A Preliminary Triadic Formalism of Volunteer Relations in Generative Relational Theory

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Abstract

This paper develops a preliminary triadic formalism for volunteer and support relations within Generative Relational Theory (GR). The addition of a third history-bearing subject or relational position can change the formal character of a support relation even when its retained pairwise records remain fixed. A third position can witness, mediate, translate, authorize, broker, publicly recognize, coordinate, review, or otherwise reorganize the relation between the first two. The paper therefore studies when a full triadic configuration contains relational distinctions that are not recoverable from a declared family of pairwise restrictions. Three GR subject-spacetimes are represented within a coupled history-bearing configuration; ontological reality remains suspended, while relational and manifestation realities retain historically variable mappings. The construction extends dyadic manifestation, epistemic access, theory-of-mind, action-provenance, evaluative, authorization, asymmetry, algebraic, and historical records to a triadic setting. It introduces pairwise shadows, reconstruction criteria, third-position functions, route-sensitive epistemic records, witnessing and publicity, reputation, accountability, brokerage, coalition, distributed provenance, and emergent triadic structures. A relation-valued equivalence gap distinguishes pairwise sufficiency from model-relative nonrecoverability without treating thirdness as a scalar or metaphysical primitive. Labelled 2-complex and spin-foam-inspired realizations remain optional representation layers. The contribution is a formal basis for distinguishing pairwise-sufficient triadic organization from pairwise-nonrecoverable structure while preserving the epistemic, normative, empirical, and ontological limits required by later work on institutions, justice, ethics, power, and historical relational dynamics.

Keywords: volunteer relations; Generative Relational Theory; triadic formalism; pairwise recoverability; third-position functions

Discussion Paper Note

This manuscript is the second formal paper in a broader Generative Relational research programme on volunteer and aid relations. Its immediate task is to identify what becomes formally available when the minimal dyad is enlarged by one further history-bearing subject or relational position. The added position may be occupied by a person, group, organization, institution, community, state, technical system, or another relational being at the declared analytical resolution. The triadic restriction is methodological: it is intended to reveal structures that cannot be represented adequately by treating the system as three independent dyads, before larger networks and institutional fields are introduced.

The paper inherits the three-level reality discipline, history-bearing subject-spacetimes, manifestation and epistemic distinctions, action provenance, authorization, typed asymmetry, local algebraic dynamics, and historical revision architecture developed for the dyadic case. Those structures are treated as reusable formal resources rather than as conclusions that settle the triadic problem. The new burden concerns triadic organization that may exceed a declared pairwise representation: mediation, witnessing, public recognition, accountability, indirect trust, reputation, brokerage, coalition, triadic closure, and other structures whose significance can depend on simultaneous organization across three positions.

The mathematical resources remain representational. Pairwise projections, reconstruction maps, labelled 2-complexes, local algebras, categorical diagrams, spin-foam-inspired histories, and related structures do not establish the ultimate ontology of social or volunteer relations. GR preserves ontological suspension while asking which representations make triadic dependence, higher-order relational structure, and historical transformation formally inspectable.

The paper also keeps formal nonrecoverability separate from metaphysical irreducibility. A triadic diagnostic can contain information absent from a declared family of pairwise records without establishing that no richer microscopic account could recover it. The intended contribution is therefore model-relative and revisable: it supplies criteria for asking when the third position changes the formal object of analysis rather than merely adding another pairwise relation.

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Status. This discussion paper contains a deeply revised preliminary triadic formal inquiry within the wider GR volunteer-relations programme. Revision Round 1B has been completed across Sections 1–24, followed by an initial whole-manuscript synchronization pass. The definitions, representations, examples, and proposed criteria of triadic nonrecoverability remain open to revision, including further cross-paper notation and literature review. Circulation at this stage does not constitute peer review, empirical validation, or normative settlement.

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Statement on the use of language models. The exploratory discussions and preparation of this paper involved OpenAI's ChatGPT. ChatGPT supported exploratory dialogue, formal reconstruction, argumentative criticism, drafting in LaTeX, and compilation and consistency checks. External scholarly references, when later introduced, will be verified from online sources before insertion. The author selected the research questions, directed and approved the theoretical commitments and epistemic status of the claims, and bears sole responsibility for the manuscript, including its definitions, formal constructions, arguments, conclusions, and errors. Authorship credit remains with the human author.

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1 Introduction

Volunteer and support relations become formally different once a third position is present. In a dyadic description, a supporter and a supportee may already require substantial structure: historically constituted subject boundaries, manifestation and epistemic access, action provenance, evaluative judgment, consent and refusal, asymmetry, and the possibility that repeated interaction generates trust, recognition, dependence, or other relational objects. A third subject does more than increase the number of such pairwise records. The third position can witness an action between the first two, mediate their disagreement, relay or filter information, authorize a procedure, translate manifestations, stabilize a reputation, create accountability, broker access, coordinate a coalition, or alter the public meaning of conduct that would have had a different relational trajectory in private. The resulting configuration can therefore contain information about the organization of the three positions that is absent from the pairwise relations considered separately.

This possibility supplies the conceptual motivation for the present paper. A simple extension from two subjects to three might begin by recording the dyads AB, AC, and BC and then treating the triad as their union. Such a representation is sufficient when the substantive question is exhausted by those pairwise structures. It becomes insufficient when the relation between A and B depends on how C is simultaneously positioned relative to both, when C observes or mediates an A–B event, when an A–C relation changes the meaning or credibility of a B–C manifestation, or when an action jointly generated by A and C reorganizes the continuation space available to B. The formal problem is therefore not simply how to append a third node. It is how to distinguish pairwise-sufficient triadic configurations from configurations whose retained relational distinctions are pairwise nonrecoverable under a declared representation and equivalence structure.

Generative Relational Theory (GR) makes this problem especially salient because the three subjects are not treated as pre-existing point entities. At a declared analytical resolution κ, the labels A, B, and C refer to history-bearing relational subject-spacetimes represented provisionally by subconfigurations $\mathcal{K}^{\kappa}_{A}$, $\mathcal{K}^{\kappa}_{B}$, and $\mathcal{K}^{\kappa}_{C}$ within a coupled triadic configuration $\mathcal{K}^{\kappa}_{ABC}$. The subject subconfigurations may overlap, share historically constituted material, and change their represented boundaries across time or resolution. The labels identify tracked positions within the model; they do not assert an ultimate metaphysics of three self-contained substances. The same discipline applies to a third position occupied by an organization, institution, community, state, or technical system. What matters for the formalism is whether the represented formation can be tracked as a sufficiently persistent and generatively consequential relational being within the selected analysis.

The triadic paper retains GR's three-level reality architecture. Claims about ultimate ontological reality remain suspended. Relational reality concerns the historically constituted and causally or generatively efficacious structures that the model seeks to represent. Manifestation reality concerns the context-dependent forms through which those structures become presented, expressed, observable, recorded, symbolized, or actionable. The addition of C makes the relation between these levels more complex because a manifestation can be addressed to one subject, observed by another, reinterpreted by a third, and subsequently fed back into the relational configuration. A private request from B to A and the same request witnessed by C need not have the same later effects. A statement from A may acquire a different meaning when translated by C. A refusal may become publicly consequential when C records or recognizes it. The triadic architecture must therefore track not only three subject-relative reconstructions but also the routes through which manifestation and interpretation pass among them.

The present paper inherits several formal disciplines established for the dyadic case while treating the triadic extension as a new research object. Observation remains distinct from intervention. Manifestation remains distinct from the relational structure manifested. Theory-of-mind records remain subject-relative reconstructions rather than transparent access to another subject. Need, value, aesthetic judgment, and normative judgment remain separately typed. Consent, refusal, standing, and authority remain distinct. Difference, asymmetry, power, and injustice remain non-equivalent analytical categories. Algebraic, field-theoretic, categorical, and spin-foam-inspired structures remain conditional modeling resources

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under ontological suspension. These inheritances reduce unnecessary redefinition, but they do not determine which triadic structures exist or whether the pairwise records are sufficient to reconstruct them.

The central formal question is therefore one of pairwise recoverability. Let the full triadic configuration at resolution κ be denoted by $\mathcal{K}^{\kappa}_{ABC}$. Suppose the model supplies three well-typed restriction or projection operations that retain the pairwise structures relevant to the analysis. The resulting pairwise shadow is introduced schematically in Equation 1.

$$\mathsf{P}^{\kappa}_{2}(\mathcal{K}^{\kappa}_{ABC}) := \big(\mathsf{r}^{\kappa}_{AB}(\mathcal{K}^{\kappa}_{ABC}), \mathsf{r}^{\kappa}_{AC}(\mathcal{K}^{\kappa}_{ABC}), \mathsf{r}^{\kappa}_{BC}(\mathcal{K}^{\kappa}_{ABC})\big). \tag{1}$$

Equation 1 is deliberately abstract. The restrictions r^{κ}_{ij} may later act on labelled complexes, epistemic records, local algebras, procedures, histories, or another declared carrier. Their role is to record what remains visible when the full three-positioned configuration is viewed only through its pairwise components.

A triadic structure becomes formally interesting when pairwise agreement does not guarantee equivalence of the full configurations relative to the retained criterion. Let $\sim_{\kappa}$ denote the declared equivalence relation on full triadic configurations and let $\sim_{2,\kappa}$ denote pairwise equivalence induced by the three retained restrictions. The minimal nonrecoverability condition is stated in Equation 2.

$$\mathcal{K}^{\kappa}_{ABC} \sim_{2,\kappa} \widetilde{\mathcal{K}}^{\kappa}_{ABC} \quad \text{and} \quad \mathcal{K}^{\kappa}_{ABC} \not\sim_{\kappa} \widetilde{\mathcal{K}}^{\kappa}_{ABC}. \tag{2}$$

Equation 2 does not assert metaphysical irreducibility. It states that, relative to the chosen pairwise records and equivalence criterion, two full configurations can remain distinguishable even though all retained pairwise projections agree. A later section will refine this condition through diagnostic-relative criteria and explicit reconstruction obligations. The point at the introduction is narrower: pairwise-nonrecoverable triadic organization names a possible surplus of retained relational distinction over a declared pairwise representation, not a property attached to the number three itself.

The distinction is important for volunteer relations because several familiar third-position functions are naturally expressed in this form. Mediation concerns more than the sum of A–C and B–C interaction when C transforms the admissible procedure or interpretive relation between A and B. Witnessing becomes triadically consequential when C's observation or acknowledgment changes later expectations, accountability, reputation, or the manifestation conditions under which the A–B relation proceeds. Brokerage concerns a structure in which access between A and B depends on C's control of a communicative, material, institutional, or interpretive path. Indirect trust arises when A's orientation toward B depends partly on a trust relation with C and on C's manifestation concerning B. Coalition appears when two positions coordinate in a way whose consequence for the third cannot be reconstructed by assigning independent pairwise actions. Public recognition arises when the third position changes a private relational event into one with a wider record, audience, or standing. These structures need not occur in every triad, but the formalism must be capable of distinguishing their presence from an ordinary collection of dyads.

The third position also alters epistemic structure. In the dyadic case, one subject reconstructs another through manifestations, observations, memories, and historically conditioned interpretation. In the triadic case, knowledge can travel indirectly. A may model B partly through a manifestation produced by C; C may model the relation between A and B rather than either subject in isolation; B may act on a belief about what C believes concerning A; and all three may distinguish private, shared, witnessed, publicly acknowledged, or strategically concealed material. Higher-order theory-of-mind therefore becomes indexed not only by depth but also by route. The difference between A's direct reconstruction of B and A's reconstruction of B through C can matter even when the final propositional content appears similar, because provenance, credibility, standing, and later revisability may differ.

A related change concerns manifestation and publicness. A manifestation accessible to all three subjects is not automatically a public norm or common-knowledge object, yet the third position makes new degrees of publicity formally available. A private A–B interaction can become witnessed, jointly acknowledged, recorded, reputationally consequential, or institutionally legible through C. These distinctions matter for support because gratitude, refusal, dependency, competence, vulnerability, or alleged misconduct can acquire different relational force when they are manifested before a third position. The

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paper will therefore treat witnessing, acknowledgment, reputation, and accountability as typed triadic objects rather than as informal commentary added after the dyadic analysis.

Action provenance likewise becomes more complex. A triadic action may be initiated by one subject, executed by another, authorized by a third, jointly generated by two, manifested to all three, and interpreted differently by each. A mediator can generate an A–B procedure without executing the substantive support action. An organization occupying C can authorize or constrain an action performed by A toward B. A witness can alter the later consequence of an action without having generated or executed it. The paper therefore extends provenance records to include third-position contribution, route-sensitive generation, and action families whose source cannot be represented adequately by a single actor label.

The same extension applies to normative and evaluative formation. A and B may disagree about B-relative need or the value of an intervention while C introduces another judgment, translates one side's reasons, supplies procedural standing, or stabilizes a category through recognition. The presence of C can widen the available normative grammar, but it can also narrow it through gatekeeping, reputational pressure, interpretive authority, or coalition. The triadic model will therefore preserve the separation among evaluative formation, normative validity, authority, and power. Third-party agreement does not settle moral correctness. Public recognition does not establish legitimacy. Mediation does not guarantee neutrality. Accountability can be enabling or coercive. The formalism records these structures before later ethical and justice papers evaluate them.

The mathematical architecture remains layered. The paper begins from history-bearing triadic configurations and partial restriction maps before introducing stronger algebraic structure. Local algebras can represent admissible quantities and transformations on selected relational regions. Category-theoretic structures become useful when mediation, translation, or route composition requires explicit typing across heterogeneous local domains. A labelled 2-complex supplies a particularly natural discrete realization because pairwise relations can be represented along interfaces while higher-order historical structure can be supported on faces or distributed subcomplexes. A face is not identified automatically with a social triad; the cellular architecture provides a place where a full-triad event or compatibility condition can be represented when the chosen model warrants that assignment. Spin-foam-inspired history language may then organize alternative labelled histories and boundary data without transferring quantum-gravitational ontology.

The paper has six principal objectives. First, it extends the history-bearing dyadic architecture to three tracked GR subject-spacetimes while preserving the three-level reality discipline and model-relative subject boundaries. Second, it formalizes pairwise shadows and criteria of pairwise sufficiency or nonrecoverability. Third, it develops a typed taxonomy of third-position functions, including mediation, witnessing, translation, brokerage, public recognition, accountability, coalition, and indirect trust. Fourth, it constructs route-sensitive epistemic and theory-of-mind records for direct and indirect access. Fifth, it develops triadic procedure, provenance, authorization, asymmetry, emergence, and local algebraic composition without collapsing them into pairwise sums. Sixth, it supplies labelled 2-complex and finite triadic realizations that make the proposed distinctions formally inspectable.

The intended contribution is correspondingly limited and foundational. The paper does not claim that all social or volunteer relations are irreducibly triadic, that every third party improves a dyadic relation, or that mediation, witnessing, reputation, or accountability are inherently beneficial. It does not derive a theory of justice, ethics, institutional legitimacy, or power from the mere presence of three positions. It does not model arbitrary networks by extrapolating from $n = 3$, and it does not treat a third institution-like subject as equivalent to the full institutional environment in which it is embedded. The triadic restriction is a controlled enlargement of the dyadic case designed to identify the first class of structures whose formal meaning depends on simultaneous organization beyond a single pair.

The paper also retains the verification discipline of the wider GR programme. A formally defined nonrecoverability witness, mediation map, witness record, coalition structure, or face-supported higher-order relation establishes only what follows within a declared model. Empirical use requires justified mapping from observed interactions to subject boundaries, pairwise restrictions, third-position functions, event histories, and manifestation records. Normative use requires independent criteria appropriate to

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justice, ethics, responsibility, or legitimacy. Ontological interpretation remains suspended. A result that disappears under reasonable changes of pairwise restriction, resolution, or representation should be reported as representation-sensitive rather than elevated into a general property of volunteer relations.

The manuscript proceeds by first clarifying the analytical scope and the reducibility problem specific to the triad. It then states the dyadic structures inherited by the paper and defines the coupled three-subject record, pairwise restrictions, and the three-level reality architecture. A comprehensive triadic concept inventory separates dyadic carryovers from genuinely new objects. Subsequent sections develop third-position functions, indirect epistemic routes and recursive theory of mind, witnessing and publicity, reputation and recognition, accountability and authorization, brokerage and coalition, triadic procedure and action provenance, typed asymmetry, and emergent relational structure. Later formal sections develop local algebraic and categorical composition, labelled 2-complex realization, spin-foam-inspired histories, and finite triadic illustrations. The final sections examine limitations and research interfaces. The guiding question throughout is whether the full triadic configuration contains relational organization that a declared family of pairwise records cannot recover, and what that difference permits the volunteer-relations programme to represent.

2 Triadic Volunteer Relations: Analytical Scope, Reducibility Burden

This section specifies the local research object that the remainder of the paper will formalize. Its role is to distinguish the presence of three tracked subjects or relational positions from the stronger claim that the resulting configuration contains pairwise-nonrecoverable triadic organization relative to the declared model. The objective is therefore twofold: to delimit which volunteer/support situations belong to the present paper and to state the burden carried by any claim that a triadic configuration either can or cannot be reconstructed from its pairwise records. The discussion proceeds through the local meaning of a third position, the inheritance of the dyadic support relation, the distinction between three-subject presence and triadic structure, the pairwise-shadow problem, the functions that make a third position relationally consequential, the methodological boundary of the paper, and the formal obligations generated by that boundary. The method is conceptual and reconstructive. No irreducibility claim is accepted merely because three subjects are present, and no pairwise reduction is accepted merely because three dyads can be written down.

Table 1 summarizes the principal burdens concealed by the ordinary description that a third party enters a volunteer/support relation. The table is intentionally pre-formal. It identifies the distinctions that later sections must type more precisely without deciding in advance which mathematical structure is required in every application.

2.1 Third-Position Presence: Triadic Relevance

The minimal domain of this paper contains three tracked GR subjects or relational positions, denoted A, B, and C. The labels identify model-relative historical objects rather than self-contained substances. At a declared resolution κ, the paper will later represent them through history-bearing subconfigurations within a coupled triadic configuration. The present section uses the labels more lightly: A and B name the two positions already sufficient to state a dyadic support relation, while C names an additional position whose presence may or may not change the formal character of that relation.

The phrase *third position* is therefore more general than *third person*. In some applications C can be another volunteer, recipient, family member, peer, translator, observer, coordinator, mediator, or professional. In others, the tracked position can be occupied by an organization, community, institution, public authority, platform, or other formation that the model treats as a sufficiently persistent and generatively efficacious relational being. The paper does not require all such cases to share one ontology or one mechanism. It requires only that the selected third position be trackable within the declared relational history and that its effects be typed rather than inferred from a label.

Table 1: Principal formal burdens introduced by a third position in a volunteer/support relation.

Apparent term	**Hidden formal question**	**Reason the question remains open**
Third subject	What permits C to be tracked as one relational being across the relevant history?	The third position may be a person, organization, institution, community, state, or technical formation whose represented boundary is model-relative.
Third party	Is C merely present, or does C alter the A–B relation?	Presence alone does not establish mediation, witnessing, brokerage, publicity, accountability, or any other triadic function.
Pairwise relation	Which information survives in the AB, AC, and BC restrictions?	Pairwise restrictions can omit route, ordering, publicness, common acknowledgment, or higher-order organization.
Triadic structure	What is retained by the full configuration that is absent from the pairwise shadow?	Triadic novelty is meaningful only relative to declared carriers, restrictions, diagnostics, and equivalence criteria.
Mediation	Does C merely communicate with both sides, or does C change the admissible A–B procedure?	Two pairwise conversations need not reconstruct the mediating transformation of the relation between the first two positions.
Witnessing	Does observation by C change later expectations, reputation, accountability, or interpretation?	A witnessed event can have different relational consequences from an otherwise similar private event.
Translation	Does C relay information, transform manifestation, or alter intelligibility?	The route $A \to C \to B$ can preserve, distort, enrich, or reframe information in ways not captured by endpoint records alone.
Brokerage	Does access between A and B depend on C?	A third position can condition who can reach whom, when, and through which channels.
Public recognition	Does C convert a private relation into a socially consequential one?	Recognition by a third position can stabilize status, reputation, standing, or shared expectations.
Accountability	Who can call whom to account, relative to what witnessed or recorded history?	Accountability can depend on a third position's observation, memory, standing, or authorization.
Coalition	Is a coordinated relation among three positions reducible to three independent dyads?	Joint alignment or exclusion can depend on a configuration-level pattern absent from isolated pairwise records.

The presence of C is analytically weaker than triadic relevance. A third participant can coexist with an A–B support relation while remaining irrelevant to the retained object of study. For example, C may be physically present yet neither observe nor alter the interaction; C may have independent relations with A and B that do not condition their mutual relation; or the selected research question may deliberately ignore every consequence of C's presence. Such a case belongs to a three-subject description without thereby requiring a full-triad nonrecoverability analysis.

Triadic relevance begins when the represented state, meaning, possibility, or consequence of one relation depends on the simultaneous organization of all three positions. The dependence can be epistemic, procedural, normative, reputational, material, institutional, or historical. If C witnesses an A–B act and the witnessing changes what can later be denied, remembered, blamed, praised, or publicly recognized, then the third position is relationally consequential. If C translates a request from B to A and the translation changes the intelligible content available to A, the route through C becomes part of the epistemic history. If C controls access by which A can reach B, the third position becomes part of the continuation

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structure. If A and C jointly coordinate an intervention whose meaning for B depends on their alignment, the resulting structure cannot be described adequately by naming only the three participant labels.

The present paper therefore distinguishes four levels: three-subject presence, third-position function, full-triad organization, and pairwise nonrecoverability. Three-subject presence records the existence of three tracked positions. A third-position function records a specified way in which one position participates relative to a focal pair. Full-triad organization records a configuration-level relation, procedure, audience, route, or historical coupling whose definition refers to the coupled ABC record; such an object can still be recoverable from sufficiently rich pairwise records. Pairwise nonrecoverability is the stronger condition in which the distinctions retained by the full analysis cannot be reconstructed from the declared pairwise representation. This four-level distinction allows the formalism to represent cases in which C matters and a full-triad object exists while the overall analysis remains pairwise sufficient, as well as cases in which the same kind of third-position function yields a nonrecoverability witness under a different representation or diagnostic.

2.2 Dyadic Inheritance: Triadic Extension

Paper I established a substantial formal burden before the dyadic statement "A supports B" could be treated as more than ordinary-language shorthand. Subject persistence, relational roles, manifestation, epistemic access, theory of mind, action provenance, interactive procedure, need and value judgment, authorization, typed asymmetry, local algebraic dynamics, historical revision, and emergent relational structure were all kept distinct. The triadic paper inherits those disciplines for each pairwise relation. It does not reset the analysis to three point actors joined by three untyped edges.

The inheritance is structural rather than duplicative. The AB restriction can contain an A–B support procedure; the AC restriction can contain a witness or coordination relation; the BC restriction can contain translation, trust, dependence, or another history. Each dyad retains the possibility of distinct manifestation records, epistemic reconstructions, role assignments, action provenance, authorization states, asymmetries, and emergent objects. The triadic question begins only after these pairwise structures have been made available.

The extension introduces a new problem because a relation that is pairwise well typed can acquire a different status when embedded in the triad. Consent manifested by B to A can become publicly consequential when C observes and records it. A private refusal can acquire institutional force when C has standing to recognize it. A claim by A about B can gain or lose credibility when C supplies independent testimony. A transfer from A to B can become reputationally visible when C publicizes it. A request routed through C can differ from a direct request even when the final propositional content appears similar. The pairwise records therefore remain necessary while becoming potentially insufficient.

This point also affects role assignment. The labels supporter, supportee, mediator, witness, translator, broker, coordinator, or authority are local relational positions rather than complete identities. C need not remain in one third-position role. The same subject can witness one episode, mediate another, broker access later, and become a co-supporter in a subsequent procedure. Likewise, A and B can exchange roles or occupy several dimensions simultaneously. The triadic formalism must therefore represent role trajectories and function-specific participation without converting the triangle into a static role taxonomy.

The three-level GR architecture is inherited in the same way. Ontological reality remains suspended. Relational reality concerns the history-bearing structures through which the three positions become sufficiently persistent and efficacious to affect later possibilities. Manifestation reality concerns how those structures become presented or accessible to one, two, or all three positions. The third subject increases the number of possible manifestation routes. A manifestation can be directed from A to B while being accessible to C; generated by C from an earlier B manifestation; addressed to the public subset $\{A, C\}$ while excluded from B; or transformed by C before becoming available to A. The formalism must therefore track routes and accessibility without treating the triad as three independent dyadic channels.

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2.3 Pairwise Shadow: Reconstruction Burden

The central analytical burden of the paper can be stated as a reconstruction problem. Let $\mathfrak{C}_3^\kappa$ denote the ambient class of full triadic configurations admitted at resolution κ. Let $\mathfrak{D}_{P_2}^\kappa \subseteq \mathfrak{C}_3^\kappa$ denote the domain on which all three declared pairwise restrictions are defined, and let $\mathfrak{D}_3^\kappa \subseteq \mathfrak{D}_{P_2}^\kappa$ be the comparison domain selected for a particular reducibility question. The introduction's schematic pairwise-shadow operation is therefore a partial map on the ambient configuration class and a total map on the retained comparison domain.

At this scope-setting stage, before pairwise equivalence classes have been defined, the strongest exact-shadow reconstruction claim can be written using a raw descriptor. Let $\sim_\kappa$ be the declared equivalence on full triadic configurations. A raw reconstruction map has the form shown in Equation 3.

$$\mathsf{Rec}_{2,\mathrm{raw}}^\kappa : \mathsf{P}_2^\kappa(\mathfrak{D}_3^\kappa) \rightharpoonup \mathfrak{D}_3^\kappa/\!\sim_\kappa . \tag{3}$$

Equation 3 asks whether an exact realized pairwise descriptor determines the retained full-equivalence class. It is deliberately preliminary. Section 6 introduces equivalence relations on the pairwise records themselves and replaces the raw descriptor by a quotient-level shadow class; that quotient criterion is the paper's definitive pairwise-sufficiency test.

The preliminary exact-shadow burden is stated in Equation 4.

$$\mathsf{Rec}_{2,\mathrm{raw}}^\kappa(\mathsf{P}_2^\kappa(X)) = [X]_{\sim_\kappa} \qquad \text{for every } X \in \mathfrak{D}_3^\kappa. \tag{4}$$

Equation 4 is therefore a scope-level exact-shadow requirement, not yet the final quotient-level notion of pairwise sufficiency. A model that treats the triad as exhausted by its dyads should ultimately specify which pairwise records are retained, which compatibility conditions make a triple admissible, which pairwise distinctions count as equivalent, which reconstruction rule is used, and which full-configuration differences are ignored by the chosen equivalence. Without those declarations, "the triad is just three dyads" is a modeling presumption rather than a demonstrated reduction.

The converse burden applies to claims of triadic novelty. A full configuration should not be called irreducibly triadic merely because it is represented with three labels or one triangular diagram. The model should exhibit some retained structure that survives in the full configuration while remaining unrecoverable from the pairwise shadow. Equation 2 gives the minimal witness form. Later sections will refine it by specifying the diagnostic family, the admissible comparison class, and the substantive meaning of the difference.

This reconstruction framing keeps both directions open. Some volunteer configurations may be pairwise sufficient. For example, if C independently supports A and B while none of the three relations conditions another, the pairwise records may recover everything the selected analysis retains. Other configurations can fail pairwise reconstruction because a witness relation, mediation procedure, public acknowledgment, brokerage path, or coalition structure contributes information only at the three-position level. The paper is therefore not committed to triadic novelty in every three-subject system. It provides a language for identifying when the stronger claim is earned.

2.4 Third-Position Functions: Simultaneous Conditioning

Several recurring third-position functions motivate the formal extension. The later taxonomy will type them more carefully, but their analytical roles can already be distinguished. Mediation concerns a third position that changes the procedure, interpretive relation, or admissible continuation between the first two positions. Witnessing concerns a third position whose observation, acknowledgment, or record changes the later relational significance of an event. Translation concerns a third position that transforms manifestation or interpretation across an epistemic route. Brokerage concerns a third position that conditions access, resources, information, or participation between the other two. Public recognition concerns a third position whose acknowledgment contributes to reputation, standing, status, or shared expectation. Accountability concerns a third position that makes a claim, action, promise, refusal, or harm answerable

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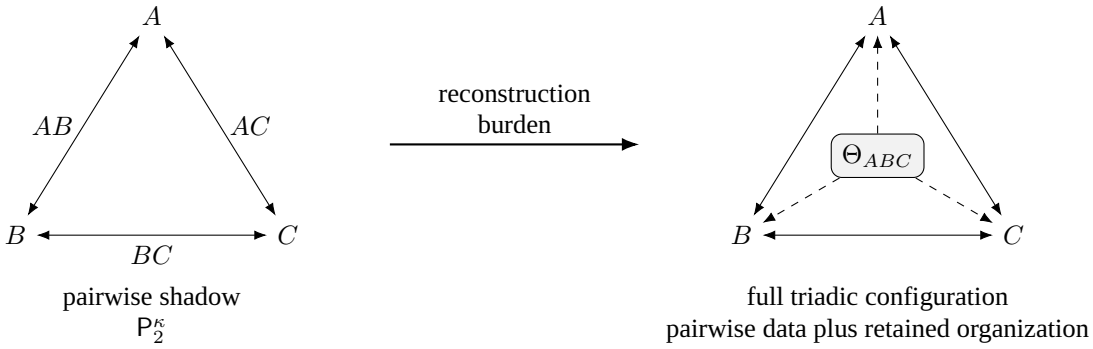

Figure 1: Analytical distinction between the pairwise shadow and a full triadic configuration. The placeholder Θ_{ABC} denotes only a possible retained three-position structure; later sections replace it with typed epistemic, procedural, normative, algebraic, or geometric constructions where justified.

within a broader relational field. Coalition and coordination concern configurations in which alignment among two positions changes the possibilities of the third.

The common feature is simultaneous conditioning. The A–C and B–C relations are not merely two independent channels when the state of one conditions how the other operates and both jointly alter the A–B relation. A mediator trusted by A but distrusted by B occupies a different triadic position from one trusted by both even if the pairwise labels "mediator–A" and "mediator–B" appear superficially similar. A witness whose acknowledgment is recognized by A but not by B generates a different publicity structure from a witness whose standing is jointly accepted. A broker who controls the only admissible route between A and B differs from a broker who merely offers one optional route among many. The triadic object therefore depends on how the pairwise relations are composed, constrained, and interpreted together.

This simultaneous conditioning can occur without centralized control. Three participants can generate a coordination convention through repeated adjustment; a reputation can stabilize because C repeatedly relays information between A and B; a shared norm can emerge because each participant models the reactions of the other two; a refusal can gain force because the third position makes it commonly acknowledged; or an exclusionary coalition can emerge through mutual reinforcement among pairwise expectations. The paper therefore does not define triadic structure by the presence of an authoritative third party. Thirdness can be distributed, procedural, epistemic, symbolic, or historical.

Figure 1 summarizes the distinction between a pairwise shadow and a full triadic configuration. The left panel represents the three retained dyadic restrictions. The right panel adds a configuration-level triadic relation Θ_{ABC}, used here only as a schematic placeholder for whatever higher-order organization later sections make explicit. The figure does not assert that every triad contains such an object.

2.5 Triadic Volunteer Domain: Methodological Boundary

The paper uses *triadic volunteer relation* for a modeled configuration in which volunteer/support-relevant processes involve three tracked positions and the research question retains at least one relation among those positions that can affect the support process. The term does not require that all three participants be volunteers, that one participant permanently occupy a supportee role, or that the third position be neutral. C can support, oppose, translate, mediate, certify, obstruct, witness, coordinate, or otherwise participate. Likewise, A and B can exchange support roles or jointly orient action toward C. The triadic designation refers to the architecture of the retained relation, not to a fixed moral division of labour.

This scope includes several families of situations. One family contains a supporter, supportee, and mediator. Another contains a supporter, supportee, and witness whose observation changes later accountability or reputation. A third contains two supporters coordinating around one supportee. A fourth contains one supporter and two supportees whose relation to one another affects allocation, comparison,

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or recognition. A fifth contains a volunteer, recipient, and organization whose procedures condition access and authorization. A sixth contains a person-to-person support relation mediated by a technical or institutional system that is modeled as a third relational being. These families are analytically different, and later sections avoid treating one as the universal triadic prototype.

The boundary excludes larger structures when their additional positions are required for the mechanism being studied. A donor–organization–volunteer–recipient chain is quadradic at the retained level if the donor's independent position matters. A public volunteer programme involving community, state, organization, volunteer, and recipient cannot be compressed into one triad without specifying which positions are aggregated and what information that aggregation discards. The triadic model can still serve as a local restriction inside such systems, but it should not be presented as a complete representation of the wider structure.

The boundary also excludes full institutional theory. The paper can model a third position occupied by an organization or authority, but it does not yet provide a general theory of organizational persistence, collective agency, legal personhood, bureaucratic procedure, or institutional legitimacy. Likewise, it can represent public recognition or accountability without supplying a complete political theory of publicity, democratic authorization, or sanction. These questions belong to later papers or domain-specific applications.

The methodological boundary is therefore minimal but nontrivial. Three positions are sufficient to introduce route dependence, third-party observation, mediation, brokerage, public acknowledgment, coalition, and pairwise nonrecoverability. They are still few enough that the provenance of each relation, manifestation route, and procedure can be inspected directly. Paper II uses this balance to isolate structures that are unavailable in the dyadic model without yet entering the combinatorial and institutional complexity of general networks.

2.6 Reducibility Failure Modes

A pairwise reduction can fail for several distinct reasons, and the paper should not collapse them into one undifferentiated notion of "higher-order interaction." Table 2 records the main families that later sections will formalize. The categories are analytical rather than exhaustive.

The table separates epistemic, procedural, normative, reputational, and geometric failures because they require different evidence and different mathematics. A route-sensitive epistemic effect can be represented without invoking topological structure. A face-level 2-complex distinction can exist even when the application carries no public-reputation mechanism. A coalition can matter without any mediator. Treating all of these as one "triadic effect" would reproduce the same overcompression that the paper seeks to avoid in its pairwise analysis.

The reducibility burden also depends on resolution. At a coarse resolution, a three-person support episode may be pairwise sufficient because only resource transfers are retained. At a finer resolution, the same episode may become triadically nonrecoverable because witness status, translation route, or public acknowledgment is added to the model. A claim of triadic novelty should therefore always be indexed by the retained representation and diagnostic family. The paper treats changes across resolutions as representation changes rather than evidence that one description has discovered the metaphysical essence of the relation.

2.7 Formal Obligations of the Minimal Triad

The analytical scope developed above yields a set of obligations for the remainder of the paper. These obligations state what a triadic formalism must be able to distinguish before stronger claims about mediation, publicity, accountability, coalition, or emergence can be evaluated.

First, the formalism must inherit the dyadic distinctions without duplicating them mechanically. Each pairwise restriction should preserve the subject, manifestation, epistemic, action, evaluative, authorization, asymmetry, algebraic, and historical records relevant to the selected analysis. Triadic novelty can be assessed only after the pairwise baseline is sufficiently explicit.

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Table 2: Principal ways in which a triadic volunteer configuration can exceed a declared pairwise representation.

Failure family	**Triadic information**	**Illustrative volunteer relation**
Route-sensitive epistemic structure	Who learns what through whom, with what transformation or filtering	B's request reaches A only through C's translation.
Witness/publicity structure	Which event is observed, acknowledged, or recorded by a third position	C's witnessing makes an A–B refusal publicly consequential.
Procedural mediation	How C changes the admissible procedure between A and B	A dispute proceeds through a mediating protocol unavailable in the private dyad.
Brokerage/access structure	Whether C controls or conditions the route connecting A and B	Access to a support resource depends on C's gatekeeping or introduction.
Reputational structure	How information about one dyad changes expectations in another	C's report about A changes B's later trust in A.
Accountability structure	Which third position can preserve evidence or call conduct to account	A promise becomes answerable because C records and recognizes it.
Coalitional structure	How alignment of two positions changes the possibility set of the third	A and C coordinate a support plan that constrains B's options.
Higher-order normative structure	Which judgments or expectations are jointly stabilized across all three positions	A shared rule becomes operative only after three-way acknowledgment.
Geometric / compositional	Face-level, cycle-level, or higher-order incidence not determined by edge labels alone	Three pairwise-compatible edge histories admit distinct labelled face structures.

Second, the formalism must define the full triadic carrier independently of its pairwise shadow. The coupled configuration must have a declared domain, historical resolution, subject subconfigurations, and admissible transformations. The three dyads cannot serve simultaneously as the definition of the whole and the data from which the whole is later reconstructed.

Third, the formalism must define pairwise restrictions explicitly. The maps r^{κ}_{AB}, r^{κ}_{AC}, and r^{κ}_{BC} must state what they retain and what they discard. A restriction of a labelled 2-complex, an epistemic record, a procedure, and a local algebra need not be the same mathematical operation.

Fourth, the formalism must state the equivalence criterion relative to which pairwise sufficiency or failure is evaluated. Literal equality is usually too strong; unrestricted observational equivalence can be too weak. The later model must identify which full-configuration distinctions remain analytically consequential.

Fifth, a claim of pairwise sufficiency must provide a reconstruction rule or an equivalent recoverability argument. A claim of triadic novelty must provide a witness to nonrecoverability relative to the declared shadow and equivalence. Neither conclusion follows from the number of subjects alone.

Sixth, the formalism must separate third-position identity from third-position function. A subject can change functions across time, several subjects can instantiate similar functions in different models, and one function can be distributed across multiple relations. Mediation, witnessing, translation, brokerage, public recognition, accountability, and coordination therefore require typed relations or transformations rather than actor labels.

Seventh, the formalism must preserve route-sensitive epistemic structure. Information moving $A \to C \to B$ can differ from information moving directly $A \to B$, even when the terminal content is superficially similar. Accessibility, translation, concealment, testimony, credibility, and higher-order theory of mind must remain route-aware where the application requires them.

Eighth, the formalism must represent publicity and acknowledgment without equating them with mere observability. An event observed by C, an event known by C to be known by A, and an event jointly acknowledged among A, B, and C occupy different epistemic and relational statuses. Reputation

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and accountability depend on such distinctions.

Ninth, the formalism must retain action and procedure provenance at the triadic level. A support action executed by A can be requested by B, authorized by C, jointly planned by A and C, or retrospectively attributed to a coalition. Initiation, execution, authorization, generation, witnessing, and authorship-like attribution should remain separable.

Tenth, the formalism must type triadic asymmetry and dependency without collapsing them into one hierarchy. C can possess informational advantage while lacking authority; A and C can jointly control resources while depending on B for local knowledge; B can possess refusal standing while lacking exit capacity. Pairwise and triadic asymmetries can therefore interact without forming one scalar ranking.

Eleventh, the formalism must allow emergent triadic structures while keeping emergence model-relative. Public reputation, three-way trust configurations, brokerage, coalition, common acknowledgment, accountability, and shared norms can become sufficiently persistent and efficacious to require representation. Their emergence should be witnessed through declared histories or diagnostics rather than assumed from terminology.

Twelfth, the formalism must preserve the three-level GR architecture and its historical variability. Every major triadic concept should admit, where meaningful, a relational realization, manifestation structure, subject-relative reconstruction, and trajectory. Ontological suspension remains in force throughout.

Thirteenth, the formalism must keep geometric and algebraic realizations conditional. A triangular drawing is not yet a 2-complex face with substantive meaning. A local algebra does not automatically factor into three actor algebras. A category of mediation morphisms, a higher-order operator, or a spin-foam-inspired history structure must be introduced only after the retained relational problem requires the corresponding mathematics.

Fourteenth, the formalism must retain empirical and normative separations. Pairwise nonrecoverability in a formal model does not establish that a concrete volunteer relation is empirically triadic, morally legitimate, unjust, powerful, democratic, or institutionally desirable. Those stronger interpretations require additional evidence and premises.

These obligations define the reducibility burden of the minimal triad. They also explain the architecture of the sections that follow. The next section identifies which dyadic records are inherited without redefinition and constructs the coupled triadic record on which pairwise restrictions, reconstruction criteria, and later third-position functions can be defined.

3 Coupled Triadic Record: Dyadic Inheritance

This section constructs the bookkeeping object on which the later triadic formalism will act. Its role is to inherit the dyadic distinctions established in Paper I while preventing the triadic configuration from being defined as a mere ordered triple of dyads. The objective is to specify a coupled three-subject record, identify which dyadic record families can be restricted from it, and state the compatibility conditions required before the three restrictions can be treated as shadows of one common history. The discussion proceeds through the coupled triadic carrier, typed inheritance of dyadic records, component-wise restriction families, pairwise-shadow compatibility, partial records, cross-dyad consistency, and the historical revision of the coupled record. The method is formal and reconstructive. No common vector space, probability space, metric, or additive aggregation is assumed over the heterogeneous components introduced below.

Table 3 summarizes the principal record families inherited from Paper I and the new triadic burden attached to each family. The table is organizational rather than ontological: an application can omit a record family whose corresponding structure has not been articulated.

3.1 Coupled Triadic Carrier

At historical stage t and analytical resolution κ, let $\mathcal{K}^{\kappa}_{ABC,t}$ denote a declared coupled triadic relational-spacetime representation. The three tracked subject-spacetimes are represented by embeddings or other

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Table 3: Dyadic record families inherited by the triadic formalism and the additional burden created by coupling three positions.

Record family	**Dyadic inheritance**	**Triadic burden**
Subject-spacetime	History-bearing coupled carrier with tracked subject subconfigurations	Three embeddings must coexist in one declared carrier, with pairwise overlaps and interfaces compatible with the full configuration.
Three-level reality	Relational, manifestation, and subject-relative reconstruction records	One manifestation can be routed through or jointly accessible to several positions, and cross-level mappings can depend on third-position history.
Epistemic access	Directional access, uptake, memory, confidence, concealment, acknowledgment	Indirect routes and third-party testimony can alter what is knowable or jointly acknowledged without changing every pairwise endpoint record.
Theory of mind	Finite-depth recursive modeling of another subject	Each subject can model two others and their mutual models, creating route- and order-sensitive higher-order structures.
Procedure	Partially ordered interaction history with requests, offers, refusals, feedback, revision	Procedure elements can involve one, two, or three positions, and a third position can alter admissibility or interpretation of an otherwise dyadic step.
Evaluation	Need, value, aesthetic, normative, and shared evaluative records	The triad can contain third-party evaluations, public standards, reputation effects, and jointly generated distinctions that are not pairwise intersections.
Authorization	Request, permission, consent, refusal, standing, authority, withdrawal	A third position can witness, certify, authorize, constrain, or contest the operative status of another relation.
Asymmetry	Typed directional comparison without scalar collapse	Pairwise asymmetries can compose, offset, or jointly condition a third relation without yielding one total hierarchy.
Local algebra	Region-indexed algebraic carriers and typed local transformations	Pairwise subalgebras need not factorize or glue uniquely into a triadic algebraic carrier.
Historical revision	Typed revision of heterogeneous stage records	Revision can change one dyad while altering the interpretation, accessibility, or admissibility of the other two.

declared structure-preserving inclusions. In the subcomplex specialization used throughout the present paper, the embeddings are stated in Equation 5.

$$\iota_A^\kappa : \mathcal{K}_{A,t}^\kappa \hookrightarrow \mathcal{K}_{ABC,t}^\kappa, \qquad \iota_B^\kappa : \mathcal{K}_{B,t}^\kappa \hookrightarrow \mathcal{K}_{ABC,t}^\kappa, \qquad \iota_C^\kappa : \mathcal{K}_{C,t}^\kappa \hookrightarrow \mathcal{K}_{ABC,t}^\kappa. \tag{5}$$

Equation 5 supplies a common carrier without requiring disjointness, exhaustiveness, or unique subject boundaries. The historical stage is carried explicitly by the subject and coupled carriers; the stage index on ι_i^κ is suppressed unless the embedding rule itself changes through history. Pairwise shared histories can overlap, a configuration-level region can be generated only through three-position interaction, and some background structure can belong to none of the three focal subject images while remaining part of the coupled carrier.

The coupled carrier also requires declared relational-interface structure. Let $\mathcal{J}_{ABC,t}^\kappa$ denote the retained family of interface regions through which transformations, manifestations, or constraints can become consequential across the tracked positions. The family can contain pairwise interfaces $\mathcal{J}_{AB,t}^\kappa$, $\mathcal{J}_{AC,t}^\kappa$, and $\mathcal{J}_{BC,t}^\kappa$ when those are meaningful, together with regions whose interpretation is genuinely

configuration-level. A compact subject-spacetime record is given in Equation 6.

$$\mathfrak{S}^{\kappa}_{ABC,t} := \left(\mathcal{K}^{\kappa}_{ABC,t}, \iota^{\kappa}_{A}, \iota^{\kappa}_{B}, \iota^{\kappa}_{C}, \mathcal{J}^{\kappa}_{ABC,t}\right). \tag{6}$$

Equation 6 is the carrier-level part of the triadic record. It does not yet identify mediation, witnessing, brokerage, coalition, or any other third-position function. Those interpretations require additional typed structure. A triangular drawing of the three subject labels is therefore insufficient to determine $\mathfrak{S}^{\kappa}_{ABC,t}$.

The full carrier also permits pairwise overlaps. For $XY \in \{AB, AC, BC\}$, define the retained subject overlap in Equation 7.

$$\mathcal{O}^{\kappa}_{XY,t} := \iota^{\kappa}_{X}(\mathcal{K}^{\kappa}_{X,t}) \cap \iota^{\kappa}_{Y}(\mathcal{K}^{\kappa}_{Y,t}). \tag{7}$$

Equation 7 can yield empty or nonempty overlaps independently across the three pairs. The triad may also contain a three-way overlap $\mathcal{O}^{\kappa}_{ABC,t}$ when the chosen representation admits one. Its existence is a representational fact, not by itself a proof of pairwise-nonrecoverable social structure. Conversely, triadic organization can exist even when the three subject images have no literal common cell, because mediation, route dependence, public acknowledgment, or compositional structure can be supported across distinct interfaces.

3.2 Inherited Dyadic Records

Paper I organized a dyadic stage through heterogeneous typed records rather than a common state vector. The triadic formalism preserves that discipline. For each pair $XY \in \{AB, AC, BC\}$, let $\mathfrak{D}^{\kappa}_{XY,t}$ denote the class of admissible dyadic stage records of the type established in Paper I. A realized dyadic record can include a coupled pairwise carrier, cross-level record, epistemic-access record, finite-depth theory-of-mind record, procedure prefix, evaluative record, authorization record, typed asymmetry record, and local algebraic record.

The triadic model does not create three independent copies of those records and then declare their product to be the whole. Instead, the full configuration supplies a common historical context from which pairwise records can be restricted when the required restriction operation is defined. The distinction is stated schematically in Equation 8.

$$D^{\kappa}_{ABC,t} \not\equiv \left(D^{\kappa}_{AB,t}, D^{\kappa}_{AC,t}, D^{\kappa}_{BC,t}\right) \quad \text{by definition.} \tag{8}$$

Equation 8 is a methodological constraint. It leaves open the possibility that a later reconstruction theorem or application-specific rule makes the full record recoverable from the triple of dyadic restrictions. What it excludes is building that conclusion into the definition of the triad.

The same discipline applies to each inherited component. A pairwise evaluative record can be obtained by retaining the A- and B-relative judgments relevant to the AB restriction, but a triadic evaluative record can also contain C-relative assessments or a jointly stabilized three-position norm. A pairwise epistemic record can retain what A and B know of one another, while the full record can retain whether C's testimony changes that knowledge. A pairwise procedure can record A's request and B's response, while the full procedure can retain that C's certification makes the response institutionally operative. The inherited dyadic grammar therefore supplies a baseline, while the triadic record remains the primary carrier for configuration-level structure.

3.3 Typed Component Restrictions

A single projection symbol cannot be assumed to act identically on every record family. Restricting a 2-complex, an epistemic ledger, an authorization record, and a local algebra are different mathematical operations. The paper therefore treats pairwise restriction as a family indexed by component type.

Let $\mathcal{T}_{\text{rec}}$ denote the retained record-type set. Fix $\tau \in \mathcal{T}_{\text{rec}}$ and $XY \in \{AB, AC, BC\}$. The corresponding partial restriction map has the form in Equation 9.

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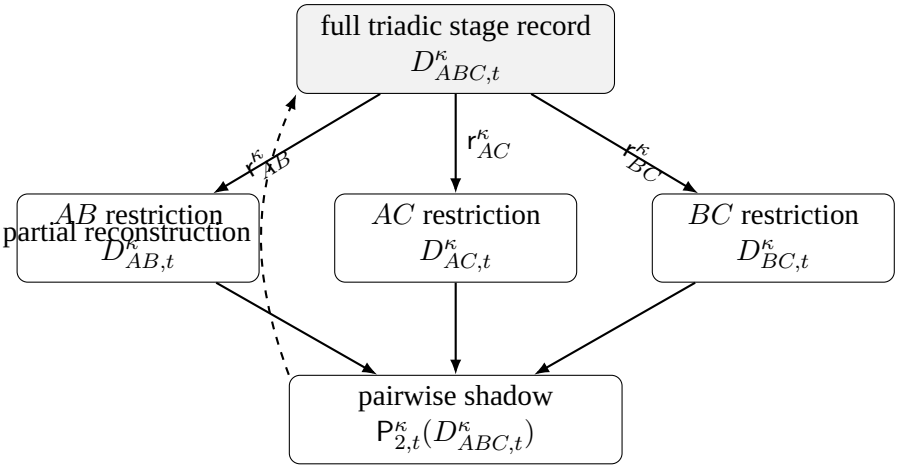

Figure 2: Coupled triadic record and typed dyadic inheritance. The three pairwise records are restrictions of a declared full record rather than independent primitives. The dashed arrow represents the later reconstruction problem and is intentionally partial.

$$\mathsf{r}^{\tau,\kappa}_{XY} : \mathfrak{R}^{\tau,\kappa}_{ABC,t} \rightharpoonup \mathfrak{R}^{\tau,\kappa}_{XY,t}, \tag{9}$$

where $\mathfrak{R}^{\tau,\kappa}_{ABC,t}$ is the admissible class of triadic records of type τ and $\mathfrak{R}^{\tau,\kappa}_{XY,t}$ is the corresponding pairwise class. Equation 9 is partial because a record can contain an intrinsically triadic component for which no faithful pairwise restriction has been declared. When a coarse shadow is nevertheless required, the application must state what is discarded.

The schematic pairwise-shadow operation P^{κ}_{2} introduced in Section 2 can now be read as an assembled family of typed restrictions rather than as one universal projection. For an admissible full record X on which the three assembled restrictions are defined, set

$$\mathsf{P}^{\kappa}_{2}(X) := \Big(\mathsf{r}^{\kappa}_{AB}(X), \mathsf{r}^{\kappa}_{AC}(X), \mathsf{r}^{\kappa}_{BC}(X)\Big). \tag{10}$$

Equation 10 defines the generic descriptor used by the later reducibility analysis. Each assembled map r^{κ}_{XY} is itself an organized family of the component-specific maps in Equation 9. Historical indices can be suppressed on the restriction symbol when the stage is carried by the argument; if the restriction rule itself changes through history, the stage index must be restored explicitly.

For a full stage record $D^{\kappa}_{ABC,t}$, the specialization is written in Equation 11.

$$\mathsf{P}^{\kappa}_{2,t}(D^{\kappa}_{ABC,t}) := \mathsf{P}^{\kappa}_{2}(D^{\kappa}_{ABC,t}) = \Big(\mathsf{r}^{\kappa}_{AB}(D^{\kappa}_{ABC,t}), \mathsf{r}^{\kappa}_{AC}(D^{\kappa}_{ABC,t}), \mathsf{r}^{\kappa}_{BC}(D^{\kappa}_{ABC,t})\Big). \tag{11}$$

Equation 11 therefore links the stage-indexed ledger to the configuration-level notation of Section 6; it hides no claim that the component restriction operations are algebraically identical.

Figure 2 summarizes the architecture. The full triadic stage record is primary. Each pairwise record is a typed restriction of that record where the relevant components are defined. The dotted return arrows indicate the reconstruction problem developed later: the three shadows need not determine the full record.

3.4 Pairwise Compatibility

A triple of dyadic records is not automatically the shadow of one admissible triadic history. Pairwise records can conflict where they overlap in subject history, manifestation, timing, label assignment, authorization, or local algebraic structure. The reconstruction rule introduced in Section 2.3 therefore requires a compatibility domain.

Let $\mathfrak{D}^{\kappa}_{AB,t} \times \mathfrak{D}^{\kappa}_{AC,t} \times \mathfrak{D}^{\kappa}_{BC,t}$ be the unconstrained product of admissible dyadic-record classes. Define the pairwise-compatibility subset in Equation 12.

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$$\mathfrak{C}^{\kappa}_{2,t} \subseteq \mathfrak{D}^{\kappa}_{AB,t} \times \mathfrak{D}^{\kappa}_{AC,t} \times \mathfrak{D}^{\kappa}_{BC,t}. \tag{12}$$

Membership in $\mathfrak{C}^{\kappa}_{2,t}$ means only that the three pairwise records satisfy the declared compatibility checks needed to be candidates for one common triadic reconstruction. Equation 12 does not imply that a reconstruction exists or is unique.

Compatibility can require several distinct checks. The A-history represented in the AB record should agree, up to the declared tracking equivalence, with the A-history represented in the AC record over their common retained interval. The same applies to B and C. If a manifestation is recorded as public to all three positions, its pairwise restrictions should not assign mutually inconsistent content unless the model explicitly represents perspective-relative manifestation. If the AB and AC restrictions both assign a local algebra to the same retained A-region, their algebraic restrictions should admit the declared comparison. If two pairwise procedure records assign incompatible orderings to a shared event, the discrepancy must either be represented as uncertainty or disqualify the triple from the compatibility domain.

A generic compatibility predicate is introduced in Equation 13.

$$\mathsf{Compat}^{\kappa}_{2,t}\big(D_{AB}, D_{AC}, D_{BC}\big) = 1 \quad \Longleftrightarrow \quad \big(D_{AB}, D_{AC}, D_{BC}\big) \in \mathfrak{C}^{\kappa}_{2,t}. \tag{13}$$

Equation 13 is a bookkeeping device, not a universal algorithm. Each application must specify which component-level equalities, equivalences, overlap maps, uncertainty tolerances, or gluing conditions constitute compatibility.

The compatibility domain is required to contain every realized shadow of an admitted full stage record. Thus, whenever $D^{\kappa}_{ABC,t}$ lies in the domain of all three assembled restrictions, the consistency condition in Equation 14 must hold.

$$\mathsf{P}^{\kappa}_{2,t}(D^{\kappa}_{ABC,t}) \in \mathfrak{C}^{\kappa}_{2,t}. \tag{14}$$

Equation 14 is an admissibility requirement on the restriction scheme. A compatibility domain that excludes the actual shadow of an admitted full record is mis-specified relative to that record.

This distinction prevents an important logical shortcut. Failure of compatibility is not evidence of triadic emergence. It can simply mean that the proposed pairwise records are mutually inconsistent. Triadic nonrecoverability becomes meaningful only after the pairwise shadow is itself coherent enough to be a candidate projection of a full configuration.

3.5 Integrated Triadic Stage Record

The coupled carrier and inherited record families can now be collected into one typed stage ledger. Let $\mathfrak{D}^{\kappa}_{ABC,t}$ denote the class of admissible integrated triadic stage records at historical stage t and resolution κ. This stage-specific class is one possible source of the abstract comparison domain $\mathfrak{D}^{\kappa}_{3}$ introduced in Section 2.3: a reducibility analysis can take $\mathfrak{D}^{\kappa}_{3}$ to be a selected subset of one stage class, a union of comparable stage classes, or a class of history records, provided the restriction and equivalence structures are declared on that domain. A realized record $D^{\kappa}_{ABC,t} \in \mathfrak{D}^{\kappa}_{ABC,t}$ is written schematically in Equation 15.

$$D^{\kappa}_{ABC,t} := \Big(\mathfrak{S}^{\kappa}_{ABC,t}, L_{ABC,t}, \mathfrak{E}_{ABC,t}, \mathfrak{T}^{(\le d_t)}_{ABC,t}, \Gamma^{\kappa,\le t}_{ABC}, \mathfrak{V}^{\kappa}_{ABC,t}, \mathfrak{U}^{\kappa}_{ABC,t}, \mathscr{A}^{\kappa}_{ABC,t}, \mathfrak{L}^{\mathrm{alg},\kappa}_{ABC,t}\Big). \tag{15}$$

The components in Equation 15 have the same broad analytical roles as their dyadic predecessors while admitting three-position content. $L_{ABC,t}$ is the triadic cross-level record connecting relational and manifestation structures. $\mathfrak{E}_{ABC,t}$ retains epistemic access, uptake, memory, disclosure, concealment, and acknowledgment relevant to the three positions. $\mathfrak{T}^{(\le d_t)}_{ABC,t}$ contains the retained finite-depth theory-of-mind objects. $\Gamma^{\kappa,\le t}_{ABC}$ is the realized triadic procedure prefix. $\mathfrak{V}^{\kappa}_{ABC,t}$ retains evaluative objects, $\mathfrak{U}^{\kappa}_{ABC,t}$ authorization-related records, $\mathscr{A}^{\kappa}_{ABC,t}$ typed asymmetry and dependency structure, and $\mathfrak{L}^{\mathrm{alg},\kappa}_{ABC,t}$ the optional local algebraic specialization.

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Equation 15 is a typed ledger rather than a state vector. No addition, norm, measure, topology, or ordering on the whole record is assumed. An application can omit components that have not been articulated. For example, a simple witness case can require epistemic and manifestation records without requiring a local algebraic specialization. A brokerage case can require route and access structure before any shared normative object exists. A coalition case can require action provenance and dependency records while leaving higher-order ToM unspecified.

The ToM component requires one additional caution. With three subjects, finite-depth theory of mind can contain A's model of B, A's model of C, A's model of B's model of C, and many further nested forms. The present section records these objects only as a retained finite-depth family. Section 9 develops the recursion and route sensitivity explicitly. The integrated record therefore leaves the internal grammar of $\mathfrak{T}^{(\leq d_t)}_{ABC,t}$ abstract at this stage.

3.6 Cross-Dyad Consistency: Subject Tracking, Contextual Divergence

Dyadic inheritance creates shared-subject consistency requirements, but it should not force all appearances of a subject across different relations to be identical records. A can manifest differently to B and C, disclose different information, occupy different relational positions, and support different locally relevant histories. The triadic model therefore distinguishes subject continuity from context-invariant manifestation.

Let $\widehat{\mathcal{K}}^{\kappa}_{A|AB,t}$ denote the A-history retained in the AB restriction and $\widehat{\mathcal{K}}^{\kappa}_{A|AC,t}$ the A-history retained in the AC restriction. The model can require a declared tracking relation

$$\widehat{\mathcal{K}}^{\kappa}_{A|AB,t} \sim^{\kappa}_{A,t} \widehat{\mathcal{K}}^{\kappa}_{A|AC,t} \tag{16}$$

without requiring literal equality. Equation 16 states that the two pairwise records are admissible reconstructions of the same tracked subject history relative to the model. Context-dependent manifestations, memories, or role assignments can remain different.

The same distinction applies to epistemic and evaluative records. B can believe that A acts from care while C attributes the same action to reputation-seeking. Those records conflict only if the model has already declared them to be descriptions of one perspective-independent object that must agree. Otherwise they are distinct subject-relative reconstructions of a common event. Pairwise compatibility therefore concerns coherence of typing and shared-history tracking, not forced convergence of beliefs or judgments.

Cross-dyad consistency also allows genuine disagreement to become triadically consequential. If C observes that A gives B one account of an action and gives C another, the discrepancy itself can become a new triadic object. If B and C hold conflicting reconstructions of A's intention and each knows the other disagrees, the later procedure can depend on the disagreement. Such structures should not be erased by an overstrong compatibility rule.

Table 4 records representative distinctions between consistency requirements and permissible divergence.

3.7 Partial Records: Articulation Growth

The integrated record in Equation 15 is intentionally permissive. A triadic relation can exist before every component required for later analysis has become articulated. C can witness an action before the three subjects possess a shared vocabulary for the event. A mediator can begin translating before mutual standing is stabilized. A public record can exist before the participants converge on its normative significance. A coalition can begin to form before any participant explicitly represents the coalition as such.

Let $\mathrm{dom}(D^{\kappa}_{ABC,t})$ denote the subset of record types actually articulated at stage t. Historical development can enlarge, shrink, or reorganize this articulation domain. A schematic articulation relation is stated in Equation 17.

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Table 4: Representative cross-dyad consistency requirements and forms of permissible divergence.

Object	Typical consistency requirement	Permissible divergence
Tracked subject history	Pairwise reconstructions must satisfy the declared subject-tracking relation	Different resolutions, manifestations, local roles, and accessible histories
Shared event	Event identity/order must be compatible where the records claim to track the same occurrence	Different interpretations, confidence, salience, or remembered detail
Public manifestation	Public content must satisfy the declared manifestation equivalence	Different uptake, inference, affective response, or normative judgment
Local algebraic region	Overlapping pairwise restrictions must admit the declared algebraic comparison/gluing map	Different nonoverlapping local carriers or representation choices
Authorization record	Same claimed authorization event must have compatible scope and historical location	Different beliefs about legitimacy, prudence, or moral force
Action provenance	Shared factual provenance claims must be trackable to one history when so declared	Different self-attributions, third-party attributions, and unresolved causal interpretations

$$\mathrm{dom}(D^{\kappa}_{ABC,t}) \rightsquigarrow \mathrm{dom}(D^{\kappa'}_{ABC,t'}) \tag{17}$$

for a later stage t'. Equation 17 is deliberately relation-valued. Later articulation can add a shared concept, differentiate a previously conflated role, remove a now-abandoned classification, or change the resolution at which a distinction is available. More articulated does not automatically mean more accurate or normatively better.

This feature is especially important for triadic novelty. A configuration can be pairwise sufficient under an early coarse record and later become nonrecoverable after a witness role, reputational object, or accountability relation becomes articulated. The change can reflect a richer representation of a stable underlying history, a genuine relational transformation, or both. The three-level and empirical sections later in the paper must keep these interpretations separate.

3.8 Historical Revision of the Triadic Record

The coupled record changes through history. Let $t \preceq t'$ and allow the analytical resolution to change from κ to κ'. Define the typed triadic revision relation in Equation 18.

$$\mathsf{TriRev}^{\kappa' \leftarrow \kappa}_{t' \leftarrow t} \subseteq \mathfrak{D}^{\kappa}_{ABC,t} \times \mathfrak{D}^{\kappa'}_{ABC,t'}. \tag{18}$$

Membership in the relation in Equation 18 means that the later record is admitted as a historically continuous revision of the earlier one relative to the model's tracking rules. The relation is not assumed functional. One earlier record can support several later continuations; one later record can admit several defensible reconstructions of its provenance.

Triadic revision can redistribute information across the pairwise shadows. A witness statement by C can revise the AB epistemic record while leaving the AC material relation unchanged. A mediation event can change the admissible AB procedure through transformations supported partly on AC and BC. A public acknowledgment can create a reputational object whose later effects appear in all three pairwise restrictions even though the object was generated at the configuration level. A coalition can alter the continuation family available to B while strengthening the A–C relation. Historical revision therefore cannot generally be represented as three independent dyadic updates.

The interaction between revision and restriction should be declared rather than presumed. In some applications, restricting a revised full record and revising a previously restricted pairwise record can agree up to the pairwise equivalence. In others, the order can matter because triadic information discarded by

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the early restriction later affects the dyad. A compatibility condition for the first regime is stated in Equation 19.

$$\mathsf{r}^{\kappa'}_{XY}(D^{\kappa'}_{ABC,t'}) \sim_{XY,t'} \mathsf{Rev}_{XY,t'\leftarrow t}\big(\mathsf{r}^{\kappa}_{XY}(D^{\kappa}_{ABC,t})\big), \tag{19}$$

when both sides are defined. Equation 19 is a condition to be tested, not a universal law. Its failure can itself be informative: it indicates that the later pairwise record depends on configuration-level history lost by the earlier pairwise restriction.

The distinction supplies a first formal route to triadic path dependence. Two full histories can possess pairwise-equivalent records at an earlier stage while generating different later pairwise relations because their discarded triadic organization differs. Section 6 turns this intuition into a sharper pairwise-sufficiency and nonrecoverability analysis.

3.9 Triadic Structure: Formal Interface

The coupled triadic record provides the common domain required by the rest of the paper. The carrier component supplies the history-bearing configuration; the three-level, epistemic, ToM, procedure, evaluative, authorization, asymmetry, and algebraic records retain the heterogeneous structures inherited from Paper I; the typed restriction family defines what is visible in each dyadic shadow; the compatibility domain separates coherent shadows from mutually inconsistent triples; and the revision relation permits all of these structures to change through history.

The construction also clarifies what has not yet been claimed. Equation 15 does not establish that any particular component is pairwise nonrecoverable. Equation 12 does not guarantee reconstruction. Equation 18 does not impose one dynamical law. The present section supplies the ledger and restriction architecture within which later claims can be tested.

The next section applies the three-level GR reality discipline directly to the triadic configuration. It will distinguish relationally real three-position structures from their manifestations and from A-, B-, and C-relative reconstructions, and it will examine how the mapping among those records changes when mediation, witnessing, translation, publicity, or other third-position functions enter the history.

4 Three-Level Reality in Triadic Configurations

This section applies the three-level GR reality discipline to the coupled triadic record introduced in Section 3. Its role is to prevent the addition of a third position from collapsing relational structure, manifestation, and subject-relative reconstruction into one enlarged information set. The objective is to specify how ontological suspension, relational reality, manifestation reality, and epistemic reconstruction are retained when three history-bearing subjects can observe, mediate, translate, witness, or publicly represent one another's relations. The discussion proceeds through the three-level discipline, relational triadic structure, manifestation multiplicity, three-perspective reconstruction, third-position cross-level mediation, level-relative pairwise shadows, and historical cross-level trajectories. The method is formal and reconstructive. The section does not yet define mediation, witnessing, publicity, or reputation in their full substantive forms; it supplies the cross-level architecture within which those later concepts can be typed.

Table 5 summarizes the additional burden created by the third position. The table should be read as a set of distinctions to preserve rather than as a claim that every triadic case realizes every listed structure.

Figure 3 gives the general architecture used in the section. The suspended ontological level remains outside the modeled causal network. The lower architecture contains one retained relational domain, one manifestation domain, three subject-relative reconstruction channels, and a historically indexed feedback relation. The third position enlarges the internal organization of these lower levels; it does not create a fourth reality level.

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Table 5: Three-level distinctions inherited from the dyadic formalism and the additional burden created by a third position.

Analytical object	**Dyadic discipline**	**Triadic burden**
Ontological status	Ultimate ontology remains suspended	The presence of a third subject supplies no warrant for reifying "thirdness" as an ultimate ontology.
Relational reality	Historically persistent and efficacious dyadic structures are tracked	Configuration-level structures can depend on three-position organization even when every pairwise relation is separately intelligible.
Manifestation reality	Relational structures admit context-dependent manifestations	One relational event can have different manifestations for A, B, and C, and a manifestation can circulate through a third position before becoming accessible to another.
Epistemic reconstruction	A and B reconstruct relational structure from accessible manifestation-history records	A, B, and C can reconstruct one another and one another's relations through direct and indirect routes with different histories.
Cross-level feedback	Manifestation can contribute to later relational stabilization	Witnessing, certification, translation, publication, or repetition by C can make an A–B manifestation consequential for later triadic structure.
Pairwise restriction	Dyadic relational and manifestation records can be restricted by declared maps	Relational, manifestation, and epistemic pairwise shadows can have different recoverability properties and should not be conflated.
Historical mapping	Manifestation and reconstruction maps can change through history	Third-position participation can alter the mapping itself, including which manifestations become public, credible, legible, or actionable.

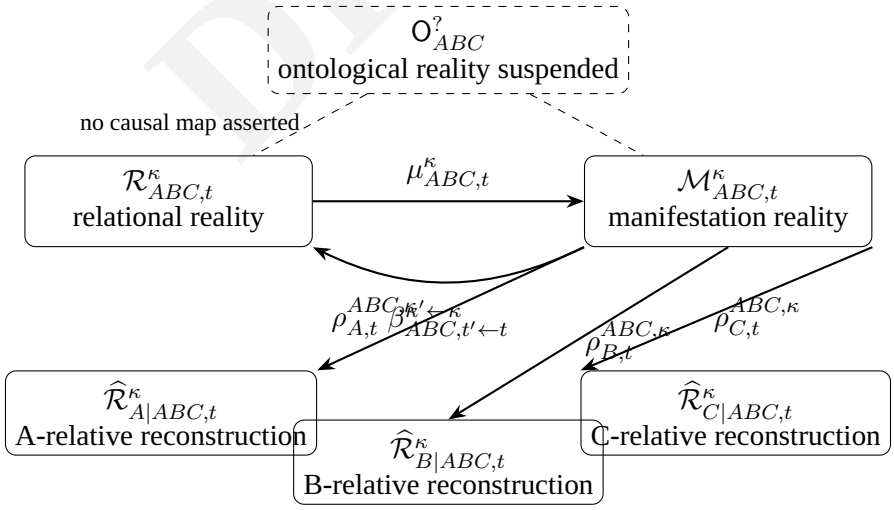

Figure 3: Three-level architecture for the coupled triadic formalism. Ontological reality remains suspended. Relational and manifestation realities are modeled separately, and A-, B-, and C-relative reconstruction maps act on subject-accessible manifestation-history records. Third-position mediation, witnessing, translation, and publicity later modify the internal route structure of the lower architecture rather than adding a new reality level.

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4.1 Three-Level Discipline

The three-level distinction is inherited from Paper I without alteration in its basic epistemic status. Ultimate ontology remains suspended; relational reality is the principal positive level at which the model tracks historically constituted and causally or generatively efficacious structures; manifestation reality records the context-dependent forms through which those structures become presented, expressed, accessible, recorded, or actionable. The triadic extension changes the organization of the relational and manifestation levels, not the status of the suspended level.

Let $\mathsf{O}^{?}_{ABC}$ denote the ontological-status marker for the triadic model. The symbol is not a latent state space and carries no internal dynamics in the present paper. In particular, the triadic formalism assumes no map $\mathsf{O}^{?}_{ABC} \to \mathcal{R}^{\kappa}_{ABC,t}$ and no map $\mathsf{O}^{?}_{ABC} \to \mathcal{M}^{\kappa}_{ABC,t}$. Mediation, witnessing, public recognition, coalition, or any other three-position structure can therefore be analyzed formally without being elevated into a claim about the ultimate constitution of social reality.

This restraint is especially important because triadic structure can appear mathematically more "complete" than dyadic structure. A triangular graph, a 2-simplex, a three-way overlap, or a nonfactorizable algebraic term can each provide a useful representation. None establishes that ultimate relational reality is simplicial, combinatorial, algebraic, or intrinsically ternary. Paper II uses such structures only when the substantive triadic question requires the corresponding mathematical property.

At the modeled levels, let $\mathcal{R}^{\kappa}_{ABC,t}$ denote the relational structures retained by the triadic representation at stage t and resolution κ, and let $\mathcal{M}^{\kappa}_{ABC,t}$ denote the corresponding class of retained manifestations. The basic cross-level relation is stated in Equation 20.

$$\mu^{\kappa}_{ABC,t} \subseteq \mathcal{R}^{\kappa}_{ABC,t} \times \mathcal{M}^{\kappa}_{ABC,t}. \tag{20}$$

Equation 20 is relation-valued rather than functional. One relational structure can manifest differently to A, B, and C; several manifestations can jointly express one retained relation; one manifestation can be compatible with several candidate relational reconstructions; and some relational structures can remain effective without a presently accessible manifestation. This many-to-many structure is already possible in the dyad. The triadic burden is that the accessibility, circulation, interpretation, and later efficacy of a manifestation can depend on the third position.

The pair $(\mathcal{R}^{\kappa}_{ABC,t}, \mathcal{M}^{\kappa}_{ABC,t})$ should therefore not be read as an objective/subjective split. A manifestation can itself become causally efficacious, and a relational structure can remain partly inaccessible to every focal subject. The distinction records analytical function: relational reality tracks retained relational organization; manifestation reality tracks the forms through which that organization becomes available in a context.

4.2 Relational Triadic Structure

A relationally triadic object is not defined merely by the participation of three named subjects. It is an object whose retained relational organization is specified at the coupled ABC level and whose later effects can depend on that organization. Section 2 introduced pairwise nonrecoverability as one route to making this claim testable. The present subsection identifies the reality-level location of the candidate object: if a mediator position, witness relation, public acknowledgment, coalition, or accountability structure is treated as sufficiently persistent and efficacious under the model, it belongs to $\mathcal{R}^{\kappa}_{ABC,t}$ rather than to a subject's private reconstruction of the triad.

This distinction permits relational reality to exceed what any one subject knows. Suppose C witnesses an A–B support action and the witness relation later changes which accounts of the action are treated as credible. A can underestimate the witness role, B can misunderstand C's later use of the information, and C can possess an incomplete model of A's intention. The witness relation can nevertheless become relationally real in the GR sense if it is retained as historically identifiable and generatively consequential. Its existence in the model does not require epistemic consensus.

Relationally triadic structure can also be distributed. It need not be localized in one three-way overlap region of $\mathcal{K}^{\kappa}_{ABC,t}$. A mediation relation can be supported by an AC communication, a BC communica-

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tion, and an institutional rule that makes C's intervention consequential for the AB procedure. A public reputation effect can be generated by several pairwise manifestations whose joint historical organization changes later expectations. The relational object can therefore have a triadic type while its cellular or procedural support is distributed across several interfaces.

For later use, let $\mathcal{R}^{(3),\kappa}_{ABC,t}$ denote the subclass of relational structures whose retained type is explicitly triadic under the current model. The inclusion is stated in Equation 21.

$$\mathcal{R}^{(3),\kappa}_{ABC,t} \subseteq \mathcal{R}^{\kappa}_{ABC,t}. \tag{21}$$

Equation 21 is a typing convention, not a theorem of irreducibility. Membership means that the model currently treats the object as configuration-level and three-position. Section 6 will impose the stronger pairwise-sufficiency tests needed before a nonrecoverability claim is made.

The distinction also leaves room for retyping through history. A relation initially represented as dyadic can later acquire a triadic type after C becomes a persistent witness, mediator, certifier, translator, or coalition partner. Conversely, a historically triadic structure can dissolve into a pairwise relation if the third-position organization ceases to be consequential. The relational type is therefore historically tracked rather than fixed by the names of the participants.

4.3 Manifestation Multiplicity

The third position creates new manifestation patterns even before any specifically triadic norm or institution is formed. A single relational event can produce different manifestations to the three subjects. Some manifestations can be accessible to only one subject, some to a pair, and some to all three. A manifestation can also be re-presented: C can repeat, translate, summarize, certify, distort, or publicly record a manifestation first produced within the A–B relation.

For each $i \in \{A, B, C\}$, let $\mathcal{M}^{\text{acc},ABC,\kappa}_{i,t} \subseteq \mathcal{M}^{\kappa}_{ABC,t}$ denote the manifestation material accessible to subject i at stage t under the triadic configuration. The jointly accessible subset is defined in Equation 22.

$$\mathcal{M}^{\text{joint},\kappa}_{ABC,t} := \mathcal{M}^{\text{acc},ABC,\kappa}_{A,t} \cap \mathcal{M}^{\text{acc},ABC,\kappa}_{B,t} \cap \mathcal{M}^{\text{acc},ABC,\kappa}_{C,t}. \tag{22}$$

Equation 22 records common accessibility only. It does not define publicity, joint acknowledgment, shared interpretation, or common knowledge. A document can be accessible to all three while none knows that the others have read it. A statement can be made before all three while each interprets it differently. A public-status object requires additional procedural or relational criteria developed later in Section 10.

The distinction between original manifestation and re-presentation is also necessary. Let $m \in \mathcal{M}^{\kappa}_{ABC,t}$ be a manifestation associated through Equation 20 with a retained relational structure r. A third-position transformation can generate a later manifestation m'_C that is about or derived from m without being identical to it. The relation is stated schematically in Equation 23.

$$m \xrightarrow[\text{C-mediated}]{\chi_C} m'_C, \qquad m, m'_C \in \mathcal{M}^{\kappa,\leq t'}_{ABC}. \tag{23}$$

Equation 23 introduces χ_C only as a typed manifestation-level mediation record. It can later be specialized as translation, testimony, certification, quotation, publication, or another declared operation. The double arrow is mnemonic for a typed relation whose exact domain and codomain will depend on the later specialization; it should not be read as a linear operator at this stage.

Re-presentation matters because a later subject can gain access to the represented relation through m'_C while lacking direct access to m. B can learn of A's earlier statement through C. A can learn how B described an intervention through C's report. A community observer represented by C can turn a previously local act into a recorded public event. The epistemic route and the manifestation route therefore become part of the history even when the underlying event remains unchanged.

Manifestation multiplicity can also create divergence between pairwise shadows. An A–B restriction can contain the original manifestation but omit C's later certification. An A–C restriction can contain the

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certification process without retaining B's direct reception. The B–C restriction can contain B's response to C's report. The three shadows can therefore be individually coherent while no single one preserves the manifestation history through which the triadic relation became effective.

4.4 Three-Perspective Reconstruction

The manifestation level becomes epistemically consequential through subject-relative reconstruction. With three subjects, the paper retains one reconstruction channel for each focal perspective while leaving the internal theory-of-mind recursion to Section 9. Let $H_i^{\leq t}$ denote the retained history available to subject i. The triadic reconstruction maps are defined in Equation 24.

$$\rho_{i,t}^{ABC,\kappa} : \mathcal{M}_{i,t}^{\text{acc},ABC,\kappa} \times H_i^{\leq t} \rightharpoonup \widehat{\mathcal{R}}_{i|ABC,t}^{\kappa}, \qquad i \in \{A, B, C\}. \tag{24}$$

Equation 24 preserves two disciplines from Paper I. First, the map is partial: accessible manifestation does not guarantee a determinate reconstruction. Second, the target is an epistemic representation of relational structure, not relational reality itself. The hat notation marks the subject-relative model.

The third position makes reconstruction route-sensitive. A can reconstruct B partly from B's direct manifestations and partly from C's testimony about B. The two routes can converge, diverge, or remain incomparable. A can also model the relation between B and C, and that model can alter A's interpretation of B's conduct toward A. These possibilities remain inside the epistemic record $\mathfrak{E}_{ABC,t}$ and the ToM record $\mathfrak{T}_{ABC,t}^{(\leq d_t)}$ introduced in Section 3.5; they do not create a new reality level.

A useful separation is therefore

$$\mathcal{R}_{ABC,t}^{\kappa} \neq \mathcal{M}_{ABC,t}^{\kappa} \neq \widehat{\mathcal{R}}_{A|ABC,t}^{\kappa}, \widehat{\mathcal{R}}_{B|ABC,t}^{\kappa}, \widehat{\mathcal{R}}_{C|ABC,t}^{\kappa} \quad \text{in general.} \tag{25}$$

Equation 25 is a categorical separation rather than an assertion that the displayed objects share one ambient space in which inequality is numerically evaluated. It states that the formalism must not identify a relational object, its manifestations, and the three subject-relative reconstructions by default.

Three perspectives also permit disagreement to become mutually represented. C can know that A and B disagree about the meaning of an action; A can know that C knows of the disagreement; B can be unaware that the disagreement has become visible to C. Such structures will later require finite-depth recursive ToM. The present section retains the more basic requirement that every higher-order epistemic object be generated through historically available manifestation and reconstruction routes rather than through transparent access to another subject's internal state.

4.5 Third-Position Cross-Level Mediation

A distinctive triadic possibility appears when the third position affects the relation between relational and manifestation realities themselves. C can make an otherwise local A–B relation legible, rename it, translate its manifestations, authenticate a record, introduce a category through which it is interpreted, or suppress a manifestation from a relevant audience. The third position can therefore modify the cross-level mapping without changing every lower-level relational component in the same way.

To preserve this distinction, define the triadic cross-level record by Equation 26.

$$L_{ABC,t} := \left(\mathsf{O}_{ABC}^{?}; \mathcal{R}_{ABC,t}^{\kappa}, \mathcal{M}_{ABC,t}^{\kappa}, \mu_{ABC,t}^{\kappa}, \{\mathcal{M}_{i,t}^{\text{acc},ABC,\kappa}, \rho_{i,t}^{ABC,\kappa}\}_{i\in\{A,B,C\}} \right). \tag{26}$$

Equation 26 supplies the content of the $L_{ABC,t}$ component already used in Equation 15. The semicolon separates the suspended ontological marker from the modeled relational, manifestation, and reconstruction components. The record is typed and heterogeneous; no common metric or vector structure is assumed.

A third-position cross-level intervention can change one or more components of Equation 26. A translator can change which manifestation is intelligible to B without changing the event being described. A

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witness can change the credibility or persistence of a manifestation. A certifier can make a record actionable within a procedure. A broker can determine which manifestations travel between otherwise weakly connected positions. A public archive can preserve a manifestation beyond the interaction that generated it. Each case can modify the effective mapping between relational structure and later accessibility.

The manifestation-feedback relation therefore also receives a triadic form. For $t \preceq t'$, define

$$\beta^{\kappa' \leftarrow \kappa}_{ABC,t' \leftarrow t} \subseteq \mathcal{M}^{\kappa}_{ABC,t} \times \mathcal{R}^{\kappa'}_{ABC,t'}. \tag{27}$$

Membership $(m, r') \in \beta^{\kappa' \leftarrow \kappa}_{ABC,t' \leftarrow t}$ means that manifestation m contributes, under the retained model, to the later relational structure r'. Equation 27 permits the relevant causal-generative route to involve C. A witnessed refusal can later stabilize a public boundary; a translated request can reorganize the A–B procedure; a recorded act of support can contribute to A's reputation; a certified statement can alter B's standing. The section does not yet decide which of these relations are normatively justified.

Third-position mediation of the cross-level mapping also creates a possible source of asymmetry. C can possess greater capacity than A or B to make a manifestation legible, durable, public, or institutionally actionable. That difference can later matter for power or justice, but the present section treats it only as a structural possibility. Typed asymmetry is developed later in Section 14.

4.6 Level-Relative Pairwise Shadows

Pairwise sufficiency can differ across the relational, manifestation, and epistemic parts of the triadic record. A configuration can be pairwise sufficient at one level and nonrecoverable at another. The paper therefore should not infer full triadic reducibility from the fact that one layer can be reconstructed from its pairwise restrictions.

Let $\mathsf{P}^{\kappa}_{2,R}$ denote the pairwise-shadow operation restricted to relational-level records, and let $\mathsf{P}^{\kappa}_{2,M}$ denote the corresponding operation for manifestation-level records. Let $\mathsf{P}^{\kappa}_{2,E}$ retain the subject-relative epistemic records associated with the three pairwise restrictions. These typed shadows are stated schematically in Equation 28.

$$\mathsf{P}^{\kappa}_{2,\ell} : \mathfrak{D}^{\kappa}_{ABC,t} \rightharpoonup \mathfrak{C}^{\kappa,\ell}_{2,t}, \qquad \ell \in \{R, M, E\}. \tag{28}$$

Equation 28 introduces three type-specific shadow families. Their codomains $\mathfrak{C}^{\kappa,\ell}_{2,t}$ are the compatible pairwise records appropriate to the selected layer; they need not be identical mathematical classes.

Several combinations are possible. A triadic relational structure can be nonrecoverable even when every manifestation currently retained by the model is pairwise reproducible. In that case, the manifestation record underdescribes the relational organization. Conversely, a relational configuration can be pairwise recoverable while its manifestation history is not, because the same relational relations have been made public, translated, or archived through a configuration-level route. Epistemic nonrecoverability can arise when A's knowledge depends on B knowing that C witnessed an event, even if the underlying relational event itself is pairwise specifiable.

The section therefore adopts the non-entailments stated in Equation 29.

$$\mathsf{Suff}_{2,R} \not\Longrightarrow \mathsf{Suff}_{2,M}, \qquad \mathsf{Suff}_{2,M} \not\Longrightarrow \mathsf{Suff}_{2,E}, \qquad \mathsf{Suff}_{2,E} \not\Longrightarrow \mathsf{Suff}_{2,R}, \tag{29}$$

where the sufficiency symbols in Equation 29 are placeholders for the typed reconstruction predicates developed formally in Section 6. The equation records only the absence of a general entailment among the levels. Particular applications can impose stronger coupling assumptions and recover some implications.

This separation is important for empirical work. Observed manifestations can be pairwise while the relational hypothesis is triadic; alternatively, a highly public three-person manifestation can be generated by relations that remain pairwise. The empirical calibration problem must therefore state which level is being reconstructed from which data rather than treating observed three-person interaction as direct evidence of relational triadic irreducibility.

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4.7 Historical Cross-Level Trajectories

The triadic cross-level record is historically variable. Let $(I, \preceq)$ be the retained historical index set. The cross-level trajectory is defined by Equation 30.

$$\mathbf{L}_{ABC,I} := \big(L_{ABC,t}\big)_{t\in I}. \tag{30}$$

Equation 30 allows changes in the relational domain, manifestation domain, manifestation relation, accessibility sets, and reconstruction maps. The trajectory can therefore represent more than changes in what A, B, or C believe. It can represent changes in what can be manifested, which manifestation routes are available, which third-position interventions are consequential, and which structures later become relationally stabilized.

A simple witness history illustrates the distinction. At t_0, A and B can interact without C having access to the event. At t_1, C witnesses a later action. At t_2, C's account becomes accessible to another subject. At t_3, the account is incorporated into a durable record that changes future expectations or standing. Across these stages, the original A–B relation, its manifestation, C's reconstruction, and the later relational effect of C's account occupy different positions in $\mathbf{L}_{ABC,I}$. Treating the whole history as one static "three-person interaction" would erase the generative sequence through which thirdness became consequential.

Historical variation can also remove triadic organization. A mediation route can become unnecessary after A and B acquire a shared vocabulary. A witness role can lose relevance after an event is independently recorded. A public reputation structure can decay. A coalition can dissolve. The cross-level trajectory must therefore permit both formation and loss of configuration-level structure.

This history-sensitive architecture also clarifies why the same manifested arrangement can carry different relational meanings at different stages. C's presence can initially be incidental, later become evidential, and later still become institutionally authoritative. The visible configuration "A, B, C are present" remains similar while the relational role of C changes. Triadic type therefore depends on historical organization rather than visual cardinality alone.

The mappings themselves can change. For $t \preceq t'$, the conditions

$$\mu^{\kappa}_{ABC,t} \neq \mu^{\kappa'}_{ABC,t'}, \qquad \rho^{ABC,\kappa}_{i,t} \neq \rho^{ABC,\kappa'}_{i,t'} \quad \text{for some } i \in \{A, B, C\} \tag{31}$$

can hold even when selected relational beings remain trackable across the two stages. Equation 31 captures changes in manifestation convention, interpretive competence, institutional legibility, disclosure practice, or other cross-level conditions. The inequalities are structural non-identity statements; no metric is implied.

The history in Equation 30 must remain compatible with the triadic revision relation introduced in Equation 18. A later cross-level record can be historically continuous with an earlier one without preserving every manifestation, access route, or reconstruction rule. Section 17 will return to the composition and revision of these histories after the paper has developed the substantive third-position structures that populate them.

4.8 Triadic Reality: Formal Interface

The three-level architecture places the later triadic formalism on a common epistemic footing. Ontological reality remains suspended. Relational reality contains the history-bearing structures that the model treats as sufficiently persistent and generatively consequential. Manifestation reality contains context-dependent forms through which those structures become accessible or actionable. A-, B-, and C-relative reconstruction maps generate epistemic models from accessible manifestation-history records without adding a fourth reality level.

The triadic extension contributes three additional disciplines. First, manifestation and epistemic access are route-sensitive: a subject can know through another subject rather than through a direct manifestation. Second, the third position can alter the cross-level mapping itself by witnessing, translating, cer-

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tifying, preserving, publicizing, or otherwise re-presenting a manifestation. Third, pairwise sufficiency must be evaluated level by level; recoverability at the manifestation level does not settle recoverability at the relational or epistemic level.

These distinctions now permit the paper to sharpen the kinematic representation of the three history-bearing subjects. Section 5 returns to the subject-spacetime carrier, defines pairwise restrictions more explicitly at the geometric level, and prepares the structural comparison required for Section 6's pairwise-sufficiency and triadic-nonrecoverability tests.

5 History-Bearing Subject Spacetimes: Pairwise Restriction Structure

This section sharpens the geometric carrier of the triadic formalism. Its role is to distinguish the full history-bearing subject-spacetime configuration from the three pairwise geometric restrictions that later enter the reducibility tests. The objective is to define how the tracked subject-spacetimes of A, B, and C are embedded in one coupled carrier, how pairwise subconfigurations are selected without presuming that they are autonomous dyads, how shared-subject history is compared across restrictions, and how configuration-level interface structure can survive even when no literal three-way overlap exists. The discussion proceeds through the triadic subject carrier, admissible pairwise restriction, shared-subject tracking, interface structure, restriction compatibility, resolution change, and historical restriction trajectories. The method remains formal and reconstructive. No claim of pairwise nonrecoverability is made in this section; Section 6 will ask whether the pairwise restrictions defined here suffice to recover the full structure.

The geometric burden is easy to understate. If one begins directly with three dyadic complexes, the full triadic configuration has already been replaced by its shadows. Conversely, if one begins with one triangular diagram, the notation can suggest a three-way relation even when the retained structure is only three pairwise links. The present section therefore starts from one coupled carrier and makes every pairwise restriction an explicit, typed operation on that carrier.

For readers without prior algebraic-topology training, the geometric convention used here is modest. A finite 2-complex is represented by vertices, edges, and faces together with incidence or boundary data telling how higher-dimensional cells attach to lower-dimensional ones. A subcomplex retains cells together with the boundary cells required by those attachments. The construction below uses only this finite incidence structure; it does not assume a metric, smooth manifold, relativistic causal structure, or physical spacetime interpretation.

Table 6 summarizes the principal geometric objects introduced below. The table separates subject carriers, pairwise induced carriers, overlap records, interaction-support regions, and restriction operators because they play different formal roles.

5.1 Coupled Triadic Subject Carrier

At historical stage t and analytical resolution κ, the model retains one coupled labelled complex $\mathcal{K}^{\kappa}_{ABC,t}$. The subject-spacetimes of A, B, and C are not introduced as three independent containers subsequently connected by edges. They are tracked subconfigurations of the common history-bearing carrier. In the subcomplex specialization used here, their embeddings are recorded in Equation 32.

$$\iota_i^{\kappa} : \mathcal{K}^{\kappa}_{i,t} \hookrightarrow \mathcal{K}^{\kappa}_{ABC,t}, \qquad i \in \{A, B, C\}. \tag{32}$$

Equation 32 repeats the carrier-level commitment of Section 3 in a form suited to geometric restriction. The three images need not be disjoint, exhaustive, or represented at identical internal resolutions. A focal subject can share retained historical material with one or both others, and some cells of the coupled carrier can belong to no focal subject-spacetime while still representing background institutions, artifacts, conventions, or interface structure relevant to the triadic relation.

A convenient carrier record is therefore

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Table 6: Principal subject-spacetime and restriction objects in the triadic formalism.

Object	Analytical role	Distinction preserved
$\mathcal{K}^{\kappa}_{ABC,t}$	Coupled history-bearing triadic carrier	Full configuration versus aggregate of pairwise carriers
ι^{κ}_{i}	Subject embedding for $i \in \{A, B, C\}$	Focal subject history versus isolated point actor
$\mathcal{K}^{\kappa}_{XY\|ABC,t}$	Pairwise carrier induced from the full triad	Pairwise shadow versus autonomous standalone dyad
$\mathcal{O}^{\kappa}_{XY,t}$	Pairwise subject-overlap record	Shared represented material versus pairwise interface
$\mathcal{O}^{\kappa}_{ABC,t}$	Literal three-way subject overlap, when defined	Common cellular overlap versus triadic organization
$\mathcal{J}^{\kappa}_{XY\|ABC,t}$	Pairwise interaction-support region inside the triadic history	Context-conditioned dyadic coupling versus subject overlap
$\mathcal{J}^{\kappa}_{ABC,t}$	Configuration-level interaction-support region	Triadic support candidate versus proof of irreducibility
$\mathrm{g}^{\kappa}_{XY,t}$	Typed geometric restriction	Full carrier versus induced pairwise subcomplex
$\chi^{XY,XZ,\kappa}_{i,t}$	Shared-subject comparison map/relation	Subject continuity versus literal equality across restrictions

$$\mathfrak{S}^{\kappa}_{ABC,t} := \Big(\mathcal{K}^{\kappa}_{ABC,t}, \{\iota^{\kappa}_{i}\}_{i\in\{A,B,C\}}, \mathcal{J}^{\kappa}_{ABC,t}\Big), \tag{33}$$

where $\mathcal{J}^{\kappa}_{ABC,t}$ is a declared configuration-level interaction-support region. Equation 33 does not define $\mathcal{J}^{\kappa}_{ABC,t}$ as a pairwise-nonrecoverable relation. It records the cells, subregions, or distributed support on which a later application locates configuration-level interaction. Whether the represented content of that support is recoverable from the pairwise restrictions remains a later question.

The distinction prevents a common geometric shortcut. A 2-simplex can encode one higher-order relation, yet not every social triad should be represented by one face. Conversely, triadic organization can be distributed across several faces, edges, labels, and historical continuations without one distinguished three-way cell. The formalism therefore treats cellular dimension and social arity as separate modeling choices.

5.2 Admissible Pairwise Geometric Restriction

A pairwise restriction must produce a valid retained subcomplex rather than an arbitrary deletion of the third subject label. Let $\mathrm{Cell}(\mathcal{K}^{\kappa}_{ABC,t})$ denote the cells of the full complex. For $XY \in \{AB, AC, BC\}$, let $S^{\kappa}_{XY,t}$ be a declared retained cell set. The pairwise restriction is admissible only when the selected set is closed under the relevant boundary operation, as stated in Equation 34.

$$c \in S^{\kappa}_{XY,t} \quad \Longrightarrow \quad \partial c \subseteq S^{\kappa}_{XY,t} \qquad \text{for every retained cell with defined boundary.} \tag{34}$$

Equation 34 is the minimal subcomplex-closure requirement. It prevents a pairwise shadow from retaining a face while silently deleting edges or vertices required by that face's boundary. An application using a different combinatorial carrier can replace the condition by the corresponding closure rule.

When the retained set satisfies Equation 34, define the induced pairwise subcomplex by

$$\mathcal{K}^{\kappa}_{XY|ABC,t} := \mathcal{K}^{\kappa}_{ABC,t}\big|_{S^{\kappa}_{XY,t}}, \qquad XY \in \{AB, AC, BC\}. \tag{35}$$

The vertical bar in Equation 35 marks restriction from the full triadic carrier. This notation is deliberately different from a standalone dyadic model $\mathcal{K}^{\kappa,\mathrm{stand}}_{XY,t}$ that might have been constructed without C. In general,

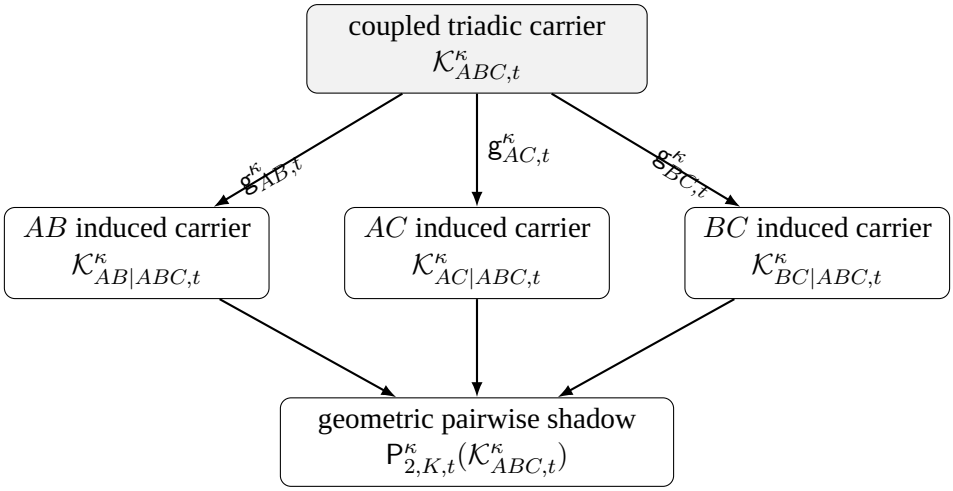

Figure 4: Geometric pairwise restrictions of the coupled triadic carrier. Each pairwise complex is induced from the full configuration through a typed restriction operation. The triple of induced complexes forms the geometric pairwise shadow used later in the reconstruction analysis.

$$\mathcal{K}^{\kappa}_{XY|ABC,t} \not\equiv \mathcal{K}^{\kappa,\mathrm{stand}}_{XY,t} \tag{36}$$

as a definitional matter. Equation 36 does not assert that the two must differ in every application. It states that the induced pairwise shadow inherits its boundary, labels, retained history, and admissible continuations from the full triadic context, whereas a standalone dyadic model can make different modeling choices.

To keep the geometric restriction typed, let $\mathfrak{K}^{\kappa}_{3,t}$ be the admitted class of triadic labelled complexes and let $\mathfrak{K}^{XY,\kappa}_{2,t}$ be the admitted class of pairwise labelled complexes for the retained pair XY. The geometric restriction operator is then

$$\mathsf{g}^{\kappa}_{XY,t} : \mathfrak{K}^{\kappa}_{3,t} \rightharpoonup \mathfrak{K}^{XY,\kappa}_{2,t}, \qquad \mathsf{g}^{\kappa}_{XY,t}(\mathcal{K}^{\kappa}_{ABC,t}) = \mathcal{K}^{\kappa}_{XY|ABC,t}, \tag{37}$$

when an admissible retained cell set and label restriction have been declared. Equation 37 is partial because some triadic representations may not admit a faithful restriction under the selected pairwise type. A coarse shadow can still be introduced, but the discarded structure must then be recorded explicitly.

Figure 4 summarizes the relation among the full carrier and the three induced pairwise complexes. The central complex remains primary; the pairwise carriers are derived restrictions rather than independent primitives.

The geometric shadow appearing in Figure 4 is collected in Equation 38.

$$\mathsf{P}^{\kappa}_{2,K,t}(\mathcal{K}^{\kappa}_{ABC,t}) := \Big(\mathcal{K}^{\kappa}_{AB|ABC,t}, \mathcal{K}^{\kappa}_{AC|ABC,t}, \mathcal{K}^{\kappa}_{BC|ABC,t}\Big). \tag{38}$$

Equation 38 is only the carrier-level part of the broader pairwise shadow defined in Section 3. Labels, epistemic records, procedure records, authorization records, and algebraic carriers require their own typed restrictions.

5.3 Shared-Subject Tracking Across Restrictions

The same focal subject appears in two different pairwise restrictions. A appears in both $AB \mid ABC$ and $AC \mid ABC$; B appears in $AB \mid ABC$ and $BC \mid ABC$; C appears in $AC \mid ABC$ and $BC \mid ABC$. The two representations of the same focal subject need not be literally identical because each restriction can retain different contextual history, boundary cells, and local resolution.

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Let $\mathcal{K}^{\kappa}_{i|XY,t}$ denote the representation of focal subject i retained inside the relevant pairwise complex. For distinct $j, k \neq i$, define a declared shared-subject comparison relation

$$\chi^{ij,ik,\kappa}_{i,t} \subseteq \mathcal{K}^{\kappa}_{i|ij,t} \times \mathcal{K}^{\kappa}_{i|ik,t}. \tag{39}$$

Membership in Equation 39 records admissible historical correspondence under the identity and resolution criteria of the model. A stronger application can use an isomorphism, embedding span, refinement map, or another structure-preserving comparison. The relation form is retained here because literal one-to-one correspondence can fail under coarse-graining, splitting, merging, or asymmetric contextual detail.

Pairwise compatibility requires enough shared-subject correspondence to treat the restrictions as shadows of one common triadic history. A minimal consistency condition is stated schematically in Equation 40.

$$\mathsf{Track}^{\kappa}_{i}\big(\mathcal{K}^{\kappa}_{i|ij,t}, \mathcal{K}^{\kappa}_{i|ik,t}; \chi^{ij,ik,\kappa}_{i,t}\big) = 1, \qquad i \in \{A, B, C\}. \tag{40}$$

Equation 40 does not require context-invariant manifestation, belief, role, or judgment. A can manifest differently to B and C while still being tracked as the same focal subject. The condition concerns historical identity at the carrier level, not agreement about what A means, intends, or ought to do.

This distinction becomes important when a pairwise shadow is later reconstructed. If the AB and AC restrictions contain incompatible A-histories, the failure is a shadow-consistency problem rather than evidence for a higher-order triadic relation. Section 6 therefore applies nonrecoverability tests only after the shared-subject and other compatibility conditions have been satisfied.

5.4 Overlap Structure, Interaction Support

Literal subject overlap and relational interaction support are different objects. For each pair XY, the pairwise subject overlap is inherited from Section 3:

$$\mathcal{O}^{\kappa}_{XY,t} := \iota^{\kappa}_{X}(\mathcal{K}^{\kappa}_{X,t}) \cap \iota^{\kappa}_{Y}(\mathcal{K}^{\kappa}_{Y,t}). \tag{41}$$

When a literal three-way overlap is meaningful in the selected representation, define

$$\mathcal{O}^{\kappa}_{ABC,t} := \iota^{\kappa}_{A}(\mathcal{K}^{\kappa}_{A,t}) \cap \iota^{\kappa}_{B}(\mathcal{K}^{\kappa}_{B,t}) \cap \iota^{\kappa}_{C}(\mathcal{K}^{\kappa}_{C,t}). \tag{42}$$

Equations 41 and 42 describe shared represented material. They do not define the relational interface. An A–B interaction can be mediated through a region that lies in neither subject image; a public record can connect all three subjects without being part of any subject-spacetime; and a third-position witness relation can be distributed across several cells rather than concentrated in one triple overlap.

For each pair XY, let $\mathcal{J}^{\kappa}_{XY|ABC,t}$ denote the interaction-support region retained for that pair inside the triadic history. Let $\mathcal{J}^{\kappa}_{ABC,t}$ denote the configuration-level support introduced in Equation 33. These objects satisfy no general identity with the overlap sets.

Literal three-way overlap is neither sufficient nor necessary, in general, for pairwise nonrecoverability. A common cell can be a convenient representation of a three-position event while remaining completely recoverable from pairwise labels. Conversely, a nonrecoverable route, publicity condition, mediation record, or compositional structure can be supported without any cell shared by all three subject-spacetimes. This statement is intentionally made in prose here because the formal nonrecoverability predicate is defined only after the equivalence structure is introduced in Section 6; Section 5 should not introduce an undefined shorthand for that later property.

The same caution applies to $\mathcal{J}^{\kappa}_{ABC,t}$. Declaring a configuration-level support region identifies where the model will inspect possible triadic organization. It does not establish that the retained object is irreducible to pairwise structure. The support region can therefore be used as the domain of a later diagnostic without making the diagnostic true by construction.

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5.5 Restriction Compatibility: Context-Loss Ledger

The three induced pairwise complexes must be mutually compatible before they can be candidates for one full reconstruction. Carrier-level compatibility includes at least shared-subject tracking, label consistency on common retained cells, and compatible boundary identification where pairwise restrictions overlap geometrically.

Let $\mathcal{K}^{\kappa}_{XY|ABC,t} \cap \mathcal{K}^{\kappa}_{XZ|ABC,t}$ denote the represented common region of two pairwise restrictions when such an intersection is defined inside the full carrier. The restricted label maps should agree there up to the declared label-equivalence relation $\sim^{\kappa}_{\lambda,t}$. A schematic compatibility condition is

$$\lambda^{\kappa}_{XY,t}\big|_R \sim^{\kappa}_{\lambda,t} \lambda^{\kappa}_{XZ,t}\big|_R, \qquad R := \mathcal{K}^{\kappa}_{XY|ABC,t} \cap \mathcal{K}^{\kappa}_{XZ|ABC,t}, \tag{43}$$

The condition applies whenever the compared restrictions and label comparison are defined. Equation 43 permits perspective-sensitive labels when the model explicitly types them as such. It excludes only contradictions in records that claim to encode one shared carrier-level datum.

Context loss occurs when restriction removes structure that can later affect the retained pair. Define the geometric discarded region for pair XY by

$$\mathcal{L}^{K,\kappa}_{XY,t} := \mathrm{Cell}(\mathcal{K}^{\kappa}_{ABC,t}) \setminus S^{\kappa}_{XY,t}. \tag{44}$$

Equation 44 is a bookkeeping set, not a measure of information loss. Some discarded cells can be irrelevant to the later XY history; others can contain precisely the witness, mediation, archival, or coordination structure that later alters the pairwise relation. The next section therefore asks not how large $\mathcal{L}^{K,\kappa}_{XY,t}$ is, but whether two full configurations that agree on all retained pairwise shadows can nevertheless differ under a declared triadic diagnostic.

5.6 Resolution Change: Restriction Ordering

Triadic restriction can be performed before or after a change of representational resolution. The two procedures need not agree automatically. Let

$$q^{\kappa\to\kappa'}_{ABC} : \mathfrak{K}^{\kappa}_3 \rightharpoonup \mathfrak{K}^{\kappa'}_3, \qquad q^{\kappa\to\kappa'}_{XY} : \mathfrak{K}^{XY,\kappa}_2 \rightharpoonup \mathfrak{K}^{XY,\kappa'}_2 \tag{45}$$

Equation 45 identifies the full-carrier and pairwise resolution maps whose order will be compared below.

be declared resolution-changing maps for the full carrier and the pairwise carrier, respectively. A restriction/coarse-graining compatibility regime satisfies

$$\mathsf{g}^{\kappa'}_{XY} \circ q^{\kappa\to\kappa'}_{ABC} \sim^{\kappa'}_{XY} q^{\kappa\to\kappa'}_{XY} \circ \mathsf{g}^{\kappa}_{XY} \tag{46}$$

on the common domain where both sides are defined. Equation 46 is a testable compatibility condition, not a universal law.

Failure of Equation 46 can have several meanings. Coarse-graining the full triad can merge cells that were separately assigned to different pairwise histories. Pairwise restriction can remove the very configuration-level structure needed to decide how coarse cells should be identified. A fine representation can contain a witness path that disappears under one pairwise coarse-graining while remaining encoded through a configuration-level label. Such failures are representation-sensitive and should not be interpreted immediately as substantive triadic emergence. They first indicate that the chosen restriction and resolution procedures do not commute under the declared equivalence.

This resolution discipline is particularly important for finite examples. A coarse triangular network can make every interaction appear three-way, while a finer 2-complex can reveal that all relevant histories decompose into pairwise episodes. The reverse can also occur: fine pairwise records can look locally complete while their coarse configuration-level organization retains a public or institutional structure not present in any one pair. Section 6 therefore treats nonrecoverability relative to a declared resolution family rather than as an invariant metaphysical property.

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5.7 Historical Restriction Trajectories

The pairwise restrictions are themselves historical objects. Let $(I, \preceq)$ be the retained historical index set. For each pair XY, define the induced pairwise carrier trajectory by

$$\mathbf{K}_{XY|ABC,I} := \left(\mathcal{K}^{\kappa_t}_{XY|ABC,t}\right)_{t\in I}. \tag{47}$$

Equation 47 permits both the full carrier and the selected resolution to vary with history. A pairwise interface can appear, disappear, or be reorganized. C can begin as an incidental background subject and later become a witness or mediator whose participation changes which AB cells must be retained. Conversely, a previously necessary mediation route can become redundant after A and B develop direct mutual intelligibility.

Historical evolution and pairwise restriction can also fail to commute. Let $\mathsf{U}^{\kappa'\leftarrow\kappa}_{ABC,t'\leftarrow t}$ denote a declared carrier-level historical update relation or partial map, and let $\mathsf{U}^{\kappa'\leftarrow\kappa}_{XY,t'\leftarrow t}$ denote the corresponding pairwise update when it exists. A commuting regime satisfies

$$\mathsf{g}^{\kappa'}_{XY,t'} \circ \mathsf{U}^{\kappa'\leftarrow\kappa}_{ABC,t'\leftarrow t} \sim^{\kappa'}_{XY,t'} \mathsf{U}^{\kappa'\leftarrow\kappa}_{XY,t'\leftarrow t} \circ \mathsf{g}^{\kappa}_{XY,t} \tag{48}$$

where both compositions are defined. Equation 48 is the carrier-level counterpart of the record-level compatibility condition in Equation 19. Its failure means that evolving the full triadic history and then restricting can produce a different pairwise carrier from restricting early and evolving only the pairwise shadow.

That failure is one geometric route to triadic path dependence. It can occur because a configuration-level event supported partly outside the early XY shadow later changes the XY boundary, labels, interface, or continuation structure. The present section does not yet classify such a failure as pairwise nonrecoverability. Section 6 will distinguish inconsistency, representation sensitivity, and diagnostic-relative nonrecoverability.

5.8 Subject-Spacetime Restriction: Formal Interface

The subject-spacetime construction now supplies a geometric foundation for the reducibility problem. The full carrier $\mathcal{K}^{\kappa}_{ABC,t}$ remains primary. The three induced complexes $\mathcal{K}^{\kappa}_{AB|ABC,t}$, $\mathcal{K}^{\kappa}_{AC|ABC,t}$, and $\mathcal{K}^{\kappa}_{BC|ABC,t}$ are typed restrictions of that carrier. Shared-subject comparison relations ensure that repeated appearances of A, B, and C can be tracked across pairwise shadows without requiring literal equality. Overlap records remain distinct from interaction-support regions. Configuration-level support does not itself establish triadic irreducibility. Restriction can lose context, and restriction can fail to commute with coarse-graining or historical update.

These distinctions sharpen the burden carried by Section 2. The next section can now formulate pairwise sufficiency on a declared compatible shadow family, define reconstruction relative to an equivalence criterion, and identify several kinds of triadic nonrecoverability without confusing them with incompatible records, arbitrary deletion, or representation artifacts.

6 Triadic Nonrecoverability: Pairwise Sufficiency Criteria

This section gives the formal core of the paper's reducibility test. Its role is to convert the qualitative claim that a triad may contain more structure than its three dyadic restrictions into a model-relative reconstruction criterion. The objective is to distinguish four questions that are easily conflated: whether three pairwise records are mutually compatible, whether at least one full triadic configuration realizes them, whether all realizing configurations are equivalent for the selected analysis, and whether a retained diagnostic can be computed from pairwise information alone. The discussion proceeds through pairwise equivalence, a relation-valued nonrecoverability witness, reconstruction, diagnostic factorization, finite ambiguity classes, historical variability, and a minimal labelled-complex example. The method is formal

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Table 7: Distinct outcomes of the pairwise reconstruction problem.

Outcome	Formal condition	Interpretation
Incompatible shadow	Pairwise records fail the declared compatibility conditions	No admissible common triadic reconstruction is licensed by the retained data.
Unrealized compatible shadow	Pairwise records pass local checks but lie outside the realized image of the full restriction map	Compatibility is necessary but insufficient for existence.
Multiple equivalent realizations	More than one full realization exists, but all belong to one retained full-equivalence class	Representation is nonunique while the selected analysis remains pairwise sufficient.
Triadic nonrecoverability	One pairwise-shadow class contains at least two full configurations that are not equivalent under the retained criterion	Pairwise information does not determine the selected full structure.
Diagnostic nonfactorizability	A declared full diagnostic differs inside one pairwise-shadow class	The diagnostic contains information unavailable from the retained pairwise records alone.

and deliberately nonmetric. No scalar "amount of thirdness," residual norm, or metaphysical irreducibility is assumed.

The section builds on three objects already introduced. Section 2.3 supplied the scope-level exact-shadow reconstruction map $\mathsf{Rec}^{\kappa}_{2,\mathrm{raw}}$. Section 3.4 distinguished compatible triples of dyadic records from arbitrary triples. Section 5 then defined geometric pairwise restrictions of one history-bearing triadic carrier. The present section replaces the raw reconstruction burden by a quotient-level criterion and asks when the retained pairwise shadow class determines the retained full-equivalence class.

Table 7 previews the distinctions developed below. In particular, failure of pairwise compatibility is not a witness of triadic nonrecoverability; it means that the proposed shadows do not yet form a coherent reconstruction problem.

6.1 Pairwise Equivalence Induced by Restriction

The schematic equivalence $\sim_{2,\kappa}$ introduced in the Introduction requires an explicit construction. Let $\mathfrak{D}^{\kappa}_3 \subseteq \mathfrak{C}^{\kappa}_3$ be the retained class of full triadic configurations on which the reduction question is posed. For each pair $XY \in \{AB, AC, BC\}$, let $\approx_{XY,\kappa}$ be the declared equivalence on the corresponding pairwise record class. The equivalence can be literal equality, resolution-relative isomorphism, equality of a selected diagnostic record, or another application-justified criterion.

For $X, Y \in \mathfrak{D}^{\kappa}_3$, define pairwise equivalence by Equation 49.

$$X \sim_{2,\kappa} Y \quad \Longleftrightarrow \quad \mathsf{r}^{\kappa}_{XY}(X) \approx_{XY,\kappa} \mathsf{r}^{\kappa}_{XY}(Y) \quad \text{for every } XY \in \{AB, AC, BC\}. \tag{49}$$

Equation 49 makes the pairwise criterion componentwise and typed. A geometric restriction can be compared geometrically, an epistemic restriction epistemically, and an algebraic restriction algebraically. The model need not force heterogeneous pairwise components into one common metric before asking whether the retained shadows agree.

For raw pairwise triples $s = (s_{AB}, s_{AC}, s_{BC})$ and $s' = (s'_{AB}, s'_{AC}, s'_{BC})$, write $s \approx_{2,\kappa} s'$ when $s_{XY} \approx_{XY,\kappa} s'_{XY}$ for every retained pair XY. With this product equivalence, Equation 49 is equivalent to $\mathsf{P}^{\kappa}_2(X) \approx_{2,\kappa} \mathsf{P}^{\kappa}_2(Y)$. The comparison domain $\mathfrak{D}^{\kappa}_3$ is, by the scope convention of Section 2.3, contained in the common definedness domain of the three restrictions. Its realized shadows therefore lie in the compatibility domain selected for the current record type.

The full configuration carries a separate declared equivalence $\sim_{\kappa}$. Two logically distinct relations between $\sim_{\kappa}$ and $\sim_{2,\kappa}$ must be kept separate. Pairwise sufficiency will require the implication $X \sim_{2,\kappa}$

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$Y \Rightarrow X \sim_\kappa Y$: configurations identified by the pairwise representation must be identified by the retained full analysis. The reverse implication, $X \sim_\kappa Y \Rightarrow X \sim_{2,\kappa} Y$, is stronger than reconstruction requires. It says that the pairwise descriptor does not preserve distinctions already discarded by the full-equivalence criterion.

When the latter property is wanted, it is stated as the optional quotient-invariance condition in Assumption 6.1.

Assumption 6.1 (Pairwise-shadow invariance under full equivalence)**.** For every $X, Y \in \mathfrak{D}_3^\kappa$,

$$X \sim_\kappa Y \quad \Longrightarrow \quad X \sim_{2,\kappa} Y. \tag{50}$$

Equation 50 is needed only when the pairwise-shadow descriptor is intended to descend to the quotient $\mathfrak{D}_3^\kappa / \sim_\kappa$. Its failure does not by itself make reconstruction inconsistent: the pairwise records may simply retain distinctions that the full analysis has chosen to ignore. The nonrecoverability problem concerns the converse direction. If $X \sim_{2,\kappa} Y$ while $X \not\sim_\kappa Y$, then the pairwise shadow has conflated configurations that the selected full analysis still distinguishes. The next subsection records that gap without converting it into a scalar residual.

6.2 Equivalence-Gap Relation

Define the pairwise/full equivalence-gap relation on $\mathfrak{D}_3^\kappa$ by Equation 51.

$$\mathcal{G}_{3|2}^\kappa := \Big\{ (X, Y) \in \mathfrak{D}_3^\kappa \times \mathfrak{D}_3^\kappa : X \sim_{2,\kappa} Y \ \wedge \ X \not\sim_\kappa Y \Big\}. \tag{51}$$

Equation 51 is relation-valued. It records which distinctions are lost by the selected pairwise representation while remaining relevant to the full configuration. This choice follows the same formal discipline used elsewhere in the GR programme: when no justified arithmetic of difference has been supplied, loss of discrimination should be represented as a relation rather than as an invented magnitude.

The definition immediately yields a precise criterion for pairwise sufficiency.

Definition 6.2 (Pairwise sufficiency)**.** The retained pairwise representation is *pairwise sufficient* on $\mathfrak{D}_3^\kappa$ relative to $(\sim_\kappa, \sim_{2,\kappa})$ when

$$\mathcal{G}_{3|2}^\kappa = \varnothing. \tag{52}$$

Equation 52 does not require literal uniqueness of the reconstructed full object. Several full configurations can possess one pairwise shadow while remaining equivalent under $\sim_\kappa$. Pairwise sufficiency therefore concerns recoverability of the distinctions retained by the analysis, not recovery of every representational detail.

Conversely, a pair $(X, Y) \in \mathcal{G}_{3|2}^\kappa$ is a *triadic nonrecoverability witness*. The witness is model-relative in three senses. It depends on the domain $\mathfrak{D}_3^\kappa$, the pairwise restriction/equivalence structure, and the full equivalence $\sim_\kappa$. Changing any of those can create or remove the witness without changing the underlying observed episode.

The relation also clarifies why "higher order" and "triadic" should not be used as synonyms for complexity. A full configuration can be complicated while $\mathcal{G}_{3|2}^\kappa$ remains empty. A simple finite configuration can have nonempty $\mathcal{G}_{3|2}^\kappa$ if one retained three-position distinction disappears under every selected dyadic restriction.

6.3 Reconstruction Criterion

The relation-valued criterion can now be connected to the scope-level reconstruction language of Section 2.3. Let $\mathfrak{C}_2^\kappa$ denote the declared compatibility domain of raw pairwise descriptors at the resolution used for the reducibility test. The product equivalence $\approx_{2,\kappa}$ is induced by the three component equivalences in Equation 49. The ambient compatible shadow-class space is

$$\overline{\mathfrak{C}}_2^\kappa := \mathfrak{C}_2^\kappa / \approx_{2,\kappa} . \tag{53}$$

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Equation 53 can contain compatible shadow classes that are not realized by any admitted full configuration.

For $X \in \mathfrak{D}_3^\kappa$, define the realized shadow-class map by

$$q_2^\kappa(X) := [\mathsf{P}_2^\kappa(X)]_{\approx_{2,\kappa}} \in \overline{\mathfrak{C}}_2^\kappa. \tag{54}$$

Equation 54 discards only distinctions already declared irrelevant at the pairwise level. The realized shadow-class space is then

$$\mathfrak{S}_2^\kappa := q_2^\kappa(\mathfrak{D}_3^\kappa) \subseteq \overline{\mathfrak{C}}_2^\kappa. \tag{55}$$

Equation 55 separates existence from uniqueness: compatible classes in $\overline{\mathfrak{C}}_2^\kappa \setminus \mathfrak{S}_2^\kappa$ are unrealized, while classes in $\mathfrak{S}_2^\kappa$ have at least one admitted full realization.

A reconstruction at the retained level is therefore a map on realized shadow classes,

$$\overline{\mathsf{Rec}}_2^\kappa : \mathfrak{S}_2^\kappa \rightharpoonup \mathfrak{D}_3^\kappa/\!\sim_\kappa . \tag{56}$$

Equation 56 gives the retained reconstruction target after quotienting by full equivalence. It refines the exact-shadow map of Equation 3 by quotienting the pairwise descriptor itself.

The following proposition connects the equivalence-gap relation with this quotient-level reconstruction.

Proposition 6.3 (Reconstruction criterion)**.** *On the retained domain $\mathfrak{D}_3^\kappa$, the following statements are equivalent:*

1. $\mathcal{G}_{3|2}^\kappa = \varnothing$*;*
2. $X \sim_{2,\kappa} Y$ *implies* $X \sim_\kappa Y$ *for all* $X, Y \in \mathfrak{D}_3^\kappa$*;*
3. *the rule*

$$\overline{\mathsf{Rec}}_2^\kappa\big(q_2^\kappa(X)\big) := [X]_{\sim_\kappa} \tag{57}$$

is well defined on $\mathfrak{S}_2^\kappa$.

Proof. Statement 1 is equivalent to Statement 2 by the definition of $\mathcal{G}_{3|2}^\kappa$ in Equation 51. If Statement 2 holds, any two full configurations with the same pairwise-shadow class are $\sim_\kappa$-equivalent, so Equation 57 does not depend on the representative and Statement 3 follows. Conversely, if Equation 57 is well defined, two configurations in the same pairwise-shadow class must be assigned the same full-equivalence class, which gives Statement 2. □

Corollary 6.4 (Coincidence under two-sided equivalence compatibility)**.** *If pairwise sufficiency holds on $\mathfrak{D}_3^\kappa$ and Assumption 6.1 also holds, then*

$$X \sim_{2,\kappa} Y \quad \Longleftrightarrow \quad X \sim_\kappa Y \qquad \text{for all } X, Y \in \mathfrak{D}_3^\kappa. \tag{58}$$

Equation 58 gives the stronger regime in which the realized pairwise-shadow classes and the retained full-equivalence classes carry exactly the same discrimination. Pairwise sufficiency alone does not require this two-sided coincidence; the pairwise representation is allowed to preserve distinctions that the full analysis ignores.

Proposition 6.3 sharpens the scope statement in Equation 4. The scope statement used exact raw shadows; the proposition uses the quotient-level pairwise equivalence that the paper actually intends for reducibility claims. Pairwise sufficiency is therefore not a claim that the three dyads literally regenerate every cell, manifestation, or historical annotation of the original triad. It is the claim that the retained pairwise shadow class determines the retained full-equivalence class.

The proposition also exposes the burden of a reduction claim. One must specify the pairwise equivalences, the full equivalence, and the domain on which the canonical reconstruction is meant to be well defined. A reconstruction obtained only after silently discarding a triadic distinction has changed $\sim_\kappa$ rather than demonstrated pairwise sufficiency under the original criterion.

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6.4 Diagnostic Factorization

The equivalence-gap criterion is general, but many applications ask a narrower question: can a particular retained diagnostic be computed from the pairwise shadows? Let d be a typed diagnostic

$$d : \mathfrak{D}_3^\kappa \rightharpoonup \mathcal{Z}_d, \tag{59}$$

Equation 59 isolates a typed full-triad diagnostic for the factorization test.

where $\mathcal{Z}_d$ is an application-specific codomain. The diagnostic can be Boolean, categorical, order-valued, algebraic, set-valued, or another declared type. No scalar codomain is required.

The diagnostic is pairwise-factorizable on a retained subdomain $\mathfrak{U} \subseteq \mathfrak{D}_3^\kappa$ when there exists a partial map

$$\overline{d}_2 : q_2^\kappa(\mathfrak{U}) \rightharpoonup \mathcal{Z}_d \quad \text{such that} \quad d(X) = \overline{d}_2(q_2^\kappa(X)) \tag{60}$$

for every $X \in \mathfrak{U}$ in the declared domain of d. Equation 60 gives a direct criterion for whether the diagnostic contains information beyond the selected pairwise records.

Definition 6.5 (Diagnostic-relative triadic nonrecoverability)**.** A diagnostic d is *triadically nonrecoverable relative to the selected pairwise shadow* on $\mathfrak{U}$ when there exist $X, Y \in \mathfrak{U}$ satisfying

$$X \sim_{2,\kappa} Y \qquad \text{and} \qquad d(X) \neq d(Y). \tag{61}$$

Equation 61 is often the most practical witness form. Later sections can ask whether publicity status, mediation structure, brokerage route, accountability, coalition membership, or a face-supported relational label supplies such a diagnostic.

The connection to full nonrecoverability depends on the diagnostic's relation to $\sim_\kappa$. Suppose d respects full equivalence, meaning

$$X \sim_\kappa Y \implies d(X) = d(Y). \tag{62}$$

Then any witness to Equation 61 is automatically a member of $\mathcal{G}_{3|2}^\kappa$. A diagnostic that does not satisfy Equation 62 may still be useful descriptively, but disagreement in that diagnostic does not by itself establish nonrecoverability relative to the declared full-equivalence criterion.

Pairwise sufficiency also has a useful consequence in the other direction.

Proposition 6.6 (Factorization under pairwise sufficiency)**.** *If $\mathcal{G}_{3|2}^\kappa = \varnothing$ on $\mathfrak{U}$ and d is constant on $\sim_\kappa$-equivalence classes, then d admits a factorization of the form in Equation 60 on $\mathfrak{U}$.*

Proof. Define $\overline{d}_2(q_2^\kappa(X)) := d(X)$. Pairwise sufficiency implies that any two representatives of the same pairwise-shadow class are $\sim_\kappa$-equivalent. Since d is constant on such classes, the definition is independent of the representative. □

Proposition 6.6 gives later substantive sections a convenient falsification route. To show that the retained pairwise representation is insufficient for a specific triadic object, it is enough to exhibit a well-typed, full-equivalence-respecting diagnostic whose value fails to factor through the pairwise shadow.

6.5 Ambiguity Classes

The reconstruction problem can also be stated through the set of full-equivalence classes compatible with one ambient pairwise shadow class. For $s \in \overline{\mathfrak{C}}_2^\kappa$, define the ambiguity class in Equation 63.

$$\mathsf{Amb}_2^\kappa(s) := \Big\{ [X]_{\sim_\kappa} : X \in \mathfrak{D}_3^\kappa,\ q_2^\kappa(X) = s \Big\}. \tag{63}$$

Equation 63 separates existence from uniqueness at the analytical level. If $s \notin \mathfrak{S}_2^\kappa$, then $\mathsf{Amb}_2^\kappa(s) = \varnothing$ and the compatible shadow class has no admitted realization in the retained domain. If the ambiguity

class contains one full-equivalence class, the shadow is sufficient for that case even when several representational realizations inhabit the class. If it contains two or more distinct full-equivalence classes, the shadow is pairwise nonrecoverable under the selected criteria.

This formulation is especially useful in finite examples. It permits a model to enumerate admissible reconstructions without inventing a distance among them. The ambiguity class can also carry explanatory information. Two admissible reconstructions can differ by a witness/publicity relation, a mediation order, a face label, a route-sensitive epistemic object, or another typed feature. Later sections can therefore characterize what the pairwise representation leaves open rather than merely declaring that reconstruction is nonunique.

The paper deliberately avoids defining an "amount of triadic information" by the cardinality or diameter of $\mathsf{Amb}_2^{\kappa}(s)$. Such measures may become useful in finite probabilistic or information-theoretic specializations, but they require additional structure and would answer a stronger question than the present formalism needs.

6.6 Resolution-Sensitive Historical Variability

Pairwise sufficiency is indexed by both historical stage and analytical resolution. Let κ_t denote the retained resolution at stage t, and let $\mathfrak{D}_{3,t}^{\kappa_t}$ be the corresponding comparison domain. The time-indexed equivalence-gap relation is

$$\mathcal{G}_{3|2,t}^{\kappa_t} := \Big\{(X_t, Y_t) : X_t \sim_{2,\kappa_t} Y_t \ \wedge \ X_t \not\sim_{\kappa_t} Y_t\Big\}. \tag{64}$$

Equation 64 permits triadic nonrecoverability to appear, disappear, or change type across a relational history. For example, a private support episode can initially be pairwise sufficient. A later witnessing event may create a public acknowledgment structure whose historical significance cannot be reconstructed from the previously retained dyads. Conversely, a triadically consequential mediation episode can later lose relevance under a coarser analysis that retains only resource transfers.

A trajectory of reducibility status can therefore be written schematically as

$$\mathbf{G}_{3|2,I} := \big(\mathcal{G}_{3|2,t}^{\kappa_t}\big)_{t\in I}. \tag{65}$$

Equation 65 is a history of relation-valued witnesses, not a monotone accumulation law. Pairwise sufficiency at one stage need not persist, and once nonrecoverability appears it need not remain. This nonmonotonicity is consistent with the wider GR treatment of evolving manifestation rules, subject boundaries, admissible operations, and historical continuations.

Resolution change adds a second source of variability. Let $c_{\kappa\to\kappa'}$ be a declared coarse-graining or comparison map. Pairwise sufficiency at a coarse resolution does not imply sufficiency at a finer resolution, and fine-resolution nonrecoverability can disappear after coarse-graining if the distinguishing object is intentionally discarded. A robust claim of triadic structure should therefore report whether its witness survives a declared family of reasonable representation changes.

Third-position relevance is also weaker than nonrecoverability throughout the trajectory. A witness, mediator, or broker can alter one pairwise relation while the full configuration remains reconstructible from all three dyads. The paper reserves the stronger language of triadic nonrecoverability for cases satisfying the criteria above.

6.7 Finite Face-Label Witness

A minimal labelled-complex example makes the criterion concrete without attributing social meaning to a face by default. Consider two finite 2-complex configurations X and Y. Each contains vertices A, B, C, the same three oriented edges e_{AB}, e_{AC}, e_{BC}, and one face f_{ABC} attached to the same edge cycle. The three pairwise restrictions retain only the corresponding vertices, edge, and pairwise labels. Suppose those restrictions are identical under the declared pairwise equivalences, while the full face label differs:

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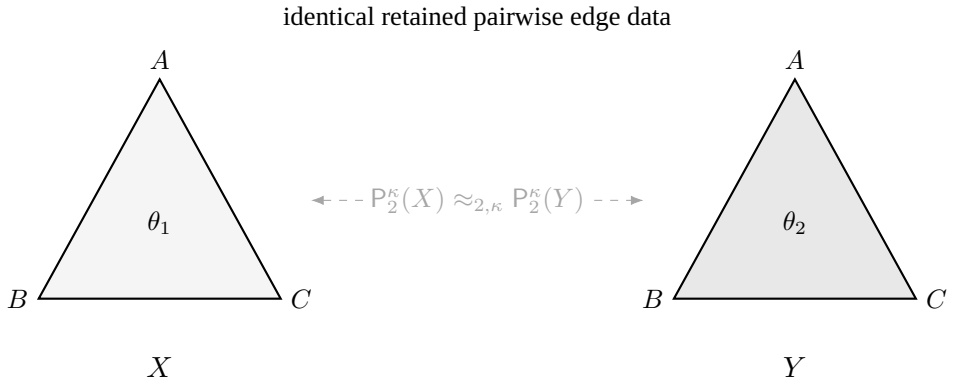

Figure 5: Finite witness of pairwise nonrecoverability. The pairwise restrictions retain identical edge-level data, while the full 2-complexes differ in a retained face label. The face has no intrinsic social interpretation; θ_1 and θ_2 acquire substantive meaning only in a later application.

$$\lambda_X(f_{ABC}) = \theta_1, \qquad \lambda_Y(f_{ABC}) = \theta_2, \qquad \theta_1 \neq \theta_2. \tag{66}$$

If $\sim_\kappa$ is defined to preserve the retained face label, Equation 66 yields

$$X \sim_{2,\kappa} Y \qquad \text{and} \qquad X \not\sim_\kappa Y, \tag{67}$$

Equation 67 supplies the finite pair that witnesses nonrecoverability under the selected face-label criterion.

so $(X, Y) \in \mathcal{G}^\kappa_{3|2}$ by Equation 51. Figure 5 depicts the construction.

The example is intentionally minimal. It shows only that pairwise edge data need not determine a retained face-level label. It does not show that every social triad deserves a face, that every face encodes a genuinely social thirdness, or that face labels are empirically identifiable. A later application might use θ to encode a jointly acknowledged procedure, a mediation status, a three-position authorization condition, or another higher-order record, but that semantic step requires separate justification.

The example also demonstrates the diagnostic formulation. Let d_f return the retained face label. Then $d_f(X) \neq d_f(Y)$ while $X \sim_{2,\kappa} Y$, so d_f fails pairwise factorization under Equation 60. If the full-equivalence relation preserves d_f, the diagnostic difference witnesses membership in $\mathcal{G}^\kappa_{3|2}$.

6.8 Scope of the Nonrecoverability Claim

The formal result of this section is deliberately narrow. Triadic nonrecoverability means that a declared pairwise representation fails to determine a declared full-equivalence class or diagnostic. It does not establish absolute irreducibility, metaphysical emergence, causal superiority of the triad, or normative significance. A richer pairwise shadow can remove a witness by retaining information previously discarded. A coarser full-equivalence relation can also remove it by declaring the distinction analytically irrelevant. Both changes are legitimate when explicitly justified.

The criterion nevertheless supplies a useful discipline for the rest of the paper. A later claim that witnessing, mediation, public acknowledgment, reputation, accountability, coalition, or another structure is pairwise nonrecoverable should be accompanied by one of three things: a nonempty equivalence-gap witness, failure of a canonical reconstruction, or a typed diagnostic that cannot factor through the retained pairwise shadow. The stronger the substantive claim, the more explicit the restriction and equivalence structure should become.

This discipline also separates several questions that later applications may answer differently. A third position can be practically important while pairwise sufficiency still holds. A triad can be pairwise nonrecoverable while the distinguishing structure has no moral significance. A model can be formally

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nonrecoverable while empirical observations remain too weak to identify which full reconstruction is realized. A normatively important third-party function can be represented pairwise if the chosen dyadic records are rich enough. The paper therefore treats pairwise reducibility as one formal axis among several, not as a universal ranking of relational depth.

Section 7 now turns from the reducibility criterion to the concept inventory required by the remainder of the paper. The inventory will separate dyadic carryovers from objects whose typing becomes specifically triadic, including third-position functions, route-indexed epistemic objects, publicity, reputation, accountability, coalition, and higher-order emergence.

7 Minimal Triadic System: Conceptual Object Inventory

This section consolidates the conceptual objects required by the minimal triadic study before the paper develops their dynamics in detail. Its role is classificatory: it distinguishes inherited dyadic object types, objects whose content becomes conditioned by a third position, and objects whose formal type is defined at the full-triad level. It also records each family's formal status, three-level burden, pairwise-restriction behavior, and historical provenance. The inventory is intentionally compact because Sections 8–15 provide the substantive definitions.

Inclusion in the inventory does not imply equal formal standing or pairwise nonrecoverability. A subject-spacetime, testimony, translation, reputation record, accountability claim, coalition record, and emergent public norm belong to different carriers. Likewise, a full-triad-typed object can remain reconstructible from sufficiently rich pairwise records. The inventory therefore separates triadic relevance, full-triad typing, historical emergence, and pairwise nonrecoverability.

7.1 Inventory Architecture

At resolution κ, an object can enter the local model as a relative primitive, a derived object, or a relationally emergent object. This formal-status classification is independent of its inheritance class. Paper II uses three inheritance classes: a *dyadic carryover*, already available inside a pairwise restriction; a *triadically contextualized object*, whose type remains dyadic or subject-relative while its content, accessibility, or efficacy depends on the third position; and a *full-triad-typed object*, whose declared carrier or defining condition is stated at the coupled ABC level. Full-triad typing does not imply pairwise nonrecoverability.

Every retained object must also answer the restriction question introduced in Sections 3–6: what survives in the AB, AC, and BC restrictions, and does the selected full meaning factor through the corresponding pairwise-shadow class? The audit schema is stated in Equation 68.

$$\mathsf{Audit}^{\kappa}_{3,t}(X) := \Big(\text{type}(X), \text{status}_{\kappa}(X), \text{inheritance}_3(X), \Lambda^{\kappa}_{ABC,t}(X), \mathsf{P}^{\kappa}_{2,X}(X), \text{hist}_t(X)\Big), \tag{68}$$

Here $\Lambda^{\kappa}_{ABC,t}(X)$ denotes the object's three-level and perspective-relative profile, $\mathsf{P}^{\kappa}_{2,X}$ its type-appropriate pairwise restriction when defined, and $\text{hist}_t(X)$ its retained provenance or trajectory record. Equation 68 is a ledger format rather than a claim that all object families inhabit one product space.

Table 8 summarizes the principal families used by the remainder of the paper.

7.2 Core Object Families

The subject-history family contains the tracked subconfigurations $\mathcal{K}^{\kappa}_{A,t}$, $\mathcal{K}^{\kappa}_{B,t}$, and $\mathcal{K}^{\kappa}_{C,t}$ inside $\mathcal{K}^{\kappa}_{ABC,t}$, together with their interfaces, manifestations, and historical continuations. Relational positions such as supporter, supportee, mediator, witness, translator, broker, reviewer, authorizer, coalition member, or excluded participant are not subject identities. Their occupancy can change across stages, and subjects can disagree about which position another subject occupies.

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Table 8: Master inventory of conceptual object families in the minimal triadic formalism.

Object family	Inheritance class	Default formal status	Principal triadic burden
Subject, history, position	Carryover / contextualized	Relative primitive / derived	Shared-subject tracking, role relativity, third-position context
Restriction, shadow, reconstruction	Full-triad typed	Derived	Definedness, compatibility, ambiguity, pairwise sufficiency, nonrecoverability
Manifestation, epistemic access, ToM	Carryover / contextualized	Derived	Three perspectives, indirect routes, route provenance, recursive modeling
Third-position functions	Full-triad typed	Derived	Mediation, witnessing, translation, brokerage, standing/provenance
Publicity, reputation, recognition	Contextualized / full-triad typed	Derived / emergent	Audience, acknowledgment, durability, circulation, public attribution
Accountability, standing, authorization	Contextualized / full-triad typed	Derived / emergent	Claim/answerability routes, review standing, authorization, enforcement
Coalition, alignment, exclusion	Full-triad typed	Derived / emergent	Coordination, route dependence, closure, gatekeeping, typed exclusion
Action, procedure, provenance	Carryover / contextualized / full-triad typed	Derived	Distributed generation, authorization, execution, witnessing, review
Evaluation, normativity, asymmetry	Carryover / contextualized	Derived / emergent	Three-perspective disagreement, typed dependency, normative uptake
Trust, public norm, partnership	Contextualized / full-triad typed	Emergent where criteria are met	Persistence, organization, efficacy, pairwise recoverability
Algebraic, categorical, geometric objects	Realization layer	Derived	Composition, gluing, distributed support, representation dependence

The restriction-reconstruction family contains typed pairwise restrictions and shadows, compatibility domains, pairwise equivalence, ambiguity classes, reconstruction maps, diagnostic factorization, and the equivalence gap $\mathcal{G}^{\kappa}_{3|2}$. These objects diagnose what a selected dyadic decomposition preserves. Context loss and pairwise nonrecoverability remain distinct: a restriction can discard material while the retained full-equivalence class remains uniquely reconstructible.

The epistemic and third-position-function families contain manifestation access, uptake, belief, confidence, memory, testimony, translation, route provenance, finite-depth ToM, mediation, witnessing, brokerage, recording, publicization, review, and coordination. Route provenance is retained separately from transmitted content. Function is retained separately from subject identity, and evidence is retained separately from standing or authority.

The public-procedural family contains audience-indexed publicity, acknowledgment, durable public records, reputation, recognition, accountability claims, responses, review, standing, authorization, determination, correction, appeal, and enforcement. Public visibility does not by itself establish acknowledgment or common knowledge; one attribution does not by itself constitute reputation; witness access does not by itself create review standing; and a determination does not by itself justify enforcement.

The positional and evaluative family contains brokerage, gatekeeping, alignment, coalition, typed exclusion, dependency, need and value judgments, aesthetic and normative judgments, refusal/exit/appeal capacities, and typed asymmetry. These objects can point in different directions across dimensions and therefore should not be compressed into one scalar hierarchy. Power and injustice remain later analytical questions.

The final family contains procedure elements, action individuation, distributed provenance, third-conditioned and joint action, trust, stable third-position channels, public norms, role ecology, and partnership. Later algebraic, categorical, cellular, and spin-foam-inspired constructions provide optional realizations. Emergence requires independently declared persistence, organization, and efficacy criteria; realization supplies a representation rather than an ontology.

Figure 6 summarizes the classification while keeping inheritance, emergence, and realization as distinct dimensions.

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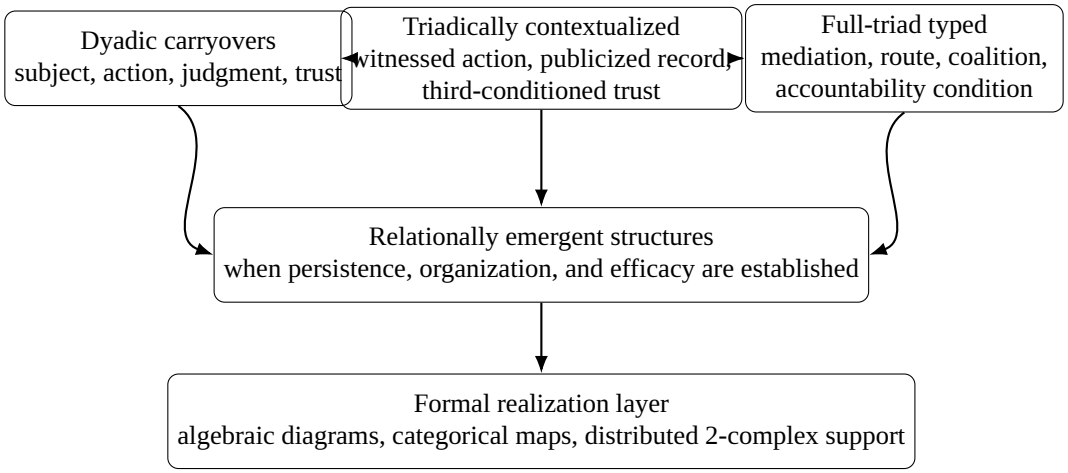

Figure 6: Inventory architecture for the minimal triadic formalism. Inheritance class, historical emergence, and mathematical realization are distinct classifications. Full-triad typing does not imply pairwise nonrecoverability.

7.3 Triadic Concept Audit

Before a later concept carries substantive weight, the manuscript should state its formal type; inheritance class; relative primitive, derived, or emergent status; relational and manifestation profiles; relevant A-, B-, and C-relative reconstructions; pairwise restrictions and any context loss; recoverability criterion; historical provenance and revision conditions; dynamical consequences; and the empirical or normative questions that remain open.

The dependency pattern is summarized in Equation 69.

$$\begin{aligned} \text{typed object} &\rightsquigarrow \text{three-level profile} \rightsquigarrow \text{pairwise restrictions} \\ &\rightsquigarrow \text{recoverability test} \rightsquigarrow \text{historical/dynamical role.} \end{aligned} \tag{69}$$

The arrows in Equation 69 denote analytical dependence rather than a universal causal sequence. A concept can be formally available without being empirically identified, historically emergent while remaining pairwise recoverable, or pairwise nonrecoverable without carrying normative importance. Section 8 begins the substantive construction with third-position function types.

8 Third-Position Functions: Mediation, Witnessing, Translation, Brokerage

This section develops the first substantive family of triadic operations after the inventory of Section 7. Its role is to formalize what a third position can *do* relative to a focal dyadic relation without turning the third subject into a fixed role or assuming that every third-party intervention is beneficial, neutral, authoritative, or irreducibly triadic. The objective is to distinguish mediation, witnessing, translation, and brokerage by their input/output structure, epistemic burden, historical provenance, and possible effects on later relations. The method is typed and relational. A function is assigned only after a focal pair, a third position, a historical stage, and the relevant domains of applicability have been declared.

Let $XY \in \{AB, AC, BC\}$ denote a focal pair and let Z denote the remaining subject. The notation $Z \mid XY$ therefore means "Z relative to the focal relation XY" and does not assign Z a permanent social identity. A can occupy a third position relative to B–C at one stage while simultaneously participating directly in A–B and A–C relations. The same subject can also perform more than one third-position function relative to the same focal pair.

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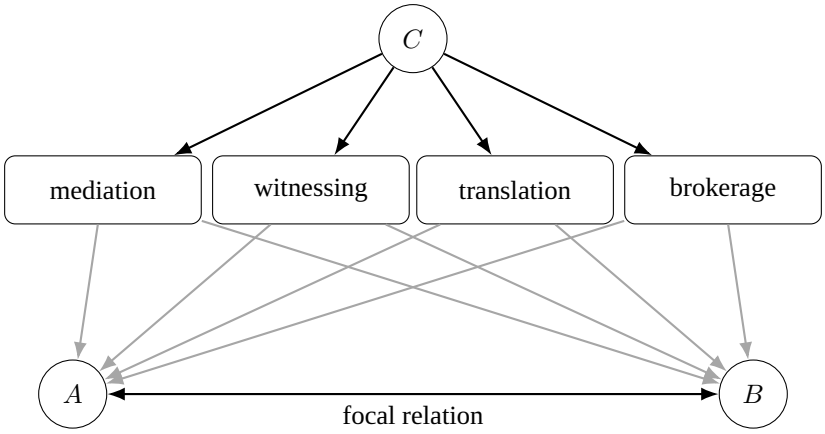

Figure 7: Core third-position function types relative to a focal A–B relation. The diagram shows functional orientation only. A function can be partial, contested, ineffective, or normatively unjustified, and one subject can perform several functions in one historical episode.

8.1 Relative Third-Position Function Record

A third-position function is modeled as a typed, partially defined transformation or relation that uses Z-relative access, standing, capability, or position to alter a retained aspect of the focal XY configuration. The function may act on manifestation, epistemic, procedural, evaluative, authorization, or relational records. It need not alter all of them.

For a function type ϕ relative to $Z \mid XY$, define a schematic function record by Equation 70.

$$\mathfrak{F}^{\kappa,\phi}_{Z|XY,t} := \Big(\mathcal{D}^{\kappa,\phi}_{Z|XY,t}, \mathcal{C}^{\kappa,\phi}_{Z|XY,t}, \Phi^{\kappa,\phi}_{Z|XY,t}, \Pi^{\kappa,\phi}_{Z|XY,t}, \mathsf{St}^{\kappa,\phi}_{Z|XY,t}\Big), \tag{70}$$

where $\mathcal{D}$ is the domain on which the function is defined, $\mathcal{C}$ its codomain, Φ the operative transformation or typed relation, Π the retained provenance record, and St a standing/authorization record when the application requires one. Equation 70 is intentionally generic. Different function types can have different carriers, and Φ need not be a linear operator or a total map.

Three cautions follow immediately. First, *function* and *subject* are distinct. C can mediate one episode and merely observe another. Second, functional effectiveness and normative legitimacy are distinct. A mediator can reshape a procedure without possessing justified authority to do so. Third, triadic relevance and pairwise nonrecoverability are distinct. A C-mediated change can remain fully reconstructible from sufficiently rich A–C and B–C records.

Figure 7 depicts the four core functions relative to a focal A–B relation.

8.2 Mediation Structure

Mediation concerns a third-position transformation of the interaction conditions between X and Y. The mediated object can be a communication channel, a negotiation state, an authorization procedure, a disagreement record, an action plan, or another typed component of the focal relation. Mediation is therefore broader than message relay and narrower than arbitrary third-party intervention.

Let $\Gamma^{\kappa}_{XY,t}$ denote a focal procedure record and let $\mathfrak{G}^{\kappa}_{XY}$ denote the class of admissible procedure records at the retained resolution. A C-mediated procedure transformation can be written as the partial map in Equation 71.

$$\mathsf{M}^{\kappa}_{C|AB,t} : \mathcal{D}^{\kappa}_{\mathsf{M},t} \rightharpoonup \mathfrak{G}^{\kappa}_{AB}, \qquad \mathcal{D}^{\kappa}_{\mathsf{M},t} \subseteq \mathfrak{G}^{\kappa}_{AB} \times \mathfrak{R}^{\mathsf{M},\kappa}_{C,t}, \tag{71}$$

where $\mathfrak{R}^{\mathsf{M},\kappa}_{C,t}$ is the declared carrier of C-relative mediation resources and $c_t \in \mathfrak{R}^{\mathsf{M},\kappa}_{C,t}$ denotes the resource record used in a realized mediation. The carrier can include C's access, translation resources,

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procedural standing, retained commitments, and prior history with A and B; it is not a scalar mediator attribute.

Equation 71 makes mediation generative when the output procedure differs in ways that alter later admissible interaction. Examples include opening a clarification path, introducing a shared vocabulary, separating a disputed claim from a disputed procedure, creating a turn-taking rule, or making refusal communicable without terminating the whole relation. The same form can also represent restrictive mediation that closes routes, privileges one manifestation regime, or changes the conditions under which one subject can participate.

A mediation record should therefore retain at least the provenance of the mediation request or initiation, the aspects of the focal procedure that C is permitted or able to transform, and the later consequences of that transformation. Neutrality is not built into the definition. Nor does mediation imply successful agreement. A mediation can be formally well typed even when the output retains disagreement or when one subject rejects the mediator's standing.

A stronger mediation claim can be stated when C's intervention changes the pairwise continuation family. Let $\mathrm{Cont}^{\kappa}_{AB,t}(\Gamma)$ denote the retained admissible continuations of a procedure state Γ. A mediation is continuation-changing when Equation 72 holds.

$$\mathrm{Cont}^{\kappa}_{AB,t}\Big(\mathsf{M}^{\kappa}_{C|AB,t}(\Gamma^{\kappa}_{AB,t}, c_t)\Big) \not\approx_{\mathrm{cont},\kappa} \mathrm{Cont}^{\kappa}_{AB,t}(\Gamma^{\kappa}_{AB,t}), \tag{72}$$

for a declared continuation-equivalence relation $\approx_{\mathrm{cont},\kappa}$. Equation 72 does not evaluate the change as better or worse. It records that the mediation has altered the represented procedural horizon.

8.3 Witnessing Structure

Witnessing begins with access but requires more than simultaneous presence. A third subject can observe an event without later retaining, attesting to, or making actionable any record of it. Conversely, a later witness record can depend on a durable manifestation such as a recording or document even when C did not perceive every underlying relational event directly. Paper II therefore distinguishes *witness access*, *retention*, *attestation*, and *later uptake*.

For an event or action object $e \in \mathfrak{E}^{\kappa,\mathrm{evt}}_{AB,t}$, define a C-relative witness record by Equation 73.

$$\mathfrak{W}^{\kappa}_{C|AB,t}(e) := \Big(\mathsf{Acc}^{\kappa}_{C,t}(e), \mathsf{Ret}^{\kappa}_{C,t}(e), \mathsf{Att}^{\kappa}_{C,t}(e), \mathsf{Prov}^{\kappa}_{C,t}(e)\Big), \tag{73}$$

where Acc records the relevant manifestation access, Ret the retained representation or record, Att an attestation object if one is produced, and Prov the provenance needed to interpret how the witness record formed. The components can be partial or absent. A nonempty access component alone does not establish a completed witness function.

Witnessing can become triadically consequential when the record changes what A or B can later claim, contest, remember, authorize, or make public. The witness can also alter behavior prospectively: A and B may act differently because C is present or because later attestation is expected. That prospective effect belongs to the coupled triadic history rather than to the later testimony alone.

The formalism keeps evidential access separate from authority. C can possess a witness record without standing to review an A–B decision. C can possess review standing while lacking direct access to the underlying event. Later accountability sections will therefore compose witness records with standing and authorization records only when the application explicitly provides those links.

Witness disagreement is also admissible. C's retained record can differ from A's or B's reconstruction, and later revision can alter any of the three. Paper II does not privilege the third position merely because it is external to the focal pair.

8.4 Translation Structure

Translation is a typed transformation between manifestation or interpretive regimes. It is relevant when A and B can each possess meaningful expressions that are not directly interoperable under the current

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dyadic reconstruction rules. The mismatch can concern natural language, professional vocabulary, institutional categories, symbolic conventions, accessibility formats, or another application-specific code.

Let $\mathcal{M}^{\alpha}_{A,t}$ and $\mathcal{M}^{\beta}_{B,t}$ be two manifestation carriers. A C-mediated translation is represented by the partial map

$$\mathsf{T}^{\alpha\Rightarrow\beta}_{C:t} : \mathcal{D}^{\alpha}_{\mathsf{T},t} \rightharpoonup \mathcal{M}^{\beta}_{B,t}, \qquad \mathcal{D}^{\alpha}_{\mathsf{T},t} \subseteq \mathcal{M}^{\alpha}_{A,t} \times \mathfrak{R}^{\mathsf{T}}_{C,t}, \tag{74}$$

Equation 74 types the C-mediated translation from a source manifestation regime into the target regime, where $\mathfrak{R}^{\mathsf{T}}_{C,t}$ collects the C-relative resources used by the translation. These can include competence, contextual memory, a lexicon, a jointly recognized convention, or another typed interpretive record. The notation $\alpha \Rightarrow \beta$ identifies the translation direction and does not imply invertibility.

Translation preserves neither full identity nor all distinctions by default. To make preservation explicit, let $\equiv^{\delta}_{\alpha,t}$ denote the source-side equivalence relevant to a retained distinction type δ, and let $\equiv^{\delta}_{\beta,t}$ denote the corresponding target-side equivalence. A translation preserves δ on its domain only if

$$m_1 \equiv^{\delta}_{\alpha,t} m_2 \implies \mathsf{T}^{\alpha\Rightarrow\beta}_{C:t}(m_1, c_t) \equiv^{\delta}_{\beta,t} \mathsf{T}^{\alpha\Rightarrow\beta}_{C:t}(m_2, c_t), \tag{75}$$

for the admissible C-relative resource record c_t. Equation 75 is one-sided. Distinct source manifestations can legitimately collapse under a coarse target vocabulary, and target distinctions can be introduced by contextual explanation rather than literal lexical correspondence. A later application must therefore declare which distinctions the translation is expected to preserve.

Translation can also be contested at three levels. A and B can disagree about the relational content, C's manifested wording or symbol, or the interpretation of the translated expression. The existence of a translation map therefore does not establish successful mutual understanding.

Historically, repeated translation can create new shared vocabulary. Once a translation convention stabilizes and becomes directly available to A and B, a formerly triadic translation route can become pairwise usable. Third-position necessity can thus diminish through the success of earlier third-position activity.

8.5 Brokerage Structure

Brokerage concerns an indirect route through which access, information, resources, recognition, authorization, or another typed object passes between X and Y via Z. Unlike mediation, which transforms the conditions of interaction, brokerage emphasizes route structure and the position of Z within that route. Unlike translation, brokerage need not change the representation carried along the route.

For object type χ, let the two brokerage legs be partial maps $\beta^{\kappa,\chi}_{A\to C,t} : \mathcal{D}^{\kappa,\chi}_{AC,t} \rightharpoonup \mathcal{C}^{\kappa,\chi}_{AC,t}$ and $\beta^{\kappa,\chi}_{C\to B,t} : \mathcal{D}^{\kappa,\chi}_{CB,t} \rightharpoonup \mathcal{C}^{\kappa,\chi}_{CB,t}$. The brokered route is the partial composite in Equation 76.

$$\mathsf{B}^{\kappa,\chi}_{C|AB,t} := \beta^{\kappa,\chi}_{C\to B,t} \circ \beta^{\kappa,\chi}_{A\to C,t}. \tag{76}$$

Its realized domain is the subset of $\mathcal{D}^{\kappa,\chi}_{AC,t}$ whose first-leg outputs lie in $\mathcal{D}^{\kappa,\chi}_{CB,t}$. Equation 76 represents transmission without implying that C controls passage.

Let $\mathsf{Reach}^{\kappa,\chi,\mathrm{direct}}_{A\to B,t}$ and $\mathsf{Reach}^{\kappa,\chi}_{A\to C\to B,t}$ be the declared direct- and brokered-route reachability predicates. The route is C-dependent when Equation 77 holds.

$$\mathsf{RouteDep}^{\kappa,\chi}_{C|AB,t} \quad \Longleftrightarrow \quad \neg\mathsf{Reach}^{\kappa,\chi,\mathrm{direct}}_{A\to B,t} \ \wedge \ \mathsf{Reach}^{\kappa,\chi}_{A\to C\to B,t}. \tag{77}$$

Equation 77 records route dependence only. Section 12 adds the stronger gatekeeping condition by requiring C to possess a declared capacity to condition passage. Route dependence alone does not establish control, coercion, domination, or injustice.

Brokerage can be symmetric, asymmetric, exclusive, competitive, or redundant. C can broker A's access to B while B has an independent direct route to A. Several routes can coexist. A broker can widen access by connecting otherwise separated positions or narrow access by filtering what passes. The historical effect depends on which route conditions become stabilized.

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Table 9: Comparison of the four core third-position function types.

Function	Primary object transformed	Distinction preserved	Typical failure or contest
Mediation	Procedure / interaction conditions	Facilitation versus agreement; function versus legitimacy	Standing dispute, biased procedure, closed continuation
Witnessing	Access, retention, attestation	Observation versus testimony; evidence versus authority	Partial access, memory conflict, contested credibility
Translation	Manifestation / interpretive carrier	Content versus representation; preservation relative to declared distinctions	Loss, distortion, vocabulary mismatch, contested interpretation
Brokerage	Route / access / transmission	Relay versus gatekeeping; access versus control	Route dependence, filtering, exclusion, hidden alternatives

8.6 Functional Overlap, Composition, Migration

The four function types are analytically distinct but can overlap in one episode. A mediator can translate an expression, witness the resulting agreement, and later broker access to a resource required for implementation. A witness can also become a reviewer. A translator can acquire de facto brokerage importance because A and B lack another interoperable route.

Composition must therefore be typed. Let $\Phi^{\phi_1}_{C|AB} : D_1 \rightharpoonup C_1$ and $\Phi^{\phi_2}_{C|AB} : D_2 \rightharpoonup C_2$ be partial function maps. When C_1 and D_2 have a declared interface, the ordered composite is stated in Equation 78.

$$\Phi^{\phi_2 \circ \phi_1}_{C|AB} := \Phi^{\phi_2}_{C|AB} \circ \Phi^{\phi_1}_{C|AB}, \qquad \mathrm{Dom}(\Phi^{\phi_2 \circ \phi_1}) = \{x \in D_1 : \Phi^{\phi_1}_{C|AB}(x) \in D_2\}. \tag{78}$$

Equation 78 makes the definedness condition explicit rather than treating the two function records as automatically composable. The reversed ordering need not be defined and, when defined, need not yield an equivalent result. Translation before mediation can produce a different procedure from mediation conducted before the contested terms have been translated. Witnessing before publicization differs from publicization of a later reconstructed account.

Functional occupancy can also migrate. C can initially mediate an A–B disagreement and later cease to be necessary after A and B develop a shared procedure. B can become the mediator of a later A–C conflict. An institution can replace an individual witness function by maintaining a durable public record. The paper therefore tracks function histories rather than assigning fixed "mediator" or "broker" identities.

8.7 Three-Level Profiles: Pairwise Recoverability

Each third-position function inherits the three-level discipline of Section 4. At the relational level, the model asks whether the function has a sufficiently specified place in the coupled triadic organization. At the manifestation level, it asks how the function becomes visible or actionable through utterances, records, procedures, spatial arrangements, interface design, or other manifestations. A-, B-, and C-relative reconstructions can then disagree about whether the function occurred, what it accomplished, and whether C possessed the relevant standing.

Table 9 summarizes the four core functions.

None of the four functions is triadically nonrecoverable by definition. A mediation can be fully encoded in rich A–C and B–C procedural records. A witness structure can be pairwise recoverable when C's access and attestation are fully retained in those restrictions. Translation and brokerage can likewise factor through pairwise routes. The stronger claim arises only when a retained function-specific

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diagnostic fails to factor through the pairwise shadow or when the corresponding configurations populate the equivalence gap $\mathcal{G}^{\kappa}_{3|2}$.

For a function type ϕ, let $d^{\kappa}_{\phi} : \mathfrak{D}^{\kappa}_{3} \to \mathcal{Y}^{\kappa}_{\phi}$ be a full-equivalence-respecting diagnostic. Function-specific triadic nonrecoverability is witnessed when

$$X \sim_{2,\kappa} Y \qquad \text{and} \qquad d^{\kappa}_{\phi}(X) \neq d^{\kappa}_{\phi}(Y), \tag{79}$$

under the diagnostic conditions of Section 6. Equation 79 prevents the paper from calling a function "pairwise nonrecoverable" merely because it involves C. The claim must remain relative to the retained pairwise representation.

8.8 Historical Function Trajectories

Third-position functions can emerge, stabilize, lose relevance, migrate, or become institutionalized. The retained function trajectory over historical interval I is defined in Equation 80.

$$\mathbf{F}^{\kappa,\phi}_{Z|XY,I} := \left(\mathfrak{F}^{\kappa,\phi}_{Z|XY,t}\right)_{t\in I}. \tag{80}$$

Equation 80 can change through revised access, altered competence, new procedures, changing standing, loss of trust, improved direct communication, or reorganization of the coupled carrier.

Several trajectories are analytically distinct. A *scaffolding trajectory* occurs when a third-position function helps create conditions under which the focal dyad can later perform the relevant operation directly. A *stabilization trajectory* occurs when repeated third-position activity becomes a durable part of the relation. A *dependency trajectory* occurs when later interaction increasingly requires passage through Z. A *displacement trajectory* occurs when the function migrates to another subject, institution, or technical interface. A *dissolution trajectory* occurs when the function ceases to be relevant.

These trajectories are not normatively ranked by the formalism. Stable mediation can be beneficial or restrictive; direct dyadic communication can represent increased autonomy or loss of a valuable review mechanism; broker dependence can create access or generate exclusion. The formal contribution is to make the trajectories distinguishable enough for later ethical, political-economic, and historical analysis.

The present section has therefore supplied a typed functional vocabulary for third-position activity without yet developing the detailed epistemic recursion produced by indirect routes. Section 9 takes the next step by formalizing how direct and mediated routes alter belief formation, theory of mind, route confidence, and higher-order mutual modeling.

9 Route-Sensitive Epistemic Modeling: Recursive Theory of Mind

This section develops the epistemic consequences of the third-position functions formalized in Section 8. Its role is to represent how a subject can form beliefs, interpretations, and theory-of-mind records through more than one route while preserving the distinction among route provenance, transmitted content, subject-relative reconstruction, and the modeled subject itself. The objective is to make indirect access and higher-order modeling explicit without granting the third position privileged access to relational reality. The discussion proceeds from typed epistemic routes to route-indexed reconstruction, route comparison, recursive theory of mind, route-conditioned confidence, and historical revision. The method is formal and relational; no universal probabilistic model of belief or reliability is assumed.

Let $i, j, k \in \{A, B, C\}$ be distinct subjects. A route from j to i may be direct, $j \to i$, or mediated through k, $j \to k \to i$. More complicated histories can be represented later by composition, but the one-intermediate-subject case is sufficient to expose the specifically triadic structure. The route notation records how a manifestation or epistemically relevant object becomes available. It does not imply that the transmitted content originates entirely in j, that k is neutral, or that i reconstructs the same object that j intended to manifest.

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9.1 Epistemic Route Records

A route should retain more than an ordered list of subjects. It must identify the object type being transmitted or made accessible, the manifestation carriers used at each step, the transformations applied along the route, and the provenance needed to interpret the result. For a route $\gamma : j \rightsquigarrow i$, define the route record in Equation 81.

$$\mathfrak{R}^{\kappa,\gamma}_{i\leftarrow j,t} := \left(\gamma, \chi, \mathcal{D}^{\kappa,\chi}_{\gamma,t}, \mathcal{C}^{\kappa,\chi}_{\gamma,t}, \Psi^{\kappa,\chi}_{\gamma,t}, \Pi^{\kappa,\chi}_{\gamma,t}\right), \tag{81}$$

where χ is the retained content type, $\mathcal{D}^{\kappa,\chi}_{\gamma,t}$ and $\mathcal{C}^{\kappa,\chi}_{\gamma,t}$ are the route domain and codomain, $\Psi^{\kappa,\chi}_{\gamma,t}$ is the partially defined route transformation, and $\Pi^{\kappa,\chi}_{\gamma,t}$ is the route provenance record. The route transformation can include uptake, translation, filtering, summarization, authentication, or another declared operation. Equation 81 therefore separates the route itself from the content that traverses it.

For the mediated route $B \to C \to A$, let $\psi^{\kappa,\chi}_{C\leftarrow B,t} : D_{CB} \rightharpoonup C_{CB}$ and $\psi^{\kappa,\chi}_{A\leftarrow C,t} : D_{AC} \rightharpoonup C_{AC}$ be the two route-leg transformations. The operative transformation is the partial composite in Equation 82.

$$\Psi^{\kappa,\chi}_{A\leftarrow C\leftarrow B,t} := \psi^{\kappa,\chi}_{A\leftarrow C,t} \circ \psi^{\kappa,\chi}_{C\leftarrow B,t}, \qquad \mathrm{Dom}(\Psi) = \{x \in D_{CB} : \psi^{\kappa,\chi}_{C\leftarrow B,t}(x) \in D_{AC}\}. \tag{82}$$

Equation 82 makes the route-leg compatibility requirement explicit. The direct route $\Psi^{\kappa,\chi}_{A\leftarrow B,t}$ and the mediated route in Equation 82 can coexist. Their outputs need not be identical or comparable under one scalar discrepancy.

Route availability is itself historical. A direct route may be absent at one stage and available later after a shared vocabulary forms. Conversely, institutional or interpersonal changes can make a previously direct route inaccessible, leaving a brokered or translated route as the only retained pathway.

9.2 Route-Indexed Reconstruction

Section 4 introduced subject-relative reconstruction maps. The present section refines that architecture by indexing reconstruction to the route through which manifestations become available. Let $\widehat{\mathcal{R}}^{\kappa,\gamma}_{i,t}$ denote the class of i-relative reconstructions formed through route γ. A route-indexed reconstruction relation is written schematically as

$$\rho^{\kappa,\gamma}_{i,t} \subseteq \left(\mathcal{C}^{\kappa,\chi}_{\gamma,t} \times H_i^{\leq t}\right) \times \widehat{\mathcal{R}}^{\kappa,\gamma}_{i,t}, \tag{83}$$

where $H_i^{\leq t}$ records the retained interpretive history of subject i. Equation 83 remains relational rather than functional because the same routed manifestation can support several admissible reconstructions.

This route index prevents a common collapse. A can receive similar propositional content directly from B and indirectly through C while treating the two records differently because the histories, translation operations, or standing of the routes differ. Conversely, two routes can carry different surface forms that A reconstructs as equivalent relative to a declared content criterion.

For two routes $\gamma_1, \gamma_2 : j \rightsquigarrow i$, define route-relative reconstruction equivalence by

$$\widehat{r}_i^{\gamma_1} \approx^{\chi}_{i,t} \widehat{r}_i^{\gamma_2} \quad \Longleftrightarrow \quad \text{the two reconstructions are equivalent under the declared criterion for type } \chi. \tag{84}$$

Equation 84 introduces no metric. A later application can supply an ordinal, logical, probabilistic, semantic, or another comparison structure if warranted.

9.3 Route Agreement, Divergence, Provenance

When several routes are available, their comparison can reveal agreement, divergence, or underdetermination. None of those outcomes determines truth by itself. A direct route can be mistaken, a translated route can correct an ambiguity, and several agreeing routes can share the same source error.

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For a route family $\Gamma^{\kappa,\chi}_{i\leftarrow j,t}$, define the route-reconstruction set

$$\mathsf{Rec}^{\kappa,\chi}_{i\leftarrow j,t} := \left\{ \widehat{r}_i^{\gamma} \;\middle|\; \gamma \in \Gamma^{\kappa,\chi}_{i\leftarrow j,t},\ \widehat{r}_i^{\gamma} \in \widehat{\mathcal{R}}^{\kappa,\gamma}_{i,t} \right\}. \tag{85}$$

The set in Equation 85 can contain mutually equivalent reconstructions, several nonequivalent candidates, or only one defined record. Route agreement therefore means agreement relative to a declared equivalence; route divergence means the retained reconstructions occupy more than one equivalence class.

Provenance should remain attached even when route outputs agree. Let $\Pi(\gamma)$ denote the provenance record of route γ. The pair

$$\left([\widehat{r}_i^{\gamma}]_{\approx^{\chi}_{i,t}}, \Pi(\gamma) \right) \tag{86}$$

Equation 86 keeps reconstructed content and route provenance jointly available without identifying them.

preserves the distinction between reconstructed content and the history through which it arrived. This distinction matters in volunteer relations because testimony, translation, public record, professional review, and direct self-report can carry similar content while creating different later obligations, trust effects, or contestability.

Route divergence can also concern provenance rather than content. A and B can agree about what C communicated while disagreeing about whether C translated, summarized, filtered, or independently reinterpreted the source. Such disagreement belongs to the epistemic architecture even when the final proposition is shared.

9.4 Epistemic Triangulation: Situated Third-Position Access

The presence of several routes permits a form of triangulation, but Paper II uses the term cautiously. Triangulation means comparison among differently generated reconstructions, not automatic convergence on an objective truth. The third position can add an independent manifestation route, supply contextual information, or expose inconsistency between A's and B's accounts; it can also reproduce shared bias or introduce new distortion.

Let d^{χ} be a diagnostic defined on the route-reconstruction sets under comparison, with codomain equivalence $\approx_{d,\chi}$. A route family is *informationally nonredundant* relative to d^{χ} when removing one route changes the retained diagnostic class. If Γ is finite and $\gamma \in \Gamma$, the condition is stated in Equation 87.

$$d^{\chi}\left(\mathsf{Rec}^{\Gamma}_{i\leftarrow j}\right) \not\approx_{d,\chi} d^{\chi}\left(\mathsf{Rec}^{\Gamma\setminus\{\gamma\}}_{i\leftarrow j}\right), \tag{87}$$

Equation 87 says that the route contributes retained information under the chosen diagnostic. It does not imply that the contribution is correct or beneficial.

Triangulation can therefore support error detection, confidence revision, or identification of unresolved alternatives without producing a unique final belief. This is especially important when C witnesses an event that A and B interpret differently, or when C translates a manifestation that A and B cannot compare directly.

9.5 Route-Sensitive First-Order Theory of Mind

Paper I treated theory of mind as a derived model formed from accessible manifestations and retained history. The triadic case adds route provenance to that construction. Let $\mathfrak{T}^{(1),\kappa,\gamma}_{i\triangleright j,t}$ denote the class of first-order ToM records in which i models j using manifestations available through route γ. The route-sensitive ToM construction relation is

$$\mathsf{ToM}^{(1),\kappa,\gamma}_{i\triangleright j,t} \subseteq \widehat{\mathcal{R}}^{\kappa,\gamma}_{i,t} \times \mathfrak{T}^{(1),\kappa,\gamma}_{i\triangleright j,t}. \tag{88}$$

Equation 88 preserves underdetermination. A's direct reconstruction of B can support one family of ToM records while C-mediated material supports another. Neither family is privileged by the formalism.

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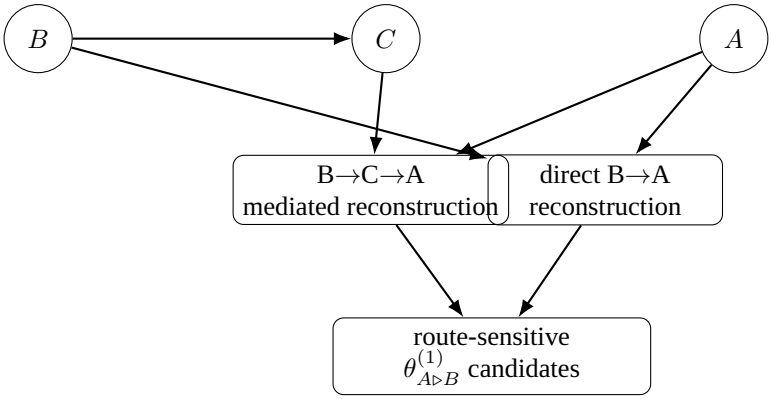

Figure 8: Route-sensitive first-order theory of mind. A can construct models of B from direct B–A manifestations and from material routed through C. The diagram records route provenance and candidate-model generation; it does not rank the routes or imply convergence.

A route-sensitive ToM profile can retain modeled belief, intention, preference, affect, evaluative judgment, normative judgment, or confidence as separately typed components. Missing components remain undefined rather than zero. A later empirical application can also record whether a ToM component depends primarily on direct interaction, witness testimony, translation, reputation, or another route.

Figure 8 summarizes direct and mediated routes into A's model of B.

9.6 Recursive Triadic Theory of Mind

The third position increases the number of meaningful recursive targets. A can model B, model C, model C's model of B, model B's model of C, or model how C interprets B's model of A. The formalism should generate these objects recursively rather than introducing each as a primitive.

Let $\mathfrak{X}^{\kappa}_{0,t}$ be the retained base target family. For distinct $i, j \in \{A, B, C\}$, let $\mathbf{T}^{\gamma}_{i\triangleright j,t}$ denote a typed ToM-construction operation defined on a subset of the current target family and indexed by the route provenance γ of the evidence used. The finite-depth grammar is

$$\mathfrak{X}^{\kappa}_{n+1,t} := \mathfrak{X}^{\kappa}_{n,t} \cup \left\{ \mathbf{T}^{\gamma}_{i\triangleright j,t}(x) \;\middle|\; x \in \mathfrak{X}^{\kappa}_{n,t},\ x \in \mathrm{Dom}(\mathbf{T}^{\gamma}_{i\triangleright j,t}) \right\}, \tag{89}$$

for a declared set of admissible subject pairs and route indices. Equation 89 permits arbitrary finite nesting but does not imply unlimited psychological recursion.

The route index matters because $\mathbf{T}^{B\to A}_{A\triangleright B}(x)$ and $\mathbf{T}^{B\to C\to A}_{A\triangleright B}(x)$ can be distinct objects even when they concern the same target content. A can therefore model B's intention differently depending on whether the evidence comes directly from B or through C.

Several recursive forms are especially relevant to volunteer relations: A's model of C's model of B; A's model of B's model of C's standing; C's model of how B interprets A's support action; and B's model of whether C believes B's refusal. These forms can condition communication, mediation, public acknowledgment, and accountability without requiring a universal game-theoretic interpretation.

9.7 Route Confidence, Credibility, Standing

Confidence, credibility, and standing must remain distinct. Confidence concerns a subject's commitment to a reconstruction or ToM record. Credibility is a route- or source-relative evaluative object concerning whether a manifestation or testimony is treated as reliable under a declared criterion. Standing concerns whether a subject is recognized as entitled to perform a procedure, make a claim, review an action, or issue an authoritative determination.

For route γ, Paper II therefore permits separate records

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$$\Big(\mathsf{Conf}^{\gamma}_{i,t}, \mathsf{Cred}^{\gamma}_{i,t}, \mathsf{Stand}^{\gamma}_{i,t}\Big), \tag{90}$$

Equation 90 keeps confidence, credibility, and standing as separate route-relative records without placing them on one common numerical scale. A can regard C's testimony as credible while denying C review standing. A can possess high confidence in a direct interpretation while later learning that the route was incomplete. Institutional standing can be high while credibility in one specific episode is contested.

These records are historically revisable. Repeated successful translation can increase route credibility; exposure of selective filtering can decrease it; formal appointment can change standing; withdrawal or conflict of interest can remove standing without changing the factual content of an earlier witness record.

9.8 Epistemic Asymmetry across Routes

Triadic epistemic asymmetry is not exhausted by comparing how much A, B, and C know. The structure can concern route access, route multiplicity, interpretive capacity, provenance visibility, or the ability to compare direct and mediated records. A may possess direct access to B while C has only mediated access; C may possess a durable record unavailable to A; B may know the source of a translated message while A sees only the translated manifestation.

For content type χ, define a route-access profile

$$\mathfrak{A}^{\kappa,\chi}_{ABC,t} := \Big(\Gamma^{\kappa,\chi}_{A,t}, \Gamma^{\kappa,\chi}_{B,t}, \Gamma^{\kappa,\chi}_{C,t}\Big), \tag{91}$$

where each $\Gamma^{\kappa,\chi}_{i,t}$ is the family of routes through which i can access content of type χ. Equation 91 is set-valued and does not define an epistemic ranking. Route multiplicity can provide redundancy, contradiction, or additional burden rather than uniformly greater knowledge.

A route-sensitive asymmetry claim therefore needs a declared dimension. Examples include one subject having access to an independent witness route, one subject controlling the only translation route, or one subject lacking provenance information for a manifestation available to the other two. These structures later intersect with brokerage and power but are not themselves equivalent to power or injustice.

9.9 Pairwise Recoverability of Epistemic Routes

Many indirect epistemic structures are recoverable from rich pairwise records. If A–C records preserve C's testimony and B–C records preserve C's source access, then the route $B \to C \to A$ can be reconstructible from the two dyads. Pairwise recoverability therefore remains an empirical/formal question rather than an automatic consequence of route length.

Let $d^{\kappa}_{\text{route}}$ be a full-equivalence-respecting diagnostic of the route-indexed epistemic structure. Route-level triadic nonrecoverability is witnessed when

$$X \sim_{2,\kappa} Y \qquad \text{and} \qquad d^{\kappa}_{\text{route}}(X) \neq d^{\kappa}_{\text{route}}(Y). \tag{92}$$

A witness for Equation 92 could arise when all three pairwise records preserve local message contents but fail to retain which combination of route provenance and higher-order acknowledgment made a particular belief actionable in the full triad. The paper will return to such structures when publicity and accountability are introduced.

9.10 Historical Epistemic-Route Trajectories

Route structures evolve. The retained epistemic-route trajectory for content type χ is defined in Equation 93.

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Table 10: Principal route-sensitive epistemic objects in the minimal triadic formalism.

Object	Formal role	Distinction preserved	Historical question
Epistemic route record	Typed path and transformation	Content versus route/provenance	Which paths appear, disappear, or become direct?
Route-indexed reconstruction	Subject-relative interpretation	Reconstruction versus manifestation	Does one route revise another?
Route-reconstruction set	Candidate family	Agreement versus underdetermination	Does route multiplicity narrow or widen candidates?
Route-sensitive ToM	Model of another subject	Target content versus evidence route	How does mediation reshape subject modeling?
Triadic ToM recursion	Higher-order subject modeling	Nested model versus target subject	Which recursive levels become procedurally relevant?
Confidence, credibility, standing	Separate evaluative records	Commitment versus reliability versus entitlement	How are routes re-evaluated through history?
Route-access profile	Set-valued access structure	Multiplicity versus epistemic ranking	Does route access diversify or concentrate?

$$\mathbf{E}^{\kappa,\chi}_{ABC,I} := \left(\mathfrak{A}^{\kappa,\chi}_{ABC,t}, \{\rho^{\kappa,\gamma}_{i,t}\}_{i,\gamma}, \{\mathsf{ToM}^{\gamma}_{i\triangleright j,t}\}_{i,j,\gamma}\right)_{t\in I}. \tag{93}$$

Equation 93 can change because direct communication becomes possible, a translator becomes unnecessary, a witness record becomes public, a brokered route is closed, a subject revises its credibility judgments, or a new shared interpretive convention emerges.

Several historically important patterns follow. A *route-shortening trajectory* occurs when a mediated route helps establish later direct access. A *route-lock-in trajectory* occurs when interaction becomes increasingly dependent on one intermediary. A *route-diversification trajectory* occurs when several independent paths become available. A *route-collapse trajectory* occurs when previously available paths disappear or become unusable. A *route-reinterpretation trajectory* occurs when the same stored manifestation acquires a different epistemic role because provenance, standing, or contextual information changes.

These patterns can alter ToM without changing the underlying focal event. A later testimony can revise A's model of B's earlier intention; a new translation can alter how B reconstructs A's prior explanation; discovery of an omitted route can change confidence in a formerly stable account. The relevant history therefore includes not only beliefs and manifestations but also the architecture through which those objects became mutually accessible.

Table 10 summarizes the principal route-sensitive objects introduced in this section.

The route-sensitive architecture developed here supplies the epistemic basis for the next section. Publicity, acknowledgment, reputation, and recognition depend not only on what becomes manifest but also on who can access it, through which routes, under which provenance conditions, and with what higher-order awareness of the other subjects' access.

10 Publicity, Acknowledgment, Reputation, Recognition

This section develops the public and recognitional structures made available by the route-sensitive epistemic architecture developed above. Its role is to distinguish broad accessibility from publicity, publicity from acknowledgment, acknowledgment from common knowledge, individual attribution from reputation, and recognition from approval or legitimacy. The objective is to formalize how an A–B event can acquire a different relational status when it is presented to a declared audience, jointly acknowledged, durably recorded, reputationally circulated, or publicly recognized through C or through another triadic route. The method remains typed and historical. No public record is assumed to be truthful merely because it is public, and no reputation or recognition structure is treated as normatively valid merely because it becomes stable.

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The minimal triad is sufficient to expose these distinctions because an event can be available to A and B privately while C remains outside the relevant audience, can become visible to C without becoming publicly addressed, or can be presented to all three under one shared audience structure. Those cases can have similar local manifestations while differing in acknowledgment, future accountability, reputation, and role stabilization.

10.1 Audience-Indexed Publicity

Publicity is modeled relative to a declared audience rather than as an absolute property. Let $\mathcal{U} \subseteq \{A, B, C\}$ with $|\mathcal{U}| \geq 2$ be an audience retained by the model, and let $m \in \mathcal{M}^{\kappa}_{ABC,t}$ be a manifestation object. Section 4 already distinguishes common accessibility from stronger public structure. The present section adds the shared-presentation relation $\mathsf{Pres}^{\kappa}_{\mathcal{U},t}(m)$, meaning that m is presented, posted, announced, displayed, recorded, or otherwise made manifest *to the audience $\mathcal{U}$ as such* under the selected model.

Define audience-indexed publicity by Equation 94.

$$\mathsf{Pub}^{\kappa}_{\mathcal{U},t}(m) \quad \Longleftrightarrow \quad \mathsf{Pres}^{\kappa}_{\mathcal{U},t}(m) \ \wedge \bigwedge_{i \in \mathcal{U}} \big(m \in \mathcal{M}^{\mathrm{acc},ABC,\kappa}_{i,t}\big). \tag{94}$$

Equation 94 is stronger than coincident accessibility because it retains a common presentation relation. A document can be technically accessible to A, B, and C through three independent routes without being presented to them as a shared audience. Conversely, a public announcement can satisfy Equation 94 even when no subject has yet actually attended to or acknowledged it.

Publicity therefore remains distinct from uptake. Let $\mathsf{Upt}^{\kappa}_{i,t}(m)$ denote the subject-relative uptake relation. The implication

$$\mathsf{Pub}^{\kappa}_{\mathcal{U},t}(m) \ \not\Longrightarrow \ \bigwedge_{i \in \mathcal{U}} \mathsf{Upt}^{\kappa}_{i,t}(m) \tag{95}$$

Equation 95 records that a public manifestation can remain unnoticed, unread, or uninterpreted by part of its audience.

Publicity is also route-sensitive. A may encounter a public record directly while B accesses it through C's summary. The public status concerns the retained audience/presentation structure; the reconstruction formed by each subject still depends on route, history, and interpretation.

10.2 Acknowledgment Structure

Acknowledgment requires a further subject-relative act or state: the model records that a subject has taken up a manifestation or claim and treats it as acknowledged under the selected procedure. Acknowledgment can concern the content itself, the occurrence of an event, the existence of a public record, or the fact that another subject has acknowledged it. These objects must remain typed.

For $i \in \mathcal{U}$, let $\mathsf{Ack}^{\kappa}_{i,t}(m)$ denote individual acknowledgment of manifestation m. The first finite audience-level condition is joint first-order acknowledgment,

$$\mathsf{JAck}^{(1),\kappa}_{\mathcal{U},t}(m) \quad \Longleftrightarrow \quad \bigwedge_{i \in \mathcal{U}} \mathsf{Ack}^{\kappa}_{i,t}(m). \tag{96}$$

Equation 96 does not imply that any subject knows that the others acknowledge m. A second structure records higher-order awareness. Define the acknowledgment graph

$$\mathcal{G}^{\mathrm{ack},\kappa}_{\mathcal{U},t}(m) := \big(\mathcal{U}, E^{\mathrm{ack},\kappa}_{\mathcal{U},t}(m)\big), \tag{97}$$

Equation 97 represents finite higher-order awareness through a directed acknowledgment graph, where $(i, j) \in E^{\mathrm{ack},\kappa}_{\mathcal{U},t}(m)$ means that i has a retained model in which j acknowledges m. A complete directed

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acknowledgment graph can therefore represent finite mutual awareness at the selected depth without silently upgrading the structure to common knowledge.

Higher-order acknowledgment can be generated recursively through the ToM grammar of Section 9.6. Paper II does not assume the infinite iteration usually associated with strong common-knowledge conditions. The level of acknowledgment required by a procedure must be declared locally. A consent procedure can require that A and B each acknowledge a condition and each represent the other's acknowledgment, while a public notice can require only audience-level availability.

The distinction among publicity, acknowledgment, and higher-order awareness is summarized by the non-entailments

$$\mathsf{Pub}_{\mathcal{U}}(m) \;\not\Longrightarrow\; \mathsf{JAck}^{(1)}_{\mathcal{U}}(m) \;\not\Longrightarrow\; \text{common knowledge of } m. \tag{98}$$

Equation 98 is central to later accountability analysis: a publicly available rule, complaint, refusal, or support record need not have been acknowledged by every subject to whom later responsibility is attributed.

10.3 Durable Public Records

Publicity becomes historically consequential when manifestations persist, can be revisited, or alter later procedures. Paper II therefore distinguishes a transient public manifestation from a durable public record. For a public manifestation m, define the record

$$\mathfrak{D}^{\mathrm{pub},\kappa}_{\mathcal{U},t}(m) := \Big(m, \mathcal{U}, \Pi_t(m), \mathsf{Ret}_t(m), \mathsf{Rev}_t(m), \mathsf{Stat}_t(m)\Big), \tag{99}$$

Equation 99 records the provenance, retention, revision, and status components of a durable public record, where $\Pi_t(m)$ records provenance, $\mathsf{Ret}_t(m)$ the retention mechanism, $\mathsf{Rev}_t(m)$ the available revision/retraction relation, and $\mathsf{Stat}_t(m)$ the current record status under the selected procedure. Possible statuses can include active, contested, corrected, superseded, or withdrawn when the application defines those categories.

Durability is not truth. A false or misleading record can persist. Nor does retraction erase every historical consequence of prior circulation. A corrected public record can continue to shape reputation, memory, or institutional action because earlier manifestations have already entered other histories.

The existence of a durable record can change later epistemic routes. A witness-dependent account can become directly accessible through a shared archive. A private disagreement can acquire a reviewable trace. A support action that initially had only A- and B-relative interpretations can later become the object of C-relative evaluation once a record is available.

10.4 Reputation Records

Reputation is not identified with one person's opinion. Paper II treats a reputation record as an audience-indexed historical structure in which attributions concerning a subject circulate, persist, and can affect later possibilities. Let $x \in \{A, B, C\}$ be the subject of reputation and let χ denote the attribution type, such as reliability, competence, generosity, intrusiveness, trustworthiness, or another application-specific descriptor.

For audience $\mathcal{U}$, define the reputation record

$$\mathfrak{Rep}^{\kappa,\chi}_{x,\mathcal{U},t} := \Big(\{\mathsf{Attr}^{\kappa,\chi}_{i\to x,t}\}_{i\in\mathcal{U}}, \mathsf{Circ}^{\kappa,\chi}_{\mathcal{U},t}, \mathsf{Prov}^{\kappa,\chi}_{\mathcal{U},t}, \mathsf{Hist}^{\le t,\chi}_{x,\mathcal{U}}\Big), \tag{100}$$

where $\mathsf{Attr}_{i\to x}$ is an audience member's attribution, Circ records the circulation structure through which attributions become available, Prov retains provenance, and Hist records the retained historical accumulation. Equation 100 does not require agreement among audience members and does not convert the attributions into a scalar score.

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A single judgment by C becomes reputationally relevant only when the model supplies a route by which it enters a broader or durable attribution structure. Thus

$$\mathsf{Attr}^{\kappa,\chi}_{C\to A,t} \;\not\Longrightarrow\; \mathfrak{Rep}^{\kappa,\chi}_{A,\mathcal{U},t} \tag{101}$$

without additional circulation or historical conditions. Equation 101 prevents the word *reputation* from being used as shorthand for any third-party opinion.

Reputation can later satisfy a relational-emergence criterion if it becomes sufficiently persistent, organized, and efficacious to alter access, trust, role assignment, invitation, scrutiny, or other continuations. That stronger status is deferred to Section 15. The present section supplies the record from which such a claim could later be assessed.

10.5 Recognition Structure

Recognition concerns a typed relational positioning through which one subject or audience treats another subject, claim, contribution, role, status, capacity, refusal, vulnerability, or authorship as countable in a specified way. Recognition is not restricted to praise. A subject can be recognized as a participant, as the originator of an idea, as competent to refuse, as entitled to speak, as responsible for an action, or as occupying another declared position. Recognition can also be negative or restrictive when the model tracks classifications that constrain future possibilities.

Let χ denote the object of recognition. A directed recognition record is

$$\mathsf{Recog}^{\kappa,\chi}_{i\rightsquigarrow j,t} \in \mathfrak{R}^{\kappa,\chi}_{i\rightsquigarrow j,t}, \tag{102}$$

Equation 102 types recognition as a directed and object-specific relational record, where the carrier $\mathfrak{R}^{\kappa,\chi}_{i\rightsquigarrow j,t}$ is application-specific. The notation does not imply approval, legitimacy, or symmetry.

Mutual recognition concerning type χ requires two directed records and a declared cross-direction compatibility predicate $\mathsf{CompatRec}^{\kappa,\chi}_{ij,t}$. The condition is stated in Equation 103.

$$\mathsf{MRec}^{\kappa,\chi}_{ij,t} \quad\Longleftrightarrow\quad \mathsf{CompatRec}^{\kappa,\chi}_{ij,t}\Big(\mathsf{Recog}^{\kappa,\chi}_{i\rightsquigarrow j,t}, \mathsf{Recog}^{\kappa,\chi}_{j\rightsquigarrow i,t}\Big). \tag{103}$$

Equation 103 avoids comparing records from different directed carriers by an undeclared equivalence. It does not require identical roles. A and B can mutually recognize each other as participants while retaining different responsibilities or capacities.

Public recognition adds an audience relation. For subject j and audience $\mathcal{U}$, let

$$\mathsf{PRec}^{\kappa,\chi}_{\mathcal{U}\rightsquigarrow j,t} := \Big(\mathsf{Pub}^{\kappa}_{\mathcal{U},t}(m_{\mathrm{rec}}), \{\mathsf{Recog}^{\kappa,\chi}_{i\rightsquigarrow j,t}\}_{i\in\mathcal{U}}, \Pi(m_{\mathrm{rec}})\Big) \tag{104}$$

Equation 104 combines audience-level publicity with directed recognition and provenance and denotes a public-recognition record when the manifestation m_{rec} and the audience-level recognition relations are defined. A public title, attribution of authorship, acknowledgment of refusal, or recognition of volunteer contribution can therefore become a triadic object without being reduced to one subject's private judgment.

Misrecognition is represented by disagreement between a recognition record and another retained relational or evaluative criterion; it is not defined as simple absence of recognition. Competing recognition records can coexist, and later evidence or procedures can revise them.

10.6 Reputation, Recognition, Role Feedback

Publicity, reputation, and recognition become generatively important when they alter later relational possibilities. A public attribution can change who is trusted, who receives an invitation, whose refusal is believed, which subject is treated as a competent mediator, or whether a support action is interpreted as generous, intrusive, obligatory, or ordinary.

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Let Q_t be a retained publicity, reputation, or recognition record and let $\mathcal{R}^{\kappa'}_{ABC,t'}$ be a later relational configuration. A cross-level feedback relation can be written as

$$\beta^{\mathrm{pub},\kappa'\leftarrow\kappa}_{t'\leftarrow t} \subseteq \mathfrak{Q}^{\kappa}_{t} \times \mathcal{R}^{\kappa'}_{ABC,t'}, \tag{105}$$

where membership $(Q_t, r_{t'})$ means that the retained public or recognitional structure contributes to the later relational organization under the selected model. Equation 105 is a relational feedback claim, not an assertion that public discourse mechanically causes every later role.

Feedback can stabilize or destabilize roles. Repeated public recognition of A as a helper can strengthen a supporter position. Public recognition of B as a competent participant can weaken a fixed supportee position. Reputation can create path dependence by making later interpretations more likely to begin from an inherited attribution. Correction and retraction can reopen those structures without guaranteeing complete reversal.

10.7 Contestation, Correction, Retraction

Public and reputational structures require explicit revision pathways because historical persistence can otherwise be mistaken for validity. Let Q_t be a public record, reputation attribution, or recognition record. A contestation relation

$$\mathsf{Cont}^{\kappa}_{t} \subseteq \mathfrak{Q}^{\kappa}_{t} \times \mathfrak{C}^{\kappa}_{t} \tag{106}$$

Equation 106 types a challenge to an existing public or reputational record.

pairs the contested object with a challenge record in a declared challenge carrier $\mathfrak{C}^{\kappa}_{t}$. A later correction or retraction map can then update the public record while preserving provenance,

$$\mathsf{Corr}^{\kappa'\leftarrow\kappa}_{t'\leftarrow t} : \mathcal{D}_{\mathsf{Corr}} \rightharpoonup \mathfrak{Q}^{\kappa'}_{t'}. \tag{107}$$

The map in Equation 107 should not silently overwrite the history through which the earlier record produced effects. A corrected reputation can coexist with residual distrust generated by the earlier public attribution. A retracted recognition can leave institutional traces. Historical analysis therefore needs both current status and provenance.

The availability of correction is itself a relational property. A subject can have formal permission to contest a public record while lacking effective access to the relevant audience or procedure. Later accountability and authority sections will separate these capacities more fully.

10.8 Three-Level Profiles

Publicity, reputation, and recognition each require three-level analysis. Their ultimate ontology remains suspended. At relational reality, the model asks whether a public, reputational, or recognitional structure has become sufficiently stable and efficacious to shape later relations. At manifestation reality, the structure appears through announcements, records, titles, rankings, acknowledgments, citations, visible roles, interface labels, public refusals, or other context-dependent forms. A-, B-, and C-relative reconstructions can disagree about both the manifestation and the underlying relational significance.

Table 11 summarizes the principal distinctions.

Figure 9 depicts the main generative sequence while preserving the possibility of feedback and contestation.

10.9 Pairwise Recoverability, Triadic Publicity

Publicity can be pairwise recoverable when the three pairwise records jointly preserve the audience, presentation, acknowledgment, and provenance structure needed by the retained diagnostic. A public manifestation is not automatically triadically nonrecoverable merely because all three subjects can access it.

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Table 11: Publicity, acknowledgment, reputation, and recognition in the minimal triadic formalism.

Object	Core condition	Distinction preserved	Typical historical change
Publicity	Shared presentation to declared audience plus accessibility	Public availability versus actual uptake	Audience expansion, restriction, archival persistence
Acknowledgment	Subject-relative uptake treated as acknowledged	Uptake versus higher-order awareness	Confirmation, revision, withdrawal, forgotten acknowledgment
Durable public record	Retained public manifestation with provenance/status	Persistence versus truth	Correction, supersession, retraction
Reputation record	Circulating historical attributions about a subject	Opinion versus audience-level reputation	Accumulation, polarization, repair, decay
Recognition record	Typed relational positioning or attribution	Recognition versus approval/legitimacy	Stabilization, contestation, role revision
Public recognition	Recognition manifested to declared audience	Private recognition versus public status	Institutionalization, correction, displacement

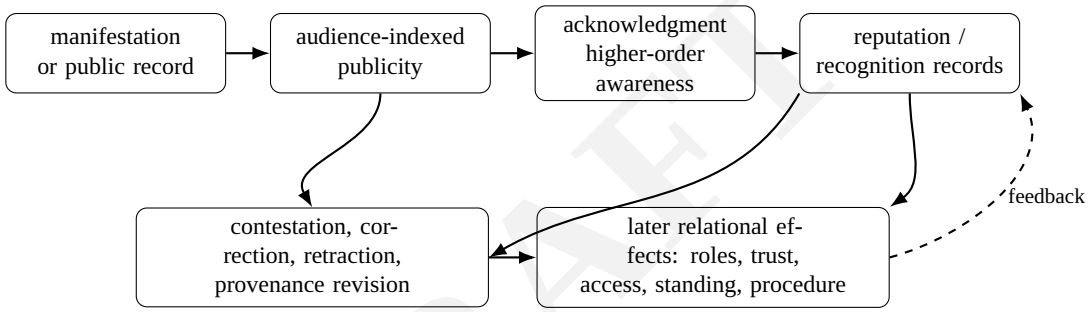

Figure 9: Publicity and recognition architecture. Public presentation, acknowledgment, reputation, and recognition are analytically distinct. Public or reputational structures can feed back into later roles and procedures, while contestation and correction can revise current status without erasing historical provenance.

A stronger triadic publicity object can arise when the model retains a common-audience presentation relation that disappears under all pairwise restrictions. Let $d^{\kappa}_{\mathrm{pub}}$ be a full-equivalence-respecting diagnostic of that audience-level structure. Triadic nonrecoverability is witnessed when

$$X \sim_{2,\kappa} Y \qquad \text{and} \qquad d^{\kappa}_{\mathrm{pub}}(X) \neq d^{\kappa}_{\mathrm{pub}}(Y). \tag{108}$$

A finite witness can be constructed when X and Y contain the same pairwise manifestations and pairwise acknowledgment relations, but Y additionally retains a full-audience presentation object stating that the manifestation was issued to A, B, and C as one public audience. If the selected pairwise restrictions discard that audience object, Equation 108 can hold. The example would establish nonrecoverability relative to that representation, not an absolute metaphysical property of publicity.

Reputation and public recognition admit the same test. A reputational diagnostic can be pairwise factorable if all circulation and attribution structure is retained in A–B, A–C, and B–C records. It can become nonrecoverable when a full-audience or higher-order acknowledgment condition contributes to the retained reputational state while remaining absent from every pairwise restriction.

10.10 Historical Trajectories: Publicity, Recognition

Publicity and recognition are historical processes rather than one-time labels. Let

$$\mathbf{Q}^{\kappa}_{ABC,I} := \Big(\mathsf{Pub}^{\kappa}_{\mathcal{U},t}, \mathcal{G}^{\mathrm{ack},\kappa}_{\mathcal{U},t}, \mathfrak{D}^{\mathrm{pub},\kappa}_{\mathcal{U},t}, \mathfrak{Rep}^{\kappa}_{x,\mathcal{U},t}, \mathsf{PRec}^{\kappa}_{\mathcal{U}\rightsquigarrow x,t}\Big)_{t\in I} \tag{109}$$

Equation 109 denotes the retained trajectory of public, acknowledgment, reputation, and recognition records across the interval, with some type indices suppressed for readability.

Several patterns matter. A *publicization trajectory* occurs when a private or pairwise-shared manifestation enters a wider audience. A *durabilization trajectory* occurs when a transient manifestation becomes a retained record. A *reputational accumulation trajectory* occurs when repeated attributions become historically organized. A *recognitional stabilization trajectory* occurs when a role or status attribution becomes increasingly consequential for later procedure. A *correction trajectory* occurs when contestation changes current public status while preserving the history of prior circulation. A *depublicization trajectory* occurs when access or audience scope narrows, although previously copied or remembered manifestations can remain operative elsewhere.

These trajectories can interact with support relations in both directions. A public reputation can condition whether A is invited to support B. A support action can generate new public recognition for A. Public recognition of B's competence can shift the relation away from a fixed beneficiary role. A contested public account of refusal can alter later authorization and accountability. None of these trajectories is normatively ranked by the present formalism.

The public and recognitional architecture developed here supplies the record structure required by the next section. Accountability, standing, authorization, and enforcement depend not only on what occurred but also on which records became public, who acknowledged them, which reputational and recognition structures stabilized, and which subjects acquired standing to demand an answer or alter later procedure.

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11 Accountability, Standing, Authorization, Enforcement

This section develops the procedural structures through which a third position can make a prior action, judgment, refusal, public record, or role assignment answerable to another subject or to a declared review process. Its role is to separate accountability from publicity, evidence, blame, liability, authorization, and enforcement. The objective is to specify the typed records required when C can receive a claim about an A–B relation, require or invite an answer, review a response, authorize a continuation, or alter later procedure. The method remains formal and reconstructive. Paper II does not infer moral responsibility, legal liability, legitimate authority, or justified punishment merely because an accountability procedure is formally defined.

Section 10 supplied public records, acknowledgment, reputation, recognition, contestation, and correction. Those structures can support accountability, but none is sufficient by itself. A public complaint does not automatically give C standing to review it. A witness record does not automatically authorize a sanction. B's consent to one A–B support action does not automatically authorize C to enforce a later rule. The present section therefore adds standing, answerability, review, authorization, determination, and enforcement as separately typed objects.

11.1 Accountability Record

An accountability relation is organized around a declared object of answerability. The object can be an action, omission, procedure, judgment, public statement, resource transfer, role assignment, refusal, or another locally defined event or record. Let x denote the subject whose conduct or decision is placed under review, let y denote the subject or position raising the claim, and let z denote a third-position reviewer when one exists.

Define the accountability record by

$$\mathfrak{Acct}^{\kappa}_{z:y\rightsquigarrow x,t}(q) := \Big(q, \mathsf{Clm}^{\kappa}_{y\rightsquigarrow x,t}, \mathsf{Ans}^{\kappa}_{x,t}, \mathsf{St}^{\mathrm{rev},\kappa}_{z,t}, \mathsf{Revw}^{\kappa}_{z,t}, \mathsf{Det}^{\kappa}_{z,t}, \Pi^{\mathrm{acct}}_{t}\Big). \tag{110}$$

Here q is the retained object of answerability, Clm the claim or challenge record, Ans the response record, St^{rev} the review-standing record, Revw the review relation or procedure, Det a determination record when the procedure produces one, and Π^{acct} the provenance of the accountability process.

The components in Equation 110 are partial. A claim can exist without a response, a response without an authorized reviewer, or a review without a final determination. Accountability is therefore not defined as a completed linear pipeline.

Accountability is also broader than fault attribution. A support action can be reviewable because a procedure requires explanation even when no wrongdoing is alleged. A mediator can be asked to explain a translation choice without being blamed. A public record can be corrected because it is inaccurate while no moral culpability is assigned. The formalism therefore keeps answerability distinct from blameworthiness, liability, and punishability.

11.2 Claim, Notice, Answerability

A claim becomes procedurally relevant only after the model states what it concerns and how it reaches the subject expected to answer. Let $\mathfrak{Q}^{\kappa}_{\text{acct}}$ be the class of accountability objects and let $\mathfrak{C}^{\kappa}_{\text{clm}}$ be a claim carrier. A typed claim relation is

$$\mathsf{Claim}^{\kappa}_{y\rightsquigarrow x,t} \subseteq \mathfrak{Q}^{\kappa}_{\text{acct}} \times \mathfrak{C}^{\kappa}_{\text{clm}}. \tag{111}$$

The claim carrier can preserve allegation, question, request for explanation, request for correction, appeal, or another application-specific subtype. Equation 111 does not assume that every claim is warranted.

Notice is separate from claim existence. Let $\mathsf{Not}^{\kappa}_{x,t}(c)$ record that claim object c has become accessible to x under the declared notice procedure. A public claim can exist without effective notice to x; conversely, x can receive a private accountability claim that is not public.

Answerability is represented as a procedural relation between a claim and a response domain. Let

$$\mathsf{Resp}^{\kappa}_{x,t} : \mathcal{D}^{\kappa}_{\mathsf{Resp},x,t} \rightharpoonup \mathfrak{C}^{\kappa}_{\text{ans}} \tag{112}$$

Equation 112 represents the partial response procedure available to the subject placed under answerability when the application supplies such a procedure. Its domain can include the claim, notice record, retained evidence, and relevant historical context. A response can acknowledge, deny, clarify, contest, correct, defer, or refuse to answer under the local procedure.

The existence of a response does not establish satisfactory answerability. Whether the response addresses the claim, whether the claimant can reply, and whether a reviewer can request further material are separate procedural questions. Paper II therefore treats answerability as a structure of admissible responses and review relations rather than as a binary variable.

11.3 Standing Types

Standing concerns whether a subject or position is recognized by the model as entitled to perform a declared procedural function. Standing is not the same as epistemic access, credibility, authority, or moral correctness. C can be a highly credible witness while lacking review standing. B can possess claim standing concerning an A–B action while lacking authority to impose a consequence on A.

Paper II distinguishes the standing types

$$\mathcal{T}_{\text{stand}} := \{\mathsf{claim}, \mathsf{witness}, \mathsf{review}, \mathsf{authorize}, \mathsf{enforce}, \mathsf{appeal}\}. \tag{113}$$

Equation 113 fixes the distinct procedural standing types retained by the model.

For $\tau \in \mathcal{T}_{\text{stand}}$, let

$$\mathsf{St}^{\tau,\kappa}_{i,t}(q) \in \mathfrak{S}^{\tau,\kappa}_{\text{stand},i,t}(q) \tag{114}$$

Equation 114 keeps standing indexed by subject, object, and procedural function and denotes the standing record of subject or position i concerning object q and function type τ. The carrier is application-specific

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and need not be binary. It can distinguish granted, disputed, conditional, lapsed, delegated, or another declared status.

The key non-entailments are

$$\begin{aligned} \mathsf{Cred}_{i,t}(q) &\not\Rightarrow \mathsf{St}^{\mathsf{review}}_{i,t}(q), \\ \mathsf{St}^{\mathsf{review}}_{i,t}(q) &\not\Rightarrow \mathsf{St}^{\mathsf{enforce}}_{i,t}(q), \\ \mathsf{St}^{\mathsf{claim}}_{i,t}(q) &\not\Rightarrow \mathsf{St}^{\mathsf{authorize}}_{i,t}(q). \end{aligned} \tag{115}$$

Equation 115 prevents the formalism from converting one procedural relation into another merely because the same subject occupies several roles. Standing can arise from different provenance bases: grant by an affected subject, prior agreement, institutional rule, jointly acknowledged procedure, or another declared source. Paper II records those bases without deciding their legitimacy by notation.

11.4 Third-Position Authorization

Authorization in the triadic setting must distinguish B's authorization of an A–B action from C's authorization of a procedure, review, continuation, or consequence. The dyadic authorization statuses inherited from Paper I therefore remain available, but the triadic formalism adds source and object indices.

Let u be an authorization record issued or recognized through C concerning object q. Write

$$\mathfrak{U}^{\kappa,\tau}_{C\curvearrowright q,t} := \Big(\tau, q, \mathrm{Scope}(u), \mathrm{Oper}(u), \mathsf{St}^{\mathsf{authorize},\kappa}_{C,t}(q), \Pi_t(u)\Big). \tag{116}$$

Here τ is the retained authorization type, $\mathrm{Scope}(u)$ the covered objects, $\mathrm{Oper}(u)$ the historical domain on which the model treats the authorization as operative, and $\Pi_t(u)$ its provenance.

Equation 116 does not collapse third-position authorization into B's consent. The two can coexist, conflict, or concern different objects. For an A–B support action a, the formalism permits

$$\mathfrak{U}^{\kappa,\mathsf{consent}}_{B\curvearrowright a,t} \quad \text{and} \quad \mathfrak{U}^{\kappa,\mathsf{authorize}}_{C\curvearrowright a,t} \tag{117}$$

Equation 117 keeps focal-subject consent and third-position authorization as separate records. A domain-specific model can require one, both, neither, or another combination before action continuation is admitted.

Third-position authorization can also concern the reviewer rather than the focal action. C can be authorized to review A's action without being authorized to choose B's support plan. An appeal body can be authorized to reconsider C's determination without inheriting every other function C possessed.

11.5 Review, Determination

Review is a typed transformation of a claim-response-evidence record into a later procedural record. Let

$$\mathsf{Revw}^{\kappa}_{z,t} : \mathcal{D}^{\kappa}_{\mathsf{Revw},z,t} \rightharpoonup \mathfrak{D}^{\kappa}_{\mathrm{det}} \tag{118}$$

Equation 118 represents review as a partial transformation from retained evidence and procedure into a determination record; its domain can contain claim, notice, response, witness, route provenance, public record, standing, and other retained evidence. The output class $\mathfrak{D}^{\kappa}_{\mathrm{det}}$ contains determination records such as sustained claim, rejected claim, unresolved claim, correction required, procedure reopened, or another application-specific result.

A determination is not assumed to be true merely because it is procedurally generated. Review can be underdetermined, biased, incomplete, or later revised. Nor does a determination automatically create enforcement authority.

The review map can itself be route-sensitive. C can review direct A–B records, testimony routed through another position, translated material, or public records whose provenance differs. The epistemic route architecture of Section 9 therefore remains active inside accountability.

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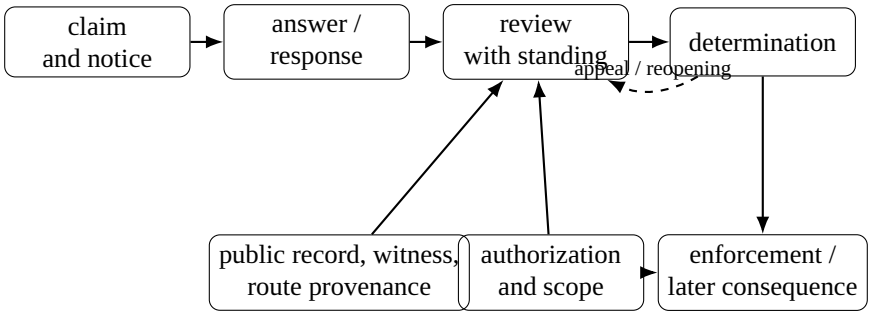

Figure 10: Triadic accountability architecture. Claim, response, review, determination, authorization, and enforcement are typed separately. Public records and witness/provenance material can enter review, while appeal or reopening can revise a prior determination. The diagram shows one admissible procedural organization rather than a universal sequence.

11.6 Enforcement, Remedy, Consequence

Enforcement is modeled as a procedure that changes the admissible continuation family, access conditions, records, resource relations, role relations, or another declared component after an enforcement basis has been established. The term is intentionally broader than physical coercion and narrower than any causal consequence.

Let d be a determination or rule-trigger record and let $\mathsf{St}_{z,t}^{\mathsf{enforce},\kappa}(d)$ be the enforcement-standing record. An enforcement transformation is

$$\mathsf{Enf}_{z,t}^{\kappa} : \mathcal{D}_{\mathsf{Enf},z,t}^{\kappa} \rightharpoonup \mathfrak{D}_{ABC,t+}^{\kappa'}. \tag{119}$$

The notation $t+$ marks a later procedural stage rather than a universal discrete-time successor. Possible outputs include correction of a record, suspension of one action path, restoration of access, creation of a new review obligation, role restriction, resource transfer, or another typed consequence.

Paper II distinguishes enforcement, remedy, and sanction-like consequence. Enforcement names the operative transformation. Remedy is an application-level evaluation or classification of an output as addressing a prior harm, defect, or claim. A sanction-like consequence is an output that restricts, burdens, or penalizes a subject under a declared regime. Neither remedy nor sanction follows from Equation 119 without extra evaluative structure.

The formalism also permits non-enforcement. A claim can be sustained while no enforcement action is authorized. A public correction can occur without role restriction. A refusal to enforce can itself become a new accountability object.

11.7 Accountability Procedure Architecture

The preceding objects can be assembled into a partially ordered accountability procedure. One common sequence is claim, notice, response, review, determination, and consequence. Paper II does not treat that sequence as universal or mandatory. Some procedures permit informal correction before review; some permit appeal; some allow public response before determination; some terminate after acknowledgment.

A schematic path is

$$\mathsf{claim} \rightsquigarrow \mathsf{notice} \rightsquigarrow \mathsf{response} \rightsquigarrow \mathsf{review} \rightsquigarrow \mathsf{determination} \rightsquigarrow \mathsf{consequence}, \tag{120}$$

Equation 120 displays one admissible ordering of accountability stages; the arrows indicate procedural succession rather than logical entailment. Branching, return, appeal, withdrawal, settlement, or termination edges can be added locally.

Figure 10 summarizes the architecture.

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11.8 Three-Level Accountability Profiles

Accountability objects inherit the three-level discipline. Their ultimate ontology remains suspended. At relational reality, the model can retain a stable answerability, review, or enforcement structure that affects later possibilities. At manifestation reality, accountability appears through complaints, hearings, notices, reports, appeals, public statements, correction labels, access restrictions, or other context-dependent forms. A-, B-, and C-relative reconstructions can disagree about whether a claim was made, whether notice was effective, whether C had standing, whether an answer was adequate, and what a determination means.

For accountability object Q, define the schematic profile

$$\Lambda^{\mathrm{acct},\kappa}_{ABC,t}(Q) := \Big(Q^{\mathsf{R}}_t, Q^{\mathsf{M}}_t, \widehat{Q}_{A,t}, \widehat{Q}_{B,t}, \widehat{Q}_{C,t}\Big), \tag{121}$$

Equation 121 separates relational, manifestation, and subject-relative accountability records, with ontological status separately marked as suspended. The equation does not imply that manifestation or reconstruction fully determines the relational structure.

11.9 Pairwise Recoverability of Accountability

Many accountability procedures are reconstructible from pairwise records. An A–C record can retain C's review standing and A's response, while B–C retains B's claim and C's notice. If the model also preserves the required provenance and compatibility relations, the full accountability procedure can factor through the pairwise shadow.

Triadic nonrecoverability requires a retained accountability diagnostic that depends on a full three-position condition absent from every pairwise restriction. Let $d^{\kappa}_{\mathrm{acct}}$ be a full-equivalence-respecting diagnostic. A witness satisfies

$$X \sim_{2,\kappa} Y \qquad \text{and} \qquad d^{\kappa}_{\mathrm{acct}}(X) \neq d^{\kappa}_{\mathrm{acct}}(Y). \tag{122}$$

For example, X and Y can preserve the same pairwise claim, response, review, and authorization records while differing in a retained full-triad condition requiring that C's determination be jointly acknowledged by A and B before enforcement becomes operative. If the pairwise restrictions discard that joint condition, Equation 122 can hold.

11.10 Authority, Legitimacy: Non-Entailments

The formalism does not derive legitimacy from procedural stability. A review body can be stable, publicly recognized, and effective while remaining normatively contested. Likewise, an enforcement transformation can be well typed and historically efficacious while lacking a justified authority basis under a later ethical or legal theory.

Paper II therefore keeps the following distinctions explicit:

$$\begin{aligned} \mathsf{publicity} &\not\Rightarrow \mathsf{standing}, \\ \mathsf{standing} &\not\Rightarrow \mathsf{authority}, \\ \mathsf{authority\ record} &\not\Rightarrow \mathsf{legitimacy}, \\ \mathsf{determination} &\not\Rightarrow \mathsf{justified\ enforcement}. \end{aligned} \tag{123}$$

Equation 123 is a scope discipline for the present paper. Later justice, ethics, and political-economy papers can supply stronger evaluative criteria.

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Table 12: Principal accountability objects in the minimal triadic formalism.

Object	Formal role	Distinction preserved	Historical question
Claim / notice	Initiates answerability relation	Claim existence versus effective notice	Which claims become procedurally visible?
Response	Typed answer or contest	Response versus adequacy or truth	Can subjects revise or supplement answers?
Standing	Entitlement to a declared procedural function	Credibility versus review/enforcement entitlement	Who gains, loses, or delegates standing?
Authorization	Scope-limited operative basis	Consent versus third-position authorization	Does authorization widen, lapse, or conflict?
Review / determination	Procedure producing a retained outcome	Review result versus truth/legitimacy	How are determinations appealed or revised?
Enforcement	Transformation of later admissible records	Enforcement versus remedy or punishment	Which consequences become operative?

11.11 Historical Accountability Trajectories

Accountability structures evolve through history. Let

$$\mathbf{A}^{\mathrm{acct},\kappa}_{ABC,I} := \left(\mathfrak{Acct}^{\kappa}_t, \mathsf{St}^{\kappa}_t, \mathfrak{U}^{\kappa}_t, \mathsf{Revw}^{\kappa}_t, \mathsf{Enf}^{\kappa}_t\right)_{t\in I} \tag{124}$$

Equation 124 denotes the retained accountability trajectory across history, with subject and object indices suppressed where the context is clear.

Several patterns are especially relevant. An *institutionalization trajectory* occurs when ad hoc third-position review becomes a durable procedure. A *standing-expansion trajectory* occurs when additional subjects acquire claim, review, or appeal standing. A *standing-contraction trajectory* occurs when those routes narrow. An *appeal trajectory* introduces a new review layer after an earlier determination. A *capture trajectory* occurs when review or enforcement becomes increasingly dependent on one subject or coalition. A *bypass trajectory* occurs when subjects route around an established accountability structure. A *repair trajectory* occurs when correction, changed standing, or revised authorization reopens a procedure after prior failure.

These trajectories can alter the relational system even without formal sanctions. The expectation of review can change how support actions are documented. Public answerability can modify role behavior. A credible appeal route can make refusal or complaint practically usable. Conversely, inaccessible or captured accountability can make a nominal right procedurally ineffective.

Table 12 summarizes the principal objects and distinctions developed in this section.

The accountability architecture now provides the procedural and authorization basis required by the next section. Brokerage, coalition, alignment, and exclusion become formally richer once one subject can control routes, two subjects can coordinate relative to a third, and public or accountability structures can alter which continuations remain accessible.

12 Brokerage, Coalition, Alignment, Exclusion

This section develops the positional and coordination structures made available by route-sensitive epistemics, publicity, and accountability. Its role is to formalize intermediation, pairwise coordination relative to a third position, typed alignment, and dimension-specific exclusion. Brokerage is not identified with power, coalition with hostility, alignment with agreement, or exclusion with injustice. Those stronger interpretations require additional conditions. The method is structural and historical: each object is typed by its carrier, provenance, scope, and effect on later continuations.

Section 8 introduced brokerage as an indirect route and distinguished route provision from gatekeeping. The present section treats brokerage more broadly as a relational position that can interact with

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coalition, alignment, and exclusion. Section 11 also showed that control over review, authorization, or enforcement routes can become procedurally consequential. These earlier objects now become components of a larger triadic organization.

12.1 Brokerage Position Record

A brokerage position exists relative to a declared object type and route architecture. Let χ denote the object that can pass between A and B, such as information, access, recognition, authorization, resources, or another typed item, and let $\Gamma^{\kappa,\chi}_{AB,t}$ be the retained family of A–B routes for that type. A C-relative brokerage position is represented by

$$\mathfrak{B}^{\kappa,\chi}_{C|AB,t} := \Big(\Gamma^{\kappa,\chi}_{AB,t}, \Gamma^{\kappa,\chi}_{A\to C\to B,t}, \mathsf{Acc}^{\kappa,\chi}_{C,t}, \mathsf{Filt}^{\kappa,\chi}_{C,t}, \mathsf{Alt}^{\kappa,\chi}_{AB,t}, \Pi^{\kappa,\chi}_{C|AB,t}\Big). \tag{125}$$

Here Acc records C's route access, Filt any retained filtering or selection relation, Alt the alternative routes available to A and B, and Π the historical provenance of the brokerage position. Equation 125 does not require C to control the route. It records the information needed to determine whether C merely participates in transmission, facilitates access, or occupies a more restrictive gatekeeping position.

Let $\mathsf{Dir}^{\kappa,\chi}_{AB,t}$ denote the family of direct or non-C-mediated routes available to the focal pair, and let $\mathsf{Cond}^{\kappa,\chi}_{C,t}$ be the declared condition-setting relation through which C can alter passage along the brokered route. Building on the route-dependence condition of Equation 77, a strong C-relative gatekeeping condition is

$$\mathsf{Gate}^{\kappa,\chi}_{C|AB,t} \quad\Longleftrightarrow\quad \mathsf{RouteDep}^{\kappa,\chi}_{C|AB,t} \wedge \mathsf{Dir}^{\kappa,\chi}_{AB,t} = \varnothing \wedge \mathrm{Dom}(\mathsf{Cond}^{\kappa,\chi}_{C,t}) \neq \varnothing. \tag{126}$$

Equation 126 is intentionally restrictive: route dependence is necessary but not sufficient for gatekeeping; the model must also represent C's capacity to condition passage. Weaker forms of brokerage dependence can occur when direct alternatives exist but are slower, less credible, less legible, institutionally disfavored, or practically costly. Such cases should be represented through explicit route conditions rather than forced into the binary gatekeeping predicate.

Brokerage can also be multiplex. C can broker information while A independently controls resource access, and B can mediate recognition between A and C. Paper II therefore does not define a single global broker role for the triad.

12.2 Coalition Records

A coalition is a historically situated coordination structure among at least two positions relative to some retained object, procedure, or continuation. In the minimal triad, the most elementary coalition is pair-relative: A and C coordinate concerning an object that also affects B. Coalition is stronger than coincident preference but weaker than complete identity of interests.

Let $S \subseteq \{A, B, C\}$ with $|S| = 2$ or 3, and let q be the object of coordination. Define a coalition record by

$$\mathfrak{C}^{\kappa}_{S,t}(q) := \Big(S, q, \mathsf{Coord}^{\kappa}_{S,t}(q), \mathsf{Commit}^{\kappa}_{S,t}(q), \mathsf{Proc}^{\kappa}_{S,t}(q), \Pi^{\kappa}_{S,t}(q)\Big). \tag{127}$$

Here Coord records the coordinated orientation or action relation, Commit the retained commitment structure when one exists, Proc the procedure through which coordination is maintained, and Π the coalition provenance.

The coalition object in Equation 127 does not require unanimity across all relevant dimensions. A and C can coordinate on a review procedure while disagreeing about B's need. B and C can coordinate on a resource transfer while disagreeing about the public description of that action. Coalition must therefore be indexed by q and by historical scope.

A coalition can also be tacit or emergent, but Paper II does not infer coalition from correlated behavior alone. Repeated parallel action can result from common constraints, shared information, or independent judgment. Coalition requires a declared coordination criterion strong enough for the application.

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Let $\mathcal{S}_2 := \{\{A,B\},\{A,C\},\{B,C\}\}$ denote the three dyadic coalition candidates. A coalition-position record is

$$\mathfrak{C}^{\mathrm{pos},\kappa}_{ABC,t}(q) := \left\{ S \in \mathcal{S}_2 \;\middle|\; \mathfrak{C}^{\kappa}_{S,t}(q) \text{ is defined} \right\}. \tag{128}$$

Equation 128 allows zero, one, two, or all three pairwise coalitions to be active under different criteria. The existence of one coalition does not imply a unified opposition by the remaining subject.

12.3 Typed Alignment

Alignment is a weaker and more general object than coalition. It records compatibility or correspondence relative to a declared dimension. Candidate dimensions include evaluative judgment, procedure, information, authorization, strategy, role expectation, recognition, or anticipated continuation.

For dimension δ and subjects i, j, let the typed alignment relation be

$$\mathsf{Aln}^{\kappa,\delta}_{ij,t} \subseteq \mathfrak{X}^{\kappa,\delta}_{i,t} \times \mathfrak{X}^{\kappa,\delta}_{j,t}. \tag{129}$$

For records x_i and x_j in the corresponding carriers, write $\mathsf{Aln}^{\kappa,\delta}_{ij,t}(x_i, x_j)$ as shorthand for $(x_i, x_j) \in \mathsf{Aln}^{\kappa,\delta}_{ij,t}$. Equation 129 does not assume equality, symmetry of influence, or shared motivation.

The full triadic alignment profile is

$$\mathfrak{L}^{\kappa,\delta}_{ABC,t} := \left(\mathsf{Aln}^{\kappa,\delta}_{AB,t}, \mathsf{Aln}^{\kappa,\delta}_{AC,t}, \mathsf{Aln}^{\kappa,\delta}_{BC,t} \right). \tag{130}$$

Equation 130 preserves the possibility of nontransitive or dimension-sensitive alignment. A can align with B on procedure and with C on evaluation, while B and C remain unaligned on both. Likewise, pairwise alignment on one dimension need not generate a three-way coalition.

A triadic alignment condition can be added only where the application supplies a higher-order compatibility predicate $\mathsf{TriAln}^{\kappa,\delta}_{ABC,t}(x_A, x_B, x_C)$. Even when all three pairwise predicates hold on one triple of records, the full condition need not follow:

$$\mathsf{Aln}^{\kappa,\delta}_{AB,t}(x_A, x_B) \wedge \mathsf{Aln}^{\kappa,\delta}_{AC,t}(x_A, x_C) \wedge \mathsf{Aln}^{\kappa,\delta}_{BC,t}(x_B, x_C) \qquad \not\Longrightarrow \mathsf{TriAln}^{\kappa,\delta}_{ABC,t}(x_A, x_B, x_C). \tag{131}$$

Equation 131 is retained by default because the full-triad criterion can depend on joint procedural, temporal, or provenance structure lost by pairwise comparison.

12.4 Exclusion Profiles

Exclusion is not treated as one undifferentiated state. A subject can be excluded from information while retaining procedural standing, excluded from authorization while retaining public recognition, or excluded from a coalition while maintaining direct access to both coalition members. The retained dimension must therefore be explicit.

An illustrative family of exclusion dimensions is

$$\begin{aligned} \Delta_{\mathrm{ex}} := \{&\mathsf{information}, \mathsf{communication}, \mathsf{procedure}, \mathsf{resource}, \mathsf{recognition}, \\ &\mathsf{authorization}, \mathsf{publicity}, \mathsf{coalition}, \mathsf{appeal}\}. \end{aligned} \tag{132}$$

For subject i and dimension $\delta \in \Delta_{\mathrm{ex}}$ from Equation 132, define an exclusion relation

$$\mathsf{Excl}^{\kappa,\delta}_{i,t} \subseteq \mathfrak{R}^{\kappa,\delta}_{ABC,t} \times \mathfrak{X}^{\kappa,\delta}_{i,t}, \tag{133}$$

Equation 133 keeps exclusion indexed by relational dimension and subject-relative restriction: the first component specifies the relevant relational structure and the second the access, participation, or position being restricted. The relation is application-specific and need not be binary.

The formalism distinguishes exclusion from nonparticipation. A subject can decline to join a coalition without being excluded from it. A subject can lack information because no one possesses it rather

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than because a route is restricted. A subject can be outside an audience because the event is private rather than because a public procedure has denied access. Exclusion therefore requires a relational criterion identifying the relevant opportunity or route and the way it is unavailable to the subject.

A restrictive exclusion can be explicit, such as denied access to a review procedure, or generated indirectly through route architecture. If C controls the only translation route by which B can participate in an A–C procedure, then C's filtering policy can generate communication exclusion even without an explicit prohibition.

12.5 Coalition-Exclusion Coupling

Coalitions can create exclusion without being defined by exclusion. A and C can coordinate on a support plan while continuing to provide B full information and refusal capacity. Conversely, a coalition can reorganize procedure so that B loses access to deliberation, review, or recognition. The relevant relation is therefore conditional.

Let $\mathfrak{C}^{\kappa}_{S,t}(q)$ be a coalition record and let $\mathsf{Excl}^{\kappa,\delta}_{i,t}$ be an exclusion relation for the nonmember $i \notin S$. A coalition-exclusion coupling record is

$$\mathfrak{X}^{\kappa,q,\delta}_{S\triangleright i,t} := \left(\mathfrak{C}^{\kappa}_{S,t}(q), \mathsf{Excl}^{\kappa,\delta}_{i,t}, \Pi^{\kappa,q,\delta}_{S\triangleright i,t}\right) \tag{134}$$

when the model retains a historical relation connecting coalition activity to the exclusion structure. Equation 134 avoids inferring exclusion merely from coalition membership.

The same coalition can have inclusionary effects in another dimension. A and C can coordinate to translate B's expression, thereby increasing B's communicative access while also excluding B from a separate planning stage. Multi-dimensional effects remain typed.

12.6 Brokerage, Coalition, Procedural Control

Brokerage becomes structurally stronger when it combines with coalition or procedural standing. Suppose C brokers B's access to A and simultaneously belongs to an A–C coalition concerning the procedure through which B seeks access. The route and coalition records can then condition each other.

Define a route-procedure coupling by

$$\mathfrak{P}^{\kappa,\chi,q}_{C|AB,t} := \left(\mathfrak{B}^{\kappa,\chi}_{C|AB,t}, \mathfrak{C}^{\kappa}_{\{A,C\},t}(q), \mathsf{St}^{\tau_{\text{proc}},\kappa}_{C,t}(q)\right), \tag{135}$$

where $\tau_{\text{proc}} \in \mathcal{T}_{\text{stand}}$ is the specific procedural standing type used by the application (for example review, authorization, enforcement, or appeal), and the third component is $\mathsf{St}^{\tau_{\text{proc}},\kappa}_{C,t}(q)$. Equation 135 does not define power. It identifies a compound structural configuration in which access, coalition, and procedure intersect.

A later power analysis can ask whether the combined structure persistently shapes B's continuation space, whether alternatives remain available, whether B can contest the route, and whether the configuration self-stabilizes. Those questions are deliberately deferred. Paper II only ensures that the relevant components remain separately visible.

12.7 Triadic Closure

Triadic closure is used here in a restricted structural sense. The term denotes a history in which previously absent or weak relations among the three subjects become sufficiently articulated that all three pairwise interfaces participate in one coupled configuration. It does not imply harmony, trust, or social cohesion.

Let $\mathsf{Def}^{\kappa}_{XY,t}$ denote the predicate that the retained XY interface is sufficiently defined, and let $\mathsf{Coupled}^{\kappa}_{ABC,t}$ state that the three retained interfaces are jointly realized by one admitted full configuration. A minimal closure predicate is

$$\mathsf{Cl}^{\kappa}_{ABC,t} \quad \Longleftrightarrow \quad \mathsf{Coupled}^{\kappa}_{ABC,t} \wedge \mathsf{Def}^{\kappa}_{AB,t} \wedge \mathsf{Def}^{\kappa}_{AC,t} \wedge \mathsf{Def}^{\kappa}_{BC,t}. \tag{136}$$

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Equation 136 is only a structural threshold; the coupling predicate prevents three mutually incompatible dyadic interfaces from being called a closed triad. A closed triad can contain conflict, exclusion, brokerage dependence, or incompatible alignments. Stronger notions of closure can require trust, shared procedure, or higher-order recognition, but those conditions must be stated explicitly.

Closure can alter brokerage. When A and B acquire a direct relation after previously communicating only through C, C's brokerage position can weaken even though C remains part of the triad. Conversely, a direct relation can become ineffective while a brokered route becomes dominant.

12.8 Coalition Switching, Positional Dynamics

A defining feature of the minimal triad is that the same subject can move between coalition, broker, mediator, outsider, or contested positions over time. Position dynamics should therefore be represented historically rather than as fixed actor attributes.

Define the retained positional record by

$$\mathfrak{P}^{\kappa}_{ABC,t} := \Big(\mathfrak{C}^{\mathrm{pos},\kappa}_{ABC,t}, \{\mathfrak{B}^{\kappa,\chi}_{Z|XY,t}\}, \{\mathsf{Excl}^{\kappa,\delta}_{i,t}\}, \{\mathsf{AIn}^{\kappa,\delta}_{ij,t}\}\Big). \tag{137}$$

Equation 137 collects the active positional structures at stage t. A historical positional trajectory is

$$\mathbf{P}^{\kappa}_{ABC,I} := \big(\mathfrak{P}^{\kappa}_{ABC,t}\big)_{t\in I}. \tag{138}$$

Coalition switching occurs when the set in Equation 128 changes across historical stages. Brokerage migration occurs when another subject becomes the operative intermediary for a retained object type. Exclusion reversal occurs when a previously excluded subject gains access or standing. Alignment retyping occurs when coordination persists but moves to a different dimension.

These changes can be path dependent. A former coalition can leave reputational residue. A former broker can retain informational advantage after direct routes appear. A subject previously excluded from review can remain distrustful after access is restored. The present formalism records these histories without reducing them to one scalar positional score.

12.9 Three-Level Profiles

Brokerage, coalition, alignment, and exclusion each require three-level analysis. Their ultimate ontology remains suspended. At relational reality, the model asks whether the relevant route, coordination, alignment, or exclusion structure is sufficiently organized and efficacious to shape later possibilities. At manifestation reality, these structures can appear through meeting invitations, routing rules, shared messages, coalition statements, access lists, seating arrangements, interface permissions, public endorsements, or other context-dependent forms. A-, B-, and C-relative reconstructions can disagree about whether a coalition exists, whether C is gatekeeping, whether B was excluded, or whether an apparent alignment is merely coincident behavior.

Table 13 summarizes the principal distinctions.

Figure 11 depicts one illustrative configuration in which C brokers A–B access while A and C coordinate on a procedure affecting B. The diagram is not a universal model of coalition.

12.10 Pairwise Recoverability

Brokerage and pairwise coalition records are often recoverable from rich dyadic restrictions. An A–C record can retain their coalition, while B–C can retain the brokered route and B-relative access. Triadic nonrecoverability arises only when the selected full diagnostic depends on a joint relation among those pairwise components that is not preserved by the restrictions.

Let $d^{\kappa}_{\mathrm{pos}}$ be a full-equivalence-respecting diagnostic of brokerage/coalition/exclusion organization. A witness satisfies

$$X \sim_{2,\kappa} Y \qquad \text{and} \qquad d^{\kappa}_{\mathrm{pos}}(X) \neq d^{\kappa}_{\mathrm{pos}}(Y). \tag{139}$$

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Table 13: Brokerage, coalition, alignment, and exclusion in the minimal triadic formalism.

Object	Core formal role	Distinction preserved	Historical question
Brokerage position	Indirect route and alternatives	Brokerage versus gatekeeping/control	Do direct alternatives appear or disappear?
Coalition record	Coordination concerning object q	Coordination versus agreement on all dimensions	Which coalitions form, dissolve, or switch?
Alignment relation	Dimension-specific compatibility	Alignment versus coalition or common motive	Does alignment persist across dimensions?
Exclusion relation	Typed restriction of access/participation	Exclusion versus nonparticipation or absence	Which forms of access become unavailable?
Triadic closure	All three pairwise interfaces defined	Closure versus harmony or trust	Does closure weaken broker dependence or intensify coordination?

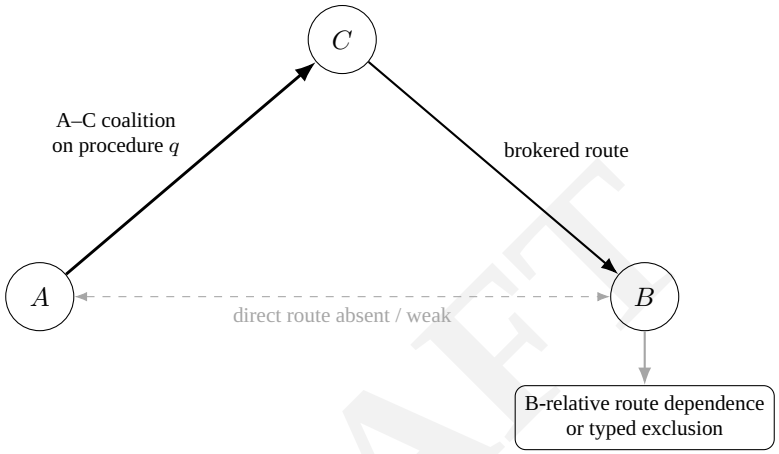

Figure 11: Illustrative coupling of brokerage, coalition, and exclusion. C brokers an A–B route while A and C coordinate on a procedure affecting B. The figure identifies a compound structure only; whether it constitutes power, domination, or injustice requires additional criteria.

For example, X and Y can preserve the same A–C coalition, the same B–C route, and the same A–B access record while differing in whether the A–C coalition is the procedural condition that activates C's filtering of B's route. If pairwise restrictions retain the components but discard their joint conditional coupling, Equation 139 can hold.

The result remains representation-relative. A richer pairwise record that preserves the coupling can restore recoverability.

12.11 Historical Brokerage-Coalition Trajectories

The positional trajectory in Equation 138 supports several historically relevant patterns. A *brokerage-decay trajectory* occurs when direct A–B routes become viable and C's intermediation loses structural importance. A *brokerage-lock-in trajectory* occurs when repeated use of C's route makes alternatives less effective or less credible. A *coalition-stabilization trajectory* occurs when episodic coordination acquires durable procedure and expectation. A *coalition-switching trajectory* occurs when the active pair changes relative to the same or a new object. An *exclusion-expansion trajectory* occurs when restrictions spread across additional dimensions. An *inclusion-repair trajectory* occurs when access, participation, or standing is restored after prior exclusion.

These trajectories can interact. Brokerage lock-in can facilitate coalition stabilization; coalition switching can alter public reputation; reputational change can affect who is accepted as mediator; accountability reform can weaken gatekeeping; triadic closure can either reduce intermediary dependence or create new coalition possibilities.

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The section therefore supplies the positional architecture needed before Paper II turns to action and procedure themselves. Section 13 will integrate mediation, brokerage, coalition, acknowledgment, authorization, and accountability into the provenance and execution structure of triadic action.

13 Triadic Procedure: Action Provenance, Distributed Generation

This section integrates the third-position functions, route-sensitive epistemic structure, publicity, accountability, brokerage, and coalition records developed in Sections 8–12 into a common procedure and action-provenance architecture. Its role is to explain how a volunteer action can remain visibly dyadic while being generated, translated, authorized, brokered, coordinated, witnessed, or reviewed through a triadic history. The objective is to separate execution from generation, generation from attribution, authorization from authorship-like attribution, and a procedure from the action records that arise inside it. The method is typed and historical. No subject receives sole responsibility merely because that subject performs the visible execution, and no distributed provenance record is itself a moral or legal theory of responsibility.

The minimal triad makes these distinctions unavoidable. B can request support, C can translate the request, A can propose an intervention, C can mediate scope, B can consent, C can authorize one procedural step, A can execute the material action, and all three can later reconstruct the provenance differently. A shorthand such as "A helped B" can therefore remain useful at one descriptive level while omitting structures that matter for interpretation, accountability, and later relational change.

13.1 Triadic Procedure Record

Let I_Γ be a finite set of retained procedure-element indices and let $\prec_\Gamma$ be a strict partial order. For each $j \in I_\Gamma$, let ζ_j be a typed triadic procedure element. Define the procedure record by Equation 140.

$$\Gamma^{\kappa}_{ABC} := \Big(I_\Gamma, \prec_\Gamma, \{\zeta_j\}_{j\in I_\Gamma}, \mathsf{Part}^{\kappa}_{ABC}, \Pi^{\kappa}_{\Gamma}\Big), \tag{140}$$

where $\mathsf{Part}^{\kappa}_{ABC}$ records the subject or position participation relevant to the retained elements and Π^{κ}_{Γ} records procedure-level provenance. Equation 140 is a bookkeeping structure rather than a global state vector. Procedure elements can belong to different formal types and need not act on one common numerical carrier.

A triadic procedure can contain inherited dyadic elements such as request, offer, clarification, consent, refusal, execution, feedback, and withdrawal together with specifically triadic elements such as mediation, witnessing, translation, brokerage, public acknowledgment, accountability review, appeal, coalition coordination, and third-position authorization. The core family used in Paper II is summarized by

$$\begin{aligned}\mathcal{T}^{(3)}_{\mathrm{proc}} := \{&\mathsf{req}, \mathsf{off}, \mathsf{clr}, \mathsf{neg}, \mathsf{cons}, \mathsf{ref}, \mathsf{exec}, \mathsf{fb},\\ &\mathsf{med}, \mathsf{wit}, \mathsf{trans}, \mathsf{brok}, \mathsf{pub}, \mathsf{ack}, \mathsf{revw}, \mathsf{appeal}, \mathsf{coal}, \mathsf{authz}, \mathsf{term}\}.\end{aligned} \tag{141}$$

Equation 141 is an analytical vocabulary, not a universal protocol. A particular support relation can omit most of these elements, repeat one element, or add a domain-specific element when the distinction has a declared role.

The partial order matters because several events can be incomparable or only partly ordered. C can begin documenting an episode while A and B continue negotiation. A public record can appear after execution but before feedback. An accountability review can begin while an action is still producing consequences. The model should not impose a total temporal order when the retained evidence supports only procedural precedence.

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13.2 Local Procedure Maps, Composability

Each procedure element ζ_j has a partial local map

$$\Theta_{\zeta_j}^{\kappa' \leftarrow \kappa} : \mathcal{D}_{\zeta_j}^{\kappa} \rightharpoonup \mathcal{C}_{\zeta_j}^{\kappa'}, \tag{142}$$

Equation 142 types each procedure element by source and target record classes determined by the element type. A mediation element can act on a procedure record, a translation element on a manifestation carrier, a publicization element on an audience/access structure, and an enforcement element on a continuation family. Paper II therefore does not assume one universal procedure state space.

For $j \prec_{\Gamma} k$, the ordered composite is defined only on

$$\mathcal{D}_{\zeta_k \circ \zeta_j} := \left\{ x \in \mathcal{D}_{\zeta_j}^{\kappa} \;\middle|\; \Theta_{\zeta_j}^{\kappa' \leftarrow \kappa}(x) \in \mathcal{D}_{\zeta_k}^{\kappa'} \right\}. \tag{143}$$

Equation 143 makes the definedness domain of an ordered two-step procedure explicit.

Undefined composition is substantively meaningful. A refusal can remove an execution step from the later domain. A translation can make a previously undefined clarification available. A coalition can create a coordination route that did not exist before. A public determination can open an appeal domain. A withdrawal can close an authorization-dependent continuation.

When a realized trace $\gamma = (\zeta_1, \ldots, \zeta_n)$ is linearly ordered and fully composable on a retained domain, the procedure composite is

$$\Theta_{\gamma} := \Theta_{\zeta_n} \circ \cdots \circ \Theta_{\zeta_1}. \tag{144}$$

Equation 144 is a specialization. The procedure remains meaningful even when no single global composite exists across heterogeneous elements.

13.3 Action Individuation in a Triadic Procedure

A procedure can contain several relational processes that later become individuated as one action, several actions, or no action at all under the selected criterion. Let η denote a retained process episode inside Γ_{ABC}^{κ}. An action-individuation relation is written

$$\mathsf{Ind}_{3,t}^{\text{act},\kappa} \subseteq \mathfrak{P}_{ABC,t}^{\kappa,\text{proc}} \times \mathfrak{A}_{ABC,t}^{\kappa,\text{act}}, \tag{145}$$

where $\mathfrak{P}_{ABC,t}^{\kappa,\text{proc}}$ is the retained class of procedure-supported process episodes and $\mathfrak{A}_{ABC,t}^{\kappa,\text{act}}$ the class of action records.

The relation in Equation 145 can be many-to-many. One negotiation-plus-transfer episode can be treated as one coordinated action at a coarse resolution while a finer model separates translation, authorization, material transfer, and public acknowledgment. Conversely, several visible acts can form one retained action when their identity depends on one continuing procedure and purpose.

The addition of C can also change action identity without changing A's visible movement. The same material transfer by A to B can be represented differently when one history includes B's request translated by C and another includes C's unilateral authorization against B's refusal. Paper II does not declare those actions numerically different by default; it requires an explicit action-equivalence criterion.

Let $\sim_{\text{act},\kappa}$ be the declared action-equivalence relation. Then

$$\mathfrak{a}_1 \sim_{\text{act},\kappa} \mathfrak{a}_2 \tag{146}$$

Equation 146 states the triadic action equivalence used for comparison.

means only that the two action records preserve the distinctions the application treats as relevant. An outcome-only analysis can identify actions that a provenance-sensitive analysis distinguishes.

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13.4 Distributed Provenance Record

The provenance of a triadic action records retained generative antecedents rather than merely the subject who executes it. Let $\mathfrak{a}$ be an action record and let $\mathfrak{B}^{\kappa}_{ABC,[t,t']}$ be the class of admissible provenance bearers. The class can contain subject-spacetimes, role positions, coalitions, procedure elements, public records, authorization records, or other explicitly retained structures.

Define the provenance-type family

$$\mathcal{T}^{(3)}_{\text{prov}} := \{\mathsf{request}, \mathsf{initiate}, \mathsf{propose}, \mathsf{mediate}, \mathsf{translate}, \mathsf{broker}, \mathsf{coordinate}, \mathsf{authorize}, \mathsf{generate}, \mathsf{execute}, \mathsf{witness}, \mathsf{record}, \mathsf{review}\}. \tag{147}$$

Equation 147 fixes the provenance roles that can contribute to one action history. The distributed provenance relation is

$$\Pi^{\kappa}_{3,t'}(\mathfrak{a}) \subseteq \mathfrak{B}^{\kappa}_{ABC,[t,t']} \times \mathcal{T}^{(3)}_{\text{prov}}. \tag{148}$$

Equation 148 allows one bearer to contribute in several ways and several bearers to share one provenance type. It also prevents the visible executor from absorbing every generative antecedent into one actor label.

For example, B can request an action, C can translate the request, A can propose an implementation, C can mediate scope, B can consent, A can execute, and C can later witness the outcome. The final action can still be described as an A-to-B support action at one level, while Equation 148 retains the richer history required for later responsibility or power analysis.

Provenance can include non-subject structures. A standing rule, public record, or coalition commitment can be causally or generatively relevant even when no subject explicitly invokes it at the moment of execution. Such inclusion requires a declared criterion; the provenance relation should not become an unbounded list of every prior condition.

13.5 Attribution Types, Authorization Distinction

Provenance and attribution remain distinct. Provenance records retained generative antecedents. Attribution records how the action is assigned for description, authorship-like recognition, coordination, accountability, or another declared purpose.

Let the attribution family be

$$\mathcal{T}^{(3)}_{\text{attr}} := \{\mathsf{init}, \mathsf{exec}, \mathsf{gen}, \mathsf{author}, \mathsf{joint}\}. \tag{149}$$

Equation 149 fixes the distinct attribution roles retained for later comparison.

For action $\mathfrak{a}$, define

$$\mathsf{Attr}^{\kappa}_{3,t'}(\mathfrak{a}) \subseteq \mathfrak{B}^{\kappa}_{ABC,[t,t']} \times \mathcal{T}^{(3)}_{\text{attr}}. \tag{150}$$

Equation 150 relates retained provenance bearers to specific action-attribution types.

The tag author denotes authorship-like social attribution, not authorization. Authorization is represented separately by records such as $\mathfrak{U}^{\kappa,\tau}_{C\curvearrowright q,t}$ from Section 11.4. This distinction prevents the notation from treating "authorized by C" and "authored by C" as the same relation.

Execution likewise does not imply sole generation or sole authorship-like attribution:

$$\mathsf{exec}(i,\mathfrak{a}) \not\Longrightarrow \mathsf{gen}(i,\mathfrak{a}) \wedge \mathsf{author}(i,\mathfrak{a}). \tag{151}$$

Equation 151 is especially important for volunteer relations in which one subject performs the visible transfer while another subject supplied the request, translation, plan, or coordination that made the action possible.

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13.6 Triadic Action Types: Third-Conditioned, Joint Generation

Not every action with triadic provenance is a three-subject joint action. Paper II therefore distinguishes *third-conditioned dyadic execution* from *joint triadic generation.*

Let $\mathsf{ExecSubj}(\mathfrak{a})$ be the set of subjects retained as executors and let $\mathsf{GenSubj}(\mathfrak{a})$ be the set of subjects retained as generative contributors under the declared provenance criterion. An action is third-conditioned relative to focal dyad A–B when

$$\mathsf{ExecSubj}(\mathfrak{a}) \subseteq \{A, B\}, \qquad C \in \mathsf{GenSubj}(\mathfrak{a}). \tag{152}$$

Equation 152 captures cases in which C mediates, translates, authorizes, brokers, or coordinates an action whose visible execution remains inside A–B.

A stronger joint-triadic criterion is application-relative. Let $\mathsf{Ack}^{\text{joint}}_{ABC}(\mathfrak{a})$ denote a declared acknowledgment predicate stating that the three positions treat $\mathfrak{a}$ as belonging to a common joint-action procedure. Paper II uses the restricted necessary condition

$$\mathsf{Joint}^{\kappa}_{ABC}(\mathfrak{a}) \quad \Longrightarrow \quad \{A, B, C\} \subseteq \mathsf{GenSubj}(\mathfrak{a}) \ \wedge \ \mathsf{Ack}^{\text{joint}}_{ABC}(\mathfrak{a}), \tag{153}$$

where $\mathsf{Ack}^{\text{joint}}_{ABC}$ is a declared joint-action acknowledgment condition. Equation 153 is a necessary condition within this paper's restricted use, not a universal theory of joint action. It does not require all three subjects to execute the same physical operation.

A coalition action is not automatically joint triadic action. A and C can coordinate an action affecting B while B remains outside its generation. Conversely, B can contribute generatively to an action while not performing the visible execution. Coalition membership, joint generation, and execution must therefore remain separately typed.

13.7 Action Transformation, Consequence

A realized triadic action acts on the coupled history-bearing configuration through a typed partial transformation. Let $\mathfrak{K}^{\kappa}_{ABC}$ denote the retained class of triadic configurations at resolution κ. Define

$$\Phi^{\kappa' \leftarrow \kappa}_{\mathfrak{a}} : \mathcal{D}^{\kappa}_{\mathfrak{a}} \rightharpoonup \mathfrak{K}^{\kappa'}_{ABC}, \qquad \mathcal{D}^{\kappa}_{\mathfrak{a}} \subseteq \mathfrak{K}^{\kappa}_{ABC}. \tag{154}$$

The action can change only a focal dyadic region, alter a third-position route, modify public or accountability records, or reconfigure the continuation family of the entire triad. Equation 154 does not imply that every action is globally consequential.

The minimal consequence record is the ordered pair

$$\mathsf{Eff}^{\kappa' \leftarrow \kappa}_{3}(\mathfrak{a}; t, t') := \left(\mathcal{K}^{\kappa}_{ABC,t}, \mathcal{K}^{\kappa'}_{ABC,t'}\right). \tag{155}$$

Equation 155 is not a difference vector. Later analyses can derive typed consequences such as changed trust, new brokerage dependence, public recognition, altered coalition structure, reduced exit capacity, new accountability standing, or changed role positions.

13.8 Perspective-Relative Provenance Reconstruction

The retained provenance record should remain distinct from the subjects' own reconstructions. Let

$$\widehat{\Pi}^{\kappa}_{i,t'}(\mathfrak{a}), \qquad i \in \{A, B, C\}, \tag{156}$$

Equation 156 denotes the A-, B-, and C-relative provenance reconstructions formed from accessible manifestations, route history, ToM, public records, and later feedback.

The three reconstructions can diverge from one another and from the analyst's retained $\Pi^{\kappa}_{3,t'}(\mathfrak{a})$. A can describe an intervention as A's independent initiative, B can treat it as B-requested support, and C

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can reconstruct it as the product of mediation and coalition coordination. None of those reconstructions becomes exhaustive merely because it is sincerely held.

A discrepancy relation can be retained without forcing a scalar distance:

$$\delta^{\kappa}_{\Pi,t}\left(\widehat{\Pi}^{\kappa}_{i,t}(\mathfrak{a}),\widehat{\Pi}^{\kappa}_{j,t}(\mathfrak{a})\right). \tag{157}$$

Equation 157 can distinguish disagreement about initiation, authorship-like attribution, authorization, translation, mediation, or another provenance type. The type of disagreement should be retained because later responsibility questions can treat them differently.

13.9 Retrospective Provenance Revision: Accountability Interface

Later consequences, public records, testimony, appeal, or accountability review can revise the retained provenance of an earlier action. Let $\mathcal{H}^{\mathrm{upd}}_{[t',t'']}$ contain later evidence and interpretive changes relevant to action reconstruction. A provenance-revision map is

$$\mathsf{Rev}^{\kappa''\leftarrow\kappa'}_{\Pi,t''\leftarrow t'} : \mathcal{D}_{\mathsf{Rev}_\Pi} \rightharpoonup \mathfrak{P}^{\kappa''}_{\Pi,ABC}, \tag{158}$$

Equation 158 represents retrospective provenance revision on a domain containing the earlier provenance record together with admissible update material. Revision preserves provenance history: an earlier attribution can remain part of the historical record even when a later reconstruction supersedes it.

Accountability can therefore operate on both actions and their provenance. A review can ask whether C had standing to authorize, whether a translation changed the object of B's consent, whether a coalition structured the action without B's participation, or whether A's execution was constrained by a procedure. Paper II does not answer the normative responsibility question at this stage; it supplies the typed objects that make such questions possible.

13.10 Continuation Families, Procedure Effects

Let h be a realized triadic procedure prefix and let $\mathfrak{Z}^{(3),\kappa}_{\mathrm{proc}}$ be the class of candidate next elements. Define the continuation family

$$\mathrm{Cont}^{\kappa}_{3,\mathrm{proc}}(h) := \left\{\zeta \in \mathfrak{Z}^{(3),\kappa}_{\mathrm{proc}} \;\middle|\; \Theta_\zeta \text{ is defined on the record produced by } h\right\}. \tag{159}$$

Equation 159 defines which procedure elements remain admissible after a realized procedure prefix.

A procedure element can alter the continuation family without changing the material situation immediately. Public acknowledgment can make review available. Translation can make consent interpretable. A coalition can open one coordinated action family while closing another. An accountability determination can create an appeal branch. A witness record can make later public contestation possible.

Figure 12 summarizes one illustrative action-generation history.

13.11 Three-Level Profiles: Action, Procedure

Procedure and action provenance inherit the three-level architecture. Ultimate ontology remains suspended. At relational reality, the model can retain action, procedure, authorization, coalition, and provenance structures that shape later possibilities. At manifestation reality, these structures appear through requests, messages, signatures, records, visible execution, meeting formats, public statements, and other context-dependent forms. A-, B-, and C-relative reconstructions can disagree about what action occurred, whose action it was, who authorized it, or whether the procedure was joint.

For action $\mathfrak{a}$, define the profile

$$\Lambda^{\mathrm{act},\kappa}_{ABC,t}(\mathfrak{a}) := \left(\mathfrak{a}^{\mathsf{R}}_t, \mathrm{Man}_t(\mathfrak{a}), \widehat{\mathfrak{a}}_{A,t}, \widehat{\mathfrak{a}}_{B,t}, \widehat{\mathfrak{a}}_{C,t}, \Pi^{\kappa}_{3,t}(\mathfrak{a})\right), \tag{160}$$

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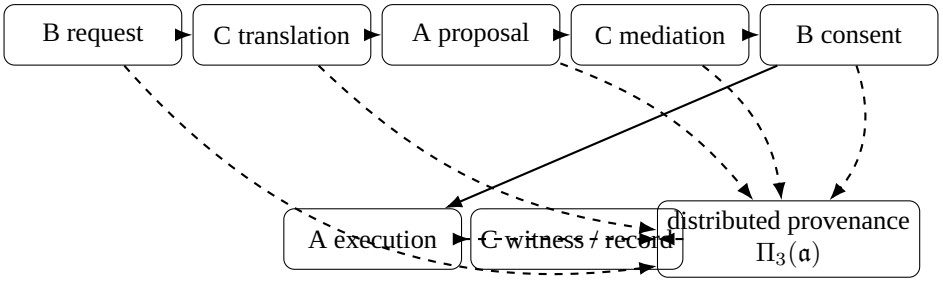

Figure 12: Illustrative triadic action provenance. The visible execution can remain A-to-B while request, translation, proposal, mediation, consent, witnessing, and recording contribute different provenance components. Dashed arrows indicate contribution to the retained provenance record rather than a universal causal law.

with ontological status separately suspended. Equation 160 places provenance alongside the action profile without treating provenance as a fourth reality level.

13.12 Pairwise Recoverability: Procedure, Provenance

Many triadic procedures remain reconstructible from rich pairwise records. The A–B record can preserve request, consent, and execution; A–C can preserve proposal, mediation, and authorization; B–C can preserve translation and witness records. If the coupling relations among those elements are retained through compatible restrictions, the full action provenance can factor through the pairwise shadow.

A stronger nonrecoverability claim requires a full-triad diagnostic. Let d^{κ}_{Π} be a full-equivalence-respecting diagnostic of action provenance or procedural coupling. Triadic nonrecoverability is witnessed when

$$X \sim_{2,\kappa} Y \qquad \text{and} \qquad d^{\kappa}_{\Pi}(X) \neq d^{\kappa}_{\Pi}(Y). \tag{161}$$

For example, X and Y can preserve the same three pairwise records while differing in whether C's translation and mediation jointly constitute the condition under which B's consent refers to the action A eventually executes. If the pairwise restrictions retain each local component but discard the higher-order procedural coupling, Equation 161 can hold.

The result is representation-relative. A richer pairwise record can restore recoverability. Paper II therefore does not equate distributed provenance with irreducible triadic provenance.

13.13 Historical Procedure-Provenance Trajectories

Procedure and provenance change historically. Let

$$\mathbf{G}^{\mathrm{proc},\kappa}_{ABC,I} := \left(\Gamma^{\kappa}_{ABC,t}, \Pi^{\kappa}_{3,t}, \mathsf{Attr}^{\kappa}_{3,t}, \mathsf{Cont}^{\kappa}_{3,\mathrm{proc}}\right)_{t\in I} \tag{162}$$

Equation 162 denotes the retained trajectory of procedure, provenance, attribution, and continuation structure, with action indices suppressed where the context is clear.

Several patterns matter. A *procedural simplification trajectory* occurs when an earlier mediator, translator, or broker becomes unnecessary. A *procedural institutionalization trajectory* occurs when ad hoc mediation or accountability becomes a stable procedure. A *provenance redistribution trajectory* occurs when generative contributions migrate among subjects or institutions. A *joint-generation trajectory* occurs when repeated dyadic support develops into stable three-position co-production. A *procedural capture trajectory* occurs when one subject or coalition increasingly controls the domains in which later actions are defined. A *provenance reopening trajectory* occurs when later evidence changes the retained understanding of an earlier action.

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Table 14: Triadic procedure and action-provenance objects in the minimal formalism.

Object	Formal role	Distinction preserved	Historical question
Procedure record	Partial order of typed elements	Procedure versus one linear script	Which elements appear, repeat, branch, or disappear?
Action individuation	Process-to-action relation	Process episode versus action identity	Does later context change action segmentation?
Distributed provenance	Generative antecedent relation	Execution versus generation	Which subjects/structures contribute to the action?
Attribution record	Social/descriptive assignment	Authorship-like attribution versus authorization	How are action roles reattributed?
Third-conditioned action	Dyadic execution with third-position generation	Visible dyadic form versus triadic genesis	Does C remain necessary to later action?
Joint triadic action	Declared three-position generation criterion	Jointness versus coalition or co-presence	Does co-production stabilize?
Consequence record	Ordered configuration comparison	Effect versus scalar welfare difference	Which relational structures change later?

These trajectories can transform relational roles even when material actions remain similar. A support action initially attributed to A can later be recognized as jointly generated. A mediator can become a broker, then become unnecessary. A supportee can become a coordinator of later volunteer action. Historical provenance therefore belongs to the object of relational analysis rather than serving only as retrospective metadata.

Table 14 summarizes the principal distinctions developed in this section.

The procedure-provenance architecture now provides the basis for the next section's asymmetry analysis. Once contribution, execution, authorization, brokerage, review, and coalition participation are separately typed, Paper II can ask how epistemic access, dependency, route control, recognition, standing, and generative capacity are distributed across all three positions without collapsing them into one hierarchy.

14 Typed Triadic Asymmetry: Dependency Structure

This section develops the asymmetry layer of the minimal triadic volunteer formalism. Its role is to extend the typed dyadic comparison discipline without reducing the three-position configuration to three independent inequalities. The objective is to distinguish ordinary pairwise asymmetry from route-conditioned, coalition-conditioned, and higher-order dependencies whose consequences depend on the third position. The discussion covers typed comparison frames, pairwise asymmetry profiles, triangular dependency, third-position modulation, coalition and gatekeeping effects, three-level realization, pairwise recoverability, historical revision, and interfaces with later power and justice analysis. The method remains formal and reconstructive. Paper II classifies relational structure; it does not infer domination, injustice, legitimacy, or moral failure merely from asymmetry.

Sections 12 and 13 already supplied several candidate sources of asymmetry: route access, translation capacity, gatekeeping, coalition membership, review standing, authorization, distributed provenance, and differential control over later procedure. These structures can point in different directions at the same time. A can possess material resources, B can retain decisive refusal standing, and C can control the only review route. The resulting triad should therefore be represented as a typed family of directional relations rather than as one ranking $A \succ B \succ C$.

14.1 Typed Comparison Frames

Let $\mathcal{I}^{3,\kappa}_{\text{asym}}$ denote the family of relational dimensions retained at resolution κ. The family can include epistemic-route access, information, resources, competence, translation, recognition, reputation, dependency, brokerage, refusal capacity, exit capacity, appeal access, standing, authority, coalition access,

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generative capacity, and normative influence. The list is open and application-specific.

For each $\alpha \in \mathcal{I}_{\text{asym}}^{3,\kappa}$ and historical stage t, let $\mathcal{P}_{\alpha,t}^{ABC,\kappa}$ be the class of retained subject- or position-indexed records relevant to dimension α. The comparison domain is defined in Equation 163.

$$\mathcal{D}_{\alpha,t}^{ABC,\kappa} \subseteq \mathcal{P}_{\alpha,t}^{ABC,\kappa} \times \mathcal{P}_{\alpha,t}^{ABC,\kappa}. \tag{163}$$

Equation 163 supplies the domain used by the triadic comparison frame,

$$\mathfrak{F}_{\alpha,t}^{ABC,\kappa} := \left(\mathcal{P}_{\alpha,t}^{ABC,\kappa}, \mathcal{D}_{\alpha,t}^{ABC,\kappa}, \simeq_{\alpha,t}^{ABC,\kappa}, \chi_{\alpha,t}^{ABC,\kappa}\right), \tag{164}$$

where $\simeq_{\alpha,t}^{ABC,\kappa}$ is the retained equivalence criterion and $\chi_{\alpha,t}^{ABC,\kappa}$ a directional-consequence criterion. Equation 164 does not require a numerical order. Definedness remains prior to symmetry or asymmetry. If a pair of position records lies outside $\mathcal{D}_{\alpha,t}^{ABC,\kappa}$, the present model does not classify that pair as equal, unequal, symmetric, or asymmetric in dimension α.

For $(x, y) \in \mathcal{D}_{\alpha,t}^{ABC,\kappa}$, define typed difference by

$$\mathsf{Diff}_{\alpha,t}^{ABC,\kappa}(x, y) \quad \Longleftrightarrow \quad x \not\simeq_{\alpha,t}^{ABC,\kappa} y. \tag{165}$$

Equation 165 defines typed difference inside the comparison frame. A difference becomes asymmetry only when the selected dimension supplies a directional consequence:

$$\mathsf{Asym}_{\alpha,t}^{ABC,\kappa}(x, y) \quad \Longleftrightarrow \quad (x, y) \in \mathcal{D}_{\alpha,t}^{ABC,\kappa} \ \wedge \ \mathsf{Diff}_{\alpha,t}^{ABC,\kappa}(x, y) \ \wedge \ \chi_{\alpha,t}^{ABC,\kappa}(x, y). \tag{166}$$

Equation 166 preserves the distinction among incomparability, difference, and asymmetry. A difference in language preference is not automatically interpretive asymmetry. It becomes asymmetry-relevant only when it changes access, translation burden, contestability, reachability, or another declared consequence.

14.2 Pairwise Asymmetry Shadow

The first layer of triadic asymmetry is the collection of pairwise asymmetry records induced by the full configuration. For each pair $XY \in \{AB, AC, BC\}$, let $\mathscr{A}_{XY|ABC,t}^{\kappa}$ denote the asymmetry profile obtained after the relevant restriction of the full record. Define the pairwise asymmetry shadow by

$$\mathsf{P}_{2,\mathscr{A},t}^{\kappa}\left(\mathscr{A}_{ABC,t}^{\kappa}\right) := \left(\mathscr{A}_{AB|ABC,t}^{\kappa}, \mathscr{A}_{AC|ABC,t}^{\kappa}, \mathscr{A}_{BC|ABC,t}^{\kappa}\right). \tag{167}$$

Equation 167 records the three restricted asymmetry profiles but does not by itself establish that they are sufficient to reconstruct the selected full-triad asymmetry structure. Two full configurations can contain the same pairwise asymmetries while differing in how those asymmetries are coupled through route conditions, coalition rules, shared procedures, or higher-order acknowledgment.

The orientation of pairwise asymmetry can also differ across dimensions. A can be advantaged relative to B in resource access while B has greater refusal capacity relative to A. C can possess superior procedural standing while remaining dependent on A for factual information. Paper II therefore does not infer a transitive subject order from the three pairwise profiles.

14.3 Triangular Dependency Record

Dependency is modeled through the effect of another position or relation on the continuation family available to a focal subject. Because such claims are counterfactual relative to a declared model, Paper II makes the suppression rule explicit.

For subject i, dimension α, and another subject j, let $\ominus_j^{\alpha}$ be a declared counterfactual suppression operation that removes or neutralizes only the j-mediated contribution relevant to dimension α, while

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retaining the rest of the comparison regime as specified by the application. Let $\mathrm{Cont}_{i,t}^{\kappa,\alpha}(X)$ denote the retained continuation family of i in configuration X. Define i's dependency on j in dimension α by

$$\mathsf{Dep}_{i\leftarrow j,t}^{\kappa,\alpha}(X) \quad \Longleftrightarrow \quad \mathrm{Cont}_{i,t}^{\kappa,\alpha}(X) \not\approx_{\mathrm{cont},\alpha}^{\kappa} \mathrm{Cont}_{i,t}^{\kappa,\alpha}\big(\ominus_{j}^{\alpha} X\big), \tag{168}$$

where $\approx_{\mathrm{cont},\alpha}^{\kappa}$ is the declared continuation equivalence. Equation 168 does not imply that j intentionally created the dependency or that the dependency is unjustified. It states that the selected continuation structure changes under the declared suppression.

Let $\mathcal{I}_{\mathrm{dep}}^{\kappa} \subseteq \mathcal{I}_{\mathrm{asym}}^{3,\kappa}$ be the dimensions for which the application has supplied a suppression operation and continuation comparison. The triangular dependency profile is the indexed family

$$\mathscr{D}_{ABC,t}^{\kappa} := \left\{ \mathsf{Dep}_{i\leftarrow j,t}^{\kappa,\alpha} \;\middle|\; i,j \in \{A,B,C\},\ i \neq j,\ \alpha \in \mathcal{I}_{\mathrm{dep}}^{\kappa} \right\}. \tag{169}$$

Equation 169 permits chains, cycles, mutual dependencies, and dimension-specific reversals. B can depend on A for resources, A can depend on C for authorization, and C can depend on B for first-person evidence. Such a cycle is not reducible to one ranking of the three subjects.

14.4 Third-Position Modulation of Pairwise Asymmetry

A specifically triadic structure appears when the asymmetry of a focal pair changes with the state or function of the third position. Let s_C and s'_C be two retained C-position records relevant to dimension α. A C-modulated A–B asymmetry exists when

$$\mathsf{Asym}_{\alpha,t}^{AB|C=s_C,\kappa}(x_A,x_B) \not\equiv \mathsf{Asym}_{\alpha,t}^{AB|C=s'_C,\kappa}(x_A,x_B), \tag{170}$$

under a declared comparison of the asymmetry records. Equation 170 is not an arithmetic inequality; it means that the asymmetry's definedness, orientation, consequence, or type changes when the retained C-condition changes.

Several earlier constructions instantiate this pattern. A–B informational asymmetry can shrink when C translates a manifestation. A–B route dependency can grow when C becomes the only broker. A–B procedural asymmetry can change when C receives review standing. A–B recognition asymmetry can change after C's public record alters whose account is treated as credible. The third position can therefore modulate a dyadic asymmetry without becoming the sole source of it.

This structure is distinct from simple addition of an A–C or B–C asymmetry. The triadic object is the dependence of the focal A–B comparison on C's position or function.

14.5 Brokerage, Gatekeeping, Coalition Asymmetry

Brokerage and coalition structures generate asymmetry when they alter access, continuation, or condition-setting capacity in a directional manner. The brokerage record $\mathfrak{B}_{k|ij}^{\kappa,\chi}$ from Section 12 can become asymmetry-relevant when k possesses a route position unavailable to i or j. The stronger gatekeeping relation adds condition-setting capacity and can therefore create a distinct procedural asymmetry.

A coalition can also create a higher-order comparison between one subject's available continuations and those generated through coordinated action by the other two. Let $\mathfrak{C}_{ij|k,t}^{\kappa,\chi}$ be a coalition record and let $\mathrm{Cont}_{k,t}^{\kappa,\chi}$ be the third subject's continuation family. A coalition-conditioned asymmetry is present when the coalition changes that family directionally under the selected criterion:

$$\chi_{\mathrm{coal},t}^{\kappa,\chi}\left(\mathrm{Cont}_{k,t}^{\kappa,\chi}, \mathrm{Cont}_{k,t}^{\kappa,\chi}[\mathfrak{C}_{ij|k,t}^{\kappa,\chi}]\right). \tag{171}$$

Equation 171 does not classify the coalition as hostile or illegitimate. A coalition can increase B's options by coordinating support or reduce B's procedural influence by making one plan difficult to contest. The direction and normative interpretation must be declared separately.

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Coalition asymmetry and pairwise asymmetry need not coincide. A and C can each possess only modest individual influence over B while their coordinated procedure produces a strong collective constraint. Conversely, two strong pairwise relations can fail to coordinate and produce little joint effect.

14.6 Standing, Authorization, Enforcement Asymmetry

Section 11 distinguished claim, review, authorization, enforcement, and appeal standing. Their asymmetric distribution can alter who can initiate, block, reopen, or terminate a procedure. Paper II therefore treats standing asymmetry as a typed relation rather than as a generic authority hierarchy.

For standing type $\tau \in \mathcal{T}_{\text{stand}}$, define the subject-position profile

$$\mathscr{S}^{\tau,\kappa}_{ABC,t}(q) := \Big(\mathsf{St}^{\tau,\kappa}_{A,t}(q), \mathsf{St}^{\tau,\kappa}_{B,t}(q), \mathsf{St}^{\tau,\kappa}_{C,t}(q)\Big). \tag{172}$$

The three components in Equation 172 need not lie on a common numerical scale. A model can compare whether each subject possesses standing of a declared kind, whether the statuses are compatible, and what directional consequence follows.

Authorization and enforcement asymmetry remain distinct. C can possess review standing but not enforcement standing. B can possess decisive consent/refusal standing concerning an A–B intervention while C has authority over one institutional continuation. A can possess execution capacity without authorization standing. The formalism therefore preserves

$$\text{standing asymmetry} \not\Rightarrow \text{global authority hierarchy}. \tag{173}$$

Equation 173 states that local procedural standing does not supply global authority across other dimensions.

14.7 Recognition, Reputation, Interpretive Asymmetry

Publicity and reputation create asymmetry when one subject's manifestations, claims, or roles receive different uptake, circulation, credibility, or recognitional effect from those of another subject. Let $\mathfrak{Rep}^{\kappa,\chi}_{i,\mathcal{U},t}$ and $\mathsf{PRec}^{\kappa,\chi}_{\mathcal{U}\rightsquigarrow i,t}$ be the reputation and public-recognition records from Section 10. A comparison can be defined over these records without assuming a scalar reputation score.

Interpretive asymmetry can be more basic. One subject can possess the vocabulary or translation route required to make an experience publicly legible, while another subject's manifestation remains difficult to articulate within the retained public scheme. The asymmetry can therefore concern not only who is believed but who can make a claim available in a form that enters the procedure at all.

Recognition asymmetry, reputation asymmetry, and credibility asymmetry should remain distinct. A subject can be publicly recognized as a participant while still being treated as less credible on one factual claim. A strong reputation can make a manifestation more readily believed without changing formal standing. These distinctions become especially important in later justice and power analysis.

14.8 Asymmetry Families in the Minimal Triad

Table 15 summarizes the principal dimensions retained by Paper II. The table identifies formal comparison objects and directional consequences rather than normative failures.

14.9 Three-Level Asymmetry Profiles

Triadic asymmetry inherits the three-level reality discipline. Ultimate ontology remains suspended. At relational reality, asymmetry concerns a typed organization that produces directionally different consequences. At manifestation reality, the asymmetry can appear through unequal speaking time, interface permissions, different routes, visible resource control, public titles, coalition membership, review roles,

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Table 15: Principal typed asymmetry families in the minimal triadic volunteer relation.

Dimension	Comparison object	Directionally consequential difference can concern
Epistemic-route	Route access, provenance, candidate reconstructions	Which subject can access, compare, or revise a better-supported reconstruction
Informational	Manifestation and record access	Who possesses information required for later judgment, consent, review, or action
Resource	Material, temporal, infrastructural, organizational resources	Who can initiate, sustain, refuse, or redirect a support procedure
Competence / translation	Skill, expertise, interpretive and translation capacity	Who can execute, assess, or render a manifestation mutually intelligible
Recognition / reputation	Public recognition, circulation, credibility, legibility	Whose claim, refusal, contribution, or role more readily enters later procedure
Brokerage / gatekeeping	Route dependence and condition-setting	Who can open, filter, condition, or block an access route
Standing / authority	Claim, review, authorization, enforcement, appeal standing	Who can demand an answer, authorize continuation, enforce, or reopen procedure
Coalition / exclusion	Coordinated commitments and typed exclusion records	Whether coordinated positions alter the third subject's accessible continuations
Refusal / exit / appeal	Effective capacities and procedural routes	Who can contest, leave, or obtain reconsideration without disproportionate loss
Generative capacity	Ability to contribute to later relation formation	Whose contribution more strongly reshapes future action/procedure possibilities
Normative influence	Uptake of evaluations and norms	Whose judgment more readily stabilizes as a shared or publicly actionable norm

or other context-dependent forms. A-, B-, and C-relative reconstructions can disagree about whether the asymmetry exists, what dimension it belongs to, and how consequential it is.

For asymmetry object $Q_{\alpha,t}$, define the profile

$$\Lambda^{\text{asym},\kappa}_{ABC,t}(Q_{\alpha,t}) := \left(Q^{\mathsf{R}}_{\alpha,t}, Q^{\mathsf{M}}_{\alpha,t}, \widehat{Q}_{A,\alpha,t}, \widehat{Q}_{B,\alpha,t}, \widehat{Q}_{C,\alpha,t}\right), \tag{174}$$

Equation 174 separates relational asymmetry, its manifestations, and subject-relative reconstructions, with ontological status separately suspended. Manifestation and reconstruction can be asymmetric even when the underlying relational structure is not, and a relational asymmetry can remain poorly manifested or unrecognized.

Figure 13 summarizes the analytical architecture.

14.10 Multidimensional Triadic Profile

The full asymmetry structure is represented as an indexed family rather than a vector with assumed commensurability:

$$\mathscr{A}^{\kappa}_{ABC,t} := \left\{ \mathfrak{A}^{ABC,\kappa}_{\alpha,t} \;\middle|\; \alpha \in \mathcal{I}^{3,\kappa}_{\text{asym}} \right\}. \tag{175}$$

Equation 175 implies no addition, norm, inner product, or total order. Cross-dimensional mechanisms can nevertheless be modeled explicitly. Gatekeeping can create informational dependency. Reputation can affect whose testimony is treated as credible. Resource asymmetry can alter exit capacity. Appeal standing can counteract one procedural dependency without eliminating the underlying resource asymmetry.

The formalism therefore avoids cancellation language unless an application supplies a common scale. Greater refusal capacity for B does not numerically cancel greater resources for A. Strong review standing for C does not offset C's dependence on A for evidence. Oppositely oriented asymmetries can coexist and jointly shape later trajectories.

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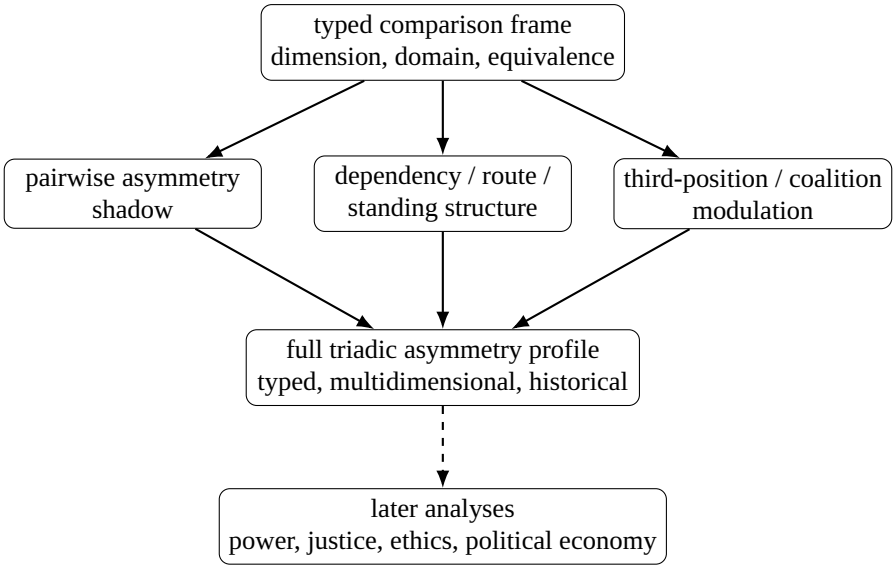

Figure 13: Typed triadic asymmetry architecture. Pairwise asymmetry, dependency, and third-position or coalition modulation contribute to a full triadic profile without being arithmetically aggregated. The dashed arrow marks an interface to later power and normative analysis rather than an automatic entailment.

14.11 Pairwise Recoverability of Triadic Asymmetry

The pairwise asymmetry shadow in Equation 167 can be sufficient when the three restricted profiles retain all information needed by the selected full diagnostic. A specifically triadic asymmetry claim requires a stronger witness.

Let d^{κ}_{asym} be a full-equivalence-respecting diagnostic of the full asymmetry/dependency organization. Triadic asymmetry nonrecoverability is witnessed when

$$X \sim_{2,\kappa} Y \qquad \text{and} \qquad d^{\kappa}_{\text{asym}}(X) \neq d^{\kappa}_{\text{asym}}(Y). \tag{176}$$

A finite witness can arise when X and Y preserve identical A–B, A–C, and B–C asymmetry profiles but differ in whether C's authorization condition modulates A–B dependency, or whether an A–C coalition jointly conditions B's continuation family. If the pairwise restrictions retain each local asymmetry but discard the coupling rule, Equation 176 can hold.

The claim remains representation-relative. Richer pairwise records can restore recoverability. Paper II therefore does not equate a complicated triangle of asymmetries with irreducible thirdness.

14.12 Historical Trajectories: Asymmetry, Dependency

Triadic asymmetry is historically revisable. Let

$$\mathbf{A}^{\text{asym},\kappa}_{ABC,I} := \left(\mathscr{A}^{\kappa}_{ABC,t}, \mathscr{D}^{\kappa}_{ABC,t}, \mathsf{P}^{\kappa}_{2,\mathscr{A},t}\right)_{t\in I} \tag{177}$$

Equation 177 defines the retained trajectory of typed asymmetry and dependency structures across the historical interval.

Several transformations matter. A route asymmetry can disappear after direct communication forms. A translation asymmetry can persist while its practical consequences shrink. Gatekeeping dependence can intensify through route lock-in or decline through bypass. Coalition formation can amplify one asymmetry while generating countervailing standing for the excluded subject. Public recognition can change credibility asymmetry. Appeal rights can reduce procedural asymmetry without changing resource distri-

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bution. A support relation can also reverse roles over time, moving a former supportee into a coordination or mediator position.

The trajectory is therefore nonmonotone and multidimensional. Paper II does not assume that all asymmetries converge toward equality or that persistence indicates legitimacy. Historical provenance remains part of the retained structure because similar present distributions can arise through different histories of delegation, learning, exclusion, public recognition, or dependency.

14.13 Difference, Asymmetry, Power, Injustice

The formal architecture preserves a strict separation among typed difference, asymmetry, dependency, power relevance, and normative diagnosis. The non-entailments are

$$\mathsf{Diff}_\alpha \not\Rightarrow \mathsf{Asym}_\alpha, \qquad \mathsf{Asym}_\alpha \not\Rightarrow \mathsf{Power}, \qquad \mathsf{Dep}_\alpha \not\Rightarrow \mathsf{Injustice}. \tag{178}$$

Equation 178 does not deny that these relations can be connected. Power analysis requires additional conditions concerning persistent trajectory shaping, option-space deformation, institutionalization, or other mechanisms. Injustice requires independent normative criteria concerning legitimacy, recognition, distribution, procedure, contestability, responsibility, or another evaluative domain.

The triadic setting sharpens this separation because a third position can reduce one asymmetry while creating another. Mediation can improve communication while increasing dependence on the mediator. Public review can strengthen contestability while concentrating enforcement authority. Coalition support can increase B's material options while reducing B's procedural participation. Such mixed structures should remain visible rather than being compressed into one verdict.

Section 14 has therefore supplied the comparison and dependency architecture needed for the next stage of the paper. Section 15 will ask when repeated triadic interaction yields relational structures that satisfy model-relative persistence, organization, and efficacy criteria, including stable mediation channels, indirect trust, public recognition, accountability regimes, coalition structures, and triadic partnership.

15 Emergent Triadic Relational Structures

This section develops the emergent-structure layer of the minimal triadic volunteer formalism. Its role is to identify when recurrent three-position organization can be treated as a relationally real object of the model rather than as a transient event, a temporary alignment, a private judgment, or an analyst's shorthand. The objective is deliberately formal and model-relative. Paper II does not derive a general metaphysics of publicness, indirect trust, mediation, reputation, accountability, coalition, shared norm, or partnership. It specifies conditions under which such structures can be tracked across history, possess enough organization to remain identifiable through local variation, and condition later possibilities, interpretations, procedures, or actions.

The construction extends the emergence discipline of Paper I while adding an independent question that is specific to Paper II: whether an emergent triadic structure is recoverable from the three pairwise restrictions. Emergence and pairwise nonrecoverability are therefore separate axes. A stable mediation channel can be emergent while remaining fully reconstructible from rich A–C and B–C records. Conversely, a one-time full-audience acknowledgment event can be pairwise nonrecoverable without becoming a persistent emergent structure. The stronger claim of an emergent and pairwise-nonrecoverable triadic object requires both burdens to be met.

The section proceeds through a general emergence criterion and witness record, then develops mediated and indirect trust, stable mediation and brokerage channels, publicness and reputation, accountability regimes, coalition and coordination structures, shared vocabulary and public norms, role ecologies and triadic partnership, three-level profiles, pairwise recoverability, and historical dissolution or retyping. It closes by stating the interface to the local algebraic and categorical dynamics developed in Section 16.

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15.1 Triadic Emergence Criterion, Witness Records

Let X be a candidate relational object represented at the coupled ABC level, let $I = [t_0, t_1]_{\preceq}$ be a retained historical interval, and let $H^{\kappa}_{ABC,I}$ be the corresponding triadic history at resolution κ. Let χ denote the specification under which tracking, organization, and efficacy are assessed. Paper II stipulates the model-relative emergence criterion in Equation 179.

$$\mathsf{Emerg}^{\kappa}_{3,\chi}\big(X; H^{\kappa}_{ABC,I}\big) \iff \mathsf{TriScope}^{\kappa}_{\chi}(X) \wedge \mathsf{Pers}^{\kappa}_{\chi}(X; I) \wedge \mathsf{Org}^{\kappa}_{\chi}(X; I) \wedge \mathsf{Effic}^{\kappa}_{\chi}(X; I). \tag{179}$$

The four terms in Equation 179 have distinct roles. $\mathsf{TriScope}^{\kappa}_{\chi}$ records that the object is typed relative to the coupled triadic configuration rather than merely being an isolated dyadic record. $\mathsf{Pers}^{\kappa}_{\chi}$ records historical trackability across a nontrivial interval. $\mathsf{Org}^{\kappa}_{\chi}$ records that the retained relations possess enough organization to identify X through local variation. $\mathsf{Effic}^{\kappa}_{\chi}$ records that X changes at least one later family of continuations, expectations, routes, interpretations, procedure states, authorization conditions, role relations, or admissible transformations.

Equation 179 is a stipulated criterion for this paper. It is not a theorem that persistence, organization, and efficacy exhaust every philosophical account of emergence. Nor does $\mathsf{TriScope}$ imply pairwise nonrecoverability. A triadic object can be defined over the coupled configuration while still being reconstructible from its pairwise shadows.

The evidence retained by the model is collected in a witness record:

$$\mathcal{W}^{3,\chi,\kappa}_{X}(I) := \Big(\mathcal{T}^{\chi,\kappa}_{X}(I), \mathcal{O}^{\chi,\kappa}_{X}(I), \mathcal{E}^{\chi,\kappa}_{X}(I), \mathcal{M}^{\chi,\kappa}_{X}(I), \Pi^{\chi,\kappa}_{X}(I)\Big), \tag{180}$$

Equation 180 collects the witnesses used to justify scope, persistence, organization, and efficacy, where $\mathcal{T}_X$ contains tracking witnesses, $\mathcal{O}_X$ relational-organization witnesses, $\mathcal{E}_X$ efficacy witnesses, $\mathcal{M}_X$ manifestation witnesses, and Π_X the provenance needed to interpret how the candidate structure formed and changed. Manifestations are evidential and generative material; they are not identical to the emergent relational object.

Several non-entailments follow immediately:

$$\begin{aligned} \mathsf{repetition} &\not\Rightarrow \mathsf{emergence}, \\ \mathsf{public\ visibility} &\not\Rightarrow \mathsf{public\ relational\ structure}, \\ \mathsf{three\ matching\ judgments} &\not\Rightarrow \mathsf{shared\ triadic\ norm}, \\ \mathsf{stable\ role\ label} &\not\Rightarrow \mathsf{stable\ role\ relation}. \end{aligned} \tag{181}$$

Equation 181 prevents emergence from becoming a synonym for frequency, visibility, agreement, or naming.

15.2 Emergence: Pairwise Recoverability

The reducibility formalism of Section 6 introduces a second axis independent of Equation 179. For $X \in \mathfrak{D}^{\kappa}_{3}$, define the configuration-relative nonrecoverability predicate by

$$\mathsf{NR}^{\kappa}_{3|2}(X) \quad \iff \quad \big|\mathsf{Amb}^{\kappa}_{2}\big(q^{\kappa}_{2}(X)\big)\big| > 1. \tag{182}$$

Equation 182 states that the pairwise-shadow class of X admits more than one retained full-equivalence class. Paper II therefore distinguishes four model-relative regimes summarized in Table 16.

The table blocks two common inferences. Historical stability does not prove irreducible thirdness, and pairwise nonrecoverability does not prove historical emergence. A later claim can therefore state precisely whether it concerns persistence, higher-order recoverability, or both.

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Table 16: Emergence and pairwise recoverability are independent analytical axes.

Regime	Emergence status	Pairwise status	Illustrative interpretation
Transient / recoverable	not emergent	recoverable	One mediation episode fully reconstructible from A–C and B–C records
Transient / nonrecoverable	not emergent	nonrecoverable	One full-audience acknowledgment event whose common-audience object is discarded by all pairwise restrictions
Emergent / recoverable	emergent	recoverable	Stable mediation or accountability channel reconstructible from sufficiently rich dyadic records
Emergent / nonrecoverable	emergent	nonrecoverable	Persistent full-triad public norm or coordination structure whose operative coupling is absent from every pairwise restriction

15.3 Mediated Trust, Indirect Trust

Trust in the triadic system can involve direct reliance, reliance on an intermediary, or reliance on another subject partly through an intermediary-dependent route. These structures should not be collapsed into one transitive chain.

For trust domain d, let $\mathsf{Tr}^{\kappa,d}_{A\rightsquigarrow B,t}$ denote a direct or focal trust relation inherited from Paper I. Let $\gamma = B \to C \to A$ be a route whose use materially contributes to A's later reliance on B. A mediated-trust record can be written

$$\mathfrak{Tr}^{\kappa,d,\gamma}_{A\rightsquigarrow B|C,t} := \Big(\mathsf{Tr}^{\kappa,d}_{A\rightsquigarrow B,t}, \mathfrak{R}^{\kappa,\gamma}_{A\leftarrow B,t}, \mathfrak{F}^{\kappa,\phi}_{C|AB,t}, \Pi^{\mathrm{tr}}_{t}\Big), \tag{183}$$

Equation 183 keeps direct and mediated trust components separate inside one triadic trust record, where $\mathfrak{R}$ is the route record, $\mathfrak{F}$ the relevant C-function record when one exists, and Π^{tr} the provenance of the trust relation.

Trust is not transitive. In particular,

$$\mathsf{Tr}^{d_1}_{A\rightsquigarrow C} \wedge \mathsf{Tr}^{d_2}_{C\rightsquigarrow B} \not\Longrightarrow \mathsf{Tr}^{d_3}_{A\rightsquigarrow B} \tag{184}$$

Equation 184 states explicitly that two trust relations do not generate a third by transitivity alone.

for arbitrary domains d_1, d_2, d_3. A can trust C's translation while distrusting B's competence, or trust C as a witness while remaining uncertain about B's future conduct. Conversely, A can come to trust B through repeated C-mediated evidence without developing broad trust in C outside the relevant route.

A mediated trust structure becomes emergent only when Equation 179 is satisfied. Candidate efficacy witnesses include willingness to accept information via a mediated route, reduced verification cost, willingness to enter a procedure that would otherwise be refused, or increased readiness to rely on B because C supplies stable translation, witnessing, or review. Such trust can be misplaced or exploited; emergence does not supply a normative verdict.

Distrust remains separately typed. A stable expectation that a C-mediated route is selective, unreliable, or captured can reorganize later procedure even when A and B retain direct trust in another domain.

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15.4 Stable Mediation, Translation, Brokerage Channels

A single mediation, translation, or brokerage event is a procedure element. A stable channel is a different object: subjects orient later interaction to the expectation that a third-position function will remain available and consequential.

For function type $\phi \in \{\mathsf{med}, \mathsf{trans}, \mathsf{brok}, \mathsf{wit}\}$, define a candidate channel record

$$\mathfrak{Ch}^{\kappa,\phi}_{Z|XY,t} := \Big(\{\mathfrak{F}^{\kappa_s,\phi}_{Z|XY,s}\}_{s\preceq t}, \mathsf{Avail}^{\phi}_{Z|XY,t}, \mathsf{Exp}^{\phi}_{XY\rightsquigarrow Z,t}, \Pi^{\phi}_{Z|XY,t}\Big), \tag{185}$$

Equation 185 defines the candidate channel whose historical stability must be justified by the emergence criterion, where Avail records future availability under the selected model and Exp records stabilized expectations that the channel can be invoked or will condition later interaction.

A stable channel changes the relation even before it is used. A credible mediation channel can make disagreement easier to express because subjects expect clarification or review rather than immediate rupture. A stable translation channel can widen the manifestations that subjects are willing to offer. A brokerage channel can lower search costs while simultaneously creating dependency. A stable witness or recording channel can change prospective behavior because later attestation is expected.

Channel stabilization can therefore be enabling, constraining, or both. It can also generate its own displacement. Successful repeated translation can create a shared vocabulary that makes the translator less necessary. Stable brokerage can generate direct contacts that later bypass the broker. An emergent channel should therefore be tracked historically rather than treated as a permanent institutional identity.

15.5 Emergent Publicness, Reputation, Recognition

Section 10 defined one-time publicity, acknowledgment, durable public records, reputation records, and recognition records. The present section asks when those structures become relationally efficacious across later history.

A candidate publicness record is written

$$\mathsf{Publ}^{\kappa}_{\mathcal{U},t} := \Big(\mathsf{Pub}^{\kappa}_{\mathcal{U},t}, \mathcal{G}^{\mathrm{ack},\kappa}_{\mathcal{U},t}, \mathfrak{D}^{\mathrm{pub},\kappa}_{\mathcal{U},t}, \mathsf{Exp}^{\mathrm{pub}}_{\mathcal{U},t}\Big), \tag{186}$$

Equation 186 records the publicness components that can become historically organized and efficacious, where $\mathsf{Exp}^{\mathrm{pub}}$ records the historically stabilized expectation that relevant manifestations will be presented, reviewable, or remembered relative to audience $\mathcal{U}$.

Publicness becomes emergent when subjects orient later action to the audience structure itself. A volunteer can document an intervention because future review is expected; B can formulate a refusal differently because it will enter a shared record; C can withhold a reputational judgment until a public correction procedure is complete. In those cases the public relation conditions later possibilities before any new public event occurs.

Reputation becomes emergent when audience-indexed attributions persist, organize later interpretation, and affect access, trust, invitation, scrutiny, or role assignment. Public recognition becomes emergent when recognitional records stabilize a subject's standing, role, authorship-like attribution, or participation conditions. Neither process proves accuracy. A false reputation can be relationally efficacious, and a public recognition regime can stabilize a problematic role.

The distinction between publicness and publicity is therefore historical: publicity can be an episode, while publicness is a persistent orientation of later relation to a shared audience or record structure.

15.6 Emergent Accountability Regimes

Section 11 represented claim, notice, response, standing, review, determination, appeal, authorization, and enforcement as separate procedural objects. A recurrent organization of those elements can itself become an emergent accountability regime.

Define a candidate accountability-regime record by

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$$\mathfrak{R}^{\text{acct},\kappa}_{ABC,t} := \Big(\mathcal{T}_{\text{stand},t}, \Gamma^{\kappa}_{\text{acct},t}, \mathfrak{D}^{\text{pub},\kappa}_{\mathcal{U},t}, \mathsf{Appeal}^{\kappa}_{t}, \mathsf{Enf}^{\kappa}_{t}, \Pi^{\text{acct}}_{t}\Big). \tag{187}$$

Equation 187 collects the components of a candidate emergent accountability regime.

The record becomes emergent only when the subjects' later behavior is conditioned by the regime rather than by isolated review events. Candidate efficacy witnesses include anticipatory documentation, increased practical usability of complaint or refusal, routinized appeal, changed authorization behavior, or altered expectations concerning correction and answerability.

A stable accountability regime need not be legitimate. It can be inaccessible, captured, biased, selectively enforced, or dependent on one coalition. Emergence therefore does not collapse the distinctions established in Equation 123. The relational fact that a review structure organizes later conduct is separate from the normative question whether its standing or enforcement is justified.

15.7 Coalition, Coordination, Role Ecology

Coalition formation can remain episodic or become a recurrent structural pattern. A stable coalition is not simply repeated agreement; it requires historically organized coordination and efficacy under the criterion of Equation 179. Let

$$\mathfrak{C}^{\text{em},\kappa,\chi}_{ij|k,t} := \Big(\mathfrak{C}^{\kappa,\chi}_{ij|k,t}, \mathsf{Exp}^{\text{coord},\chi}_{ij,t}, \mathsf{Cont}^{\chi}_{ABC,t}, \Pi^{\chi}_{ij|k,t}\Big) \tag{188}$$

Equation 188 defines the candidate emergent coalition record whose persistence and efficacy are evaluated historically. The expectation term records that later coordination is anticipated, while the continuation term records the domain in which the coalition conditions future possibilities.

Repeated triadic interaction can also generate a broader role ecology: supporter, supportee, mediator, translator, witness, reviewer, broker, coordinator, or public-recognition roles become mutually conditioned. Let

$$\mathscr{R}^{\text{role},\kappa}_{ABC,t} := \Big\{ \varrho^{\kappa,\tau}_{i,t} \;\Big|\; i \in \{A,B,C\},\ \tau \in \mathcal{T}^{3}_{\text{role}} \Big\} \tag{189}$$

Equation 189 collects the interacting role-position records retained at one historical stage. The ecological language indicates mutual conditioning, not biological analogy. C's stabilization as a mediator can change A's and B's expectations of one another; B's later emergence as a coordinator can weaken a fixed supportee role; a stable reviewer role can change how A documents support actions.

Role stabilization does not imply fixed subject identity. The same subject can migrate among roles, occupy several roles simultaneously, or cease to occupy a role when the relevant channel dissolves.

15.8 Shared Vocabulary, Public Norms

A triadic shared vocabulary can emerge when expressions become recurrently usable across all three subject positions and reduce translation or clarification burdens. Let $\mathcal{V}^{\kappa}_{ABC,t}$ be a candidate vocabulary record. Its emergence requires more than identical words: the retained expressions must have enough shared operational use to affect later interpretation or procedure.

A public or triadically shared norm requires a stronger condition than three matching private judgments. Let $N^{\kappa}_{ABC,t}$ be a candidate norm object. Paper II requires at least a retained common normative object, a history of orientation by the three positions, and efficacy in later judgment or action. Schematically,

$$\mathsf{SharedNorm}^{\kappa}_{ABC,t}(N) \Longrightarrow \mathsf{Orient}^{\kappa}_{A,t}(N) \wedge \mathsf{Orient}^{\kappa}_{B,t}(N) \wedge \mathsf{Orient}^{\kappa}_{C,t}(N) \wedge \mathsf{Effic}^{\kappa}_{N,t}. \tag{190}$$

Equation 190 states necessary structure for the restricted use of "shared norm" in this paper. It does not require complete agreement about interpretation, reasons, scope, or validity. The subjects can orient to the same norm while contesting its application.

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A public norm can be more specific: the norm is also presented, acknowledged, or procedurally invoked relative to a declared audience. Publicity can make the norm contestable and reviewable, but public status does not establish normative validity.

15.9 Triadic Partnership, Co-Production

Triadic partnership is treated as a candidate emergent structure of sustained co-participation. It is not the absence of role differences and does not require equal resources, equal competence, identical authority, or symmetric contribution.

Let $\mathsf{Part}^{\kappa}_{ABC,t}$ denote a candidate partnership record. Paper II requires that all three subjects possess practically consequential routes for contribution, correction, and revision of the shared procedure, even when those routes differ by domain. A restricted condition is

$$\mathsf{Part}^{\kappa}_{ABC,t} \Longrightarrow \bigwedge_{i\in\{A,B,C\}} \Big(\mathsf{Contrib}^{\kappa}_{i,t} \wedge \mathsf{Correct}^{\kappa}_{i,t} \wedge \mathsf{Revise}^{\kappa}_{i,t}\Big), \tag{191}$$

where each predicate is typed by the relevant domain and practical efficacy criterion. Equation 191 is a necessary condition for the restricted partnership concept used here, not a universal social theory of partnership.

Partnership differs from coalition. A coalition can coordinate two subjects relative to a third, whereas triadic partnership requires consequential participation by all three. Partnership also differs from joint action: one jointly generated action can occur without a persistent partnership, while an established partnership can persist through periods in which no jointly generated action occurs.

Let π_S, π_R, and π_M denote supporter-, supportee-, and mediator-like role positions, and let $\rightsquigarrow^{\text{role}}_{\chi}$ be a declared role-tracking relation. One possible trajectory is stated in Equation 192.

$$\big(\pi_S, \pi_R, \pi_M\big)_{t_0} \rightsquigarrow^{\text{role}}_{\chi} \mathsf{Part}^{\kappa'}_{ABC,t_1}. \tag{192}$$

Equation 192 represents a historical reorganization from earlier differentiated role positions toward the restricted partnership record. The reverse transition is also possible under crisis, specialization, route collapse, or loss of capacity.

15.10 Three-Level Emergence Profiles

Every emergent structure inherits the three-level discipline of Section 4. Ultimate ontology remains suspended. At relational reality, the model retains the persistent and efficacious structure warranted by Equation 179. At manifestation reality, the structure can appear through routines, public records, meeting procedures, role labels, access permissions, repeated translation, recurring review, joint planning, silence, timing, or spatial organization. A-, B-, and C-relative reconstructions can disagree about whether the structure exists or what it means.

For emergent object E_X, define the profile

$$\Lambda^{\text{em},\kappa}_{ABC,t}(E_X) := \Big(E^{\mathsf{R}}_{X,t}, \mathrm{Man}^{\kappa}_{t}(E^{\mathsf{R}}_{X,t}), \widehat{E}_{X,A,t}, \widehat{E}_{X,B,t}, \widehat{E}_{X,C,t}\Big), \qquad \mathrm{Ont}(E_X) = \mathsf{suspended}. \tag{193}$$

Equation 193 permits relational persistence together with manifestational change and epistemic disagreement. A mediation channel can remain operative after explicit mediation language disappears. A reputational structure can persist through changing public manifestations. A coalition can remain effective while one member denies the coalition label. A partnership can be relationally stable even when outside observers continue to describe the positions through older supporter/supportee categories.

Table 17 summarizes selected emergent structures and their model-relative witnesses.

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Table 17: Selected emergent triadic relational structures. The table lists candidate witnesses and non-equivalences rather than universal substantive definitions.

Structure	Candidate persistence/efficacy witness	Important non-equivalence
Mediated / indirect trust	later reliance changes because a route, witness, translation, or review relation persists	not transitive trust and not trust in every domain
Stable mediation / translation channel	subjects orient later interaction to expected availability of the third-position function	not one repeated service event or permanent identity
Publicness	later action is organized by expected shared audience or record structure	not one public manifestation or common knowledge
Reputation / public recognition	circulating attributions or recognition alter access, trust, scrutiny, standing, or role	not accuracy, approval, or legitimacy
Accountability regime	claim/review/appeal structures condition behavior before disputes occur	not justified authority or fair enforcement
Stable coalition / brokerage structure	recurrent coordination or route dependence shapes later continuations	not hostility, power, or injustice by itself
Shared vocabulary / public norm	recurrent common usage or orientation reduces translation cost or structures later judgment	not identical semantics or consensus
Role ecology	mediator, supporter, reviewer, broker, and related positions mutually condition later expectations	not fixed subject identity
Triadic partnership	all three positions have consequential routes for contribution, correction, and revision	not equality in every dimension or one joint act

15.11 Pairwise Recoverability of Emergent Structures

An emergent triadic object can remain pairwise recoverable. The emergence criterion and the nonrecoverability criterion should therefore be reported separately. Let d_E^κ be a full-equivalence-respecting diagnostic of an emergent structure. Pairwise nonrecoverability is witnessed when

$$X \sim_{2,\kappa} Y \qquad \text{and} \qquad d_E^\kappa(X) \neq d_E^\kappa(Y). \tag{194}$$

A stable mediation channel can fail Equation 194 because its history is reconstructible from A–C and B–C records. A public norm can satisfy it when the operative object includes a full-audience acknowledgment condition absent from every pairwise restriction. A partnership can be recoverable when all three pairwise records retain the necessary contribution and revision routes, or nonrecoverable when a higher-order joint coordination condition is discarded by all three projections.

Figure 14 summarizes the architecture developed in this section.

15.12 Dissolution, Reopening, Retyping

Emergence does not imply permanence. A structure that satisfies Equation 179 on one interval can cease to satisfy persistence, organization, or efficacy on a later interval. A mediation channel can dissolve after direct communication develops. Reputation can decay or fragment across audiences. An accountability regime can lose practical efficacy. A coalition can dissolve, a role ecology can reorganize, and a public norm can be reopened through contestation.

Let E_X be an emergent structure on interval I and let I' be a later retained interval. A candidate dissolution condition is

$$\mathsf{Emerg}_{3,\chi}^{\kappa}(E_X; H_{ABC,I}^{\kappa}) \wedge \neg \mathsf{Emerg}_{3,\chi'}^{\kappa'}(E_X; H_{ABC,I'}^{\kappa'}) \Longrightarrow \mathsf{Diss}_3^{\chi,\chi'}(E_X; I'), \tag{195}$$

provided that χ and χ' support an application-justified comparison. Equation 195 does not require every manifestation to disappear. A public role label can persist after the associated standing loses efficacy; a review ritual can remain after practical appeal becomes impossible; a coalition label can persist after coordination ends.

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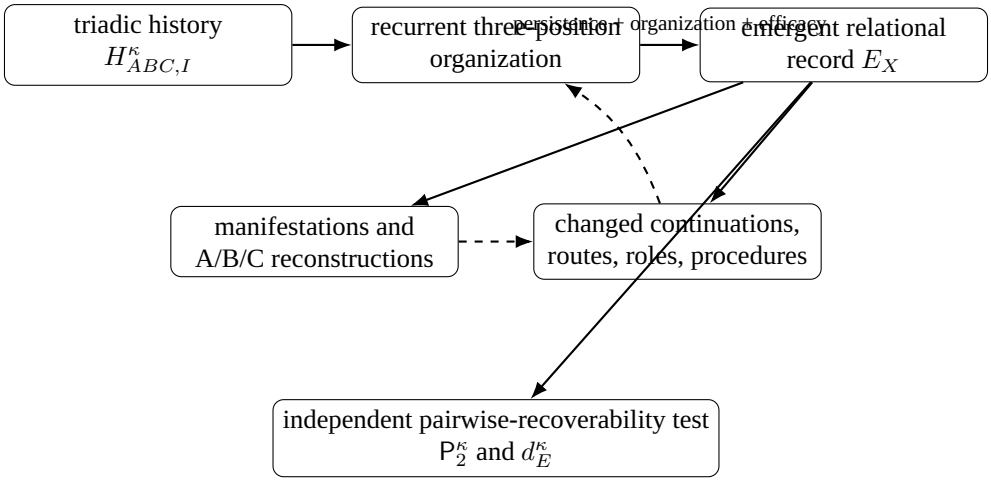

Figure 14: Emergent triadic relational architecture. Historical organization becomes an emergent record only under the declared persistence, organization, and efficacy criteria. Pairwise recoverability is tested independently and does not follow from emergence.

Retyping is broader than dissolution. A broker can become a mediator, a mediation channel can become a routinized accountability mechanism, a support relation can become partnership, or a public reputation can become institutionally encoded standing. The formalism permits such transitions only when an explicit bridge relates the old and new types.

Reopening is especially important for norms, recognition, and accountability. Stabilization can narrow the continuations subjects treat as available. New evidence, a new route, changed capacity, or a revised public record can reopen previously stabilized interpretations and thereby increase generative possibility. Reopening is therefore not automatically structural failure.

15.13 Algebraic Dynamics: Categorical Interface

The emergent structures developed above remain typed historical records. Section 16 will represent their dynamical consequences through local algebraic and categorical constructions. The interface must preserve the distinction between an emergent relation and any one operator, morphism, or algebraic carrier used to represent it.

An emergent mediation channel can constrain which procedure maps are admissible. A public norm can alter selection among available transformations. A coalition can introduce a composite action family. An accountability regime can add review and appeal morphisms. A partnership can change which joint transformations are defined. These consequences can be represented algebraically only after their domains, codomains, composition rules, and locality conditions are stated.

The same emergent object can therefore have several mathematical realizations. One finite model may encode a stable mediation channel as a family of partial maps, another as a span or cospan of procedure objects, and another as a local algebra of admissible transformations. Paper II does not infer that relational reality is fundamentally algebraic or categorical because one representation is useful.

Section 15 has therefore supplied a model-relative criterion for triadic relational emergence and a typed family of candidate structures. Mediated trust, stable third-position channels, publicness, reputation, public recognition, accountability regimes, coalitions, shared vocabulary, public norms, role ecologies, and triadic partnership become emergent only when persistence, organization, and efficacy are historically warranted. Pairwise recoverability remains an independent question, manifestations and subject-relative reconstructions remain distinct from relational structures, and none of the emergent objects is granted normative validity by formal stability alone.

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16 Typed Composition in Local Triadic Dynamics

This section develops a mathematical realization layer for the local triadic structures constructed in Sections 8–15. Its role is to represent supported transformations, pairwise restrictions, reconstruction from pairwise data, and composition across heterogeneous carriers while preserving the distinction between substantive GR objects and their mathematical realizations. The objective is to make four requirements explicit: local support, typed restriction, controlled gluing, and composition across maps whose carriers need not share one algebraic structure. The discussion therefore proceeds from regional algebraic assignments to pairwise algebraic shadows, gluing, dynamical lifts, restriction–dynamics compatibility, categorical composition, mediated diagram shapes, functorial representation, optional operator-algebraic specialization, and algebraic recoverability. The method is representation-conscious throughout: algebraic structure is used where a declared product or linear structure performs analytical work, while categorical language is used where the primary requirement is well-typed composition among heterogeneous objects and maps.

The construction inherits the ontological restraint and first-definition discipline of Paper I. A mediation step, translation, witness route, publicization event, coalition-conditioned action, review, or accountability transformation is identified first as a substantive GR object with a declared source, target, support, historical stage, and resolution. A mathematical carrier is introduced only after those features have been specified. Algebraic or categorical adequacy therefore concerns preservation of selected relational distinctions. It does not establish that volunteer relations possess an ultimately algebraic or categorical ontology.

Only limited mathematical background is required at the general level. An algebra supplies a carrier with declared internal operations, especially a product when composition or combination is analytically meaningful. A category supplies typed objects and morphisms, identity morphisms, and associative composition. A span and a cospan are finite diagrams with a shared source or shared target. Stronger structures, including linearity, involution, C^*-norms, Hilbert-space representations, pullbacks, pushouts, and completely positive maps, enter only as separately stated specializations.

16.1 Regional Algebraic Assignments

Let $\mathfrak{R}^{\kappa}_{ABC,t}$ denote the declared family of admissible support regions in the coupled history-bearing carrier $\mathcal{K}^{\kappa}_{ABC,t}$. Depending on the application, a region can track a subject-supported subconfiguration, a pairwise interface, a mediation support, a public-record support, a coalition-relevant region, or another locally justified part of the retained triadic configuration. Let $\mathbf{Alg}^{\kappa}_{ABC,t}$ denote the declared class of algebraic carriers admitted by the selected realization. A regional algebraic assignment is the map stated in Equation 196.

$$\mathsf{A}^{\kappa}_{ABC,t} : \mathfrak{R}^{\kappa}_{ABC,t} \longrightarrow \mathbf{Alg}^{\kappa}_{ABC,t}, \qquad R \longmapsto \mathcal{A}^{\kappa}_{ABC,t}(R). \tag{196}$$

Equation 196 assigns mathematical structure to a retained region; it does not identify the region with an algebra. The elements and operations of $\mathcal{A}^{\kappa}_{ABC,t}(R)$ require an application-level interpretation. They can encode, for example, retained local observables, symbolic records, constraints, generators, or transformation descriptors when the selected algebra makes those objects compositional.

When $R_1 \subseteq R_2$ and the realization supports an isotony-type inclusion, the model may supply the embedding in Equation 197.

$$\iota^{\kappa}_{R_1R_2,t} : \mathcal{A}^{\kappa}_{ABC,t}(R_1) \hookrightarrow \mathcal{A}^{\kappa}_{ABC,t}(R_2). \tag{197}$$

Equation 197 is optional. A context-sensitive representation can instead require a quotient, a partial comparison map, or another typed interface. The regional assignment in Equation 196 therefore carries no primitive assumption that larger regions contain smaller-region algebras as literal subalgebras.

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For the reconstruction problem, suppose that the whole retained triadic support and the three induced pairwise supports belong to the chosen region system. Define the corresponding shorthand carriers by Equation 198.

$$\begin{aligned} \mathcal{A}^{\text{full},\kappa}_{ABC,t} &:= \mathcal{A}^{\kappa}_{ABC,t}(\mathcal{K}^{\kappa}_{ABC,t}), \\ \mathcal{A}^{\text{pair},\kappa}_{XY|ABC,t} &:= \mathcal{A}^{\kappa}_{ABC,t}(\mathcal{K}^{\kappa}_{XY|ABC,t}), \qquad XY \in \{AB, AC, BC\}. \end{aligned} \tag{198}$$

Equation 198 is notation for one selected algebraic assignment on already defined geometric carriers. It does not imply that the induced pairwise algebra is identical to an algebra independently assigned to a standalone dyadic model.

The full-triad carrier also need not factorize into three pairwise carriers. The default nonfactorization commitment is recorded in Equation 199.

$$\mathcal{A}^{\text{full},\kappa}_{ABC,t} \not\equiv \mathcal{A}^{\text{pair},\kappa}_{AB|ABC,t} \otimes \mathcal{A}^{\text{pair},\kappa}_{AC|ABC,t} \otimes \mathcal{A}^{\text{pair},\kappa}_{BC|ABC,t}. \tag{199}$$

Equation 199 states a modeling restraint rather than an impossibility theorem. Shared histories, overlapping supports, public records, relation-specific constraints, and higher-order compatibility conditions can obstruct a useful tensor factorization. A tensor-product specialization remains available when its independence and coupling assumptions are separately justified.

16.2 Pairwise Algebraic Shadows

The geometric restrictions of Section 5 induce algebraic restrictions only when the selected representation supplies corresponding maps. For each $XY \in \{AB, AC, BC\}$, let the partial algebraic restriction be the map in Equation 200.

$$\rho^{\text{alg},\kappa}_{XY,t} : \mathcal{D}^{\text{alg},\kappa}_{XY,t} \subseteq \mathcal{A}^{\text{full},\kappa}_{ABC,t} \rightharpoonup \mathcal{A}^{\text{pair},\kappa}_{XY|ABC,t}. \tag{200}$$

Equation 200 requires only a declared typed restriction. A stronger specialization may require $\rho^{\text{alg},\kappa}_{XY,t}$ to preserve addition, multiplication, involution, norm, or another selected structure. Those properties are not inherited merely from the word "restriction."

The common domain on which all three pairwise algebraic restrictions are defined is introduced in Equation 201.

$$\mathcal{D}^{\kappa}_{2,\text{alg},t} := \bigcap_{XY \in \{AB,AC,BC\}} \mathcal{D}^{\text{alg},\kappa}_{XY,t}. \tag{201}$$

For $a \in \mathcal{D}^{\kappa}_{2,\text{alg},t}$, the algebraic pairwise shadow is defined in Equation 202.

$$\mathsf{P}^{\kappa}_{2,\text{alg},t}(a) := \Big(\rho^{\text{alg},\kappa}_{AB,t}(a), \rho^{\text{alg},\kappa}_{AC,t}(a), \rho^{\text{alg},\kappa}_{BC,t}(a)\Big). \tag{202}$$

Equation 202 is a realization of the record-level pairwise-shadow architecture developed in Sections 3 and 6. It remains subordinate to that broader architecture. In particular, an algebraic shadow that is sufficient for one diagnostic does not establish pairwise sufficiency for the entire integrated triadic record.

Partial definedness remains substantively meaningful. A public-audience record can be represented on a full-triad carrier while only some of its manifestations admit pairwise restrictions. The formalism therefore preserves undefinedness rather than replacing it with an algebraic zero unless the selected realization explicitly assigns that interpretation.

16.3 Compatibility and Algebraic Gluing

Reconstruction requires a compatibility domain for raw pairwise algebraic descriptors. For each pair XY, let $\approx^{\kappa}_{XY,\text{alg},t}$ be the declared equivalence on the corresponding pairwise algebraic carrier. Let $\mathfrak{C}^{\kappa}_{2,\text{alg},t}$ be

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the selected subset of $\mathcal{A}^{\text{pair},\kappa}_{AB|ABC,t} \times \mathcal{A}^{\text{pair},\kappa}_{AC|ABC,t} \times \mathcal{A}^{\text{pair},\kappa}_{BC|ABC,t}$ whose members satisfy the shared-subject, overlap, and typing checks required to be candidates for one full-triad realization. Membership in this compatibility domain does not guarantee that a full realization exists.

Let $\mathbf{a}_2 = (a_{AB}, a_{AC}, a_{BC}) \in \mathfrak{C}^{\kappa}_{2,\text{alg},t}$. Let $\sim^{\kappa}_{\text{alg},t}$ denote the declared full-equivalence relation on the retained full algebraic comparison domain. The algebraic gluing set is defined in Equation 203.

$$\text{Glue}^{\text{alg},\kappa}_{ABC,t}(\mathbf{a}_2) := \left\{ a \in \mathcal{D}^{\kappa}_{2,\text{alg},t} \;\middle|\; \rho^{\text{alg},\kappa}_{XY,t}(a) \approx^{\kappa}_{XY,\text{alg},t} a_{XY} \text{ for every } XY \in \{AB, AC, BC\} \right\}. \quad (203)$$

Equation 203 poses a gluing problem without assuming a sheaf condition or a unique-gluing theorem. The three reconstruction cases are stated in Equation 204.

$$\begin{aligned} \text{Glue}^{\text{alg},\kappa}_{ABC,t}(\mathbf{a}_2) = \varnothing \qquad & \text{no admitted full realization,} \\ \left|\text{Glue}^{\text{alg},\kappa}_{ABC,t}(\mathbf{a}_2)/{\sim^{\kappa}_{\text{alg},t}}\right| = 1 \qquad & \text{one retained full-equivalence class,} \\ \left|\text{Glue}^{\text{alg},\kappa}_{ABC,t}(\mathbf{a}_2)/{\sim^{\kappa}_{\text{alg},t}}\right| > 1 \qquad & \text{multiple retained full-equivalence classes.} \end{aligned} \quad (204)$$

Equation 204 separates existence from uniqueness. The third case is an algebraic witness of pairwise nonrecoverability relative to the selected restriction maps and equivalences. Its substantive interpretation still requires showing that the distinction between the surviving full-equivalence classes corresponds to a relational feature relevant to the research question.

A useful stronger specialization is available when the four algebraic carriers are vector spaces over a common field and the three restriction maps are linear on a common linear domain. The pairwise-invisible subspace is then defined in Equation 205.

$$\mathcal{N}^{\text{alg},\kappa}_{3|2,t} := \ker \rho^{\text{alg},\kappa}_{AB,t} \cap \ker \rho^{\text{alg},\kappa}_{AC,t} \cap \ker \rho^{\text{alg},\kappa}_{BC,t}. \quad (205)$$

Equation 205 identifies directions erased by all three selected linear restrictions. A nonzero $n \in \mathcal{N}^{\text{alg},\kappa}_{3|2,t}$ does not by itself establish substantive nonrecoverability. A witness additionally requires admitted full elements a and $a + n$ for which the retained full equivalence distinguishes the two. Under those conditions, their pairwise algebraic shadows coincide while their retained full classes differ.

16.4 Algebraic Lifts of Substantive Transformations

Section 13.7 introduced a class $\mathfrak{K}^{\kappa}_{ABC}$ of retained triadic configurations and typed partial action transformations between resolutions. The present section uses the same carrier convention for a generic substantive transformation. Let the transformation be the partial map in Equation 206.

$$\Phi^{\kappa' \leftarrow \kappa}_{t' \leftarrow t} : \mathcal{D}^{\kappa}_{\Phi} \subseteq \mathfrak{K}^{\kappa}_{ABC} \rightharpoonup \mathfrak{K}^{\kappa'}_{ABC}. \quad (206)$$

Equation 206 can represent mediation, translation, publicization, review, coalition-conditioned action, revision, enforcement, or another previously typed procedure element. The time indices identify the retained source and target stages; they do not require the ambient configuration class to be newly defined at every stage.

Let $R \in \mathfrak{R}^{\kappa}_{ABC,t}$ be the algebraically represented support of Φ, and let $R' \in \mathfrak{R}^{\kappa'}_{ABC,t'}$ be its tracked target support. An algebraic lift, when the model supplies one, is the map in Equation 207.

$$\alpha^{\kappa}_{R,t}[\Phi] : \mathcal{D}^{\text{alg}}_{\Phi,R} \subseteq \mathcal{A}^{\kappa}_{ABC,t}(R) \rightharpoonup \mathcal{A}^{\kappa'}_{ABC,t'}(R'). \quad (207)$$

Equation 207 preserves the distinction between the substantive transformation and its mathematical realization. The lift can forget provenance, audience, standing, authorization, or historical context unless those features are represented in its carrier. It need not be injective, surjective, invertible, linear, multiplicative, or norm preserving. Each stronger property requires separate assumptions.

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Table 18: Principal operation families available to the local triadic realization.

Operation family	Local analytical role	Typical retained target
Mediation	Transform interaction conditions or procedure	Procedure state and continuation family
Translation	Transform a manifestation or interpretive carrier	Route-indexed manifestation and reconstruction input
Witness / record	Produce or retain attestation	Public record, later review input, provenance
Publicization	Modify audience and accessibility structure	Publicity, acknowledgment possibilities, reputation input
Brokerage / gatekeeping	Reconfigure route availability or condition passage	Route-access and dependency records
Coalition coordination	Generate or restrict coordinated transformation families	Joint plan, role ecology, continuation family
Review / enforcement	Transform accountability or admissible continuation records	Determination, appeal, correction, access or role state
Revision / reopening	Alter retained judgment, norm, role, or historical interpretation	Cross-level and historical record families

Table 18 summarizes operation families that can admit such lifts in suitable applications.

Two lifts that are extensionally equal on a local algebra can still realize different substantive GR transformations when the representation omits distinctions in provenance, standing, audience, authorization, or coalition history. Equality of mathematical maps therefore establishes equivalence only relative to the information retained by the selected realization.

16.5 Restriction–Dynamics Compatibility

A local consistency question concerns the order of transformation and pairwise restriction. Let $\alpha^{\kappa}_{ABC,t}[\Phi]$ be a full-triad algebraic lift and let $\alpha^{\kappa}_{XY,t}[\Phi_{XY}]$ be a declared pairwise lift on the induced XY carrier. The two composites can be compared only on inputs for which both routes are defined. Their common comparison domain is stated in Equation 208.

$$\begin{aligned}\mathcal{D}_{\mathrm{cov},XY}(\Phi) := \big\{ a \in \mathcal{D}^{\kappa}_{2,\mathrm{alg},t} : a \in \mathrm{Dom}(\alpha^{\kappa}_{ABC,t}[\Phi]),\\ \alpha^{\kappa}_{ABC,t}[\Phi](a) \in \mathrm{Dom}(\rho^{\mathrm{alg},\kappa'}_{XY,t'}),\\ \rho^{\mathrm{alg},\kappa}_{XY,t}(a) \in \mathrm{Dom}(\alpha^{\kappa}_{XY,t}[\Phi_{XY}])\big\}.\end{aligned} \tag{208}$$

For $a \in \mathcal{D}_{\mathrm{cov},XY}(\Phi)$, restriction–dynamics compatibility is the equivalence in Equation 209.

$$\rho^{\mathrm{alg},\kappa'}_{XY,t'}\big(\alpha^{\kappa}_{ABC,t}[\Phi](a)\big) \approx^{\kappa'}_{XY,\mathrm{alg},t'} \alpha^{\kappa}_{XY,t}[\Phi_{XY}]\big(\rho^{\mathrm{alg},\kappa}_{XY,t}(a)\big). \tag{209}$$

Equation 209 states compatibility only relative to the declared target equivalence. Exact equality is a stronger specialization. The corresponding truth-valued predicate is defined in Equation 210.

$$\begin{aligned}\mathsf{Cov}^{\kappa'\leftarrow\kappa}_{XY}(\Phi; a) \quad \Longleftrightarrow \quad & \rho^{\mathrm{alg},\kappa'}_{XY,t'}\big(\alpha^{\kappa}_{ABC,t}[\Phi](a)\big)\\ & \approx^{\kappa'}_{XY,\mathrm{alg},t'} \alpha^{\kappa}_{XY,t}[\Phi_{XY}]\big(\rho^{\mathrm{alg},\kappa}_{XY,t}(a)\big).\end{aligned} \tag{210}$$

A failure of $\mathsf{Cov}^{\kappa'\leftarrow\kappa}_{XY}$ has more than one possible interpretation. It can expose a modeling inconsistency, or it can indicate that the full transformation depends on a third-position, public, coalition, or accountability condition that the pairwise restriction intentionally discards. The algebraic calculation alone does not choose between these interpretations; the substantive model must identify which information differs across the two routes.

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16.6 Categorical Organization of Heterogeneous Carriers

Several triadic transformations act on carriers with different internal structures. Translation can act on manifestation records, review on accountability records, coalition coordination on procedure records, and publicization on audience structures. Their common mathematical requirement is typed composition rather than membership in one common algebra. For a fixed stage and resolution, let $\mathbf{Dyn}^{\kappa}_{ABC,t}$ denote a model-declared category whose objects are selected typed carriers and whose morphisms are selected total or partial transformations between them.

The category declaration carries a closure obligation. For each object X, the retained identity map id_X is a morphism; whenever $f : X \rightharpoonup Y$ and $g : Y \rightharpoonup Z$ are retained morphisms, their ordinary partial-map composite is retained whenever the model admits that procedural composition; and the selected composition is associative on its inherited definedness domains. This requirement is summarized in Equation 211.

$$\mathbf{Dyn}^{\kappa}_{ABC,t} = \big(\mathrm{Obj}(\mathbf{Dyn}^{\kappa}_{ABC,t}), \mathrm{Mor}(\mathbf{Dyn}^{\kappa}_{ABC,t}), \circ, \mathrm{id}\big). \tag{211}$$

Equation 211 is a representational organization. An application can choose a smaller category of local algebras, a category of procedure records, or a larger typed category with declared interfaces among several carrier families. Calling the structure a category commits the model to the identity and associativity requirements just stated; a collection of typed arrows that is not closed under the selected composition should instead be treated as a diagram or graph of partial transformations.

A realized procedure trace is a composable path when each target carrier matches the next source carrier. The path is stated in Equation 212.

$$X_0 \xrightarrow{f_1} X_1 \xrightarrow{f_2} \cdots \xrightarrow{f_n} X_n. \tag{212}$$

The corresponding composite is defined in Equation 213.

$$f_\gamma := f_n \circ \cdots \circ f_1 : \mathrm{Dom}(f_\gamma) \subseteq X_0 \rightharpoonup X_n, \tag{213}$$

where $\mathrm{Dom}(f_\gamma)$ contains precisely the inputs whose successive images remain inside the domain of every later map. Equation 213 permits one compositional notation for heterogeneous procedural steps without treating those steps as operators on a common vector space.

16.7 Mediation Shapes through Spans and Cospans

Third-position functions often generate span- or cospan-shaped diagrams. Let X_A and X_B be A- and B-relative carriers and let X_C be a carrier associated with a C-mediated resource, interpretation, record, or procedure position. A span is the diagram in Equation 214.

$$X_A \xleftarrow{f} X_C \xrightarrow{g} X_B. \tag{214}$$

A cospan is the diagram in Equation 215.

$$X_A \xrightarrow{u} X_C \xleftarrow{v} X_B. \tag{215}$$

Equation 214 can represent a C-relative carrier that supplies two typed outputs, while Equation 215 can represent A- and B-relative inputs entering a shared mediation, review, or record carrier. Arrow direction receives substantive meaning only after the corresponding maps have been interpreted. The formalism therefore avoids treating every span as a pullback or every cospan as a pushout. Those terms are reserved for cases in which the selected category possesses and the application justifies the corresponding universal property.

Figure 15 summarizes the separation among full-triad algebraic content, pairwise restrictions, the gluing problem, and a categorical mediation shape.

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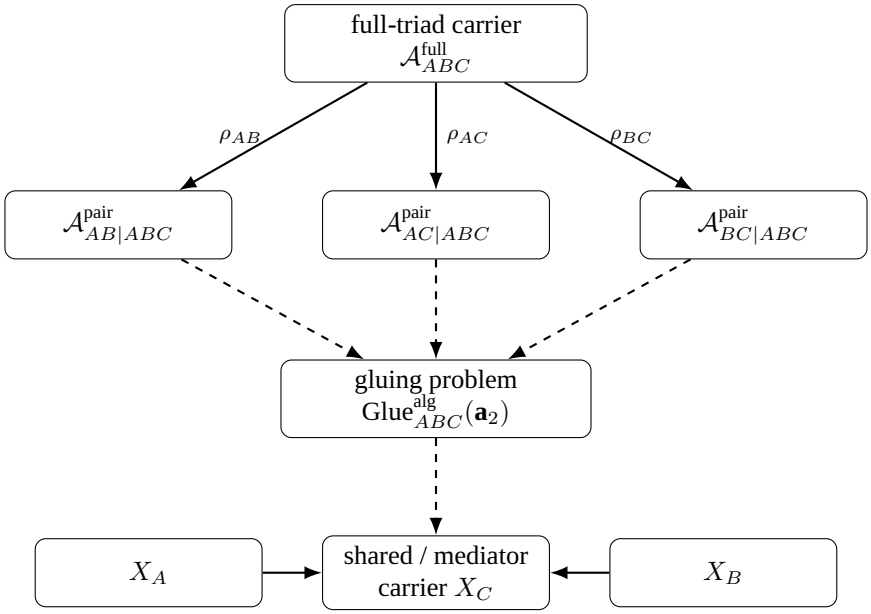

Figure 15: Typed local architecture for the triadic realization. Full-triad algebraic content restricts to three induced pairwise carriers. A compatible pairwise triple defines a gluing problem whose solution set can be empty, unique up to the retained full equivalence, or nonunique. The lower cospan illustrates a distinct categorical mediation shape. Dashed arrows indicate reconstruction or representational interfaces rather than automatic algebra homomorphisms.

16.8 Route Agreement and Contextual Noncommutativity

Categorical diagrams can compare the retained outcomes of different procedural routes. Suppose X is a source carrier, Y is a target carrier, and the two typed routes are those in Equation 216.

$$X \xrightarrow{f} M \xrightarrow{g} Y, \qquad X \xrightarrow{h} N \xrightarrow{k} Y. \tag{216}$$

On their common definedness domain, exact route agreement is stated in Equation 217.

$$g \circ f = k \circ h. \tag{217}$$

When literal equality is stronger than the application requires, let $\approx_Y$ be a declared target equivalence. Equivalence-relative route agreement is stated in Equation 218.

$$g \circ f \approx_Y k \circ h. \tag{218}$$

Failure of Equation 218 can encode translation dependence, procedural order effects, route-specific public status, coalition-conditioned access, or another contextual difference represented in Y. The categorical comparison itself records route dependence; it does not identify the substantive source of that dependence.

A commutator becomes available only in the stronger specialization in which both route transformations are linear endomorphisms of one common vector-space or algebraic carrier. For such endomorphisms α and β, define the commutator in Equation 219.

$$[\beta, \alpha] := \beta \circ \alpha - \alpha \circ \beta. \tag{219}$$

A nonzero value in Equation 219 witnesses order sensitivity in that mathematical specialization. Its interpretation as mediation dependence, reputational sequencing, review-order effects, or another social mechanism requires an independent mapping back to the substantive GR record.

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16.9 Functorial Representation of Relational Dynamics

The distinction between substantive transformations and mathematical realizations can be organized by a functor when both sides possess compatible categorical structure. Let $\mathbf{RelDyn}^{\kappa}_{ABC}$ be a model-declared category whose objects are retained substantive relational record classes and whose morphisms are admissible GR transformations. Let $\mathbf{MathDyn}^{\kappa}_{ABC}$ be a category of selected mathematical carriers and admissible maps. A representation functor is the mapping in Equation 220.

$$\mathcal{F}^{\kappa}_{ABC} : \mathbf{RelDyn}^{\kappa}_{ABC} \longrightarrow \mathbf{MathDyn}^{\kappa}_{ABC}. \tag{220}$$

Equation 220 is warranted only when identities and composition are preserved. For composable substantive transformations f and g, the required preservation conditions are stated in Equation 221.

$$\mathcal{F}^{\kappa}_{ABC}(\mathrm{id}_X) = \mathrm{id}_{\mathcal{F}^{\kappa}_{ABC}(X)}, \qquad \mathcal{F}^{\kappa}_{ABC}(g \circ f) = \mathcal{F}^{\kappa}_{ABC}(g) \circ \mathcal{F}^{\kappa}_{ABC}(f). \tag{221}$$

If Equation 221 fails for the proposed realization, the mapping should be described as a weaker representation or translation rather than as a functor. Even when functoriality holds, $\mathcal{F}^{\kappa}_{ABC}$ need not be faithful, full, or essentially surjective. Distinct relational transformations can collapse to one mathematical map when the representation forgets provenance, audience, standing, or historical context; conversely, the mathematical category can contain objects with no retained substantive interpretation.

This interface provides a practical adequacy criterion. A realization intended to distinguish consent from authorization, witnessing from review standing, or brokerage from gatekeeping must preserve those distinctions at the level required by the research question. Internal mathematical consistency alone does not establish representational adequacy.

16.10 Optional Operator-Algebraic States

A stronger operator-algebraic specialization can be useful when a local carrier is chosen to be a unital C^*-algebra. In that case, a mathematical state is a positive normalized linear functional. The specialization is stated in Equation 222.

$$\omega^{\kappa}_{R,t} : \mathcal{A}^{\kappa}_{ABC,t}(R) \longrightarrow \mathbb{C}, \qquad \omega^{\kappa}_{R,t}(a^*a) \geq 0, \qquad \omega^{\kappa}_{R,t}(1) = 1. \tag{222}$$

Equation 222 defines a mathematical state of the selected C^*-algebra. It is distinct from A's, B's, or C's substantive belief, value judgment, affective state, or relational position. Hilbert-space and GNS-type representations, when useful, occur only after the algebra and mathematical state have been selected.

Open-system specializations can similarly use positive or completely positive maps when their domains, codomains, and interpretation warrant those structures. A mediation, review, or publicization process does not acquire the substantive meaning of a quantum channel merely because a completely positive map can represent selected mathematical effects. Historically changing roles, partial definedness, and changing carrier systems can require more general maps between distinct algebras.

The general triadic formalism imposes no primitive unitary dynamics and no fixed global Hilbert-space carrier. Noninvertibility, partiality, and changes in the family of admissible carriers are compatible with the history-bearing and open character of the modeled relations.

16.11 Algebraic Recoverability

The algebraic realization supplies a finite specialization of the general pairwise-recoverability problem. Let $\mathcal{D}^{\kappa}_{\mathrm{alg},t} \subseteq \mathcal{D}^{\kappa}_{2,\mathrm{alg},t}$ be the retained algebraic comparison domain, and let $\sim^{\kappa}_{\mathrm{alg},t}$ be its declared full equivalence. Pairwise algebraic equivalence is induced by the three component restrictions according to Equation 223.

$$a \sim^{\kappa}_{2,\mathrm{alg},t} b \quad \Longleftrightarrow \quad \rho^{\mathrm{alg},\kappa}_{XY,t}(a) \approx^{\kappa}_{XY,\mathrm{alg},t} \rho^{\mathrm{alg},\kappa}_{XY,t}(b) \text{ for every } XY \in \{AB, AC, BC\}. \tag{223}$$

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Table 19: Typed algebraic and categorical structures in the local triadic realization.

Structure	Formal role	Main restraint	Diagnostic use
Regional algebraic assignment	Represent selected local content	Region and algebra remain distinct objects	Support-sensitive realization
Pairwise algebraic restriction	Realize pairwise shadows	Restriction can be partial and structure-specific	Recoverability and context loss
Compatibility domain / gluing set	Test full realization of pairwise data	Compatibility does not guarantee existence or uniqueness	Empty, unique-class, or nonunique reconstruction
Algebraic lift	Realize a substantive GR transformation	Map properties require separate assumptions	Local dynamical effect
Restriction–dynamics compatibility	Compare transformation and restriction order	Failure requires substantive interpretation	Context-sensitive dynamics
Category of dynamics	Organize heterogeneous typed maps	Category axioms and closure must be satisfied	Composability and path structure
Span / cospan	Represent mediated diagram shape	Universal properties require separate proof	Mediation and shared-carrier organization
Representation functor	Preserve selected compositional structure	Functoriality must be verified	Representation adequacy and information loss

The retained algebraic representation is pairwise sufficient on $\mathcal{D}^{\kappa}_{\mathrm{alg},t}$ when the implication in Equation 224 holds.

$$a \sim^{\kappa}_{2,\mathrm{alg},t} b \quad \Longrightarrow \quad a \sim^{\kappa}_{\mathrm{alg},t} b \qquad \text{for all } a, b \in \mathcal{D}^{\kappa}_{\mathrm{alg},t}. \tag{224}$$

A direct algebraic nonrecoverability witness is therefore the pair described in Equation 225.

$$a \sim^{\kappa}_{2,\mathrm{alg},t} b \qquad \text{and} \qquad a \not\sim^{\kappa}_{\mathrm{alg},t} b. \tag{225}$$

Equation 225 is the algebraic counterpart of the equivalence-gap criterion in Section 6. Nonunique gluing under Equation 203 can supply such a witness. Under the linear specialization, a pairwise-invisible direction from Equation 205 can also supply a witness when two admitted full elements separated by that direction remain distinct under the full equivalence.

A diagnostic can provide another witness when it respects full equivalence. Let $d^{\kappa}_{\mathrm{alg},t} : \mathcal{D}^{\kappa}_{\mathrm{alg},t} \rightharpoonup \mathcal{Z}_{\mathrm{alg}}$ respect full equivalence, so that $a \sim^{\kappa}_{\mathrm{alg},t} b$ implies equality of the diagnostic values wherever both are defined. If $a \sim^{\kappa}_{2,\mathrm{alg},t} b$ while $d^{\kappa}_{\mathrm{alg},t}(a) \neq d^{\kappa}_{\mathrm{alg},t}(b)$, then the selected diagnostic cannot factor through the pairwise shadow on that comparison pair. The diagnostic conclusion remains representation-relative and gains substantive significance only through its interpretation in the GR record.

Table 19 summarizes the principal structures developed in this section.

16.12 Historical Interface

The structures developed above are indexed by a retained stage and resolution. Historical development can change the region system, algebraic assignment, restriction domains, pairwise equivalences, gluing possibilities, admissible morphisms, and even the object and morphism families of $\mathbf{Dyn}^{\kappa}_{ABC,t}$. Consequently, ordered composition inside one fixed categorical realization and historical change in the realization itself are different formal problems.

A coalition can create a transformation that was previously unavailable. Withdrawal can remove a route. A shared vocabulary can replace a mediated communication path with a direct map. An accountability regime can introduce review and appeal morphisms. A public record can change which relational distinctions remain action-guiding and therefore which mathematical carriers are adequate. The historical object is thus richer than a product of fixed local operators.

Section 16 has established a typed local realization of the triadic formalism. Full-triad algebraic carriers are not presumed to factorize into dyads; pairwise descriptors enter an explicit compatibility and

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gluing problem; substantive transformations remain distinct from their algebraic lifts; heterogeneous procedural maps compose only under declared typing conditions; and pairwise recoverability is restated at the algebraic layer without promoting that layer to ontology. Section 17 develops the corresponding historical problem, including changing records, changing continuation families, changes in admissible categorical structure, and the relation between historical revision and pairwise restriction.

17 Typed Historical Composition in Triadic Relations

This section develops the historical-composition layer of the triadic volunteer formalism. Its role is to connect stage-indexed triadic records into histories in which subjects, routes, procedures, public records, standing, coalitions, asymmetries, emergent structures, continuation families, and mathematical carriers can change while remaining historically trackable. The objective is to distinguish several sources of temporal difference that a single state-update notation would conflate: relational transition, representational revision, sequence-sensitive composition, delayed manifestation, retrospective reconstruction, pairwise information loss, branch-dependent development, and carrier change. The discussion proceeds from an enriched typed stage ledger to admissible revision histories, continuation-family change, temporal lag, time-indexed reconstruction, historical pairwise shadows, path dependence, feedback, categorical carrier revision, and persistence. The method is relational and representation-conscious. Paper II assumes neither one fixed global state space nor one universal dynamical law across the history.

The historical layer extends the revision relation $\mathsf{TriRev}_{t' \leftarrow t}^{\kappa' \leftarrow \kappa}$ introduced in Section 3.8. Sections 8–16 subsequently introduced route-sensitive epistemic records, public and accountability structures, positional and coalition records, action provenance, typed asymmetries, emergent structures, and local categorical dynamics. Historical composition must therefore retain those later structures without retroactively changing the meaning of the earlier integrated record $D_{ABC,t}^{\kappa}$.

17.1 Integrated Historical Stage Ledger

Let $\mathcal{J}_{ABC,t}^{\kappa}$ be the finite or otherwise explicitly retained index family of additional record types articulated at stage t after the base record $D_{ABC,t}^{\kappa}$ has been formed. For each $\tau \in \mathcal{J}_{ABC,t}^{\kappa}$, let $\mathcal{X}_{\tau,t}^{\kappa}$ be the declared carrier for records of type τ, and let $X_{\tau,t}^{\kappa} \in \mathcal{X}_{\tau,t}^{\kappa}$ be the retained record when that component is articulated. The enriched stage ledger is defined in Equation 226.

$$\mathbb{D}_{ABC,t}^{\kappa} := \left(D_{ABC,t}^{\kappa}; (X_{\tau,t}^{\kappa})_{\tau \in \mathcal{J}_{ABC,t}^{\kappa}} \right). \tag{226}$$

Equation 226 is a typed record extension, not a Cartesian state vector. The additional family can retain, for example, route records from Section 9, publicness or reputation records from Section 10, accountability structures from Section 11, brokerage or coalition records from Section 12, provenance structures from Section 13, emergent relational objects from Section 15, or selected objects and morphisms from $\mathbf{Dyn}_{ABC,t}^{\kappa}$. Each retained component keeps the type, provenance, definedness conditions, and equivalence relations established in its source section.

Let $\widetilde{\mathfrak{D}}_{ABC,t}^{\kappa}$ denote the class of admissible enriched ledgers at stage t. Its construction is stated in Equation 227.

$$\widetilde{\mathfrak{D}}_{ABC,t}^{\kappa} := \left\{ \left(D; (X_\tau)_{\tau \in \mathcal{J}_{ABC,t}^{\kappa}} \right) \;\middle|\; \begin{array}{l} D \in \mathfrak{D}_{ABC,t}^{\kappa}, \quad X_\tau \in \mathcal{X}_{\tau,t}^{\kappa}, \\ \text{declared cross-component typing conditions hold} \end{array} \right\}. \tag{227}$$

Equation 227 makes admissibility explicit without imposing a common metric, topology, norm, probability law, or arithmetic across the heterogeneous entries. The articulation family $\mathcal{J}_{ABC,t}^{\kappa}$ can change historically. A public record can exist before a reputation object is retained; a coalition can exist before a shared norm stabilizes; an accountability claim can appear before review standing is settled. Greater articulation therefore records a finer or differently organized representation, not automatically a more accurate or normatively preferable one.

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17.2 Typed Sources of Historical Change

Historical comparison requires a distinction between change in modeled relational reality and change in the record through which that reality is represented. Section 4 defined $\mathcal{R}^{\kappa}_{ABC,t}$ as the relational structures retained at stage t. A substantive relational-transition relation is defined in Equation 228.

$$\mathsf{RelChg}^{\kappa' \leftarrow \kappa}_{t' \leftarrow t} \subseteq \mathcal{R}^{\kappa}_{ABC,t} \times \mathcal{R}^{\kappa'}_{ABC,t'}. \tag{228}$$

Membership $(r_t, r_{t'}) \in \mathsf{RelChg}^{\kappa' \leftarrow \kappa}_{t' \leftarrow t}$ means that the selected model admits $r_{t'}$ as a historically continuous later relational configuration of r_t under its declared tracking rules. Equation 228 does not claim that the whole past has changed, and it does not identify relational change with a subject's later reinterpretation.

Historical revision of the enriched representation is separately typed by Equation 229.

$$\mathsf{RecRev}^{\kappa' \leftarrow \kappa}_{t' \leftarrow t} \subseteq \widetilde{\mathfrak{D}}^{\kappa}_{ABC,t} \times \widetilde{\mathfrak{D}}^{\kappa'}_{ABC,t'}. \tag{229}$$

Equation 229 refines the coarser relation TriRev from Equation 18. A selected realization can embed the base record into the enriched ledger and require every admitted RecRev pair to project to an admitted TriRev pair on the components retained by $D^{\kappa}_{ABC,t}$. This projection requirement preserves continuity with the earlier formalism while allowing the later record types to participate in revision.

The two relations in Equations 228 and 229 can vary independently within the limits of the selected interpretation. Newly recovered testimony can revise the modeled provenance of an earlier action without changing the historical occurrence of that action. A later public apology can change a relational trajectory while leaving the factual content of an earlier record intact. A new translation can change access to an earlier refusal while the refusal itself remains historically fixed. The formalism therefore treats revision of representation, revision of interpretation, and modeled relational transition as distinct analytical possibilities.

17.3 Compositional Histories of Revision

Historical composition is defined over typed stage-resolution pairs so that changing analytical resolution is not suppressed. Let a retained history index be the sequence in Equation 230.

$$\gamma := \big((t_0, \kappa_0), (t_1, \kappa_1), \ldots, (t_n, \kappa_n)\big), \qquad t_0 \preceq t_1 \preceq \cdots \preceq t_n. \tag{230}$$

Suppose every adjacent revision relation in Equation 230 is declared and compositionally compatible. The composite revision relation associated with γ is defined in Equation 231.

$$\mathsf{RecRev}_{\gamma} := \mathsf{RecRev}^{\kappa_n \leftarrow \kappa_{n-1}}_{t_n \leftarrow t_{n-1}} \circ \cdots \circ \mathsf{RecRev}^{\kappa_1 \leftarrow \kappa_0}_{t_1 \leftarrow t_0} \subseteq \widetilde{\mathfrak{D}}^{\kappa_0}_{ABC,t_0} \times \widetilde{\mathfrak{D}}^{\kappa_n}_{ABC,t_n}. \tag{231}$$

Equation 231 uses ordinary relation composition. No revision step is assumed functional or invertible. Several later ledgers can remain admissible from one earlier ledger, and one later ledger can admit several historically defensible provenance paths.

For an initial ledger $D_0 \in \widetilde{\mathfrak{D}}^{\kappa_0}_{ABC,t_0}$, define its endpoint family along γ by Equation 232.

$$\mathrm{End}_{\gamma}(D_0) := \left\{ D_n \in \widetilde{\mathfrak{D}}^{\kappa_n}_{ABC,t_n} \;\middle|\; (D_0, D_n) \in \mathsf{RecRev}_{\gamma} \right\}. \tag{232}$$

Let $\sim^{\mathrm{hist}}_{\chi}$ be a declared equivalence relation on comparable terminal ledgers, and let q^{hist}_{χ} denote the corresponding quotient map. For any subset S of its domain, write $q^{\mathrm{hist}}_{\chi}[S] := \{q^{\mathrm{hist}}_{\chi}(x) : x \in S\}$ for the image set of retained equivalence classes. Two admissible histories γ_1 and γ_2 from the same initial ledger exhibit retained sequence sensitivity when Equation 233 holds.

$$q^{\mathrm{hist}}_{\chi}\big[\mathrm{End}_{\gamma_1}(D_0)\big] \neq q^{\mathrm{hist}}_{\chi}\big[\mathrm{End}_{\gamma_2}(D_0)\big]. \tag{233}$$

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Equation 233 compares retained equivalence classes of reachable outcomes rather than treating the revision relations themselves as points in an unspecified metric space. Translation before authorization can therefore differ historically from translation introduced only after disputed execution; review before publicization can differ from review performed after reputational polarization; acknowledgment before coalition stabilization can differ from the same acknowledgment after a coalition has already conditioned participation. A commutator test remains a special case only when the relevant transformations are linear endomorphisms of one common carrier.

17.4 Continuation-Family Revision

Historical generativity includes changes in what remains possible after an event. Let $\mathfrak{N}^{\kappa}_{ABC,t}$ be the declared class of typed next-step descriptors at stage t, including only the procedure elements, actions, route changes, role transitions, review steps, or other continuations retained by the application. The continuation assignment is the partial map in Equation 234.

$$\mathrm{Cont}^{\kappa}_{ABC,t} : \widetilde{\mathfrak{D}}^{\kappa}_{ABC,t} \rightharpoonup \mathcal{P}\big(\mathfrak{N}^{\kappa}_{ABC,t}\big). \tag{234}$$

Equation 234 does not treat the elements of $\mathfrak{N}^{\kappa}_{ABC,t}$ as numerically commensurable. It collects the currently admitted next moves while preserving their types.

Because the next-step carrier can itself change through history, comparison across stages requires a declared cross-stage equivalence $\approx^{t,t'}_{\mathrm{cont},\chi}$ on the continuation families being compared. For an admitted revision pair $(D_t, D_{t'}) \in \mathsf{RecRev}^{\kappa' \leftarrow \kappa}_{t' \leftarrow t}$, a history-sensitive continuation change is stated in Equation 235.

$$\mathrm{Cont}^{\kappa}_{ABC,t}(D_t) \not\approx^{t,t'}_{\mathrm{cont},\chi} \mathrm{Cont}^{\kappa'}_{ABC,t'}(D_{t'}). \tag{235}$$

Equation 235 is a structural comparison, not a welfare ranking. The later family can become wider, narrower, retyped, or incomparable. A mediation episode can establish direct A–B communication and reduce later dependence on C. A public record can make a review path available. A coalition can close one route while enabling another. An accountability repair can restore practical usability to a previously ineffective refusal procedure. The historical object therefore includes both realized events and the changing continuation structure generated around them.

17.5 Typed Temporal Lags in Historical Effects

Relational consequence and subject-accessible manifestation can occur at different historical stages. Let $\mathfrak{E}^{\mathrm{evt},\kappa}_{ABC,t}$ denote the declared class of retained event objects at stage t. A delayed-effect relation from an earlier event to a later relational object is defined in Equation 236.

$$\mathsf{DelayEff}^{\kappa' \leftarrow \kappa}_{t' \leftarrow t} \subseteq \mathfrak{E}^{\mathrm{evt},\kappa}_{ABC,t} \times \mathcal{R}^{\kappa'}_{ABC,t'}. \tag{236}$$

Membership $(e_t, r_{t'}) \in \mathsf{DelayEff}^{\kappa' \leftarrow \kappa}_{t' \leftarrow t}$ means that the retained historical model treats the later relational object $r_{t'}$ as conditioned by event e_t under the declared evidential or mechanistic interpretation. Equation 236 is application-specific and does not by itself establish a universal causal law.

Subject-relative manifestation is tested against the accessibility sets introduced in Section 4. For subject i, an effect at t' can lack an accessible manifestation at that stage even though the effect is already retained relationally. This possibility is stated in Equation 237.

$$\begin{aligned} &(e_t, r_{t'}) \in \mathsf{DelayEff}^{\kappa' \leftarrow \kappa}_{t' \leftarrow t} \\ &\qquad \wedge \quad \Big\{ m \in \mathcal{M}^{\mathrm{acc},ABC,\kappa'}_{i,t'} \;\Big|\; (r_{t'}, m) \in \mu^{\kappa'}_{ABC,t'} \Big\} = \varnothing. \end{aligned} \tag{237}$$

Equation 237 permits a reputational structure to alter C's later willingness to mediate before A has access to a manifestation of that change, or a coalition to condition B's available routes before B can

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reconstruct the coalition as an organized relational object. The converse temporal pattern is also possible: a manifestation can appear before the evidence is sufficient to classify the corresponding relational structure as persistent or efficacious.

17.6 Time-Indexed Epistemic Reconstruction

A history-bearing formalism should preserve the time at which a representation became available. Let Q identify a declared object type, let $\mathcal{Q}_{t'}^{\kappa'}$ be its candidate carrier at a later stage, and let $\mathsf{Pred}_{i,t}^{\kappa}(Q;t') \subseteq \mathcal{Q}_{t'}^{\kappa'}$ be the prediction family retained for subject i at stage t. Let $\widehat{Q}_{i,t''}^{\mathrm{retro}}$ be an admissible retrospective reconstruction formed at $t'' \succeq t'$ after additional manifestations or records have become accessible. The prediction-comparison record is defined in Equation 238.

$$\mathfrak{P}_{i;t,t',t''}^{\kappa,\kappa'}(Q) := \Big(\mathsf{Pred}_{i,t}^{\kappa}(Q;t'), Q_{t'}, \widehat{Q}_{i,t''}^{\mathrm{retro}}\Big). \tag{238}$$

Equation 238 keeps prospective expectation, later retained object, and retrospective reconstruction in different fields. A can later attribute an action to coalition influence after a public record becomes available while A's earlier prediction genuinely lacked that structure. B can retrospectively classify C's earlier mediation as gatekeeping after route dependence becomes visible. Later evidence can justify revision without being projected backward as information that the subject possessed earlier.

Retrospective reconstruction can remain plural even after subjects receive a common later record. Different route histories, standing, trust, interpretive competence, and prior expectations can support different reconstructions for A, B, and C. Historical revision therefore preserves both the time of manifestation and the perspective from which a reconstruction is formed.

17.7 Historical Compatibility of Pairwise Shadows

The historical relation between full-triad revision and pairwise restriction is one of the principal new questions of Paper II. The base shadow $\mathsf{P}_{2,t}^{\kappa}$ from Section 3 acts on $D_{ABC,t}^{\kappa}$. Historical comparison of the enriched ledger requires an explicit extension rather than silently applying the old map to a new domain. Let the extended pairwise shadow be the partial map in Equation 239.

$$\widetilde{\mathsf{P}}_{2,t}^{\kappa} : \widetilde{\mathfrak{D}}_{ABC,t}^{\kappa} \rightharpoonup \widetilde{\mathfrak{C}}_{2,t}^{\kappa}, \tag{239}$$

where $\widetilde{\mathfrak{C}}_{2,t}^{\kappa}$ is the declared compatibility class for the pairwise restrictions of all retained enriched components for which such restrictions are defined. Equation 239 is required to agree with $\mathsf{P}_{2,t}^{\kappa}$ on the base components inherited from $D_{ABC,t}^{\kappa}$.

Let the induced pairwise historical revision relation be defined by Equation 240.

$$\widetilde{\mathsf{Rev}}_{2,t'\leftarrow t}^{\kappa'\leftarrow\kappa} \subseteq \widetilde{\mathfrak{C}}_{2,t}^{\kappa} \times \widetilde{\mathfrak{C}}_{2,t'}^{\kappa'}. \tag{240}$$

Two typed routes now connect a full enriched ledger at t to a pairwise ledger at t'. They are displayed in Equation 241.

$$\widetilde{\mathsf{P}}_{2,t'}^{\kappa'} \circ \mathsf{RecRev}_{t'\leftarrow t}^{\kappa'\leftarrow\kappa} \qquad \text{and} \qquad \widetilde{\mathsf{Rev}}_{2,t'\leftarrow t}^{\kappa'\leftarrow\kappa} \circ \widetilde{\mathsf{P}}_{2,t}^{\kappa}. \tag{241}$$

To compare these relation-valued routes, let $\approx_{2,\chi}^{t'}$ be the declared equivalence on target pairwise ledgers. For a relation $R \subseteq X \times \widetilde{\mathfrak{C}}_{2,t'}^{\kappa'}$, define its target family at $x \in X$ by $R[x] := \{y : (x,y) \in R\}$. The two routes are historically compatible when their target families determine the same $\approx_{2,\chi}^{t'}$-classes for every source ledger in their common domain. Historical noncommutativity is therefore witnessed by a source ledger D_t satisfying Equation 242.

$$q_{2,\chi}^{t'}\Big[\big(\widetilde{\mathsf{P}}_{2,t'}^{\kappa'} \circ \mathsf{RecRev}_{t'\leftarrow t}^{\kappa'\leftarrow\kappa}\big)[D_t]\Big] \neq q_{2,\chi}^{t'}\Big[\big(\widetilde{\mathsf{Rev}}_{2,t'\leftarrow t}^{\kappa'\leftarrow\kappa} \circ \widetilde{\mathsf{P}}_{2,t}^{\kappa}\big)[D_t]\Big], \tag{242}$$

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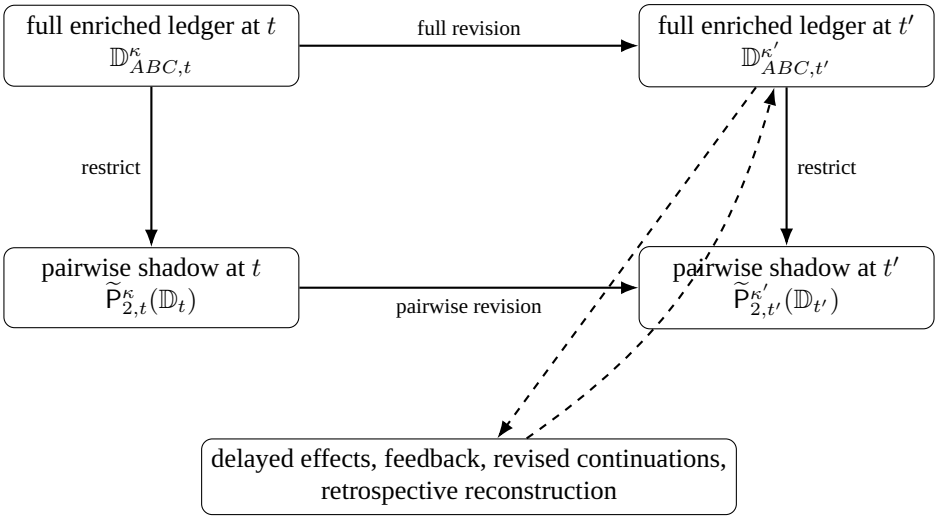

Figure 16: Typed historical composition in the triadic formalism. Full-ledger revision and induced pairwise revision can yield different pairwise outcome classes when early restriction removes historically generative structure. Delayed effects, feedback, changing continuation families, and retrospective reconstruction can condition later records without implying that the earlier event itself is rewritten.

where $q^{t'}_{2,\chi}$ is the quotient map induced by $\approx^{t'}_{2,\chi}$, and the bracket notation denotes the image of a target family under that quotient map. Equation 242 states that restricting first and revising later can retain different pairwise outcome classes from revising the full configuration before restriction. The discrepancy can arise when a triadic public, mediation, coalition, provenance, or accountability structure conditions later A–B development even though the earlier pairwise shadow discarded that structure. The equation does not by itself prove substantive triadic irreducibility; mis-specified restrictions or pairwise revision rules remain alternative explanations that must be audited.

Figure 16 summarizes the two historical routes and the additional lag and feedback structures that can condition later records.

17.8 Branch-Dependent Historical Differentiation

Path dependence in the present framework means that admissible intermediate organization can affect the retained classes of later outcomes. The criterion should accommodate relation-valued revision rather than presuppose one terminal state for each path. Let γ_1 and γ_2 be histories beginning from the same admitted ledger D_0 and terminating in comparable stage-resolution classes. Path dependence relative to χ is witnessed when Equation 243 holds.

$$q^{\text{hist}}_{\chi}\big[\text{End}_{\gamma_1}(D_0)\big] \neq q^{\text{hist}}_{\chi}\big[\text{End}_{\gamma_2}(D_0)\big]. \tag{243}$$

Equation 243 can be witnessed by route dependence, reputation, coalition structure, accountability standing, action provenance, asymmetric continuation families, or emergent partnership even when a coarser material description of the endpoint is equivalent. The same resource transfer can therefore belong to different historical classes when one route preserves practical refusal, another route creates reputational dependency, or another stabilizes direct communication that removes a mediator's earlier necessity.

Branch selection is one source of such differentiation. Several continuations can be admissible at a stage, and a locally modest event can select a branch that later reorganizes the route system. A translator can help establish direct A–B communication and thereby reduce later translation dependence. A public accusation can condition later reputation even after a factual correction. A coalition formed for one episode can later become an accountability bloc. Paper II records these historical possibilities without

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assuming attractors, hysteresis, bifurcations, or criticality; those stronger dynamical concepts require additional structure supplied by a specific application or later paper.

17.9 Self-Conditioning Historical Feedback

Triadic history becomes self-conditioning when an earlier retained component contributes to a later component that in turn alters still later generative conditions. Let τ be a declared component type and let $\mathcal{Q}_t^{\tau}$, $\mathcal{Q}_{t'}^{\tau'}$, and $\mathcal{Q}_{t''}^{\tau''}$ be the corresponding stage-specific carriers. A generic typed feedback relation is defined in Equation 244.

$$\mathsf{Feed}_{t'' \leftarrow t' \leftarrow t}^{\tau'' \leftarrow \tau' \leftarrow \tau} \subseteq \mathcal{Q}_t^{\tau} \times \mathcal{Q}_{t'}^{\tau'} \times \mathcal{Q}_{t''}^{\tau''}, \qquad t \preceq t' \preceq t''. \tag{244}$$

Equation 244 is deliberately relation-valued. It can represent a witness record that contributes to later reputation, which then changes the credibility granted to later testimony; a mediation success that creates direct communication, which later dissolves brokerage dependence; or repeated accountability review that changes how subsequent actions are documented before disputes occur. The three component types may differ because social feedback often changes the kind of object through which later interaction is conditioned.

Feedback can also cross the relational and manifestation levels. A public label can contribute to stabilization of a relational role; the stabilized role can then alter the manifestations expected in later encounters. A repeated public norm can condition which proposals are treated as routine or contestable. Such cross-level efficacy remains compatible with ontological suspension because the formalism models retained relational and manifestation dynamics without inferring an ultimate metaphysical substrate.

17.10 Carrier Revision across Categorical Realizations

Section 16 introduced $\mathbf{Dyn}_{ABC,t}^{\kappa}$ as a model-declared category only when the selected objects and morphisms satisfy the required identity, closure, and associativity conditions. Historical development can change that category. A new appeal route can add a morphism, direct communication can replace a mediation cospan, a changed role ontology can require different objects, or a coalition can make a previously admitted partial map unavailable.

A minimal cross-stage object bridge is the typed relation in Equation 245.

$$\mathsf{Br}_{t' \leftarrow t}^{\kappa' \leftarrow \kappa} \subseteq \mathrm{Obj}\big(\mathbf{Dyn}_{ABC,t}^{\kappa}\big) \times \mathrm{Obj}\big(\mathbf{Dyn}_{ABC,t'}^{\kappa'}\big). \tag{245}$$

Equation 245 tracks which earlier mathematical carriers are represented as continuing, splitting, merging, or otherwise corresponding to later carriers. The bridge need not be functional, invertible, or structure preserving.

When the selected historical realization supplies a genuine functor that maps both objects and morphisms while preserving identities and composition, the stronger carrier map in Equation 246 can be used.

$$\mathcal{H}_{t' \leftarrow t}^{\kappa' \leftarrow \kappa} : \mathbf{Dyn}_{ABC,t}^{\kappa} \longrightarrow \mathbf{Dyn}_{ABC,t'}^{\kappa'}. \tag{246}$$

Equation 246 is optional and carries the ordinary functorial obligations stated in Section 16.9. When morphisms appear, disappear, split, or become incomparable in a way that no selected functor preserves, the weaker bridge in Equation 245 should be retained. This distinction prevents substantive historical change from being hidden inside one fixed mathematical universe.

17.11 Tracking Continuity under Divergent Reconstruction

Historical revisability permits persistence without literal identity and continuity without epistemic convergence. Let $\mathcal{X}_t^{\tau}$ and $\mathcal{X}_{t'}^{\tau'}$ be declared carriers for a tracked relational object type, and let $\mathsf{Trk}_{t' \leftarrow t}^{\chi}$ be the application-specific tracking relation. The shorthand used in this section is defined by Equation 247.

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Table 20: Selected component families of triadic historical revision. The table records typed historical change rather than a common numerical dynamics.

Retained component	Historical revision	Illustrative triadic consequence
Subject/carrier structure	boundary, overlap, role, support region, or continuation change	one subject becomes mediator, broker, reviewer, or partner
Cross-level mapping	manifestation, audience, reconstruction, or re-presentation rule changes	the same event becomes public, legible, or differently interpretable
Epistemic routes / ToM	route access, translation, credibility, nesting, or model revision	direct access replaces mediation or route lock-in develops
Publicity / reputation	audience, record status, circulation, correction, or recognition changes	later trust, invitation, standing, or scrutiny becomes conditioned
Accountability / authorization	claim, standing, review, appeal, scope, or enforcement changes	previously unavailable contestation or correction becomes usable
Brokerage / coalition	route dependence, gatekeeping, coordination, membership, or exclusion changes	continuations concentrate, diversify, close, or reopen
Provenance / procedure	action segmentation, generation, attribution, or procedural order changes	later responsibility or jointness is reconstructed differently
Asymmetry / emergence	dependency, third-position modulation, persistence, or retyping changes	one asymmetry attenuates while another becomes systemically important
Algebraic/categorical carrier	local algebra, restriction, gluing, object, or morphism family changes	the selected mathematical realization itself changes across stages

$$x_t \rightsquigarrow_{\chi}^{\mathrm{trk}} x_{t'} \quad \Longleftrightarrow \quad (x_t, x_{t'}) \in \mathsf{Trk}^{\chi}_{t' \leftarrow t} \subseteq \mathcal{X}_t^{\tau} \times \mathcal{X}_{t'}^{\tau'}. \tag{247}$$

Equation 247 can track continuity through changes in manifestation, role label, resolution, local algebra, or pairwise shadow when the declared criterion regards those changes as compatible with one continuing relational object. The tracking relation can also fail for superficially similar manifestations when their provenance or relational histories diverge.

Let $\widehat{x}_{i,t'}$ denote subject i's retained reconstruction of the tracked object at stage t', and let $\sim_{\chi}^{\mathrm{int}}$ be the declared interpretive-equivalence relation. Persistent relational continuity can coexist with divergent reconstructions according to the pattern in Equation 248.

$$\begin{aligned} x_t \rightsquigarrow_{\chi}^{\mathrm{trk}} x_{t'} \quad &\wedge \quad \widehat{x}_{A,t'} \not\sim_{\chi}^{\mathrm{int}} \widehat{x}_{B,t'} \\ &\wedge \quad \widehat{x}_{A,t'} \not\sim_{\chi}^{\mathrm{int}} \widehat{x}_{C,t'}. \end{aligned} \tag{248}$$

Equation 248 gives one sufficient pattern of continuing relation with subject-relative disagreement; an application can add the B–C comparison when its diagnostic requires pairwise divergence among all three reconstructions. A public correction can therefore stabilize one procedural status while leaving reputation contested, and a mediation channel can persist relationally while different subjects disagree about whether its continued role constitutes support, dependency, or gatekeeping.

Historical revision can also reopen previously stabilized structures. A coalition can dissolve, public recognition can fragment across audiences, standing can be revoked, a shared vocabulary can lose coordination capacity, or a mediator can become obsolete after direct communication stabilizes. Reopening does not erase the earlier period of emergence. It is a later historical property that must be represented by the tracking and revision architecture rather than by retrospectively denying that the earlier structure existed.

17.12 Historical Revision Inventory

Table 20 summarizes the principal component families whose historical change can be represented by the architecture developed in this section.

The inventory in Table 20 shows why historical composition is broader than numerical state updating. A triadic history can generate new object types, new routes, new review standing, new pairwise

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restrictions, or a new categorical carrier. Historical revision can therefore change both retained values and the grammar of distinctions that the representation can articulate.

17.13 Geometric Realization Interface

The historical architecture developed in this section remains representation-general. Section 18 supplies one labelled 2-complex realization, and that realization must preserve the distinctions established here: base stage records and enriched historical ledgers, relational transition and representational revision, full-ledger revision and pairwise restriction, realized events and continuation families, delayed relational effects and accessible manifestations, and local transformation and carrier change.

A labelled 2-complex can represent selected historical changes through relabeling on a fixed cellular carrier, extension by new cells, reconfiguration of incidence relations, or topology-sensitive modification. The substantive model must determine which regime is warranted. Social vocabulary such as "network," "triangle," "path," or "history" does not by itself justify a particular cellular operation.

Section 17 has therefore supplied a typed architecture for composing triadic histories. Revision is relation-valued and can change analytical resolution; sequence sensitivity is evaluated through reachable equivalence classes; continuation families can change; relational effects can precede their accessible manifestations; retrospective reconstruction remains time-indexed; pairwise restriction can lose historically generative information; branch-dependent histories can diverge; feedback can change object type; mathematical carrier categories can themselves be revised; and persistence can coexist with divergent reconstruction or later reopening. Section 18 now asks how these historical distinctions can be realized geometrically without treating every triangular cell as evidence of a substantive triadic relation.

18 Typed Labelled 2-Complex Realization

This section constructs a discrete geometric realization of the triadic volunteer formalism. Its role is to place the stage-indexed relational records developed in Sections 5–17 on a finitely inspectable cellular carrier without changing their substantive meaning. The objective is to specify how subject histories, pairwise restrictions, third-position functions, manifestation records, procedure and provenance, typed asymmetries, emergent structures, and historical revision can be represented by a labelled 2-complex while keeping the substantive ledger distinct from its geometric encoding. The discussion proceeds through the realization relation, typed cell labels, subject and pairwise subcomplexes, support assignments, localized and distributed triadic structure, cellular recoverability, action and provenance supports, historical transformation, restriction–revision compatibility, resolution change, continuation families, and interfaces with the algebraic realization. The method is representational and reconstructive. The labelled 2-complex is an optional finite carrier; it is not proposed as the ultimate ontology of social relations.

Ontological suspension therefore remains active. A successful cellular encoding can show that a selected formal distinction is representable and inspectable on the chosen carrier. It does not establish that persons are vertices, relations are edges, higher-order social facts are faces, or historical reality is fundamentally cellular. Alternative carriers remain admissible when they preserve the relevant GR distinctions more faithfully.

18.1 Cellular Carrier and Realization Relation

Let the oriented combinatorial 2-complex used at stage t and resolution κ be defined in Equation 249.

$$\mathcal{K}^{\kappa}_{ABC,t} := \left(V^{\kappa}_{ABC,t}, E^{\kappa}_{ABC,t}, F^{\kappa}_{ABC,t}, \partial^{\kappa}_{ABC,t}\right), \tag{249}$$

where V, E, and F are the retained vertices, edges, and faces and ∂ contains the incidence, orientation, and attaching data required by the selected combinatorial model. The complex can be finite or locally finite when the application supplies the corresponding local finiteness conditions. Cell dimension has no fixed social semantics. A vertex can encode an event marker or a bookkeeping site rather than a

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person; an edge can encode a route or tracking relation rather than an interpersonal relation; a face can encode a compatibility condition, joint procedure, or another higher-order record rather than a temporal interval.

Let $\mathbf{C}^{\kappa}_{ABC,t}$ be the class of admissible labelled cellular records built on carriers of the form in Equation 249. A selected cellular record is defined in Equation 250.

$$\mathfrak{C}^{\kappa}_{ABC,t} := \left(\mathcal{K}^{\kappa}_{ABC,t}, \Lambda^{\kappa}_{ABC,t}, \mathsf{Supp}^{\kappa}_{ABC,t}, \mathsf{Hist}^{\kappa}_{ABC,t}\right) \in \mathbf{C}^{\kappa}_{ABC,t}. \tag{250}$$

The components of Equation 250 respectively contain the cell carrier, typed label families, declared support assignments, and the historical ordering or tracking information attached to selected cells or regions. They are representation-layer objects and do not replace the enriched substantive ledger $\mathbb{D}^{\kappa}_{ABC,t}$ introduced in Equation 226.

The distinction between substantive record and cellular representation is made explicit by the realization relation in Equation 251.

$$\mathsf{Real}^{\kappa}_{ABC,t} \subseteq \widetilde{\mathfrak{D}}^{\kappa}_{ABC,t} \times \mathbf{C}^{\kappa}_{ABC,t}. \tag{251}$$

A pair $(\mathbb{D}, \mathfrak{C}) \in \mathsf{Real}^{\kappa}_{ABC,t}$ means that $\mathfrak{C}$ is an admitted cellular realization of the distinctions retained by $\mathbb{D}$ under the declared interpretation rules. The relation is allowed to be many-to-many. One substantive ledger can admit several cellular realizations, and several formally different cellular records can represent substantively equivalent ledgers. This flexibility prevents arbitrary segmentation choices from becoming part of the substantive ontology by definition.

18.2 Typed Cell Labels and Semantic Domains

Cell labels must preserve the typing discipline already used for epistemic, procedural, authorization, asymmetry, public, provenance, and emergent records. Let $\mathcal{T}^{\kappa}_{\mathrm{cell},t}$ be the retained family of semantic label types. For $\tau \in \mathcal{T}^{\kappa}_{\mathrm{cell},t}$ and $p \in \{0,1,2\}$, the corresponding partial cell-label map is defined in Equation 252.

$$\lambda^{\kappa}_{p,\tau,t} : \mathsf{Cell}_p\big(\mathcal{K}^{\kappa}_{ABC,t}\big) \rightharpoonup \mathcal{L}^{\kappa}_{p,\tau,t}. \tag{252}$$

The family $\Lambda^{\kappa}_{ABC,t}$ in Equation 250 consists of the label maps in Equation 252 that are actually articulated by the selected realization. A role label, manifestation label, authorization status, public-record label, asymmetry descriptor, and local-algebra identifier therefore remain in distinct codomains unless the application supplies a justified comparison map. Co-location on one cell does not identify the corresponding concepts.

The semantic type also constrains interpretation. If a cell receives a label $\ell \in \mathcal{L}^{\kappa}_{p,\tau,t}$, the model must state what record type τ means, which substantive component it realizes, and which distinctions are discarded by the encoding. Cell dimension alone supplies none of those meanings. This requirement is especially important for faces: two-dimensional support can represent a higher-order coupling, but two-dimensionality is not itself a criterion of triadicity.

18.3 Subject Subcomplexes and Pairwise Cellular Restrictions

The subject-spacetime carriers of Section 5 can be represented by tracked subcomplexes. For each $i \in \{A, B, C\}$, let the cellular embedding be defined in Equation 253.

$$\iota^{\kappa}_{i,t} : \mathcal{K}^{\kappa}_{i,t} \hookrightarrow \mathcal{K}^{\kappa}_{ABC,t}, \qquad i \in \{A, B, C\}. \tag{253}$$

The three images in Equation 253 need not partition the coupled complex. They can overlap, share historical material, or depend on contextual cells assigned to no single subject carrier. A pairwise interface likewise need not equal the set-theoretic intersection of two subject images.

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For each pair $XY \in \{AB, AC, BC\}$, let $S^{\kappa}_{XY,t}$ be the declared retained cell family used by the pairwise cellular restriction. When the selected combinatorial model admits an incidence closure, the induced pairwise carrier is defined in Equation 254.

$$\mathcal{K}^{\kappa}_{XY|ABC,t} := \left\langle S^{\kappa}_{XY,t} \right\rangle_{\partial}. \tag{254}$$

Here $\langle S \rangle_{\partial}$ denotes the smallest retained subcomplex containing S under the declared incidence closure. The resulting pairwise carrier is extracted from the triadic representation. It is therefore distinct by construction from a standalone dyadic model, even when the two later satisfy a declared equivalence.

The cellular restriction map for pair XY is the partial map in Equation 255.

$$\mathsf{r}^{\kappa}_{XY,\mathrm{cell},t} : \mathbf{C}^{\kappa}_{ABC,t} \rightharpoonup \mathbf{C}^{\kappa}_{XY|ABC,t}, \tag{255}$$

where the restriction inherits only the cells, labels, support assignments, and historical tracking data admitted by the selected rule. On their common domain, the three component restrictions assemble into the cellular pairwise shadow in Equation 256.

$$\mathsf{P}^{\kappa}_{2,\mathrm{cell},t}(\mathfrak{C}) := \Big(\mathsf{r}^{\kappa}_{AB,\mathrm{cell},t}(\mathfrak{C}), \mathsf{r}^{\kappa}_{AC,\mathrm{cell},t}(\mathfrak{C}), \mathsf{r}^{\kappa}_{BC,\mathrm{cell},t}(\mathfrak{C})\Big). \tag{256}$$

Equation 256 realizes the generic pairwise-shadow architecture of Section 3; it does not redefine pairwise sufficiency at the substantive level. A well-formed cellular realization should also respect the substantive restriction scheme: if $\mathfrak{C}$ realizes $\mathbb{D}$ and both pairwise restrictions are defined, the restricted cellular record should realize the corresponding restricted substantive record under the selected interpretation equivalence.

18.4 Concept Supports across Relational, Manifestation, and Epistemic Records

A conceptual object can depend on several cells without being identical with any one cell. For every record object X represented by a selected realization $\mathfrak{C}$, let its cellular support be defined in Equation 257.

$$\mathrm{supp}_{\mathfrak{C}}(X) \subseteq \mathsf{Cell}\big(\mathcal{K}^{\kappa}_{ABC,t}\big). \tag{257}$$

The support in Equation 257 is model-relative. It need not be a subcomplex; an application that requires incidence closure can use $\langle \mathrm{supp}_{\mathfrak{C}}(X) \rangle_{\partial}$. Distinct cellular realizations of the same substantive ledger can assign different supports while preserving the declared substantive equivalence.

The three-level GR discipline places an important restriction on this support language. Ontological reality remains suspended and therefore receives no positive cellular representation by default. The cellular carrier represents retained relational objects, manifestations, and epistemic reconstructions where the model has articulated them. For a relational object X^{R}, a manifestation m, and subject i's epistemic reconstruction $\widehat{X}_i$, the separately typed supports are stated in Equation 258.

$$\begin{aligned} S^{\mathsf{R},\mathfrak{C}}_{X} &:= \mathrm{supp}_{\mathfrak{C}}(X^{\mathsf{R}}), \\ S^{\mathsf{M},\mathfrak{C}}_{m} &:= \mathrm{supp}_{\mathfrak{C}}(m), \\ S^{\mathsf{E},\mathfrak{C}}_{\widehat{X}_i} &:= \mathrm{supp}_{\mathfrak{C}}(\widehat{X}_i). \end{aligned} \tag{258}$$

Equation 258 is a list of distinct representational supports rather than a chain of identities. A public norm can have distributed relational support, a short utterance as one manifestation support, and different A-, B-, and C-relative reconstruction supports. The epistemic support is not a third ontological layer; it is a record of a subject-relative reconstruction formed through manifestation access and interpretation.

Table 21 summarizes recurrent support choices and the corresponding interpretive cautions.

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Table 21: Illustrative support assignments in the labelled triadic 2-complex realization.

Substantive object	Candidate cellular support	Interpretive discipline
Subject history	tracked subject subcomplex	subject identity is not one vertex or one fixed boundary
Pairwise interface	declared connecting region or induced pairwise subcomplex	interface need not equal literal set intersection
Mediation / translation	mixed-dimensional support across focal relation and third position	function is not identical with one mediator-labelled cell
Publicity / acknowledgment	audience, route, and history supports	shared visibility does not by itself establish common knowledge
Accountability	claim, response, review, determination, and appeal supports	procedural organization can be branching or partial
Coalition / exclusion	distributed coordination and route-support cells	coalition membership and exclusion remain dimension-specific
Action provenance	request, mediation, authorization, execution, witness, and review supports	visible execution does not exhaust generation
Emergent trust / norm / partnership	distributed support across recurrent episodes	one compact label cannot establish emergence by itself
Local algebra	admissible region R paired with $\mathcal{A}^{\kappa}_{ABC,t}(R)$	algebra is an additional representation layer

18.5 Localized and Distributed Triadic Support

A full-triad object can admit either localized or distributed cellular support. Let T be a retained object whose substantive type is scoped to the coupled triadic configuration. A localized face realization is defined in Equation 259.

$$\mathrm{supp}_{\mathfrak{C}}(T) = \{f\}, \qquad f \in F^{\kappa}_{ABC,t}, \qquad \lambda^{\kappa}_{2,\tau_T,t}(f) \in \mathcal{L}^{\kappa}_{2,\tau_T,t}. \tag{259}$$

Equation 259 is appropriate only when the interpretation rule justifies one face as the support of the retained full-triad record. A public recognition status, a joint procedural closure condition, or a higher-order compatibility record can sometimes satisfy that requirement.

A distributed realization uses the support pattern in Equation 260.

$$\mathrm{supp}_{\mathfrak{C}}(T) = S_T, \qquad S_T \subseteq \mathsf{Cell}(\mathcal{K}^{\kappa}_{ABC,t}), \qquad |S_T| > 1. \tag{260}$$

The family S_T in Equation 260 can contain cells of several dimensions and can cross several pairwise regions. Reputation, accountability, coalition structure, partnership, and historically changing third-position functions often require such distributed support because their meaning depends on several coupled episodes.

The choice between Equations 259 and 260 is therefore representational. Triadic scope is determined by the substantive typing and retained coupling of the object, not by whether the geometric realization happens to use one triangular face.

18.6 Cellular Pairwise Recoverability

The cellular realization inherits the pairwise-recoverability question of Section 6. Let $\mathcal{D}^{\kappa}_{\mathrm{cell},t} \subseteq \mathbf{C}^{\kappa}_{ABC,t}$ be a declared comparison domain on which all three cellular restrictions are defined, and let $\sim^{\kappa}_{\mathrm{cell},t}$ be the selected full-equivalence relation. The induced pairwise cellular equivalence is defined in Equation 261.

$$\mathfrak{C} \sim^{\kappa}_{2,\mathrm{cell},t} \mathfrak{C}' \quad \Longleftrightarrow \quad \mathsf{r}^{\kappa}_{XY,\mathrm{cell},t}(\mathfrak{C}) \sim^{\kappa}_{XY,\mathrm{cell},t} \mathsf{r}^{\kappa}_{XY,\mathrm{cell},t}(\mathfrak{C}') \quad \text{for each } XY \in \{AB, AC, BC\}. \tag{261}$$

The cellular representation is pairwise sufficient on $\mathcal{D}^{\kappa}_{\mathrm{cell},t}$ when Equation 262 holds.

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$$\mathfrak{C} \sim^{\kappa}_{2,\mathrm{cell},t} \mathfrak{C}' \quad \Longrightarrow \quad \mathfrak{C} \sim^{\kappa}_{\mathrm{cell},t} \mathfrak{C}' \qquad \text{for all } \mathfrak{C}, \mathfrak{C}' \in \mathcal{D}^{\kappa}_{\mathrm{cell},t}. \tag{262}$$

A cellular nonrecoverability witness is therefore a pair satisfying Equation 263.

$$\mathfrak{C} \sim^{\kappa}_{2,\mathrm{cell},t} \mathfrak{C}' \qquad \text{and} \qquad \mathfrak{C} \not\sim^{\kappa}_{\mathrm{cell},t} \mathfrak{C}'. \tag{263}$$

A pairwise-invisible face label can supply one finite witness to Equation 263; so can distributed support that is fragmented by every pairwise restriction in a way that destroys the selected coupling criterion. The witness establishes nonrecoverability for the declared cellular representation and equivalences. A substantive triadic claim additionally requires that the distinguishing cellular structure realize a defensible substantive distinction in $\widetilde{\mathfrak{D}}^{\kappa}_{ABC,t}$ rather than an arbitrary encoding artifact.

18.7 Action and Provenance Supports

Let $\mathfrak{a}$ be an individuated action from Section 13. The selected action support is defined in Equation 264.

$$S^{\mathfrak{C}}_{\mathfrak{a},t} := \mathrm{supp}_{\mathfrak{C}}(\mathfrak{a}). \tag{264}$$

The distributed provenance relation $\Pi^{\kappa}_{3,t}(\mathfrak{a})$ can contain several bearers and provenance types. If each represented provenance entry u has an admitted cellular support, the provenance support is defined in Equation 265.

$$S^{\mathfrak{C}}_{\Pi(\mathfrak{a}),t} := \bigcup_{u \in \Pi^{\kappa}_{3,t}(\mathfrak{a})} \mathrm{supp}_{\mathfrak{C}}(u). \tag{265}$$

Equation 265 is defined only for provenance entries included in the cellular realization. The action support and provenance support can differ because a compact execution event can depend on a wider history of request, translation, mediation, authorization, coalition, witness, or review.

A cellular action transformation represents the substantive action map of Equation 154 only after a realization interface has been declared. The representation-layer map is stated in Equation 266.

$$\widehat{\Phi}^{\kappa' \leftarrow \kappa}_{\mathfrak{a}} : \widehat{\mathcal{D}}^{\kappa}_{\mathfrak{a}} \rightharpoonup \mathbf{C}^{\kappa'}_{ABC,t'}, \qquad \widehat{\mathcal{D}}^{\kappa}_{\mathfrak{a}} \subseteq \mathbf{C}^{\kappa}_{ABC,t}. \tag{266}$$

Compatibility with the substantive action requires that an admitted cellular transformation connect realizations of substantive records that are themselves related by the relevant action transformation or historical revision rule. Equation 266 therefore encodes a realized effect on the carrier rather than defining the action's substantive meaning.

18.8 Cellular Historical Episodes and Tracking

Section 17 defined historical composition independently of one geometric carrier. A cellular realization associates retained historical markers with cell regions. The family of finite historical supports is defined in Equation 267.

$$\mathsf{H}^{\kappa}_{ABC,t}(\mathfrak{C}) \subseteq \mathcal{P}_{\mathrm{fin}}\Big(\mathsf{Cell}(\mathcal{K}^{\kappa}_{ABC,t})\Big). \tag{267}$$

Let $\preceq_{H,\mathfrak{C}}$ be the declared precedence relation on the retained historical supports. A cellular episode is the partially ordered family in Equation 268.

$$\gamma_{\mathfrak{C}} := \Big((C_j)_{j \in J}, \preceq_{H,\mathfrak{C}}\Big), \qquad C_j \in \mathsf{H}^{\kappa}_{ABC,t}(\mathfrak{C}). \tag{268}$$

Equation 268 allows mixed-dimensional episode supports and partial order. Independent events therefore need not be forced into a total temporal sequence merely for convenience of representation.

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A special history-preserving growth regime can use literal subcomplex inclusion. Its condition is stated in Equation 269.

$$t \preceq t' \quad \Longrightarrow \quad \mathcal{K}^{\kappa}_{ABC,\leq t} \hookrightarrow \mathcal{K}^{\kappa}_{ABC,\leq t'}. \tag{269}$$

Equation 269 is a restricted representation regime. The historical formalism of Section 17 also permits relabelling, changed tracking, regrouping, resolution change, or replacement of represented carriers. Such representational revision does not imply that an earlier historical event has been ontologically rewritten.

18.9 Cellular Transformation Regimes

A cellular update can change several distinct features of the representation. To keep those changes separate, let the declared transformation type be selected from the family in Equation 270.

$$\tau_{\widehat{\Phi}} \in \{\mathsf{Relabel}, \mathsf{Extension}, \mathsf{Reattach}, \mathsf{TopDiag}\}. \tag{270}$$

A Relabel transformation changes retained labels while preserving the selected cell and incidence data. An Extension transformation adds retained cells while preserving the earlier complex through a declared embedding. A Reattach transformation changes incidence or attaching data and can therefore alter which cellular compositions and interfaces are available. The TopDiag label is reserved for cases in which the application declares a topological diagnostic and observes a change in that diagnostic across the represented carriers.

Let I_{top} be such a declared diagnostic. The corresponding witness condition is stated in Equation 271.

$$I_{\mathrm{top}}\big(\mathcal{K}^{\kappa}_{ABC,t}\big) \neq I_{\mathrm{top}}\big(\mathcal{K}^{\kappa'}_{ABC,t'}\big). \tag{271}$$

Equation 271 establishes a difference under the selected diagnostic. Equality of one diagnostic does not establish topological equivalence, and a diagnostic change does not by itself establish a social phase transition, moral transformation, or ontological topology of the relation. This narrower terminology prevents ordinary role revision or changed attribution from being redescribed as topology change without a mathematical basis.

18.10 Historical Ledger Revision on Cellular Carriers

The enriched record-revision relation $\mathsf{RecRev}^{\kappa' \leftarrow \kappa}_{t' \leftarrow t}$ from Equation 229 acts on substantive ledgers. A cellular realization can carry its own revision relation, defined in Equation 272.

$$\mathsf{CellRev}^{\kappa' \leftarrow \kappa}_{t' \leftarrow t} \subseteq \mathbf{C}^{\kappa}_{ABC,t} \times \mathbf{C}^{\kappa'}_{ABC,t'}. \tag{272}$$

The two revision layers are connected through the realization relation rather than identified. A coherent lifted revision quadruple is defined in Equation 273.

$$\begin{aligned} \mathsf{LiftRev}^{\kappa' \leftarrow \kappa}_{t' \leftarrow t} := \Big\{ (\mathbb{D}_t, \mathfrak{C}_t, \mathbb{D}_{t'}, \mathfrak{C}_{t'}) \;\Big|\; & (\mathbb{D}_t, \mathfrak{C}_t) \in \mathsf{Real}^{\kappa}_{ABC,t}, \\ & (\mathbb{D}_t, \mathbb{D}_{t'}) \in \mathsf{RecRev}^{\kappa' \leftarrow \kappa}_{t' \leftarrow t}, \\ & (\mathfrak{C}_t, \mathfrak{C}_{t'}) \in \mathsf{CellRev}^{\kappa' \leftarrow \kappa}_{t' \leftarrow t}, \\ & (\mathbb{D}_{t'}, \mathfrak{C}_{t'}) \in \mathsf{Real}^{\kappa'}_{ABC,t'} \Big\}. \end{aligned} \tag{273}$$

Equation 273 prevents a change in the drawing or cellular segmentation from silently becoming a substantive historical change. It also allows a substantive revision to have more than one adequate cellular realization.

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18.11 Pairwise Restriction and Cellular Revision

The historical compatibility problem in Section 17.7 has a cellular specialization. Because the cellular revision is relation-valued, the comparison should use reachable target families rather than formal composition symbols alone. Partial cellular restriction maps are viewed here through their graph relations when composed with revision relations. Let the pairwise cellular revision relation be defined in Equation 274.

$$\mathsf{CellRev}^{\kappa' \leftarrow \kappa}_{2,t' \leftarrow t} \subseteq \mathbf{C}^{\kappa}_{2,t} \times \mathbf{C}^{\kappa'}_{2,t'}, \tag{274}$$

where $\mathbf{C}^{\kappa}_{2,t}$ is the declared compatibility class for the three pairwise cellular records. Two relations connect a full cellular record at t to a pairwise cellular record at t'. They are defined in Equation 275.

$$\begin{aligned} \mathcal{R}_{\text{full}\to 2} &:= \mathsf{P}^{\kappa'}_{2,\text{cell},t'} \circ \mathsf{CellRev}^{\kappa' \leftarrow \kappa}_{t' \leftarrow t}, \\ \mathcal{R}_{2\to\text{pair}} &:= \mathsf{CellRev}^{\kappa' \leftarrow \kappa}_{2,t' \leftarrow t} \circ \mathsf{P}^{\kappa}_{2,\text{cell},t}. \end{aligned} \tag{275}$$

For a relation $R \subseteq X \times Y$, write $R[x] := \{y \in Y : (x,y) \in R\}$. Let $q^{t'}_{2,\text{cell},\chi}$ be the quotient map induced by the declared target pairwise equivalence. A cellular restriction–revision discrepancy is witnessed by a source $\mathfrak{C}_t$ satisfying Equation 276.

$$q^{t'}_{2,\text{cell},\chi}\big[\mathcal{R}_{\text{full}\to 2}[\mathfrak{C}_t]\big] \neq q^{t'}_{2,\text{cell},\chi}\big[\mathcal{R}_{2\to\text{pair}}[\mathfrak{C}_t]\big]. \tag{276}$$

Equation 276 means that restricting the cellular record before revision and restricting after full cellular revision can produce different pairwise outcome classes. One possible explanation is that early restriction discarded a full-triad support region that later became relevant to the pairwise reconstruction. Alternative explanations include mis-specified restriction rules, inconsistent cellular revision rules, or changes in the interpretation map. The discrepancy is therefore a diagnostic rather than a standalone proof of substantive triadic irreducibility.

18.12 Resolution Change and Pairwise Compatibility

Resolution change should act on complete labelled cellular records rather than only on bare support sets. Let the declared coarse-graining operation from fine resolution κ_f to coarse resolution κ_c be the partial map in Equation 277.

$$\mathsf{Q}^{f\to c}_{ABC} : \mathbf{C}^{\kappa_f}_{ABC,t} \rightharpoonup \mathbf{C}^{\kappa_c}_{ABC,t}. \tag{277}$$

The map in Equation 277 can collapse cells, aggregate labels, discard distinctions, or replace support assignments according to explicit rules. It is called coarse-graining only relative to the declared resolution scheme; the framework does not infer that the finer representation is ontologically superior.

For each pair XY, let $\mathsf{Q}^{f\to c}_{XY}$ be the corresponding pairwise coarse-graining operation where defined. Pairwise restriction is strongly compatible with coarse-graining on a cellular record $\mathfrak{C}$ when the two routes determine equivalent pairwise outputs as stated in Equation 278.

$$\mathsf{Q}^{f\to c}_{XY}\Big(\mathsf{r}^{\kappa_f}_{XY,\text{cell}}(\mathfrak{C})\Big) \sim^{\kappa_c}_{XY,\text{cell}} \mathsf{r}^{\kappa_c}_{XY,\text{cell}}\Big(\mathsf{Q}^{f\to c}_{ABC}(\mathfrak{C})\Big), \tag{278}$$

for every $\mathfrak{C}$ in the common domain of the two routes. Failure of Equation 278 can reflect an inconsistent resolution scheme or a distinction that disappears under coarse-graining.

Where the coarse-graining operation also induces a cell-support pushforward $\mathsf{Q}^{f\to c}_{*}$, one can test whether a represented object X is support-compatible across resolutions using Equation 279.

$$\mathsf{Q}^{f\to c}_{*}\Big(\text{supp}_{\mathfrak{C}_f}(X)\Big) \simeq_{X,\kappa_c} \text{supp}_{\mathfrak{C}_c}(X), \qquad \mathfrak{C}_c = \mathsf{Q}^{f\to c}_{ABC}(\mathfrak{C}_f). \tag{279}$$

A triadic nonrecoverability result can therefore be resolution-sensitive. A coarse representation can merge the structure that distinguished two full-triad records, while a finer representation can preserve it.

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Conversely, a fine representation can contain distinctions intentionally excluded from the analytical question. Claims about pairwise recoverability should report the resolution family and equivalence criterion under which they are stable.

18.13 Cellular Boundaries and Continuation Realizations

The generative orientation of GR requires attention to admissible continuations as well as realized history. Let $\mathbf{B}^{\kappa}_{ABC,t}$ denote the declared class of admissible cellular boundary descriptors at stage t. The incoming-boundary extraction is the partial map in Equation 280.

$$\mathsf{b}^{\kappa}_{-,t} : \mathbf{C}^{\kappa}_{ABC,t} \rightharpoonup \mathbf{B}^{\kappa}_{ABC,t}. \tag{280}$$

For a boundary descriptor $\sigma \in \mathbf{B}^{\kappa}_{ABC,t}$, the cellular extension family is defined in Equation 281.

$$\mathrm{Ext}_{\mathrm{cell},3,\kappa}(\sigma) := \left\{ \mathfrak{C}' \in \mathbf{C}^{\kappa}_{ABC,t'} \;\middle|\; \mathsf{b}^{\kappa}_{-,t'}(\mathfrak{C}') \simeq_{\Sigma} \sigma \right\}. \tag{281}$$

Equation 281 is a family of represented continuations. It is neither a probability distribution nor a normative ranking. A realization is informative only if changes in this family can be related back to the substantive continuation assignment $\mathrm{Cont}^{\kappa}_{ABC,t}$ developed in Section 17.4.

If a defensible event-conditioned comparison is available, let $\sigma^{(e)}_{t'}$ and $\sigma^{(e^{\circ})}_{t'}$ be the two boundary descriptors generated by the declared comparison construction. Here e° denotes the comparison condition rather than an automatically available "absence of e" world. Event e has a represented generative effect when Equation 282 holds.

$$\mathrm{Ext}_{\mathrm{cell},3,\kappa'}\big(\sigma^{(e)}_{t'}\big) \not\simeq_{\mathrm{Ext}} \mathrm{Ext}_{\mathrm{cell},3,\kappa'}\big(\sigma^{(e^{\circ})}_{t'}\big). \tag{282}$$

Equation 282 can represent a change in available mediation, refusal, review, direct communication, coalition, or partnership paths without imposing a common welfare scalar on those possibilities.

18.14 Emergent Structures on Distributed Cellular Support

An emergent triadic object X satisfying the criterion of Section 15 can be realized on distributed cellular support. Let $\mathcal{W}^{3,\chi,\kappa}_{X}(I)$ be the substantive witness record from Equation 180. Define $\mathcal{J}^{\mathrm{wit},\chi,\kappa}_{X}(I)$ as the index set of retained witness items appearing in its tracking, organization, efficacy, manifestation, and provenance components. A cellular realization of those witness items uses the support association in Equation 283.

$$\mathsf{SuppWit}^{\kappa}_{X,\mathfrak{C}} : \mathcal{J}^{\mathrm{wit},\chi,\kappa}_{X}(I) \rightharpoonup \mathcal{P}\Big(\mathsf{Cell}(\mathcal{K}^{\kappa}_{ABC})\Big). \tag{283}$$

Equation 283 attaches cellular regions to the retained witness items where the application represents them. A compact visualization label such as "partnership" can summarize the resulting structure, but the emergence claim remains grounded in the substantive witness record and its history rather than in the compact label itself.

This distinction permits several geometric encodings of the same emergent object. Recurrent mediation episodes can realize a stable mediation channel; public attributions and later role consequences can realize reputation; contribution, correction, and revision episodes can realize partnership. The cellular geometry makes those support relations inspectable without supplying the emergence criterion on its own.

18.15 Regional Algebra Interface

The local algebraic realization of Section 16 can be attached to admissible cellular regions. For a cellular record $\mathfrak{C}$, let $\mathsf{Reg}(\mathfrak{C})$ denote the declared regions on which an algebra is assigned. The interface is the partial assignment in Equation 284.

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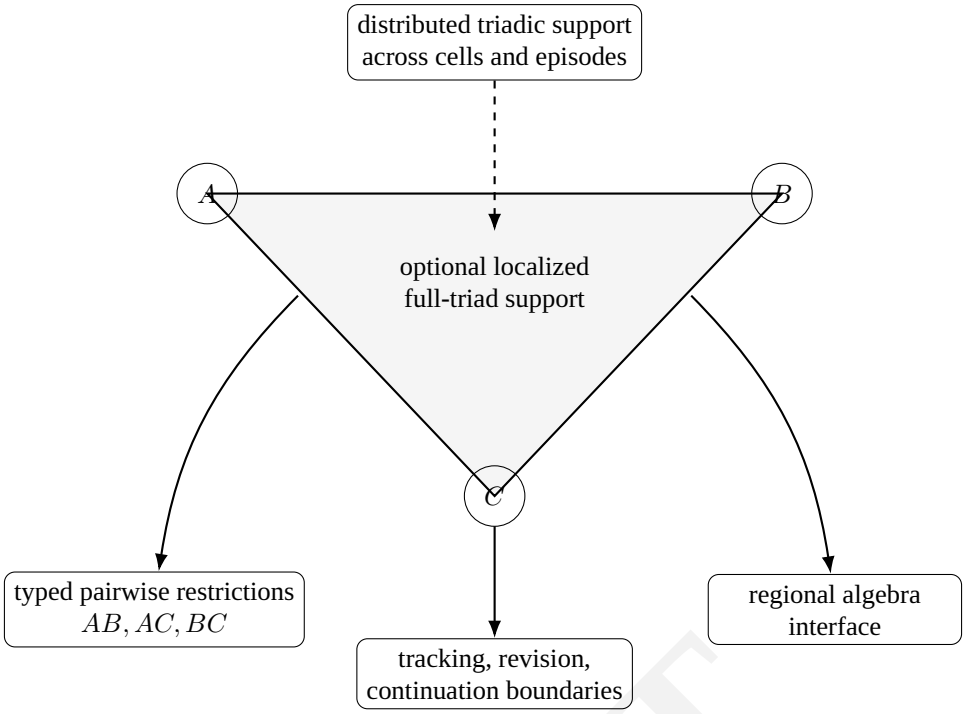

Figure 17: Typed labelled 2-complex realization of the triadic formalism. A full-triad object can have localized or distributed cellular support. Pairwise restrictions, historical tracking and continuation, and regional algebraic representations remain separately typed interfaces. The diagram is a representational architecture rather than a claim that triadic social reality is literally triangular or cellular.

$$\mathfrak{A}^{\kappa}_{\mathfrak{C},t} : \mathsf{Reg}(\mathfrak{C}^{\kappa}_{ABC,t}) \rightharpoonup \mathbf{Alg}^{\kappa}_{ABC,t}, \qquad R \longmapsto \mathcal{A}^{\kappa}_{ABC,t}(R). \tag{284}$$

Equation 284 reuses the regional algebra already defined in Section 16. It does not infer an algebra from geometry alone. A cellular transformation and an algebraic lift are compatible only when the application supplies comparison maps showing that the transformed cellular region and the transformed algebraic carrier represent the same retained substantive update up to the declared equivalence.

The geometric and algebraic layers therefore provide complementary realizations. The cellular carrier identifies represented support, incidence, and historical tracking; the algebraic layer represents selected compositional structure on admitted regions. Neither layer is required to be complete, faithful, or ontologically privileged.

Figure 17 summarizes the separation among full-triad support, pairwise restrictions, distributed historical support, and the algebraic interface.

18.16 Finite Inspectability and Representation Limits

The labelled 2-complex realization makes selected parts of the triadic formalism finitely inspectable. A finite application can enumerate cells, specify incidence and attaching data, assign typed labels, identify subject and pairwise regions, attach supports for mediation, publicity, accountability, coalition, procedure, provenance, and emergence, encode a historical partial order, and test pairwise restriction, coarse-graining, or selected topological and algebraic diagnostics.

Finite inspectability remains distinct from empirical adequacy. Cellular segmentation depends on the selected representation. Labels are theory-mediated encodings, incidence retains only selected relational structure, and support assignments can omit background institutions or longer histories. A formally valid pairwise-invisible label can also lack empirical justification in a concrete case. The realization relation in

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Equation 251 therefore carries an interpretive burden: the analyst must explain why a cellular distinction corresponds to a defensible distinction in the substantive ledger.

The same restraint applies to structural change. A change in incidence, homology, connectivity, or another cellular diagnostic establishes a change in the represented carrier under the chosen construction. It does not independently establish a moral improvement, social phase transition, or ultimate topology of the volunteer relation. Conversely, substantively important change can occur through revised standing, interpretation, manifestation, or authority while the chosen cellular carrier remains topologically unchanged.

The section therefore preserves the order of analysis used throughout Paper II. Substantive relational records and their three-level distinctions are specified first; a cellular realization is selected second; geometric diagnostics are applied third; empirical and normative interpretation remain additional tasks. Section 18 supplies the finite carrier required by the stronger history-level specialization in Section 19, where incoming and outgoing boundary records, internal labelled histories, gluing, pairwise history shadows, and optional history weights are introduced.

19 Typed Spin-Foam-Inspired Triadic Histories

This section develops an optional history-level realization of the labelled 2-complex architecture constructed in Section 18. Its role is to represent finite triadic histories with declared incoming and outgoing boundaries, internally labelled cellular structure, pairwise history restrictions, partial gluing, optional history weights, and cross-resolution comparison while preserving the distinction between substantive GR history and its mathematical encoding. The objective is to make historical comparison inspectable without importing physical spin-foam semantics into the social model. The discussion proceeds from substantive history traces and their realization relation to typed boundary histories, internal labels, alternative-history families, pairwise shadows, gluing, optional weighting, algebraic and categorical boundary transport, resolution change, and limits. The method is representational and reconstructive: the spin-foam-inspired object is one finite carrier for selected historical distinctions, not a claim about the ultimate ontology of persons, relations, or social spacetime.

The structural analogy is deliberately narrow. A physical spin-foam model uses labelled 2-complex histories with lower-dimensional boundary data and model-specific label and amplitude structures. Paper II retains only the abstract idea that a bounded labelled 2-complex can carry an internal history and can sometimes compose with another compatible bounded history. The present labels, restrictions, equivalences, and weights receive GR-specific meanings only through explicit interpretation rules. Quantum probability, superposition, physical discreteness, a universal Hilbert space, and quantum-gravitational ontology do not follow from the use of this carrier.

The triadic specialization adds a further requirement. A full history can contain retained public, coalition, provenance, mediation, recognition, or accountability structure that disappears under every selected pairwise restriction. History comparison must therefore preserve the pairwise-sufficiency discipline established in Section 6. Boundary equivalence, history equivalence, and pairwise equivalence remain separately declared throughout the section.

19.1 History Realization over Substantive Ledger Traces

Section 17 defined enriched stage ledgers and relation-valued historical revision. The present realization begins from those objects rather than replacing them. Fix a finite retained stage sequence at one declared comparison resolution κ,

$$\gamma := (t_0, t_1, \dots, t_n), \qquad t_0 \prec t_1 \prec \cdots \prec t_n. \tag{285}$$

Equation 285 is a finite history index. Histories whose analysis changes resolution can be brought into this specialization only after a comparison-resolution scheme or an explicitly indexed family of history realizations has been declared.

A substantive ledger trace along γ is defined in Equation 286.

$$\widetilde{\mathfrak{H}}^{\kappa}_{ABC,\gamma} := \Big\{ (D_0, \dots, D_n) \;\Big|\; D_j \in \widetilde{\mathfrak{D}}^{\kappa}_{ABC,t_j},\ (D_j, D_{j+1}) \in \mathsf{RecRev}^{\kappa\leftarrow\kappa}_{t_{j+1}\leftarrow t_j} \text{ for } 0 \le j < n \Big\}. \quad (286)$$

Equation 286 retains the order and typed revision relations already established in Section 17. It does not assert that the revision relation is functional, invertible, or generated by one global dynamical law.

Let $\mathbf{B}^{\kappa}_{ABC,-}$ and $\mathbf{B}^{\kappa}_{ABC,+}$ be declared classes of incoming and outgoing history-boundary descriptors. A bounded cellular history record is defined in Equation 287.

$$\mathfrak{h} := \Big(\mathcal{K}^{\kappa}_{ABC,\mathfrak{h}}, \Lambda^{\kappa}_{ABC,\mathfrak{h}}, \mathcal{S}^{\kappa}_{ABC,\mathfrak{h}}, \mathcal{O}^{\kappa}_{ABC,\mathfrak{h}}, \jmath^{\mathfrak{h}}_{-}, \jmath^{\mathfrak{h}}_{+} \Big). \quad (287)$$

Here $\mathcal{K}^{\kappa}_{ABC,\mathfrak{h}}$ is the finite oriented 2-complex carrier, $\Lambda^{\kappa}_{ABC,\mathfrak{h}}$ is the family of typed labels, $\mathcal{S}^{\kappa}_{ABC,\mathfrak{h}}$ is the family of declared support assignments, and $\mathcal{O}^{\kappa}_{ABC,\mathfrak{h}}$ retains the historical ordering or tracking data used by the selected realization. The boundary maps $\jmath^{\mathfrak{h}}_{-}$ and $\jmath^{\mathfrak{h}}_{+}$ identify admitted boundary descriptors with boundary subrecords of the cellular history. Pairwise shadows are derived later; they are not primitive components of Equation 287.

For declared boundary descriptors $\Sigma_- \in \mathbf{B}^{\kappa}_{ABC,-}$ and $\Sigma_+ \in \mathbf{B}^{\kappa}_{ABC,+}$, let $\mathbf{H}^{\kappa}_{ABC}[\Sigma_-, \Sigma_+]$ denote the class of well-formed bounded history records satisfying the selected boundary, incidence, label, support, and tracking requirements. Boundary matching is expressed through the equivalence condition in Equation 288.

$$\jmath^{\mathfrak{h}}_{-}(\Sigma_-) \simeq_{\partial,3,\kappa} \partial_- \mathfrak{h}, \qquad \jmath^{\mathfrak{h}}_{+}(\Sigma_+) \simeq_{\partial,3,\kappa} \partial_+ \mathfrak{h}. \quad (288)$$

Equation 288 uses a declared representational boundary equivalence. Changing that equivalence can change the admitted history class and must therefore be reported when histories are compared.

The common carrier for boundary-typed histories is the disjoint union in Equation 289.

$$\mathbf{H}^{\kappa,\mathrm{adm}}_{ABC} := \bigsqcup_{\substack{\Sigma_- \in \mathbf{B}^{\kappa}_{ABC,-} \\ \Sigma_+ \in \mathbf{B}^{\kappa}_{ABC,+}}} \mathbf{H}^{\kappa}_{ABC}[\Sigma_-, \Sigma_+]. \quad (289)$$

Equation 289 preserves boundary typing while providing one domain for restriction, gluing, weighting, and transport maps.

The distinction between substantive history and cellular history is made explicit by the realization relation in Equation 290.

$$\mathsf{HistReal}^{\kappa}_{ABC,\gamma} \subseteq \widetilde{\mathfrak{H}}^{\kappa}_{ABC,\gamma} \times \mathbf{H}^{\kappa,\mathrm{adm}}_{ABC}. \quad (290)$$

A pair $(\mathbf{D}, \mathfrak{h}) \in \mathsf{HistReal}^{\kappa}_{ABC,\gamma}$ means that $\mathfrak{h}$ is an admitted bounded cellular realization of the distinctions retained by the substantive ledger trace $\mathbf{D} = (D_0, \dots, D_n)$ under the declared interpretation rules. The relation can be many-to-many. Several cellular histories can realize one substantive trace, and formally different substantive traces can admit equivalent cellular realizations when the chosen representation forgets distinctions. This many-to-many structure prevents representational multiplicity from being mistaken for substantive historical multiplicity.

19.2 Typed Historical Support on Internal Cells

The internal labels inherit the typed discipline of Section 18.2. Let $\mathcal{L}^{\kappa}_{p,\tau}$ be the declared label carrier for cell dimension p and record type τ, and write $\mathrm{Cell}_p(\mathcal{K})$ for the set of p-cells of a finite complex. For every label family retained by $\mathfrak{h}$, the typed map is stated in Equation 291.

$$\lambda^{\kappa}_{p,\tau,\mathfrak{h}} : \mathcal{D}^{\kappa}_{p,\tau,\mathfrak{h}} \subseteq \mathrm{Cell}_p\big(\mathcal{K}^{\kappa}_{ABC,\mathfrak{h}}\big) \rightharpoonup \mathcal{L}^{\kappa}_{p,\tau}. \quad (291)$$

Equation 291 retains partial definedness and semantic typing. Procedure labels, manifestation labels, authorization states, public records, asymmetry descriptors, subject-relative reconstructions, and algebra

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Table 22: Spin-foam-inspired structural resources in the typed triadic history realization.

Retained structural resource	Interpretation discipline in Paper II
Bounded labelled 2-complex	finite carrier for selected triadic history distinctions; no physical-discreteness claim
Boundary descriptors	retained comparison data linked to cellular boundaries by declared equivalence; no quantum-state semantics
Typed cell labels and supports	GR-specific records with separately declared codomains and interpretation rules
Alternative cellular realizations	multiple encodings of one substantive trace are allowed and remain distinct from alternative substantive histories
Alternative substantive histories	distinct admissible ledger traces can share retained boundaries; comparison requires an explicit substantive criterion
Pairwise history shadows	three induced dyadic history records used for recoverability tests; incompleteness is an empirical/formal question
History gluing	partial composition under explicit boundary and carrier compatibility conditions
History weights	optional model data whose cost, plausibility, salience, or other semantics must be declared independently
Resolution change	representation-level comparison across cellular granularities; finer resolution carries no automatic ontological priority

identifiers belong to distinct codomains unless a comparison map has been separately justified. Their placement on one cell does not identify the corresponding objects.

Historical support can be localized or distributed. A retained triadic object may be represented on one face when the chosen semantics warrants such localization, while another object may depend on a family of cells distributed across several episodes. The existence of full-triad historical content therefore places no general requirement that one face carry the entire triadic record. Support remains a declared association whose adequacy is judged against the substantive history trace through Equation 290.

The three-level GR distinction also remains active inside the history realization. Relational-reality records, manifestation records, and subject-relative epistemic reconstructions receive separately typed labels or supports. Their historical co-occurrence can be analytically important without collapsing the three record types into one another.

Table 22 summarizes the structural resources retained in the specialization.

19.3 Sources of Historical Multiplicity

Two kinds of multiplicity should be separated before historical comparison begins. For a fixed substantive trace $\mathbf{D} \in \widetilde{\mathfrak{H}}^{\kappa}_{ABC,\gamma}$, the realization fiber is defined in Equation 292.

$$\mathsf{RealFib}^{\kappa}_{H}(\mathbf{D}) := \left\{ \mathfrak{h} \;\middle|\; (\mathbf{D}, \mathfrak{h}) \in \mathsf{HistReal}^{\kappa}_{ABC,\gamma} \right\}. \tag{292}$$

Distinct members of Equation 292 can differ because of segmentation, subdivision, label placement, or another representational choice while still realizing the same substantive history. Such multiplicity is evidence about representation, not by itself evidence of multiple social histories.

Counterfactual or branch-sensitive analysis requires a different construction. The model-declared family of substantively admissible traces sharing the retained boundary conditions is stated in Equation 293.

$$\mathfrak{A}^{\kappa}_{ABC,\gamma}[\Sigma_-, \Sigma_+] \subseteq \left\{ \mathbf{D} \in \widetilde{\mathfrak{H}}^{\kappa}_{ABC,\gamma} \;\middle|\; \exists\, \mathfrak{h} \in \mathbf{H}^{\kappa}_{ABC}[\Sigma_-, \Sigma_+] \text{ with } (\mathbf{D}, \mathfrak{h}) \in \mathsf{HistReal}^{\kappa}_{ABC,\gamma} \right\}. \tag{293}$$

Membership in Equation 293 must be justified by the substantive model. Boundary similarity alone does not generate counterfactual admissibility. A comparison between a history containing mediation event e and a comparison history without that event is meaningful only when the construction of both histories preserves the other distinctions required by the research question.

This separation supports two distinct questions. Representational robustness asks whether one substantive result survives several admitted realizations in $\mathsf{RealFib}^{\kappa}_{H}(\mathbf{D})$. Historical sensitivity asks whether different traces in $\mathfrak{A}^{\kappa}_{ABC,\gamma}[\Sigma_-, \Sigma_+]$ yield different later role structures, continuation families, reputation

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records, accountability states, or recoverability properties. The two questions should not be inferred from one another.

19.4 Pairwise Boundary Shadows

Full triadic boundary descriptors can themselves be restricted to pairwise boundary descriptors. For each $XY \in \{AB, AC, BC\}$ and each boundary orientation $\varepsilon \in \{-, +\}$, let the partial restriction be defined in Equation 294.

$$\sigma^{\kappa}_{XY,\varepsilon} : \mathbf{B}^{\kappa}_{ABC,\varepsilon} \rightharpoonup \mathbf{B}^{\kappa}_{XY|ABC,\varepsilon}. \tag{294}$$

On the common domain of the three restrictions, the pairwise boundary shadow is defined in Equation 295.

$$\mathsf{P}^{\kappa}_{2,\partial}(\Sigma_\varepsilon) := \Big(\sigma^{\kappa}_{AB,\varepsilon}(\Sigma_\varepsilon), \sigma^{\kappa}_{AC,\varepsilon}(\Sigma_\varepsilon), \sigma^{\kappa}_{BC,\varepsilon}(\Sigma_\varepsilon)\Big). \tag{295}$$

Equation 295 collects the three retained pairwise boundary records without asserting that they reconstruct the full boundary.

Let $\sim_{\partial,2,\kappa}$ be the equivalence induced by the three pairwise restrictions and let $\sim_{\partial,3,\kappa}$ be the selected equivalence on full boundary descriptors. Boundary-level nonrecoverability is witnessed by the relation stated in Equation 296.

$$\Sigma \sim_{\partial,2,\kappa} \Sigma' \qquad \text{and} \qquad \Sigma \not\sim_{\partial,3,\kappa} \Sigma'. \tag{296}$$

Equation 296 is a boundary specialization of the general equivalence-gap logic. A retained public-audience condition, triadic authorization state, or other higher-order boundary record can supply such a difference when the selected pairwise restrictions omit it.

19.5 Pairwise History Recoverability

Pairwise history shadows are derived from full bounded histories. Let $\mathbf{H}^{\kappa}_{XY|ABC}$ denote the declared class of induced pairwise bounded histories. The history restriction is the partial map in Equation 297.

$$\mathsf{r}^{\kappa}_{XY,H} : \mathbf{H}^{\kappa,\mathrm{adm}}_{ABC} \rightharpoonup \mathbf{H}^{\kappa}_{XY|ABC}, \qquad XY \in \{AB, AC, BC\}, \tag{297}$$

where $\mathbf{H}^{\kappa,\mathrm{adm}}_{ABC}$ is the disjoint union of the admitted boundary-typed full-history classes. Each restriction inherits only the cells, labels, supports, boundary data, and tracking relations retained by the selected pairwise rule.

On the common domain of all three restrictions, the pairwise history shadow is defined in Equation 298.

$$\mathsf{P}^{\kappa}_{2,H}(\mathfrak{h}) := \Big(\mathsf{r}^{\kappa}_{AB,H}(\mathfrak{h}), \mathsf{r}^{\kappa}_{AC,H}(\mathfrak{h}), \mathsf{r}^{\kappa}_{BC,H}(\mathfrak{h})\Big). \tag{298}$$

Equation 298 is a history-level realization of the pairwise-shadow architecture used throughout Paper II. It does not imply that the shadow is sufficient for the full history.

Let $\sim_{H,3,\kappa}$ be a declared equivalence on comparable full histories, and let $\approx_{2,H,\kappa}$ be the declared equivalence on comparable pairwise history shadows. The induced pairwise equivalence on full histories is defined by $\mathfrak{h} \sim_{H,2,\kappa} \mathfrak{h}'$ exactly when $\mathsf{P}^{\kappa}_{2,H}(\mathfrak{h}) \approx_{2,H,\kappa} \mathsf{P}^{\kappa}_{2,H}(\mathfrak{h}')$. A history-level nonrecoverability witness satisfies Equation 299.

$$\mathfrak{h} \sim_{H,2,\kappa} \mathfrak{h}' \qquad \text{and} \qquad \mathfrak{h} \not\sim_{H,3,\kappa} \mathfrak{h}'. \tag{299}$$

The corresponding history-level equivalence gap is defined in Equation 300.

$$\mathcal{G}^{\kappa}_{H,3|2} := \left\{(\mathfrak{h}, \mathfrak{h}') \;\middle|\; \mathfrak{h} \sim_{H,2,\kappa} \mathfrak{h}',\ \mathfrak{h} \not\sim_{H,3,\kappa} \mathfrak{h}'\right\}. \tag{300}$$

Equation 300 is representation-relative. A nonempty gap can reflect a genuine retained higher-order distinction only when the interpretation relation connects that distinction to the substantive history trace. A gap generated solely by arbitrary cellular segmentation should instead be treated as a representation artifact.

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19.6 Boundary-Compatible Partial History Composition

History composition requires more than pairwise boundary resemblance. Let $\mathsf{Comp}_{\partial,3,\kappa}(\mathfrak{h}_2,\mathfrak{h}_1)$ be the declared composability predicate. It can require full-boundary equivalence, compatible subject tracking, compatible label and support typing, consistent public or authorization records, and any algebraic or categorical bridge needed by the selected realization.

The partial gluing operation on admitted histories is defined in Equation 301.

$$\#_{\partial,3,\kappa} : \mathbf{H}^{\kappa,\mathrm{adm}}_{ABC} \times \mathbf{H}^{\kappa,\mathrm{adm}}_{ABC} \rightharpoonup \mathbf{H}^{\kappa,\mathrm{adm}}_{ABC}. \tag{301}$$

Take $\mathfrak{h}_1$ from boundary Σ_0 to Σ_1 and $\mathfrak{h}_2$ from boundary Σ_1' to Σ_2. Equation 301 is defined only when $\mathsf{Comp}_{\partial,3,\kappa}(\mathfrak{h}_2,\mathfrak{h}_1)$ holds. Under that condition, the output is a history from Σ_0 to Σ_2 up to the retained boundary equivalence.

Undefined composition is an informative outcome. It can record inconsistent subject tracking, incompatible authorization states, contradictory public records, pairwise restrictions that cannot be jointly inherited, or a failure to bridge the mathematical carriers attached to the intermediate boundary. The model should identify which composability condition fails rather than replace undefinedness by an artificial neutral element.

Associativity is an additional property, not part of the meaning of gluing. If both parenthesized composites are defined and the selected boundary identifications are coherent, the realization can impose the equivalence in Equation 302.

$$(\mathfrak{h}_3\#\mathfrak{h}_2)\#\mathfrak{h}_1 \sim_{H,3,\kappa} \mathfrak{h}_3\#(\mathfrak{h}_2\#\mathfrak{h}_1). \tag{302}$$

Equation 302 should be asserted only on the common definedness domain. Parenthesization can remain historically relevant when an intermediate gluing changes which boundary data are retained for the next composition.

19.7 Pairwise Restriction of Glued Histories

Full-history gluing and pairwise restriction define two routes whenever both routes are well typed. For pair XY, the outputs to be compared are stated in Equation 303.

$$\mathsf{r}^{\kappa}_{XY,H}\big(\mathfrak{h}_2\#\mathfrak{h}_1\big) \qquad \text{and} \qquad \mathsf{r}^{\kappa}_{XY,H}(\mathfrak{h}_2)\#_{XY,\kappa}\mathsf{r}^{\kappa}_{XY,H}(\mathfrak{h}_1). \tag{303}$$

Let $\mathcal{D}^{\kappa}_{\mathsf{GlueCov},XY}$ be the common domain on which the full gluing, both pairwise restrictions, and the pairwise gluing operation are defined. The covariance predicate is defined in Equation 304.

$$\mathsf{GlueCov}^{\kappa}_{XY}(\mathfrak{h}_2,\mathfrak{h}_1) \iff \mathsf{r}^{\kappa}_{XY,H}\big(\mathfrak{h}_2\#\mathfrak{h}_1\big) \sim_{H,XY,\kappa} \mathsf{r}^{\kappa}_{XY,H}(\mathfrak{h}_2)\#_{XY,\kappa}\mathsf{r}^{\kappa}_{XY,H}(\mathfrak{h}_1), \tag{304}$$

for $(\mathfrak{h}_2,\mathfrak{h}_1) \in \mathcal{D}^{\kappa}_{\mathsf{GlueCov},XY}$. Failure of Equation 304 has several possible interpretations. The restriction or gluing rules can be inconsistent, or full gluing can depend on a retained triadic boundary condition that the pairwise route intentionally discards. The substantive interpretation must identify which information differs between the routes.

19.8 Optional Weighted History Records

A history family requires no numerical weight. When an application introduces weights, their carrier, algebraic operations, and substantive meaning must be stated independently. Let $\mathbb{W}$ be the selected weight carrier. A history-weight map is defined in Equation 305.

$$w_{\kappa} : \mathbf{H}^{\kappa,\mathrm{adm}}_{ABC} \longrightarrow \mathbb{W}. \tag{305}$$

The value $w_{\kappa}(\mathfrak{h})$ can encode cost, accessibility, model support, heuristic salience, plausibility, or another declared quantity. The term "weight" supplies no probability, welfare, credibility, or physical-amplitude semantics on its own.

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If $\mathbb{W}$ carries a composition operation $\odot$ that is defined for the weights of composable histories, a realization can impose Equation 306.

$$w_\kappa(\mathfrak{h}_2 \# \mathfrak{h}_1) = w_\kappa(\mathfrak{h}_2) \odot w_\kappa(\mathfrak{h}_1). \tag{306}$$

Equation 306 is optional and applies only on the defined gluing domain. Ordinary multiplication should be used only when the selected carrier and interpretation justify it.

Aggregation across an alternative family requires a second operation and a convergence or finiteness condition appropriate to that operation. Let $\mathfrak{H}^\kappa_{ABC}[\Sigma_-, \Sigma_+] \subseteq \mathbf{H}^\kappa_{ABC}[\Sigma_-, \Sigma_+]$ be the declared cellular history family selected for aggregation. If $\oplus$ is available, the aggregate record can be defined by Equation 307.

$$\mathcal{Z}_{3,\kappa}(\Sigma_-, \Sigma_+) := \bigoplus_{\mathfrak{h} \in \mathfrak{H}^\kappa_{ABC}[\Sigma_-, \Sigma_+]} w_\kappa(\mathfrak{h}), \tag{307}$$

where the aggregation is required to be defined on the selected family. Equation 307 should be called a probability, amplitude, partition function, or expectation only after the additional mathematical and interpretive requirements associated with those terms have been supplied.

19.9 Weight Aggregation over Pairwise Shadow Fibers

Pairwise shadows induce fibers of full histories. Because the history comparison already uses declared equivalence, the fiber should be defined relative to that equivalence rather than literal syntactic equality. For a retained pairwise shadow s, define the fiber in Equation 308.

$$\mathsf{Fib}^\kappa_{2,H}(s) := \left\{ \mathfrak{h} \in \mathbf{H}^{\kappa,\mathrm{adm}}_{ABC} \;\middle|\; \mathsf{P}^\kappa_{2,H}(\mathfrak{h}) \approx_{2,H,\kappa} s \right\}. \tag{308}$$

A fiber containing histories from more than one $\sim_{H,3,\kappa}$-class supplies a direct history-level witness of pairwise nonrecoverability, provided the surviving full distinction has a substantive interpretation through the realization relation.

If $\mathbb{W}$ supports the required aggregation, the shadow-fiber weight is defined in Equation 309.

$$W_{2,\kappa}(s) := \bigoplus_{\mathfrak{h} \in \mathsf{Fib}^\kappa_{2,H}(s)} w_\kappa(\mathfrak{h}). \tag{309}$$

Equation 309 is an aggregate over a representation-relative fiber. It becomes a probability marginal only under a separately defined normalized positive probability model.

19.10 Typed Boundary Transport Realizations

A bounded history can carry a mathematical transport between selected boundary carriers when the application supplies such a realization. Suppose the incoming and outgoing boundaries are assigned admitted algebraic carriers $\mathcal{A}^\kappa(\Sigma_-)$ and $\mathcal{A}^\kappa(\Sigma_+)$. A history-dependent algebraic transport is the partial map in Equation 310.

$$\mathcal{T}^{\mathrm{alg}}_{\mathfrak{h}} : \mathcal{D}^{\mathrm{alg}}_{\mathfrak{h}} \subseteq \mathcal{A}^\kappa(\Sigma_-) \rightharpoonup \mathcal{A}^\kappa(\Sigma_+). \tag{310}$$

Equation 310 is a mathematical lift of selected history effects. It is not identical with the substantive history, the boundary descriptor, or any subject's reconstruction of that history.

For composable histories whose algebraic lifts are defined on compatible domains, transport can satisfy the composition condition in Equation 311.

$$\mathcal{T}^{\mathrm{alg}}_{\mathfrak{h}_2 \# \mathfrak{h}_1} \simeq_{\mathcal{A}} \mathcal{T}^{\mathrm{alg}}_{\mathfrak{h}_2} \circ \mathcal{T}^{\mathrm{alg}}_{\mathfrak{h}_1}. \tag{311}$$

Equation 311 is evaluated on the common inherited domain. Failure can reflect an inadequate lift, a change of algebraic carrier, or a substantive effect that the selected algebraic representation does not preserve.

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Categorical transport requires an explicitly selected category. Let $\mathbf{BdyDyn}^{\kappa}_{ABC}$ be a model-declared category of boundary carriers and admitted boundary transformations. A history realization can then be assigned a morphism as stated in Equation 312.

$$\mathcal{T}^{\mathrm{cat}}_{\mathfrak{h}} \in \mathrm{Hom}_{\mathbf{BdyDyn}^{\kappa}_{ABC}}\big(B_-, B_+\big). \tag{312}$$

If partial maps are required, $\mathbf{BdyDyn}^{\kappa}_{ABC}$ must be chosen so that its morphism class and composition law satisfy the closure conditions described in Section 16.6. A mere collection of partially defined arrows should not be called a category without those conditions.

Subject-relative epistemic reconstruction remains a separate historical object. A, B, and C can form different reconstructions of the same realized boundary transition even when $\mathcal{T}^{\mathrm{alg}}_{\mathfrak{h}}$ or $\mathcal{T}^{\mathrm{cat}}_{\mathfrak{h}}$ is fixed. The mathematical transport therefore cannot be used as a synonym for belief update, interpretation, acknowledgment, or retrospective understanding.

19.11 Cross-Resolution History Comparison

Resolution change should act on complete bounded histories rather than only on isolated support sets. Let the declared history coarse-graining operation be the partial map in Equation 313.

$$\mathsf{Q}^{f\to c}_{H} : \mathbf{H}^{\kappa_f,\mathrm{adm}}_{ABC} \rightharpoonup \mathbf{H}^{\kappa_c,\mathrm{adm}}_{ABC}. \tag{313}$$

The map can collapse cells, aggregate labels, suppress intermediate episodes, revise support assignments, or replace boundary descriptors according to an explicit resolution rule. Calling the operation coarse-graining does not make the finer history ontologically prior.

When the history coarse-graining also induces a support pushforward, support compatibility for a retained object X is stated in Equation 314.

$$\mathsf{Q}^{f\to c}_{H,*}\Big(\mathrm{supp}_{\kappa_f}(X;\mathfrak{h}_f)\Big) \simeq_{X,\kappa_c} \mathrm{supp}_{\kappa_c}\Big(X;\mathsf{Q}^{f\to c}_{H}(\mathfrak{h}_f)\Big). \tag{314}$$

Equation 314 extends the support-compatibility discipline of Section 18.12 to complete histories.

Pairwise restriction and history coarse-graining define two further routes. Let $\mathsf{Q}^{f\to c}_{H,2}$ be the induced coarse-graining on pairwise history shadows where it exists. Strong route compatibility is stated in Equation 315.

$$\mathsf{P}^{\kappa_c}_{2,H}\Big(\mathsf{Q}^{f\to c}_{H}(\mathfrak{h}_f)\Big) \approx_{2,H,\kappa_c} \mathsf{Q}^{f\to c}_{H,2}\Big(\mathsf{P}^{\kappa_f}_{2,H}(\mathfrak{h}_f)\Big), \tag{315}$$

for every $\mathfrak{h}_f$ in the common domain of both routes. Failure of Equation 315 can reflect an inconsistent resolution scheme or a higher-order distinction that is erased differently by restriction and coarse-graining.

If history weights are intended to survive resolution change, the model can additionally impose Equation 316.

$$w_c(\mathfrak{h}_c) \simeq_W \bigoplus_{\mathfrak{h}_f:\, \mathsf{Q}^{f\to c}_{H}(\mathfrak{h}_f)\sim_{H,3,\kappa_c}\mathfrak{h}_c} w_f(\mathfrak{h}_f). \tag{316}$$

Equation 316 is optional and requires the relevant aggregation to be mathematically defined. Stability of a substantive conclusion across several justified resolutions strengthens representational robustness; it does not convert the representation into an ontological proof.

19.12 History Generation Interface

The present section takes the substantive history traces and admitted bounded cellular realizations as given. A stronger generative theory would specify rules that create, remove, or recombine the combinatorial carriers themselves. In the triadic setting, such rules could represent the formation or dissolution of a third position, the creation of a public-audience interface, institutionalization of review, coalition formation, or the appearance of a new mediation route.

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A group-field-theory-inspired construction is one possible mathematical analogy for such a future layer, but Paper II does not develop it. A genuine generative theory would require new carrier spaces, interaction kernels or rewriting rules, symmetry assumptions, and an explicit account of what cell creation or deletion represents in relation to history-bearing GR subject-spacetimes. The present spin-foam-inspired layer therefore remains a history realization rather than a history-generation theory.

The label grammar can also change historically. The present formalism assumes that the relevant label carriers have already been declared. Historical systems can generate new categories of recognition, review, mediation, public standing, or accountability. A future generative extension may therefore need rules for the emergence and revision of admissible label types as well as rules for changing the carrier complexes.

19.13 Three-Level Interpretation Limits

The spin-foam-inspired realization preserves the three-level GR architecture. At relational reality, the substantive history trace retains historically constituted relations, roles, procedures, and structures selected by the analysis. At manifestation reality, boundary and internal manifestation records encode context-dependent presentations, expressions, records, or events. Subject-relative reconstructions remain epistemic objects linked to accessible manifestations. Ontological reality remains suspended.

The bounded cellular history is therefore a representation of retained relational history rather than a declaration of ultimate social ontology. A history weight likewise acquires no moral, welfare, credibility, probabilistic, or physical interpretation without a separately justified interpretive map. Pairwise nonrecoverability at the cellular level establishes only that the selected representation preserves a full-history distinction erased by the selected pairwise restrictions; substantive significance still depends on the history-realization relation and the underlying evidence.

Several further limits remain. The admitted history family depends on selected boundaries, temporal segmentation, resolution, and equivalence criteria. Internal labels can omit tacit, embodied, institutional, environmental, or long-horizon relations. Boundary gluing depends on subject-tracking and compatibility conventions. Alternative cellular realizations can differ without corresponding to distinct substantive histories. Counterfactual alternative histories require independent substantive justification. Weight systems can be mathematically coherent while empirically unsupported. Finite bounded histories also leave open the problem of indefinitely extended continuation.

Figure 18 summarizes the resulting architecture.

The contribution of the section is consequently bounded but useful. It supplies a typed history-level carrier in which substantive ledger traces remain distinct from their cellular realizations; incoming and outgoing boundaries are explicit; full histories can be restricted to pairwise shadows; gluing is partial and condition-sensitive; weights and boundary transports are optional additional structures; and recoverability can be tested across resolution changes. Section 20 turns to finite triadic examples that test whether these distinctions remain operational in concrete volunteer and support configurations.

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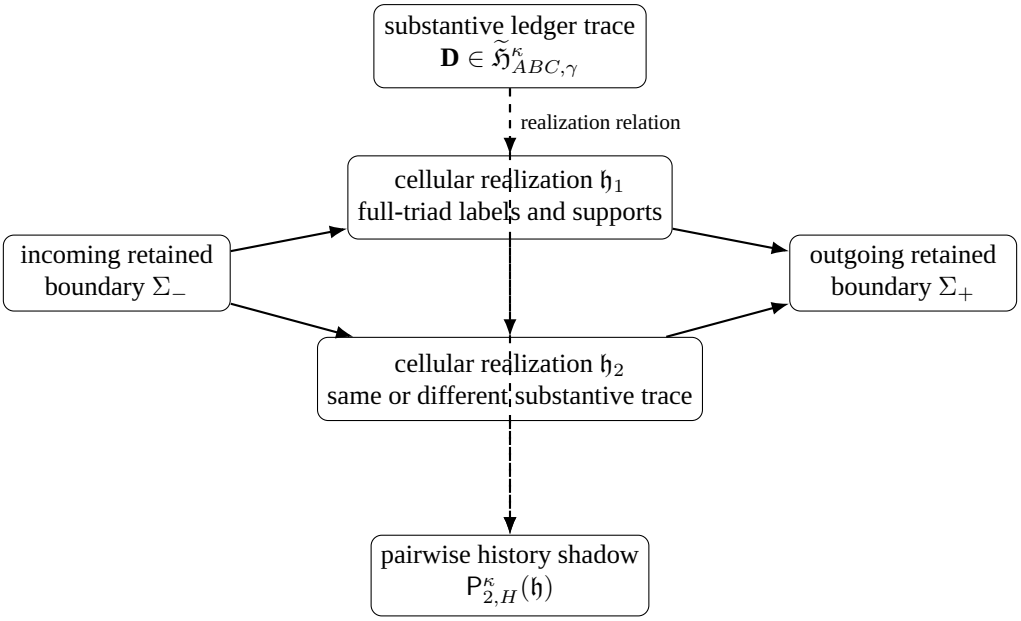

Figure 18: Typed spin-foam-inspired triadic history realization. A substantive ledger trace can admit several cellular realizations, while distinct substantively admissible traces can also share retained boundary conditions. Pairwise history shadows are derived from the cellular histories and can erase full-triad distinctions. The realization relation keeps representational multiplicity separate from substantive historical multiplicity.

20 Finite Triadic Illustrations

This section subjects the formal architecture developed in Sections 4–19 to six finite diagnostic histories. Its role is to test whether distinctions introduced separately in the manuscript remain coherent when several of them occur in one bounded case. The objective is not to supply empirical evidence for the framework. It is to expose typing failures, hidden equivalence assumptions, and overstrong inferences before the formalism is carried into later philosophical, empirical, or normative work. The method is constructive: each illustration declares a finite substantive history fragment, identifies the local distinction being tested, and states only the recoverability, procedural, or emergence conclusion licensed by that specification.

The six cases treat mediated requested support, witnessed support with public acknowledgment, accountability review by C, brokered support with a gatekeeping condition, a finite equivalence gap, and repeated interaction with role reorganization. The cases are deliberately heterogeneous. One exhibits local pairwise sufficiency, one exhibits a nonempty equivalence gap, and the others test distinctions that are logically independent of the reducibility question. None is offered as a universal volunteer script, a statistical generalization, or a completed ethical judgment.

Figure 19 previews the six cases. Solid arrows denote retained procedural or historical continuation. Dashed arrows indicate a mediated, public, review, gatekeeping, or full-triad relation whose formal meaning is stated in the corresponding subsection.

Table 23 identifies the local burden of each case. The cases are not ranked, and success on one diagnostic does not entail success on another.

20.1 Finite Diagnostic Protocol

All six illustrations inherit the same background discipline. A, B, and C are history-bearing subject-spacetimes inside one coupled triadic carrier; supporter, supportee, translator, witness, reviewer, broker, and partner-like positions are relational functions rather than fixed identities. Relational reality, man-

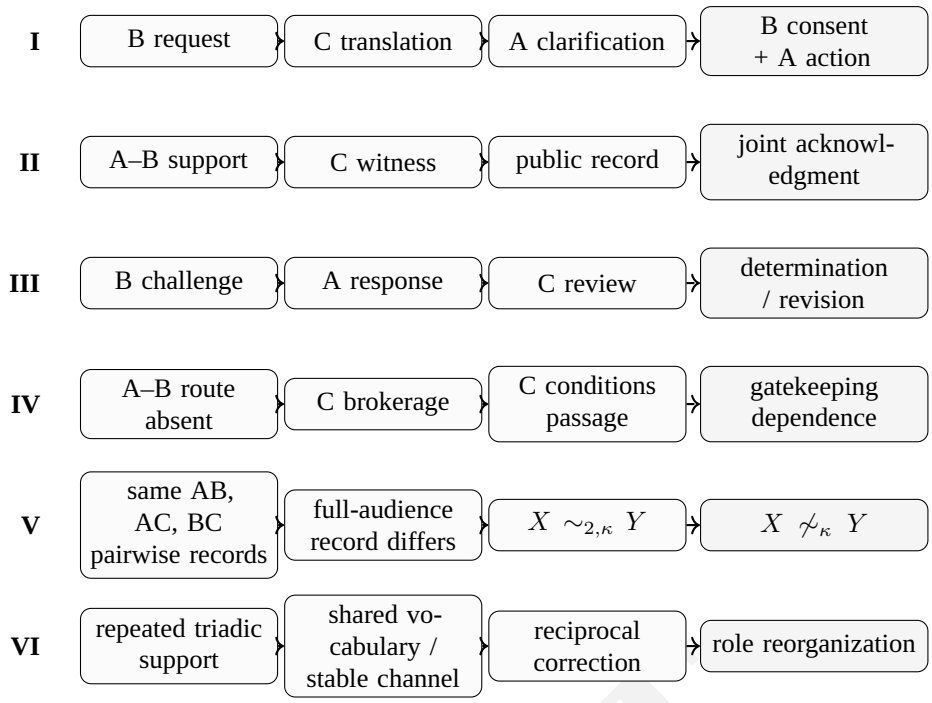

Figure 19: Six finite triadic diagnostic histories. The cases are independent test fragments rather than a developmental sequence. Dashed arrows mark mediated, public, review, gatekeeping, or full-triad relations whose interpretation is declared locally.

Table 23: Formal burden carried by the six finite triadic illustrations.

Case	Retained history pattern	Principal diagnostic burden
I	request, translation, clarification, consent, execution	third-position translation can be historically consequential while the declared local representation remains pairwise sufficient
II	support, witness, public record, joint acknowledgment	witnessing, publicity, acknowledgment, and reputation input remain separately typed around one A–B action
III	challenge, response, C-review, determination	review standing, determination, and enforcement capacity remain procedurally distinct
IV	C-mediated resource route with condition-setting	brokerage becomes gatekeeping only when route dependence and operative condition-setting are both retained
V	pairwise-equivalent records with a differing full-audience record	one explicit finite witness of pairwise nonrecoverability relative to the declared full-triad equivalence
VI	repeated support, stabilized channel, reciprocal correction, role revision	persistence can support emergence and partnership-like reorganization while functional asymmetries continue to migrate

ifestation reality, and subject-relative reconstruction remain separately typed, while ultimate ontology remains suspended. Every third-position function carries route or procedural provenance. Pairwise restrictions are derived from the full record and are never assumed sufficient by construction. Formal well-typedness supplies no moral, legal, or political verdict by itself.

For illustration $r \in \{1, \dots, 6\}$, let the retained stage sequence be $\gamma_r = (t_{r,0}, \dots, t_{r,n_r})$. The substantive history fragment used by the illustration is a member of the history carrier introduced in Section 19.1, as stated in Equation 317.

$$\mathbf{D}_r = \big(D_{r,0}, \dots, D_{r,n_r}\big) \in \widetilde{\mathfrak{H}}^{\kappa}_{ABC,\gamma_r}. \tag{317}$$

Equation 317 keeps the examples inside the enriched ledger and relation-valued revision architecture of Sections 17 and 19. Only components needed for the local test are written explicitly; omitted components remain unspecified.

Reducibility claims are localized as well. Let $\mathfrak{U}^{\kappa}_r \subseteq \mathfrak{D}^{\kappa}_3$ be the finite comparison domain declared

for illustration r. Define the local equivalence-gap relation by Equation 318.

$$\mathcal{G}^{\kappa}_{r,3|2} := \mathcal{G}^{\kappa}_{3|2} \cap \big(\mathfrak{U}^{\kappa}_{r} \times \mathfrak{U}^{\kappa}_{r}\big). \tag{318}$$

Equation 318 prevents a single finite illustration from establishing global pairwise sufficiency or global nonrecoverability. Case I will have an empty local gap under its declared representation; Case V will contain one explicit witness. The other cases do not require a reducibility result.

A cellular realization can be attached when it carries analytical work. If $\mathfrak{h}_r$ is used, adequacy is asserted only through the history-realization relation of Equation 290. The finite examples therefore do not identify a substantive history with one privileged 2-complex segmentation, and they require no spin-foam weight unless a later application supplies one.

20.2 Mediated Requested Support through a Translation Route

The first illustration represents a requested support action whose execution depends historically on a third-position translation route. B manifests a request m^{α}_{B} in source regime α. A lacks a direct reconstruction rule that makes the request actionable under the retained procedure. C applies the translation map defined in Equation 74, A asks for clarification, B confirms the clarified proposal, and A executes support action $\mathfrak{a}_1$.

The realized procedure trace is stated in Equation 319.

$$\gamma^{\mathrm{proc}}_{1} = \big(\zeta^{B}_{\mathrm{req}}, \zeta^{C}_{\mathrm{trans}}, \zeta^{A}_{\mathrm{clr}}, \zeta^{B}_{\mathrm{cons}}, \zeta^{A}_{\mathrm{exec}}(\mathfrak{a}_1)\big). \tag{319}$$

Equation 319 makes translation part of the realized procedure rather than retrospective narrative metadata.

Choose an admissible C-relative translation resource record $c_{t_1} \in \mathfrak{R}^{\mathsf{T}}_{C,t_1}$ and suppose the source pair lies in the domain of the translation map. The realized translation instance is given by Equation 320.

$$\mathsf{T}^{\alpha\Rightarrow\beta}_{C:t_1}\big(m^{\alpha}_{B}, c_{t_1}\big) = m^{\beta}_{B|C}. \tag{320}$$

Equation 320 does not assert semantic identity between m^{α}_{B} and $m^{\beta}_{B|C}$. Which distinctions the translation is required to preserve remains governed by the translation-preservation specification of Section 8.4.

At execution stage t_4, the distributed provenance relation can retain at least the contributions in Equation 321.

$$\{(B, \mathsf{request}), (C, \mathsf{translate}), (A, \mathsf{execute})\} \subseteq \Pi^{\kappa}_{3,t_4}(\mathfrak{a}_1). \tag{321}$$

Equation 321 shows that an action manifested as A-to-B support can possess distributed triadic provenance.

Now choose the pairwise restrictions so that the A–C shadow retains C's translated manifestation and its route identifier, the B–C shadow retains B's source request and C's translation provenance, and the A–B shadow retains clarification, consent, and execution. Let $\mathfrak{U}^{\kappa}_{1}$ contain only the finite full configurations admitted by this specification. Suppose configurations with equivalent three pairwise shadows are also equivalent under the selected full criterion. The local result is stated in Equation 322.

$$\mathcal{G}^{\kappa}_{1,3|2} = \varnothing. \tag{322}$$

Equation 322 is a local recoverability result, not a theorem about translation in general. It demonstrates that historical importance of a third position and pairwise nonrecoverability are logically distinct.

The same case also admits historical scaffolding. Repeated translations can stabilize a shared A–B vocabulary. If direct communication later becomes possible, C's translation function can become unnecessary in that domain while the earlier C-mediated history remains part of the relation's provenance.

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20.3 Witnessed Support with Public Acknowledgment

The second illustration keeps the material A–B support action fixed while changing its third-position manifestation and audience structure. A performs action $\mathfrak{a}_2$ with B's retained authorization. C witnesses the event, contributes an attestation to a shared record, and the record is presented to audience $\mathcal{U} = \{A, B, C\}$. All three then acknowledge the record under the selected first-order criterion.

At stage t_1, the C-relative witness record is stated in Equation 323.

$$\mathfrak{W}^{\kappa}_{C|AB,t_1}(\mathfrak{a}_2) = \Big(\mathsf{Acc}^{\kappa}_{C,t_1}(\mathfrak{a}_2), \mathsf{Ret}^{\kappa}_{C,t_1}(\mathfrak{a}_2), \mathsf{Att}^{\kappa}_{C,t_1}(\mathfrak{a}_2), \mathsf{Prov}^{\kappa}_{C,t_1}(\mathfrak{a}_2)\Big). \tag{323}$$

Equation 323 provides witness access, retention, attestation, and provenance. It does not itself satisfy the audience-indexed publicity criterion.

Let m_2^{pub} be the later shared record at stage t_2. The public and acknowledgment conditions are stated in Equation 324.

$$\mathsf{Pub}^{\kappa}_{\mathcal{U},t_2}\big(m_2^{\mathrm{pub}}\big) \quad \wedge \quad \mathsf{JAck}^{(1),\kappa}_{\mathcal{U},t_2}\big(m_2^{\mathrm{pub}}\big). \tag{324}$$

Equation 324 is stronger than C's witnessing and stronger than coincident accessibility. It remains weaker than an unbounded common-knowledge condition.

C's attestation can become an input to A's audience-indexed reputation record only when the circulation, provenance, and historical conditions of Equation 100 are also supplied. The case therefore does not infer reputation from one witness judgment. It shows the ordered distinction among witness access, witness attestation, audience-level presentation, joint acknowledgment, and possible later reputational circulation.

This distinction also matters for later accountability. A privately retained witness record may support review without creating a public record, while a public record may exist without C having adequate witness provenance. The two routes remain separately inspectable.

20.4 Third-Position Accountability Review

The third illustration begins after an A–B support action. B challenges one procedural aspect while leaving the material occurrence of support uncontested. A responds. C possesses operative review standing under the retained procedure and no operative enforcement standing for the same object. C reviews the record and issues a determination that clarification should be reopened.

The accountability fragment at stage t_2 is stated in Equation 325.

$$\mathfrak{Acct}^{\kappa}_{C:B\rightsquigarrow A,t_2}(q_3) = \Big(q_3, \mathsf{Clm}^{\kappa}_{B\rightsquigarrow A,t_2}, \mathsf{Ans}^{\kappa}_{A,t_2}, \mathsf{St}^{\mathrm{rev},\kappa}_{C,t_2}, \mathsf{Revw}^{\kappa}_{C,t_2}, \mathsf{Det}^{\kappa}_{C,t_2}, \Pi^{\mathrm{acct}}_{t_2}\Big). \tag{325}$$

Equation 325 instantiates the accountability record of Section 11.1. The local standing specification classifies C's review-standing record as operative and supplies no operative enforcement-standing basis for q_3. Because standing records are typed and can be graded or contested, the distinction is stated through the declared local interpretation rather than by treating standing itself as a Boolean predicate.

Suppose C's review map is defined on the retained claim-response-evidence record and produces determination d_3^{reopen}. The finite review result is stated in Equation 326.

$$\mathsf{Revw}^{\kappa}_{C,t_2}(q_3, \mathsf{evid}_3) = d_3^{\mathrm{reopen}}, \qquad d_3^{\mathrm{reopen}} \in \mathfrak{D}^{\kappa}_{\mathrm{det}}. \tag{326}$$

Equation 326 records a determination output without identifying it with truth, consensus, or enforcement.

Let h_{3,t_2} be the realized procedure prefix after the determination. Under the local rule, reopening clarification is admitted while a sanction-like continuation is unavailable. Equation 327 states the procedural distinction.

$$\zeta_{\mathrm{clr}} \in \mathrm{Cont}^{\kappa}_{3,\mathrm{proc}}(h_{3,t_2}), \qquad \zeta_{\mathrm{sanction}} \notin \mathrm{Cont}^{\kappa}_{3,\mathrm{proc}}(h_{3,t_2}). \tag{327}$$

Equation 327 makes the practical effect of the review visible without converting review standing into enforcement authority.

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A and B can still disagree about the relational meaning of the earlier action after C's procedurally valid determination. The finite case therefore separates reviewability, determination, continuation change, epistemic agreement, and legitimacy.

20.5 Brokerage Dependence with a Gatekeeping Condition

The fourth illustration tests the boundary between brokerage and gatekeeping. A seeks to route resource-type object r to B. At stage t_0, the selected model retains no direct A–B route for r, while a brokered A–C–B route exists. C also possesses an operative condition-setting relation on that brokered route.

The local route-dependence and condition-setting specification is stated in Equation 328.

$$\mathsf{RouteDep}^{\kappa,r}_{C|AB,t_0} \quad \wedge \quad \mathsf{Dir}^{\kappa,r}_{AB,t_0} = \varnothing \quad \wedge \quad \mathrm{Dom}\big(\mathsf{Cond}^{\kappa,r}_{C,t_0}\big) \neq \varnothing. \tag{328}$$

Equation 328 satisfies the strong gatekeeping criterion of Equation 126. The resulting local classification is stated in Equation 329.

$$\mathsf{Gate}^{\kappa,r}_{C|AB,t_0}. \tag{329}$$

Equation 329 is stronger than the existence of a brokered route because C can condition passage and no direct alternative is retained at the selected stage.

Suppose the operative condition is a documentation requirement and let $h_{4,1}$ denote the procedure prefix after the requirement is imposed but before it is satisfied. Let $h_{4,2}$ denote the prefix after satisfactory documentation is retained. The passage continuation changes as stated in Equation 330.

$$\zeta_{\text{pass}}(r) \notin \mathrm{Cont}^{\kappa}_{3,\text{proc}}(h_{4,1}), \qquad \zeta_{\text{pass}}(r) \in \mathrm{Cont}^{\kappa}_{3,\text{proc}}(h_{4,2}). \tag{330}$$

Equation 330 records the operative consequence of the condition without introducing an unjustified ordering over whole continuation families.

The case establishes gatekeeping under the declared route architecture. It does not establish coercion, domination, or injustice. Those interpretations require additional trajectory-shaping and normative criteria. Gatekeeping can also dissolve historically: if a direct A–B route for r later becomes available, the strong criterion can fail even while C remains relevant as a witness, reviewer, translator, or another third-position function.

20.6 Pairwise-Equivalent Configurations with a Full-Audience Difference

The fifth illustration provides one explicit finite witness of pairwise nonrecoverability. Select two full configurations $X, Y \in \mathfrak{U}^{\kappa}_5$ whose three pairwise restrictions are equivalent under the componentwise criterion of Section 6.1. In both configurations, A and B agree on a support action, C witnesses it, and each pair retains the same local manifestation and acknowledgment data. The selected full representation additionally contains an audience-level presentation component that distinguishes one shared presentation to $\{A, B, C\}$ from three separate pairwise presentations.

Let $\mathsf{AudPres}^{\kappa}_{ABC}(Z)$ denote, only for this finite example, the component of full configuration Z containing presentation objects addressed to $\{A, B, C\}$ as one audience. Choose p_{ABC} so that the full-audience difference is stated in Equation 331.

$$\mathsf{AudPres}^{\kappa}_{ABC}(X) = \varnothing, \qquad \mathsf{AudPres}^{\kappa}_{ABC}(Y) = \{p_{ABC}\}. \tag{331}$$

Equation 331 defines the one retained full-triad distinction used by the example. The extractor is local to the diagnostic case and introduces no universal ontology of publicity.

The selected pairwise restrictions discard that audience-level component while preserving the three pairwise records. Their equivalence is stated in Equation 332.

$$\begin{aligned} \mathsf{r}^{\kappa}_{AB}(X) &\approx_{AB,\kappa} \mathsf{r}^{\kappa}_{AB}(Y), \\ \mathsf{r}^{\kappa}_{AC}(X) &\approx_{AC,\kappa} \mathsf{r}^{\kappa}_{AC}(Y), \\ \mathsf{r}^{\kappa}_{BC}(X) &\approx_{BC,\kappa} \mathsf{r}^{\kappa}_{BC}(Y). \end{aligned} \tag{332}$$

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Equation 332 yields the induced pairwise equivalence in Equation 333.

$$X \sim_{2,\kappa} Y. \tag{333}$$

Let the declared full-equivalence relation preserve the existence of the shared-audience presentation component. The full configurations then satisfy Equation 334.

$$X \not\sim_{\kappa} Y. \tag{334}$$

Equations 333 and 334, together with $X, Y \in \mathfrak{U}_5^{\kappa}$, give the local gap witness in Equation 335.

$$(X, Y) \in \mathcal{G}_{5,3|2}^{\kappa}. \tag{335}$$

Equation 335 is representation-relative. A richer pairwise encoding could carry a common event identifier or another sufficient cross-pair linkage and thereby remove the local gap. The example therefore demonstrates a declared loss of discrimination, not metaphysical irreducibility of publicity.

A cellular history-level witness requires an additional step. Suppose substantive traces $\mathbf{D}_X$ and $\mathbf{D}_Y$ containing the relevant configurations admit cellular realizations $\mathfrak{h}_X$ and $\mathfrak{h}_Y$ through Equation 290, and suppose the full-audience distinction survives the interpretation relation rather than arising from arbitrary segmentation. If the pairwise history shadows are equivalent while the full histories are not, the history-level result is stated in Equation 336.

$$\mathfrak{h}_X \sim_{H,2,\kappa} \mathfrak{h}_Y, \qquad \mathfrak{h}_X \not\sim_{H,3,\kappa} \mathfrak{h}_Y \quad \Longrightarrow \quad (\mathfrak{h}_X, \mathfrak{h}_Y) \in \mathcal{G}_{H,3|2}^{\kappa}. \tag{336}$$

Equation 336 keeps the Section 19 distinction between a substantive higher-order difference and a merely representational cellular difference.

20.7 Repeated Triadic Support with Role Reorganization

The sixth illustration tests persistence, emergence, and migration of third-position functions. A, B, and C participate in four bounded support episodes over interval $I = \{t_0, \ldots, t_8\}$. Early episodes depend on C for translation and mediation. Later episodes use a shared vocabulary, B increasingly corrects A's local assumptions, A and C both rely on B's first-person information, and C's function shifts toward occasional review.

Let E_{chan} denote a candidate persistent communication/mediation channel and E_{voc} a candidate shared-vocabulary record. Under declared witness specifications χ_{chan} and χ_{voc}, suppose the emergence predicates in Equation 337 are satisfied.

$$\mathsf{Emerg}_{3,\chi_{\text{chan}}}^{\kappa}\big(E_{\text{chan}}; H_{ABC,I}^{\kappa}\big) \quad \wedge \quad \mathsf{Emerg}_{3,\chi_{\text{voc}}}^{\kappa}\big(E_{\text{voc}}; H_{ABC,I}^{\kappa}\big). \tag{337}$$

Equation 337 records persistence, organization, and efficacy relative to the declared witnesses. It does not claim permanence or ontological fundamentality.

The role structure can also change. At t_0, retain differentiated supporter-, supportee-, and mediator-like positions, written (π_S, π_R, π_M). At t_8, suppose all three subjects possess consequential routes for contribution, correction, and revision under the restricted partnership criterion of Section 15.9. The candidate trajectory is stated in Equation 338.

$$(\pi_S, \pi_R, \pi_M)_{t_0} \rightsquigarrow_{\chi_P}^{\text{role}} \mathsf{Part}_{ABC,t_8}^{\kappa'}. \tag{338}$$

Equation 338 represents historical reorganization. It does not imply equality in competence, resources, authority, recognition, or contribution.

To display function migration without treating importance as a scalar, define a case-specific necessity marker $\delta_{C,\phi}^{(6)}(t) \in \{0, 1\}$, where value 1 means that the selected procedure at stage t requires C to perform

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function type ϕ for the retained route to remain available. This marker is local to the finite test. The migration pattern is stated in Equation 339.

$$\delta^{(6)}_{C,\mathsf{translate}}(t_0) = 1, \qquad \delta^{(6)}_{C,\mathsf{translate}}(t_8) = 0, \qquad \delta^{(6)}_{C,\mathsf{review}}(t_0) = 0, \qquad \delta^{(6)}_{C,\mathsf{review}}(t_8) = 1. \tag{339}$$

Equation 339 records domain-specific procedural necessity, not a magnitude of C's importance. The example allows successful translation scaffolding to reduce later translation dependence while the evolving relation creates a new review function.

Neither emergence nor partnership settles reducibility. The stabilized channel and role reorganization can be pairwise recoverable under one representation and nonrecoverable under another. A separate application of Equation 318 is required before either conclusion is drawn.

20.8 Comparative Diagnostics across the Six Illustrations

The six finite histories test different interfaces of the triadic architecture. Case I shows that third-position translation can be historically constitutive of an action history while remaining pairwise recoverable under sufficiently rich restrictions. Case II separates witness access, attestation, audience-level publicity, acknowledgment, and later reputational circulation. Case III separates review standing, determination, continuation change, enforcement capacity, and consensus. Case IV distinguishes brokerage from gatekeeping through route dependence plus operative condition-setting. Case V supplies one explicit pairwise/full equivalence-gap witness. Case VI tests persistence, emergence, partnership-like reorganization, and migration of third-position functions.

Several structural conclusions follow within the limited scope of the test suite. Triadic organization is richer than a triangle graph because relevant objects can be route-indexed, audience-indexed, procedure-indexed, support-distributed, or history-indexed. Materially similar A–B actions can therefore belong to different full histories when their translation, witness, publicity, accountability, brokerage, or role-provenance structures differ. Pairwise nonrecoverability remains a separate diagnostic: substantial third-position involvement does not entail a nonempty equivalence gap, while one bounded full-audience distinction can generate a local gap without being persistent or emergent.

The cases also test the representational restraint established in Sections 18 and 19. A finite case may admit several cellular realizations, and successful cellular verification would establish only consequences of the chosen encoding. It would not establish empirical adequacy, normative validity, or a literal cellular ontology of social relations. Conversely, a cellular difference cannot serve as a substantive triadic witness unless the realization relation connects the difference to the retained substantive history.

The section therefore functions as a finite formal test suite for Paper II. The examples show that the current architecture can represent mediation, publicness, accountability, gatekeeping, local pairwise nonrecoverability, emergence, and role reorganization without collapsing those objects into one generic notion of thirdness. Section 21 turns from the diagnostic cases to the philosophical implications of third positions, mediation, publicness, and relational order, while later papers retain the stronger questions of justice, ethics, power, psychoanalysis, aesthetics, and political economy.

21 Philosophical Interpretation of Triadic Relational Architecture

This section develops the philosophical interpretation of the formal architecture established in Paper II. Its role is downstream of the construction and diagnostic sections: it asks what can reasonably be inferred from the typed triadic model once the distinctions among third-position function, pairwise recoverability, mediation, publicness, reputation, accountability, emergence, and history have already been made explicit. The objective is to articulate a limited philosophy of triadic relational organization without converting model-relative results into claims about ultimate ontology or treating formal nonrecoverability as a proof of metaphysical irreducibility. Table 24 previews the three-level status of the major objects interpreted below. The discussion then develops indexed third positions, model-relative relational sufficiency, mediation, publicness, reputational identity, subject constitution, distributed relational order,

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Table 24: Three-level status of major triadic philosophical objects. Relational-reality claims remain model-relative, manifestation forms are context-dependent, and ultimate ontology is suspended throughout Paper II.

Object	Relational-reality interpretation	Manifestation-reality forms	Ontological status
Third-position function	historically situated function relative to a focal relation, with typed access, capacity, standing, or route conditions	mediation acts, testimony, translations, brokerage episodes, review participation	ultimate status suspended
Mediation channel	organized relation that can alter later continuations, procedures, or interpretive possibilities	meetings, messages, translations, turn-taking rules, clarification procedures	ultimate status suspended
Publicness; reputation	audience-indexed presentation, durable circulation, or recognition structure that can condition later action	notices, records, acknowledgments, labels, repeated attributions, public responses	ultimate status suspended
Relational order; shared norm	historically stabilized organization that conditions routes, expectations, procedures, judgments, or revisions	routines, role labels, standing rules, review practices, recurring vocabulary, public invocations	ultimate status suspended

norm stabilization, and epistemic situatedness. The method is interpretive and reconstructive: philosophical claims are constrained by the formal distinctions already established and remain revisable where the model leaves alternatives open.

A second discipline governs the entire section. The three-level GR architecture introduced in Section 4 remains in force. Ultimate ontological reality is suspended. Positive claims in the present section concern relational reality when a structure is historically trackable, sufficiently organized, and generatively or causally efficacious under the selected model. Manifestation reality concerns the context-dependent forms through which such structures become presented, accessible, expressed, recorded, enacted, or interpreted. An observer's or participant's reconstruction of a structure remains an epistemic object and is not automatically identical with either relational reality or manifestation reality.

21.1 Three-Level Status of Triadic Objects

The philosophical significance of the triadic formalism depends on keeping the three GR levels separate. A mediator role, public record, reputation structure, accountability procedure, coalition, or shared norm can be treated as relationally real within the model when the relevant historical and efficacy burdens are met, especially the emergence criterion in Equation 179. The same structure can possess many manifestations and can be reconstructed differently by A, B, C, or an external analyst. None of those modeling successes establishes the ultimate ontology of the social world. The table is analytical rather than exhaustive; each row inherits the more precise typing and historical conditions defined in the corresponding formal sections.

The distinction matters because manifestation-level stability can exceed relational warrant, and relational persistence can survive manifestational change. A role label can remain publicly visible after the relation that once sustained it has dissolved. Conversely, a mediation channel can remain practically effective even when participants cease to describe it explicitly as mediation. The three-level architecture therefore supports philosophical interpretation while blocking a direct inference from appearance, naming, or successful mathematical representation to ultimate ontology.

21.2 Relational Status of Third Positions

Section 8 defines a third position relative to a focal pair through the indexing form $Z \mid XY$. The philosophical consequence is that Paper II does not require a permanent social kind called "the third party." A subject occupies a third position only relative to a declared focal relation, historical stage, and

retained function. C can mediate A–B while A simultaneously occupies a third position relative to B–C in another domain. The position can migrate, disappear, divide among several functions, or become unnecessary after the relation changes.

At relational reality, third-position structure therefore concerns organized relations and functions rather than an intrinsic property of a subject. At manifestation reality, the same structure can appear through a mediator title, a witness statement, a translated document, a brokerage route, a review meeting, or another context-specific form. Ontological suspension prevents either the role label or the formal index $Z \mid XY$ from being treated as evidence that social reality is ultimately composed of triadic positions.

This relational interpretation also prevents normative inflation. A third position can widen a continuation family, create a route for correction, or make an event publicly reviewable. It can also narrow access, intensify surveillance, stabilize dependence, or condition participation through gatekeeping. The strong gatekeeping condition in Equation 126 already demonstrates that occupying an intermediate route is insufficient for the stronger power-like claim. Third-position organization is therefore generative in the limited sense that it can alter relational possibilities; its ethical or political quality requires separate evaluation.

The same restraint applies to irreducibility. Third-position relevance and pairwise nonrecoverability are distinct. A mediator can be historically consequential while the full configuration remains recoverable from sufficiently rich A–C and B–C records. Conversely, a full-audience object can be pairwise nonrecoverable even when no enduring mediator identity exists. Paper II therefore uses "third position" as a relational index and reserves stronger claims for the specific formal burdens that establish them.

21.3 Model-Relative Sufficiency of Pairwise Description

The equivalence-gap analysis of Section 6 supports a philosophy of relational scale selection rather than a universal doctrine of dyadic insufficiency. The relation $\mathcal{G}^{\kappa}_{3|2}$ records whether the selected pairwise shadow loses distinctions that remain relevant under the selected full-equivalence criterion. Pairwise sufficiency therefore depends on the comparison domain, analytical resolution κ, retained restrictions, and declared equivalences. A change in any of those can change the recoverability judgment without changing the historical episode itself.

This result places a methodological burden on relational theory. The unit of analysis should be chosen according to the distinctions the inquiry intends to preserve. A dyadic description can be fully adequate for a question about direct resource transfer while remaining inadequate for a question about common-audience acknowledgment, distributed provenance, coalition structure, or a history in which a third-position intervention changed later possibilities. A richer unit is justified when a specified diagnostic fails to factor through the coarser representation, not merely because a larger network exists around the focal relation.

At relational reality, the point concerns which structures are retained as efficacious within the chosen model. At manifestation reality, the same episode can present itself as apparently dyadic because only A and B are visibly acting while a third-position route, public record, or standing relation conditions what their action means or what can follow. Ontological suspension again limits the conclusion: model-relative failure of pairwise recovery establishes a representational burden for the inquiry, not a proof that reality is ultimately constituted by irreducible triads.

This interpretation also clarifies why the finite illustrations in Section 20 use local comparison domains. One case can establish local sufficiency or provide a local nonrecoverability witness, but it cannot settle the proper relational scale for every volunteer relation. The philosophical lesson is therefore one of disciplined adequacy: relational granularity should be justified by the questions, distinctions, and historical effects the analysis is meant to retain.

21.4 Mediation as Continuation Reorganization

Mediation has a stronger philosophical significance in Paper II than the transmission of information through an intermediary. Section 8 permits a mediation to change the continuation family of the focal

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relation under Equation 72. Once that condition is met, mediation participates in the organization of later possibility. It can create a clarification route, alter the form in which refusal becomes communicable, reorganize turn-taking, establish a vocabulary, separate substantive disagreement from procedural disagreement, or modify the conditions under which A and B can continue to interact.

At relational reality, mediation is therefore potentially constitutive of a later A–B relation when the mediated procedure has persistent or efficacious consequences. At manifestation reality, mediation can appear as a conversation, translated message, facilitated meeting, revised form, procedural pause, or other local intervention. These manifestations do not by themselves establish an enduring mediation structure; persistence and efficacy remain historical questions. Ultimate ontology remains suspended, so the language of constitution describes model-relative relational generation rather than an assertion that mediation is an ontological substance.

The historical architecture of Section 17 adds an important temporal consequence. Mediation can be self-displacing. Repeated translation can generate a shared vocabulary; facilitated clarification can establish direct A–B procedures; a temporary review route can teach participants how to reopen disputed questions without C. In such cases, successful third-position activity can reduce the later necessity of the third position. The opposite trajectory is also possible: recurrent mediation can become a durable channel, and a relation can become practically dependent on C's continuing availability.

Neither trajectory is normatively decisive. Self-displacement can accompany abandonment or loss of useful support, while durable mediation can preserve access for subjects who continue to need translation, accessibility, or procedural assistance. The ethically relevant questions concern authorization, competence, practical alternatives, dependence, revisability, and the effects on the subjects' continuation families. Paper II supplies the architecture in which those later evaluations can be stated without defining mediation itself as either emancipatory or paternalistic.

21.5 Publicness as Audience-Indexed Relational Structure

Section 10 defines publicity relative to a declared audience and a shared presentation relation in Equation 94. This formal choice has a direct philosophical implication: publicness is not exhausted by the number of observers who happen to possess access to the same manifestation. A public object exists, within the model, when a manifestation is organized as presented to an audience as such. Uptake, acknowledgment, higher-order awareness, common knowledge, truth, and legitimacy remain separate questions.

At relational reality, publicness can become historically efficacious when audience structure, durable records, acknowledgment routes, or public recognition condition later action. A refusal entered into a reviewable record can alter what later claims are procedurally available. A support action publicly attributed to A can affect future expectations even if the material transfer is unchanged. A correction can revise the status of a record while leaving earlier reputational effects partly intact. At manifestation reality, these structures can appear through notices, posts, meeting minutes, testimonies, archives, labels, public ceremonies, or informal circulation. No particular manifestation form is identical with publicness itself.

The non-entailments formalized in Equation 98 are philosophically important because they block a common collapse between publicity and shared belief. A public realm, in the restricted sense available to Paper II, involves organized audience relations and histories of presentation, acknowledgment, correction, and possible invocation. It does not require that each participant actually attends to the object, agrees about it, or possesses an infinite hierarchy of mutual knowledge.

Publicness can also be pairwise recoverable or pairwise nonrecoverable. A durable public record may be reconstructible from rich pairwise archives. A full-audience acknowledgment object can instead supply a nonrecoverability witness when every pairwise restriction discards the operative common-audience structure. The philosophical status of publicness therefore should not be identified with irreducible thirdness. Publicness is one family of relational organization whose recoverability must be tested under the selected representation.

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21.6 Reputational Formation of Relational Identity

The reputation record defined in Equation 100 separates a circulating historical structure from one person's isolated attribution. This distinction supports a relational account of identity formation without reducing identity to public opinion. A repeated attribution can become relationally consequential when circulation, persistence, provenance, and later effects stabilize it sufficiently to change access, scrutiny, trust, standing, role assignment, or expected conduct.

Volunteer relations make the mechanism visible. Public recognition of A as "the helper" can stabilize expectations that A should continue to provide support. Public identification of B as "the person in need" can make a context-specific support relation appear to define B more generally. Recognition of B's competence, contribution, refusal, or later independence can destabilize that earlier position. None of these trajectories requires the reputation to be accurate, deserved, consensual, or normatively legitimate in order for it to become generatively efficacious.

At relational reality, the relevant object is the organized reputation or recognition structure and its historical effects. At manifestation reality, it can appear through praise, criticism, labels, introductions, rankings, stories, records, invitations, exclusions, or patterns of deference. A subject's own reconstruction of those manifestations can diverge from the relational structure that has become effective in other subjects' actions. Ultimate ontology remains suspended; the claim is that such historically constituted relational objects can be modeled as causally or generatively relevant without asserting that social identity is nothing more than reputation.

The distinction is especially important for the later subject-formation paper. Paper II establishes conditions under which role attributions can leave the private dyad, circulate through third positions or audiences, and feed back into later relational possibilities. Whether and how subjects internalize, resist, reinterpret, strategically use, or transform those positions requires a richer theory of co-formation than the present triadic formalism supplies.

21.7 Subject Distinction in Triadic Constitution

Paper I already treats A and B as history-bearing relational subject-spacetimes rather than isolated points. Paper II extends the same commitment without dissolving the distinctions needed for responsibility, consent, standing, action provenance, or refusal. A, B, and C remain separately trackable subjects in the formalism, yet the conditions under which each subject interprets and acts can be partly generated through their relations.

A third position makes that relational constitution more visible. A mediator can change the vocabulary through which A interprets B. A witness can alter A's anticipation of later accountability. A public audience can change the practical significance of B's refusal. A coalition can alter the routes that A experiences as available. A translation can make a previously inaccessible manifestation actionable without making C the author of B's underlying expression. The relational environment therefore participates in the constitution of practical and interpretive positions while preserving subject distinction.

The three-level architecture is again decisive. At relational reality, the claim concerns effective relations among history-bearing subjects and the continuation structures those relations help generate. At manifestation reality, self–other positioning can appear through forms of address, role labels, spatial arrangements, translated expressions, public expectations, procedural turns, or patterns of response. Ontological suspension blocks the stronger claim that the self is ultimately nothing beyond relation. Paper II requires only the more limited proposition that independent formal trackability does not entail independent historical constitution.

This limited relational nonseparability also preserves accountability. Explaining that A's action was generated through a reputational, institutional, or mediation structure does not automatically remove A's responsibility, and identifying C's constitutive role does not automatically assign C responsibility for every downstream action. Responsibility remains a typed normative and procedural question. The philosophical gain is that action can be attributed without pretending that its conditions were generated by the visible actor alone.

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21.8 Distributed Formation of Relational Order

The emergent structures of Section 15 provide a restrained route to a broader concept of relational order. Paper II need not introduce a new aggregate "order object" to make this point. If routes, public records, standing, review procedures, coalitions, shared vocabularies, role ecologies, or public norms satisfy the declared persistence, organization, and efficacy conditions, their mutual historical conditioning can produce a durable pattern of possibilities and expectations that no single command exhausts.

At relational reality, distributed order refers to that historically stabilized organization and its effects on later continuations. At manifestation reality, it can appear through routines, procedural forms, repeated meeting structures, informal role expectations, access rules, public records, recurring vocabulary, spatial arrangements, or institutionalized sequences of review. Different manifestations can sustain similar relational organization, and similar-looking routines can belong to very different relational orders when provenance, standing, practical alternatives, or continuation effects differ. Ultimate ontology remains suspended; "order" is an analytical synthesis of relationally efficacious organization, not a claim about the final furniture of social reality.

The historical composition developed in Section 17 adds path dependence to this interpretation. An initially modest route or record can condition later branch availability; a temporary coalition can become a recurrent accountability bloc; a public attribution can persist after correction; a mediation channel can either disappear through successful direct coordination or become institutionally indispensable. Relational order is therefore better treated as historically generated organization than as a static arrangement of roles.

Distributed formation carries no automatic democratic or emancipatory implication. An exclusionary coalition, captured review procedure, gatekeeping route, or reputational hierarchy can also become persistent and efficacious. The later justice and political-economy papers therefore need to ask how an order was generated, whose revisions remain practically consequential, which continuations it opens or closes, and whether its persistence depends on extractive or exclusionary relations. Paper II establishes the structural possibility of distributed order while leaving its normative appraisal open.

21.9 Historical Stabilization of Normative Objects

The shared-norm condition in Equation 190 provides a more precise philosophical basis for discussing norm formation than a simple sequence from private judgment to public agreement. Paper II requires a retained normative object, orientation by the relevant positions, and efficacy in later judgment or action. Complete agreement about interpretation, justification, scope, or validity is unnecessary. A norm-like structure can therefore become relationally consequential while remaining contested.

Third positions can participate in this stabilization in several ways. Witnessing can preserve a disputed event for later invocation. Translation can make an evaluative distinction shareable across manifestation regimes. Public presentation can make a candidate norm contestable by a declared audience. Review standing can create a route through which the norm is applied, challenged, or revised. Repetition across history can then stabilize orientation to the normative object. None of these mechanisms is individually necessary or sufficient for norm formation across every domain, and Paper II does not claim that every norm requires a literal three-person origin.

At relational reality, a norm-like object is warranted when the retained historical structure is sufficiently persistent, organized, and efficacious under the selected criteria. At manifestation reality, the same object can appear through explicit rules, recurring phrases, precedents, routines, sanctions, permissions, role expectations, or shared evaluative vocabulary. Ontological suspension remains in place, and normative validity remains distinct from relational existence. A harmful or unjust norm can be relationally real; a morally compelling judgment can fail to stabilize into an effective relational norm.

This distinction is important for the later justice and ethics papers. Those papers can evaluate the genesis, authority, scope, revisability, and substantive quality of norms without first denying their relational efficacy. Paper II supplies a way to describe how normative objects can be historically generated and causally operative while keeping the questions of ultimate ontology and justified normativity separately

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open.

21.10 Epistemic Situatedness of Third Positions

The route-sensitive epistemic architecture of Section 9 gives the third position additional routes without granting an Archimedean standpoint. C can possess information unavailable to A or B, but C also has a route history, manifestation access, interpretive resources, theory-of-mind errors, interests, role investments, and standing limits. A witness can misremember, a translator can collapse distinctions, a mediator can privilege one vocabulary, a reviewer can possess standing without adequate evidence, and a public audience can amplify a false attribution.

At relational reality, epistemic routes, witness relations, translation resources, and review structures can be real and consequential parts of the triadic configuration. At manifestation reality, evidence and claims become available through particular records, utterances, documents, observations, translations, or public presentations. Subject-relative reconstruction remains a further epistemic layer: access to a manifestation does not guarantee correct reconstruction of the relational object to which it may be evidence. Ontological suspension prevents epistemic success from becoming a shortcut to ultimate ontology.

The philosophical value of a third-party procedure therefore lies in the possibility of additional routes, comparison, contestation, correction, and revision, not in an assumed view from nowhere. Multiple situated routes can improve an inquiry under appropriate conditions while still reproducing shared error, institutional bias, informational dependence, or correlated interpretation. Later epistemic-justice and governance work can therefore evaluate the quality and distribution of routes rather than equating third-party status with neutrality.

This point also clarifies why triadic modeling does not automatically outrank dyadic or first-person access. A direct A–B manifestation can be more informative than a distorted mediated route; C's testimony can be indispensable in another case; a durable public record can later outperform all three recollections for a narrow procedural question while remaining incomplete for lived experience. Methodological pluralism therefore applies inside the triadic architecture itself.

21.11 Programme Interfaces

The philosophical interpretation developed in this section is intentionally limited to the middle layer supplied by Paper II. The subject co-formation paper can examine how repeated triadic relations alter subject boundaries, practical horizons, role identifications, and capacities. The psychoanalytic paper can investigate whether third positions, audiences, reputational gazes, and recurring mediation structures participate in desire, lack, repetition, rescue positioning, or gratitude expectations. The aesthetics paper can examine how timing, distance, tact, publicity, translation, interruption, and presentation become aesthetically salient before or alongside explicit normative judgment.

The justice paper can evaluate distributive, procedural, recognitional, epistemic, and relational consequences of triadic structures. The ethics paper can examine permission, competence, paternalism, refusal, exit, restraint, and alterity tolerance under third-position intervention. The power and political-economy paper can ask when brokerage, standing, resource routes, coalition structure, public recognition, or institutional mediation become trajectory-shaping, extractive, or difficult to revise. The historical-dynamics paper can develop trust formation, institutionalization, attractor-like stabilization, path dependence, dissolution, and long-run role transformation using the historical objects established in Sections 17–19.

Paper II therefore supports a limited philosophical conclusion. A focal A–B relation can be reorganized by historically situated third-position functions, audience structures, public records, reputational circulation, accountability procedures, coalitions, shared vocabularies, norms, and other relational objects. Some of these structures remain recoverable from rich pairwise descriptions; others satisfy a declared nonrecoverability criterion. Some persist and become emergent relational objects; others remain transient. Their manifestations can change while their relational effects persist, and the same manifestations can support different relational interpretations depending on history and provenance. None of these model-relative results settles ultimate ontology or normative validity.

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The next section returns from philosophical interpretation to verification discipline. It distinguishes mechanically checkable properties of the formal system from empirical calibration, identifies the dependence of results on representation and data quality, and states the limitations that remain in the present minimal triadic model.

22 Verification Architecture for Triadic Relational Models

This section specifies how claims made with the triadic formalism should be checked, calibrated, and delimited. Its role is methodological: the preceding sections developed typed relational objects, pairwise reconstruction criteria, historical dynamics, mathematical realizations, and finite illustrations; the present section states what would count as a successful use of those resources and what stronger conclusions still require independent argument. The objective is to keep five burdens distinct: formal coherence, representation robustness, empirical identification, normative appraisal, and ontological interpretation. The discussion proceeds from formal auditing and reconstruction checks to finite reproducibility, representation sensitivity, empirical coding and identifiability, normative and ontological limits, boundary adequacy, recurrent failure modes, and a reusable verification protocol. The method is diagnostic rather than foundational. A model may succeed at one burden while remaining weak, underdetermined, or deliberately silent at another.

Table 25 summarizes the five burdens. Their order reflects the workflow of an application rather than an epistemic hierarchy. Formal correctness is necessary for a formally stated claim, but it does not by itself establish empirical adequacy, normative validity, or ontology. Conversely, an empirically informative qualitative reconstruction need not employ every mathematical realization developed in this paper.

Verification is also level-sensitive within the three-level GR architecture. Manifestation-level coding can be checked against records and observations; relational-reality reconstructions can be assessed for coherence, explanatory adequacy, and empirical identification relative to those manifestations; ultimate ontological reality remains suspended rather than becoming another object of model verification. This division prevents observational fit from being redescribed as direct access to relational or ultimate reality.

22.1 Formal Well-Formedness and Dependency Audit

Formal verification begins with well-formedness. Every retained object should be traceable to a declared carrier, every map or relation should have an explicit domain and codomain, and every composition should be evaluated only where the relevant operations are defined. The same discipline applies to stage indices, analytical resolution, subject or position indices, route indices, audience indices, and the three GR reality levels. This requirement is especially important in the triadic setting because mediation, translation, review, brokerage, coalition, publicness, and historical revision can live on different carriers and need not share one common state space.

The audit should therefore record at least the following information for every nontrivial formal object used in an application:

1. the carrier on which the object is defined and the equivalence under which comparison is made;
2. the source, target, partiality, and realized domain of each map or relation;
3. the historical stage and analytical resolution attached to the object;
4. the subject, third-position, route, audience, role, or function indices required by the object;
5. the GR reality level represented by the object and any cross-level interpretation rule;
6. the domain on which a proposed composition or comparison is actually defined;
7. the earlier definitions on which the object depends.

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Table 25: Distinct burdens in the verification architecture.

Burden	Primary question	Typical evidence or test	Stronger conclusion not licensed by success alone
Formal coherence	Are objects, maps, restrictions, equivalences, and compositions well typed and mutually consistent?	type audit, definedness audit, reconstruction checks, finite calculation	empirical truth
Representation robustness	Does the retained conclusion survive reasonable changes of boundary, resolution, coding, restriction, or mathematical carrier?	sensitivity analysis across an admitted representation family	representation-independent ontology
Empirical identification	Do observed materials discriminate the relational feature being claimed?	coding protocol, provenance, triangulation, candidate-model comparison, discriminating observations	normative validity
Normative appraisal	Which independent ethical, political, legal, or justice criteria evaluate the represented relation?	declared normative premises and domain-specific argument	descriptive prevalence or causal efficacy
Ontological interpretation	What, if anything, does the successful representation imply about the ultimate nature of the represented reality?	separate metaphysical argument under explicit assumptions	identification of the formal carrier with ultimate ontology

The final item is a dependency audit rather than a mere notation audit. For example, a claim about gatekeeping should depend on the previously defined brokerage and condition-setting criteria rather than on a newly introduced proxy for centrality; a claim about public reputation should inherit the audience, circulation, and attribution structures already defined; and a claim about historical noncommutativity should use the relation-valued revision architecture rather than silently replace it with deterministic functions. Formal consistency therefore includes consistency with the manuscript's own derivational architecture.

A formally valid expression can still be substantively weak. Correct typing shows that the claim is expressible inside the declared model; it does not show that the model has been coded from evidence appropriately or that the represented relation satisfies any normative criterion.

22.2 Pairwise Reconstruction and Nonrecoverability Audit

The defining formal burden of Paper II concerns decomposition into pairwise records. Section 6 already separates compatibility, existence of a full realization, uniqueness up to the retained full equivalence, and diagnostic factorization. Verification should reuse that architecture directly rather than introduce a second reconstruction vocabulary.

For an admitted full configuration $X \in \mathfrak{D}_3^\kappa$, the audit begins with its pairwise-shadow class $q_2^\kappa(X)$ and the associated ambiguity class $\mathsf{Amb}_2^\kappa(q_2^\kappa(X))$ from Equation 63. The interpretation is then determined by the already established cases: an empty ambiguity class records lack of an admitted full realization for the tested shadow class; one retained full-equivalence class supports pairwise sufficiency for that case; and two or more distinct retained full-equivalence classes support pairwise nonrecoverability. A specific nonrecoverability claim should additionally identify either a pair in the equivalence-gap relation $\mathcal{G}_{3|2}^\kappa$ or a full-equivalence-respecting diagnostic that fails to factor through the pairwise shadow.

This audit prevents three recurrent overclaims. Third-party involvement alone is insufficient evidence of nonrecoverability. Loss of raw information under restriction is insufficient when the discarded distinctions are irrelevant under the selected full-equivalence criterion. Nonunique mathematical realization is also insufficient when the competing realizations represent one retained substantive configuration. The verification target is the research-relevant distinction preserved by the full model and lost by the selected pairwise representation.

Reconstruction claims should also be retested under reasonable changes of resolution and restriction

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policy. A witness that disappears once a pairwise record retains an event identifier, audience tag, or provenance link may still be valid for the original representation, but its scope should be reported as representation-sensitive.

22.3 Finite Reproducibility and Executable Checks

Finite instantiations provide a second formal burden: reproducibility of consequences after the encoding has been fixed. The labelled 2-complex and bounded-history realizations make many checks mechanically inspectable, including typing, incidence consistency, pairwise restriction, equivalence-class construction, gluing or composability conditions, route reachability, and finite ambiguity classes.

A reproducible finite case should therefore publish or preserve enough information to reconstruct the declared carriers, labels, restrictions, equivalences, procedure records, route/provenance records, standing records, historical transitions, and any realization relation used to connect substantive records to mathematical carriers. Two researchers who accept the same finite encoding should be able to reproduce the same formal outputs.

Finite reproducibility has a deliberately limited evidential meaning. A program can verify that a chosen encoding contains a nonempty equivalence gap, that two histories satisfy a declared boundary-composition rule, or that one route is condition-setting under the model's graph structure. The calculation does not establish that the original social episode was coded correctly. It also does not convert an illustrative finite case into empirical confirmation. The finite layer verifies consequences of an encoding; the empirical layer evaluates the warrant for that encoding.

22.4 Representation Robustness

A triadic distinction can be formally valid and still depend heavily on one convenient representation. Robustness should therefore be assessed relative to a declared family of admissible representations rather than asserted globally. Let $\mathfrak{R}_\chi$ be the family admitted by a representation specification χ, let Q be a research conclusion with conclusion space $\mathcal{C}_Q$, and let Verd_Q be the possibly partial verdict map from representations to conclusions. Representation robustness is defined in Equation 340.

$$\mathsf{Rob}_\chi(Q) \quad \Longleftrightarrow \quad \mathsf{Verd}_Q(\mathfrak{r}_1) \approx_Q \mathsf{Verd}_Q(\mathfrak{r}_2) \quad \text{for every } \mathfrak{r}_1, \mathfrak{r}_2 \in \mathrm{Dom}(\mathsf{Verd}_Q) \cap \mathfrak{R}_\chi. \tag{340}$$

Equation 340 deliberately uses an equivalence on conclusions rather than a numerical distance among representations. The admissibility specification χ carries the substantive burden: it should state which changes count as reasonable alternatives for the research question. An excessively narrow family can manufacture apparent robustness, while an excessively broad family can erase distinctions the application intentionally treats as constitutive.

At least five sensitivity families are relevant to triadic applications. *Boundary sensitivity* concerns which external subjects, institutions, or publics are omitted. *Resolution sensitivity* concerns refinement, coarse-graining, and temporal segmentation. *Restriction sensitivity* concerns what pairwise shadows retain. *Semantic sensitivity* concerns coding rules for mediation, publicity, reputation, standing, coalition, gatekeeping, and provenance. *Carrier sensitivity* concerns whether the same substantive conclusion survives translation among the relational ledger, algebraic/categorical realization, labelled 2-complex, and bounded-history representations where such translations are defined.

Robustness failure is itself informative. It identifies the representational commitments carrying the result. The appropriate repair is usually to narrow the claim, report the dependency, or collect evidence capable of discriminating among the competing encodings.

22.5 Empirical Coding and Candidate Relational Models

Empirical use begins with observed or recorded material rather than pre-labelled GR objects. Interviews, field notes, messages, organizational records, surveys, interaction logs, documents, or sensor traces re-

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quire segmentation and interpretation before they can support claims about mediation, publicity, review standing, coalition, reputation, or relational emergence. The empirical interface should therefore preserve uncertainty and alternative codings rather than force every observation through a deterministic map.

Let $\mathcal{D}^{\text{obs}}$ denote the retained observational material under empirical specification ζ, and let $\widehat{\mathcal{M}}_{ABC,\zeta}$ denote admissible coded manifestation records. The coding interface is the relation in Equation 341.

$$\mathsf{Code}_{\zeta}^{(3)} \subseteq \mathcal{D}^{\text{obs}} \times \widehat{\mathcal{M}}_{ABC,\zeta}. \tag{341}$$

Equation 341 permits one observational record to support several admissible codings and permits a coding to remain undefined when the available material is insufficient. The specification ζ should record sampling, segmentation, temporal window, coder rules, missing-data treatment, uncertainty conventions, and the analytical resolution at which manifestation records are formed.

Coded manifestations then enter a second relation connecting manifestation-level evidence to candidate relational models. Let $\widehat{\mathfrak{R}}_{ABC,\zeta}$ denote the admitted candidate relational models. The reconstruction interface is defined in Equation 342.

$$\mathsf{Recon}_{\zeta}^{(3)} \subseteq \widehat{\mathcal{M}}_{ABC,\zeta} \times \widehat{\mathfrak{R}}_{ABC,\zeta}. \tag{342}$$

The two relations should remain distinct. Coding concerns how observed material is organized into manifestation-level records. Reconstruction concerns which relational-reality models are compatible with those records under the stated interpretive assumptions. This separation preserves the three-level GR architecture: relational reality is not read directly from a transcript, log, photograph, or public record.

For one observational body $d \in \mathcal{D}^{\text{obs}}$, the resulting candidate-model set is defined in Equation 343.

$$\mathsf{Cand}_{\zeta}(d) := \left\{ R \in \widehat{\mathfrak{R}}_{ABC,\zeta} : \exists m \in \widehat{\mathcal{M}}_{ABC,\zeta} \text{ with } (d,m) \in \mathsf{Code}_{\zeta}^{(3)} \text{ and } (m,R) \in \mathsf{Recon}_{\zeta}^{(3)} \right\}. \tag{343}$$

Equation 343 makes empirical underdetermination visible rather than treating it as noise around one privileged reconstruction. Triadic coding carries specific burdens: the researcher should state what evidence supports assigning C a mediation, witness, brokerage, review, or authorization function; what turns simultaneous visibility into public presentation; what circulation history supports a reputational claim; what evidence distinguishes coordination from coincident alignment; and what supports gatekeeping rather than mere route dependence or scarcity.

22.6 Identifiability and Discriminating Evidence

Empirical identification concerns whether the retained observations distinguish the relational feature claimed by the analysis. Let q be a declared diagnostic on candidate relational models, with comparison equivalence $\approx_q$. The diagnostic is identified at observational body d under ζ when all empirically admissible candidates agree on the diagnostic up to $\approx_q$, as stated in Equation 344.

$$\mathsf{Ident}_{\zeta}(q;d) \quad \Longleftrightarrow \quad q(R_1) \approx_q q(R_2) \quad \text{for every } R_1, R_2 \in \mathsf{Cand}_{\zeta}(d) \cap \mathrm{Dom}(q). \tag{344}$$

Equation 344 is diagnostic-relative. The evidence can identify whether C possessed review standing while leaving C's motive underdetermined, or identify that a record was publicly circulated while leaving its reputational consequence unresolved. Identification of one object therefore should not be generalized to the entire relational model.

When an application supplies a forward empirical model Pred_{ζ} from relational candidates to predicted manifestation records, observational equivalence can also be defined. For models in the domain of that forward model, Equation 345 states the criterion.

$$R_1 \sim_{\text{obs},\zeta} R_2 \quad \Longleftrightarrow \quad \mathsf{Pred}_{\zeta}(R_1) \approx_{\mathcal{M}}^{\zeta} \mathsf{Pred}_{\zeta}(R_2). \tag{345}$$

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Equation 345 is optional because many qualitative applications will not possess a justified forward prediction operator. In those cases, identifiability should be argued through the candidate set and concrete discriminating evidence rather than simulated by notation.

Several triadic distinctions are especially vulnerable to observational equivalence. Mediation and brokerage can generate similar message-flow traces. A public record and several coordinated private records can appear similar in a static archive. Coalition and coincident alignment can yield the same immediate action. Gatekeeping and simple scarcity can both produce route dependence. Formal review standing and de facto influence can be confounded. Reputation and multiple independent attributions can appear similar when circulation histories are absent. Empirical claims should therefore identify which observations would discriminate among these candidate structures and report when the available evidence does not supply them.

The pairwise-sufficiency question has an additional empirical burden. A formal equivalence gap matters empirically only after the coding and reconstruction procedures warrant the full-triad distinction and the pairwise restrictions used to diagnose its loss. Formal nonrecoverability cannot compensate for an empirically unsupported full-triad coding.

22.7 Normative Appraisal and Ontological Suspension

Normative appraisal is a separate research burden. Paper II can represent consent procedures, publicity, recognition, standing, authorization, accountability, coalition, exclusion, asymmetry, dependence, and emergent norms. The presence, stability, effectiveness, or public recognition of those objects does not establish their legitimacy, justice, permissibility, or ethical success. Such conclusions require an independently stated normative framework and arguments appropriate to the later ethics, justice, law, or political-economy questions.

The same separation runs in the opposite direction. A normatively defensible position can remain socially unstable, institutionally weak, or publicly unrecognized. Stability, prevalence, institutionalization, and pairwise recoverability therefore serve descriptive or structural roles rather than functioning as normative criteria by themselves. Section 21 already preserves this separation for publicness, recognition, order, and norm formation; the present section turns it into an application-level requirement.

Ontological interpretation carries another independent burden. The labelled 2-complex, regional algebra, category-theoretic diagrams, and spin-foam-inspired bounded histories are representational resources whose usefulness depends on explicit interpretation rules. Successful use of one carrier can support claims about formal adequacy for a selected task. It does not by itself establish that social reality is fundamentally cellular, algebraic, categorical, discrete, or quantum. The three-level GR framework therefore continues to suspend ultimate ontology while permitting substantive claims at relational reality and context-sensitive claims at manifestation reality.

A later metaphysical argument could propose stronger commitments, but those commitments would require premises beyond successful model fit or formal elegance. Ontological restraint is consequently part of the verification architecture rather than a disclaimer added after the mathematics.

22.8 Triadic Boundary Adequacy and External Explanatory Burden

The minimal triad is an analytical boundary. It does not imply causal closure around A, B, and C. Volunteer relations can depend on organizations, families, donors, governments, platforms, professional communities, legal rules, infrastructures, markets, and wider publics. The relevant verification question is therefore whether the chosen triadic boundary carries enough of the explanatory burden for the research claim being made.

Boundary adequacy should be challenged whenever an omitted position or structure determines a relation represented internally as though it were generated by A, B, and C alone. C's review standing may derive from an organization. A coalition can depend on a funding regime. Reputation can require a wider audience and circulation infrastructure. B's practical ability to refuse can depend on alternatives supplied outside the triad. A public record can become consequential because an external institution

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Table 26: Selected failure modes and repair strategies for triadic applications.

Failure mode	Diagnostic condition	Appropriate repair
Type or definedness failure	an object, map, relation, or composition is used outside its declared carrier or domain	repair typing, restrict the domain, or leave the operation undefined
Dependency failure	a later claim introduces a proxy that bypasses the manuscript's established derivation	reconnect the claim to prior definitions or justify a new specialization explicitly
Restriction failure	a pairwise projection is undefined or inconsistent with the retained full object	revise the restriction scheme or narrow the reconstruction claim
Compatibility or realization failure	pairwise records do not admit the asserted common full configuration	distinguish incompatibility from unrealized compatibility and revise the candidate set
Nonrecoverability overreach	third-party involvement or raw information loss is treated as irreducibility without an equivalence-gap or factorization witness	supply the declared witness or use weaker language
Route/provenance collapse	direct and mediated access or execution and action genesis are merged	restore route and provenance records
Function/identity collapse	mediation, witnessing, brokerage, review, or authorization is treated as a permanent subject identity	model the function as a relationally indexed position
Publicity/reputation collapse	visibility, public presentation, acknowledgment, circulation, reputation, and recognition are conflated	restore audience and circulation distinctions
Standing/authorization collapse	influence, access, credibility, standing, authorization, review, and enforcement are conflated	restore the corresponding procedural types
Coalition or gatekeeping overreach	alignment becomes coalition, or route centrality becomes discretionary control without the stronger criteria	apply the established coordination and condition-setting tests
Representation fragility	the conclusion changes across reasonable boundaries, resolutions, codings, or carriers	report the dependency, narrow the scope, or gather discriminating evidence
Empirical underdetermination	materially different relational models remain in the candidate set	retain multiple reconstructions or collect evidence that separates them
Normative overreach	prevalence, efficacy, recognition, or stable formation is treated as legitimacy or justice	introduce independent normative premises
Ontological overreach	a useful mathematical realization is identified with ultimate social ontology	return to ontological suspension or supply a separate metaphysical argument
Boundary leakage	omitted institutional, public, temporal, or fourth-position structure carries essential explanatory burden	widen the model, encode a boundary condition, or narrow the claim
Illustrative overreach	finite calculability is treated as empirical confirmation	separate reproducibility of consequences from warrant for the encoding

recognizes it. In such cases, the appropriate repair is to widen the represented system, model the external structure as an explicit boundary condition, or narrow the explanatory claim.

The same discipline applies temporally. A short observational window can make a third-position intervention appear decisive even when its enabling conditions were produced earlier, and a bounded history can miss delayed consequences that become visible only later. Boundary verification therefore includes social, institutional, spatial, and temporal boundary choices.

22.9 Failure Modes and Appropriate Repairs

Table 26 summarizes recurrent failure modes. The final column emphasizes repair rather than merely naming error because several failures are resolved by narrowing scope or improving evidence rather than abandoning the formalism.

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The table also clarifies what falsifiability means for this preliminary formalism. A particular instantiation can fail because it is ill typed, unreconstructible, representation-fragile, empirically underidentified, normatively unsupported, or drawn around an inadequate boundary. These are distinct failure conditions. Preserving that distinction allows later research to revise one layer without discarding results that remain valid at another.

22.10 Reusable Verification Protocol

A later application of the triadic formalism should record the following information whenever the corresponding object is used:

1. the research question, analytical resolution, historical window, triadic boundary, focal pair, and known external interfaces;
2. the carrier, type, domain, codomain, partiality, indices, and three-level status of each major formal object;
3. the dependency path connecting later diagnostics to the definitions from which they are derived;
4. the three pairwise restrictions, their component equivalences, and the compatibility conditions imposed on their joint shadow;
5. the ambiguity class, equivalence-gap witness, or diagnostic-factorization test supporting any claim of pairwise insufficiency;
6. the route and provenance records supporting indirect epistemic, action, mediation, or accountability claims;
7. the audience, acknowledgment, circulation, and attribution criteria supporting publicity, reputation, or recognition claims;
8. the standing, authorization, review, enforcement, coordination, condition-setting, and scope criteria supporting procedural, coalition, or gatekeeping claims;
9. the persistence, organization, and efficacy evidence supporting any emergent triadic structure;
10. the finite encoding and reproducible calculations used to test formal consequences where such an encoding is available;
11. the admitted family of alternative representations and the sensitivity of the retained conclusion to boundary, resolution, restriction, semantics, and carrier choice;
12. the empirical specification ζ, including observation sources, segmentation, coding rules, uncertainty handling, and manifestation-level coding alternatives;
13. the candidate relational models supported by the coded manifestations and the evidence that discriminates among them;
14. observationally equivalent or otherwise empirically indistinguishable alternatives that remain unresolved;
15. the independent normative premises required for any claim of legitimacy, justice, obligation, permissibility, dignity, or ethical success;
16. the ontological status assigned to the formal language and the conditions under which the application would be narrowed, revised, or rejected.

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The protocol defines the methodological status of Paper II. The manuscript supplies a formal vocabulary and a family of representational tools for third-position function, pairwise recoverability, route-sensitive epistemics, publicity, accountability, coalition, asymmetry, emergence, historical revision, and bounded-history realization. It does not supply a uniquely privileged empirical coding procedure, a complete normative theory, or a final social ontology. Those burdens remain explicit interfaces to later empirical and theoretical work.

Section 23 therefore has a narrower task: it maps the verified formal resources into the wider volunteer/aid research programme and states which later papers inherit, reinterpret, or extend them. The conclusion then summarizes the contribution of Paper II at the level actually established by the manuscript.

23 Programme-Level Research Interfaces

This section specifies the research interface through which the triadic formalism can be reused by the subsequent volunteer/aid papers. Its role is architectural rather than additive: the section identifies which commitments should remain stable across the programme, which Paper II constructions are available for reuse, which substantive questions remain open, and which later findings would justify backward revision of the formal foundations. The discussion therefore introduces no new ontology and no new mathematical layer. Formal availability provides an analytical resource for later inquiry; the substantive interpretation, justification, and empirical adequacy of that resource remain tasks of the paper that uses it.

A programme interface is treated here as a four-part research contract. It contains *inherited commitments* that preserve conceptual continuity, *reusable constructions* that later papers may adopt when their research object requires them, *new burdens* that Paper II intentionally leaves unresolved, and *revision triggers* indicating when later analysis should feed back into the foundations. This distinction limits two opposite risks. Unrestricted inheritance would make later papers depend on every provisional notation or representation introduced in Paper II. Unrestricted reinvention would allow the same terms to acquire incompatible meanings across the series without an explicit account of the change.

Table 27 summarizes these four interface components.

Table 27: Research-contract components governing programme-level inheritance from Paper II.

Interface component	Programme role	Expected practice in later papers
Inherited commitment	Preserve a distinction that carries foundational or cross-paper meaning	Retain the distinction unless a later paper gives an explicit reason for revising it
Reusable construction	Supply an already-defined formal object, diagnostic, or representation	Reuse when analytically appropriate; specialize, weaken, or replace it when the new research object requires another representation
New burden	Mark a substantive question deliberately left open by Papers I–II	Supply the relevant metaphysical, psychoanalytic, aesthetic, normative, political-economic, historical, or empirical argument
Revision trigger	Identify evidence or analysis that exposes an inadequacy in an inherited assumption	Revise the affected foundation and state which downstream claims or definitions require synchronization

23.1 Cross-Paper Commitments

The strongest programme-level inheritances concern distinctions rather than particular symbols. Ontological reality remains suspended; relational reality remains distinct from manifestation reality; and subject-relative reconstruction remains epistemic rather than forming an additional reality level. A, B, and C remain history-bearing subject-spacetimes at the declared analytical resolution. Their tracked identities remain distinct from the relational positions they may occupy, including supporter, supportee, mediator, witness, translator, broker, reviewer, audience member, or coalition participant.

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Several Paper II distinctions should also travel forward when their objects remain relevant. Third-position relevance is distinct from pairwise nonrecoverability. Pairwise recoverability remains relative to the declared representation, restriction policy, resolution, and equivalence criterion. Action execution remains distinct from distributed action provenance. Standing, authorization, legitimacy, and normative validity remain separately typed. Asymmetry, dependency, brokerage, gatekeeping, exclusion, and enforcement supply possible ingredients of later power or justice analysis without individually settling those analyses. Emergence remains distinct from pairwise nonrecoverability. Mathematical realization remains distinct from substantive GR interpretation.

These commitments constrain meaning without freezing notation. A later paper may replace a cellular realization with another carrier, refine an emergence criterion, enlarge the set of positions, or introduce a stronger account of subject persistence. The paper should then state which earlier distinction is preserved, which representation has changed, and whether any earlier result depended on the abandoned specialization. This procedure treats backward revision as part of an open research programme rather than as an exceptional failure.

23.2 Subject Co-Formation Interface

Paper III develops the programme's metaphysics of subject co-formation and therefore receives the strongest metaphysical handoff from the formal foundations. Papers I–II already provide history-bearing subjects and model-relative tracking across stages. They also supply relational-change records, recognition, reputation, third-position functions, distributed action provenance, role ecology, and historical revision. These constructions make persistent relational change describable without yet deciding when the tracked subject itself has been reconstituted.

The new burden of Paper III is therefore a criterion of subject co-formation. A change in a role, expectation, reputation record, or continuation family may remain a change in the relation incident on a relatively stable tracked subject. Other changes may reorganize practical horizons, self-reconstruction, boundary conditions, value orientation, capacities, or the historical conditions under which the subject can be reidentified. Paper III should specify which changes are sufficient for the stronger co-formation claim and which remain relational modifications within a continuing subject model.

Triadicity sharpens this task because subject formation can be mediated through third positions. Public recognition of A as a helper may enter A's later self-reconstruction; repeated reliance on B's first-person knowledge may alter how B's supportee position is stabilized or resisted; C may migrate from translator to reviewer and thereby acquire a different relational history. These possibilities make third-position histories relevant to subject co-formation while preserving ontological suspension and the model-relative character of subject tracking.

A backward-revision trigger arises if Paper III shows that a Paper II tracking rule treats a substantively important subject transformation as mere state change, or conversely treats ordinary relational variation as identity change. Such a result would justify revising the subject-tracking interface in Papers I–II rather than forcing the metaphysical paper to inherit an inadequate persistence criterion.

23.3 Psychoanalytic Position Interface

Paper IV, *Relational Positions in Volunteer Aid: A Preliminary Psychoanalytic Formalization*, inherits the formal distinction between subject identity and relational position together with action provenance, recursive theory of mind, public recognition, reputation, audience structure, mediation, role stabilization, and historical repetition. The triadic architecture adds analytically relevant third-position phenomena such as anticipated observation, public confirmation of a helper role, witness-dependent repetition, coalition-based identification, and situations in which B is encountered partly through A's relation to C or to a wider audience.

The psychoanalytic paper carries a different explanatory burden from Paper II. Desire, lack, fantasy, identification, gratitude, rescue positioning, repetition, and alterity tolerance require psychoanalytic argument; their presence cannot be inferred from a witness record, a public audience, a stabilized supporter

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position, or repeated action alone. The formal records can locate where a psychoanalytic interpretation may attach, while the psychoanalytic theory supplies the mechanism and interpretive warrant.

The interface also preserves the possibility of mixed genesis. Supporter-side subjective investment can coexist with genuine concern for B, and concern for B can itself become entangled with recognition, desire, identification, or repetition. Paper IV can therefore analyze coupled motives and positions without reducing volunteering either to pure altruism or to disguised self-interest.

A revision trigger would arise if psychoanalytic analysis reveals a recurrent provenance or position distinction that the current formal language cannot express without collapsing subject, role, fantasy, and action genesis. In that case the appropriate response is to enrich the formal foundation explicitly rather than encode the missing distinction through an improvised proxy.

23.4 Aesthetic Support Interface

Paper V, *A Preliminary Generative Relational Aesthetics of Volunteer Support*, inherits the distinction between relational and manifestation reality, pre-explicit manifestation structure, explicit evaluative records, audience-indexed publicness, translation, witnessing, recognition, role manifestation, and historical revision. These objects allow the aesthetic inquiry to study how support is manifested before articulated aesthetic judgment is formed.

The triadic extension is especially relevant because the aesthetic form of support need not be private to A and B. A gesture may be experienced as tactful by B, awkward by C, dignified in one public presentation, and humiliating in another. Witnessing can change the manifested form of giving; mediation can reorganize timing, distance, tone, or pacing; recognition can stabilize a relational style; and translation can concern sensibility and presentation as well as lexical content.

Paper V is responsible for supplying the substantive aesthetic concepts and evaluative criteria required for claims concerning fittingness, tact, restraint, dignity, rhythm, delicacy, intrusion, awkwardness, cultivated sensibility, and aesthetic disagreement. Aesthetic fittingness and normative legitimacy remain distinct evaluative questions. Their interaction is an object for later analysis rather than a relation fixed by the formal foundation.

A backward-revision trigger arises if aesthetic analysis shows that the current manifestation architecture is too coarse to represent recurrent pre-explicit forms that materially organize later judgment. Such a finding would justify refining manifestation records while retaining the three-level reality distinction.

23.5 Relational Justice Interface

Paper VI, *A Preliminary Theory of Relational Justice for Volunteer Aid*, inherits procedure, epistemic routes, recognition, standing, accountability, coalition, exclusion, typed asymmetry, dependency, publicness, action provenance, and historical continuation families. These constructions make several justice questions expressible without deciding them in advance. B may possess nominal standing to challenge A before C while lacking practical access to the procedure. C may hold review standing whose legitimacy remains disputed. Public recognition may distribute status unevenly. A coalition between A and C may reorganize B's access to deliberation or appeal. A formally open route may remain practically unusable.

Paper VI supplies the normative criteria required to evaluate such structures as distributively, procedurally, recognitionally, epistemically, or relationally just. The dimensions need not collapse into one scalar measure. The paper should also state how conflicts among dimensions are interpreted and how historical structure affects the assessment. An arrangement that performs adequately on each isolated dyadic comparison may still contain a full-triad condition that is relevant to justice once the wider procedure, audience, or coalition structure is retained.

The justice interface therefore inherits formal distinctions while leaving normative authority open. A standing relation records who can make a claim within a procedure; it does not establish that the procedure gives the right persons adequate standing. Recognition records can become relationally efficacious without being deserved. Stable accountability can exist within an unjust institutional arrangement.

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A revision trigger arises if justice analysis requires a distinction that the current records systematically erase, such as the difference between formally available and substantively exercisable standing. The relevant formal object should then be refined rather than asking the normative theory to work with a known loss of information.

23.6 Volunteer Ethics Interface

Paper VII, *Volunteer Intervention: A Preliminary Generative Relational Ethics*, inherits epistemic uncertainty, route-sensitive reconstruction, translation, mediation, witnessing, consent and refusal records, standing, authorization, action provenance, accountability, public consequences, and revisability. The triadic setting makes intermediary conduct a direct ethical object because C can alter what A and B can know, authorize, refuse, disclose, contest, or later revise.

The new burden concerns duties, permissions, competence, restraint, confidentiality, paternalism, responsibility, withdrawal, and supererogation under uncertainty. C's formal capacity to translate, mediate, publicize, testify, broker, review, or intervene does not itself answer whether C ought to exercise that capacity. Likewise, witnessing can create morally relevant knowledge without automatically generating one determinate duty, and publicity can increase accountability while also exposing B to privacy loss, reputational pressure, or unwanted recognition.

Distributed provenance makes responsibility especially important. An action can be requested by B, interpreted or translated by C, proposed and executed by A, and later reviewed through another procedure. Paper VII should determine which forms of contribution ground which responsibilities under the relevant ethical theory. The answer may vary across action types, competence conditions, foreseeable consequences, and opportunities for refusal or revision.

A backward-revision trigger arises if ethical analysis repeatedly requires provenance distinctions that the existing generation/execution/authorization/witnessing architecture cannot represent. The formal layer should then be expanded before substantive responsibility is assigned through an inadequate proxy.

23.7 Relational Power Interface

Paper VIII, *Relational Power in Volunteer Aid: A Preliminary Political Economy*, receives a particularly dense triadic inheritance. Brokerage, gatekeeping, coalition, reputation, recognition, standing, authorization, exclusion, route dependence, typed asymmetry, dependency, publicness, and distributed action provenance all provide candidate mechanisms through which resources and institutions can shape volunteer relations.

These constructions permit power analysis without treating power as a primitive scalar. A donor, organization, state, platform, volunteer coordinator, or professional body can occupy a C-like position that conditions access, defines recognized need, certifies claims, controls visibility, allocates standing, or shapes feasible coalitions. Symbolic and reputational effects can feed back into material access, while resource and labor structures can alter which epistemic or procedural routes remain viable.

Paper VIII requires stronger criteria for a power claim. Relevant candidates include persistence, trajectory-shaping capacity, control over alternatives, resource and labor structure, institutional reproduction, differential revision capacity, symbolic or material accumulation, and extractive organization. Brokerage, gatekeeping, coalition, dependency, or asymmetry can contribute to power without individually constituting a sufficient general criterion for it.

The revision interface is bidirectional. If Paper VIII shows that an apparently local triadic object depends systematically on an institutional field, resource circuit, or historically reproduced infrastructure outside the minimal triad, the correct response may be to enlarge the modeled boundary. The minimal triad should not absorb wider structural power by assigning every external condition to C.

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23.8 Historical Dynamics Interface

Paper IX, *Volunteer Action in Relational System Formation: A Preliminary Generative Relational Account of Historical Dynamics*, inherits the widest dynamic architecture. Continuation families, historical revision, restriction/revision noncommutativity, delayed consequences, path dependence, role migration, emergent trust, public norms, partnership, coalition switching, brokerage decay or lock-in, history composition, and alternative full-triad histories provide a vocabulary for asking how local support relations participate in wider historical change.

The new burden is one of scale and mechanism. Paper IX should specify how local action contributes to nucleation, propagation, stabilization, transformation, or dissolution of a broader relational system. Candidate mechanisms include trust formation, path-dependent audience construction, role ecology, institutionalization of mediation and accountability, heterogeneous value generation, coalition diffusion, and the extension of local practices into wider public or democratic relations. The historical significance of an episode follows from such mechanisms and their propagation conditions, rather than from the local episode's moral salience or material outcome alone.

Paper IX also provides a major backward-revision test for the foundations. If longer histories reveal state variables, memory structures, or scale transitions that cannot be represented by the current continuation and revision architecture, the historical paper should identify where the formal compression fails. The foundational papers can then be revised without requiring the historical theory to remain inside an inadequate finite-stage representation.

23.9 Programme Inheritance Map

Table 28 summarizes the principal Paper II contributions available to each later inquiry together with the new analytical burden that remains outside Paper II's scope. The table describes availability rather than mandatory importation. A later paper should inherit only the constructions required by its research object while preserving the cross-paper commitments stated earlier in this section.

Table 28: Programme-level research interfaces available from Paper II.

Later paper	Reusable Paper II constructions	New analytical burden
P3 Subject co-formation	historical subject tracking, role ecology, recognition, reputation, third-position histories, distributed provenance	criteria for subject reconstitution, changing boundaries, practical horizons, values, and capacities
P4 Psychoanalytic positions	relational positions, audience structure, recursive ToM, public confirmation, repetition histories, provenance	desire, lack, fantasy, identification, repetition mechanisms, alterity tolerance, mixed genesis
P5 Aesthetics	manifestation structure, public presentation, translation, witnessing, recognition, historical revision	fittingness, tact, dignity, rhythm, intrusion, sensibility, aesthetic validity and disagreement
P6 Justice	standing, accountability, recognition, coalition, exclusion, asymmetry, epistemic routes, continuation access	distributive, procedural, recognitional, epistemic, and relational justice criteria
P7 Ethics	uncertainty, mediation, witnessing, consent/refusal, authorization, provenance, publicity, revisability	duties, permissions, competence, restraint, paternalism, responsibility, confidentiality, withdrawal
P8 Political economy	brokerage, gatekeeping, coalition, reputation, standing, dependency, exclusion, route control, provenance	institutional power, resources, labor, accumulation, reproduction, trajectory-shaping mechanisms
P9 Historical dynamics	path dependence, continuation families, historical revision, role migration, emergence, public norms, history composition	nucleation, propagation, institutionalization, scale transition, system formation and dissolution

Cross-paper continuity is strongest for concepts that participate in several later inquiries: three-level reality, history-bearing subject tracking, manifestation, route provenance, action provenance, standing, authorization, recognition, publicness, dependency, asymmetry, coalition, trust, shared norm, generative

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capacity, and role stabilization. When a later paper changes the meaning or type of one of these concepts, the revision should state which earlier properties are retained, weakened, extended, or rejected. This practice allows conceptual development without silent equivocation.

23.10 Programme Dependency Structure

The programme dependency structure is best understood as graded research inheritance rather than strict logical entailment. Paper I supplies the minimal dyadic object language. Paper II adds third-position and pairwise-recoverability architecture. Paper III develops subject co-formation. Paper IV builds principally on the dyadic and subject-formation foundations while drawing on triadic audience and recognition structures where relevant. Paper V likewise depends primarily on the reality-level and subject-formation architecture while using triadic manifestation and publicness when the aesthetic question requires them. Papers VI and VII draw on the metaphysical, psychoanalytic, and aesthetic analyses in addition to the formal foundations. Paper VIII depends strongly on the triadic structural vocabulary and on the later accounts of subject formation, position, and manifestation. Paper IX integrates the formal, metaphysical, aesthetic, normative, and political-economic results into historical analysis.

This graded structure matters because Paper II should not become an obligatory bottleneck for every later argument. A dyadic aesthetic problem can remain dyadic. A psychoanalytic account of A's relation to B need not manufacture C merely because Paper II exists. Conversely, a justice or power problem that depends on public review, brokerage, coalition, or third-party standing should inherit the corresponding triadic distinctions rather than redescribe them as if the relation were dyadic.

Backward revision is equally important. Paper III may show that one subject-tracking rule is too rigid; Paper IV may expose a missing provenance distinction; Paper V may require a finer manifestation record; Paper VI may distinguish nominal from effective standing; Paper VIII may reveal that a local triad is embedded in a larger institutional field; and Paper IX may expose scale transitions absent from the finite-stage dynamics. Such findings should be recorded as revision triggers with an explicit dependency audit. The programme thereby remains cumulative without assuming that the earliest formalization has final authority over later substantive inquiry.

23.11 Higher-Arity Extension Interface

Paper II stops at three focal positions because the minimal triad is sufficient to expose mediation, audience, review, coalition, brokerage, and pairwise-recoverability problems that cannot be represented inside a single focal dyad. The boundary is methodological rather than ontological. A fourth subject, organization, platform, public, institution, or legal regime may carry essential explanatory structure in a concrete case.

A higher-arity extension can reuse the same recoverability discipline without presupposing a general simplicial or higher-category social ontology. The analyst first declares the retained full configuration, then specifies a family of lower-arity restrictions, the resolution at which they are compared, and the equivalence criteria used for the lower- and higher-arity objects. The substantive question is whether configurations that remain equivalent under all declared lower-arity shadows can nevertheless differ under the retained full-configuration criterion. A positive witness establishes only representation-relative nonrecoverability for that declared comparison problem.

The same discipline applies to model expansion. If an apparently triadic phenomenon changes substantially when a fourth position is added, the analyst should determine whether the earlier triad was an adequate coarse-graining, a boundary-conditioned local model, or an under-specified representation. The higher-arity extension therefore continues the verification logic of Section 22 rather than converting numerical arity into a metaphysical hierarchy.

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23.12 Programme-Level Open Problems

The formal foundations leave several substantive problems intentionally open. The later papers will need to determine when repeated relational change constitutes subject transformation; how desire, lack, fantasy, and identification interact with genuine concern for another subject; how pre-explicit manifestation becomes cultivated aesthetic judgment; which conditions make a support relation just; which duties and permissions govern intervention under uncertainty; when brokerage, standing, resource control, institutional position, and dependency become relational power; and how local support relations nucleate, diffuse, stabilize, transform, or dissolve wider relational systems.

Several cross-cutting problems also remain. Publicness can support accountability and common knowledge while also creating surveillance, reputational pressure, or exclusion. Heterogeneous values and norms can become relationally efficacious without becoming commensurable on one scalar. Formal models can gain explanatory power while remaining underidentified by available evidence. Minimal dyadic or triadic boundaries can omit institutions or publics that carry the relevant generative burden. These are programme-level research questions rather than omissions to be silently resolved inside the formal foundation.

The interface architecture therefore completes Paper II's handoff by specifying continuity without closure. Later papers receive a disciplined vocabulary, reusable diagnostics, and explicit revision paths while retaining responsibility for their own substantive theories and methods. Section 24 can consequently synthesize Paper II as a completed preliminary triadic foundation whose results remain open to backward revision from the programme it enables.

24 Foundational Synthesis of Triadic Volunteer Relations

This section consolidates the contribution of Paper II at the level established by the preceding formal, interpretive, and methodological analyses. Its role is synthetic: it identifies the reusable architecture of the minimal triadic model, clarifies the scope of the strongest results, and specifies the programme boundary at which later substantive inquiries begin. The synthesis introduces no additional ontology, normative criterion, or empirical law.

Paper II extends the dyadic foundations of Paper I by asking what becomes analytically available when a third history-bearing subject or relational position enters a support relation. The extension is deliberately more demanding than the addition of a third actor label. A third position can participate in mediation, translation, witnessing, brokerage, public presentation, review, authorization, coordination, coalition, condition-setting, or distributed action provenance. These functions can reorganize epistemic routes, continuation possibilities, accountability structures, public status, role configurations, and historical development even when the visibly executed support act remains concentrated in an A–B relation.

The resulting architecture distinguishes numerical three-subject presence from analytically relevant third-position structure. A, B, and C remain tracked GR subject-spacetimes within a coupled history-bearing configuration, while third-position functions are indexed relative to a focal relation and historical stage. Equation 70 supplies the corresponding typed function record, and Equation 80 permits those functions to change through history. The model can therefore represent C as a mediator relative to A–B while A occupies another third position relative to B–C, and it can represent the migration or disappearance of such functions without converting supporter, supportee, mediator, witness, broker, reviewer, or coalition member into exhaustive identities.

The formal core of the paper is the pairwise-recoverability problem. Pairwise equivalence is induced by the declared restriction scheme in Equation 49, while the relation-valued equivalence gap is defined in Equation 51. A nonempty gap identifies configurations that are indistinguishable under the retained pairwise representation while remaining distinct under the selected full-triad criterion. Pairwise sufficiency is correspondingly defined by the emptiness condition in Equation 52. These objects provide a disciplined test of representation-relative recoverability. Their scope depends on the comparison domain, restriction rules, equivalence criteria, and analytical resolution; they do not establish a metaphysical doc-

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trine of intrinsically irreducible triads.

The same discipline organizes route-sensitive epistemics. Direct access, mediated access, translation, testimony, public records, and other routes can support different reconstruction histories even when they concern similar content. Equation 86 keeps route provenance distinct from reconstructed content, while Equation 88 permits theory-of-mind records to inherit that route sensitivity. A third position can therefore enlarge, filter, redirect, or contest epistemic access without acquiring an epistemically privileged standpoint. The model retains the possibility that A, B, and C are differently informed, differently situated, and differently reliable across domains.

Public and procedural structures require a further set of distinctions. Audience-indexed publicity is defined separately from visibility in Equation 94; durable reputation is represented through historical circulation in Equation 100; and accountability, standing, authorization, review, and enforcement remain separately typed. These distinctions allow the same material intervention to acquire different relational consequences when it is witnessed, publicly recorded, jointly acknowledged, reviewed, authorized, contested, or incorporated into a reputational history. They also prevent procedural existence, public uptake, or recognition from functioning as shortcuts to legitimacy or normative validity.

Brokerage, gatekeeping, coalition, alignment, exclusion, and triangular dependency supply another layer of triadic organization. Brokerage records route structure, while the stronger gatekeeping criterion in Equation 126 additionally requires the relevant condition-setting capacity. Coalition requires coordination beyond coincident preference, and alignment remains dimension-specific. These constructions establish a formal substrate for later political-economic and power analysis while leaving the stronger claims about domination, exploitation, justice, or legitimate authority to the papers that introduce the required substantive criteria.

Action provenance is correspondingly distributed. Equation 148 permits request, translation, proposal, mediation, authorization, execution, witnessing, recording, and review to have distinct sources within one action history. Visible execution therefore need not identify the complete generative history of an intervention. The distinction matters for later responsibility analysis because execution, generation, authorization, attribution, and review can be distributed across different positions and can change over time.

Historical composition extends the same architecture beyond a single stage. Section 17 distinguishes relational change from revision of the retained record, models continuation families with explicit domains, and represents path dependence through reachable historical alternatives. Equation 243 gives the resulting history-sensitive criterion. Third-position functions can scaffold a relation and later disappear, stabilize into durable mediation or accountability channels, become locked into brokerage positions, or participate in coalition switching and role reorganization. The emergence criterion of Equation 179 remains analytically separate from pairwise nonrecoverability: an emergent structure can remain pairwise recoverable, while a nonrecoverable episode can remain historically transient.

The geometric, algebraic, categorical, and spin-foam-inspired layers provide optional realizations of these substantive distinctions. The cellular realization relation in Equation 251 explicitly separates a substantive GR configuration from a labelled 2-complex realization. The history-realization relation in Equation 290 similarly separates a substantive relational history from a bounded cellular history. This separation permits multiple mathematical realizations of one substantive configuration and prevents representational multiplicity from being mistaken for multiple social histories. The mathematical carriers can therefore support gluing, restriction, coarse-graining, boundary comparison, or history composition without settling ultimate social ontology.

The finite illustrations then serve as diagnostic tests of the architecture. They show that mediation can be consequential while remaining pairwise recoverable; witnessing and public acknowledgment can alter later records; typed accountability can exist without sanction; brokerage becomes gatekeeping only under stronger route and condition-setting requirements; a full-audience component can furnish a local nonrecoverability witness; and repeated triadic interaction can reorganize functions without requiring a fixed third-party identity. Their evidential role is formal operability: they demonstrate that the distinctions can be instantiated coherently inside finite models, while leaving empirical identification to

independently justified data and coding procedures.

The philosophical interpretation developed in Section 21 follows from these formal distinctions without converting them into ontology. Thirdness is treated as a relationally indexed position; mediation can reorganize possible continuations rather than merely transmit content; publicness depends on audience structure and history; reputation and recognition can become causally efficacious relational structures; and relational order can be distributed across mediation, accountability, coordination, publicness, and norm-like formations. Throughout these interpretations, ultimate ontological reality remains suspended, relational reality carries the strongest positive structural claims, and manifestation reality records context-sensitive presentation and accessibility.

The verification architecture specifies the burden attached to any application of the model. Representation robustness is defined in Equation 340; empirical material enters through the coding relation of Equation 341; and candidate relational reconstructions are generated through Equation 342. Diagnostic-relative identification then permits some relational objects to be better supported than others within the same episode. Formal coherence, representation robustness, empirical identification, normative appraisal, and ontological interpretation consequently remain distinct research burdens. A formally successful reconstruction does not by itself verify an empirical coding, justify a normative conclusion, or establish a final ontology.

These limits also define the boundary of the minimal triad. A fourth subject, wider institution, platform, legal regime, funding structure, public, or earlier historical condition may carry explanatory work that cannot responsibly be compressed into A, B, and C. In such cases, the appropriate response is to widen the represented system, introduce an explicit boundary condition, or narrow the claim. The triadic model is therefore a controlled analytical enlargement of the dyadic case rather than a proposed universal scale for social relations.

Within that scope, Paper II establishes a reusable formal middle layer for the volunteer/aid research programme. Its principal distinctions include third-position function versus subject identity, third-party involvement versus pairwise nonrecoverability, route versus content, publicity versus visibility, reputation versus isolated attribution, standing versus authorization, coalition versus alignment, brokerage versus gatekeeping, execution versus distributed provenance, emergence versus nonrecoverability, and mathematical realization versus substantive interpretation. These distinctions can be inherited selectively by later inquiries without requiring those inquiries to reproduce every carrier or notation used here.

The programme-level interfaces in Section 23 therefore mark the next stage of research. Paper III can examine when relational change becomes subject co-formation; Paper IV can investigate psychoanalytic relational positions; Paper V can develop the aesthetics of supporting; Papers VI and VII can introduce independent criteria for justice and intervention ethics; Paper VIII can analyze relational power and political economy; and Paper IX can investigate trust, path dependence, scale transition, and wider historical system formation. Later papers may also require backward revision when they reveal inadequate assumptions about subject tracking, manifestation, provenance, standing, institutional boundaries, or scale.

Paper II therefore closes a foundational stage without closing the research programme. Its main achievement is a disciplined language for making the generative significance of third positions formally inspectable while preserving historical revisability, representation dependence, empirical underdetermination, normative independence, and ontological restraint. That architecture provides the basis on which the programme can now move from the minimal structure of triadic relations toward more substantively committed analyses of subject formation, aesthetics, justice, ethics, power, and historical relational dynamics.

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