

Relational Equivalence In Heterogeneous Co-Creation

A Generative-Relational Inquiry

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Abstract

Cooperative activity increasingly joins participants with different histories, forms of embodiment, material dependencies, and capacities for generating further possibilities. This paper investigates the conditions under which heterogeneous collaborative configurations can be treated as equivalent for a specified operational purpose while retaining differences relevant to explanation and ethical judgment. It proposes a locally scoped reachability approach: under explicit composability conditions, mutual accessibility between configurations induces equivalence classes, and their quotient supports a partial order. Equivalent task outcomes alone imply a weaker relation than mutual configuration reachability. This mathematical construction offers a selective account of feasible cooperation while preserving the need for separate inquiry into participant identities and histories. The inquiry then distinguishes joint attainability from warranted symbiosis and considers a stronger form of co-creation in which collective activity alters available actions, constraints, and the relations by which configurations are compared. By separating representational equivalence, generative transformation, and normative evaluation, the paper develops a conditional generative-relational framework for studying heterogeneous cooperation across human, technological, and ecological settings.

Keywords: heterogeneous collaboration; relational equivalence; preorder; generative possibility; co-creation; symbiosis.

Notice On The Local Scope Of Definitions. Definitions, formal distinctions, and mathematical constructions introduced in this paper are local to its inquiry into heterogeneous collaborative configurations. Their interpretation in other generative-relational writings requires independent assessment of domains, resources, admissibility conditions, and the purposes of comparison. Mathematical propositions will be conditional on explicitly stated assumptions; normative judgments require their own premises.

Working Manuscript And Authorship. This is a preliminary working manuscript, version 1.0, containing the front matter and all nine sections, with a completed internal mathematical and editorial revision. It has received neither external peer review nor author approval for public release. OpenAI's ChatGPT assisted with exploratory conceptual discussion, source discovery, drafting, and XeLaTeX preparation. Wanhong Huang directs the research, reviews its claims, and bears responsibility for the final manuscript and its dissemination. The bibliography verification record states the sources and level of checking undertaken.

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1 Introduction

Cooperation often proceeds across enduring differences. A person, an artificial system, a robot, and a living organism can contribute to a shared undertaking through distinct capacities, material conditions, and histories. Their contributions can become mutually responsive even where their modes of sensing, acting, maintaining themselves, or receiving institutional recognition diverge. Such collaborations invite an elementary question of explanation: which relational similarities are sufficient for accomplishing a common task, and which differences remain significant for understanding how that task becomes possible? The present inquiry approaches this problem through generative-relational (GR) analysis, treating cooperation as an evolving arrangement of participants, resources, actions, and constraints. Its particular concern is the coexistence of operational equivalence and historical or generative heterogeneity.

A familiar description of collaboration starts with participants who possess different capabilities and then traces the outcomes of their interaction. This approach supports practical comparisons, yet it may quietly stabilize the very relations that cooperative activity changes. A joint undertaking can alter access to tools, create shared conventions, establish new channels of interpretation, and expand or contract what its participants can subsequently accomplish. The resulting relations may differ in kind from those that characterized the initial configuration. The question therefore extends beyond the contribution of each participant to a fixed goal. It concerns the transformation of the conditions under which future goals, actions, and comparisons become possible.

Scientific and social inquiry already offers important resources for this question. Star and Griesemer (1989) examine cooperation among participants occupying different social worlds in the early development of a natural-history museum, emphasizing standardized methods and boundary objects that support coordination across divergent viewpoints. Their historical account gives a concrete reason to examine the practical work that sustains a shared object across differences. Suchman (2007) investigates the situated enactment of human-machine agency and the sociomaterial conditions through which similarities and differences become consequential. These approaches resist overly simple descriptions of a collaboration through the capacities attributed to its participants in isolation. Their contributions also remind us that the vocabulary of the subject, agent, or collaborator requires care when applied across heterogeneous forms of existence.

Biological symbiosis offers a distinct, historically developed register of relational change. Margulis (1981) examines the evolutionary significance of symbiotic associations in a theory of cellular evolution. The biological mechanisms in that account are specific to living systems and offer no automatic model of human-machine interaction. Nevertheless, symbiotic histories make visible a conceptual possibility of broader interest: an association can change the organization and future capacities of the participants rather than merely combine capacities already fully specified in advance. In the social and institutional register, Ostrom (2010) examines diverse arrangements of collective governance and their varying outcomes, providing reasons to distinguish cooperation's organizational form from its benefits, burdens, and sustainability. The present inquiry places these traditions in dialogue while maintaining the integrity of their different objects and methods.

The mathematical device considered here is a locally specified relation of reachability. It is important to select its domain precisely. A human participant and a robot are heterogeneous subjects or systems; their individual identities alone give little guidance about the reachability of a joint project. A more tractable domain consists of *collaborative configurations*: specified arrangements of participants, available instruments, relevant environmental conditions, institutional permissions, and other resources. Under an explicit protocol, one configuration may be accessible from another through a sequence of admissible changes. Where the resulting reachability relation is reflexive and transitive, it is a preorder.

Mutual reachability then determines an equivalence relation, and the associated quotient supports a partial order. These are established results of order theory. Their value in this paper lies in making the assumptions and representational consequences of a cooperative comparison explicit.

For example, consider a marine observation task involving a researcher, a mobile robot, and a community responsible for access to a coastal site. Several distinct arrangements may permit the same specified observations to be collected or interpreted, even though the work follows different paths and depends on different material, technical, and institutional conditions. Equality of selected outcomes supports an outcome-based comparison. The mutual-reachability classification developed here makes a stronger, different claim: the configurations themselves belong to one class only where admissible paths connect them in both directions. Each comparison can assist planning when its criterion is made explicit. Yet the internal differences remain relevant when evaluating upkeep, control over measurements, disturbance to living beings, the distribution of risk, or the conditions supporting subsequent inquiry. This example is schematic: its purpose is to expose the kinds of distinctions a model would need to preserve, rather than to report an empirical result.

The same example reveals a stronger possibility. Cooperation may create a new calibration procedure, a durable knowledge resource, a revised permission arrangement, or an additional action that becomes available in later work. An account that only places the initial configurations in a fixed reachability diagram would overlook this structural change. The collaborative process may transform the accessible state space, the relation of permitted transitions, and even the criteria by which configurations count as comparable. Consequently, a generative-relational treatment must attend to changes in the *structure of comparison* as well as changes in the positions occupied within that structure. Later sections will distinguish modifications occurring on a fixed domain from those requiring partial mappings between changing domains. This distinction is especially important when new participants, practices, or forms of joint activity enter the inquiry.

A related caution concerns the language of equivalence. Mutual accessibility is relative to the chosen operations and constraints. It can hold among heterogeneous configurations whose constituent participants have sharply different histories, material dependencies, generative possibilities, and normative situations. The quotient of a preorder retains selected reachability judgments while compressing distinctions internal to each class. Analyses of responsibility, care, exploitation, consent, or welfare may require a return to the richer configuration records. Similarly, joint attainability beyond a specified individual baseline can be established without settling whether the cooperation is mutually beneficial. A shared achievement might arise from pooling resources, from an asymmetrical dependence, or from an arrangement that transfers environmental costs to affected parties. These alternatives demand different evidence about how the achievement was produced and sustained. Claims about desirable symbiosis thus require evaluative standards and evidence concerning affected parties.

These distinctions matter especially in discussions of artificial intelligence and nonhuman participation. The analytical inclusion of human, artificial, robotic, or ecological configurations expresses a commitment to studying heterogeneous relations. It leaves questions about consciousness, legal status, moral standing, and the distinctive interests of living beings to appropriately justified forms of inquiry. A model of shared operational reachability supplies evidence about a particular practical relation. Its implications for rights, duties, distributive fairness, and ecological compatibility must be argued on their respective grounds. Such care preserves the usefulness of formal comparison while acknowledging the material asymmetries through which collaboration is enacted.

The paper develops a bounded set of contributions. It first distinguishes heterogeneous participants from the collaborative configurations on which a reachability relation is defined. It then develops the preorder and quotient construction as a locally conditioned analytical instrument, with attention to the

information retained and compressed. Next, it specifies a comparison between joint and individual reachable possibilities under common baselines, while examining alternative explanations of apparently novel joint achievements. It subsequently studies co-creation that changes admissible relations and their domains, and considers the additional evidential and normative requirements of warranted symbiosis. The formal results serve to discipline these distinctions; the philosophical contribution lies in their application to historically situated and evolving cooperation.

This inquiry is situated alongside two related GR investigations while maintaining an independent object. Work on symbolic equivalence examines differences lost through contextual classification, and work on generative precedence examines the assumptions required to interpret order relations. The present paper brings those insights to collaborative relations in which the conditions of reachability themselves are subject to transformation. It approaches the existence and moral status of heterogeneous participants with suitable ontological openness. The immediate task is to clarify the changing configurations through which cooperation becomes possible, so that later judgments of shared generativity and responsibility can rest on explicit and revisable grounds.

2 Heterogeneous Subjects And Relational Configurations

The unit of analysis matters whenever cooperation crosses differences in embodiment, historical development, access to resources, or institutional standing. A participant can be described through its capacities, its position within an activity, or its relations to other participants. Each description serves a particular explanatory purpose. In a generative-relational (GR) inquiry, these purposes should be made explicit before the analyst assigns equivalence or precedence to the objects under comparison. The present section therefore distinguishes the participants implicated in cooperation from the collaborative configurations through which their activities become possible. This distinction gives the subsequent reachability model a precise domain while retaining the relational histories that a selective comparison may compress.

The term *heterogeneous participant* is used here as an analytical designation for entities or collectives entering a specified cooperative relation. It may include persons, organizations, technological systems, and living organisms, where the chosen research problem warrants their inclusion. The designation describes participation in an inquiry rather than a single theory of agency. Persons may exercise deliberation and bear recognized rights; organizations act through distributed procedures and authorized representatives; robots depend on engineered interfaces and maintenance; living organisms have physiological dependencies and welfare considerations appropriate to their kinds of life. Grouping these cases for relational study leaves their distinct properties and standing available for independent examination. A participant's role in one activity can also coexist with other roles, commitments, and dependencies beyond that activity.

A *collaborative configuration*, by contrast, concerns an arrangement through which participants, instruments, environmental conditions, and permissions jointly support particular activities. Such a configuration can change even while the named participants remain the same. Access to a shared dataset may be withdrawn, a robot may acquire a calibrated sensor, a working group may negotiate a new procedure, or a marine environment may impose revised operating constraints. Conversely, configurations involving different participants may support comparable tasks under a specified outcome criterion even when transitions between those configurations remain unavailable. The relational unit therefore needs more detail than a roster of actors, while remaining selective enough for analysis. This follows the wider insight, illustrated in Star and Griesemer (1989), that cooperation across distinct social worlds depends on concrete arrangements for coordination and translation.

Definition 2.1 (Local Collaborative Configuration). Fix an inquiry context κ specifying an activity, a comparison purpose, and a provisional boundary of relevant conditions. Let S_κ denote a set of participant referents and C_κ a set of collaborative configurations. A participation incidence relation $I_\kappa \subseteq S_\kappa \times C_\kappa$ records which referents participate in which configurations. Each $c \in C_\kappa$ is associated with a selective description $D_\kappa(c)$ of participant roles, available means and capacities, material and ecological conditions, applicable permissions or obligations, and accessible relational provenance. The selection and granularity of these records are specified by κ and remain open to revision.

This definition creates an explicit separation between a participant referent $s \in S_\kappa$ and a configuration $c \in C_\kappa$. The incidence relation indicates participation; its mathematical form alone says little about influence, responsibility, ownership, consent, or the quality of engagement. Those features can be represented in $D_\kappa(c)$ when relevant evidence permits, or assessed through additional relations suited to the question. The label $D_\kappa(c)$ should accordingly be read as an inquiry record, with varying degrees of completeness, rather than as a complete inventory of reality. A history may survive through documentation, testimony, institutional memory, or material traces; some of its differences may remain inaccessible. The resulting limitation belongs to the specified record and evidential conditions.

The choice of configurations as the domain also avoids giving a universal ranking to heterogeneous participants. Suppose two research teams differ in personnel, instruments, and institutional authority, yet each can perform a specified measurement under a given protocol. Their configurations become comparable at the level of that measurement outcome; whether they are mutually reachable requires evidence of admissible reconfiguration paths. An outcome comparison concerns what each arrangement can accomplish; a reachability comparison additionally concerns which arrangements are accessible from one another. It would leave open the participants' broader capacities, historical relations, and claims to recognition. In particular, the equivalence that later arises from mutual reachability concerns selected configuration states. Its transfer to any claim about subject identity, autonomy, or moral standing would require an additional argument.

An arrangement includes relations of dependence as well as present resources. A coastal research project may depend on a community's continuing authorization, a laboratory's calibration records, a robot's battery and repair infrastructure, and local ecological conditions. The apparent capability of the assembled team can change when one of these dependencies changes. Material support also bears a history: a trained operator, a maintained instrument, and a negotiated access agreement reflect earlier labor and interactions. Suchman (2007) emphasizes the practical and sociomaterial specificity of human-machine configurations. For the present framework, this supports treating assigned roles and attributed capabilities as situated descriptions whose relevance must be assessed within the actual cooperative arrangement.

A further distinction concerns *the capacity to obtain an outcome* and *the conditions through which that capacity is sustained*. One configuration may produce a useful dataset by drawing heavily on a participant's unpaid work or a fragile ecological resource. Another may produce a similar dataset while distributing maintenance duties and protecting future access. Their immediate operational results can coincide even though their conditions of continued generativity differ. A later reachability analysis can legitimately focus on a selected outcome, provided the record preserves a path back to these differences whenever they matter for explanation or judgment. The relation between cooperative achievement and warranted symbiosis will receive separate treatment in Section 7.

The institutional dimension deserves equal attention. Cooperative capacities may depend on rules of admission, authority to operate equipment, ownership of shared records, and procedures for resolving disputes. Such arrangements change the feasible set of actions and the bargaining positions of participants. Ostrom (2010) offers a developed account of institutional diversity in complex governance

systems; its relevance here is to the importance of specifying practical rules and organizational conditions. This paper draws a narrower conclusion: a configuration model that records only technical capabilities may provide an inadequate basis for interpreting cooperation whose feasibility depends on institutions. Recording the relevant institutional conditions remains a matter of empirical inquiry, with the appropriate level of detail determined by the intended comparison.

The inclusion of living organisms introduces additional care in interpretation. Biological associations can transform conditions for growth, survival, and future reproduction, as illustrated by research on symbiosis in cellular evolution (Margulis, 1981). Human-machine collaboration, ecological coexistence, and biological symbiosis exhibit different mechanisms and normative stakes. The same configuration vocabulary can help locate their relational dependencies, while the substantive accounts of physiology, learning, responsibility, and ecological welfare remain specific to each domain. Their comparison is therefore an invitation to differentiated inquiry, with the practical and moral relevance of each difference established through appropriate evidence.

The context κ includes a bounded task and comparison purpose, but its practical boundary can evolve. Collaborative configurations may acquire participants, lose access to resources, create procedures, or develop new means of describing their own histories. A change in $D_\kappa(c)$ may reflect improved documentation, an altered physical arrangement, or both; these cases require distinct evidential interpretation. Similarly, the identification of a participant across successive configurations can require continuity criteria appropriate to that participant and inquiry. The present construction permits such criteria to be stated locally while leaving broader questions of subject continuity and generative identity open. This flexibility will matter when the paper turns from reachability within a fixed domain to cooperation that transforms the domain itself.

Remark 2.2 (Participation And Normative Standing). A participation incidence relation and a selective configuration record establish descriptive relations for a particular inquiry. Claims about agency, interests, consent, responsibility, and normative standing depend on additional substantive premises and evidence appropriate to the participants concerned.

These distinctions prepare a disciplined form of relational comparison. The domain C_κ specifies what will be compared, I_κ records participation, and D_κ preserves selected conditions through which a cooperative arrangement is constituted. The next step is to define which transitions between configurations count as admissible under the context and how their finite composition yields a reachability relation. A preorder and its quotient can then be studied on a clearly identified domain, with the historical and generative differences internal to its classes remaining available to further inquiry where evidence supports them.

3 Local Reachability And Equivalence

The preceding section established collaborative configurations as the domain of comparison. A relation between these configurations can now express the feasibility of moving from one cooperative arrangement to another under specified conditions. The purpose is selective: a reachability account records attainable transitions for an inquiry while leaving the identities, historical situations, and generative capacities of the participating entities open to further description. The present section develops the elementary preorder and quotient construction, then identifies the conditions required for its operational interpretation. The underlying mathematical results are familiar from transition-system analysis and order theory; their contribution here is to make the assumptions of heterogeneous collaborative comparison explicit (cf. Baier and Katoen (2008)).

An admissible transition can involve reassignment of a task, transfer of access to an instrument, revision of an operating permission, or a physical modification of a shared environment. Whether a transition is available depends on the selected activity and on conditions such as technical feasibility, institutional authorization, safety, environmental capacity, and resources. Accordingly, admissibility is a property of a specified relational inquiry, rather than an unrestricted assertion that any configuration can be produced from another. The question is also prospective. A feasible transition can remain unrealized, while the historical record may preserve an actual transition whose conditions are subsequently unavailable.

Definition 3.1 (Admissible Collaborative Reachability). Let C_κ be the collaborative-configuration domain defined for inquiry context κ , and let $R_\kappa \subseteq C_\kappa \times C_\kappa$ denote its locally admissible one-step transitions. A finite R_κ -path from c to d is a sequence $c = c_0, c_1, \dots, c_n = d$ with $(c_{i-1}, c_i) \in R_\kappa$ for $1 \leq i \leq n$; the length-zero path is permitted. Define contextual reachability by

$$c \lesssim_\kappa d \iff \text{there exists a finite admissible } R_\kappa\text{-path from } c \text{ to } d. \quad (1)$$

For this construction, the configuration domain is sufficiently specified that any two admissible finite paths sharing the same intermediate configuration can be concatenated as a feasible path. Relevant remaining resources, permissions, and elapsed time are included in the state description whenever they affect this condition. This is an explicit adequacy assumption for the operational interpretation.

Allowing the length-zero path gives $c \lesssim_\kappa c$. Concatenation of finite paths through the same sufficiently specified intermediate state gives $a \lesssim_\kappa c$ whenever $a \lesssim_\kappa b$ and $b \lesssim_\kappa c$. Thus $\lesssim_\kappa$ is reflexive and transitive: it is the reflexive–transitive closure of R_κ , or the smallest preorder containing that relation. This statement concerns paths admitted by the construction. Its applicability to a cooperative system depends on the adequacy of C_κ and R_κ to the operative constraints of that system.

Resource limits illustrate why compatibility matters. Suppose a robot can reach one worksite using most of its battery, while travel from that worksite to another is feasible for a fully charged robot. Joining those transitions merely by the worksite’s name creates an apparent route that may exceed the available energy. Recording battery state, access permissions, and relevant timing within the collaborative configuration allows the relation to distinguish the two cases. A fixed total time horizon introduces a similar requirement: reachability defined separately by “within one hour” need not be transitive across successive stages. A cumulative horizon can instead be represented through remaining-time state variables or treated with an explicitly bounded path definition, whose algebraic properties must then be checked anew.

The preorder permits two distinct configurations to be reachable from each other. The following construction organizes such mutual accessibility into classes without treating the constituent arrangements as identical in other respects.

Proposition 3.2 (Mutual Reachability And Quotient Partial Order). For the preorder $\lesssim_\kappa$ of Definition 3.1, define

$$c \sim_\kappa d \iff (c \lesssim_\kappa d) \wedge (d \lesssim_\kappa c). \quad (2)$$

Then $\sim_\kappa$ is an equivalence relation. On the quotient $Q_\kappa = C_\kappa / \sim_\kappa$, the relation

$$[c]_\kappa \leq_\kappa [d]_\kappa \iff c \lesssim_\kappa d \quad (3)$$

is well-defined and is a partial order.

Proof Sketch. Reflexivity and transitivity follow from the preorder, while symmetry of $\sim_\kappa$ is immediate

from its definition. If $c' \sim_{\kappa} c$, $d \sim_{\kappa} d'$, and $c \preceq_{\kappa} d$, transitivity gives $c' \preceq_{\kappa} d'$, establishing independence from representatives. Reflexivity and transitivity descend to the quotient. Finally, $[c]_{\kappa} \leq_{\kappa} [d]_{\kappa}$ and $[d]_{\kappa} \leq_{\kappa} [c]_{\kappa}$ imply $c \sim_{\kappa} d$ and hence $[c]_{\kappa} = [d]_{\kappa}$. $\square$

In a finite directed transition graph, the equivalence classes correspond to strongly connected components, and their condensation graph is acyclic. The quotient relation captures the same broad reachability pattern at the class level. In a collaborative interpretation, one class can contain several configurations that permit return transitions among them, although their participants, dependencies, costs, and histories differ. Two configurations may be mutually reachable only through an expensive reconfiguration, repeated maintenance, or a transfer of authority. The quotient faithfully records reciprocal reachability under κ ; an assessment of equitable reciprocity or sustainable cooperation calls for further information.

For example, suppose one coastal observation configuration places instrument operation with a scientific team, while another transfers operating authority to a local organization under an agreed procedure. If each arrangement can be restored from the other under the admitted permissions and resources, they belong to one mutual-reachability class. This formal equivalence concerns the stipulated operational pathway. The participants may retain distinct expertise, decision-making powers, responsibilities, and vulnerability to resource loss. Treating the class as a compact planning unit remains compatible with preserving those differences in the more detailed record introduced in Section 2.

A further representational limitation arises when the compared configurations are themselves coarse descriptions. Let $\pi : \tilde{C} \rightarrow C_{\kappa}$ map detailed operational states into configuration labels. Suppose a detailed state a can move to u , and a distinct detailed state v can move to b , while $\pi(u) = \pi(v) = m$. At the label level these edges appear as $\pi(a) \rightarrow m \rightarrow \pi(b)$, although the detailed record may provide no admissible transition from u to v . Reachability inferred by joining these coarse edges can therefore exceed the paths supported by the detailed system. The distinction matters when a single collaborative label conceals resource levels, changing permissions, or different histories of access. An operationally sound quotient requires an appropriately specified transition system. For a further abstraction to preserve path claims, its projected paths must admit suitable lifts to detailed operational paths, or an independently defended abstraction criterion must establish the reachability claims of interest.

Remark 3.3 (Operational Scope And Historical Difference). Mutual-reachability equivalence is relative to the selected configurations, admissible operations, and compositional conditions. Its quotient preserves the preorder's reachability judgments among classes. Participant histories, path-specific costs, institutional asymmetries, and generative possibilities may require the underlying configuration records and further inquiry.

These results make local equivalence available as a disciplined instrument for studying heterogeneous cooperation. They also reveal its selective character: the same formal class can encompass materially and historically different pathways. The next section examines the differences retained within and between cooperative classes, especially when questions of provenance, responsibility, and generativity require greater resolution than the quotient supplies.

4 Residual Difference Within Cooperative Classes

Mutual reachability makes a selective comparison possible: configurations that can be reached from one another under a specified transition system belong to the same equivalence class. The quotient partial order introduced in Section 3 retains the reachability relations between these classes. Its explanatory economy also creates a question of resolution. What aspects of a cooperative arrangement remain available at the class level, and which require attention to its particular configurations, transitions, and

histories? This distinction matters when an inquiry moves from operational feasibility to the conditions through which cooperative possibilities were established, sustained, or altered.

Let $q_\kappa : C_\kappa \rightarrow Q_\kappa$ denote the quotient map $q_\kappa(c) = [c]_\kappa$. For any $c, d \in C_\kappa$, the construction gives

$$c \lesssim_\kappa d \iff q_\kappa(c) \leq_\kappa q_\kappa(d). \quad (4)$$

Thus a question confined to whether one configuration can reach another under the stated assumptions retains an exact answer through the quotient. A question about the path taken, the means used, the participants exposed to costs, or the provenance of a recorded outcome requires a richer object of inquiry. The difference concerns the information carried by the representation, rather than a defect in the mathematical quotient.

A precise test determines when a property of individual configurations may be assigned directly to their mutual-reachability classes.

Proposition 4.1 (Criterion For Class-Level Attribution). Let Y be a set of possible descriptions and $f : C_\kappa \rightarrow Y$ a property or descriptor of collaborative configurations. There exists a function $\bar{f} : Q_\kappa \rightarrow Y$ satisfying $f = \bar{f} \circ q_\kappa$ if and only if $f(c) = f(d)$ whenever $c \sim_\kappa d$. When it exists, $\bar{f}$ is unique.

Proof Sketch. If $f = \bar{f} \circ q_\kappa$, then equivalent configurations have the same quotient image and hence the same f -value. Conversely, if f is constant on every equivalence class, define $\bar{f}([c]_\kappa) = f(c)$. Constancy makes this definition independent of the representative. Surjectivity of q_κ gives uniqueness. $\square$

The criterion separates genuinely class-invariant statements from attributions that depend on a representative. A class may have a well-defined position in the reachability order while containing configurations with distinct maintenance burdens, distributions of decision authority, environmental exposures, or records of prior labor. Assigning any of these descriptors to the entire class requires checking the criterion or supplying an explicitly stated aggregation rule. An aggregate such as the range of costs over configurations in a class, the existence of a member satisfying a requirement, or the minimum cost of reaching a specified target class can itself be a meaningful class-level descriptor under a stated cost and path protocol. Its interpretation must retain the distinction between a property shared by every member and a property asserted of some members or selected paths.

There is a second limit to quotient information. Even complete knowledge of the reachability pre-order can leave the one-step transition structure and path-specific burdens underdetermined.

Proposition 4.2 (Identical Reachability With Different Path Structures). Let $C = \{a, b, c\}$. Consider one transition relation with edges $a \rightarrow b$, $b \rightarrow c$, and $c \rightarrow a$, and another with an edge $x \rightarrow y$ for every distinct $x, y \in C$. Their reflexive-transitive closures coincide, and both yield the same single-class quotient order. Nevertheless, their one-step transitions differ: with unit cost per edge, the shortest path from a to c has length two in the first system and length one in the second.

Proof Sketch. Each directed graph is strongly connected, so every configuration reaches every other and the quotient consists of one class. In the cycle, the route from a to c passes through b ; the complete directed graph contains the direct edge $a \rightarrow c$. $\square$

The example is deliberately elementary. It shows that equality of reachability judgments, even on the same named configurations, leaves open differences in intermediate operations and path lengths. In collaborative systems, these differences may correspond to additional authorization stages, transport, maintenance, or exposure to ecological disturbance. Actual costs may depend on resources and circumstances much more richly than unit edge weights. Recording such information calls for a transition- or path-sensitive description, together with the constraints that make its composition meaningful. A quo-

tient can then serve as a navigational summary while remaining linked to the operational detail required for practical decisions.

Historical provenance requires a further distinction between *configuration* and *occurrence*. Let E_κ denote recorded occurrences, with $\chi_\kappa : E_\kappa \rightarrow C_\kappa$ assigning a configuration description to each occurrence. Distinct occurrences can have the same configuration image, even before quotienting. One dataset may have been produced through an openly negotiated sharing arrangement, another through a procedure whose authorization later became contested; their present equipment, accessible outputs, and permitted transitions might nevertheless admit the same operational description. A provenance record can therefore be attached to occurrences and their relations. Its reduction to present configuration labels or equivalence classes is justified only when the corresponding factorization condition holds. This extends the situated attention to practices and arrangements found in Suchman (2007) without assigning any universal historical meaning to a particular technical state.

Remark 4.3 (Occurrence-Level Provenance And Class Resolution). If $h_\kappa : E_\kappa \rightarrow H_\kappa$ records a selected aspect of provenance, a unique class-level record $\bar{h}_\kappa : (q_\kappa \circ \chi_\kappa)(E_\kappa) \rightarrow H_\kappa$ satisfying $h_\kappa = \bar{h}_\kappa \circ q_\kappa \circ \chi_\kappa$ exists precisely when that aspect of provenance is constant on every fibre of $q_\kappa \circ \chi_\kappa$. The domain here consists of classes represented in the occurrence record. Where this condition fails, the occurrence-level records remain relevant even when the configurations share an operational class.

This observation has particular force in heterogeneous cooperation. A shared result can emerge through unequally distributed contributions, including field observation, technical maintenance, translation across professional communities, organizational access, and continuing stewardship. Institutional arrangements can make these contributions visible in different ways. The historical study of coordination across social worlds by Star and Griesemer (1989) provides a useful point of contact: mechanisms that support a common cooperative undertaking may coexist with differences among the participating practices and interests. For the present analysis, the evidential task is to trace the particular arrangements that support the outcome and to preserve the possibility that their significance differs among participants.

Responsibility introduces a distinct evaluative level. Operational mutuality describes available movements between configurations. A claim about responsibility additionally concerns such matters as who had authority to decide, which obligations were in force, what consequences were reasonably foreseeable, and how actions and omissions were connected to particular outcomes. The appropriate inquiry may accordingly use occurrence records, institutional rules, and participant-specific evidence even when its operational planning uses quotient classes. Ethical assessment also needs substantive standards appropriate to the participants concerned; mathematical accessibility alone supplies neither an allocation of responsibility nor a judgment of welfare.

A useful practice is therefore to treat quotienting as a *controlled change of descriptive resolution*. The class order answers questions about mutual attainability and comparative reachability. Representative-level descriptions answer questions about specific configurations; transition and path records answer questions about operational means and burdens; occurrence-level records assist historical attribution. Where a conclusion changes with the selected representative or history, the analyst can retain these finer records or formulate an explicitly qualified class-level summary. The generative-relational contribution lies in maintaining a justified passage between these resolutions, so that operational equivalence supports coordinated inquiry while the heterogeneous conditions of cooperation remain available for explanation and judgment.

Such distinctions also prepare the paper's next step. Having established what an existing reachability class preserves, we can ask whether collective participation enables outcomes beyond those available

under suitably matched alternative arrangements. That comparison concerns the expansion of generative possibilities and requires its own counterfactual conditions, developed in Section 5.

5 Collective Expansion Of Reachable Possibilities

The preceding sections characterized cooperative arrangements through locally admissible transitions, their reachability preorder, and the equivalence classes formed by mutual accessibility. These structures describe possibilities within a specified transition system. A different question arises when participants enter a new relation: under what conditions can their joint activity be said to make further possibilities accessible? Answering this question requires a comparison across arrangements, including alternatives in which the same participants, material means, or institutional supports are organized differently. The relevant claim concerns a difference between feasible outcome sets under an explicitly chosen comparison protocol. Its explanatory significance depends on the adequacy of that protocol and on evidence about the capacities attributed to each arrangement.

A familiar illustration comes from collaborative environmental inquiry. A coastal organization may hold situated knowledge and legitimate access to a study area; a technical team may have instruments and calibration expertise; a data steward may maintain procedures for preserving and sharing measurements. Their cooperation can make a particular observation record attainable under agreed conditions. The observation's attainability, however, also depends on whether a comparison arrangement could employ substitute instruments, obtain authorized access through another route, or purchase equivalent specialist services. A description of what occurred jointly establishes an actual history. A claim about the additional possibilities attributable to joint participation further requires a justified account of the available alternatives. Research on coproduction likewise locates cooperative achievements within specific institutional arrangements rather than deriving them solely from the identities of the contributing parties (Ostrom (1996)).

The counterfactual baseline therefore deserves independent specification. A comparison can contrast joint participation with individual activity, a pre-existing institutional arrangement, a simpler coalition, or a realistically available technical substitute. Each baseline answers a different question. Comparing a research consortium with one isolated observer can reveal the value of pooled capacities under that arrangement; comparing the consortium with an established network of contractors may test a stronger claim about distinctively collaborative possibilities. In either case, a common task, observation criterion, initial situation, and sufficiently comparable resource envelope are needed. Material resources include equipment, energy, labor, expertise, and access rights; relevant ecological conditions and time limits also shape which paths are feasible. The same declared budget may support different activities where permissions, transaction costs, and obligations differ, so the comparison must document these differences rather than treating nominal resource equality as complete comparability.

A bounded comparison is particularly useful here. Section 3 defined a preorder through finite-path reachability in a configuration system whose state records the constraints needed for composability. A claim about what a group can achieve *within a specified horizon* introduces an additional restriction. Such bounded attainability need not itself be transitive: two successive tasks, each possible within one day, may jointly require two days. The definition below consequently uses feasible paths under a common protocol while retaining the preorder of Section 3 for its separate structural purpose.

Definition 5.1 (Comparative Collective Attainability). Fix a comparison protocol η specifying a task, a reference initial situation, a time horizon, a resource-accounting rule, the relevant scenario-specific permission conditions, and a common domain Ω of outcome descriptions. Let J denote a proposed

joint arrangement and let $\mathcal{B}$ be a specified family of alternative arrangements. For each $s \in \{J\} \cup \mathcal{B}$, let (C_s, R_s, c_s^0) be a scenario-specific configuration system with initial configuration c_s^0 , let $\mathcal{P}_s^\eta(c_s^0)$ be the finite R_s -paths from c_s^0 satisfying the protocol, and let $\rho_s : C_s \rightarrow \Omega$ describe the comparable outcome of a terminal configuration. Define

$$\mathcal{A}_s(\eta) = \{\rho_s(\text{end}(p)) : p \in \mathcal{P}_s^\eta(c_s^0)\}, \quad \mathcal{A}_{\mathcal{B}}(\eta) = \bigcup_{s \in \mathcal{B}} \mathcal{A}_s(\eta). \quad (5)$$

A witness of collective expansion relative to $\mathcal{B}$ is an outcome $\omega \in \mathcal{A}_J(\eta) \setminus \mathcal{A}_{\mathcal{B}}(\eta)$. An inclusive expansion holds when $\mathcal{A}_{\mathcal{B}}(\eta) \subsetneq \mathcal{A}_J(\eta)$. These predicates are relative to the scenario specifications, the outcome descriptions, and the comparison protocol.

The family $\mathcal{B}$ records alternative *scenarios*, each with its own feasible pathways. Its union means that an outcome is accessible in at least one specified alternative. It makes no assertion that the outcomes in this union can all be produced simultaneously, or that the resources required by different alternatives can be spent more than once. By contrast, the joint arrangement must possess an admissible path to each outcome attributed to $\mathcal{A}_J(\eta)$. The common codomain Ω makes outcome comparison possible across scenario-specific configuration spaces; a change in its descriptive granularity may change whether a witness of expansion appears. If all outcomes are recorded simply as “a dataset,” joint calibration and individually acquired measurements may appear identical. A finer outcome record can distinguish coverage, reliability, authorization, and continued accessibility when those features are relevant to the inquiry. Such refinements must be justified by the research purpose and supporting evidence.

A witness of expansion is a deliberately modest criterion. It establishes the existence of a possibility available in the scenario designated as joint and absent from the chosen alternatives as modeled. Interpreting the difference as a consequence of cooperation additionally requires defensible matching and causal or mechanistic evidence. It leaves open whether the joint arrangement also preserves the possibilities of the alternatives. The stronger inclusion condition settles that further set-theoretic question but remains tied to the same comparison protocol.

Proposition 5.2 (Collective Expansion With Foregone Alternatives). There exist finite transition scenarios sharing the same outcome domain, initial outcome, and one-step comparison horizon, for which a joint arrangement has a witness of collective expansion relative to its baseline, while neither its attainable-outcome set nor the baseline set contains the other. Consequently, the existence of a witness implies neither inclusive expansion nor retention of every alternative possibility.

Proof Sketch. Let $\Omega = \{u, v, w\}$ and consider two scenarios with a common one-step horizon, identical initial outcome, and equal formal operation budgets. In the baseline scenario, the initial configuration has outcome u and admits a one-step transition to a configuration with outcome v . In the joint scenario, the initial configuration likewise has outcome u and admits a one-step transition to a configuration with outcome w . Length-zero paths are feasible in both systems. Therefore $\mathcal{A}_{\mathcal{B}}(\eta) = \{u, v\}$ and $\mathcal{A}_J(\eta) = \{u, w\}$. The outcome w witnesses collective expansion, while v remains exclusive to the baseline. The two sets are incomparable under inclusion. $\square$

This example is a mathematical construction, with no empirical claim attached to its three named outcomes. Its interpretation nevertheless matters. A jointly operated observation system might enable a new synchronized measurement while requiring the retirement of an older independent sampling procedure. The new record may be valuable, and the retired capability may also have served a distinct scientific or community purpose. The existence of one newly accessible outcome describes a gain in the selected possibility space; the complete comparison also attends to retained and foregone options. A research group may further ask whether a previously unavailable *sequence* of measurements has become

possible. Such a question requires an outcome description rich enough to preserve sequence structure, or a direct comparison of feasible paths, since a terminal-outcome map can compress distinctions established in Section 4.

The same framework also separates expansion in kind from improvement in degree. Suppose two arrangements can each produce an observation record with the same descriptive content. The joint arrangement may achieve it with less energy, shorter delay, lower error, or greater operational resilience. Relative to an outcome domain that records only the observation's content, their attainable sets can coincide. A performance-sensitive evaluation may instead attach costs and risks to paths, or include quality thresholds in the outcome definition. This yields evidence of efficiency or robustness under specified metrics, without forcing an artificial claim of a newly attainable type of outcome. Conversely, a seemingly unique outcome may cease to be unique when the baseline is strengthened to include a feasible substitute or when additional time is allowed. Such sensitivity is analytically informative: it locates precisely which relational conditions support the claimed collective advantage.

A distinction also arises between *model-implied* and *empirically established* expansion. The set difference in Definition 5.1 is mathematically determinate once the scenario systems and maps have been supplied. Establishing those systems as adequate accounts of an actual cooperative setting requires observations, technical testing, documentary evidence, or defensible causal assumptions. Actual outcomes under one arrangement typically leave some alternative outcomes unobserved. The counterfactual literature accordingly stresses the specification of interventions and assumptions permitting identification from observed evidence (Hernán and Robins (2020)). Here the formal definition supplies a comparison target; it does not, by itself, identify the effects of cooperation from observational records. Where evidence is incomplete, the analyst can state possible witnesses conditionally, compare multiple credible baselines, and record which assumptions determine the result.

Collective attainability also depends on a continuing network of relations. An outcome may require permission to access a site, the labor of calibrating a sensor, the preservation of a trusted data record, and the ecological conditions enabling an observation. Once these enabling relations are recognized, "joint capability" becomes a description of a sustained arrangement rather than a simple sum of personal traits. A change in one relation can shift the attainable set even where the roster of participants remains constant. This connects the present inquiry to the institutional variability emphasized by Ostrom (2010) and to the situated organization of technical practice considered by Suchman (2007). Such connections motivate further investigation; they supply no universal equation by which cooperation must enlarge a possibility set.

Finally, comparative expansion and warranted symbiosis answer different questions. A coalition can produce an additional scientific dataset while transferring disproportionate maintenance costs to one participant, weakening local control over data, or intensifying disturbance to a living environment. Even inclusive expansion across operational outcomes leaves the distribution of costs, consent, dependence, recognition, and longer-term ecological effects to be evaluated. A substantive claim of beneficial cooperation consequently requires explicit evaluative criteria and evidence about the conditions through which outcomes are obtained and sustained. The present section establishes a conditional account of *additional attainability*; Section 7 will examine the further grounds of reciprocal and ecologically responsible relations.

The comparison has so far treated each scenario as possessing a specified configuration space and transition relation over the selected horizon. Heterogeneous cooperation can also alter the repertoire of available operations, the permissions through which activities become admissible, or the categories through which outcomes are described. In that case, the structure used for the comparison itself changes. Section 6 turns to these transformations and the conditions under which reachability relations drawn

from different stages can be related with appropriate mathematical and interpretive care.

6 Transformation Of Cooperative Structures

Section 5 compared the attainable outcomes of cooperative arrangements whose configuration systems were specified under a common protocol. A further possibility deserves separate treatment. Through collaboration, participants can alter the conditions that determine which operations are available in the first place. They may develop a shared instrument, negotiate a new access rule, establish a maintenance practice, or create a role through which previously separated activities become coordinated. Such changes concern the organization of future activity as well as the outcomes of an existing activity. Within a generative-relational inquiry, they invite a study of how cooperative structures themselves become objects of collective action.

An operational structure can change in several analytically distinct respects. New equipment may make a previously infeasible transition executable; an agreement may authorize an operation that was technically possible but institutionally excluded; a changing environment may remove an earlier option; and an emerging division of labor may establish new intermediate configurations. A revised description can also register capacities or relationships that an earlier model left implicit. These developments have different evidential implications. The first examples concern changes in material or institutional conditions; the last may concern a change in the analyst's resolution. In practice, material activity and descriptive revision may occur together. The inquiry therefore benefits from recording both the transformation being claimed and the basis for relating its successive descriptions.

Consider a coastal observation project whose initial arrangement permits collection, local calibration, and storage of measurements through separate organizations. Subsequent cooperation may establish a jointly maintained calibration facility and a negotiated protocol for exchanging instruments and data. The participating organizations then acquire routes through which a measurement can be checked, repaired, and reused. Describing this development simply as the addition of an outcome to an attainable set misses a consequential feature: the newly established facility and protocol can change which *future transitions* are feasible. Their continuing maintenance, labor requirements, and authorization conditions are part of that transformation. Ostrom's analysis of institutional diversity is useful here because it treats the rules structuring an action situation as themselves susceptible to collective modification (Ostrom (2005, chap. 1)). Suchman's treatment of human-machine configurations likewise directs attention to materially enacted arrangements and the practices through which they change (Suchman (2007, chap. 15)). These sources motivate the domain of inquiry; the formal comparison developed below is specific to the present paper.

The distinction between changes *within* a fixed system and changes *of* the system is relative to the chosen model boundary. If the configuration state includes current permissions, equipment inventories, and a representation of governing rules, then a rule revision may appear as an ordinary transition within a richer state space. A more economical account may instead compare two stage-specific transition systems. Both can be legitimate representations for different questions. The first offers continuity at finer descriptive resolution; the second makes the altered repertoire of operations salient. Choosing between them calls for a reasoned account of the resources, records, and explanatory purpose at stake, rather than an assumption that one form is universally prior.

Definition 6.1 (Stage-Relative Cooperative Structure And Translation). For each descriptive stage $i \in \{\alpha, \beta\}$, let $S_i = (C_i, R_i, \kappa_i)$ specify a collaborative-configuration domain C_i , an admissible one-step relation $R_i \subseteq C_i \times C_i$, and the contextual conditions κ_i supporting its interpretation. Write $\preceq_i = R_i^*$

for the reflexive–transitive reachability relation under the composability assumptions of Definition 3.1, and let $Q_i = C_i / \sim_i$ be its mutual-reachability quotient. A *partial stage translation* is a function $\tau : D \rightarrow C_\beta$ on a stated comparison domain $D \subseteq C_\alpha$. It is *forward reachability preserving* when

$$c, d \in D \wedge c \lesssim_\alpha d \implies \tau(c) \lesssim_\beta \tau(d). \quad (6)$$

The stage labels identify descriptions selected for comparison. The translation’s domain, intended meaning, and evidential support are part of the specification; the definition itself asserts neither a material transformation nor a unique global temporal ordering of stages.

The use of a partial translation reflects an ordinary feature of evolving collaboration. An older configuration may have a recognizable counterpart after the introduction of a new participant or instrument, while a newly constituted task may have no predecessor at the earlier stage. Conversely, a discontinued procedure may lack a later counterpart. A map that is defined only for genuinely comparable configurations expresses this limitation more accurately than an arbitrary identification of all elements of two domains. A translation may also compress differences: two previously distinguishable arrangements might receive the same later descriptor. A defensible comparison therefore states how configurations are matched and which aspects of their operation the matching preserves.

The preservation condition in Definition 6.1 concerns *complete reachability* among configurations in D . Checking that mapped one-step edges remain admissible is sufficient only when the relevant paths can be transported coherently, including their intermediate states. A path between two members of D may travel through configurations outside D , for which τ has no assigned value. In that case, the existence of a corresponding path at stage β requires a separate argument. This caveat parallels Section 3’s requirement that resource and permission states support path composition. It also prevents a discontinuity in a translated historical record from being disguised as a continuous operational route.

A forward-preserving translation has a useful mathematical consequence for the class descriptions introduced earlier. The comparison domain inherits the earlier full-system reachability relation restricted to its members; it therefore retains relations whose witnessing paths pass outside that domain. It lets comparative analysis move from individual configurations to operational equivalence classes, while marking the precise sense in which structural information may be retained.

Proposition 6.2 (Order-Preserving Transport Of Cooperative Classes). Suppose $\tau : D \rightarrow C_\beta$ satisfies Definition 6.1 and is forward reachability preserving. Let $Q_{\alpha,D} = D / (\sim_\alpha|_D)$ carry the partial order induced by the restriction of $\lesssim_\alpha$ to D , where restricted reachability may have paths passing through intermediate configurations outside D . Then

$$\bar{\tau} : Q_{\alpha,D} \longrightarrow Q_\beta, \quad \bar{\tau}([c]_{\alpha,D}) = [\tau(c)]_\beta \quad (7)$$

is well-defined and order preserving. The assumptions permit distinct source classes to be mapped to one target class and permit new target classes or comparabilities outside the translation’s image.

Proof Sketch. If $c \sim_\alpha d$ with $c, d \in D$, then $c \lesssim_\alpha d$ and $d \lesssim_\alpha c$. Forward preservation implies $\tau(c) \lesssim_\beta \tau(d)$ and $\tau(d) \lesssim_\beta \tau(c)$, so their images belong to the same target class. Thus $\bar{\tau}$ is independent of the chosen representative. If $[c]_{\alpha,D} \leq [d]_{\alpha,D}$, then $c \lesssim_\alpha d$, and forward preservation yields $[\tau(c)]_\beta \leq [\tau(d)]_\beta$. Injectivity, surjectivity, and reflection of order require further hypotheses. $\square$

The proposition states a conditional fact about representational transport. Its conclusion is deliberately asymmetric. A source relation can survive under translation even when the target contains additional paths, and a target can identify classes that were previously distinguishable. A reverse comparison would require its own matching and preservation conditions. The result thereby supports a disciplined

use of operational equivalence across stages while preserving the possibility that the equivalence classes themselves change. In particular, a later class cannot generally be interpreted as the continued identity of an earlier class merely because a translation sends one to the other. Its members, available paths, and historical formation may have changed.

A simple fixed-domain case makes this structural change visible. Let $C = \{a, b, c\}$ represent three collaborative configurations, and initially admit the transitions $a \rightarrow b$ and $b \rightarrow c$. Their reachability order has three distinct classes arranged in a chain. Suppose a genuinely new, repeatably feasible operation $c \rightarrow a$ becomes available under a revised cooperative protocol, with the necessary reset resources and permissions included in the configuration description. The revised transition system is strongly connected: each configuration reaches the others, and the quotient now has a single class. Here the new relation changes the operational equivalence structure, even though the set of named configurations remains fixed. An assessment based only on which terminal configuration names exist at either stage would overlook this altered capacity for cycling among arrangements.

Remark 6.3 (Reachability Changes Under Revised Operations). For a fixed configuration domain C under consistent path-composability semantics, if $R_\alpha \subseteq R_\beta$, then $R_\alpha^* \subseteq R_\beta^*$ and $\sim_\alpha \subseteq \sim_\beta$. Under this pure addition of admissible transitions, existing mutual-reachability classes can merge and cannot split. Withdrawal of operations may instead separate formerly equivalent configurations. When configuration domains or state descriptions change, comparisons of class membership require an explicit translation such as Definition 6.1; simple inclusion of transition relations is then inapplicable.

This monotonicity observation is elementary but useful for evaluating claims of cooperative development. A new permitted operation can increase reachability while also changing the quotient's number and membership of classes. Such a merger is a refinement of operational possibilities, rather than evidence that histories, agents, or normative obligations have become identical. Equally, a cooperative institution may increase some capacities while withdrawing others, for example by replacing unrestricted instrument access with a safer and more resource-conscious shared protocol. Its revised relation need have no set-inclusion ordering with its predecessor. The possibility of both gains and losses was already visible in Section 5's attainable-outcome comparison; here it concerns the operations that support future attainability.

Structural change also invites a question of attribution. A new edge in a model can represent actual improvements in equipment, newly granted legal access, acquired expertise, or an analyst's belated recognition of an existing procedure. The appearance of a transition in a later diagram is compatible with each of these explanations. Stronger evidence can connect the difference to documented practices: agreements establishing permissions, technical demonstrations of the newly feasible operation, maintenance records showing sustained availability, or counterfactual comparisons with arrangements lacking the cooperative intervention. Where such evidence is incomplete, the appropriate claim concerns a *mod-elled* change in reachability; a material or causal account remains a further research task. This distinction is especially important where institutional changes redistribute who can authorize a transition or who can bear its costs.

An evolving relation may also modify its own conditions of revision. Participants can establish procedures for changing operating permissions, training new members, or replacing instruments. These are operations whose objects include the rules and resources governing other operations. They can be represented explicitly within a sufficiently rich configuration state, or through a documented succession of stage-relative models. Either choice makes a concrete modeling commitment about the level at which change is tracked. The layered approach to rules and action situations in Ostrom (2005) supplies a relevant institutional comparison, while the present reachability construction offers a particular formal

vocabulary for recording which transitions remain available after a revision.

At a wider scale, heterogeneous co-creation may produce novel configuration types. A cooperative network that develops a shared monitoring service, for example, can give rise to tasks, records, access obligations, and maintenance relationships that were outside the older model's domain. The significance of this development is partly captured by the new configurations and their links to prior arrangements. It may also require an enlarged account of what counts as an outcome, a resource, or a participant. Here the stage translation can legitimately remain partial, and the analyst may preserve a separate narrative or provenance record explaining how the new descriptions arose. The emergence of a new model category alone establishes a change of representation; establishing a new practical capacity calls for evidence that the corresponding activity is feasible under actual constraints.

The generative-relational implication is therefore specific and testable in principle. Cooperative activity can affect both the realizable paths through a given configuration structure and the structure of permissible paths itself. A responsible explanation identifies the participating practices that sustain the change, states the comparison mapping where stages differ, and separates representational revision from demonstrated transformation in operational conditions. This approach enriches Section 5's comparison of outcomes by attending to the jointly created means through which further outcomes become available. It also leaves space for asymmetries: the new structure may be sustained by unequal labor, may depend on revocable permissions, or may expose particular participants and environments to continuing burdens.

Whether such transformations deserve the stronger description of *symbiosis* depends on further judgments about who benefits, who bears risks, which capacities remain viable, and how decisions and obligations are distributed. These are matters of warranted normative evaluation rather than consequences of an order-preserving map or an enlarged reachability relation. Section 7 develops the conditions under which the cooperative structures examined here may support claims of reciprocity, welfare, and ecologically responsible coexistence.

7 Conditions For Warranted Symbiosis

The preceding sections have established several ways in which collaborative configurations can be compared. Mutual reachability yields operational equivalence under specified transition rules. A joint arrangement can make an outcome attainable beyond the opportunities offered by selected alternatives. Cooperation can also modify the rules, resources, and configuration domains through which subsequent possibilities arise. Each result concerns an important aspect of collective activity. A further inquiry is required to determine when the underlying relationships deserve an affirmative ethical assessment. The expression *warranted symbiosis* will here designate an evidence-supported judgment of beneficial, responsible coexistence under explicitly justified standards. Its use calls for attention to the participating subjects, the affected environment, and the history through which the arrangement is maintained.

Terminological care is especially important. In biological research, *symbiosis* has several established usages, including a broad description of sustained interspecific association whose consequences may be mutualistic, commensal, or parasitic. The diversity of usage and the challenge of comparing effects on different partners are discussed by Neubauer et al. (2024), who also propose a more particular terminological convention for plant-microbe research. A biological relationship can accordingly be classified as symbiotic without an affirmative ethical endorsement of its effects. When the present inquiry extends the vocabulary to human institutions, technological systems, and ecological collaborations, the extension is an expressly proposed analogy. The additional adjective *warranted* carries a normative task: it

calls for reasons supporting the arrangement's benefits, legitimate conditions, and sustainable continuation. The analogy supplies neither a biological identity among its participants nor a settled account of the moral standing of artificial agents.

Consider a coastal observation project jointly sustained by residents, researchers, instrument maintainers, and a local ecosystem. Reliable measurements may depend on permission to enter particular sites, continued technical upkeep, and habitat conditions allowing observations to take place. The arrangement can generate scientific benefits even while concentrating uncompensated maintenance labor, restricting the community's access to the resulting records, or disturbing sensitive habitats. Such consequences occupy different registers. Measurement quality concerns an outcome; access and decision power concern governance; habitat disturbance concerns ecological effects; and the recognition of responsibility concerns a historically situated relationship. A reachability description helps identify feasible paths through this arrangement. Evaluating those paths requires evidence about who carries them and what their continuation entails.

An ethical appraisal begins with the identification of affected parties. The roster of formally named collaborators is often narrower than the set of people and living systems exposed to a project's consequences. Employees maintaining infrastructure, local communities dependent on the site, and animals affected by field operations can bear significant consequences even where they lack authority over project decisions. Identifying their interests requires domain-appropriate evidence and a stated account of which interests receive consideration. For human participants, agency, adequately informed participation, contestability, and realistic opportunities to revise or leave an arrangement may be relevant. For nonhuman animals, welfare and ecological conditions require species-appropriate assessment and safeguards. For current artificial systems, obligations borne by human operators and institutions can be assessed independently of contested questions about machine experience or moral status. These distinctions allow common relational analysis alongside different normative standards.

The comparison also needs a temporal and distributional scope. An arrangement may create immediate gains through the gradual depletion of shared resources, increased workloads, or deferred maintenance. Benefits measured at the organizational level can coexist with concentrated exposure to risk among particular participants. A credible appraisal therefore records relevant baselines, the interval of assessment, mechanisms through which continuing costs arise, and the position of parties with limited bargaining power. Social-ecological systems research provides an important parallel: Ostrom (2009) develops a framework for investigating how interacting resource, governance, actor, and ecological conditions bear on sustainability. The present approach draws on that concern with linked conditions while requiring its own case-specific standards of welfare and responsibility.

Definition 7.1 (Contextual Warrant For Beneficial Symbiosis). Fix a cooperative arrangement J , a time interval H , a comparison protocol η , and a documented set A_η of directly participating and materially affected parties. Let ν_η specify independently defended evaluative standards, including applicable considerations of welfare and harm, distribution of benefits and burdens, effective participation and redress, responsibility for sustaining the arrangement, and ecological viability. Let E_η denote the evidence available for these assessments. A claim that J constitutes *warranted beneficial symbiosis relative to* $(H, \eta, \nu_\eta, E_\eta)$ is supported to the extent that the evidence substantiates the applicable standards, material uncertainties and excluded effects are disclosed, and affected parties have appropriate opportunities to challenge the record or its criteria. The warrant remains qualified by the adequacy of those standards to the parties and consequences concerned. The definition supplies a structure for a contextual warrant; it assigns neither universal weights to these considerations nor a presumption of equal normative standing among heterogeneous participants.

This definition is deliberately evaluative rather than a theorem about cooperative graphs. Its content depends on reasons that precede the application of a mathematical reachability relation. Ethical standards can conflict, and a case may warrant only a qualified judgment. For example, a short-term research opportunity may be valuable while ecological monitoring remains incomplete. The appropriate conclusion can identify the supported benefits, the unresolved risks, and the conditions for continuation. In this form, the framework promotes the legibility of a normative argument without imposing a single quantitative score on unlike forms of value. The diversity of ways in which people value natural systems, emphasized by the IPBES assessment of nature's values (IPBES (2022)), provides a further reason to document the standards and participation procedures through which ecological evaluations are made.

The logical independence of operational success and ethical appraisal can be exhibited directly. Take a fixed set of collaborative configurations C with an admissible transition relation R , and suppose that each feasible path carries records of who performed labor, who approved access, what material was consumed, and what effects were incurred. The projection onto C and R may discard precisely this information. A project that employs a fair and revisable division of maintenance duties and an otherwise identical project sustained through involuntary uncompensated work may display the same admissible configuration transitions. Their observed paths can also culminate in the same measurements. The difference resides in the burden and governance annotations and in the histories supporting those transitions. A mathematical quotient constructed from R^* remains the same under these distinct annotations, provided the annotations are external to the stipulated admissibility relation. If an authorization difference changes which transitions are feasible, the operational models must reflect that difference instead.

Proposition 7.2 (Independence Of Reachability And Normative Appraisal). There exist cooperative descriptions with identical configuration sets, transition relations, reachability preorders, and quotient partial orders, yet with different recorded distributions of benefit, burden, and authority. Consequently, these order-theoretic data alone determine neither satisfaction of a specified ethical standard nor the warrant for beneficial symbiosis.

Proof Sketch. Let $C = \{a, b\}$, let $R = \{(a, b)\}$, and take the same initial configuration and realized transition in two arrangements. Their reflexive-transitive closures and quotient orders coincide. Enrich each arrangement with an annotation λ on the realized transition. In the first, λ_1 records an agreed and fairly compensated task; in the second, λ_2 records a coercively imposed task borne by a vulnerable participant. Under an evaluative standard that distinguishes legitimate participation and imposed burdens, the two annotations support different appraisals while the reachability structures remain identical. The construction establishes the claimed logical independence. $\square$

The proposition uses a hypothetical contrast. It demonstrates the information required for an ethical inference rather than supplying empirical evidence that particular collaborations have the stipulated features. An investigator might enlarge the configuration descriptions so that compensation, authorization, and institutional constraints become explicit state variables. Such enlargement can make part of the relevant difference visible within an enriched operational model. The ethical interpretation of those variables, including the justification of what constitutes coercion or adequate compensation, still rests on independently stated normative premises. Thus increased descriptive resolution is valuable, while responsibility for ethical judgment remains with accountable deliberation and evidence.

Reciprocity warrants similar care. Different participants may contribute different kinds of resources, and the comparison of those contributions often lacks a natural common unit. A person can contribute knowledge and consent; a research organization can supply instruments; an automated service can supply computation; an ecosystem can sustain life processes on which a project depends. Some contributions are intentional acts, while others are enabling conditions. Reciprocity among people may involve

recognition, fair terms, participation in rule revision, and reliable commitments across time. Ecological responsibility can instead require habitat safeguards and restraints on extraction. Mechanical symmetry of contribution or equal financial return provides an inadequate universal test for these relations. The relevant standards should make clear who can undertake obligations, who can assert claims, and whose interests must be represented through other procedures.

Remark 7.3 (Reciprocity Across Unequal Dependencies). An affirmative appraisal of reciprocity may accommodate asymmetric contributions, benefits, vulnerabilities, and capacities to undertake obligations. Its warrant depends on defensible terms of participation, the handling of power and dependency, and arrangements for recognition, contestation, and repair, as appropriate to the parties involved. Operational equivalence, symmetric transition relations, and equal shares of a measured output each leave these matters open.

Power relations become especially consequential where the parties' options to exit or contest a rule differ. A collaboration may be formally voluntary and yet depend on access to wages, medical care, infrastructure, or permissions controlled by another party. The ability to refuse an assignment can then vary sharply even among participants who occupy comparable operational roles. Long-term dependency also affects the quality of reciprocal commitments: a powerful partner may be able to withdraw crucial resources at short notice, whereas a less powerful partner has invested in capacities useful only within the relationship. Documenting such dependencies makes the arrangement's generative conditions visible beyond the set of reachable configurations. An assessment can examine whose approval changes the rules, who has access to evidence, whose complaints receive review, and which practical remedies exist after harm.

Responsibility in such cases can be distributed without dissolving into anonymity. Young (2006) develops a social-connection account of responsibilities arising through structural processes, emphasizing shared forward-looking work to address injustice. Its scope concerns moral agents implicated in social arrangements. The present inquiry takes this as a relevant approach to collective responsibility, rather than as an equation assigning equivalent moral duties to every component of a heterogeneous system. Human participants and institutions may hold different decision-making powers, legal obligations, and capacities for redress. Instruments and artificial systems may figure centrally in a causal or organizational account while the responsibility for their deployment, maintenance, and oversight remains subject to independent ethical and institutional analysis.

An ecologically credible account also examines what sustains a cooperative trajectory. Additional measurement capacity can improve environmental understanding and at the same time intensify energy use, material extraction, habitat disturbance, or electronic waste. Even when a particular intervention creates a net gain under one valuation, its effects across species, places, and future periods may remain uneven. Ecological viability consequently invites scrutiny of material flows, renewal and depletion rates, thresholds of disturbance, and the continued conditions of life affected by the work. Data from such assessments can support a qualified decision while uncertainties are openly recorded. The standing of ecological values, including their relation to human interests and to nonhuman welfare, calls for plural and carefully justified evaluation rather than an assumed single metric.

This account permits provisional judgments and revision. A cooperative arrangement may deserve support when its available evidence indicates significant shared benefits and adequate safeguards, coupled with effective monitoring and procedures for responding to emerging harms. Its warrant may weaken if new records reveal displaced burdens, unexpected ecological losses, or failures of governance. Conversely, parties may improve an arrangement by revising compensation, redistributing authority, adding habitat protections, or creating accessible complaint procedures. These revisions can affect the

transition structure analyzed in Section 6 as well as the normative evidence evaluated here. An affirmative concept of symbiosis can therefore remain responsive to changing relations without presuming that an initially favorable judgment guarantees its future continuation.

The contribution of this section is to articulate the conditions under which the reachability and structural-transformation results of Sections 3–6 can enter a responsible account of cooperative life. Equivalence classes summarize a particular form of operational accessibility; comparative outcome sets identify relative possibilities; stage translations track changes in cooperative structure. Evaluative protocols then ask what these arrangements mean for persons, other living beings, and the material systems on which they depend. The disciplines of proof, empirical identification, and normative justification each retain their own standards, while their findings may inform one another. Section 8 develops how this combined approach can guide the study, design, and governance of heterogeneous co-creation.

8 Implications For Heterogeneous Co-Creation

The preceding analysis offers a way to examine cooperative arrangements through their feasible transformations, the differences compressed by operational equivalence, and the conditions under which collective activity changes the structure of subsequent possibilities. Translating this account into practice requires a sequence of accountable judgments. The relevant questions concern which configurations matter for a project, how available transitions are established, which differences are preserved in the record, what cooperation makes possible under a specified comparison, and how the benefits and burdens of these possibilities are assessed. These questions arise together in the design of scientific collaborations, autonomous technical systems, institutional partnerships, and human relationships with shared environments. Their answers may be recorded with varying degrees of precision according to the consequences of the activity.

The initial design task is to identify the cooperative undertaking and its affected relations at an appropriate resolution. In a marine observation program, for example, a collaborative configuration may include field researchers, coastal residents, mobile sensors, a data-processing service, the permissions governing site access, equipment energy states, and environmentally sensitive operating areas. These components enter the description through different roles. A resident or researcher can have reasons, authority, and obligations; a sensing instrument has operational capacities and maintenance requirements; an animal population has conditions of welfare and ecological vulnerability; and an institutional rule determines which actions are permitted. A common configuration model makes these dependencies available for joint inquiry while preserving the distinct kinds of evidence and justification appropriate to each. The description should also identify affected parties whose interests extend beyond the roster of formally recognized collaborators.

An operational model is useful insofar as its transitions correspond to coherent feasible activities. Section 3’s reachability construction provides one option when state descriptions retain the resources and permissions required for path composition. A planner could examine whether a configuration with locally governed sensor access is reachable from a laboratory-governed one, whether a maintenance procedure enables return to an earlier arrangement, or whether an emergency stop remains feasible after a device enters a restricted area. The resulting equivalence classes summarize reciprocal operational access. Their use in planning should be accompanied by sufficient details about the steps, costs, and permissions by which configurations are connected. A shortest-path result under estimated energy consumption, for instance, carries different evidential significance from a witnessed field operation, and both differ from a negotiated institutional permission.

This distinction suggests a practical rule for selecting abstraction levels. A quotient representation can support high-level planning when the decision concerns class-level reachability. A question about who authorized a route, who maintained a sensor, or who bore a disturbance requires access to the underlying occurrences and annotations. The criterion established in Proposition 4.1 determines when an attributed descriptor is constant on each equivalence class. Where a descriptor varies within a class, a reviewable system can preserve representative-level records or refine the model’s resolution. Such selective abstraction makes compact cooperative maps compatible with accountability for the histories they summarize.

Data provenance is particularly important where a cooperative process crosses institutional and technical systems. The World Wide Web Consortium’s PROV-O recommendation provides terms for describing entities, activities, agents, and relationships of use, generation, association, and attribution (W3C (2013)). This vocabulary can help represent how a measurement was collected, transformed by a model, reviewed by a researcher, and included in a shared report. The recommendation provides a provenance representation; the completeness and reliability of a particular record remain empirical matters. In the present framework, provenance entries may be connected to configurations and paths while retaining time-specific permissions, uncertainty about attribution, and records of consequential changes. Their purpose is to make claims about a collaborative process open to examination, rather than to infer ethical responsibility automatically from a machine-readable relation.

The comparison of prospective alternatives supplies a further discipline. Section 5 defines joint and baseline attainable-outcome sets under an explicit protocol η . For a proposed collaborative intervention, the relevant baseline might be a human-operated survey, a limited sensor deployment, or an existing arrangement for community observations. The comparison should maintain a common outcome description and account for differences in initial conditions, duration, permissions, and resources. A demonstrated outcome in $\mathcal{A}_J(\eta) \setminus \mathcal{A}_B(\eta)$ can witness an additional possibility under those assumptions. The design inquiry must then examine whether other baseline possibilities were relinquished, whether the difference is robust to credible alternative scenarios, which pathways have been witnessed, and what additional evidence supports the feasibility of modeled paths. Where field trials, natural experiments, or historical comparisons are unavailable, the result should be presented as a scenario-relative prospective judgment.

A program capable of improving its own cooperative procedures raises an additional challenge. Participants may revise the rules for accessing data, establish a new maintenance practice, or create a previously unavailable channel for ecological intervention. Section 6 treats such changes through stage-specific structures and explicit translations between their configuration domains. For practice, a record of the change should distinguish new physical operations, revised institutional permissions, altered resource conditions, and improved descriptive resolution. It should document which previous capacities remain available and which transition paths depend on the new arrangement. This makes it possible to ask whether the collective activity has expanded the means of generating further possibilities, rather than merely improving performance on a fixed task. The evidence can include equipment tests, rule amendments, observed work practices, and the continued availability of resources supporting the new paths.

These requirements can be assembled into a bounded procedure without prescribing one universal measure of cooperative success.

Definition 8.1 (Reviewable Cooperative Inquiry Record). For a cooperative inquiry context κ and assessment protocol η , a *reviewable cooperative inquiry record* consists of linked documentation of (i) the participating and materially affected relations, with the relevant configuration domain and admissibility conditions; (ii) the feasible transitions, realized occurrences, and provenance needed to support claims about paths and responsibilities; (iii) the compared scenarios, outcome descriptions, and evidence for prospective or realized attainability; (iv) revisions in operational structures, with explicit comparison mappings where configuration domains differ; and (v) the applicable evaluative standards ν_η , affected-party participation procedures, and evidence bearing on welfare, burdens, redress, and ecological viability. Its claimed conclusions are accompanied by a statement of their domain, uncertainties, and opportunities for review. This is a proposed research-and-design record, whose appropriate level of detail and accessibility depend on the stakes, participant rights, confidentiality obligations, and purposes of the inquiry.

The record is intended to connect distinct forms of inquiry while preserving their standards of justification. A path may be established by a transition-system calculation, a provenance claim by an activity log and corroborating testimony, a counterfactual comparison by explicit modeling assumptions and appropriate evidence, and an ethical conclusion by reasoned deliberation under defensible standards. Agreement among these layers strengthens a cooperative explanation; disagreement identifies a matter requiring investigation. An institution may consequently revise its transition model after observing repeated equipment failures, reconsider an outcome comparison after discovering an excluded baseline, or alter its governance procedures after affected participants identify a concentrated burden. The record supports such revision without requiring an initial determination of every participant's ultimate ontological status.

Institutional participation deserves a more specific treatment than the assignment of named agents to tasks. Permission to access a site, consent to a use of records, authority to revise a rule, and an effective means of contesting a decision can affect both reachability and the warrant for cooperation. The CARE Principles for Indigenous Data Governance, articulated by Carroll et al. (2020), provide an important domain-specific account of collective benefit, authority to control, responsibility, and ethics in the governance of Indigenous data. They illustrate how information-sharing arrangements can require respect for collective rights and locally grounded authority alongside technical interoperability. Their scope is specific to Indigenous peoples and their data governance; applying them to a particular partnership requires attention to the communities concerned and their own decision-making institutions. More generally, the present framework asks each collaboration to identify whose authorization is necessary, who may revise the terms of use, and which arrangements make participation and redress practically effective.

For artificial systems, a comparable practical task concerns the allocation of control and the verification of delegated operations. A model may propose survey routes, detect changes in habitat measurements, or coordinate robotic instruments. Its recommendations can influence the actions available to human participants, especially where the system controls access to information or acts at operational speed. The NIST Artificial Intelligence Risk Management Framework organizes voluntary risk-management work through governance, contextual mapping, measurement, and management, with attention to the system life cycle (NIST (2023)). Those practices can inform the present inquiry's procedures for testing constraints, documenting failures, allocating human oversight, and revising deployment permissions. Their use here remains selective: the GR distinction between configuration equivalence and generative conditions addresses an explanatory question, while the risk framework addresses practical management of AI systems. Responsibility for deployment and governance must be specified through applicable human and institutional roles; an AI system's participation in a cooperative configuration by itself settles

neither legal accountability nor contested questions of machine moral status.

Ecological relations impose further design requirements. In the marine example, a new sensor route may improve detection of environmental changes while increasing disturbance to marine animals or using materials whose maintenance and disposal carry environmental costs. An appropriate review can compare habitat exposure, energy use, expected device loss, repair practices, and the distribution of informational benefits across affected communities. Monitoring thresholds and intervention procedures can be incorporated into admissible operations where evidence justifies them. Their ethical adequacy continues to depend on species- and context-appropriate standards. The social-ecological perspective of Ostrom (2009) and the plural valuation concerns identified by IPBES (2022) are relevant here: the state of an ecosystem, the rules governing resource use, and the interests recognized in evaluation interact over time. Treating ecological viability as a continuing condition of cooperation gives practical content to the warranted-symbiosis inquiry of Section 7.

A design may also require procedures for recovering from failures and repairing relations. Consider an automated observation service that produces useful scientific records while repeatedly restricting community access to raw measurements. Correcting an access-control mechanism can restore certain permitted paths, yet a trustworthy repair may additionally require explanation of the previous restriction, appropriate remedy, and renewed agreement on future uses. Likewise, removing an environmentally disruptive operation may reduce the system's immediate set of feasible actions while improving the conditions sustaining its longer-term operation. These examples show why evaluating cooperative transformation solely by the number of reachable configurations can obscure the direction and distribution of its consequences. The record should instead support comparison among operational changes, the histories generating them, and their normative justification.

Remark 8.2 (Reviewability Across Structural Change). When cooperative activity changes a configuration domain, admissible relation, or institutional permission, the relevant inquiry record should preserve a justified relation between earlier and later descriptions, document the supporting occurrences and evidence, and revisit the associated counterfactual and normative assessments. An order-preserving stage translation, where available, provides one component of this comparison. Continued reachability, historical continuity, and the warrant for beneficial symbiosis require their respective evidence and standards.

The framework consequently suggests a research program with several distinct empirical questions. Investigators can study which operational distinctions are lost when collaborative activities are compressed into equivalence classes; whether more complete provenance changes attributions of responsibility; which collective interventions produce robust additional attainable outcomes under plausible baselines; and whether governance revisions preserve ecological and social conditions required for future cooperation. These questions are open to case studies, participatory assessment, controlled technical comparisons where appropriate, and formal model validation. Their conclusions should distinguish demonstrated transitions from inferred opportunities, and localized findings from broader analogies among different kinds of subjects.

The implication for generative-relational inquiry is practical as well as conceptual. Cooperative structures can be used to plan feasible actions, compare possibilities, and understand transformations in the conditions of joint production. Their evaluation also calls for records through which individual histories, unequal dependencies, and affected interests remain visible. A disciplined connection among mathematical reachability, historical evidence, counterfactual analysis, and normative review allows the study of co-creation to remain both explanatory and accountable. The final section will consolidate the resulting scope of relational equivalence and identify the questions that remain open for subsequent

research.

9 Conclusion

Heterogeneous cooperation joins participants whose histories, material conditions, and practical capacities may differ even when their actions converge on a shared task. This inquiry has developed a way to compare such arrangements while retaining the differences relevant to explanation and judgment. Its principal claim is that relational equivalence has a legitimate and productive scope: it can describe reciprocal accessibility among collaborative configurations under stated operational conditions. The same inquiry must remain attentive to the histories, possibilities, and dependencies through which those configurations are produced and sustained. In this sense, equivalence and heterogeneity are compatible dimensions of cooperative analysis. Their relation depends on the resolution and purpose of the description.

The mathematical foundation is deliberately modest. Given a configuration space and a coherently composable relation of admissible transitions, finite-path reachability yields a preorder. Mutual reachability then induces an equivalence relation whose quotient carries a partial order. These constructions give useful summaries of operational accessibility. Their application also depends on specifying resources, permissions, and other state variables required to compose feasible paths. The quotient preserves the reachability judgments encoded by the preorder; it supplies a more compact representation by identifying members of each mutual-reachability class. The proof of a partial order establishes the properties of that representation under its assumptions. The further question of what it explains about actual cooperation is answered through the adequacy of those assumptions and the evidence supporting them.

Several consequences follow from taking this boundary seriously. Configurations occupying the same mutual-reachability class can differ in path costs, provenance, participants' exposure to risk, and the distribution of authority. A class-level attribution is therefore warranted only where the relevant property is constant across the class or where an explicit aggregation rule supplies the intended meaning. Moreover, a record of realized occurrences can preserve distinctions that disappear when paths are projected into configuration labels. Historical continuity, operational equivalence, and the identities of participating subjects consequently demand different descriptive resources. Maintaining these distinctions permits collective achievements to be understood alongside the particular contributions and conditions through which they arose.

The argument extends beyond the classification of existing cooperative configurations. Under an explicit counterfactual protocol, a joint arrangement may display additional attainable outcomes relative to specified alternatives. Attribution of that difference to the cooperative intervention relies on defensible causal or mechanistic evidence. Such a result identifies a comparative possibility and invites inquiry into its enabling relations. The stronger claim that cooperation retains every baseline possibility requires separate inclusion of attainable-outcome sets. Claims about efficiency, realized causal effects, or improved welfare require further measures and evidence. This separation makes the concept of collective generativity more discriminating: it can identify newly available opportunities while allowing their costs, exclusions, and practical significance to be examined independently.

Co-creation can also change the conditions under which further opportunities arise. Participants may develop instruments, revise operating rules, establish shared maintenance practices, or introduce new kinds of collaborative configurations. Comparing the resulting cooperative structures requires attention to the domains and admissible relations appropriate to each stage. A forward reachability-preserving

translation provides a conditional means of transporting equivalence classes and their ordering across such descriptions. Actual transformation, however, is established through evidence of changed operations, resources, practices, or permissions. The distinction between an altered system and an altered account of a system is particularly consequential when an apparent expansion depends on a newly recognized capability or a newly authorized action. Generative change thus becomes an object of inquiry in its own right, with the mechanisms sustaining it made available for investigation.

These descriptive results acquire their ethical significance through additional standards. A wider field of feasible actions can coexist with concentrated burdens, fragile dependencies, and ecological damage. The proposed concept of warranted symbiosis therefore concerns the grounds for judging a cooperative relation beneficial and responsible, including the distribution of its effects, legitimate participation, possibilities for contestation and repair, and the viability of affected living systems. Different participants may require different forms of consideration: institutional accountability, human consent and welfare, ecological safeguards, and careful treatment of unresolved questions concerning artificial agents. A shared operational class provides a basis for comparison; a defensible appraisal draws on evidence and normative reasoning appropriate to those relations.

The reviewable cooperative inquiry record proposed here links these levels without collapsing their methods. Transition models support claims about admissible paths; historical documentation supports claims about realized activities and provenance; matched comparisons support bounded inferences about collective possibilities; and deliberative procedures address the justification and revision of cooperative terms. In practical settings, these elements can guide the design of partnerships, autonomous services, research infrastructures, and environmental observation arrangements. Their use remains sensitive to the scope of a particular task, the standards governing its participants, and the quality of available evidence. A record that can be challenged and revised offers an institutional resource for responsible collaboration, subject to legitimate limits on access and disclosure.

Further research can proceed in several connected directions. Formal work can examine how uncertainty, stochastic transitions, resource depletion, and changing participant sets affect reachability and equivalence. Empirical and historical inquiry can test whether more detailed records of provenance and access change interpretations of apparently equivalent cooperative arrangements, and can investigate which interventions genuinely alter future operational possibilities. Participatory and ecological inquiry can assess how alternative governance structures distribute benefits and burdens over time. These approaches may yield different accounts of the same cooperative setting; their productive comparison depends on keeping each account's evidential and normative commitments explicit. Questions concerning the ontological character of heterogeneous subjects and the foundations of generative order remain open to independent philosophical investigation.

Relational equivalence, then, offers a selective means of organizing the accessible structures of cooperation. Its explanatory value increases when joined to an account of historical difference, counterfactual possibility, structural transformation, and warranted evaluation. Heterogeneous subjects can co-create pathways through which further activities become possible, while retaining distinct identities, dependencies, and claims upon one another. Studying these relations calls for mathematical precision, historical attentiveness, and responsible judgment together. Such an inquiry can recognize common possibilities while preserving the conditions through which their creation and continuation matter.

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