

“Qiutong Cunyi” in Diplomacy

Invariance in the Topology and Geometry of Coupled Generative Systems

Wanhong Huang

huangwanhong@serendip.ngo

Abstract

A diplomatic formula of the twentieth century holds that parties should seek what they have in common while preserving what they do not. Read as a claim about structure it asks what two parties can hold together without either adopting the other's terms. This paper develops that reading in the mathematics of composite systems. A pair is given a joint state on a product of their state spaces, the operations each party may perform on its own side generate a group acting on that state, and the common part is identified with the functions constant on the orbits of that action while the preserved difference is identified with the coordinates the action moves. Nothing is coarsened and no shared description is built. Five results follow. Each party can compute the complete invariant from its own description alone, so long as the pair stands in relation to nothing beyond it. The possible common parts carry an order that is partial and a distance that is total, the distance extending the order and deciding every pair it leaves open while being one extension among many that disagree about one such pair in eight. The states lying over a given common part form a manifold whose Euler characteristic counts the distinguishable ways the parties have of describing themselves, and it falls to its minimum both where the common part is uniform and where it is trivial, so it is maximal strictly between the two ends. A party that enlarges its own capacity to describe leaves the common part where it stood and multiplies that characteristic. And the common part has no resting place under a generator that does not change with time, while a generator that dissipates gives it one which forgets the initial condition and moves toward the uniform end as the coupling grows against the dissipation, at the cost of the completeness of the invariants. The formalism is used for its invariant theory and no physical claim is made; the scope section states in five particulars what is used and what is withheld.

Keywords: diplomacy; invariants; composite systems; coexistence; generative relational theory

Draft. This is a working draft. It is circulated for comment, and its arguments, formulations, and open questions are subject to revision. Correction and criticism are welcome at huangwanhong@serendip.ngo.

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Use of generative artificial intelligence. Generative AI assistants (Anthropic’s Claude and OpenAI’s ChatGPT) were used extensively in preparing this manuscript: for literature survey, for drafting prose from the author’s specifications, for adversarial review of the argument, for the computations and code underlying the illustrations, for consistency and citation auditing, and for typesetting. Formulations and objections arose in the course of those exchanges as well as from the author. Every claim, argument, and citation was reviewed and decided by the author, who is responsible for the content and for any errors. Bibliographic and mathematical details are to be verified against published sources before reliance.

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A note on the standing of this paper

This paper borrows a body of mathematics built for a subject it is not about. The mathematics of composite systems was developed for physics, and I use a part of it, the part concerning what is preserved when each of two parties transforms its own side. I make no claim that a state, a ministry, or a negotiating party is a system of the kind that mathematics was built for, and Section 2.2 says as exactly as I can what is used and what is withheld. A reader who suspects that borrowing of this kind is a way of dressing a thin argument is right to suspect it, and the best I can offer is that the result in Section 6 is a statement about group actions and would stand if every word about diplomacy were struck out.

Three things in the paper may be wrong in different ways. The identification of the common part with the invariants of a local action is a proposal about how to read a formula, and a reader may hold that the formula means something else. The position taken in Section 7 about what messages can establish is a commitment I argue for and do not prove, and the sections around it survive its rejection. And the statement in Section 9 about the strata is a claim about a quotient that I have computed for small cases; it is the part most likely to be wrong in detail, and where a computation supports it the computation is reported.

The formula the title names belongs to a diplomatic tradition and to a particular occasion in it. I use it as a name for a structural question and not as a topic in the history of that tradition, and Section 3 records the provenance in one paragraph so that the borrowing is visible.

Corrections and objections are welcome at the address on the title page.

1 INTRODUCTION

This paper concerns what two parties may hold together without either adopting the other's terms. Its objective is to give that question a formal statement, to identify the common part with the invariants of an action each party performs on its own side, to say what operations performed separately can and cannot establish, and to describe the space in which the possible common parts are arranged. The method is to take a body of mathematics developed for composite systems, to use the part of it that concerns invariance under transformations of one component, and to state at each point what is used and what is withheld.

The question has a name in the diplomatic vocabulary of the twentieth century. The formula 求同存异, rendered here as the seeking of what is held in common together with the preservation of what is not, is associated with the Asian-African Conference of 1955 and has been in use since. Read as a maxim of forbearance it says little that a practitioner does not already know. Read as a claim about structure it asks something sharper: what can two parties hold in common when neither will adopt the other's way of describing matters, and what is it that they hold?

Two readings of the common part are available before any formalism, and the paper parts from both. On the first, the common part is the overlap of what the two already hold, which is generally thin and sometimes empty. On the second, it is a description both can state, arrived at by each giving up the distinctions the other cannot make. The second reading is the one a companion paper develops at length, representing a joint description as a common factor of the two parties' dynamics and establishing that such a description collapses distinctions at least one party can make (Huang, 2026a). It is a coarsening, and it is bought with the difference the formula was to have preserved.

A third reading is available and is the one this paper develops. Suppose each party may transform its own side, re-describing its own affairs in whatever terms it chooses, and suppose these transformations generate a group acting on the joint state of the pair. Then a quantity constant on the orbits of that action is one both parties compute in their own coordinates and agree upon, and they agree upon it without either having adopted the other's coordinates and without any shared description having been built. The common part is identified here with such quantities, and the preserved difference with the coordinates the action moves.

What that identification buys is worth stating plainly, since it is the paper's reason for existing. A common factor is constructed and lossy: to have one, each party gives something up. An invariant is neither. It is already there, it costs the parties nothing, and it is indifferent to how either of them chooses to describe its own side. If the formula is read as asking for a shared description, the second half of it is in tension with the first. If it is read as asking for an invariant, the two halves are consistent, since the invariance is defined with respect to exactly the freedom the second half preserves.

The mathematics of composite systems supplies the setting. A pair of parties is given a joint state on a product of their state spaces; the operations each may perform alone generate a group acting on that state; and the invariants of that action are the objects the paper is about. This body of mathematics was developed for physics, and the paper uses a part of it without claiming that any party is a system of the kind it was built for. Section 2 states what is used and what is withheld, and the results of Section 6 are statements about group actions that would stand if every word about diplomacy were struck out.

Four further matters are treated and each is separable from the others.

The first concerns what can be established. Operations that each party performs alone cannot alter an invariant of the joint action, by definition. Whether messages passing between the parties can is a question on which the formalism admits two answers, and they differ in what they require of the diplomatic claim. Section 7 sets out both, states which the paper adopts, and gives the grounds. A reader who declines the position retains everything in Sections 6 and 9.

The second concerns the space in which the possible common parts lie. The quotient of joint states by the action of local transformations is not a manifold; it carries strata, and passing from one to another is not a matter of degree. Section 9 treats this, and it bears on questions posed in the form of how much two parties share, which may be asking for a number where the object has parts of different dimension.

The third concerns what lies over the base. The states carrying a given common part form a manifold of flags, one point of which is a way a party has of describing its own side together with an order upon its distinctions. That manifold has an Euler characteristic, which is a topological invariant, and the characteristic counts the distinguishable ways the parties have of describing themselves. Section 11 establishes that it falls from $d!$ to 1 as the coefficients of the common part come to coincide, so the measure attaching to the second clause of the formula is extinguished exactly where the first is satisfied completely. The base itself is contractible and carries no topology, so everything of that kind lies in the fibre and in how the fibre degenerates.

The fourth concerns the two ends of the interval. At one end no invariant remains beyond the trivial and the parties hold nothing in common; at the other the joint state is fixed by its invariants alone and the parties differ in coordinates and in nothing else, which the companion account of relational crystallisation records as a failure and not as an ideal (Huang, 2026b). The characteristic is d at the first and 1 at the second, and $d!$ between them, so a requirement that an arrangement stand strictly between the two ends is a requirement that the measure of preserved difference not be at a minimum. The account selects no point within the interval, since the characteristic is constant across it.

Two results attach to the motion the pair undergoes. Under a generator that does not change with time the induced motion on the space of possible common parts has no attracting

set, so a pair does not settle anywhere, and the strength of the coupling governs how quickly the common part moves and not how far it moves. Adding a term that dissipates changes both: the motion acquires an attracting point which does not depend on where the pair began, and the point moves toward the uniform end as the coupling grows against the dissipation. Read with the collapse above, a relation strongly coupled and little dissipating comes to rest near the point at which preserved difference is smallest, without any party intending it. What the dissipative setting costs is the completeness of the invariants, and Section 8 states the price where it is incurred.

Two further results attach to what the parties may do. A party may enlarge its own space of descriptions, and Section 15 establishes that doing so leaves the common part exactly where it stood and multiplies the characteristic of the fibre: what a party produces by developing its own capacity to describe is preserved difference, and it is nothing else. It may also raise a ceiling that the pair never occupies, since raising the rank is an operation upon the pair and not upon either of them. And the possible common parts carry two structures that answer two questions, an order that is partial and a distance that is total; Section 13 shows that the distance extends the order and contradicts it nowhere, that it decides every pair the order leaves open, and that it is one extension among many which disagree with one another about one such pair in eight.

Three limits are declared at the outset. The formalism is used for its invariant theory and no interpretation of it is asserted, so questions that belong to that interpretation are outside the paper. The identification of the common part with an invariant is a proposal about how to read a formula and not a finding about the formula. And every numerical statement is produced by computation before the proposition reporting it is written, with the computation accompanying the paper.

The exposition proceeds as follows. Section 2 fixes the scope and the standing of the formalism, Section 3 the formula and the readings already available, and Section 4 the prior formulations. Section 5 gives the formal setting and Section 6 the invariants. Section 7 treats what may be established and Section 8 the dynamics, both where nothing is lost and where something is. Section 9 describes the space of possible common parts, Section 10 the states lying over it, Section 11 the degeneration of those states and the collapse of their characteristic, Section 12 the geometry of the base, Section 13 the order and the distance it carries, and Section 14 the map that carries a state to the parties' own descriptions. Section 15 treats what a party's own generation does. Section 16 treats the two degenerate ends, Section 17 the computations, and Section 18 the correspondence with the companion papers. Sections 19 through 21 state the constraint form of the account, its boundaries, and the questions it opens.

The paper belongs to a series applying generative relational theory to governance, and it is the fourth of a set on diplomacy.

2 SCOPE, THE COMPANION PAPERS, AND THE STANDING OF THE FORMALISM

This section fixes what the paper undertakes. Its objective is to state the object, to say exactly which part of a borrowed mathematics is used and which part is withheld, to record what is carried from the companion papers, and to state the bounds observed in treating a formula that belongs to a diplomatic tradition.

2.1 *The Object of the Account*

The object is the structure of what two parties may hold in common while each retains its own way of describing its own affairs. The object is a property of a pair and not of either party, and the level is that of the mathematics: the paper asks what follows from a given structure and leaves to others the question of which structure any actual pair exhibits.

Three things lie outside. The paper offers no procedure for identifying the invariants of an actual relation, and Section 20 states why it could not. It offers no criterion by which an arrangement is to be assessed, since the companion papers hold criteria of that kind and this one adds none. And it takes no position on the history or the politics of the formula it names, which Section 3 treats only so far as is needed to say what is being read.

2.2 *The Mathematics Used and the Interpretation Withheld*

The mathematics of composite systems was developed for physics. What this paper uses is the part concerning invariance: a state on a product of two spaces, a group of transformations acting on one component at a time, and the functions constant on the orbits of that group. That apparatus is linear algebra together with the theory of group actions, and it carries no physical commitment.

Four things are withheld and each is named so that a reader may check that the paper keeps to it.

No claim is made that a party, a ministry, or a negotiating apparatus is a physical system of the kind the formalism was built for. The formalism is borrowed for its invariant theory and the borrowing is the whole of the relation.

No claim is made about the statistical structure the formalism carries in its original setting. The questions about correlations that distinguish that setting from a classical one are not raised here, no result below depends on their answer, and Section 20 records what would be needed before they could be.

No dynamical law is asserted. Where the paper treats evolution, at Section 8, the generators are stipulated, including the non-conservative one used there, and the results concern what a stipulated class of evolutions preserves and where it comes to rest. No claim is made that any party evolves under a generator of either kind.

And no measurement theory is used. The reduced description available to a party is taken as the description obtained by disregarding the other party's side, and nothing turns on how such a description would be produced by an act.

2.3 *The Results Carried from the Companion Papers*

Three results are used and not re-derived. The representation of a joint description as a common factor of two parties' dynamics, together with the result that such a description collapses distinctions at least one party can make, belongs to the companion treatment of knowledge among differently situated parties (Huang, 2026a). The identification of a state in which the parties differ in labels alone as a failure mode belongs to the treatment of relational crystallisation (Huang, 2026b). And the account of a slow variable that carries the terms of a coupling, moved by the parties' states and by nothing acting on it directly, belongs to the treatments of coupled systems and of the gift (Huang, 2026a,c).

2.4 *The Political Bounds Observed*

The formula this paper names is associated with a particular conference and with a particular diplomatic tradition, and it remains in use. Three bounds are observed in handling it.

The formula is treated as a name for a structural question, and the paper takes no position on any use that has been made of it. No state is named, and no episode is characterised. And the results are stated for pairs of parties in general, so that nothing in the paper bears on any actual relation without an argument the paper does not supply.

3 THE FORMULA AND ITS RECEIVED READINGS

This section treats the formula the paper is named for. Its objective is to record its provenance, to set out the renderings available in English, to give the two readings of the common part that are available before any formalism, and to state the residue both of them leave.

3.1 *The Formula and Its Provenance*

The formula 求同存异 is associated with the Asian-African Conference held at Bandung in 1955 and with the diplomatic tradition that conference belongs to. It has remained in use since, in settings where parties with substantial disagreements have sought terms on which to proceed together. The association is taken here from ordinary usage and is not documented, since the paper makes no claim about the history of the formula and would lose nothing if the attribution were corrected.

The paper takes the formula as a name for a question and not as a topic in the history of that tradition. What is used is the shape of the formula: it has two clauses, the second is not

a concession attached to the first, and an arrangement satisfying only one of them satisfies neither. Everything below follows from taking that shape seriously.

3.2 The Renderings Available in English

Several renderings are in circulation and they differ in what they commit the reader to. Seeking commonality while preserving difference is the most literal and is the least committal. Convergence without homogenisation states the second clause as a limit upon the first. Coexistence through shared invariants states a mechanism, and is the rendering this paper's argument ends in place of the one it begins from.

The rendering used in the subtitle, relational coherence amid difference, avoids the word common in its first clause. The avoidance is deliberate: a reader who hears the first clause as a search for shared substance has already been given the reading Section 3.3 sets aside.

3.3 The Reading of the Common Part in Terms of Overlap

On the first available reading, what the parties hold in common is what each already holds: the intersection of their positions, their interests, or their beliefs. Diplomacy on this reading is the work of finding that intersection and building upon it.

Two difficulties attend the reading and both are familiar in practice. The intersection is generally thin, and where the parties differ deeply it is empty, so the reading yields nothing exactly where it is most needed. And the intersection is not stable, since each party continues to move, so what was held in common ceases to be held without either party having done anything toward the other.

3.4 The Reading of the Common Part in Terms of a Shared Description

On the second reading, what the parties hold in common is a description both can state. This is the reading that underlies an agreed text, and it is developed formally in a companion paper, which represents such a description as a common factor of the two parties' dynamics (Huang, 2026a).

The reading is more powerful than the first, since a description both can state exists where an intersection of positions does not. It has one cost, established there and used here. A description both can state makes only distinctions both can make, so it is coarser than each of them, and arriving at it requires at least one party to give up a distinction it can draw. The second clause of the formula is what is spent in satisfying the first.

3.5 *The Residue Both Readings Leave*

Claim 3.1. (The residue). *Both available readings identify the common part with something the parties possess or construct: an intersection of what they already hold, or a description arrived at by each giving something up. Neither considers a common part that is possessed by neither party, constructed by neither, and indifferent to how each describes its own side.*

Claim 3.1 fixes what the remainder of the paper supplies. A quantity of that kind is an invariant of a group action, the group being generated by what each party may do to its own side, and Section 6 identifies the common part with such quantities. What makes the identification worth proposing is that the invariance is defined with respect to exactly the freedom the second clause of the formula preserves, so the two clauses cease to be in tension.

4 PRIOR FORMULATIONS

This section surveys the formulations under which coexistence amid difference has already been treated, and the one prior use of a formalism of the kind this paper borrows. Its objective is to record what each settles, to concede the borrowing that has been attempted before, and to state the residue.

4.1 *Convergence and Socialisation in the Study of International Relations*

One body of work treats parties as coming to share what they did not share before. States are held to acquire norms through participation in institutions, to internalise standards they first complied with under pressure, and to be socialised into the practices of a community (Checkel, 2005; Johnston, 2008).

The literature is careful about the difference between compliance and internalisation, and it supplies mechanisms for each. What it treats is movement toward a shared condition, so its object is the first clause of the formula and its measure of success is how far the second clause has been given up.

4.2 *Differentiated Integration in the Study of European Institutions*

A second body of work treats arrangements in which parties participate to differing depths, so that an order is held together without its members holding the same obligations (Schimmelfennig and Rittberger, 2015). The vocabulary of differentiated integration was developed for exactly the condition of union without uniformity.

What that literature supplies is a taxonomy of institutional forms and an account of when each is chosen. What it does not supply is an account of what the parties hold in common

when their obligations differ, since the shared element there is an institution and the question of this paper is what is shared where no institution is available.

4.3 Agonistic Accounts of Coexistence Under Disagreement

A third body of work holds that a political order rests on a form of conflict that is contained without being resolved, and that arrangements aiming at consensus suppress what they cannot remove (Mouffe, 2000; Connolly, 1995). The second clause of the formula has a defender here, and a strong one.

What these accounts establish is that the preservation of difference is a condition of an order and not a residue left over from an incomplete agreement. What they leave open is the structure of what holds the parties together while the difference is preserved, which they describe in terms of a shared ethos and do not formalise.

4.4 Formalisms of This Kind Already Applied to Social Theory

A formalism of the kind this paper borrows has been applied to social theory before, and at length. The proposal that social systems be understood in terms drawn from quantum theory, including the suggestion that the relevant states are not classical, has been developed as a general programme (Wendt, 2015). The programme has been contested, and the contest concerns exactly what is at issue here, which is whether the borrowing is a claim about the subject or a use of a mathematics.

The present paper is not in that programme and the difference is stated plainly. That programme asserts a claim about what social systems are. This paper asserts none, uses the invariant theory of a group action, and withholds the interpretation, as Section 2.2 records in four particulars. A reader who rejects the programme need not on that account reject anything below, and a reader who accepts it will find that the present paper does not depend on it.

The mathematics itself is standard and is drawn from the literature on composite systems (Nielsen and Chuang, 2010; Horodecki et al., 2009).

4.5 The Common Factor Developed in the Companion Paper

A companion paper takes the second reading of Section 3.4 and develops it, representing a joint description as a common factor of two parties' dynamics and deriving the properties of that representation (Huang, 2026a). That treatment establishes that such a description is coarser than each party's own, that the order of such descriptions has no greatest element in general, and that a description once available may cease to be so through a party's own evolution.

The present paper parts from it at the first step. A common factor is constructed and is bought with a distinction some party can make. What follows here is offered as the alternative

that the second clause of the formula invites, and Section 6.6 states the relation between the two accounts.

4.6 *The Residue Left by the Foregoing Formulations*

Claim 4.1. (The residue). *The foregoing formulations treat movement toward a shared condition, the institutional forms that hold parties of differing obligation together, the place of unresolved conflict in an order, and the properties of a description two parties can both state. What none of them treats is a common part that neither party possesses, that neither constructs, and that is defined by its indifference to how each party describes its own side.*

5 THE FORMAL SETTING

This section fixes the objects the remainder of the paper works with. Its objective is to give each party a state space, to give the pair a joint state, to say what a party may do to its own side, and to define the description a party has of the pair from where it stands.

5.1 *The State Space of a Party*

Each party is assigned a finite-dimensional complex vector space with an inner product, written $\mathcal{H}_A$ for one party and $\mathcal{H}_B$ for the other, of dimensions d_A and d_B . A basis of $\mathcal{H}_A$ is a way the first party has of describing its own affairs, and different bases are different such ways.

Two features of the choice are stated here and defended at Section 20. The spaces are finite because a description that a party could give is finite. And the spaces are not assumed to be of the same dimension, since nothing requires that two parties describe their affairs at the same resolution.

5.2 *The Composite and the States It Admits*

The pair is assigned the tensor product $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$, and a state of the pair is a positive operator ρ on that space with $\text{tr } \rho = 1$. A state of the form $\rho_A \otimes \rho_B$ is a product state; a convex combination of product states is separable; and states that are neither are the remaining ones.

The choice of the tensor product is the substantive one in this section, and what it carries is that the pair admits states that are not determined by states of the parts. That is the formal content of the claim, argued elsewhere in this series, that a relation is not reducible to the relata.

5.3 The Local Group and the Redescription a Party May Perform

A party may re-describe its own side. Taking such a redescription to preserve the inner product, the transformations available to the first party form the unitary group $U(\mathcal{H}_A)$, and those available to the second form $U(\mathcal{H}_B)$. The pair of them acts on a state of the composite by

$$\rho \longmapsto (U_A \otimes U_B) \rho (U_A \otimes U_B)^\dagger, \quad U_A \in U(\mathcal{H}_A), U_B \in U(\mathcal{H}_B). \quad (1)$$

Write $G = U(\mathcal{H}_A) \times U(\mathcal{H}_B)$ for the group so obtained and call it the local group.

Equation (1) is what the second clause of the formula is taken to license. Each party may transform its own side by any element of its own factor, and neither party's choice constrains the other's.

5.4 The Reduced Description Available to a Party

The description the first party has of the pair, from where it stands, is obtained by disregarding the second party's side. Formally it is the partial trace $\rho_A = \text{tr}_B \rho$, and it is the unique operator on $\mathcal{H}_A$ agreeing with ρ on every quantity that concerns the first party alone.

Two properties are used below. The reduced description discards the correlations between the parties, so it is not in general recoverable from the pair and the pair is not in general recoverable from it. And it transforms under the local group by $\rho_A \mapsto U_A \rho_A U_A^\dagger$, so the second party's choice of description leaves it untouched.

6 INVARIANTS UNDER LOCAL REDESCRIPTION

This section states the paper's central identification and the results that support it. Its objective is to define the invariants of the local action, to record what is known about them, to give the reading of the formula they supply, and to state the relation between this account and one in which a joint description is constructed.

6.1 The Orbits of the Local Action and the Functions Constant on Them

Two states of the pair lie on the same orbit of the local group when one is carried to the other by a transformation of the form $U_A \otimes U_B$. Two states on one orbit differ in how the parties have chosen to describe their own sides and in nothing else.

Definition 6.1. (Invariant). *A function on states of the pair is an invariant of the local action when it is constant on each orbit, so that $f((U_A \otimes U_B) \rho (U_A \otimes U_B)^\dagger) = f(\rho)$ for every U_A and U_B .*

Two properties follow immediately and are the reason for the definition. An invariant is computed by either party in whatever coordinates that party has chosen, and the value obtained does not depend on the choice. And an invariant is unchanged by anything either party does to its own side alone, so no party can move it by redescription.

6.2 The Complete Invariant in the Case of a Pure Joint State

Where the state of the pair is pure, written $\rho = |\psi\rangle\langle\psi|$, the invariants are known completely. Every such state admits a decomposition

$$|\psi\rangle = \sum_{i=1}^r \sqrt{\lambda_i} |a_i\rangle \otimes |b_i\rangle, \quad \lambda_i > 0, \quad \sum_i \lambda_i = 1, \quad (2)$$

with $\{|a_i\rangle\}$ orthonormal in $\mathcal{H}_A$ and $\{|b_i\rangle\}$ orthonormal in $\mathcal{H}_B$; the number r is the Schmidt rank and the λ_i are the Schmidt coefficients (Nielsen and Chuang, 2010).

Proposition 6.2. (Completeness). *Two pure states of the pair lie on the same orbit of the local group if and only if their multisets of Schmidt coefficients agree. The Schmidt coefficients are therefore a complete invariant of the local action on pure states, and every invariant of that action is a function of them.*

The bases $\{|a_i\rangle\}$ and $\{|b_i\rangle\}$ in Equation (2) are not invariant and are moved by the action. They are what each party has chosen; the coefficients are what neither has.

6.3 The Invariant Carried by a Party's Own Description

A further property of the pure case is used throughout the paper and is worth stating separately, since it is stronger than the coordinate-independence that Definition 6.1 supplies.

Proposition 6.3. (Sufficiency of one side). *Where the state of the pair is pure, the reduced description held by either party has the Schmidt coefficients as its spectrum. Each party can therefore compute the complete invariant of the pair from its own description alone, without any information from the other and without either party having adopted the other's coordinates.*

Proposition 6.3 is what makes the identification of the following subsection worth proposing. It is not merely that the two parties would agree on a value if each computed it. It is that each already holds what is needed to compute it, and that what each holds is its own description in its own terms.

6.4 The Reading of the Two Clauses

Claim 6.4. (The reading). *Read through the present setting, the first clause of the formula asks for the invariants of the local action and the second preserves the coordinates that action moves. The two clauses concern different objects, the first concerns what no party may alter by redescription and the second concerns exactly the freedom to redescribe, so satisfying one imposes no cost upon the other.*

Claim 6.4 is a proposal about how to read a formula and is not a finding about the formula, and Section 20 records what would be needed before it could be more than that. What may be said for it is that it removes a tension the other readings carry. On the reading of Section 3.4 the second clause is spent in satisfying the first, since a description both can state is coarser than each. Here nothing is spent, because the invariance is defined with respect to the very freedom the second clause preserves.

6.5 The Invariance Displayed

The content of Definition 6.1 may be shown. Four hundred states were obtained from one state by transformations that each party performed on its own side, so that by Definition 6.1 every one of them carries the same common part. Figure 1 displays what moved and what did not.

The left panel shows the first vector of one party's description for each of the four hundred states, plotted on the sphere of such vectors at $d = 2$. The transformations carry it everywhere: the party's description of its own side is different in every one of the four hundred cases. The middle panel shows the larger coefficient of the common part for the same four hundred states, on the whole range the base admits, from the uniform point at one half to the product point at one. It does not move from 0.72. The right panel shows the same quantity again with its mean subtracted and the scale enlarged by a factor of 10^{16} , where the four hundred values are seen to span ten units, which is the precision of the arithmetic and not a variation of the common part.

6.6 The Case of a Mixed Joint State and What It Costs

The completeness of Proposition 6.2 holds for pure states of the pair and fails otherwise. Where ρ is mixed, its reduced descriptions do not determine its orbit, the invariant theory of the local action is substantially harder, and no complete set of invariants of the kind Equation (2) supplies is available in general (Horodecki et al., 2009).

Claim 6.5. (The condition of self-sufficiency). *The complete invariant is available to a party from its own description alone exactly in the case where the state of the pair is pure. A state of the pair that is mixed is what obtains where the pair stands in relation*

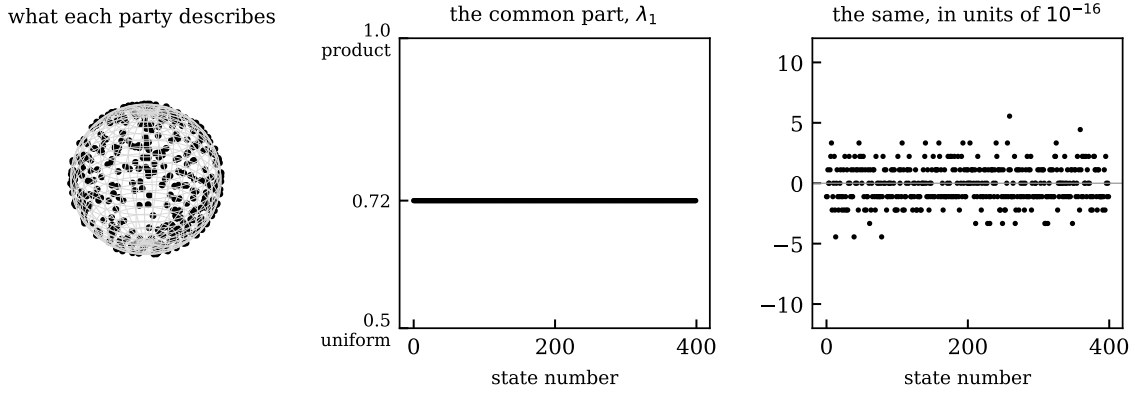


Figure 1: Four hundred states obtained from one state by transformations each party performs on its own side, at $d = 2$. Left: the first vector of one party's description for each state, which covers the sphere. Middle: the larger coefficient of the common part for the same states, drawn over the whole range the base admits, which does not move. Right: the same quantity with its mean subtracted and the scale enlarged by 10^{16} , where the spread is ten units and is the precision of the arithmetic. The figure displays Definition 6.1 in a computed instance.

to something beyond it, so the self-sufficiency of Proposition 6.3 is a property of a pair considered in isolation and is lost where a third party is involved.

Claim 6.5 is stated as a property of the formalism and its reading is offered as a reading. If the involvement of a third party is what makes the joint state of a pair mixed, then what two parties can establish between themselves, from their own sides and without conferring, is smaller in a world of many parties than in a world of two. The paper does not establish that the antecedent holds of any actual arrangement.

6.7 The Distance from a Constructed Shared Description

The relation between this account and the companion one may now be stated exactly.

A common factor is an object built by the parties, at a cost, and it is coarser than either of them. An invariant is not built, costs nothing, and is indifferent to how either party describes its own side. The two are not rivals at the level of the mathematics, since a pair may have both, and they are alternatives at the level of the reading: an account that identifies the common part with a common factor must treat the second clause of the formula as a limit upon the first, and an account that identifies it with an invariant need not.

What the common factor supplies and the invariant does not is a description in which the parties may write something down. An invariant is a number and an agreed text is not, so an arrangement that must record its terms will construct a common factor whatever the present account says about what the parties hold. Section 18 treats the two together.

7 THE REACH OF OPERATIONS PERFORMED SEPARATELY

This section treats what the parties can and cannot bring about. Its objective is to state what operations each party performs alone leave untouched, to set out two cases that differ in what messages between the parties can do, to say what each case requires of the claim about diplomacy, and to state the position the paper takes together with the grounds for it.

7.1 *Operations Each Party Performs Alone*

An operation confined to one party's side acts on the joint state by an element of the local group, or more generally by a channel acting on one factor. By Definition 6.1 no such operation alters an invariant, so the following holds without argument.

Proposition 7.1. (Redescription changes nothing). *Every invariant of the local action is unchanged by operations that the parties perform on their own sides, however elaborate and however many. The common part is therefore not something a party can produce by working upon itself.*

Proposition 7.1 is the formal statement of something the received accounts of diplomacy record in other terms: a party cannot establish a relation by preparing itself for one.

7.2 *The Case in Which Messages Establish the Common Part*

Suppose the common part is a correlation between what the two parties hold. Messages then establish it directly. A party that reports its own condition gives the other a description that depends on it, and after the report the two descriptions are correlated where before they were independent. Repeated correspondence increases the correlation, and nothing further is required.

On this case the answer to the question of the section is that operations performed separately establish nothing and messages establish everything. The formula is then satisfied by an exchange of letters conducted with sufficient care.

7.3 *The Case in Which Messages Do Not Establish It*

Suppose now that the common part is of a kind that operations performed separately, together with messages passing between the parties, cannot increase. The formalism of composite systems has a standard result of exactly this shape: quantities of a certain class do not increase under operations confined to one side together with communication between the sides (Horodecki et al., 2009).

On this case the answer is different. Whatever the parties do apart, and whatever they tell one another, the common part is unchanged; establishing it requires an operation performed upon the pair.

7.4 *The Commitment Each Case Requires*

The two cases are not alternative formalisms for one claim. They are different claims and each carries a commitment.

The first requires that whatever two parties come to hold in common be reproducible by correspondence conducted well enough. A practice that treats presence, visits, and acts as doing something letters cannot do is then a practice responding to a cost, since what the acts achieve could have been achieved otherwise at greater expense.

The second requires that some of what parties hold in common be unreachable by correspondence at any expense. A practice that insists on presence is then responding to a structural condition and not to a cost, and a party that declines to settle a matter by letter is not being ceremonious.

7.5 *The Position Taken and the Grounds for It*

The paper takes the second case, and the grounds are drawn from practice in place of from the formalism.

Diplomatic practice sustains, at considerable expense, a class of acts whose work is not reproduced by correspondence: the visit, the attendance, the delivery made in person, the gift, the refusal to conclude a matter in writing. A companion paper treats one member of that class at length and finds that what it achieves is achieved through an operation performed at the receiving end and occasioned by an act (Huang, 2026c). The practice behaves as though something is established by acts that is not established by reports of them.

Two qualifications belong with the position and neither is small. The inference runs from a practice to a structure, and a reader may hold that the practice reflects the cost of verification, the value of a costly signal, or an inherited ceremony, in which case the first case obtains and the practice is explained without any structural claim. And the formalism was not consulted in reaching the position; it was consulted afterwards, and what it supplies is a setting in which the position can be stated exactly.

Claim 7.2. (What the position commits the account to). *The account holds that some of what two parties hold in common is not established by operations each performs alone together with reports exchanged between them, and that establishing it requires an act upon the pair. The holding is argued from practice and is not proved, and Sections 6 and 9 stand without it.*

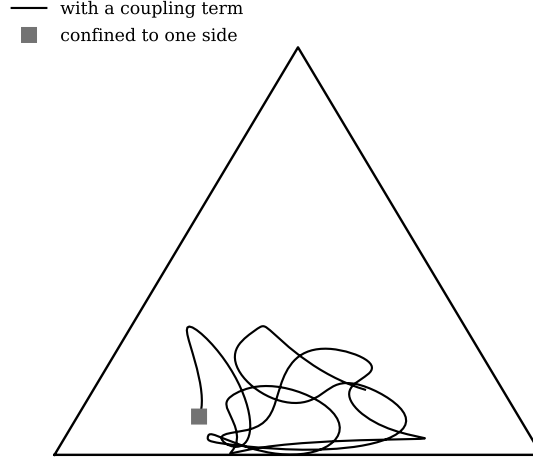


Figure 2: The Schmidt coefficients of one state, drawn on the base at $d = 3$, evolving under two generators. The square marks the whole trajectory produced by a generator confined to the two sides, which does not move. The curve is the trajectory produced by the same generator with a coupling term added, from the circle. The figure displays a computed instance of Proposition 8.1.

8 THE DYNAMICS OF THE JOINT STATE

This section treats change over time. Its objective is to state what evolution generated on the parties' own sides preserves, what a generator acting on the pair permits, and what trajectories are available to parties acting separately.

8.1 Evolution Under Generators Confined to One Side

Let the joint state evolve under a generator that is a sum of a term acting on the first party's factor and a term acting on the second's, so that the evolution takes the form $U_A(t) \otimes U_B(t)$. Each party evolves as it would have evolved alone, and the pair evolves as the two of them do.

Proposition 8.1. (Separate evolution is confined to an orbit). *Under a generator with no term coupling the two factors, the joint state remains on the orbit of the local group at which it began. Every invariant is constant along such an evolution, and the trajectory of the pair in the space of invariants is a single point.*

Figure 2 displays the proposition in a computed instance. Under a generator confined to the two sides the coefficients of a randomly drawn state moved by at most 7.8×10^{-16} over the interval, which is the precision of the arithmetic; under the same generator with a coupling term added they moved by 0.2301.

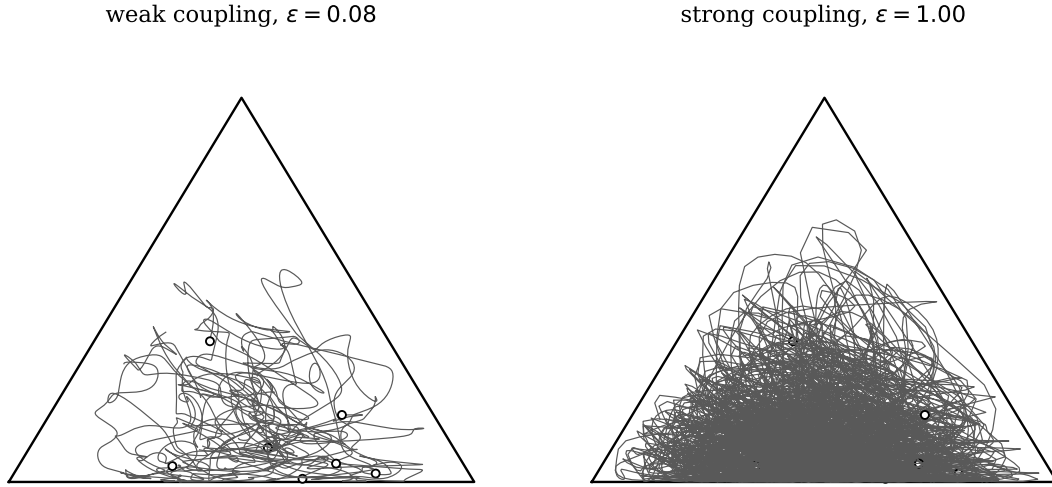


Figure 3: Paths traced on the base at $d = 3$ by seven initial states, under one generator with a coupling term at two strengths. Circles mark the initial points. The largest excursion of a coefficient from its initial value was 0.3109 on the left and 0.3740 on the right, with medians of 0.2116 and 0.3377. The figure displays the behaviour of the model.

Proposition 8.1 is Proposition 7.1 extended in time and it is worth stating separately for what it excludes. Two parties may each change a great deal, at length, and by their own lights transform themselves entirely, while what they hold in common has not moved at all.

8.2 Evolution Under a Generator Acting on the Pair

A generator carrying a term that acts on both factors at once moves the joint state off its orbit, and the invariants change. What such a term represents in the setting of this paper is an interaction: something done that is not the doing of either party by itself.

The rate at which the invariants change under such a generator depends on the strength of the coupling term, and where that strength is small the invariants move slowly against the parties' own motion. That separation of rates is the one used in the companion treatments of coupled systems, in which the terms of a relation move on a slower scale than the states of the parties and are moved by them (Huang, 2026a,c).

8.3 The Motion Induced on the Space of Possible Common Parts

The invariants move under a coupling term, so the pair traces a path in the space of Section 9. That space is two-dimensional at $d = 3$ and the paths may be drawn in it, which Figure 3 does for seven initial states at two strengths of the coupling.

Two features of the picture are worth stating and neither was anticipated by the prose.

Proposition 8.2. (No resting point). *Under a generator that does not change with time, the motion induced on the space of possible common parts has no attracting set and no point of that space is fixed by it. The paths are recurrent.*

The proposition follows from the evolution preserving the norm and was checked. Over a horizon of two hundred units a path returned to within 0.0006 of its initial spectrum, against a maximum excursion of 0.3284. What follows is that a pair evolving under a fixed coupling does not settle anywhere in the space of possible common parts, and an account in which a relation comes to rest requires something the present setting does not contain.

Proposition 8.3. (Strength sets the rate). *The strength of the coupling term governs how quickly the common part moves and not how far it moves. Weakening the coupling delays the excursion and does not bound it.*

Computed at four strengths, with the coupling scaled by 0.04, 0.08, 0.25 and 1.00: the maximum excursions were 0.3820, 0.3508, 0.4358 and 0.3284, differing by less than a factor of two across a twenty-five-fold change in the coupling, while the time taken to reach half that excursion fell from 22.98 to 10.45, 3.43 and 0.25. A weakly coupled pair explores what a strongly coupled pair explores, and takes longer about it.

Both propositions hold for a generator that does not change with time and in the absence of anything dissipative. What a setting with dissipation would give is not treated here and is recorded at Section 21.

8.4 The Motion Where Something Is Dissipated

Propositions 8.2 and 8.3 hold where nothing is lost. A setting in which something is lost may be had by adding to the generator a term that is not conservative, which in the formalism of composite systems takes a standard form and which in the field-theoretic construction of the companion paper would be a term of the action that does not conserve what the rest of it conserves (Huang, 2026a). The construction is not repeated here and what is used is its effect.

Taking each party to be carried, at a rate γ , toward a fixed state of its own, and the coupling to act at a strength ε , the motion on the base was computed for six initial states at each of two ratios.

Proposition 8.4. (A resting point, and the loss of the initial condition). *Where something is dissipated, the motion induced on the space of possible common parts has an attracting point, and the point does not depend on the state the pair began in. A pair evolving under such a generator forgets where it started.*

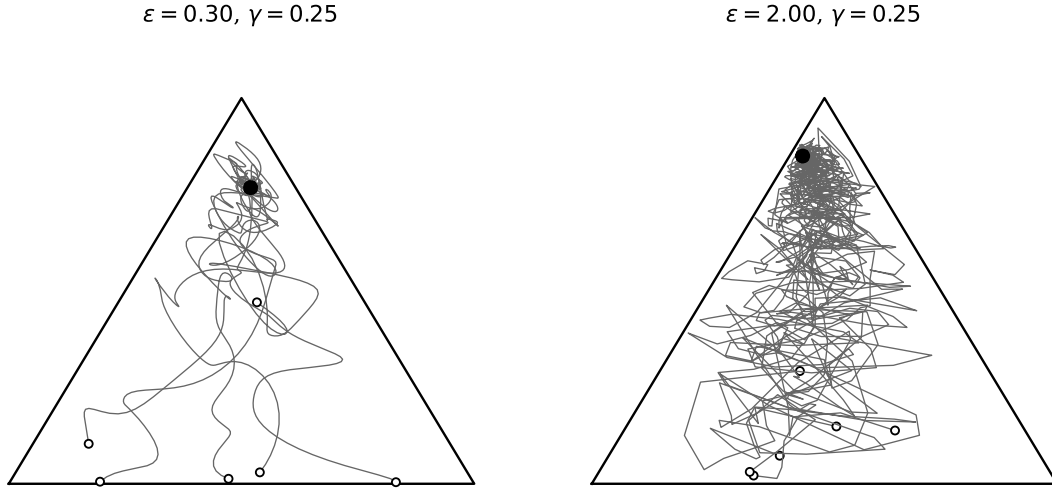


Figure 4: Paths on the base at $d = 3$ under a generator with a coupling term and a term that dissipates, from six initial states at each of two coupling strengths. Open circles mark the initial points and filled circles the endpoints, which coincide. The figure displays the behaviour of the model.

The six paths at each ratio ended at one point to the precision of the computation. At $\varepsilon = 0.30$ with $\gamma = 0.25$ the common endpoint was $(0.4204, 0.3237, 0.2559)$, and at $\varepsilon = 2.00$ with the same γ it was $(0.4192, 0.2975, 0.2833)$. Figure 4 shows the paths.

Proposition 8.4 inverts the closed case in a respect worth marking. There the invariants were fixed by the initial state and no local action moved them. Here they are fixed by the generator and the initial state is forgotten. A relation that dissipates holds what its coupling and its losses determine, and not what it began with.

Proposition 8.5. (The resting point and the ratio). *The attracting point depends on the coupling and the dissipation through their ratio. As the ratio rises the point moves toward the uniform end of the base, and as it falls the point moves toward the end at which the parties hold nothing in common.*

Computed at $\gamma = 0.25$, the resting spectra were

$$\begin{aligned}
 \varepsilon/\gamma = 0.4 : & \quad (0.6156, 0.2033, 0.1811) \\
 \varepsilon/\gamma = 1.2 : & \quad (0.4204, 0.3237, 0.2559) \\
 \varepsilon/\gamma = 2.4 : & \quad (0.3859, 0.3339, 0.2802) \\
 \varepsilon/\gamma = 6.0 : & \quad (0.4172, 0.2995, 0.2833) \\
 \varepsilon/\gamma = 16.0 : & \quad (0.3951, 0.3192, 0.2857).
 \end{aligned}$$

The movement toward uniformity is present and it saturates; the computed points approach

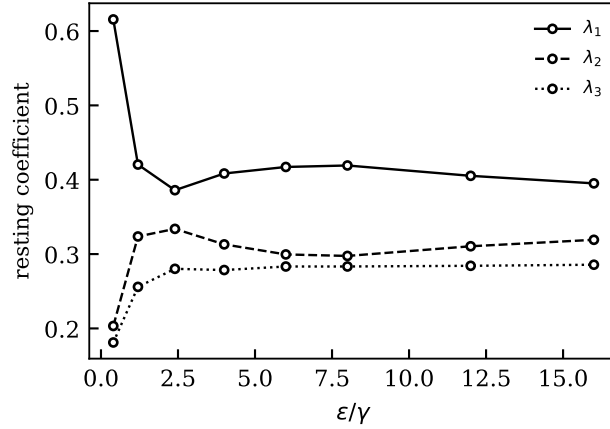


Figure 5: The three coefficients of the resting spectrum, plotted against the ratio of the coupling strength to the dissipation rate, at $d = 3$ with $\gamma = 0.25$. The figure reports the computed endpoints.

the uniform spectrum and do not reach it. Figure 5 shows the coefficients against the ratio.

Read together with Section 11, Proposition 8.5 has a consequence that neither section has alone. The characteristic of the fibre falls to its minimum at the uniform end, and a rising ratio of coupling to dissipation carries the resting point toward that end. A relation strongly coupled and little dissipating therefore rests near the point at which the measure of preserved difference is smallest, and does so without any party intending it.

8.5 The Price of Admitting Dissipation

What the preceding subsection buys is paid for, and the price should be stated where it is incurred.

A dissipative evolution does not preserve purity, and the results of Section 6 were established for pure states of the pair. Proposition 6.2, which makes the coefficients a complete invariant, and Proposition 6.3, which lets a party compute that invariant from its own side, both fail once the joint state is mixed, as Claim 6.5 records.

Claim 8.6. (What dissipation costs). *A setting that admits dissipation admits a resting point and loses the completeness of the invariants. The spectrum of a party's own description remains defined and remains invariant under redescription; what it no longer does is determine the orbit.*

The two settings are therefore not one setting with a parameter. The closed one answers what is held in common and by whom it may be computed; the dissipative one answers where a relation comes to rest and forgets what it began with. Sections 9 through 15 belong to the first, and Section 21 records the question of what they become in the second.

8.6 *The Trajectories Available to Parties Acting Separately*

Taking Propositions 7.1 and 8.1 together, the trajectories available to parties who act only upon themselves are confined to a level set of every invariant. The space of such trajectories is large, since an orbit of the local group is large, and it contains no path between one value of an invariant and another.

Two readings are recorded and each is a reading. An arrangement in which the parties reform themselves, each in its own way and at length, is an arrangement in which the common part is where it was. And a party that believes it has changed a relation by changing itself has, on this account, changed its position within an orbit.

9 THE ORBIT SPACE AND ITS STRATA

This section describes the space in which the possible common parts lie. Its objective is to identify that space, to record its strata, to state the order that the possible common parts carry, and to draw the consequence for questions posed in terms of how much two parties share. Every numerical statement below was computed before the proposition reporting it was written, and the computation accompanies the paper.

9.1 *The Quotient by Local Redescription*

By Proposition 6.2 two pure states of the pair lie on one orbit exactly when their Schmidt coefficients agree, so the quotient of pure states by the local group is the set of possible multisets of Schmidt coefficients. Writing $d = \min(d_A, d_B)$ and ordering the coefficients, that set is

$$\Delta_d^\downarrow = \left\{ \lambda \in \mathbb{R}^d : \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_d \geq 0, \sum_i \lambda_i = 1 \right\}, \quad (3)$$

an ordered simplex of dimension $d - 1$.

The space of possible common parts is therefore a simplex, and a common part is a point in it. For two parties each describing their affairs in three terms the simplex has dimension two; in five terms, dimension four.

9.2 *The Strata and the Faces That Carry Them*

The Schmidt rank of a state is the number of its nonzero coefficients, and the states of a given rank form a stratum of the quotient. In the simplex of Equation (3) the stratum of rank r is the relative interior of a face of dimension $r - 1$: for $d = 3$ the ranks one, two and three occupy faces of dimensions zero, one and two, and for $d = 5$ the ranks one through five occupy faces of dimensions zero through four.

Two properties follow. The rank cannot exceed the smaller of the two parties' dimensions,

so a party describing its affairs in few terms bounds the rank of whatever the pair may hold, however finely the other party describes its own. And a path in the simplex that reaches a face has lost a coefficient, which is a change of stratum and not a change of magnitude.

9.3 *The Order the Possible Common Parts Carry*

A natural order on the simplex is majorisation: one common part stands above another when the partial sums of its ordered coefficients dominate the other's. The order is standard, it is the order in which conversions of the kind treated at Section 7 are possible, and it is partial.

Proposition 9.1. (The order is partial). *Majorisation does not order the simplex totally. Common parts exist that stand in neither relation to one another, so that neither is above the other and they are not equal.*

The proposition is established by exhibition and its frequency was measured. In dimension three the ordered coefficients (0.50, 0.45, 0.05) and (0.60, 0.20, 0.20) have partial sums (0.50, 0.95, 1.00) and (0.60, 0.80, 1.00), so neither dominates the other. Sampling pairs from the simplex, the proportion of incomparable pairs was 24.1 per cent in dimension three, 37.8 per cent in dimension four, and 45.6 per cent in dimension five, over two thousand pairs at each dimension.

The correspondence with a result obtained in the companion paper by an unrelated route is worth recording. There the joint descriptions available to two parties were found to carry no greatest element, with several maximal descriptions in 6.5 per cent of small cases and 24.5 per cent of slightly larger ones (Huang, 2026a). Here the possible common parts are found to be partially ordered, with incomparability rising in the same direction as the dimension. The two accounts reach the absence of a single best common part from different starting points.

9.4 *The Consequence for Questions Posed in Terms of Degree*

A question asking how much two parties share asks for a number. What the present section supplies is a point in a simplex, and a number is a function of that point.

Proposition 9.2. (A scalar measure conflates incomparable parts). *Any scalar measure of the common part is a function on the simplex, so it assigns one value to distinct points. Points exist that receive the same value under such a measure and that stand in neither relation under majorisation, so a scalar measure identifies common parts that the order does not compare.*

The proposition was checked in dimension three. The ordered coefficients

$$(0.500, 0.300, 0.200) \quad \text{and} \quad (0.514, 0.246, 0.240)$$

carry the same entropy to four decimal places, at 1.0297, and neither majorises the other. A party comparing two relations by such a measure would report them equal in what is shared while the order that governs what can be converted into what does not compare them at all.

What follows is a caution and not a prohibition. Scalar measures are useful and the paper uses one in its illustrations. What they may not carry is the weight of a claim that one relation shares more than another, since that claim is about an order the measure does not represent.

10 THE FIBRATION OVER THE ORBIT SPACE

This section turns from the base to what lies over it. Its objective is to record that the base carries no topology of its own, to identify the orbit through a state, to name the space that orbit is, and to say what a point of that space is in the reading the paper offers. The dimensions reported were computed before the statements reporting them were written, by taking the rank of the infinitesimal action, and the computation accompanies the paper.

10.1 *The Base and the Topology It Does Not Carry*

The orbit space of Equation (3) is an intersection of half-spaces with a simplex, hence convex, hence contractible. Its homotopy groups vanish and its homology is that of a point.

The observation is worth stating because it fixes where anything topological can come from. Nothing about the space of possible common parts, taken by itself, has topological content. What has such content is the collection of states lying over each point of it, and the way that collection changes as the point moves.

10.2 *The Orbit Through a State and Its Dimension*

Fix a pure state of the pair and consider its orbit under the local group. The dimension of that orbit was computed for spaces of equal dimension d by taking the rank of the map that sends an element of the Lie algebra of the local group to the corresponding infinitesimal motion of the state.

Where the Schmidt coefficients are distinct and all positive, the orbit has dimension $2d^2 - d - 1$ within a projective space of dimension $2d^2 - 2$, leaving a quotient of dimension $d - 1$ as Equation (3) requires. The computed dimensions were 5, 14 and 27 at $d = 2, 3, 4$, against ambient dimensions 6, 16 and 30.

10.3 *The Stabiliser and the Space the Orbit Is*

The subgroup fixing a state with distinct positive coefficients consists of the transformations that multiply the two Schmidt bases by opposite phases, so it is a maximal torus. The orbit is

accordingly the quotient of the local group by that torus, and the quotient of a unitary group by a maximal torus is the manifold of complete flags.

Definition 10.1. (The fibre and the space of a party's descriptions). *The fibre of the quotient map over a point of the orbit space is the orbit of any state carrying that point's coefficients. Where the coefficients are distinct and positive, that orbit is a bundle over the product of the two parties' manifolds of complete flags, with a torus for its own fibre. The space of one party's descriptions compatible with the given common part is that party's manifold of complete flags.*

The distinction between the orbit and the space of one party's descriptions must be kept, since the measure used below belongs to the second and not to the first. The stabiliser of a state with distinct positive coefficients is a torus of rank d inside a group of rank $2d$, so the orbit carries a torus factor and its own Euler characteristic vanishes. At $d = 2$ the orbit is five-dimensional, which shows it directly. What the following section measures is the space of a party's descriptions, whose characteristic does not vanish.

10.4 The Fibration Displayed at the Smallest Dimension

At $d = 2$ the whole arrangement may be drawn. The base is the interval running from the point at which the two coefficients are equal to the point at which one carries the whole weight. Over the interior of that interval the space of a party's descriptions is a two-sphere, of Euler characteristic two, and over the uniform point it is a single point, of characteristic one. Figure 6 shows it.

10.5 The Reading of a Point of the Fibre

A complete flag in a party's space is an ordered chain of subspaces, and it is determined by an ordered orthonormal basis taken up to phases. In the terms of Section 5.1 that is a way the party has of describing its own affairs, together with an order upon the descriptive distinctions it makes.

Claim 10.2. (The two halves and the two spaces). *The base of the fibration carries what the parties hold in common and the fibre carries the ways in which each of them describes its own side. The first clause of the formula concerns a point of the base and the second concerns the fibre over it.*

Claim 10.2 is the reason the fibration is the right object for the question. The two clauses were shown at Claim 6.4 to concern different objects; here the two objects are exhibited, and they are the two parts of one space.

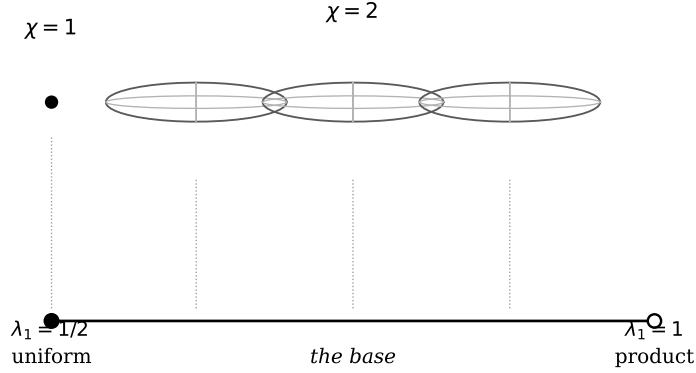


Figure 6: The arrangement at $d = 2$. The base is the interval below; the spaces drawn above it are the descriptions available to one party at the points beneath them. Over the interior that space is a two-sphere and over the uniform point it is a single point. The figure displays the objects and reports no measurement.

11 THE DEGENERATION OF THE FIBRE AND THE COLLAPSE OF ITS CHARACTERISTIC

This section states what happens to the fibre as the point in the base moves to a wall. Its objective is to record the change in dimension, to give the Euler characteristic of the fibre as a function of the point, and to state the result the paper was extended in order to reach.

11.1 The Shrinking of the Orbit at a Coincidence

Where two Schmidt coefficients coincide, the subgroup fixing the state is larger, since a rotation mixing the two corresponding directions leaves the state unchanged. The orbit is smaller by the difference in the dimensions of the two subgroups.

Computation gives the difference exactly. At $d = 3$ the orbit falls from 14 to 12 when one coincidence is present, and at $d = 4$ from 27 to 25, a shrinkage of 2 in each case, which is the dimension of the unitary group of a plane less that of its diagonal torus. Where all coefficients coincide the orbit has dimension $d^2 - 1$, computed as 3, 8 and 15 at $d = 2, 3, 4$, and the subgroup fixing the state is the whole unitary group acting diagonally, so the fibre is a single point.

11.2 The Euler Characteristic of the Fibre

The fibre over a point of the base is a manifold of partial flags, determined by how the coefficients at that point are grouped: coefficients of equal value are not distinguished by the state, and the directions carrying them may be mixed freely.

Proposition 11.1. (The characteristic of the space of a party's descriptions). *Let*

the coefficients at a point of the base take values with multiplicities $k_1, \dots, k_m$ summing to d , counting the vanishing coefficients as one class. The space of one party's descriptions compatible with that point is a manifold of partial flags, and its Euler characteristic is

$$\chi = \frac{d!}{k_1! k_2! \cdots k_m!}. \quad (4)$$

The orbit of the pair over that point carries a torus factor and has Euler characteristic zero, so the measure is taken of the first space and not of the second.

The group whose order the generic value counts is worth naming, since the paper computes it throughout. The value $d!$ is the order of the group of permutations of d objects, the ordered simplex of Equation (3) is a fundamental domain for the action of that group on unordered spectra, and the characteristic over a point counts the images of a description under the subgroup that the coefficients at that point leave free. A braid group stands over that permutation group and would appear where the exchange of coefficients along a path is tracked and not only its endpoint; the coefficients here are real and ordered, so paths among them carry no braiding, and Section 21 records what a setting admitting it would require.

The values were computed for small dimensions. At $d = 3$ they are 6, 3 and 1 for the patterns $1 + 1 + 1$, $2 + 1$ and 3 ; at $d = 4$ they are 24, 12, 6, 4 and 1; at $d = 5$ they are 120, 60, 30, 20, 10, 5 and 1.

Figure 7 shows the base at $d = 3$ with the characteristic marked on each of its parts.

Two features of the figure are worth marking. The collapse is driven by coincidence among the coefficients and not by the vanishing of one, since a single vanishing coefficient forms a class of its own and leaves the characteristic unchanged. And the collapse is not gradual: the characteristic takes integer values that divide $d!$ and it changes only at the walls.

11.3 The Result

Claim 11.2. (The collapse). *The Euler characteristic of the space of a party's descriptions is a topological invariant, it counts the distinguishable ways the parties have of describing their own sides, and it falls from $d!$ where the common part is generic to 1 where the common part is uniform. The preservation of difference has a topological measure, and that measure vanishes to its minimum exactly where the common part is at its most symmetric.*

Claim 11.2 is a statement about a group action and a quotient, and it holds whatever is made of the reading. What the reading adds is the observation that the two clauses of the formula, having been shown to concern different objects at Claim 6.4, are found here to be in a definite relation after all: the second is not spent in satisfying the first, and it is nonetheless extinguished at the one point where the first is satisfied completely.

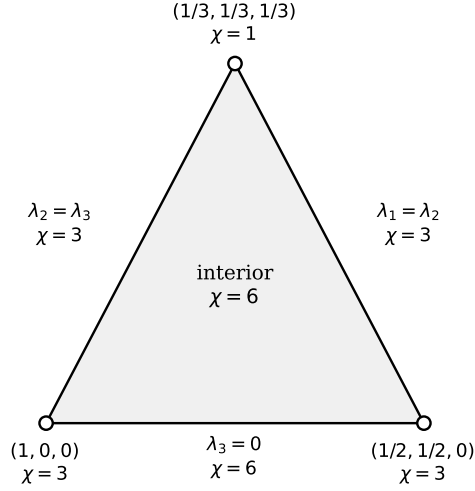


Figure 7: The orbit space at $d = 3$, with the Euler characteristic of the fibre marked on each part. The characteristic is 6 in the interior and on the edge where the smallest coefficient vanishes, 3 on each edge where two coefficients coincide and at the two corresponding corners, and 1 at the point where all three coincide. The figure reports the values computed from Equation (4).

The companion account of relational crystallisation identifies a condition in which parties differ in labels alone and treats it as a failure (Huang, 2026b). That condition is the uniform point of the base, and Claim 11.2 supplies for it a measure that is zero-dimensional and an argument that requires no normative premise.

12 THE SPHERICAL GEOMETRY OF THE BASE

This section gives the base a geometry. Its objective is to identify the base with a familiar geometric object, to state the distance that identification supplies, and to record the quantities computed from it.

12.1 The Square-Root Map and the Sphere

Sending a point of the base to the vector of square roots of its coordinates carries the ordered simplex onto the ordered part of the positive orthant of the unit sphere in $\mathbb{R}^d$, since $\sum_i \lambda_i = 1$ becomes $\sum_i (\sqrt{\lambda_i})^2 = 1$. The image inherits the geometry of the sphere, and the distance between two points of the base is the angle between their images,

$$\theta(\lambda, \mu) = \arccos \left(\sum_i \sqrt{\lambda_i \mu_i} \right), \quad (5)$$

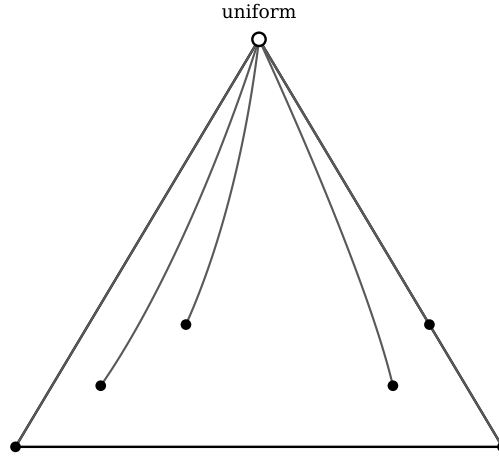


Figure 8: Geodesics of the distance of Equation (5) at $d = 3$, drawn from the uniform point to six others. The paths are great circles on the sphere carried back to the base by squaring. The figure displays the geometry of the model.

which is the Fisher–Rao distance and is standard.

12.2 The Quantities the Geometry Supplies

Three quantities were computed. Two points of the base at which different single coefficients carry the whole weight lie at angle $\pi/2$, at every dimension, so the orthant has diameter $\pi/2$. A point of that kind lies at angle $\arccos(1/\sqrt{d})$ from the uniform point, computed as 0.7854, 0.9553, 1.0472 and 1.1071 at $d = 2, 3, 4, 5$. And the walls of the orthant are the images of the walls of the simplex, so the strata of Section 9 and the boundary of the orthant are the same object seen twice.

Figure 8 shows geodesics of that distance drawn from the uniform point, with lengths from 0.2937 to 0.9553 for the six endpoints displayed.

The geometry therefore supplies what the order of Section 9.3 does not: a distance between any two possible common parts, and a notion of one being nearer to the uniform point than another. What that supply costs is the subject of the next section.

13 THE ORDER AND THE METRIC

This section holds the two structures of the preceding sections together. Its objective is to say what question each answers, to establish that the metric never contradicts the order, to show that it decides everything the order leaves open, and to state what follows for any claim that one relation shares more than another. The proportions reported were computed before the

statements reporting them were written.

13.1 *The Question Each Structure Answers*

The majorisation order of Section 9.3 answers an operational question: whether one possible common part can be carried into another by what the parties are able to do. It is partial, and where it is silent there is no operation of the relevant kind in either direction.

The distance of Equation (5) answers a comparative question: how far apart two possible common parts lie. It is total, and it assigns a number to every pair.

Both are structures on the same space and neither is a substitute for the other. What follows records how they stand to one another.

13.2 *The Metric Agrees Wherever the Order Speaks*

Proposition 13.1. (Extension). *Where one point of the base majorises another, it lies no nearer the uniform point in the distance of Equation (5). The distance to the uniform point therefore extends the majorisation order and contradicts it nowhere.*

The proposition follows from the concavity of $\sum_i \sqrt{\lambda_i}$ under the ordering by majorisation, and it was checked. Of 14 908, 12 637 and 11 277 majorisation-comparable pairs sampled at $d = 3, 4, 5$, the distance to the uniform point disagreed with the order in no case.

13.3 *The Metric Decides Everything the Order Leaves Open*

Proposition 13.2. (Totality). *The distance to the uniform point separates every pair of distinct points that the majorisation order leaves incomparable. Of the pairs on which the order is silent, the distance is silent on none.*

The proposition was checked at three dimensions. Of 1005, 1519 and 1743 incomparable pairs sampled at $d = 3, 4, 5$, the distance ranked all of them. The pair of Section 9.3 supplies an instance: the points (0.50, 0.45, 0.05) and (0.60, 0.20, 0.20) are incomparable, and the second lies nearer the uniform point at 0.2706 against 0.3907, the two standing at 0.3248 from one another.

13.4 *The Cost of the Decision*

A structure that decides everything the order leaves open has supplied something the order does not contain, and the question is what.

Proposition 13.3. (The extension is one among many). *Every function on the base that reverses the majorisation order extends it, and there are many such functions. Two of*

- ordered by majorisation
- incomparable, ranked alike
- incomparable, ranked oppositely

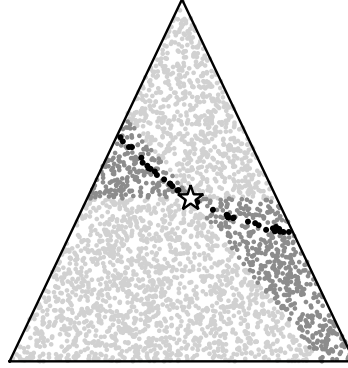


Figure 9: Points of the base at $d = 3$ compared with the reference $(0.55, 0.30, 0.15)$, marked by a star. Of 3000 sampled points, 2320 are ordered against the reference by majorisation, 643 are incomparable and are ranked alike by the distance and by the entropy, and 37 are incomparable and are ranked oppositely by the two. The figure reports the sample and no claim beyond it.

them will in general rank incomparable points differently, so a judgment that one relation shares more than another is a judgment relative to the function chosen.

The proposition was checked against a second such function. Taking the entropy of the coefficients as the alternative, the distance to the uniform point and the entropy ranked incomparable pairs oppositely in 12.3 per cent of cases at $d = 3$, 12.4 per cent at $d = 4$, and 11.4 per cent at $d = 5$. Figure 9 shows the points at which the two disagree about a fixed reference.

13.5 The Consequence for a Claim About Degree

The three propositions together fix what may be said. Where the order speaks, a claim that one relation shares more than another is operational and any extension will agree with it. Where the order is silent, such a claim is a report of the measure chosen, and about one incomparable pair in eight will be decided the other way by an equally reasonable measure.

This supersedes the caution of Section 9.4, which held that a scalar measure conflates incomparable parts. The more exact statement is that a scalar measure separates them, and separates them in a way that another scalar measure will contradict.

14 THE MOMENTUM MAP AND THE CONVEXITY WITHHELD

This section records one further identification and states what it does not supply. Its objective is to name the map that carries a state to the pair of reduced descriptions, to say what that identification places the account within, and to record why the theorem usually associated with it is not available here.

14.1 *The Map That Carries a State to the Parties' Own Descriptions*

The action of the local group on the space of states preserves the natural symplectic form, and the map assigning to a state the pair of its reduced descriptions is the momentum map of that action. The orbit space of Section 9 is then the reduction of the state space by the group.

The identification is exact and it is worth stating for what it names. Each party's own description of the pair, defined at Section 5.4 by disregarding the other party's side, is the momentum of the group of redescrptions. What a party holds of the relation is the conserved quantity of the freedom the second clause of the formula preserves.

14.2 *The Theorem That Is Not Available Here*

Momentum maps of this kind carry a convexity theorem: the image of the map is a convex polytope. That theorem is the source of most of what is known about which combinations of reduced descriptions are jointly possible.

It supplies nothing here, and the reason should be stated in place of passed over. For a pure state of a pair, the two reduced descriptions have the same spectrum, so the image of the momentum map is carried by a single ordered spectrum and the polytope degenerates to the base of Equation (3). The convexity theorem in its substantial form requires three or more parties, or states of the pair that are not pure, and neither is treated in this paper.

Claim 14.1. (What the identification does and does not give). *The momentum map identifies each party's own description as the conserved quantity of the local action, which is a statement about the setting. It does not supply a polytope of jointly possible descriptions, since for a pure state of two parties that polytope is the base itself. An account of three or more parties would have one, and Section 21 records the question.*

15 GENERATION AND THE CEILING IT RAISES

This section treats what happens when a party comes to make distinctions it did not make before. Its objective is to represent that as an operation on the setting, to state what it does to the common part, to state what it does to the ceiling and to the fibre, and to distinguish the

two kinds of generation the setting admits. The values reported were computed before the statements reporting them were written.

15.1 The Enlargement of a Party's Own Space

Nothing in Section 5 requires the spaces to be fixed. A party that comes to draw a distinction it did not draw before is a party whose space of descriptions has grown, and the growth is represented by an isometric embedding of the old space in a new one.

Definition 15.1. (Generation). *A generation on one side is an isometric embedding $\iota : \mathcal{H}_A \hookrightarrow \mathcal{H}'_A$ with $d'_A > d_A$, carrying each state of the pair to its image under $\iota \otimes \text{id}$. It is performed by one party and is not an element of the local group, since it does not act within a fixed space.*

The distinction from a redescription is exact and is the reason the operation is treated separately. A redescription rearranges what a party can already say. A generation enlarges what it can say.

15.2 The Common Part Under an Enlargement

Proposition 15.2. (The common part does not move). *An enlargement performed by one party leaves the Schmidt coefficients of the joint state unchanged, so the point of the base is where it was. Every invariant of Section 6 takes the value it took before.*

The proposition follows from the embedding carrying the decomposition of Equation (2) term by term and adding directions in which the state has no weight. It is the counterpart, for generation, of Proposition 8.1 for evolution: a party that develops itself has not by that act altered what it holds in common with anyone.

15.3 The Ceiling and What Raises It

Section 9.2 records that the rank of the common part cannot exceed the smaller of the two parties' dimensions. An enlargement changes that bound.

Proposition 15.3. (The ceiling). *The maximal rank available to the pair is $\min(d_A, d_B)$. An enlargement performed by the party of smaller dimension raises it; an enlargement performed by the other party does not.*

The asymmetry is worth marking. Whether a party's own generation raises the ceiling depends on the other party's dimension, and that is not something the generating party can settle from its own side. Computed instances: spaces of dimensions three and three admit rank at most three, and so do spaces of dimensions four and three, and five and three.

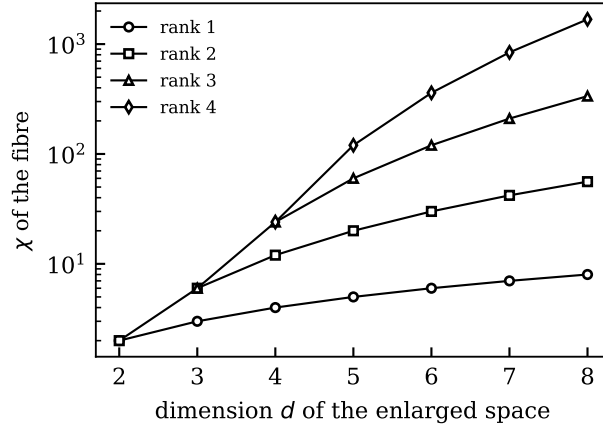


Figure 10: The Euler characteristic of the space of a party's descriptions over a fixed common part, as the space of one party is enlarged, for states of rank one through four. The vertical scale is logarithmic. At rank three the values run 6, 24, 60, 120, 210 and 336 for dimensions three through eight; at rank one they run 2 through 8. The figure reports values computed from Equation (4).

15.4 The Characteristic of the Fibre Under an Enlargement

The common part does not move under an enlargement and the fibre over it does. The state acquires directions in which it has no weight, so its multiplicity pattern gains a class, and Equation (4) gives a larger value.

Proposition 15.4. (The measure of difference grows). *An enlargement performed by one party leaves the point of the base fixed and multiplies the Euler characteristic of the space of a party's descriptions over it.*

Computed values. A state of rank three carried from a space of dimension three into one of dimension four takes the characteristic from 6 to 24; carried into dimension five it reaches 60, and into dimension six, 120. A state of rank two carried from dimension two through dimensions three, four and five takes the characteristic from 2 through 6, 12 and 20. Where the state occupies the whole space the characteristic is $d!$, which is 2, 6, 24, 120 and 720 at $d = 2$ through 6.

Figure 10 shows the growth for four ranks.

Claim 15.5. (What generation does to the two clauses). *Generation performed by one party leaves the first clause of the formula exactly where it stood and multiplies the measure attaching to the second. What a party produces by developing its own capacity to describe is preserved difference, and it is nothing else.*

Claim 15.5 stands beside Claim 11.2 and inverts it. There the measure of difference collapsed as the common part became uniform, with nothing done by either party. Here the

measure grows with what one party does, and the common part does not move.

15.5 *The Two Kinds of Generation*

The setting now admits two operations that produce something and they differ in what they produce and in who can perform them.

An enlargement is performed by one party, alone, and raises what the pair may hold without altering what it holds. An operation acting on the pair, of the kind Section 7 treats, alters what the pair holds and cannot be performed by either party alone.

Claim 15.6. (Generation that does not reach the relation). *A party may raise the ceiling by its own generation and the pair may never occupy the room so made. Nothing in the setting carries a raised ceiling into a risen rank, since raising the rank is an operation upon the pair. A party that has developed itself and finds the relation unchanged has met a structural feature of the setting and not a failure of effort.*

Claim 15.6 is the strongest thing this section supplies to the reading. It also states a condition under which the two kinds are related: generation on one side is what makes a subsequent joint operation able to reach further than it could have reached before, so the two are ordered in time and neither substitutes for the other.

16 THE TWO DEGENERATE ENDS

This section treats the two conditions at which the account says something unambiguous. Its objective is to describe each, to record what the fibre is over each, and to state the result that follows for an arrangement standing between them.

16.1 *The End at Which Nothing Is Held in Common*

Where the joint state is a product, the Schmidt rank is one and a single coefficient carries the whole weight. The invariants take their extreme values and none of them distinguishes one such state from another, so the parties hold nothing in common beyond what any pair of parties holds.

The fibre over that point is a partial flag manifold whose Euler characteristic is d , since the multiplicity pattern has one coefficient and one class of $d - 1$ vanishing coefficients. Computed values are 2, 3, 4, 5 and 6 at $d = 2$ through 6.

16.2 *The End at Which the Parties Differ in Coordinates Alone*

Where the coefficients are all equal, the joint state is fixed by its invariants and the parties differ in nothing the local group does not move. The fibre is a single point, of Euler characteristic 1,

at every dimension.

The companion account of relational crystallisation treats this condition as a failure and not as an ideal (Huang, 2026b). The present account supplies for it a measure that has fallen to its minimum, and the supply requires no normative premise.

16.3 *The Interval and the Refusal to Select Within It*

Proposition 16.1. (The characteristic is maximal in the interior). *The Euler characteristic of the space of a party's descriptions takes the value d at the product end and 1 at the uniform end, and $d!$ at a point where the coefficients are distinct and positive. For d greater than two it is therefore maximal strictly between the two ends.*

Computed values at $d = 3, 4, 5, 6$: at the product end 3, 4, 5 and 6; at the uniform end 1 in each case; and at a generic interior point 6, 24, 120 and 720.

Proposition 16.1 gives the formula a reading it does not have on either of the readings of Section 3. The requirement that an arrangement stand between the two ends is not a compromise between two goods, and it is the condition under which the measure attaching to the second clause is not at a minimum. Both ends minimise it and the interior does not.

What the account does not do is select a point within the interval. The characteristic takes the same value at every point where the coefficients are distinct and positive, so it separates the interior from the two ends and orders nothing within it. Section 19 states why the account declines to supply an ordering by other means.

17 THE COMPUTATIONS AND THEIR STANDING

This section records what was computed, by what means, and what each computation establishes. Its objective is to keep the numerical statements of the paper together in one place and to say plainly what they do and do not support.

17.1 *The Quantities Computed*

Four computations were performed and each is reported at the point where it is used. Orbit dimensions were obtained by taking the rank of the map carrying an element of the Lie algebra of the local group to the corresponding infinitesimal motion of a state, at $d = 2, 3, 4$ and for generic, once degenerate, and fully degenerate spectra. Euler characteristics were evaluated from Equation (4) for every multiplicity pattern at d up to five, and for the enlargements of Section 15. Frequencies of incomparability under majorisation, and of disagreement between two extensions of that order, were estimated by sampling from the base at $d = 3, 4, 5$. Distances were evaluated from Equation (5). Trajectories on the base were obtained by evolving

randomly drawn states under stipulated generators and taking the spectrum at each step, first without and then with a term that dissipates.

17.2 The Reach of Each Computation

The dimension computations check statements that also follow from the structure, and their role is confirmation. Where they and the prose disagreed the prose was corrected.

The characteristics are exact and follow from Equation (4), so the computation there is arithmetic and not evidence.

The trajectory computations establish what they display and no more. That six paths reached one point is a fact about one generator drawn at random and one dissipator chosen by the author, and it is consistent with Proposition 8.4 without establishing it in general.

The frequencies are estimates from finite samples of a distribution the author chose, namely the uniform distribution on the base. They report what that distribution gives and they support no claim about how the possible common parts of actual relations are distributed, which is a question the paper does not raise.

17.3 The Standing of the Numbers

No quantity in this paper was measured from any record of any relation. The numbers describe a construction, they were computed before the statements reporting them were written, and the code accompanies the paper so that a reader may check them or replace the author's choices with others.

18 THE CORRESPONDENCE WITH THE COMPANION PAPERS

This section states how the objects of this paper stand to those of the three companion papers on diplomacy. Its objective is to identify the counterparts, to record where two accounts reach one conclusion by different routes, and to say where they part.

18.1 The Common Factor and the Local Invariant

The companion treatment of knowledge among differently situated parties represents a joint description as a common factor of two parties' dynamics (Huang, 2026a). Section 6.6 states the relation: a common factor is built at a cost and is coarser than each party's own description, while an invariant is not built, costs nothing, and is indifferent to how either party describes its own side.

The two accounts reach one conclusion by routes that are unrelated, and the coincidence is recorded at Section 9.3. There the joint descriptions available to two parties were found by enumeration to carry no greatest element. Here the possible common parts were found to be

ordered only partially, with incomparability rising as the parties describe their affairs more finely. Neither account supplies a single best common part, and neither derives that from the other.

18.2 The Reduced Description and the Sites of a Passage

The companion anatomy of injustice treats a claim passing between parties and identifies the points at which it may fail (Huang, 2026d). Two of its objects have counterparts here. What a party construes from its own side is the reduced description of Section 5.4, which Section 14 identifies as the momentum of the local action. And what that account calls a shared description, whose coarseness may leave a divergence unregistrable, is the common factor of the companion paper and not the invariant of this one.

The difference is worth marking for what it does not remove. An invariant is indifferent to redescription and is therefore not subject to the failure that account identifies at the site of registration. It is also unavailable as a place to record anything, so it supplies no remedy for that failure.

18.3 The Slow Variable and the Two Kinds of Change

The companion treatment of the gift represents the terms of a relation as a slow variable moved by the parties' states and by nothing acting on it directly (Huang, 2026c). Section 8 carries the same separation of rates, with the invariants moving under a coupling term and not at all under generators confined to one side.

Two of that paper's claims have counterparts here. Proposition 8.1 states in this setting what that paper states of accumulation: change on one side alone does not reach the terms. And Claim 15.6's account of a raised ceiling that the pair never occupies is the counterpart of that paper's finding that a practice may leave a deposit which the relation does not take up.

19 THE CONSTRAINT FORM OF THE ACCOUNT

This section states the form in which the account's normative content is held. Its objective is to fix the relation between the mathematics and any reason for action, to say why no ordering of arrangements is supplied, and to record the sentence the account must remain able to refuse.

19.1 The Wall Between a Structure and a Justification

The account describes a structure. That an arrangement stands at one point of the base and not another, that its fibre has one characteristic and not another, and that one operation is available to a party while another is not are statements about a construction. None of them is offered as a reason for an arrangement or for the treatment of any party.

The separation requires maintenance here more than in the companion papers, since the vocabulary is unusually inviting. A characteristic that is larger sounds better than one that is smaller, and nothing in the mathematics supports that. Where a normative statement appears in this paper it is imported, and it is attributed at the point of use.

19.2 The Grounds for Supplying No Ordering

Three grounds tell against an ordering of arrangements by the quantities of this paper.

The order that the setting does supply is partial, by Proposition 9.1, and Proposition 13.3 establishes that the extensions completing it disagree with one another on the pairs that matter. An ordering would therefore be a choice among extensions presented as a finding.

The characteristic of the fibre separates the interior of the base from its two ends and takes one value throughout the interior, by Proposition 16.1, so it supplies no ordering within the interval where arrangements ordinarily stand.

And an ordering would require a quantity summed or compared across parties. The invariants of this paper belong to a pair and not to a party, and the account admits no operation that would trade one party's position against another's.

19.3 The Sentence the Account Must Remain Able to Refuse

A test of the account is whether it can refuse the following sentence: that where the parties' descriptive capacities differ, the party of smaller dimension should adopt the larger party's terms, since the rank available to the pair is bounded by the smaller and the bound would thereby be raised.

The account refuses it, and three features secure the refusal. Adopting another party's terms is not an enlargement in the sense of Definition 15.1, since it replaces a space in place of embedding it, and Proposition 15.2 does not apply to it. The bound of Proposition 15.3 is a statement about what is available and carries no recommendation that it be raised. And the second clause of the formula is exactly what such an adoption spends, so a reading of the formula cannot recommend it without contradicting the half of it that Section 6 was written to preserve.

The sentence is refusable because nothing in the account values a larger invariant, a higher rank, or a greater characteristic. A reader who finds the account recommending any of these has found an importation, and it is not the paper's.

20 THE BOUNDARIES OF THE UNDERTAKING

This section states the boundaries within which the account holds. Each is given as a property of the undertaking together with what would have to be established for it to be moved.

20.1 *The Restriction to Pure States of a Pair*

The completeness of Proposition 6.2 and the sufficiency of Proposition 6.3 hold for pure states of the pair and fail otherwise, as Claim 6.5 records. A state of the pair that is mixed is what obtains where the pair stands in relation to something beyond it, so the account is an account of two parties considered in isolation.

Moving this boundary requires the invariant theory of the local action on mixed states, which is substantially harder and for which no complete set of invariants is available in general (Horodecki et al., 2009).

20.2 *The Two Settings and What Each Answers*

Sections 6 through 15 treat a pair whose joint state is pure and whose evolution loses nothing, and Sections 8.4 and 8.5 treat one that dissipates. Claim 8.6 records that these are two settings and not one setting with a parameter, since the completeness of the invariants holds in the first and fails in the second.

The boundary this fixes is that no result of the closed setting may be carried into the dissipative one without an argument, and the paper supplies none. A reader who takes the dissipative setting to be the realistic one should take Sections 9 through 15 as an account of a limit in which nothing is lost.

20.3 *The Restriction to Two Parties*

Everything above concerns a pair. Three or more parties admit forms of correlation with no counterpart in the bipartite case, the invariant theory is different, and Section 14 records that the convexity theorem becomes substantial only there.

The restriction is not a simplification that could be removed by taking one party to stand for several, since the grouping of parties is itself a choice that alters the local group.

20.4 *The Finiteness and the Choice of the Local Group*

The spaces are finite-dimensional and the local group was taken to consist of transformations preserving the inner product. Both are choices. A party whose redescrptions do not preserve an inner product would generate a larger group, the orbits would be larger, and the invariants would be fewer.

What would have to be established to move this boundary is which transformations a party may in fact perform upon its own description, and the paper supplies no way of finding out.

20.5 *The Reading, Which Remains a Proposal*

The identification of the common part with the invariants of the local action, at Claim 6.4, is a proposal about how to read a formula. It is not established by anything in the mathematics, which would hold if the formula had never been uttered, and it is not established by anything in the diplomatic record, which the paper does not consult.

What may be said for it is recorded at Section 6.4 and at Proposition 16.1: it removes a tension the other readings carry, and it supplies the requirement that an arrangement stand between two ends with a measure that is at a minimum at each of them. A reader who declines the identification keeps the mathematics and loses the paper's subject.

20.6 *The Interpretation Withheld*

Section 2.2 names five particulars in which the borrowed formalism is used and its interpretation withheld. The boundary they jointly fix is that no question belonging to that interpretation is raised here, and no result depends on how such a question would be answered.

A reader who holds that the formalism cannot be used without its interpretation has an objection the paper does not answer. What the paper offers against it is that the results of Sections 6, 9, 11 and 13 are statements about a group acting on a space, and that each of them would stand if the formalism had been arrived at without any physical theory at all.

21 QUESTIONS OPENED BY THE ACCOUNT

This account identifies a structure that the received readings of the formula leave undescribed: a common part that neither party possesses and neither constructs, defined by its indifference to how each describes its own side, together with a space of descriptions over it whose size is measurable and whose collapse is exact. If that structure is real, the following questions arise, and the account answers none of them.

Q1. Claim 6.4 identifies the common part with the invariants of the local action. What in a diplomatic record would count for or against that identification, given that both parties compute such an invariant from their own side and neither would ordinarily report it?

Q2. Claim 6.5 holds that the completeness of the invariants fails where the pair stands in relation to something beyond it. Is what two parties can establish between themselves in fact smaller in a world of many parties, and by what observation would the difference appear?

Q3. Section 7 takes the position that some of what parties hold in common is not established by operations each performs alone together with reports exchanged between them. What evidence would distinguish that position from the alternative, on which the practice of presence responds to a cost?

Q4. Claim 11.2 holds that the measure of preserved difference collapses as the common part

becomes uniform. Do arrangements in which the parties have come to differ in labels alone in fact exhibit a reduced range of distinguishable positions, and how would that range be counted?

Q5. Proposition 13.3 holds that two extensions of the majorisation order will disagree about one incomparable pair in eight. Which extension do practitioners use when they judge one relation to share more than another, and are they aware that they have chosen one?

Q6. Proposition 15.4 holds that a party's own generation multiplies the measure attaching to preserved difference and leaves the common part where it was. Is there a practice of developing one's own descriptive capacity in order to make a larger relation possible, and does it precede the interactions that would realise it?

Q7. Claim 15.6 holds that a raised ceiling may never be occupied. What distinguishes a pair that takes up the room made by one party's generation from one that does not?

Q8. Proposition 16.1 holds that the characteristic is maximal strictly between the two ends and constant across the interior. Is there a finer invariant that orders the interior, and would it order it in a way any party would recognise?

Q9. Section 14 records that the convexity theorem becomes substantial only for three or more parties. What does the polytope of jointly possible descriptions look like for three parties, and does it have a reading in the terms of this paper?

Q9b. Claim 8.6 holds that admitting dissipation buys a resting point and costs the completeness of the invariants. What survives of Sections 9 through 15 in the dissipative setting, and is there an invariant there that plays the part the coefficients play here?

Q9c. Section 11 records that the coefficients are real and ordered, so paths among them carry no braiding. What would a setting in which they braid look like, and would the exchange of coefficients along a path carry anything a diplomatic reading could use?

Q10. Section 20 records that the local group was taken to preserve an inner product. Which transformations may a party in fact perform upon its own description, and what would the invariants be if the group were larger?

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