

A Preliminary Dyadic Formalism of Volunteer Relations in Generative Relational Theory

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Abstract

This paper develops a preliminary dyadic formalism for volunteer and support relations within Generative Relational Theory (GR). The ordinary claim that one subject supports another already presupposes subject individuation, relational history, manifestation, epistemic access, action attribution, need and value judgment, authorization, and normative interpretation. The paper reconstructs these conditions before treating “support,” “need,” “good,” “beneficiary,” or “consent” as settled categories. At a declared analytical resolution, the dyad is represented by two history-bearing GR subject-spacetimes within a coupled relational configuration. A labelled 2-complex supplies one candidate discrete representation; spin-foam-inspired histories and operator-algebraic realizations enter only as optional model layers under ontological suspension. The formalism distinguishes ontological, relational, and manifestation reality and treats relational–manifestation mappings as historically variable. Mutual manifestation and recursive theory-of-mind structures provide the epistemic interface, while action is reconstructed as a derived object with provenance, attribution, manifestation, and subject-relative epistemic profiles. Need, value, aesthetic, and normative judgments remain separately typed and potentially divergent. Typed partial maps and local algebraic structures then support the analysis of procedure, consent and refusal, asymmetry, feedback, revision, and emergent structures such as trust and recognition. Finite dyadic illustrations and a layered verification architecture clarify what follows from the formal construction and what still requires empirical, normative, or ontological argument. The contribution is a philosophical-formal foundation for later work on triadic relations, subject co-formation, psychoanalytic structure, aesthetics, justice, ethics, political economy, power, and historical relational dynamics.

Keywords: volunteer relations; Generative Relational Theory; dyadic formalism; relational epistemology; action provenance

Discussion Paper Note

This manuscript belongs to a broader Generative Relational research programme on volunteer and aid relations. Its immediate task is foundational. It develops the minimal dyadic architecture required before later papers address triadic mediation, subject co-formation, psychoanalytic dynamics, the aesthetics of support, relational justice, volunteer ethics, political economy, power, and historical trust formation.

The manuscript is intentionally comprehensive in conceptual depth while restricted to two focal subjects. The restriction does not imply that real volunteer relations are ordinarily isolated dyads. It creates a minimal domain in which the relations among subject history, manifestation, epistemic access, theory of mind, action provenance, evaluative formation, consent, refusal, asymmetry, and relational emergence can be stated before third-party institutions and wider social systems are introduced.

The mathematical resources used in the paper are modeling languages. A labelled 2-complex, spin-foam-inspired history, local operator algebra, field representation, category, groupoid, or dynamical system does not establish the ultimate ontology of volunteer relations. GR retains ontological suspension while asking whether these structures permit precise, revisable, and formally inspectable representations of historically generated relational processes.

The paper also distinguishes formal construction from empirical validation and normative settlement. A formally coherent representation can clarify what must be specified for an aid relation to be modeled, yet it does not by itself show that a concrete volunteer setting instantiates the model, that a support action is ethically justified, or that a manifested need corresponds to a normatively valid claim. Those questions remain available for later empirical, interpretive, and normative work.

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Status. This discussion paper presents a complete revised version of a preliminary formal inquiry into dyadic volunteer relations in Generative Relational Theory. Definitions, representations, examples, and research interfaces remain open to revision as the wider programme and future empirical applications develop. Public circulation does not constitute peer review, empirical validation, or normative settlement.

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Statement on the use of language models. The exploratory discussions and preparation of this paper involved OpenAI’s ChatGPT. ChatGPT supported exploratory dialogue, formal reconstruction, argumentative criticism, source discovery followed by website verification, drafting in $\text{\LaTeX}$, and compilation and consistency checks. The author selected the research questions, directed and approved the theoretical commitments and epistemic status of the claims, and bears sole responsibility for the manuscript, including its definitions, formal constructions, arguments, conclusions, and errors. Authorship credit remains with the human author.

1 Introduction

Volunteer action is often described through a compact relational form: one subject supports another. The description is practically useful, yet it conceals a substantial theoretical burden. Before an intervention can be identified as support, a theory must already have distinguished the relevant subjects, identified an event as an action, attributed that action to one or more sources, interpreted a condition as a need, formed some judgment concerning what may count as good for the affected subject, and determined how request, permission, refusal, competence, and responsibility bear on the interaction. The apparent simplicity of the dyad therefore depends upon prior relational, epistemic, normative, and historical articulation. This paper develops a preliminary formal architecture for making that articulation explicit within Generative Relational Theory (GR).

The motivation is philosophical before it is technical. Concepts such as volunteer, helper, supporter, beneficiary, need, assistance, consent, and good commonly enter social analysis after their practical meanings have already stabilized. That stabilization can make the concepts appear primitive even when their applicability depends upon a longer process of recognition, interpretation, role formation, communication, institutional inheritance, and repeated interaction. A volunteer can believe that an action benefits another subject while the other subject rejects the action, interprets its meaning differently, or understands the relevant need through a different normative vocabulary. Two observers can agree that a material transfer occurred while disagreeing about whether the event constituted aid, intrusion, obligation, reciprocity, compensation, care, or control. A subject can intentionally perform an action while possessing only a partial account of the history through which that action became desirable, available, or intelligible. These possibilities indicate that the dyadic relation cannot be adequately represented by attaching a scalar value to an already constituted action between already constituted actors.

The paper is adjacent to several established literatures without reducing its object to any one of them. Sociological research commonly treats volunteering as freely given time directed toward another person, group, or cause and distinguishes it from more spontaneous helping, while functional approaches show that sustained volunteering can serve heterogeneous motives rather than one unitary altruistic function [15, 3]. Care ethics places caring practices within moral and political relations, and relational-autonomy scholarship emphasizes that agency and autonomy are socially constituted rather than adequately represented by self-sufficient individuals [14, 10]. Work on epistemic injustice and recognition further shows why credibility, interpretive resources, and socially mediated recognition can matter to normative and relational analysis [5, 8]. Paper I does not attempt to synthesize these traditions substantively. Its narrower contribution is formal: it asks which typed objects and historical relations must be available before a dyadic support interaction can be represented without silently fixing its subjects, motives, needs, meanings, authority, or value.

GR supplies a distinctive starting point for this reconstruction. The subjects conventionally denoted by A and B are not treated as point-like entities whose identities precede their histories and relations. At a selected analytical resolution, each subject is represented as a history-bearing relational spacetime whose present capacities, vulnerabilities, expectations, interpretations, and available continuations depend partly upon accumulated relational structure. The dyad is consequently embedded in a coupled configuration containing the histories of both subjects together with the interfaces through which they become mutually manifest and capable of affecting one another. A labelled 2-complex provides one candidate discrete representation of such a history-bearing configuration. Spin-foam-inspired structures can provide a more specialized realization of labelled histories where their additional architecture is analytically useful. These representations remain formal resources under ontological suspension rather than claims that social subjects possess a quantum-gravitational ontology.

This treatment also changes the status of the labels supporter and supportee. They identify relational positions within a historically situated configuration rather than exhaustive identities of the participating subjects. A subject can occupy a supporter position in one interaction, a supportee position in another, several positions simultaneously, or a position whose meaning changes through the relation itself. Repeated interaction can stabilize a role, weaken it, reverse it, or reorganize the relation so that an initially

asymmetric helper-recipient pattern becomes a more reciprocal form of participation. The formalism must therefore be able to represent the formation and transformation of roles without identifying a subject with the role that the subject temporarily occupies.

The same requirement applies to action. A support action should not enter the formalism as an atomic primitive whose origin, attribution, and meaning are transparent. A relational process can become individuated as an action only after some structure permits the event to be distinguished, tracked, attributed, and interpreted. The action can be initiated by the would-be supporter, elicited by a request or manifestation from the supportee, jointly negotiated, generated through an established role, mediated by a norm, triggered by affect, sedimented through repeated practice, or distributed across a wider relational structure. Its executor, initiator, generator, and publicly recognized author can consequently differ. The actor's own account of why the action occurred can also differ from the other subject's attribution and from a later reconstruction of the action's generative history. Paper I therefore treats action provenance and action epistemology as components of the dyadic formal problem rather than as psychological supplements added after the action has been defined.

The epistemic problem is equally central. A history-bearing subject does not ordinarily gain transparent access to the relational state of another subject. The other becomes available through manifestations: speech, gesture, bodily condition, silence, documents, requests, refusals, institutional classifications, recurring practices, material traces, and other context-dependent forms. Observation of a manifestation remains distinct from knowledge of the relation that produced it. Interpretation adds further structure, and theory-of-mind reasoning adds another layer of modeled beliefs, intentions, preferences, expectations, and judgments. The supporter may therefore act on a model of the supportee rather than on direct access to the supportee's relational condition. The supportee simultaneously constructs models of the supporter, including models of the supporter's intentions, competence, reliability, and normative orientation. Higher-order structures arise when each subject models how the other subject models the relation. A dyadic formalism that omits these recursive epistemic relations risks converting uncertainty, disagreement, and misrecognition into hidden assumptions of the model.

The paper connects this epistemic architecture with GR's distinction among ontological reality, relational reality, and manifestation reality. Claims concerning ultimate ontological reality remain suspended. Relational reality provides the principal positive level of analysis: historically constituted structures can be treated as relationally real when the adopted model identifies them as sufficiently persistent and causally or generatively efficacious within the relevant domain. Manifestation reality concerns the context-dependent forms through which such structures become presented, expressed, observable, institutionally legible, or actionable. The relation between relational and manifestation realities is neither assumed to be one-to-one nor treated as historically fixed. One relational structure can support several manifestations, similar manifestations can be compatible with different relational structures, and a manifested classification or role can later become part of the relational structure when it acquires sufficient persistence and efficacy. The mapping between relational and manifestation levels is therefore itself a possible object of historical change.

This three-level architecture applies throughout the paper. Trust, for example, can be analyzed as a historically stabilized relational structure while its manifestations vary across reassurance, disclosure, delegated discretion, reduced monitoring, or silent reliance. Consent can be manifested through words, signatures, conduct, or withdrawal, while its relational significance depends upon competence, information, refusal capacity, dependency, and the history of the interaction. A role can manifest through a title or recurrent behavior while remaining relationally unstable. A norm can manifest through a rule statement, convention, sanction, or vocabulary while its relational formation remains contested. The paper therefore asks, for each major concept, how its relational realization, manifestation, epistemic accessibility, and historical trajectory should be distinguished. Ontological suspension remains in place throughout that analysis.

Need and good require particular care because volunteer action is commonly justified through them. A condition experienced or manifested by the supportee does not arrive already labeled as a need. The supportee can form a judgment concerning the condition, the supporter can form a different judgment,

and interaction can produce a provisional shared judgment that neither side possessed at the beginning. These judgments can draw upon bodily states, preferences, capabilities, prior commitments, professional knowledge, cultural expectations, legal categories, practical constraints, and historically stabilized norms. The resulting need judgments can converge, remain partially aligned, or conflict. Their formation should remain distinguishable from the further question of whether a need claim is normatively valid.

Goodness judgments have a similar structure. The supporter can judge that an action is good for the supportee while the supportee evaluates the same action differently. An intervention can provide an immediate material benefit while altering autonomy, dependence, dignity, trust, role structure, or later generative capacity. An action can also benefit the supporter by generating meaning, connection, recognition, competence, or a stable relational position. These possibilities do not reduce volunteering to self-interest. They show that the normative object is richer than a one-directional transfer whose value can be read directly from the intention of the helper or from one short-term consequence. The paper accordingly treats goodness judgments as typed and subject-relative evaluative objects before later work asks how competing judgments should be assessed.

Aesthetic and affective formation belong to the same minimal architecture. Support can be manifested through timing, pacing, tone, gesture, spatial arrangement, restraint, delicacy, reciprocity, silence, or the form in which a resource is offered. Such manifested structures can become salient before either subject articulates an explicit judgment that the interaction is fitting, dignified, graceful, awkward, intrusive, humiliating, excessive, cold, or otherwise aesthetically charged. Explicit aesthetic vocabularies can subsequently reorganize attention and alter later manifestations. The present paper preserves this recursive possibility without reducing normative judgment to aesthetic response. A later paper will examine the aesthetics of volunteer support directly; the current formalism requires only enough structure to represent pre-explicit differentiation, explicit judgment, and their possible historical co-formation.

The formal programme is therefore layered. At the highest level, the paper inherits a GR commitment to make the assumptions of a model visible before stronger mathematical structure is introduced. A global metric, additive comparison, probability distribution, or Hilbert space is not treated as a free starting point for heterogeneous relational objects. At a lower and more concrete level, the dyadic relation is represented through history-bearing configurations, admissible regions, relational interfaces, manifestations, epistemic constructions, and typed partial transformations. Local algebraic structure is introduced where composition requires it. Operator-algebraic representations, state functionals, open-system maps, and representation-dependent Hilbert-space constructions become available only after their domains and interpretive roles have been specified. Field-theoretic language is reserved for cases in which region-indexed data satisfy coherent restriction, overlap, transport, and realization conditions. This ordering permits the paper to use mathematical resources from operator algebra, field theory, discrete geometry, and spin-foam-inspired history models without identifying any one of them with the final ontology of volunteer relations.

Partiality is especially important in the minimal dyad. A request can be meaningful only under some communicative conditions. Consent can be undefined when the relevant object of consent has not been specified. A transformation available before an intervention can cease to be available after the intervention changes the relational configuration. A comparison between the subjects can be meaningful in one dimension and unavailable in another. An observed silence can support several interpretations without selecting one automatically. The formalism therefore treats domains of applicability as part of the structure rather than assuming that every operator acts globally. This discipline also separates undefinedness from refusal, prohibition, ignorance, and zero effect, which carry different relational meanings.

The dyad is minimal in actor count while remaining conceptually nontrivial. With only two focal subjects, the analysis already encounters subject history, role formation, private and shared manifestations, recursive theory of mind, action provenance, consent, refusal, uncertainty, competing judgments of need and good, aesthetic differentiation, normative disagreement, asymmetry, feedback, and emergent structures such as trust, recognition, reciprocity, dependence, standing, and obligation. Introducing a third participant would add mediation, witness, public recognition, brokerage, indirect trust, accountability, coalition, and other irreducibly triadic structures. Those extensions deserve a separate formal treatment.

Restricting Paper I to the dyad therefore protects the analytical boundary while retaining enough complexity to test whether GR can represent the interaction of subject, knowledge, action, normativity, and history within one coherent architecture.

Six objectives guide the construction. The paper builds a minimal dyadic GR system for volunteer and support relations; represents the focal subjects as history-bearing relational spacetimes rather than point actors; applies the three-level GR architecture systematically across the dyad; reconstructs action as a provenance-bearing derived object; formalizes mutual manifestation, recursive theory of mind, evaluative formation, authorization, feedback, and revision; and develops a typed partial-map and local-algebra architecture capable of supporting later emergent structures. These objectives yield four principal contributions: a three-level dyadic formal architecture, a history-bearing account of subject and action, a relational epistemology of support, and a compositional strategy linking relational spacetime, local algebraic dynamics, ordered interaction histories, and model-relative emergence. The concept inventory and type discipline serve as infrastructure for the wider volunteer-relation programme.

The scope remains deliberately foundational. Paper I does not claim an empirical theory of volunteer behavior, a completed moral theory of aid, a just-distribution criterion, a general psychology of helpers, or a final ontology of social action. The formal constructions are candidate representations whose adequacy depends on typing, application-specific interpretation, empirical identification where relevant, and later normative argument. Subsequent papers can therefore revise the carriers, enrich subject formation, develop psychoanalytic and aesthetic hypotheses, introduce explicit justice and ethics criteria, incorporate political-economic structure, and model longer historical dynamics without treating the present architecture as closed.

The remainder of the manuscript proceeds by progressively specifying the dyadic architecture. The next sections clarify the analytical scope and formal burden of volunteer relations, provide the mathematical and GR background required by the construction, and develop the three-level reality framework. The paper then defines history-bearing subject spacetimes and supplies a comprehensive inventory of conceptual objects in the minimal dyad. Subsequent sections formalize mutual manifestation, epistemic access, recursive theory of mind, action genesis, interactive procedures, evaluative and normative formation, consent and refusal, and typed relational asymmetries. The later formal sections develop local algebraic dynamics, historical composition, cross-level revision, and emergent dyadic structures before presenting a labelled 2-complex realization and a spin-foam-inspired specialization. Finite illustrations test the internal architecture, while the discussion and limitation sections identify the interfaces to later metaphysical, psychoanalytic, aesthetic, ethical, political-economic, and historical work. The resulting programme treats the statement that one subject supports another as the beginning of formal inquiry rather than as a primitive description whose conceptual structure can be assumed in advance.

2 Dyadic Volunteer Relations: Analytical Scope, Formal Burden

This section specifies the local research object that the remainder of the paper will formalize. Its role is to separate the ordinary-language intelligibility of volunteer support from the stronger commitments required by a generative-relational model. The objective is therefore neither to provide a universal sociological definition of volunteering nor to settle whether a particular intervention is ethically justified. The section instead identifies the minimal relational conditions under which a two-subject support interaction can become an object of formal inquiry. The discussion proceeds through the local meaning of volunteer support, the provisional status of supporter and supportee positions, the derived character of aid-related categories, the individuation of interaction and action, the dyadic boundary, and the formal obligations generated by that boundary. The method is conceptual and reconstructive: apparently simple terms are unpacked into the relational distinctions that later sections will type more precisely.

Table 1 summarizes the principal burdens carried by the ordinary statement that one subject supports another. The table is intentionally pre-formal. It does not yet assign mathematical types to the listed objects. Its purpose is to show why the statement cannot serve as the primitive description of the model without silently fixing questions that the paper intends to examine.

Table 1: Principal formal burdens concealed by the ordinary description that one subject supports another.

Apparent term	Hidden formal question	Reason the question remains open
Subject	What permits a participant to be tracked as the same relational being across an interaction history?	GR treats subject identity as history-bearing and representation-dependent rather than as a primitive point label.
Supporter / supportee	What relation assigns these positions, and how stable are they?	A role can be provisional, disputed, reciprocal, reversed, or dissolved through later interaction.
Action	What event counts as one action, who generates it, and who is its author?	Execution, initiation, intention, attribution, and generative provenance can diverge.
Support	What makes an action support-oriented rather than merely causal, helpful-looking, intrusive, or role-compliant?	Support is partly an evaluative and relational classification whose applicability can differ across perspectives.
Need	Whose judgment identifies a condition as requiring support?	The supportee, supporter, and later joint inquiry can generate different need judgments.
Good	Relative to which subject, relation, horizon, and evaluative structure is an outcome treated as good?	Material benefit, autonomy, dignity, dependence, trust, and later generative capacity can vary independently.
Consent	What exactly is authorized, under what capacities, and for how long?	Request, permission, compliance, endorsement, refusal, and withdrawal are distinct relational events.
Outcome	Which consequences belong to the action, and over what historical horizon?	Immediate, delayed, indirect, reversible, and relation-changing effects can differ.
Relation	What persists after the local act has ended?	Trust, expectation, obligation, dependence, recognition, and role structure can be generated by the interaction itself.

2.1 Volunteer Support: Local Analytical Object

The expression *volunteer support* is used here as a local analytical designation for interactions whose participants, attributions, and normative status remain partly open to inquiry. Sociological accounts often characterize volunteering through freely given time directed toward another person, group, or cause, while also distinguishing it from adjacent helping practices [15]. The present paper adopts no universal sociological definition. It retains only the relational features needed to formalize a support-oriented dyad across humanitarian, community, educational, health, mutual-aid, religious, emergency, cultural, advocacy, companionship, and related settings.

The first retained feature is directed practical relevance. Some event or process is taken, by at least one participant or by the analytical model, to bear upon another participant’s condition, possibilities, capacities, vulnerabilities, or relation to the surrounding configuration. This practical relevance does not establish that the event is beneficial. It identifies a candidate domain in which support-related judgment can arise. A well-intended intervention can be rejected, a requested intervention can produce harmful consequences, and a materially useful action can stabilize an objectionable dependency. The formal object therefore begins with support orientation or support attribution, while successful support remains a later evaluative question.

The second retained feature is partial voluntariness on the side provisionally occupying the supporter position. Volunteer motivation is empirically plural rather than exhausted by one altruistic function [3]. A participant can also act under moral expectation, institutional pressure, reputational incentive, religious commitment, friendship, professional identity, emergency circumstance, or accumulated role obligation

while still describing the action as voluntary. Paper I therefore avoids a scalar essence of voluntariness and requires instead that the provenance of participation remain representable.

The third retained feature is relational openness. A support interaction can change the participants and the relation through which support becomes possible. Repeated assistance can generate trust, dependence, recognition, resentment, obligation, reciprocity, competence, or a revised understanding of need. A requested act can become the beginning of a durable relation; an established relation can make later support possible without explicit request; a role initially understood as helper-recipient can later become reciprocal participation. The object of analysis is therefore not merely a transfer at one moment. It is a historically situated interaction whose consequences can include changes in the conditions of later interaction.

Remuneration and the boundary between volunteering and adjacent prosocial practices remain secondary to the present formal problem. Unpaid participation is important to ordinary usage, yet payment status alone does not settle provenance, role pressure, epistemic access, consent, or relational consequence. Courteous gestures, caregiving, solidarity, donation, emergency intervention, and professional support can overlap with the modeled structure without being classified identically in every domain. Paper I therefore asks a prior question: when an interaction is treated as volunteer support, which relational objects must be available for that treatment to become formally intelligible? Political-economic and domain-specific classification questions are deferred.

2.2 Supporter-Supportee Position Structure

The labels *supporter* and *supportee* refer in this paper to relational positions rather than complete identities of the participating subjects. This distinction is necessary because the same subject can occupy different support positions across relations, across stages of one relation, or simultaneously within different dimensions of the same interaction. A person can provide technical knowledge while receiving emotional support; a recipient of material assistance can possess superior local knowledge; a volunteer can depend upon the supportee for recognition, access, interpretation, or belonging. The relational positions therefore identify a local organization of the interaction rather than a total hierarchy between two persons.

A supporter position is provisionally present when a participant is recognized, by self-attribution, other-attribution, shared procedure, or analytical reconstruction, as contributing an action or resource oriented toward a support-related transformation. A supportee position is provisionally present when another participant is recognized as the focal subject whose condition, possibilities, capacities, or relation is expected to be affected in a support-relevant way. Both descriptions remain revisable. The paper will later distinguish self-recognition, other-recognition, jointly stabilized role assignment, and institutional manifestation of a role. At the current stage, the distinction merely prevents the role label from being identified with the subject itself.

The position structure can be asymmetric without being exhaustively hierarchical. A supporter can possess greater access to one resource while the supportee possesses greater authority concerning local meaning, personal preference, embodied experience, or the practical feasibility of an intervention. Different relational dimensions can therefore assign different directional structures to the same dyad. The paper treats these dimensions separately rather than deriving a single global ranking of the participants. This typed treatment is important for later analysis of knowledge, competence, dependency, refusal capacity, generative capacity, standing, and authority.

Role formation can also precede explicit agreement. A participant can be treated as a helper because of dress, institutional affiliation, prior reputation, physical position, group expectation, or repeated conduct before either side has articulated the role. Conversely, a participant can explicitly claim a supporter role that the other side does not recognize. Manifestation and relational stabilization can therefore diverge. An apparent helper can fail to acquire relational standing, while a person without a visible volunteer label can become a stable source of support through repeated interaction. The three-level analysis developed later will make these distinctions explicit.

Role persistence likewise requires explanation. Repetition can stabilize expectations so that later actions are interpreted through an already available supporter-supportee grammar. Such stabilization can reduce coordination costs, preserve expertise, and make assistance easier to initiate. It can also narrow the relation when either subject becomes difficult to recognize outside the established role. A supportee who develops new capacities can continue to be treated through an obsolete dependency model; a supporter can become invested in remaining necessary; both subjects can reproduce a role because it has become the easiest shared interpretation of the relation. Paper I does not treat these possibilities as inevitable. It requires a formal architecture capable of representing them when they occur.

The position structure can therefore change over historical time. A relation can begin with a sharply asymmetric allocation of resources or competence, develop reciprocal recognition, and later reorganize into partnership. It can also move in the opposite direction, with an initially reciprocal relation becoming more dependent or role-fixed. Role dissolution can itself be a meaningful outcome. In some support settings, success can reduce the practical necessity of the supporter position; in others, continuing companionship or community participation can remain valuable even after the original need has changed. The formalism should preserve both trajectories rather than build perpetual support or planned exit into the definition of volunteering.

2.3 Derived Categories: Aid, Need, Good, Beneficiary

Four ordinary categories deserve special restraint because they can silently place a normative conclusion inside the description of the interaction. The categories are *aid*, *need*, *good*, and *beneficiary*. Each can become useful after the relevant relational conditions have been specified. None is required as an unexplained primitive of the minimal dyad.

The term *aid* ordinarily suggests that an intervention has a support-related orientation and some defensible relation to the condition of the affected subject. The same manifested behavior can nevertheless receive different classifications. A transfer of money can be interpreted as emergency assistance, gift, compensation, inducement, dependency-producing support, symbolic recognition, or unwanted charity. A physical intervention can be rescue under one set of conditions and intrusion under another. These distinctions can depend on request, competence, role, prior obligation, cultural meaning, timing, power, and the history of the relation. Paper I therefore begins from candidate support actions and later asks how aid-related classifications are generated and revised.

A *need* likewise requires a judgment structure. Bodily condition, material scarcity, legal vulnerability, informational disadvantage, social isolation, impaired capability, or another state can provide evidence relevant to need without carrying a complete normative interpretation by itself. The supportee can judge that a condition calls for assistance; the supporter can form a different judgment; a professional or institution can supply a vocabulary through which the condition becomes legible; repeated interaction can alter what either side recognizes as important. The paper consequently distinguishes the condition under study from judgments that classify the condition as a need. This separation also keeps open cases in which the supportee's self-interpretation deserves strong epistemic authority while still permitting later analysis of emergency, impaired capacity, misinformation, or conflicting obligations.

The category *good* introduces a further evaluative layer. An intervention can improve one dimension while worsening another. Material security can increase while autonomy contracts; immediate distress can decline while long-term dependency grows; an awkward intervention can be materially useful while damaging dignity or trust; a burdensome intervention can be experienced negatively in the short term while preserving an important future capability. These possibilities do not require a single welfare metric. They require the formalism to keep the evaluated dimensions visible and to identify whose judgment is being represented. A supporter's judgment that an intervention is good for the supportee is therefore distinct from the supportee's own judgment, from a relational evaluation of the dyad, and from any later normative assessment that attempts to adjudicate among them.

The term *beneficiary* compresses these difficulties into a role noun. It can be administratively convenient, especially where organizations allocate resources or report programme outcomes. Within the

present dyadic formalism, however, calling B a beneficiary before benefit has been specified can turn an intended outcome into an assumed property of the subject. The paper therefore prefers the more neutral term *supportee* for the provisional relational position. A supportee may later be shown, under declared evaluative criteria, to benefit from an intervention. The formalism also permits the supporter to benefit, the relation to benefit, or different dimensions of benefit to diverge. This neutrality is methodological rather than rhetorical: it preserves the evaluative questions that later sections are designed to represent.

The derived status of these categories does not imply that ordinary participants must suspend practical judgment until a complete formal analysis has been performed. Volunteer practice often requires action under time pressure, uncertainty, partial information, and incomplete agreement. The formal claim is narrower. A theory that wishes to explain or evaluate the interaction should distinguish the practical use of a category from the conditions that justify or stabilize that category within the model. The paper's formal architecture is intended to make those conditions explicit without requiring practical agents to become formal theorists.

2.4 Interaction Procedure, Action Individuation

A minimal volunteer relation contains more structure than a single act. Even a short interaction can include encounter, observation, request, question, clarification, offer, refusal, negotiation, execution, feedback, correction, withdrawal, or acknowledgment. The order of these events can matter. An unsolicited intervention followed by consent is not automatically equivalent to prior consent followed by intervention; a clarification received before action can alter the meaning of the action; a refusal after partial execution can create a different relational history from refusal before any intervention occurs. The formalism therefore requires a procedural layer in addition to an action layer.

Procedure and action should remain distinct. A procedure is an ordered or partially ordered organization of relational events through which some local question becomes actionable, contested, revised, or settled. An action is one event or transformation individuated within that organization and attributed to one or more sources. The same physical movement can occupy different procedural roles depending on what preceded it, what it responds to, and what expectations surround it. Conversely, one support-oriented procedure can be realized through different concrete actions while preserving enough relational structure to count as the same kind of procedure for a chosen analysis.

Action individuation therefore requires a boundary rule. A theory must decide when a process begins, when it ends, which micro-events belong to it, which subject or subjects generate it, and what counts as the relevant unit for later evaluation. These decisions are model-relative. A donation can be treated as one transfer event, as the visible endpoint of a longer deliberative process, as one moment in a recurring relationship, or as part of an institutional chain. Paper I will later represent action provenance so that these alternative resolutions can be stated rather than silently conflated.

Attribution introduces a second boundary. The person who physically executes an action need not be the sole generator of the action. A supportee can initiate the process through a request; a supporter can formulate the practical response; a shared norm can structure the available options; an institution can constrain the form of execution; repeated interaction can make one response habitual. The categories of executor, initiator, generator, and author can consequently diverge. This divergence is especially important when responsibility, praise, blame, or authority is later assessed. Paper I treats it first as a problem of action ontology and epistemic attribution rather than as an already settled moral allocation.

The epistemic status of the action can also differ across the dyad. Both participants can observe the same manifested event while disagreeing about its meaning. One participant can know that an action occurred while remaining uncertain about its intention or provenance. A supporter can describe an act as spontaneous even when a later reconstruction identifies a long history of role formation and expectation; a supportee can attribute strategic motives that the supporter rejects; either side can remain partly unaware of the consequences generated by the act. Action occurrence, action provenance, action meaning, and action evaluation therefore require separate representation.

These distinctions support a broader GR point. An action is not merely a movement emitted by a pre-

existing actor toward another actor. It can be a historically generated relational event whose individuation, attribution, and significance depend upon the configuration in which it occurs. That configuration can itself be changed by the event. The paper's later operator language will formalize this possibility only after the relevant domains and relational carriers have been specified.

2.5 Dyadic Boundary, Deferred Structures

The paper restricts its focal system to two subjects in order to expose the minimal relational architecture with as little actor-level complexity as possible. This boundary is methodological. It does not imply that real volunteer relations are ordinarily isolated from organizations, families, communities, law, markets, professional institutions, infrastructures, or public authority. Such structures frequently shape who can volunteer, who is recognized as needing support, which actions are permitted, which resources are available, and which interpretations become credible. The present paper treats those wider formations only where they have already become part of the history or relational structure of A, B, or the dyadic interface.

The dyadic boundary excludes irreducibly triadic mechanisms from the core formalism. A third participant can witness an interaction, mediate disagreement, certify competence, enforce a promise, redistribute reputation, broker information, translate between normative vocabularies, or convert a private act into a publicly accountable event. These functions cannot always be reduced to properties of an A-B relation without loss of structure. A separate paper in the research programme therefore develops the triadic extension.

The boundary also postpones organizational and political-economic mechanisms whose primary object is a network or institution rather than the minimal dyad. Donor chains, volunteer labour substitution, organizational governance, state responsibility, market effects, professional regulation, public budgeting, and institutional power can shape dyadic interaction while requiring additional subjects, resources, or levels of organization for adequate analysis. Paper I preserves interfaces to those questions without incorporating them into the primitive architecture.

Several later philosophical questions are likewise deferred even though the dyadic formalism must leave room for them. Subject co-formation requires an account of how repeated support changes the constitution of A and B. Psychoanalytic analysis requires a fuller treatment of lack, desire, identification, repetition, and helper-supportee positioning. Aesthetic analysis requires a theory of how manifested form, sensibility, fittingness, dignity, and shared style become evaluatively consequential. Justice and ethics require criteria for legitimacy, responsibility, paternalism, distribution, standing, and permissible intervention. Political-economic analysis requires a theory of resources, unpaid labour, symbolic capital, and relational power. Historical analysis requires a longer account of trust formation, relational nucleation, path dependence, and heterogeneous value generation. The present paper contributes the dyadic language on which those analyses can later operate.

This division of labour is important for claim discipline. A formal structure capable of representing consent does not yet supply a complete ethics of consent. A model capable of representing dependence does not establish that dependence is unjust. A representation of action provenance does not determine whether a mixed motive undermines moral worth. A trust variable or relational object does not by itself explain how trust should be cultivated. Paper I aims to make these later questions formally articulable while preserving their independent argumentative burdens.

2.6 Formal Obligations of the Minimal Dyad

The analytical scope developed above yields a set of research obligations that guide the remainder of the manuscript. These obligations are stronger than a list of topics because they specify what the formalism must be able to distinguish if it is to represent the minimal dyad without collapsing central questions into notation.

First, the formalism must represent subject persistence without treating subject identity as a timeless primitive. A and B must be trackable across the modeled interaction while their boundaries, internal

histories, roles, and capacities remain open to refinement. Later sections satisfy this obligation through history-bearing subject-spacetime representations and explicit resolution choices.

Second, the formalism must separate subjects from relational positions. Supporter and supportee positions must be assignable, contestable, revisable, and historically transformable. A role should be capable of changing without requiring the subject itself to disappear, and a subject should be able to occupy several role dimensions without being reduced to any one of them.

Third, the formalism must distinguish relational structure from manifestation. A manifested request, refusal, title, gesture, silence, or visible transfer cannot automatically be identified with the complete relational object it expresses. The mappings through which relational structures become manifest, and through which subjects interpret those manifestations, must themselves be representable as partial and historically variable.

Fourth, the formalism must preserve epistemic asymmetry. A's representation of B, B's representation of A, and higher-order theory-of-mind constructions must remain distinct from the relational subjects they concern. Confidence, uncertainty, false belief, concealment, and revision should be representable without converting them into defects of one common omniscient state description.

Fifth, the formalism must derive action from a richer event structure. Action individuation, provenance, attribution, intention, manifestation, and consequence require separate types or mappings where the later mathematics demands them. The same manifested event must be capable of receiving different attributions or interpretations without forcing the model to choose one perspective as the primitive truth of the action.

Sixth, the formalism must derive need and goodness judgments rather than importing them as untyped facts. It must identify whose judgment is represented, which relational evidence and normative resources are available, which temporal horizon is retained, and how disagreement or revision can occur. The same discipline applies to aesthetic and explicit normative judgments.

Seventh, the formalism must represent request, permission, consent, refusal, endorsement, withdrawal, refusal capacity, and exit capacity as distinguishable relational objects. This obligation is necessary because formal agreement can carry different significance under different information, dependency, competence, and power conditions.

Eighth, the formalism must preserve historical order. Procedures and actions can change the domains on which later actions are defined. Ordered composition, partial composability, memory dependence, and revision must therefore be available before stronger assumptions such as commutativity, reversibility, or Markovian evolution are introduced.

Ninth, the formalism must allow relational emergence. Trust, recognition, reciprocity, expectation, obligation, dependence, standing, authority, and shared norms can become sufficiently persistent and efficacious to require representation at the relational level. Their emergence should be derived from declared histories or constitutive conditions rather than inserted as unexplained scalar attributes of one participant.

Tenth, the formalism must remain compatible with the three-level GR architecture. Every major concept should admit a declared ontological status, relational realization, manifestation structure, epistemic accessibility, and historical trajectory where those distinctions are meaningful. Ontological suspension limits the metaphysical claims of the model while leaving relational and manifestation dynamics available for substantive analysis.

These obligations define the formal burden of the minimal dyad. They also explain why the later mathematical architecture is layered. A single vector state containing "all relevant information" would hide many of the distinctions that the paper seeks to preserve, while a single scalar utility or benefit measure would settle evaluative questions before their objects had been specified. The paper therefore proceeds incrementally. Mathematical structure is introduced only when a later section has identified the relational work that the structure must perform. The next section supplies the mathematical background required for that construction at a level intended to remain accessible to readers with general undergraduate preparation in mathematics and physics.

Table 2: Mathematical structures introduced for the dyadic formalism. The third column states the restricted role adopted in Paper I rather than the full mathematical or physical meaning of each structure.

Structure	General mathematical role	Role in the present paper
Partial map	Transformation defined only on a specified subdomain	Configuration-sensitive observation, interpretation, request, refusal, action, and revision
Semigroup action	Associative composition without requiring inverses	History-bearing composition of generative mechanisms when reversibility is not assumed
Equivalence relation	Declared criterion for treating objects as equivalent	Typed comparison without presupposing a metric or scalar distance
Labelled 2-complex	Vertices, edges, faces, incidence, and labels	Candidate discrete carrier for subject histories, interfaces, and coupled relational structure
Spin-foam architecture	Labelled two-dimensional histories between boundary spin networks	Structural precedent for labelled histories; quantum-geometric semantics are not transferred
$*$ -algebra / C^* -algebra	Algebraic composition with involution and analytic norm structure	Optional local carrier for observables, transformations, evaluations, or generators when assumptions justify it
Algebraic state	Positive normalized linear functional on a C^* -algebra	Derived evaluation context rather than a primitive global state vector
GNS representation	Hilbert-space representation induced by an algebraic state	Optional route to operator calculations after algebra and evaluation have been specified
Local algebraic net	Region-indexed family of compatible algebras	Algebra-first organization of relationally local formal objects
Discrete field assignment	Local data assigned across cells or regions	Optional distributed representation after local carriers and coherence rules are defined
Homology	Algebraic diagnostic of cycles and boundaries	Optional diagnostic for selected structural changes in a represented relational complex

3 Dyadic GR Dynamics: Mathematical Foundations

This section supplies the mathematical background required by the dyadic formalism developed later in the paper. Its role is pedagogical and architectural. The intended reader is assumed to know elementary set theory, functions, linear algebra, and the general idea of a dynamical system, while no prior familiarity is assumed with partial maps, semigroups, cell complexes, spin foams, operator algebras, algebraic field theory, or homology. The discussion therefore proceeds from the weakest structures used by the model toward stronger structures that become available only under additional assumptions. The method is reconstructive: each mathematical object is introduced together with the specific analytical burden that motivates its later use, and physical precedents are kept distinct from their GR interpretation.

Table 2 summarizes the principal structures introduced in this section and the narrower role for which Paper I retains them. The table is deliberately asymmetric in detail. Some structures, such as partial maps and ordered composition, belong close to the minimal formal core. Others, such as Hilbert-space representations and homological diagnostics, are optional specializations that become relevant only when later assumptions justify them.

3.1 Partial Maps, Definedness, Composition

A total function presupposes that every element of its declared domain admits an output. Volunteer relations frequently violate that assumption at the level of formal description. A request may be meaningful only when a communication channel exists; consent may be unavailable when the relevant option has not been presented; a refusal may be impossible when no exit route exists; an interpretation may be undefined when the manifestation required for that interpretation has not occurred. The formalism therefore treats definedness as part of the mathematical data rather than as an afterthought.

Let X and Y be sets. A partial map from X to Y consists of a specified applicability domain $D_f \subseteq X$ together with an ordinary function from that domain into Y . The notation used throughout the paper is stated in Equation 1.

$$f : X \rightharpoonup Y, \quad f|_{D_f} : D_f \longrightarrow Y, \quad D_f \subseteq X. \quad (1)$$

Equation 1 distinguishes an undefined input $x \notin D_f$ from an input $x \in D_f$ whose image has a substantively special value. The distinction will later separate, for example, a refusal from the absence of a well-formed opportunity to refuse. It also prevents the symbol 0 from being used as a generic placeholder for unavailable operations when the relevant carrier has no justified zero element.

Partial maps compose only where the first output lies inside the applicability domain of the second map. Let $f : X \rightharpoonup Y$ and $g : Y \rightharpoonup Z$. The natural domain of the composite is defined in Equation 2.

$$D_{g \circ f} := \{x \in D_f \mid f(x) \in D_g\}, \quad (g \circ f)(x) := g(f(x)) \quad (x \in D_{g \circ f}). \quad (2)$$

Equation 2 makes failure of composition formally informative. An earlier interaction can remove an option, alter a role, change the relevant interface, or otherwise move the dyad outside the domain required by a later operation. Such failure should be recorded as a change in admissible relational structure rather than repaired by silently extending every operation to a universal domain.

A richer algebra of partial maps may later be useful. Restriction categories, for example, internalize information about the domain on which a morphism is defined and provide an abstract language for categories of partial maps [4]. Paper I does not require a full restriction-category construction at the background stage. The relevant lesson is narrower: definedness can be represented compositionally, and a model need not convert every context-sensitive operation into a total function before analysis begins.

3.2 Semigroup Composition, Historical Irreversibility

Repeated volunteer interactions require an algebra of composition even when no inverse operation is available. A semigroup supplies the minimal standard structure needed for this purpose. A semigroup is a set $\mathcal{M}$ equipped with an associative binary operation. Writing the product of $m_1, m_2 \in \mathcal{M}$ as $m_1 m_2$, associativity is stated in Equation 3.

$$(m_1 m_2) m_3 = m_1 (m_2 m_3) \quad (m_1, m_2, m_3 \in \mathcal{M}). \quad (3)$$

Equation 3 concerns regrouping of an already ordered composition; it does not assert that $m_1 m_2 = m_2 m_1$. A semigroup also need not contain an identity or inverses. These omissions are useful in history-bearing social models because a later operation need not undo an earlier relational change.

Let S denote a carrier of configurations at an abstract modeling layer. An action of the semigroup $\mathcal{M}$ on S assigns a transformed configuration to each pair (m, x) . The compatibility condition with semigroup multiplication is stated in Equation 4.

$$\alpha : \mathcal{M} \times S \longrightarrow S, \quad \alpha(m_1 m_2, x) = \alpha(m_1, \alpha(m_2, x)). \quad (4)$$

Equation 4 provides a highest-level compositional template. Paper I will later replace the bare carrier S by richer history-bearing relational configurations and will frequently use partial maps whose source and target structures can change. The semigroup language therefore supplies a constraint on composition rather than a claim that every volunteer interaction belongs to one fixed global semigroup.

Noninvertibility and irreversibility should also remain distinct. A semigroup permits noninvertible mechanisms, but noninvertibility by itself does not specify which relational distinctions are lost, preserved, or newly generated. Later GR constructions can add an equivalence relation, a history structure, or another diagnostic when a substantive claim about irreversible loss is required. The present paper therefore uses semigroup composition as a minimal historical resource while leaving stronger irreversibility claims to explicitly defined structures.

3.3 Relational Comparison, Equivalence Structure

Many later concepts require comparison without requiring a numerical distance. Two manifestations may count as equivalent for one purpose while remaining different for another; two subject positions may be interchangeable relative to one retained role structure and non-interchangeable relative to another. The formalism therefore begins comparison through declared equivalence rather than through an assumed metric.

Let X be a set of objects available to a local analysis. An equivalence relation $\sim_\kappa$ on X is reflexive, symmetric, and transitive. The index κ records the relational content retained by the comparison. The three defining conditions are collected in Equation 5.

$$\begin{aligned} x &\sim_\kappa x, \\ x &\sim_\kappa y \implies y \sim_\kappa x, \\ x &\sim_\kappa y \text{ and } y \sim_\kappa z \implies x \sim_\kappa z. \end{aligned} \tag{5}$$

Equation 5 does not determine the content of κ . An application may retain role structure, historical provenance, action manifestation, outcome type, or another declared family of relations. The same pair of objects can therefore be equivalent relative to κ_1 and non-equivalent relative to κ_2 . This indexed structure is essential for the later treatment of action identity. Two materially similar transfers, for example, may be equivalent when the analysis retains only transferred resources while becoming non-equivalent when consent or provenance is included.

A metric $d : X \times X \rightarrow \mathbb{R}_{\geq 0}$ can be added when a meaningful quantitative geometry has been justified, but it is stronger than the equivalence structure required at the present stage. In particular, a metric imposes numerical comparability and symmetry of distance. Paper I therefore does not use distance notation for epistemic, normative, or relational discrepancies unless the corresponding carrier and comparison rule have been independently specified.

3.4 Discrete Complexes, Labelled 2-Complexes

A graph records vertices and edges. The dyadic GR programme requires a richer carrier because histories can involve higher-order incidence relations, closed interaction patterns, shared interfaces, and local regions whose internal organization should remain visible. A two-dimensional cell complex supplies one candidate discrete representation. Standard treatments of CW complexes and cellular homology provide the underlying topological vocabulary [7].

For the purposes of this paper, a finite labelled 2-complex is represented schematically by the tuple in Equation 6.

$$\mathcal{K} := (V, E, F, \partial, \lambda). \tag{6}$$

In Equation 6, V is a set of vertices, E a set of edges, F a set of faces, and ∂ records the incidence or attachment information connecting higher-dimensional cells to lower-dimensional cells. The label map λ assigns model-dependent information to selected cells or composite regions. The tuple is intentionally schematic: Paper I does not fix one universal social meaning for a vertex, edge, or face.

A later dyadic realization can use vertices for localized relational events or updates, edges for transition or dependence structure, and faces for higher-order compositional or compatibility relations. Another realization may distribute meanings differently. The formal commitment is therefore to incidence structure plus explicitly declared labels, not to a universal correspondence between cell dimension and social meaning.

The distinction between incidence and labeling is analytically important. A change in λ can alter the represented meaning, status, or local parameter of a cell while preserving the underlying complex. Attaching, deleting, identifying, or splitting cells can instead alter the incidence structure itself. The later volunteer formalism uses this separation to distinguish changes of interpretation or role labels from stronger changes in the represented relational history.

A region $R \subseteq \mathcal{K}$ will denote a declared subcomplex or another explicitly admissible local portion of the represented complex. The word local is relational rather than necessarily spatial. A region may correspond to a communication interface, a shared practice, a segment of interaction history, a role-bearing subconfiguration, or another bounded structure selected by the model.

3.5 Spin-Foam History Architecture

Spin foams provide a physical precedent for representing histories with labelled two-dimensional complexes. In the standard construction, spin networks are graphs labelled by group representations and intertwining data, while a spin foam is a two-dimensional complex whose faces and edges carry corresponding labels; amplitudes can then be associated with the resulting history [1]. The present paper borrows the combinatorial architecture while leaving the quantum-gravitational interpretation behind.

This restriction is substantive. A labelled GR 2-complex does not become a quantum spacetime merely because a spin-foam model uses a similar combinatorial carrier. A GR label can represent a relational role, a manifestation type, a constraint, a historical marker, a local algebraic choice, or another application-defined object. A GR history weight, if introduced later, need not be a quantum amplitude. The mathematical precedent supplies a way to organize local labels across a compositional history; the physical semantics remain domain-specific.

Spin-foam architecture is useful for the dyadic project because it keeps boundary structure and internal history conceptually distinct. A subject-spacetime may be represented at a chosen boundary while an intervening labelled complex records alternative or realized interaction histories. Refinement can expose internal relational structure that a coarser representation compresses. These possibilities are representational resources, not evidence that a human subject or volunteer relation possesses the ontology of a spin foam.

The paper therefore uses the following hierarchy. A GR subject is first treated as a history-bearing relational being under ontological suspension. A labelled 2-complex is one candidate representation of that history-bearing structure. A spin-foam-inspired model is a stronger optional instantiation in which additional labeling and history-composition rules are adopted. Later group-field-theoretic language, if used at all, would add still stronger assumptions concerning the generation or recombination of such combinatorial histories and is not part of the minimal background required here.

3.6 Algebraic Carriers, Operators, Involutions

The word operator is used throughout the paper in a mathematically limited sense. An operator is a map acting on a specified structured domain. The domain may be a vector space, an algebra, a Hilbert space, a set of relational configurations, or another object for which the map is well typed. Calling a transformation an operator therefore does not determine its ontology or imply quantum mechanics.

An algebra provides a carrier on which addition, scalar multiplication, and an internal product are defined compatibly. Let $\mathcal{A}$ be an algebra over $\mathbb{C}$. For $a, b \in \mathcal{A}$, the product ab can represent sequential composition, multiplication of observables, or another model-defined operation. The present paper does not assign a universal substantive interpretation to the product.

A $*$ -algebra additionally carries an involution $a \mapsto a^*$ satisfying conjugate linearity, $(ab)^* = b^*a^*$, and $(a^*)^* = a$. A C^* -algebra adds a complete norm compatible with the involution. Its characteristic norm identity is stated in Equation 7 [2].

$$\|a^*a\| = \|a\|^2 \quad (a \in \mathcal{A}). \quad (7)$$

Equation 7 supplies substantial analytic structure. Paper I therefore does not assume a C^* -algebra whenever the word operator appears. Finite associative algebras, algebras of maps, partial morphisms, or other carriers can be sufficient for particular parts of the dyadic model. The C^* specialization becomes useful when positivity, norm control, state functionals, or representation theory contributes to a later construction.

The ordering of assumptions matters. A relational action should first be given a domain, codomain, and interpretation. Only then should the paper ask whether the relevant family of actions closes under an algebraic product, possesses an involution, admits a norm, or satisfies the C^* identity. This order prevents mathematical convenience from deciding substantive questions about the volunteer relation before those questions have been articulated.

3.7 Algebraic States, Hilbert-Space Representations

A state in operator-algebraic mathematics is not the same object as the full historical state of a GR subject. To avoid that ambiguity, Paper I reserves the term *algebraic state* for an evaluation functional on an algebra and uses relational-configuration language for the history-bearing dyadic object.

Let $\mathcal{A}$ be a unital C^* -algebra. An algebraic state is a positive normalized linear functional $\omega : \mathcal{A} \rightarrow \mathbb{C}$. Positivity means that $\omega(a^*a) \geq 0$ for all $a \in \mathcal{A}$, and normalization means $\omega(1) = 1$. These conditions are collected in Equation 8.

$$\omega : \mathcal{A} \longrightarrow \mathbb{C}, \quad \omega(a^*a) \geq 0, \quad \omega(1) = 1. \quad (8)$$

Equation 8 allows evaluation to be introduced after the algebraic structure has been specified. This is consistent with the algebra-first strategy used in operator-algebraic mathematical physics, where abstract algebras can be studied prior to selecting one privileged Hilbert-space realization [6, 2].

The Gelfand–Naimark–Segal construction provides a standard route from an algebraic state to a Hilbert-space representation. Under the usual C^* -algebraic assumptions, the construction associates ω with a cyclic representation $(\pi_\omega, \mathcal{H}_\omega, \Omega_\omega)$ satisfying Equation 9 [2].

$$\omega(a) = \langle \Omega_\omega, \pi_\omega(a) \Omega_\omega \rangle_{\mathcal{H}_\omega} \quad (a \in \mathcal{A}). \quad (9)$$

Equation 9 explains why a Hilbert space should not be the primitive container for the complete dyadic model. Different evaluation states can induce different representations of the same abstract algebra, and some relational transformations may be more naturally expressed as maps between algebras before any common Hilbert-space implementation is chosen. Paper I therefore treats Hilbert-space operators as derived calculational tools where useful, not as definitions of the subjects or their relational reality.

3.8 Local Algebraic Assignment, Open-System Maps

The dyadic relation is historically extended and internally structured. A single global algebra can therefore be unnecessarily strong as an initial assumption. The algebra-first alternative is to assign formal structure to declared relational regions and then state how those local structures fit together. Algebraic quantum field theory provides a mature physical example of region-indexed observable algebras, although Paper I does not inherit relativistic locality or the physical ontology of quantum fields [6].

Let $\mathfrak{R}(\mathcal{K})$ be a declared family of admissible regions in a represented complex $\mathcal{K}$. A local algebraic assignment associates each retained region R with an algebra $\mathcal{A}_\mathcal{K}(R)$. When $R_1 \subseteq R_2$ and the model treats the smaller algebra as part of the larger one, the corresponding embedding is stated in Equation 10.

$$R_1 \subseteq R_2 \implies \iota_{R_1 R_2} : \mathcal{A}_\mathcal{K}(R_1) \hookrightarrow \mathcal{A}_\mathcal{K}(R_2). \quad (10)$$

Equation 10 is an assumption to be justified by a particular model, not a universal law of GR. Some volunteer relations may contain contextual structures that do not embed cleanly across scales. In those cases the later formalism may require quotient maps, translations, gluing data, partial morphisms, or a category of local algebras rather than one simple inclusion net.

Open-system dynamics motivate another layer of flexibility. A transformation need not be an automorphism of one fixed algebra. It may be a noninvertible endomorphism, a positive or completely positive map in an appropriate C^* -algebraic specialization, or a morphism between distinct local algebras when the represented relational carrier changes. Paper I keeps the generic notation in Equation 11 until a stronger class of maps is justified.

$$\alpha_R^{\mathcal{K} \rightarrow \mathcal{K}'} : \mathcal{A}_{\mathcal{K}}(R) \rightarrow \mathcal{A}_{\mathcal{K}'}(R'). \quad (11)$$

Equation 11 permits source and target algebras to differ. That typing will later become important when an interaction changes the relevant subject boundary, interface, role structure, or represented history strongly enough that one fixed algebra no longer supplies an adequate carrier.

3.9 Discrete Field Assignments, Interface Transport

The term field is used in this paper only after a relational domain and a family of local carriers have been specified. At the present background level, a discrete field is simply an assignment of local mathematical data across selected cells or regions together with any comparison or transport rules required by the model. The word does not imply a physical field ontology.

Let $C_p(\mathcal{K})$ denote a chosen family of p -cells in a complex $\mathcal{K}$, and let X_c denote the admissible carrier associated with a cell c . A general cell-supported field assignment is stated in Equation 12.

$$\varphi : c \in C_p(\mathcal{K}) \mapsto \varphi(c) \in X_c. \quad (12)$$

Equation 12 deliberately leaves the carriers X_c open. They can be scalar spaces, vector spaces, algebras, spaces of maps, finite label sets, or other structures. A field representation earns its role only when the local data can be compared coherently across the relational domain.

Suppose an oriented interface e connects a source region R_s to a target region R_t . A comparison or transport map between their local carriers can be written as in Equation 13.

$$\tau_e : X_{R_s} \rightarrow X_{R_t}. \quad (13)$$

Equation 13 represents translation or comparison of local data; it does not by itself represent a substantive support action. Later sections will distinguish infrastructure that permits two regional descriptions to be compared from a generative operator that changes the relational configuration. This distinction prevents representational compatibility from being confused with causal influence.

A coherent field architecture additionally requires restriction or overlap conditions. Paper I will introduce those conditions only when a substantive object, such as a distributed recognition or interaction structure, actually requires field-level representation. Region-indexed data that lack such coherence should remain region-indexed data rather than being renamed a field.

3.10 Ordered Composition, Sequence Sensitivity

Historical order enters the dyadic model through composition. Let $\Phi_1, \dots, \Phi_n$ be a sequence of well-typed partial transformations whose successive domains and codomains permit composition. The ordered history map associated with the sequence is defined in Equation 14.

$$\Phi_\gamma := \Phi_n \circ \Phi_{n-1} \circ \dots \circ \Phi_1, \quad \gamma = (\Phi_1, \dots, \Phi_n). \quad (14)$$

Equation 14 does not require a universal scalar time coordinate. The order can be inherited from the interaction history, an event partial order, or another declared sequencing structure. An earlier transformation may change the domain on which a later transformation is defined, so the history can be path-dependent even before a linear state space is introduced.

Sequence sensitivity should first be stated without subtraction. Let Φ and Ψ be two transformations such that both ordered compositions are defined on a common retained set D . The order-sensitive subset is defined in Equation 15.

$$\Omega_{\Psi, \Phi} := \{x \in D \mid (\Psi \circ \Phi)(x) \not\sim_\kappa (\Phi \circ \Psi)(x)\}. \quad (15)$$

Equation 15 detects order effects relative to the declared equivalence $\sim_\kappa$. If $\Omega_{\Psi, \Phi}$ is nonempty, the two orders produce relationally distinguishable outcomes under the retained comparison. This criterion remains meaningful when subtraction is unavailable.

A commutator becomes legitimate only under stronger assumptions. If Φ and Ψ are linear operators acting on a common vector or algebraic space and both products are defined, the commutator is stated in Equation 16.

$$[\Psi, \Phi] := \Psi \circ \Phi - \Phi \circ \Psi. \quad (16)$$

Equation 16 provides a calculable specialization of sequence sensitivity; it is not a stylistic synonym for complexity or historicity. Paper I will use noncommutativity only when the common linear structure required by the subtraction has been explicitly established.

3.11 Topological Diagnostics, Representation Limits

The labelled 2-complex separates labels from incidence structure. A later interaction can therefore alter a label while preserving the complex, change incidence while preserving selected topological invariants, or produce a stronger structural change detected by homological or other topological diagnostics. Homology supplies one standard and calculable diagnostic for finite complexes [7].

Let $C_q(\mathcal{K})$ denote the q -th chain group generated by oriented q -cells of a finite complex. Boundary maps $\partial_q : C_q(\mathcal{K}) \rightarrow C_{q-1}(\mathcal{K})$ satisfy the chain condition in Equation 17.

$$\partial_q \circ \partial_{q+1} = 0. \quad (17)$$

Equation 17 implies that every boundary is a cycle. The q -th homology group is defined in Equation 18.

$$H_q(\mathcal{K}) := \ker \partial_q / \text{im } \partial_{q+1}. \quad (18)$$

Equation 18 groups cycles that differ by boundaries. In a finite model over a field of coefficients, the dimension of $H_q(\mathcal{K})$ gives the corresponding Betti number. A change in a Betti number is sufficient to establish that the represented complexes differ topologically with respect to that invariant. Equality of a selected list of Betti numbers is not sufficient to prove topological equivalence, because homology is not a complete invariant.

Paper I includes this background for scope control rather than because every volunteer interaction should be treated topologically. Most dyadic changes will be representable at weaker levels: updated labels, changed admissible transformations, modified role assignments, revised local algebras, or altered continuation structures. Topological language should be reserved for cases in which the represented incidence structure itself changes and the chosen diagnostic detects that change.

3.12 Dyadic Formal Hierarchy

The mathematical resources introduced above occupy different levels of commitment. Paper I will therefore use a layered construction rather than one universal formalism. The hierarchy is methodological: weaker structures remain available when stronger structures are neither necessary nor justified.

At the most abstract level, the paper needs objects, relations, partial operations, and a rule for composition. History-bearing subjects then require a relational carrier capable of representing internal and shared histories; a labelled 2-complex is one candidate. Local algebras become useful when families of admissible quantities or transformations require compositional structure. Algebraic states and Hilbert-space representations appear only when evaluation and operator calculations require them. Field structure appears only when local data satisfy restriction, overlap, and transport conditions. Topological diagnostics enter only when the relational carrier itself changes strongly enough to make incidence or topology part of the research object.

The resulting hierarchy can be summarized schematically without asserting that every later model must traverse every level. The sequence is stated in Equation 19.

$$\begin{aligned} \text{partial relational structure} &\rightsquigarrow \text{history-bearing complex} \rightsquigarrow \text{local algebraic structure} \\ &\rightsquigarrow \text{evaluation/representation} \rightsquigarrow \text{field or topological specialization.} \end{aligned} \quad (19)$$

Equation 19 is a dependency ordering rather than a universal temporal process. A finite dyadic example may stop at partial maps and a labelled complex; another may require a local algebra but no field; a later history-ensemble model may use spin-foam-inspired structure without employing a C^* -algebra. The adequacy of a formalism depends on the relational object, the question, the retained scale, and the distinctions that the model must preserve.

This discipline also fixes the role of natural-science inspiration in the paper. Spin foams, algebraic field theory, operator algebras, open-system maps, and topology provide mature mathematical precedents for compositional histories, local structure, representation dependence, and structural change. Their use in GR is selective and conditional. Formal similarity does not establish shared ontology, and a mathematical construction earns its place only when the volunteer relation independently requires the structure that the construction supplies. The next section will state the GR commitments that determine which of these mathematical resources are substantively admissible for the dyadic model.

4 Generative Relational Foundations

This section states the Generative Relational Theory (GR) commitments required by the dyadic volunteer formalism before the paper introduces the dedicated three-level reality architecture in the following section. Its role is foundational rather than exhaustive: the section identifies the relational, historical, generative, and representational commitments that constrain later definitions of subject, action, need, judgment, and support. The discussion proceeds through relational constitution, historical persistence, generative possibility, subject individuation, relational spacetime, partiality, relational efficacy, and formal pluralism. The method is reconstructive. Each commitment is stated only at the strength needed by Paper I, and stronger metaphysical or mathematical readings remain open unless later sections supply additional argument.

Table 3 summarizes the principal commitments and the formal consequences they impose on the present paper. The table should be read as a constraint map rather than as a compressed ontology. The mathematical representations in the third column remain conditional on the analytical task and on the additional assumptions stated where they are used.

4.1 Relational Constitution, Analytical Individuation

GR begins from the proposition that a relational being is not exhausted by an inventory of intrinsic properties taken at one instant. The analytical identity of a retained being depends partly on the relations through which it emerged, the relations that currently sustain or constrain it, the distinctions by which it remains trackable, and the transformations that remain available from its present configuration. Paper I adopts this proposition as a modeling commitment concerning the level at which its objects are described. It does not require the stronger metaphysical claim that relations are the sole constituents of ultimate reality.

This commitment changes the formal meaning of the symbols A and B . In ordinary dyadic notation, two letters can suggest two objects that exist independently and are subsequently connected by a relation. The present paper uses the letters only as convenient names for history-bearing relational objects whose formal representations are developed in Section 6. The relation between them is therefore not an edge added after two complete subjects have been defined. Their current boundaries, capacities, roles, expectations, and possibilities can already depend on relations that predate the local support episode.

Table 3: Generative Relational commitments retained by Paper I and their immediate formal consequences.

GR commitment	Analytical meaning	Formal consequence for Paper I
Relational constitution	A retained being is analyzed through the relations that constitute, sustain, differentiate, and connect it	Subjects are represented through history-bearing relational configurations rather than primitive point actors
Historicity	Present relational structure carries a trajectory of prior formation whose consequences can remain active	Histories, ordered transformations, persistence criteria, and path dependence must remain representable
Generativity	Relational structure matters partly through the possibilities, constraints, distinctions, and later relations it helps produce	The model must represent admissible continuations and changes in later possibility, not only present descriptive state
Ontological suspension	A useful representation does not settle ultimate ontology	Spacetime, field, algebra, operator, and 2-complex language remain modeling resources with explicit scope
Partial individuation	Subject boundaries and relational positions can depend on history, scale, and analytical resolution	Boundaries, interfaces, overlaps, refinement, and coarse-graining remain explicit model choices
Definedness	A relational operation may be meaningful only on part of the represented domain	Partial maps and applicability domains precede assumptions of total action
Relational efficacy	A structure becomes analytically consequential through persistent causal or generative effects in the retained relational domain	Relational reality cannot be reduced to a private description or manifestation alone
Formal pluralism	Different relational questions preserve different structures	Mathematical tools are selected by typed analytical burden rather than by one universal GR formalism

Analytical individuation is consequently model-relative. A person can be treated as one effective subject for one question and as a finer relational configuration for another. A legal office, family role, professional identity, bodily capacity, linguistic repertoire, or accumulated relation of trust may be internal detail at one resolution and part of the explicit subject representation at another. The appropriate boundary depends on which differences and transformations the analysis needs to preserve.

The paper uses *relational position* for a location within a represented configuration whose meaning is supplied by its relations, permissions, dependencies, expectations, and available transformations. A relational position is therefore distinct from the being that occupies it. The supporter and supportee labels introduced in Section 2 name positions within a dyadic analysis; they do not define the subjects themselves. This distinction leaves open histories in which the positions are negotiated, shared, reversed, weakened, or dissolved while the tracked subjects remain analytically continuous under a declared identification rule.

4.2 Historical Persistence, Trajectory Dependence

Historicity gives the relational commitment a temporal dimension. A present configuration can contain structures whose current effect depends on earlier events, repeated interactions, inherited classifications, accumulated expectations, or previously stabilized constraints. GR therefore treats history as part of the object of analysis whenever removing the relevant trajectory would erase distinctions needed to explain present relational possibilities.

Paper I does not require one primitive scalar time coordinate for every construction. Section 3 introduced ordered composition and the possibility of partial historical order. The present GR commitment is weaker and more general: a history-bearing object must admit some defensible structure of persistence

and succession sufficient to distinguish earlier and later relational organization where the argument requires that distinction.

Let $H_{\leq \lambda}$ denote the retained history available at a historical index λ in a declared ordering. The notation is schematic and does not assume that the history is a real-valued time series. A relational configuration at λ can then be written provisionally as $\mathcal{C}_\lambda[H_{\leq \lambda}]$ to record that its present organization is history-bearing. This dependence is stated in Equation 20.

$$\mathcal{C}_\lambda = \mathcal{C}_\lambda[H_{\leq \lambda}]. \quad (20)$$

Equation 20 is bookkeeping rather than a universal functional law. It records that the current configuration can depend on retained historical structure. The exact mathematical type of $H_{\leq \lambda}$ is supplied by the later instantiation: an event order, a labelled subcomplex, an interaction record, an institutional history, or another history-bearing object may be appropriate.

Historical persistence also requires criteria of tracking. Repeated use of the same label does not by itself establish that one relational object has persisted. An application should state which relations, records, material continuities, institutional identifications, bodily continuities, or other structures warrant treating a later object as the continuation of an earlier one. This requirement is especially important for support relations because role labels can persist while their relational content changes, and relational content can persist while its manifestation changes.

4.3 Generative Possibility, Continuation Structure

The term *generative* indicates that the significance of a relational configuration is partly prospective. A relation matters not only through what it presently records, but also through the distinctions, constraints, capacities, roles, and continuations that it helps make possible. Paper I therefore requires a formal language capable of representing changes in later possibility without presupposing one scalar measure of generative capacity.

A minimal representation begins with a configuration $\mathcal{C}$ and a declared family of admissible continuations. Let $\text{Ext}(\mathcal{C})$ denote the extension family retained by the model. Equation 21 gives the generic form.

$$\text{Ext}(\mathcal{C}) := \{(\mathcal{C}', \iota) \mid \iota : \mathcal{C} \rightarrow \mathcal{C}' \text{ is an admissible continuation map}\}. \quad (21)$$

Equation 21 intentionally leaves the continuation map partial and the carrier type open. A later 2-complex specialization can interpret ι as an embedding or another history-preserving map; a stronger history-reconfiguring regime may instead require maps for which no fixed-past embedding exists. The present section needs only the weaker point that generativity can be represented through the organization of admissible continuation rather than through a single intrinsic quantity stored in a subject.

This perspective makes several later distinctions possible. An interaction can preserve the set of formally imaginable continuations while changing their practical accessibility. It can preserve accessibility while changing the transformations available inside those continuations. It can also alter the subject boundary or relational carrier strongly enough that the relevant continuation family itself must be reconstructed. These changes occupy different formal levels and should not be compressed into one scalar “amount of support” or “amount of capacity.”

Generativity can include preservation as well as novelty. A support action may generate a new relation, preserve a fragile capacity, reopen a previously unavailable possibility, stabilize a shared vocabulary, or maintain a distinction that would otherwise be lost. The formal architecture must therefore remain sensitive to production, preservation, reopening, redirection, and closure as different possible consequences of relational action.

4.4 Relational Being, Subject Tracking

Paper I treats the subject as a relational being at the declared analytical scale. The term does not denote a metaphysically self-sufficient substance, and it also does not dissolve the distinction between subjects into an undifferentiated whole. A subject remains analytically distinguishable when the model supplies enough historical and relational structure to track it across the transformations relevant to the research question.

A provisional tracking relation can be written as in Equation 22. Let $\mathcal{S}_\lambda$ and $\mathcal{S}_{\lambda'}$ denote subject representations at two comparable historical indices. The partial relation $\chi_{\lambda\lambda'}$ records a model-justified continuation where one is available.

$$\chi_{\lambda\lambda'} : \mathcal{S}_\lambda \rightarrow \mathcal{S}_{\lambda'}. \quad (22)$$

Equation 22 allows tracking to fail when a boundary dissolves, when coarse-graining removes the distinctions needed for identification, or when the later configuration does not support the earlier subject category. The partiality prevents the notation from assuming in advance that every subject persists through every transformation.

This restraint becomes important for mutual support. A later interaction can change a subject's capacities, self-understanding, social roles, or practical horizon while the model continues to track the subject under a broader criterion. Another interaction can alter the constitutive relations so substantially that the earlier criterion becomes inadequate. Paper I keeps those possibilities formally available while leaving the deeper metaphysics of subject continuity to the later paper on subject co-formation.

4.5 Relational Spacetime, Scale Selection

The word *spacetime* in GR names a representation in which relational extension and historical structure are modeled together. It does not require a Lorentzian manifold, a primitive metric, or a physical interpretation of social relations. Section 3 introduced labelled 2-complexes as one candidate discrete carrier because vertices, edges, faces, labels, and incidence relations can jointly represent local events, transitions, higher-order compatibility, and historical composition.

Paper I therefore uses relational spacetime in a deliberately model-relative sense. A represented spacetime is a structured historical domain within which subjects, interfaces, actions, manifestations, and continuations can be localized at a chosen resolution. The same substantive relation can admit several representations. A coarse model can treat a stable subject as one effective region; a finer model can expose internal histories required to explain an action or judgment. The choice of scale is an analytical commitment and must remain visible whenever a conclusion depends on it.

Resolution change can alter which relations appear local, which subject boundaries are visible, and which interactions count as direct. A dyadic relation that appears bilateral at one scale can contain institutional, technological, linguistic, or historical mediations at another. Paper I retains the dyadic boundary as a declared analytical cut rather than as a claim that no third-party structure participates in the subjects' histories. The later triadic paper will relax that cut when third-party relations become part of the immediate formal object.

The 2-complex and spin-foam-inspired structures introduced in Section 3 therefore occupy a representation layer below the GR commitment itself. GR requires a history-bearing relational object; it does not require that every adequate representation be a 2-complex. A 2-complex becomes useful when incidence, higher-order relation, gluing, refinement, or topology-sensitive history must be represented explicitly. Other mathematical carriers remain admissible when they preserve the relevant relational structure more economically.

4.6 Partiality, Relational Interfaces

GR treats definedness as substantive when the availability of a relation or transformation depends on relational articulation. A request can be meaningful only after a communicative channel has formed.

A refusal can have practical force only where it can be expressed or otherwise enacted. A consent procedure can require capacities and distinctions that were absent earlier. An interpretation can become available only after a manifestation enters a subject's accessible domain. These are relational conditions of applicability, not merely programming exceptions.

The paper therefore adopts partiality near the formal foundation. Let Φ be a relational operation whose applicability domain is $\mathcal{D}_\Phi$. The generic type is given in Equation 23.

$$\Phi : \mathcal{D}_\Phi \rightarrow \mathcal{C}, \quad \mathcal{D}_\Phi \subseteq \mathcal{C}. \quad (23)$$

Equation 23 does not yet specify whether Φ is an observation, communication, judgment, support action, or revision. Later sections assign those types separately. The present GR commitment is that applicability belongs to the object whenever definedness depends on relational history or structure.

Relational interfaces provide a corresponding localization of interaction. An interface need not be a physical boundary. It can be a communication relation, a shared language, an institutional permission, a bodily channel, a technical platform, a role expectation, or another structure through which two retained regions become mutually consequential. The formalism should therefore distinguish the constitutive support of a subject or relation from the interface through which a particular transformation propagates.

This distinction also limits causal simplification. An action attributed to A can be generated through an interface whose prior structure includes contributions from B , shared history, institutions, norms, or material arrangements. Treating the action as “located in A ” can therefore be convenient at one level while remaining incomplete as an account of its generative provenance. Section 10 develops this point explicitly.

4.7 Relational Efficacy, Persistence Criteria

The present paper needs a criterion for treating some modeled structure as more than a transient description. GR uses relational efficacy for this purpose. A retained structure becomes analytically significant when it persists sufficiently across the comparisons relevant to the model and when its presence contributes causally or generatively to later relational consequences. Persistence and efficacy are therefore distinct requirements: a pattern can recur without shaping later possibilities, and a powerful local event can be consequential without remaining stable as a continuing structure.

Paper I will later apply this distinction to trust, recognition, dependence, role stabilization, standing, and other emergent dyadic objects. At the present foundational stage, the criterion can be stated schematically. Let X_λ denote a candidate relational structure at index λ , let $\mathcal{C}_X$ denote the declared family of historical comparisons used to track it, and let $\mathcal{T}_X$ denote a declared family of perturbative or counterfactual tests relevant to its consequences. A relational-reality claim about X requires an application-specific identification rule that supports persistence across enough of $\mathcal{C}_X$ and at least one consequential difference under the retained tests in $\mathcal{T}_X$.

This criterion is deliberately model-relative. It does not establish that relational reality is exhausted by whatever one model can track, and it does not elevate empirical persistence into ultimate ontology. It supplies a disciplined reason for retaining an object inside the relational description: the structure remains sufficiently identifiable and makes a difference to the subsequent relational history under the selected analytical conditions.

Causal and generative efficacy should likewise remain distinct where the application requires the distinction. A causal effect can concern an immediate change in an observed configuration. Generative efficacy concerns contribution to the later conditions under which relations, distinctions, roles, interpretations, or continuations become available. One event can possess both forms of efficacy, one, or neither under a particular representation.

4.8 Formal Pluralism, Representation Discipline

GR does not prescribe one mathematical language for every relational problem. Partial maps are appropriate where definedness is central. Semigroups and noninvertible maps are useful where historical

composition and irreversibility matter. Groups can become appropriate for symmetry once globally articulated reversible transformations exist. Operator algebras become useful where local composition, evaluation, or open-system dynamics require their structure. Labelled 2-complexes are useful where higher-order historical incidence must be retained. Field representations become appropriate only after local data satisfy coherence conditions. These tools answer different questions and carry different assumptions.

The resulting pluralism is constrained rather than eclectic. A formalism earns its place when the analytical object independently requires the structure that the formalism supplies. A commutator is warranted only after two ordered compositions are defined on a common linear carrier. A field is warranted only after local realization and coherence conditions have been established. A topological claim is warranted only after the represented carrier and the relevant invariant or equivalence have been specified. A Hilbert-space operator is warranted only after the algebraic or state assumptions needed for the representation have been stated.

This discipline produces a hierarchy between substantive GR objects and their mathematical realizations. Paper I will repeatedly distinguish a relational object from a representation, a representation from a derived calculational realization, and a formal consequence from an empirical or normative conclusion. The distinction protects the paper from two opposite errors: using mathematical vocabulary metaphorically without specifying structure, and allowing a mathematically convenient structure to determine the substantive ontology by notation alone.

The same discipline governs empirical interpretation. Observed data do not arrive pre-labelled as GR subjects, relational regions, manifestations, actions, norms, or local algebras. An empirical application would require declared coding and identification rules that map observations into the formal objects. Alternative defensible mappings can yield different instantiated models. Representation sensitivity is therefore part of the evidential problem rather than a defect that can be removed simply by making the mathematics more elaborate.

4.9 Three-Level Reality Interface

The commitments above prepare the three-level reality architecture without duplicating it. Relational constitution identifies the main positive analytical domain. Historicity requires trajectories of subjects, relations, and mappings to remain visible. Generativity requires the model to retain future possibility and not only present description. Ontological suspension prevents the relational model from being read as a final metaphysics. Relational efficacy explains why some historically stabilized structures deserve treatment as analytically real within the model. Manifestation, however, introduces a further distinction that cannot be reduced to these commitments alone.

A relational structure can be causally or generatively efficacious without being exhausted by the way it appears in one context. The same relational structure can acquire several manifestations, and similar manifestations can be compatible with different relational structures. Conversely, manifestations can become generatively consequential and thereby participate in later relational stabilization. Paper I therefore requires a dedicated architecture for relational reality, manifestation reality, and their historical mappings.

The following section develops that architecture. It formalizes the distinction among ontological reality, relational reality, and manifestation reality; states the many-to-many and historically variable relations between relational and manifestation levels; clarifies why subject-relative representations and theory-of-mind structures do not constitute a fourth reality level; and establishes the cross-level audit applied to the major concepts of the dyadic system.

5 Three-Level Reality: Structure, Mapping, Trajectory

This section develops the three-level reality architecture used throughout the remainder of the dyadic volunteer formalism. Its role is to separate three questions that are easily conflated when a support relation

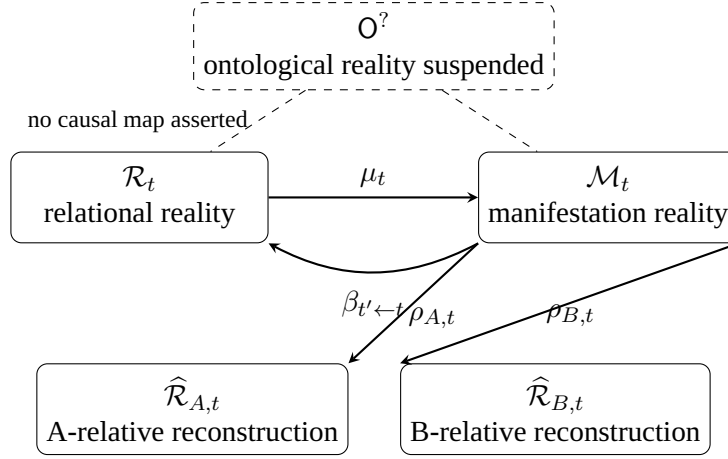


Figure 1: Three-level GR architecture for the dyadic formalism. $O^?$ marks ontological suspension rather than a modeled state space. The relation μ_t connects retained relational structures with admissible manifestations at historical index t . Feedback relations $\beta_{t' \leftarrow t}$ can connect manifested forms with later relational structures. Subject-relative reconstruction maps $\rho_{A,t}$ and $\rho_{B,t}$ act on accessible manifestation-history records and produce models of relational structure rather than a fourth reality level.

is described in ordinary language: what ultimate reality may be, which historically stabilized relational structures are retained as causally or generatively efficacious, and how those structures become presented or accessible in a particular context. The objective is to make these levels formally distinguishable while preserving their historical interaction. The discussion proceeds through ontological suspension, relational reality, manifestation reality, cross-level correspondence, subject-relative epistemic reconstruction, historical mapping trajectories, manifestation feedback, and a generic concept-audit scheme. The method is reconstructive and typed. The formal objects introduced below organize the analysis without converting the three levels into a known metaphysical hierarchy.

Figure 1 summarizes the architecture. The upper element marks ontological suspension and carries no modeled causal arrow to the lower levels. The lower part of the figure contains the relational and manifestation levels, the subject-relative reconstruction channels, and the possibility that manifested forms can contribute to later relational stabilization. Each arrow in the lower part corresponds to a relation or partial map defined in this section.

5.1 Ontological Suspension

GR leaves the ultimate ontology of the modeled domain open. Paper I therefore uses the symbol $O^?$ only as a status marker for suspended ontological commitment. It is not a set of hidden states, a latent variable space, or a mathematical carrier on which later operators act. The question mark is essential: the notation records that the present formalism does not determine whether the ultimate basis of the volunteer relation is material, mental, informational, biological, symbolic, relational, or otherwise constituted.

This restraint has a direct formal consequence. No map of the form $O^? \rightarrow \mathcal{R}_t$ or $O^? \rightarrow \mathcal{M}_t$ is assumed. Introducing such a map would already specify a structure for the suspended level and a rule connecting it with the modeled levels. Paper I lacks grounds for that stronger commitment. The formal analysis therefore begins positively at the relational and manifestation levels while allowing later philosophical work to ask whether a deeper ontology can or should be supplied.

Ontological suspension also limits the interpretation of mathematical success. A labelled 2-complex can represent historical incidence without establishing that ultimate reality is combinatorial. An operator algebra can organize composable transformations without establishing that a volunteer relation is fundamentally operator-algebraic. A spin-foam-inspired realization can encode labelled histories without establishing a quantum-gravitational ontology. The same discipline applies to categories, fields, stochastic processes, and other mathematical resources introduced later.

Suspension is consequently an active methodological rule rather than an absence of analysis. It requires the paper to state which claims are made at the relational level, which concern manifestation, which depend on a selected mathematical realization, and which remain outside the present model. This separation allows the formalism to become more explicit without allowing mathematical notation to settle an ontological question by default.

5.2 Relational Reality

Relational reality is the principal positive analytical level of the present framework. At a historical index t , let $\mathcal{R}_t$ denote the class of relational structures retained by the dyadic model at the selected scale and under the identification criteria stated by the application. The elements of $\mathcal{R}_t$ can include subject configurations, roles, dependencies, trust relations, obligations, norms, action structures, interfaces, or other historically constituted objects once the paper has supplied criteria for tracking them.

Membership in $\mathcal{R}_t$ does not mean that the formalism has discovered an ultimate substance. It means that the adopted model treats the structure as sufficiently identifiable, historically persistent for the analytical horizon, and causally or generatively efficacious under the declared comparison rules. Section 4 introduced those criteria abstractly. The present section places the resulting objects inside the three-level architecture.

Relational reality is therefore stronger than private belief and weaker than final ontology. A relation can be real in the present GR sense even when one subject misunderstands it, when neither subject possesses a complete description of it, or when its manifestations vary across contexts. Trust can shape later disclosure without either subject naming the relation as trust. A role can structure permissible conduct before a participant articulates its normative basis. A dependency can constrain available continuations even when its visible manifestations appear ordinary.

The relational level is also historically indexed. The notation $\mathcal{R}_t$ should not be read as a static inventory whose members remain fixed while only their numerical values change. A later index can contain new relational beings, lose previously retained structures, alter subject boundaries, or reorganize the criteria through which persistence is tracked. Paper I therefore permits

$$\mathcal{R}_t \neq \mathcal{R}_{t'} \quad \text{for some } t \preceq t'. \quad (24)$$

Equation 24 records structural change at the relational level. The inequality is set- or class-level non-identity, not a numerical difference. Later sections must specify whether a particular transition changes membership, incidence, admissible transformations, labels, or another declared component of the relational representation.

5.3 Manifestation Reality

Manifestation reality concerns the context-dependent forms through which relational structures become presented, expressed, observable, recorded, symbolized, institutionally legible, or practically actionable. Let $\mathcal{M}_t$ denote the class of manifestations retained by the model at historical index t . Possible manifestations include utterances, gestures, bodily expressions, visible transfers, silence, delay, documents, titles, legal classifications, interface states, rituals, spatial arrangements, or other forms through which relational structure becomes accessible in a context.

Manifestation reality is not treated as illusion. A manifested form can have direct practical consequences. A spoken refusal can stop an intervention. A visible expression can redirect attention. A legal title can coordinate expectations. A public act of gratitude can alter later recognition. The distinction from relational reality concerns analytical role rather than a hierarchy in which manifestation is merely unreal appearance.

The relation between the relational and manifestation levels is generally many-to-many. Paper I therefore begins with a manifestation relation rather than a global manifestation function. The relation is defined in Equation 25.

$$\mu_t \subseteq \mathcal{R}_t \times \mathcal{M}_t. \quad (25)$$

Membership $(r, m) \in \mu_t$ means that m is an admissible manifestation of r under the retained context at t . Equation 25 permits one relational structure to admit several manifestations and one manifested form to remain compatible with several relational structures. A relation may manifest through speech in one setting and through habitual conduct in another. Conversely, the same visible gesture can express care, obligation, strategic display, fear, or convention under different histories. Manifestational similarity therefore does not establish relational equivalence.

The relation μ_t is model-relative. It depends on the context, resolution, coding rules, and historically available forms of expression. A manifestation that is salient in one setting can be unavailable in another, and a previously unavailable mode of expression can become stabilized through learning, institutionalization, or technological change. The manifestation relation itself must therefore remain historical.

5.4 Cross-Level Correspondence

The manifestation relation in Equation 25 separates two kinds of non-identity that matter throughout the paper. First, a relational object is not exhausted by any one of its manifestations. Second, a manifested form does not uniquely identify the relational structure that produced or sustained it. These two failures of one-to-one correspondence are analytically important for support relations because action, need, consent, trust, and role are frequently inferred from manifested conduct.

For a relational object $r \in \mathcal{R}_t$, define its retained manifestation family by Equation 26.

$$\text{Man}_t(r) := \{m \in \mathcal{M}_t \mid (r, m) \in \mu_t\}. \quad (26)$$

Equation 26 does not require the family to be nonempty. A relational structure can be retained by a model even when the current observational context supplies no manifestation accessible to the subjects or investigator. The family can also contain several mutually different forms. This flexibility will later matter for hidden action, unarticulated need, tacit trust, and role structures that operate without explicit naming.

For a manifestation $m \in \mathcal{M}_t$, the compatible relational family is defined in Equation 27.

$$\text{Rel}_t(m) := \{r \in \mathcal{R}_t \mid (r, m) \in \mu_t\}. \quad (27)$$

Equation 27 records underdetermination at the manifestation interface. A single manifested object can remain compatible with several relational interpretations. A signature can manifest informed consent, formal compliance under pressure, routine administrative completion, or another relation depending on the surrounding history. The existence of several compatible relational structures does not imply that all are equally well supported; it means only that manifestation alone is insufficient for unique identification under the retained model.

Table 4 illustrates the distinction for several concepts that recur later. The table is schematic. Section 7 will provide the comprehensive concept inventory and the stronger type audit.

The table also shows why three-level analysis cannot be replaced by a vocabulary of hidden essence and visible indicator. Some manifested forms become relationally real in their own right when they acquire persistence and efficacy. A title can begin as a manifestation of authority and later participate in reproducing the authority relation. A repeated ritual can manifest a role and also stabilize the role. The levels therefore classify analytical status and function; they are not mutually exclusive containers into which each object must be placed exactly once.

5.5 Subject-Relative Epistemic Reconstruction

A and B do not receive $\mathcal{R}_t$ directly. Each subject encounters only a historically conditioned subset of manifestation reality together with memories, prior interpretations, and other retained epistemic resources. The paper therefore distinguishes manifestation from subject-relative reconstruction.

Table 4: Illustrative three-level profiles for recurrent dyadic concepts. Ontological status remains suspended in every row; the listed relational and manifestation forms are examples rather than exhaustive definitions.

Concept	Relational-level object	Possible manifestations
Subject	History-bearing relational configuration tracked under declared identity criteria	Speech, body, records, roles, actions, names, interface traces
Action	Historically generated relational transformation with attribution and consequence structure	Gesture, transfer, utterance, document, silence, delay, visible intervention
Trust	Persistent relational structure affecting disclosure, reliance, or admissible continuation	Reassurance, delegated discretion, reduced checking, disclosure, reliance
Consent	Historically situated authorization structure subject to capacity and scope conditions	Explicit assent, signature, conduct, refusal, withdrawal, silence under declared conventions
Norm	Stabilized normative structure shaping judgment or practice	Rule statement, recurrent practice, sanction, vocabulary, ritual, institutional procedure

For $i \in \{A, B\}$, let $\mathcal{M}_{i,t}^{\text{acc}} \subseteq \mathcal{M}_t$ denote the manifestations accessible to subject i at index t . Let $\mathcal{H}_{i,t}$ denote the retained historical record that subject i can use in the reconstruction process. The epistemic input domain is defined in Equation 28.

$$\mathcal{E}_{i,t} := \mathcal{M}_{i,t}^{\text{acc}} \times \mathcal{H}_{i,t}. \quad (28)$$

The Cartesian product in Equation 28 is a bookkeeping structure. It states that present manifestations and retained history can jointly condition interpretation; it does not imply statistical independence, equal weighting, or a complete psychological theory of inference.

Let $\hat{\mathcal{R}}_{i,t}$ denote the class of relational models or reconstructions available to subject i within the formal description. A subject-relative reconstruction is represented by the partial map in Equation 29.

$$\rho_{i,t} : \mathcal{E}_{i,t} \rightarrow \hat{\mathcal{R}}_{i,t}. \quad (29)$$

Equation 29 is partial because some manifestation-history records can remain uninterpretable, ambiguous, or outside the subject’s available conceptual distinctions. Later theory-of-mind constructions will refine the codomain and introduce models of another subject’s beliefs, preferences, intentions, and judgments. At the present stage, the essential point is that the result of $\rho_{i,t}$ is a subject-relative model of relational structure rather than relational reality itself.

The reconstruction architecture prevents a recurrent category error. If B expresses pain, asks for assistance, or remains silent, A encounters manifestations whose relational meaning depends on context and history. A’s later claim that B needs help belongs to a derived epistemic and evaluative structure. It should not be identified with B’s relational condition merely because A’s interpretation is sincere or confident. The same distinction applies symmetrically to B’s interpretation of A’s action or intention.

Subject-relative reconstruction does not create a fourth reality level. The accessible manifestations, retained memories, inferential practices, and resulting models are themselves relationally situated structures. The hat notation marks epistemic representation, not an additional ontological stratum. This convention will be used later when the paper distinguishes a subject from another subject’s model of that subject.

5.6 Historical Mapping Trajectories

The three-level architecture is historical at more than one location. Relational objects can change, manifestations can change, and the correspondence between the two can change. Subject-relative interpretive

capacities can also change. A formalism that tracks only $\mathcal{R}_t$ while holding manifestation and reconstruction rules fixed would therefore omit part of the generative process that Paper I seeks to analyze.

Let $(I, \preceq)$ be the historical index set introduced for the local analysis. For each $t \in I$, define the cross-level record L_t by Equation 30.

$$L_t := (\mathcal{R}_t, \mathcal{M}_t, \mu_t, \mathcal{M}_{A,t}^{\text{acc}}, \mathcal{M}_{B,t}^{\text{acc}}, \rho_{A,t}, \rho_{B,t}). \quad (30)$$

Equation 30 is a typed analytical record, not a vector state. Its components belong to different mathematical classes and no addition, norm, or metric on L_t is assumed. The purpose of the record is to keep several trajectories visible at once.

A cross-level history over I is the indexed family defined in Equation 31.

$$\mathbf{L}_I := (L_t)_{t \in I}. \quad (31)$$

Equation 31 permits change in every component of Equation 30. A trust relation can persist while its manifestations shift from explicit reassurance to tacit reliance. A subject can learn to interpret a previously ambiguous gesture. A new institutional interface can make a previously hidden dependency visible. A shared vocabulary can make a need articulable that previously lacked an accepted manifestation. Conversely, a changing institution can make an earlier manifestation lose its interpretive force.

The history of the mapping relation is therefore part of the research object. The condition $\mu_t \neq \mu_{t'}$ can hold even when a retained relational object remains identifiable across $t \preceq t'$. Likewise, $\rho_{i,t} \neq \rho_{i,t'}$ can hold when a subject acquires new interpretive competence, vocabulary, experience, or trust. These differences are again structural rather than numerical unless a later specialization supplies a comparison structure.

This point is especially important for aesthetics and norm formation. A manifested structure can become salient before a subject possesses an explicit vocabulary for judging it. Later articulation can stabilize a category that changes subsequent attention and manifestation. Paper I does not yet supply a general theory of that recursion, but Equation 31 provides a place in which the relevant changes can be represented without forcing them into a fixed one-way causal sequence.

5.7 Manifestation Feedback

Manifestation can become generatively consequential. A manifested title can alter deference; a public refusal can change later expectations; a repeated form of gratitude can stabilize a helper role; an interface can make one mode of response easier and another harder. The formalism therefore requires a way to represent influence from manifestation at one historical index into relational structure at a later index without identifying every manifestation with an independent relational being.

For $t \preceq t'$, define a manifestation-feedback relation by Equation 32.

$$\beta_{t' \leftarrow t} \subseteq \mathcal{M}_t \times \mathcal{R}_{t'}. \quad (32)$$

Membership $(m, r') \in \beta_{t' \leftarrow t}$ means that m contributes causally or generatively to r' under the retained model. Equation 32 is deliberately relational rather than functional. One manifestation can contribute to several later relational structures, several manifestations can participate jointly in one later structure, and many manifestations can leave no retained relational trace. The definition also leaves the mechanism open. Repetition, institutionalization, symbolic recognition, memory, embodied learning, and technical mediation can all supply different routes of feedback.

A manifestation that repeatedly participates in later relational organization can itself become trackable as a relational being. The transition should be stated through the persistence and efficacy criteria of Section 4, not by automatically promoting every manifested object into $\mathcal{R}_{t'}$. This qualification is important for symbols. A symbol can begin as a context-dependent manifestation and later acquire sufficient historical persistence and generative efficacy to become part of the relational structure through which subsequent interactions are organized.

Feedback also makes the two lower levels recursively coupled. Relational structure constrains available manifestations through μ_t ; manifestations can contribute to later relational structure through $\beta_{t' \leftarrow t}$. The pair of relations does not define a closed deterministic loop. It provides a typed interface through which later sections can represent particular feedback mechanisms while preserving contingency, partiality, and historical change.

5.8 Three-Level Concept Audit

The three-level architecture becomes operational only if it can be applied repeatedly to the concepts used in the dyadic model. Paper I therefore introduces a generic concept profile. Let X denote a concept type such as subject, action, need, trust, consent, norm, judgment, recognition, or power. At historical index t , let $X_t^R \subseteq \mathcal{R}_t$ denote the relational realizations retained for that concept, and let $X_t^M \subseteq \mathcal{M}_t$ denote its retained manifestations. The manifestation relation restricted to the concept type is

$$\mu_t^X := \mu_t \cap (X_t^R \times X_t^M). \quad (33)$$

Equation 33 keeps the global three-level architecture and the concept-specific analysis compatible. It does not require that every concept have a manifestation at every index or that every manifestation be accessible to both subjects.

Let $\hat{X}_{A,t}$ and $\hat{X}_{B,t}$ denote the subject-relative reconstructions of X available through the maps in Equation 29. The concept profile is recorded in Equation 34.

$$\Lambda_t(X) := (\mathcal{O}_X^?; X_t^R, X_t^M, \mu_t^X, \hat{X}_{A,t}, \hat{X}_{B,t}). \quad (34)$$

In Equation 34, $\mathcal{O}_X^?$ is a status marker indicating that the ultimate ontology of X remains suspended. It is not another component of the same mathematical type as the remaining entries. The semicolon records that distinction. The profile is therefore a typed analytical ledger rather than an element of a presumed product vector space.

The historical trajectory of the concept is the indexed family in Equation 35.

$$\Lambda_I(X) := (\Lambda_t(X))_{t \in I}. \quad (35)$$

Equation 35 makes historical change part of the concept analysis itself. For a concept such as trust, the relational structure, its manifestations, each subject's reconstruction, and the correspondence among these components can all change. For action, the relation can change as an event becomes attributable, publicly manifested, differently interpreted, and later reclassified in light of consequences. For consent, a manifested assent can acquire a different relational meaning when refusal capacity, information, or role structure changes.

The generic profile does not replace concept-specific definitions. It supplies a recurring audit. When a later section introduces a major concept, the manuscript should state, where relevant: the concept's suspended ontological status; its relational realization criteria; its manifestation forms; the manifestations accessible to A and B; the subject-relative reconstructions produced from those manifestations; the historical trajectory of each component; and any feedback by which manifestation participates in later relational stabilization or transformation.

The comprehensive concept inventory in Section 7 will apply this discipline systematically. Before that inventory can be built, the paper must specify the history-bearing subject-spacetimes whose relational and manifestation structures the audit will track. Section 6 develops those subject-spacetimes and the coupled dyadic configuration in which supporter and supportee positions become available.

6 History-Bearing Subject Spacetimes

This section specifies the subject-level objects required by the dyadic formalism. Its role is to replace the convenient prose labels A and B with history-bearing GR representations that remain compatible

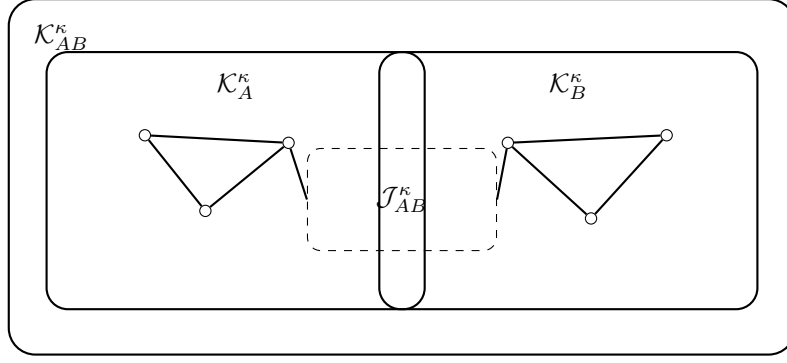


Figure 2: Schematic coupled subject-spacetimes in the dyadic model. Each focal subject is represented by a history-bearing subconfiguration inside the coupled spacetime $\mathcal{K}_{AB}^\kappa$. The relational interface $\mathcal{J}_{AB}^\kappa$ can include coupling structure beyond literal overlap, while shared relational material may also occur in $\mathcal{O}_{AB}^\kappa$. The diagram is representational and carries no claim that subjects possess a unique cellular decomposition.

with the three-level architecture of Section 5. The objective is not to identify a person, organization, or other participant with one mathematical object in an ontological sense. It is to state what the model must retain when a focal subject is represented as a relational spacetime rather than as a point actor. The construction proceeds through subject indexing, coupled dyadic spacetime, shared history and interfaces, representational boundaries, historical tracking, relational positions, and three-level subject trajectories. The method is formal and reconstructive: each object is introduced with an explicit modeling role, while stronger claims about ultimate subject ontology remain suspended.

The central commitment can be stated compactly. The labels A and B name focal subject-spacetimes at a declared analytical resolution. They do not denote pre-existing substances to which relations are subsequently attached. Their formal representatives are history-bearing portions of a wider relational spacetime, and the coupling through which volunteer interaction becomes possible belongs to the modeled object from the beginning. The principal subject-spacetime objects introduced below are consolidated in Table 5 after their definitions.

Figure 2 gives a schematic view of the architecture developed in this section. The diagram emphasizes three points: each focal subject has internal historical structure, the two subject-spacetimes can share relational material, and the dyadic interface can contain coupling structure that is not reducible to the literal overlap of the two subject representations.

6.1 Subject Labels, History-Bearing Representation

The symbols A and B are retained because a dyadic analysis requires stable references in prose and notation. Their use does not make the corresponding subjects primitive objects. A subject label instead indexes a history-bearing relational representation selected for a particular analytical purpose and resolution. The same person or organization can therefore admit several compatible or partially compatible GR representations without one of those representations being declared ontologically fundamental.

Let κ denote a declared representational resolution. A history-bearing subject-spacetime representative for focal subject $i \in \{A, B\}$ is a labelled subcomplex

$$\mathcal{K}_i^\kappa = (V_i^\kappa, E_i^\kappa, F_i^\kappa, \partial_i^\kappa, \lambda_i^\kappa), \quad i \in \{A, B\}. \quad (36)$$

Equation 36 reuses the labelled 2-complex architecture introduced in Section 3. The cells need not have universal meanings. Depending on the application, vertices can represent locally individuated events or updates, edges can encode admissible continuations or dependencies, faces can encode higher-order compatibility or compositional relations, and labels can record model-specific relational types, capacities, constraints, manifestations, or other retained data.

Definition 6.1 (History-bearing subject-spacetime). At representational resolution κ , a history-bearing subject-spacetime is a relationally tracked subconfiguration $\mathcal{K}_i^\kappa$ whose retained internal structure is sufficient for the local analysis to identify historically accumulated relations, present interfaces, and admissible continuations relevant to focal subject i . The label i indexes the tracking convention; it does not assert a pre-relational substance underlying the representation.

The definition places the analytical burden on tracking and retained structure. If a model collapses the whole subject into a single vertex, that coarse representation is admissible only when the internal history omitted by the collapse is irrelevant to the question being asked. When a support relation depends on a prior promise, learned competence, accumulated trust, institutional membership, earlier injury, or a history of refusal, the corresponding structure must be retained at a resolution that can represent its later effects.

This point also limits ordinary state-vector intuition. A list of current attributes can be useful as a derived summary, yet such a list does not replace $\mathcal{K}_i^\kappa$ when the current capacities of the subject depend on the path through which they were generated. Paper I consequently treats compact state-like variables as projections or summaries of a richer history-bearing object whenever historical structure matters to the dyadic relation.

6.2 Coupled Dyadic Spacetime

A volunteer/support relation is represented inside a coupled spacetime rather than between two isolated containers. Let $\mathcal{K}_{AB}^\kappa$ be a declared coupled region at resolution κ that retains the portions of A, B, and their relational coupling required by the local research question. No uniqueness or order-theoretic minimality of this coupled region is assumed; a different admissible boundary or resolution can support a different coupled representation. The subject-spacetime representatives are related to the coupled object through embeddings or other declared structure-preserving inclusions. In the subcomplex specialization used here, the relation is stated in Equation 37.

$$\iota_A^\kappa : \mathcal{K}_A^\kappa \hookrightarrow \mathcal{K}_{AB}^\kappa, \quad \iota_B^\kappa : \mathcal{K}_B^\kappa \hookrightarrow \mathcal{K}_{AB}^\kappa. \quad (37)$$

Equation 37 is a representational inclusion. It does not require the two images to be disjoint, exhaustive, or uniquely recoverable from the coupled complex. The dyadic spacetime itself is recorded by the typed object in Equation 38.

$$\mathfrak{D}_{AB}^\kappa := (\mathcal{K}_{AB}^\kappa, \iota_A^\kappa, \iota_B^\kappa, \mathcal{J}_{AB}^\kappa), \quad (38)$$

where $\mathcal{J}_{AB}^\kappa \subseteq \mathcal{K}_{AB}^\kappa$ is a declared relational-interface region. The interface contains the retained cells, relations, or composite regions through which a change supported in one subject-spacetime can become consequential to the other or to their coupled configuration. The interface may include shared material, connecting cells, jointly generated structures, or a model-defined neighborhood required for composition.

Definition 6.2 (Coupled dyadic spacetime). A coupled dyadic spacetime at resolution κ is a record of the form in Equation 38 in which the two focal subject-spacetimes are embedded in a common history-bearing configuration and the model declares an interface sufficient to represent their retained modes of interaction.

The qualifier dyadic concerns the number of focal subject-spacetimes, not the claim that the relation is socially isolated. Language, law, infrastructure, inherited norms, technical artifacts, and institutional conditions can appear as labels, regions, constraints, or background relational structures inside $\mathcal{K}_{AB}^\kappa$. They are not modeled as a third focal subject in Paper I unless the analysis assigns them an independent subject trajectory, epistemic position, or action role. That stronger move belongs to the triadic extension developed in later work.

6.3 Shared Histories, Relational Interfaces

The coupled representation permits shared relational history without collapsing A and B into one subject. When the images of the subject-spacetimes overlap, define the retained overlap by Equation 39.

$$\mathcal{O}_{AB}^\kappa := \iota_A^\kappa(\mathcal{K}_A^\kappa) \cap \iota_B^\kappa(\mathcal{K}_B^\kappa). \quad (39)$$

Equation 39 may be empty. When it is nonempty, the overlap can represent relational material that the adopted model does not assign exclusively to one focal subject. Examples include a jointly created routine, a shared symbolic vocabulary, a common institutional history, a remembered interaction, or another structure whose later efficacy depends on both trajectories. The overlap does not establish identity of A and B. It records that the chosen decomposition is relational rather than a partition into mutually exclusive substances.

The relational interface $\mathcal{J}_{AB}^\kappa$ is broader than the overlap. A communication channel can connect two non-overlapping subject histories; a promise can be represented through cells relating otherwise distinct subconfigurations; an intervention can occur at a boundary region; and a joint procedure can generate new cells that belonged to neither subject representation before the interaction. Paper I therefore keeps $\mathcal{O}_{AB}^\kappa$ and $\mathcal{J}_{AB}^\kappa$ formally distinct.

Remark 6.3 (Overlap and interface). The condition $\mathcal{O}_{AB}^\kappa \neq \emptyset$ is neither necessary nor sufficient for a support relation. Coupling can occur through an interface with no shared subcomplex, and a shared history can persist without any current support action. The interface is defined by the relational mechanism under study, while overlap records shared represented material.

This distinction becomes important for self-other analysis. GR can preserve analytic distinction between A and B while allowing their histories to be entangled, partially overlapping, and mutually generative. The formalism therefore avoids two extremes: an atomistic model in which subjects are fully separable before interaction, and an identity claim in which the distinction between the two focal trajectories disappears. The dyadic architecture requires distinguishability for tracking while permitting relational non-separability in the retained history.

6.4 Subject Boundaries, Representational Resolution

A subject boundary is a modeling commitment. The body, a legal identity, an organizational chart, an account identifier, a remembered biography, and a network of obligations can suggest different boundaries for the same focal subject. GR therefore requires the analysis to state which relations are treated as internal to $\mathcal{K}_i^\kappa$, which belong to the dyadic interface, and which remain in the surrounding coupled spacetime.

Resolution change makes this dependence explicit. Let κ_f and κ_c denote finer and coarser representations of the same focal analysis, with κ_f retaining at least as much internal detail as κ_c . A declared coarse-graining is represented by the partial maps in Equation 40.

$$q_i^{f \rightarrow c} : \mathcal{K}_i^{\kappa_f} \rightharpoonup \mathcal{K}_i^{\kappa_c}, \quad q_{AB}^{f \rightarrow c} : \mathcal{K}_{AB}^{\kappa_f} \rightharpoonup \mathcal{K}_{AB}^{\kappa_c}. \quad (40)$$

Equation 40 treats $q_i^{f \rightarrow c}$ and $q_{AB}^{f \rightarrow c}$ as partial structure-preserving morphisms between represented complexes, rather than as functions whose arguments are whole complexes treated as points of another state space. Where the selected coarse-graining warrants it, these morphisms induce maps on cells, subcomplexes, labels, or support families. Coarse-graining can be partial because some fine structures can lack a meaningful coarse representative under a selected scheme. When both subject-level and coupled morphisms are defined, the embeddings should be compatible with coarse-graining on the retained domain. The compatibility condition is stated in Equation 41.

$$q_{AB}^{f \rightarrow c} \circ \iota_i^{\kappa_f} = \iota_i^{\kappa_c} \circ q_i^{f \rightarrow c}, \quad i \in \{A, B\}, \quad (41)$$

whenever both compositions in Equation 41 are defined. Failure of this condition can indicate that the coarse representation loses relational information needed to identify the focal subject consistently inside the dyadic configuration.

Boundary choice also interacts with manifestation. A subject can manifest through a body, a document, a digital account, an authorized representative, or an institutional role whose spatial or material boundary differs from the relational subject boundary. Section 5 therefore prevents manifestation boundaries from being substituted automatically for relational boundaries. The visible source of an utterance can be localized while the relational structure that gives the utterance standing extends across a longer history or a wider institutional complex.

A useful formal claim should accordingly report its resolution dependence. Trust identified at one scale can decompose into several distinct relations at a finer scale. A support action that appears to be directly dyadic can reveal institutional mediation after refinement. Conversely, fine distinctions can be intentionally suppressed when they do not affect the research question. The purpose of resolution control is not to maximize detail; it is to make explicit which distinctions the formal result depends on.

6.5 Historical Tracking, Structural Reconfiguration

The use of a stable label A across a history requires an identification rule. GR does not infer subject persistence from repeated notation alone. Let $t \preceq t'$ be two indices in the historical order. A tracking relation for focal subject i is defined in Equation 42.

$$\tau_{t' \leftarrow t}^i \subseteq \mathcal{K}_{i,t}^\kappa \times \mathcal{K}_{i,t'}^\kappa. \quad (42)$$

Membership $(x, x') \in \tau_{t' \leftarrow t}^i$ means that the adopted model treats the later relational material x' as historically continuous with, descended from, or otherwise trackable from x for the purpose of identifying focal subject i . A relation is used instead of a total function because historical reconfiguration can include splitting, merging, disappearance, or emergence of represented components.

Definition 6.4 (Subject persistence criterion). A focal subject label i is retained across $t \preceq t'$ only when the application supplies a tracking relation $\tau_{t' \leftarrow t}^i$ satisfying the identity criteria declared for the local analysis. The criteria can depend on causal continuity, memory, institutional recognition, embodied continuity, relational obligations, documentary linkage, or another application-appropriate structure.

The definition permits persistence without structural stasis. The later subject-spacetime can differ substantially from the earlier one while still being tracked as the same focal subject under the chosen criteria. Volunteer interaction itself can contribute to that change. New competencies can be acquired, obligations can be generated, trust can be formed, roles can be revised, and relational interfaces can be reorganized. The resulting subject trajectory is therefore a history of reconfiguration rather than a sequence of snapshots of an invariant container.

Tracking can also fail. If a coarse organization divides into successor organizations, if an anonymous participant cannot be connected across episodes, or if a transformation changes the represented structure beyond the chosen identity criterion, the model should not force a persistence claim merely to preserve notation. Such failures are informative because later judgments about responsibility, consent, trust, or accumulated obligation can depend on which historical continuities remain warranted.

6.6 Supporter-Supportee Position Structure

The words supporter and supportee designate relational positions inside the dyadic configuration. They are not synonyms for A and B . Let $\mathcal{P}_t^\kappa$ denote the set or structured family of relational positions distinguished by the model at historical index t and resolution κ . The assignment of focal subjects to positions is represented by a relation rather than a function, as stated in Equation 43.

$$\varrho_t^\kappa \subseteq \{A, B\} \times \mathcal{P}_t^\kappa. \quad (43)$$

Equation 43 allows one subject to occupy several positions and allows a position to be jointly instantiated where the substantive model supports that interpretation. Let π_S and π_R denote, respectively, the supporter and supportee positions used in the minimal orientation of Paper I. A configuration can satisfy the provisional assignment in Equation 44.

$$(A, \pi_S) \in \varrho_t^\kappa, \quad (B, \pi_R) \in \varrho_t^\kappa. \quad (44)$$

Equation 44 does not define A as a supporter or B as a supportee. It records a historically situated role relation within one configuration. Later interaction can add reciprocal support, reverse the positions, dissolve the distinction, or generate another position such as co-participant. The role relation itself therefore has a trajectory, represented by Equation 45.

$$\varrho_I^\kappa := (\varrho_t^\kappa)_{t \in I}. \quad (45)$$

The history in Equation 45 is analytically important because the meaning of an action can depend on the role structure from which it emerged. The same transfer, request, silence, or refusal can have different significance when it occurs in a newly formed support relation, an entrenched helper-recipient relation, a reciprocal partnership, or a relation whose earlier role assignment has already been contested.

Role attribution also belongs to the three-level architecture. A supporter position can be relationally stabilized while manifesting through a title, repeated behavior, spatial arrangement, language, or expectation. A and B can reconstruct the role differently, and those reconstructions can feed back into later behavior. The role relation in Equation 43 therefore remains distinct from its manifestations and from either subject's belief about who is supporting whom.

6.7 Three-Level Subject Trajectories

The subject-spacetime construction can now be connected directly to the concept audit of Section 5. At the relational level, the focal subject is represented by the tracked history-bearing configuration $\mathcal{K}_{i,t}^\kappa$. At the manifestation level, only selected manifestations of that configuration become available in the dyadic situation. Within subject-relative epistemic reconstruction, A and B form models from those manifestations and from retained history. Ultimate subject ontology remains suspended.

Let Subj_i denote the concept type corresponding to focal subject i . Its generic profile is the specialization of Equation 34 shown in Equation 46.

$$\Lambda_t(\text{Subj}_i) = \left(\text{O}_{\text{Subj}_i}^?; \mathcal{K}_{i,t}^\kappa, M_{i,t}, \mu_t^{\text{Subj}_i}, \widehat{\text{Subj}_{i,t}}^A, \widehat{\text{Subj}_{i,t}}^B \right). \quad (46)$$

In Equation 46, $M_{i,t}$ is the retained manifestation family through which focal subject i becomes accessible in the dyadic context. The subject-relative reconstructions can contain beliefs about identity, need, intention, competence, role, reliability, vulnerability, or other later-defined objects. The profile does not imply that either reconstruction exhausts the relational subject-spacetime.

The summary in Table 5 records the analytical distinctions preserved by these objects.

Historical interaction can change every retained component except the suspended ontological status marker. The subject-spacetime can be reconfigured, its manifestations can change, the manifestation relation can change, and each participant's reconstruction can change. A volunteer relation can therefore be subject-forming even before Paper I develops the stronger metaphysics of subject co-formation reserved for later work. The present section establishes only the formal capacity to record such change.

The subject-spacetime construction supplies the carriers needed by the remainder of Paper I. The next section can therefore build a comprehensive concept inventory without treating the focal subjects, their roles, or their manifestations as interchangeable objects. That inventory will classify which concepts are primitive relative to the dyadic model, which are derived from previously defined structures, and which should be treated as relationally emergent only after additional historical conditions are satisfied.

Table 5: Principal subject-spacetime objects in the minimal dyadic formalism.

Object	Analytical role	Principal distinction preserved
$\mathcal{K}_i^\kappa$	History-bearing subject-spacetime representative	Subject history versus point actor
$\mathcal{K}_{AB}^\kappa$	Coupled dyadic relational spacetime	Dyadic relation versus two isolated containers
$\mathcal{J}_{AB}^\kappa$	Model-declared relational interface	Coupling structure versus subject-internal structure
$\mathcal{O}_{AB}^\kappa$	Shared represented material	Historical overlap versus subject identity
$q_i^{f \rightarrow c}$	Resolution-changing map	Coarse subject description versus fine internal history
$\tau_{t' \leftarrow t}^i$	Historical tracking relation	Persistence of subject label versus structural stasis
ϱ_t^κ	Subject-position relation	Focal subject versus supporter/supportee role
$\Lambda_t(\text{Subj}_i)$	Three-level subject profile	Relational subject versus manifestation and reconstruction

7 Minimal Dyadic System: Conceptual Object Inventory

This section consolidates the conceptual objects required by the minimal dyadic study before the paper turns to detailed epistemic, procedural, evaluative, and algebraic constructions. Its role is classificatory and formal: the section records what the model may need to represent, identifies which objects can be taken as relative primitives at the present level, distinguishes objects derived from earlier structure from objects that require relational emergence, and states the three-level and historical questions that accompany each family. The objective is not to define every later object completely in one place. It is to prevent later sections from introducing action, need, consent, trust, judgment, or power through notation that silently assumes their formal status. The method is a typed inventory. Each object family is related to the history-bearing subject-spacetimes of Section 6 and to the three-level profile developed in Section 5.

The inventory is deliberately broader than the set of symbols used immediately after this section. A comprehensive dyadic formalism must distinguish objects that can coexist in one support relation even when a particular finite illustration suppresses some of them. At the same time, inclusion in the inventory does not imply equal ontological or formal standing. A desire, an observed utterance, a request, a judgment of need, a support action, and an emergent trust relation are different kinds of objects. Treating them as coordinates of one global state vector would therefore add a substantial commensurability assumption that the present paper has not earned.

7.1 Inventory Architecture

The classification uses three formal-status categories relative to the present analytical layer. A *relative primitive* is an object that Paper I accepts as supplied by the local instantiation rather than deriving it from another object already formalized in this manuscript. A *derived object* is generated, inferred, attributed, or constructed through declared relations or maps from previously available objects. An *emergent object* requires a stronger historical condition: the model must establish enough persistence, relational organization, and causal or generative efficacy for the object to be treated as a relationally real structure rather than as a transient description. Definition 7.1 records this usage.

Definition 7.1 (Relative formal status). For a concept type X represented at resolution κ , its Paper I formal status is written schematically as

$$\sigma_\kappa(X) \in \{\text{RP}, \text{D}, \text{E}\}, \quad (47)$$

where RP denotes a relative primitive of the selected instantiation, D a derived object, and E a relationally emergent object. The classification is model-relative and does not assign an ultimate ontological category to X .

Equation 47 is intentionally weaker than a universal ontology of concepts. An intention can be supplied as a relative primitive in a coarse behavioral model while becoming a derived object in a richer psychological model. Trust can be treated as a reported variable in an empirical dataset while remaining relationally emergent in the GR interpretation of that dataset. The status ledger therefore records where the present formal derivation begins, not what reality is ultimately made of.

Every retained concept also inherits the three-level audit introduced in Section 5. For a concept X , the analysis should specify the relational object X_t^R when such an object is warranted, the manifestations through which it becomes accessible, the subject-relative reconstructions formed by A and B, and the historical trajectories of those relations. Ultimate ontological status remains suspended. The inventory additionally records whether the object is primarily descriptive, epistemic, evaluative, procedural, transformative, or emergent. These roles can overlap, but they should not be collapsed by notation.

Table 6 summarizes the principal families. The later tables unpack them into more specific objects and indicate the default formal status used by Paper I.

7.2 Subject-History Objects

The first family contains the carriers already established in Section 6. The focal subjects are represented through $\mathcal{K}_A^\kappa$ and $\mathcal{K}_B^\kappa$ inside the coupled configuration $\mathcal{K}_{AB}^\kappa$. Their interface $\mathcal{J}_{AB}^\kappa$, shared represented material $\mathcal{O}_{AB}^\kappa$, historical tracking relations $\tau_{t' \leftarrow t}^i$, and role relation ϱ_t^κ are likewise available before the later epistemic and evaluative layers are built.

Within Paper I these objects function as relative primitives or immediately derived structural objects. Their substantive content is supplied by the instantiation, while their admissible relations are constrained by the GR architecture. They include the historical ordering used by the model, the selected resolution, subject boundaries, relational interfaces, and the supporter/supportee positions π_S and π_R . None of these objects establishes that the corresponding subject, boundary, or role is ontologically fundamental.

The role-position family also includes later positions that can become relevant inside the dyad: requester, responder, observer, evaluator, decision-maker, refusal-holder, exit-capable participant, partner, co-participant, and other locally justified positions. These positions are derived from the relational configuration and can change over time. A subject can occupy several positions simultaneously, and the same position can be differently manifested or differently recognized by A and B.

7.3 Capacity-State Objects

A dyadic support relation depends on subject conditions that need not be reducible to one state vector. Paper I therefore retains a family of capacity and state-like objects whose exact mathematical carriers remain application-dependent. The family can include bodily or material condition, practical capacity, cognitive capacity, communicative capacity, interpretive capacity, normative capacity, affective state, attention, desire, preference, intention, expectation, memory, prior commitment, vulnerability, available resources, refusal capacity, exit capacity, revision capacity, and generative capacity.

Several distinctions are essential. Capacity differs from realized influence or realized action. Preference differs from desire. Intention differs from the history that generated an action. Memory differs from the historical structure itself. Vulnerability differs from a judgment that intervention is required. Generative capacity differs from a scalar amount of power. These distinctions allow a later model to ask

Table 6: Master inventory of conceptual object families in the minimal dyadic formalism.

Object family	Default status	Principal formal role	Main three-level or historical burden
Subject, history, position	RP/D	Carrier, tracking, role structure	Subject manifestation, boundary, persistence, role trajectory
Capacity, state, orientation	RP/D	Local subject condition	Manifestation, inferability, historical modification
Manifestation, epistemic access	D	Accessibility and reconstruction	Relational object versus manifested form versus subject model
Theory of mind, communication	D	Recursive mutual modeling	Perspective dependence, depth, confidence, revision
Need, value, aesthetic, normative judgment	D/E	Evaluative formation	Pre-explicit structure, articulation, disagreement, stabilization
Authorization, procedure	D	Request, consent, refusal, participation	Capacity conditions, manifestation, scope, withdrawal
Action, event, consequence	D	Historical transformation	Provenance, attribution, manifestation, side-relative knowledge
Counterfactual, prediction	D	Possible continuation comparison	Horizon, uncertainty, prediction error, revision
Asymmetry, misalignment	D	Typed relational comparison	Dimension, definedness, history, non-scalar comparison
Feedback, revision	D	Recursive update	Interpretation of feedback, changed mappings, changed operators
Trust, recognition, dependence, shared norm	E	Relationally real dyadic organization	Persistence, efficacy, manifestation, emergence trajectory
Algebraic, operator, field objects	D	Mathematical realization	Support region, typing, coherence, representation dependence

whether one subject possesses a relevant capacity, whether that capacity becomes manifest, whether the other subject recognizes it, and whether it becomes effective in the interaction.

The three-level audit is especially important here because many state-like objects are only indirectly accessible. A bodily condition can have relational efficacy before A recognizes it. B can manifest distress without endorsing A's interpretation of that manifestation. A declared preference can be a manifestation of B's present judgment while leaving open the wider relational history through which that preference formed. The epistemic sections that follow therefore operate on manifestations and reconstructions rather than assuming transparent access to the retained subject condition.

Table 7: Capacity and state-like objects retained by the minimal dyadic model.

Object	Default status	Distinction preserved	Typical trajectory question
Bodily/material condition	RP	Condition versus manifestation or diagnosis	What changes physically or materially across support?
Practical capacity	RP/D	Ability versus realized action	Does support enlarge or narrow later feasible action?
Cognitive/interpretive capacity	RP/D	Competence versus conclusion	Does mutual intelligibility develop across interaction?
Communicative capacity	RP/D	Expressive ability versus message content	Do new forms of expression become available?
Desire, preference	RP/D	Orientation versus normative validity	How are orientations formed, revised, or stabilized?
Intention	D	Self-attributed aim versus full action provenance	Does later feedback alter self-understanding of action?
Affect, attention	RP/D	Pre-explicit salience versus articulated judgment	How does attention become historically organized?
Expectation, memory	D	Anticipation/retention versus actual history	How does repeated interaction alter prediction and recall?
Vulnerability	D	Exposure or fragility versus need judgment	Which relational conditions make vulnerability consequential?
Resources	RP	Available means versus legitimate authority	How do resource relations change dependence or access?
Refusal/exit capacity	D	Formal permission versus effective capacity	Can B contest, refuse, or leave as the relation evolves?
Revision capacity	D	Ability to update versus actual revision	Can either side revise models, judgments, and roles?
Generative capacity	D/E	Ability to contribute to later formation versus realized influence	Does interaction widen or narrow future participation?

Table 7 records the main state-like families and the distinctions that Paper I preserves.

7.4 Manifestation-Epistemic Objects

The manifestation and epistemic family contains the objects through which one subject becomes partially accessible to the other. Its minimal members include manifestation tokens or families, observability rela-

tions, actual observation, representation, interpretation, belief, knowledge claims, confidence, memory of manifestation, disclosure, concealment, strategic presentation, accidental unavailability, and joint acknowledgment. These objects are derived from the coupled spacetime, manifestation relation, historical context, and subject-relative reconstruction maps introduced in Section 5.

The inventory distinguishes at least five questions that ordinary language often compresses into the verb *know*: whether an object can manifest to a subject, whether the subject encounters the manifestation, whether the manifestation is represented, whether an interpretation is formed, and whether the resulting belief is treated as justified or sufficiently reliable for the local purpose. Paper I does not require one universal epistemology that reduces these stages to scalar certainty. It requires the stages to remain available for separate modeling.

For a retained object X , the accessibility pattern can therefore differ between A and B. A private intention can be accessible to A through self-report while remaining inferential for B. A bodily consequence can be experienced by B before A receives any manifestation of it. A public action can be visible to both while its significance remains differently interpreted. A jointly observed event can become jointly acknowledged only after a further communicative act. These cases motivate the explicit epistemic architecture developed in the later section on mutual manifestation and epistemic access.

7.5 Theory-of-Mind Objects

Theory-of-mind objects are derived subject models constructed from manifestations, prior history, interpretive resources, and other retained epistemic inputs. The basic forms are $T_A(B)$ and $T_B(A)$. Their internal contents can include modeled beliefs, preferences, intentions, needs, competencies, values, emotions, roles, and expectations. Higher-order objects such as $T_A(T_B(A))$ become available when the local model requires one subject's representation of the other's representation.

The inventory also includes ToM depth, confidence, false belief, underdetermined belief, strategic manipulation of belief, and revision after feedback. Higher-order recursion is not treated as automatically infinite in practical modeling. A finite dyadic application can truncate the hierarchy at the depth required by the question, provided that the truncation is stated. ToM objects remain relationally situated epistemic constructions rather than direct copies of another subject-spacetime.

A further distinction concerns self-modeling. A can possess a self-representation of intention, motive, role, or competence that differs from a later relational reconstruction of the action's generative provenance. B can likewise misrecognize or revise B's own state. The inventory therefore permits self-ToM-like or self-model objects without granting either subject infallible access to its own generative history.

7.6 Communication Objects

Communication supplies manifestations that can reorganize the epistemic and procedural structure of the dyad. Candidate objects include request, invitation, offer, question, clarification, statement, proposal, warning, advice, refusal, counterproposal, gratitude, complaint, explanation, correction, apology, acknowledgment, silence, delay, withdrawal, and nonverbal signaling. Their formal type depends on the application: a communication can be represented as an event, a manifestation, a partially defined transformation, or a structured message object carried by an interface.

A message and its interpretation remain distinct. A request can be manifested clearly while the scope of the request remains disputed. Silence can be observed while its meaning remains underdetermined. Gratitude can manifest appreciation, social convention, relief, strategic politeness, or another relational state depending on the history and context. Communication therefore enters the later formalism through typed manifestation and interpretation rather than through a universal semantic map.

Communication can also change the conditions of later communication. A clarification can create a shared vocabulary. A refusal can alter role expectations. Repeated successful translation can increase trust and lower the cost of later explanation. Conversely, repeated misinterpretation can narrow the set

of signals treated as credible. These second-order effects make communication part of the historical generativity of the dyad.

7.7 Evaluative-Judgment Objects

The evaluative family contains need judgments, practical evaluations, goodness judgments, aesthetic judgments, affectively mediated distinctions, explicit normative judgments, and judgments about other judgments. These objects are derived. A need is not inserted as a primitive property of B, and an action is not labeled good merely because A intends support. The later evaluative section will specify how these judgments are typed and how disagreement is represented.

Need-related objects include B's self-judged need, A's judgment of B's need, and a jointly stabilized need judgment where interaction supports one. The three can diverge. Need can also change over time, remain hidden, be misclassified, or concern different dimensions such as immediate material condition, practical capacity, relational participation, or longer-term generative possibility.

Goodness-related objects include A's judgment that an action is good for B, B's judgment that it is good for B, judgments concerning effects on A, and judgments concerning the relation itself. Short-term benefit, long-term capability, autonomy, dependence, dignity, trust, and reversibility can enter such evaluations without being forced into one scalar utility function.

Aesthetic and affective objects include fittingness, harmony, dignity, discomfort, admiration, aversion, awkwardness, shame, gratitude, resentment, meaningfulness, and other locally justified distinctions. Some of these can arise through pre-explicit manifestation structure before an explicit aesthetic proposition is articulated. Their later stabilization can in turn reshape attention and manifestation. Explicit normative modalities can include required, permitted, optional, supererogatory, discouraged, forbidden, legitimate, illegitimate, justified, and unjustified. Paper I keeps formation of these judgments separate from their validity.

Judgments themselves can become objects. A can evaluate B's refusal, B can evaluate A's judgment of need, and either subject can form a judgment concerning the other's judgment of the relation. This recursive availability will later support analysis of paternalism, disagreement, recognition, and normative revision without requiring those later evaluations to be introduced as primitives.

7.8 Authorization-Procedure Objects

The authorization family distinguishes request, permission, consent, acquiescence, compliance, endorsement, refusal, withdrawal of consent, refusal capacity, exit capacity, competence, information conditions, standing, authority, and role-based permission. The distinction is necessary because the manifestations of these objects can look similar while their relational status differs. A spoken "yes" can manifest consent, compliance, uncertainty, politeness, or pressure depending on the historical configuration. Silence can accompany permission in one practice and have no authorizing significance in another.

Interactive procedure objects organize these authorizations through time. The minimal repertoire includes initial encounter, inquiry, assessment, mutual clarification, negotiation, consent procedure, joint planning, support execution, monitoring, feedback, correction, repeated interaction, handover, withdrawal, termination, and post-action reflection. The procedure is not assumed to be a fixed pipeline. Stages can be omitted, repeated, reordered, jointly produced, or rendered undefined by an earlier transformation.

Standing and authority require particular restraint. A's judgment that an action would benefit B does not itself generate authority to intervene. B's request can generate a reason for action without determining every later procedural permission. Professional role, emergency circumstance, prior agreement, institutional mandate, or another relation can modify standing, but those relations must be represented explicitly. The later ethics and justice papers will evaluate such structures; Paper I supplies the objects required for that evaluation.

7.9 Action-Event-Consequence Objects

The action family is central because Paper I treats action as a derived GR object rather than as an atomic primitive. The inventory distinguishes an underlying relational process or event, its individuation as an action, attribution to one or more subjects or relations, generative provenance, intention, execution, manifestation, interpretation, and relational consequence. The detailed construction appears in the later section on action genesis, provenance, attribution, and epistemic profiles.

Candidate action types include observation, inquiry, asking, offering, giving, transferring resources, teaching, advising, accompanying, caring, protecting, mediating, advocating, representing, coordinating, facilitating access, withholding, refusing, withdrawing, correcting, repairing, apologizing, terminating support, and transitioning toward partnership. The list is functional rather than moral. The same type can be beneficial, harmful, ambiguous, or differently evaluated depending on provenance, manifestation, consent, consequence, and history.

Consequences are separate objects. They can be intended or unintended, foreseen or unforeseen, direct or indirect, immediate or delayed, reversible or practically irreversible. A consequence can affect B, A, the coupled relation, later observability, future action families, refusal capacity, exit capacity, trust, role structure, or the manifestation relation itself. The distinction between action and consequence prevents retrospective outcome knowledge from being read back into the action's initial epistemic profile.

Action attribution also remains open. The executor, initiator, generator, and author of one support event can differ. B can initiate a process through a request while A executes the visible action. A jointly negotiated plan can have no adequate single-agent generator even when one subject performs the final physical transfer. An institution can structure the action family while remaining outside the focal dyad at the current resolution. Paper I records these possibilities without dissolving the dyadic boundary into a complete institutional model.

7.10 Counterfactual-Prediction Objects

Support decisions frequently rely on possible futures. The inventory therefore includes subject-relative predictions of continuation under a candidate action, continuation without the action, and continuation under alternative actions. These are derived epistemic objects rather than realized histories. They require a declared horizon, a criterion for comparing continuations, and a statement of uncertainty.

Prediction can concern material effect, practical capacity, dependence, autonomy, trust, role stability, reversibility, or another retained dimension. A and B can form different counterfactuals from the same manifestation history. Later realized history can diverge from both. Prediction error then becomes part of the feedback and revision architecture rather than evidence that the original action was irrational by definition.

The counterfactual family also provides a bridge to generative possibility. A subject can judge an action partly by how it changes admissible or effectively reachable continuations. Paper I keeps such judgments distinct from a substantive claim that the predicted future is objectively best.

7.11 Asymmetry-Misalignment Objects

The dyad can contain multiple typed asymmetries. Candidate dimensions include knowledge, information access, resources, competence, institutional status, dependency, recognition, communication, interpretive capacity, refusal capacity, exit capacity, generative capacity, authority, and realized normative influence. The inventory treats each asymmetry as dimension-specific. A relation can be symmetric in one dimension and asymmetric in another.

Misalignment differs from asymmetry. Misalignment concerns a discrepancy between objects that are intended to correspond or that become analytically compared, such as B's relational condition and A's model of B, B's self-judged need and A's judgment of B's need, A's intention and B's inferred intention, A's goodness judgment and B's goodness judgment, manifested consent and inferred consent,

or predicted consequence and realized consequence. The paper does not use subtraction notation unless the relevant carriers support subtraction or another declared discrepancy operation.

The asymmetry and misalignment families can interact. Epistemic asymmetry can increase the chance of need-judgment misalignment. Resource asymmetry can reduce effective refusal capacity. Recognition asymmetry can influence whose interpretation becomes stabilized. These relations remain hypotheses or derived mechanisms until the corresponding maps and historical evidence are specified.

7.12 Feedback-Revision Objects

Feedback objects record later manifestations or relational changes that become available for revising earlier models and judgments. B's reported experience of an intervention, A's observation of an unintended consequence, a refusal, a change of trust, a new request, or a later reinterpretation can all function as feedback. The feedback event is distinct from A's or B's interpretation of it.

Revision can affect ToM, need judgment, goodness judgment, normative judgment, role assignment, counterfactual expectations, admissible action family, trust, recognition, refusal capacity, and later communication. A system with feedback but no revision capacity can repeatedly reproduce the same misalignment. A system with revision capacity can still fail to revise if manifestations are inaccessible, distrusted, or translated through a rigid interpretive frame.

The inventory also allows revision of the cross-level relations themselves. Interaction can change how B manifests distress, how A interprets silence, how consent is signaled, or which expressions count as credible. This possibility is one reason the history of μ_t , $\rho_{A,t}$, and $\rho_{B,t}$ belongs to the formal object rather than to an external commentary on the model.

7.13 Emergent Relational Objects

Some of the most important concepts in volunteer relations should normally enter as emergent relational structures. Candidate objects include trust, distrust, recognition, mutual recognition, reciprocity, dependence, mutual dependence, expectation, reliability, obligation, standing, authority, legitimacy, role stabilization, helper-supportee patterning, partnership, shared evaluative vocabulary, and shared norm. Their presence is not inferred from one isolated manifestation.

The emergence requirement is historical. A model should state the relations through which the candidate structure is constituted, the horizon over which it persists, the manifestations through which it can be investigated, and the causal or generative consequences that make it relevant to later interaction. Trust, for example, can be associated with repeated reliance, disclosure, reduced monitoring, accepted vulnerability, or other relations, but none of those manifestations alone universally defines trust. A shared norm can organize later judgment even amid disagreement, but coincidence of two private judgments does not automatically create a shared normative structure.

Emergent objects can also become inputs to later generation. Trust can alter which requests are credible. A stabilized helper-supportee role can change which actions appear natural or expected. A shared vocabulary can make previously inaccessible distinctions easier to articulate. Dependence can reshape refusal or exit capacity. The inventory therefore treats emergence as a change in the later generative environment rather than as the appearance of an inert label.

Table 8 summarizes the main emergent candidates and the evidence burden attached to each.

7.14 Algebraic-Operator-Field Objects

The final family contains mathematical realization objects. These are derived from the substantive GR architecture rather than treated as substantive concepts by themselves. Candidate objects include local algebras $\mathcal{A}(R)$, algebra embeddings or restriction-compatible maps, states or evaluation functionals, typed partial transformations, generators, operator representations, comparison transports, regional realization maps, quasi-local completions, compatible field families, path composites, cycle diagnostics, and optional topological invariants.

Table 8: Candidate emergent relational objects and their minimum evidential burden in Paper I.

Candidate object	Required relational burden	Historical or manifestation burden
Trust / distrust	Structured reliance, vulnerability, expectation, or credibility relation	Persistence beyond one signal; context-sensitive manifestations
Recognition	Relational positioning of the other as a subject, role-holder, partner, recipient, or another locally defined status	Recurrent treatment or uptake; possible misrecognition and revision
Reciprocity	Historically linked responsive contributions across the dyad	Tracking of response relations; need not imply equal exchange
Dependence	Later possibilities materially conditioned by retained relation to the other	Persistence, alternatives, exit conditions, changing intensity or type
Standing / authority	Relationally effective capacity to make claims, authorize, decide, or intervene	Source, scope, contestability, manifestation, possible withdrawal
Obligation	Stabilized normative relation connecting claim, role, history, or commitment to later action	Formation history, recognition, contestability, revision
Role stabilization	Recurrent supporter/supportee or other position structure	Persistence, reproduction, manifestation, reversal/dissolution conditions
Partnership	Jointly sustained relation with reciprocal participation in relevant generation	Shared procedures, mutual recognition, role transformation
Shared vocabulary	Commonly usable distinctions that organize mutual interpretation	Historical acquisition, translation, revision, exclusions
Shared norm	Relationally stabilized normative structure affecting later judgment or practice	Formation, manifestation, disagreement, efficacy, revisability

Operator families later used by Paper I can include manifestation/readout maps, attention-shifting maps, recognition transformations, communication maps, request or refusal transformations, evaluative-judgment generators, support/intervention transformations, feedback maps, revision maps, and withdrawal or termination transformations. Their names do not guarantee that they are linear operators on one Hilbert space. The mathematical type must be derived from the carrier on which each transformation acts.

Field-level objects require an additional coherence burden. Local data become a field representation only when the selected regions possess compatible carriers, realization maps, restriction structure, overlap consistency, interface comparison or transport, and whatever gluing or compatible-family construction the model requires. A labelled 2-complex can provide the history-bearing carrier, while a spin-foam-inspired realization can supply one more specialized labelled-history architecture. Neither mathematical choice changes the three-level status of the substantive social concepts by itself.

7.15 Inventory Closure Rules

The concept inventory is finite at the level of families but recursively open at the level of generated objects. A judgment can become the object of another judgment. A manifestation can concern an earlier representation. A ToM model can contain a model of another ToM model. A role can become the object of refusal or endorsement. A shared norm can alter the criteria through which later manifestations

are interpreted. Paper I therefore needs a closure discipline rather than an attempt to enumerate every possible higher-order object.

Let $\mathcal{I}_0^\kappa$ denote the retained finite inventory of base concept types at resolution κ . Let $\mathfrak{G}^\kappa$ denote the family of well-typed generation operations admitted by the model, including representation, interpretation, ToM construction, evaluation, judgment, attribution, historical tracking, and state-changing transformation where their domains are defined. The recursively generated inventory is stated schematically in Equation 48.

$$\mathcal{I}_{n+1}^\kappa := \mathcal{I}_n^\kappa \cup \{G(x_1, \dots, x_m) \mid G \in \mathfrak{G}^\kappa, (x_1, \dots, x_m) \in \text{Dom}(G) \cap (\mathcal{I}_n^\kappa)^m\}, \quad n \geq 0. \quad (48)$$

Equation 48 does not assert that every formally generable object is analytically useful. It records a grammar: higher-order objects become admissible only through operations whose domains, codomains, and interpretation have been declared. The full finite-depth inventory used by one application is then a selected subset of $\bigcup_{n \geq 0} \mathcal{I}_n^\kappa$, with recursion truncated by the research question and available evidence.

This closure rule prevents two opposite errors. The first is uncontrolled symbol proliferation, in which every psychologically or normatively conceivable object receives an independent primitive symbol. The second is premature reduction, in which higher-order structures are declared impossible because the initial inventory did not list them explicitly. A finite basis plus typed generation rules preserves both tractability and conceptual openness.

7.16 Concept Audit Protocol

The inventory supplies a recurring protocol for the remainder of the manuscript. Before a major concept is used in a formal claim, the paper should be able to answer the following questions: what is the object's relative formal status; what mathematical type carries it; at which GR reality level is the substantive claim made; through which manifestations can the object become accessible; how can A and B reconstruct it differently; what historical tracking is required; whether the object changes the represented configuration; which operations can act on it; what assumptions permit composition; and what remains undetermined.

Table 9 records these fields. The table is a writing and verification device as much as a mathematical one. It provides a test against the accidental reintroduction of point-like subjects, transparent action, primitive need, scalar power, or universal commensurability later in the paper.

The resulting inventory makes the minimal dyad comprehensive without converting it into an undifferentiated master state. Subject-spacetimes, manifestations, models, judgments, actions, procedures, asymmetries, and emergent relations remain separately typed while participating in one historical system. The next section can therefore develop mutual manifestation and epistemic access with a clear account of which objects are being accessed and which objects are being constructed through that access.

Table 9: Concept audit fields applied before a major Paper I object enters formal argument.

Audit field	Required question
Relative status	Is the object a relative primitive, derived construction, or relationally emergent structure?
Formal type	What set, relation, map, algebraic element, functional, family, or other carrier contains it?
Reality level	Is the substantive claim relational, manifestational, or explicitly left under ontological suspension?
Manifestation	Through which context-dependent forms can the object become accessible?
Subject-relative reconstruction	How can A and B model or interpret the object differently?
Historical tracking	What permits the object or its role to be identified across the relevant trajectory?
Dynamical role	Does the object describe, evaluate, authorize, transform, or emerge from the configuration?
Definedness	On which configurations or regions is the relevant operation meaningful?
Composition	Which domain/codomain or compatibility conditions permit later operations to compose?
Uncertainty and limitation	What remains underdetermined, representation-dependent, or reserved for later papers?

8 Mutual Manifestation: Epistemic Access, Observation, Reconstruction

This section develops the epistemic interface through which the two history-bearing subject-spacetimes of Section 6 become partially accessible to one another. Its role is to refine the three-level architecture of Section 5 by separating manifestation that exists in the retained model from manifestation that is accessible to a particular subject, manifestation that is actually taken up, and relational structure that is reconstructed from that uptake. The objective is to make mutual access explicit without treating either subject as transparent to the other or to itself. The discussion proceeds through directional manifestation, accessibility, observation and uptake, shared and private visibility, reconstruction and confidence, memory, disclosure and concealment, joint acknowledgment, self-manifestation, and historical change in the access architecture. The method is typed and reconstructive. Epistemic terminology is used at the level required by the dyadic formalism; a complete philosophical theory of knowledge is outside the scope of Paper I.

Figure 3 summarizes the architecture. A relational object can admit several manifestations. Only some manifestations are accessible to a given subject, and only some accessible manifestations are actually registered or taken up. Subject-relative reconstruction then acts on uptake together with retained history. The figure represents a factorization of analytical stages rather than a universal psychological sequence.

8.1 Directional Mutual Manifestation

Mutual manifestation begins from a simple asymmetry of access: each subject-spacetime can become manifest to the other only through selected forms available in the coupled configuration. The existence of a coupled relational spacetime $\mathcal{K}_{AB,t}^\kappa$ does not imply that A has direct access to $\mathcal{K}_{B,t}^\kappa$ or that B has direct access to $\mathcal{K}_{A,t}^\kappa$. Each side encounters context-dependent manifestations that may reveal, obscure, compress, or reorganize selected relational features.

Let Subj_A and Subj_B denote the relational subject objects tracked at the current resolution. Their

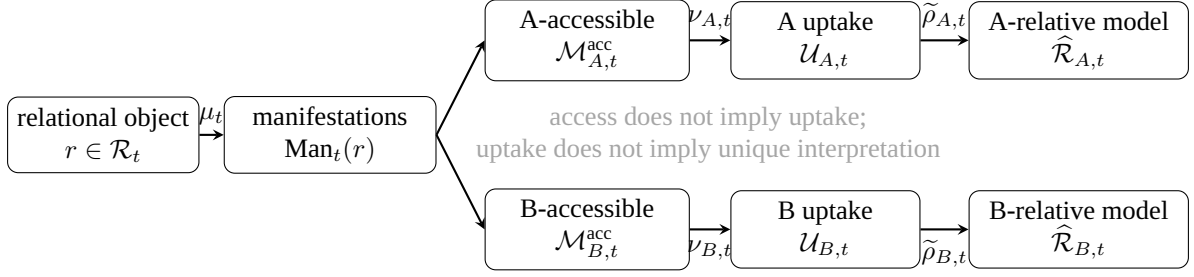


Figure 3: Epistemic-access architecture for a relational object. The manifestation relation μ_t supplies admissible manifestations. Subject-relative accessibility selects manifestations available to A or B. Observation/uptake maps $\nu_{A,t}$ and $\nu_{B,t}$ produce registered epistemic inputs, and reconstruction maps $\tilde{\rho}_{A,t}$ and $\tilde{\rho}_{B,t}$ generate subject-relative models using those inputs together with retained history. The arrows do not imply that each stage is total, deterministic, or temporally sequential in every application.

retained manifestation families are supplied by Equation 26. To record the direction in which a subject becomes accessible, define the directional accessible-manifestation family in Equation 49.

$$\mathcal{M}_{i \rightarrow j,t} := \text{Man}_t(\text{Subj}_i) \cap \mathcal{M}_{j,t}^{\text{acc}}, \quad i, j \in \{A, B\}, i \neq j. \quad (49)$$

Equation 49 records manifestations of subject i that are accessible to subject j under the retained model. The intersection is defined at the manifestation level. It does not assert that the relational subject itself is contained in the other subject's epistemic domain. A bodily expression, a message, an institutional credential, a refusal, a delay, or a visible transfer can belong to $\mathcal{M}_{i \rightarrow j,t}$ while the relational structure that gives that manifestation its meaning remains only partially reconstructed.

Mutual manifestation requires neither equal bandwidth nor symmetric form. The families $\mathcal{M}_{A \rightarrow B,t}$ and $\mathcal{M}_{B \rightarrow A,t}$ can differ in size, type, persistence, reliability, and historical depth. B may disclose extensive information about vulnerability while A discloses little about intention. A may possess institutional records concerning B while B has access mainly to A's immediate conduct. The resulting asymmetry belongs to the access architecture and should not be converted automatically into a moral or power judgment. Work on epistemic injustice shows why credibility and shared interpretive resources can later acquire ethical and political significance, but the present formalism first records the structure of access and interpretation without inferring injustice from difference alone [5].

The directional family also leaves room for manifestations generated jointly. A jointly written plan, a shared routine, a jointly enacted gesture, or a jointly maintained record need not belong exclusively to either directional family. Such manifestations can be attached to the coupled relation or to a shared region of $\mathcal{K}_{AB,t}^\kappa$. Paper I therefore treats mutual manifestation as a structure of directional and shared accessibility rather than as two independent streams emitted by isolated subjects.

8.2 Accessibility, Observation, Uptake

Accessibility and observation are different formal conditions. Section 5.5 introduced $\mathcal{M}_{i,t}^{\text{acc}}$ as the manifestations accessible to subject i . Accessibility states that a manifestation is available through the retained channels and conditions of the model. It does not imply that the subject attends to it, perceives it, records it, or treats it as relevant.

To retain this distinction, let $\mathcal{U}_{i,t}$ denote a class of subject-relative uptake records. An uptake record can represent attention, perceptual registration, receipt of a message, retrieval of a document, or another model-specific form in which an accessible manifestation becomes part of the subject's current epistemic input. The observation/uptake map is introduced in Equation 50.

$$\nu_{i,t} : \mathcal{M}_{i,t}^{\text{acc}} \rightarrow \mathcal{U}_{i,t}. \quad (50)$$

The partiality of Equation 50 is substantive. A manifestation can be accessible while remaining unattended, masked by competing signals, below a perceptual threshold, excluded by an interpretive routine, or otherwise absent from the subject’s effective uptake. A visible facial expression need not be noticed. A written refusal can be received but not read. A public record can be available but practically inaccessible because the subject lacks the competence or time needed to retrieve it.

The map $\nu_{i,t}$ also prevents the word *observation* from carrying more epistemic force than the formalism supports. Uptake need not produce accurate identification, belief, or knowledge. A subject can register an utterance while misunderstanding its language. A subject can notice silence while lacking a basis for classifying its meaning. The formal sequence therefore separates accessibility, uptake, interpretation, and later epistemic standing.

Section 5.5 introduced the coarser reconstruction map $\rho_{i,t} : \mathcal{E}_{i,t} \rightarrow \widehat{\mathcal{R}}_{i,t}$ with $\mathcal{E}_{i,t} = \mathcal{M}_{i,t}^{\text{acc}} \times \mathcal{H}_{i,t}$. Where the application requires explicit observation structure, this map can be factorized through uptake. Let $\tilde{\rho}_{i,t}$ denote a reconstruction map whose immediate manifestation input is an uptake record. The factorization is stated in Equation 51.

$$\rho_{i,t}(m, h) = \tilde{\rho}_{i,t}(\nu_{i,t}(m), h) \quad (51)$$

whenever $m \in \text{Dom}(\nu_{i,t})$ and the right-hand side is defined. Equation 51 is an optional refinement of the earlier architecture rather than a replacement for it. Applications that do not need to distinguish accessibility from actual uptake can retain the coarser map $\rho_{i,t}$.

8.3 Shared, Private, Hidden Manifestations

The dyadic model requires a vocabulary for asymmetric access without assuming that every manifestation has a unique owner. For $m \in \mathcal{M}_t$, define its accessibility profile by Equation 52.

$$\mathbf{V}_t(m) := \{i \in \{A, B\} \mid m \in \mathcal{M}_{i,t}^{\text{acc}}\}. \quad (52)$$

The set $\mathbf{V}_t(m)$ has four possible values in the strict dyadic model: $\{A, B\}$, $\{A\}$, $\{B\}$, or $\emptyset$. These values correspond respectively to jointly accessible, A-only accessible, B-only accessible, and currently inaccessible manifestations. The classification concerns accessibility, not observation or interpretation.

Actual uptake can be tracked separately. Define the uptake profile in Equation 53.

$$\mathbf{U}_t(m) := \{i \in \{A, B\} \mid m \in \text{Dom}(\nu_{i,t})\}. \quad (53)$$

Equation 53 permits $\mathbf{V}_t(m) = \{A, B\}$ while $\mathbf{U}_t(m) = \{A\}$, for example when both subjects could observe a manifested event but only A attends to it. It also permits a manifestation to be accessible to B but not interpretable by B after uptake. Shared observability therefore remains weaker than shared understanding.

Several forms of hiddenness should remain distinct. A relational object r can have no retained manifestation at the modeled stage, so that $\text{Man}_t(r) = \emptyset$. Alternatively, r can have manifestations while every retained manifestation is inaccessible to both subjects. A third possibility is asymmetric access, where only one subject can encounter the relevant manifestation. A fourth is failed uptake despite accessibility. A fifth is successful uptake followed by unresolved interpretation. Table 10 distinguishes these cases.

This taxonomy matters because the same manifested absence can arise through different mechanisms. A subject can remain unaware of a need because the need has no accessible manifestation, because an accessible signal is not noticed, because the signal is noticed but not understood, or because several interpretations remain viable. A dyadic ethics that treats every failure to respond as the same epistemic failure would therefore erase distinctions that the formal architecture makes available.

Table 10: Distinct failure modes between relational structure and subject-relative reconstruction.

Condition	Formal signature	Analytical interpretation
Unmanifested	$\text{Man}_t(r) = \emptyset$	No retained manifestation of the relational object in the modeled context
Manifested but inaccessible	$m \in \text{Man}_t(r)$ and $V_t(m) = \emptyset$	Manifestation exists in the model but is unavailable to A and B
Asymmetrically accessible	$ V_t(m) = 1$	Only one subject has access to the manifestation
Accessible but not taken up	$i \in V_t(m)$ and $i \notin U_t(m)$	Available manifestation fails to enter current epistemic uptake
Taken up but uninterpreted	$m \in \text{Dom}(\nu_{i,t})$ while $\tilde{\rho}_{i,t}$ is undefined	Subject registers the manifestation without an admissible relational reconstruction
Interpreted but underdetermined	$\tilde{\rho}_{i,t}$ admits several retained models	Evidence supports more than one relational reconstruction

8.4 Reconstruction, Belief, Confidence

A subject-relative reconstruction is not automatically a belief, and a belief is not automatically knowledge. Paper I requires only enough epistemic structure to distinguish the object reconstructed, the subject's commitment to that reconstruction, and the strength or warrant of the commitment where the application supplies such information. The paper therefore avoids building a complete epistemology into the dyadic formalism.

Let q denote a proposition, classification, or structured claim extracted from a subject-relative reconstruction. Examples include claims that B is in pain, that A intends to assist, that a refusal is sincere, or that a role relation is present. The model can attach a subject-relative epistemic-status record $S_{i,t}(q)$ to q . At minimum, the record may distinguish whether the claim is entertained, accepted, rejected, unresolved, or unavailable. Applications can add richer structures for evidence and justification.

Confidence should also be separated from truth and warrant. Let $\mathcal{C}_{i,t}$ denote a model-specific confidence carrier. The paper does not assume that $\mathcal{C}_{i,t} = [0, 1]$ or that confidence is metrically comparable across subjects. When confidence is represented, use the partial map in Equation 54.

$$c_{i,t} : \hat{\mathcal{R}}_{i,t} \rightarrow \mathcal{C}_{i,t}. \quad (54)$$

Equation 54 permits a later finite instantiation to choose probabilities, ordinal levels, intervals, verbal categories, or another carrier appropriate to the evidence. The general formalism requires only that the interpretation of the codomain be stated. High confidence then records strong commitment within the chosen representation; it does not by itself establish that the reconstruction is accurate or justified.

This distinction is important for support relations because epistemic overconfidence can be relationally consequential even when the original interpretation was reasonable. A may possess limited manifestations of B's condition, form one plausible reconstruction, assign it high confidence, and intervene before seeking clarification. B can simultaneously interpret A's confidence as expertise, arrogance, care, or institutional authority. The later normative analysis therefore needs access to both the reconstruction and the epistemic stance attached to it.

A reconstruction can also be explicitly plural. Let $\mathcal{Q}_{i,t}(m, h) \subseteq \hat{\mathcal{R}}_{i,t}$ denote the retained candidate set compatible with an uptake-history record when the application wishes to preserve ambiguity rather than select one model. This candidate-set representation is useful where several explanations remain live. It prevents a computational convenience such as maximum-likelihood selection from being mistaken for the subject's or modeler's exclusive epistemic conclusion.

8.5 Manifestation Records: Memory, Forgetting, Distortion

Current uptake is only one source of epistemic reconstruction. Section 5.5 therefore included a historical record $\mathcal{H}_{i,t}$. The record is not identical with the objective relational history of the dyad. It contains whatever traces the model treats as available to subject i : memories, stored messages, documents, learned routines, remembered refusals, institutional records, previous interpretations, or other retained materials.

To distinguish historical occurrence from subject-relative retention, let $u \in \mathcal{U}_{i,t}$ be an uptake record at index t and let $t \preceq t'$. A memory-retention map can be represented by Equation 55.

$$\text{Mem}_{i,t' \leftarrow t} : \mathcal{U}_{i,t} \rightarrow \mathcal{H}_{i,t'}. \quad (55)$$

The map is partial because some uptake is forgotten or never stored in a form available to later reconstruction. It also need not preserve the uptake token literally. A later memory can be compressed, reclassified, merged with other episodes, or retained mainly through an affective or practical trace. The formalism therefore does not identify memory with perfect archival persistence.

Distortion can be represented only relative to a comparison structure. If the model provides a tracking relation between an earlier uptake object and its later retained representation, it can ask whether the later record preserves selected content. Without such a relation, the paper should not write a numerical memory error by default. This restraint parallels the broader GR rule that discrepancy requires a declared comparison type.

Memory changes the meaning of apparently identical manifestations. A phrase encountered after repeated reliability can be reconstructed differently from the same phrase encountered during an initial meeting. A delayed response can manifest ordinary busyness in one history and withdrawal in another. Historical epistemic records therefore enter reconstruction constitutively rather than as optional background commentary.

8.6 Disclosure, Concealment, Strategic Legibility

Subjects do not merely receive manifestations. They can participate in shaping which manifestations become available to the other side. Disclosure, concealment, selective presentation, simplification, translation, and strategic legibility can alter the access architecture without requiring a change in the underlying relational object of interest.

At the present stage, these processes are represented conservatively as changes in accessibility structure rather than as fully specified action operators. Let $\mathcal{M}_{j,t}^{\text{acc}}$ and $\mathcal{M}_{j,t'}^{\text{acc}}$ be the manifestation sets accessible to subject j before and after a retained interaction. A disclosure can enlarge accessibility in a relevant region, while concealment can reduce or redirect it. The relation in Equation 56 records this change without assuming monotonicity.

$$\mathcal{M}_{j,t}^{\text{acc}} \rightsquigarrow \mathcal{M}_{j,t'}^{\text{acc}}. \quad (56)$$

The wavy arrow in Equation 56 is descriptive shorthand rather than a defined dynamical map. Later sections on action genesis and local algebraic dynamics will supply typed transformations when the mechanism of disclosure or concealment is part of the substantive model.

Disclosure is broader than verbal self-report. B can disclose vulnerability by permitting observation, granting document access, accepting accompaniment, or changing a routine so that A can witness a relevant condition. A can disclose intention by explaining a plan, exposing decision criteria, sharing uncertainty, or making the limits of competence visible. Conversely, concealment can be active or incidental. Privacy, shame, fear, institutional confidentiality, translation difficulty, role expectations, interface design, or absence of a shared vocabulary can all restrict legibility without deliberate deception.

Strategic legibility adds another layer. A subject can choose manifestations partly because of how the other is expected to interpret them. Such behavior already anticipates the theory-of-mind structures developed in the next section. At the present stage, the important point is that the manifestation relation and the accessibility sets can be affected by historically generated expectations about another subject's

reconstruction process. The dyad therefore contains an epistemic feedback loop before any explicit strategic game is assumed.

8.7 Joint Acknowledgment

Joint accessibility does not guarantee that the two subjects know that they are attending to the same manifestation. Support procedures often require a stronger structure: each side must possess some manifestation that the other has received, understood, or accepted a relevant object. Examples include confirming a request, repeating a plan, signing a jointly reviewed document, acknowledging a refusal, or explicitly checking that an instruction was understood.

Paper I uses *joint acknowledgment* for this stronger but still finite requirement. It does not use the term *common knowledge* at this stage, because common knowledge normally invokes an indefinitely iterated epistemic condition. Higher-order recursion belongs to the theory-of-mind architecture of the next section.

Let $m \in \mathcal{M}_t$ be a manifestation that is jointly taken up, so that $U_t(m) = \{A, B\}$. A first-order joint acknowledgment requires additional manifestations a_A and a_B through which A and B respectively make their uptake of m available to the other side. The condition is stated schematically in Equation 57.

$$\text{Ack}_{AB,t}^{(1)}(m) \implies U_t(m) = \{A, B\} \text{ and } a_A \in \mathcal{M}_{A \rightarrow B,t}, a_B \in \mathcal{M}_{B \rightarrow A,t}, \quad (57)$$

where the local interpretation rules identify a_A and a_B as acknowledgments of uptake of m . Equation 57 gives a necessary structural pattern rather than a complete semantic definition. The meaning of an acknowledgment remains context dependent. A nod can acknowledge receipt in one practice and merely indicate politeness in another.

Joint acknowledgment is especially important for consent and procedure. A spoken assent that both parties hear is weaker than a jointly acknowledged agreement if the parties attach incompatible meanings to the assent. A refusal can be manifest and jointly heard while one side fails to acknowledge its normative significance. The distinction allows later sections to separate public manifestation, mutual uptake, shared interpretation, and normative authorization.

8.8 Self-Manifestation: Reflective Access, Self-Misinterpretation

The dyadic model should not assume that a subject possesses transparent access to its own relational constitution. A can observe some manifestations of A, remember some earlier intentions, and articulate some motives, yet the generative provenance of A's action can include habits, role expectations, affective responses, institutional conditions, or relational tensions that are not fully represented in A's self-description. The same applies to B.

For $i \in \{A, B\}$, define the self-accessible manifestation family by Equation 58.

$$\mathcal{M}_{i \rightarrow i,t} := \text{Man}_t(\text{Subj}_i) \cap \mathcal{M}_{i,t}^{\text{acc}}. \quad (58)$$

Equation 58 permits a subject to reconstruct itself from manifestations and retained history in the same broad architecture used for reconstruction of the other. The self-directed case differs because some internal or embodied traces may be accessible only to the subject, while other aspects of subject constitution can remain unavailable to explicit reflection.

Self-reconstruction should therefore remain separate from generative provenance. A later action section will distinguish the history through which an action becomes generated from A's account of why A acted. The epistemic-access architecture already makes room for that distinction: self-interpretation is one relationally situated reconstruction, not a privileged identity map from subject reality to perfect self-knowledge.

This restraint is also relevant to sincerity. A subject can sincerely report a motive while the model retains additional historical conditions that contributed to the action. Sincerity concerns the relation between a self-report and the subject's current self-model. It does not establish that the self-model exhausts

the action's provenance. Paper I can therefore analyze mixed or partially opaque motivation without treating every incomplete self-explanation as deception.

8.9 Epistemic Access Dynamics

The preceding structures are historical. Interaction can alter which manifestations exist, which are accessible, which are taken up, how they are remembered, and how they are reconstructed. The access architecture should therefore be treated as part of the evolving dyadic configuration rather than as a fixed observational shell placed around an otherwise dynamic relation.

For subject i , define the local epistemic-access record in Equation 59.

$$\mathfrak{E}_{i,t} := (\mathcal{M}_{i,t}^{\text{acc}}, \mathcal{U}_{i,t}, \nu_{i,t}, \mathcal{H}_{i,t}, \tilde{\rho}_{i,t}, c_{i,t}). \quad (59)$$

Equation 59 is a typed record rather than a vector state. Its components need not admit addition, scalar comparison, or one common topology. The dyadic epistemic record is the ordered pair defined in Equation 60.

$$\mathfrak{E}_{AB,t} := (\mathfrak{E}_{A,t}, \mathfrak{E}_{B,t}). \quad (60)$$

A later interaction can transform $\mathfrak{E}_{AB,t}$ by changing access, uptake, history, interpretation, or confidence. Learning a shared vocabulary can enlarge the domain of $\tilde{\rho}_{i,t}$. Repeated interaction can make previously ambiguous manifestations easier to interpret. Betrayal can reduce the credibility assigned to a familiar signal. A newly established privacy boundary can shrink access while improving the legitimacy of the relation. Epistemic development is therefore neither monotone increase nor convergence by default.

The distinction between access and reconstruction also clarifies the meaning of epistemic disagreement. Two subjects can reconstruct the same jointly observed manifestation differently because their histories, conceptual resources, confidence structures, or role expectations differ. Conversely, they can arrive at similar reconstructions from different manifestation sets. Agreement and disagreement therefore require analysis of provenance as well as comparison of final beliefs.

The next section develops theory-of-mind structures on top of this access architecture. ToM requires more than observing another subject. It requires a subject-relative model of the other's epistemic, intentional, preferential, or evaluative states, often together with models of how the other represents oneself. The present section supplies the manifestation, uptake, memory, and reconstruction structures that make those higher-order models formally available without granting transparent access to either subject.

9 Recursive Theory-of-Mind Structure

This section develops the theory-of-mind (ToM) layer built on the manifestation and epistemic-access architecture of Section 8. Its role is to represent how one history-bearing GR subject can form a partial model of another subject's beliefs, intentions, preferences, affects, evaluations, or judgments, and how such modeling can itself become recursively nested. The objective is formal rather than psychological: Paper I does not propose a complete cognitive theory of mindreading, nor does it treat ToM as transparent access to another subject. The discussion proceeds through first-order model construction, typed ToM content, recursive depth, belief divergence, model multiplicity, anticipatory manifestation, historical revision, and the limits that the ToM layer passes to the later action-genesis analysis.

The term *theory of mind* is used here in the broad inferential sense associated with attribution of mental states that are not directly observable and that can support prediction or interpretation of another subject's behavior [12]. Developmental work on false-belief and second-order-belief tasks supplies established examples in which one subject must represent another subject's belief as distinct from one's own epistemic position, and in which a subject can represent one person's belief about another person's belief [16, 11]. Paper I borrows only that structural lesson. It does not infer that every support relation

uses explicit propositional mindreading, that ToM is one psychologically unified mechanism, or that a formal nested model establishes the empirical cognitive process by which a particular volunteer or supportee reasons.

9.1 Theory-of-Mind Construction

Section 5.5 introduced a subject-relative relational reconstruction $\widehat{\mathcal{R}}_{i,t}$ generated from accessible manifestations and retained history. A ToM model is a more selective derived object. It concerns the part of a reconstruction that the model attributes to another subject's epistemic, intentional, preferential, affective, evaluative, or normative condition. The construction therefore begins after manifestation, accessibility, uptake, and reconstruction have already been distinguished.

Let $i, j \in \{A, B\}$ with $i \neq j$. Let $\mathfrak{T}_{i \triangleright j, t}^{(1)}$ denote the class of admissible first-order ToM records in which subject i models subject j at historical index t . The symbol $i \triangleright j$ marks the modeling direction; it does not denote a power relation or causal influence. Because one relational reconstruction can support more than one admissible model of another subject, Paper I begins with a relation rather than a single-valued function. The first-order ToM construction relation is stated in Equation 61.

$$\text{ToM}_{i \triangleright j, t}^{(1)} \subseteq \widehat{\mathcal{R}}_{i, t} \times \mathfrak{T}_{i \triangleright j, t}^{(1)}. \quad (61)$$

Membership $(\widehat{r}_i, \theta_{i \triangleright j}) \in \text{ToM}_{i \triangleright j, t}^{(1)}$ means that the subject-relative reconstruction $\widehat{r}_i$ supports $\theta_{i \triangleright j}$ as an admissible model of j under the retained interpretive rules. Equation 61 preserves underdetermination: the same manifestation-history record can support several candidate ToM records, and no candidate need be available when the relevant evidence or conceptual structure is insufficient.

A deterministic specialization is available only when the application supplies a rule selecting at most one retained ToM record. In that case, Equation 61 can be represented by a partial map $\vartheta_{i \triangleright j, t}^{(1)} : \widehat{\mathcal{R}}_{i, t} \rightarrow \mathfrak{T}_{i \triangleright j, t}^{(1)}$. The relational formulation remains the default because it does not hide model multiplicity behind an arbitrary selection rule.

The ToM layer remains inside relational and manifestation analysis. A ToM record is a relationally situated model formed by one subject from manifestations, history, and interpretive resources. It is not a fourth reality level and is not identified with the modeled subject's relational reality. In particular, the existence of $\theta_{A \triangleright B}$ does not imply that B's relevant states have been fully or correctly recovered.

9.2 First-Order Mental-State Models

A first-order ToM record should preserve heterogeneity among the kinds of states or judgments attributed to the other subject. Paper I therefore does not place beliefs, intentions, preferences, affects, aesthetic judgments, and normative judgments into one primitive numerical vector. Let $\text{Typ}_{j, t}^{\text{ToM}}$ denote the model-declared family of ToM content types retained for subject j at index t . A typical application can include labels such as belief, intention, preference, desire, affect, evaluation, normative judgment, and confidence, but the list is application-specific and need not be exhaustive.

For each $\tau \in \text{Typ}_{j, t}^{\text{ToM}}$, let $\widehat{\mathcal{Q}}_{i \triangleright j, t}^{\tau}$ denote the class of i -relative models of j -states of type τ . The hat and modeling-direction indices are essential: an element of $\widehat{\mathcal{Q}}_{i \triangleright j, t}^{\tau}$ is not the target state itself. It is a model available to i concerning a target state attributed to j .

A first-order ToM record can then be represented as a typed partial profile. Let $D_{i \triangleright j, t}^{(1)} \subseteq \text{Typ}_{j, t}^{\text{ToM}}$ be the set of content types for which the subject-relative model is defined. A ToM profile is the map in Equation 62.

$$\theta_{i \triangleright j, t}^{(1)} : D_{i \triangleright j, t}^{(1)} \longrightarrow \bigsqcup_{\tau \in \text{Typ}_{j, t}^{\text{ToM}}} \widehat{\mathcal{Q}}_{i \triangleright j, t}^{\tau}, \quad \theta_{i \triangleright j, t}^{(1)}(\tau) \in \widehat{\mathcal{Q}}_{i \triangleright j, t}^{\tau}. \quad (62)$$

The disjoint union in Equation 62 is a typing device. It keeps heterogeneous content classes distinct while permitting one record to collect them. The profile need not contain every type. A can model B's

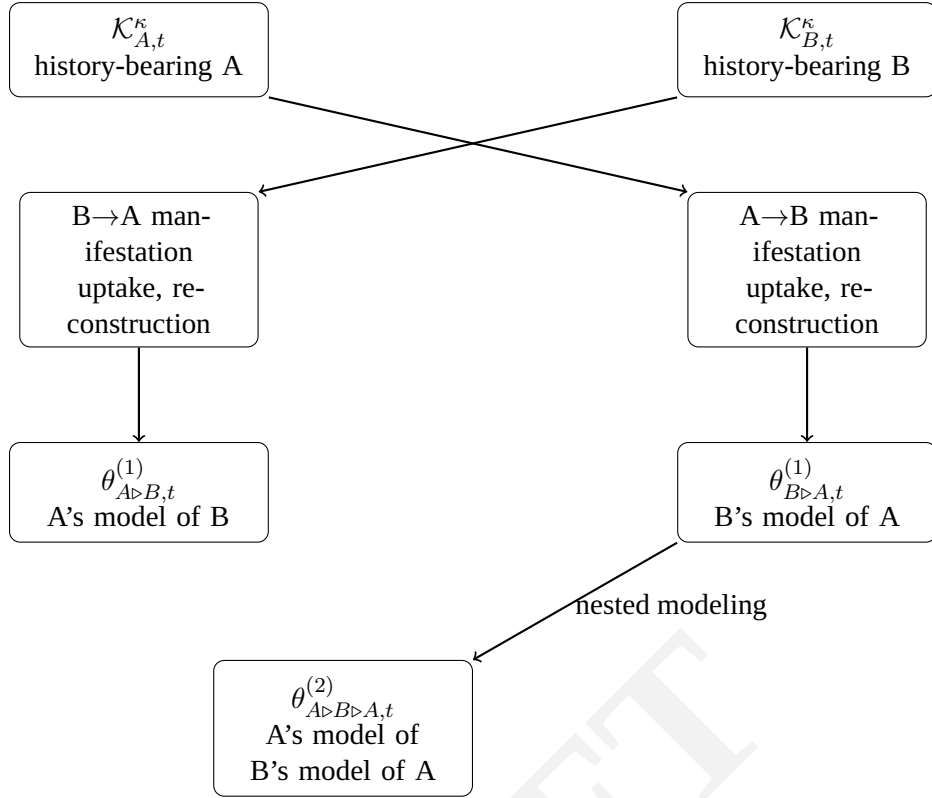


Figure 4: Schematic recursive theory-of-mind architecture. First-order ToM records model the other subject through manifestation, uptake, history, and reconstruction. A higher-order object can model another subject’s model of oneself. The diagram records modeling direction and recursive structure; it does not assert transparent access, correctness, or one universal psychological mechanism.

immediate intention while lacking a stable model of B’s normative judgment; B can model A’s expressed plan while remaining uncertain about A’s affective state. Missing components are undefined rather than zero.

The target types can also differ in formal structure. A preference model can be ordinal in one application, a belief model can use a finite epistemic-status set, and an affect model can remain qualitative. Paper I therefore does not assume a common metric or common probability scale across the components of Equation 62. Any such structure must be introduced locally and justified by the substantive question.

Figure 4 summarizes the directional and recursive architecture. The lower arrows represent manifestation and first-order modeling; the upper nested arrow represents A’s model of B’s model of A. The diagram is schematic and does not imply that ToM formation is temporally linear or deterministic.

9.3 Typed Theory-of-Mind Content

The ToM profile becomes analytically useful only when its content is related to the dyadic concepts already distinguished in Section 7. Several types are especially relevant to support relations.

A belief-type component can model what A takes B to accept, reject, or regard as unresolved. An intention-type component can model what A takes B to be trying to do. A preference-type component can model B’s inferred ordering or selection among alternatives where the application supports such an ordering. An affect-type component can model fear, gratitude, discomfort, shame, trust-related vulnerability, or another affective state at the level of representation available to A. An evaluative component can model what A takes B to regard as beneficial, harmful, fitting, humiliating, or otherwise valuable. A normative component can model what A takes B to regard as permitted, required, forbidden, legitimate, or inappropriate.

Table 11: Illustrative theory-of-mind content types in the dyadic support model.

Content type	Example modeled object	Distinction preserved
Belief / epistemic status	A models whether B accepts a proposition about the situation	B’s modeled belief versus A’s own belief or available evidence
Intention	A models what B is trying or planning to do	Intention versus observed behavior or later consequence
Preference / desire	A models B’s selection or attraction among alternatives	Preference versus normative endorsement or need judgment
Affect	A models fear, gratitude, shame, discomfort, trust, or another affective condition	Affect versus public manifestation and versus evaluative judgment
Goodness / need judgment	A models what B takes to be beneficial or needed	B’s modeled self-evaluation versus A’s evaluation of B
Normative judgment	A models what B treats as permitted, required, forbidden, or legitimate	Modeled normativity versus consent, compliance, or preference
Model of A	A models how B represents A’s intention, role, competence, or trustworthiness	First-order modeling versus higher-order recursive ToM

These components differ both epistemically and normatively. A can infer that B prefers an action without inferring that B judges it good. A can infer that B consents while remaining uncertain whether B endorses the action. A can infer that B is grateful while being wrong about the affect manifested by politeness. The ToM layer should therefore preserve the distinctions among preference, need judgment, goodness judgment, consent, affect, and normative evaluation rather than compress them into a generic representation of “what B wants.”

Table 11 records several content families and the distinctions they preserve. The table is illustrative rather than exhaustive.

The final row is the bridge to recursive modeling. Once A models B’s representation of A, the object of A’s ToM is itself a subject-relative model. This recursion can become relevant before any explicit strategic behavior occurs. A volunteer can alter an explanation because A anticipates that B may otherwise misread A’s intention; B can phrase a refusal differently because B expects A to interpret an unqualified refusal as hostility. The formalism therefore requires higher-order models while retaining finite scope.

9.4 Recursive Depth

Higher-order ToM should be generated by the closure rules of Section 7.15 rather than introduced as an unlimited family of primitives. Let $\mathfrak{X}_{0,t}^\kappa$ denote the retained base objects that can serve as ToM targets at resolution κ . These can include subject objects, state-like objects, judgments, intentions, or first-order epistemic records. Let $\mathbf{T}_{i>j,t}$ denote the typed ToM-construction operation when subject i models an object attributed to subject j and the operation is defined.

Define the finite-depth target grammar recursively by Equation 63.

$$\mathfrak{X}_{n+1,t}^\kappa := \mathfrak{X}_{n,t}^\kappa \cup \{ \mathbf{T}_{i>j,t}(x) \mid i \neq j, x \in \mathfrak{X}_{n,t}^\kappa, x \in \text{Dom}(\mathbf{T}_{i>j,t}) \}, \quad n \geq 0. \quad (63)$$

Equation 63 states a grammar of admissible nesting. It does not claim that subjects actually compute every object in $\mathfrak{X}_{n,t}^\kappa$. A first-order object can represent A’s model of B’s intention. A second-order object can represent A’s model of B’s model of A’s intention. A third-order object could represent B’s model of that second-order object if the application and evidence justify it. The experimental study of second-order belief attribution provides one familiar instance of such nested content [11], while Paper I uses the

recursion as a formal possibility rather than an empirical developmental claim.

For a generated ToM object x , define its ToM depth $d_{\text{ToM}}(x)$ as the minimum number of applications of ToM-construction operations required to generate x from the retained base inventory. This depth is syntactic relative to the declared grammar and resolution. It is not a scalar measure of social intelligence.

A concrete model should specify a finite retained depth d_{max} . Truncation can reflect the research question, evidential limits, cognitive assumptions, or computational tractability. Paper I does not infer an infinite hierarchy merely because the grammar permits arbitrary finite recursion. In particular, joint acknowledgment from Section 8.7 should not be silently upgraded to common knowledge by assuming unbounded iteration.

Recursive depth can also be heterogeneous across content types. A can have a second-order model of B's belief about A while possessing only a first-order model of B's affect. A procedure can require mutual acknowledgment of consent without requiring either side to model the other's model of every relevant preference. The depth parameter should therefore remain attached to specific modeled content rather than to a single global ToM level of the subject.

9.5 Belief Divergence: False-Belief Representation

One of the principal reasons to distinguish ToM from ordinary reconstruction is that a subject can model another subject as holding an epistemic position different from one's own. Classic false-belief work operationalized this separation by requiring a participant to represent another person's belief despite possessing information that the other person lacks [16]. Paper I adopts the structural distinction while avoiding a premature truth predicate on every modeled claim.

Let q be a proposition or structured claim for which subject i has an epistemic-status record $S_{i,t}(q)$ as introduced in Section 8.4. Let $\hat{S}_{j,t}^i(q)$ denote i 's model of j 's epistemic status concerning q . If the application supplies a comparison relation $\sim_{i,t}^{\text{ep}}$ on these status objects, then modeled epistemic divergence is defined by Equation 64.

$$\text{Div}_{i>j,t}^{\text{ep}}(q) \iff \hat{S}_{j,t}^i(q) \not\sim_{i,t}^{\text{ep}} S_{i,t}(q). \quad (64)$$

Equation 64 does not yet establish that j is wrong. It states that i models j as occupying an epistemic position distinct from i 's own. Calling the modeled belief *false* requires an additional criterion connecting q with the relational or empirical structure used as the paper's reference. In some finite illustrations such a criterion can be available; in other social cases the relevant proposition can remain contested or underdetermined.

This distinction is important in support relations. A can believe that a resource is available while correctly modeling B as unaware of it. B can know that a prior refusal has been withdrawn while modeling A as still acting on the old refusal. A can judge that B misunderstands A's intention while lacking decisive evidence that A's own self-model exhausts that intention's provenance. The formalism therefore permits epistemic divergence without automatically granting one side privileged access to relational reality.

Divergence can also concern value and normativity. A can model B as regarding an intervention as humiliating while A regards it as respectful. Such disagreement is not a false-belief case merely because the two judgments differ. The ToM architecture must preserve the type of the modeled content before any correctness criterion is applied.

9.6 Confidence, Underdetermination, Model Multiplicity

A ToM record can be detailed while remaining weakly supported. Conversely, a sparse model can be highly warranted within a narrow domain. The formalism therefore keeps model content separate from confidence and from evidential support. Section 8.4 introduced subject-relative confidence carriers without assuming a common scalar interval. The same discipline applies to ToM components.

For a retained ToM record $\theta \in \mathfrak{T}_{i \triangleright j, t}^{(1)}$, let $c_{i \triangleright j, t}^{\text{ToM}}(\theta)$ be defined only when the application supplies a confidence carrier suitable for the model. A high confidence value, when one exists, is a fact about i 's commitment to the model under the chosen representation. It is not a guarantee of correctness.

Model multiplicity is more fundamental. Define the candidate set supported by a reconstruction $\hat{r}_i$ by Equation 65.

$$\text{Cand}_{i \triangleright j, t}^{(1)}(\hat{r}_i) := \left\{ \theta \in \mathfrak{T}_{i \triangleright j, t}^{(1)} \mid (\hat{r}_i, \theta) \in \text{ToM}_{i \triangleright j, t}^{(1)} \right\}. \quad (65)$$

When the set in Equation 65 has more than one retained member, the current evidence and interpretive structure underdetermine the ToM record relative to the model. A later procedure can reduce the set by asking a question, observing a response, checking a record, or revising the interpretive vocabulary. It can also enlarge the candidate set when new information exposes an earlier simplification.

Underdetermination should remain visible because premature collapse to one model can generate paternalistic or otherwise consequential errors. A may observe B's silence and retain several admissible interpretations: assent, uncertainty, fear, fatigue, politeness, strategic delay, or lack of comprehension. Choosing one interpretation as if it were given by the manifestation itself hides the inferential step through which later support action becomes justified in A's own model.

The same point applies to self-models. A can be uncertain about B's intention while B is also uncertain about B's own mixed motivation. The ToM architecture should not treat another subject's first-person report as infallible access to ultimate provenance, even though such a report can carry distinctive epistemic importance. Section 8.8 already distinguished sincerity from exhaustive self-knowledge; the ToM layer preserves that distinction when one subject interprets another's self-description.

9.7 Anticipatory Modeling, Strategic Manifestation

Recursive ToM can become generatively active when a subject chooses or modulates manifestations partly in light of an anticipated reconstruction by the other subject. This does not require deception or game-theoretic optimization. Ordinary explanation, politeness, tact, reassurance, clarification, and refusal can already depend on a model of how the other is likely to interpret a manifestation.

Consider B's manifestation toward A. Section 8.6 treated strategic legibility at the manifestation level. The ToM layer can now state the missing conditioning object. Let $\theta_{B \triangleright A \triangleright B, t}^{(2)}$ denote a retained second-order model in which B represents A's model of B. Let $\mathcal{P}_{B \rightarrow A, t}$ denote the application-defined class of presentation options available to B. A ToM-conditioned presentation map can be represented by Equation 66.

$$\zeta_{B \rightarrow A, t} : \mathcal{P}_{B \rightarrow A, t} \times \mathfrak{T}_{B \triangleright A \triangleright B, t}^{(2)} \rightarrow \mathcal{M}_{B \rightarrow A, t}. \quad (66)$$

Equation 66 is a possible specialization, not a universal law of manifestation. Its role is to represent cases in which B's available presentation and B's model of A's likely interpretation jointly condition the manifested form. A parallel map can be defined for A toward B.

The map also distinguishes strategic manifestation from strategic falsification. B can select a clearer wording because B expects A to misunderstand an indirect refusal. A can state uncertainty explicitly because A expects B to overestimate A's competence. A volunteer can reduce visible displays of gratitude-seeking because A anticipates that B may otherwise experience the relation as morally pressuring. These are strategic in the weak sense that another subject's anticipated reconstruction helps shape manifestation.

More problematic cases remain available. A can present an intervention in a way designed to make refusal costly, or B can conceal a preference because B predicts that disclosure will change A's willingness to help. The formalism does not evaluate these strategies here. It records how recursive epistemic modeling can become part of the generative provenance of later manifestation and action.

9.8 Historical Revision of Theory-of-Mind Models

ToM records are historical objects. Repeated interaction can stabilize a model, expose its limitations, or change the types of states that one subject is capable of attributing to the other. The same manifested behavior can therefore support different ToM reconstructions at different stages of the dyad because the manifestation relation, memory record, interpretive vocabulary, and prior ToM history have changed.

Let $t \preceq t'$. Let $\mathcal{H}_{i,[t,t']}^{\text{upd}}$ denote the application-defined record of new uptake, feedback, remembered outcomes, corrections, and other materials available to subject i between the two indices. A ToM-revision map is represented in Equation 67.

$$\text{Rev}_{i \triangleright j, t' \leftarrow t}^{\text{ToM}} : \mathfrak{T}_{i \triangleright j, t}^{(\leq d)} \times \mathcal{H}_{i,[t,t']}^{\text{upd}} \rightarrow \mathfrak{T}_{i \triangleright j, t'}^{(\leq d')}. \quad (67)$$

The depth bounds d and d' can differ. Interaction can make higher-order modeling newly relevant, or it can simplify a relation so that previously required recursive modeling becomes unnecessary. Equation 67 is partial because revision can fail: evidence can remain ambiguous, a subject can ignore feedback, a conceptual distinction can be unavailable, or a retained model can resist change for historical reasons.

Revision is not necessarily convergence. A and B can receive the same event and revise in different directions because they occupy different histories and roles. Repeated interaction can increase mutual intelligibility in one domain while deepening disagreement in another. A can become more accurate about B's preferences while becoming more confident in an inaccurate model of B's gratitude. B can learn A's procedural habits while remaining uncertain about A's motivation. ToM development therefore has a typed trajectory rather than a single direction of improvement.

Historical revision also permits ToM to participate in relational emergence. Reliable prediction of another subject's responses can contribute to trust, while repeated model failure can contribute to caution or distrust. A shared vocabulary can reduce ambiguity in need judgments. A history of misrecognition can make later manifestations harder to interpret even when their immediate content is clear. These later relational consequences belong to the later analysis of emergent dyadic structure; the present section provides the epistemic structures through which they can become generated.

9.9 Theory-of-Mind Limits, Action-Genesis Interface

The ToM formalism has a limited role in Paper I. It represents subject-relative models that can become relevant to later evaluation and action. It does not establish that one side correctly knows the other, that explicit mindreading is psychologically necessary for every volunteer interaction, or that deeper recursion is normatively superior. Some support can proceed through established routines, professional protocols, embodied responsiveness, or jointly stabilized practices with relatively little explicit ToM modeling. Other interactions can depend strongly on inferred intention, preference, fear, trust, or anticipated interpretation.

Several constraints should remain visible. First, ToM is manifestation-dependent and historically conditioned. Second, ToM content is typed; belief, intention, preference, affect, need judgment, and normative judgment are not interchangeable. Third, nested models can be underdetermined and should be truncated at a finite depth appropriate to the application. Fourth, confidence in a ToM model is distinct from accuracy. Fifth, a subject's self-report is epistemically important without becoming an infallible map of action provenance. Sixth, recursive modeling can alter manifestation and thereby enter the generative history of the relation.

These constraints define the interface to the next section on action genesis. A later support action can depend on A's model of B, A's model of B's model of A, and B's corresponding structures. The action should nevertheless be derived from a broader provenance that also includes relational history, role structure, available procedures, affective conditions, norms, material constraints, and other generative sources. ToM is therefore one component of action genesis rather than a complete explanation of volunteer behavior.

The transition is especially important for normative analysis. If A acts because A models B as needing, preferring, consenting to, or valuing a particular intervention, the formalism can ask how that model

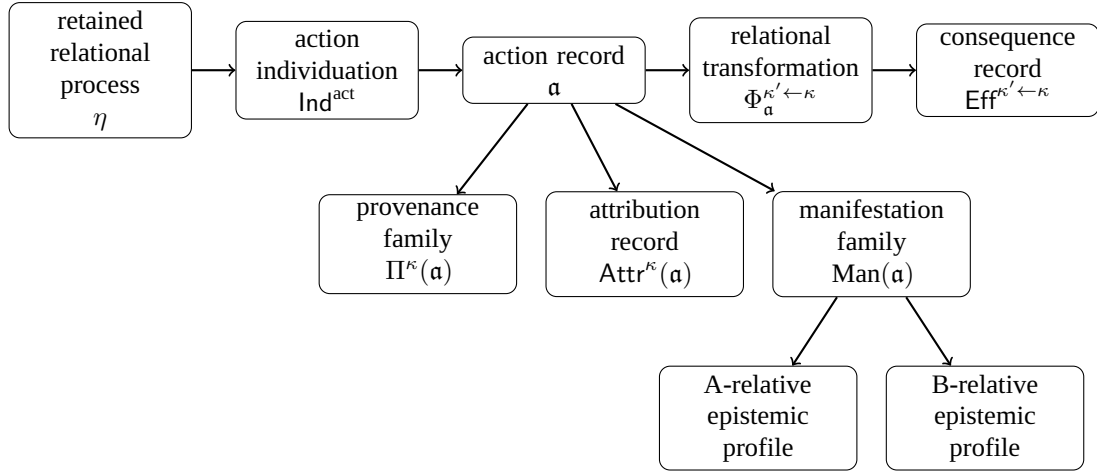


Figure 5: Action-genesis architecture for the dyadic formalism. A retained relational process can be individuated as an action record under declared criteria. Provenance and attribution are derived from that record without being identified with one another. Manifestations support distinct A-relative and B-relative epistemic profiles, while the action can also be represented through a typed relational transformation whose later consequences become available for feedback and provenance revision.

was generated, which manifestations supported it, what alternatives remained admissible, how confident A was, and whether B’s later feedback revised it. The next section uses those resources to construct action as a history-bearing, attributed, provenance-sensitive relational object rather than as an already transparent primitive.

10 Action Genesis: Provenance, Attribution, Epistemic Profiles

This section develops the action layer required by the dyadic volunteer formalism. Its role is to replace the ordinary shorthand in which an already individuated actor performs an already individuated action on another already individuated subject. The objective is to represent action as a derived historical object whose genesis can involve a relational process, attribution, intention, role structure, prior interaction, manifestation, and subject-relative reconstruction. The discussion proceeds through action individuation, provenance, attribution roles, intentional structure, manifestation and epistemic profiles, action equivalence, consequence separation, and historical revision. The method is typed and reconstructive. Paper I does not propose a complete metaphysics or psychology of action; it specifies the minimum structure required before a support event can be treated as an action with a defensible relational and epistemic profile.

Figure 5 summarizes the section’s architecture. The arrows in the figure record formal dependence or model construction rather than one universal psychological sequence. A support action can be jointly negotiated, procedurally routine, affectively responsive, or historically sedimented without passing through the same explicit deliberative stages. The architecture therefore keeps the objects distinct while allowing their relations to vary across applications.

10.1 Relational Process, Action Individuation

The formalism begins one level earlier than an action label. Let $[t, t']$ denote a retained historical interval or ordered segment, with no requirement that t and t' belong to a universal scalar time. Let $\mathfrak{P}_{AB, [t, t']}^{\kappa}$ denote the class of relational process records retained in the coupled dyadic representation at resolution κ . An element $\eta \in \mathfrak{P}_{AB, [t, t']}^{\kappa}$ can record a temporally or partially ordered pattern of manifestations, bodily changes, communications, material transfers, local transformations, or other relational occurrences selected by the model. The notation does not imply that every process is already an action.

Let $\mathfrak{A}_{AB,[t,t']}^\kappa$ denote the class of candidate action records available to the same analysis. Action individuation is represented by the relation in Equation 68.

$$\text{Ind}_{[t,t']}^{\text{act},\kappa} \subseteq \mathfrak{P}_{AB,[t,t']}^\kappa \times \mathfrak{A}_{AB,[t,t']}^\kappa. \quad (68)$$

Membership $(\eta, \mathfrak{a}) \in \text{Ind}_{[t,t']}^{\text{act},\kappa}$ means that the process record η supports $\mathfrak{a}$ as an action record under the individuation criteria adopted by the model. Equation 68 is deliberately relational rather than functional. One extended process can support several action individuations, and several local process fragments can be grouped into one action when the retained purpose, procedure, or attribution rule warrants that grouping.

This non-uniqueness is analytically important. A sequence consisting of a request, a pause, a verbal confirmation, and a transfer can be represented as four local events, as one negotiated support action, or as a larger procedure containing several actions. The appropriate individuation depends on what the analysis needs to preserve. A study of consent may retain the request and confirmation separately. A study of resource transfer may treat the sequence as one action. A study of relational repair may treat the same sequence as only one segment of a longer history.

Actionhood also requires attribution structure. Paper I uses the word *action* when the model supplies enough relational structure to connect a retained process with one or more subject positions, dyadic positions, or jointly generated sources in a manner relevant to agency, responsibility, coordination, or practical interpretation. Intention is one possible component of that structure, but the definition does not require every action to be the product of explicit deliberation. Habitual, reactive, routine, jointly improvised, and role-mediated actions remain admissible.

The three-level architecture applies already at this stage. The ultimate ontology of action remains suspended. At the relational level, $\mathfrak{a}$ is a historically situated action record whose identity depends on declared tracking and individuation criteria. At the manifestation level, only selected aspects of that action become visible through gesture, speech, transfer, document, silence, delay, bodily movement, or another manifested form. The relational action record and its manifestations should therefore remain distinct throughout the later analysis.

10.2 Generative Provenance

Action individuation answers what the model treats as an action. Provenance addresses a different question: which retained antecedent structures materially participate in the generation, selection, stabilization, or execution of that action? The answer need not be reducible to one actor's reported motive. A support action can depend on prior requests, learned routines, role expectations, trust, affect, institutional procedure, material opportunity, earlier promises, or the anticipated response of the other subject.

Let $\mathfrak{H}_{AB}^{\leq t'}(\kappa)$ denote the family of admissible historical subrecords drawn from the coupled dyadic history available up to t' at resolution κ . The provenance relation is stated in Equation 69.

$$\text{Prov}_{t'}^\kappa \subseteq \mathfrak{A}_{AB,[t,t']}^\kappa \times \mathfrak{H}_{AB}^{\leq t'}(\kappa). \quad (69)$$

For an action record $\mathfrak{a}$, its retained provenance family is defined by Equation 70.

$$\Pi_{t'}^\kappa(\mathfrak{a}) := \left\{ h \in \mathfrak{H}_{AB}^{\leq t'}(\kappa) \mid (\mathfrak{a}, h) \in \text{Prov}_{t'}^\kappa \right\}. \quad (70)$$

Equation 70 defines provenance as a family of retained historical contributors rather than a complete causal explanation. The model can require stronger causal tests for particular applications, but Paper I does not infer causation from temporal precedence, correlation, or retrospective narrative alone. The provenance family records which antecedent structures the model has reason to retain when explaining how the action became available or enacted.

Several recurrent provenance patterns are summarized in Table 12. The categories are non-exclusive. One action can be elicited by B, mediated by a stabilized norm, shaped by A's affective state, and jointly

Table 12: Illustrative provenance patterns for dyadic support actions. The patterns are analytical descriptions rather than mutually exclusive causal classes.

Pattern	Generative emphasis	Illustrative relational structure
Self-initiated	A's retained intention, attention, or judgment contributes without a prior request by B	A notices a manifestation and offers support before B asks
Supportee-elicited	B's request, disclosure, refusal, or other manifestation materially changes A's action family	B asks for help and A responds
Jointly negotiated	The action is selected through reciprocal clarification, proposal, revision, or agreement	A and B co-determine timing, form, or limits of support
Historically sedimented	Earlier interaction stabilizes an expectation or routine that makes later action available	Repeated cooperation creates a familiar support practice
Norm-mediated	A retained norm, obligation, promise, code, or shared expectation participates in action genesis	A acts partly because a role or promise has acquired normative force
Role-mediated	A stabilized relational position changes which actions are expected or practically available	A occupies a supporter position whose routines structure response
Affect-mediated	Affective salience contributes to attention, urgency, restraint, or selection	Concern, discomfort, gratitude, fear, or empathy alters action selection
Distributed	No single subject-level source adequately explains the action's genesis	A procedure, interface, shared routine, and both subjects jointly generate the enacted result

negotiated through interaction. The purpose of the taxonomy is to prevent the shorthand “A chose to help B” from erasing the heterogeneous relations that participated in the action's formation.

The distributed case is especially important for GR. The dyadic model retains A and B as analytically distinguishable subjects while allowing the genesis of an action to belong partly to the coupled configuration. A request can originate with B, the available response can depend on a shared procedure, A can execute the visible transfer, and the final form can emerge only after negotiation. Asking for one unique internal source in A would misdescribe the modeled history.

Provenance is itself historically revisable. Later information can reveal that an apparent self-initiated action was elicited by an earlier unnoticed signal, or that a supposedly joint decision was structured by an asymmetry that one side had not recognized at the time. Section 10.7 formalizes this revision without treating later knowledge as automatically exhaustive or infallible.

10.3 Attribution Roles

Provenance should remain distinct from attribution. Provenance records retained generative antecedents. Attribution records how an action is assigned to subjects, positions, procedures, or relational structures for purposes such as description, coordination, accountability, or later evaluation. A process can have distributed provenance while still possessing a clear executor, and an action can be publicly attributed to A even when B initiated the procedure.

Let $\mathfrak{B}_{AB,[t,t']}^\kappa$ denote the class of admissible attribution bearers retained by the dyadic model over the relevant interval. The class can contain subject-spacetimes, relational positions, jointly constituted dyadic structures, or explicitly retained procedural objects. Let $\mathcal{T}_{\text{attr}}$ denote a finite family of attribution types. A minimal useful family is stated in Equation 71.

$$\mathcal{T}_{\text{attr}} := \{\text{init}, \text{exec}, \text{gen}, \text{auth}\}, \quad (71)$$

where the tags denote initiation, execution, generation, and authorship-like social attribution, respectively. The last tag does not by itself assign moral or legal responsibility. It records a model-relative attribution that may later become normatively relevant.

For an action record α , the attribution relation is stated in Equation 72.

$$\text{Attr}_{t'}^{\kappa}(\alpha) \subseteq \mathfrak{B}_{AB,[t,t']}^{\kappa} \times \mathcal{T}_{\text{attr}}. \quad (72)$$

Equation 72 permits several bearers to share one attribution type and one bearer to carry several types. If B requests assistance, A executes a transfer, and the form of the transfer is jointly negotiated, then B can be retained as an initiator, A as an executor, and the coupled procedure as one contributor to generation. A later institution-level analysis may add other bearers, but Paper I keeps the focal attribution record inside the dyadic boundary unless external structure is necessary to type the action coherently.

The separation among attribution types also blocks a common normative shortcut. Execution does not imply sole authorship; initiation does not imply control over final form; generative contribution does not imply responsibility for every consequence. Later ethics and justice papers can ask how responsibility should track these roles. Paper I supplies the typed distinctions needed before such evaluation begins.

Attribution is subject-relative as well as model-relative. A can attribute an action to A while B treats it as jointly generated, or B can attribute a support event to an impersonal procedure that A experiences as a personal choice. These attribution judgments belong to the epistemic profiles developed below. The relation in Equation 72 records the analyst's retained attribution structure under declared evidence and criteria; it does not erase the subjects' own possibly divergent attributions.

10.4 Intentional Structure

Intention is one possible component of action genesis, yet it should not be treated as the exhaustive source of action or as an immediately transparent internal fact. A support action can be deliberate, habitual, reactive, affectively triggered, role-guided, strategically planned, or jointly improvised. Several such modes can coexist within one history. Paper I therefore treats intentionality as a typed relationally reconstructed structure rather than a binary property attached to an action.

Let $i \in \{A, B\}$. When the model retains an intentional structure associated with subject i and action α , denote the relationally retained intention record by $I_{i,t'}^R(\alpha)$. The superscript R marks its status inside the relational model rather than a claim about ultimate mental ontology. An intention record can be supported by first-person report, prior commitment, action sequence, repeated practice, or other evidence admitted by the application.

The subject's self-description should remain distinct from the retained relational record. Let $m_{i,t'}^{\text{int}}(\alpha)$ denote an admissible manifestation through which i describes or displays an intention. When this manifestation is defined, it belongs to $\mathcal{M}_{t'}$. Let $\hat{I}_{i,t'}^i(\alpha)$ denote i 's self-reconstruction, and let $\hat{I}_{i,t'}^j(\alpha)$ denote the other subject j 's reconstruction of i 's intention. The distinctions are summarized schematically in Equation 73.

$$I_{i,t'}^R(\alpha), \quad m_{i,t'}^{\text{int}}(\alpha), \quad \hat{I}_{i,t'}^i(\alpha), \quad \hat{I}_{i,t'}^j(\alpha). \quad (73)$$

Equation 73 is a list of separately typed objects, not an equality chain. A sincere self-description can remain incomplete because mixed motivation, habitual action, affective influence, or historically sedimented roles are only partly available to reflective access. The other subject can also infer intention incorrectly from a manifestation. These possibilities extend the distinction between self-manifestation and exhaustive self-knowledge developed in Section 8.8.

The formalism therefore avoids the reduction “A intended to help” $\Rightarrow$ “the action's provenance is fully explained.” Intention can be an important generative component while role structure, procedural availability, B's request, A's ToM model, material opportunity, and prior history also participate. Conversely, an action can remain attributable to A even when its immediate execution is routine and A performs little explicit deliberation at the moment of enactment.

This separation will matter later for normative evaluation. Benevolent intention, beneficial consequence, valid consent, competent execution, and relationally appropriate support are different objects. Paper I does not rank them here. It ensures that none is smuggled into the definition of another.

10.5 Manifestation, Epistemic Profiles

An action record at the relational level can have several manifestations. A resource transfer can manifest through visible movement, a bank record, a message, a change in access, or another context-dependent trace. Some manifestations can be accessible to both subjects, others only to one, and some relevant aspects of the action can remain unmanifested in the retained observational context. Section 5 already provides the general manifestation relation μ_t ; the present subsection specializes that architecture to action.

Let $\alpha^R \in \mathcal{R}_t$ denote a retained relational action record. Its manifestation family is defined through the general construction of Section 5.4 and written here as $\text{Man}_t(\alpha^R)$. A and B can have different accessible subfamilies, different uptake, and different reconstructions of the same relational action. The action's epistemic profile should therefore be indexed by subject and content type rather than compressed into one statement that the action is "known."

Let $\mathcal{T}_E^{\text{act}}$ denote the finite set of epistemic content types retained for the action analysis. A useful core family is stated in Equation 74.

$$\mathcal{T}_E^{\text{act}} := \{\text{occ}, \text{attr}, \text{prov}, \text{int}, \text{mean}, \text{eff}\}, \quad (74)$$

where the tags denote occurrence, attribution, provenance, intention, interpreted meaning, and expected or reconstructed effect. For subject i , the action-relative epistemic profile is the typed family in Equation 75.

$$\mathbf{E}_{i,t'}^{\text{act}}(\alpha) := \{E_{i,t'}^\tau(\alpha)\}_{\tau \in \mathcal{T}_E^{\text{act}}}, \quad (75)$$

with each component $E_{i,t'}^\tau(\alpha)$ taking values in the carrier appropriate to its type. Equation 75 is therefore an indexed heterogeneous family rather than a vector in a common numerical space. An occurrence component can be binary, modal, or evidential; an attribution component can be a relation over bearers and attribution types; a provenance component can be a family of historical subrecords; an effect component can be a counterfactual or predictive structure. No common arithmetic is assumed.

The profile permits several distinctions that ordinary descriptions suppress. B can know that a transfer occurred while remaining uncertain why A performed it. A can know that A acted while having only a partial reconstruction of the action's own provenance. A and B can agree on execution and disagree on authorship. They can share the visible manifestation and disagree on its meaning. They can agree on intention while predicting different consequences. These are different epistemic configurations even when the same action record is retained.

Subject-relative provenance models are especially important. Let $\hat{\Pi}_t^A(\alpha)$ and $\hat{\Pi}_t^B(\alpha)$ denote A's and B's reconstructed provenance families. In general, the paper permits the relation stated in Equation 76.

$$\Pi_{t'}^\kappa(\alpha), \quad \hat{\Pi}_{t'}^A(\alpha), \quad \hat{\Pi}_{t'}^B(\alpha) \quad \text{to remain mutually non-identical.} \quad (76)$$

Equation 76 avoids declaring one subject's interpretation to be the action's complete origin story. The retained provenance family is itself model-relative and revisable. The two subject-relative reconstructions can nevertheless be compared with it under declared criteria. This structure provides a formal home for provenance disagreement without requiring the paper to psychologize every support action.

The same architecture applies when one subject does not classify the underlying process as an action at all. A repeated delay, tone of voice, spatial repositioning, or habitual nonresponse can be experienced by B as an agentively consequential act while A treats it as background behavior. The action-individuation relation in Equation 68 and the subject-relative profiles in Equation 75 permit this divergence to be represented rather than resolved by stipulation.

10.6 Action Identity, Equivalence

Two manifested behaviors can look similar while differing in provenance, attribution, intention, or relational consequence. Conversely, one relational action can manifest differently across contexts or observers. Action identity must therefore be indexed by the structure retained in the comparison.

Let χ denote a declared action-comparison specification. Depending on the research question, χ can retain process structure, attribution, provenance, intention, manifestation, procedure, consequence, or another typed subset of the action record. Define the action-equivalence relation $\sim_\chi^{\text{act}}$ on a retained action class by Equation 77.

$$\alpha_1 \sim_\chi^{\text{act}} \alpha_2 \iff \alpha_1 \text{ and } \alpha_2 \text{ preserve the action structure retained by } \chi. \quad (77)$$

Equation 77 is schematic until an application supplies the retained structure and the corresponding test. It prevents an apparent physical or manifestational similarity from settling action identity in advance. Two transfers of the same amount can be equivalent relative to a material-transfer specification and non-equivalent relative to a provenance specification when one follows an explicit request and the other is imposed without consultation.

The same discipline applies to action manifestation. If $m_1 = m_2$ under a coarse observational coding, the underlying action records can still belong to different $\sim_\chi^{\text{act}}$ classes once history, role, consent, or provenance is retained. Conversely, two different manifested forms can count as equivalent action realizations under a specification that preserves their relational function and history. Action comparison is therefore a typed analytical decision rather than a consequence of surface resemblance alone.

Action equivalence can also change with historical resolution. Later evidence can reveal a provenance difference that an earlier coding could not represent. A finer resolution can split one earlier action class into several classes, while coarse-graining can merge distinctions that matter for another question. This resolution dependence mirrors the subject-spacetime discipline of Section 6: action identity is tracked within a declared representation rather than treated as an immutable metaphysical atom.

10.7 Consequence Separation, Historical Revision

Action and consequence remain separate objects. The action record describes the individuated and attributed relational process. Consequence concerns what later changes become associated with the action under a declared comparison and historical horizon. The distinction prevents later outcome knowledge from being read back into the action's initial intention, epistemic profile, or normative status.

Let $\mathfrak{R}_{AB}^\kappa$ denote the class of admissible coupled dyadic spacetime representations at resolution κ . A realized action can be associated with a typed partial transformation as stated in Equation 78.

$$\Phi_a^{\kappa' \leftarrow \kappa} : \mathcal{D}_a^{\kappa' \leftarrow \kappa} \rightarrow \mathfrak{R}_{AB}^{\kappa'}, \quad \mathcal{D}_a^{\kappa' \leftarrow \kappa} \subseteq \mathfrak{R}_{AB}^\kappa. \quad (78)$$

If the source configuration lies in the domain of Equation 78 and the action is realized, denote its image by $\mathcal{K}_{AB,t'}^{\kappa'}$. The equation does not require the action to modify every part of the dyad, nor does it require a fixed topology. The map can preserve the carrier while changing labels or admissible continuations, or it can connect distinct carriers when the representation warrants stronger structural change.

Before an application defines a metric or subtraction on configurations, the minimal consequence record should remain an ordered comparison. Define the action-effect record by Equation 79.

$$\text{Eff}^{\kappa' \leftarrow \kappa}(\alpha; t, t') := \left(\mathcal{K}_{AB,t}^\kappa, \mathcal{K}_{AB,t'}^{\kappa'} \right). \quad (79)$$

Equation 79 is not a difference vector. A later analysis can derive typed consequence descriptors from the ordered pair: a change in B's practical capacity, a change in trust, a new dependency, a role transition, a changed manifestation relation, or a change in the admissible action family. Each descriptor requires its own comparison structure.

Consequences can be temporally extended and causally distributed. An immediate material benefit can coexist with a later dependency. A support action can improve B's short-term access while reducing

B's later exit capacity. A refused intervention can alter trust even when no material transfer occurs. An apparently minor clarification can change the interpretive map through which later requests become legible. The action-effect record is therefore only the starting point for consequence analysis, not a scalar welfare measure.

Later consequences can also revise the reconstruction of action provenance. Let $t' \preceq t''$ and let $\mathcal{H}_{[t', t'']}^{\text{upd}}$ denote the retained update record containing later feedback, outcome information, newly accessible manifestations, and other evidence relevant to action reconstruction. A provenance-revision map is represented in Equation 80.

$$\text{Rev}_{t'' \leftarrow t'}^{\text{Prov}} : \mathfrak{Prov}_{t'}^{\kappa} \times \mathcal{H}_{[t', t'']}^{\text{upd}} \rightarrow \mathfrak{Prov}_{t''}^{\kappa}, \quad (80)$$

where $\mathfrak{Prov}_s^{\kappa}$ denotes the class of admissible provenance records available at historical index s . Equation 80 permits later reconstruction without implying that later interpretation is automatically more accurate. New evidence can correct an earlier model, expose further ambiguity, or generate competing provenance reconstructions.

Historical revision applies to action identity as well. A process initially classified as a unilateral intervention can later be reconstructed as partly elicited by an earlier manifestation that A had not consciously registered. A supposedly consensual action can later be reclassified when previously inaccessible pressure becomes visible. These revisions do not change the historical occurrence of the manifested event; they change the relational and epistemic structures through which that event is individuated, attributed, and interpreted.

10.8 Procedure Interface

The action formalism developed above supplies the unit required by the next section on interactive procedures. A dyadic support relation rarely consists of one isolated action. Requests, questions, offers, refusals, clarifications, agreements, transfers, monitoring, corrections, and withdrawals can form an ordered history in which the availability and meaning of one action depend on earlier actions.

The present section establishes four constraints that the procedure layer must preserve. First, each action has an individuation rule and need not coincide with one visible event. Second, provenance can be distributed across the dyadic history. Third, attribution roles can differ across initiation, execution, generation, and authorship-like social assignment. Fourth, A and B can possess different epistemic profiles of the same action and can revise those profiles after feedback.

These constraints make procedure composition more demanding than concatenating action labels. A later action can be undefined because an earlier refusal closed its domain. A clarification can change the provenance of a later choice by making one preference newly legible. A consent manifestation can be interpreted differently by A and B. A joint plan can reassign attribution roles before execution. The next section therefore treats interactive procedure as a typed composition of partially defined action records whose domains, manifestations, epistemic profiles, and relational consequences evolve through history.

11 Interactive Procedure Structure

This section develops the procedure layer of the dyadic volunteer formalism. Its role is to represent support as a historically ordered interaction rather than as one isolated action. The objective is to specify how encounters, inquiries, requests, offers, clarifications, negotiations, authorizations, refusals, interventions, monitoring, feedback, corrections, withdrawals, handovers, and terminations can be assembled without presuming that every support relation follows one fixed sequence. The discussion proceeds through procedure records, typed procedure elements, partial composability, branching continuations, joint participation, authorization-related elements, feedback, procedure-dependent meaning, and three-level trajectory. The method remains structural and typed. Section 11 does not yet decide whether a procedure is ethically justified or whether an expressed consent is normatively valid; those questions require the evaluative and authorization structures developed in later sections.

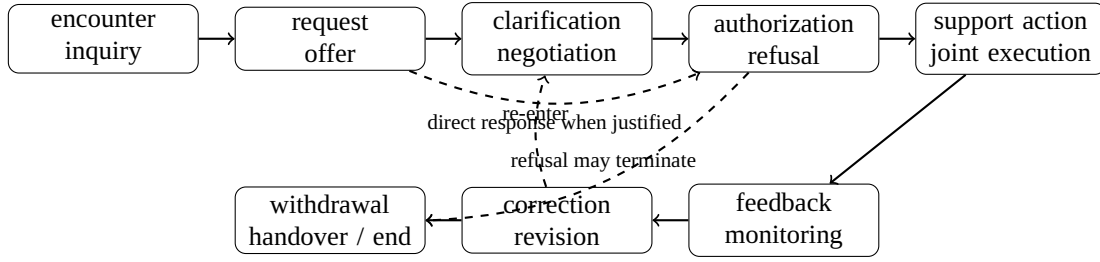


Figure 6: Schematic procedure architecture for a dyadic support relation. Solid arrows show one admissible ordered pattern, while dashed arrows indicate that clarification, refusal, revision, and termination can redirect the realized history. The diagram is not a universal protocol: individual applications can omit, repeat, reorder, or refine elements when their domains and procedural meanings permit.

A procedure is therefore treated as more than a list of action labels. Each local element inherits the distinctions established in Sections 8–10: manifestations can be only partially accessible, A and B can model the same event differently, action provenance can be distributed, and the domain of a later action can depend on what earlier interactions have already changed. Figure 6 summarizes the resulting architecture before the individual components are defined.

Figure 6 displays one structured family of possibilities rather than a mandatory pipeline. A requested emergency intervention may contain little negotiation. A continuing companionship relation may involve repeated cycles of inquiry and feedback. A refusal can end one action family while leaving another available. A clarification can alter the meaning of an earlier request and thereby change the provenance of a later action. The formal task is consequently to represent admissible ordering and redirection without converting one familiar procedural script into a universal model of support.

11.1 Procedure Records, Historical Order

An interactive procedure is represented at a declared resolution and historical window. Let I_Γ be a finite set indexing retained procedure elements and let $\prec_\Gamma$ be a strict partial order on I_Γ . The order records procedural precedence: $j \prec_\Gamma k$ means that the element indexed by j is treated as antecedent to the element indexed by k in the retained procedure history. Incomparable indices permit the representation of locally concurrent, independently ordered, or unresolved elements without forcing them into an artificial scalar timeline.

For each $j \in I_\Gamma$, let ζ_j denote the corresponding typed procedure-element record. The realized procedure record is defined in Equation 81.

$$\Gamma_{AB}^\kappa := (I_\Gamma, \prec_\Gamma, \{\zeta_j\}_{j \in I_\Gamma}). \quad (81)$$

Equation 81 is a bookkeeping structure rather than a global state vector. The elements ζ_j can belong to different formal types. One may be an action record from Section 10; another may be a manifestation update, a request record, a refusal record, a feedback record, or a procedure-level status change. The partial order records their retained historical relation without assuming that their payloads inhabit one common numerical or algebraic space.

A linearly ordered procedure is a specialization. If the application supplies a total order $j_1 \prec_\Gamma \dots \prec_\Gamma j_n$, the realized trace can be written $\gamma = (\zeta_{j_1}, \dots, \zeta_{j_n})$. The distinction between Γ and γ is useful because a procedure can contain partially ordered elements even when one observer later narrates them as a sequence. The reconstruction should preserve the weaker order actually supported by the model.

Historical segmentation remains resolution-dependent. One analysis can treat a verbal request, a pause, a clarification, and a confirmation as four procedure elements. Another can retain them as one negotiated unit. The procedure record inherits the action-individuation discipline of Section 10.1: seg-

Table 13: Core procedure-element types in the minimal dyadic support formalism. The listed functions specify procedural roles without assigning normative validity.

Type	Procedural function	Formal significance
Encounter / inquiry	Open an interaction or seek information	Can change accessibility, attention, or the later action domain without yet producing support
Request / offer	Make a sought or proposed intervention manifest	Creates a procedure-relative object for interpretation, response, acceptance, modification, or refusal
Clarification / negotiation	Modify content, scope, timing, conditions, or interpretation	Can alter provenance, action identity, expected consequence, and admissible continuation
Authorization / refusal	Register a procedure-relevant permission, rejection, or boundary manifestation	Changes which later actions are procedurally available; full normative validity is deferred
Execution	Realize a retained support action or joint plan	Invokes a typed action transformation and creates later consequence records
Monitoring / feedback	Generate information concerning process, effect, experience, or changing preference	Supplies material for epistemic and procedural revision
Correction	Revise plan, meaning, action, or procedure state	Can reopen earlier stages or replace later continuations
Withdrawal / handover / termination	Reduce, transfer, or close a procedure-relative role or action family	Alters continuation structure and can change role, dependency, or responsibility relations

mentation is an explicit representational choice whose adequacy depends on the distinctions the research question needs to preserve.

11.2 Procedure Elements, Local Functions

Procedure elements are distinguished by their formal role in the dyadic history. Let $\mathcal{T}_{\text{proc}}$ denote the finite family of procedure types retained by Paper I. A core type family is stated in Equation 82.

$$\mathcal{T}_{\text{proc}} := \{\text{enc, inq, req, off, clr, neg, auth, ref, exec, mon, fb, corr, with, hand, term}\}. \quad (82)$$

Equation 82 uses tags for encounter, inquiry, request, offer, clarification, negotiation, authorization, refusal, execution, monitoring, feedback, correction, withdrawal, handover, and termination. The list is intentionally finite and non-exhaustive. An application can refine one tag or add a domain-specific type when the added distinction has a declared analytical role.

Let $\text{type}_{\text{proc}}(\zeta_j) \in \mathcal{T}_{\text{proc}}$ denote the type assigned to a procedure element. Table 13 summarizes the role of the principal types. The table describes analytical functions rather than normative verdicts. A request can be mistaken, a refusal can be justified, an offer can be intrusive, and a termination can be appropriate or harmful depending on relations developed later in the paper.

A procedure type does not determine one mathematical carrier. A request may be represented by a communicative action record together with its manifestation and epistemic uptake. Monitoring may be represented by an observation or measurement record. A refusal may contain an action record and a role-relative boundary update. Handover may require a transformation of role positions and admissible continuations. The common feature is procedural placement, not a shared arithmetic.

Procedure elements can also have several functions at once. A clarification can simultaneously disclose previously private information, revise a need judgment, alter a proposed action, and change the perceived standing of the speaker. The formalism therefore allows one ζ_j to carry several typed payloads while retaining one procedure-type tag for the local role emphasized by the analysis.

11.3 Partial Composability

Procedure structure depends on more than temporal order. A later element must also be well typed relative to the situation produced by earlier elements. Section 10 already represented actions by partial transformations. The procedure layer generalizes the same discipline to heterogeneous local elements.

For each retained procedure element ζ_j , let Θ_{ζ_j} denote its procedure-level partial map between the record classes relevant to that element. Its source and target are written explicitly in Equation 83.

$$\Theta_{\zeta_j} : \mathcal{D}_{\zeta_j} \rightarrow \mathcal{C}_{\zeta_j}, \quad (83)$$

where $\mathcal{D}_{\zeta_j}$ is the declared domain of procedural applicability and $\mathcal{C}_{\zeta_j}$ is the corresponding codomain. These record classes can contain relational, manifestation, epistemic, or role-relative data according to the element type. Equation 83 does not require all procedure elements to act on one universal carrier.

For two historically ordered elements ζ_j and ζ_k with $j \prec_{\Gamma} k$, the composition $\Theta_{\zeta_k} \circ \Theta_{\zeta_j}$ is admissible only where the output of the first map lies in the domain of the second, as stated in Equation 84.

$$\mathcal{D}_{\zeta_k \circ \zeta_j} := \{x \in \mathcal{D}_{\zeta_j} \mid \Theta_{\zeta_j}(x) \in \mathcal{D}_{\zeta_k}\}. \quad (84)$$

Equation 84 makes procedural failure formally informative. A refusal can remove a proposed intervention from the later domain. A clarification can make a previously undefined action sufficiently specified to become available. A withdrawal can close a role-dependent action family. A disclosure can add information required for an informed choice. Undefined composition is therefore not automatically an error in the formalism; it can represent a substantive change in what the relation permits or makes intelligible.

When a linear trace $\gamma = (\zeta_1, \dots, \zeta_n)$ is fully composable on a retained domain, its procedure composite is stated in Equation 85.

$$\Theta_{\gamma} := \Theta_{\zeta_n} \circ \dots \circ \Theta_{\zeta_2} \circ \Theta_{\zeta_1}. \quad (85)$$

Equation 85 is a derived specialization rather than the definition of procedure itself. A procedure record can remain meaningful even when no single composite exists across all of its heterogeneous elements. Later sections can compose selected subfamilies only after common source-target structure has been established.

The same caution applies to identity-like steps. Waiting, maintaining availability, or leaving a proposal open can be procedurally meaningful even when the represented relational carrier changes little. Modeling such an interval by an identity map is justified only when the retained record genuinely remains invariant at the selected resolution. Procedural inactivity should not be equated automatically with formal identity.

11.4 Branching Continuations, Termination

A support procedure ordinarily permits several possible continuations. A request can be accepted, refused, clarified, or redirected. An offer can be modified before execution. Monitoring can lead to continuation, correction, withdrawal, or handover. The formalism therefore retains branching rather than treating the realized history as the only path that ever existed.

Let h denote a realized procedure prefix and let $\mathfrak{Z}_{\text{proc}}^{\kappa}$ denote the retained class of candidate next elements. Define the admissible continuation family in Equation 86.

$$\text{Cont}_{\text{proc}}^{\kappa}(h) := \{\zeta \in \mathfrak{Z}_{\text{proc}}^{\kappa} \mid \Theta_{\zeta} \text{ is defined on the record produced by } h\}. \quad (86)$$

Equation 86 records availability rather than desirability. Several continuations can be formally available while only some are ethically permissible, practically feasible, or mutually acceptable. The later evaluative sections add those distinctions.

A realized element can change the continuation family. Let h' extend h by one defined element. It is possible that $\text{Cont}_{\text{proc}}^{\kappa}(h')$ contains actions unavailable after h , or excludes actions that were previously available. This change gives procedural history generative force: interaction reorganizes the space of subsequent interaction.

Termination is therefore typed. A procedure can end because its objective has been achieved, because B refuses further support, because A withdraws, because competence limits require referral, because the relevant need judgment is revised, because the relationship changes category, or because the procedural domain becomes undefined. These endings should not be represented by one undifferentiated terminal symbol when their later relational consequences differ.

Handover deserves separate attention. In a strict dyadic analysis, a full transfer to a third party lies beyond the modeled system. Paper I can nevertheless represent the local dyadic component: A and B can generate a termination or transfer-intention record whose external realization is deferred to the triadic framework. This preserves the dyadic boundary without erasing the procedural significance of transition planning.

11.5 Joint Participation Structure

The word *joint* has several meanings that should remain distinct. A procedure element can be jointly enacted, jointly observable, jointly acknowledged, or jointly generated. These properties can coincide, yet none follows automatically from another.

Let $\mathcal{T}_J := \{\text{enact, obs, ack, gen}\}$ denote the retained jointness types. For a procedure element ζ , define the jointness record in Equation 87.

$$\text{Joint}^{\kappa}(\zeta) := \{\tau \in \mathcal{T}_J \mid \zeta \text{ satisfies the declared criterion for jointness type } \tau\}. \quad (87)$$

Joint enactment concerns execution by both subject positions. Joint observability concerns access by both sides at the manifestation level. Joint acknowledgment requires a stronger epistemic relation in which each side has evidence that the other treats the retained element as part of the shared procedure. Joint generation concerns provenance: the procedure element materially depends on contributions from both sides even when only one subject performs the visible execution.

These distinctions prevent several common conflations. A transfer performed by A can be jointly generated after negotiation even though B does not execute the physical transfer. A public refusal can be jointly observable without being jointly endorsed. A jointly enacted plan can be misunderstood by both sides at the level of provenance. A quiet routine can be jointly generated and enacted while lacking explicit acknowledgment as a formal procedure.

Jointness also has a historical trajectory. Repeated interaction can transform a unilateral offer pattern into a jointly negotiated routine. Conversely, a procedure that began collaboratively can become one-sided when information, dependence, or role expectations change. Procedure analysis should therefore track jointness as a time-indexed relational structure rather than assign it permanently to the dyad.

11.6 Request, Offer, Negotiation, Authorization

Requests and offers create candidate objects of response. A request by B does not by itself establish that the requested object is a genuine need, that the requested intervention is good for B, or that A has a duty to provide it. An offer by A likewise does not establish that B needs the proposed support, that B welcomes it, or that A possesses standing to intervene. Their immediate procedural role is narrower: they introduce or modify a candidate action family within the dyadic interaction.

Let α^{prop} denote a proposed action record whose execution has not yet been realized. A request or offer can make such a proposal manifest together with scope conditions such as timing, duration,

resources, role, limits, or expected effect. Clarification can revise these conditions. Negotiation can generate a new proposal whose provenance is distributed across the preceding exchange.

The history of proposal formation matters to later action identity. Suppose an initial offer α_0^{prop} is modified through clarification and negotiation into α_1^{prop} . The executed action can be equivalent to the initial offer under a coarse material-transfer specification and non-equivalent under a procedure-sensitive specification that retains timing, conditions, or authority. Section 10.6 therefore applies to procedure-generated actions as well as isolated ones.

Authorization-related manifestations enter at this stage, but their normative interpretation is deferred. A verbal “yes,” a signature, a nod, continued participation, silence, or a platform click can function as a procedure-relevant manifestation only under an explicit interpretation rule. Section 11 records how such manifestations affect continuation structure; the later authorization section will distinguish request, permission, consent, acquiescence, compliance, endorsement, refusal, and withdrawal and will examine the capacities on which their normative significance depends.

Refusal has positive procedural structure. It can close one proposed action while preserving others, request clarification, modify scope, or terminate the procedure. Treating refusal merely as the absence of consent would lose these generative consequences. The same point applies to withdrawal: ending a prior authorization can actively reorganize what later action remains admissible.

11.7 Feedback, Correction, Handover

Support procedures can remain revisable after execution begins. Monitoring and feedback generate new manifestations concerning effect, burden, misunderstanding, changing preference, capacity, or relational meaning. These manifestations do not enter the model as transparent truth. They pass through the accessibility, uptake, reconstruction, and ToM structures already developed in Sections 8 and 9.

Let ζ_{fb} be a feedback element and let α be an earlier action record. A feedback relation can associate the element with one or more retained targets as stated schematically in Equation 88.

$$\text{Target}^{\text{fb}} \subseteq \mathcal{Z}_{\text{proc}}^{\kappa} \times (\mathcal{A}_{AB}^{\kappa} \cup \mathcal{Z}_{\text{proc}}^{\kappa}). \quad (88)$$

Equation 88 permits feedback to concern an action, an earlier interpretation, a procedure rule, or another local element. The displayed union is a tagged analytical union; it does not assert a shared algebraic carrier for actions and procedure records.

Correction is broader than reversing an earlier action. A correction can alter a future plan, revise an interpretation, replace a communication channel, narrow the scope of support, add safeguards, redistribute execution, or terminate the intervention. Some corrections can restore a previous relational possibility; others create a new procedure state that has no exact historical predecessor. The procedure formalism therefore does not equate correction with inversion.

Feedback can also change action provenance retrospectively. A later disclosure can show that B’s earlier silence reflected fear rather than indifference. A monitoring record can reveal that A’s intervention produced an effect neither subject expected. A correction can then modify both the procedure and the reconstructed meaning of earlier elements. Historical revision remains epistemically defeasible: later interpretation can improve the model or introduce new uncertainty.

Handover or closure can itself require feedback. A competent termination process may need to communicate role change, preserve relevant records, clarify residual commitments, and confirm which forms of contact remain available. Paper I does not derive a universal closure protocol. It identifies closure as a relationally consequential procedure object whose form can affect later trust, dependency, expectation, and role persistence.

11.8 Procedure-Dependent Meaning

The meaning of a local action can depend on the procedure history in which it occurs. The same material transfer can express solidarity after a request, paternalism after an ignored refusal, compensation after

acknowledged harm, routine reciprocity inside an established practice, or emergency intervention under another history. Procedure therefore supplies context to action interpretation without making interpretation arbitrary.

Let $h_{\Gamma}^{\leq j}$ denote the retained procedure prefix available to subject i when element ζ_j occurs. A procedure-conditioned meaning map can be represented schematically by Equation 89.

$$\text{Mean}_i^{\kappa}(\zeta_j \mid h_{\Gamma}^{\leq j}) \in \text{Mean}_{i,j}^{\kappa}, \quad (89)$$

where $\text{Mean}_{i,j}^{\kappa}$ is the carrier of meanings available to subject i under the adopted representation. Equation 89 is intentionally weak. It does not assume that meaning is a scalar, that it is uniquely determined by the prefix, or that A and B possess the same carrier of interpretive possibilities.

Two procedure prefixes can therefore yield distinct interpretations of a locally similar element. If h and h' preserve different request, refusal, role, or trust histories, then the model can retain distinct outputs $\text{Mean}_i^{\kappa}(\zeta \mid h)$ and $\text{Mean}_i^{\kappa}(\zeta \mid h')$ without treating the local manifestation as changed. This gives formal content to procedure-sensitive meaning.

Procedure also shapes aesthetics. Timing, pacing, turn-taking, interruption, delay, tact, restraint, repetition, and closure can become manifestation-level structures that are aesthetically salient before either subject articulates a reflective judgment. Section 11 records only the procedural substrate. The later evaluative section will consider how such structures can contribute to aesthetic differentiation and normative judgment without identifying the two.

11.9 Three-Level Procedure Trajectory

The three-level architecture applies to the procedure as a whole. The ultimate ontology of procedure remains suspended. At the relational level, Γ_{AB}^{κ} records a historically consequential organization of interactions whose ordering can enable, close, or transform later possibilities. At the manifestation level, selected procedure elements appear through utterances, gestures, documents, pauses, interface events, transfers, refusals, acknowledgments, and other context-dependent forms. A and B then reconstruct the procedure through subject-relative epistemic histories.

Let $\text{Man}_t(\Gamma)$ denote the manifestation family retained for a procedure at historical stage t , and let $\hat{\Gamma}_t^A$ and $\hat{\Gamma}_t^B$ denote the corresponding A-relative and B-relative reconstructions. The procedure profile is stated in Equation 90.

$$\Lambda_t^{\text{proc}}(\Gamma) := \left(\Gamma_t^R, \text{Man}_t(\Gamma_t^R), \hat{\Gamma}_t^A, \hat{\Gamma}_t^B \right), \quad (90)$$

with ultimate ontological status marked separately by the suspension convention of Section 5. Equation 90 is a heterogeneous profile. Its components are not coordinates of one state vector.

The profile can change even when some visible procedure elements repeat. A familiar phrase can acquire a different relational meaning after trust changes. A previously acceptable delay can become threatening after a rupture. A formal consent screen can retain the same manifestation while the surrounding dependency structure changes. Conversely, a stable relational procedure can acquire new manifestations when language, interface, disability accommodation, or shared practice changes.

The mappings among these components also possess histories. Repeated interaction can make B's brief response increasingly legible to A; a conflict can destabilize an earlier interpretive convention; a jointly developed vocabulary can reduce the ambiguity of later negotiation. Procedure history therefore modifies not only what happens next but also how later elements can become manifest and intelligible.

11.10 Evaluative-Formation Interface

The procedure layer supplies the historical substrate for the next section on need, value, aesthetic judgment, and normative judgment. A judgment concerning support can depend on more than the executed

intervention. How the relation arrived at that intervention can affect what each subject takes the intervention to mean, whether a need judgment remains credible, whether an offer appears fitting, whether a refusal is recognized, and whether later feedback can reopen the procedure.

Three consequences follow for the next stage. First, need judgments can be procedure-sensitive: inquiry, disclosure, clarification, and revision can change what A and B take B to need. Second, goodness judgments can be procedure-sensitive: the same material outcome can be evaluated differently when produced through requested, negotiated, imposed, or withdrawn support. Third, aesthetic and normative judgments can respond to procedure-level manifestation such as timing, tact, reciprocity, interruption, humiliation, or restraint.

The procedure architecture therefore prevents outcome-only evaluation from exhausting the dyadic relation. It also prevents procedure from determining value automatically. A carefully negotiated action can still be harmful; an emergency intervention with limited prior dialogue can be justified under some conditions; an aesthetically graceful interaction can still contain problematic power. The next section keeps these evaluative dimensions typed and distinct while examining how they are generated within the history made available by the procedure record.

12 Need, Value, Aesthetic Judgment, Normative Judgment

This section develops the evaluative layer of the minimal dyadic volunteer formalism. Its role is to specify how judgments concerning need, value, aesthetic significance, and normativity can be generated without treating any of them as transparent properties already attached to B, to an action, or to a procedure. The objective is to preserve four distinctions at once: relational condition versus subject-relative judgment, manifestation versus interpretation, one evaluative family versus another, and judgment formation versus judgment validity. The discussion proceeds through a general typing scheme, need judgment, value and goodness judgment, pre-explicit aesthetic differentiation, articulated normative judgment, disagreement and higher-order evaluation, shared evaluative formation, and historical revision. The method is reconstructive and typed. Paper I supplies a formal architecture for evaluative formation; it does not provide a complete moral theory, a complete aesthetics, or a universal metric of welfare.

The procedure architecture of Section 11 makes the evaluative problem historically situated. A request, refusal, intervention, consequence, delay, or correction can become an object of judgment only through a retained relation in which some manifestations are accessible and some reconstructions are available. The same material transfer can therefore receive different evaluations when its provenance, procedure, timing, role structure, or later consequence differs. Conversely, two subjects can evaluate different aspects of one relation while appearing verbally to agree. Figure 7 previews the resulting architecture. Its arrows indicate admissible dependence or feedback paths rather than one universal psychological sequence.

12.1 Evaluative Domains, Judgment Typing

Paper I treats an evaluative judgment as a derived object produced under a declared subject position, historical stage, resolution, target, and evaluative family. Let

$$\mathcal{T}_{\text{eval}} := \{\text{need, val, aes, norm}\} \quad (91)$$

be the four evaluative families retained in this section. Equation 91 is an analytical partition for the present formalism. It does not claim that every philosophical theory must separate the four families in exactly this manner.

For subject position $i \in \{A, B\}$, historical stage t , resolution κ , and evaluative family $\tau \in \mathcal{T}_{\text{eval}}$, let $\mathcal{D}_{i,t}^{\tau,\kappa}$ denote the admissible input records for that judgment family and let $\mathcal{V}_{i,t}^{\tau,\kappa}$ denote its output carrier. The general judgment relation is stated in Equation 92.

$$\mathbf{J}_{i,t}^{\tau,\kappa} \subseteq \mathcal{D}_{i,t}^{\tau,\kappa} \times \mathcal{V}_{i,t}^{\tau,\kappa}. \quad (92)$$

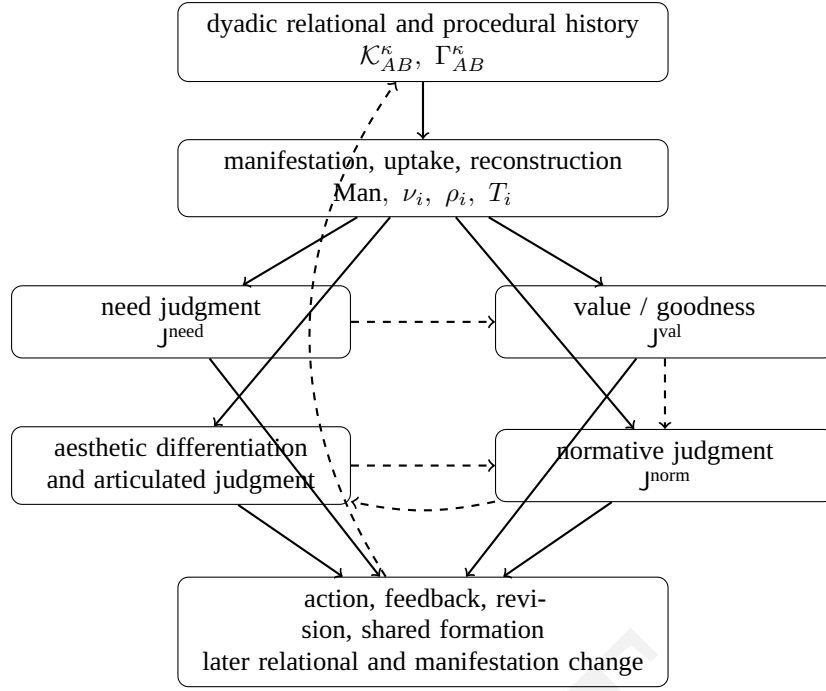


Figure 7: Evaluative-formation architecture in the minimal dyad. Relational and procedural history becomes partially available through manifestation, uptake, reconstruction, and theory-of-mind structure. Need, value, aesthetic, and normative judgments remain separately typed. Dashed arrows represent possible cross-family influence or historical feedback, not universal entailment or a fixed causal order.

If $(d, v) \in J_{i,t}^{\tau,\kappa}$, the model records that input record d supports evaluative output v for subject i under the retained judgment rule. Equation 92 is relational rather than functional because the same evidence can remain compatible with several judgments, a subject can suspend judgment, and an application can deliberately represent underdetermination. A functional specialization is permitted only when the local model supplies a rule selecting one output.

The input record d can include subject-relative reconstructions, procedure history, action provenance, counterfactual expectations, prior norms, memory, affective differentiation, or other objects already defined in the manuscript. These elements need not share one numerical carrier. The output v can likewise be qualitative, ordered, set-valued, algebraic, modal, or otherwise structured. No scalar utility, universal preference order, or common metric is assumed by Equation 92.

The judgment relation also preserves the three-level distinction. Ultimate ontology of need, value, beauty, or normativity remains suspended. At the relational level, the model can identify historically consequential conditions, practices, roles, or normative structures. At the manifestation level, subjects encounter utterances, bodily states, gestures, timing, documents, transfers, refusals, and other presentations. A judgment is then a relationally situated construction that can itself become causally or generatively effective. It is not a fourth reality level.

Table 14 summarizes the principal burden of each family. The rows overlap in application without becoming interchangeable. A judgment that B needs shelter can support a judgment that providing shelter has positive value, yet it does not by itself establish a duty on A. An aesthetically fitting manner of support can affect dignity or trust, yet aesthetic fittingness does not settle moral permissibility.

12.2 Need Judgment

Need is among the most easily naturalized terms in support discourse. Paper I therefore treats it as a judgment family rather than an intrinsic label already attached to B. B can be hungry, isolated, injured, unable to access a service, exposed to risk, or constrained in a practical capacity, yet whether a retained

Table 14: Evaluative families retained in Paper I and the distinctions that prevent their collapse into one generic value variable.

Family	Primary analytical object	Possible output structure	Distinction preserved
Need	A condition judged to call for some form of support, remedy, accommodation, or preserved capacity	Typed need descriptor, urgency, domain, horizon, uncertainty	Relational condition is distinct from A's or B's judgment that it constitutes a need
Value / goodness	Significance of an action, consequence, continuation, or relation for a declared target and dimension	Qualitative evaluation, partial order, multi-dimensional profile, local scalar where justified	Benefit for B is distinct from benefit for A, the relation, or another horizon
Aesthetic	Fittingness, dignity, tact, rhythm, harmony, tension, awkwardness, or related aesthetic salience	Pre-explicit differentiation and/or articulated aesthetic judgment	Manifestation-level aesthetic organization can precede explicit judgment
Normative	Requirement, permission, prohibition, legitimacy, responsibility, or another norm-governed status	Modal or reason-bearing judgment record	Normative formation is distinct from validity, authority, consent, and enforcement

condition counts as a need for the present support relation depends on the criteria, horizon, alternatives, and subject-relative reconstruction used by the analysis. The formalism can represent urgent and comparatively direct cases without making every need judgment equally contestable.

Let $\mathcal{D}_{i \rightsquigarrow B, t}^{\text{need}, \kappa}$ denote the input records through which subject i evaluates a possible need of B. The directional index $i \rightsquigarrow B$ identifies the judging position and the target of the judgment; it does not imply authority over B. Let $\mathcal{V}_{B, t}^{\text{need}, \kappa}$ denote a carrier of typed need descriptors. The need-judgment relation is stated in Equation 93.

$$\mathcal{J}_{i \rightsquigarrow B, t}^{\text{need}, \kappa} \subseteq \mathcal{D}_{i \rightsquigarrow B, t}^{\text{need}, \kappa} \times \mathcal{V}_{B, t}^{\text{need}, \kappa}, \quad i \in \{A, B\}. \quad (93)$$

Equation 93 gives formal room to distinguish A's judgment of B's need from B's self-judgment. For a common retained input object d or for historically corresponding inputs, the outputs can agree, diverge, remain underdetermined, or fail to be comparable because the two subjects employ different need categories. A divergence does not by itself show that either subject is irrational or that one side possesses final epistemic priority.

A need descriptor can be typed by the dimension in which support is considered. Let $\Xi_{\text{need}}^{\kappa}$ denote the retained family of need dimensions. Depending on the application, elements can concern immediate material condition, bodily safety, practical capability, access, communication, relational participation, accommodation, or longer-horizon generative possibility. Paper I does not assume that these dimensions can be ranked by one common measure. The inclusion of capability and substantive possibility is compatible with capability-oriented approaches that resist reducing well-being to resources alone, while the present formalism does not import a complete capability theory [13].

Need judgments are also horizon-sensitive. A short-horizon intervention can reduce immediate distress while weakening a longer-horizon capacity; a temporary dependency can be justified during emergency care while becoming problematic if indefinitely reproduced. The judgment carrier may therefore include an explicit horizon tag h when the application needs one. The inclusion of h records the scope of the judgment rather than turning time into a universal scalar parameter.

The three-level audit is essential. A relationally consequential condition of B can exist without becoming manifest to A. A manifestation can be accessible while A does not notice it. A can observe a manifestation and reconstruct it inaccurately. B can manifest a need strategically, incompletely, indirectly, or through a vocabulary shaped by prior institutions and relations. The formal object of need judgment therefore concerns a historical relation among condition, manifestation, uptake, reconstruction,

and evaluation rather than direct reading of an inner state.

A jointly stabilized need judgment requires an additional historical burden. Agreement between A and B at one moment is weaker than a need judgment produced through mutual clarification, reciprocal correction, and later retained use. Section 12.7 formalizes that distinction after the other evaluative families have been introduced.

12.3 Goodness Judgment, Value Profiles

Value judgment concerns the significance of an action, consequence, continuation, or relation for a declared target and evaluative dimension. The term *value* is used broadly in this section and does not imply price, utility, magnitude, or commensurability. A goodness judgment is one target-typed form of value judgment, for example A's judgment that a candidate intervention would be good for B over a specified horizon.

Let $\mathfrak{D}_{\text{val},t}^\kappa$ denote the class of evaluable objects retained at stage t . Its elements can include an action record α , a consequence record, a procedure segment, a continuation family, a role configuration, or the dyadic relation itself. Let $q \in \{A, B, AB\}$ denote the declared beneficiary or relational target, where AB refers to the coupled dyadic relation rather than to a third subject. For judging position i , the typed value relation is stated in Equation 94.

$$J_{i \rightsquigarrow q,t}^{\text{val},\kappa} \subseteq \mathcal{D}_{i \rightsquigarrow q,t}^{\text{val},\kappa} \times \mathcal{V}_{q,t}^{\text{val},\kappa}. \quad (94)$$

The input domain in Equation 94 includes the evaluable object together with whatever history, prediction, consequence, or comparison the application requires. The output carrier can preserve several dimensions instead of aggregating them. Let $\mathcal{B}_{\text{val}}^\kappa$ denote the retained evaluative dimensions and let $\mathcal{H}_{\text{val}}^\kappa$ denote the retained horizons. A subject-relative value profile for evaluable object x is defined schematically in Equation 95.

$$\text{ValProf}_{i \rightsquigarrow q,t}^\kappa(x) := \{(\beta, h, v) \mid \beta \in \mathcal{B}_{\text{val}}^\kappa, h \in \mathcal{H}_{\text{val}}^\kappa, v \text{ is retained as the } (\beta, h)\text{-evaluation of } x\}. \quad (95)$$

Equation 95 is deliberately set-valued. One action can be evaluated positively in an immediate material dimension and negatively in a longer-horizon dependency dimension. Another can impose a short-term burden while enlarging later practical capacity. The profile permits such mixed structure without assuming that the dimensions admit addition or one common ranking.

This target typing also blocks a recurrent ambiguity in volunteer discourse. The following judgments are formally different: an action is judged good for B; the action is judged good for A; the action is judged good for the continuing relation; the action is judged socially admirable; the action is judged morally required. Paper I does not infer one from another. In particular, a support activity can generate meaning, belonging, recognition, or identity for A while also benefiting B. The existence of A-relative value does not show that B is merely instrumental, and B-relative benefit does not show that the action's provenance is normatively unproblematic.

Counterfactual comparison can enter a value judgment when the application provides comparable continuation records. Let $x^{(1)}$ and $x^{(2)}$ denote two admissible continuations or action-conditioned futures. A relation $x^{(1)} \succ_{i \rightsquigarrow q}^{\beta,h} x^{(2)}$ may be introduced only after the comparison dimension β , horizon h , judging position i , and target q have been specified. The paper does not assume that these local comparison relations combine into a total order.

The distinction between value formation and value validity remains open. A's historically stabilized judgment that a procedure is beneficial for B can become causally effective by guiding action, shaping later attention, or influencing B's opportunities. Its efficacy does not prove that the judgment is correct. B's disagreement likewise does not automatically settle the matter. The later ethics and justice papers will address stronger evaluative standards; the present section makes the objects and their disagreements formally visible.

12.4 Pre-Explicit Aesthetic Differentiation, Articulated Judgment

Aesthetic structure enters the dyad before the formalism requires an explicit proposition of taste. The procedure developed in Section 11 can already exhibit timing, pacing, interruption, silence, distance, restraint, tact, symmetry, tension, repetition, and closure. Such manifestation-level organization can become salient to A or B before either subject articulates a judgment such as “fitting,” “awkward,” “dignified,” “intrusive,” “graceful,” or “humiliating.” The formal claim is therefore weaker than saying that an aesthetic judgment exists before judgment. A pre-explicit differentiation can be operative before explicit aesthetic articulation.

Let $\mathcal{M}_{i,t}^{\text{aes},\kappa}$ denote the manifestation records available to subject i that the local analysis retains as potentially aesthetically salient. Let $\mathcal{S}_{i,t}^{\text{aes},\kappa}$ denote a carrier of pre-explicit aesthetic differentiations. The differentiation relation is stated in Equation 96.

$$\text{Diff}_{i,t}^{\text{aes},\kappa} \subseteq \mathcal{M}_{i,t}^{\text{aes},\kappa} \times \mathcal{S}_{i,t}^{\text{aes},\kappa}. \quad (96)$$

An element $s \in \mathcal{S}_{i,t}^{\text{aes},\kappa}$ need not be a proposition or a verbally expressible reason. It can record a retained distinction in salience, ease, tension, attraction, aversion, proportion, rhythm, dignity, awkwardness, or another locally specified aesthetic dimension. Paper I remains agnostic about whether such differentiation is bodily, cultural, learned, material, perceptual, symbolic, or multiply generated in a particular case. Its role is to prevent explicit language from being treated as the only possible beginning of aesthetic formation.

Articulated aesthetic judgment requires additional structure. Let $\mathcal{V}_{i,t}^{\text{aes},\kappa}$ denote the carrier of explicit or otherwise stabilized aesthetic judgments available to subject i . The articulation relation is stated in Equation 97.

$$\text{Art}_{i,t}^{\text{aes},\kappa} \subseteq \mathcal{S}_{i,t}^{\text{aes},\kappa} \times \mathfrak{H}_i^{\leq t} \times \mathcal{V}_{i,t}^{\text{aes},\kappa}, \quad (97)$$

where $\mathfrak{H}_i^{\leq t}$ denotes the retained subject-relative history relevant to articulation. Equation 97 records that a pre-explicit differentiation can participate in later aesthetic judgment through a history of vocabulary, exposure, role, comparison, memory, and interaction. It does not state that every differentiation becomes articulated or that articulation is uniquely determined by the earlier manifestation.

The reverse relation is equally important. Once an aesthetic vocabulary has stabilized, it can reorganize later attention and manifestation. A and B can learn that certain forms of interruption feel intrusive, that a certain pace permits dignity, or that a ritualized form of giving creates unwanted distance. These learned distinctions can alter later procedure, which changes the manifestation structure on which future judgments are based. Aesthetic formation is therefore potentially recursive.

This recursive structure creates a distinctive research possibility for support relations. Two interventions can have similar material consequences while differing markedly in the manner through which support is manifested. One can preserve privacy, pacing, reciprocity, and dignity; another can publicly expose dependence or produce a humiliating asymmetry of presentation. Paper I does not infer moral wrongness from aesthetic awkwardness or moral legitimacy from aesthetic harmony. It records aesthetic manifestation as a separate relationally consequential dimension that can later interact with trust, recognition, consent, and normativity.

12.5 Normative Judgment, Retained Reasons

Normative judgment concerns what a subject takes to be required, permitted, prohibited, justified, legitimate, responsible, appropriate, or otherwise norm-governed within a declared scope. Paper I treats these judgments as historically generated relational objects rather than as outputs of one universal moral calculus. The formalism must allow disagreement among moral theories, social norms, professional standards, role expectations, legal rules, and locally generated dyadic commitments without collapsing them into one modality.

Let $\mathcal{V}_{i,t}^{\text{norm},\kappa}$ denote the carrier of normative judgment records available to subject i . A record can include a modality, scope, target, retained reasons, exception conditions, confidence, or another locally required field. The normative judgment relation is stated in Equation 98.

$$J_{i,t}^{\text{norm},\kappa} \subseteq \mathcal{D}_{i,t}^{\text{norm},\kappa} \times \mathcal{V}_{i,t}^{\text{norm},\kappa}. \quad (98)$$

The carrier $\mathcal{V}_{i,t}^{\text{norm},\kappa}$ can include judgments such as required, permitted, optional, supererogatory, discouraged, forbidden, legitimate, illegitimate, justified, or unjustified where the application defines those terms. The list is illustrative rather than exhaustive, and Paper I does not assume that every normative framework recognizes the same modalities or relations among them.

Normative judgments can draw on earlier evaluative objects without being reducible to them. Let $\mathfrak{B}_{i,t}^{\text{norm},\kappa}(x)$ denote the family of retained antecedent objects that subject i treats as relevant to a normative judgment concerning x . Candidate members can include a need judgment, a B-relative value profile, an aesthetic judgment concerning dignity or humiliation, a promise, a role relation, a prior refusal, a shared norm, a forecast of harm, or another historically available reason-bearing object. The record is defined schematically in Equation 99.

$$\mathfrak{B}_{i,t}^{\text{norm},\kappa}(x) := \{y \mid y \text{ is retained by } i \text{ as relevant to the normative judgment of } x\}. \quad (99)$$

Equation 99 is a provenance record for judgment, not an inference theorem. A need judgment can provide a reason to consider intervention without entailing permission to intervene. A positive value judgment can support a reason without creating standing. An aesthetic judgment of fittingness can shape how a procedure is received without settling legitimacy. A prior promise can create a role-relative reason while remaining defeasible under changed conditions. These distinctions are central to the later section on consent, refusal, standing, and authority.

Formation and validity remain separate. A normative judgment can be historically stabilized, widely shared, institutionally manifested, and causally efficacious while still being open to criticism. Conversely, a weakly recognized judgment can later prove better supported. Paper I therefore represents normative formation and revision without defining a universal validity operator whose output would settle the later ethical inquiry.

12.6 Disagreement, Higher-Order Evaluation

The dyadic architecture requires a way to represent disagreement without assuming a numerical distance between judgments. For each evaluative family τ , let $\simeq_t^{\tau,\kappa}$ denote a declared equivalence criterion on comparable judgment records at stage t . Two selected judgments j_A and j_B are evaluatively aligned relative to τ when $j_A \simeq_t^{\tau,\kappa} j_B$; they are evaluatively divergent when that equivalence fails. The divergence condition is stated in Equation 100.

$$\text{Div}_{AB,t}^{\tau,\kappa}(j_A, j_B) \iff j_A \not\simeq_t^{\tau,\kappa} j_B. \quad (100)$$

Equation 100 is typed. A and B can agree about need while disagreeing about value, agree about material value while disagreeing about dignity, or agree that an action is beneficial while disagreeing about whether it is permitted. The relation also depends on what the comparison retains. Two judgments can be equivalent regarding immediate material benefit and non-equivalent regarding long-horizon dependency.

Disagreement can concern the object or the judgment itself. Because judgment records belong to the recursively generated inventory of Section 7.15, A can form a normative judgment concerning B's refusal, B can evaluate A's need judgment, and either side can evaluate the other's confidence or reasons. Such higher-order evaluation is analytically important for paternalism. A can believe both that B rejects an intervention and that B's rejection is mistaken. The formalism records the second judgment without allowing it to erase the first.

Higher-order evaluation also interacts with ToM. A's judgment of B's judgment may be based on $T_A(B)$ rather than on direct access to B's evaluative state. A can therefore disagree with a judgment B

never actually held, or attribute reasons that B does not endorse. The distinction between a relationally manifested disagreement and a reconstructed disagreement should remain explicit in empirical applications.

A further asymmetry concerns which judgment becomes consequential. A and B can possess equally articulated but differently effective evaluations if one side controls resources, procedure, visibility, or later action. The later section on typed relational asymmetry will treat such influence differences as typed asymmetries rather than as properties of judgment content itself.

12.7 Shared Evaluative Formation

Coincidence between two judgments is weaker than a shared evaluative object. If A and B independently judge that a proposed action is acceptable, their outputs can satisfy an equivalence criterion without any joint formation. A shared judgment or shared evaluative vocabulary requires a historical relation through which the dyad participates in generating, stabilizing, or revising the object together.

Let $\mathfrak{J}_{A,t}^{\tau,\kappa}$ and $\mathfrak{J}_{B,t}^{\tau,\kappa}$ denote the retained judgment-record classes for evaluative family τ , and let $\mathfrak{J}_{AB,t}^{\tau,\kappa}$ denote the class of candidate shared evaluative objects. Let $\mathfrak{H}_{AB,\text{eval}}^{\leq t}$ denote the retained dyadic histories relevant to evaluative formation. The joint-formation relation is stated in Equation 101.

$$\text{JointForm}_t^{\tau,\kappa} \subseteq \mathfrak{H}_{AB,\text{eval}}^{\leq t} \times \mathfrak{J}_{A,t}^{\tau,\kappa} \times \mathfrak{J}_{B,t}^{\tau,\kappa} \times \mathfrak{J}_{AB,t}^{\tau,\kappa}. \quad (101)$$

Membership $(h, j_A, j_B, j_{AB}) \in \text{JointForm}_t^{\tau,\kappa}$ means that the application treats j_{AB} as a jointly generated evaluative object supported by history h and by the retained contributions represented by j_A and j_B . Equation 101 does not require j_{AB} to equal either input judgment or their intersection. Interaction can generate a new distinction, compromise, vocabulary, or standard that neither side possessed in the same form beforehand.

This possibility is especially important for aesthetic and normative formation. Repeated support can create a shared sense of pace, tact, privacy, reciprocity, or acceptable intervention. A phrase, gesture, or pause can acquire a dyad-specific meaning through use. A jointly stabilized evaluative vocabulary can then alter how later manifestations are perceived. The resulting object is relationally real only when the model supplies persistence and generative efficacy criteria of the kind developed in Section 7.13.

Shared formation does not require complete consensus. A norm can be shared enough to organize disagreement. A vocabulary can be jointly usable while A and B contest particular applications. A procedure can contain a recognized right of refusal even when the subjects disagree over whether one refusal was well founded. The relevant shared object is the relation that structures later interpretation or action, not unanimous private belief.

The emergence of a shared judgment can also change subject-level evaluative capacity. A distinction first generated in interaction can later become part of A's or B's individual repertoire. This feedback supports the later subject-co-formation paper but is already admissible in Paper I as a historical change in the evaluative environment.

12.8 Evaluative Trajectory, Revision

Evaluative judgment changes through history. New manifestations can reveal a previously hidden condition; consequences can disconfirm a prediction; B can revise a self-judged need; A can discover that an apparently helpful action increased dependency; a previously awkward interaction can become intelligible after clarification; a shared norm can lose authority after repeated contestation. Paper I therefore represents evaluative formation through revisable records rather than one irreversible conclusion.

Let $\mathfrak{V}_{AB,t}^{\kappa}$ denote the heterogeneous evaluative record retained at stage t . It collects the selected A-relative, B-relative, and shared need, value, aesthetic, and normative judgment objects without treating them as coordinates of one vector. A schematic record is stated in Equation 102.

$$\mathfrak{V}_{AB,t}^{\kappa} := \left(\mathfrak{J}_{A,t}^{\text{need}}, \mathfrak{J}_{B,t}^{\text{need}}, \mathfrak{J}_{A,t}^{\text{val}}, \mathfrak{J}_{B,t}^{\text{val}}, \mathfrak{J}_{A,t}^{\text{aes}}, \mathfrak{J}_{B,t}^{\text{aes}}, \mathfrak{J}_{A,t}^{\text{norm}}, \mathfrak{J}_{B,t}^{\text{norm}}, \mathfrak{J}_{AB,t}^{\text{shared}} \right)_{\kappa}. \quad (102)$$

Equation 102 is bookkeeping. The components remain differently typed. The notation records which evaluative objects are retained together for one stage of analysis; it does not define addition, distance, probability, or a common state space over them.

Let $t \preceq t'$ denote the retained historical order. An evaluative revision relation is stated in Equation 103.

$$\text{Rev}_{t' \leftarrow t}^{\text{eval}, \kappa} \subseteq \mathfrak{V}_{AB, t}^{\kappa} \times \mathfrak{V}_{AB, t'}^{\kappa}. \quad (103)$$

The revision relation can be partial, nonunique, and history-conditioned. Several later evaluative records can remain admissible after the same feedback event because A and B can interpret evidence differently. Revision can affect the judgment content, confidence, scope, comparison horizon, vocabulary, or the relation through which later judgments become manifest.

The three-level trajectory is broader than judgment change alone. A relational condition can change; its manifestation can change; the subject's interpretive map can change; the evaluative vocabulary can change; and the resulting judgment can alter later procedure. An action can therefore modify the very conditions through which its future consequences will be evaluated. This recursive possibility is a central GR feature of the dyad.

The section also preserves nonrevision as a meaningful historical outcome. A subject can retain a judgment despite contrary feedback because the feedback is inaccessible, distrusted, reinterpreted, normatively discounted, or outweighed by another retained reason. A shared norm can remain stable because disagreement occurs inside rather than outside its grammar. Formal revisability means that change is representable and its conditions can be stated; it does not imply that every judgment should or will change.

12.9 Authorization Interface

The evaluative architecture prepares the next section on consent, refusal, standing, and authority. Need, positive value, aesthetic fittingness, and normative judgment can each contribute reasons within a support relation, yet none alone determines whether A is authorized to intervene. The distinction can be stated directly:

$$J_{A \rightsquigarrow B, t}^{\text{val}, \kappa}(x) \text{ positive} \not\Rightarrow \text{Authority}_{A \rightsquigarrow B, t}^{\kappa}(x). \quad (104)$$

Equation 104 is a formal non-entailment claim inside the architecture of Paper I. It says that an A-relative judgment of B-relative benefit does not by itself generate authority. Additional relations can include B's request or consent, emergency conditions, standing supplied by a role, a prior commitment, competence, information conditions, institutional authorization, or another locally justified structure. Their normative adequacy remains a later question.

The same discipline applies in the opposite direction. B's refusal remains a procedure-relative object whose manifestation can have relational consequences even when A judges the refusal harmful. A can retain a higher-order judgment that B is mistaken while still representing the refusal and its effect on the later action domain. This separation is necessary if the formalism is to describe paternalism, disagreement, emergency intervention, and respect for refusal without deciding those ethical problems by notation.

Section 12 has therefore supplied four evaluative families, their subject-relative typing, their three-level status, their possible disagreement, their shared formation, and their historical revision. The next section can now examine authorization-related structures without treating need, goodness, aesthetic fittingness, or moral judgment as substitutes for consent, standing, or authority.

13 Consent, Refusal, Standing, Authority

This section develops the authorization-related layer of the minimal dyadic volunteer formalism. Its role is to separate procedural and relational structures that are easily compressed into a single word such as

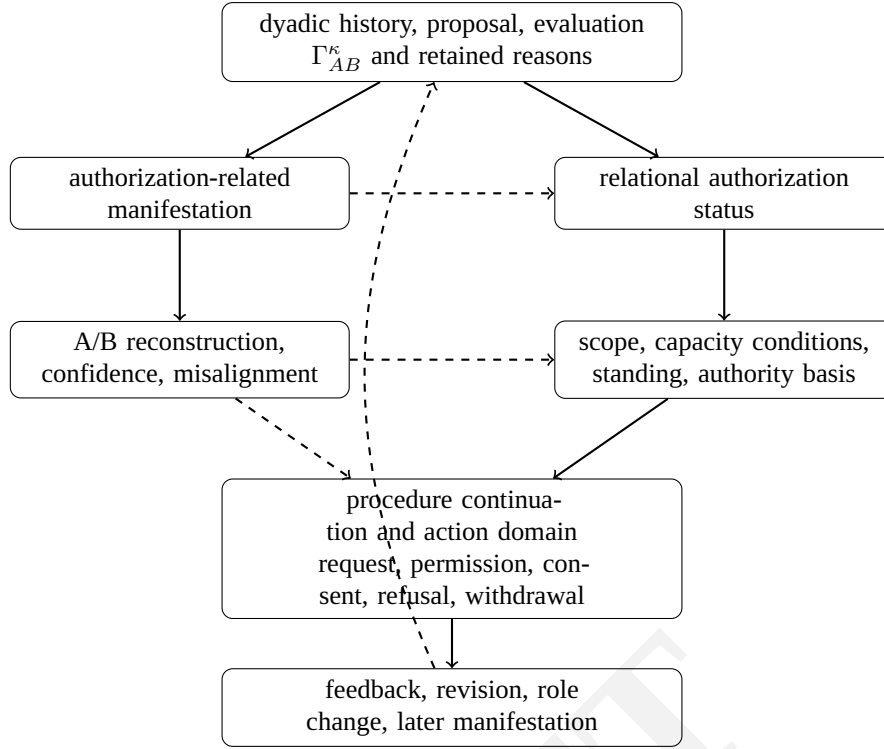


Figure 8: Authorization architecture in the minimal dyad. Manifestations and relational authorization statuses are distinguished, while subject-relative reconstruction can remain misaligned with either. Scope, capacity conditions, standing, and authority basis constrain the procedure continuation and action domain. Dashed arrows mark possible interpretive or historical dependence rather than automatic entailment.

“consent” even though they carry different generative consequences. The objective is to distinguish request, permission, consent, acquiescence, compliance, endorsement, refusal, withdrawal, standing, and authority while preserving their three-level reality status, subject-relative epistemic access, scope, historical trajectory, and relation to later action. The discussion proceeds through authorization-status typing, manifestation and reconstruction, scope and duration, capacity conditions, refusal and withdrawal, relational standing, intervention authority, boundary conditions, and historical revision. The method remains formal and reconstructive. Paper I identifies objects and relations that later ethical and justice analyses can evaluate; it does not provide a complete theory of valid consent, legitimate authority, emergency ethics, or legal capacity.

The evaluative architecture of Section 12 supplies reasons and judgments without converting them automatically into authorization. A may judge that an intervention would benefit B, B may judge that assistance is needed, both may regard a procedure as aesthetically fitting, and the dyad may share a norm favoring mutual aid. None of these facts alone determines whether a particular intervention is requested, permitted, consented to, refused, or within A’s authority. Conversely, an authorization-related manifestation can be present while its relational meaning remains uncertain. A spoken “yes” can manifest consent, acquiescence, compliance, politeness, misunderstanding, or pressure depending on the retained history and procedure. Figure 8 previews the resulting architecture.

13.1 Authorization Status Typing

The present formalism begins by separating a family of authorization-related statuses rather than defining one binary variable. Let

$$\mathcal{T}_{\text{auth}} := \{\text{request, permission, consent, acquiescence, compliance, endorsement, refusal, withdrawal}\} \quad (105)$$

Table 15: Authorization-related statuses in the minimal dyad. The descriptions are analytical roles used in Paper I and do not claim universal legal definitions.

Status	Restricted analytical role	Distinction preserved	Possible procedural consequence
Request	A subject invites or asks another subject to consider a specified action or response	Request is neither proof of need nor blanket authorization for arbitrary implementation	Introduces or modifies a candidate action family
Permission	A subject allows a specified action within a retained scope	Permission need not include positive endorsement or initiation	Makes one action or class procedurally available under the adopted scheme
Consent	A subject participates in authorizing a specified intervention or procedure under declared scope and assessment conditions	Consent is typed by object, history, capacity, information, and revocability	Supports an authorization basis for later action
Acquiescence	A subject does not actively resist or accepts continuation without the stronger positive structure assigned to consent	Nonresistance is not automatically consent or endorsement	May preserve continuation while leaving authorization status contested
Compliance	Conduct conforms to a request, rule, proposal, or pressure	Behaviorally conforming action does not establish consent	Can permit execution while preserving disagreement over authorization
Endorsement	A subject positively evaluates or supports the proposed action, procedure, or norm	Positive evaluation is distinct from permission, consent, and authority	Can strengthen shared formation without itself fixing scope
Refusal	A subject rejects, blocks, contests, or declines a proposed action or procedure within a declared scope	Refusal is a positive relational event rather than mere absence of consent	Reconfigures later continuations and can generate alternatives
Withdrawal	A subject terminates or narrows a previously retained authorization or participation status	Earlier authorization remains historical while later authorization changes	Revises the continuation and action domain prospectively

be the retained status family. Equation 105 is an analytical vocabulary for Paper I rather than a universal legal or moral taxonomy. Applications can refine the family, omit statuses that are irrelevant, or distinguish further subtypes when the local practice requires them.

For $\tau \in \mathcal{T}_{\text{auth}}$, subject positions $i, j \in \{A, B\}$, historical stage t , and resolution κ , let $\mathfrak{U}_{i \rightsquigarrow j, t}^{\tau, \kappa}(x)$ denote an analytical authorization-status record concerning object x . Let $\mathfrak{U}_{AB, t}^{\kappa}$ denote the retained class of such records at stage t . The target x can be a proposed action, procedure segment, disclosure, transfer, monitoring practice, role transition, or another locally defined object. The notation does not assert that the status is stored inside i as an intrinsic property. It records a relation whose constitution can depend on subject history, procedure, manifestation, capacity conditions, and the object to which the status applies.

Table 15 states the restricted analytical role assigned to each status. The distinctions are especially important because several of them can share similar manifestations. Continued participation can be compliance, acquiescence, or consent. A request can carry strong endorsement or can be made under perceived necessity. A refusal can reject one action while inviting another. Withdrawal can revise an earlier consent without erasing the historical fact that the earlier authorization existed.

The formalism does not derive one status from another unless an application explicitly supplies the rule. In particular, the present architecture contains no default entailments from compliance to consent,

from permission to endorsement, or from request to unrestricted authorization. This independence is stated schematically in Equation 108.

$$\text{compliance} \not\Rightarrow \text{consent}, \quad (106)$$

$$\text{permission} \not\Rightarrow \text{endorsement}, \quad (107)$$

$$\text{request}(x) \not\Rightarrow \text{consent}(x') \quad \text{for arbitrary } x'. \quad (108)$$

Equation 108 is a statement about the typing discipline of Paper I. A particular practice can establish stronger local implications, yet those implications then belong to the model and require explicit scope conditions. The equation prevents a formalism from converting one observable response into a stronger authorization status merely because such a conversion is convenient.

13.2 Manifestation, Reconstruction, Misalignment

Authorization statuses participate in the three-level architecture developed in Section 5. Their ultimate ontological status remains suspended. At relational reality, the model can retain historically situated relations of requesting, permitting, consenting, refusing, withdrawing, or authorizing. At manifestation reality, those relations can appear through utterance, signature, gesture, platform interaction, bodily orientation, continued participation, silence, delay, or another context-dependent form. A and B then reconstruct the manifestation through their own histories, language, ToM structure, and procedural expectations.

For an authorization-related object U , the generic three-level profile of Section 5.8 can be specialized as stated in Equation 109.

$$\Lambda_t(U) := \left(\mathcal{O}_U^?; U_t^R, U_t^M, \mu_t^U, \hat{U}_{A,t}, \hat{U}_{B,t} \right), \quad (109)$$

where $\mathcal{O}_U^?$ marks ontological suspension, U_t^M is the retained manifestation family, and μ_t^U is the concept-specific restriction of the manifestation relation in the sense of Equation 33. Equation 109 does not imply one-to-one correspondence among the components. One relational consent structure can have several manifestations. One manifestation can remain compatible with several relational statuses. A and B can reconstruct the same manifestation differently. Historical revision can later change either reconstruction without changing the original recorded manifestation.

This architecture makes authorization misalignment a typed research object. Let $\delta^{\text{auth},\kappa}$ denote a declared discrepancy relation rather than numerical subtraction. Candidate comparisons include

$$\delta^{\text{auth},\kappa} \left(U_t^R, \hat{U}_{A,t} \right), \quad \delta^{\text{auth},\kappa} \left(U_t^R, \hat{U}_{B,t} \right), \quad \delta^{\text{auth},\kappa} \left(\hat{U}_{A,t}, \hat{U}_{B,t} \right). \quad (110)$$

Equation 110 permits cases in which A takes B's conduct to be consent while B understands the same conduct as reluctant compliance, or B believes a refusal was explicit while A reconstructs it as a request for clarification. The formalism records these possibilities without deciding the correct interpretation by notation. Evidence, procedure, language, history, and later clarification determine how an application adjudicates the competing reconstructions.

Silence deserves particular care. Silence can be a manifestation, yet its relational meaning depends on the procedure. A practice can assign silence a defined opt-in or opt-out status, while another practice can treat silence as nonresponse. A dependency relation can make silence practically safer than refusal. A language barrier can make silence epistemically ambiguous. Paper I therefore does not assign silence a universal authorization meaning.

The same caution applies to signatures, checkboxes, and interface clicks. Their formal visibility can exceed their relational determinacy. A signed record can supply strong evidence that a manifestation occurred while leaving questions about information, scope, dependency, or reconstruction open. Manifestation-level explicitness is therefore analytically valuable without being identical to relational validity.

13.3 Scope, Duration, Revocability

Authorization is object- and history-sensitive. A subject can permit one action while refusing another, consent to one procedure stage while withholding consent to later monitoring, or authorize an intervention under conditions that cease to hold after new information appears. Paper I therefore attaches scope and temporal structure to every authorization record for which those distinctions matter.

For an authorization-status record u , let

$$\text{Scope}^\kappa(u) \subseteq \mathfrak{D}_{\text{auth}}^\kappa \quad (111)$$

be the set or structured family of objects covered by that record, where $\mathfrak{D}_{\text{auth}}^\kappa$ can contain actions, procedure segments, disclosures, monitoring practices, role transitions, or other locally defined objects. Equation 111 deliberately does not make scope a numerical magnitude.

An executed action α' inherits the authorization status of a proposed action α only when the application provides a declared relation connecting α' to the authorized scope. A coarse material description can treat two actions as equivalent while a procedure-sensitive description distinguishes them because timing, publicity, duration, or data use changed. Authorization transfer therefore depends on the action-equivalence criteria developed in Section 10.6.

Temporal operativeness can likewise be represented without assuming one universal clock. Let I be the historical index set retained by the application. Define the operative domain of u in Equation 112. The term operative is descriptive of the modeled procedure and deliberately avoids treating temporal inclusion as proof of morally or legally valid consent.

$$\text{Oper}^\kappa(u) \subseteq I. \quad (112)$$

The set $\text{Oper}^\kappa(u)$ records the historical stages at which the model treats u as operative. It can be bounded by a stated duration, procedure condition, role relation, new information, completion criterion, or withdrawal event. The construction permits nonmonotone histories: an authorization can lapse and later be renewed under a new record rather than being treated as one unchanged status extending indefinitely.

Revocability is represented procedurally through the possibility of a withdrawal record rather than by assuming that every authorization has identical withdrawal conditions. Let u_t be an authorization record and $w_{t'}$ a later withdrawal record. A withdrawal-sensitive update relation is stated schematically in Equation 113.

$$\text{Upd}_{t' \leftarrow t}^{\text{withdraw}, \kappa} \subseteq \mathfrak{U}_{AB, t}^\kappa \times \mathfrak{U}_{AB, t'}^\kappa, \quad (113)$$

where the later record can narrow, terminate, or otherwise revise the earlier operative scope. Equation 113 preserves historical provenance: withdrawal changes later authorization without rewriting the earlier relational history as though no prior authorization had existed.

13.4 Consent Capacity, Refusal Capacity, Exit Capacity

The manifestation of consent or refusal can be separated from the conditions under which that manifestation receives stronger normative significance. Paper I does not define a universal threshold for valid consent. It instead constructs a typed assessment record that later ethical, legal, clinical, or institutional applications can populate according to their own standards. Relational-autonomy scholarship provides an important adjacent precedent for treating autonomy and agency as socially conditioned rather than reducible to isolated choice, but the present section remains formally neutral among competing normative theories of autonomy [10].

Let $\mathcal{T}_C^{\text{auth}} := \{\text{communicative, interpretive, deliberative, refusal, exit, revision}\}$ index the capacity dimensions retained for the minimal architecture. For subject i , let the authorization-capacity record be

$$\mathfrak{C}_{i, t}^{\text{auth}, \kappa} := \left\{ C_{i, t}^{\alpha, \kappa} \right\}_{\alpha \in \mathcal{T}_C^{\text{auth}}}. \quad (114)$$

Equation 114 does not require the components to be scalar or commensurable. Communicative capacity concerns the ability to manifest a position through the available interface. Interpretive capacity concerns the ability to understand the retained object, consequences, or procedure at the resolution relevant to the decision. Deliberative capacity concerns the ability to form or revise a choice under the adopted model. Refusal and exit capacities concern whether rejection or departure can be practically enacted. Revision capacity concerns whether later feedback can reopen an earlier position.

Information and dependency require separate records because they are relational conditions rather than capacities stored inside one subject. Let $\mathcal{I}_{i,t}^\kappa(x)$ denote the information record available to i concerning object x , and let $\mathcal{D}_{AB,t}^{\text{dep},\kappa}$ denote the retained dependency/pressure structure relevant to authorization. The assessment basis for an authorization status can therefore include

$$\mathfrak{B}_{i,t}^{\text{auth},\kappa}(x) := \left(\mathcal{C}_{i,t}^{\text{auth},\kappa}, \mathcal{I}_{i,t}^\kappa(x), \mathcal{D}_{AB,t}^{\text{dep},\kappa}, \Gamma_{AB,\leq t}^\kappa \right). \quad (115)$$

Equation 115 is an analytical input record. It does not itself declare consent valid or invalid. A domain-specific theory can later define which components are necessary, which thresholds matter, which forms of dependency defeat or merely qualify authorization, and how uncertainty should be handled.

The distinction between formal permission to refuse and effective refusal capacity is especially important. A procedure can state that B may refuse while practical dependence, communication barriers, fear of relational loss, or lack of alternatives makes refusal difficult to enact. Conversely, B can retain substantial practical refusal capacity even in a setting without an explicit formal refusal protocol. Paper I therefore keeps normative permission, manifestation of refusal, and effective refusal capacity as separate objects.

Exit capacity is similarly broader than the physical possibility of leaving. Exiting a support relation can require alternative resources, informational access, social support, institutional substitution, or the ability to bear transition costs. A later political-economy analysis can examine those structures in depth. The present section records exit as a typed capacity because its availability changes the relational significance of continued participation.

13.5 Refusal, Withdrawal, Continuation Revision

Refusal is generative because it changes the history that remains available. The procedure architecture of Section 11 already permits a realized element to alter $\text{Cont}_{\text{proc}}^\kappa(h)$. A refusal record r can remove one proposed action, introduce a demand for clarification, redirect support toward another form, or terminate the current procedure. Withdrawal can similarly revise an authorization that previously made an action available.

A refusal-sensitive procedure should therefore specify an update rule on continuation families rather than equate refusal with a missing positive signal. Let h be the procedure prefix before refusal and $h \cdot r$ the prefix extended by a recognized refusal. The update is represented schematically in Equation 116.

$$\mathcal{U}_{r,h}^{\text{ref},\kappa} : \text{Cont}_{\text{proc}}^\kappa(h) \rightarrow \text{Cont}_{\text{proc}}^\kappa(h \cdot r). \quad (116)$$

Equation 116 is partial because the model can leave unresolved how a refusal transforms every continuation. It also avoids assuming simple set contraction. A refusal can make the originally proposed action unavailable while opening new alternatives, such as clarification, delay, referral, self-directed support, or a different role configuration.

A recognized refusal does not require A to agree with B's evaluative reasons. A can retain a judgment that refusal is harmful, mistaken, or based on incomplete information while the model still records the refusal as a relational event. This separation is essential for later analysis of paternalism. Higher-order disagreement does not erase the first-order authorization status.

Refusal can itself be misrecognized. B can intend a refusal while A interprets the manifestation as hesitation. A can understand a refusal accurately while later institutional manifestation records only "service not completed." Repeated misrecognition can change trust, disclosure, and future refusal capacity.

The history of refusal therefore belongs to the dyadic relational trajectory rather than to one isolated decision point.

Withdrawal extends the same logic across time. A subject can withdraw consent to continued monitoring, data use, contact, accompaniment, or another continuing procedure while preserving authorization for a narrower residual relation. The formalism therefore supports partial withdrawal and scope revision rather than requiring an all-or-nothing termination of the dyad.

13.6 Relational Standing

Standing concerns who is recognized within the modeled relation as eligible to initiate, request, authorize, refuse, contest, interpret, or decide a particular class of objects. It is a relational position rather than a scalar quantity. A can have standing to offer assistance without standing to decide B's values. B can have standing to refuse an intervention affecting B while lacking standing to authorize A's use of a third party's resources. A professional or institutional role can confer standing within one procedure without creating general authority over adjacent domains.

Let $\mathcal{T}_{\text{stand}}$ denote a locally declared family of standing types, such as request, offer, authorize, refuse, contest, or decide. A standing relation is represented in Equation 117.

$$\text{Stand}_t^\kappa \subseteq \{A, B\} \times \mathcal{T}_{\text{stand}} \times \mathfrak{D}_{\text{auth}}^\kappa. \quad (117)$$

A triple $(i, s, x) \in \text{Stand}_t^\kappa$ records that subject position i is treated by the adopted model as having standing of type s concerning object x at historical stage t . Equation 117 is descriptive of the modeled relational order. Whether that standing is morally or legally justified remains a separate question.

Standing can be generated through several histories: direct participation, prior agreement, role assignment, expertise, institutional recognition, ownership of a resource, responsibility for a dependent process, or another relation specified by the application. Different sources can coexist and conflict. A model should therefore retain standing provenance where later contestation or authority depends on how the standing was formed.

Standing also has manifestation and epistemic dimensions. An institutional title can manifest standing without exhausting it. A can possess standing that B does not recognize, or B can attribute standing to A that the formal procedure does not grant. Such misalignment can shape compliance, deference, refusal, and later trust even before a dispute becomes explicit.

13.7 Intervention Authority

Authority is stronger than capability and more specific than general standing. Paper I uses the term for a modeled relational status in which a subject is treated, under a declared scheme and basis, as authorized to initiate, decide, or execute a specified action affecting another subject or the coupled relation. The definition is deliberately model-relative so that the formalism can represent consent-based authority, role-based authority, prior commitments, institutional authorization, emergency conditions, or other locally specified bases without declaring them equally legitimate.

Let $\mathfrak{A}_{AB,t}^\kappa$ be the action-record family available at stage t and let $\mathfrak{B}_{\text{authority},t}^\kappa$ be the retained class of authority bases. Define the authority relation in Equation 118.

$$\text{Authority}_{A \rightsquigarrow B,t}^\kappa \subseteq \mathfrak{A}_{AB,t}^\kappa \times \mathfrak{B}_{\text{authority},t}^\kappa. \quad (118)$$

If $(\mathfrak{a}, b) \in \text{Authority}_{A \rightsquigarrow B,t}^\kappa$, the adopted model records basis b for treating A as authorized with respect to action $\mathfrak{a}$ at that stage. The basis can include one or more authorization records, relevant standing, role relations, prior commitments, competence requirements, institutional rules, emergency conditions, or another declared source. Equation 118 does not establish moral legitimacy merely by membership. For concise proposition-like statements, define $\text{Auth}_{A \rightsquigarrow B,t}^\kappa(\mathfrak{a})$ to mean that at least one retained basis b satisfies $(\mathfrak{a}, b) \in \text{Authority}_{A \rightsquigarrow B,t}^\kappa$. Let $\text{ExecCap}_{A,t}^\kappa(\mathfrak{a})$ denote the proposition that A has the retained practical capacity to execute $\mathfrak{a}$.

Several non-equivalences are essential. Execution capacity does not generate authority, and authority does not guarantee execution competence. Recognition by B can support practical authority without proving legitimacy. Institutional authorization can exist while B contests it. Conversely, B's request can supply an important basis without authorizing every action that A regards as responsive to the request. These separations are summarized in Equation 119.

$$\text{ExecCap}_{A,t}^{\kappa}(\alpha) \not\Rightarrow \text{Auth}_{A \rightsquigarrow B,t}^{\kappa}(\alpha), \quad \text{Auth}_{A \rightsquigarrow B,t}^{\kappa}(\alpha) \not\Rightarrow \text{ExecCap}_{A,t}^{\kappa}(\alpha). \quad (119)$$

Equation 119 complements Equation 104. Capability, positive evaluation, standing, and authority occupy different formal roles. A later ethical theory can ask when they should align and what follows when they do not.

Authority is also scope-sensitive. Authority to provide transport does not automatically include authority to disclose B's information. Authority to coordinate one procedure does not automatically include authority to redefine B's need. A role can authorize monitoring during one interval and cease when the role dissolves. The same standing record can therefore support several action-specific authority relations rather than one global authority attribute attached to A.

13.8 Authorization Boundary Conditions

Several cases place pressure on a consent-centered architecture without justifying the opposite move of treating consent as dispensable. Emergency conditions, temporarily limited communicative capacity, prior directives, strong dependency, conflicting standing claims, and rapidly changing risk can produce support situations in which current explicit consent is absent, delayed, ambiguous, or contested. Paper I represents these as boundary conditions requiring additional basis records rather than as exceptions that erase the authorization problem.

An emergency record, for example, can enter $\mathfrak{B}_{\text{authority},t}^{\kappa}$ as a model-defined condition that changes the procedure's continuation family and the reasons available to A. The existence of that record does not by itself prove that intervention is justified. The later ethics paper must specify the threshold, proportionality, reversibility, competence, and review conditions appropriate to such intervention. Paper I needs only enough structure to represent the case without pretending that ordinary consent is present.

Prior commitments can similarly persist into later stages while their interpretation becomes contested. A previous request, promise, standing agreement, or advance instruction can contribute to later authority basis even when the present manifestation is unavailable. The model must then track provenance, scope, historical continuity, and any later evidence of revision. Treating the earlier record as timeless would discard precisely the history that makes the question difficult.

Dependency can alter the relational significance of apparent authorization without automatically invalidating it. A may control a resource B urgently needs; B may fear losing a relationship; A may rely on B's continued participation for identity, status, or project success. These dependencies can affect refusal and exit capacities, manifestation strategy, and the interpretation of compliance. Paper I records the dependency structure as part of the authorization-assessment basis and leaves its normative diagnosis to later work.

The architecture also permits disagreement over authority itself. B can deny that A has standing to decide, while A's institution recognizes such standing. A can believe that emergency conditions authorize intervention, while B rejects the classification of the situation as an emergency. The disagreement is not a defect in the formalism. It is a relational object that later justice, ethics, and power analyses need to explain and evaluate.

13.9 Authorization Trajectories

Authorization status changes through history. New information can revise consent, a role can expire, refusal can become easier after alternative support appears, trust can make clarification less costly, an

Table 16: Three-level and historical audit for selected authorization concepts.

Concept	Relational-reality object	Manifestation examples	Historical questions
Consent	Scope-sensitive authorization relation supported by declared conditions	Verbal agreement, signature, gesture, interface selection, continued participation	Was scope revised, withdrawn, renewed, or reinterpreted? Did capacity or information conditions change?
Refusal	Relational rejection or boundary-setting event concerning a defined object	“No,” withdrawal, nonparticipation, blocking gesture, written refusal	Was refusal recognized? Did it alter later continuations? Did repeated treatment change refusal capacity?
Standing	Relational eligibility to initiate, authorize, contest, or decide within a specified domain	Role title, recognized responsibility, procedure position, customary participation	How was standing generated, recognized, contested, transferred, or dissolved?
Authority	Action-specific authorization status supported by a declared basis	Directive, approved intervention, institutional act, accepted role exercise	Did the basis remain in force? Did action scope change? Did later contestation alter effective authority?

earlier misunderstanding can be repaired, or repeated disregard of refusal can reduce later willingness to disclose. Authorization therefore belongs to a trajectory rather than a single isolated decision.

A generic authorization-revision relation is stated in Equation 120.

$$\text{Rev}_{t' \leftarrow t}^{\text{auth}, \kappa} \subseteq \mathfrak{U}_{AB, t}^{\kappa} \times \mathfrak{U}_{AB, t'}^{\kappa}. \quad (120)$$

Equation 120 can represent renewal, withdrawal, narrowing, expansion, reinterpretation, or replacement of an earlier status. The relation is partial because some earlier records remain historically relevant without having one well-defined successor. It is nonunique because A and B can retain different reconstructions of what changed.

The trajectory is also cross-level. A relational consent status can change while the original signed manifestation remains physically unchanged. A later verbal withdrawal can modify relational authorization while an institutional interface continues to display the earlier permission. Conversely, a manifestation can change while the relational status remains stable, such as when a new accessibility format expresses the same retained permission. The history of authorization therefore includes the history of mappings between relational reality and manifestation reality.

Repeated authorization practice can become generative. A dyad that repeatedly clarifies scope and recognizes refusal can develop a shared vocabulary that makes later negotiation easier. A dyad that repeatedly treats compliance as consent can stabilize a contrary interpretive pattern. These practices can become relationally real when they persist and shape later continuation. Paper I records the possibility without treating persistence as normative validation.

Table 16 summarizes the three-level and historical audit for the central authorization concepts.

13.10 Typed-Asymmetry Interface

The authorization architecture prepares the next section on typed relational asymmetry. Consent, refusal, standing, and authority can each be distributed asymmetrically without those asymmetries sharing one meaning. A can possess greater informational access while B possesses stronger refusal standing. A can control resources while B retains formal authority over one decision. A can hold institutional standing while lacking interpretive access to B’s experience. A relation can therefore contain intersecting asymmetries whose consequences differ by dimension.

Three distinctions will matter immediately. First, authorization asymmetry is not identical to power. Unequal standing or authority can be temporary, reciprocal across domains, or limited to a narrow procedure without producing persistent trajectory-shaping effects. Second, asymmetry is not identical to injustice. Some role differentiation can support care, expertise, responsibility, or coordination. Third, formal symmetry does not establish effective symmetry. A and B can possess nominally equal rights of refusal while differing substantially in communicative, exit, or resource capacity.

The dyadic formalism must therefore type the relevant dimension before making a stronger claim. The next section will classify epistemic, informational, resource, competence, recognition, interpretive, dependency, communicative, refusal-capacity, exit-capacity, authority, generative-capacity, and normative-influence asymmetries while preserving the distinctions established here.

Section 13 has thus separated authorization statuses, manifestation, reconstruction, scope, temporal operativeness, capacity conditions, refusal, withdrawal, standing, authority, and historical revision. The result is a formal interface for later normative analysis: Paper I can represent when and how an intervention becomes requested, permitted, refused, or authority-supported without making those statuses substitutes for moral legitimacy, relational justice, or power analysis.

14 Typed Relational Asymmetry

This section develops the asymmetry layer of the minimal dyadic volunteer formalism. Its role is to classify relational differences that can become directionally consequential without compressing them into one scalar hierarchy or reading them immediately as power, injustice, domination, or moral failure. The objective is to specify when an A–B comparison is meaningful, when the comparison remains undefined, which relational dimension is being compared, and how asymmetry can be represented across epistemic, material, interpretive, authorization-related, and generative domains. The discussion proceeds through comparison frames, typed difference, directional consequence, an asymmetry taxonomy, three-level realization, historical trajectories, multidimensional composition, and interfaces with later power and normative analysis. The method remains formal and reconstructive. Paper I supplies a typed language for asymmetry; later papers will determine which asymmetries are legitimate, unjust, power-relevant, or historically self-reinforcing in particular domains.

Section 13 already showed why equal-looking procedural forms can coexist with unequal refusal capacity, exit capacity, access, standing, or authority. Earlier epistemic sections likewise distinguished manifestation access, observation, reconstruction, confidence, and theory-of-mind structure. These differences should not be aggregated before the formalism identifies what kind of relation differs and what consequence follows from the difference. Figure 9 summarizes the analytical sequence used in this section.

14.1 Typed Comparison Frames

A relational asymmetry claim is meaningful only relative to a declared comparison dimension. Let $\mathcal{I}_{\text{asym}}^\kappa$ denote the family of relational dimensions retained at resolution κ . Typical dimensions in the present dyad include epistemic access, information availability, resources, competence, recognition, interpretation, dependency, communication, refusal capacity, exit capacity, authority, generative capacity, and normative influence. The list is open because different volunteer settings can require additional dimensions.

For each $\alpha \in \mathcal{I}_{\text{asym}}^\kappa$ and historical stage t , let $\mathcal{P}_{\alpha,t}^\kappa$ denote the retained collection of positions or position-indexed records relevant to comparison in dimension α . Let $\mathcal{D}_{\alpha,t}^\kappa \subseteq \mathcal{P}_{\alpha,t}^\kappa \times \mathcal{P}_{\alpha,t}^\kappa$ be the domain on which the comparison is defined. The local comparison frame is stated in Equation 121.

$$\mathfrak{F}_{\alpha,t}^\kappa := (\mathcal{P}_{\alpha,t}^\kappa, \mathcal{D}_{\alpha,t}^\kappa, \simeq_{\alpha,t}^\kappa, \chi_{\alpha,t}^\kappa). \quad (121)$$

In Equation 121, $\simeq_{\alpha,t}^\kappa$ is the retained equivalence criterion for the dimension and $\chi_{\alpha,t}^\kappa$ is a directional-consequence criterion introduced below. The frame does not require numerical ordering. It supplies

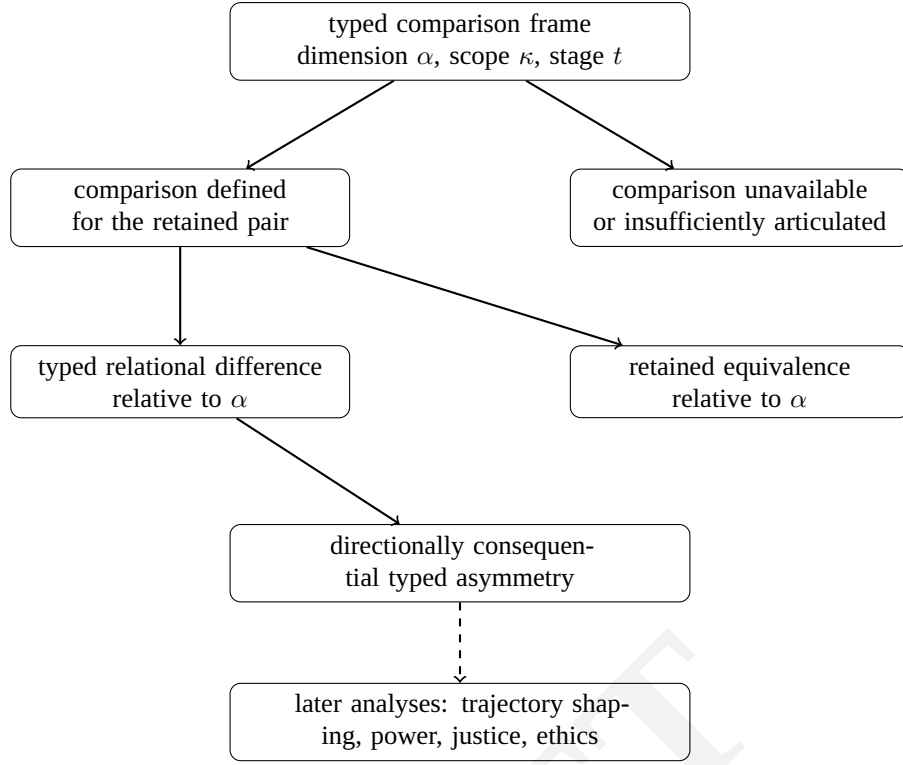


Figure 9: Typed-asymmetry architecture for the minimal dyad. A comparison first requires a declared relational dimension and a domain on which the comparison is meaningful. Defined comparison can yield retained equivalence or typed difference. A difference becomes asymmetry only when the adopted dimension supplies a directional consequence. The dashed arrow marks an interface to later power and normative analyses rather than an automatic entailment.

only the minimum structure needed to ask whether two retained positions can be compared, whether they remain equivalent for the local question, and whether any identified difference is directionally consequential.

Definedness is analytically prior to symmetry or asymmetry. If $(x, y) \notin \mathcal{D}_{\alpha, t}^{\kappa}$, the present model does not classify the pair as symmetric, asymmetric, or equally situated in dimension α . The comparison may be unavailable because the relevant position has not been articulated, a manifestation is inaccessible, the role structure is unresolved, a translation rule is missing, or the two records belong to incomparable domains. This case preserves the partiality discipline established in Sections 3 and 4.

14.2 Relational Difference, Directional Consequence, Asymmetry

A defined comparison can reveal difference without yet establishing asymmetry. For $(x, y) \in \mathcal{D}_{\alpha, t}^{\kappa}$, define the typed relational-difference predicate in Equation 122.

$$\text{Diff}_{\alpha, t}^{\kappa}(x, y) \iff x \not\prec_{\alpha, t}^{\kappa} y. \quad (122)$$

Equation 122 is deliberately nonmetric. It records a failure of retained equivalence and does not assign a numerical distance, sign, magnitude, or ranking. A pair can therefore be different without one position being “more” of the relevant property than the other.

Asymmetry requires an additional directional condition. Let $\chi_{\alpha, t}^{\kappa}(x, y)$ denote the proposition that the relational organization of x and y produces directionally different consequences, capacities, constraints, dependencies, reachability, or later generative possibilities in dimension α . The typed asymmetry predicate is defined in Equation 123.

$$\text{Asym}_{\alpha,t}^{\kappa}(x, y) \iff (x, y) \in \mathcal{D}_{\alpha,t}^{\kappa} \wedge \text{Diff}_{\alpha,t}^{\kappa}(x, y) \wedge \chi_{\alpha,t}^{\kappa}(x, y). \quad (123)$$

Equation 123 separates three questions that are often compressed into one. The comparison must be meaningful, retained equivalence must fail, and the failure must have a directional consequence in the specified dimension. A difference in preferred language, for example, is not automatically an epistemic asymmetry. It becomes asymmetry-relevant only when the difference changes access, interpretive reach, communicative burden, or another declared consequence.

The orientation of an asymmetry is also typed. A relation can favor A in one dimension and B in another. For this reason Paper I does not introduce one primitive order $A \succ B$. Where a domain does justify an order, that order can be added locally to the corresponding comparison frame. The general architecture remains compatible with nonordered, partially ordered, set-valued, or structurally described asymmetries.

14.3 Asymmetry Families in the Minimal Dyad

Table 17 summarizes the principal dimensions retained by the present paper. The entries are not diagnoses of failure. They identify different relational questions that should remain formally distinguishable even when several occur together.

The distinctions in Table 17 prevent several category errors. Informational asymmetry is not identical to epistemic asymmetry because access to information does not guarantee justified interpretation. Competence asymmetry is not identical to authority because ability to perform an action does not determine who may authorize it. Dependency asymmetry is not identical to resource asymmetry because dependence can arise through recognition, care, trust, institutional continuity, or other relations even where material holdings are similar. Generative-capacity asymmetry is broader than immediate influence because it concerns the capacity to shape later formation, while normative influence concerns a more specific effect on reasons, categories, expectations, or norms.

14.4 Epistemic, Informational, Interpretive Asymmetry

The epistemic architecture of Sections 8 and 9 permits several asymmetries that should remain distinct. A can observe a manifestation unavailable to B, B can possess first-person access to an experience that A can only infer, A can possess specialized knowledge relevant to interpretation, and both can hold different confidence structures regarding the same event. The resulting relation can therefore contain intersecting asymmetries rather than one direction of epistemic advantage.

Let $\mathfrak{E}_{i,t}^{\kappa}$ be the epistemic-access record introduced earlier for subject i . For a declared epistemic dimension α_{ep} , the comparison frame $\mathfrak{F}_{\alpha_{\text{ep}},t}^{\kappa}$ may compare selected components of $\mathfrak{E}_{A,t}^{\kappa}$ and $\mathfrak{E}_{B,t}^{\kappa}$ without requiring the entire records to live in one vector space. The comparison can be relation-valued. A model may record that A has documentary access unavailable to B while B has experiential access unavailable to A. Neither fact produces a complete epistemic ranking.

Interpretive asymmetry becomes especially important when one subject's vocabulary, institutional language, or conceptual resources determine the forms in which the other's experience must be expressed. A can possess substantial competence in a professional vocabulary while B possesses richer access to the lived situation. The resulting asymmetry can shift during clarification, translation, learning, or repeated interaction. The formalism therefore treats interpretive asymmetry as historical and revisable rather than as an intrinsic superiority of one subject.

Recognition can mediate these asymmetries. A contribution can be observable and intelligible yet receive little weight because the speaker lacks recognized standing or credibility. Conversely, a highly recognized subject can have limited access to the underlying experience while still shaping how the relation is interpreted. Recognition asymmetry therefore belongs neither wholly to epistemic access nor wholly to authority; it can condition both.

Table 17: Principal typed asymmetry families in the minimal dyadic volunteer relation.

Dimension	Comparison object	Directionally consequential difference can concern
Epistemic	Belief, confidence, evidential access, reconstruction quality	Which subject can form or revise a better-supported model of a retained object
Informational	Availability, visibility, disclosure, record access	Which subject can access information required for later judgment or action
Resource	Material, temporal, infrastructural, financial, organizational resources	Which subject can initiate, sustain, refuse, or exit a procedure because of unequal means
Competence	Skill, expertise, procedural ability, domain-specific knowledge	Which subject can execute or assess a relevant action under the retained criteria
Recognition	Credibility, social acknowledgment, role recognition, legibility	Which subject's claims or manifestations are more readily received as relevant or authoritative
Interpretive	Vocabulary, translation capacity, conceptual access, hermeneutic resources	Which subject can render experience intelligible within the shared or dominant interpretive scheme
Dependency	Reliance on resources, relation continuity, care, access, or support	Which subject bears greater cost if the relation, resource, or procedure is withdrawn
Communicative	Ability to express, clarify, negotiate, question, or contest	Which subject can more effectively shape the manifest record of the interaction
Refusal capacity	Practical ability to reject a proposal without disproportionate loss	Whether formally available refusal has comparable effective availability
Exit capacity	Practical ability to leave the relation or procedure	Whether continued participation is equally revisable for both sides
Authority	Action-specific authorization, standing, decision scope	Which subject is treated as able to authorize, decide, or direct a specified action
Generative capacity	Capacity to create, stabilize, revise, transmit, or reopen relational structures	Which subject can more strongly alter the conditions of later relation formation
Normative influence	Capacity to shape reasons, evaluative categories, expectations, or shared norms	Which subject can more strongly affect what later counts as appropriate, relevant, or justified

14.5 Capacity, Dependency, Authorization Asymmetry

Several asymmetries concern what subjects can practically do within the relation. Resource access, competence, refusal capacity, exit capacity, standing, and authority are related but nonidentical. Their separation is necessary because a support relation can be formally reciprocal in one domain while practically one-sided in another.

Let $\mathfrak{C}_{i,t}^\kappa$ denote a retained family of subject-relative capacities introduced in Section 7. For a capacity type c , the model can define a comparison relation $\preceq_{c,t}^\kappa$ only when the capacity domain supports such an ordering. When no defensible scalar or order exists, the model should retain a structured comparison instead. Equation 124 records this conditional use.

$$c \text{ admits a justified order at } (t, \kappa) \implies \preceq_{c,t}^\kappa \text{ may be added to } \mathfrak{F}_{c,t}^\kappa. \quad (124)$$

Equation 124 prevents the notation from presupposing commensurability. Refusal capacity, for example, can depend on several heterogeneous relations: communication ability, alternatives, resource security, anticipated retaliation, emotional dependence, legal standing, and trust. A scalar index can be useful in an application, yet Paper I does not treat such an index as primitive.

Dependency asymmetry requires the same restraint. Let $\text{Dep}_{i \rightsquigarrow j,t}^\kappa(x)$ denote a typed record that sub-

ject position i depends on j with respect to relational object x . The reverse record can coexist. Mutual dependence is therefore compatible with asymmetry when the objects, costs, or consequences of dependence differ. One subject can depend on material support while the other depends on role continuity, recognition, information, or participation. The later psychoanalytic and political-economy papers will examine these configurations more deeply.

Authorization asymmetry is similarly domain-specific. A may have authority to execute a medical or technical task while B retains stronger standing concerning refusal. B may authorize access to personal information while A controls the institutional channel through which that authorization becomes effective. The asymmetry profile should therefore preserve the action type, scope, and historical basis of authority rather than represent authority as a global attribute.

14.6 Generative Capacity, Normative Influence

Generative-capacity asymmetry concerns unequal capacity to create, stabilize, transmit, revise, or reopen relational structures. The object is broader than immediate control. A subject can have little authority over one decision while possessing substantial capacity to shape the language, routines, expectations, or infrastructures through which later decisions become possible. Another subject can have formal standing while limited capacity to preserve an alternative interpretation after the immediate procedure ends.

Let $\mathfrak{G}_{i,t}^\kappa$ denote the retained generative-capacity record for subject position i . Paper I does not assume that $\mathfrak{G}_{i,t}^\kappa$ is numerical. A typed asymmetry in generative capacity is present only when a declared comparison identifies a directional difference in the capacity to affect later formation. This condition is stated in Equation 125.

$$\text{Asym}_{\text{gen},t}^\kappa(A, B) \iff \text{Asym}_{\alpha_{\text{gen},t}}^\kappa(\mathfrak{G}_{A,t}^\kappa, \mathfrak{G}_{B,t}^\kappa). \quad (125)$$

Equation 125 is intentionally formal. It does not tell the analyst which subject has greater generative capacity, whether the asymmetry is beneficial, or whether it should be reduced. Expertise can produce asymmetry that enlarges the other's later capacities through teaching or translation. The same expertise can become restrictive when it controls the terms through which alternative experience becomes intelligible. Historical trajectory therefore matters to evaluation.

Normative-influence asymmetry is a more specific case. It concerns unequal capacity to shape which reasons, categories, expectations, distinctions, or evaluative vocabularies become stabilized in the dyad. A repeated support relation can make one subject's interpretation of need easier to articulate and the other's more costly to express. A professional vocabulary can enable precise coordination while simultaneously setting the terms through which disagreement must be translated. Paper I records the asymmetry; the later justice and ethics papers will ask whether the resulting formation remains contestable, revisable, and generative of the other subject's capacities.

14.7 Three-Level Asymmetry Profiles

Typed asymmetry participates in the three-level architecture of Section 5. The ultimate ontological status of an asymmetry remains suspended. At relational reality, the model can retain a historically efficacious difference in access, dependence, authority, recognition, or another typed dimension. At manifestation reality, that asymmetry can appear through titles, turn-taking, spatial arrangement, access credentials, response latency, resource control, institutional procedure, bodily deference, silence, or other contextual forms. A and B can then reconstruct the asymmetry differently.

For a typed asymmetry object $\text{Asym}_{\alpha,t}^\kappa$, define the subject-relative three-level profile in Equation 126.

$$\Lambda_t^\kappa(\text{Asym}_\alpha) := \left(\text{O}_{\text{Asym}_\alpha}^\kappa ; \text{Asym}_{\alpha,t}^{R,\kappa}, \text{Man}_t^\kappa(\text{Asym}_\alpha), \widehat{\text{Asym}}_{\alpha,t}^{A,\kappa}, \widehat{\text{Asym}}_{\alpha,t}^{B,\kappa} \right). \quad (126)$$

Equation 126 uses a leading ontological-status marker, separated by the semicolon, while the relational asymmetry, its manifestations, and subject-relative reconstructions remain the modeled entries of the profile. The relational asymmetry, its manifestations, and the two subject-relative reconstructions

can all diverge. A resource asymmetry can be manifest and mutually acknowledged. An interpretive asymmetry can be relationally consequential while poorly visible. A formal authority symmetry can coexist with a manifested pattern of deference that produces practical asymmetry. Manifestation can therefore reveal, obscure, exaggerate, or partially reverse the appearance of a relational asymmetry.

The mapping can also change over time. A relation once manifested through explicit instruction can later be reproduced through habitual anticipation. A refusal-capacity asymmetry can become less visible after B stops attempting refusal. A recognition asymmetry can weaken as repeated interaction produces shared vocabulary. The history of manifestation is therefore part of the asymmetry trajectory rather than merely evidence attached after the fact.

14.8 Asymmetry Trajectories

A static comparison captures only one stage of a relational process. Volunteer relations can generate, amplify, attenuate, reverse, or relocate asymmetry. Repeated support can increase B's practical capacity and reduce resource dependence. It can also increase role dependence, interpretive deference, or institutional reliance. A's expertise can initially create strong competence asymmetry and later reduce that asymmetry through teaching. Conversely, repeated delegation can increase asymmetry if one subject becomes progressively less able to act without the other.

Let $\mathfrak{A}_{\alpha,t}^{\kappa}$ denote the retained asymmetry record in dimension α at stage t . A generic historical relation between asymmetry records is stated in Equation 127.

$$\text{Rev}_{\alpha,t' \leftarrow t}^{\text{asym},\kappa} \subseteq \mathfrak{A}_{\alpha,t}^{\kappa} \times \mathfrak{A}_{\alpha,t'}^{\kappa}. \quad (127)$$

Equation 127 can represent persistence, attenuation, amplification, reversal, disappearance, or retyping. The relation is not assumed to be monotone. An asymmetry can narrow in one dimension while expanding in another, and the same event can have opposite effects on different dimensions.

Historical provenance is therefore part of the asymmetry record. Two dyads can display similar present authority distributions while having different histories of request, teaching, coercion, emergency response, repeated coordination, or mutual delegation. Those histories can affect trust, contestability, revision cost, and later generative capacity even when the current snapshot appears similar. Paper I keeps provenance available without treating origin as a complete normative verdict.

14.9 Multidimensional Asymmetry Profiles

The dyad normally contains several asymmetry dimensions simultaneously. Paper I represents the combined structure as an indexed family rather than a vector whose components are assumed commensurable. The multidimensional asymmetry profile is defined in Equation 128.

$$\mathcal{A}_{AB,t}^{\kappa} := \{ \mathfrak{A}_{\alpha,t}^{\kappa} \mid \alpha \in \mathcal{I}_{\text{asym}}^{\kappa} \}. \quad (128)$$

Equation 128 is a family notation rather than an arithmetic vector. No addition, norm, inner product, or total ordering is implied. Those structures may be introduced in a later application only when the retained dimensions admit them and when the resulting comparison has a substantive interpretation.

Dimensions can interact without being reducible to one another. Resource asymmetry can reduce exit capacity. Recognition asymmetry can amplify interpretive influence. Competence asymmetry can be associated with temporary role differentiation while simultaneously creating dependence. Strong refusal standing can counterbalance resource dependence in one procedure without eliminating the dependency itself. These interactions can be represented through typed maps among asymmetry records when an application supplies the mechanism.

The possibility of cross-dimensional interaction also prevents simple cancellation language. Greater authority for B does not numerically cancel greater resources for A unless a model establishes a common scale and a rule of aggregation. A relation can remain deeply asymmetric even when different dimensions point in opposite directions. The profile preserves that structure for later analysis.

14.10 Difference, Asymmetry, Power, Injustice

The formal architecture requires a strict separation among relational difference, typed asymmetry, power relevance, and normative diagnosis. The first two are defined in Equations 122 and 123. Power requires an additional account of persistent trajectory shaping, while injustice requires independent evaluative criteria concerning legitimacy, distribution, recognition, procedure, responsibility, contestability, or another normative domain.

The non-entailments are summarized in Equation 129.

$$\text{Diff}_\alpha \not\Rightarrow \text{Asym}_\alpha, \quad \text{Asym}_\alpha \not\Rightarrow \text{Power}, \quad \text{Asym}_\alpha \not\Rightarrow \text{Injustice}. \quad (129)$$

Equation 129 does not deny that the relations can be connected. It records the additional burden required for each stronger claim. A competence asymmetry can support effective care without generating domination. A resource asymmetry can become power-relevant when it persistently reshapes B's later possibilities. A formally equal arrangement can still be unjust if its history or effective capacities systematically exclude one subject. The later papers on justice, ethics, and political economy will add the premises needed for those evaluations.

Authority deserves separate emphasis. Authority is itself one typed relational structure and can be distributed asymmetrically, yet authority does not exhaust power. A may possess narrow procedural authority while B controls resources that determine whether the procedure can continue. A may possess recognized expertise while B's refusal remains decisive. Power analysis must therefore examine how several asymmetries compose historically rather than infer power from one role label.

14.11 Local-Algebraic Interface

Typed asymmetry is not itself an operator. It is a relational structure that can condition which operators are admissible, how local transformations are interpreted, which continuation families remain available, or how different subject-relative records are compared. The next section will develop the local algebraic architecture required to represent such dynamics.

For a retained region $R \subseteq \mathcal{K}_{AB,t}^\kappa$, let $\mathcal{A}_t^\kappa(R)$ denote the history- and resolution-indexed local algebraic carrier developed in Section 15. A typed asymmetry record can enter the specification of an admissible transformation family without becoming an element of the algebra by default. The conditional family is written schematically in Equation 130.

$$\text{Adm}_t^\kappa(R \mid \mathcal{A}_{AB,t}^\kappa) \subseteq \text{Part}(\mathcal{A}_t^\kappa(R), \mathcal{A}_{t'}^{\kappa'}(R')), \quad (130)$$

where $\text{Part}(X, Y)$ denotes the retained class of partial maps from X to Y . Equation 130 states only that the admissible dynamical family may depend on the current asymmetry profile. It does not identify asymmetry with an operator, field value, generator, or scalar parameter. A later specialization can make that dependence explicit when the substantive mechanism warrants it.

Section 14 has therefore supplied a typed comparison architecture for the minimal dyad. The resulting framework preserves undefined comparison, nonmetric difference, directional asymmetry, multidimensional profiles, three-level manifestation, historical revision, and strict separation from power and injustice. The next section will use these distinctions to construct the local algebraic dynamics through which observation, recognition, communication, support, feedback, and revision can act on the history-bearing dyadic configuration.

15 Local Algebraic Dynamics

This section develops the local algebraic layer of the dyadic volunteer formalism. Its role is to give the substantive objects constructed in Sections 6–14 a compositional mathematical realization without identifying subjects, actions, judgments, or asymmetries with algebraic elements by stipulation. The objective is to specify where local algebraic carriers are assigned, how relational transformations admit

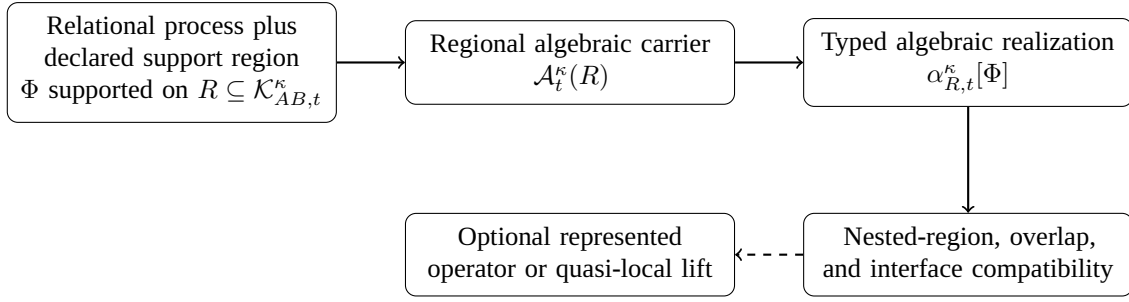


Figure 10: Local algebraic realization of a dyadic GR transformation. Each arrow is a modeling step rather than an ontological identity. Algebraic and represented-operator levels are admitted only after the substantive relational process, support region, and compatibility conditions are specified.

algebraic lifts, which operation families represent readout, judgment-conditioned action, support, refusal, feedback, and revision, and which compatibility conditions are required before local maps can be assembled across a coupled history-bearing spacetime. The discussion proceeds from regional carriers to typed dynamical lifts, epistemic and judgment-conditioned operation families, nested-region and overlap coherence, algebraic-state and represented-operator specializations, open-system maps, ordered composition, and quasi-local assembly. The method is algebra-first and representation-conscious: stronger structures are introduced only after their domains, codomains, and substantive interpretations have been declared.

The mathematical background in Section 3 already separated a local algebra from a global state space, an algebraic state from a history-bearing GR subject, and sequence sensitivity from a commutator. Section 14.11 then supplied a direct interface from the typed asymmetry record $\mathcal{A}_{AB,t}^{\kappa}$ to an admissible family of partial transformations. The present section makes that interface explicit. Figure 10 summarizes the resulting representation ladder. A substantive relational process is identified first; only then is an algebraic carrier selected and a well-typed local dynamical map introduced. Hilbert-space operators and field-like structures remain later or optional realizations.

15.1 Regional Algebraic Carriers

Let $\mathfrak{R}(\mathcal{K}_{AB,t}^{\kappa})$ denote the declared family of admissible regions of the coupled subject-spacetime at historical stage t and resolution κ . A region can lie mainly within the retained representation of A, mainly within B, on the interaction interface $\mathcal{J}_{AB}^{\kappa}$ introduced in Section 6, or across a shared or overlapping subconfiguration. The formalism does not require every region to have the same substantive role.

Let $\mathbf{Alg}_t^{\kappa}$ denote the class of algebraic carriers admitted by the selected local realization. A local algebraic assignment is the typed map

$$\mathbf{A}_t^{\kappa} : \mathfrak{R}(\mathcal{K}_{AB,t}^{\kappa}) \longrightarrow \mathbf{Alg}_t^{\kappa}, \quad R \longmapsto \mathcal{A}_t^{\kappa}(R). \quad (131)$$

Equation 131 is an assignment of mathematical structure. It does not state that the region itself is an algebra. The elements of $\mathcal{A}_t^{\kappa}(R)$ must be declared by the local model. Depending on the question, they can encode retained observables, evaluative quantities, constraints, generators, symbolic records, admissible local transformations, or another class of objects whose product and involution have an explicit interpretation.

The same subject-spacetime can support several algebraic assignments for different analytical tasks. For example, a communication model can use an algebra generated by message or signaling operations, while a later open-system specialization can use a C^* -algebra of bounded observables. These are distinct representations unless an explicit morphism relates them. Paper I therefore avoids a universal symbol such as “the algebra of A” or “the algebra of B.” The local carrier is indexed by region, history, resolution, and retained analytical purpose.

When $R_1 \subseteq R_2$ and the model supplies compatible inclusion, the isotony-type embedding introduced

in Section 3.8 becomes history- and resolution-indexed as stated in Equation 132.

$$R_1 \subseteq R_2 \implies \iota_{R_1 R_2}^{t, \kappa} : \mathcal{A}_t^\kappa(R_1) \hookrightarrow \mathcal{A}_t^\kappa(R_2). \quad (132)$$

Equation 132 is an assumption of the selected algebraic realization, not a general property of every GR region system. Context-dependent descriptions can require quotient maps, partial embeddings, contextual translation, or a more general categorical diagram. The stronger embedding form is retained where it is useful and warranted.

A further restraint concerns factorization. Paper I makes no primitive assumption that a coupled-region algebra factorizes as $\mathcal{A}_t^\kappa(R_A) \otimes \mathcal{A}_t^\kappa(R_B)$. Shared histories, overlapping subject-spacetime regions, institutional mediation, and relation-specific objects can make such a tensor factorization inappropriate. A tensor-product specialization remains available where the substantive model supplies sufficiently independent component algebras and a justified coupling construction.

15.2 Dynamical Lift of Relational Transformations

Sections 10 and 11 treated an action or procedure element first as a relationally identified transformation of the coupled GR configuration. The algebraic layer should preserve that order of dependence. Let $\mathfrak{R}_{AB,t}^\kappa$ and $\mathfrak{R}_{AB,t'}^{\kappa'}$ denote the admitted classes of coupled dyadic configurations at the source and target stages. A generic retained relational transformation has the typed form

$$\Phi_{t' \leftarrow t}^{\kappa' \leftarrow \kappa} : \mathcal{D}_{\Phi}^{\kappa' \leftarrow \kappa} \subseteq \mathfrak{R}_{AB,t}^\kappa \rightarrow \mathfrak{R}_{AB,t'}^{\kappa'}. \quad (133)$$

Equation 133 gives the retained relational transformation or transformation schema at the configuration level and is consistent with the action-specific specialization in Equation 78. Suppose $R \subseteq \mathcal{K}_{AB,t}^\kappa$ is a support region for the part of Φ represented algebraically and $R' \subseteq \mathcal{K}_{AB,t'}^{\kappa'}$ is its tracked target region. An algebraic lift is then a typed map

$$\alpha_{R,t}^\kappa[\Phi] : \mathcal{D}_{\Phi,R}^{\text{alg}} \subseteq \mathcal{A}_t^\kappa(R) \rightarrow \mathcal{A}_{t'}^{\kappa'}(R'). \quad (134)$$

Equation 134 distinguishes the substantive transformation Φ from its algebraic realization $\alpha_{R,t}^\kappa[\Phi]$. The latter is not defined merely by assigning a symbol. A model must specify which algebraic elements represent the retained local content, how the relational transformation acts on that content, and which features of the action are omitted by the lift.

The lift need not be injective, surjective, linear, invertible, multiplicative, or norm preserving. These properties belong to stronger specializations. The minimal requirement is typing plus a declared correspondence between the relational action and the algebraic effect. Where several algebraic lifts reproduce the same retained relational record, the representation is non-unique and should be treated accordingly.

This distinction is especially important when the history-bearing carrier changes. If $\mathcal{K}_{AB,t}^\kappa$ and $\mathcal{K}_{AB,t'}^{\kappa'}$ are represented by distinct complexes or by different region systems, then the algebraic lift naturally connects distinct local algebras. Writing the map as an endomorphism of one fixed $\mathcal{A}$ would suppress the structural change. The typed form in Equation 134 therefore remains the default.

15.3 Readout, Manifestation, Epistemic Operations

Observation and manifestation require algebraic structure of a different kind from state-changing support action. Section 8 distinguished accessibility, uptake, reconstruction, and belief formation before any local algebra was imposed. The present subsection gives those operations a mathematical interface without identifying them with intervention.

Let $\mathcal{Y}_{i,t}^\kappa(R)$ denote the retained output carrier for manifestations or readout records available to subject $i \in \{A, B\}$ in region R . A local readout map has the type

$$\text{Read}_{i,t,R}^\kappa : \mathcal{D}_{i,t,R}^{\text{read}} \subseteq \mathcal{A}_t^\kappa(R) \rightarrow \mathcal{Y}_{i,t}^\kappa(R). \quad (135)$$

Equation 135 produces a subject-relative readout or manifestation record. It does not by itself alter the source algebraic object. If observation in a particular model is interventionist, the intervention must be represented separately through a dynamical map and composed with the readout operation.

The distinction can be stated through a product-type observation instrument when both pieces are required. Let $\alpha_{i,t,R}^{\text{obs}}$ denote the algebraic effect of an observation procedure on the represented local configuration, when such an effect is defined. A joint instrument can then be written as

$$\text{Instr}_{i,t,R} := (\text{Read}_{i,t,R}^{\kappa}, \alpha_{i,t,R}^{\text{obs}}). \quad (136)$$

Equation 136 preserves the difference between what is recorded and what is changed by the procedure. This distinction prevents an epistemic map from being silently treated as a causal support operator.

A later subject-relative reconstruction can use the readout record together with historical and interpretive context, as developed in Sections 8 and 9. The algebraic layer does not replace those epistemic objects. It supplies typed local inputs from which such objects may be constructed.

15.4 Judgment-Conditioned Operator Generation

Section 12 distinguished need, value, aesthetic, and normative judgments from the actions that may follow them. The local algebraic architecture should preserve this distinction. Let $\mathfrak{J}_{i,t}^{\kappa}$ denote a retained family of evaluative or normative judgments available to subject i at stage t , and let $\mathfrak{Q}_{i,t}^{\kappa}$ denote the relevant contextual record, including applicable procedure, authorization status, ToM record, and typed asymmetry information. A judgment-conditioned generation relation can be written as

$$\text{GenOp}_{i,t}^{\kappa} : \mathfrak{J}_{i,t}^{\kappa} \times \mathfrak{Q}_{i,t}^{\kappa} \rightarrow \text{Adm}_{i,t}^{\kappa}, \quad (137)$$

where $\text{Adm}_{i,t}^{\kappa}$ is the currently admissible family of typed local transformations. Equation 137 does not claim that every judgment produces an action. It represents one route through which a judgment can modify the action family available to the subject.

The distinction among judgment, admissibility, selection, and execution is important. A positive value judgment can make one intervention salient while authority remains absent. A normative judgment can classify an action as required while the subject lacks the competence or resources to perform it. A refusal by B can remove a previously admissible support operation from A's retained continuation family. These cases are represented by changes in $\text{Adm}_{i,t}^{\kappa}$ rather than by treating a judgment as an executed operator.

When an application supplies a selection rule, one can add a partial map

$$\text{Sel}_{i,t}^{\kappa} : \text{Adm}_{i,t}^{\kappa} \times \mathfrak{Q}_{i,t}^{\kappa} \rightarrow \text{Exec}_{i,t}^{\kappa}, \quad (138)$$

where $\text{Exec}_{i,t}^{\kappa}$ denotes the family of transformations selected for possible execution. Execution remains a further relational event. Equation 138 therefore prevents judgment, choice, and action from being identified by notation alone.

15.5 Support, Refusal, Revision Operator Families

The minimal dyad requires several operation families with different substantive roles. Table 18 summarizes the principal families retained at this stage. Their membership can overlap in a concrete model, but their analytical roles should remain distinguishable.

For an operation family F, let

$$\text{Adm}_{F,t}^{\kappa}(R) \subseteq \text{Part}(\mathcal{A}_t^{\kappa}(R), \mathcal{A}_{t'}^{\kappa'}(R')) \quad (139)$$

denote the currently admissible typed maps in that family. Equation 139 is intentionally broad. A support operation can act within one retained algebra, between algebras attached to different regions, or

Table 18: Principal local operation families in the dyadic algebraic architecture. The table records analytical roles rather than one universal algebraic type.

Operation family	Local analytical role	Typical relational target
Readout / manifestation	Produce an accessible or recordable local output	Manifestation and epistemic-access records
Attention / recognition	Modify salience, tracking, or retained relational identification	Subject-relative reconstruction, role recognition
Communication	Add, remove, or transform messages and signaling relations	Interaction interface and manifestation structure
Request / offer / refusal	Reconfigure candidate continuations or authorization-relevant procedure	Admissible procedure and action family
Support / intervention	Transform a retained part of the coupled relational configuration	B-relative, A-relative, or dyadic continuation structure
Feedback / correction	Modify later interpretation or action conditions in response to realized consequences	ToM, evaluative records, procedure, admissibility
Revision	Update a judgment, role, mapping, or local transformation family	Cross-level and historical records
Withdrawal / termination	End or narrow a retained procedure or authorization domain	Later continuation and role structure

between algebraic carriers associated with different history complexes. A refusal operation can transform the admissible family without directly changing a resource record. A revision operation can alter an interpretive or evaluative carrier while preserving the represented material configuration.

The family labels therefore do not determine the mathematical properties of their members. A readout map can be nonlinear. A support operation can be noninvertible. A revision map can be partial. A communication map can be represented by a $*$ -homomorphism in one application and by a non-homomorphic map in another. The substantive role and the selected carrier determine which stronger conditions are appropriate.

The action-provenance analysis in Section 10 also carries into this layer. Two algebraic maps that produce the same local output can represent different GR actions when their provenance, authorization history, or role attribution differs. Algebraic extensional equality is therefore not automatically action identity. A model that intentionally quotients over provenance should state that equivalence explicitly.

15.6 Nested-Region Compatibility

Local dynamical maps become regionally coherent only when they interact consistently with the inclusion structure of the local algebras. Let $R_1 \subseteq R_2$ and suppose both regions support the required embeddings and dynamical lifts. Write $\alpha_{R_j}[\Phi] : \mathcal{A}_t^\kappa(R_j) \rightarrow \mathcal{A}_{t'}^{\kappa'}(R'_j)$ for $j = 1, 2$ for the corresponding local maps. Exact nested-region covariance requires the diagram in Equation 140 to commute on the common domain of definition.

$$\iota_{R'_1 R'_2}^{t', \kappa'} \circ \alpha_{R_1}[\Phi] = \alpha_{R_2}[\Phi] \circ \iota_{R_1 R_2}^{t, \kappa}. \quad (140)$$

Equation 140 states that applying the local dynamics before embedding and embedding before applying the regional dynamics give the same retained result. This is a substantive coherence requirement. A contextual effect can violate it because the larger region contains information absent from the smaller one. In such cases the model should retain the contextual dependence rather than force exact covariance.

Approximate compatibility can be defined only after a comparison structure on the two routes has been supplied. Paper I does not introduce a universal norm for that purpose. A finite application can use a norm, edit distance, divergence, or typed equivalence according to the carrier. The comparison tool must be justified independently.

15.7 Overlap Coherence

Nested regions are only one form of locality. Two regions can overlap without either containing the other. Let $R_{12} = R_1 \cap R_2$ be an admissible overlap and suppose restriction maps from the two larger regional descriptions to the overlap have been defined. Write $\rho_{1 \rightarrow 12}^{\text{alg}}$ and $\rho_{2 \rightarrow 12}^{\text{alg}}$ for the retained restriction operations on the post-transformation algebras. Exact overlap coherence of the local dynamics is stated in Equation 141.

$$\rho_{1 \rightarrow 12}^{\text{alg}}(\alpha_{R_1}[\Phi](x_1)) \sim_{\kappa'} \rho_{2 \rightarrow 12}^{\text{alg}}(\alpha_{R_2}[\Phi](x_2)), \quad (141)$$

for compatible inputs x_1, x_2 representing the same retained overlap content. Equation 141 uses the declared equivalence $\sim_{\kappa'}$ rather than literal equality so that representationally different but relationally equivalent descriptions can remain coherent.

Overlap failure can have several meanings. It can indicate an inconsistent local fit, a missing translation rule, a substantive contextual effect, a change in subject or region boundaries, or insufficient resolution. The formalism therefore treats overlap failure as diagnostic information rather than automatically as an error to be removed.

Where a family of local descriptions admits gluing, a regional object can be reconstructed from compatible local pieces. Paper I does not assume unique gluing in the general dyad. Shared histories, hidden context, and role-dependent interpretation can make several regional reconstructions compatible with the same local data. The stronger gluing property should be introduced only in the later field realization if the selected carriers support it.

15.8 Algebraic States, Represented Operators

The local algebraic architecture permits an evaluation layer after the algebra has been specified. In a C^* -algebraic specialization, let

$$\omega_{t,R}^{\kappa} : \mathcal{A}_t^{\kappa}(R) \longrightarrow \mathbb{C} \quad (142)$$

be an algebraic state in the sense reviewed in Section 3.7. Equation 142 is a mathematical evaluation functional. It should not be conflated with A's or B's substantive value judgment from Section 12. A local algebraic state can be used to calculate expectations or induce a representation, while the meaning of those quantities in the volunteer relation requires a separate interpretation map.

Where the assumptions reviewed in Section 3.7 hold, the GNS construction supplies a representation-dependent Hilbert-space realization. A local algebraic transformation can then admit an implementation

$$\alpha_R[\Phi] \rightsquigarrow V_{R,\omega}[\Phi] \quad \text{on } \mathcal{H}_{\omega}, \quad (143)$$

when the required compatibility conditions are satisfied. Equation 143 preserves the ordering established earlier: the Hilbert-space operator is a calculational realization derived from the algebraic carrier and evaluation context, not the primitive container of the dyadic spacetime.

15.9 Open-System Specializations

Volunteer relations are naturally open-system problems: the dyad exchanges information, resources, norms, and material effects with a larger environment even when Paper I brackets third parties analytically. A local transformation should therefore not be required to be reversible or unitary. In an appropriate C^* - or W^* -algebraic specialization, positive or completely positive maps can provide one mathematically controlled representation of effective subsystem dynamics, as discussed in Section 3.8.

Let $\mathcal{A}$ and $\mathcal{B}$ be retained local algebras. A completely positive map can be written schematically as

$$\mathcal{E} : \mathcal{A} \longrightarrow \mathcal{B}, \quad (144)$$

with positivity and complete positivity interpreted in the standard operator-algebraic sense when those assumptions are selected. Equation 144 supplies a possible effective dynamical form. It does not imply that the volunteer relation is a quantum system or that all support processes preserve a physically interpreted density matrix.

The same restraint applies to semigroup dynamics. If a one-parameter family $\{\mathcal{E}_s\}_{s \geq 0}$ is justified and acts on a stable algebraic domain, then a semigroup law can be imposed. Historical memory, changing domains, role revision, and topology change can instead require non-Markovian families, cocycles, partial morphisms, or maps between different algebras. The general Paper I architecture remains broader than the semigroup specialization.

15.10 Ordered Algebraic Composition

The interaction history of Section 11 induces ordered composition of the algebraic lifts whenever their domains and codomains are compatible. Let $\alpha_1, \dots, \alpha_n$ be local dynamical maps representing an ordered sequence of retained dyadic transformations. Their composite is

$$\alpha_\gamma := \alpha_n \circ \dots \circ \alpha_2 \circ \alpha_1. \quad (145)$$

Equation 145 retains the procedural order. An earlier support, refusal, clarification, or feedback operation can alter the domain on which a later operation is defined. Failure of composition is therefore itself a possible historical result rather than merely a technical inconvenience.

Sequence sensitivity should first be evaluated through the general equivalence-based criterion introduced in Equation 15. A commutator becomes available only if two transformations act linearly on a common algebraic domain and both orders are defined. In that specialization,

$$[\alpha_2, \alpha_1] = \alpha_2 \circ \alpha_1 - \alpha_1 \circ \alpha_2. \quad (146)$$

A nonzero value in Equation 146 records order sensitivity of the selected algebraic realization. It does not by itself establish a substantive causal mechanism. The interpretation must be traced back through the algebraic lifts to the corresponding relational operations.

Closed procedure cycles can also be represented when the composed maps return to a comparable local algebra. Such a cycle can support a residual map or holonomy-like diagnostic in later work. Paper I leaves the detailed cycle analysis to the historical-composition and field-realization sections because a meaningful residual requires a declared equivalence, representation choice, and substantive interpretation of the cycle.

15.11 Quasi-Local Assembly

A larger algebraic carrier can be derived when the retained region family is directed and the embeddings in Equation 132 are compatible. In a normed C^* -algebraic specialization, the local algebras can generate a quasi-local algebra through the completion of their directed union,

$$\mathcal{A}_{t,\text{ql}}^\kappa := \overline{\bigcup_{R \in \mathcal{K}_{AB,t}^\kappa} \mathcal{A}_t^\kappa(R)}^{\|\cdot\|}, \quad (147)$$

where $R \in \mathcal{K}_{AB,t}^\kappa$ denotes the declared family of retained finite or relatively local regions. Equation 147 is available only when the algebra class, directedness, embeddings, and norm completion are justified.

Compatible local dynamical maps can induce a quasi-local transformation under additional boundedness and consistency assumptions. Write the induced map schematically as

$$\alpha_{t,\text{ql}}^\kappa : \mathcal{A}_{t,\text{ql}}^\kappa \rightarrow \mathcal{A}_{t',\text{ql}}^{\kappa'}. \quad (148)$$

Equation 148 remains a derived object. The source and target quasi-local algebras can differ when the relational carrier, resolution, or admissible region family changes. A global endomorphism is only one special case.

The quasi-local construction should also remain distinct from a field representation. A quasi-local algebra assembles compatible local algebraic carriers through embeddings and completion. A field representation requires local field data together with restriction, overlap, comparison, and gluing structure appropriate to the field object. The two constructions can interact, but neither implies the other.

15.12 Historical-Dynamics Interface

The local algebraic layer now provides the transformation language required by the next stage of the dyadic formalism. At historical stage t , the coupled configuration supplies a region family, local carriers, admissible operation families, subject-relative readout maps, judgment-conditioned generation rules, and typed algebraic lifts. After one or more operations are executed, these structures can themselves change. The local dynamics is therefore history-indexed rather than fixed once for the entire model.

A compact algebraic record can be written as

$$\mathfrak{L}_{AB,t}^{\text{alg},\kappa} := \left(\{\mathcal{A}_t^\kappa(R)\}_R, \{\iota_{RR'}^{t,\kappa}\}, \{\text{Adm}_{F,t}^\kappa(R)\}_{F,R}, \{\text{Read}_{i,t,R}^\kappa\}_{i,R} \right), \quad (149)$$

with the understanding that each displayed family is restricted to the regions and maps actually defined by the model. Equation 149 is a bookkeeping object, not a new ontological layer.

A relational transformation history can then induce a revision relation

$$\mathfrak{L}_{AB,t}^{\text{alg},\kappa} \xrightarrow{\Gamma_{t \rightarrow t'}} \mathfrak{L}_{AB,t'}^{\text{alg},\kappa'}, \quad (150)$$

where $\Gamma_{t \rightarrow t'}$ can change the local algebras, embeddings, admissible operator families, readout structure, or resolution index. Equation 150 is intentionally more general than an automorphism of one fixed algebra. The next section will place this revision inside the broader historical composition of subject-spacetimes, cross-level mappings, procedure, feedback, and changing interpretive structures.

Section 15 has therefore supplied a local compositional realization of the dyadic GR architecture without reducing the volunteer relation to operator algebra. Regional carriers, dynamical lifts, readout maps, judgment-conditioned operation generation, support/refusal/revision families, locality conditions, represented-operator specializations, open-system maps, ordered composition, and quasi-local assembly occupy distinct formal roles. Their common purpose is to make relational transformations precise enough to compose and revise while retaining the historical, three-level, epistemic, and normative distinctions established earlier in the manuscript.

16 Historical Composition: Cross-Level Revision, Memory, Path Dependence

This section develops the historical-composition layer of the dyadic volunteer formalism. Its role is to assemble the history-bearing subject-spacetime, three-level reality architecture, epistemic records, theory-of-mind structures, procedure, evaluative formation, authorization, typed asymmetry, and local algebraic dynamics into one revisable trajectory description. The objective is to represent how earlier interactions condition later possibilities without reducing the dyad to a fixed state vector or a single Markovian evolution law. The discussion proceeds through integrated stage records, ordered historical composition, memory and delayed consequences, prediction error, changing continuation families, cross-level mapping revision, path dependence, feedback, and persistence under revision. The method remains typed and reconstructive: historical composition is represented only where the relevant source, target, tracking, and comparison structures have been declared.

Figure 11 summarizes the architecture. A historical stage contains several heterogeneous records that describe one coupled dyadic configuration at a declared resolution. Interaction can revise several of those records simultaneously. The resulting later stage then changes the conditions under which subsequent manifestations, interpretations, judgments, authorizations, and actions become possible.

16.1 Integrated Dyadic Stage Records

The previous sections introduced several records at different formal levels. Historical composition requires a way to retain them together without pretending that they form coordinates of one common state

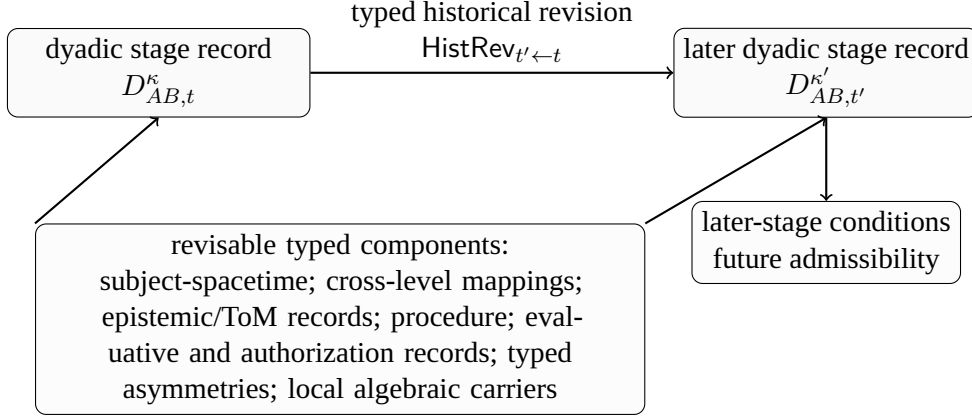


Figure 11: Historical-composition architecture for the dyadic formalism. The stage records are typed ledgers rather than vectors. Historical revision can alter several heterogeneous components at once. The later record also conditions which future manifestations, interpretations, judgments, authorizations, and actions remain available.

space. Let $\mathfrak{D}_{AB,t}^\kappa$ denote the class of admissible integrated dyadic stage records at historical index t and resolution κ . A realized record $D_{AB,t}^\kappa \in \mathfrak{D}_{AB,t}^\kappa$ can have the typed form stated in Equation 151.

$$D_{AB,t}^\kappa := \left(\mathcal{K}_{AB,t}^\kappa, L_t, \mathfrak{E}_{AB,t}, \mathfrak{T}_{AB,t}^{(\leq d_t)}, \Gamma_{AB}^{\kappa, \leq t}, \mathfrak{V}_{AB,t}^\kappa, \mathfrak{U}_{AB,t}^\kappa, \mathfrak{A}_{AB,t}^\kappa, \mathfrak{L}_{AB,t}^{\text{alg}, \kappa} \right). \quad (151)$$

Equation 151 collects the coupled subject-spacetime, cross-level record, epistemic-access record, retained finite-depth ToM structure, realized procedure prefix, evaluative record, authorization record, multidimensional asymmetry record, and local algebraic record. The notation is organizational. These entries are differently typed, and the paper assumes no addition, norm, probability measure, or common topology on the whole tuple.

The ToM component in Equation 151 is itself a bookkeeping family. Define

$$\mathfrak{T}_{AB,t}^{(\leq d_t)} := \left(\mathfrak{T}_{A \triangleright B,t}^{(\leq d_A)}, \mathfrak{T}_{B \triangleright A,t}^{(\leq d_B)} \right), \quad d_t := \max\{d_A, d_B\}, \quad (152)$$

when the retained application supplies finite depths d_A and d_B . Equation 152 does not require equal recursion depth or comparable confidence on the two sides.

The procedure component $\Gamma_{AB}^{\kappa, \leq t}$ denotes the restriction of the realized procedure record in Equation 81 to elements retained no later than stage t under the procedure's partial order. This restriction can itself be nonunique when the historical indexing of the broader model is coarser than the internal procedure order. The application should then state the tracking rule used to align the two descriptions.

A stage record can omit a component whose corresponding structure has not been articulated. An early interaction can lack a shared evaluative record, a well-defined authorization status, or a local algebraic specialization. The integrated architecture therefore permits partial records. Completeness of Equation 151 is a modeling achievement rather than a starting assumption.

16.2 Typed Historical Revision

Historical change should be represented more generally than as an endomorphism of one fixed state space. Let $t \preceq t'$ and allow the analytical resolution to change from κ to κ' . A typed historical-revision relation is defined in Equation 153.

$$\text{HistRev}_{t' \leftarrow t}^{\kappa' \leftarrow \kappa} \subseteq \mathfrak{D}_{AB,t}^\kappa \times \mathfrak{D}_{AB,t'}^{\kappa'}. \quad (153)$$

Membership $(D_{AB,t}^\kappa, D_{AB,t'}^{\kappa'}) \in \text{HistRev}_{t' \leftarrow t}^{\kappa' \leftarrow \kappa}$ means that the later record is admitted as a historically continuous revision of the earlier record under the tracking and reconstruction rules of the model.

Equation 153 is relational rather than functional because one earlier record can support several later reconstructions and one later record can admit several defensible provenance accounts.

The revision relation can act unevenly across the components of Equation 151. A material support action can change $\mathcal{K}_{AB,t}^\kappa$ while leaving a role label temporarily unchanged. A clarification can leave the represented material configuration stable while revising L_t , $\mathfrak{E}_{AB,t}$, and $\mathfrak{V}_{AB,t}^\kappa$. A public refusal can change manifestation, authorization, asymmetry, and admissible operator families at once. Historical composition therefore concerns coordinated revision across heterogeneous records rather than one privileged carrier.

The direction $t \preceq t'$ supplies historical order, not a universal scalar duration. Two histories can contain incomparable events or use different temporal resolutions. The partial-order discipline of Section 11 therefore remains active at the integrated level.

16.3 Ordered Composition, Historical Segmentation

A dyadic history can contain several relational transformations whose effects depend on their order. Let $\gamma = (\Phi_1, \dots, \Phi_n)$ be a linearly ordered specialization of a retained procedure fragment such that every consecutive source-target composition is defined. The composite relational transformation is stated in Equation 154.

$$\Phi_\gamma := \Phi_n \circ \dots \circ \Phi_2 \circ \Phi_1. \quad (154)$$

Equation 154 records a composable subhistory. The full procedure need not admit one global composite. A refusal can close the domain of a later intervention, a role transition can change the subject position to which a later request is addressed, and a change in manifestation mapping can alter the epistemic meaning of an otherwise similar signal. In such cases, failure of composition is substantive information about the history.

Historical segmentation also matters. A model can treat a request and clarification as two transformations, one negotiated episode, or a larger interaction unit. Different segmentations can preserve different provenance and order-sensitive distinctions. Paper I therefore requires the segmentation rule to remain explicit when a historical composite is used for explanation or comparison.

Sequence sensitivity is retained at the general relational level before any algebraic commutator is invoked. For two histories γ and γ' built from the same multiset of local events under different admissible orders, define a typed history comparison only after an equivalence criterion $\sim_\chi^{\text{hist}}$ has been stated. Order sensitivity is present when Equation 155 holds.

$$\Phi_\gamma(D) \not\sim_\chi^{\text{hist}} \Phi_{\gamma'}(D), \quad (155)$$

for a stage record D on which both composites are defined. Equation 155 is broader than the commutator in Equation 146. It can compare histories whose retained outputs are relational, procedural, epistemic, or authorization records rather than elements of one linear algebra.

16.4 Memory, Delayed Consequences

Historical influence can persist without remaining continuously manifest. Section 8 represented subject-relative history through $\mathcal{H}_{i,t}$, while Section 10 separated action from later consequences. The present layer makes their interaction explicit.

For subject $i \in \{A, B\}$ and $t \preceq t'$, let the memory-revision relation be defined by Equation 156.

$$\text{MemRev}_{i,t' \leftarrow t} \subseteq \mathcal{H}_{i,t} \times \mathcal{H}_{i,t'}. \quad (156)$$

Equation 156 can represent retention, forgetting, reclassification, compression, or reinterpretation of historical material. It does not require memory to preserve a literal copy of the earlier record. A later reconstruction can therefore depend on a history that has itself been transformed by subsequent interaction.

Delayed consequences require a separate historical relation. Let $\mathfrak{A}_{AB,[s,t]}^\kappa$ be the retained action class over an earlier interval and let $\mathcal{R}_{t'}$ be the later relational domain. Define the delayed-consequence relation in Equation 157.

$$\text{DelayEff}_{t' \leftarrow [s,t]}^\kappa \subseteq \mathfrak{A}_{AB,[s,t]}^\kappa \times \mathcal{R}_{t'}, \quad t \preceq t'. \quad (157)$$

Membership $(\alpha, r') \in \text{DelayEff}_{t' \leftarrow [s,t]}^\kappa$ records that the application retains action α as contributing to later relational structure r' . The relation does not by itself establish exclusive causation. A later dependency, trust relation, capability change, or role stabilization can be jointly generated by several actions, background conditions, and intervening events.

Delayed consequence and memory can interact. A consequence can become legible only after a later event supplies a vocabulary or comparison point. Conversely, a remembered action can acquire new evaluative meaning after an unanticipated outcome occurs. Historical analysis should therefore distinguish delayed relational effect from delayed recognition of an effect.

16.5 Prediction Error, Retrospective Reconstruction

Support action is often selected under uncertainty about future trajectories. Let $\hat{\mathcal{F}}_t^i(\alpha)$ denote the family of future continuations that subject i retains as plausible for action α at stage t . The notation records a subject-relative predictive family rather than a probability distribution unless an application supplies probabilities.

When a later continuation $h_{t'}$ becomes available, a prediction-comparison relation can be introduced. Define the relation in Equation 158.

$$\text{PredCmp}_{i,t' \leftarrow t} \subseteq \hat{\mathcal{F}}_t^i(\alpha) \times \mathfrak{H}_{AB}^{\leq t'}(\kappa). \quad (158)$$

Equation 158 leaves the comparison type open. One application can test whether the realized continuation belonged to the earlier admissible family; another can compare expected and realized capability changes; another can record a qualitative surprise relation. A scalar prediction error is introduced only when the forecast and outcome carriers support the required arithmetic.

Retrospective reconstruction can revise several earlier objects. The action's provenance family can change through Equation 80; a need judgment can be revised through Equation 103; an authorization interpretation can change through Equation 120; and the action-equivalence class can split when later evidence reveals a distinction absent from the earlier resolution. Later knowledge therefore alters present reconstruction of the past without changing the fact that the earlier manifestation occurred under its original historical conditions.

The distinction between occurrence and reconstruction is essential. A later judgment that an intervention was harmful does not imply that A initially intended harm. A later discovery of pressure can change the retained authorization analysis without implying that every participant consciously represented that pressure at the earlier stage. Historical revision changes the model's relational and epistemic record of the event while preserving temporal provenance.

16.6 Changing Continuation Families

Generativity enters the historical layer through changes in what can happen next. Let $\mathcal{C}_{AB,t}^\kappa$ denote the retained continuation family available to the coupled dyad at stage t , with each continuation typed by the application. The family can include possible procedure branches, action families, role transitions, disclosure options, exit routes, or other future relational developments.

A realized historical segment can transform the continuation family. Let $\Xi_{t' \leftarrow t}$ denote the induced continuation-revision relation defined in Equation 159.

$$\Xi_{t' \leftarrow t} \subseteq \mathcal{C}_{AB,t}^\kappa \times \mathcal{C}_{AB,t'}^{\kappa'}. \quad (159)$$

Equation 159 permits expansion, contraction, substitution, branching, or reclassification of later possibilities. A successful clarification can create a jointly acceptable support option that was previously unavailable. A refusal can close one intervention route while opening negotiation or exit. Repeated dependency can leave nominal options present while changing their effective accessibility. A new shared vocabulary can create previously unavailable forms of need articulation.

The local algebraic analogue is the revision of admissible operator families. The family $\text{Adm}_{\mathbb{F},t}^{\kappa}(R)$ introduced in Section 15 can change when the continuation structure changes. The broader continuation relation in Equation 159 remains primary at the substantive level; operator-family revision is one representation of that change when an algebraic carrier has been selected.

16.7 Cross-Level Mapping Revision

Historical composition changes not only relational content but also the mappings through which relational structures become manifest and interpreted. Section 5.6 represented the cross-level record by L_t . The present section adds an explicit revision relation between such records.

Let $\mathfrak{L}_t$ denote the class of admissible cross-level records of the form in Equation 30. Define the cross-level revision relation in Equation 160.

$$\text{CrossRev}_{t' \leftarrow t} \subseteq \mathfrak{L}_t \times \mathfrak{L}_{t'}. \quad (160)$$

Equation 160 can alter $\mathcal{R}$, $\mathcal{M}$, μ , subject-relative access sets, and reconstruction maps together or separately. A stable trust relation can acquire quieter manifestations. A new accessibility technology can alter manifestation without changing the underlying authorization status. Repeated interaction can change $\rho_{A,t}$ so that B's brief response becomes more legible to A. A rupture can have the opposite effect and make formerly familiar manifestations ambiguous.

Manifestation feedback through $\beta_{t' \leftarrow t}$ can also participate in the revision. A repeated public role label can stabilize a supporter-supportee pattern; a visible refusal can alter later standing and authorization expectations; a shared ritual can make a previously implicit norm more durable. Historical cross-level revision therefore includes both changes in mapping and changes generated partly by manifested forms themselves.

The same manifested form can consequently change meaning across history. Let m be trackable across t and t' , and let $h_{i,t} \in \mathcal{H}_{i,t}$ and $h_{i,t'} \in \mathcal{H}_{i,t'}$ denote the retained histories paired with that manifestation in the two reconstruction domains. The relation

$$\rho_{i,t}(m, h_{i,t}) \not\sim_{\chi}^{\text{int}} \rho_{i,t'}(m, h_{i,t'}) \quad (161)$$

can hold under a declared interpretive-equivalence criterion $\sim_{\chi}^{\text{int}}$, whenever both reconstruction terms are defined. Equation 161 supplies a minimal formal representation of historical meaning change without assuming that the manifested form itself changed; the retained histories and reconstruction maps can both contribute to the shift.

16.8 Path Dependence, Historical Contingency

Path dependence in the present framework means that historically admissible order and intermediate structure can affect later dyadic records. The concept does not require every event to be irreversible or every sequence to diverge permanently. It requires only that distinct admissible histories can leave different retained consequences at a later comparison point.

Let γ_1 and γ_2 be two admissible histories connecting comparable initial and later analytical windows. A path-dependent distinction is present relative to specification χ when Equation 162 holds.

$$D_{AB,t'}^{\gamma_1} \not\sim_{\chi}^{\text{hist}} D_{AB,t'}^{\gamma_2}. \quad (162)$$

Table 19: Selected components of dyadic historical revision. The entries identify typed forms of change rather than a common numerical dynamics.

Retained component	Historical revision	Illustrative dyadic consequence
Subject-spacetime	boundary, label, interface, continuation, or carrier change	altered capacity, role, accessibility, or coupling
Cross-level mapping	manifestation or reconstruction relation changes	the same relational structure becomes differently visible or intelligible
Epistemic/ToM record	access, memory, confidence, nesting, or interpretation changes	later support is based on a revised model of the other
Procedure	branch, order, closure, handover, or termination changes	earlier refusal or clarification changes later admissibility
Evaluative record	need, value, aesthetic, or normative judgment revises	later consequences alter the meaning of earlier support
Authorization	scope, withdrawal, standing, or authority basis changes	a previously available action becomes unauthorized or contested
Asymmetry record	dimension, direction, persistence, or comparison frame changes	an epistemic advantage attenuates, reverses, or becomes power-relevant later
Algebraic carrier	local algebra, embedding, operator family, or readout changes	the mathematical representation itself must be updated across history

Equation 162 can be witnessed by different trust structures, role positions, authorization statuses, evaluative vocabularies, asymmetry profiles, or continuation families even when a coarse material outcome is similar. A requested transfer and an imposed transfer can deliver the same resource while generating different later expectations. A clarification offered before refusal can produce a different trust trajectory from the same clarification offered after repeated boundary violations.

Historical contingency also enters through branch selection. Several continuations can be admissible at one stage, and a locally modest event can select a branch whose later self-reinforcement magnifies its significance. Paper I records this possibility without claiming that every dyad possesses attractors or near-critical dynamics. Those stronger dynamical interpretations require additional assumptions and belong primarily to later papers.

16.9 Feedback, Self-Conditioning History

Feedback becomes especially important when the consequences of earlier interaction modify the conditions of later interaction. The dyad then exhibits self-conditioning history: previous relational outcomes become part of the environment through which later actions are generated, interpreted, or authorized.

Let Q_t denote any retained historical component for which a later value $Q_{t'}$ participates in generating a still later component $Q_{t''}$. A generic second-order feedback relation is stated in Equation 163.

$$\text{Feed}_{t'' \leftarrow t' \leftarrow t}^Q \subseteq Q_t \times Q_{t'} \times Q_{t''}, \quad t \preceq t' \preceq t''. \quad (163)$$

Equation 163 is a schematic typed relation. In a trust application, successful recognition of refusal can increase later disclosure, which then improves later need reconstruction. In an authorization application, repeated disregard of expressed boundaries can reduce later disclosure and make apparent compliance less informative. In an aesthetic application, repeated tactful interaction can stabilize a shared sensibility that changes the manifestation of later offers. In an asymmetry application, one subject's accumulated interpretive advantage can change the conditions under which later disagreement becomes legible.

Table 19 summarizes several historically revisable components and the kinds of change represented in this section.

The examples in Table 19 show why feedback should not be reduced to numerical error correction. Feedback can create new distinctions, close an action domain, reorganize role structure, change a mani-

festation grammar, or alter the algebra through which later transformations are represented.

16.10 Revision, Persistence, Divergence

Revisability does not imply convergence. A and B can receive the same later manifestation and revise in different directions because their histories, trust relations, normative vocabularies, or ToM structures differ. A later shared record can also remain contested. Historical composition therefore requires criteria for persistence that do not assume eventual consensus.

For a tracked relational object $x_t \in \mathcal{R}_t$ and candidate successor $x_{t'} \in \mathcal{R}_{t'}$, let $\rightsquigarrow_{\chi}^{\text{trk}}$ denote the declared tracking relation. Persistence is represented by Equation 164.

$$x_t \rightsquigarrow_{\chi}^{\text{trk}} x_{t'}. \quad (164)$$

Equation 164 states continuity under specification χ without requiring literal identity of manifestation, label, location, or local algebraic carrier. Trust can persist through a change in its manifestations; a subject role can persist through a change in institutional title; a norm can persist through reinterpretation. Conversely, two superficially similar manifestations can fail the tracking relation when their relational histories diverge.

Subject-relative reconstruction can diverge even when relational tracking remains stable. If $x_t \rightsquigarrow_{\chi}^{\text{trk}} x_{t'}$ while $\hat{x}_t^A$ and $\hat{x}_{t'}^B$ belong to different interpretive classes, the formalism records persistence together with epistemic disagreement. This combination is common in contested support relations: the same continuing relation can be understood as care by one subject and control by the other.

Historical revision can also reopen distinctions previously stabilized. A shared vocabulary can later fragment, a role can lose legitimacy, a consent record can be withdrawn, or a dependency can become newly visible. Such reopening is compatible with the earlier metamodel's insistence that stronger claims about irreversible collapse depend on the selected carrier and congruence. Paper I therefore treats persistence, loss, and recovery as representation-relative unless a particular instantiation establishes stronger results.

16.11 Emergent-Structure Interface

The historical-composition layer prepares the next section on emergent dyadic relational structures. Trust, recognition, reciprocity, dependence, expectation, obligation, authority, shared vocabulary, shared norm, role stabilization, and partnership are not introduced merely because repeated events occur. Their emergence requires criteria for persistence, relational efficacy, and generative consequence across histories of the kind developed above.

The integrated record in Equation 151 provides the evidential and formal context in which such criteria can be stated. An emergent trust relation, for example, can depend on repeated acknowledgment, memory, revised ToM, procedure history, authorization practice, and changed continuation families. A helper-supportee pattern can depend on role tracking, repeated manifestation, attribution, asymmetry, and feedback. A shared norm can depend on evaluative co-formation and later practice that reproduces its interpretive grammar.

Section 16 has therefore supplied a general historical architecture rather than a universal law of dyadic evolution. It represents integrated stage records, typed revision, ordered composition, delayed consequences, prediction comparison, changing continuation families, cross-level mapping trajectories, path dependence, feedback, persistence, and divergence. The next section can now ask when recurrent historical organization becomes sufficiently stable and generatively efficacious to count as an emergent relational structure within the GR model.

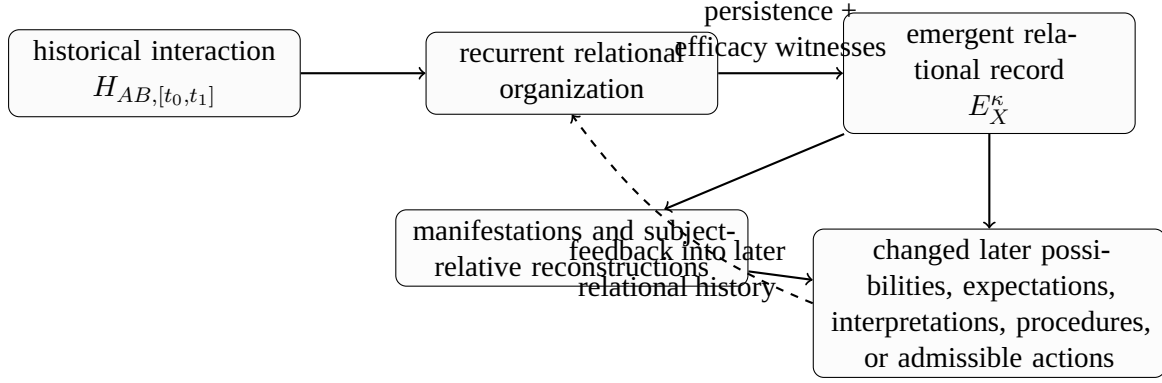


Figure 12: Emergent-structure architecture for the minimal dyad. Repetition becomes an emergent relational structure only after persistence, organization, and causal or generative efficacy are warranted under a declared specification. Manifestations and reconstructions provide access to the structure without exhausting it.

17 Emergent Dyadic Relational Structures

This section develops the emergent-structure layer of the minimal dyadic volunteer formalism. Its role is to identify when recurrent historical organization can be treated as a relationally real object of the model rather than as a transient event, a private judgment, or a convenient summary label. The objective is limited and formal. The section does not derive a general metaphysics of trust, recognition, reciprocity, dependence, obligation, legitimacy, or partnership. It specifies criteria under which such objects can be represented as historically persistent and causally or generatively efficacious structures inside the coupled dyad. The discussion proceeds through a general emergence criterion, then examines trust, expectation, recognition, reciprocity, dependence, shared vocabulary, shared norm, role stabilization, standing, authority, obligation, legitimacy attribution, and partnership. It closes with three-level profiles, dissolution and reopening, and the interface to the labelled 2-complex realization developed in the next section.

The construction builds directly on the formal-status classification in Section 7 and the historical-composition architecture in Section 16. Repetition alone does not establish emergence. A recurrent manifestation can remain epiphenomenal, a repeated judgment can remain private, and a frequently used label can fail to organize later interaction. The additional burden is relational efficacy across history. A candidate emergent object must be trackable under a declared specification, sufficiently organized to survive local variation, and capable of conditioning later possibilities, interpretations, procedures, or actions.

Figure 12 summarizes the architecture used in this section. Historical interaction supplies recurrent organization. Persistence and efficacy witnesses support an emergent relational record. That record can then alter later continuation families while acquiring several possible manifestations and subject-relative reconstructions.

17.1 Emergence Criteria, Witness Records

Let X denote a candidate dyadic relational object and let $I = [t_0, t_1]_{\preceq}$ denote a retained historical interval in the partial historical order. Let $H_{AB,I}$ denote the dyadic history retained on that interval. Paper I represents emergence through a model-relative predicate rather than a scalar threshold. Within the present formalism, Equation 165 stipulates the criterion used to classify a retained dyadic object as emergent relative to specification χ .

$$\text{Emerg}_\chi(X; H_{AB,I}) \iff \text{Pers}_\chi(X; I) \wedge \text{Org}_\chi(X; I) \wedge \text{Effic}_\chi(X; I), \quad (165)$$

where χ denotes the specification under which tracking, organization, and efficacy are assessed.

The three predicates have distinct roles. Pers_χ records that candidate instances can be tracked across a nontrivial historical interval. Org_χ records that the retained relation possesses enough structure to be identified as more than an accidental sequence of similar events. Effic_χ records that the relation changes at least one later family of possibilities, interpretations, expectations, procedure states, evaluative records, authorization conditions, or admissible transformations.

Equation 165 is deliberately local to the present formalism. It does not assert that persistence, organization, and efficacy exhaust every philosophical account of emergence. It supplies the minimum burden required before Paper I treats a dyadic structure as relationally real rather than as a transient manifestation or an analyst's shorthand. Depending on the application, the efficacy term can be causal, generative, constraining, enabling, or interpretive. The model must state which consequence witnesses the claim.

A witness record collects the historical evidence used by the model. Let

$$\mathcal{W}_X^\chi(I) := (\mathcal{T}_X^\chi(I), \mathcal{O}_X^\chi(I), \mathcal{E}_X^\chi(I), \mathcal{M}_X^\chi(I)), \quad (166)$$

Equation 166 collects the witnesses retained for an emergence assessment. Here $\mathcal{T}_X^\chi(I)$ records tracking relations, $\mathcal{O}_X^\chi(I)$ records relational organization, $\mathcal{E}_X^\chi(I)$ records efficacy witnesses, and $\mathcal{M}_X^\chi(I)$ records manifestations available during the interval. The manifestation component is evidential and historical; it is not itself a necessary ontological constituent of the emergent relational object. A trust relation, for example, can remain relationally efficacious while explicit declarations of trust disappear from manifestation reality.

Several non-entailments follow immediately. Repetition does not imply emergence. Shared observation does not imply shared structure. Coincident private judgments do not imply a jointly generated norm. A frequent role label does not by itself establish a stable role relation. An explicit declaration of trust does not by itself establish trust. These separations prevent the concept of emergence from becoming a synonym for frequency, visibility, agreement, or naming.

17.2 Trust, Distrust, Expectation

Trust is represented here as a typed and directional relational object. Let $d \in \mathcal{D}_{\text{tr}}$ denote a trust domain, such as truthful disclosure, confidentiality, respect for refusal, competence in a task, reliability of follow-through, or responsiveness to feedback. A candidate trust record is written

$$\text{Tr}_{i \rightsquigarrow j, t}^{\kappa, d}, \quad i, j \in \{A, B\}, \quad i \neq j. \quad (167)$$

The arrow indexes the direction in which one subject relies on another in domain d . Equation 167 is not a scalar trust score. A subject can trust another with confidential information while distrusting the other's technical competence, or trust competence while expecting poor respect for refusal. Domain typing therefore precedes any attempt at aggregation.

A trust record becomes emergent under Equation 165 when a history of interaction makes the relation persistent and consequential for later conduct. Possible efficacy witnesses include disclosure that would otherwise be withheld, reduced verification cost, willingness to accept a handover, increased readiness to request clarification, or a broadened continuation family. These consequences need not be normatively desirable in every setting. Trust can be misplaced, overextended, or exploited.

Distrust is not defined as a negative numerical value of trust. Let $\text{DTr}_{i \rightsquigarrow j, t}^{\kappa, d}$ record a historically stabilized expectation that reliance in domain d is unsafe, unreliable, or requires additional verification. Trust and distrust can coexist across different domains and can even coexist within one domain when evidence remains ambivalent. Treating them as distinct typed structures permits the model to represent cautious reliance, partial disclosure, or bounded cooperation without forcing one net score.

Expectation is broader than trust. A subject can reliably expect an unwanted response without trusting the responder. Let $\text{Exp}_{i \rightsquigarrow j, t}^\kappa(q)$ denote a stabilized anticipation by i concerning a response type or relational feature q associated with j . Expectations become relationally significant when they condition action before the anticipated event occurs. Repeated disregard of refusal can generate an expectation of non-responsiveness that changes what B later discloses. Repeated tactful clarification can generate an

expectation that ambiguity will be handled without penalty. The historical relation thereby becomes part of the generative environment of later action.

17.3 Recognition, Mutual Recognition

Recognition concerns the relational standing through which a subject, capacity, claim, role, experience, or boundary becomes treated as intelligible and consequential within the dyad. Recognition-theoretic social philosophy provides an adjacent tradition in which recognition is constitutive of social relations and conflict; Paper I uses a narrower typed relation and leaves the fuller normative theory open [8]. Let $q \in \mathcal{Q}_{\text{rec}}$ denote the recognition content retained by the application. A directional recognition record is written $\text{Rec}_{i \triangleright j, t}^{\kappa, q}$, where the record states that i relates to j through recognition content q . The content can concern j as a subject, as a competent participant, as a bearer of a refusal, as an interpreter of their own experience, or as an authorized contributor to a joint procedure. The formalism therefore avoids reducing recognition to praise or positive affect.

Recognition can be manifested through listening, turn-taking, acknowledgment, respect for a boundary, inclusion in planning, attribution of competence, institutional naming, or practical uptake of a claim. Similar manifestations can carry different relational meanings, and recognition can remain relationally real through forms that are not verbally articulated. The three-level distinction of Section 5 is therefore essential to its interpretation.

Mutual recognition requires more than the conjunction of two isolated recognition judgments. A minimal candidate condition is given in Equation 168.

$$\text{MRec}_{AB, t}^{\kappa, q_A, q_B} \implies \text{Rec}_{A \triangleright B, t}^{\kappa, q_A} \wedge \text{Rec}_{B \triangleright A, t}^{\kappa, q_B} \wedge \text{Couple}_{\text{rec}}^{\kappa}(A, B, t), \quad (168)$$

where $\text{Couple}_{\text{rec}}^{\kappa}$ records that the two directional relations participate in one historically coupled interaction structure. Equation 168 does not require $q_A = q_B$ and does not imply symmetry of role, competence, authority, or dependence. Mutual recognition can therefore coexist with typed asymmetry.

Misrecognition can also stabilize relationally. A repeatedly imposed beneficiary label, a persistent assumption of incompetence, or a helper identity attributed despite refusal can become historically efficacious even when one subject rejects it. Paper I therefore treats recognition and misrecognition through provenance, manifestation, reconstruction, and consequence rather than by assuming that recognition is necessarily accurate or normatively valid.

17.4 Reciprocity, Dependence

Reciprocity is represented as patterned responsiveness rather than equality of exchange. Let $\mathcal{A}_{AB, I}$ denote the retained family of dyadic actions over interval I . A reciprocity record $\text{Recip}_{AB}^{\kappa}(I)$ is warranted when earlier contributions, refusals, disclosures, or accommodations by one subject systematically enter the generation of later responses by the other. The relevant structure can be represented schematically by Equation 169.

$$a_i(t) \rightsquigarrow_{\chi}^{\text{gen}} a_j(t') \quad \text{for recurrent admissible pairs } t \prec t', \quad (169)$$

where $\rightsquigarrow_{\chi}^{\text{gen}}$ records generative relevance under specification χ . Equation 169 does not require equal magnitude, immediate return, identical goods, or symmetric roles. A supportee's disclosure can generate later restraint by the supporter; the supporter's reliable presence can generate later willingness by the supportee to correct an error. These are reciprocal structures without being exchanges of commensurable quantities.

Dependence was introduced as a typed asymmetry dimension in Section 14. The emergent-structure layer adds historical stabilization. Let $\text{Dep}_{i \rightsquigarrow j, t}^{\kappa, x}$ denote dependence by i on j with respect to relational object or capability x . A persistent dependence becomes relationally significant when removal or withdrawal of the relevant relation changes i 's continuation family, effective cost, access, or capacity in a way that persists beyond one isolated event.

Mutual dependence is compatible with asymmetry. B can depend on A for material access while A depends on B for role continuity, recognition, local knowledge, or participation. These dependencies can be differently replaceable and can carry different exit consequences. Paper I therefore preserves the vector of typed dependencies rather than compressing the relation into one label such as “dependent relationship.”

Reciprocity and dependence can interact without being identical. Recurrent reciprocal responsiveness can reduce one dependency by transferring competence or widening alternatives. It can also deepen dependence by making one subject increasingly central to the other’s routines, identity, information, or access. The later political-economy and subject-formation papers will examine those trajectories more fully.

17.5 Standing, Authority, Obligation, Legitimacy Attribution

Section 13 treated standing and authority as typed authorization objects at a given historical stage. The present section asks when those objects become historically stabilized relations. Repeated procedures can make one subject the recurrent interpreter, gatekeeper, advocate, coordinator, or decision bearer even without a formal office. A standing relation is emergent when that recurrent position remains trackable and consequential across episodes.

Authority requires the same historical qualification. A single permission to perform one act does not create a general authority relation. Conversely, repeated delegation, acknowledged expertise, or stable procedural responsibility can generate an authority structure whose effects extend to later action generation. The resulting authority remains scope-sensitive and revocable; historical stabilization does not convert it into unlimited authority.

Obligation requires particular caution because normative formation and normative validity remain distinct. Paper I uses $\text{OblRec}_{i \rightsquigarrow j, t}^{\kappa}(q)$ for a *relational obligation record*: a historically stabilized structure in which action or restraint of type q is treated within the dyad as due, expected, claimable, criticizable, or otherwise normatively consequential. The notation records a relationally effective normative structure. It does not prove that the obligation is morally or legally justified.

The same restraint applies to legitimacy. Let $\text{LegitAttr}_{i, t}^{\kappa}(X)$ denote a subject-relative attribution of legitimacy to relational object X , and let $\text{LegitRec}_{AB, t}^{\kappa}(X)$ denote a historically stabilized dyadic legitimacy-recognition structure when the attribution becomes jointly consequential. The existence of LegitRec does not entail normative legitimacy. It records that legitimacy claims have become part of the relational environment through which later conduct is interpreted or authorized.

These distinctions preserve a general principle of the paper:

$$\text{historical stabilization} \not\Rightarrow \text{normative validity.} \quad (170)$$

Equation 170 applies to obligations, authority, legitimacy attributions, role expectations, and shared norms. A relational structure can be stable and causally efficacious while remaining contestable, unjustified, or harmful.

17.6 Shared Vocabulary, Shared Norm

A shared vocabulary is more than simultaneous possession of the same words. Let $\mathcal{V}_{AB, t}^{\kappa}$ denote a historically stabilized set of distinctions, terms, gestures, or interpretive conventions that both subjects can use to coordinate some part of the relation. Its emergence requires recurrent mutual usability, not complete semantic identity. A word such as “help,” “enough,” “stop,” or “later” can acquire dyad-specific significance through repeated interaction even when each subject retains a different wider conceptual background.

The shared-vocabulary record becomes especially important when it reduces translation cost or opens new forms of mutual correction. A newly stabilized distinction can allow B to articulate a previously difficult refusal or allow A to ask about a need without imposing a prior category. In that sense, vocabulary can be generative: it changes which later relations and judgments become articulable.

A shared norm requires a stronger structure. Let $\mathcal{N}_{AB,t}^\kappa$ denote a candidate dyadic norm record. Coincident private judgments, such as $N_{A,t}(q) = N_{B,t}(q)$, do not by themselves establish $\mathcal{N}_{AB,t}^\kappa$. A shared norm requires a history in which the normative distinction participates in mutual expectation, criticism, justification, coordination, or revision. A candidate implication is stated in Equation 171.

$$\text{Emerg}_\chi(\mathcal{N}_{AB}^\kappa; H_{AB,I}) \implies \text{JointOrient}_\chi(\mathcal{N}_{AB}^\kappa; I), \quad (171)$$

where JointOrient_χ records that both subjects' later conduct or judgment is organized in relation to the same retained normative object, whether through endorsement, reliance, contestation, criticism, or negotiated revision. Shared normativity therefore does not require consensus. A norm can be shared enough to structure disagreement.

The emergent norm can also exceed the initial private repertoires. Interaction can generate a dyad-specific rule or evaluative distinction that neither subject previously held in the same form. Paper I records this possibility without treating novelty as a necessary condition of shared normative emergence. Translation, stabilization, recombination, and revision can be generative even when their materials have prior histories.

17.7 Role Stabilization, Partnership

Supporter and supportee were introduced in Section 6 as relational positions rather than subject identities. Repeated interaction can nevertheless stabilize those positions. Let $\pi_{S,t}^\kappa$ and $\pi_{R,t}^\kappa$ denote supporter and supportee positions at stage t . A role pattern becomes emergent when the positions are historically trackable and when occupying them conditions later expectation, interpretation, authorization, or action even in the absence of a new explicit assignment.

A stabilized helper-supportee pattern can be useful, temporary, constraining, or self-reinforcing. It can support reliable coordination in recurrent care. It can also narrow the space in which B is recognized as a contributor or in which A can withdraw without identity disruption. Paper I does not diagnose either trajectory normatively. It records role stabilization so that later metaphysical, psychoanalytic, ethical, and power analyses have a typed historical object to examine.

Partnership is treated as a distinct emergent possibility rather than as the negation of support. Let $\text{Part}_{AB,t}^\kappa$ denote a candidate partnership record. Its emergence requires a recurrent structure of co-participation in which both subjects possess practically consequential routes for contribution, correction, and revision of the relation. Partnership does not require equal resources, identical competence, symmetric authority, or disappearance of support. It requires that the dyad no longer be adequately represented by one-directional generation of relevant procedure and meaning.

A possible role trajectory is therefore

$$(\pi_S^\kappa, \pi_R^\kappa)_{t_0} \rightsquigarrow_\chi^{\text{role}} \text{Part}_{AB,t_1}^{\kappa'}, \quad (172)$$

Equation 172 records one candidate role trajectory. Here $\rightsquigarrow_\chi^{\text{role}}$ is a declared role-tracking relation rather than a universal developmental law. The reverse transition is also possible. A partnership can become more role-differentiated during crisis, loss of capacity, or task specialization. The formalism therefore treats role structure as historically revisable rather than normatively ordered along one direction.

17.8 Three-Level Emergence Profiles

Every emergent object inherits the three-level discipline of Section 5. Ultimate ontology remains suspended. At relational reality, the model retains the persistent and efficacious structure warranted by Equation 165. At manifestation reality, the structure can appear through speech, silence, routine, spatial organization, turn-taking, delegation, access, withdrawal, bodily orientation, timing, documents, role labels, or other contextual forms. A and B can then reconstruct the same emergent relation differently.

Table 20: Selected emergent dyadic relational structures. The table lists model-relative witnesses and cautions; it does not provide universal substantive definitions.

Structure	Candidate historical/efficacy witness	Important non-equivalence
Trust / distrust	reliance, disclosure, verification change, guarded continuation	not a scalar attitude or explicit declaration
Expectation	anticipatory adjustment before later interaction	not identical to trust
Recognition	later claims, capacities, roles, or boundaries receive consequential uptake	not necessarily approval or accuracy
Mutual recognition	coupled directional recognition records	not symmetric roles or equal authority
Reciprocity	earlier contributions recurrently condition later responses	not equal exchange
Dependence	withdrawal changes continuation, access, cost, or capability	not reducible to resource inequality
Standing / authority	recurrent procedural position conditions later admissibility	not unlimited permission or legitimacy
Obligation record	actions/restraints become claimable, criticizable, or expected	not proof of moral or legal validity
Shared vocabulary	recurrent mutual usability changes intelligibility or translation cost	not identical private semantics
Shared norm	both subjects orient later judgment/action to one normative object	not consensus or coincident opinion
Role pattern	supporter/supportee positions condition later interaction without fresh assignment	not fixed subject identity
Partnership	both subjects possess consequential routes for contribution and revision	not equality in every dimension

Let $E_{X,t}^R$ denote the relationally retained emergent object, $\text{Man}_t(E_{X,t}^R)$ its manifestation family in the sense of Equation 26, and $\hat{E}_{X,t}^A, \hat{E}_{X,t}^B$ the subject-relative reconstructions. A generic three-level profile is stated in Equation 173.

$$\Lambda_t(E_X) = (E_{X,t}^R, \text{Man}_t(E_{X,t}^R), \hat{E}_{X,t}^A, \hat{E}_{X,t}^B), \quad \text{Ont}(E_X) = \text{suspended}. \quad (173)$$

Equation 173 permits relational persistence together with manifestational change and epistemic disagreement. Trust can persist after explicit reassurance becomes unnecessary. Dependence can intensify while its visible manifestations become less obvious. A shared norm can remain efficacious while the subjects disagree about its interpretation. A partnership can be relationally stable even when an external observer continues to label one subject “volunteer” and the other “beneficiary.”

Table 20 summarizes the principal emergent structures retained in Paper I and the kinds of witnesses that can support them.

17.9 Dissolution, Reopening, Transformation

Emergence does not imply permanence. A relation that satisfied Equation 165 on one interval can cease to satisfy the persistence or efficacy criteria on a later interval. Trust can dissolve after a severe breach, a shared vocabulary can lose mutual usability, a role can become obsolete, dependence can disappear after new alternatives arise, and a shared norm can be reopened through disagreement or changed circumstances.

Paper I therefore represents dissolution through failure of historical continuation under a declared tracking criterion rather than through an assumed scalar decay law. Let $E_{X,t}^R$ be an emergent relational object and let $I' = [t_1, t_2]_{\leq}$ be a later interval. A candidate dissolution condition is stated in Equation 174.

$$\text{Emerg}_X(E_X; H_{AB,[t_0,t_1]}) \wedge \neg \text{Emerg}_{X'}(E_X; H_{AB,I'}) \implies \text{Diss}^{X,X'}(E_X; I'), \quad (174)$$

provided that χ and χ' support an application-justified comparison. Equation 174 does not require literal disappearance of every manifestation. A title can remain after authority loses efficacy; a ritual can continue after trust dissolves; a helper label can persist after the practical support relation has ended. Conversely, manifestations can disappear while the relational structure persists.

Transformation is broader than dissolution. An emergent role pattern can become a partnership, a dependency can become reciprocal, a shared norm can be reinterpreted, or trust in one domain can migrate into a different domain. Such transitions require tracking across changing types or specifications. The historical formalism therefore permits retyping when the application supplies an explicit bridge rather than forcing every emergent object to retain one fixed label throughout its history.

Reopening is especially important for normative and recognition structures. A stabilized interpretation can become contestable again when new manifestation, new vocabulary, changed capacity, or revised historical understanding enters the dyad. Reopening is therefore not simply structural failure. It can increase generative capacity by restoring alternatives that earlier stabilization had made difficult to articulate.

17.10 Geometric Realization Interface

The emergent structures developed above remain abstract records of historically organized dyadic relations. The next section places the wider Paper I architecture on a labelled 2-complex. That realization requires decisions about where subject histories, interfaces, actions, manifestations, and emergent records are represented geometrically or combinatorially.

An emergent object need not correspond to one vertex, edge, or face. Trust can depend on a distributed history of interactions; a shared norm can be supported by several events and manifestations; a role pattern can span multiple regions and times. The geometric realization must therefore preserve the distinction between a relational structure and any single local carrier used to encode evidence for it.

A later labelled realization can attach emergent records to appropriate cells, subcomplexes, interfaces, or higher-order metadata only after the representational role has been stated. The construction should also permit the same emergent relation to have several manifestations and should permit a relational object to persist through local changes in the underlying complex when a declared tracking map supports that persistence.

Section 17 has therefore supplied a model-relative criterion for relational emergence and a typed family of emergent dyadic structures. Trust, distrust, expectation, recognition, reciprocity, dependence, standing, authority, obligation records, legitimacy attributions, shared vocabulary, shared norm, role stabilization, and partnership become available as formal objects only when persistence, organization, and efficacy are historically warranted. Their manifestations and subject-relative reconstructions remain distinct from the relational structures themselves, and their stability does not establish normative validity. The next section can now instantiate the dyadic architecture on a labelled 2-complex while preserving those distinctions.

18 Labelled 2-Complex Realization

This section provides a finite labelled 2-complex realization of the dyadic architecture developed in Sections 6–17. Its role is representational: locality, incidence, historical support, shared structure, and alternative continuation become explicit without identifying the complex with ultimate ontology. The construction integrates the mathematical carrier introduced in Section 3 with the subject subcomplexes of Section 6, then assigns typed cellular supports to selected actions, manifestations, procedures, and emergent records. A 2-complex is useful because the dyad contains higher-order historical relations that cannot be represented faithfully by two nodes and one edge while keeping cell semantics application-dependent.

Figure 13 previews the construction. The upper and lower cellular chains depict retained portions of the A and B subject-spacetimes, vertical and diagonal interface cells depict interaction support, and

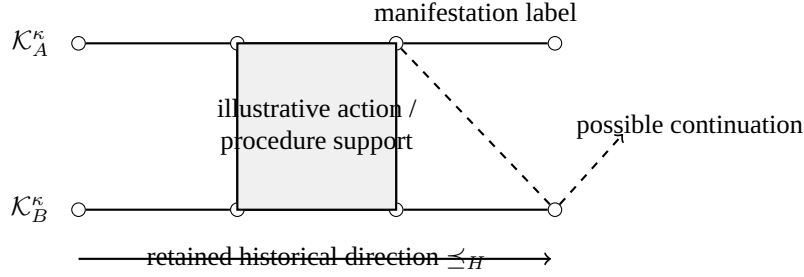


Figure 13: Schematic labelled 2-complex realization of the minimal dyad. Cellular chains represent retained portions of the A and B subject-spacetimes; interface cells and a two-dimensional face support a coupled episode. Labels and support regions encode model-dependent relational or manifestation information. The cellular drawing does not imply that subjects are literally composed of such cells or that historical order is intrinsic to every 2-complex.

a two-dimensional face records a higher-order relation among several local events. The shaded face is only an illustrative support region for a dyadic action or procedure episode. A manifestation label can be attached to one boundary cell while the relational consequence is supported by a larger region. The figure therefore separates cellular support from conceptual identity.

18.1 Dyadic Realization Record

Let $\mathcal{K}_{AB}^\kappa$ be the coupled dyadic complex at resolution κ , with subject embeddings ι_A^κ and ι_B^κ already defined by Equation 37. The cellular carrier is represented by vertices V^κ , edges E^κ , faces F^κ , and incidence data ∂^κ . The present realization also requires cell labels, a declared historical-order relation on the cells or regions used as historical markers, and the subject/interface regions needed by the dyadic analysis. A convenient realization record is stated in Equation 175.

$$\mathfrak{C}_{AB}^\kappa := \left(V^\kappa, E^\kappa, F^\kappa, \partial^\kappa, \lambda^\kappa, \preceq_H^\kappa; \iota_A^\kappa, \iota_B^\kappa, \mathcal{J}_{AB}^\kappa, \mathcal{O}_{AB}^\kappa \right). \quad (175)$$

Equation 175 combines objects introduced separately earlier in the paper. The first five entries describe the labelled 2-complex. The relation $\preceq_H^\kappa$ supplies the historical ordering required by the application; it is not inferred from incidence alone. The maps ι_A^κ and ι_B^κ locate the retained subject-spacetimes inside the coupled carrier. The region $\mathcal{J}_{AB}^\kappa$ retains the relational interface, while $\mathcal{O}_{AB}^\kappa$ retains shared represented material where the chosen resolution supports such overlap. The record can be weakened when an application does not need every component.

Historical order requires particular care. A bare 2-complex specifies incidence, not universal physical or social time. The model can use a partial order on event-bearing cells, a sequence of declared slices, causal annotations, archival ordering, or another justified historical relation. Paper I therefore treats $\preceq_H^\kappa$ as additional modeled structure. This choice preserves the distinction between combinatorial adjacency and historical precedence.

For later typing it is useful to collect the cells without forcing them into one numerical coordinate space. Define the disjoint cell carrier by Equation 176.

$$\text{Cell}(\mathcal{K}_{AB}^\kappa) := V^\kappa \sqcup E^\kappa \sqcup F^\kappa. \quad (176)$$

Equation 176 permits a conceptual object to be supported by cells of different dimensions while preserving their type. A support region can therefore consist of a vertex, an edge family, several faces, or a mixed cellular set when the application requires it.

Table 21: Illustrative realization guide for the labelled dyadic 2-complex. Assignments remain application-dependent and do not establish one universal cell semantics.

Conceptual object	Candidate cellular support	Realization caution
Subject history	tracked subcomplex $\mathcal{K}_i^\kappa$	subject identity is not one cell or a fixed boundary
Dyadic interface	connecting cells or declared interface subcomplex $\mathcal{J}_{AB}^\kappa$	interface need not equal literal overlap
Action / procedure episode	local mixed-dimensional support region	cellular support is not identical with action provenance
Manifestation	labeled event-bearing cells or boundary region	manifestation does not exhaust relational reality
ToM / epistemic reconstruction	metadata, labels, or separate derived record linked to source cells	belief content should not be confused with the cells that supplied evidence
Authorization status	typed labels or relation on procedure-support cells	an observed “yes” need not equal consent
Emergent trust / norm	distributed support across several historical cells or subcomplexes	persistence and efficacy cannot be reduced to one local label
Local algebra	region assignment $R \mapsto \mathcal{A}_\kappa(R)$	algebraic carrier is an additional representational layer

18.2 Cell Semantics, Typed Labels

Paper I does not assign one universal social meaning to vertex, edge, or face. The realization instead fixes a cell semantics locally. In one finite example, a vertex may record an interaction update, an edge may record an admissible continuation or dependency, and a face may record a higher-order compatibility among several events. In another example, vertices may correspond to procedure stages while faces represent shared institutional or normative constraints. The interpretation must be declared before the cellular data are used analytically.

Let $\text{Cell}_p(\mathcal{K}_{AB}^\kappa)$ denote the retained cells of dimension $p \in \{0, 1, 2\}$. Labels are typed by cell dimension through partial maps stated in Equation 177.

$$\lambda_p^\kappa : \text{Cell}_p(\mathcal{K}_{AB}^\kappa) \rightarrow \mathcal{L}_p^\kappa, \quad p \in \{0, 1, 2\}. \quad (177)$$

In Equation 177, $\mathcal{L}_p^\kappa$ is an application-defined label carrier. Partiality allows some cells to remain unlabeled with respect to a particular analytical dimension. A label can encode a role type, manifestation class, authorization status, asymmetry marker, local algebra identifier, historical tag, evidential provenance, or another typed datum. Distinct label families should be used when the codomains have different semantics. A role label and a normative judgment label, for example, should not be placed in one undifferentiated alphabet merely because both are attached to cells.

Labels also need not exhaust the conceptual object they help represent. A cell carrying the manifestation label “refusal” records a manifested event under a declared coding rule. It does not by itself establish the relational authorization status developed in Section 13. Likewise, a cell labeled “trust expression” is evidence or manifestation relevant to a trust analysis; it is not identical with the emergent trust record defined in Section 17. The distinction between relational object, manifestation, and encoding remains active after geometric realization.

Table 21 summarizes recurrent realization choices and the principal cautions attached to them.

18.3 Subject Subcomplexes, Relational Interfaces

The realization retains the subject-spacetime discipline of Section 6. The images $\iota_A^\kappa(\mathcal{K}_A^\kappa)$ and $\iota_B^\kappa(\mathcal{K}_B^\kappa)$ need not partition $\mathcal{K}_{AB}^\kappa$. They can overlap, share background cells, or be connected through cells assigned

to the interface. The coupled complex can also contain retained contextual structure that belongs to neither focal subject subcomplex but remains necessary for the dyadic relation.

The interface $\mathcal{J}_{AB}^\kappa$ is therefore represented by a declared support region rather than by the set-theoretic intersection alone. A local interaction can cross from an A-supported cell to a B-supported cell through connecting edges, share a face whose boundary contains cells from both histories, or depend on a contextual region that makes the interaction meaningful. The representation should preserve whichever of these structures matters to the application.

A useful distinction is between *shared historical material* and *interaction support*. Shared language, a prior institution, a remembered event, or a recurring practice may be encoded inside $\mathcal{O}_{AB}^\kappa$ even when no current interaction occurs there. Conversely, a new action can be supported on $\mathcal{J}_{AB}^\kappa$ without becoming a long-term shared object. This distinction prevents geometric overlap from functioning as a universal proxy for relational intimacy, common knowledge, or mutual recognition.

Subject boundaries can change across history. If an interaction generates a shared vocabulary, a joint routine, a delegated role, or a new persistent dependency, the later coupled realization can contain cells or labels that were absent from the earlier subject/interface decomposition. Such a change does not require the model to decide whether the resulting structure is “inside” either subject in an ultimate sense. It requires only that the later representation state how the new region is tracked and how it affects the continuation of the dyad.

18.4 Concept Support, Action Support, Manifestation Support

The term *support* is used in this subsection in a geometric sense and should not be confused with volunteer support as the substantive object of the paper. A conceptual record can depend on several cells without being identical with any one of them. Define a model-relative support assignment by Equation 178.

$$\text{supp}_\kappa(X) \subseteq \text{Cell}(\mathcal{K}_{AB}^\kappa), \quad (178)$$

where X is a retained conceptual object for which the application supplies a cellular realization. Equation 178 does not require the support set to form a subcomplex. If closure under incidence is analytically required, the application can replace the cell set by the subcomplex generated by it under the declared incidence closure.

Action support deserves a more specific construction. Let α be an individuated action with relational transformation Φ_α as introduced in Equation 78. Its cellular realization can be represented by an action-support set S_α^κ together with a partial transformation between admissible labelled complexes. Let $\mathbf{C}_\kappa \subseteq \mathcal{R}_{AB}^\kappa$ denote the subclass of coupled dyadic configurations that admit the labelled 2-complex realization selected in this section. The realized action map is stated in Equation 179.

$$\widehat{\Phi}_\alpha^{\kappa' \leftarrow \kappa} : \widehat{\mathcal{D}}_\alpha^{\kappa' \leftarrow \kappa} \subseteq \mathbf{C}_\kappa \rightarrow \mathbf{C}_{\kappa'}, \quad S_\alpha^\kappa = \text{supp}_\kappa(\alpha). \quad (179)$$

Equation 179 permits the action to alter labels, admissible cells, incidence relations, or the resolution itself, depending on the declared transformation type. The map remains a realization of the substantive action record; it does not replace the provenance $\Pi(\alpha)$, the epistemic profiles of the action, or the evaluative and authorization records attached to it.

Manifestation support is separately typed. Let m be a manifestation belonging to the manifestation family of some relational object X . A manifestation-support assignment $S_m^{\mathbf{M},\kappa}$ identifies the cells through which that manifestation is represented, while $S_X^{\mathbf{R},\kappa}$ identifies the cells retained as support for the relational object. The two supports can differ, as stated in Equation 180.

$$S_m^{\mathbf{M},\kappa} = \text{supp}_\kappa(m), \quad S_X^{\mathbf{R},\kappa} = \text{supp}_\kappa(X), \quad S_m^{\mathbf{M},\kappa} \neq S_X^{\mathbf{R},\kappa}. \quad (180)$$

The final relation in Equation 180 is a warning against identifying the manifestation carrier with the relational carrier. A spoken refusal may be localized to one event-bearing cell while the relational effects of refusal reorganize a much larger procedure history. A trust manifestation can occur in one disclosure event while the emergent trust record is supported by a longer recurrent pattern.

18.5 Historical Ordering, Cellular Episodes

Historical composition in Section 16 was defined independently of one geometric carrier. The 2-complex realization now associates retained episodes with cell supports while preserving that abstraction. Let $H^\kappa \subseteq \mathcal{P}_{\text{fin}}(\text{Cell}(\mathcal{K}_{AB}^\kappa))$ be the retained family of finite cell supports used as historical markers. A one-cell marker is represented by a singleton support. The relation $\preceq_H^\kappa$ orders these supports whenever the application warrants precedence. A cellular episode is a finite historically ordered family stated in Equation 181.

$$\gamma^\kappa = (C_1, \dots, C_n; \preceq_H^\kappa), \quad C_j \subseteq \text{Cell}(\mathcal{K}_{AB}^\kappa). \quad (181)$$

Equation 181 allows each historical marker C_j to be a mixed-dimensional support region rather than one vertex. This is useful when one procedure stage contains several coupled events or when one evaluative episode depends on a face-level compatibility relation. The historical order can remain partial: two cells or regions can be incomparable when the model lacks a justified sequence relation.

Faces acquire a specific but limited analytical role in this realization. A face can encode a higher-order relation whose meaning depends on a loop, compatibility condition, co-participation pattern, or compositional constraint among several boundary cells. The paper does not require every face to represent a temporal interval. A face can support a procedural relation, a shared practice, a compatibility condition, or another model-defined two-dimensional relational unit. This flexibility is one reason a 2-complex is more expressive than a bare dyadic graph.

Ordered histories can be represented through successive subcomplexes when the application uses growth without retroactive reconfiguration. Let $\mathcal{K}_{\leq t}^\kappa$ denote the subcomplex retained up to stage t . One history-preserving regime satisfies the inclusion pattern stated in Equation 182.

$$t \preceq_H t' \implies \mathcal{K}_{\leq t}^\kappa \hookrightarrow \mathcal{K}_{\leq t'}^\kappa. \quad (182)$$

Equation 182 is a special regime rather than a general GR law. Later sections and future papers may permit historical reinterpretation, relabeling of retained cells, replacement of tracking maps, or stronger reconfiguration in which no literal subcomplex inclusion survives. The realization therefore distinguishes a history-preserving extension from a history-reconstructing transformation.

18.6 Transformation Regimes on the Complex

A realized dyadic action or historical update can modify the labelled complex at several strengths. Distinguishing them prevents every intervention from being described as a topology change and prevents a label update from being mistaken for structural reconfiguration.

The weakest regime changes labels while preserving the cell sets and incidence data. An observed role transition, revised epistemic reconstruction, or updated authorization record may be representable this way when the underlying retained carrier remains adequate. A second regime extends the complex by adding cells and incidence relations while preserving the earlier history as a subcomplex, as in Equation 182. A third regime changes incidence among retained cells or replaces part of the carrier, thereby altering which local compositions and interfaces are available. A fourth, stronger regime changes a topological diagnostic of the represented complex. Homology can witness some such changes, while equality of selected homology groups does not establish complete topological equivalence.

These regimes can be summarized by the transformation-type record in Equation 183.

$$\tau_a^\kappa \in \{\text{Label, Extension, Incidence, Topology}\}, \quad (183)$$

where the four labels denote analytical classes, not exhaustive physical mechanisms. An application can refine the taxonomy when several changes occur together. The strongest class should be asserted only when an explicit invariant or structural comparison supports it.

This distinction is also useful for the three-level framework. A manifestation can change while the relational carrier remains stable; a relational relation can change while the chosen manifestation labels

remain superficially similar; and a stronger history can require reconfiguration of the carrier itself. The geometric representation therefore permits cross-level trajectories without identifying every change in manifestation with a change in incidence or topology.

18.7 Refinement, Coarse-Graining, Representation Compatibility

Resolution dependence was introduced for subject-spacetimes in Equation 40. The 2-complex realization extends that requirement to action supports, manifestation supports, interfaces, and emergent records. Let $q_{AB}^{f \rightarrow c}$ be a declared coarse-graining map from a finer complex $\mathcal{K}_{AB}^{\kappa_f}$ to a coarser complex $\mathcal{K}_{AB}^{\kappa_c}$. A conceptual record X is representation-compatible across the two resolutions when the application supplies an admissible comparison between the coarse-grained fine support and the support assigned directly at the coarse level.

Where the coarse-graining morphism $q_{AB}^{f \rightarrow c}$ induces a partial pushforward on retained cell supports, denote that induced map by $q_{AB,*}^{f \rightarrow c}$. A generic compatibility condition is stated in Equation 184.

$$q_{AB,*}^{f \rightarrow c}(\text{supp}_{\kappa_f}(X)) \simeq_{X,\kappa_c} \text{supp}_{\kappa_c}(X), \quad (184)$$

where $\simeq_{X,\kappa_c}$ is an application-defined support-equivalence relation. Equation 184 does not require literal equality. Fine cells can collapse into one coarse region, labels can aggregate, and a distributed fine-scale support can become one coarse marker. The comparison must preserve the distinction relevant to X .

Refinement can also expose structure that was absent at the coarse scale. A single coarse “support action” cell may refine into a request, clarification, refusal-sensitive revision, execution, and feedback episode. A coarse trust label may refine into several reliance and verification patterns. Such refinement does not imply that the fine representation is ontologically more real. It states that the finer model retains distinctions the coarse model intentionally suppressed.

Representational robustness can then be assessed by asking whether a conclusion survives a reasonable family of refinements and coarse-grainings. If the classification of an intervention as jointly generated, trust-forming, authority-dependent, or topology-changing disappears under small reasonable changes of resolution, the conclusion should be reported as representation-sensitive.

18.8 Alternative Continuations, Boundary Data

The generative orientation of GR requires the realized complex to retain possible continuations as well as the history already represented. A finite 2-complex can encode one realized history while a continuation family records alternative admissible extensions from a declared boundary. Let Σ_t^κ denote the retained boundary data at stage t . Let $\mathbf{C}_\kappa$ again denote the admissible class of labelled dyadic complexes. The extension family is defined in Equation 185.

$$\text{Ext}_\kappa(\Sigma_t^\kappa) := \left\{ \mathcal{K}' \in \mathbf{C}_\kappa \mid \partial_- \mathcal{K}' \simeq_\Sigma \Sigma_t^\kappa \right\}. \quad (185)$$

In Equation 185, ∂_- denotes the incoming boundary selected by the local history convention and $\simeq_\Sigma$ is the boundary-equivalence criterion retained by the application. The construction is deliberately broader than a quantum transition amplitude. It records which labelled continuations are admitted by the model.

Support actions, refusals, revised judgments, changing trust, or new authority relations can change the extension family. The generative effect of an event can therefore be represented by a change in admissible continuation rather than by a scalar change in welfare or utility. If $\mathbf{a}$ occurs at stage t , the later family can be compared with the counterfactual family that the model retains without that event. A generic comparison is stated in Equation 186.

$$\text{Ext}_{\kappa'}(\Sigma_{t'}^{\kappa'} \mid \mathbf{a}) \not\sim_{\text{Ext}} \text{Ext}_{\kappa'}(\Sigma_{t'}^{\kappa'} \mid \neg \mathbf{a}), \quad (186)$$

where $\simeq_{\text{Ext}}$ is a declared equivalence on continuation families. Equation 186 is meaningful only when the application supplies a defensible counterfactual construction. It provides a geometric interface to the trajectory and counterfactual objects introduced earlier without requiring all continuations to share one scalar weight.

Alternative continuations also clarify the difference between possibility and realized history. A later cell can be absent because it was never admissible, because it remained admissible but unrealized, because a prior action closed that branch, or because the model failed to retain the distinction. These cases should not be represented by the same null symbol.

18.9 Emergent Records on Distributed Cellular Support

The emergent structures of Section 17 should generally be realized on distributed cellular support. Trust, shared vocabulary, obligation records, or partnership are historical structures whose witnesses can span several events, faces, or interface regions. A single cell label can summarize such a structure for convenience, yet the emergence claim must remain traceable to the broader history that warrants it.

Let E_X^κ be an emergent relational record satisfying Equation 165. A distributed realization consists of a support family $\mathcal{S}_X^\kappa$ and a witness association ω_X^κ stated in Equation 187.

$$\mathcal{S}_X^\kappa \subseteq \mathcal{P}(\text{Cell}(\mathcal{K}_{AB}^\kappa)), \quad \omega_X^\kappa : \mathcal{S}_X^\kappa \rightarrow \mathcal{W}_X^\chi, \quad (187)$$

where $\mathcal{W}_X^\chi$ is the witness-record type associated with the emergence specification χ . Equation 187 allows several cellular regions to contribute evidence for one emergent object and allows different realizations to support the same abstract record. The notation ω_X^κ denotes a witness association here and should not be confused with an operator-algebraic state functional; the local context keeps the types distinct.

To avoid that collision in later implementations, a computational realization may rename the witness association w_X^κ . The conceptual point is that emergence is supported by a structured history, not localized by fiat. When a compact visualization attaches a trust or partnership label to one region, the model should retain a traceable link to the episodes and relations that justify that label.

Manifestation and relational support can again diverge. An emergent shared norm may be manifested through one phrase, routine, or ritual, while its relational support spans repeated criticism, correction, reliance, and expectation. A partnership may be publicly represented by no special title at all. The 2-complex realization should therefore preserve both the distributed relational support and any local manifestation carriers selected for observation.

18.10 Finite Realizability, Model Limits

The labelled 2-complex makes the dyadic formalism finitely inspectable. A finite example can enumerate cells, specify incidence, assign typed labels, locate subject and interface regions, attach action and manifestation supports, define a historical partial order, and calculate selected algebraic or topological diagnostics. Such a construction can test internal typing and composability. It does not by itself establish that the selected cells are the empirically correct decomposition of a real volunteer relation.

Several limitations should remain explicit. First, cellular decomposition is representation-dependent. The same episode can admit several reasonable segmentations. Second, labels are theory-laden encodings whose empirical interpretation requires evidence. Third, cell incidence captures only the relations deliberately retained by the model. Fourth, a finite complex can omit background institutions, material conditions, or historical relations that remain causally important. Fifth, an apparently stable cellular pattern can arise from different relational histories. The geometric representation therefore increases inspectability while leaving empirical identification and ontological interpretation separate.

The realization also preserves the paper's three-level discipline. Relational reality is represented through persistent configurations with declared causal or generative efficacy. Manifestation reality is represented through contextual labels, boundary events, or local carriers. Subject-relative reconstructions

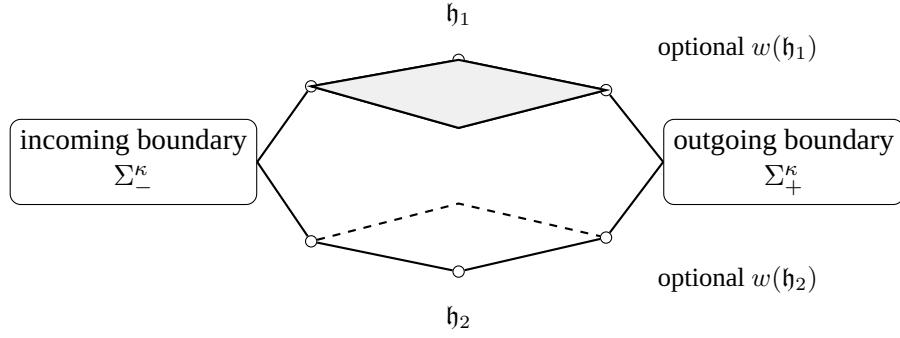


Figure 14: Spin-foam-inspired representation of alternative dyadic histories between declared boundaries. The internal labelled 2-complexes h_1 and h_2 are candidate relational histories compatible with the selected boundary data. Optional history weights require an independently defined carrier and interpretation. The diagram does not assert quantum superposition, physical spacetime discreteness, or a quantum-gravitational ontology.

remain derived records linked to the manifestations from which they are formed. Ultimate ontology remains suspended. The use of a cellular carrier does not alter these epistemic statuses.

The present section has therefore supplied a concrete discrete carrier for the abstract Paper I architecture. Subject histories, interfaces, actions, manifestations, procedure episodes, asymmetries, local algebraic regions, and emergent relational records can now be assigned typed cellular support while remaining distinct from that support. Historical order is declared rather than inferred from incidence, representation can be refined or coarse-grained, actions can alter labels or stronger structural features, and boundary data can support alternative continuation families. The next section adds a stronger optional specialization inspired by spin-foam history models, where boundary structures, internal labelled histories, and history-level weights can be represented with additional mathematical commitments.

19 Spin-Foam-Inspired History Realization

This section develops an optional history-level specialization of the labelled 2-complex realization introduced in Section 18. Its role is to make boundary data, alternative internal histories, partial gluing, and optional history weights explicit. Spin foams supply the structural precedent of labelled two-dimensional histories with lower-dimensional boundary data [1]; Paper I borrows that architecture only. Quantum geometry, Planck-scale discreteness, Hilbert-space completeness, and quantum probability are not inferred from the representation. The discussion therefore treats spin-foam-inspired history structure as a model-relative specialization under ontological suspension.

Figure 14 previews the specialization. A retained incoming boundary Σ_-^κ and outgoing boundary Σ_+^κ can be connected by more than one admissible labelled history. The two internal complexes shown in the figure represent alternative relational continuations rather than simultaneous physical worlds. Their labels can encode procedure stages, relational positions, manifestation types, local algebra identifiers, or other GR-specific data. Optional history weights are attached only after a weight carrier and interpretation have been declared.

19.1 Boundary History Record

The specialization begins by separating boundary data from the internal history that connects them. Let Σ_-^κ denote a retained incoming boundary and Σ_+^κ a retained outgoing boundary at resolution κ . These boundaries can contain subject-state labels, relational positions, manifestation records, local algebra identifiers, or other data selected by the application. Their role is model-relative: they mark the information held fixed when candidate internal histories are compared.

Let $\mathfrak{H}_{AB}^\kappa(\Sigma_-^\kappa, \Sigma_+^\kappa)$ denote the family of admissible labelled histories connecting the two boundaries. Each history $\mathfrak{h}$ contains a labelled 2-complex together with explicit boundary identifications. A convenient history record is stated in Equation 188.

$$\mathfrak{h} := (\mathcal{K}_{AB,\mathfrak{h}}^\kappa, j_-^\mathfrak{h}, j_+^\mathfrak{h}, \lambda_{E,\mathfrak{h}}^\kappa, \lambda_{F,\mathfrak{h}}^\kappa) \in \mathfrak{H}_{AB}^\kappa(\Sigma_-^\kappa, \Sigma_+^\kappa). \quad (188)$$

In Equation 188, $j_-^\mathfrak{h}$ and $j_+^\mathfrak{h}$ identify the selected incoming and outgoing boundary structures with boundary subcomplexes of $\mathcal{K}_{AB,\mathfrak{h}}^\kappa$. The maps $\lambda_{E,\mathfrak{h}}^\kappa$ and $\lambda_{F,\mathfrak{h}}^\kappa$ are typed edge and face labels. The notation resembles the edge- and face-labelling architecture familiar from spin-foam models, but the codomains are GR-specific and need not be representation labels of a Lie group.

Boundary equality is generally too strong. Two histories can agree on the distinctions retained by the dyadic analysis while differing in irrelevant representational detail. The admissibility condition can therefore use a boundary-equivalence relation $\simeq_{\partial,\kappa}$:

$$j_-^\mathfrak{h}(\Sigma_-^\kappa) \simeq_{\partial,\kappa} \partial_- \mathcal{K}_{AB,\mathfrak{h}}^\kappa, \quad j_+^\mathfrak{h}(\Sigma_+^\kappa) \simeq_{\partial,\kappa} \partial_+ \mathcal{K}_{AB,\mathfrak{h}}^\kappa. \quad (189)$$

Equation 189 permits coarse-grained or manifestation-equivalent boundary data to be compared without requiring literal cell identity. The retained boundary criterion must nevertheless be fixed before the history family is interpreted, because changing the criterion can change which histories count as admissible alternatives.

19.2 Internal Labels, GR Semantics

The spin-foam precedent becomes useful only after the label semantics are separated from their physical source. In gravitational spin-foam models, labels carry representation-theoretic information and participate in the definition of amplitudes [1]. In the present dyadic specialization, internal labels are typed records whose meaning is supplied by the volunteer-relation model. A face label may encode a procedure episode, a retained relational constraint, an emergent-record witness, or a local algebra class. An edge label may encode an interface state, role transition, manifestation type, or compatibility datum. No one-to-one correspondence with physical spin or intertwiner labels is assumed.

Let the edge and face label carriers be $\mathcal{L}_E^\kappa$ and $\mathcal{L}_F^\kappa$. The typed label maps are stated in Equation 190.

$$\lambda_{E,\mathfrak{h}}^\kappa : E(\mathcal{K}_{AB,\mathfrak{h}}^\kappa) \rightarrow \mathcal{L}_E^\kappa, \quad \lambda_{F,\mathfrak{h}}^\kappa : F(\mathcal{K}_{AB,\mathfrak{h}}^\kappa) \rightarrow \mathcal{L}_F^\kappa. \quad (190)$$

Partiality in Equation 190 allows the model to retain unlabeled cells where no relevant classification is justified. Several independent label maps can be used when one cell carries multiple kinds of information. This is preferable to constructing one large label alphabet that silently combines manifestation, normative, epistemic, and relational objects.

The three-level reality discipline remains active at the label level. A manifestation label records a context-dependent presentation or observation code. A relational label records a model-relative relational structure. An epistemic label records a subject-relative reconstruction. Ultimate ontology remains outside the label system. The same cell can therefore carry several typed labels without those labels becoming ontologically interchangeable.

Table 22 summarizes the structural resources borrowed from the spin-foam precedent and the stronger meanings that Paper I declines to inherit automatically.

19.3 Alternative Histories, Dyadic Continuations

The principal analytical advantage of the history family is that a selected dyadic boundary can admit several internally different continuations. A request may be followed by immediate support, clarification, refusal, negotiation, delayed assistance, or withdrawal. Two histories can reach boundary-equivalent

Table 22: Spin-foam-inspired resources retained by the dyadic GR specialization and stronger interpretations left unasserted.

Retained structural resource	Interpretation discipline in Paper I
Labelled 2-complex history	used as a discrete relational-history carrier; no claim of fundamental physical discreteness
Boundary structures	used to hold selected dyadic data fixed across history comparison; not assumed to be quantum states
Edge and face labels	typed GR records with application-defined codomains; not automatically group representations or intertwiners
Alternative internal histories	admissible relational continuations between declared boundaries; no automatic superposition claim
History weights	optional model-relative comparison data; probability or amplitude semantics require additional assumptions
History gluing	composition along compatible boundaries; physical locality and quantum composition laws are not inherited automatically
Refinement and coarse-graining	comparison of representational resolutions; no claim that refinement reveals a more fundamental ontology

later configurations while differing in provenance, procedure, trust formation, authority use, or manifestation. Treating them as distinct histories permits the model to preserve those differences even when a coarse endpoint comparison would collapse them.

The alternative-history family is therefore broader than a set of trajectories in one fixed numerical state space. Let

$$\mathfrak{H}_{AB}^{\kappa}[\Sigma_-, \Sigma_+] := \{h \mid h \text{ satisfies Equations 188–189}\}. \quad (191)$$

The family in Equation 191 can contain histories with different numbers of cells, different internal labels, or different incidence structure. A history need not be rejected merely because it cannot be placed pointwise beside another history. Comparison can instead use typed history invariants, boundary-equivalence data, procedure summaries, emergent-record witnesses, or application-defined morphisms between complexes.

This structure is especially useful for counterfactual support analysis. Suppose h_a includes an intervention a while h_{-a} retains a comparable continuation without that intervention. The paper can compare the resulting relational structures, manifestations, epistemic reconstructions, or emergent records without first assigning a scalar welfare difference. The comparison remains conditional on the defensibility of the paired-history construction.

Alternative histories also permit epistemic distinctions. A may assign high plausibility to one continuation family while B anticipates another. The realized history can later revise both anticipatory models. These subject-relative expectations should remain separate from the model’s admissible-history family itself.

19.4 History Composition, Boundary Gluing

History composition becomes available when an outgoing boundary of one history is compatible with the incoming boundary of another. Let $h_1 \in \mathfrak{H}_{AB}^{\kappa}(\Sigma_0, \Sigma_1)$ and $h_2 \in \mathfrak{H}_{AB}^{\kappa}(\Sigma'_1, \Sigma_2)$, and suppose $\Sigma_1 \simeq_{\partial, \kappa} \Sigma'_1$. A partial gluing operation can then be defined:

$$\#_{\partial, \kappa} : \mathfrak{H}_{AB}^{\kappa}(\Sigma'_1, \Sigma_2) \times \mathfrak{H}_{AB}^{\kappa}(\Sigma_0, \Sigma_1) \rightarrow \mathfrak{H}_{AB}^{\kappa}(\Sigma_0, \Sigma_2), \quad (h_2, h_1) \mapsto h_2 \# h_1, \quad (192)$$

The gluing in Equation 192 is defined only when the retained boundary data and label constraints are compatible.

The order in Equation 192 is historical: h_1 precedes h_2 . Gluing can fail because subject tracking is inconsistent, authorization states conflict, local algebra assignments do not match, boundary labels are

incompatible, or the selected resolution does not preserve the required interface. Undefined composition is therefore a substantive modeled outcome.

Associativity should also be stated only when the gluing convention supports it. For three compatible histories, a model may impose

$$(\mathfrak{h}_3 \# \mathfrak{h}_2) \# \mathfrak{h}_1 \simeq_{H, \kappa} \mathfrak{h}_3 \# (\mathfrak{h}_2 \# \mathfrak{h}_1), \quad (193)$$

where $\simeq_{H, \kappa}$ is a declared history-equivalence relation. Equation 193 is weaker than literal equality and accommodates harmless differences in cellular presentation. Applications in which parenthesization changes retained structure, or in which the required intermediate boundary identifications are incompatible, should not impose this condition.

19.5 History Weights, Interpretive Discipline

A spin-foam-inspired model may attach weights to alternative histories, yet the existence of a history family does not require weights. When weights are useful, their carrier and interpretation must be stated before aggregation. Let $\mathfrak{H}_{AB}^{\kappa, \text{adm}}$ denote the disjoint union of the boundary-typed history families admitted by the application, so that each history retains its source and target boundary type. Let $(\mathbb{W}, \odot)$ be a model-specific composition carrier and let

$$w_\kappa : \mathfrak{H}_{AB}^{\kappa, \text{adm}} \longrightarrow \mathbb{W}. \quad (194)$$

Equation 194 assigns a declared weight to each admissible history. The value $w_\kappa(\mathfrak{h})$ may represent accessibility, support, cost, plausibility, heuristic salience, or another declared quantity. It is not a probability merely because it is called a weight.

If the model requires multiplicative compatibility with history gluing, it may impose

$$w_\kappa(\mathfrak{h}_2 \# \mathfrak{h}_1) = w_\kappa(\mathfrak{h}_2) \odot w_\kappa(\mathfrak{h}_1), \quad (195)$$

where Equation 195 applies only to defined compositions. The operation $\odot$ need not be ordinary multiplication unless the chosen carrier warrants it.

Aggregation over alternative histories requires still more structure. If $\mathbb{W}$ also carries an operation $\oplus$ compatible with the intended interpretation, one may define an aggregate history record

$$\mathcal{Z}_\kappa(\Sigma_-, \Sigma_+) := \bigoplus_{\mathfrak{h} \in \mathfrak{H}_{AB}^{\kappa}[\Sigma_-, \Sigma_+]} w_\kappa(\mathfrak{h}), \quad (196)$$

provided the family and aggregation are mathematically well defined. Equation 196 should not be called a probability, amplitude, or partition function unless the application supplies the corresponding semantics. Complex-valued weights would introduce further interpretive obligations, including how observable or decision-relevant quantities are obtained from them. Paper I therefore keeps $\mathbb{W}$ abstract.

This restraint is especially important for volunteer relations. A complex history weight can be mathematically convenient without implying quantum cognition, quantum agency, or literal interference among social histories. The formal value of the construction lies in its capacity to retain and compose multiple histories under explicit rules.

19.6 Boundary Algebra Transport

The history realization can be connected to the local algebraic architecture of Section 15. Suppose the incoming and outgoing boundaries support algebraic carriers $\mathcal{A}^\kappa(\Sigma_-)$ and $\mathcal{A}^\kappa(\Sigma_+)$. When the selected realization supplies a history-dependent algebraic transport, represent it by the typed partial map

$$\mathcal{T}_\mathfrak{h} : \mathcal{D}_\mathfrak{h} \subseteq \mathcal{A}^\kappa(\Sigma_-) \rightharpoonup \mathcal{A}^\kappa(\Sigma_+), \quad (197)$$

Equation 197 defines the boundary-algebra transport associated with a retained history. The domain $\mathcal{D}_{\mathfrak{h}}$ records which boundary-algebraic objects can be transported through the selected history. The map may encode the cumulative effect of communication, support, refusal, revision, changing authorization, or another history-bearing transformation.

When two histories glue and the corresponding algebraic domains are compatible, transport should satisfy a compositional condition of the form

$$\mathcal{T}_{\mathfrak{h}_2 \# \mathfrak{h}_1} \simeq_{\mathcal{A}} \mathcal{T}_{\mathfrak{h}_2} \circ \mathcal{T}_{\mathfrak{h}_1}, \quad (198)$$

on the common defined domain. Equation 198 supplies a bridge between cellular history composition and operator/algebraic dynamics without requiring every history to act on one universal Hilbert space.

Subject-relative epistemic reconstructions can be transported separately. A's reconstruction of the incoming boundary and B's reconstruction of the same boundary can differ, and each can be revised after the realized history. These epistemic updates are not automatically identical with $\mathcal{T}_{\mathfrak{h}}$, which acts on the model's algebraic carrier rather than on a subject's private reconstruction unless the application deliberately identifies those domains.

19.7 Refinement, Coarse-Graining, History Equivalence

A spin-foam-inspired history can be refined by subdividing cells, introducing intermediate procedure stages, or replacing a coarse relational label with several more specific labels. It can be coarse-grained by suppressing distinctions that are irrelevant to the current question. The formal challenge is to preserve the analytical object while permitting the cellular description to change.

Let $r : \mathfrak{h}_f \rightarrow \mathfrak{h}_c$ be a coarse-graining map from a fine history to a coarse history. For a concept X retained across both resolutions, require a comparison condition

$$r_*(\text{supp}_{\kappa_f}(X; \mathfrak{h}_f)) \simeq_{X, \kappa_c} \text{supp}_{\kappa_c}(X; \mathfrak{h}_c). \quad (199)$$

Equation 199 extends the support-compatibility requirement of Section 18 to complete histories. A coarse history can suppress internal distinctions while preserving the relational conclusion relevant to X .

A further compatibility rule is needed if history weights are intended to survive changes of resolution. One possibility aggregates the fine histories mapped to a common coarse history:

$$w_c(\mathfrak{h}_c) \simeq_W \bigoplus_{\mathfrak{h}_f : r(\mathfrak{h}_f) = \mathfrak{h}_c} w_f(\mathfrak{h}_f), \quad (200)$$

where $\simeq_W$ is an application-defined weight equivalence. Equation 200 is optional and requires the aggregation structure introduced above. It should not be imposed when weights have no interpretation that is stable under coarse-graining.

This comparison supplies a robustness test. If an apparent support effect, trust transition, or role change exists only at one highly specific cellular decomposition, the model should report that dependence. Conversely, an emergent relation that remains identifiable across several justified resolutions has stronger representational robustness, though still not ontological proof.

19.8 History Generation Interface

The present section takes an admissible family of labelled histories as given. A stronger future formalism could ask how the complexes themselves are generated, recombined, or removed. One possible direction is a group-field-theory-inspired construction in which elementary relational cells are created and glued by field-like combinatorial rules. Paper I does not develop such a theory, because doing so would require new carrier spaces, interaction kernels, symmetry assumptions, and an interpretation of what cell creation means for subject and relation formation.

The distinction is useful nonetheless. The current spin-foam-inspired layer is a *history realization*: it represents and compares admissible labelled histories. A stronger history-generation layer would specify rules whose outputs are the combinatorial histories themselves. Such rules could become relevant when the research object is subject-boundary formation, changing numbers of relational positions, topology change, or the appearance and disappearance of interaction interfaces. Those questions exceed the minimal dyadic realization developed here and are reserved for later GR formal work.

A related future question concerns endogenous label grammars. Paper I currently assumes that the label carriers $\mathcal{L}_E^\kappa$ and $\mathcal{L}_F^\kappa$ are available to the model. In historically evolving normative systems, the categories used to label action, need, authority, or recognition can themselves change. A history-generation framework may eventually need to generate not only new cells but also new admissible label types. The normative-formation and subject-co-formation papers provide natural interfaces for that extension.

19.9 Three-Level Status, Formal Limits

The spin-foam-inspired realization retains the three-level reality architecture established in Section 5. At the relational level, a labelled history represents persistent and generatively consequential relations at the selected resolution. At the manifestation level, selected boundary and internal labels represent context-dependent expressions, records, or observable events. Subject-relative reconstructions remain epistemic objects linked to those manifestations. Ontological reality remains suspended.

The specialization therefore does not identify the model's labelled 2-complex with ultimate reality. It also does not identify a history weight with moral value, welfare, probability, credibility, or physical amplitude without an additional interpretive map. The same numerical weight could receive different meanings in different models, and different carriers can represent the same qualitative history ordering. The formalism should expose those choices rather than hide them.

Several additional limitations remain. First, the admissible history family depends on the boundary and resolution chosen by the model. Second, internal labels can underdescribe tacit, embodied, institutional, or environmental relations. Third, history comparison can be underdetermined when several reconstructions fit the available manifestations. Fourth, gluing can depend on conventions for boundary equivalence and subject tracking. Fifth, a mathematically elegant weight system can remain empirically unsupported. Sixth, finite spin-foam-inspired histories do not by themselves solve the problem of open-ended future continuation.

The contribution of this section is consequently bounded. It supplies a richer history-level realization in which boundary data, alternative labelled histories, partial composition, optional weights, algebraic transport, and cross-resolution comparison can be made explicit while preserving the philosophical commitments of Paper I. The next section turns from representation to finite dyadic illustrations. Those examples will use small histories to test whether the preceding distinctions remain operational when support is requested, unsolicited, differently interpreted, delayed in consequence, or capable of transforming relational roles.

20 Finite Dyadic Illustrations

This section tests the formal architecture developed in Sections 5–19 on five deliberately small dyadic histories. Its role is diagnostic rather than evidential: each illustration asks whether the distinctions among subject-spacetime, manifestation, epistemic reconstruction, theory of mind, action provenance, procedure, evaluation, authorization, asymmetry, historical revision, and emergence remain operational when several layers are active at once. The objective is to expose where the formalism distinguishes cases that an outcome-only description would collapse together. The discussion proceeds through requested support with procedural agreement, unsolicited support with evaluative divergence, jointly generated action with provenance divergence, delayed consequence with epistemic revision, and repeated support with role reorganization. The method is finite and constructive. Every illustration uses a bounded set of

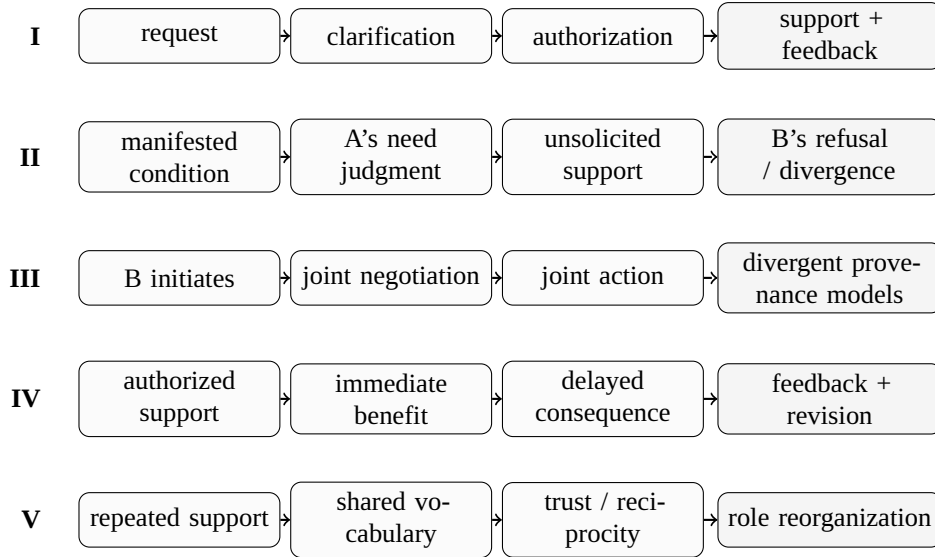


Figure 15: Five finite dyadic illustrations. The histories are diagnostic test cases rather than a developmental sequence. Solid arrows indicate retained procedural or historical continuation. Dashed arrows mark divergence or delayed access that requires a subject-relative or history-sensitive distinction.

Table 23: Formal purpose of the five finite dyadic illustrations.

Case	Retained history pattern	Principal formal distinction tested
I	request, clarification, authorization, execution, feedback	agreement across procedure, need judgment, value judgment, and action-specific authorization without identifying those records
II	observed condition, unilateral need judgment, unsolicited action, refusal	positive A-relative support judgment can coexist with B-relative evaluative divergence and absent authorization basis
III	B initiation, joint negotiation, joint execution	action occurrence and jointness can be shared while provenance reconstructions and attribution roles diverge
IV	authorized action, immediate benefit, delayed consequence, later feedback	relational consequence can precede A's manifestation access and can revise later value judgments and admissible continuations
V	repeated support, shared vocabulary, reciprocal correction, role revision	finite repeated history can satisfy model-relative emergence criteria and reorganize supporter/supportee roles without implying equality

retained events and labels; no example is offered as empirical evidence, a universal volunteer script, or a completed ethical judgment.

A finite illustration is understood here as a model fragment with finitely many retained procedure elements, manifestations, action records, and history stages. Finiteness is therefore representational. It does not imply that the represented subjects or their actual histories are finite. Let $I_r = \{t_0, \dots, t_{n_r}\}$ denote the retained historical indices of illustration r , and let Γ_r^κ denote its finite procedure record. For each example, only the components of the integrated dyadic record in Equation 151 that are needed for the local distinction are displayed. Omitted components remain unspecified rather than being assigned default values.

Figure 15 previews the five histories. The diagrams show retained procedural relations rather than literal psychological sequences. A dashed branch marks either disagreement, delayed access, or historical reorganization whose exact meaning is stated in the corresponding subsection.

Table 23 summarizes the formal distinction emphasized by each example. The table does not assign normative rankings among the cases.

20.1 Finite Illustration Protocol

The illustrations use a common protocol so that their differences arise from the represented history rather than from incompatible notation. First, the subjects are represented by history-bearing subconfigurations $\mathcal{K}_A^\kappa$ and $\mathcal{K}_B^\kappa$ inside a coupled dyadic configuration $\mathcal{K}_{AB}^\kappa$. Second, the supporter and supportee labels refer to relational positions at the retained stage, not fixed identities. Third, every action appears inside a procedure record and carries a provenance record in the sense of Section 10. Fourth, evaluative and authorization records remain separately typed. Fifth, manifestations and subject-relative reconstructions can diverge. Sixth, later stages are compared through typed historical revision rather than subtraction of global state vectors.

Each illustration can be realized on a small labelled 2-complex, yet the cellular description is suppressed unless it clarifies the local point. A procedure element can be supported on one or more edges or faces, and a complete finite history can be regarded as an element of a boundary-conditioned history family $\mathfrak{H}_{AB}^\kappa[\Sigma_-, \Sigma_+]$. The examples therefore remain compatible with Sections 18 and 19 without requiring spin-foam-specific labels or history weights.

The examples also use no primitive scalar welfare function. Where a positive or negative evaluative descriptor is convenient, it is treated as a local element of an explicitly declared value carrier. Let

$$\mathcal{V}_{\text{toy}}^{\text{val}} := \{v^+, v^-, v^\pm, v^?\}, \quad (201)$$

where the symbols denote, respectively, a locally positive judgment, a locally negative judgment, a mixed profile, and an unresolved judgment under the retained dimension and horizon. Equation 201 is a finite pedagogical carrier. It is not a universal utility scale, and no arithmetic is defined on it.

Likewise, the illustrations use the minimal authorization carrier $\mathcal{U}_{\text{toy}} := \{\text{req, perm, con, ref, with}\}$ for request, permission, consent, refusal, and withdrawal. The richer distinctions of Section 13, including acquiescence, compliance, endorsement, standing, and authority basis, remain available when needed. The toy carrier merely keeps the finite examples readable.

A final discipline concerns verdicts. The illustrations can state that a retained authorization basis is absent, that A and B disagree, or that a delayed consequence changes a later value profile. They do not infer a complete ethical verdict from those facts. The later justice and ethics papers will supply additional standards for legitimacy, duty, paternalism, harm, and supererogation.

20.2 Requested Support: Procedural Agreement

The first illustration represents a comparatively aligned support history. B manifests a request for a bounded action $\mathfrak{a}_s$. A asks one clarifying question concerning scope. B confirms the clarified proposal. A executes the action, and B later provides feedback that the requested local objective was achieved. The realized procedure trace is

$$\gamma_1 = (\zeta_{\text{req}}^B, \zeta_{\text{clr}}^A, \zeta_{\text{auth}}^B, \zeta_{\text{exec}}^A(\mathfrak{a}_s), \zeta_{\text{fb}}^B), \quad (202)$$

Equation 202 records the retained procedure trace for the first illustration, with the displayed order understood as a linearly ordered specialization of the procedure record in Equation 81.

The request and clarification generate historically corresponding need inputs d_A and d_B . Suppose both subjects retain the same local need descriptor n_s after clarification:

$$(d_A, n_s) \in \mathcal{J}_{A \rightsquigarrow B, t_1}^{\text{need}, \kappa}, \quad (d_B, n_s) \in \mathcal{J}_{B \rightsquigarrow B, t_1}^{\text{need}, \kappa}. \quad (203)$$

Equation 203 records local agreement, not identity of the subjects' evidential histories. A's judgment is reconstructed through B's manifestation and clarification; B's self-judgment can draw on experiential information unavailable to A.

Let the short-horizon value dimension be β_s and let h_s denote the retained horizon. Suppose both A and B assign v^+ to the proposed action for B:

$$(\beta_s, h_s, v^+) \in \text{ValProf}_{A \rightsquigarrow B, t_2}^\kappa(\mathfrak{a}_s) \cap \text{ValProf}_{B \rightsquigarrow B, t_2}^\kappa(\mathfrak{a}_s). \quad (204)$$

The intersection in Equation 204 is only a concise notation for a shared descriptor under the declared dimension and horizon. It does not imply that the full value profiles are identical.

Let u_s be B's consent record covering the clarified action. Then

$$\alpha_s \in \text{Scope}^\kappa(u_s), \quad t_2 \in \text{Oper}^\kappa(u_s), \quad (205)$$

Equation 205 records the scope and temporal-operativeness conditions retained for the authorization in this illustration, and suppose the local model retains u_s as one basis for $\text{Auth}_{A \rightsquigarrow B, t_2}^\kappa(\alpha_s)$. The point of the example is that request, need agreement, positive value judgment, consent, and authority are separately represented even when they align in one simple history.

The feedback element ζ_{fb}^B confirms the local objective and does not require revision of the action's authorization scope. A historical update can therefore preserve the main records while adding consequence and feedback information. Nothing in the example yet warrants an emergent trust relation under Equation 165; one successful episode can be evidence in a later trust history without satisfying persistence by itself.

At the three-level scale, the request and consent are manifestations whose relational meaning is comparatively well aligned under the adopted procedure. Ontological reality remains suspended. The illustration therefore shows a useful limiting case: the full architecture does not disappear when disagreement is absent. Its role is to make explicit which agreements are actually being claimed.

20.3 Unsolicited Support: Evaluative Divergence

The second illustration keeps the material outcome locally favorable while introducing disagreement concerning need, value, and authorization. A observes a manifestation m_B and reconstructs it as evidence for a need descriptor n_A . A then executes an unsolicited action α_u intended to improve B's immediate access to a resource. B receives the resource but interprets the action as intrusive and refuses continuation of the same support mode.

The initial epistemic chain is represented by

$$m_B \xrightarrow{\nu_A, t_0} o_A \xrightarrow{\rho_A, t_0} \hat{x}_B^A \rightsquigarrow (d_A, n_A) \in J_{A \rightsquigarrow B, t_0}^{\text{need}, \kappa}. \quad (206)$$

The dashed final relation in Equation 206 denotes judgment formation rather than logical entailment. B's self-judgment at the same stage can retain a different descriptor n_B or remain undefined because B does not classify the condition as a support-relevant need in the same way.

The value profiles can diverge across dimensions. Let β_{mat} denote an immediate material-access dimension and β_{ag} an agency/dignity-sensitive dimension. A retains

$$(\beta_{\text{mat}}, h_s, v^+) \in \text{ValProf}_{A \rightsquigarrow B, t_1}^\kappa(\alpha_u), \quad (207)$$

while B retains a mixed profile

$$\{(\beta_{\text{mat}}, h_s, v^+), (\beta_{\text{ag}}, h_s, v^-)\} \subseteq \text{ValProf}_{B \rightsquigarrow B, t_1}^\kappa(\alpha_u). \quad (208)$$

Equations 207–208 show why one positive material effect does not settle the evaluative structure of the action.

Suppose the retained history contains no request, permission, consent, role basis, prior commitment, or emergency condition that the local model accepts as an authority basis for α_u . Then the model records

$$\neg \text{Auth}_{A \rightsquigarrow B, t_1}^\kappa(\alpha_u) \quad (209)$$

under that declared specification. Equation 209 is a local formal statement, not a universal moral verdict. A different application could introduce an emergency basis or another authority rule and obtain a different result.

B's later refusal u_{ref} has positive procedural consequences. If h_1 denotes the procedure prefix before refusal and h_2 the prefix after refusal, then a candidate continuation relation is

$$\text{Cont}_{\text{proc}}^{\kappa}(h_2) \subsetneq \text{Cont}_{\text{proc}}^{\kappa}(h_1), \quad (210)$$

Equation 210 records the modeled contraction in admissible continuations after refusal, where the excluded branch contains repetition of the same unsolicited action under the retained authorization rule. Other forms of interaction can remain available.

This illustration therefore separates material consequence, B-relative value, A-relative value, authorization, and procedural continuation. It also demonstrates that the same manifested transfer can be relationally reconstructed as helpful by A and intrusive by B without requiring the formalism to erase either reconstruction.

20.4 Joint Action: Provenance Divergence

The third illustration keeps action occurrence and joint participation comparatively clear while allowing disagreement about how the action was generated. B initiates a request. A and B negotiate the details and jointly execute an action $\mathfrak{a}_j$. Both later acknowledge that the action occurred and that both participated. The remaining disagreement concerns provenance and authorship-like attribution.

Let the action-attribution record preserve three different roles:

$$\text{Init}(\mathfrak{a}_j) = B, \quad \text{Exec}(\mathfrak{a}_j) = \{A, B\}, \quad \text{Gen}(\mathfrak{a}_j) = AB. \quad (211)$$

Equation 211 states that B initiated the procedure, execution was joint, and the realized action was generated through a coupled negotiation history. The notation AB is a dyadic generative tag, not a third subject.

Let the retained generative provenance record be

$$\Pi^{\kappa}(\mathfrak{a}_j) = \{r_B, c_A, n_{AB}, \pi_{S/R}, h_{AB}^{<t}\}_{\chi}, \quad (212)$$

where r_B is B's request manifestation, c_A is A's clarifying contribution, n_{AB} is the jointly stabilized proposal generated through negotiation, $\pi_{S/R}$ denotes the previously stabilized supporter/supportee role structure retained by the model, and $h_{AB}^{<t}$ is the earlier dyadic history. The subscript χ marks the provenance specification; Equation 212 is a finite selected record rather than an exhaustive causal decomposition.

A's self-reconstruction can emphasize responsive support:

$$\widehat{\Pi(\mathfrak{a}_j)}^A = \{r_B, c_A, n_{AB}\}, \quad (213)$$

Equation 213 records A's provenance reconstruction. B's reconstruction can additionally treat the previously stabilized helper role as motivationally relevant:

$$\widehat{\Pi(\mathfrak{a}_j)}^B = \{r_B, c_A, n_{AB}, \pi_{S/R}\}. \quad (214)$$

Equation 214 records B's provenance reconstruction. The two reconstructions therefore satisfy

$$\widehat{\Pi(\mathfrak{a}_j)}^A \neq \widehat{\Pi(\mathfrak{a}_j)}^B, \quad (215)$$

Equation 215 makes the provenance divergence explicit even though both subjects acknowledge the same executed action and the same joint participation.

The illustration makes three points. First, action agreement does not imply provenance agreement. Second, execution, initiation, generation, and authorship-like interpretation can diverge without contradiction. Third, neither subject-relative provenance reconstruction is automatically identified with the model's retained provenance record. A can lack reflective access to some of the historical structures that

helped generate the action, while B can overattribute importance to a role history that the adopted model later revises. Provenance remains an inspectable, revisable record rather than a hidden essence.

The same structure can be carried by a small 2-complex with the request and clarification on one part of the interface, the negotiated action on a later face, and the provenance labels distributed across the history. The action therefore need not be located at one cell to possess a history-dependent generative record.

20.5 Delayed Consequence: Epistemic Revision

The fourth illustration tests the separation among relational consequence, manifestation, subject-relative access, and later revision. B requests an action α_d , the action is authorized and executed, and both subjects initially judge the short-horizon outcome positively. A delayed relational consequence c_{del} later develops. B encounters the consequence before A receives any manifestation that makes it epistemically available.

Let $t_1 < t_2 < t_3$ be retained stages. The action is executed at t_1 . At t_2 , the delayed consequence belongs to the relational record:

$$c_{\text{del}} \in \mathcal{R}_{t_2}^{\kappa}, \quad (216)$$

Equation 216 records the delayed relational consequence, while the directional manifestation structure satisfies

$$m_{\text{del}}^B \in \mathcal{M}_{\text{self}, B, t_2}^{\kappa}, \quad m_{\text{del}}^A \notin \mathcal{M}_{B \rightarrow A, t_2}^{\kappa}. \quad (217)$$

The first component of Equation 217 records that B has a manifestation channel relevant to the consequence; the second records that no corresponding manifestation is yet available to A through the B-to-A direction retained by the model. The notation does not imply infallible B self-knowledge.

At t_3 , B communicates feedback ζ_{fb}^B that makes a manifestation of the delayed consequence accessible to A. A's reconstruction is then revised:

$$\hat{c}_{\text{del}, A, t_2} = \perp_{ho}, \quad \hat{c}_{\text{del}, A, t_3} \in \hat{\mathcal{R}}_{A, t_3}^{\kappa}, \quad (218)$$

Equation 218 records A's later epistemic revision, where $\perp_{\rho}$ denotes absence of a retained reconstruction rather than a false proposition.

Suppose A's original value profile contained

$$(\beta_{\text{mat}}, h_s, v^+) \in \text{ValProf}_{A \rightsquigarrow B, t_1}^{\kappa}(\alpha_d), \quad (219)$$

Equation 219 records A's earlier value profile, and the later feedback supports a new longer-horizon dependency descriptor v^- in dimension β_{dep} . The revised profile can contain both records:

$$\{(\beta_{\text{mat}}, h_s, v^+), (\beta_{\text{dep}}, h_{\ell}, v^-)\} \subseteq \text{ValProf}_{A \rightsquigarrow B, t_3}^{\kappa}(\alpha_d). \quad (220)$$

Equation 220 records the revised value profile. The later judgment does not rewrite the earlier manifestation history. It revises what A can now infer about the action's consequences over a longer horizon.

The continuation family can also change. Let h_{t_1} and h_{t_3} denote procedure prefixes before and after the delayed feedback. If repeated execution of α_d is removed pending clarification, then

$$\text{Cont}_{\text{proc}}^{\kappa}(h_{t_3}) \subsetneq \text{Cont}_{\text{proc}}^{\kappa}(h_{t_1}). \quad (221)$$

Equation 221 records the resulting revision in admissible continuations. The contraction is historically generated by new information and revised evaluation, not by a timeless prohibition built into the original action type.

This example shows why consequence and knowledge of consequence must be separated. It also shows how feedback can alter value judgment, ToM, and later admissible action families while preserving the fact that the original action was requested and authorized under the earlier record.

20.6 Repeated Support: Role Reorganization

The fifth illustration uses a finite repeated history to test emergent relational structure and role change. Suppose A and B complete three bounded support episodes over interval $I = \{t_0, \dots, t_6\}$. Each episode contains a request or jointly generated proposal, clarification, execution, and feedback. Across the episodes, B develops greater practical competence in the relevant task, A increasingly relies on B's correction of local conditions, and the pair stabilizes a small shared vocabulary for timing, refusal, and revision.

Let $\mathcal{V}_{AB,I}^\kappa$ denote the candidate shared vocabulary generated by the repeated episodes. Suppose the terms or gestures {stop, later, check} become mutually usable in the sense of Section 17.6. A finite witness record can include repeated successful correction episodes and later use without renewed explanation. Under a declared specification χ_V , suppose

$$\text{Emerg}_{\chi_V}(\mathcal{V}_{AB,I}^\kappa; H_{AB,I}) \quad (222)$$

Equation 222 records the candidate shared-vocabulary emergence condition, which holds because persistence, organization, and efficacy are all witnessed on the interval.

The same history can support a typed trust relation. Let d_{ref} be the domain “respect for refusal and correction.” If B increasingly relies on A to stop or revise action when the shared signal is used, and this reliance changes later disclosure and procedure, then a candidate trust record

$$\text{Trust}_{B \rightsquigarrow A, I}^\kappa(d_{\text{ref}}) \quad (223)$$

Equation 223 records the domain-specific trust relation retained in the fifth illustration and can satisfy the emergence criterion under specification χ_T . The existence of this trust record does not imply trust in every domain.

Repeated feedback also changes the role structure. At the beginning of the interval, the retained roles are well approximated by supporter/supportee positions $(\pi_S^\kappa, \pi_R^\kappa)$. By the final episodes, B contributes local knowledge, proposes procedure changes, corrects A's assumptions, and performs parts of the task that were initially executed only by A. Let $\text{Part}_{AB,t_6}^\kappa$ denote the partnership record defined in Section 17.7. A candidate role trajectory is

$$(\pi_S^\kappa, \pi_R^\kappa)_{t_0} \rightsquigarrow_{\chi_P}^{\text{role}} \text{Part}_{AB,t_6}^\kappa. \quad (224)$$

Equation 224 does not state that support has disappeared or that all asymmetries have been eliminated. A can retain greater technical competence or resource access while the procedure becomes genuinely co-participatory in the dimensions retained by χ_P .

The three-level trajectory can also change. Early partnership-relevant manifestations may consist of A asking questions and B answering them. Later manifestations can include B initiating corrections, A deferring to B's local knowledge, and both subjects using the shared vocabulary without explicit negotiation. The relational partnership record can therefore become more stable even while the visible supporter/supportee labels remain unchanged in an external institutional setting.

This finite example illustrates how a dyadic history can generate objects not contained in any one episode. Shared vocabulary, domain-specific trust, reciprocal correction, and partnership are supported by distributed historical evidence. Their emergence remains model-relative and reversible; a later rupture can dissolve trust or restore a more differentiated role pattern.

20.7 Cross-Illustration Comparison: Formal Lessons

The five illustrations can now be compared without introducing a global ranking. The first case shows that agreement does not collapse typed distinctions: request, need judgment, value judgment, consent, and authority remain separate even when they align. The second shows that a locally beneficial material effect can coexist with evaluative disagreement and absent authority under a declared specification. The third shows that shared action occurrence is compatible with provenance disagreement. The fourth shows that relational consequences can precede one subject's epistemic access and can revise later action

families. The fifth shows that repeated finite interaction can generate relational structures whose support is distributed across history rather than located in one event.

Several formal lessons follow. First, the minimal dyad cannot be represented adequately by one action arrow $A \rightarrow B$. Every illustration requires a history-bearing configuration together with some combination of manifestation, reconstruction, evaluation, authorization, and revision. Second, subject-relative disagreement is not an error state to be removed by notation. It is often part of the object being modeled. Third, action identity and action value are history-sensitive. Two materially similar transfers can differ because one was requested and clarified while another was unsolicited and later refused. Fourth, the three-level architecture has operational consequences: relational change, manifestation, and reconstruction can occur at different stages. Fifth, emergence is a historical predicate rather than a synonym for repetition.

The examples also show the value of formal restraint. None required a global metric, one scalar welfare function, a universal probability model, or a global Hilbert space. The toy value carrier in Equation 201 was sufficient to distinguish local positive, negative, mixed, and unresolved judgments. The procedure records used finite partial orders. The delayed-consequence example required only partial manifestation and reconstruction relations. The role-reorganization example used the emergence criterion without assigning a numerical trust score.

A final observation concerns verification. Each illustration can be implemented as a finite labelled 2-complex and checked for internal typing, procedure order, support assignment, history composition, and declared comparison relations. Such implementation would verify consequences of the chosen model, not establish empirical adequacy. The same example could admit several reasonable cellular realizations, and robustness across those realizations is a separate question.

Section 20 therefore functions as a finite formal test suite for Paper I. The examples demonstrate that the architecture can distinguish aligned support, unsolicited intervention, provenance disagreement, delayed consequence, and emergent role change while preserving three-level reality and historical revision. The next section turns to the philosophical discussion opened by these distinctions, including relational lack, supporter-side and coupled genesis, the aesthetics of supporting, and research interfaces that require fuller treatment in subsequent papers.

21 Philosophical Implications: Relational Lack, Self–Other Formation, Aesthetic Support

This section draws several preliminary philosophical implications from the dyadic formal architecture developed in Sections 2–20. Its role is exploratory rather than constitutive. The formalism established earlier does not require a psychoanalytic theory of volunteering, a completed metaphysics of subject formation, a theory of aesthetic value, or a substantive ethics of aid. It does, however, expose questions that become difficult to formulate when supporter, supportee, action, need, and motivation are treated as already individuated objects. The objective here is therefore to identify a small family of research hypotheses opened by the formalism, state their present limits, and locate them within the subsequent GR volunteer research programme. The discussion proceeds through relational lack, alternative patterns of action genesis, self–other relational nonseparability, a psychoanalytic research interface, the aesthetic form of support manifestation, and several normative questions that the present paper intentionally leaves unsettled.

The hypotheses in this section should be read under three restrictions. First, none is a general explanation of volunteer action. Requested support, professionalized volunteering, emergency response, solidarity, companionship, collective action, and other forms of volunteering can have different generative structures. Second, no hypothesis below licenses a reduction of support to hidden self-interest. A support action can be genuinely responsive to B while also participating in A's subject formation, relational stabilization, or search for meaning. Third, the language of lack, desire, fittingness, or relational nonseparability is not introduced here as an ultimate ontology. The three-level discipline of Section 5 re-

Table 24: Preliminary discussion hypotheses opened by the dyadic formalism. The entries identify research directions rather than conclusions established by Paper I.

Research interface	Preliminary dyadic insight	Later destination
Relational lack	Some support actions may partly organize a relational tension or unfilled position expressed through the would-be supporter or the coupled dyad.	Subject co-formation; psychoanalytic formalization
Action genesis	Supportee-centered, supporter-centered, and coupled genesis should remain distinguishable heuristic possibilities.	Subject co-formation; psychoanalytic formalization
Self–other relation	A and B remain analytically distinguishable while their formation can be historically entangled and mutually conditioning.	Subject metaphysics; ethics
Aesthetic support	Manifestation-level timing, tact, pacing, spatial form, restraint, or awkwardness may become aesthetically salient before explicit judgment.	Generative relational aesthetics of support
Normative assessment	Mixed motivation, relational investment, role stabilization, and alterity tolerance can matter without being settled by material benefit alone.	Relational justice; volunteer ethics
Historical formation	Repeated support may generate trust, shared vocabulary, role reorganization, or path-dependent relational structures.	Historical dynamics; political economy and power

mains in force: ultimate ontology is suspended, relational structures are modeled through their historical efficacy, and manifestations remain context-dependent forms of presentation and access.

Table 24 summarizes the hypotheses and their status. The table separates questions that can already be stated inside the dyadic formalism from claims whose substantive development belongs to later papers.

21.1 Relational-Lack Hypothesis

The action-provenance architecture in Section 10 permits a question that is obscured by a simple need-response model. A conventional representation might begin with B’s need, followed by A’s recognition of that need and a support action. The dyadic formalism does not reject that pattern. It shows only that the provenance of an action can include more than one source and that A’s self-description of the provenance need not exhaust the generative history. This opens a further possibility: in some cases, a support action may be generated partly by a lack, tension, incompleteness, or unstable relational position that is expressed through the would-be supporter.

The phrase *relational lack* is intentionally provisional. It should not be read as a primitive variable already admitted to the ontology in Section 7. Nor should it be understood automatically as a measurable deficit, a pathology, or an absent commodity that another subject can supply. The notation $\ell_{AB,t}^\kappa$ serves only as a research placeholder for a model-relative relational tension under specification κ at historical stage t . The placeholder acquires no substantive formal type in the present paper. Its purpose is to mark a future question: can a coupled configuration contain an unfilled, unstable, or recurrent relational position whose organization contributes to the genesis of volunteer action?

A weak schematic hypothesis is stated in Equation 225.

$$\ell_{AB,t}^\kappa \rightsquigarrow \text{PosReorg}_{AB,t}^\kappa \rightsquigarrow \Pi^\kappa(\mathbf{a}), \quad (225)$$

where $\text{PosReorg}_{AB,t}^\kappa$ is a placeholder for reorganization of relational positions and $\Pi^\kappa(\mathbf{a})$ is the action-provenance record developed in Section 10. The double arrow in Equation 225 is deliberately nonfunctional. Paper I does not claim that relational lack determines action, that every support action contains such a lack, or that the relation can be represented by one causal chain.

The hypothesis is most useful as a caution against interpreting volunteering only through B’s independently given need. A may enter volunteer activity while seeking relation, recognition, meaning,

responsibility, continuity of self, opportunities to exercise a valued capacity, or a position in which being needed becomes intelligible. These possibilities can coexist with genuine responsiveness to B. Care ethics is relevant to this coexistence because it treats caring as a moral and political practice embedded in relations and distributions of responsibility rather than as a private sentiment alone [14]. A meal can relieve hunger while the act of preparing and offering it also participates in A's formation as a caregiver. Repeated accompaniment can reduce B's isolation while also reorganizing A's own relational world. A support relation can therefore have more than one generative consequence without becoming morally fraudulent for that reason.

The three-level framework further complicates the hypothesis. A relational tension need not manifest as an explicit statement such as "I need to be needed." Its manifestation can be indirect, distributed, or absent from reflective self-description. Conversely, a manifested desire to help need not correspond to one stable relational structure. The supporter may interpret the same history differently at later times. A later psychoanalytic analysis would therefore need to distinguish at least the relational structure under study, its manifestations, A's reconstruction of those manifestations, B's reconstruction of them, and the later reinterpretation of the action's provenance.

The point is consequently narrower than the claim that volunteering is "really" about the supporter. A relation can be partly generated through A's position without being reducible to A. The next subsection makes this distinction explicit by separating three heuristic patterns of genesis.

21.2 Supporter-Centered, Supportee-Centered, Coupled Genesis

The dyadic model suggests three heuristic orientations for later analysis. They identify where a retained model places the dominant generative burden; they are not mutually exclusive empirical types and need not partition all volunteer relations.

In a *supportee-centered* pattern, B's condition and its manifestations provide the principal retained trigger for action. A observes or receives a request, constructs a need judgment, forms an evaluative and normative response, and acts. This pattern is closest to the ordinary aid narrative and remains fully compatible with the GR formalism. Its main complication is epistemic: the B-relative condition, its manifestation to A, A's reconstruction, and B's own judgment remain distinct records.

In a *supporter-centered* pattern, A's historical position contributes substantially to the genesis of action. The contribution can involve an established role identity, a search for meaning, a normative self-understanding, accumulated guilt, a desire for recognition, a repeated pattern of care, or another relationally constituted structure. The term supporter-centered is therefore weaker than self-interested. It identifies a provenance in which A's history carries significant explanatory weight.

A *coupled* pattern places the generative source in the evolving relation itself. The action becomes available through an interaction whose structure cannot be adequately assigned to either subject as sole origin. A request can become meaningful only after earlier trust; A's willingness to offer support can depend on B's previous correction; B's willingness to accept can depend on A's demonstrated respect for refusal; a shared vocabulary can make a later action possible without either subject possessing that vocabulary before the relationship formed. In such cases, the relation generates conditions that later appear as capacities, expectations, or intentions of the individual participants.

The three patterns can be summarized schematically in Equation 228.

$$\text{Gen}_B : \mathcal{K}_B \rightsquigarrow \Pi^\kappa(\mathbf{a}), \quad (226)$$

$$\text{Gen}_A : \mathcal{K}_A \rightsquigarrow \Pi^\kappa(\mathbf{a}), \quad (227)$$

$$\text{Gen}_{AB} : H_{AB, \leq t}^\kappa \rightsquigarrow \Pi^\kappa(\mathbf{a}). \quad (228)$$

The arrows in Equation 228 indicate heuristic provenance orientation rather than causal sufficiency. A concrete history can satisfy more than one orientation.

The coupled case is especially important for GR because it resists a model in which relations merely connect subjects who were fully constituted beforehand. Sections 6, 16, and 17 permit the subject configurations, relation, and admissible continuations to change together. An interaction can alter what A

can become, what B can become, and what future relation between them is possible. This makes the language of mutual constitution available without erasing the analytical distinction between A and B.

21.3 Self–Other Relational Nonseparability

The discussion of supporter-side genesis can become misleading if A and B are treated as sharply bounded substances whose internal needs merely interact externally. Paper I has used subject-spacetime notation precisely to avoid that presupposition. At a declared resolution, $\mathcal{K}_A^\kappa$ and $\mathcal{K}_B^\kappa$ can be analytically distinguished, historically tracked, and assigned different manifestations, capacities, responsibilities, and authorization records. Their coupled configuration $\mathcal{K}_{AB}^\kappa$ can nevertheless contain shared history, interface structure, jointly generated vocabulary, distributed trust records, and emergent relations that cannot be located inside either subject alone.

A cautious philosophical formulation is therefore:

$$\boxed{\text{self and other are distinguishable without being relationally separable.}} \quad (229)$$

Equation 229 is a methodological proposition for the present programme, not an identity claim. It does not state $A = B$, dissolve responsibility, or imply that every boundary between persons is artificial. The formalism continues to require subject-relative manifestations, consent, refusal, standing, authority, and asymmetry. Those distinctions would be unintelligible if the subjects were simply identical.

The proposition instead concerns constitution. Some parts of what A can intelligibly be at t_1 can depend on the history with B before t_1 . The same holds in the other direction. A role such as supporter can require a supportee position to become meaningful. A shared vocabulary exists only through a history of mutual uptake. Trust is directional yet relationally constituted. A refusal can reorganize A's later understanding of support, while A's response to the refusal can change B's willingness to disclose. The boundary remains analytically usable even as the history crossing that boundary becomes part of each subject's later configuration.

This qualification matters for the phrase “whose need is being met?” In some cases the question is well formed and indispensable. A concrete material need of B can be identified and evaluated without ambiguity. In other cases, especially those involving companionship, identity, shared practice, or long-term mutual formation, a binary allocation of need to A or B can underdescribe the process. The relation may generate or transform the relevant needs, capacities, and meanings as it develops. The question can then require reformulation: which relational conditions are being generated, stabilized, or altered, and how do those changes affect each subject's later possibilities?

The expression 自他无二 can motivate this direction of inquiry, but Paper I does not formalize it as metaphysical identity. The formal commitment remains weaker and more inspectable: subject distinction and relational entanglement can coexist. A subsequent metaphysics paper can ask how far that claim should be strengthened.

21.4 Psychoanalytic Research Interface

The relational-lack hypothesis creates a natural interface with psychoanalytic theories of lack, desire, symbolic position, repetition, and identification. Lacan's account of the subject, repetition, drive, and symbolic structure is one relevant resource for the later paper, without being adopted here as GR ontology [9]. Paper I does not develop that interface into a psychoanalytic model. It records several questions whose formal statement is now possible because action provenance, role position, manifestation, self-reconstruction, and repeated dyadic history have already been separated.

One question concerns the helper position itself. A history can stabilize a role in which A becomes intelligible to A through being useful, needed, rescuing, protecting, teaching, or repairing. The role may be healthy, contingent, institutionally supported, or problematic; Paper I assigns no default evaluation. The psychoanalytic question is whether the persistence of the role sometimes performs subject-organizing work that exceeds the immediate needs of B.

A second question concerns successful support. If A's subject stability becomes strongly invested in the persistence of the helper position, then a reduction in B's need can alter more than the practical task. It can destabilize the relation through which A's role has been sustained. A later paper can ask whether such a structure helps explain over-helping, resistance to handover, disappointment when B becomes independent, or moralized demands for gratitude. The current formalism supplies several places at which these possibilities could become visible: action provenance, role trajectory, authorization history, feedback response, trust/distrust, and the continuation family after refusal.

A third question concerns repetition. Distinct support relationships can reproduce a similar helper-supportee pattern even when the concrete B changes. If the pattern is historically recurrent across A's relations, a later model may need to represent a higher-order relational regularity rather than analyze each dyad independently. Such an extension exceeds the strict two-subject boundary of Paper I and belongs naturally to the psychoanalytic and subject-formation papers.

A fourth question concerns remainder. The formal architecture has deliberately avoided assuming that every lack is a measurable deficit that can be restored by transferring an element. A later psychoanalytic treatment may ask whether some relational tensions are produced through symbolization, role formation, or the very constitution of the subject and therefore cannot be represented adequately as missing quantities. This question should remain open until the relevant mathematical carrier and philosophical commitments are stated. The present paper does not turn a psychoanalytic metaphor into a primitive operator.

The interface is therefore methodological. Psychoanalytic concepts can place pressure on action-provenance and subject-formation models by asking whether articulated motives exhaust the generative structure of action, whether relational roles participate in the production of desire, and whether repetition can preserve a relation that explicit normative judgment would otherwise revise. A later paper can test those possibilities without retroactively converting the dyadic formalism into psychoanalytic ontology.

21.5 Aesthetic Form, Support Manifestation

Section 12 introduced aesthetic judgment as a distinct evaluative family and allowed pre-explicit manifestation structure to precede explicit articulation. The finite and procedural sections make the philosophical significance of that move clearer. Support has a manner of appearing and unfolding. Timing, tone, pacing, spatial distance, bodily orientation, silence, tact, restraint, interruption, sequencing, symmetry, asymmetry, delicacy, abruptness, or excess can shape how the action is experienced before either subject can formulate an explicit aesthetic judgment.

The relevant hypothesis is not that a completed aesthetic judgment exists before judgment. It is that manifestation reality can possess aesthetically salient organization before reflective articulation. A preliminary recursive path is stated in Equation 230.

$$\mathcal{R}_t \xrightarrow{\mu_t} \mathcal{M}_t^{\text{aes}} \rightsquigarrow (E_{A,t}, E_{B,t}) \rightsquigarrow E_{AB,t'} \rightsquigarrow \mathcal{M}_{t''}, \quad t \preceq t' \preceq t'', \quad (230)$$

where $\mathcal{M}_t^{\text{aes}}$ denotes manifestations that become aesthetically salient under the retained analysis and E denotes aesthetic-evaluative records. The arrows after manifestation remain schematic because Paper I does not specify a universal psychological sequence.

This possibility matters because two interventions with similar material effects can differ substantially in relational form. A support action can be materially useful while manifesting as humiliating, intrusive, performative, hurried, or excessively controlling. Another action can preserve comparable material efficacy while manifesting tact, proportion, respect for pace, or a form of restraint that allows B to retain initiative. These differences can alter later trust, disclosure, refusal, gratitude, resentment, and willingness to continue the relation. Aesthetic form can therefore become generatively relevant without being reducible to ethical validity.

The reverse direction is equally important. Once A and B have stabilized a vocabulary of what feels tactful, dignified, awkward, cold, warm, excessive, or fitting, that vocabulary can reshape later attention. A gesture that was previously unnoticed can become salient; a pause can become legible as respect for

deliberation; a familiar form of giving can become experienced as patronizing after the role relation changes. The aesthetic history of the dyad can therefore alter the manifestation grammar through which later support is interpreted.

The present paper should not collapse aesthetics into ethics. An elegant or harmonious support procedure can still be unjust, manipulative, or unauthorized. An awkward emergency intervention can be ethically justified. The research question is narrower: how does the aesthetic organization of manifestation participate in the relational history of support, and when does it alter epistemic, normative, or affective conditions of later interaction? A dedicated GR aesthetics paper can develop dignity, humiliation, tact, intrusion, rhythm, restraint, excess, shared sensibility, and aesthetic asymmetry in greater depth.

21.6 Normative Research Questions

The discussion above generates normative questions that Paper I is designed to expose rather than settle. The first concerns mixed motivation. If A's action both benefits B and contributes to A's subject formation, the mere presence of A-relative benefit does not show that the action is morally defective. Many sustained relations generate value for more than one participant. The relevant ethical questions concern the structure of the relation: whether B's agency is respected, whether refusal remains effective, whether A can tolerate role change, whether material or symbolic dependence is manufactured unnecessarily, and whether the action remains revisable when B's judgment diverges.

The second concerns supporter self-investment. A strong investment in the helper role can become normatively significant when it changes the continuation family available to B. If A resists handover, discounts refusal, interprets independence as ingratitude, or repeatedly reconstructs B as needing support in order to preserve A's own position, the issue is no longer merely motivational impurity. The investment has entered relational reality through its effects on authorization, interpretation, and future possibilities.

The third concerns paternalism. Section 13 already separated a positive A-relative value judgment from authority to intervene. The present discussion adds a provenance question: when A's own relational investment contributes to the judgment that B "needs" intervention, the epistemic and normative burden of inquiry may increase. A later ethics paper can ask whether mature support requires a capacity to revise not only one's judgment of B but also one's own supporter identity.

The fourth concerns alterity tolerance. The formal distinction between B and A's reconstruction of B makes room for a demanding ethical possibility: good support may require A to tolerate B's resistance to the relational position A had expected B to occupy. B can refuse the action, reinterpret its meaning, reject gratitude expectations, alter the shared vocabulary, or leave the relationship. The capacity of the support relation to remain generative under such divergence may be a more informative ethical criterion than agreement alone.

The fifth concerns the success condition of support. Some support relations may reasonably aim at the disappearance of the supporter/supportee asymmetry; others, such as companionship or long-term communal practice, may aim at durable relation rather than role elimination. The formalism therefore does not impose "making oneself unnecessary" as a universal criterion. A later justice and ethics programme should distinguish relations in which successful support reduces dependence from relations in which successful support transforms the quality of interdependence.

21.7 Research Programme Interfaces

The discussion-level hypotheses clarify why Paper I is foundational rather than exhaustive. The dyadic formalism supplies typed objects and histories that later papers can specialize without redefining the basic architecture each time.

The subject co-formation paper can develop how $\mathcal{K}_A$, $\mathcal{K}_B$, and $\mathcal{K}_{AB}$ co-evolve, including role internalization, boundary change, practical-horizon expansion, and migration of shared values back into subject structure. The psychoanalytic paper can develop relational lack, desire, helper/supportee positioning, repetition, gratitude, refusal, and subject investment in asymmetry. The aesthetics paper can

develop manifestation-level form, shared sensibility, dignity, humiliation, tact, rhythm, restraint, and the recursive formation of aesthetic judgment. The justice paper can evaluate distributive, procedural, recognitional, epistemic, and relational dimensions of aid. The ethics paper can develop duties of inquiry, revisability, restraint, competence, refusal, exit, and alterity tolerance.

The political-economy and power paper can extend the dyad into resource, institutional, symbolic-capital, and unpaid-labor structures while preserving the typed distinction between asymmetry and power established in Section 14. The historical-dynamics paper can study trust formation, relational nucleation, path dependence, role attractors, heterogeneous value generation, and the possible democratic significance of shared volunteer practice. The triadic formalism can introduce organizations, communities, witnesses, mediators, states, and third-party accountability that the dyadic boundary has deliberately excluded.

Paper I therefore closes its philosophical discussion with a limited claim. The apparently simple relation in which one subject supports another can be generated by more than an independently given need and an already constituted helper. Subjects, roles, manifestations, judgments, authorizations, and actions can be historically co-produced, and their later meanings can depend on the trajectories through which they formed. That claim does not decide why people volunteer, what good support ultimately is, or which volunteer institutions are just. It identifies a formal and philosophical domain in which those questions can be asked without presupposing the answers in the initial notation.

22 Formal Verification, Empirical Interface, Model Limitations

This section specifies how the dyadic formalism should be checked, instantiated, challenged, and limited once its conceptual and mathematical architecture has been assembled. Its role is epistemic and methodological: the preceding sections define a family of typed objects, historical relations, partial transformations, evaluative records, algebraic realizations, and finite illustrations, while the present section states which forms of success those constructions can and cannot establish. The objective is to separate type correctness, formal consistency, finite reproducibility, representation robustness, empirical identification, normative validity, and ontological interpretation. The discussion proceeds through a layered verification architecture, formal and computational obligations, representation sensitivity, empirical calibration, identifiability, normative and ontological limits, the dyadic boundary, and explicit failure conditions. The method is diagnostic. A formalism becomes more useful when it becomes easier to determine where it fails, which conclusions depend on modeling choices, and which claims require evidence or argument beyond mathematical coherence.

A central discipline of the paper is that different standards of adequacy answer different questions. A well-typed operator can still represent the wrong social process. A finite computation can be reproducible while the empirical coding that produced its inputs remains contestable. A robust descriptive model can leave normative legitimacy unresolved. A mathematically successful spin-foam-inspired realization does not establish that volunteer relations possess quantum-gravitational ontology. The verification architecture must therefore prevent success at one layer from being silently promoted into success at another.

Figure 16 summarizes this separation. The central path runs from typing and formal consistency through finite realization, representation robustness, and empirical identification. Normative validation and ontological interpretation are shown as distinct research interfaces rather than as later rungs that follow automatically from empirical adequacy.

22.1 Verification Layers, Epistemic Separation

Let $\mathcal{M}^\kappa$ denote a particular instantiated dyadic model at resolution κ . Paper I associates with that model a verification record rather than a single score. Write

$$\mathfrak{V}_\chi(\mathcal{M}^\kappa) := (T_\chi, C_\chi, F_\chi, R_\chi, E_\chi, N_\chi, O_\chi), \quad (231)$$

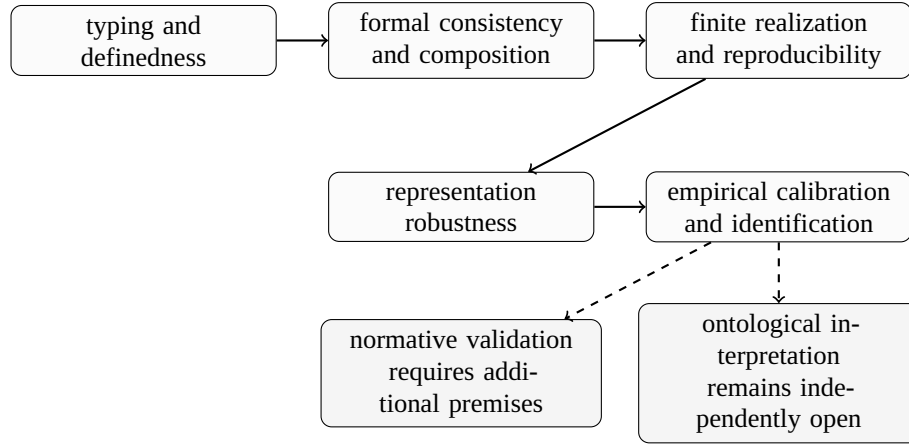


Figure 16: Verification architecture for the dyadic formalism. The central sequence records increasingly demanding descriptive and empirical obligations. Normative validity and ontological interpretation remain distinct interfaces: neither follows automatically from formal consistency, reproducibility, robustness, or empirical fit.

where χ is the declared verification specification. The components record, respectively, typing and definedness, formal consistency, finite reproducibility, representation robustness, empirical identification, normative validation, and ontological interpretation. Equation 231 is a bookkeeping device. It does not impose an ordering on the components and does not define an aggregate measure of model quality.

The first four components can often be assessed within the formal research programme itself. T_χ asks whether every object has the declared type, domain, codomain, and level of reality. C_χ asks whether the stated compositions, equivalences, restrictions, histories, and propositions are mutually coherent under their assumptions. F_χ asks whether finite instances can be reproduced and checked. R_χ asks whether retained conclusions survive an admitted family of reasonable representational choices.

The remaining components require distinct forms of inquiry. E_χ concerns the mapping between observations and the formal objects, including identifiability and comparison with alternatives. N_χ concerns the validity of normative conclusions and therefore requires substantive ethical, political, legal, or other evaluative premises beyond the existence of a formal judgment record. O_χ concerns ontological interpretation and remains constrained by the ontological suspension adopted throughout the paper.

The separation is deliberately strict. In particular,

$$C_\chi = \text{adequate} \not\Rightarrow E_\chi = \text{adequate}, \quad E_\chi = \text{adequate} \not\Rightarrow N_\chi = \text{adequate}, \quad (232)$$

and neither descriptive adequacy nor normative defensibility entails a determinate claim about ultimate ontology. Equation 232 prevents formal explicitness from functioning as a substitute for empirical evidence or normative argument.

22.2 Type Correctness, Compositional Consistency

The first verification obligation concerns type discipline. Paper I has introduced concept families whose members belong to different formal domains: subject-spacetimes, manifestations, epistemic reconstructions, ToM objects, evaluative judgments, authorization records, asymmetry records, local algebras, operators, history complexes, and emergent relations. A valid formula must preserve those distinctions. An operation defined on a manifestation record cannot be applied to a relational object unless an explicit map relates the two. A partial transformation cannot be used outside its declared domain. An action-provenance relation cannot be subtracted from a trust record merely because both are associated with one interaction.

For a finite expression $\mathcal{E}$ built from the paper's symbols, let $\text{TypeOK}_\chi(\mathcal{E})$ mean that every primitive and derived operation in $\mathcal{E}$ is applied to an object in its declared domain, that every composition has

matching interfaces, and that every equality or equivalence compares objects whose types admit the retained comparison. The predicate is intentionally syntactic-relative-to-semantics: a formally well-typed expression can remain substantively inappropriate if the empirical interpretation of its symbols is poor.

Composition requires an additional check. Consider two partial transformations $\Phi_1 : \mathcal{D}_1 \rightarrow \mathcal{C}_1$ and $\Phi_2 : \mathcal{D}_2 \rightarrow \mathcal{C}_2$. The composite $\Phi_2 \circ \Phi_1$ is defined only on configurations $x \in \mathcal{D}_1$ for which $\Phi_1(x) \in \mathcal{D}_2$. A verification pass must therefore test the realized composition domain rather than merely confirm that both transformations exist in the model. The same burden applies to procedure composition, history gluing, interface transport, algebra embeddings, and cross-level reconstruction.

Historical tracking adds another layer. A comparison between X_t and $X_{t'}$ requires a declared tracking relation or continuation criterion. Repeated labels do not establish persistence. If a subject boundary, role, trust relation, or norm is claimed to persist, the model must state what makes the later object count as the continuation of the earlier one. This requirement is especially important under refinement, coarse-graining, role change, and topology-sensitive history reconstruction.

The formal consistency audit should therefore include at least the following checks: symbol definition before use; domain and codomain; total versus partial status; resolution index; reality-level status; subject-relative epistemic index where relevant; historical tracking relation; composition domain; equivalence relation used for comparison; and assumptions needed by any stronger algebraic specialization. These obligations operationalize the concept-audit protocol introduced in Section 7.

22.3 Finite Reproducibility

The second verification layer asks whether the formal objects can be instantiated and checked on finite examples. Section 20 supplied five conceptual test histories. A stronger computational realization can encode a finite labelled 2-complex, local manifestation records, partial procedure maps, action-provenance relations, authorization states, and local algebraic transformations in data structures or matrices where the chosen carrier supports them.

Finite reproducibility is useful because it exposes mistakes that remain difficult to detect in symbolic prose. A partial map can be evaluated on every retained domain element. Two action orders can be compared directly. A proposed gluing can be checked for boundary compatibility. A local algebra embedding can be tested for homomorphism properties. A finite history family can be enumerated. A coarse-graining map can be checked against a declared equivalence. These checks do not require the substantive model to be empirically true.

Let $\mathcal{I}_\chi^{\text{fin}}$ denote a finite instantiation and let $\mathcal{Q}$ denote a formally stated query, proposition, or diagnostic. For two independent executions r_1 and r_2 under the declared encoding, write their outputs as $\text{Out}_{r_1}(\mathcal{Q})$ and $\text{Out}_{r_2}(\mathcal{Q})$. The reproducibility condition can then be written schematically as

$$\text{Repr}_\chi(\mathcal{Q}; \mathcal{I}_\chi^{\text{fin}}) \iff \text{Out}_{r_1}(\mathcal{Q}) \approx_\chi^{\mathcal{Q}} \text{Out}_{r_2}(\mathcal{Q}). \quad (233)$$

Equation 233 states the finite reproducibility condition relative to the declared encoding. The equivalence $\approx_\chi^{\mathcal{Q}}$ allows representationally irrelevant differences such as ordering of stored records or alternative identifiers where the mathematical result is unchanged.

Reproducibility should remain separate from empirical calibration. A perfectly reproducible computation can implement an empirically unmotivated encoding. Conversely, an empirically well-motivated qualitative study can resist reduction to a convenient finite matrix. The value of finite checking is narrower: it establishes that the declared formal consequences actually follow in the chosen finite representation.

22.4 Representation Robustness

Many central objects in Paper I depend on representational choices made before calculation begins. These include the resolution κ , subject boundaries, the retained dyadic window, cell decomposition, action individuation, manifestation coding, equivalence criteria, ToM content types, value dimensions,

trust domains, local algebra classes, history boundaries, refinement rules, and the threshold or witness conditions used for emergence. A conclusion that changes under small and substantively reasonable changes in these choices should be reported as representation-sensitive.

Let $\mathfrak{P}_\chi = \{p_1, \dots, p_m\}$ denote an admitted family of representation specifications. Each $p \in \mathfrak{P}_\chi$ induces an instantiated model $\mathfrak{M}_p$ and a retained conclusion Q_p . A nonmetric robustness predicate can be defined by

$$\text{Rob}_\chi(Q; \mathfrak{P}_\chi) \iff Q_p \approx_Q^\chi Q_{p'} \quad \text{for all admitted } p, p' \in \mathfrak{P}_\chi, \quad (234)$$

where $\approx_Q^\chi$ is a declared equivalence relation on the conclusion type. Equation 234 avoids assuming a numerical distance among representations. If a metric or ordered discrepancy is substantively justified in a particular application, a stronger sensitivity measure may be introduced there.

Robustness can fail in informative ways. A trust-emergence result may depend on whether one episode is coded as a correction or as a new action. An asymmetry classification may disappear when the retained dimension changes. A provenance reconstruction may change when an earlier interaction enters the historical window. A role-stabilization claim may depend on the temporal resolution used to distinguish repeated manifestations. Such failures do not automatically invalidate the model; they identify which conclusions belong to the chosen representation rather than to a broader equivalence class of reasonable encodings.

At least four families of sensitivity deserve explicit reporting in later applications. *Boundary sensitivity* concerns subject and dyadic-system boundaries. *Resolution sensitivity* concerns refinement and coarse-graining. *Semantic sensitivity* concerns coding of manifestations, actions, judgments, and roles. *Formal sensitivity* concerns the choice of algebra, equivalence, ordering, topology, or emergence criterion. These families need not be reducible to one robustness score.

22.5 Empirical Calibration

Empirical application begins with manifestations and records, not with pre-labelled GR objects. Interviews, observed interactions, documents, logs, institutional records, sensor measurements, or other data sources do not arrive already partitioned into subject-spacetimes, need judgments, action-provenance structures, authorization states, trust relations, local algebras, or emergent norms. Those objects have to be constructed through a declared empirical specification.

This distinction can be represented by a two-stage calibration architecture. Let $\mathcal{D}^{\text{obs}}$ denote the retained observed material. A coding map first produces a manifestation/data record,

$$\chi_\zeta : \mathcal{D}^{\text{obs}} \rightarrow \widehat{\mathcal{M}}_\zeta, \quad (235)$$

Equation 235 defines the first empirical coding stage. Here ζ denotes the empirical specification, including sampling, segmentation, coding rules, uncertainty handling, and resolution. A reconstruction relation then maps coded manifestations into one or more admissible relational-model candidates,

$$\rho_\zeta^{\text{emp}} : \widehat{\mathcal{M}}_\zeta \rightrightarrows \widehat{\mathfrak{R}}_{AB, \zeta}. \quad (236)$$

The double arrow in Equation 236 is important. The data may support more than one relational reconstruction. Empirical calibration should preserve that multiplicity rather than force every manifestation pattern into one hidden-state estimate.

The calibration architecture mirrors the three-level discipline of Section 5. Observed manifestations do not become relational reality simply because they are measured reliably. A signed consent form, for example, can be observed with high confidence while the relational status of consent remains contestable because refusal capacity, information, dependence, or scope remain uncertain. A repeated supportive gesture can be manifest while the action's provenance remains underdetermined. An expressed claim of trust can be recorded while the relational efficacy criterion for emergent trust remains unsatisfied.

Empirical use therefore requires a declared evidence policy for each formal object. The policy should state what observations count as manifestations, which inferences are used to construct relational records,

how subject-relative interpretations are represented, which uncertainties remain unresolved, and what observations could challenge the proposed reconstruction. The formalism becomes empirically useful only when these mappings are explicit enough to be criticized and revised.

22.6 Observational Equivalence, Identifiability

The distinction between relational and manifestation realities creates an unavoidable identifiability problem. Different relational configurations can produce similar manifestations, while one relational configuration can produce different manifestations across contexts. An empirical model should therefore report when the available observations fail to distinguish among materially different relational reconstructions.

Let $\mathcal{I}_1$ and $\mathcal{I}_2$ denote two instantiated relational models under the same empirical specification ζ . Let $\mathcal{O}_\zeta$ denote the forward manifestation/observation map used to derive predicted observation records. Define observational equivalence by

$$\mathcal{I}_1 \sim_{\text{obs}, \zeta} \mathcal{I}_2 \iff \mathcal{O}_\zeta(\mathcal{I}_1) \approx_{\mathcal{M}}^\zeta \mathcal{O}_\zeta(\mathcal{I}_2), \quad (237)$$

where $\approx_{\mathcal{M}}^\zeta$ is the declared equivalence on predicted manifestation records. If $\mathcal{I}_1$ and $\mathcal{I}_2$ differ on a relational object relevant to the research question while satisfying Equation 237, that object is not identifiable from the retained observations under ζ .

Several objects in Paper I are especially vulnerable to non-identifiability. Action provenance can remain ambiguous even when the action itself is observed. A supporter role can be publicly manifest while the subject's self-identification with that role remains unclear. Trust and strategic compliance can produce similar short-term behavior. Consent and acquiescence can share a manifestation. Shared vocabulary and merely coordinated wording can be difficult to distinguish. A history-generated norm and coincident private judgments can look similar in a single snapshot.

Identifiability can improve through additional historical, comparative, or intervention evidence, but the formalism does not assume that every latent relational object can eventually be recovered. Some distinctions may remain underdetermined relative to the available manifestations. Reporting that underdetermination is a successful outcome of the model, not a failure of formalization.

22.7 Normative Validation Limits

Paper I formalizes normative judgments, shared norms, standing, authority, obligation records, legitimacy attributions, and authorization histories. The existence of these objects inside a model does not establish that their content is normatively correct. The paper has maintained the distinction between normative formation and normative validity precisely to prevent that inference.

Let $\text{Form}_{N, \chi}(X)$ denote that a normative object X has been successfully formed, tracked, or reconstructed under specification χ , and let $\text{Valid}_{N, \eta}(X)$ denote a validity judgment under an independently declared normative framework η . The formal separation is

$$\text{Form}_{N, \chi}(X) \not\Rightarrow \text{Valid}_{N, \eta}(X). \quad (238)$$

Equation 238 records the non-entailment between normative formation and normative validity. A stable shared norm can be unjust. A widely recognized authority can lack legitimacy. A formally valid consent procedure can remain ethically problematic under a richer account of dependency or information. A trust relation can support harmful coordination. A role can be mutually recognized while constraining later possibilities in objectionable ways.

Conversely, a normatively defensible action can remain weakly manifested, socially contested, or relationally unstable. Normative validity therefore cannot be inferred from prevalence, persistence, institutionalization, shared vocabulary, or relational efficacy alone. Later ethics and justice papers must supply the substantive criteria that Paper I deliberately withholds.

This limit also applies to the finite illustrations. The examples demonstrate that the formal architecture can distinguish agreement, divergence, provenance ambiguity, delayed consequences, and role

reorganization. They do not rank the examples morally. Any later ranking must specify the normative dimensions retained, the relation among them, the standing of affected subjects, and the reasons that justify the comparison.

22.8 Ontological Interpretation Limits

The dyad's mathematical realization remains subject to ontological suspension. A labelled 2-complex can represent history-bearing relational organization without implying that social reality is fundamentally cellular. A local algebra can organize admissible quantities or transformations without implying that relational reality is fundamentally a C^* -algebra. A spin-foam-inspired history family can represent alternative labelled histories without implying quantum geometry. An operator representation can make composition calculable without proving that action is ontologically an operator.

The same restraint applies when a formal structure predicts or organizes observations well. Empirical fit can support the adequacy of a representation for a research purpose. It does not establish identity between the represented social process and the mathematical ontology from which the formal tool was historically borrowed. The inferential gap can be stated schematically as

$$\text{Adeq}_\chi(\mathfrak{F}, \mathcal{D}) \not\Rightarrow \text{OntId}(\mathfrak{F}, \mathcal{O}), \quad (239)$$

where $\mathfrak{F}$ is a formal representation, $\mathcal{D}$ is the retained empirical domain, and $\mathcal{O}$ denotes ultimate ontology. Equation 239 is not a theorem about metaphysics; it records the methodological discipline adopted by the GR programme.

Ontological suspension is compatible with substantive comparison among representations. One representation can preserve history better, expose action provenance more clearly, support better finite checking, or fit observations more robustly than another. Such comparisons concern analytical adequacy. They leave open whether any successful formalism captures the ultimate mode of being of the represented relation.

22.9 Dyadic Boundary, External Structure

The minimal dyad is an analytical boundary, not a claim of causal closure. Real volunteer relations can be embedded in families, organizations, communities, markets, states, professional regimes, infrastructures, cultural traditions, legal orders, donor structures, and broader histories. Those external relations can shape the capacities, manifestations, norms, resources, and action families available to A and B.

The dyadic model can represent such influence only indirectly unless the external structure is explicitly added. Let $\mathcal{K}_{AB}^\kappa$ denote the retained dyadic subconfiguration inside a wider represented spacetime $\mathcal{K}^\kappa$. Its external interface can be recorded schematically as

$$\partial_{\text{ext}} \mathcal{K}_{AB}^\kappa := \text{Int}_\kappa(\mathcal{K}_{AB}^\kappa, \mathcal{K}^\kappa \setminus \mathcal{K}_{AB}^\kappa), \quad (240)$$

where Int_κ is an application-specific interface construction. Equation 240 does not require a set-theoretic complement interpretation when the represented complex uses overlapping substructures; the notation marks the external relations omitted by the local dyadic closure.

A dyadic conclusion becomes fragile when omitted external structure carries the generative burden of the phenomenon being explained. Apparent A-to-B authority may actually derive from an organization. B's refusal capacity may depend on material alternatives supplied by a third party. A shared vocabulary may have been inherited from a community rather than generated inside the dyad. Reputational effects can depend on observers. Resource asymmetry can be produced by institutions. In such cases the dyadic model remains useful as a local projection, but a triadic or wider representation is required for explanation.

The boundary therefore generates a reporting obligation. An empirical application should state which external relations are held fixed, ignored, summarized as background conditions, or explicitly represented through interface variables. Paper II will relax the dyadic restriction by introducing third-party positions and genuinely triadic structures.

Table 25: Selected failure conditions for the dyadic formalism.

Failure mode	Diagnostic condition	Consequence for the model
Type failure	operation applied outside its declared domain or to an incompatible object type	expression is formally invalid before substantive interpretation
Definedness failure	partial transformation is treated as total or a composition is used outside its realized domain	procedure/history claim is unsupported
Tracking failure	persistence is inferred from repeated naming without a historical continuation relation	subject, role, norm, trust, or other trajectory claim is underdetermined
Level collapse	relational structure is identified directly with one manifestation or reconstruction	three-level architecture is violated
ToM overreach	A's reconstruction of B is treated as B's actual state without independent warrant	epistemic claim exceeds available access
Provenance overreach	action genesis is assigned uniquely where several provenance reconstructions remain admissible	authorship/motivation claim is non-identifiable
Authorization overreach	consent, standing, or authority is inferred from one manifestation while scope/capacity/dependency remain unresolved	intervention status is insufficiently supported
Commensurability overreach	typed asymmetries or value records are aggregated without a justified common order or metric	scalar ranking is formally unearned
Algebraic overreach	C^* -, W^* -, Hilbert-space, field, or commutator structure is introduced without the required assumptions	borrowed mathematics exceeds the GR object actually defined
Emergence overreach	persistence, organization, or efficacy witness is missing	candidate trust/norm/role/partnership remains a summary label rather than an emergent record
Representation fragility	retained conclusion changes across reasonable encodings without being reported	claim is more representation-dependent than stated
Empirical underdetermination	distinct relational models remain observationally equivalent	latent relational object is not identified by the retained evidence
Normative overreach	formation, agreement, efficacy, or persistence is treated as validity	ethical/justice conclusion requires independent premises
Ontological overreach	successful mathematical representation is identified with ultimate ontology	claim exceeds ontological suspension
Dyadic leakage	omitted third-party or institutional relations carry essential explanatory structure	local dyad is descriptively incomplete for the research question
Illustrative overreach	finite toy example is treated as empirical confirmation	calculability is mistaken for evidence

22.10 Failure Conditions, Falsifiability

A preliminary formal programme should state how its claims can fail. Table 25 summarizes recurrent failure modes. Some are formal, some representational, some empirical, and some interpretive. Their separation matters because different failures require different repairs.

The table makes the formalism falsifiable in a broad methodological sense: proposed instantiations

can fail typing, coherence, robustness, identifiability, or empirical comparison. Domain-specific empirical hypotheses can be falsified more narrowly once they specify predicted manifestations, admissible alternatives, and rejection criteria. Paper I itself does not claim a universal empirical law whose falsification could be reduced to one statistical test.

Failure can also be productive. If a field realization fails overlap coherence, the model may need only regional records. If action provenance remains non-identifiable, the empirical conclusion can be restricted to manifested action and attributed meaning. If a trust-emergence criterion fails, the model can retain repeated expectation without calling it trust. If a 2-complex encoding is highly representation-sensitive, another history representation may be preferable. The framework is designed so that such narrowing is available without abandoning the entire programme.

22.11 Verification Protocol, Programme Interface

The preceding analysis can be condensed into a reusable protocol for later applications. For each instantiated dyadic model, the researcher should record:

1. the declared analytical resolution, subject boundary, historical window, and external interfaces;
2. the type, domain, codomain, partiality, and reality-level status of every major object;
3. the tracking relations used to compare objects across history;
4. the composition domains and equivalence relations used by procedures, operators, histories, and refinements;
5. the finite realization or calculation used to check stated formal consequences, where such a realization is available;
6. the admitted family of reasonable representation choices and the robustness of retained conclusions across that family;
7. the empirical coding map from observations to manifestation records and the reconstruction relation from manifestations to candidate relational models;
8. identifiability limits, including observationally equivalent alternatives;
9. the normative premises required for any claim of legitimacy, justice, obligation, permissibility, or ethical success;
10. the ontological status of the formal language, including which interpretations remain suspended;
11. the failure conditions and observations that would narrow, revise, or reject the proposed instantiation.

The protocol clarifies the research status of Paper I. The manuscript supplies a formal vocabulary and a layered verification architecture. It does not supply one universally privileged empirical coding, one complete normative theory, or one final ontology. Its strongest contribution lies in making those later commitments explicit enough that they can be compared rather than hidden inside apparently simple terms such as “support,” “need,” “consent,” “trust,” or “good.”

The remaining sections therefore have a narrower role. The next section will locate the present architecture within the wider volunteer/aid research programme and identify the formal objects inherited by later papers. The conclusion will then summarize the dyadic formalism without converting its preliminary representational achievements into empirical, normative, or ontological closure.

Table 26: Research interfaces for the formal, metaphysical, psychoanalytic, and aesthetic extensions.

Later paper	Principal Paper I inheritance	New burden
Triadic formalism	subject-spacetimes, manifestation, ToM, procedure, provenance, asymmetry, emergence	third position, mediation, witness, brokerage, public recognition, accountability, genuinely triadic structure
Subject co-formation	history-bearing subjects, role trajectories, feedback, emergent relations, cross-level revision	criteria for subject reconstitution, boundary change, value migration, co-evolution, generative-capacity transformation
Psychoanalytic inquiry	action provenance, relational positions, self/other reconstruction, supporter-side and coupled genesis hypotheses	lack, desire, fantasy, identification, repetition, symbolic investment, alterity tolerance
Aesthetic support	manifestation structure, pre-explicit differentiation, aesthetic judgments, historical revision	aesthetic salience, sensibility, relational form, cultivated judgment, shared aesthetic vocabulary, aesthetic validity

23 Subsequent Volunteer-Relation Research Interfaces

This section specifies how later papers can inherit the dyadic formalism without treating formal availability as substantive completion. Its role is architectural. Paper I has supplied typed objects for history-bearing subjects, three-level reality, manifestation, epistemic reconstruction, theory of mind, action provenance, procedure, evaluative formation, authorization, asymmetry, local algebraic dynamics, historical composition, emergence, and finite realization. Later inquiries may reuse those objects while adding their own explanatory or normative commitments.

For each later paper P_j , let the research interface at resolution κ be

$$\mathcal{I}_{1 \rightarrow j}^\kappa := (\mathcal{H}_j^\kappa, \mathcal{A}_j^\kappa, \mathcal{U}_j), \quad (241)$$

where $\mathcal{H}_j^\kappa$ contains inherited objects and methodological disciplines, $\mathcal{A}_j^\kappa$ contains the additional structures required by the later inquiry, and $\mathcal{U}_j$ records questions intentionally unresolved by Paper I. Equation 241 is organizational rather than algebraic. Its governing discipline is

$$\text{formal availability} \not\Rightarrow \text{substantive resolution}. \quad (242)$$

Equation 242 states the scope restriction that governs every later interface. A trust record therefore does not settle justified trust, an authority record does not settle legitimate authority, and an aesthetic judgment record does not settle aesthetic value.

Tables 26 and 27 summarize the principal inheritances and new burdens.

23.1 Triadic Relational Extension

Paper II introduces a third history-bearing subject or position whose presence can mediate, witness, authorize, translate, broker, or publicly recognize the relation between the first two. Pairwise dyadic records remain reusable, but the new inquiry must identify structures not recoverable from the three pairwise projections alone, including mediation, reputation, triadic closure, third-party accountability, coalition, and indirect trust. The point of the extension is therefore genuinely triadic organization rather than an additional node attached to an unchanged dyadic grammar.

23.2 Subject Co-Formation

Paper III inherits the history-bearing subject-spacetimes, role trajectories, feedback structure, emergent relations, and cross-level revision maps. Its new burden is metaphysical and dynamical: it must state when a relational change counts as a transformation of the subject rather than merely a change in a relation incident on an otherwise stable subject. It must also analyze co-evolution of subject boundaries,

Table 27: Research interfaces for the normative, political-economic, and historical-dynamical extensions.

Later paper	Principal Paper I inheritance	New burden
Relational justice	need/value judgments, authorization, recognition, dependency, standing, typed asymmetry, history	distributive, procedural, recognitional, epistemic, and relational justice criteria
Volunteer ethics	provenance, uncertainty, ToM, consent/refusal, authority, consequence, revision	duties, permissions, restraint, supererogation, competence limits, responsibility, paternalism, ethical revisability
Political economy	resources, dependence, authority, generative capacity, algebraic/history structures	donors, organizations, labor, institutions, symbolic/material capital, accumulation, field-level power mechanisms
Historical dynamics	path dependence, continuation families, trust/recognition records, emergent roles, 2-complex histories	relational nucleation, propagation, stabilization, institutionalization, long-run trust dynamics, heterogeneous value formation

practical horizons, values, and generative capacities while preserving the distinction between model-relative subject tracking and ultimate ontology.

23.3 Psychoanalytic Relational Positioning

Paper IV inherits provenance, role positions, self/other reconstruction, repetition-sensitive history, and the preliminary hypotheses of relational lack and coupled genesis. It must add a theory of desire, fantasy, identification, symbolic positioning, repetition, and the relation between articulated motive and less accessible generative structure. The later paper can therefore test whether helper/supportee positions perform subject-organizing work without converting every volunteer action into disguised self-interest or importing psychoanalytic concepts as unexamined primitives.

23.4 Aesthetic Support Relations

Paper V inherits manifestation structure, pre-explicit aesthetic differentiation, explicit aesthetic judgments, and their recursive historical relation. It must develop the substantive aesthetics of timing, tact, restraint, rhythm, spatial form, dignity, awkwardness, intrusion, and cultivated sensibility, together with criteria for aesthetic validity and disagreement. The central open question is how relational form becomes aesthetically salient before explicit articulation and how stabilized aesthetic vocabularies later reorganize perception and support practice.

23.5 Relational Justice

Paper VI inherits the typed records needed to formulate justice questions: need and value judgments, procedure, recognition, authorization, dependency, standing, asymmetry, and historical consequence. It must supply normative criteria for distributive, procedural, recognitional, epistemic, and relational justice and specify how heterogeneous considerations are related without presupposing one scalar justice function. A favorable outcome in one B-relative dimension can therefore remain compatible with problematic dependency, recognition, or refusal conditions until a substantive justice theory evaluates the combination.

23.6 Volunteer Ethics

Paper VII inherits action provenance, epistemic uncertainty, theory of mind, consent/refusal, standing, authority, consequence, and revisability. It must determine duties, permissions, supererogation, restraint, competence limits, responsibility, paternalism, and conditions of appropriate withdrawal. The formal non-entailments already established in Paper I become constraints on ethical reasoning: a favorable supporter-relative judgment concerning B does not itself confer authority, and capacity to act does not itself establish permission.

23.7 Political-Economic Power Structure

Paper VIII expands the dyad to donors, organizations, labor relations, states, markets, infrastructures, symbolic and material capital, credentialing, access control, and accumulation. Paper I supplies local records of resources, dependence, authority, generative capacity, and trajectory change; the later paper must explain when these differences become persistent power mechanisms and how volunteer support re-distributes capacities and burdens across a larger institutional configuration. The visible A–B interaction is therefore treated as locally focal rather than politically or economically self-contained.

23.8 Historical Relational Dynamics

Paper IX inherits ordered composition, path dependence, changing continuation families, memory, delayed consequence, feedback, emergent trust, shared vocabulary, role stabilization, and labelled histories. Its new burden is stronger temporal mechanism: relational nucleation, amplification, diffusion, stabilization, institutionalization, dissolution, heterogeneous value formation, and the passage from local dyadic history to wider relational systems. Long-run significance may therefore exceed immediate material effect, but the mechanism and scale of that amplification must be established rather than inferred from the dyad alone.

23.9 Programme-Level Dependency Structure

The resulting programme is connected by formal inheritance without forming one deductive chain. A schematic dependency relation is

$$\begin{aligned} P_1 &\rightsquigarrow P_2, \\ P_1 &\rightsquigarrow P_3 \rightsquigarrow P_4, \\ (P_1, P_3) &\rightsquigarrow P_5, \\ (P_1, P_3, P_4, P_5) &\rightsquigarrow P_6, P_7, \\ (P_2, P_3, P_4, P_5) &\rightsquigarrow P_8, \\ (P_1, P_2, P_3, P_5, P_6, P_7, P_8) &\rightsquigarrow P_9, \end{aligned} \tag{243}$$

Equation 243 records the programme-level inheritance structure, where $\rightsquigarrow$ denotes research dependence or inheritance rather than logical entailment. Later papers may revise an inherited object, reject a provisional representation, or expose a limitation in Paper I. The programme therefore requires conceptual stability without rigidity: shared terms should retain their typed distinctions unless a later inquiry gives an explicit reason to modify them.

This interface architecture permits the conclusion to synthesize the dyadic formalism without reopening the substantive programmes reserved for later work.

24 Conclusion

This section consolidates the formal architecture developed for the minimal dyadic volunteer relation. Its role is synthetic: it restates the principal analytical commitments, the strongest formal results, and the

limits that remain after the full construction. No new ontology, normative criterion, or dynamical law is introduced here.

The paper began from a compact proposition that ordinarily appears transparent:

$$A \xrightarrow{a} B. \quad (244)$$

Equation 244 seems to require only an actor, an action, and a recipient. The analysis has shown that it already presupposes subject individuation, action attribution, supportee positioning, judgments of need and value, some authorization structure, and partial epistemic access to a historically constituted relation. Paper I has made those presuppositions explicit rather than treating them as primitives.

The first contribution is the subject-and-reality architecture. The focal subjects are represented at a declared resolution through history-bearing subconfigurations $\mathcal{K}_A^\kappa$ and $\mathcal{K}_B^\kappa$ within a coupled dyadic configuration $\mathcal{K}_{AB}^\kappa$. Supporter and supportee are therefore relational positions rather than exhaustive identities. Their boundaries, overlaps, and continuities are representation-dependent and historically trackable. The same architecture is governed by the three GR reality levels: ultimate ontology remains suspended; relational reality carries the positive model of persistent and causally or generatively efficacious structures; manifestation reality records the context-dependent forms through which those structures become expressed, accessible, or actionable. Subject-relative reconstruction is built from manifestations and history without being promoted to a fourth reality level. The mapping between relational and manifestation realities can itself change across the dyadic trajectory.

The second contribution is a relational epistemology of support. Mutual visibility was separated from uptake, interpretation, belief, confidence, memory, joint acknowledgment, and recursive theory of mind. A and B therefore act through historically conditioned reconstructions of one another rather than through transparent access. The same discipline extends to action: an action was reconstructed through process individuation, attribution, intention, execution, initiation, provenance, manifestation, consequence, and retrospective revision. The provenance record $\Pi^\kappa(\alpha)$ can encode self-initiated, supportee-elicited, jointly negotiated, norm-mediated, role-mediated, affectively conditioned, historically sedimented, or distributed genesis without requiring one hidden motive to exhaust the explanation.

The third contribution is the procedural and evaluative architecture. Support was represented as a partially ordered and only partially composable history of encounter, inquiry, request, offer, clarification, negotiation, authorization-related manifestation, refusal, execution, monitoring, feedback, correction, withdrawal, handover, and termination. Need, goodness, aesthetic judgment, and normative judgment were kept distinct while remaining historically coupled. A's judgment of B's need need not coincide with B's self-judgment; a positive value judgment can vary by subject, target, dimension, and horizon; manifestation can become aesthetically salient before explicit articulation; and a stabilized norm can remain normatively contestable. Consent, refusal, standing, and authority were then represented as typed relations whose scope and operative conditions depend on capacities, information, dependency, history, and later revision.

These distinctions support a disciplined treatment of asymmetry. Information, epistemic access, competence, resources, recognition, interpretation, dependency, communication, refusal capacity, exit capacity, authority, generative capacity, and normative influence can differ independently. The paper therefore retains the non-entailment chain

$$\text{difference} \not\Rightarrow \text{typed asymmetry} \not\Rightarrow \text{power} \not\Rightarrow \text{injustice}, \quad (245)$$

Equation 245 therefore requires stronger political or normative claims to supply additional structure rather than inherit it from mere difference.

The fourth contribution is compositional. The model does not require one primitive global phase space, Hilbert space, metric, or utility scale. Local regions can receive algebraic carriers where the application warrants them; substantive transformations can be lifted only after their domains and codomains are specified; and stronger C^* -, W^* -, GNS-, completely-positive-, or quasi-local structures enter only as justified specializations. Historical sequence sensitivity is meaningful before a commutator is. Ordered

composition therefore carries the primary burden, while noncommutativity is introduced only after both orderings inhabit a common algebraic structure.

The fifth contribution is the integration of history and emergence. The integrated stage record permits subject-spacetime, manifestation mappings, epistemic access, theory of mind, procedure, evaluative records, authorization, asymmetry, continuation families, and local algebraic structure to change together or separately. Memory can be revised, consequences delayed, predictions corrected, and admissible operations reorganized. Trust, distrust, expectation, recognition, reciprocity, dependence, standing, authority, obligation records, shared vocabulary, shared norms, role stabilization, and partnership can then be classified as model-relative emergent structures when the declared specification establishes sufficient persistence, organization, and causal or generative efficacy. Emergence remains historical: such structures can weaken, dissolve, reopen, or change type.

The labelled 2-complex and spin-foam-inspired sections supplied concrete realizations of this architecture. They provide subject subcomplexes, interfaces, cellular supports, boundary data, alternative continuations, refinement, coarse-graining, labelled internal histories, and partial gluing. Their role is representational. A 2-complex is not asserted to be the ontology of a person; a spin-foam-inspired history does not make a volunteer relation quantum gravitational; and a history weight acquires probabilistic or quantum meaning only if additional assumptions justify that interpretation. The finite illustrations showed that requested support, unsolicited intervention, joint action, delayed consequence, and repeated role reorganization can all be represented while preserving these distinctions and without imposing one global welfare scalar.

The discussion developed several hypotheses only to the point required by the foundational architecture. Relational lack, supporter-centered genesis, supportee-centered genesis, coupled genesis, self–other relational nonseparability, and the aesthetics of supporting remain interfaces for later papers. The formulation that self and other are distinguishable without being relationally separable preserves both relational constitution and analytical distinction. It permits later metaphysical and psychoanalytic inquiry without reducing volunteering to supporter self-interest or erasing the difference between A and B.

The verification architecture sets the corresponding limits. Formal consistency does not establish empirical identification; reproducibility inside a finite realization does not remove coding or boundary uncertainty; relational stability does not establish justice; and successful mathematical representation does not settle ontology. The dyadic boundary is likewise analytical rather than causally closed. Organizations, communities, states, markets, families, professions, technologies, and wider histories can condition every component of the dyad even when they are not represented as focal subjects.

Within those limits, Paper I has completed its intended foundational task. Equation 244 can now be read as a compressed manifestation of a history-bearing, three-level, epistemically partial, procedurally articulated, evaluatively revisable, asymmetry-sensitive, and compositionally dynamic relational system. The contribution is therefore a disciplined starting language rather than a completed theory of volunteering. By making the assumptions surrounding subject, action, knowledge, value, authorization, asymmetry, history, and emergence explicit and revisable, the paper provides a common formal basis on which the subsequent triadic, metaphysical, psychoanalytic, aesthetic, justice, ethics, political-economic, and historical studies can build or against which they can argue.

References

- [1] John C. Baez. “Spin Foam Models”. In: *Classical and Quantum Gravity* 15 (1998), pp. 1827–1858. arXiv: gr-qc/9709052.
- [2] Ola Bratteli and Derek W. Robinson. *Operator Algebras and Quantum Statistical Mechanics 1: C*- and W*-Algebras, Symmetry Groups, Decomposition of States*. 2nd ed. Springer, 1987. DOI: 10.1007/978-3-662-02520-8.

- [3] E. Gil Clary et al. "Understanding and Assessing the Motivations of Volunteers: A Functional Approach". In: *Journal of Personality and Social Psychology* 74.6 (1998), pp. 1516–1530. DOI: 10.1037/0022-3514.74.6.1516.
- [4] J. R. B. Cockett and Stephen Lack. "Restriction Categories I: Categories of Partial Maps". In: *Theoretical Computer Science* 270.1–2 (2002), pp. 223–259. DOI: 10.1016/S0304-3975(00)00382-0.
- [5] Miranda Fricker. *Epistemic Injustice: Power and the Ethics of Knowing*. Oxford University Press, 2007. DOI: 10.1093/acprof:oso/9780198237907.001.0001.
- [6] Rudolf Haag and Daniel Kastler. "An Algebraic Approach to Quantum Field Theory". In: *Journal of Mathematical Physics* 5.7 (1964), pp. 848–861. DOI: 10.1063/1.1704187.
- [7] Allen Hatcher. *Algebraic Topology*. Cambridge University Press, 2002. ISBN: 9780521795401.
- [8] Axel Honneth. *The Struggle for Recognition: The Moral Grammar of Social Conflicts*. Trans. by Joel Anderson. Polity Press, 1995.
- [9] Jacques Lacan. *The Four Fundamental Concepts of Psychoanalysis*. Ed. by Jacques-Alain Miller. Trans. by Alan Sheridan. New York: W. W. Norton, 1978.
- [10] Catriona Mackenzie and Natalie Stoljar, eds. *Relational Autonomy: Feminist Perspectives on Autonomy, Agency, and the Social Self*. Oxford University Press, 2000. DOI: 10.1093/oso/9780195123333.001.0001.
- [11] Josef Perner and Heinz Wimmer. "'John Thinks That Mary Thinks That...' Attribution of Second-Order Beliefs by 5- to 10-Year-Old Children". In: *Journal of Experimental Child Psychology* 39.3 (1985), pp. 437–471. DOI: 10.1016/0022-0965(85)90051-7.
- [12] David Premack and Guy Woodruff. "Does the Chimpanzee Have a Theory of Mind?" In: *Behavioral and Brain Sciences* 1.4 (1978), pp. 515–526. DOI: 10.1017/S0140525X00076512.
- [13] Amartya Sen. *Development as Freedom*. Oxford University Press, 1999. ISBN: 9780198297581.
- [14] Joan C. Tronto. *Moral Boundaries: A Political Argument for an Ethic of Care*. New York: Routledge, 1993.
- [15] John Wilson. "Volunteering". In: *Annual Review of Sociology* 26 (2000), pp. 215–240. DOI: 10.1146/annurev.soc.26.1.215.
- [16] Heinz Wimmer and Josef Perner. "Beliefs about Beliefs: Representation and Constraining Function of Wrong Beliefs in Young Children's Understanding of Deception". In: *Cognition* 13.1 (1983), pp. 103–128. DOI: 10.1016/0010-0277(83)90004-5.