrestricted records would improve predictive accuracy. Responsible reconstruction requires logging the provenance and lawful scope of each evidence request.

6.2 Structural Inference, Dynamical Identification, and Causal Questions

At the structural level, an AI system may compare transaction identifiers, deliveries, interface types, timestamps, and shared inputs to construct a typed candidate multigraph. Unresolved duplicates, incompatible units, broken provenance chains, and impossible time orderings become auditable exceptions. At the dynamical level, the question is different: which conditional law predicts how inventories, productive capacity, and participation evolve under a declared intervention? Established research on sparse identification illustrates how selected dynamical equations can sometimes be recovered from measurements under model assumptions [3]. Research on nonlinear-network observability separately addresses whether an assumed system's internal states can in principle be distinguished through measurements [14]. Neither establishes identification of a complete economic network or the institutional meaning of its relations.

For an exploratory model, write x_t for a declared physical and operational state, $a_t \in \mathcal{I}$ for an authorized intervention under examination, and u_t for unobserved disturbances. The distinction between the structural transition and its learned surrogate is recorded in Equation 23.

$$x_{t+1} = F(x_t, a_t, u_t), \quad \hat{x}_{t+1} = \hat{F}_\theta(\hat{x}_t, a_t), \quad t = 0, \dots, T-1. \quad (23)$$

The forecast $\hat{F}_\theta$ approximates a defined target over a declared evaluation domain and need not identify F uniquely. Some state variables, such as available spare parts, may be measured intermittently;

others, such as participant confidence or willingness to cooperate, may require a historically and contextually qualified interpretation. An estimated transition can be valuable for scheduling without thereby establishing a causal explanation. Interventions may change authorization, participants' behavior, prices, or information disclosure, moving the system outside the data-generating regime. Scholarship on causal representation learning emphasizes these general identification and transfer questions [20]; the present economic and institutional application is a new proposal.

An inference system should consequently attach a declared task to each output: retrospective accounting, descriptive anomaly identification, conditional forecasting, counterfactual comparison under an explicit causal model, or a suggestion for institutional review. The standard of evidence rises when the output affects an individual's access to essential goods or their ability to contest a determination. An observational correlation between a maintenance request and a later decline in food production, for example, leaves alternative explanations involving equipment age, delivery delays, energy constraints, and shared bottlenecks. A model-assisted counterfactual requires its own assumptions about those mechanisms and cannot inherit causal authority from predictive accuracy alone.

6.3 Certified Numerical Envelopes under Historical Ambiguity

Point predictions obscure distinctions among histories compatible with the same available records. For a declared scalar material query $g : H_T \rightarrow \mathbb{R}$, the analyst may instead report a range of values over $\mathfrak{H}(o; v, K)$. Equation 24 defines the compatible-history envelope whenever the set is nonempty and the extrema are finite. Infima and suprema allow a noncompact candidate space; computable approximations would need their own optimization certificates.

$$I_g(o; v, K) := \left[\inf_{h \in \mathfrak{H}(o; v, K)} g(h), \sup_{h \in \mathfrak{H}(o; v, K)} g(h) \right]. \quad (24)$$

Proposition 6.1 (Conditional inclusion of an executed-history query). *Suppose $\mathfrak{H}(o; v, K)$ in Equation 22 is nonempty, g is real-valued and bounded on that set, and the actual executed history h^* belongs to $\mathfrak{H}(o; v, K)$. Then $g(h^*) \in I_g(o; v, K)$ as defined in Equation 24. If a coarse numerical estimator $\hat{g}$ is certified to satisfy $|g(h) - \hat{g}(\Pi(h))| \leq \varepsilon$ for every h in this compatible set, the actual query additionally satisfies the same bound at h^* .*

Proof. Membership of h^* in the compatibility set implies that its query value lies between the infimum and supremum defining I_g . The second statement follows by applying the stipulated uniform coarse-estimation inequality to that particular member. $\square$

Proposition 6.1 makes a modest but useful distinction. The envelope is a *conditional identification region*, whose validity depends on the true-history inclusion premise. It is not a confidence interval with a stated frequentist coverage probability. An incomplete ledger, misspecified conservation equation, manipulated timestamp, or incorrectly excluded candidate event can invalidate inclusion. A wide but supported range may therefore be a more defensible result than a narrow and unsupported point estimate. The distinction also applies to an empty compatibility set: inconsistency among recorded constraints calls for an audit of evidence and assumptions, rather than a manufactured numerical answer.

Joint evidential coverage and approximation. When the observation record and a fitted coarse estimator are random, two separate assertions may be available: coverage of the executed history by the compatibility set, and validity of a uniform approximation certificate on that set. These assertions need not be independent. The following result states what their declared error probabilities can establish, without converting the set-valued identification region into an unconditional guarantee.

Proposition 6.2 (Joint coverage of a reconstructed numerical query). *Fix a probability law $\mathbb{P}$ over the generation of an executed history h^* , observations o , and any randomness of the fitted coarse estimator. Let $E_H = \{h^* \in \mathfrak{H}(o; v, K)\}$, and let E_C be the event that $\mathfrak{H}(o; v, K)$ is nonempty and the numerical*

error inequality $|g(h) - \hat{g}(\Pi(h))| \leq \varepsilon$ holds uniformly for all members of that set, with its query bounds finite and Π and $\hat{g}$ defined throughout that set. Treat the joint assertion as false whenever its envelope or estimator is unavailable. If $\mathbb{P}(E_H) \geq 1 - \alpha_H$ and $\mathbb{P}(E_C) \geq 1 - \alpha_C$, then the joint numerical assertions in Equation 25 hold with probability at least $\max\{0, 1 - \alpha_H - \alpha_C\}$.

$$\begin{aligned} \mathbb{P}(g(h^*) \in I_g(o; v, K), \\ |g(h^*) - \hat{g}(\Pi(h^*))| \leq \varepsilon) \geq \max\{0, 1 - \alpha_H - \alpha_C\}. \end{aligned} \quad (25)$$

Proof. On $E_H \cap E_C$, Proposition 6.1 applies to the realized compatible set and gives both assertions in Equation 25. The union bound gives $\mathbb{P}(E_H \cap E_C) \geq 1 - \mathbb{P}(E_H^c) - \mathbb{P}(E_C^c) \geq 1 - \alpha_H - \alpha_C$. The probability is nonnegative, which yields the displayed lower bound. $\square$

The probability in Proposition 6.2 is meaningful only with a defensible generating law, measurement process, and certification procedure for the two stated events. Coverage inferred from an exchangeable historical sample cannot automatically be transferred to strategically concealed records, novel interventions, or an altered authorization regime. The union bound does not assume independence; it may be conservative when failures coincide. Neither event supplies a probability that a claimant's recorded legal status is correct, and neither verifies that a role can exercise its review rights. Operational access is tested separately in Equation 20. Where E_H is uncalibrated, the deterministic proposition remains a conditional statement and no numerical coverage probability follows.

In some applications g itself is a forecast depending on an unobserved disturbance process or an intervention. The compatibility set must then include the relevant uncertainty about transition models, rather than only reconstruct past events. Section 3 provided bounds for a declared weighted-path surrogate; such a bound can contribute to an error budget for a specific query when its assumptions hold. It does not confer a uniform accuracy guarantee on a different query such as next-period equipment failure or the legal validity of a request. Material accounting, model uncertainty, truncation, and claim status retain separate validation channels.

6.4 Prediction Calibration, Drift, and Error Budgets

Statistical calibration addresses the relation between reported predictive uncertainty and observed outcomes over a specified evaluation population. Guo and colleagues analyze calibration of neural-network classification confidences and practical post-processing methods in the datasets they study [8]. Angelopoulos and Bates describe conformal prediction methods that can construct prediction sets with finite-sample *marginal* coverage guarantees under exchangeability-related assumptions [2]. These precedents are relevant to measurable outputs of an AI production system, such as next-period delivery volume, while leaving institutional correctness and substantive fairness for separate examination.

One illustrative split-conformal procedure fixes a predictor using training observations independent of a calibration sample. Let $r_1, \dots, r_n$ be absolute residual scores for exchangeable calibration pairs and let $r_{(k)}$ be their k th order statistic, with a convention of an infinite threshold when $k > n$. Equation 26 defines a proposed interval for a new prediction $\hat{y}(x)$.

$$\begin{aligned} k &= \lceil (n+1)(1-\alpha) \rceil, & q_\alpha &= r_{(k)}, \\ C_\alpha(x) &= [\hat{y}(x) - q_\alpha, \hat{y}(x) + q_\alpha]. \end{aligned} \quad (26)$$

Under the conventional exchangeability and scoring conditions, the associated finite-sample theorem yields marginal coverage at least $1 - \alpha$ for a fresh exchangeable test pair. The guarantee concerns outcomes drawn under that sampling regime. It does not give uniform coverage for every rare claim type, every participant, or every future regime. In highly coupled time series, indiscriminate reshuffling can defeat the relevant assumption. A deployed system should therefore specify calibration periods, spatial grouping, dependence diagnostics, error-rate reporting by relevant subgroup where legitimate, and recalibration or abstention triggers. Reported predictive performance should distinguish calibration

error, interval width, operational loss, and decision consequences. Coverage cannot substitute for a decision about who is permitted to impose the modeled action.

For an entirely separate deterministic certification setting, let E_{recon} , E_{model} , E_{trunc} , and E_{scale} be valid *pathwise upper bounds* for a fixed query and horizon, under mutually compatible definitions and a traceable chain of intermediate targets. The triangle inequality gives the accounting statement in Equation 27.

$$|y^* - \hat{y}^{\text{coarse}}| \leq E_{\text{recon}} + E_{\text{model}} + E_{\text{trunc}} + E_{\text{scale}}. \quad (27)$$

Each term in Equation 27 needs a documented intermediate quantity and valid bound. In particular, a confidence interval or conformal prediction radius cannot silently be added as though it were a deterministic error certificate. If a constituent claim is probabilistic, one must state an event on which the component bound holds and account for the joint failure probability, for example through a union bound across declared events. Overlapping sources of uncertainty may make an additive upper bound conservative; treating correlated failures as independent can make a purportedly tighter bound invalid. The analyst should also distinguish sensor-level noise, missing-not-at-random data, model misspecification, intervention shift, and contested normative status. These are different kinds of uncertainty with potentially different remedies.

6.5 Rare Obligations, Anomalies, and Review-Sensitive Exceptions

A model that optimizes average material prediction error may pay little attention to a rare but consequential omission. A single undisclosed shared-energy bottleneck can invalidate parallel-route reliability claims. A low-frequency equipment claim can carry a valid deadline and procedural right even when its associated material throughput is negligible compared with the network total. Anomaly detection may help discover inconsistencies in record continuity, event identifiers, material balances, service times, and reported authorizations; it cannot independently decide the legal or ethical standing of the detected object.

The proposed workflow therefore maintains two distinct escalation channels. The *material exception channel* flags missing or inconsistent meter readings, impossible inventories, unsupported timing, threshold-sensitive capacity, and departures from a certified intervention domain. The *normative exception channel* preserves named claims, disputed or unknown statuses, versions, notice deadlines, access restrictions, and evidence needed for authorized review. A rare assertion need not be classified as invalid because the predictive model assigns it low likelihood, and a high anomaly score does not justify an automated sanction. A review system may require a more restrictive evidence-access policy than a material forecasting system; the ability to retrieve records and an authorized procedure for correcting them remain relevant acceptance tests.

Table 6 summarizes the analytical tasks and their distinct forms of verification. It is a proposed research workflow, not a factual account of existing production infrastructure.

The separation in Table 6 also guards against misplaced authority. Automated discovery can suggest that one period’s energy balance has an unaccounted unit; a competent institution determines whether that discrepancy warrants investigation and whether disclosure of related records is permissible. The same institution may be an inappropriate sole auditor if it controls the measurement protocol or benefits from the omission. Independent scrutiny and opportunities to challenge classification rules are part of the proposed governance interface rather than consequences of predictive accuracy.

6.6 Two-Period Energy and Maintenance Reconstruction

The common worked example in Sections 2.7 and 5.6 begins with six kilowatt-hours available in period one. Authenticated deliveries allocate two to agriculture A , one to logistics L , and two to maintenance M ; a complete record shows one kilowatt-hour unused. For the present *partial-observation variant*, suppose the period meter records six kilowatt-hours and the three delivery receipts establish the five executed kilowatt-hours, while the disposition record for the remaining one kilowatt-hour is missing.

Table 6: AI-assisted reconstruction tasks, proposed outputs, and independent conditions for accepting the outputs in a rights-sensitive production-network analysis.

Task	Proposed AI-assisted output	Independent validation or escalation condition
Record linkage	Typed event graph with provenance and duplicate resolution	Verified identifiers, units, timestamps and accountable correction
Historical ambiguity	Compatible-history set or query envelope	Truth-inclusion assumptions, complete constraint inventory and evidence access
Operational forecast	Dated quantitative prediction with uncertainty report	Declared intervention domain, dependency-aware validation and drift checks
Scale certification	Effective parameters and bounded material-query error	Certified reconstruction, model, truncation and scale terms
Claim monitoring	Named statuses, missing records and review exceptions	Authorized status source, procedural standing, access and version control
Governance support	Inspectable alternatives and requests for human/institutional review	Competent authority, plural appraisal, reversibility and contestability

The observed residual equals one kilowatt-hour, but it is compatible with multiple completions: unused capacity, measured loss under a specified accounting convention, or an additional delivery with a missing receipt. The analyst retains the candidate statuses and requests an inventory or meter reconciliation before certifying the unused amount. It would be incorrect to add one unit of inferred use to the original fully observed account as though both scenarios described the same verified history.

Assume that a versioned register independently authenticates claim c_M as *disputed*, with a recorded request for a replacement component before period two. Its status is preserved exactly in the protected-query interface; no forecast or imputed energy flow can convert that status into a determination of validity. Suppose, for illustration, that a specified engineering model assigns period-two deliverable food capacity of seven units when the component is timely installed and four units when the required maintenance is absent, all other modeled inputs held fixed. The current evidence does not establish timely installation. On these stipulated alternatives, the compatible-history envelope for next-period capacity is $[4, 7]$ units. That bound is conditional on the completeness of the alternatives and the correctness of the engineering model. Other equipment failures or substitutes would require an expanded set of scenarios.

An AI might separately predict six units of food delivery with a calibrated interval under a maintained observational distribution. For an illustrative calibration sample of size $n = 19$ and nominal miscoverage $\alpha = 0.1$, Equation 26 uses the eighteenth ordered residual. If that residual happened to be 0.8 units, the resulting interval around six would be $[5.2, 6.8]$. Those numerical choices are stipulated and do not amount to an empirically calibrated result. Moreover, the interval’s marginal coverage statement and the $[4, 7]$ conditional engineering envelope answer different questions and rely on different premises. A regulator or operator should not select whichever interval is shorter without examining its coverage domain and underlying uncertainties.

The example separates four records that might otherwise be merged: the observed first-period energy balance, the unresolved disposition of one unit, the forecast of second-period production, and the authenticated *disputed* claim status of c_M . The first can be reconciled by event-level evidence; the second may require fine-scale inspection; the third calls for model calibration and intervention-domain checks; the fourth requires preserved provenance and an authorized dispute process. An apparent financial saving from simplifying the energy schedule cannot settle the legitimacy of the claim or establish successful monetary dissolution. It also cannot establish ecological renewal, which would require a distinct dated account of the relevant environmental capacity.

6.7 Research Validation and Institutional Limits

The proposed AI-assisted reconstruction system can be evaluated through controlled falsification tasks before any institutional use. One task would construct fully observed synthetic histories with typed event identities, intentionally hide selected receipts, and test whether the reconstructed ambiguity set contains the hidden truth under its stipulated noise model. A second would inject correlated energy outages and shared-maintenance bottlenecks, testing whether an algorithm recognizes its failure to certify independence or future feasibility. A third would vary the authorization-register version while fixing the material ledger, testing whether exact-status queries detect the difference. A fourth would stratify prospective forecast errors by operating regime and query to reveal instability in aggregate calibration statistics. Such exercises would evaluate a specified model and test environment, not prove universal identifiability or justice.

Institutional evaluation adds questions inaccessible to prediction tests alone. Relevant participants need an intelligible account of which evidence the AI used, the rules authorizing collection and access, the means of challenging a mistaken event match, and the procedure for appealing or revising a material decision. Long-running systems require versioned reports of sensor changes, protocol revisions, newly excluded states, and departures from their original certificate domain. The adequacy of retention and transparency must be considered alongside confidentiality, cybersecurity and legitimate interests of affected subjects. If claim ownership, legal personhood, or standards of fairness remain contested, those issues enter the institutional forum as such. An AI can document the competing characterizations and assist with fact-finding without treating its own forecast as an institutional judgment.

Section 6 thus provides an epistemic interface for subsequent governance analysis. A reconstruction can be conditionally certified for a bounded material query, or supplied with measured statistical uncertainty under a defensible sampling model, while still requiring exact-status protection and accessible procedures for unresolved claims. The central empirical question is whether a real reconstruction pipeline can meet these separate tests at feasible cost under changing, heterogeneous production networks. The next section examines which planning and protocol-adaptation powers could justifiably be delegated to such a system, and how those powers could remain reviewable and reversible.

7 AI-Assisted Governance and Protocol Adaptation

A scale-dependent model of productive value circuits becomes institutionally consequential when its analyses influence the rules by which operators obtain resources, assign priorities, discharge commitments, or coordinate emergencies. This section examines the governable interface between AI-assisted reconstruction and the legitimate adoption of revised protocols. Its objective is to formulate a bounded procedure for proposing, authorizing, evaluating, executing, monitoring, and revising cooperative rules in a hypothetical AI-operated production network. The analysis combines a typed delegation model, a conditional robust-feasibility proposition, a plural decision procedure, and a two-period maintenance example. It distinguishes information furnished by AI from the authority to change institutional obligations. The procedure is a proposed research construction, not an observed governance system or a self-validating criterion of justice.

7.1 Delegated Authority and the Object of Protocol Change

A production network may maintain separate protocols for dispatching energy, reserving repair capacity, certifying deliveries, publishing evidence, and hearing appeals. A protocol change can alter the order in which operators receive scarce inputs, the evidence needed to complete a transfer, the circumstances under which emergency use is permitted, and the institution that reviews disputed claims. Those changes have different legal and normative standing. An update to a forecast model may be an authorized technical maintenance operation; withdrawal of an entitlement or the cancellation of a contested obligation can require a distinct procedure and cannot be justified merely by a lower estimated computational cost.

Let v identify a complete, versioned protocol and let $s = (x, \mathcal{C}, \mathcal{U}, \mathcal{V}, \mathcal{P})$ be a typed operative state. The physical component x contains dated stocks, capacities, and productive conditions. The other components hold commitments and claims, authorizations, review standing, and evidence provenance, in the sense of Section 2. The state is intentionally not a homogeneous numerical vector. Define $A_v(s)$ as the set of actions recorded as permissible for specified principals, agents, circumstances, and times, subject to any higher-order institutional limits. An AI operating a machine may be an execution agent under such a delegation even when the rights holder and the reviewing institution are human or collective bodies. The framework leaves possible AI legal and moral standing unresolved.

An AI-produced protocol proposal is represented by a versioned dossier. Equation 28 lists a minimal research schema rather than a universal legal form.

$$\rho = (v, v', D_\rho, \mathcal{I}_\rho, Q_\rho, \mathcal{E}_\rho, \mathcal{R}_\rho, \tau_\rho). \quad (28)$$

Here v' is the proposed successor version, D_ρ its domain of applicable histories and operating circumstances, $\mathcal{I}_\rho$ its action/intervention class, Q_ρ its protected numerical and exact queries, $\mathcal{E}_\rho$ its evidence and error certificates, $\mathcal{R}_\rho$ its review, challenge, and fallback provisions, and τ_ρ its expiry or reassessment rule. The dossier must identify the people or institutions whose authorization is needed, their scope of competence, and the procedure for resolving objections. A cryptographic signature may help authenticate a recorded decision; it does not establish that the signatory possessed the relevant institutional power or exercised it legitimately.

The admissible roles should be differentiated: a *proposer* constructs alternatives, a *verifier* inspects their certificates, an *authorizer* applies the controlling institutional rules, an *executor* performs approved actions, and a *reviewer* examines disputes or procedural errors. One organization may lawfully hold several roles, but their consolidation introduces identifiable conflict-of-interest risks. Hohfeld's distinctions among powers and claims [10] and Schlager and Ostrom's account of differentiated resource rights [19] provide useful conceptual comparisons. They do not specify the allocation of these roles in any particular AI production regime. Nor does technological independence of an AI model imply that its operator is an institutionally independent reviewer.

7.2 Constrained Proposals, Forecasts, and Protected Questions

A proposed allocation schedule can be screened against material conservation, date-dependent capacities, permitted interventions, and the availability of authentic evidence. The candidate search may employ a fine event history or one of the partial abstraction maps of Section 4. A coarse candidate enters the subsequent approval stage only under a query-specific certificate valid for the affected horizon and intervention. A convenient operational score, such as forecast food delivery or average maintenance delay, remains a descriptive or planning observable. The admissibility conditions include independent institutional questions about claim status, delegated powers, affected stakeholders, and opportunities to challenge the proposed schedule.

For an observed record o_t and active protocol version v , let $\mathfrak{H}_t = \mathfrak{H}(o_t; v, K)$ be the compatible-history set of Equation 22, assumed nonempty for the certificate under discussion. Let $U_v(h, a)$ denote a specified set of disturbances or unresolved material conditions after action a in compatible history h . Define a *robustly screened action* relative to the declared model and safety domain through Equation 29.

$$\begin{aligned} A_v^{\text{scr}}(\mathfrak{H}_t) := \{ & a \in \mathcal{I}_v : a \in A_v(h) \quad \text{for all } h \in \mathfrak{H}_t, \\ & F_v(h, a, u) \in K_v \quad \text{for all } h \in \mathfrak{H}_t, u \in U_v(h, a) \}. \end{aligned} \quad (29)$$

The state-transition notation $F_v(h, a, u)$ in Equation 29 denotes the successor *operative state* given the end-state of a compatible dated history h ; it does not identify a complete historical record with an instantaneous state. The set K_v contains explicitly selected, version-specific operational constraints, such as nonnegative inventory, minimum reserve, and conditions on protected-record handling. Its definition cannot by itself settle whether a minimum reserve or distribution rule is morally adequate. Checking the

universal quantifiers requires suitable finite encodings, sound relaxations, or other defensible certificates; an AI assertion that a plan is safe is not a certificate of the predicate.

Different purposes warrant different screening regimes. A mandatory reserve may require a worst-case guarantee over a declared uncertainty set; an exploratory production forecast can be presented with a statistical interval and its assumptions. Distribution-free marginal coverage claims from conformal methods, where applicable [2], do not deliver per-action worst-case feasibility for each claimant or each network regime. Likewise, a certificate of approximate numerical preservation cannot substitute for an exact answer about whether an authorization is current or a dispute is pending. For those queries, an implementation needs the authenticated resolver and versioned access routes defined in Sections 4 and 5.

The screening stage may produce several admissible alternatives, none of which possesses a uniquely justified overall score. A proposal should report constraints assumed, rejected alternatives, unresolved assumptions, expected beneficiaries and burdens, and the reversibility of any physical consequences. Interventions involving irreversible disposal, forced deprivation, or changing legal standing require stronger authorization than switching among genuinely optional delivery windows. A technically feasible solution therefore has the status of an inspectable proposal awaiting the relevant institutional decision.

7.3 Conditional Safety of Monitored Protocol Execution

A modest formal guarantee is possible when an authorized control mechanism checks its domain before each action. The result is a controlled-invariance statement conditional on the reliability of the compatible-history model and the enforceability of its execution gate. It is deliberately narrower than a claim that automated governance is legitimate or safe in general.

Proposition 7.1 (Conditional preservation under a monitored action gate). *Fix a protocol version v and a set of operative states K_v . Consider a finite execution sequence $t = 0, \dots, T$ with actual states s_t . Suppose: (i) $s_0 \in K_v$; (ii) at each $t < T$ the evidence model provides a nonempty set $\mathfrak{H}_t$ containing a history whose terminal operative state is s_t ; (iii) the execution gate accepts only a duly authorized action $a_t \in A_v^{\text{scr}}(\mathfrak{H}_t)$; (iv) the true disturbance u_t belongs to $U_v(h_t, a_t)$ for at least one such history h_t ending in s_t , and the actual successor equals $F_v(h_t, a_t, u_t)$; and (v) the gate cannot be bypassed by the modeled executor. Then $s_t \in K_v$ for $0 \leq t \leq T$. If the construction of K_v and the transition rule additionally enforce preservation of a declared family of protected-record queries, those particular query properties persist for the modeled sequence, subject to any explicitly authorized status transitions.*

Proof. The initial state satisfies $s_0 \in K_v$. Assume $s_t \in K_v$ for some $t < T$. Hypotheses (ii)–(iv) provide a compatible history h_t ending in s_t , a screened action a_t , and an included disturbance u_t . Equation 29 therefore places $F_v(h_t, a_t, u_t) = s_{t+1}$ in K_v . The gate condition ensures that the modeled execution follows this transition. Finite induction establishes the first conclusion. The statement about protected queries follows only if their persistence is among the declared state-transition invariants, with the allowed authenticated changes specified in advance. $\square$

Proposition 7.1 isolates what a verified action gate can establish *within* its assumptions. It gives no guarantee when the true history is excluded from $\mathfrak{H}_t$, the disturbance exceeds U_v , a hidden transfer bypasses the gate, the safe set omits a relevant ecological dependency, or the institutional permissions are incorrectly recorded. A persistent record of a dispute, even when exact, is different from a finding that the dispute has been adjudicated fairly. Computational tractability can also restrict the screening to conservative subsets of feasible actions; a planner must report this loss of options rather than present the screening set as the entire space of legitimate arrangements.

A change from v to v' requires its own migration assessment. A new protocol may legitimately alter operational capacities or the classification of protected records, and a state safe under v need not remain within $K_{v'}$. For a permitted migration map $M_{v,v'}$, Equation 30 specifies two separate acceptance conditions on a declared transition domain $D_{v,v'}$.

$$\begin{aligned} M_{v,v'}(s) \in K_{v'}, \quad q_{v'}(M_{v,v'}(s)) &= T_{v,v'}^{\text{auth}}(q_v(s)), \\ s \in D_{v,v'}, \quad q &\in Q_{\text{protected}}. \end{aligned} \tag{30}$$

Here $\tau_{v,v'}^{\text{auth}}$ is the authorized change in query semantics or status under the applicable procedure; for a protected unchanged claim it is the identity. The first condition is material and operational; the second protects declared status and procedural semantics. Version compatibility alone does not confer lawful authority to change a claim. Equally, an authorized change of claim status does not demonstrate the physical feasibility of the associated resource transfer. An audit should preserve the pre-migration record, decision basis, identity mappings, and accessible review channel.

7.4 Plural Appraisal and Institutionally Justified Prioritization

Formal screening often leaves multiple schedules. A network may choose between earlier food delivery, preventive equipment replacement, ecological repair, and a buffer against future energy interruptions. Their benefits and burdens can fall on different participants, times, and kinds of productive capacity. An AI planner can enumerate tradeoffs, identify Pareto-undominated alternatives under selected metrics, and disclose assumptions about scenario probabilities. The choice of which participants count, which rights are non-negotiable, and how burdens may be traded requires a separate normative and institutional appraisal.

One possible representation maintains a finite family of declared evaluators $\mathcal{J}$. Each evaluator j supplies a context-specific partial comparison $\succeq_j$ over a defined set $\mathcal{F}$ of technically feasible and institutionally admissible proposals. Equation 31 records a conservative unanimity relation useful for exposing, rather than resolving, plural disagreement.

$$a \succeq_{\mathcal{J}} b \iff a \succeq_j b \text{ for each } j \in \mathcal{J}, \quad a, b \in \mathcal{F}. \quad (31)$$

The relation in Equation 31 can be incomplete: some feasible choices remain incomparable. Its use assumes each participating comparison is well-defined for the options concerned. It has no automatic majoritarian, constitutional, or distributive authority. The selection of $\mathcal{J}$, rules for abstention, and treatment of absent or unrepresented affected subjects are prior political questions. Amartya Sen’s account of comparative public reasoning is a relevant philosophical reference for resisting premature reduction to a single maximand [21]; the present intersection relation is a newly stipulated formal device, not Sen’s formula.

Institutions can adopt a transparent method for cases left incomparable: reserve scarce capacity for urgent needs under an expressly enacted priority rule, rotate nonurgent slots, appoint an authorized mediator, or require consent among specified parties. Any such method must disclose its scope, exception procedure, and conditions for review. In a network whose operational labour is conducted largely by AI, participation may include human beneficiaries, trustees of common resources, organizations controlling infrastructure, and possibly AI subjects should relevant standing be recognized. The model does not determine who qualifies by computational ability. Treating a model-generated consensus among several AI agents as social authorization would silently assume the legitimacy of their mandates and representation.

Elinor Ostrom’s work on polycentric governance provides a comparative institutional perspective on multiple centers of collective decision-making [17]; it does not imply that fragmented administration automatically eliminates domination. A system may decentralize energy dispatch while centralizing protocol updates or dispute certification. Independent mechanisms for challenging an unfavorable classification matter where the certifier benefits from the classification. The proposed governance architecture therefore records both the locations of decision power and the means by which affected parties can question its use. Normative disagreement may persist even after these procedures are implemented.

7.5 Monitoring, Reversibility, and Protocol-Version Accountability

Protocol adaptation occurs over time. The institution may authorize a pilot for a limited period, learn from its consequences, and then retain, revise, or revoke the delegation. A proposed implementation needs indicators for boundary-stock errors, missed deliveries, claims awaiting review, calibration drift, and observed concentration of authority. These indicators serve different roles: a rising forecast error can

Table 7: Stages of AI-assisted protocol revision and the independently required institutional checks. The table describes a proposed research protocol rather than an operational standard.

Stage	AI-assisted or computational output	Independent permission, evidence, and procedural requirement
Problem identifica- tion	Dated shortages, drift indicators, unresolved commitments	Evidence access and classification rules; identification of affected subjects
Candidate design	Feasible schedules, uncertainty sets, effective-scale maps	Scope of delegated action and protected-query selection
Technical screening	Robust material constraints, sensitivity and error certificates	Authenticated measurements and valid operating domain
Institutional adoption	Inspectable reasons, alternatives, estimated burdens	Competent authorization, plural appraisal, recorded objections
Execution and moni- toring	Action ledger, deviations, exact-status audit and alerts	Gate enforceability, lawful status changes, independent review
Revision or suspen- sion	Updated versions, fallback options and discrepancy reports	Feasible rollback, procedural appeal and remedies for realized harm

justify model recalibration, while discovery of an unreviewed claim-status mutation can justify suspension or institutional investigation. Their thresholds and consequences must be specified by accountable authorities before operation, subject to lawful emergency exceptions.

Define an audit record for executed or rejected changes as a linked collection of the preceding protocol version, signed authorization, tested uncertainty domain, affected observables, certificate versions, proposed and actually executed actions, material outputs, status-preserving query results, objections, and review outcomes. When legal confidentiality limits broad disclosure, a designated authorized reviewer may need a protected retrieval route rather than publication of every primary record. The possibility of access control also requires the research programme to ask whether nominally available remedies are usable in practice, including their cost, language, latency, and independence from the authority under challenge.

Operational reversibility must be distinguished from *institutional reversibility*. A delivery priority can sometimes be reset before execution; energy already consumed, irreversible soil damage, or an emergency deprivation may be impossible to undo physically. A disputed status change may be corrected through an authorized procedure while leaving consequences requiring a separate remedy. A fallback protocol therefore requires a versioned, feasible operational plan, sufficient reserve or alternative capacity where relevant, and a lawful route for reinstating previously recognized claims or obtaining institutional review. Returning the software configuration to an earlier commit is insufficient when the earlier material state has disappeared.

The National Institute of Standards and Technology’s AI Risk Management Framework distinguishes organizational governance, contextual mapping, measurement, and management of AI risk, emphasizing context-sensitive and continuing risk practices [15]. This voluntary risk-management precedent neither certifies the hypothetical protocol architecture nor settles the public-law conditions of delegation in an AI-operated production network. The proposed procedure adds domain-specific concerns about scarcity, simultaneous claims on productive capacity, and formally protected access to unresolved obligations. A governance review should document where technical monitoring failed to detect a socially or legally significant consequence, since an unchanged quantitative score can coexist with newly discovered abuses of institutional power.

Table 7 separates performance evidence from the exercise of institutional power at each stage. The stages can overlap in a real process; the order represents a minimal analytic decomposition. The table also makes explicit where an apparently well-performing model might be operating outside its delegated domain. An institution that originally authorized demand forecasting may not have delegated authority to modify resource rights, even if the same AI system can technically recommend both actions.

7.6 Two-Period Maintenance Protocol and Contested Claim

Consider the illustrative system employed in Section 6.6. In the first period, a verified meter records six kilowatt-hours of available energy. Authenticated receipts record two delivered to agriculture A , one to logistics L , and two to maintenance M . The remaining unit has an unresolved physical disposition in the partial-observation variant, consistent with the alternative interpretations considered in that section. A separate authenticated register describes c_M , a claim regarding a replacement component before period two, as *disputed*. For this example, assume the operative protocol permits adjustments to future delivery times but does not authorize the dispatch system to discharge c_M , deny an appeal, or compel the transfer of another participant's materials.

The AI planner identifies two possible second-period coordination proposals. Proposal a expedites the food-delivery route while deferring an optional maintenance slot; proposal b reserves an authorized service window for equipment checks and reschedules a nonurgent delivery inside its applicable deadline. Assume, as stipulations rather than empirical estimates, that the engineer's conditional capacity model gives seven units of food when an essential replacement is timely installed and four when it is absent. Neither the five recorded first-period deliveries nor the one-unit residual proves whether that replacement will occur. Accordingly the planner must retain the Section 6.6 capacity envelope $[4, 7]$ when the evidence has not narrowed the admissible alternatives.

Suppose a specific protected service rule requires at least four units of second-period food capacity within the modeled horizon. Under the complete stipulated set of two capacity outcomes, both proposals satisfy that numerical floor; neither receives a certificate for delivery of seven units. A different, legitimately enacted rule could require a higher minimum, in which case the same uncertainty envelope may yield no robustly screened proposal without additional information, reserves, or substitute capacity. Importantly, any conclusion that a or b belongs to the screened set also requires an independent check of its energy schedule, stock identity, authority record and admissible disturbances. The illustrative capacity figures alone are insufficient for Equation 29.

The maintenance claimant may prefer b , whereas an authorized food-delivery steward may prefer a within the permitted deadline. Their disagreement is represented as an unresolved institutional choice, even when a throughput score assigns one proposal a higher number. The operative rule for resolving it may be consultation, a temporary precautionary reservation, or a determination by a competent body. Throughout, c_M remains recorded as disputed until an authenticated procedure changes that status. The new dispatch version may reorganize how repair reservations appear in a coarse network dashboard; its protected-query resolver must continue to return the authentic claim status and the authorized means of review. A report that merely describes the two proposals as energy-equivalent would obscure this procedural condition.

The example also exposes a concrete form of protocol capture. If the AI planner and the sole claim-status certifier are controlled by the same energy provider, the provider may have incentives to classify maintenance claims as minor or delay evidence access. An independent review route and separation of authority could mitigate some such risks under a suitable institutional arrangement. The comparison is a hypothesis for future evaluation; it does not prove that one particular arrangement is immune to strategic behavior. A second-period shortage, unexpected equipment failure, or newly authenticated claim record may invalidate the proposal's original certificate and trigger a documented reauthorization or fallback review.

7.7 Scope of Model-Assisted Institutional Adaptation

The proposed governance interface connects three forms of accountability. Technical accountability concerns whether the chosen model and constrained plan behave as certified over their stated domains. Documentary accountability concerns preservation of event identity, claim provenance, authorization changes, and audit access. Political and legal accountability concerns the legitimacy of the powers exercised, the recognition of affected parties, justification amid plural values, and the effectiveness of challenge and remedy. Their interaction is the object of the proposed normative-renormalization analysis. Each may

fail separately: a physically adequate plan can be unauthorized, a valid authorization can rely on poor forecasts, and accurate records can be unavailable to a person entitled to contest them.

Operational protocol screening should also test the review access that an approved action depends upon. In particular, the exact-status resolver may continue to return a valid record while a version migration breaks a deadline-sensitive filing channel. Equation 20 specifies one separate, role- and deadline-dependent test for such access. A transition can preserve every numerical member of K_v while failing that interface test; consequently the invariant proposition below is neither a sufficient test of operational contestability nor a certificate of lawful rights modification. In any implementation that declares procedural accessibility to be part of K_v , the state and transition model must explicitly represent the necessary routing, deadlines, and service availability before the invariance result applies.

The formal result in Proposition 7.1 is best read as a conditional invariant for one specified execution environment. It depends on true-history inclusion, disturbance coverage, gate enforceability, correctly specified state constraints, and continued verification after protocol changes. The plural comparison relation in Equation 31 records specified agreements among evaluators and leaves institutional selection open. Neither construction demonstrates that monetary claims disappear when production is coordinated with fewer payments. Existing liabilities, obligations to external suppliers, access to reserve resources, and authority to distribute scarce productive capacity may preserve monetary functions or cause re-monetization under conditions examined in Paper II.

Future tests should therefore vary the integrity and independence of the authorization channel, the costs of dispute access, the quality of uncertainty sets, and the degree to which competing evaluators influence selection. An important falsification case would maintain apparently satisfactory energy and throughput scores while suppressing a claimant's ability to obtain a hearing. Another would insert an unmodeled common-mode energy failure and test whether the monitoring system appropriately withdraws its certificate. These tests would not adjudicate justice automatically, but they could reveal when a purportedly rights-sensitive coordination architecture fails its own declared protections. The following section examines the related distribution of power over evidence, standards, and certification in greater detail.

8 Epistemic Power, Abstraction Capture, and Critical Failures

Scale-dependent representations of productive value circuits acquire political and institutional significance when their outputs guide access to energy, maintenance, information, and review. This section examines the concentration of power over those representations and the failures that can emerge when aggregate accuracy coexists with neglected claims or poorly modeled interdependence. Its objective is to specify distinguishable mechanisms of epistemic control, establish a conditional limit on sample-based detection of rare omissions, and formulate an inspectable response to critical failures. The analysis combines institutional comparison, a finite probability proposition, a coupled two-period example, and candidate audit procedures. The proposed mechanisms are hypotheses and design objects for future investigation in AI-operated production networks; neither their prevalence nor their normative permissibility is established by the formal constructions.

8.1 Control over Representation and the Distribution of Epistemic Power

A distributed economy may rely on many autonomous productive operators while depending on a small set of entities to define common data schemas, train circuit-reconstruction models, authorize abstraction maps, certify error estimates, and operate access to claim records. The resulting concentration concerns the ability to influence what becomes visible to coordinators and reviewers. It can persist when the material network itself contains alternative suppliers and when money ceases to mediate a substantial share of its transactions. Relational accounts of power through dependence provide one relevant conceptual precedent [6]; the present analysis extends that vocabulary provisionally to model-governed interfaces

among hypothetical AI operators. A dependence relation here is conditional on available alternatives, enforceable access rules, and the real costs of using them.

For a protocol version v , distinguish four capacities: selecting the admissible evidence and training record $\mathcal{E}_v$; selecting the query and classification vocabulary Q_v ; certifying a scale map α_v and its reported errors; and determining access to resolution and review $\mathcal{R}_v$. Their holders may differ. A producer can submit an event without being able to change the certified event type; a claimant can possess review standing while facing a costly or unavailable fine-scale resolver; a verifier can attest to the numerical accuracy of a dashboard while lacking jurisdiction to alter a maintenance obligation. These capacities are analytically separable even if one institution lawfully exercises several of them. The distinction between formal institutional power, an agent’s operational capability, and the practical capacity to exercise a right remains essential.

The present paper uses *abstraction capture* for a conditional institutional failure mode in which the governance of evidence, representation, certification, or review systematically favors a subset of interests while the resulting summary continues to satisfy its declared tests. Capture may arise from strategic manipulation, inherited classification biases, exclusive infrastructure, poorly designed delegation, or dependence on a single technical provider. Establishing actual capture in a concrete institution would require evidence concerning control, differential burdens, feasible alternatives, and relevant justificatory standards. Numerical inaccuracy alone is insufficient to establish capture; a numerically accurate but selectively scoped representation can be captured as well. Conversely, disagreement with a justified collective decision does not by itself demonstrate capture.

This analysis is compatible with a plurality of institutional forms. Polycentric arrangements can supply overlapping capacities for rule-making and monitoring [16, 17], yet the existence of several nominal certifiers need not yield substantive independence if they all depend on one data repository, model provider, or credentialing authority. The normative concern is therefore the effective distribution of access, correction, and review, with explicit attention to affected participants and the costs of exercising those opportunities.

8.2 Endogenous Metrics, Selective Observability, and Certification Scope

A governance dashboard may report delivery rates, resource utilization, delay, computational burden, and predicted regeneration. These quantities have practical analytical uses. Their selection also influences behavior: an operator evaluated primarily on delivered food may defer maintenance whose benefits accrue in later periods; a supplier evaluated on settled transactions may classify disputed deliveries as completed; a certificate optimized for average forecast error may fail to expose a rare, consequential rights-status error. The possibility of feedback between indicators and behavior warrants treating indicator design as a revisable institutional choice, rather than as a neutral observation of a fixed environment.

Let X_v be a declared class of fine-scale operative states, let $O_v : X_v \rightarrow Y_v$ be the published observation/aggregation interface, and let Q_v^{aud} be the questions used in certification. A certificate may establish that each selected numerical question $q \in Q_v^{\text{aud}}$ is answered within its specified tolerance on a tested domain. Its meaning depends on the question set, data provenance, intervention class, time horizon, and evidence that the domain actually covers the proposed use. An omitted claim-status question may remain unresolved even when the energy and throughput tests are exact. Section 4 established the query-relative abstraction principle; the institutional problem here is who has the effective authority to delimit and revise that query set.

Two distinct failure mechanisms follow. Under *scope omission*, a class of affected claims never enters the audited query family. Under *evidence exclusion*, a formally protected question exists, but the observation model or authenticated resolver fails to include the records needed to answer it. An exact protected-query statement presupposes a functioning resolver and a legitimate, adequately specified protected set, as analyzed in Section 5. A certificate cannot repair these missing premises by increasing the precision of other numerical outputs. A model may also pass tests against an earlier distribution and become unreliable when incentives, technology, institutional powers, or resource pathways change;

Table 8: Illustrative mechanisms of abstraction capture and critical failure in multiscale production governance. The observations are audit prompts rather than sufficient evidence of misconduct.

Mechanism	Evidence relevant to evaluation	Candidate response and independent limit
Query-set selection	Missing claim types; certification restricted to averages; omitted time horizons	Challengeable query registry and affected-party participation; standing remains institutionally determined
Evidence access concentration	Unavailable fine histories; asymmetric retrieval delays; rejected provenance links	Audited access paths and independent record custody; disclosure constrained by justified privacy and security rules
Metric-induced displacement	Improved reported throughput with deferred renewal or increasing future deficits	Multi-period stress cases and material-state checks; proxy choice remains contestable
Certification dependence	Nominally separate verifiers using one model, dataset, or authority	Trace shared dependencies, diversify where justified, and record correlated error assumptions
Rare-claim invisibility	Small claimant classes absent from routine samples or exception reports	Targeted sampling and guaranteed complaint channels; random sampling alone lacks complete coverage
Critical regime change	Abrupt failure after a threshold crossing; outdated uncertainty set or permissions	Revalidation, fallback, emergency review, and prospective rollback; irreversible effects require remedies

observational calibration must retain its stated temporal and distributional scope.

The evidential significance of certification is likewise bounded. A verifier’s independence is not established by using a different neural-network architecture if all models inherit identical absent records. Signatures and immutable hashes can help authenticate versions of inputs and decisions while leaving open whether a submission was excluded upstream or whether the authorized party had a viable route to contest a classification. The NIST AI Risk Management Framework provides an independently developed risk-management comparator for context-sensitive governance, measurement, and ongoing management [15]. The certificate, claim-resolver, and protocol structures proposed here are original research constructions, not institutional requirements inferred from that framework.

8.3 Structural Failure Modes and Audit Evidence

Table 8 distinguishes failure mechanisms by their objects of control, possible empirical signs, and corresponding candidate safeguards. The table separates symptoms from conclusive findings. A common supplier may be efficient and lawful, for example, while producing a common-mode vulnerability that becomes serious under particular outages or restrictions. An audit should consider both the benefits of standardization and the costs of exclusion; a recommendation to duplicate infrastructure requires a material feasibility and proportionality assessment of its own.

The table also points to a distinction between *availability of appeal in a data schema* and *effective review*. A resolver may reproduce the exact recorded status of a claim while withholding the associated reasons, deadlines, translation support, or authorized route for obtaining relevant evidence. Meaningful procedural access depends on the affected party’s practical circumstances, the competence of the reviewing body, and governing rights. Hirschman’s distinction between voice and exit [9] is a useful comparative lens; whether either channel is viable in an AI-intensive economy depends on its institutional design. A technical audit should record these questions as separate findings, with appropriate uncertainty, rather than infer legitimacy from a passing interface test.

8.4 Sampling Limits for Rare but Protected Claims

Rare events create a particularly sharp distinction between statistical visibility and normative importance. Suppose an auditing institution samples past transactions to detect a type of omitted claim. The sampling probability reflects the observation procedure and the distribution represented in the audit population; it has no necessary relationship to the claim’s validity, the gravity of its possible suppression, or the magnitude of material throughput associated with it. A modest formal statement makes the coverage limitation explicit.

Proposition 8.1 (Conditional detection coverage of independent transaction samples). *Fix an audit population and a reproducible binary indicator marking a specified omission class. Assume n observations are independent draws from an unchanged distribution in which each draw belongs to that class with probability p , where $0 < p < 1$. If the auditor detects the class whenever at least one sampled observation belongs to it and identification is accurate, then the probability of at least one detection is the expression in Equation 32. To attain a detection probability at least $1 - \delta$ for $0 < \delta < 1$, a sufficient and necessary integer sample-size threshold under these assumptions is given in Equation 33.*

$$P_{\text{detect}}(n, p) = 1 - (1 - p)^n. \quad (32)$$

$$n \geq \left\lceil \frac{\log \delta}{\log(1 - p)} \right\rceil. \quad (33)$$

Proof. The specified class is absent from each independent observation with probability $1 - p$. Independence implies a joint absence probability $(1 - p)^n$, giving Equation 32 by complementation. The inequality $(1 - p)^n \leq \delta$ is equivalent to $n \log(1 - p) \leq \log \delta$. Since $\log(1 - p) < 0$, division reverses the inequality, and integer sample size yields Equation 33. $\square$

For an illustrative incidence $p = 0.001$, a thousand independent draws detect the class with probability approximately 0.632; attaining a probability at least 0.95 requires approximately three thousand such draws. These figures describe detection *conditional on reliable recognition*. They establish neither the incidence of real economic wrongs nor the sufficiency of any audit to secure a right. The derivation ceases to apply directly when omissions are clustered, auditors rely on a common incomplete source, the class is deliberately concealed, the event population changes, or the chosen detector misclassifies the relevant rights issue. A class absent from the observation frame has effectively zero sampling probability even if the true affected population contains many cases.

Proposition 8.1 supports a narrow design implication: sampling should be complemented by event-level complaint routes, claimant-initiated review, targeted tests for known vulnerable classes, and procedures for amending the monitored question set. A low-incidence finding may call for closer consideration when the event carries a legally protected claim or a substantial interperiod burden; a high-incidence statistical anomaly is not automatically a rights violation. Protective thresholds and resource priorities require a legitimate procedure to define the stakes and burdens of further investigation. Repeated auditing also has costs: retention, confidentiality, expert attention, and the possible chilling effect of comprehensive surveillance belong in the evaluative record.

8.5 Critical Transitions and Coupled Dependence Across Periods

Network descriptions can fail discontinuously when an overlooked dependency crosses a threshold. Threshold-cascade and interdependent-network studies provide established structural precedents under their respective assumptions [22, 4]. They do not show that an actual AI economic network possesses the same critical parameters or that a particular distribution rule is just. The present thought experiment instead uses a transparent counterexample to expose the risk of relying on aggregate first-period adequacy as evidence of future productive viability.

Retain the two-period illustrative energy account used in Sections 2 and 7. A source provides six kilowatt-hours in period one. Agriculture receives two, logistics one, and a maintenance process two;

one remains unallocated. Separately, a replacement-part claim c_M is under review. Let $z = 1$ denote that a physically adequate replacement part arrives by the next operating deadline, and $z = 0$ denote that it does not. Under one expressly stipulated production scenario, effective period-two capacity obeys Equation 34.

$$K_2 = 7 - 3(1 - z), \quad z \in \{0, 1\}, \quad d_2 = 6. \quad (34)$$

Here d_2 is a hypothetical six-unit demand threshold in the same output-capacity units as K_2 , rather than a kilowatt-hour quantity. A timely part permits capacity seven; an untimely part leaves capacity four. Consequently, the indicator of meeting the specified period-two demand jumps when the part-arrival condition changes. This stipulated discontinuity is a simple model of a lumpy maintenance dependency, not an estimated law of agricultural output. The first-period two-kilowatt-hour maintenance allocation can have been physically consumed without completing repair when the required part has not arrived. Conversely, the dispute's recorded status alone does not determine whether the part physically arrives: authorization, supplier availability, and performance are independently modeled conditions.

A planning-level summary that reports five kilowatt-hours used, one remaining, and an expected period-two capacity may conceal the timing-sensitive part condition. If c_M has been classified as a low-value exception because its current monetary magnitude is small, a certification rule based solely on mean energy accounting may pass despite the potential output shortfall. A correct fine-scale model can instead condition the next-period forecast on part arrival, stock and deadline evidence, and the allowed interventions. The model must also retain the claim's dispute history and review pathway independently of the production forecast. The two questions may interact institutionally through permitted allocations; their answers require different tests.

The dependence problem is aggravated when the same certifier controls the data used by alternative planners. Correlated records can produce near-identical errors across apparently diverse AI forecasts. A change in the underlying supplier network, access rights, or production technology may invalidate a previously tested uncertainty set. The conditional invariance result in Proposition 7.1 offers no guarantee after its true-state coverage, disturbance, or authorization premises cease to hold. In practical research, detecting such premise failure is a separate monitoring task; a halted or reverted action can prevent some further harms while leaving earlier consumption, lost output, and wrongful exclusion to be addressed through other processes.

8.6 Capture-Sensitive Review, Fallback, and Institutional Remedies

A capture-sensitive evaluation would predeclare a small number of decision objects while allowing justified additions to protected questions. For a contemplated protocol, the record would identify its affected population, material boundary, intervention horizon, certified error queries, exact claim and authorization questions, known shared information dependencies, and institutions responsible for classification, certification, execution, and appeal. The reviewer would inspect whether a nominally alternative model uses independent observations or only a different transformation of the same restricted dataset. Model revisions would receive versioned provenance so that a later dispute can recover which rules and evidence were operative at the time of an actual transfer.

Challenge procedures require attention to asymmetrical resources. A participant may lack the computational means to reconstruct the full network, legal capacity to retrieve restricted evidence, or a practical alternative supplier. Accordingly, the proposed arrangement would test sample-based error certificates together with accessible complaint intake, independent review of declared exclusions, purpose-limited disclosure, and restoration of finer-grained records where authorized. A successful exact-query test concerns the correctness of a recorded status; a rights-protective finding additionally requires examination of notice, access, timeliness, remedy, and the independence of the decision-maker. The independent scrutiny of power has affinity with normative concerns about non-domination [18], while the specific multi-scale audit protocol remains a proposal of the present paper.

Fallback arrangements should be specified before exceptional failures occur. Candidate provisions include automatic suspension of a coarse-map certificate when its domain assumptions fail; referral of

disputed allocations to an authorized reviewer; temporary reliance on a narrower, better evidenced action set; maintenance of a read-only audit trace when switching protocol versions; and correction or compensation under applicable rules after an irreversible loss. These are different measures. Reverting a forecasting model is often computationally feasible; recovering delivered resources, restoring depleted ecological conditions, and undoing an exclusion can be materially impossible or normatively incomplete. The costs and powers needed to operate an audit or fallback system must themselves be evaluated, since an overly expensive safeguard may concentrate authority among the few actors able to maintain it.

A useful stress-testing programme would generate histories in which overall throughput stays constant while claims are selectively omitted, certification evidence becomes inaccessible, or one maintenance dependency fails at a deadline. It would then compare resource feasibility, reconstruction error, exact-query recovery, effective complaint access, exposure to correlated failures, and costs of additional review. The interpretation of these outputs remains plural: one institution may prioritize uninterrupted essential supply, another may insist on stricter authorization before a transfer, and affected participants may disagree about acceptable temporary burdens. The evaluator should document such conflicts and decision procedures, rather than collapse them into a presumed objective justice score. Highly capable AI may assist in producing counterexamples and auditing graph interfaces, yet legitimate authority over the resulting decision remains an independent institutional object.

8.7 Scope of Critical-Failure Analysis within Normative Renormalization

The analysis identifies three coupled conditions for a defensible multiscale research programme. A numerical certificate requires adequate questions, coverage of the relevant domain, and independently inspectable evidence. Protected normative questions require authenticated and practically accessible routes to the corresponding claim histories, with procedures for correcting exclusions and contested classifications. Stable coordination requires renewed appraisal when network dependencies, participants, data access, or governing rules change. These conditions are analytically distinct and historically revisable; passing one class of checks supplies no general guarantee for the others.

Section 8.4 established only a conditional sampling formula, while the maintenance example showed how a low-frequency or low-magnitude omitted condition can have a discontinuous productive consequence under stipulated assumptions. Neither construction demonstrates the empirical prevalence of abstraction capture, identifies an optimal audit architecture, or resolves the legal standing of nonhuman participants. Those matters call for comparative institutional research, realistic data and stakeholder participation, and evaluation of alternative accountability arrangements.

The next section develops a multi-period evaluation using overlapping productive circuits, differentiated claim status, and explicit failure scenarios. Its purpose is to compare candidate scales and governance protocols under a shared set of stipulated physical assumptions while keeping competing normative judgments and the political economy of certification visible. The possibility of distributed monetary dissolution remains conditional on effective production, regenerative continuation, and justified institutional relations, rather than on the computational elegance of the abstract network alone.

9 Multi-Period Evaluation of Normatively Constrained Coarse-Graining

A multiscale description of productive circuits becomes practically consequential when a planner uses it to decide whether production can continue, which resources should be reserved, and which institutional commitments require review. This section constructs a finite, three-period comparison of normatively constrained coarse-graining in a hypothetical five-operator food network. Its objective is to bring the paper's attenuation, reconstruction, rights-preservation, and governance interfaces into one inspectable analytical setting. The method is a stipulated counterfactual comparison: it specifies event-unique physical accounts, alternative maintenance-part histories, a limited monetary-access measure, protected claim queries, and distinct certification tests. It then establishes a conditional comparison result for approximated evaluation vectors and identifies the evidence that would be required to move beyond the con-

Table 9: Stipulated event-unique electricity ledger for the three-period five-operator illustration; all entries are kilowatt-hours, with electricity generation recorded once per period.

Period	A	B	L	M	Residual	Generated	Modeling role
1	2	0	1	2	1	6	Existing food and maintenance baseline
2	2	2	1	1	0	6	Processing and scheduled inspection
3	2	2	1	1	0	6	Continued food and maintenance services

structured example. The numerical quantities illustrate representational and institutional distinctions; they are neither observed economic data nor estimates of the effects of AI coordination.

9.1 Five-Operator Boundary and Dated Resource Accounts

Let the modeled boundary contain five operational roles: agriculture A , food processing B , logistics L , electricity supply E , and equipment maintenance M . The roles are distinct event and resource-accounting interfaces, although ownership and ultimate legal responsibility may be concentrated or distributed among other principals. Agriculture provides a dated material input to processing; logistics transports completed food; electricity enables specified processes; maintenance supports equipment availability across periods. Processing and maintenance may participate in overlapping circuit descriptions through common electricity generation and shared productive infrastructure. For example, a projection $A \rightarrow B \rightarrow L \rightarrow A$ may describe food production together with a stipulated productive return, while $A \rightarrow M \rightarrow A$ describes a maintenance-service excursion. The return edges require their own typed event interpretations; geometric closure alone supplies no physical regeneration. Energy deliveries from E can support several of these descriptions without becoming multiple generation occurrences. An event identifier, rather than the number of circuits in which an occurrence appears, determines whether an energy delivery is counted in the ledger.

The evaluation horizon is $t \in \{1, 2, 3\}$, with six kilowatt-hours of available electricity in each period. The first period retains the fully observed baseline of Section 2.7: two kilowatt-hours delivered to A , one to L , two to M , and one left unused. For the subsequent two periods, stipulate two kilowatt-hours for A , two for B , one for L , and one for M . The latter maintenance allocations finance scheduled inspections or service effort; their physical occurrence does not, by itself, establish that a separately needed replacement component has arrived. These are alternative time-indexed configurations of one model, and the later allocations are additional scenario assumptions rather than changes retroactively attributed to the original example.

For each t , let G_t denote a uniquely registered energy-generation occurrence, u_{it} its dated allocation to operator i , and r_t its unallocated residual. Equation 35 states the constructed boundary balance.

$$G_t = \sum_{i \in \{A, B, L, M\}} u_{it} + r_t, \quad (G_1, G_2, G_3) = (6, 6, 6) \text{ kWh.} \quad (35)$$

Each allocation is nonnegative and counted once; meter loss, imports, storage carryover, and exports are stipulated to be zero for this illustrative accounting boundary. A practical use would require separate tests of those assumptions, energy conversion, and within-period deadlines. Table 9 gives the complete numerical ledger. Processing uses no separately metered electricity in period one within the declared boundary; its first-period food output is treated as enabled by other stipulated capacities and inputs. This simplifying convention avoids silently adding a processing demand to the six-kilowatt-hour baseline.

The resource identities in Table 9 do not assert that the agriculture, processing, and logistics chain can produce an arbitrary output. Grain availability, processing throughput, component inventory, authorized land and infrastructure use, transportation deadlines, and ecological regeneration remain independent conditions. The example provisionally assumes that other necessary inputs are sufficient for the explicitly stated output capacities. In particular, an electricity-balanced history can coexist with a material shortfall when a necessary replacement part is delayed.

9.2 Maintenance Timing, Production Continuity, and Regenerative Scope

Maintain the replacement-component request c_M from Sections 2.7 and 8.5. Its historical register carries an authenticated identifier, a status of *disputed*, a versioned evidentiary record, and an available review pathway under the illustrative institution's rules. Let $z_t = 1$ indicate physical installation of an adequate component before the production deadline for period t , and $z_t = 0$ indicate its absence. For the limited three-period demonstration, assume that once installed the component remains operable through period three, so $z_3 \geq z_2$. Separately stipulate a target food-output capacity of six comparable food units per period in periods two and three, and the conditional equipment capacity in Equation 36.

$$K_t = 4 + 3z_t, \quad y_t = \min\{6, K_t\}, \quad d_t = 6, \quad t \in \{2, 3\}. \quad (36)$$

Here K_t , y_t , and d_t are food-capacity or food-output quantities, not electricity quantities. The function y_t presumes the other dated material, processing, and logistical constraints are satisfied. The difference between four and seven is stipulated to model a discontinuous equipment dependency; it is not a calibrated engineering relationship. No analogous quantity is stipulated for ecological renewal, so a positive result for y_t cannot establish durable soil or environmental regeneration.

Equation 36 can be evaluated for three component-arrival histories. Timely receipt yields $(z_2, z_3) = (1, 1)$ and $(y_2, y_3) = (6, 6)$. A delayed component that arrives for period three yields $(0, 1)$ and $(4, 6)$, leaving a two-unit shortfall against the period-two target. An absent component through the horizon yields $(0, 0)$ and $(4, 4)$, leaving two units unmet in each period. All three histories are compatible with the identical electricity totals in Table 9. They differ in a dated, causally important material input. The classification of c_M as disputed is held fixed across these physical histories; neither arrival nor absence decides its legal merits.

For prospective certification from an observation o , fix a proposed protocol π , a version v , and a decision horizon. Let $\mathfrak{H}(o; \pi, v)$ be the nonempty set of possible future histories compatible with authenticated evidence *available at the certification time*, declared engineering premises, available alternative suppliers, and the interventions permitted by π under v . A different protocol can generate a different set of feasible future histories even when the initial observation is identical. The robust target-capacity screen for the two future periods is given by Equation 37.

$$\underline{\Delta}(o; \pi, v) = \inf_{h \in \mathfrak{H}(o; \pi, v)} \min_{t \in \{2, 3\}} (K_t(h) - d_t), \quad \underline{\Delta}(o; \pi, v) \geq 0 \quad \text{for a capacity-floor certificate.} \quad (37)$$

If the evidence and the proposed protocol admit both timely and delayed component histories, the infimum equals -2 under the stipulated model and the proposed certificate must be withheld. A positive ex ante certificate would require additional justified premises, such as an enforceable and physically credible delivery guarantee that excludes the delayed history from the certified possibility set. *After* the relevant deadline, authenticated evidence of timely installation can restrict an ex post reconstruction to the timely history, yielding a margin of 1 in both periods; such a retrospective finding cannot retroactively certify an earlier decision. Successful capacity screening still requires separate verification of material execution, legally permissible interventions, and the validity of the evidence model. The test does not select the just allocation among feasible options.

9.3 Scale Maps, Exact Claims, and Audit Access

The three scales used in this evaluation are an event-scale record s^0 , an operational record s^1 , and a planning record s^2 . At the event scale, each generation and delivery occurrence has a unique identifier, a timestamp, an energy unit, an authorization reference, and evidence provenance. The operational scale groups allocations by dated productive function, while retaining part-receipt timing as a certified decision variable. The planning scale may aggregate the energy account to total consumption and residual balances, subject to a separately resolvable query for component arrival and the exact record of c_M . These transformations are partial maps: a certificate for a period-total query does not automatically authorize their use for a finer-grained maintenance or claim-review decision.

Table 10: Candidate scale-dependent decision interfaces and their separate numerical, physical and normative validation requirements.

Scale	Accessible productive information	Certified question and limitation	Protected exception and review interface
Event s^0	Meter and receipt identities, dated component evidence	Event-unique balance; evidentiary completeness must be assessed	Full versioned c_M record and authorized access
Operational s^1	Functional energy totals, dated component state	Timely maintenance and capacity for declared interventions	Authenticated claim-status token and review locator
Planning s^2	Period aggregate, uncertainty envelope, decision horizon	Period totals and robust floor only when the relevant set of histories is covered	Exact claim query, material bottleneck query and escalation path

For the stipulated institutional protocol, let Q_{ex} include the recorded status of c_M at a stated version, the affected claimant’s recorded opportunity for review, the identity of the rule authorizing a status revision, and the dated fact of component receipt when that event has been authenticated. The latter is a protected historical factual query, while the first three concern institutional records. A claim resolver is assessed for both correct answers and practically accessible, timely review as distinguished in Section 5. A failure to retrieve the record is logged as an exception requiring investigation; it is not converted into the answer *discharged*. For declared numerical queries, the scale map may report certified intervals or error tolerances instead of complete histories.

Table 10 distinguishes the information available at each scale and the decisions for which it is provisionally usable. The column on protected exceptions is essential because a physically accurate planning dashboard can still be institutionally incomplete. The table describes a design to be evaluated, not evidence that an actual administrative system would deliver equivalent safeguards.

A status-free planning map collapses two event histories with identical energy accounts but different c_M statuses. Proposition 2.1 establishes why that map cannot answer the specified exact-status query for both. A capacity-blind map creates an analogous practical problem: the same six-kilowatt-hour total corresponds to both adequate and inadequate future food capacity. Consequently, a planning certificate must name its query, time horizon, recognized exceptions, accessible evidence, and the authority responsible for its use. These specifications express what a certificate claims to preserve; a real institution would additionally need independent evaluation of whether they are lawful and substantively appropriate.

9.4 Monetary Reliance and the Limits of Apparent Substitution

The economic interpretation of these histories requires an additional distinction between a medium of payment and the resources needed to complete production. Define a narrow *local monetary access-gate count* $g(h)$ as the number of dated, indispensable access transactions within the declared boundary for which the stipulated protocol requires monetary settlement before the essential input can be obtained. This is an intentionally coarse diagnostic rather than a monetary aggregate, welfare index, or measure of the monetary economy as a whole. Other monetary functions, including valuation, external debt, purchasing power, accounting, and obligations beyond the boundary, remain separate research objects. A history with $g(h) = 0$ can depend on imported goods purchased elsewhere with money, so the reported count cannot establish systemic monetary dissolution.

Consider four protocol-history pairings. In a *timely nonmonetary arrangement*, an authorized reciprocal delivery provides the part before period two; the energy balance and future capacity hold, with no declared local monetary gate. In an *authorized monetary fallback*, the institution purchases an alternative part through one required monetary settlement; capacity is the same, but $g(h) = 1$. In a *delayed nonmonetary arrangement*, the part is received only for period three, producing a period-two shortfall despite $g(h) = 0$. In a *status-erasing coarse representation*, physical delivery is stipulated timely and

Table 11: Constructed three-period protocol comparisons under the common energy ledger. Food outputs are model units; the access-gate count concerns only declared essential-input transactions inside the boundary.

Protocol-history pairing	z_2, z_3	y_2, y_3	Shortfall	$g(h)$	Exact claims	Reading
Timely nonmonetary	1, 1	6, 6	0	0	Pass	Local access without a payment gate
Authorized monetary fall-back	1, 1	6, 6	0	1	Pass	Timely access by paid alternative
Delayed nonmonetary	0, 1	4, 6	2	0	Pass	Less monetary gating, weaker continuity
Status-erasing coarse view	1, 1	6, 6	0	0	Fail	Correct totals, invalid claim interface

the numerical accounts are accurate, but the summary silently deletes c_M 's disputed status and review pathway; this candidate map fails the declared exact-query requirement. The latter is a representation and certification failure, not a distinct claim that physical delivery itself ceased.

The comparison in Table 11 makes these dimensions visible. In all four constructed pairings, c_M is actually recorded as disputed in the authoritative event ledger. Even when an alternative part is purchased, whether that purchase discharges or changes the original claim requires a separate institutional act and cannot be inferred from the arrival event. The table stipulates that the claimed review pathway is available in the first three cases; it is omitted from the status-erasing representation in the fourth.

Here shortfall is the sum of unmet target units in periods two and three. The operational effectiveness of the monetary fallback is stipulated rather than economically explained. Its external funding, distributive burdens, relative prices, and bargaining power are therefore unresolved. Likewise, timely nonmonetary replacement requires actual component availability, appropriate access rights, reliable timing, and regenerative support for subsequent periods. The delayed case shows why eliminating one settlement step can coincide with less production. The status-erasing case shows why maintaining output alone does not establish the legitimacy of the abstraction or of any rule that relies on it.

9.5 Certified Numerical Comparison under Plural Criteria

A research evaluation can compare candidate protocols without imposing one scalar justice function. Let Π be a finite set of candidate policies within the declared authority for consideration and Ω a fixed set of common initial records and exogenous scenarios. For $\omega \in \Omega$, let $h^\pi(\omega)$ denote the policy-specific realized or simulated history under candidate π . Define $f_j(\pi, \omega)$ as an oriented numerical performance criterion evaluated on $h^\pi(\omega)$, for j in a declared finite index set J , where larger values denote improvement according to that particular criterion. The same background scenario ω is used to compare the candidate policies, while their consequent productive histories may differ. Final institutional admissibility remains subject to the exact-query and procedural gates. Examples include negative production shortfall, verified slack over a protected capacity floor, negative local monetary access gates, or negative audit cost. Their scales and units remain distinct. A numerical summary supplies $\hat{f}_j(\pi, \omega)$ for the policy-specific history $h^\pi(\omega)$ with a certified error $\varepsilon_j \geq 0$, where the certificate applies uniformly to the declared $\Pi \times \Omega$. The common scenario fixes only exogenous conditions and the initial information cutoff; effects of the policy on participants' choices, prices, alternative suppliers, or disclosure behavior belong to $h^\pi(\omega)$ and may therefore differ across policies. A comparison that treats those endogenous responses as identical assumes an additional invariant mechanism that requires separate justification.

Equation 38 distinguishes quantitative certification from exact and procedural institutional gates.

$$\mathcal{E}(\pi, \omega) = ((f_j(\pi, \omega))_{j \in J}, Q_{\text{ex}}(\pi, \omega), R(\pi, \omega)), \quad |f_j(\pi, \omega) - \hat{f}_j(\pi, \omega)| \leq \varepsilon_j. \quad (38)$$

The component Q_{ex} contains the applicable protected exact-query outcomes, while R records whether the prescribed evidence access, notice, and review arrangements are operable for the relevant parties.

Their contents need not have a common numerical metric. In the present institutional experiment, the decision rule may reject a protocol for failing a required exact query even when its quantitative vector is otherwise favorable. Other institutions may define different protected queries through an authorized, contestable process; the structure cannot specify those commitments in place of the affected community.

Proposition 9.1 (Conditional criterionwise comparison under certified aggregation). *Fix $\pi_a, \pi_b \in \Pi$ and the declared common exogenous scenario set Ω , using the policy-specific histories $h^{\pi_a}(\omega)$ and $h^{\pi_b}(\omega)$ for each $\omega \in \Omega$. Assume that, for every $\omega \in \Omega$ and every $j \in J$, both estimated performance values satisfy the error guarantee in Equation 38 with the same nonnegative tolerance ε_j . If the summaries satisfy Equation 39 for every such ω and j , then π_a strictly outperforms π_b on each oriented numerical criterion throughout Ω .*

$$\hat{f}_j(\pi_a, \omega) - \hat{f}_j(\pi_b, \omega) > 2\varepsilon_j, \quad j \in J, \quad \omega \in \Omega. \quad (39)$$

This conclusion makes no claim about admissibility under Q_{ex} , effective review R , or the selection and legitimacy of J .

Proof. For each declared criterion and shared scenario, the first certificate gives $f_j(\pi_a, \omega) \geq \hat{f}_j(\pi_a, \omega) - \varepsilon_j$ and the second gives $f_j(\pi_b, \omega) \leq \hat{f}_j(\pi_b, \omega) + \varepsilon_j$. Subtracting yields a difference bounded below by the estimated margin minus $2\varepsilon_j$, which is positive by Equation 39. The reasoning applies separately to each j and ω , without comparing units across criteria. Exact-query standing and procedural adequacy are separate components of Equation 38 and are not inferred from the numerical inequality. $\square$

Proposition 9.1 supplies a deliberately narrow sufficient condition. If the upper error bounds depend on policy and history, the sum of their respective certified bounds replaces $2\varepsilon_j$. When the declared estimates have no adequate uniform error certificate, a favorable dashboard difference supplies no analogous result. When two policies improve different criteria, they remain incomparable under the criterionwise order until an authorized institution supplies a defensible prioritization procedure. Moreover, selecting the criteria or controlling which background scenarios enter Ω can itself constitute epistemic power, as analyzed in Section 8.

The constructed timely nonmonetary and monetary fallback arrangements have identical stipulated food outcomes and distinguishable access-gate counts. The numerical information therefore supports a limited comparison on local payment dependence, conditional on equivalent component availability and other material premises. A comparison that silently changes Ω , the initial record, or the observation cutoff between policies would lack the common counterfactual basis required by Proposition 9.1. It supplies neither a strict improvement across every possible criterion nor a basis for declaring the paid alternative illegitimate. Conversely, the delayed nonmonetary case sacrifices two units of food output while avoiding a local monetary gate. A scalar index formed by arbitrarily adding these quantities would obscure their distinct units and normative roles. The research programme calls for explicitly recorded tradeoffs and reviewable choices.

9.6 Certification Drift, Reflexive Power, and the Evaluation Horizon

Successive abstraction certificates should be tested against changes in the productive network rather than treated as permanent properties of its representation. A model calibrated during regular supply can lose coverage after an energy-supplier failure, an altered repair technology, changed authorization, strategic exclusion of evidence, or the emergence of an alternative product. A three-period evaluation is especially vulnerable to unmodeled effects beyond its terminal date. A maintenance schedule that supports period-three output may exhaust spare parts or soil capacity in period four. Accordingly the experimental record should report a temporal boundary, assumed terminal productive stocks, known omitted regenerative conditions, and the events that trigger recertification. A certificate of local continuity provides limited evidence about longer-run reproduction.

The same institutional actor may propose the coarse map, operate a payment channel, decide whether component evidence is visible, and administer claims. That combination can alter both the measured numerical advantage and the capacity of a claimant to dispute the record. A capture-sensitive evaluation would disclose shared certification dependencies, permit authorized independent access to fine-grained evidence, and document changes to the query vocabulary. Additional certifiers provide meaningful evidential independence only when their access and failure modes support that claim. Redundancy across correlated copies of one ledger is weaker than the number of nominal reviewers suggests.

The proposed method can also alter participants' conduct. If an AI planner preferentially funds activities recognized by an aggregate regeneration metric, operators may reclassify their events to obtain access, or defer work whose productive value becomes apparent only later. Such effects are hypotheses requiring observation and comparison. A more extensive research design could randomize or otherwise identify the effects of proposed decision-support interfaces where ethically and institutionally permissible, combine process tracing with authenticated ledger audits, and include qualitative evaluation of procedural access by affected parties. It should track model monitoring costs, privacy and confidentiality, switching capacity, and whether independent review changes outcomes rather than merely producing additional records.

In this setting the word *renormalization* names an exploratory family of partial, successive, question-relative abstractions with declared invariants and validity domains. Obtaining useful coarse descriptions is an operational possibility; demonstrating a stable group or semigroup of transformations, general scale invariance, or a universal normative fixed point would require further definitions and results. The current three-period example provides a reproducible target for testing selected interfaces. It makes no claim to an established physical renormalization theory for institutional normativity.

9.7 Evaluation Results and Research Implications

The three-period construction yields several conditional findings. Event-unique accounting preserves the stipulated six-kilowatt-hour totals across overlapping circuit descriptions. Component timing distinguishes physically continuous from deficient histories with identical electricity records. A status-bearing resolver distinguishes live disputes from other institutional statuses, while procedural review requires additional practical access. A local count of required monetary settlement gates can fall to zero even when future food demand is unmet, and a payment-based fallback can maintain production under one permitted counterfactual. The criterionwise comparison proposition identifies when numerical dominance survives bounded approximation; it does not transform competing normative criteria into one automatic ranking.

The examples together establish a research architecture rather than a policy endorsement. Candidate scales can be evaluated against the same declared event boundary, intervention horizon, evidence assumptions, rights-sensitive query set, and institutional authorization rules. Ex ante certificates should be distinguished from ex post histories, and a plausible physical history from one that is legally or procedurally acceptable. For hypothetical AI-operated economies, more extensive data and computational capacity may make some of these tests easier to perform; effective challenge, legitimate rule selection, and distribution of control over the evaluation infrastructure remain political-economic questions. The concluding section will specify the modest claims supported by the analysis and the empirical, mathematical, and institutional inquiries still required to evaluate normative renormalization in practice.

10 Conclusion: Conditions of Legitimate Multi-Scale Coordination

The inquiry into normative renormalization concerns the conditions under which a productive network may be represented at several levels of resolution while retaining the information necessary for materially feasible, epistemically accountable, and institutionally justified decisions. This concluding section identifies the results established by the formal constructions, states the limits of the three-period economic example, and specifies a research agenda for evaluating scale-dependent coordination in hypothetical AI-operated production networks. Its method is a synthesis of conditional mathematical results,

constructed counterexamples, and institutional requirements. The proposed framework remains a preliminary research programme rather than an empirically validated method or a general theory of social renormalization.

The motivating economic transition is the possible reduction of monetary dependence in some interconnected production circuits. Increased AI productive competence, improved shared data, and greater capability to coordinate materials directly may permit some access relations to be settled through nonmonetary arrangements. They also leave finite energy, infrastructural bottlenecks, temporal interdependence, ownership, authority, and contestable value distributions in place. A tractable description of these relations can support decision-making, provided that its convenience is assessed alongside its omissions and the practical consequences of the decisions it authorizes. The research question is consequently broader than the compression of a network dataset: it concerns the legitimacy of decision-specific simplification when the simplified representation participates in the organization of productive life.

10.1 Conditional Mathematical and Representational Findings

The first set of results concerns the relationship between recurrent circuit descriptions and their aggregate influence. A finite production graph can support indefinitely repeated walks without implying indefinitely realizable production. The weighted-path surrogate in Section 3 treats the contribution of such descriptions through specified weights, path counts, and attenuation factors. Proposition 3.1 gives a sufficient condition for a bounded high-order numerical tail. The resulting effective depth depends on the target observable, the error tolerance, and the stipulated growth and attenuation bounds. Correlated dependencies, changes of network regime, and a growing number of paths can invalidate a convenient geometric estimate. A small surrogate weight accordingly describes a restricted numerical property; it does not establish that the represented activity is materially dispensable or that the associated claimant has lost standing.

A second result concerns the organization of admissible abstraction maps. Section 4 models changes of scale as partial, typed transformations whose domains and supported queries are declared. Event-unique material accounts, relevant inventory boundaries, dated capacities, and authenticated source identities must be checked with respect to the intended use. Proposition 4.1 states a conditional composition guarantee for compatible numerical interfaces. Its additive error estimate permits controlled propagation of specified tolerances through successive stages. The estimate depends on valid certificates for the relevant domains and observables; it neither establishes the adequacy of the observable nor certifies decisions involving another time horizon or intervention class. The associated dynamic error analysis is made explicit in Proposition 4.2, which requires both the fine-state projections and the iterated coarse predictions to remain within the certified domain at every modeled step. A valid initial certificate alone does not extend the guarantee to an extrapolation beyond that domain.

The categorical counterpart is especially significant. Proposition 2.1 establishes that an abstraction merging fine states with different answers to a protected exact query cannot reconstruct both answers through a single deterministic resolver. Section 5 extends the corresponding design problem to recorded permissions, claims, duties, versioned determinations, and opportunities for review. Proposition 5.1 describes conditional preservation of protected query results across authorized interfaces. Accurate status retrieval is analytically distinct from the claimant's capacity to reach evidence, receive notice, submit an objection, and obtain a reasoned decision. Procedural effectiveness is therefore evaluated beyond the equality of record fields.

A third result concerns the possible contribution of advanced AI to reconstruction and coordination. Proposition 6.1 bounds the value of a stipulated numerical observable when the true history belongs to the declared set of histories compatible with the evidence and the material model. The statement depends on coverage of that set. Statistical coverage from a calibrated predictive procedure and identification of a causal intervention effect require their own assumptions. Proposition 7.1 similarly provides a monitored safety conclusion only where uncertainty sets, transition conditions, action screening, and execution controls satisfy its hypotheses. These are useful components of a proposed decision-support architecture,

while institutional authority over the meaning and use of a certificate remains an independent question.

Finally, Proposition 8.1 makes the weakness of random discovery for rare events explicit under independent sampling assumptions. Section 8 uses this result to distinguish statistical assurance from access to a particular unresolved claim. Proposition 9.1 supplies a limited numerical comparison result when the estimated margin exceeds the specified error allowance for each oriented criterion on the declared domain. It neither selects a final distributive rule nor resolves conflicts among incommensurable normative criteria. Together, the propositions clarify what can be proved within the proposed formalism and which forms of legitimacy require additional evidence and judgment.

10.2 Material Continuity, Monetary Access, and Generative Value Return

The three-period example in Section 9 connects those distinctions to the economic question motivating this series. Its five operator roles carry out agricultural production, processing, logistics, electricity provision, and maintenance. The stipulated event-unique ledger records eighteen kilowatt-hours generated across three periods. Identical energy totals can coexist with different maintenance histories and production capacities. Timely nonmonetary component provision and an authorized monetary fallback both sustain the stipulated output target. A delayed nonmonetary provision leaves the local monetary access-count at zero while producing a temporary food shortfall. The constructed status-erasing coarse view retains satisfactory physical totals but fails the declared protected claim-status query, as shown in Table 11.

These cases distinguish *local monetary substitution*, *productive continuity*, and *justified value return*. The local gate count records required monetary settlements for essential inputs within a chosen boundary. It does not cover external purchasing, credit, valuation, interregional dependencies, monetary liabilities, or all the functions served by money. Its reduction supports a narrow claim about particular transactions. A stronger hypothesis of distributed monetary dissolution would need evidence that alternative arrangements remain effective across overlapping jurisdictions, production timescales, supply interruptions, and the obligations they create. Increased nonmonetary coordination may coexist with partial re-monetization, negotiated compensation, or monetary fallbacks where access and scarcity continue to require them.

Productive regeneration poses a second limit. Maintenance that restores one machine's capacity may leave ecological or social capacities unexamined. An apparently closed material circuit may rely on an external supply of energy, depleted soils, unpaid care, public infrastructure, or knowledge accumulated outside the selected ledger boundary. Such enabling conditions affect the durability and justice of the process even where their contributions are difficult to quantify. The present paper therefore treats generative value return as a substantive economic and normative inquiry, informed by traditions of generative justice, rather than as a numerical synonym for gross throughput, graph recurrence, or reduced settlement frequency. Retaining differentiated accounts of ecological renewal, social access, productive autonomy, and institutional contribution may be more informative than forcing them into one equivalent unit.

The counterfactual of extensive AI labour does not eliminate these questions. Operators may possess technical capacity while their principals, affected communities, or other institutions possess the authority to grant access and to challenge use. Whether an AI itself holds legal or moral standing remains open under the assumptions of this programme. The proposed formalism can record positions and competing claims without deciding the underlying law or moral ontology through its syntax. The persistence or transformation of monetary institutions is, on this account, conditional on material conditions and historically generated production relations, including relations over informational and procedural infrastructure.

10.3 Institutional Conditions for Responsible Scale Reduction

A candidate scale reduction becomes institutionally usable only relative to declared tasks and governing arrangements. A certificate should identify the exact and numerical queries it supports, the period and geographic boundary, the relevant intervention class, resource units, authentication assumptions, available evidence, error budgets, protected exceptions, and recertification triggers. These items permit another

Table 12: Proposed validation domains for decision-specific abstraction in AI-operated production networks. Passing a domain-specific test does not establish the remaining domains.

Domain	Candidate validation question	Failure or escalation condition
Physical accounting	Are dated inventories, event identities, units, and shared capacities consistent?	Unmatched input, duplicate occurrence, infeasible schedule
Numerical prediction	Are error certificates applicable to the declared query, horizon, and interventions?	Uncovered history, distribution change, certificate drift
Protected records	Can specified claim, permission, and provenance statuses be retrieved at the correct version?	Omitted record, unauthorized status alteration, inaccessible evidence
Procedural standing	Can affected parties obtain notice, reasons, review, and effective correction?	Ineffective or blocked contestation, unavailable reviewer
Institutional authority	Are model approval, protocol migration, and execution powers authorized and accountable?	Concentrated unreviewable control, unapproved policy change
Regenerative continuity	Do measured productive and ecological capacities remain adequate across the relevant timescales?	Deferred repair, externalized depletion, terminal-horizon blind spot

reviewer to determine what a result actually covers. They also make visible the limits of transferring a planning-scale aggregate to a claim-resolution or emergency-allocation decision. The certificate is a record of bounded assurance under its stated conditions, rather than an unconditional warrant to implement a policy.

Table 12 consolidates the independent validation dimensions developed throughout the paper. Its organization is deliberately plural. Correct physical totals, reliable predictions, preserved query answers, accessible review, and accountable institutional authority answer different questions. A positive finding in one column of an operational assessment should not silently compensate for a failure in another. Institutions may adopt context-specific tradeoffs for some numerical objectives, while preserving categorical protections selected through an authorized and reviewable process. The choice and revision of those protections require attention to affected interests and historically constituted rights.

Institutional responsibility therefore concerns both the numerical integrity of a summary and the distribution of power over the summary itself. Model owners may select data sources, decide which exceptions are represented, define admissible queries, certify aggregate results, and determine when a contested claim is visible to reviewers. Concentration across these roles can create epistemic and practical dependency even if physical production is distributed across many AI operators. Independent access to relevant evidence, diversified verification capacity, transparent version histories, and credible fallback procedures are candidate counterweights. They require substantive governance arrangements and continuing resources, not merely additional signatures on an otherwise identical data product.

Different standards of justice may yield incompatible evaluations of one productive protocol. A system that improves aggregate output could worsen a participant’s bargaining position; a protocol favoring local autonomy might impose higher short-run coordination costs; a protective procedure may delay an urgent delivery while preventing an unauthorized deprivation. Section 5 represents such differences without presupposing a universally shared ranking. Decision support can disclose alternative evaluations, evidential limits, and the institutional process by which a particular action is selected. The legitimacy of that selection remains situated in the relevant legal and political arrangements, including the ability to challenge their operation.

10.4 Computational Assistance, Institutional Reflexivity, and Failure Boundaries

Advanced AI could enlarge the practical domain over which heterogeneous production relations become observable. Candidate uses include identifying recurring dependency patterns, reconciling time-indexed event identities, checking resource ledgers, estimating bounded truncation errors, distinguishing plausible histories, and exposing changes in productive capacity or unresolved commitments. Such uses depend on the quality and accessibility of evidence, appropriate identification assumptions, computational resources, and institutional permission. A sophisticated learner operating over an incomplete or strategically curated ledger may produce a precise answer to an inadequately specified question. The proposed benefit of AI is therefore a conditional improvement in selected epistemic and coordination tasks.

A scale map is also capable of changing the network it describes. Once a planning summary affects component access, compensation, or maintenance priorities, actors may adjust their practices and reporting. Such adaptation can alter the distribution of observed events, the reliability of an earlier error certificate, and the incentives to reveal disputed obligations. Continuous monitoring and revised models may be helpful, yet model revision can itself change the categories through which claims are recognized. A responsible governance design must therefore preserve traceability between former and current versions and specify procedures for reconsidering decisions made under an obsolete classification.

The paper examines failure cases in Sections 3, 6, and 8. Correlated pathways can defeat an independence-based fragility estimate; switching regimes can invalidate a stable transfer-matrix intuition; a candidate-history set that excludes the true history can defeat its own reconstruction envelope; and several certifiers sharing one compromised registry can fail together. Rare obligations may be missed by aggregate audits despite their seriousness to the relevant claimant. These constructions demonstrate the importance of qualifying claims by their assumptions, maintaining fine-scale escalation paths, and recording when a certificate is insufficient to support an action.

The possibility of immediate loss also affects the design of review. Post hoc correction may be inadequate after an irreversible resource transfer, a missed food-delivery interval, or destruction of evidence. Protected operations may therefore require stronger advance checks, human or otherwise authorized institutional oversight, safe fallback modes, and procedural remedies calibrated to the potential harm. The appropriate combination of controls depends on the legal and material setting. In this paper those arrangements remain proposals for assessment; there is no empirical evidence here that an AI governance architecture implements them reliably.

10.5 Research Design and Independent Validation

The next stage of inquiry should move progressively beyond constructed ledgers while preserving their reproducibility. A useful initial benchmark would contain authenticated event identities, time-indexed inventories, production capacities, alternative suppliers, access rules, claim histories, and independently specified disturbances. Baseline methods would include event-level planning, fixed-depth truncation, unconstrained network aggregation, and the proposed rights-sensitive partial maps. Comparison should track accuracy, computational and communication cost, numerical error coverage, resource conservation, production shortfalls, missed critical events, recoverability of protected queries, and actual procedural accessibility. The benchmark should disclose which features are synthetic and which correspond to independently verified economic or administrative records.

The mathematical programme requires further development in several directions. One concerns attenuation bounds under correlated walks, heterogeneous weights, shared bottlenecks, and endogenous responses to abstraction. Another concerns classes of typed scale maps whose composition preserves declared observables and protected query interfaces while remaining computationally tractable. A third concerns domain shift: detecting when a certificate no longer covers newly authorized participants, altered resource technologies, or revised rules. Future work might examine minimax certificates and robust intervention classes, while establishing the computational burden and additional conservatism such guarantees introduce. These questions should be investigated by explicit propositions, counterexamples, and

algorithms before general claims about multiscale efficiency are advanced.

The empirical programme would ask whether additional AI-assisted reconstruction and summarization improve decisions over credible alternatives. Candidate studies could examine resource dispatch, maintenance planning, environmental accounting, or cooperative procurement in domains where consent, lawful data access, and safe experimentation are feasible. Investigators would need to separate prediction accuracy from causal effects on continuity, monetary reliance, bargaining power, and equitable value return. Historical process analysis and qualitative accounts from affected parties would supply forms of evidence that meter records alone cannot produce. The costs of independent auditing, appeals, protected storage, and system migration should be incorporated in performance comparisons rather than treated as costless safeguards.

The institutional programme would examine who is authorized to define the protected query vocabulary, what constitutes practical access to review, how conflicts among heterogeneous normative standards are adjudicated, and which remedies remain available when a coarse model has generated a harmful decision. It would also compare arrangements with differing distributions of control over data provision, independent verification, appeals, and protocol authorization. These arrangements can be assessed for concentrated epistemic power even when aggregate throughput remains stable. The unresolved legal personality of AI actors, the distinction between operators and principals, and cross-jurisdictional duties demand separate jurisprudential analysis rather than inference from an economic production graph.

A prospective evaluation should include explicit stopping and revision rules. In counterfactual policy comparisons, the initial evidence and exogenous scenario boundary should be fixed across candidates, while allowing their authorized actions to generate different histories. Evidence acquired after a delivery or deadline should inform retrospective reconstruction and future recertification, while remaining distinguishable from evidence available when an earlier decision was made. A model whose evidence coverage has become uncertain, whose exact-query resolver is inaccessible, or whose proposed action exceeds its delegated authority should be routed for renewed assessment before operational reliance. The performance criteria and escalation thresholds should be publicly or institutionally justified to the degree appropriate to confidentiality and affected interests. Such a procedure would test whether the proposed abstraction aids collective learning and reason-giving, in addition to supporting efficient calculation. Its success cannot be inferred solely from a favorable synthetic benchmark.

10.6 Relationship to the Research Programme and the Meaning of Renormalization

The present paper complements two distinct investigations within the AI Monetary Relations programme. Paper II considers the conditional and spatially distributed decline of monetary settlement across productive relations, including historical power, value circulation, and economic fragility. Paper III supplies typed grammatical and event-semantic tools for describing candidate circuits, recursive walks, and overlapping executed histories. Paper IV asks which effective descriptions of those histories could remain useful when the cost of examining their full resolution becomes prohibitive and when access to claims and review must remain intact. This relation is methodological: the grammar does not prove monetary dissolution, and monetary substitution does not establish the legitimacy of an abstraction built to coordinate it.

The term *normative renormalization* is accordingly used with a restricted and explicitly provisional meaning. It refers to a research programme on successive, task-relative transformations of productive-network descriptions under specified numerical and protected-query conditions. The introductory numerical certificate in Equation 2 deliberately measures only commensurable numerical observables. Preservation of disputed statuses, authorization, evidence provenance and usable review proceeds through distinct interfaces and institutional tests. The paper borrows the problem of multiscale representation from mathematical and physical inquiry and develops a separate institutional interpretation for value circuits. It has not demonstrated a renormalization group, a universal scale-invariant law of economic production, or a common normative fixed point. Any stronger mathematical use of the term would require a specified transformation space, composition rules, admissible transformations, invariants, and analysis of their em-

pirical relevance. The present partial maps and certificates constitute an initial formal vocabulary for that inquiry.

The distinction between *descriptive disappearance* and *institutional discharge* remains central. A low-weight path may vanish from a numerical approximation while the underlying material occurrence remains recorded. An unresolved claim may be absent from a planning dashboard but continue to exist in the authoritative dossier. A validly discharged obligation, conversely, depends on an appropriate act under a governing rule and a reviewable evidential history. Similarly, money may cease to function as a prerequisite for one local productive access route while remaining significant elsewhere as a settlement instrument, accounting convention, store of claims, or means through which power is exercised. Keeping these processes distinct prevents a convenient coarse description from becoming an unexamined declaration about social or economic reality.

10.7 Concluding Perspective on Accountable Coordination

Normative renormalization presents a possible way to approach the tension between large-scale epistemic tractability and the historical specificity of productive and normative relations. Within the assumptions developed here, high-order attenuation may justify finite-depth numerical treatment; typed partial maps may carry bounded query errors across scales; protected interfaces may retain declared claims and versioned statuses; and AI-assisted reconstruction may enlarge the information available to decision-makers. Each possibility depends on limited conditions. Their conjunction requires appropriate resources, reliable evidence, independent validation, accessible contestation, and legitimate institutional authority.

The three-period example illustrates why these conditions matter. Materially identical energy records can support divergent productive futures, local nonmonetary access can coexist with an unmet output target, and a financially effective fallback can leave the original disputed obligation unresolved. A coarse map may convey the right physical total while silently excluding the information needed for a justified claim decision. Conversely, a carefully scoped abstraction may reduce the burden of routine coordination while retaining fine-grained access for consequential exceptions. These are analytically distinct outcomes that should remain visible in subsequent mathematical and empirical work.

A decision-specific validation report should additionally distinguish a deterministic history-inclusion premise from a statistically calibrated probability of that premise. Proposition 6.2 shows how declared reconstruction and approximation failure events can be combined with a conservative union bound, while providing no coverage guarantee when either premise lacks a credible generating law. For institutional practice, the exact status of a dispute remains a separate question from the ability to file a timely challenge: Equation 20 states a proposed access test that a numerical error certificate cannot replace. These distinctions identify distinct failure mechanisms and justify separate evidence, review, and recertification budgets.

The programme's working conclusion is therefore conditional and revisable: multiscale representations may contribute to future AI-assisted production coordination when their material validity, epistemic coverage, protected query semantics, review procedures, and institutional authorization are separately tested and continually monitored. The question of whether any such system contributes to a more generative, equitable, and sustainable political economy requires evidence about its history, its distribution of power and value, and the experiences of those affected by its decisions. That question remains open to further theoretical, empirical, and institutional investigation.

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