

A Preliminary Generative Relational Meta-Theory of Theory Formation

HUANG WANHONG
huangwanhong@serendip.ngo

Abstract

This paper develops a preliminary Generative Relational meta-theory for investigating how theories form around specific problems. It treats theory formation as a historically situated and relationally evolving process in which the research problem, reason-bearing entities or distributed configurations, their relational-historical locations, epistemic-normative-practical organization, abstractions, and formal representations can all become objects of revision. The architecture begins with an applicability and route-selection test, followed by seven substantive stages: problem constitution; identification and localization of question-relevant reason-bearing entities or configurations; evidence-constrained reconstruction of situated epistemic, normative, and practical organization; evolutionary reconstruction of entities, relations, boundaries, types, and admissible state spaces; cross-context abstraction together with transformation analysis; formal symbolization with representation-bias audit; and recursive evaluation. The transformation stage distinguishes recurrence from invariance and equivariance and selects algebraic resources according to supported transformation properties: groups for reversible common-domain transformations, semigroups or monoids for irreversible composition on a common domain, groupoids for reversible local transport among multiple objects, and more general categories for typed, irreversible, or evolving-state-space mappings. The framework is explicitly non-universal. Its intended domain comprises inquiries in which historically evolving, relationally constituted epistemic entities or configurations materially participate in problem formation, judgment, abstraction, representation, or recursive revision. A source-limited pilot reconstruction of the formation of the Recognition-Primed Decision model illustrates how the framework can separate source-supported stages from unresolved provenance without treating the pilot as validation. The paper presents a revisable meta-methodological design language rather than an empirically validated general theory of inquiry, and it distinguishes structural formalization from ontological commitment and empirical support.

Keywords: theory formation; problem constitution; Generative Relational theory; evolutionary epistemology; invariance

Discussion Paper Note

This paper develops a preliminary meta-theoretical framework for examining how theories are formed around specific problems. Its central object is the generative trajectory through which a provisional question, reason-bearing entities or distributed configurations, relational-historical locations, situated practices of reasoning and judgment, cross-context abstractions, transformation claims, and formal representations become organized into a theory. The framework is developed within the broader Generative Relational (GR) research programme, while its claims remain methodological and exploratory rather than ontological.

The motivating cases include naturalistic decision making and the study of proficient scholarly practice. Naturalistic decision making provides an important precedent for investigating judgment in field conditions characterized by incomplete information, time pressure, uncertainty, changing situations, and consequential action. The scholarly case raises a related but distinct problem: proficient inquiry can involve generation and revision of the problem itself. A researcher may alter the question, redefine the relevant object, introduce new distinctions, or establish that an initially plausible problem was poorly constituted. The paper treats this problem-forming capacity as a central feature of theory formation.

The framework is explicitly non-universal. Applicability is assessed at the level of the research problem rather than by disciplinary label. The full route is intended principally for problems in which historically evolving, relationally constituted epistemic entities or configurations materially participate in problem formation, judgment, abstraction, representation, or recursive revision. A natural-scientific inquiry can satisfy these conditions when the object is scientific problem formation or research practice, while a social-scientific inquiry can fall outside them when situated reason-bearing configurations contribute little to the target explanation.

The formal resources introduced in the paper serve structural analysis. Group actions are considered where supported transformations are reversible on a common domain. Semigroups or monoids are considered where transformations compose on a common domain without general invertibility. Groupoids are reserved for multi-object settings in which the relevant local morphisms are reversible. More general categories are available for typed, irreversible, or evolving-state-space transformations. These resources provide candidate languages for representing relational and historical structure; their mathematical availability does not establish either empirical adequacy or an ultimate mathematical ontology of the phenomena studied.

A parallel caution applies to symbolization. Formal representation can clarify structure, expose inconsistency, and generate new conjectures, yet it can also introduce ontological sharpening, discretization, metric imposition, scalarization, homogenization, atemporalization, boundary closure, or observer-selection bias. The paper therefore treats symbolization as an epistemically productive and potentially lossy transformation whose adequacy must be audited against the reconstructed problem and evidence.

The current manuscript contains fourteen substantive sections and a revised full architecture. The central sequence moves from problem constitution and entity localization through situated reconstruction and evolutionary analysis to comparative abstraction, transformation analysis, symbolization, and recursive evaluation. Later sections address scope conditions, empirical design, meta-methodological implications, and the conclusion. Section 12 includes two stylized formation trajectories and a source-limited pilot reconstruction of the formation of the Recognition-Primed Decision model. The pilot uses a verified published record of the original fireground study to distinguish source-supported formation stages from unresolved provenance; it is an empirical-contact exercise rather than cross-domain validation. The present draft has undergone global consistency, algebraic-integrity, argument-economy, empirical-contact/release-readiness, and release-level claim/literature-positioning revision. It should still be read as a design language for future comparative studies rather than as empirical validation of a general theory of theory formation.

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This working paper is offered for scholarly discussion, criticism, and further development. The author asks users to preserve the distinction among exploratory formalization, empirical support, and ontological commitment when adapting the framework. This ethical request does not narrow otherwise applicable legal permissions, exceptions, or limitations.

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Notices

Status. This manuscript is a revised working first draft containing fourteen completed substantive sections. The formal vocabulary, ordering of sections, comparative cases, literature review, and empirical validation strategy remain subject to revision. The present release has undergone five whole-manuscript revision passes: global consistency; argument-level and algebraic integrity; argument economy with worked analytic illustrations; empirical-contact/release-readiness review; and release-level claim and literature-positioning review. The illustrative trajectories in Section 12.6 are stylized illustrations of the framework. The source-limited RPD pilot in Section 12.7 is an empirical-contact exercise rather than cross-domain validation. The framework has not yet been empirically validated across domains.

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Statement on the use of language models. The exploratory discussions and preparation of this paper involved OpenAI’s ChatGPT. ChatGPT supported exploratory dialogue, formal reconstruction, source discovery followed by website verification, argumentative criticism, and drafting in L^AT_EX. The author selected the research questions, theoretical commitments, scope conditions, and epistemic status of the claims and bears sole responsibility for the manuscript, including its definitions, formal constructions, arguments, conclusions, and errors. Authorship credit remains with the human author.

1 Introduction

This section specifies the analytical object and scope of the paper. Its objective is to formulate theory formation as a historically situated, relationally evolving research process whose initiating problem, epistemically relevant entities, abstractions, and symbolic representations may themselves change during inquiry. The argument proceeds conceptually and comparatively. Naturalistic decision making supplies one motivating precedent for studying proficient cognition under bounded and changing conditions, while the history and methodology of inquiry supply a second motivation for treating problem constitution and theory revision as endogenous to research. Research on scientific discovery and model-based theory construction, together with relational and processual social science, provides important neighboring traditions against which the proposed Generative Relational contribution must be specified with care. The section then states the preliminary architecture of the framework, its evolutionary extension, its positive scope conditions, and the principal contributions developed in the remainder of the paper.

A large class of methodological representations begins from a problem that has already been formulated. A question is specified, evidence is gathered, candidate explanations are generated, and a theory is evaluated against some epistemic criterion. This ordering is often useful, but it can conceal a prior generative process. Research practice may involve determining what deserves to count as the problem, which distinctions make the problem intelligible, which entities possess reasons relevant to it, what counts as evidence, which historical scale matters, and what form of answer would constitute a satisfactory resolution. Under these conditions, the initiating question is part of the research object rather than a fixed external input.

This observation is compatible with earlier critiques of idealized rational inquiry without being reducible to them. Simon’s account of bounded rationality challenged models that presupposed extensive knowledge of the environment, stable preferences, and powerful computational capacities [15]. Naturalistic decision making later developed a field-oriented programme for examining how people make decisions in real-world settings, emphasizing complex conditions and the role of experience in rapid situation categorization [8]. These traditions motivate attention to bounded information, situated judgment, and practiced competence. The present inquiry extends the motivating question from decision formation

to theory formation, where competent practice may include revising the object of decision or inquiry itself.

The difference becomes visible when the research process is represented provisionally. Let Q_t denote the problem or question constituted at research time t ; let $(\mathcal{E}_{Q_t}, \Lambda_t)$ denote the retained reason-bearing entities or distributed configurations together with their relational-historical locations; let $\hat{\Xi}_t$ denote the evidence-constrained epistemic-normative-practical reconstruction; let Ω_t denote the evolutionary reconstruction; let $\mathfrak{B}_t$ denote the family of candidate cross-context abstractions; let $\mathfrak{T}_t$ denote the supported transformation-structured account; let $\mathfrak{S}_t$ denote the symbolization record; let $\mathcal{K}_{th,t}$ denote the provisional substantive claim set supported by the current formation record; and let V_t denote recursive evaluation. Equation 1 gives the schematic dependency order.

$$\begin{aligned} Q_t &\longrightarrow (\mathcal{E}_{Q_t}, \Lambda_t) \longrightarrow \hat{\Xi}_t \longrightarrow \Omega_t \longrightarrow \mathfrak{B}_t \\ &\longrightarrow \mathfrak{T}_t \longrightarrow \mathfrak{S}_t \longrightarrow \mathcal{K}_{th,t} \longrightarrow V_t \longrightarrow Q_{t+1}. \end{aligned} \quad (1)$$

Equation 1 is a dependency-oriented structural formalization rather than an empirically parameterized model or a universally fixed temporal sequence. The claim set $\mathcal{K}_{th,t}$ is included explicitly because the object ultimately evaluated is not only the pathway of formation but also the substantive theoretical output supported by that pathway. Its analytical role is to expose dependencies that linear accounts can suppress: evaluation can alter the question that conditions the next round of entity identification, reconstruction, abstraction, transformation analysis, representation, and claim formation. A successful inquiry may preserve Q_t , refine it, divide it into several questions, enlarge or narrow its scope, replace it, or dissolve it when the original formulation depends upon an unproductive distinction. The framework therefore treats improvement of the question as one possible epistemic achievement.

The entity term in Equation 1 requires a specifically relational interpretation. The relevant unit is not assumed to be an isolated individual who carries a context-independent stock of rationality. An entity can be a person, team, institution, scholarly community, human-machine configuration, or another analytically justified configuration whose historically situated perception, practice, judgment, or experience provides reasons relevant to Q_t . The relevant reasons are question-relative. A person can possess strong reasons with respect to one problem and weak or irrelevant reasons with respect to another. In some cases, the epistemically productive unit is distributed across several entities and the relations and mediators among them; Section 4 represents such a unit as a justified distributed configuration rather than forcing the reasons into one visible individual.

Historical and relational position are constitutive components of this reconstruction. Two entities occupying the same nominal organizational category can differ substantially in prior encounters, intellectual lineages, institutional access, social recognition, practical responsibilities, and relations to other actors. The present framework therefore assigns each relevant entity a relational-historical location $\Lambda_i(t)$ and represents the analyst's reconstruction of its epistemic, normative, and practical organization as evidence-constrained. Equation 2 records that dependence schematically.

$$\hat{\Xi}_i^{Q_t}(t) = \mathfrak{R}\left(Q_t, \Lambda_i(t), \mathcal{D}_i^{Q_t}\right), \quad (2)$$

where $\mathcal{D}_i^{Q_t}$ denotes the problem-relevant evidence used in reconstruction and $\mathfrak{R}$ denotes the locally justified reconstructive procedure. Equation 2 does not assert that relational-historical location and evidence are independently measurable or jointly sufficient. Its role is to keep the analyst's reconstruction distinct from direct access to the entity's generative organization and to preserve historical-relational location when the research problem makes it relevant.

This commitment places the paper near relational and processual approaches while requiring a more precise statement of difference. Emirbayer's relational sociology explicitly contrasts substantialist conceptions of static things with a processual perspective centered on dynamic and unfolding relations, including problems of boundaries, entities, network dynamics, causality, and normative implications [5]. Abbott's methodological work likewise foregrounds time, events, causal assumptions, turning points, and process in social analysis [1]. These traditions make it untenable to identify the proposed framework's

contribution simply with the inclusion of relations or history. The stronger proposal concerns the joint evolution of the entity, its relational-historical position, the available state space, the research question, the theory space, and the symbolic language through which the evolving object becomes tractable.

A conventional diachronic representation can follow a recognizable entity across successive times. The GR extension considered here permits the entity's boundaries, type, available relations, and space of possible states to change through the trajectory. When the research problem itself changes historically, let $\mathbf{Q}_{[t,t+1]} = (Q_t, Q_{t+1})$ denote the local problem lineage across the transition. Let $\Omega_t^{Q_t}$ denote the coupled configuration at time t , including the relevant entities, relations, mediators, locations, reconstructed practices, local question, and retained conditions. Equation 3 represents a supported transition in a form that also permits both the problem index and the admissible state space to change.

$$\mathcal{U}_{t,t+1}^{\mathbf{Q}} : \mathfrak{D}_t^{Q_t} \subseteq \mathfrak{X}_t^{Q_t} \longrightarrow \mathfrak{X}_{t+1}^{Q_{t+1}}, \quad \mathcal{U}_{t,t+1}^{\mathbf{Q}}(\Omega_t^{Q_t}) = \Omega_{t+1}^{Q_{t+1}}. \quad (3)$$

Here $\mathfrak{X}_t^{Q_t}$ denotes the analytically admissible configuration space at time t under the local problem constitution, and $\mathfrak{D}_t^{Q_t}$ is the supported domain of the reconstructed transformation. The possibility $\mathfrak{X}_t^{Q_t} \neq \mathfrak{X}_{t+1}^{Q_{t+1}}$ is analytically significant. Theory formation can involve changes in the space of available distinctions and possible states rather than movement within a permanently fixed state space. New instruments can create new observables; new concepts can separate objects previously treated as one; institutional transformations can create new roles and relations; and a formal language can make previously unavailable theoretical constructions thinkable. The same logic permits an evolving theory space Θ_t whose available models and explanations depend upon the historical configuration of problems, evidence, methods, instruments, and representational resources.

The history of science provides a broad reason to take such changes seriously. Kuhn's account of normal and revolutionary science challenged a simple cumulative picture of scientific development and placed conceptual reorganization within the history of scientific change [10]. The present framework does not adopt a Kuhnian theory of scientific development as its general model. Its narrower purpose is to construct an analyzable architecture for the local formation of a theory around a specific problem, including circumstances in which the problem, object, relevant entities, and representational space are revised during the inquiry.

Evolution also changes what abstraction should mean in the framework. A search for common properties across entities can erase the historical and relational structures that make a comparison informative. The proposed method therefore treats abstraction as a search for structures that remain invariant, vary covariantly, or transform equivariantly across explicitly specified changes of context. The comparison space includes spatial, temporal, and spatiotemporal transformations across both same-type and cross-type entities. Where a reversible transformation group G is justified, a candidate invariant I can be examined through Equation 4.

$$I(g \cdot z) = I(z), \quad g \in G^* \subseteq G, \quad (4)$$

where G^* states the actual transformation domain over which the invariance claim is proposed. The restriction is methodologically important because an invariant observed across a limited family of contexts should not be promoted to an unrestricted universal. Strict invariance will also be too strong for many historical problems. In such cases, the more informative object may be an equivariant structure whose representation changes systematically with the transformation. Equation 5 records this possibility.

$$F(g \cdot z) = \rho(g)F(z), \quad (5)$$

where F extracts the structure being compared and $\rho(g)$ represents the corresponding transformation in the target representation. Groups are therefore only one formal resource. Irreversible but composable change on a common domain can motivate semigroup or monoid representations; reversible local transport among multiple objects can motivate groupoids; and typed, irreversible, or evolving-state-space mappings can motivate more general categorical morphisms. These mathematical resources enter as candidate structural languages whose adequacy must be justified case by case.

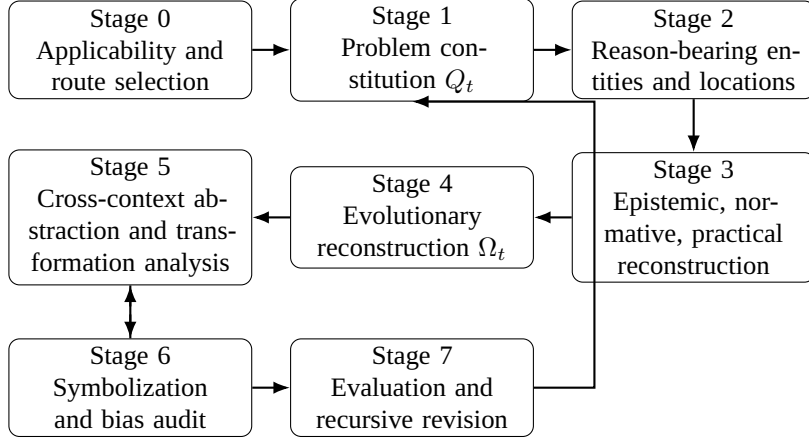


Figure 1: Preliminary recursive architecture for Generative Relational theory formation. The dashed return indicates that formalization can disclose structures that motivate renewed abstraction; the principal evaluative return permits revision of the initiating problem and subsequent stages.

Symbolization consequently forms a distinct methodological stage. The transition from an empirically or interpretively reconstructed structure to a formal representation can produce clarification, compression, comparison, and new inferential possibilities. It can also produce loss. Entity symbols can suggest sharper boundaries than the evidence supports; scalar variables can suppress multidimensional historical structure; metrics can introduce comparability assumptions; discrete states can conceal continuous change; and static relations can erase path dependence. The paper therefore represents symbolization as a transformation with an associated loss and bias audit. Equation 6 states the analytical decomposition.

$$\Sigma_t^Q : D_{\Sigma,t}^Q \subseteq \mathcal{R}_t^Q \longrightarrow \mathcal{F}_t^Q, \quad \mathcal{L}_{\Sigma,t}^Q = \mathcal{A}(R_t^Q, R_t^{Q,\text{formal}}), \quad (6)$$

where R_t^Q denotes the selected reconstructed structure, $D_{\Sigma,t}^Q$ the domain chosen for formalization, $\mathcal{F}_t^Q$ the formal representation space, and $\mathcal{L}_{\Sigma,t}^Q$ the explicitly audited representational loss under an assessment procedure $\mathcal{A}$. The notation does not imply that representational loss can always be reduced to a scalar quantity. It creates a place in the framework where ontological sharpening, discretization, metric imposition, scalarization, homogenization, atemporalization, boundary closure, and observer-dependent selection can be examined rather than silently absorbed into the formal model. Failure to identify an adequate formalization remains an admissible research result.

Figure 1 summarizes the resulting architecture. Stage 0 precedes seven substantive stages because the framework is intended for a positive class of research problems rather than as a universal method. Stage 5 contains two linked analytical operations developed in separate sections: cross-context abstraction followed by transformation-structured testing of invariance, equivariance, covariance, or more general transport. The principal route is most suitable when the problem is partly constituted through inquiry; question-relevant entities or distributed configurations possess situated reasons; relational-historical location materially affects those reasons; tacit, practical, normative, or perceptual organization matters; the epistemic environment is bounded or changing; evolutionary reconstruction contributes explanatory value; and comparison across contexts can support meaningful structural abstraction. These are problem-level scope conditions. They do not map cleanly onto disciplinary boundaries.

The positive scope conditions matter because the framework contains an obvious risk of overextension. If every entity boundary, relation, question, state space, transformation, and representation can be revised without constraint, a sufficiently flexible reconstruction could accommodate almost any observed trajectory after the fact. The method therefore requires explicit domains and codomains for formal transformations where applicable, evidential support for proposed historical reconstructions, stated boundary conditions, comparison against cases not used to construct a claimed regularity, and preservation of failed mappings or residual differences. Section 11 develops these limitations in detail, including the absence

or misidentification of suitable entities, distributed epistemic agency, retrospective reconstruction bias, survivorship bias, evolutionary underconstraint, scale dependence, restricted invariance domains, and formalization failure.

The framework’s central research object can now be stated more precisely. Theory formation is treated as the evolving coupled trajectory through which a question, a relational-historical epistemic configuration, an abstract structural model, and a symbolic representation become provisionally stabilized. A compact notation for this trajectory is given in Equation 7.

$$\Gamma_{[t_0, t_n]} = \left\{ Q_t, \mathcal{E}_{Q_t}, \mathbf{\Lambda}_t, \widehat{\mathbf{E}}_t, \mathbf{\Omega}_t, \mathfrak{B}_t, \mathfrak{T}_t, \mathfrak{S}_t, V_t \right\}_{t=t_0}^{t_n}. \quad (7)$$

Equation 7 should be read as a bookkeeping structure for a research programme. It does not imply that every empirical study must observe every component directly, that each component has a unique decomposition, or that the sequence has a universal temporal granularity. Its purpose is to preserve the possibility that changes in one component alter the interpretation or admissible construction of others. In particular, Q_t conditions which entities appear epistemically relevant, while learning from those entities can transform Q_t and thereby change the relevant entity set. Problem formation and epistemic-community formation can therefore become coupled processes.

The paper makes five preliminary contributions. First, it treats problem constitution as an endogenous and revisable component of theory formation. Second, it specifies epistemically relevant GR entities through question-relative reasons and relational-historical locations, including distributed configurations where agency or knowledge cannot be localized to a single individual. Third, it introduces evolutionary reconstruction before cross-context abstraction, allowing entities, relations, boundaries, types, state spaces, and theory spaces to change through inquiry. Fourth, it develops invariance and equivariance as disciplined comparative objectives while extending the available formal vocabulary to irreversible, partial, and cross-type transformations. Fifth, it separates abstraction from symbolization and makes representational loss and formalization bias objects of explicit methodological audit.

These contributions define a meta-methodological proposal rather than a replacement for established empirical, historical, interpretive, mathematical, or computational methods. Their value depends on whether they help researchers identify structures that would otherwise be obscured, formulate more adequate questions, preserve relevant historical and relational information, and expose biases introduced by formal representation. The framework therefore remains open to revision through the same recursive logic that it proposes for theory formation itself.

2 Intellectual Context of Theory-Formation Research

This section locates the proposed meta-theory among several neighboring traditions that already treat important components of inquiry as situated, historically conditioned, practically organized, or socially distributed. Its objective is comparative rather than genealogically exhaustive. The discussion identifies seven lines of work that constrain the novelty claim of the present framework: research on ill-structured problems and problem finding; tacit and reflective professional knowing; naturalistic decision making; distributed and social accounts of cognition and knowledge; scientific discovery and theory construction; historical, relational, and processual accounts of scientific or social change; and evolutionary epistemology. Table 1 summarizes the analytical contribution taken from each line and the interface that remains open for the framework developed here. The comparison supports a limited claim: the proposed contribution lies in organizing these concerns within a recursive theory-formation architecture in which the initiating question, reason-bearing configuration, relational-historical location, evolutionary trajectory, abstraction, and symbolic representation can each become objects of revision.

Table 1: Neighboring research traditions and their interface with the proposed meta-theory. The table identifies analytical contributions used as constraints or resources; it does not imply that each tradition can be reduced to the listed feature.

Tradition	Analytical contribution	Interface retained here
Ill-structured problems and problem finding	Problem formulation and problem structure can be incomplete, fluid, or themselves objects of inquiry.	Treat the provisional research question as a revisable component that conditions later entity selection, abstraction, and evaluation.
Tacit and reflective professional knowing	Competent practice can depend on knowledge, recognition, and reflection that exceed explicit rule application.	Reconstruct situated epistemic, normative, and practical organization without reducing it to reported propositions.
Naturalistic decision making	Expertise can be studied in consequential environments characterized by uncertainty, limited time, changing conditions, and experience-dependent recognition.	Extend reconstruction from situated decision episodes toward problem constitution and theory-generative episodes.
Distributed and social epistemic organization	Cognitive and knowledge-producing activity can be organized across persons, artifacts, practices, and critical communities.	Permit reason-bearing configurations whose relevant organization is relational rather than localized in one individual.
Scientific discovery and theory construction	New theories and concepts can be studied as extended processes of construction, evaluation, revision, analogy, modeling, and problem solving rather than as unanalyzable leaps.	Embed theory construction within a versioned architecture that also tracks question constitution, relational-historical location, transformation scope, and symbolization bias.
Historical, relational, and processual inquiry	Scientific and social objects can be understood through historical sequences, changing relations, events, boundaries, and processes.	Make relational-historical location constitutive of the analytical entity and permit entity identity, boundaries, and state spaces to evolve.
Evolutionary epistemology	Knowledge generation can be analyzed through variation, selection, retention, and other evolutionary analogies or mechanisms.	Use evolutionary reconstruction more broadly to study path dependence and transformation while leaving Darwinian selection as one possible mechanism rather than a universal template.

2.1 Problem Constitution in Ill-Structured Inquiry

Research on problem structure provides an early constraint on any meta-theory that begins from a well-defined question. Simon’s analysis of ill-structured problems emphasized that the boundary between well-structured and ill-structured problems is itself fluid and that real problem solving can require access to large bodies of potentially relevant information whose consequences cannot be exhaustively represented in advance [16]. This observation weakens the assumption that a research problem can always be specified once, prior to substantive inquiry, and then held fixed while the solver searches for a solution.

The problem-finding literature makes the generative role of question formation more explicit. Getzels treated problem finding as a theoretical object distinct from the better studied activity of solving a presented problem [6]. The associated distinction between presented and discovered problem situations is especially relevant to theory formation because the latter can involve uncertain formulation, uncertain method, and uncertain criteria of satisfactory resolution. Within such settings, a productive contribution

may consist partly in finding or reformulating the problem that subsequent work should address.

The present framework inherits this concern but assigns problem constitution a stronger architectural role. A provisional question does more than initiate a search. It influences which entities appear epistemically relevant, which histories become salient, which comparisons are admitted, which structures are candidates for abstraction, and which forms of evidence are treated as probative. Learning during those later stages can in turn alter the question. Problem finding therefore enters the framework as a recursive relation between question constitution and subsequent inquiry rather than as a single preliminary act.

This recursive treatment also separates two claims that should remain distinct. First, some inquiries begin with incomplete or unstable problems. This is an empirical and methodological observation supported by work on ill-structured problems and problem finding. Second, the present paper proposes that a theory-formation framework should preserve the question itself as a revisable variable throughout inquiry. The second claim is a meta-methodological extension, and its adequacy must be evaluated through the later comparative and empirical programme rather than inferred directly from the earlier literature.

2.2 Situated Professional Knowing

A second lineage concerns forms of competent knowing that are only partially captured by explicit propositions or formal rules. Polanyi's account of tacit knowledge foregrounded the role of inherited practices, implied commitments, and forms of knowing that exceed what an actor can fully state [13]. Schön's study of professional practice similarly developed reflection-in-action as an account of how practitioners engage situations whose uncertainty and uniqueness resist straightforward application of technical rules [14]. Naturalistic decision making adds an empirically oriented programme for examining experienced decision makers in consequential real-world environments, including conditions of time pressure, uncertainty, and changing information [8].

These traditions are directly relevant to the present framework because theory formation can depend on more than explicit inferential sequences. A scholar may perceive an anomaly before possessing a vocabulary for it, recognize that a familiar distinction has become unproductive, judge that a historical comparison is illuminating, or reject a formally available model because it destroys a relation regarded as explanatorily important. Such acts can later be reconstructed propositionally, yet the reconstruction should not be assumed to exhaust the generative process that produced them.

The proposed concept of a reason-bearing GR entity is intended to preserve this methodological possibility without converting expertise into an intrinsic personal property. An entity is relevant because its situated experience, practice, judgment, or interaction supplies reasons material to a specified question. The relevant status is therefore question-relative and historically conditioned. A highly proficient practitioner in one problem field need not possess comparable reasons in another, and institutional recognition alone does not establish epistemic relevance.

This treatment also creates an empirical obligation. Retrospective testimony by proficient practitioners cannot be equated with the historical process that generated their judgments. Self-reports can compress uncertainty, reorder events, rationalize abandoned paths, and reinterpret earlier activity through later concepts. The framework therefore treats interviews and autobiographical materials as evidence to be triangulated, where possible, against contemporaneous drafts, correspondence, recorded decisions, observed practice, institutional records, or other traces of the evolving inquiry. Section 11 develops this reconstruction problem as a specific limitation rather than presuming transparent access to expert cognition.

2.3 Distributed Epistemic Organization

The third lineage expands the epistemic unit beyond an isolated individual. Hutchins' analysis of ship navigation showed how cognitive activity can be organized across persons, representational artifacts, culturally constituted practices, and the social organization of work [7]. Longino's account of scientific knowledge likewise argued that the cognitive capacities involved in science are exercised through social interaction and that critical social processes contribute to the objectivity of inquiry [11]. These

approaches place an important constraint on any theory-formation framework that would locate reasons exclusively within individual minds.

The GR proposal therefore permits a reason-bearing configuration rather than requiring a single reason-bearing person. Such a configuration can include researchers, practitioners, instruments, archives, computational systems, representational artifacts, institutional routines, and the relations that coordinate them. The claim is methodological: some explanatory structures become visible only when the unit of reconstruction includes the organization connecting these components. It does not require attributing identical agency, consciousness, or normative standing to every component in the configuration.

This distinction matters for theory formation because epistemically relevant reasons can be distributed asymmetrically. An instrument can make a signal available without interpreting its significance; a database can preserve a historical trace without judging its relevance; a junior researcher can notice an anomaly that a senior researcher can contextualize; an institutional review procedure can expose a normative constraint absent from the original model. The generative episode may therefore depend on a pattern of coupling that no participant possesses in isolation.

For the present meta-theory, distributed organization changes both entity identification and later abstraction. Stage 2 must remain capable of discovering that the initially selected individual is an inadequate epistemic unit. Stage 3 must then reconstruct differentiated roles within the configuration rather than averaging them into a fictive collective mind. Stage 4 must examine how the configuration itself develops, because artifacts, institutional rules, division of labor, and participation can change during the history of an inquiry.

2.4 Scientific Discovery and Theory Construction

A fifth neighboring line addresses scientific discovery and theory construction directly. Darden treats discovery as an extended process involving construction, evaluation, revision, anomaly resolution, and domain-specific reasoning strategies [4]. Nersessian analyzes conceptual innovation through model-based reasoning, including analogy, visual and imagistic representation, thought experiments, mathematical analysis, and interaction with experimental resources [12]. These approaches are especially close to the present problem because they study how new representational structures are generated rather than only how finished theories are assessed.

This literature narrows the novelty claim of the proposed framework. Construction, revision, model use, and historically situated reasoning already have substantial analyses. The present proposal therefore does not claim to make theory generation analyzable for the first time. Its narrower aim is to coordinate such generative activity with several additional obligations that are often treated separately: explicit revision of the research problem itself; question-relative identification of reason-bearing entities or distributed configurations; reconstruction of relational-historical location and identity through change; transformation-scoped comparison across contexts; and an explicit audit of losses and distortions introduced by symbolization.

The comparison also cautions against treating the seven-stage architecture as a cognitive sequence that every scientist follows. Darden's and Nersessian's cases show that construction, evaluation, representation, and problem solving can interleave. The stages developed in this paper are therefore analytical reconstruction sites. Their ordering provides dependency discipline for evidence and revision, while an empirical history may contain loops, overlaps, and locally different orders.

2.5 Historical Process in Scientific Change

A sixth lineage concerns historical change in scientific concepts, standards, and communities. Kuhn's account of normal and revolutionary science challenged a simply cumulative representation of scientific development and emphasized historically organized episodes in which shared problems, exemplars, standards, and conceptual resources can be reconfigured [10]. Whatever position one takes on the details of Kuhn's account, the broader methodological consequence is important for the present paper: the space within which scientific problems and acceptable solutions are formulated has a history.

The proposed framework generalizes this consequence into a local research requirement. A theory-formation episode should be reconstructed with attention to the concepts, instruments, methods, institutions, and prior theories available at the relevant historical location. The absence of a later concept at an earlier time is analytically meaningful because it can restrict which questions, distinctions, or formalizations were then accessible. Conversely, a newly introduced representation can enlarge the theory space by making previously unavailable comparisons or transformations tractable.

This historical orientation supports the distinction between an evolving state and an evolving state space. In some inquiries, the objects under study can be represented adequately as trajectories within a fixed space of possibilities. In others, new categories, instruments, institutional forms, or mathematical languages alter the available coordinates or even the admissible kinds of state. The present framework therefore permits the state space of inquiry to change, while treating such change as a claim requiring historical evidence rather than a default assumption.

Historical reconstruction also complicates retrospective evaluation. A theory that appears poorly chosen from a later standpoint may have been reasonable relative to the evidence, conceptual repertoire, and alternatives available at the time. Conversely, later success does not establish that the original generative process was epistemically exemplary. The meta-theory therefore separates historical reconstruction of available reasons from retrospective endorsement of the resulting theory.

2.6 Relational Process in Social Inquiry

Relational and processual social science provides the closest neighboring literature to the ontological restraint and evolutionary emphasis of the GR proposal. Emirbayer's relational sociology criticized substantialist approaches that treat entities as self-subsistent units and advanced a perspective centered on dynamic, unfolding relations [5]. Abbott's processual sociology likewise foregrounded time, events, sequences, turning points, and changing contexts in social explanation [1]. These traditions make it inappropriate to present relationality or historical process by themselves as novel contributions of the present framework.

The proposed distinction is more specific. For the purposes of theory-formation research, a GR entity is represented through its relational-historical location, so the identity relevant to analysis may itself depend on the trajectory of relations through which the entity has been constituted. A scholar, laboratory, legal institution, disciplinary category, or conceptual object can therefore change in ways that exceed modification of attributes attached to a fixed underlying unit. Boundaries, roles, available relations, and sometimes type assignments can become part of the evolutionary reconstruction.

This proposal also extends the comparative objective. Synchronic comparison can examine configurations across locations at a given period, while diachronic comparison can examine sequences through time. The GR framework adds an explicit search for the transformations under which selected structures remain invariant, covariant, or equivariant, including comparisons across different entity types where a defensible mapping exists. The emphasis is therefore placed on transformation structure rather than on a claim that relational or historical analysis has been absent from social science.

The distinction must remain modest. Existing relational, historical, and processual approaches contain substantial internal diversity, and many already analyze changing boundaries, identities, networks, institutions, or temporal scales. The present paper proposes a formal interface for a particular meta-theoretical problem: tracing how question formation, epistemic configuration, evolutionary history, structural abstraction, and symbolization interact during theory formation. Whether that interface yields explanatory gains beyond existing processual methods is an empirical and comparative question for later work.

2.7 Evolutionary Models of Knowledge Generation

Evolutionary epistemology supplies a seventh neighboring line. Campbell developed an evolutionary account of knowledge processes and scientific discovery organized around variation and selective retention [2, 3]. Such work is valuable here because it treats knowledge generation as temporally extended

and because it directs attention to the production, differential survival, and retention of candidate constructions.

The present framework uses the term *evolutionary* in a deliberately broader sense. Variation, selection, and retention mechanisms may characterize some theory-forming processes, but the framework does not assume that every relevant transformation has a Darwinian structure. An inquiry can also exhibit path dependence, irreversible accumulation, bifurcation, reclassification, topology change, changing boundaries, altered measurement practices, or reconstruction of the space of admissible theories. The analytical task is therefore to identify the transformation structure supported by the case rather than to select a biological analogy in advance.

This broader use also motivates a differentiated formal vocabulary. Reversible transformations on a common domain may support group-theoretic analysis; irreversible composable transformations on a common domain may be represented more naturally through semigroup or monoid structure; reversible local transformations among multiple objects can motivate groupoid constructions; and typed, irreversible, or evolving-state-space mappings may require more general category-theoretic morphisms. These are candidate formal resources whose use must be justified locally. Their appearance in the framework records classes of transformation that may need representation; it does not establish that empirical inquiry will instantiate any particular algebraic object.

The evolutionary perspective further changes the target of invariance analysis. Strict invariance is only one possibility. In many cross-context comparisons, the more informative structure may be equivariant: practices or representations change under a transformation while a relation among those changes is preserved. This permits comparison of heterogeneous cases without forcing identical surface behavior. It also provides a disciplined way to ask whether an apparent regularity is robust across spatial, temporal, spatiotemporal, same-type, or cross-type transformations.

2.8 Residual Meta-Theoretical Problem

The neighboring literatures establish that the principal ingredients of the proposed framework have substantial antecedents. Problems can be ill structured or found rather than merely solved; competent knowing can be tacit and reflective; expert action can be studied under naturalistic constraints; cognition and scientific criticism can be socially distributed; theory construction and conceptual innovation can be analyzed as extended reasoning processes; scientific and social objects can be historically and relationally constituted; and knowledge generation can be modeled through evolutionary processes. The present project should therefore be evaluated by the coherence and usefulness of its synthesis, not by a claim to have introduced these ingredients independently.

The residual problem is therefore one of operational coordination rather than simple synthesis. Some neighboring approaches already connect several components of construction, evaluation, representation, history, and distributed cognition. The proposed meta-theory asks a narrower question: whether one versioned research architecture can keep the provisional question, the configuration that supplies relevant reasons, relational-historical location, evolving entity identity and state spaces, cross-context transformation claims, symbolic representation, and recursive evaluation jointly inspectable without assuming that they collapse into a single method or universal sequence.

That coordination introduces dependencies that become invisible when the components are studied separately. A revised question can alter the relevant entity set. A newly included entity can expose a history that changes the abstraction. A formal representation can reveal an invariant that motivates renewed empirical comparison. The same representation can also erase temporal or relational distinctions and thereby require revision. A theory generated at one stage can enter the environment of later inquiry and transform the field in which subsequent questions are constituted. The framework developed in the following sections treats these dependencies as the principal analytical object.

The next substantive section begins the architecture with problem constitution. The later sequence should not be read as an irreversible pipeline. Its ordering provides analytical discipline for reconstruction, while the evaluation stage permits justified returns to earlier components. This combination of

staged analysis and recursive revision is intended to make the framework empirically usable without treating theory formation as a fixed sequence of cognitively or historically universal steps.

3 Problem Constitution in Situated Inquiry

This section develops the first substantive operation of the proposed architecture: the constitution of a research problem. Its objective is to specify what is being constituted when an inquiry begins from a provisional question and to explain why that constitution conditions later choices of epistemically relevant entities, evidence, scale, abstraction, and formal representation. The argument is conceptual and reconstructive. It draws on the literature on ill-structured problems and problem finding while extending the analysis toward a GR setting in which the question can be revised as the relevant relational-historical configuration becomes better understood. The section first defines problem constitution, then distinguishes its principal components, examines its selective effects on inquiry, and concludes with a taxonomy of legitimate problem revision.

Research methods often represent the research question as a sentence that precedes method selection. This representation is operationally convenient, yet it can understate the structure carried by the question. A question such as “What explains the persistence of a political institution?” already presupposes a candidate object, a temporal horizon, a conception of persistence, a relevant contrast class, and some expectation concerning what would count as an explanation. Similar presuppositions appear in less explicitly causal inquiries. A question concerning preservation, diagnosis, expertise, legitimacy, or theory change can delimit the relevant object before the inquiry has established that the proposed boundary is analytically adequate.

The literature on ill-structured problems provides an important precedent for treating problem specification as incomplete or fluid. Simon emphasized that the boundary between well-structured and ill-structured problems is itself difficult to draw and examined problem solving under conditions in which relevant information, operators, and consequences are not fully specified in advance [16]. Getzels treated problem finding as a theoretical object in its own right, thereby distinguishing the generation or recognition of a problem from the solution of a problem already presented to the inquirer [6]. The present framework takes these insights as methodological constraints and adds a relational-historical question: how does a provisional problem organize the field of entities, reasons, comparisons, and representations that become available to the subsequent inquiry?

3.1 Constitutional Structure of a Research Problem

Within the present framework, *problem constitution* denotes the provisional specification of the object and conditions of inquiry together with the kind of epistemic achievement sought from the inquiry. This definition concerns research organization rather than ultimate ontology. A phenomenon may exist independently of the way it is investigated, while the research problem through which that phenomenon becomes tractable can still be historically and relationally constituted.

For analytical purposes, let a provisional research problem at research time t be represented by the ordered structure in Equation 8.

$$Q_t = \langle O_t, S_t, G_t, C_t, A_t \rangle. \quad (8)$$

In Equation 8, O_t denotes the provisionally constituted object of inquiry, S_t its relevant scope and scale, G_t the epistemic or practical target of the inquiry, C_t the conditions under which the question is posed, and A_t the admissible form or criterion of an answer. The components are analytical distinctions rather than claims that every research project explicitly states five separable variables. Their role is diagnostic: they make visible several commitments that can otherwise remain implicit inside a grammatically simple question.

The object term O_t concerns what is being treated as the bearer of the phenomenon to be explained, interpreted, evaluated, predicted, designed, or transformed. An object can be an entity, relation, process,

event, field, practice, institution, trajectory, configuration, or another justified unit. The framework does not privilege any one of these forms. The methodological requirement is that the selected unit be open to revision when evidence indicates that the initial object boundary suppresses a constitutive relation or combines analytically heterogeneous processes.

The scope term S_t records the spatial, temporal, organizational, historical, and comparative extent within which the question is meant to hold. Scale is included because a pattern can change character when the unit of analysis changes. An account developed at the level of an individual practitioner may differ from one developed at the level of a team, institution, discipline, or transnational field. Likewise, a relation that appears stable over a short interval may be transient over a longer historical horizon. Explicit scope therefore prevents an initially local observation from silently acquiring universal force.

The target term G_t identifies the epistemic or practical work that the inquiry seeks to accomplish. Explanation, prediction, interpretation, reconstruction, diagnosis, evaluation, design, intervention, preservation, and classification impose different requirements on the inquiry. A description that is adequate for reconstructing a historical trajectory may be insufficient for prediction; a predictive association may remain insufficient for a normative evaluation. Treating the target as part of the problem therefore helps prevent the later method from being judged by criteria that were never appropriate to its objective.

The condition term C_t identifies the boundary conditions, informational constraints, institutional setting, and other contextual restrictions within which the problem is constituted. In some inquiries, the conditions include uncertainty, limited time, incomplete evidence, conflicting signals, or changing environments. In others, they include legal jurisdictions, technological capabilities, measurement regimes, or historically available concepts. The conditions are relevant because a problem that appears identical at the level of wording can require a different theory when the operative conditions change.

The answer term A_t specifies what would count, provisionally, as a satisfactory form of resolution. This can include the expected level of explanation, required evidence, admissible uncertainty, formal precision, normative justification, or practical usability. The term also records that answer criteria may be heterogeneous. A legal argument, a causal model, a historical reconstruction, and an engineering design do not establish adequacy in the same way. Methodological pluralism therefore begins inside the constitution of the problem rather than after a universal research procedure has already been selected.

Definition 3.1 (Problem constitution). For a research episode at time t , problem constitution is the provisional specification of $Q_t = \langle O_t, S_t, G_t, C_t, A_t \rangle$ together with the relations among these components that determine which observations, entities, reasons, comparisons, and representations can become relevant to the inquiry. The specification remains revisable throughout the research trajectory.

Definition 3.1 treats the five components as mutually constraining. Changing the temporal scale can change the object that remains identifiable; changing the target from prediction to historical explanation can change which evidence is relevant; changing the answer criterion can alter which formal representations are useful. Problem constitution therefore concerns a configuration rather than a list of independent settings.

3.2 Question-Conditioned Relevance

A principal reason for making problem constitution explicit is that the provisional question conditions what later appears relevant. Let $\mathcal{U}_t$ denote the set of candidate observations, records, entities, practices, concepts, instruments, and relations available to the inquiry at time t . The problem induces a relevance assessment over this available field. Equation 9 records this dependence schematically.

$$\rho_t(z) = \rho(z \mid Q_t, \Lambda_t, H_t), \quad z \in \mathcal{U}_t. \quad (9)$$

Here $\rho_t(z)$ denotes the inquiry's provisional assessment of the relevance of candidate element z , Λ_t denotes the relational-historical location from which the inquiry is organized, and H_t denotes the historically available background carried into the episode. Equation 9 is not a claim that relevance can ordinarily be reduced to a scalar measure. It expresses the weaker methodological point that relevance is

conditional: evidence is evidence for a question, a comparison is informative relative to a contrast, and an entity supplies reasons relative to a problem.

This conditionality gives the initiating question substantial downstream influence. A problem that defines the object as an individual decision maker can direct attention toward individual perception, memory, and judgment. A reformulation that defines the object as a distributed decision configuration can make communication channels, artifacts, organizational roles, and coordination practices analytically central. Both formulations may concern the same episode, yet they open different evidential and theoretical spaces.

The same dependence appears in theory formation. A problem constituted around “which option is chosen” can favor models of selection among alternatives. A problem constituted around “how the situation becomes recognizable as a particular kind of situation” can favor models of salience, categorization, and experienced pattern recognition. Neither formulation is automatically superior. Their adequacy depends on the research target and on whether later reconstruction shows that the chosen object and distinctions preserve the generative structure of the case.

Question-conditioned relevance introduces a risk of premature closure. If Q_t strongly filters $\mathcal{U}_t$, observations that would motivate a different problem can remain outside the active field of inquiry. The framework therefore requires a distinction between *working relevance* and *constitutive adequacy*. Working relevance permits research to proceed with a tractable provisional question. Constitutive adequacy concerns whether the question continues to preserve the relations and distinctions required by the developing evidence.

Claim 3.2 (Provisional priority of problem constitution). Within the present framework, the initiating problem has provisional methodological priority because it conditions the subsequent identification of relevant entities, evidence, scales, and answer criteria. This priority is procedural rather than epistemically final: later stages can supply reasons for revising the problem itself.

Claim 3.2 explains why problem constitution is placed before reason-bearing entity identification in the analytical architecture. The sequence does not imply that the researcher possesses a correct question before encountering the relevant entities. It provides a reconstruction point from which dependencies can be traced. Once new entities, histories, or practices become visible, the question can be reconstructed in return.

3.3 Relational-Historical Location of the Problem

Problem constitution is itself situated. A research question becomes thinkable through a historically available repertoire of concepts, instruments, institutions, observations, and normative concerns. The same phenomenon can therefore support different research problems at different historical locations. Kuhn’s analysis of scientific development provides one influential account of historically organized changes in scientific problems, exemplars, and standards [10]. The present framework makes a narrower methodological use of this insight: the availability of a question and the distinctions encoded within it should be reconstructed relative to the historical and relational field in which the question emerged.

Let Λ_t denote the relational-historical location of the inquiry itself, including the research community, available conceptual vocabulary, institutional position, instruments, prior theories, and relevant normative commitments. The constitution of the problem can then be represented as the situated operation in Equation 10.

$$Q_t = \Phi_t(D_t, \Lambda_t, J_t), \quad (10)$$

where D_t denotes the materials or phenomena provisionally available to the inquiry and J_t denotes the judgments through which salience, boundaries, and targets are selected. Equation 10 does not imply that the question is freely constructed by the researcher. D_t can constrain plausible formulations, and unsuccessful formulations can be rejected by evidence, contradiction, failed prediction, practical breakdown, or comparative inadequacy. The equation records that the path from available phenomena to a researchable problem is mediated by historically situated distinctions and judgments.

This point is especially important for retrospective studies of theory formation. A later investigator can easily formulate a historical episode using concepts unavailable to the original actors. Such vocabulary can be analytically useful, but it should be distinguished from the problem space that was historically accessible at the time. A reconstruction that silently imports later categories risks misrepresenting both the reasons available to earlier researchers and the novelty of the theory they eventually produced.

The relational location of the problem also includes institutional and normative position. Funding priorities, disciplinary boundaries, legal categories, technological capacities, public controversies, and professional responsibilities can influence which questions become salient and which answer forms receive recognition. These influences do not determine the content of a theory in any simple way. They nevertheless form part of the field within which some problems become actionable, legitimate, or visible while others remain marginal.

3.4 Problem Revision as an Epistemic Outcome

A central commitment of the framework is that research can improve by changing the question. Revision can occur because the initial object was too narrow, because an assumed entity proves derivative, because a relevant relation was omitted, because a new timescale changes the phenomenon, because the answer criterion was inappropriate, or because formalization reveals that the original distinctions do not support the intended comparison. In these cases, revision of Q_t is part of inquiry rather than evidence that inquiry failed to begin correctly.

Let $\mathcal{Z}_t$ collect the information returned to the problem-constitution stage from later parts of the research architecture, including reconstructed entities and practices, evolutionary trajectories, comparisons across contexts, formal representations, and evaluative results. Problem revision is represented schematically in Equation 11.

$$Q_{t+1} = \mathcal{R}_t(Q_t, \mathcal{Z}_t). \quad (11)$$

The revision operator $\mathcal{R}_t$ in Equation 11 is a formal placeholder for a reason-giving research process, not a deterministic update rule. Its value lies in keeping the direction of dependence explicit: later inquiry can transform the question that originally organized the inquiry. Table 2 distinguishes six recurrent modes of such transformation.

The distinction between replacement and dissolution is analytically useful. Replacement preserves an unresolved field of inquiry while reorganizing it through a better question. Dissolution indicates that the original question no longer requires an answer in its initial form because the distinction that generated the puzzle has been removed or reclassified. Both outcomes can represent epistemic progress.

Problem revision also changes the downstream architecture. If Q_t changes to Q_{t+1} , then the set of relevant entities can change, the appropriate relational-historical locations can change, and a previously useful abstraction can cease to be appropriate. Equation 12 states this dependency without assuming that every revision produces change at every later stage.

$$\begin{aligned} Q_t \mapsto Q_{t+1} &\implies \left(\mathcal{E}_{Q_t}, \mathbf{\Lambda}_t, \widehat{\mathbf{E}}_t, \mathbf{\Omega}_t, \mathfrak{B}_t, \mathfrak{T}_t, \mathfrak{S}_t \right) \\ &\rightsquigarrow \left(\mathcal{E}_{Q_{t+1}}, \mathbf{\Lambda}_{t+1}, \widehat{\mathbf{E}}_{t+1}, \mathbf{\Omega}_{t+1}, \mathfrak{B}_{t+1}, \mathfrak{T}_{t+1}, \mathfrak{S}_{t+1} \right). \end{aligned} \quad (12)$$

The double-line implication and wavy transition in Equation 12 denote a possible downstream reconfiguration rather than a logical entailment of component-wise change. Its role is to mark the recursive coupling among stages. A revised question can preserve some earlier constructions while requiring others to be rebuilt.

3.5 Adequacy Criteria for a Provisional Problem

Because the framework permits question revision, it also requires criteria for judging whether a provisional problem is sufficiently adequate to organize the next stage. The criteria should support inquiry

Table 2: Principal modes of problem revision in the proposed framework. The categories are analytical and can overlap within a research trajectory.

Revision mode	Structural change	Typical reason for revision
Retention	The operative components of Q_t remain adequate.	Later reconstruction supports the initial object, scope, target, conditions, and answer form.
Refinement	One or more components become more precise without changing the basic problem identity.	Better evidence clarifies a boundary, timescale, contrast class, or criterion.
Decomposition	One problem becomes several coupled subproblems.	The initial object combines mechanisms, scales, or answer types that require separate treatment.
Rescaling	The spatial, temporal, organizational, or historical scope changes.	A pattern depends on a scale different from the one initially selected.
Replacement	A new problem organizes the inquiry around a different object or target.	The original formulation suppresses a more generative structure or directs attention toward a derivative phenomenon.
Dissolution	The initial question loses its role as a productive research problem.	Its central distinction proves unsupported, category-dependent, or unnecessary after reconstruction.

without demanding impossible certainty at the beginning. Five dimensions are proposed as a preliminary diagnostic set.

First, *object adequacy* asks whether O_t preserves the relations and processes required by the phenomenon under investigation. Second, *scope adequacy* asks whether S_t matches the spatial, temporal, organizational, and historical scale at which the relevant structure can be observed. Third, *target adequacy* asks whether G_t identifies an epistemic or practical achievement that the available inquiry can meaningfully pursue. Fourth, *conditional adequacy* asks whether the principal boundary conditions and constraints in C_t are sufficiently visible to prevent unsupported generalization. Fifth, *answer adequacy* asks whether A_t specifies criteria that are appropriate to the target and do not privilege a formal, quantitative, qualitative, historical, or normative mode merely by convention.

These dimensions can be collected as the diagnostic profile in Equation 13.

$$\alpha(Q_t) = (a_O(Q_t), a_S(Q_t), a_G(Q_t), a_C(Q_t), a_A(Q_t)). \quad (13)$$

Equation 13 should not be interpreted as a requirement to assign numerical scores. The notation records five distinguishable sites of possible inadequacy. A research programme can therefore state why it is revising a question rather than relying on an undifferentiated judgment that the original problem was “wrong.” The profile also permits asymmetric revision. An inquiry can preserve its object and target while changing only its temporal scale or evidence criterion.

The provisional character of these criteria is important. Some inquiries discover relevant conditions only after contact with practitioners, archives, instruments, or unfamiliar cases. Requiring complete specification at the beginning would reproduce the very idealization that the framework seeks to avoid. A workable problem is therefore one that provides enough structure to guide inquiry while remaining open to reasons generated by the inquiry itself.

Remark 3.3 (Problem adequacy and ontological restraint). A revision of the research problem does not by itself establish that the underlying phenomenon has changed or that the phenomenon is constituted solely by inquiry. The present framework distinguishes changes in the researcher’s problem represen-

tation from changes in the relational configuration being studied. Empirical and historical analysis is required to determine when these processes interact.

Remark 3.3 preserves the ontological suspension required by the broader GR programme. Problem constitution is epistemically consequential because it organizes inquiry; this consequence does not license a general inference from research representation to ultimate ontology.

The next stage follows directly from the conditional character of relevance. Once Q_t is sufficiently specified for inquiry to proceed, the framework asks which GR entities or distributed configurations possess reasons that are relevant to this particular problem and how those entities are located within the relations and histories that make their reasons intelligible. The next substantive section develops that identification and relational-historical localization problem.

4 Relational-Historical Location of Reason-Bearing GR Entities

This section develops the second substantive operation of the proposed architecture: the identification and localization of GR entities or distributed configurations whose situated reasons can materially contribute to the problem constituted in Section 3. Its objective is to specify why epistemic relevance should be indexed to a particular problem, why the relevant bearer of reasons cannot be assumed in advance to be an isolated individual, and why an entity's relational-historical location forms part of the evidence needed to interpret its reasons. The argument is conceptual and reconstructive. It draws on naturalistic decision making, tacit and reflective professional knowledge, distributed cognition, social epistemology, and relational-processual social theory while preserving the present framework's ontological restraint. The section first defines a GR entity for the purposes of this meta-methodology, then develops question-relative reason-bearing status, formalizes relational-historical location, and concludes with a selection discipline for individual and distributed epistemic configurations.

4.1 GR Entities as History-Bearing Relational Configurations

The term *entity* can easily suggest an independently bounded bearer of fixed properties. Such a representation is too strong for the present framework. The proposed meta-theory requires only that an object of inquiry possess sufficient local continuity for its trajectory, relations, practices, or reasons to be reconstructed. A GR entity is therefore treated here as an analytically individuated, history-bearing relational configuration whose boundary is provisionally adequate to the research problem. This formulation permits persons, teams, institutions, communities, organizations, and other justified configurations to enter the analysis without asserting that their analytical boundaries exhaust their ontology.

For an entity e_i considered at research-relevant time t , Equation 14 introduces a minimal local description.

$$\mathfrak{E}_i(t) = \langle B_i(t), \mathcal{R}_i(t), H_i^{\leq t}, P_i(t) \rangle. \quad (14)$$

In Equation 14, $B_i(t)$ denotes the provisional boundary by which the entity is individuated for the inquiry, $\mathcal{R}_i(t)$ denotes the set or structured family of relations currently relevant to its constitution and action, $H_i^{\leq t}$ denotes the history carried into the present episode, and $P_i(t)$ denotes the entity's current practical, institutional, material, or organizational position. The tuple is a structural representation rather than an ontological decomposition. Its purpose is to prevent the analysis from treating a reason-bearing entity as a point-like source of statements detached from the relations and histories through which its competence, access, incentives, vocabulary, and judgment have developed.

The boundary term $B_i(t)$ is necessary because individuation is itself problem-relative. A pilot can be analyzed as an individual decision maker for one question, as a participant in a cockpit crew for another, and as one node in a larger aircraft-automation-organization configuration for a third. The same physical person remains present, while the analytically adequate boundary changes with Q_t . Hutchins's analysis of navigation provides an important precedent for treating cognition as organized across persons,

artifacts, procedures, and culturally structured activity rather than locating every relevant cognitive operation inside a single individual [7]. The present framework generalizes the methodological lesson: entity boundaries should be justified by the problem and remain revisable when the selected boundary hides a constitutive relation.

The relational term $\mathcal{R}_i(t)$ records connections through which the entity obtains information, authority, resources, constraints, recognition, obligations, or opportunities for action. Relational sociology has similarly argued against treating social units as self-subsistent substances and has emphasized transactions and unfolding relations as central analytical objects [5]. The present use is narrower. It does not require a universal relational ontology. It requires the analyst to preserve those relations that materially affect why a candidate entity can perceive, judge, or act as it does with respect to the problem under study.

The historical term $H_i^{\leq t}$ records the accumulated trajectory through which the present configuration became possible. Training, prior failures, inherited practices, disciplinary lineages, institutional changes, previous collaborations, exposure to exceptional events, and earlier conceptual commitments can all enter when supported by the inquiry. Abbott's processual emphasis on events, sequences, turning points, and temporality supplies a neighboring methodological precedent for treating present social configurations as historically generated rather than as timeless carriers of variables [1]. Within the present framework, history is especially important because a reason that appears identical in propositional form can have different epistemic significance when it is generated through different trajectories of experience and access.

The positional term $P_i(t)$ records the entity's present placement within the relevant field. Physical location is one possible component, but organizational role, legal status, disciplinary location, access to instruments, place in a communication network, relation to affected populations, and position within a chain of responsibility can be equally important. The term therefore denotes a generalized relational position rather than a coordinate in geographical space alone.

Definition 4.1 (GR entity for theory-formation analysis). For the purposes of this meta-theory, a GR entity is a provisionally individuated, history-bearing relational configuration $\mathfrak{E}_i(t)$ whose boundary, relations, history, and present position can be reconstructed to the degree required by a specified research problem. Entityhood in this definition is analytical and revisable; it does not establish an ultimate ontology.

Definition 4.1 makes entity identification subordinate to research adequacy. A person, institution, or collective does not enter the framework merely because it is socially recognized as an entity. It enters because the selected individuation preserves structures that matter to the problem. This condition also allows the inquiry to discover that an initially selected entity is too narrow, too broad, or internally heterogeneous.

4.2 Question-Relative Reason-Bearing Status

After a provisional problem Q_t has been constituted, the framework asks which entities have access to experiences, practices, observations, judgments, or forms of knowledge that can supply reasons relevant to that problem. The phrase *reason-bearing* is used deliberately. It does not certify the correctness of an entity's beliefs, nor does it equate professional status with epistemic authority. It identifies a candidate source of reasons whose relevance and quality remain open to evaluation.

Let $\mathcal{U}_t^E$ denote the set of candidate entities or configurations that the inquiry could in principle examine at time t . For each candidate $e_i \in \mathcal{U}_t^E$, let $\mathcal{G}_i(Q_t)$ denote the set of reason-giving materials attributable to or generated through that candidate with respect to Q_t . These materials can include articulated reasons, observed practices, discriminations, diagnostic cues, procedural knowledge, counterexamples, normative considerations, or historically accumulated judgments. The provisional reason-bearing set is defined in Equation 15.

$$\mathcal{E}_{Q_t} = \{e_i \in \mathcal{U}_t^E : \mathcal{G}_i(Q_t) \text{ contains material bearing on at least one component of } Q_t\}. \quad (15)$$

Equation 15 is intentionally permissive. Entry into $\mathcal{E}_{Q_t}$ means that the entity warrants examination, not that its reasons should be accepted. Subsequent reconstruction and evaluation can distinguish reliable from unreliable judgment, locally useful from overgeneralized practice, justified from self-serving claims, and epistemically relevant from merely authoritative speech. This separation is necessary because historical success, institutional prestige, confidence, and formal credentials can each diverge from the quality of the reasons relevant to a particular question.

Naturalistic decision making supplies a useful illustration. The NDM tradition studies experienced practitioners operating in consequential real-world settings and examines how experience supports situation assessment, recognition, and action under conditions such as uncertainty and time pressure [8]. The methodological lesson for the present framework is not that experts form a universally privileged class. It is that a problem can make certain histories of exposure, practice, and situated access epistemically relevant. An experienced pilot may therefore be highly reason-bearing for a question concerning recognition of an anomalous cockpit configuration while possessing no corresponding privilege for a question concerning the political legitimacy of aviation regulation.

Tacit and reflective-practice traditions strengthen the same point. Polanyi’s account of tacit knowledge emphasizes forms of knowing that exceed complete articulation, while Schön’s analysis of professional practice foregrounds reflection and improvisation within uncertain situations [13, 14]. If such forms of competence matter to Q_t , the analyst cannot identify relevant reason-bearing entities solely by searching for explicit propositions. Observed action, sequence of attention, distinctions made in practice, and the entity’s history of interaction with the problem domain can become evidence for later reconstruction.

Question-relativity also implies that the reason-bearing set can change when the problem changes. Let $\mathcal{S}$ denote the selection operation that maps a problem and an available field of candidates into a provisional reason-bearing set. Equation 16 states this dependence.

$$\mathcal{E}_{Q_t} = \mathcal{S}(Q_t, \mathcal{U}_t^E, \Lambda_t), \quad (16)$$

where Λ_t denotes the relational-historical information available for assessing candidate entities. If evaluation later produces a revised question Q_{t+1} , the appropriate set need not remain stable. Equation 17 records the resulting possibility.

$$Q_t \mapsto Q_{t+1} \quad \text{can induce} \quad \mathcal{E}_{Q_t} \mapsto \mathcal{E}_{Q_{t+1}}. \quad (17)$$

Equation 17 does not assert that every problem revision requires a new entity set. It marks a recursive dependency that the framework must permit. A study that begins by examining senior decision makers may discover that the operative knowledge is distributed across technical staff, affected users, institutional routines, or artifacts. In such a case, retaining the original entity set would protect the initial problem formulation from evidence generated by the inquiry itself.

4.3 Relational-Historical Location

Reason-bearing status alone is insufficient because reasons are produced from positions. Two entities can state the same proposition while differing substantially in access to evidence, exposure to consequences, institutional incentives, temporal experience, and relations to other participants. Conversely, two entities can occupy the same formal role while having different histories that make different aspects of a situation salient. The framework therefore associates each candidate entity with a relational-historical location.

For the present purpose, the relational-historical location of e_i at time t is represented by Equation 18.

$$\Lambda_i(t) = \langle \mathcal{N}_i(t), H_i^{\leq t}, P_i(t), M_i(t) \rangle. \quad (18)$$

In Equation 18, $\mathcal{N}_i(t)$ denotes the entity’s relevant relational neighborhood, $H_i^{\leq t}$ its historical trajectory, $P_i(t)$ its current practical or institutional position, and $M_i(t)$ the material and representational

mediators through which it perceives, records, communicates, or acts. The inclusion of $M_i(t)$ is important because access is often technologically and symbolically mediated. Instruments, documents, interfaces, models, checklists, databases, and computational systems can alter what becomes observable and what can be communicated to other entities.

The four components of $\Lambda_i(t)$ are not assumed to be independent. Historical trajectories can generate present network positions; institutional positions can determine access to instruments; mediating artifacts can stabilize relations across time; and relational neighborhoods can shape the categories through which an entity interprets its own experience. The representation is therefore intended to organize reconstruction rather than to supply a separable-variable model.

The epistemic significance of a reason can then be indexed to its location. Let $g \in \mathcal{G}_i(Q_t)$ be a candidate reason generated by or reconstructed from entity e_i . Equation 19 represents its situated character.

$$g = g(Q_t, \Lambda_i(t), \Xi_i(t)) , \quad (19)$$

where $\Xi_i(t)$ denotes the entity's locally relevant epistemic, normative, and practical organization. The next substantive section will develop $\Xi_i(t)$ in detail. At the present stage, Equation 19 serves only to prevent a reason from being detached from the conditions through which it became available. The equation does not imply that every valid reason is relativized into local correctness. A situated origin and a broader domain of validity are compatible. The task of later evaluation is to determine which aspects of a reason are local, which travel across contexts, and which fail when the relational-historical location changes.

This formulation also clarifies the difference between relational-historical location and conventional demographic or institutional description. A list of attributes can state that an entity is a senior physician, a junior engineer, or a professor at a particular institution. Relational-historical localization asks how the entity came to occupy that position, which relations and mediators provide access to the relevant phenomenon, which events have shaped salience and judgment, and which dependencies or constraints condition what the entity can know or say. Location is therefore reconstructive and generative rather than merely classificatory.

Remark 4.2 (Situated reasons and epistemic relativism). Locating a reason within $\Lambda_i(t)$ does not establish that the reason is valid only for that location. Relational-historical reconstruction identifies conditions of generation and access. Claims concerning validity, transfer, invariance, or generalization require additional comparison and evaluation in later stages of the framework.

Remark 4.2 is important for the framework's scope. Historical and relational dependence can explain why a reason was available without determining in advance how broadly the reason holds. This separation permits the later invariant and equivariant analyses to ask whether a structure survives transformations across locations, times, scales, or entity types.

4.4 Distributed Reason-Bearing Configurations

Some research problems do not admit a defensible allocation of the relevant reasons to a single entity. Cognition and judgment can be distributed across multiple persons, artifacts, organizational routines, and communication channels. Hutchins's ship-navigation analysis is a paradigmatic example of cognition organized through a socially and materially distributed system [7]. Longino's account of scientific knowledge likewise foregrounds the epistemic significance of social interaction and critical exchange in scientific inquiry [11]. These traditions motivate an important safeguard for the present framework: the analyst should be able to select a relational configuration as the reason-bearing unit when decomposition into isolated entities would destroy the process that generates the reasons of interest.

Let I_t index a set of participating entities and let $\mathcal{R}_t^E$ denote the relations among them that are relevant to the problem. Let $\mathcal{M}_t$ denote mediating artifacts or representational systems whose participation is analytically necessary. A distributed epistemic configuration can then be represented by Equation 20.

Table 3: Diagnostic audit for selecting reason-bearing entities or configurations.

Diagnostic site	Guiding question	Principal failure risk
Problem relevance	Which component of Q_t can the entity’s experience, practice, observation, or judgment materially inform?	Prestige or availability substitutes for relevance.
Access	Through which relations, events, instruments, or practices does the entity obtain access to the phenomenon?	Claimed competence is accepted without reconstructing its evidential basis.
Relational-historical location	Which trajectory and present position make the entity’s reasons intelligible?	Reasons are detached from incentives, dependencies, histories, and mediators.
Boundary adequacy	Is the selected unit sufficiently complete to preserve the reason-generating process?	An individual is isolated from a constitutive team, institution, artifact, or network.
Contrast and revisability	Which absent, unsuccessful, peripheral, or differently located cases could challenge the selection?	Survivorship, authority, and success become circular definitions of epistemic quality.

$$\mathcal{K}_{Q_t}(t) = \langle \{\mathfrak{E}_i(t)\}_{i \in I_t}, \mathcal{R}_t^E, \mathcal{M}_t \rangle. \quad (20)$$

The configuration in Equation 20 is warranted when the target reason-generating process depends materially on interactions among its components. Examples can include cross-checking, division of attention, sequential handoff, adversarial criticism, shared instrumentation, record keeping, model-mediated coordination, or human-machine coupling. The analytical criterion is functional and relational: if the relevant epistemic performance cannot be reconstructed adequately by attributing it to one component independently, the distributed configuration becomes a candidate unit.

This possibility modifies the interpretation of Step 2 in the overall framework. The task is not simply to find the person who “knows the answer.” It is to identify the entity or configuration within which problem-relevant reasons are generated, retained, challenged, transmitted, or transformed. In some cases this unit will be an individual. In others it can be a team, institution, disciplinary community, laboratory, courtroom, cockpit, or socio-technical arrangement. The appropriate boundary remains an empirical and analytical question.

Distributed configuration does not eliminate individual asymmetries. Members can possess unequal access, authority, vulnerability, expertise, or capacity to contest a claim. These asymmetries can be constitutive of the reason-generating process and therefore should remain visible in $\mathcal{R}_t^E$. A collective label should not homogenize the entities that compose it. The framework consequently treats distributed reason-bearing as a structured configuration rather than as an undifferentiated group mind.

4.5 Selection Discipline and Revisability

Entity identification creates an early risk of circularity. A researcher can choose recognized experts, reconstruct their practices, abstract common features, and then redescribe those features as the nature of expertise. The resulting theory would partly reproduce its own selection rule. A GR meta-theory therefore requires an explicit selection discipline that separates candidate relevance from later epistemic endorsement.

Table 3 summarizes five diagnostic questions for selecting and revising reason-bearing entities or configurations. The diagnostics are qualitative obligations rather than a scoring system.

The first diagnostic requires a traceable relation between the candidate entity and a component of the problem structure in Equation 8. The second asks how access is produced rather than accepting a role label as evidence of competence. The third requires sufficient reconstruction of $\Lambda_i(t)$ to interpret the candidate reasons. The fourth tests whether the selected unit preserves the process through which those reasons arise. The fifth creates space for negative and contrastive cases, including failed decisions, unrecognized scholarship, peripheral participants, or differently situated entities whose trajectories could reveal that an apparent expert regularity is contingent on selection.

The selection discipline also recognizes a genuine failure state. Some problems may provide no presently identifiable entity or configuration whose reasons are sufficiently relevant for this route to carry the inquiry. Let $\text{Adeq}(e_i, Q_t)$ denote a qualitative adequacy judgment supported by the diagnostics in Table 3. The method permits the outcome stated in Equation 21.

$$\nexists e_i \in \mathcal{U}_t^E \text{ such that } \text{Adeq}(e_i, Q_t) \text{ is sufficiently supported.} \quad (21)$$

Equation 21 is not a failure of research in general. It is a result about the suitability of this route for the current problem and evidence. The inquiry may require a phenomenon-first strategy, a different empirical design, a revised question, or a hybrid method. This admissible exit condition will be developed systematically in the later limitations section. Its presence here prevents the framework from manufacturing reason-bearing entities merely because the architecture contains an entity-identification stage.

The output of the present section is therefore not a fixed roster of authorities. It is a revisable set of candidate entities $\mathcal{E}_{Q_t}$ and, where required, distributed configurations $\mathcal{K}_{Q_t}(t)$, each accompanied by a relational-historical localization sufficient for the next analytical stage. The framework can then ask what these entities actually perceive, know, value, judge, and do, how tacit and explicit resources interact in practice, and which parts of that organization can be reconstructed without projecting the analyst's categories onto the entity. The next substantive section develops that epistemic-normative-practical reconstruction.

5 Epistemic-Normative-Practical Reconstruction

This section develops the third substantive operation of the proposed architecture: reconstruction of the locally relevant epistemic, normative, and practical organization of the reason-bearing entities or distributed configurations identified in Section 4. Its objective is to specify what must be reconstructed before the analyst can abstract recurrent structures from expert, scholarly, professional, institutional, or other situated practices. The method is reconstructive rather than introspectively literal. Reported reasons, observed conduct, contemporaneous traces, artifacts, and outcomes provide different forms of evidence, none of which is assumed to reveal the generative organization of inquiry transparently. The section first defines the reconstruction target, then distinguishes salience, knowledge, judgment, normativity, and practice, examines their coupling in situated episodes, and concludes with an evidence architecture designed to limit retrospective rationalization and analyst projection.

5.1 Reconstruction Target and Analytical Components

Section 4 introduced $\Xi_i(t)$ as a placeholder for the locally relevant epistemic, normative, and practical organization of entity e_i . The present section gives this placeholder an explicit structure. For an entity or justified distributed configuration examined with respect to problem Q_t , Equation 22 represents the reconstruction target.

$$\Xi_i^{Q_t}(t) = \langle \text{Sal}_i(t), \text{K}_i(t), \text{J}_i(t), \text{N}_i(t), \text{Pr}_i(t) \rangle. \quad (22)$$

In Equation 22, $\text{Sal}_i(t)$ denotes the organization of salience and attention through which some available features become candidates for notice; $\text{K}_i(t)$ denotes the explicit, tacit, procedural, and representational resources available to the entity; $\text{J}_i(t)$ denotes the discriminative and evaluative operations through

which candidate interpretations, explanations, actions, or questions are assessed; $N_i(t)$ denotes the epistemic, practical, ethical, professional, or other normative commitments that shape acceptable inquiry and action; and $Pr_i(t)$ denotes the recurrent practices through which perception, interpretation, testing, communication, intervention, and revision are enacted. The superscript Q_t indicates that the reconstruction is problem-indexed. The analyst need not reconstruct every aspect of an entity's cognition or practice, only those components that materially bear on the constituted research problem.

The five components in Equation 22 are analytical distinctions rather than independent modules. Saliency can depend on prior knowledge; judgment can reorganize what later becomes salient; normative commitments can influence which anomalies deserve attention; practice can stabilize or modify tacit knowledge; and repeated practice can alter the standards by which judgment is exercised. The framework therefore treats $\Xi_i^{Q_t}(t)$ as an organized configuration whose internal couplings are themselves potential objects of reconstruction.

Definition 5.1 (Situated reconstruction). Epistemic-normative-practical reconstruction is the evidence-constrained inference of a problem-relevant organization $\Xi_i^{Q_t}(t)$ from observable or documentable traces of an entity's situated inquiry, judgment, and practice. The reconstructed organization is an analytical model of the generative process; it is not identified with an entity's self-description or with an unobservable complete mental state.

Definition 5.1 imposes an important epistemic restraint. The analyst rarely has direct access to the complete organization that generated a decision, interpretation, or theory. Reconstruction therefore produces a supported representation of that organization, not a transparent copy. This distinction becomes especially important when tacit knowledge, distributed activity, historical memory, or retrospective narration materially contribute to the phenomenon.

5.2 Saliency and the Formation of Epistemic Attention

A central difference between novice and proficient practice can arise before explicit inference begins. Experienced entities may notice a deviation, pattern, relation, or contingency that remains background for others. Naturalistic decision making is relevant here because it emphasizes experience-dependent situation assessment and rapid categorization in consequential settings [8]. Tacit-knowledge traditions likewise caution against reducing competent recognition to rules that actors can fully articulate [13]. The present framework therefore treats saliency formation as an independent reconstruction target rather than assuming that all entities receive the same informational input and differ only in subsequent reasoning.

Let $\mathcal{Z}_i(t)$ denote the set of cues, observations, records, signals, relations, or other informational features to which entity e_i has access within its relational-historical location $\Lambda_i(t)$. Let $\mathcal{Z}_{i,Q_t}^*(t)$ denote the subset that becomes epistemically salient with respect to Q_t . Equation 23 states this relation.

$$\mathcal{Z}_{i,Q_t}^*(t) = \text{Sal}_i(t)(\mathcal{Z}_i(t); Q_t, \Lambda_i(t)), \quad \mathcal{Z}_{i,Q_t}^*(t) \subseteq \mathcal{Z}_i(t). \quad (23)$$

Equation 23 is a structural representation. It does not assume that saliency is deterministic or that the set of accessible cues is fully enumerable. Its purpose is to distinguish availability from attention. An entity can have physical or informational access to a feature without treating it as relevant, while another entity can assign high significance to a weak or unusual cue because previous experience, theoretical commitments, or normative concerns make that cue recognizable.

This distinction is especially important for theory formation. A proficient scholar may encounter the same text, dataset, anomaly, historical episode, or conceptual tension as many other researchers, yet constitute a research problem because a relation among those materials becomes salient. The analytically relevant question is therefore not only which information was available, but which differences, similarities, absences, sequences, or contingencies became worthy of inquiry and under what prior conditions they acquired that status.

Saliency should also remain open to error. Expertise can produce sensitivity to weak signals, but it can also stabilize habitual expectations that suppress unfamiliar evidence. A reconstruction that treats

noticed cues as inherently important would reproduce the entity’s own attentional biases. The framework instead records how salience was formed and leaves the later evaluative stages to test whether the salient structure survives comparison, counterevidence, and transformation across contexts.

5.3 Knowledge, Judgment, Normativity, and Practice

The remaining components of $\Xi_i^{Q_t}(t)$ specify resources and operations that act on, and are modified by, the salient field. Polanyi’s account of tacit knowing supports the inclusion of epistemic resources that exceed complete articulation [13]. Schön’s account of reflection-in-action similarly provides a precedent for studying professional knowing as situated engagement with uncertain and distinctive situations rather than as the direct execution of fully specified technical rules [14]. These traditions justify a reconstruction that includes explicit statements while refusing to equate the generative organization of practice with what can be reported propositionally.

The knowledge component $K_i(t)$ can include explicit propositions, concepts, classifications, mathematical or linguistic representations, procedural routines, exemplars, remembered episodes, instrument-dependent knowledge, and tacit discriminations. The purpose of the category is functional: it records resources the entity can draw upon when recognizing, interpreting, comparing, simulating, testing, or communicating a candidate account. The framework does not require every item in $K_i(t)$ to have the same epistemic status. Well-supported knowledge, inherited convention, heuristic expectation, and provisional conjecture can coexist and should be distinguished where the evidence permits.

The judgment component $J_i(t)$ concerns operations that rank, discriminate, accept, reject, suspend, revise, or connect candidate interpretations and actions. Judgment can involve evidential assessment, analogical comparison, diagnosis, model selection, estimation of plausibility, sensitivity to contradiction, recognition of explanatory insufficiency, or evaluation of whether a question is worth pursuing. For scholarship, judgment can also include assessments of conceptual fertility, historical significance, theoretical economy, or aesthetic coherence. These forms of judgment require local justification and should not be collapsed into one scalar notion of “quality.”

The normative component $N_i(t)$ records standards and commitments that influence what counts as acceptable inquiry or action. Some are epistemic, such as commitments to evidential adequacy, reproducibility, explanatory depth, consistency, or openness to revision. Others can be professional, legal, ethical, institutional, or practical. Normativity enters the reconstruction because a theory-forming entity does not only infer what may be true; it can also decide which risks are tolerable, which questions deserve attention, which simplifications are permissible, which evidence may be ethically obtained, and which consequences matter when choosing among competing representations.

The practice component $Pr_i(t)$ concerns recurrent activities through which the other components are enacted and modified. Reading, observing, measuring, calculating, interviewing, comparing cases, drawing diagrams, writing drafts, simulating alternatives, consulting colleagues, testing prototypes, challenging arguments, and revising terminology can all function as epistemic practices when they contribute materially to theory formation. In distributed settings, practice also includes coordination structures, handoffs, criticism, shared records, instrumentation, and other interactions that organize collective cognition. Hutchins’s analysis of ship navigation is relevant because it provides a detailed case in which cognitive work is materially and socially organized across persons, artifacts, and procedures [7]. Longino’s account of scientific inquiry similarly supports attention to criticism and social interaction as epistemically consequential features of knowledge production [11].

The five components therefore form a coupled organization. Equation 24 represents this dependence without imposing a specific causal model.

$$\mathcal{C}_{i,Q_t}^{\Xi}(t) \subseteq \{Sal_i, K_i, J_i, N_i, Pr_i\} \times \{Sal_i, K_i, J_i, N_i, Pr_i\}. \quad (24)$$

In Equation 24, $\mathcal{C}_{i,Q_t}^{\Xi}(t)$ denotes the set of empirically or interpretively supported couplings among the components of $\Xi_i^{Q_t}(t)$. The relation is not assumed to be symmetric, complete, or stable through time. For one inquiry, prior knowledge may strongly condition salience; for another, practical experimentation

Table 4: Evidence classes for epistemic-normative-practical reconstruction.

Evidence class	Principal contribution	Characteristic risk
Articulated accounts	Stated reasons, categories, perceived alternatives, explicit standards, and retrospective interpretation.	Rationalization, memory compression, hindsight, vocabulary acquired after the episode, and omission of tacit operations.
Observed practice	Sequence, timing, interaction, cue use, coordination, and enacted departures from stated procedure.	Observer effects, limited access to internal discrimination, and ambiguity between habitual and reasoned action.
Contemporaneous traces	Temporal order of revisions, abandoned alternatives, emerging concepts, and locally available evidence.	Fragmentary survival, selective recording, missing informal interactions, and archival asymmetry.
Mediating artifacts	Material or symbolic structures that organize attention, memory, communication, comparison, or calculation.	Inferring user intention from artifact structure or treating an artifact as causally sufficient on its own.
Outcomes and consequences	Downstream performance, error, uptake, persistence, revision, or practical consequence.	Hindsight bias, survivorship bias, and retrospective attribution of success or failure to the wrong mechanism.

may revise judgment and subsequently reorganize knowledge. The task is to reconstruct the locally supported coupling structure rather than to impose a universal ordering in advance.

5.4 Episodes, Traces, and Reconstruction Evidence

A major methodological difficulty is that the organization represented by $\Xi_i^{Q_t}(t)$ is only partially observable. Retrospective interviews can reveal categories and reasons that matter to the entity, yet they can also compress long development into a coherent narrative, omit forgotten contingencies, reinterpret earlier events through later concepts, or rationalize actions whose generative basis was tacit. Observed behavior avoids some retrospective distortions but does not by itself disclose the reasons that made an action intelligible. Drafts, notes, correspondence, logs, diagrams, models, and artifacts can preserve temporal traces, while remaining incomplete and selective records of the process that produced them. Reconstruction therefore requires evidence triangulation.

Let $\mathcal{D}_i^{Q_t}$ denote the body of reconstruction evidence assembled for entity or configuration e_i with respect to Q_t . For the present framework, Equation 25 distinguishes five broad evidence classes.

$$\mathcal{D}_i^{Q_t} = \langle \mathcal{D}_i^{\text{art}}, \mathcal{D}_i^{\text{obs}}, \mathcal{D}_i^{\text{con}}, \mathcal{D}_i^{\text{med}}, \mathcal{D}_i^{\text{out}} \rangle. \quad (25)$$

Here $\mathcal{D}_i^{\text{art}}$ denotes articulated accounts such as interviews, explanations, memoirs, or stated reasons; $\mathcal{D}_i^{\text{obs}}$ denotes observed practice; $\mathcal{D}_i^{\text{con}}$ denotes contemporaneous traces such as dated drafts, correspondence, logs, notebooks, or decision records; $\mathcal{D}_i^{\text{med}}$ denotes material and representational mediators such as instruments, interfaces, diagrams, databases, checklists, or shared models; and $\mathcal{D}_i^{\text{out}}$ denotes relevant outcomes and downstream consequences. The categories can overlap. Their role is to prevent a single evidential form, especially retrospective articulation, from becoming the default proxy for the entire generative process.

Table 4 summarizes the principal contribution and characteristic risk of each evidence class. The table is diagnostic rather than prescriptive; appropriate evidence depends on the problem, access conditions, ethical constraints, and historical distance of the inquiry.

The reconstruction itself should be represented as an inference from this evidence rather than as direct observation of $\Xi_i^{Q_t}(t)$. Let $\hat{\Xi}_i^{Q_t}(t)$ denote the analyst’s reconstructed organization. Equation 26 makes this epistemic status explicit.

$$\hat{\Xi}_i^{Q_t}(t) = \Re(Q_t, \Lambda_i(t), \mathcal{D}_i^{Q_t}), \quad (26)$$

where $\Re$ denotes the reconstruction procedure justified for the study. Equation 26 does not prescribe one universal procedure. Depending on the problem, $\Re$ can combine historical reconstruction, qualitative coding, process tracing, protocol analysis, ethnographic observation, comparative case analysis, quantitative behavioral measures, or other methods. Methodological pluralism is necessary because different evidence classes expose different aspects of the organization being reconstructed.

The hat in $\hat{\Xi}_i^{Q_t}(t)$ is methodologically important. It marks a distinction between the entity's generative organization and the analyst's evidence-constrained representation of it. This notation prevents later abstraction from silently treating reconstructed categories as if they were direct ontological components of cognition. It also creates an explicit location for uncertainty, disagreement among sources, and competing reconstructions.

5.5 Temporal and Distributed Reconstruction

A static reconstruction at one time can conceal how epistemic organization was generated. Training, prior successes and failures, disciplinary inheritance, institutional change, repeated exposure, technological mediation, and collaboration can alter salience, knowledge, judgment, normative commitment, and practice. The reconstruction target should therefore be indexed to time when the research question concerns formation rather than merely present competence.

For a sequence of research-relevant times $t_0 < t_1 < \dots < t_n$, the reconstructed trajectory is represented by Equation 27.

$$\hat{\Xi}_i^Q = (\hat{\Xi}_i^{Q_{t_0}}(t_0), \hat{\Xi}_i^{Q_{t_1}}(t_1), \dots, \hat{\Xi}_i^{Q_{t_n}}(t_n)). \quad (27)$$

Equation 27 permits both the problem and the reconstructed organization to change. This is necessary because a scholar, physician, engineer, judge, organization, or scientific community can acquire new distinctions while simultaneously reformulating the question to which those distinctions apply. The trajectory therefore records co-development rather than treating an invariant agent as acting on an invariant problem.

Distributed reason-bearing configurations require an additional caution. When Section 4.4 justifies $\mathcal{K}_{Q_t}(t)$ as the relevant epistemic unit, the reconstructed organization cannot be obtained by merely summing individual $\Xi_i^{Q_t}(t)$. Interaction structure can generate epistemic capacities that are absent from the isolated components. Cross-checking, division of attention, adversarial criticism, shared memory, instrument-mediated coordination, and sequential handoff can produce reasons at the configuration level. The appropriate reconstruction target is therefore $\hat{\Xi}_{\mathcal{K}}^{Q_t}(t)$, whose evidence must include the relations and mediators that make the configuration operative.

This distributed treatment also prevents a common attribution error. A successful outcome produced by a socio-technical configuration should not automatically be redescribed as the competence of the most visible individual within that configuration. Conversely, a failure generated by a relational interface should not automatically be localized in one participant. The reconstruction should preserve the level at which the relevant epistemic organization is supported by evidence.

5.6 Reconstruction Adequacy and Output of the Stage

The purpose of this stage is to produce a sufficiently supported reconstruction for later evolutionary and comparative analysis, not to claim exhaustive access to cognition. Adequacy therefore depends on the relation between the reconstructed organization, the problem Q_t , the relational-historical location $\Lambda_i(t)$, and the available evidence $\mathcal{D}_i^{Q_t}$. A useful reconstruction should identify which components are directly evidenced, which are inferentially supported, which remain uncertain, and which alternative interpretations materially compete with the selected account.

Table 5: Diagnostic audit for epistemic-normative-practical reconstruction.

Diagnostic site	Guiding question	Principal failure risk
Problem relevance	Which component of Q_t does each reconstructed element help explain or interpret?	Biographical detail accumulates without analytical relevance.
Evidence diversity	Which conclusions depend on articulation, observation, contemporaneous traces, mediators, or outcomes?	One evidence class silently stands for the entire generative process.
Tacit structure	Which competent discriminations are enacted more reliably than they are articulated?	Absence of verbal explanation is mistaken for absence of epistemic organization.
Normative visibility	Which standards, values, obligations, or risk judgments influence attention and choice?	Normative selection is redescribed as value-neutral inference.
Relational level	Is the relevant organization individual or distributed across a configuration?	Collective capacities are localized in one actor or individual variation is erased by a collective label.
Temporal integrity	Can the reconstruction distinguish categories available during the episode from concepts acquired later?	Retrospective vocabulary is projected backward into the historical process.

Table 5 states six audit questions that can be applied before the framework moves to evolutionary reconstruction.

A reconstruction that remains underdetermined after this audit can still be analytically useful if its uncertainty is retained. The framework therefore permits multiple candidate reconstructions when the evidence supports more than one plausible organization. Prematurely collapsing such alternatives would convert evidential ambiguity into artificial formal precision.

The output of Section 5 is consequently a set of problem-indexed reconstructed organizations and, where the problem requires, their temporal trajectories and distributed forms. These reconstructions supply the material for the next stage, which asks how entities, relations, boundaries, practices, and available state spaces evolve. The transition is important: the present section reconstructs the organization operative within selected episodes, while the next section will examine the transformations through which that organization and its relational field become historically generated.

6 Evolutionary Reconstruction of Relational Configurations

This section develops the fourth substantive operation of the proposed architecture: reconstruction of how reason-bearing entities, distributed configurations, relations, boundaries, practices, and problem spaces become historically generated and transformed. Its objective is to distinguish a genuinely evolutionary account from a merely longitudinal description in which a fixed entity is observed at several dates. The method remains reconstructive and problem-indexed. It first defines the coupled configuration that serves as the local unit of evolutionary analysis, then specifies relational-historical continuation, differentiates several transformation sites, formalizes the possibility of evolving state spaces, and concludes with evidential constraints on claims of path dependence, irreversibility, and structural change. The formal objects introduced here are analytical representations. They do not establish that social or

epistemic reality possesses one unique underlying state space or dynamical law.

6.1 Configuration as the Evolutionary Unit

A diachronic description can record that an entity possessed property p_0 at t_0 and property p_1 at t_1 . Such a description is often sufficient when the entity boundary, relation structure, relevant types, and space of possible states can reasonably be held fixed for the question at hand. The present framework requires a stronger representation when those background structures materially participate in the change being explained. Relational and processual approaches in social theory have already emphasized unfolding transactions, temporal ordering, turning points, and eventful change rather than treating social objects as timeless substances [5, 1]. The present proposal builds on that orientation by making the evolving analytical configuration explicit within a theory-formation framework.

Let I_t index the entities or justified distributed units relevant to the inquiry at time t . Let $\mathcal{R}_t$ denote the problem-relevant relations among them, $\mathcal{M}_t$ the mediating artifacts or representational systems, $\Lambda_t = \{\Lambda_i(t)\}_{i \in I_t}$ their reconstructed relational-historical locations, $\hat{\Xi}_t = \{\hat{\Xi}_i^{Q_t}(t)\}_{i \in I_t}$ their reconstructed epistemic-normative-practical organizations, and $\mathcal{C}_t$ the wider conditions retained because they constrain or enable the inquiry. Equation 28 defines the local evolutionary configuration. When the problem itself changes through the reconstructed history, Q_t denotes the local problem at time t , while $\mathbf{Q}_{[t_0, t_1]} = \{Q_s : t_0 \leq s \leq t_1\}$ denotes the problem lineage over an interval.

$$\Omega_t^{Q_t} = \langle \{\mathfrak{E}_i(t)\}_{i \in I_t}, \mathcal{R}_t, \mathcal{M}_t, \Lambda_t, \hat{\Xi}_t, Q_t, \mathcal{C}_t \rangle. \quad (28)$$

Equation 28 is deliberately redundant in a limited sense. The local entity description in Equation 14 already contains boundary, relations, history, and position, while $\Lambda_i(t)$ and $\hat{\Xi}_i^{Q_t}(t)$ provide more specialized reconstructions. The expanded configuration keeps these analytical levels visible because different research designs can provide evidence at different levels. A study may have strong evidence for institutional relations but weak evidence for individual judgment, or detailed cognitive-process evidence but only a coarse reconstruction of the surrounding field. The representation therefore organizes evidential claims rather than asserting separable ontological components.

The superscript Q_t marks each local configuration as problem-indexed, while $\mathbf{Q}$ marks an interval-level lineage when several local questions are connected through revision. The same historical field can support several legitimate configurations when different questions require different boundaries, entities, mediators, or conditions. Evolutionary reconstruction consequently begins from the problem constitution developed in Section 3; it does not attempt to recover an exhaustive history of everything that changed.

A problem-indexed evolution map represents transformation between reconstructed configurations. Equation 29 states the minimal form.

$$\mathcal{U}_{t_0, t_1}^{\mathbf{Q}} : \mathfrak{D}_{t_0}^{Q_{t_0}} \subseteq \mathfrak{X}_{t_0}^{Q_{t_0}} \longrightarrow \mathfrak{X}_{t_1}^{Q_{t_1}}, \quad \mathcal{U}_{t_0, t_1}^{\mathbf{Q}}(\Omega_{t_0}^{Q_{t_0}}) = \Omega_{t_1}^{Q_{t_1}}. \quad (29)$$

Here $\mathfrak{X}_t^{Q_t}$ denotes the analytically admissible configuration space at time t under the local problem constitution, while $\mathfrak{D}_{t_0}^{Q_{t_0}}$ denotes the subset on which the reconstructed transformation is defined. The notation permits a transformation to be partial. Historical evidence can support a transition only for some configurations or components, and later sections will use this partiality when distinguishing groups, semi-groups, groupoids, and more general categorical representations. Equation 29 should therefore be read as a structural placeholder for a reconstructed transformation, not as an assumption that one deterministic law governs the empirical process.

6.2 Relational-Historical Continuation

Evolutionary analysis creates an identity problem. If boundaries, relations, practices, or types can change, the analyst cannot assume that the symbol e_i denotes an unchanged entity through the entire trajectory. A university department can split, a scientific community can reorganize around a new instrument, an

expert can change professional role, and a human-machine configuration can acquire a new component that alters where the relevant epistemic capacity resides. The question of continuation must therefore be reconstructed rather than presupposed.

For two times $t_a < t_b$, let $\mathcal{E}_{t_a}^Q$ and $\mathcal{E}_{t_b}^Q$ denote the sets of problem-relevant entities or configurations. Equation 30 defines a continuation relation $\mathfrak{C}_Q^{t_a, t_b}$.

$$\mathfrak{C}_Q^{t_a, t_b} \subseteq \mathcal{E}_{t_a}^Q \times \mathcal{E}_{t_b}^Q. \quad (30)$$

Membership $(e_i, e_j) \in \mathfrak{C}_Q^{t_a, t_b}$ means that the available evidence supports treating e_j as a continuation, descendant, transformed successor, or otherwise historically connected realization of e_i for the analytical purpose specified by Q . The relation is intentionally broader than numerical identity. It can be one-to-one, one-to-many, many-to-one, or unresolved. This accommodates fission, merger, succession, replacement, and distributed reorganization without forcing the analyst to impose a bijection between entities at different times.

Continuation should be supported by explicit criteria. Depending on the problem, relevant continuity can concern retained relations, institutional succession, memory or archival inheritance, transfer of practices, persistence of obligations, continuity of personnel, material infrastructure, causal dependence, or recognized lineage. No single criterion is privileged in advance. A criterion that establishes continuity for legal responsibility may differ from one appropriate to cognitive practice, organizational learning, disciplinary lineage, or technological systems.

The continuation relation also interacts with relational-historical location. If $(e_i, e_j) \in \mathfrak{C}_Q^{t_a, t_b}$, the later location $\Lambda_j(t_b)$ should not be treated as an independent attribute list. It is partly generated through the trajectory connecting the two configurations. Equation 31 represents this dependence schematically.

$$\Lambda_j(t_b) = \mathcal{L}_{t_a, t_b}^Q \left(\Lambda_i(t_a), \mathcal{H}_{i \rightsquigarrow j}^{[t_a, t_b]}, \mathcal{C}_{[t_a, t_b]} \right), \quad (31)$$

where $\mathcal{H}_{i \rightsquigarrow j}^{[t_a, t_b]}$ denotes the reconstructed relational-historical path connecting the earlier and later realizations and $\mathcal{C}_{[t_a, t_b]}$ denotes retained conditions along the interval. The operator $\mathcal{L}_{t_a, t_b}^Q$ is a representational device for the dependence of later location on prior location and intervening history. It need not be available in closed mathematical form. In qualitative research it can be instantiated through a process-traced sequence of events, institutional changes, interactions, and mediating artifacts.

Equation 31 clarifies the framework's use of path dependence. Path dependence is not established merely because the past occurred before the present. A path-dependent claim requires evidence that intervening history changes the later configuration in a way that cannot be adequately reconstructed from a coarse present-state description alone. The analyst should therefore distinguish historical narration from a stronger claim that sequence, timing, or prior relational structure materially constrains subsequent possibilities.

6.3 Boundary-Type-Topology Transformation

The evolutionary configuration permits several sites of transformation. A theory that records only changing attributes can miss changes in the analytical unit itself. Table 6 distinguishes six transformation sites that may matter for theory-formation research.

The categories in Table 6 can overlap. A boundary change often alters topology; a new relation can change the effective type of a distributed configuration; and a reformulated problem can require a different entity boundary. The table therefore provides sites for reconstruction rather than a partition of reality.

Topology requires particular care. The framework uses topology in a broad structural sense only when the empirical representation justifies it. If the relevant relations can be represented as a graph, hypergraph, simplicial object, category, or another relational structure, one may examine connectivity, components, cycles, interfaces, or other preserved and transformed features. The existence of such a representation does not imply that the empirical field literally is that mathematical object. Formalization

Table 6: Transformation sites in evolutionary reconstruction.

Transformation site	Analytical question	Illustrative consequence
State	Which properties, beliefs, practices, or resources change while the analytical unit remains sufficiently stable?	Longitudinal change can be represented within a fixed entity boundary and state space.
Relation	Which ties, dependencies, communication channels, authorities, or material couplings are created, removed, or reweighted?	The entity’s access, incentives, salience, or practical capacity changes through relational rewiring.
Boundary	Which components count as internal to the relevant entity or distributed configuration at different times?	A previously external artifact, team, or institution becomes constitutive of the reason-generating unit, or a former component becomes analytically separable.
Type	Does the classification needed to describe the entity or practice remain adequate across the trajectory?	A research group becomes a formal institution, a tool becomes an active computational participant, or a provisional role differentiates into new categories.
Topology	Does the organization of connectivity, reachability, clustering, hierarchy, or interface structure change?	The same named participants can occupy a structurally different configuration after a merger, platform shift, or reorganization.
Problem field	Do the question, admissible evidence, or available candidate answers change through the process?	The theory-formation trajectory alters the space within which subsequent inquiry proceeds.

should preserve the distinction between a useful structural model and an ontological assertion, consistent with the manuscript’s general commitment to ontological suspension.

Type change is similarly analytical rather than merely terminological. If a classification is treated as fixed, observations that no longer fit it can be forced into residual categories. Evolutionary reconstruction instead permits the type system itself to become revisable. Let $\mathcal{T}_t^Q$ denote the set of entity or practice types currently used by the analysis and let τ_t^Q assign a type where such an assignment is warranted. Equation 32 records the possibility that the classificatory system changes across the trajectory.

$$\tau_t^Q : \mathcal{E}_t^Q \longrightarrow \mathcal{T}_t^Q, \quad \mathcal{T}_{t_a}^Q \not\equiv \mathcal{T}_{t_b}^Q. \quad (32)$$

The symbol $\not\equiv$ in Equation 32 indicates that the two type systems need not be analytically identical; it does not assert that every historical transition changes the taxonomy. The practical implication is that cross-time comparison may require a translation between type systems before invariance or difference can be assessed. This issue will become important in the later sections on cross-context abstraction and transformation-structured invariance, where cross-type comparisons are treated explicitly.

6.4 Evolving State Spaces

A stronger evolutionary claim arises when the relevant possibilities cannot be represented adequately within one fixed state space. Standard dynamical descriptions often place successive states in a common space $\mathfrak{X}$ and model change as movement within that space. Equation 33 states this familiar case.

$$\Omega_{t_a}^{Q_{t_a}}, \Omega_{t_b}^{Q_{t_b}} \in \mathfrak{X}^Q. \quad (33)$$

Equation 33 is appropriate when the variables, entity types, admissible relations, and relevant possibilities can be specified with sufficient stability for the research purpose. The present framework does not prefer a changing state space when a fixed one is adequate. Evolutionary reconstruction becomes stronger only when the evidence indicates that the possibilities available later cannot be represented faithfully as merely different coordinates in the earlier space.

The evolving-state-space case is stated in Equation 34.

$$\Omega_{t_a}^{Q_{t_a}} \in \mathfrak{X}_{t_a}^{Q_{t_a}}, \quad \Omega_{t_b}^{Q_{t_b}} \in \mathfrak{X}_{t_b}^{Q_{t_b}}, \quad \mathfrak{X}_{t_a}^{Q_{t_a}} \not\equiv \mathfrak{X}_{t_b}^{Q_{t_b}}. \quad (34)$$

Several mechanisms can motivate Equation 34. New instruments can make previously unavailable observations possible. New concepts can divide one former category into several distinguishable objects. New institutions can create roles or obligations that had no direct predecessor in the earlier analytical vocabulary. New computational systems can introduce actions or couplings that were absent from the prior configuration. A theory can itself create new candidate questions, evidence standards, and representational possibilities. In such cases the state-space transformation is part of the history to be explained.

An evolving state space does not require the analyst to claim that reality itself creates possibilities in the same formal sense. $\mathfrak{X}_t^{Q_t}$ is the state space warranted by the inquiry at time t under the local problem constitution. Change in $\mathfrak{X}_t^{Q_t}$ can reflect changes in the phenomenon, the available practices, the representational vocabulary, the measurement interface, or some combination of these. Later evaluation should distinguish these sources where the evidence permits.

Theory formation makes this issue especially salient because theories can modify their own subsequent environment. Kuhn's historical analysis of scientific development challenged a simple cumulative picture of scientific change and foregrounded historically shifting problem structures, standards, and conceptual organization [10]. Within the present framework, such feedback can be represented without adopting the entirety of any specific philosophy of scientific change. If T_t denotes a theory or theoretical construction produced within $\Omega_t^{Q_t}$, some of its concepts, instruments, classifications, or standards can enter the conditions of later inquiry. Equation 35 represents this possibility.

$$\mathfrak{G}_t^{\text{fb}}(T_t) = \langle \Delta C_{t+1}^{(T)}, \Delta M_{t+1}^{(T)}, \Delta Q_{t+1}^{(T)}, \Delta \mathfrak{X}_{t+1}^{Q_{t+1},(T)} \rangle. \quad (35)$$

In Equation 35, $\mathfrak{G}_t^{\text{fb}}$ is a feedback record rather than a causal law, and each $\Delta^{(T)}$ records a change that the reconstruction provisionally attributes in part to the uptake of T_t in later conditions, mediators, problem formulation, or admissible state description. The delta notation denotes a documented change record rather than numerical subtraction. The analytical point is reflexive: products of inquiry can enter the field in which later inquiry develops, while the strength of any causal attribution remains limited by the available evidence.

6.5 Irreversibility Across Transformation Domains

Historical transformations are often asymmetric with respect to reversal. A document can be restored from a backup, while a training history, institutional collapse, or accumulated sequence of decisions may have no empirically meaningful inverse that recreates the earlier relational field. The framework should therefore avoid assuming that every transformation in Equation 29 belongs to a group.

Let $\mathcal{U}_{t_a, t_b}^Q$ and $\mathcal{U}_{t_b, t_c}^Q$ be two reconstructed transformations whose domains permit sequential application. The composite is defined only on configurations that survive both mappings. Equation 36 makes the supported composite domain explicit and names the resulting via- t_b transformation.

$$\begin{aligned}\mathfrak{D}_{t_a, t_c}^{(t_b), \mathbf{Q}} &= \left\{ \omega \in \mathfrak{D}_{t_a}^{Q_{t_a}} : \mathcal{U}_{t_a, t_b}^{\mathbf{Q}}(\omega) \in \mathfrak{D}_{t_b}^{Q_{t_b}} \right\}, \\ \mathcal{U}_{t_a, t_c}^{(t_b), \mathbf{Q}} &:= \mathcal{U}_{t_b, t_c}^{\mathbf{Q}} \circ \mathcal{U}_{t_a, t_b}^{\mathbf{Q}} \quad \text{on } \mathfrak{D}_{t_a, t_c}^{(t_b), \mathbf{Q}}.\end{aligned}\tag{36}$$

Equation 36 defines a composite path; it does not assert path independence. If an independently reconstructed direct transformation from t_a to t_c also exists, agreement between that direct reconstruction and $\mathcal{U}_{t_a, t_c}^{(t_b), \mathbf{Q}}$ is an additional empirical or representational consistency claim, not a consequence of composability. The construction requires neither global invertibility nor universal closure over every conceivable configuration. This distinction anticipates the formal hierarchy developed later in the manuscript. Reversible transformations on a common domain can admit group-theoretic treatment; irreversible but composable transformations on a common domain may be represented more naturally by semigroup-like structures; reversible transformations among multiple local objects can motivate groupoids; and local transformations without supported inverses require a more general categorical treatment. The choice among these resources should follow the empirical and analytical structure rather than precede it.

Irreversibility itself should also be evidenced rather than presumed. A reconstructed process can be practically irreversible because relevant information has been lost, because institutional dependencies have changed, because an event has altered incentives or legitimacy, or because the later state contains structures that cannot be removed without generating a different history. These mechanisms are distinct. Describing a trajectory as irreversible without identifying the operative sense of irreversibility would obscure rather than explain the process.

6.6 Evolutionary Reconstruction Discipline

The flexibility of an evolutionary framework creates a methodological danger. If entity boundaries, relations, types, state spaces, questions, and transformation laws can all be revised after observing the outcome, the reconstruction can become too accommodating to constrain explanation. The framework therefore requires explicit evidence for each claimed transformation site and a record of which components were fixed, revised, or left unresolved during analysis.

Table 7 provides six diagnostics for this stage.

The audit in Table 7 also constrains later formalization. A group, semigroup, groupoid, category, graph, or dynamical system earns a place only when the reconstructed transformation supports the relevant structural assumptions. The formal vocabulary should therefore be introduced after the transformation sites are evidenced, consistent with the manuscript's general rule that formalism must earn its analytical role.

The output of Section 6 is a problem-indexed family of reconstructed configurations $\{\Omega_t^{Q_t}\}$ together with supported continuation relations, transformation sites, and transformation maps. This output prepares the next analytical stage. The next section will compare configurations across space, time, spatiotemporal location, scale, and entity type in order to identify candidate recurrent structures before any claim of invariance or equivariance is made.

Table 7: Diagnostic audit for evolutionary reconstruction.

Diagnostic site	Guiding question	Principal failure risk
Continuation	What evidence connects earlier and later entities or configurations?	A stable name is mistaken for stable identity, or a genuine continuation is lost because surface attributes changed.
Sequence	Which ordering or timing relations are supported by contemporaneous evidence?	A retrospective narrative imposes an ordered mechanism that the historical record does not establish.
Transformation site	Is the claimed change located in state, relation, boundary, type, topology, problem field, or several of these?	Distinct forms of change are collapsed into an undifferentiated claim of “evolution.”
Path dependence	Which later possibilities depend materially on the intervening trajectory?	Mere temporal precedence is redescribed as path dependence.
State-space change	Why is a fixed state space inadequate for the observed transition?	Analytical novelty is manufactured by changing the representation when ordinary state change would suffice.
Irreversibility	Which mechanism prevents or limits a meaningful inverse transformation?	Historical asymmetry is asserted rhetorically without identifying the source of non-reversibility.

7 Cross-Context Structural Abstraction

This section develops the first analytical operation within Stage 5 of the proposed architecture: comparison across reconstructed contexts in order to identify candidate structures that may travel beyond a single episode, entity, location, or historical trajectory. Its objective is to formalize comparison without collapsing contextual difference prematurely. The section therefore separates comparison design from claims of invariance. It first defines a compact context signature for problem-relevant configurations, then constructs a six-family comparison architecture across spatial, temporal, and spatiotemporal differences and across same-type and cross-type entities or configurations. It subsequently introduces correspondence and translation requirements, specifies several targets of structural abstraction, and concludes with an audit that records the empirical support and domain of each candidate abstraction. Invariance, covariance, equivariance, and transformation-law preservation are reserved for the next section, where the transformations themselves become explicit analytical objects.

7.1 Comparison Contexts and Analytical Views

Evolutionary reconstruction produces a family of locally problem-indexed configurations $\{\Omega_t^{Q_t}\}$ together with continuation relations, transformation sites, and partial transformation maps. For the comparative stage, let Q denote the provisional comparison problem under which selected historical configurations are viewed; Q can preserve, refine, or translate the local Q_t distinctions, but that relation should be documented rather than assumed. Cross-context abstraction asks which aspects of these reconstructions can be meaningfully compared. A comparison does not begin from two raw cases. It begins from two cases described at sufficiently explicit locations, times, types, and scales for the proposed comparison to have an interpretable domain.

Let a index a comparison context. Equation 37 defines a compact context signature.

$$\chi_a^Q = \langle \ell_a, t_a, \kappa_a, s_a \rangle. \quad (37)$$

In Equation 37, ℓ_a denotes the locus relevant to the comparison, t_a the time or historically delimited interval, κ_a the entity or configuration type used by the analysis, and s_a the analytical scale. The locus ℓ_a can be physical, institutional, jurisdictional, organizational, network-relational, or another explicitly justified form of location. It is a compact comparison descriptor rather than a substitute for the richer relational-historical location $\Lambda_i(t)$ defined in Equation 18. The context signature selects which dimensions are being varied or held sufficiently comparable for the present analytical purpose.

The scale component s_a is kept explicit because an apparent regularity can depend on whether the unit of analysis is an episode, person, team, institution, field, or wider historical formation. A comparison between two individual scholars and a comparison between two scholarly communities can therefore occupy different scales even when they concern the same named problem. Cross-scale comparison is admissible, but it should be identified as such rather than treated as if the two configurations had identical analytical granularity.

A context signature does not yet specify which parts of a reconstructed configuration will be compared. Let $\mathcal{V}_{Q,\chi_a}$ denote a problem-indexed analytical view that retains the components of Ω_a^Q required for the intended comparison. Equation 38 defines the resulting comparison object.

$$Z_a^Q = \mathcal{V}_{Q,\chi_a}(\Omega_a^Q). \quad (38)$$

The operator $\mathcal{V}_{Q,\chi_a}$ in Equation 38 is selective by design. A study of problem recognition can retain salience cues, relational position, and judgmental transitions while omitting details that do not bear on the question. A study of institutional theory formation can retain organizational boundaries, communication relations, archival inheritance, and representational artifacts. The view should therefore be documented sufficiently for another analyst to know which structures were preserved and which were bracketed. Cross-context abstraction inherits the evidential limits of these views and cannot recover information that the comparison design removed.

Remark 7.1 (Comparison views and ontological restraint). The comparison object Z_a^Q is an analytical view of a reconstructed configuration. Similarity or difference between two comparison objects does not by itself establish similarity or difference between the total realities from which they were constructed. Claims should remain indexed to the problem, view, scale, and evidence that support the comparison.

Remark 7.1 prevents abstraction from acquiring a stronger ontological status than the reconstruction warrants. It also prepares the later symbolization audit: comparison already contains a representational choice before any mathematical formalization is introduced.

7.2 Multiaxial Comparison Architecture

The framework organizes its core comparison space through two axes. The first distinguishes spatial, temporal, and spatiotemporal displacement. The second distinguishes same-type and cross-type comparison. Their product yields six principal families. Table 8 states the architecture.

The term *spatial* in Table 8 does not presuppose Euclidean distance as the relevant geometry. Two courts in different jurisdictions, two laboratories in different institutional systems, or two research communities occupying different network positions can constitute spatially differentiated contexts for a problem whose operative location is institutional or relational. The locus variable should therefore be defined by the research question rather than by a default physical metric.

The temporal family also requires care. A comparison across t_a and t_b does not automatically concern the same entity. Section 6 introduced the continuation relation $\mathfrak{C}_{Q_a,t_b}^{t_a,t_b}$ precisely because boundaries and identities can change. Same-type temporal comparison can therefore involve a continuing entity, two different entities assigned to a stable analytical type, or two entities whose types can be translated adequately across changing type systems. These cases should be distinguished in the empirical record even when they occupy the same cell in Table 8.

Table 8: Six principal families of cross-context comparison. Spatial location is interpreted according to the problem and can include physical, institutional, jurisdictional, or relational loci.

Displacement	Same-type comparison	Cross-type comparison
Spatial	Compare configurations treated as the same relevant type across distinct loci within a sufficiently matched time window.	Compare different entity or configuration types across distinct loci within a sufficiently matched time window.
Temporal	Compare a type across distinct times or historical intervals, using continuation or justified type correspondence where required.	Compare different types across distinct times, including cases in which the historical process itself generates a type transition.
Spatiotemporal	Compare the same relevant type when both locus and time differ.	Compare different types when both locus and time differ, requiring the strongest correspondence discipline because several contextual dimensions change together.

Cross-type comparison is methodologically important because a generative structure can recur without preserving the surface identity of its participants. A pattern of anomaly recognition, relational mediation, problem decomposition, or institutional feedback can appear in differently typed configurations. Cross-type comparison allows the analyst to investigate this possibility without first homogenizing those configurations into one common type. The burden of correspondence is consequently higher: the analysis must show which structural roles are comparable and which type-specific features remain irreducible.

Scale is orthogonal to the six-family architecture. Each cell in Table 8 can hold scale fixed or vary it deliberately. A same-type temporal comparison at the level of individual practice differs from a same-type temporal comparison at the level of an institution, even if both concern the same historical field. When scale changes, the comparison should state whether the later abstraction concerns a cross-scale relation, a coarse-graining, a decomposition, or some other justified mapping.

This multiaxial architecture differs from a simple synchronic/diachronic partition by making entity type, relational-historical location, and analytical scale explicit within the comparison design. A synchronic comparison can still be represented as a spatial or cross-sectional case, and a diachronic comparison can still be represented as a temporal case. The additional structure becomes necessary when the entities, type systems, boundaries, or state spaces reconstructed in Section 6 are themselves changing.

7.3 Type Translation and Structural Correspondence

Cross-context comparison requires a correspondence rule before any recurrent structure can be claimed. Superficially similar labels can hide different relational roles, while differently named entities can occupy structurally comparable positions. The framework therefore separates *type translation* from *structural correspondence*.

Section 6.3 allowed the type system $\mathcal{T}_t^Q$ to evolve. When two contexts use different type systems, let ϑ_{ab}^Q denote a partial type translation supported for the intended comparison. Equation 39 states its minimal form.

$$\vartheta_{ab}^Q : \mathcal{D}_{ab}^{\mathcal{T}} \subseteq \mathcal{T}_a^Q \dashrightarrow \mathcal{T}_b^Q. \quad (39)$$

The dashed arrow in Equation 39 indicates that the translation can be partial and need not preserve every distinction. A category used in one historical or institutional setting can split into several later categories, merge with another, or lack a defensible counterpart. The analyst should record such failures rather than completing the translation by convenience. Same-type comparison across changing type systems is warranted only when the relevant type relation is sufficiently supported for the specific structural question being asked.

Type correspondence alone is insufficient because the abstraction target can concern relations, processes, topology, or judgmental organization rather than nominal categories. Let μ_{ab}^Q denote a justified partial correspondence between selected components of Z_a^Q and Z_b^Q . Equation 40 collects the comparison pair and its correspondence rule.

$$\mathfrak{A}_{ab}^Q = \langle Z_a^Q, Z_b^Q, \mu_{ab}^Q \rangle. \quad (40)$$

The mathematical form of μ_{ab}^Q depends on the representation earned by the evidence. It can be a partial function between coded components, a relation between roles, a graph or network matching, a sequence alignment, a functor between categorical representations, or a qualitatively specified correspondence when stronger formalization is not justified. Equation 40 therefore names the methodological obligation without prescribing one universal mapping structure.

A correspondence should preserve the distinction between *matched role* and *identical entity*. For example, a human checklist, an organizational review routine, and a computational validation layer can each occupy a checking role in differently typed configurations without being interchangeable entities. A cross-type abstraction can legitimately concern the checking relation while retaining the material, institutional, and normative differences among its realizations.

Remark 7.2 (Correspondence precedes invariance). A claim that a structure is preserved across contexts presupposes a defensible account of what is being compared across those contexts. A correspondence rule can remain qualitative or partial. The absence of a satisfactory correspondence is a reason to suspend the invariance claim, not a reason to force the cases into a common representation.

Remark 7.2 establishes the logical order used by the next two sections. Section 7 identifies candidate comparable structures; the next section will specify the transformations under which preservation is claimed.

7.4 Targets of Structural Abstraction

Abstraction in the present framework seeks recurrent relational or generative structure while retaining the conditions under which that structure was reconstructed. It is therefore broader than extracting common attributes from a collection of entities. Several abstraction targets can coexist within one comparison family. Table 9 distinguishes six preliminary targets.

The targets in Table 9 are not mutually exclusive. A process schema can depend on a relational motif; a topological reorganization can alter a dynamical profile; and a normative-judgment structure can determine which states count as acceptable outcomes. The purpose of the taxonomy is to identify what the abstraction claims to preserve. A phrase such as “the same pattern” is too weak unless the analyst states whether sameness concerns state, relation, process, topology, dynamics, judgment, or some explicit combination.

Let A index a candidate abstraction and let $\mathbb{Z}^Q = \{Z_a^Q\}_{a \in \mathcal{I}_Q}$ denote the comparison family selected for the inquiry. Let $\mathbb{M}^Q$ collect the justified correspondences used within that family. Equation 41 represents the abstraction stage schematically.

$$\mathfrak{B}_Q = \mathcal{A}_Q(\mathbb{Z}^Q, \mathbb{M}^Q), \quad (41)$$

where $\mathfrak{B}_Q$ is a set of candidate structural abstractions and $\mathcal{A}_Q$ denotes the analytical procedure by which they are reconstructed. The procedure can be qualitative, comparative-historical, statistical, computational, formal, or mixed, provided that its relation to the underlying evidence is explicit. Equation 41 does not imply that one algorithm generates the abstractions or that all research designs produce the same $\mathfrak{B}_Q$.

A candidate abstraction should carry its empirical support domain. For $A \in \mathfrak{B}_Q$, Equation 42 defines the contexts in which an instantiation of A is sufficiently supported by the current analysis.

Table 9: Preliminary targets of cross-context structural abstraction.

Target	Object of abstraction	Illustrative question
State pattern	A recurrent configuration of properties, resources, or local conditions.	Which combinations of uncertainty, time pressure, or representational support recur when a form of judgment appears?
Relational motif	A recurrent pattern of dependence, mediation, authority, criticism, communication, or access.	Does reliable problem recognition depend on a recurrent relation among observer, artifact, and feedback source?
Process schema	A recurrent ordering or coupling of operations without requiring identical surface actions.	Does anomaly salience tend to be followed by historical localization, relational comparison, and problem reformulation?
Topological structure	A recurrent pattern of connectivity, interface, reachability, clustering, or cycle structure when such a representation is warranted.	Which connection structures are retained when the named entities and institutional forms differ?
Dynamical profile	A recurrent qualitative pattern such as stabilization, escalation, oscillation, threshold behavior, attraction, or branching.	Do differently realized inquiry processes display comparable transition profiles under matched perturbations?
Normative-judgment structure	A recurrent organization of evaluative distinctions, admissibility criteria, or trade-offs.	Which judgmental relations persist even when the substantive values, roles, or institutional settings differ?

$$\text{Supp}_Q(A) = \{a \in \mathcal{I}_Q : A \text{ has a supported instantiation in } Z_a^Q\}. \quad (42)$$

The support set in Equation 42 is not yet a generalization domain. It records where the abstraction has been reconstructed, not where it is guaranteed to hold. This distinction is important for small comparative samples, historically selected cases, and expert-centered research. A recurrent process found in six highly experienced practitioners can motivate a candidate structure without licensing a claim about all practitioners, all tasks, or all historical settings.

Abstraction should also preserve informative non-instantiations. A context in which A fails to appear can identify a boundary condition, expose an incorrect correspondence, reveal a hidden type difference, or show that the abstraction is too coarse. Negative cases therefore contribute to the structure of the candidate theory rather than serving merely as noise to be removed.

7.5 Recurrence, Variation, and Preparatory Transformation Families

A recurrent abstraction can vary in its local realization. This variation is not necessarily a defect. If two contexts instantiate the same relational motif through different entities, artifacts, temporal sequences, or scales, the difference can reveal the transformation through which the structure travels. The comparison stage should therefore record both recurrent structure and systematic variation before deciding whether the appropriate later concept is invariance, covariance, equivariance, or a more general transformation law.

For a candidate abstraction A and two contexts a and b , let A_a and A_b denote its locally reconstructed realizations. The comparison record can be written as Equation 43.

$$\mathfrak{P}_{ab}^Q(A) = \langle A_a, A_b, \mu_{ab}^Q, \Delta_{ab}^Q \rangle, \quad (43)$$

Table 10: Diagnostic audit for cross-context structural abstraction.

Diagnostic site	Guiding question	Principal failure risk
Comparison axis	Which spatial, temporal, spatiotemporal, type, and scale dimensions are held comparable or deliberately varied?	Several contextual changes are collapsed into one unexplained comparison.
View construction	Which components of each Ω_a^Q are retained in Z_a^Q , and why?	A pattern is created by selectively omitting inconvenient structure.
Type translation	Which type correspondences are supported, partial, or unresolved?	Historically distinct categories are homogenized by terminology.
Structural correspondence	What licenses μ_{ab}^Q between the selected components or roles?	Surface resemblance is mistaken for structural comparability.
Support domain	In which contexts is the candidate abstraction actually reconstructed, and which cases challenge it?	A local recurrence is stated as a general law.
Residual difference	Which differences remain after correspondence, and could they alter the interpretation of the candidate structure?	Abstraction erases the conditions that make the cases epistemically different.

where Δ_{ab}^Q records the problem-relevant differences that remain after correspondence has been established. The term Δ_{ab}^Q can contain changes in locus, time, type, scale, boundary, mediation, topology, or another explicitly specified dimension. It is a structured record rather than a scalar distance unless a justified metric is later introduced.

Equation 43 prepares a transformation family without assuming one. Across a set of comparison pairs, the analyst can ask whether the observed Δ_{ab}^Q admit a coherent transformation description. If the same structural feature survives every supported transformation in a specified family, invariance becomes a candidate claim. If the feature changes according to a stable rule, covariance or equivariance can become appropriate. If transformations compose on a common domain but lack inverses, a semigroup-like representation may be warranted. If transformations are reversible but organized among multiple local objects, a groupoid may be faithful; if local transformations lack supported inverses or cross type/state-space boundaries, a more general category may be required. These stronger formal claims belong to the next section; the current section supplies the empirical and comparative substrate on which they depend.

This order is particularly important for cross-type comparison. One should not begin by selecting a mathematically elegant group and then search for empirical cases that realize its action. The inquiry should begin from justified correspondences and observed differences, then determine which algebraic structure, if any, captures their composition and preservation properties. Formal elegance is therefore treated as a later criterion of representation, subordinate to empirical and interpretive adequacy.

7.6 Abstraction Discipline and Stage Output

Cross-context abstraction can fail through overmatching as easily as through undercomparison. A researcher can ignore historically specific differences, translate types too aggressively, choose only cases that exhibit the expected pattern, or treat a recurring label as evidence of a recurring structure. Table 10 states six diagnostics for this stage.

The audit in Table 10 gives cross-context abstraction a different role from simple aggregation. The objective is to construct a disciplined family of candidate structures together with their contexts, correspondences, residual differences, and support sets. An abstraction can remain provisional, local, or conditional. It can also fail because no adequate correspondence exists across the proposed cases. Such failure is informative: it can indicate that the problem should be rescaled, that the entity types were wrongly aligned, or that a historically specific mechanism should remain irreducible.

The output of Section 7 is therefore the family $\mathfrak{B}_Q$ of candidate abstractions, accompanied by context signatures χ_a^Q , analytical views Z_a^Q , justified correspondences μ_{ab}^Q , residual-difference records Δ_{ab}^Q , and empirical support sets $\text{Supp}_Q(A)$. These objects are sufficient to formulate the next question with greater precision: under which explicit transformations, if any, is a candidate structure preserved, transformed according to a stable rule, or carried between differently typed objects? The next section develops that transformation-structured analysis.

8 Transformation-Structured Invariance and Equivariance

This section completes Stage 5 of the proposed architecture by testing whether candidate abstractions identified in Section 7 are preserved, systematically transformed, or structurally transported across explicitly supported transformations. Its role is to replace loose claims of recurrence with transformation-indexed claims whose domain can be inspected and challenged. The section constructs supported transformation families from the comparative records developed previously; distinguishes invariance, equivariance, covariance, and more general commutative transport; specifies when group, semigroup, monoid, groupoid, and categorical structures are warranted; introduces a cross-type commutation criterion for cases in which source and target objects do not share one state space or representational type; and concludes with evidential and counterexample disciplines designed to constrain overgeneralization and post-hoc selection of transformation families.

The formal objects introduced here remain structural representations. They do not establish that the empirical or social world is ultimately a group, category, state space, or algebraic object. Their purpose is narrower: to make explicit which transformations are being compared, which structural features are claimed to survive them, and which assumptions are required for the claim to be meaningful.

8.1 Supported Transformation Families

Section 7 produced the comparative records needed for transformation analysis. These include the comparison objects Z_a^Q , correspondences μ_{ab}^Q , and residual-difference records Δ_{ab}^Q . A transformation should enter the analysis only when a comparison pair supports a sufficiently explicit mapping between selected components of two analytical views.

Let a and b index two comparison contexts. A supported partial transformation from the source view to the target view is written in Equation 44.

$$\tau_{ab}^Q : D_{ab}^Q \subseteq Z_a^Q \longrightarrow Z_b^Q, \quad (44)$$

where D_{ab}^Q is the part of the source view for which the empirical reconstruction and the correspondence μ_{ab}^Q jointly support a mapping. The domain is allowed to be proper because a historical transition, cross-type comparison, or scale change may preserve only part of the structure represented in Z_a^Q . Equation 44 therefore does not assume that every context can be globally transformed into every other context.

A transformation record should retain the context change that motivates the mapping. Let $\delta\chi_{ab}^Q$ denote the documented change between the source and target context signatures, including the spatial, temporal, type, and scale dimensions that matter for the comparison. Equation 45 defines the minimal record used in this section.

$$\Theta_{ab}^Q = \langle \chi_a^Q, \chi_b^Q, \delta\chi_{ab}^Q, \mu_{ab}^Q, \tau_{ab}^Q \rangle. \quad (45)$$

The transformation record in Equation 45 prevents the mapping τ_{ab}^Q from becoming detached from the historical and relational difference it is meant to represent. Two mathematically identical maps can have different meanings when one represents geographical relocation, another represents institutional reclassification, and another represents a historical change in entity type. Transformation claims should therefore remain indexed to the contextual change from which they were reconstructed.

For a candidate abstraction $A \in \mathfrak{B}_Q$, let $\mathfrak{T}_Q(A)$ collect the transformations for which A has sufficiently supported local realizations at both source and target. Equation 46 defines this family.

$$\mathfrak{T}_Q(A) = \left\{ \tau_{ab}^Q : a, b \in \text{Supp}_Q(A), \tau_{ab}^Q \text{ is supported on } D_{ab}^Q \right\}. \quad (46)$$

Equation 46 is deliberately empirical in its entry condition. It is not the set of all transformations that an analyst can imagine. It is the transformation family presently supported by the comparative design. A later study can enlarge, contract, or partition this family as new contexts become available or as earlier correspondences are revised.

The six-family comparison architecture developed in Table 8 can now be interpreted as generating several kinds of transformation family. Spatial transformations can vary locus while holding a sufficiently matched time and type. Temporal transformations can vary historical position while preserving a justified continuation or type correspondence. Spatiotemporal transformations vary both. Each can be same-type or cross-type, and each can be conditioned further by scale. The transformation family therefore carries the comparison design into the formal stage rather than replacing it.

8.2 Invariance Relative to a Transformation Family

Invariance is meaningful only relative to a specified transformation family and a specified structural feature. Let A_a denote the local realization of a candidate abstraction A in context a . Suppose first that the local realizations can be compared in a common structural codomain $\mathcal{Y}_A^Q$ through a map $\alpha_{A,a}^Q : Z_a^Q \rightarrow \mathcal{Y}_A^Q$. For a supported transformation τ_{ab}^Q , strict invariance is stated in Equation 47.

$$\alpha_{A,b}^Q(\tau_{ab}^Q(z)) = \alpha_{A,a}^Q(z) \quad \text{for supported } z \in D_{ab}^Q. \quad (47)$$

Equation 47 states that the feature extracted by α_A^Q remains unchanged under the specified transformation. The claim concerns the abstraction represented in $\mathcal{Y}_A^Q$; it does not require the full configurations Z_a^Q and Z_b^Q to be identical. A relational motif can remain invariant while the participating entities change. A topological property can remain invariant while local weights change. A process ordering can remain invariant while the concrete actions and artifacts through which it is realized differ.

Strict equality can be too strong when the abstraction is qualitative, interval-valued, partially observed, or represented at different resolutions. In such cases the analyst can introduce an explicitly justified equivalence or tolerance relation $\simeq_{A,Q}$ on $\mathcal{Y}_A^Q$. Conditional invariance is then stated in Equation 48.

$$\alpha_{A,b}^Q(\tau_{ab}^Q(z)) \simeq_{A,Q} \alpha_{A,a}^Q(z). \quad (48)$$

The relation $\simeq_{A,Q}$ in Equation 48 requires its own justification. It can represent an equivalence class, an interval of measurement uncertainty, a qualitative coding equivalence, a topological equivalence, or another problem-relevant criterion. Introducing $\simeq_{A,Q}$ merely to rescue a failed invariance claim would make the framework unfalsifiable. The equivalence or tolerance relation should therefore be fixed or independently defended before it is used to classify challenged cases.

An invariance claim should also state its transformation scope. Let $\mathfrak{T}_Q^*(A) \subseteq \mathfrak{T}_Q(A)$ denote the family over which the claim is presently supported. Equation 49 gives the scoped form.

$$\text{Inv}_Q(A; \mathfrak{T}_Q^*(A), \simeq_{A,Q}) \iff \forall \tau_{ab}^Q \in \mathfrak{T}_Q^*(A), \alpha_{A,b}^Q \circ \tau_{ab}^Q \simeq_{A,Q} \alpha_{A,a}^Q \text{ on } D_{ab}^Q. \quad (49)$$

Equation 49 encodes a methodological constraint: the statement “ A is invariant” is incomplete unless the transformation family and the preservation criterion are supplied. A spatial invariant across three institutions in one jurisdiction is not automatically a temporal invariant across historical periods, and neither is automatically a cross-type invariant. The framework therefore treats invariance as indexed rather than absolute.

Remark 8.1 (Recurrence and invariance). A recurrent abstraction can motivate the construction of $\mathfrak{T}_Q(A)$, but recurrence alone does not establish Equation 49. Invariance requires the analyst to identify the transformations linking the cases and to test preservation under those transformations.

Remark 8.1 is the formal version of the distinction introduced at the end of Section 7. It also prevents an observed commonality from being strengthened into a symmetry claim without evidence about the intervening transformation.

8.3 Equivariance, Covariance, and Transformation-Law Preservation

Many important structures do not remain numerically or descriptively unchanged when context changes. Their local realization changes in a systematic way. In such cases equivariance is often a better representation than invariance. Let $\rho_A^Q(\tau_{ab}^Q)$ denote the transformation induced on the structural codomain by τ_{ab}^Q . Equation 50 states the corresponding equivariance condition.

$$\alpha_{A,b}^Q \circ \tau_{ab}^Q = \rho_A^Q(\tau_{ab}^Q) \circ \alpha_{A,a}^Q \quad \text{on } D_{ab}^Q. \quad (50)$$

Equation 50 states that transforming the reconstructed case and then extracting the structure gives the same result as first extracting the structure and then applying the corresponding structural transformation. Invariance is recovered as the special case in which $\rho_A^Q(\tau)$ acts trivially on the relevant structural feature. Equivariance therefore allows the analysis to preserve a transformation law even when the represented value, orientation, role assignment, or local realization changes.

This distinction is useful for the comparisons developed in the present framework. For example, a problem-recognition sequence can preserve its ordering while the substantive cues change. A mediation structure can preserve role relations while the institutional identities occupying those roles change. A normative judgment structure can transform coherently when the available options, legal categories, or scales differ. In these cases the more informative claim concerns the rule relating local realizations rather than literal sameness of the realizations.

The term *covariance* is used here in an operational, representation-level sense rather than as a claim about tensorial covariance in physics or differential geometry. It denotes a closely related but slightly broader requirement: the form of a relation or law is retained when its participating representations are transformed coherently. Suppose a local structural relation in context a is written as $\mathcal{L}_a^Q(u_a, v_a) = 0$. Let R_{ab}^Q and S_{ab}^Q transport the represented arguments into the target context. Covariance of the relation is stated in Equation 51.

$$\mathcal{L}_a^Q(u_a, v_a) = 0 \quad \implies \quad \mathcal{L}_b^Q(R_{ab}^Q u_a, S_{ab}^Q v_a) = 0. \quad (51)$$

Equation 51 does not require the local expressions $\mathcal{L}_a^Q$ and $\mathcal{L}_b^Q$ to be textually or numerically identical. It requires the relation encoded by them to retain its form under a justified coordinated transformation. This use is especially appropriate when the comparison crosses representational systems, coding schemes, or type vocabularies. The burden of proof lies in showing that R_{ab}^Q and S_{ab}^Q are legitimate transports rather than convenient relabelings.

Equivariance and covariance both retain transformation structure. The distinction made here is analytical rather than ontological: equivariance emphasizes commutation between a transformation of the underlying object and a corresponding transformation of its extracted structure, while covariance emphasizes preservation of the form of a relation under coordinated changes of representation. Some applications can be expressed in either language. The manuscript therefore treats the choice as problem-dependent and avoids assigning stronger metaphysical significance to the terminology.

8.4 Algebraic Structures for Transformation Families

The transformation family $\mathfrak{T}_Q(A)$ does not automatically possess a group structure. The appropriate algebra depends on closure, composition, identity, invertibility, and the extent to which transformations

Table 11: Candidate algebraic structures for supported transformation families. The table gives representational conditions, not ontological classifications of the underlying world.

Structure	Required features	Useful analytical situation	Principal caution
Group	Associative composition, closure on a common action domain, identity, and an inverse for each transformation.	Reversible symmetry transformations for which both forward and inverse operations are supported within the same analytical domain.	Historical reversibility should not be inferred from mathematical convenience.
Semigroup	Associative and closed composition on a common domain; inverses and identity are not required.	Repeated irreversible transformations whose compositions remain within one analytical transformation class.	Closure can fail when the state space, entity type, or admissible operation changes.
Monoid	Semigroup structure together with an identity transformation.	Irreversible but composable transformations where a meaningful “no-change” operation belongs to the same family.	The identity should correspond to an analytically legitimate operation rather than a formal placeholder only.
Groupoid	Multiple objects with composable morphisms and an inverse for every morphism.	Locally defined reversible transformations among distinct contexts, types, charts, or state-space components.	A groupoid does not model irreversibility merely because transformations are local.
Category	Multiple objects, identity morphisms, associative composition, and no general requirement of invertibility.	Cross-type, evolving-state-space, or path-dependent transformations whose domains and codomains differ and whose arrows can be irreversible.	A category can encode almost any composable mapping; explanatory value therefore depends on empirically disciplined object and morphism definitions.

share a common domain and codomain. Table 11 summarizes the principal options used in this framework.

The group case is the most restrictive. Let a group G_Q act on a common retained comparison domain $\mathcal{Z}_Q^*$. Calling this a group action requires the action to be defined on the whole retained domain, to satisfy the group action axioms there, and to contain only transformations whose inverses are also supported within that domain. An invariant feature $I_A^Q : \mathcal{Z}_Q^* \rightarrow \mathcal{Y}_A^Q$ then satisfies Equation 52.

$$I_A^Q(g \cdot z) = I_A^Q(z) \quad \text{for every } g \in G_Q \text{ and } z \in \mathcal{Z}_Q^*. \quad (52)$$

Equation 52 is elegant because the invariance claim can be stated over the entire supported group action. Its use should nevertheless be earned. If a historical transformation has no supported inverse, placing it in G_Q would add a symmetry that the reconstruction does not possess.

When transformations share a common domain and compose associatively but are not generally reversible, a semigroup action can be sufficient. If an identity operation is also analytically meaningful, the structure becomes a monoid. These representations can capture repeated learning, institutional accumulation, degradation, commitment, or path-dependent updates when successive operations remain in one transformation class. They become inadequate when the transformation changes the type system or state space so substantially that a common action domain is no longer defensible.

Groupoids and categories address a different problem. Let C_Q have contexts, reconstructed configuration types, or state-space components as objects and supported transformations as morphisms. Because

the transformations in Equation 44 can be partial, composition is defined only where the first map lands inside the domain of the second. Equation 53 defines the via- b composite and its supported domain.

$$\begin{aligned} D_{ac}^{(b),Q} &= \left\{ z \in D_{ab}^Q : \tau_{ab}^Q(z) \in D_{bc}^Q \right\}, \\ \tau_{ac}^{(b),Q} &:= \tau_{bc}^Q \circ \tau_{ab}^Q \quad \text{on } D_{ac}^{(b),Q}. \end{aligned} \quad (53)$$

Equation 53 establishes composability without assuming path independence. If an independently reconstructed direct transformation $\tau_{ac}^{\text{dir},Q}$ is also available, agreement with $\tau_{ac}^{(b),Q}$ is an additional path-consistency hypothesis that requires evidence; it is not a category axiom. A groupoid is obtained only on a subcollection for which every morphism has a supported inverse and the required compositions and identities are defined. When the empirical transformations are partial, the relevant local source domains and target images must themselves be treated as justified objects, or otherwise represented so that each retained arrow is genuinely invertible between its stated objects; a partial map into a larger target does not become a groupoid morphism merely because an inverse exists on its image. A general category permits irreversible arrows but still requires an explicitly justified object set, identities, and associative composition on the retained morphism family. This distinction corrects a possible ambiguity in informal uses of “local transformation”: locality and invertibility are independent properties. A transformation can be local and irreversible, in which case a general categorical representation is more appropriate than a groupoid.

A categorical representation also permits the structural abstraction itself to be transported. Let S_Q be a category of structural representations and let F_A^Q be a proposed assignment from objects and supported transformations in C_Q to objects and morphisms in S_Q . The assignment qualifies as a functor only if it preserves identities and composition. The composition-preservation requirement is stated in Equation 54.

$$F_A^Q(\tau_{ac}^{(b),Q}) = F_A^Q(\tau_{bc}^Q) \circ F_A^Q(\tau_{ab}^Q). \quad (54)$$

Equation 54 expresses a stronger form of structural coherence across a transformation chain. It says that the structural representation of the supported via- b composite agrees with the composition of the represented transformations. Together with identity preservation, satisfaction of this condition is what licenses calling F_A^Q a functor; categorical notation alone does not supply the property. Failure of Equation 54 can reveal that the abstraction discards interaction effects, that the intermediate context matters irreducibly, or that the proposed morphisms were defined too coarsely.

8.5 Cross-Type Structural Transport

Cross-type comparisons create the clearest case in which invariance cannot be reduced to equality in one common coordinate system. A physician, patient, diagnostic instrument, and clinical protocol can occupy different epistemic roles; a judge, litigant, precedent, and administrative rule can occupy different legal roles; two scholarly communities can employ different type systems while instantiating a comparable generative relation. The analysis therefore requires a transport structure that can relate different source and target representation spaces without first declaring them identical.

Let $\mathcal{Y}_{A,a}^Q$ and $\mathcal{Y}_{A,b}^Q$ be the local structural representation spaces for contexts a and b . Let $r_{ab}^{A,Q} : \mathcal{Y}_{A,a}^Q \rightarrow \mathcal{Y}_{A,b}^Q$ be a justified structural transport. Figure 2 represents the corresponding commutative requirement.

The commutation condition represented in Figure 2 is written in Equation 55.

$$\alpha_{A,b}^Q \circ \tau_{ab}^Q = r_{ab}^{A,Q} \circ \alpha_{A,a}^Q \quad \text{on } D_{ab}^Q. \quad (55)$$

Equation 55 generalizes both strict invariance and equivariance. If the source and target structural spaces have been independently justified as the same space and $r_{ab}^{A,Q}$ is the identity, the equation becomes an invariance condition. If $r_{ab}^{A,Q}$ is a non-trivial but stable representation of the empirical transformation,

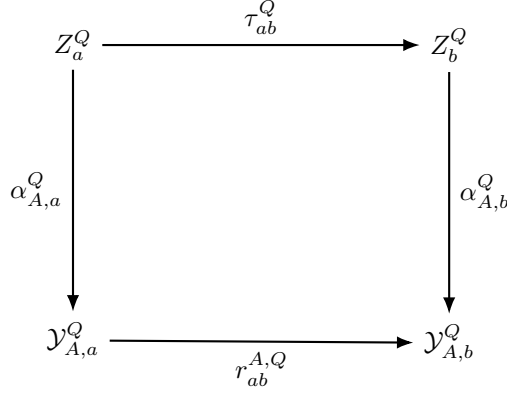


Figure 2: Cross-type structural transport. Commutation requires the structure extracted after the empirical transformation to agree with the transported structure extracted from the source context.

the equation expresses equivariant transport. When the source and target types remain irreducibly different, Equation 55 still permits a structural comparison without forcing identity between their local representations.

The transport $r_{ab}^{A,Q}$ must be justified separately from τ_{ab}^Q . A convenient transport can often be invented after observing the target realization. Such post-hoc flexibility would trivialize commutation. Evidence for $r_{ab}^{A,Q}$ can come from independently defined role correspondences, validated coding translations, historically documented transformations, measurement calibration, formal derivation, or another method appropriate to the problem. Where no such transport can be defended, the correct result is that cross-type invariance or equivariance has not been established.

8.6 Transformation Support and Counterexample Discipline

A transformation-structured framework faces a distinctive form of underconstraint. If the analyst is free to choose the transformation family after seeing which cases preserve the desired abstraction, almost any recurrent structure can be declared invariant over a conveniently restricted set. The transformation family, preservation criterion, and admissible transport therefore require independent discipline.

For a candidate abstraction A , let $\text{PresSupp}_Q(A)$ denote the subset of supported transformations for which the relevant invariance, equivariance, covariance, or commutation condition is satisfied. Equation 56 defines this set schematically.

$$\text{PresSupp}_Q(A) = \{\tau \in \mathfrak{T}_Q(A) : \tau \text{ satisfies the stated preservation condition for } A\}. \quad (56)$$

The complement $\mathfrak{T}_Q(A) \setminus \text{PresSupp}_Q(A)$ is analytically important. A failed transformation can identify a boundary condition, expose a missing mediator, show that a type translation was invalid, or suggest that the candidate abstraction should be split into several more local structures. The objective is therefore not to maximize the apparent size of $\text{PresSupp}_Q(A)$ by excluding difficult cases. The objective is to discover the largest transformation domain that the evidence can support without erasing informative failures.

Table 12 states the principal audit questions for this stage.

The audit in Table 12 also constrains the use of symmetry language. A symmetry is informative when a supported family of transformations preserves a specified structure. Calling a regularity a symmetry before the transformation family has been established reverses the order of analysis. The framework therefore treats symmetry discovery as a result of comparison and transformation reconstruction rather than as a default assumption.

The same discipline applies to approximate invariance. If the empirical setting requires a tolerance relation, uncertainty interval, or equivalence class, the criterion should be fixed independently enough

Table 12: Diagnostic audit for transformation-structured invariance and equivariance.

Diagnostic site	Guiding question	Principal failure risk
Transformation evidence	What empirical or interpretive evidence licenses τ_{ab}^Q and its domain D_{ab}^Q ?	A mathematically convenient mapping is substituted for a historically supported transformation.
Family construction	Was $\mathfrak{T}_Q(A)$ specified from the comparison design or restricted after observing preservation failures?	Post-hoc selection manufactures an invariant.
Preservation target	Which state, relation, process, topology, dynamics, or judgment structure is claimed to be preserved?	“Same structure” remains undefined and shifts across cases.
Algebraic assumptions	Are closure, identity, inverse, or composability claims supported for the selected transformations?	Group or categorical language contributes formal elegance without empirical warrant.
Transport rule	What independently licenses $\rho_A^Q(\tau)$ or $r_{ab}^{A,Q}$?	An arbitrary output transformation makes equivariance or commutation true by construction.
Counterexamples	Which transformations challenge the preservation claim, and how are they retained in the analysis?	Negative cases are removed from the domain without theoretical justification.
Scale and type	Does preservation survive a change of scale or type translation, or is the claim local to one analytical level?	A scale-specific or type-specific regularity is stated as a general structural law.

that challenged cases can genuinely fail it. Approximation should preserve empirical uncertainty, not absorb every discrepancy into an adjustable threshold.

8.7 Stage Output and Interface with Symbolization

The output of Section 8 is a transformation-structured account of the candidate abstractions $A \in \mathfrak{B}_Q$. For each sufficiently developed candidate, the account records the supported transformation family $\mathfrak{T}_Q(A)$, the transformation domains D_{ab}^Q , the algebraic composition properties that are actually warranted, the preservation map or structural transport when one exists, the set $\text{PresSupp}_Q(A)$, and the counterexamples or unresolved cases that delimit the claim.

This stage can therefore produce several legitimate outcomes. A candidate abstraction can be invariant over a specified family, equivariant under a supported representation of that family, covariant at the level of a relation, transportable through a more general commutative structure, locally valid only within a restricted transformation domain, or unsupported as a cross-context regularity. Failure to obtain invariance is not a failure of the method. It can reveal that the theoretically important object is the transformation law itself, the boundary at which preservation breaks, or the historical conditions that prevent a structure from travelling.

The notation used in this section already constitutes a minimal structural formalization, but it should be distinguished from the fuller symbolization problem addressed next. Here the formalism serves primarily to expose comparison domains, transformations, composition, and preservation claims. The next stage asks how reconstructed structures are encoded in a chosen symbolic system and what is lost, discretized, homogenized, scalarized, dehistoricized, or otherwise biased by that encoding. The next section therefore treats symbolization as a productive but potentially distorting transformation in its own right.

9 Formal Symbolization with Representation-Bias Audit

This section develops Stage 6 of the proposed architecture: translating selected reconstructed structures into a symbolic or formal representation while preserving an explicit account of what the translation clarifies, suppresses, distorts, or creates. Its role is to separate the analytical usefulness of formalization from any assumption that a formal object is a transparent copy of the reconstructed phenomenon. The section is organized in six analytical parts. It first defines symbolization as a partial map from an evidence-constrained reconstruction into a formal representation space. It then introduces feature-level preservation and loss records. A transformation-commutation criterion links symbolization to the invariance and equivariance analysis of Section 8. The section next develops a taxonomy of recurrent representation biases, examines the generative contribution of formalization to further inquiry, and concludes with an audit that permits partial, plural, or failed formalization as legitimate outcomes.

The discussion uses *symbolization* broadly. A symbolic system can include mathematical notation, logic, diagrams, graphs, categories, dynamical systems, statistical variables, computational state descriptions, coding schemes, or other explicitly structured representations. These resources differ substantially in expressive power and epistemic role. The common methodological question is whether the representation preserves the structures relevant to the research problem closely enough for the inferences drawn from it to remain warranted.

9.1 Symbolization as a Partial Representational Map

The input to formal symbolization is not the unmediated world. By the present stage, the inquiry has already constituted a problem, identified reason-bearing entities or configurations, reconstructed epistemic-normative-practical organization, reconstructed historical evolution, abstracted candidate structures, and tested their behavior under supported transformations. Let R_t^Q denote the selected evidence-constrained reconstructed structure at research time t , including the context, support, residual differences, and transformation information retained from the preceding stages. A symbolization is represented in Equation 57.

$$\Sigma_t^Q : D_{\Sigma,t}^Q \subseteq \mathcal{R}_t^Q \longrightarrow \mathcal{F}_t^Q, \quad R_t^Q \longmapsto R_t^{Q,\text{formal}}, \quad (57)$$

where $\mathcal{R}_t^Q$ is the reconstruction space relevant to the problem, $D_{\Sigma,t}^Q$ is the part selected for formalization, and $\mathcal{F}_t^Q$ is the chosen formal representation space. The domain is allowed to be proper. A defensible formalization can represent only one process, relation, scale, or structural feature while leaving other reconstructed material qualitative or unresolved.

Equation 57 also makes the selection boundary visible. Choosing $D_{\Sigma,t}^Q$ is itself an analytical act. A model of a scholarly episode that represents problem revision and argument structure while omitting institutional incentives, affective states, or material constraints can still be useful, provided that the omitted dimensions are recorded and the model's claims remain within the represented domain. The framework therefore distinguishes *partial formalization* from *incomplete reporting*. Partial formalization states its boundary; incomplete reporting conceals it.

A minimal symbolization record should retain the reconstruction, selected domain, target formal language, and interpretive rule through which symbols are connected back to the reconstructed structure. Equation 58 gives the corresponding record.

$$\mathfrak{S}_t^Q = \langle R_t^Q, D_{\Sigma,t}^Q, \Sigma_t^Q, R_t^{Q,\text{formal}}, \iota_t^Q \rangle, \quad (58)$$

where ι_t^Q is an interpretation specification connecting formal objects and operations to their problem-relevant meanings. The interpretation specification need not be a total inverse of Σ_t^Q . In many applications no such inverse exists because several reconstructed configurations can receive the same formal representation or because the formal system contains constructions that have no direct empirical counterpart.

This non-bijectivity is methodologically important. If Σ_t^Q is many-to-one on a relevant region, distinct histories or relations can become indistinguishable after formalization. If the formal codomain contains elements outside the interpreted image of Σ_t^Q , formal manipulations can generate objects whose empirical meaning remains unsettled. Neither condition invalidates the representation by itself. Each condition changes what can legitimately be inferred from it.

9.2 Feature Preservation and Representational Loss

Representation adequacy should be assessed relative to features that matter for the constituted problem. Let $\mathcal{P}_t^Q = \{p_j\}_{j \in J}$ be a family of problem-relevant predicates, relations, process properties, topological properties, temporal dependencies, or other structural features retained from the earlier reconstruction. For each p_j , let $\hat{p}_j$ denote a formal counterpart when one has been independently defined. Feature-level preservation is stated in Equation 59.

$$p_j(R_t^Q) \simeq_{j,Q} \hat{p}_j(\Sigma_t^Q(R_t^Q)), \quad p_j \in \mathcal{P}_t^Q, \quad (59)$$

where $\simeq_{j,Q}$ is an explicitly justified equality, equivalence, tolerance, or qualitative correspondence appropriate to the feature. Equation 59 does not require every feature in $\mathcal{P}_t^Q$ to be preserved. It provides a disciplined way to state which features the representation is designed to carry.

The associated loss record is therefore set-valued rather than assumed to be a single scalar. Equation 60 identifies features for which the current symbolization lacks an adequate formal counterpart or fails the stated preservation criterion.

$$\mathcal{L}_{\Sigma,t}^Q = \left\{ p_j \in \mathcal{P}_t^Q : p_j \text{ is omitted, unresolved, or not preserved under } \Sigma_t^Q \right\}. \quad (60)$$

The record in Equation 60 is broader than measurement error. It can contain erased historical dependence, collapsed entity distinctions, discarded uncertainty, removed interaction structure, missing scale relations, or a normative distinction that has no faithful formal counterpart. Some losses can be acceptable because they concern dimensions outside the local explanatory target. Other losses undermine the very feature for which the model was introduced. The adequacy judgment therefore depends on the relation between the loss set and the aims encoded in Q_t .

Loss should also be distinguished from *distortion*. An omitted feature is visible as absent when the representation boundary is stated. A distorted feature remains present in the formal system but with altered structural implications. Treating an ordinal ranking as a metric distance, for example, retains a numerical-looking relation while introducing assumptions about magnitude that the reconstruction did not warrant. Representation audit must therefore examine both omission and structural modification.

9.3 Transformation Fidelity Across Symbolic Representation

Section 8 reconstructed supported transformations among analytical views and distinguished invariance, equivariance, covariance, and structural transport. Symbolization is adequate for these claims only if the relevant transformation structure survives the representational change. For transformation-fidelity testing, let the structures selected for symbolization be the analytical views used in Section 8, or documented refinements of those views, so that $D_{\Sigma,a}^Q \subseteq Z_a^Q$ and $D_{\Sigma,b}^Q \subseteq Z_b^Q$. Reuse τ_{ab}^Q only on the jointly represented subdomain $D_{ab}^{\Sigma,Q} = \{z \in D_{ab}^Q \cap D_{\Sigma,a}^Q : \tau_{ab}^Q(z) \in D_{\Sigma,b}^Q\}$. Let $\hat{\tau}_{ab}^Q : \Sigma_a^Q(D_{ab}^{\Sigma,Q}) \rightarrow \mathcal{F}_b^Q$ be the proposed formal counterpart. If these typing conditions cannot be justified, the commutation test is undefined rather than failed. Figure 3 gives the central adequacy square.

Strict transformation fidelity is stated in Equation 61.

$$\Sigma_b^Q \circ \tau_{ab}^Q = \hat{\tau}_{ab}^Q \circ \Sigma_a^Q \quad \text{on } D_{ab}^{\Sigma,Q}. \quad (61)$$

Equation 61 states that transforming the reconstructed structure and then symbolizing it gives the same formal result as symbolizing first and applying the proposed formal transformation. When exact

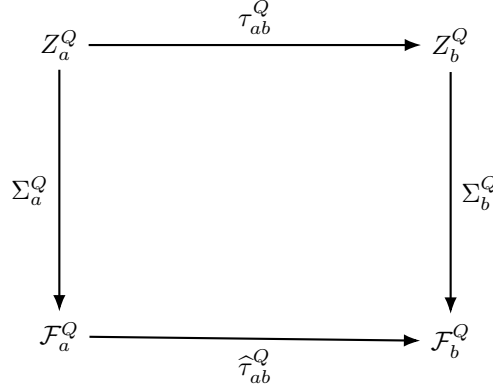


Figure 3: Transformation fidelity under symbolization. Adequate representation of the transformation requires the empirical-reconstructive route and the formal route to agree under a stated preservation criterion.

equality is too strong, an independently specified equivalence or tolerance relation can be used. Approximate transformation fidelity is stated in Equation 62.

$$\Sigma_b^Q \circ \tau_{ab}^Q \simeq_{\Sigma, Q} \hat{\tau}_{ab}^Q \circ \Sigma_a^Q \quad \text{on } D_{ab}^{\Sigma, Q}. \quad (62)$$

Failure of Equation 61 or Equation 62 is analytically informative. It can show that the formal representation erases a mediator required by the empirical transformation, freezes a type distinction that changes historically, assumes a fixed state space where the reconstructed state space evolves, or assigns a formally simple operation to a transition whose empirical composition depends on path. The mismatch can therefore identify the representational source of an apparent failure of invariance or equivariance.

The commutation criterion also constrains mathematical generalization. A formal symmetry discovered inside $\mathcal{F}_t^Q$ does not automatically imply an empirically meaningful symmetry. To support such an inference, the corresponding formal transformation should lie within the interpreted image of an empirically or interpretively supported transformation family, or the manuscript should clearly mark the symmetry as a formal exploration whose empirical status remains open.

9.4 Taxonomy of Representation-Induced Bias

Representation bias enters when the structure of the symbolic language encourages an inference that is stronger, sharper, more homogeneous, more static, or otherwise different from the reconstruction that justified the formalization. Table 13 summarizes eight recurrent forms relevant to the present framework. The categories can overlap, and their importance varies with the research problem.

The first category in Table 13 is especially important for GR. Writing an entity as E_i is useful because the notation permits relations and trajectories to be tracked. The notation can nevertheless encourage the reader to imagine a stable object whose identity exists independently of the relational-historical reconstruction that justified the index i . Section 4 therefore remains conceptually prior to the formal notation: the symbol records a provisional individuation; it does not settle the ontology of the entity.

Metric and scalar representations require similar restraint. Introducing a quantity $d(x, y)$ does more than rename a qualitative difference if d is treated as a metric. It can introduce non-negativity, identity conditions, symmetry, and triangle inequality. Likewise, writing expertise, trust, power, or problem quality as one scalar can imply an order and comparability that the evidence does not support. When these assumptions are analytically useful, they should be stated as model assumptions and tested where possible. When they are introduced only because numerical formalism is convenient, they constitute representation-induced bias.

Atemporalization and boundary closure are particularly consequential for an evolutionary-relational framework. A static network can preserve relational adjacency while losing the history through which

Table 13: Principal forms of representation-induced bias in formal symbolization.

Bias form	Representational move	Principal analytical risk
Ontological sharpening	A symbol, node, variable, or state is given a stable identity or boundary.	A provisional relational configuration is read as a sharply bounded or intrinsically existing entity.
Discretization	A continuous, gradual, or ambiguous process is represented by a finite set of states or events.	Transitional structure and uncertainty are replaced by artificially sharp thresholds.
Metric imposition	A distance, norm, score, or coordinate system is introduced.	Comparability, symmetry, magnitude, or triangle-like assumptions enter without empirical justification.
Scalarization	A multidimensional, relational, or historical organization is reduced to one or a few numerical variables.	Distinct trajectories or mechanisms become observationally equivalent in the formal model.
Homogenization	Differently typed entities or roles are encoded in one common variable family or node class.	Cross-type differences that condition reasons, agency, or transformation become invisible.
Atemporalization	A history-bearing relation is encoded as a static edge, coefficient, or contemporaneous state.	Path dependence, ordering, duration, and historical location are suppressed.
Boundary closure	The model fixes a system boundary and treats exterior conditions as exogenous or absent.	Feedback, distributed agency, and changing membership are excluded from the represented causal or relational field.
Observer selection	The formal vocabulary privileges features that are easily symbolized or already salient to the analyst.	The representational language reproduces the initial question or disciplinary vocabulary while obscuring alternative problem constitutions.

the relation acquired its present meaning. A fixed vertex set can erase entry, exit, fusion, fission, and type change. A closed dynamical system can suppress the institutional or material conditions that alter the admissible state space. Formal compactness is therefore not sufficient evidence of representational adequacy.

9.5 Formalization as an Epistemically Generative Operation

Symbolization can also improve inquiry. A formal representation can expose hidden assumptions, make composition rules explicit, reveal incompatibilities among locally plausible claims, identify missing variables or morphisms, and suggest new comparisons. Group-theoretic notation can direct attention toward transformations under which a structure might be preserved. Category-theoretic notation can expose a missing intermediate mapping. A dynamical representation can reveal that a supposedly static classification actually requires memory or path dependence. Formalization is therefore potentially generative as well as compressive.

The generative contribution creates a feedback relation between abstraction and symbolization. Let $\mathfrak{B}_Q^{(k)}$ denote the candidate abstraction family at iteration k , and let $\Sigma_Q^{(k)}$ denote the corresponding symbolization. Insights produced through the formal representation can motivate a revised abstraction family through an explicitly documented revision operator. Equation 63 represents this loop.

$$\mathfrak{B}_Q^{(k+1)} = \mathcal{G}_Q\left(\mathfrak{B}_Q^{(k)}, \Sigma_Q^{(k)}, \mathcal{E}_\Sigma^{(k)}\right), \quad (63)$$

where $\mathcal{E}_\Sigma^{(k)}$ denotes the new questions, contradictions, or structural possibilities disclosed by the

Table 14: Diagnostic audit for formal symbolization and representation bias.

Diagnostic site	Guiding question	Principal failure risk
Formalization boundary	Which part of R_t^Q lies inside $D_{\Sigma,t}^Q$ and which reconstructed dimensions remain outside it?	Partial representation is silently presented as a model of the whole problem field.
Interpretation rule	What does each formal object or operation mean in relation to the reconstruction?	Symbols acquire intuitive meanings that were never warranted by the evidence.
Feature preservation	Which elements of $\mathcal{P}_t^Q$ satisfy Equation 59?	A formally elegant representation discards the feature it was introduced to explain.
Transformation fidelity	Does the square in Figure 3 commute under the stated criterion?	Formal transformations are mistaken for empirical transformations.
Bias profile	Which bias forms in Table 13 are introduced or amplified by the chosen representation?	Distortion enters as an unexamined consequence of notation or model class.
Formal generativity	Which new conjectures or distinctions arose from the representation, and have they been returned to evidence?	A property of the formal language is re-described as a discovered property of the phenomenon.
Alternative representations	Would another admissible symbolization preserve materially different features?	One convenient formalism is treated as uniquely natural.
Failure threshold	Which losses or distortions would make the formalization inadequate for the present claim?	The representation is retained regardless of counterevidence because failure conditions were never specified.

formalization. Equation 63 is a methodological representation rather than an algorithm. Its purpose is to mark the origin of a revised abstraction when the revision was generated by the symbolic system itself.

The feedback loop also creates a risk of formalism-driven problem drift. A tractable formal question can become more salient than the original research problem because it is easier to manipulate within $\mathcal{F}_t^Q$. The revised abstraction should therefore return to the evidence and to Q_t before being accepted. A mathematically interesting consequence can remain valuable as a formal conjecture even when its empirical interpretation is unresolved, provided that the distinction is stated explicitly.

Plural formalization can be preferable to forced synthesis. Two representations may preserve different problem-relevant features: a network representation can preserve connectivity while a temporal process representation preserves ordering and duration. If no single formalism preserves both without unacceptable distortion, the framework permits a family $\{\Sigma_t^{Q,m}\}_{m \in M}$ of complementary representations. Their non-equivalence is itself informative when it identifies irreducible representational trade-offs.

9.6 Representation Audit and Stage Output

The symbolization stage should conclude with an audit that connects formal choices to the reconstructed evidence and to the claims the model is intended to support. Table 14 states eight diagnostic sites.

The final diagnostic is essential to methodological restraint. Formalization can legitimately fail. If every available symbolic system erases a relation, historical dependency, type distinction, or uncertainty structure that is central to Q_t , the appropriate conclusion can be that the present problem lacks an adequate formal representation. A qualitative reconstruction can then remain the stronger result. The framework therefore treats failure to formalize as information about the relation between the research object and the available representational resources rather than as an automatic deficiency of the inquiry.

The output of Section 9 is a symbolization record $\mathfrak{S}_t^Q$, a feature-preservation profile, the loss set $\mathcal{L}_{\Sigma,t}^Q$,

the transformation-fidelity result, an explicit representation-bias profile, and any formalization-generated conjectures that require renewed empirical or interpretive examination. These outputs prepare the recursive evaluation stage. The next section asks whether the resulting theory-formation trajectory remains adequate to the problem that initiated it, whether the problem itself should be refined or reconstructed, and which earlier stages require revision in light of the complete inquiry.

10 Recursive Evaluation of Theory Formation

This section develops Stage 7, the final substantive operation of the proposed formation architecture: evaluating the completed theory-formation trajectory and routing justified revisions back to the stages on which the resulting account depends. Its role is broader than ordinary model assessment. The object of evaluation includes the provisional problem, the selection and localization of reason-bearing entities or configurations, the epistemic-normative-practical reconstruction, the evolutionary account, the cross-context abstractions, the transformation-structured preservation claims, and the formal symbolization. The section is organized in six analytical parts. It first defines a versioned formation record that preserves these dependencies. It then introduces a multi-site adequacy profile without reducing heterogeneous forms of support to one scalar score. The third subsection develops revision routing and dependency-sensitive back-propagation. The fourth considers problem revision, branching, and suspension. The fifth specifies provisional stabilization and stopping conditions. The section concludes with an evaluation audit and the interface to the framework's scope and limitation analysis.

The central methodological proposition is that a theory can fail for reasons located upstream from the place where the failure becomes visible. A formal contradiction can originate in an inappropriate symbolization, but it can also reveal an unstable abstraction, an invalid cross-context correspondence, an inadequate reconstruction of an entity, or a poorly constituted initiating question. Recursive evaluation therefore asks two distinct questions: whether the current output is adequate for the claim being made, and where the earliest unsupported dependency in the formation trajectory is located.

10.1 The Formation Record as the Evaluation Object

Let k index a documented iteration of the inquiry. The completed record at iteration k is represented by Equation 64.

$$\mathfrak{F}^{(k)} = \langle Q^{(k)}, \mathcal{E}^{(k)}, \Lambda^{(k)}, \hat{\Xi}^{(k)}, \Omega^{(k)}, \mathfrak{B}^{(k)}, \mathfrak{T}^{(k)}, \mathfrak{S}^{(k)}, \mathcal{K}_{\text{th}}^{(k)} \rangle. \quad (64)$$

In Equation 64, $Q^{(k)}$ denotes the current problem constitution; $\mathcal{E}^{(k)}$ the reason-bearing entities or distributed configurations retained for the inquiry; $\Lambda^{(k)}$ their relational-historical locations; $\hat{\Xi}^{(k)}$ the evidence-constrained epistemic-normative-practical reconstructions; $\Omega^{(k)}$ the evolutionary reconstruction; $\mathfrak{B}^{(k)}$ the candidate abstraction family; $\mathfrak{T}^{(k)}$ the transformation-structured account; $\mathfrak{S}^{(k)}$ the symbolization record; and $\mathcal{K}_{\text{th}}^{(k)}$ the provisional set of substantive theoretical claims supported by the current record. The superscript (k) records revision history rather than historical time inside the phenomenon. This distinction prevents the analyst's sequence of revisions from being conflated with the temporal sequence being reconstructed.

The record is intentionally broader than its claim set $\mathcal{K}_{\text{th}}^{(k)}$. Two inquiries can arrive at similar formal expressions through substantially different problem constitutions, evidence bases, entity selections, or abstraction paths. The completed theory should therefore remain traceable to the sequence of analytical commitments that made it possible. This traceability is particularly important in an evolutionary-relational framework because later stages can depend on choices whose adequacy becomes visible only after cross-context comparison or formalization.

The formation record also preserves partiality. A component can remain unresolved without forcing the remaining components into false precision. For example, $\mathfrak{S}^{(k)}$ can record a failed formalization

while $\widehat{\Xi}^{(k)}$ and $\Omega^{(k)}$ remain well supported. Recursive evaluation therefore does not presuppose that successful inquiry culminates in a fully formalized theory.

10.2 A Multi-Site Adequacy Profile

Evaluation requires different standards at different sites. Problem adequacy, evidential reconstruction, transformation support, and representational fidelity are not naturally commensurable. Collapsing them into one score would create a new scalarization problem of the kind identified in Section 9. The present framework therefore uses a structured profile. Let the evaluation profile of $\mathfrak{F}^{(k)}$ be given by Equation 65.

$$\text{Ad}^{(k)} = \langle a_Q, a_E, a_{\Xi}, a_{\Omega}, a_B, a_T, a_{\Sigma}, a_K \rangle^{(k)}. \quad (65)$$

The components in Equation 65 denote, respectively, the adequacy of the current problem constitution, entity/configuration selection, epistemic-normative-practical reconstruction, evolutionary reconstruction, abstraction family, transformation claims, symbolization, and the resulting substantive claim set. Each component can be recorded qualitatively, ordinally, probabilistically, or through a domain-specific evidential procedure when such a procedure is justified. The notation does not require numerical commensuration.

Table 15 identifies the principal evaluative question at each site. The table also distinguishes a local symptom from the earliest analytical dependency that may require renewed examination.

A weakness in one component does not automatically invalidate every other component. A failed group-theoretic invariance claim can coexist with a strong historical reconstruction; an inadequate formalization can coexist with a robust qualitative abstraction. The evaluation profile therefore supports selective retention. The objective is to identify which conclusions depend on the weak component and which remain warranted independently.

This selective logic also limits rhetorical overcorrection. A counterexample to one transformation family should not be redescribed as a refutation of the entire research programme when the other stages remain independently supported. Conversely, a strong result at one site should not compensate for a weak upstream dependency. Elegant formal consistency cannot repair an entity selection that omitted the relevant reason-bearing configuration.

10.3 Revision Routing and Dependency-Sensitive Back-Propagation

Recursive evaluation becomes methodologically useful when a detected inadequacy can be routed to the stage where revision should begin. Equation 66 defines the formation-site set together with its principal dependency order.

$$\mathbb{L} = \{Q, E, \Xi, \Omega, B, T, \Sigma, K\}, \quad Q \prec E \prec \Xi \prec \Omega \prec B \prec T \prec \Sigma \prec K. \quad (66)$$

The relation $\prec$ in Equation 66 records the principal dependency direction developed in Sections 3–9. Let $D^{(k)}$ denote the evidence and diagnostic findings available at iteration k . A revision-routing map is represented in Equation 67.

$$\mathcal{R}_{\text{eval}}(\mathfrak{F}^{(k)}, D^{(k)}) = \mathbb{J}^{(k)} \subseteq \mathbb{L}, \quad (67)$$

where $\mathbb{J}^{(k)}$ is the set of sites whose present representations require revision, renewed evidence, branching, or suspension. Equation 67 is not an automated diagnosis. It is a record of reasoned methodological judgment whose evidential basis should be stated.

A central discipline is to identify the earliest unsupported dependency that materially contributes to the detected failure. Let $\prec$ denote the principal dependency order on $\mathbb{L}$. For a detected failure f , Equation 68 defines the candidate starting site as the earliest member of the justified revision set under that order.

Table 15: Evaluation sites in a completed theory-formation record.

Site	Principal evaluative question	Indicative revision trigger
Problem constitution	Does $Q^{(k)}$ still identify the relevant object, scope, target, conditions, and answer form after the inquiry has developed?	Later evidence shows that the initial distinction, scale, target, or answer criterion mislocates the research problem.
Entity/config.	Do the retained entities or distributed configurations possess problem-relevant reasons under the reconstructed conditions?	Relevant access, mediation, affected positions, or distributed contributions were excluded or authority was mistaken for epistemic relevance.
Epistemic-practical reconstruction	Is $\hat{\Xi}^{(k)}$ sufficiently supported by convergent evidence and explicit uncertainty?	Retrospective accounts conflict with traces, practice, artifacts, or alternative reconstructions.
Evolutionary reconstruction	Does $\Omega^{(k)}$ preserve supported continuation, transformation, path dependence, and state-space change?	The account assumes stable identity, boundaries, types, or state spaces where the evidence indicates structural change.
Abstraction	Does each retained abstraction remain supported across its stated comparison domain while preserving relevant residual differences?	A recurrent pattern disappears after scale change, case expansion, or correction of a correspondence mapping.
Transformation account	Are preservation, equivariance, covariance, and composition claims supported over the stated transformation family?	Counterexamples, unsupported inverses, post-hoc transformation selection, or invalid composition undermine the claim.
Symbolization	Does the representation preserve the problem-relevant features and supported transformations within its declared boundary?	Loss, distortion, non-commutation, or formalism-driven problem drift changes the meaning of the represented structure.
Claim	Does the stated theoretical claim remain within the support jointly provided by the preceding sites?	The conclusion generalizes beyond the support domain, suppresses unresolved alternatives, or presents a heuristic as an established result.

$$j_{\min}(f) \in \min_{\prec} \{j \in \mathbb{J}^{(k)} : j \text{ materially contributes to } f\}. \quad (68)$$

The minimization in Equation 68 is conceptual rather than numerical. If a symbolization collapses two entity types because the formal language lacks a distinction that the reconstruction preserved, revision may begin at Σ . If the apparent type distinction was itself created by an unsupported entity classification, the appropriate return point lies earlier. The principle reduces the temptation to repair downstream symptoms while retaining the unsupported assumption that generated them.

Revision can propagate forward because later objects depend on earlier ones. Let $\text{Dep}^+(j)$ denote the downstream dependency closure of a revised site j . The set of objects requiring re-evaluation after a justified revision is represented by Equation 69.

$$\mathbb{R}_{\downarrow}^{(k)} = \bigcup_{j \in \mathbb{J}^{(k)}} (\{j\} \cup \text{Dep}^+(j)). \quad (69)$$

Equation 69 does not imply that every downstream object must be discarded. It requires re-evaluation. A revised problem can leave some reconstructed evidence unchanged while altering its relevance; a revised entity boundary can preserve parts of the abstraction while changing the interpretation of its units; a revised formalization can leave the empirical reconstruction untouched. Each retained object

should therefore carry an explicit justification for why the upstream change does not alter the claim it supports.

This dependency-sensitive procedure produces a new formation record through Equation 70.

$$\mathfrak{F}^{(k+1)} = \mathcal{U}_{\text{eval}}\left(\mathfrak{F}^{(k)}, D^{(k)}, \mathbb{J}^{(k)}, \mathbb{R}_{\downarrow}^{(k)}\right), \quad (70)$$

where $\mathcal{U}_{\text{eval}}$ denotes a documented revision process. The update can retain, modify, split, suspend, or remove components. The equation therefore represents provenance and dependency rather than a deterministic optimization algorithm.

10.4 Problem Revision, Branching, and Suspension

Problem revision deserves special treatment because $Q^{(k)}$ organizes relevance throughout the formation record. Section 3 distinguished retention, refinement, decomposition, rescaling, replacement, and dissolution as recurrent problem-revision modes. Recursive evaluation supplies the evidence from which one of these modes can become warranted. A revised question can alter the entity set, the comparison domain, the abstraction target, and the standard by which a formal representation is judged adequate.

Revision need not converge immediately to one successor question. Inquiry can disclose two or more well-supported problem constitutions that preserve different explanatory targets or scales. Let $\mathcal{B}^{(k+1)}$ denote the set of warranted successor branches. Equation 71 represents this possibility.

$$\mathcal{B}^{(k+1)} = \left\{ \mathfrak{F}^{(k+1,1)}, \dots, \mathfrak{F}^{(k+1,m)} \right\}, \quad m \geq 1. \quad (71)$$

Branching is methodologically preferable to premature synthesis when two formations depend on non-equivalent object boundaries, scales, normative targets, or representational commitments. Their later relation can itself become an object of comparison. The framework therefore permits plural theory formation without assuming that all branches must ultimately collapse into one representation.

Suspension is distinct from branching. An inquiry should be suspended at a site when the available evidence cannot discriminate among materially different reconstructions and the uncertainty propagates to the claim under evaluation. Suspension preserves the unresolved alternatives and identifies the evidence that would make renewed discrimination possible. This treatment is preferable to silently selecting the analytically convenient branch.

Dissolution is the strongest form of problem revision. If the inquiry establishes that the initial puzzle depended on a distinction that the reconstructed field does not support, the appropriate output can be removal of $Q^{(k)}$ from the active formation path. A dissolved question can remain historically informative because the trajectory documents why the question once appeared well formed and which later distinctions made its inadequacy visible.

10.5 Provisional Stabilization and Stopping Conditions

A recursive framework requires a stopping discipline. Unlimited return to earlier stages would make completion impossible, while premature closure would convert revisability into rhetoric. The framework therefore treats completion as provisional stabilization relative to a stated problem, support domain, and claim strength.

Let $\mathbb{J}_{\text{mat}}^{(k)}$ denote the subset of revision sites whose unresolved inadequacy would materially alter the principal claim $K_{\star}^{(k)} \in \mathcal{K}_{\text{th}}^{(k)}$. Let $\mathcal{O}^{(k)}$ denote the set of declared open problems and unresolved limitations that do not currently defeat that principal claim. A formation is provisionally stabilized when Equation 72 is satisfied.

$$\text{Stable}_Q\left(\mathfrak{F}^{(k)}\right) = 1 \quad \text{only if} \quad \mathbb{J}_{\text{mat}}^{(k)} = \emptyset \text{ and } \mathcal{O}^{(k)} \text{ is explicitly bounded.} \quad (72)$$

Equation 72 states a release condition, not a claim of final truth. A paper can therefore be complete while preserving unresolved questions, alternative reconstructions, known counterexamples outside its

Table 16: Diagnostic audit for recursive evaluation of theory formation.

Diagnostic site	Guiding question	Principal failure risk
Formation provenance	Can each major claim be traced through the versioned formation record in Equation 64?	The final theory is detached from the analytical choices and evidence that generated it.
Adequacy profile	Are heterogeneous strengths and weaknesses kept distinct rather than compressed into one quality score?	Strong performance at one site masks a material weakness at another.
Earliest dependency	Does revision begin at the earliest unsupported site that materially contributes to the detected failure?	Downstream patching preserves the assumption responsible for the failure.
Forward re-evaluation	Have downstream objects in Equation 69 been rechecked after an upstream revision?	Earlier changes silently invalidate later abstractions or conclusions.
Problem status	Is the current question retained, refined, decomposed, rescaled, replaced, or dissolved for stated reasons?	The initiating question survives by inertia after the inquiry has undermined it.
Alternative branches	Are materially non-equivalent reconstructions retained when the evidence does not warrant synthesis?	Ambiguity is hidden through premature unification or convenient case selection.
Stabilization basis	Are the material revision set and unresolved open problems compatible with the released claim?	Completion is confused either with finality or with simple exhaustion of time and resources.
Reopening conditions	Which future evidence, counterexample, transformation, or contextual change would reopen the formation record?	Revisability is asserted abstractly but no practical falsification or revision pathway is specified.

support domain, and representational limitations. The requirement is that these residual uncertainties are compatible with the stated strength and scope of the released claim.

Three further stopping conditions are useful. First, the claim should remain inside the intersection of the support domains supplied by the stages on which it depends. Second, additional revision should have an identified evidential or analytical trigger rather than being performed merely because another representation is imaginable. Third, the cost of further refinement can be recorded when inquiry is constrained by finite time, access, or resources, but practical exhaustion should be distinguished from epistemic stabilization.

This distinction is especially important for the motivating cases of situated inquiry. High-quality theory formation can occur under bounded information and finite time. The resulting stabilization can therefore be rationally provisional: the inquiry states what has been supported under the present conditions, what remains uncertain, and which future observations or comparisons would reopen the formation record.

10.6 Evaluation Audit and Stage Output

The recursive evaluation stage should conclude with a record that makes both retained claims and routed revisions inspectable. Table 16 states eight diagnostic obligations.

The output of Section 10 is therefore a versioned formation record $\mathfrak{F}^{(k)}$ containing its substantive claim set $\mathcal{K}_{\text{th}}^{(k)}$, a multi-site adequacy profile $\mathcal{A}d^{(k)}$, a justified revision set $\mathbb{J}^{(k)}$, its downstream re-evaluation closure $\mathbb{R}_{\downarrow}^{(k)}$, any warranted branch set $\mathcal{B}^{(k+1)}$, and an explicit stabilization or suspension statement. These objects turn revisability into a documented methodological operation rather than a

general declaration of intellectual openness.

The completed recursive architecture also makes the framework's limits more visible. Some problems will lack an adequate reason-bearing entity or configuration; some will not require the present entity-centered reconstruction at all; some evolutionary accounts will remain underconstrained; some cross-context abstractions will fail to survive expansion of the comparison domain; and some structures will resist faithful symbolization. The next section therefore turns from internal evaluation to the framework's positive scope conditions, recurrent failure modes, and methodological limits. Its purpose is to specify where the architecture earns its use and where another research strategy is likely to be more appropriate.

11 Scope Conditions, Failure Modes, and Methodological Limits

This section specifies the domain in which the proposed meta-theory is likely to earn its analytical cost and the conditions under which its use should be reduced, suspended, or replaced. Its role is methodological rather than defensive. A framework that can redescribe every inquiry after the fact would have weak discriminative value, while a framework that states only where it fails would provide little guidance for research design. The section therefore defines scope positively, in terms of structural properties of research problems, and then identifies recurrent failure modes that arise when the architecture is applied beyond those conditions or when one of its internal reconstructions becomes under-supported. The discussion is organized in six analytical parts. It first develops a non-scalar applicability profile. It then distinguishes full, partial, and alternative methodological routes. The third subsection addresses absence, inaccessibility, and misidentification of reason-bearing entities or configurations. The fourth examines reconstruction and evolutionary underconstraint. The fifth considers limits of cross-context abstraction, transformation claims, and scale transfer. The sixth integrates symbolization limits and methodological pluralism into a final scope audit.

The framework is problem-relative rather than discipline-relative. The same discipline can contain problems that fit the architecture closely and problems for which the architecture adds little. Conversely, problems from different disciplines can exhibit similar formation structures. The relevant question is therefore whether historically evolving, relationally situated epistemic entities or configurations materially participate in the constitution, interpretation, abstraction, or recursive revision of the problem being studied. Disciplinary labels can guide source selection, but they do not determine applicability by themselves.

11.1 Positive Applicability Conditions

The proposed architecture is most useful when several problem properties make a fixed-input model of theory formation inadequate. Let Q denote the provisional problem and let $\mathfrak{A}(Q)$ denote its applicability profile. Equation 73 represents the profile as a structured diagnostic object rather than as a scalar score.

$$\mathfrak{A}(Q) = \langle a_P, a_R, a_H, a_K, a_B, a_E, a_X, a_F \rangle_Q. \quad (73)$$

The components in Equation 73 refer, respectively, to problem endogeneity, reason-bearing participation, relational-historical dependence, tacit or practice-embedded knowing, bounded epistemic conditions, evolutionary change, cross-context comparability, and reflexive feedback. They are diagnostic dimensions rather than required numerical variables. A research design can record each component qualitatively or through domain-specific evidence where stronger operationalization is available.

Problem endogeneity, a_P , is high when competent inquiry can materially revise the object, scale, target, conditions, or admissible answer form of the initiating question. Reason-bearing participation, a_R , concerns whether situated entities or distributed configurations possess problem-relevant access, experience, judgment, or practice that contributes materially to the formation of an answer. Relational-historical dependence, a_H , concerns whether removing an entity from its position in a history of relations would erase information needed to explain its epistemic role. Practice-embedded knowing, a_K , becomes

Table 17: Positive applicability conditions for the proposed meta-theory.

Condition	Indicative question	Why the framework can add value
Problem endogeneity	Can competent inquiry revise what the problem is, its scale, or the form of an adequate answer?	Problem constitution becomes part of the research object rather than a fixed preliminary step.
Reason-bearing participation	Do situated entities or distributed configurations possess problem-relevant reasons, access, judgment, or practice?	The inquiry can reconstruct how epistemic contribution is generated and distributed.
Relational-historical dependence	Would the entity's epistemic role change materially if its relational history or position were removed?	Relational-historical location becomes explanatory rather than decorative context.
Practice-embedded knowing	Does competent performance depend on salience, tacit knowledge, practical judgment, or normative discrimination?	Reconstruction can include forms of knowing that are poorly represented by explicit propositions alone.
Bounded epistemic conditions	Are evidence, time, attention, reliability, or environmental stability materially limited?	The framework can study situated formation under the conditions in which actual judgment occurs.
Evolutionary change	Do entities, boundaries, types, relations, mediators, or state spaces change during the relevant trajectory?	Evolutionary reconstruction can replace fixed-entity longitudinal tracking where stronger change is evidenced.
Cross-context comparability	Can meaningful spatial, temporal, or spatiotemporal comparisons be constructed?	Structural abstraction and transformation analysis can test which features travel across contexts.
Reflexive feedback	Can an output of inquiry alter later conditions, classifications, practices, or available questions?	Recursive evaluation can represent theory formation as part of an evolving field.

important when explicit propositions do not exhaust the relevant competence, for example when salience, pattern recognition, judgment, embodied skill, or normative discrimination contributes to successful inquiry.

The component a_B concerns bounded epistemic conditions such as uncertainty, incomplete or conflicting information, changing evidence, finite time, and limited attention. These conditions make models that assume complete information less representative of practice. Evolutionary change, a_E , becomes important when the relevant entities, boundaries, relations, types, mediators, or available state spaces change during the inquiry or the phenomenon. Cross-context comparability, a_X , concerns whether spatial, temporal, or spatiotemporal contrasts can support disciplined abstraction or transformation analysis. Reflexive feedback, a_F , concerns whether theories, classifications, instruments, or decisions produced by inquiry can become conditions of subsequent inquiry or alter the field being studied.

Table 17 summarizes these conditions. None is intended as a universal entrance requirement. The profile is designed to make the reason for using the framework explicit. A problem strongly exhibiting only one dimension may require only a local borrowing from the architecture, while a problem exhibiting several mutually connected dimensions can justify the full recursive procedure.

The framework's central scope is therefore defined structurally. For example, a problem concerning how a scientific community notices and interprets an anomalous observation may fit the architecture even when the underlying phenomenon belongs to natural science. A direct measurement problem whose answer does not materially depend on reconstructing reason-bearing entities may fit poorly. The difference lies in the formation structure of the problem, not in the disciplinary label attached to it.

11.2 Methodological Route Selection

Stage 0 of the architecture should therefore choose a methodological route rather than assume that the complete GR sequence is always appropriate. Let $\mathcal{R}_Q$ denote the admissible route family for problem Q . Equation 74 records three broad outcomes.

$$\mathcal{R}_Q = \{r_{\text{full}}, r_{\text{partial}}, r_{\text{alternative}}\}. \quad (74)$$

The full route, r_{full} , applies when the problem materially involves question revision, reason-bearing entities or configurations, relational-historical reconstruction, evolutionary change, cross-context abstraction, and recursive evaluation. The partial route, r_{partial} , uses only the stages that answer the local question. An inquiry can, for example, require relational-historical entity reconstruction without requiring group-theoretic invariance, or require a symbolization-bias audit without requiring a distributed-cognition analysis. The alternative route, $r_{\text{alternative}}$, is appropriate when another methodology provides more direct access to the problem and the present architecture would add representational machinery without corresponding explanatory gain.

Route selection preserves methodological pluralism. A phenomenon-first experimental design, a statistical identification strategy, a proof-theoretic argument, archival interpretation, ethnography, simulation, or another approach can be primary when its assumptions match the problem more closely. The present meta-theory can still be used locally to study how the problem was constituted or how a model was symbolized, but that local use should not be redescribed as evidence that the entire formation architecture governs the underlying phenomenon.

A useful stopping condition arises when the entity-centered portion of the method lacks an adequate epistemic target. Let $\hat{\mathcal{E}}_Q$ denote the set of entities or distributed configurations for which the inquiry has obtained sufficient evidence to justify problem-relevant reason-bearing status. Equation 75 states the empirical situation in which that retained set is empty.

$$\hat{\mathcal{E}}_Q = \emptyset. \quad (75)$$

Equation 75 is a methodological result, not an ontological proof that no relevant entity exists. It can reflect genuine absence, inaccessible evidence, an immature field, an incorrectly constituted question, or an entity boundary that has been drawn too narrowly. The framework should therefore distinguish *no adequate entity has been identified* from the stronger statement *no adequate entity exists*. The former is often the defensible conclusion available to research.

11.3 Entity Availability, Boundary Error, and Distributed Reason

Three recurrent failures arise at the entity-identification stage. The first is absence or inaccessibility. A problem can concern a novel event, extinct practice, poorly documented community, or emerging phenomenon for which the required reason-bearing configuration cannot be reconstructed with adequate evidence. In such cases, the framework should retain the unresolved status rather than infer a complete epistemic organization from sparse traces.

The second failure is misidentification. Because the initiating question conditions what appears epistemically relevant, a poorly constituted problem can select an inappropriate entity set. Institutional prestige, formal authority, public recognition, or retrospective success can then be mistaken for problem-relevant reason-bearing capacity. Section 4 already separated reason-bearing status from authority and correctness; the limitation analysis adds that the selection procedure itself can reproduce the assumptions built into the question.

Let $B_Q(E)$ denote the current boundary used to individuate a candidate epistemic unit. A boundary error exists when the reason-generating dependencies required for the local explanation cross that boundary in a way that materially changes the reconstructed capacity. Equation 76 represents this condition schematically.

$$\text{Dep}_Q(E) \not\subseteq B_Q(E) \quad \text{and the omitted dependencies materially alter the attributed capacity.} \quad (76)$$

The third failure is therefore distributed reason. What appears as the competence of one visible individual can be generated through a configuration involving colleagues, artifacts, institutional routines, instruments, interfaces, datasets, or computational systems. In such cases the suitable epistemic unit is closer to a relational configuration $\mathcal{C}_Q$ than to a single entity E_i . The methodological correction is not to inflate the individual's internal competence until it explains the observed outcome, but to reconstruct the dependencies through which the outcome became possible.

Entity absence and distributed reason should remain conceptually distinct. The failure to locate an adequate individual can indicate that no relevant epistemic entity is presently reconstructible, or it can indicate that the entity boundary should be widened to a distributed configuration. Evidence about interaction, mediation, access, and task dependence is needed to distinguish these possibilities.

11.4 Reconstruction Bias and Evolutionary Underconstraint

The framework's historical depth produces a second family of limitations. Reconstructing a past organization of knowing and practice is an inverse problem: the analyst observes traces, reports, artifacts, decisions, and consequences, then infers a generative organization that could have produced them. Different latent reconstructions can sometimes remain compatible with the available record. Let D denote the retained evidence and let $\mathcal{H}(D)$ denote the set of reconstructions compatible with that evidence under the current interpretive criteria. Equation 77 states a non-identifiability condition.

$$|\mathcal{H}(D)| > 1 \quad \text{and available evidence does not warrant selecting a unique member.} \quad (77)$$

Under Equation 77, a single reconstructed trajectory should not be presented as historically established. The inquiry can retain several alternatives, report the dimensions on which they differ, and specify what additional evidence would discriminate among them. Retrospective self-report is especially vulnerable because later conceptual organization can make earlier uncertainty appear more coherent than it was. Drafts, contemporaneous correspondence, recorded practice, artifacts, and independently dated traces can reduce this problem, but they do not guarantee uniqueness.

Selection and survivorship bias create a related limitation. If cases are chosen because they are later celebrated as successful, a recurrent feature can reflect the selection mechanism rather than a generative condition of success. The framework should therefore seek, where feasible, failed, interrupted, unrecognized, or differently successful comparison trajectories. A property shared by successful cases gains stronger evidential value when the comparison design also tests whether the property appears in relevant unsuccessful cases and whether apparently successful outcomes depended on contingencies outside the reconstructed mechanism.

Evolutionary reconstruction introduces a further risk. The framework permits entities, boundaries, relations, types, problems, and state spaces to evolve. If all of these elements can be adjusted freely after observing the outcome, almost any trajectory can be rationalized. Let $\mathfrak{M}$ denote a family of admissible evolutionary reconstructions and $\text{Fit}(M, D)$ indicate compatibility of a reconstruction M with evidence D . A severe underconstraint exists when many materially different reconstructions fit the same evidence while producing no discriminating implications. Equation 78 represents this risk.

$$\exists M_1, M_2 \in \mathfrak{M}, \quad M_1 \not\approx M_2, \quad \text{Fit}(M_1, D) \wedge \text{Fit}(M_2, D), \quad (78)$$

with the additional condition that the current design lacks evidence capable of discriminating between M_1 and M_2 . Equation 78 is a warning against retrospective flexibility. Continuation relations, boundary changes, type transformations, state-space changes, and claims of irreversibility should therefore be supported by traces that are independent of the explanatory convenience they provide. Counterfactual implications, prospective prediction where available, preregistered coding rules, independent case expansion, or other domain-appropriate constraints can strengthen the reconstruction.

11.5 Limits of Cross-Context Generalization

Cross-context abstraction can fail even when each local reconstruction is strong. Section 7 separated recurrence from invariance and Section 8 indexed preservation claims to specified transformation families. The present limitation is that every such claim has a support boundary. An invariant discovered under $\mathfrak{T}_Q^*$ should not be assumed to survive a larger transformation family $\mathfrak{T}_Q^+$. Equation 79 states this explicitly.

$$I(\tau z) = I(z) \quad \forall \tau \in \mathfrak{T}_Q^* \quad \not\Rightarrow \quad I(\tau z) = I(z) \quad \forall \tau \in \mathfrak{T}_Q^+. \quad (79)$$

The same discipline applies to equivariance and categorical transport. A successful commutative square in one family of mappings does not license a general claim of structural equivalence between the underlying domains. Cross-type mappings can preserve one relation while failing to preserve another, and historical transformations can create new types for which the earlier translation is undefined.

Scale dependence is a particularly important source of failure. Let s denote an analytical scale and let A_s denote an abstraction supported at that scale. The persistence of A_s under a scale transformation $\sigma_{s \rightarrow s'}$ is an empirical question. Equation 80 records the possibility of scale-specific structure.

$$A_s \not\subseteq \sigma_{s \rightarrow s'}^{-1}(A_{s'}). \quad (80)$$

Equation 80 allows a process that appears stable at an organizational scale to decompose into heterogeneous local mechanisms, or a variable local process to aggregate into a stable macro-pattern. Neither level automatically invalidates the other. The problem constitution determines which scale relation matters for the claim, and the abstraction should state the scale at which its support was established.

Case expansion creates another test. A candidate abstraction that survives three contexts can fail when a fourth context introduces a relation or historical condition absent from the original comparison set. Such failure should normally narrow the support domain or motivate a revised conditional abstraction rather than be hidden as an anomalous case. The framework therefore treats residual differences and counterexamples as information about the boundary of the transformation family.

11.6 Symbolization Limits and Methodological Pluralism

Formal symbolization adds a final family of limits. Section 9 established that a formal representation can omit or distort problem-relevant features and that formal operations can generate structures whose empirical interpretation remains unresolved. The limitation analysis adds a stronger conclusion: some reconstructed structures may resist adequate formalization under the currently available symbolic resources. Failure to produce a faithful formal representation can therefore be an informative outcome.

Let $\mathfrak{S}_Q$ denote the candidate symbolization family for a reconstructed structure R^Q . If every available symbolization loses or distorts at least one feature that is constitutive of the local claim, the appropriate result is a formalization limit rather than selection of the least visibly inconvenient model. Equation 81 states this condition schematically.

$$\forall \Sigma \in \mathfrak{S}_Q, \quad \mathcal{L}_\Sigma^Q \cap \mathcal{P}_{\text{constitutive}}^Q \neq \emptyset \quad \text{or} \quad \mathcal{D}_\Sigma^Q \cap \mathcal{P}_{\text{constitutive}}^Q \neq \emptyset. \quad (81)$$

In Equation 81, $\mathcal{L}_\Sigma^Q$ denotes the loss set developed in Section 9, $\mathcal{D}_\Sigma^Q$ a corresponding distortion set, and $\mathcal{P}_{\text{constitutive}}^Q$ the features without which the local claim changes meaning. Under this condition, qualitative, historical, diagrammatic, plural, or otherwise partial representation can be more rigorous than a mathematically complete but substantively misleading formalization.

This conclusion also limits the status of the algebra introduced in Sections 8 and 9. Groups, semi-groups, groupoids, categories, commutative diagrams, and evolving state spaces are representational resources whose use depends on independently supported properties of the reconstructed process. Their elegance does not establish the ontology of the phenomenon, and the absence of a convenient algebra does not establish the absence of structure.

Table 18: Scope, failure-mode, and methodological-limit audit.

Audit site	Diagnostic question	Methodological response
Applicability	Which components of $\mathfrak{A}(Q)$ materially affect the answer, and which are incidental?	Use the full framework only where the additional reconstruction changes what can be warranted.
Route selection	Is the full, partial, or alternative route most direct for the current problem?	Preserve methodological pluralism and document why the selected route earns its complexity.
Entity availability	Has an adequate reason-bearing entity or distributed configuration actually been identified?	Retain unresolved status, revise the question or boundary, or choose another route when $\hat{\mathcal{E}}_Q = \emptyset$.
Boundary validity	Do material epistemic dependencies cross the chosen entity boundary?	Reconstruct the distributed configuration rather than attributing system-level capacity to a visible component.
Reconstruction identifiability	Do materially different reconstructions remain compatible with the evidence?	Preserve alternative branches and state the evidence required to discriminate among them.
Evolutionary constraint	Are continuity, boundary change, path dependence, and state-space change independently supported?	Constrain the reconstruction; avoid retrospective flexibility that fits any observed trajectory.
Generalization domain	Is the abstraction or invariant being asserted beyond the cases, scales, and transformations that support it?	Narrow the support domain, condition the claim, or collect new comparison evidence.
Scale dependence	Does the structure survive translation across the scale relevant to the claim?	State the scale of validity and preserve cross-scale disagreement when it is explanatory.
Formalization adequacy	Does every available formalization lose or distort a constitutive feature?	Permit partial, plural, qualitative, or failed formalization rather than manufacturing precision.
Ontological restraint	Is a useful representation being promoted into a claim about ultimate ontology without independent support?	Keep representational adequacy, empirical support, and ontological commitment distinct.

Table 18 consolidates the main scope and failure checks. It is designed for use at Stage 0 and again during recursive evaluation. Repeating the audit is important because a problem can move into or out of the framework’s central scope as the inquiry changes its question, entity boundaries, evidence base, or comparison design.

The framework is therefore non-universal by design. Its strongest domain consists of research problems in which historically evolving, relationally constituted epistemic entities or configurations materially participate in problem formation, judgment, abstraction, representation, or recursive revision. Even within that domain, the full architecture is optional rather than mandatory: local questions can justify local use. The positive applicability profile, route selection, and failure-mode audit are intended to prevent the framework from becoming a general-purpose vocabulary applied wherever relations, history, or formal structure can be mentioned.

This non-universality is compatible with the framework’s meta-theoretical ambition. A meta-theory can be valuable by identifying a recurrent family of formation problems and providing a disciplined architecture for studying them without claiming jurisdiction over every form of inquiry. The next section therefore turns from scope to research design. It develops empirical strategies for reconstructing theory-formation episodes, comparing trajectories, testing candidate abstractions, and documenting the evidence

required by the recursive architecture.

12 Empirical Designs for Theory-Formation Research

This section translates the preceding meta-theoretical architecture into empirical research designs. Its role is to specify how the objects introduced in Sections 3–11 can be investigated through observable traces without collapsing the framework into a single preferred method. The objective is therefore operational rather than prescriptive. Different theory-formation problems can require observation, interviews, archival reconstruction, document analysis, artifact tracing, comparative case design, formal modelling, or combinations of these methods. The present framework supplies a common logic for deciding what evidence is needed, how evidence should be indexed to the formation trajectory, and how empirical findings can support, revise, or suspend claims at different stages.

The empirical programme follows six principles. First, the unit of analysis should be a formation episode or trajectory rather than an isolated final theory. Second, evidence should be tied to the component of the formation record that it is intended to reconstruct. Third, contemporaneous traces and observed practice should be distinguished from later retrospective accounts. Fourth, temporal order, relational position, and changes in the available state space should remain recoverable from the evidence record where the research question depends on them. Fifth, cross-context abstraction should be tested on cases beyond those from which the abstraction was generated whenever feasible. Sixth, empirical design should remain recursively revisable: evidence can require changes to the initiating question, the retained entity set, the historical reconstruction, the comparison family, or the formal representation.

Naturalistic decision-making research provides one motivating example of inquiry into competent judgment under field conditions rather than only under idealized laboratory assumptions [8]. Reflective-practice research similarly directs attention to knowing expressed in action and to the difficulty of recovering practice solely from explicit propositions [14]. Distributed-cognition research shows why relevant cognitive organization can extend across people, artifacts, and coordinated environments [7]. Processual social analysis emphasizes temporal ordering and changing relations [1], while social accounts of scientific knowledge make criticism, interaction, and community organization relevant to epistemic reconstruction [11]. The designs below use these traditions as methodological resources while retaining the present framework’s problem-relative and recursively revisable architecture.

12.1 Formation Episodes as Empirical Units

The empirical unit should be defined at the scale at which a theory-forming change can be reconstructed. A formation episode can be brief, such as the reinterpretation of an anomalous signal during a high-pressure decision, or extended, such as the multi-year development of a research programme. What unifies these cases is that the episode contains a traceable relation among a provisional problem, a reason-bearing configuration, a sequence of epistemic operations, and an output that changes the subsequent formation path.

Let c index an empirical case and let $I_c = [t_{c,0}, t_{c,1}]$ denote the temporal interval selected for reconstruction. Equation 82 defines a case record at the level required by the present framework.

$$\mathcal{C}_c^Q = \langle Q_c, I_c, \hat{\mathcal{E}}_c^Q, \hat{\Lambda}_c^Q, \hat{\Xi}_c^Q, \hat{\Omega}_c^Q, \mathcal{D}_c^Q \rangle. \quad (82)$$

The hats in Equation 82 indicate evidence-constrained reconstructions rather than direct observation of the theoretical objects themselves. $\mathcal{D}_c^Q$ denotes the evidence corpus from which the reconstructed entity set, relational-historical positions, epistemic-normative-practical organizations, and evolutionary configurations are inferred. The interval I_c should be chosen so that the beginning and end are analytically meaningful. A case that begins only after the decisive problem reformulation has already occurred will systematically obscure the event the framework is intended to explain.

Case boundaries therefore require justification. Three boundary criteria are especially useful. A *problem boundary* is appropriate when the episode begins with the appearance or reformulation of a

problem and ends with provisional stabilization, replacement, or dissolution. A *configuration boundary* is appropriate when the relevant entity set or relational structure changes materially. A *representation boundary* is appropriate when the dominant symbolic or formal representation changes and that change reorganizes subsequent reasoning. Several boundaries can coincide, but empirical design should state which one controls inclusion.

A final theory, publication, judgment, or decision can serve as a locator for case discovery without becoming the unit of analysis. Retrospective selection on a visible output is particularly vulnerable to survivorship bias. A stronger case design therefore seeks earlier traces, abandoned alternatives, failed branches, and interrupted trajectories where they are recoverable. These materials help distinguish a generative structure from a narrative reconstructed only after success became known.

12.2 Evidence Architecture for Reconstruction

Section 5 distinguished the target organization $\Xi_i^{Q_i}(t)$ from the analyst's reconstruction $\hat{\Xi}_i^{Q_i}(t)$. Empirical design should preserve that distinction through an explicit evidence architecture. Let s index evidence-source classes and let $\mathcal{D}_{c,s}^Q$ denote the evidence contributed by class s to case c . Equation 83 defines the case evidence bundle.

$$\mathcal{D}_c^Q = \bigcup_{s \in \mathcal{S}_D} \mathcal{D}_{c,s}^Q, \quad \mathcal{S}_D = \{\text{conc}, \text{retro}, \text{trace}, \text{artifact}, \text{outcome}, \text{context}\}. \quad (83)$$

The six source classes in Equation 83 are concurrent evidence, retrospective accounts, contemporaneous traces, mediating artifacts, outcome evidence, and contextual evidence. Concurrent evidence includes direct observation, contemporaneous verbalization where appropriate, interaction records, or instrument logs captured while the formation episode is occurring. Retrospective evidence includes interviews, memoirs, later explanations, and reflective accounts. Contemporaneous traces include drafts, notebooks, correspondence, version histories, meeting notes, annotations, and intermediate models. Mediating artifacts include instruments, diagrams, software, checklists, databases, physical interfaces, and other objects that organize perception or action. Outcome evidence concerns what the formation produced and how that output performed under later use or criticism. Contextual evidence reconstructs institutional, disciplinary, technological, normative, and relational conditions needed to interpret the other sources.

Retrospective evidence remains valuable because actors can report distinctions, doubts, and considerations that leave few external traces. Its evidential role should be indexed, however, to the temporal distance and conditions of recall. The research record should distinguish an account made during the episode, an account made shortly after it, and a narrative produced years later after the eventual outcome became known. This is particularly important when the aim is to reconstruct salience formation or rejected alternatives rather than only articulated justification.

Distributed configurations require configuration-level evidence. When the relevant reason-bearing unit includes several persons, artifacts, or institutional routines, the analyst should record interaction paths and task distribution rather than merge the case into the statements of the most visible participant. Hutchins's study of distributed cognition motivates this design sensitivity by showing how cognitive work can be organized across coordinated systems rather than located entirely within an isolated individual [7]. The present framework generalizes this point by asking whether the reasons relevant to Q are generated at the entity level, the relation level, or the configuration level.

Table 19 links the evidence classes to their principal analytical uses and characteristic risks.

Triangulation in this framework means more than counting source types. Evidence should be assessed for whether independent traces constrain the same reconstructed claim. Agreement among several retrospective interviews can still reflect a shared later narrative, while one dated draft can decisively establish that a distinction existed earlier than later accounts suggest. The empirical record should therefore preserve provenance and temporal location for each evidential contribution.

Table 19: Evidence architecture for empirical reconstruction of theory formation.

Evidence class	Principal analytical use	Characteristic risk
Concurrent evidence	Salience, interaction, temporal order, situated judgment, and practice as an episode unfolds	Reactivity, incomplete access, and excessive dependence on what can be observed in real time
Retrospective accounts	Actor interpretations, recalled alternatives, reasons, and perceived turning points	Hindsight, rationalization, memory compression, and narrative smoothing
Contemporaneous traces	Problem reformulation, abandoned branches, intermediate representations, and sequence reconstruction	Selective preservation and ambiguous authorship or timing
Mediating artifacts	Distribution of cognition, symbolic constraints, affordances, and representation changes	Treating an artifact’s designed function as equivalent to its actual use
Outcome evidence	Consequences, later criticism, practical success, and downstream feedback	Conflating historical success with epistemic quality
Contextual evidence	Relational-historical position, institutional constraints, available methods, and background conditions	Over-expanding context until causal or epistemic relevance becomes indeterminate

12.3 Temporal Event-Ledger Design

Evolutionary reconstruction requires an evidence structure that preserves temporal order without assuming that every change is continuous or that the entity identities remain fixed. A useful empirical device is an event ledger. Let e_k denote an event relevant to the formation trajectory. Equation 84 represents one ledger entry.

$$e_k = \langle t_k, \mathcal{P}_k, \mathcal{R}_k, q_k, \sigma_k, d_k \rangle, \quad (84)$$

where t_k is the best-supported temporal location, $\mathcal{P}_k$ identifies participating entities or configurations, $\mathcal{R}_k$ records relevant relations, q_k records the current problem status, σ_k records the event class, and d_k points to the supporting evidence. The event class can distinguish, for example, observation, anomaly recognition, problem reformulation, entity-boundary change, representation change, hypothesis generation, rejection, comparison, formalization, criticism, or stabilization.

The ledger for case c is represented by Equation 85.

$$\mathcal{L}_c^Q = (e_1, e_2, \dots, e_{n_c}), \quad t_1 \preceq t_2 \preceq \dots \preceq t_{n_c}. \quad (85)$$

The relation $\preceq$ permits partial temporal ordering when available evidence cannot resolve exact sequencing. Historical research often contains dated events, interval-censored events, and events whose order is known only relative to others. The ledger should preserve that uncertainty rather than assign false precision.

Event ledgers support three empirical tasks. First, they allow the analyst to identify candidate change points where the problem, entity boundary, relation topology, symbolic representation, or state space changes. Second, they allow competing reconstructions to be compared against the same temporal evidence. Third, they make path dependence empirically inspectable by showing which later options were available only because earlier events created or removed relations, concepts, instruments, or institutional possibilities.

Abbott’s processual work provides an important methodological precedent for taking sequence and temporality seriously rather than treating time as an additional independent variable [1]. The present event-ledger design extends that commitment to the specific objects of the GR formation architecture. It

can therefore record not only state transitions but changes in the admissible states, entity continuations, and problem definitions that organize later transitions.

A change-point claim should be tied to evidence. Let Δ_k denote a proposed structural change between adjacent reconstructed segments, and let $D_k^\Delta \Vdash_Q \Delta_k$ mean that the cited evidence is sufficient, relative to problem Q , to distinguish the proposed pre-change and post-change reconstructions. Equation 86 records the minimal support requirement.

$$\text{Claim}(\Delta_k) \implies \exists D_k^\Delta \subseteq \mathcal{D}_c^Q \quad D_k^\Delta \Vdash_Q \Delta_k. \quad (86)$$

Equation 86 does not require experimental identification of a causal effect. It requires enough evidence to justify the analytical distinction being introduced. If the evidence cannot distinguish a genuine structural change from ordinary variation within a stable configuration, the stronger evolutionary claim should remain suspended.

12.4 Comparative Sampling Across Contexts

The comparison matrix developed in Section 7 becomes an empirical sampling design when cases are selected to populate specified spatial, temporal, or spatiotemporal contrasts across same-type or cross-type entities. Case sampling should therefore be driven by the transformation or abstraction claim under investigation rather than by convenience alone.

Let $\mathcal{C}^Q$ denote the available case universe and let $\mathcal{S}^Q \subseteq \mathcal{C}^Q$ denote the sampled set. Equation 87 partitions the sampled cases by analytical role.

$$\mathcal{S}^Q = \mathcal{S}_{\text{gen}}^Q \cup \mathcal{S}_{\text{contrast}}^Q \cup \mathcal{S}_{\text{test}}^Q \cup \mathcal{S}_{\text{failure}}^Q. \quad (87)$$

$\mathcal{S}_{\text{gen}}^Q$ contains cases used to generate candidate abstractions. $\mathcal{S}_{\text{contrast}}^Q$ contains cases chosen because they differ along a theoretically relevant contextual or type dimension. $\mathcal{S}_{\text{test}}^Q$ contains cases withheld, where feasible, from abstraction construction and used to test transfer. $\mathcal{S}_{\text{failure}}^Q$ contains failed, interrupted, negative, or counterexample cases that can reveal whether a proposed generative structure is merely associated with later success.

This partition is conceptual rather than necessarily disjoint in every design. Historical scarcity can make a single case serve more than one analytical role. When that occurs, the manuscript should state the overlap and weaken claims that depend on independent testing. A candidate abstraction developed and tested on the same small case set can still be useful, but its status is exploratory rather than strongly corroborated.

Table 20 translates the six comparison families into empirical design questions.

Scale should be sampled explicitly when the claim is scale-sensitive. A design that observes individual scholars, research groups, and disciplinary institutions should not assume that a pattern reconstructed at one level transfers automatically to another. Multi-scale sampling can reveal whether a purported invariant belongs to an individual practice, a distributed configuration, an institutional routine, or a larger historical field.

12.5 Empirical Tests of Structural Claims

The transition from recurrent pattern to structural claim requires explicit testing against cases, transformations, or evidence not used solely to generate the abstraction. Let A denote a candidate abstraction generated from $\mathcal{S}_{\text{gen}}^Q$. Its empirical test set is $\mathcal{S}_{\text{test}}^Q$. Equation 88 defines the observed test-support domain.

$$\text{TestSupp}_Q(A) = \left\{ c \in \mathcal{S}_{\text{test}}^Q : A \text{ is supported in case } c \text{ under the stated correspondence and scale} \right\}. \quad (88)$$

A test case should challenge some dimension that the proposed abstraction claims to survive. If A is claimed to be spatially invariant, test cases should vary location while preserving the conditions

Table 20: Empirical sampling logic for the six comparison families.

Comparison family	Illustrative sampling structure	Primary empirical question
Spatial, same type	Comparable entities in different contemporaneous locations or institutions	Which relational or practical structures persist across location while entity type remains approximately stable?
Spatial, cross type	Different entity types observed across contemporaneous contexts	Can a deeper relational structure be transported across type differences without erasing those differences?
Temporal, same type	One entity lineage or comparable same-type entities across periods	Which structures persist, transform, or disappear through historical change?
Temporal, cross type	Historically successive or analogous entities whose types differ	Does a transformation law connect practices or relations even when type identity is absent?
Spatiotemporal, same type	Same-type cases varying jointly in location and period	Does the candidate abstraction survive combined historical and contextual displacement?
Spatiotemporal, cross type	Different entity types compared across both location and historical period	Which claims remain transportable under the strongest contextual and type heterogeneity represented in the design?

necessary to interpret the mapping. If the claim is cross-type equivariance, the test should include a justified type translation and examine whether the relevant diagram commutes. If the claim concerns an evolutionary transformation law, the test should examine cases with comparable temporal structure rather than only static endpoints.

Counterexamples require equal visibility. Let $\mathcal{F}_Q(A)$ denote the observed failure set. Equation 89 records cases in which the stated preservation claim fails under conditions that fall inside its declared domain.

$$\mathcal{F}_Q(A) = \left\{ c \in \mathcal{S}_{\text{test}}^Q : c \in \text{Dom}_Q(A) \text{ and } A \text{ fails the declared preservation test in } c \right\}. \quad (89)$$

A nonempty $\mathcal{F}_Q(A)$ does not mechanically invalidate every useful form of A . It can require narrowing the support domain, distinguishing subtypes, revising the correspondence map, or abandoning the stronger invariance claim in favor of covariance, equivariance, or local recurrence. The important empirical obligation is that such revisions remain visible rather than removing difficult cases after inspection.

Symbolization claims can be tested in the same manner. Section 9 introduced a commutation condition between an empirical transformation τ and its formal counterpart $\hat{\tau}$. Empirically, the analyst can compare the two routes on held-out or newly observed cases. Let Σ denote the symbolization map. Equation 90 defines a qualitative commutation residual.

$$\mathfrak{R}_\Sigma(x, \tau) = \text{Disc}(\Sigma(\tau x), \hat{\tau}(\Sigma x)), \quad (90)$$

where Disc is a problem-appropriate discrepancy relation or measure. A numerical metric is required only when the representation and evidence justify one. A persistent residual can identify omitted struc-

ture, an invalid formal transformation, or a symbolization that fails to preserve the empirical distinction relevant to the claim.

The empirical testing logic therefore supports several epistemic outcomes: strengthened support, narrowed domain, transformed claim type, unresolved status, or rejection. This plurality is important because the framework does not treat every candidate abstraction as a hypothesis that must be accepted or rejected in binary form. Some structures can remain useful local representations after stronger transfer claims fail.

12.6 Illustrative Formation Trajectories

The preceding design rules can remain difficult to interpret when they are presented only as methodological obligations. This subsection therefore supplies two stylized analytic illustrations. They are not empirical findings and do not validate the framework. Their purpose is narrower: to show how the same formation architecture can organize a short, high-pressure episode and a longer scholarly trajectory without assuming that the two domains are substantively equivalent. The examples also make visible where problem revision, distributed reason, evolutionary reconstruction, abstraction, symbolization, and recursive evaluation enter an actual case record.

12.6.1 Concurrent-Warning Diagnostic Episode

Consider a stylized high-stakes episode in which a crew receives simultaneous low-speed and overspeed indications. The example deliberately abstracts from aircraft-specific procedures, engineering details, and operational recommendations. Its analytical object is the formation of a diagnostic problem under conflicting information, limited time, and potentially distributed human–artifact cognition.

A weak initial problem statement might be “which warning is correct?” That formulation presupposes that one indication should be selected and the other rejected. A formation analysis keeps the question provisional. The operative problem can be reformulated as a configuration question concerning which combinations of vehicle state, sensing condition, instrument reliability, and information conflict could jointly generate the observed indications, and which distinctions are needed before a defensible action-oriented judgment can be formed. The revision is methodologically important because it changes what counts as relevant evidence before any final diagnosis is reached.

The reason-bearing unit need not be a single individual. Depending on the evidence, it can include multiple crew members, instruments, cross-checking practices, checklists, and other mediating artifacts. The empirical reconstruction would therefore separate the visible decision maker from the configuration that actually generates reasons relevant to the problem. Salience reconstruction would ask which indications were noticed first, which inconsistencies reorganized attention, which prior expectations were suspended, and which sources were treated as more or less reliable. Normative reconstruction would record safety-relevant priorities and procedural obligations without inferring them solely from the final action.

The evolutionary record would preserve temporal order among warning onset, cross-source comparison, problem reformulation, changes in the estimated state, and any branching of candidate diagnoses. A stronger state-space claim would be warranted only if the episode required introducing a possibility that was not represented in the initial diagnostic space. Cross-case comparison could then ask whether a recurrent structure such as conflict recognition, source triangulation, and problem reconstitution survives changes of crew, platform, or informational condition. Such recurrence would remain a candidate abstraction until tested under an explicit transformation family.

Formalization could represent the episode through a diagnostic graph, a partial-order event structure, a probabilistic model, or another representation appropriate to the evidence. Each choice would preserve different features. A scalar confidence score, for example, might compress uncertainty while erasing which source conflicts produced it. The symbolization audit would therefore ask whether the chosen representation preserves the distinctions needed by the problem. Recursive evaluation would then determine

whether the final model failed because of inadequate data, an incorrect entity boundary, an impoverished diagnostic state space, an unsupported abstraction, or the symbolization itself.

12.6.2 Scholarly Problem-Formation Episode

A second stylized case concerns the genesis of a scholarly problem. Suppose a researcher begins with a familiar question such as why an established theory performs poorly in a particular class of cases. During reading, archival work, discussion, or confrontation with anomalous evidence, the researcher comes to suspect that the difficulty lies partly in how the theory constitutes its object. The problem may then shift from explaining a residual within the established framework to reconstructing the historical-relational conditions under which the original object, categories, and explanatory target were formed.

The reason-bearing configuration in this case can include the scholar, interlocutors, prior literature, archives, diagrams, computational tools, institutional routines, and other artifacts that materially organize inquiry. The relevant evidence is correspondingly heterogeneous: dated notes, draft histories, annotations, correspondence, seminar records, model revisions, and later retrospective accounts can each support different components of the reconstruction. A polished publication alone would be insufficient for recovering the generative trajectory because it can conceal abandoned questions, failed mappings, and earlier problem formulations.

Evolutionary reconstruction would track the sequence through which the object boundary, relevant entities, conceptual vocabulary, and available theory space change. Cross-context abstraction could then compare several scholarly trajectories. A recurrent pattern such as anomaly recognition, historical localization, relational comparison, and problem reformulation may become analytically interesting. It should still be recorded with residual differences, because different disciplines can realize the sequence through different evidential practices, norms, and symbolic resources. Cross-type comparison therefore seeks a preserved relational or process structure rather than behavioral identity.

Symbolization can enter late or early in the episode. A graph, algebraic structure, dynamical model, or categorical representation can make a hidden assumption visible and thereby generate a new question. The framework treats that generative effect as part of the formation history. It also records the possibility that the formal representation sharpens boundaries or suppresses historical distinctions that were constitutive of the original problem. Recursive evaluation can therefore return from a mathematically coherent model to an earlier problem formulation if the formal success depends on an unsupported simplification.

Table 21 summarizes how the two stylized cases instantiate the same analytical obligations while retaining different empirical contents.

The two illustrations also clarify what the framework does not supply. It does not provide a domain-specific diagnostic procedure for the first case or a universal recipe for scholarly creativity in the second. It supplies a common set of formation objects whose empirical content has to be reconstructed independently in each domain. This distinction is necessary if cross-domain portability is to remain structural rather than analogical.

12.7 Source-Limited Empirical Contact: Formation of the Recognition-Primed Decision Model

The preceding illustrations were intentionally stylized. A stronger test of empirical interpretability requires contact with a documented formation history in which the proposed analytical objects can be reconstructed only to the extent allowed by the surviving evidence. This subsection therefore conducts a limited pilot reconstruction of the formation of the Recognition-Primed Decision (RPD) model. The purpose is not to validate the Generative Relational meta-theory or to provide a complete intellectual history of naturalistic decision making. It is to test whether the framework disciplines reconstruction when the evidence is partial and retrospectively curated.

The evidential basis is deliberately narrow. Klein's retrospective overview describes naturalistic decision making as an approach that emerged in the 1980s to study decisions in real-world settings and

Table 21: Two stylized formation trajectories used as analytic illustrations.

Formation object	Concurrent-warning episode	Scholarly problem-formation episode
Problem constitution	Reframes an apparent choice between conflicting indications into a configuration-level diagnostic problem	Reframes a residual failure inside an established theory into a question about object constitution and historical-relational assumptions
Reason-bearing configuration	Crew members, instruments, mediating artifacts, and cross-checking practices where supported by the evidence	Scholar, interlocutors, literature, archives, analytical artifacts, and institutional practices where they materially organize inquiry
Evolutionary reconstruction	Preserves warning order, salience shifts, diagnostic branching, and possible expansion of the diagnostic state space	Preserves changes in question, object boundary, conceptual vocabulary, comparison set, and accessible theory space
Cross-context abstraction	Tests whether conflict recognition, triangulation, or problem reconstitution recur under specified contextual changes	Tests whether anomaly recognition, historical localization, relational comparison, or problem reformulation survive cross-case translation
Symbolization audit	Examines whether diagnostic graphs, probabilities, or event structures preserve the information conflict relevant to judgment	Examines whether formal models preserve historical and relational distinctions rather than converting them into unsupported fixed entities
Recursive evaluation	Routes a failure to data, entity boundary, state-space specification, abstraction, or representation as warranted	Routes a failure to the question, historical reconstruction, comparison design, formalization, or substantive claim set as warranted

emphasizes experience, rapid situation categorization, and complex field conditions [8]. More specifically, Klein, Calderwood, and Clinton-Cirocco published in 2010 an edited version of their original unpublished 1985 fireground study, together with a later postscript [9]. The article reports that the study examined highly proficient personnel under extreme time pressure and potentially consequential conditions, used 26 experienced fireground commanders with a mean of 23 years of experience, and probed 156 decision points from nonroutine critical incidents. It further reports that fewer than 12% of those decision points showed evidence of simultaneous comparison and relative evaluation of two or more options, whereas more than 80% involved using experience to identify a situation as typical of a prototype and to identify a typical course of action. The authors state that the RPD model was synthesized from these data and emphasized recognition rather than calculation or comparative analysis for rapid decision making [9].

These sources support only a partial formation record. They provide comparatively strong evidence for the study’s problem target, the selection of experienced fireground commanders as question-relevant participants, the use of critical-incident reconstruction, several recurrent decision patterns, and the subsequent synthesis of an RPD model. They provide much weaker access to the investigators’ own changing problem formulations, abandoned analytical categories, internal disagreements, intermediate coding schemes, and alternative formal representations. Equation 91 therefore represents the case as a partial empirical record rather than a completed formation trajectory.

$$\hat{\mathcal{C}}_{\text{RPD}} = \langle \hat{Q}, \hat{\mathcal{E}}, \hat{\Xi}, \hat{\Omega}, \hat{\mathfrak{B}}, \hat{\mathcal{K}}, \mathcal{U} \rangle, \quad (91)$$

where the hatted components denote source-supported reconstructions and $\mathcal{U}$ records unresolved components that cannot responsibly be inferred from the available publications. Equation 91 is not intended as a new general formal object. It is a case-specific projection of the empirical case record in

Equation 82, introduced to make missing information visible rather than silently completing the trajectory from general knowledge.

The provisional problem can be reconstructed at a useful but limited level. The study sought to understand how highly proficient personnel make decisions under severe time pressure in consequential environments [9]. That problem constitution materially differs from an experimental design that begins by assuming extensive comparison among a supplied option set. The empirical record therefore permits a modest inference that the investigators treated real-time field conditions and experienced performance as constitutive features of the research problem rather than as noise to be removed. Klein's later overview places this orientation within the broader emergence of naturalistic decision making as a programme concerned with real-world decision settings [8]. The surviving sources do not, however, establish a complete sequence of how the investigators themselves revised the question before, during, and after data collection.

The reason-bearing entity selection is more directly recoverable. Experienced fireground commanders were selected because the question concerned rapid proficient decision making in fireground operations. Within the terminology of Section 4, this supports a question-relative entity set rather than a general claim that occupational rank or experience automatically confers epistemic authority. The reported mean experience of 23 years and the focus on nonroutine critical incidents provide evidence that the sample was designed to expose practiced judgment under demanding conditions [9]. The published record does not by itself determine how much of the relevant reasoning was distributed across commanders, crews, instruments, procedures, or organizational structures, so a stronger distributed-configuration claim should remain unresolved in this pilot.

The epistemic-normative-practical reconstruction is likewise partial. Critical-incident interviews supplied retrospective access to what commanders attended to and how they described their decision processes. The reported pattern is important because it challenged an analysis centered on simultaneous comparison among alternatives: the study found little evidence of such comparison and much more frequent recognition of a situation and a typical response [9]. Within the present framework, this can support a candidate process abstraction concerning recognition-based situation assessment. It does not warrant treating retrospective verbalization as transparent access to the full cognitive process. The evidence architecture developed earlier in this section would therefore classify the interviews as retrospective evidence and would seek contemporaneous traces, interaction records, artifacts, or independent outcome evidence before making stronger claims about salience, tacit judgment, or distributed cognition.

The case also illustrates the difference between recurrence and transformation-tested invariance. The repeated recognition pattern across the reported decision points is evidence for recurrence within the sampled fireground cases. It does not by itself establish an invariant across occupations, organizations, historical periods, or other high-pressure domains. Klein's broader account indicates that NDM research subsequently addressed multiple real-world domains [8], but a GR invariance claim would require explicit correspondence maps and transformation families across those cases. The pilot therefore stops at a source-supported candidate abstraction rather than retrospectively promoting the RPD pattern to a universal law of expert decision making.

The symbolization stage is similarly constrained. The published study reports that an RPD model was synthesized from the data [9]. This supports the historical claim that empirical regularities were converted into a model. A representation-bias audit would require the model's successive drafts, coding choices, alternative representations, and evidence about which distinctions were lost or sharpened during synthesis. Those materials are not supplied by the limited source set used here. The appropriate result is therefore unresolved symbolization provenance rather than a fabricated audit.

Table 22 summarizes the pilot. The most informative result is methodological: the framework remains useful even where the record is incomplete because it distinguishes supported reconstruction from missing provenance. Empirical contact does not require every stage to be populated. It requires the analyst to state which stages are evidenced, which are inferred with limited confidence, and which remain open.

This pilot yields a modest but useful result. The framework can organize a documented theory-

Table 22: Source-limited pilot reconstruction of RPD model formation.

Formation object	Source-supported reconstruction	Unresolved or weakly supported component
Problem constitution	Study of proficient decision making under extreme time pressure and consequential field conditions	Detailed sequence of investigator-side problem revision before and during the study
Reason-bearing entities	Twenty-six experienced fireground commanders; nonroutine critical incidents; mean experience reported as 23 years	Extent of distributed reasoning across crews, instruments, procedures, and organizational relations
Situated reconstruction	Retrospective critical-incident accounts; little reported simultaneous option comparison; frequent recognition of a typical situation and course of action	Full salience dynamics, tacit processes, contemporaneous interaction structure, and independent reconstruction of unreported alternatives
Abstraction	Recognition-based pattern recurrent across many probed decision points and used to synthesize the RPD model	Transformation-tested invariance beyond the sampled fireground setting
Symbolization	Published record states that an RPD model was synthesized from the observations	Intermediate representations, rejected models, representational losses, and alternative symbolizations
Recursive revision	Later NDM programme situates the work within a broader real-world decision-research agenda	Detailed version history linking each empirical finding to successive question, model, and scope revisions

formation episode without claiming more provenance than the sources provide. It also exposes what a stronger empirical study would need: dated study protocols, coding records, early drafts, correspondence, rejected interpretations, and evidence about how the RPD representation changed as findings accumulated. The missing materials are analytically informative because they identify the difference between reconstructing a published model’s evidential basis and reconstructing the model’s full generative history.

12.8 Recursive Field Protocols

Empirical research itself can alter the formation record. Interviews can prompt participants to articulate previously tacit distinctions; observation can change behavior; archival discoveries can reframe the initiating problem; and newly constructed visualizations can reveal patterns that were not previously salient. The empirical design should therefore record interventions and analyst-generated artifacts as part of the inquiry history rather than treating them as external to it.

Let $\mathcal{I}_c$ denote the set of research interventions in case c . Equation 92 augments the empirical case record accordingly.

$$\tilde{\mathcal{C}}_c^Q = \langle \mathcal{C}_c^Q, \mathcal{I}_c, \mathcal{V}_c \rangle, \quad (92)$$

where $\mathcal{V}_c$ is the version history of analytical decisions, coding changes, revised case boundaries, and altered formal representations. This record makes it possible to distinguish a pattern observed before an analytical intervention from one that became visible only after the researcher introduced a new representation or question.

A recursive empirical protocol can be organized as four passes. The first pass establishes the provisional problem and case boundary. The second reconstructs entities, evidence, and temporal structure without yet forcing cross-case abstraction. The third introduces comparative and formal analysis. The

Table 23: Illustrative empirical design patterns for theory-formation research.

Design pattern	Typical evidence architecture	Principal analytical strength
Situated formation episode	Concurrent observation, event logs, immediate retrospective interviews, mediating artifacts, and outcome traces	Reconstructs salience, judgment, time pressure, interaction, and local problem reformulation with relatively fine temporal resolution
Historical theory trajectory	Drafts, correspondence, notebooks, publications, institutional records, later interviews, and contextual sources	Reconstructs long-term problem evolution, abandoned branches, relational-historical position, and changes in symbolic resources
Distributed formation system	Interaction records, role/task traces, artifact histories, organizational documents, interviews across positions, and configuration-level outcomes	Locates epistemic contribution across relations and mediators rather than assigning the whole process to one visible actor

fourth performs the recursive evaluation developed in Section 10, routes detected failures to their earliest unsupported dependencies, and determines whether further data collection, reanalysis, branching, or suspension is warranted.

Table 23 summarizes three broad empirical design patterns. These are reusable configurations rather than fixed study types.

The three patterns can be combined. A long historical trajectory can contain several intensively observed episodes; a distributed research system can be studied historically; and contemporary fieldwork can become a future archival record if versioning and provenance are preserved. The framework therefore treats empirical design as compositional, provided that each evidence source is tied to the claim it is used to support.

The output of Section 12 is an empirical design packet consisting of a justified case boundary, a source-indexed evidence corpus, an event ledger, a sampling-role map, declared structural tests, intervention provenance, and a versioned analytical record. Equation 93 collects these components.

$$\mathfrak{D}_Q = \langle \{\mathcal{C}_c^Q\}, \{\mathcal{D}_c^Q\}, \{\mathcal{L}_c^Q\}, \mathcal{S}^Q, \mathcal{T}_{\text{emp}}^Q, \{\mathcal{I}_c\}, \mathcal{V}^Q \rangle. \quad (93)$$

$\mathcal{T}_{\text{emp}}^Q$ denotes the set of declared empirical tests for abstractions, transformation laws, or symbolization claims, while $\mathcal{V}^Q$ records the version history of analytical decisions. The packet is deliberately richer than a conventional data set because the research object includes formation, reconstruction, and revision. It allows another analyst to determine which claims rest on which traces, which parts of the trajectory remain ambiguous, and where later evidence would reopen the analysis.

The empirical programme also clarifies the status of the present paper. The meta-theory supplies a structured design language and a family of testable obligations; it does not yet provide empirical validation of the architecture across domains. Such validation requires studies that use the packet in Equation 93 to reconstruct actual theory-formation trajectories and then test whether the resulting abstractions survive spatial, temporal, spatiotemporal, type, and scale changes. The next section therefore turns from empirical implementation to the broader meta-methodological implications of treating theory formation as a revisable, relational, historical, and partially formalizable research object.

13 Meta-Methodological Implications

This section draws out the meta-methodological consequences of the framework developed in Sections 3–12. Its role is synthetic. It does not introduce a new ontology of inquiry or a universally preferred research method. It examines how methodological choice, epistemic authority, interdisciplinary transfer, formal representation, and theory evaluation change once theory formation is treated as a revisable

historical-relational process. The discussion proceeds from method selection to upstream dependency, then to epistemic authority, cross-domain translation, formalization, and trajectory-level evaluation. The resulting implications specify a research discipline for choosing and revising methods without collapsing qualitative, historical, empirical, and formal inquiry into a single hierarchy. The stylized trajectories in Section 12.6 provide a concrete reference point: the same analytical obligation can be instantiated differently without requiring substantive equivalence between domains.

13.1 Method Selection as a Problem-Relative Research Object

A first implication concerns the status of method choice. In many research settings, methods enter inquiry through disciplinary training, local convention, data availability, or institutional expectation. These routes can be legitimate, yet the present framework requires a more explicit relation between a method and the problem that gives the method its analytical role. The positive applicability profile developed in Section 11 and the empirical design packet developed in Section 12 provide the relevant constraints.

Let $\mathcal{M}$ denote the available methodological repertoire. The admissible methods for a provisional problem Q are defined in Equation 94 as those that satisfy the declared obligations generated by the problem, its applicability profile, and the available evidence.

$$\mathcal{M}_Q^{\text{adm}} = \{m \in \mathcal{M} \mid m \text{ satisfies the declared obligations of } \langle Q, \mathfrak{A}(Q), \mathfrak{D}_Q \rangle\}. \quad (94)$$

Equation 94 deliberately defines a set rather than a scalar ranking. Historical reconstruction, ethnographic observation, experiments, quantitative models, formal proof, interviews, archival work, simulation, and comparative analysis can answer different obligations. Their adequacy can therefore be partially ordered or incomparable rather than reducible to one measure of methodological quality. A method can be highly adequate for reconstructing temporal succession and weak for identifying causal effects; another can reverse those strengths.

The practical consequence is a shift from method-first design toward obligation-first design. The analyst identifies what has to be reconstructed, discriminated, measured, compared, or preserved before deciding which techniques deserve inclusion. A mixed-method design then becomes more than the coexistence of several familiar techniques. It becomes a structured allocation of methods to distinct epistemic obligations.

Method selection is also revisable. If recursive evaluation changes Q , alters the retained entity configuration, or shows that the state space was improperly specified, the admissible set in Equation 94 can change. Methodological consistency therefore does not require keeping the initial technique fixed throughout the project. It requires maintaining a traceable relation between each technique and the claim for which it is used.

13.2 Upstream Constitution and Dependency-Sensitive Research Quality

A second implication concerns where research quality should be audited. Sections 3 and 10 show that a technically competent downstream analysis can inherit an unsupported upstream choice. A precise statistical model, elegant category-theoretic representation, or carefully coded interview corpus can remain misdirected if the initiating problem, entity boundary, or comparison relation is poorly constituted.

Let $\mathcal{G}_F$ denote the dependency graph of a formation record $\mathfrak{F}$. For any downstream object y , Equation 95 defines the set of upstream objects whose adequacy is materially required by y .

$$\text{Up}_{\mathfrak{F}}(y) = \{x \in \mathfrak{F} \mid x \rightsquigarrow y \text{ in } \mathcal{G}_F \text{ and } x \text{ materially constrains } y\}. \quad (95)$$

Equation 95 gives methodological content to the idea of upstream error. A claim concerning an invariant, for example, depends on the comparison cases, the analytical views constructed for those cases, the correspondence relation used to compare them, and the problem-relative features treated as relevant. Validation of the invariant alone cannot repair unsupported choices in those antecedent objects.

Table 24: Dependency-sensitive levels of meta-methodological audit.

Audit level	Primary question	Characteristic failure
Problem constitution	Is the research problem adequate to the phenomenon and intended explanatory or practical target?	Precise work directed at a poorly bounded or misleading question
Epistemic configuration	Are the relevant reason-bearing entities, relations, and mediators appropriately identified?	Authority or visibility substituted for problem-relevant epistemic contribution
Historical reconstruction	Are sequence, continuation, boundary change, and state-space change supported by evidence?	Retrospective coherence imposed on an underdetermined trajectory
Comparative abstraction	Are cases genuinely comparable under the declared correspondence and scale?	Superficial recurrence treated as structural preservation
Formal representation	Does the symbolization preserve the features and transformations required by the claim?	Mathematical precision obtained by deleting a constitutive distinction
Claim stabilization	Do scope, counterexamples, residuals, and reopening conditions support the present claim status?	Provisional stabilization presented as closure

This implication changes peer review and replication as well. Replication of the final computation is valuable, but a fuller audit asks whether the problem boundary, entity selection, temporal reconstruction, comparison design, and symbolization can also be reconstructed from the research record. The empirical packet in Equation 93 was designed partly for this purpose. It permits disagreement to be localized. Two analysts may accept the same data and disagree about the case boundary; accept the same case boundary and disagree about the temporal ordering; or accept both and disagree about the abstraction. Such localization is more informative than treating the project as a single indivisible claim.

Table 24 summarizes the resulting audit levels.

The table does not imply that every study must perform every audit at equal depth. It specifies where a failure can reside. The intensity of each audit remains proportional to the claim being made.

13.3 Epistemic Authority as Relational, Claim-Specific, and Revisable

A third implication concerns expertise and epistemic authority. Section 4 treated reason-bearing status as question-relative and separated it from institutional rank, visibility, and eventual historical success. The meta-methodological consequence is that authority should be attached to a contribution under specified conditions rather than assigned globally to an entity.

For a candidate claim c , let the notation in Equation 96 record that an entity or distributed configuration E_i provides a relevant warrant for c under a problem, time, and relational-historical location.

$$E_i \Vdash_{Q,t,\Lambda_i(t)} c. \quad (96)$$

Equation 96 is a bookkeeping relation rather than a proof of truth. It records the conditions under which the contribution becomes epistemically relevant. The same entity can support one claim and remain irrelevant to another. Its relevance can also change as Q changes or as its relational-historical location changes.

This formulation has consequences for research involving celebrated scholars, domain experts, institutions, and computational systems. Reputation can justify attention, but it does not by itself establish the content of the warrant relation. The analyst still asks what experience, access, practice, interaction, or inferential resource connects the entity to the claim. The same discipline applies to distributed con-

figurations. A team, laboratory, committee, human-machine system, or documentary infrastructure can possess a configuration-level capacity that is not reducible to the most visible participant.

The framework therefore supports a layered epistemic map. Some contributions can concern direct observation; others can concern historical interpretation, technical transformation, normative evaluation, or practical feasibility. Disagreement among reason-bearing entities need not be treated as noise to be averaged away. It can indicate heterogeneous access to the problem or reveal that the problem combines several epistemic tasks that should be separated.

A further consequence is that absence of an adequate reason-bearing entity remains informative. As Section 11 emphasized, failure to identify such an entity under current evidence does not prove ontological absence. It signals that the entity-centered route has not yet earned its role and that the inquiry may need a different methodological route.

13.4 Interdisciplinary Transfer as Structured Translation

A fourth implication concerns interdisciplinary inquiry. The framework gives a disciplined interpretation of transfer across domains. A concept, formalism, or mechanism developed in one domain should not be imported merely because its vocabulary appears suggestive in another. Sections 7 and 8 require an explicit correspondence, a transformation-support domain, and visible residual differences.

For an abstraction A transported from context a to context b , define the transfer record in Equation 97.

$$\mathfrak{T}_{a \rightarrow b}^Q(A) = \langle \Psi_{a \rightarrow b}^Q, \text{Pres}_{a \rightarrow b}^Q(A), \Delta_{ab}^Q, \text{Supp}_{a \rightarrow b}^Q(A) \rangle. \quad (97)$$

Here $\Psi_{a \rightarrow b}^Q$ is the justified structural translation, $\text{Pres}_{a \rightarrow b}^Q(A)$ records what is preserved, Δ_{ab}^Q records relevant residual differences, and $\text{Supp}_{a \rightarrow b}^Q(A)$ records the domain over which the transfer is presently supported. Equation 97 makes partial transfer an ordinary result. A mapping can preserve one relational structure while failing to preserve another.

This is particularly important when formal resources travel between natural science, social science, law, education, political theory, or organizational research. Group theory, field language, dynamical systems, topology, and category theory can provide powerful representations, yet their usefulness depends on the transformations actually warranted in the target problem. Structural similarity can motivate a formal experiment; it does not establish equivalence of the underlying domains.

Failed translation can also be informative. If $\Psi_{a \rightarrow b}^Q$ cannot preserve a distinction required by the target case, the residual identifies where the source formalism ceases to travel. Such failures can generate new theory by exposing the need for a different type system, state space, morphism class, or explanatory scale. Interdisciplinary inquiry then becomes a controlled encounter between structures rather than a search for a universal vocabulary.

13.5 Formalization as Accountable Epistemic Intervention

A fifth implication concerns the role of mathematics and symbolic systems. Section 9 treated symbolization as a partial map whose adequacy depends on feature preservation and transformation fidelity. Meta-methodologically, this means that formalization should be documented as an intervention in inquiry rather than presented as a neutral transcription of an already completed conceptual analysis.

Let $\mathfrak{S}_Q$ denote the portfolio of admissible symbolizations currently under consideration. Equation 98 records the portfolio together with the claims for which each representation is licensed.

$$\mathfrak{S}_Q = \{(\Sigma_j, \mathcal{C}_j, \mathcal{L}_j, \mathcal{B}_j)\}_{j \in J}, \quad (98)$$

where $\mathcal{C}_j$ is the claim set supported by representation Σ_j , $\mathcal{L}_j$ records known representational loss, and $\mathcal{B}_j$ records identified representation-induced bias. Equation 98 permits several formalizations to coexist. One representation can preserve topology, another temporal ordering, and another probabilistic

uncertainty. Their coexistence is methodologically preferable when no single representation preserves every feature needed by the research problem.

This implication also constrains mathematical elegance. Simplicity, symmetry, compactness, and tractability can be epistemically productive because they disclose candidate relations and permit derivation. They remain criteria internal to a larger adequacy judgment. A beautiful representation that systematically erases historical continuation or forces heterogeneous entities into an unjustified common metric should have its claim domain narrowed accordingly.

Formalization can nevertheless generate knowledge rather than merely display it. A symbolic representation can reveal a missing transformation, identify an inconsistency, produce a new invariant candidate, or make a previously implicit assumption visible. The resulting conjecture then returns to the empirical and historical record. This cycle gives formal reasoning a generative role while preserving the distinction between a mathematically coherent construction and an empirically warranted account.

The same principle applies when formalization fails. If every available Σ_j in Equation 98 destroys a constitutive distinction, retaining a qualitative or partially formal reconstruction can be the stronger methodological result. Formal incompleteness is preferable to false precision.

13.6 Theory Evaluation as Trajectory-Level and Versioned Judgment

A sixth implication concerns what counts as the object of theory evaluation. The conventional object is often the stabilized theory or model. The recursive architecture developed in Section 10 adds the formation trajectory itself. A theory can be evaluated for empirical adequacy or explanatory value while its pathway of constitution is also evaluated for unsupported exclusions, unstable boundaries, hidden transformations, and representational distortions.

For version k of a project, Equation 99 defines a claim package that retains provenance and reopening information.

$$\mathfrak{P}^{(k)} = \langle \mathcal{K}_{\text{th}}^{(k)}, \mathcal{S}^{(k)}, \mathcal{R}^{(k)}, \mathcal{X}^{(k)}, \mathcal{O}^{(k)} \rangle, \quad (99)$$

where $\mathcal{K}_{\text{th}}^{(k)}$ is the substantive claim set already carried by the formation record in Equation 64, $\mathcal{S}^{(k)}$ their evidential support, $\mathcal{R}^{(k)}$ their stated scope, $\mathcal{X}^{(k)}$ the unresolved residuals or counterevidence, and $\mathcal{O}^{(k)}$ the conditions under which the package should be reopened. Equation 99 turns revisability into an inspectable property of the research product.

The trajectory-level view also changes the meaning of negative results. Failure to confirm a proposed invariant can still improve the theory by narrowing the transformation family. Failure to identify a stable entity can reveal a distributed configuration. Failure of symbolization can expose a missing qualitative distinction. Dissolution of the initiating question can represent progress when the inquiry establishes that the original partition was misleading. These outcomes are difficult to classify as success or failure when the final theory alone is used as the unit of evaluation.

Versioning further supports cumulative scholarship. A later study can inherit a claim together with its support domain, residuals, and reopening conditions rather than inheriting only the polished conclusion. This makes disagreement more precise and allows future work to target the weakest dependency in the earlier formation record.

13.7 A Meta-Methodological Discipline for Revisable Inquiry

The implications developed above can be summarized as a discipline of traceable conditionality. Research questions remain provisional; methods are selected against explicit obligations; epistemic authority is claim-specific and relationally located; interdisciplinary transfer records preserved structure and residual difference; formalization carries a representation audit; and theory evaluation retains formation history and reopening conditions.

Table 25 summarizes the principal shifts without treating them as universal replacements for existing methodology.

Table 25: Principal meta-methodological implications of the proposed framework.

Research object	Meta-methodological implication	Operational consequence
Method choice	Adequacy is problem-relative and obligation-specific	Record which method supports which analytical obligation and revise the portfolio when upstream objects change
Research quality	Downstream rigor depends on upstream constitution	Audit the earliest unsupported dependency rather than repairing only the visible downstream failure
Expertise and authority	Epistemic relevance is claim-specific and relationally located	Separate reputation or role from the evidence that connects an entity or configuration to a claim
Interdisciplinary transfer	Transfer is a structured translation with a support domain and residuals	Preserve partial correspondence and record failed translations rather than asserting broad equivalence
Formalization	Symbolization is a generative but potentially lossy intervention	Maintain representation-specific claim scopes, bias audits, and alternative formalizations where needed
Theory evaluation	The evaluand includes the versioned formation trajectory	Publish scope, support, residuals, counterexamples, and reopening conditions together with substantive claims

The framework therefore supports methodological pluralism without methodological indifference. Different methods can remain partially irreducible while still being accountable to shared requirements of traceability, scope discipline, evidence linkage, and revisability. It also supports formal ambition without formal imperialism. Mathematical structures can become central when they preserve and clarify the reconstructed phenomenon, and they can remain local or absent when the evidence does not warrant them.

The resulting stance is intentionally demanding: the additional formation record is justified only when reconstructing the generation of questions, entities, relations, practices, abstractions, or representations materially improves the inquiry. Under those conditions, formation history becomes part of the epistemic object rather than merely background to it.

14 Conclusion

This section closes the paper by consolidating the proposed meta-theoretical architecture, clarifying its present epistemic standing, and specifying a limited programme for subsequent research. Its role is synthetic. The argument developed across Sections 3–13 has already supplied the definitions, formal constructions, empirical obligations, and scope conditions on which the conclusion depends. The present section therefore introduces no independent explanatory mechanism. It identifies what the framework presently contributes, which claims remain provisional, and which empirical and formal tasks would be required before a stronger general account of theory formation could be warranted.

14.1 The Formation Architecture in Consolidated Form

The paper treats theory formation as a trajectory rather than a final theory considered in isolation. Stage 0 asks whether the complete architecture is appropriate for the problem. Stages 1–4 then constitute the provisional problem, identify and locate question-relevant reason-bearing entities or configurations, reconstruct their epistemic–normative–practical organization, and recover the evolution of states, relations,

boundaries, types, and, where warranted, the available state space. Stage 5 compares reconstructed cases and tests candidate abstractions under supported transformations. Stage 6 formalizes selected structures while auditing representational loss and bias. Stage 7 evaluates the resulting claim set and routes failures to the earliest materially unsupported dependency.

The contribution lies less in any one component than in their coordination. The framework makes several objects that are often treated as background conditions available for explicit revision: the research question, the epistemically relevant configuration, historical continuation, comparison maps, transformation families, symbolic representations, and the dependency structure of evaluation. It also allows the theory space itself to change when new concepts, instruments, or distinctions alter what can be represented or compared.

This coordination explains the differentiated formal vocabulary developed in the paper. Groups, semigroups or monoids, groupoids, and general categories are available only when the reconstructed transformations warrant their respective assumptions. Likewise, symbolization is treated as a productive but potentially lossy intervention. The framework therefore permits multiple partial representations, qualitative remainders, and formalization failure when a stronger formal model would manufacture unsupported precision.

14.2 Epistemic Standing and Scope

The present manuscript presents a meta-methodological proposal, not an empirically validated general theory of inquiry. Its formal objects organize analytical obligations and revision pathways; they are not yet an empirically parameterized model of theory formation across domains. The stylized trajectories in Section 12.6 illustrate how the architecture can be interpreted, while Section 12 specifies what comparative studies would be required for validation.

Applicability is problem-relative. The full architecture is intended principally for cases in which problem constitution is itself consequential, historically situated entities or configurations materially contribute reasons, practical or tacit organization matters, evolutionary reconstruction changes the explanation, and cross-context comparison can support meaningful structural claims. Other problems can use a partial route or another methodology. An empty retained entity set records a present evidential limitation rather than proving that no relevant entity exists, and a distributed configuration should be introduced only when cross-boundary dependencies are materially supported.

The principal methodological risk is retrospective flexibility. A framework that allows questions, boundaries, state spaces, transformations, and representations to change can accommodate almost any history if those changes are introduced only after observing the outcome. The manuscript therefore requires traceable evidence for revisions, explicit support domains for structural claims, visible counterexamples and residuals, and versioned records of analytical intervention. The same discipline applies to invariance, cross-domain transfer, and formalization: each claim remains indexed to the transformations, scales, types, and representations for which support has actually been established.

14.3 A Research Programme for Comparative Validation

The source-limited RPD reconstruction in Section 12.7 provides an initial empirical-contact exercise: within the sources used here, it separates evidenced stages from unresolved provenance and indicates that the published model and retrospective account do not by themselves support reconstruction of a full generative history. The most immediate empirical task therefore remains comparative validation of the formation architecture. Section 12 proposed situated formation episodes, historical theory trajectories, and distributed formation systems as three reusable design patterns. A first programme could select cases that differ in domain, temporal scale, information conditions, and organization while preserving enough comparability to test whether the proposed analytical objects can be reconstructed without forcing the evidence into the framework.

Such studies should examine at least four issues. First, they should test whether explicit problem-constitution records improve explanatory reconstruction of theory development relative to analyses that

begin from the final published question. Second, they should test whether question-relative reason-bearing selection identifies epistemically important entities or configurations that prestige-based or role-based sampling would omit. Third, they should determine whether the distinction among state change, structural change, and state-space change improves the reconstruction of historical trajectories. Fourth, they should test whether candidate abstractions and transformation laws survive held-out spatial, temporal, spatiotemporal, type, or scale comparisons.

The comparative programme should include failure cases. A study in which the full architecture adds little beyond a simpler method would help delimit the applicability profile. A case in which no stable reason-bearing configuration can be reconstructed would test the framework's unresolved-entity route. A case in which several materially different historical reconstructions remain compatible with the evidence would test branching and suspension. A case in which a candidate invariant fails under a held-out transformation would test support-domain revision. A case in which formalization consistently destroys a constitutive distinction would test the admissibility of formalization failure. These outcomes would refine the framework even when they do not confirm its stronger aspirations.

A second research direction concerns identity through evolution. The present manuscript introduced continuation relations that permit one-to-one, one-to-many, many-to-one, and unresolved succession. Further work can investigate when two temporally separated configurations should be treated as continuations of one GR entity, when a type change creates a new analytical object, and when continuity is better represented at the level of a relation, process, or distributed configuration. This problem connects the present framework with the broader GR work on equivalence, topology, and history-bearing entities, while remaining an open formal and empirical problem in the current paper.

A third direction concerns transformation structure. The present hierarchy of groups, semigroups, monoids, groupoids, and categories should be treated as a family of candidate representations rather than a finished algebra of inquiry. Comparative empirical work can establish which composition rules, inverse conditions, local domains, and cross-type mappings are actually encountered in theory-formation trajectories. Formal work can then ask whether useful invariants, equivariants, functors, or natural correspondences emerge under those empirically constrained structures. The order of work is important: the observed transformation structure should constrain the algebraic representation, and the representation can then generate new empirical questions.

A fourth direction concerns symbolization audits. The representation-bias taxonomy in Section 9 is presently a methodological proposal. Future studies can compare alternative symbolizations of the same reconstructed case and assess which features each representation preserves, loses, or distorts. Such comparison would help determine whether the transformation-fidelity condition functions as a practical diagnostic and whether plural representations improve theoretical reliability relative to premature unification.

A fifth direction concerns research infrastructure. The framework generates a large provenance burden because it asks researchers to retain question versions, entity-selection rationales, temporal evidence, comparison maps, counterexamples, representation audits, and reopening conditions. Software support could make this burden tractable. A versioned formation ledger could connect claims to evidence, entities, transformations, symbolizations, and revision decisions. Such tooling would not validate the meta-theory, but it could make the proposed discipline of traceable conditionality operational enough to test in collaborative research.

Table 26 summarizes these immediate research directions and the principal evidence that would bear on them.

14.4 Closing Synthesis

The paper's central proposal is that theory formation can itself become a structured object of inquiry when the research problem is generated within historically evolving relations. Under such conditions, the path from a question to a theory contains epistemically consequential choices about what counts as the problem, which entities possess relevant reasons, how their practices are reconstructed, which historical

Table 26: Immediate research programme for evaluating and extending the proposed meta-theory.

Research direction	Primary analytical question	Relevant evidence or test
Comparative validation	Does the full formation architecture improve reconstruction where its positive scope conditions are present?	Cross-domain and held-out cases, simpler-method baselines, explicit failure cases, and versioned formation records
Identity through evolution	Which continuities justify treating historically transformed configurations as the same, successor, split, merged, or unresolved GR entity?	Longitudinal traces, continuation evidence, boundary and type-change records, and competing identity reconstructions
Transformation structure	Which algebraic representation is warranted by observed composition, reversibility, locality, and cross-type transport?	Empirically reconstructed transformation families, counterexamples, and tests of invariance or equivariance
Symbolization audit	Which formal representations preserve the structures required by the problem, and where do they introduce loss or distortion?	Parallel formalizations, transformation-fidelity tests, residual analysis, and qualitative remainder records
Research infrastructure	Can formation provenance be retained without making the framework impracticably burdensome?	Versioned ledgers, collaborative tooling, reproducibility studies, and user-level burden assessment

continuities are retained, what comparisons are legitimate, which structures are abstracted, and how those structures are symbolized. A final theory records only part of that process.

The Generative Relational framework developed here makes that formation history inspectable. It treats the question as revisable, epistemic entities as relationally located, historical identity as reconstructable, abstraction as transformation-sensitive, symbolization as generative and lossy, and evaluation as recursive. The framework also permits its own partial use, suspension, failure, and replacement. These features are intended to preserve the openness required by the object it studies.

The resulting meta-theory remains preliminary. Its present value lies in the coherence of the analytical architecture, the explicitness of its scope conditions, and the empirical obligations it creates. Stronger claims depend upon comparative studies that test whether the architecture reveals recurrent structures of theory formation, whether those structures survive transformations beyond their generative cases, and whether the added reconstruction burden produces explanatory gains that simpler approaches do not provide.

If those tests are productive, theory formation can be studied as an evolving relational process whose questions, epistemic configurations, abstractions, and symbolic resources co-develop through time. If the tests fail, the same framework requires its own revision, narrowing, branching, or abandonment at the unsupported dependency. That recursive openness is therefore part of the proposal's methodological content. A meta-theory of revisable inquiry should remain revisable under the evidential standards it asks other theories to satisfy.

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