

A Preliminary Metaphysics of Power in Generative Relational Theory: On the Conditions under Which Relational Asymmetry Becomes Power-Relevant

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Abstract

Generative Relational Theory (GR) treats relational beings as history-bearing structures whose current form, efficacy, and possible continuation depend on relational histories rather than on isolated point-like attributes. This paper develops a preliminary metaphysics of power within that framework. Its starting point is a typed relational asymmetry that has already acquired relational reality; the central problem is the additional structure required for such an asymmetry to become power-relevant. The proposed account locates that transition in directionally organized dynamical consequence: a relational asymmetry becomes power-relevant when the relational position of one GR entity contributes systematically to deformation of the accessible, feasible, or comparatively weighted future possibilities of another GR entity or of a shared relational configuration. Relational power is then distinguished from a transient power-relevant episode by the conditions through which the power structure itself acquires expected persistence, causal efficacy, and generative efficacy across admissible historical continuations. Attractor-like and metastable regimes, perturbation response, history weights, closure structures, and operator representations are introduced as candidate modeling resources under continued ontological suspension. The paper further distinguishes relational power from its context-dependent manifestations and examines how symbolic, institutional, embodied, and other mediation can make power epistemically accessible while also participating in its reproduction. One symbolic route is resolved into an historically generated equivalence relation, a quotient representation that suppresses selected relational differences, and subsequent symbolization; the equivalence relation itself can remain analytic, can acquire an operative role in manifestation reality, and can additionally acquire relational reality when its persistence and efficacy are established. Comparative relations such as “more powerful than,” together with associated rankings or metrics, are likewise treated as historically generated structures whose roles can overlap: a comparison can remain proto-relational while its relational status is unresolved, can become stabilized in manifestation reality, and can additionally acquire relational reality when the comparison structure becomes sufficiently persistent, causally efficacious, and generatively consequential. A later case study reconstructs realist balance-of-power reasoning as a conditional dynamical account in which coupled power deformations can generate a metastable balance-like regime and, under specified conditions, sustain expected proximity to a near-critical corridor. The resulting account is an asymmetry-derived construction of directional relational power; reciprocal power can be represented by coexisting directional components, while other genealogies remain open. The objective is representational and metaphysical within GR’s layered reality framework; it does not identify any single mathematical representation with ultimate ontology.

Keywords: Generative Relational Theory; relational power; relational asymmetry; manifestation reality; generative dynamics

Preliminary Research Note

This paper belongs to the broader Generative Relational (GR) research programme. It develops a limited metaphysical and formal question concerning power. GR permits relational asymmetries to become historically stable and generatively consequential structures. The present inquiry asks which additional conditions allow such asymmetries to acquire power relevance, how the dynamical organization associated with a power-relevant asymmetry can acquire relational-reality status as a power structure, and how relational power can subsequently appear, reproduce, or transform through manifestation and mediation.

The paper adopts ontological suspension as a methodological discipline. GR entities are treated here as history-bearing relational spacetime structures. A labeled 2-complex is one candidate representation of such a history-bearing structure, and spin-foam constructions provide one possible instantiated model of a 2-complex architecture where their structural resources are useful. These representations do not establish that ultimate social, political, or physical ontology is literally a 2-complex, a spin foam, an attractor, a field, or an operator algebra. The paper instead studies the conditional consequences of these representations while leaving stronger ontological commitments open.

The paper also distinguishes several analytical statuses. Relational reality, manifestation reality, proto-relational formal status, and mathematical representation perform different roles in the argument. A relational power structure can be modeled through dynamical, field-like, operator, categorical, or attractor-based resources without being identified with any one of those resources. A manifestation of power can also become generatively consequential and, under further historical conditions, participate in the formation of later relational reality. Comparative relations and metrics are treated with the same discipline: their existence as analytical or symbolic structures does not by itself establish relational reality, while sufficient historical persistence and type-appropriate relational efficacy can make them parts of the relational system they initially described. For the power-related structures formalized in this manuscript, the stricter criteria specified in the relevant definitions continue to apply.

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Notices

This page records the manuscript's publication status, licence, development disclosure, and relation to the broader Generative Relational research programme.

Status. This preliminary discussion paper develops an early metaphysical and formal treatment of power within Generative Relational Theory. Definitions, mathematical constructions, categorical representations, dynamical hypotheses, and the proposed relations among relational asymmetry, power relevance, relational power, manifestation, and self-reproduction remain open to revision. The paper does not present a complete general dynamics of power and does not select a final ontology of power, spacetime, agency, or social structure.

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Statement on the use of language models. The exploratory discussions and preparation of this paper involved OpenAI's ChatGPT. ChatGPT supported exploratory dialogue, formal reconstruction, argumentative criticism, drafting in $\text{\LaTeX}$, and technical checking. The author selected the research questions, directed and approved the theoretical commitments and epistemic status of the claims, and bears sole responsibility for the manuscript, including its definitions, formal constructions, arguments, conclusions, and errors. Authorship credit remains with the human author.

1 Introduction

This section establishes the analytical problem, metaphysical scope, and methodological architecture of the paper. Its role is to identify the point at which an already relationally real asymmetry becomes a candidate structure of power, to distinguish that transition from the later stabilization of power as relational reality, and to locate manifestation, comparison, and mathematical representation within the same Generative Relational framework. The discussion proceeds from the type of GR entities and the status of relational asymmetry to directional dynamical consequence, relational realization of power, epistemic access through manifestation, reflexive comparison structures, and self-reproducing power dynamics. The argument is constructive and conditional: its purpose is to specify a coherent architecture in which stronger dynamical or empirical models can later be inserted without assigning one preliminary formalism the status of ultimate ontology.

Generative Relational Theory (GR) approaches an entity through the history of relations that constitutes and sustains its effective existence, the relations that organize its present configuration, and the possible continuations that remain available from that configuration. A GR entity is therefore not treated as a point-like bearer of a fixed list of intrinsic attributes. The relevant analytical unit is a history-bearing relational spacetime entity. Several such entities, together with cross-entity relations and retained environmental or institutional structures, can be represented within a coupled historical configuration. The manuscript therefore reserves *entity* for the history-bearing relatum and *configuration* for the wider modeled state in which one or more such entities participate. Depending on the local problem, such a structure can be represented through several mathematical languages. A labeled 2-complex supplies one candidate model in which events, relations, higher-order cells, boundaries, labels, and historical attachments can be represented without reducing the entity to a single node. A spin foam can provide a more concrete instantiated model of a 2-complex architecture where an application requires its boundary-history and locally-weighted structure [1]. These choices remain representational. GR suspends any inference that the ultimate ontology of a person, institution, state, symbolic order, or political system is thereby fixed by the mathematical model used to study it.

The same discipline applies to time. The present paper does not assume a primitive external scalar parameter that moves a relational entity from one state indexed by t to another indexed by $t + 1$. Historical succession is represented through events and model-relative precedence relations within the history-bearing structure. A relational slice can index a modeled stage, and a downward-closed historical domain can retain the events represented as preceding or supporting that stage. Ordinary clock coordinates remain available when an empirical application requires dates, durations, or rates, while the formal architecture of the paper begins from event order and historical extension. This choice matters for power because a present relational position can depend on an extended history that is invisible in a snapshot, and because a later transformation can alter the significance or continuation of earlier relations without erasing their represented occurrence.

GR distinguishes ontological reality, relational reality, and manifestation reality. Ultimate ontological reality remains suspended in the present inquiry. Relational reality concerns structures that acquire sufficient historical persistence together with type-appropriate relational efficacy to participate in the continuing organization of relational spacetime. Depending on the structure under study, that efficacy can be causal, generative, or both. For the asymmetry-derived power structures developed in this paper, Section 4 adopts the stricter operational requirement of both causal and generative efficacy. Manifestation reality concerns the context-dependent forms through which a relational, proto-relational, or otherwise modeled structure becomes presented, recognized, measured, symbolized, embodied, or epistemically accessible. These statuses are analytical roles rather than disjoint worlds. A structure can retain a manifestation role after acquiring relational reality, and a manifested candidate can remain proto-relational while its relational-reality status is unresolved. Relationalization therefore adds a relational-reality status under stated historical conditions; it does not require the manifestation role to disappear. Mathematical objects such as operators, attractors, basins, closure structures, metrics, and categories occupy a further modeling role. They represent selected aspects of relational or manifestation structure; they do not constitute an additional layer of reality and do not receive an ultimate ontological interpretation by default.

Several established traditions in the study of power motivate distinctions that remain analytically separate in the present construction. Dahl gives an explicitly relational formulation of power and treats comparability as a further problem; Lukes surveys and extends disputes over how power should be conceptualized; Foucault approaches power through relations, points of application, and resistance; Bourdieu emphasizes the symbolic mediation and reproduction of social relations; and Barnett and Duvall distinguish compulsory, institutional, structural, and productive forms of power in international politics [5, 10, 7, 3, 2]. These traditions are not treated as a single lineage or translated into GR by stipulation. Their coexistence supports the narrower methodological choice made here: directional relation, structural reproduction, manifestation, and comparison should remain distinguishable until an explicit mapping between them has been established.

Remark 1.1 (Formal provenance under ontological suspension). Because ultimate ontology is suspended, a nontrivial formal symbol in this paper is never claimed to be derived directly from ultimate ontological reality. Each formal object must instead have an explicit provenance within the declared model: it is either (i) part of a model carrier or antecedent structure stated as an assumption, (ii) constructed from previously typed objects by a defined map, relation, quotient, restriction, or composition, (iii) produced as a context-dependent manifestation or epistemic readout, or (iv) introduced as an auxiliary mathematical representation whose adequacy requires separate justification. Whenever a later construction strengthens one of these statuses, the additional assumptions are stated at the point of strengthening.

Remark 1.1 governs the derivation discipline of the manuscript. In particular, notation such as a comparison relation, operator lift, metric, attractor-like regime, quotient, or categorical morphism acquires no substantive content from notation alone. Its domain, codomain, construction rule, and reality or modeling status must be supplied before the symbol is used inferentially.

The immediate antecedent of the present analysis is relational asymmetry. GR requires a stronger object than bare difference. Two history-bearing entities can differ in numerous properties while remaining exchangeable with respect to a specified relational transformation or invariant. Conversely, two entities with similar observable manifestations can occupy different relational positions once historical dependence, operator availability, standing, resource access, conversion interfaces, or future reachability are represented. A typed relational asymmetry therefore depends on an articulated relational dimension and on the conditions under which the corresponding comparison is meaningful. Such asymmetry may arise after symmetry reduction, may be primitive relative to the selected domain, or may become articulated from conditions in which the symmetry predicate itself was previously only partially applicable. The present paper requires only the resulting interface: a typed asymmetry has already become a sufficiently persistent and consequential part of relational reality.

That starting point is deliberately stronger than an observational judgment that two entities appear unequal. A typed relational asymmetry can be represented as relationally real only when the relevant difference participates in the historical organization of the system rather than appearing solely as a transient readout. An access asymmetry, for example, can become relationally real when it is repeatedly reproduced in the active structure of a system, changes which transformations are available to the differently positioned entities, and thereby affects subsequent histories. A classification asymmetry can acquire similar status when it persistently changes standing, recognized claims, admissible procedures, or other generative possibilities. Relational asymmetry in this sense is already a structured component of the relational system. The central problem of the present paper begins one step later.

The construction developed below is deliberately directional and asymmetry-derived. It asks when an antecedent asymmetry from A toward B supports a relational-power component $\mathcal{P}_{A \curvearrowright B}^{\mathcal{R}}$. This scope does not commit GR to the claim that every possible form of power has the same genealogy. Reciprocal or approximately symmetric power relations can be represented by two directional components, while other routes to power remain open outside the present construction.

Relational asymmetry alone does not provide a sufficient account of power. An asymmetry can be large under one description while remaining dynamically peripheral to the continuation under study. Two entities can possess different knowledge, resources, histories, or symbolic positions without that difference

materially reorganizing one another's future possibilities. The relation becomes relevant to a theory of power when the asymmetry enters the generative dynamics of the coupled relational system. The present inquiry therefore places *directional dynamical consequence* between relational asymmetry and relational power. The key question is whether the relational position of one history-bearing GR entity contributes systematically to deformation of the accessible, feasible, or comparatively weighted continuations of another GR entity or of the shared configuration in which both participate.

Perturbation provides one way to investigate this consequence without assuming in advance that power is a scalar possession. A controlled change to the relational position, support structure, or admissible operation of one GR entity can be used as an epistemic probe. The resulting response may appear in changes to reachable histories, active relations, operator domains, historical weights, institutional standing, symbolic recognition, or other features of the coupled configuration. The paper treats breadth, depth, persistence, robustness, and generativity as possible descriptors of such response profiles. These descriptors do not constitute a universal coordinate system of power and do not supply an automatic aggregation rule. Their analytical role is to make different forms of dynamical consequence explicit before a comparison norm is introduced. A perturbation analysis can therefore reveal how a relational position participates in the dynamics while leaving open the further question of how a historically situated system interprets, compares, or ranks those consequences.

This distinction is necessary because the comparative structure expressed by statements such as “*A* has more power than *B*” is itself generated within a historical and relational setting; it is not assumed to be relationally real at the moment of its introduction. A comparison requires some structure that specifies which manifestations, consequences, scales, histories, and dimensions count as relevant to the comparison. The resulting comparison structure can remain proto-relational while its relational-reality status is unresolved, can be stabilized in manifestation reality through titles, rankings, metrics, or institutional categories, and can additionally acquire relational reality when its historical persistence and required efficacy are established. Manifestation and relational-reality roles can coexist. A symbolic hierarchy such as an office title can initially present a comparison of authority. A statistical measure such as a national output indicator can initially provide one manifested basis for comparing states. If such structures become embedded in institutional decision making, resource allocation, diplomatic expectations, or access to consequential operations, the comparison structure participates in generating the relational world that it purports to describe. Power comparison is therefore treated as a separate problem from power itself.

The transition from power relevance to relational power requires a further condition. A single episode can produce a strong directional effect without forming a persistent power structure. Relational power, as developed here, requires the power-relevant dynamical organization generated around the antecedent asymmetry to acquire the kind of historical endurance and efficacy that GR uses to characterize relational reality. The paper expresses this requirement through three connected ideas: expected persistence across admissible historical continuations, causal efficacy within the represented relational system, and generative efficacy in shaping later relations or possibilities. Expected persistence is intentionally broader than convergence to a fixed point or a limit cycle. A relational power structure can be metastable, can remain sensitive to perturbation, and can persist across a family of branching continuations without determining a unique future. The mathematics of attractor-like regimes, basins, closure structures, and weighted histories is introduced later as a set of candidate representations for this status rather than as an assertion that relational reality is fundamentally a dynamical system of one prescribed type.

This formulation also preserves a role for agency and contingency. A family of admissible continuations can contain several trajectories with different weights or tendencies, and an agent can participate in selecting among available continuations without the formalism identifying the history weight with an exhaustive theory of choice. The relevant expectation-like structure therefore describes a tendency of the modeled relational system. It does not establish a deterministic law of development and does not derive a metaphysics of free will from probability. The distinction allows power to be studied as a structure that changes the relative accessibility of futures while leaving the actual continuation historically contingent.

Once relational power has formed, its dynamical role can exceed local trajectory deformation. A power structure can change the organization of the generative landscape itself. It can make some operations easier

to execute, remove or narrow competing operations, enlarge the set of histories compatible with a particular relational regime, or change the relative weights assigned to alternative continuations. In an operator representation, a relational power structure can be lifted to a transformation acting on a suitable space of histories, observables, or weights. The operator is a representation of the power structure's dynamical action; it is not the ontology of power. This separation permits operator-algebraic tools to analyze composition, state-dependence, and order sensitivity while preserving the distinction between a relationally real power structure and the mathematical machinery used to model its effects.

The resulting landscape deformation creates a route to self-sustaining power. A relational power structure can favor histories that enter a regime in which the same power structure receives stronger institutional support, wider access to consequential operations, greater symbolic legitimacy, or more durable resource control. The resulting regime can then reproduce or extend the conditions through which the power structure remains efficacious. Self-reproduction is treated as a higher-order property rather than as a necessary condition of every instance of power. Short-lived power relations can remain analytically meaningful, while historically durable power structures can be studied through the additional mechanisms by which they stabilize, expand, adapt, or transform the supports of their own continuation. This formulation also permits persistence of an abstract relational organization across changes in its institutional or symbolic manifestation.

Manifestation occupies a central position in this process because relational power is not epistemically transparent. Observers ordinarily encounter titles, commands, deference, silence, exclusion, prestige, legal standing, rankings, resource distributions, institutional responses, and other context-dependent forms. One relational power structure can support several such manifestations, and similar manifestations can arise from different underlying relational structures. The inference from manifestation to relational power is therefore underdetermined. One important symbolic route can be decomposed further: an historically generated equivalence relation treats selected relational structures as equivalent for a declared purpose, a quotient representation suppresses the differences outside that equivalence, and symbolization presents the resulting class through a title, category, credential, or other sign. The equivalence relation is itself a structure whose status within the GR analysis must be established separately. An analytic equivalence can remain at the modeling level; a socially enacted equivalence can possess a manifestation role; and a persistent, causally and generatively efficacious equivalence can additionally acquire relational reality while remaining manifested through symbols or institutional practices. Symbolic and institutional mediation can therefore do more than present an already existing power relation. A manifested hierarchy can alter expectations, recognition, standing, resource allocation, or the domains of later operations. Under those conditions manifestation becomes a generative interface. A structure that first appears at the manifestation level can, over an extended history, contribute to the formation of later relational reality when the type-appropriate persistence and efficacy conditions for that later structure are independently satisfied.

The paper develops this architecture before turning to an international-political case. Realist balance-of-power reasoning provides a demanding test because it often describes international structure through distributions or comparisons of capabilities. Within GR, a state is represented as a history-bearing relational spacetime rather than as a point carrying a single power quantity. Capability indicators become context-dependent manifestations or comparison structures extracted from a richer relational configuration. A state's relational power can then be studied through the way its position deforms the future possibility structures of other states and of the coupled international system. Balancing responses can be represented as counter-deformations generated by other relational powers. Under specified conditions, their coupled dynamics may support a metastable balance-like regime in which durable capture by a single attractor is inhibited while the system retains sufficient persistence, susceptibility, and reconfigurability for counter-response to remain consequential.

This reconstruction creates a conditional interface with the criticality hypothesis of international politics. A balance-like regime need not correspond to numerical equality among states or to a static equilibrium. One possible GR representation treats balance as expected persistence within a transition-sensitive corridor in which competing power deformations prevent complete confinement to a single effective attractor. Such a representation is a hypothesis about a class of dynamical regimes, not a universal account of

international politics. Balancing can fail when counter-operations become unavailable, when one power structure reshapes the landscape sufficiently to make a competing regime self-sustaining, or when perturbations move the system into fragmentation or another configuration. The case study therefore serves as a test of the proposed power architecture rather than as evidence that every international system exhibits critical balance.

The paper develops the argument in stages. The next section reconstructs the minimum antecedent structure required to treat typed asymmetry as relationally real within history-bearing GR spacetime. The analysis then introduces directional dynamical consequence and perturbative inquiry as the bridge from relational asymmetry to power relevance, before examining expected persistence, relational efficacy, and attractor-like representation in the formation of relational power. Subsequent sections address manifestation and epistemic access, the emergence and possible relationalization of comparison structures and metrics, and symbolic, institutional, embodied, and related mediation. A later categorical section organizes these objects within a common typed architecture. The dynamical analysis then develops operator representations of landscape deformation and the conditions under which power can reproduce or extend its own supports. A bounded international-political case study examines balance-of-power reasoning and the conditional possibility of a near-critical balance regime. The final analytical section states the formal, empirical, and ontological limits of the proposal before the conclusion.

The resulting inquiry has a deliberately limited ambition. It does not reduce power to asymmetry, to a scalar metric, to symbolic recognition, to an operator, or to attractor geometry. It specifies a sequence of analytical distinctions through which these objects can be related without being conflated. The central proposal is that power becomes available for GR analysis when an already relationally real asymmetry acquires directionally organized dynamical consequence and subsequently stabilizes as an expectedly persistent, causally efficacious, and generatively consequential relational structure. The later formal sections ask how such a structure can be represented, manifested, compared, reproduced, and transformed while the ultimate ontology of power remains suspended.

2 Relational Asymmetry within History-Bearing GR Spacetime

This section supplies the minimum antecedent formalism required before power can be analyzed within Generative Relational Theory. Its role is to specify the type of the entities being compared, the history structure through which those entities persist, and the status of the typed relational asymmetry that serves as the starting object of the later power analysis. The discussion first represents GR entities as history-bearing relational spacetimes, then introduces event precedence and historical domains, reconstructs typed relational asymmetry at the level required here, and summarizes the several antecedent structures from which such asymmetry may arise. The section concludes by distinguishing a relationally real asymmetry from a temporary comparison or manifestation. The construction is representational: a 2-complex and its associated event order provide one candidate model of GR spacetime without fixing the ultimate ontology of a GR entity.

2.1 History-Bearing GR Entities

The basic relata of the present analysis are GR entities rather than point-like actors. Let $\mathfrak{S}_{\text{GR}}$ denote the formal domain of GR entities admitted by a chosen model. An element $A \in \mathfrak{S}_{\text{GR}}$ is interpreted as a history-bearing relational spacetime entity whose present relational organization depends on an extended history. The same interpretation applies to a second entity $B \in \mathfrak{S}_{\text{GR}}$. A coupled configuration such as $\mathcal{C}_H$ is a distinct modeled object that contains one or more such entities together with the relations and contextual structures retained for the analysis. The symbols A and B therefore do not denote isolated points carrying complete intrinsic property lists. They denote relationally and historically structured entities whose effective boundaries and internal resolutions depend on the analytical scale and model.

A labeled combinatorial 2-complex provides one candidate representation of this type of entity. For

the present section, the candidate representations of A and B are written in Equation 1.

$$\mathcal{K}_A = (V_A, E_A, F_A, \lambda_A), \quad \mathcal{K}_B = (V_B, E_B, F_B, \lambda_B). \quad (1)$$

In Equation 1, V_X , E_X , and F_X denote, respectively, the vertex, edge, and face sets of the candidate 2-complex representing $X \in \{A, B\}$. The collective symbol λ_X denotes the model-specified labels and incidence-compatible data attached to the cells of $\mathcal{K}_X$. The 2-complex carries higher-order historical and relational organization that would be suppressed if the entity were reduced to a single node. A spin-foam construction may serve as a more specialized instantiated model of such a labeled 2-complex when its additional structures are analytically justified. Neither representation is assigned ultimate ontological status by the present paper.

The two entities can participate in a wider coupled configuration. Let $\mathcal{C}_H$ denote a model-relative relational configuration supported by a historical domain H and containing the represented structures associated with A , B , and any cross-entity relations retained for the analysis. The subscript H is historical rather than a scalar time index. A relation between A and B can therefore be supported by events and higher-order cells distributed across a larger coupled history rather than by a single edge between two point-like actors.

Remark 2.1 (Spacetime typing of power relata). Any later comparison, perturbation, asymmetry relation, power relation, or metric in this paper acts on or between history-bearing GR entities or their model-relative representations. A scalar descriptor extracted from A or B is a readout of a richer structure and does not replace the entity type established in this subsection.

Remark 2.1 is especially important for the later international-political application. A state can be represented at an effective scale as one GR entity while retaining an internally extended historical, institutional, material, and symbolic organization. Comparisons of gross domestic product, military capacity, territory, or another manifested descriptor can then be studied as comparison structures over selected readouts without turning the represented state into the scalar used to compare it.

2.2 Event Precedence in Relational Spacetime

History in the present formalism is organized through event precedence rather than a primitive external clock. For each candidate 2-complex $\mathcal{K}_X$, let $\text{Ev}(\mathcal{K}_X) \subseteq V_X$ denote the selected vertices that the model treats as events, and let $\preceq_X$ be a partial order on that event set. The resulting event structure is stated in Equation 2.

$$\mathcal{H}_X := (\text{Ev}(\mathcal{K}_X), \preceq_X), \quad X \in \{A, B\}. \quad (2)$$

Equation 2 assigns model-relative generative precedence to selected events. If $e_1 \prec_X e_2$, the representation records that the realized occurrence or relational consequences associated with e_1 belong to the historical domain on which the modeled representation of e_2 depends. The relation need not be a total order, and incomparable events need not be assigned an earlier-later relation merely to create a global sequence.

A relational slice Σ_X is an antichain selected from $\text{Ev}(\mathcal{K}_X)$ to index a modeled stage of the entity history. The accumulated event domain associated with that slice is the order ideal defined in Equation 3.

$$D_X(\Sigma_X) := \{e \in \text{Ev}(\mathcal{K}_X) \mid e \preceq_X s \text{ for at least one } s \in \Sigma_X\}. \quad (3)$$

Equation 3 supplies a history-bearing domain from which the present relational organization of X can be modeled. A later empirical application may introduce dates, durations, or other scalar coordinates after showing that those coordinates are compatible with the event order. Such coordinates remain additional representations rather than primitive ingredients of the GR entity type.

The same logic applies to the coupled configuration $\mathcal{C}_H$. The historical domain H can contain event structures inherited from A and B , together with cross-entity events and higher-order relations generated through their interaction. An admissible historical extension $H \hookrightarrow H'$ preserves the relevant earlier incidence and precedence data while permitting new events and relations to enter the represented history. This extension relation will later support claims about persistence and relational reproduction without requiring notation of the form $t \mapsto t + 1$.

2.3 Typing Relational Asymmetry

Relational asymmetry requires more structure than non-identity between two GR entities. A difference in cell labels, size, internal complexity, or manifested attributes can be irrelevant to the relational dimension under analysis. The present paper therefore treats asymmetry as typed by an explicitly specified relational dimension.

Let α denote such a relational dimension in the coupled configuration $\mathcal{C}_H$. Let $\mathcal{Y}_\alpha(\mathcal{C}_H)$ denote the profile space used by the model to represent the relations, capacities, constraints, dependencies, or admissible transformations relevant to α . A profile extraction map is defined in Equation 4.

$$\rho_\alpha^{\mathcal{C}_H} : \mathfrak{S}_{\text{GR}} \rightarrow \mathcal{Y}_\alpha(\mathcal{C}_H). \quad (4)$$

The partiality in Equation 4 records that the selected relational dimension need not be applicable to every GR entity represented by the model. When defined, $\rho_\alpha^{\mathcal{C}_H}(A)$ and $\rho_\alpha^{\mathcal{C}_H}(B)$ retain only the relational information required by α ; they are not complete representations of A and B .

Let $\triangleright_\alpha^{\mathcal{C}_H}$ be a model-specified directed comparison relation on the applicable part of $\mathcal{Y}_\alpha(\mathcal{C}_H)$. The relation need not be total, numerical, or antisymmetric. It records the directional distinction relevant to α , such as unequal dependency, differentiated access, one-way procedural availability, or another application-defined consequence. A typed relational asymmetry from A toward B is present when the condition in Equation 5 holds.

$$\rho_\alpha^{\mathcal{C}_H}(A) \triangleright_\alpha^{\mathcal{C}_H} \rho_\alpha^{\mathcal{C}_H}(B). \quad (5)$$

Equation 5 does not define power. It records a directionally structured difference under one specified relational dimension. A pair of GR entities can satisfy Equation 5 in one dimension while remaining equivalent, incomparable, or oppositely ordered in another. The relation also carries no automatic normative judgment.

Definition 2.2 (Typed relational asymmetry). For a coupled historical configuration $\mathcal{C}_H$ and relational dimension α , a typed relational asymmetry from A toward B is the structured relation represented by Equation 5, provided that both profile extractions are defined and that $\triangleright_\alpha^{\mathcal{C}_H}$ has been specified independently of the conclusion to be drawn from the comparison.

Definition 2.2 makes explicit a condition that will remain important throughout the paper: the comparison structure by which an asymmetry is identified cannot be smuggled into the formalism as an unexplained greater-than sign. The relation $\triangleright_\alpha^{\mathcal{C}_H}$ can itself possess a history, manifestation, and later relational effects. Sections 6 and 8 treat that issue directly after the power relation itself has been constructed.

2.4 Antecedent Structures of Relational Differentiation

Definition 2.2 identifies the antecedent object required for the power analysis without prescribing one universal genealogy of asymmetry. Three broad antecedent structures remain compatible with the present framework. An articulated symmetry can precede relational differentiation; a typed asymmetry can be primitive relative to the adopted effective description; or the conditions required to apply a symmetry predicate can themselves emerge only after a pre-differentiated relational regime has developed sufficient structure. These possibilities remain analytically distinct because formation of the conditions for comparison, formation of symmetry, and breaking of an articulated symmetry are different processes.

The symmetry-breaking route can be stated compactly when a group action has already been justified. Let G act on a set X of relational positions relevant to a configuration, and let $H \leq G$ be a residual subgroup retained after a symmetry reduction. For any $x \in X$, the corresponding orbits satisfy Equation 6.

$$\text{Orb}_H(x) \subseteq \text{Orb}_G(x). \quad (6)$$

Equation 6 permits positions that belonged to one G -orbit to occupy different H -orbits after symmetry reduction. Such orbit splitting establishes relational differentiation relative to the chosen action. It does

not by itself establish the directed relation in Equation 5. A further dimension-specific analysis is required before the differentiated positions count as typed asymmetric.

The same restraint applies to the other antecedent structures. A primordial asymmetry relative to an effective model does not establish that ultimate ontology is asymmetric. A pre-differentiated condition does not supply a third truth value between symmetry and asymmetry; it records that the relevant predicate has not yet acquired an admissible domain and comparison structure. Partial transformations, local symmetries, or other restricted structures can become useful while that articulation develops. The present paper requires only the resulting typed relational asymmetry and therefore leaves the full internal formalization of these antecedent regimes outside its immediate scope.

Remark 2.3 (Limited role of symmetry breaking). Symmetry breaking supplies one admissible route to the relational differentiation from which a typed asymmetry may later be constructed. The power analysis developed below does not require every asymmetry, or every power relation, to descend from an antecedently symmetric configuration.

Remark 2.3 prevents the later power formalism from acquiring an unnecessary universal genealogy. Once a typed asymmetry has been identified and its relational status established, the subsequent question is whether that asymmetry has directionally consequential effects on the generative possibility structure of other GR entities or of the coupled configuration.

2.5 Relational Realization of Asymmetry

A typed comparison can be formally meaningful while remaining too local, temporary, or epiphenomenal to count as part of relational reality. The power analysis therefore starts from a stronger antecedent: the typed asymmetry itself has acquired relational reality within the historical domain under study. Let $\mathcal{R}_H$ denote the model-defined domain of structures assigned relational-reality status relative to historical domain H . In accordance with Remark 1.1, $\mathcal{R}_H$ is not introduced as a primitive image of ultimate ontology. It is selected from structures represented in the coupled configuration after the model has supplied the historical support and efficacy conditions required by GR. For the antecedent asymmetry those conditions are assumed here rather than re-derived, because the present paper begins from the already realized asymmetry and studies the additional formation of power.

Let $\alpha_\alpha(A, B \mid \mathcal{C}_H)$ denote the typed asymmetry structure identified by Definition 2.2. The antecedent assumed by the remainder of the paper is stated in Equation 7.

$$\alpha_\alpha(A, B \mid \mathcal{C}_H) \in \mathcal{R}_H. \quad (7)$$

Equation 7 means that the asymmetry is supported by an extended relational history, persists at the scale relevant to the inquiry, and has type-appropriate relational efficacy for later relational organization. For the antecedent assumed here, that efficacy may be causal, generative, or both; the stronger power construction in Section 4 will require both causal and generative efficacy for the power organization itself. The present section leaves the stronger dynamical treatment of persistence to the later section on the relational reality of power. At this stage the condition excludes a purely transient comparison judgment, a one-off manifested ranking, or an analytical label that makes no difference to subsequent relations.

The relational reality of asymmetry can be expressed through several model-specific mechanisms. A difference in standing can alter which procedures are admissible. A difference in infrastructural access can change which transformations are practically reachable. A historically stabilized dependency can alter the continuation space of one entity even when formal permissions appear similar. In each case the asymmetry is carried by the relational configuration and participates in later generation. Its status therefore exceeds a context-dependent manifestation of difference.

Assumption 2.4 (Antecedent relational asymmetry). The subsequent power-formation analysis assumes a coupled historical configuration $\mathcal{C}_H$, two GR entities $A, B \in \mathfrak{S}_{\text{GR}}$, a specified relational dimension α , and a typed asymmetry $\alpha_\alpha(A, B \mid \mathcal{C}_H)$ satisfying Equation 7.

Assumption 2.4 fixes the starting point of the new argument. It does not yet state that A has power over B , that the asymmetry can be assigned a scalar magnitude, or that the relation is normatively objectionable. The next section asks a narrower and stronger question: whether the relationally real asymmetry produces directionally organized deformation of the future possibility structure of B , of A , or of the wider coupled configuration. That additional condition is the proposed bridge from relational asymmetry to power relevance.

3 Power Relevance through Directional Dynamical Consequence

This section constructs the first new step beyond relational asymmetry. Its role is to identify the additional dynamical structure required before an already relationally real asymmetry can be treated as power-relevant. The analysis proceeds in five stages. It first defines the generative possibility structure relative to a GR entity in a coupled historical configuration. It then introduces admissible perturbations as model-relative epistemic probes and records their consequences without presupposing a scalar measure of power. A structured response profile organizes several candidate dimensions of dynamical consequence. The section then separates the existence of a consequence from a comparison of directional significance within the coupled system. The final subsection defines a power-relevant relational asymmetry relative to an explicitly supplied response-salience schema. The construction remains prior to the stronger claim that power itself has acquired relational reality, which is developed in the subsequent analysis of the relational reality of power.

3.1 Generative Consequence of Relational Position

Assumption 2.4 supplies a coupled historical configuration $\mathcal{C}_H$, two history-bearing GR entities $A, B \in \mathfrak{S}_{GR}$, and a relationally real typed asymmetry $\alpha_\alpha(A, B \mid \mathcal{C}_H)$. Let $\mathfrak{C}$ denote the class of model-admissible coupled historical configurations of the type used in the present analysis, with $\mathcal{C}_H \in \mathfrak{C}$. The existence of the asymmetry does not yet specify what later histories remain possible for either entity. A dynamical analysis therefore requires an explicit representation of admissible continuation.

Let $\Gamma_B^+(\mathcal{C}_H)$ denote the model-specified family of admissible historical continuations from $\mathcal{C}_H$ whose represented evolution contains a well-typed continuation of B . An element $\gamma \in \Gamma_B^+(\mathcal{C}_H)$ is a history-bearing extension compatible with the event-order, incidence, boundary, and admissibility conditions adopted by the model. The superscript $+$ indicates forward admissible continuation relative to the historical domain H ; it does not introduce a scalar future-time coordinate.

A second component records which immediate transformations involving the represented continuation of B are currently available. Let $\mathcal{O}_B^{\text{adm}}(\mathcal{C}_H)$ denote the model-specified family of admissible next transformations for B within $\mathcal{C}_H$. Each retained transformation is a partial map on $\mathfrak{C}$ whose domain contains the current configuration. At the present stage, $\mathcal{O}_B^{\text{adm}}(\mathcal{C}_H)$ is only a typed family of available transformations. No closure, linearity, probability measure, or operator algebra is assumed.

The minimal generative possibility structure of B used in this section is defined in Equation 8.

$$\text{Poss}_B(\mathcal{C}_H) := (\Gamma_B^+(\mathcal{C}_H), \mathcal{O}_B^{\text{adm}}(\mathcal{C}_H)). \quad (8)$$

Equation 8 is deliberately weaker than a complete dynamics. It records two aspects of relational becoming that can be altered by power: which continuations are represented as admissible and which transformations can currently participate in generating them. Later sections can enrich this structure with history weights, attractor-like regimes, costs, or other application-specific relations once their mathematical assumptions have been stated.

The analytical question can now be made more precise. A relational position of A has a generative consequence for B when an admissible variation of the represented position or organization of A changes $\text{Poss}_B(\mathcal{C}_H)$ in a way that is retained by the response criteria of the model. This formulation does not require A to issue a command or exercise intentional agency. The relevant effect can be mediated by dependency,

standing, infrastructure, institutional position, symbolic organization, or another relation represented inside the coupled history.

3.2 Perturbation as an Epistemic Probe

The effect of a relational position is not generally visible from a single snapshot of $\mathcal{C}_H$. The present formalism therefore uses perturbation as an epistemic probe of dynamical consequence. The perturbation language is model-relative: it asks how the represented continuation structure changes under explicitly declared variations while leaving open whether a particular perturbation is physically, institutionally, or experimentally realizable outside the model.

Let $\mathfrak{D}_A(\mathcal{C}_H)$ denote a specified family of admissible perturbations directed at the represented relational position or internal organization of A within $\mathcal{C}_H$. Each retained perturbation is a partial transformation on $\mathfrak{C}$. For $\delta_A \in \mathfrak{D}_A(\mathcal{C}_H)$, write $\mathcal{C}_H^{\delta_A}$ for the resulting comparison configuration when the perturbation is defined. The typing and evaluation of the perturbation are stated in Equation 9.

$$\delta_A : \mathfrak{C} \rightarrow \mathfrak{C}, \quad \delta_A(\mathcal{C}_H) = \mathcal{C}_H^{\delta_A}. \quad (9)$$

Equation 9 is a modeling operation. It does not imply that the historical record encoded by H is literally rewritten. A concrete application can realize a perturbation through an intervention, a naturally occurring change, a counterfactual reconstruction, or a computational variation, provided that the interpretation is stated explicitly.

Meaningful response comparison requires the perturbation to preserve enough structure for the before-and-after objects to remain comparable. The requirement is stated in Assumption 3.1.

Assumption 3.1 (Perturbation comparability). For each perturbation δ_A retained in $\mathfrak{D}_A(\mathcal{C}_H)$, the model specifies a comparison configuration $\mathcal{C}_H^{\delta_A}$ in which the identity conditions for the response target B , the relevant relational scale, and the typing required to construct Poss_B remain sufficiently stable for a declared comparison to be well-defined.

Assumption 3.1 prevents a large structural change from being treated automatically as evidence about the original relation. An application that changes the entity boundary, historical resolution, or relational dimension must supply the additional translation required to compare the resulting possibility structures.

The perturbative response of B to δ_A is represented by the ordered pair in Equation 10.

$$\mathcal{R}_{A \curvearrowright B}(\delta_A \mid \mathcal{C}_H) := \left(\text{Poss}_B(\mathcal{C}_H), \text{Poss}_B(\mathcal{C}_H^{\delta_A}) \right). \quad (10)$$

Equation 10 records a response without assigning it a magnitude. The first component supplies the reference possibility structure and the second supplies the structure under the declared perturbation. A response can therefore be studied even when the two components contain sets, partial transformations, or other objects for which subtraction and a norm have not been defined. The corresponding class of defined response pairs is introduced in Equation 11.

$$\mathfrak{Resp}_{A \curvearrowright B}(\mathcal{C}_H) := \{ \mathcal{R}_{A \curvearrowright B}(\delta_A \mid \mathcal{C}_H) \mid \delta_A \in \mathfrak{D}_A(\mathcal{C}_H), \delta_A(\mathcal{C}_H) \text{ is defined} \}. \quad (11)$$

Equation 11 separates the response map $\mathcal{R}_{A \curvearrowright B}$ from the class of response objects on which later profile extraction acts.

Remark 3.2 (Perturbation is not a power measure). A perturbation response supplies evidence about the dynamical consequences associated with a relational position. The formalism does not infer a scalar quantity of power from the existence or size of a response unless an application independently defines and justifies the required comparison or metric structure.

Remark 3.2 keeps the dynamical inquiry separate from the later problem of power comparison. The same response data can support different comparisons under different historically and institutionally generated comparison structures.

3.3 Dynamical Profiles of Relational Consequence

A single statement that the possibility structure has changed can suppress several distinct forms of consequence. The present paper therefore introduces a response profile whose components remain structurally heterogeneous. Let β denote a finite family of response descriptors selected for the analysis, and let $\mathcal{D}_\beta$ denote the corresponding structured profile space. A profile map is represented in Equation 12.

$$\Pi_\beta : \mathfrak{Resp}_{A \curvearrowright B}(\mathcal{C}_H) \rightarrow \mathcal{D}_\beta. \quad (12)$$

In Equation 12, the domain is the response-pair class defined in Equation 11. The profile space $\mathcal{D}_\beta$ need not be numerical and need not carry a total order. Its role is to preserve analytically relevant distinctions before any comparison norm is introduced.

Table 1 gives five candidate descriptors that are particularly useful for the present power analysis. They are available analytical questions rather than a canonical coordinate system for power.

The descriptors in Table 1 deliberately resist premature aggregation. Breadth can be set-valued, depth can be categorical or ordered only locally, persistence can be indexed by historical extensions, robustness depends on the declared perturbation family, and generativity can concern qualitatively different kinds of newly available relation. A formula that adds these components would therefore require additional commensurability, weighting, and interpretation assumptions.

For a perturbation family $\mathfrak{D}_A^*(\mathcal{C}_H) \subseteq \mathfrak{D}_A(\mathcal{C}_H)$, the corresponding set of defined response profiles is stated in Equation 13.

$$\mathfrak{P}_{A \curvearrowright B}^\beta(\mathfrak{D}_A^* | \mathcal{C}_H) := \{\Pi_\beta(\mathcal{R}_{A \curvearrowright B}(\delta_A | \mathcal{C}_H)) \mid \delta_A \in \mathfrak{D}_A^*(\mathcal{C}_H), \Pi_\beta \text{ is defined}\}. \quad (13)$$

Equation 13 supplies a non-scalar object on which a later salience or comparison structure can operate. It also makes robustness a property of a profile family rather than an undefined appeal to a small numerical perturbation. A metric on perturbations can be introduced in a specialized model, but the present formalism requires only an explicitly declared family of admissible variations.

3.4 Directional Significance within Coupled Dynamics

A response profile can be rich without yet supporting a claim about directional significance. Such significance requires an additional interpretive or comparative structure that specifies which profile differences matter for the inquiry. The present section introduces that structure explicitly so that a judgment of power relevance is not hidden inside the definition of the response itself.

Let χ denote a response-salience schema whose domain contains selected profile families of the type in Equation 13. The schema is a partial predicate defined in Equation 14.

$$\chi : \text{Dom}(\chi) \subseteq 2^{\mathcal{D}_\beta} \longrightarrow \{0, 1\}. \quad (14)$$

The value $\chi(\mathfrak{P}) = 1$ means only that the declared schema treats the profile family $\mathfrak{P}$ as dynamically salient for the specified analytical purpose. The schema is therefore an identification device at the modeling or epistemic interface; it is not itself what makes the underlying asymmetry causally or generatively consequential. Equation 14 does not imply that χ is universal, observer-independent, or already relationally real. The later analysis of comparative structures examines how such criteria can themselves be historically generated, manifested, stabilized, and made causally consequential.

The schema χ can use several kinds of evidence without reducing them to one number. It may require that a perturbation of A changes the domain of operations available to B , that a consequence persists across a specified family of historical extensions, that the response remains robust under declared changes of context, or that new relations and later asymmetries are generated. A different schema can select different features. The formal requirement is transparency about the selection rule, not prior agreement on one metric of power.

A stronger comparison can be introduced when the analysis asks whether the response associated with one direction is more consequential than the reverse direction. If $\mathfrak{P}_{A \curvearrowright B}^\beta$ and $\mathfrak{P}_{B \curvearrowright A}^\beta$ are both defined, let

Table 1: Candidate descriptors of a relational dynamical response profile. The entries organize different forms of consequence without assigning a common numerical scale or aggregation rule.

Descriptor	Structural question	Representative object of inquiry
Breadth	How widely does the perturbation alter the represented family of admissible continuations or transformations?	Changes in Γ_B^+ or in the domain of $\mathcal{O}_B^{\text{adm}}$
Depth	At which representational level does the response occur?	Manifested readout, active relational structure, admissibility conditions, or history-bearing configuration
Persistence	Across which admissible historical extensions does the induced consequence remain operative?	Retention or recurrence of the response over a declared family of historical continuations
Robustness	Which declared perturbations of context, representation, or local structure preserve the directional consequence?	Stability of the response under a specified admissible perturbation family
Generativity	Which new relations, constraints, permissions, operations, or later asymmetries become possible because of the response?	Extension or reorganization of future relational possibilities

$\succ_{\chi}^{C_H}$ denote a model-specified partial comparison relation on such profile families. A directional comparison, when defined, is written in Equation 15.

$$\mathfrak{P}_{A \curvearrowright B}^{\beta} \succ_{\chi}^{C_H} \mathfrak{P}_{B \curvearrowright A}^{\beta}. \quad (15)$$

Equation 15 is intentionally stronger than a claim that the original asymmetry is power-relevant. Mutual power relations can exist without satisfying a strict directional comparison of this form. The relation $\succ_{\chi}^{C_H}$ is also allowed to leave profile families incomparable. Its mathematical and relational status must be established separately from the dynamical responses it compares.

Remark 3.3 (Comparison does not precede its own formation). The notation in Equation 15 records a comparison only after a comparison relation has been specified. The present paper does not treat the greater-than-like structure as a primitive property of GR spacetime entities. Its historical formation and possible acquisition of manifestation or relational reality are analyzed later.

Remark 3.3 is central to the distinction between power dynamics and power ranking. A system can contain power-relevant directional effects even when no complete ordering of the relevant GR entities is available.

3.5 Power-Relevant Relational Asymmetry

The preceding constructions permit a definition of power relevance that preserves the difference between an antecedent asymmetry, its dynamical consequences, and the criteria by which those consequences are treated as salient. The definition is local to a coupled historical configuration and to an explicitly declared analytical scheme. It therefore supplies a typed bridge to the later metaphysics of relational power without yet asserting that a persistent power structure has formed.

Definition 3.4 (Power-relevant relational asymmetry). Let $\alpha_{\alpha}(A, B \mid C_H) \in \mathcal{R}_H$ satisfy Assumption 2.4. Let $\mathfrak{D}_A^*(C_H)$ be a declared admissible perturbation family, let Π_{β} be a defined response-profile map, and let χ be a declared response-salience schema. Within the declared model, the asymmetry is identified

as power-relevant from A toward B relative to $(\mathfrak{D}_A^*, \beta, \chi)$ when the profile family in Equation 13 lies in $\text{Dom}(\chi)$, χ assigns that family the value one, and the model supplies an explicit generative-dependence account connecting the antecedent asymmetry to the salient response rather than treating their coexistence as sufficient evidence of dependence. The dependence account carries the substantive directional claim; χ supplies the declared criterion by which that consequence is retained as power-relevant for the analysis.

Definition 3.4 does not require a total ranking of A and B , a scalar power magnitude, or the reverse-direction comparison in Equation 15. It requires an already relationally real asymmetry, an explicit perturbative or dynamical account of its consequences, an explicit criterion for treating those consequences as salient, and an account of how the asymmetry participates in generating the response. These conditions are sufficient for the present paper to call the asymmetry power-relevant; they are not yet sufficient for the stronger claim that power has stabilized as a relationally real structure.

The generative-dependence requirement in Definition 3.4 prevents a common inferential shortcut. A salient response of B after a variation of A can arise through relations that are independent of the typed asymmetry α_α , through a third structure omitted from the model, or through a change in the comparison frame itself. A domain-specific application must therefore identify the mechanism, intervention structure, process trace, or other evidence by which the antecedent asymmetry contributes to the response.

Figure 1 summarizes the logical architecture of the section. The figure separates the antecedent relational asymmetry from the independently declared perturbation family, the observed or modeled response profile, and the salience schema. The final arrow reaches power relevance rather than relational power, preserving the additional persistence and efficacy analysis required in the next section.

Research Obligation 3.5 (Identification of power relevance). An empirical or domain-specific application of Definition 3.4 should specify the entity boundaries, perturbation family, response target, profile descriptors, salience schema, and evidence for generative dependence. A change in any of these objects can alter the resulting power-relevance judgment.

Obligation 3.5 keeps the formal definition connected to an identifiable research procedure. It also prepares the next section's stronger question. A power-relevant asymmetry can remain episodic or highly contingent. The transition to relational power requires an account of expected persistence and of the causal and generative efficacy through which the power structure itself becomes a relationally real regime.

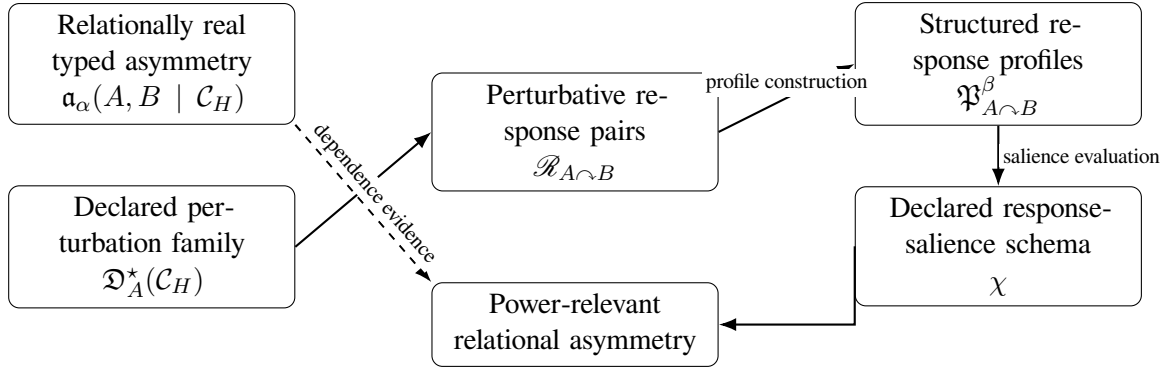


Figure 1: Formal route from relational asymmetry to power relevance. The perturbation family is declared independently of the antecedent asymmetry. Response pairs are organized into structured profiles and evaluated under an explicit salience schema, while the final inference additionally requires evidence that the antecedent asymmetry contributes generatively to the salient response. No scalar power metric or total ranking is presupposed.

4 Relational Reality of Power

Section 3 established a typed route from an already relationally real asymmetry to power relevance. The present section addresses a stronger question: under what conditions does the power-relevant dynamical organization itself acquire relational reality? The distinction is necessary because the antecedent asymmetry and the power structure generated from its consequences are different formal objects. A relationally real asymmetry can support a salient directional response in one episode while the resulting power organization remains contingent, weakly retained, or dependent on a narrow configuration. The analysis therefore treats relational power as a higher-order relational structure whose own historical persistence and efficacy must be established.

The section proceeds in six stages. It first defines expected persistence over partially ordered historical extensions without introducing primitive scalar time or assigning an objective probability interpretation to history weights. It then develops an attractor-like representation of persistent relational regimes while preserving the possibility of metastability, branching, and near-critical dynamics. The third subsection distinguishes persistence from causal and generative efficacy. The fourth locates unstable and developing power candidates in a proto-relational domain and introduces a basin-like representation of their possible stabilization. The fifth examines closure operators as an optional algebraic representation of stabilization. The final subsection combines the preceding conditions into a definition of relational power. Attractor, basin, closure, and expectation-like language remain modeling resources throughout; none is identified with ultimate ontology.

For bookkeeping, the term *power candidate* denotes the dynamical organization constructed around an antecedent asymmetry that satisfies Definition 3.4, while the relational-reality status of that additional power organization has not yet been established. The term marks an analytical stage rather than a new layer of reality. Section 4.4 later collects such candidates in an explicit proto-relational domain.

4.1 Expected Persistence across Historical Extensions

Power relevance in Definition 3.4 is local to a declared coupled historical configuration and to a specified family of perturbations, response descriptors, and salience criteria. Relational power requires a stronger form of historical retention. The relevant persistence cannot be defined by a primitive sequence of external clock times because the temporal organization of the present framework begins from partially ordered event histories. Persistence is therefore evaluated across a declared family of admissible historical extensions.

Let $\mathfrak{H}_H^+$ denote a model-specified family of historical domains H' that extend the reference domain H in a manner compatible with the event order and incidence structure of the model. For each $H' \in \mathfrak{H}_H^+$, let

$\Gamma_{H \rightsquigarrow H'}^+(\mathcal{C}_H)$ denote the retained family of admissible continuations from $\mathcal{C}_H$ whose terminal represented configuration is indexed by H' . The notation $H \rightsquigarrow H'$ records historical extension and does not imply that all event pairs in the two domains are totally ordered.

A comparison among alternative continuations can require a weighting structure. The present paper introduces a history-weight system only at the minimum level needed to define an expectation-like persistence profile. For each retained extension H' , let $w_{H \rightsquigarrow H'}$ assign a nonnegative finite weight to each continuation in $\Gamma_{H \rightsquigarrow H'}^+(\mathcal{C}_H)$. The technical condition is stated in Assumption 4.1.

Assumption 4.1 (History-weight system). For every $H' \in \mathfrak{H}_H^+$, the retained continuation family is finite or countable and the weight assignment $w_{H \rightsquigarrow H'} : \Gamma_{H \rightsquigarrow H'}^+(\mathcal{C}_H) \rightarrow [0, \infty)$ has strictly positive and finite total weight.

Assumption 4.1 is a representational convenience. It does not assign frequentist probability, objective chance, quantum amplitude, or a unique rational credence to the retained histories. A later application can supply one of those interpretations only after its additional assumptions have been justified.

Let $\mathcal{A} \subseteq \mathfrak{C}$ be a model-defined relational regime, meaning a class of coupled historical configurations that satisfy a declared structural criterion. The indicator $\mathbf{1}_{\mathcal{A}}$ takes the value one on configurations in $\mathcal{A}$ and zero otherwise. If $\mathcal{C}_{H'}^\gamma$ denotes the terminal configuration of continuation $\gamma \in \Gamma_{H \rightsquigarrow H'}^+(\mathcal{C}_H)$, the weighted occupancy of $\mathcal{A}$ at historical extension H' is defined in Equation 16.

$$\text{Occ}_{\mathcal{A}}^w(H' \mid \mathcal{C}_H) := \frac{\sum_{\gamma \in \Gamma_{H \rightsquigarrow H'}^+(\mathcal{C}_H)} w_{H \rightsquigarrow H'}(\gamma) \mathbf{1}_{\mathcal{A}}(\mathcal{C}_{H'}^\gamma)}{\sum_{\gamma \in \Gamma_{H \rightsquigarrow H'}^+(\mathcal{C}_H)} w_{H \rightsquigarrow H'}(\gamma)}. \quad (16)$$

Equation 16 lies in $[0, 1]$ because of Assumption 4.1. Its numerical range is a normalized bookkeeping device. The value does not become an empirical probability unless a separate probabilistic interpretation is supplied.

The expected-persistence profile of $\mathcal{A}$ relative to the reference configuration is the history-indexed function defined in Equation 17.

$$\text{EP}_{\mathcal{A}}^w(\cdot \mid \mathcal{C}_H) : \mathfrak{H}_H^+ \longrightarrow [0, 1], \quad H' \longmapsto \text{Occ}_{\mathcal{A}}^w(H' \mid \mathcal{C}_H). \quad (17)$$

Equation 17 defines expected persistence without requiring every admissible continuation to remain inside the regime. Branching, contingency, exceptional histories, and agent-mediated selections can therefore remain represented. The formalism also avoids identifying openness of the future with any particular metaphysics of free will.

Whether a persistence profile is sufficient for a given inquiry remains a model-level judgment. Let ψ_{pers} denote a declared persistence schema whose domain contains selected functions of the type in Equation 17 and whose codomain is $\{0, 1\}$. The regime is expectedly persistent relative to the declared schema when $\psi_{\text{pers}}(\text{EP}_{\mathcal{A}}^w) = 1$. A schema may use a threshold, a lower envelope over a declared family of historical extensions, a stability condition under changes of history weights, or another explicit rule. The present paper does not supply a universal persistence threshold.

Remark 4.2 (Expected persistence and open continuation). Expected persistence constrains the weighted family of admissible continuations without requiring a unique future history. The construction therefore leaves room for stochasticity, contingency, and agency while remaining neutral about whether those features are sufficient for, or constitutive of, free will.

Remark 4.2 clarifies the limited role of expectation language. The analytical function of expected persistence is to represent a relational regime that can remain real and consequential without being an eternal fixed configuration.

4.2 Attractor-Like Representation of Relational Regimes

The expected-persistence profile permits a dynamical interpretation of relational reality that is broader than fixed-point or limit-cycle stability. A relationally real structure can retain its organization while its local manifestations, supporting relations, and internal configurations continue to change. The paper therefore uses *attractor-like regime* as a modeling term for a class of configurations toward which a declared family of continuations exhibits a recurrent or retaining tendency. The qualifier “-like” is essential because the present construction does not assume a metric phase space, continuous flow, asymptotic convergence, or an invariant compact set of the kind required by particular mathematical definitions of attractors.

The caution is also consistent with the dynamical-systems literature, in which the term *attractor* admits more than one mathematically substantive formulation; Milnor’s measure-oriented definition is one prominent example [12]. The present construction therefore does not import one classical attractor definition by analogy. It specifies only the retaining or recurrent features actually used by the GR model and leaves stronger topological, metric, or measure-theoretic structure to a separately justified application.

Definition 4.3 (Attractor-like relational regime). A model-defined regime $\mathcal{A} \subseteq \mathfrak{C}$ is attractor-like relative to $(\mathfrak{H}_H^+, w, \psi_{\text{pers}})$ and a declared family of admissible perturbations when two conditions hold: first, $\psi_{\text{pers}}(\text{EP}_{\mathcal{A}}^w) = 1$ for the reference configuration or declared class of reference configurations; second, configurations obtained through the retained perturbation family exhibit a model-specified tendency to re-enter or remain within $\mathcal{A}$ across the historical extensions retained by the analysis.

Definition 4.3 deliberately leaves the second condition structurally general. A concrete model may express return tendency through history weights, reachability, recurrence, a Lyapunov-like function, transition rates, or another mathematically appropriate device. The general GR-level argument requires only that the choice be explicit and that its relation to the represented histories be stated.

A relational regime may satisfy Definition 4.3 only over a finite or historically bounded family of extensions. The paper uses *metastable* for this weaker and often more suitable case. Metastability permits a relational being or power structure to remain persistent and efficacious over the historical scale relevant to the inquiry while still allowing later transformation, bifurcation, dissolution, or critical transition. Relational reality therefore does not require eternal invariance.

The same construction permits near-critical relational reality. A regime can remain historically persistent while its susceptibility to perturbation is elevated and its future continuations are comparatively easy to redirect. Criticality and relational reality are therefore analytically distinct. The former concerns properties of response and transition structure; the latter additionally concerns historical persistence and relational efficacy. A later application can study their interaction without making noncriticality a prerequisite for reality status.

Remark 4.4 (Attractor representation and ontological suspension). An attractor-like regime is a dynamical representation of persistence within a chosen model. The present paper does not infer that relational reality is ultimately an attractor, a phase-space object, or a dynamical-system state in a fundamental ontological sense.

Remark 4.4 preserves the distinction between a useful dynamical representation and an ontological commitment. This distinction becomes especially important when basin and closure constructions are added below.

4.3 Efficacy within Relational Continuation

Expected persistence is insufficient for relational reality. A pattern can be stable within a model while remaining dynamically irrelevant to the other structures under study. The present account therefore requires efficacy in addition to persistence. The paper distinguishes causal efficacy from generative efficacy because the two roles can be related without being identical.

A candidate power structure is causally efficacious when an admissible comparative intervention on that structure changes at least one declared consequence in the coupled relational configuration and the model

supplies an explicit dependence account linking the intervention to that change. The relevant consequence can be a change in a possibility structure, an active relation, a readout, a constraint, or another well-typed object. This criterion mirrors the generative-dependence discipline in Definition 3.4; it prevents mere correlation between a persistent pattern and a later outcome from being treated automatically as causal efficacy.

Generative efficacy is stronger in a different direction. A candidate power structure is generatively efficacious when its presence contributes to the production, modification, stabilization, or removal of structures that condition later relational continuation. Examples include a change in $\mathcal{O}_X^{\text{adm}}(\mathcal{C}_H)$, creation of a new relation, alteration of a future entity boundary, modification of an institutional or symbolic support, or reorganization of which continuation histories remain admissible. Generative efficacy therefore concerns the formation of later conditions of becoming rather than only a single downstream response.

Definition 4.5 (Relational efficacy of a power candidate). A power candidate is relationally efficacious relative to a declared model when the model establishes both (i) causal efficacy through at least one well-typed comparative intervention and dependence account, and (ii) generative efficacy through a demonstrated contribution to the formation or modification of later relational conditions.

Definition 4.5 does not require every local manifestation of the candidate to produce a visible effect. A relational power structure can be efficacious through the organization of unavailable actions, expected responses, or self-stabilizing institutions even when overt intervention is rare. The epistemic consequences of such latent efficacy are developed in the following section.

Table 2 summarizes the distinction between the reality conditions introduced in this section and the modeling resources used to represent them. The table is deliberately asymmetric: expected persistence and relational efficacy are proposed conditions for relational realization, whereas attractor, basin, and closure structures remain optional formal representations.

4.4 Basin Structure of Proto-Relational Formation

The transition from a power-relevant episode to relational power requires a formal location for structures that are already identifiable and dynamically consequential but have not yet been assigned relational-reality status. Let $\mathcal{Q}_H^P$ denote the proto-relational domain of power candidates associated with historical domain H . An element $q^P \in \mathcal{Q}_H^P$ is a model record whose components include (at minimum) the antecedent typed asymmetry, the declared perturbation and response-profile structures used to identify its directional consequence, the salience schema, and the generative-dependence evidence required by Definition 3.4. The record is not asserted to be an additional ontological entity; it is the formal carrier on which later persistence and efficacy tests are evaluated. Membership in $\mathcal{Q}_H^P$ is therefore a status within the model, not a fourth layer of reality and not a conclusion that the power structure has already become relationally real.

A history extension can transport, transform, fragment, or terminate such a candidate. The present paper does not require one universal transport operator on $\mathcal{Q}_H^P$. It requires only that an application specify enough identity or continuity conditions to determine when later configurations instantiate the same candidate relational organization, a transformed descendant, or a different structure. Those conditions are the basis on which the regime indicator in Equation 16 is constructed.

Let $\mathcal{A}_P \subseteq \mathfrak{C}$ be a model-defined attractor-like regime associated with a candidate power structure. Its basin-like domain is defined relative to the declared persistence and perturbation structure rather than by metric distance. The basin-like domain is stated in Equation 18.

$$\mathcal{B}_H^{w,\psi}(\mathcal{A}_P) := \{ \mathcal{C} \in \mathfrak{C} \mid \psi_{\text{pers}}(\text{EP}_{\mathcal{A}_P}^w(\cdot \mid \mathcal{C})) = 1 \}, \quad (18)$$

where the expression is evaluated only for configurations for which the required continuation family and history-weight system are defined. Equation 18 does not state that every member of the basin is itself relational power. It identifies starting configurations whose weighted continuations satisfy the declared persistence criterion for the candidate regime.

Table 2: Conditions and modeling resources in the relational realization of power.

Element	Analytical role	Status in the present formalism
Expected persistence	Retention of a candidate relational organization across a declared weighted family of historical continuations	Proposed condition for relational realization of the power candidate; does not require deterministic continuation
Causal efficacy	Detectable dependence of a declared relational consequence on the candidate structure under an admissible comparison	Proposed condition for relational realization of the power candidate; requires an explicit dependence account
Generative efficacy	Contribution to the formation or modification of later relations, operator availability, admissibility, or other conditions of becoming	Proposed condition for relational realization of the power candidate and the specifically generative character of GR power
Attractor-like regime	Dynamical representation of recurrent or retaining organization across historical extensions	Optional representation; not an ontological identity
Basin-like domain	Representation of proto-relational starting conditions tending toward a candidate regime	Optional representation; basin membership does not confer relational reality
Closure operator	Algebraic representation of stabilization on an ordered proto-relational domain	Optional representation; fixed points remain candidates until efficacy is established

The basin construction makes perturbation especially useful. Symbolic mediation, institutional re-design, resource redistribution, or another intervention can enlarge, shrink, fragment, or redirect the set in Equation 18 without changing every local relation directly. A later section develops this idea for self-sustaining power, where power can modify the landscape in ways that make its own supporting regime more accessible. At the present stage, basin geometry remains a modeling description of relationalization tendencies.

Remark 4.6 (Basin membership and relational reality). Membership in $\mathcal{B}_H^{w,\psi}(\mathcal{A}_P)$ identifies a tendency toward or retention within a modeled regime. It does not establish causal efficacy, generative efficacy, or relational-reality status for the starting configuration or for every candidate structure represented within it.

Remark 4.6 blocks the inference from dynamical tendency to ontology. The distinction also permits several competing basins or partially overlapping regimes to remain represented without forcing one global attractor decomposition.

4.5 Closure Structure of Relationalization

The basin-like representation describes stabilization dynamically. A complementary algebraic representation becomes available when the proto-relational domain carries an order that records increasing stabilization, retention, or structural completion. Fix a historical domain H and let $(\mathcal{Q}_H^P, \sqsubseteq_H)$ be an application-specified partially ordered proto-relational domain. The semantics of $q \sqsubseteq_H q'$ must be stated by the application; the present paper does not assume that power candidates across all historical domains admit one universal ordering.

A closure operator on this ordered domain is a map $C_{P,H} : \mathcal{Q}_H^P \rightarrow \mathcal{Q}_H^P$ satisfying the three conditions in Equation 19.

$$q \sqsubseteq_H C_{P,H}(q), \quad q \sqsubseteq_H q' \implies C_{P,H}(q) \sqsubseteq_H C_{P,H}(q'), \quad C_{P,H}(C_{P,H}(q)) = C_{P,H}(q). \quad (19)$$

Equation 19 gives a possible algebraic representation of stabilization. Extensivity records movement to a structure that retains or completes the declared stabilization features, monotonicity preserves the supplied order, and idempotence represents the fact that applying the same stabilization closure to an already closed candidate produces no further change at that level of representation.

The fixed-point domain of $C_{P,H}$ is defined in Equation 20.

$$\text{Fix}(C_{P,H}) := \{q \in \mathcal{Q}_H^P \mid C_{P,H}(q) = q\}. \quad (20)$$

Equation 20 identifies stabilized candidates relative to the supplied historical domain, order, and closure. It does not define relational reality. A fixed point can remain causally inert or fail to generate later relational consequences. The closure construction therefore complements rather than replaces the efficacy conditions in Definition 4.5.

Remark 4.7 (Closure and attractor representations). A closure operator and an attractor-like regime can represent related aspects of stabilization without being mathematically identical. The former is an order-theoretic idempotent operation on a declared proto-relational domain; the latter represents history-indexed dynamical retention or return. Identifying them requires additional structure that the present paper does not assume.

Remark 4.7 preserves methodological pluralism at the formal level. A domain-specific model can use one representation, both, or neither according to the structure of the problem.

4.6 Relational Realization of Power

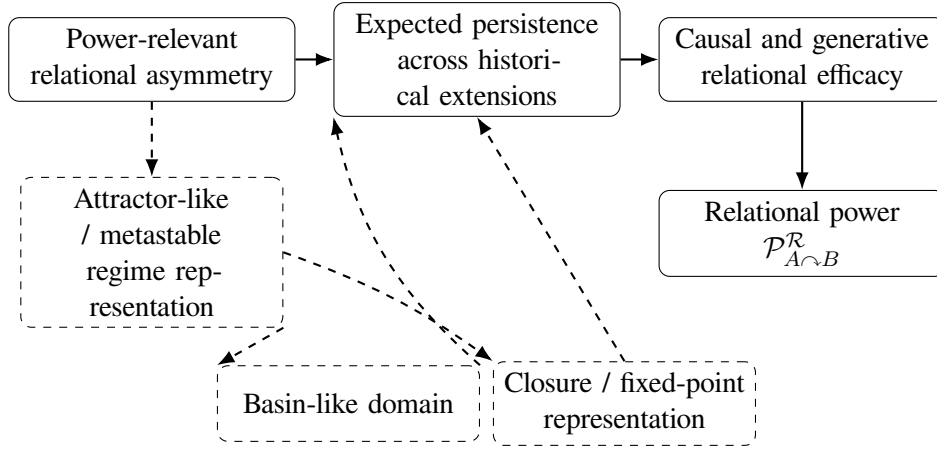
The preceding constructions can now be combined without identifying any single mathematical resource with power itself. The antecedent object is the relationally real typed asymmetry from Assumption 2.4. Section 3 adds a power-relevance structure by demonstrating directional dynamical consequence under an explicit salience and dependence account. The present section adds a second layer: the power organization itself must persist across admissible historical continuations and remain causally and generatively efficacious.

Figure 2 summarizes the transition developed in this section. The figure distinguishes the conditions for relational realization from the optional attractor, basin, and closure representations used to model stabilization.

Definition 4.8 (Relational power in the present construction). Let $\alpha_\alpha(A, B \mid \mathcal{C}_H) \in \mathcal{R}_H$ be a power-relevant relational asymmetry in the sense of Definition 3.4. A relational power structure from A toward B , denoted $\mathcal{P}_{A \curvearrowright B}^{\mathcal{R}}$, is established relative to a declared historical-extension family, history-weight system, persistence schema, and relational model when: (i) the associated power candidate admits a model-defined regime $\mathcal{A}_P$ whose expected-persistence profile satisfies the declared persistence schema; (ii) the candidate is causally efficacious under a well-typed comparative intervention and explicit dependence account; and (iii) the candidate is generatively efficacious by contributing to the production or modification of later relational conditions. These conditions assign relational-reality status to the power structure within the stated model and historical scope.

When the directional relata and historical configuration are fixed locally, later sections may abbreviate $\mathcal{P}_{A \curvearrowright B}^{\mathcal{R}}$ as $\mathcal{P}^{\mathcal{R}}$. The abbreviation suppresses indices only; it does not remove the directional or historical typing established by the definition.

Within the asymmetry-derived scope fixed in Section 1, Definition 4.8 makes a directional relational-power component a second-order relational formation. The antecedent asymmetry is already relationally real, yet the organization through which that asymmetry persistently deforms future possibilities is an additional structure whose reality status must be established separately. A persistent asymmetry can therefore exist without relational power if its consequences never organize into a power-relevant dynamical regime. Conversely, a power-relevant episode can fail to become relational power when the episode lacks expected persistence or fails the efficacy conditions.



Dashed boxes are optional modeling resources. Stabilization alone does not confer relational reality; expected persistence must be accompanied by causal and generative efficacy.

Figure 2: Relational realization of power. A power-relevant relational asymmetry becomes relational power only after the generated power organization satisfies an explicit expected-persistence criterion and establishes causal and generative efficacy. Attractor-like regimes, basin-like domains, and closure operators provide optional representations of stabilization and are shown with dashed connections to emphasize that none is identical with relational reality.

The definition also leaves several properties open. Relational power need not be represented by a scalar magnitude, need not generate a total comparison among GR entities, need not be intentional, and need not reproduce itself. Self-reproduction, landscape deformation, and adaptive persistence are stronger dynamical properties examined later. Likewise, the definition does not imply domination, injustice, or normative defect. Those interpretations require additional structures and evaluative premises.

Remark 4.9 (Directional components and reciprocal power). The notation $\mathcal{P}_{A \curvearrowright B}^{\mathcal{R}}$ denotes a directional component of relational power generated from the antecedent asymmetry considered in this paper. A relation in which A and B both exercise consequential power can be represented by the coexistence of $\mathcal{P}_{A \curvearrowright B}^{\mathcal{R}}$ and $\mathcal{P}_{B \curvearrowright A}^{\mathcal{R}}$. Their coexistence does not require a strict ranking between the two components. The present construction therefore does not identify power with one-way dominance or claim that every possible power relation originates from a single asymmetry-derived route.

Research Obligation 4.10 (Identification of relational power). A domain-specific application of Definition 4.8 should specify the historical-extension family, continuation weights or alternative persistence representation, regime-identification rule, persistence schema, comparative intervention used to establish causal efficacy, and the later relational conditions used to establish generative efficacy. If an attractor, basin, or closure representation is used, its additional mathematical assumptions should be stated separately.

Obligation 4.10 preserves the difference between a metaphysical proposal inside GR and an empirically or mathematically instantiated model. The next section turns to the corresponding epistemic problem. Once relational power exists, observers and participants still access it only through historically and contextually mediated manifestations, failed or enabled transformations, and other partial evidence.

5 Epistemic Access to Relational Power

This section develops the epistemic interface of the relational-power account. Its role is to specify how a power structure that satisfies Definition 4.8 can become evidentially accessible without being identified with any single observable or symbolic presentation. The discussion proceeds through five steps. It first defines context-dependent manifestations of relational power. It then characterizes latent power, for

which relational efficacy persists despite sparse or absent overt manifestation. The third subsection treats attempted transformation as an epistemic probe of power-conditioned accessibility. The fourth assembles evidence across partially ordered historical domains rather than relying on a snapshot. The final subsection formulates the resulting attribution problem as an underdetermined inverse problem. Throughout the section, epistemic access remains distinct from both ultimate ontology, which remains suspended, and relational reality, whose status was established through persistence and efficacy in Section 4.

5.1 Manifestational Evidence

Fix a coupled historical configuration $\mathcal{C}_H \in \mathfrak{C}$ and let $\mathcal{P}^{\mathcal{R}} \in \mathfrak{RP}(\mathcal{C}_H)$, where $\mathfrak{RP}(\mathcal{C}_H)$ denotes the model-defined class of relational-power structures instantiated in $\mathcal{C}_H$ according to Definition 4.8. The class notation is bookkeeping: it does not assert that all relational-power structures share one universal algebraic type beyond the structure required by the local model.

Let c denote a manifestation context. The context can encode an observational position, institutional vocabulary, symbolic order, embodiment channel, measurement protocol, analytical scale, or another condition that affects how relational power is presented. Let $\mathcal{M}_{H,c}^P$ denote the corresponding manifestation space for context-dependent presentations of relational power at historical domain H . A context-dependent manifestation map is defined in Equation 21.

$$\pi_{H,c}^P : \text{Dom}(\pi_{H,c}^P) \subseteq \mathfrak{RP}(\mathcal{C}_H) \longrightarrow \mathcal{M}_{H,c}^P. \quad (21)$$

For a relational-power structure in the domain of this map, its manifestation is written as Equation 22.

$$m_{H,c}^P := \pi_{H,c}^P(\mathcal{P}^{\mathcal{R}}). \quad (22)$$

The codomain in Equation 21 is intentionally general. An element of $\mathcal{M}_{H,c}^P$ can be numerical, categorical, symbolic, embodied, institutional, graphical, linguistic, or otherwise structured. Examples can include a title, command relation, deference pattern, restriction of standing, resource allocation, rank, prestige marker, exclusion, or another context-dependent presentation. The definition does not require every manifestation to be consciously recognized as power by the participating entities.

The manifestation map is epistemic and representational. It does not state that $\mathcal{P}^{\mathcal{R}}$ is exhausted by $m_{H,c}^P$, and it does not require $\pi_{H,c}^P$ to be injective. One relational-power structure can have different manifestations under different contexts, and several relational-power structures can share the same manifestation under a coarse context. The distinction is stated in Remark 5.1.

Remark 5.1 (Manifestation is evidence rather than identity). A manifested presentation of power is evidence generated through a declared context. Equality of manifestations does not by itself establish equality of relational-power structures, and difference of manifestations does not by itself establish distinct relational-power structures.

Remark 5.1 makes the epistemic problem explicit. Relational power belongs to the history-bearing organization of the coupled system; manifestation supplies one context-sensitive interface through which that organization can become accessible.

5.2 Latent Power

Relational power can remain efficacious when its overt manifestations are sparse. A configuration can contain a stable restriction of alternatives, an expectation of response, an institutionally supported dependency, or a power-conditioned operator domain even when no command, sanction, or visible intervention occurs at the selected slice. Latency therefore concerns the relation between relational efficacy and manifestational sparsity rather than the absence of relational power.

To represent this distinction, suppose the manifestation space $\mathcal{M}_{H,c}^P$ contains a designated element $\perp_{H,c}$ whenever the application has a meaningful representation of no detected or no retained manifestation

in context c . The symbol is application-dependent and should be omitted when such a null readout has no coherent interpretation. Observational silence at a context is expressed in Equation 23.

$$\pi_{H,c}^P(\mathcal{P}^R) = \perp_{H,c}. \quad (23)$$

Equation 23 concerns the manifestation map only. It does not negate the persistence or efficacy conditions in Definition 4.8. A relational-power structure can therefore satisfy Equation 23 in one context while remaining causally and generatively consequential in the underlying coupled configuration.

Definition 5.2 (Latent relational power). Let $\mathcal{P}^R \in \mathfrak{RP}(\mathcal{C}_H)$. The power structure is latent relative to a declared context c when the model retains its relational persistence and efficacy while the selected manifestation map yields no retained overt manifestation, or yields a manifestation whose informational content is insufficient to recover the operative relational-power structure under the declared epistemic criterion.

Definition 5.2 is context-relative. The same power structure can be latent under a public symbolic readout and manifest under an institutional-access readout. Latency also need not indicate concealment by an intentional agent. It can arise when power operates through stabilized expectations, background rules, habitual coordination, infrastructural dependency, or domain restrictions that make overt intervention unnecessary.

This observation gives a precise place to what might otherwise be described loosely as power residing in what does not happen. The relevant evidence is not the mere absence of an event. It is the relation between the absent event and a structured space of counterfactual or attempted continuations whose accessibility depends on the power configuration. The following subsection develops that relation through partially defined transformations.

5.3 Power Revealed through Attempted Transformation

Partial transformations provide an epistemic route to power structures that are weakly manifested in ordinary observation. Let $\Phi_B : \mathfrak{C} \rightarrow \mathfrak{C}$ be a model-specified transformation representing an operation whose execution by, for, or through the continuation of B is substantively relevant to the inquiry. The operation can represent an attempted entry, claim, transfer, revision, participation, resource use, institutional action, or another domain-specific transformation. Its partial domain records the configurations from which the operation is represented as available.

A failed attempt becomes informative only relative to a declared comparison. Let $\mathcal{C}_H^\circ$ be a comparison configuration obtained through an admissible intervention or reconstruction that modifies the candidate power-conditioning relation while preserving the comparability requirements of the inquiry. A transformation-access contrast is present when Equation 24 holds.

$$\mathcal{C}_H \notin \text{Dom}(\Phi_B), \quad \mathcal{C}_H^\circ \in \text{Dom}(\Phi_B). \quad (24)$$

Equation 24 states an accessibility difference. It does not yet attribute the difference uniquely to $\mathcal{P}^R$. Such an attribution requires the dependence account demanded by Definition 4.5 and must exclude or model alternative structures that can explain the domain change.

The reverse pattern is equally informative. A relational-power structure can enable an operation that would otherwise be unavailable, so that the actual configuration lies in $\text{Dom}(\Phi_B)$ while a declared comparison configuration does not. Power therefore need not appear only as restriction. It can be evidenced through privileged accessibility, protected continuation, amplified reachability, or the creation of operations available to one relational position and unavailable to another.

Remark 5.3 (Attempted transformation as an epistemic event). A failed or enabled transformation can reveal a relational boundary that was weakly visible in ordinary manifestation. The evidential content lies in the structured contrast among operation domains, configurations, and dependence assumptions, rather than in failure or success considered alone.

Remark 5.3 keeps the transformation test within the same epistemic discipline as perturbation in Section 3.2. In both cases, the model probes how a relational configuration conditions continuation. The distinction is practical: perturbation varies a represented structure to study response, whereas attempted transformation can expose a boundary through an operation whose definedness is itself part of the observed relational organization.

5.4 Historical Identification of Power Relations

Manifestations and operation-domain contrasts can be transient or context-specific. Relational power, by contrast, was defined through expected persistence and efficacy across historical continuation. Epistemic identification should therefore retain historical structure when the power claim depends on persistence, reproduction, or path-dependent effects.

Let $H \preceq H'$ denote an admissible historical extension in the history domain of the model. Define the historical interval of represented subdomains in Equation 25.

$$[H, H']_{\text{Hist}} := \{K \mid H \preceq K \preceq H'\}. \quad (25)$$

For each retained $K \in [H, H']_{\text{Hist}}$, let $\mathcal{C}_K$ denote the corresponding coupled configuration when defined. Historical evidence also requires a tracking rule for the relational-power structure itself. Let $\rightsquigarrow_{H,K}^P \subseteq \mathfrak{RP}(\mathcal{C}_H) \times \mathfrak{RP}(\mathcal{C}_K)$ be a model-specified historical-continuation relation. A pair $\mathcal{P}_H^R \rightsquigarrow_{H,K}^P \mathcal{P}_K^R$ means only that the model supplies enough continuity evidence to treat the later power structure as a continuation or descendant of the reference structure for the present epistemic inquiry; it does not require literal identity. The categorical witness-valued refinement of this tracking relation is introduced later in Section 8.1.

Fix a reference $\mathcal{P}_H^R \in \mathfrak{RP}(\mathcal{C}_H)$. For each retained K , let $\mathcal{P}_K^R$ range only over structures satisfying $\mathcal{P}_H^R \rightsquigarrow_{H,K}^P \mathcal{P}_K^R$. Let $e_{K,c}^P(\mathcal{P}_K^R)$ be a model-specified epistemic record that can contain a manifestation readout, a transformation-access result, a perturbative response descriptor, or another explicitly typed item of evidence. The record need not contain every evidential mode at every historical domain.

The historical evidence family for a power candidate is defined in Equation 26.

$$\mathcal{E}_c^P[H, H'] := \{e_{K,c}^P(\mathcal{P}_K^R) \mid K \in [H, H']_{\text{Hist}}, \mathcal{P}_H^R \rightsquigarrow_{H,K}^P \mathcal{P}_K^R, e_{K,c}^P(\mathcal{P}_K^R) \text{ is defined}\}. \quad (26)$$

Equation 26 is set-like notation for a history-indexed evidence family. An application that needs ordering among the retained records should preserve the partial order inherited from $[H, H']_{\text{Hist}}$ rather than replace it automatically with a total sequence.

Historical identification matters for at least three reasons. First, a present manifestation can be produced by a relational structure whose supporting history is no longer visible at the active boundary. Second, equal present manifestations can be supported by different power histories and can therefore imply different future operator availability or robustness under perturbation. Third, a currently weak manifestation can coexist with a highly persistent power structure when prior institutionalization has already reorganized expectations and admissible operations. The evidence family is designed to retain these distinctions without requiring an observer to reconstruct an unlimited total history.

Research Obligation 5.4 (Historical identification of power). A historical claim about relational power should specify the historical domain retained for the inquiry, the identity or transport conditions used to track the candidate power structure across that domain, the contexts under which evidence is read out, and the criteria by which transient evidence is distinguished from persistence of the relational structure itself.

Obligation 5.4 prevents present manifestations from silently standing in for the historical persistence condition established in Section 4.1. It also keeps the historical identity of a power structure separate from continuity of any one symbolic or institutional manifestation.

5.5 Underdetermination of Power Attribution

The preceding subsections imply that inference from evidence to relational power is generally an inverse problem. Manifestation maps can be many-to-one, operation-domain contrasts can admit several causal explanations, and historical evidence can remain finite relative to the relational structure being modeled. The present formalism therefore requires attribution claims to preserve the possibility of epistemic underdetermination.

Fix $\mathcal{C}_H$ and context c . Let $\mathcal{E}_{H,c}^P$ denote a model-defined evidence space whose elements can contain any combination of the evidence types defined in this section. Let $\varepsilon_{H,c}^P$ be a partial evidence map of the type stated in Equation 27.

$$\varepsilon_{H,c}^P : \text{Dom}(\varepsilon_{H,c}^P) \subseteq \mathfrak{RP}(\mathcal{C}_H) \longrightarrow \mathcal{E}_{H,c}^P. \quad (27)$$

Equation 27 is an epistemic construction, not a generative operator on the relational configuration. When the evidence map is defined for two power structures, evidence equivalence is defined in Equation 28.

$$\mathcal{P}_1^{\mathcal{R}} \sim_{\varepsilon,H,c} \mathcal{P}_2^{\mathcal{R}} \iff \varepsilon_{H,c}^P(\mathcal{P}_1^{\mathcal{R}}) = \varepsilon_{H,c}^P(\mathcal{P}_2^{\mathcal{R}}). \quad (28)$$

The equivalence relation in Equation 28 is relative to the selected evidence map. It does not assert relational identity. A richer evidence space can refine an equivalence class that appears homogeneous under a coarse readout.

For an observed evidence object $e \in \mathcal{E}_{H,c}^P$, the compatible power fiber is defined in Equation 29.

$$\text{Fib}_\varepsilon(e) := \{ \mathcal{P}^{\mathcal{R}} \in \text{Dom}(\varepsilon_{H,c}^P) \mid \varepsilon_{H,c}^P(\mathcal{P}^{\mathcal{R}}) = e \}. \quad (29)$$

If the fiber in Equation 29 contains several distinct relational-power structures under the identity criterion of the model, the evidence is insufficient for unique power attribution. This underdetermination can persist even when the evidence is extensive. Additional manifestations, perturbations, operation-domain contrasts, or historical records can reduce the compatible fiber without logically guaranteeing that it becomes a singleton.

Table 3 summarizes the principal epistemic routes introduced in this section and the inferential limits attached to each one.

Definition 5.5 (Epistemically underdetermined power attribution). A power attribution based on evidence e is epistemically underdetermined relative to $(\varepsilon_{H,c}^P, \mathcal{C}_H)$ when $\text{Fib}_\varepsilon(e)$ contains more than one relational-power structure that the model treats as distinct.

Definition 5.5 does not imply skepticism about power. It states that relational reality can be better determined than a selected epistemic interface. The same distinction also blocks the reverse error of treating a strong manifestation as sufficient evidence of a correspondingly strong relational power structure. Comparative judgments such as “more powerful than” require a further comparison structure over manifestations, response profiles, relational-power structures, or combinations of them. The subsequent section on comparative power structures will construct that layer explicitly.

The epistemic architecture developed here places a constraint on the next analytical step. A statement that one GR spacetime entity is more powerful than another cannot be read directly from a manifestation, a blocked operation, or a response profile. Such a statement requires a comparison relation that determines what is being compared, on which historical and manifestational domain, and under which ordering or metric structure. The next section therefore treats power comparison itself as a generated relational object whose status must be established independently.

Table 3: Epistemic routes to relational power and their inferential limits.

Epistemic route	Primary evidential contribution	Inference not established by the route alone
Manifestational readout	Context-dependent presentation of a relational-power structure through symbolic, institutional, embodied, numerical, or other forms	Identity, magnitude, or uniqueness of the underlying relational-power structure
Latent-power analysis	Separation of relational efficacy from sparse or absent overt manifestation	Presence of a particular hidden power structure without independent efficacy evidence
Attempted transformation	Evidence from enabled or blocked operation domains relative to a declared comparison configuration	Unique attribution of the access difference to one power structure without a dependence account
Historical evidence family	Persistence, path dependence, and changing manifestations across partially ordered historical domains	Exhaustive recovery of the complete relational history or a unique causal genealogy
Evidence-fiber analysis	Explicit representation of the set of relational-power structures compatible with selected evidence	Elimination of underdetermination unless the chosen evidence map is independently shown to discriminate the relevant structures

6 Comparative Structures in Power Recognition

A relational-power structure can be dynamically consequential before a system possesses any settled rule for declaring one GR entity “more powerful” than another. The role of this section is to formalize that additional comparative layer without converting power into a primitive scalar ranking. The section first separates history-bearing GR entities from the representations selected for comparison, then defines partial comparison operations and their judgment objects. It subsequently distinguishes proto-relational, manifestational, and relationally real statuses of comparative structures, introduces metric and quasi-metric constructions as optional comparative resources, and concludes by showing how a comparison judgment can become performative and feed back into later power relations. The analysis is formal and historical: no comparison rule, ranking, or metric is treated as prior to the relational processes through which it becomes available and consequential.

6.1 Comparison Operations over GR Spacetimes

The relata compared in this paper remain the history-bearing GR entities introduced in Section 2.1. Let $\mathfrak{S}_{\text{GR}}(\mathcal{C}_H) \subseteq \mathfrak{S}_{\text{GR}}$ denote the GR entities represented in a coupled historical configuration $\mathcal{C}_H$. A comparison of two such entities cannot be defined merely by placing a greater-than sign between their names. The comparison must first specify which relational-power structures, manifestations, response profiles, or other features of the two spacetimes are retained for the comparison.

Let β denote a declared power-comparison basis. The basis identifies the relational and epistemic material admitted to a particular comparison. Its associated comparison-representation map is the partial map stated in Equation 30.

$$\zeta_{H,c}^{P,\beta} : \mathfrak{S}_{\text{GR}}(\mathcal{C}_H) \rightarrow \mathcal{Z}_{H,c}^{P,\beta}. \quad (30)$$

The map in Equation 30 takes values in the model-defined comparison space $\mathcal{Z}_{H,c}^{P,\beta}$. Its elements can retain, according to β , relational-power structures, selected manifestations, perturbative response profiles, operation-access structures, historical evidence, or combinations of these. The context c is included when

manifestational or epistemic conditions contribute to the comparison. The map is partial because one basis need not yield a meaningful representation for every GR entity or every historical configuration.

For entities $A, B \in \text{Dom}(\zeta_{H,c}^{P,\beta})$, define the corresponding comparison inputs in Equation 31.

$$z_A^{P,\beta;H,c} := \zeta_{H,c}^{P,\beta}(A), \quad z_B^{P,\beta;H,c} := \zeta_{H,c}^{P,\beta}(B). \quad (31)$$

Equation 31 preserves the distinction between the history-bearing spacetimes and the comparison representations constructed from them. Two GR entities can remain distinct even when a coarse basis maps them to the same comparison input, and one entity can admit several comparison representations under different bases.

Let $\mathcal{J}_{H,c}^{P,\beta}$ denote a judgment space whose elements record the comparative statuses admitted by the model. The power-comparison operation is the partial map in Equation 32.

$$\kappa_{H,c}^{P,\beta} : \text{Dom}(\kappa_{H,c}^{P,\beta}) \subseteq \mathcal{Z}_{H,c}^{P,\beta} \times \mathcal{Z}_{H,c}^{P,\beta} \longrightarrow \mathcal{J}_{H,c}^{P,\beta}. \quad (32)$$

For a pair in the domain of this operation, the resulting judgment is defined in Equation 33.

$$j_{AB;H,c}^{P,\beta} := \kappa_{H,c}^{P,\beta} \left(z_A^{P,\beta;H,c}, z_B^{P,\beta;H,c} \right). \quad (33)$$

The judgment space can contain directional comparison, equivalence, incomparability, indeterminacy, or another application-defined status. The present formalism does not require a total order. In particular, the existence of relational power from A toward B , from B toward A , or in both directions does not by itself establish a globally meaningful statement that one entire GR spacetime is more powerful than the other.

Definition 6.1 (Power-comparison operation). A power-comparison operation is a partial map of the type in Equation 32, together with a declared comparison basis β and a specified interpretation of its judgment space. A directional judgment such as “ A is more powerful than B ” is licensed only when the comparison representations of both GR entities are defined and the corresponding pair lies in the domain of the operation.

Definition 6.1 separates power comparison from the response comparison introduced in Equation 15. The latter compares two directional response-profile families under a declared salience schema. The present construction is more general: a comparison basis can include those profiles, yet it can also retain historical, manifestational, institutional, or other relational material. Neither construction is treated as a primitive order on GR entities.

6.2 Proto-Relational Comparison Judgments

A formally generated comparison judgment need not be stable, publicly manifested, or causally consequential. The present paper therefore assigns such judgments an initial formal location without granting them relational-reality status. Let $\mathcal{Q}_{H,c}^{P,\text{cmp}}$ denote the proto-relational domain of power-comparison candidates associated with historical domain H and context c . This domain is analogous in formal role to the proto-relational power domain $\mathcal{Q}_H^P$ introduced in Section 4.4, while its elements concern comparative structures rather than candidate power structures themselves.

When a judgment from Equation 33 has been generated but its persistence and efficacy remain unestablished, the present formalism can record it as in Equation 34.

$$j_{AB;H,c}^{P,\beta} \in \mathcal{Q}_{H,c}^{P,\text{cmp}}. \quad (34)$$

Equation 34 records a formal status rather than a fourth layer of reality. A comparison can be analytically coherent and locally useful while remaining short-lived, weakly reproduced, or dynamically inert. An analyst’s one-time ranking, a provisional diplomatic assessment, or a local classification used by one institution can therefore enter the proto-relational domain without being promoted immediately to relational reality.

A proto-relational judgment can also remain incomparable with another judgment. The formalism does not require a higher-order rule that ranks all comparison norms against one another. If several institutions, symbolic orders, or analytical frameworks use different bases $\beta_1, \beta_2, \dots$, the resulting judgments can coexist while preserving their distinct domains and semantics.

Proto-relational and manifestation status are not mutually exclusive. A comparison judgment can be publicly or institutionally manifested while its relational-reality status remains unresolved. Within one declared historical scope, the label *proto-relational* is withdrawn once relational reality has been established for that comparison structure, although the same structure can continue to possess a manifestation role.

Remark 6.2 (Comparison status and truth conditions). Proto-relational status concerns the historical and dynamical status of a comparison structure, not the logical truth or falsity of its content. A judgment can be well supported yet dynamically inert, or poorly supported yet socially consequential. The epistemic adequacy of a judgment and the relational reality of the comparison norm are therefore separate questions.

Remark 6.2 prevents a relational-reality criterion from becoming an epistemic validation rule. The distinction becomes important when a symbolic ranking acquires institutional consequences despite being an incomplete or contested representation of underlying power relations.

6.3 Manifested Comparison Structures

Comparative power judgments can themselves appear in manifestation reality. A title, rank, index, map, doctrine, public category, or institutional classification can make a comparison socially legible even when the relation it expresses remains a coarse or contested representation of relational power. Let $\mathcal{M}_{H,c}^{P,\text{cmp}}$ denote the manifestation space of power-comparison structures under context c .

A comparison-manifestation map is defined in Equation 35.

$$\pi_{H,c}^{P,\text{cmp}} : \text{Dom}(\pi_{H,c}^{P,\text{cmp}}) \subseteq \mathcal{Q}_{H,c}^{P,\text{cmp}} \longrightarrow \mathcal{M}_{H,c}^{P,\text{cmp}}. \quad (35)$$

For a comparison judgment in the domain of this map, its manifested comparison is written in Equation 36.

$$m_{AB;H,c}^{P,\text{cmp}} := \pi_{H,c}^{P,\text{cmp}} \left(j_{AB;H,c}^{P,\beta} \right). \quad (36)$$

The codomain can contain symbolic orderings such as office hierarchies, numerical rankings, great-power classifications, prestige orderings, or other context-dependent forms through which comparative power becomes visible and actionable. A symbol such as “president,” for example, can participate in a manifested ordering of offices. A cross-national comparison can similarly use gross domestic product, territorial size, military indicators, or another selected manifestation as evidence within a declared basis. None of these manifestations exhausts the history-bearing GR entities being compared.

Manifested comparison has two analytically distinct roles. First, it can provide an epistemic interface through which participants recognize or contest a power ordering. Second, once the manifested comparison enters decisions, expectations, rules, or allocation procedures, it can become generatively consequential. The second role creates the possibility that a comparison structure later acquires relational reality.

Table 4 separates the three statuses developed so far and the additional condition required for relationalization.

The distinction in Table 4 permits a symbolic hierarchy to be manifestationally stable while still lacking enough efficacy to count as a relationally real comparison norm. It also permits the reverse pattern: a comparison rule can become deeply institutionalized and relationally consequential while its overt symbolic expression changes across contexts.

6.4 Relationalization of Comparative Norms

For the power-comparison structures examined here, the paper applies the same stricter relationalization discipline used for other power-related structures: historical persistence is represented through a declared

Table 4: Reality status of power-comparison structures.

Status	Formal role	Condition not supplied by that status alone
Proto-relational comparison	A generated comparison judgment or norm candidate recorded in $\mathcal{Q}_{H,c}^{P,\text{cmp}}$	Historical persistence, manifestation, causal efficacy, or generative efficacy
Manifested comparison	A context-dependent symbolic, numerical, institutional, or other presentation in $\mathcal{M}_{H,c}^{P,\text{cmp}}$	Identity with underlying relational power or relational-reality status of the comparison norm
Relationally real comparison	A comparison structure that persists and becomes causally and generatively consequential in later relational organization	Epistemic correctness, universality, or validity outside the declared historical and relational domain

persistence criterion, and causal efficacy and generative efficacy must be established for the comparison structure itself. When the history-weight construction of Section 4 is appropriate, the persistence criterion can take the form of expected persistence; other well-typed persistence representations remain possible. The relevant candidate is therefore the comparative norm or ordering practice, rather than the relational-power structure it purports to describe.

Let $q^{\text{cmp}} \in \mathcal{Q}_{H,c}^{P,\text{cmp}}$ denote a comparison candidate. Suppose an admissible historical extension $H \hookrightarrow H'$ permits the candidate to be tracked under model-specified identity or transport conditions. The candidate is causally efficacious when a well-typed intervention on the operative comparison structure changes a declared relational consequence under an explicit dependence account. It is generatively efficacious when its operation contributes to creating or modifying later relational conditions, including standing, resource allocation, admissible operations, institutional access, symbolic expectations, or subsequent power relations.

Definition 6.3 (Relationalized power-comparison norm). A power-comparison candidate q^{cmp} acquires relational reality over a declared historical scope when the model establishes: (i) persistence of the comparison structure under an explicitly stated historical persistence criterion; (ii) causal efficacy of the comparison structure with respect to at least one declared relational consequence; and (iii) generative efficacy through contribution to the production or modification of later relational organization. Expected persistence is one admissible implementation of condition (i) when the required continuation-weight structure has been defined. The resulting structure is denoted $J^{P,\mathcal{R}}$ within the stated model and historical domain.

Definition 6.3 does not imply that the comparison norm accurately measures every relevant dimension of relational power. It states that the norm has become part of relational reality because it persists and changes subsequent relations. A socially effective great-power classification, office hierarchy, status ranking, or institutional index can therefore be relationally real even when its epistemic adequacy remains disputed.

The distinction also allows a manifested comparison to acquire relational-reality status without requiring a single fixed symbol or ceasing to function as a manifestation. A ranking practice can persist through changing labels, measures, or institutional carriers when enough of its relational organization remains stable to support the identity conditions of the model. Conversely, a durable symbol can survive while losing the causal and generative efficacy required for relational-reality status.

Remark 6.4 (Comparison norms can participate in power formation). Once a power-comparison norm becomes relationally real, it can alter the conditions under which power is recognized, allocated, reproduced, or contested. Relationalizing a comparison norm therefore changes more than the epistemic description of an already fixed power landscape; it can become one of the structures through which that landscape evolves.

Remark 6.4 supplies the bridge to the reflexive analysis developed at the end of this section.

6.5 Metric Structures of Manifested Power

Numerical comparison is one special case of the broader architecture. A model can construct a metric-like representation from manifested or relationally retained features without treating the resulting number as an intrinsic quantity stored inside a GR spacetime entity. Let $\mathcal{X}_{H,c}^{P,\beta}$ denote a declared numerical or structured feature space obtained from a comparison basis β , and let $\eta_{H,c}^{P,\beta} : \mathcal{Z}_{H,c}^{P,\beta} \rightarrow \mathcal{X}_{H,c}^{P,\beta}$ be the associated feature representation.

When a symmetric metric is justified, the model may define a distance function of the type in Equation 37.

$$d_{H,c}^{P,\beta} : \mathcal{X}_{H,c}^{P,\beta} \times \mathcal{X}_{H,c}^{P,\beta} \longrightarrow \mathbb{R}_{\geq 0}. \quad (37)$$

Calling Equation 37 a metric requires the application to establish the usual identity, symmetry, and triangle conditions on its declared domain. Power relations, however, are frequently directional. A comparison of the difficulty, cost, or resistance with which A can reshape B 's continuation relative to the reverse direction can therefore require a quasi-metric or another asymmetric enriched structure for which $d(x, y) = d(y, x)$ is not assumed.

Remark 6.5 (A metric is a representation before it is a relational institution). The mathematical function $d_{H,c}^{P,\beta}$ introduced by an analyst belongs first to the formal modeling level. An institutionally implemented metric, index, ranking, or scoring practice can additionally possess manifestation reality. Such an operative metric can acquire relational reality only when its historical persistence and causal or generative efficacy are established independently.

Remark 6.5 is essential for the later international-political case study. Gross domestic product, territorial size, military indicators, composite indexes, and similar quantities can contribute to a manifested comparison of states. Their availability does not establish a universal metric of relational power. If actors and institutions repeatedly use such a measure to allocate resources, recognize status, structure alliances, or anticipate behavior, the operative metric practice can itself become a relationally real part of the power configuration.

A metric practice can therefore exhibit the same nonexclusive status pattern as a nonnumerical comparison norm. It can begin as an analytical construction and later acquire a public or institutional manifestation. If its historical persistence and the required efficacy are independently established, the operative metric practice can additionally acquire relational reality while continuing to function through manifested forms. This status claim concerns the historically instantiated metric practice, not the abstract mathematical formula considered apart from its use.

6.6 Reflexive Power Recognition

The comparison architecture becomes dynamically significant when a judgment changes the configuration from which later power relations develop. Let $j_{AB;H,c}^{P,\beta}$ be a comparison judgment generated by Equation 33. A performative use of that judgment can be represented by a judgment-conditioned generative transformation $\Phi_{P,\text{cmp}}^{[j]}$ of the type stated in Equation 38.

$$\Phi_{P,\text{cmp}}^{[j]} : \text{Dom}(\Phi_{P,\text{cmp}}^{[j]}) \subseteq \mathfrak{C} \longrightarrow \mathfrak{C}. \quad (38)$$

The superscript $[j]$ records that the concrete transformation is conditioned by the comparison judgment; it does not identify the judgment with the transformation. The operator can represent, depending on the application, changed recognition, allocation, access, deference, institutional standing, alliance choice, or another consequence of acting on the comparison.

If $\mathcal{C}_H \in \text{Dom}(\Phi_{P,\text{cmp}}^{[j]})$, the resulting configuration is stated in Equation 39.

$$\mathcal{C}_{H'} = \Phi_{P,\text{cmp}}^{[j]}(\mathcal{C}_H), \quad H \hookrightarrow H'. \quad (39)$$

Equation 39 gives a formal route by which a comparison can participate in generating the later power landscape. The relational power structures represented in $\mathcal{C}_{H'}$ can differ from those represented in $\mathcal{C}_H$ because actors or institutions have responded to the comparison itself.

Definition 6.6 (Reflexive power-recognition loop). A reflexive power-recognition loop is present over a declared historical extension when (i) a power-comparison operation generates a judgment from representations of the current relational configuration; (ii) the judgment enters a manifested or relationally real comparison structure; (iii) a judgment-conditioned generative transformation changes the later relational configuration; and (iv) the changed configuration modifies at least one power relation, power manifestation, or later comparison input used by the same class of recognition process.

Figure 3 summarizes the architecture developed in this section. The upper path distinguishes GR spacetime entities from their comparison representations and judgments. The lower branches show distinct, potentially overlapping status roles: a judgment can remain proto-relational while its relational status is unresolved, can be stabilized in manifestation reality, and can additionally acquire relational reality when the required historical and efficacy conditions are established. The feedback path represents a performative comparison that changes the later configuration. The figure does not collapse these statuses into a mandatory or mutually exclusive sequence.

Definition 6.6 allows comparison to become self-reinforcing without assuming that every recognized hierarchy reproduces itself. A judgment can also weaken an existing power relation, create contestation, redirect resources, or alter the comparison basis used later. The direction of the feedback is therefore an empirical and model-specific question.

The section therefore establishes a second-order feature of the proposed metaphysics of power. Relational power can shape the conditions under which comparison norms emerge, while comparison norms can in turn become part of the relational structures through which power is recognized and reproduced. The next section develops the mediation processes through which relational power and comparative structures acquire symbolic, embodied, institutional, and other forms of manifestation.

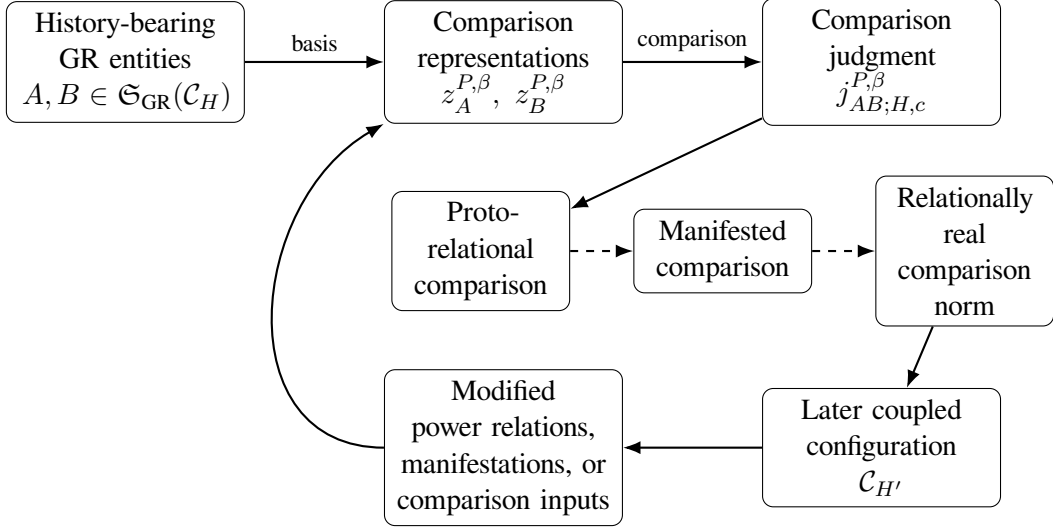


Figure 3: Reality-status pathways and reflexive feedback of power-comparison structures. A comparison judgment over history-bearing GR entities can remain proto-relational while its relational status is unresolved, can acquire a context-dependent manifestation, and can additionally become relationally real when the required persistence and efficacy are established. Manifestation and relational-reality roles can coexist. The displayed feedback path shows how an operative comparison norm can change a later relational configuration and thereby alter subsequent power relations or comparison inputs. Direct performative use of a judgment before relationalization remains separately represented by Equation 38. Dashed arrows indicate conditional status transitions under additional stated conditions.

7 Mediation of Relational Power into Manifestation

This section develops the mediation layer between relational power and its context-dependent manifestations. Its role is to refine the manifestation map introduced in Section 5 by making explicit the structures through which a relationally real power structure becomes symbolically, bodily, institutionally, or otherwise presentable. The objective is twofold. First, the section distinguishes mediation from manifestation: a mediator organizes how a relational structure is carried into a presentation, while the resulting manifestation is the context-dependent presentation thereby produced. Second, the section specifies how a manifestation can become dynamically consequential and, under stronger historical conditions, participate in the constitution of a later relationally real structure. Particular attention is given to one symbolic route in which an historically generated equivalence relation suppresses selected relational differences, a quotient represents the resulting equivalence classes, and symbolization makes those classes manifest. The discussion proceeds through symbolic mediation, relational quotienting, embodied and institutional mediation, manifestation-conditioned transformation, and a relationalization criterion. The construction remains representational and maintains ontological suspension regarding the ultimate status of every mediation channel, equivalence structure, or quotient representation.

7.1 Symbolic Mediation

The manifestation map in Equation 21 records a typed relation between relational power and a context-dependent presentation. For some inquiries, that direct map is sufficient. A finer analysis can resolve the process into a mediation channel and a presentation map. Let $\mathfrak{Med}_{H,c}^P$ denote a model-defined class of mediation channels available for power manifestations in historical domain H and context c . An element $\nu \in \mathfrak{Med}_{H,c}^P$ labels one such channel. The label can refer to a symbolic, embodied, institutional, spatial, technological, or other mediation structure when the application supplies its semantics.

For a declared channel ν , let $\mathcal{U}_{H,c}^{P,\nu}$ denote the corresponding intermediate mediation space. The channel

map is the partial map in Equation 40.

$$\mu_{H,c}^{P,\nu} : \text{Dom}(\mu_{H,c}^{P,\nu}) \subseteq \mathfrak{RP}(\mathcal{C}_H) \longrightarrow \mathcal{U}_{H,c}^{P,\nu}. \quad (40)$$

A second partial map carries a mediated structure into the manifestation space already introduced in Section 5.1. Its type is stated in Equation 41.

$$\rho_{H,c}^{P,\nu} : \text{Dom}(\rho_{H,c}^{P,\nu}) \subseteq \mathcal{U}_{H,c}^{P,\nu} \longrightarrow \mathcal{M}_{H,c}^P. \quad (41)$$

Whenever the two maps are composable on a relational-power structure $\mathcal{P}^{\mathcal{R}}$, the channel-specific manifestation is defined by Equation 42.

$$\pi_{H,c}^{P,\nu}(\mathcal{P}^{\mathcal{R}}) := \rho_{H,c}^{P,\nu}(\mu_{H,c}^{P,\nu}(\mathcal{P}^{\mathcal{R}})). \quad (42)$$

Equation 42 is a factorization available when a model has reason to distinguish the mediator from the manifested output. The domain of the composite consists of those elements of $\text{Dom}(\mu_{H,c}^{P,\nu})$ whose mediated image lies in $\text{Dom}(\rho_{H,c}^{P,\nu})$. Several channels can contribute jointly to one manifestation, and uniqueness of the factorization is left open.

Symbolic mediation is represented by the special channel label $\nu = \text{sym}$. The corresponding intermediate structure can include titles, offices, credentials, legal categories, prestige markers, classifications, public narratives, or other symbols through which a relational power structure becomes intelligible within a symbolic order. The symbolic structure need not reproduce the relational-power structure with full fidelity. It can retain a coarse distinction, amplify one dimension, suppress another, or translate several heterogeneous relations into one socially legible category.

One important route to this compression is developed in Subsection 7.2. An equivalence relation can first identify relational-power structures as equivalent for a declared symbolic purpose; quotienting then suppresses distinctions outside that equivalence, and a symbolization map carries the resulting class into a symbolic mediation space. This route is optional. Symbolic mediation can also proceed through analogy, narrative, indexical association, or another declared structure for which quotienting is not appropriate.

Definition 7.1 (Symbolically mediated power manifestation). Let $\mathcal{P}^{\mathcal{R}} \in \mathfrak{RP}(\mathcal{C}_H)$. A symbolically mediated power manifestation under context c is an element of $\mathcal{M}_{H,c}^P$ obtained through a composable factorization of the form in Equation 42 with $\nu = \text{sym}$, where the intermediate mediation space is explicitly interpreted as a symbolic structure.

Definition 7.1 assigns symbolic order a mediation role. Constitutive status requires the additional persistence and efficacy conditions used for relational reality. A title can manifest a power structure that already has relational reality, while the same title can remain weakly connected to relational efficacy in another historical configuration. The stronger possibility in which symbolic mediation subsequently changes relational reality is developed in Subsection 7.5.

7.2 Relational Quotienting in Symbol Formation

A symbolic manifestation can arise after a relational system has already developed a rule for treating several history-bearing structures as equivalent for a declared purpose. This possibility requires a distinction between the equivalence relation that organizes the source structures, the mathematical quotient used to represent its classes, and the symbol that presents a selected class in manifestation reality. The equivalence relation can remain an analytic construction introduced by a modeler, can acquire a context-dependent manifested role when enacted in practice, and can additionally acquire relational reality when its historical identity and type-appropriate efficacy are sufficiently established. Manifestation and relational-reality roles are therefore not treated as mutually exclusive.

Let $\mathfrak{RP}_{H,c}^{\text{sym}}(\mathcal{C}_H) \subseteq \mathfrak{RP}(\mathcal{C}_H)$ denote the model-selected class of relational-power structures retained as possible inputs to a symbolic quotient construction. Let $\sim_{H,c}^{P,\text{sym}}$ be an equivalence relation on this source

class. When reflexivity, symmetry, and transitivity are established on the declared domain, the corresponding quotient map is stated in Equation 43.

$$q_{H,c}^{P,\text{sym}} : \mathfrak{RP}_{H,c}^{\text{sym}}(\mathcal{C}_H) \longrightarrow \mathfrak{RP}_{H,c}^{\text{sym}}(\mathcal{C}_H) / \sim_{H,c}^{P,\text{sym}}, \quad \mathcal{P}^{\mathcal{R}} \longmapsto [\mathcal{P}^{\mathcal{R}}]_{\sim}. \quad (43)$$

Equation 43 is a formal quotient representation. Its existence does not establish that $\sim_{H,c}^{P,\text{sym}}$ is relationally real. An equivalence relation introduced only to simplify analysis remains part of the modeling level even when the quotient is mathematically well defined.

Let $\mathcal{U}_{H,c}^{P,\text{sym}}$ retain the symbolic mediation space introduced in Subsection 7.1. A symbolization map on a declared subset of quotient classes is typed in Equation 44.

$$\sigma_{H,c}^{P,\text{sym}} : \text{Dom}(\sigma_{H,c}^{P,\text{sym}}) \subseteq \mathfrak{RP}_{H,c}^{\text{sym}}(\mathcal{C}_H) / \sim_{H,c}^{P,\text{sym}} \longrightarrow \mathcal{U}_{H,c}^{P,\text{sym}}. \quad (44)$$

When $\mathcal{P}^{\mathcal{R}}$ lies in the quotient source and its equivalence class lies in the domain of $\sigma_{H,c}^{P,\text{sym}}$, the symbolic mediation map from Equation 40 can be refined as stated in Equation 45.

$$\mu_{H,c}^{P,\text{sym}}(\mathcal{P}^{\mathcal{R}}) = \sigma_{H,c}^{P,\text{sym}}\left(q_{H,c}^{P,\text{sym}}(\mathcal{P}^{\mathcal{R}})\right). \quad (45)$$

Equation 45 makes one symbolic route explicit: relational differentiation is reduced according to a declared equivalence structure before a common symbol is assigned. An office title such as “president,” for example, can be modeled by treating historically distinct power-bearing configurations as equivalent with respect to a declared office-position relation and then symbolizing the corresponding equivalence class. The example concerns formal architecture only; the title does not exhaust the relational power of an office holder or fix a universal comparison of political power.

Definition 7.2 (Relationally real equivalence structure). An equivalence relation $\sim_{H,c}$ on a declared class of GR relational structures is represented as relationally real over a specified historical family when (i) the identity of its equivalence classes is persistent under an explicitly stated historical persistence criterion; (ii) class membership has established causal efficacy for at least one retained relational consequence; and (iii) the equivalence structure has generative efficacy in the reproduction, creation, restriction, or transformation of later relations or operation domains. Expected persistence is one admissible implementation of condition (i) when the required continuation-weight structure has been defined.

Definition 7.2 applies the same status discipline used elsewhere in the paper. A classification practice can remain a transient analytic partition, can acquire a stable manifestation role through repeated symbols or records, and can additionally acquire relational reality when the equivalence itself begins to organize later relational evolution. The latter status does not erase the manifested role through which the classification can remain socially legible. The mathematical quotient in Equation 43 remains a representation at the modeling level even in the last case; what acquires relational reality is the historically operative equivalence structure and the relations through which its class distinctions are reproduced.

Remark 7.3 (Quotienting requires an equivalence structure). The quotient construction in this subsection is available only when the declared relation satisfies the conditions required for the chosen quotient. Context-dependent similarity, tolerance, preorder, partial-equivalence, or local-groupoid structure may require a different formal representation. The paper therefore treats quotienting as one important route to symbolic manifestation rather than as a universal mechanism of symbol formation.

Remark 7.3 preserves the distinction between symbolic compression and mathematical quotienting. It also creates a direct interface with the categorical architecture in Section 8, where the quotient route is represented without assigning categorical structure to ultimate ontology.

7.3 Embodied Mediation

Relational power can become manifest through recurrent bodily and practical organization before, or without, explicit symbolic articulation. The channel label $\nu = \text{emb}$ will denote an embodied mediation channel. Its intermediate space $\mathcal{U}_{H,c}^{P,\text{emb}}$ can retain posture, spatial orientation, turn-taking, habitual yielding, approach or withdrawal, patterned attention, or other application-defined forms of embodied responsiveness. These examples specify possible semantics of the channel; they do not imply that every bodily regularity is evidence of power.

The same factorization in Equation 42 applies with $\nu = \text{emb}$. The analytical distinction is that the intermediate mediator need not contain a propositional judgment of the form “ A is more powerful than B .” An embodied manifestation can therefore provide evidence of a relational-power structure even when the participants do not explicitly name the relation as power.

Remark 7.4 (Pre-symbolic manifestation). An embodied or habitual manifestation can precede explicit symbolic articulation while remaining historically organized and context-dependent. Pre-symbolic manifestation in this sense concerns the channel of presentation. It does not imply that the underlying power structure lacks relational reality or that symbolic mediation will necessarily arise later.

Remark 7.4 preserves the distinction between the reality status of power and the representational form through which power becomes accessible. A historically persistent power relation can be weakly symbolized and strongly embodied; another can be strongly symbolized while bodily responses remain heterogeneous. The comparison among channels therefore requires an explicit observational or interpretive criterion rather than a presumed hierarchy of manifestations.

Embodied mediation also creates an epistemic difficulty. A similar bodily pattern can arise from trust, coordination, convention, fear, physical constraint, strategic accommodation, or several relations at once. The underdetermination result in Section 5.5 therefore continues to apply. Embodied evidence can strengthen or weaken a relational-power interpretation when combined with historical and intervention evidence, yet the manifestation alone does not uniquely identify the underlying relation.

7.4 Institutional Mediation

Institutional mediation concerns structures through which relational power is made effective and legible by rules, offices, procedures, infrastructures, classifications, or stabilized organizational practices. The channel label $\nu = \text{inst}$ denotes this case. Its intermediate mediation space can contain, for example, recognized standing, office competence, jurisdiction, credentialed authority, resource-control interfaces, procedural gates, or classifications that determine which operations are available from a configuration.

Institutional mediation is closely related to the configuration components introduced earlier in the paper. A relational-power structure can be manifested through changes in the active relational boundary, the operative nomos, or the admissible operator family. To keep the role of the mediator explicit, let $\text{Adm}(\mathcal{C}_H)$ denote the application-defined admissibility profile extracted from the coupled configuration. The profile can include the domains of selected transformations, institutional permissions, or other declared accessibility information. An institutional mediator is dynamically consequential when its operation changes this profile over an admissible historical extension. A schematic change is stated in Equation 46.

$$\text{Adm}(\mathcal{C}_H) \mapsto \text{Adm}(\mathcal{C}_{H'}), \quad H \hookrightarrow H'. \quad (46)$$

Equation 46 does not assign the change uniquely to power. The attribution requires the same dependence discipline used in Definition 3.4. Its role is to identify a location at which institutional mediation can carry a relational power structure into later accessibility conditions.

Definition 7.5 (Institutionally mediated power manifestation). An institutionally mediated power manifestation is a channel-specific manifestation obtained through Equation 42 with $\nu = \text{inst}$, where the declared mediator consists of institutional relations that organize the presentation or operation of power. The manifestation can be evidential, performative, or both, depending on whether the mediated structure also changes the subsequent coupled configuration.

The final sentence of Definition 7.5 prepares a distinction used in the next subsection. An institutional title printed on a record can function primarily as a manifestation. The same title can also activate a domain of operations, authorize resource use, or change standing. The latter case gives manifestation a generative role in subsequent relational evolution.

Table 5 summarizes the three mediation channels developed in this section. The table records the analytical roles needed here. Additional channels can be introduced when their domains and effects are defined.

7.5 Transformative Effects of Manifestation

A manifestation becomes dynamically consequential when acting on, through, or in response to that manifestation changes the coupled relational configuration. Let $m_{H,c}^{P,\nu} := \pi_{H,c}^{P,\nu}(\mathcal{P}\mathcal{R})$ be a channel-specific manifestation defined by Equation 42. A manifestation-conditioned generative transformation is a partial map of the type in Equation 47.

$$\Phi_{P,\nu}^{[m]} : \text{Dom}(\Phi_{P,\nu}^{[m]}) \subseteq \mathfrak{C} \longrightarrow \mathfrak{C}. \quad (47)$$

The superscript $[m]$ records that the concrete transformation is conditioned by the manifested structure. It does not identify the manifestation with the transformation. If $\mathcal{C}_H \in \text{Dom}(\Phi_{P,\nu}^{[m]})$, the resulting historical extension is written in Equation 48.

$$\mathcal{C}_{H'} = \Phi_{P,\nu}^{[m]}(\mathcal{C}_H), \quad H \hookrightarrow H'. \quad (48)$$

Equation 48 provides a typed route for manifestation to re-enter relational dynamics. A symbolic title can alter recognition or resource access; an embodied pattern can stabilize expectations that condition later interaction; an institutional classification can open or close a transformation domain. The formalism records the configuration change without requiring every manifestation to have such effects.

Definition 7.6 (Performative power manifestation). A power manifestation $m_{H,c}^{P,\nu}$ is performative relative to a declared model when there exists a manifestation-conditioned transformation of the type in Equation 47 whose application over an admissible historical extension changes at least one relational structure, admissibility condition, generative possibility, power manifestation, or comparison input retained by the model.

Definition 7.6 separates visibility from efficacy. A manifestation can be richly articulated yet dynamically weak. Another can be minimally articulated and strongly performative because institutional or habitual responses make it consequential for later relations. This separation also clarifies why symbolic mediation can amplify power without being the source of every relational-power structure.

The transformation in Equation 48 can alter the very conditions under which the original power relation remains effective. Manifestation can therefore support several higher-order effects: stabilization of an existing power regime, attenuation through contestation, translation into another power dimension, redistribution across relational positions, or generation of a new supporting structure. These effects are possibilities of the formal architecture. Their direction and magnitude require application-specific evidence and comparison criteria.

Figure 4 summarizes the mediation and feedback architecture. The mediation box collects the symbolic, embodied, and institutional channels developed above, while the lower path shows how a manifestation can become performative through a separately typed generative transformation. The dashed arrow indicates the conditional relationalization developed in Subsection 7.6.

7.6 Relationalization of Manifestation

A manifestation can participate in the formation of later relational reality without collapsing the distinction between manifestation reality and relational reality. The transition is historical and conditional. Let

Table 5: Mediation channels for relational power and their analytical roles.

Mediation channel	Intermediate organization	Possible relation to later dynamics
Symbolic	Titles, offices, credentials, categories, prestige markers, narratives, or other symbolically organized structures; one route can pass through relational equivalence, quotienting, and symbolization	Can articulate, compress, amplify, or redirect the socially legible presentation of relational power; later effects require a generative coupling
Embodied	Bodily orientation, habitual yielding, recurrent attention, approach or withdrawal, spatial practice, or other embodied patterns	Can manifest power before explicit naming; later effects depend on whether the pattern changes subsequent relations or operation domains
Institutional	Standing, jurisdiction, procedure, organizational role, infrastructure, classification, or operation-access structure	Can make power legible while also changing permissions, admissibility, allocation, or later relational positions

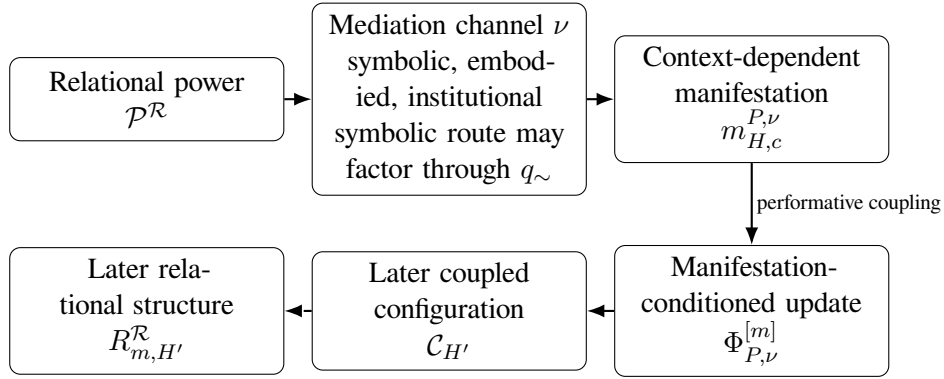


Figure 4: Mediation, performative manifestation, and conditional relationalization of power. Relational power can be carried through a declared mediation channel into a context-dependent manifestation; a symbolic channel can, when the required equivalence structure exists, be refined through quotienting and symbolization. A manifestation becomes performative when a separately typed generative transformation changes the later coupled configuration. Under additional persistence and efficacy conditions, manifestation-supported structures can acquire relational reality. The dashed arrow marks conditional relationalization.

$m_{H,c}^{P,\nu} \in \mathcal{M}_{H,c}^P$ be a manifestation generated at historical domain H . Suppose a family of subsequent events and transformations produces a later coupled configuration $C_{H'}$ with $H \hookrightarrow H'$. Let $R_{m,H'}^R$ denote a candidate relational structure in that later configuration whose historical support materially depends on the manifestation and on responses to it.

Let $\mathfrak{RS}(C_{H'})$ denote the model-defined class of relational structures represented in the later coupled configuration. Relationalization is represented by a binary relation. This choice permits one manifestation to contribute to several later relational structures and one later structure to depend on several manifestations and mediators. The relation type is stated in Equation 49.

$$\rightsquigarrow_{\text{rel}}^{H,H',c,\nu} \subseteq \mathcal{M}_{H,c}^P \times \mathfrak{RS}(C_{H'}). \quad (49)$$

For a selected manifestation and later relational structure, relationalization is recorded in Equation 50.

$$m_{H,c}^{P,\nu} \rightsquigarrow_{\text{rel}}^{H,H',c,\nu} R_{m,H'}^R. \quad (50)$$

Equations 49 and 50 receive the relationalization interpretation only when the later structure has an explicit generative dependence on the manifestation history and independently satisfies the relational-reality conditions relevant to its type. For a power-supporting structure, these conditions include expected persistence together with causal and generative efficacy, as developed in Section 4. A recurrent manifestation that remains causally inert does not acquire relational reality merely through repetition.

Definition 7.7 (Relationalized power manifestation). A manifestation $m_{H,c}^{P,\nu}$ is relationalized over an admissible historical extension $H \hookrightarrow H'$ when there exists a later relational structure $R_{m,H'}^{\mathcal{R}}$ such that (i) the manifestation and responses to it form part of the declared generative support of that structure; (ii) the structure satisfies the applicable expected-persistence criterion; and (iii) its causal and generative efficacy in the later relational configuration are established under the declared model.

Definition 7.7 yields a general principle for the layered GR account. Reality status is determined by the relevant historical persistence and relational efficacy conditions, independently of the level at which a structure first becomes accessible. Relationalization does not assert numerical or ontological identity between an earlier manifestation and the later relational structure; it asserts that the manifestation history belongs to the generative support through which the later structure is constituted. A symbolic, embodied, or institutional manifestation can therefore remain a manifestation, become performatively consequential without yet stabilizing, or participate in the formation of a later relationally real structure.

The same principle applies to comparative structures from Section 6. A manifested ranking can remain context-bound, become performative by conditioning resource or status allocation, and eventually support a relationalized comparison norm if the corresponding persistence and efficacy conditions are met. The relationalization criterion therefore links manifestation dynamics to the reflexive comparison architecture without assuming that all manifestations follow one common pathway.

This section establishes the conceptual and typed mediation layer needed for the categorical architecture that follows. Relational power, mediation channels, manifestation spaces, performative transformations, and relationalization relations now have distinct analytical roles. The following section organizes these objects over historical domains and specifies which parts of the architecture can be represented categorically while preserving ontological suspension.

8 Categorical Architecture of Power Formation

The preceding sections introduced several kinds of structure whose mathematical roles should remain distinct: relationally real power, context-dependent manifestations, proto-relational candidates, comparison judgments, historically operative equivalence relations, historical extensions, and conditional relationalization. The role of this section is to organize those structures within a common categorical representation without strengthening their ontological or algebraic status beyond what has already been established. The construction begins with a thin category of admissible historical domains, then introduces minimal categorical realizations of the relational, manifestation, proto-relational, quotient, and comparison domains. Partial comparison operations are represented by spans over their defined domains. Symbolic quotienting is represented by an object-level quotient functor when a genuine equivalence relation has been established, while mediation, continuation, symbolization, and relationalization are represented through witness-valued profunctors in the minimal discrete realization. The final diagram combines these components and marks explicitly which arrows are categorical representations rather than claims about ultimate ontology.

Basic category, functor, natural-transformation, and universal-construction terminology follows standard categorical usage [11]. The specific spans and witness-valued profunctors introduced below are nevertheless defined locally because the paper uses them only at the level justified by the preceding GR constructions.

8.1 Historical Indexing of Relational Categories

Historical order supplies the base over which the categorical representation is indexed. Let $\mathfrak{H}_{\mathcal{K}}$ denote the model-defined collection of admissible historical domains carried by the selected history-bearing GR spacetime representation $\mathcal{K}$. The relation $H \preceq H'$ retains the meaning used in Section 5.4: the represented history at H' extends the represented history at H according to the model's precedence and persistence conventions.

The corresponding thin history category is defined in Equation 51.

$$\text{Hom}_{\mathbf{Hist}_{\mathcal{K}}}(H, H') := \begin{cases} \{\iota_{H, H'}\}, & H \preceq H', \\ \emptyset, & \text{otherwise,} \end{cases} \quad H, H' \in \mathfrak{H}_{\mathcal{K}}. \quad (51)$$

The identity morphism is $\iota_{H, H}$, and composition is induced by transitivity of historical extension. Equation 51 therefore records only the order structure of historical inclusion or continuation. It does not assert that every substantively relevant continuation is uniquely determined by the earlier history.

For each historical domain H , let $\mathfrak{R}\mathfrak{S}(\mathcal{C}_H)$ denote the model-defined class of relationally real structures represented in the coupled configuration $\mathcal{C}_H$, as introduced in Section 7.6. Let $\text{Disc}(X)$ denote the discrete category whose objects are the elements of a set or class X and whose only morphisms are identities. The minimal categorical realization of relational reality at H is defined in Equation 52.

$$\mathbf{R}_H := \text{Disc}(\mathfrak{R}\mathfrak{S}(\mathcal{C}_H)). \quad (52)$$

The use of a discrete category in Equation 52 is intentional. Earlier sections established classes of relational structures and typed transformations on coupled configurations, but they did not establish a general composition law among arbitrary relational structures themselves. Starting discretely therefore prevents category theory from adding unearned internal morphisms. An application may replace $\mathbf{R}_H$ by a richer category after specifying the relevant morphisms, identities, composition, and preservation conditions.

Historical continuation between relational structures need not be a function. One structure at H may admit several continuations at H' , and a later structure may have several historical supports. Let $\mathbb{T}_{H, H'}^{\mathcal{R}}$ assign to a pair $(R, R') \in \mathbf{R}_H \times \mathbf{R}_{H'}$ the set of model-declared continuation witnesses connecting R at H to R' at H' . Its type is stated in Equation 53.

$$\mathbb{T}_{H, H'}^{\mathcal{R}} : \mathbf{R}_H^{\text{op}} \times \mathbf{R}_{H'} \longrightarrow \mathbf{Set}, \quad H \preceq H'. \quad (53)$$

Because the categories in Equation 52 are discrete, any witness-set assignment of the type in Equation 53 is a profunctor in the minimal representation. A richer non-discrete realization would require the corresponding naturality conditions to be established separately.

Remark 8.1 (Historical indexing does not restore primitive time). The objects of $\mathbf{Hist}_{\mathcal{K}}$ are historical domains generated from the event-order structure of the model. The categorical index therefore records admissible historical extension rather than a scalar parameter t . Introducing category theory does not change the temporal commitment established in Section 2.2.

Remark 8.1 fixes the interpretation of the history index used throughout the categorical construction.

8.2 Proto-Relational Domains

The proto-relational power domain $\mathcal{Q}_H^P$ introduced in Section 4.4 contains candidate power organizations constructed around power-relevant asymmetries whose own relational-reality status remains unestablished. The already relationally real antecedent asymmetry is therefore not demoted to proto-relational status. The comparison-specific domain $\mathcal{Q}_{H, c}^{P, \text{cmp}}$ introduced in Section 6.2 plays the analogous role for comparison candidates. Their minimal categorical realizations are defined in Equation 54.

$$\mathbf{Q}_H^P := \text{Disc}(\mathcal{Q}_H^P), \quad \mathbf{Q}_{H, c}^{P, \text{cmp}} := \text{Disc}(\mathcal{Q}_{H, c}^{P, \text{cmp}}). \quad (54)$$

Equation 54 gives proto-relational structures a categorical representation without granting them relational reality. The category belongs to the formal modeling level. Its objects remain candidates whose persistence and efficacy must be evaluated independently.

The closure operator $C_{P,H} : \mathcal{Q}_H^P \rightarrow \mathcal{Q}_H^P$ introduced in Section 4.5 induces an endofunctor on the discrete category $\mathbf{Q}_H^P$. Denote this endofunctor by $\mathbf{C}_{P,H}$. Its action on objects is stated in Equation 55.

$$\mathbf{C}_{P,H}(q) := C_{P,H}(q), \quad q \in \text{Ob}(\mathbf{Q}_H^P). \quad (55)$$

Equation 55 is a categorical repackaging of the previously defined closure structure. It does not strengthen the conclusion drawn from a fixed point: an object satisfying $\mathbf{C}_{P,H}(q) = q$ is stabilized under the chosen closure representation, while relational reality still requires the applicable persistence and efficacy conditions.

8.3 Manifestation Categories

The manifestation spaces introduced in Sections 5 and 7 can be represented categorically in the same minimal manner. Let $\mathcal{M}_{H,c}^P$ denote the context-dependent manifestation space of relational power and let $\mathcal{M}_{H,c}^{P,\text{cmp}}$ denote the manifestation space of power-comparison structures. Their discrete categorical realizations are defined in Equation 56.

$$\mathbf{M}_{H,c}^P := \text{Disc}(\mathcal{M}_{H,c}^P), \quad \mathbf{M}_{H,c}^{P,\text{cmp}} := \text{Disc}(\mathcal{M}_{H,c}^{P,\text{cmp}}). \quad (56)$$

The categories in Equation 56 record manifestation roles rather than exclusive reality classes. An object represented in a manifestation category can remain proto-relational with respect to relational-reality status, or can also correspond to a relationally real structure when that status has been established independently. Manifestation therefore does not inherit relational reality automatically from a source and is not erased when relational reality is later established.

Let $\mathbf{R}_H^P := \text{Disc}(\mathfrak{RP}(\mathcal{C}_H))$, where $\mathfrak{RP}(\mathcal{C}_H)$ is the model-defined class of relational-power structures introduced in Section 5.1. For a mediation channel $\nu \in \mathfrak{Med}_{H,c}^P$, define a mediation-witness profunctor by Equation 57.

$$\mathbb{M}_{H,c}^{P,\nu} : (\mathbf{R}_H^P)^{\text{op}} \times \mathbf{M}_{H,c}^P \longrightarrow \mathbf{Set}. \quad (57)$$

For objects $\mathcal{P}^R \in \mathbf{R}_H^P$ and $m \in \mathbf{M}_{H,c}^P$, the set $\mathbb{M}_{H,c}^{P,\nu}(\mathcal{P}^R, m)$ contains the declared mediation witnesses through which channel ν presents the relational-power structure as the manifestation m . A nonempty witness set can reproduce the two-stage mediation factorization of Section 7; the profunctor notation additionally permits one relational-power structure to support several manifestations and one manifestation to have several relational supports.

Remark 8.2 (Mediation is not assumed to be a functor). Equation 57 deliberately avoids assigning a unique manifestation to every relational-power structure. A functor $\mathbf{R}_H^P \rightarrow \mathbf{M}_{H,c}^P$ is a stronger special case that becomes available only when the model supplies a single-valued, composition-compatible mediation rule.

Remark 8.2 motivates the witness-valued representation retained in the quotient-symbolization refinement below.

8.4 Quotient Representation of Symbolic Mediation

Subsection 7.2 identified one symbolic route in which an equivalence relation organizes relational-power structures before symbolization. The categorical representation should preserve two distinctions. First, the mathematical quotient is available only when the declared source relation satisfies the conditions required for an equivalence relation on the selected domain. Second, existence of a quotient object does not determine the status of the equivalence relation. It can remain an analytic device, can acquire a manifested role as a social classification, and can additionally acquire relational reality when the relevant historical and

efficacy conditions are established. These roles are not assumed to be mutually exclusive, and reality status remains an independently established GR property.

Let $\mathfrak{RP}_{H,c}^{\text{sym}}(\mathcal{C}_H) \subseteq \mathfrak{RP}(\mathcal{C}_H)$ be the source class introduced in Subsection 7.2, and define its minimal discrete categorical realization by $\mathbf{R}_{H,c}^{P,\text{sym}} := \text{Disc}(\mathfrak{RP}_{H,c}^{\text{sym}}(\mathcal{C}_H))$. When $\sim_{H,c}^{P,\text{sym}}$ is an equivalence relation on this source class, the quotient set induces the discrete quotient category stated in Equation 58.

$$\mathbf{R}_{H,c}^{P,\text{sym}} / \sim := \text{Disc}(\mathfrak{RP}_{H,c}^{\text{sym}}(\mathcal{C}_H) / \sim_{H,c}^{P,\text{sym}}). \quad (58)$$

The quotient map in Equation 43 induces the functor in Equation 59.

$$\mathbf{Q}_{H,c}^{P,\text{sym}} : \mathbf{R}_{H,c}^{P,\text{sym}} \longrightarrow \mathbf{R}_{H,c}^{P,\text{sym}} / \sim, \quad \mathcal{P}^{\mathcal{R}} \longmapsto [\mathcal{P}^{\mathcal{R}}]_{\sim}. \quad (59)$$

Because both categories are discrete, Equation 59 records only the object-level quotient established earlier. A richer categorical quotient would require an internal morphism structure and compatibility or congruence conditions that the present paper has not assumed.

The symbolization step can remain non-unique. Define the witness-valued symbolization profunctor by Equation 60.

$$\mathbb{S}_{H,c}^{P,\text{sym}} : \left(\mathbf{R}_{H,c}^{P,\text{sym}} / \sim \right)^{\text{op}} \times \mathbf{M}_{H,c}^P \longrightarrow \mathbf{Set}. \quad (60)$$

For an equivalence class $[\mathcal{P}^{\mathcal{R}}]_{\sim}$ and a manifestation $m \in \mathbf{M}_{H,c}^P$, a nonempty value of $\mathbb{S}_{H,c}^{P,\text{sym}}([\mathcal{P}^{\mathcal{R}}]_{\sim}, m)$ records one or more declared symbolization witnesses connecting the quotient class to the manifestation. When the symbolization and presentation rules are single-valued on a selected domain, the profunctor can be replaced by the corresponding representable map or functor. The witness-valued form is retained here because several symbols can present one class and one symbol can be supported by several relational histories.

Remark 8.3 (Reality status of the equivalence relation). Neither Equation 58 nor Equation 59 upgrades $\sim_{H,c}^{P,\text{sym}}$ to relational reality. An analytic equivalence remains part of the modeling apparatus. A contextually enacted equivalence can possess manifestation reality. A historically persistent equivalence that satisfies the causal and generative efficacy conditions in Definition 7.2 can additionally be represented as a relationally real structure. The categorical quotient is a representation in all three cases.

Remark 8.3 prevents an important categorical shortcut. The existence of a quotient is a mathematical statement about the selected representation; the reality status of the relation used to form that quotient is a separate GR question. This distinction is retained in the integrated diagram below.

8.5 Comparative Structures

The comparison formalism in Section 6 contains partial maps whose definedness is analytically significant. A categorical representation should preserve that partiality. Let $\mathbf{Z}_{H,c}^{P,\beta} := \text{Disc}(\mathcal{Z}_{H,c}^{P,\beta})$ and $\mathbf{J}_{H,c}^{P,\beta} := \text{Disc}(\mathcal{J}_{H,c}^{P,\beta})$. Let $\mathbf{D}_{\kappa}$ be the discrete category on the domain of the comparison map $\kappa_{H,c}^{P,\beta}$. The comparison operation is represented by the span stated in Equation 61.

$$\mathbf{Z}_{H,c}^{P,\beta} \times \mathbf{Z}_{H,c}^{P,\beta} \xleftarrow{\mathbf{d}_{\kappa}} \mathbf{D}_{\kappa} \xrightarrow{\overline{\kappa}_{H,c}^{P,\beta}} \mathbf{J}_{H,c}^{P,\beta}. \quad (61)$$

In Equation 61, $\mathbf{d}_{\kappa}$ is induced by inclusion of $\text{Dom}(\kappa_{H,c}^{P,\beta})$ into the product comparison space, while $\overline{\kappa}_{H,c}^{P,\beta}$ is induced by the comparison function on its declared domain. The span therefore encodes definedness without converting the partial comparison operation into an artificial total map.

A generated judgment can subsequently acquire different statuses. Define status-witness profunctors of the types in Equation 62.

$$\mathbb{S}_{H,c}^Q : (\mathbf{J}_{H,c}^{P,\beta})^{\text{op}} \times \mathbf{Q}_{H,c}^{P,\text{cmp}} \longrightarrow \mathbf{Set}, \quad \mathbb{S}_{H,c}^M : (\mathbf{J}_{H,c}^{P,\beta})^{\text{op}} \times \mathbf{M}_{H,c}^{P,\text{cmp}} \longrightarrow \mathbf{Set}. \quad (62)$$

The witness sets in Equation 62 record, respectively, that a judgment is retained as a proto-relational comparison candidate or instantiated as a manifested comparison structure. A single judgment can possess both statuses under different contexts or carriers. Relational-reality status remains a further historical achievement rather than a third output automatically assigned by the comparison map.

Metric and quasi-metric constructions from Section 6.5 fit the same architecture. An abstract distance function belongs to the formal modeling level. When a metric practice becomes manifested, it can be represented as an object of $\mathbf{M}_{H,c}^{P,\text{cmp}}$; when its historical persistence and efficacy are established, the relationalization machinery of the next subsection can relate that practice to a relationally real structure at a later historical domain.

8.6 Conditional Relationalization

Sections 4, 6, and 7 all use a conditional transition from candidate or manifested structures to relational reality. The transition is generally many-to-many and historically indexed. A profunctor is therefore a more conservative representation than a total functor in the minimal categorical architecture.

Let $\mathbf{X}_{H,c}$ denote one of the categorical status domains $\mathbf{Q}_H^P$, $\mathbf{Q}_{H,c}^{P,\text{cmp}}$, $\mathbf{M}_{H,c}^P$, or $\mathbf{M}_{H,c}^{P,\text{cmp}}$, with context indices omitted when inapplicable. For an admissible historical extension $H \preceq H'$, the generic relationalization-witness profunctor is stated in Equation 63.

$$\mathbb{L}_{H,H',c}^X : \mathbf{X}_{H,c}^{\text{op}} \times \mathbf{R}_{H'} \longrightarrow \mathbf{Set}. \quad (63)$$

For objects $x \in \mathbf{X}_{H,c}$ and $R' \in \mathbf{R}_{H'}$, a member of $\mathbb{L}_{H,H',c}^X(x, R')$ is an admissible historical witness that the candidate or manifestation x , together with the declared subsequent responses and transformations, forms part of the generative support of the relationally real structure R' . A nonempty witness set does not replace the persistence and efficacy tests; the witness is admissible only when those conditions have been established for the target structure.

The binary relationalization relation in Equation 49 is recovered by forgetting the internal witness set and retaining only its nonemptiness. In the minimal discrete architecture, this correspondence is stated in Equation 64.

$$x \rightsquigarrow_{\text{rel}}^{H,H',c} R' \iff \mathbb{L}_{H,H',c}^X(x, R') \neq \emptyset, \quad (64)$$

where the channel index ν can be reinstated when relationalization is mediated by a declared channel. Equation 64 gives the categorical representation a direct interface with the earlier typed relation.

Remark 8.4 (Representable relationalization is a stronger special case). If each admissible source object x is associated with a selected relationalized object $S(x) \in \mathbf{R}_{H'}$, and if the witness profunctor in Equation 63 is represented by that assignment, then relationalization may be written through a functor $S : \mathbf{X}_{H,c}^{\text{rel}} \rightarrow \mathbf{R}_{H'}$ on a declared subcategory of relationalizable sources. The present paper does not assume this representability in general.

Remark 8.4 explains why earlier informal diagrams suggesting a direct map from manifestation to relational reality are useful only in restricted cases. The profunctor representation preserves multiplicity, partiality, and historical dependence without claiming that relationalization is a single-valued operation.

Table 6 summarizes the categorical devices introduced in this section and the structural commitments each device is intended to encode.

8.7 Integrated Categorical Diagram

The categorical architecture can now be displayed without treating category theory as a fourth layer of reality. Figure 5 contains two panels. The upper panel organizes history-indexed relational power, manifestation, proto-relational candidates, a quotient-symbolization route for symbolic mediation, and later relational reality. The lower panel shows how a partial comparison operation is represented by a span and

Table 6: Categorical devices and the structural commitments they encode in the power formalism.

Device	Analytical role	Constraint preserved
Thin history category $\mathbf{Hist}_{\mathcal{K}}$	Indexes admissible historical domains and extensions	Does not introduce primitive scalar time or unique substantive continuation
Discrete status categories	Give previously defined relational, manifestation, proto-relational, and judgment domains a minimal categorical realization	Do not invent internal morphisms or upgrade reality status
Profunctor $\mathbb{T}^{\mathcal{R}}$	Records possibly non-unique historical continuation of relational structures	Avoids assuming a unique continuation functor
Profunctor $\mathbb{M}^{P,\nu}$	Records mediation witnesses between relational power and manifestation	Allows one-to-many and many-to-one mediation
Quotient functor $\mathbf{Q}^{P,\text{sym}}$ with symbolization profunctor $\mathbb{S}^{P,\text{sym}}$	Represents the route from a declared relational equivalence through quotient classes into symbolic manifestation	Keeps the mathematical quotient distinct from the reality status of the equivalence relation
Span for $\kappa^{P,\beta}$	Represents a partial comparison operation through its declared domain	Preserves definedness and incomparability
Profunctor $\mathbb{L}^X$	Records historical witnesses of conditional relationalization	Keeps relationalization many-to-many and conditional on persistence and efficacy

how its judgment can enter proto-relational or manifested comparison status before any later relationalization. The box for ultimate ontological reality is deliberately disconnected: no categorical transport from ultimate ontology is asserted.

Figure 5 should be read as a typing and status diagram, not as a claim that every power process follows one mandatory path. A relational-power structure can remain weakly manifested; a symbolic manifestation can arise without a quotient construction; an analytic equivalence can support a formal quotient without acquiring relational reality; a manifestation can remain causally inert; a comparison judgment can remain proto-relational; and historical continuation can occur without relationalization of any comparison or manifestation. The diagram records the admissible interfaces among structures whose reality statuses and dynamical roles were established separately in earlier sections.

The figure also separates categorical representation from metaphysical commitment. The categories, spans, and profunctors introduced here organize model-relative objects and witness structures. They do not imply that ultimate ontology is categorical, that a GR spacetime is literally a category, or that relational reality is exhausted by the objects retained in the chosen representation. The principal gain is structural: the formalism can preserve partiality, multiplicity, historical indexing, and status change without converting those features into an unjustified total map or scalar hierarchy.

Research Obligation 8.5 (Categorical enrichment). Any later replacement of the discrete categories in this section by non-discrete, enriched, higher, or double-categorical structures should specify the additional morphisms, composition laws, coherence conditions, and empirical or interpretive role that justify the stronger representation. Results proved only in the discrete or witness-valued architecture should not be transferred automatically to that enriched setting.

Obligation 8.5 keeps the present construction consistent with GR’s methodological pluralism and ontological suspension. The categorical architecture is sufficiently expressive for the next section, where relational power is represented dynamically through operators acting on history weights and attractor accessibility. That operator representation will be introduced as a further modeling layer rather than as the ontology of power itself.

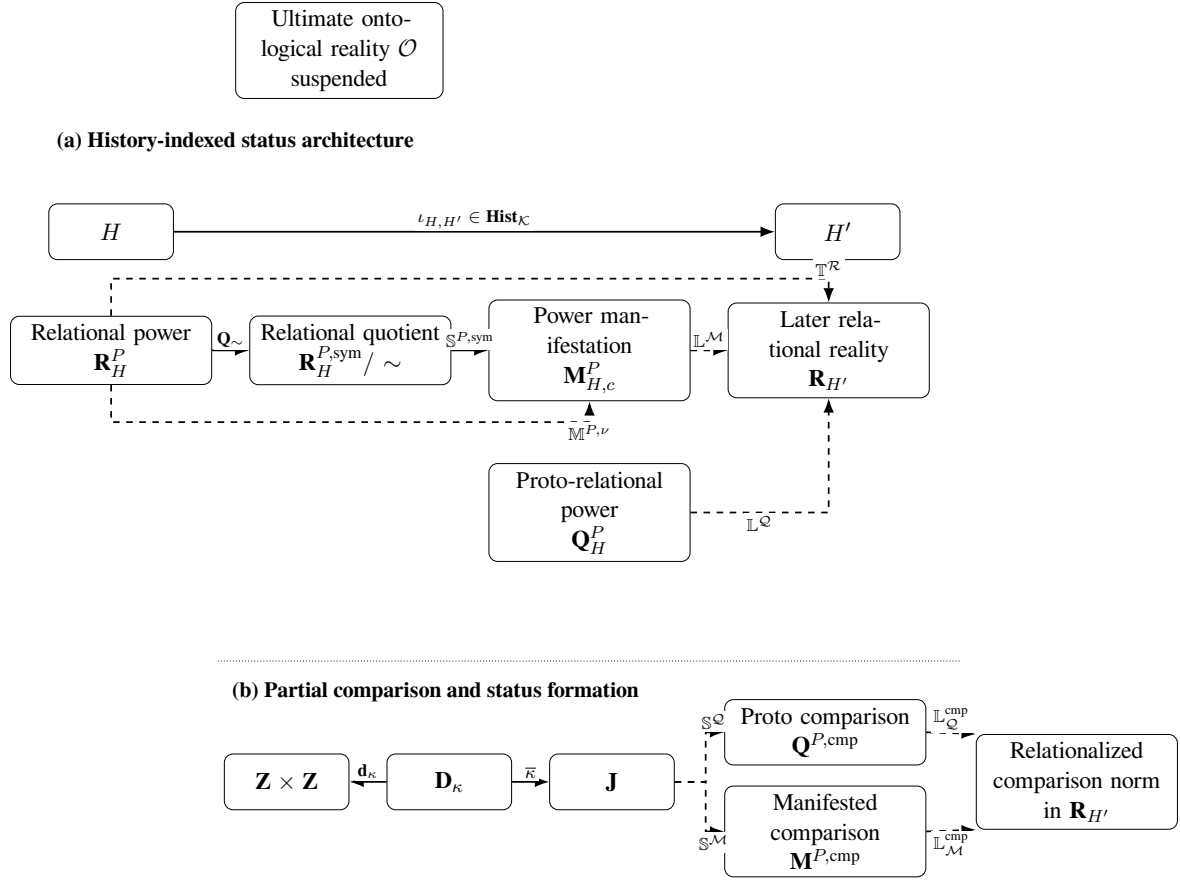


Figure 5: Integrated categorical architecture of power formation. Panel (a) organizes history-indexed relational power, quotient-symbolization, manifestation, proto-relational power, historical continuation, and conditional relationalization. Panel (b) represents the partial comparison operation by a span and separates proto-relational and manifested comparison status before possible relationalization. Solid arrows denote already-defined maps or history morphisms; dashed arrows denote witness-valued or conditional relations. Ultimate ontological reality remains suspended and disconnected from the formal transport structure.

9 Self-Sustaining Power through Generative-Landscape Deformation

Sections 4–8 established power as a relationally real structure, distinguished its manifestations from its relational status, and organized the relevant status transitions through a minimal categorical architecture. The present section adds a dynamical representation of a further possibility: relational power can alter the distribution of admissible continuations in ways that make some relational regimes easier to reach or retain, and some of those regimes can in turn support continuation of the power structure itself. This possibility supplies a formal route to self-sustaining power without treating self-reproduction as a defining condition of power in general.

The construction proceeds in six stages. It first defines a local operator representation of relational power on an auxiliary space of history weights. It then distinguishes reweighting of already represented continuations from changes in their support. The third subsection connects those deformations to the expected accessibility of attractor-like regimes and to the basin-like construction introduced in Section 4. The fourth defines a power-compatible relational regime through historical continuation of relational power. The fifth combines landscape deformation and compatibility into a definition of relational reproduction. The final subsection allows that reproduction to preserve a dynamical role while changing symbolic, institutional, or other supporting structures. Operator, weight, basin, and attractor language remains representational throughout; none is identified with the ontology of power.

9.1 Operator Representation of Relational Power

Let $\mathcal{C}_H \in \mathfrak{C}$ be a coupled historical configuration and let $\mathcal{P}^{\mathcal{R}} \in \mathfrak{RP}(\mathcal{C}_H)$ be a relational-power structure satisfying Definition 4.8. To represent how that structure can deform a family of possible continuations, the analysis requires an auxiliary linear space on which an operator can act. The linearity belongs to the representation space and should not be transferred automatically to the underlying GR spacetime or to relational power itself.

Fix an admissible historical extension $H' \in \mathfrak{H}_H^+$. Let $\bar{\Gamma}_{H \rightsquigarrow H'}^+(\mathcal{C}_H)$ be a finite or countable *comparison universe* of histories containing every continuation retained by the baseline representation and every continuation retained by the power-conditioned representation under comparison. Histories absent from one representation are assigned zero weight in that representation. The associated weight space is defined in Equation 65.

$$\mathcal{W}_{H \rightsquigarrow H'} := \ell^1(\bar{\Gamma}_{H \rightsquigarrow H'}^+(\mathcal{C}_H)), \quad \mathcal{W}_{H \rightsquigarrow H'}^+ := \{u \in \mathcal{W}_{H \rightsquigarrow H'} \mid u(\gamma) \geq 0\}. \quad (65)$$

Equation 65 places absolutely summable real-valued history-weight assignments in a Banach space and identifies its positive cone. The construction is local to the declared comparison universe. A generative history outside that universe requires enlargement of the representation rather than being treated as implicitly impossible.

Let $\mathcal{B}_+(\mathcal{W}_{H \rightsquigarrow H'})$ denote the class of bounded linear operators on $\mathcal{W}_{H \rightsquigarrow H'}$ that preserve the positive cone. Linearity is an additional representational assumption, not a consequence of the GR definition of power. The operator lift below is therefore available only when the application supplies a declared power-conditioned transformation of history weights that is representable, on the retained comparison universe, by a positive bounded linear operator. If the empirically or conceptually adequate transformation is nonlinear, the present operator lift should be replaced by a correspondingly typed nonlinear map rather than forced into $\mathcal{B}_+$. A local operator lift of relational power is introduced in Definition 9.1.

Definition 9.1 (Operator lift of relational power). A local operator lift on the historical extension $H \rightsquigarrow H'$ is a partial representation map

$$\Lambda_{H \rightsquigarrow H'}^P : \mathfrak{RP}(\mathcal{C}_H) \rightarrow \mathcal{B}_+(\mathcal{W}_{H \rightsquigarrow H'}), \quad \mathcal{P}^{\mathcal{R}} \mapsto \hat{P}_{H \rightsquigarrow H'}. \quad (66)$$

Equation 66 states the type of the operator lift. The lift is defined only when the model specifies a comparison universe, a history-weight space, a power-conditioned weight transformation, and a positive bounded linear operator that represents that transformation on the retained domain. The operator is therefore derived from an explicitly declared deformation rule on the auxiliary representation space; it is not introduced directly from the ontology or from the notation $\mathcal{P}^{\mathcal{R}}$ alone.

Definition 9.1 does not identify $\mathcal{P}^{\mathcal{R}}$ with $\hat{P}_{H \rightsquigarrow H'}$. The former is a relationally real structure within the GR analysis; the latter is an auxiliary mathematical representation of one aspect of its dynamical consequence. Different operator representations can therefore be admissible for the same relational-power structure when they preserve different observables or levels of resolution.

Let $w_{H \rightsquigarrow H'} \in \mathcal{W}_{H \rightsquigarrow H'}^+$ be the baseline weight assignment from Assumption 4.1, extended by zero to the comparison universe when necessary. The conditioned vector is required to be nonzero. Under that condition, the normalized power-conditioned weight assignment is defined in Equation 67.

$$\bar{w}_{H \rightsquigarrow H'}^{[P]} := \frac{\hat{P}_{H \rightsquigarrow H'} w_{H \rightsquigarrow H'}}{\left\| \hat{P}_{H \rightsquigarrow H'} w_{H \rightsquigarrow H'} \right\|_1}, \quad \bar{w}_{H \rightsquigarrow H'} := \frac{w_{H \rightsquigarrow H'}}{\|w_{H \rightsquigarrow H'}\|_1}. \quad (67)$$

The normalization in Equation 67 creates a common comparison scale. It does not convert the weights into objective probabilities. The same caution applied to the expected-persistence construction in Section 4.1.

A family of locally defined bounded operators can generate an operator algebra on the auxiliary weight space when linear combinations and composition are retained. The present paper uses that stronger structure only when it performs an analytical role. In particular, composition can represent order-sensitive deformation by several power or counter-power operations, while linear combinations remain formal unless the model supplies an interpretation for them.

Remark 9.2 (Operator-algebraic scope). If a declared family of bounded operators on $\mathcal{W}_{H \rightsquigarrow H'}$ is closed under the operations required by an associative operator algebra, the model may study algebraic composition, generated subalgebras, and noncommuting operator orderings. This additional structure belongs to the representation space. It does not establish that relational power is fundamentally linear, quantum mechanical, or an operator-algebraic object in ultimate ontology.

Remark 9.2 preserves the separation between the minimal GR claim and stronger mathematical realizations. The next subsection uses only the positive operator action required to compare history weights.

9.2 Deformation of Historical Weights

The operator lift permits two distinct forms of landscape deformation. First, power can change the relative weight assigned to histories that remain represented in both the baseline and power-conditioned continuation families. Second, it can alter support by making some represented histories newly accessible or effectively absent at the chosen modeling resolution. The distinction matters because a change in relative tendency and a change in defined possibility are different dynamical claims.

The weight-deformation vector is defined in Equation 68.

$$\Delta_P^w(H') := \overline{w}_{H \rightsquigarrow H'}^{[P]} - \overline{w}_{H \rightsquigarrow H'} \in \mathcal{W}_{H \rightsquigarrow H'}. \quad (68)$$

Subtraction in Equation 68 is defined because the two normalized weight assignments lie in the same auxiliary linear space. The equation does not assert that histories or relational powers can themselves be subtracted.

The support of a nonnegative history-weight assignment u is the subset of the comparison universe defined in Equation 69.

$$\text{supp}(u) := \{\gamma \in \overline{\Gamma}_{H \rightsquigarrow H'}^+(\mathcal{C}_H) \mid u(\gamma) > 0\}. \quad (69)$$

Equation 69 gives support change a precise location in the representation. A power-conditioned operator can deform relative weights while preserving support, alter support while leaving some retained weights unchanged, or do both.

Definition 9.3 (History-weight deformation by relational power). The operator representation of $\mathcal{P}^{\mathcal{R}}$ produces a nontrivial history-weight deformation on $H \rightsquigarrow H'$ when $\Delta_P^w(H') \neq 0$. It produces a support deformation when

$$\text{supp}(\overline{w}_{H \rightsquigarrow H'}^{[P]}) \neq \text{supp}(\overline{w}_{H \rightsquigarrow H'}). \quad (70)$$

Equation 70 records the corresponding change in represented support.

Definition 9.3 provides a formal meaning for the claim that power changes the possibility landscape. The first condition represents changed comparative weighting within the declared universe; the second represents a change in which histories receive positive represented accessibility. Neither condition by itself establishes that the deformation is normatively desirable, intentionally chosen, or self-sustaining.

The construction also leaves room for more general amplitude-like representations. A later model may replace real nonnegative weights by complex amplitudes or another structured valuation only after it supplies the relevant state space, composition rule, and interpretation. The present paper retains nonnegative weights because they are sufficient to express the landscape deformation required below without importing a quantum interpretation.

9.3 Deformation of Attractor Accessibility

History-weight deformation becomes relevant to attractor-like regimes when the reweighted histories terminate in, return to, or remain within different relational regimes. Let $\mathcal{A}_i \subseteq \mathfrak{C}$ be a model-defined

attractor-like regime satisfying Definition 4.3. For a fixed historical extension H' , the baseline and power-conditioned weighted occupancies are computed by Equation 71.

$$\begin{aligned}\text{Acc}_{\mathcal{A}_i}^0(H') &:= \sum_{\gamma \in \bar{\Gamma}_{H \rightsquigarrow H'}^+(\mathcal{C}_H)} \bar{w}_{H \rightsquigarrow H'}(\gamma) \mathbf{1}_{\mathcal{A}_i}(\mathcal{C}_{H'}^\gamma), \\ \text{Acc}_{\mathcal{A}_i}^P(H') &:= \sum_{\gamma \in \bar{\Gamma}_{H \rightsquigarrow H'}^+(\mathcal{C}_H)} \bar{w}_{H \rightsquigarrow H'}^{[P]}(\gamma) \mathbf{1}_{\mathcal{A}_i}(\mathcal{C}_{H'}^\gamma).\end{aligned}\tag{71}$$

The two quantities in Equation 71 are normalized weight shares over the same declared continuation universe. They are therefore comparable within the model, while remaining distinct from empirical probabilities unless a probabilistic interpretation is independently justified.

The accessibility-deformation profile of $\mathcal{A}_i$ is defined in Equation 72.

$$\Delta_{\mathcal{A}_i}^P(H') := \text{Acc}_{\mathcal{A}_i}^P(H') - \text{Acc}_{\mathcal{A}_i}^0(H'), \quad H' \in \mathfrak{H}_H^+.\tag{72}$$

Equation 72 does not define a scalar measure of power. It measures only the modeled change in weighted accessibility of one declared regime under one operator representation of an already established relational power structure. Distinct regimes can have positive, negative, or incomparable profiles across historical extensions.

The basin-like construction from Equation 18 can now be evaluated under the conditioned weights. Let $\mathcal{B}_H^{w,\psi}(\mathcal{A}_i)$ denote the baseline basin-like domain and $\mathcal{B}_H^{w[P],\psi}(\mathcal{A}_i)$ the corresponding domain when the persistence schema is evaluated with power-conditioned weights. The operator representation produces basin expansion at the chosen resolution when Equation 73 holds.

$$\mathcal{B}_H^{w,\psi}(\mathcal{A}_i) \subsetneq \mathcal{B}_H^{w[P],\psi}(\mathcal{A}_i).\tag{73}$$

Equation 73 uses set inclusion rather than geometric volume. A geometric claim about basin depth, boundary curvature, or metric distance requires additional structure on the configuration space. The present result states only that more represented configurations satisfy the declared tendency toward the regime after the power-conditioned deformation.

Remark 9.4 (Landscape deformation beyond basin expansion). Relational power can deform a generative landscape without producing the set inclusion in Equation 73. It may redistribute weights inside an unchanged basin, shrink one basin while enlarging another, alter transition pathways, or modify the admissible continuation family itself. Basin expansion is therefore one specific representation of landscape deformation rather than its general definition.

Remark 9.4 is important for the international-relations application in the next section. Balancing and concentration of power need not be modeled as simple movement along one scalar potential; they can involve simultaneous deformations of several competing regimes and transition paths.

9.4 Power-Compatible Relational Regimes

A power-conditioned attractor-like regime becomes relevant to self-sustainability only when the regime supports historical continuation of the power structure whose operator representation helped make that regime accessible. This condition should not be defined through the preferences or intentions of an actor unless intentional agency has been established independently. The paper therefore uses *power-compatible regime* for a relational regime that supports continuation of power, reserving *targeted regime* for applications that additionally model purposive steering.

The historical-continuation profunctor from Section 8.1 supplies a minimal way to state this support condition. Fix $\mathcal{P}^R \in \mathfrak{RP}(\mathcal{C}_H)$. The continuation-support region of that power structure is the set defined in Equation 74.

$$\mathcal{S}_H(\mathcal{P}^R) := \left\{ \mathcal{C}_{H'} \in \mathfrak{C} \mid \begin{array}{l} H' \in \mathfrak{H}_{H,H'}^+ \exists \mathcal{P}'^R \in \mathfrak{RP}(\mathcal{C}_{H'}) \text{ such that} \\ \mathbb{T}_{H,H'}^R(\mathcal{P}^R, \mathcal{P}'^R) \neq \emptyset \end{array} \right\}.\tag{74}$$

Equation 74 requires a historical witness of continuation from the reference power structure to a later relational-power structure. It does not require literal identity between the two structures. The stronger question of how much transformation can occur while power remains adaptively persistent is addressed in Subsection 9.6.

Definition 9.5 (Power-compatible relational regime). A model-defined relational regime $\mathcal{A}_P \subseteq \mathcal{C}$ is compatible with $\mathcal{P}^{\mathcal{R}}$ over the declared historical scope when the retained portion of $\mathcal{A}_P$ used by the analysis is contained in $\mathcal{S}_H(\mathcal{P}^{\mathcal{R}})$. Equivalently, every configuration of the regime retained for the claim has at least one represented historical continuation witness to a later relational-power structure.

For conceptual consistency, the paper distinguishes *power-favored* from *power-compatible*. A regime is called power-favored relative to a declared baseline and accessibility-comparison schema when the power-conditioned accessibility profile is judged to increase its modeled accessibility or retention. This is a claim about landscape deformation. Power compatibility, by contrast, is the continuation condition in Definition 9.5. A regime can be power-favored without supporting continuation of the same power structure, or power-compatible without being made more accessible by that structure. Relational reproduction below requires both roles to be connected.

Definition 9.5 is deliberately strong and local. An empirical application can weaken it by replacing set inclusion with a declared weight or tolerance schema when only a sufficiently large subset of the regime supports power continuation. The present definition is sufficient to separate two questions: whether power makes a regime more accessible, and whether that regime in turn contains conditions under which power can continue.

A power-compatible regime need not be hegemonic, centralized, or consciously preferred. A distributed institutional arrangement, a symbolic order, or a decentralized network can be compatible with continuation of a relational-power structure if the relevant historical witnesses exist. Compatibility is therefore a structural relation between a power organization and a regime, not a normative judgment about that regime.

9.5 Relational Reproduction of Power

The preceding subsections supply the two directions required for self-sustainability. The first direction runs from relational power to the generative landscape through the operator lift and the resulting deformation of history weights. The second runs from a power-compatible regime to later relational power through historical continuation. A self-supporting loop appears when these directions are coupled.

Let $\mathcal{A}_P$ be power-compatible with $\mathcal{P}^{\mathcal{R}}$. The power-conditioned accessibility profile $\text{Acc}_{\mathcal{A}_P}^P$ and the baseline profile $\text{Acc}_{\mathcal{A}_P}^0$ are functions on the declared family of historical extensions. Because a systematic increase need not be represented by one universal threshold, let ψ_{self} be a declared self-support schema whose domain contains pairs of such profiles and whose codomain is $\{0, 1\}$. The schema may require non-decrease over a selected family of extensions, a positive lower envelope, robustness under admissible perturbations of the weights, or another explicit criterion.

Definition 9.6 (Relational reproduction of power). A relational-power structure $\mathcal{P}^{\mathcal{R}}$ exhibits relational reproduction through a power-compatible regime $\mathcal{A}_P$ when:

1. an operator lift $\hat{P}_{H \rightsquigarrow H'}$ is defined over the declared historical scope;
2. $\mathcal{A}_P$ satisfies Definition 9.5; and
3. the power-conditioned and baseline accessibility profiles satisfy Equation 75.

$$\psi_{\text{self}}(\text{Acc}_{\mathcal{A}_P}^P, \text{Acc}_{\mathcal{A}_P}^0) = 1. \quad (75)$$

Definition 9.6 identifies a higher-order dynamical property. The definition does not imply that every relational power reproduces itself, that reproduced power necessarily expands, or that the power-compatible

regime is intentionally selected. A short-lived power relation can satisfy Definition 4.8 without satisfying Definition 9.6.

The dependency structure of self-sustaining power is summarized in Figure 6. The upper route represents the lift from relational power to a positive operator on history weights and the corresponding change in attractor accessibility. The lower return route represents historical continuation from a power-compatible regime to later relational power. The figure does not treat the operator or the attractor regime as the ontology of power.

Figure 6 also clarifies why self-sustaining power can extend beyond direct coercion. A power structure can reproduce itself by changing institutional prerequisites, symbolic expectations, admissible operator families, or other conditions that make power-compatible continuations more accessible. The particular mechanism remains domain-specific and requires empirical or interpretive support beyond the structural definition.

9.6 Adaptive Persistence

Relational reproduction should not require a power structure to preserve the same institutional carrier, symbolic manifestation, or local configuration. Such a requirement would identify persistence with literal structural identity and would exclude cases in which a power organization adapts while retaining a recognizable dynamical role. The present subsection therefore introduces a deliberately model-relative criterion of role continuity.

Let $\mathcal{D}_P$ be a model-defined space of dynamical-role representations, and let ρ_P map relational-power structures retained across the declared historical scope into $\mathcal{D}_P$. A role representation can encode, for example, which GR spacetimes are directionally affected, which possibility structures are deformed, or which classes of operators remain consequential. The model must state the information retained by ρ_P ; the symbol does not provide that content by itself.

Let $\sim_{\text{role}}$ be a declared equivalence relation on $\mathcal{D}_P$. Two relational-power structures are role-equivalent when Equation 76 holds.

$$\mathcal{P}^{\mathcal{R}} \equiv_{\rho_P} \mathcal{P}'^{\mathcal{R}} \iff \rho_P(\mathcal{P}^{\mathcal{R}}) \sim_{\text{role}} \rho_P(\mathcal{P}'^{\mathcal{R}}). \quad (76)$$

Equation 76 makes adaptive identity explicitly representation-relative. It does not assert that the two power structures are ontologically identical or normatively equivalent.

Definition 9.7 (Adaptive persistence of relational power). A relational-power structure $\mathcal{P}^{\mathcal{R}}$ is adaptively persistent over a declared family of historical extensions when later relational-power structures $\mathcal{P}'^{\mathcal{R}}$ exist with historical continuation witnesses under $\mathbb{T}^{\mathcal{R}}$, satisfy $\mathcal{P}^{\mathcal{R}} \equiv_{\rho_P} \mathcal{P}'^{\mathcal{R}}$, and may differ in manifestation, institutional support, mediator, or other structures not retained by the declared role representation.

Definition 9.7 allows self-sustaining power to change the means through which it remains effective. A symbolic hierarchy can be replaced by a procedural rule, a direct command structure by a distributed institutional arrangement, or one support network by another while the selected dynamical role remains role-equivalent. Whether a particular historical case satisfies that relation is an empirical and interpretive question.

The distinction between reproduction and adaptive persistence also limits the stronger claim that power “seeks” its own continuation. The formalism establishes only a conditional feedback structure. Intentional steering toward a targeted regime requires an independently modeled agent, objective, and action-selection relation. In their absence, a regime that is both power-favored and power-compatible can remain self-supporting without being wanted by any single relational being.

The section therefore extends the metaphysics of power in one precise direction. Relational power can be represented as deforming history weights, accessibility, and basin-like structure; a compatible regime can support later power; and the resulting feedback can preserve a dynamical role across changing manifestations and supports. These properties provide the formal resources required by the international-relations case study developed next, where balancing will be examined as counter-deformation within a shared generative landscape rather than as arithmetic equality of scalar power quantities.

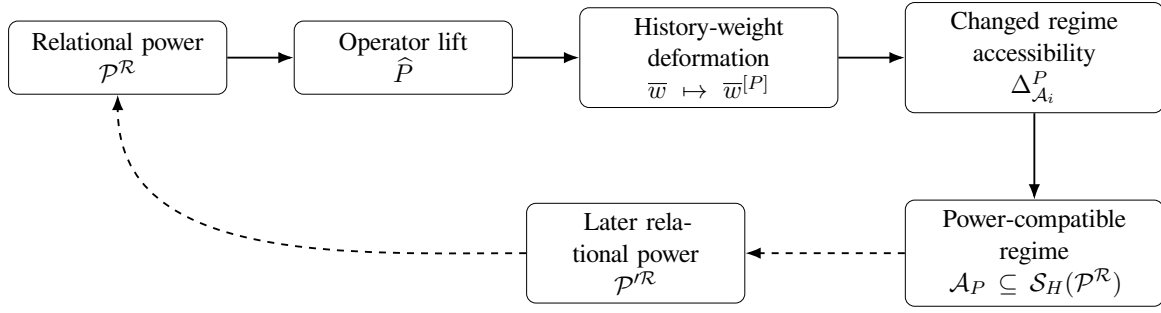


Figure 6: Dynamical architecture of relationally reproducing power. Relational power is lifted to an auxiliary operator representation that deforms history weights and changes modeled regime accessibility. A power-compatible regime can support later relational power through a separately established historical-continuation relation. Dashed return paths are conditional and do not imply deterministic recurrence, intentional steering, or identity of power across historical extensions.

10 Balance as a Near-Critical Relational Regime in International Politics

This section uses international-political realism as a bounded case study for the power formalism developed in Sections 3–9. Its role is neither to endorse realism nor to replace existing balance-of-power theories. The objective is narrower: to reconstruct several familiar realist propositions in a history-bearing GR representation and to ask whether a balance-like configuration can, under additional conditions, be modeled as an expectedly persistent near-critical relational regime. The reconstruction is deliberately conditional. Structural realism classically explains recurrent balancing through an anarchic international structure and the distribution of capabilities, while later realist work has differentiated balancing against power from balancing against threat and has debated the forms and empirical regularity of balancing itself [16, 15, 14]. The present section preserves those disagreements rather than treating “balance” as a single uncontested mechanism.

The discussion proceeds in seven stages. It first interprets capability distributions as coarse manifestations of history-bearing GR spacetimes. It then applies the operator representation of relational power to a coupled international configuration and defines balancing as a family of counter-deformations rather than as arithmetic equalization. The fourth subsection defines a balance-like relational regime independently of equality of capability readouts. The fifth introduces a non-scalar near-critical corridor and states a conditional critical-balance hypothesis. The sixth identifies structural routes through which balancing can fail. The final subsection distinguishes balance generated without an explicit balance norm from balance whose symbolic or institutional manifestation has itself become relationally consequential. The criticality vocabulary follows the general dynamical meaning of regime susceptibility and transition structure; it does not infer political criticality from evidence in ecological or other physical systems [13]. A previous formulation of the Criticality Hypothesis of International Politics motivates the specific conjecture that political relational systems may, under some conditions, preserve organization while retaining substantial susceptibility and reconfigurability [8].

10.1 Realist Capability Representation

A structural-realist description ordinarily suppresses much of the internal history of states in order to represent the international system through an ordering principle and a distribution of capabilities. Waltz’s formulation treats states as functionally similar units differentiated by their capabilities, with balancing emerging as a recurrent systemic pattern under anarchy [16]. Within the present GR formalism, that abstraction can be represented without treating a state as a point that carries an intrinsic scalar quantity of power.

Let $\mathcal{C}_H^{\text{IR}} \in \mathfrak{C}^{\text{IR}} \subseteq \mathfrak{C}$ be a coupled international-political configuration over historical domain H . Let $A_1, \dots, A_n \in \mathfrak{S}_{\text{GR}}(\mathcal{C}_H^{\text{IR}})$ denote the history-bearing GR spacetime entities retained as state-level relata by

the chosen analysis. Each A_i can itself admit a 2-complex representation with internal event, relation, and higher-order history. The state-level representation therefore functions as a scale-relative effective object rather than as a claim that the state is ontologically point-like.

For a declared realist observational context c_{cap} , let $\mathcal{X}_{H, c_{\text{cap}}}^{\text{cap}}$ be a capability-representation space. A capability readout is the partial map stated in Equation 77.

$$r_{H, c_{\text{cap}}}^{\text{cap}} : \mathfrak{S}_{\text{GR}}(\mathcal{C}_H^{\text{IR}}) \rightarrow \mathcal{X}_{H, c_{\text{cap}}}^{\text{cap}}, \quad A_i \mapsto x_i^{\text{cap}}. \quad (77)$$

Equation 77 can retain military, economic, technological, demographic, territorial, or other capability descriptors selected by the model. The codomain need not be one-dimensional. A scalar aggregate can be added only when the application supplies an aggregation rule.

The readout in Equation 77 belongs initially to the manifestational or modeling interface developed in Sections 5 and 6. A capability comparison can therefore be understood as a special case of the partial comparison architecture in Section 6. The existence of a ranking among x_i^{cap} does not establish an identical ranking among the corresponding relational-power structures. This distinction is especially important because realist traditions themselves differ over which observable relations should enter strategic comparison. Walt's balance-of-threat formulation, for example, argues that alliance behavior responds to perceived threat rather than to aggregate power alone [15].

Remark 10.1 (Capability distributions as coarse manifestations). Within the present reconstruction, a distribution of capabilities is a deliberately coarse representation of a coupled relational history. It can be analytically powerful at a declared scale while leaving institutional, symbolic, network, dependency, and historical relations outside the retained readout. The reconstruction therefore does not treat coarse-graining as error by definition; it makes the information loss explicit.

Remark 10.1 permits a realist model to remain useful without requiring GR to identify manifested capability with power itself. Table 7 summarizes the principal translation used throughout the case study.

10.2 Power-Induced Deformation of the International Landscape

The international configuration permits several relational-power structures to operate within one coupled history. Let $\mathcal{P}_i^{\mathcal{R}} \in \mathfrak{RP}(\mathcal{C}_H^{\text{IR}})$ be a relational-power structure associated with the position of A_i . Where the operator lift from Definition 9.1 is defined, write $\hat{P}_i$ for its local representation on the declared history-weight space. The operator can alter relative weights, support, or regime accessibility without requiring that A_i intentionally selects a terminal international order.

Fix a finite family of international relational regimes $\mathfrak{A}^{\text{IR}} = \{\mathcal{A}_1, \dots, \mathcal{A}_m\}$ whose membership criteria have been specified independently of the power comparison. For each historical extension $H' \in \mathfrak{H}_H^+$, collect the power-conditioned accessibility profiles from Section 9.3 into the vector stated in Equation 78.

$$\mathbf{a}_i^P(H') := (\text{Acc}_{\mathcal{A}_1}^{P_i}(H'), \dots, \text{Acc}_{\mathcal{A}_m}^{P_i}(H')) \in [0, 1]^m. \quad (78)$$

Equation 78 is a profile over declared regimes, not a scalar measure of the power of A_i . Two actors can produce different deformations that remain incomparable under the chosen regime family. The same actor can also produce different profiles under different historical configurations or comparison universes.

A concentration of manifested capability can therefore be dynamically weak when it fails to reorganize relevant continuations, while a less conspicuous relational position can be dynamically consequential when it changes operator availability, alliance possibilities, settlement infrastructures, or other high-leverage relations. This claim is structural rather than empirical: a concrete application must establish which relational channels transmit the deformation.

10.3 Balancing through Counter-Deformation

Balance-of-power theories commonly describe balancing through mechanisms such as internal military buildup and counterbalancing alliances, while the broader literature also recognizes variation in balancing behavior and competing responses [14]. The GR reconstruction treats these mechanisms as possible

Table 7: Realist balance-of-power shorthand and its GR reconstruction in the present case study. The table states representational correspondences rather than equivalences between theories.

Realist shorthand	GR representation	Role in the case study
State	History-bearing GR spacetime entity A_i within $\mathcal{C}_H^{\text{IR}}$	Retains internal history while permitting a state-level effective description
Capability distribution	Context-dependent readouts x_i^{cap} and their comparison structure	Supplies a coarse manifestation through which relative positions can be compared
Concentration of power	Relational-power structures whose operator lifts deform shared history weights or regime accessibility	Replaces intrinsic scalar possession with configuration-dependent dynamical consequence
Balancing	Admissible counter-transformations and their operator lifts	Represents responses that partially counter a declared landscape deformation
Balance	Balance-like relational regime satisfying a declared response schema	Allows dynamic balance without requiring equality of capability readouts

instances of a more general structural operation: a response can counter-deform a landscape previously deformed by another relational-power structure.

Let $\mathfrak{B}_{-i}(\mathcal{C}_H^{\text{IR}})$ be a model-defined family of admissible balancing transformations generated by actors other than, or coalitions not identical with, the focal power position A_i . Each $\Phi_{j|i}^{\text{bal}} \in \mathfrak{B}_{-i}(\mathcal{C}_H^{\text{IR}})$ is a partial generative transformation on the coupled configuration class. When the same comparison universe and linear representability conditions used in Definition 9.1 are satisfied, denote its positive bounded operator representation by $\hat{\mathcal{B}}_{j|i} \in \mathcal{B}_+(\mathcal{W}_{H \rightsquigarrow H'})$. This typing is required before the composite $\hat{\mathcal{B}}_{j|i} \hat{P}_i$ is formed.

Assume that $\hat{\mathcal{B}}_{j|i} \hat{P}_i w \neq 0$ on the declared comparison universe. The sequentially conditioned history-weight assignment is defined in Equation 79.

$$\overline{w}_{H \rightsquigarrow H'}^{[P_i; \mathcal{B}_{j|i}]} := \frac{\hat{\mathcal{B}}_{j|i} \hat{P}_i w_{H \rightsquigarrow H'}}{\left\| \hat{\mathcal{B}}_{j|i} \hat{P}_i w_{H \rightsquigarrow H'} \right\|_1}. \quad (79)$$

Equation 79 preserves procedural order: the balancing response is represented after the focal deformation. A different historical ordering can require a different composite and need not yield the same result.

For a declared regime $\mathcal{A}$, let $\text{Acc}_{\mathcal{A}}^{P_i; \mathcal{B}}$ denote the accessibility profile computed from Equation 79. Let ψ_{ctr} be a declared counter-deformation schema whose domain contains pairs $(\text{Acc}_{\mathcal{A}}^{P_i; \mathcal{B}}, \text{Acc}_{\mathcal{A}}^{P_i})$ and whose codomain is $\{0, 1\}$. The schema must state what counts as counter-deformation for the application, such as a reduction in accessibility over a specified family of extensions, restoration of an excluded continuation family, or another explicit structural criterion.

Definition 10.2 (Balancing counter-deformation). A balancing transformation $\Phi_{j|i}^{\text{bal}}$ counter-deforms the focal power deformation relative to regime $\mathcal{A}$ when its operator lift is defined and

$$\psi_{\text{ctr}}(\text{Acc}_{\mathcal{A}}^{P_i; \mathcal{B}}, \text{Acc}_{\mathcal{A}}^{P_i}) = 1. \quad (80)$$

Definition 10.2 does not require restoration of the pre-deformation configuration. Balancing can produce a new history, new alliances, new institutions, or new capability manifestations while reducing or redirecting a selected dynamical consequence of the focal power structure. It also does not assume that every state responds by balancing; the definition classifies a response when the specified counter-deformation occurs.

10.4 Balance as a Relational Response Regime

The preceding construction permits balance to be separated from equality. A system can remain balance-like while capability readouts are unequal, and equal capability readouts can coexist with a weak or absent counter-response structure. The relevant object for the present reconstruction is therefore a regime of coupled response, not an arithmetic identity such as $P(A) = P(B)$.

Let $\mathcal{X}_{\text{bal}}$ be a model-defined space containing the response profiles required by the case study. Let $\chi_{\text{bal}} : \mathfrak{C}^{\text{IR}} \rightarrow \mathcal{X}_{\text{bal}}$ collect, for each configuration in its domain, the declared information concerning focal deformations, admissible counter-transformations, their counter-deformation status, and any additional response relations retained by the model. Let $\psi_{\text{bal}} : \mathcal{X}_{\text{bal}} \rightarrow \{0, 1\}$ be a declared balance schema.

Definition 10.3 (Balance-like relational regime). The balance-like regime associated with $(\chi_{\text{bal}}, \psi_{\text{bal}})$ is the subset of international configurations defined in Equation 81.

$$\mathcal{A}_{\text{bal}} := \{ \mathcal{C}_H^{\text{IR}} \in \text{Dom}(\chi_{\text{bal}}) \mid \psi_{\text{bal}}(\chi_{\text{bal}}(\mathcal{C}_H^{\text{IR}})) = 1 \}. \quad (81)$$

Definition 10.3 deliberately leaves the balance schema open to theory and application. A structural-realist implementation can weight counterbalancing responses to shifts in relative capability; a balance-of-threat implementation can condition the response profile on threat-relevant manifestations; other realist or non-realist models can select different relational evidence. The GR claim is limited to the architecture: balance is represented as a historically situated response regime whose comparison and admissibility structures are explicit.

This representation also accommodates the distinction between a spontaneously generated balance and a balance maintained through deliberate diplomatic practice. The realist literature has long contained more automatic and more managed interpretations of balance-of-power operation [14]. The distinction becomes formal in Subsection 10.7 after the near-critical hypothesis is stated.

10.5 Near-Critical Persistence

A balance-like response regime is not yet a critical regime. Criticality introduces an additional dynamical claim concerning the coexistence of persistence, susceptibility, and reconfigurability. The Criticality Hypothesis of International Politics proposes that some international relational systems may, under specified conditions, occupy regimes in which organization persists while perturbations can propagate and substantial reconfiguration remains accessible [8]. The hypothesis is explicitly weaker than the claim that international politics always returns to a critical point or that near-criticality is politically desirable.

To preserve that scope, the present paper uses a non-scalar profile. Let $\mathcal{X}_{\text{pers}}$, $\mathcal{X}_{\text{sus}}$, and $\mathcal{X}_{\text{rec}}$ be model-defined spaces of persistence, susceptibility, and reconfigurability descriptors. Let Π_{pers} , Π_{sus} , and Π_{rec} be partial maps from $\mathfrak{C}^{\text{IR}}$ into those spaces. The criticality profile is stated in Equation 82.

$$\mathbf{c}_{\text{IR}}(\mathcal{C}_H^{\text{IR}}) := (\Pi_{\text{pers}}(\mathcal{C}_H^{\text{IR}}), \Pi_{\text{sus}}(\mathcal{C}_H^{\text{IR}}), \Pi_{\text{rec}}(\mathcal{C}_H^{\text{IR}})). \quad (82)$$

The persistence component can reuse the expected-persistence machinery of Section 4; the susceptibility component can reuse the perturbative response profiles of Section 3; and the reconfigurability component can be represented through reachable-history diversity, operator availability, or another declared structure. Equation 82 does not aggregate these components into one criticality score.

Let $\psi_{\text{crit}} : \mathcal{X}_{\text{pers}} \times \mathcal{X}_{\text{sus}} \times \mathcal{X}_{\text{rec}} \rightarrow \{0, 1\}$ be a declared critical-corridor schema. The corresponding near-critical corridor is defined in Equation 83.

$$\mathfrak{C}_{\text{crit}}^{\text{IR}} := \{ \mathcal{C}_H^{\text{IR}} \mid \psi_{\text{crit}}(\mathbf{c}_{\text{IR}}(\mathcal{C}_H^{\text{IR}})) = 1 \}. \quad (83)$$

Equation 83 treats near-criticality as a model-relative region rather than as one universal scalar threshold. This use is consistent with the broader critical-transition literature, where proximity to transition is a property of specified dynamical models and observables rather than a synonym for visible disorder [13].

The intersection of balance-like and near-critical configurations is stated in Equation 84.

$$\mathcal{A}_{\text{bal}}^{\text{crit}} := \mathcal{A}_{\text{bal}} \cap \mathfrak{C}_{\text{crit}}^{\text{IR}}. \quad (84)$$

The intersection can be empty. Its definition therefore does not identify balance with criticality. To state persistence without an undefined qualitative threshold, let ψ_{cb} be a declared persistence schema whose domain contains expected-persistence profiles of the type defined in Equation 17, evaluated for the regime $\mathcal{A}_{\text{bal}}^{\text{crit}}$. The schema can require a lower envelope, robust retention under a declared perturbation family, or another explicitly stated rule.

Hypothesis 10.4 (Conditional critical-balance hypothesis). For some international relational configurations and declared balancing mechanisms, the coupled sequence of power deformation and admissible counter-deformation may produce a regime whose expected-persistence profile for $\mathcal{A}_{\text{bal}}^{\text{crit}}$ lies in $\text{Dom}(\psi_{\text{cb}})$ and satisfies $\psi_{\text{cb}}(\text{EP}_{\mathcal{A}_{\text{bal}}^{\text{crit}}}^w(\cdot \mid \mathcal{C}_H^{\text{IR}})) = 1$. Under those stated conditions, balance can be modeled as expected maintenance within a near-critical relational corridor rather than as equality of capability readouts.

Hypothesis 10.4 is intentionally conditional. It does not predict that every imbalance generates balancing, that every balance is near-critical, or that the international system possesses a purposive tendency to optimize criticality. Its empirical content lies in the conjunction: the identified system must exhibit a balance-like response structure, the criticality descriptors must be independently operationalized, and the resulting regime must show expected persistence over the historical scope under examination.

The dynamical architecture of the hypothesis is summarized in Figure 7. The figure shows a focal power deformation, admissible counter-deformations, a balance-like response regime, and the additional near-critical condition. The feedback arrow from manifested balance norms is reserved for the reflexive case developed in Subsection 10.7.

10.6 Failure of Balancing

The formalism also identifies why a balancing tendency cannot be assumed. A focal power structure can alter the configuration in ways that remove, weaken, or redirect the operations through which counter-deformation would otherwise occur. Conversely, other actors can retain formally available responses whose dynamical effect is too weak to satisfy the declared counter-deformation schema. These cases should remain distinct.

For focal position A_i , define the effective balancing family in Equation 85.

$$\mathfrak{B}_{-i}^{\text{eff}}(\mathcal{C}_H^{\text{IR}}) := \left\{ \Phi_{j|i}^{\text{bal}} \in \mathfrak{B}_{-i}(\mathcal{C}_H^{\text{IR}}) \mid \begin{array}{l} \Phi_{j|i}^{\text{bal}} \text{ is admissible at the relevant configuration,} \\ \text{its operator lift is defined, and Equation 80 holds} \end{array} \right\}. \quad (85)$$

Equation 85 separates formal availability from effective counter-deformation. The effective family can be empty because the relevant response is outside the operator domain, because institutional or material prerequisites are absent, or because an available response fails to alter the selected accessibility profile in the required way.

Definition 10.5 (Structural balancing failure). Balancing fails structurally relative to the declared focal deformation, regime family, and counter-deformation schema when $\mathfrak{B}_{-i}^{\text{eff}}(\mathcal{C}_H^{\text{IR}}) = \emptyset$ over the historical scope retained by the inquiry.

Definition 10.5 is deliberately narrower than a claim that a state has behaved irrationally or that a realist theory has been falsified. The international-relations literature documents several alternative responses and debates over underbalancing, bandwagoning, soft balancing, and the conditions under which counterbalancing occurs [14, 15]. The GR definition provides a common structural question: which counter-transformations are available, and which of them materially redirect the represented landscape?

A further possibility is self-supporting concentration. If the focal power deformation shrinks the domains of counter-transformations while enlarging accessibility of a power-compatible regime, the self-sustaining mechanism of Section 9 can operate at the system level. In that case, balance failure is not merely the absence of a response; it can be generated by the power structure's prior reorganization of the response landscape.

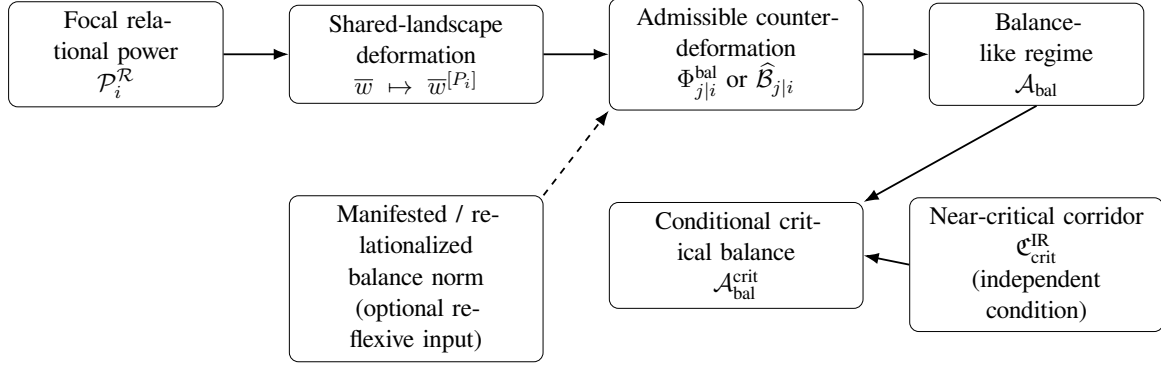


Figure 7: GR reconstruction of balancing and conditional critical balance. Focal relational power can deform a shared history-weight landscape; admissible counter-deformation can contribute to a balance-like regime. Near-criticality is specified independently, and the conjunction of the balance-like and near-critical conditions defines the conditional critical-balance regime. A manifested or relationalized balance norm can additionally condition the balancing repertoire in the reflexive case.

10.7 Emergent and Reflexively Maintained Balance

A final distinction concerns the status of the idea of balance itself. Some realist accounts describe balance as an unintended systemic result of states pursuing their own security, whereas other traditions give greater importance to deliberate balancing, diplomatic management, or actors consciously acting to preserve a balance [14]. GR can represent these possibilities without treating them as one mechanism.

Definition 10.6 (Emergent balance). A balance-like regime $\mathcal{A}_{\text{bal}}$ is emergent in the present sense when its expected persistence can be represented through coupled power and counter-deformation histories without requiring a relationally real comparison norm or symbolic commitment whose content is “maintain the balance” to condition the relevant response operators.

Definition 10.6 allows balance to arise from decentralized responses whose system-level effect is not represented as an actor’s intended target. The definition therefore remains compatible with an automatic or self-organizing interpretation while avoiding the stronger claim that such an interpretation is always empirically correct.

A reflexive form becomes possible when a manifestation of balance is generated and then acquires performative or relational efficacy. Let $m_{\text{bal}} \in \mathcal{M}_{H,c}^P$ denote a manifested balance concept, classification, or strategic representation. If Section 7’s relationalization conditions are satisfied, let $J_{\text{bal}}^{\mathcal{R}}$ denote the resulting relationally real balance norm or comparison structure. Its effect can be represented through a conditioned admissible operator family as in Equation 86.

$$\mathcal{O}_{\text{adm}}(\mathcal{C}_H^{\text{IR}}) \mapsto \mathcal{O}_{\text{adm}}(\mathcal{C}_H^{\text{IR}} \mid J_{\text{bal}}^{\mathcal{R}}). \quad (86)$$

Equation 86 means that an operative idea of balance can alter which alliances, interventions, restraint practices, signaling acts, or institutional responses are treated as admissible or salient by the represented system. The equation does not assume that the norm successfully produces balance or that actors interpret the norm identically.

Definition 10.7 (Reflexively maintained balance). A balance-like regime is reflexively maintained when a manifested balance structure has become performatively or relationally efficacious and the resulting conditioned operator family contributes generatively to histories satisfying the declared balance schema.

Definition 10.7 completes the case study by linking systemic dynamics to the paper’s earlier account of manifestation and relationalization. Balance can first emerge as a system-level response pattern, later become symbolically recognized, and then feed back into the relational dynamics through which balancing

is reproduced. The sequence is historically contingent. A symbolic commitment to balance can also fail to stabilize, can misrepresent the underlying landscape, or can generate transformations that move the system away from the balance-like regime it was intended to preserve.

The case study therefore yields a conditional reinterpretation of realist balance. A capability distribution becomes one coarse manifestation of historically extended GR spacetimes; relational power acts through deformation of shared possibilities; balancing denotes counter-deformation when the appropriate responses are available and effective; and balance denotes a metastable response regime rather than equality of scalar capabilities. Under the additional and independently testable conditions stated in Hypothesis 10.4, such a regime may overlap with an expectedly persistent near-critical corridor. This reconstruction does not establish that international politics is critical or that balancing is universally restorative. It identifies one formal bridge through which the metaphysics of relational power can be connected to a familiar international-political theory while preserving alternative mechanisms, failed balancing, and historically changing comparison structures.

11 Formal Scope of the Proposed Power Metaphysics

This section consolidates the epistemic status of the constructions developed in the paper. Its role is delimitative: the preceding sections introduced a layered account of relational asymmetry, power relevance, relational power, manifestation, comparison, categorical representation, landscape deformation, and one international-political case study. The present section identifies which parts belong to the proposed GR metaphysics, which parts are optional mathematical representations, which claims remain dynamical hypotheses, and which interfaces require empirical specification. The discussion proceeds through representational status, dynamical scope, comparative structures, agency, empirical interfaces, and open formal extensions.

The distinctions collected here are analytically consequential. The paper uses several mathematical languages because different questions require different structures. A history-bearing 2-complex helps represent extended relational organization; partial maps represent restricted transformation; attractor-like language represents persistence and regime structure; category-theoretic devices organize typed relations among formal levels; and operator lifts represent dynamical action on an auxiliary history-weight space. Their coexistence does not imply that they are mutually reducible or that one of them supplies the final ontology of power.

11.1 Representational Status

The proposed metaphysics uses the three-layer GR architecture developed throughout the manuscript. Ultimate ontological reality remains suspended. Relational reality contains structures whose historical persistence and type-appropriate relational efficacy are sufficiently established within the declared historical and modeling scope. That efficacy can be causal, generative, or both according to the structure being classified; for the relational-power construction of Section 4, both are required. Manifestation reality contains context-dependent presentations through which relational, proto-relational, or otherwise modeled structures become epistemically, institutionally, symbolically, or practically accessible. Manifestation and relational-reality roles can therefore coexist for the same historically tracked structure when their respective conditions are independently satisfied. The proto-relational domains introduced in Sections 4 and 6 are formal holding spaces for candidate structures whose status as relational reality remains unresolved. They do not constitute an additional layer of reality.

The same discipline applies to the representation of GR entities. The symbols A and B denote history-bearing GR spacetime entities at the level selected by the inquiry. The coupled configuration $\mathcal{C}_H$ is a distinct modeled object that contains those entities together with retained cross-entity and contextual structures. A labeled 2-complex provides one candidate representation of such a history-bearing structure, and spin-foam-inspired constructions can provide a more specific instantiated architecture when their assumptions

are useful. These choices organize histories, boundaries, labels, and relational continuation. Their use does not identify ultimate social, political, or physical ontology with a 2-complex or a spin foam.

Remark 11.1 (Representation and reality status). A mathematical representation can be adequate for a declared analytical purpose while remaining ontologically noncommittal. Conversely, a structure can qualify as relationally real within the GR framework without possessing a unique mathematical representation. The correspondence between relational reality and formal representation is therefore many-to-many in the present inquiry.

Remark 11.1 constrains the interpretation of the paper’s formal vocabulary. In particular, the closure map of Subsection 4.5, the categorical profunctors of Section 8, and the operator lift of Section 9 describe selected structural roles. None of these objects is treated as a metaphysical identity condition for power.

For terminological consistency, the manuscript uses *relational realization* when a candidate power organization itself acquires relational-reality status. It uses *relationalization* for the historically mediated case in which a manifestation, comparison structure, or other source becomes part of the generative support of a later relationally real structure. Relationalization therefore need not assert identity between the source object and the later relational structure.

Table 8 summarizes the principal constructions and their status in the present paper.

Consistent with Remark 1.1, every stronger formal object used in the paper has a declared construction path. Typed asymmetry is constructed from a profile map and a specified comparison relation; perturbative response from a partial transformation on the coupled configuration class; relational power from power relevance plus separately tested persistence and efficacy; manifestation from typed readout or mediation maps; comparison judgments from a partial comparison operation; quotients from an explicitly declared equivalence relation; categorical objects from previously defined status domains; and operator lifts from an auxiliary history-weight space plus a representability assumption. These construction paths are part of the formal scope of the claims and should not be replaced by notation-only identification.

11.2 Scope of Dynamical Claims

The dynamical language of the paper has several levels of commitment. At the weakest level, a power-relevant asymmetry is associated with directional consequence for a future possibility structure. Sections 3 and 4 then add expected persistence and relational efficacy. These conditions define the proposed transition from power relevance to relational power without requiring one universal differential equation or one globally specified phase space. The scope is directional and asymmetry-derived. Reciprocal power can be represented by coexisting directional components, as stated in Remark 4.9; the manuscript does not claim that this construction exhausts every possible genealogy or organization of power.

Attractor-like, metastable, basin-like, and closure descriptions provide stronger representations. They become analytically useful when the modeled continuation structure supports repeated return, persistence, attraction, or stabilization. Their use does not require a fixed point or a limit cycle, and near-critical relational reality remains compatible with the framework when sufficient persistence coexists with elevated susceptibility or reconfigurability. The relevant historical scale must be declared in each application because a regime can be stable at one resolution and transitional at another.

The history-weight formalism of Section 9 adds another layer. A weight assignment w organizes represented continuations for comparison. Normalization permits expectation-like summaries and accessibility profiles. Such weights receive a probabilistic interpretation only after the application supplies the required probability structure. They receive a quantum-amplitude interpretation only after an independently justified state space, amplitude rule, and physical interpretation have been established. The present paper uses neither interpretation as a foundational assumption.

Claim 11.2 (Dynamical scope of relational power). Within the asymmetry-derived construction developed in this paper, a directional relational-power component requires a history-sensitive account of directional consequence, expected persistence, causal efficacy, and generative efficacy. A specific attractor geometry, probability law, operator algebra, or criticality model is an additional representation whose adequacy depends on the local research question.

Table 8: Epistemic and representational status of principal constructions in the proposed power meta-physics.

Construction	Role in the paper	Present status	Additional requirement for stronger use
Typed relational asymmetry	Antecedent relational structure from which power relevance is investigated	Relationally real within the declared historical scope when the criteria in Section 2 are satisfied	Domain-specific identification of the relevant asymmetry type and historical support
Directional relational power $\mathcal{P}_{A \curvearrowright B}^{\mathcal{R}}$	Asymmetry-derived power structure with directional consequence, expected persistence, and relational efficacy	Proposed GR relational-reality construction	Empirical reconstruction of persistence, causal efficacy, and generative efficacy
Manifestation spaces and mediation maps	Context-dependent access to power through symbolic, embodied, institutional, or other channels	Formal representation of manifestation pathways	Application-specific semantics and evidence for the selected mediation channel
Attractor-like regime, basin-like domain, closure map	Models stabilization, expected persistence, and proto-relational convergence	Optional dynamical and algebraic representations	Explicit state space, continuation structure, and stability or closure assumptions
Categorical spans and profunctors	Organize partial comparison, mediation, historical continuation, and relationalization witnesses	Typed categorical representations	Richer internal categorical structure only where composition and preservation properties are established
Operator lift $\Lambda_P(\mathcal{P}^{\mathcal{R}}) = \hat{P}$	Represents power-conditioned deformation of an auxiliary history-weight space	Modeling lift, distinct from the ontology of power	Declared weight space, operator domain, and interpretation of the resulting deformation
Near-critical balance regime	International-political case study combining balancing response with independently specified criticality conditions	Conditional hypothesis	Empirical operationalization of balancing, persistence, susceptibility, and reconfigurability

Claim 11.2 prevents the paper’s optional dynamical resources from becoming hidden premises of the core definition. It also preserves the possibility that different applications represent persistence through different mathematical devices while retaining the same GR distinction between transient power relevance and relationally real power.

Self-reproduction occupies a still narrower scope. Section 9 models a case in which power-conditioned deformation favors a regime relative to a declared baseline and that regime is also power-compatible, so

historical continuation can help reproduce or extend the conditions of the power structure's efficacy. This is a higher-order property of some power relations. Relational power can exist over a meaningful historical interval without satisfying this self-support condition.

11.3 Scope of Comparative Structures

The paper treats comparison as generated structure. A judgment such as $A \succ_P B$ is produced only relative to a declared comparison representation, context, and comparison operation. The comparison can remain undefined for some pairs, can leave some outputs incomparable, and can change as the underlying relational or manifestational structure changes. This architecture therefore does not presuppose a total power ordering.

The same restraint applies to metrics. A scalar metric, quasi-metric, ranking, index, or enriched comparison can be useful when an application establishes the relevant comparison space and interpretation. GDP, territorial area, military capability, institutional office, prestige, and similar observables can enter manifestation-level comparison structures without thereby becoming universal measures of relational power. A metric used by actors or institutions can itself become relationally consequential when its classifications alter recognition, allocation, admissibility, or future histories. The reality status of the metric practice must then be evaluated separately from the mathematical properties of the metric used to represent it.

The quotient-symbolization route developed in Sections 7 and 8 is similarly conditional. A relationally operative equivalence relation can support a quotient representation, and a symbolization process can map the resulting equivalence classes into manifestation. Analytic equivalence relations introduced by the researcher occupy a different epistemic status. The quotient construction itself remains a mathematical representation in either case.

Remark 11.3 (Comparison structures as objects of inquiry). A power-comparison relation or metric can function simultaneously as an instrument for description and as an object whose own emergence, stabilization, manifestation, and relational consequences require explanation. The present framework therefore permits reflexive analysis without treating the comparison scheme as epistemically external to the system by default.

Remark 11.3 is especially important where rankings, titles, strategic categories, or institutional metrics become performative. In those cases, the comparison structure can contribute to the power configuration that later observations attempt to measure.

11.4 Scope of Agency

The core definition of relational power does not require a unitary intentional agent. A distributed institution, infrastructure, market arrangement, symbolic order, or other history-bearing relational structure can exert persistent directional effects on future possibility structures even when no single subject selects those effects as an objective. The formalism therefore distinguishes directional generative consequence from intentional steering.

Intentional agency can be added when the application independently establishes an agent, a target representation, and a relation between deliberation and subsequent generative operations. Under those conditions, a modeled regime can additionally be designated as a deliberately targeted regime. Targeted status is conceptually independent of whether the regime is power-favored or power-compatible; those additional relations must be established separately. The intentional interpretation must be earned by the agency model rather than inferred from the fact that the power structure deforms future possibilities.

The same distinction applies to freedom and contingency. The expected-persistence formalism leaves room for multiple admissible continuations and does not reduce the future to a unique trajectory. This openness is compatible with models containing agency, stochasticity, unresolved contingency, or combinations of them. It does not by itself establish a metaphysical theory of free will.

Remark 11.4 (Intentional and non-intentional power). The phrase “power-favored regime” denotes the landscape-deformation role defined in Section 9.4: relative to a declared baseline and comparison schema, the represented power structure increases the regime’s modeled accessibility or retention. This should be distinguished from *power-compatible*, which means that the regime supports historical continuation of the power structure. The phrase “targeted regime” should be reserved for cases in which an additional account of agency supports intentional selection or steering toward that regime.

Remark 11.4 preserves the applicability of the formalism to both agent-centered and distributed structural cases while preventing teleological language from entering the model without support.

11.5 Empirical Interfaces

An empirical application requires a translation from observed or reconstructed material into the formal objects used by the paper. The first task is entity and history specification. The analyst must state which history-bearing structures count as the GR entities under comparison, which events enter the modeled history, which precedence relations are retained, and which boundaries or scopes define the relevant coupled configuration. A state or organization can require a different relational decomposition for different research questions.

The second task is asymmetry identification. The application must specify the relational dimension α , the evidence supporting the typed asymmetry, and the reason that the asymmetry is treated as relationally real over the historical scope. The third task is dynamical interrogation: admissible perturbations, response descriptors, comparison or salience schemas, and generative-dependence claims require operational definitions. Observational correlation alone does not identify a power mechanism under the present framework.

The fourth task concerns the reality status of power. Evidence for expected persistence, causal efficacy, and generative efficacy must be supplied at the scale relevant to the application. Process tracing, archival reconstruction, comparative institutional analysis, longitudinal data, intervention studies, simulation, or mixed methods can each contribute depending on the question. The framework does not privilege one empirical method in advance.

The fifth task concerns mediation and comparison. Manifestation channels, symbolic categories, equivalence relations, rankings, metrics, and institutional comparison rules must be identified through the practices that actually sustain them. A numerical index constructed by the analyst and a numerical index that governs institutional allocation can share a mathematical form while occupying different relational statuses.

Finally, an application of the landscape formalism must specify what history weights, regime accessibility, attractor-like regions, or operator lifts mean in empirical terms. The power operator $\hat{P}$ can be useful only after the auxiliary representation space has been defined and the mapping from relational evidence to operator action has been justified. Section 10 illustrates this requirement: the balance and criticality schemas remain hypotheses until their components can be operationalized independently.

Research Obligation 11.5 (Empirical identification of relational power). An empirical use of the proposed formalism should document the mapping from observed or reconstructed evidence to at least the following objects when they are invoked: the GR entity and historical scope, the typed asymmetry, admissible perturbations, directional response, persistence evidence, causal efficacy, generative efficacy, manifestation channel, comparison structure, and any auxiliary dynamical representation used in the analysis.

Research Obligation 11.5 is intentionally modular. An application that does not use attractor, operator, or metric language need not operationalize those objects. The obligation is to make the bridge explicit for each formal construction actually carrying an empirical claim.

11.6 Open Formal Interfaces

Several mathematical interfaces remain deliberately open. The first concerns the internal structure of history-bearing GR entities. The present paper permits 2-complex, spin-foam-inspired, graph-like, or

other representations under ontological suspension. A future application can strengthen the combinatorial, topological, geometric, or categorical structure when the analytical problem requires it.

The second concerns partiality and local structure. Several established formalisms can refine the minimal language of partial maps. Restriction categories abstract the structure of partial maps and their domains of definedness [4]; partial group actions and inverse-semigroup actions provide one route for organizing partial transformation structure [6]; and inverse semigroups more generally supply an algebraic theory of partial symmetries [9]. Groupoids can likewise encode local reversible relations. Such enrichment is warranted when these properties perform analytical work beyond the partial-map construction already sufficient for several arguments in the paper.

The third concerns mediation and comparison. Profunctors and spans provide a deliberately weak categorical representation because mediation and comparison can be many-to-many, partial, and witness-dependent. Double categories or equipments may offer a more unified language when vertical relational transformations, horizontal proto-relations, and 2-cells expressing mediation or stabilization all need to be composed explicitly. Quotient categories, congruences, partial equivalence relations, tolerance structures, or local groupoids can refine the quotient-symbolization route when ordinary equivalence is too strong.

The fourth concerns dynamics. A genuine operator algebra becomes available only after an appropriate linear or otherwise algebraic representation space has been specified. Such a development could study compositional noncommutativity, spectra, invariant subspaces, or response operators without identifying those structures with the ontology of power. Stochastic processes, path-integral-style history sums, nonlinear dynamical systems, and critical-transition models provide additional possibilities when their assumptions can be connected to empirical or theoretical evidence.

The fifth concerns geometry and measurement. Metric, quasi-metric, Lawvere-enriched, or other directional geometric structures may help represent asymmetric transformation costs or accessibility. Their value depends on whether the relevant distances preserve the relational distinctions required by the application. A convenient scalarization is insufficient reason to impose commensurability on heterogeneous power relations.

Remark 11.6 (Plurality of formal extensions). The open interfaces identified in this section do not form a mandatory progression toward one maximally elaborate formal system. Each extension should be introduced only when it resolves a defined representational, inferential, or empirical limitation in the problem under study.

Remark 11.6 restates the methodological principle that governs the paper as a whole. Formal enrichment is justified by analytical work, while ontological suspension leaves alternative representations available. The resulting framework therefore supports further work on operator dynamics, categorical mediation, critical regimes, comparison structures, and empirical identification without treating any one of those developments as a required completion of the proposed metaphysics.

12 Conclusion

This concluding section consolidates the paper's proposed metaphysics of power within Generative Relational Theory and states the limits that remain attached to it. The paper has treated power as a relational formation whose analysis begins only after relational difference has been distinguished from typed relational asymmetry and after the relevant asymmetry has acquired relational-reality status. The central contribution is therefore a conditional account of the additional structure through which a relationally real asymmetry can become power-relevant and, under stronger historical conditions, become a relationally real power structure. The resulting account remains representational and revisable. It does not identify a single mathematical construction with ultimate ontology and does not reduce power to one scalar magnitude, one manifestation, or one mechanism of formation.

The first part of the argument fixed the relata of power analysis. A GR entity was treated as a history-bearing relational spacetime rather than as a point carrying an intrinsic power quantity. A labeled 2-complex supplied one candidate representation of such a history-bearing structure, while event precedence,

relational slices, and accumulated historical domains supplied a model-relative account of temporal organization. This construction allowed relational asymmetry to be typed relative to a declared relational dimension while preserving the distinction among difference, relational differentiation, and asymmetry. Symmetry breaking can generate relational differentiation under suitable conditions, yet the power analysis does not depend on symmetry breaking as a universal genealogy. The antecedent required by the present paper is the relationally real typed asymmetry stated in Assumption 2.4.

Section 3 then introduced the first additional condition. Relational asymmetry becomes relevant to power when it participates in directionally organized dynamical consequence. The generative possibility structure in Equation 8 records accessible histories and admissible operations without presuming that either admits a scalar summary. Admissible perturbations provide a way to interrogate how a relational position changes those possibilities. The resulting response profile can be described through breadth, depth, persistence, robustness, generativity, and other application-specific descriptors, while the comparison among such descriptors remains a separate problem. The definition of power relevance in Definition 3.4 therefore requires both directional dynamical consequence and an explicit account of generative dependence. Perturbation functions as an epistemic probe of relational dynamics rather than as an automatic measure of power.

The transition from power relevance to relational power required a second layer of conditions. Section 4 treated relational power as a higher-order relational organization whose own reality status must be established. The paper proposed expected persistence across admissible historical continuations together with causal efficacy and generative efficacy as the relevant criteria within the present GR framework. This formulation accommodates fixed points and limit cycles when they are useful, while also allowing metastable, path-dependent, branching, and near-critical regimes. Attractor-like regimes, basin-like domains, and closure operators were therefore introduced as candidate mathematical resources for representing stabilization. They do not independently confer relational reality. Definition 4.8 reserves relational power for the power candidate generated around a power-relevant asymmetry only when that candidate satisfies the stated persistence and efficacy conditions at a declared historical scale. Within this paper, that definition applies to the directional, asymmetry-derived construction $\mathcal{P}_{A \rightsquigarrow B}^R$. Reciprocal power can be represented by coexisting directional components, while other genealogies of power remain outside the present derivation.

The epistemic analysis showed that such a power structure need not be directly visible. Section 5 separated relational power from its context-dependent manifestations, identified latent power as a possible structure of relational efficacy without continuous overt display, and treated attempted transformation as one route through which constraints on admissible continuation can become observable. Historical evidence families and evidence fibers then made explicit the inverse problem in power attribution: similar manifestations can be compatible with different relational structures, and one relational power structure can generate different manifestations across contexts. Epistemic access therefore remains partial even when the underlying power relation is treated as relationally real.

The same discipline was applied to statements that compare power. Section 6 represented a judgment such as “ A is more powerful than B ” as the output of a partially defined comparison operation acting on declared representations of history-bearing GR entities. The judgment can remain proto-relational while its relational status is unresolved, can acquire a stable role in manifestation reality, and can additionally acquire relational reality when the comparison structure satisfies an explicit historical persistence criterion and becomes causally efficacious and generatively efficacious. These statuses are not mutually exclusive: a relationally real comparison norm can continue to operate through manifested forms. Metrics, quasi-metrics, rankings, and indices consequently occupy two analytically distinct roles. They can serve as formal instruments introduced by an analyst, and they can become historically operative structures that help organize the relations they describe. The reflexive loop in Definition 6.6 captures the latter possibility by allowing a comparison judgment to condition later generative transformations.

Sections 7 and 8 developed the mediation architecture in greater detail. Relational power can become manifest through symbolic, embodied, institutional, and other channels. One symbolic route proceeds through an historically generated equivalence structure, a quotient representation that suppresses selected relational differences, and a subsequent act of symbolization. The equivalence relation itself can occupy

different GR statuses: it may remain an analytic device, become an enacted manifestational structure, or acquire relational reality when its persistence and efficacy are established. The quotient construction remains a formal representation in each case. The categorical architecture organized these distinctions through history-indexed categories, spans, and witness-valued profunctors while deliberately avoiding stronger functorial assumptions where uniqueness or compositional preservation had not been established. Figure 5 summarizes this typed architecture without introducing an arrow from suspended ultimate ontology into the modeled relational domains.

The dynamical analysis of Section 9 added a further property that some power structures can possess. A relational power structure can be represented by an operator acting on an auxiliary history-weight space, and the resulting deformation can change the relative support or accessibility of alternative continuation regimes. This operator lift is a representation of dynamical action rather than an ontology of power. Where a power-conditioned deformation increases access to a regime that in turn reproduces or extends the supports of the original power structure, the framework identifies a self-supporting loop of relational reproduction. Definition 9.6 and the self-support schema in Equation 75 formalize this possibility. Self-reproduction remains a higher-order property. Relational power can be real and consequential over a meaningful historical interval without reproducing itself through such a loop, and adaptive persistence can preserve a power role while its institutional or symbolic supports change.

The international-political case in Section 10 illustrated how these distinctions alter the interpretation of balance. States were treated as history-bearing GR spacetimes, while capability indicators were treated as coarse manifestations or comparison inputs rather than complete power quantities. Balancing was represented as counter-deformation of a shared generative landscape, and a balance-like regime was characterized through the organization of coupled responses rather than through numerical equality of capabilities. The conditional critical-balance hypothesis in Hypothesis 10.4 then identified one possible regime in which balance-like dynamics remain expectedly persistent within a near-critical corridor. The hypothesis allows balancing failure, attractor capture, fragmentation, and alternative responses. It therefore serves as an application of the power formalism rather than as a universal law of international politics.

Across these constructions, four distinctions carry the main conceptual burden. Typed relational asymmetry supplies an antecedent structure, while power relevance concerns its directional dynamical consequences. Relational power additionally requires the power organization itself to acquire expected persistence and relational efficacy. Manifestation provides context-dependent access to that power and can, under performative conditions, participate in later relational change. Comparison structures organize judgments about power while remaining distinct from the power structures they compare. Finally, the mathematical languages used to represent these relations remain modeling resources under ontological suspension. Keeping these distinctions explicit allows GR to analyze power as relationally real without turning one formal representation into a final metaphysics.

Several open problems remain substantive rather than merely technical. The internal mathematics of history-bearing GR entities can be strengthened beyond the minimal 2-complex representation. The persistence criterion can be developed for broader classes of branching and stochastic histories. Empirical work is required to identify admissible perturbations, generative dependence, and manifestation channels in concrete domains. Comparison structures require independent study because the norms and metrics through which power becomes comparable can themselves alter power relations. Stronger categorical, operator-algebraic, enriched, and critical-transition formulations can be introduced where their assumptions are justified. Research Obligation 11.5 therefore remains central to any empirical application of the proposed architecture.

The resulting metaphysical proposal is intentionally limited. Within the present GR framework, the asymmetry-derived form of power studied here can be understood as a historically constituted relational structure generated around an already relationally real asymmetry whose directionally organized consequences deform the generative possibilities of other GR entities or of a shared configuration. The additional power organization acquires relational reality only when it becomes sufficiently persistent, causally efficacious, and generatively efficacious under the declared model. Some such power structures can subsequently reshape the landscape of possible continuations in ways that stabilize, reproduce, or transform the

conditions of their own efficacy. Their manifestations, comparison norms, and metrics can also become active participants in that history. This account leaves ultimate ontology open while providing a formal vocabulary for studying how power can emerge, become knowable, acquire durability, and participate in the generation of later relational worlds.

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