

A Preliminary Taxonomy of Preservation in Generative Relational Theory

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Abstract

This paper develops a preliminary taxonomy of preservation within Generative Relational (GR) theory. Preservation is treated as a typed and historically situated claim concerning which aspects of a represented object, relation, capacity, lineage, or dynamical organization remain sufficiently continuous, recoverable, or reproducible across changes of relational spacetime. The inquiry distinguishes preservation from bare equality and from equivalence considered without historical or provenance structure. It also distinguishes preservation claims made at the level of manifestation, relational reality, and mathematical representation while maintaining GR's suspension of ultimate ontological commitments. The proposed taxonomy is organized around preservation targets, historical modes, retained relational content, comparison relations, representational resolution, and preservation modes. Six principal target families are developed: manifestational traces, relational structures, generative capacities, generative conditions, historical-lineage and regeneration structures, and dynamical regimes. Preservation modes include continuous retention, historically continuous transformation, dormant recoverability, reconstructive recovery, recovery-mechanism regeneration, and dynamical-regime reproduction. A companion GR taxonomy of equivalence supplies comparison resources; the present paper asks the additional preservation question of what those relations are intended to retain across a declared history. Cross-target lifting is made bridge-dependent, and preservation results are represented through typed profiles that distinguish satisfaction, failure, and unresolved applicability. A separate judgment architecture distinguishes structural preservation identification from aesthetic, epistemic, normative, practical, and political-economic valuation. Language preservation provides a bounded stress test in which corpora, living capacities, generative conditions, intergenerational transmission, revival, lineage, and dynamical continuity can diverge. The paper also proposes a Generative Relational Preservation Investigation as a reporting procedure for domain-specific inquiry. The framework remains preliminary and classificatory: domain-specific normative duties, legal and governance arrangements, empirical calibration, and engineering implementation require additional analysis.

Keywords: Generative Relational Theory; preservation; equivalence; historical continuity; generative capacity; dynamical structure

Preliminary Research Note

This manuscript belongs to the broader Generative Relational (GR) research programme. Its immediate object is the concept of preservation across historically changing relational systems. The paper develops a taxonomy before advancing a domain-specific theory of language preservation, a general theory of generativity preservation, or a normative account of preservation duties. Its principal task is to distinguish several claims that ordinary preservation language can place under one label: retention of manifested traces, continuity of relational organization, preservation of generative capacity, maintenance or regeneration of enabling conditions, recoverability after interruption, and reproduction of selected dynamical characteristics.

The inquiry adopts ontological suspension. Ultimate ontological identity, continuity, or preservation is therefore not inferred from manifestation, structural similarity, historical tracking, functional equivalence, recoverability, topological invariance, attractor structure, or another formal relation developed in the manuscript. Mathematical structures are used as conditional representations whose domains, retained distinctions, and inferential limits must remain explicit.

Formal economy is a governing constraint. The manuscript inherits history-bearing GR configurations, labelled 2-complex representations, manifestation maps, partial transformations, state-dependent operation schemas, admissible historical extensions, local algebraic structures, dynamical composition, and comparison relations from preceding GR work. A companion GR taxonomy of equivalence supplies a broader library of comparison relations. The present paper introduces new notation only where preservation requires additional structure beyond equivalence, especially a declared preservation target, historical or provenance relation, retained content, and preservation mode.

The present release contains the complete planned architecture, Sections 1–17, together with three manuscript-wide revisions covering architectural consistency, terminology and literature interfaces, and logical/formal integrity. The taxonomy, language stress test, investigation procedure, boundary analysis, and conclusion are therefore available for cross-sectional review as one argument. External literature is inserted only after source verification and recording in the manuscript’s reference ledger.

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Status. This working draft contains Sections 1–17 and completes the planned manuscript architecture. Four manuscript-wide revisions have aligned the formal architecture, terminology, literature interfaces, structural titles, cross-references, presentation, logical/typing discipline, countermodel coverage, and notation economy of the preservation taxonomy. The manuscript remains preliminary: domain-specific validation, additional empirical examples, release-oriented copyediting, and possible venue-specific compression require further work.

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Statement on the use of language models. The exploratory discussions and preparation of this paper involved OpenAI’s ChatGPT. ChatGPT supported exploratory dialogue, formal reconstruction, argumentative criticism, drafting in $\text{\LaTeX}$, and technical checking. The author selected the research questions, directed and approved the theoretical commitments and epistemic status of the claims, and bears sole responsibility for the manuscript, including its definitions, formal constructions, taxonomy, arguments, conclusions, and errors. Authorship credit remains with the human author.

1 Introduction

This section establishes the analytical object, scope, and methodological architecture of the preservation taxonomy developed in this paper. Its role is to separate preservation from adjacent ideas such as identity, equivalence, persistence, recovery, and dynamical resemblance before those ideas are assigned more specific formal relations. The discussion proceeds from preservation as a Generative Relational problem to the historical structure of preservation claims, the interface with the companion GR taxonomy of equivalence, the status of preservation across GR's reality and representation layers, and a bounded language example that exposes several distinctions needed by the general framework. The method is classificatory and formal. It develops a vocabulary for well-typed preservation claims, while domain-specific normative justification, empirical calibration, legal duty, and engineering implementation remain downstream inquiries.

1.1 Preservation as a Generative Relational Problem

The word *preservation* appears to identify a simple relation between an earlier object and a later condition of that object. In historically evolving systems, however, several materially different claims can receive the same description. A written corpus can survive after the community that produced it has ceased to use the corresponding language. A social practice can retain recognizable symbols while the relations that previously gave those symbols their practical role have changed. A technical capacity can disappear from current use while remaining reconstructible from surviving knowledge, tools, and practitioners. A later system can reproduce a dynamical pattern associated with an earlier system while differing in components, organization, and genealogy. Each case can be called preservation in ordinary language, although the preserved object, historical relation, and required comparison differ substantially.

Generative Relational Theory makes these differences especially important because GR treats the relevant entities and relations as historically constituted and generatively consequential. A GR entity is represented at a selected analytical scale through the relations and histories that support its present organization and through the continuations that remain available from that organization. Preservation cannot therefore be reduced in advance to equality between two snapshots. The later configuration can contain new relations, revised boundaries, additional history, transformed manifestations, altered operation domains, and new generative possibilities while remaining continuous with selected structures of the earlier configuration. Conversely, a later configuration can reproduce a highly similar surface state without inheriting the historical or relational structure that motivated the preservation claim.

The paper consequently treats preservation as a typed comparison problem whose specification begins before any conclusion that something has been preserved. A well-formed preservation claim must identify at least the object whose continuation matters, the relational or representational level on which that object is being described, the historical or spacetime relation connecting the compared configurations, the content retained for comparison, the relation used to evaluate that content, and the mode through which preservation is claimed. Section 4 develops these components separately because different applications can fix them in different ways.

This architecture also clarifies why preservation is broader than persistence. Persistence ordinarily concerns continued presence or retention through an interval or family of historical extensions. Preservation can include persistence, yet several preservation claims concern interruption, recovery, reconstruction, transfer to a different relational environment, or reproduction of selected dynamics after the original configuration has ceased to remain operative. A theory that identifies preservation only with continuous persistence would exclude such cases by definition. The present paper therefore keeps persistence as one preservation mode among several possible modes.

Preservation is also narrower than unrestricted similarity. Two independently formed systems can be structurally isomorphic, functionally substitutable, behaviourally similar, or dynamically conjugate without either preserving the other. Similarity becomes preservation-relevant only after the model states why the relation connects the compared systems to a declared object of retention across a historical,

genealogical, reconstructive, or representational relation. This additional requirement gives preservation a directional and provenance-sensitive character that equivalence considered abstractly need not possess.

The same distinction applies to copying and reproduction. A copy can preserve selected manifested content while leaving the relational history of the source outside the copied structure. A reconstructed practice can recover an operation family while acquiring a new institutional location and new historical supports. A simulated dynamical regime can reproduce a selected attractor or transition pattern while lacking the genealogy through which the earlier regime formed. The paper treats these possibilities as analytically distinct cases and records them through typed preservation profiles.

The taxonomy is therefore intended to support typed preservation profiles that retain distinctions hidden by an unqualified preservation predicate. A preservation profile records which object classes and which levels of organization satisfy the declared preservation conditions and which remain absent, transformed, or unresolved. One application may establish strong continuity of manifestation and weak continuity of relational structure. Another may establish loss of the earlier relational configuration together with recoverability of a generative capacity. A third may establish only reproduction of selected dynamical characteristics. The profile form keeps these claims available simultaneously and reduces the risk that a successful preservation result at one level is generalized silently to another.

1.2 Historical Change and the Scope of Preservation Claims

Historical extension is constitutive of the preservation problem. A living relational system acquires new events, relations, distinctions, constraints, and possibilities as its history develops. The preservation target therefore cannot be assumed to remain numerically identical to its earlier representation. Even when the represented entity remains recognizable, its active relations and future possibilities can change sufficiently that a literal state-preservation criterion becomes inappropriate to the research object.

This feature creates a first distinction among state preservation, historical continuity, and preservation of a transformation regime. State preservation compares selected configurations at two historical locations. Historical continuity additionally asks whether the later configuration can be tracked as a continuation, descendant, transfer, or reconstruction of the earlier object under declared identity conditions. Preservation of a transformation regime concerns the structure through which changes themselves occur, including the continued availability of relevant operations, revision procedures, transmission mechanisms, or dynamical characteristics. These claims can coincide in special cases while remaining analytically separable.

Historical change also affects the conditions under which a preserved capacity can operate. A linguistic competence, craft practice, institutional procedure, scientific method, or other generative capacity is exercised within a relational environment. That environment can contain other participants, resources, material supports, classifications, infrastructures, recognized roles, and inherited expectations. A later historical configuration can retain the competence while removing many of the conditions that previously made the competence effective. Conversely, a later system can recreate a suitable environment after direct possession of the capacity has declined. Preservation of capacity and preservation of conditions therefore require separate entries in the taxonomy.

The distinction becomes still sharper when the conditions themselves are historical products. A condition relevant to generation at one relational slice can depend on earlier interactions, institutional settlements, environmental changes, or accumulated conventions. Reconstructing a previous condition set exactly can be impossible or undesirable because later history has added relations that cannot be removed without changing the object of inquiry. Preservation analysis therefore needs criteria that can tolerate historically appropriate transformation while retaining enough structure for the later system to remain comparable to the earlier target.

This problem motivates a graded family of preservation modes. At the strongest end, a capacity and its principal enabling conditions remain continuously operative across the historical domain retained by the model. A weaker mode permits temporary non-actualization while maintaining an admissible route through which an adequately corresponding capacity can be recovered. A further weakening permits

changes in the operation family itself, provided the later system can regenerate a mechanism capable of restoring the relevant generative function. The weakest major mode considered in the present programme allows the original grammar and conditions to differ substantially while requiring reproducibility of selected dynamical characteristics. Sections 11 and 12 formalize these modes after the relevant carriers and comparison relations have been specified.

The historical relation connecting two configurations is therefore part of the preservation claim. Ordinary historical continuation, migration into a different relational environment, interruption followed by reconstruction, and branching into several descendants should not be treated as interchangeable. A diaspora language, for example, can preserve a lineage while acquiring new contact relations and institutional conditions. A reconstructed language can recover substantial grammar and vocabulary after a discontinuity in fluent transmission. A branching language family can preserve different subsets of an earlier structure in different descendants. The taxonomy must permit these cases without assuming that only one later object can inherit preservation-relevant continuity.

Provenance matters for the same reason. Two later systems can satisfy the same structural comparison relation while having different histories of derivation. If one was historically generated through transformations of the earlier system and the other was independently constructed to match selected characteristics, the two preservation claims have different genealogical content. A preservation analysis can still classify both cases, but the historical witness connecting the first pair should remain available as part of the preservation claim alongside the coarse equivalence test.

The paper accordingly treats historical comparison as a typed relation within the preservation specification. Sections 4 and 9 distinguish continuous historical tracking, reconstructive linkage, relational relocation, and branching descent. The purpose is not to supply a universal metaphysics of identity through time. It is to state enough historical structure that a preservation claim reveals which continuity it actually establishes.

1.3 Interface with the GR Equivalence Taxonomy

A companion GR study develops a taxonomy of equivalence across individual reality levels, cross-level structures, historical evolutions, and dynamical regimes. That work supplies an important formal interface for the present paper because every preservation claim requires some relation through which earlier and later structures are compared. Equality, structural equivalence, isomorphism, symmetry-relative interchangeability, simulation, bisimulation, approximate similarity, and agreement of selected diagnostics can each become relevant under different preservation specifications.

The preservation problem nevertheless adds structure that an equivalence relation alone does not provide. Equivalence classifies a relation among objects under a declared carrier and retained content. Preservation asks whether such a relation is being used to retain a specified object, capacity, condition, lineage, or dynamical organization across a declared historical or reconstructive relation. Two systems can therefore satisfy a strong equivalence relation while remaining unrelated by preservation, and a preservation claim can remain meaningful under a weaker comparison relation when the retained object does not require stronger equivalence.

This difference can be illustrated by structural isomorphism. Suppose two institutional configurations are isomorphic relative to a selected network representation. The isomorphism establishes a strong structural correspondence at that resolution. It does not establish that one configuration historically continues, reconstructs, or preserves the other. A preservation claim requires additional information about the target of retention and the relation connecting the two configurations. The same logic applies to dynamical equivalence. Two systems can display equivalent attractor organization without sharing a lineage. The dynamical relation becomes preservation-relevant only when the analysis identifies attractor organization as a preservation target and supplies the historical or reconstructive relation under which its later realization counts as retention or reproduction.

The preservation taxonomy therefore uses equivalence relations as comparison resources within a broader preservation specification. This choice also prevents a hierarchy of preservation strength from

being read directly from a hierarchy of mathematical relation strength. An isomorphism can be formally stronger than approximate behavioural similarity while being preservation-irrelevant to a target defined by historical transmission. Conversely, a relatively weak diagnostic agreement can be sufficient for a preservation claim whose object is deliberately restricted to one dynamical characteristic. The relevant strength of preservation depends jointly on the target, retained content, comparison relation, historical mode, and resolution.

The interface with equivalence also clarifies the role of partiality. A comparison relation need not be defined for every pair of configurations. A preservation claim can therefore fail at the level of applicability before it fails at the level of comparison. An earlier and later system may lack a justified correspondence of entities, relations, histories, or representational scales sufficient to pose the intended preservation test. Treating such a case as simply “not preserved” would collapse undefined comparison into a negative result. The paper therefore retains definedness as part of the preservation specification.

Historical changes can also alter the equivalence structures used by preservation analysis. A language community can revise what counts as standard usage. An institution can change which roles are treated as interchangeable. A scientific field can revise the criteria by which methods are classified as equivalent. Preservation criteria can therefore evolve along with the objects they classify. The taxonomy distinguishes changes in the preserved object from changes in the comparison relation and from changes in the judgment by which a comparison is treated as sufficient for preservation.

This last distinction is especially important for generative systems. A system can preserve the capacity to generate new distinctions while changing the equivalence classes through which earlier distinctions were organized. In such a case, insisting on preservation of the earlier equivalence structure could conflict with preservation of the system’s transformative generativity. The taxonomy should therefore permit a preservation claim to retain a higher-order capacity for revision even when lower-order classifications change.

The preservation paper consequently depends on the equivalence taxonomy without duplicating it. The companion study asks which comparison relations are well formed and which structures they preserve. The present inquiry asks which of those comparison relations are recruited by preservation claims, which object is intended to continue through the comparison, and which historical mode makes the comparison preservation-relevant. This division of labor allows later GR papers on language, generativity, ethics, law, governance, and engineering to state their preservation assumptions with greater precision.

1.4 Reality Status and Formal Provenance

GR distinguishes ontological reality, relational reality, and manifestation reality, with mathematical representation serving as an analytical layer external to that three-layer reality distinction. Preservation claims can be posed relative to each of these statuses, but their evidential and inferential roles differ. The taxonomy therefore records reality status before stronger conclusions are drawn from a preservation relation.

Ultimate ontological reality remains suspended in the present inquiry. A model can conditionally ask which transformations would preserve an ontological structure if such a structure were specified, yet the paper does not infer ontological preservation from equality of manifestations, persistence of relational organization, algebraic isomorphism, topological invariance, or another modeled correspondence. This restraint follows the broader GR methodology in which formal tools provide conditional representations without settling ultimate ontology.

Relational-reality preservation concerns structures that participate in the continuing organization of a history-bearing relational system. Depending on the local GR construction, the relevant criteria can include historical persistence, causal efficacy, generative efficacy, or another type-appropriate form of relational consequence. A relational structure can remain preserved through changing manifestations when its historically tracked organization continues to shape later relations. It can also cease to be preserved while familiar manifestations remain visible if those manifestations no longer correspond to the earlier relational organization.

Manifestation preservation concerns context-dependent presentation. Texts, symbols, names, rituals, interfaces, rankings, recordings, or other manifested forms can remain stable across historical change. Such stability is analytically important because many practical preservation programmes act first on manifestations. The taxonomy nevertheless treats manifestation preservation as its own claim. A preserved recording can provide evidence for an earlier language without preserving the living relations through which the recorded expressions were generated. A retained title can preserve a symbolic presentation of an institution while the institution's effective relations change. Conversely, a relational structure can persist while its symbols, vocabulary, or interfaces change.

Mathematical and computational structures supply a further representational interface. A labelled 2-complex can represent history-bearing incidence and higher-order relations. A local algebra can represent admissible observables, transformations, constraints, or generators. An operator family can represent state-dependent action. An attractor-like regime can represent persistence or recurrence. Homology can diagnose selected topological differences. These structures can support precise preservation tests while remaining representations of selected features. Preservation of a mathematical invariant is therefore a formally meaningful result whose relation to relational preservation requires an explicit bridge.

Formal provenance is especially important when preservation notation is introduced. A symbol denoting a preservation target must identify the carrier from which that target is obtained. A comparison relation must have a stated domain and mathematical status. A historical transport or tracking relation must specify what warrants continuity across the compared domains. A dynamical signature must be derived from a defined dynamics through a declared extractor. A judgment operator or tolerance relation must state the basis on which selected differences are retained or ignored. These requirements prevent notation from creating a preservation object that the preceding model has not supplied.

The paper also distinguishes preservation identification from preservation valuation. A formal or historically grounded procedure can establish that a later configuration preserves a selected structure under a declared relation. That result does not establish that preservation is desirable, obligatory, just, aesthetically valuable, or politically legitimate. Normative, aesthetic, epistemic, historical, and practical judgments can each provide reasons for valuing preservation, yet those reasons belong to an additional evaluative layer. Section 13 formalizes this distinction so that the taxonomy remains usable even in cases where preservation itself is contested.

The converse separation is equally important. A community or institution can value a structure whose preservation cannot be established under the available evidence. Value does not supply missing historical continuity, operational capacity, or dynamical correspondence by itself. The taxonomy therefore allows preservation status and preservation value to remain independently uncertain. This structure also supplies an interface for later GR work on language extinction, cultural revival, future generations, preservation burdens, and governance under disagreement.

1.5 Language Preservation as a Motivating Domain

Language provides a demanding motivating domain because several preservation objects that appear closely connected in ordinary use can diverge under historical change. A language can be documented extensively while losing fluent speakers. Fluent speakers can remain while intergenerational transmission weakens. Grammar and vocabulary can survive in teaching materials while institutional domains of use contract. A revitalized language can regain speakers through new educational and technological conditions while differing in pronunciation, vocabulary, register, or social function from earlier forms. These cases make language useful for testing the taxonomy without making the paper a complete theory of language preservation.

Corpus preservation provides the clearest initial distinction. Written texts, audio recordings, dictionaries, grammatical descriptions, and annotated corpora can preserve valuable manifestations and traces of earlier linguistic practice. They can also support later recovery. Their preservation does not establish that a living generative capacity remains actualized. Fluent linguistic generation involves capacities to produce novel utterances, recognize context-sensitive appropriateness, participate in repair, interpret

historically accumulated expressions, generate new terminology, and respond to interaction in ways that can become part of later usage. Some of these capacities depend on relations that a corpus records only partially.

The difference becomes visible in ethnographic and anthropological practice. Researchers often learn aspects of language and culture through sustained participation in a community, where recorded outputs form only one part of the evidential setting. Shared activity, trust, kinship, environment, correction, silence, humor, ritual, conflict, and accumulated interaction can alter what an expression means and which distinctions become salient. The example does not establish that every linguistic fact requires co-experience. It shows that preservation of symbolic traces and preservation of the relational conditions under which further linguistic knowledge can emerge are distinct analytical targets.

Language also illustrates the difficulty of preserving generative conditions. Conditions relevant to linguistic generation include speakers, interlocutors, families, schools, workplaces, media, public institutions, economic opportunities, technologies, places, and relations with other languages. These conditions evolve. A preservation programme cannot ordinarily restore the complete relational environment of an earlier historical period. Attempting to freeze that environment could conflict with the continuing history of the community itself. The preservation question therefore shifts from exact restoration of conditions toward identification of which condition classes, relational functions, or recovery mechanisms matter for the selected preservation target.

Language revival introduces a further distinction between lineage and state similarity. A later community can reconstruct substantial parts of a language after an interruption in transmission. The revived system can develop new vocabulary, new institutional functions, and new relations with other languages. A preservation taxonomy should be able to classify the relation without deciding by terminology alone whether the revived language is numerically identical to the earlier language, a descendant, a reconstruction, or a new system shaped by inherited material. Different preservation targets can support different answers. Manifested vocabulary, generative capacity, genealogy, and social conditions need not have the same preservation status.

Artificial intelligence makes the distinction sharper. A model trained on surviving linguistic material can generate novel strings that conform to many recorded patterns. Such production may preserve or reconstruct selected manifestations and can support learning or revitalization. The existence of generated output does not by itself establish preservation of a living relational lineage, community-based transmission, or historically situated generative conditions. The taxonomy should permit these achievements to be described precisely without treating them as either complete preservation or complete failure.

The language case also exposes the relation between preservation and power. Educational systems, labor markets, administrative rules, media infrastructure, digital platforms, and AI support can alter the costs and opportunities associated with different languages. A community's preference for one language can therefore emerge within a relational field in which generative capacities are distributed unevenly. The present paper does not evaluate such distributions normatively. It records the analytical consequence that a preservation claim concerning capacity or conditions can require a broader relational carrier than the language's symbolic forms alone.

These examples motivate the paper's layered taxonomy. Language preservation can refer to traces, relational structure, generative capacity, enabling conditions, transmission mechanisms, recoverability, historical lineage, or selected dynamical characteristics. The categories can overlap, and successful preservation in one category does not automatically establish success in another. Section 14 returns to these cases after the target and mode taxonomies have been formally developed.

1.6 Manuscript Architecture

The manuscript is organized in seventeen sections. Section 2 positions the preservation taxonomy relative to verified literature on cultural and linguistic safeguarding, heritage continuity, ecological resilience and restoration, identity through change, recoverability, and dynamical persistence. Section 3 establishes the formal GR foundations required by the taxonomy, including history-bearing relational spacetimes,

historical extension, reality and representation status, retained relational content, comparison relations inherited from the GR equivalence programme, partial definedness, and preservation witnesses.

Section 4 introduces the general architecture of a preservation specification and separates preservation target, reality or representation status, historical mode, retained content, comparison relation, representational resolution, and preservation mode. Sections 5–10 develop the six principal target families: manifestational traces, relational structures, generative capacities, generative conditions, historical-lineage and regeneration structures, and dynamical regimes.

Section 11 compares preservation modes across target classes. Section 12 develops bridge-qualified cross-target lifting and typed preservation profiles. Section 13 separates preservation identification from aesthetic, epistemic, normative, practical, and political-economic valuation. Section 14 uses language preservation as a bounded stress test. Section 15 proposes a structured Generative Relational Preservation Investigation for domain-specific use. Section 16 states epistemic, formal, empirical, ontological, and normative boundaries. Section 17 consolidates the taxonomy and identifies interfaces for later work on language, generativity, ethics, governance, law, anthropology, and engineering.

The resulting architecture places the taxonomy before a general theory of preservation value or intervention. Its central objective is to make preservation claims sufficiently explicit that later GR inquiries can state which object is retained, which historical relation supports the claim, which comparison relation is being used, and which preservation mode has actually been established. Preservation thereby becomes a structured research question whose missing indices can be identified before stronger theoretical or practical conclusions are drawn.

2 Literature Interfaces for Preservation Analysis

This section positions the proposed Generative Relational preservation taxonomy relative to several bodies of work that already use preservation-adjacent concepts with materially different objects and criteria. Its role is a scoped comparison across several domains: cultural safeguarding, language revitalization, ecological resilience and restoration, philosophical criteria of identity, and dynamical persistence. Each illuminates a different part of the preservation problem. The objective is to identify the distinctions that can be inherited without collapsing their disciplinary meanings into one vocabulary. The discussion proceeds from cultural and linguistic safeguarding to historical recreation, ecological resilience and recovery, identity criteria across change, reconstructive continuation, and dynamical regime structure. The resulting synthesis identifies a formal gap that motivates the GR taxonomy: existing literatures often specify valuable preservation practices or domain-specific continuity criteria, while the present inquiry requires a typed account of which object is retained, across which history, under which comparison relation, and in which preservation mode.

2.1 Cultural and Linguistic Safeguarding

Cultural-heritage and language-revitalization frameworks provide a direct starting point because they already resist a purely archival interpretation of preservation. The 2003 UNESCO Convention for the Safeguarding of the Intangible Cultural Heritage defines intangible cultural heritage through practices, representations, expressions, knowledge, skills, associated artefacts, and cultural spaces that communities recognize as part of their heritage. Its definition explicitly links transmission across generations with continuing recreation in response to environment, interaction with nature, and history. The Convention also defines safeguarding through a family of measures that includes identification, documentation, research, preservation, protection, promotion, transmission, and revitalization within a multi-operation safeguarding architecture [1]. This architecture is important for the present inquiry because it places continuity, viability, transmission, and recreation within one safeguarding field while leaving their internal relations open.

The Convention also locates language in a differentiated position. Language is identified as a vehicle of oral traditions and expressions within a wider field of intangible heritage, and safeguarding measures

are expected to involve communities, groups, and relevant individuals in identifying, maintaining, transmitting, and managing heritage [1]. The resulting framework therefore supplies several analytically separable objects: manifested expressions, knowledge and skills, transmission practices, community participation, and the conditions through which heritage remains viable. A GR preservation taxonomy can use this separation without converting the Convention's legal and policy framework into a general metaphysics of preservation.

UNESCO's 2003 expert-group document on language vitality makes a parallel distinction within linguistic preservation. The document treats language endangerment through several factors, including intergenerational transmission, number and proportion of speakers, domains of language use, responses to new domains and media, and the availability of materials for education and literacy [2]. The same document distinguishes documentation from vitality and emphasizes meaningful contemporary roles for endangered languages. These criteria show that a language can be richly documented while losing domains of use or intergenerational transmission. They also show that speaker number, educational material, social domain, and technological adaptation are distinct dimensions of linguistic continuation, each requiring its own evidential interpretation.

Language-shift and revitalization scholarship broadens this analysis beyond documentation. Fishman's account of reversing language shift gives intergenerational continuity a central analytical role and treats maintenance as a problem situated within threatened language-in-society configurations [9]. The Cambridge handbook edited by Austin and Sallabank organizes endangered-language research across endangerment, documentation, responses, and challenges, while its introductory and sociolinguistic treatments connect language shift to political, social, educational, and economic conditions [10]. Grenoble and Whaley likewise describe revitalization as a family of community-specific efforts whose design depends on historical, economic, social, political, attitudinal, and institutional conditions [3]. More recent revitalization syntheses extend the field across homes, educational institutions, documentation, literacy, digital media, community-based practice, and language change during revitalization [11]. Together, these frameworks move the unit of analysis beyond an archived code: revitalization can alter competence, use domains, intergenerational interaction, institutional support, technological access, and the practical settings in which linguistic capacities are exercised.

These sources support a first taxonomic lesson. Linguistic and cultural preservation claims can concern at least four different objects: symbolic traces, active capacities, transmission relations, and enabling social conditions. The literatures do not require these objects to share one preservation criterion. Documentation can succeed while transmission weakens; revitalization can increase use while transforming earlier forms; institutional support can expand without reproducing earlier relational conditions. The GR taxonomy developed later in this paper therefore treats these as candidate preservation targets whose relations require explicit statement.

2.2 Heritage Continuity and Historical Recreation

The cultural-heritage interface also clarifies why preservation need not mean historical immobility. UNESCO's definition describes intangible cultural heritage as intergenerationally transmitted and continuously recreated in relation to environment, interaction, and history [1]. The Nara Document on Authenticity adds a complementary heritage principle: authenticity judgments depend on cultural context, the evolution of heritage through time, and multiple sources of information such as form, materials, use, traditions, setting, and other internal or external factors [13]. These frameworks permit continuity to coexist with modification of manifestations, practices, contextual relations, and the evidential basis through which authenticity is assessed. Safeguarding can therefore operate through active renewal while historical continuity remains a separately articulated claim.

This combination has two consequences for the preservation taxonomy. First, a claim of preservation can target a lineage or relational organization whose manifestations change. A performance practice, oral tradition, craft, or linguistic register can retain historical continuity while its vocabulary, material support, institutional location, or public function changes. Second, the historical process through which

a later form arises can matter independently of present-state similarity. A newly invented practice may reproduce many observable features of an earlier practice without possessing the same transmission history, whereas a historically continuous practice can depart substantially from earlier manifestations while remaining recognized as a continuation by its participants.

The distinction motivates a separation among state similarity, historical continuity, and reconstructive relation. A snapshot comparison asks whether two represented states resemble one another under a declared criterion. A historical-continuity claim asks whether the later structure is connected to the earlier one through an admissible lineage of transmission, transformation, or relational succession. A reconstructive claim asks whether an interrupted or weakened structure has been reconstituted through surviving traces, capacities, descendants, or newly assembled conditions. Existing heritage frameworks make room for all three possibilities, although they do not supply one general formal relation that orders them.

For GR, the important inheritance is the idea that historical recreation can itself belong to preservation. This inheritance remains bounded by the paper's ontological restraint. The fact that a community recognizes continuity through recreation does not by itself settle every question of relational identity, legal authenticity, or normative legitimacy. It does establish that exact state identity is too narrow to function as the default preservation criterion for historically living systems.

2.3 Ecological Resilience and Restoration

Ecology supplies a different preservation vocabulary centred on persistence under disturbance, regime structure, adaptive capacity, and recovery. Holling's classic distinction between resilience and stability shifted attention from constancy around a single equilibrium toward the capacity of ecological systems to persist while absorbing change and to occupy alternative stability domains [4]. Walker and colleagues distinguish resilience, adaptability, and transformability, linking resilience to retained function, structure, identity, and feedbacks while reserving transformability for the capacity to create a fundamentally new system [12]. Later social-ecological resilience work further emphasizes nonlinear dynamics, thresholds, uncertainty, adaptation, and transformation across scales [5]. This literature therefore offers several preservation-relevant relations: persistence within a regime, adaptation that modifies trajectories while retaining selected organization, and transformation that changes the governing system itself.

This interface is especially useful for the GR distinction between configuration preservation and dynamical-regime preservation. A system can leave a particular local state while remaining within a wider basin or regime. Conversely, a system can retain many visible components while crossing a threshold into a different organization of feedbacks and reachable futures. A preservation claim that refers to resilience therefore requires specification of the disturbance family, system boundary, retained function or organization, and regime criterion. These requirements parallel the GR insistence that preservation predicates receive explicit domains and retained content.

Ecological restoration adds a second orientation: recovery relative to a reference model does not require the recreated system to duplicate every historical component. The Society for Ecological Restoration's international standards describe ecological restoration as assisting the recovery of degraded ecosystems and organize practice around reference ecosystems, recovery trajectories, attributes of ecosystem recovery, and continuing adaptive management [6]. Restoration thus connects an earlier or reference condition to a later trajectory through graded recovery along a reference-guided trajectory.

The ecological literature also exposes a tension that recurs in the preservation taxonomy. Resilience of a present regime and recovery toward a reference regime can point in different directions. A degraded system can be resilient in the sense of remaining within an undesirable regime, while a restoration programme can seek a structural transition toward another regime. Preservation therefore cannot be defined by persistence alone. The object and direction of persistence must be specified before resilience can function as a preservation criterion.

This lesson carries into non-ecological cases without importing ecological ontology. A language, institution, knowledge practice, or technical capacity can exhibit robust continuation of some structures

while losing others. The formal analogy is useful only after the relevant system boundary, perturbations, recovery target, and comparison relation have been independently justified.

2.4 Identity Criteria across Change

Philosophical work on identity provides a complementary interface because preservation claims frequently use the language of the same object surviving change. The identity literature distinguishes numerical identity from qualitative similarity and treats criteria of identity as standards by which sameness is judged. It also distinguishes synchronic criteria from diachronic criteria that concern persistence across time [7]. These distinctions help prevent a preservation claim from moving silently between literal identity, resemblance, and continuity.

For GR, the relevance is methodological. A preservation inquiry can ask whether an object at one historical slice is numerically identical to an object at another slice, yet many preservation claims require weaker relations. A language can change vocabulary while remaining treated as historically continuous; a restored ecosystem can approximate a reference structure without being numerically identical to an earlier ecosystem; a reconstructed practice can be historically derivative without preserving uninterrupted activity. The preservation predicate therefore needs an explicitly selected relation whose strength is adequate to the local research question.

The identity literature also clarifies why criteria must be typed by object class and historical scope. A diachronic identity criterion for one class of objects does not automatically transfer to another, and an equivalence relation can supply a criterion for a selected description without becoming identity in a richer description [7]. This point closely matches the GR equivalence programme, where equivalence is indexed by retained relational content and representational resolution. The preservation taxonomy adds historical and modal structure by asking whether such a relation is being used to establish continuous retention, descendant continuity, reconstruction, recoverability, or another preservation mode.

The present paper therefore treats identity as one limiting case within a broader preservation architecture. Literal identity can be appropriate where the modeled carrier and individuation conditions justify it. Structural, functional, generative, historical, and dynamical preservation require weaker or differently typed relations. A general taxonomy should keep those relations distinct enough that a strong identity claim is not inferred from a weaker preservation success.

2.5 Recoverability and Reconstructive Continuation

Recoverability becomes relevant when continuous possession of a structure or capacity is too demanding. Language revitalization already supplies practical examples: transmission can weaken severely while documentation, partial competence, pedagogical resources, community commitment, or institutional support remain available for later revitalization [2, 3]. Ecological restoration supplies an analogous structure in which a degraded system can enter a recovery trajectory relative to a reference model after substantial departure from that reference [6]. These literatures motivate a preservation mode in which the present configuration retains an admissible route to a target condition even when the target is not currently actualized.

The resulting relation differs from simple persistence. Continuous retention asks whether the selected structure remains present across the relevant history. Recoverability asks whether a later configuration with the required structure can be reached from the present one through an admissible family of transformations. Reconstructive continuation adds provenance: the later structure can depend on surviving records, partial capacities, descendants, institutions, or material supports inherited from the earlier system. These distinctions are central to language revival, ecosystem restoration, reconstruction of technical practices, and preservation of knowledge that can temporarily fall out of active use.

Recoverability also introduces dependence on the recovery mechanism. A target can remain theoretically reconstructible while the practical mechanism for reconstruction disappears. Conversely, a system can lose a currently executable capacity while retaining teachers, tools, archives, algorithms, institutions, or relational supports capable of regenerating it. Sections 7, 8, and 9 therefore distinguish preservation

of a target capacity from preservation of the conditions and mechanisms through which that capacity can be regained.

The literature reviewed here does not supply one universal mathematical definition of recoverability, and the paper does not impose one at the literature-interface stage. In a finite dynamical model it may be represented by reachability. In a historical or ethnographic application it may require a defensible lineage of transmission and reconstruction. In an engineering system it may depend on retained specifications, materials, competencies, and executable procedures. The common analytical point is weaker: recoverability is a distinct preservation mode whose target, admissible path, and criterion of adequate reconstruction require explicit specification.

2.6 Dynamical and Structural Persistence

Dynamical-systems language provides a further weakening of preservation claims. Attractor concepts describe forms of recurrent or asymptotically organizing behavior, although the mathematical literature contains several inequivalent definitions suited to different purposes. Milnor's measure-oriented formulation is one influential example [8]. Resilience theory likewise uses basin structure and regime shifts to characterize persistence and transformation under perturbation [4, 5]. These formalisms permit a system to preserve selected dynamical characteristics even when its constituent states continue to change.

For the GR programme, this literature motivates preservation targets such as attractor organization, basin structure, recurrence, transition connectivity, susceptibility, fixed structures, cycle-return characteristics, or selected topological invariants. None of these targets is presumed to be universally appropriate. Their relevance depends on the represented system and on the analytical reason for preserving them. A model can preserve an attractor type while changing local parameters; preserve a transition graph while changing the identities of local states; preserve selected homological characteristics while changing labels and weights; or reproduce a metastable regime through a different generative grammar and different material conditions.

Dynamical persistence is therefore potentially weaker than preservation of the generating mechanism. Two systems can exhibit similar attractor or transition structures while possessing different operators, histories, or constitutive supports. This distinction is important for the later idea of dynamical-structural reproducibility: a preservation claim can concern the possibility of reproducing a selected regime without requiring reconstruction of the original grammar or conditions that generated it.

The dynamical interface also reinforces the need for representational caution. Attractors, basins, phase spaces, and stability notions require mathematical assumptions that may be unavailable in a qualitative or historically open relational system. GR can use attractor-like, metastable, reachability, and topological structures as conditional representations while keeping their formal prerequisites explicit. The preservation taxonomy should therefore classify dynamical-regime preservation as one possible target family, with other preservation targets retaining independent analytical status.

2.7 Positioning of the GR Preservation Taxonomy

The literatures reviewed in this section reveal a recurrent structural pattern. Preservation-adjacent inquiries differ along several dimensions at once: the object can be a trace, practice, capacity, social domain, ecosystem function, historical lineage, identity relation, recovery trajectory, or dynamical regime; the historical relation can involve uninterrupted continuation, adaptation, reconstruction, or transformation; and the comparison relation can range from identity to structural, functional, or dynamical similarity. Existing frameworks often treat these distinctions within a domain-specific purpose. The present GR inquiry seeks a cross-domain taxonomy that keeps the distinctions visible before a substantive preservation judgment is made.

Table 1 summarizes the principal interfaces retained from the literature. The table does not rank the traditions or translate them into one common ontology. It identifies the preservation dimensions that motivate later formal separation.

Table 1: Literature interfaces retained for the GR preservation taxonomy. Each tradition supplies a different preservation-relevant structure; the final column states the limited role inherited by the present paper.

Literature interface	Preservation-relevant structure	Role in the present taxonomy
Intangible cultural heritage	Viability, transmission, community recreation, documentation, revitalization	Separates manifested traces, living practice, transmission, and historical recreation
Language shift and revitalization	Intergenerational continuity, use domains, speaker communities, documentation, education, digital media, community practice	Motivates distinct targets for capacity, use conditions, transmission, reconstruction, and recovery
Ecological resilience and transformability	Persistence under disturbance, basins, regime shifts, adaptive capacity, transformation	Separates regime retention, adaptive change, and system transformation
Ecological restoration	Reference conditions, recovery trajectories, graded recovery, adaptive management	Motivates reconstructive and trajectory-based preservation modes
Identity theory	Numerical identity, qualitative similarity, criteria of diachronic identity	Clarifies the difference between identity, similarity, and persistence criteria
Dynamical systems	Attractors, recurrence, stability, basin and return structure	Supplies candidate dynamical preservation targets under explicit modeling assumptions

The resulting research gap is narrower than a claim that prior work lacks concepts of preservation. The reviewed traditions contain sophisticated preservation, safeguarding, resilience, recovery, identity, and persistence frameworks. The gap addressed here concerns their coordination inside GR. A preservation claim in the present framework should specify the target object, reality or representation status, historical scope, retained content, comparison relation, representational resolution, and preservation mode. The comparison relation can be inherited from the companion GR equivalence taxonomy where appropriate; preservation adds the question of what that relation is intended to retain across a declared history.

This positioning also fixes the paper’s normative boundary. UNESCO safeguarding frameworks contain explicit value commitments; language revitalization programmes are often motivated by community goals; ecological restoration specifies desired recovery directions; and resilience research can distinguish desirable from undesirable regimes only through an additional evaluative specification. The GR preservation taxonomy does not convert those domain-specific values into one general duty of preservation. Its immediate task is prior and classificatory: to make the object and mode of preservation explicit enough that normative, legal, political, empirical, or engineering inquiries can later evaluate the appropriate claim.

3 Formal Foundations for GR Preservation

This section establishes the minimum formal architecture required before preservation claims are classified. Its role is to specify the history-bearing carrier, the historical relations through which two represented configurations become candidates for a preservation comparison, the GR status at which the target is described, the retained content selected for comparison, and the partial comparison relation applied to that content. The construction proceeds from labelled relational spacetime and event-ordered history to status-indexed preservation targets, retained-content projections, comparison carriers, applicability conditions, and candidate preservation witnesses. The method is formal and representational. It inherits

Table 2: Core notation used throughout the GR preservation taxonomy. Local definitions and domain restrictions remain controlling where a symbol is specialized later.

Symbol	Role	Typing or first formal location
$K = (V, E, F, \partial, \lambda)$	History-bearing relational-spacetime carrier	Equation 1
X_Σ, X'_Σ	Compared represented targets at declared historical slices	Sections 3.1–3.2
ℓ, ℓ'	Relational, manifestational, or representational status indices	Section 3.3
κ, ϱ	Retained-content basis and representational resolution	Equation 6
$\Pi_{\kappa, \varrho; \Sigma}^\ell, z_{X; \kappa, \varrho, \Sigma}^\ell$	Retained-content projection and its retained comparison representation	Equations 6–7
$\text{Hist}_h(X, X'), \eta$	Historical-witness class selected by mode h , and one witness	Definition 3.2 and Section 4.3
$T_{\kappa, \varrho}^\eta, \mathbf{r}$	Historical comparison transport and declared comparison relation	Equations 8–9
$\mathfrak{P} =$ $(\theta, \ell, \ell', h, \kappa, \mathbf{r}, \varrho, \mu)$	Complete structural preservation specification	Definition 4.1
$M_\mu^{\mathfrak{P}}(X, X', \eta)$	Grounded preservation-mode predicate	Equation 17
A, W	Comparison-evaluable witness class and comparison-successful candidate-witness class	Section 3.6
$\text{App}_{\mathfrak{P}}^{\text{pres}}, \text{Pres}_{\mathfrak{P}}^{\text{str}}$	Full applicability test and partial Boolean structural-preservation predicate	Equations 18–20
$\nu_{\mathfrak{P}}, \mathbf{Prof}_{\mathfrak{P}}$	Three-valued typed evaluation and typed preservation profile	Equations 121–122

GR’s ontological suspension and introduces no general preservation predicate, preservation mode, or normative valuation at this stage.

For release-level readability, Table 2 collects the symbols that recur across the formal architecture. The table serves as a navigation aid; the equations and definitions cited in the third column remain authoritative. Sans-serif symbols are used mainly for predicates, typed relations, and transports; calligraphic symbols mainly denote carriers, families, and domains; fraktur symbols denote specifications or status-indexed structures; and bold symbols denote profile-level objects where introduced.

3.1 History-Bearing GR Spacetimes

Preservation in GR concerns objects whose present organization can depend on extended relational history. A point-like state description is therefore insufficient as the general carrier of the taxonomy. The

present paper adopts a labelled two-dimensional complex as one candidate representation of a history-bearing relational spacetime. The representation is stated in Equation 1.

$$K = (V, E, F, \partial, \lambda). \quad (1)$$

Here V , E , and F denote the retained vertex, edge, and face sets; ∂ records incidence and attachment data; and λ collects application-defined labels on cells or composite regions. A vertex can represent an event or another localized relational unit, an edge can represent a transition or dependence, and a face can retain a higher-order compositional or compatibility structure. These interpretations are model-relative. Equation 1 supplies a carrier on which preservation questions can be represented; it does not assign cellular structure to ultimate ontology.

A preservation analysis can select one or more history-bearing subconfigurations inside K . Let $X \subseteq K$ denote a represented object at a declared resolution. Depending on the application, X can represent a language community, institution, practice, body of knowledge, technical capacity, ecosystemic organization, or another relational formation. The inclusion notation states only that the information retained for X is carried by a selected part of the modeled relational spacetime. It does not require X to possess a unique boundary, a disjoint decomposition from other objects, or a scale-independent cell structure.

The choice of a two-complex is useful for preservation because different proposed preservation targets can inhabit different parts of the same history-bearing representation. Manifested traces can be attached as labels or records; relational structures can be represented through incidence and higher-order attachments; operation domains can be associated with regions or interfaces; and later dynamical sections can attach local algebraic or field data where those constructions are justified. The common carrier therefore supports comparison among preservation claims without treating the carrier itself as the object that every claim must preserve.

Remark 3.1 (Representational role of the carrier). The labelled complex in Equation 1 is a candidate formal representation under ontological suspension. Preservation of the represented complex, preservation of a structure represented on the complex, and preservation of the relational formation represented by that structure are distinct claims unless an application establishes the required correspondence.

Remark 3.1 prevents a topological, combinatorial, or algebraic invariance from acquiring preservation significance by notation alone. Later sections may use such invariances as diagnostics, while the preservation target and retained content remain independently specified.

3.2 Historical Extensions of Relational Slices

The temporal organization of the taxonomy begins with event order; no primitive universal scalar time is required by the present formalism. Let $\text{Ev}(K) \subseteq V$ be the vertices selected as events for a given application, and let $\preceq_K$ be a partial precedence relation on that event set. The resulting event structure is defined in Equation 2.

$$\mathcal{H}_K := (\text{Ev}(K), \preceq_K). \quad (2)$$

If $e_1 \preceq_K e_2$, the representation records that the occurrence or relational consequences associated with e_1 belong to the historical domain retained as antecedent to, or supportive of, e_2 . The relation can leave events incomparable. A global total order is unnecessary for the preservation questions considered here.

A relational slice Σ is represented by an antichain in $\mathcal{H}_K$ when the selected application admits such a slicing. Its accumulated event domain is the order ideal stated in Equation 3.

$$D_K(\Sigma) := \{e \in \text{Ev}(K) \mid e \preceq_K s \text{ for at least one } s \in \Sigma\}. \quad (3)$$

The notation $K_{\leq \Sigma}$ denotes the model-relative subcomplex supported by the accumulated event domain in Equation 3 together with the incidence-compatible cells and labels retained by the application. The notation therefore refers to a history-bearing modeled stage that retains antecedent structure.

A later preservation comparison requires more than two independently indexed slices. The model must supply a relation through which the two represented histories become comparable. In a fixed-history-extension regime, this relation can be an incidence- and label-compatible embedding of the form stated in Equation 4.

$$\iota_{\Sigma\Sigma'} : K_{\leq\Sigma} \hookrightarrow K_{\leq\Sigma'}. \quad (4)$$

Equation 4 is appropriate when the later representation retains the earlier modeled history while adding new events, relations, or labels. It is deliberately insufficient as a universal account of preservation. Migration, branching descent, reconstruction after interruption, revised historical representation, or transformations that change topology can require a weaker historical correspondence than embedding.

The present paper therefore uses *historical witness* as the general term for the structure that makes a preservation comparison historically or genealogically meaningful. Let $\text{Hist}(X_\Sigma, X'_{\Sigma'})$ denote the application-defined class of admissible witnesses connecting two represented targets. An element $\eta \in \text{Hist}(X_\Sigma, X'_{\Sigma'})$ can be an extension embedding, a tracking relation, a relocation correspondence, a documented reconstruction lineage, or another well-typed provenance structure. Section 4 distinguishes the principal historical-witness subclasses used by the preservation specification.

Definition 3.2 (Historical witness). For represented targets X_Σ and $X'_{\Sigma'}$, a historical witness is a model-specified element $\eta \in \text{Hist}(X_\Sigma, X'_{\Sigma'})$ that supplies the provenance, tracking, extension, descent, relocation, or reconstructive relation required to compare the second target as historically connected to the first for the declared inquiry.

Definition 3.2 does not establish preservation. It establishes only that the comparison is more than an ahistorical resemblance between independently selected systems. Two unrelated configurations can later satisfy a strong structural equivalence while lacking a historical witness of the type required by a preservation claim.

3.3 Reality-Representation Status

The object selected for preservation must also be typed by its status within the GR analysis. GR distinguishes ultimate ontological reality, relational reality, and manifestation reality, while mathematical representations serve as modeling structures external to that three-layer reality distinction. Preservation claims can therefore refer to materially different objects even when they use similar vocabulary.

Ultimate ontological reality remains suspended in the present inquiry. A statement that an ontological structure itself has been preserved can be considered conditionally when a domain-specific metaphysics supplies that structure, although the present taxonomy does not derive such a claim from relational or manifestational evidence. The formal development therefore concentrates on relational, manifestational, and representational preservation targets while retaining ontological preservation as a conditional possibility.

Let $\mathfrak{R}_H$ denote the model-defined class of structures assigned relational-reality status relative to a retained historical domain H . Membership in $\mathfrak{R}_H$ requires the persistence and type-appropriate relational efficacy supplied by the relevant GR application. The notation is a status assignment inside the declared model; it is not a direct image of ultimate ontology.

For a manifestation context c , let $\mathfrak{M}_{H,c}$ denote the corresponding manifestation space and let the context-dependent manifestation map have the partial type stated in Equation 5.

$$\pi_{H,c} : \text{Dom}(\pi_{H,c}) \subseteq \mathfrak{R}_H \longrightarrow \mathfrak{M}_{H,c}. \quad (5)$$

The map in Equation 5 can represent symbolic, linguistic, institutional, numerical, embodied, archival, or other context-dependent presentations. It need not be injective. Several relational structures can share one manifested presentation, and one relational structure can admit different manifestations under different contexts.

Mathematical representations occupy a further analytical role. Let $\mathfrak{Q}_H$ denote a model-defined class of formal representations available over historical domain H , including complexes, algebras, operator

families, categories, dynamical signatures, or other structures introduced with explicit provenance. A representation map can be partial and many-to-one. Equality or isomorphism inside $\mathfrak{Q}_H$ does not by itself establish equality or preservation inside $\mathfrak{R}_H$.

For later bookkeeping, the present paper uses the status index $\ell \in \{\text{rel}, \text{man}, \text{rep}\}$ for relational, manifestational, and representational targets. Ontological claims are discussed separately because GR does not provide a general extraction map from ultimate ontology into the formalism.

Remark 3.3 (Cross-status preservation claims). A preservation result at one status does not automatically lift to another. Manifestational continuity can coexist with relational discontinuity; relational continuity can coexist with substantial manifestational change; and representational invariance can remain compatible with several relational interpretations. Any cross-status inference requires an additional mapping and justification.

Remark 3.3 supports the distinction between corpus-level preservation and living relational or generative preservation developed in Section 14.

3.4 Retained Relational Content

A preservation claim rarely compares the complete information represented in two historical configurations. The analysis usually retains selected relations, capacities, manifestations, histories, interfaces, or dynamical characteristics while leaving other differences outside the comparison. The retained content therefore needs an explicit formal location.

Let κ index the content retained for a particular preservation inquiry, and let ϱ denote the declared representational resolution. For a target of status ℓ , the retained-content projection is the partial map stated in Equation 6.

$$\Pi_{\kappa, \varrho; \Sigma}^{\ell} : \text{Dom}(\Pi_{\kappa, \varrho; \Sigma}^{\ell}) \dashrightarrow \mathcal{Z}_{\kappa, \varrho; \Sigma}^{\ell}. \quad (6)$$

The carrier $\mathcal{Z}_{\kappa, \varrho; \Sigma}^{\ell}$ contains only the material selected by κ at resolution ϱ . Depending on the application, its elements can retain a relational substructure, a manifestation profile, an operation family, a genealogy, a continuation structure, a dynamical signature, or a combination of these. The partial arrow records that a proposed retained-content basis need not be meaningful for every target or every historical domain.

For X_{Σ} in the domain of the projection, define its retained comparison representation by Equation 7.

$$z_{X; \kappa, \varrho, \Sigma}^{\ell} := \Pi_{\kappa, \varrho; \Sigma}^{\ell}(X_{\Sigma}). \quad (7)$$

Equation 7 preserves the distinction between the preservation target and the representation used to compare it. Two targets can receive the same retained representation under a coarse κ while differing in other relations that become visible under a finer basis or resolution.

The content index κ should be fixed before the preservation comparison is evaluated. Choosing κ after observing the desired result would allow the taxonomy to manufacture preservation by discarding every inconvenient difference. The same discipline applies to resolution. A preservation claim that survives only under aggressive coarse-graining can remain useful, yet the coarse-graining belongs to the claim's specification and should remain visible.

Definition 3.4 (Retained preservation content). Retained preservation content is the relational, manifestational, generative, historical, dynamical, or representational material selected by κ and extracted at declared resolution ϱ through a partial projection of the form in Equation 6. The retained content determines what the later comparison is permitted to treat as preservation-relevant.

Definition 3.4 makes preservation explicitly selective without treating selectivity as arbitrary. The analytical purpose, evidence, and later judgment structure must justify the retained-content basis.

3.5 Comparison Relations Inherited from GR Equivalence

The companion GR taxonomy of equivalence supplies a family of comparison relations that can be reused once the preservation carrier and retained content have been specified. The present paper does not reproduce that taxonomy. It requires only an interface broad enough to admit equality, isomorphism, interchangeability, simulation, bisimulation, approximate similarity, diagnostic agreement, and other well-typed relations where their assumptions are satisfied.

A historical witness can be necessary before the retained representations at two slices inhabit directly comparable carriers. Let $\eta \in \text{Hist}(X_\Sigma, X'_{\Sigma'})$. When the application supplies an induced comparison transport, write the partial map as in Equation 8.

$$T_{\kappa, \varrho}^\eta : \mathcal{Z}_{\kappa, \varrho; \Sigma}^\ell \dashrightarrow \mathcal{Z}_{\kappa, \varrho; \Sigma'}^{\ell'}. \quad (8)$$

The map in Equation 8 can transport retained relational positions, relabel corresponding structures, carry an operation family into a later representation, or realize another declared correspondence. It need not be invertible. In cases where the two retained representations already occupy a common carrier, $T_{\kappa, \varrho}^\eta$ can be the appropriate identity or inclusion map.

Let r denote a declared comparison relation on the applicable target carrier. The corresponding comparison statement is written in Equation 9.

$$T_{\kappa, \varrho}^\eta(z_{X; \kappa, \varrho, \Sigma}^\ell) \ r \ z_{X'; \kappa, \varrho, \Sigma'}^{\ell'}. \quad (9)$$

Equation 9 does not require r to be an equivalence relation in the strict mathematical sense. A simulation preorder can be directional; an approximate relation can depend on a tolerance; a diagnostic agreement relation can preserve only selected readouts. The expression records the comparison relation actually justified by the local analysis.

Remark 3.5 (Equivalence as a resource for preservation). A relation r that holds in Equation 9 establishes a comparison at the retained level. It becomes preservation-relevant only when the historical witness, preservation target, and later preservation-mode conditions connect that comparison to a declared claim of retention, continuation, recovery, reconstruction, or reproduction.

Remark 3.5 states the formal boundary between the present paper and the equivalence taxonomy. Preservation uses comparison relations; it is not identified with the existence of such a relation.

3.6 Applicability under Partial Definedness

Definedness is a substantive component of the preservation problem. A language can lack enough surviving speakers for a fluency comparison to be meaningful, a reconstructed practice can lack a warranted lineage map, and two dynamical regimes can lack a common diagnostic carrier. Under such conditions, the appropriate result is an undefined preservation test relative to the proposed specification, recorded explicitly as unresolved applicability.

For a proposed comparison based on $(\ell, \ell', \kappa, \varrho, \eta, r)$, define the applicability predicate in Equation 10.

$$\begin{aligned} \text{App}_{\ell, \ell', \kappa, \varrho, \eta, r}(X_\Sigma, X'_{\Sigma'}) = 1 \quad &\iff \quad X_\Sigma \in \text{Dom}(\Pi_{\kappa, \varrho; \Sigma}^\ell), \\ &X'_{\Sigma'} \in \text{Dom}(\Pi_{\kappa, \varrho; \Sigma'}^{\ell'}), \\ &z_{X; \kappa, \varrho, \Sigma}^\ell \in \text{Dom}(T_{\kappa, \varrho}^\eta), \\ &(T_{\kappa, \varrho}^\eta(z_{X; \kappa, \varrho, \Sigma}^\ell), z_{X'; \kappa, \varrho, \Sigma'}^{\ell'}) \in \text{Dom}(r). \end{aligned} \quad (10)$$

Equation 10 separates three outcomes that ordinary preservation language often conflates. A comparison can be defined and satisfy the relation; it can be defined and fail the relation; or the comparison can be undefined because one of its required objects, transports, or relations lacks an admissible domain.

The third outcome is particularly important in GR because relational articulation can itself be historically generated. A comparison unavailable at one slice can become meaningful later after positions,

distinctions, operation domains, or historical records have become sufficiently articulated. Conversely, an earlier comparison can cease to be meaningful after migration, fragmentation, topology change, or loss of the supporting interface. The taxonomy therefore retains partiality and represents unresolved applicability explicitly.

Where a richer partial-algebraic representation is useful, applicability can be internalized through restriction idempotents or partial identities associated with the relevant projections and transports. The general taxonomy does not require that stronger machinery. Equation 10 provides the minimum explicit definedness condition needed by the later preservation specification.

For later use, separate comparison definedness from comparison success. The comparison-evaluable historical witness class is defined in Equation 11.

$$A_{\kappa, \varrho, r}^{\ell, \ell'}(X_{\Sigma}, X'_{\Sigma'}) := \{ \eta \in \text{Hist}(X_{\Sigma}, X'_{\Sigma'}) \mid \text{App}_{\ell, \ell', \kappa, \varrho, \eta, r}(X_{\Sigma}, X'_{\Sigma'}) = 1 \}. \quad (11)$$

Membership in Equation 11 states only that the retained-content comparison can be evaluated along the witness. It does not state that the comparison relation succeeds. This distinction allows a defined comparison failure to produce preservation value 0 while unresolved applicability remains separate.

3.7 Preservation Witnesses across Historical Domains

The preceding constructions can now be assembled into a formal object that remains weaker than a completed preservation judgment. The historical witness supplies provenance; the retained-content projections state what is being compared; the comparison transport relates historical carriers when needed; and the relation r determines the comparison satisfied by the retained content. Their conjunction provides a *candidate preservation witness*.

For two represented targets, define the witness class in Equation 12.

$$W_{\kappa, \varrho, r}^{\ell, \ell'}(X_{\Sigma}, X'_{\Sigma'}) := \{ \eta \in A_{\kappa, \varrho, r}^{\ell, \ell'}(X_{\Sigma}, X'_{\Sigma'}) \mid T_{\kappa, \varrho}^{\eta}(z_{X; \kappa, \varrho, \Sigma}^{\ell} \text{ } r \text{ } z_{X'; \kappa, \varrho, \Sigma'}^{\ell'}) \}. \quad (12)$$

The class in Equation 12 is therefore a subset of the evaluable class in Equation 11. An element of the candidate class certifies two limited facts. First, the compared objects are connected by an admissible historical or provenance relation for the inquiry. Second, their declared retained contents satisfy the selected comparison relation after the required transport. Neither fact determines which preservation mode is being claimed or whether the comparison is sufficient to identify the later object as preserved in the substantive sense relevant to the application.

Definition 3.6 (Candidate preservation witness). A candidate preservation witness for X_{Σ} and $X'_{\Sigma'}$, relative to $(\ell, \ell', \kappa, \varrho, r)$ is an element of the witness class in Equation 12. The witness records a historically grounded retained-content comparison and remains prior to the preservation-mode and judgment conditions introduced later.

Definition 3.6 prevents two opposite shortcuts. A strong equivalence without provenance remains an equivalence result and lacks the historical witness required for preservation. A documented historical lineage without preservation of the declared content remains a provenance result and lacks the retained-content comparison required for preservation. Section 4 adds preservation targets and modes, and Section 11 determines how combinations of provenance and retained-content comparison support continuous retention, historically continuous transformation, dormant recoverability, reconstructive recovery, recovery-mechanism regeneration, or dynamical-regime reproduction.

Table 3 summarizes the formal objects introduced in this section and the inferences they do not establish by themselves.

The foundational architecture can now support the taxonomy without treating preservation as a primitive predicate. Section 4 adds the preservation target class, historical comparison mode, preservation mode, and resolution discipline needed to form a complete structural preservation specification. Sections 5–10 instantiate that specification for manifestations, relational structure, generative capacity, generative conditions, historical lineage, and dynamical regimes.

Table 3: Formal objects supplied before the preservation taxonomy. The final column records the inference deliberately withheld at the foundational stage.

Formal object	Analytical role	Inference not established by the object alone
$K = (V, E, F, \partial, \lambda)$	History-bearing relational spacetime carrier	Ultimate ontology or preservation of the represented formation
$\mathcal{H}_K$ and $K_{\leq \Sigma}$	Event precedence and accumulated modeled history	One universal scalar time or uninterrupted identity
$\eta \in \text{Hist}(X_\Sigma, X'_{\Sigma'})$	Provenance, tracking, continuation, relocation, or reconstruction witness	Preservation of any selected content
Status index ℓ	Distinguishes relational, manifestational, and representational targets	Cross-status equivalence or ontological identity
$\Pi_{\kappa, \varrho; \Sigma}^\ell$	Extracts retained content at a declared resolution	Adequacy of the retained-content basis or value of preserving it
$T_{\kappa, \varrho}^\eta$	Transports retained content into a comparable later carrier	Invertibility, losslessness, or preservation
r	Declared equality, equivalence, simulation, similarity, or diagnostic relation	Historical continuity or preservation significance
App and A	Record whether the proposed retained-content comparison is defined along a witness	Success of the comparison relation or preservation
W	Retains evaluable witnesses whose selected comparison relation succeeds	Preservation mode or preservation value
W	Combines provenance and retained-content comparison	Preservation mode, preservation-identification judgment, or normative value

4 Taxonomic Architecture of GR Preservation

This section assembles the formal objects established in Section 3 into the general architecture used by the preservation taxonomy. Its role is to distinguish the dimensions that specify a preservation claim before Sections 5–10 instantiate those dimensions for manifestations, relational structures, generative capacities, generative conditions, historical lineages, and dynamical regimes. The objective is classificatory and formal. The section introduces a preservation specification, defines preservation-target classes and historical comparison modes, separates preservation modes from target classes, extends the definedness discipline to the full specification, and states a local implication relation for comparing preservation claims. Preservation identification and preservation value remain outside the structural tuple and are developed in Section 13.

4.1 Preservation Specifications

Section 3 supplied historically grounded retained-content comparisons without yet determining what kind of preservation claim those comparisons support. A complete structural specification requires additional indices for the class of target under investigation, the historical mode through which the comparison is organized, and the preservation mode asserted by the inquiry. These indices are combined with the status, retained-content, comparison, and resolution data already introduced.

Let Θ denote the model-defined index family of preservation-target classes, let $\mathfrak{H}$ denote the index family of historical comparison modes, and let $\mathfrak{M}$ denote the index family of preservation modes. For $\theta \in \Theta$, status indices ℓ, ℓ' , historical mode $h \in \mathfrak{H}$, retained-content basis κ , comparison relation r , resolution ϱ , and preservation mode $\mu \in \mathfrak{M}$, the structural preservation specification is defined in Equation 13.

$$\mathfrak{P} := (\theta, \ell, \ell', h, \kappa, r, \varrho, \mu). \quad (13)$$

Equation 13 is a specification schema whose components can be dependent in a concrete application. The target class can constrain the meaningful comparison relations, a historical mode can restrict the admissible witness class, and a preservation mode can require structures that some target classes do not possess. The later definedness construction records these dependencies explicitly.

For each target index θ , status ℓ , and relational slice Σ , let $\mathcal{T}_\theta^\ell(\Sigma)$ denote the model-defined class of preservation targets of type θ available at that status and slice. A pair $(X_\Sigma, X'_{\Sigma'})$ is eligible for specification $\mathfrak{P}$ only when the first and second targets belong to the corresponding target classes at their declared statuses. The target carrier is therefore historical and status-indexed; the taxonomy does not assume one timeless set containing every object that can be discussed as preserved.

The structural specification also keeps the two status indices ℓ and ℓ' visible. Many preservation claims compare objects at the same status, such as relational organization at two historical slices. Other inquiries compare a manifested or representational record with a later relationally realized structure. Such cross-status claims require an explicit mapping or witness supplied by the application. Equation 13 permits the typing while leaving the validity of the cross-status bridge to the later applicability test.

Definition 4.1 (Structural preservation specification). A structural preservation specification is a tuple of the form in Equation 13 together with the target carriers, historical-witness subclass, retained-content projections, comparison transport, comparison relation, and mode predicate required to evaluate that tuple on a declared pair of history-bearing targets.

Definition 4.1 identifies the displayed tuple as an index of a formal construction. A specification becomes evaluable only after the objects indexed by its entries have been supplied.

The separation between structural specification and judgment is intentional. A model can establish that a pair satisfies a formally declared preservation specification while participants, institutions, or analysts dispute whether that specification captures the preservation concept relevant to their inquiry. Section 13 assigns that higher-order issue to preservation-identification judgment and separates it from preservation-value judgment.

4.2 Preservation Targets

The first taxonomic dimension concerns the object whose continuity, recoverability, or reproducibility is being asserted. Ordinary preservation language can apply the same noun to materially different target structures. A preserved language can mean preserved recordings, continued relational practice, retained linguistic competence, viable conditions of use, recoverable transmission, or reproducible generative dynamics. The taxonomy therefore treats target selection as an explicit operation.

The present paper uses six principal target families as the minimum architecture for the target-specific analysis. They are indexed schematically in Equation 14.

$$\Theta_{\text{core}} := \{\theta_{\text{man}}, \theta_{\text{rel}}, \theta_{\text{cap}}, \theta_{\text{cond}}, \theta_{\text{hist}}, \theta_{\text{dyn}}\}. \quad (14)$$

The six indices in Equation 14 denote manifestational traces, relational structures, generative capacities, generative conditions, historical-lineage or regeneration structures, and dynamical regimes, respectively. Sections 5–10 develop these families in detail.

The family in Equation 14 is not a partition of relational reality. A single history-bearing formation can supply targets in several classes simultaneously. A language community can have manifested corpora, relational institutions, distributed generative capacities, enabling conditions, transmission histories, and

dynamical organization. A target index states which object is being tested in one preservation claim; it does not assert that the other target classes are absent.

The target classes also differ in mathematical type. A manifestational target can be a record or context-indexed presentation. A relational target can be a subcomplex, relation family, interface structure, or another historically efficacious organization. A generative-capacity target can be represented through partial operation families or state-dependent schemas. A condition target can contain support, admissibility, material, institutional, or environmental structures. A historical target can concern transmission, descent, reconstruction, or recoverability. A dynamical target can concern transition organization, attractor-like regimes, cycle structure, fixed structures, or selected topological diagnostics. The common word *target* therefore names a shared analytical role across heterogeneous carrier types.

Remark 4.2 (Target-class overlap). Membership in one preservation-target family does not exclude membership in another. Cross-target relations require separate bridge assumptions. In particular, successful preservation of a manifestation target does not by itself establish preservation of a relational, generative, historical, or dynamical target supported by the same broader formation.

Remark 4.2 supplies an important non-implication refined by the target-specific sections and the lifting analysis in Section 12. The taxonomy records divergence among preservation claims through typed components and leaves scalar aggregation to a separate evaluative layer.

4.3 Historical Modes of Comparison

The second new taxonomic dimension concerns the historical relation under which two targets are compared. Section 3.2 introduced the general historical-witness class $\text{Hist}(X_\Sigma, X'_{\Sigma'})$. The present subsection refines that class by mode so that continuation, relocation, reconstruction, and branching are not represented by one undifferentiated provenance relation.

For each $h \in \mathfrak{H}$, let $\text{Hist}_h(X_\Sigma, X'_{\Sigma'})$ denote the subclass of historical witnesses admitted under mode h . The minimum historical-mode family used in the taxonomy is stated in Equation 15.

$$\mathfrak{H}_{\text{core}} := \{h_{\text{rep}}, h_{\text{cont}}, h_{\text{reloc}}, h_{\text{recon}}, h_{\text{branch}}\}. \quad (15)$$

The indices in Equation 15 designate same-history representational comparison, historical continuation, relational relocation, reconstruction after interruption, and branching descent, respectively.

The representational mode h_{rep} concerns two representations of overlapping historical material where the preservation question is primarily representational. A coarse-grained and a refined model of the same retained history can enter this mode when the application supplies the required correspondence. The historical-continuation mode h_{cont} covers cases in which the later target is tracked as a continuation or descendant through a retained historical sequence. The relocation mode h_{reloc} concerns continuation across a change of relational environment, territory, institutional setting, or another surrounding space-time structure. Diasporic language change is one possible application.

The reconstruction mode h_{recon} applies when active continuity has been interrupted and the later target is produced through a documented reconstructive relation to surviving traces, archives, partial practitioners, or other inherited material. The branching mode h_{branch} applies when one earlier target has more than one historically defensible descendant. Branching prevents the preservation taxonomy from treating historical continuity as a function that must assign one unique later object to each earlier one.

These modes classify provenance; preservation strength remains a separate dimension. A relocation can preserve a target strongly or weakly. A reconstruction can recover a generative capacity while failing to restore earlier social conditions. A branching descendant can preserve selected structures while transforming others. Historical mode therefore remains distinct from preservation mode.

Definition 4.3 (Historical comparison mode). A historical comparison mode h is an index selecting a declared witness subclass $\text{Hist}_h(X_\Sigma, X'_{\Sigma'}) \subseteq \text{Hist}(X_\Sigma, X'_{\Sigma'})$ whose members satisfy the provenance conditions associated with the selected historical relation.

Definition 4.3 permits additional modes in specialized applications. Historical reconfiguration with topology change, mediated reconstruction, or composite migration-and-revival histories can be represented by refined subclasses when the corresponding provenance criteria are available.

4.4 Preservation Modes

Historical mode identifies how two targets are connected; preservation mode identifies what kind of retention, continuation, recovery, or reproduction is being asserted. This distinction is central to the taxonomy. A historical continuation can support continuous retention, gradual transformation, or loss followed by later recovery. A reconstruction history can support partial capacity recovery without preserving earlier relational conditions. Preservation mode therefore supplies a separate axis.

Let $\mathfrak{M}$ denote the model-defined family of preservation modes. The core family anticipated by the later synthesis is stated in Equation 16.

$$\mathfrak{M}_{\text{core}} := \{\mu_{\text{ret}}, \mu_{\text{trans}}, \mu_{\text{dorm}}, \mu_{\text{rec}}, \mu_{\text{meta}}, \mu_{\text{dyn}}\}. \quad (16)$$

The indices in Equation 16 designate continuous retention, historically continuous transformation, dormant recoverability, reconstructive recovery, recovery-mechanism regeneration, and dynamical-regime reproduction. Section 11 compares these modes systematically after the target-specific sections supply their formal content.

At the present stage, each preservation mode is represented by a partial mode predicate. The predicate must retain the pair being compared as well as the historical witness, because several modes depend on properties of the source, the later target, and the intervening history simultaneously. For a specification $\mathfrak{P}$ with mode μ , define the grounded mode predicate as in Equation 17.

$$M_{\mu}^{\mathfrak{P}} : \text{Dom}(M_{\mu}^{\mathfrak{P}}) \subseteq \{(X_{\Sigma}, X'_{\Sigma'}, \eta) \mid \eta \in \text{Hist}_h(X_{\Sigma}, X'_{\Sigma'})\} \longrightarrow \{0, 1\}. \quad (17)$$

The domain in Equation 17 contains the historically grounded comparison records required by the selected mode. A retention predicate can inspect continuous availability across an identified historical corridor; a transformation predicate can inspect a sequence of admissible descendants; a reconstructive predicate can inspect the source, interruption, reconstruction witness, and recovered target. The triple type therefore avoids treating the same symbol alternately as a predicate of a witness and as a predicate of a target pair. Dormant-recoverability, meta-recovery, and dynamical-reproduction predicates can likewise access the pair and witness while using the target-specific reachability structures introduced later.

The partiality in Equation 17 is substantive. A dynamical-reproduction mode cannot be evaluated when no dynamical signature has been defined. Recovery cannot be evaluated when no admissible continuation or reconstruction class has been supplied. The taxonomy therefore leaves unsupported target-mode combinations undefined and preserves that undefinedness in later profiles.

Remark 4.4 (Preservation mode and historical mode). Historical mode classifies the provenance relation between the compared targets. Preservation mode classifies the kind of retention, transformation, recovery, regeneration, or reproduction asserted relative to that provenance relation. Neither mode determines the other in the general taxonomy.

Remark 4.4 is especially important for later language applications. Intergenerational continuation, migration, revival, and reconstruction can each support several preservation modes depending on the selected target and retained content.

4.5 Resolution and Partiality

The preservation specification inherits the resolution parameter ϱ and partial-definedness discipline from Section 3.4 and Section 3.6. Resolution can change the result of a preservation comparison because

coarse descriptions suppress distinctions that finer descriptions retain. A preservation claim is therefore incomplete when its success depends materially on resolution and the resolution is left implicit.

The full specification also introduces new definedness conditions beyond the retained-content comparison. Let $\text{Tar}_\theta^\ell(X_\Sigma)$ denote the predicate that X_Σ is an admissible target of type θ at status ℓ , and let $\text{ModeAdm}(\theta, h, \mu)$ denote the model-declared compatibility of the target, historical mode, and preservation mode. The applicability of the complete preservation specification is defined in Equation 18.

$$\begin{aligned} \text{App}_{\mathfrak{P}}^{\text{pres}}(X_\Sigma, X'_{\Sigma'}) = 1 &\iff \text{Tar}_\theta^\ell(X_\Sigma) = 1, \\ &\text{Tar}_\theta^{\ell'}(X'_{\Sigma'}) = 1, \\ &\text{ModeAdm}(\theta, h, \mu) = 1, \\ &\exists \eta \in \text{Hist}_h(X_\Sigma, X'_{\Sigma'}) \text{ such that} \\ &\quad \eta \in \mathbf{A}_{\kappa, \theta, r}^{\ell, \ell'}(X_\Sigma, X'_{\Sigma'}), \\ &\quad (X_\Sigma, X'_{\Sigma'}, \eta) \in \text{Dom}(\mathbf{M}_\mu^{\mathfrak{P}}). \end{aligned} \tag{18}$$

Equation 18 tests applicability. Its positive output establishes joint evaluability of the selected target type, historical mode, retained-content comparison, and mode predicate on at least one historical witness; it does not require the retained-content comparison to succeed. Structural preservation is evaluated separately using the successful comparison-witness subclass $\mathbf{W}$.

Structural preservation is a partial Boolean function whose domain is exactly the applicability set. This typing prevents an inapplicable comparison from being read as a negative preservation result. Define

$$\begin{aligned} \text{Dom}(\text{Pres}_{\mathfrak{P}}^{\text{str}}) &:= \left\{ (X_\Sigma, X'_{\Sigma'}) \mid \text{App}_{\mathfrak{P}}^{\text{pres}}(X_\Sigma, X'_{\Sigma'}) = 1 \right\}, \\ \text{Pres}_{\mathfrak{P}}^{\text{str}} : \text{Dom}(\text{Pres}_{\mathfrak{P}}^{\text{str}}) &\longrightarrow \{0, 1\}. \end{aligned} \tag{19}$$

For a pair in this domain, structural satisfaction is defined in Equation 20.

$$\begin{aligned} \text{Pres}_{\mathfrak{P}}^{\text{str}}(X_\Sigma, X'_{\Sigma'}) = 1 &\iff \exists \eta \in \text{Hist}_h(X_\Sigma, X'_{\Sigma'}) \cap \mathbf{W}_{\kappa, \theta, r}^{\ell, \ell'}(X_\Sigma, X'_{\Sigma'}) \\ &\text{with } \mathbf{M}_\mu^{\mathfrak{P}}(X_\Sigma, X'_{\Sigma'}, \eta) = 1. \end{aligned} \tag{20}$$

Equation 20 is the first general preservation predicate in the paper, and its scope is deliberately limited. The superscript *str* marks satisfaction of a declared structural specification. It does not establish that the specification is the appropriate preservation criterion for a community, legal system, scientific inquiry, or normative theory. Those higher-order identification and value questions are treated separately in Section 13.

The partial-function typing makes the three outcomes exact. If Equation 18 holds and at least one admissible grounded witness satisfies the mode predicate, the value is 1. If applicability holds and every admissible grounded witness has mode value 0, the value is 0. If applicability fails, $\text{Pres}_{\mathfrak{P}}^{\text{str}}$ is undefined on that pair. Later profiles encode this third case by $\perp$; the value 0 is reserved for defined structural failure.

Definition 4.5 (Structural preservation satisfaction). A pair of history-bearing targets structurally satisfies preservation specification $\mathfrak{P}$ when it lies in the domain in Equation 19 and the existential grounded-witness condition in Equation 20 holds.

Definition 4.5 gives the later taxonomy a common evaluation form while allowing the substantive content of r , Hist_h , and $\mathbf{M}_\mu^{\mathfrak{P}}$ to vary by target class.

4.6 Cross-Taxonomy Implication Discipline

Preservation claims often appear to form an intuitive strength hierarchy. Continuous retention can seem stronger than recoverability, and generative-capacity preservation can seem stronger than preservation of

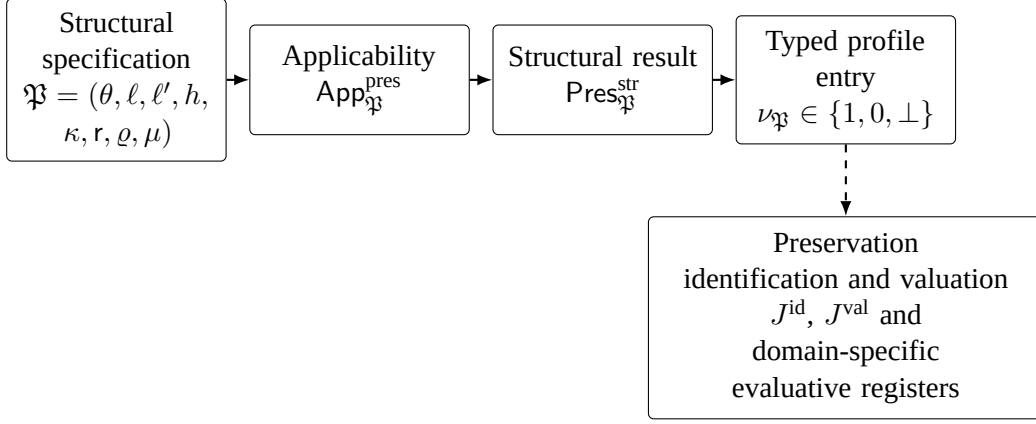


Figure 1: Evaluation architecture of a GR preservation claim. Structural specification, applicability, and structural satisfaction remain distinct from typed profile construction and later preservation-identification or value judgment. The dashed arrow marks evaluative dependence without collapsing judgment into the structural predicate.

manifested traces. Such intuitions can be valid under specified bridge assumptions, although the general taxonomy does not supply a universal chain. Different target classes retain different objects, and different modes can be incomparable when their domains or historical requirements differ.

A local implication relation can nevertheless be defined after a comparison domain has been fixed. Let $\mathcal{D}$ be a declared class of target pairs on which two structural specifications $\mathfrak{P}_1$ and $\mathfrak{P}_2$ are both applicable. Structural implication on $\mathcal{D}$ is defined in Equation 21.

$$\mathfrak{P}_1 \implies_{\mathcal{D}} \mathfrak{P}_2 \iff \forall (X_{\Sigma}, X'_{\Sigma'}) \in \mathcal{D}, \quad \text{Pres}_{\mathfrak{P}_1}^{\text{str}}(X_{\Sigma}, X'_{\Sigma'}) = 1 \implies \text{Pres}_{\mathfrak{P}_2}^{\text{str}}(X_{\Sigma}, X'_{\Sigma'}) = 1. \quad (21)$$

In Equation 21, both preservation predicates are evaluated on the displayed target pair. The relation is domain-relative. Changing the target class, retained content, resolution, historical mode, or bridge assumptions can remove an implication that held on a narrower domain.

The implication relation also distinguishes formal weakening from cross-target lifting. If $\mathfrak{P}_1$ and $\mathfrak{P}_2$ concern the same target family and differ only by retained content or comparison strength, an implication can sometimes be established directly. If they concern different target families, such as manifestational traces and generative capacity, a bridge map or substantive theorem is required before preservation of one can imply preservation of the other. Section 12 develops those lifting questions after the target-specific taxonomies are available.

Remark 4.6 (Absence of a universal preservation ladder). The core target classes and preservation modes are not assumed to form one total order of preservation strength. Implication is established locally through Equation 21 after the comparison domain and bridge assumptions have been supplied.

Remark 4.6 prevents several premature conclusions. Corpus preservation can support later recovery without implying current generative-capacity preservation. Dynamical-regime reproduction can preserve selected system characteristics while using a different grammar and different enabling conditions. Historical lineage can remain strong while a particular operational capacity disappears. The later taxonomy therefore produces typed preservation profiles whose component statuses remain visible; scalar scoring is outside the structural architecture.

Figure 1 summarizes the evaluation architecture established across this section and Sections 12–13. The figure separates the structural specification and its applicability test from the later typed profile and judgment layers.

Table 4 summarizes the components of the structural preservation specification and the role assigned to each one.

Table 4: Components of a structural preservation specification in GR. The components jointly type a preservation claim while leaving preservation identification and preservation value to later judgment.

Component	Representative notation	Analytical role	Principal question deferred
Target class	θ	Selects the object whose preservation is tested	Whether preservation of another target class follows
Status	ℓ, ℓ'	Locates targets in relational, manifestational, or representational analysis	Ultimate ontological identity
Historical mode	h	Selects the provenance or continuity witness family	Normative legitimacy of that historical relation
Retained content	κ	Specifies which structure is preservation-relevant	Whether the selected content deserves preservation
Comparison relation	r	States the equality, equivalence, similarity, simulation, or diagnostic relation used	Universal adequacy of the relation
Resolution	ϱ	Records representational scale and retained distinctions	Scale-independent preservation
Preservation mode	μ	Specifies retention, transformation, recovery, regeneration, or reproduction	Value or desirability of that mode

The structural architecture established in this section permits preservation claims to be written with enough information to distinguish object, provenance, comparison, and mode. Sections 5–10 instantiate the target dimension of the taxonomy. Their task is to identify which formal objects, comparison relations, and preservation modes become meaningful for each target family while retaining the definedness and implication discipline established here.

5 Manifestational Preservation

This section develops the first target-specific branch of the preservation taxonomy. Its role is to formalize preservation claims whose primary targets are context-dependent manifestations and records derived from those manifestations. The objective is to distinguish active manifestation, archival trace, multi-context profile, and corpus-level preservation while retaining the historical, resolution, and definedness structure established in Sections 3 and 4. The discussion proceeds from context-indexed manifestations to recording operations, multi-context comparison, corpus correspondence, manifestational change under relational continuity, and the inferential limits of trace-based preservation. The analysis is deliberately modest: manifestational preservation can be substantively important while remaining formally weaker than claims concerning relational structure, generative capacity, generative conditions, historical lineage, or dynamical regime.

5.1 Context-Relative Manifestations

The manifestation map in Equation 5 assigns a context-dependent presentation to a relationally represented structure whenever the selected context renders that structure legible. Manifestational preservation takes the resulting presentation as its target; the complete relational structure that supports it belongs to a separate target class.

Fix a historical domain H , a relational slice Σ , and a context c for which $X_\Sigma \in \text{Dom}(\pi_{H,c})$. The active manifestation of X_Σ in context c is defined in Equation 22.

$$m_{X;\Sigma,c} := \pi_{H,c}(X_\Sigma) \in \mathfrak{M}_{H,c}. \quad (22)$$

Equation 22 makes the context index part of the target. Equality of two displayed strings, images, institutional labels, numerical readouts, or other presentations therefore does not erase the contexts through which those presentations were generated.

Let $\mathcal{C}_\Sigma$ denote a model-declared family of manifestation contexts available at slice Σ . The class of active manifestational targets is represented by the tagged disjoint union in Equation 23.

$$\mathcal{T}_{\theta_{\text{man}}}^{\text{man}}(\Sigma) := \bigsqcup_{c \in \mathcal{C}_\Sigma} \text{Im}(\pi_{H,c}). \quad (23)$$

The disjoint union in Equation 23 retains context provenance even when two contexts produce extensionally equal presentations. A context-neutral quotient can be introduced later when an application has independent reasons to identify such presentations.

A single-context preservation comparison is obtained by setting the preservation target in Equation 13 to θ_{man} and the relevant status indices to manifestation status. The retained-content projection can select all or part of the presentation, and the comparison relation can be equality, structural equivalence, approximate similarity, diagnostic agreement, or another relation justified by the manifestation carrier. The historical witness remains required whenever the claim concerns preservation across distinct historical slices; simultaneous descriptions can be compared without that diachronic witness.

Definition 5.1 (Context-relative manifestational preservation). A pair of context-indexed manifestations exhibits context-relative manifestational preservation under specification $\mathfrak{P}_{\text{man}}$ when $\theta = \theta_{\text{man}}$, the relevant target status is manifestational, the specification is applicable in the sense of Equation 18, and the structural preservation predicate in Equation 20 is satisfied for the retained manifestation content.

Definition 5.1 intentionally delegates the substantive comparison relation to the application. Exact textual identity can be appropriate for one archival inquiry, while phonological similarity, institutional-role correspondence, image-feature agreement, or another typed relation can be appropriate for another. The preservation claim remains indexed by that choice.

5.2 Symbolic-Archival Traces

An archive ordinarily stores a record produced from a manifestation. The complete manifestation event and the underlying relational structure remain distinct possible targets. Recording therefore introduces an additional transformation whose domain, retained information, and provenance should remain visible.

Let a denote a recording or acquisition protocol applicable to manifestations in context c . The associated trace-extraction map is the partial map stated in Equation 24.

$$\tau_{c,a} : \text{Dom}(\tau_{c,a}) \subseteq \mathfrak{M}_{H,c} \dashrightarrow \mathfrak{A}_{H,c,a}, \quad (24)$$

where $\mathfrak{A}_{H,c,a}$ is the record space generated under the declared protocol. The map can encode transcription, audio or video recording, photography, symbolic classification, numerical measurement, database entry, or another explicitly specified recording operation. Partiality records cases in which a manifestation is available but the selected protocol does not yield a well-typed record.

For $m_{X;\Sigma,c} \in \text{Dom}(\tau_{c,a})$, the derived trace is defined in Equation 25.

$$q_{X;\Sigma,c,a} := \tau_{c,a}(m_{X;\Sigma,c}). \quad (25)$$

The trace in Equation 25 can be supplemented by provenance metadata. Let $\nu_{X;\Sigma,c,a}$ denote the metadata retained by the application, such as source identification, historical anchor, context, acquisition protocol, location at the adopted resolution, or declared uncertainty. The corresponding archival package is defined in Equation 26.

$$\mathfrak{a}_{X;\Sigma,c,a} := (q_{X;\Sigma,c,a}, \nu_{X;\Sigma,c,a}). \quad (26)$$

Equation 26 distinguishes recorded content from the information required to interpret that record historically. The metadata can remain incomplete, and that incompleteness should remain explicit in the archival representation.

The composite map from relational target to archival trace is stated in Equation 27.

$$\mathcal{A}_{H,c,a} := \tau_{c,a} \circ \pi_{H,c} : \text{Dom}(\mathcal{A}_{H,c,a}) \subseteq \mathfrak{R}_H \dashrightarrow \mathfrak{A}_{H,c,a}. \quad (27)$$

The map in Equation 27 is frequently many-to-one. A manifestation context can collapse relational distinctions, and the recording protocol can collapse further distinctions inside the manifestation. Archival preservation can therefore be exact at the level of q while remaining underdetermined with respect to the structure from which q was generated.

Remark 5.2 (Trace derivation and preservation status). An archival package derived through Equation 26 can be treated as a manifestationally derived representational target when the preservation inquiry concerns storage, copying, migration of file format, or another operation on the record itself. Its representational preservation status should remain distinct from preservation of the manifestation event and from preservation of the relational structure represented through that event.

Remark 5.2 permits archives to receive strong preservation claims at their own target level without turning those claims into stronger cross-status conclusions.

5.3 Multi-Context Manifestational Continuity

A single manifestation context can provide a narrow or distorted view of a history-bearing structure. Preservation analysis can therefore retain a family of contexts when continuity across several presentations is relevant. The contexts available at two historical slices need not be literally identical. A later institution, technology, linguistic register, measurement protocol, or social setting can require a declared correspondence between contexts.

Let $\mathcal{C}_\Sigma$ and $\mathcal{C}_{\Sigma'}$ denote the available context families at two slices. A context correspondence is a partial map of the form stated in Equation 28.

$$\chi_{\Sigma\Sigma'} : \mathcal{C}_\Sigma \dashrightarrow \mathcal{C}_{\Sigma'}. \quad (28)$$

The map in Equation 28 records which later context is treated as comparable with an earlier context. A context can fall outside the domain when no defensible later counterpart has been established.

For a historical witness η and a matched pair c and $c' = \chi_{\Sigma\Sigma'}(c)$, let $L_{c \rightarrow c'}^\eta$ denote a declared transport or translation between the corresponding manifestation carriers when such a map is required. The context-level preservation condition is stated in Equation 29.

$$L_{c \rightarrow c'}^\eta(m_{X;\Sigma,c}) \mathfrak{r}_{c,c'}^{\text{man}} m_{X';\Sigma',c'}. \quad (29)$$

Equation 29 permits the two manifestation spaces to differ. The transport can represent orthographic conversion, format migration, measurement translation, institutional relabeling, or another well-typed comparison interface. It can be omitted when both manifestations already occupy a common carrier.

Let $\mathcal{C}_\kappa \subseteq \mathcal{C}_\Sigma$ denote the context family retained by the preservation specification. Multi-context manifestational preservation requires the retained contexts to lie in the domain of the context correspondence and the context-level relations to hold, as stated in Equation 30.

$$\text{MC}_\kappa(X_\Sigma, X'_{\Sigma'}; \eta, \chi) = 1 \iff \mathcal{C}_\kappa \subseteq \text{Dom}(\chi_{\Sigma\Sigma'}) \text{ and } \forall c \in \mathcal{C}_\kappa, \text{ Equation 29 holds.} \quad (30)$$

Equation 30 uses universal retention only over the context family explicitly selected by κ . An application can choose a smaller required family, a partially ordered family of contexts, or a graded comparison architecture when its evidence warrants a different structure. The present construction makes the retained family explicit before the preservation result is evaluated.

Multi-context continuity can strengthen a manifestational preservation claim by reducing dependence on one readout. It still remains a manifestational result. Several contexts can reproduce the same underlying collapse of relational distinctions, and contexts selected from one institutional or linguistic environment can share systematic limitations. Cross-status lifting therefore requires additional evidence even when Equation 30 is satisfied over a rich context family.

5.4 Corpus-Level Preservation

A corpus assembles multiple traces together with indexing, metadata, and internal organization. Modeling a corpus as an indexed family preserves duplicate records, repeated events, retained ordering information, and heterogeneous provenance.

Let I_Σ be the corpus index family at slice Σ . A corpus is represented schematically in Equation 31.

$$\mathcal{C}_\Sigma := (\mathbf{a}_i)_{i \in I_\Sigma}, \quad \mathbf{a}_i = (q_i, \nu_i). \quad (31)$$

Equation 31 permits different records to use different manifestation contexts or acquisition protocols, provided that their provenance retains enough information for the comparison being attempted. A corpus can also contain internal relations among records, such as temporal ordering, speaker relation, document hierarchy, cross-reference, or another declared structure. Such relations enter the retained-content basis κ when they are preservation-relevant.

Let $I_{\kappa, \varrho} \subseteq I_\Sigma$ denote the indices retained at resolution ϱ . A corpus correspondence witness is a relation $\xi \subseteq I_\Sigma \times I_{\Sigma'}$ whose retained source domain contains $I_{\kappa, \varrho}$ and whose matched records satisfy declared content and metadata relations. The conditions are stated in Equation 32.

$$\begin{aligned} \text{CorpW}_{\kappa, \varrho}(\xi) = 1 \iff & I_{\kappa, \varrho} \subseteq \text{Dom}_1(\xi), \\ & \forall i \in I_{\kappa, \varrho}, \exists j \in I_{\Sigma'} \text{ with } (i, j) \in \xi, \\ & q_i \mathbf{r}_q q'_j \text{ and } \nu_i \mathbf{r}_\nu \nu'_j \text{ for at least one such } j. \end{aligned} \quad (32)$$

Here $\text{Dom}_1(\xi)$ denotes the source-side domain of the correspondence. The relations $\mathbf{r}_q$ and $\mathbf{r}_\nu$ can differ: a file can undergo format migration while preserving declared content, and metadata can be enriched while remaining historically compatible with an earlier provenance record.

Definition 5.3 (Corpus-level preservation). A later corpus $\mathcal{C}_{\Sigma'}$ preserves the retained corpus content of $\mathcal{C}_\Sigma$ relative to $(\kappa, \varrho, \mathbf{r}_q, \mathbf{r}_\nu)$ when there exists a corpus correspondence witness ξ satisfying Equation 32 and the historical or representational mode declared by the preservation specification supplies the required provenance for that correspondence.

Definition 5.3 covers exact copying as a special case while also admitting format migration, re-indexing, metadata enrichment, and other transformations whose retained content remains equivalent under the declared relations. It does not require the source corpus to remain the active manifestation system of a living relational formation.

The formal separation between corpus and relational structure produces an immediate inferential result.

Proposition 5.4 (Non-injective trace obstruction). *Let $\mathcal{A}_{H,c,a}$ be the composite relational-to-trace map in Equation 27. Suppose there exist relational targets $X, Y \in \text{Dom}(\mathcal{A}_{H,c,a})$ that are distinct under a declared relational comparison while $\mathcal{A}_{H,c,a}(X) = \mathcal{A}_{H,c,a}(Y)$. Then equality of the resulting archival traces is insufficient to establish that the underlying relational targets are equal or equivalent under that relational comparison. A cross-target implication from archival-trace preservation to relational preservation therefore requires an additional bridge condition that excludes or resolves the non-injectivity relevant to the claim.*

Proof. The assumed equality of traces assigns the same archival value to X and Y , while the declared relational comparison distinguishes them. Any inference rule that used trace equality alone to establish the relational comparison would therefore assign the same relational status to a pair that the declared relational relation treats as distinct. The trace map lacks the information required for that inference. A stronger bridge condition, richer evidence, or finer manifestation/recording map is required. $\square$

Proposition 5.4 formalizes one limitation of corpus-based preservation without diminishing the value of archival work. A corpus can preserve rare manifestations, support later reconstruction, and provide evidence for historical analysis. Those roles concern trace, recoverability, or later bridge constructions and remain distinct from the stronger conclusion that the source relational or generative system has itself been preserved.

5.5 Manifestational Change under Relational Continuity

Preservation can also be obscured when the manifestation changes substantially while the underlying relational lineage remains historically continuous. A living system can adopt a new script, recording technology, institutional label, public interface, measurement protocol, or representational convention. Raw manifestation identity would classify such a case as change even when a well-supported historical witness tracks the relational structure through that change.

Let $\eta \in \text{Hist}_{h_{\text{cont}}}(X_\Sigma, X'_{\Sigma'})$ be a historical-continuation witness for relational targets whose continuity is independently supported. Let c and c' be manifestation contexts at the two slices. A manifestation-transport map associated with the historical and contextual bridge has the type stated in Equation 33.

$$\Lambda_{c \rightarrow c'}^\eta : \mathfrak{M}_{H,c} \dashrightarrow \mathfrak{M}_{H',c'}. \quad (33)$$

The map can encode a change of script, interface, unit, classification convention, archival format, or another transformation for which the application supplies a defensible translation rule. Manifestational continuity under transformation is represented by Equation 34.

$$\Lambda_{c \rightarrow c'}^\eta (m_{X;\Sigma,c}) \text{ r}_{\text{man}} m_{X';\Sigma',c'}. \quad (34)$$

Equation 34 permits substantial surface difference while retaining a declared manifestational relation after transport. The historical witness and transport map perform different roles. The witness supports provenance across the history; the transport specifies how the two manifestation carriers are compared.

Definition 5.5 (Covariant manifestational preservation). Covariant manifestational preservation holds relative to $(\eta, c, c', \Lambda_{c \rightarrow c'}^\eta, \text{r}_{\text{man}})$ when the historical and contextual transports are defined and Equation 34 is satisfied on the retained manifestation content.

Definition 5.5 gives manifestational preservation a form compatible with historical evolution. It also prevents raw sameness from becoming the only manifestational criterion. A transformed manifestation can preserve a declared presentation-level structure, while an unchanged manifestation can coexist with deeper relational change.

The converse pattern is equally important. An institution can retain the same title, a ritual can retain the same visible sequence, or a language archive can retain the same corpus while the relational organization supporting those manifestations changes substantially. In such cases, manifestation identity remains genuine at its own level and the cross-status preservation question remains open.

Table 5: Principal forms of manifestational preservation and their inferential scope. Each form can support a valid preservation claim at its declared target level while leaving stronger cross-target conclusions unresolved.

Preservation form	Primary target	Representative witness or relation	Additional conclusion left open
Single-context manifestation	Context-indexed presentation	Historical witness plus declared manifestation comparison	Relational continuity or generative capacity
Archival trace	Recorded presentation and provenance package	Trace relation and provenance compatibility	Identity of the relational source
Multi-context profile	Family of matched context presentations	Context correspondence, transports, and context-level comparisons	Exhaustiveness of the relational description
Corpus-level preservation	Indexed family of archival packages	Corpus correspondence witness ξ	Living transmission, capacity, or enabling conditions
Covariant manifestation	Presentation transformed across historical contexts	Historical witness η and manifestation transport $\Lambda_{c \rightarrow c'}^\eta$	Preservation of every relational property of the lineage

5.6 Inferential Boundaries of Trace Preservation

The constructions above support a precise boundary around manifestational preservation. They identify several preservation claims that can be strong within the manifestation or archival target class while remaining insufficient for stronger target classes. Table 5 summarizes the principal forms developed in this section and the additional conclusion that each form leaves open.

The first boundary concerns cross-target implication. Let $\mathfrak{P}_{\text{man}}$ be a manifestational preservation specification and $\mathfrak{P}_{\text{rel}}$ a relational-structural specification defined on a common comparison domain $\mathcal{D}$. Proposition 5.4 supplies one sufficient reason for the implication in Equation 35 to fail.

$$\mathfrak{P}_{\text{man}} \not\Rightarrow_{\mathcal{D}} \mathfrak{P}_{\text{rel}} \quad \text{in the absence of a bridge that resolves the relevant manifestational collapse.} \quad (35)$$

Equation 35 uses the domain-relative implication relation defined in Equation 21. It is not a universal statement that manifestation evidence is useless for relational preservation. It states that a bridge theorem or additional evidence is required before the implication can be licensed on the selected domain.

The same discipline applies to generative capacity and generative conditions. A preserved archive can become part of a later recovery mechanism, although corpus preservation does not establish that the operations, competences, institutions, interaction partners, environments, or other conditions needed for current generation remain available. Sections 7 and 8 formalize those stronger targets separately.

A reverse implication is also unavailable in general. Relational continuity can coexist with changed manifestations, as Equation 34 demonstrates, and generative capacities can remain operative while their public or technological presentation changes. Preservation profiles therefore report manifestational status alongside other target classes and reserve cross-target inference for explicit bridge assumptions.

Remark 5.6 (Manifestational preservation as a complete local claim). A preservation claim can be complete relative to a manifestational target even when it is weak relative to the wider relational system. Exact preservation of an archive, a legally significant document, a recording, or another context-dependent presentation can be the full object of an inquiry. The taxonomy treats inferential weakness across target classes separately from adequacy within the declared target class.

Remark 5.6 prevents the taxonomy from treating manifestational preservation as intrinsically inferior. The relevant question is whether the preservation target matches the purpose of the inquiry. The next section moves to relational-structural preservation, where the target changes from presentations and traces to historically organized relations and interfaces themselves.

6 Relational-Structural Preservation

This section develops preservation claims whose primary targets are historically organized relational structures. Manifestations and archival traces remain distinct target classes developed in Section 5. Its role is to specify which relations, interfaces, incidence patterns, role structures, and historically efficacious organizations can be retained across changing GR configurations without requiring literal identity of the complete carrier. The objective is to distinguish relational-structural preservation from both manifestational continuity and the later preservation of generative capacities or conditions. The discussion proceeds from relational targets and retained interfaces to structural comparison, relational-reality continuity, resolution change, and historically continuous transformation. The method is formal and model-relative. Structural isomorphism, relational equivalence, refinement covariance, and local continuity are treated as candidate comparison resources whose preservation significance depends on the declared target, retained content, historical witness, and preservation mode.

6.1 Relational Configurations as Preservation Targets

A relational-structural preservation claim takes as its primary object a represented organization of relations inside a history-bearing GR spacetime. The target can include incidence relations among cells, typed dependencies, interfaces among subconfigurations, role relations, admissibility-supporting connections, or another relational organization selected by the application. The target is therefore richer than a manifestation while remaining narrower than the complete information carried by the surrounding spacetime.

Let $X_\Sigma \subseteq K_{\leq \Sigma}$ be a represented history-bearing target and let κ_{rel} denote retained relational content at resolution ϱ . The relational comparison object is the specialization of the retained-content projection in Equation 6 stated in Equation 36.

$$S_{\kappa_{\text{rel}}, \varrho}^{\text{rel}}(X_\Sigma) := \Pi_{\kappa_{\text{rel}}, \varrho; \Sigma}^{\text{rel}}(X_\Sigma) \in \mathcal{Z}_{\kappa_{\text{rel}}, \varrho; \Sigma}^{\text{rel}}. \quad (36)$$

Equation 36 introduces no universal tuple for relational structure. The retained object can be a subcomplex, a typed relation family, an incidence structure, an interface graph, a dependency system, a categorical diagram, or another well-typed representation. The index κ_{rel} records which relations and attachments are relevant to the preservation claim, while ϱ records the representational scale at which those relations are distinguished.

The distinction between the represented target X_Σ and the extracted structure $S_{\kappa_{\text{rel}}, \varrho}^{\text{rel}}(X_\Sigma)$ is substantive. One history-bearing formation can support several relational preservation inquiries at different resolutions or under different retained content. A language community, for example, can be represented at one resolution through speaker-to-speaker interaction, at another through institutions and educational interfaces, and at a third through a wider multilingual ecology. Preservation at one resolution does not settle preservation at another.

Definition 6.1 (Relational-structural preservation target). A relational-structural preservation target is a history-bearing GR target X_Σ together with a declared retained relational basis κ_{rel} and resolution ϱ for which the projection in Equation 36 is defined.

Definition 6.1 keeps the preservation object explicit. It prevents an application from treating preservation of one selected relation family as preservation of the entire formation.

6.2 Retained Relational Interfaces

Relational preservation can depend on the relations retained inside a target and on the interfaces through which that target participates in a wider configuration. An internally similar subconfiguration can occupy a substantially different relational position after migration, institutional reorganization, boundary change, or replacement of surrounding interfaces. The preservation basis must therefore state whether interface structure belongs to the retained content.

Let $\mathcal{E}_\kappa(X_\Sigma)$ denote the model-declared family of retained typed relations internal to, incident on, or otherwise constitutive of the selected target. Let $\mathcal{I}_\kappa(X_\Sigma; K_{\leq \Sigma})$ denote the retained interface family connecting the target to the wider modeled spacetime. The interface-enriched relational object is defined in Equation 37.

$$\hat{S}_{\kappa, \varrho}^{\text{rel}}(X_\Sigma) := \left(S_{\kappa, \varrho}^{\text{rel}}(X_\Sigma), \mathcal{E}_\kappa(X_\Sigma), \mathcal{I}_\kappa(X_\Sigma; K_{\leq \Sigma}) \right). \quad (37)$$

Equation 37 is an optional refinement. An application that does not require a separate interface representation can absorb the retained interfaces into $S_{\kappa, \varrho}^{\text{rel}}$. The explicit form is useful when internal organization and external relational position can vary independently.

A historical witness $\eta \in \text{Hist}_h(X_\Sigma, X'_{\Sigma'})$ can induce partial correspondences among retained relations and interfaces. Write these correspondences as in Equation 38 whenever the application supplies them.

$$T_\eta^{\mathcal{E}} : \mathcal{E}_\kappa(X_\Sigma) \dashrightarrow \mathcal{E}_{\kappa'}(X'_{\Sigma'}), \quad T_\eta^{\mathcal{I}} : \mathcal{I}_\kappa(X_\Sigma; K_{\leq \Sigma}) \dashrightarrow \mathcal{I}_{\kappa'}(X'_{\Sigma'}; K'_{\leq \Sigma'}). \quad (38)$$

The transports in Equation 38 can be embeddings, relabellings, quotient-compatible maps, typed relation correspondences, or other model-defined structures. Their partiality records cases in which a relation disappears, an interface splits, or a later structure lacks a defensible predecessor.

Remark 6.2 (Internal structure and relational position). Preservation of internal relational structure and preservation of relational position in a wider spacetime are distinct claims. Equality or isomorphism of the internal projection in Equation 36 does not establish preservation of the interface family in Equation 37 unless the retained-content basis includes that interface information.

Remark 6.2 is especially relevant to relocation. A practice, institution, or language can retain substantial internal organization while entering a new economic, legal, technological, or ecological environment that changes the relations through which its later continuation is generated.

6.3 Structural Isomorphism in Relational Comparison

The equivalence taxonomy supplies several comparison relations that can be used to compare relational structures. The present section specializes those relations to preservation without treating one of them as universally privileged. Exact structural isomorphism can be appropriate when the selected relations and incidence data admit a reversible structure-preserving correspondence. Weaker relational equivalence can be appropriate when the preservation claim retains only specified functions, typed relations, quotient structures, or diagnostic features.

Let S and S' denote relational comparison objects of the form in Equation 36. Suppose a model defines a class of admissible structure-preserving maps between such objects. A structural isomorphism is represented by an admissible map $\phi : S \rightarrow S'$ with admissible inverse $\phi^{-1} : S' \rightarrow S$ such that Equation 39 holds.

$$\phi^{-1} \circ \phi = \text{id}_S, \quad \phi \circ \phi^{-1} = \text{id}_{S'}. \quad (39)$$

The substantive meaning of “structure-preserving” must be supplied by the retained relational basis. Equation 39 does not determine whether labels, directions, weights, histories, interfaces, or higher-order attachments belong to the preserved structure.

A weaker relational equivalence is written as $S \simeq_{\kappa, \varrho}^{\text{rel}} S'$ when the application supplies an equivalence relation on the relevant relational comparison carrier. Structural isomorphism can imply this equivalence under an application-defined bridge, although the implication is not built into the notation. Likewise,

simulation, bisimulation, homomorphic correspondence, orbit equivalence, or diagnostic agreement can serve as the comparison relation r when their assumptions match the preservation question.

The preservation significance of any such comparison remains historical. Two independently generated institutions can be isomorphic as organizational graphs without one preserving the other. A structurally equivalent language-use network can arise in two unrelated populations. The historical witness remains necessary for preservation in the sense formalized by this paper.

Proposition 6.3 (Ahistorical structural equivalence is insufficient for preservation). *Let X_Σ and $Y_{\Sigma'}$ be relational-structural targets whose retained structures satisfy a declared relation r , including the special case of structural isomorphism. If $\text{Hist}_h(X_\Sigma, Y_{\Sigma'}) = \emptyset$ for the historical mode required by preservation specification $\mathfrak{P}$, then $\text{Pres}_{\mathfrak{P}}^{\text{str}}(X_\Sigma, Y_{\Sigma'})$ is undefined or fails the applicability condition, irrespective of the strength of r .*

Proof. Equation 18 requires a historical witness in $\text{Hist}_h(X_\Sigma, Y_{\Sigma'})$ before structural preservation satisfaction can be evaluated. When the required witness class is empty, the applicability condition is not satisfied. The retained structural relation therefore remains an equivalence result without acquiring preservation status under $\mathfrak{P}$. $\square$

Proposition 6.3 states formally the distinction that motivated the paper's interface with the equivalence taxonomy. Preservation adds historical or provenance structure to a comparison relation; it does not replace the comparison relation itself.

6.4 Relational Reality across Historical Extension

Relational-structural preservation can be stated at the representational level or at the level of relational reality. The second claim carries an additional status requirement. A represented relation can remain formally identifiable while losing the persistence or efficacy required for relational-reality status, and a newly efficacious relational structure can arise while sharing only limited structure with its predecessor.

Let $X_\Sigma \in \mathfrak{R}_H$ and $X'_{\Sigma'} \in \mathfrak{R}_{H'}$ be targets assigned relational-reality status under their respective historical domains. Endpoint relational-reality preservation requires both status assignments together with a preservation specification whose relational retained content is satisfied. The endpoint condition is stated in Equation 40.

$$X_\Sigma \in \mathfrak{R}_H, \quad X'_{\Sigma'} \in \mathfrak{R}_{H'}, \quad \text{Pres}_{\mathfrak{P}_{\text{rel}}}^{\text{str}}(X_\Sigma, X'_{\Sigma'}) = 1. \quad (40)$$

Equation 40 is stronger than relational equivalence of two projected structures because each endpoint must independently satisfy the relational-reality criteria adopted by the GR application. It is weaker than a claim of uninterrupted relational continuity across every intermediate historical stage.

For preservation modes that require continuous or historically continuous retention, an application can specify a historical corridor $\mathfrak{C}_\eta$ associated with witness η . The corridor is a finite chain, directed family, or other model-defined collection of intermediate targets whose ordering is compatible with the retained history. Relational-status continuity along the corridor is expressed schematically by Equation 41.

$$X_{\Sigma_s} \in \mathfrak{R}_{H_s} \quad \text{for each retained } \Sigma_s \in \mathfrak{C}_\eta. \quad (41)$$

Equation 41 does not require the same manifestation, label set, or local configuration at every stage. It requires the tracked relational organization to remain inside the status domain selected by the application while the preservation mode separately specifies which retained structural relations must continue to hold.

Remark 6.4 (Relational status and relational identity). Persistence of relational-reality status does not by itself establish preservation of one relational structure. Distinct structures can each be relationally real. Preservation additionally requires the historical witness and retained-content comparison specified by $\mathfrak{P}_{\text{rel}}$.

The distinction in Remark 6.4 prevents relational reality from functioning as an identity criterion. Relational reality states that a structure is sufficiently historically persistent and efficacious within the adopted model; preservation asks which later structure is connected to which earlier structure and what content is retained through that connection.

6.5 Structural Refinement and Coarse-Graining

Relational preservation is resolution-sensitive. A coarse model can preserve a role pattern or interface graph while finer internal relations change substantially. Conversely, a fine representation can change through local refinement while the coarse relational organization remains stable. Preservation claims therefore need a rule for transporting relational content across representational resolution.

Let $S^{(f)}$ be a fine relational representation and $S^{(c)}$ a coarser representation. A declared coarse-graining map is written in Equation 42.

$$Q_{fc} : S^{(f)} \dashrightarrow S^{(c)}. \quad (42)$$

The map in Equation 42 can identify cells, aggregate relation types, suppress internal histories, quotient role distinctions, or otherwise remove structure according to the declared resolution change. It need not be invertible.

Suppose $T_\eta^{(f)}$ and $T_\eta^{(c)}$ are historical transports induced by the same witness at fine and coarse resolution. Exact refinement covariance of preservation is represented by the commutative condition in Equation 43.

$$Q'_{fc} \circ T_\eta^{(f)} = T_\eta^{(c)} \circ Q_{fc} \quad (43)$$

whenever both compositions are defined. Equation 43 states that transporting the fine structure historically and then coarse-graining gives the same retained coarse result as coarse-graining first and then applying the coarse historical transport. Approximate covariance can be used when an application supplies a discrepancy measure and tolerance.

Coarse preservation does not generally lift to fine preservation. The obstruction is immediate when coarse-graining collapses distinctions.

Proposition 6.5 (Non-injective coarse-graining obstruction). *Let $Q_{fc} : S^{(f)} \rightarrow S^{(c)}$ be a non-injective coarse-graining map. Equality or another comparison relation defined solely on $Q_{fc}(S^{(f)})$ cannot, without an additional lifting assumption, establish preservation of the fine relational structure.*

Proof. Because Q_{fc} is non-injective, there exist distinct fine structures $s_1 \neq s_2$ with $Q_{fc}(s_1) = Q_{fc}(s_2)$. A preservation test performed only on the coarse images cannot distinguish s_1 from s_2 . Therefore successful coarse comparison is compatible with more than one fine relational structure, and a fine-structure preservation conclusion requires additional information or a justified lifting relation. $\square$

Proposition 6.5 is the relational-structural analogue of the trace obstruction in Proposition 5.4. Both results identify loss of distinctions as a barrier to lifting a weaker preservation claim into a stronger one.

6.6 Relational Continuity under Configuration Change

A historically living relational system can change at each stage while preserving a selected organization through a chain of locally justified correspondences. Relational continuity is therefore better represented by composable historical transports than by equality of endpoint configurations. This formulation allows role replacement, institutional revision, migration, local relation turnover, or representational refinement while retaining a declared relational lineage.

The finite historically ordered family selected from a witness corridor is stated in Equation 44.

$$X_{\Sigma_0}, X_{\Sigma_1}, \dots, X_{\Sigma_n} \quad (44)$$

For each adjacent pair, let T_i denote the induced transport on retained relational structure. The composed relational transport is stated in Equation 45.

$$T_{0 \rightsquigarrow n} := T_{n-1} \circ \cdots \circ T_1 \circ T_0, \quad (45)$$

whenever every adjacent codomain-domain pair is well typed. The composition in Equation 45 makes the intervening historical route part of the preservation representation. An endpoint relation can be recovered from the composite when the selected comparison relation is compatible with composition, while the historical route remains richer than the endpoint result.

Definition 6.6 (Historically continuous relational preservation). A relational-structural target exhibits historically continuous relational preservation over a declared corridor when: (i) adjacent targets are connected by admissible historical witnesses; (ii) the corresponding retained relational transports are defined and composable; (iii) each adjacent comparison satisfies the declared relational preservation criterion; and (iv) the selected preservation mode treats the resulting corridor as an admissible continuous transformation history.

Definition 6.6 permits substantial local change. Its continuity lies in the existence of a historically connected sequence of admissible relational transformations, allowing the concrete configuration to change across the sequence.

Proposition 6.7 (Local relational continuity and endpoint comparison). *Suppose the retained relational comparison relation r_{rel} is transitive on the transported comparison carrier and is compatible with each local historical transport along the corridor in Definition 6.6. If every adjacent pair satisfies r_{rel} , then the transported first and final retained structures satisfy r_{rel} at the endpoint.*

Proof. Compatibility places each adjacent comparison in a common transported carrier on which r_{rel} is defined. Repeated application of transitivity to the finite sequence of adjacent comparisons yields the endpoint comparison. $\square$

The converse of Proposition 6.7 does not follow in general. Endpoint equivalence can hold between structures connected by a history that passes through discontinuity, loss, replacement, or later reconstruction. It can also hold between unrelated systems. Endpoint equivalence therefore cannot reconstruct historically continuous preservation without additional evidence concerning the intervening corridor.

Corollary 6.8 (Endpoint equivalence does not determine relational lineage). *A successful endpoint relational comparison is insufficient, by itself, to establish historically continuous relational preservation in the sense of Definition 6.6.*

Corollary 6.8 gives the relational branch of the taxonomy a central distinction: preservation can be path-sensitive even when the comparison relation is evaluated at endpoints. Historical continuity therefore carries information that equivalence alone does not preserve.

Table 6 summarizes the relational-structural preservation forms developed in this section and the inferential boundaries attached to each one.

Relational-structural preservation therefore occupies a distinct position in the taxonomy. It is stronger than preservation of selected manifestations when the retained target contains relations that the manifestation suppresses, yet it remains independent of later generative-capacity and generative-condition targets. A relational structure can persist while a capability becomes unavailable, and a capability can sometimes be reconstructed within a substantially altered relational structure. The next two target-specific sections develop those further distinctions through generative-capacity and generative-condition preservation.

Table 6: Relational-structural preservation forms and their principal inferential boundaries.

Preservation form	Retained relational object	Inference not established by the form alone
Internal structural preservation	Selected internal relations, incidence, or role structure	Preservation of interfaces or wider relational position
Interface-enriched preservation	Internal structure together with declared external interfaces	Preservation of generative capacity or generative conditions
Structural isomorphism	Reversible structure-preserving correspondence on retained relational content	Historical lineage or preservation status without a historical witness
Relational-reality endpoint preservation	Retained relational structure whose endpoints both satisfy relational-reality criteria	Uninterrupted relational continuity across intermediate history
Resolution-covariant preservation	Relational content compatible with declared refinement or coarse-graining maps	Fine-grained preservation when coarse-graining is non-injective
Historically continuous relational preservation	Composable local relational correspondences along a declared historical corridor	Normative value, desirability, or preservation of later generative capacities

7 Generative-Capacity Preservation

This section develops preservation claims whose primary target is a capacity to generate transformations, expressions, distinctions, decisions, artifacts, or other later relational structure. Its role is to separate preservation of generative capacity from preservation of the relational configuration that currently carries that capacity and from preservation of the conditions that make the capacity executable. The objective is to specify a minimal formal carrier for generative grammar, make partial definedness and state dependence explicit, distinguish schema-level availability from present operational availability, and identify comparison relations through which capacities can be tracked across historical change. The discussion proceeds from distributed capacity targets to partial operation families, state-dependent schemas, operational availability, historical capacity equivalence, and dormant capacity. The method is formal and model-relative. The section does not assume that a generative capacity is a context-independent operator, that every capacity is continuously executable, or that preservation of capacity establishes preservation of its enabling conditions.

7.1 Generative Grammar as a Distributed Capacity

A generative-capacity preservation claim requires a target richer than an output set. Two systems can produce overlapping outputs while differing in the operations through which those outputs are generated, the distinctions available to participants, or the compositions through which later novelty becomes possible. The present section therefore treats a generative grammar as a distributed family of operation types and compositional relations available within a history-bearing GR configuration.

Fix a relational slice Σ and an admissible region $R \in \mathcal{R}(K_\Sigma)$. Let $\mathcal{P}_\Sigma(R)$ denote the model-defined family of generative operation types retained at that region. An operation type is a typed capacity descriptor such as formation, interpretation, correction, classification, revision, composition, negotiation, or another application-defined generative act. Membership in $\mathcal{P}_\Sigma(R)$ records that the model recognizes the operation type as part of the local generative repertoire. It does not yet state that the operation can

be executed in the current configuration.

The composition structure among operation types is represented by a partial compositional relation $\text{Comp}_\Sigma(R)$. For $p, q, r \in \mathcal{P}_\Sigma(R)$, a record $(p, q; r) \in \text{Comp}_\Sigma(R)$ means that the model treats an execution of p followed by an execution of q as admitting a composite operation of type r on some declared domain. The relation is partial because some compositions can be undefined, context-sensitive, or unavailable at the selected resolution.

The local capacity grammar is defined in Equation 46.

$$\Gamma_\Sigma^{\text{cap}}(R) := (\mathcal{P}_\Sigma(R), \text{Comp}_\Sigma(R)). \quad (46)$$

Equation 46 is intentionally weaker than an executable dynamics. It records the types of generative acts and their declared compositional organization before the concrete realization, domain, or admissibility of those acts has been supplied. This separation permits the taxonomy to distinguish a retained capacity grammar from the conditions under which the grammar is presently usable.

For a retained region family $\mathcal{R}_{\text{cap}}(K_\Sigma) \subseteq \mathcal{R}(K_\Sigma)$, the distributed capacity target is stated in Equation 47.

$$\mathbf{\Gamma}_\Sigma^{\text{cap}} := \{ \Gamma_\Sigma^{\text{cap}}(R) \mid R \in \mathcal{R}_{\text{cap}}(K_\Sigma) \}. \quad (47)$$

The distributed form in Equation 47 permits different regions to support different operation types and different compositional structures. A language, institution, scientific practice, or craft can therefore possess a heterogeneous generative organization without being compressed into one globally uniform grammar. A later field-like construction can be introduced when local capacity data satisfy restriction, overlap, or gluing conditions, although such additional structure is not required for the present taxonomy.

Remark 7.1 (Capacity grammar and substantive interpretation). The notation $\Gamma_\Sigma^{\text{cap}}(R)$ does not identify a generative capacity with linguistic syntax, a formal grammar in computer science, or a fixed algebra of operators. It is a GR-level preservation target whose substantive interpretation is supplied by the application. Its content can include linguistic, epistemic, institutional, technical, artistic, or other generative operation types.

7.2 Definedness of Partial Operation Families

The presence of an operation type in a capacity grammar is weaker than present executability. GR already treats applicability domains as substantive relational structure. Generative-capacity preservation therefore needs to retain the distinction between a capacity type represented by the model and a concrete operation whose action is defined on the current configuration.

Let Ω denote the model-defined configuration class at the resolution retained by the inquiry. A simple local specialization assigns to each operation type $p \in \mathcal{P}_\Sigma(R)$ a partial transformation of the form stated in Equation 48.

$$\Phi_{p,R} : D_{p,R} \longrightarrow \Omega, \quad D_{p,R} \subseteq \Omega. \quad (48)$$

Membership $\omega \in D_{p,R}$ means that the relational description contains enough articulation for the operation represented by $\Phi_{p,R}$ to act coherently at configuration ω . A configuration outside $D_{p,R}$ does not thereby falsify the capacity type p ; the concrete operation is undefined there under the selected representation.

The defined operation family at configuration ω is stated in Equation 49.

$$\mathcal{P}_\Sigma^{\text{def}}(R \mid \omega) := \{ p \in \mathcal{P}_\Sigma(R) \mid \omega \in D_{p,R} \}. \quad (49)$$

Equation 49 permits loss or contraction of present capacity to occur through domain change even when the operation-type family itself remains represented. If $D_{p,R}$ contracts along history, fewer configurations support the corresponding operation. If the domain expands, a previously latent operation type can become defined across a wider relational regime.

Where local reversibility is substantively justified, inverse-semigroup or restriction-categorical structure can internalize the domain information carried externally by $D_{p,R}$. The present section does not

require those stronger constructions. Its minimum requirement is that definedness remain visible and that preservation claims do not convert a partial capacity into a total operation by notation alone.

Definition 7.2 (Schema-level capacity availability). An operation type p has schema-level availability at region R and slice Σ when $p \in \mathcal{P}_\Sigma(R)$. Schema-level availability records membership in the represented generative repertoire and does not establish definedness or operational execution at a particular configuration.

Definition 7.3 (Defined capacity availability). An operation type p has defined capacity availability at (ω, R, Σ) when $p \in \mathcal{P}_\Sigma^{\text{def}}(R \mid \omega)$ in the sense of Equation 49.

Definitions 7.2 and 7.3 give the preservation taxonomy two distinct states before execution is considered. A later system can preserve the represented capacity type while changing the relational domain on which that capacity is defined.

7.3 State-Dependent Generative Schemas

A fixed partial map remains too strong for many historically evolving generative systems. An operation called by the same type can be realized differently after earlier relations, classifications, institutional rules, or learned distinctions change the configuration. The capacity formalism therefore permits a state-dependent schema whose output is itself a partial transformation.

Let $\text{PMap}(\Omega)$ denote the model-defined class of typed partial transformations on or between the relevant configuration carriers. For each $p \in \mathcal{P}_\Sigma(R)$, a state-dependent generative schema is a partial map of the form stated in Equation 50.

$$F_{p,R} : \Omega \dashrightarrow \text{PMap}(\Omega), \quad \Phi_{p,R}^{[\omega]} := F_{p,R}(\omega) \quad (50)$$

whenever the schema is defined at ω . The concrete transformation $\Phi_{p,R}^{[\omega]}$ selected by Equation 50 can have its own domain of execution. The two levels of partiality are distinct: the model may lack a defined realization for operation type p at ω , or it may define a concrete transformation whose execution domain excludes the current local state to which the operation would otherwise be applied.

State dependence gives historical change a direct route into generative capacity. Let ω' be a later configuration reached through an admissible historical transformation. Even when p remains in the operation-type family, Equation 51 can hold.

$$F_{p,R}(\omega) \not\equiv_{\text{op}} F_{p,R'}(\omega'), \quad (51)$$

where $\equiv_{\text{op}}$ is a declared comparison relation on concrete operations. Equation 51 records transformation of the realization of a retained operation type. It does not establish loss of the capacity. Whether the later realization remains preservation-equivalent depends on the retained content and comparison relation selected by the preservation specification.

The state-dependent refinement permits the local capacity grammar to be enriched as stated in Equation 52.

$$\widehat{\Gamma}_\Sigma^{\text{cap}}(R) := \left(\mathcal{P}_\Sigma(R), \{F_{p,R}\}_{p \in \mathcal{P}_\Sigma(R)}, \text{Comp}_\Sigma(R) \right). \quad (52)$$

Equation 52 is the primary formal target used by the remainder of this section. The admissibility, material support, institutional support, and wider enabling environment of the realized operations are treated separately in Section 8.

Remark 7.4 (Historical change of a retained grammar). Preservation of $\widehat{\Gamma}^{\text{cap}}$ does not require constant concrete transformations. A living generative capacity can preserve operation types and relevant compositional organization while the state-dependent realizations of those types change through historical learning, institutional revision, technological change, or another represented process.

7.4 Operational Availability

The enriched grammar in Equation 52 describes represented capacity. Present operation additionally depends on whether a concrete realization is defined and whether the current relational configuration admits its execution. The taxonomy therefore distinguishes capacity preservation from operational availability while retaining a formal interface between them.

Let $\text{Adm}_{\Sigma,R}(\Phi \mid \omega) \in \{0, 1\}$ be a model-defined admissibility predicate for a concrete operation Φ at configuration ω . The predicate is introduced here only as an interface to the conditions of generation developed in Section 8; its substantive determinants remain outside the present capacity target. Executable availability is defined in Equation 53.

$$E_{p,R,\Sigma}^{\text{exec}}(\omega) = 1 \iff \begin{cases} F_{p,R}(\omega) \text{ is defined,} \\ \omega \in \text{Dom}(F_{p,R}(\omega)), \\ \text{Adm}_{\Sigma,R}(F_{p,R}(\omega) \mid \omega) = 1. \end{cases} \quad (53)$$

Equation 53 defines current operational availability; ontological existence is outside the scope of this predicate. A capacity can remain represented in the grammar while failing one of the execution conditions at the current configuration.

The distinction yields a local implication that is useful for preservation analysis.

Proposition 7.5 (Executable availability implies represented availability). *If $E_{p,R,\Sigma}^{\text{exec}}(\omega) = 1$, then $p \in \mathcal{P}_{\Sigma}(R)$ and the state-dependent schema $F_{p,R}$ is defined at ω .*

Proof. Equation 53 is evaluated only for an operation type p retained by the enriched grammar. Its first execution condition requires $F_{p,R}(\omega)$ to be defined. The stated consequences therefore follow directly from the typing of Equation 53. $\square$

The converse of Proposition 7.5 fails without additional conditions. A represented operation type can be undefined at the current configuration, can produce a concrete operation whose execution domain excludes that configuration, or can be blocked by the current admissibility structure. This asymmetry between represented capacity and executable availability is central to dormant preservation developed in Subsection 7.6.

A preservation inquiry can also retain a family of executable operation types as its capacity target. The current executable repertoire is defined in Equation 54.

$$\mathcal{P}_{\Sigma}^{\text{exec}}(R \mid \omega) := \{p \in \mathcal{P}_{\Sigma}(R) \mid E_{p,R,\Sigma}^{\text{exec}}(\omega) = 1\}. \quad (54)$$

Changes in Equation 54 can therefore arise through alteration of the capacity grammar, alteration of operation domains, or alteration of enabling conditions. The next section separates the third source from the capacity target itself.

7.5 Capacity Equivalence across Historical Change

Generative-capacity preservation requires a comparison relation that can track capacity through historical change without requiring literal equality of operation labels or concrete realizations. The comparison can be exact, structural, functional, simulation-based, or approximate according to the retained content selected by the preservation specification. The present section introduces an explicit capacity correspondence whose stronger specialization preserves operation types and their state-dependent action up to declared transports.

Let $\widehat{\Gamma}_{\Sigma}^{\text{cap}}(R)$ and $\widehat{\Gamma}_{\Sigma'}^{\text{cap}}(R')$ be local capacity grammars connected by an admissible historical witness $\eta \in \text{Hist}_h(X_{\Sigma}, Y_{\Sigma'})$. A capacity correspondence consists of a partial operation-type map

$$u_{\eta} : \mathcal{P}_{\Sigma}(R) \dashrightarrow \mathcal{P}_{\Sigma'}(R') \quad (55)$$

and a configuration transport Q_η defined on the retained comparison domain. The correspondence is composition-compatible when every retained composable pattern $(p, q; r)$ in the source that lies inside the declared domain is carried to a composable pattern $(u_\eta(p), u_\eta(q); u_\eta(r))$ in the target.

A stronger dynamical compatibility condition is stated in Equation 56 whenever the relevant operations and transports are defined.

$$Q_\eta \circ F_{p,R}(\omega) \text{ } r_{\text{op}} \text{ } F_{u_\eta(p),R'}(Q_\eta(\omega)) \circ Q_\eta. \quad (56)$$

The relation r_{op} in Equation 56 is supplied by the preservation specification. Exact equality, conjugacy, simulation, bisimulation, agreement of selected outputs, or another typed relation can be appropriate in different applications. The equation therefore does not impose one universal notion of capacity identity.

Definition 7.6 (Generative-capacity preservation). Let $\mathfrak{P}$ be a preservation specification with target class $\theta = \text{cap}$. A pair of history-indexed generative-capacity targets exhibits generative-capacity preservation when: (i) the required historical witness exists; (ii) the retained-content projections of the two capacity grammars are defined at the declared resolution; (iii) a capacity correspondence of the type in Equation 55 is defined on the retained operation family; and (iv) the declared comparison relation, including Equation 56 where required by $\mathfrak{P}$, satisfies the preservation criterion of $\text{Pres}_{\mathfrak{P}}^{\text{str}}$.

Definition 7.6 permits substantial historical change. A later capacity can preserve a functionally or structurally corresponding generative organization while using different labels, implementations, participants, or concrete operations. The retained content determines which of those differences matter for the preservation claim.

The definition also preserves a boundary with the next target class. A capacity correspondence can remain valid even when the environment supporting execution has changed. Conversely, similar environments do not establish preservation of a capacity grammar. The relation between the two targets requires a separate coupling analysis and explicit bridge assumptions.

Proposition 7.7 (Capacity preservation and executable repertoire). *Generative-capacity preservation in the sense of Definition 7.6 does not, by itself, imply equality or equivalence of the executable repertoires $\mathcal{P}_{\Sigma}^{\text{exec}}(R \mid \omega)$ and $\mathcal{P}_{\Sigma'}^{\text{exec}}(R' \mid \omega')$.*

Proof. The preservation definition compares retained capacity grammars and their declared operation correspondences. Executable availability in Equation 53 additionally depends on execution domains and the admissibility predicate. Two preserved capacity grammars can therefore be embedded in different generative conditions that yield different executable repertoires. $\square$

Proposition 7.7 formalizes the distinction that motivates the separate condition target in Section 8: preserving the ability represented by a grammar and preserving the conditions under which that ability can presently operate are different preservation claims.

7.6 Dormant Generative Capacity

Strong preservation can silently require continuous execution or continuous availability of a capacity. Living generative systems often violate that requirement. A language can retain speakers with a competence that is rarely exercised in public institutions. A craft capacity can remain embodied in practitioners while the relevant material or workplace is temporarily unavailable. A scientific procedure can remain technically understood while an instrument is out of service. The preservation taxonomy therefore needs a status for capacities that remain historically represented while being operationally inactive at a selected configuration.

Definition 7.8 (Dormant generative capacity). An operation type p is dormant at (ω, R, Σ) when: (i) p has schema-level availability in the sense of Definition 7.2; (ii) the model retains a historically identified state-dependent schema $F_{p,R}$ for the operation type; and (iii) $E_{p,R,\Sigma}^{\text{exec}}(\omega) = 0$.

Table 7: Generative-capacity preservation statuses and their principal inferential boundaries.

Capacity status	Retained object	Inference not established by the status alone
Schema-level availability	Operation type in the represented generative repertoire	Definedness, admissibility, or current execution
Defined availability	Operation type with a defined local realization/domain	Present admissibility or successful execution
Executable availability	Defined and admissible operation at the selected configuration	Historical persistence or future retention
Generative-capacity preservation	Historically connected correspondence of retained capacity grammars	Preservation of enabling generative conditions
Dormant capacity	Historically retained capacity schema with current operational inactivity	Recoverability after loss of the retained capacity schema
Capacity equivalence under transport	Declared correspondence of operation types, compositions, and selected action	Identity of implementation, manifestation, participants, or surrounding relational structure

Definition 7.8 deliberately groups several causes of inactivity. The schema can be undefined at the present configuration, its realized operation can lack a current execution domain, or the operation can be inadmissible under the present conditions. Section 8 differentiates these causes where the surrounding conditions are the preservation target.

Dormancy is also distinct from mere non-use. An operation type can be executable according to Equation 53 even when no participant happens to execute it during the observed interval. The taxonomy therefore separates current exercise, executable availability, and dormant represented capacity.

A dormant capacity can still participate in a preservation claim. Suppose a historical witness and capacity correspondence establish preservation of the enriched grammar from Σ to Σ' , while one corresponding operation type is executable at Σ and dormant at Σ' . The capacity target can remain preserved even though operational preservation fails at the later slice. This pattern is stated in Equation 57.

$$\text{Pres}_{\mathfrak{P}_{\text{cap}}}^{\text{str}}(\hat{\Gamma}_{\Sigma}^{\text{cap}}, \hat{\Gamma}_{\Sigma'}^{\text{cap}}) = 1, \quad E_{p,R,\Sigma}^{\text{exec}}(\omega) = 1, \quad E_{u_{\eta}(p),R',\Sigma'}^{\text{exec}}(\omega') = 0. \quad (57)$$

Equation 57 gives a direct formal rejection of strong preservation as the only preservation mode for capacity. Historical preservation can retain a capacity target while allowing temporary operational inactivity.

Dormancy does not establish future recoverability after the capacity grammar itself has been lost. Once the operation type, state-dependent schema, or required compositional structure ceases to be historically identifiable in the current capacity target, the preservation question moves beyond dormant retention to reconstruction or recovery. Those modes are developed in Section 9.

Table 7 summarizes the capacity distinctions developed in this section. Their common purpose is to prevent the phrase “the capacity still exists” from carrying an unexamined mixture of representation, definedness, execution, continuity, and recoverability. Generative-capacity preservation concerns the historically tracked ability to generate under a declared comparison relation. The surrounding conditions that make the ability executable form a distinct preservation target, developed in Section 8.

8 Generative-Condition Preservation

This section develops preservation claims whose primary target is the historically situated condition structure under which a generative capacity can become defined, admissible, executable, and consequential. Its role is to separate preservation of a generative grammar from preservation of the relational, institutional, environmental, material, and infrastructural supports through which that grammar can be realized. The objective is to derive condition targets from the existing GR configuration, distinguish constitutive support from local manifestations of support, state the additional assumptions required for a field-like condition representation, and formalize historical change in conditions without equating preservation with frozen context. The discussion proceeds from condition extraction and support structure to relational, institutional, environmental, and material conditions, local condition fields, historical condition comparison, and capacity–condition coupling. The method is formal and model-relative. Generative conditions are treated as historically evolving relational structures, and preservation of those conditions is not assumed to require identity of their concrete realization across historical extension.

8.1 Conditions of Generative Realization

Section 7 distinguished schema-level capacity from defined and executable availability. The distinction implies a corresponding preservation target. A capacity grammar can remain historically identifiable while the relational configuration ceases to support the conditions under which one or more operation types are defined or admissible. Generative-condition preservation therefore concerns the support structures on which realization of a retained capacity depends. The capacity grammar remains a distinct preservation target.

Let ω_Σ denote the model-defined relational configuration at slice Σ , let $R \in \mathcal{R}(K_\Sigma)$ be a retained region, and let κ_{cond} specify the relational content treated as condition-relevant for the inquiry. The condition projection is a specialization of the retained-content machinery of Section 3.4. Its local form is stated in Equation 58.

$$C_{\Sigma, \kappa_{\text{cond}}, \varrho}^{\text{gen}}(R \mid \omega_\Sigma) := \Pi_{\kappa_{\text{cond}}, \varrho}^{\text{cond}}(\omega_\Sigma; R). \quad (58)$$

Equation 58 defines a local generative-condition record by projection from an already represented history-bearing configuration. The codomain of $\Pi_{\kappa_{\text{cond}}, \varrho}^{\text{cond}}$ is application-defined. It can retain relational supports, institutional or nomic structures, environmental relations, material resources, interface conditions, admissibility-relevant labels, or other structures for which the application supplies a generative-dependence account. The projection is partial because some configurations or resolutions may not support the condition distinctions required by the selected basis.

The index κ_{cond} is substantive. A condition record is never intended to contain every relation surrounding a generative process. It contains the relational content selected as relevant to realization of a declared capacity family. Two condition analyses can therefore project different records from the same ω_Σ when they study different generative capacities or different resolutions.

Definition 8.1 (Generative-condition target). A generative-condition target at (Σ, R, ϱ) is a defined value of the projection in Equation 58, together with the declared condition basis κ_{cond} and the capacity family relative to which the projected structure is treated as condition-relevant.

Definition 8.1 prevents the word *condition* from becoming a residual category containing everything outside the capacity grammar. An application must identify which relations or supports enter the condition target and why changes in those structures can alter definedness, admissibility, execution, or another declared aspect of generative realization.

The resulting target is historical. A later condition record can contain additional relations or resources generated after the reference slice, can lose earlier supports, or can reorganize their roles. Condition preservation therefore cannot generally be reduced to equality of the records in Equation 58.

8.2 Relational-Institutional Support

Generative conditions frequently depend on relations whose role is distributed across a history-bearing configuration. A speaker can retain linguistic competence while losing interlocutors or institutional domains of use. A craft practice can retain embodied technique while losing apprenticeship relations. A scientific procedure can remain conceptually understood while institutional access to an instrument, laboratory, dataset, or authorization structure changes. These cases motivate a support representation that identifies condition-relevant regions without treating support as a scalar quantity.

For a retained capacity family $\mathcal{P}_\Sigma^*(R) \subseteq \mathcal{P}_\Sigma(R)$, let $\text{Supp}_\Sigma^{\text{cond}}(\mathcal{P}^*)$ denote the model-defined family of regions whose declared modification can alter the definedness, admissibility, or executable realization of at least one retained operation type. The support family is stated in Equation 59.

$$\text{Supp}_\Sigma^{\text{cond}}(\mathcal{P}^*) \subseteq \mathcal{R}(K_\Sigma). \quad (59)$$

Equation 59 is a typed support declaration; individual necessity is an additional claim that requires separate justification. Some capacities can be multiply realized, supported by redundant institutions, or sustained through several partially substitutable relational paths. A minimal support set need not exist, and different resolutions can yield different support descriptions.

Institutional support is represented inside the same relational architecture. When the label structure λ_Σ or an application-defined subcomplex identifies legal, educational, organizational, economic, symbolic, or technical institutions, the corresponding institutionally typed support family can be written as $\text{Supp}_\Sigma^{\text{inst}}(\mathcal{P}^*) \subseteq \text{Supp}_\Sigma^{\text{cond}}(\mathcal{P}^*)$. The subtype is introduced only when the application supplies the institutional typing. An institution is therefore not added as an external variable detached from GR spacetime; it is represented through historically organized relational structure already carried by the configuration.

Relational support and institutional support are analytically distinct. Both are represented through relational structures. Institutional analysis selects codified roles, rules, and organized supports as a specific class. Institutional support can be implemented through relations among persons, artifacts, rules, infrastructures, and symbolic classifications. Conversely, relations that are not institutionally codified can remain decisive for generative realization. The preservation specification must therefore state which support classes are retained and which transformations are allowed within them.

Remark 8.2 (Support and continuous realization). Preservation of a condition-support family does not require uninterrupted exercise of the corresponding capacity. It concerns retention or transformation of the structures selected as enabling conditions. Current execution remains a separate status determined by the coupling between capacity and conditions.

Remark 8.2 keeps condition preservation distinct from the executable-capacity status of Section 7.4. A supporting institution can remain available while a capacity is temporarily unused, and a capacity can remain dormant even while some supporting structures survive.

8.3 Environmental-Material Conditions

Some generative processes depend materially on environments that are not well described as institutional support. Ecological knowledge can depend on continued access to a landscape and to recurrent interactions with species, seasons, soils, or water systems. Craft capacity can depend on material properties, tools, and workspaces. Linguistic practices can depend on places, embodied routines, communication media, and shared activities through which expressions acquire situated significance. These conditions belong to the preservation taxonomy when an application establishes their relation to generative realization.

The existing support construction can represent these structures without imposing a universal boundary between system and environment. Let τ_{cond} be an application-defined typing map on the retained condition-support family. When defined, it can classify selected support regions by roles such as relational, institutional, environmental, material, technical, or other domain-specific types. The typed

condition-support profile is stated in Equation 60.

$$\tau_{\text{cond}} : \text{Supp}_{\Sigma}^{\text{cond}}(\mathcal{P}^*) \dashrightarrow \mathcal{Y}_{\text{cond}}, \quad (60)$$

where $\mathcal{Y}_{\text{cond}}$ is the model-defined condition-type space. Equation 60 does not require the types to be mutually exclusive. A school can simultaneously function as an institution, a material site, a technical infrastructure, and a recurrent social environment. A landscape can also acquire institutional or symbolic organization. The typing records analytical roles, while the underlying condition target remains relationally embedded.

This representation has two consequences for preservation. First, relocation can preserve some condition roles while changing their concrete carriers. A linguistic community can recreate domains of interaction after migration without reproducing the earlier physical environment. Second, some condition roles can be weakly substitutable while others can depend on historically specific relations that are difficult to reconstruct. The taxonomy therefore assigns substitution, necessity, and adequacy to application-specific comparison relations. Environmental or material identity becomes preservation-relevant only when selected by the specification.

Caution 8.3 (External-condition language). The distinction between a generative capacity and its conditions is analytical. A condition represented as external at one resolution can become part of the internal relational organization of a finer-grained history-bearing entity. Claims about internal and external conditions should therefore state the entity boundary and resolution used by the model.

Caution 8.3 is especially important for language and culture. Socially learned competence can appear as an individual capacity at one scale while the relations through which that competence is reproduced appear only at a wider community scale. The preservation taxonomy permits both descriptions when their boundary conventions remain explicit.

8.4 Local Condition Fields

A distributed condition target can sometimes admit a field-like representation. Such a representation is useful when generative realization depends on locally varying support conditions that must be compared across regions. Following the algebra-first and field-realization discipline used elsewhere in GR, the field structure is introduced only after local condition records and compatibility maps have been specified.

For each retained region R in a condition domain $\mathcal{R}_C(K_{\Sigma})$, let $V_C(R)$ be a model-defined local carrier for condition data. A local realization map is stated in Equation 61.

$$\Xi_R^C : V_C(R) \dashrightarrow \mathcal{C}_{\Sigma}(R), \quad \Xi_R^C(c_R) = C_{\Sigma, \kappa_{\text{cond}}, \varrho}^{\text{gen}}(R \mid \omega_{\Sigma}), \quad (61)$$

where $\mathcal{C}_{\Sigma}(R)$ denotes the class of well-typed local condition records admitted by the selected model. A datum $c_R \in V_C(R)$ becomes a local condition-field value only relative to the realization map in Equation 61.

For nested regions $R_1 \subseteq R_2$, suppose the carrier architecture supplies a partial restriction map $\sigma_{R_2 R_1}^C : V_C(R_2) \dashrightarrow V_C(R_1)$. Compatible local values satisfy Equation 62.

$$\sigma_{R_2 R_1}^C(c_{R_2}) = c_{R_1}. \quad (62)$$

When overlapping regions also satisfy the declared overlap-consistency rule, a compatible family of local condition data can be written as Equation 63.

$$\mathbf{C}_{\Sigma} := \{c_R\}_{R \in \mathcal{R}_C(K_{\Sigma})}. \quad (63)$$

Equation 63 defines a local condition-field representation only under the realization and compatibility assumptions just stated. The notation does not assert that generative conditions form a physical field

or that every condition target decomposes into local values. Failure of restriction or overlap compatibility is a formal obstruction to the selected field representation and can indicate irreducible contextual dependence at the chosen regional decomposition.

The field form is useful for preservation because a later system can preserve a distributional or relational pattern of enabling conditions without preserving each local carrier identically. A preservation specification can compare condition fields through exact restriction-compatible isomorphism, covariant transport, approximate regional agreement, or another relation warranted by the application. The specific comparison belongs to the preservation specification; the field definition supplies only the coherence conditions for the distributed representation.

8.5 Condition Change across Historical Extension

Generative conditions are historically variable. New institutions can arise, technologies can change, environments can be transformed, and a community can accumulate additional relations that alter the role of earlier supports. A preservation theory that requires identity of condition records across historical extension would therefore classify ordinary historical development as preservation failure even when generative realization remains robust.

Let $\eta \in \text{Hist}_h(X_\Sigma, X'_{\Sigma'})$ be an admissible historical witness and let $C_\Sigma^{\text{gen}}(R)$ and $C_{\Sigma'}^{\text{gen}}(R')$ abbreviate the corresponding local condition targets under fixed declared bases and resolutions. A condition transport is a partial map of the form stated in Equation 64.

$$T_{\eta,R}^C : C_\Sigma^{\text{gen}}(R) \dashrightarrow C_{\Sigma'}^{\text{gen}}(R'). \quad (64)$$

The map in Equation 64 need not preserve every component. Its role is to carry the source condition information retained by the preservation specification into the target comparison carrier. The declared comparison relation r_C then determines whether the transported source and target conditions agree at the required level.

Definition 8.4 (Generative-condition preservation). Let $\mathfrak{P}$ be a preservation specification with target class $\theta = \theta_{\text{cond}}$. A pair of history-indexed generative-condition targets exhibits generative-condition preservation when the required historical witness and condition transport are defined, the retained condition content satisfies the declared comparison relation r_C , and the selected preservation-mode predicate satisfies the structural-preservation criterion of Definition 4.5.

Definition 8.4 permits several preservation strengths. Exact retention can require a condition transport that preserves the selected record isomorphically. Covariant preservation can permit the concrete support structure to change while preserving its declared relational role. Functional condition preservation can compare the generative consequences supported by two generative-condition profiles while permitting their internal composition to differ. Approximate preservation can tolerate bounded differences under a declared discrepancy. The taxonomy does not identify these relations unless bridge assumptions have been supplied.

Historical extension can also increase the condition structure. Suppose the later system contains the transported earlier support together with newly generated condition-relevant structure. Such an extension can preserve the earlier target while enlarging the range or robustness of generative realization. Conversely, new conditions can displace earlier supports while maintaining a preservation-relevant function. Preservation is therefore compatible with accumulation, substitution, and reorganization when the retained-content basis and comparison relation license those transformations.

Remark 8.5 (Condition preservation and frozen context). Generative-condition preservation does not generally require reproduction of the complete historical context of the reference system. The target is the condition structure selected by the preservation specification. A stronger claim of contextual identity requires an independently justified retained-content basis and comparison relation.

Remark 8.5 formalizes the limitation that motivated the present target class. Historical conditions can evolve continuously, and an expanding relational history cannot be preserved by requiring every later configuration to remain identical to an earlier one.

8.6 Capacity-Condition Coupling

The separation of capacity and condition is analytically useful, although realized generativity depends on their coupling. Preserving a capacity grammar while losing its enabling conditions can produce dormancy. Preserving a condition structure while losing the relevant capacity grammar can leave the same environment unable to generate the earlier operation family. A further difficulty remains: even componentwise preservation of capacity and condition need not preserve their joint realization when the rule coupling them changes.

Let $\mathcal{G}_R$ denote the class of local capacity targets admitted at region R , let $\mathcal{C}_R$ denote the corresponding class of local condition targets, and let $\mathcal{U}_R^{\text{op}}$ be a declared ambient carrier containing every operation type that can appear in the compared local capacity targets at the adopted resolution. The ambient carrier is bookkeeping structure; a particular capacity target still selects its own represented operation family. A local capacity–condition realization map is stated in Equation 65.

$$J_R : \mathcal{G}_R \times \mathcal{C}_R \dashrightarrow 2^{\mathcal{U}_R^{\text{op}}}, \quad (65)$$

where $2^{\mathcal{U}_R^{\text{op}}}$ is the family of operation-type subsets on the common comparison carrier. For a capacity target Γ_R and condition target C_R , the value $J_R(\Gamma_R, C_R)$ is the operation family that the selected model treats as executable or otherwise generatively realizable under their joint configuration. Equation 65 is derived at the analytical interface between the capacity statuses of Section 7 and the condition structures of the present section. It is partial because some pairs can lack a well-defined joint realization.

The current executable repertoire from Section 7.4 is a specialization of Equation 65 when the target records retain the schema definedness, operation domains, and admissibility relations required to evaluate execution. The joint-map formulation adds one important possibility: the same capacity and condition descriptions can interact differently after the coupling architecture changes.

Suppose historical witness η supplies a capacity transport U_η^Γ , a condition transport T_η^C , and an operation-family transport $U_\eta^\mathcal{P}$. Capacity–condition coupling is preservation-compatible when Equation 66 holds on the retained domain.

$$U_\eta^\mathcal{P}(J_R(\Gamma_R, C_R)) \text{ } r_{\mathcal{P}} \text{ } J_{R'}(U_\eta^\Gamma(\Gamma_R), T_\eta^C(C_R)). \quad (66)$$

The relation $r_{\mathcal{P}}$ in Equation 66 is declared by the application. It can compare operation sets, typed transition families, realizability profiles, or another generative consequence. The equation states a compatibility requirement; it does not assume that the joint realization factorizes into independent actions of capacity and condition.

Proposition 8.6 (Componentwise preservation and joint generativity). *Preservation of a generative-capacity target and preservation of a generative-condition target do not, by themselves, imply preservation of the jointly realized generative repertoire. Such an implication requires a bridge condition such as the coupling compatibility in Equation 66.*

Proof. The two componentwise preservation claims constrain the transported capacity and condition targets under their respective comparison relations. They do not constrain the target-side joint map $J_{R'}$. Two historically connected systems can therefore contain preservation-equivalent capacity and condition components while assigning different executable or realizable operation families to the same corresponding pair. Equation 66, or another declared bridge relation on the joint realization, is required to exclude that case. $\square$

Proposition 8.6 supplies a non-reduction result for the present taxonomy. Preservation of capacity and preservation of condition can both hold while their joint realization changes. The relational rule that makes those components jointly operative can itself be historically generated and can therefore become preservation-relevant.

Table 8: Generative-condition preservation objects and their principal inferential boundaries.

Condition object	Preservation-relevant content	Inference not established by the object alone
Local condition record	Selected condition-relevant projection of a history-bearing configuration	Exhaustive specification of all enabling relations
Condition-support family	Regions whose declared modification can alter realization of retained capacities	Minimality, necessity, or unique causal sufficiency of each support
Typed institutional, environmental, and material support	Analytical roles assigned to support regions	Scale-independent internal/external boundary or mutual exclusivity of types
Local condition field	Compatible distributed condition data with realization and restriction structure	Physical field ontology or universal local decomposability
Historical condition preservation	Retained condition content under historical transport and declared comparison	Identity of concrete context or preservation of capacity
Capacity-condition coupling	Joint rule connecting capacity and condition targets to realized operations	Preservation from componentwise capacity and condition equivalence alone

Example 8.7 (Finite coupling countermodel). Let the source and target regions each admit one capacity target g , one condition target c , and one operation type p . Let the historical transports identify g , c , and p across the two regions, and let the component comparison relations be equality. Capacity preservation and condition preservation therefore both succeed. Define the two joint realization maps by Equation 67.

$$J_R(g, c) = \{p\}, \quad J_{R'}(g, c) = \emptyset. \quad (67)$$

Equation 67 leaves both component targets unchanged while changing their jointly realized repertoire. The example therefore witnesses Proposition 8.6 in the smallest finite carrier needed for the distinction. Coupling compatibility in Equation 66 excludes precisely this pattern when a positive joint-generativity lift is claimed.

Example 8.7 makes the coupling obstruction explicit without introducing additional target types: the preserved components are identical, and the preservation-relevant change occurs only in the joint realization rule.

Table 8 summarizes the condition structures introduced in this section. Their common function is to make explicit that the environment of generation is itself a history-bearing preservation target. The analysis also establishes the limit of the present target class: conditions can be preserved, transformed, substituted, or redistributed while the historical lineage through which a later system inherits or reconstructs those conditions remains a separate question. The next section therefore turns from condition structure to transmission, descent, interruption, reconstruction, and recoverability across extended histories.

9 Historical-Lineage Preservation

This section develops preservation claims whose primary target is the historically organized lineage through which a later relational formation is connected to an earlier one. Its role is to distinguish preserva-

tion of genealogy, transmission, relocation, reconstructive provenance, recoverability, and branching descent from preservation of a contemporaneous configuration, capacity grammar, or generative-condition profile. The objective is to formalize historical continuity without assuming uninterrupted identity, to represent interruption and reconstruction without retroactively converting them into continuous descent, and to distinguish ordinary recoverability from the stronger case in which the mechanisms of recovery must themselves be regenerated. The discussion proceeds from historical tracking and genealogical corridors to transmission, relocation, interruption, capacity and condition recoverability, recovery-mechanism regeneration, and branching descendants. The method is formal and model-relative. Historical lineage is represented through typed witness composition and provenance structure; it does not receive an ultimate ontological interpretation.

9.1 Genealogical Continuity in Historical Tracking

Section 3.2 introduced historical witnesses as the structures that make preservation comparisons historically meaningful, and Section 4.3 distinguished continuation, relocation, reconstruction, and branching modes. Historical-lineage preservation promotes the provenance relation itself to the preservation target. The relevant question combines endpoint retained-content comparison with identification of a defensible chain or network of historical descent connecting them.

Let $\mathfrak{S} = (I, \preceq_I)$ be a model-defined partially ordered family of relational slices retained for a lineage analysis. For each $i \in I$, let X_i denote the target tracked at that slice when such a target is defined. For comparable indices $i \preceq_I j$, let $\text{Tr}_{ij}(X_i, X_j)$ denote the application-defined class of historical tracking witnesses that identify X_j as a continuation, transformed descendant, relocation, or other historically connected successor of X_i under the declared mode. A lineage corridor over a selected subdomain $I_0 \subseteq I$ is stated in Equation 68.

$$\mathcal{L}_{I_0}(X) := (\{X_i\}_{i \in I_0}, \{\eta_{ij} \in \text{Tr}_{ij}(X_i, X_j)\}_{i \preceq_I j}), \quad (68)$$

where the witness family retained in Equation 68 is required only on the comparison pairs selected by the local model. The construction does not require a total order, a witness between every pair of slices, or literal identity of the tracked target.

A lineage corridor becomes compositionally coherent when successive tracking witnesses compose consistently with the longer-range witness supplied by the model. For $i \preceq_I j \preceq_I k$, the condition is stated in Equation 69 whenever all displayed compositions are defined.

$$\eta_{jk} \circ \eta_{ij} \text{ r}_{\text{hist}} \eta_{ik}. \quad (69)$$

The relation r_{hist} in Equation 69 is a declared witness-equivalence relation. Exact equality can be appropriate when tracking maps are canonical. Other applications can require agreement only on retained provenance, boundary, or identity-supporting data.

Definition 9.1 (Historically tracked lineage). A historically tracked lineage is a lineage corridor of the form in Equation 68 whose retained witness compositions satisfy the declared coherence relation in Equation 69 on the comparison domain selected by the model.

Definition 9.1 gives historical continuity a positive formal carrier. It also separates lineage from endpoint equivalence. Two endpoint targets can be structurally equivalent while no tracked lineage connects them. Conversely, a well-documented lineage can persist while the endpoint target changes substantially under a retained structural comparison.

Proposition 9.2 (Endpoint equivalence and genealogical continuity). *A retained-content equivalence between two endpoint targets does not, by itself, establish historically tracked lineage in the sense of Definition 9.1. A lineage claim requires independently supplied tracking witnesses and their declared coherence.*

Proof. The endpoint comparison constrains the retained representations of the two targets. It contains no witness object of the type required by Equation 68. Two independently generated targets can therefore satisfy the same endpoint equivalence while lacking any admissible historical tracking relation. The missing provenance cannot be derived from endpoint equivalence alone. $\square$

Proposition 9.2 places the historical-lineage target on the same inferential footing as earlier target classes. Structural similarity and genealogy can support one another in a preservation specification, yet neither substitutes for the other.

9.2 Transmission Structures

Historical descent often depends on mechanisms that carry capacities, practices, distinctions, records, or conditions into later relational configurations. Intergenerational language transmission, apprenticeship, institutional training, scientific education, ritual participation, and technical replication provide different substantive examples. The taxonomy treats these cases through a transmission structure only after the application identifies what is transmitted, between which historical carriers, and with what effect on the later target.

Let $\mathcal{U}_i$ denote the selected transmission-relevant content at slice i . Depending on the inquiry, $\mathcal{U}_i$ can contain capacity schemas, condition records, manifested traces, procedural distinctions, or combinations of these. A local transmission operation between historically comparable slices is a partial map of the form stated in Equation 70.

$$\tau_{ij} : \mathcal{U}_i \dashrightarrow \mathcal{U}_j, \quad i \preceq_I j. \quad (70)$$

The partiality in Equation 70 records that some source content can fail to have a meaningful transmitted counterpart. Transmission can also be many-stage, distributed across several participants or institutions, and mediated by artifacts or infrastructures. The map therefore represents the retained transmission relation while allowing execution to be distributed across multiple participants, institutions, artifacts, or infrastructures.

The relation between transmission and preservation requires a later target effect. A transmission operation is preservation-relevant when its image enters the retained structure used to identify or realize the later preservation target. This condition is stated in Definition 9.3.

Definition 9.3 (Transmission relevance). A transmission operation τ_{ij} is preservation-relevant for target class θ when: (i) it is supported by an admissible historical witness between the relevant slices; (ii) its transmitted image is identifiable in the later history; and (iii) that image contributes to the retained content, capacity, condition, or other structure on which the later preservation specification for θ is evaluated.

Definition 9.3 avoids treating the presence of a record, lesson, or communication event as sufficient evidence of successful preservation. A transmission attempt can occur without establishing later uptake, and later uptake can transform what was transmitted. Historical lineage can therefore preserve a genealogy of transformation, with exact-copy continuity appearing only as a special case.

Transmission can also be recursive. A later participant can become a new transmitter, an institution can revise the curriculum through which it reproduces itself, and a language community can transmit practices for generating new expressions in addition to a fixed lexicon. These cases motivate the later distinction between recovery of a capacity and regeneration of a mechanism capable of producing or restoring capacities.

9.3 Relational Relocation

Relational continuity can cross a change of surrounding spacetime. A linguistic community can migrate, an institution can move into a new jurisdiction, a craft practice can be transplanted into a different material environment, and a technical system can be re-instantiated on different infrastructure. These cases

preserve some historical relations while replacing substantial portions of the generative conditions identified in Section 8. The relocation mode h_{reloc} therefore requires a historical relation that is weaker than inclusion of one complete condition structure into another.

Let K_Σ and $K'_{\Sigma'}$ be the source and target GR spacetime representations. A relocation witness is represented by a tuple of the form stated in Equation 71.

$$\eta_{\text{reloc}} := (\rho_K, \rho_X, \rho_\partial, \rho_H), \quad (71)$$

where ρ_K records the model-defined correspondence between retained carrier material, ρ_X tracks the preservation target, ρ_∂ tracks declared interfaces or boundaries, and ρ_H records the provenance relation connecting the relocation event to the accumulated history. The components can be partial and need not form an isomorphism of complete complexes.

Definition 9.4 (Relational relocation). A historical transition from $X_\Sigma \subseteq K_\Sigma$ to $X'_{\Sigma'} \subseteq K'_{\Sigma'}$ is a relational relocation relative to a preservation specification when an admissible witness of the form in Equation 71 connects the retained target and provenance data across the change of surrounding GR spacetime, while allowing declared parts of the environmental, institutional, material, or relational condition structure to differ.

Definition 9.4 makes relocation distinct from both ordinary continuation and reconstruction. It retains an active historical passage of the target through a changed relational environment. Reconstruction, developed next, permits a stronger interruption in which the target is no longer continuously instantiated across the selected history.

The distinction is important for generative preservation. A capacity can migrate successfully while some earlier conditions disappear, new conditions arise, and the capacity itself changes through adaptation. Relocation can therefore support historically continuous transformation without satisfying strong preservation of either grammar or conditions.

9.4 Interruption and Reconstruction

Historical preservation claims often concern systems whose active lineage has been interrupted. A language can lose fluent speakers before later revitalization efforts, a craft can cease ordinary practice while tools and records survive, or an institution can be dissolved and later reconstituted from legal, archival, and personal continuities. The reconstruction mode h_{recon} represents these cases without treating the interruption as if continuous active retention had occurred.

Let $\Sigma_a \prec \Sigma_b \prec \Sigma_c$ mark three retained slices. An interruption interval for target class θ is present when the reference target is identified at Σ_a , no target satisfying the selected active-continuation criterion is identified over the retained interval containing Σ_b , and a later candidate target appears at Σ_c . The pattern is stated schematically in Equation 72.

$$X_{\Sigma_a} \in \mathcal{T}_\theta(\Sigma_a), \quad \mathcal{T}_\theta^{\text{act}}(\Sigma_b) = \emptyset, \quad \tilde{X}_{\Sigma_c} \in \mathcal{T}_\theta(\Sigma_c). \quad (72)$$

The empty active-target set in Equation 72 is relative to the declared model and observation window. It does not establish metaphysical nonexistence of every possible remnant. Surviving traces, partial capacities, institutions, environmental supports, or archival structures can remain available even while the active target criterion fails.

A reconstruction witness must therefore state how the later candidate depends on surviving historical material. Let $\mathcal{R}_{a:c}$ denote the retained reconstruction resources connecting the earlier and later slices. These resources can include traces, partial practitioners, transmitted fragments, institutional records, material artifacts, or historically connected condition structures. A reconstruction witness is represented in Equation 73.

$$\eta_{\text{recon}} = (\mathcal{R}_{a:c}, \text{RecProc}_{a:c}, \zeta_{a:c}), \quad (73)$$

where $\text{RecProc}_{a:c}$ is the declared reconstruction procedure or process family and $\zeta_{a:c}$ records the provenance relation by which the procedure and resources generate or support the later candidate target.

Definition 9.5 (Reconstructive lineage). A later target $\tilde{X}_{\Sigma_c}$ has reconstructive lineage from X_{Σ_a} when an interruption pattern of the form in Equation 72 is retained, an admissible reconstruction witness of the form in Equation 73 is available, and the later target satisfies the retained-content comparison required by the preservation specification under historical mode h_{recon} .

Definition 9.5 does not classify reconstruction as weaker or stronger than every continuous-lineage claim. The two modes preserve different historical information. A reconstruction can recover a highly similar grammar while lacking uninterrupted social transmission. A continuously transmitted system can preserve lineage while changing its grammar substantially. Their relation depends on the target and retained content, and the taxonomy encodes no universal ranking between them.

9.5 Recoverability of Generative Capacity

Reconstruction describes an achieved later relation. Recoverability concerns a possibility available before such recovery has occurred. The distinction is central to the preservation taxonomy because strong preservation can fail while the system remains inside a historically reachable region from which a relevant generative capacity can be restored.

Fix a reference capacity target Γ^* and a declared capacity comparison relation r_Γ . For a configuration ω , let $\hat{\Gamma}^{\text{cap}}(\omega)$ denote the enriched capacity target when it is defined. The capacity-recovery target class is defined in Equation 74.

$$\mathfrak{T}_{\text{rec}}^\Gamma(\Gamma^*) := \left\{ \omega \mid \hat{\Gamma}^{\text{cap}}(\omega) \ r_\Gamma \ \Gamma^* \right\}. \quad (74)$$

Equation 74 is comparison-relative. The target class can encode exact grammar recovery, functional recovery, dynamical action equivalence, or another declared capacity relation.

Let $\mathcal{H}_h^+(\omega)$ denote the family of finite admissible historical continuations or reconstructive paths whose initial configuration is ω under historical mode h . For $\gamma \in \mathcal{H}_h^+(\omega)$, write $\text{end}(\gamma)$ for its terminal configuration. Capacity recoverability is defined by reachability of the class in Equation 74. The recovery region is stated in Equation 75.

$$\text{Rec}_h^\Gamma(\Gamma^*) := \left\{ \omega \mid \exists \gamma \in \mathcal{H}_h^+(\omega) \text{ with } \text{end}(\gamma) \in \mathfrak{T}_{\text{rec}}^\Gamma(\Gamma^*) \right\}. \quad (75)$$

Definition 9.6 (Recoverable generative capacity). A reference capacity Γ^* is recoverable from configuration ω relative to (h, r_Γ) when $\omega \in \text{Rec}_h^\Gamma(\Gamma^*)$ in the sense of Equation 75.

Definition 9.6 separates three statuses. A capacity can be presently executable, dormant while its grammar remains represented, or absent from the current capacity target while remaining recoverable through an admissible historical path. Recoverability is therefore a weaker mode than continuous operational availability and a different mode from achieved reconstruction.

Recoverability is also path-relative. One admissible history can restore a target capacity while another closes the relevant route. A preservation claim about recoverability must therefore state the admissible history family and the conditions under which the recovery path is available.

9.6 Recoverability of Generative Conditions

Generative conditions can also be recoverable, although their historical character makes exact restoration especially demanding. A former institution can be rebuilt, an ecological relation can sometimes be restored, a technical infrastructure can be re-established, and an interactional setting can be approximated. Yet a later condition structure necessarily enters a longer history than the earlier one, and some relations can be irreversibly lost. The condition-recovery relation should therefore be defined at the retained level. Equality of complete context constitutes a separate and substantially stronger claim.

Fix a reference condition target C^* and comparison relation r_C . The condition-recovery target and recovery-region constructions are stated in Equations 76 and 77.

$$\mathfrak{T}_{\text{rec}}^C(C^*) := \left\{ \omega \mid C^{\text{gen}}(\omega) \ r_C \ C^* \right\}, \quad (76)$$

$$\text{Rec}_h^C(C^*) := \left\{ \omega \mid \exists \gamma \in \mathcal{H}_h^+(\omega) \text{ with } \text{end}(\gamma) \in \mathfrak{T}_{\text{rec}}^C(C^*) \right\}. \quad (77)$$

Equations 76 and 77 define the condition recovery target and its recovery region. Here $C^{\text{gen}}(\omega)$ abbreviates the condition target extracted under fixed declared basis and resolution when the projection is defined.

Definition 9.7 (Recoverable generative conditions). A reference condition target C^* is recoverable from configuration ω relative to (h, r_C) when $\omega \in \text{Rec}_h^C(C^*)$.

Definition 9.7 makes condition recoverability explicit through the region in Equation 77. Condition recoverability need not imply capacity recoverability. Restoring a school, habitat, laboratory, communication medium, or institutional domain does not restore a lost competence by itself. The converse also fails: a reconstructed capacity can be realized in conditions that differ substantially from the earlier ones. The capacity–condition coupling of Section 8.6 is therefore also relevant to recovery claims.

A stronger joint-recovery claim can be posed by defining a target class for triples (Γ, C, J) containing capacity, condition, and their realization coupling. The present section leaves that composite target to the cross-taxonomy synthesis because requiring recovery of all three components can be substantially stronger than either capacity or condition recoverability alone.

9.7 Recovery-Mechanism Regeneration

A natural attempt to weaken recoverability is to require only the ability to recover the ability to recover. Under a fixed transition structure, however, this formulation adds no new reachability content. The distinction becomes substantive only when the family of recovery operations or transition rules can itself change historically. This subsection makes that condition explicit.

Let $\Omega_{\mathcal{F}}$ be the configuration space on which a fixed family $\mathcal{F}$ of admissible transitions acts, let $T \subseteq \Omega_{\mathcal{F}}$, and let $\text{Rec}_{\mathcal{F}}(T)$ denote the backward-reachable region of T under the reflexive-transitive finite-path closure generated by $\mathcal{F}$. The fixed-family collapse result is stated in Proposition 9.8.

Proposition 9.8 (Collapse of iterated recoverability under fixed transition structure). *Let $T \subseteq \Omega_{\mathcal{F}}$. If reachability under fixed transition family $\mathcal{F}$ is the reflexive-transitive closure of finite admissible composition, then*

$$\text{Rec}_{\mathcal{F}}(\text{Rec}_{\mathcal{F}}(T)) = \text{Rec}_{\mathcal{F}}(T). \quad (78)$$

Proof. Reflexivity gives $\text{Rec}_{\mathcal{F}}(T) \subseteq \text{Rec}_{\mathcal{F}}(\text{Rec}_{\mathcal{F}}(T))$. For the reverse inclusion, let x reach some $y \in \text{Rec}_{\mathcal{F}}(T)$. By definition, y reaches a target in T . Transitivity of the fixed-family reachability relation composes the two histories, so $x \in \text{Rec}_{\mathcal{F}}(T)$. $\square$

Equation 78 states the fixed-family collapse established by Proposition 9.8. Second-order recoverability therefore cannot be obtained merely by applying the same recovery operator twice. A genuinely different preservation target arises when the available grammar, transition family, or recovery mechanism is itself historically mutable.

Let the extended recovery state be a pair $\Xi = (\omega, \mathcal{F})$ containing the current relational configuration and its represented family of recovery-relevant operations. Let Θ be a family of admissible higher-order historical transformations that can change both components. Define the mechanism-capable class for target T in Equation 79.

$$\mathfrak{M}_{\text{cap}}(T) := \{(\omega, \mathcal{F}) \mid \omega \in \text{Rec}_{\mathcal{F}}(T)\}. \quad (79)$$

A recovery mechanism is regenerable from Ξ_0 when a higher-order history generated by Θ reaches the class in Equation 79. The corresponding region is defined in Equation 80.

$$\text{MetaRec}_{\Theta}(T) := \{\Xi_0 \mid \exists \Gamma \in \mathcal{H}_{\Theta}^+(\Xi_0) \text{ with } \Gamma(\Xi_0) \in \mathfrak{M}_{\text{cap}}(T)\}. \quad (80)$$

Definition 9.9 (Recovery-mechanism regeneration). Recovery-mechanism regeneration for target class T is preservation of membership in, or reachable access to, the higher-order region $\text{MetaRec}_{\Theta}(T)$ when the historical dynamics represented by Θ can alter the recovery-relevant operation family itself.

Definition 9.9 gives formal content to a preservation mode weaker than current recoverability in one respect and stronger in another. The target need not be recoverable under the current operation family, yet the system must retain a historically admissible route by which a recovery-capable mechanism can be generated. The additional structure is therefore historical plasticity of the grammar or recovery architecture, which is distinct from another iteration of fixed reachability.

9.8 Branching Lineage Multiplicity

Historical continuity need not select one unique successor. Languages split, institutions generate successor organizations, practices differentiate into schools or traditions, and reconstructed systems can produce several historically defensible continuations. The branching historical mode h_{branch} therefore treats descendant multiplicity as part of the lineage structure and supports branch-relative preservation claims for several successors.

For reference target X_Σ and later slice family $J \subseteq I$, define the admissible descendant family in Equation 81.

$$\text{Desc}_h(X_\Sigma; J) := \{Y_{\Sigma'} \mid \Sigma' \in J, \text{Hist}_h(X_\Sigma, Y_{\Sigma'}) \neq \emptyset\}. \quad (81)$$

Equation 81 is provenance-based. It does not require every descendant to satisfy the same preservation specification. A later branch can preserve one target class strongly and another weakly, while a second branch exhibits the opposite profile.

Let $\mathfrak{P}_1, \dots, \mathfrak{P}_m$ be a declared family of structural preservation specifications applicable to descendants of X_Σ . The branch preservation profile of a descendant Y is defined in Equation 82.

$$\mathbf{P}_X(Y) := (\text{Pres}_{\mathfrak{P}_1}^{\text{str}}(X, Y), \dots, \text{Pres}_{\mathfrak{P}_m}^{\text{str}}(X, Y)), \quad (82)$$

whenever the component predicates are defined. Undefined components remain explicitly unresolved and retain the value $\perp$.

Definition 9.10 (Branch-relative preservation). A preservation claim is branch-relative when it is evaluated between a reference target and one member of an admissible descendant family such as Equation 81, with the branch-specific historical witness and preservation specification retained in the evaluation.

Branch-relative preservation prevents an early lineage from being treated as if it possesses one uniquely correct later continuation. It also permits several descendants to preserve different aspects of the source. A language can split into descendants that retain different lexical, grammatical, social, or dynamical characteristics. An institution can generate legal and practical successors whose preservation profiles differ by target class. The taxonomy therefore represents multiplicity as a structural feature of lineage through one-to-many branch relations and branch-relative profiles.

Table 9 summarizes the structures developed in this section. Their common role is to make lineage itself available as a preservation target while keeping transmission, relocation, reconstruction, recoverability, and branching formally distinct. The analysis also establishes a limit of lineage-based preservation. A system can retain or regain a defensible lineage while its long-run dynamical organization changes substantially; conversely, a later system can reproduce selected dynamical characteristics without recovering the earlier lineage, grammar, or conditions. The next section therefore turns to dynamical-regime preservation as a weaker and differently typed preservation target.

Table 9: Historical-lineage preservation structures and their principal inferential boundaries.

Lineage structure	Preservation-relevant content	Inference not established by the structure alone
Historically tracked lineage	Coherent family of provenance/tracking witnesses across retained slices	Endpoint structural equivalence or unchanged manifestation
Transmission structure	Historically supported carrying of selected content into later targets	Successful uptake, exact copying, or normative legitimacy
Relational relocation	Active historical passage across changed surrounding spacetime	Preservation of earlier generative conditions in their concrete form
Reconstructive lineage	Later target derived through documented resources and reconstruction procedure after interruption	Uninterrupted continuity or identity of the reconstructed system
Capacity recoverability	Reachability of a preservation-relevant capacity target	Current execution or achieved recovery
Condition recoverability	Reachability of a preservation-relevant condition target	Capacity recovery or restoration of complete historical context
Recovery-mechanism regeneration	Higher-order reachability of a state with a recovery-capable operation family	Recovery under the current grammar or fixed transition structure
Branching descent	Multiple historically admissible descendants with branch-specific preservation profiles	Unique successor identity or one common preservation strength across branches

10 Dynamical-Regime Preservation

This section develops preservation claims whose primary target is a selected organization of dynamics. Manifestations, relational configurations, capacity grammars, generative-condition profiles, and lineages remain distinct target classes. Its role is to formalize the weakest major preservation target introduced by the taxonomy: a later or historically reachable system can preserve a declared dynamical regime even when the concrete structures that realize that regime have changed. The objective is to specify which dynamical characteristics can serve as preservation targets, how those characteristics are extracted and compared, and which inferential limits follow from the coarseness of the selected signature. The discussion proceeds through dynamical signatures, attractor and basin organization, metastability, transition and reachability structure, ordered composition and cycles, algebraic fixed structures, topological diagnostics, critical-response characteristics, and dynamical-structural reproducibility. The method remains representation-relative. Dynamical structures are mathematical resources for preservation analysis and are not assigned ultimate ontological status.

10.1 Dynamical Signatures as Preservation Targets

A dynamical preservation claim requires a carrier on which change is represented and a declared rule for extracting the characteristics to be compared. The carrier can be a configuration space, a family

of generative-possibility structures, a local algebraic net, or another object whose transformations have already been typed by the model. The taxonomy therefore begins with a minimal dynamical record that can be instantiated on several carrier types without assuming one universal phase space.

For a target X_Σ represented at resolution ϱ , let $\mathcal{X}_\Sigma^{(\varrho)}$ denote the model-defined carrier of dynamically comparable states or configurations, and let $\mathcal{F}_\Sigma^{(\varrho)}$ denote the retained family of partial or total transformations acting on that carrier or between its typed subdomains. Let $\mathcal{H}_\Sigma^{+,(\varrho)}$ denote the admissible finite histories generated by well-typed compositions of those transformations. The local dynamical record is stated in Equation 83.

$$\mathbf{D}_\Sigma^{(\varrho)}(X) := (\mathcal{X}_\Sigma^{(\varrho)}, \mathcal{F}_\Sigma^{(\varrho)}, \mathcal{H}_\Sigma^{+,(\varrho)}). \quad (83)$$

Equation 83 is deliberately weaker than a complete dynamical system. It does not require a metric, probability measure, differentiable flow, global time parameter, or one common algebraic carrier. The record collects only the structures required to state the selected dynamical diagnostics.

Let β_{dyn} denote a declared dynamical-signature basis with index set I_β . For each $i \in I_\beta$, let d_i be a partial diagnostic whose domain contains the dynamical records for which that diagnostic is meaningful. The corresponding dynamical-signature map is defined in Equation 84.

$$\mathfrak{D}_{\beta_{\text{dyn}}, \varrho} : \text{Dom}(\mathfrak{D}_{\beta_{\text{dyn}}, \varrho}) \dashrightarrow \prod_{i \in I_\beta} \mathcal{Z}_i, \quad \mathbf{D} \mapsto (d_i(\mathbf{D}))_{i \in I_\beta}. \quad (84)$$

The codomains $\mathcal{Z}_i$ in Equation 84 can be structurally heterogeneous. A selected signature can include attractor-like regimes, basin relations, reachability preorders, cycle-return classes, fixed subalgebras, homological data, response profiles, or other application-defined diagnostics. No aggregation into one scalar is implied.

Definition 10.1 (Dynamical-signature target). A dynamical-signature target is a value $\mathfrak{D}_{\beta_{\text{dyn}}, \varrho}(\mathbf{D})$ of the partial map in Equation 84, together with the declared basis β_{dyn} , resolution ϱ , and comparison relation used to determine preservation-relevant agreement.

Definition 10.1 keeps the preservation claim tied to the information actually retained. A change of signature basis can change the preservation judgment even when the underlying relational history is unchanged. A coarse signature can therefore classify two dynamically distinct systems as equivalent relative to its diagnostic content.

Proposition 10.2 (Non-injective dynamical-signature obstruction). *If $\mathfrak{D}_{\beta_{\text{dyn}}, \varrho}$ is non-injective on a class of dynamical records, equality or declared equivalence of their dynamical signatures does not, by itself, establish identity or preservation of the underlying carrier, capacity grammar, generative conditions, or historical lineage.*

Proof. Non-injectivity supplies distinct dynamical records $\mathbf{D}_1$ and $\mathbf{D}_2$ with the same retained signature. Any structures that distinguish $\mathbf{D}_1$ from $\mathbf{D}_2$ while lying outside the signature basis remain unrecovered by the signature comparison. A stronger preservation conclusion therefore requires an additional bridge from signature agreement to the target class in question. $\square$

Proposition 10.2 is the principal inferential boundary of the present section. Dynamical preservation can be a complete preservation objective for a declared dynamical target while remaining intentionally weaker than preservation of the concrete structures that realize it.

10.2 Attractor-Basin Organization

Attractor language provides one way to describe recurrent or retaining organization without requiring literal state identity. The term has several mathematically substantive definitions in dynamical-systems theory; Milnor's measure-oriented formulation is one prominent example [8]. The present taxonomy

therefore uses *attractor-like regime* only after the local model states the retention or return condition it actually requires.

Let $\mathcal{A} \subseteq \mathcal{X}_\Sigma^{(\varrho)}$ be a model-defined regime and let $\psi_{\mathcal{A}}$ be a partial return–retention schema acting on admissible history families. The retained return predicate is defined in Equation 85.

$$\text{Ret}_{\mathcal{A}}(x) := \psi_{\mathcal{A}}(\mathcal{H}_\Sigma^{+,(\varrho)}(x), \mathcal{A}) \in \{0, 1\}, \quad (85)$$

whenever the schema is defined. The value one means only that the declared model treats the admissible histories from x as satisfying its return or retention criterion relative to $\mathcal{A}$.

The corresponding basin-like domain is stated in Equation 86.

$$\mathcal{B}_\psi(\mathcal{A}) := \left\{ x \in \mathcal{X}_\Sigma^{(\varrho)} \mid \text{Ret}_{\mathcal{A}}(x) = 1 \right\}. \quad (86)$$

Definition 10.3 (Attractor–basin preservation target). An attractor–basin preservation target is the declared dynamical structure

$$\mathfrak{D}_{\text{AB}} := (\mathcal{A}, \mathcal{B}_\psi(\mathcal{A}), \text{Ret}_{\mathcal{A}}), \quad (87)$$

considered at a specified resolution and under an explicit comparison relation for regimes, basin-like domains, and return predicates.

Equation 87, used in Definition 10.3, permits preservation of retaining organization even when the states inside the regime change. A later system can preserve the number, relational arrangement, accessibility, or qualitative role of selected attractor-like regimes while realizing them through different local operations. Conversely, preservation of one regime does not establish preservation of the full dynamical landscape.

Historical comparison can require a transport between source and target dynamical carriers. Let U_η^{dyn} denote such a partial transport under historical witness η . An attractor–basin comparison is well typed only when the transported source regime and the later regime lie in a common declared comparison domain. This requirement prevents similarity of labels such as “stable” or “attractor” from serving as a preservation relation without a mapping between the underlying carriers.

10.3 Metastable Regimes

Some preservation targets concern persistence over a historically relevant scale while permitting later escape, bifurcation, or dissolution. Metastability is useful for this purpose because a language practice, institution, ecological regime, or relational organization can remain persistent over a selected historical window without being invariant over every admissible continuation.

Let $\mathcal{W}$ denote a declared family of historical extensions or perturbation-relative continuations on which persistence is evaluated. Let $m_{\mathcal{A}}$ be a model-defined retention profile with codomain $\mathcal{Y}_m$. The profile is stated in Equation 88.

$$m_{\mathcal{A}} : \mathcal{W} \dashrightarrow \mathcal{Y}_m. \quad (88)$$

The codomain in Equation 88 can contain binary retention statuses, weighted occupancies, recurrence descriptors, or another quantity justified by the local model. No universal lifetime or probability interpretation is assumed.

Let ψ_{meta} be a declared schema that evaluates whether the profile in Equation 88 exhibits sufficient retention on $\mathcal{W}$. Let $\mathcal{E}_{\mathcal{A}}$ denote the application-defined family of admissible escape histories or perturbations that leave the retained regime. Metastability is defined in Definition 10.4.

Definition 10.4 (Metastable preservation target). A regime $\mathcal{A}$ is metastable relative to $(\mathcal{W}, \psi_{\text{meta}})$ when $\psi_{\text{meta}}(m_{\mathcal{A}}) = 1$ and the model admits a nonempty escape family $\mathcal{E}_{\mathcal{A}}$ under the larger continuation or perturbation scope retained for the analysis. The corresponding preservation target is the selected retention profile together with the declared escape structure.

Definition 10.4 makes the historical scale part of the preservation claim. Two regimes can preserve comparable metastable organization even when one persists for a different clock duration, provided the comparison relation is defined on the history scale actually retained by the model. A stronger duration-preservation claim requires an independently justified temporal coordinate and comparison rule.

Metastable preservation can therefore survive substantial internal change. The preserved characteristic can be a corridor of retention, a pattern of return after disturbance, or an organization of escape channels. This target is especially useful when requiring fixed-point or asymptotic invariance would exceed the empirical or historical scope of the inquiry.

10.4 Transition-Reachability Structure

Generative dynamics can be preserved at the level of possible transitions even when local operation labels or internal implementations change. This subsection represents that target through one-step transition and finite reachability relations.

For a carrier $\mathcal{X}$ and retained transformation family $\mathcal{F}$, define the one-step transition relation in Equation 89.

$$x \rightsquigarrow_{\mathcal{F}} y \iff \exists \Phi \in \mathcal{F} \text{ such that } x \in \text{Dom}(\Phi) \text{ and } \Phi(x) = y. \quad (89)$$

Let $\rightsquigarrow_{\mathcal{F}}^*$ denote the reflexive transitive closure of Equation 89. The finite reachable set from x is defined in Equation 90.

$$\text{Reach}_{\mathcal{F}}(x) := \{y \in \mathcal{X} \mid x \rightsquigarrow_{\mathcal{F}}^* y\}. \quad (90)$$

The reachability target can retain the full directed transition relation, only its transitive closure, selected reachability classes, or a coarser quotient under a declared equivalence. A comparison relation r_{reach} can therefore be based on graph isomorphism, simulation, bisimulation, preorder equivalence, or another relation whose assumptions have been stated by the application.

Definition 10.5 (Reachability preservation). A historical comparison exhibits transition–reachability preservation when the selected source and target transition structures are connected by the required historical witness, the relevant transition or reachability objects are defined at the declared resolutions, and they satisfy the comparison relation r_{reach} specified by the preservation claim.

Definition 10.5 permits a later system to preserve the organization of accessible futures while changing the mechanisms that implement individual transitions. The converse also fails: preservation of many local operation labels does not guarantee preservation of reachability when their domains or compositions have changed.

10.5 Cycle Structure of Ordered Composition

Reachability records whether transitions are possible, while ordered composition records how sequences of transformations act. Historical order can matter when the first operation changes the domain, carrier, or state on which a later operation is instantiated. The resulting compositional structure can therefore serve as a preservation target independently of the individual local maps.

Let $\gamma = (\Phi_1, \dots, \Phi_n)$ be a finite composable path in the retained dynamical record. Its ordered composite is defined in Equation 91.

$$\Phi_{\gamma} := \Phi_n \circ \dots \circ \Phi_2 \circ \Phi_1. \quad (91)$$

Equation 91 is defined only on the subset for which each intermediate output lies in the domain of the next transformation. Failure of one ordering while another remains defined is itself a sequence-sensitive feature.

A closed cycle requires a return to a retained base carrier. When the terminal carrier differs representationally from the initial carrier but the preservation model supplies a closure identification ζ_c , the cycle return map is stated in Equation 92.

$$A_c := \zeta_c \circ \Phi_{\gamma_c}. \quad (92)$$

The closure identification in Equation 92 is part of the cycle specification and cannot be inferred from endpoint similarity alone.

Let $\mathcal{C}_\beta$ be a declared family of preservation-relevant cycles and let $[A_c]_{r_c}$ denote the comparison class of each return map under the selected cycle relation. The cycle signature is defined in Equation 93.

$$\mathfrak{C}_\beta := \{[A_c]_{r_c} \mid c \in \mathcal{C}_\beta\}. \quad (93)$$

Definition 10.6 (Cycle-structure preservation). Cycle-structure preservation holds relative to a declared cycle family when historically comparable systems admit a well-typed correspondence among the retained cycles and their return maps satisfy the declared cycle comparison relation after the required transport or closure identifications.

Definition 10.6 allows preservation of mesoscopic compositional organization even when isolated edge maps differ. Noncommutative diagnostics can be added only when both orderings are defined on a common algebraic or linear domain. A nonzero commutator then records an order effect in that representation; it is not assumed merely because the system is historically complex.

10.6 Algebraic Diagnostics of Fixed Structures

An operator-algebraic realization can supply preservation targets that summarize what a family of transformations leaves invariant. These targets become available only when the relevant local or quasi-local algebras and algebra maps have already been defined.

Let $\mathcal{A}$ be a retained algebra and let $\alpha_c : \mathcal{A} \rightarrow \mathcal{A}$ be an algebra endomorphism representing the return action of cycle c . The fixed structure of one return map is defined in Equation 94.

$$\text{Fix}(\alpha_c) := \{a \in \mathcal{A} \mid \alpha_c(a) = a\}. \quad (94)$$

For a declared cycle family $\mathcal{C}_\beta$, the common fixed structure is stated in Equation 95.

$$\mathcal{F}_{\text{cyc}} := \bigcap_{c \in \mathcal{C}_\beta} \text{Fix}(\alpha_c). \quad (95)$$

When every α_c is a unital algebra endomorphism, the object in Equation 95 is a fixed subalgebra. In weaker map classes it should be interpreted only as the common fixed set justified by the model.

Additional algebraic diagnostics can include conjugacy classes of return maps, spectra in finite-dimensional realizations, invariant subspaces, or commutators of selected transformations. Let Π_{alg} denote the partial extractor that assembles the subset of these diagnostics warranted by the application. The resulting algebraic signature is stated in Equation 96.

$$\mathfrak{D}_{\text{alg}} := \Pi_{\text{alg}}(\mathcal{A}, \{\alpha_c\}_{c \in \mathcal{C}_\beta}). \quad (96)$$

Equation 96 is intentionally partial. Spectral data require a suitable representation, commutators require common composable domains, and fixed-subalgebra language requires sufficient algebraic closure.

Remark 10.7 (Algebraic preservation and map identity). Preservation of $\mathcal{F}_{\text{cyc}}$, a spectrum, or another algebraic diagnostic does not generally imply equality of the transformations that generated it. The diagnostic is the preservation target only when the preservation specification explicitly selects that level of information.

Remark 10.7 keeps the algebraic target aligned with Proposition 10.2. System-level invariants can be preservation-relevant without functioning as lossless encodings of the underlying dynamics.

10.7 Topological-Homological Characteristics

The labelled 2-complex used as a GR spacetime representation also permits topological diagnostics of the carrier on which the dynamics is organized. Such diagnostics can enter a dynamical preservation signature when cycles, connectivity, or admissible transitions depend on the topology of the represented complex. They remain representation-level objects and should not be identified with the ontology of the preserved system.

For a finite complex K , let $C_q(K)$ denote its q -th chain group with boundary map $\partial_q : C_q(K) \rightarrow C_{q-1}(K)$ satisfying $\partial_q \circ \partial_{q+1} = 0$. The homology group is defined in Equation 97.

$$H_q(K) := \ker \partial_q / \text{im } \partial_{q+1}. \quad (97)$$

Let $\beta_q(K)$ denote the corresponding Betti number when the coefficient choice makes the homology finite-dimensional, and let $\chi(K)$ denote the Euler characteristic when defined. For a selected degree set Q , a topological diagnostic signature is stated in Equation 98.

$$\mathfrak{T}_Q(K) := (\{H_q(K)\}_{q \in Q}, \{\beta_q(K)\}_{q \in Q}, \chi(K)), \quad (98)$$

with components omitted when they are outside the declared model.

A stronger historical comparison becomes available when a carrier transformation induces a chain map $f_\# : C_\bullet(K) \rightarrow C_\bullet(K')$. The induced maps on homology are written as Equation 99.

$$f_{*,q} : H_q(K) \longrightarrow H_q(K'). \quad (99)$$

Isomorphism of selected induced maps can support a stronger preservation claim than equality of Betti numbers alone. When no chain map or historical correspondence is defined, the comparison should remain at the level of the selected diagnostics.

Caution 10.8 (Topological diagnostic scope). Agreement of selected homology groups, Betti numbers, Euler characteristic, or other incomplete invariants does not establish general topological equivalence of the represented carriers. A preservation claim should state the invariant actually retained and the historical or representational map, if any, that supports its comparison.

Caution 10.8 is especially important for dynamical-structural preservation. A later system can pre-serve selected topological characteristics while changing finer incidence, labels, local maps, or relational histories that the chosen invariant does not record.

10.8 Critical-Response Characteristics

A dynamical signature can also preserve how a system responds to perturbation. High susceptibility, slow recovery, amplification, structural transition, and continuing reconfigurability are different response characteristics and need not coincide. The taxonomy therefore treats critical-response preservation through an explicit perturbation family and a declared response extractor; criticality labels are meaningful only after those structures have been specified.

Let $\Delta_\beta(X)$ be a model-specified perturbation family for the dynamical target X , and let Resp_β be a partial response map. The response family is defined in Equation 100.

$$\mathcal{P}_\beta^\Delta(X) := \{\text{Resp}_\beta(\delta \mid X) \mid \delta \in \Delta_\beta(X), \text{Resp}_\beta(\delta \mid X) \text{ is defined}\}. \quad (100)$$

A partial diagnostic map Π_{crit} can then extract the response characteristics actually used by the model. The resulting critical-response signature is stated in Equation 101.

$$\mathfrak{D}_{\text{crit}}(X) := \Pi_{\text{crit}}(\mathcal{P}_\beta^\Delta(X)). \quad (101)$$

Depending on the application, Π_{crit} can retain susceptibility-like response, recovery descriptors, amplification, transition margins, or combinations of these. Each component requires its own assumptions and scale.

Definition 10.9 (Critical-response preservation). Critical-response preservation holds relative to a declared perturbation family and diagnostic extractor when historically comparable dynamical targets yield signatures of the form in Equation 101 that satisfy the selected comparison relation after the required transport between their response spaces.

Definition 10.9 does not assert that the systems are physically critical or that one universal criticality threshold exists. It preserves only the response organization selected by the model. This scope is compatible with preservation of near-critical, metastable, or transition-sensitive regimes without converting those terms into universal GR properties.

10.9 Dynamical-Structural Reproducibility

The preceding subsections permit the weakest major preservation proposal developed in this paper. The original grammar, generative conditions, local operators, and even parts of the represented carrier can change, while a later system or a historically reachable descendant reproduces a declared dynamical signature. The preservation claim remains stronger than bare dynamical equivalence because a GR preservation specification also retains the historical or provenance relation by which the later realization is connected to the source.

Fix a reference dynamical signature $\mathfrak{D}^*$ and a declared comparison relation r_D . The realized dynamical target class is defined in Equation 102.

$$\mathfrak{T}_{\text{dyn}}(\mathfrak{D}^*) := \{ \omega \mid \mathfrak{D}_{\beta_{\text{dyn}}, \varrho}(\mathbf{D}(\omega)) \ r_D \ \mathfrak{D}^* \}, \quad (102)$$

whenever the signature map is defined. Equation 102 can represent strict dynamical equivalence, approximate signature agreement, preservation of selected invariant classes, or another relation justified by the application.

For historical mode h , define the dynamical reproduction region in Equation 103.

$$\text{DynRec}_h(\mathfrak{D}^*) := \{ \omega \mid \exists \gamma \in \mathcal{H}_h^+(\omega) \text{ with } \text{end}(\gamma) \in \mathfrak{T}_{\text{dyn}}(\mathfrak{D}^*) \}. \quad (103)$$

Equation 103 is a reachability construction whose target is a dynamical signature. Particular grammars and generative-condition profiles remain separate targets.

Definition 10.10 (Dynamical-structural reproducibility preservation). Let X_Σ supply reference signature $\mathfrak{D}^*$ and let $Y_{\Sigma'}$ be a historically or reconstructively connected later target under mode h . Dynamical-structural reproducibility preservation holds relative to $(\beta_{\text{dyn}}, \varrho, r_D, h)$ when the preservation specification retains the required provenance witness and either: (i) $Y_{\Sigma'}$ realizes a dynamical record in $\mathfrak{T}_{\text{dyn}}(\mathfrak{D}^*)$; or (ii) the later configuration remains inside $\text{DynRec}_h(\mathfrak{D}^*)$ under a preservation mode that targets continued reproducibility even when current realization is absent.

Definition 10.10 distinguishes realized reproduction from preserved reproducibility. The second case permits a later system to lack the reference regime at the active slice while retaining a historically admissible route to reproduce it. This is weaker than preservation of the original grammar or conditions, yet it remains stronger than an analyst discovering an unrelated dynamical analogue.

Proposition 10.11 (Provenance boundary of dynamical reproduction). *Agreement of dynamical signatures between two systems does not constitute preservation in the present taxonomy when no historical, reconstructive, relocation, branching, or other declared provenance witness connects the later system to the reference target. In that case the formal result is dynamical equivalence or analogy relative to the selected signature.*

Proof. The structural preservation architecture of Section 4 requires a historical mode and an admissible historical witness in addition to target comparison. Signature agreement supplies the comparison component but does not supply the missing provenance component. The preservation predicate is therefore under-specified until a declared history relation is established. $\square$

Table 10: Dynamical-regime preservation targets and their principal inferential boundaries.

Dynamical target	Preservation-relevant structure	Inference not established by the target alone
Dynamical signature	Declared family of heterogeneous diagnostics at a selected resolution	Identity of the underlying carrier or completeness of the retained dynamics
Attractor–basin organization	Regime, basin-like domain, and explicit return/retention schema	Classical attractor structure under an unstated metric or measure
Metastable regime	Retention profile over a declared historical scale and admissible escape structure	Eternal invariance or a universal lifetime
Transition–reachability structure	One-step transitions, finite reachability, or selected quotient thereof	Equality of local mechanisms or preservation of operation labels
Cycle structure	Ordered path composites and selected return-map classes	Equality of individual interface maps or absence of sequence sensitivity
Algebraic fixed structure	Fixed sets/subalgebras, selected spectra, conjugacy classes, or commutators when defined	Identity of the maps generating those diagnostics
Topological signature	Selected homology, Betti, Euler, or induced-map information	General topological equivalence from incomplete invariants
Critical-response signature	Declared perturbation-response characteristics at stated scale	Universal criticality status or one domain-independent threshold
Dynamical-structural reproducibility	Current or historically reachable reproduction of the declared signature with provenance	Preservation of the original grammar, conditions, lineage detail, or relational configuration

Proposition 10.11 prevents dynamical-structural preservation from collapsing into a free-standing similarity judgment. Together with Proposition 10.2, it also clarifies the weakness of the target: even a fully justified dynamical preservation claim does not automatically establish preservation of manifestation, relational structure, capacity, conditions, or lineage beyond the provenance required by the selected historical mode.

Table 10 summarizes the target classes developed in this section. Their common feature is a shift from preserving the concrete realization to preserving selected organization of change, recurrence, reachability, composition, invariance, topology, or response. The resulting target can be substantially weaker than preservation of capacity or condition while remaining analytically meaningful when the declared objective concerns dynamical behavior. The next section turns from target classes to preservation modes across those targets and asks how continuous retention, transformation, dormancy, reconstruction, regeneration of recovery mechanisms, and dynamical reproduction relate without being forced into one universal strength ordering.

11 Preservation Modes across Target Classes

Sections 5–10 developed six preservation-target families and identified several historically distinct ways in which those targets can remain available, change, disappear, return, or be reproduced. The role of the present section is to synthesize those results at the level of preservation mode. Its objective is to specify the formal burden carried by continuous retention, historically continuous transformation, dormant recoverability, reconstructive recovery, recovery-mechanism regeneration, and dynamical-regime repro-

duction; to identify the target classes on which each mode is naturally defined; and to state the bridge assumptions required before one mode can imply another. The discussion is comparative and formal. Preservation modes organize different relations between history and target realization; they do not form a universal scalar ranking of preservation strength.

The six core mode labels introduced in Equation 16 are retained throughout this section. A mode label therefore abbreviates a partial predicate of the type defined in Equation 17. The present section supplies a more explicit interpretation of those predicates after the target-specific constructions of Sections 5–10 have become available.

To avoid introducing a new primitive target map, the shorthand $\mathsf{T}_{\kappa, \varrho; \Sigma}^{\theta, \ell}$ used in this section denotes the target-specific extraction already supplied by the target typing and retained-content projection $\Pi_{\kappa, \varrho; \Sigma}^{\ell}$ of Section 3.4. When a target-specific section has introduced a more informative extractor, such as a capacity grammar, condition projection, or dynamical-signature map, T denotes that established specialization.

11.1 Continuous Retention

Continuous retention is the preservation mode with the strongest requirement of uninterrupted target availability along the selected historical corridor. The mode does not require literal equality of successive realizations. It requires the preservation-relevant target to remain instantiated at every retained stage and successive realizations to satisfy the declared local comparison relation under a coherent historical witness.

Let $\mathcal{L}_{I_0}(X)$ be a historically tracked lineage corridor in the sense of Equation 68. For target class θ , write $T_i := \mathsf{T}_{\kappa, \varrho}^{\theta, \ell_i}(X_i)$ for the preservation-relevant target extracted at slice i when the extraction is defined. For a declared local witness η_{ij} , use the comparison transport $T_{\kappa, \varrho}^{\eta_{ij}}$ of Equation 8. Let r_{ret} be the local comparison relation selected for retention. The corridor-retention predicate is stated in Equation 104.

$$\begin{aligned} \text{Ret}_{\kappa, \varrho, r_{\text{ret}}}^{\theta}(\mathcal{L}_{I_0}(X)) &= 1 \\ \iff T_i &\text{ is defined for every retained } i \in I_0, \\ \eta_{ij} \in \mathsf{Tr}_{ij}(X_i, X_j) &\text{ satisfies the retained historical-coherence conditions,} \\ T_{\kappa, \varrho}^{\eta_{ij}}(T_i) \mathrel{r_{\text{ret}}} T_j &\text{ for every declared local comparison } i \prec j. \end{aligned} \tag{104}$$

Equation 104 is evaluated only on the local comparison graph selected by the model. A dense historical record can compare many pairs of slices, while a sparse reconstruction can retain only adjacent or otherwise evidenced comparisons.

Definition 11.1 (Continuous-retention mode). A preservation specification is in continuous-retention mode μ_{ret} when its selected target remains instantiated throughout the declared historical corridor and the corridor satisfies Equation 104 under the retained target, comparison, resolution, and provenance criteria.

Definition 11.1 permits gradual representational change. A manifested form can change through a covariant context transport; a relational structure can change while selected relations and interfaces remain continuously trackable; a generative capacity can alter its concrete state-dependent realization while the corresponding operation family remains identifiable. Continuous retention therefore concerns uninterrupted preservation-relevant availability while permitting admissible configuration change.

The mode is nevertheless demanding. If the selected target disappears from the represented history at an intermediate slice, continuous retention fails even when a later reconstruction produces an endpoint that is highly similar to the earlier target. That later case belongs to reconstructive recovery or another mode with an explicit interruption structure.

11.2 Historically Continuous Transformation

Historically continuous transformation retains a compositional lineage while permitting the target itself to change through locally admissible transformations. The mode is useful when the preservation objective

concerns continuity of descent, organization, or function through change across a historically connected sequence of comparisons.

Let $i_0 \prec i_1 \prec \dots \prec i_n$ be a finite retained chain inside a lineage corridor, with target realizations $T_k := T_{\kappa_k, \varrho_k}^\theta(X_{i_k})$. For each adjacent pair, let $\chi_k : T_k \dashrightarrow T_{k+1}$ be the declared target-level transformation induced or witnessed by the historical relation between the corresponding GR configurations. The transformation corridor is stated in Equation 105.

$$T_0 \xrightarrow{\chi_0} T_1 \xrightarrow{\chi_1} \dots \xrightarrow{\chi_{n-1}} T_n, \quad \chi_{n-1} \circ \dots \circ \chi_0 \text{ defined on the retained source domain.} \quad (105)$$

The arrows in Equation 105 are formal maps with declared domains; they are not shorthand for an ordinary-language sequence. Each χ_k can represent covariant manifestation change, relational reorganization, capacity adaptation, condition change, transmission-mediated revision, or another target-specific transformation whose typing has already been established.

Definition 11.2 (Historically continuous transformation mode). A preservation specification is in historically continuous transformation mode μ_{trans} when the target admits a corridor of the form in Equation 105, the corresponding GR historical witnesses remain compositionally coherent, and every retained local transformation satisfies the continuity conditions declared for the target class even though a fixed source-to-each-slice retention comparison need not hold.

Definition 11.2 makes the distinction from continuous retention structural. Continuous retention asks whether the selected target remains preservation-equivalent under a fixed retained criterion across the corridor. Historically continuous transformation asks whether the changing target remains connected through admissible descendant transformations whose local continuity is itself preservation-relevant.

A conditional relation between the two modes can be stated when the same retained-content basis and a transitive comparison relation apply throughout the corridor.

Proposition 11.3 (Local retention and endpoint comparison). *Assume a transformation corridor of the form in Equation 105 in which every adjacent pair satisfies one common transitive comparison relation r on one common retained target type. Then the source and endpoint satisfy $T_0 r T_n$.*

Proof. The adjacent comparisons give $T_0 r T_1$, $T_1 r T_2$, and so forth through $T_{n-1} r T_n$. Repeated transitivity yields $T_0 r T_n$. $\square$

Proposition 11.3 gives one route from local continuity to endpoint comparison. Endpoint comparison alone does not reconstruct the intervening transformation corridor, as already established for relational lineage in Section 6. Historically continuous transformation therefore preserves information that an endpoint equivalence can omit.

11.3 Dormant Recoverability

Dormant recoverability addresses a different pattern: the preservation-relevant target remains historically represented, while its active or executable realization is temporarily unavailable and a declared route to renewed realization remains open. The mode is most naturally defined for targets that possess a meaningful distinction between representation and activation, including generative capacities and some condition or institutional structures. It remains undefined for target classes lacking such a distinction.

For target class θ , let $\text{Act}_\theta(T, \omega) \in \{0, 1\}$ be a partial activation predicate supplied by the local model. For a generative capacity, $\text{Act}_\theta = 1$ can specialize to executable availability of the kind represented by $E_{p, R, \Sigma}^{\text{exec}}$. For an institutional support condition, activation can concern present operation of the support relation. The predicate has no universal semantics outside the target-specific model.

Fix a reference target T^* and comparison relation r_θ . For a history γ , write $T_\gamma := T^\theta(\text{end}(\gamma))$ whenever the target extraction is defined, using the endpoint notation established in Section 9.5. The

active recovery region is defined in Equation 106.

$$\text{ActRec}_h^\theta(T^*) := \left\{ \omega \mid \begin{array}{l} \exists \gamma \in \mathcal{H}_h^+(\omega), \quad T_\gamma \text{ is defined,} \\ T_\gamma r_\theta T^*, \quad \text{Act}_\theta(T_\gamma, \text{end}(\gamma)) = 1 \end{array} \right\}. \quad (106)$$

Equation 106 asks for reachability of an active realization of the preservation-relevant target. It is therefore stronger than the mere statement that a dormant schema remains represented.

Definition 11.4 (Dormant recovery). A preservation specification is in dormant-recoverability mode μ_{dorm} at configuration ω when: (i) the selected target remains historically identified and satisfies the declared retained-content relation to the reference target; (ii) its target-specific activation predicate is defined and equals zero at ω ; and (iii) $\omega \in \text{ActRec}_h^\theta(T^*)$ under the declared historical mode and recovery family.

Definition 11.4 separates dormant recoverability from two neighboring statuses. Dormancy without condition (iii) leaves later activation unresolved. Recoverability after the target grammar or target structure itself has disappeared belongs to the reconstructive or ordinary recovery constructions of Section 9. The dormant mode therefore retains the target while relaxing continuous active realization.

This mode also clarifies why continuous active use is an unnecessarily strong preservation criterion in many generative systems. A language variety can retain speakers and a learnable grammar while lacking a current institutional domain of use; a scientific technique can remain competently represented while temporarily unused; an institutional procedure can remain legally and technically available while no current event activates it. Whether such cases satisfy dormant recoverability depends on the target-specific activation and recovery structure; non-use alone is insufficient.

11.4 Reconstructive Recovery

Reconstructive recovery applies after an interruption in the preservation-relevant target or its active lineage. The mode combines the reconstruction historical mode h_{recon} with achieved recovery of a declared target. Its formal burden is therefore greater than endpoint similarity: the later realization must be historically connected to inherited material through a reconstruction witness, and the recovered target must satisfy the retained comparison criterion.

Let X_{Σ_a} be the reference target, let Σ_b denote an interruption domain in which the selected target no longer satisfies the required continuity condition, and let $\tilde{X}_{\Sigma_c}$ be the later reconstructed target. Denote by $\eta_{ac}^{\text{recon}} \in \text{Hist}_{h_{\text{recon}}}(X_{\Sigma_a}, \tilde{X}_{\Sigma_c})$ the declared reconstruction witness. For target extractor T^θ and comparison relation r_{rec} , achieved reconstructive recovery is stated in Equation 107.

$$\eta_{ac}^{\text{recon}} \text{ is admissible,} \quad T^\theta(\tilde{X}_{\Sigma_c}) r_{\text{rec}} T^\theta(X_{\Sigma_a}). \quad (107)$$

Definition 11.5 (Reconstructive recovery). A preservation specification is in reconstructive-recovery mode μ_{rec} when an interruption of the selected target or continuity criterion is retained in the modeled history, an admissible reconstruction witness connects the later realization to the inherited source material, and Equation 107 holds at the recovered target.

Definition 11.5 concerns achieved recovery. It should therefore remain distinct from membership in a recovery region such as $\text{Rec}_h^\Gamma(\Gamma^*)$ or $\text{Rec}_h^C(C^*)$, which states only that a suitable recovery history remains available. The distinction is analogous to the difference between reachable and reached states in a transition system.

Reconstructive recovery and historically continuous transformation are generally incomparable. The former records an interruption followed by a provenance-supported reconstruction. The latter records an uninterrupted transformation corridor. A reconstruction can recover a highly similar capacity after lineage discontinuity, while a continuous lineage can transform so substantially that the endpoint fails a stringent reconstruction-equivalence criterion.

11.5 Recovery-Mechanism Regeneration

Recovery-mechanism regeneration moves the preservation target one level upward. The relevant structure is no longer only a capacity, condition, or other target to be recovered; it includes the historically mutable family of operations by which recovery can become possible. Section 9.7 established that iterated recoverability collapses under a fixed reflexive and transitive transition structure. The mode is therefore distinctive only when the recovery grammar or recovery architecture itself can change.

Let T be the target class to be recovered, let $\Xi = (\omega, \mathcal{F})$ be the extended recovery state, and let Θ be the higher-order transformation family introduced in Equation 80. The mode predicate is naturally evaluated through the higher-order region $\text{MetaRec}_\Theta(T)$.

Definition 11.6 (Recovery-mechanism-regeneration mode). A preservation specification is in recovery-mechanism-regeneration mode μ_{meta} when the selected extended state Ξ belongs to $\text{MetaRec}_\Theta(T)$ under a historically admissible higher-order transformation family, and the provenance structure identifies the regenerated recovery architecture as a descendant or reconstruction of the preservation-relevant source system under the declared historical mode.

Definition 11.6 allows present recoverability to fail. The mode can hold when $\omega \notin \text{Rec}_{\mathcal{F}}(T)$ while a higher-order history changes $\mathcal{F}$ into some $\mathcal{F}'$ for which the later configuration enters $\text{Rec}_{\mathcal{F}'}(T)$. The preserved object is therefore historical plasticity of the recovery architecture.

A conditional implication from ordinary recoverability to meta-recoverability can be obtained only after the higher-order model includes the current recovery-capable state as an admissible endpoint. One sufficient condition is stated in Proposition 11.7.

Proposition 11.7 (Recoverability under identity access). *Suppose Θ admits a zero-change or identity higher-order history at every extended state. Let $\mathfrak{M}_{\text{cap}}(T)$ be the mechanism-capable class of Equation 79. If $\omega \in \text{Rec}_{\mathcal{F}}(T)$, then $(\omega, \mathcal{F}) \in \text{MetaRec}_\Theta(T)$.*

Proof. The premise $\omega \in \text{Rec}_{\mathcal{F}}(T)$ places $(\omega, \mathcal{F})$ in $\mathfrak{M}_{\text{cap}}(T)$. The admitted identity higher-order history reaches that same mechanism-capable state, so the extended state belongs to $\text{MetaRec}_\Theta(T)$ by Equation 80. $\square$

The converse of Proposition 11.7 fails in the intended cases: meta-recoverability can hold precisely because a currently inadequate recovery family can later be transformed into a recovery-capable one. The proposition therefore records a conditional embedding of ordinary recoverability into the higher-order mode and leaves the reverse implication open to the specified recovery dynamics.

11.6 Dynamical-Regime Reproduction

Dynamical-regime reproduction is the core mode whose preservation target is a selected organization of dynamics. The concrete grammar, conditions, or relational carrier that originally realized the regime belong to separate target classes. Section 10 supplied the signature map $\mathfrak{D}_{\beta_{\text{dyn}, \mathcal{Q}}}$, the target class $\mathfrak{T}_{\text{dyn}}(\mathfrak{D}^*)$, and the reachability region $\text{DynRec}_h(\mathfrak{D}^*)$. The present subsection identifies their mode-level role across the taxonomy.

Two statuses should remain visible. A later target can currently realize the declared dynamical signature, or it can preserve a route from which that signature remains reproducible. Define the reproduction-status pair in Equation 108.

$$\mathbf{R}_{\text{dyn}}(\omega; \mathfrak{D}^*) := \left(\mathbf{1}_{\mathfrak{T}_{\text{dyn}}(\mathfrak{D}^*)}(\omega), \mathbf{1}_{\text{DynRec}_h(\mathfrak{D}^*)}(\omega) \right), \quad (108)$$

where each indicator is evaluated only when the corresponding target and reachability structures are defined. Equation 108 distinguishes realized reproduction $(1, 1)$ from latent reproducibility $(0, 1)$; the pair $(0, 0)$ records failure of both retained statuses under the selected model.

Definition 11.8 (Dynamical-regime-reproduction mode). A preservation specification is in dynamical-regime-reproduction mode μ_{dyn} when the selected provenance witness connects the later configuration to the reference system and the later configuration has reproduction status $(1, 1)$ or $(0, 1)$ in Equation 108, according to whether the local specification requires current reproduction or continued reproducibility of the declared signature.

Definition 11.8 deliberately permits changes in the lower-level realization. The later system can use different operation schemas, different generative conditions, different local algebraic carriers, or a structurally modified 2-complex while preserving or regaining the selected dynamical signature. Such freedom is the reason this mode can be weaker than capacity or condition preservation. It is also the reason the signature, comparison relation, resolution, and provenance witness must remain explicit.

Dynamical-regime reproduction should not be conflated with reconstructive recovery. Reconstruction seeks a target related to inherited source material under a reconstruction history and a declared target comparison. Dynamical reproduction can accept a substantially different realization when its selected dynamical organization satisfies the preservation specification. One reconstructed target can therefore fail dynamical reproduction, while another historically transformed system can reproduce the relevant regime without reconstructing the earlier grammar.

11.7 Implication Boundaries among Preservation Modes

The six core preservation modes differ in the historical information they retain and in the target structures required for their evaluation. Their relations therefore form a domain-dependent implication network with no built-in total ordering. Equation 21 already defines implication between complete structural specifications. For mode-level comparison, fix target class θ , historical domain $\mathcal{D}$, and a declared bridge-assumption family $\mathcal{B}$. The corresponding mode implication is stated in Equation 109.

$$\begin{aligned}
\mu_a &\implies_{\theta, \mathcal{D}, \mathcal{B}} \mu_b \\
&\iff \forall (X, Y) \in \mathcal{D}, \\
&\quad \left[\text{App}_{\mathfrak{p}_a}^{\text{pres}}(X, Y) = 1 \wedge \text{App}_{\mathfrak{p}_b}^{\text{pres}}(X, Y) = 1 \wedge \mathcal{B}(X, Y) = 1 \wedge \text{Pres}_{\mathfrak{p}_a}^{\text{str}}(X, Y) = 1 \right] \\
&\implies \text{Pres}_{\mathfrak{p}_b}^{\text{str}}(X, Y) = 1.
\end{aligned} \tag{109}$$

where $\mathcal{B}(X, Y) = 1$ abbreviates satisfaction of the assumptions required to align the target extractors, history witnesses, retained contents, resolutions, and comparison relations of the two specifications. The use of complete structural specifications in Equation 109 is deliberate: the grounded mode predicate of Equation 17 depends on a witness and cannot be compared coherently at pair level after the witness information has been discarded.

Several general boundaries follow from the constructions above.

Proposition 11.9 (Absence of a universal total order on core preservation modes). *The formal definitions of μ_{ret} , μ_{trans} , μ_{dorm} , μ_{rec} , μ_{meta} , and μ_{dyn} do not induce a universal total implication order across the target classes of the present taxonomy.*

Proof. It is enough to exhibit one pair of core modes whose definitions permit countermodels in both implication directions. Consider continuous retention and dormant recoverability on a target class with a defined activation predicate. A history in which the target remains instantiated and active at every retained stage can satisfy continuous retention while failing dormant recoverability because the later target is not inactive. Conversely, a history in which the target remains represented, is inactive at the later slice, and has an admissible path to renewed activation can satisfy dormant recoverability while failing continuous retention whenever the selected corridor contains an interval in which the retention criterion is not satisfied. Thus neither mode universally implies the other. A universal total implication order would have to compare this pair in at least one direction, so no such order follows from the core definitions alone. The other constructions in this section provide further incomparabilities, including current

Table 11: Core preservation modes across target classes. The modes retain different historical information and therefore do not form a universal strength hierarchy.

Mode	Minimum preservation structure	Characteristic historical status	Principal information not established by the mode alone
Continuous retention	Target instantiated throughout a tracked corridor with local retained-content comparison	Uninterrupted availability	Literal identity, unchanged manifestation, or normative value
Historically continuous transformation	Coherent descendant transformations with local continuity criteria	Uninterrupted transformation lineage	Fixed endpoint equivalence under a stronger comparison
Dormant recoverability	Historically retained inactive target plus reachable active realization	Temporary non-actualization without target loss	Continuous active use or recovery after loss of the target grammar itself
Reconstructive recovery	Interruption, reconstruction witness, and achieved target comparison	Recovery after represented discontinuity	Uninterrupted descent or restoration of every earlier condition
Recovery-mechanism regeneration	Higher-order path to a recovery-capable operation family	Recovery architecture itself changes historically	Current recoverability or achieved target recovery
Dynamical-regime reproduction	Current or reachable reproduction of a declared dynamical signature with provenance	Realization can change substantially	Preservation of original grammar, conditions, fine structure, or complete lineage

recoverability versus recovery-mechanism regeneration and dynamical reproduction versus lower-level target preservation. □

Proposition 11.9 does not prevent local implication results. Proposition 11.3 gives one under transitivity; Proposition 11.7 gives another under higher-order identity access. Additional applications can prove further implications after the relevant target and bridge structures have been fixed.

Table 11 summarizes the six modes and their principal formal burdens. The table is comparative, and row position carries no preservation-strength ordering.

Table 11 also clarifies why preservation assessment should eventually be profile-based. One historical system can exhibit continuous retention of a lineage, dormant recoverability of one capacity, loss of one enabling condition, and reproduction of a selected dynamical regime at the same later slice. A second system can exhibit a different combination. The next section therefore moves from preservation modes considered individually to cross-taxonomy lifting and typed preservation profiles, where results across target classes can be assembled without collapsing them into one scalar preservation score.

12 Cross-Taxonomy Lifting and Preservation Profiles

Sections 5–11 established preservation targets and modes whose formal burdens differ by object type, historical structure, resolution, and comparison relation. The role of the present section is to organize re-

lations among those branches without converting the taxonomy into a universal hierarchy. Its objective is to define cross-target lifting through explicit bridge assumptions, to identify recurrent obstructions to such lifting, to extend the preservation analysis across representational resolutions, and to assemble results into typed preservation profiles. The discussion proceeds from manifestation-to-relational and relational-to-generative lifting to capacity-condition coupling, historical-to-dynamical lifting, refinement, counterexample classes, and profile construction. The method is formal and comparative. A lift is licensed only on a declared domain and through an independently stated bridge; preservation at one target class therefore remains locally complete even when no stronger cross-target conclusion is available.

Equation 21 defined implication between complete structural preservation specifications on a domain where both are applicable. Cross-taxonomy lifting adds an explicit bridge family because the source and target specifications can concern different mathematical objects. Let $\mathfrak{P}_a$ and $\mathfrak{P}_b$ be two structural preservation specifications, and let $\mathcal{D}_{ab}$ be a declared class of pairs contained in $\text{Dom}(\text{Pres}_{\mathfrak{P}_a}^{\text{str}}) \cap \text{Dom}(\text{Pres}_{\mathfrak{P}_b}^{\text{str}})$. The same symbols (X, Y) index the broader formations from which the source and target specifications can extract different typed preservation targets; they do not require the two specifications to compare identical mathematical carriers. A bridge family $\mathcal{B}_{a \Rightarrow b}$ contains the additional maps, identifications, sufficiency conditions, evidential relations, or compatibility assumptions required to connect the source target class to the target class. Its satisfaction on a pair is written $\mathcal{B}_{a \Rightarrow b}(X, Y) = 1$ only on the declared bridge domain. The bridge-qualified lift is stated in Equation 110.

$$\begin{aligned} \mathfrak{P}_a &\xrightarrow[\mathcal{B}_{a \Rightarrow b}]{\mathcal{D}_{ab}} \mathfrak{P}_b \\ \iff &\quad \forall (X, Y) \in \mathcal{D}_{ab}, \\ &[\text{Pres}_{\mathfrak{P}_a}^{\text{str}}(X, Y) = 1 \wedge \mathcal{B}_{a \Rightarrow b}(X, Y) = 1] \implies \text{Pres}_{\mathfrak{P}_b}^{\text{str}}(X, Y) = 1. \end{aligned} \tag{110}$$

Equation 110 uses $\mathcal{B}_{a \Rightarrow b}(X, Y) = 1$ as shorthand for satisfaction of the declared bridge family on the pair. The bridge can contain several typed conditions and need not reduce to one Boolean test in a concrete implementation. The notation records that cross-target implication has acquired additional premises beyond the source preservation result.

Definition 12.1 (Cross-target preservation lift). A cross-target preservation lift from $\mathfrak{P}_a$ to $\mathfrak{P}_b$ is a relation of the form in Equation 110 on a declared domain $\mathcal{D}_{ab}$ contained in the domains of both structural preservation predicates, together with a bridge family whose assumptions are sufficient to connect the source target, historical witness, retained content, resolution, and comparison structure to those required by the target specification.

Definition 12.1 makes lifting a theorem- or assumption-bearing relation that requires an explicit bridge between target classes. Sections 12.1–12.4 instantiate this discipline for recurrent transitions in the taxonomy.

12.1 Manifestation-to-Relational Lifting

Manifestational preservation supplies context-indexed presentations, archival traces, multi-context profiles, or corpora. Relational-structural preservation concerns historically organized relations and interfaces. Proposition 5.4 and Equation 35 already establish that the first target class does not generally determine the second. A positive lift therefore requires information showing that the retained manifestation distinguishes, or otherwise determines at the declared resolution, the relational differences relevant to the target specification.

Let $M_{\kappa_m, \varrho}$ denote the retained manifestational extractor on a comparison domain $\mathcal{D}_{mr}$, and let $S_{\kappa_r, \varrho}$ denote the relational-target extractor. Let r_m and r_r be the corresponding comparison relations. A manifestation-to-relational discriminating bridge is stated in Equation 111.

$$M_{\kappa_m, \varrho}(X) \ r_m \ M_{\kappa_m, \varrho}(Y) \implies S_{\kappa_r, \varrho}(X) \ r_r \ S_{\kappa_r, \varrho}(Y), \quad (X, Y) \in \mathcal{D}_{mr}. \tag{111}$$

Equation 111 is stronger than successful manifestation comparison on one observed pair. It states that, on the declared lifting domain, the retained manifestational comparison is sufficiently discriminating for the relational content selected by κ_r . An application can establish the bridge through injectivity on a restricted fiber, additional provenance, multi-context triangulation, an independently verified reconstruction map, or another domain-specific argument.

Proposition 12.2 (Manifestation-to-relational lifting under a discriminating bridge). *Let $\mathfrak{P}_{\text{man}}$ and $\mathfrak{P}_{\text{rel}}$ be applicable on $\mathcal{D}_{mr}$. Suppose their historical witnesses and resolutions are compatible and Equation 111 holds for the retained manifestational and relational targets. Then manifestational preservation lifts to relational-structural preservation on $\mathcal{D}_{mr}$ whenever the remaining mode predicates of $\mathfrak{P}_{\text{rel}}$ are satisfied by the transported witness.*

Proof. Manifestational preservation supplies a historical witness satisfying the source mode and the relation r_m on the retained manifestation targets. Equation 111 transports that successful comparison to r_r on the relational targets. Compatibility of the historical witness, resolution, and target mode places the same transported history inside the witness class required by $\mathfrak{P}_{\text{rel}}$. Equation 20 then yields relational-structural preservation. $\square$

Proposition 12.2 does not make archival or corpus preservation inherently weak. It identifies the additional burden required when an inquiry seeks to infer preservation of relational structure from a manifestational record. When Equation 111 is unavailable, manifestational preservation remains a complete result relative to its own target class in the sense of Remark 5.6.

12.2 Relational-to-Generative Lifting

Relational continuity can support generative capacity and generative conditions while remaining formally distinct from both. A preserved interface, role structure, or institutional relation does not determine which operation schemas remain available or which environmental supports remain sufficient for generation. Relational-to-generative lifting therefore requires a map or relation showing how the retained relational structure determines the selected generative target at the adopted resolution.

Let $S_{\kappa_r, \varrho}$ be a relational target. For $q \in \{\text{cap}, \text{cond}\}$, let $G_{\kappa_q, \varrho}^q$ be a partial extractor for the corresponding capacity or condition target. A local relational-to-generative bridge has the type stated in Equation 112.

$$G_{\kappa_r \Rightarrow \kappa_q, \varrho}^q : \text{Dom}(G^q) \subseteq S_{\kappa_r, \varrho} \dashrightarrow \mathcal{T}_{\kappa_q, \varrho}^q, \quad (112)$$

where $S_{\kappa_r, \varrho}$ is the retained relational carrier and $\mathcal{T}_{\kappa_q, \varrho}^q$ is the capacity- or condition-target carrier. The map in Equation 112 represents a substantive dependence claim: the selected generative target can be constructed or identified from the retained relational structure on the declared domain.

Fix historical witness η . Let T_η^{rel} denote retained relational transport, and let T_η^q denote the corresponding generative-target transport. Equation 113 states historical compatibility of the bridge.

$$G_{\Sigma'}^q \circ T_\eta^{\text{rel}} \, r_q \, T_\eta^q \circ G_\Sigma^q \quad (113)$$

whenever both routes are defined. Equation 113 allows equality, isomorphism, functional equivalence, or another declared comparison relation r_q according to the target class.

Proposition 12.3 (Relational-generative lifting). *Suppose a relational preservation specification is structurally satisfied by (X, Y) through witness η . Assume that the selected generative target is defined by Equation 112, that the bridge respects the retained relational comparison, and that Equation 113 holds along η . Preservation of the corresponding generative target then follows whenever its mode predicate is satisfied on the transported target.*

Proof. Relational structural preservation supplies an admissible historical witness and a successful comparison on the retained relational targets. By assumption, the partial bridge map G^q is defined on those

targets and respects their comparison relation. Equation 113 identifies, up to r_q , the result of first transporting the relational target and then extracting the generative target with the result of first extracting and then transporting the generative target. The transported witness therefore satisfies the target-side generative comparison. If the generative mode predicate is also defined and satisfied on that grounded witness, Definition 4.5 yields preservation of the selected generative target. $\square$

Proposition 12.3 is conditional by design. A relational system can preserve a social role pattern while competence disappears, or preserve competence-relevant relations while material conditions become inadequate. A successful lift therefore reports an identified dependence between selected relational structure and selected generative content. It does not establish that every generative capacity or condition is reducible to relational structure at the adopted model resolution.

12.3 Capacity-to-Condition Coupling

Sections 7 and 8 deliberately separated capacity from condition. Their combination still does not determine realized generativity because the coupling between the two can change. Cross-taxonomy synthesis therefore requires a derived target for the operation family jointly realized by a capacity–condition pair.

For local capacity target Γ_R and condition target C_R , Section 8.6 defined the joint realization map J_R in Equation 65. Define the derived realized-generativity target in Equation 114.

$$\mathcal{R}_R^{\text{gen}}(\Gamma_R, C_R) := J_R(\Gamma_R, C_R), \quad (114)$$

whenever J_R is defined. Equation 114 does not add a seventh primitive target family to Equation 14. It is a derived cross-taxonomy target used to test whether separately preserved components remain jointly generative.

Let U_η^Γ , T_η^C , and U_η^P be the transports used in Equation 66. A joint-generativity lift is available when capacity preservation, condition preservation, and coupling compatibility hold together. The resulting implication is stated in Proposition 12.4.

Proposition 12.4 (Capacity–condition lift to realized generativity). *Let the capacity and condition preservation specifications be structurally satisfied through historically compatible witnesses. Assume that the capacity–condition realization maps are defined and that Equation 66 holds under the declared operation-family comparison. The derived realized-generativity targets in Equation 114 then satisfy that same comparison.*

Proof. The two componentwise preservation results supply corresponding capacity and condition targets after historical transport. Equation 66 states that applying the earlier joint realization and transporting its operation family agrees, under r_P , with realizing the transported capacity and condition in the later system. The two values in Equation 114 therefore satisfy the declared preservation comparison. $\square$

Proposition 12.4 supplies the positive bridge complementing Proposition 8.6. Componentwise preservation becomes sufficient for the derived generative target only after the coupling relation has been retained. A change in that coupling can therefore be preservation-relevant even when both component targets remain individually preserved.

12.4 Historical-to-Dynamical Lifting

Historical-lineage preservation supplies provenance, descent, transmission, relocation, reconstruction, or branching structure. Dynamical-regime preservation supplies selected organization of transition, recurrence, composition, invariance, topology, or response. Provenance alone does not determine dynamics. A strong lineage can continue while its dynamical organization changes, and a later branch can reproduce an earlier dynamical regime through substantially different local mechanisms.

Let $\eta \in \text{Hist}_h(X, Y)$ be a lineage witness, and let $\mathfrak{D}_X$ and $\mathfrak{D}_Y$ denote the selected dynamical signatures extracted from the corresponding dynamical records. A historical-to-dynamical bridge is represented by a partial signature transport in Equation 115.

$$T_\eta^{\mathfrak{D}} : \text{Dom}(T_\eta^{\mathfrak{D}}) \subseteq \mathcal{Z}_{\text{dyn}, X} \dashrightarrow \mathcal{Z}_{\text{dyn}, Y}, \quad T_\eta^{\mathfrak{D}}(\mathfrak{D}_X) r_D \mathfrak{D}_Y. \quad (115)$$

The map in Equation 115 is additional structure on the lineage. It can be induced by tracked transition graphs, transported cycle classes, continuation-compatible response diagnostics, or another domain-specific construction. A lineage witness that carries no dynamical information need not admit such a map.

Proposition 12.5 (Historical-to-dynamical lifting). *Suppose $\mathfrak{P}_{\text{hist}}$ is satisfied by (X, Y) through witness η . If the selected dynamical signatures are defined, Equation 115 holds, and the dynamical preservation mode required by $\mathfrak{P}_{\text{dyn}}$ is satisfied at Y or through its declared reproduction region, then $\mathfrak{P}_{\text{hist}}$ lifts to $\mathfrak{P}_{\text{dyn}}$ under the bridge containing those assumptions.*

Proof. The satisfied historical specification supplies the provenance witness η . The bridge assumption in Equation 115 transports the selected source signature into the target signature carrier and establishes the declared signature comparison r_D . The remaining premise supplies the mode-specific current-realization or reproducibility condition required by the dynamical specification. Hence the historical witness, retained dynamical comparison, and dynamical mode predicate jointly satisfy the structural-preservation conditions for $\mathfrak{P}_{\text{dyn}}$. $\square$

The converse to Proposition 12.5 remains unavailable in general. Proposition 10.11 requires provenance before dynamical equivalence counts as preservation, while preservation of a dynamical signature can coexist with loss of detailed lineage information. Historical-to-dynamical lifting therefore combines provenance with signature transport; it does not identify those objects.

12.5 Preservation under Refinement

Preservation profiles can change when the representational resolution changes. Section 6.5 established the issue for relational structure through the coarse-graining map Q_{fc} and Proposition 6.5. The same problem arises across the target taxonomy whenever a resolution map suppresses distinctions used by a preservation specification.

Let ϱ_f and ϱ_c denote finer and coarser resolutions, and let Q_{fc}^θ be a declared resolution map on target class θ . For historical witness η , preservation covariance across resolution is represented by Equation 116.

$$Q_{fc, \Sigma'}^\theta \circ T_\eta^{\theta, f} r_c T_\eta^{\theta, c} \circ Q_{fc, \Sigma}^\theta, \quad (116)$$

whenever the target transports and resolution maps are defined. Equation 116 generalizes Equation 43: historical transport and resolution change give preservation-compatible results under the coarse comparison r_c .

A second condition is required for the comparison relation itself. The fine comparison is resolution-compatible downward when Equation 117 holds.

$$a r_f b \implies Q_{fc}^\theta(a) r_c Q_{fc}^\theta(b). \quad (117)$$

Proposition 12.6 (Downward preservation projection). *Let fine and coarse preservation specifications concern the same target family and historical mode. If Equations 116 and 117 hold, and the coarse mode predicate is preserved by the resolution map, then structural preservation at resolution ϱ_f implies structural preservation at resolution ϱ_c on the declared domain.*

Proof. Fine structural preservation supplies a fine-resolution witness whose transported retained targets satisfy r_f and whose fine mode predicate succeeds. Equation 116 transports that witness data consistently through the resolution map. Equation 117 then converts the successful fine comparison into the

required coarse comparison. By assumption, the coarse mode predicate is preserved under the same resolution projection. The projected grounded witness therefore satisfies the complete coarse structural specification. $\square$

Proposition 12.6 establishes only the fine-to-coarse direction under the stated covariance and comparison assumptions. The reverse direction requires a lifting condition. If Q_{fc}^θ is non-injective relative to preservation-relevant distinctions, coarse success can be compatible with several fine targets. Fine preservation can then be recovered only through additional data such as a section, a restricted injectivity condition, a discriminating auxiliary record, or a bridge theorem that controls the relevant fibers. Preservation under refinement is consequently asymmetric in many practical models: fine evidence can project to a justified coarse claim while coarse evidence remains insufficient for a fine claim.

Example 12.7 (Coarse preservation without fine preservation). Let the fine retained-content carrier be $\mathcal{Z}_f = \{a_0, a_1\}$ and the coarse carrier be $\mathcal{Z}_c = \{c\}$. Let the resolution map satisfy $Q_{fc}(a_0) = Q_{fc}(a_1) = c$, and use equality as both the fine and coarse comparison relation. Suppose a historically admissible pair has retained fine targets a_0 and a_1 and satisfies the same mode predicate at both resolutions. The decisive comparison is stated in Equation 118.

$$a_0 \neq a_1, \quad Q_{fc}(a_0) = c = Q_{fc}(a_1). \quad (118)$$

Equation 118 gives defined fine failure together with coarse success. The pair is therefore a concrete countermodel to an unqualified coarse-to-fine implication. A section, restricted injectivity, or other fiber-resolving bridge removes the ambiguity by supplying information that the coarse carrier has discarded.

Example 12.7 gives a finite fiber-collapse model for Proposition 12.6. It also separates defined fine failure from absent fine-resolution applicability.

12.6 Counterexample Boundaries of Cross-Taxonomy Implication

Cross-target lifting is strengthened by explicit records of failure. A failed implication and an undefined target comparison carry different information. The first supplies a counterexample to a proposed lift; the second identifies an applicability obstruction that prevents the proposed implication from being tested on the selected pair.

For structural specifications $\mathfrak{P}_a$ and $\mathfrak{P}_b$ on domain $\mathcal{D}$, define the counterexample class in Equation 119.

$$\text{CE}_{\mathcal{D}}(\mathfrak{P}_a, \mathfrak{P}_b) := \{(X, Y) \in \mathcal{D} \mid \text{App}_{\mathfrak{P}_a}^{\text{pres}}(X, Y) = 1, \quad \text{App}_{\mathfrak{P}_b}^{\text{pres}}(X, Y) = 1, \\ \text{Pres}_{\mathfrak{P}_a}^{\text{str}}(X, Y) = 1, \quad \text{Pres}_{\mathfrak{P}_b}^{\text{str}}(X, Y) = 0\}. \quad (119)$$

where all predicates are evaluated on the displayed pair. A nonempty class in Equation 119 disproves the unqualified implication $\mathfrak{P}_a \implies_{\mathcal{D}} \mathfrak{P}_b$.

The corresponding applicability-obstruction class is stated in Equation 120.

$$\text{UN}_{\mathcal{D}}(\mathfrak{P}_a, \mathfrak{P}_b) := \{(X, Y) \in \mathcal{D} \mid \text{App}_{\mathfrak{P}_a}^{\text{pres}}(X, Y) = 1, \text{Pres}_{\mathfrak{P}_a}^{\text{str}}(X, Y) = 1, \text{App}_{\mathfrak{P}_b}^{\text{pres}}(X, Y) \neq 1\}. \quad (120)$$

Equation 120 does not show that target preservation fails. It records that the target specification lacks the objects, witnesses, comparison relation, resolution, or mode structure required for evaluation.

Several counterexample families are already supplied by the preceding taxonomy. Manifestational collapse can place a pair in $\text{CE}(\mathfrak{P}_{\text{man}}, \mathfrak{P}_{\text{rel}})$. Non-injective coarse-graining can yield coarse relational preservation together with failure of fine relational preservation. A relational lineage can continue while a selected capacity disappears. Capacity and condition preservation can coexist with failure of joint generativity when the coupling changes. Historical lineage can persist while its dynamical signature changes. Conversely, dynamical-structural reproducibility can hold while grammar, condition, or fine relational preservation fails. These patterns are summarized in Table 12.

Table 12: Representative cross-taxonomy lifting relations and recurrent obstruction patterns.

Proposed lift	Bridge required for a positive result	Representative obstruction without the bridge
Manifestation $\Rightarrow$ relational structure	Discriminating manifestation or independent relational reconstruction	Trace or corpus collapse hides relationally distinct sources
Relational structure $\Rightarrow$ generative target	Covariant generative extraction from retained relational content	Stable roles or interfaces coexist with loss of competence or enabling conditions
Capacity + condition $\Rightarrow$ realized generativity	Preserved capacity–condition coupling	Components remain equivalent while executable repertoire changes
Historical lineage $\Rightarrow$ dynamical regime	Signature transport or reproduction bridge along the lineage	Genealogy persists while transition, cycle, attractor, or response structure changes
Coarse $\Rightarrow$ fine preservation	Fiber-resolving lift, section, or restricted injectivity	Coarse-graining collapses fine preservation-relevant distinctions
Dynamical regime $\Rightarrow$ grammar/condition	Identification theorem connecting signature to underlying realization	Non-injective dynamical signatures admit distinct underlying systems

Table 12 gives the lifting analysis a falsifiable form inside a declared model. A proposed bridge can be rejected by finding a pair that satisfies its source preservation specification while failing the target specification under conditions where both are defined. Undefinedness directs attention to missing articulation or evidence.

12.7 Typed Preservation Profiles

The target and mode analyses permit several preservation claims to be evaluated simultaneously on the same historical pair. Their results should retain specification type and definedness. A binary scalar such as “preserved” would erase which target survived, which mode was used, and which comparisons remained unavailable.

Let $\mathbb{V}_{\text{pres}} := \{1, 0, \perp\}$, where 1 denotes structural satisfaction, 0 denotes defined structural failure, and $\perp$ denotes inapplicability or unresolved definedness relative to the proposed specification. For a preservation specification $\mathfrak{P}$, define its typed evaluation in Equation 121.

$$\nu_{\mathfrak{P}}(X, Y) := \begin{cases} 1, & \text{App}_{\mathfrak{P}}^{\text{pres}}(X, Y) = 1 \text{ and } \text{Pres}_{\mathfrak{P}}^{\text{str}}(X, Y) = 1, \\ 0, & \text{App}_{\mathfrak{P}}^{\text{pres}}(X, Y) = 1 \text{ and } \text{Pres}_{\mathfrak{P}}^{\text{str}}(X, Y) = 0, \\ \perp, & \text{App}_{\mathfrak{P}}^{\text{pres}}(X, Y) \neq 1. \end{cases} \quad (121)$$

Equation 121 extends the three-way discipline of Section 4.5 to profile construction. The symbol $\perp$ records unresolved applicability, while defined preservation failure is recorded separately by 0.

Let $\mathfrak{P} = (\mathfrak{P}_1, \dots, \mathfrak{P}_m)$ be an ordered family of declared preservation specifications. The typed preservation profile is defined in Equation 122.

$$\mathbf{Prof}_{\mathfrak{P}}(X, Y) := ((\mathfrak{P}_1, \nu_{\mathfrak{P}_1}(X, Y)), \dots, (\mathfrak{P}_m, \nu_{\mathfrak{P}_m}(X, Y))). \quad (122)$$

The specification labels remain part of Equation 122. Two vectors with the same pattern of zeros and ones can express materially different preservation results when their target classes, historical modes, retained contents, resolutions, or preservation modes differ.

For bookkeeping, the profile can be decomposed into the three specification sets in Equation 123.

$$\begin{aligned}\text{Sat}(X, Y) &:= \{\mathfrak{P}_i \mid \nu_{\mathfrak{P}_i}(X, Y) = 1\}, \\ \text{Fail}(X, Y) &:= \{\mathfrak{P}_i \mid \nu_{\mathfrak{P}_i}(X, Y) = 0\}, \\ \text{Und}(X, Y) &:= \{\mathfrak{P}_i \mid \nu_{\mathfrak{P}_i}(X, Y) = \perp\}.\end{aligned}\tag{123}$$

Equation 123 preserves the distinction between contrary evidence and absent applicability. A language-preservation application, for example, can report successful corpus preservation, failed current capacity preservation, unresolved fine-grained condition preservation, and successful dynamical-regime reproducibility without compressing those results into one score.

Definition 12.8 (Typed preservation profile). A typed preservation profile is the family in Equation 122 evaluated on a declared pair of history-bearing targets. Its interpretation retains the complete structural specifications attached to its components and the three-valued status in Equation 121.

Definition 12.8 supplies the principal synthesis object of the taxonomy. Cross-target lifting can enrich the profile by establishing additional satisfied components, while counterexamples and undefined components remain visible. The profile itself carries no scalar aggregation rule and no ranking of target classes. Any later judgment that one profile represents “better” preservation requires an additional comparison basis and belongs to the judgment structure developed in Section 13.

The separation between structural profile and judgment is especially important when preservation objectives conflict. One intervention can improve trace preservation while weakening living generative conditions; another can support current generative capacity while accepting substantial manifestational change; a reconstruction can restore a dynamical regime while preserving only a partial lineage. The taxonomy records these configurations before aesthetic, epistemic, normative, practical, or political-economic valuation is introduced. Section 13 therefore turns from the structural question of which preservation specifications are satisfied to the distinct question of how preservation criteria and preservation values are formed and assessed.

13 Judgment Structure of Preservation Claims

Sections 4–12 developed preservation as a typed structural inquiry: a declared target, historical mode, retained content, comparison relation, resolution, and preservation mode jointly determine whether a structural preservation specification is applicable and satisfied. The role of the present section is to separate that structural result from the further judgments through which a preservation claim becomes accepted, contested, valued, prioritized, or rejected in a historically situated setting. Its objective is to distinguish preservation identification from preservation valuation, to formalize sufficient similarity without assuming one universal metric, to represent preservation criteria as historically generated structures, and to distinguish aesthetic, epistemic, normative, practical, and political-economic valuation. The method is classificatory and formal. The section introduces judgment maps and judgment spaces as additional structures over the preservation profiles already derived; it does not infer normative obligation or aesthetic value from formal preservation alone.

The distinction is necessary because the structural predicate $\text{Pres}_{\mathfrak{P}}^{\text{str}}$ answers a conditional question: whether a pair satisfies the preservation specification $\mathfrak{P}$ that has already been declared. It does not determine whether $\mathfrak{P}$ is the preservation criterion that a community, legal system, research programme, archive, or engineering project should use. It also does not determine whether successful preservation under that criterion has positive value. The same structurally satisfied profile can therefore support different historically generated judgments, and one judgment context can treat several structurally different profiles as sufficiently preservation-equivalent for its purpose.

13.1 Preservation Identification

Preservation identification concerns the inferential step from structural evidence to a contextually licensed statement that a selected object counts as preserved. The step is distinct from the calculation of $\nu_{\mathfrak{P}}$ in Equation 121. The typed evaluation records whether one already specified preservation relation is satisfied, fails, or remains undefined. Identification judgment selects which structural specifications, evidence, tolerances, and historical burdens are sufficient for the preservation claim being made.

Let $\mathcal{E}_c^{\text{pres}}$ denote a context-indexed evidence space whose elements can contain preservation specifications, typed preservation profiles, provenance witnesses, uncertainty records, resolution information, and other evidence admitted by context c . Let $\mathcal{J}_c^{\text{id}}$ be a judgment space containing the identification statuses permitted by that context. A preservation-identification operation is the partial map stated in Equation 124.

$$J_{\beta,c}^{\text{id}} : \text{Dom}(J_{\beta,c}^{\text{id}}) \subseteq \mathcal{E}_c^{\text{pres}} \dashrightarrow \mathcal{J}_c^{\text{id}}, \quad (124)$$

where β denotes the declared identification basis. The basis can retain, for example, one target class, several profile components, a required historical witness, a tolerance rule, or a hierarchy of evidential burdens. The partiality records that some evidence packages remain insufficiently articulated for the requested identification.

Definition 13.1 (Preservation-identification judgment). A preservation-identification judgment is the result of a partial operation of the type in Equation 124, together with an explicit identification basis β and interpretation of the judgment space $\mathcal{J}_c^{\text{id}}$. The judgment licenses a preservation description only within the historical, evidential, and contextual scope specified by that basis.

Definition 13.1 permits a judgment space richer than a binary decision. A legal register can use recognized and unrecognized statuses; an archive can use preserved, partially preserved, unresolved, and lost; a scientific analysis can retain incomparable preservation profiles; and a community process can leave identification open while evidence remains disputed. The taxonomy therefore does not impose one universal codomain on $J_{\beta,c}^{\text{id}}$.

The separation from structural satisfaction can be stated directly. Let $e_{\mathfrak{P}}(X, Y)$ be an evidence record containing $\mathfrak{P}$ and its typed evaluation. Structural success $\nu_{\mathfrak{P}}(X, Y) = 1$ makes one positive fact available to the identification operation. It does not force $J_{\beta,c}^{\text{id}}(e_{\mathfrak{P}}(X, Y))$ to classify the whole object as preserved when β requires additional target classes, stronger provenance, finer resolution, or a different preservation mode. Conversely, an identification context can adopt a deliberately coarse criterion under which a profile with unresolved fine-grained components still counts as preserved for a stated purpose. The identification basis is therefore an explicit component of the epistemic architecture and requires its own justification.

13.2 Tolerance in Sufficient Similarity

Many preservation judgments concern sufficient continuity or similarity, with literal identity representing only one possible criterion. The formal burden is to state what differences remain admissible for the preservation purpose. A universal metric is unnecessary and often unavailable because preservation targets can be symbolic, relational, categorical, algebraic, topological, or dynamically structured.

Let a and b be comparison objects in a target-specific domain. A discrepancy representation is a partial map of the type stated in Equation 125.

$$\Delta_{\beta} : \text{Dom}(\Delta_{\beta}) \subseteq \mathcal{Z}_{\beta} \times \mathcal{Z}_{\beta} \dashrightarrow \mathcal{D}_{\beta}, \quad (125)$$

where $\mathcal{D}_{\beta}$ is a discrepancy space appropriate to the retained structure. A preservation context c supplies an admissible-discrepancy region $\mathcal{A}_{\beta,c} \subseteq \mathcal{D}_{\beta}$. Sufficient similarity is then represented in Equation 126.

$$a \approx_{\beta,c} b \iff \Delta_{\beta}(a, b) \in \mathcal{A}_{\beta,c}, \quad (126)$$

whenever the discrepancy map is defined.

Equation 126 separates the representation of difference from the acceptance region used by the judgment context. A metric specialization is available when $\mathcal{Z}_\beta$ carries a justified distance d_β . For threshold $\varepsilon_{\beta,c} \geq 0$, the familiar form is stated in Equation 127.

$$a \approx_{\beta,c}^\varepsilon b \iff d_\beta(a, b) \leq \varepsilon_{\beta,c}. \quad (127)$$

The metric form in Equation 127 is one specialization of the more general discrepancy formulation. Structural edit classes, permitted diagrammatic defects, interval-valued discrepancies, simulation defects, retained invariant classes, or multi-component tolerances can instantiate Equation 126 without a scalar distance.

Remark 13.2 (Tolerance is criterion-relative). A tolerance region $\mathcal{A}_{\beta,c}$ records which represented differences a declared preservation context accepts. It does not establish that the excluded differences are ontologically important or that the included differences are normatively harmless.

Remark 13.2 is especially important for language, heritage, and ecological applications. A pronunciation shift can be acceptable under one preservation purpose and central under another. A coarse dynamical signature can remain stable while a community-valued distinction disappears. The preservation taxonomy therefore treats tolerance as typed by target, basis, history, and context.

13.3 Historically Generated Preservation Criteria

Preservation criteria are themselves historically situated relational structures. A distinction counted as essential at one historical stage can become negotiable later; a previously peripheral feature can become preservation-relevant after political conflict, technological change, new knowledge, or intergenerational reinterpretation. GR therefore should not model the preservation criterion as an external timeless evaluator when the criterion participates in the relational history under study.

Let $\mathcal{K}_{\Sigma,c}^{\text{pres}}$ denote a model-relative criterion configuration at relational slice Σ and context c . A criterion configuration can include the retained target basis, comparison relation, tolerance structure, evidential burden, and institutional or community procedures used to form preservation judgments. Its schematic form is stated in Equation 128.

$$\mathcal{K}_{\Sigma,c}^{\text{pres}} := (\beta_{\Sigma,c}, r_{\Sigma,c}, \mathcal{A}_{\Sigma,c}, \mathcal{B}_{\Sigma,c}^{\text{ev}}), \quad (128)$$

where $\mathcal{B}_{\Sigma,c}^{\text{ev}}$ denotes the declared evidential burden. Equation 128 is a bookkeeping structure whose components can be refined or reorganized by a domain-specific theory of judgment.

For a historical witness $\eta : \Sigma \rightsquigarrow \Sigma'$, a criterion-update map can be introduced when the model has enough evidence to track revision of the preservation criterion. The map is stated in Equation 129.

$$\Theta_\eta^{\text{crit}} : \mathcal{K}_{\Sigma,c}^{\text{pres}} \dashrightarrow \mathcal{K}_{\Sigma',c'}^{\text{pres}}, \quad (129)$$

where the target context c' can differ from c . The update can preserve some criterion components, revise others, or leave part of the criterion undefined while a dispute remains unresolved.

Definition 13.3 (Historically generated preservation criterion). A preservation criterion is historically generated within the present GR model when its retained-content basis, comparison structure, tolerance, or evidential burden is represented as part of a criterion configuration of the form in Equation 128 and at least one of those components is historically formed or revised through a traceable relation such as Equation 129.

Definition 13.3 does not imply that every criterion is arbitrary or that historical origin determines validity. It locates the criterion inside the same relational history that generates institutions, classifications, expertise, technologies, and community practices. Epistemic adequacy and normative legitimacy remain separate evaluations.

A criterion can also become generatively consequential. Suppose a manifested judgment or institutional classification changes funding, access, teaching, conservation effort, archival selection, or admissible forms of innovation. Let j be such a judgment and let Ψ_j^J denote the corresponding partial judgment-conditioned transformation on the later relational configuration. The reflexive relation is represented in Equation 130.

$$\Psi_j^J : X_\Sigma \dashrightarrow X_{\Sigma'}, \quad X_{\Sigma'} = \Psi_j^J(X_\Sigma), \quad (130)$$

whenever the transformation is defined. Equation 130 makes explicit that preservation judgment can become part of the generative conditions of later preservation. A declaration of authenticity, endangerment, protected status, or recoverability can therefore alter the future object to which subsequent preservation judgments are applied.

13.4 Aesthetic Valuation

Aesthetic valuation concerns qualities of form, expression, resonance, style, beauty, and historically situated sensibility. Such qualities can contribute materially to preservation decisions in language, ritual, art, architecture, landscape, and related domains. The taxonomy keeps aesthetic valuation distinct from preservation identification and from normative obligation.

Let $\mathcal{J}_c^{\text{aest}}$ denote a context-relative aesthetic-judgment space. An aesthetic valuation is represented by the partial map in Equation 131.

$$J_{\beta_A, c}^{\text{aest}} : \text{Dom}(J_{\beta_A, c}^{\text{aest}}) \subseteq \mathcal{E}_c^{\text{pres}} \dashrightarrow \mathcal{J}_c^{\text{aest}}. \quad (131)$$

The basis β_A states which manifestations, relations, histories, or generative characteristics enter the aesthetic judgment. The codomain need not be numerical or totally ordered. Incomparability, ambivalence, disagreement, and context dependence can remain represented.

Aesthetic judgment can become historically consequential without becoming a normative judgment by definition. A community's experience that one form carries expressive or historical resonance can alter transmission and institutional support. Conversely, a form can be aesthetically disfavored while remaining normatively protected for reasons of justice, rights, knowledge, or future option value. The formal separation prevents one evaluative register from being inserted silently into another.

13.5 Epistemic Valuation

Epistemic valuation concerns the contribution of preservation to knowledge, evidence, inferential access, interpretive possibility, reconstruction, and future inquiry. The preservation target can be valued because its loss would remove information already known to be significant, because it supports forms of situated knowledge that are difficult to externalize, or because maintaining recoverability preserves access to questions that cannot yet be fully specified.

Let $\mathcal{J}_c^{\text{epi}}$ denote an epistemic-judgment space. An epistemic valuation is the partial map stated in Equation 132.

$$J_{\beta_E, c}^{\text{epi}} : \text{Dom}(J_{\beta_E, c}^{\text{epi}}) \subseteq \mathcal{E}_c^{\text{pres}} \dashrightarrow \mathcal{J}_c^{\text{epi}}. \quad (132)$$

The basis β_E can include trace completeness, provenance, discriminating capacity, reconstructibility, access to situated practice, uncertainty, and the preservation of alternative explanatory or interpretive possibilities.

Historical valuation can be represented as a distinct coordinate when the preservation inquiry needs to evaluate continuity, provenance, testimony, memory, or lineage independently of epistemic usefulness. Let $J_{\beta_H, c}^{\text{hist}}$ denote such a judgment. The taxonomy does not identify historical value with epistemic value: a lineage can be historically significant while adding little new information to a selected inquiry, and a newly reconstructed dataset can be epistemically valuable while possessing weak lineage continuity.

Epistemic valuation also remains distinct from structural recoverability. A target can be formally recoverable while the evidence needed to justify the recovery is weak, inaccessible, or too coarse for a

particular inquiry. Conversely, an epistemically valuable archive can remain insufficient to reconstruct a living generative capacity. The distinction retains the session's original motivation for separating corpus preservation from relational and generative preservation.

13.6 Normative Valuation

Normative valuation concerns reasons, permissions, duties, claims, prohibitions, responsibilities, and legitimacy associated with preservation, transformation, recovery, or termination. The preservation taxonomy supplies objects on which normative reasoning can operate; it does not derive a normative theory from those objects. A structurally satisfied preservation profile can be normatively endorsed, tolerated, criticized, or opposed depending on additional premises concerning persons, communities, future participants, rights, harms, justice, authority, and other normative considerations.

Let $\mathcal{J}_c^{\text{norm}}$ denote a normative-judgment space appropriate to the selected theory or institutional context. A normative valuation is represented in Equation 133.

$$J_{\beta_N, c}^{\text{norm}} : \text{Dom}(J_{\beta_N, c}^{\text{norm}}) \subseteq \mathcal{E}_c^{\text{pres}} \times \mathcal{N}_c \dashrightarrow \mathcal{J}_c^{\text{norm}}, \quad (133)$$

where $\mathcal{N}_c$ denotes the declared normative premises, principles, procedures, or nomic structures admitted by the analysis. Including $\mathcal{N}_c$ in the domain makes the dependence on normative premises explicit.

Equation 133 blocks a naturalistic shortcut. Neither $\text{Pres}_{\mathfrak{P}}^{\text{str}} = 1$ nor $\nu_{\mathfrak{P}} = 0$ produces an obligation by formal transformation alone. A community can have reasons to discontinue a preserved practice; a lost practice can generate reasons for reconstruction; preservation can conflict with autonomy or other claims; and irreversible loss can generate intergenerational concerns even where current use is limited. Those arguments require normative content beyond the structural taxonomy.

The historically generated character of normative criteria also requires care. A nomos can become relationally real, institutionally powerful, and generatively consequential while remaining normatively contestable. Historical persistence and efficacy therefore describe the status of a norm within the relational system; they do not establish its legitimacy.

13.7 Practical-Political-Economic Valuation

Practical valuation concerns feasibility, resources, institutional capacity, technical constraints, reversibility, timing, and opportunity costs. Political-economic valuation adds the distribution of resources and burdens, the structures through which support is allocated, and the power relations that shape institutional viability. These considerations become important when several preservation profiles are simultaneously possible but require different expenditures or distribute generative capacity differently.

Let $\mathcal{J}_c^{\text{prac}}$ and $\mathcal{J}_c^{\text{pe}}$ denote practical and political-economic judgment spaces. Their corresponding maps are stated in Equation 134.

$$\begin{aligned} J_{\beta_P, c}^{\text{prac}} &: \mathcal{E}_c^{\text{pres}} \times \mathcal{R}_c \dashrightarrow \mathcal{J}_c^{\text{prac}}, \\ J_{\beta_{PE}, c}^{\text{pe}} &: \mathcal{E}_c^{\text{pres}} \times \mathcal{R}_c \times \mathcal{Q}_c \dashrightarrow \mathcal{J}_c^{\text{pe}}, \end{aligned} \quad (134)$$

where $\mathcal{R}_c$ represents the relevant resource and implementation conditions and $\mathcal{Q}_c$ represents the declared distributional or power structure. The maps remain partial because cost, feasibility, distribution, and power can be poorly identified or incommensurable across alternatives.

Political-economic judgment is not reducible to monetary cost. A language-preservation programme can redistribute educational time, institutional recognition, technological support, professional opportunity, symbolic authority, or access to AI infrastructure. An ecological preservation programme can shift land use and livelihood conditions. An archival programme can determine whose records become searchable and whose histories remain weakly represented. These distributional consequences can in turn modify the generative conditions analyzed in Section 8.

The reflexive structure of Equation 130 therefore applies strongly to practical and political-economic judgments. Allocation decisions based on one preservation profile can alter the later preservation profile

itself. Preservation policy becomes part of the evolving relational system and can alter the later object, its relations, and its preservation conditions.

13.8 Independence of Preservation from Value

The preceding judgment types can now be collected without aggregating them. Let Ξ_J denote the selected family of value registers. One possible family is $\Xi_J = \{\text{aest}, \text{epi}, \text{hist}, \text{norm}, \text{prac}, \text{pe}\}$. For evidence package e , the typed judgment profile is defined in Equation 135.

$$\mathbf{J}_{\beta,c}(e) := \left(J_{\beta_I,c}^{\text{id}}(e), \{J_{\beta_\xi,c}^\xi(e)\}_{\xi \in \Xi_J} \right), \quad (135)$$

whenever the displayed components are defined. Equation 135 preserves the type of each judgment. It supplies no rule for adding aesthetic value to epistemic value, converting practical feasibility into normative legitimacy, or ranking several registers on one common scale.

Proposition 13.4 (Independence of preservation and value). *Within the present taxonomy, structural preservation satisfaction does not entail positive valuation in any register $\xi \in \Xi_J$, and positive valuation in a register ξ does not entail structural preservation satisfaction, unless an additional bridge principle is introduced.*

Proof. Fix any structural model in which $\text{Pres}_{\mathfrak{P}}^{\text{str}}(X, Y) = 1$. The valuation maps are additional partial structures whose defining equations impose no axiom that fixes their sign from the structural predicate. The same structural model can therefore be extended by two admissible valuation assignments on the relevant evidence package, one with positive J^ξ and one with non-positive J^ξ , while leaving every structural object unchanged. Hence structural preservation does not entail positive valuation in the class of models defined here. Conversely, fix an evidence package to which a positive value is assigned. Because the valuation definitions impose no axiom forcing applicability or structural satisfaction of $\mathfrak{P}$, models with the same positive valuation can differ in whether the structural preservation claim is satisfied, failed, or undefined. Positive valuation therefore does not entail structural preservation. Either implication requires an additional bridge axiom or substantive argument restricting the admissible model class. $\square$

Proposition 13.4 formalizes a central boundary of the paper. A preserved structure can be judged harmful, obsolete, burdensome, or normatively objectionable. A lost or currently unrecoverable structure can remain aesthetically, historically, epistemically, or normatively valuable. A community can value a language while choosing not to use it in one domain, or value a historical practice while rejecting its reconstruction. Structural preservation and preservation value therefore occupy different analytical layers.

The value registers can also disagree with one another. Aesthetic value can support preservation while practical feasibility is weak. Epistemic value can support archival retention while normative reasons favor limiting active reproduction. Political-economic considerations can reveal that one preservation strategy transfers burdens to a group that did not select the preservation objective. The taxonomy records those conflicts while leaving aggregation to an explicitly justified evaluative procedure.

Table 13 completes the separation required before the language application. Preservation identification, preservation value, and implementation judgment can interact historically and can become mutually consequential, while their inferential roles remain distinct. Section 14 therefore uses language preservation as a stress test without assuming that corpus survival, relational continuity, generative capacity, community valuation, normative reasons, or practical feasibility coincide.

Table 13: Judgment types associated with preservation claims and the inferential boundaries retained by the taxonomy.

Judgment type	Primary object of assessment	Inference not supplied by that judgment alone
Identification	Whether declared structural evidence is sufficient to count as preservation in context	Positive value, legitimacy, or obligation
Aesthetic	Expressive, formal, experiential, or stylistic value	Normative duty or historical continuity
Epistemic	Knowledge, evidence, interpretive access, reconstructibility, and future inquiry	Normative priority over competing claims
Historical	Provenance, memory, testimony, continuity, and lineage significance	Present utility or normative legitimacy
Normative	Reasons, duties, permissions, claims, legitimacy, and responsibility	Empirical feasibility or structural preservation status
Practical	Feasibility, resources, implementation capacity, reversibility, and timing	Justice of burden allocation or legitimacy
Political-economic	Distribution of resources, opportunities, burdens, authority, and generative support	Normative resolution of those distributions

14 Language Preservation as a Taxonomic Stress Test

Sections 5–13 developed the preservation taxonomy without fixing one substantive domain as its privileged model. The role of the present section is to test that architecture against language preservation, where archival traces, living competence, social conditions, transmission, migration, revival, and historical change can diverge sharply. Its objective is to show how several preservation specifications can apply to one language-related history without collapsing them into a single preserved-or-lost judgment. The discussion proceeds through corpus preservation, capacity-condition decoupling, intergenerational transmission, diasporic transformation, revival after interruption, dynamical continuity, and typed preservation profiles. The method is illustrative and classificatory. Language provides a bounded stress test for the taxonomy, while empirical theories of language endangerment and normative accounts of linguistic rights remain separate inquiries.

The application begins from a model-relative GR representation of a language-bearing relational configuration. Let $L_{\Sigma, \varrho}$ denote the linguistic relational material retained at slice Σ and resolution ϱ within a wider history-bearing configuration X_{Σ} . The symbol can include speakers, linguistic practices, institutional domains, manifested forms, transmission relations, and other structures selected by the local analysis. It does not define a language ontologically as a tuple of components. The selected linguistic material can support several distinct preservation targets under the general specification $\mathfrak{P}$ of Section 4.

UNESCO’s language-vitality framework already separates intergenerational transmission, domains

of use, new domains and media, educational materials, and documentation, while its intangible-cultural-heritage framework treats safeguarding through viability, transmission, documentation, and continuing recreation [2, 1]. Research on language shift and revitalization further places intergenerational continuity, community context, education, social domains, documentation, and technological settings inside historically specific language-in-society configurations [9, 10, 3, 11]. The GR taxonomy uses these distinctions as a demanding application domain: several dimensions that can be coordinated in practice remain analytically separable in the preservation claim itself.

14.1 Corpus without Living Generativity

A corpus is a paradigmatic manifestational preservation target. Section 5.4 represented a corpus through context-dependent manifestations, recording maps, provenance data, and a corpus correspondence witness. The language case shows both the strength and the limit of that construction. A large corpus can preserve pronunciations, lexical forms, texts, conversations, grammatical examples, and other traces with high fidelity while leaving open whether the relations and capacities that generated those traces remain active.

For a language-related configuration $L_{\Sigma, \varrho}$, let $\mathcal{C}_L(L_{\Sigma, \varrho})$ denote the corpus object obtained through the manifestational and archival construction of Section 5. A corpus-preservation specification $\mathfrak{P}_L^{\text{corp}}$ can therefore be satisfied between slices Σ and Σ' even when a distinct capacity specification $\mathfrak{P}_L^{\text{cap}}$ fails. The pattern is stated in Equation 136.

$$\nu_{\mathfrak{P}_L^{\text{corp}}}(L_{\Sigma}, L_{\Sigma'}) = 1, \quad \nu_{\mathfrak{P}_L^{\text{cap}}}(L_{\Sigma}, L_{\Sigma'}) = 0. \quad (136)$$

Equation 136 is a typed possibility with explicitly limited empirical scope. It records the case in which selected manifestations remain preserved while the relevant generative grammar, operation family, or competence no longer satisfies the declared capacity comparison.

The non-injective trace obstruction in Proposition 5.4 explains why exact corpus equality cannot repair this gap by itself. Recording can collapse distinctions among interactional histories, social relations, pragmatic contexts, tacit corrections, or other relational material that does not survive in the selected trace map. A corpus can therefore provide strong evidence for manifestational preservation while remaining insufficient evidence for relational, capacity, condition, or lineage preservation unless a cross-target bridge of the type developed in Section 12.1 is separately established.

This distinction does not diminish the role of documentation. UNESCO's language-vitality analysis explicitly treats documentation as relevant while distinguishing it from vitality factors such as intergenerational transmission and domains of use [2]. Within the present taxonomy, that institutional distinction receives a formal interpretation: documentation can satisfy a preservation specification whose target class differs from the targets used to assess living generativity.

14.2 Grammar without Generative Conditions

A second stress case preserves a language-related generative capacity while weakening the conditions under which that capacity can be exercised. Section 7 distinguished schema-level, defined, and executable availability. A speaker population can retain lexical, grammatical, pragmatic, or compositional capacities while later institutions and environments provide few domains in which those capacities are executable or socially consequential.

Let $\widehat{\Gamma}_{L, \Sigma}^{\text{cap}}(R)$ denote the language-related capacity grammar inherited from Equation 52, and let $C_{L, \Sigma}^{\text{gen}}(R)$ denote the associated condition target inherited from Definition 8.1. A stylized capacity-preserved/condition-degraded pattern is represented in Equation 137.

$$\nu_{\mathfrak{P}_L^{\text{cap}}} = 1, \quad \nu_{\mathfrak{P}_L^{\text{cond}}} = 0, \quad (137)$$

where both evaluations are understood relative to their declared histories, resolutions, and retained content. Equation 137 can describe a model in which competence survives while educational, occupational, institutional, technological, or interactional supports no longer satisfy the selected condition criterion.

The pattern also illustrates why language preservation cannot be reduced to counting persons who retain a capacity. Operational availability depends on whether state-dependent schemas can be instantiated inside the current configuration. A fluent speaker can preserve a capacity while lacking interlocutors in one domain, lacking institutional permission in another, or lacking the technical infrastructure required for a new medium. These are condition-level differences whose preservation status must be evaluated separately from the capacity grammar.

The coupling result in Proposition 8.6 adds a further caution. Even when both capacity and conditions are separately preserved, their relation can change. A later environment can contain schools, media, and fluent speakers while the coupling among those structures supports a different repertoire of use, correction, innovation, or social uptake. Language-related generativity is therefore not represented by the additive presence of a grammar and a context; the joint realization map must also remain compatible with the preservation claim.

14.3 Conditions without Fluent Capacity

The converse stress case retains substantial enabling infrastructure while the relevant linguistic capacity has weakened or disappeared. A community, institution, or state can preserve classrooms, archives, dictionaries, digital platforms, designated public domains, or funding mechanisms while lacking enough speakers or practitioners who instantiate the target competence. The generative-condition profile can therefore remain comparatively strong while capacity preservation fails.

A stylized profile of this form is stated in Equation 138.

$$\nu_{\mathfrak{P}_L^{\text{cond}}} = 1, \quad \nu_{\mathfrak{P}_L^{\text{cap}}} = 0. \quad (138)$$

Equation 138 separates infrastructure from competence. It also clarifies why the same educational material can function differently across histories. In one configuration, materials participate in active transmission among speakers and learners; in another, they remain available as a reconstruction resource after the corresponding living capacity has ceased to be instantiated.

The distinction gives the taxonomy a place for preservation programmes whose practical achievement lies in retaining future recovery options even when current fluency is absent. A condition target can be preservation-relevant because it keeps teaching resources, interaction spaces, or institutional support available for a later recovery trajectory. That possibility is evaluated through the lineage and recoverability machinery of Section 9; it should not be redescribed as current capacity merely because the infrastructure exists.

The language case also shows why a condition specification can contain undefined components. An institution can document available funding and educational materials while the interactional conditions required for peer use, household transmission, or technical innovation remain poorly known. A typed preservation profile should retain $\perp$ for those components, leaving missing articulation explicitly unresolved.

14.4 Intergenerational Transmission

Intergenerational transmission provides a particularly demanding test because it connects capacity, conditions, lineage, and historical transformation. UNESCO identifies intergenerational language transmission as a central factor in language vitality, while Fishman’s reversing-language-shift framework places intergenerational continuity at the centre of threatened-language maintenance [2, 9]. Within GR, transmission is represented as a historically situated operation that produces or sustains later capacity through a traceable relation among participants, institutions, and inherited resources.

Let $\mathsf{T}_{\Sigma \rightsquigarrow \Sigma'}^L$ be a language-related transmission operation of the type introduced in Equation 70. The target of transmission need not reproduce the source grammar literally. A preservation-relevant transmission can support a later capacity grammar $\hat{\Gamma}_{L, \Sigma'}^{\text{cap}}$ that differs from its antecedent while satisfying a declared historical comparison. The relevant pattern is stated in Equation 139.

$$\mathsf{T}_{\Sigma \rightsquigarrow \Sigma'}^L : \hat{\Gamma}_{L, \Sigma}^{\text{cap}} \dashrightarrow \hat{\Gamma}_{L, \Sigma'}^{\text{cap}}, \quad \hat{\Gamma}_{L, \Sigma'}^{\text{cap}} \text{ r } \mathcal{H}_{\Sigma \rightsquigarrow \Sigma'}(\hat{\Gamma}_{L, \Sigma}^{\text{cap}}), \quad (139)$$

where $\mathcal{H}_{\Sigma \rightsquigarrow \Sigma'}$ denotes the declared historical transport appropriate to the comparison. Equation 139 expresses preservation through historically continuous transformation, allowing the transmitted capacity to change while lineage remains traceable.

The formulation accommodates ordinary linguistic change. New lexical items, altered pronunciation, new registers, borrowed expressions, changing politeness systems, or revised technical vocabularies can enter the transmitted language while a lineage remains identifiable. Strong preservation of every antecedent feature could therefore conflict with the historical process through which a living language continues to generate new forms.

Transmission is also relationally broader than a parent-child channel. The model can retain peer groups, schools, apprenticeships, media communities, religious institutions, professional networks, or digital interaction as transmission-supporting relations when the application supplies the relevant history. The GR taxonomy classifies their preservation role through the target and mode actually supported by evidence; it does not assign one institution a universal priority.

14.5 Diasporic Transformation

Diasporic language histories test preservation across relocation. Section 9.3 distinguished relational relocation from ordinary historical continuation by requiring a witness that connects a source configuration to a later configuration embedded in a different relational environment. Language is well suited to this mode because speakers can retain substantial capacity and lineage while the surrounding institutional, demographic, material, and contact-language conditions change.

Let L_{Σ}^A denote selected language-related material in source environment A and $L_{\Sigma'}^B$, the tracked descendant in environment B . A relocation witness $\mathcal{R}_{A \rightsquigarrow B}^L$ can support a preservation claim even when the condition targets differ substantially. The typed pattern is represented in Equation 140.

$$\nu_{\mathfrak{P}_L^{\text{lin}}} = 1, \quad \nu_{\mathfrak{P}_L^{\text{cond}}} \in \{0, \perp\}, \quad \nu_{\mathfrak{P}_L^{\text{cap}}} \in \{1, 0, \perp\}, \quad (140)$$

where the three statuses depend on the actual evidence and declared preservation specifications. Equation 140 does not assert that diasporic language histories typically take one form. It shows that lineage continuity is logically compatible with substantial reorganization of conditions and with several possible capacity outcomes.

This case also clarifies the difference between preservation of a language and preservation of one historical ecology of use. A relocated community can preserve a linguistic lineage while generating new domains, borrowing from surrounding languages, reorganizing register, or changing the social distribution of use. Treating every such change as loss would silently privilege source-environment conditions as the preservation target. The taxonomy requires that target choice to be stated explicitly.

Diasporic transformation can also create branching descendants. Distinct communities can carry historically defensible lineages from a common source while developing different capacities, conditions, manifestations, and dynamical regimes. Definition 9.10 allows those branches to receive separate preservation profiles without requiring one uniquely authentic successor.

14.6 Revival after Interruption

Revival places the strongest pressure on simple continuity language. A later community can regain use of a language-related grammar or construct a new active linguistic system from archives, partial speakers, educational materials, related varieties, or institutional support after an interruption in ordinary transmission. Language-revitalization research emphasizes that such programmes operate under historically specific social, political, economic, educational, and technological conditions [3, 11]. The taxonomy therefore treats revival as a reconstructive preservation problem whose adequacy depends on declared reconstruction witnesses and comparison relations.

Let Σ_a index a slice at which the target capacity is active, let Σ_b belong to an interruption interval, and let Σ_c index a later reconstructed regime. A revival claim can be represented through the reconstructive-

lineage machinery of Definition 9.5. The minimal pattern is stated in Equation 141.

$$\nu_{\mathfrak{P}_L}^{\text{cap}}(\Sigma_a, \Sigma_b) = 0, \quad \exists \mathcal{W}_{\Sigma_a \rightsquigarrow \Sigma_c}^{\text{rec}}, \quad \nu_{\mathfrak{P}_L}^{\text{cap}}(\Sigma_a, \Sigma_c) = 1, \quad (141)$$

where the final positive evaluation uses the declared reconstruction witness and comparison relation. Equation 141 permits interruption without requiring that the reconstructed capacity be numerically identical to the antecedent capacity.

The reconstructive case should be separated from mere archival survival. A rich corpus can make revival more feasible while leaving revival unrealized. Conversely, a later active system can emerge through several resources whose relation to the antecedent language requires explicit historical and comparison arguments. The existence of a reconstruction programme therefore does not, by itself, settle whether the result counts as preservation, transformation, branching descent, or the formation of a distinct language-related system.

Recovery-mechanism regeneration sharpens the case further. If the original teaching practices, institutional supports, or speaker networks have disappeared, a later history may first have to generate a new mechanism capable of producing competent speakers. The distinction developed in Section 9.7 then matters: higher-order preservation concerns the regeneration of a recovery grammar, not merely a second application of the same fixed recovery operations.

14.7 Dynamical Continuity under Linguistic Change

A final stress case asks whether selected dynamical characteristics can remain reproducible while linguistic manifestations, grammar, conditions, and even local relational structures change. This is the weakest preservation target developed in Section 10. In a language application, a dynamical signature can be chosen to retain only those interactional or transmission characteristics that matter for the inquiry, such as recurrent correction cycles, stable learner-to-speaker pathways, innovation uptake, domain-switching structure, or another explicitly defined transition pattern.

Let $\mathfrak{D}_L^*$ be a declared language-related dynamical signature. Dynamical-structural reproducibility at a later configuration is represented by membership in the target class of Equation 102. A language-related instance is stated in Equation 142.

$$L_{\Sigma'} \in \mathfrak{T}_{\text{dyn}}^L(\mathfrak{D}_L^*) \iff \mathfrak{D}_L(\mathbf{D}(L_{\Sigma'})) \mathbf{r}_D \mathfrak{D}_L^*. \quad (142)$$

Equation 142 can hold even when the later language-related configuration differs strongly from the reference configuration in vocabulary, institutions, media, or local interaction structure.

The result should be interpreted narrowly. Dynamical-regime preservation does not establish preservation of linguistic identity, historical lineage, corpus content, grammar, or social conditions. Proposition 10.2 blocks that reverse inference when the signature map is non-injective. A later system can reproduce the selected dynamics while being substantially different in every omitted respect.

The language case nevertheless shows why this weak preservation mode can be analytically useful. A preservation programme can care about maintaining the capacity for ongoing transmission, correction, innovation, or adaptive domain formation even while allowing many surface and structural features to change. The relevant claim is then explicitly dynamical. Its adequacy depends on whether the selected signature captures what the preservation inquiry intends to retain, a question returned to the judgment structure of Section 13.

14.8 Preservation Profiles for Language Cases

The preceding stress cases can now be assembled through the typed preservation profile of Definition 12.8. Let the language-specific profile basis contain corpus/manifestation, relational structure, generative capacity, generative conditions, lineage/recoverability, and dynamical regime. The corresponding profile is stated in Equation 143.

$$\mathbf{V}_L := \left(\nu_{\text{corp}}, \nu_{\text{rel}}, \nu_{\text{cap}}, \nu_{\text{cond}}, \nu_{\text{lin}}, \nu_{\text{dyn}} \right) \in \{1, 0, \perp\}^6. \quad (143)$$

Table 14: Stylized language-preservation profiles illustrating divergence among preservation targets. The symbols 1, 0, and $\perp$ denote satisfied, defined-but-failed, and undefined or unresolved specifications.

Stress case	Corpus	Relational structure	Capacity	Conditions	Lineage	Dynamics
Archive-rich interruption	1	$\perp$	0	0/ $\perp$	1	$\perp$
Capacity with domain contraction	1	1/ $\perp$	1	0	1	0/ $\perp$
Infrastructure with lost fluency	1	1/ $\perp$	0	1	1/ $\perp$	$\perp$
Transformative transmission	1/ $\perp$	1	1	1	1	1/ $\perp$
Diasporic descendant	1/ $\perp$	1/ $\perp$	1/0/ $\perp$	0/ $\perp$	1	1/ $\perp$
Reconstructive revival	1	1/ $\perp$	1	1/ $\perp$	1	1/ $\perp$
Dynamical reproduction only	0/ $\perp$	0/ $\perp$	0/ $\perp$	0/ $\perp$	0/ $\perp$	1

Equation 143 is a typed status vector, not a numerical score. The entries are not assumed to have equal importance, and the profile has no built-in aggregation rule.

Table 14 gives stylized stress cases. The entries indicate logically coherent profile patterns under appropriately chosen specifications; they are not empirical classifications of named communities or languages. Their role is to demonstrate that the preservation taxonomy distinguishes cases that a single preserved/lost label would compress.

The slash-separated entries in Table 14 mark alternatives that depend on the declared specification and evidence; they are not probability values. For example, a diaspora case can preserve generative capacity in one domain, lose it in another, or remain underdetermined at the chosen resolution. A revival can reconstruct a capacity while leaving the relation to earlier relational structures contested. The table therefore illustrates typed partiality while leaving case-specific profile assignment to evidence and specification.

The stress test yields three conclusions for the taxonomy. First, language preservation is structurally plural: corpus, relational structure, capacity, conditions, lineage, and dynamics can vary independently enough that one binary status is usually insufficient. Second, historical mode matters: continuous transmission, relocation, interruption, and reconstruction can support different preservation claims even when endpoint manifestations are similar. Third, the value of a preservation profile remains a separate judgment. A community, institution, or researcher can value one component more strongly than another, and those priorities require the judgment architecture of Section 13; the structural profile alone carries no evaluative ordering.

Language therefore serves as one stress-test domain within the broader preservation taxonomy. The taxonomy can represent corpus-rich loss of living capacity, preserved competence under weakened conditions, enabling infrastructure without fluent capacity, transformative transmission, diasporic continuity, reconstructive revival, and dynamical reproduction within one typed architecture. Section 15 converts that architecture into a procedure that specifies which preservation target, historical scope, comparison relation, mode, and profile are being claimed in a concrete inquiry.

15 Generative Relational Preservation Investigation

Sections 4–14 developed a preservation taxonomy and showed that one historical formation can support several preservation claims with different targets, modes, resolutions, and evidential burdens. The role of the present section is to convert that classificatory architecture into a reproducible research procedure. Its objective is to specify the minimum sequence through which a concrete preservation inquiry identifies its investigation object, types the preservation target, fixes the relevant reality or representation status, establishes historical scope and tracking, selects retained content and a comparison relation, identifies an admissible preservation mode, constructs a typed preservation profile, and reports the formal and

Table 15: Stages of a Generative Relational Preservation Investigation. Each stage records a distinct specification or evidential burden before a preservation conclusion is reported.

Stage	Investigation task	Principal formal object	Required record
1	Investigation object and preservation target	Target class θ and history-bearing carrier	Entity boundary, target extraction, resolution
2	Reality-status specification	Status indices ℓ, ℓ'	Relational, manifestational, or representational status and any cross-status bridge
3	Historical scope and tracking	Historical mode h and witness family Hist_h	Historical window, tracking rule, provenance evidence
4	Retained-content selection	Basis κ and retained-content projections	Included distinctions, excluded distinctions, sensitivity alternatives
5	Comparison-relation selection	Relation r at resolution ϱ	Domain, semantics, tolerance or equivalence criterion
6	Preservation-mode identification	Mode μ and predicate $M_\mu^{\mathfrak{P}}$	Mode admissibility and mode-specific evidence
7	Typed preservation profile	Prof$\mathfrak{P}$	Satisfied, failed, and undefined specifications
8	Formal and empirical reporting	GRPI report bundle	Evidence, bridge assumptions, uncertainty, unresolved components

empirical basis of the resulting claims. The method is procedural and model-relative. The procedure, termed a *Generative Relational Preservation Investigation* (GRPI), organizes already defined GR objects and preservation specifications; it does not add a new reality layer or replace domain-specific empirical methods.

Table 15 summarizes the eight stages developed in this section. Their order records inferential dependence; empirical work can iterate among stages when revisions are reported explicitly. Evidence collected at a later stage can motivate revision of an earlier specification, provided that the revision and its consequences are reported explicitly.

15.1 Preservation Target of the Investigation Object

A preservation inquiry begins by identifying the history-bearing formation whose preservation claims are under investigation and by selecting the target class on which each claim will operate. This step is necessary because an ordinary-language object label can contain several formally distinct preservation targets. A study of one language, institution, ecological practice, archive, or technical system can concern manifestations, relational organization, generative capacity, enabling conditions, historical lineage, or dynamical regime. The broader formation supplies the investigation object; the index $\theta \in \Theta_{\text{core}}$ from Equation 14 selects the target used in one preservation specification.

Let $\mathcal{D}_{\text{inv}}$ denote the declared class of history-bearing target pairs retained by an investigation at resolution ϱ . The target family used by the inquiry is a finite subset $\Theta_{\text{inv}} \subseteq \Theta$ for which the required target carriers are defined on at least part of $\mathcal{D}_{\text{inv}}$. The registered target basis is stated in Equation 144.

$$\Theta_{\text{inv}} := \{\theta_1, \dots, \theta_m\} \subseteq \Theta, \quad \mathcal{D}_{\text{inv}} \subseteq \bigcup_{i=1}^m \mathcal{T}_{\theta_i}(\Sigma) \times \mathcal{T}_{\theta_i}(\Sigma'). \quad (144)$$

Equation 144 is schematic because the target carriers remain status-indexed in the full specification. Its

analytical role is to require the inquiry to state which target classes it intends to evaluate. A single-target study can set $m = 1$; a preservation-profile study ordinarily uses several target classes.

Target selection also requires an explicit boundary and extraction rule. A language-related investigation, for example, can retain a corpus as a manifestational target, a distributed competence structure as a capacity target, and a transmission network as part of a lineage target. The same empirical materials can contribute evidence to several target extractions while the extracted objects remain formally distinct.

Research Obligation 15.1 (Investigation-object and target registration). A domain-specific GRPI should state the modeled entity or formation boundary and the historical and representational resolution. It should also state the preservation-target class or classes selected for evaluation and the rule by which each target is extracted from the broader history-bearing configuration. A preservation conclusion should be indexed by that registration so that the ordinary-language object label does not conceal target multiplicity.

Obligation 15.1 prevents a later result concerning one target, such as corpus preservation, from silently becoming a claim about the whole language, institution, practice, or other formation.

15.2 Reality-Status Specification

The second stage records the analytical status of the compared targets. Section 3.3 distinguished relational reality, manifestation reality, and mathematical representation while maintaining ontological suspension. A GRPI therefore carries the status indices ℓ and ℓ' from Equation 13 into the empirical or formal specification of each target pair.

Same-status comparisons are often the least demanding. A relational target at slice Σ can be compared with a historically tracked relational target at Σ' , and a corpus recorded under one manifestation context can be compared with a later corpus under a declared recording transport. Cross-status investigations require an additional bridge. A manifested trace does not become a relational target merely because both refer to the same ordinary-language object, and a mathematical attractor or algebra does not acquire relational-reality status merely because it represents a persistent pattern.

For a cross-status specification, let $\mathcal{B}_{\ell \Rightarrow \ell'}$ denote the declared bridge family connecting the source and target statuses. The status-registration condition is stated in Equation 145.

$$\text{StatAdm}_{\text{inv}}(\ell, \ell'; X_{\Sigma}, X'_{\Sigma'}) = 1 \quad \Longrightarrow \quad \begin{cases} \ell = \ell', & \text{or} \\ \mathcal{B}_{\ell \Rightarrow \ell'}(X_{\Sigma}, X'_{\Sigma'}) = 1. \end{cases} \quad (145)$$

Equation 145 states an admissibility requirement for any cross-status comparison in a GRPI. The content of $\mathcal{B}_{\ell \Rightarrow \ell'}$ depends on the inquiry. It can contain an empirically warranted reconstruction, a manifestation-to-relational discriminating bridge of the type developed in Section 12.1, or another explicitly justified relation.

Ontological claims require additional restraint. A GRPI can conditionally ask what would count as preservation under an assumed ontological model, although the investigation should record that assumption separately from relational, manifestational, and representational conclusions. The procedure therefore preserves the ontological suspension governing the wider GR programme.

15.3 Historical Scope and Tracking

Preservation is historically typed. The third stage specifies which slices, branches, relocation histories, interruptions, or reconstruction episodes belong to the inquiry and which evidence supports tracking across them. The historical mode h selects the relevant witness family Hist_h , while the investigation adds an evidence-supported restriction so that formally imaginable historical witnesses are distinguished from historically warranted ones.

Let $\mathcal{E}$ denote the evidence package retained by an inquiry and let $\text{Supp}_{\mathcal{E}}(\eta)$ be a partial predicate recording whether $\mathcal{E}$ supplies the provenance, continuity, or reconstruction evidence required to use

witness η . The evidence-supported historical family is defined in Equation 146.

$$\text{Hist}_h^{\text{inv}}(X_\Sigma, X'_{\Sigma'} \mid \mathcal{E}) := \{ \eta \in \text{Hist}_h(X_\Sigma, X'_{\Sigma'}) \mid \text{Supp}_{\mathcal{E}}(\eta) = 1 \}. \quad (146)$$

Equation 146 gives the procedure an empirical interface without treating evidence as self-interpreting. The predicate $\text{Supp}_{\mathcal{E}}$ must be instantiated by the domain. Archival provenance, institutional records, participant testimony, longitudinal observation, genealogical evidence, version history, ecological tracking, or another evidence family can support different witness types.

The historical scope should also state its boundaries. An inquiry can compare two endpoints, a finite corridor of intermediate slices, a branching descendant family, or a reconstructive history containing an interruption. Different scopes can support different preservation conclusions even when the endpoint manifestations agree. The distinction between endpoint equivalence and historically continuous preservation established in Section 6.6 therefore becomes a reporting requirement at this stage.

Research Obligation 15.2 (Historical tracking and provenance). A GRPI that makes a continuation, relocation, reconstruction, or branching preservation claim should report the historical domain retained by the inquiry, the tracking or descendant rule, the evidence supporting the retained witnesses, and the points at which tracking is unresolved, reconstructed, or continuously observed.

Obligation 15.2 preserves the difference between a formally admissible witness and an empirically supported historical claim.

15.4 Retained-Content Selection

The fourth stage specifies the relational content κ that the comparison is intended to retain. Preservation conclusions can change when different distinctions, operations, interfaces, histories, or dynamical characteristics are selected. Retained content should therefore be chosen and documented as part of the investigation design before interpretive conclusions are drawn from the comparison.

A GRPI can use one retained-content basis or a finite sensitivity family. Let $\mathcal{K}_{\text{inv}}$ denote the set of retained-content bases evaluated by the inquiry. The sensitivity family is stated in Equation 147.

$$\mathcal{K}_{\text{inv}} := \{ \kappa_1, \dots, \kappa_s \}. \quad (147)$$

Equation 147 permits a study to ask whether a preservation result is stable under several defensible content selections. The family does not imply that every κ_j is equally appropriate. Each basis requires a substantive rationale tied to the research question and the target class.

For every κ_j , the investigation should record both included and excluded content. A language study that retains lexical distinctions while excluding pragmatic interaction, a heritage study that retains architectural form while excluding use relations, or a dynamical study that retains attractor type while excluding local transition rates makes a bounded preservation claim. Explicit exclusions make that scope inspectable.

Sensitivity across $\mathcal{K}_{\text{inv}}$ can itself become informative. If preservation succeeds only under a coarse basis that removes distinctions regarded as consequential under a finer basis, the result should be reported as resolution- or content-sensitive, with its scope restricted to the successful specification. Section 12.5 supplies the corresponding formal discipline for resolution change.

15.5 Comparison-Relation Selection

The fifth stage selects the comparison relation r appropriate to the retained target and content. The companion GR equivalence taxonomy supplies a library of possible relations, including identity, structural equivalence, isomorphism, interchangeability, simulation-like relations, approximate similarity, and diagnostic agreement. The preservation investigation uses such relations only after their domain and interpretation have been declared.

Let $\mathcal{R}_{\text{inv}}(\theta, \kappa, \varrho)$ denote the finite family of comparison relations admitted by an investigation for target class θ , retained content κ , and resolution ϱ . The registered comparison family is written in Equation 148.

$$\mathcal{R}_{\text{inv}}(\theta, \kappa, \varrho) := \{r_1, \dots, r_q\}, \quad \text{Dom}(r_i) \subseteq \mathcal{T}_\theta \times \mathcal{T}_\theta. \quad (148)$$

Equation 148 records that comparison definedness belongs to the investigation. A metric tolerance requires a defined metric and threshold; isomorphism requires a specified structured carrier; simulation requires a transition structure; diagnostic agreement requires a declared diagnostic family. The symbol r therefore gains no substantive content from notation alone.

When several comparison relations are defensible, the inquiry can report them separately or establish implications among them on a declared domain. A stronger relation should not be selected merely because it sounds more faithful to the past. Some targets preserve function under substantial structural change, while others make fine structural retention central to the inquiry. The comparison relation should follow the retained-content question.

Research Obligation 15.3 (Comparison-relation transparency). For every preservation specification evaluated in a GRPI, the comparison relation should be stated together with its domain, retained content, resolution, interpretation, tolerance or equivalence criterion when applicable, and the evidential or formal procedure by which the relation is evaluated.

Obligation 15.3 makes the comparison criterion auditable independently of whether the resulting preservation claim succeeds.

15.6 Preservation-Mode Identification

The sixth stage identifies the preservation mode whose historical semantics are being tested. Section 11 distinguished continuous retention, historically continuous transformation, dormant recoverability, reconstructive recovery, recovery-mechanism regeneration, and dynamical-regime reproduction. A concrete inquiry can evaluate more than one mode for the same target because the modes retain different historical information.

For target class θ and historical mode h , define the investigation-admissible mode family in Equation 149.

$$\mathfrak{M}_{\text{inv}}(\theta, h) := \{\mu \in \mathfrak{M} \mid \text{ModeAdm}(\theta, h, \mu) = 1\}. \quad (149)$$

Equation 149 prevents an inquiry from applying a preservation mode whose required structures are absent from the target or historical relation. A reconstructive-recovery claim, for example, requires an interruption and a reconstruction witness; continuous retention requires a corridor on which the target remains continuously available; recovery-mechanism regeneration requires the higher-order mechanism structure developed in Section 9.7.

Mode identification should also separate current state from preserved possibility. A dynamical regime can be absent at the active slice while remaining reproducible, and a capacity can be operationally inactive while remaining recoverable. The procedure therefore reports the mode predicate $M_\mu^{\mathfrak{P}}$ together with the target comparison and preserves the distinctions among successful modes in the final report.

Where several admissible modes apply, their results remain separate profile components. The implication analysis in Section 11.7 supplies no total ordering among those modes.

15.7 Typed Preservation Profile

The seventh stage assembles the registered specifications into a typed preservation profile. Let $\mathfrak{P}_{\text{inv}} = (\mathfrak{P}_1, \dots, \mathfrak{P}_n)$ be the finite family formed from the investigation's selected target classes, statuses, historical modes, retained contents, comparison relations, resolutions, and preservation modes. The direct investigation profile is defined in Equation 150.

$$\mathbf{Prof}_{\text{inv}}^{\text{dir}}(X, Y) := \mathbf{Prof}_{\mathfrak{P}_{\text{inv}}}(X, Y), \quad (150)$$

where each component is evaluated through Equation 121. Equation 150 therefore retains the three statuses 1, 0, and $\perp$ for satisfied, failed, and undefined or unresolved specifications.

Cross-target lifting can add derived profile components after the direct evaluations are available. Such components should remain distinguishable from direct evaluations because they inherit the assumptions of the bridge family used in the lift. Let $\text{Src}(\mathfrak{P}_i)$ denote the reporting tag associated with component $\mathfrak{P}_i$. The minimum source-tag family is stated in Equation 151.

$$\text{Src}(\mathfrak{P}_i) \in \{\text{direct}, \text{lift}, \text{unresolved}\}. \quad (151)$$

A component tagged direct is evaluated from its own target, witness, and comparison structures. A component tagged lift is inferred through a bridge-qualified result of the type defined in Section 12. The tag unresolved records cases in which the intended support route has not been established. Equation 151 is a reporting convention that records evidential provenance for each profile component.

The investigation profile should remain non-aggregated unless a later judgment procedure independently defines an aggregation rule. A GRPI can therefore report successful trace preservation, failed current capacity preservation, unresolved fine-grained condition preservation, and successful dynamical reproducibility in one record. Section 13 supplies the separate judgment architecture required before any claim that one profile is preferable to another.

15.8 Formal-Empirical Reporting

The final stage records enough information for the preservation claim to be reconstructed, criticized, and revised. A GRPI report should distinguish formal satisfaction from empirical support and both from preservation value. Formal satisfaction concerns the typed predicates developed in this paper. Empirical support concerns whether the modeled target, historical witness, retained content, and comparison relation are warranted by evidence. Preservation identification and value concern the judgment structures of Section 13.

For reporting purposes, define a GRPI report bundle by Equation 152.

$$\mathfrak{R}_{\text{GRPI}} := \left(\mathcal{O}_{\text{inv}}, \mathcal{D}_{\text{inv}}, \mathfrak{P}_{\text{inv}}, \mathcal{E}, \mathcal{B}_{\text{inv}}, \mathbf{Prof}_{\text{inv}}^{\text{dir}}, \mathcal{U}_{\text{inv}} \right). \quad (152)$$

In Equation 152, $\mathcal{O}_{\text{inv}}$ records the investigation object and boundary, $\mathcal{D}_{\text{inv}}$ the comparison domain, $\mathfrak{P}_{\text{inv}}$ the registered preservation specifications, $\mathcal{E}$ the evidence package, $\mathcal{B}_{\text{inv}}$ the bridge assumptions used for cross-status or cross-target inference, $\mathbf{Prof}_{\text{inv}}^{\text{dir}}$ the direct typed profile, and $\mathcal{U}_{\text{inv}}$ the unresolved applicability, tracking, evidence, or model components. Derived profile components and their source tags can be appended to the report after the corresponding bridge arguments have been stated.

The bundle in Equation 152 is a methodological record. It is not an ontological representation of the preserved object, and it does not require every domain to encode evidence in one common data structure. Its role is to make the inferential path of a preservation claim visible.

Research Obligation 15.4 (Formal and empirical preservation reporting). A reported GR preservation result should identify the investigation object and boundary, preservation target, status, historical mode and witness evidence, retained content, comparison relation, resolution, preservation mode, applicability result, structural preservation result, profile source, bridge assumptions, unresolved components, and the distinction between formal satisfaction, empirical support, and later value judgment.

Obligation 15.4 supplies a minimum transparency standard for domain-specific applications. A study can remain qualitative, ethnographic, historical, computational, or mixed-method while satisfying the obligation, provided that the evidence and interpretive procedures supporting the formal objects are stated at the level appropriate to the inquiry.

The GRPI procedure therefore converts the preservation taxonomy into a research workflow without assigning one method privileged access to preservation. Historical and ethnographic inquiry can establish provenance and relational continuity; archival work can characterize manifested traces; computational

models can test dynamical or topological signatures; institutional analysis can identify generative conditions; and formal work can determine which implications follow once the relevant objects are defined. Their results enter one typed investigation record while remaining methodologically plural. Section 16 examines the epistemic, normative, and formal boundaries that continue to constrain such investigations even after their specifications and reporting structures have been made explicit.

16 Boundaries of Preservation Inference

Sections 3–15 constructed a typed preservation taxonomy and a procedure for applying it to concrete inquiries. The role of the present section is to delimit the conclusions that this architecture can support. Its objective is to separate representational adequacy from relational reality, formal satisfaction from empirical identification, preservation from equivalence considered without historical provenance, and structural preservation from normative endorsement. The discussion proceeds through representation dependence, empirical identification burdens, ontological suspension, two recurrent inferential boundaries, and a set of open formal problems. The method is critical and scope-defining: the section preserves the preceding constructions while specifying the additional assumptions required when a preservation claim is extended beyond the level at which it was established.

16.1 Representation Dependence

Every preservation specification in the present paper is evaluated through a declared representation. A history-bearing GR spacetime may be represented at different resolutions, with different target extractions, regional decompositions, manifestation contexts, algebraic carriers, dynamical diagnostics, or comparison relations. A preservation result should therefore identify which features are invariant under admitted changes of representation and which features remain tied to the selected model.

Let $\mathcal{M}$ and $\mathcal{M}'$ denote two admissible representations of overlapping relational material, and let $Q_{\mathcal{M} \rightarrow \mathcal{M}'}$ be a declared translation, refinement, coarse-graining, or re-expression map between the retained target carriers. For a preservation specification $\mathfrak{P}$ in representation $\mathcal{M}$, write $Q_*\mathfrak{P}$ for the induced specification in $\mathcal{M}'$ whenever the target, history, retained content, relation, and mode admit transport. Representation covariance of the structural preservation predicate is stated in Equation 153.

$$\text{Pres}_{\mathfrak{P}}^{\text{str}, \mathcal{M}}(X, X') = \text{Pres}_{Q_*\mathfrak{P}}^{\text{str}, \mathcal{M}'}(Q_{\mathcal{M} \rightarrow \mathcal{M}'} X, Q_{\mathcal{M} \rightarrow \mathcal{M}'} X'), \quad (153)$$

whenever both sides are defined. Equation 153 is a property to be established for a declared representation change; it is not a general theorem of the taxonomy. Failure can arise because the new representation removes preservation-relevant distinctions, changes the domain of a comparison relation, alters the historical tracking rule, or replaces the target by a different effective object.

The resolution analysis in Sections 6.5 and 12.5 supplies one important special case. A fine-grained preservation result can descend to a coarser representation when the coarse-graining and comparison structures satisfy the required covariance conditions. The reverse direction generally requires additional fiber information. Representation dependence therefore has substantive consequences: a preservation claim can remain valid at one resolution while being undefined, underdetermined, or false at another.

Remark 16.1 (Representation-relative conclusions). A preservation result establishes a relation within the declared model, target extraction, historical scope, and comparison architecture. Stability under another representation requires a separate covariance or lifting argument. Numerical, algebraic, topological, or dynamical agreement obtained in one representation should therefore be reported together with the representation on which it was established.

Remark 16.1 is especially important for dynamical-structural preservation. Equality of selected spectra, homology groups, attractor classifications, or cycle-return diagnostics can be robust under some representation changes and highly sensitive to others. The taxonomy supplies a place to state that dependence; it does not convert a representation-dependent diagnostic into a representation-independent essence.

16.2 Empirical Identification Burdens

Formal well-typedness does not identify the empirical objects that populate a preservation specification. A concrete GRPI must decide which observations support the entity boundary, historical witness, target extraction, retained content, comparison relation, preservation mode, and cross-target bridges used in the analysis. The empirical burden therefore concerns the mapping from evidence to formal objects as much as the subsequent evaluation of those objects.

Let $\mathcal{E}_{\text{emp}}$ denote an empirical evidence space and let $\mathcal{Q}_{\text{form}}$ denote the class of formal preservation records admitted by a selected model. An identification map has the partial type stated in Equation 154.

$$\mathcal{I}_{\text{emp}} : \text{Dom}(\mathcal{I}_{\text{emp}}) \subseteq \mathcal{E}_{\text{emp}} \dashrightarrow \mathcal{Q}_{\text{form}}. \quad (154)$$

The partiality in Equation 154 records cases in which available evidence does not support a sufficiently articulated formal assignment. An archive can be too sparse to establish a lineage witness; an ethnographic record can reveal a practice without identifying its full generative grammar; an institutional dataset can measure outcomes while leaving the relational mechanism underdetermined.

Empirical identification can also be many-to-one or one-to-many. For an evidence object $e \in \mathcal{E}_{\text{emp}}$, define the compatible formal fiber in Equation 155.

$$\text{Fib}_{\mathcal{I}}(e) := \{q \in \mathcal{Q}_{\text{form}} \mid q \text{ is compatible with } e \text{ under the declared identification rule}\}. \quad (155)$$

A non-singleton fiber represents empirical underdetermination: several model specifications remain compatible with the evidence. Several preservation specifications can fit the same evidence because different historical reconstructions, target boundaries, or retained-content choices remain compatible with the record.

Research Obligation 16.2 (Empirical identification record). An empirical preservation study should report the evidence used to identify each nontrivial formal object, the alternative assignments that remain materially plausible, and the observations or assumptions through which those alternatives are excluded, retained, or left unresolved. A formally evaluable preservation predicate does not by itself satisfy this identification burden.

Obligation 16.2 preserves methodological pluralism. Historical archives, participant observation, interviews, institutional records, computational models, material traces, and quantitative data can identify different parts of a preservation specification. Their contributions need not be reduced to one evidential scale, although the bridge from each evidence type to the formal object it supports should remain explicit.

16.3 Ontological Suspension

The preservation taxonomy operates under the ontological suspension adopted by the wider GR programme. A labelled 2-complex can represent history-bearing relational spacetime; a local algebra can represent admissible operations or observables; a field can represent distributed condition or deformation data; an attractor-like regime can represent historical retention; and homology can diagnose selected topological structure. These constructions provide conditional mathematical languages for preservation analysis. Their usefulness does not establish that the preserved phenomenon is ultimately a 2-complex, algebra, field, attractor, or topological object.

The distinction applies equally to reality status. Relational reality and manifestation reality are analytical statuses inside the GR framework. A mathematical representation can supply evidence for a relational-reality claim only after the historical persistence and type-appropriate efficacy conditions of the represented structure have been established. Likewise, a contextually stable manifestation can remain insufficient to identify the relational organization that supports it.

Caution 16.3 (Formal representation and ontological preservation). Preservation of a mathematical representation, or of selected invariants within that representation, does not by itself establish preservation

of ultimate ontological reality. An ontological-preservation claim requires an independently stated ontological model together with a justified bridge from the formal representation to the ontology assumed by that model.

Caution 16.3 also limits negative conclusions. A change in a selected representation can show that preservation has failed relative to that representation and retained content. It does not establish that every possible representation, relational description, or ontological interpretation has lost continuity. The scope of both positive and negative preservation claims should therefore remain indexed by the model that supports them.

16.4 Equivalence without Preservation

The companion GR equivalence taxonomy supplies relations that compare structures at one or several levels. The present paper adds historical and preservation-mode requirements because equivalence by itself does not identify a preserved descendant, reconstruction, relocation, or reproducible regime. The boundary can be stated directly using the preservation witness architecture of Sections 3.5 and 3.7.

Let X and Y satisfy a declared comparison relation $X \text{ r } Y$. If the preservation specification additionally requires a historical or provenance witness $\eta \in \text{Hist}_h(X, Y)$ and the relevant witness family is empty, then the comparison result remains an equivalence claim without satisfying that preservation specification. The condition is summarized in Equation 156.

$$X \text{ r } Y, \quad \text{Hist}_h(X, Y) = \emptyset \implies \text{App}_{\mathfrak{P}}^{\text{pres}}(X, Y) \neq 1, \quad \nu_{\mathfrak{P}}(X, Y) = \perp \quad (156)$$

for any $\mathfrak{P}$ whose declared historical mode h requires such a witness. Equation 156 follows the partial-function discipline of Equations 19 and 121: an empty required witness family makes the preservation specification inapplicable on that pair; missing provenance therefore does not become a defined negative result. A value 0 is reserved for cases in which applicability holds and the preservation test itself fails.

Proposition 16.4 (Equivalence insufficiency). *Suppose a preservation specification $\mathfrak{P}$ requires a comparison relation r together with a historical witness of type h . Satisfaction of r is insufficient for structural-preservation satisfaction when the required witness is absent or unresolved.*

Proof. The structural preservation architecture is conjunctive in the conditions registered by $\mathfrak{P}$. The comparison relation supplies one condition, while the historical mode requires a typed witness. Satisfaction of the first cannot supply an element of the second. If the required witness family is empty, Equation 18 fails and the structural predicate is undefined on the pair; if historical evidence remains unresolved, applicability likewise remains unresolved. In neither case can comparison alone produce the satisfied value. $\square$

Proposition 16.4 prevents a recurring shortcut in preservation discourse. A newly engineered language model can reproduce a corpus distribution, an independent institution can realize an isomorphic organizational graph, or an unrelated dynamical system can possess a conjugate attractor structure. Those equivalences can be analytically important without establishing historical preservation of the earlier language, institution, or dynamical lineage.

Example 16.5 (Isomorphic systems without preservation provenance). Let X and Y be disjoint copies of the same two-vertex, one-edge labelled graph, and let r be labelled-graph isomorphism. The comparison succeeds because an isomorphism exists. Now choose a historical-continuation specification for which $\text{Hist}_{h_{\text{cont}}}(X, Y) = \emptyset$. Equation 156 then assigns the preservation profile entry $\perp$; neither defined structural value 1 nor 0 applies. The finite example separates structural resemblance from provenance: the graphs can be indistinguishable under the selected comparison while remaining unrelated by the historical mode required for preservation.

Example 16.5 gives a finite instance of the undefinedness side of the three-valued preservation logic: the comparison relation succeeds while the required provenance structure remains absent.

16.5 Preservation without Normative Endorsement

Section 13 separated preservation identification from valuation. The boundary has direct normative significance. A preservation specification can be structurally satisfied and correctly identified while the resulting preservation remains normatively contested, aesthetically disfavored, practically burdensome, politically unequal, or epistemically unimportant for the inquiry at hand.

Let $\mathcal{V}^N$ denote a declared normative valuation map whose domain contains selected preservation results. Structural preservation and positive normative valuation are independent unless an additional normative principle links them. The non-entailment is stated in Equation 157.

$$\text{Pres}_{\mathfrak{P}}^{\text{str}}(X, X') = 1 \not\Rightarrow \mathcal{V}^N(\mathfrak{P}, X, X') > 0. \quad (157)$$

Equation 157 records a logical boundary. It does not assert that preservation lacks normative value. It requires the value claim to be argued through an additional ethical, legal, political, aesthetic, epistemic, or practical framework.

The converse boundary also remains important. A community, institution, or analyst can value a lost or transformed generative capacity and seek its reconstruction even when the taxonomy classifies the current state as non-preservation relative to a strong lineage criterion. Normative reasons can therefore motivate preservation, restoration, revival, compensation, or redesign without altering the structural classification of what has already occurred.

Remark 16.6 (Governance remains downstream). The taxonomy can identify which target, mode, history, and comparison relation a policy intends to preserve. It does not determine who possesses legitimate authority to choose that target, how preservation burdens should be distributed, or when transformation or termination should be permitted. Those questions belong to the later normative and governance programme.

Remark 16.6 preserves the ordering of the wider research programme: ontological and formal clarification supplies objects for normative analysis, while normative analysis supplies criteria that practical governance and engineering can subsequently instantiate.

16.6 Open Formal Problems

The taxonomy leaves several formal questions open. These problems arise from the same features that motivated the framework: partial definedness, evolving histories, changing representations, heterogeneous targets, and historically generated comparison criteria. Table 16 summarizes the principal unresolved areas retained by the present paper.

The final problem in Table 16 is especially important for generativity. A fixed state space or fixed operation family can represent generation only within an already articulated domain. Transformative generativity can modify the distinctions, operators, or carrier through which later possibilities become expressible. A mature preservation theory may therefore require morphisms between model classes, not only trajectories inside one model class.

The open problems also delimit the role of the present taxonomy. The framework supplies a language for separating targets, histories, modes, comparison relations, profiles, and judgments. It does not provide a complete dynamics of generativity, a universal theory of historical identity, or an exhaustive decision procedure for preservation. Those stronger projects remain available as later GR developments built on the distinctions established here.

Table 16: Open formal problems retained by the present preservation taxonomy. The entries identify directions for further GR work; the current definitions retain their stated scope without depending on prior resolution of these problems.

Problem family	Formal issue	Required development
Evolving comparison criteria	r , κ , and tolerances can change historically	History-indexed comparison transport and criteria for continuity of the criterion itself
Changing operator families	Recovery mechanisms can disappear or be regenerated	Higher-order dynamics on $(\omega, \mathcal{F})$ and typed comparison of evolving grammar families
Branching lineages	Several descendants can satisfy different preservation specifications	Non-scalar comparison of preservation profiles over branching histories
Cross-resolution stability	Fine and coarse preservation can diverge	Refinement/coarse-graining covariance and explicit fiber-sensitive recovery conditions
Field realizability	Local capacity or condition records need not glue globally	Obstruction theory for partial restriction, overlap, and contextual gluing structures
Topology-changing histories	Carrier type can change across reconstruction or evolution	Typed morphisms between complexes and algebras with declared invariant transport
Uncertainty and evidence	Historical witnesses and target extractions can remain underdetermined	Set-valued or probabilistic identification layers that preserve alternative compatible formalizations
Preservation of future novelty	Future generative distinctions may be unavailable to present enumeration	Formal treatment of open-ended grammar and carrier expansion without presupposing a closed possibility space

17 Conclusion

This section consolidates the preservation taxonomy developed across the manuscript and identifies the analytical interfaces that the taxonomy supplies for later Generative Relational research. Its role is synthetic. The objective is to restate the paper’s principal distinctions at the level established by the preceding sections, connect the target-specific analyses to the preservation-profile and investigation architectures, and locate the resulting framework within the wider language and generativity programme. The discussion proceeds through typed historical preservation, target-mode profiles, generativity and language interfaces, and downstream normative and practical research. The section introduces no new preservation category and does not strengthen the ontological or normative status of the formal constructions developed earlier.

17.1 Typed Historical Analysis of Preservation

The paper began from a simple source of ambiguity: statements that something has been “preserved” can refer to materially different objects, histories, comparison relations, and standards of continuity. A corpus can remain available after fluent use declines. A relational structure can continue while its manifestations change. A generative capacity can remain historically represented while current conditions prevent its execution. A later reconstruction can recover selected operations after an interruption. A new

realization can reproduce a dynamical regime without recovering the earlier grammar, conditions, or lineage. Ordinary preservation vocabulary can describe each of these cases while leaving the analytical relation underspecified.

The taxonomy responds by treating preservation as a typed historical inquiry. The structural preservation specification introduced in Equation 13 registers the preservation target, source and target status, historical mode, retained content, comparison relation, resolution, and preservation mode before a preservation result is evaluated. This architecture gives preservation a different formal role from equality or equivalence considered alone. Comparison relations supply the machinery by which retained content can be related. Historical or provenance witnesses supply the connection through which that comparison becomes preservation-relevant. Preservation modes then state the form of retention, transformation, recovery, regeneration, or reproducibility being asserted.

This structure permits exact preservation, approximate preservation, recoverability, reconstruction, and dynamical reproduction to occupy one framework without being collapsed into one predicate. It also retains undefinedness as a substantive outcome. A preservation question can fail because a declared comparison is false, or remain unresolved because the target, historical witness, applicability domain, or comparison relation has not become sufficiently articulated. The three-valued discipline used later in the typed preservation profile keeps these cases distinct.

The distinction among ontological reality, relational reality, manifestation reality, and mathematical representation remains integral to this architecture. Ultimate ontology remains suspended. Relational preservation concerns historically persistent and efficacious organization within the adopted GR model. Manifestational preservation concerns context-dependent presentations and traces. Mathematical structures such as labelled 2-complexes, partial transformations, local algebras, fields, attractor-like regimes, cycle maps, and topological invariants provide representations and diagnostics whose adequacy depends on the local preservation question. Their formal preservation can support a preservation analysis without becoming an identity claim about ultimate ontology.

The result is a framework in which preservation can remain historically open. A living system can acquire new events, relations, distinctions, and conditions while selected structures remain continuous. Preservation therefore need not require literal recurrence of an earlier state. The framework can describe continuity through transformation, interruption followed by reconstruction, migration into a new relational environment, branching descendants, and reproducibility of selected system characteristics. These modes preserve different information and can become comparable only after target-specific bridge assumptions are stated.

17.2 Target-Mode Profiles in Cross-Taxonomy Relations

Sections 5–10 developed six principal target families: manifestations and traces, relational structures, generative capacities, generative conditions, historical-lineage or regeneration structures, and dynamical regimes. Their separation is one of the main outcomes of the taxonomy. A single history-bearing formation can supply preservation targets in several families at once, while the preservation status of those targets can diverge.

Manifestational preservation captures symbolic, archival, corpus-level, and other context-dependent records. Its formal limits become visible when the manifestation or recording map is non-injective. Equal traces can then be compatible with distinct relational structures. Relational-structural preservation shifts the target toward historically organized relations, interfaces, and incidence structures. Endpoint structural equivalence can support this analysis while remaining weaker than a historically witnessed lineage through intermediate configurations.

Generative-capacity preservation adds partial operation families, state-dependent schemas, and operational availability. This layer makes the existence of a capacity explicitly dependent on definedness and admissibility. Generative-condition preservation separates the supports under which capacities become realizable, including relational, institutional, environmental, material, and technical conditions. The joint-realization construction of Section 8 shows that separately preserved capacity and conditions

remain insufficient for preservation of realized generativity when their coupling changes.

Historical-lineage preservation then introduces transmission, relocation, interruption, reconstruction, recoverability, and branching descent. One formal result from that analysis is especially important for the hierarchy of preservation modes: under a fixed reflexive and transitive recovery relation, iterated recoverability collapses to ordinary recoverability. A genuinely additional meta-recovery layer therefore requires the recovery-relevant grammar or mechanism family itself to change and later become regenerable. The higher-order state construction records that stronger possibility without presenting it as an automatic property of every generative system.

Dynamical-regime preservation supplies the weakest major preservation target developed in the paper. The target can consist of selected attractor-like organization, basin structure, metastability, transition relations, ordered composition, cycle residuals, fixed structures, topological characteristics, or perturbation-response signatures. Dynamical-structural reproducibility permits those characteristics to reappear under different carriers, grammars, conditions, or local realizations when the declared signature comparison is satisfied. Historical or reconstructive provenance remains separately required before dynamical equivalence is classified as preservation.

Section 11 reorganized these target-specific results by preservation mode. The retained modes include continuous retention, historically continuous transformation, dormant recoverability, reconstructive recovery, recovery-mechanism regeneration, and dynamical-regime reproduction. They preserve different kinds of historical information, so their implications remain local to a target class, historical domain, and bridge structure.

The cross-taxonomy construction in Section 12 makes that locality explicit. A preservation result at one target class can support another only through a declared bridge. Manifestational preservation can lift toward relational preservation when the manifestation architecture is sufficiently discriminating and historically aligned. Relational preservation can support generative-capacity or condition preservation when the relevant extraction maps and transports are compatible. Fine-grained preservation can support coarser preservation under suitable covariance, while coarse preservation cannot generally recover distinctions collapsed by the coarse-graining map.

These relations motivate the typed preservation profile in Equation 122. The profile records satisfied, failed, and undefined specifications together with their complete target, historical, resolution, relation, and mode indices. Preservation becomes a multidimensional result whose structural architecture contains no built-in scalar aggregation. This architecture can describe cases in which archival traces survive, current capacity fails, conditions remain partially available, lineage remains reconstructible, and selected dynamics remain reproducible. A later comparison among such profiles requires an additional judgment structure and cannot be inferred from the profile alone.

17.3 Generativity-Lineage-Language Interfaces

The language stress test in Section 14 illustrates why the taxonomy matters for generative systems. Language preservation can concern corpora, fluent competence, relational conditions of use, intergenerational transmission, diasporic continuation, revival after interruption, or dynamical characteristics of linguistic generation. The categories can support one another while remaining formally distinct.

A corpus can preserve a large manifestation domain and provide important resources for later recovery. The corpus alone does not establish current living generativity when the recording architecture omits relations through which context-sensitive use, correction, trust, pragmatic distinction, or innovation becomes effective. Conversely, a community can retain fluent capacity while many earlier manifestations, institutional domains, or material conditions change. Preservation therefore becomes more informative when the target class and mode are stated explicitly.

The same structure applies to generativity more broadly. A generative capacity can remain available while its earlier conditions disappear. Conditions can remain while competence declines. A transmission mechanism can preserve a route by which new competent participants arise even as the grammar changes historically. Recovery can restore selected capacities after interruption. A later system can reproduce

selected dynamical characteristics while possessing a different lineage. The taxonomy supplies terms for each relation without deciding in advance which one should receive normative priority.

The distinction between grammar or capacity and generative conditions also supports later GR work on open-ended generation. The present formalism can represent state-dependent schemas and evolving operator families, while Section 16.6 records the limit of any model whose possibility space is fixed in advance. Transformative generativity can alter the distinctions, carriers, or operation families through which future possibilities become expressible. Preservation of such open-ended capacity may therefore require morphisms between model classes and criteria for continuity of evolving grammars in addition to trajectories inside any one articulated state space.

The bounded language application also clarifies the role of anthropology and ethnography in the later programme. A preservation investigation can require evidence about situated practice, shared history, institutional support, interaction, and transmission that a corpus does not fully encode. Historical and ethnographic methods can therefore contribute provenance and relational evidence, while formal methods characterize which preservation claims follow after those objects have been specified. The GRPI procedure in Section 15 is designed to retain that methodological plurality.

17.4 Normative-Practical Research Interfaces

The preservation taxonomy deliberately stops before a general ethics of preservation. Section 13 separates structural preservation from identification and valuation. A pair of structures can satisfy a preservation specification while participants dispute whether that specification captures the continuity that matters to them. A correctly identified preservation result can also receive different aesthetic, epistemic, historical, normative, practical, or political-economic valuations. These judgment channels remain additional structures whose criteria can themselves develop historically.

This separation provides an interface for later work on language rights, multilingual democracy, intergenerational responsibility, authority over standardization, preservation burdens, and termination. A future normative inquiry can begin by specifying which preservation target and mode it values. It can then ask who has standing to define the relevant criterion, how present participants and future persons enter the judgment, which asymmetries shape the available options, and how preservation costs are distributed. The taxonomy supplies the object of that normative argument without determining its conclusion.

The same architecture supports a political economy of generativity. Educational systems, labor markets, media, scientific institutions, digital platforms, and AI systems can alter generative conditions, operation domains, transmission paths, and the comparative accessibility of different linguistic or cultural futures. Those effects can be represented as changes in preservation-relevant targets without converting economic usefulness, institutional support, or technological availability into a complete preservation metric. Later work on power over generativity can use the existing GR power formalism to study who or what shapes the grammars, conditions, infrastructures, and reachable regimes through which future generation occurs.

Practical research also receives a more precise target. Engineering infrastructure for language preservation can be designed around a declared preservation profile, with corpus accumulation serving as one component among several possible preservation targets. An archive can prioritize manifested traces; a transmission platform can support capacity and lineage; a community infrastructure can sustain conditions of use; an adaptive technical system can support revision and innovation; a recovery-oriented architecture can retain evidence and mechanisms needed for later reconstruction. The taxonomy allows these projects to state which preservation achievements they seek and which remain outside their scope.

The Generative Relational Preservation Investigation defined in Equation 152 provides the corresponding methodological interface. A domain-specific inquiry can register its object, comparison domain, preservation specifications, evidence, bridge assumptions, direct profile, and unresolved components. Formal, historical, ethnographic, institutional, and computational methods can contribute different evidence to the same typed record. The procedure does not eliminate disagreement, underdetermination, or normative conflict. It makes their location visible within the preservation claim.

The paper’s principal contribution is therefore classificatory infrastructure for a larger research programme. Preservation becomes a family of explicitly typed historical relations whose targets, modes, resolutions, comparison structures, and judgments can be distinguished before stronger claims are made. This framework creates a common vocabulary for later GR inquiries into language, culture, generativity, power, ethics, law, governance, anthropology, education, and engineering while retaining the possibility that different domains require different preservation relations.

The remaining work is correspondingly plural. Formal research can develop evolving grammars, cross-resolution stability, topology-changing histories, and open-ended generativity. Empirical and interpretive research can identify how preservation targets are instantiated in concrete communities and institutions. Normative research can investigate authority, value, burden, justice, and termination. Practical research can design infrastructures that support the selected preservation targets while leaving room for historical transformation. The taxonomy supplies a disciplined starting point for those inquiries by requiring preservation claims to state what is being preserved, through which historical relation, under which comparison, and with what inferential scope.

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